# WRITTEN PROBLEMS:

Ans 1.)

a) $\forall x \forall y [\text{Integers}(x) \wedge \text{Integers}(y)$
$\Rightarrow (x+y \Leftrightarrow y+x)]$

$\forall x \forall y \; \text{Integers}(x) \wedge \text{Integers}(y)$
$\Rightarrow (x \cdot y \Leftrightarrow y \cdot x)$

b) $\forall x \forall y [\text{Integers}(x) \wedge \text{Integers}(y)$
$\Rightarrow \text{Integers}(x+y)]$

$\forall x \forall y [\text{Integers}(x) \wedge \text{Integers}(y)$
$\Rightarrow \text{Integers}(x \cdot y)]$

c) $\forall x \forall y \forall z [\text{Integers}(x) \wedge \text{Integers}(y) \wedge \text{Integers}(z)$
$\Rightarrow (x \cdot (y+z) \Leftrightarrow x \cdot y + x \cdot z)]$

d) $\forall x \forall y \forall z [\text{Integers}(x) \wedge \text{Integers}(y) \wedge \text{Integers}(z)$
$\Rightarrow (x + (y+z) \Leftrightarrow (x+y)+z)]$

$\forall x \forall y \forall z [\text{Integers}(x) \wedge \text{Integers}(y) \wedge \text{Integers}(z)$
$\Rightarrow (x \cdot (y \cdot z) \Leftrightarrow (x \cdot y) \cdot z)]$

e) $\forall_x$ [Integers $(x) \Rightarrow (x + 0 \Leftrightarrow x)$]

$\forall_x$ [Integers $(x) \Rightarrow (x \cdot 1 \Leftrightarrow x)$]

Ans2. $\alpha = \forall_x (P(x) \vee Q(x))$

$\beta = \forall_x P(x) \vee \forall_x Q(x)$

Let's Assume, domain of $x$ is Integers $\mathbb{Z}$
$= \{\ldots, -3, -2, -1, 0, 1, 2, 3 \ldots\}$

$P(x)$: $x$ is non-negative Integer.

$Q(x)$: $x$ is Negative Integer.

$\alpha = \forall_x (P(x) \vee Q(x))$ is True, because all integers are either Negative integer or Non-negative integer.

$\beta = \forall_x P(x) \vee \forall_x Q(x)$ is false, because not all integers are non-negative or negative.

I gave an example of a model where $\alpha$ is True, but $\beta$ is false.

So, $\boxed{\alpha \models \beta \text{ is False}}$

Ans 3.1 ii) By forward chaining:

Assumption a to i are given in question

By <a> and <d>: In 'or' statement, if one proposition true, then it is true

owns (HanSOLO, FALCON) $\Rightarrow$ visits (HanSOLO, kenobi)

$\therefore$ visits (HanSOLO, kenobi) . . . . (j)

By <j> and <e>:

visits (HanSOLO, kenobi) $\Rightarrow$ wise (HanSOLO)

$\therefore$ wise (HanSOLO) . . . . (k)

By <b> and <d>:

$\neg$Happy (Leia) $\Rightarrow$ visits (Leia, kenobi)

$\therefore$ visits (Leia, kenobi) . . . . (L)

By <L> and <e>:

visits (Leia, kenobi) $\Rightarrow$ wise (Leia)

$\therefore$ wise (Leia) . . . . . (M)

By <a> and <f>

visits (HanSOLO, kenobi) $\wedge$ owns (HanSOLO, FALCON)
$\Rightarrow$ Teaches (kenobi, HanSOLO)

$\therefore$ Teaches (Kenobi, HanSoLo) - - - - - (N)

By $\langle$a) and $\langle$N$\rangle$

Owns (HanSoLo, FALCON) $\wedge$ Teaches (Kenobi, HanSoLo)
$\Rightarrow$ JoinsRebel (HanSoLo)

$\therefore$ JoinsRebel (HanSoLo) - - - - $\langle$O$\rangle$

By $\langle$b$\rangle$ and $\langle$C$\rangle$ . . . .

Loves (Leia, HanSoLo) $\wedge$ $\neg$ Happy (Leia)
$\Rightarrow$ DeclareLove (Leia, HanSoLo)

DeclareLove (Leia, HanSoLo) - - - - - $\langle$P$\rangle$

By $\langle$N$\rangle$, $\langle$P$\rangle$ and $\langle$M$\rangle$ :

Teaches (Kenobi, HanSoLo) $\wedge$ Wise (Leia)
$\wedge$ DeclareLove (Leia, HANSOLO)

$\Longrightarrow$ HasFriend (HanSoLo, CHEWBACCA)

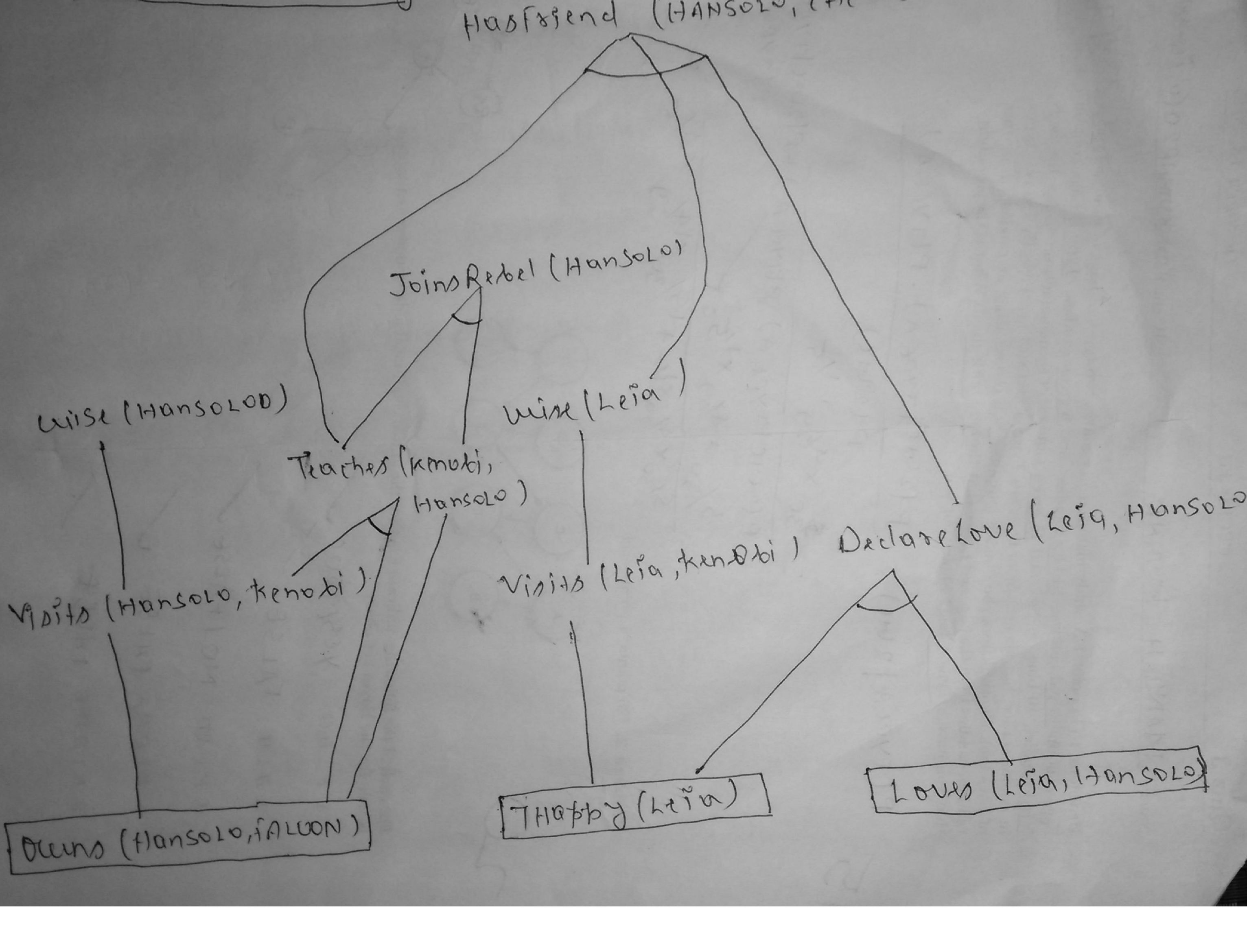
$\therefore$ HasFriend (HanSoLo, CHEWBACCA) . . . . $\langle$Q$\rangle$

By $\langle$Q$\rangle$

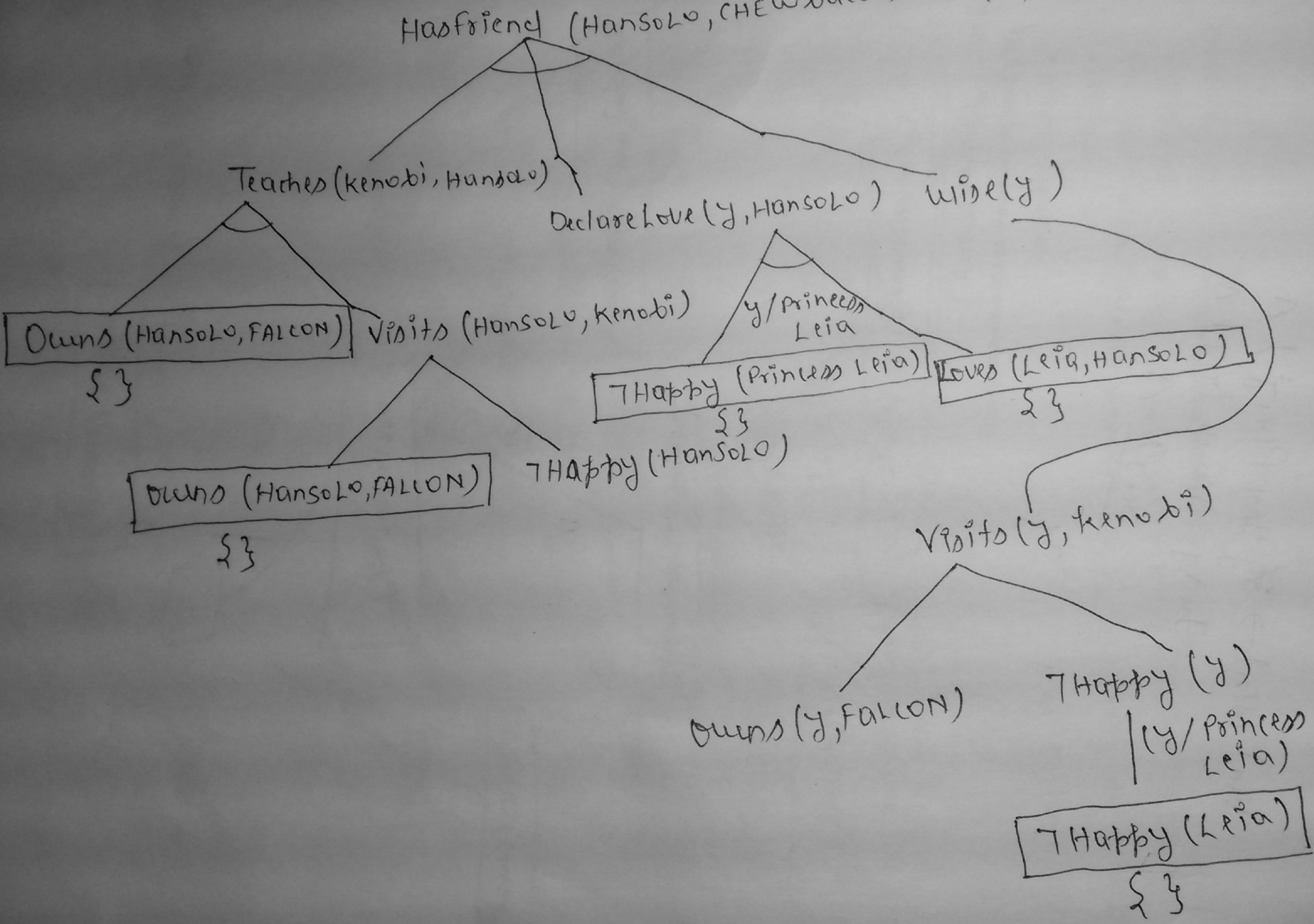
$\boxed{\text{HasFriend (HanSoLo, CHEWBACCA)}}$

Proved.

Forward chaining:



Ans 3. i) By Backward chaining: {x/HanSolo}



To prove contradiction, Assume Exist (zeus)

**Ans 4:** a) able (zeus) ∧ willing (zeus)
⇒ Prevents (zeus, EVIL)

b) ¬able (zeus) ⇒ Impotent (zeus)

c) ¬willing (zeus) ⇒ malevolent (zeus)

d) ¬Prevents (zeus, EVIL)

e) Exist (zeus) ⇒ ¬Impotent (zeus) ∧ ¬Malevolent (zeus)

By (a): By applying modus ponen & De morgan's rule:

(¬able (zeus) ∨ ¬willing (zeus) ∨ Prevents (zeus, EVIL))

By (b):

[(able (zeus) ∨ Impotent (zeus))]

By resolution on able (zeus)

(¬willing (zeus) ∨ Impotent (zeus) ∨ Prevents (zeus, Evil))

By (c):

willing (zeus) ∨ malevolent (zeus)

By resolution on willing (zeus)

Impotent (zeus) ∨ malevolent (zeus) ∨ Prevents (zeus, Evil)

Impotent (Zeus) ∨ Malevolent (Zeus) ∨ Prevents (Zeus, EVIL)

By <d>

¬Prevents (Zeus, EVIL)

(Impotent (Zeus) ∨ Malevolent (Zeus))

By <e>:

¬Exist (Zeus) ∨
¬Impoten (Zeus) ∧ ¬Malevolent (Zeus)

By applying De Morgan's Law:

¬Exist (Zeus) ∨
¬(Impotent (Zeus) ∨ Malevolent (Zeus))

¬Exist (Zeus)

Exist (Zeus)
(Assumption)

$\square$

∵ our Assumption Exist (zeus) Led to contradiction as conclusion was unsatisfiable.

∴ Exist (zeus) is false.

∴ $\boxed{\neg \text{Exist}(\text{zeus})}$ Proved.

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Ans 5.

a) To Prove,

$\forall x\,(P(x) \Rightarrow P(x))$ Valid.

Let's Assume,

$\neg \forall x\,(P(x) \Rightarrow P(x))$ is valid.

$\therefore \neg \forall x\,(\neg P(x) \vee P(x))$ is valid (By modus Ponen)

$\Rightarrow \exists x\,(P(x) \wedge \neg P(x))$ is valid

.... (By Demorgan's theorem)

This is false, as there no x exist for which $P(x) \wedge \neg P(x)$ is true.

$\therefore \exists x\,(P(x) \wedge \neg P(x))$ unsatisfiable.

$\therefore$ By contradiction,

$\therefore \forall x\,(P(x) \Rightarrow P(x))$ is valid.

$\therefore$ $\boxed{\forall x\,(P(x) \Rightarrow P(x))}$

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b) To prove:

$(\neg \exists x\, P(x)) \Rightarrow (\forall x \neg P(x))$ valid.

Let's Assume:

$\neg((\neg \exists_x P(x)) \Rightarrow (\forall x \neg P(x))$ is valid.

$\therefore \neg(\neg \exists x \neg P(x) \vee \forall x \neg P(x))$ is valid.

$\therefore \neg(\neg\neg(\forall x\, P(x)) \vee \forall x \neg P(x))$ is valid.

$\therefore \neg(\forall x\, P(x) \vee \forall x \neg P(x))$ is valid.

$\Rightarrow \neg \forall x\, (P(x) \vee \neg P(x))$ is valid.

$\Rightarrow \exists x\, (\neg P(x) \wedge P(x))$ is valid.

But,
$\exists x\, (\neg P(x) \wedge P(x))$ is unsatisfiable.

So, our assumption is false.

$\therefore$ By contradiction,

$(\neg \exists x\, P(x)) \Rightarrow (\forall x \neg P(x))$ is valid.

(c.) To Prove:-

$(\forall_x (P(x) \vee q(x))) \Rightarrow ((\forall_x P(x)) \vee (\exists_x q(x))))$
is valid.

Let's Assume:

$\neg (\forall_x (P(x) \vee q(x))) \Rightarrow ((\forall_x P(x)) \vee (\exists_x q(x))))$
is valid.

Let's Assume:

$A = \forall_x (P(x) \vee q(x)))$

$B = ((\forall_x P(x)) \vee (\exists_x q(x)))$

So, $\neg (A \Rightarrow B)$ is valid.

$\therefore \neg (\neg A \vee B)$ is valid.

$\therefore (\neg(\neg A) \wedge \neg B)$ is valid.

Now,

$\neg B = \neg ((\forall_x P(x)) \vee (\exists_x q(x))))$

$= (\exists_x \neg P(x)) \wedge (\forall_x \neg q(x))$

By Skolmenize, I can write:

$: (\exists_x \neg P(x)) \wedge (\forall y \neg q(y))$

$= \forall y \exists x (\neg P(x) \wedge \neg g(y))$

$= \forall y \exists x \neg (P(x) \vee g(y))$

As $\forall y$ can take any value.
$\therefore$ Lets take $y = x$

$\therefore \neg B = \exists x \neg (P(x) \vee g(x))$

$\neg(\neg A) = \neg [\forall x \neg (P(x) \vee g(x))]$

$= [\exists x (P(x) \vee g(x))]$

$\therefore (\neg(\neg A) \wedge \neg B)$ is valid.

$\therefore \exists x [(P(x) \vee g(x)) \wedge \neg (P(x) \vee g(x))]$ is valid.

But this is false, as this is unsatisfiable because $(P \wedge \neg P)$ is always false.

$\therefore$ By contradiction:

$(\forall x (P(x) \vee g(x))) \Rightarrow ((\forall x P(x)) \vee (\exists x g(x)))$

is Valid.

Hence, proven.