Department of Mathematics
Assignment Sheet (Module-1)
Applied Mathematics III (BMAS 0109)

Q.1 If $y=\sqrt{x+2}$, find y_n.

Q2 Find the n^{th} differential co-efficient of $e^x sin^3 x$.

Q.3 If $u=\log\sqrt{x^2+y^2+z^2}$ show that $\left(x^2+y^2+z^2\right)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)=1$.

Q.4 If $x=e^r\cos\theta,\ y=e^r sin\theta$ show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=e^{-2r}\left(\frac{\partial^2 u}{\partial r^2}+\frac{\partial^2 u}{\partial \theta^2}\right)$.

Q5 Verify Euler's theorem for the functions:

(i) $u=\left(x^{1/2}+y^{1/2}\right)(x^n+y^n)$

(ii) $u=sin^{-1}\frac{x}{y}+tan^{-1}\frac{y}{x}$.

Q6 Show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=2tan(u)$, where $u=sin^{-1}\left(\frac{x^3+y^3+z^3}{ax+by+cz}\right)$

Q7 If $u=xlog(xy)$, where $x^3+y^3+3xy=1$, find $\frac{du}{dx}$.

Q8 Find the first six terms of the expansion of the function $e^x log(1+y)$ in a Taylor series in the neighbourhood of the point (0,0).

Q9 Expand $\left(x^2y+\sin y+e^x\right)$ in powers of $(x-1)$ and $(y-\pi)$.

Q10 If $u=x\sin y$ and $v=y\sin x$, then find $\frac{\partial(u,v)}{\partial(x,y)}$.

Q11. If $u=xyz, v=x^2+y^2+z^2, w=x+y+z$, then compute the Jacobian $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.

Q12. A rectangular box, open at the top, is to have a given capacity. Find the dimension of the box requiring least material for its construction.

Q13. The sum of three positive numbers is constant. Prove that their product is maximum when they are equal.

Q.14 Using Beta and Gamma functions evaluate the followings:

(a) $\int_0^\infty \frac{x^c}{c^x}\ dx=\frac{\Gamma(c+1)}{(\log c)^{c+1}}; c>1$.

(b) $\int_0^2 (8-x)^{-1/3}\ dx$, (c) $\int_0^\infty \frac{dx}{1+x^6}$, (d) $\int_0^{\pi/2}\tan^n x\ dx$

Q.15 Using Beta and Gamma functions prove the followings:

(a) $\int_0^{\infty} x^{-1/2}\ e^{-x^2}\ dx\ \times \int_0^{\infty} x^2\ e^{-x^4}\ dx = \frac{\pi}{4\sqrt{2}}$.

(b) $\beta(m,n) = \beta(m+1,n) + \beta(m,n+1)$,

Q.16 Evaluate the following integrals:

(a) $\int_0^{\pi/2} \int_0^{a\cos\theta} r\sqrt{a^2 - r^2}\ dr\, d\theta$, (b) $\iint r^2 dr\, d\theta$, over the area between the circles $r = a\cos\theta$ and $r = 2a\cos\theta$.

Q.17 Evaluate the $\iint e^{2x+3y}\ dx\, dy$, over the triangle bounded by $x = 0$, $y = 0$ and $x + y = 1$.

Q.18 Evaluate (a) $\int_0^3 \int_0^1 (x^2 + 3y^2)\ dy\, dx$ (b) $\int_0^1 \int_0^x e^{y/x}\ dy\, dx$

Q.19 Use Trapezoidal rule to evaluate $\int_0^{\infty}\ x^3\, dx$ considering five sub-intervals.

Q.20 Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using

(a) Simpson's one-third rule taking h = 1/4

(b) Simpson's three-eight rule taking h = 1/6 .

Q.21 If u = (2 V^6 – 5 V), find the percentage error in u at V=1 if error in V is 0.5.

Q.22 Suppose 1.414 is used as an approximation to $\sqrt{2}$. Find the absolute and relative errors.