Department of Mathematics
Tute Sheet (Module-1)
ENGINEERING CALCULUS (BMAS 0109)

Q.1 If $u=\log\left(\frac{x^2}{y}\right)$, then find $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$.

Q.2 If $w=f(x,y)$, where $x=e^u\cos v, y=e^u\sin v$, show that

$y\frac{\partial w}{\partial u}+x\frac{\partial w}{\partial v}=e^{2u}\frac{\partial w}{\partial y}$.

Q.3 Evaluate $\iint e^{2x+3y}\,dx\,dy$, over the triangle bounded by $x=0, y=0$ and $x+y=1$.

Q.4 If $u=\log(x^3+y^3+z^3-3xyz)$, show that

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}.$$

Q.5 If $u=\tan^{-1}\frac{x^3+y^3}{x-y}$, prove that

(i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$

(ii) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\sin 4u-\sin 2u=2\cos 3u\,.\sin u$

Q.6 If the curves $f(x,y)=0$ and $\phi(x,y)=0$ touch each other, show that at the point of contact $\frac{\partial f}{\partial x}.\frac{\partial \phi}{\partial y}-\frac{\partial f}{\partial y}.\frac{\partial \phi}{\partial x}=0$.

Q.7 Expand $e^x\sin y$ in powers of x and y as far as the terms of third-degree.

Q.8 If $u=\log\left(\frac{x^4+y^4}{x+y}\right)$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=3$.

Q.9 If $u=x+2y+z, v=x-2y+3z$ and $w=2xy-xz+4yz-2z^2$, show that they are not independent. Find the relation between u, v and w. (Ans. $u^2-v^2=4w$)

Q.10 If u, v, w are the roots of the cubic equation $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ in λ, then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

Q.11 Find the extreme values of the function $x^3 + y^3 - 3axy$.

Q.12 Find the maximum and minimum distances of the point $(3, 4, 12)$ from the sphere $x^2 + y^2 + z^2 = 1$.

Q.13 If $x^x\, y^y\, z^z = c$, show that at $x = y = z$

$$\frac{\partial^2 z}{\partial x\, \partial y} = -(x \log e\, x)^{-1}.$$

Q.14 If $u = e^{x\,y\,z}$, prove that $\frac{\partial^3 u}{\partial x\, \partial y\, \partial z} = (1 + 3xyz + x^2y^2z^2)\, u$.

Q.15 If $u = r^m$ and $r^2 = x^2 + y^2 + z^2$, prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = m\,(m+1)\, r^{m-2}.$$

Q.16 The temperature T at any point (x, y, z) in space is $T(x, y, z) = K\, x\, y\, z^2$, where K is heat constant. Drive the highest temperature on the surface of a sphere with center at origin and radius 4.

Q.17 Determine the dimensions of a rectangular box which is open at the top such that the least material is required for construction of the box. The capacity of box is given as 256 cubic feet approximately.

Q.18 Using Beta and Gamma functions evaluate the following:

$\int_0^\infty x^{1/4}\; e^{-\sqrt{x}}\, dx.$

Q19 Evaluate $\int_0^1 x^5 \left(1 - x^3\right)^{10} dx$ using Beta function.

Q20 Evaluate $\iint x^2 y^2 dxdy$ over the circle $x^2 + y^2 = 1$.

Q21. Evaluate $\int_0^{\log 2} \int_0^{x} \int_0^{x+\log y} e^{x+y+z} dzdydx.$

Q.22 Find the solution of $\int f(x)dx$ using the Trapezoidal rule.

x	**f(x)**
0.0	**1.0000**
0.1	**0.9975**
0.2	**0.9900**
0.3	**0.9776**
0.4	**0.8604**