Department of Mathematics
MTL 601 (Probability and Statistics)
Tutorial Sheet No. 4

1. If random samples of size three and drawn without replacement from the population consisting of four numbers $4, 5, 5, 7$. Find sample mean X for each sample and make sampling distribution of X. Calculate the mean and standard deviation of this sampling distribution. Compare your calculations with population parameters.

2. Assume that a school district has 10,000 6th graders. In this district, the average weight of a 6th grader is 80 pounds, with a standard deviation of 20 pounds. Suppose you draw a random sample of 50 students. What is the probability that the average weight of a sampled student will be less than 75 pounds?

3. Find the probability that of the next 120 births, no more than 40% will be boys. Assume equal probabilities for the births of boys and girls. Assume also that the number of births in the population (N) is very large, essentially infinite.

4. Suppose a Taxi Service receives on the average 8 requests per hour. How many car should he keep in order that 90% of the requests are met?

5. A Manufacturer claims that at most 10% of his products are defective. To test this claim 18 units are inspected, and the claim is accepted if at most 2 are defectives.

   (i) What is the probability that his claim will be accepted if actually 20% products are defective.

   (ii) What is the probability that his claim will be rejected if actually 5% products are defective.

6. Calculate the probability that, for a random sample of 5 values taken from a $N(100, 252)$ population

   (a) $\overline{X}$ will be between 80 and 120

   (b) S will exceed 41.7

7. Independent random samples of size n1 and n2 are taken from the normal populations $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ respectively

   (a) Write down the sampling distributions of $\overline{X_1}$ and $\overline{X_2}$ and hence determine the sampling distribution of $\overline{X_1} - \overline{X_2}$, the difference between the sample means.

   (b) Now assume that $\sigma_1^2 = \sigma_2^2 = \sigma^2$,

      (i) Express the sampling distribution of $\overline{X_1} - \overline{X_2}$ in standard normal form.

      (ii) State the sampling distribution of $\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{\sigma^2}$.

8. Let $X_1, X_2, \ldots, X_n$ be a random sample from normal distributed population with mean μ and variance σ^2. Let s^2 be the sample variance. Prove that $\frac{(n-1)s^2}{\sigma^2}$ has χ^2 distribution with $n-1$ degrees of freedom.

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