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NPTEL

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Courses » Introduction to Probability Theory and Stochastic Processes

Announcements **Course** Ask a Question Progress Mentor FAQ

Unit 13 - Week 11

Course outline

Assignment 11

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2018-10-17, 23:59 IST.

Each of the following questions has four options out of which one or more options can be correct. Individual marks are mentioned corresponding to each question. In the case of multiple answers, no partial marks will be awarded if all the correct choices are not selected. 0 marks for questions not attempted.

1) Let $\{X(t), t \geq 0\}$ be a continuous time Markov Chain with state space $S = \{0,\ 1,\ 2\}$ and ***2 points***

$$P = \begin{bmatrix} -1 & 2 & -1 \\ \frac{1}{4} & -1 & \frac{3}{4} \\ 1 & 0 & 0 \end{bmatrix},\ Q = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 0.25 & 0.5 & 0.25 \end{bmatrix}$$

Which of the above mentioned two matrix could be a possible infinitesimal generator matrix for $\{X(t), t \geq 0\}$?

- [ ] Both P and Q
- [ ] Only P
- [ ] Only Q
- [ ] Neither P nor Q

No, the answer is incorrect.
Score: 0

Accepted Answers:
Neither P nor Q

2) Women arrive in a bank according to a Poisson process with rate λ and men arrive in the same bank according to a Poisson process with rate μ. The arrival of men and women are independent. The probability that the first arrival in the queue is a women is ***2 points***

- $\frac{\mu}{\lambda+\mu}$
- $\frac{\mu}{\lambda-\mu}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\frac{\lambda}{\lambda+\mu}$

3) Consider the random telegraph signal, denoted by $X(t)$, jumps between two states, 0 and 1, according to the following rules. At time $t = 0$, the signal $X(t)$ start with equal probability for the two states, i.e., ***2 points***

$P(X(0) = 0) = P(X(0) = 1) = 1/2$ and let the switching times be decided by a Poisson process $\{Y(t), t \geq 0\}$ with parameter λ independent of $X(0)$. At time t, the signal $X(t) = \frac{1}{2}\left(1 - (-1)^{X(0)+Y(t)}\right), t > 0.$. Then,

- $E(X(t)) = \frac{1}{4}$ for all t
- $E(X(t)) = \frac{t}{2}$ for all t
- $E(X(t)) = \frac{t}{4}$ for all t
- $E(X(t)) = \frac{1}{2}$ for all t

No, the answer is incorrect.
Score: 0

Accepted Answers:
$E(X(t)) = \frac{1}{2}$ for all t

4) For an irreducible, positive recurrent, time homogeneous continuous-time Markov chain, consider the following statements: ***2 points***

1. Limiting distribution does not exist.

2. Stationary distribution does not exist.

3.Limiting distribution and stationary distribution both exists and are same.

Choose the correct option based on the above three statements.

- All three statements are true.
- Statement 1 is always true but 2 and 3 may or may not be true.
- Statement 2 and 3 are always true but 3 is not true.

Only statement 3 is true.

No, the answer is incorrect.
Score: 0

Accepted Answers:
Only statement 3 is true.

5) Consider a birth and death process $\{X(t), t \geq 0\}$ such that $\lambda_i = \lambda$ for all i. Further, suppose $\mu_i = 0, i = 1, 2, \ldots$ and initially $X(0) = 0$. Let $X(t)$ denotes the state of the system at time t. The probability $P(X(0.5) = 0)$ when $\lambda = 1$ is given by ***2 points***

- 0.61
- 0.39
- 0.21
- 0.79

No, the answer is incorrect.
Score: 0

Accepted Answers:
0.61

6) Consider a birth and death process $\{X(t), t \geq 0\}$ such that $\lambda_i = \lambda$ for all i. Further, suppose $\mu_i = 0, i = 1, 2, \ldots$ and initially $X(0) = 0$. Let $X(t)$ denotes the state of the system at time t. Find the probability $P(X(5) = 3, X(2) = 1)$ when $\lambda = 1$ is ***2 points***

- 0.01
- 0.06
- 0.1
- 0.9

No, the answer is incorrect.
Score: 0

Accepted Answers:
0.06

7) Consider a birth and death process $\{X(t), t \geq 0\}$ such that $\lambda_i = \lambda$ for all i. Further, suppose $\mu_i = 0, i = 1, 2, \ldots$ and initially $X(0) = 0$. Let $X(t)$ denotes the state of the system at time t. The expected number in the system at time t, i.e., $E(X(t))$ is ***2 points***

- λ
- λ^2
- λt
- $1 - \lambda t$

No, the answer is incorrect.
Score: 0

Accepted Answers:
λt

8) The backward and forward Kolmogorov equations for a Continuous time markov chain $\{X(t),\ t \geq 0\}$ with transition probabilities matrix $P(t)$ and infinitesimal generator ***2 points***

matrix Q are given by

Both the equations are $P'(t) = P(t)Q$

$P'(t) = QP(t), P'(t) = P(t)Q$ respectively

$P'(t) = P(t)Q, P'(t) = QP(t)$ respectively

Both the equations are $P'(t) = QP(t)$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$P'(t) = QP(t), P'(t) = P(t)Q$ *respectively*

9) Two communication satellites are placed in orbit. The lifetime of the the satellite is exponential distribution with mean $\frac{1}{\mu}$. If one satellite fails, its replacement is sent up. Assume that there is only one repair team.The time necessary to prepare and send up a replacement is exponential distribution with mean $\frac{1}{\lambda}$. Let $X(t)$ is the number of satellites not in the orbit at time t. Assume $\{X(t),\ t \geq 0\}$ is a Markov process with state space $\{0, 1, 2\}$. The limiting distribution of the markov chain $\{X(t),\ t \geq 0\}$ is given by ***2 points***

$\pi_0 = \pi_1 = \pi_2 = \frac{1}{4}$

$\pi_0 = \frac{1}{1+2\rho+2\rho^2},\ \pi_1 = 2\rho\pi_0,\ \pi_2 = 2\rho^2\pi_0,\ \text{where } \rho = \frac{\lambda}{\mu}$

$\pi_0 = \frac{1}{1+2\rho+2\rho^2},\ \pi_1 = 2\rho\pi_0,\ \pi_2 = 2\rho^2\pi_0,\ \text{where } \rho = \frac{\mu}{\lambda}$

$\pi_0 = \frac{1}{1+\rho+\frac{\rho^2}{2}},\ \pi_1 = \rho\pi_0,\ \pi_2 = \frac{\rho^2}{2}\pi_0,\ \text{where } \rho = \frac{\mu}{\lambda}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\pi_0 = \frac{1}{1+2\rho+2\rho^2},\ \pi_1 = 2\rho\pi_0,\ \pi_2 = 2\rho^2\pi_0,\ \text{where } \rho = \frac{\mu}{\lambda}$

10) Assume that individuals remain healthy for an exponential time with mean $\frac{1}{\lambda}$ before becoming sick. Assume also that it takes an exponential time to recover from sick to healthy again with mean sick time of $\frac{1}{\mu}$. If the individual starts healthy at time 0, then we are interested in the probabilities of being sick and healthy in future times. Let state 0 denote the healthy state and state 1 denote the sick state. Let (π_0, π_1) denotes the stationary distribution. Then, which of the following is/are TRUE? ***2 points***

$\pi_0 = \pi_1 = \frac{1}{2}$

$\pi_0 = \frac{\mu}{\lambda+\mu},\ \pi_1 = \frac{\lambda}{\lambda+\mu}$

$\pi_0 = \frac{\lambda}{\lambda+\mu},\ \pi_1 = \frac{\mu}{\lambda+\mu}$

- [ ] $\pi_0 = \frac{\mu}{\lambda - \mu}$, $\pi_1 = \frac{\lambda}{\lambda - \mu}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\pi_0 = \frac{\mu}{\lambda + \mu}$, $\pi_1 = \frac{\lambda}{\lambda + \mu}$

Previous Page   End