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Courses » Introduction to Probability Theory and Stochastic Processes

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Unit 8 - Week 6

Assignment 6

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2018-09-12, 23:59 IST.

Each of the following questions has four options out of which one or more options can be correct. Individual marks are mentioned corresponding to each question. In the case of multiple answers, no partial marks will be awarded if all the correct choices are not selected. 0 marks for questions not attempted.

1) Let $X_1, X_2, \ldots, X_n$ are i.i.d. random variables with a common probability density function given by *2 points*

$$f(x) = \begin{cases} 3x^2, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Which of the following is the pdf of the random variable Y where $Y = \min\{X_1, X_2, \ldots, X_n\}$

- [ ] $$g(y) = \begin{cases} 3ny^{3n-1}, & 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$
- [ ] $$g(y) = \begin{cases} 1-(1-y^3)^n, & 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$
- [ ] $$g(y) = \begin{cases} (1-y^3)^n, & 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$
- [ ] $$g(y) = \begin{cases} 3ny^2(1-y^3)^{n-1}, & 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

No, the answer is incorrect.
Score: 0

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$$f(x,y) = \begin{cases} e^{-y}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases}$$

The conditional pdf of Y given $X = x$ is

$$g(y \mid x) = \begin{cases} xe^{-y}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases}$$

$$g(y \mid x) = \begin{cases} e^{y-x}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases}$$

$$g(y \mid x) = \begin{cases} e^{x-y}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases}$$

$$g(y \mid x) = \begin{cases} ye^{-x}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases}$$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$$g(y \mid x) = \begin{cases} e^{x-y}, & 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases}$$

3) Let (X, Y) be two-dimensional random variable with joint pdf is given by ***2 points***

$$f(x,y) = \begin{cases} \frac{1}{2}\, y^2 e^{-x}, & 0 < y < x < \infty \\ 0, & \text{otherwise} \end{cases}$$

Then, $P(Y < 1 \mid X = 3)$ equals

$\frac{1}{81}$

$\frac{1}{27}$

$\frac{1}{9}$

$\frac{1}{3}$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$\frac{1}{27}$

4) For each fixed $\lambda > 0$, let X be a Poisson distributed random variable with parameter λ. ***2 points***
Suppose λ itself is a random variable following a gamma distribution with pdf

$$f(\lambda) = \begin{cases} \frac{1}{\Gamma(n)} \lambda^{n-1} e^{-\lambda}, & \lambda > 0 \\ 0, & \text{otherwise} \end{cases}$$

where n is a fixed positive constant. The pmf of the random variable X is given by

- [ ] $P(X=k) = \frac{\Gamma(k+n)}{\Gamma(n)\Gamma(k)} \left(\frac{1}{2}\right)^{k+n-1}, \quad k = 0, 1, \ldots$
- [ ] $P(X=k) = \frac{\Gamma(k+n)}{\Gamma(n)\Gamma(k+1)} \left(\frac{1}{2}\right)^{k+n}, \quad k = 0, 1, \ldots$
- [ ] $P(X=k) = \frac{\Gamma(k+n+1)}{\Gamma(n)\Gamma(k+1)} \left(\frac{1}{2}\right)^{k+n+1}, \quad k = 0, 1, \ldots$
- [ ] $P(X=k) = \frac{\Gamma(k+n+1)}{\Gamma(n)\Gamma(k+1)} \left(\frac{1}{2}\right)^{k+n}, \quad k = 0, 1, \ldots$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$P(X=k) = \frac{\Gamma(k+n)}{\Gamma(n)\Gamma(k+1)} \left(\frac{1}{2}\right)^{k+n}, \quad k = 0, 1, \ldots$

5) Let $X_1, X_2, \ldots, X_5$ be i.i.d random variables each having uniform distributions in the interval (0,1). Which of the following statements are true? ***2 points***

- [ ] $P(\frac{1}{4} < \min(X_1, X_2, \ldots, X_5) < \frac{3}{4}) = 0.24$
- [ ] $P(\frac{1}{4} < \max(X_1, X_2, \ldots, X_5) < \frac{3}{4}) = 0.24$
- [ ] $P(\max(X_1, X_2, \ldots, X_5) > \frac{1}{2}) = 0.97$
- [ ] $P(\min(X_1, X_2, \ldots, X_5) < \frac{3}{4}) = 0.75$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$P(\frac{1}{4} < \min(X_1, X_2, \ldots, X_5) < \frac{3}{4}) = 0.24$
$P(\frac{1}{4} < \max(X_1, X_2, \ldots, X_5) < \frac{3}{4}) = 0.24$
$P(\max(X_1, X_2, \ldots, X_5) > \frac{1}{2}) = 0.97$

6) Suppose you participate in a chess tournament in which you play until you lose a game. Suppose you are a very average player, each game is equally likely to be a win, a loss or a tie. You collect 2 points for each win, 1 point for each tie and 0 points for each loss. The outcome of each game is independent of the outcome of every other game. Let X_i be the number of points you earn for ith game and let Y equal the total number of points earned in the tournament. ***2 points***

Which of the following statements are true?

- [ ] $E(Y) = 3$

- [ ] The MGF of Y is $M_Y(t) = \frac{M_X(t)}{1-pM_X(t)}$, where $p = \frac{1}{3}$
- [ ] The number of games played by you is a Geometric random variable
- [ ] The MGF of Y is $M_Y(t) = \frac{pM_X(t)}{1-(1-p)M_X(t)}$ where $p = \frac{1}{3}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$E(Y) = 3$
The number of games played by you is a Geometric random variable
The MGF of Y *is* $M_Y(t) = \frac{pM_X(t)}{1-(1-p)M_X(t)}$ *where* $p = \frac{1}{3}$

7) Let (X, Y) be a two-dimensional random variable with joint pdf ***2 points***

$$f(x,y) = \begin{cases} 1, & 0 < x, y < 1 \\ 0, & \text{otherwise} \end{cases}$$

The pdf of $Z = X + Y$ is given by

- [ ] $$g(z) = \begin{cases} z, & 0 \le z \le 1 \\ 2 - z & 1 \le z \le 2 \\ 0, & \text{otherwise} \end{cases}$$
- [ ] $$g(z) = \begin{cases} z, & 0 \le z \le 1 \\ 0, & \text{otherwise} \end{cases}$$
- [ ] $$g(z) = \begin{cases} 2 - z & 1 \le z \le 2 \\ 0, & \text{otherwise} \end{cases}$$
- [ ] $$g(z) = \begin{cases} z, & 0 \le z \le 1 \\ z^2 & 1 \le z \le 20, \quad \text{otherwise} \end{cases}$$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$$g(z) = \begin{cases} z, & 0 \le z \le 1 \\ 2 - z & 1 \le z \le 2 \\ 0, & \text{otherwise} \end{cases}$$

8) A random sample of size 4 is drawn from a population that has a uniform distribution on the interval (0,5). The resulting order statistics are $X_{(1)}, X_{(2)}, X_{(3)}$ and $X_{(4)}$. Which of the following statements are true? ***1 point***

- [ ] CDF of the 3rd order statistic $X_{(3)}$ is $\frac{20x^3}{625} - \frac{3x^4}{625}$
- [ ] $P(X_{(3)} > 2) = 0.72$
- [ ]

$P(X_{(3)} > 2) = 0.82$

☐ $P(X_{(4)} > 4) = 0.70$

No, the answer is incorrect.
Score: 0

Accepted Answers:

CDF of the 3rd order statistic $X_{(3)}$ *is* $\frac{20x^3}{625} - \frac{3x^4}{625}$

$P(X_{(3)} > 2) = 0.82$

9) Suppose that the two-dimensional integer-valued discrete random variable (X, Y) has joint pmf **1 point**

$$P(X = x, Y = y) = \begin{cases} c(2x + y), & 0 \le x \le 2, 0 \le y \le 3 \\ 0, & \text{otherwise} \end{cases}$$

Which of the following statements are true?

☐ value of c is $\frac{1}{42}$

☐ value of c is $\frac{1}{7}$

☐ $P(Y \le 2 \mid X = 1) = \frac{9}{14}$

☐ $P(Y = 1 \mid X = 2) = \frac{5}{22}$

No, the answer is incorrect.
Score: 0

Accepted Answers:

value of c is $\frac{1}{42}$

$P(Y \le 2 \mid X = 1) = \frac{9}{14}$

$P(Y = 1 \mid X = 2) = \frac{5}{22}$

10)The number of pages N in a fax transmission has geometric distribution with mean 4. The number of bits k in a fax page also has geometric distribution with mean 10^5 bits independent of any other page and the number of pages. Find the probability distribution of total number of bits in fax transmission **1 point**

☐ $Geometric(1 - \frac{1}{4 \times 10^5})$

☐ $Geometric(\frac{1}{4 \times 10^5})$

☐ $Exponential(1 - \frac{1}{4 \times 10^5})$

☐ $Geometric(1 - \frac{1}{10^5})$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$Geometric(\frac{1}{4 \times 10^5})$

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