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NPTEL

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Courses » Introduction to Probability Theory and Stochastic Processes

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Unit 12 - Week 10

Course outline

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Assignment 10

The due date for submitting this assignment has passed.
As per our records you have not submitted this assignment.

Due on 2018-10-10, 23:59 IST.

Each of the following questions has four options out of which one or more options can be correct. Individual marks are mentioned corresponding to each question. In the case of multiple answers, no partial marks will be awarded if all the correct choices are not selected. 0 marks for questions not attempted.

1) Consider a Markov chain $\{X_n, n = 0, 1, 2, \ldots\}$ with state space $S = \{-10, -9, \ldots, 0, \ldots, 9, 10\}$ with one-step transition probabilities *2 points*

$$P_{i,i+1} = P_{i,i-1} = 0.5 \; \forall \; -9 \leq i \leq 9$$

$$P_{10,9} = P_{10,10} = P_{-10,-10} = P_{-10,-9} = 0.5$$

Which one of the following is TRUE?

- [ ] The chain is reducible.
- [ ] Chain has a unique stationary distribution
- [ ] $P_{0,0}^{(2)} = \frac{1}{4}$
- [ ] Period of state 0 is 2

No, the answer is incorrect.
Score: 0

Accepted Answers:
Chain has a unique stationary distribution

2) Consider a Markov chain $\{X_n, n = 0, 1, 2, \ldots\}$ with state space $S = \{0, 1, 2, 3, 4\}$ and one-step transition probability matrix P given by *2 points*

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$$

- [ ] All states are recurrent.
- [ ] States 0, 1, 2, 4 are transient and state 3 is recurrent.
- [ ] States 0, 1, 2, 3 are recurrent and state 4 is transient.
- [ ] Only state 3 is recurrent.

No, the answer is incorrect.
Score: 0

Accepted Answers:
States 0, 1, 2, 3 are recurrent and state 4 is transient.

3) The one-step transition probability matrix of a discrete time Markov chain $\{X_n, n = 0, 1, 2, \ldots\}$ having three states $S = \{1, 2, 3\}$ is given by ***2 points***

$$P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$$

and the initial distribution is

$$\pi(0) = (0.7, 0.2, 0.1)$$

What is the value of $P(X_3 = 1, X_2 = 2, X_1 = 3, X_0 = 1)$?

- [ ] 0.0048
- [ ] 0.0672
- [ ] 0.0572
- [ ] 0.5

No, the answer is incorrect.
Score: 0

Accepted Answers:
0.0672

4) Let $\{X_n, n = 0, 1, 2, \ldots\}$ be a Markov chain with state space $S = \{1, 2, 3, \ldots, 14\}$ such that one-step transition probabilities are given by ***2 points***

$p_{ij} > 0$ if $|i - j|$ is even

$p_{ij} = 0$ if $|i - j|$ is odd

Which of the following is FALSE?

- [ ] Chain has exactly 2 stationary distributions.
- [ ] $P_{2,3}^{(4)} = 0$
- [ ] Chain is aperiodic.
- [ ] There are exactly two closed communicating classes.

No, the answer is incorrect.
Score: 0

Accepted Answers:
Chain has exactly 2 stationary distributions.

5) Consider a Markov chain $\{X_n, n = 0, 1, 2, \ldots\}$ with state space $S = \{1, 2, 3, 4\}$ and transition probability matrix **2 points**

$$P = \begin{pmatrix} \frac{1}{8} & \frac{1}{4} & \frac{1}{8} & \frac{1}{2} \\ \frac{3}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{3}{4} & 0 & \frac{1}{4} \\ \frac{1}{8} & 0 & \frac{7}{8} & 0 \end{pmatrix}$$

Which one of the following is a stationary distribution for the Makov chain?

- $(\frac{1}{3}, 0, \frac{1}{3}, \frac{1}{3})$
- $(\frac{1}{3}, 0, 0, \frac{2}{3})$
- $(0, \frac{1}{4}, \frac{1}{2}, \frac{1}{4})$
- $(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$

6) Consider a Markov chain $\{X_n, n = 0, 1, 2, \ldots\}$ with state space $S = \{0, 1, 2\}$ and one-step transition matrix **2 points**

$$P = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \end{pmatrix}$$

Which of the following is FALSE?

- $\lim_{n\to\infty} P_{0,2}^{(n)} = \frac{1}{3}$
- All states are positive recurrent.
- Period of state 1 is 1.
- $(\frac{1}{2}, 0, \frac{1}{2})$ is a stationary distribution

No, the answer is incorrect.
Score: 0

Accepted Answers:
$(\frac{1}{2}, 0, \frac{1}{2})$ *is a stationary distribution*

7) Consider a Markov chain $\{X_n, n = 0, 1, 2, \ldots\}$ with state space $S = \{1, 2\}$. Which one of the following can be TRUE? **2 points**

- The chain has unique stationary distribution.
- The chain has no stationary distribution.

$P(X_n = 1 \mid X_0 = 1)$ does not converges as $n \to \infty$

Both the states are transient states.

No, the answer is incorrect.
Score: 0

Accepted Answers:
The chain has unique stationary distribution.
$P(X_n = 1 \mid X_0 = 1)$ does not converges as $n \to \infty$

8) Let X_n be result of nth toss of a fair coin such that $X_n = 1$ if head occurs and 0 if tail occurs. Let ***2 points***

$$S_n = \sum_{i=1}^{n} X_i$$

Let Y_n be the remainder when S_n is divided by 2. Which of the following is/are true?

The state space of $\{Y_n, n = 0, 1, 2, \ldots\}$ is $\{0, 1, \ldots, 9\}$

There is exactly one closed communicating class.

The Markov Chain $\{Y_n, n = 0, 1, 2, \ldots\}$ has exactly three stationary distributions.

The Markov chain $\{Y_n, n = 0, 1, 2, \ldots\}$ is irreducible but not aperiodic.

No, the answer is incorrect.
Score: 0

Accepted Answers:
There is exactly one closed communicating class.

9) Consider a Markov chain $\{X_n, n = 0, 1, 2, \ldots\}$ with state space $S = \{0, 1, 2, 3, 4\}$ and the one-step transition matrix ***2 points***

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.25 & 0.75 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 \\ 0.25 & 0.25 & 0 & 0.25 & 0.25 \\ 0 & 0 & 0 & 0.5 & 0.5 \end{pmatrix}$$

Which of the following is/are true?

$\sum_{n=1}^{\infty} P_{2,2}^{(n)} = \infty$

$\lim_{n \to \infty} P_{2,3}^{(n)} = 1$

$\lim_{n \to \infty} P_{2,3}^{(n)} = 0$

$\sum_{n=1}^{\infty} P_{2,2}^{(n)} < \infty$

No, the answer is incorrect.
Score: 0

Accepted Answers:

$\lim_{n\to\infty} P_{2,3}^{(n)} = 0$

$\sum_{n=1}^{\infty} P_{2,2}^{(n)} < \infty$

10) Consider a Markov chain $\{X_n, n = 0, 1, 2, \ldots\}$ with state space $S = \{0, 1, 2, 3, 4\}$ and the one-step transition matrix **2 points**

$$P = \begin{pmatrix} 0.8 & 0 & 0 & 0.2 & 0 \\ 0 & 0.25 & 0 & 0 & 0.75 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0 & 0 & 0.8 & 0 \\ 0 & 0.75 & 0 & 0 & 0.25 \end{pmatrix}$$

Which of the following is/are TRUE?

- [ ] Chain is aperiodic
- [ ] Chain has infinitely many stationary distributions
- [ ] $\lim_{n\to\infty} P_{3,0}^{(n)} = 0.5$
- [ ] $(\frac{3}{8}, \frac{1}{8}, 0, \frac{3}{8}, \frac{1}{8})$ is a stationary distribution.

No, the answer is incorrect.
Score: 0

Accepted Answers:
Chain is aperiodic
Chain has infinitely many stationary distributions
$\lim_{n\to\infty} P_{3,0}^{(n)} = 0.5$
$(\frac{3}{8}, \frac{1}{8}, 0, \frac{3}{8}, \frac{1}{8})$ *is a stationary distribution.*

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