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NPTEL

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Courses » Introduction to Probability Theory and Stochastic Processes

Announcements **Course** Ask a Question Progress Mentor FAQ

Unit 4 - Week 2

Course outline

Assignment 2

The due date for submitting this assignment has passed. **Due on 2018-08-15, 23:59 IST.**
As per our records you have not submitted this assignment.

Each of the following questions has four options out of which one or more options can be correct. Individual marks are mentioned corresponding to each question. In case of multiple answers no partial marks will be awarded if all the correct choices are not selected. 0 marks for questions not attempted.

1) Let $\Omega = \{a, b, c, d\}$ be a sample space and F be a sigma field over Ω defined as $F = \{\phi, \Omega, \{a, b\}, \{c, d\}\}$. Consider the function $X : \Omega \to R$ defined as $X(a) = X(b) = 1,\ X(c) = 1,\ X(d) = 2$ then, which of the following statements are TRUE. ***1 point***

- X is a random variable with respect to F
- X is not a random variable with respect to F
- The minimal sigma-field with respect to which X is a random variable is $\{\phi, \Omega, \{d\}, \{c\}, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \}$
- The minimal sigma-field with respect to which X is a random variable is $\{\phi, \Omega, \{d\}, \{a, b, c\}\}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
X is not a random variable with respect to F
The minimal sigma-field with respect to which X is a random variable is $\{\phi, \Omega, \{d\}, \{a, b, c\}\}$

2) Consider the random variable X that represents the number of people who are hospitalized or die in a single head-on collision on the road in front of a particular spot in a year. The distribution of such random variables are typically obtained from historical data. Without getting into the statistical aspects involved, let us suppose that the cumulative distribution function of X is as follows: ***1 point***

x	0	1	2	3	4	5	6	7	8	9	10
$F(x)$	0.250	0.546	0.898	0.932	0.955	0.972	0.981	0.989	0.995	0.998	1.0

Then identify the TRUE statements.

$P(X \leq 5/X > 2) = \frac{37}{51}$

$P(X \geq 5) = 0.028$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$P(X \leq 5/X > 2) = \frac{37}{51}$

3) An urn contains cards numbered $1, 2, \ldots, 100$. Let X be the least number on the 50 cards drawn randomly without replacement from the urn. Then the $P(X \geq \frac{5}{2})$ is *2 points*

$\frac{1}{2}$

$\frac{148}{198}$

$\frac{49}{50}$

$\frac{49}{198}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\frac{49}{198}$

4) Let $\Omega = [0, 1]$. Define the random variable $X : \Omega \to R$ by *2 points*

$$X(w) = \begin{cases} w, & 0 \leq w \leq \frac{1}{2} \\ w - \frac{1}{2}, & \frac{1}{2} < w \leq 1 \end{cases}.$$

For any interval $I \subseteq [0, 1]$, define $P(I) = \int_I 2x\, dx$.
Then, which of the following statements are TRUE.

The probability density function of X is $f_X(x) = 2x,\ 0 \leq x \leq 1$

The probability density function of X is $f_X(x) = 4x + 1,\ 0 \leq x \leq \frac{1}{2}$

$P(X < \frac{1}{2} |X > \frac{1}{4}) = 0.5$

$P(X < \frac{1}{3} |X > \frac{1}{4}) = \frac{13}{45}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
The probability density function of X is $f_X(x) = 4x + 1,\ 0 \leq x \leq \frac{1}{2}$
$P(X < \frac{1}{3} |X > \frac{1}{4}) = \frac{13}{45}$

5) The distribution function of a random variable X is given as *1 point*

$$F(x) = \begin{cases} 1 - \frac{9}{x^2}, & x > 3 \\ 0 & \text{othervise} \end{cases}$$

Identify the FALSE statements.

$P(X \leq 5) = \frac{16}{25}$

$P(X > 8) = \frac{9}{64}$

$f_X(x) = \frac{18}{x^3}$, $x > 3$

$f_X(x) = \frac{3}{x^3}$, $x > 3$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$f_X(x) = \frac{3}{x^3}$, $x > 3$

6) Let X be a random variable such that $P(X = 2) = \frac{1}{4}$ and its distribution function is given by *2 points*

$$F_X(x) = \begin{cases} 0, & x < -3 \\ \alpha(x+3), & -3 \leq x < 2 \\ \dfrac{3}{4}, & 2 \leq x < 4 \\ \beta x^2, & 4 \leq x < 8/\sqrt{3} \\ 1, & x \geq 8/\sqrt{3} \end{cases}.$$

If 2 is the only jump discontinuity of F. Then identify the TRUE statements.

$\alpha = \frac{1}{10}$, $\beta = \frac{3}{16}$

$\alpha = \frac{1}{10}$, $\beta = \frac{3}{64}$

$P(X < 3/X \geq 2) = \frac{1}{2}$

$P(X > 3/X \geq 2) = \frac{1}{4}$

No, the answer is incorrect.
Score: 0

Accepted Answers:
$\alpha = \frac{1}{10}$, $\beta = \frac{3}{64}$
$P(X < 3/X \geq 2) = \frac{1}{2}$

7) Consider a random experiment of choosing a point in the annular disc of inner radius r_1 and outer radius r_2. Let X be the distance of chosen point from the center of annular disc. Then identify the TRUE statements. *2 points*

$f_X(x) = \frac{2x}{r_2 - r_1}$, $r_1 \leq x \leq r_2$

If $r_1 = 5$ and $r_2 = 12$, then $P(X > 7) = \frac{24}{119}$

If $r_1 = 5$ and $r_2 = 10$, then $P(5.5 \leq X \leq 7) = \frac{1}{4}$

$f_X(x) = \frac{2x}{r_2^2 - r_1^2}$, $r_1 \leq x \leq r_2$

No, the answer is incorrect.
Score: 0

Accepted Answers:

If $r_1 = 5$ and $r_2 = 10$, then $P(5.5 \leq X \leq 7) = \frac{1}{4}$

$f_X(x) = \frac{2x}{r_2^2 - r_1^2}$, $r_1 \leq x \leq r_2$

8) Let X be a random variable with cumulative distribution function given by: ***2 points***

$$F_X(x) = \begin{cases} 0, & x < 1 \\ \dfrac{1}{25}, & 1 \leq x < 2 \\ \dfrac{x^2}{10}, & 2 \leq x < 3 \\ 1, & x \geq 3 \end{cases}.$$

If $F_X(x) = \alpha F_d(x) + \beta F_c(x)$, where α and β are constants such that $\alpha + \beta = 1$, F_d is the cumulative discrete distribution function and F_c is the cumulative continuous distribution function of some discrete and continuous random variables X_d and X_c respectively. Then, identify the TRUE statements.

- [ ] $\alpha = \frac{2}{5}$ and $\beta = \frac{3}{5}$
- [ ] The probability density function of X_c is $f(x) = \frac{2x}{5}$, $2 < x < 3$
- [ ] $\alpha = \frac{1}{5}$ and $\beta = \frac{4}{5}$
- [ ] The probability mass function of X_d is $P(X_d = 1) = \frac{1}{10}$, $P(X_d = 2) = \frac{9}{10}$

No, the answer is incorrect.
Score: 0

Accepted Answers:

The probability density function of X_c is $f(x) = \frac{2x}{5}$, $2 < x < 3$

9) Let X be a continuous random variable with pdf ***2 points***

$$f_X(x) = \begin{cases} \frac{x^3}{64}, & 0 \leq x \leq 4 \\ 0, & \text{elsewhere} \end{cases}$$

Then the probability density function of the random variable $\min(\sqrt{X}, 2 - \sqrt{X})$ is

- ( ) $$f_Y(y) = \begin{cases} \frac{y^6}{32}, & 0 \leq y \leq 1 \\ \frac{(2-y)^6}{32}, & 1 \leq y \leq 2 \\ 0, & \text{elsewhere} \end{cases}$$
- ( ) $$f_Y(y) = \begin{cases} \frac{y^7}{32}, & 0 \leq y \leq 1 \\ \frac{(2-y)^7}{32}, & 1 \leq y \leq 2 \\ 0, & \text{elsewhere} \end{cases}$$
- ( ) $$f_Y(y) = \begin{cases} \frac{y^7}{32}, & 0 \leq y \leq 2 \\ 0, & \text{elsewhere} \end{cases}$$
- ( ) $$f_Y(y) = \begin{cases} \frac{7y^6}{128}, & 0 \leq y \leq 2 \\ 0, & \text{elsewhere} \end{cases}$$

No, the answer is incorrect.

Score: 0

Accepted Answers:

$$f_Y(y) = \begin{cases} \frac{y^7}{32}, & 0 \le y \le 1 \\ \frac{(2-y)^7}{32}, & 1 \le y \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

10) Suppose that X is a continuous random variable with pdf $f_X(x) = 2x$ for $0 < x < 1$. ***2 points***

Define

$$Y = \begin{cases} X, & X < \frac{3}{4} \\ 1, & \frac{3}{4} \le X \end{cases}.$$

Then, which of the following statements are TRUE

- [ ] The distribution of Y is continuous.
- [ ] The distribution of Y is either discrete or mixed type.
- [ ] $f_Y(y) = 2y,\ 0 \le y \le 1$
- [ ] $P(Y = 1) = \frac{7}{16}$

No, the answer is incorrect.

Score: 0

Accepted Answers:

The distribution of Y is either discrete or mixed type.

$P(Y = 1) = \frac{7}{16}$

Previous Page

End