Department of Mathematics
MTL 601 (Probability and Statistics)
Tutorial Sheet No. 6

1. Suppose that the variance of the speeds with which men and women can perform a certain task are respectively $\sigma_1 = 12$ seconds and $\sigma_2 = 15$ seconds. If 20 men and 25 women required on the average 29.4 seconds and 32.5 seconds to perform the given task, obtain a 0.95 confidence interval for the difference between the true average times it takes men and women to perform the task.

2. Four determinants of the pH of a certain solution were 7.90, 7.94, 7.91, 7.93. If μ is the "true" mean of the pH of the solution and assuming normality of determination of pH of the solution, find 99 percent confidence limits of μ.

3. Suppose a manufacturer regularly tests received consignments of yarn to check the average count or linear density (in text). Experience has shown that standard count tests on specimens chosen at random from a delivery of a certain type of yarn usually have a coefficient of variation of 3%. A normal 35-tex yarn is to be tested. How many tests are to be required to make the 95% confidence limits equal to + or - 0.5?

4. A retailer buys garments of the same style from two manufacturers and suspects that the variation in the masses of the garments produced by the two makers is different. A sample of size 20 was therefore chosen from a batch of garments produced by the first manufacturer and weighed. The resulting sample variance was $s_1^2 = 25.0(grams)^2$. A sample of size 25 was chosen from a consignment sent by the second manufacturer, the sample variance being $s_2^2 = 14.1(grams)^2$. Compute the 95% confidence interval for the ratio of the variances.

5. Let $X_1, X_2, \ldots, X_50$ denote a random sample of size 50 from a normal distribution $N(\theta, 100)$. Find a uniformly most powerful critical region of size $\alpha = 0.10$ for testing $H_0 : \ \theta = 50$ against $H_1 : \ \theta > 50$.

6. Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with the following pdf

$$f(x;\theta) = \begin{cases} \theta x^{\theta-1} & 0 < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

where $\theta > 0$. Find a sufficient statistics for θ and show that a uniformly most powerful test of $H_0 : \ \theta = 6$ against $H_1 : \ \theta < 6$ is based on this statistics.

7. If $X_1, X_2, \ldots, X_n$ is a random sample from a beta distribution with parameters $\alpha = \beta = \theta > 0$, find a best critical region for testing $H_0 : \ \theta = 1$ against $H_1 : \ \theta = 2$.

8. In a certain chemical process, it is very important that a particular solution that is to be used as a reactant have a pH of exactly 8.20 . A method for determining pH that is available for solutions of this type is known to give measurements that are normally distributed with a mean equal to the actual pH and with a standard deviation of 0.02 . Suppose 10 independent measurements yielded the following pH values:

pH values	8.18	8.17	8.16	8.15	8.17	8.21	8.22	8.16	8.19	8.18

   (a) What conclusion can be drawn at the $\alpha = 0.10$ level of significance?
   (b) What about at the $\alpha = 0.05$ level of significance?

9. The life of a certain electrical equipment is normally distributed. A random sample of lives of twelve such equipments has a standard deviation of 1.3 years. Test the hypothesis that the standard deviation is more than 1.2 years at 10% level of significance.

10. Consider the strength of a synthetic fibre that is possible affected by the percentage of cotton in the fibre. Five levels of this percentage are considered with five observations at each level. The data is shown in the Table 1.

    Use the F-test, with $\alpha = 0.05$ to see if there are differences in the breaking strength due to the percentages of cotton used.

Table 1: Strength of a synthetic fibre for Exercise 10

15	7	7	15	11	9
20	12	17	12	18	18
25	14	18	18	19	19
30	19	25	22	19	23
35	7	10	11	15	11

Table 2: Data for Exercise 14

Day	Sun	Mon	Tue	Wed	Thur	Fri	Sat
Number of Earthquakes (f_i)	156	144	170	158	172	148	152

11. It is desired to determine whether there is less variability in the marks of Probability and Statistics course by IITD students than in that by IITB students. If independent random samples of size 10 of the two IIT's yield $s_1 = 0.025$ and $s_2 = 0.045$, test the hypothesis at the 0.05 level of significance.

12. Elongation measurements are made on 10 pieces on steel, 5 of which are treated with method A (aluminium only) and the remaining 5 are method B (aluminium plus calcium). The results obtained are as under:

Method A	78	29	25	23	30
Method B	34	27	30	26	23

Test the hypothesis that (a) $\sigma_A^2 = \sigma_B^2$ (b) $\mu_B - \mu_A = 10\%$ at 2% level of significance by choosing approximate alternatives.

13. Suppose the weekly number of accidents over a 60-week period in Delhi is as follows:

1	0	0	1	3	4	0	2	1	4	2	2	0	0	5	2	1	3	0	1
1	8	0	2	0	1	9	3	3	5	1	3	2	0	7	0	0	0	1	3
3	3	1	6	3	0	1	2	1	2	1	1	0	0	2	1	3	0	0	2

Test the hypotheses that the number of accidents in a week has a Poisson distribution. Assume $\alpha = 0.05$. ($\chi^2_{3,0.05} = 7.81; \chi^2_{4,0.05} = 9.48$).

14. A study was investigated to see if Southern California earthquakes of at least moderate size (having values of at least 4.4 on the Richter Scale) are more likely to occur on certain days of the week than on others. The catalogs yielded the following data on 1100 earthquakes: Test at the 5% level of significance, the hypotheses that an earthquake is equally likely to occur on any of the 7 days of the week.

15. Consider the data of Table 3 that corresponds to 60 rolls of a die: Test the hypotheses that the die is fair ($P_i = \frac{1}{6}, \quad i = 1, \ldots, 6$), at 0.5% level of significance.

Table 3: Data for Exercise 15

4	3	3	1	2	3	4	6	5	6
2	4	1	3	4	5	3	4	3	4
3	3	4	5	4	5	6	4	5	1
6	3	6	2	4	6	4	6	3	5
6	3	6	2	4	6	4	6	3	2
5	4	6	3	3	3	5	3	1	4

Table 4: Data for Exercise 16

	Accident	No Accident
Cellular Phone	22	278
No Phone	26	374

Table 5: Data for Exercise 17

	Smokers	Non Smokers
Lung cancer	62	14
No Lung Cancer	9938	19986

16. A sample of 300 cars having cellular phones and one of 400 cars without phones are tracked for 1 year. Table 4 gives the number of cars involved in accidents over that year. Use the above to test the hypotheses that having a cellular phone in your car and being involved in an accident are independent. Use the 5 percent level of significance.

17. A randomly chosen group of 20,000 nonsmokers and one of 10,000 smokers were followed over a 10-year period. The following data of Table 5 relate the numbers of them that developed lung cancer during the period. Test the hypotheses that smoking and lung cancer are independent. Use the 1% level of significance.

18. Use the 10% level of significance to perform a hypotheses test to see if there is any evidence of a difference between the Channel A viewing area and Channel B viewing area in the proportion of residents who viewed a news telecast by both the channels. A simple random sample of 175 residents in the Channel A viewing area and 225 residents in the Channel B viewing area is selected. Each resident in the sample is asked whether or not he/she viewed the news telecast. In the Channel A telecast, 49 residents viewed the telecast, while 81 residents viewed the Channel B telecast.

19. Three kinds of lubricants are being prepared by a new process. Each lubricant is tested on a number of machines, and the result is then classified as acceptable or non-acceptable. The data in the Table 6 represents the outcome of such an experiment. Test the hypotheses that the probability p of a lubricant resulting in an acceptable outcome is the *same* for all three lubricants. Test at the 5% level of significance.

20. Twenty five men between the ages of 25 and 30, who were participating in a well-known heart study carried out in New Delhi, were randomly selected. Of these, 11 were smokers and 14 were not. The following data refers to readings of their systolic blood pressure.

    Use the data of Table 7 to test the hypothesis that the mean blood pressures of smokers and nonsmokers are the same at 5% level of significance.

21. To examine the effects of pets and friends in stressful situations, researchers recruited 45 people to participate in an experiment and data is shown in Table 8. Fifteen of the subjects were randomly assigned to each of the 3 groups to perform a stressful task alone (Control Group), with a good friend present, or with their dog present. Each subjects mean heart rate during the task was recorded. Using ANOVA method, test the appropriate hypotheses at the $\alpha = 0.05$ level to decide if the mean heart rate differs between the groups.

Table 6: Data for Exercise 19

	Lubricant 1	Lubricant 2	Lubricant 3
Acceptable	144	152	140
Non-acceptable	56	48	60
Total	200	200	200

Table 7: Data for Exercise 20

Smokers	Nonsmokers
124	130
134	122
136	128
125	129
133	118
127	122
135	116
131	127
133	135
125	120
118	122
	120
	115
	123

Table 8: Data for Exercise 21

	n	Mean	SD
Control	15	82.52	9.24
Pets	15	73.48	9.97
Friends	15	91.325	8.34

22. A fisheries researcher wishes to conclude that there is a difference in the mean weights of 3 species of fish (A,B,C) caught in a large lake. The data is shown in Table 9. Using ANOVA method, test the hypotheses at $\alpha = 0.05$ level.

Table Values

P($\mathbf{Z}$ is a standard normal distribution $\geq Z_\alpha$) $= \alpha$

P(χ^2 r.v. with n degrees of freedom $\geq \chi^2_{n,\alpha}$) $= \alpha$

P($\mathbf{t}$ r.v. with n degrees of freedom $\geq t_{n,\alpha}$) $= \alpha$

P($\mathbf{F}$ r.v. with n_1 and n_2 degrees of freedom $\geq F_{n_1,n_2,\alpha}$) $= \alpha$

$Z_{0.025} = 1.96; Z_{0.05} = 1.645; Z_{0.0764} = 1.43; Z_{0.01} = 2.33; Z_{0.035} = 1.81$

$\chi^2_{9,0.05} = 16.917; \chi^2_{6,0.05} = 12.6; \chi^2_{5,0.05} = 11.1; \chi^2_{4,0.05} = 9.48;$

$\chi^2_{3,0.05} = 7.81; \chi^2_{2,0.05} = 5.99; \chi^2_{9,0.95} = 3.334$

$t_{8,0.025} = 2.31; t_{9,0.025} = 2.26; t_{10,0.025} = 2.22$

$F_{9,11,0.025} = 0.1539; F_{10,15,0.025} = 3.5217; F_{15,10,0.025} = 3.0602$

Note: If above table values are not matched, please leave the answer without numerical.

Table 9: Mean weights of 3 species of fish for Exercise 22

A	B	C
1.5	1.5	6
4	1	4.5
4.5	4.5	4.5
3	2	5.5