Lecture 2: Immediate RL and Bandits

B. Ravindran

Reinforcement Learning

- Familiar models of machine learning
  - Learning from data
- How did you learn to cycle?
  - Trial and error!
  - Falling down hurts!
  - Evaluation, not instruction
  - Reinforcement Learning
- Walk, Talk, etc.

Immediate Reinforcement

- The payoff accrues immediately after an action is chosen
- One key question - the dilemma between exploration and exploitation
- *Bandit problems* encapsulate 'Explore vs Exploit'

The Explore-Exploit Dilemma

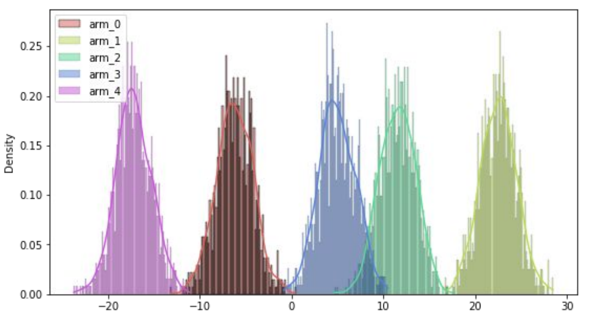
- Explore to find profitable actions
- Exploit to act according to the best observations already made

- Always exploiting might not be optimal
- Always exploring might not be optimal either

- Hence, there is an explore-exploit dilemma

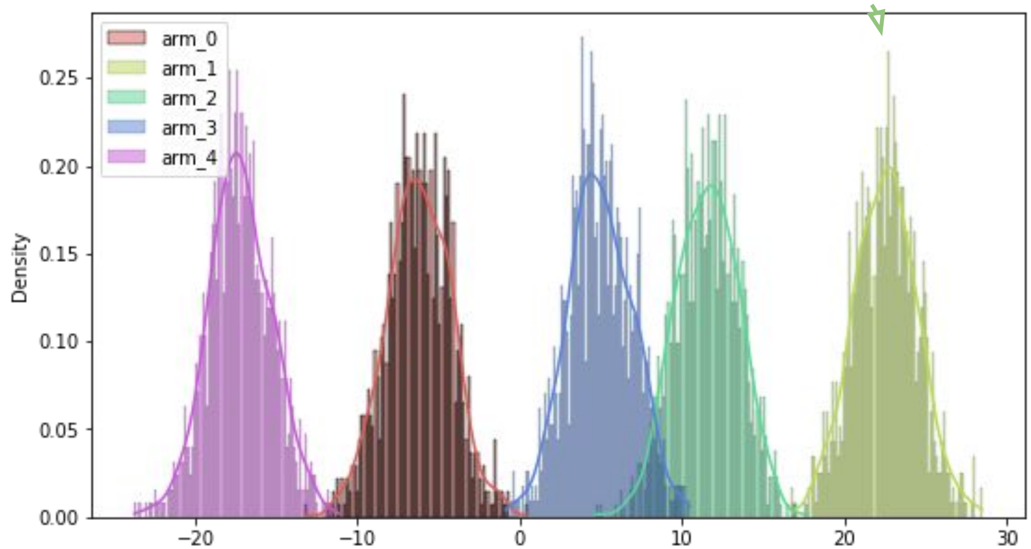
Multi-arm Bandits

- n-arm bandit problem is to learn to preferentially select a particular action (arm) from a set of n actions (1, 2, 3, ... , *n*)
- Each selection results in a reward derived from the respective probability distribution
- Arm *i* has a reward distribution with mean μ_i and $\mu^* = \max\{\mu_i\}$



Objective

- ❑ Identify the correct arm eventually



Traditional Approaches

- Let $r_{i,k}$ be the reward sample acquired when i^{th} arm is selected for the k^{th} time

- Define:

$$Q_t(a) \doteq \frac{\text{sum of rewards when } a \text{ taken prior to } t}{\text{number of times } a \text{ taken prior to } t}$$

$$Q_t(a) = \frac{\sum_{i=1}^{t-1} R_i \cdot \mathbb{1}_{A_i=a}}{\sum_{i=1}^{t-1} \mathbb{1}_{A_i=a}}$$

$$A_t \doteq \underset{a}{\arg\max}\, Q_t(a)$$ (greedy action)

Traditional Approaches

$$Q_t(a) = \frac{\sum_{i=1}^{t-1} R_i \cdot \mathbb{1}_{A_i=a}}{\sum_{i=1}^{t-1} \mathbb{1}_{A_i=a}}$$

N_A Number of times arm A is sampled

$$NewEstimate \leftarrow OldEstimate \; + \; StepSize \left[Target - OldEstimate \right]$$

$$Q_{n+1} \quad = \quad Q_n + \alpha \Big[R_n - Q_n \Big]$$

- Setting $\alpha = \frac{1}{N_A}$ yields the same average

Traditional Approaches

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- Setting $\alpha = \frac{1}{N_A}$ yields the same average

Traditional Approaches

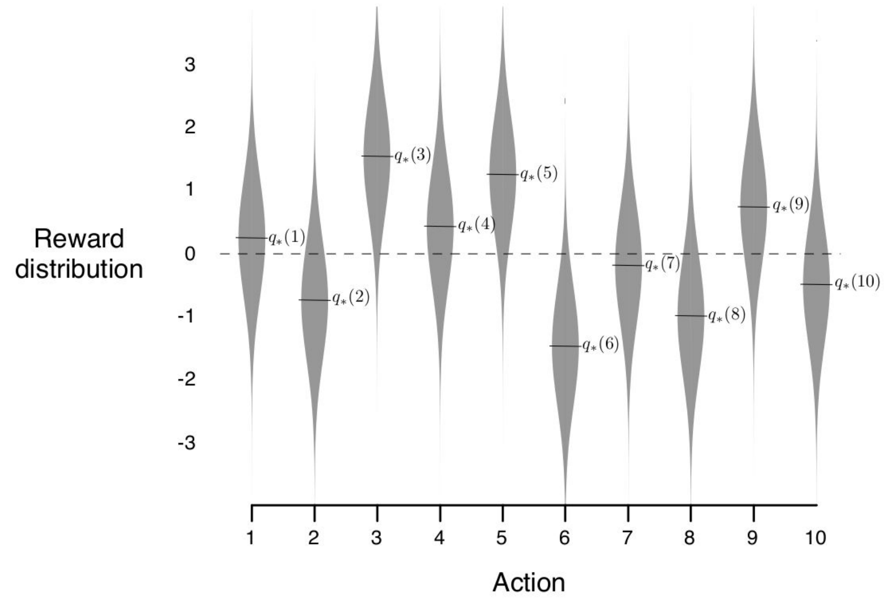
- **Epsilon Greedy :** Select arm $a^* = \arg\max_a Q_t(a)$ with probability $1 - \epsilon$ and select any arbitrary arm with probability ϵ

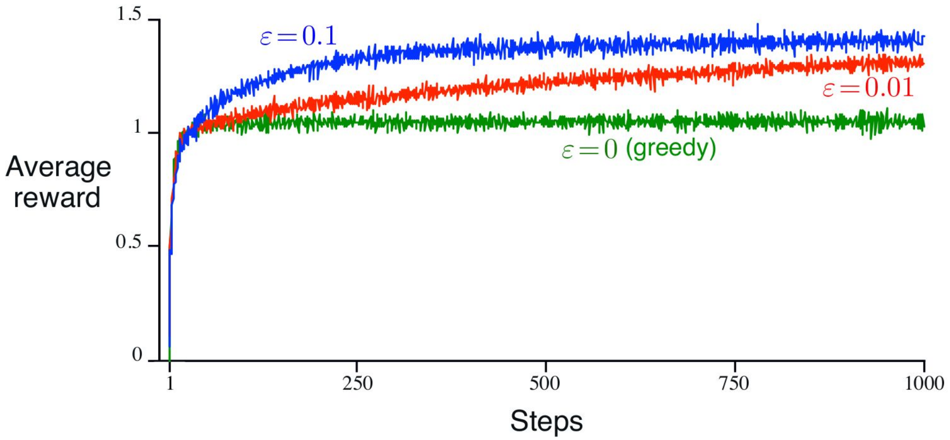
- **Softmax :** Select arms with probability proportional to the current value estimates

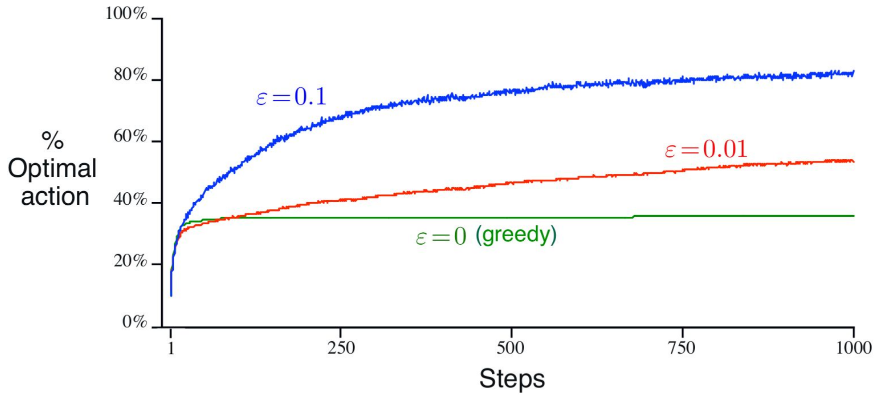
$$\Pr\{A_t = a\} \doteq \frac{e^{(Q_t(a)\ /\ \tau)}}{\sum_{b=1}^{k} e^{(Q_t(b)\ /\ \tau)}}$$

- Asymptotic Convergence guarantees

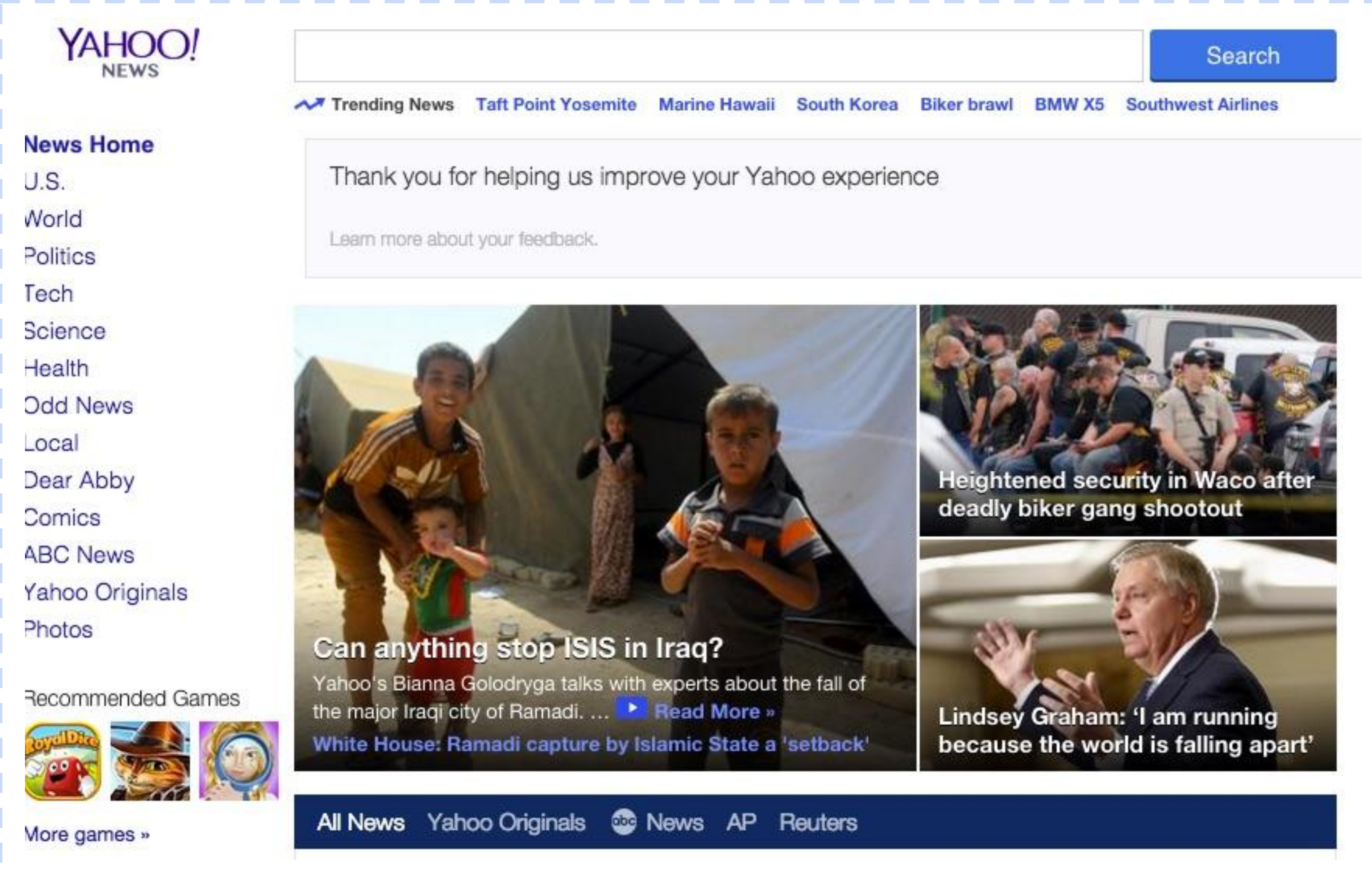
ϵ-Greedy Example





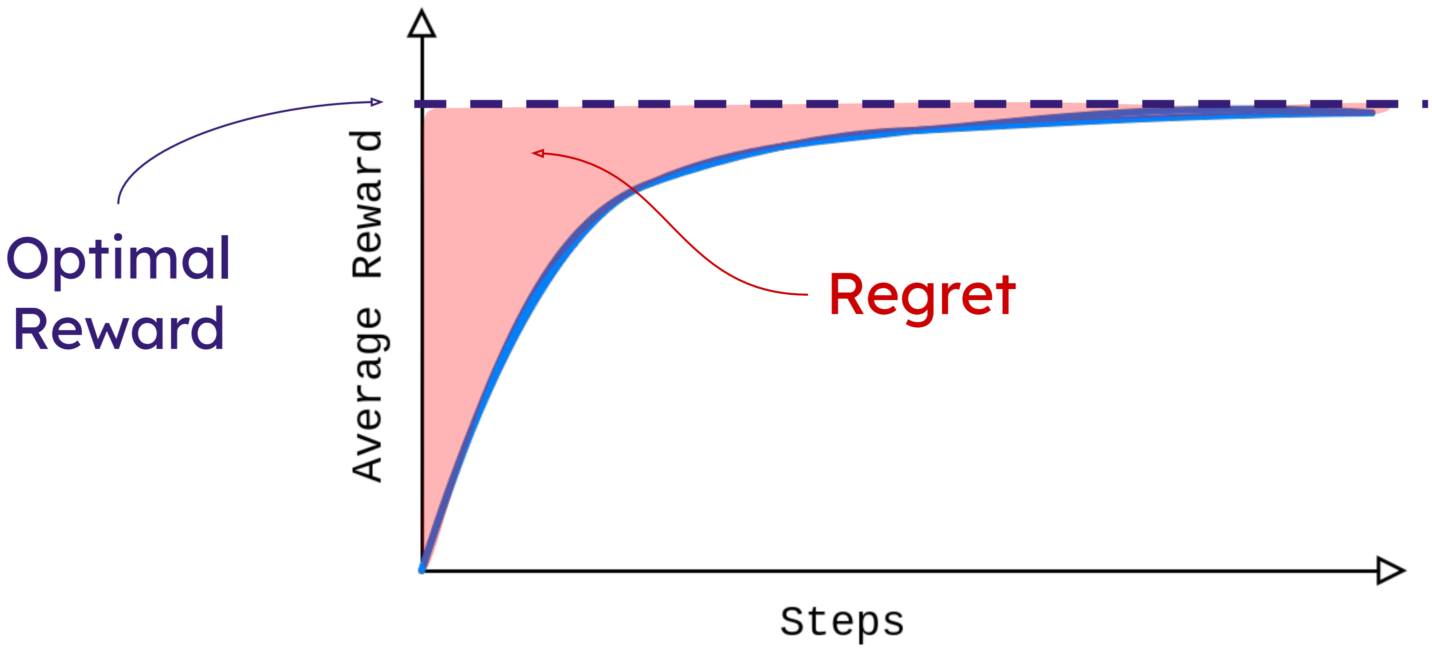


Customization



Objective

- ❑ Identify the correct arm eventually
- ❑ Maximize the total rewards obtained
  - ❑ Minimize regret (= loss) while learning



Objective

- Identify the correct arm eventually
- Maximize the total rewards obtained
  - Minimize regret (= loss) while learning

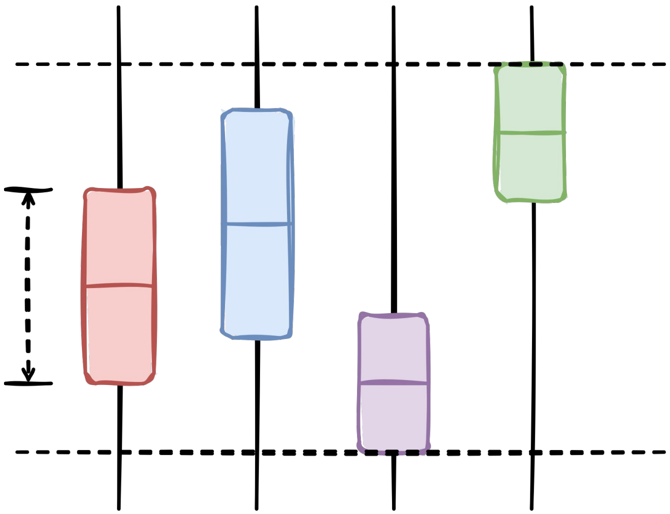
- Probably Approximately Correct (PAC) frameworks
  - Identification of an ε-optimal arm with probability $1 - \delta$
  - ε-Optimal: Mean of the selected arm satisfies
  - Minimize sample complexity: Order of samples required for such an arm identification

Other Approaches

- Median Elimination (Even-Dar et al., 2006)
- Upper Confidence Bounds (UCB) (Auer et al., 1998, 2010)
- Thompson Sampling (Chappelle & Li, 2001, Agrawal & Goyal, 2012)

UCB

- ε-greedy action selection forces the non-greedy actions to be tried
- With no preference for arms that are nearly greedy or particularly uncertain
- Opt for non-greedy actions based on their potential for optimality & consider estimate uncertainties



$$A_t = \arg\max_a \left[Q_t(a) + c\sqrt{\frac{\ln t}{N_t(a)}} \right]$$

UCB

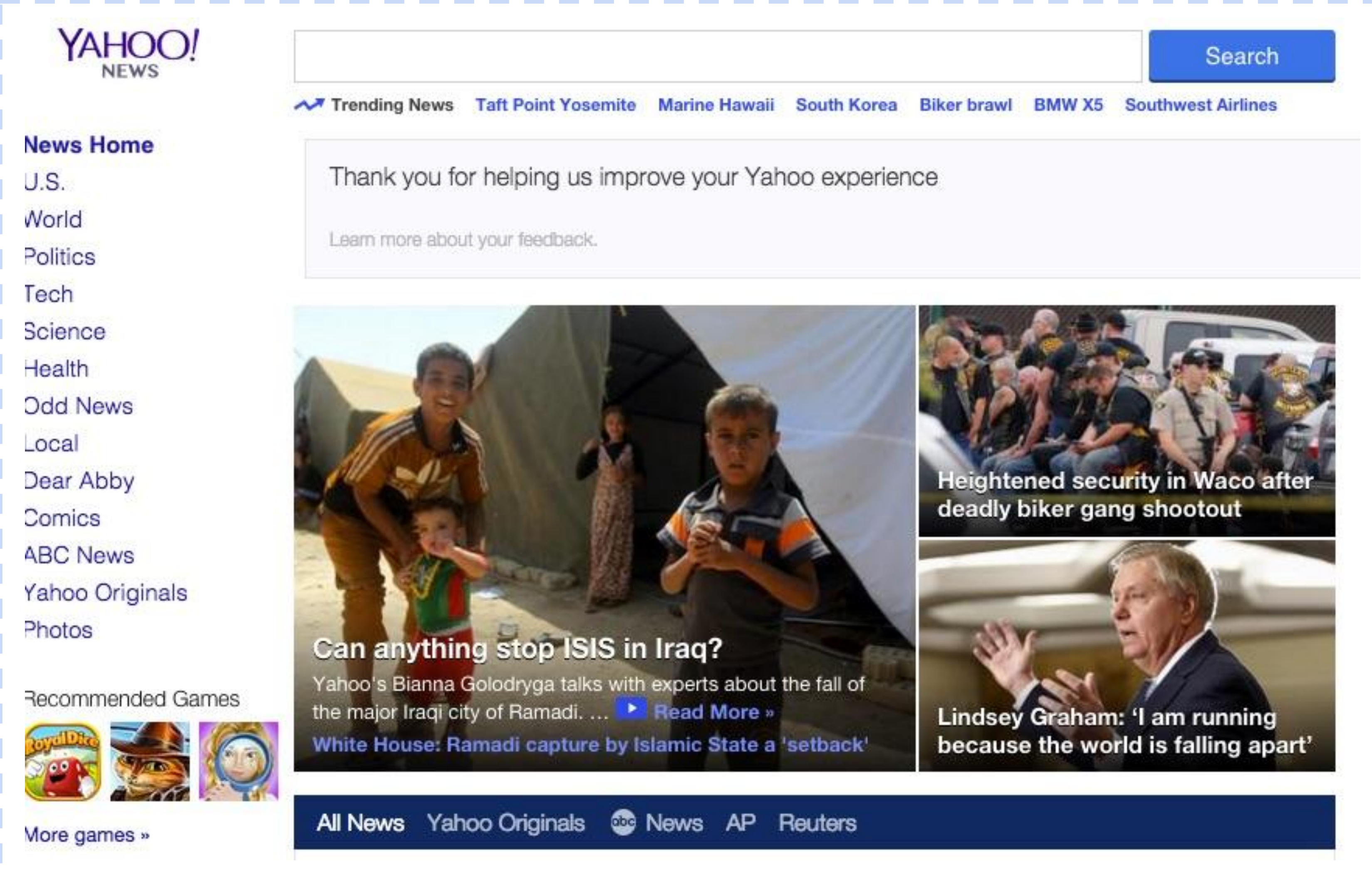
$$A_t = \arg\max_a \boxed{\left[Q_t(a) + c\sqrt{\frac{\ln\ t}{N_t(a)}} \right]}$$

Upper bound on true $Q_t^*(a)$

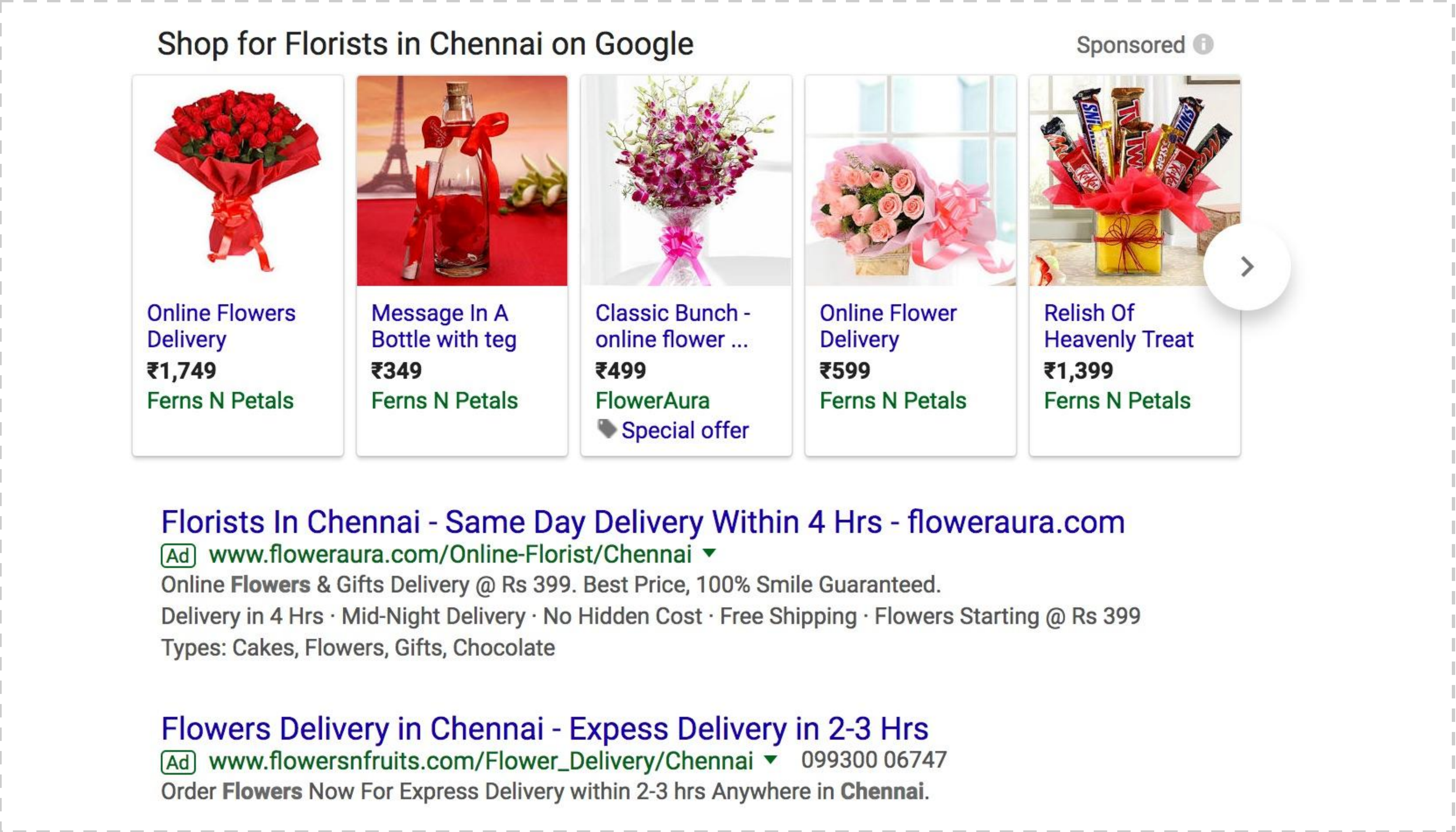
Uncertainty in the estimate of $Q_t(a)$

- c > 0 - controls the degree of exploration
- Sub-optimal arm *j* played fewer than $\dfrac{8\ \ln t}{\Delta_j^2}$ times
- Further improvements focus on reducing the constants

Customization



Ad Selection



Contextual Bandits

- Different ads for different users
  - One bandit for each user!
- Hard to train - Need several rounds of experience with same user

- Assume that the parameters of the reward distributions themselves are determined by a set of hyperparameters
  - Typical assumption is a linear parameterization of the expectation

Contextual Bandits

- Assume that each user is represented by a set of features
  - Can be joint features of user and arm
- The “statistic” used for choosing arms is now dependent on these features
- Could correspond to the presence or absence of different signals

LinUCB

- ❑ One of the more popular contextual bandit algorithms
- ❑ *Predicted expected reward* assumed to be a linear function of the features
- ❑ Use ridge regression to fit parameters
- ❑ Can derive upper confidence bounds for the regression fit
- ❑ Use UCB like action selection

- ❑ Gives better performance with lesser “training” data