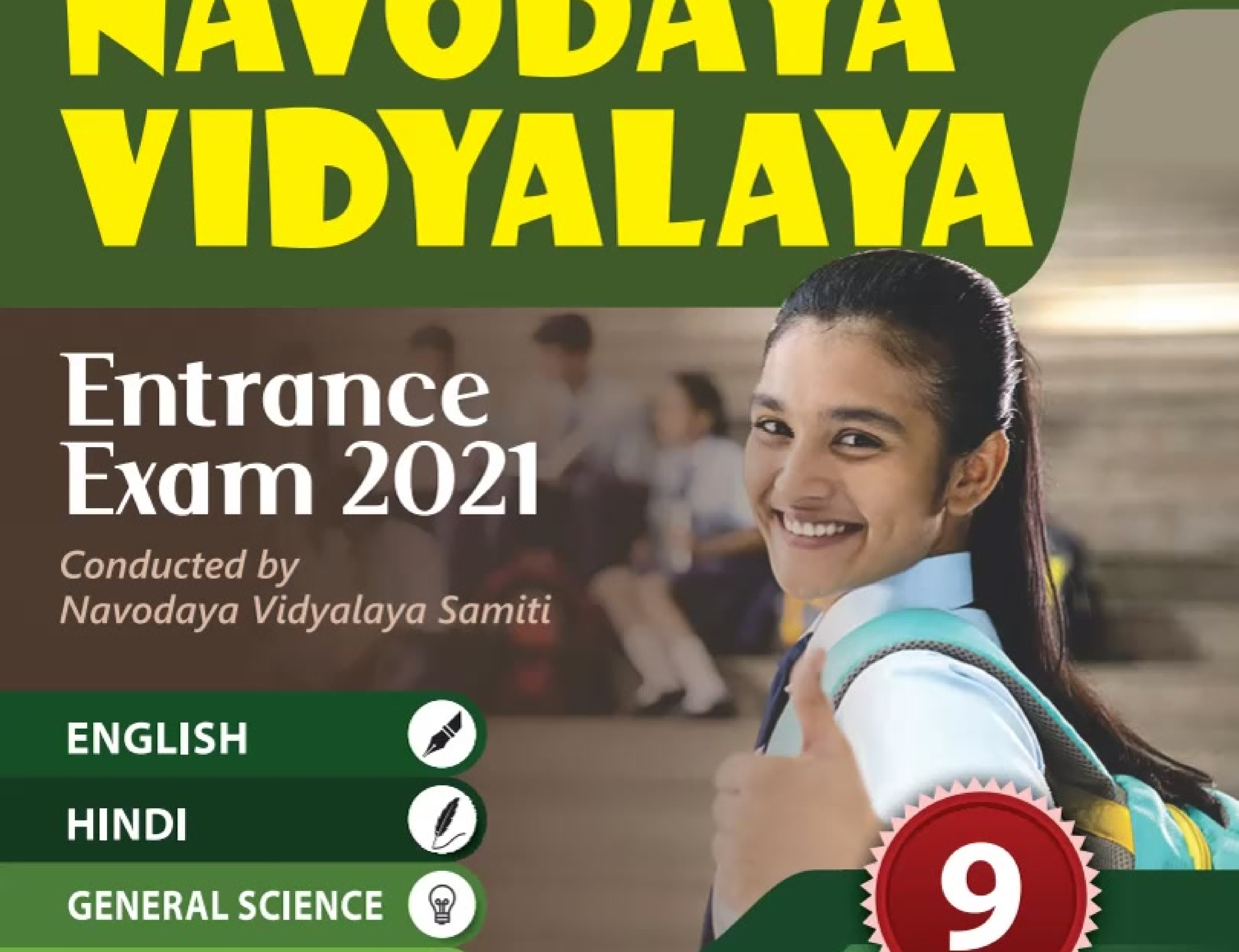
arihant

JAWAHAR NAVODAYA VIDYALAYA

Entrance Exam 2021

Conducted by Navodaya Vidyalaya Samiti



ENGLISH

HINDI

GENERAL SCIENCE

MATHEMATICS

9

With Solved Paper 2020

JAWAHAR NAVODAYA VIDYALAYA

Entrance Exam 2021

Conducted by
Navodaya Vidyalaya Samiti

JAWAHAR NAVODAYA VIDYALAYA

Entrance Exam 2021

Conducted by
Navodaya Vidyalaya Samiti

Complied & Edited by
Arihant 'Expert Team'

9

arihant
ARIHANT PUBLICATIONS (INDIA) LTD.

arihant

Arihant Publications (India) Ltd.

卐 **Administrative & Production Offices**

Regd. Office

'Ramchhaya' 4577/15, Agarwal Road, Darya Ganj, New Delhi -110002
Tele: 011- 47630600, 43518550

Head Office

Kalindi, TP Nagar, Meerut (UP) - 250002, Tel: 0121-7156203, 7156204

卐 **Sales & Support Offices**

Agra, Ahmedabad, Bengaluru, Bareilly, Chennai, Delhi, Guwahati,
Hyderabad, Jaipur, Jhansi, Kolkata, Lucknow, Nagpur & Pune.

卐 **ISBN** 978-93-24196-57-6

PO No : TXT-XX-XXXXXXX-X-XX

Published by Arihant Publications (India) Ltd.
For further information about the books published by Arihant, log on to
www.arihantbooks.com or e-mail at info@arihantbooks.com

Follow us on

Contents

Jawahar Navodaya Vidyalayas Selection Test

SYLLABUS

ENGLISH
Comprehension (Unseen Passage), Word and Sentence Structure, Spelling, Rearranging jumbled words , Passivation, Use of degrees of comparison, Modal auxiliaries , Use of prepositions , Tense forms, Reported speech

हिन्दी

(क) भाषिक अनुप्रयोग और व्याकरणिक कुशलताएँ

वर्ण विचार/वर्तनी विवेक, शब्दभेद (स्त्रोत/उत्पत्ति), पर्याय/विलोम, शब्द विवेक (शब्द प्रयोग में सूक्ष्म अंतर), पद भेद (व्याकरणिक कोटि) की पहचान, पद परिचय, अशुद्ध वाक्य को शुद्ध करना, वाक्य रचनान्तरण (सरल/संयुक्त/मिश्र), मुहावरा, लोकोक्ति

(ख) अपठित बोधात्मक प्रश्न

GENERAL SCIENCE
Food – Crop Production and Management; Microorganism; Food Preservation, Materials I – Synthetic fibers; Plastics; Metals and Non – metals, Material II – Coal and Petroleum; Refining of Petroleum; Fossil Fuels; Combustion and Flame, Living / Non living; Cell structure and Function; Conservation of Plants and Animals – Wildlife, Sanctuary and National Parks, Reproduction – Asexual and Sexual, Reproduction, Reaching the age of adolescence, Force – Frictional Force; Gravitational Force; Thrust and Pressure, Light – Reflection of Light; Multiple Reflection; Human eye; Care of the Eyes; Sound; Human ears; Loudness and Pitch, Audible and inaudible Sounds, Chemical Effects of Electric Current; Electroplating, Natural Phenomena – Lightning; Earthquakes, Pollution of Air and Water, Solar system; Stars and Constellations

MATHEMATICS
Rational Numbers, Squares and Square Roots, Cubes and Cube Roots, Exponents and Powers, Direct and Inverse Proportions, Comparing Quantities (Percentage, Profit and Loss, Discount, Simple and Compound Interest), Algebraic Expressions and Identities including Factorization, Linear Equations in One Variable, Understanding Quadrilaterals (Parallelogram, rhombus, rectangle, square, kite) , Mensuration: *(a) Area of plane figures* , *(b) Surface area and volume of cube, cuboid and cylinder* , Data Handling (Bar graph, pie chart, organizing data, probability)

JAWAHAR
NAVODAYA
VIDYALAYA

MATHEMATICS

CHAPTER

01

NUMBER SYSTEM
(RATIONAL NUMBERS)

In this chapter, we study the various types of numbers, properties of rational numbers, simplification and test for divisibility.

Digits The symbols 0,1, 2, 3, 4, 5, 6, 7, 8, 9 are known as digits in Hindu Arabic System.

Numbers or Numerals A mathematical symbol which represent the digits, are known as numbers or numerals.

Types of Numbers There are some types of numbers, which are given below

(i) **Natural Numbers** Those numbers which are used for counting, are known as natural numbers. These are denoted by N.

e.g. $N = 1, 2, 3, \ldots$

Here 1 is the first and smallest natural number.

(ii) **Whole Numbers** If 0 is included in natural numbers, then these numbers are known as whole numbers. These numbers are denoted by W.

e.g. $W = 0, 1, 2, 3, \ldots$

(iii) **Integers** All whole numbers and their negative numbers are known as integers. These numbers are denoted by I.

$\therefore\ I = 0, \pm 1, \pm 2, \pm 3, \ldots$

Here, 1, 2, 3, … are positive integers, denoted by I^+, and

$-1, -2, -3, \ldots$ are negative integers, denoted by I^-.

Here 0 is neither positive nor negative integer.

(iv) **Rational Numbers** Numbers in the form of $\frac{p}{q}$, where $p, q \in I$ and $q \neq 0$, are known as rational numbers. It is denoted by Q.

e.g. $\frac{2}{3}, \frac{5}{6}, 6, \frac{-4}{5}$, etc.

(v) **Irrational Numbers** Numbers which can not be expressed in the form of $\frac{p}{q}$, where $p, q \in I$ and $q \neq 0$, are known as irrational numbers.

e.g. $\pi, \sqrt{2}, \sqrt{6}, \sqrt[3]{11}$, etc.

(vi) **Real Numbers** Those numbers which are either rational or irrational, are known real numbers. It is denoted by R.

All natural, whole, integer, rational and irrational numbers are real numbers.

e.g. $2, 0, -5, \frac{1}{2}, \sqrt{6}$, etc.

(vii) **Even Numbers** Those numbers which are divisible by 2, are known as even numbers.

e.g., 2, 4, 6, ...

Note 2 is smallest even number.

(viii) **Odd Numbers** Those numbers, which are not divisible by 2, are known as odd numbers.

e.g. 1, 3, 5, 7, ...

(ix) **Prime Numbers** Those numbers which are divisible by 1 and the number itself, are known as prime numbers.

e.g. 2, 3, 5, 7, 11, 13, ...

Here 2 is the only even prime number.

(x) **Composite Numbers** Those numbers which are divisible by at least one number except 1 and the number itself, are known as composite numbers.

e.g. 12, 8 and 15 etc, are composite numbers.

Example 1 Which of the following is not true?

(a) 11 is prime number

(b) 2/5 is rational number

(c) ± 8 is the real number

(d) $\frac{6}{0}$ is rational number

Sol. *(d)* $\frac{6}{0}$ is not true, because $\frac{6}{0}$ is not rational number, as in its denominator, (p/q form) $q = 0$, which is wrong.

Example 2 Sum of first five prime numbers is

(a) 25 (b) 26 (c) 27 (d) 28

Sol. *(d)* We know that, first five prime numbers are 2, 3, 5, 7, 11

$\therefore$ Their sum $= 2 + 3 + 5 + 7 + 11 = 28$

Properties of Rational Numbers

(i) **Closure Property** The sum and multiplication of two rational numbers are rational numbers.

(a) $2 + \frac{2}{7} = \frac{16}{7}$ [rational number]

(b) $2 \times \frac{2}{7} = \frac{4}{7}$ [rational number]

(ii) **Commutative Property** If a and b are any rational numbers, then we have

(a) $a + b = b + a$ [for addition]

(b) $a \times b = b \times a$ [for multiplication]

(iii) **Associative Property** If a, b and c are three any rational numbers, then we have,

(a) $(a + b) + c = a + (b + c)$ [for addition]

(b) $(a \times b) \times c = a \times (b \times c)$ [for multiplication]

(iv) **Distributive Property** If a, b, c are any rational numbers, then we have

$$a(b + c) = ab + ac,$$

which is called distributive property of multiplication over addition.

(v) **Additive Identity** If a is any rational number, then we have

$$a + 0 = a,$$

i.e. adding a number 0 to any number, is equal to the number itself. Therefore, '0' is called additive identity.

(vi) **Multiplicative Identity** If a is any rational number, then we have

$$a \times 1 = a$$

i.e. multiplying a number 1 to any number, is equal to the number itself. Therefore, 1 is called multiplicative identity.

Rational Numbers Between any Two Given Rational Numbers

Between any two given rational numbers, there are countless or infinite rational numbers.

To find rational numbers between any two given rational numbers, we can use the idea of mean.

Thus, we can say that, 'if a and b are two rational numbers, then $\frac{a+b}{2}$ will be a rational number, such that $a < \frac{a+b}{2} < b$.

e.g. A rational number between 2 and 3 is $\frac{2+3}{2} = \frac{5}{2}$.

Here, we find that, $2 < \frac{5}{2} < 3$.

Example 3 A rational number between $\frac{2}{3}$ and $\frac{3}{4}$ is

(a) 15/24 (b) 17/24 (c) 13/24 (d) 5/24

Sol. *(b)* We find the mean of $\frac{2}{3}$ and $\frac{3}{4}$.

i.e. $\left(\frac{2}{3}+\frac{3}{4}\right) \div 2 = \left(\frac{8+9}{12}\right) \div 2$

$= \frac{17}{12} \div 2 = \frac{17}{12} \times \frac{1}{2} = \frac{17}{24}$

Simplification

To solve any expression or to simplify, we have many operations, i.e. Brackets, Addition, Subtraction, Multiplication, Division, etc.

We follow the rule of VBODMAS, i.e.

V → Vinculum or bar '—'
B → Brackets, i.e. ( ), { }, [ ]
O → Of
D → Division, i.e. ÷
M → Multiplication, i.e. '×'
A → Addition, i.e. '+'
S → Subtraction, i.e. '–'

To solve brackets, we follow the order

(i) ( ), circular or small bracket
(ii) { }, curly or middle bracket
(iii) [ ], square or big bracket

Note In the absence of any bracket or operation, their is no change in order to solve the expression.

Example 4 $\frac{3}{4}$ of $\frac{2}{7}$ of $\frac{1}{5}$ of 560 = ?

(a) 28 (b) 24 (c) 32 (d) 36

Sol. *(b)* We have, $\frac{3}{4}$ of $\frac{2}{7}$ of $\frac{1}{5}$ of 560

$= \frac{3}{4} \times \frac{2}{7} \times \frac{1}{5} \times 560 = 24$

Example 5 The value of

$1 \div [1 + 1 \div \{1 + 1 \div (1 + 1 \div 2)\}]$ is

(a) $\frac{5}{8}$ (b) $\frac{8}{5}$ (c) $\frac{2}{5}$ (d) $\frac{3}{8}$

Sol. *(a)* We have,

$$1 \div [1 + 1 \div \{1 + 1 \div (1 + 1 \div 2)\}]$$
$$= 1 \div \left[1 + 1 \div \left\{1 + 1 \div \left(1 + \frac{1}{2}\right)\right\}\right]$$
$$= 1 \div \left[1 + 1 \div \left\{1 + 1 \div \frac{3}{2}\right\}\right]$$
$$= 1 \div \left[1 + 1 \div \left\{1 + \frac{2}{3}\right\}\right]$$
$$= 1 \div \left[1 + 1 \div \frac{5}{3}\right] = 1 \div \left[1 + \frac{3}{5}\right] = 1 \div \frac{8}{5} = \frac{5}{8}$$

Test for Divisibility

Generally, to check the divisibility of one number by another, we normally do actual division and see whether remainder is zero or not. But sometimes we use direct condition for divisibility, which is as shown below

- **Divisible by 2** When the last digit of a number is either 0 or even, then the number is divisible by 2.

  e.g. 12, 86, 472, 520, 1000 etc. are divisible by 2.
- **Divisible by 3** When the sum of the digits of a number is divisible by 3, then the number is divisible by 3.

  e.g. 1233 as sum of digits

  $1 + 2 + 3 + 3 = 9$, which is divisible by 3, so 1233 must be divisible by 3.

- **Divisible by 4** When the number made by last two digits of a number is divisible by 4, then that particular number is divisible by 4. Apart from this, the number having two or more zeros at the end, is also divisible by 4.

  e.g. 6428 is divisible by 4 as the number made by its last two digits i.e., 28 is divisible by 4.
- **Divisible by 5** Numbers having 0 or 5 at the end are divisible by 5.

  e.g. 45, 4350, 135, 14850 etc. are divisible by 5 as they have 0 or 5 at the end.
- **Divisible by 6** When a number is divisible by both 3 and 2, then that particular number is divisible by 6 also.

  e.g. 18, 36, 720, 1440 etc. are divisible by 6 as they are divisible by both 3 and 2.
- **Divisible by 7** A number is divisible by 7 when the difference between twice the digit at ones place and the number formed by other digits is either zero or a multiple of 7.

  e.g. 658 is divisible by 7 because $65-2\times 8=65-16=49$. As 49 is divisible by 7, the number 658 is also divisible by 7.
- **Divisible by 8** When the number made by last three digits of a number is divisible by 8, then the number is also divisible by 8. Apart from this, if the last three or more digits of a number are zeros, then the number is divisible by 8.

  e.g. 2256. As 256 (the last three digits of 2256) is divisible by 8, therefore 2256 is also divisible by 8.
- **Divisible by 9** When the sum of all the digits of a number is divisible by 9, then the number is also divisible by 9.

  e.g. 936819 as sum of digits $9+3+6+8+1+9=36$, which is divisible by 9. Therefore, 936819 is also divisible by 9.
- **Divisible by 10** When a number ends with zero, then it is divisible by 10.

  e.g. 20, 40, 150, 123450, 478970 etc. are divisible by 10 as these all end with zero.
- **Divisible by 11** When the sums of digits at odd and even places are equal or differ by a number divisible by 11, then the number is also divisible by 11.

  e.g. 217382 Let us see

  Sum of digits at odd places $=2+7+8=17$

  Sum of digits at even places $=1+3+2=6$

  Difference $=17-6=11$

  Clearly, 217382 is divisible by 11.
- **Divisible by 12** A number which is divisible by both 4 and 3 is also divisible by 12.

  e.g. 2244 is divisible by both 3 and 4. Therefore, it is also divisible by 12.
- **Divisible by 25** A number is divisible by 25 when its last 2 digits are divisible by 25.

  e.g. 500, 1275, 13550 are divisible by 25 as last 2 digits of these numbers are divisible by 25.

Example 6 Which of the following number is not divisible by 3?

(a) 75 (b) 52 (c) 63 (d) 42

Sol. *(b)* $\because$ Sum of digits, we have

$75=7+5=12$ (divisible by 3)

$52=5+2=7$ (not divisible by 3)

$63=6+3=9$ (divisible by 3)

$42=4+2=6$ (divisible by 3)

$\therefore$ 52 is not divisible by 3, since its sum is not divisible by 3.

PRACTICE EXERCISE

1. A number in the form $\frac{p}{q}$ is said to be a rational number, if

(a) p, q are integers
(b) p, q are integers and $q \neq 0$
(c) p, q are integers and $p \neq 0$
(d) p, q are integers and $p \neq 0$, also $q \neq 0$

2. The numerical expression $\frac{3}{8} + \frac{(-5)}{7} = \frac{-19}{56}$ shows that

(a) rational numbers are closed under addition
(b) rational numbers are not closed under addition
(c) rational numbers are closed under multiplication
(d) addition of rational numbers is not commutative

3. Which of the following is not true?

(a) rational numbers are closed under addition
(b) rational numbers are not closed under subtraction
(c) rational numbers are closed under multiplication
(d) rational numbers are closed under division

4. Between two given rational numbers, we can find

(a) one and only one rational number
(b) only two rational numbers
(c) only ten rational numbers
(d) infinitely rational numbers

5. The product $\frac{3}{4}, \frac{2}{5}$ and $\frac{25}{3}$ is

(a) 5/2 (b) 2/5 (c) 3/5 (d) 5/3

6. Smallest 3-digit prime number is

(a) 103 (b) 107 (c) 101 (d) 109

7. What should be added to $-\frac{5}{7}$ to get $-\frac{3}{2}$?

(a) $-\frac{11}{14}$ (b) $\frac{11}{14}$
(c) $\frac{14}{11}$ (d) $-\frac{14}{11}$

8. Find the additive identity for the rational number

(a) 0 (b) 1 (c) 2 (d) 3

9. Which statement is true?

(a) $-5 + 3 \neq 3 + (-5)$
(b) $\frac{-8}{12} = \frac{10}{-15}$
(c) 2 is not natural number
(d) 17 is not prime number

10. What should be subtracted from $-\frac{3}{4}$ to make $\frac{2}{3}$?

(a) $\frac{-17}{12}$ (b) $\frac{17}{12}$
(c) $\frac{12}{9}$ (d) $\frac{11}{12}$

11. A rational number between $-\frac{3}{5}$ and $\frac{1}{4}$ is

(a) $\frac{7}{40}$ (b) $-\frac{7}{40}$
(c) $\frac{9}{40}$ (d) $-\frac{9}{40}$

12. The value of $\frac{3}{5} + \frac{3}{5} + \ldots$ upto 25 times is

(a) 25 (b) 10 (c) 15 (d) 35

13. $8\frac{1}{4} + 8\frac{1}{2} + ? = 20\frac{1}{8}$

(a) $8\frac{1}{4}$ (b) $3\frac{5}{8}$
(c) $3\frac{3}{8}$ (d) $8\frac{5}{9}$

14. Which of the following is correct?

(a) $a + 0 = b$ (b) $-a \times b = b \times (-a)$
(c) $a - b = b - a$ (d) $\frac{a}{b} = \frac{b}{a}$

15. The value of $2\frac{4}{5} \div 3\frac{1}{2}$ of $\frac{4}{5}$ is

(a) 0 (b) 1
(c) 2 (d) 3

16. By division algorithm, which of the following is correct?
(a) $41 = 7 \times 5 + 6$ (b) $56 = 5 \times 11 + 2$
(c) $30 = 5 \times 8 - 5$ (d) $25 = 5 \times 4 + 4$

17. If $157x234$ is divisible by 3, then the digit at the place of x is
(a) 0 (b) 1 (c) 2 (d) 4

18. By which number, 91476 is not divisible?
(a) 11 (b) 7 (c) 3 (d) 8

19. If a number $573xy$ is divisible by 90, then the value of $x + y$ is
(a) 13 (b) 3 (c) 8 (d) 6

20. Which number is divisible by 5 and 9 both?
(a) 585 (b) 285 (c) 389 (d) 560

21. Which number is not divisible by 6?
(a) 270 (b) 385
(c) 312 (d) 432

22. Which number is divisible by 5 and 25 both?
(a) 2170 (b) 5125
(c) 3107 (d) 4115

23. What least number should be subtracted from 1365 to get a number exactly divisible by 25?
(a) 15 (b) 5 (c) 10 (d) 20

24. Which of the following number is divisible by 9?
(a) 4621 (b) 2834 (c) 9216 (d) 1560

25. By how much $\frac{3}{4}$th of 52 is lesser than $\frac{2}{3}$rd of 99?
(a) 27 (b) 33 (c) 39 (d) 66

26. The value of K, where $31K2$ is divisible by 6, is
(a) 1 (b) 2
(c) 3 (d) 7

Answers

1	(b)	2	(a)	3	(d)	4	(d)	5	(a)	6	(c)	7	(a)	8	(a)	9	(b)	10	(a)
11	(b)	12	(c)	13	(c)	14	(b)	15	(b)	16	(a)	17	(c)	18	(d)	19	(b)	20	(a)
21	(b)	22	(b)	23	(a)	24	(c)	25	(a)	26	(c)								

Hints and Solutions

1. A number in the form $\frac{p}{q}$ is said to be a rational number, if p and q are integers and $q \neq 0$.

2. We have, $\frac{3}{8} + \left(\frac{-5}{7}\right) = \frac{-19}{56}$

$\because \frac{3}{8}$ and $\frac{-5}{7}$ are rational numbers and their addition is $\frac{-19}{56}$, which is also a rational number.

Hence, the rational numbers are closed under addition.

3. Rational numbers are not closed under division.

As, 1 and 0 are the rational numbers but $\frac{1}{0}$ is not defined.

4. We can find infinite rational numbers between any two given rational numbers.

5. We have, $\frac{3}{4} \times \frac{2}{5} \times \frac{25}{3} = \frac{1}{2} \times \frac{1}{5} \times 25 = \frac{5}{2}$

6. 101 is not divisible by any of the prime numbers 2, 3, 5, 7, 11.

$\therefore$ 101 is smallest three-digit prime number.

7. Let x should be added to $-\frac{5}{7}$ to get $-\frac{3}{2}$.

Then, $\frac{-5}{7} + x = \frac{-3}{2}$

$\Rightarrow \quad x = \frac{-3}{2} + \frac{5}{7} = \frac{-21+10}{14} = -\frac{11}{14}$

Hence, $-\frac{11}{14}$ should be added.

8. The additive identity for the rational number is 0.

9. By options,

(a) $-5+3=-2$

and $3+(-5)=-2$, which are equal.

(b) $\frac{-8}{12}=\frac{10}{-15} \Rightarrow \frac{-2}{3}=\frac{-2}{3}$, which is true.

(c) 2 is a natural number

(d) 17 is also a prime number.

10. Let x be subtracted.

Then, $\frac{-3}{4}-x=\frac{2}{3}$

$\Rightarrow \frac{-3}{4}-\frac{2}{3}=x$

$\Rightarrow x=\frac{-9-8}{12}=\frac{-17}{12}$

Hence, $-\frac{17}{12}$ should be subtracted.

11. The rational number between $-\frac{3}{5}$ and $\frac{1}{4}$

is $\frac{\frac{-3}{5}+\frac{1}{4}}{2}=\frac{-12+5}{20\times 2}=-\frac{7}{40}$

$\left[\because \text{ A rational number between two rational number} = \frac{\text{Sum of rational number}}{2}\right]$

12. $\frac{3}{5}+\frac{3}{5}+ \ldots\ldots$ upto 25 times

$$=\frac{3}{5}\times 25=3\times 5=15$$

13. $8\frac{1}{4}+8\frac{1}{2}+?=20\frac{1}{8}$

$$\Rightarrow ?=20\frac{1}{8}-8\frac{1}{4}-8\frac{1}{2}$$

$$=(20-8-8)+\left(\frac{1}{8}-\frac{1}{4}-\frac{1}{2}\right)$$

$$=4+\frac{1-2-4}{8}$$

$$=4+\frac{-5}{8}=\frac{27}{8}=3\frac{3}{8}$$

14. $-a\times b=b\times(-a)$

Because multiplication of two numbers in any order are same.

15. We have, $2\frac{4}{5}\div 3\frac{1}{2}$ of $\frac{4}{5}$

$$=\frac{14}{5}\div\frac{7}{2}\times\frac{4}{5}$$

$$=\frac{14}{5}\div 7\times\frac{2}{5}$$

$$=\frac{14}{5}\div\frac{14}{5}=1$$

16. By option (a),

$$\text{RHS}=7\times 5+6=35+6=41=\text{LHS}$$

$\therefore \quad 41=7\times 5+6$ is correct.

17. $\because$ Sum of digits $=1+5+7+x+2+3+4$

$$=22+x$$

This addition is divisible by 3, if 2 is at the place of x,

i.e. $\quad 22+x=22+2=24$, divisible by 3.

So, $\quad x=2$

18. $\because$ We have, to check

divisible by 11 91476 = Sum of odd places digits − Sum of even places digits

$=(9+4+6)-(1+7)$

$=19-8=11$, which is divisible by 11.

Divisible by 7 $=19476=194-2(76)=194-152$

$=42$, which is divisible by 7.

Divisible by 2 given number is even number, hence it is divisible by 2.

Divisible by 8 $91476 \rightarrow 476$ is not divisible by 8.

Hence, 91476 is not divisible by 8.

19. We know that, if any number divisible by 90, i.e. divisible by 9 and 10.

So, to make number $573xy$ divisible by 10, we have to put $y=0$ to make unit digit 0, (i.e. 0 unit digit number divisible by 10), i.e. $y=0$.

Now, to also make $573x0$ divisible by 9,

$5+7+3+x+10=15+x=15+\underline{3}=18$,

i.e. divisible by 3.

So, $\quad x=3$

$\therefore \quad x+y=3+0=3$

20. By option (a),

We have, 585, divisible by 5 because unit digit is 5, and sum of digits $=5+8+5=18=1+8=9$ which is also divisible by 9.

21. We know that, if any number divisible by 2 and 3, it is also divisible by 6.

$\therefore$ By option (b), 385 is not divisible by 2.

Hence, it is also not divisible by 6.

22. We know that,

Number is divisible by 5, if last digit is 5 or 0.

And divisible by 25, if last two digits is divisible by 25.

Hence, 5125 is only number divisible by 5 and 25 both.

23.
$$\begin{array}{r} 25)\,1365\,(54 \\ \underline{125} \\ 115 \\ \underline{100} \\ \underline{15} \end{array}$$

$\therefore$ Required number is 15.

24. We have, $4621 = 4 + 6 + 2 + 1 = 13$, not divisible by 9

$2834 = 2 + 8 + 3 + 4 = 17$, not divisible by 9

$9216 = 9 + 2 + 1 + 6 = 18$, divisible by 9

$1560 = 1 + 5 + 6 + 0 = 12$, not divisible by 9

Hence, 9216 is only number divisible by 9.

25. $\therefore$ Required answer

$$= \frac{2}{3} \times 99 - \frac{3}{4} \times 52$$
$$= 2 \times 33 - 3 \times 13$$
$$= 66 - 39 = 27$$

26. The number $31K2$ is divisible by 6, it mean it is divisible by 2 and 3 both.

Here, unit digit is 2 (even), so this number is divisible by 2.

Now, for 3, first we have to add all the digits.

$\therefore \; 3 + 1 + K + 2 = 6 + K$

$= 6 + 3 = 9,$

for $K = 3$, it is divisible by 3.

Hence, $K = 3$

CHAPTER

02

SQUARE AND SQUARE ROOTS

In this chapter, we study the squares of number and their properties. Also finding the square root of numbers and decimal numbers by prime factorisation and long division method.

Square

The square of a number is the product of the number itself, i.e. $a \times a = a^2$.

Perfect Square

A given number is said to be a perfect square, if it can be expressed as the product of two equal factors. A natural number 'n' is a perfect square if $n = m^2$ for any natural number m.

e.g. $4 = 2^2$ or 2×2 and $25 = 5^2$ is a perfect square.

Squares from 1 to 20 numbers

Numbers	Squares	Numbers	Squares
1	1	11	121
2	4	12	144
3	9	13	169
4	16	14	196
5	25	15	225
6	36	16	256
7	49	17	289
8	64	18	324
9	81	19	361
10	100	20	400

Properties of Square

(i) The number of zeroes at the end of a perfect square is always even.

(ii) A perfect square leaves a remainder 0 or 1 on division by 3.

(iii) The number ending with an odd number of zeroes is never a perfect square.

(iv) Squares of even numbers are always even.

(v) Squares of odd numbers are always odd.

Example 1 Which are of the following will have odd unit digit?

(a) $(32)^2$ (b) $(35)^2$ (c) $(64)^2$ (d) $(68)^2$

Sol. (b) We know that that square of odd number in an odd number

The number $(35)^2$ have odd unit digit.

Pythagorean Triplet

In a triplet (m, n, p) of three natural numbers m, n and p is called a Pythagorean triplet, if $m^2 + n^2 = p^2$. It is easy to prove that for any natural number m greater than 1, $(2m, m^2 - 1, m^2 + 1)$ is a Pythagorean triplet.

Example 2 Find the value of $(13)^2 + 3 \times 7$.

(a) 190 (b) 189 (c) 191 (d) 192

Sol. (a) $(13)^2 + 3 \times 7 = 169 + 21 = 190$

Square Root

The square root of the number x is the number which multiplied by itself gives x as the product. It is denoted by the symbol $\sqrt[2]{\ }$ or $\sqrt{\ }$.

e.g. If $y = x^2$, then we call, x is a square root of y,

i.e. $x = \pm\sqrt{y}$.

Properties of Square Root

(i) If the unit digit of a number is 2, 3, 7 or 8, then it does not have a square root.

(ii) Square root of even number is even.

(iii) Square root of odd number is odd.

Square Root of a Perfect Square by the Prime Factorisation Method

The following steps are given below.

I. Resolve the given number into prime factors.

II. Make pairs of similar factors.

III. Choose one prime from each pair and multiply all primes.

Thus, the product obtained is the square root of given number.

Example 3 The square root of 1764 is

(a) 41 (b) 43 (c) 42 (d) 40

Sol. (c) By prime factorisation method,

2	1764
2	882
3	441
3	147
7	49
7	7
	1

$$\therefore \quad \sqrt{1764} = \sqrt{\overline{2 \times 2} \times \overline{3 \times 3} \times \overline{7 \times 7}}$$
$$= 2 \times 3 \times 7 = 42$$

Square Root of a Perfect Square by Long Division Method

If it is not easy to evaluate square root using prime factorisation method, then we use division method.

The steps of this method can be easily understood with the help of following examples.

e.g. Find the square root of 18769.

Step **I** In the given number, mark off the digits in pairs starting from the unit digit. Each pair and the remaining one digit (if any) is called a period.

Step **II** Choose a number whose square is less than or equal to 1. Here, $1^2 = 1$, on subtracting, we get 0 (zero) as remainder.

Step **III** Bring down the next period, i.e. 87. Now, the trial divisor is $1 \times 2 = 2$ and trial dividend is 87. So, we take 23 as divisor and put 3 as quotient. The remainder is 18 now.

	137
1	$\overline{1}\,\overline{87}\,\overline{69}$
	1
23	87
	69
267	1869
	1869
	×

Step **IV** Bring down the next period, which is 69. Now, trial divisor is $13 \times 2 = 26$ and trial dividend is 1869. So, we take 267 as dividend and 7 as quotient. The remainder is 0.

Step **V** The process (processes like III and IV) goes on till all the periods (pairs) come to an end and we get remainder as 0 (zero) now.

Hence, the required square root $= 137$

Example 4 2025 plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and the number of plants in each now.

(a) 55, 45 (b) 45, 45 (c) 35, 35 (d) 36, 36

Sol. *(b)* Let the number of rows be x.

Then, number of plants in a row $= x$

So, number of plants to be planted in a garden

$$x \times x = x^2$$

According to the question,

Total number of plants to be planted = 2025

$\therefore \quad x^2 = 2025$

3	2025
3	675
3	225
3	75
5	25
5	2
	1

$\Rightarrow \quad x = \sqrt{2025} = \sqrt{3 \times 3 \times 3 \times 3 \times 5 \times 5}$

$= 3 \times 3 \times 5 = 45$

Hence, the number of rows is 45 and the number of plants in each row is 45.

Formula for Finding the Number of Digits in the Square Root of a Perfect Square

If any perfect square number contains 'n' digits. then, its square root will contain $\frac{n}{2}$ digits, when n is even and $\frac{n+1}{2}$ digits, when n is odd.

e.g. Square root of 64 is 8. [$\because n = 2$ i.e. even]

Also, square root of 144 is 12. [$\because n = 3$ i.e. odd]

Square Root of Product of Numbers and Rational Number

(i) The square root of product of integers is the square root of integer by taking separately, for any integer a and b, we have

$$\sqrt[2]{ab} = \sqrt[2]{a}\ \sqrt[2]{b}$$

(ii) The square root of rational number $\frac{a}{b}$ is

$$\sqrt[2]{\frac{a}{b}} = \frac{\sqrt[2]{a}}{\sqrt[2]{b}}$$

Square Root of Number in Decimal Form

Make the number of decimal places even by affixing zero, if necessary. Now, mark bars and find out the square root by the long division method. Put the decimal point in the square root as soon as the integral part is completed.

Example 5 Find the square root of 176.252176.

(a) 13.276 (b) 13.801 (c) 13.295 (d) 13.218

Sol. *(a)* By long division method,

	13.276
1	$\overline{1}\,\overline{76}.\overline{25}\,\overline{21}\,\overline{76}$
	1
23	76
	69
262	725
	524
2647	20121
	18529
26546	159276
	159276
	×

$\therefore \quad \sqrt{176.252176} = 13.276$

PRACTICE EXERCISE

1. The value of $(15)^2 + (8)^2 + 2$ is
   (a) 289 (b) 291 (c) 293 (d) 295

2. Which of the following cannot be a perfect square?
   (a) 841 (b) 529
   (c) 198 (d) All of these

3. Which of the following cannot be a digit in the unit place of a perfect square ?
   (a) 0 (b) 1 (c) 5 (d) 7

4. The square root of 73.96 is
   (a) 8.6 (b) 86
   (c) 0.86 (d) None of these

5. If $x = \sqrt{3018 + \sqrt{36 + \sqrt{169}}}$, then the value of x is
   (a) 55 (b) 44 (c) 63 (d) 42

6. $\sqrt{12} + \sqrt{24}$ is equal to
   (a) $2\sqrt{3} + 3\sqrt{2}$ (b) $4\sqrt{3} + \sqrt{6}$
   (c) $\sqrt{7} + 2\sqrt{3}$ (d) $2\sqrt{6} + 2\sqrt{3}$

7. Which one of the following will have even unit digit?
   (a) $(43)^2$ (b) $(37)^2$
   (c) $(63)^2$ (d) $(34)^2$

8. If the area of an equilateral triangle is $24\sqrt{3}\ \text{m}^2$, then its perimeter is
   (a) $12\sqrt{6}$ m (b) $9\sqrt{6}$ m
   (c) $8\sqrt{3}$ m (d) $4\sqrt{3}$ m

9. The value of $(301)^2 - (300)^2$ is
   (a) 1 (b) 601
   (c) 106 (d) 100

10. A General arranges his soldiers in rows to form a perfect square. He finds that in doing so, 60 soldiers are left out. If the total number of soldiers be 8160. Then, the number of soldiers in each row is
    (a) 90 (b) 91
    (c) 92 (d) 80

11. The greatest six digit number which is a perfect square is
    (a) 998004 (b) 998006
    (c) 998049 (d) 998001

12. What is that fraction which when multiplied by itself gives 227.798649 ?
    (a) 15.093 (b) 15.099
    (c) 14.093 (d) 9.0019

13. In a triplet $(6, a, 10)$ what value of 'a' will make it a Pythagorean triplet ?
    (a) 4 (b) 16
    (c) 8 (d) 5

14. If a number is increased by two times, then the square of the number will increase
    (a) two times (b) three times
    (c) four times (d) five times

15. Two numbers are in the ratio of 9 : 7. If the difference of their square is 288, then the smaller of the number is
    (a) 21 (b) 24
    (c) 27 (d) 28

16. The number of digits in the square root of 298116 is
    (a) 4 (b) 5
    (c) 3 (d) 6

17. If $\sqrt{2401} = \sqrt{7^x}$, then the value of x is
    (a) 3 (b) 4
    (c) 5 (d) 6

18. The least number to be added to 269 to make it a perfect square is
    (a) 31 (b) 16
    (c) 17 (d) 20

19. If $\sqrt{18225} = 135$, then the value of $\sqrt{18225} + \sqrt{182.25} + \sqrt{1.8225} + \sqrt{0.018225}$ is
    (a) 1.49985 (b) 14.985
    (c) 149.985 (d) 1499.85

20. The expression
$$\sqrt{\frac{0.85\times(0.105+0.024-0.008)}{0.022\times0.25\times1.7}}$$
simplifies to

(a) $\sqrt{11}$ (b) $\sqrt{1.1}$
(c) 11 (d) $\sqrt{0.011}$

21. The value of $\sqrt{\frac{16}{36}+\frac{1}{4}}$ is

(a) $\frac{2}{5}$ (b) $\frac{1}{3}$ (c) $\frac{5}{3}$ (d) $\frac{5}{6}$

22. If $\frac{52}{x}=\sqrt{\frac{169}{289}}$, then the value of x is

(a) 52 (b) 58 (c) 62 (d) 68

23. A square board has an area of 144 sq units. How long is each side of the board?

(a) 11 units (b) 12 units
(c) 13 units (d) 14 units

24. If $\sqrt{1+\frac{25}{144}}=1+\frac{x}{12}$, then x is equal to

(a) 1 (b) 2 (c) 5 (d) 9

25. The value of $\frac{\sqrt{80}-\sqrt{112}}{\sqrt{45}-\sqrt{63}}$ is

(a) $\frac{3}{4}$ (b) $1\frac{3}{4}$
(c) $1\frac{1}{3}$ (d) $1\frac{7}{9}$

26. The least number which is added to 17420 will make it a perfect square is

(a) 3 (b) 5 (c) 9 (d) 4

27. If $\sqrt{0.09\times0.09\times x}=0.09\times0.09\times\sqrt{z}$, then the value of $\frac{x}{z}$

(a) 0.0081 (b) 0.810
(c) 0.801 (d) 8.09

Answers

1	(b)	2	(c)	3	(d)	4	(a)	5	(a)	6	(d)	7	(d)	8	(a)	9	(b)	10	(a)
11	(d)	12	(a)	13	(c)	14	(c)	15	(a)	16	(c)	17	(b)	18	(d)	19	(c)	20	(a)
21	(d)	22	(d)	23	(b)	24	(a)	25	(c)	26	(d)	27	(a)						

Hints and Solutions

1. $(15)^2+(8)^2+2=225+64+2=291$

2. We know that, a number ending with digits 2, 3, 7 or 8 can never be a perfect square. So, 198 cannot be written in the form of a perfect square.

3. Digit 7 cannot be a place of a perfect square.

4. By using long division method,

$$\begin{array}{r|l} & 8.6 \\ \hline 8 & \overline{73}.\overline{96} \\ +8 & 64 \\ \hline 166 & 996 \\ & 996 \\ \hline & \times \end{array}$$

Hence, $\sqrt{73.96}=8.6$

5. According to question,

$$\sqrt{3018+\sqrt{36+\sqrt{169}}}=\sqrt{3018+\sqrt{36+13}}$$
$$=\sqrt{3018+7}=\sqrt{3025}=55$$

6. $\sqrt{12}+\sqrt{24}=\sqrt{2\times2\times3}+\sqrt{2\times2\times6}$
$$=2\sqrt{3}+2\sqrt{6}$$

7. We know that, the square of even number is even.

$\therefore$ The number $(34)^2$ has even unit digit.

8. Let the side of an equilateral triangle be a m.

Since, the area of an equilateral triangle
$$=24\sqrt{3}\,\text{m}^2$$

$\Rightarrow \quad \frac{\sqrt{3}}{4}a^2 = 24\sqrt{3} \;\Rightarrow\; a^2 = 96$

$\Rightarrow \quad a = 4\sqrt{6}$ m

9. $(301)^2 - (300)^2$

$= (301 + 300)(301 - 300)$

$[\because a^2 - b^2 = (a+b)(a-b)]$

$= (601) \times 1$

$= 601$

10. Total number of soldiers arranged

$= 8160 - 60 = 8100$

Since, the number of soldiers in each row is equal to number of rows.

$\therefore$ Number of soldiers in each row

$= \sqrt{8100}$

$= \sqrt{9 \times 9 \times 10 \times 10} = 90$

11. The greatest six digit number = 999999

	999
9	99 99 99
	81
189	1899
	1701
1989	19899
	17901
	1998

$\times$

$\therefore$ The greatest number of six digit which is a perfect square

$= 999999 - 1998$

$= 998001$

12. Let the fraction be x.

Then, $\quad x^2 = 227.798649$

$\therefore \quad x = \sqrt{227.798649} = 15.093$

$[\because$ using long divison method$]$

13. $6^2 = 36,\; a^2 = a^2, 10^2 = 100$

By Pythagoraem triplet, $6^2 + a^2 = 10^2$

$\Rightarrow \quad a^2 = 10^2 - 6^2$

$\Rightarrow \quad a^2 = 100 - 36 = 64$

$\Rightarrow \quad a = \sqrt{64} = 8$

14. Let the number be y.

If the number is increased by two times it becomes $2y$.

Square of the number $= (2y)^2 = 4y^2$

$\therefore$ The number will be increased by four times.

15. Let the number be $9x$ and $7x$.

According to the question,

$81x^2 - 49x^2 = 288$

$\Rightarrow \quad x^2 = \frac{288}{32} \;\Rightarrow\; x^2 = 9 \;\Rightarrow\; x = 3$

$\therefore$ The smaller number is 21.

16.

2	298116
2	149058
3	74529
3	24843
7	8281
7	1183
13	169
13	13
	1

$\Rightarrow \quad \sqrt{298116} = 2 \times 3 \times 7 \times 13$

$= 546$

$\therefore$ Number of digits = 3

17. $\quad \sqrt{2401} = \sqrt{7^x}$

$\Rightarrow \quad 2401 = 7^x$

$\Rightarrow \quad 7^4 = 7^x$

$\therefore \quad x = 4$

18. We know, $256 < 269 < 289$

$\Rightarrow (16)^2 < 269 < (17)^2$

$\therefore$ Number to be added $= (17)^2 - 269$

$= 289 - 269 = 20$

19. $\sqrt{18225} + \sqrt{182.25} + \sqrt{1.8225} + \sqrt{0.018225}$

$= \sqrt{18225} + \sqrt{\frac{18225}{100}} + \sqrt{\frac{18225}{10000}} + \sqrt{\frac{18225}{1000000}}$

$= 135 + \frac{135}{10} + \frac{135}{100} + \frac{135}{1000}$

$= 135 + 13.5 + 1.35 + 0.135$

$= 149.985$

20. $\sqrt{\dfrac{0.85\times(0.105+0.024-0.008)}{0.022\times0.25\times1.7}}$

$$=\sqrt{\frac{0.85\times0.121}{0.022\times0.25\times1.7}}$$

$$=\sqrt{\frac{85\times121\times10}{22\times25\times17}}=\sqrt{11}$$

21. $\sqrt{\dfrac{16}{36}+\dfrac{1}{4}}=\sqrt{\dfrac{16+9}{36}}$

$$=\sqrt{\frac{25}{36}}=\frac{5}{6}$$

22. Given, $\dfrac{52}{x}=\sqrt{\dfrac{169}{289}}$

$\Rightarrow \quad x=\dfrac{52\times17}{13}=68$

23. Given, area of square board $=144$ sq units

$\therefore \quad (\text{side})^2=144 \quad [\because \text{area of square}=(\text{side})^2]$

$\Rightarrow \quad (\text{side})^2=12$

$\Rightarrow \quad \text{side}=12$ units

Hence, the length of each side of the board is 12 units.

24. Given, $\sqrt{1+\dfrac{25}{144}}=1+\dfrac{x}{12}$

$\Rightarrow \quad \sqrt{\dfrac{169}{144}}=1+\dfrac{x}{12}$

$\Rightarrow \quad \dfrac{13}{12}=1+\dfrac{x}{12}$

$\Rightarrow \quad \dfrac{x}{12}=\dfrac{1}{12}\Rightarrow x=1$

25. $\dfrac{4\sqrt{5}-4\sqrt{7}}{3\sqrt{5}-3\sqrt{7}}=\dfrac{4(\sqrt{5}-\sqrt{7})}{3(\sqrt{5}-\sqrt{7})}=\dfrac{4}{3}=1\dfrac{1}{3}$

26. Since, 17420 lies between 131^2 and 132^2.

Now, $(132)^2=17424$

Hence, it should be 4 added.

27. $\sqrt{0.09\times0.09\times x}=0.09\times0.09\times\sqrt{z}$

$\Rightarrow \quad 0.09\times\sqrt{x}=0.09\times0.09\times\sqrt{z}$

$\Rightarrow \quad \dfrac{\sqrt{x}}{\sqrt{z}}=0.09$

$\Rightarrow \quad \dfrac{x}{z}=(0.09)^2$ (squaring both sides)

$\Rightarrow \quad \dfrac{x}{z}=0.0081$

CHAPTER

03

CUBE AND CUBE ROOTS

In this chapter, we study the cubes of numbers and their properties, and cube roots by prime factorisation method.

Cube

The cube of a number is the product of the number itself thricely.

e.g. If x is a non-zero number, then $x \times x \times x = x^3$ is called cube of x.

The cube of rational number is the cube of the numerator divided by the cube of denominator. e.g. Cube of $\frac{4}{5}$ is $\frac{64}{125}$.

Perfect Cube

A natural number n is said to be a perfect cube if there is an integer m such that $n = m \times m \times m$.

Cubes from 1 to 15 numbers

Numbers	Cubes
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512

Numbers	Cubes
9	729
10	1000
11	1331
12	1728
13	2197
14	2744
15	3375

Properties of Cube of Numbers

(i) Cubes of all even natural numbers are always even.

(ii) Cubes of all odd natural numbers are always odd.

(iii) Cubes of negative integers are always negative.

(iv) For any rational number $\frac{a}{b}$, we have

$$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$$

Example 1 Find the value of $(11)^3 + 4(7)^3 - 5$.

(a) 2697 (b) 2698 (c) 2699 (d) 3000

Sol. *(b)* $(11)^3 + 4(7)^3 - 5 = 1331 + 4(343) - 5$

$= 1331 + 1372 - 5 = 2698$

Example 2 Which one of the following will have odd unit digit

(a) $(24)^3$ (b) $(64)^3$

(c) $(27)^3$ (d) $(52)^3$

Sol. *(c)* We knwo cube of odd number is odd number.

$\therefore$ Number $(27)^3$ have odd unit digit

Cube Root

If n is perfect cube for any integer m i.e. $n = m^3$, then m is called the cube root of n and it is denoted by $m = \sqrt[3]{n}$.

Cube Root of a Perfect Cube by Prime Factorisation

The following steps are given below.

I. Factorise the given number into prime factors.

II. Make triples of similar factors or arrange them in group of three equal factors at a time.

III. Choose one prime from each pair and multiply all primes.

Example 3 Find the cube root of 74088.

(a) 40 (b) 47 (c) 42 (d) 45

Sol. *(c)* Resolving the given number, we get

2	74088
2	37044
2	18522
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$74088 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{7 \times 7 \times 7}$

$\therefore \quad \sqrt[3]{74088} = 2 \times 3 \times 7 = 42$

Cube Root of a Negative Cube

If a is a positive integer, then $-a$ is a negative integer.

We know that, $(-a)^3 = -a^3$

So, $\sqrt[3]{-a^3} = -a$

In general, we have $\sqrt[3]{-x} = -\sqrt[3]{x}$

Cube Root of Product of Numbers and Rational Number

(i) The cube root of product of integers is the cube root of integer by taking separately.

For any two integer a and b, we have

$$\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$$

(ii) Cube Root of Rational Number $\frac{a}{b}$ is $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

Example 4 The simplified form of $\sqrt[3]{125 \times 64}$ is

(a) 20 (b) 40 (c) 60 (d) 80

Sol. *(a)* $125 \times 64 = \underline{5 \times 5 \times 5} \times \underline{4 \times 4 \times 4}$

$\therefore$ LHS $= \sqrt[3]{125 \times 64} = \sqrt[3]{(5 \times 4)^3} = (5 \times 4) = 20$

Now, $\sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5} = 5$

and $\sqrt[3]{64} = \sqrt[3]{4 \times 4 \times 4} = 4$

$\therefore \quad$ RHS $= \sqrt[3]{125} \times \sqrt[3]{64} = (5 \times 4) = 20$

$\Rightarrow \quad$ LHS = RHS

Hence, $\sqrt[3]{125 \times 64} = \sqrt[3]{125} \times \sqrt[3]{64}$

Example 5 Find the cube root of 4.096.

(a) 1.6 (b) 165

(c) 1.75 (d) None of these

Sol. (a) $\sqrt[3]{4.096} = \sqrt[3]{\frac{4096}{1000}} = \frac{\sqrt[3]{4096}}{\sqrt[3]{1000}}$

$\therefore \quad 4096 = \underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{2\times2\times2}$

$\therefore \quad \sqrt[3]{4096} = 2\times2\times2\times2 = 16$

Also, $\sqrt[3]{1000} = \sqrt[3]{10\times10\times10} = 10$

So, $\frac{\sqrt[3]{4096}}{\sqrt[3]{1000}} = \frac{16}{10} = 1.6$

Hence, $\sqrt[3]{4.096} = 1.6$

Important Points

(i) If 1, 4, 5, 6 and 9 in the unit place, then cube of that number given the same digit in the unit place.

(ii) 3 in the unit place have cube with 7 in the unit place.

(iii) 7 in the unit place have cube with 3 in the unit place.

(iv) 2 in the unit place have cube with 8 in the unit place.

(v) 8 in the unit place have cube with 2 in the unit place.

Example 6 Difference of two perfect cubes is 189. If the cube root of the smaller of the two numbers is 3, then find the cube root of the larger number.

(a) 5 (b) 6 (c) 7 (d) 8

Sol. (b) Given difference of two perfect cube = 189 and cube root of the smaller number = 3

$\therefore$ Cube of smaller number $= (3)^3 = 27$

Let cube root of the larger number be x.

Then, cube of larger number $= x^3$

According to the question,

$$x^3 - 27 = 189$$

$\Rightarrow \quad x^3 = 189 + 27 \Rightarrow x^3 = 216$

$\Rightarrow \quad x = \sqrt[3]{216} = \sqrt[3]{6\times6\times6}$

$\therefore \quad x = 6$

Hence, the cube root of the larger number is 6.

PRACTICE EXERCISE

1. Which of the following is not a perfect cube ?
   (a) 1000000
   (b) 216
   (c) 10000
   (d) None of the above

2. Which of the following perfect cube is the cube of an even number ?
   (a) 343 (b) 2197
   (c) 216 (d) 1331

3. Which of the following perfect cube is the cube of an odd number ?
   (a) 1728 (b) 512
   (c) 729 (d) 1000

4. If a number is increased by four times, then cube of the number will increase
   (a) 64 times (b) 69 times
   (c) 729 times (d) 1000 times

5. The value of $\sqrt[3]{\frac{27}{125}}$ is
   (a) $\frac{3}{5}$ (b) $\frac{3}{25}$
   (c) $\frac{9}{25}$ (d) not appropriate data

6. The unit place of cube roots of 117649 is
   (a) 4 (b) 9
   (c) 49 (d) None of these

7. The smallest number by which 3087 may be multiplied so that the product is a perfect cube, is

(a) 6 (b) 5
(c) 4 (d) 3

8. The smallest by which 392 may be divided so that the quotient is a perfect cube, is

(a) 50 (b) 51
(c) 49 (d) 62

9. In a number pattern, 8, 27, 64, x, the value of x will be

(a) 125 (b) 216 (c) 100 (d) 115

10. The cube of 31.2 is

(a) 3037.1328 (b) 3037.1381
(c) 30371.328 (d) 30371328

11. The sum of the cubes of first three natural number is equal to

(a) $(1+2+3)^3$ (b) $(1+2+3)^2$
(c) 3^3 (d) 3^2

12. The cube root of $\frac{-343}{1331}$ is

(a) $\frac{7}{11}$ (b) $\frac{-7}{11}$ (c) $\frac{11}{7}$ (d) $\frac{-11}{7}$

13. If one side of a cube is 33 m, then the volume of the cube is

(a) 35937 (b) 35936
(c) 3934 (d) None of these

14. Three numbers are in the ratio of $1:2:3$. The sum of their cubes is 121500. The numbers are

(a) 15, 30, 45 (b) 30, 45, 15
(c) 7, 14, 21 (d) None of these

15. If the volume of a cubical box is $35.937\ \text{m}^3$, what is the length of its one side ?

(a) 3.3 m (b) 6.6 m
(c) 3.6 m (d) None of these

16. $(-216\times729)^{1/3}$

(a) 54 (b) -54 (c) -45 (d) 45

17. The value of $\sqrt[3]{0.064}+\sqrt[3]{27}-\sqrt[3]{729}$ is

(a) 12.4 (b) -2 (c) 5.6 (d) -5.6

18. The value of $\frac{\sqrt[3]{8}}{\sqrt{16}}\div\sqrt{\frac{100}{49}}\times\sqrt[3]{125}$ is

(a) 7 (b) $1\frac{3}{4}$ (c) $\frac{7}{100}$ (d) $\frac{4}{7}$

19. The value of $\sqrt{\sqrt[3]{0.004096}}$ is

(a) 0.4 (b) 0.04
(c) 0.16 (d) 4

20. The value of $(27\times-2744)^{1/3}$ is

(a) -40 (b) -42
(c) -22 (d) -32

21. The value of $\frac{(73)^3+(53)^3}{73\times73-73\times53+53\times53}$ is

(a) 126 (b) 216 (c) 162 (d) -126

22. What is the smallest number in which we multiply by 392, we get the factors in the form of cube ?

(a) 5 (b) 6
(c) 7 (d) 8

23. The value of $\sqrt[3]{\frac{27}{64}}$ is

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{5}{4}$ (d) $\frac{5}{3}$

24. If $\sqrt{\frac{x}{0.0064}}=\sqrt[3]{0\ .008}$, then the value of x is

(a) 0.256 (b) 0.0256
(c) 0.000256 (d) 0.00256

25. The smallest number in which we multiply by 1800, we get the cube. Then, the sum of digit is

(a) 2 (b) 3 (c) 6 (d) 8

26. If cube root of 175616 is 56, then the value of $\sqrt[3]{175.616}+\sqrt[3]{0.175616}+\sqrt[3]{0.000175616}$ is

(a) 0.168 (b) 6.216
(c) 6.116 (d) 62.16

27. The cube root of 0.000001 is

(a) 0.1 (b) 0.01
(c) 0.316 (d) 0.031

Answers

1	(c)	2	(c)	3	(c)	4	(a)	5	(a)	6	(b)	7	(d)	8	(c)	9	(a)	10	(c)
11	(b)	12	(b)	13	(a)	14	(a)	15	(a)	16	(b)	17	(d)	18	(b)	19	(a)	20	(b)
21	(a)	22	(c)	23	(a)	24	(c)	25	(c)	26	(b)	27	(b)						

Hints and Solutions

1. $\sqrt[3]{1000000} = 100$ is a perfect cube.

$\sqrt[3]{216} = 6$ is a perfect cube.

$\sqrt[3]{10000}$ = not a perfect cube.

2. 216 is the cube of an even number because cube of an even number is always even.

3. 729 is the cube of an odd number because the cube of odd number is always odd.

4. Let the number be x.

After increasing 4 times the number $= 4x$

Cube of the number $= (4x)^3 = 64x^3$

$\therefore$ The cube of a number increases by 64 times.

5. $\sqrt[3]{\dfrac{27}{125}} = \sqrt[3]{\dfrac{3\times3\times3}{5\times5\times5}} = \dfrac{3}{5}$

6. $\sqrt[3]{117649} = 49$

$\therefore$ Unit place is 9.

7. Writing 3087 as a product of a prime factors, we have

$$\begin{array}{r|l} 3 & 3087 \\ \hline 3 & 1029 \\ \hline 7 & 343 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$\therefore \quad 3087 = 3\times3\times\overline{7\times7\times7}$

Clearly, to make it a perfect cube it must be multiplied by 3.

8. Writing 392 as a product of prime factors, we have

$$\begin{array}{r|l} 2 & 392 \\ \hline 2 & 196 \\ \hline 2 & 98 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$\therefore \quad 392 = 2\times2\times2\times7\times7$

Clearly, to make it perfect cube it must be divided by (7×7) i.e., 49.

9. Here, we see that $8 = 2^3$, $27 = 3^3$, $64 = 4^3$.

It means given pattern is a cube of consecutive natural number.

$\therefore \quad x = 5^3 = 125$

10. $31.2\times31.2\times31.2 = 30371.328$

11. $1^3 + 2^3 + 3^3 = 1 + 8 + 27$

$= 36 = (6)^2 = (1+2+3)^2$

12. $\sqrt[3]{\dfrac{-343}{1331}} = \dfrac{\sqrt[3]{-343}}{\sqrt[3]{1331}} = \dfrac{\sqrt[3]{-7\times-7\times-7}}{\sqrt[3]{11\times11\times11}} = \dfrac{-7}{11}$

13. Volume of cube $= (33)^3 = 35937$

14. Let the number be x, $2x$ and $3x$.

$$\begin{aligned} \therefore \quad (x)^3 + (2x)^3 + (3x)^3 &= 121500 \\ \Rightarrow \quad 1x^3 + 8x^3 + 27x^3 &= 121500 \\ \Rightarrow \quad 36x^3 &= 121500 \\ \Rightarrow \quad x^3 &= \frac{121500}{36} \\ \Rightarrow \quad x^3 &= 3375 \\ \Rightarrow \quad x &= \sqrt[3]{3375} \\ \Rightarrow \quad x &= \sqrt[3]{15\times15\times15} \\ \Rightarrow \quad x &= 15 \end{aligned}$$

$\therefore$ The numbers are $x = 15$, $2x = 30$, $3x = 45$.

15. $\because$ Volume of a cube $= (\text{side})^3$

$$\begin{aligned} (\text{side})^3 &= 35.937 \\ \Rightarrow \quad \text{side} &= \sqrt[3]{35.937} \\ \Rightarrow \quad \text{side} &= \sqrt[3]{3.3\times3.3\times3.3} \\ \Rightarrow \quad \text{side} &= 3.3\,\text{m} \end{aligned}$$

16. $(-216 \times 729)^{1/3} = (-216)^{1/3} \times (729)^{1/3}$

$= -\underline{(6 \times 6 \times 6)}^{1/3} \times (\underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3})^{1/3}$

$= -6 \times 3 \times 3 = -54$

17. $\sqrt[3]{0.064} + \sqrt[3]{27} - \sqrt[3]{729}$

$\because \quad \sqrt[3]{0.064} = \sqrt[3]{\underline{0.4 \times 0.4 \times 0.4}} = 0.4$

$\sqrt[3]{27} = \sqrt[3]{\underline{3 \times 3 \times 3}} = 3$

$\sqrt[3]{729} = \sqrt[3]{\underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}}$

$= 3 \times 3 = 9$

$\therefore \quad \sqrt[3]{0.064} + \sqrt[3]{27} - \sqrt[3]{729} = 0.4 + 3 - 9 = -5.6$

18. $\frac{\sqrt[3]{8}}{\sqrt{16}} \div \sqrt{\frac{100}{49}} \times \sqrt[3]{125} = \frac{2}{4} \times \frac{7}{10} \times 5$

$= \frac{7}{4} = 1\frac{3}{4}$

19. $\sqrt{\sqrt[3]{0.004096}} = \sqrt{\sqrt[3]{(0.16)^3}} = \sqrt{0.16} = 0.4$

20. $(27 \times -2744)^{1/3} = (27)^{1/3} \times (-2744)^{1/3}$

$= 3 \times -14 = -42$

21. $\frac{(73)^3 + (53)^3}{73 \times 73 - 73 \times 53 + 53 \times 53}$

$= \frac{(73 + 53)(73 \times 73 - 73 \times 53 + 53 \times 53)}{73 \times 73 - 73 \times 53 + 53 \times 53}$

$= 73 + 53 = 126$

22.

2	392
2	196
2	98
7	49
7	7
	1

The divisible number of $392 = 2 \times 2 \times 2 \times 7 \times 7$. It is clear that on multiplying by 7, we get 392 cube number.

23. $\sqrt[3]{\frac{27}{64}} = \sqrt[3]{\left(\frac{3}{4}\right)^3} = \frac{3}{4}$

24. $\sqrt{x} = \sqrt{0.0064} \times \sqrt[3]{0.008}$

$\Rightarrow \sqrt{x} = 0.08 \times 0.2 = 0.016$

$\Rightarrow \quad x = 0.000256$

25. $1800 = 2^3 \times 3^2 \times 5^2$

It is clear that we have to multiply by 15 to get the sum of digit is 6.

26. $\sqrt[3]{175.616} + \sqrt[3]{0.175616} + \sqrt[3]{0.000175616}$

$= 5.6 + 0.56 + 0.056 = 6.216$

27. $(0.000001)^{1/3} = \sqrt[3]{0.000001}$

$= \sqrt[3]{\frac{1}{1000000}} = \frac{1}{100}$

$= 0.01$

CHAPTER

04

EXPONENTS /POWERS AND SURD

In this chapter, we study the positive and negative exponents with their laws of exponents and also surds with their laws of exponent.

Exponential Form

The repeated multiplication of the same non-zero rational number a with itself in the form of a^n {i.e., $a \times a \times$ .... $\times a \times$ (n times) $= a^n$}, where a is called the base and n is an integer called the exponent or index. This type of representation of a number is called the exponential form of the given number. e.g. $6 \times 6 \times 6 = 6^3$

Here, 6 is the base and 3 is the exponent and we read it as "6 raised to the power of 3".

Rational Exponents

A rational exponents represent both an integer exponent and nth root.

e.g. $a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

Negative Integral Exponents

For any non-zero integer a, we have

$$a^{-n} = \frac{1}{n} \text{ or } a^{-n} \times a^n = 1$$

So, a^{-n} is the multiplicative inverse or reciprocal of a^n and *vice-versa*.

e.g. $(5)^{-2} = \frac{1}{5^2}$

Example 1 Find the multiplicative inverse of 10^{-5}.

(a) 10^4 (b) 10^5
(c) 10^6 (d) None of these

Sol. *(b)* We have, $10^{-5}=\frac{1}{10^5}$

Reciprocal of $\frac{1}{10^5}=10^5$

$\therefore$ Multiplicative inverse of 10^{-5} is 10^5

$[\because\ 10^{-5}\times 10^5=10^0=1]$

Laws of Exponent

I. If a and b be any real numbers and m, n be positive integers, then

(i) $a^m\times a^n=a^{m+n}$

(ii) $a^m\div a^n=a^{m-n}, a\neq 0$

(iii) $(a^m)^n=a^{mn}$ (iv) $(ab)^n=a^nb^n$

(v) $\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}$ (vi) $(a)^0=1,\ a\neq 0$

II. If a and b be any real numbers and m, n be negative integers, then

(i) $a^{-m}\times a^{-n}=\frac{1}{a^m}\times\frac{1}{a^n}$

$$=\frac{1}{a^m\times a^n}=\frac{1}{a^{m+n}}=a^{-(m+n)}$$

(ii) $a^{-m}\div a^{-n}=\frac{1}{a^m}\div\frac{1}{a^n}=\left(\frac{1}{a^m}\times\frac{a^n}{1}\right)$

$$=\frac{a^n}{a^m}=a^{n-m}=a^{-m-(-n)}$$

(iii) $(a^{-m})^{-n}=\left[\frac{1}{(a^{-m})}\right]^n$

$$=(a^m)^n=a^{mn}=a^{(-m)(-n)}$$

(iv) $(ab)^{-n}=\frac{1}{(ab)^n}=\frac{1}{a^n\times b^n}$

$$=\frac{1}{a^n}\times\frac{1}{b^n}=a^{-n}\times b^{-n}$$

(v) $\left(\frac{a}{b}\right)^{-n}=\frac{1}{\left(\frac{a}{b}\right)^n}=\frac{1}{\frac{a^n}{b^n}}=\frac{b^n}{a^n}=\frac{a^{-n}}{b^{-n}}$

Example 2 $\left(\frac{5}{7}\right)^8\div\left(\frac{4}{5}\right)^8$ is equal to

(a) $\left(\frac{5}{7}/\frac{4}{5}\right)^8$ (b) $\left(\frac{5}{7}\times\frac{4}{5}\right)^8$

(c) $\left(\frac{5}{7}/\frac{4}{5}\right)^0$ (d) None of these

Sol. *(a)* $\frac{(5/7)^8}{(4/5)^8}=\left(\frac{5/7}{4/5}\right)^8$ $\left[\because\frac{a^m}{b^m}=\left(\frac{a}{b}\right)^m\right]$

$$=\left(\frac{5}{7}/\frac{4}{5}\right)^8$$

Example 3 The value of

$(7^{-1}-8^{-1})^{-1}-(3^{-1}-4^{-1})^{-1}$ is

(a) 44 (b) 56 (c) 68 (d) 12

Sol. *(a)* Using law of exponents, $a^{-m}=\frac{1}{a^m}$

[$\because a$ is non-zero integer]

$\therefore$ $(7^{-1}-8^{-1})^{-1}-(3^{-1}-4^{-1})^{-1}$

$$=\left(\frac{1}{7}-\frac{1}{8}\right)^{-1}-\left(\frac{1}{3}-\frac{1}{4}\right)^{-1}=\left(\frac{1}{56}\right)^{-1}-\left(\frac{1}{12}\right)^{-1}$$

$$=56-12=44$$

Example 4 Evaluate $\left(\frac{625}{81}\right)^{-1/4}$

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{1}{5}$ (d) $\frac{5}{2}$

Sol. *(a)* $\left(\frac{625}{81}\right)^{-1/4}=\left(\frac{81}{625}\right)^{1/4}=\left(\frac{3^4}{5^4}\right)^{1/4}$

$$=\left[\left(\frac{3}{5}\right)^4\right]^{1/4}=\frac{3}{5}$$

Example 5 Simplify $\left(\frac{x^b}{x^c}\right)^a\times\left(\frac{x^c}{x^a}\right)^b\times\left(\frac{x^a}{x^b}\right)^c$

(a) 0 (b) 1 (c) -1 (d) 2

Sol. *(b)* Given expression

$$=(x^{b-c})^a\times(x^{c-a})^b\times(x^{a-b})^c$$
$$=x^{a(b-c)+b(c-a)+c(a-b)}$$
$$=x^0=1$$

Surd or Radicals

If $\sqrt[n]{a}$ is irrational, where a is a rational number and n is a positive integer, then $\sqrt[n]{a}$ or $a^{1/n}$ is called a surd or radical of order n and a is called the radicand.

- A surd of order 2 is called a quadratic or square surd.
- A surd of order 3 is called a cubic surd.
- A surd of order 4 is called a biquadratic surd.

Surd in Simplest Form

A surd in its simplest form has

(i) the smallest possible index of this radical.

(ii) no fraction under the radical sign.

(iii) no factor of the form b^n, where b is rational, under the radical sign of index n.

Note Let n be a positive integer and a be a real number.

(i) If a is irrational, then $\sqrt[n]{a}$ is not a surd.

(ii) If a is rational, then $\sqrt[n]{a}$ is a surd.

Laws of Surd

As surds can be expressed with fractional exponent, the laws of indices are therefore, applicable to surd.

(i) $(\sqrt[n]{a})^n = a$ (ii) $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$

(iii) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$

(iv) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

(v) $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$

Example 6 The index form of $\sqrt[9]{\left(\frac{4}{5}\right)^3}$ is

(a) $\left(\frac{4}{5}\right)^{1/3}$ (b) $\left(\frac{4}{5}\right)^3$ (c) $\left(\frac{4}{5}\right)^{1/2}$ (d) $\left(\frac{4}{5}\right)^{1/27}$

Sol. *(a)* $\sqrt[9]{\left(\frac{4}{5}\right)^3} = \left[\left(\frac{4}{5}\right)^3\right]^{1/9} = \left(\frac{4}{5}\right)^{3/9} = \left(\frac{4}{5}\right)^{1/3}$

$[\because (a^m)^n] = a^{mn}$

Example 7 If $3^x = 5^y = 75^z$, then the value of z is

(a) $\frac{xy}{(2x+y)}$ (b) $\frac{xy}{x+2y}$

(c) $\frac{xy}{x-y}$ (d) $\frac{xy}{x-2y}$

Sol. *(a)* Let $3^x = 5^y = (75)^z = k$

Then, $3 = k^{1/x}, 5 = k^{1/y}$ and $75 = k^{1/z}$

Now, $75 = 3 \times 5^2$

$\Rightarrow k^{1/z} = k^{1/x} \cdot k^{2/y}$

$\Rightarrow k^{1/z} = k^{\left(\frac{1}{x}+\frac{2}{y}\right)}$

$\therefore \frac{1}{z} = \frac{1}{x} + \frac{2}{y}$

$\Rightarrow z = \frac{xy}{(2x+y)}$

PRACTICE EXERCISE

1. The value of $\frac{5^0 + 2^1}{3^2 + 8^0}$ is

(a) $\frac{3}{10}$ (b) $\frac{5}{3}$

(c) $\frac{2}{5}$ (d) $\frac{3}{5}$

2. The multiplicative inverse of 10^{-100} is

(a) 10 (b) 100

(c) 10^{100} (d) 10^{-100}

3. The value of $\frac{5}{(121)^{-1/2}}$ is

(a) -55 (b) $\frac{1}{55}$ (c) $-\frac{1}{55}$ (d) 55

4. The value of $3 \times 9^{-3/2} \times 9^{1/2}$ is

(a) $\frac{1}{3}$ (b) 3

(c) 27 (d) $-\frac{1}{3}$

5. The simplified form of $(-4)^5 \div (-4)^8$ is

(a) $\frac{1}{4^3}$ (b) $\frac{1}{(-4)^3}$

(c) $\frac{1}{4^4}$ (d) None of these

6. The value of $\left(\frac{1}{2^3}\right)^2$ is

(a) $\frac{1}{62}$ (b) $\frac{1}{64}$

(c) $\frac{1}{32}$ (d) None of these

7. Evaluate $(-3)^4 \times \left(\frac{5}{3}\right)^4$

(a) 5^7 (b) 5^6

(c) 5^3 (d) 5^4

8. Evaluate, $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$

(a) -1 (b) -2

(c) -3 (d) -4

9. If $5^a = 3125$, then the value of 5^{a-3} is

(a) 625 (b) 25

(c) 5 (d) 225

10. The standard form of 0.0000078 is

(a) 78×10^{-6} (b) 78×10^{6}

(c) 78×10^{-5} (d) None of these

11. If $3^x = \frac{1}{9}$, the value of x is

(a) 2 (b) -2 (c) 1/2 (d) 1

12. The value of $(12^2 + 5^2)^{1/2}$ is

(a) 11 (b) 13 (c) 12 (d) 15

13. The value of $(0.000064)^{5/6}$ is

(a) $\frac{32}{100000}$ (b) $\frac{16}{10000}$

(c) $\frac{16}{100000}$ (d) None of these

14. The value of $\left[\left(\frac{25}{9}\right)^{5/2}\right]^{3/5}$ is

(a) $\frac{25}{27}$ (b) $\frac{125}{27}$

(c) $\frac{25}{9}$ (d) None of these

15. The value of $\left(-\frac{1}{125}\right)^{-2/3}$ is

(a) 5 (b) 25

(c) -25 (d) None of these

16. The value of $\frac{(81)^{1/3} \times (576)^{1/3}}{(64)^{2/3} \times (27)^{2/3}}$ is

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$

(c) $\frac{5}{8}$ (d) None of these

17. The value of $\frac{2^{10+n} \times 4^{3n-5}}{2^{4n+1} \times 2^{3n-1}}$ is

(a) 0 (b) 1

(c) 3 (d) 4

18. If $9\sqrt{x} = \sqrt{12} + \sqrt{147}$, then the value of x is

(a) 1 (b) 2

(c) 3 (d) 4

Answers

1	(a)	2	(c)	3	(d)	4	(a)	5	(b)	6	(b)	7	(d)	8	(a)	9	(b)	10	(a)
11	(b)	12	(b)	13	(a)	14	(b)	15	(b)	16	(a)	17	(b)	18	(c)				

Hints and Solutions

1. $\frac{5^0+2^1}{3^2+8^0}=\frac{1+2}{9+1}=\frac{3}{10}$

2. For multiplicative inverse, let a be the multiplicative inverse of 10^{-100}.

[$\because$ If a is multiplicative inverse of b then $a\times b=1$]

$\therefore\ a\times 10^{-100}=1$

$$\Rightarrow\quad a=\frac{1}{10^{-100}}\frac{1}{\frac{1}{10^{100}}}=10^{100}\quad\left[\because a^{-m}=\frac{1}{a^m}\right]$$

3. $\frac{5}{(121)^{-1/2}}=5\times 121^{1/2}$

$$=5\times(11^2)^{1/2}=5\times 11=55$$

4. $3\times 9^{-3/2}\times 9^{1/2}=3\times\left(3^{2\times-\frac{3}{2}}\right)\times\left(3^{2\times\frac{1}{2}}\right)$

$$=3\times(3)^{-3}\times 3=3\times\left(\frac{1}{3}\right)^3\times 3$$

$$=3\times\frac{1}{27}\times 3=\frac{1}{3}$$

5. We have, $(-4)^5\div(-4)^8$

$$=\frac{(-4)^5}{(-4)^8}=\frac{1}{(-4)^8\times(-4)^{-5}}\qquad\left[\because a^m=\frac{1}{a^{-m}}\right]$$

$$=\frac{1}{(-4)^{8-5}}=\frac{1}{(-4)^3}\qquad[\because a^m\times a^n=a^{m+n}]$$

which is the required form.

6. We have, $\left(\frac{1}{2^3}\right)^2$

$$=\frac{(1)^2}{(2^3)^2}\qquad\left[\because\left(\frac{a}{b}\right)^m=\frac{a^m}{b^m}\right]$$

$$=\frac{1}{2^6}=\frac{1}{64}\qquad[\because(a^m)^n=a^{m\times n}]$$

7. We have, $(-3)^4\times\left(\frac{5}{3}\right)^4$

$$=(-1\times 3)^4\times\left(\frac{5}{3}\right)^4\qquad[\because -a=-1\times a]$$

$$=(-1)^4\times 3^4\times\frac{5^4}{3^4}$$

$$\left[\because(a\times b)^m=a^m\times b^m,\left(\frac{a}{b}\right)^m=\frac{a^m}{b^m}\right]$$

$$=1\times 5^4=(5)^4\qquad[\because(-1)^4=1]$$

which is the required form.

8. We have, $\left\{\left(\frac{1}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{-1}$

$$=\left\{\frac{(1)^{-1}}{(3)^{-1}}-\frac{(1)^{-1}}{(4)^{-1}}\right\}^{-1}\qquad\left[\because\left(\frac{a}{b}\right)^m=\frac{a^m}{b^m}\right]$$

$$=\left\{\frac{3}{1}-\frac{4}{1}\right\}^{-1}\qquad\left[\because a^{-m}=\frac{1}{a^m}\right]$$

$$=(3-4)^{-1}=(-1)^{-1}\qquad\left[\because a^{-m}=\frac{1}{a^m}\right]$$

$$=\frac{1}{(-1)^1}$$

$$=\frac{1}{-1}=-1$$

9. Given, $5^a=3125$

$\Rightarrow\quad 5^a=5^5$

On comparing, we get

$a=5$

$\therefore\quad 5^{a-3}=5^{5-3}=25$

10. According to question,

$$0.000078=\frac{78}{1000000}=78\times 10^{-6}$$

11. $\because\quad 3^x=\frac{1}{9}$

$\therefore\quad 3^x=\left(\frac{1}{3}\right)^2$

or $\quad 3^x=3^{-2}$

On comparing both sides, we get $x=-2$

12. $(12^2+5^2)^{1/2}=(144+25)^{1/2}$

$$=(169)^{1/2}$$

$$=(13^2)^{1/2}=13$$

13. $(0.000064)^{5/6} = \left(\frac{64}{1000000}\right)^{5/6}$

$= \left[\left\{\left(\frac{2}{10}\right)^6\right\}^{1/6}\right]^5 = \left(\frac{2}{10}\right)^5 = \frac{32}{100000}$

14. $\left[\left(\frac{25}{9}\right)^{5/2}\right]^{3/5} = \left[\left\{\left(\frac{5}{3}\right)^2\right\}^{5/2}\right]^{3/5}$

$= \left[\left(\frac{5}{3}\right)^5\right]^{3/5} = \left(\frac{5}{3}\right)^3 = \frac{125}{27}$

15. $\left(-\frac{1}{125}\right)^{-2/3} = \left[\left(-\frac{1}{5} \times -\frac{1}{5} \times -\frac{1}{5}\right)^{-1/3}\right]^2$

$= (-5)^2 = 25$

16. $\frac{(81)^{1/3} \times (576)^{1/3}}{(64)^{2/3} \times (27)^{2/3}} = \frac{(3^4)^{1/3} \times (2^6 \times 3^2)^{1/3}}{(4^3)^{2/3} \times (3^3)^{2/3}}$

$= \frac{3^{4/3} \times 2^2 \times 3^{2/3}}{4^2 \times 3^2} = \frac{3^2 \times 2^2}{4^2 \times 3^2} = \frac{9 \times 4}{16 \times 9} = \frac{1}{4}$

17. $\frac{2^{10+n} \times 4^{3n-5}}{2^{4n+1} \times 2^{3n-1}} = \frac{2^{10+n} \times 2^{6n-10}}{2^{4n+1} \times 2^{3n-1}}$

$= \frac{2^{10+n+6n-10}}{2^{4n+1+3n-1}} = \frac{2^{7n}}{2^{7n}} = 1$

18. $9\sqrt{x} = \sqrt{12} + \sqrt{147}$

$= \sqrt{2 \times 2 \times 3} + \sqrt{3 \times 7 \times 7}$

$= 2\sqrt{3} + 7\sqrt{3}$

$\Rightarrow \quad 9\sqrt{x} = 9\sqrt{3} \quad \Rightarrow \quad x^{1/2} = 3^{1/2}$

On comparing both sides, we get $x = 3$