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JAWAHAR NAVODAYA VIDYALAYA

Entrance Exam 2021

Conducted by
Navodaya Vidyalaya Samiti

ENGLISH

HINDI

GENERAL SCIENCE

MATHEMATICS

With Solved Paper 2020

9

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PRACTICE SETS **1-20**

SYLLABUS

ENGLISH

Comprehension (Unseen Passage), Word and Sentence Structure, Spelling, Rearranging jumbled words , Passivation, Use of degrees of comparison, Modal auxiliaries , Use of prepositions , Tense forms, Reported speech

हिन्दी

(क) भाषिक अनुप्रयोग और व्याकरणिक कुशलताएँ

वर्ण विचार/वर्तनी विवेक, शब्दभेद (स्त्रोत/उत्पत्ति), पर्याय/विलोम, शब्द विवेक (शब्द प्रयोग में सूक्ष्म अंतर), पद भेद (व्याकरणिक कोटि) की पहचान, पद परिचय, अशुद्ध वाक्य को शुद्ध करना, वाक्य रचनान्तरण (सरल/संयुक्त/मिश्र), मुहावरा, लोकोक्ति

(ख) अपठित बोधात्मक प्रश्न

GENERAL SCIENCE

Food – Crop Production and Management; Microorganism; Food Preservation, Materials I – Synthetic fibers; Plastics; Metals and Non – metals, Material II – Coal and Petroleum; Refining of Petroleum; Fossil Fuels; Combustion and Flame, Living / Non living; Cell structure and Function; Conservation of Plants and Animals – Wildlife, Sanctuary and National Parks, Reproduction – Asexual and Sexual, Reproduction, Reaching the age of adolescence, Force – Frictional Force; Gravitational Force; Thrust and Pressure, Light – Reflection of Light; Multiple Reflection; Human eye; Care of the Eyes; Sound; Human ears; Loudness and Pitch, Audible and inaudible Sounds, Chemical Effects of Electric Current; Electroplating, Natural Phenomena – Lightning; Earthquakes, Pollution of Air and Water, Solar system; Stars and Constellations

MATHEMATICS

Rational Numbers, Squares and Square Roots, Cubes and Cube Roots, Exponents and Powers, Direct and Inverse Proportions, Comparing Quantities (Percentage, Profit and Loss, Discount, Simple and Compound Interest), Algebraic Expressions and Identities including Factorization, Linear Equations in One Variable, Understanding Quadrilaterals (Parallelogram, rhombus, rectangle, square, kite) , Mensuration: *(a) Area of plane figures , (b) Surface area and volume of cube, cuboid and cylinder* , Data Handling (Bar graph, pie chart, organizing data, probability)

CHAPTER

10

LINES AND TRIANGLES

In this chapter, we study the Line, Triangle their types and properties etc.

Lines and Triangles are the branch of plane geometry. In which we study the two intersecting lines forms different types of angles and three intersecting lines forms a different types of triangles.

Definitions Related to Lines and Angles

Line Segment

A line segment is a portion (or part) of a line. It has two end points.

It has a definite length. Distance between P and Q is called length of the line segment PQ.

Ray

A ray extends indefinitely in one direction. This is shown by an arrow *ie*,

P is called initial point. It has no definite length.

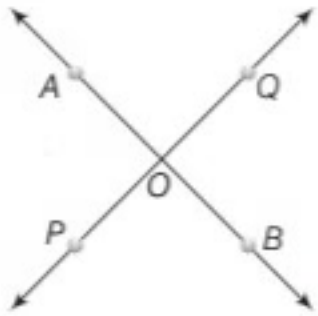
Line

A line segment PQ when extended indefinitely in both the directions is called a line.

- Line has no end point.
- Line has no definite length.
- Line is a set of infinite points.

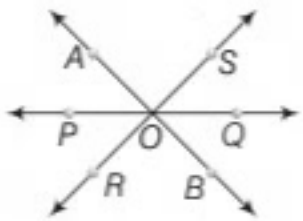
Intersecting Lines

Two lines having a common point, are called intersecting lines. This common point is called point of intersection. In the figure, 'O' is the common point.



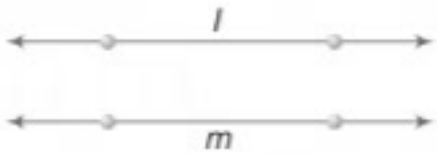
Concurrent Lines

Three or more lines in a plane which are intersecting at the same point, are called concurrent lines.



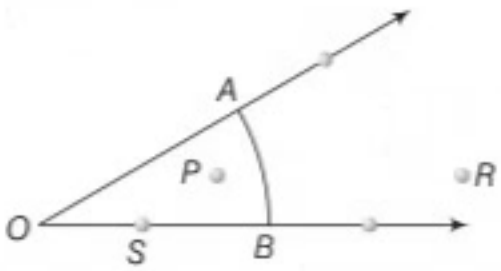
Parallel Lines

Two lines in a plane which do not intersect anywhere in a plane are called parallel lines.



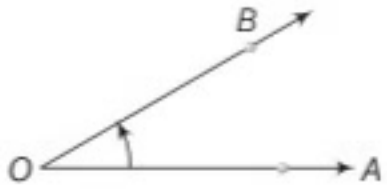
Angles

A figure consisting of two rays join with end points is called an angle. In the figure, $\angle AOB$ is a angle with rays OA and OB.

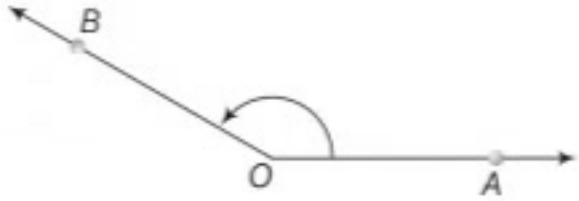


Classification of Angles

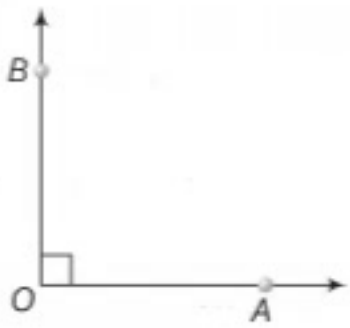
(i) **Acute Angle** An angle between 0° and 90° (less than 90°) is called acute angle.



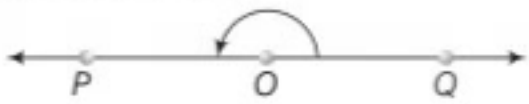
(ii) **Obtuse Angle** An angle between 90° and 180° (greater than 90°) is called obtuse angle.



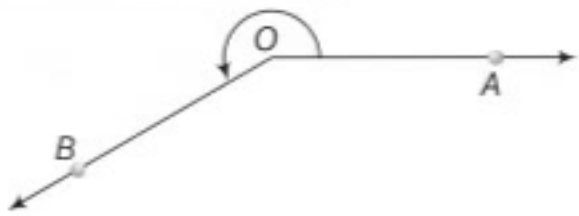
(iii) **Right Angle** An angle equal to 90° is called right angle.



(iv) **Straight Angle** An angle equal to 180° is called straight angle.



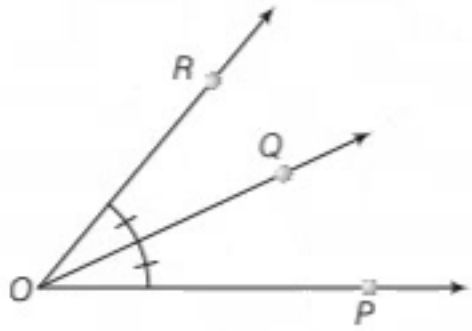
(v) **Reflex Angle** An angle between 180° and 360° is called reflex angle.



(vi) **Complete Angle** An angle equal to 360° is called complete angle.

(vii) **Bisector of an Angle** A ray OQ is called the bisector of $\angle POR$, if

$$\angle POQ = \angle ROQ.$$



$$\therefore \quad \angle POQ = \angle QOR = \frac{1}{2}\angle POR$$

(viii) **Complementary Angles** Two angles are said to be complementary, if their sum is 90°.

Complementary angles are complement of each other.

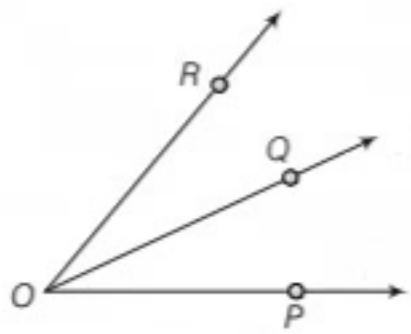
Complement of θ is (90° − θ).

(ix) **Supplementary Angles** Two angles are said to be supplementary, if their sum is 180°.

Supplementary angles are supplement of each other.

Supplement of θ is (180° − θ).

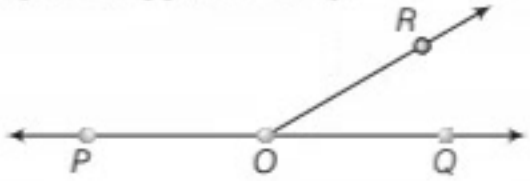
(x) **Adjacent Angles** Two angles are said to be adjacent, if they have a common vertex.



They have a common arm and their non-common arms are on either side of the common arm.

Here, $\angle POQ$ and $\angle ROQ$ are adjacent angles and have the same vertex O, a common arm OQ, the non- common arms OP, OR on either side of OQ.

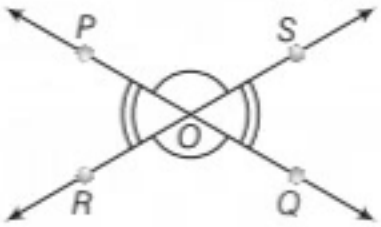
(xi) **Linear Pair** Two angles are said to form a linear pair of angles, if they are adjacent angles and they are supplementary.



$$\therefore \quad \angle POR + \angle QOR = 180°$$

If a ray stands on a line, then the sum of the adjacent angles so formed is 180°.

(xii) **Vertically Opposite Angles** If two lines PQ and RS intersects at a point O, then the pair of $\angle POR$ and $\angle QOS$ or pair of $\angle POS$ and $\angle ROQ$ is said to be a pair of vertically opposite angles.



Vertically opposite angles are always equal.

i.e. $\angle POR = \angle QOS$ and $\angle POS = \angle ROQ$

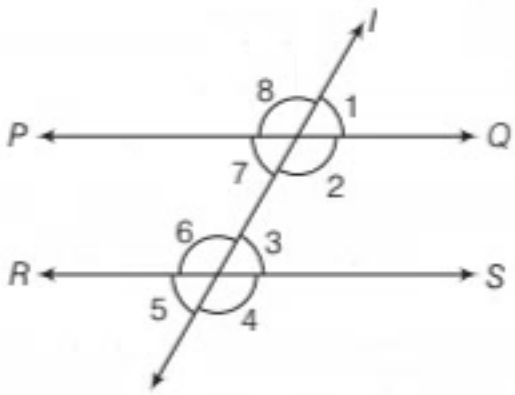
Properties of Parallel Lines

Let PQ and RS be two parallel lines, cut by a transversal l.

Then,

- Angles $\angle 1$, $\angle 8$, $\angle 5$ and $\angle 4$ are the exterior angles and $\angle 2$, $\angle 3$, $\angle 6$ and $\angle 7$ are the interior angles.
- Pairs of corresponding angles are equal

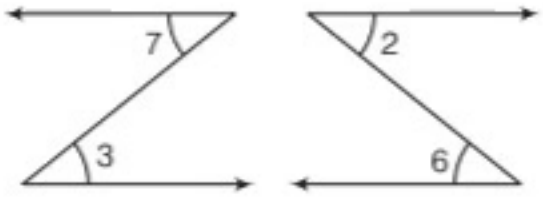
  i.e. $\angle 1 = \angle 3$, $\angle 2 = \angle 4$, $\angle 7 = \angle 5$, $\angle 8 = \angle 6$



- The sum of co-interior angles on the same side of transversal is 180°.

  i.e., $\angle 2 + \angle 3 = 180°$ and $\angle 7 + \angle 6 = 180°$

- The pairs of opposite angles of transversal line is said to be alternate angles.



i.e., $\angle 7 = \angle 3$

$\angle 2 = \angle 6$

- The vertical opposite angles are equal, i.e.

$\angle 1 = \angle 7$; $\angle 2 = \angle 8$

and $\angle 3 = \angle 5$; $\angle 4 = \angle 6$

Example 1 Find the measure of an angle which is 28° more than its complement.

(a) 58° (b) 59°
(c) 60° (d) 61°

Sol. (*b*) Let measure of the required angle be $x°$.

Then, measure of its complement $= 90° - x$

$\therefore \quad x - (90° - x) = 28°$

$\Rightarrow \quad 2x = 118°$

$\Rightarrow \quad x = 59°$

Hence, the measure of the required angle is 59°.

Example 2 Find the measure of an angle, which is 32° less than its supplement.

(a) 74° (b) 73°
(c) 72° (d) 71°

Sol. (*a*) Let the measure of the required angle be x.

Then, measure of its supplement $= (180° - x)$

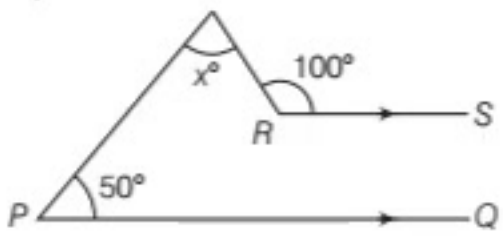
$\therefore \quad (180° - x) - x = 32°$

$\Rightarrow \quad 180° - 32° = 2x$

$\Rightarrow \quad 2x = 148°$

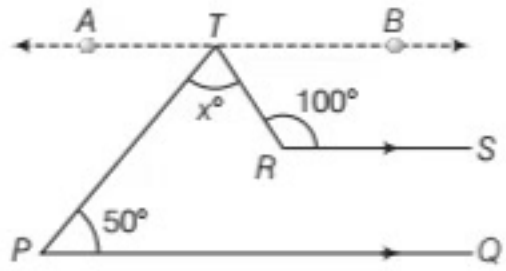
$\Rightarrow \quad x = 74°$

Example 3 In the given figure, $PQ \parallel RS$. The value of $x°$, is



(a) 120° (b) 50°
(c) 135° (d) 140°

Sol. (*b*) Draw $AB \parallel PQ$



$\angle ATP = \angle TPQ = 50°$

[$\because$ alternate interior angles]

$\angle BTR + \angle TRS = 180°$

[$\because$ sum of interior angles]

$\Rightarrow \quad \angle BTR = 80°$ and $\angle ATP + x + \angle BTR = 180°$

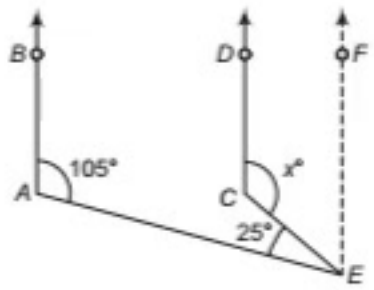
[$\because$ AB is the straight line]

$\Rightarrow \quad 50° + x + 80° = 180°$

$\therefore \quad x = 50°$

Example 4 In the given figure, $AB \parallel CD$, the value of x is

(a) 120° (b) 130° (c) 132° (d) 134°



Sol. (*b*) Let $CD \parallel EF$. Then, $x + \angle CEF = 180°$

[$\because$ sum of interior angles]

$\Rightarrow \quad \angle CEF = 180° - x$

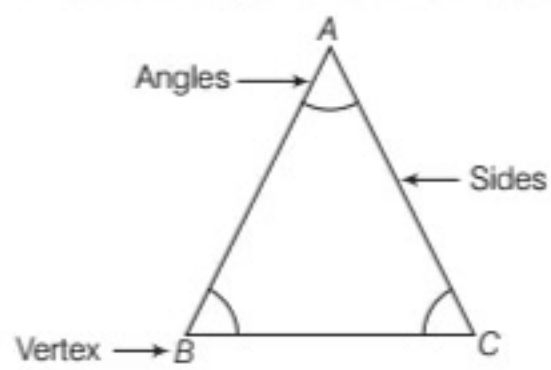
But $\quad \angle BAE + \angle AEF = 180°$

$\Rightarrow 105° + 25° + (180° - x) = 180°$

$\therefore \quad x = 130°$

Triangle

A closed figure which has three sides, three angles and three vertices, is called a triangle.



Properties of Triangle

- The sum of three angles of a triangle is equal to 180°.
- The sum of lengths of any two sides of triangle is greater than the length of third side.
- In a triangle, an exterior angle is equal to the sum of the two interior opposite angles.

Types of Triangle

Different types of triangle, according to their sides and angles are given below

I. According to their Sides

(i) **Scalene Triangle** If all sides are different in lengths called scalene triangle. In which all angles are different.

(ii) **Isosceles Triangle** If any two sides are equal in length called isosceles triangle. In which opposite angles of equal sides are also equal.

(iii) **Equilateral Triangle** If all three sides are equal in length called equilateral triangle. In which all three angles of triangles are equal to 60°.

II. According to their Angles

(i) **Acute Angled Triangle :** If each angle of triangle is less than 90°. It is called an acute angled triangle.

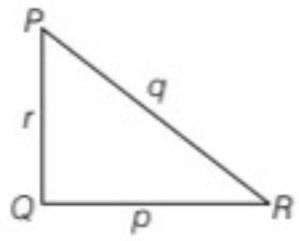
(ii) **Right Angled Triangle :** If any one angle of a triangle is 90°. It is called right angled triangle.

(iii) **Obtuse Angled Triangle :** If any one angle of triangle is greater than 90°. It is called an obtuse angled triangle.

Pythagoras Theorem

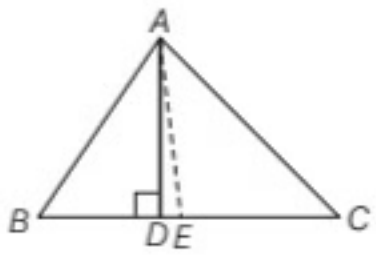
In a right angle triangle, the square of the hypotenuse equals the sum of the square of its other sides.

i.e., $q^2 = p^2 + r^2$



Altitude and Median of a Triangle

A line segment from a vertex of triangle, perpendicular to the line containing the opposite side is called an **altitude** of a triangle. In the figure AD is the altitude of a triangle.



A line segment that joins a vertex of a triangle to the mid-point of the opposite side is called median of triangle. In a figure, AE is the **median** of a triangle.

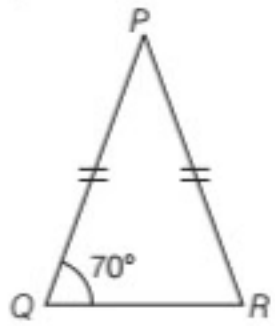
Note : Every triangle has three altitudes and three medians, which are drawn from each vertex.

Example 5 ΔPQR is an isosceles triangle with $PQ = PR$. If $\angle Q = 70°$, then the other angles of a triangle, are

(a) 40°, 70° (b) 35°, 75°
(c) 55°, 55° (d) None of these

Sol. (*a*) In ΔPQR, we have,

$$PQ = PR$$



i.e., $\angle Q = \angle R = 70°$

Since, $\angle P + \angle Q + \angle R = 180°$

[the sum of three angles of triangle]

$\therefore$ $\angle P + 140° = 180°$

$\Rightarrow$ $\angle P = 180° - 140° = 40°$

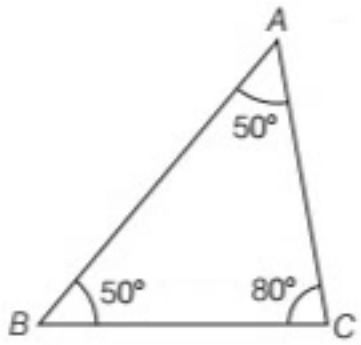
Hence, the required angles of triangle are

$\angle P = 40°$ and $\angle R = 70°$.

Example 6 In ΔABC, $\angle A = 50°$, $\angle B = 50°$, $\angle C = 80°$, which two sides of this triangle are equal.

(a) $AB = AC$ (b) $AC = BC$
(c) $AB = BC$ (d) None of these

Sol. (*b*) Since, $\angle A = \angle B = 50°$



Therefore, the sides opposite these angles must be equal.
The side opposite $\angle A$ is BC and the side opposite $\angle B$ is AC

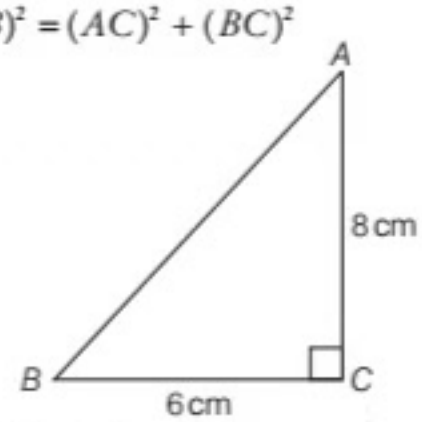
$\therefore$ In $\Delta ABC, AC = BC$

Example 7. The length of the sides of a right angle triangle are 6 cm and 8 cm. What is the length of the hypotenuse ?

(a) 11 cm (b) 10 cm (c) 9 cm (d) 8 cm

Sol. (*b*) Using Pythagoras theorem in ΔABC

$\therefore \quad (AB)^2 = (AC)^2 + (BC)^2$



$\Rightarrow \quad (AB)^2 = (8)^2 + (6)^2$

$\Rightarrow \quad (AB)^2 = 64 + 36$

$= 100$

$\Rightarrow \quad AB = 10$ cm

Hence, the length of the hypotenuse is 10 cm.

PRACTICE EXERCISE

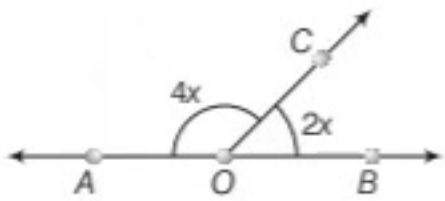
1. An angle is 14° more than its complement. Then, its measure is
(a) 166° (b) 86° (c) 76° (d) 52°

2. The measure of an angle is twice the measure of its supplementary angle. Then, its measure is
(a) 120° (b) 60° (c) 100° (d) 90°

3. How many least number of distinct points required to determine a unique line ?
(a) One (b) Two (c) Three (d) Infinite

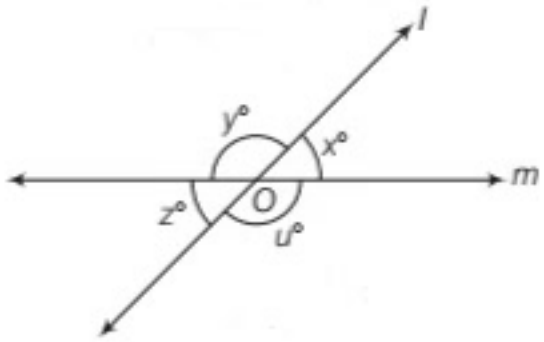
4. If OA and OB are opposite rays; a ray OC inclined. If one of the angle is 75°, then the measurement of the second angle is
(a) 105° (b) 70°
(c) 15° (d) None of these

5. In figure, $\angle AOC$ and $\angle BOC$ form a linear pair. Then, the value of x is



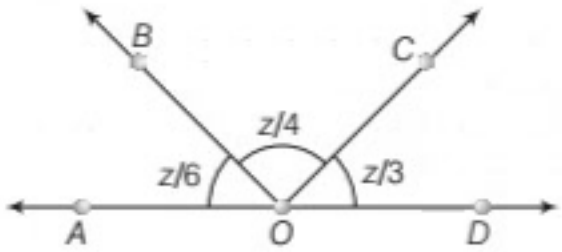
(a) 15° (b) 40° (c) 25° (d) 30°

6. Lines l and m intersect at O, forming angles as shown in figure. If $x = 45°$, then values of y, z and u are respectively



(a) 45°, 135°, 135°
(b) 135°, 135°, 45°
(c) 135°, 45°, 135°
(d) 115°, 45°, 115°

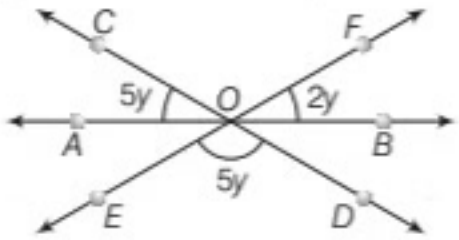
7. The value of z (in degrees), in the given figure is



(a) 180° (b) 216° (c) 240° (d) 40°

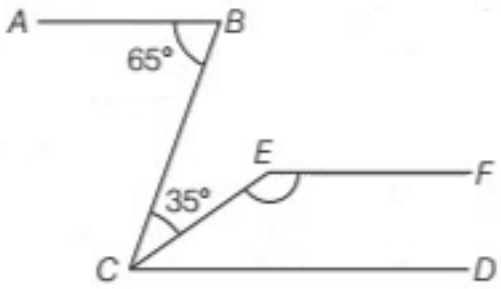
8. Which of the following statements is false?
(a) A line segment can be produced to any desired length
(b) Through a given point, only one straight line can be drawn
(c) Through two given points, it is possible to draw one and only one straight line
(d) Two straight lines can intersect in only one point

9. In the figure, the value of y is



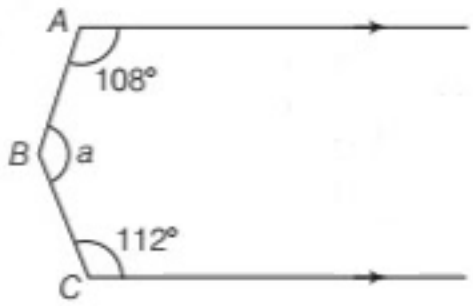
(a) 25° (b) 35°
(c) 15° (d) 40°

10. AB and CD are two parallel lines. The points B and C are joined such that $\angle ABC = 65°$. A line CE is drawn making angle of 35° with the line CB, EF is drawn parallel to AB, as shown in figure, then $\angle CEF$ is



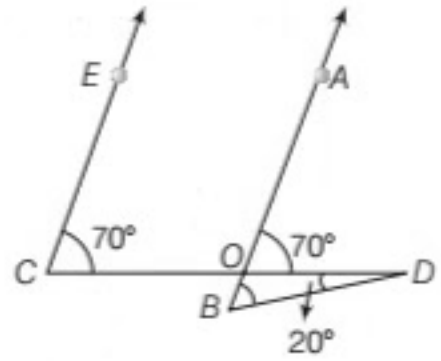
(a) 160° (b) 155°
(c) 150° (d) 145°

11. In the given figure, ray $A \parallel$ ray C, the value of 'a' is



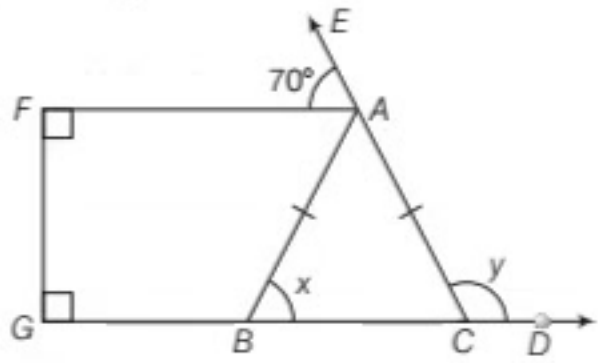
(a) 120° (b) 140°
(c) 90° (d) 150°

12. In the given figure, if $EC \parallel AB$, $\angle ECD = 70°$, $\angle BDO = 20°$, then $\angle OBD$ is



(a) 70° (b) 60° (c) 50° (d) 20°

13. In the given figure, the value of x and y are respectively

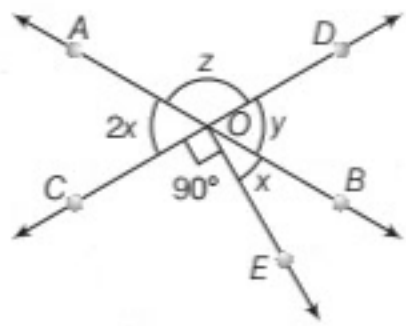


(a) 70°, 110° (b) 110°, 70°
(c) 120°, 60° (d) 70°, 90°

14. The earth makes a complete rotation about its axis in 24 h. Through what angle will it turn in 3 h 20 min?
(a) 60° (b) 50°
(c) 70° (d) 90°

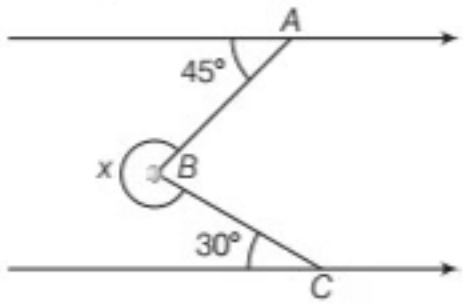
15. An angle is $\frac{2}{3}$rd of its complement and $\frac{1}{4}$th of its supplement, then the angle is
(a) 46° (b) 56°
(c) 36° (d) 40°

16. In the given figure, if $\angle COE = 90°$, then the value of x is



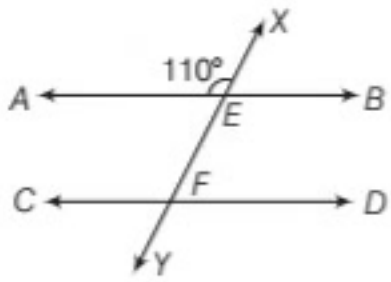
(a) 120° (b) 60°
(c) 45° (d) 30°

17. The value of x, in the figure is



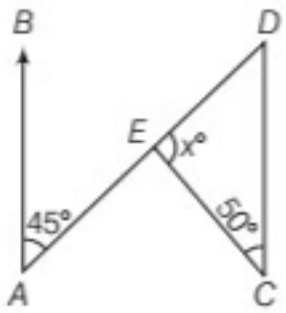
(a) 75° (b) 185° (c) 285° (d) 245°

18. In the given figure, AB and CD are two parallel lines. A line XY meets the lines AB and CD at E and F respectively. If $\angle XEA = 110°$, then $\angle EFD$ is



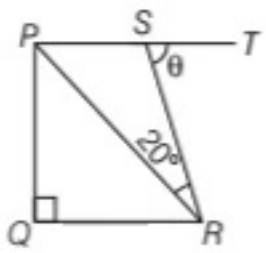
(a) 110° (b) 70° (c) 80° (d) 45°

19. In the given figure,
$AB || CD$, $\angle BAE = 45°$, $\angle DCE = 50°$ and $\angle CED = x°$, then the value of x is



(a) 85° (b) 95°
(c) 130° (d) 135°

20. In the trapezium $PQRS$, $QR || PS$, $\angle Q = 90°$, $PQ = QR$ and $\angle PRS = 20°$. If $\angle TSR = \theta$, then value of θ is

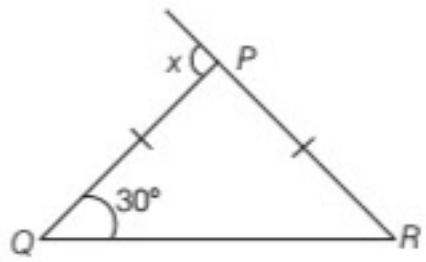


(a) 75° (b) 55°
(c) 65° (d) 45°

21. If ΔABC is an isosceles with $AB = AC$, if $\angle A = 80°$, then $\angle ABC$ will be
(a) 100° (b) 80°
(c) 50° (d) 40°

22. In ΔABC, $BC = CA$, its two equal angles are
(a) $\angle B = \angle C$
(b) $\angle A = \angle B$
(c) $\angle A = \angle C$
(d) $\angle A = \angle B = \angle C$

23. The value of x in figure, where ΔPQR is an isosceles with $PQ = PR$ will be



(a) 30° (b) 60°
(c) 90° (d) 150°

24. The length of the sides BC and AC of a right angled ΔABC are 3 cm and 4 cm, the length of hypotenuse will be
(a) 5 cm (b) 6 cm
(c) 14 cm (d) 10 cm

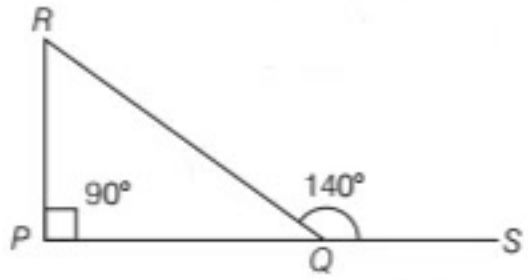
25. If the square of the hypotenuse (in cm) of an isosceles right triangle is 200 then the length of each side will be
(a) 15 cm (b) 200 cm
(c) 10 cm (d) None of these

26. In ΔABC all sides are of same length, then each angle will be
(a) 50° (b) 90°
(c) 60° (d) 180°

27. In a ΔABC, $AB = 11$ cm, $BC = 60$ cm and $AC = 61$ cm. What type of ΔABC ?
(a) Acute angled triangle
(b) Right angled triangle
(c) Obtuse angled triangle
(d) None of the above

28. The total number of triangles formed in a rectangle are
(a) 4 (b) 8
(c) 6 (d) 3

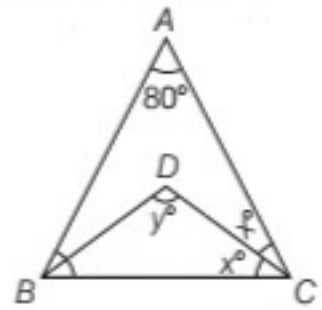
29. The value of $\angle PRQ$ in the given triangle is



(a) 50°
(b) 140°
(c) 90°
(d) 60°

30. ABC is a triangle such that $AB = 10$ and $AC = 3$. The side BC is
(a) equal to 7
(b) less than 7
(c) greater than 7
(d) None of the above

31. In the given figure, $\angle A = 80°$, $\angle B = 60°$, $\angle C = 2x°$ and $\angle BDC = y°$. BD and CD bisect angles B and C respectively. The value of x and y are respectively



(a) 10°, 160° (b) 15°, 70°
(c) 20°, 130° (d) 20°, 125°

32. If one of the angles of a triangle is greater than each of the two remaining angles by 30°, then the angles of the triangle are
(a) 40°, 40°, 100° (b) 50°, 50°, 80°
(c) 30°, 30°, 120° (d) 35°, 35°, 110°

Answers

1	(d)	2	(a)	3	(b)	4	(a)	5	(d)	6	(c)	7	(c)	8	(b)	9	(c)	10	(c)
11	(b)	12	(c)	13	(a)	14	(b)	15	(c)	16	(d)	17	(c)	18	(b)	19	(a)	20	(c)
21	(c)	22	(b)	23	(b)	24	(a)	25	(c)	26	(c)	27	(b)	28	(b)	29	(a)	30	(c)
31	(c)	32	(b)																

Hints and Solutions

1. Let the angle be x, then its complement be $(90° - x)$.

$\therefore \quad x = (90° - x) + 14° \Rightarrow 2x = 104°$

$\therefore \quad x = \frac{104°}{2} = 52°$

2. Let the angle be x, then its supplementary be $(180° - x)$.

$\therefore \quad x = 2(180° - x) \Rightarrow x = 360° - 2x$

$\Rightarrow \quad 3x = 360°$

$\therefore \quad x = 120°$

3. One and only one straight line passes through two distinct points.

4. Since, OA and OB are opposite rays.
So, AB is a line.
Sum of the two angles $= 180°$

$\therefore$ Second angle $= 180° - 75° = 105°$

5. Since, $\angle AOC + \angle BOC = 180°$ [linear pair]

$\Rightarrow \quad 4x + 2x = 180°$

$\Rightarrow \quad 6x = 180°$

$\therefore \quad x = 30°$

6. $\because \quad x = z$ [vertically opposite angles]

$\Rightarrow \quad x = 45° \Rightarrow z = 45°$

and $y + x = 180°$ [linear pair]

$\Rightarrow \quad y = 180° - 45°$

$\therefore \quad y = 135°$

Also, $y = u$ [$\because$ vertically opposite angles]

$\Rightarrow \quad u = 135°$

$\therefore \quad y = 135°,$
$z = 45°,$
$u = 135°$

7. We have, $\frac{z}{6}+\frac{z}{4}+\frac{z}{3}=180°$ [$\because$ AOD is a line]

$\Rightarrow \quad \frac{2z+3z+4z}{12}=180°$

$\Rightarrow \quad \frac{9z}{12}=180°$

$\therefore \quad z=\frac{180°\times 12}{9}=240°$

8. Since, an infinite number of straight lines can be drawn through a given point.
Hence, (b) is false statement.

9. Since, OA, OB are opposite rays.

$\therefore \angle AOC+\angle COF+\angle FOB=180°$

$\Rightarrow \quad 5y+5y+2y=180°$

[$\because$ $\angle COF=\angle EOD$ vertical opposite angle]

$\Rightarrow \quad 12y=180° \Rightarrow y=\frac{180°}{12°}=15°$

10. Since, $AB \parallel CD$

$\Rightarrow \quad \angle BCD=\angle ABC=65°$

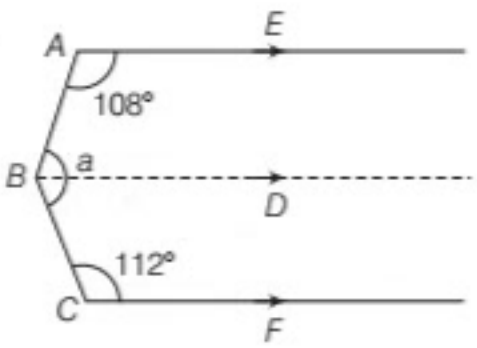
But, $\angle ECD=65°-\angle BCE$

$=65°-35°=30°$

Now, $\angle CEF+\angle ECD=180°$ [sum of co-interior angles is 180°; since $CD \parallel EF$]

$\therefore \quad \angle CEF=180°-30°=150°$

11.



Draw ray $AE \parallel$ ray $BD \parallel$ ray CF

Then, $\angle EAB+\angle ABD=180°$

[the sum of co-interior angles is 180°]

$\Rightarrow \quad 180°+\angle ABD=180°$

$\Rightarrow \quad \angle ABC=180°-108°$

$=72°$ ...(i)

and $\angle FCB+\angle CBD=180°$

$\Rightarrow \quad 112°+\angle CBD=180°$

$\Rightarrow \quad \angle CBD=180°-112°$

$=68°$ ...(ii)

Now, adding Eqs, (i) and (ii), we have

$\angle ABD+\angle CBD=72°+68°$

$\therefore \quad \angle ABC=140°$

Hence, $a=140°$

12. $\because \quad \angle AOD=\angle ECO$ [$\because EC \parallel AB$]

$\Rightarrow \quad \angle AOD=70°$

So, $\angle BOD=110°$ [$\because$ AOB is the line]

In ΔBOD, $\angle OBD+\angle BOD+\angle ODB=180°$

$\Rightarrow \quad \angle OBD=180°-(110°+20°)$

$\therefore \quad \angle OBD=50°$

13. In ΔABC, $\angle ABC=\angle ACB=x$

[Angles opposite to equal sides are equal]

and $x+y=180°$

Now, $\angle EAF=\angle ACB=70°$

[corresponding angles]

$\therefore \quad x=70°$

$\Rightarrow \quad y=180°-70°$ [$\because$ linear pair]

$\therefore \quad y=110°$

14. In 24 h, earth covers an angle = 360°

In 1 h, earth covers an angle $=\frac{360°}{24}$

In 3 h, 20 min earth will cover an angle

$=\frac{360°}{24}\times(3 \text{ h } 20 \text{ min})$

$=\frac{360°}{24}\times\left(3+\frac{20}{60}\right)=\frac{360°}{24}\times\frac{10}{3}=50°$

15. Let angle be x, then its complement be $(90°-x)$.

$\therefore \quad \frac{2}{3}(90°-x)=x \Rightarrow 180°-2x=3x$

$\therefore \quad x=36°$

16. Since, $\angle BOD=\angle AOC$

[$\because$ vertically opposite angles]

$\Rightarrow \quad 2x=y$

Now, $\angle COE+\angle EOB+\angle BOD=180°$

[$\because$ COD is line or linear pair]

$\Rightarrow \quad 90°+x+2x=180°$

$\Rightarrow \quad 3x=90°$

$\therefore \quad x=30°$

17. Here, $\angle ABC = 45° + 30° = 75°$

[let ray $A \parallel$ ray C]

$\Rightarrow \quad x = 360° - \angle ABC = 360° - 75°$

$\therefore \quad x = 285°$

18. $\because \angle XEA = \angle BEF = 110°$

[vertically opposite angles]

$\therefore \quad \angle BEF + \angle EFD = 180°$ [co-interior angles]

$\Rightarrow \quad \angle EFD = 180° - 110° = 70°$

19. In a given figure, $AB \parallel CD$

$\therefore \quad \angle BAD = \angle ADC = 45°$ [alternate angle]

In ΔECD, $x° + 50° + 45° = 180° \Rightarrow x = 85°$

20. $\because PQ = QR \Rightarrow \angle QPR = \angle QRP = 45°$

and $PS \parallel QR$, $\angle SPR = \angle QRP = 45°$

[alternate angles]

$\therefore \theta = \angle PRS + \angle SPR = 20° + 45°$

$= 65°$ [by exterior angle theorem of ΔPSR]

21. Given, $\angle A = 80°$ and $AB = AC$

Therefore, $\angle B = \angle C = x$

[Opposite angles to equal sides are equal]

In ΔABC, $\angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad 80° + x + x = 180°$

$\Rightarrow \quad 2x = 100°$

$\therefore \quad x = 50°$

22. Since in a ΔABC, $BC = CA$, therefore $\angle A = \angle B$

23. Since, $PQ = PR$

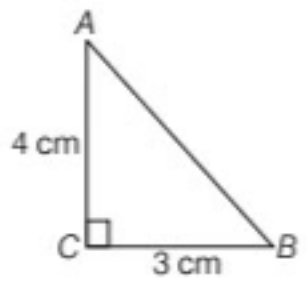
i.e., $\angle Q = \angle R = 30°$

Now, $\angle P = 180° - (30° + 30°) = 120°$

$\therefore \quad x = 180° - CP$ [linear pair]

$x = 180° - 120° = 60°$

24. Given $AC = 4$ cm, $BC = 3$ cm, $AB = ?$



By pythagoras theorem,

$(AB)^2 = (AC)^2 + (BC)^2$

$= 4^2 + 3^2 = 16 + 9 = 25$

$\therefore \quad AB = \sqrt{25} = 5$ cm

25. Let the sides of a right isosceles triangle be x and x.

$\therefore \quad H^2 = x^2 + x^2 \Rightarrow 200 = 2x^2$

$\Rightarrow \quad x^2 = 100 \Rightarrow x = 10$ cm

26. Since, all sides of a triangle are of same length, therefore all angles are of equal, say x.

$\therefore \quad 3x = 180°$

$\Rightarrow \quad x = 60°$

27. Now, $AB^2 + BC^2 = 11^2 + 60^2$

$= 121 + 3600 = 3721$

and $AC^2 = (61)^2 = 3721$

$\because \quad AC^2 = AB^2 + BC^2$

Hence, ΔABC is right angled triangle.

28. The total number of triangle formed in a rectangle are 8.

29. $\because PQR$ is a line.

$\therefore \quad \angle PQR = 180° - 140° = 40°$

and in ΔPQR, $\angle RPQ + \angle PQR + \angle PRQ = 180°$

$\angle PRQ = 180° - (90° + 40°)$

$= 180° - 130° = 50°$

30. Since, the sum of any two sides of a triangle is greater than the third side, so BC must be greater than 7, then $AC + BC > AB$.

31. In the given figure,

$\angle C = 180° - \angle A - \angle B$

$\Rightarrow 2x = 180° - 80° - 60° = 40°$

$\therefore \quad x = 20°$

Also, $\angle B = 60° \Rightarrow \angle DBC = \frac{1}{2} \times 60° = 30°$

In ΔBDC, $\angle DBC + y + x = 180°$

$\Rightarrow 30° + y + 20° = 180° \Rightarrow y = 130°$

32. Since, $(x + 30°) + x + x = 180°$

$\Rightarrow \quad 3x = 150° \Rightarrow x = 50°$

$\therefore$ Angles are $50°, 50°, 80°$.

CHAPTER

11

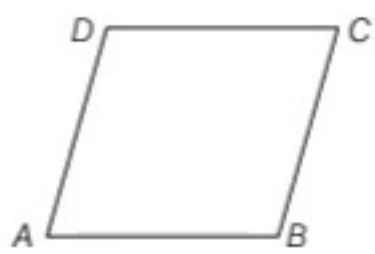
QUADRILATERALS

(PARALLELOGRAM, RHOMBUS, RECTANGLE, SQUARE, KITE)

In this chapter, we study the quadrilateral and their types. Also study the circle and cyclic quadrilaterals with their important results.

Quadrilateral

A plane figure which is made up of joining any four non-collinear points, is called quadrilateral.



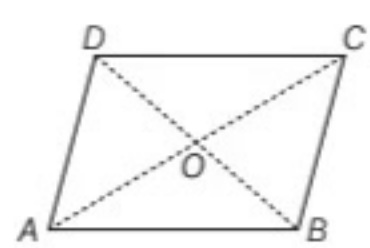
The sum of all angles of a quadrilateral is 360°.
i.e. $\angle A + \angle B + \angle C + \angle D = 360°$.

Types of Quadrilateral

Some types of quadrilateral are as given below

(i) Parallelogram

A quadrilateral is a parallelogram, if its both pairs of opposite sides are parallel.



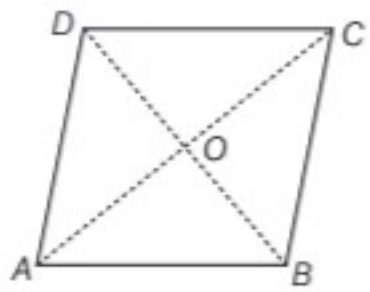
In figure, quadrilateral $ABCD$ is a parallelogram because $AB \parallel DC$ and $AD \parallel BC$.

- Opposite sides are equal, i.e. $AB = CD$ and $AD = BC$.

- Opposite angles are equal i.e. $\angle A = \angle C$ and $\angle B = \angle D$.
- Diagonals bisect each other, i.e. $AO = OC$, $OD = OB$.

(ii) Rhombus

A parallelogram in which all the sides are equal is called a rhombus.

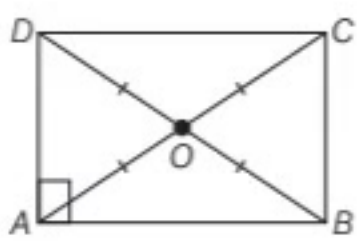


In figure, $ABCD$ is rhombus in which $AB \parallel DC$ and $AD \parallel BC$ and $AB = BC = CD = DA$.

- In rhombus diagonal bisects each other but they are not equal.

(iii) Rectangle

A parallelogram in which each angle is a right angle and opposite sides are equal is called rectangle.

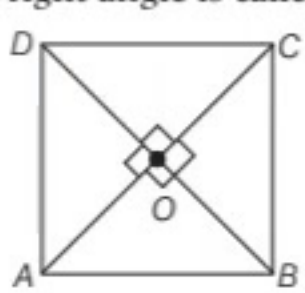


In figure, $ABCD$ is a rectangle in which $AB = DC$ and $AD = BC$, also $\angle A = \angle B = \angle C = \angle D = 90°$.

- In rectangle diagonals bisect each other and they are equal, i.e. $AO = OB = OC = OD$

(iv) Square

A parallelogram having all sides equal and each angle equal to a right angle is called a square.



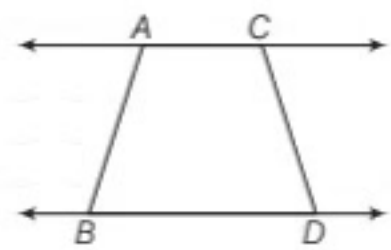
In figure, $ABCD$ is a square in which $AB = BC = CD = DA$ and $\angle A = \angle B = \angle C = \angle D = 90°$.

- In a square diagonals bisect each other at 90°and they are equal, i.e. $AO = OB = OC = OD$.

(v) Trapezium

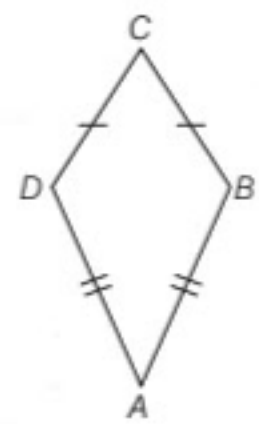
A quadrilateral in which one pair of opposite sides is parallel, is called a trapezium.

In trapezium $ABCD$, sides AC and BD are parallel to each other, but AB and CD neither parallel nor equal.



(vi) Kite

A quadrilateral which has two pairs of equal adjacent sides but unequal opposite sides, is a kite.

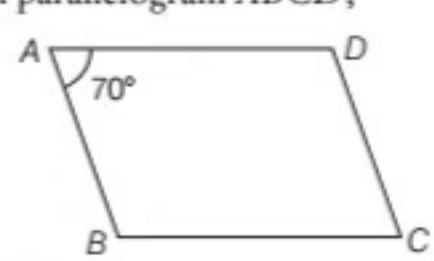


In figure, $ABCD$ is a kite in which $AB = AD$ and $BC = CD$ but $AD \neq BC$ and $AB \neq CD$.

Example 1 $ABCD$ is a parallelogram in which $\angle A = 70°$, the remaining angles of parallelogram are

(a) 110°, 65°, 115° (b) 110°, 70°, 110°
(c) 110°, 50°, 130° (d) None of these

Sol. *(b)* In parallelogram $ABCD$,



$$AD \parallel BC$$

$\therefore \quad \angle A + \angle B = 180°$

$\Rightarrow \quad \angle B = 180° - 70° = 110°$

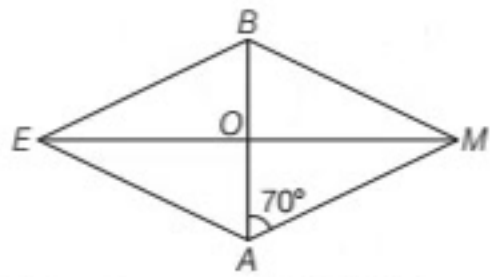
and $\angle C = \angle A$ [opposite angles]

$\therefore$ $\angle C = 70°$

and $\angle D = \angle B = 110°$ [opposite angles]

Hence, the angles B, C, D are $110°, 70°, 110°$ respectively.

Example 2 In rhombus $BEAM$, $\angle AME$ and $\angle AEM$ are



(a) 20°, 20° (b) 30°, 30°
(c) 35°, 35° (d) 40°, 40°

Sol. *(a)* Given, $\angle BAM = 70°$

We know that, in rhombus, diagonals bisect each other at right angles.

$\therefore$ $\angle BOM = \angle BOE = \angle AOM = \angle AOE = 90°$

Now, in ΔAOM,

$\angle AOM + \angle AMO + \angle OAM = 180°$

[angle sum property of triangle]

$\Rightarrow$ $90° + \angle AMO + 70° = 180°$

$\Rightarrow$ $\angle AMO = 180° - 90° - 70°$

$\Rightarrow$ $\angle AMO = 20° = \angle AME$

Also, $AM = BM = BE = EA$

In ΔAME, we have,

$AM = EA$

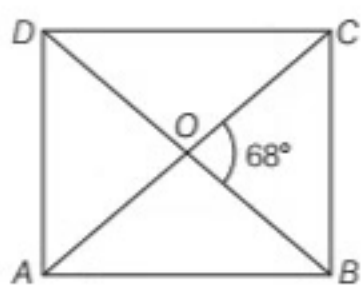
$\therefore$ $\angle AME = \angle AEM = 20°$

[$\because$ equal sides make equal angles]

Example 3 The diagonals of a rectangle $ABCD$ intersect in O, if $\angle BOC = 68°$, find $\angle ODA$.

(a) 55° (b) 56°
(c) 57° (d) 58°

Sol. *(b)* Given, $\angle BOC = 68°$ [given]



Then, $\angle AOD = 68°$ [vertically opposite angles]

$\because$ $OA = OD$

[$\because$ diagonals bisects each other]

$\therefore$ $\angle ODA = \angle OAD$

$\angle ODA + \angle OAD + \angle AOD = 180°$

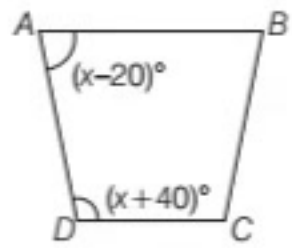
[sum of angles of a ΔAOD]

$2\angle ODA + 68° = 180°$

$\Rightarrow$ $2\angle ODA = 180° - 68° = 112°$

$\Rightarrow$ $\angle ODA = 56°$

Example 4 The value of x in the trapezium $ABCD$ given below is



(a) 80° (b) 82°
(c) 84° (d) 85°

Sol. *(a)* Given, a trapezium $ABCD$ in which

$\angle A = (x - 20)°$, $\angle D = (x + 40)°$

Since, in a trapezium, the angles on either side of the base are supplementary, therefore

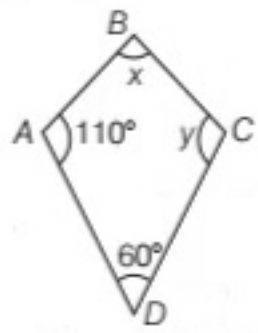
$(x - 20°) + (x + 40°) = 180°$

$\Rightarrow$ $2x + 20° = 180°$

$\Rightarrow$ $2x = (180° - 20°) = 160°$

$\therefore$ $x = 80°$

Example 5 The values of x and y in the following kite, are



(a) 80° (b) 70°
(c) 82° (d) 84°

Sol. *(a)* In a kite, one pair of opposite angles are equal.

$\therefore$ $y = 110°$

Now, by the angle sum property of a quadrilateral, we have

$\angle A + \angle B + \angle C + \angle D = 360°$

$$110° + x + 110° + 60° = 360°$$
$$\Rightarrow \quad x = 360° - 280°$$
$$\therefore \quad x = 80°$$

Polygons

A polygon is a closed plane figure bounded by straight lines.

Types of Polygons

Polygons are as following four types

1. **Convex polygon** A polygon in which none of its interior angles is more than 180°, is called convex polygon.
2. **Concave polygon** A polygon in which atleast one angle is more than 180°, is called concave polygon.
3. **Irregular polygon** A polygon in which all the sides or angles are not of the same measure, is called an irregular polygon.
4. **Regular polygon** A regular polygon has all its sides and angles equal.

   (i) Each exterior angle of a regular polygon
   $$= \frac{360°}{\text{Number of sides}}$$
   (ii) Each interior angle = 180° − Exterior angle

   (iii) Sum of all interior angles $= (2n - 4) \times 90°$
   $$= (n-2) \times 180°$$
   (iv) Sum of all exterior angles = 360°

   (v) Number of diagonals of polygon of n sides
   $$= \frac{n(n-3)}{2}$$

Example 6 Each interior angle of a regular polygon is 144°. Find the interior angle of a regular polygon which has double the number of sides as the first polygon.

(a) 170° (b) 160°
(c) 162° (d) 180°

Sol. *(c)* ∵ Each interior angle of polygon = 144°

$$\Rightarrow \quad \frac{(2n-1)90°}{n} = 144°$$
$$\Rightarrow \quad 180n - 360° = 144n$$
$$\Rightarrow \quad 36n = 360°$$
$$\Rightarrow n = 10°$$

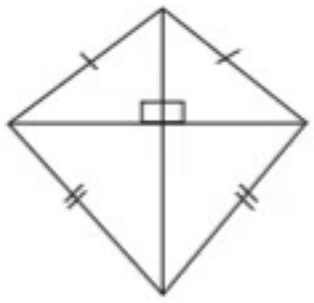
According to the given condition,

Total sides of a new polygon $= 2 \times 10 = 20$

Each interior angle of new polygon
$$= \frac{(2 \times 20 - 4) \times 90°}{20}$$
$$= \frac{36 \times 9}{2} = 162°$$

Kite

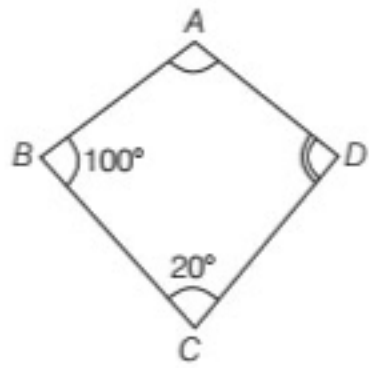
A kite is a quadrilateral whose four sides can be grouped into two pairs of equal- length sides that are adjacent to each other



Properties of Kite

- Its two diagonals are at right angles to each other.
- There is a pair of equal opposite angles.

Example 7 Calculate the measures of the unmarked angles of the kite ABCD.



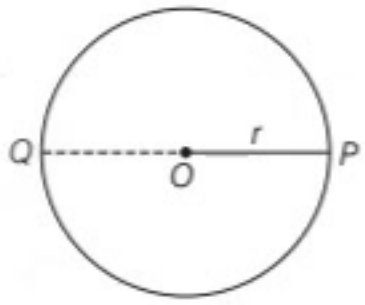
(a) 170° and 120° (b) 100° and 100°
(c) 100° and 140° (d) 110° and 130°

Sol. *(c)* We know that, $\angle ABC = \angle ADC = 100°$

∵ Sum of four angles of kite = 360°
$$\angle ABC + \angle BCD + \angle ADC + \angle BAD = 360°$$
$$100° + 20° + 100° + \angle BAD = 360°$$
$$\angle BAD = 360° - 220°$$
$$\angle BAD = 140°$$

Circle

A circle is a set of those points in a plane, which are at a given constant distance from a given fixed point in the plane.



- The fixed point O is called the centre of the circle.
- The constant distance r is called the radius of the circle.
- A circle can have many radii measure and all the radii of a circle are of equal length.
- The line PQ is a diameter (d) of a circle and

$$d = PQ = 2 \times \text{Radius} = 2r$$

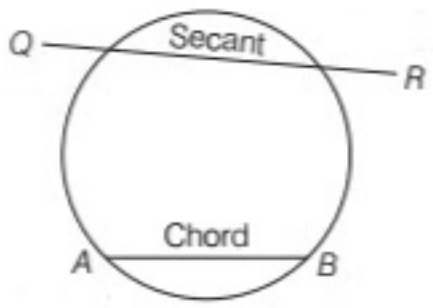
(i) Arc of the Circle

A continuous part of a circle is called an arc of the circle.

(ii) Chord

A line segment joining any two points on the circle is called its chord.

In a figure, AB is a chord.



Note Biggest chord of a circle is its diameters.

(iii) Secant

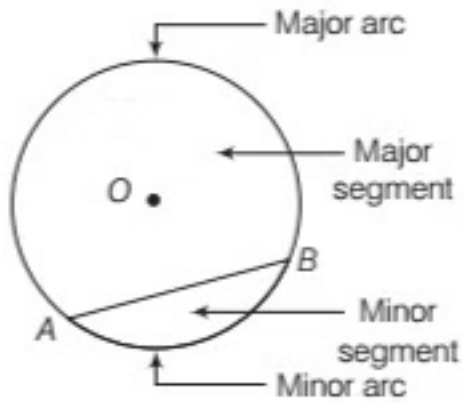
A line which intersect a circle in two distinct points is called a secant of the circle.

In above figure, QR is a secant.

(iv) Semicircle

A diameter divides the circle into two equal arcs, each of these two arcs is called a semicircle.

An arc whose length is less than the arc of a semicircle is called a minor arc, otherwise it is called a major arc.



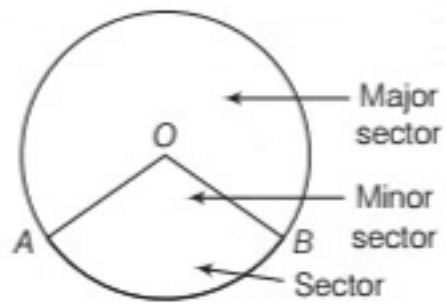
(v) Segment

If AB be a chord of the circle then AB divides the circular region into two parts, each part is called a segment of the circle.

The segment containing the minor arc is called the minor segment and the segment containing the major arc is called the major segment.

(vi) Central Angle

If $C\ (O, r)$ be any circle, then any angle whose vertex is centre of circle is called a central angle.



The degree measure of an arc is the measure of the central angle containing the arc.

(vii) Sector

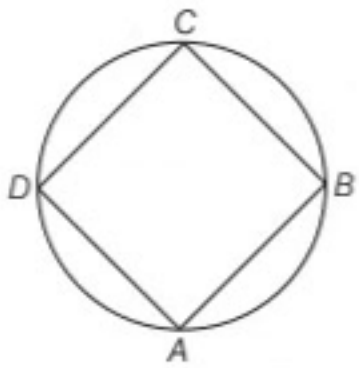
A sector is that region of a circle $C\ (O, r)$ which lies between an arc and the two radii joining the extremities of the arc to the centre.

(viii) Quadrant

One fourth of a circular region is called a quadrant.

(ix) Cyclic Quadrilateral

If all four vertices of a quadrilateral lie on a circle, then such a quadrilateral is called a cyclic quadrilateral.



The sum of opposite angles of a cyclic quadrilateral is 180°, i.e.,

$$\angle A + \angle C = 180°$$

and $$\angle B + \angle D = 180°$$

Some Important Results Related to Circles

(i) An infinite number of circles can pass through the two points.

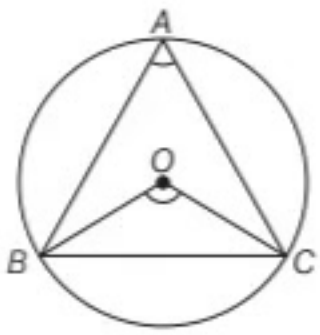
(ii) There is one and only one circle passing through three non-collinear points.

(iii) Angles in the same segment of a circle are equal.

(iv) The perpendicular from the centre to any chord bisects the chord.

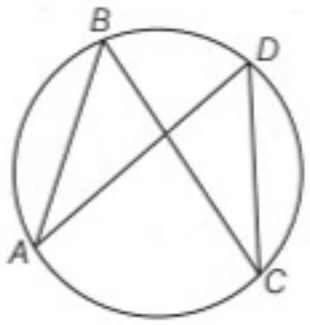
(v) The line joining the centre of the circle to the mid-point of any chord of a circle, is perpendicular to the chord.

(vi) The angle subtended by an arc at the centre of a circle is double the angle subtended by it at any point on the remaining part of the circle.



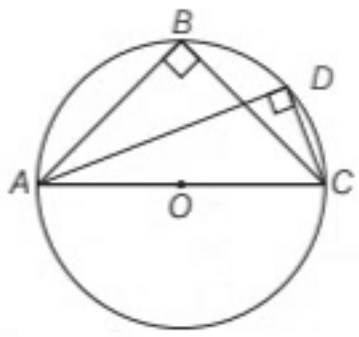
i.e. $$\angle BOC = 2\angle BAC$$

(vi) Angles in the same segment of a circle are equal.



Here, $$\angle ABC = \angle ADC$$

(vii) The angle made in a semicircle is always a right angle.

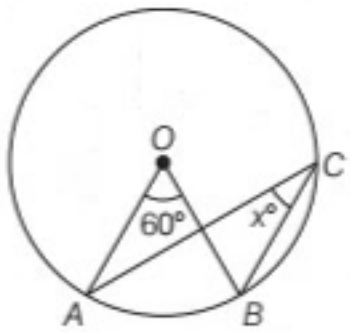


Here, $$\angle ABC = 90°$$

and $$\angle ADC = 90°$$

(viii) If the sum of any pair of opposite angles of a quadrilateral is 180°, then the quadrilateral is cyclic.

Example 6 The value of $x°$ in the figure is



(a) 20° (b) 100°
(c) 60° (d) 30°

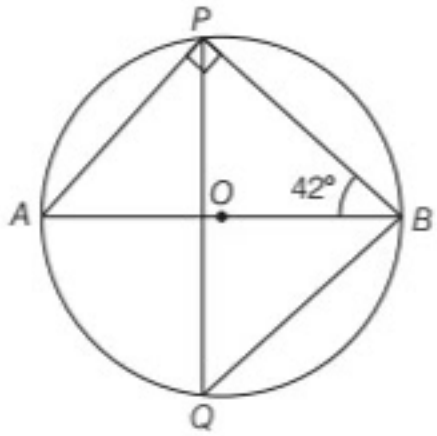
Sol. *(d)* We know that, the angle subtended by an arc at the centre of a circle is double the angle subtended by it any point of the remaining part of the circle.

$\therefore$ $$\angle AOB = 2(\angle ACB)$$

$\Rightarrow$ $$60° = 2\angle ACB$$

$\Rightarrow$ $$\angle ACB = 30°$$

Example 7 In figure, the value $\angle PQB$, where O is the centre of the circle, is



(a) 47° (b) 48° (c) 49° (d) 50°

Sol. *(b)* In ΔAPB,

$\angle APB = 90°$ [angle in a semi-circle]

$\angle PBA = 42°$ [given]

Now, $\angle PAB + \angle APB + \angle PBA = 180°$

$\Rightarrow \quad \angle PAB + 90° + 42° = 180°$

$\Rightarrow \quad \angle PAB = 180° - 132° = 48°$

Since, we know that the angle subtended by an arc in the same segment are equal.

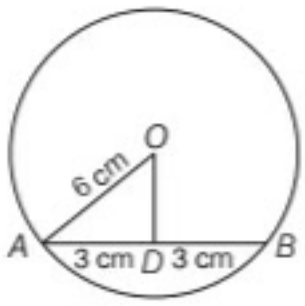
$\therefore \quad \angle PQB = \angle PAB = 48°$

Example 8 The radius of a circle is 6 cm and the length of one of its chords is 6 cm. The distance of the chord from the centre is

(a) $3\sqrt{5}$ cm (b) $3\sqrt{2}$ cm

(c) $3\sqrt{3}$ cm (d) $3\sqrt{6}$ cm

Sol. *(c)* Let AB be a chord of a circle with centre O and radius 6 cm such that $AB = 6$ cm.



From O, draw $OD \perp AB$. Join OA

Clearly,

$$AD = \frac{1}{2} AB = 3 \text{ cm and } OA = 6 \text{ cm.}$$

Now, in right angle ΔODA,

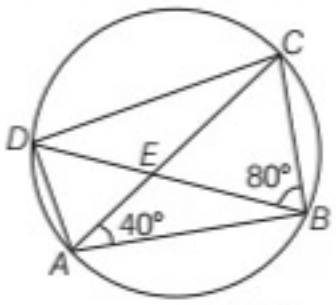
$$OD = \sqrt{OA^2 - AD^2}$$

[using Pythagoras theorem]

$$= \sqrt{6^2 - 3^2} = \sqrt{27} = 3\sqrt{3} \text{ cm}$$

Hence, the distance of the chord from the centre is $3\sqrt{3}$ cm.

Example 9 In figure, if $\angle DBC = 80°$ and $\angle BAC = 40°$, then the value of $\angle BCD$ is



(a) 50° (b) 60°

(c) 70° (d) 80°

Sol. *(b)* Given,

$\angle DBC = 80°$ and $\angle BAC = 40°$

Consider the chord CD, we find that $\angle CBD$ and $\angle CAD$ are angles in the same segment of the circle.

$\therefore \quad \angle CBD = \angle CAD$

$\Rightarrow \quad 80° = \angle CAD$

$\Rightarrow \quad \angle CAD = 80°$

Now, $\angle BAD = \angle BAC + \angle CAD$

$\Rightarrow \quad \angle BAD = 40° + 80° = 120°$ ...(i)

Since, $ABCD$ is a cyclic quadrilateral.

$\therefore \quad \angle BAD + \angle BCD = 180°$

$\Rightarrow \quad 120° + \angle BCD = 180°$ [using Eq. (i)]

$\Rightarrow \quad \angle BCD = 60°$

PRACTICE EXERCISE

1. A quadrilateral has three acute angles each measuring $75°$, the measure of fourth angle is

(a) $145°$ (b) $135°$ (c) $125°$ (d) $130°$

2. The measures of the four angles of a quadrilateral are in the ratio of $1:2:3:4$. What is the measure of fourth angle?

(a) $144°$ (b) $135°$ (c) $125°$ (d) $150°$

3. The diagonals of a rectangle $ABCD$ cut at O. OAL is an equilateral triangle drawn so that B and L are on the same side of AC. If $\angle ACD = 30°$, then the angles of ΔALB are

(a) $60°$, $60°$ and $60°$
(b) $30°$, $30°$ and $120°$
(c) $30°$, $60°$ and $120°$
(d) Cannot be determined

4. The sum of two opposite angles of a parallelogram is $130°$. All the angles of parallelogram are

(a) $65°, 65°, 115°, 115°$ (b) $145°, 135°, 35°, 45°$
(c) $90°, 130°, 80°, 60°$ (d) $40°, 140°, 80°, 110°$

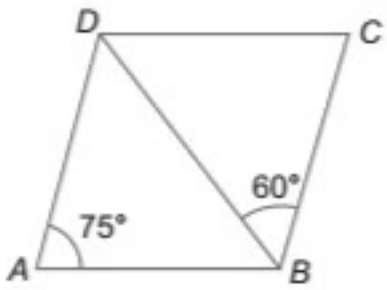
5. The length of the diagonal of a rectangle whose sides are 12 cm and 5 cm, is

(a) 17 cm (b) 13 cm (c) 25 cm (d) 14 cm

6. In a quadrilateral $ABCD$, if AO and BO be the bisectors of $\angle A$ and $\angle B$ respectively, $\angle C = 70°$ and $\angle D = 30°$, then $\angle AOB$ is

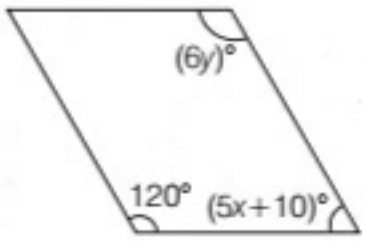
(a) $40°$ (b) $50°$ (c) $80°$ (d) $100°$

7. In the given figure, $ABCD$ is a parallelogram in which $\angle DAB = 75°$ and $\angle DBC = 60°$. Then, $\angle BDC$ is equal to



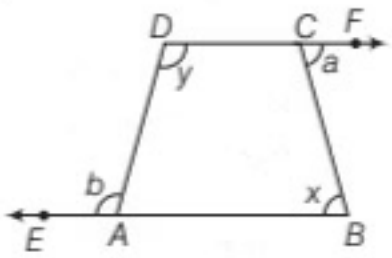
(a) $75°$ (b) $45°$
(c) $60°$ (d) $55°$

8. The values of x and y in the following parallelogram is



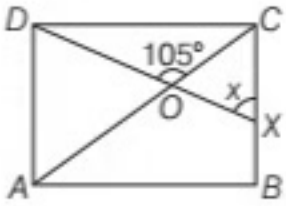
(a) $10°, 20°$ (b) $60°\ 80°$
(c) $90°, 110°$ (d) None of these

9. The sides BA and DC of quadrilateral $ABCD$ are produced as shown in figure. Then, which of the following statement is correct?



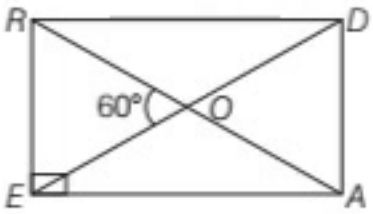
(a) $2x + y = a + b$ (b) $x + \frac{y}{2} = \frac{a+b}{2}$
(c) $x + y = a + b$ (d) $x + a = y + b$

10. In the given figure, $ABCD$ is a square. A line segment DX cuts the side BC at X and the diagonal AC at O such that $\angle COD = 105°$, $\angle OCX = 45°$ and $\angle OXC = x$. The value of x is



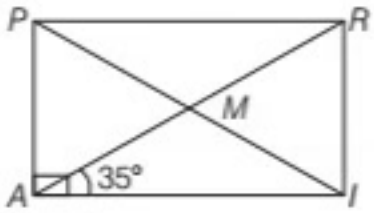
(a) $40°$ (b) $60°$ (c) $80°$ (d) $85°$

11. In rectangle $READ$, the values of $\angle EAR$, $\angle RAD$ and $\angle ROD$ are respectively



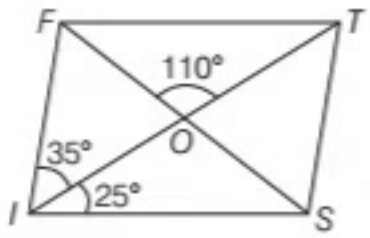
(a) $30°, 60°, 120°$ (b) $40°, 60°, 110°$
(c) $30°, 40°, 110$ (d) None of these

12. In rectangle *PAIR*, the values of $\angle ARI$, $\angle RMI$ and $\angle PMA$ are



(a) 60°, 70°, 70°
(b) 55°, 70°, 70°
(c) 60°, 80°, 80°
(d) None of these

13. In parallelogram *FIST*, the value of $\angle OST$ is

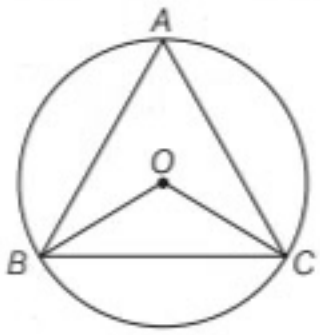


(a) 70° (b) 72°
(c) 75° (d) 80°

14. The external angle of a regular polygon is 45°. Find the sum of all the internal angles of it.
(a) 1082°
(b) 1080°
(c) 1085°
(d) 1090°

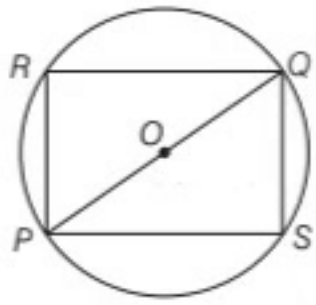
15. Find the number of non overlapping triangles can be formed in 9 sided polygon by joining the vertices.
(a) 5 (b) 7
(c) 6 (d) 4

16. An equilateral ΔABC is inscribed in a circle with centre *O*. Then, $\angle BOC$ is equal to



(a) 120° (b) 75°
(c) 180° (d) 160°

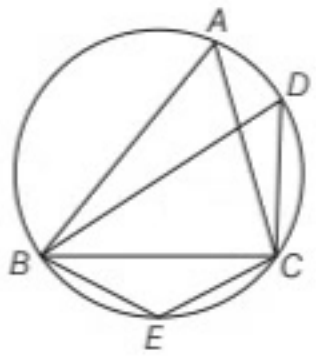
17. In the adjoining figure, *POQ* is the diameter of the circle, *R* and *S* are any two points on the circle. Then,



(a) $\angle PRQ > \angle PSQ$ (b) $\angle PRQ < \angle PSQ$
(c) $\angle PRQ = \angle PSQ$ (d) $\angle PRQ = \frac{1}{2}\angle PSQ$

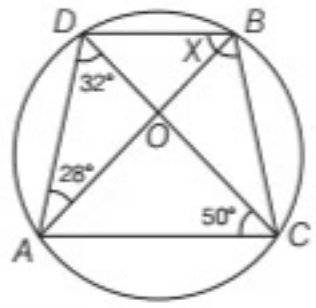
18. In a circle with centre *O* and radius 5 cm, *AB* is a chord of length 8 cm. If $OM \perp AB$, then the length of *OM* is
(a) 4 cm (b) 5 cm
(c) 3 cm (d) 2 cm

19. In the adjoining figure, ΔABC is an isosceles triangle with $AB = AC$ and $\angle ABC = 50°$. Then, $\angle BDC$ is



(a) 110° (b) 90° (c) 80° (d) 70°

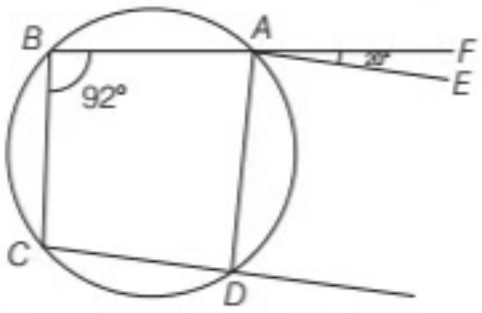
20. If *O* is the centre of the circle, then *x* is



(a) 72° (b) 62° (c) 82° (d) 52°

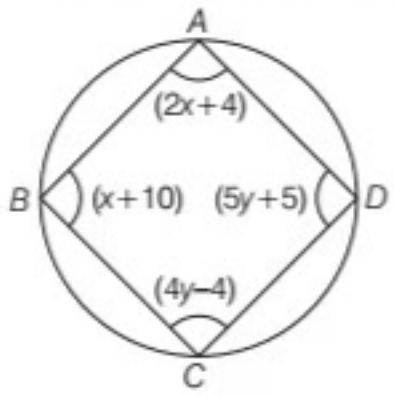
21. In a cyclic quadrilateral *ABCD*, if $\angle B - \angle D = 60°$, then the measure of the smaller of the two is
(a) 60° (b) 40° (c) 38° (d) 30°

22. In the given figure, $ABCD$ is a cyclic quadrilateral. AE is drawn parallel to CD and BA is produced. If $\angle ABC = 92°$ and $\angle FAE = 20°$, then $\angle BCD$ is equal to



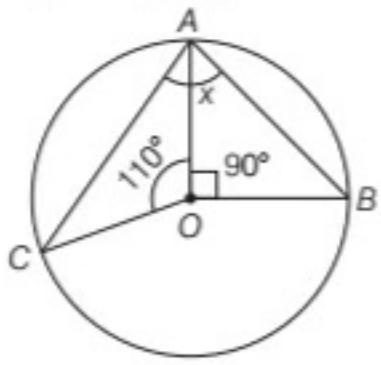
(a) 88° (b) 98° (c) 108° (d) 72°

23. The values of x and y in the figure are measure of angles, then $x + y$ is equal to



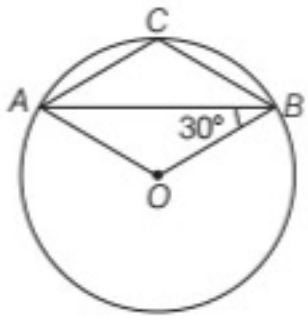
(a) 90° (b) 85° (c) 75° (d) 65°

24. If O is the centre of the circle, the value of x in the adjoining figure, is



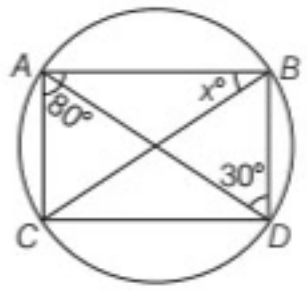
(a) 80° (b) 70° (c) 60° (d) 50°

25. In the given figure, O is centre, then $\angle ACB$ is



(a) 60° (b) 120°
(c) 75° (d) 90°

26. In the following figure, the value of $x°$ is



(a) 60° (b) 90°
(c) 70° (d) 40°

27. In the figure, O is the centre and AOC is the diameter of the circle. BD is chord and OB and CD are joined D is joined to A. If $\angle AOB = 120°$, then the value of x is

(a) 30° (b) 40°
(c) 50° (d) 60°

Answers

1	(b)	2	(a)	3	(b)	4	(a)	5	(b)	6	(d)	7	(b)	8	(a)	9	(c)	10	(b)
11	(a)	12	(b)	13	(c)	14	(b)	15	(b)	16	(a)	17	(c)	18	(c)	19	(c)	20	(c)
21	(a)	22	(c)	23	(d)	24	(a)	25	(b)	26	(c)	27	(a)						

Hints and Solutions

1. Since, $\angle A + \angle B + \angle C + \angle D = 360°$

$\therefore \quad 75° + 75° + 75° + \angle D$

$\Rightarrow \quad 225° + \angle D = 360°$

$\Rightarrow \quad \angle D = 360° - 225° = 135°$

2. Let the angles be x, $2x$, $3x$ and $4x$.

$\because \quad x + 2x + 3x + 4x = 360°$

$\Rightarrow \quad 10x = 360°$

$\Rightarrow \quad x = 36°$

$\therefore$ Fourth angle $= 4 \times 36° = 144°$

3. $\because OA = OB$

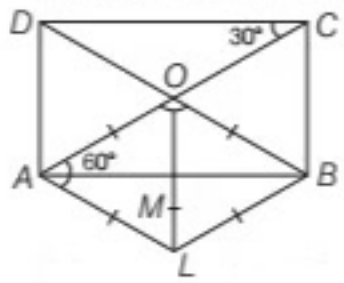
[$\because$ the diagonals of a rectangle bisect each other]

Also, $OA = OL \Rightarrow AOBL$ is a rhombus.

Since, $\Delta\, AOL$ is an equilateral triangle.

$\therefore \quad \angle AOL = 60°$

$\Rightarrow \quad \angle AOB = 2\angle AOL = 120°$



Since, $CD \parallel AB$

$\Rightarrow \quad \angle DCA = \angle CAB = 30°$

and $\quad \angle OAL = 60°$

$\Rightarrow \quad \angle BAL = 60° - 30° = 30° = \angle ABL$

$\therefore$ In $\Delta\, ALB$,

$\angle ALB = 120°$, $\angle ABL = 30°$, $\angle LAB = 30°$

4. Let $\quad \angle A + \angle C = 130°$, then

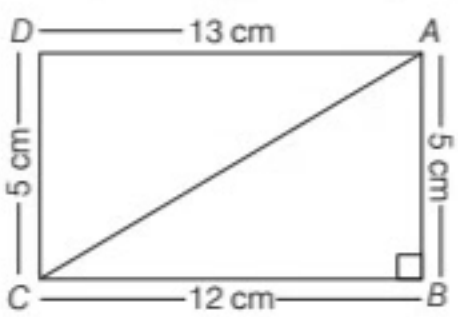
$\angle B + \angle D = 360° - 130°$

$= 230°$

$\therefore$ Angles $= \dfrac{130°}{2}, \dfrac{230°}{2} = 65°, 115°$

Hence, all angles are 65°, 65°, 115°, 115°.

5.



Since, here $\angle ABC = 90°$

So, by pythagoras theorem,

diagonal $AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{5^2 + 12^2}$

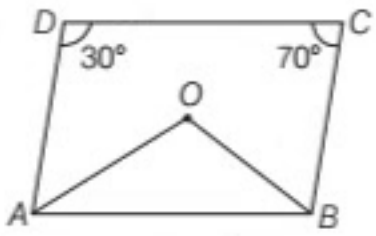
$= \sqrt{25 + 44}$

$= \sqrt{169} = 13$ cm

6. Since, $\quad \angle A + \angle B + \angle C + \angle D = 360°$

$\therefore \quad \angle A + \angle B = 360° - (130° + 70°)$

$= 360° - 200° = 160°$



$\Rightarrow \quad \dfrac{1}{2}(\angle A + \angle B) = 80°$

So, $\quad \angle OAB + \angle ABO = 80°$

$\therefore \quad \angle AOB = 180° - (\angle OAB + \angle ABO)$

$= (180° - 80°) = 100°$

7. $\angle C = \angle A = 75°$

[opposite angles of a parallelogram are equal]

$\therefore \quad \angle BDC = 180° - (60° + 75°)$

$= 180° - 135° = 45°$

8. In a parallelogram, adjacent angles are supplementary.

$\therefore \quad 120° + (5x + 10)° = 180°$

$\Rightarrow \quad 5x + 10° + 120° = 180°$

$\Rightarrow \quad 5x = 180° - 130°$

$\Rightarrow \quad 5x = 50°$

$\Rightarrow \quad x = 10°$

Also, opposite angles are equal in a parallelogram.

Therefore, $\quad 6y = 120° \Rightarrow y = 20°$

9. Since, $\quad \angle A + b = 180° \Rightarrow \angle A = 180° - b$

Also, $\quad \angle C + a = 180°$ [linear pair]

$\Rightarrow \quad \angle C = 180° - a$

But $\quad \angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow \quad (180° - b) + x + (180° - a) + y = 360°$

$\therefore \quad x + y = a + b$

10. Given, $\angle COD = 105°$ and $\angle OCX = 45°$

$$\angle COD + \angle COX = 180°$$

$$\Rightarrow \quad \angle COX = 180° - \angle COD$$

$$= 180° - 105° = 75°$$

In ΔOCX, $\angle OCX + \angle COX + \angle OXC = 180°$

$$\Rightarrow \quad 45° + 75° + x = 180°$$

$$\therefore \quad x = 180° - 120° = 60°$$

11. Given, a rectangle *READ*, in which

$$\angle ROE = 60°$$

$\therefore \quad \angle EOA = 180° - 60° = 120°$ [linear pair]

Now, in ΔEOA, $\angle OEA = \angle OAE = 30°$

[$\because$ $OE = OA$ and equal sides make equal angles]

$\therefore \quad \angle EAR = 30°$, $\angle RAD = 90° - \angle EAR = 60°$

and $\angle ROD = \angle EOA = 120°$

12. Given, $\angle RAI = 35°$

$\therefore \quad \angle PRA = 35°$

[$PR \parallel AI$ and AR is transversal]

Now, $\angle ARI = 90° - \angle PRA = 90° - 35° = 55°$

$\because \quad AM = IM$, $\angle MIA = \angle MAI = 35°$

In ΔAMI, $\angle RMI = \angle MAI + \angle MIA = 70°$

[exterior angle]

Also, $\angle RMI = \angle PMA$

$\Rightarrow \quad \angle PMA = 70°$ [vertically opposite angles]

13. Given, $\angle FIS = 60°$

Now, $\angle FTS = \angle FIS = 60°$

[$\because$ opposite angles of a parallelogram are equal]

Now, $FT \parallel IS$ and TI is a transversal, therefore

$\angle FTO = \angle SIO = 25°$ [alternate angles]

$\therefore \quad \angle STO = \angle FTS - \angle FTO = 60° - 25° = 35°$

Also, $\angle FOT + \angle SOT = 180°$ [linear pair]

$\Rightarrow \quad 110° + \angle SOT = 180°$

$\Rightarrow \quad \angle SOT = 180° - 110° = 70°$

In ΔTOS, $\angle TSO + \angle OTS + \angle TOS = 180°$

[angle sum property of triangle]

$\therefore \quad \angle OST = 180° - (70° + 35°) = 75°$

14. External angle of any polygen

$$\frac{360°}{n} = 45° \Rightarrow n = \frac{360°}{45°} \Rightarrow n = 8°$$

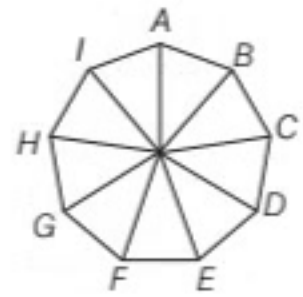
$\therefore$ Every interior angle of regular polygon

$= 180° -$ External angle

$= 180° - 45° = 135°$

$\therefore$ Sum of interior angles of it $= 8 \times 135° = 1080°$

15. Hence total number of non overlapping triangles can be formed in 9 sided polygon is 7.



16. We know that, the angle subtended by an arc at the centre of a circle is double the angle subtended by it any point on the remaning part of the circle.

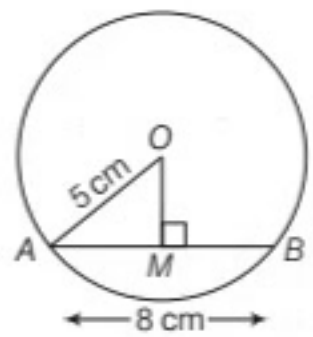
$$\angle BOC = 2\angle A$$

$$= 2 \times 60°$$

$$\therefore \quad \angle BOC = 120°$$

17. $\angle PRQ = \angle PSQ = 90°$ [each angle in semi-circle]

18. $\because OA = 5$ cm



and $\quad AM = \frac{1}{2}AB = \frac{1}{2} \times 8 = 4$ cm

$$\therefore \quad OM = \sqrt{OA^2 - AM^2}$$

$$= \sqrt{5^2 - 4^2} = 3 \text{ cm}$$

19. Since, $\quad AB = AC$

$$\Rightarrow \quad \angle ACB = \angle ABC = 50°$$

In ΔABC,

$$\angle BAC = 180° - (50° + 50°)$$

$$= 80°$$

$$\therefore \quad \angle BDC = \angle BAC = 80°$$

[angle in the same segment]

20. In ΔDAC, $\angle ADC + \angle DCA + \angle CAD = 180°$

$$\Rightarrow \quad \angle CAD = 180° - 32° - 50° = 98°$$

Now, $\angle CAD + \angle CBD = 180°$

[opposite angles of a quadrilateral]

$$\therefore \quad x = 180° - 98° = 82°$$

21. Since, $\angle B + \angle D = 180°$

[sum of opposite angles of a cyclic quadrilateral]

$\angle B - \angle D = 60°$ [given]

$\Rightarrow$ $\angle B = 120°$ and $\angle D = 60°$

$\therefore$ Required smaller angle $\angle D = 60°$

22. $\angle B + \angle D = 180°$

$\Rightarrow$ $\angle D = 180° - 92° = 88°$

Now, $\angle DAE = \angle D = 88°$ [$\because AE \parallel CD$]

$\Rightarrow$ $\angle FAD = 88° + 20° = 108°$

$\therefore$ $\angle BCD = \angle FAD = 108°$

23. Since, $ABCD$ is a cyclic quadrilateral.

$\angle B + \angle D = 180°$

and $\angle A + \angle C = 180°$

$\Rightarrow$ $x + 10 + 5y + 5 = 180°$

$x + 5y = 165°$ ...(i)

and $2x + 4 + 4y - 4 = 180°$

$\Rightarrow$ $2x + 4y = 180°$ ...(ii)

Solving Eqs.(i) and (ii), we get

$x = 40°$ and $y = 25°$

$\therefore$ $x + y = 40° + 25° = 65°$

24. $\angle COB = 360° - (\angle COA + \angle BOA)$

$= 360° - (110° + 90°)$

$= 160°$

$\therefore$ $x = \frac{1}{2} \times \angle COB$ [by theorem]

$= \frac{1}{2} \times 160° = 80°$

25. In the given figure, $OA = OB$ (radius of circle)

$\Rightarrow$ $\angle OAB = \angle OBA = 30°$

$\therefore$ $\angle AOB = 180° - 60° = 120°$

$\therefore$ Major $\angle AOB = 360° - 120° = 240°$

$\Rightarrow$ $\angle ACB = \frac{1}{2} \times 240° = 120°$

[angle subtended in the arc is half of that subtended at the centre.]

26. In a given figure,

$\angle ADB = \angle ACB = 30°$

[angle subtended in the same segment]

In ΔABC, $\angle x° = 180° - (\angle ACB + \angle CAB)$

$= 180° - (30° + 80°) = 70°$

27. In a given figure,

$\angle COB = 180° - 120° = 60°$ [$\because COA$ is a line]

$\therefore$ $x = \frac{1}{2} \angle COB$

[angle subtended in the arc is half of that subtended at the centre.]

$= \frac{1}{2} \times 60° = 30°$

CHAPTER

12

AREA OF PLANE FIGURES

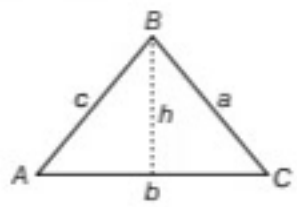
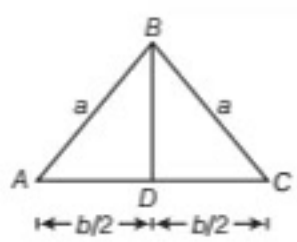
In this chapter, we study the Area of Plane Figures, triangle and its type Square, circle, semi-circle, Trapezium, Hexagon etc.

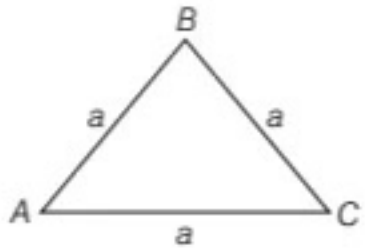
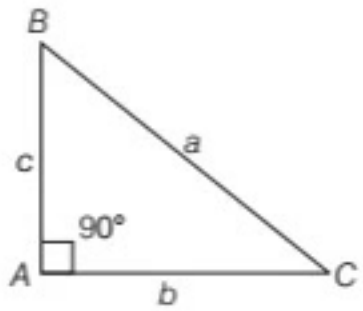
Plane figures are the flat shape in two dimensions, having length and width (breadth).

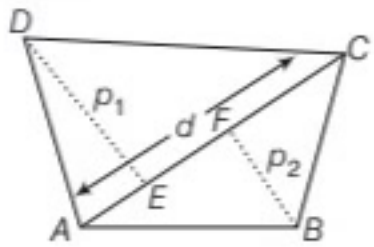
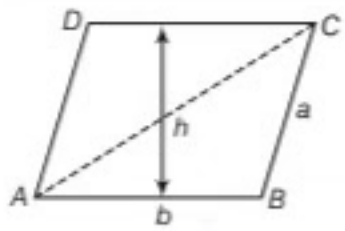
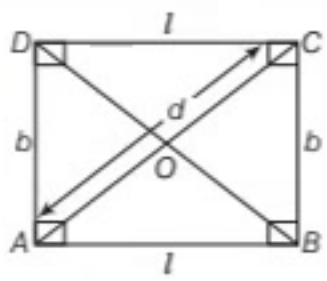
Perimeter The length of boundary of a simple closed figure is known as perimeter.

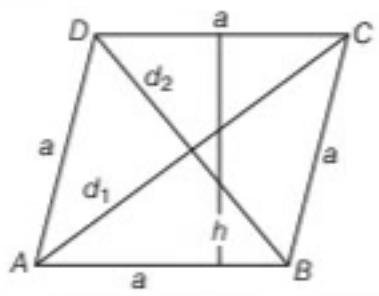
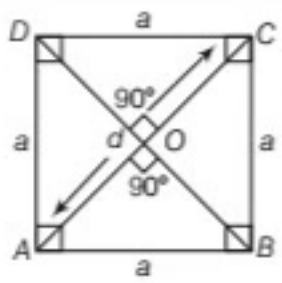
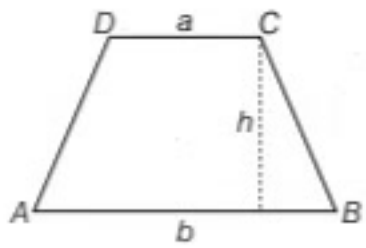
Area The measure of region enclosed in a simple closed curve is called area of closed curve.

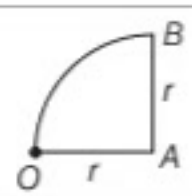
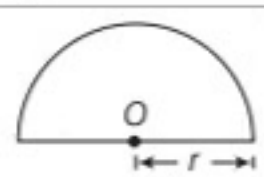
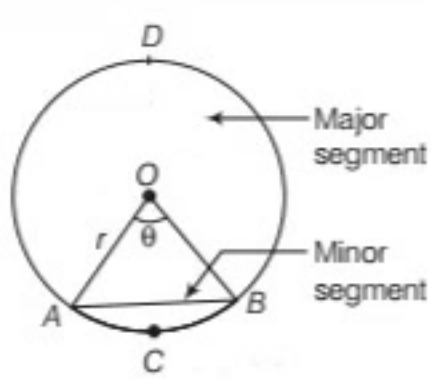
Perimeter and Area of Plane Figures

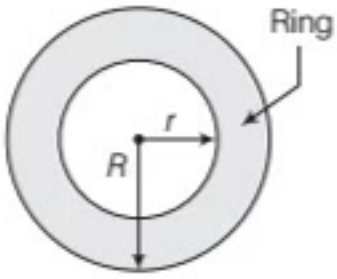
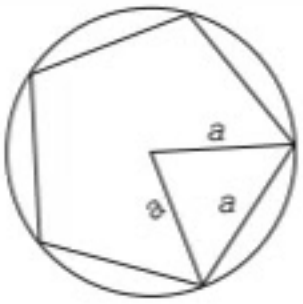
	Type	Figure	Perimeter (P)	Area (A)
1	Triangle		$P = a + b + c$ $= 2s$	$A = \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} bh$ $= \sqrt{s(s-a)(s-b)(s-c)}$
2	Isosceles Triangle		$P = 2a + b$	$A = \frac{b}{4}\sqrt{4a^2 - b^2}$

	Type	Figure	Perimeter (P)	Area (A)
3	Equilateral Triangle		$P = 3a$	$A = \frac{\sqrt{3}}{4}a^2$
4	Right Angled Triangle		$P = a + b + c = 2s$	$A = \frac{1}{2}bc$

	Type	Figure	Diagonal	Perimeter (P)	Area (A)
5	Quadrilateral	 Diagonal $AC = d$; DE, BF are two perpendiculars drawn on the diagonal (AC) and p_1, p_2 are lengths of the two perpendiculars	$BD \neq AC$	$P = AB + BC + CD + DA$	$A = \frac{1}{2} \times d \times (p_1 + p_2)$
6	Parallelogram	 b = base, h = perpendicular distance between the base and its opposite side	$BD \neq AC$	$P = 2(a + b)$	$A = b \times h = 2\times$ (area of $\Delta\, ABD$) (or $\Delta\, BCD$)
7	Rectangle	 l = length, b = breadth d = diagonal	$BD = AC$ Also, $AO = OC$ $= OD = OB$ and $d^2 = l^2 + b^2$	$P = 2(l + b)$	$A = l \times b$ $= l \times \sqrt{d^2 - l^2}$ $= b \times \sqrt{d^2 - b^2}$

	Type	Figure	Diagonal	Perimeter (P)	Area (A)
8	Rhombus		$BD \neq AC$ and $d_1^2 + d_2^2 = 4a^2$	$P = 4a = 2\sqrt{d_1^2 + d_2^2}$	$A = \frac{1}{2} \times d_1 \; x d_2$ $= a \times h$
9	Square	 a = length of side d = diagonal	$BD = AC$ and $OA = OB$ $= OC = OD$ and $d = a\sqrt{2}$	$P = 4a = 2d\sqrt{2}$	$A = a^2 = \frac{d^2}{2}$
10	Trapezium			P = sum of all sides	$A = \frac{1}{2}(a + b) \times h$

	Type	Figure	Perimeter (or Circumference)	Area
11	Quadrant		$P = 2r + \frac{\pi r}{2}$	$A = \frac{1}{4}\pi r^2$
12	Semi-circle		$P = (\pi r + 2r)$	$A = \frac{1}{2}\pi r^2$
13	Circle		Circumference of circle, $P = 2\pi r = \pi d$ Length of arc $AB = 2\pi r \times \frac{\theta}{360°}$	$A = \pi r^2$

	Type	Figure	Perimeter (or Circumference)	Area
14	Ring		$P = 2\pi R + 2\pi r$	$A = \pi R^2 - \pi r^2$ $= \pi(R^2 - r^2)$
15	Hexagon inscribed in a circle		$P = 6(a)$	$A = 6 \times \frac{\sqrt{3}a^2}{4}$

Conversion of Units

$100 \text{ mm}^2 = 1 \text{ cm}^2$ $\quad$ $100 \text{ cm}^2 = 1 \text{ dm}^2$

$100 \text{ dm}^2 = 1 \text{ m}^2$ $\quad$ $10000 \text{ cm}^2 = 1 \text{ m}^2$

$1 \text{ acre} = 100 \text{ m}^2$ $\quad$ $1 \text{ hectare} = 10000 \text{ m}^2$

$1 \text{ hectare} = 100 \text{ acres}$ $\quad$ $100 \text{ hectare} = 1 \text{ km}^2$

Example 1 The base of a triangular field is three times its altitude. If the cost of cultivating the field at ₹ 240 per hectare ₹ 3240, find its base and height.

(a) 900 m, 300 m (b) 910 m, 310 m
(c) 905 m, 315 m (d) None of these

Sol. (a) Area of the field $= \frac{\text{Total cost}}{\text{Rate}}$

$= \frac{3240}{240}$ hectare

$= 13.5 \times 10000 \text{ m}^2$

$= 135000 \text{ m}^2$

Let the height be x metre.

Then, base = $3x$ metre

$\therefore \quad \frac{1}{2} \times x \times 3x = 135000$

$\Rightarrow \quad x^2 = 90000$

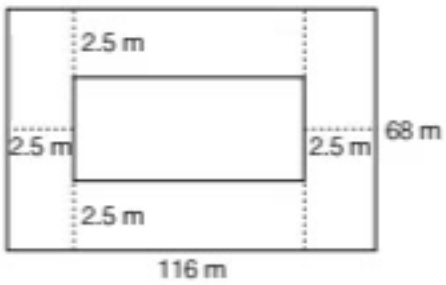
$\Rightarrow \quad x = 300 \text{ m}$

Hence, base is 900 and height is 300 m.

Example 2 A rectangular grassy plot is 116 m by 68 m. It has a gravel path 2.5 m wide all round it on the inside. Find the area of the path.

(a) 895 m^2 (b) 900 m^2
(c) 910 m^2 (d) 890 m^2

Sol. (*a*) Area of the plot $= (116 \times 68) = 7888 \text{ m}^2$



Area of the plot excluding the path

$= (116 - 5) \times (68 - 5)$

$= 111 \times 63 = 6993 \text{ m}^2$

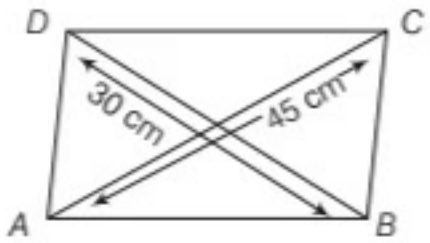
$\therefore$ Required area of path $= 7888 - 6993 = 895 \text{ m}^2$

Example 3 The floor of a building consists of 3000 tiles, which are rhombus shaped and each of its diagonals are 45 cm and 30 cm in length. Find the total cost of polishing the floor, if the cost per sq metre is ₹ 4.

(a) ₹ 800 (b) ₹ 810 (c) ₹ 820 (d) ₹ 850

Sol. (b) Area of one rhombus shaped tile $= \frac{1}{2} \times d_1 \times d_2$

$= \frac{1}{2} \times 45 \times 30 = 45 \times 15 = 675 \text{ cm}^2$



Now, floor of a building consists of 3000 tiles of such kind.

$\therefore$ Area of floor = Number of tiles × Area of one tile

$$= 3000 \times 675$$

$$= 2025000\,\text{cm}^2 = \frac{2025000}{10000}\,\text{m}^2$$

$$= 2025\,\text{m}^2 \quad \left[\begin{array}{l}\because 1\text{ m} = 100\text{ cm} \\ \therefore\ 1\text{m}^2 = 10000\text{ cm}^2\end{array}\right]$$

$\because$ Rate of polishing the floor = ₹4 per m^2

$\therefore$ Total cost of polishing the floor = ₹ 4×202.5

= ₹810

Hence, the total cost of polishing the floor is ₹810.

Example 4 The area of trapezium is $450\,\text{m}^2$ and the distance between two parallel sides is 10 m and one of the parallel side 15 m. Find the other parallel side.

(a) 75 m (b) 80 m
(c) 85 m (d) 90 m

Sol. (*a*) Given, one of the parallel sides of the trapezium, a=15 m and height (h) = 10 m

Let another side be b m.

Then, area of trapezium = 450 m^2

$\therefore$ Area of trapezium $= \frac{1}{2}h(a+b)$

$$\Rightarrow \quad 450 = \frac{1}{2} \times 10 \times (15+b)$$

$$\Rightarrow \quad \frac{450 \times 2}{10} = 15 + b$$

$$\Rightarrow \quad 90 = 15 + b$$

$$\therefore \quad b = 90 - 15 = 75$$

Hence, the other parallel side of trapezium is 75m.

Example 5 If the area of a semi-circular field is $30800\,\text{m}^2$, then find the perimeter of the field.

(a) 705 m (b) 710 m
(c) 700 m (d) 720 m

Sol. (*d*) Let the radius of the field be r.

Then, $\frac{1}{2}\pi r^2 = 30800$

$$\Rightarrow \quad \frac{1}{2} \times \frac{22}{7} \times r^2 = 30800$$

$$\Rightarrow \quad r^2 = 30800 \times 2 \times \frac{7}{22} = 19600$$

$$\Rightarrow \quad r = 140\text{ m}$$

$\therefore$ The perimeter of the field $= \pi r + 2r$

$$= \frac{22}{7} \times 140 + 2 \times 140$$

$$= 440 + 280 = 720\text{m}$$

Example 6. In a circle of radius 35 cm, an arc subtends an angle of 72° at the centre. Find the length of the arc and the area of the sector.

(a) 660 cm^2 (b) 700 cm^2 (c) 770 cm^2 (d) 750 cm^2

Sol. (c) Length of the arc $= \frac{2\pi r\theta}{360°}$

$$= 2 \times \frac{22}{7} \times 35 \times \frac{72}{360} = 44\text{ cm}$$

Area of the sector $= \frac{\pi r^2\theta}{360°}$

$$= \frac{22}{7} \times 35 \times 35 \times \frac{72}{360} = 770\text{ cm}^2$$

Example 7. A regular hexagon is inscribed in a circle of radius 5 cm. The area of regular hexagon inscribed in a circle is

(a) $\frac{75}{2}\text{ cm}^2$ (b) $75\sqrt{3}\text{ cm}^2$
(c) $\frac{75\sqrt{3}}{2}\text{ cm}^2$ (d) None of these

Sol. (c) Area of hexagon inscribed in a circle

$$6 \times \frac{\sqrt{3}}{4}(a)^2 = 6\frac{\sqrt{3}}{4} \times (5)^2 = \frac{75}{2}\sqrt{3}\text{ cm}^2$$

PRACTICE EXERCISE

1. The area of an equilateral triangle with side 10 cm is

(a) $15\sqrt{3}$ cm^2 (b) $25\sqrt{3}$ cm^2
(c) $5\sqrt{3}$ cm^2 (d) $35\sqrt{3}$ cm^2

2. An isosceles right angled triangle has area 200 cm^2. The length of its hypotenuse is

(a) $15\sqrt{2}$ cm (b) $\frac{10}{\sqrt{2}}$ cm
(c) $10\sqrt{2}$ cm (d) $20\sqrt{2}$ cm

3. The diagonal of a square field measures 50 m. The area of square field is

(a) 1250 m^2 (b) 1200 m^2
(c) 1205 m^2 (d) 1025 m^2

4. The sum of the length of two diagonals of a square is 144 cm, then the perimeter of square is

(a) 144 cm (b) $72\sqrt{2}$ cm
(c) $144\sqrt{2}$ cm (d) None of these

5. The circumference of a circle is 176 m. Then, its area is

(a) 2464 m^2 (b) 2164 m^2
(c) 2346 m^2 (d) 2246 m^2

6. The area of a rhombus whose one side and one diagonal measure 20 cm and 24 cm respectively, is

(a) 364 cm^2 (b) 374 cm^2
(c) 384 cm^2 (d) 394 cm^2

7. The area of the largest circle that can be drawn inside a square of side 14 cm in length, is

(a) 84 cm^2 (b) 96 cm^2
(c) 104 cm^2 (d) 154 cm^2

8. The least number of square slabs that can be fitted in a room 10.5 m long and 3 m wide, is

(a) 12 (b) 13
(c) 14 (d) 15

9. The length of a rectangle is 2 cm more than its breadth and the perimeter is 48 cm. The area of the rectangle (in cm^2) is

(a) 96 (b) 28
(c) 143 (d) 144

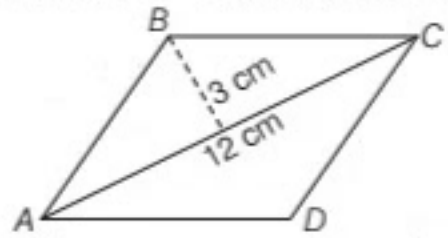
10. In a circle of radius 42 cm, an arc subtends an angle of 72° at the centre. The length of the arc is

(a) 52.8 cm (b) 53.8 cm
(c) 72.8 cm (d) 79.8 cm

11. If the side of a square be increased by 50%, then the per cent increase in area is

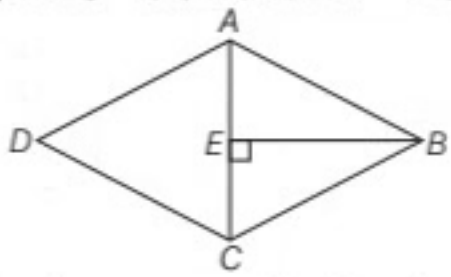
(a) 50 (b) 100 (c) 125 (d) 150

12. The figure $ABCD$ is a quadrilateral, in which $AB = CD$ and $BC = AD$. Its area is



(a) 72 cm^2 (b) 36 cm^2
(c) 24 cm^2 (d) 18 cm^2

13. What is the area of the rhombus $ABCD$ below, if $AC = 6$ cm and $BE = 4$ cm?



(a) 36 cm^2 (b) 16 cm^2
(c) 24 cm^2 (d) 13 cm^2

14. The area of a parallelogram is 60 cm^2 and one of its altitude is 5 cm. The length of its corresponding side is

(a) 12 cm (b) 6 cm (c) 4 cm (d) 2 cm

15. If the ratio of the areas of two square is 4 : 1, then the ratio of their perimeter is

(a) 2 : 1 (b) 1 : 2
(c) 1 : 4 (d) 4 : 1

16. The inner circumference of a circular park is 440 m. The track is 14 m wide. The diameter of the outer circle of the track is

(a) 168 m (b) 169 m (c) 144 m (d) 108 m

17. A wire is in the form of a circle of radius 42 cm. It is bent into a square. The side of the square is

(a) 33 cm (b) 66 cm
(c) 78 cm (d) 112 cm

18. The perimeter of a trapezium is 52 cm and its each non-parallel side is equal to 10 cm with its height 8 cm. It area is

(a) 124 cm^2 (b) 118 cm^2
(c) 128 cm^2 (d) 112 cm^2

19. The areas of two circles are in the ratio 49 : 64. Find the ratio of their circumferences.

(a) 7 : 8 (b) 5 : 8 (c) 5 : 3 (d) 5 : 9

20. The diameter of the wheels of a bus is 140 cm. How many revolutions per minute must a wheel make in order to move at a speed of 66 km/h ?

(a) 200 (b) 250 (c) 300 (d) 350

21. The length of the sides of a triangle are in the ratio 3 : 4 : 5 and its perimeter is 144 cm. The area of the triangle is

(a) 684 cm^2 (b) 664 cm^2
(c) 764 cm^2 (d) 864 cm^2

22. The difference between the sides at right angles in a right angled triangle is 14 cm. The area of the triangle is 120 cm^2. The perimeter of the triangle is

(a) 68 cm (b) 64 cm
(c) 60 cm (d) 58 cm

23. The area of the quadrilateral whose sides measures 9 cm, 40 cm, 28 cm and 15 cm and in which the angle between the first two sides is a right angle, is

(a) 206 cm^2 (b) 306 cm^2
(c) 356 cm^2 (d) 380 cm^2

24. If three sides of a triangle are 6 cm, 8 cm and 10 cm, then the altitude of the triangle using the largest side as base will be

(a) 8 cm (b) 6 cm
(c) 4.8 cm (d) 4.4 cm

25. The area of a circle is 13.86 hectares. The cost of fencing it at the rate of 60 paise per metre is

(a) ₹ 784.00 (b) ₹ 788.00
(c) ₹ 792.00 (d) ₹ 796.00

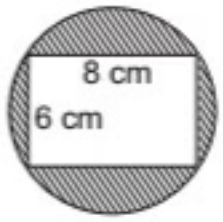
26. If the diagonal of a rectangle is 13 cm and its perimeter is 34 cm, then its area will be

(a) 442 cm^2 (b) 260 cm^2
(c) 60 cm^2 (d) 20 cm^2

27. The cross-section of a canal is in the shape of a trapezium. The canal is 15 m wide at the top and 9 m wide at the bottom. The area of cross-section is 720 m^2, the depth of the canal is

(a) 58.4 m (b) 58.6 m
(c) 58.8 m (d) 60 m

28. In the adjacent figure, find the area of the shaded region.

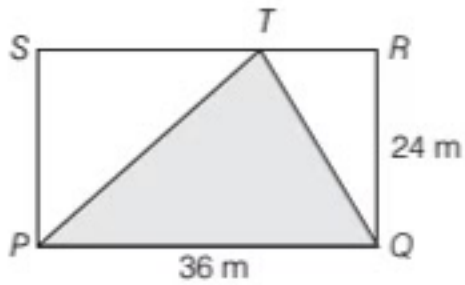


(a) 15.28 cm^2
(b) 61.14 cm^2
(c) 30.57 cm^2
(d) 40.76 cm^2

29. A square and an equilateral triangle have equal perimeters. If the area of the equilateral triangle is $16\sqrt{3}$ cm^2, then the side of the square is

(a) 4 cm (b) $4\sqrt{2}$ cm
(c) $6\sqrt{2}$ cm (d) 6 cm

30. find the figure area of the shaded portion. In the following figure,



(a) 433m^2 (b) 432 m^2
(c) 434 m^2 (d) None of these

31. A regular hexagon is inscribed in a circle of radius 8 cm. The perimeter of the regular hexagon is

(a) 48 cm (b) 50 cm
(c) 52 cm (d) 54 cm

32. Find the area of regular hexagon inscribed in a circle of radius 10 cm.

(a) $140\sqrt{3}$ cm^2 (b) $150\sqrt{3}$ cm^2
(c) $120\sqrt{3}$ cm^2 (d) None of these

Answers

1	(b)	2	(d)	3	(a)	4	(c)	5	(a)	6	(c)	7	(d)	8	(c)	9	(c)	10	(a)
11	(c)	12	(b)	13	(c)	14	(a)	15	(a)	16	(a)	17	(b)	18	(c)	19	(a)	20	(b)
21	(d)	22	(c)	23	(b)	24	(c)	25	(c)	26	(c)	27	(d)	28	(c)	29	(d)	30	(b)

Hints and Solutions

1. Area of equilateral triangle $= \frac{\sqrt{3}}{4}(\text{side})^2$

$$= \frac{\sqrt{3}}{4} \times 10 \times 10 = 25\sqrt{3} \text{ cm}^2$$

2. Area of an isosceles right angled triangle

$$= \frac{1}{2}(\text{side})^2 = 200 \text{ cm}^2$$

$\therefore$ side = 20 cm

$\therefore$ Hypotenuse $= \sqrt{a^2 + a^2} = \sqrt{2}a = 20\sqrt{2}$ cm

3. Area of square $= \frac{1}{2} \times (\text{diagonal})^2$

$$= \frac{1}{2} \times 50 \times 50 = 1250 \text{ m}^2$$

4. Length of a diagonal of square $= \frac{144}{2} = 72$ cm

Side of square $= \frac{\text{length of diagonal}}{\sqrt{2}} = \frac{72}{\sqrt{2}}$ cm

$\therefore$ The perimeter of square $= 4a = 4 \times \frac{72}{\sqrt{2}}$

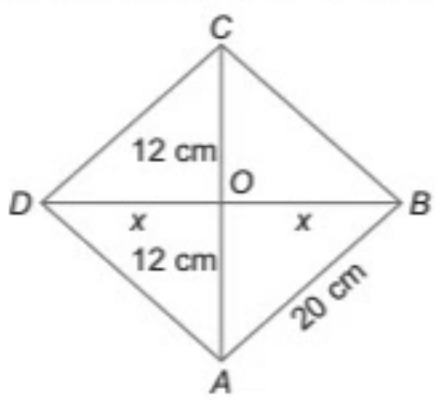
$$= 144\sqrt{2} \text{ cm}$$

5. Circumference of circle $= 2\pi r = 176$ m

$\Rightarrow$ $r = \frac{176 \times 7}{2 \times 22} = 28$ m

$\therefore$ Area $= \pi r^2 = \frac{22}{7} \times 28 \times 28 = 2464 \text{ m}^2$

6. Let the other diagonal be $2x$. In ΔAOB,



$$(20)^2 = (12)^2 + x^2$$

$\Rightarrow$ $x^2 = 256$

$\Rightarrow$ $x = 16$ cm

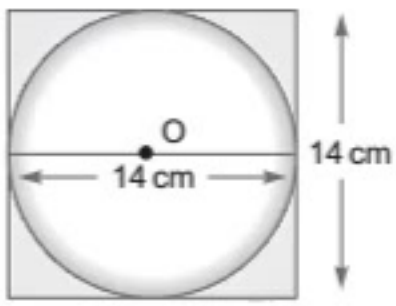
$\therefore$ Other diagonal $= 2x = 32$ cm

$\therefore$ Area $= \frac{1}{2} \times d_1 d_2$

$$= \frac{1}{2} \times 24 \times 32$$

$$= 384 \text{ cm}^2$$

7. Diameter of circle = side of square = 14 cm



$\Rightarrow$ $r = 7$ cm

$\therefore$ Area of circle, $\pi r^2 = \frac{22}{7} \times 7 \times 7 = 154 \text{ cm}^2$

8. Side of the greatest square tile = GCM of the length and breadth of the room

GCM of 10.5 and 3 = 1.5 m.

$\therefore$ Area of room $= 10.5 \times 3 \text{ m}^2$

$\therefore$ Number of tiles needed $= \frac{10.5 \times 3}{1.5 \times 1.5} = 14$ tiles

9. Let length = x cm and breadth = $(x - 2)$ cm

$\therefore$ $2[x + (x - 2)] = 48$

$\Rightarrow$ $4x - 4 = 48$

$\Rightarrow$ $x = \frac{52}{4} = 13$ cm

$\therefore$ Length = 13 cm and breadth = 11 cm

Hence, area $= l \times b$

$$= 13 \times 11 = 143 \text{ cm}^2$$

10. Length of an arc

$$= 2\pi r \times \frac{\theta}{360°} = \frac{2 \times 22 \times 42 \times 72°}{7 \times 360°}$$

$$= \frac{264}{5} = 52.8 \text{ cm}$$

11. Let the original side of a square be 'a'.

$\therefore$ Area of square $= a^2$

Now, new side $= a + \frac{a}{2} = \frac{3a}{2}$

$\Rightarrow$ New area $= \frac{9a^2}{4}$

$\therefore$ Increase in area $= \frac{9a^2}{4} - a^2 = \frac{5a^2}{4}$

$\therefore$ Percent increase in area

$$= \frac{5a^2}{4a^2} \times 100 = 125\%$$

12. It is clear from the figure that, quadrilateral $ABCD$ is a parallelogram. The diagonal AC of the given parallelogram $ABCD$ divides it into two triangles of equal areas.

Area of the $\Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$

$$= \frac{1}{2} \times 12 \times 3 = 18 \text{ cm}^2$$

$\therefore$ Area of the parallelogram $ABCD$

$$= 2 \times \text{Area of } \Delta ABC$$

$$= 2 \times 18 = 36 \text{ cm}^2$$

13. The diagonal AC of the rhombus $ABCD$ divides it into two triangles of equal areas.

Now, area of $\Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$

$$= \frac{1}{2} \times 4 \times 6 = 12 \text{ cm}^2$$

$\therefore$ Area of the rhombus $ABCD = 2 \times$ Area of ΔABC

$$= 2 \times 12 = 24 \text{ cm}^2$$

14. We know that,

Area of a parallelogram = Side × Altitude

$\Rightarrow$ $a \times h = 60 \Rightarrow a \times 5 = 60$

$\Rightarrow$ $a = \frac{60}{5}$

$\therefore$ $a = 12$ cm

15. Let the sides of the two square be a and b.

$\therefore$ Ratio of their areas

$$\frac{a^2}{b^2} = \frac{4}{1}$$

or $\left(\frac{a}{b}\right)^2 = \left(\frac{2}{1}\right)^2$

$\Rightarrow$ $\frac{a}{b} = \frac{2}{1}$

$\therefore$ $a : b = 2 : 1$

16. Inner circumference of a park

$$= 2\pi r = 440 \text{ m}$$

$\Rightarrow$ $r = \frac{440}{2 \times 22} \times 7 = 70$ m

Width of track = 14 m

$\Rightarrow$ Radius of outer circle

$$= (70 + 14) = 84 \text{ m}$$

$\therefore$ Diameter of outer circle $= 2 \times 84 = 168$ m

17. Circumference of circle $= 2\pi r$

$$= 2 \times \frac{22}{7} \times 42 = 264 \text{ cm}$$

$\therefore$ Length of wire = 264 cm

Now, wire is bent into a square.

$\therefore$ Perimeter of square = length of wire = 264 cm

$\Rightarrow$ $4 \times$ side of square = 264

$\therefore$ Side of square $= \frac{264}{4} = 66$ cm

18. Given, perimeter of a trapezium is 52 cm and each non-parallel side is of 10 cm.

Then, sum of its parallel sides

$$= 52 - (10 + 10)$$

$= 52 - 20 = 32$ cm

$\therefore$ Area of the trapezium $= \frac{1}{2}(a + b) \times h$

$$= \frac{1}{2} \times 32 \times 8$$

$[\because h = 8 \text{ cm and } a + b = 32 \text{ cm}]$

$$= 128 \text{ cm}^2$$

19. Given, the area of two circles are in the ratio 49 : 64.

Area of a circle $= \pi r^2$

Let area of the first circle $= \pi r_1^2$

and area of the second circle $= \pi r_2^2$

According to the question, $\frac{49}{64} = \frac{\pi r_1^2}{\pi r_2^2}$

$\Rightarrow \quad \frac{49}{64} = \frac{r_1^2}{r_2^2}$

$\Rightarrow \quad \frac{(7)^2}{(8)^2} = \frac{r_1^2}{r_2^2} \Rightarrow \left(\frac{7}{8}\right)^2 = \left(\frac{r_1}{r_2}\right)^2$

$\therefore \quad r_1 = 7$ and $r_2 = 8$

The ratio of circumferences of these two circles

$$= \frac{2\pi r_1}{2\pi r_2} = \frac{r_1}{r_2} = \frac{7}{8}$$

[$\because$ circumference of circle $= 2\pi r$]

Hence, required ratio is 7 : 8.

20. Distance covered by wheel in one minute

$$= \left(\frac{66 \times 1000 \times 100}{60}\right) = 110000 \text{ cm}$$

Circumference of wheel

$$= \left(2 \times \frac{22}{7} \times 70\right) = 440 \text{ cm}$$

$\therefore$ Number of revolutions in 1 min

$$= \left(\frac{110000}{440}\right) = 250$$

21. Given, perimeter of triangle = 144 cm

$\therefore$ Sides of triangle are

$$a = \frac{3}{3+4+5} \times 144 = 36 \text{ cm}$$

and $b = 48$ cm,

$c = 60$ cm

Now, $s = \frac{a+b+c}{2} = \frac{36+48+60}{2}$

$= 72$ cm

$\therefore$ Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{72 \times 36 \times 24 \times 12} = 72 \times 12$

$= 864 \text{ cm}^2$

22. Let the sides containing right angled be x cm and $(x - 14)$ cm.

$\therefore$ Area $= \left[\frac{1}{2} x \times (x - 14)\right] \text{cm}^2$

$\Rightarrow \quad \frac{1}{2} x(x - 14) = 120 \quad [\because \text{area} = 120 \text{ cm}^2]$

$\Rightarrow \quad x^2 - 14x - 240 = 0$

$\Rightarrow \quad (x - 24)(x + 10) = 0 \Rightarrow x = 24$

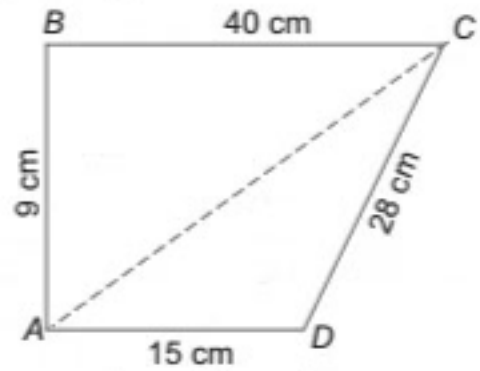
and $\quad x \neq -10$

Other side $= 24 - 14 = 10$ cm

$\Rightarrow$ Hypotenuse $= \sqrt{24^2 + 10^2} = \sqrt{676} = 26$ cm

$\therefore \quad$ Perimeter $= (24 + 10 + 26) = 60$ cm.

23. Applying Pythagoras theorem in ΔABC we get,



$$9^2 + 40^2 = AC^2$$

$\Rightarrow \quad AC = \sqrt{1681} = 41$ cm

$\therefore$ Area of quadrilateral

$=$ area of ΔABC + area of ΔADC

$= \frac{1}{2}(9 \times 40) + \sqrt{42 \times 1 \times 14 \times 27}$

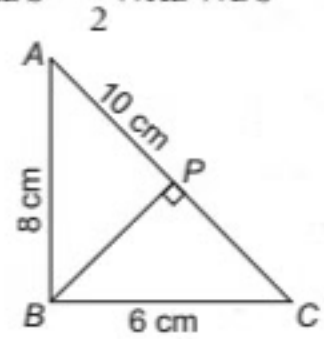
$[\because s = \frac{15 + 28 + 41}{2} = 42 \text{ cm}]$

and area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= 180 + 14 \times 3 \times 3$

$= 180 + 126 = 306 \text{ cm}^2$

24. Area of $\Delta ABC = \frac{1}{2} \times AB \times BC$



$\Rightarrow \quad \frac{1}{2} \times 8 \times 6 = \frac{1}{2} \times AC \times BP$

$\Rightarrow \quad \frac{1}{2} \times 48 = \frac{1}{2} \times 10 \times BP$

$\Rightarrow \quad BP = 4.8 \text{ cm}$

25. Since, 1 hec $= 10000 \text{ m}^2$

Also, $\pi r^2 = 13.86 \times 10000$

$\Rightarrow \quad r = \sqrt{\frac{138600}{22} \times 7} = 210 \text{ m}$

Circumference of circle $= 2\pi r = 1320\text{m}$

$\therefore$ Total cost of fencing $= 1320 \times 0.60 =$ ₹792

26. Since, $2(l + b) = 34 \Rightarrow l + b = 17$

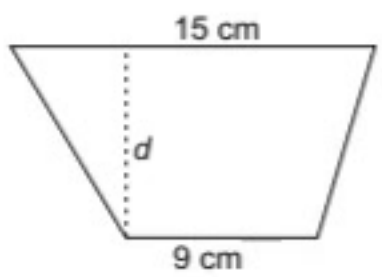
and $\sqrt{l^2 + b^2} = 13 \Rightarrow l^2 + b^2 = 169$

On solving, we get $l = 12$ and $b = 5$

$\therefore$ Area of rectangle $5 = l \times b$

$= 12 \times 5 = 60 \text{ cm}^2$

27. Area of cross-section of canal $= \frac{1}{2}(15 + 9) \times d$



$\Rightarrow \quad 720 = \frac{1}{2} \times 24 \times d$

$\Rightarrow \quad d = 60 \text{ m}$

28. Diameter of circle $= \sqrt{6^2 + 8^2} = 10$ cm

$\therefore$ Area of cricle $= \pi (5)^2 = 25\pi \text{ cm}^2$

$= 25 \times \frac{22}{7} = 78.57 \text{ cm}^2$

and area of rectangle $= 8 \times 6 = 48$ cm

$\therefore$ Shaded area $= 78.57 - 48$

$= 30.57 \text{ cm}^2$

29. Area of equilateral triangle $= \frac{\sqrt{3}}{4} a^2$

$\Rightarrow \quad \frac{\sqrt{3}}{4} a^2 = 16\sqrt{3} \Rightarrow a^2 = 64 \Rightarrow a = 8$ cm

Since, perimeter of square = perimeter of an equilateral triangle

$\Rightarrow \quad 4x = 3 \times 8 \quad \Rightarrow \quad x = 6$ cm

30. Area of the shaded portion = Area of ΔPTQ

$\because$ Area of a triangle $= \frac{1}{2} \times$ Base $\times$ Height

So, in ΔPTQ, RQ = Height

$\therefore$ Area of $\Delta PTQ = \frac{1}{2} \times 36 \times 24 = 18 \times 24 = 432 \text{ m}^2$

31. The perimeter of regular hexagon

$= 6 \times$ radius of a circle $= 6 \times 8 = 48$ cm

32. Area of regular hexagon inscribed in a circle

$= \frac{6\sqrt{3}}{4}(r)^2$

$= \frac{6\sqrt{3}}{4} \times (10)^2 = 150\sqrt{3} \text{ cm}^2$

CHAPTER

13

SURFACE AREA AND VOLUME

This chapter is very important for entrance. In this chapter, we study the surface area and volume of various solid figures such as cuboid, cube, cylinder, cone, sphere etc.

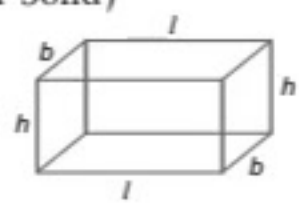
Any figure bounded by one or more surfaces is called a **solid figure**. Hence, a solid figure must have length, breadth (width) and thickness (depth or height).

Surface Area The surface area is the sum of all the areas of all the shapes that cover the surface of the object.

Volume The amount of space occupied by a solid is called its volume.

Surface Area and Volume of Plane Figures

Cuboid (Rectangular Solid)



Let length, breadth and height are respectively l, b and h.

Total number of faces = 6

Surface Area $(SA) = 2(bh + lh)$

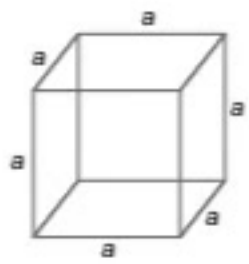
Base Area $(B) = lb$

Total $(C + 2B) = 2(bh + lh + lb)$

Volume $(V) = l \times b \times h = \sqrt{A_1 A_2 A_3}$

where, A_1, A_2 and A_3 are areas of base, side and end face respectively.

Cube



It has six equal faces of each side (edge) a.

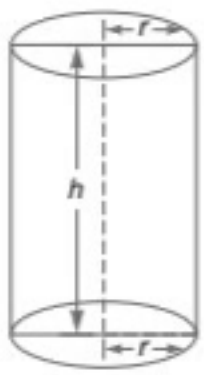
Surface Area $(SA) = 4a^2$

Base Area $(B) = a^2$

Total $(C + 2B) = 6a^2$

Volume $(V) = a^3$

Cylinder



Let the radius of base and height be respectively r and h.

Surface Area (SA)

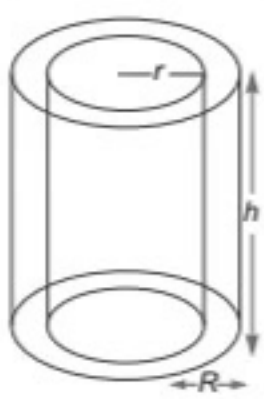
$= (\text{base perimeter}) \times (\text{height}) = 2\pi rh$

Base Area $(B) = \pi r^2$

Total $(C + 2B) = 2\pi r(h + r)$

Volume $(V) = \pi r^2 h$

Hollow cylinder



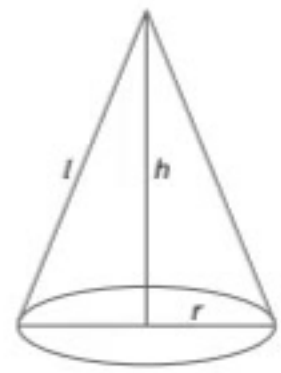
Surface Area $(SA) = 2\pi Rh + 2\pi rh$

Base Area $(B) = \pi(R^2 - r^2)$

Total $(C + 2B) = 2\pi h(R + r) + 2\pi(R^2 - r^2)$

Volume $(V) = \pi(R^2 - r^2)h$

Cone (Right Circular)



Let the radius of base, altitude and slant height be respectively r, h and l.

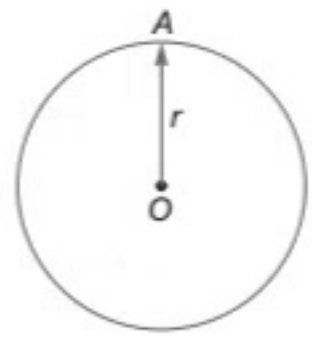
Surface Area $(SA) = \pi rl$, where $l = \sqrt{h^2 + r^2}$

Base Area $(B) = \pi r^2$

Total $(C + 2B) = \pi r(l + r)$

Volume $(V) = \frac{1}{3}$ (Base area) $\times$ Altitude $= \frac{1}{3}\pi r^2 \times h$

Sphere



Let radius of sphere be r.

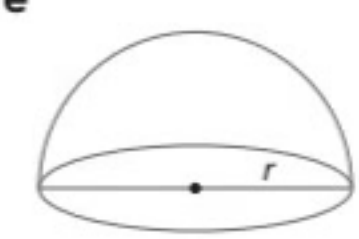
Surface Area $(SA) = 4\pi r^2$

Base Area $(B) =$ No Base

Total $(C + 2B) = 4\pi r^2$

Volume $(V) = \frac{4}{3}\pi r^3$

Hemisphere



Surface Area $(SA) = 2\pi r^2$

Base Area $(B) = \pi r^2$

Total $(C + 2B) = 3\pi r^2$

Volume $(V) = \frac{2}{3}\pi r^3$

Example 1 The volume and surface area of a cube sides measures 4 cm are respectively
(a) 64 cm^3, 96 cm^2 (b) 64 cm^3, 80 cm^2
(c) 64 cm^3, 90 cm^2 (d) 60 cm^3, 96 cm^2

Sol. *(a)* Volume $= a^3 = (4)^3 = 64\ cm^3$

and surface area $= 6a^2$

$$= 6 \times 4 \times 4 = 96\ cm^2$$

Example 2 How many bricks each measuring 25 cm × 11.5 cm × 6 cm will be needed to construct a wall 8 m long, 6 m high and 22.5 cm thick ?
(a) 6262 (b) 6260 (c) 6624 (d) 6520

Sol. *(b)* Number of bricks required

$$= \frac{\text{Volume of wall in cm}^3}{\text{Volume of 1 brick in cm}^3}$$

$$= \frac{800 \times 600 \times 22.5}{25 \times 11.5 \times 6} = 6260$$

Example 3 If the radius of a cylinder is increased from 6 cm to 14 cm and the surface area of it kept same. If its height is 5 cm, what will be its new height?
(a) $\frac{15}{7}$ cm (b) $\frac{15}{8}$ cm
(c) $\frac{17}{7}$ cm (d) None of these

Sol. *(a)* When $r = 6$ cm and $h = 5$ cm, then surface area of cylinder $S_1 = 2\pi rh$

$$= 2\pi \times 6 \times 5 = 60\pi\ cm^2$$

When $r_1 = 14$ cm and height h_1 cm, then the surface area of cylinder

$$S_2 = 2\pi r_1 h_1 = 2\pi \times 14 h_1$$

According to the given condition,

$$S_1 = S_2$$

$$\Rightarrow \quad 60\pi = 28\pi h_1$$

$$\Rightarrow \quad h_1 = \frac{60}{28} = \frac{15}{7}\ cm$$

Example 4 The volume and curved surface area of a cylinder of length 60 cm with diameter of the base 7 cm are respectively
(a) 2310 cm^3, 1320 cm^2 (b) 2410 cm^3, 1320 cm^2
(c) 2310 cm^3, 1350 cm^2 (d) 2410 cm^3, 1350 cm^2

Sol. *(a)* Volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 3.5 \times 3.5 \times 60 = 2310\ cm^3$$

and curved surface area $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 3.5 \times 60 = 1320\ cm^2$$

Example 5 The slant height, volume and curved surface area of a cone of base radius 21 cm and height 28 cm are respectively
(a) 35 cm 12936 cm^3, 2310 cm^2
(b) 35 cm 12930 cm^3, 2320 cm^2
(c) 36 cm 12940 cm^3, 2325 cm^2
(d) None of the above

Sol. *(a)* Slant height, $l = \sqrt{r^2 + h^2} = \sqrt{(21)^2 + (28)^2}$

$$= \sqrt{1225} = 35\ cm$$

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 21 \times 21 \times 28 = 12936\ cm^3$$

and curved surface area of cone $= \pi rl$

$$= \frac{22}{7} \times 21 \times 35 = 2310\ cm^2$$

Example 6 A circus tent is cylindrical to a height of 4 m and conical above it. If its diameter is 105 m and its slant height is 40 m. The total area of the canvas required (in m^2) is
(a) 7928 m^2 (b) 7920 m^2
(c) 7923 m^2 (d) None of these

Sol. *(b)* Total area of canvas $= (2\pi rh + \pi rl)$

$$= \left(2 \times \frac{22}{7} \times \frac{105}{2} \times 4 + \frac{22}{7} \times \frac{105}{2} \times 40\right)$$

$$= 1320 + 6600 = 7920\ m^2$$

Example 7 The volume and total surface area of a hemisphere of diameter 21 cm are respectively
(a) 2420 cm^3, 1038 cm^2
(b) 2422 cm^3, 1039 cm^2
(c) 2425.5 cm^3, 1039.5 cm^2
(d) None of the above

Sol. (c) Volume of hemisphere $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}$$

$$= 2425.5 \text{ cm}^3$$

and total surface area of hemisphere

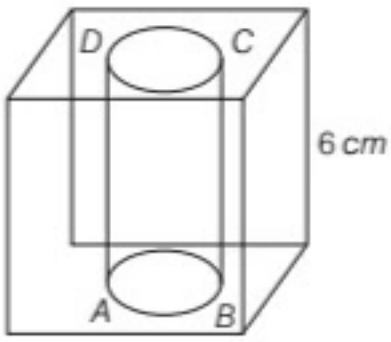
$$= 3\pi r^2 = 3 \times \frac{22}{7} \times 10.5 \times 10.5$$

$$= 1039.5 \text{ cm}^2$$

Example 8 Find the volume of cylinder which is exactly fit into a cube of side 6 cm.

(a) 171 cm^3 (b) 169.71 cm^3
(c) 173 cm^3 (d) None of the above

Sol. (*b*)



Here, height of cylinder = 6 cm

Radius of cylinder $= \frac{6}{2} = 3$ cm

Now, volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times (3)^2 \times 6$$

$$= \frac{1188}{7} = 169.71 \text{ cm}^3$$

PRACTICE EXERCISE

1. The volume of a cuboid is 440 cm^3, the area of its base is 88 cm^2, then its height is
(a) 5 cm (b) 10 cm (c) 11 cm (d) 6 cm

2. The surface area of a cube is 486 sq m, then its volume is
(a) 729 m^3 (b) 781 m^3 (c) 625 m^3 (d) 879 m^3

3. Rectangular sand box is 5 m wide and 2 m long. How many cubic metres of sand are needed to fill the box upto a depth of 10 cm?
(a) 1 m^3 (b) 10 m^3
(c) 100 m^3 (d) 1000 m^3

4. If the height of a cylinder becomes 1/4 of the original height and the radius is doubled, then which of the following will be true?
(a) Volume of the cylinder will be doubled
(b) Volume of the cylinder will remain unchanged
(c) Volume of the cylinder will be halved
(d) Volume of the cylinder will be $\frac{1}{4}$ of the original volume

5. A cube whose side is 5 cm will have surface area is equal to
(a) 125 cm^2 (b) 50 cm^2
(c) 100 cm^2 (d) None of these

6. The maximum length of a pencil that can be kept in a rectangular box of dimensions 8 cm × 6 cm × 2 cm is
(a) $2\sqrt{54}$ cm (b) $2\sqrt{26}$ cm
(c) $2\sqrt{14}$ cm (d) $2\sqrt{13}$ cm

7. The sum of the length, breadth and depth of a cuboid is 20 cm and its diagonal is $4\sqrt{5}$ cm, then its surface area is
(a) 400 cm^2 (b) 420 cm^2
(c) 300 cm^2 (d) 320 cm^2

8. How many 6 m cubes can be cut from a cuboid measuring 18 m × 15 m × 8 m ?
(a) 8 (b) 9 (c) 10 (d) 7

9. The ratio of radii of two cylinders is 1 : 2 and heights are in the ratio 2 : 3. The ratio of their volumes is
(a) 1 : 6 (b) 1 : 9 (c) 1 : 3 (d) 2 : 9

10. Two cubes have volumes in the ratio 1 : 64. The ratio of the areas of a face of first cube to that of the other is
(a) 1 : 4 (b) 1 : 8 (c) 1 : 16 (d) 1 : 32

11. If the volumes of two cubes are in the ratio 8 :1, then ratio of their edges is
(a) 2:1 (b) 4:1 (c) $2\sqrt{2}:1$ (d) 8:1

12. The total surface area of a right circular cylinder whose height is 15 cm and the radius of the base is 7 cm, is
(a) 968 cm^2 (b) 2310 cm^2
(c) 488 cm^2 (d) 1860 cm^2

13. The outer dimensions of a closed wooden box are 10 cm × 8 cm × 7 cm. Thickness of the wood is 1 cm. The total cost of wood required to make box, if 1 cm^3 of wood cost ₹ 2 is
(a) ₹ 540 (b) ₹ 640 (c) ₹ 740 (d) ₹ 780

14. The diameter of a right circular cone is 12 m and the slant height is 10 m. The total surface area of cone is
(a) $\frac{2412}{7}$ m^2 (b) $\frac{2312}{7}$ m^2
(c) $\frac{2112}{7}$ m^2 (d) $\frac{2012}{7}$ m^2

15. The dimensions of a field are 12 m × 10 m. A pit 5 m long, 4 m wide and 2 m deep is dug in one corner of the field and the earth removed has been evenly spread over the remaining area of the field. The level of the field is raised by
(a) 30 cm (b) 35 cm (c) 38 cm (d) 40 cm

16. A plate of metal 1 cm thick, 9 cm broad, 81 cm long is melted into a cube. The difference in the surface area of two solids is
(a) 1152 cm^2 (b) 1150 cm^2
(c) 1052 cm^2 (d) 1050 cm^2

17. The sum of the radius of the base and the height of a cylinder is 37 m. If the total surface area of the solid cylinder is 1628 m^2. The circumference of base of cylinder is
(a) 11 m (b) 22 m (c) 33 m (d) 44 m

18. The volume of a metallic cylindrical pipe is 770 cm^3. Its length is 14 cm and its external radius is 9 cm. Then, its thickness is
(a) 1 cm (b) 1.5 cm
(c) 2 cm (d) 2.5 cm

19. A 20 m deep well with diameter 14 m is dug up and the earth from digging is spread evenly to form a platform 22m ×14 m. The height of platform is
(a) 10 m (b) 15 m
(c) 20 m (d) 25 m

20. The circumference of the base of a 9 m high wooden solid cone is 44 m. The slant height of the cone is
(a) $\sqrt{120}$ m (b) $\sqrt{130}$ m
(c) $\sqrt{150}$ m (d) $7\sqrt{5}$ m

21. How many metres of cloth 50 m wide will be required to make a conical tent, the radius of whose base is 7 m and whose height is 24 m ?
(a) 9 m (b) 11 m (c) 12 m (d) 13 m

22. It is required to make a hollow cone 24 cm high whose base radius is 7 cm. The area of sheet required including the base is
(a) 700 cm^2 (b) 704 cm^2
(c) 708 cm^2 (d) 710 cm^2

23. The radius of a sphere whose surface area is 154 cm^2, is
(a) 3.5 cm (b) 3.6 cm
(c) 3.7 cm (d) None of these

24. A hemispherical bowl made of brass has inner diameter 10.5 cm. The cost of tin plating it on the inside at the rate of ₹ 16 per 100 cm^2 is
(a) ₹ 28 (b) ₹ 27.72
(c) ₹29.27 (d) ₹28.52

25. If the volume of a sphere is double that of the other sphere, then the ratio of their radii is
(a) $2\sqrt{2}:1$ (b) $\sqrt[3]{2}:1$
(c) $1:\sqrt[3]{2}$ (d) 2:1

26. The internal and external diameters of a hollow hemispherical vessel are 24 cm and 25 cm respectively. The total area to be painted is

(a) $\frac{13211}{7}$ cm^2 (b) $\frac{26961}{14}$ cm^2
(c) $\frac{6961}{14}$ cm^2 (d) $\frac{16951}{14}$ cm^2

27. Three cubes each of side 10 cm are joined end to end. The surface area of the resultant figure is

(a) 1400 cm^2 (b) 1500 cm^2
(c) 1450 cm^2 (d) 1550 cm^2

28. Height of a solid cylinder is 10 cm and diameter 8 cm. Two equal conical holes have been made from its both ends. If the diameter of the hole is 6 cm and height 4 cm. The volume of remaining portion is

(a) 24π cm^3 (b) 36π cm^3
(c) 72π cm^3 (d) 136π cm^3

29. The length, breadth and height of a room are in the ratio of 3 : 2 : 1. If its volume be 1296 m^3, its breadth is

(a) 12 m (b) 18 m
(c) 16 m (d) 24 m

30. The diameters of two cones are equal. If their slant height be in the ratio 5 : 7, the ratio of their curved surface areas is

(a) 25 : 7 (b) 25 : 49
(c) 5 : 49 (d) 5 : 7

31. A rectangular paper 11 cm by 8 cm can be exactly wrapped to cover the curved surface of a cylinder of height 8 cm. The volume of the cylinder is

(a) 66 cm^3 (b) 77 cm^3
(c) 88 cm^3 (d) 12 cm^3

32. A cylindrical tube open at both ends is made of metal. The internal diameter of the tube is 11.2 cm and its length is 21 cm. The metal everywhere is 0.8 cm thick. The volume of the metal is

(a) 316 cm^3 (b) 310 cm^3
(c) 306.24 cm^3 (d) 280.52 cm^3

33. A solid right circular cylinder of radius 8 cm and height 2 cm is melted into a right circular cone with radius of the base 8 cm. Its height is

(a) 5 cm (b) 6 cm
(c) 5.75 cm (d) 6.25 cm

34. A hemispherical bowl is made from a metal sheet having thickness 0.3 cm. The inner radius of the bowl is 24.7 cm. The cost of polishing its outer surface at the rate of ₹ 4 per 100cm^2 is (take $\pi = 3.14$)

(a) ₹ 159 (b) ₹ 157
(c) ₹ 160 (d) ₹ 165

35. If the radius of a cylinder is increased from 7 m to 10 m and the surface area of it kept same. If its height is 4 m, then new height will be

(a) 2.8 m (b) 3.1 m
(c) 3.6 m (d) 3.3 m

36. Find the volume of the cone which is exactly fit in the cube of side 8 cm.

(a) 133 cm^3 (b) 134 cm^3
(c) 135 cm^3 (d) None of these

Answers

1	(a)	2	(a)	3	(a)	4	(b)	5	(c)	6	(b)	7	(d)	8	(c)	9	(a)	10	(c)
11	(a)	12	(a)	13	(b)	14	(c)	15	(d)	16	(a)	17	(d)	18	(a)	19	(a)	20	(b)
21	(b)	22	(b)	23	(a)	24	(b)	25	(b)	26	(b)	27	(a)	28	(d)	29	(a)	30	(d)
31	(b)	32	(c)	33	(b)	34	(b)	35	(a)	36	(c)								

Hints and Solutions

1. Height $= \dfrac{\text{Volume of the cuboid}}{\text{Area of its base}}$

$= \dfrac{440}{88} = 5\text{ cm}$

2. Let edge of cube be a.

Surface area of the cube $= 6a^2$

$\therefore \quad 6a^2 = 486$

$\Rightarrow \quad a^2 = 81$

$\Rightarrow \quad a = 9$

$\therefore$ Volume of the cube $= (\text{edge})^3$

$= (9)^3 = 729\text{ m}^3$

3. Sand needed to fill the tank

$= \left(5 \times 2 \times \dfrac{10}{100}\right) = 1\text{ m}^3$

4. We know that, the volume of a cylinder having base radius r and height h is $V = \pi r^2 h$

Now, if new height is $\dfrac{1}{4}$th of the original height and the radius is doubled, i.e.

$h' = \dfrac{1}{4}h$ and $r' = 2r$, then

New volume, $V' = \pi(2r)^2 \times \dfrac{1}{4}h = 4\pi r^2 \times \dfrac{1}{4}h$

$= \pi r^2 h = V$

Hence, the new volume of cylinder is same as the original volume.

5. Now, surface area of cube $= 4(\text{side})^2$

$= 4 \times (5)^2 = 100\text{ cm}^2$

6. Length of longest pencil

= diagonal of the box

$= \sqrt{8^2 + 6^2 + 2^2} = \sqrt{104} = 2\sqrt{26}$ cm

7. Given, $\quad l + b + h = 20\text{ cm}$

and $\quad \sqrt{l^2 + b^2 + h^2} = 4\sqrt{5}$

$\therefore$ Surface area $= 2(lb + bh + hl)$

$= (l + b + h)^2 - (l^2 + b^2 + h^2)$

$= (20)^2 - (4\sqrt{5})^2$

$= 400 - 80 = 320\text{ cm}^2$

8. Volume of cube $= 6 \times 6 \times 6\text{ m}^3$

Volume of cuboid $= 18 \times 15 \times 8\text{ m}^3$

$\therefore$ Required number of cube

$= \dfrac{\text{Volume of cuboid}}{\text{Volume of cube}}$

$= \dfrac{18 \times 15 \times 8}{6 \times 6 \times 6} = 10$

9. Let r_1, r_2 be radii of two cylinders and h_1, h_2 be their heights.

Then, $\dfrac{r_1}{r_2} = \dfrac{1}{2}$ and $\dfrac{h_1}{h_2} = \dfrac{2}{3}$

$\therefore \quad \dfrac{V_1}{V_2} = \dfrac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = \left(\dfrac{r_1}{r_2}\right)^2 \times \dfrac{h_1}{h_2} = \left(\dfrac{1}{2}\right)^2 \times \dfrac{2}{3}$

$= \dfrac{1}{4} \times \dfrac{2}{3} = \dfrac{1}{6} = 1:6 = \dfrac{1}{4} \times \dfrac{2}{3} = \dfrac{1}{6} = 1{:}6$

10. Let a and b be the edges of the two cubes, respectively.

Then, according to the question,

$a^3 : b^3 = 1 : 64 \qquad [\because \text{ volume of cube} = (\text{edge})^3]$

$\Rightarrow \quad \dfrac{a^3}{b^3} = \dfrac{1}{64}$

$\Rightarrow \quad \left(\dfrac{a}{b}\right)^3 = \left(\dfrac{1}{4}\right)^3$

$\Rightarrow \quad \dfrac{a}{b} = \dfrac{1}{4} \qquad$ [taking cube roots on both sides]

Now, ratio of areas, $\left(\dfrac{a}{b}\right)^2 = \left(\dfrac{1}{4}\right)^2$

$[\because \text{ surface area of cube} = 6 \times (\text{edge})^2]$

$\Rightarrow \quad \dfrac{a^2}{b^2} = \dfrac{1}{16}$

$\therefore \quad a^2 : b^2 = 1 : 16$

11. Let the edges of cubes be x and y, then volumes are x^3 and y^3 respectively.

$\therefore \quad \dfrac{x^3}{y^3} = \dfrac{8}{1}$

$\Rightarrow \quad \dfrac{x}{y} = \dfrac{2}{1}$

12. Total surface area of right circular cylinder

$$= 2\pi r\,(h + r) = 2 \times \frac{22}{7} \times 7\,(15 + 7)$$

$$= 2 \times 22 \times 22 = 968 \text{ cm}^2$$

13. External volume of the box

$$= 10 \times 8 \times 7 = 560 \text{ cm}^3$$

Internal dimensions as thickness of wood

$$= 1 \text{ cm}$$

Internal length $= 10 - 2 = 8$ cm,

breadth $= 8 - 2 = 6$ cm, height $= 7 - 2 = 5$ cm

$\therefore$ Internal volume $= 8 \times 6 \times 5 = 240 \text{ cm}^2$

$\Rightarrow$ Volume of wood

$$= \text{External volume} - \text{Internal volume}$$

$$= 560 - 240 = 320 \text{ cm}^3$$

$\therefore$ Total cost of wood required to make the box

$$= 320 \times 2 = ₹\ 640$$

14. Total surface area $= \pi r\,(l + r)$

$$= \frac{22}{7} \times 6 \times (10 + 6) = \frac{2112}{7} \text{ m}^2$$

15. Area of the field = Length × Breadth

$$= 12 \times 10 = 120 \text{ m}^2$$

Area of the pit's surface $= 5 \times 4 = 20 \text{ m}^2$

Area on which the earth is to be spread

$$= 120 - 20 = 100 \text{ cm}^2$$

Volume of earth dug out $= 5 \times 4 \times 2 = 40 \text{ m}^3$

$\therefore$ Level of field raised $= \dfrac{40}{100} = \dfrac{2}{5}$ m

$$= \frac{2}{5} \times 100 = 40 \text{ cm}$$

16. Let the edge of the cube be 'x'.

Then, volume of the cube is

$$x^3 = 9 \times 81 \times 1 \text{ cm}^3 = 729 \text{ cm}^3$$

$\therefore$ $x = \sqrt[3]{729} = 9$ cm

Surface area of metal plate

$$= 2\,(81 \times 9 + 9 \times 1 + 1 \times 81) = 2 \times (819)$$

$$= 1638 \text{ cm}^2$$

Total surface area of the cube

$$= 6\,(\text{edge})^2 = 6\,(9)^2 = 486 \text{ cm}^2$$

$\therefore$ Difference of surface area of two solids

$$= 1638 - 486 = 1152 \text{ cm}^2$$

17. Given, $r + h = 37$ m

and total surface area $= 1628 \text{ m}^2$

$= 2\pi r\,(h + r) = 1628 \text{ m}^2 \Rightarrow 2\pi r\,(37) = 1628$

$\Rightarrow$ $r = \dfrac{1628 \times 7}{2 \times 22 \times 37} = 7$

$\therefore$ Circumference of its base $= 2\pi r$

$$= 2 \times \frac{22}{7} \times 7 = 44 \text{ m}$$

18. External radius, $R = 9$ cm

Internal radius, r cm.

Length of pipe $= 14$ cm

Since, volume of pipe $= 770 \text{ cm}^3$

$\Rightarrow$ Volume of hollow cylinder $= 770$

$\therefore$ $r(R^2 - r^2)h = 770$

$\Rightarrow$ $81 - r^2 = \dfrac{770 \times 7}{22 \times 14} = 17$

$\Rightarrow$ $r^2 = 64 \Rightarrow r = 8$ cm

$\therefore$ Thickness $= R - r = 9 - 8 = 1$ cm

19. Volume of earth dug out from the well

$$\pi r^2 h = \frac{22}{7} \times \left(\frac{14}{2}\right)^2 \times 20$$

$$= 22 \times 7 \times 20 \text{ m}^3$$

Let height of platform be h m.

$\therefore$ Volume of platform $= 22 \times 14 \times h$

$\Rightarrow$ $22 \times 14 \times h = 22 \times 7 \times 20$

$\Rightarrow$ $h = \dfrac{22 \times 7 \times 20}{22 \times 14} = 10$ m

20. Since, circumference of cone $= 44$ m

$\Rightarrow$ $2\pi r = 44 \Rightarrow r = \dfrac{44}{2\pi} = 7$m

$\therefore$ Slant height $= \sqrt{r^2 + h^2} = \sqrt{49 + 81}$

$$= \sqrt{130} \text{ m}^3$$

21. Slant height $= \sqrt{r^2 + h^2} = \sqrt{24^2 + 7^2}$

$$= \sqrt{576 + 49}$$

$$= \sqrt{625} = 25$$

Curved surface area $= \pi rl = \frac{22}{7} \times 7 \times 25 = 550 \text{ m}^2$

Since, width of cloth = 50 m

$\therefore$ Length of required cloth $= \frac{550}{50} = 11$ m

22. Given, $h = 24$ cm, $r = 7$ cm

Now, slant height

$$l = \sqrt{r^2 + h^2}$$
$$= \sqrt{24^2 + 7^2} = \sqrt{576 + 49}$$
$$= \sqrt{625} = 25 \text{ cm}$$

$\therefore$ Area of metal sheet required

= Total surface area of cone

$$= \pi r (l + r) = \frac{22}{7} \times 7 (7 + 25)$$
$$= 22 (32) = 704 \text{ cm}^2$$

23. Let the radius of the sphere be r cm.

Surface area of the sphere $= 154 \text{ cm}^2$

$\therefore \quad 4\pi r^2 = 154$

[$\because$ surface area of a sphere $= 4\pi r^2$]

$$\Rightarrow \quad 4 \times \frac{22}{7} \times r^2 = 154$$
$$\Rightarrow \quad r^2 = \frac{154 \times 7}{22 \times 4} = 12.25$$
$$\Rightarrow \quad r = \sqrt{12.25} = 3.5 \text{ cm}$$

Hence, the radius of the sphere is 3.5 cm.

24. We have, inner diameter = 10.5 cm

$\therefore$ Inner radius $(r) = \frac{10.5}{2} = 5.25$ cm

Curved surface area of hemispherical bowl of inner side $= 2\pi r^2 = 2 \times \frac{22}{7} \times (5.25)^2$

$$= 2 \times \frac{22}{7} \times 5.25 \times 5.25$$
$$= 173.25 \text{ cm}^2$$

$\because$ Cost of tin plating on inside for 100 cm^2

= ₹16

$\therefore$ Cost of tin plating on the inside for 173.25 cm^2

$$= \frac{16 \times 173.25}{100} = ₹\ 27.72$$

25. Let r_1 and r_2 be the radii and V_1, V_2 be its volume of spheres respectively.

Given, $V_1 = 2V_2 \Rightarrow \frac{V_1}{V_2} = 2$

$$\therefore \quad \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{2}{1} \Rightarrow \frac{r_1^3}{r_2^3} = \frac{2}{1}$$
$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{\sqrt[3]{2}}{1}$$

26. Internal radius $(r) = 12$ cm

and external radius $(R) = \frac{25}{2}$ cm

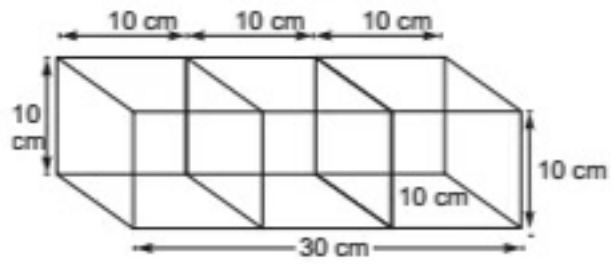
$\therefore$ Area to be painted

= Internal area + External area + Area of edge

$$2\pi r^2 + 2\pi R^2 + \pi (R^2 - r^2)$$
$$= 2 \times \frac{22}{7} \times 12 \times 12 + 2 \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2} + \frac{22}{7}\left(\frac{25}{2} \times \frac{25}{2} - 12 \times 12\right)$$
$$= \frac{6336}{7} + \frac{6875}{7} + \frac{539}{14}$$
$$= \frac{26422}{14} + \frac{539}{14} = \frac{26961}{14} \text{ cm}^2$$

27. If three cubes each of side 10 cm are joined, then a cuboid will be formed of dimensions

30 cm × 10 cm × 10 cm



$\therefore$ Surface area of the cuboid $= 2[lb + bh + hl]$

$$= 2[30 \times 10 + 10 \times 10 + 30 \times 10]$$
$$= 2[300 + 100 + 300] = 2[700] = 1400 \text{ cm}^2$$

28. Volume of cylinder $= \pi(4)^2 \times 10 = 160\pi \text{ cm}^3$

Volume of one cone $= \frac{1}{3} \times \pi \times 3^2 \times 4$

$= 12\pi \text{ cm}^3$

$\therefore$ Volume of both cones $= 24\pi \text{ cm}^3$

$\therefore$ Volume of remaining portion

$= 160\pi - 24\pi = 136\pi \text{ cm}^3$

29. Let the sides be $3x, 2x$ and $1x$.

$\therefore$ Volume $= l \times b \times h$

$\Rightarrow \quad 1296 = 3x \times 2x \times x$

$\Rightarrow \quad 6x^3 = 1296$

$\Rightarrow \quad x^3 = 216 \Rightarrow x = 6$

$\therefore$ Breadth $= 2 \times 6 = 12$ m

30. Ratios of two curved surface area

$= C_1 : C_2 = \pi r l_1 : \pi r l_2$

$= l_1 : l_2 = 5 : 7$

31. $\therefore$ Surface area of cylinder = Area of paper

$\Rightarrow \quad 2\pi rh = l \times b$

$\Rightarrow \quad 2\pi r \times 8 = 11 \times 8$

$\Rightarrow \quad 2\pi r = 11 \Rightarrow r = \frac{11 \times 7}{2 \times 22} = \frac{7}{4}$

$\therefore$ Volume $= \pi \left(\frac{7}{4}\right)^2 \times 8 = 77 \text{cm}^3$

32. Volume of metal $= \pi [R^2 - r^2] h$

$= \frac{22}{7} [6^2 - (5.6)^2] \times 21$

$= 66 \times 4.64$

$= 306.24 \text{ cm}^3$

33. Volume of circular cylinder $= \pi(8)^2(2) = 128\pi$ cm

Volume of right circular cone $= \frac{1}{3}\pi r^2 h$

$\therefore \quad \frac{1}{3}\pi r^2 h = 128\pi$

$\Rightarrow \quad \frac{1}{3} \times \pi \times (8)^2 \times h = 128\pi$

$\Rightarrow \quad h = 6$ cm

34. Given, inner radius of the hemispherical bowl

$= 24.7$ cm

Thickness of metal sheet $= 0.3$ cm

Now, outer radius of the hemispherical bowl

$= 24.7 + 0.3 = 25$ cm

$\therefore$ Outer surface area of the hemispherical bowl

$= 2\pi r^2$

$= 2 \times 3.14 \times (25)^2 = 157 \times 25 = 3925 \text{ cm}^2$

Now, cost of polishing $100 \text{ cm}^2 =$ ₹ 4

$\therefore$ Cost of polishing 3925 cm^2

$= \frac{4 \times 3925}{100} =$ ₹ 157

35. When radius $r = 7$ m and height $h = 4$ m,

then surface area of cylinder, $S_1 = 2\pi rh$

$= 2\pi \times 7 \times 4 = 56\pi \text{m}^2$

When radius $r_1 = 10$ m and height $= h_1$ m,

then surface area of new cylinder,

$S_2 = 2\pi r_1 h_1 = 2\pi \times 10 \times h_1$

$= 20\pi h_1$

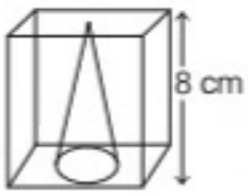
According to the given condition,

$S_1 = S_2$

$\Rightarrow \quad 56\pi = 20\pi \times h_1$

$\Rightarrow \quad h_1 = \frac{56}{20} = 2.8$ m

36. Here, height of cone $h = 8$ cm



and radius of cone $r = \frac{8}{2} = 4$ cm

Now, volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (4)^2 \times 8$

$= \frac{2816}{21} = 135 \text{ cm}^3$

CHAPTER

14

DATA HANDLING

In this chapter, we study the data in frequency distribution table, representation of there data in various types of graph such as Bar graph Pie chart, Line graph etc. and also study probability of event.

Data

A collection of numerical facts regarding a particular type of information is called data.

(i) **Primary Data** The data collected actually in the process of investigation by the investigator is known as primary data.

(ii) **Secondary Data** Data which is already collected by other persons is called secondary data.

e.g. As investigator collects data related to industries through the government publications.

Organising Data

The initial form of data is in unorganised form, i.e. raw data form. To organise the given data in systematic manner, we use a term frequency.

Frequency

The number of times a particular observation occurs is called frequency.
e.g. The frequency distribution table is shown below.

Subjects	Tally marks	Number of students
Art	卌 II	7
Mathematics	卌	5
Science	卌 I	6
English	IIII	4

Here we see that, the number of students corresponding to each subject are the frequencies.

Grouping Data

Sometimes, the given data is very large, so we make a group. The data arranged in each group is known as class-interval (or class).

Each of group 0-10, 10-20, 20-30, etc, is called class-interval. The upper value of a class-interval is called its upper class limit and the lower value of the class-interval is called its lower class limit.

Here, in the class interval 10-20,
the upper class limit =20

and the lower class limit =10.

Some more related terms of grouping data are given below.

(i) **Width** The difference between the upper class limit and lower class limit of a class is called the width or size of the class-interval.

(ii) **Tally marks** Frequency distribution table involves the tally marks for counting purpose. We use bar (i.e.| |) known as tally marks.

(iii) **Class Frequency** The frequency of a class in a continuous frequency distribution is called as class frequency.

(iv) **Class Marks** It is the mid-value of the class interval.

Class mark

$$= \frac{\text{Lower limit of class + Upper limit of class}}{2}$$

(v) **Range** It is the difference between the highest and the lowest values of the given observation on data.

Example 1 The first and second class interval of a grouped data are 6-12, 12-18. Its fifth class interval is

(a) 20-26 (b) 30-36 (c) 24-30 (d) 31-37

Sol. *(b)* Given, class intervals are 6-12 and 12-18

Here, width of the class is $12 - 6$ i.e. 6

$\therefore$ Then, next class intervals will be 18-24, 24-30, 30-36 etc.

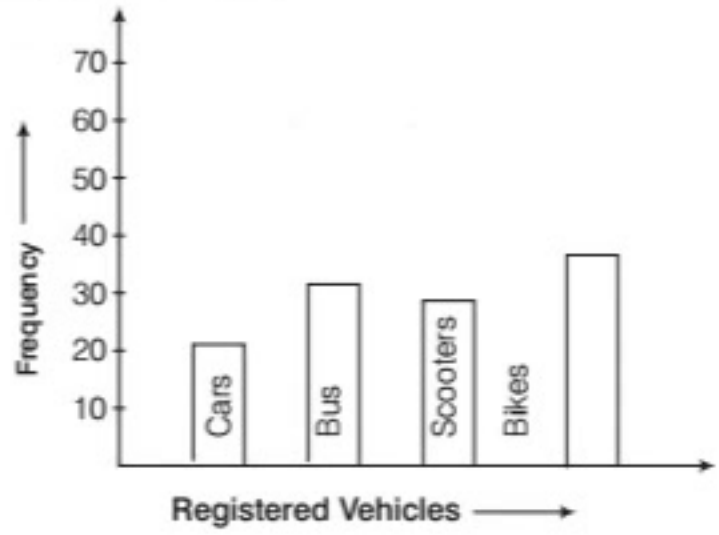
Hence, fifth class interval is 30-36.

Types of Graphical Data

The following types of groups are given below

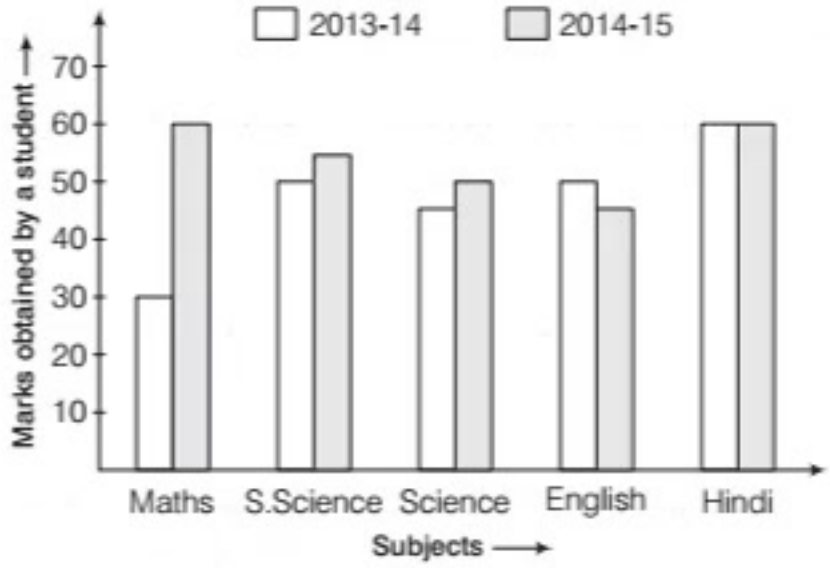
1. Bar Graph

A pictorial representation of numerical data in the form of rectangles (or bars) of uniform width and various heights is called a bar graph, where the commodity taken on the horizontal axis and the heights of the bars (rectangles) show the frequency of the commodity. e.g. The registration of vehicles versus number of vehicles shown through the bar graph.



2. Double Bar Graph

A bar graph showing two sets of data simultaneously is called a double bar graph. It is useful for the comparison of the data.

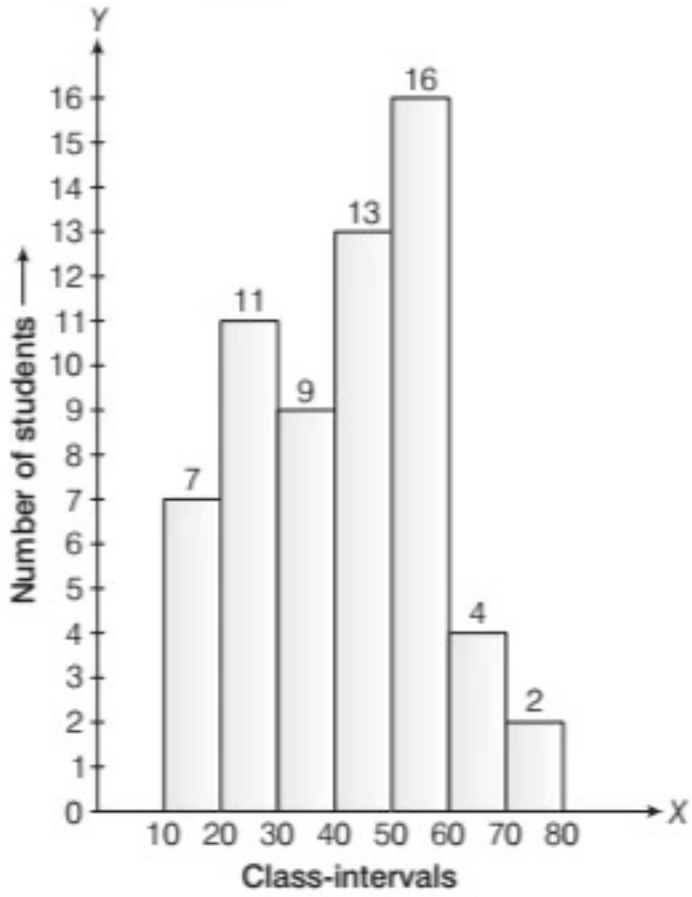


3. Histogram Graph

A histogram is the graphical representation of a grouped frequency distribution in exclusive form with continuous classes in the form of rectangles with class-intervals as bases and the corresponding frequencies as heights.

There is no gap between any two consecutive rectangles.

The shape of the graph is shown below.



4. Pie Chart

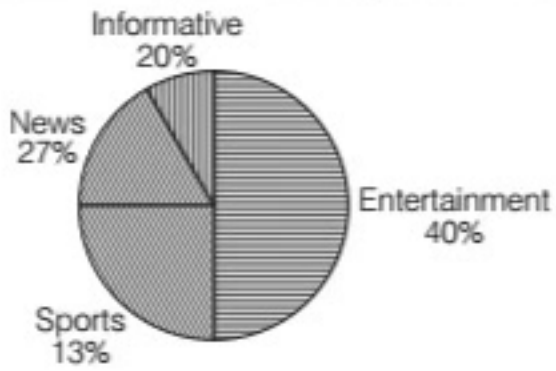
In a pie chart, the various observations or components are represented by the sectors of a circle and the whole circle represents the sum of the values of all components.

The central angle for a component is given by

Central angle for a component

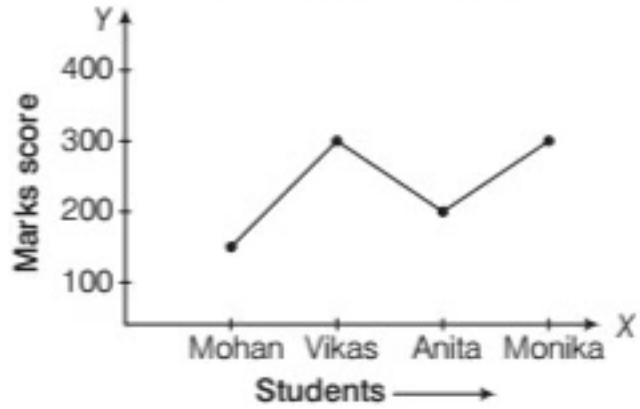
$$= \left(\frac{\text{Value of the component}}{\text{Sum of the values of all components}} \times 360° \right)$$

e.g. The following pie graph shows the percentage of viewers watching different types of TV channels.

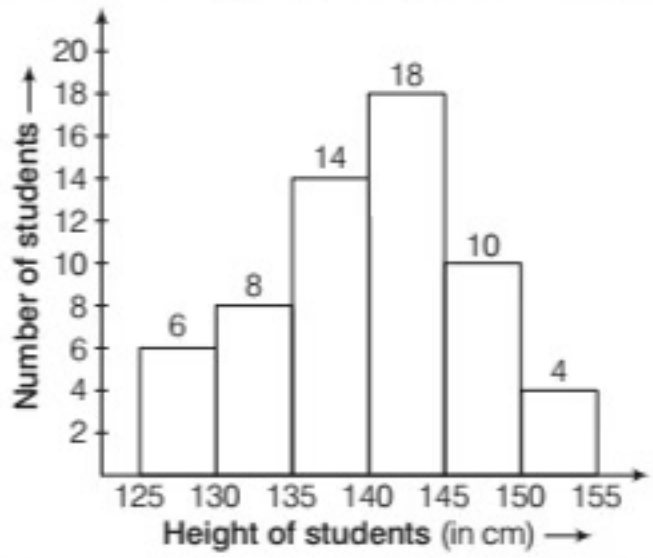


5. Line Graph

A line graph is a graph, which is used to display data that changes continuously over period of time. e.g. The marks score by the students shown through the line graph as given below.



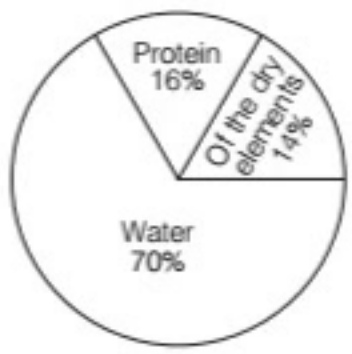
Example 2 In a histogram, the number of students having height less than 140 cm is



(a) 32 (b) 28 (c) 40 (d) 34

Sol. *(b)* Number of students who have height less than 140 cm = 6 + 8 + 14 = 28.

Example 3 The following pie chart gives the distribution of constituents in the human body. The central angle of the sector showing the distribution of protein and other constituents is



(a) 108° (b) 54°
(c) 30° (d) 216°

Sol. *(a)* Distribution of protein and other constituents in human body = 16 + 14 = 30%

Central angle of the sector showing the distribution of protein and other constituents

$$= \frac{30}{100} \times 360° = 108°$$

Experiment

An operation which can produce some well-defined outcomes is called an experiment.

Random Experiment

An experiment in which the outcomes cannot be predicted exactly in advance and all possible outcomes are known, is called random experiment. e.g. On tossing a coin, head or tail are the two outcomes, but you cannot predict, which outcome you will get.

Event

Each outcome of an experiment or a collection of outcomes make an event.

e.g.

(i) Throwing a die and getting each of the outcomes 1, 2, 3, 4, 5 or 6 is an event.

(ii) Tossing a coin and getting a head or a tail is an event.

Negation of an event Negation of an event A is 'not A', which occurs only when A does not occur. It is denoted by '$\overline{A}$'.

Probability of an Event

Let E be an event. Then, the probability of occurrence of E is defined as

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

Note The probability of any event lies between 0 to 1.

Important Points

(i) In tossing a coin, all possible outcomes are $\{H, T\}$.

(ii) In tossing 2 coins, all possible outcomes are $\{HH, HT, TH, TT\}$.

(iii) On rolling a dice, all possible outcomes are $\{1, 2, 3, 4, 5, 6\}$.

(iv) In drawing a card from a well-shuffled deck of 52 cards, it has 4 suits (spades, clubs, hearts and diamonds), each having 13 cards.

In out of 52 cards, 26 cards are black and 26 cards are red.

(a) Cards of spades and clubs are black cards.

(b) Cards of hearts and diamonds are red cards.

(c) Kings, queens and jacks (or knaves) are known as face cards.

Thus, there are in 12 face cards.

Example 4 The two coins are tossed together. The probability of getting two heads together is?

(a) $\frac{1}{4}$ (b) $\frac{2}{4}$ (c) $\frac{1}{2}$ (d) 1

Sol. *(a)* When two coins are tossed together, all possible outcomes are – $\{HH, HT, TH, TT\}$

$\therefore$ Number of all possible outcomes = 4

Possibility of getting two heads together $= \frac{1}{4}$

Example 5 A dice is thrown. What is the probability of getting a prime number?

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

Sol. *(b)* In throwing a die, all possible outcomes are 1, 2, 3, 4, 5, 6.

$\therefore$ Number of all possible outcomes = 6

Prime numbers are 2, 3, 5.

Number of prime numbers = 3

$\therefore P(\text{getting a prime number}) = \frac{3}{6} = \frac{1}{2}$

Example 6 From a well-shuffled deck of 52 cards, one card is drawn at random. What is the probability that the card drawn is an ace?

(a) $\frac{2}{13}$ (b) $\frac{3}{13}$ (c) $\frac{1}{13}$ (d) $\frac{5}{13}$

Sol. *(c)* Total number of cards = 52

Total number of possible outcomes = 52.

Number of all aces = 4.

$\therefore \quad P(\text{getting an ace}) = \frac{4}{52} = \frac{1}{13}$

PRACTICE EXERCISE

1. A graph that displays data that changes continuously over periods of time is
   (a) bar graph (b) pie chart
   (c) histogram (d) line graph

2. The range of the data 30, 61, 55, 56, 60, 20, 26, 46, 28, 56 is
   (a) 26 (b) 30
   (c) 41 (d) 61

3. Which of the following is not a random experiment?
   (a) Tossing a coin
   (b) Rolling a die
   (c) Choosing a card from a deck of 52 cards
   (d) Throwing a stone to the roof of a building

4. Tally marks are used to find
   (a) class intervals
   (b) range
   (c) frequency
   (d) upper limit

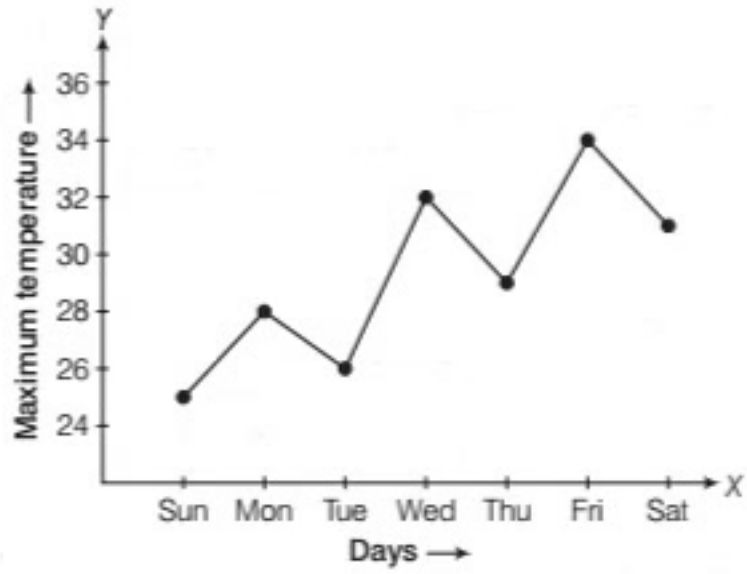
5. In a frequency distribution with classes 0-10, 10-20 etc., the size of the class intervals is 10. The lower limit of fourth class is
   (a) 40 (b) 50
   (c) 20 (d) 30

6. Monthly salary of a person is ₹ 15000. The central angle of the sector representing his expenses on food and house rent on a pie chart is 60°. The amount he spends on food and house rent, is
   (a) ₹ 5000 (b) ₹ 2500
   (c) ₹ 6000 (d) ₹ 9000

7. The first and second class interval of a grouped data are 5-15, 15-25, 25-35. Its sixth class interval is
   (a) 65-70 (b) 44-55
   (c) 45-50 (d) 55-65

8. The class mark of the interval 13.5-18.5 is
   (a) 15 (b) 16
   (c) 14 (d) 13

Directions (Q. Nos. 9-11) *Study the graph and answer the questions that follow.*

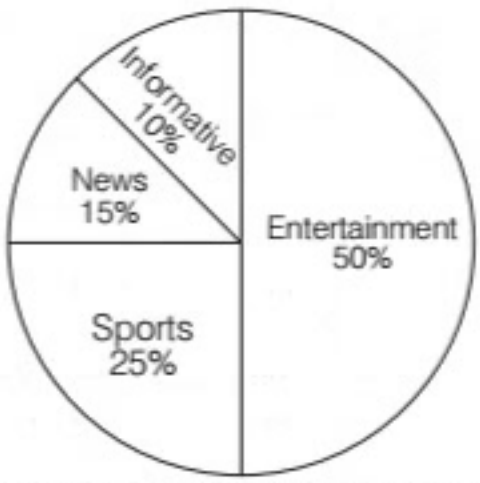


9. On which day the temperature was least?
   (a) Sunday
   (b) Monday
   (c) Friday
   (d) None of the above

10. On which day the temperature was 31°C?
    (a) Monday (b) Saturday
    (c) Tuesday (d) Friday

11. Which was the hottest day?
    (a) Monday (b) Friday
    (c) Tuesday (d) Saturday

Directions (Q.Nos. 12 and 13) *A pie chart is given below.*



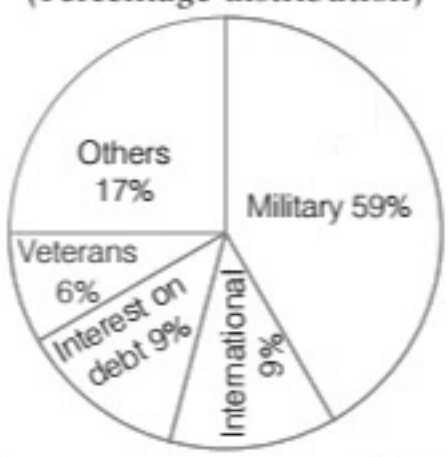
Viewers watching different types of channels on TV

12. Which type of programmes are viewed the most?
(a) Entertainment viewers
(b) Informative viewers
(c) Sports viewers
(d) None of the above

13. Which two types of programmes have number of viewers equal to those watching sports channels?
(a) News and Sports
(b) News and Informative
(c) News and Entertainment
(d) None of the above

Directions (Q. Nos. 14 and 15) *Study the following pie chart and answer the questions that follow.*

National budget expenditure in the year 2012
(Percentage distribution)

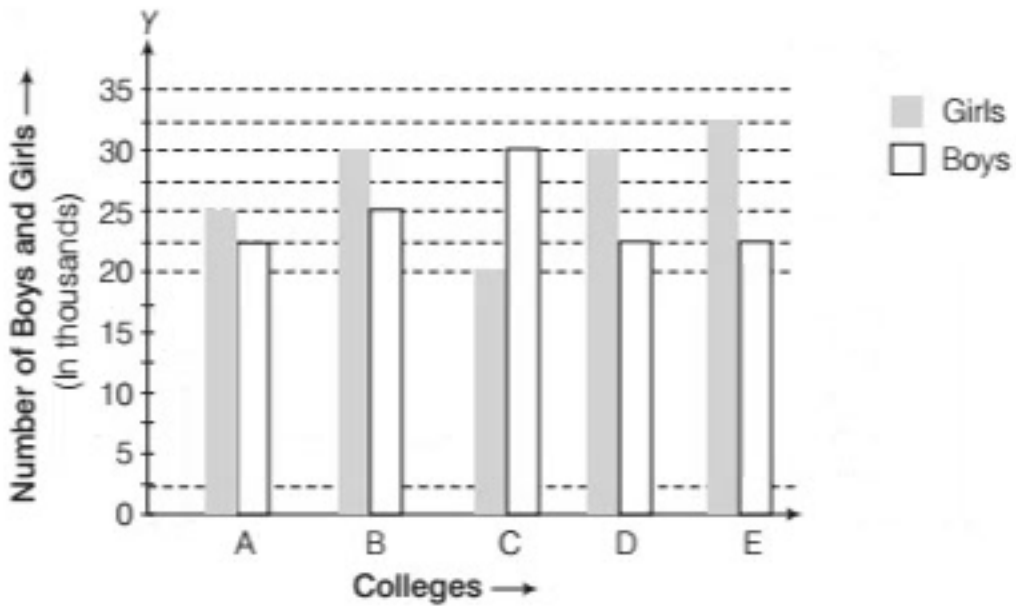


14. In year 2012, if India had a total expenditure of ₹ 120 billion, then how many billions did it spend on interest on debt?
(a) ₹ 10.8 billion
(b) ₹ 11.2 billion
(c) ₹ 11.50 billion
(d) ₹ 11.70 billion

15. If ₹ 9 billion were spent in year 2012 for veterans, then what would have been the total expenditure for that year (in billions)?
(a) ₹ 160 billion
(b) ₹ 165 billion
(c) ₹ 150 billion
(d) ₹ 170 billion

Directions (Q. Nos. 16 and 17) *Study the following graph carefully and answer the questions given below.*

Total Number of Boys and Girls in Various Colleges (Number in Thousands)



16. What is the average number of girls from all the colleges together?
(a) 25000 (b) 27500
(c) 27000 (d) 25500

17. What is the difference between the total number of girls and the total number of boys from all the colleges together?
(a) 13500 (b) 14000
(c) 15000 (d) 16000

18. A dice is throw, find the probability of getting not even number?
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{5}$ (d) $\frac{1}{3}$

19. What is the probability of choosing a vowel from the alphabets?
(a) $\frac{21}{26}$ (b) $\frac{5}{26}$ (c) $\frac{1}{26}$ (d) $\frac{3}{26}$

20. A coin is tossed 12 times and the outcomes are observed as shown below

The chance of occurrence of head is
(a) $\frac{1}{2}$ (b) $\frac{5}{12}$ (c) $\frac{7}{12}$ (d) $\frac{5}{7}$

21. Ram puts some buttons on the table. There were 4 blue, 7 red, 3 black and 6 white buttons in all. All of a sudden, a cat jumped on the table and knocked out one button on the floor. What is the probability that the button on the floor is blue?
(a) $\frac{7}{20}$ (b) $\frac{3}{5}$ (c) $\frac{1}{5}$ (d) $\frac{1}{4}$

22. Two coins are tossed simultaneously. What is the probability of getting one head and one tail?
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{2}{3}$

23. A bag contains 3 white and 2 red balls. One ball is drawn at random. What is the probability that the ball drawn is red?
(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

24. A dice is thrown. What is the probability of getting 6?
(a) 1 (b) $\frac{1}{6}$
(c) $\frac{5}{6}$ (d) None of these

25. A dice is thrown. What is the probability of getting an even number?
(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{5}{6}$ (d) $\frac{1}{6}$

26. From a well-shuffled deck of 52 cards, one card is drawn at random. What is the probability that the drawn card is a queen?
(a) $\frac{1}{4}$ (b) $\frac{1}{52}$ (c) $\frac{1}{13}$ (d) $\frac{1}{26}$

27. From a well-shuffled deck of 52 cards, one card is drawn at random. What is the probability that the drawn card is a black 6?
(a) $\frac{3}{26}$ (b) $\frac{1}{26}$ (c) $\frac{1}{13}$ (d) $\frac{1}{52}$

28. Numbers 1 to 10 are written on ten separate slips (one slip of one number), kept in a box and mixed well. One slip is chosen from the box without looking into it. What is the probability of getting a number less than 6?
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

Answers

1	(d)	2	(c)	3	(d)	4	(c)	5	(d)	6	(b)	7	(d)	8	(b)	9	(a)	10	(b)
11	(b)	12	(a)	13	(b)	14	(a)	15	(c)	16	(b)	17	(c)	18	(b)	19	(b)	20	(b)
21	(c)	22	(b)	23	(d)	24	(b)	25	(a)	26	(c)	27	(b)	28	(a)				

Hints and Solutions

1. Line graph is an important way to represent and compare the data which varies continuously.

A line graph displays the relation between two varying quantities.

In a line graph, we connect all the points by a line segment while in bar graph and histogram, we use rectangles of uniform width.

2. Range of data

= Maximum value – Minimum value

= 61 – 20 = 41

3. Tossing a coin, rolling a die and choosing a card from a deck of 52 cards are the random experiments, as we don't have an idea about the output of these experiments. But if we throw a stone to the roof of a building, we know that output, it will fall on the ground.

4. Tally marks are used to find the frequency of the observations.

5. Given classes are 0-10 and 10-20.

As, the class of given classes is 10, so the next classes will be 20-30 and 30-40.

As, the fourth class is 30-40.

Hence, the lower limit of fourth class is 30.

6. Central angle of the sector representing his expenses on food and house rent on a pie chart

$= 60°$

Part of the monthly salary he is expending on food and house rent $= \frac{60°}{360°} = \frac{1}{6}$

Hence, the amount he spends on food and house rent $= \frac{1}{6} \times$ Monthly salary

$= \frac{1}{6} \times 15000 =$ ₹ 2500

7. Given class intervals are 5-15, 15-25 and 25-35

Here, width of the class is 15-5 i.e. 10.

∴ The next class intervals will be

35-45, 45-55, 55-65 etc

Hence, sixth class interval is 55-65.

8. Class mark $= \frac{\text{Lower limit} + \text{Upper limit}}{2}$

$= \frac{13.5+18.5}{2} = \frac{32.0}{2} = 16$

9. On Sunday, the temperature was 25°C. So, it is least temperature in the week.

10. On Saturday, the temperature was 31°C.

11. On Friday, the temperature was maximum i.e. 34°C. Hence, it is the hottest day of the week.

12. From the given pie chart, we have the following table :

Types of viewers	Percentage
Sports viewers	25
News viewers	15
Informative viewers	10
Entertainment viewers	50

Since, percentage of entertainment is highest, so entertainment programme are viewed the most.

13. Here, the number of viewers watching news = 15%

Number of viewers watching informative = 10%

∴ Sum of number of viewers watching news and informative = (15 + 10)% = 25%

= Number of viewers watching sports

Hence, news and informative programmes have number of viewers equal to those watching sports channel.

14. Total expenditure = ₹ 120 billion

∴ Expenditure of interest on debt = 9% of 120

$= \frac{9}{100} \times 120 =$ ₹ 10.8 billion

15. ₹ 9 billion were spent for veterans. 6% of the total expenditure was spent on veterans in the year 2012.

Hence, the total expenditure

$= \frac{9}{6} \times 100 =$ ₹ 150 billion

16. Total number of girls

= (25+ 30+ 20+ 30+ 32.5) thousands

= 137.5×1000=137500

Average number of girls $= \frac{137500}{5} = 27500.$

17. Total number of boys

$=(22.5+25+30+22.5+22.5)$ thousands

$=(122.5\times1000)=122500$

Total number of girls $=137500$

Required difference $=(137500-122500)=15000$.

18. The probability of getting not even number

= Probability of getting odd number

$=\frac{3}{6}=\frac{1}{2}$ [$\because$ Odd numbers in a die are 1, 3, 5]

19. Total number of alphabets = 26

Total number of vowels = 5

$\therefore$ Probability of choosing a vowel from the alphabets $=\frac{\text{Total number of vowels}}{\text{Total number of alphabets}}=\frac{5}{26}$

20. Total number of times a coin is tossed = 12

Total number of occurrence of head = 5

$\therefore$ The chance of occurrence of head

$=\frac{\text{Number of times head appeared}}{\text{Number of times a coin is tossed}}=\frac{5}{12}$

21. Total number of buttons $=4+7+3+6=20$

$\therefore$ Probability that button on the floor is blue

$=\frac{\text{Number of blue buttons}}{\text{Total number of buttons}}=\frac{4}{20}=\frac{1}{5}$

22. Total number of outcomes $=2\times2=4$

Number of favourable outcomes

$=2$ [i.e. (H, T), (T, H)]

$\therefore$ Probability of getting one head and one tail

$=\frac{2}{4}=\frac{1}{2}$

23. Total number of balls $=3+2=5$ balls

Number of favourable outcomes = Number of red balls = 2

$\therefore$ Probability of getting a red ball $=\frac{2}{5}$

24. Total number of outcomes in a die = 6

Number of favourable outcomes = 1

$\therefore$ P (getting 6) $=\frac{1}{6}$

25. Total number of outcomes in a die = 6

Number of favourable outcomes = 3

[$\because$ even numbers are 2, 4, 6]

$\therefore P$ (getting even number) $=\frac{3}{6}=\frac{1}{2}$

26. Total number of outcomes in a deck of cards

= 52

Number of favourable outcomes (queen) = 4

$\therefore P$ (getting queen) $=\frac{4}{52}=\frac{1}{13}$

27. Total number of outcomes = 52

Number of favourable outcomes = 2

$\therefore P$ (getting card of black 6) $=\frac{2}{52}=\frac{1}{26}$

28. Total number of outcomes = 10

Number of favourable outcomes = 5

$\therefore P$ (getting number less than 6) $=\frac{5}{10}=\frac{1}{2}$

CHAPTER

09

LINEAR EQUATIONS IN ONE VARIABLE

In this chapter, we study one and two variable types linear equations and various word problems by using linear equations.

Equation

An equation is an equality involving variables and it has an equality sign (=). The expression on the left of the equality sign is the Left Hand Side (LHS) and expression on the right of the equality sign is the Right Hand Side (RHS).

Linear Equations

The equations, in which the highest power of the variable appearing in the expression is 1, are called linear equations.

Linear Equations in One Variable

The expression, which form the equation contains only one variable and the highest power of the variable appearing in the equation is 1, are called linear equations in one variable.

e.g. $2x - 5 = 3$ and $\frac{5}{3}x + 3 = \frac{1}{2}$

Note The graph of the linear equation is always a straight line.

Properties of Linear Equations

(i) If same number is added or subtracted to both the sides of an equation, the equality remains the same.

(ii) If same number is multiplied to both the sides of an equation, the equality remains the same.

(iii) If both sides are divided by a same non-zero number, the equality remains the same.

Example 1 Solve $2(x-3)-(5-3x) = 3(x+1)-4(2+x)$

(a) 2 (b) 1
(c) 3 (d) 0

Sol. *(b)* Given,

$$2(x-3)-(5-3x)=3(x+1)-4(2+x)$$

$\Rightarrow$ $2x-6-5+3x=3x+3-8-4x$

$\Rightarrow$ $5x-11=-x-5 \Rightarrow 6x=6$

$\Rightarrow$ $x=1$

Example 2 If $\frac{2x+1}{3x-1}=\frac{2x+1}{3x+2}$, the value of x is

(a) $-\frac{1}{2}$ (b) 0
(c) $\frac{1}{3}$ (d) $-\frac{2}{3}$

Sol. *(a)* We have, $\frac{2x+1}{3x-1}=\frac{2x+1}{3x+2}$

$\Rightarrow$ $(2x+1)(3x+2)=(2x+1)(3x-1)$

$\Rightarrow$ $6x^2+4x+3x+2=6x^2-2x+3x-1$

$\Rightarrow$ $6x^2+7x+2=6x^2+x-1$

$\Rightarrow$ $7x-x=-1-2 \Rightarrow 6x=-3$

$\therefore$ $x=-\frac{3}{6}=-\frac{1}{2}$

Solving Word Problems by Using Linear Equation

I. Firstly, denote the unknown quantity by any letters, say x, y, z, etc.

II. Translate the statements of the problem into mathematical statements.

III. Using the conditions given in the problem, form the equation.

IV. Solve the equation for the unknown quantity.

Example 3 The digit in the ten's place of a two-digit number is 3 more than the digit in the unit's place. Let the digit at unit's place be b. Then, the number is

(a) $11b+30$ (b) $10b+30$
(c) $11b+3$ (d) $10b+3$

Sol. *(a)* Let digit at unit's place be b.

Then, digit at ten's place $=(3+b)$

$\therefore$ Number $=10(3+b)+b$

$=30+10b+b=11b+30$

Example 4 Thirty one is added 63 to twice a whole number gives. The number is

(a) 15 (b) 16 (c) 17 (d) 18

Sol. *(b)* Let whole number be x.

Then, $31+2x=63$

$\Rightarrow$ $2x=32 \Rightarrow x=16$

Example 5 A boy is now one-third as old as his father. Twelve years hence, he will be half as old as his father. The present age of the boy is

(a) 12 yr (b) 20 yr (c) 36 yr (d) 25 yr

Sol. *(a)* Let the present age of the father be x yr and that of the son be $\frac{1}{3}x$ yr.

After twelve years,

Age of father $=(x+12)$ yr

and age of son $=\left(\frac{1}{3}x+12\right)$ yr

According to given condition,

$$\frac{1}{3}x+12=\frac{1}{2}(x+12)$$

$\Rightarrow$ $\frac{x+36}{3}=\frac{x+12}{2}$

$\Rightarrow$ $2x+72=3x+36 \Rightarrow x=36$

$\therefore$ Age of the boy $=\frac{x}{3}=\frac{36}{3}=12$ yr.

PRACTICE EXERCISE

1. Linear equation in one variable has
(a) only one variable with any power
(b) only one term with a variable
(c) only one variable with power 1
(d) only constant term

2. Which of the following is a linear expression?
(a) x^2+1 (b) $y+y^2$
(c) 4 (d) $1+z$

3. A linear equation in one variable has
(a) only one solution
(b) two solutions
(c) more than two solutions
(d) no solution

4. If $8x-3=25+17x$, then x is
(a) a fraction
(b) an integer
(c) a rational number
(d) cannot be solved

5. If $7x:63=1:9$, then x is equal to
(a) 1 (b) 2
(c) 3 (d) -1

6. If $\frac{3x+6}{8}-\frac{11x-8}{24}+\frac{x}{3}=\frac{3x}{4}-\frac{x+7}{24}$, then the value of x is
(a) -3 (b) 3/2
(c) 3 (d) 1/3

7. If $\sqrt{3}x-2=2\sqrt{3}+4$, then the value of x is
(a) $2(1-\sqrt{3})$ (b) $2(1+\sqrt{3})$
(c) $1+\sqrt{3}$ (d) $1-\sqrt{3}$

8. If $a(x-a^2)-b(x-b^2)=0$, then x is equal to
(a) $\frac{(-a+b)(a^2+ab+b^2)}{(a+b)}$ (b) $\frac{a^3+b^3}{(a-b)}$
(c) $\frac{a^3-b^3}{a+b}$ (d) a^2+ab+b^2

9. If one number is thrice the other and their sum is 20, then the numbers are
(a) 5, 15 (b) 4, 12
(c) 3, 9 (d) 6, 18

10. If the sum of five consecutive even numbers is 340, then the smallest numbers will be
(a) 62 (b) 64 (c) 66 (d) 68

11. The sum of 4 consecutive odd numbers is 56. The smallest number is
(a) 12 (b) 11 (c) 13 (d) 14

12. The sum of the two numbers is 11 and their product is 30, then the numbers are
(a) 8, 3 (b) 9, 2 (c) 7, 4 (d) 6, 5

13. Divide 60 into two parts, so that three times the greater may be exceed 100 by as much as 8 times the less falls short of 200. What is the greater part ?
(a) 36 (b) 24 (c) 20 (d) 25

14. The ages of two persons differ by 20 yr. If 5 yr ago, the elder one be 5 times as old as the younger one, their present ages are
(a) 50 yr, 30 yr (b) 28 yr, 5 yr
(c) 20 yr, 10 yr (d) 30 yr, 10 yr

15. Given that $\frac{4p+9q}{p}=\frac{5q}{p-q}$ and p and q are both positive. The value of $\frac{p}{q}$ is
(a) $\frac{5}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{3}{5}$

16. A fraction becomes 4/5 when 5 is added to its numerator and 1 is subtracted from its denominator. It becomes 1/2 when 3 and 5 are subtracted from its numerator and denominator. The numerator of the fraction will be
(a) 4 (b) 11 (c) 10 (d) 9

17. The total cost of 6 books and 4 pencils is ₹ 34 and that of 5 books and 5 pencils is ₹ 30. The cost of each book and pencil (in ₹) respectively is
(a) 1 and 5 (b) 5 and 1
(c) 6 and 1 (d) 1 and 6

18. The age of a man is 4 times that of his son. Five years ago, the man was nine times as old as his son was at that time. The present age of the man is
(a) 28 yr (b) 45 yr (c) 32 yr (d) 48 yr

19. In a class $\frac{3}{5}$ of the students are girls and rest are boys. If $\frac{2}{9}$ of the girls and $\frac{1}{4}$ of the boys are absent. What part of the total number of students are present ?
(a) $\frac{23}{30}$ (b) $\frac{23}{36}$ (c) $\frac{18}{49}$ (d) $\frac{17}{25}$

20. If a sum of ₹ 275 is to be divided between Ram and Shyam so that Ram gets more than 3/4th of what Shyam gets, then the share of Ram will be
(a) ₹ 200 (b) ₹ 175 (c) ₹ 160 (d) ₹ 100

21. Find a number such that if 5,15 and 35 are added to it, the product of the first and third results may be equal to the square of the second.
(a) 10 (b) 7 (c) 6 (d) 5

22. The length of a rectangle is 8 cm more than its breadth. If the perimeter of the rectangle is 68 cm, its length and breadth are respectively.
(a) 21 cm, 13 cm (b) 21 cm, 32 cm
(c) 20 cm, 10 cm (d) 13 cm, 15 cm

23. In a two digit number, the tens digit is twice the unit digit. When the digits are reversed, the new number formed is 18 less than the original number. The original number is
(a) 45 (b) 42
(c) 43 (d) 41

24. A man has a certain number of chickens and goats. Their head count is 30. If the total number of their legs is 84, what is the ratio between the number of chickens and goats ?
(a) 1 : 2 (b) 2 : 3
(c) 3 : 2 (d) 3 : 4

Answers

1	(c)	2	(d)	3	(a)	4	(c)	5	(a)	6	(c)	7	(b)	8	(d)	9	(a)	10	(b)
11	(b)	12	(d)	13	(a)	14	(d)	15	(c)	16	(b)	17	(b)	18	(c)	19	(a)	20	(b)
21	(d)	22	(a)	23	(b)	24	(c)												

Hints and Solutions

1. Linear equation in one variable has only one variable with power 1.

2. We know that, the algebraic expression in one variable having the highest power of the variable as 1, is known as the linear expression.
Here, $1 + z$ is the only linear expression, as the power of the variable z is 1.

3. A linear equation in one variable has only one solution.

4. Given, $8x - 3 = 25 + 17x$
$\Rightarrow \quad 8x - 17x = 25 + 3$
$\Rightarrow \quad -9x = 28$
$\therefore \quad x = \frac{-28}{9}$
Hence, x is a rational number.

5. $\frac{7x}{63} = \frac{1}{9} \Rightarrow x = \frac{63}{9 \times 7} = 1$

6. Given, $\frac{3x+6}{8} - \frac{11x-8}{24} + \frac{x}{3} = \frac{3x}{4} - \frac{x+7}{24}$
$\Rightarrow \quad \frac{3(3x+6) - (11x-8) + 8x}{24} = \frac{6(3x) - (x+7)}{24}$
$\Rightarrow \quad 9x + 18 - 11x + 8 + 8x = 18x - x - 7$

$\Rightarrow \quad 26 + 6x = 17x - 7$

$\Rightarrow \quad 11x = 33$

$\Rightarrow \quad x = 3$

7. Given, $\sqrt{3}x - 2 = 2\sqrt{3} + 4$

$\therefore \quad \sqrt{3}x = 2\sqrt{3} + 6$

$\Rightarrow \quad x = \frac{2\sqrt{3}+6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 2(1+\sqrt{3})$

8. Since, $\quad ax - a^3 - bx + b^3 = 0$

$\therefore \quad (a-b)x - (a^3 - b^3) = 0$

$\Rightarrow (a-b)[x - (a^2 + b^2 + ab)] = 0$

$\Rightarrow \quad x = a^2 + b^2 + ab$

9. Let the one number be x and number be $3x$.

According to the given condition,

$x + 3x = 20 \Rightarrow x = 5$

$\therefore$ Other number is $(3x) = 3 \times 5 = 15$

10. Let the smallest number be x.

Then,

$x + (x+2) + (x+4) + (x+6) + (x+8) = 340$

$\Rightarrow \quad 5x + 20 = 340$

$\Rightarrow \quad 5x = 320$

$\therefore \quad x = 64$

11. Let four consecutive odd number be

$x+1, x+3, x+5, x+7.$

Then, $(x+1) + (x+3) + (x+5) + (x+7) = 56$

$4x = 40 \Rightarrow x = 10$

$\therefore$ The smaller odd number is $x+1 = 10+1 = 11$

12. Let numbers be x and $11 - x$.

Since, product $\quad x(11-x) = 30 \quad$ [given]

$\Rightarrow \quad x^2 - 11x + 30 = 0$

$\Rightarrow \quad (x-5)(x-6) = 0$

$\Rightarrow \quad x = 5, 6$

13. Let two parts be x and $60 - x$.

Then, $\quad 3x - 100 = 200 - 8(60 - x)$

$\Rightarrow \quad 3x - 100 = 200 - 480 + 8x$

$\Rightarrow \quad 8x - 3x = 480 - 200 - 100$

$\Rightarrow \quad 5x = 180$

$\therefore \quad x = \frac{180}{5} = 36$

Thus, another number $60 - 36 = 24$.

$\therefore$ Greater number $= 36$

14. Let their ages be x and $(x - 20)$ yr.

5 yr ago, their ages are $(x - 5)$ and $(x - 20 - 5)$ yr.

According to the given condition,

$5(x - 20 - 5) = (x - 5)$

$\Rightarrow \quad 5x - 125 = x - 5$

$\Rightarrow \quad 4x = 120 \Rightarrow x = 30$

Hence, their present ages are 30 yr and 10 yr.

15. Given, $\quad \frac{4p + 9q}{p} = \frac{5q}{p - q}$

$\Rightarrow \quad \frac{4\left(\frac{p}{q}\right) + 9}{\left(\frac{p}{q}\right)} = \frac{5}{\left(\frac{p}{q}\right) - 1}$

[on dividing by q in numerator and denominator]

$\Rightarrow 4\left(\frac{p}{q}\right)^2 + 9\left(\frac{p}{q}\right) - 4\left(\frac{p}{q}\right) - 9 = 5\left(\frac{p}{q}\right)$

$\Rightarrow \quad 4\left(\frac{p}{q}\right)^2 - 9 = 0 \Rightarrow \left(\frac{p}{q}\right)^2 = \left(\frac{3}{2}\right)^2$

$\therefore \quad \frac{p}{q} = \frac{3}{2}$

16. Let the fraction be $\frac{x}{y}$.

Then, $\frac{x+5}{y-1} = \frac{4}{5} \Rightarrow 5x - 4y = 29 \quad$ ...(i)

and $\frac{x-3}{y-5} = \frac{1}{2} \quad \Rightarrow \quad 2x - y = 1 \quad$ ...(ii)

On solving Eqs. (i) and (ii), we get $x = 11$

17. Let CP of a book be ₹ x and CP of pencil be ₹y.

Then, $\quad 6x + 4y = 34 \quad$ ...(i)

and $\quad 5x + 5y = 30 \quad$ ...(ii)

On solving Eqs. (i) and (ii), we get

$x = 5$ and $y = 1$.

18. Let the son's age be x yr, then man age's be $4x$.

Five years ago, their ages are $(x - 5)$ yr and $(4x - 5)$ yr respectively.

According to the given condition,

$(4x - 5) = 9(x - 5)$

$\Rightarrow \quad 5x = 40 \quad \Rightarrow \quad x = 8$ yr

Hence, the man's age $= 4x = 4 \times 8 = 32$ yr.

19. Let the number of students be x.

Then, number of girls $= \frac{3}{5}x$ and number of boys

$= \frac{2}{5}x$

Number of girls present $= \frac{7}{9} \times \frac{3}{5}x = \frac{7x}{15}$ and

Number of boys present $= \frac{3}{4} \times \frac{2}{5}x = \frac{3x}{10}$

$\therefore$ Total students present $= \left(\frac{7x}{15} + \frac{3x}{10}\right) = \frac{23}{30}x$

20. Let Shyam's share be ₹ x.

Then, Ram's share $= \left(x + \frac{3x}{4}\right) =$ ₹ $\frac{7x}{4}$

$\therefore \quad x + \frac{7x}{4} = 275 \Rightarrow x = 100$

$\therefore$ Ram's share $= \frac{7}{4} \times 100 =$ ₹ 175

21. Let the number be x.

Then, first number $= (x+5)$

second number $= (x + 15)$

and third number $= (x+35)$

According to the question,

$(x + 5)(x + 35) = (x + 15)^2$

$\Rightarrow \quad 175 + 40x = 225 + 30x$

$\Rightarrow \quad 10x = 50$

$\therefore \quad x = 5$

22. Let breadth of rectangle be x.

Then, its length $= (x + 8)$ cm

$\therefore$ Perimeter of rectangle $= 2[x + (x + 8)]$

$= 2(2x + 8) = 4x + 16$

$\therefore \quad 4x + 16 = 68$ [given]

$\Rightarrow \quad 4x = 52$

$\Rightarrow \quad x = 13$

So, breadth of rectangle $= 13$ cm

and length $= 13 + 8 = 21$ cm

23. Let unit digit $= x$

Then, tens digit $= 2x$

$\therefore$ Original number $= 10 \times 2x + x = 20x + x = 21x$

New Number $= 10 \times x + 2x$

$= 10x + 2x = 12x$

Now, $21x - 12x = 18$

$\Rightarrow \quad 9x = 18$

$\Rightarrow \quad x = 2$

$\therefore$ Unit digit $= 2$ and Tens digit $= 2 \times 2 = 4$

Thus, required number $= 10 \times 4 + 2 = 42$

24. Let the number of chickens and goat be x and

According to the question,

$x + y = 30$ ...(i)

and $2x + 4y = 84$

$\Rightarrow \quad x + 2y = 42$ ...(ii)

On solving Eqs. (i) and (ii), we get

$x = 18$ and $y = 12$

$\therefore$ Required ratio $= 18 : 12 = 3 : 2$