JAWAHAR NAVODAYA VIDYALAYA

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Entrance Exam 2021

Conducted by Navodaya Vidyalaya Samiti

- ENGLISH
- HINDI
- GENERAL SCIENCE
- MATHEMATICS

9

With Solved Paper 2020

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Contents

SYLLABUS

ENGLISH

Comprehension (Unseen Passage), Word and Sentence Structure, Spelling, Rearranging jumbled words , Passivation, Use of degrees of comparison, Modal auxiliaries , Use of prepositions , Tense forms, Reported speech

हिन्दी

(क) भाषिक अनुप्रयोग और व्याकरणिक कुशलताएँ

वर्ण विचार/वर्तनी विवेक, शब्दभेद (स्त्रोत/उत्पत्ति), पर्याय/विलोम, शब्द विवेक (शब्द प्रयोग में सूक्ष्म अंतर), पद भेद (व्याकरणिक कोटि) की पहचान, पद परिचय, अशुद्ध वाक्य को शुद्ध करना, वाक्य रचनान्तरण (सरल/संयुक्त/मिश्र), मुहावरा, लोकोक्ति

(ख) अपठित बोधात्मक प्रश्न

GENERAL SCIENCE

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MATHEMATICS

Rational Numbers, Squares and Square Roots, Cubes and Cube Roots, Exponents and Powers, Direct and Inverse Proportions, Comparing Quantities (Percentage, Profit and Loss, Discount, Simple and Compound Interest), Algebraic Expressions and Identities including Factorization, Linear Equations in One Variable, Understanding Quadrilaterals (Parallelogram, rhombus, rectangle, square, kite) , Mensuration: *(a) Area of plane figures* , *(b) Surface area and volume of cube, cuboid and cylinder* , Data Handling (Bar graph, pie chart, organizing data, probability)

JAWAHAR
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VIDYALAYA

MATHEMATICS

CHAPTER

01

NUMBER SYSTEM (RATIONAL NUMBERS)

In this chapter, we study the various types of numbers, properties of rational numbers, simplification and test for divisibility.

Digits The symbols 0,1, 2, 3, 4, 5, 6, 7, 8, 9 are known as digits in Hindu Arabic System.

Numbers or Numerals A mathematical symbol which represent the digits, are known as numbers or numerals.

Types of Numbers There are some types of numbers, which are given below

(i) **Natural Numbers** Those numbers which are used for counting, are known as natural numbers. These are denoted by N.

e.g. $N = 1, 2, 3, \ldots$

Here 1 is the first and smallest natural number.

(ii) **Whole Numbers** If 0 is included in natural numbers, then these numbers are known as whole numbers. These numbers are denoted by W.

e.g. $W = 0, 1, 2, 3, \ldots$

(iii) **Integers** All whole numbers and their negative numbers are known as integers. These numbers are denoted by I.

$\therefore \; I = 0, \pm 1, \pm 2, \pm 3, \ldots$

Here, 1, 2, 3, ... are positive integers, denoted by I^+, and

$-1, -2, -3, \ldots$ are negative integers, denoted by I^-.

Here 0 is neither positive nor negative integer.

(iv) **Rational Numbers** Numbers in the form of $\frac{p}{q}$, where $p, q \in I$ and $q \neq 0$, are known as rational numbers. It is denoted by Q.

e.g. $\frac{2}{3}, \frac{5}{6}, 6, \frac{-4}{5}$, etc.

(v) **Irrational Numbers** Numbers which can not be expressed in the form of $\frac{p}{q}$, where $p, q \in I$ and $q \neq 0$, are known as irrational numbers.

e.g. $\pi, \sqrt{2}, \sqrt{6}, \sqrt[3]{11}$, etc.

(vi) **Real Numbers** Those numbers which are either rational or irrational, are known real numbers. It is denoted by R.

All natural, whole, integer, rational and irrational numbers are real numbers.

e.g. $2, 0, -5, \frac{1}{2}, \sqrt{6}$, etc.

(vii) **Even Numbers** Those numbers which are divisible by 2, are known as even numbers.

e.g., 2, 4, 6, ...

Note 2 is smallest even number.

(viii) **Odd Numbers** Those numbers, which are not divisible by 2, are known as odd numbers.

e.g. 1, 3, 5, 7, ...

(ix) **Prime Numbers** Those numbers which are divisible by 1 and the number itself, are known as prime numbers.

e.g. 2, 3, 5, 7, 11, 13, ...

Here 2 is the only even prime number.

(x) **Composite Numbers** Those numbers which are divisible by at least one number except 1 and the number itself, are known as composite numbers.

e.g. 12, 8 and 15 etc, are composite numbers.

Example 1 Which of the following is not true?

(a) 11 is prime number

(b) 2/5 is rational number

(c) ± 8 is the real number

(d) $\frac{6}{0}$ is rational number

Sol. *(d)* $\frac{6}{0}$ is not true, because $\frac{6}{0}$ is not rational number, as in its denominator, (p/q form) $q = 0$, which is wrong.

Example 2 Sum of first five prime numbers is

(a) 25 (b) 26 (c) 27 (d) 28

Sol. *(d)* We know that, first five prime numbers are 2, 3, 5, 7, 11

$\therefore$ Their sum $= 2 + 3 + 5 + 7 + 11 = 28$

Properties of Rational Numbers

(i) **Closure Property** The sum and multiplication of two rational numbers are rational numbers.

(a) $2 + \frac{2}{7} = \frac{16}{7}$ [rational number]

(b) $2 \times \frac{2}{7} = \frac{4}{7}$ [rational number]

(ii) **Commutative Property** If a and b are any rational numbers, then we have

(a) $a + b = b + a$ [for addition]

(b) $a \times b = b \times a$ [for multiplication]

(iii) **Associative Property** If a, b and c are three any rational numbers, then we have,

(a) $(a + b) + c = a + (b + c)$ [for addition]

(b) $(a \times b) \times c = a \times (b \times c)$ [for multiplication]

(iv) **Distributive Property** If a, b, c are any rational numbers, then we have

$$a(b + c) = ab + ac,$$

which is called distributive property of multiplication over addition.

(v) **Additive Identity** If a is any rational number, then we have

$$a + 0 = a,$$

i.e. adding a number 0 to any number, is equal to the number itself. Therefore, '0' is called additive identity.

(vi) **Multiplicative Identity** If a is any rational number, then we have

$$a \times 1 = a$$

i.e. multiplying a number 1 to any number, is equal to the number itself. Therefore, 1 is called multiplicative identity.

Rational Numbers Between any Two Given Rational Numbers

Between any two given rational numbers, there are countless or infinite rational numbers.

To find rational numbers between any two given rational numbers, we can use the idea of mean.

Thus, we can say that, 'if a and b are two rational numbers, then $\frac{a+b}{2}$ will be a rational number, such that $a < \frac{a+b}{2} < b$.

e.g. A rational number between 2 and 3 is $\frac{2+3}{2} = \frac{5}{2}$.

Here, we find that, $2 < \frac{5}{2} < 3$.

Example 3 A rational number between $\frac{2}{3}$ and $\frac{3}{4}$ is

(a) 15/24 (b) 17/24 (c) 13/24 (d) 5/24

Sol. *(b)* We find the mean of $\frac{2}{3}$ and $\frac{3}{4}$.

i.e. $$\left(\frac{2}{3}+\frac{3}{4}\right) \div 2 = \left(\frac{8+9}{12}\right) \div 2$$
$$= \frac{17}{12} \div 2 = \frac{17}{12} \times \frac{1}{2} = \frac{17}{24}$$

Simplification

To solve any expression or to simplify, we have many operations, i.e. Brackets, Addition, Subtraction, Multiplication, Division, etc.

We follow the rule of VBODMAS, i.e.

V → Vinculum or bar '—'
B → Brackets, i.e. ( ), { }, [ ]
O → Of
D → Division, i.e. ÷
M → Multiplication, i.e. '×'
A → Addition, i.e. '+'
S → Subtraction, i.e. '–'

To solve brackets, we follow the order

(i) ( ), circular or small bracket
(ii) { }, curly or middle bracket
(iii) [ ], square or big bracket

Note In the absence of any bracket or operation, their is no change in order to solve the expression.

Example 4 $\frac{3}{4}$ of $\frac{2}{7}$ of $\frac{1}{5}$ of $560 = ?$

(a) 28 (b) 24 (c) 32 (d) 36

Sol. *(b)* We have, $\frac{3}{4}$ of $\frac{2}{7}$ of $\frac{1}{5}$ of 560

$$= \frac{3}{4} \times \frac{2}{7} \times \frac{1}{5} \times 560 = 24$$

Example 5 The value of

$1 \div [1 + 1 \div \{1 + 1 \div (1 + 1 \div 2)\}]$ is

(a) $\frac{5}{8}$ (b) $\frac{8}{5}$ (c) $\frac{2}{5}$ (d) $\frac{3}{8}$

Sol. *(a)* We have,

$$1 \div [1 + 1 \div \{1 + 1 \div (1 + 1 \div 2)\}]$$
$$= 1 \div \left[1 + 1 \div \left\{1 + 1 \div \left(1 + \frac{1}{2}\right)\right\}\right]$$
$$= 1 \div \left[1 + 1 \div \left\{1 + 1 \div \frac{3}{2}\right\}\right]$$
$$= 1 \div \left[1 + 1 \div \left\{1 + \frac{2}{3}\right\}\right]$$
$$= 1 \div \left[1 + 1 \div \frac{5}{3}\right] = 1 \div \left[1 + \frac{3}{5}\right] = 1 \div \frac{8}{5} = \frac{5}{8}$$

Test for Divisibility

Generally, to check the divisibility of one number by another, we normally do actual division and see whether remainder is zero or not. But sometimes we use direct condition for divisibility, which is as shown below

- **Divisible by 2** When the last digit of a number is either 0 or even, then the number is divisible by 2.

  e.g. 12, 86, 472, 520, 1000 etc. are divisible by 2.

- **Divisible by 3** When the sum of the digits of a number is divisible by 3, then the number is divisible by 3.

  e.g. 1233 as sum of digits

  $1 + 2 + 3 + 3 = 9$, which is divisible by 3, so 1233 must be divisible by 3.

- **Divisible by 4** When the number made by last two digits of a number is divisible by 4, then that particular number is divisible by 4. Apart from this, the number having two or more zeros at the end, is also divisible by 4.

  e.g. 6428 is divisible by 4 as the number made by its last two digits i.e., 28 is divisible by 4.
- **Divisible by 5** Numbers having 0 or 5 at the end are divisible by 5.

  e.g. 45, 4350, 135, 14850 etc. are divisible by 5 as they have 0 or 5 at the end.
- **Divisible by 6** When a number is divisible by both 3 and 2, then that particular number is divisible by 6 also.

  e.g. 18, 36, 720, 1440 etc. are divisible by 6 as they are divisible by both 3 and 2.
- **Divisible by 7** A number is divisible by 7 when the difference between twice the digit at ones place and the number formed by other digits is either zero or a multiple of 7.

  e.g. 658 is divisible by 7 because $65 - 2 \times 8 = 65 - 16 = 49$. As 49 is divisible by 7, the number 658 is also divisible by 7.
- **Divisible by 8** When the number made by last three digits of a number is divisible by 8, then the number is also divisible by 8. Apart from this, if the last three or more digits of a number are zeros, then the number is divisible by 8.

  e.g. 2256. As 256 (the last three digits of 2256) is divisible by 8, therefore 2256 is also divisible by 8.
- **Divisible by 9** When the sum of all the digits of a number is divisible by 9, then the number is also divisible by 9.

  e.g. 936819 as sum of digits $9 + 3 + 6 + 8 + 1 + 9 = 36$, which is divisible by 9. Therefore, 936819 is also divisible by 9.
- **Divisible by 10** When a number ends with zero, then it is divisible by 10.

  e.g. 20, 40, 150, 123450, 478970 etc. are divisible by 10 as these all end with zero.
- **Divisible by 11** When the sums of digits at odd and even places are equal or differ by a number divisible by 11, then the number is also divisible by 11.

  e.g. 217382 Let us see

  Sum of digits at odd places $= 2 + 7 + 8 = 17$

  Sum of digits at even places $= 1 + 3 + 2 = 6$

  Difference $= 17 - 6 = 11$

  Clearly, 217382 is divisible by 11.
- **Divisible by 12** A number which is divisible by both 4 and 3 is also divisible by 12.

  e.g. 2244 is divisible by both 3 and 4. Therefore, it is also divisible by 12.
- **Divisible by 25** A number is divisible by 25 when its last 2 digits are divisible by 25.

  e.g. 500, 1275, 13550 are divisible by 25 as last 2 digits of these numbers are divisible by 25.

Example 6 Which of the following number is not divisible by 3?

(a) 75 (b) 52 (c) 63 (d) 42

Sol. *(b)* $\because$ Sum of digits, we have

$$75 = 7 + 5 = 12 \text{ (divisible by 3)}$$
$$52 = 5 + 2 = 7 \text{ (not divisible by 3)}$$
$$63 = 6 + 3 = 9 \text{ (divisible by 3)}$$
$$42 = 4 + 2 = 6 \text{ (divisible by 3)}$$

$\therefore$ 52 is not divisible by 3, since its sum is not divisible by 3.

PRACTICE EXERCISE

1. A number in the form $\frac{p}{q}$ is said to be a rational number, if
   (a) p, q are integers
   (b) p, q are integers and $q \neq 0$
   (c) p, q are integers and $p \neq 0$
   (d) p, q are integers and $p \neq 0$, also $q \neq 0$

2. The numerical expression $\frac{3}{8} + \frac{(-5)}{7} = \frac{-19}{56}$ shows that
   (a) rational numbers are closed under addition
   (b) rational numbers are not closed under addition
   (c) rational numbers are closed under multiplication
   (d) addition of rational numbers is not commutative

3. Which of the following is not true?
   (a) rational numbers are closed under addition
   (b) rational numbers are not closed under subtraction
   (c) rational numbers are closed under multiplication
   (d) rational numbers are closed under division

4. Between two given rational numbers, we can find
   (a) one and only one rational number
   (b) only two rational numbers
   (c) only ten rational numbers
   (d) infinitely rational numbers

5. The product $\frac{3}{4}, \frac{2}{5}$ and $\frac{25}{3}$ is
   (a) 5/2 (b) 2/5 (c) 3/5 (d) 5/3

6. Smallest 3-digit prime number is
   (a) 103 (b) 107 (c) 101 (d) 109

7. What should be added to $-\frac{5}{7}$ to get $-\frac{3}{2}$?
   (a) $-\frac{11}{14}$ (b) $\frac{11}{14}$
   (c) $\frac{14}{11}$ (d) $-\frac{14}{11}$

8. Find the additive identity for the rational number
   (a) 0 (b) 1 (c) 2 (d) 3

9. Which statement is true?
   (a) $-5 + 3 \neq 3 + (-5)$
   (b) $\frac{-8}{12} = \frac{10}{-15}$
   (c) 2 is not natural number
   (d) 17 is not prime number

10. What should be subtracted from $-\frac{3}{4}$ to make $\frac{2}{3}$?
    (a) $\frac{-17}{12}$ (b) $\frac{17}{12}$
    (c) $\frac{12}{9}$ (d) $\frac{11}{12}$

11. A rational number between $-\frac{3}{5}$ and $\frac{1}{4}$ is
    (a) $\frac{7}{40}$ (b) $-\frac{7}{40}$
    (c) $\frac{9}{40}$ (d) $-\frac{9}{40}$

12. The value of $\frac{3}{5} + \frac{3}{5} + \ldots$ upto 25 times is
    (a) 25 (b) 10 (c) 15 (d) 35

13. $8\frac{1}{4} + 8\frac{1}{2} + ? = 20\frac{1}{8}$
    (a) $8\frac{1}{4}$ (b) $3\frac{5}{8}$
    (c) $3\frac{3}{8}$ (d) $8\frac{5}{9}$

14. Which of the following is correct?
    (a) $a + 0 = b$ (b) $-a \times b = b \times (-a)$
    (c) $a - b = b - a$ (d) $\frac{a}{b} = \frac{b}{a}$

15. The value of $2\frac{4}{5} \div 3\frac{1}{2}$ of $\frac{4}{5}$ is
    (a) 0 (b) 1
    (c) 2 (d) 3

16. By division algorithm, which of the following is correct?
(a) $41 = 7 \times 5 + 6$ (b) $56 = 5 \times 11 + 2$
(c) $30 = 5 \times 8 - 5$ (d) $25 = 5 \times 4 + 4$

17. If $157x234$ is divisible by 3, then the digit at the place of x is
(a) 0 (b) 1 (c) 2 (d) 4

18. By which number, 91476 is not divisible?
(a) 11 (b) 7 (c) 3 (d) 8

19. If a number $573xy$ is divisible by 90, then the value of $x + y$ is
(a) 13 (b) 3 (c) 8 (d) 6

20. Which number is divisible by 5 and 9 both?
(a) 585 (b) 285 (c) 389 (d) 560

21. Which number is not divisible by 6?
(a) 270 (b) 385
(c) 312 (d) 432

22. Which number is divisible by 5 and 25 both?
(a) 2170 (b) 5125
(c) 3107 (d) 4115

23. What least number should be subtracted from 1365 to get a number exactly divisible by 25?
(a) 15 (b) 5 (c) 10 (d) 20

24. Which of the following number is divisible by 9?
(a) 4621 (b) 2834 (c) 9216 (d) 1560

25. By how much $\frac{3}{4}$th of 52 is lesser than $\frac{2}{3}$rd of 99?
(a) 27 (b) 33 (c) 39 (d) 66

26. The value of K, where $31K2$ is divisible by 6, is
(a) 1 (b) 2
(c) 3 (d) 7

Answers

1	(b)	2	(a)	3	(d)	4	(d)	5	(a)	6	(c)	7	(a)	8	(a)	9	(b)	10	(a)
11	(b)	12	(c)	13	(c)	14	(b)	15	(b)	16	(a)	17	(c)	18	(d)	19	(b)	20	(a)
21	(b)	22	(b)	23	(a)	24	(c)	25	(a)	26	(c)								

Hints and Solutions

1. A number in the form $\frac{p}{q}$ is said to be a rational number, if p and q are integers and $q \neq 0$.

2. We have, $\frac{3}{8} + \left(\frac{-5}{7}\right) = \frac{-19}{56}$

$\because \frac{3}{8}$ and $\frac{-5}{7}$ are rational numbers and their addition is $\frac{-19}{56}$, which is also a rational number.

Hence, the rational numbers are closed under addition.

3. Rational numbers are not closed under division.

As, 1 and 0 are the rational numbers but $\frac{1}{0}$ is not defined.

4. We can find infinite rational numbers between any two given rational numbers.

5. We have, $\frac{3}{4} \times \frac{2}{5} \times \frac{25}{3} = \frac{1}{2} \times \frac{1}{5} \times 25 = \frac{5}{2}$

6. 101 is not divisible by any of the prime numbers 2, 3, 5, 7, 11.

$\therefore$ 101 is smallest three-digit prime number.

7. Let x should be added to $-\frac{5}{7}$ to get $-\frac{3}{2}$.

Then, $\frac{-5}{7} + x = \frac{-3}{2}$

$\Rightarrow \quad x = \frac{-3}{2} + \frac{5}{7} = \frac{-21+10}{14} = -\frac{11}{14}$

Hence, $-\frac{11}{14}$ should be added.

8. The additive identity for the rational number is 0.

9. By options,

(a) $-5+3=-2$

and $3+(-5)=-2$, which are equal.

(b) $\frac{-8}{12}=\frac{10}{-15} \Rightarrow \frac{-2}{3}=\frac{-2}{3}$, which is true.

(c) 2 is a natural number

(d) 17 is also a prime number.

10. Let x be subtracted.

Then, $\frac{-3}{4}-x=\frac{2}{3}$

$\Rightarrow \quad \frac{-3}{4}-\frac{2}{3}=x$

$\Rightarrow \quad x=\frac{-9-8}{12}=\frac{-17}{12}$

Hence, $-\frac{17}{12}$ should be subtracted.

11. The rational number between $-\frac{3}{5}$ and $\frac{1}{4}$

is $\dfrac{\frac{-3}{5}+\frac{1}{4}}{2}=\frac{-12+5}{20\times 2}=-\frac{7}{40}$

$\left[\because \text{ A rational number between two rational number} = \frac{\text{Sum of rational number}}{2}\right]$

12. $\frac{3}{5}+\frac{3}{5}+ \ldots\ldots$ upto 25 times

$=\frac{3}{5}\times 25=3\times 5=15$

13. $8\frac{1}{4}+8\frac{1}{2}+?=20\frac{1}{8}$

$\Rightarrow \quad ?=20\frac{1}{8}-8\frac{1}{4}-8\frac{1}{2}$

$=(20-8-8)+\left(\frac{1}{8}-\frac{1}{4}-\frac{1}{2}\right)$

$=4+\frac{1-2-4}{8}$

$=4+\frac{-5}{8}=\frac{27}{8}=3\frac{3}{8}$

14. $-a\times b=b\times(-a)$

Because multiplication of two numbers in any order are same.

15. We have, $2\frac{4}{5}\div 3\frac{1}{2}$ of $\frac{4}{5}$

$=\frac{14}{5}\div\frac{7}{2}\times\frac{4}{5}$

$=\frac{14}{5}\div 7\times\frac{2}{5}$

$=\frac{14}{5}\div\frac{14}{5}=1$

16. By option (a),

$\text{RHS}=7\times 5+6=35+6=41=\text{LHS}$

$\therefore \quad 41=7\times 5+6$ is correct.

17. $\because$ Sum of digits $=1+5+7+x+2+3+4$

$=22+x$

This addition is divisible by 3, if 2 is at the place of x,

i.e. $\quad 22+x=22+2=24$, divisible by 3.

So, $\quad x=2$

18. $\because$ We have, to check

divisible by 11 91476 = Sum of odd places digits − Sum of even places digits

$=(9+4+6)-(1+7)$

$=19-8=11$, which is divisible by 11.

Divisible by 7 $=19476=194-2(76)=194-152$

$=42$, which is divisible by 7.

Divisible by 2 given number is even number, hence it is divisible by 2.

Divisible by 8 $91476\rightarrow 476$ is not divisible by 8.

Hence, 91476 is not divisible by 8.

19. We know that, if any number divisible by 90, i.e. divisible by 9 and 10.

So, to make number $573xy$ divisible by 10, we have to put $y=0$ to make unit digit 0, (i.e. 0 unit digit number divisible by 10), i.e. $y=0$.

Now, to also make $573x0$ divisible by 9,

$5+7+3+x+10=15+x=15+\underline{3}=18$,

i.e. divisible by 3.

So, $\quad x=3$

$\therefore \quad x+y=3+0=3$

20. By option (a),

We have, 585, divisible by 5 because unit digit is 5, and sum of digits $=5+8+5=18=1+8=9$ which is also divisible by 9.

21. We know that, if any number divisible by 2 and 3, it is also divisible by 6.

$\therefore$ By option (b), 385 is not divisible by 2. Hence, it is also not divisible by 6.

22. We know that,

Number is divisible by 5, if last digit is 5 or 0.

And divisible by 25, if last two digits is divisible by 25.

Hence, 5125 is only number divisible by 5 and 25 both.

23. 25)1365(54

$$\begin{array}{r} 125 \\ \hline 115 \\ 100 \\ \hline 15 \\ \hline \end{array}$$

$\therefore$ Required number is 15.

24. We have, $4621 = 4 + 6 + 2 + 1 = 13$, not divisible by 9

$2834 = 2 + 8 + 3 + 4 = 17$, not divisible by 9

$9216 = 9 + 2 + 1 + 6 = 18$, divisible by 9

$1560 = 1 + 5 + 6 + 0 = 12$, not divisible by 9

Hence, 9216 is only number divisible by 9.

25. $\therefore$ Required answer

$$= \frac{2}{3} \times 99 - \frac{3}{4} \times 52$$

$$= 2 \times 33 - 3 \times 13$$

$$= 66 - 39 = 27$$

26. The number $31K2$ is divisible by 6, it mean it is divisible by 2 and 3 both.

Here, unit digit is 2 (even), so this number is divisible by 2.

Now, for 3, first we have to add all the digits.

$\therefore\ 3 + 1 + K + 2 = 6 + K$

$= 6 + 3 = 9,$

for $K = 3$, it is divisible by 3.

Hence, $K = 3$

CHAPTER

02

SQUARE AND SQUARE ROOTS

In this chapter, we study the squares of number and their properties. Also finding the square root of numbers and decimal numbers by prime factorisation and long division method.

Square

The square of a number is the product of the number itself, i.e. $a \times a = a^2$.

Perfect Square

A given number is said to be a perfect square, if it can be expressed as the product of two equal factors. A natural number 'n' is a perfect square if $n = m^2$ for any natural number m.

e.g. $4 = 2^2$ or 2×2 and $25 = 5^2$ is a perfect square.

Squares from 1 to 20 numbers

Numbers	Squares	Numbers	Squares
1	1	11	121
2	4	12	144
3	9	13	169
4	16	14	196
5	25	15	225
6	36	16	256
7	49	17	289
8	64	18	324
9	81	19	361
10	100	20	400

Properties of Square

(i) The number of zeroes at the end of a perfect square is always even.

(ii) A perfect square leaves a remainder 0 or 1 on division by 3.

(iii) The number ending with an odd number of zeroes is never a perfect square.

(iv) Squares of even numbers are always even.

(v) Squares of odd numbers are always odd.

Example 1 Which are of the following will have odd unit digit?

(a) $(32)^2$ (b) $(35)^2$ (c) $(64)^2$ (d) $(68)^2$

Sol. (b) We know that that square of odd number in an odd number

The number $(35)^2$ have odd unit digit.

Pythagorean Triplet

In a triplet (m, n, p) of three natural numbers m, n and p is called a Pythagorean triplet, if $m^2 + n^2 = p^2$. It is easy to prove that for any natural number m greater than 1, $(2m, m^2 - 1, m^2 + 1)$ is a Pythagorean triplet.

Example 2 Find the value of $(13)^2 + 3 \times 7$.

(a) 190 (b) 189 (c) 191 (d) 192

Sol. (a) $(13)^2 + 3 \times 7 = 169 + 21 = 190$

Square Root

The square root of the number x is the number which multiplied by itself gives x as the product. It is denoted by the symbol $\sqrt[2]{\ }$ or $\sqrt{\ }$.

e.g. If $y = x^2$, then we call, x is a square root of y,

i.e. $x = \pm\sqrt{y}$.

Properties of Square Root

(i) If the unit digit of a number is 2, 3, 7 or 8, then it does not have a square root.

(ii) Square root of even number is even.

(iii) Square root of odd number is odd.

Square Root of a Perfect Square by the Prime Factorisation Method

The following steps are given below.

I. Resolve the given number into prime factors.

II. Make pairs of similar factors.

III. Choose one prime from each pair and multiply all primes.

Thus, the product obtained is the square root of given number.

Example 3 The square root of 1764 is

(a) 41 (b) 43 (c) 42 (d) 40

Sol. (c) By prime factorisation method,

2	1764
2	882
3	441
3	147
7	49
7	7
	1

$$\therefore \quad \sqrt{1764} = \sqrt{\overline{2 \times 2} \times \overline{3 \times 3} \times \overline{7 \times 7}}$$
$$= 2 \times 3 \times 7 = 42$$

Square Root of a Perfect Square by Long Division Method

If it is not easy to evaluate square root using prime factorisation method, then we use division method.

The steps of this method can be easily understood with the help of following examples.

e.g. Find the square root of 18769.

Step I In the given number, mark off the digits in pairs starting from the unit digit. Each pair and the remaining one digit (if any) is called a period.

Step II Choose a number whose square is less than or equal to 1. Here, $1^2 = 1$, on subtracting, we get 0 (zero) as remainder.

Step **III** Bring down the next period, i.e. 87. Now, the trial divisor is $1 \times 2 = 2$ and trial dividend is 87. So, we take 23 as divisor and put 3 as quotient. The remainder is 18 now.

	137
1	$\overline{1}\,\overline{87}\,\overline{69}$
	1
23	87
	69
267	1869
	1869
	×

Step **IV** Bring down the next period, which is 69. Now, trial divisor is $13 \times 2 = 26$ and trial dividend is 1869. So, we take 267 as dividend and 7 as quotient. The remainder is 0.

Step **V** The process (processes like III and IV) goes on till all the periods (pairs) come to an end and we get remainder as 0 (zero) now.

Hence, the required square root $= 137$

Example 4 2025 plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and the number of plants in each now.

(a) 55, 45 (b) 45, 45 (c) 35, 35 (d) 36, 36

Sol. *(b)* Let the number of rows be x.

Then, number of plants in a row $= x$

So, number of plants to be planted in a garden

$$x \times x = x^2$$

According to the question,

Total number of plants to be planted = 2025

$\therefore \quad x^2 = 2025$

3	2025
3	675
3	225
3	75
5	25
5	2
	1

$$\Rightarrow \quad x = \sqrt{2025} = \sqrt{3 \times 3 \times 3 \times 3 \times 5 \times 5}$$

$$= 3 \times 3 \times 5 = 45$$

Hence, the number of rows is 45 and the number of plants in each row is 45.

Formula for Finding the Number of Digits in the Square Root of a Perfect Square

If any perfect square number contains 'n' digits. then, its square root will contain $\frac{n}{2}$ digits, when n is even and $\frac{n+1}{2}$ digits, when n is odd.

e.g. Square root of 64 is 8. $[\because n = 2$ i.e. even$]$

Also, square root of 144 is 12. $[\because n = 3$ i.e. odd$]$

Square Root of Product of Numbers and Rational Number

(i) The square root of product of integers is the square root of integer by taking separately, for any integer a and b, we have

$$\sqrt[2]{ab} = \sqrt[2]{a}\ \sqrt[2]{b}$$

(ii) The square root of rational number $\frac{a}{b}$ is

$$\sqrt[2]{\frac{a}{b}} = \frac{\sqrt[2]{a}}{\sqrt[2]{b}}$$

Square Root of Number in Decimal Form

Make the number of decimal places even by affixing zero, if necessary. Now, mark bars and find out the square root by the long division method. Put the decimal point in the square root as soon as the integral part is completed.

Example 5 Find the square root of 176.252176.

(a) 13.276 (b) 13.801 (c) 13.295 (d) 13.218

Sol. *(a)* By long division method,

	13.276
1	$\overline{1}\,\overline{76}.\overline{25}\,\overline{21}\,\overline{76}$
	1
23	76
	69
262	725
	524
2647	20121
	18529
26546	159276
	159276
	×

$\therefore \quad \sqrt{176.252176} = 13.276$

PRACTICE EXERCISE

1. The value of $(15)^2 + (8)^2 + 2$ is
   (a) 289 (b) 291 (c) 293 (d) 295

2. Which of the following cannot be a perfect square?
   (a) 841 (b) 529
   (c) 198 (d) All of these

3. Which of the following cannot be a digit in the unit place of a perfect square ?
   (a) 0 (b) 1 (c) 5 (d) 7

4. The square root of 73.96 is
   (a) 8.6 (b) 86
   (c) 0.86 (d) None of these

5. If $x = \sqrt{3018 + \sqrt{36 + \sqrt{169}}}$, then the value of x is
   (a) 55 (b) 44 (c) 63 (d) 42

6. $\sqrt{12} + \sqrt{24}$ is equal to
   (a) $2\sqrt{3} + 3\sqrt{2}$ (b) $4\sqrt{3} + \sqrt{6}$
   (c) $\sqrt{7} + 2\sqrt{3}$ (d) $2\sqrt{6} + 2\sqrt{3}$

7. Which one of the following will have even unit digit?
   (a) $(43)^2$ (b) $(37)^2$
   (c) $(63)^2$ (d) $(34)^2$

8. If the area of an equilateral triangle is $24\sqrt{3}\ \text{m}^2$, then its perimeter is
   (a) $12\sqrt{6}$ m (b) $9\sqrt{6}$ m
   (c) $8\sqrt{3}$ m (d) $4\sqrt{3}$ m

9. The value of $(301)^2 - (300)^2$ is
   (a) 1 (b) 601
   (c) 106 (d) 100

10. A General arranges his soldiers in rows to form a perfect square. He finds that in doing so, 60 soldiers are left out. If the total number of soldiers be 8160. Then, the number of soldiers in each row is
    (a) 90 (b) 91
    (c) 92 (d) 80

11. The greatest six digit number which is a perfect square is
    (a) 998004 (b) 998006
    (c) 998049 (d) 998001

12. What is that fraction which when multiplied by itself gives 227.798649 ?
    (a) 15.093 (b) 15.099
    (c) 14.093 (d) 9.0019

13. In a triplet $(6, a, 10)$ what value of 'a' will make it a Pythagorean triplet ?
    (a) 4 (b) 16
    (c) 8 (d) 5

14. If a number is increased by two times, then the square of the number will increase
    (a) two times (b) three times
    (c) four times (d) five times

15. Two numbers are in the ratio of 9 : 7. If the difference of their square is 288, then the smaller of the number is
    (a) 21 (b) 24
    (c) 27 (d) 28

16. The number of digits in the square root of 298116 is
    (a) 4 (b) 5
    (c) 3 (d) 6

17. If $\sqrt{2401} = \sqrt{7^x}$, then the value of x is
    (a) 3 (b) 4
    (c) 5 (d) 6

18. The least number to be added to 269 to make it a perfect square is
    (a) 31 (b) 16
    (c) 17 (d) 20

19. If $\sqrt{18225} = 135$, then the value of $\sqrt{18225} + \sqrt{182.25} + \sqrt{1.8225} + \sqrt{0.018225}$ is
    (a) 1.49985 (b) 14.985
    (c) 149.985 (d) 1499.85

20. The expression

$$\sqrt{\frac{0.85\times(0.105+0.024-0.008)}{0.022\times 0.25\times 1.7}}$$

simplifies to

(a) $\sqrt{11}$ (b) $\sqrt{1.1}$
(c) 11 (d) $\sqrt{0.011}$

21. The value of $\sqrt{\frac{16}{36}+\frac{1}{4}}$ is

(a) $\frac{2}{5}$ (b) $\frac{1}{3}$ (c) $\frac{5}{3}$ (d) $\frac{5}{6}$

22. If $\frac{52}{x}=\sqrt{\frac{169}{289}}$, then the value of x is

(a) 52 (b) 58 (c) 62 (d) 68

23. A square board has an area of 144 sq units. How long is each side of the board?

(a) 11 units (b) 12 units
(c) 13 units (d) 14 units

24. If $\sqrt{1+\frac{25}{144}}=1+\frac{x}{12}$, then x is equal to

(a) 1 (b) 2 (c) 5 (d) 9

25. The value of $\frac{\sqrt{80}-\sqrt{112}}{\sqrt{45}-\sqrt{63}}$ is

(a) $\frac{3}{4}$ (b) $1\frac{3}{4}$
(c) $1\frac{1}{3}$ (d) $1\frac{7}{9}$

26. The least number which is added to 17420 will make it a perfect square is

(a) 3 (b) 5 (c) 9 (d) 4

27. If $\sqrt{0.09\times 0.09\times x}=0.09\times 0.09\times\sqrt{z}$, then the value of $\frac{x}{z}$

(a) 0.0081 (b) 0.810
(c) 0.801 (d) 8.09

Answers

1	(b)	2	(c)	3	(d)	4	(a)	5	(a)	6	(d)	7	(d)	8	(a)	9	(b)	10	(a)
11	(d)	12	(a)	13	(c)	14	(c)	15	(a)	16	(c)	17	(b)	18	(d)	19	(c)	20	(a)
21	(d)	22	(d)	23	(b)	24	(a)	25	(c)	26	(d)	27	(a)						

Hints and Solutions

1. $(15)^2+(8)^2+2=225+64+2=291$

2. We know that, a number ending with digits 2, 3, 7 or 8 can never be a perfect square. So, 198 cannot be written in the form of a perfect square.

3. Digit 7 cannot be a place of a perfect square.

4. By using long division method,

$$\begin{array}{r|l} & 8.6 \\ \hline 8 & \overline{73}.\overline{96} \\ +8 & 64 \\ \hline 166 & 996 \\ & 996 \\ \hline & \times \end{array}$$

Hence, $\sqrt{73.96}=8.6$

5. According to question,

$$\sqrt{3018+\sqrt{36+\sqrt{169}}}=\sqrt{3018+\sqrt{36+13}}$$
$$=\sqrt{3018+7}=\sqrt{3025}=55$$

6. $\sqrt{12}+\sqrt{24}=\sqrt{2\times 2\times 3}+\sqrt{2\times 2\times 6}$

$$=2\sqrt{3}+2\sqrt{6}$$

7. We know that, the square of even number is even.

$\therefore$ The number $(34)^2$ has even unit digit.

8. Let the side of an equilateral triangle be a m.

Since, the area of an equilateral triangle

$$=24\sqrt{3}\,\text{m}^2$$

$\Rightarrow \quad \frac{\sqrt{3}}{4}a^2 = 24\sqrt{3} \Rightarrow a^2 = 96$

$\Rightarrow \quad a = 4\sqrt{6}$ m

9. $(301)^2 - (300)^2$

$= (301 + 300)(301 - 300)$

$[\because a^2 - b^2 = (a+b)(a-b)]$

$= (601) \times 1$

$= 601$

10. Total number of soldiers arranged

$= 8160 - 60 = 8100$

Since, the number of soldiers in each row is equal to number of rows.

$\therefore$ Number of soldiers in each row

$= \sqrt{8100}$

$= \sqrt{9 \times 9 \times 10 \times 10} = 90$

11. The greatest six digit number = 999999

	999
9	99 99 99
	81
189	1899
	1701
1989	19899
	17901
	1998

×

$\therefore$ The greatest number of six digit which is a perfect square

$= 999999 - 1998$

$= 998001$

12. Let the fraction be x.

Then, $\quad x^2 = 227.798649$

$\therefore \quad x = \sqrt{227.798649} = 15.093$

[$\because$ using long divison method]

13. $6^2 = 36$, $a^2 = a^2$, $10^2 = 100$

By Pythagoraem triplet, $6^2 + a^2 = 10^2$

$\Rightarrow \quad a^2 = 10^2 - 6^2$

$\Rightarrow \quad a^2 = 100 - 36 = 64$

$\Rightarrow \quad a = \sqrt{64} = 8$

14. Let the number be y.

If the number is increased by two times it becomes $2y$.

Square of the number $= (2y)^2 = 4y^2$

$\therefore$ The number will be increased by four times.

15. Let the number be $9x$ and $7x$.

According to the question,

$81x^2 - 49x^2 = 288$

$\Rightarrow \quad x^2 = \frac{288}{32} \Rightarrow x^2 = 9 \Rightarrow x = 3$

$\therefore$ The smaller number is 21.

16.

2	298116
2	149058
3	74529
3	24843
7	8281
7	1183
13	169
13	13
	1

$\Rightarrow \quad \sqrt{298116} = 2 \times 3 \times 7 \times 13$

$= 546$

$\therefore$ Number of digits = 3

17. $\sqrt{2401} = \sqrt{7^x}$

$\Rightarrow \quad 2401 = 7^x$

$\Rightarrow \quad 7^4 = 7^x$

$\therefore \quad x = 4$

18. We know, $256 < 269 < 289$

$\Rightarrow (16)^2 < 269 < (17)^2$

$\therefore$ Number to be added $= (17)^2 - 269$

$= 289 - 269 = 20$

19. $\sqrt{18225} + \sqrt{182.25} + \sqrt{1.8225} + \sqrt{0.018225}$

$= \sqrt{18225} + \sqrt{\frac{18225}{100}} + \sqrt{\frac{18225}{10000}} + \sqrt{\frac{18225}{1000000}}$

$= 135 + \frac{135}{10} + \frac{135}{100} + \frac{135}{1000}$

$= 135 + 13.5 + 1.35 + 0.135$

$= 149.985$

20. $\sqrt{\dfrac{0.85\times(0.105+0.024-0.008)}{0.022\times0.25\times1.7}}$

$$=\sqrt{\frac{0.85\times0.121}{0.022\times0.25\times1.7}}$$

$$=\sqrt{\frac{85\times121\times10}{22\times25\times17}}=\sqrt{11}$$

21. $\sqrt{\dfrac{16}{36}+\dfrac{1}{4}}=\sqrt{\dfrac{16+9}{36}}$

$$=\sqrt{\frac{25}{36}}=\frac{5}{6}$$

22. Given, $\dfrac{52}{x}=\sqrt{\dfrac{169}{289}}$

$\Rightarrow \quad x=\dfrac{52\times17}{13}=68$

23. Given, area of square board $=144$ sq units

$\therefore \quad (\text{side})^2=144 \quad [\because \text{area of square}=(\text{side})^2]$

$\Rightarrow \quad (\text{side})^2=12$

$\Rightarrow \quad \text{side}=12$ units

Hence, the length of each side of the board is 12 units.

24. Given, $\sqrt{1+\dfrac{25}{144}}=1+\dfrac{x}{12}$

$\Rightarrow \quad \sqrt{\dfrac{169}{144}}=1+\dfrac{x}{12}$

$\Rightarrow \quad \dfrac{13}{12}=1+\dfrac{x}{12}$

$\Rightarrow \quad \dfrac{x}{12}=\dfrac{1}{12}\Rightarrow x=1$

25. $\dfrac{4\sqrt{5}-4\sqrt{7}}{3\sqrt{5}-3\sqrt{7}}=\dfrac{4(\sqrt{5}-\sqrt{7})}{3(\sqrt{5}-\sqrt{7})}=\dfrac{4}{3}=1\dfrac{1}{3}$

26. Since, 17420 lies between 131^2 and 132^2.

Now, $(132)^2=17424$

Hence, it should be 4 added.

27. $\sqrt{0.09\times0.09\times x}=0.09\times0.09\times\sqrt{z}$

$\Rightarrow \quad 0.09\times\sqrt{x}=0.09\times0.09\times\sqrt{z}$

$\Rightarrow \quad \dfrac{\sqrt{x}}{\sqrt{z}}=0.09$

$\Rightarrow \quad \dfrac{x}{z}=(0.09)^2$ (squaring both sides)

$\Rightarrow \quad \dfrac{x}{z}=0.0081$

CHAPTER

03

CUBE AND CUBE ROOTS

In this chapter, we study the cubes of numbers and their properties, and cube roots by prime factorisation method.

Cube

The cube of a number is the product of the number itself thricely.

e.g. If x is a non-zero number, then $x \times x \times x = x^3$ is called cube of x.

The cube of rational number is the cube of the numerator divided by the cube of denominator. e.g. Cube of $\frac{4}{5}$ is $\frac{64}{125}$.

Perfect Cube

A natural number n is said to be a perfect cube if there is an integer m such that $n = m \times m \times m$.

Cubes from 1 to 15 numbers

Numbers	Cubes
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512

Numbers	Cubes
9	729
10	1000
11	1331
12	1728
13	2197
14	2744
15	3375

Properties of Cube of Numbers

(i) Cubes of all even natural numbers are always even.

(ii) Cubes of all odd natural numbers are always odd.

(iii) Cubes of negative integers are always negative.

(iv) For any rational number $\frac{a}{b}$, we have

$$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$$

Example 1 Find the value of $(11)^3 + 4(7)^3 - 5$.

(a) 2697 (b) 2698 (c) 2699 (d) 3000

Sol. *(b)* $(11)^3 + 4(7)^3 - 5 = 1331 + 4(343) - 5$

$= 1331 + 1372 - 5 = 2698$

Example 2 Which one of the following will have odd unit digit

(a) $(24)^3$ (b) $(64)^3$

(c) $(27)^3$ (d) $(52)^3$

Sol. *(c)* We knwo cube of odd number is odd number.

$\therefore$ Number $(27)^3$ have odd unit digit

Cube Root

If n is perfect cube for any integer m i.e. $n = m^3$, then m is called the cube root of n and it is denoted by $m = \sqrt[3]{n}$.

Cube Root of a Perfect Cube by Prime Factorisation

The following steps are given below.

I. Factorise the given number into prime factors.

II. Make triples of similar factors or arrange them in group of three equal factors at a time.

III. Choose one prime from each pair and multiply all primes.

Example 3 Find the cube root of 74088.

(a) 40 (b) 47 (c) 42 (d) 45

Sol. *(c)* Resolving the given number, we get

2	74088
2	37044
2	18522
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$$74088 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{7 \times 7 \times 7}$$

$$\therefore \quad \sqrt[3]{74088} = 2 \times 3 \times 7 = 42$$

Cube Root of a Negative Cube

If a is a positive integer, then $-a$ is a negative integer.

We know that, $(-a)^3 = -a^3$

So, $\sqrt[3]{-a^3} = -a$

In general, we have $\sqrt[3]{-x} = -\sqrt[3]{x}$

Cube Root of Product of Numbers and Rational Number

(i) The cube root of product of integers is the cube root of integer by taking separately.

For any two integer a and b, we have

$$\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$$

(ii) Cube Root of Rational Number $\frac{a}{b}$ is $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

Example 4 The simplified form of $\sqrt[3]{125 \times 64}$ is

(a) 20 (b) 40 (c) 60 (d) 80

Sol. *(a)* $125 \times 64 = \underline{5 \times 5 \times 5} \times \underline{4 \times 4 \times 4}$

$\therefore$ LHS $= \sqrt[3]{125 \times 64} = \sqrt[3]{(5 \times 4)^3} = (5 \times 4) = 20$

Now, $\sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5} = 5$

and $\sqrt[3]{64} = \sqrt[3]{4 \times 4 \times 4} = 4$

$\therefore$ RHS $= \sqrt[3]{125} \times \sqrt[3]{64} = (5 \times 4) = 20$

$\Rightarrow$ LHS = RHS

Hence, $\sqrt[3]{125 \times 64} = \sqrt[3]{125} \times \sqrt[3]{64}$

Example 5 Find the cube root of 4.096.

(a) 1.6 (b) 165
(c) 1.75 (d) None of these

Sol. (a) $\sqrt[3]{4.096} = \sqrt[3]{\frac{4096}{1000}} = \frac{\sqrt[3]{4096}}{\sqrt[3]{1000}}$

$\therefore \quad 4096 = \underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{2\times2\times2}\times\underline{2\times2\times2}$

$\therefore \quad \sqrt[3]{4096} = 2\times2\times2\times2 = 16$

Also, $\sqrt[3]{1000} = \sqrt[3]{10\times10\times10} = 10$

So, $\frac{\sqrt[3]{4096}}{\sqrt[3]{1000}} = \frac{16}{10} = 1.6$

Hence, $\sqrt[3]{4.096} = 1.6$

Important Points

(i) If 1, 4, 5, 6 and 9 in the unit place, then cube of that number given the same digit in the unit place.

(ii) 3 in the unit place have cube with 7 in the unit place.

(iii) 7 in the unit place have cube with 3 in the unit place.

(iv) 2 in the unit place have cube with 8 in the unit place.

(v) 8 in the unit place have cube with 2 in the unit place.

Example 6 Difference of two perfect cubes is 189. If the cube root of the smaller of the two numbers is 3, then find the cube root of the larger number.

(a) 5 (b) 6 (c) 7 (d) 8

Sol. (b) Given difference of two perfect cube = 189 and cube root of the smaller number = 3

$\therefore$ Cube of smaller number $= (3)^3 = 27$

Let cube root of the larger number be x.

Then, cube of larger number $= x^3$

According to the question,

$$x^3 - 27 = 189$$

$$\Rightarrow \quad x^3 = 189 + 27 \Rightarrow x^3 = 216$$

$$\Rightarrow \quad x = \sqrt[3]{216} = \sqrt[3]{6\times6\times6}$$

$$\therefore \quad x = 6$$

Hence, the cube root of the larger number is 6.

PRACTICE EXERCISE

1. Which of the following is not a perfect cube ?

(a) 1000000
(b) 216
(c) 10000
(d) None of the above

2. Which of the following perfect cube is the cube of an even number ?

(a) 343 (b) 2197
(c) 216 (d) 1331

3. Which of the following perfect cube is the cube of an odd number ?

(a) 1728 (b) 512
(c) 729 (d) 1000

4. If a number is increased by four times, then cube of the number will increase

(a) 64 times (b) 69 times
(c) 729 times (d) 1000 times

5. The value of $\sqrt[3]{\frac{27}{125}}$ is

(a) $\frac{3}{5}$ (b) $\frac{3}{25}$
(c) $\frac{9}{25}$ (d) not appropriate data

6. The unit place of cube roots of 117649 is

(a) 4 (b) 9
(c) 49 (d) None of these

7. The smallest number by which 3087 may be multiplied so that the product is a perfect cube, is
(a) 6 (b) 5
(c) 4 (d) 3

8. The smallest by which 392 may be divided so that the quotient is a perfect cube, is
(a) 50 (b) 51
(c) 49 (d) 62

9. In a number pattern, 8, 27, 64, x, the value of x will be
(a) 125 (b) 216 (c) 100 (d) 115

10. The cube of 31.2 is
(a) 3037.1328 (b) 3037.1381
(c) 30371.328 (d) 30371328

11. The sum of the cubes of first three natural number is equal to
(a) $(1+2+3)^3$ (b) $(1+2+3)^2$
(c) 3^3 (d) 3^2

12. The cube root of $\frac{-343}{1331}$ is
(a) $\frac{7}{11}$ (b) $\frac{-7}{11}$ (c) $\frac{11}{7}$ (d) $\frac{-11}{7}$

13. If one side of a cube is 33 m, then the volume of the cube is
(a) 35937 (b) 35936
(c) 3934 (d) None of these

14. Three numbers are in the ratio of $1:2:3$. The sum of their cubes is 121500. The numbers are
(a) 15, 30, 45 (b) 30, 45, 15
(c) 7, 14, 21 (d) None of these

15. If the volume of a cubical box is 35.937 m^3, what is the length of its one side ?
(a) 3.3 m (b) 6.6 m
(c) 3.6 m (d) None of these

16. $(-216 \times 729)^{1/3}$
(a) 54 (b) −54 (c) −45 (d) 45

17. The value of $\sqrt[3]{0.064}+\sqrt[3]{27}-\sqrt[3]{729}$ is
(a) 12.4 (b) −2 (c) 5.6 (d) −5.6

18. The value of $\frac{\sqrt[3]{8}}{\sqrt{16}} \div \sqrt{\frac{100}{49}} \times \sqrt[3]{125}$ is
(a) 7 (b) $1\frac{3}{4}$ (c) $\frac{7}{100}$ (d) $\frac{4}{7}$

19. The value of $\sqrt{\sqrt[3]{0.004096}}$ is
(a) 0.4 (b) 0.04
(c) 0.16 (d) 4

20. The value of $(27 \times -2744)^{1/3}$ is
(a) − 40 (b) − 42
(c) − 22 (d) − 32

21. The value of $\frac{(73)^3+(53)^3}{73\times73-73\times53+53\times53}$ is
(a) 126 (b) 216 (c) 162 (d) − 126

22. What is the smallest number in which we multiply by 392, we get the factors in the form of cube ?
(a) 5 (b) 6
(c) 7 (d) 8

23. The value of $\sqrt[3]{\frac{27}{64}}$ is
(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{5}{4}$ (d) $\frac{5}{3}$

24. If $\sqrt{\frac{x}{0.0064}}=\sqrt[3]{0.008}$, then the value of x is
(a) 0.256 (b) 0.0256
(c) 0.000256 (d) 0.00256

25. The smallest number in which we multiply by 1800, we get the cube. Then, the sum of digit is
(a) 2 (b) 3 (c) 6 (d) 8

26. If cube root of 175616 is 56, then the value of $\sqrt[3]{175.616}+\sqrt[3]{0.175616}+\sqrt[3]{0.000175616}$ is
(a) 0.168 (b) 6.216
(c) 6.116 (d) 62.16

27. The cube root of 0.000001 is
(a) 0.1 (b) 0.01
(c) 0.316 (d) 0.031

Answers

1	(c)	2	(c)	3	(c)	4	(a)	5	(a)	6	(b)	7	(d)	8	(c)	9	(a)	10	(c)
11	(b)	12	(b)	13	(a)	14	(a)	15	(a)	16	(b)	17	(d)	18	(b)	19	(a)	20	(b)
21	(a)	22	(c)	23	(a)	24	(c)	25	(c)	26	(b)	27	(b)						

Hints and Solutions

1. $\sqrt[3]{1000000} = 100$ is a perfect cube.

$\sqrt[3]{216} = 6$ is a perfect cube.

$\sqrt[3]{10000} =$ not a perfect cube.

2. 216 is the cube of an even number because cube of an even number is always even.

3. 729 is the cube of an odd number because the cube of odd number is always odd.

4. Let the number be x.

After increasing 4 times the number $= 4x$

Cube of the number $= (4x)^3 = 64x^3$

$\therefore$ The cube of a number increases by 64 times.

5. $\sqrt[3]{\dfrac{27}{125}} = \sqrt[3]{\dfrac{3\times3\times3}{5\times5\times5}} = \dfrac{3}{5}$

6. $\sqrt[3]{117649} = 49$

$\therefore$ Unit place is 9.

7. Writing 3087 as a product of a prime factors, we have

$$\begin{array}{r|l} 3 & 3087 \\ 3 & 1029 \\ 7 & 343 \\ 7 & 49 \\ 7 & 7 \\ & 1 \end{array}$$

$\therefore \quad 3087 = 3\times3\times\underline{7\times7\times7}$

Clearly, to make it a perfect cube it must be multiplied by 3.

8. Writing 392 as a product of prime factors, we have

$$\begin{array}{r|l} 2 & 392 \\ 2 & 196 \\ 2 & 98 \\ 7 & 49 \\ 7 & 7 \\ & 1 \end{array}$$

$\therefore \quad 392 = 2\times2\times2\times7\times7$

Clearly, to make it perfect cube it must be divided by (7×7) i.e., 49.

9. Here, we see that $8 = 2^3$, $27 = 3^3$, $64 = 4^3$.

It means given pattern is a cube of consecutive natural number.

$\therefore \quad x = 5^3 = 125$

10. $31.2\times31.2\times31.2 = 30371.328$

11. $1^3 + 2^3 + 3^3 = 1 + 8 + 27$

$= 36 = (6)^2 = (1+2+3)^2$

12. $\sqrt[3]{\dfrac{-343}{1331}} = \dfrac{\sqrt[3]{-343}}{\sqrt[3]{1331}} = \dfrac{\sqrt[3]{-7\times-7\times-7}}{\sqrt[3]{11\times11\times11}} = \dfrac{-7}{11}$

13. Volume of cube $= (33)^3 = 35937$

14. Let the number be x, $2x$ and $3x$.

$\therefore \quad (x)^3 + (2x)^3 + (3x)^3 = 121500$

$\Rightarrow \quad 1x^3 + 8x^3 + 27x^3 = 121500$

$\Rightarrow \quad 36x^3 = 121500$

$\Rightarrow \quad x^3 = \dfrac{121500}{36}$

$\Rightarrow \quad x^3 = 3375$

$\Rightarrow \quad x = \sqrt[3]{3375}$

$\Rightarrow \quad x = \sqrt[3]{15\times15\times15}$

$\Rightarrow \quad x = 15$

$\therefore$ The numbers are $x = 15$, $2x = 30$, $3x = 45$.

15. $\because$ Volume of a cube $= (\text{side})^3$

$(\text{side})^3 = 35.937$

$\Rightarrow \quad \text{side} = \sqrt[3]{35.937}$

$\Rightarrow \quad \text{side} = \sqrt[3]{3.3\times3.3\times3.3}$

$\Rightarrow \quad \text{side} = 3.3\text{ m}$

16. $(-216\times 729)^{1/3} = (-216)^{1/3}\times(729)^{1/3}$

$= -\underline{(6\times 6\times 6)}^{1/3}\times(\underline{3\times 3\times 3}\times\underline{3\times 3\times 3})^{1/3}$

$= -6\times 3\times 3 = -54$

17. $\sqrt[3]{0.064}+\sqrt[3]{27}-\sqrt[3]{729}$

$\because \quad \sqrt[3]{0.064} = \sqrt[3]{0.4\times 0.4\times 0.4} = 0.4$

$\sqrt[3]{27} = \sqrt[3]{3\times 3\times 3} = 3$

$\sqrt[3]{729} = \sqrt[3]{\underline{3\times 3\times 3}\times\underline{3\times 3\times 3}}$

$= 3\times 3 = 9$

$\therefore \quad \sqrt[3]{0.064}+\sqrt[3]{27}-\sqrt[3]{729} = 0.4+3-9 = -5.6$

18. $\dfrac{\sqrt[3]{8}}{\sqrt{16}}\div\sqrt{\dfrac{100}{49}}\times\sqrt[3]{125} = \dfrac{2}{4}\times\dfrac{7}{10}\times 5$

$= \dfrac{7}{4} = 1\dfrac{3}{4}$

19. $\sqrt{\sqrt[3]{0.004096}} = \sqrt{\sqrt[3]{(0.16)^3}} = \sqrt{0.16} = 0.4$

20. $(27\times -2744)^{1/3} = (27)^{1/3}\times(-2744)^{1/3}$

$= 3\times -14 = -42$

21. $\dfrac{(73)^3+(53)^3}{73\times 73-73\times 53+53\times 53}$

$= \dfrac{(73+53)(73\times 73-73\times 53+53\times 53)}{73\times 73-73\times 53+53\times 53}$

$= 73+53 = 126$

22.

2	392
2	196
2	98
7	49
7	7
	1

The divisible number of $392 = 2\times 2\times 2\times 7\times 7$. It is clear that on multiplying by 7, we get 392 cube number.

23. $\sqrt[3]{\dfrac{27}{64}} = \sqrt[3]{\left(\dfrac{3}{4}\right)^3} = \dfrac{3}{4}$

24. $\sqrt{x} = \sqrt{0.0064}\times\sqrt[3]{0.008}$

$\Rightarrow \sqrt{x} = 0.08\times 0.2 = 0.016$

$\Rightarrow \quad x = 0.000256$

25. $1800 = 2^3\times 3^2\times 5^2$

It is clear that we have to multiply by 15 to get the sum of digit is 6.

26. $\sqrt[3]{175.616}+\sqrt[3]{0.175616}+\sqrt[3]{0.000175616}$

$= 5.6+0.56+0.056 = 6.216$

27. $(0.000001)^{1/3} = \sqrt[3]{0.000001}$

$= \sqrt[3]{\dfrac{1}{1000000}} = \dfrac{1}{100}$

$= 0.01$

CHAPTER

04

EXPONENTS /POWERS AND SURD

In this chapter, we study the positive and negative exponents with their laws of exponents and also surds with their laws of exponent.

Exponential Form

The repeated multiplication of the same non-zero rational number a with itself in the form of a^n {i.e., $a \times a \times$ .... $\times a \times (n$ times$) = a^n$}, where a is called the base and n is an integer called the exponent or index. This type of representation of a number is called the exponential form of the given number. e.g. $6 \times 6 \times 6 = 6^3$

Here, 6 is the base and 3 is the exponent and we read it as "6 raised to the power of 3".

Rational Exponents

A rational exponents represent both an integer exponent and nth root.

e.g. $a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

Negative Integral Exponents

For any non-zero integer a, we have

$$a^{-n} = \frac{1}{n} \text{ or } a^{-n} \times a^n = 1$$

So, a^{-n} is the multiplicative inverse or reciprocal of a^n and *vice-versa*.

e.g. $(5)^{-2} = \frac{1}{5^2}$

Example 1 Find the multiplicative inverse of 10^{-5}.

(a) 10^4
(b) 10^5
(c) 10^6
(d) None of these

Sol. *(b)* We have, $10^{-5} = \frac{1}{10^5}$

Reciprocal of $\frac{1}{10^5} = 10^5$

$\therefore$ Multiplicative inverse of 10^{-5} is 10^5

$[\because\ 10^{-5} \times 10^5 = 10^0 = 1]$

Laws of Exponent

I. If a and b be any real numbers and m, n be positive integers, then

(i) $a^m \times a^n = a^{m+n}$

(ii) $a^m \div a^n = a^{m-n}, a \neq 0$

(iii) $(a^m)^n = a^{mn}$ (iv) $(ab)^n = a^n b^n$

(v) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ (vi) $(a)^0 = 1,\ a \neq 0$

II. If a and b be any real numbers and m, n be negative integers, then

(i) $a^{-m} \times a^{-n} = \frac{1}{a^m} \times \frac{1}{a^n}$

$$= \frac{1}{a^m \times a^n} = \frac{1}{a^{m+n}} = a^{-(m+n)}$$

(ii) $a^{-m} \div a^{-n} = \frac{1}{a^m} \div \frac{1}{a^n} = \left(\frac{1}{a^m} \times \frac{a^n}{1}\right)$

$$= \frac{a^n}{a^m} = a^{n-m} = a^{-m-(-n)}$$

(iii) $(a^{-m})^{-n} = \left[\frac{1}{(a^{-m})}\right]^n$

$$= (a^m)^n = a^{mn} = a^{(-m)(-n)}$$

(iv) $(ab)^{-n} = \frac{1}{(ab)^n} = \frac{1}{a^n \times b^n}$

$$= \frac{1}{a^n} \times \frac{1}{b^n} = a^{-n} \times b^{-n}$$

(v) $\left(\frac{a}{b}\right)^{-n} = \frac{1}{\left(\frac{a}{b}\right)^n} = \frac{1}{\frac{a^n}{b^n}} = \frac{b^n}{a^n} = \frac{a^{-n}}{b^{-n}}$

Example 2 $\left(\frac{5}{7}\right)^8 \div \left(\frac{4}{5}\right)^8$ is equal to

(a) $\left(\frac{5}{7} / \frac{4}{5}\right)^8$ (b) $\left(\frac{5}{7} \times \frac{4}{5}\right)^8$

(c) $\left(\frac{5}{7} / \frac{4}{5}\right)^0$ (d) None of these

Sol. *(a)* $\frac{(5/7)^8}{(4/5)^8} = \left(\frac{5/7}{4/5}\right)^8$ $\left[\because \frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m\right]$

$$= \left(\frac{5}{7} / \frac{4}{5}\right)^8$$

Example 3 The value of

$$(7^{-1} - 8^{-1})^{-1} - (3^{-1} - 4^{-1})^{-1} \text{ is}$$

(a) 44 (b) 56 (c) 68 (d) 12

Sol. *(a)* Using law of exponents, $a^{-m} = \frac{1}{a^m}$

[$\because a$ is non-zero integer]

$\therefore$ $(7^{-1} - 8^{-1})^{-1} - (3^{-1} - 4^{-1})^{-1}$

$$= \left(\frac{1}{7} - \frac{1}{8}\right)^{-1} - \left(\frac{1}{3} - \frac{1}{4}\right)^{-1} = \left(\frac{1}{56}\right)^{-1} - \left(\frac{1}{12}\right)^{-1}$$

$$= 56 - 12 = 44$$

Example 4 Evaluate $\left(\frac{625}{81}\right)^{-1/4}$

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{1}{5}$ (d) $\frac{5}{2}$

Sol. *(a)* $\left(\frac{625}{81}\right)^{-1/4} = \left(\frac{81}{625}\right)^{1/4} = \left(\frac{3^4}{5^4}\right)^{1/4}$

$$= \left[\left(\frac{3}{5}\right)^4\right]^{1/4} = \frac{3}{5}$$

Example 5 Simplify $\left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b \times \left(\frac{x^a}{x^b}\right)^c$

(a) 0 (b) 1 (c) -1 (d) 2

Sol. *(b)* Given expression

$$= (x^{b-c})^a \times (x^{c-a})^b \times (x^{a-b})^c$$
$$= x^{a(b-c) + b(c-a) + c(a-b)}$$
$$= x^0 = 1$$

Surd or Radicals

If $\sqrt[n]{a}$ is irrational, where a is a rational number and n is a positive integer, then $\sqrt[n]{a}$ or $a^{1/n}$ is called a surd or radical of order n and a is called the radicand.

- A surd of order 2 is called a quadratic or square surd.
- A surd of order 3 is called a cubic surd.
- A surd of order 4 is called a biquadratic surd.

Surd in Simplest Form

A surd in its simplest form has

(i) the smallest possible index of this radical.

(ii) no fraction under the radical sign.

(iii) no factor of the form b^n, where b is rational, under the radical sign of index n.

Note Let n be a positive integer and a be a real number.

(i) If a is irrational, then $\sqrt[n]{a}$ is not a surd.

(ii) If a is rational, then $\sqrt[n]{a}$ is a surd.

Laws of Surd

As surds can be expressed with fractional exponent, the laws of indices are therefore, applicable to surd.

(i) $(\sqrt[n]{a})^n = a$ (ii) $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$

(iii) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$

(iv) $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

(v) $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$

Example 6 The index form of $\sqrt[9]{\left(\dfrac{4}{5}\right)^3}$ is

(a) $\left(\dfrac{4}{5}\right)^{1/3}$ (b) $\left(\dfrac{4}{5}\right)^{3}$ (c) $\left(\dfrac{4}{5}\right)^{1/2}$ (d) $\left(\dfrac{4}{5}\right)^{1/27}$

Sol. *(a)* $\sqrt[9]{\left(\dfrac{4}{5}\right)^3} = \left[\left(\dfrac{4}{5}\right)^3\right]^{1/9} = \left(\dfrac{4}{5}\right)^{3/9} = \left(\dfrac{4}{5}\right)^{1/3}$

$[\because (a^m)^n] = a^{mn}$

Example 7 If $3^x = 5^y = 75^z$, then the value of z is

(a) $\dfrac{xy}{(2x+y)}$ (b) $\dfrac{xy}{x+2y}$

(c) $\dfrac{xy}{x-y}$ (d) $\dfrac{xy}{x-2y}$

Sol. *(a)* Let $3^x = 5^y = (75)^z = k$

Then, $3 = k^{1/x}, 5 = k^{1/y}$ and $75 = k^{1/z}$

Now, $75 = 3 \times 5^2$

$\Rightarrow$ $k^{1/z} = k^{1/x} \cdot k^{2/y}$

$\Rightarrow$ $k^{1/z} = k^{\left(\frac{1}{x}+\frac{2}{y}\right)}$

$\therefore$ $\dfrac{1}{z} = \dfrac{1}{x} + \dfrac{2}{y}$

$\Rightarrow$ $z = \dfrac{xy}{(2x+y)}$

PRACTICE EXERCISE

1. The value of $\dfrac{5^0 + 2^1}{3^2 + 8^0}$ is

(a) $\dfrac{3}{10}$ (b) $\dfrac{5}{3}$

(c) $\dfrac{2}{5}$ (d) $\dfrac{3}{5}$

2. The multiplicative inverse of 10^{-100} is

(a) 10 (b) 100

(c) 10^{100} (d) 10^{-100}

3. The value of $\dfrac{5}{(121)^{-1/2}}$ is

(a) -55 (b) $\dfrac{1}{55}$ (c) $-\dfrac{1}{55}$ (d) 55

4. The value of $3 \times 9^{-3/2} \times 9^{1/2}$ is

(a) $\dfrac{1}{3}$ (b) 3

(c) 27 (d) $-\dfrac{1}{3}$

5. The simplified form of $(-4)^5 \div (-4)^8$ is
(a) $\frac{1}{4^3}$ (b) $\frac{1}{(-4)^3}$
(c) $\frac{1}{4^4}$ (d) None of these

6. The value of $\left(\frac{1}{2^3}\right)^2$ is
(a) $\frac{1}{62}$ (b) $\frac{1}{64}$
(c) $\frac{1}{32}$ (d) None of these

7. Evaluate $(-3)^4 \times \left(\frac{5}{3}\right)^4$
(a) 5^7 (b) 5^6
(c) 5^3 (d) 5^4

8. Evaluate, $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$
(a) -1 (b) -2
(c) -3 (d) -4

9. If $5^a = 3125$, then the value of 5^{a-3} is
(a) 625 (b) 25
(c) 5 (d) 225

10. The standard form of 0.0000078 is
(a) 78×10^{-6} (b) 78×10^{6}
(c) 78×10^{-5} (d) None of these

11. If $3^x = \frac{1}{9}$, the value of x is
(a) 2 (b) -2 (c) 1/2 (d) 1

12. The value of $(12^2 + 5^2)^{1/2}$ is
(a) 11 (b) 13 (c) 12 (d) 15

13. The value of $(0.000064)^{5/6}$ is
(a) $\frac{32}{100000}$ (b) $\frac{16}{10000}$
(c) $\frac{16}{100000}$ (d) None of these

14. The value of $\left[\left(\frac{25}{9}\right)^{5/2}\right]^{3/5}$ is
(a) $\frac{25}{27}$ (b) $\frac{125}{27}$
(c) $\frac{25}{9}$ (d) None of these

15. The value of $\left(-\frac{1}{125}\right)^{-2/3}$ is
(a) 5 (b) 25
(c) -25 (d) None of these

16. The value of $\frac{(81)^{1/3} \times (576)^{1/3}}{(64)^{2/3} \times (27)^{2/3}}$ is
(a) $\frac{1}{4}$ (b) $\frac{3}{4}$
(c) $\frac{5}{8}$ (d) None of these

17. The value of $\frac{2^{10+n} \times 4^{3n-5}}{2^{4n+1} \times 2^{3n-1}}$ is
(a) 0 (b) 1
(c) 3 (d) 4

18. If $9\sqrt{x} = \sqrt{12} + \sqrt{147}$, then the value of x is
(a) 1 (b) 2
(c) 3 (d) 4

Answers

1	*(a)*	2	*(c)*	3	*(d)*	4	*(a)*	5	*(b)*	6	*(b)*	7	*(d)*	8	*(a)*	9	*(b)*	10	*(a)*
11	*(b)*	12	*(b)*	13	*(a)*	14	*(b)*	15	*(b)*	16	*(a)*	17	*(b)*	18	*(c)*				

Hints and Solutions

1. $\dfrac{5^0+2^1}{3^2+8^0}=\dfrac{1+2}{9+1}=\dfrac{3}{10}$

2. For multiplicative inverse, let a be the multiplicative inverse of 10^{-100}.

[$\because$ If a is multiplicative inverse of b then $a\times b=1$]

$\therefore\ a\times 10^{-100}=1$

$$\Rightarrow \quad a=\frac{1}{10^{-100}}\frac{1}{\frac{1}{10^{100}}}=10^{100}\quad\left[\because a^{-m}=\frac{1}{a^m}\right]$$

3. $\dfrac{5}{(121)^{-1/2}}=5\times 121^{1/2}$

$$=5\times(11^2)^{1/2}=5\times 11=55$$

4. $3\times 9^{-3/2}\times 9^{1/2}=3\times\left(3^{2\times-\frac{3}{2}}\right)\times\left(3^{2\times\frac{1}{2}}\right)$

$$=3\times(3)^{-3}\times 3=3\times\left(\frac{1}{3}\right)^3\times 3$$

$$=3\times\frac{1}{27}\times 3=\frac{1}{3}$$

5. We have, $(-4)^5\div(-4)^8$

$$=\frac{(-4)^5}{(-4)^8}=\frac{1}{(-4)^8\times(-4)^{-5}}\qquad\left[\because a^m=\frac{1}{a^{-m}}\right]$$

$$=\frac{1}{(-4)^{8-5}}=\frac{1}{(-4)^3}\qquad[\because a^m\times a^n=a^{m+n}]$$

which is the required form.

6. We have, $\left(\dfrac{1}{2^3}\right)^2$

$$=\frac{(1)^2}{(2^3)^2}\qquad\left[\because\left(\frac{a}{b}\right)^m=\frac{a^m}{b^m}\right]$$

$$=\frac{1}{2^6}=\frac{1}{64}\qquad[\because(a^m)^n=a^{m\times n}]$$

7. We have, $(-3)^4\times\left(\dfrac{5}{3}\right)^4$

$$=(-1\times 3)^4\times\left(\frac{5}{3}\right)^4\qquad[\because -a=-1\times a]$$

$$=(-1)^4\times 3^4\times\frac{5^4}{3^4}$$

$$\left[\because(a\times b)^m=a^m\times b^m,\left(\frac{a}{b}\right)^m=\frac{a^m}{b^m}\right]$$

$$=1\times 5^4=(5)^4\qquad[\because(-1)^4=1]$$

which is the required form.

8. We have, $\left\{\left(\dfrac{1}{3}\right)^{-1}-\left(\dfrac{1}{4}\right)^{-1}\right\}^{-1}$

$$=\left\{\frac{(1)^{-1}}{(3)^{-1}}-\frac{(1)^{-1}}{(4)^{-1}}\right\}^{-1}\qquad\left[\because\left(\frac{a}{b}\right)^m=\frac{a^m}{b^m}\right]$$

$$=\left\{\frac{3}{1}-\frac{4}{1}\right\}^{-1}\qquad\left[\because a^{-m}=\frac{1}{a^m}\right]$$

$$=(3-4)^{-1}=(-1)^{-1}\qquad\left[\because a^{-m}=\frac{1}{a^m}\right]$$

$$=\frac{1}{(-1)^1}$$

$$=\frac{1}{-1}=-1$$

9. Given, $5^a=3125$

$\Rightarrow\quad 5^a=5^5$

On comparing, we get

$a=5$

$\therefore\quad 5^{a-3}=5^{5-3}=25$

10. According to question,

$$0.000078=\frac{78}{1000000}=78\times 10^{-6}$$

11. $\because\quad 3^x=\dfrac{1}{9}$

$\therefore\quad 3^x=\left(\dfrac{1}{3}\right)^2$

or $\quad 3^x=3^{-2}$

On comparing both sides, we get $x=-2$

12. $(12^2+5^2)^{1/2}=(144+25)^{1/2}$

$$=(169)^{1/2}$$

$$=(13^2)^{1/2}=13$$

13. $(0.000064)^{5/6} = \left(\frac{64}{1000000}\right)^{5/6}$

$$= \left[\left\{\left(\frac{2}{10}\right)^6\right\}^{1/6}\right]^5 = \left(\frac{2}{10}\right)^5 = \frac{32}{100000}$$

14. $\left[\left(\frac{25}{9}\right)^{5/2}\right]^{3/5} = \left[\left\{\left(\frac{5}{3}\right)^2\right\}^{5/2}\right]^{3/5}$

$$= \left[\left(\frac{5}{3}\right)^5\right]^{3/5} = \left(\frac{5}{3}\right)^3 = \frac{125}{27}$$

15. $\left(-\frac{1}{125}\right)^{-2/3} = \left[\left(-\frac{1}{5}\times-\frac{1}{5}\times-\frac{1}{5}\right)^{-1/3}\right]^2$

$$= (-5)^2 = 25$$

16. $\frac{(81)^{1/3}\times(576)^{1/3}}{(64)^{2/3}\times(27)^{2/3}} = \frac{(3^4)^{1/3}\times(2^6\times3^2)^{1/3}}{(4^3)^{2/3}\times(3^3)^{2/3}}$

$$= \frac{3^{4/3}\times2^2\times3^{2/3}}{4^2\times3^2} = \frac{3^2\times2^2}{4^2\times3^2} = \frac{9\times4}{16\times9} = \frac{1}{4}$$

17. $\frac{2^{10+n}\times4^{3n-5}}{2^{4n+1}\times2^{3n-1}} = \frac{2^{10+n}\times2^{6n-10}}{2^{4n+1}\times2^{3n-1}}$

$$= \frac{2^{10+n+6n-10}}{2^{4n+1+3n-1}} = \frac{2^{7n}}{2^{7n}} = 1$$

18. $9\sqrt{x} = \sqrt{12} + \sqrt{147}$

$$= \sqrt{2\times2\times3} + \sqrt{3\times7\times7}$$

$$= 2\sqrt{3} + 7\sqrt{3}$$

$\Rightarrow \quad 9\sqrt{x} = 9\sqrt{3} \quad \Rightarrow \quad x^{1/2} = 3^{1/2}$

On comparing both sides, we get $x = 3$

CHAPTER

05

RATIO AND PROPORTION

In this chapter, we study the ratio and proportion with their properties and also proportion.

Ratio

The ratio of two quantities of the same kind and in the same unit is the fraction that one quantity is of the other.

Note : In the ratio $a : b$, the first term a is antecedent and second term b is consequent.

Properties of Ratios

(i) The value of a ratio remains unchanged, if each one of its terms is multiplied or divided by a same non-zero number.

(ii) If $a : b$ and $c : d$ are two ratios, then the compounded ratio is $ac : bd$.

Example 1 If $a : b = 2 : 5$, the value of $(3a + 4b) : (4a + 5b)$ is

(a) $\frac{26}{33}$ (b) $\frac{33}{26}$ (c) $\frac{44}{23}$ (d) $\frac{33}{25}$

Sol. *(a)* We have, $\frac{3a+4b}{4a+5b} = \frac{3\left(\frac{a}{b}\right)+4}{4\left(\frac{a}{b}\right)+5} = \frac{3\times\frac{2}{5}+4}{4\times\frac{2}{5}+5} = \frac{26}{33}$ $\left[\because \frac{a}{b} = \frac{2}{5} \text{ given}\right]$

Proportion

If two ratio are equal, then we can say that both ratio are in proportion .

Direct Proportion

If the values of two quantities depend on each other in such a way that, a change in one, results in a corresponding change in the other and *vice-versa*, then the two quantities are said to be in direct proportion.

In other words, if two quantities a and b vary with each other in such a manner that the ratio $\frac{a}{b}$ (or $a : b$) remains constant, i.e. $\frac{a}{b} = k$ or $a = kb$ (where, k is any positive constant), then we say that a and b vary directly with each other or a and b are in direct proportion or a and b have a direct variation.

Example 2 The variable x varies directly as y and $x = 80$ when y is 160. What is y when x is 64?

(a) 127 (b) 128 (c) 129 (d) 130

Sol. *(b)* If x varies directly as y.

$\therefore \quad x / y = k$ (constant) ...(i)

If $\quad x = 80$ and $y = 160$

$\therefore \quad \frac{x}{y} = \frac{80}{160} = \frac{1}{2} \Rightarrow k = \frac{1}{2}$

When $x = 64$, then from Eq. (i),

$\frac{64}{y} = \frac{1}{2}$ [putting the value of k]

$\Rightarrow \quad y = 64 \times 2 = 128$

Inverse Proportion

The two quantities may vary in such a way that if one increases, the other decreases and *vice-versa*, then the two quantities are said to be inverse proportion. In other words if two quantities a and b vary with each other in such a manner that the product ab remains constant and is positive, then we say that a varies inversely with b and b varies inversely with a. Thus, two quantities a and b are said to vary in inverse proportion, if there exists a relation of the type $ab = k$ between them, where k is a positive constant.

Let a, b, c and d are four quantities, then the proportional are $a : b :: c : d$.

where a, b, c, d are known as first proportional, second proportional, third proportional and fourth proportional respectively.

Note (i) In the proportion $a : b :: c : d$, a and d are extreme values and b and c are mean values. i.e. Product of means = Product of extreme.

(ii) If x is the third proportional to a, b, then $a : b :: b : x$.

Example 3 If x varies inversely as y and $y = 60$ when $x = 1.5$. Find x, when $y = 4.5$.

(a) 20 (b) 21 (c) 22 (d) 23

Sol. *(a)* If x varies inversely as y.

$\therefore \quad xy = k$ (constant) ...(i)

If $\quad x = 1.5$ and $y = 60$

$\therefore \quad xy = 1.5 \times 60 = 90$

$\Rightarrow \quad k = 90$

When $y = 4.5$, then from Eq. (i),

$4.5 \times x = k$

$\Rightarrow \quad 4.5 \times x = 90$ [putting the value of k]

$\Rightarrow \quad x = \frac{90}{4.5} = 20$

Example 4 In a camp, there is enough flour for 300 persons for 42 days. How long will the flour last, if 20 more persons join the camp?

(a) $\frac{315}{7}$ day (b) $\frac{315}{8}$ day

(c) $\frac{126}{8}$ day (d) None of these

Sol. *(b)* $\because$ For 300 persons flour is enough for 42 days.

$\therefore$ For 1 person flour enough

$= 300 \times 42 = 12600$ days

Now, 20 more persons join the camp.

So, total persons = 300 + 20 = 320

$\therefore$ For 320 persons flour enough

$= \frac{12600}{320} = \frac{315}{8}$ day

Example 5 The fourth proportional of 6, 11 and 12 is

(a) 29 (b) 23

(c) 72 (d) 22

Sol. *(d)* Let fourth proportional be x, then

$6 : 11 :: 12 : x$

[$\because$ Product of means = Product of extreme]

$\Rightarrow \quad 6 \times x = 11 \times 12$

$\Rightarrow \quad x = \frac{11 \times 12}{6} = 22$

PRACTICE EXERCISE

1. The number of teeth and the age of a person vary
   (a) directly with each other
   (b) inversely with each other
   (c) neither directly nor inversely with each other
   (d) sometimes directly and sometimes inversely with each other
2. Which of the following vary inversely with each other?
   (a) Speed and distance covered
   (b) Distance covered and taxi fare
   (c) Distance travelled and time taken
   (d) Speed and time taken
3. Both x and y are in direct proportion, then $\frac{1}{x}$ and $\frac{1}{y}$ are
   (a) in indirect proportion
   (b) in direct proportion
   (c) neither in direct nor in inverse proportion
   (d) sometimes in direct and sometimes in inverse proportion
4. If two quantities p and q vary inversely with each other, then
   (a) $\frac{p}{q}$ remains constant
   (b) $p + q$ remains constant
   (c) $p \times q$ remains constant
   (d) $p - q$ remains constant
5. The variable x is inversely proportional to y. If x increases by $p\%$, then by what per cent will y decrease?
   (a) $p\%$ (b) $2p\%$
   (c) $3p\%$ (d) None of these
6. l varies directly as m and $l = 5$, when $m = \frac{2}{3}$. Find l when $m = \frac{16}{3}$.
   (a) 38 (b) 39
   (c) 40 (d) 41
7. The fourth proportional to 3, 5 and 21 is
   (a) 35 (b) $\frac{5}{7}$
   (c) $\frac{7}{5}$ (d) None of these
8. What must be added to each term of the ratio $49:68$, so that it becomes $3:4$?
   (a) 11 (b) 10 (c) 7 (d) 8
9. A bag contains ₹600 in the form of one rupee, 50 paise and 25 paise coins in the ratio $3:4:12$. The number of 25 paise coins is
   (a) 800 (b) 900
   (c) 1205 (d) None of these
10. The cost of making an article is divided between materials, labour and overheads in the ratio $5:3:1$. If the materials cost ₹6.90, then cost of the article is
    (a) ₹12.42 (b) ₹13.20
    (c) ₹14.00 (d) None of these
11. If $\frac{a}{3} = \frac{b}{4} = \frac{c}{7}$, then $\frac{a+b+c}{c}$ is equal to
    (a) $\frac{1}{7}$ (b) $\frac{1}{2}$ (c) 7 (d) 2
12. Out of the ratios $7:20; 13:25; 17:30$ and $11:15$, the smallest one is
    (a) $10:7$ (b) $7:20$
    (c) $17:20$ (d) None of these
13. ₹770 have been divided among A, B, C in such a way that A receives $\frac{2}{9}$th of what B and C together receive. Then, A's share is
    (a) ₹140 (b) ₹154
    (c) ₹165 (d) ₹ 170
14. If x varies inversely as y and $x = 20$ when $y = 600$, find y when $x = 400$.
    (a) 30 (b) 40
    (c) 32 (d) 35

15. If $2x = 3y = 4z$, then $x : y : z$ is
(a) $6:4:3$ (b) $5:4:2$
(c) $2:3:4$ (d) None of these

16. The ratio of number of boys and girls in a school of 720 students is $7:5$. How many more girls should be admitted to make the ratio $1:1$?
(a) 160 (b) 145 (c) 120 (d) 170

17. The incomes of A and B are in the ratio $3:2$ and their expenditures in the ratio $5:3$. If each saves ₹1500, then B's income is
(a) ₹6000 (b) ₹4700
(c) ₹3000 (d) ₹7500

18. Third proportional to 9 and 12 is
(a) 16 (b) 10.5
(c) $6\sqrt{3}$ (d) None of these

19. If $a : b = 3:4$, then $(6a + b) : (4a + 5b)$ is
(a) $1:2$
(b) $3:5$
(c) $7:9$
(d) None of the above

20. The ratio of zinc and copper in a brass piece is $13:7$. How much zinc will be there in 100 kg of such a piece?
(a) 65 kg (b) 40 kg
(c) 45 kg (d) 50 kg

Answers

1	(d)	2	(d)	3	(b)	4	(c)	5	(a)	6	(c)	7	(a)	8	(d)	9	(b)	10	(a)
11	(d)	12	(b)	13	(a)	14	(a)	15	(a)	16	(c)	17	(a)	18	(a)	19	(d)	20	(a)

Hints and Solutions

1. The number of teeth and the age of a person vary sometimes directly and sometimes inversely with each other, we cannot predict about the number of teeth with exactly the age of a person. It change with person-to-person.

Hence, option (d) is correct.

2. We know that, when we increases the speed, then the time taken by vehicle decreases.

Hence, speed and time taken vary inversely with each other.

So, option (d) is correct.

3. If both x and y are in directly proportion, then $\frac{1}{x}$ and $\frac{1}{y}$ are in direct proportion.

Hence, option (b) is correct.

4. If two quantities p and q vary inversely with each other, then $p \times q$ remains constant. Since, in inverse proportion, an increase in p cause a proportional decrease in q and vice-versa.

Hence, option (c) is correct.

5. The variable x is inversely proportionls to y.

$\therefore \quad xy = k$ (constant)

Since, we know that two quantities x and y are said to be in inverse proportion, if an increase in x cause a proportional decrease in y and *vice-versa*.

So, we can say y decrease by $p\%$.

6. If l varies directly as m.

$$\therefore \quad l/m = k \text{ (constant)} \quad \ldots\text{(i)}$$

$$\text{If} \quad l = 5 \text{ and } m = \frac{2}{3}$$

$$\therefore \quad \frac{l}{m} = \frac{5}{2/3} = \frac{5}{1} \times \frac{3}{2} = \frac{15}{2}$$

$$\Rightarrow \quad k = \frac{15}{2}$$

When, $m = \frac{16}{3}$, then from Eq. (i),

$$\frac{l}{16/3} = \frac{15}{2} \quad \text{[putting the value of } k\text{]}$$

$$\Rightarrow \quad l = \frac{15}{2} \times \frac{16}{3} = 40$$

7. Let the fourth proportional be x.

$$\therefore \quad 3:5::21:x$$

$$\Rightarrow \quad x = \frac{5 \times 21}{3} = 35$$

8. Let the number be x added in the given ratio.

$\therefore \quad \frac{49+x}{68+x} = \frac{3}{4}$

$\Rightarrow \quad 4(49+x) = 3(68+x)$

$\Rightarrow \quad x = 8$

9. Given ratio is $3:4:12$.

The ratio of values in coins $= \frac{3}{1} : \frac{4}{2} : \frac{12}{4} = 3:2:3$

Value of 25 paise coins $= ₹\left(600 \times \frac{3}{8}\right) = ₹\,225$

$\therefore$ Number of these coins $= 225 \times 4 = 900$

10. If material cost be ₹5, then cost of article be ₹9.

$\therefore \quad 5:9::6.90:x$

$\Rightarrow \quad x = \frac{9 \times 6.90}{5}$

$\Rightarrow \quad x = ₹12.42$

11. Let $\frac{a}{3} = \frac{b}{4} = \frac{c}{7} = k$.

Then, $a = 3k$, $b = 4k$ and $c = 7k$.

$\therefore \quad \frac{a+b+c}{c} = \frac{3k+4k+7k}{7k} = 2$

12. Now, $\frac{7}{20} = 0.35, \frac{13}{25} = 0.52, \frac{17}{30} = 0.56$ and $\frac{11}{15} = 0.73$.

$\therefore$ The smallest value is 0.35 i.e. ratio $7:20$.

13. $\therefore \quad A:(B+C) = 2:9$

$\therefore \quad A$'s part $= ₹\left(770 \times \frac{2}{11}\right) = ₹140$

14. If x varies inversely as y.

$\therefore \quad xy = k$ (constant) ...(i)

If $\quad x = 20$ and $y = 600$

$\therefore \quad xy = 20 \times 600 = 12000$

$\Rightarrow \quad k = 12000$

When $x = 400$, then from Eq. (i),

$y \times 400 = k$

$\Rightarrow \quad y \times 400 = 12000$ [putting the value of k]

$\Rightarrow \quad y = \frac{12000}{400} = 30$

15. Let $2x = 3y = 4z = k$

Then, $x = \frac{k}{2}, y = \frac{k}{3}$ and $z = \frac{k}{4}$

$\therefore \quad x:y:z = \frac{k}{2} : \frac{k}{3} : \frac{k}{4} = \frac{1}{2} : \frac{1}{3} : \frac{1}{4} = 6:4:3$

16. Number of boys $= 720 \times \frac{7}{12} = 420$

Number of girls $= 720 - 420 = 300$

$\therefore$ Number of girls to be admitted

$= 420 - 300 = 120$

17. Let their incomes be $3x$ and $2x$ and their corresponding expenditure be $5y$ and $3y$.
According to the given conditions,

$3x - 5y = 1500$ and $2x - 3y = 1500$

On solving, we get

$x = 3000$ and $y = 1500$

Hence, B's income $= 2x = ₹\,6000$

18. Let third proportion be x.

$\therefore \quad 9:12::12:x$

$\Rightarrow \quad 9x = 12 \times 12$

$\Rightarrow \quad x = 16$

19. Now, $\frac{6a+b}{4a+5b} = \frac{6\left(\frac{a}{b}\right)+1}{4\left(\frac{a}{b}\right)+5}$

$= \frac{6 \times \frac{3}{4} + 1}{4 \times \frac{3}{4} + 5} = \frac{11}{16}$

20. Amount of zinc $= \left(100 \times \frac{13}{20}\right)$ kg

$= 65$ kg

CHAPTER

06

COMPARING QUANTITIES

In this chapter, we study about the percentage and its important rules, profit, loss and discount with their important rules. Apart from this we will also study Simple and Compound Interest.

Sometimes, we have to measure and compare our daily life quantities such as height, distance, price of commodity, rate of interest etc. The best measure of these compare quantities in the percentage.

Percentage

The meaning of term per cent is per hundred or hundred part.

e.g. Meaning of 38% is $\frac{38}{100}$ i.e. 38 parts out of 100 parts.

e.g. $\frac{30}{100} = 30\%$

Important Rules

(i) For expressing 'x' % as a fraction, then $x\% = \frac{x}{100}$.

(ii) For expressing a fraction $\frac{x}{y}$ as a per cent. Then, $\frac{x}{y} = \left(\frac{x}{y} \times 100\right)\%$

(iii) If A's income is $x\%$ more than that of B, then B's income is less than that of A by $\left[\frac{x}{100 + x} \times 100\right]\%$.

(iv) If A's income is $x\%$ less than that of B, then B's income is more than that of A by $\left[\frac{x}{100 - x} \times 100\right]\%$.

Example 1 What per cent of $\frac{2}{7}$ is $\frac{1}{35}$?

(a) 10% (b) 11% (c) 12% (d) 13%

Sol. (*a*) Required per cent $= \left[\frac{\frac{1}{35}}{\frac{2}{7}} \times 100\right]\%$

$= \left(\frac{1}{35} \times \frac{7}{2} \times 100\right)\%$

$= 10\%$

Example 2 The salary of a worker is first increased by 10% and thereafter it was reduced by 10%. What was the change in his salary ?

(a) 4% (b) 2% (c) 3% (d) 1%

Sol. (*d*) Let the salary of the worker be ₹ 100. After increase, it becomes ₹100 + 10% of 100 = ₹ 110

After decrease, it becomes ₹110 − 10% of ₹110 = ₹99

The percentage reduction = 100 − 99 = 1%

Example 3 The length of a rectangle is increased by 60%. By what per cent would the breadth be decreased to maintain the same area?

(a) $37\frac{1}{2}\%$ (b) 38% (c) 39% (d) 40%

Sol. (*a*) Let the length be 100 m and breadth be 100 m, then

New length = 160 m, new breadth = x,

then $160 \times x = 100 \times 100$

$\Rightarrow x = \frac{100 \times 100}{160} \Rightarrow x = \frac{125}{2}$

$\therefore$ Decrease in breadth $= \left(100 - \frac{125}{2}\right)\%$

$= 37\frac{1}{2}\%$

Profit and Loss

- The price at which an article or item is bought is called its **cost price** and it is denoted by CP.
- And if the price at which an article is sold is called its **selling price** and it is denoted by SP.
- If the selling price of an article is greater than cost price, then it is **profit** (SP > CP) otherwise it is **loss** (CP > SP).
- The list price of an item is known as its **marked price**. It is the price that appers on the item's tag.
- If the shopkeeper given a some rebate on an article is called **discount.**

Note : Discount is always given on a marked price.

Important Formulae

(i) Profit = SP − CP

(ii) Loss = CP − SP

(iii) Profit per cent $= \frac{\text{Profit}}{\text{CP}} \times 100$

$= \frac{\text{SP} - \text{CP}}{\text{CP}} \times 100$

(iv) Loss per cent $= \frac{\text{Loss}}{\text{CP}} \times 100$

$= \frac{\text{CP} - \text{SP}}{\text{CP}} \times 100$

(v) If there is a profit of r%, then

$\text{SP} = \frac{100 + r}{100} \times \text{CP}$

(vi) If there is a loss of r%, then

$\text{SP} = \frac{100 - r}{100} \times \text{CP}$

(vii) Discount = Marked price − Sale price

(viii) Rate of discount = Discount%

$= \frac{\text{Discount}}{\text{MP}} \times 100$

(ix) $\text{SP} = \text{MP}\frac{(100 - \text{Discount}\%)}{100}$

or $\text{MP} = \frac{100 \times \text{SP}}{100 - \text{Discount}\%}$

Example 4 A boy buys a pen for ₹ 25 and sells it for ₹ 20. Find his loss per cent.

(a) 10% (b) 20% (c) 22% (d) 23%

Sol. (*b*) Loss per cent $= \frac{\text{Loss}}{\text{CP}} \times 100$

$$= \frac{5}{25} \times 100$$

$$= 20\%$$

Example 5 A shopkeeper sells a pen set at 30% profit to another shopkeeper who sells it at a loss of 30%. If price of pen set ₹ 150, what is net profit or loss on total transaction?

(a) profit 9% (b) loss 9%
(c) profit 10% (d) loss 10%

Sol. (*b*) Cost price of pen is ₹ 150.

Initially shopkeeper sells a pen in 30% profit.

$\therefore$ SP of a pen $= 150 + 30\%$ of 150

$$= 150 + \frac{30}{100} \times 150$$

$$= 150 + 45 = ₹\,195$$

Now, shopkeeper sells a pen to another shopkeeper at an loss of 30%. Then,

SP of a pen $= 195 - \frac{30}{100} \times 195$

$$= 195 - 58.5 = ₹\,136.5$$

Now, loss % in transaction $= \frac{150 - 136.5}{150} \times 100$

$$= \frac{13.5}{150} \times 100 = \frac{1350}{150} = 9\%$$

Example 6 The marked price of a ceiling fan is ₹ 1250 and the shopkeeper allows a discount of 6% on it. Find the selling price of the fan.

(a) ₹ 1190 (b) ₹ 1175
(c) ₹ 1200 (d) ₹ 1180

Sol. (*b*) Marked price = ₹ 1250 and discount = 6%

Discount = 6% of MP

$$= (6\% \text{ of } ₹\,1250)$$

$$= ₹\left(1250 \times \frac{6}{100}\right) = ₹\,75$$

Selling price = (MP) − (discount)

$$= ₹\,(1250 - 75) = ₹\,1175$$

Hence, the selling price of the fan is ₹ 1175.

Example 7 Two successive profits of 50% and 50% is equivalent to

(a) 100% (b) 75% (c) 50% (d) 125%

Sol. (*d*) Let the cost price of an item be x. Then

After profit of 50%, SP of an item

$$= x + 50\% \text{ of } x$$

$$= x + \frac{50}{100} \times x = x + \frac{x}{2} = \frac{3x}{2}$$

Again profit of 50%, SP of an item

$$= \frac{3x}{2} + 50\% \text{ of } \frac{3x}{2}$$

$$= \frac{3x}{2} + \frac{50}{100} \times \frac{3x}{2}$$

$$= \frac{3x}{2} + \frac{3x}{4} = \frac{9x}{4}$$

Now, total profit % in whole transaction

$$= \frac{\frac{9x}{4} - x}{x} \times 100\%$$

$$= \frac{5x}{4 \times x} \times 100 = 125\%$$

Simple Interest

If the interest is paid to the lender regularly every year or half year on the same principal, then it is called simple interest.

Or The interest is said to be simple, if it is calculated on the original principal throughout the loan period.

i.e. $$\text{SI} = \frac{PRT}{100}$$

where, P is the principal amount
R is the rate of interest
T is the time period

Compound Interest

If the borrower and the lender agree to fix up a certain interval of time (say, a year or half year or a quarter of a year, etc), so that the amount at the end of an interval becomes the principal for the next interval, then the total interest over all the intervals calculated in this way is called the compound interest.

$$CI = P\left[\left(1+\frac{R}{100}\right)^T - 1\right]$$

where, P is the principal amount

T is the time period

R is the rate of interest

Note The simple interest and compound interest for one year will be same, if the rate of interest is annually.

Important Rules

- **Rule 1.** When interest is compounded annually

Then, Amount $= P\left(1+\frac{R}{100}\right)^n$

where, n is number of time for which interest is calculated,

- **Rule 2.** When interest is compounded half-yearly then, interest would be calculated after every six months,

So, $n = T \times 2$ and rate $= \frac{R}{2}$

$\therefore$ Amount $= P\left(1+\frac{R/2}{100}\right)^n$

- **Rule 3.** When interest is compounded quarterly. Then, interest would be calculated after every three months, So

$n = T \times 4$

and Rate of interest $= \frac{R}{4}$

$\therefore$ Amount $= P\left(1+\frac{R/4}{100}\right)^n$.

- **Rule 4.** When rate of interest are different for different years say R_1, R_2, R_3% for first, second and third year respectively, then

Amount $= P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$.

Note If the difference between the simple interest and compound interest on the same sum ₹ P at r % per annum for 2 yr is ₹ d, then

$$d = P\left(\frac{r}{100}\right)^2$$

When interest is compounded yearly, then on the same sum and at the same rate, then

Compound interest for the first year
= Simple interest for the first year

If A and B are the amounts of a certain sum for two consecutive year, then

Simple interest for 1 yr $= B - A$.

Example 8 The sum required to earn a monthly interest of ₹ 400 at 10% per annum at simple interest.

(a) ₹ 47000 (b) ₹ 48000 (c) ₹ 49000 (d) ₹ 50000

Sol. (*b*) Total interest amount needed in a year

$= ₹\ 400 \times 12 = ₹\ 4800$

$$\therefore \quad P = \frac{100 \times SI}{R \times T}$$

$$= \frac{4800 \times 100}{10 \times 1} = ₹\ 48000$$

Example 9 Find CI on a sum ₹ 8000 for 2 yr at 5% per annum compounded annually.

(a) ₹ 821 (b) ₹ 820
(c) ₹ 822 (d) None of these

Sol. (*b*) Given, principal $(P) = ₹\ 8000$, time $(n) = 2$ yr, rate $(R) = 5\%$

$$\therefore \quad \text{Amount}\,(A) = P\left(1+\frac{R}{100}\right)^n$$

$$= 8000\left(1+\frac{5}{100}\right)^2$$

$$= 8000\left(\frac{105}{100}\right)^2$$

$$= 8000 \times \frac{21}{20} \times \frac{21}{20}$$

$$= ₹\ 8820$$

$\therefore$ Compound Interest (AI)

$= \text{Amount}\,(A) - \text{Principal}\,(P)$

$= ₹\,(8820 - 8000)$

$= ₹\ 820$

Example 10 Find the compound interest on ₹ 5000 for 4 yr if the rate of interest is 10% per annum for the first two years and 15% for the next two years.

(a) ₹ 3001 (b) ₹ 3003
(c) ₹ 3008 (d) ₹ 3009

Sol. (*a*) Let $R_1 = 10\%$, $T_1 = 2$ yr

and $R_2 = 15\%$, $T_2 = 2$ yr

and $P = ₹\,5000$

$\therefore$ Amount $= 5000\left(1+\frac{10}{100}\right)^2\left(1+\frac{15}{100}\right)^2$

$= 5000 \times \left(\frac{110}{100}\right)^2\left(\frac{115}{100}\right)^2$

$= 5000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{23}{20} \times \frac{23}{20}$

$= ₹\,8001$ (approx)

$\therefore$ Compound interest $= 8001 - 5000$

$= ₹\,3001$

PRACTICE EXERCISE

1. If 90% of x is 315 km, then the value of x is
   (a) 325 km (b) 350 km (c) 350 m (d) 325m

2. A toy is purchased at ₹ 1500 including 5% SGST and 5% CGST. Find the actual price of the toy without GST.
   (a) ₹ 14500 (b) ₹ 1350 (c) ₹ 1500 (d) ₹ 1400

3. If 11% of a number exceeds 7% of the same by 18, the number is
   (a) 300 (b) 450 (c) 350 (d) 370

4. If *A*'s income is 30% less than *B*'s, then how much per cent is *B*'s income more than *A*'s?
   (a) $42\frac{6}{7}\%$ (b) $32\frac{1}{10}\%$ (c) 30% (d) 40%

5. A man spent 20% of his monthly earning on house rent. Out of the balance, he spent 75% on the other house expenses. If he had a balance of ₹ 250 at the end of the month, the monthly earning of the man is
   (a) ₹ 1250 (b) ₹ 1200
   (c) ₹ 1150 (d) None of these

6. If the numerator of a fraction is increased by 20% and its denominator is diminished by 10%, the value of the fraction is $\frac{16}{21}$. The original fraction is
   (a) $\frac{3}{5}$ (b) $\frac{4}{5}$
   (c) $\frac{4}{7}$ (d) None of these

7. In an examination, it is required to get 36% of maximum marks to pass. A student got 113 marks and was declared failed by 85 marks. The maximum marks are
   (a) 500 (b) 1008 (c) 640 (d) 550

8. The price of cooking oil has increased by 25%. The percentage of reduction that a family should affect in the use of cooking oil so as not to increase the expenditure on this account is
   (a) 20% (b) 15% (c) 22% (d) 25%

9. A boy who asked to find $3\frac{1}{2}\%$ of a sum of money misread the question and found $5\frac{1}{2}\%$ of it. His answer was ₹220, what would have been the correct answer ?
   (a) ₹ 140 (b) ₹ 150 (c) ₹ 157 (d) ₹ 160

10. *A*'s salary increased by 12% over last year and has become ₹ 6720. If it increased by 20% over last year's salary, then the next salary will be
    (a) ₹ 8000 (b) ₹ 8064
    (c) ₹ 7500 (d) ₹ 7200

11. The ratio of the number of boys and girls in a school is $3:2$. If 20% of the boys and 30% of the girls are scholarship holders, the percentage of the students who are not scholarship holder is
    (a) 50% (b) 72% (c) 75% (d) 76%

12. If 50% of $(x - y) = 30\%$ of $(x + y)$, then what per cent of x is y ?
(a) 25% (b) $33\frac{1}{3}\%$ (c) 40% (d) 400%

13. If 15% of 40 is greater than 25% of a number by 2, then the number is
(a) 12 (b) 16 (c) 24 (d) 32

14. If 35% of a number is 12 less than 50% of that number, then the number is
(a) 80 (b) 60 (c) 50 (d) 40

15. 60% of that the students in a school are boys. If the number of girls in the school is 300, then the number of boys is
(a) 300 (b) 450 (c) 500 (d) 750

16. Profit after selling an article for ₹ 425 is the same as loss after selling it for ₹ 355. The cost of the article is
(a) ₹ 390 (b) ₹ 405
(c) ₹ 380 (d) None of these

17. The difference between a discount of 40% on ₹ 500 and two successive discounts of 36% and 4% on the same amount is
(a) ₹ 0 (b) ₹ 2 (c) ₹ 5 (d) ₹ 7.20

18. The marked price of an article is 10% higher than the cost price. A discount of 10% is given on the marked price. In this kind of sale, the seller
(a) bears no loss, no gain
(b) gains 1%
(c) loses 1%
(d) None of the above

19. On selling an article for ₹ 240, a trader loses 4%. In order to gain 10%, he must sell that article for
(a) ₹ 264 (b) ₹ 275
(c) ₹ 250 (d) ₹ 280

20. A sweet seller declares that he sells sweets at the cost price. However, he uses a weight of 450 g instead of 500 g. His percentage profit is
(a) $11\frac{1}{9}\%$ (b) 12%
(c) 13% (d) None of these

21. A radio is sold at a gain of 16%. If it had been sold for ₹ 20 more, 20% would have been gained. The cost price of the radio is
(a) ₹ 420 (b) ₹ 410 (c) ₹ 485 (d) ₹ 500

22. A dealer sold a radio at a loss of 2.5%. Had he sold it for ₹ 100 more, he would have gained $7\frac{1}{2}\%$. In order to gain $12\frac{1}{2}\%$, he should sell it for
(a) ₹ 850 (b) ₹ 925 (c) ₹ 1000 (d) ₹ 1125

23. By selling sugar at ₹ 11.16 per kg, a man loses 7%. To gain 7%, it must be sold (per kg) at
(a) ₹ 11.24 (b) ₹ 12.84 (c) ₹ 14.64 (d) ₹ 13.24

24. If a commission of 10% is given to the marked price of a book, the publisher gains 20%. If the commission is increased to 15%, the gain is
(a) $16\frac{2}{3}\%$ (b) $13\frac{1}{3}\%$
(c) $15\frac{1}{6}\%$ (d) 15%

25. Oranges are bought at 7 for ₹ 3. At what rate per hundred must they be sold to gain 33% ?
(a) ₹ 56 (b) ₹ 60
(c) ₹ 58 (d) ₹ 57

26. A reduction of 20% in the price of oranges enables a man to buy 5 oranges more for ₹ 10. The price of an orange before reduction was
(a) 20 paise (b) 40 paise
(c) 50 paise (d) 60 paise

27. Kiran buys an article with 25% discount on the marked price. She makes a profit of 10% by selling it at ₹ 660. What was the marked price ?
(a) ₹ 900 (b) ₹ 600
(c) ₹ 700 (d) ₹ 800

28. If a shirt cost ₹ 64 after a 20% discount, what was its original price ?
(a) ₹ 76.80 (b) ₹ 80
(c) ₹ 88 (d) ₹ 86.80

29. The cost price of 19 chairs is equal to the selling price of 16 chairs. Then the gain is
(a) $3\frac{9}{17}\%$ (b) $15\frac{15}{19}\%$
(c) $18\frac{3}{4}\%$ (d) None of these

30. The ratio between the sales price and the cost price of an article is 7 : 5. What is the ratio between the profit and the cost price of that article ?
(a) 2 : 5 (b) 7 : 2
(c) 2 : 7 (d) Data inadequate

31. By selling a table for ₹ 350 instead of ₹ 400, loss percent increased by 5%. The cost price of the table is
(a) ₹ 1050 (b) ₹ 417.50 (c) ₹ 435 (d) ₹ 1000

32. A house worth ₹150000 is sold by *X* at 5% profit to *Y*. *Y* sells the house back to *X* at a 2% loss. Then, in the entire transaction
(a) *X* gains ₹ 4350 (b) *X* loses ₹ 4350
(c) *X* gains ₹ 3150 (d) *X* loses ₹ 3150

33. Find interest and amount to be paid on ₹ 15000 at 5% per annum after 2 yr.
(a) ₹ 1500, ₹ 16600 (b) ₹ 1500, ₹ 16500
(c) ₹ 1600, ₹ 16500 (d) None of the above

34. A sum of money at simple interest amount to be ₹ 1260 in 2 yr and ₹ 1350 in 5 yr, then the rate per cent per annum is
(a) 30% (b) 10% (c) 2.5% (d) 5%

35. The difference of 13% per annum and 12% of a sum in one year is ₹ 110. Then, the sum is
(a) ₹ 12000 (b) ₹ 13000 (c) ₹ 11000 (d) ₹ 16000

36. The amount of a certain sum at compound interest for 2 yr at 5% is ₹ 4410. The sum is
(a) ₹ 4000 (b) ₹ 4200 (c) ₹ 3900 (d) ₹ 3800

37. Two equal amounts of money are deposited in two banks, each at 15% per annum, for $3\frac{1}{2}$ yr and 5 yr. If the difference between their interest is ₹ 144, each sum is
(a) ₹ 460 (b) ₹ 500 (c) ₹ 640 (d) ₹ 720

38. If the simple interest on ₹ 1500 increases by ₹ 30, when the time increases by 8 yr. The rate per cent per annum is
(a) 0.5% (b) 0.25% (c) 0.75% (d) 1.25%

39. What annual payment will discharge a debt of ₹ 19350 due 4 yr hence at the rate of 5% simple interest ?
(a) ₹ 4500 (b) ₹ 5400
(c) ₹ 4000 (d) None of these

40. The simple interest on a sum of money for 3 yr at $6\frac{2}{3}\%$ per annum is ₹ 6750. The compound interest on the same sum at the same rate of interest for the same period will be
(a) ₹ 7200 (b) ₹ 7210
(c) ₹ 7120 (d) ₹ 7012

41. The difference of the compound interest and the simple interest on ₹ 60000 at 6% annually for 2 yr will be
(a) ₹ 215 (b) ₹ 216 (c) ₹ 220 (d) ₹ 250

42. The compound interest on ₹ 350 for 1 year at 4% per annum, the interest being payable half yearly, will be
(a) ₹ 364.14 (b) ₹ 365.15
(c) ₹ 14.14 (d) ₹ 15.15

43. The compound interest on ₹ 2000 for $1\frac{1}{4}$ yr is 10% per annum, the interest being payable quarterly will be
(a) ₹ 2262.81 (b) ₹ 262.81
(c) ₹ 261.81 (d) None of these

44. A sum of money doubles itself at compound interest in 15 yr. In how many years will it become eight times ?
(a) 20 yr (b) 40 yr
(c) 35 yr (d) 45 yr

45. A certain sum amounts to ₹ 7350 in 2 yr and to ₹ 8575 in 3 yr. The the rate per cent and sum are
(a) $16\frac{2}{3}\%$, ₹ 5400 (b) $16\frac{2}{3}\%$, ₹ 2400
(c) $12\frac{1}{2}\%$, ₹ 5400 (d) None of these

Answers

1	(b)	2	(b)	3	(b)	4	(a)	5	(a)	6	(c)	7	(d)	8	(a)	9	(a)	10	(d)
11	(d)	12	(a)	13	(b)	14	(a)	15	(b)	16	(a)	17	(d)	18	(c)	19	(b)	20	(a)
21	(d)	22	(c)	23	(b)	24	(b)	25	(d)	26	(c)	27	(d)	28	(b)	29	(c)	30	(a)
31	(d)	32	(a)	33	(b)	34	(c)	35	(c)	36	(a)	37	(c)	38	(b)	39	(a)	40	(b)
41	(b)	42	(c)	43	(b)	44	(d)	45	(a)										

Hints and Solutions

1. We have, 90% of $x = 315$

$$\Rightarrow \quad \frac{90}{100} \times x = 315$$

$$\Rightarrow \quad x = \frac{315 \times 100}{90}$$

$$= \frac{315 \times 10}{9} = 350$$

$$\Rightarrow \quad x = 350 \text{ km}$$

2. Now, SGST = 5% of 1500

$$\therefore \quad = \frac{5}{100} \times 1500 = ₹\,75$$

Similarly, CGST = 5% of 1500 = ₹ 75

$\therefore$ Actual price of toy = 1500 − SGST − CGST

$$= 1500 - 75 - 75 = 1500 - 150$$

$$= ₹\,1350$$

3. Let x be the given number.

Then, according to the question,

11% of x − 7% of $x = 18$

$$\Rightarrow \quad 4\% \text{ of } x = 18$$

$$\Rightarrow \quad x = \frac{18 \times 100}{4}$$

$$\Rightarrow \quad x = 450$$

4. $\therefore$ Required percentage

$$= \left[\frac{30}{(100 - 30)} \times 100\right]\%$$

$$= 42\frac{6}{7}\%$$

5. Let total income be ₹ x.

Then, 25% of (80% of x) = 250

$$\Rightarrow \quad \frac{25}{100} \times \frac{80}{100} \times x = 250$$

$$\Rightarrow \quad x = 250 \times 5 = ₹\,1250$$

6. Let the fraction be $\frac{x}{y}$.

New fraction $= \frac{120\% \text{ of } x}{90\% \text{ of } y} \Rightarrow \frac{4x}{3y} = \frac{16}{21}$

$$\therefore \quad \frac{x}{y} = \frac{16}{21} \times \frac{3}{4} = \frac{4}{7}$$

7. Let maximum marks be x.

Then, $\quad 36\%$ of $x = 113 + 85 = 198$

$$\therefore \quad x = \frac{198 \times 100}{36} = 550$$

8. Reduction in consumption $= \left[\frac{25}{100 + 25} \times 100\right]\%$

$$= 20\%$$

9. Since, $5\frac{1}{2}\% \equiv 220 \Rightarrow 3\frac{1}{2}\% \equiv \frac{220 \times (7/2)}{11/2} = ₹\,140$

10. Last year's salary $= \frac{6720 \times 100}{100 + 12} = ₹\,6000$

$\therefore$ The next year's salary $= \frac{6000 \times 120}{100} = ₹\,7200$

11. Let the number of boys and girls be $3x$ and $2x$.

Number of those who are not scholarship holder

$$= (80\% \text{ of } 3x + 70\% \text{ of } 2x)$$

$$= \frac{12x}{5} + \frac{7x}{5} = \frac{19x}{5}$$

$\therefore$ Required percentage $= \left(\frac{19x}{5} \times \frac{1}{5x} \times 100\right)\%$

$$= 76\%$$

12. Since, $\frac{50}{100}(x - y) = \frac{30}{100}(x + y)$

$$\Rightarrow \quad 5(x - y) = 3(x + y) \quad 2x = 8y$$

$$\Rightarrow \quad \frac{y}{x} \times 100 = \frac{20}{80} \times 100 = 25\%$$

13. According to the given condition,

$$\frac{15}{100} \times 40 - \frac{25}{100} x = 2 \Rightarrow x = 16$$

14. Since, $(50 - 35)\%$ of $x = 12$

$$\Rightarrow \quad x = \frac{12 \times 100}{15} = 80$$

15. Here, 40% are girls in a school.

$$\therefore \quad 40\% = 300$$

$$\Rightarrow \quad 60\% = \frac{300 \times 60}{40} = 450$$

16. Let cost price of an article be ₹ x.

According to the given condition,

$$425 - x = x - 355$$

$$\Rightarrow \quad 2x = 780 \Rightarrow x = ₹\ 390$$

17. SP at 40% discount = 60% of ₹ 500 = ₹ 300

SP after two successive discounts of 36% and 4%

$$= 96\% \text{ of } (64\% \text{ of } 500)$$

$$= \frac{96}{100} \times \frac{64}{100} \times 500 = ₹\ 307.20$$

$\therefore$ Required difference = ₹ 7.20

18. Let CP of article be Rs 100, then marked price be Rs 110.

$$\therefore \quad SP = 90\% \text{ of Rs } 110$$

$$= \frac{90}{100} \times 110 = \text{Rs } 99$$

$\therefore$ Loss per cent $= 100 - 99 = 1\%$

19. $CP = \frac{100}{96} \times 240 = ₹\ 250$

Now, $CP = ₹\ 250$, Gain = 10%

$$\therefore \quad SP = \frac{110}{100} \times 250 = ₹\ 275$$

20. Let CP of each gram be ₹ 1.

Then, CP of 450 g = ₹ 450, SP of 450 g = ₹ 500

$\therefore$ Profit per cent $= \left(\frac{50}{450} \times 100\right)\% = 11\frac{1}{9}\%$

21. Let CP of radio be ₹ x. Then,

120% of $x - 116\%$ of $x = 20$

$$\Rightarrow \quad 4\% \text{ of } x = 20$$

$$\Rightarrow \quad x = \frac{20 \times 100}{4} = ₹\ 500$$

22. Let CP of radio be ₹ x. Then,

$$107\frac{1}{2}\% \text{ of } x - 97\frac{1}{2}\% \text{ of } x = 100$$

$$\Rightarrow \quad 10\% \text{ of } x = 100$$

$$\Rightarrow \quad x = \frac{100 \times 100}{10}$$

$$\Rightarrow \quad x = ₹\ 1000$$

23. Let CP of sugar per kg be ₹ x.

$$\therefore \quad 93\% \text{ of } x = 11.16$$

$$\Rightarrow \quad x = \frac{11.16 \times 100}{93} = 12$$

So, $\quad CP = ₹\ 12$ per kg

$$\therefore \quad SP = 107\% \text{ of } ₹\ 12$$

$$= \frac{107}{100} \times 12 = ₹\ 12.84$$

24. Let CP be ₹ 100, then SP be ₹ 120.

Now, SP = ₹ 120, commission = 10%

$\therefore$ Marked price $= \frac{100}{90} \times 120 = ₹\ \frac{400}{3}$

Now, marked price $= \frac{400}{3}$, commission = 15%

$\therefore$ SP = 85% of $\frac{400}{3} = \frac{85}{100} \times \frac{400}{3} = \frac{340}{3}$

$\therefore$ Gain per cent $= \frac{340}{3} - 100 = 13\frac{1}{3}\%$

25. CP of an orange $= \frac{3}{7}$

$\therefore$ CP of 100 orange $= \frac{300}{7}$

$\Rightarrow$ SP of 100 oranges $= \frac{300}{7} \times \frac{(100 + 33)}{100} = ₹\ 57$

$$\left[\because SP = CP\left(\frac{100 + r}{100}\right)\right]$$

26. Reduced price $= \frac{10 \times 20}{5 \times 100} = ₹\ \frac{2}{5}$ per orange

$\therefore$ Original price $= \frac{2 \times 100}{5 \times 80} = \frac{1}{2} = 50$ paise

27. $\because$ SP = ₹ 660, Profit % = 10%

$\Rightarrow$ CP = ₹ 600

$$\therefore \quad MP = \frac{600 \times 100}{75} = ₹\ 800$$

28. Let the original CP be ₹ x.

$\therefore \quad x \times \frac{80}{100} = 64 \Rightarrow x = ₹\ 80$

29. $\therefore$ Gain % $= \frac{19-16}{16} \times 100$

$= \frac{3 \times 100}{16} = \frac{75}{4} = 18\frac{3}{4}\%$

30. Let SP = $7x$ and CP = $5x$

$\therefore$ Profit $= 7x - 5x = 2x$

$\therefore \frac{\text{Profit}}{\text{CP}} = \frac{2x}{5x} = \frac{2}{5}$

31. Given, 5% = ₹ 50

$\Rightarrow$ 100% $= \frac{50}{5} \times 100 =$ ₹ 1000

32. Final cost of the house

$= 150000 \times \frac{100+5}{100} \times \frac{(100-2)}{100}$

$=$ ₹154350

$\therefore$ X gains ₹ 4350.

33. $\because$ Interest $= \frac{P \times R \times T}{100}$

$= \frac{15000 \times 5 \times 2}{100} =$ ₹ 1500

$\therefore$ Amount = Principal + Interest

$=$ ₹ $(15000 + 1500) =$ ₹ 16500

34. Interest in three years $= 1350 - 1260 =$ ₹ 90

$\therefore$ Interest in one year $=$ ₹ $\frac{90}{3} =$ ₹ 30

and interest for two years = ₹ 60

and Principal = ₹ $(1260 - 60)$

$=$ ₹1200

Hence, rate of interest $= \frac{100 \times 60}{1200 \times 2} = 2.5\%$

35. Let sum = ₹ x

Then, according to given condition

$$\frac{x \times 13 \times 1}{100} - \frac{x \times 12 \times 1}{100} = 110$$

$\Rightarrow \quad \frac{x}{100} = 110$

$\Rightarrow \quad x =$ ₹ 11000

36. According to given condition,

$$4410 = P\left(1 + \frac{5}{100}\right)^2$$

$\Rightarrow \quad P = \frac{4410}{(21/20)^2}$

$= \frac{4410 \times 400}{441} =$ ₹ 4000

37. Let the sum be ₹ x. Then,

$$\frac{x \times 5 \times 15}{100} - \frac{x \times 15}{100} \times \frac{7}{2} = 144$$

$\Rightarrow \quad \frac{x}{100}\left(75 - \frac{105}{2}\right) = 144$

$\Rightarrow \quad x = \frac{144 \times 100 \times 2}{45} \Rightarrow x =$ ₹ 640

38. $t = 8$ yr

SI increase by ₹ 30 in 8 yr

$\therefore$ SI increases in 1 yr $= \frac{30}{8} = 3.75$

Let the rate be x% per annum

$$3.75 = \frac{1500 \times 1 \times x}{100}$$

$$x = 0.25\%$$

39. Let the annual instalment be ₹ x, then

$4x$ + {interest on x for $(3 + 2 + 1)$ yr} $= 19350$

$\Rightarrow \quad 4x + \frac{x \times 6 \times 5}{100} = 19350$

$\therefore \quad x =$ ₹ 4500

40. Since, $\text{SI} = \frac{PRT}{100}$

$\therefore \quad 6750 = \frac{P \times \frac{20}{3} \times 3}{100}$

$\Rightarrow \quad P = 33750$

Now, $\quad \text{CP} = 33750\left[\left(1 + \frac{20}{300}\right)^3 - 1\right]$

$= 33750\left[\frac{4096}{3375} - 1\right] =$ ₹ 7210

41. Now, simple interest for two years, $\text{SI} = \frac{PRT}{100}$

$= \frac{60000 \times 6 \times 2}{100} =$ ₹7200

and compound interest for two years $\text{CI} = A - P$

$$=P\left(1+\frac{R}{100}\right)^n-P=60000\left(1+\frac{6}{100}\right)^2-60000$$

$$=60000\left(\frac{53}{50}\right)^2-60000$$

$$=67416-60000=₹7416$$

$\therefore$ Required difference $=\text{CI}-\text{SI}$

$$=7416-7200=₹216$$

42. $$A=P\left(1+\frac{R/2}{100}\right)^{2n}$$

$$=350\times\frac{102}{100}\times\frac{102}{100}$$

$$=₹\,364.14$$

$\therefore$ Compound interest $=₹\,364.14-₹\,350$

$$=₹\,14.14$$

43. $$A=2000\left(1+\frac{10}{400}\right)^{4\times\frac{5}{4}}$$

$$=2000\left(\frac{41}{40}\right)^5=₹\,2262.81$$

$\therefore$ Compound interest $=2262.81-2000$

$$=₹262.81$$

44. $$P\left(1+\frac{R}{100}\right)^{15}=2P$$

or $$\left(1+\frac{R}{100}\right)^{15}=2 \quad \text{...(i)}$$

Now, $P\left(1+\frac{R}{100}\right)^n=8P$

$$\Rightarrow \left(1+\frac{R}{100}\right)^n=8=(2)^3=\left\{\left(1+\frac{R}{100}\right)^{15}\right\}^3$$

[from Eq. (i)]

$$\Rightarrow \left(1+\frac{R}{100}\right)^n=\left(1+\frac{R}{100}\right)^{45}$$

$$\Rightarrow n=45$$

45. SI on ₹ 7350 for 1 yr

$$=₹\,(8575-7350)=₹1225$$

$$\therefore \quad \text{Rate}=\left(\frac{100\times1225}{7350\times1}\right)\%=16\frac{2}{3}\%$$

Let the sum be ₹ x.

Then, $$x\left(1+\frac{50}{3\times100}\right)^2=7350$$

$$\Rightarrow x\times\frac{7}{6}\times\frac{7}{6}=7350$$

$$\Rightarrow x=7350\times\frac{36}{49}=5400$$

or Sum = ₹ 5400

CHAPTER

07

SPEED, TIME AND DISTANCE

In this chapter, we study speed of a person/object/train to cover a distance from a point to another point in certain time.

Speed

The rate at which a body or an object travels to cover a certain distance is called speed of body. The unit of speed is km/h and m/s.

Time

The duration in hours, minutes or seconds spent to cover a certain distance is called the time. The unit of time is hours, minute and seconds.

Distance

The length of the path travelled by any object or any person between two places is known as distance. The unit of distance is m, km, etc.

Relationship between Time, Speed and Distance

Relationship between time, distance and speed is expressed by

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \text{ or Distance} = \text{Speed} \times \text{Time}$$

This expression shown that

1. Speed is directly proportional to distance.
2. Distance and time are directly proportional.
3. Time is inversely proportional to speed.

Average Speed

When a certain distance is covered at speed A and the same distance is covered at speed B, then the average speed during the whole journey is given by $\frac{2AB}{A+B}$.

Note If a body covers a distance D_1 at S_1 km/h, D_2 at S_2 km/h, D_3 at S_3 km/h and so on upto D_n at S_n km/h then

Average speed

$$= \frac{D_1 + D_2 + D_3 + D_4 + \ldots + D_n}{\frac{D_1}{S_1} + \frac{D_2}{S_1} + \frac{D_3}{S_3} + \frac{D_4}{S_4} + \ldots + \frac{D_n}{S_n}}$$

$$\text{Average speed} \neq \frac{S_1 + S_2 + S_3 + S_4 \ldots S_n}{n}$$

Example 1 A person goes to Delhi from Mumbai at the speed of 60 km/h and comes back at the speed of 50 km/h. Calculate the average speed of the person for the entire trip.

(a) 54.75 km/h (b) 54.54 km/h
(c) 57.57 km/h (d) 75.57 km/h

Sol. *(b)* Average Speed $= \frac{2 \times 60 \times 50}{60 + 50}$

$$[\because A = 60 \text{ km/h}, B = 50 \text{ km/h}]$$

$$= \frac{6000}{110} = 54.54 \text{ km/h}$$

Example 2 A teacher reaches the school in 30 min with average speed of 15km/h. If they want to reach school in 10 minute earlier, their speed should be

(a) 80 km/h (b) 90 km/h
(c) 50 km/h (d) 75 km/h

Sol. *(b)* Let the distance covered by the teacher to reach to the school in d km with time t.

$$\because \quad \text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$15 = \frac{d}{\frac{30}{60}} \Rightarrow d = \frac{15 \times 60}{30} = 30\text{km}$$

Again using formula,

$$S = \frac{30}{\frac{(30-10)}{60}} = \frac{30 \times 60}{20} = 90 \text{ km/h}$$

Some Another Formulae

1. To convert a m/s into km/h, multiply by $\frac{18}{5}$, i.e. $a \times \frac{18}{5}$ km/h. e.g. Convert 25 m/s to km/h.

$$25 \text{ m/s} = \left(25 \times \frac{18}{5}\right) = 5 \times 18 = 90 \text{ km/h}$$

2. To convert a km/h into m/sec, multiply by $\frac{5}{18}$, i.e. $a \times \frac{5}{18}$ m/s. e.g. Convert 72 km/h into m/s

$$\therefore 72 \text{ km/h} = \left(72 \times \frac{5}{18}\right) \text{ m/s} = 4 \times 5 = 20 \text{ m/s}$$

3. When two bodies A and B are moving with speed a km/h and b km/h respectively, then the relative speed of two bodies is
   (i) $(a + b)$ km/h (if they are moving in opposite direction)
   (ii) $(a - b)$ km/h (if they are moving in same direction)
4. If a man changes his speed in the ratio $a : b$, then the ratio of time taken becomes $b : a$.
5. The distance covered by train in passing a pole or a standing man or a signal post or any other object (of negligible length) is equal to the length of the train.
6. If a train passes a stationary object (bridge, platform etc) having some length, then the distance covered by train is equal to the sum of the lengths of train and that particular stationary object which it is passing.
7. (i) If two trains of lengths x and y km are moving in opposite directions with speeds of u and v respectively, then time taken by the trains to cross each other

$$= \frac{\text{Sum of lengths}}{\text{Sum of speeds}} = \frac{(x+y)}{(u+v)}$$

   (ii) If trains are moving in same directions, then time taken by faster train to cross slower train

$$= \frac{\text{Sum of lengths}}{\text{Difference of speed}} = \frac{x+y}{u-v}$$

Here, $u > v$.

Example 3 The speed of a bus is 72 km/h. The distance covered by the bus in 5 s is
(a) 50 m (b) 74.5 m (c) 100 m (d) 60 m

Sol. *(c)* Speed of bus in m/s $= 72 \times \frac{5}{18} = 20$ m/s

$\therefore$ Distance travelled in 5 s $= 20 \times 5$ (Speed × Time)
$= 100$ m

Example 4 Two trains are running in the same direction. The speeds of two trains are 5 km/h and 15 km/h, respectively. What will be the relative speed of second train with respect to first?
(a) 10 km/h (b) 15 km/h (c) 20 km/h (d) 5 km/h

Sol. *(a)* We know that, if two trains are running in same direction, then difference in speeds is the required relative speed.

$\therefore$ Required relative speed $= 15 - 5 = 10$ km/h

Example 5 Two trains of lengths 75 m and 95 m are moving in the same direction at 9 m/s and 8 m/s, respectively. Find the time taken by the faster train to cross the slower train.
(a) 120 s (b) 170 s
(c) 140 s (d) 190 s

Sol. *(b)* According to the formula,

$$\text{Required time} = \frac{x + y}{u - v}$$

where, $x = 75$ m,
$y = 95$ m,
$u = 9$ m/s
and $v = 8$ m/s

$$\therefore \text{Required time} = \frac{75 + 95}{9 - 8} = 170 \text{ s}$$

PRACTICE EXERCISE

1. A train runs at the rate of 120 km/s. The speed is m/s is
(a) $66\frac{2}{3}$ (b) 25 (c) 30 (d) $33\frac{1}{3}$

2. An Athlete runs 200 m race in 24 s his speed (in km/h) is
(a) 20 (b) 24 (c) 30 (d) 28.5

3. A person crosses a 600 m long street in 5 min. What is his speed in km per hour?
(a) 3.6 (b) 7.2
(c) 8.4 (d) 10

4. A train is 125 m long. If the train takes 30 s to cross a tree by the railway line, then the speed of the train is
(a) 14 km/h (b) 15 km/h
(c) 16 km/h (d) 12 km/h

5. A man riding on a bicycle at a speed of 15 km/h crosses a bridge in 5 min. Find the length of the bridge.
(a) 1 km (b) 2 km
(c) $2\frac{1}{2}$ km (d) $1\frac{1}{4}$ km

6. A train 110 m long is running at the speed of 72 km/h to pass a 132 m long platform in how many times?
(a) 9.8 s (b) 12.1 s (c) 12.42 s (d) 14.3 s

7. If a man covers $10\frac{1}{5}$ km in 3 h, the distance covered by him in 5 h is
(a) 16 km (b) 15 km (c) 18 km (d) 17 km

8. A train 700 m long is running at the speed of 72 km/h. If it crosses a tunnel in 1 min, then the length of the tunnel is
(a) 650 m (b) 500 m (c) 550 m (d) 700 m

9. A car does a journey in 10 h, the first half at 21 km per hour, and the rest at 24 km per hour.The distance travelled by the car is
(a) 264 km (b) 244 km (c) 254 km (d) 224 km

10. Sunita reaches the coaching in 25 min with an average speed of 14 km/h. She want to reach school 5 min later, then her speed should be
(a) 11.67 km/h (b) 12 km/h
(c) 13 km/h (d) 13.5 kn/h

11. A man travels first 50 km at 25 km/h next 40 km at 20 km/h and then 90 km at 15 km/h. His average speed for the whole journey in (m/s)is
(a) 18 (b) 5
(c) 10 (d) 36

12. A car is ahead of a scooter by 30 km, car goes at the rate of 50 km/h and the scooter goes at the rate of 60 km/h. The scooter overtake the car after
(a) 3 h (b) 3.5 h
(c) 4 h (d) $3\frac{1}{4}$ h

13. A train passes two bridges of lengths 800 m and 400 m in 100 s and 60 s respectively. The length of the train is
(a) 80 m (b) 90 m
(c) 200 m (d) 150 m

14. Two runners cover the same distance at the rate of 15 km and 16 km per hour respectively. The distance travelled when one takes 16 min longer than the other is
(a) 64 km (b) 60 km
(c) 52 km (d) 58 km

15. Excluding stoppages, the speed of a bus is 54 km/h and including stoppages, it is 45 km/h. For how many minutes does the bus stop per hour?
(a) 9 (b) 10
(c) 12 (d) 20

16. Two persons are moving in the directions opposite to each other. The speeds of the both persons are 5 km/h and 3 km/h, respectively. Find the relative speed of the two persons in respect of each other
(a) 7 km/h (b) 3 km/h (c) 8 km/h (d) 5 km/h

17. A train 240 m long passes a pole in 24 s. How long will it take to pass a platform 650 m long?
(a) 65 sec (b) 89 sec (c) 100 sec (d) 150 sec

18. Distance between two towns P and Q is 240 km. A motor cycle rider starts from P towards Q at 8 pm at a speed of 40 km/h. At the same time another motor cycle rider starts from Q towards P at 50 km/h. At what time will they meet?
(a) 9 : 45 pm (b) 10 : 40 pm
(c) 11 pm (d) 10 : 30 pm

19. A car travels 25 km an hour faster than a bus a takes 10 hours less journey of 500 kms. The speed of the car and bus are given by respectively.
(a) 60 and 35 (b) 45 and 70
(c) 80 and 55 (d) 50 and 55

20. Two trains running in opposite directions cross a man standing on the platform in 54s and 34s respectively and they cross each other in 46 s. Find the ratio of their speeds.
(a) 3 : 2 (b) 2 : 3 (c) 5 : 3 (d) 3 : 5

Answers

1	(d)	2	(c)	3	(b)	4	(b)	5	(d)	6	(b)	7	(d)	8	(b)	9	(d)	10	(a)
11	(b)	12	(a)	13	(c)	14	(a)	15	(b)	16	(c)	17	(b)	18	(b)	19	(b)	20	(a)

Hints and Solutions

1. $120 \text{ km/h} = 120 \times \frac{5}{18} = 33\frac{1}{3}$ m/s

2. Distance = 200 m, Time = 24 s

Speed $= \frac{200}{24} \times \frac{18}{5}$ km/h = 30 km/h

3. Speed $= \frac{\text{Distance}}{\text{Time}} = \frac{600}{5 \times 60}$ m/s = 2 m/sec

$= 2 \times \frac{18}{5}$ km/hr = 7.2 km/hr

4. Speed of train $= \frac{\text{Distance}}{\text{Time}} = \frac{125}{30} \times \frac{18}{5}$

$= 15$ km/h

5. Length of bridge

= Speed × time taken to cross the bridge

$= \frac{15 \times 5}{60} = 1\frac{1}{4}$ km

6. Speed of the train = 72 km/h

$= 72 \times \frac{5}{18}$ m/s = 20 m/s

∴ Required time $= \frac{110 + 132}{20}$

$= \frac{242}{20} = 12.1$ s

7. Distance covered by man in 3h

$= 10\frac{1}{5}$ km

Distance covered by man in 1 h

$= \frac{51}{5 \times 3} = \frac{17}{5}$ km

Distance covered by man in 5 h will be

$= \frac{17}{5} \times 5 = 17$ km

8. Speed $= 72 \times \frac{5}{18} = 20$ m/s

Let the length of tunnel be x m.

Then, $\frac{700 + x}{20} = 60$

$\Rightarrow \quad x = 500$ m

9. Average speed

$= \frac{2 \times 21 \times 24}{21 + 24} = \frac{2 \times 21 \times 24}{45}$

$= \frac{2 \times 21 \times 24}{45} \times \frac{112}{5}$ km/hr

∴ Distance $= \frac{112}{5} \times 10 = 224$ km

10. Let the distanve covered by Sunita to reach the school in d km withe time t. Then

Speed $= \frac{\text{Distance}}{\text{Time}}$

$14 = \frac{d}{\frac{25}{60}}$

$\Rightarrow \quad d = \frac{14 \times 25}{60} = \frac{350}{60} = \frac{35}{6}$

Again using formula,

require speed $= \frac{\frac{35}{6}}{\frac{(25+5)}{60}} = \frac{35 \times 10}{30}$

$= \frac{35}{3} = 11.67$ km/h

11. Total distance covered

$= 50 + 40 + 90 = 180$ km

Total time taken $= \frac{50}{25} + \frac{40}{20} + \frac{90}{15} = 10$ h

∴ Average speed for the whole journey

$= \frac{180}{10} = 18 = 18 \times \frac{5}{18} = 5$ m / s

12. Distance between car and scooter = 30 km

Difference of velocity

$= 60 - 50 = 10$ km/h

So, time taken by scooter to travel 30 km extra than a car when its speed 10 km/h more

$= \frac{30}{10} = 3$ h

13. Let train is x m long.

According to the question,

$$\frac{800 + x}{100} = \frac{400 + x}{60}$$

$$4800 + 6x = 4000 + 10x \Rightarrow 4x = 800$$

$$\therefore \quad x = 200\,\text{m}$$

14. Let distance covered by runners = x km

Then, according to the question,
difference time in both runner = 6 min

$$\Rightarrow \quad \frac{x}{15} - \frac{x}{16} = \frac{6}{60} \quad [\because 1 \text{ hour} = 60 \text{ min}]$$

$$\Rightarrow \quad \frac{x}{15 \times 16} = \frac{6}{60}$$

$$\Rightarrow \quad x = \frac{15 \times 16}{10} = 64 \text{ km}$$

Hennce, distance covered by runners = 64 km

15. Speed = 54 km/hr

After stopage = 45 km/hr

$\Rightarrow$ Difference = 54 − 45 = 9 km/hr

$$\therefore \quad \text{Required time} = \frac{9}{54} \times 60 = 10 \text{ min}$$

16. We know that, the two speeds will be added, if the motions of two objects are in opposite directions.

$\therefore$ Required relative speed

$$= 5 + 3 = 8 \text{ km/h}$$

17. Speed of train $= \frac{240}{24}$ m/s = 10 m/s

$$\text{Time} = \frac{650 + 240}{10} = 89\,\text{s}$$

18. Suppose, they meet x h after 8 pm. Then, sum of distance covered by them in hours = 240 km.

$$\therefore \quad 40x + 50x = 240$$

$$\Rightarrow \quad x = \frac{240}{90} = 2 \text{ h } 40 \text{ min}$$

$$= 8 \text{ pm} + 2 \text{ h } 40 \text{ min}$$

Hence, they will meet = 10:40 pm

19. Let speed of bus = x km/hr

Speed of car = $(x + 25)$ km/hr

According to the question,

$$\frac{500}{x} - \frac{500}{x + 25} = 10$$

$\therefore$ Car x = 45 km/hr.

Bus = 45 + 25 = 70 km/hr.

20. Let the speed of two trains be x and y, respectively.

$\therefore$ Length of 1st train = $54x$

Length of the 2nd train = $34y$

According to the question,

$$\frac{54x + 34y}{x + y} = 46$$

$$\Rightarrow \quad 54x + 34y = 46x + 46y$$

$$\Rightarrow \quad 27x + 17y = 23x + 23y$$

$$\Rightarrow \quad 4x = 6y \Rightarrow 2x = 3y$$

$$\Rightarrow \quad \frac{x}{y} = \frac{3}{2}$$

$$\therefore \quad x : y = 3 : 2$$

CHAPTER

08

ALGEBRAIC EXPRESSION AND IDENTITIES

In this chapter, we study the various terms of Algebraic expression, operations on polynomials, various identities, factorisation by various method.

Constant A symbol having a fixed numerical value is called a constant.

Variable A symbol which takes various numerical values is called a variable.

e.g., $P=2(l+b)$

Here, 2 is a constant and l and b are variables.

Algebraic Expression

An algebraic expression is a combination of constants and variables connected by fundamental operations $(+, -, \times, \div)$. e.g. $2x+3$, $8a^2b + a^3b^3 - 5a$ etc .

Terms

The separated parts of an algebraic expression are called its term.

e.g. $2x$ and 3 are the terms of expression $2x+3$.

Like and Unlike Terms

The terms having same variable and the same exponents are like terms, otherwise it has, unlike terms.

e.g. In $x^2 - xy + 2x^2 + y$, x^2 and $2x^2$ are like terms and $-xy$ and y are unlike terms.

Factors

Each term in an algebraic expression is a product of one or more numbers and variables. These numbers and variables are known as the factors of that term.

e.g. $7x$ is the product of number 7 and variable x. So, 7 and x are factors of $7x$.

Coefficients

In a term of an algebraic expression, the numerical value of a term is called coefficient and any of the factors with the sign of the term is called the coefficient of the product of the other factors.

e.g. In $3xy$, the coefficient of y is $3x$, the coefficient of x is $3y$ and the coefficient of xy is 3.

Polynomial

An algebraic expression that contains one or more terms with non-zero coefficients, is called a polynomial.

e.g. $a + b + c + d$, $3xy$ and $7xy - 10$, etc. are all polynomials.

Note (i) An expression may contain a term involving rational power of a variable but in a polynomial, the power of each variable must be a non-negative integer.

(ii) The highest power of the variable (or in more than one variable, the sum of the highest power of the variable) is the degree of the polynomial.

Monomial

An algebraic expression that contains only one term, is called a monomial.

e.g. $3, -6abc$ and $5x^2y$, etc., are all monomials.

Binomial

An algebraic expression that contains two terms, is called a binomial.
e.g. $x + 5$, $x^3 + 7$ and $a^2 - 2abc$, etc. are all binomials.

Trinomial

An algebraic expression that contains three terms, is called a trinomial.

e.g. $2x - y + 3$ and $3 + xyz + x^3$, etc. are all trinomials.

Quadrinomial

An algebraic expression that contains four terms, is called a quadrinomial. e.g. $a^3 + b^3 + c^3 + 3abc$ and $ab + bc + ca + abc$, etc., are all quadrinomials.

Example 1 The example of binomial is

(a) $5+2x$ (b) $2x^2$
(c) $-2x-5y-1$ (d) $3+xy+2y^2$

Sol. *(a)* We have, $5+2x$, only this algebraic contains two terms. Hence, $5+2x$ is the example of binomial.

Fundamental Operations on Polynomials

Some operations based on polynomials are discussed below

I. **Addition of Polynomials** Polynomials can be added by arranging their like terms and combining them.

II. **Subtraction of Polynomials** Polynomials can be subtracted by arranging their like terms and by changing sign of each term of the polynomial to be subtracted and then added.

III. **Multiplication of Polynomials** We know that,

(a) The product of two factors with like signs is positive and product of unlike signs is negative.

(b) If x is any variable and m, n are positive integers, then $x^m \times x^n = x^{m+n}$

Thus, $x^3 \times x^6 = x^{(3+6)} = x^9$.

IV. (a) **Division of a Monomial by a Monomial**
We have, quotient of two monomials
= (quotient of their coefficients)
× (quotients of two monomials)

(b) **Division of a Polynomial by a Monomial**

To dividing a polynomial by a monomial, we can divide each term of the polynomial by the monomial.

e.g. $\frac{2x^2+6x+8}{2}=\frac{2x^2}{2}+\frac{6x}{2}+\frac{8}{2}$
$= x^2+3x+4$

(c) **Division of a Polynomial by a Polynomial**

The following steps are given below

(i) Firstly, arrange the terms of the dividend and divisor in descending order of their degrees.

(ii) Divide the first term of the dividend by the first term of the divisor to obtain the first term of the quotient.

(iii) Multiply all the terms of the divisor by the first term of the quotient and subtract the result from the dividend.

(iv) Consider the remainder (if any) as a new dividend and proceed as before.

(v) Repeat this process till we obtain a remainder which is either 0 or a polynomial of degree less than the degree of the divisor.

Note In polynomials, we have dividend = (divisor × quotient) + remainder

Example 2 The sum of $3x^2-4x+5$ and $9x-10$ is

(a) $3x^2+5x$ (b) $3x^2+5x-5$
(c) $5x-5$ (d) $3x^2-5x+5$

Sol. *(b)* The sum of $3x^2-4x+5$ and $9x-10$, is

$$\begin{array}{r} 3x^2-4x+5 \\ +9x-10 \\ \hline 3x^2+5x-5 \\ \hline \end{array}$$

Example 3 The product of $(3x+7y)$ and $(3x-7y)$ is

(a) $9x^2-49y^2$ (b) $39x^2-49y^2$
(c) $9x^2+49y^2$ (d) $9x^2-49y$

Sol. *(a)* $(3x+7y)\times(3x-7y)$
$= 3x(3x-7y)+7y(3x-7y)$
$=9x^2-21xy+21xy-49y^2=9x^2-49y^2$

Example 4 Find the quotient and the remainder when x^4+1 is divided by $x-1$.

(a) $x^3+x^2+x+1, 2$ (b) $x^3+x^2-x+1, 2$
(c) $x^3+x^2+x+1, 3$ (d) None of these

Sol. *(a)* Using long division method,

$$\begin{array}{r|l} & x^3+x^2+x+1 \\ \hline x-1 & x^4+1 \\ & \underline{-x^4 \mp x^3} \\ & x^3+1 \\ & \underline{-x^3 \mp x^2} \\ & x^2+1 \\ & \underline{-x^2 \mp x} \\ & x+1 \\ & \underline{-x \mp 1} \\ & 2 \end{array}$$

Hence, quotient $= x^3+x^2+x+1$ and remainder = 2.

Identity

An identity is an equality which is true for all values of the variables.

e.g. $(a+b)^2=(a)^2+2ab+(b)^2$

Some Important Identities

- $(a+b)^2=a^2+2ab+b^2$
- $(a-b)^2=a^2-2ab+b^2$
- $(a+b)(a-b)=a^2-b^2$
- $(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$
  $=a^2+b^2+c^2+2(ab+bc+ca)$
- $(a+b-c)^2=a^2+b^2+c^2+2ab-2bc-2ca$
- $(a-b+c)^2=a^2+b^2+c^2-2ab-2bc+2ca$
- $(-a+b+c)^2=a^2+b^2+c^2-2ab+2bc-2ca$
- $(a-b-c)^2=a^2+b^2+c^2-2ab+2bc-2ca$
- $(a+b)^3=a^3+b^3+3ab(a+b)$
- $(a-b)^3=a^3-b^3-3ab(a-b)$
- $a^3+b^3=(a+b)^3-3ab(a+b)$
- $a^3+b^3=(a+b)(a^2+b^2-ab)$

- $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$
- $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$
- $(x + a)(x + b) = x^2 + (a + b)x + ab$

Example 5 If $p+q=12$ and $pq=22$, then the value of p^2+q^2 is

(a) 95 (b) 105 (c) 100 (d) 110

Sol. (c) We know that,

$$(p+q)^2 = p^2 + 2pq + q^2$$

$$\Rightarrow \quad (12)^2 = p^2 + 2\times 22 + q^2 \quad [\because p + q = 12 \text{ and } pq = 22]$$

$$\Rightarrow \quad 144 = p^2 + 44 + q^2$$

$$\therefore \quad p^2+q^2 = 144 - 44 = 100$$

Example 6 If $x+\frac{1}{x}=3$, then the value of $x-\frac{1}{x}$ is

(a) $+\sqrt{5}$ (b) $-\sqrt{5}$ (c) $\pm\sqrt{5}$ (d) ± 5

Sol. (c) Given, $x+\frac{1}{x}=3$

$$\Rightarrow \quad \left(x+\frac{1}{x}\right)^2=3^2 \quad \text{[on squaring]}$$

$$\Rightarrow \quad \left(x^2+\frac{1}{x^2}+2\right)=9$$

$$\Rightarrow \quad x^2+\frac{1}{x^2}=7$$

$$\Rightarrow \quad x^2+\frac{1}{x^2}-2=7-2 \quad \text{[subtracting 2 on both sides]}$$

$$\Rightarrow \quad x^2+\frac{1}{x^2}-2=5 \quad \Rightarrow \left(x-\frac{1}{x}\right)^2=5$$

$$\Rightarrow \quad \left(x-\frac{1}{x}\right)=\pm\sqrt{5}$$

Example 7 If $4x-5z=16$ and $zx=12$, then the value of $64x^3-125z^3$ is

(a) 15600 (b) 1561
(c) 15616 (d) 15618

Sol. (c) $64x^2-125z^3=(4x)^3-(5z)^3$

$$=(4x-5z)[(4x)^2+4x\cdot 5z+(5z)^2]$$

$$=16[(4x-5z)^2+40zx+20zx]$$

$$=16[16^2+60\times 12]$$

$$=16\times 976=15616$$

Factor and Factorisation

A polynomial $g(x)$ is called **a factor** of polynomial $p(x)$, if $g(x)$ divides $p(x)$ exactly. To express polynomial as the product of polynomial of degree less than that of the given polynomial is called as **factorisation**.

Factorisation by Common Factors

A factor which occurs in each term, is called the common factor. e.g. Factorise $16x^2y+4xy$

We have, $16x^2y=2\times2\times2\times2\times x\times x\times y$

and $4xy=2\times2\times x\times y$

Here, $2\times2\times x\times y$ is common in these two terms.

Factorisation by Splitting Middle Term

Let factors of the quadratic polynomial $ax^2 + bx + c$ be $(px + q)$ and $(rx + s)$. Then,

$$ax^2 + bx + c = (px + q)(rx + s)$$

$$= prx^2 + (ps + qr)x + qs$$

On comparing the coefficients of x^2, x and constant terms from both sides, we get

$$a = pr,\ b = ps + qr \text{ and } c = qs$$

Here, b is the sum of two numbers ps and qr, whose product is $(ps)(qr) = (pr)(qs) = ac$.

Thus, to factorise $ax^2 + bx + c$, write b as the sum of two numbers, whose product is ac.

Note To factorise $ax^2 + bx - c$ and $ax^2 - bx - c$, write b as the difference of two numbers whose product is $(-ac)$.

Example 8 Factors of $2x^2 + 7x + 3$ are

(a) $(x+2)(x+1)$ (b) $(2x+1)(x+3)$
(c) $(x+3)(2x-1)$ (d) $(2x-2)(x-3)$

Sol. (b) Given polynomial is $2x^2+7x+3$

On comparing with ax^2+bx+c, we get

$$a = 2,\ b = 7 \text{ and } c = 3$$

Now, $\quad ac = 2\times 3 = 6$

So, all possible pairs of factors of 6 are 1 and 6, 2 and 3.

Clearly, pair 1 and 6 gives

$$1+6=7=b$$

$$\therefore \quad 2x^2+7x+3=2x^2+(1+6)x+3$$
$$=2x^2+x+6x+3$$
$$=x(2x+1)+3(2x+1)$$
$$=(2x+1)(x+3)$$

Factorisation by Algebraic Identities

To solve these types of question, we have to use some algebraic identities.

e.g. $x^2-(2y)^2=(x+2y)(x-2y)$

Here, we use $a^2-b^2=(a+b)(a-b)$ identity.

Example 9 Factors of $x^3+27y^3+8z^3-18xyz$ are

(a) $(x+3y+2z)(x^2+9y^2+4z^2-3xy-6yz-2xz)$
(b) $(x-3y-2z)(x^2-9y^2+4z^2+3xy+6yz+2xz)$
(c) $(x+3y-3z)(x^2-9y^2+4z^2-3xy-6yz-2xz)$
(d) None of the above

Sol. (*a*) We have, $x^3+27y^3+8z^3-18xyz$

$$=x^3+(3y)^3+(2z)^3-3(x)(3y)(2z)$$
$$=(x+3y+2z)[x^2+9y^2+4z^2-x(3y)-3y(2z)-x(2z)]$$
$$=(x+3y+2z)(x^2+9y^2+4z^2-3xy-6yz-2xz)$$

Example 10 The value of $x^4-3x^3+2x^2+x-1$ at $x=2$ is

(a) 0 (b) 1 (c) 2 (d) 3

Sol. (*b*) Let $p(x)=x^4-3x^3+2x^2+x-1$ ...(i)

On putting $x=2$ in Eq. (i), we get

$$p(2)=2^4-3\times(2)^3+2\times(2)^2+2-1$$
$$=16-24+8+1=1$$

Example 11 The value of p, if $(2x-1)$ is a factor of $2x^3+px^2+11x+p+3$ is

(a) -7 (b) 7 (c) -6 (d) 5

Sol. (*a*) Let $q(x)=2x^3+px^2+11x+p+3$

If $q(x)$ is divisible by $2x-1$, then $(2x-1)$ is a factor of $q(x)$.

$$\therefore \quad 2x-1=0$$
$$\Rightarrow \quad x=\frac{1}{2}$$

On putting $x=\frac{1}{2}$ in $q(x)$, we have

$$q\left(\frac{1}{2}\right)=2\times\left(\frac{1}{2}\right)^3+p\left(\frac{1}{2}\right)^2+11\left(\frac{1}{2}\right)+p+3=0$$
$$\Rightarrow \quad 2\times\frac{1}{8}+p\times\frac{1}{4}+\frac{11}{2}+p+3=0$$
$$\Rightarrow \quad \frac{1}{4}+\frac{p}{4}+\frac{11}{2}+p+3=0$$
$$\Rightarrow \quad \frac{1+p+22+4p+12}{4}=0$$
$$\Rightarrow \quad 5p+35=0$$
$$\Rightarrow \quad 5p=-35$$
$$\therefore \quad p=-7$$

PRACTICE EXERCISE

1. In the expression $3a^2-4ab+5b^2+7ba$ the like terms are

(a) $-4ab,\ 7ba$ (b) $3a^2,\ 5b^2$
(c) $4ab,\ 7ba$ (d) $-4ab,\ -7ba$

2. The value of the expression

$$8y^2+\frac{1}{3}x^2+\frac{1}{5}z^2-3xy+4yz$$

at $x=1$, $y=-2$ and $z=-3$ is

(a) $64\frac{2}{15}$ (b) $64\frac{1}{15}$
(c) $64\frac{3}{15}$ (d) None of these

3. $-6x^2y^2+4x^2y^2-3x^2y^2$ is equal to

(a) $6x^2y^2$ (b) $-5x^2y^2$
(c) $-6x^2y^2$ (d) $5x^2y^2$

4. The difference of $x^3 - x^2 + 2x - 19$ and $2x^3 - x^2 + 4x - 6$ is
(a) $x^3 - 2x + 13$ (b) $-x^3 + 6x^2 - 8x + 12$
(c) $-x^3 - 2x - 13$ (d) None of the above

5. What must be added to $x^2 + 4x - 6$ to get $x^3 - x^2 + 2x - 2$?
(a) $x^3 + 6x - 8$ (b) $x^3 - 2x^2 + 2x - 4$
(c) $x^3 + 2x^2 + 2x - 4$ (d) $x^3 - 2x^2 - 2x + 4$

6. What must be subtracted from $x^4 + 2x^2 - 3x + 7$ to get $x^3 + x^2 + x - 1$?
(a) $x^4 - x^3 + x^2 - 4x + 8$
(b) $x^3 + x^4 + x^2 + 4x + 8$
(c) $x^4 + x^3 - x^2 + 4x - 8$
(d) None of the above

7. The product of $2x^2 + x - 5$ and $x^2 - 2x + 3$ is
(a) $2x^4 + 3x^3 + x^2 + 13x + 15$
(b) $2x^4 - 3x^3 + x^2 - 13x - 15$
(c) $2x^3 - 3x^2 - x - 15$
(d) $2x^4 - 3x^3 - x^2 + 13x - 15$

8. The value of $(29x - 6x^2 - 28) \div (3x - 4)$ is
(a) $(2x - 7)$ (b) $(-2x + 7)$
(c) $(2x + 7)$ (d) $(7 + 2x)$

9. What should be subtracted from $p^2 - 6p + 7$ so that it may exactly be divisible by $(p - 1)$?
(a) 4 (b) −4 (c) 2 (d) −2

10. Divide $3y^4 - 3y^3 - 4y^2 - 4y$ by $y^2 - 2y$, then remainder is
(a) $4y^2$ (b) $4y$
(c) $2y$ (d) 0

11. If $P = a^4 + a^3 + a^2 - 6$,
$Q = a^2 - 2a^3 - 2 + 3a$
and $R = 8 - 3a - 2a^2 + a^3$,
then the value of $P + Q + R$ is
(a) a^4 (b) a^3
(c) $2a^4$ (d) $3a^4$

12. If the expression $px^3 + 3x^2 - 3$ and $2x^3 - 5x + p$ when divided by $x - 4$ leave the same remainder, then the value of p is
(a) 0 (b) 1
(c) 2 (d) 3

13. The value of p and q when $px^3 + x^2 - 2x - q$ is exactly divisible by $(x - 1)$ and $(x + 1)$, is
(a) 2, 1 (b) 2, 2
(c) 1, 1 (d) 1, 0

14. The expansion of $\left(\frac{x}{5} - \frac{y}{6}\right)^2$ is
(a) $\frac{x^2}{25} - \frac{y^2}{36} - \frac{xy}{15}$ (b) $\frac{x^2}{25} + \frac{y^2}{36} + \frac{xy}{15}$
(c) $\frac{x^2}{10} + \frac{y^2}{12} - \frac{xy}{15}$ (d) $\frac{x^2}{25} + \frac{y^2}{36} - \frac{xy}{15}$

15. The irreducible factorisation $(25x^2 - 9y^2)$ is
(a) $(5x - 3y)(5x + 3y)$
(b) $(5x^2 - 3y^2)$
(c) $(5x - 3y)^2$
(d) None of the above

16. If $(a + b) = 4$ and $a^2 + b^2 = 7$, then the value of ab is
(a) $-\frac{9}{2}$ (b) $\frac{9}{2}$ (c) $\frac{9}{4}$ (d) $\frac{7}{4}$

17. If $(2x - 3)(? + 6x + 9) = 8x^3 - 27$, then ? will be replaced by
(a) $-4x^2$ (b) $4x^2$
(c) $5x^2$ (d) None of these

18. If $a^2 + b^2 + c^2 = ab + bc + ca$, then the value of $a^3 + b^3 + c^3$ is
(a) $3abc$ (b) $3a^2b^2c^2$
(c) $3(abc)^3$ (d) None of these

19. If $(x + y + z) = 10$ and $x^2 + y^2 + z^2 = 40$, then $(xy + yz + zx)$ is equal to
(a) 60 (b) 30
(c) 40 (d) 20

20. If $(x - y) = 6$ and $xy = 1$, then the value of $x^3 - y^3$ is
(a) 324 (b) 432 (c) 234 (d) 322

21. Common factors between $3x^2y^3, 10x^3y^2$ and $-6x^2y^2z$ terms is
(a) $3x^2y$ (b) $3xz$
(c) x^2y^2 (d) $-x^2y^2$

22. Factors of $x^2yz + xy^2z + xyz^2$ are
(a) $xyz(x + y + z)$ (b) $xyz(x - y - z)$
(c) $xy(x + y + z)$ (d) $xz(x^2 + y + z)$

23. The factor form of $5x^2-20xy$ is
(a) $5x(x-4y)$
(b) $10x(x-2y)$
(c) $5(x^2-2y)$
(d) None of the above

24. The factor of form $8-4x-2x^3+x^4$ is
(a) $(2-x)(4-x^3)$ (b) $(2+x)(4-x^3)$
(c) $(2+x)(4+x^3)$ (d) $(2-x)(4+x^3)$

25. Factorise $3x^2+7x-6$
(a) $(x+3)(3x-2)$ (b) $(x-3)(3x+2)$
(c) $(x+3)(3x-3)$ (d) $(x+3)(3x-4)$

26. Factorise $8x^2-9x-14$
(a) $(x+2)(8x-7)$ (b) $(x-2)(8x-7)$
(c) $(x-2)(8x+7)$ (d) $(x+2)(8x+7)$

27. Factors of $1-8x^3$ are
(a) $(1-2x)(1+2x-4x^2)$
(b) $(1-2x)(1+2x+4x^2)$
(c) $(1-2x)(1-4x+4x^2)$
(d) $(1-2x)(1+4x-4x^2)$

28. Factors of x^6-y^6 are
(a) $(x^2-y^2)(x^4+y^4)$
(b) $(x^2+y^2)(x^4-x^2y^2+x^4)$
(c) $(x+y)(x-y)(x^2+xy+y^2)$
(d) $(x+y)(x-y)(x^2-xy+y^2)(x^2+xy+y^2)$

29. The factor form of $z^2+\frac{1}{z^2}+2-2z-\frac{2}{z}$ is
(a) $\left(z+\frac{1}{z}+2\right)\left(z-\frac{1}{z}\right)$ (b) $\left(z+\frac{1}{z}\right)\left(z+\frac{1}{z}-2\right)$
(c) $\left(z-\frac{1}{z}+2\right)\left(z+\frac{1}{z}\right)$ (d) $\left(z-\frac{1}{z}\right)\left(z-\frac{1}{z}-2\right)$

30. The factor form of $(a^4b^4-16c^4)$ is
(a) $4(a^2b^2+c^2)(ab-2c)(ab+2c)$
(b) $(a^2b^2-4c^2)(ab+2c)^2$
(c) $(a^2b^2+4c^2)(ab+2c)(ab-2c)$
(d) $(a^2b^2-4c^2)^2(ab+2c)(ab+4c)$

31. The factor form of $x^2-2\sqrt{3x}+3$ is
(a) $(x+\sqrt{3})^2$ (b) $(x-\sqrt{3})^2$
(c) $(x+\sqrt{3})(x-\sqrt{3})$ (d) $(x+2)(x+\sqrt{3})$

32. If $p(x)=x^3-5x^2+x-5$, then the remainder, when $p(x)$ is divided by $(x+1)$, is
(a) -11 (b) -12
(c) 11 (d) 12

33. Polynomial $x^{11}+1$ has factor
(a) $(x+1)$ (b) $(x-1)$
(c) $(x-2)$ (d) $(x-3)$

34. The factor of the polynomial $4x^3+3x^2-4x-3$ is
(a) $(x-2)$ (b) $(x-1)$
(c) $(x+1)$ (d) $(x-3)$

Answers

1	(a)	2	(a)	3	(b)	4	(c)	5	(d)	6	(a)	7	(d)	8	(b)	9	(c)	10	(d)
11	(a)	12	(b)	13	(a)	14	(d)	15	(a)	16	(b)	17	(b)	18	(a)	19	(b)	20	(c)
21	(c)	22	(a)	23	(a)	24	(a)	25	(a)	26	(c)	27	(b)	28	(d)	29	(b)	30	(c)
31	(b)	32	(b)	33	(a)	34	(b)												

Hints and Solutions

1. $-4ab$ and $7ba$ are like terms because variable parts are same in both side.

2. Let $E = 8y^2 + \frac{1}{3}x^2 + \frac{1}{5}z^2 - 3xy + 4yz$.

At $x = 1$, $y = -2$ and $z = -3$ is

$$E = 8(-2)^2 + \frac{1}{3}(1)^2 + \frac{1}{5}(-3)^2 - 3\times1(-2) + 4\times(-2)\times(-3)$$

$$= 8\times4 + \frac{1}{3} + \frac{1}{5}\times9 + 6 + 24$$

$$= 32 + \frac{1}{3} + \frac{9}{5} + 30$$

$$= \frac{32\times15 + 5 + 9\times3 + 30\times15}{15}$$

$$= \frac{480 + 5 + 27 + 450}{15} = \frac{962}{15} = 64\frac{2}{15}$$

3. $-6x^2y^2 + 4x^2y^2 - 3x^2y^2 = -5x^2y^2$.

4. Rearranging the terms of the given polynomials, changing the sign of each term of the polynomial which is to be subtracted and adding the two polynomials, we get

$$\begin{array}{r}
x^3 - x^2 + 2x - 19 \\
2x^3 - x^2 + 4x - 6 \\
- \quad + \quad - \quad + \\
\hline
-x^3 \quad\;\; - 2x \;\; - 13 \\
\hline
\end{array}$$

5.
$$\begin{array}{r}
x^3 - x^2 + 2x - 2 \\
x^2 + 4x - 6 \\
- \quad - \quad + \\
\hline
x^3 - 2x^2 - 2x + 4 \\
\hline
\end{array}$$

6.
$$\begin{array}{r}
x^4 + 0x^3 + 2x^2 - 3x + 7 \\
x^3 + x^2 + x - 1 \\
- \quad - \quad - \quad + \\
\hline
x^4 - x^3 + x^2 - 4x + 8 \\
\hline
\end{array}$$

7. Now, $(2x^2 + x - 5)\times(x^2 - 2x + 3)$

$= 2x^2(x^2 - 2x + 3) + x(x^2 - 2x + 3) - 5(x^2 - 2x + 3)$

$= 2x^4 - 4x^3 + 6x^2 + x^3 - 2x^2 + 3x - 5x^2 + 10x - 15$

$= 2x^4 - x^3(4 - 1) + x^2(6 - 5 - 2) + x(3 + 10) - 15$

[arranging like terms]

$= 2x^4 - 3x^3 - x^2 + 13x - 15$

8. Arranging the terms of the dividend and the divisor in descending power and then dividing, we get

$$\begin{array}{r|l}
 & -2x + 7 \\
\hline
3x - 4 & -6x^2 + 29x - 28 \\
 & -6x^2 + 8x \\
 & + \quad - \\
\cline{2-2}
 & 21x - 28 \\
 & 21x - 28 \\
 & - \quad + \\
\cline{2-2}
 & \times \\
\cline{2-2}
\end{array}$$

$\therefore (29x - 6x^2 - 28) \div (3x - 4) = (-2x + 7)$

9.
$$\begin{array}{r|l}
 & p - 5 \\
\hline
p - 1 & p^2 - 6p + 7 \\
 & p^2 - p \\
 & - \quad + \\
\cline{2-2}
 & -5p + 7 \\
 & -5p + 5 \\
 & + \quad - \\
\cline{2-2}
 & 2 \\
\end{array}$$

Remainder = 2

Hence, 2 is subtracted from it.

10.
$$\begin{array}{r|l}
 & 3y^2 + 3y + 2 \\
\hline
y^2 - 2y & 3y^4 - 3y^3 - 4y^2 - 4y \\
 & 3y^4 - 6y^3 \\
 & - \quad + \\
\cline{2-2}
 & 3y^3 - 4y^2 - 4y \\
 & 3y^3 - 6y^2 \\
 & - \quad + \\
\cline{2-2}
 & 2y^2 - 4y \\
 & 2y^2 - 4y \\
 & - \quad + \\
\cline{2-2}
 & 0 \\
\cline{2-2}
\end{array}$$

$\therefore$ Remainder is 0.

11. $P + Q + R = (a^4 + a^3 + a^2 - 6) + (a^2 - 2a^3 - 2 + 3a) + (8 - 3a - 2a^2 + a^3)$

$= a^4 + 0 + 0 + 0 + 0$

$= a^4$

12. Let $f(x)=px^3+3x^2-3$ and $g(x)=2x^3-5x+p$

$\therefore \quad f(4)=p(4)^3+3(4)^2-3$

and $\quad g(4)=2(4)^3-5(4)+p$

$\Rightarrow \quad f(4)=64p+45$ and $g(4)=p+108$

Since, $f(4)=g(4)\Rightarrow 64p+45=p+108$

$\Rightarrow \quad p=1$

13. Let $f(x)=px^3+x^2-2x-q=0$

$f(1)=p+1-2-q=0$

$\Rightarrow \quad p-q=1 \quad \ldots\text{(i)}$

$f(-1)=p(-1)^3+(-1)^2-2(-1)-q=0$

$\Rightarrow \quad p+q=3 \quad \ldots\text{(ii)}$

On solving Eqs. (i) and (ii), we get

$$p=2, q=1$$

14. $\left(\frac{x}{5}-\frac{y}{6}\right)^2=\left(\frac{x}{5}\right)^2-2\times\frac{x}{5}\times\frac{y}{6}+\left(\frac{y}{6}\right)^2$

[by using identity $(a-b)^2=a^2-2ab+b^2$]

$$=\frac{x^2}{25}-\frac{xy}{15}+\frac{y^2}{36}$$

$$=\frac{x^2}{25}+\frac{y^2}{36}-\frac{xy}{15}$$

15. $(25x^2-9y^2)=[(5x)^2-(3y)^2]$

$=(5x-3y)(5x+3y) \quad [\because (a^2-b^2)=(a-b)(a+b)]$

16. We have, $a+b=4$ and $a^2+b^2=7$

$\therefore \quad (a+b)^2=(4)^2$

$\Rightarrow \quad a^2+b^2+2ab=16$

$\Rightarrow \quad 7+2ab=16$

$\Rightarrow \quad 2ab=16-7=9$

$\therefore \quad ab=\frac{9}{2}$

17. Now, $\quad 8x^3-27=(2x)^3-(3)^3$

$=(2x-3)[(2x)^2+2x\times 3+3^2]$

$[\because a^3-b^3=(a-b)(a^2+ab+b^2)]$

$=(2x-3)(4x^2+6x+9)$

So, ? will be replaced by $4x^2$.

18. $\because \; a^3+b^3+c^3-3abc$

$=(a^2+b^2+c^2-ab-bc-ca)(a+b+c)$

$=[(a^2+b^2+c^2)-(ab+bc+ca)](a+b+c)$

But, $a^2+b^2+c^2=ab+bc+ca$ [given]

$\therefore \; a^3+b^3+c^3-3abc=(a+b+c)\times 0=0$

$\therefore \quad a^3+b^3+c^3=3abc$

19. $\because \; (x+y+z)=10$

On squaring both sides, we get

$$(x+y+z)^2=(10)^2$$

$\Rightarrow x^2+y^2+z^2+2xy+2yz+2zx=100$

$\Rightarrow \quad 40+2(xy+yz+zx)=100$

$\Rightarrow \quad 2(xy+yz+zx)=100-40$

$\Rightarrow \quad 2(xy+yz+zx)=60$

$\therefore \quad (xy+yz+zx)=\frac{60}{2}=30$

20. $\because (x-y)=6$

On squaring both sides, we get

$$(x-y)^2=(6)^2$$

$\Rightarrow \quad x^2+y^2-2xy=36$

$\Rightarrow \quad x^2+y^2-2\times 1=36$

$\Rightarrow \quad x^2+y^2=36+2=38$

$\because \quad x^3-y^3=(x-y)(x^2+y^2+xy)$

$\therefore \quad x^3-y^3=(6)(38+1)=6\times 39=234$

21. We have,

$3x^2y^3=3\times x\times x\times y\times y\times y,$

$10x^3y^2=2\times 5\times x\times x\times x\times y\times y$

and $\; -6x^2y^2z=-2\times 3\times x\times x\times y\times y\times z$

Here, x, x, y, y are common factors.

Hence, x^2y^2 is common factor.

22. We have, $x^2yz+xy^2z+xyz^2$

$=x\times x\times y\times z+x\times y\times y\times z+x\times y\times z\times z$

$=xyz(x+y+z)$

23. We have, $5x^2-20xy=5x(x-4y)$

24. $8-4x-2x^3+x^4$

$=4(2-x)-x^3(2-x)=(2-x)(4-x^3)$

25. We have, $3x^2+7x-6$

$=3x^2+9x-2x-6$

$=3x(x+3)-2(x+3)$

$=(3x-2)(x+3)$

26. We have, $8x^2-9x-14$

$=8x^2-16x+7x-14$

$=8x(x-2)+7(x-2)$

$=(8x+7)(x-2)$

27. We have, $1-8x^3=1^3-(2x)^3$

$=(1-2x)(1+2x+4x^2)$

$[\because a^3-y^3=(a-b)(a^2+ab+b^2)]$

28. $x^6-y^6=(x^3)^2-(y^3)^2=(x^3+y^3)(x^3-y^3)$

$[\because a^2-b^2=(a+b)(a-b)]$

$=(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$

$\begin{bmatrix} \because \quad a^3+b^3=(a+b)(a^2-ab+b^2) \\ \text{and } a^3-b^3=(a-b)(a^2+ab+b^2) \end{bmatrix}$

$=(x+y)(x-y)(x^2-xy+y^2)(x^2+xy+y^2)$

29. We have , $z^2+\frac{1}{z^2}+2-2z-\frac{2}{z}$

$=\left(z+\frac{1}{z}\right)^2-2\left(z+\frac{1}{z}\right) \quad \left[\because z^2+\frac{1}{z^2}+2=\left(z+\frac{1}{z}\right)^2\right]$

$=\left(z+\frac{1}{z}\right)\left(z+\frac{1}{z}-2\right)$

30. $(a^4b^4-16c^4)=[(a^2b^2)-(4c^2)^2]$

$=(a^2b^2+4c^2)(a^2b^2-4c^2)$

$=(a^2b^2+4c^2)[(ab)^2-(ac)^2]$

$=(a^2b^2+4c^2)(ab+2c)(ab-2c)$

31. $x^2-2\sqrt{3x}+3$

$=x^2-\sqrt{3}x-\sqrt{3}x+3$

$=x(x-\sqrt{3})-\sqrt{3}(x-\sqrt{3})$

$=(x-\sqrt{3})(x-\sqrt{3})=(x-\sqrt{3})^2$

32. Given, that

$p(x)=x^3-5x^2+x-5$

$\therefore \quad p(-1)=(-1)^3-5(-1)^2+(-1)-5$

$=-1-5-1-5=-12$

33. Let $p(x)=x^{11}+1$

$\therefore \quad p(-1)=(-1)^{11}+1=-1+1=0$

Here, $\quad x=-1$

i.e. $\quad x+1=0$

So, $(x+1)$ is factor of given polynomial.

34. We have, $p(x)=4x^3+3x^2-4x-3$

By options,

(a) $p(2)=4\times2^3+3\times2^2-4\times2-3$

$=32+12-8-3=33$

(b) $p(1)=4\times1^3+3\times1^2-4\times1-3$

$=4+3-4-3=0$

Here, by $p(1)$, i.e. $x=1 \Rightarrow (x-1)=0$

So, $(x-1)$ is a factor of given polynomial.

CHAPTER

09

LINEAR EQUATIONS IN ONE VARIABLE

In this chapter, we study one and two variable types linear equations and various word problems by using linear equations.

Equation

An equation is an equality involving variables and it has an equality sign (=). The expression on the left of the equality sign is the Left Hand Side (LHS) and expression on the right of the equality sign is the Right Hand Side (RHS).

Linear Equations

The equations, in which the highest power of the variable appearing in the expression is 1, are called linear equations.

Linear Equations in One Variable

The expression, which form the equation contains only one variable and the highest power of the variable appearing in the equation is 1, are called linear equations in one variable.

e.g. $2x - 5 = 3$ and $\frac{5}{3}x + 3 = \frac{1}{2}$

Note The graph of the linear equation is always a straight line.

Properties of Linear Equations

(i) If same number is added or subtracted to both the sides of an equation, the equality remains the same.

(ii) If same number is multiplied to both the sides of an equation, the equality remains the same.

(iii) If both sides are divided by a same non-zero number, the equality remains the same.

Example 1 Solve $2(x-3)-(5-3x) = 3(x+1)-4(2+x)$

(a) 2 (b) 1
(c) 3 (d) 0

Sol. (*b*) Given,

$$2(x-3)-(5-3x)=3(x+1)-4(2+x)$$
$$\Rightarrow 2x-6-5+3x=3x+3-8-4x$$
$$\Rightarrow 5x-11=-x-5 \Rightarrow 6x=6$$
$$\Rightarrow x=1$$

Example 2 If $\frac{2x+1}{3x-1}=\frac{2x+1}{3x+2}$, the value of x is

(a) $-\frac{1}{2}$ (b) 0
(c) $\frac{1}{3}$ (d) $-\frac{2}{3}$

Sol. (*a*) We have, $\frac{2x+1}{3x-1}=\frac{2x+1}{3x+2}$

$$\Rightarrow (2x+1)(3x+2)=(2x+1)(3x-1)$$
$$\Rightarrow 6x^2+4x+3x+2=6x^2-2x+3x-1$$
$$\Rightarrow 6x^2+7x+2=6x^2+x-1$$
$$\Rightarrow 7x-x=-1-2 \Rightarrow 6x=-3$$
$$\therefore x=-\frac{3}{6}=-\frac{1}{2}$$

Solving Word Problems by Using Linear Equation

I. Firstly, denote the unknown quantity by any letters, say x, y, z, etc.

II. Translate the statements of the problem into mathematical statements.

III. Using the conditions given in the problem, form the equation.

IV. Solve the equation for the unknown quantity.

Example 3 The digit in the ten's place of a two-digit number is 3 more than the digit in the unit's place. Let the digit at unit's place be b. Then, the number is

(a) $11b+30$ (b) $10b+30$
(c) $11b+3$ (d) $10b+3$

Sol. (*a*) Let digit at unit's place be b.

Then, digit at ten's place $=(3+b)$

$\therefore$ Number $=10(3+b)+b$

$=30+10b+b=11b+30$

Example 4 Thirty one is added 63 to twice a whole number gives. The number is

(a) 15 (b) 16 (c) 17 (d) 18

Sol. (*b*) Let whole number be x.

Then, $31+2x=63$

$\Rightarrow 2x=32 \Rightarrow x=16$

Example 5 A boy is now one-third as old as his father. Twelve years hence, he will be half as old as his father. The present age of the boy is

(a) 12 yr (b) 20 yr (c) 36 yr (d) 25 yr

Sol. (*a*) Let the present age of the father be x yr and that of the son be $\frac{1}{3}x$ yr.

After twelve years,

Age of father $=(x+12)$ yr

and age of son $=\left(\frac{1}{3}x+12\right)$ yr

According to given condition,

$$\frac{1}{3}x+12=\frac{1}{2}(x+12)$$
$$\Rightarrow \frac{x+36}{3}=\frac{x+12}{2}$$
$$\Rightarrow 2x+72=3x+36 \Rightarrow x=36$$

$\therefore$ Age of the boy $=\frac{x}{3}=\frac{36}{3}=12$ yr.

PRACTICE EXERCISE

1. Linear equation in one variable has
   (a) only one variable with any power
   (b) only one term with a variable
   (c) only one variable with power 1
   (d) only constant term

2. Which of the following is a linear expression?
   (a) $x^2 + 1$ (b) $y + y^2$
   (c) 4 (d) $1 + z$

3. A linear equation in one variable has
   (a) only one solution
   (b) two solutions
   (c) more than two solutions
   (d) no solution

4. If $8x - 3 = 25 + 17x$, then x is
   (a) a fraction
   (b) an integer
   (c) a rational number
   (d) cannot be solved

5. If $7x : 63 = 1 : 9$, then x is equal to
   (a) 1 (b) 2
   (c) 3 (d) -1

6. If $\frac{3x+6}{8} - \frac{11x-8}{24} + \frac{x}{3} = \frac{3x}{4} - \frac{x+7}{24}$, then the value of x is
   (a) -3 (b) 3/2
   (c) 3 (d) 1/3

7. If $\sqrt{3}x - 2 = 2\sqrt{3} + 4$, then the value of x is
   (a) $2(1 - \sqrt{3})$ (b) $2(1 + \sqrt{3})$
   (c) $1 + \sqrt{3}$ (d) $1 - \sqrt{3}$

8. If $a(x - a^2) - b(x - b^2) = 0$, then x is equal to
   (a) $\frac{(-a+b)(a^2+ab+b^2)}{(a+b)}$ (b) $\frac{a^3+b^3}{(a-b)}$
   (c) $\frac{a^3-b^3}{a+b}$ (d) $a^2 + ab + b^2$

9. If one number is thrice the other and their sum is 20, then the numbers are
   (a) 5, 15 (b) 4, 12
   (c) 3, 9 (d) 6, 18

10. If the sum of five consecutive even numbers is 340, then the smallest numbers will be
    (a) 62 (b) 64 (c) 66 (d) 68

11. The sum of 4 consecutive odd numbers is 56. The smallest number is
    (a) 12 (b) 11 (c) 13 (d) 14

12. The sum of the two numbers is 11 and their product is 30, then the numbers are
    (a) 8, 3 (b) 9, 2 (c) 7, 4 (d) 6, 5

13. Divide 60 into two parts, so that three times the greater may be exceed 100 by as much as 8 times the less falls short of 200. What is the greater part ?
    (a) 36 (b) 24 (c) 20 (d) 25

14. The ages of two persons differ by 20 yr. If 5 yr ago, the elder one be 5 times as old as the younger one, their present ages are
    (a) 50 yr, 30 yr (b) 28 yr, 5 yr
    (c) 20 yr, 10 yr (d) 30 yr, 10 yr

15. Given that $\frac{4p+9q}{p} = \frac{5q}{p-q}$ and p and q are both positive. The value of $\frac{p}{q}$ is
    (a) $\frac{5}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{3}{5}$

16. A fraction becomes 4/5 when 5 is added to its numerator and 1 is subtracted from its denominator. It becomes 1/2 when 3 and 5 are subtracted from its numerator and denominator. The numerator of the fraction will be
    (a) 4 (b) 11 (c) 10 (d) 9

17. The total cost of 6 books and 4 pencils is ₹ 34 and that of 5 books and 5 pencils is ₹ 30. The cost of each book and pencil (in ₹) respectively is
(a) 1 and 5 (b) 5 and 1
(c) 6 and 1 (d) 1 and 6

18. The age of a man is 4 times that of his son. Five years ago, the man was nine times as old as his son was at that time. The present age of the man is
(a) 28 yr (b) 45 yr (c) 32 yr (d) 48 yr

19. In a class $\frac{3}{5}$ of the students are girls and rest are boys. If $\frac{2}{9}$ of the girls and $\frac{1}{4}$ of the boys are absent. What part of the total number of students are present ?
(a) $\frac{23}{30}$ (b) $\frac{23}{36}$ (c) $\frac{18}{49}$ (d) $\frac{17}{25}$

20. If a sum of ₹ 275 is to be divided between Ram and Shyam so that Ram gets more than 3/4th of what Shyam gets, then the share of Ram will be
(a) ₹ 200 (b) ₹ 175 (c) ₹ 160 (d) ₹ 100

21. Find a number such that if 5,15 and 35 are added to it, the product of the first and third results may be equal to the square of the second.
(a) 10 (b) 7 (c) 6 (d) 5

22. The length of a rectangle is 8 cm more than its breadth. If the perimeter of the rectangle is 68 cm, its length and breadth are respectively.
(a) 21 cm, 13 cm (b) 21 cm, 32 cm
(c) 20 cm, 10 cm (d) 13 cm, 15 cm

23. In a two digit number, the tens digit is twice the unit digit. When the digits are reversed, the new number formed is 18 less than the original number. The original number is
(a) 45 (b) 42
(c) 43 (d) 41

24. A man has a certain number of chickens and goats. Their head count is 30. If the total number of their legs is 84, what is the ratio between the number of chickens and goats ?
(a) 1 : 2 (b) 2 : 3
(c) 3 : 2 (d) 3 : 4

Answers

1	(c)	2	(d)	3	(a)	4	(c)	5	(a)	6	(c)	7	(b)	8	(d)	9	(a)	10	(b)
11	(b)	12	(d)	13	(a)	14	(d)	15	(c)	16	(b)	17	(b)	18	(c)	19	(a)	20	(b)
21	(d)	22	(a)	23	(b)	24	(c)												

Hints and Solutions

1. Linear equation in one variable has only one variable with power 1.

2. We know that, the algebraic expression in one variable having the highest power of the variable as 1, is known as the linear expression.
Here, $1 + z$ is the only linear expression, as the power of the variable z is 1.

3. A linear equation in one variable has only one solution.

4. Given, $8x - 3 = 25 + 17x$
$\Rightarrow \quad 8x - 17x = 25 + 3$
$\Rightarrow \quad -9x = 28$
$\therefore \quad x = \frac{-28}{9}$

Hence, x is a rational number.

5. $\frac{7x}{63} = \frac{1}{9} \Rightarrow x = \frac{63}{9 \times 7} = 1$

6. Given, $\frac{3x+6}{8} - \frac{11x-8}{24} + \frac{x}{3} = \frac{3x}{4} - \frac{x+7}{24}$

$\Rightarrow \quad \frac{3(3x+6) - (11x-8) + 8x}{24} = \frac{6(3x) - (x+7)}{24}$

$\Rightarrow \quad 9x + 18 - 11x + 8 + 8x = 18x - x - 7$

$$\Rightarrow \quad 26 + 6x = 17x - 7$$
$$\Rightarrow \quad 11x = 33$$
$$\Rightarrow \quad x = 3$$

7. Given, $\sqrt{3}x - 2 = 2\sqrt{3} + 4$

$$\therefore \quad \sqrt{3}x = 2\sqrt{3} + 6$$
$$\Rightarrow \quad x = \frac{2\sqrt{3} + 6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 2(1 + \sqrt{3})$$

8. Since, $ax - a^3 - bx + b^3 = 0$

$$\therefore \quad (a - b)x - (a^3 - b^3) = 0$$
$$\Rightarrow (a - b)[x - (a^2 + b^2 + ab)] = 0$$
$$\Rightarrow \quad x = a^2 + b^2 + ab$$

9. Let the one number be x and number be $3x$.

According to the given condition,

$$x + 3x = 20 \Rightarrow x = 5$$

$\therefore$ Other number is $(3x) = 3 \times 5 = 15$

10. Let the smallest number be x.

Then,

$$x + (x + 2) + (x + 4) + (x + 6) + (x + 8) = 340$$
$$\Rightarrow \quad 5x + 20 = 340$$
$$\Rightarrow \quad 5x = 320$$
$$\therefore \quad x = 64$$

11. Let four consecutive odd number be $x+1, x+3, x+5, x+7$.

Then, $(x+1) + (x+3) + (x+5) + (x+7) = 56$

$$4x = 40 \Rightarrow x = 10$$

$\therefore$ The smaller odd number is $x + 1 = 10 + 1 = 11$

12. Let numbers be x and $11 - x$.

Since, product $x(11 - x) = 30$ [given]

$$\Rightarrow \quad x^2 - 11x + 30 = 0$$
$$\Rightarrow \quad (x - 5)(x - 6) = 0$$
$$\Rightarrow \quad x = 5, 6$$

13. Let two parts be x and $60 - x$.

Then, $3x - 100 = 200 - 8(60 - x)$

$$\Rightarrow \quad 3x - 100 = 200 - 480 + 8x$$
$$\Rightarrow \quad 8x - 3x = 480 - 200 - 100$$
$$\Rightarrow \quad 5x = 180$$
$$\therefore \quad x = \frac{180}{5} = 36$$

Thus, another number $60 - 36 = 24$.

$\therefore$ Greater number $= 36$

14. Let their ages be x and $(x - 20)$ yr.

5 yr ago, their ages are $(x - 5)$ and $(x - 20 - 5)$ yr.

According to the given condition,

$$5(x - 20 - 5) = (x - 5)$$
$$\Rightarrow \quad 5x - 125 = x - 5$$
$$\Rightarrow \quad 4x = 120 \Rightarrow x = 30$$

Hence, their present ages are 30 yr and 10 yr.

15. Given, $\dfrac{4p + 9q}{p} = \dfrac{5q}{p - q}$

$$\Rightarrow \quad \frac{4\left(\frac{p}{q}\right) + 9}{\left(\frac{p}{q}\right)} = \frac{5}{\left(\frac{p}{q}\right) - 1}$$

[on dividing by q in numerator and denominator]

$$\Rightarrow 4\left(\frac{p}{q}\right)^2 + 9\left(\frac{p}{q}\right) - 4\left(\frac{p}{q}\right) - 9 = 5\left(\frac{p}{q}\right)$$
$$\Rightarrow \quad 4\left(\frac{p}{q}\right)^2 - 9 = 0 \Rightarrow \left(\frac{p}{q}\right)^2 = \left(\frac{3}{2}\right)^2$$
$$\therefore \quad \frac{p}{q} = \frac{3}{2}$$

16. Let the fraction be $\dfrac{x}{y}$.

Then, $\dfrac{x + 5}{y - 1} = \dfrac{4}{5} \Rightarrow 5x - 4y = 29$ ...(i)

and $\dfrac{x - 3}{y - 5} = \dfrac{1}{2} \Rightarrow 2x - y = 1$ ...(ii)

On solving Eqs. (i) and (ii), we get $x = 11$

17. Let CP of a book be ₹ x and CP of pencil be ₹y.

Then, $6x + 4y = 34$ ...(i)

and $5x + 5y = 30$ ...(ii)

On solving Eqs. (i) and (ii), we get

$$x = 5 \text{ and } y = 1.$$

18. Let the son's age be x yr, then man age's be $4x$.

Five years ago, their ages are $(x - 5)$ yr and $(4x - 5)$ yr respectively.

According to the given condition,

$$(4x - 5) = 9(x - 5)$$
$$\Rightarrow \quad 5x = 40 \Rightarrow x = 8 \text{ yr}$$

Hence, the man's age $= 4x = 4 \times 8 = 32$ yr.

19. Let the number of students be x.

Then, number of girls $= \frac{3}{5}x$ and number of boys

$$= \frac{2}{5}x$$

Number of girls present $= \frac{7}{9} \times \frac{3}{5}x = \frac{7x}{15}$ and

Number of boys present $= \frac{3}{4} \times \frac{2}{5}x = \frac{3x}{10}$

$\therefore$ Total students present $= \left(\frac{7x}{15} + \frac{3x}{10}\right) = \frac{23}{30}x$

20. Let Shyam's share be ₹ x.

Then, Ram's share $= \left(x + \frac{3x}{4}\right) =$ ₹ $\frac{7x}{4}$

$\therefore \quad x + \frac{7x}{4} = 275 \Rightarrow x = 100$

$\therefore \quad$ Ram's share $= \frac{7}{4} \times 100 =$ ₹ 175

21. Let the number be x.

Then, first number $= (x+5)$

second number $= (x + 15)$

and third number $= (x + 35)$

According to the question,

$(x + 5)(x + 35) = (x + 15)^2$

$\Rightarrow \quad 175 + 40x = 225 + 30x$

$\Rightarrow \quad 10x = 50$

$\therefore \quad x = 5$

22. Let breadth of rectangle be x.

Then, its length $= (x + 8)$ cm

$\therefore$ Perimeter of rectangle $= 2[x + (x + 8)]$

$= 2(2x + 8) = 4x + 16$

$\therefore \quad 4x + 16 = 68 \quad$ [given]

$\Rightarrow \quad 4x = 52$

$\Rightarrow \quad x = 13$

So, breadth of rectangle = 13 cm

and length = 13 + 8 = 21 cm

23. Let unit digit $= x$

Then, tens digit $= 2x$

$\therefore$ Original number $= 10 \times 2x + x = 20x + x = 21x$

New Number $= 10 \times x + 2x$

$= 10x + 2x = 12x$

Now, $21x - 12x = 18$

$\Rightarrow \quad 9x = 18$

$\Rightarrow \quad x = 2$

$\therefore$ Unit digit = 2 and Tens digit $= 2 \times 2 = 4$

Thus, required number $= 10 \times 4 + 2 = 42$

24. Let the number of chickens and goat be x and

According to the question,

$x + y = 30 \quad \ldots$(i)

and $\quad 2x + 4y = 84$

$\Rightarrow \quad x + 2y = 42 \quad \ldots$(ii)

On solving Eqs. (i) and (ii), we get

$x = 18$ and $y = 12$

$\therefore$ Required ratio $= 18 : 12 = 3 : 2$

CHAPTER

10

LINES AND TRIANGLES

In this chapter, we study the Line, Triangle their types and properties etc.

Lines and Triangles are the branch of plane geometry. In which we study the two intersecting lines forms different types of angles and three intersecting lines forms a different types of triangles.

Definitions Related to Lines and Angles

Line Segment

A line segment is a portion (or part) of a line. It has two end points.

It has a definite length. Distance between P and Q is called length of the line segment PQ.

Ray

A ray extends indefinitely in one direction. This is shown by an arrow *ie*,

P is called initial point. It has no definite length.

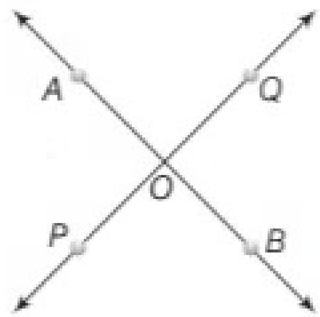
Line

A line segment PQ when extended indefinitely in both the directions is called a line.

- Line has no end point.
- Line has no definite length.
- Line is a set of infinite points.

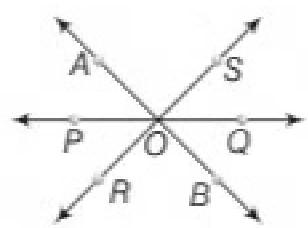
Intersecting Lines

Two lines having a common point, are called intersecting lines. This common point is called point of intersection. In the figure, ‘O’ is the common point.



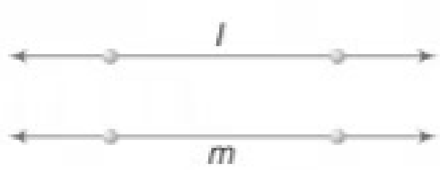
Concurrent Lines

Three or more lines in a plane which are intersecting at the same point, are called concurrent lines.



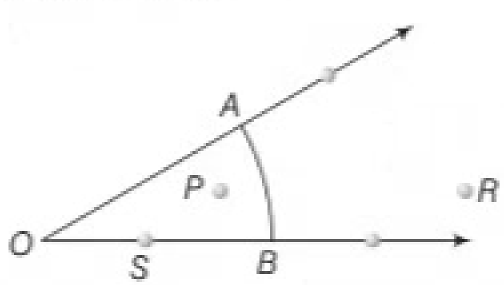
Parallel Lines

Two lines in a plane which do not intersect anywhere in a plane are called parallel lines.



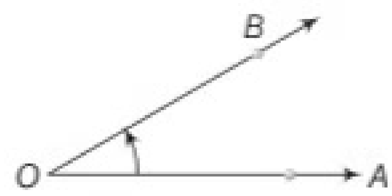
Angles

A figure consisting of two rays join with end points is called an angle. In the figure, $\angle AOB$ is a angle with rays OA and OB.

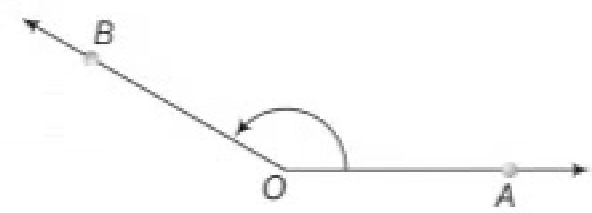


Classification of Angles

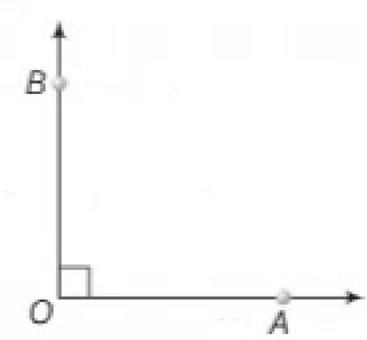
(i) **Acute Angle** An angle between $0°$ and $90°$ (less than $90°$) is called acute angle.



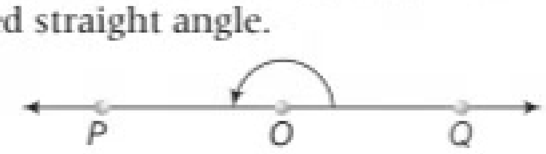
(ii) **Obtuse Angle** An angle between $90°$ and $180°$ (greater than $90°$) is called obtuse angle.



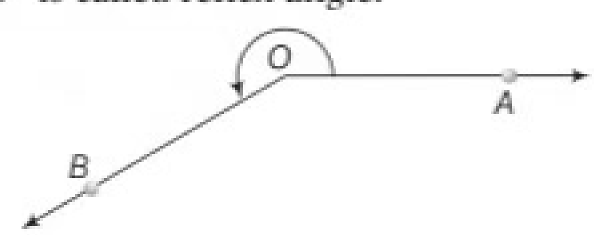
(iii) **Right Angle** An angle equal to $90°$ is called right angle.



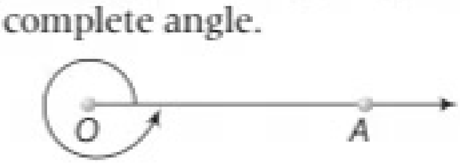
(iv) **Straight Angle** An angle equal to $180°$ is called straight angle.



(v) **Reflex Angle** An angle between $180°$ and $360°$ is called reflex angle.

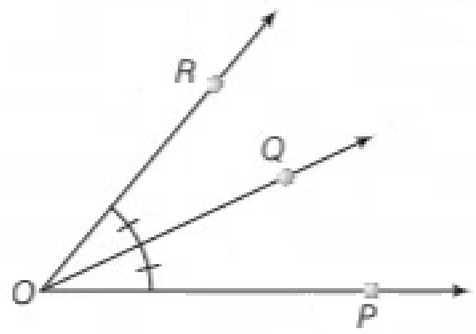


(vi) **Complete Angle** An angle equal to $360°$ is called complete angle.



(vii) **Bisector of an Angle** A ray OQ is called the bisector of $\angle POR$, if

$$\angle POQ = \angle ROQ.$$



$$\therefore \quad \angle POQ = \angle QOR = \frac{1}{2}\angle POR$$

(viii) **Complementary Angles** Two angles are said to be complementary, if their sum is $90°$.

Complementary angles are complement of each other.

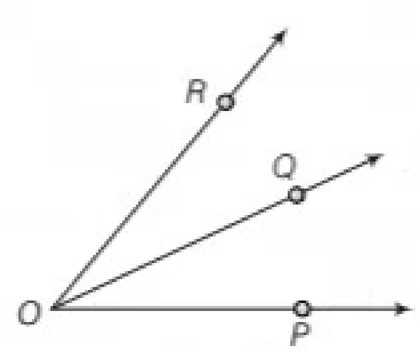
Complement of θ is $(90° - \theta)$.

(ix) **Supplementary Angles** Two angles are said to be supplementary, if their sum is $180°$.

Supplementary angles are supplement of each other.

Supplement of θ is $(180° - \theta)$.

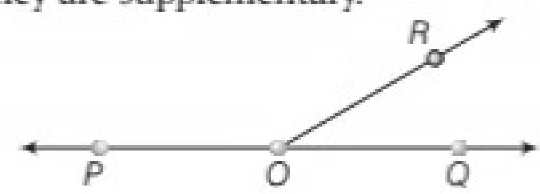
(x) **Adjacent Angles** Two angles are said to be adjacent, if they have a common vertex.



They have a common arm and their non-common arms are on either side of the common arm.

Here, $\angle POQ$ and $\angle ROQ$ are adjacent angles and have the same vertex O, a common arm OQ, the non- common arms OP, OR on either side of OQ.

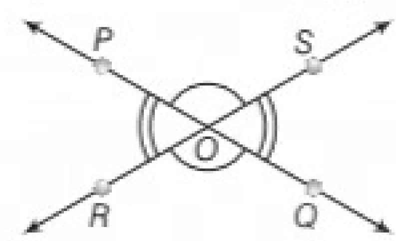
(xi) **Linear Pair** Two angles are said to form a linear pair of angles, if they are adjacent angles and they are supplementary.



$$\therefore \quad \angle POR + \angle QOR = 180°$$

If a ray stands on a line, then the sum of the adjacent angles so formed is $180°$.

(xii) **Vertically Opposite Angles** If two lines PQ and RS intersects at a point O, then the pair of $\angle POR$ and $\angle QOS$ or pair of $\angle POS$ and $\angle ROQ$ is said to be a pair of vertically opposite angles.



Vertically opposite angles are always equal.

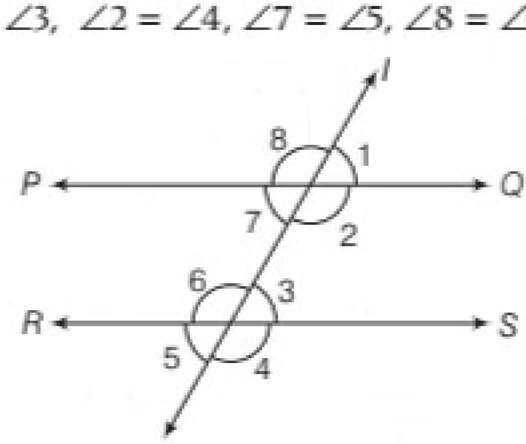
i.e. $\angle POR = \angle QOS$ and $\angle POS = \angle ROQ$

Properties of Parallel Lines

Let PQ and RS be two parallel lines, cut by a transversal l.

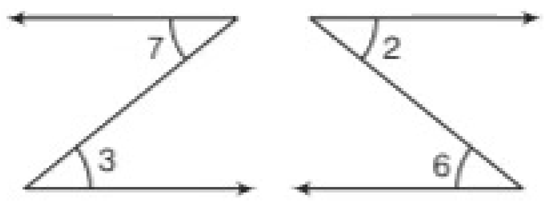
Then,

- Angles $\angle 1$, $\angle 8$, $\angle 5$ and $\angle 4$ are the exterior angles and $\angle 2$, $\angle 3$, $\angle 6$ and $\angle 7$ are the interior angles.
- Pairs of corresponding angles are equal
  i.e. $\angle 1 = \angle 3$, $\angle 2 = \angle 4$, $\angle 7 = \angle 5$, $\angle 8 = \angle 6$



- The sum of co-interior angles on the same side of transversal is $180°$.
  i.e., $\angle 2 + \angle 3 = 180°$ and $\angle 7 + \angle 6 = 180°$

- The pairs of opposite angles of transversal line is said to be alternate angles.



i.e., $\angle 7 = \angle 3$

$\angle 2 = \angle 6$

- The vertical opposite angles are equal, i.e.

$\angle 1 = \angle 7$; $\angle 2 = \angle 8$

and $\angle 3 = \angle 5$; $\angle 4 = \angle 6$

Example 1 Find the measure of an angle which is 28° more than its complement.

(a) 58° (b) 59°
(c) 60° (d) 61°

Sol. (*b*) Let measure of the required angle be $x°$.
Then, measure of its complement $= 90° - x$

$\therefore \quad x - (90° - x) = 28°$

$\Rightarrow \quad 2x = 118°$

$\Rightarrow \quad x = 59°$

Hence, the measure of the required angle is 59°.

Example 2 Find the measure of an angle, which is 32° less than its supplement.

(a) 74° (b) 73°
(c) 72° (d) 71°

Sol. (*a*) Let the measure of the required angle be x.
Then, measure of its supplement $= (180° - x)$

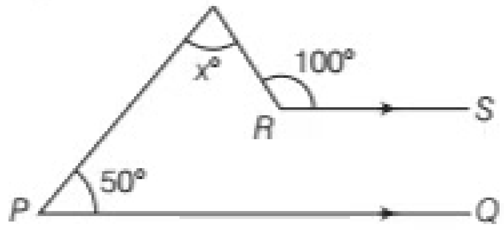
$\therefore \quad (180° - x) - x = 32°$

$\Rightarrow \quad 180° - 32° = 2x$

$\Rightarrow \quad 2x = 148°$

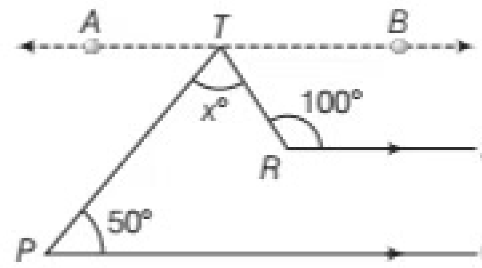
$\Rightarrow \quad x = 74°$

Example 3 In the given figure, $PQ \parallel RS$. The value of $x°$, is



(a) 120° (b) 50°
(c) 135° (d) 140°

Sol. (*b*) Draw $AB \parallel PQ$



$\angle ATP = \angle TPQ = 50°$

[$\because$ alternate interior angles]

$\angle BTR + \angle TRS = 180°$

[$\because$ sum of interior angles]

$\Rightarrow \quad \angle BTR = 80°$ and $\angle ATP + x + \angle BTR = 180°$

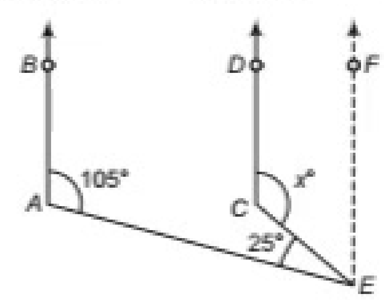
[$\because$ AB is the straight line]

$\Rightarrow \quad 50° + x + 80° = 180°$

$\therefore \quad x = 50°$

Example 4 In the given figure, $AB \parallel CD$, the value of x is

(a) 120° (b) 130° (c) 132° (d) 134°



Sol. (*b*) Let $CD \parallel EF$. Then, $x + \angle CEF = 180°$

[$\because$ sum of interior angles]

$\Rightarrow \quad \angle CEF = 180° - x$

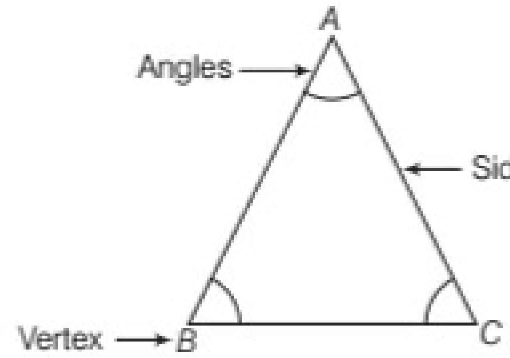
But $\quad \angle BAE + \angle AEF = 180°$

$\Rightarrow 105° + 25° + (180° - x) = 180°$

$\therefore \quad x = 130°$

Triangle

A closed figure which has three sides, three angles and three vertices, is called a triangle.



Properties of Triangle

- The sum of three angles of a triangle is equal to $180°$.
- The sum of lengths of any two sides of triangle is greater than the length of third side.
- In a triangle, an exterior angle is equal to the sum of the two interior opposite angles.

Types of Triangle

Different types of triangle, according to their sides and angles are given below

I. According to their Sides

(i) **Scalene Triangle** If all sides are different in lengths called scalene triangle. In which all angles are different.

(ii) **Isosceles Triangle** If any two sides are equal in length called isosceles triangle. In which opposite angles of equal sides are also equal.

(iii) **Equilateral Triangle** If all three sides are equal in length called equilateral triangle. In which all three angles of triangles are equal to 60°.

II. According to their Angles

(i) **Acute Angled Triangle :** If each angle of triangle is less than 90°. It is called an acute angled triangle.

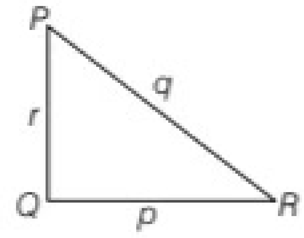
(ii) **Right Angled Triangle :** If any one angle of a triangle is 90°. It is called right angled triangle.

(iii) **Obtuse Angled Triangle :** If any one angle of triangle is greater than 90°. It is called an obtuse angled triangle.

Pythagoras Theorem

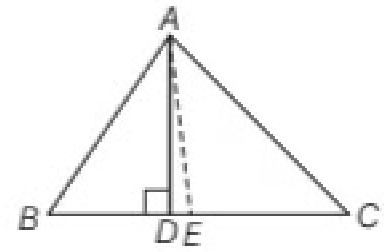
In a right angle triangle, the square of the hypotenuse equals the sum of the square of its other sides.

i.e., $q^2 = p^2 + r^2$



Altitude and Median of a Triangle

A line segment from a vertex of triangle, perpendicular to the line containing the opposite side is called an **altitude** of a triangle. In the figure AD is the altitude of a triangle.



A line segment that joins a vertex of a triangle to the mid-point of the opposite side is called median of triangle. In a figure, AE is the **median** of a triangle.

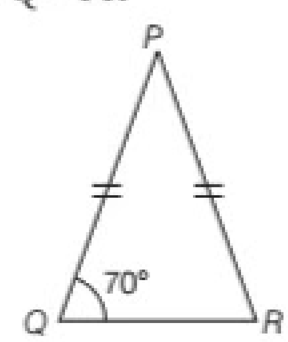
Note : Every triangle has three altitudes and three medians, which are drawn from each vertex.

Example 5 ΔPQR is an isosceles triangle with $PQ = PR$. If $\angle Q = 70°$, then the other angles of a triangle, are

(a) 40°, 70° (b) 35°, 75°
(c) 55°, 55° (d) None of these

Sol. (*a*) In ΔPQR, we have,

$$PQ = PR$$



i.e., $\angle Q = \angle R = 70°$

Since, $\angle P + \angle Q + \angle R = 180°$

[the sum of three angles of triangle]

$\therefore \quad \angle P + 140° = 180°$

$\Rightarrow \quad \angle P = 180° - 140° = 40°$

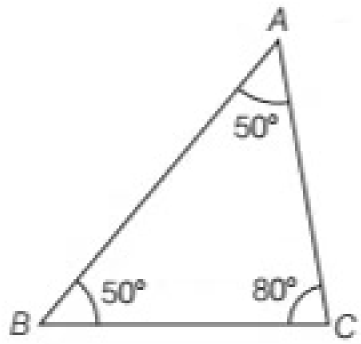
Hence, the required angles of triangle are

$\angle P = 40°$ and $\angle R = 70°$.

Example 6 In ΔABC, $\angle A = 50°$, $\angle B = 50°$, $\angle C = 80°$, which two sides of this triangle are equal.

(a) $AB = AC$ (b) $AC = BC$
(c) $AB = BC$ (d) None of these

Sol. (*b*) Since, $\angle A = \angle B = 50°$



Therefore, the sides opposite these angles must be equal.
The side opposite $\angle A$ is BC and the side opposite $\angle B$ is AC

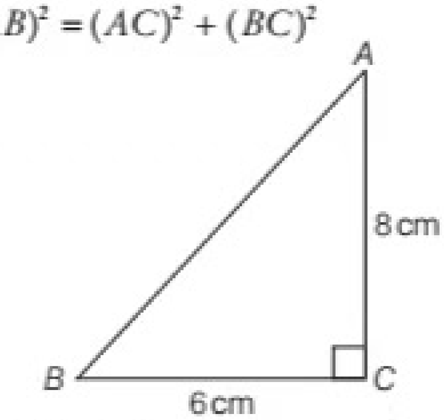
$\therefore$ In $\Delta ABC, AC = BC$

Example 7. The length of the sides of a right angle triangle are 6 cm and 8 cm. What is the length of the hypotenuse ?

(a) 11 cm (b) 10 cm (c) 9 cm (d) 8 cm

Sol. (*b*) Using Pythagoras theorem in ΔABC

$\therefore \quad (AB)^2 = (AC)^2 + (BC)^2$



$\Rightarrow \quad (AB)^2 = (8)^2 + (6)^2$

$\Rightarrow \quad (AB)^2 = 64 + 36$

$= 100$

$\Rightarrow \quad AB = 10$ cm

Hence, the length of the hypotenuse is 10 cm.

PRACTICE EXERCISE

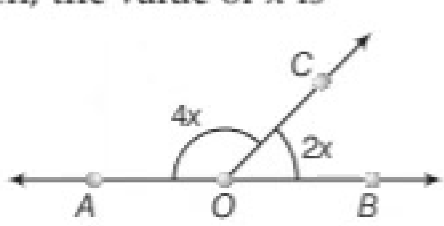
1. An angle is 14° more than its complement. Then, its measure is
(a) 166° (b) 86° (c) 76° (d) 52°

2. The measure of an angle is twice the measure of its supplementary angle. Then, its measure is
(a) 120° (b) 60° (c) 100° (d) 90°

3. How many least number of distinct points required to determine a unique line ?
(a) One (b) Two (c) Three (d) Infinite

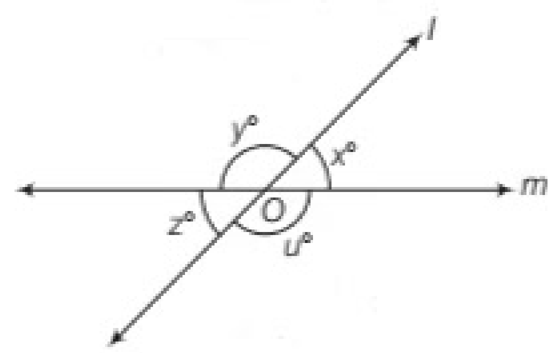
4. If OA and OB are opposite rays; a ray OC inclined. If one of the angle is 75°, then the measurement of the second angle is
(a) 105° (b) 70°
(c) 15° (d) None of these

5. In figure, $\angle AOC$ and $\angle BOC$ form a linear pair. Then, the value of x is



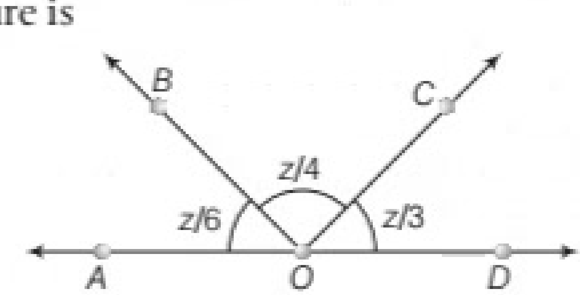
(a) 15° (b) 40° (c) 25° (d) 30°

6. Lines l and m intersect at O, forming angles as shown in figure. If $x = 45°$, then values of y, z and u are respectively



(a) 45°, 135°, 135°
(b) 135°, 135°, 45°
(c) 135°, 45°, 135°
(d) 115°, 45°, 115°

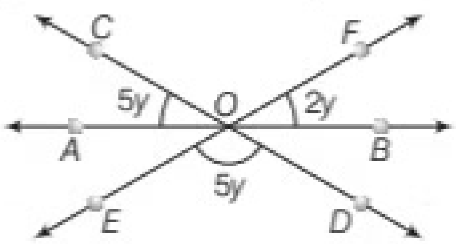
7. The value of z (in degrees), in the given figure is



(a) 180° (b) 216° (c) 240° (d) 40°

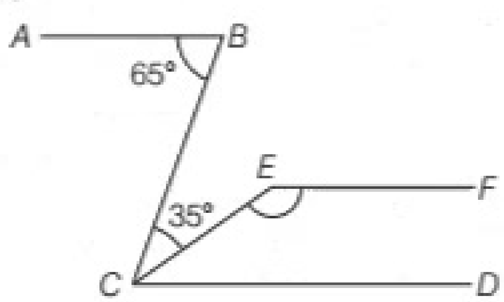
8. Which of the following statements is false?
(a) A line segment can be produced to any desired length
(b) Through a given point, only one straight line can be drawn
(c) Through two given points, it is possible to draw one and only one straight line
(d) Two straight lines can intersect in only one point

9. In the figure, the value of y is



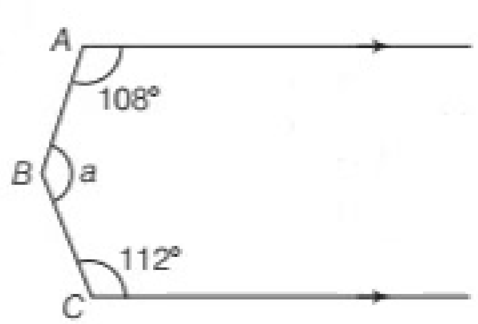
(a) 25° (b) 35°
(c) 15° (d) 40°

10. AB and CD are two parallel lines. The points B and C are joined such that $\angle ABC = 65°$. A line CE is drawn making angle of 35° with the line CB, EF is drawn parallel to AB, as shown in figure, then $\angle CEF$ is



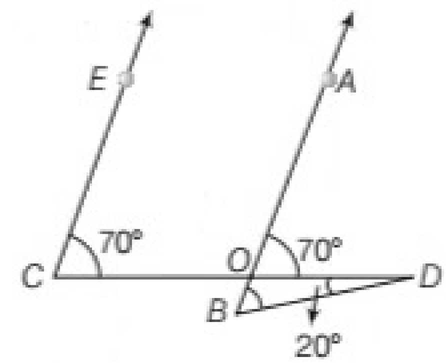
(a) 160° (b) 155°
(c) 150° (d) 145°

11. In the given figure, ray $A \parallel$ ray C, the value of 'a' is



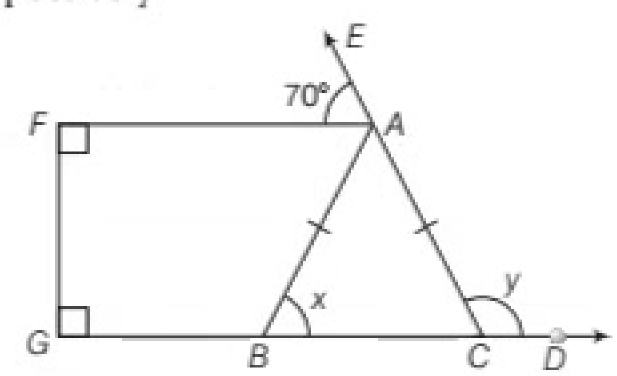
(a) 120° (b) 140°
(c) 90° (d) 150°

12. In the given figure, if $EC \parallel AB$, $\angle ECD = 70°$, $\angle BDO = 20°$, then $\angle OBD$ is



(a) 70° (b) 60° (c) 50° (d) 20°

13. In the given figure, the value of x and y are respectively

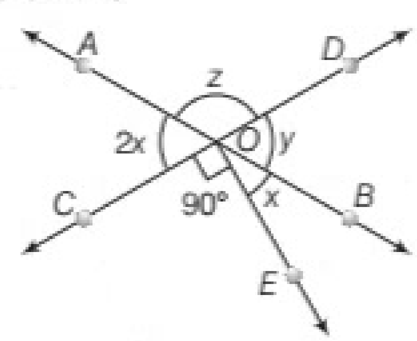


(a) 70°, 110° (b) 110°, 70°
(c) 120°, 60° (d) 70°, 90°

14. The earth makes a complete rotation about its axis in 24 h. Through what angle will it turn in 3 h 20 min?
(a) 60° (b) 50°
(c) 70° (d) 90°

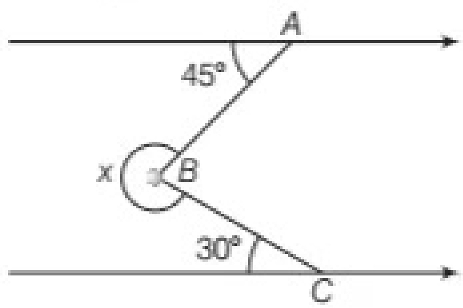
15. An angle is $\frac{2}{3}$rd of its complement and $\frac{1}{4}$th of its supplement, then the angle is
(a) 46° (b) 56°
(c) 36° (d) 40°

16. In the given figure, if $\angle COE = 90°$, then the value of x is



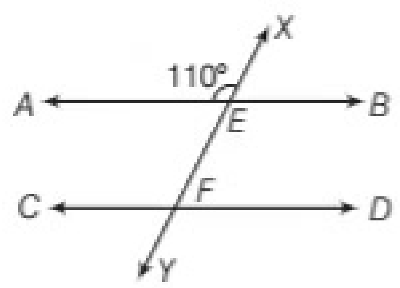
(a) 120° (b) 60°
(c) 45° (d) 30°

17. The value of x, in the figure is



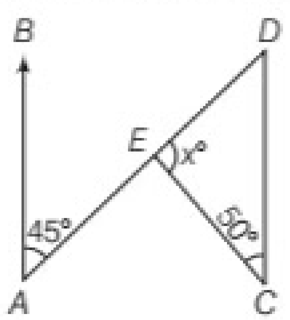
(a) 75° (b) 185° (c) 285° (d) 245°

18. In the given figure, AB and CD are two parallel lines. A line XY meets the lines AB and CD at E and F respectively. If $\angle XEA = 110°$, then $\angle EFD$ is



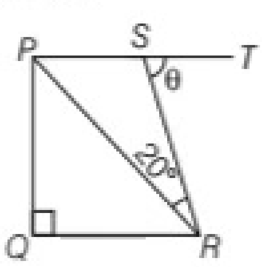
(a) 110° (b) 70° (c) 80° (d) 45°

19. In the given figure, $AB \| CD$, $\angle BAE = 45°$, $\angle DCE = 50°$ and $\angle CED = x°$, then the value of x is



(a) 85° (b) 95°
(c) 130° (d) 135°

20. In the trapezium $PQRS$, $QR \| PS$, $\angle Q = 90°$, $PQ = QR$ and $\angle PRS = 20°$. If $\angle TSR = \theta$, then value of θ is

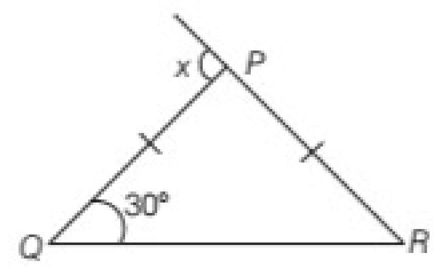


(a) 75° (b) 55°
(c) 65° (d) 45°

21. If ΔABC is an isosceles with $AB = AC$, if $\angle A = 80°$, then $\angle ABC$ will be
(a) 100° (b) 80°
(c) 50° (d) 40°

22. In ΔABC, $BC = CA$, its two equal angles are
(a) $\angle B = \angle C$
(b) $\angle A = \angle B$
(c) $\angle A = \angle C$
(d) $\angle A = \angle B = \angle C$

23. The value of x in figure, where ΔPQR is an isosceles with $PQ = PR$ will be



(a) 30° (b) 60°
(c) 90° (d) 150°

24. The length of the sides BC and AC of a right angled ΔABC are 3 cm and 4 cm, the length of hypotenuse will be
(a) 5 cm (b) 6 cm
(c) 14 cm (d) 10 cm

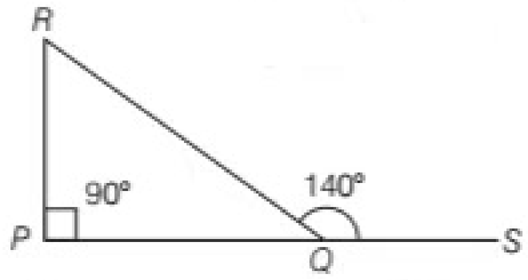
25. If the square of the hypotenuse (in cm) of an isosceles right triangle is 200 then the length of each side will be
(a) 15 cm (b) 200 cm
(c) 10 cm (d) None of these

26. In ΔABC all sides are of same length, then each angle will be
(a) 50° (b) 90°
(c) 60° (d) 180°

27. In a ΔABC, $AB = 11$ cm, $BC = 60$ cm and $AC = 61$ cm. What type of ΔABC ?
(a) Acute angled triangle
(b) Right angled triangle
(c) Obtuse angled triangle
(d) None of the above

28. The total number of triangles formed in a rectangle are
(a) 4 (b) 8
(c) 6 (d) 3

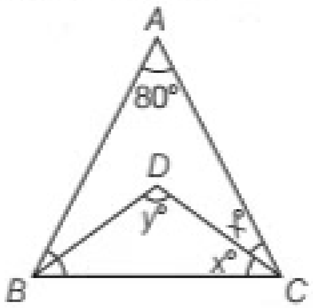
29. The value of $\angle PRQ$ in the given triangle is



(a) 50°
(b) 140°
(c) 90°
(d) 60°

30. ABC is a triangle such that $AB = 10$ and $AC = 3$. The side BC is
(a) equal to 7
(b) less than 7
(c) greater than 7
(d) None of the above

31. In the given figure, $\angle A = 80°$, $\angle B = 60°$, $\angle C = 2x°$ and $\angle BDC = y°$. BD and CD bisect angles B and C respectively. The value of x and y are respectively



(a) 10°, 160° (b) 15°, 70°
(c) 20°, 130° (d) 20°, 125°

32. If one of the angles of a triangle is greater than each of the two remaining angles by 30°, then the angles of the triangle are
(a) 40°, 40°, 100° (b) 50°, 50°, 80°
(c) 30°, 30°, 120° (d) 35°, 35°, 110°

Answers

1	(d)	2	(a)	3	(b)	4	(a)	5	(d)	6	(c)	7	(c)	8	(b)	9	(c)	10	(c)
11	(b)	12	(c)	13	(a)	14	(b)	15	(c)	16	(d)	17	(c)	18	(b)	19	(a)	20	(c)
21	(c)	22	(b)	23	(b)	24	(a)	25	(c)	26	(c)	27	(b)	28	(b)	29	(a)	30	(c)
31	(c)	32	(b)																

Hints and Solutions

1. Let the angle be x, then its complement be $(90° - x)$.

$\therefore \quad x = (90° - x) + 14° \Rightarrow 2x = 104°$

$\therefore \quad x = \dfrac{104°}{2} = 52°$

2. Let the angle be x, then its supplementary be $(180° - x)$.

$\therefore \quad x = 2\,(180° - x) \Rightarrow x = 360° - 2x$

$\Rightarrow \quad 3x = 360°$

$\therefore \quad x = 120°$

3. One and only one straight line passes through two distinct points.

4. Since, OA and OB are opposite rays.

So, AB is a line.

Sum of the two angles $= 180°$

$\therefore$ Second angle $= 180° - 75° = 105°$

5. Since, $\angle AOC + \angle BOC = 180°$ [linear pair]

$\Rightarrow \quad 4x + 2x = 180°$

$\Rightarrow \quad 6x = 180°$

$\therefore \quad x = 30°$

6. $\because \quad x = z$ [vertically opposite angles]

$\Rightarrow \quad x = 45° \Rightarrow z = 45°$

and $y + x = 180°$ [linear pair]

$\Rightarrow \quad y = 180° - 45°$

$\therefore \quad y = 135°$

Also, $y = u$ [$\because$ vertically opposite angles]

$\Rightarrow \quad u = 135°$

$\therefore \quad y = 135°,$
$z = 45°,$
$u = 135°$

7. We have, $\frac{z}{6}+\frac{z}{4}+\frac{z}{3}=180°$ [$\because$ AOD is a line]

$\Rightarrow \quad \frac{2z+3z+4z}{12}=180°$

$\Rightarrow \quad \frac{9z}{12}=180°$

$\therefore \quad z=\frac{180°\times 12}{9}=240°$

8. Since, an infinite number of straight lines can be drawn through a given point.

Hence, (b) is false statement.

9. Since, OA, OB are opposite rays.

$\therefore \ \angle AOC+\angle COF+\angle FOB=180°$

$\Rightarrow \quad 5y+5y+2y=180°$

[$\because$ $\angle COF=\angle EOD$ vertical opposite angle]

$\Rightarrow \quad 12y=180° \Rightarrow y=\frac{180°}{12°}=15°$

10. Since, $AB \parallel CD$

$\Rightarrow \quad \angle BCD=\angle ABC=65°$

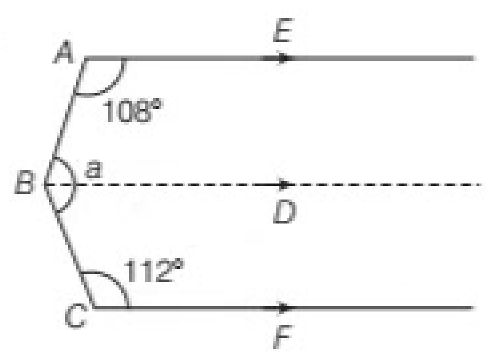
But, $\angle ECD=65°-\angle BCE$

$=65°-35°=30°$

Now, $\angle CEF+\angle ECD=180°$ [sum of co-interior angles is 180°; since $CD \parallel EF$]

$\therefore \quad \angle CEF=180°-30°=150°$

11.



Draw ray $AE \parallel$ ray $BD \parallel$ ray CF

Then, $\angle EAB+\angle ABD=180°$

[the sum of co-interior angles is 180°]

$\Rightarrow \quad 180°+\angle ABD=180°$

$\Rightarrow \quad \angle ABC=180°-108°$

$=72° \quad$ ...(i)

and $\angle FCB+\angle CBD=180°$

$\Rightarrow \quad 112°+\angle CBD=180°$

$\Rightarrow \quad \angle CBD=180°-112°$

$=68° \quad$ ...(ii)

Now, adding Eqs. (i) and (ii), we have

$\angle ABD+\angle CBD=72°+68°$

$\therefore \quad \angle ABC=140°$

Hence, $a=140°$

12. $\because \quad \angle AOD=\angle ECO$ [$\because$ $EC \parallel AB$]

$\Rightarrow \quad \angle AOD=70°$

So, $\angle BOD=110°$ [$\because$ AOB is the line]

In ΔBOD, $\angle OBD+\angle BOD+\angle ODB=180°$

$\Rightarrow \quad \angle OBD=180°-(110°+20°)$

$\therefore \quad \angle OBD=50°$

13. In ΔABC, $\angle ABC=\angle ACB=x$

[Angles opposite to equal sides are equal]

and $x+y=180°$

Now, $\angle EAF=\angle ACB=70°$

[corresponding angles]

$\therefore \quad x=70°$

$\Rightarrow \quad y=180°-70°$ [$\because$ linear pair]

$\therefore \quad y=110°$

14. In 24 h, earth covers an angle $=360°$

In 1 h, earth covers an angle $=\frac{360°}{24}$

In 3 h, 20 min earth will cover an angle

$=\frac{360°}{24}\times(3\text{ h }20\text{ min})$

$=\frac{360°}{24}\times\left(3+\frac{20}{60}\right)=\frac{360°}{24}\times\frac{10}{3}=50°$

15. Let angle be x, then its complement be $(90°-x)$.

$\therefore \quad \frac{2}{3}(90°-x)=x \Rightarrow 180°-2x=3x$

$\therefore \quad x=36°$

16. Since, $\angle BOD=\angle AOC$

[$\because$ vertically opposite angles]

$\Rightarrow \quad 2x=y$

Now, $\angle COE+\angle EOB+\angle BOD=180°$

[$\because$ COD is line or linear pair]

$\Rightarrow \quad 90°+x+2x=180°$

$\Rightarrow \quad 3x=90°$

$\therefore \quad x=30°$

17. Here, $\angle ABC = 45° + 30° = 75°$ [let ray $A \parallel$ ray C]

$\Rightarrow \quad x = 360° - \angle ABC = 360° - 75°$

$\therefore \quad x = 285°$

18. $\because \angle XEA = \angle BEF = 110°$ [vertically opposite angles]

$\therefore \quad \angle BEF + \angle EFD = 180°$ [co-interior angles]

$\Rightarrow \angle EFD = 180° - 110° = 70°$

19. In a given figure, $AB \parallel CD$

$\therefore \angle BAD = \angle ADC = 45°$ [alternate angle]

In ΔECD, $x° + 50° + 45° = 180° \Rightarrow x = 85°$

20. $\because PQ = QR \Rightarrow \angle QPR = \angle QRP = 45°$

and $PS \parallel QR$, $\angle SPR = \angle QRP = 45°$ [alternate angles]

$\therefore \theta = \angle PRS + \angle SPR = 20° + 45°$

$= 65°$ [by exterior angle theorem of ΔPSR]

21. Given, $\angle A = 80°$ and $AB = AC$

Therefore, $\angle B = \angle C = x$

[Opposite angles to equal sides are equal]

In ΔABC, $\angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad 80° + x + x = 180°$

$\Rightarrow \quad 2x = 100°$

$\therefore \quad x = 50°$

22. Since in a ΔABC, $BC = CA$, therefore $\angle A = \angle B$

23. Since, $PQ = PR$

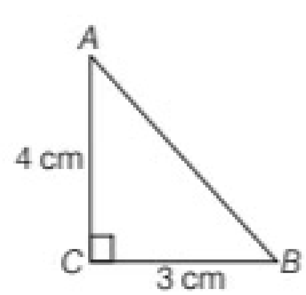
i.e., $\angle Q = \angle R = 30°$

Now, $\angle P = 180° - (30° + 30°) = 120°$

$\therefore \quad x = 180° - CP$ [linear pair]

$x = 180° - 120° = 60°$

24. Given $AC = 4$ cm, $BC = 3$ cm, $AB = ?$



By pythagoras theorem,

$(AB)^2 = (AC)^2 + (BC)^2$

$= 4^2 + 3^2 = 16 + 9 = 25$

$\therefore \quad AB = \sqrt{25} = 5\text{ cm}$

25. Let the sides of a right isosceles triangle be x and x.

$\therefore \quad H^2 = x^2 + x^2 \Rightarrow 200 = 2x^2$

$\Rightarrow \quad x^2 = 100 \Rightarrow x = 10\text{ cm}$

26. Since, all sides of a triangle are of same length, therefore all angles are of equal, say x.

$\therefore \quad 3x = 180°$

$\Rightarrow \quad x = 60°$

27. Now, $AB^2 + BC^2 = 11^2 + 60^2$

$= 121 + 3600 = 3721$

and $\quad AC^2 = (61)^2 = 3721$

$\because \quad AC^2 = AB^2 + BC^2$

Hence, ΔABC is right angled triangle.

28. The total number of triangle formed in a rectangle are 8.

29. $\because PQR$ is a line.

$\therefore \quad \angle PQR = 180° - 140° = 40°$

and in ΔPQR, $\angle RPQ + \angle PQR + \angle PRQ = 180°$

$\angle PRQ = 180° - (90° + 40°)$

$= 180° - 130° = 50°$

30. Since, the sum of any two sides of a triangle is greater than the third side, so BC must be greater than 7, then $AC + BC > AB$.

31. In the given figure,

$\angle C = 180° - \angle A - \angle B$

$\Rightarrow 2x = 180° - 80° - 60° = 40°$

$\therefore \quad x = 20°$

Also, $\angle B = 60° \Rightarrow \angle DBC = \frac{1}{2} \times 60° = 30°$

In ΔBDC, $\angle DBC + y + x = 180°$

$\Rightarrow 30° + y + 20° = 180° \Rightarrow y = 130°$

32. Since, $\quad (x + 30°) + x + x = 180°$

$\Rightarrow \quad 3x = 150° \Rightarrow x = 50°$

$\therefore$ Angles are $50°, 50°, 80°$.

CHAPTER

11

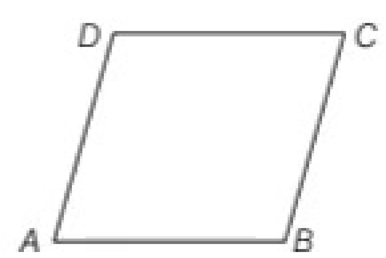
QUADRILATERALS

(PARALLELOGRAM, RHOMBUS, RECTANGLE, SQUARE, KITE)

In this chapter, we study the quadrilateral and their types. Also study the circle and cyclic quadrilaterals with their important results.

Quadrilateral

A plane figure which is made up of joining any four non-collinear points, is called quadrilateral.



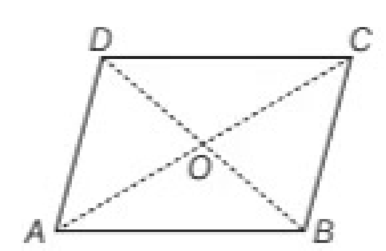
The sum of all angles of a quadrilateral is 360°.
i.e. $\angle A + \angle B + \angle C + \angle D = 360°$.

Types of Quadrilateral

Some types of quadrilateral are as given below

(i) **Parallelogram**

A quadrilateral is a parallelogram, if its both pairs of opposite sides are parallel.



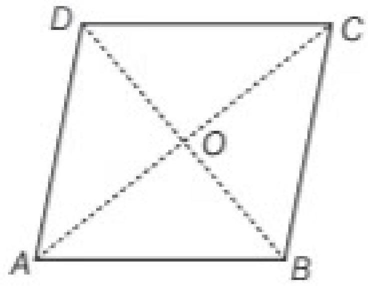
In figure, quadrilateral $ABCD$ is a parallelogram because $AB \parallel DC$ and $AD \parallel BC$.

- Opposite sides are equal, i.e. $AB = CD$ and $AD = BC$.

- Opposite angles are equal i.e. $\angle A = \angle C$ and $\angle B = \angle D$.
- Diagonals bisect each other, i.e. $AO = OC$, $OD = OB$.

(ii) Rhombus

A parallelogram in which all the sides are equal is called a rhombus.

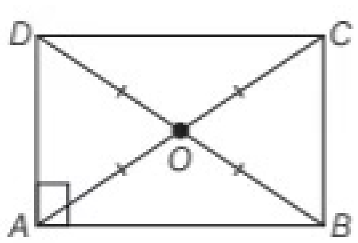


In figure, $ABCD$ is rhombus in which $AB \parallel DC$ and $AD \parallel BC$ and $AB = BC = CD = DA$.

- In rhombus diagonal bisects each other but they are not equal.

(iii) Rectangle

A parallelogram in which each angle is a right angle and opposite sides are equal is called rectangle.

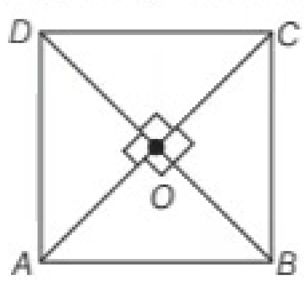


In figure, $ABCD$ is a rectangle in which $AB = DC$ and $AD = BC$, also $\angle A = \angle B = \angle C = \angle D = 90°$.

- In rectangle diagonals bisect each other and they are equal, i.e. $AO = OB = OC = OD$

(iv) Square

A parallelogram having all sides equal and each angle equal to a right angle is called a square.



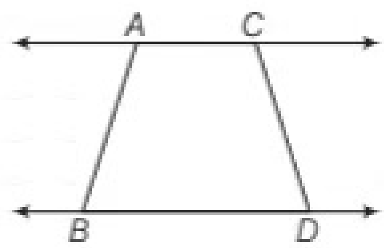
In figure, $ABCD$ is a square in which $AB = BC = CD = DA$ and $\angle A = \angle B = \angle C = \angle D = 90°$.

- In a square diagonals bisect each other at $90°$and they are equal, i.e. $AO = OB = OC = OD$.

(v) Trapezium

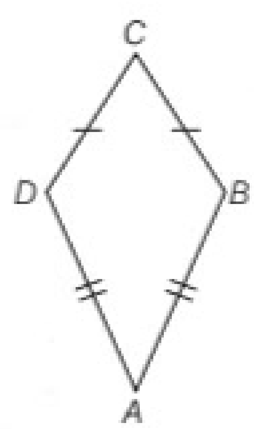
A quadrilateral in which one pair of opposite sides is parallel, is called a trapezium.

In trapezium $ABCD$, sides AC and BD are parallel to each other, but AB and CD neither parallel nor equal.



(vi) Kite

A quadrilateral which has two pairs of equal adjacent sides but unequal opposite sides, is a kite.

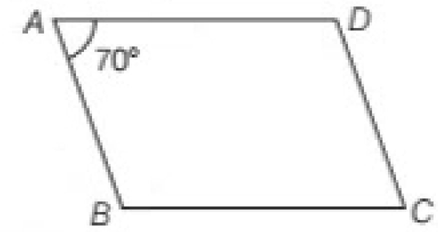


In figure, $ABCD$ is a kite in which $AB = AD$ and $BC = CD$ but $AD \neq BC$ and $AB \neq CD$.

Example 1 $ABCD$ is a parallelogram in which $\angle A = 70°$, the remaining angles of parallelogram are

(a) $110°, 65°, 115°$ (b) $110°, 70°, 110°$
(c) $110°, 50°, 130°$ (d) None of these

Sol. *(b)* In parallelogram $ABCD$,



$$AD \parallel BC$$

$$\therefore \quad \angle A + \angle B = 180°$$

$$\Rightarrow \quad \angle B = 180° - 70° = 110°$$

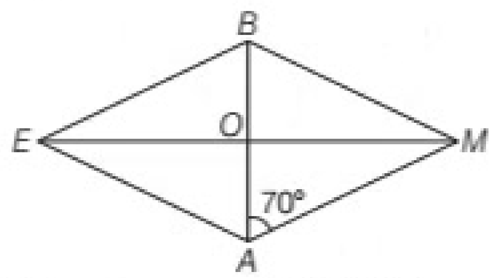
and $\quad \angle C = \angle A \quad$ [opposite angles]

$\therefore \quad \angle C = 70°$

and $\quad \angle D = \angle B = 110° \quad$ [opposite angles]

Hence, the angles B, C, D are $110°, 70°, 110°$ respectively.

Example 2 In rhombus $BEAM$, $\angle AME$ and $\angle AEM$ are



(a) 20°, 20° (b) 30°, 30°
(c) 35°, 35° (d) 40°, 40°

Sol. *(a)* Given, $\angle BAM = 70°$

We know that, in rhombus, diagonals bisect each other at right angles.

$\therefore \angle BOM = \angle BOE = \angle AOM = \angle AOE = 90°$

Now, in ΔAOM,

$\angle AOM + \angle AMO + \angle OAM = 180°$

[angle sum property of triangle]

$\Rightarrow \quad 90° + \angle AMO + 70° = 180°$

$\Rightarrow \quad \angle AMO = 180° - 90° - 70°$

$\Rightarrow \quad \angle AMO = 20° = \angle AME$

Also, $AM = BM = BE = EA$

In ΔAME, we have,

$AM = EA$

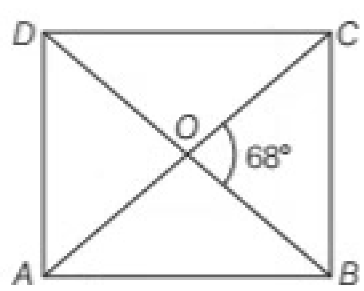
$\therefore \angle AME = \angle AEM = 20°$

[$\because$ equal sides make equal angles]

Example 3 The diagonals of a rectangle $ABCD$ intersect in O, if $\angle BOC = 68°$, find $\angle ODA$.

(a) 55° (b) 56°
(c) 57° (d) 58°

Sol. *(b)* Given, $\angle BOC = 68°$ [given]



Then, $\quad \angle AOD = 68° \quad$ [vertically opposite angles]

$\because \quad OA = OD$

[$\because$ diagonals bisects each other]

$\therefore \quad \angle ODA = \angle OAD$

$\angle ODA + \angle OAD + \angle AOD = 180°$

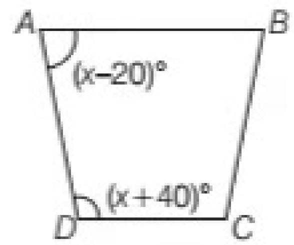
[sum of angles of a ΔAOD]

$2\angle ODA + 68° = 180°$

$\Rightarrow \quad 2\angle ODA = 180° - 68° = 112°$

$\Rightarrow \quad \angle ODA = 56°$

Example 4 The value of x in the trapezium $ABCD$ given below is



(a) 80° (b) 82°
(c) 84° (d) 85°

Sol. *(a)* Given, a trapezium $ABCD$ in which

$\angle A = (x - 20)°, \angle D = (x + 40)°$

Since, in a trapezium, the angles on either side of the base are supplementary, therefore

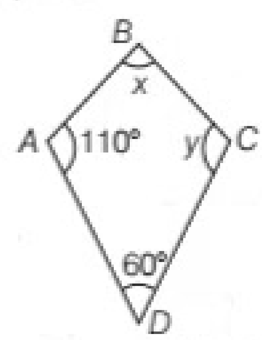
$(x - 20°) + (x + 40°) = 180°$

$\Rightarrow \quad 2x + 20° = 180°$

$\Rightarrow \quad 2x = (180° - 20°) = 160°$

$\therefore \quad x = 80°$

Example 5 The values of x and y in the following kite, are



(a) 80° (b) 70°
(c) 82° (d) 84°

Sol. *(a)* In a kite, one pair of opposite angles are equal.

$\therefore \quad y = 110°$

Now, by the angle sum property of a quadrilateral, we have

$\angle A + \angle B + \angle C + \angle D = 360°$

$$110^\circ + x + 110^\circ + 60^\circ = 360^\circ$$
$$\Rightarrow \quad x = 360^\circ - 280^\circ$$
$$\therefore \quad x = 80^\circ$$

Polygons

A polygon is a closed plane figure bounded by straight lines.

Types of Polygons

Polygons are as following four types

1. **Convex polygon** A polygon in which none of its interior angles is more than 180°, is called convex polygon.
2. **Concave polygon** A polygon in which atleast one angle is more than 180°, is called concave polygon.
3. **Irregular polygon** A polygon in which all the sides or angles are not of the same measure, is called an irregular polygon.
4. **Regular polygon** A regular polygon has all its sides and angles equal.

(i) Each exterior angle of a regular polygon
$$= \frac{360^\circ}{\text{Number of sides}}$$

(ii) Each interior angle = 180° – Exterior angle

(iii) Sum of all interior angles $= (2n-4)\times 90^\circ$
$$= (n-2)\times 180^\circ$$

(iv) Sum of all exterior angles = 360°

(v) Number of diagonals of polygon of n sides
$$= \frac{n(n-3)}{2}$$

Example 6 Each interior angle of a regular polygon is 144°. Find the interior angle of a regular polygon which has double the number of sides as the first polygon.

(a) 170° (b) 160°
(c) 162° (d) 180°

Sol. *(c)* $\because$ Each interior angle of polygon = 144°

$$\Rightarrow \quad \frac{(2n-1)90^\circ}{n} = 144^\circ$$
$$\Rightarrow \quad 180n - 360^\circ = 144n$$
$$\Rightarrow \quad 36n = 360^\circ$$
$$\Rightarrow n = 10^\circ$$

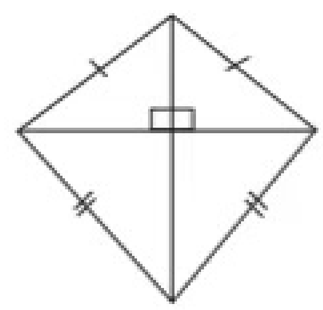
According to the given condition,

Total sides of a new polygon $= 2\times 10 = 20$

Each interior angle of new polygon
$$= \frac{(2\times 20-4)\times 90^\circ}{20}$$
$$= \frac{36\times 9}{2} = 162^\circ$$

Kite

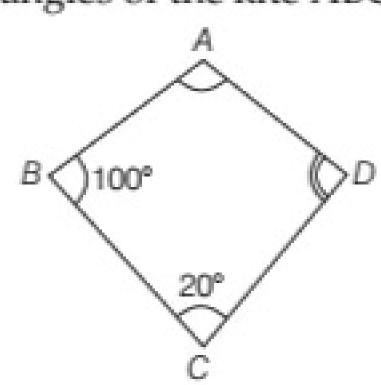
A kite is a quadrilateral whose four sides can be grouped into two pairs of equal- length sides that are adjacent to each other



Properties of Kite

- Its two diagonals are at right angles to each other.
- There is a pair of equal opposite angles.

Example 7 Calculate the measures of the unmarked angles of the kite ABCD.



(a) 170° and 120° (b) 100° and 100°
(c) 100° and 140° (d) 110° and 130°

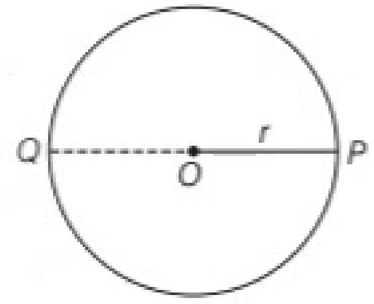
Sol. *(c)* We know that, $\angle ABC = \angle ADC = 100^\circ$

$\because$ Sum of four angles of kite = 360°

$$\angle ABC + \angle BCD + \angle ADC + \angle BAD = 360^\circ$$
$$100^\circ + 20^\circ + 100^\circ + \angle BAD = 360^\circ$$
$$\angle BAD = 360^\circ - 220^\circ$$
$$\angle BAD = 140^\circ$$

Circle

A circle is a set of those points in a plane, which are at a given constant distance from a given fixed point in the plane.



- The fixed point O is called the centre of the circle.
- The constant distance r is called the radius of the circle.
- A circle can have many radii measure and all the radii of a circle are of equal length.
- The line PQ is a diameter (d) of a circle and

$$d = PQ = 2 \times \text{Radius} = 2r$$

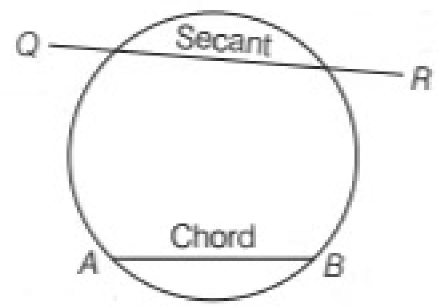
(i) Arc of the Circle

A continuous part of a circle is called an arc of the circle.

(ii) Chord

A line segment joining any two points on the circle is called its chord.

In a figure, AB is a chord.



Note Biggest chord of a circle is its diameters.

(iii) Secant

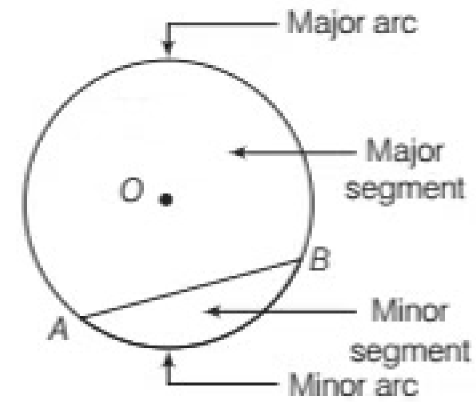
A line which intersect a circle in two distinct points is called a secant of the circle.

In above figure, QR is a secant.

(iv) Semicircle

A diameter divides the circle into two equal arcs, each of these two arcs is called a semicircle.

An arc whose length is less than the arc of a semicircle is called a minor arc, otherwise it is called a major arc.



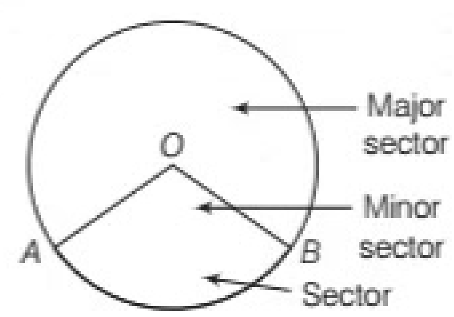
(v) Segment

If AB be a chord of the circle then AB divides the circular region into two parts, each part is called a segment of the circle.

The segment containing the minor arc is called the minor segment and the segment containing the major arc is called the major segment.

(vi) Central Angle

If $C(O, r)$ be any circle, then any angle whose vertex is centre of circle is called a central angle.



The degree measure of an arc is the measure of the central angle containing the arc.

(vii) Sector

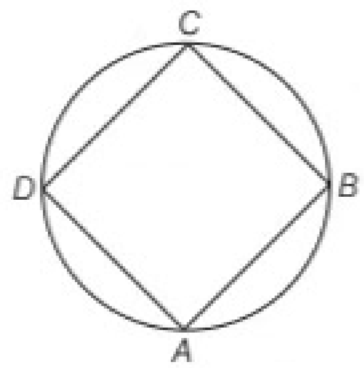
A sector is that region of a circle $C(O, r)$ which lies between an arc and the two radii joining the extremities of the arc to the centre.

(viii) Quadrant

One fourth of a circular region is called a quadrant.

(ix) Cyclic Quadrilateral

If all four vertices of a quadrilateral lie on a circle, then such a quadrilateral is called a cyclic quadrilateral.



The sum of opposite angles of a cyclic quadrilateral is 180°, i.e.,

$$\angle A + \angle C = 180°$$

and $\quad \angle B + \angle D = 180°$

Some Important Results Related to Circles

(i) An infinite number of circles can pass through the two points.

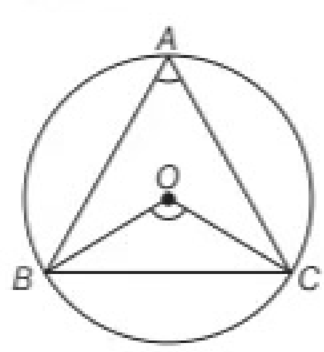
(ii) There is one and only one circle passing through three non-collinear points.

(iii) Angles in the same segment of a circle are equal.

(iv) The perpendicular from the centre to any chord bisects the chord.

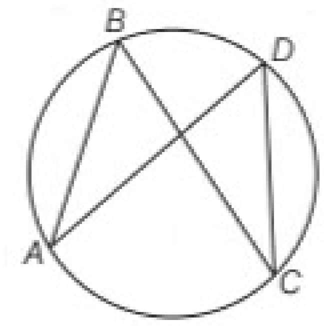
(v) The line joining the centre of the circle to the mid-point of any chord of a circle, is perpendicular to the chord.

(vi) The angle subtended by an arc at the centre of a circle is double the angle subtended by it at any point on the remaining part of the circle.



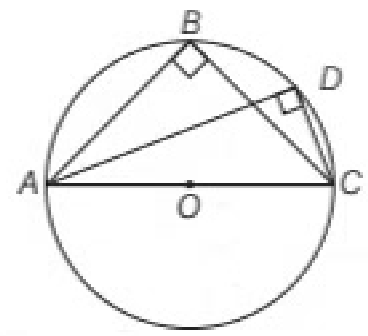
i.e. $\quad \angle BOC = 2\angle BAC$

(vi) Angles in the same segment of a circle are equal.



Here, $\quad \angle ABC = \angle ADC$

(vii) The angle made in a semicircle is always a right angle.

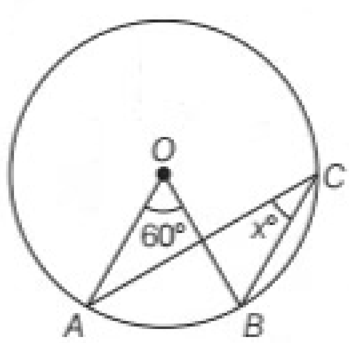


Here, $\quad \angle ABC = 90°$

and $\quad \angle ADC = 90°$

(viii) If the sum of any pair of opposite angles of a quadrilateral is $180°$, then the quadrilateral is cyclic.

Example 6 The value of $x°$ in the figure is



(a) 20° (b) 100°
(c) 60° (d) 30°

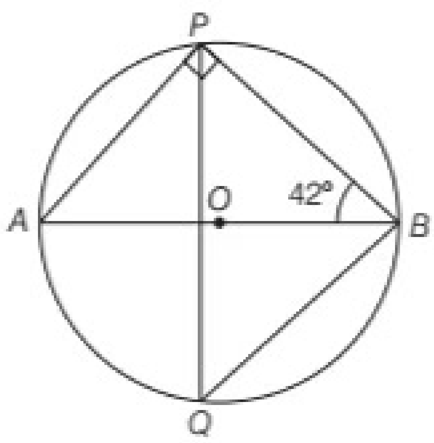
Sol. *(d)* We know that, the angle subtended by an arc at the centre of a circle is double the angle subtended by it any point of the remaining part of the circle.

$\therefore \quad \angle AOB = 2(\angle ACB)$

$\Rightarrow \quad 60° = 2\angle ACB$

$\Rightarrow \quad \angle ACB = 30°$

Example 7 In figure, the value $\angle PQB$, where O is the centre of the circle, is



(a) 47° (b) 48° (c) 49° (d) 50°

Sol. *(b)* In ΔAPB,

$$\angle APB = 90° \quad \text{[angle in a semi-circle]}$$
$$\angle PBA = 42° \quad \text{[given]}$$

Now, $\angle PAB + \angle APB + \angle PBA = 180°$

$\Rightarrow \angle PAB + 90° + 42° = 180°$

$\Rightarrow \angle PAB = 180° - 132° = 48°$

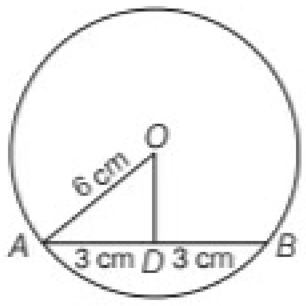
Since, we know that the angle subtended by an arc in the same segment are equal.

$\therefore \angle PQB = \angle PAB = 48°$

Example 8 The radius of a circle is 6 cm and the length of one of its chords is 6 cm. The distance of the chord from the centre is

(a) $3\sqrt{5}$ cm (b) $3\sqrt{2}$ cm
(c) $3\sqrt{3}$ cm (d) $3\sqrt{6}$ cm

Sol. *(c)* Let AB be a chord of a circle with centre O and radius 6 cm such that $AB = 6$ cm.



From O, draw $OD \perp AB$. Join OA

Clearly,

$$AD = \frac{1}{2} AB = 3 \text{ cm and } OA = 6 \text{ cm.}$$

Now, in right angle ΔODA,

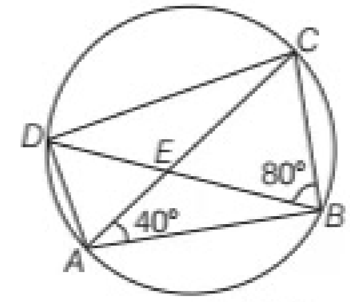
$$OD = \sqrt{OA^2 - AD^2}$$

[using Pythagoras theorem]

$$= \sqrt{6^2 - 3^2} = \sqrt{27} = 3\sqrt{3} \text{ cm}$$

Hence, the distance of the chord from the centre is $3\sqrt{3}$ cm.

Example 9 In figure, if $\angle DBC = 80°$ and $\angle BAC = 40°$, then the value of $\angle BCD$ is



(a) 50° (b) 60°
(c) 70° (d) 80°

Sol. *(b)* Given,

$$\angle DBC = 80° \text{ and } \angle BAC = 40°$$

Consider the chord CD, we find that $\angle CBD$ and $\angle CAD$ are angles in the same segment of the circle.

$\therefore \angle CBD = \angle CAD$

$\Rightarrow 80° = \angle CAD$

$\Rightarrow \angle CAD = 80°$

Now, $\angle BAD = \angle BAC + \angle CAD$

$\Rightarrow \angle BAD = 40° + 80° = 120°$ ...(i)

Since, $ABCD$ is a cyclic quadrilateral.

$\therefore \angle BAD + \angle BCD = 180°$

$\Rightarrow 120° + \angle BCD = 180°$ [using Eq. (i)]

$\Rightarrow \angle BCD = 60°$

PRACTICE EXERCISE

1. A quadrilateral has three acute angles each measuring $75°$, the measure of fourth angle is
(a) $145°$ (b) $135°$ (c) $125°$ (d) $130°$

2. The measures of the four angles of a quadrilateral are in the ratio of $1:2:3:4$. What is the measure of fourth angle?
(a) $144°$ (b) $135°$ (c) $125°$ (d) $150°$

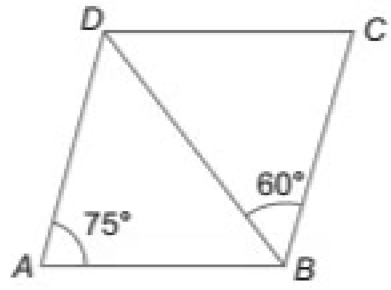
3. The diagonals of a rectangle $ABCD$ cut at O. OAL is an equilateral triangle drawn so that B and L are on the same side of AC. If $\angle ACD = 30°$, then the angles of ΔALB are
(a) $60°$, $60°$ and $60°$
(b) $30°$, $30°$ and $120°$
(c) $30°$, $60°$ and $120°$
(d) Cannot be determined

4. The sum of two opposite angles of a parallelogram is $130°$. All the angles of parallelogram are
(a) $65°, 65°, 115°, 115°$ (b) $145°, 135°, 35°, 45°$
(c) $90°, 130°, 80°, 60°$ (d) $40°, 140°, 80°, 110°$

5. The length of the diagonal of a rectangle whose sides are 12 cm and 5 cm, is
(a) 17 cm (b) 13 cm (c) 25 cm (d) 14 cm

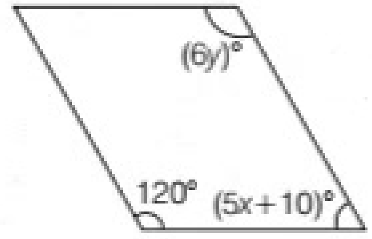
6. In a quadrilateral $ABCD$, if AO and BO be the bisectors of $\angle A$ and $\angle B$ respectively, $\angle C = 70°$ and $\angle D = 30°$, then $\angle AOB$ is
(a) $40°$ (b) $50°$ (c) $80°$ (d) $100°$

7. In the given figure, $ABCD$ is a parallelogram in which $\angle DAB = 75°$ and $\angle DBC = 60°$. Then, $\angle BDC$ is equal to



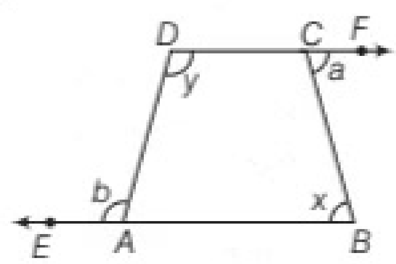
(a) $75°$ (b) $45°$
(c) $60°$ (d) $55°$

8. The values of x and y in the following parallelogram is



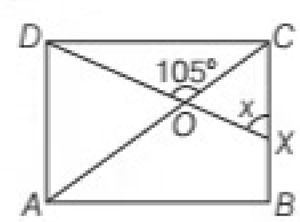
(a) $10°, 20°$ (b) $60°\ 80°$
(c) $90°, 110°$ (d) None of these

9. The sides BA and DC of quadrilateral $ABCD$ are produced as shown in figure. Then, which of the following statement is correct?



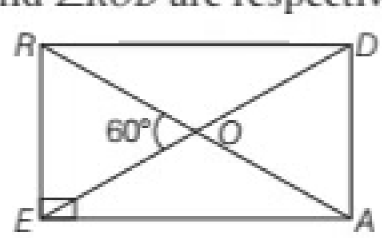
(a) $2x + y = a + b$ (b) $x + \frac{y}{2} = \frac{a+b}{2}$
(c) $x + y = a + b$ (d) $x + a = y + b$

10. In the given figure, $ABCD$ is a square. A line segment DX cuts the side BC at X and the diagonal AC at O such that $\angle COD = 105°$, $\angle OCX = 45°$ and $\angle OXC = x$. The value of x is



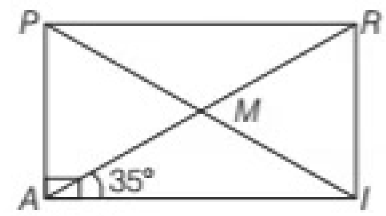
(a) $40°$ (b) $60°$ (c) $80°$ (d) $85°$

11. In rectangle $READ$, the values of $\angle EAR$, $\angle RAD$ and $\angle ROD$ are respectively



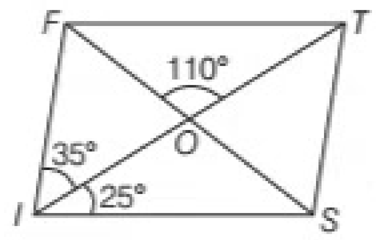
(a) $30°, 60°, 120°$ (b) $40°, 60°, 110°$
(c) $30°, 40°, 110$ (d) None of these

12. In rectangle *PAIR*, the values of $\angle ARI$, $\angle RMI$ and $\angle PMA$ are



(a) 60°, 70°, 70°
(b) 55°, 70°, 70°
(c) 60°, 80°, 80°
(d) None of these

13. In parallelogram *FIST*, the value of $\angle OST$ is

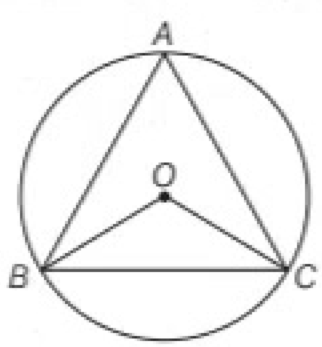


(a) 70° (b) 72°
(c) 75° (d) 80°

14. The external angle of a regular polygon is 45°. Find the sum of all the internal angles of it.
(a) 1082°
(b) 1080°
(c) 1085°
(d) 1090°

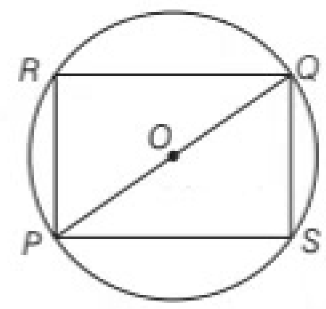
15. Find the number of non overlapping triangles can be formed in 9 sided polygon by joining the vertices.
(a) 5 (b) 7
(c) 6 (d) 4

16. An equilateral ΔABC is inscribed in a circle with centre *O*. Then, $\angle BOC$ is equal to



(a) 120° (b) 75°
(c) 180° (d) 160°

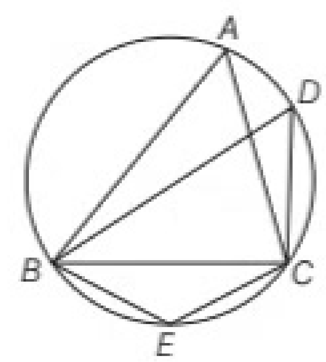
17. In the adjoining figure, *POQ* is the diameter of the circle, *R* and *S* are any two points on the circle. Then,



(a) $\angle PRQ > \angle PSQ$ (b) $\angle PRQ < \angle PSQ$
(c) $\angle PRQ = \angle PSQ$ (d) $\angle PRQ = \frac{1}{2}\angle PSQ$

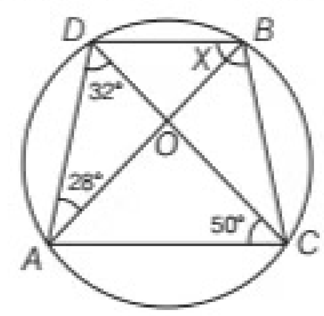
18. In a circle with centre *O* and radius 5 cm, *AB* is a chord of length 8 cm. If $OM \perp AB$, then the length of *OM* is
(a) 4 cm (b) 5 cm
(c) 3 cm (d) 2 cm

19. In the adjoining figure, ΔABC is an isosceles triangle with $AB = AC$ and $\angle ABC = 50°$. Then, $\angle BDC$ is



(a) 110° (b) 90° (c) 80° (d) 70°

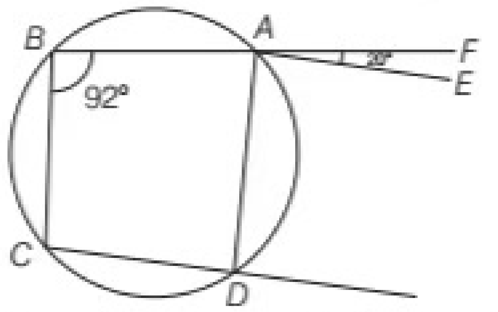
20. If *O* is the centre of the circle, then *x* is



(a) 72° (b) 62° (c) 82° (d) 52°

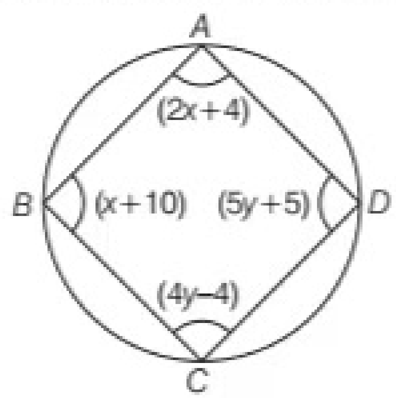
21. In a cyclic quadrilateral *ABCD*, if $\angle B - \angle D = 60°$, then the measure of the smaller of the two is
(a) 60° (b) 40° (c) 38° (d) 30°

22. In the given figure, $ABCD$ is a cyclic quadrilateral. AE is drawn parallel to CD and BA is produced. If $\angle ABC = 92°$ and $\angle FAE = 20°$, then $\angle BCD$ is equal to



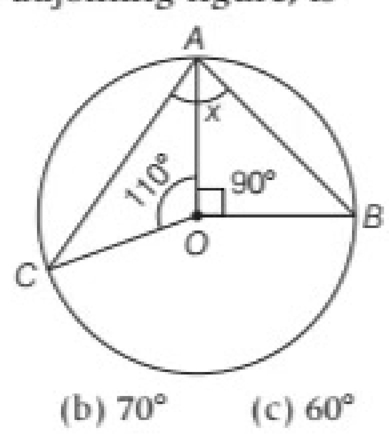
(a) 88° (b) 98° (c) 108° (d) 72°

23. The values of x and y in the figure are measure of angles, then $x + y$ is equal to



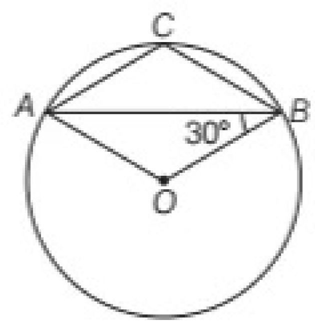
(a) 90° (b) 85° (c) 75° (d) 65°

24. If O is the centre of the circle, the value of x in the adjoining figure, is



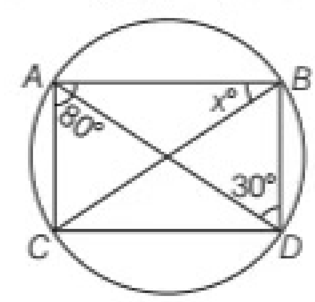
(a) 80° (b) 70° (c) 60° (d) 50°

25. In the given figure, O is centre, then $\angle ACB$ is



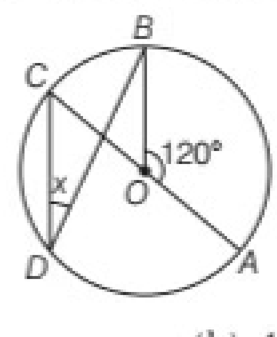
(a) 60° (b) 120°
(c) 75° (d) 90°

26. In the following figure, the value of $x°$ is



(a) 60° (b) 90°
(c) 70° (d) 40°

27. In the figure, O is the centre and AOC is the diameter of the circle. BD is chord and OB and CD are joined D is joined to A. If $\angle AOB = 120°$, then the value of x is



(a) 30° (b) 40°
(c) 50° (d) 60°

Answers

1	(b)	2	(a)	3	(b)	4	(a)	5	(b)	6	(d)	7	(b)	8	(a)	9	(c)	10	(b)
11	(a)	12	(b)	13	(c)	14	(b)	15	(b)	16	(a)	17	(c)	18	(c)	19	(c)	20	(c)
21	(a)	22	(c)	23	(d)	24	(a)	25	(b)	26	(c)	27	(a)						

Hints and Solutions

1. Since, $\angle A + \angle B + \angle C + \angle D = 360^\circ$

$\therefore \quad 75^\circ + 75^\circ + 75^\circ + \angle D$

$\Rightarrow \quad 225^\circ + \angle D = 360^\circ$

$\Rightarrow \quad \angle D = 360^\circ - 225^\circ = 135^\circ$

2. Let the angles be x, $2x$, $3x$ and $4x$.

$\because \quad x + 2x + 3x + 4x = 360^\circ$

$\Rightarrow \quad 10x = 360^\circ$

$\Rightarrow \quad x = 36^\circ$

$\therefore$ Fourth angle $= 4 \times 36^\circ = 144^\circ$

3. $\because\ OA = OB$

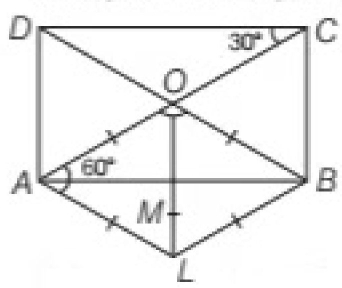
[$\because$ the diagonals of a rectangle bisect each other]

Also, $OA = OL \Rightarrow AOBL$ is a rhombus.

Since, $\Delta\ AOL$ is an equilateral triangle.

$\therefore \quad \angle AOL = 60^\circ$

$\Rightarrow \quad \angle AOB = 2\angle AOL = 120^\circ$



Since, $CD \parallel AB$

$\Rightarrow \quad \angle DCA = \angle CAB = 30^\circ$

and $\quad \angle OAL = 60^\circ$

$\Rightarrow \quad \angle BAL = 60^\circ - 30^\circ = 30^\circ = \angle ABL$

$\therefore$ In $\Delta\ ALB$,

$\angle ALB = 120^\circ,\ \angle ABL = 30^\circ,\ \angle LAB = 30^\circ$

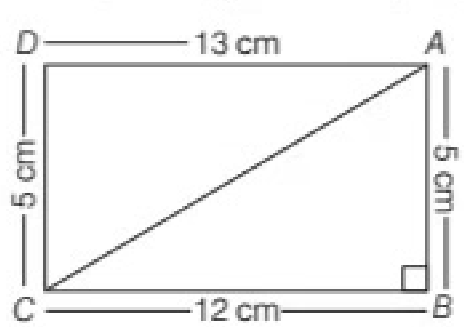
4. Let $\quad \angle A + \angle C = 130^\circ$, then

$$\angle B + \angle D = 360^\circ - 130^\circ = 230^\circ$$

$\therefore$ Angles $= \dfrac{130^\circ}{2}, \dfrac{230^\circ}{2} = 65^\circ, 115^\circ$

Hence, all angles are 65°, 65°, 115°, 115°.

5.



Since, here $\angle ABC = 90^\circ$

So, by pythagoras theorem,

diagonal $AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{5^2 + 12^2}$

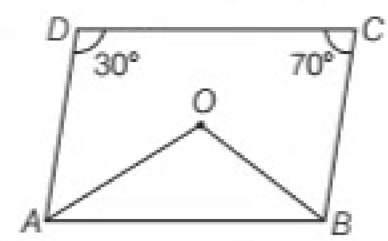
$= \sqrt{25 + 44}$

$= \sqrt{169} = 13$ cm

6. Since, $\angle A + \angle B + \angle C + \angle D = 360^\circ$

$\therefore \quad \angle A + \angle B = 360^\circ - (130^\circ + 70^\circ)$

$= 360^\circ - 200^\circ = 160^\circ$



$\Rightarrow \quad \dfrac{1}{2}(\angle A + \angle B) = 80^\circ$

So, $\quad \angle OAB + \angle ABO = 80^\circ$

$\therefore \quad \angle AOB = 180^\circ - (\angle OAB + \angle ABO)$

$= (180^\circ - 80^\circ) = 100^\circ$

7. $\angle C = \angle A = 75^\circ$

[opposite angles of a parallelogram are equal]

$\therefore \quad \angle BDC = 180^\circ - (60^\circ + 75^\circ)$

$= 180^\circ - 135^\circ = 45^\circ$

8. In a parallelogram, adjacent angles are supplementary.

$\therefore \quad 120^\circ + (5x + 10)^\circ = 180^\circ$

$\Rightarrow \quad 5x + 10^\circ + 120^\circ = 180^\circ$

$\Rightarrow \quad 5x = 180^\circ - 130^\circ$

$\Rightarrow \quad 5x = 50^\circ$

$\Rightarrow \quad x = 10^\circ$

Also, opposite angles are equal in a parallelogram.

Therefore, $\quad 6y = 120^\circ \Rightarrow y = 20^\circ$

9. Since, $\quad \angle A + b = 180^\circ \Rightarrow \angle A = 180^\circ - b$

Also, $\quad \angle C + a = 180^\circ$ [linear pair]

$\Rightarrow \quad \angle C = 180^\circ - a$

But $\quad \angle A + \angle B + \angle C + \angle D = 360^\circ$

$\Rightarrow \quad (180^\circ - b) + x + (180^\circ - a) + y = 360^\circ$

$\therefore \quad x + y = a + b$

10. Given, $\angle COD = 105°$ and $\angle OCX = 45°$

$$\angle COD + \angle COX = 180°$$

$\Rightarrow$ $\angle COX = 180° - \angle COD$

$$= 180° - 105° = 75°$$

In $\Delta OCX, \angle OCX + \angle COX + \angle OXC = 180°$

$\Rightarrow$ $45° + 75° + x = 180°$

$\therefore$ $x = 180° - 120° = 60°$

11. Given, a rectangle *READ*, in which

$$\angle ROE = 60°$$

$\therefore$ $\angle EOA = 180° - 60° = 120°$ [linear pair]

Now, in $\Delta EOA, \angle OEA = \angle OAE = 30°$

[$\because$ $OE = OA$ and equal sides make equal angles]

$\therefore$ $\angle EAR = 30°, \angle RAD = 90° - \angle EAR = 60°$

and $\angle ROD = \angle EOA = 120°$

12. Given, $\angle RAI = 35°$

$\therefore$ $\angle PRA = 35°$

[$PR \parallel AI$ and AR is transversal]

Now, $\angle ARI = 90° - \angle PRA = 90° - 35° = 55°$

$\because$ $AM = IM, \angle MIA = \angle MAI = 35°$

In $\Delta AMI, \angle RMI = \angle MAI + \angle MIA = 70°$

[exterior angle]

Also, $\angle RMI = \angle PMA$

$\Rightarrow$ $\angle PMA = 70°$[vertically opposite angles]

13. Given, $\angle FIS = 60°$

Now, $\angle FTS = \angle FIS = 60°$

[$\because$ opposite angles of a parallelogram are equal]

Now, $FT \parallel IS$ and TI is a transversal, therefore

$\angle FTO = \angle SIO = 25°$ [alternate angles]

$\therefore$ $\angle STO = \angle FTS - \angle FTO = 60° - 25° = 35°$

Also, $\angle FOT + \angle SOT = 180°$ [linear pair]

$\Rightarrow$ $110° + \angle SOT = 180°$

$\Rightarrow$ $\angle SOT = 180° - 110° = 70°$

In $\Delta TOS, \angle TSO + \angle OTS + \angle TOS = 180°$

[angle sum property of triangle]

$\therefore$ $\angle OST = 180° - (70° + 35°) = 75°$

14. External angle of any polygen

$$\frac{360°}{n} = 45° \Rightarrow n = \frac{360°}{45°} \Rightarrow n = 8°$$

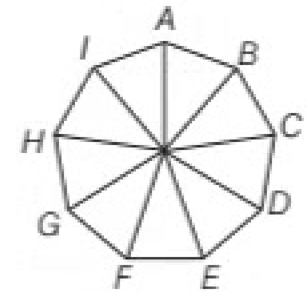
$\therefore$ Every interior angle of regular polygon

$= 180° -$ External angle

$= 180° - 45° = 135°$

$\therefore$ Sum of interior angles of it $= 8 \times 135° = 1080°$

15. Hence total number of non overlapping triangles can be formed in 9 sided polygon is 7.



16. We know that, the angle subtended by an arc at the centre of a circle is double the angle subtended by it any point on the remaning part of the circle.

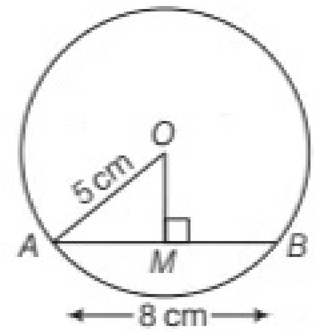
$$\angle BOC = 2\angle A$$

$$= 2 \times 60°$$

$\therefore$ $\angle BOC = 120°$

17. $\angle PRQ = \angle PSQ = 90°$ [each angle in semi-circle]

18. $\because$ $OA = 5$ cm



and $AM = \frac{1}{2} AB = \frac{1}{2} \times 8 = 4$ cm

$\therefore$ $OM = \sqrt{OA^2 - AM^2}$

$$= \sqrt{5^2 - 4^2} = 3 \text{ cm}$$

19. Since, $AB = AC$

$\Rightarrow$ $\angle ACB = \angle ABC = 50°$

In ΔABC,

$$\angle BAC = 180° - (50° + 50°)$$

$$= 80°$$

$\therefore$ $\angle BDC = \angle BAC = 80°$

[angle in the same segment]

20. In $\Delta DAC, \angle ADC + \angle DCA + \angle CAD = 180°$

$\Rightarrow$ $\angle CAD = 180° - 32° - 50° = 98°$

Now, $\angle CAD + \angle CBD = 180°$

[opposite angles of a quadrilateral]

$\therefore$ $x = 180° - 98° = 82°$

21. Since, $\angle B + \angle D = 180^\circ$

[sum of opposite angles of a cyclic quadrilateral]

$\angle B - \angle D = 60^\circ$ [given]

$\Rightarrow$ $\angle B = 120^\circ$ and $\angle D = 60^\circ$

$\therefore$ Required smaller angle $\angle D = 60^\circ$

22. $\angle B + \angle D = 180^\circ$

$\Rightarrow$ $\angle D = 180^\circ - 92^\circ = 88^\circ$

Now, $\angle DAE = \angle D = 88^\circ$ [$\because AE \parallel CD$]

$\Rightarrow$ $\angle FAD = 88^\circ + 20^\circ = 108^\circ$

$\therefore$ $\angle BCD = \angle FAD = 108^\circ$

23. Since, $ABCD$ is a cyclic quadrilateral.

$\angle B + \angle D = 180^\circ$

and $\angle A + \angle C = 180^\circ$

$\Rightarrow$ $x + 10 + 5y + 5 = 180^\circ$

$x + 5y = 165^\circ$ ...(i)

and $2x + 4 + 4y - 4 = 180^\circ$

$\Rightarrow$ $2x + 4y = 180^\circ$ ...(ii)

Solving Eqs.(i) and (ii), we get

$x = 40^\circ$ and $y = 25^\circ$

$\therefore$ $x + y = 40^\circ + 25^\circ = 65^\circ$

24. $\angle COB = 360^\circ - (\angle COA + \angle BOA)$

$= 360^\circ - (110^\circ + 90^\circ)$

$= 160^\circ$

$\therefore$ $x = \frac{1}{2} \times \angle COB$ [by theorem]

$= \frac{1}{2} \times 160^\circ = 80^\circ$

25. In the given figure, $OA = OB$ (radius of circle)

$\Rightarrow$ $\angle OAB = \angle OBA = 30^\circ$

$\therefore$ $\angle AOB = 180^\circ - 60^\circ = 120^\circ$

$\therefore$ Major $\angle AOB = 360^\circ - 120^\circ = 240^\circ$

$\Rightarrow$ $\angle ACB = \frac{1}{2} \times 240^\circ = 120^\circ$

[angle subtended in the arc is half of that subtended at the centre.]

26. In a given figure,

$\angle ADB = \angle ACB = 30^\circ$

[angle subtended in the same segment]

In ΔABC, $\angle x^\circ = 180^\circ - (\angle ACB + \angle CAB)$

$= 180^\circ - (30^\circ + 80^\circ) = 70^\circ$

27. In a given figure,

$\angle COB = 180^\circ - 120^\circ = 60^\circ$ [$\because COA$ is a line]

$\therefore$ $x = \frac{1}{2} \angle COB$

[angle subtended in the arc is half of that subtended at the centre.]

$= \frac{1}{2} \times 60^\circ = 30^\circ$

CHAPTER

12

AREA OF PLANE FIGURES

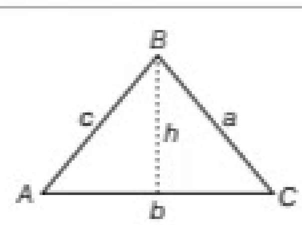
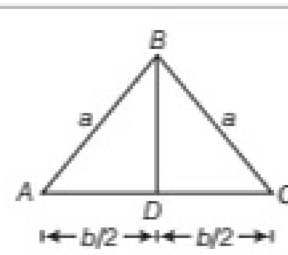
In this chapter, we study the Area of Plane Figures, triangle and its type Square, circle, semi-circle, Trapezium, Hexagon etc.

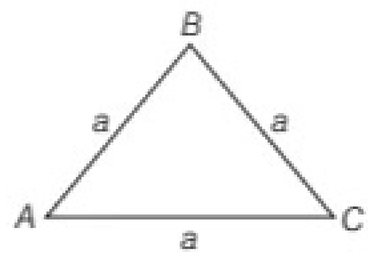
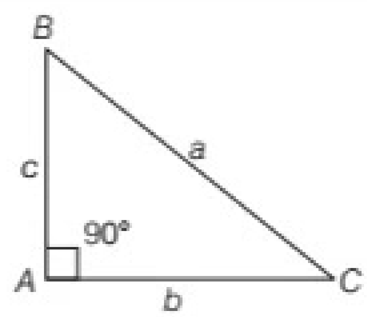
Plane figures are the flat shape in two dimensions, having length and width (breadth).

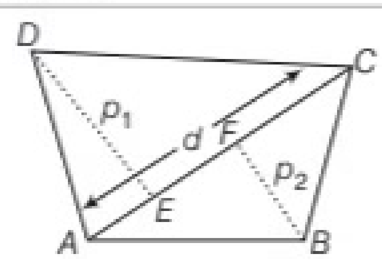
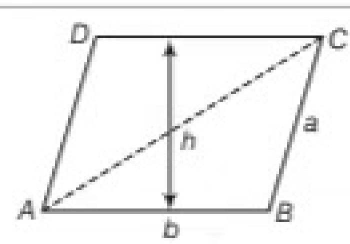
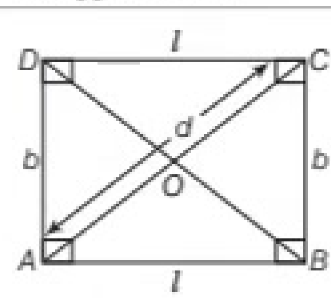
Perimeter The length of boundary of a simple closed figure is known as perimeter.

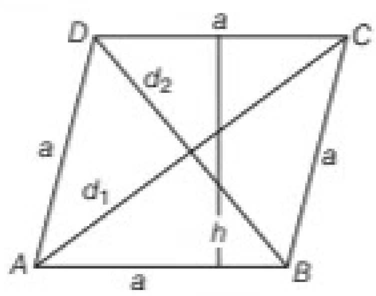
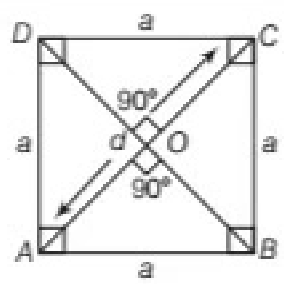
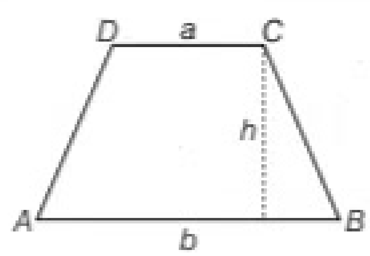
Area The measure of region enclosed in a simple closed curve is called area of closed curve.

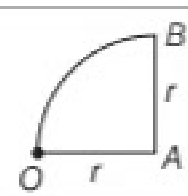
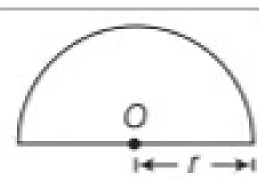
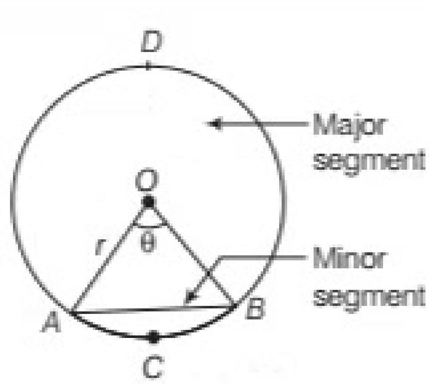
Perimeter and Area of Plane Figures

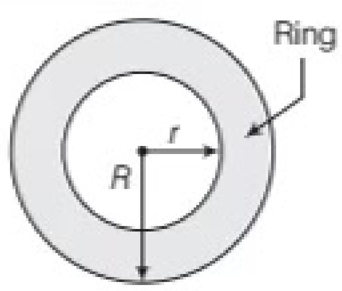
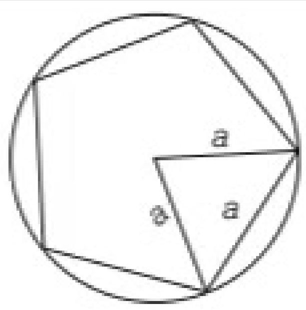
	Type	Figure	Perimeter (P)	Area (A)
1	Triangle		$P = a + b + c$ $= 2s$	$A = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} bh$ $= \sqrt{s(s-a)(s-b)(s-c)}$
2	Isosceles Triangle		$P = 2a + b$	$A = \frac{b}{4}\sqrt{4a^2 - b^2}$

	Type	Figure	Perimeter (P)	Area (A)
3	Equilateral Triangle		$P = 3a$	$A = \frac{\sqrt{3}}{4}a^2$
4	Right Angled Triangle		$P = a + b + c = 2s$	$A = \frac{1}{2}bc$

	Type	Figure	Diagonal	Perimeter (P)	Area (A)
5	Quadrilateral	 Diagonal $AC = d$; DE, BF are two perpendiculars drawn on the diagonal (AC) and p_1, p_2 are lengths of the two perpendiculars	$BD \neq AC$	$P = AB + BC + CD + DA$	$A = \frac{1}{2} \times d \times (p_1 + p_2)$
6	Parallelogram	 b = base, h = perpendicular distance between the base and its opposite side	$BD \neq AC$	$P = 2(a + b)$	$A = b \times h = 2 \times$ (area of $\Delta\, ABD$) (or $\Delta\, BCD$)
7	Rectangle	 l = length, b = breadth d = diagonal	$BD = AC$ Also, $AO = OC = OD = OB$ and $d^2 = l^2 + b^2$	$P = 2(l + b)$	$A = l \times b$ $= l \times \sqrt{d^2 - l^2}$ $= b \times \sqrt{d^2 - b^2}$

	Type	Figure	Diagonal	Perimeter (P)	Area (A)
8	Rhombus		$BD \neq AC$ and $d_1^2 + d_2^2 = 4a^2$	$P = 4a = 2\sqrt{d_1^2 + d_2^2}$	$A = \frac{1}{2} \times d_1\ x d_2$ $= a \times h$
9	Square	 a = length of side d = diagonal	$BD = AC$ and $OA = OB$ $= OC = OD$ and $d = a\sqrt{2}$	$P = 4a = 2d\sqrt{2}$	$A = a^2 = \frac{d^2}{2}$
10	Trapezium			P = sum of all sides	$A = \frac{1}{2}(a + b) \times h$

	Type	Figure	Perimeter (or Circumference)	Area
11	Quadrant		$P = 2r + \frac{\pi r}{2}$	$A = \frac{1}{4}\pi r^2$
12	Semi-circle		$P = (\pi r + 2r)$	$A = \frac{1}{2}\pi r^2$
13	Circle		Circumference of circle, $P = 2\pi r = \pi d$ Length of arc $AB = 2\pi r \times \frac{\theta}{360°}$	$A = \pi r^2$

	Type	Figure	Perimeter (or Circumference)	Area
14	Ring		$P = 2\pi R + 2\pi r$	$A = \pi R^2 - \pi r^2$ $= \pi(R^2 - r^2)$
15	Hexagon inscribed in a circle		$P = 6(a)$	$A = 6 \times \frac{\sqrt{3}a^2}{4}$

Conversion of Units

$100\ mm^2 = 1\ cm^2$ $\quad 100\ cm^2 = 1\ dm^2$

$100\ dm^2 = 1\ m^2$ $\quad 10000\ cm^2 = 1\ m^2$

$1\ acre = 100\ m^2$ $\quad 1\ hectare = 10000\ m^2$

$1\ hectare = 100\ acres$ $\quad 100\ hectare = 1\ km^2$

Example 1 The base of a triangular field is three times its altitude. If the cost of cultivating the field at ₹ 240 per hectare ₹ 3240, find its base and height.

(a) 900 m, 300 m (b) 910 m, 310 m
(c) 905 m, 315 m (d) None of these

Sol. (a) Area of the field $= \frac{\text{Total cost}}{\text{Rate}}$

$= \frac{3240}{240}$ hectare

$= 13.5 \times 10000\ m^2$

$= 135000\ m^2$

Let the height be x metre.

Then, base = $3x$ metre

$\therefore \quad \frac{1}{2} \times x \times 3x = 135000$

$\Rightarrow \quad x^2 = 90000$

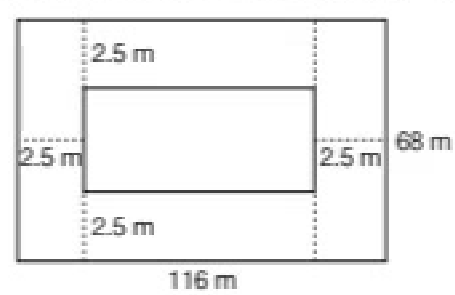
$\Rightarrow \quad x = 300$ m

Hence, base is 900 and height is 300 m.

Example 2 A rectangular grassy plot is 116 m by 68 m. It has a gravel path 2.5 m wide all round it on the inside. Find the area of the path.

(a) 895 m^2 (b) 900 m^2
(c) 910 m^2 (d) 890 m^2

Sol. (a) Area of the plot $= (116 \times 68) = 7888\ m^2$



Area of the plot excluding the path

$= (116 - 5) \times (68 - 5)$

$= 111 \times 63 = 6993\ m^2$

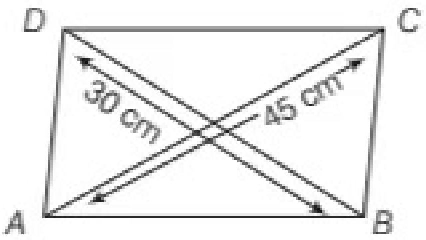
$\therefore$ Required area of path $= 7888 - 6993 = 895\ m^2$

Example 3 The floor of a building consists of 3000 tiles, which are rhombus shaped and each of its diagonals are 45 cm and 30 cm in length. Find the total cost of polishing the floor, if the cost per sq metre is ₹ 4.

(a) ₹ 800 (b) ₹ 810 (c) ₹ 820 (d) ₹ 850

Sol. (b) Area of one rhombus shaped tile $= \frac{1}{2} \times d_1 \times d_2$

$= \frac{1}{2} \times 45 \times 30 = 45 \times 15 = 675\ cm^2$



Now, floor of a building consists of 3000 tiles of such kind.

$\therefore$ Area of floor = Number of tiles × Area of one tile

$$= 3000 \times 675$$

$$= 2025000\text{cm}^2 = \frac{2025000}{10000}\text{m}^2$$

$$= 2025\text{ m}^2 \quad \left[\begin{array}{l}\because 1\text{ m} = 100\text{ cm} \\ \therefore 1\text{m}^2 = 10000\text{ cm}^2\end{array}\right]$$

$\because$ Rate of polishing the floor = ₹4 per m^2

$\therefore$ Total cost of polishing the floor = ₹ 4×202.5

= ₹810

Hence, the total cost of polishing the floor is ₹810.

Example 4 The area of trapezium is 450m^2 and the distance between two parallel sides is 10 m and one of the parallel side 15 m. Find the other parallel side.

(a) 75 m (b) 80 m
(c) 85 m (d) 90 m

Sol. (*a*) Given, one of the parallel sides of the trapezium, a=15 m and height (h) = 10 m

Let another side be b m.

Then, area of trapezium = 450 m^2

$\therefore$ Area of trapezium $= \frac{1}{2}h(a+b)$

$$\Rightarrow \quad 450 = \frac{1}{2} \times 10 \times (15+b)$$

$$\Rightarrow \quad \frac{450 \times 2}{10} = 15 + b$$

$$\Rightarrow \quad 90 = 15 + b$$

$$\therefore \quad b = 90 - 15 = 75$$

Hence, the other parallel side of trapezium is 75m.

Example 5 If the area of a semi-circular field is 30800 m^2, then find the perimeter of the field.

(a) 705 m (b) 710 m
(c) 700 m (d) 720 m

Sol. (*d*) Let the radius of the field be r.

Then, $\frac{1}{2}\pi r^2 = 30800$

$$\Rightarrow \quad \frac{1}{2} \times \frac{22}{7} \times r^2 = 30800$$

$$\Rightarrow \quad r^2 = 30800 \times 2 \times \frac{7}{22} = 19600$$

$$\Rightarrow \quad r = 140\text{ m}$$

$\therefore$ The perimeter of the field $= \pi r + 2r$

$$= \frac{22}{7} \times 140 + 2 \times 140$$

$$= 440 + 280 = 720\text{m}$$

Example 6. In a circle of radius 35 cm, an arc subtends an angle of 72° at the centre. Find the length of the arc and the area of the sector.

(a) 660 cm^2 (b) 700 cm^2 (c) 770 cm^2 (d) 750 cm^2

Sol. (c) Length of the arc $= \frac{2\pi r\theta}{360°}$

$$= 2 \times \frac{22}{7} \times 35 \times \frac{72}{360} = 44\text{ cm}$$

Area of the sector $= \frac{\pi r^2\theta}{360°}$

$$= \frac{22}{7} \times 35 \times 35 \times \frac{72}{360} = 770\text{ cm}^2$$

Example 7. A regular hexagon is inscribed in a circle of radius 5 cm. The area of regular hexagon inscribed in a circle is

(a) $\frac{75}{2}\text{ cm}^2$ (b) $75\sqrt{3}\text{ cm}^2$
(c) $\frac{75\sqrt{3}}{2}\text{ cm}^2$ (d) None of these

Sol. (c) Area of hexagon inscribed in a circle

$$6 \times \frac{\sqrt{3}}{4}(a)^2 = 6\frac{\sqrt{3}}{4} \times (5)^2 = \frac{75}{2}\sqrt{3}\text{ cm}^2$$

PRACTICE EXERCISE

1. The area of an equilateral triangle with side 10 cm is

(a) $15\sqrt{3}\text{ cm}^2$ (b) $25\sqrt{3}\text{ cm}^2$
(c) $5\sqrt{3}\text{ cm}^2$ (d) $35\sqrt{3}\text{ cm}^2$

2. An isosceles right angled triangle has area 200 cm^2. The length of its hypotenuse is

(a) $15\sqrt{2}$ cm (b) $\frac{10}{\sqrt{2}}$ cm
(c) $10\sqrt{2}$ cm (d) $20\sqrt{2}$ cm

3. The diagonal of a square field measures 50 m. The area of square field is

(a) 1250 m^2 (b) 1200 m^2
(c) 1205 m^2 (d) 1025 m^2

4. The sum of the length of two diagonals of a square is 144 cm, then the perimeter of square is

(a) 144 cm (b) $72\sqrt{2}$ cm
(c) $144\sqrt{2}$ cm (d) None of these

5. The circumference of a circle is 176 m. Then, its area is

(a) 2464 m^2 (b) 2164 m^2
(c) 2346 m^2 (d) 2246 m^2

6. The area of a rhombus whose one side and one diagonal measure 20 cm and 24 cm respectively, is

(a) 364 cm^2 (b) 374 cm^2
(c) 384 cm^2 (d) 394 cm^2

7. The area of the largest circle that can be drawn inside a square of side 14 cm in length, is

(a) 84 cm^2 (b) 96 cm^2
(c) 104 cm^2 (d) 154 cm^2

8. The least number of square slabs that can be fitted in a room 10.5 m long and 3 m wide, is

(a) 12 (b) 13
(c) 14 (d) 15

9. The length of a rectangle is 2 cm more than its breadth and the perimeter is 48 cm. The area of the rectangle (in cm^2) is

(a) 96 (b) 28
(c) 143 (d) 144

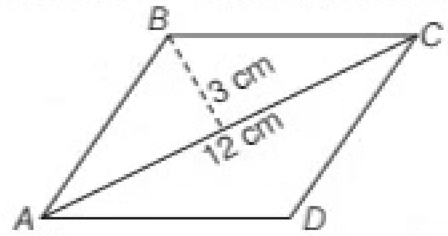
10. In a circle of radius 42 cm, an arc subtends an angle of 72° at the centre. The length of the arc is

(a) 52.8 cm (b) 53.8 cm
(c) 72.8 cm (d) 79.8 cm

11. If the side of a square be increased by 50%, then the per cent increase in area is

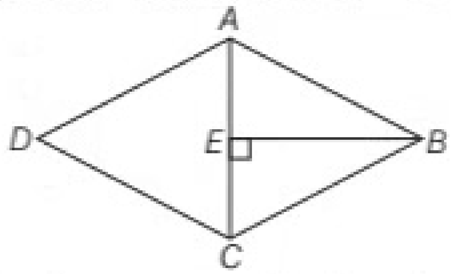
(a) 50 (b) 100 (c) 125 (d) 150

12. The figure $ABCD$ is a quadrilateral, in which $AB = CD$ and $BC = AD$. Its area is



(a) 72 cm^2 (b) 36 cm^2
(c) 24 cm^2 (d) 18 cm^2

13. What is the area of the rhombus $ABCD$ below, if $AC = 6$ cm and $BE = 4$ cm?



(a) 36 cm^2 (b) 16 cm^2
(c) 24 cm^2 (d) 13 cm^2

14. The area of a parallelogram is 60 cm^2 and one of its altitude is 5 cm. The length of its corresponding side is

(a) 12 cm (b) 6 cm (c) 4 cm (d) 2 cm

15. If the ratio of the areas of two square is $4:1$, then the ratio of their perimeter is

(a) 2 : 1 (b) 1 : 2
(c) 1 : 4 (d) 4 : 1

16. The inner circumference of a circular park is 440 m. The track is 14 m wide. The diameter of the outer circle of the track is

(a) 168 m (b) 169 m (c) 144 m (d) 108 m

17. A wire is in the form of a circle of radius 42 cm. It is bent into a square. The side of the square is

(a) 33 cm (b) 66 cm
(c) 78 cm (d) 112 cm

18. The perimeter of a trapezium is 52 cm and its each non-parallel side is equal to 10 cm with its height 8 cm. It area is
(a) 124 cm^2 (b) 118 cm^2
(c) 128 cm^2 (d) 112 cm^2

19. The areas of two circles are in the ratio 49 : 64. Find the ratio of their circumferences.
(a) 7 : 8 (b) 5 : 8 (c) 5 : 3 (d) 5 : 9

20. The diameter of the wheels of a bus is 140 cm. How many revolutions per minute must a wheel make in order to move at a speed of 66 km/h ?
(a) 200 (b) 250 (c) 300 (d) 350

21. The length of the sides of a triangle are in the ratio 3 : 4 : 5 and its perimeter is 144 cm. The area of the triangle is
(a) 684 cm^2 (b) 664 cm^2
(c) 764 cm^2 (d) 864 cm^2

22. The difference between the sides at right angles in a right angled triangle is 14 cm. The area of the triangle is 120 cm^2. The perimeter of the triangle is
(a) 68 cm (b) 64 cm
(c) 60 cm (d) 58 cm

23. The area of the quadrilateral whose sides measures 9 cm, 40 cm, 28 cm and 15 cm and in which the angle between the first two sides is a right angle, is
(a) 206 cm^2 (b) 306 cm^2
(c) 356 cm^2 (d) 380 cm^2

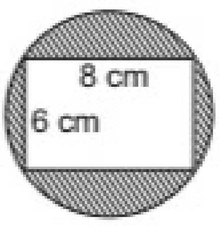
24. If three sides of a triangle are 6 cm, 8 cm and 10 cm, then the altitude of the triangle using the largest side as base will be
(a) 8 cm (b) 6 cm
(c) 4.8 cm (d) 4.4 cm

25. The area of a circle is 13.86 hectares. The cost of fencing it at the rate of 60 paise per metre is
(a) ₹ 784.00 (b) ₹ 788.00
(c) ₹ 792.00 (d) ₹ 796.00

26. If the diagonal of a rectangle is 13 cm and its perimeter is 34 cm, then its area will be
(a) 442 cm^2 (b) 260 cm^2
(c) 60 cm^2 (d) 20 cm^2

27. The cross-section of a canal is in the shape of a trapezium. The canal is 15 m wide at the top and 9 m wide at the bottom. The area of cross-section is 720 m^2, the depth of the canal is
(a) 58.4 m (b) 58.6 m
(c) 58.8 m (d) 60 m

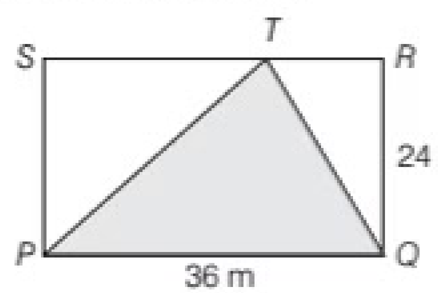
28. In the adjacent figure, find the area of the shaded region.



(a) 15.28 cm^2
(b) 61.14 cm^2
(c) 30.57 cm^2
(d) 40.76 cm^2

29. A square and an equilateral triangle have equal perimeters. If the area of the equilateral triangle is $16\sqrt{3}$ cm^2, then the side of the square is
(a) 4 cm (b) $4\sqrt{2}$ cm
(c) $6\sqrt{2}$ cm (d) 6 cm

30. find the figure area of the shaded portion. In the following figure,



(a) 433m^2 (b) 432 m^2
(c) 434 m^2 (d) None of these

31. A regular hexagon is inscribed in a circle of radius 8 cm. The perimeter of the regular hexagon is
(a) 48 cm (b) 50 cm
(c) 52 cm (d) 54 cm

32. Find the area of regular hexagon inscribed in a circle of radius 10 cm.
(a) $140\sqrt{3}$ cm^2 (b) $150\sqrt{3}$ cm^2
(c) $120\sqrt{3}$ cm^2 (d) None of these

Answers

1	*(b)*	2	*(d)*	3	*(a)*	4	*(c)*	5	*(a)*	6	*(c)*	7	*(d)*	8	*(c)*	9	*(c)*	10	*(a)*
11	*(c)*	12	*(b)*	13	*(c)*	14	*(a)*	15	*(a)*	16	*(a)*	17	*(b)*	18	*(c)*	19	*(a)*	20	*(b)*
21	*(d)*	22	*(c)*	23	*(b)*	24	*(c)*	25	*(c)*	26	*(c)*	27	*(d)*	28	*(c)*	29	*(d)*	30	*(b)*

Hints and Solutions

1. Area of equilateral triangle $= \frac{\sqrt{3}}{4}(\text{side})^2$

$$= \frac{\sqrt{3}}{4} \times 10 \times 10 = 25\sqrt{3}\ \text{cm}^2$$

2. Area of an isosceles right angled triangle

$$= \frac{1}{2}(\text{side})^2 = 200\ \text{cm}^2$$

$\therefore$ side $= 20$ cm

$\therefore$ Hypotenuse $= \sqrt{a^2 + a^2} = \sqrt{2}a = 20\sqrt{2}$ cm

3. Area of square $= \frac{1}{2} \times (\text{diagonal})^2$

$$= \frac{1}{2} \times 50 \times 50 = 1250\ \text{m}^2$$

4. Length of a diagonal of square $= \frac{144}{2} = 72$ cm

Side of square $= \frac{\text{length of diagonal}}{\sqrt{2}} = \frac{72}{\sqrt{2}}$ cm

$\therefore$ The perimeter of square $= 4a = 4 \times \frac{72}{\sqrt{2}}$

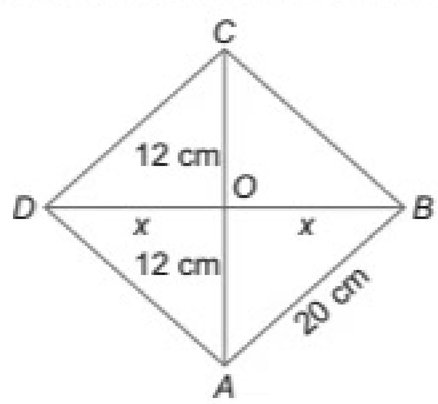
$$= 144\sqrt{2}\ \text{cm}$$

5. Circumference of circle $= 2\pi r = 176$ m

$\Rightarrow$ $r = \frac{176 \times 7}{2 \times 22} = 28$ m

$\therefore$ Area $= \pi r^2 = \frac{22}{7} \times 28 \times 28 = 2464\ \text{m}^2$

6. Let the other diagonal be $2x$. In ΔAOB,



$$(20)^2 = (12)^2 + x^2$$

$\Rightarrow$ $x^2 = 256$

$\Rightarrow$ $x = 16$ cm

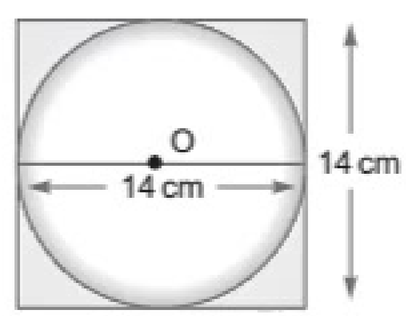
$\therefore$ Other diagonal $= 2x = 32$ cm

$\therefore$ Area $= \frac{1}{2} \times d_1 d_2$

$$= \frac{1}{2} \times 24 \times 32$$

$$= 384\ \text{cm}^2$$

7. Diameter of circle = side of square = 14 cm



$\Rightarrow$ $r = 7$ cm

$\therefore$ Area of circle, $\pi r^2 = \frac{22}{7} \times 7 \times 7 = 154\ \text{cm}^2$

8. Side of the greatest square tile = GCM of the length and breadth of the room

GCM of 10.5 and 3 = 1.5 m.

$\therefore$ Area of room $= 10.5 \times 3\ \text{m}^2$

$\therefore$ Number of tiles needed $= \frac{10.5 \times 3}{1.5 \times 1.5} = 14$ tiles

9. Let length $= x$ cm and breadth $= (x - 2)$ cm

$\therefore$ $2[x + (x - 2)] = 48$

$\Rightarrow$ $4x - 4 = 48$

$\Rightarrow$ $x = \frac{52}{4} = 13$ cm

$\therefore$ Length = 13 cm and breadth = 11 cm

Hence, area $= l \times b$

$$= 13 \times 11 = 143\ \text{cm}^2$$

10. Length of an arc

$$= 2\pi r \times \frac{\theta}{360°} = \frac{2 \times 22 \times 42 \times 72°}{7 \times 360°}$$

$$= \frac{264}{5} = 52.8 \text{ cm}$$

11. Let the original side of a square be 'a'.

$\therefore$ Area of square $= a^2$

Now, new side $= a + \frac{a}{2} = \frac{3a}{2}$

$\Rightarrow$ New area $= \frac{9a^2}{4}$

$\therefore$ Increase in area $= \frac{9a^2}{4} - a^2 = \frac{5a^2}{4}$

$\therefore$ Percent increase in area

$$= \frac{5a^2}{4a^2} \times 100 = 125\%$$

12. It is clear from the figure that, quadrilateral $ABCD$ is a parallelogram. The diagonal AC of the given parallelogram $ABCD$ divides it into two triangles of equal areas.

Area of the $\Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$

$$= \frac{1}{2} \times 12 \times 3 = 18 \text{ cm}^2$$

$\therefore$ Area of the parallelogram $ABCD$

$$= 2 \times \text{Area of } \Delta ABC$$

$$= 2 \times 18 = 36 \text{ cm}^2$$

13. The diagonal AC of the rhombus $ABCD$ divides it into two triangles of equal areas.

Now, area of $\Delta ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$

$$= \frac{1}{2} \times 4 \times 6 = 12 \text{ cm}^2$$

$\therefore$ Area of the rhombus $ABCD = 2 \times$ Area of ΔABC

$$= 2 \times 12 = 24 \text{ cm}^2$$

14. We know that,

Area of a parallelogram = Side $\times$ Altitude

$\Rightarrow$ $a \times h = 60 \Rightarrow a \times 5 = 60$

$\Rightarrow$ $a = \frac{60}{5}$

$\therefore$ $a = 12 \text{ cm}$

15. Let the sides of the two square be a and b.

$\therefore$ Ratio of their areas

$$\frac{a^2}{b^2} = \frac{4}{1}$$

or $\left(\frac{a}{b}\right)^2 = \left(\frac{2}{1}\right)^2$

$\Rightarrow$ $\frac{a}{b} = \frac{2}{1}$

$\therefore$ $a : b = 2 : 1$

16. Inner circumference of a park

$$= 2\pi r = 440 \text{ m}$$

$\Rightarrow$ $r = \frac{440}{2 \times 22} \times 7 = 70 \text{ m}$

Width of track $= 14$ m

$\Rightarrow$ Radius of outer circle

$$= (70 + 14) = 84 \text{ m}$$

$\therefore$ Diameter of outer circle $= 2 \times 84 = 168$ m

17. Circumference of circle $= 2\pi r$

$$= 2 \times \frac{22}{7} \times 42 = 264 \text{ cm}$$

$\therefore$ Length of wire $= 264$ cm

Now, wire is bent into a square.

$\therefore$ Perimeter of square = length of wire = 264 cm

$\Rightarrow$ $4 \times$ side of square $= 264$

$\therefore$ Side of square $= \frac{264}{4} = 66$ cm

18. Given, perimeter of a trapezium is 52 cm and each non-parallel side is of 10 cm.

Then, sum of its parallel sides

$$= 52 - (10 + 10)$$

$= 52 - 20 = 32$ cm

$\therefore$ Area of the trapezium $= \frac{1}{2}(a + b) \times h$

$$= \frac{1}{2} \times 32 \times 8$$

$[\because h = 8 \text{ cm and } a + b = 32 \text{ cm}]$

$$= 128 \text{ cm}^2$$

19. Given, the area of two circles are in the ratio 49 : 64.

Area of a circle $= \pi r^2$

Let area of the first circle $= \pi r_1^2$

and area of the second circle $= \pi r_2^2$

According to the question, $\frac{49}{64} = \frac{\pi r_1^2}{\pi r_2^2}$

$\Rightarrow \quad \frac{49}{64} = \frac{r_1^2}{r_2^2}$

$\Rightarrow \quad \frac{(7)^2}{(8)^2} = \frac{r_1^2}{r_2^2} \Rightarrow \left(\frac{7}{8}\right)^2 = \left(\frac{r_1}{r_2}\right)^2$

$\therefore \quad r_1 = 7$ and $r_2 = 8$

The ratio of circumferences of these two circles

$$= \frac{2\pi r_1}{2\pi r_2} = \frac{r_1}{r_2} = \frac{7}{8}$$

[$\because$ circumference of circle $= 2\pi r$]

Hence, required ratio is 7 : 8.

20. Distance covered by wheel in one minute

$$= \left(\frac{66 \times 1000 \times 100}{60}\right) = 110000 \text{ cm}$$

Circumference of wheel

$$= \left(2 \times \frac{22}{7} \times 70\right) = 440 \text{ cm}$$

$\therefore$ Number of revolutions in 1 min

$$= \left(\frac{110000}{440}\right) = 250$$

21. Given, perimeter of triangle = 144 cm

$\therefore$ Sides of triangle are

$$a = \frac{3}{3+4+5} \times 144 = 36 \text{ cm}$$

and $b = 48$ cm,

$c = 60$ cm

Now, $s = \frac{a+b+c}{2} = \frac{36+48+60}{2}$

$= 72$ cm

$\therefore$ Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{72 \times 36 \times 24 \times 12} = 72 \times 12$

$= 864 \text{ cm}^2$

22. Let the sides containing right angled be x cm and $(x - 14)$ cm.

$\therefore$ Area $= \left[\frac{1}{2} x \times (x - 14)\right] \text{cm}^2$

$\Rightarrow \quad \frac{1}{2} x(x-14) = 120 \quad$ [$\because$ area $= 120 \text{ cm}^2$]

$\Rightarrow \quad x^2 - 14x - 240 = 0$

$\Rightarrow \quad (x-24)(x+10) = 0 \Rightarrow x = 24$

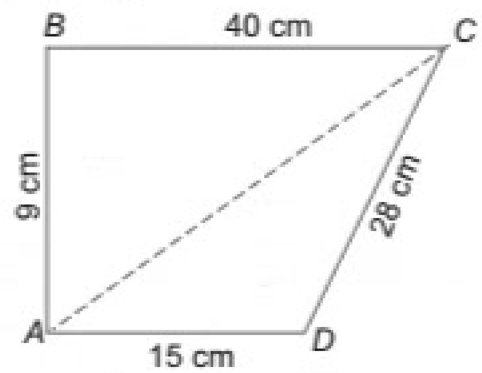
and $\quad x \neq -10$

Other side $= 24 - 14 = 10$ cm

$\Rightarrow$ Hypotenuse $= \sqrt{24^2 + 10^2} = \sqrt{676} = 26$ cm

$\therefore \quad$ Perimeter $= (24 + 10 + 26) = 60$ cm.

23. Applying Pythagoras theorem in ΔABC we get,



$$9^2 + 40^2 = AC^2$$

$\Rightarrow \quad AC = \sqrt{1681} = 41$ cm

$\therefore$ Area of quadrilateral

= area of ΔABC + area of ΔADC

$= \frac{1}{2}(9 \times 40) + \sqrt{42 \times 1 \times 14 \times 27}$

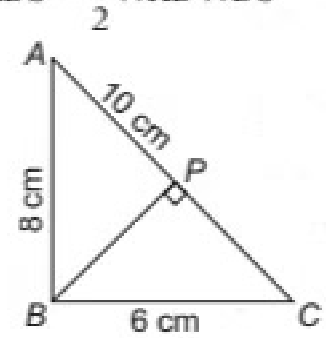
$\left[\because s = \frac{15+28+41}{2} = 42 \text{ cm}\right]$

and area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= 180 + 14 \times 3 \times 3$

$= 180 + 126 = 306 \text{ cm}^2$

24. Area of $\Delta ABC = \frac{1}{2} \times AB \times BC$



$\Rightarrow \quad \frac{1}{2} \times 8 \times 6 = \frac{1}{2} \times AC \times BP$

$\Rightarrow \quad \frac{1}{2}\times 48 = \frac{1}{2}\times 10\times BP$

$\Rightarrow \quad BP = 4.8\,\text{cm}$

25. Since, 1 hec $= 10000\ \text{m}^2$

Also, $\quad \pi r^2 = 13.86\times 10000$

$\Rightarrow \quad r = \sqrt{\frac{138600}{22}\times 7} = 210\ \text{m}$

Circumference of circle $= 2\pi r = 1320\text{m}$

$\therefore$ Total cost of fencing $= 1320\times 0.60 =$ ₹792

26. Since, $\quad 2(l+b) = 34 \Rightarrow l+b = 17$

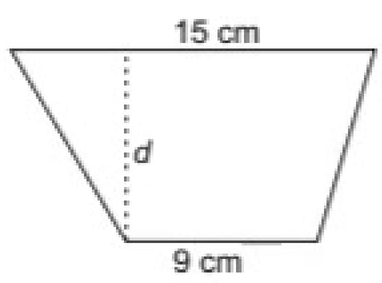
and $\sqrt{l^2+b^2} = 13 \Rightarrow l^2+b^2 = 169$

On solving, we get $l = 12$ and $b = 5$

$\therefore$ Area of rectangle $5 = l\times b$

$= 12\times 5 = 60\ \text{cm}^2$

27. Area of cross-section of canal $= \frac{1}{2}(15+9)\times d$



$\Rightarrow \quad 720 = \frac{1}{2}\times 24\times d$

$\Rightarrow \quad d = 60\,\text{m}$

28. Diameter of circle $= \sqrt{6^2+8^2} = 10\ \text{cm}$

$\therefore$ Area of cricle $= \pi\,(5)^2 = 25\pi\ \text{cm}^2$

$= 25\times\frac{22}{7} = 78.57\ \text{cm}^2$

and area of rectangle $= 8\times 6 = 48$ cm

$\therefore$ Shaded area $= 78.57 - 48$

$= 30.57\ \text{cm}^2$

29. Area of equilateral triangle $= \frac{\sqrt{3}}{4}a^2$

$\Rightarrow \quad \frac{\sqrt{3}}{4}a^2 = 16\sqrt{3} \Rightarrow a^2 = 64 \Rightarrow a = 8\ \text{cm}$

Since, perimeter of square = perimeter of an equilateral triangle

$\Rightarrow \quad 4x = 3\times 8 \quad \Rightarrow \quad x = 6\ \text{cm}$

30. Area of the shaded portion = Area of ΔPTQ

$\because$ Area of a triangle $= \frac{1}{2}\times$ Base $\times$ Height

So, in ΔPTQ, RQ = Height

$\therefore$ Area of $\Delta PTQ = \frac{1}{2}\times 36\times 24 = 18\times 24 = 432\ \text{m}^2$

31. The perimeter of regular hexagon

$= 6\times$radius of a circle $= 6\times 8 = 48$ cm

32. Area of regular hexagon inscribed in a circle

$= \frac{6\sqrt{3}}{4}(r)^2$

$= \frac{6\sqrt{3}}{4}\times(10)^2 = 150\sqrt{3}\ \text{cm}^2$

CHAPTER

13

SURFACE AREA AND VOLUME

This chapter is very important for entrance. In this chapter, we study the surface area and volume of various solid figures such as cuboid, cube, cylinder, cone, sphere etc.

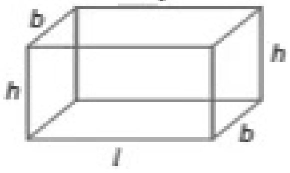
Any figure bounded by one or more surfaces is called a **solid figure**. Hence, a solid figure must have length, breadth (width) and thickness (depth or height).

Surface Area The surface area is the sum of all the areas of all the shapes that cover the surface of the object.

Volume The amount of space occupied by a solid is called its volume.

Surface Area and Volume of Plane Figures

Cuboid (Rectangular Solid)



Let length, breadth and height are respectively l, b and h.

Total number of faces = 6

Surface Area $(SA) = 2(bh + lh)$

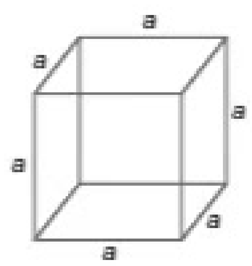
Base Area $(B) = lb$

Total $(C + 2B) = 2(bh + lh + lb)$

Volume $(V) = l \times b \times h = \sqrt{A_1 A_2 A_3}$

where, A_1, A_2 and A_3 are areas of base, side and end face respectively.

Cube



It has six equal faces of each side (edge) a.

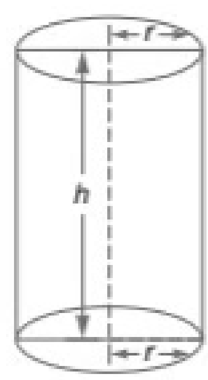
Surface Area $(SA) = 4a^2$

Base Area $(B) = a^2$

Total $(C + 2B) = 6a^2$

Volume $(V) = a^3$

Cylinder



Let the radius of base and height be respectively r and h.

Surface Area (SA)

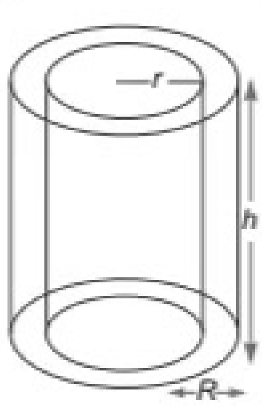
$= \text{(base perimeter)} \times \text{(height)} = 2\pi rh$

Base Area $(B) = \pi r^2$

Total $(C + 2B) = 2\pi r(h + r)$

Volume $(V) = \pi r^2 h$

Hollow cylinder



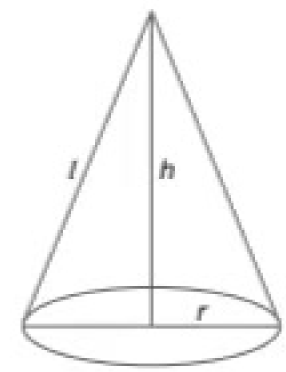
Surface Area $(SA) = 2\pi Rh + 2\pi rh$

Base Area $(B) = \pi(R^2 - r^2)$

Total $(C + 2B) = 2\pi h(R + r) + 2\pi(R^2 - r^2)$

Volume $(V) = \pi(R^2 - r^2)h$

Cone (Right Circular)



Let the radius of base, altitude and slant height be respectively r, h and l.

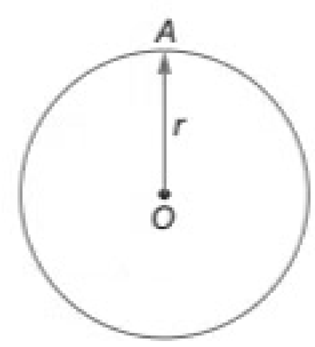
Surface Area $(SA) = \pi rl$, where $l = \sqrt{h^2 + r^2}$

Base Area $(B) = \pi r^2$

Total $(C + 2B) = \pi r(l + r)$

Volume $(V) = \frac{1}{3}$ (Base area) $\times$ Altitude $= \frac{1}{3}\pi r^2 \times h$

Sphere



Let radius of sphere be r.

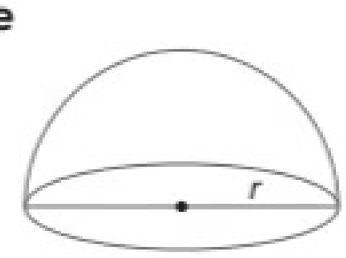
Surface Area $(SA) = 4\pi r^2$

Base Area $(B) =$ No Base

Total $(C + 2B) = 4\pi r^2$

Volume $(V) = \frac{4}{3}\pi r^3$

Hemisphere



Surface Area $(SA) = 2\pi r^2$

Base Area $(B) = \pi r^2$

Total $(C + 2B) = 3\pi r^2$

Volume $(V) = \frac{2}{3}\pi r^3$

Example 1 The volume and surface area of a cube sides measures 4 cm are respectively

(a) 64 cm^3, 96 cm^2 (b) 64 cm^3, 80 cm^2
(c) 64 cm^3, 90 cm^2 (d) 60 cm^3, 96 cm^2

Sol. *(a)* Volume $= a^3 = (4)^3 = 64 \text{ cm}^3$

and surface area $= 6a^2$

$$= 6 \times 4 \times 4 = 96 \text{ cm}^2$$

Example 2 How many bricks each measuring 25 cm × 11.5 cm × 6 cm will be needed to construct a wall 8 m long, 6 m high and 22.5 cm thick ?

(a) 6262 (b) 6260 (c) 6624 (d) 6520

Sol. *(b)* Number of bricks required

$$= \frac{\text{Volume of wall in cm}^3}{\text{Volume of 1 brick in cm}^3}$$

$$= \frac{800 \times 600 \times 22.5}{25 \times 11.5 \times 6} = 6260$$

Example 3 If the radius of a cylinder is increased from 6 cm to 14 cm and the surface area of it kept same. If its height is 5 cm, what will be its new height?

(a) $\frac{15}{7}$ cm (b) $\frac{15}{8}$ cm
(c) $\frac{17}{7}$ cm (d) None of these

Sol. *(a)* When $r = 6$ cm and $h = 5$ cm, then surface area of cylinder $S_1 = 2\pi rh$

$$= 2\pi \times 6 \times 5 = 60\pi \text{ cm}^2$$

When $r_1 = 14$ cm and height h_1 cm, then the surface area of cylinder

$$S_2 = 2\pi r_1 h_1 = 2\pi \times 14 h_1$$

According to the given condition,

$$S_1 = S_2$$

$$\Rightarrow \quad 60\pi = 28\pi h_1$$

$$\Rightarrow \quad h_1 = \frac{60}{28} = \frac{15}{7} \text{ cm}$$

Example 4 The volume and curved surface area of a cylinder of length 60 cm with diameter of the base 7 cm are respectively

(a) 2310 cm^3, 1320 cm^2 (b) 2410 cm^3, 1320 cm^2
(c) 2310 cm^3, 1350 cm^2 (d) 2410 cm^3, 1350 cm^2

Sol. *(a)* Volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 3.5 \times 3.5 \times 60 = 2310 \text{ cm}^3$$

and curved surface area $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 3.5 \times 60 = 1320 \text{ cm}^2$$

Example 5 The slant height, volume and curved surface area of a cone of base radius 21 cm and height 28 cm are respectively

(a) 35 cm 12936 cm^3, 2310 cm^2
(b) 35 cm 12930 cm^3, 2320 cm^2
(c) 36 cm 12940 cm^3, 2325 cm^2
(d) None of the above

Sol. *(a)* Slant height, $l = \sqrt{r^2 + h^2} = \sqrt{(21)^2 + (28)^2}$

$$= \sqrt{1225} = 35 \text{ cm}$$

Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 21 \times 21 \times 28 = 12936 \text{ cm}^3$$

and curved surface area of cone $= \pi rl$

$$= \frac{22}{7} \times 21 \times 35 = 2310 \text{ cm}^2$$

Example 6 A circus tent is cylindrical to a height of 4 m and conical above it. If its diameter is 105 m and its slant height is 40 m. The total area of the canvas required (in m^2) is

(a) 7928 m^2 (b) 7920 m^2
(c) 7923 m^2 (d) None of these

Sol. *(b)* Total area of canvas $= (2\pi rh + \pi rl)$

$$= \left(2 \times \frac{22}{7} \times \frac{105}{2} \times 4 + \frac{22}{7} \times \frac{105}{2} \times 40\right)$$

$$= 1320 + 6600 = 7920 \text{ m}^2$$

Example 7 The volume and total surface area of a hemisphere of diameter 21 cm are respectively

(a) 2420 cm^3, 1038 cm^2
(b) 2422 cm^3, 1039 cm^2
(c) 2425.5 cm^3, 1039.5 cm^2
(d) None of the above

Sol. (c) Volume of hemisphere $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}$$

$$= 2425.5 \text{ cm}^3$$

and total surface area of hemisphere

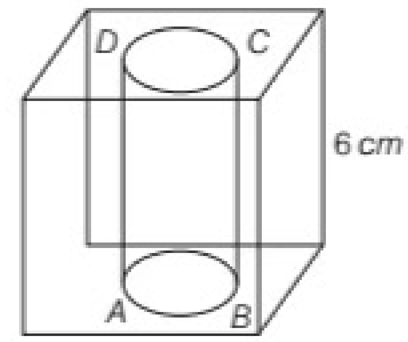
$$= 3\pi r^2 = 3 \times \frac{22}{7} \times 10.5 \times 10.5$$

$$= 1039.5 \text{ cm}^2$$

Example 8 Find the volume of cylinder which is exactly fit into a cube of side 6 cm.

(a) 171 cm^3 (b) 169.71 cm^3
(c) 173 cm^3 (d) None of the above

Sol. (*b*)



Here, height of cylinder = 6 cm

Radius of cylinder $= \frac{6}{2} = 3$ cm

Now, volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times (3)^2 \times 6$$

$$= \frac{1188}{7} = 169.71 \text{ cm}^3$$

PRACTICE EXERCISE

1. The volume of a cuboid is 440 cm^3, the area of its base is 88 cm^2, then its height is
(a) 5 cm (b) 10 cm (c) 11 cm (d) 6 cm

2. The surface area of a cube is 486 sq m, then its volume is
(a) 729 m^3 (b) 781 m^3 (c) 625 m^3 (d) 879 m^3

3. Rectangular sand box is 5 m wide and 2 m long. How many cubic metres of sand are needed to fill the box upto a depth of 10 cm?
(a) 1 m^3 (b) 10 m^3
(c) 100 m^3 (d) 1000 m^3

4. If the height of a cylinder becomes 1/4 of the original height and the radius is doubled, then which of the following will be true?
(a) Volume of the cylinder will be doubled
(b) Volume of the cylinder will remain unchanged
(c) Volume of the cylinder will be halved
(d) Volume of the cylinder will be $\frac{1}{4}$ of the original volume

5. A cube whose side is 5 cm will have surface area is equal to
(a) 125 cm^2 (b) 50 cm^2
(c) 100 cm^2 (d) None of these

6. The maximum length of a pencil that can be kept in a rectangular box of dimensions 8 cm × 6 cm × 2 cm is
(a) $2\sqrt{54}$ cm (b) $2\sqrt{26}$ cm
(c) $2\sqrt{14}$ cm (d) $2\sqrt{13}$ cm

7. The sum of the length, breadth and depth of a cuboid is 20 cm and its diagonal is $4\sqrt{5}$ cm, then its surface area is
(a) 400 cm^2 (b) 420 cm^2
(c) 300 cm^2 (d) 320 cm^2

8. How many 6 m cubes can be cut from a cuboid measuring 18 m × 15 m × 8 m ?
(a) 8 (b) 9 (c) 10 (d) 7

9. The ratio of radii of two cylinders is 1 : 2 and heights are in the ratio 2 : 3. The ratio of their volumes is
(a) 1 : 6 (b) 1 : 9 (c) 1 : 3 (d) 2 : 9

10. Two cubes have volumes in the ratio 1 : 64. The ratio of the areas of a face of first cube to that of the other is
(a) 1 : 4 (b) 1 : 8 (c) 1 : 16 (d) 1 : 32

11. If the volumes of two cubes are in the ratio 8 :1, then ratio of their edges is
(a) 2:1 (b) 4:1 (c) $2\sqrt{2}:1$ (d) 8:1

12. The total surface area of a right circular cylinder whose height is 15 cm and the radius of the base is 7 cm, is
(a) 968 cm^2 (b) 2310 cm^2
(c) 488 cm^2 (d) 1860 cm^2

13. The outer dimensions of a closed wooden box are 10 cm × 8 cm × 7 cm. Thickness of the wood is 1 cm. The total cost of wood required to make box, if 1 cm^3 of wood cost ₹ 2 is
(a) ₹ 540 (b) ₹ 640 (c) ₹ 740 (d) ₹ 780

14. The diameter of a right circular cone is 12 m and the slant height is 10 m. The total surface area of cone is
(a) $\frac{2412}{7}$ m^2 (b) $\frac{2312}{7}$ m^2
(c) $\frac{2112}{7}$ m^2 (d) $\frac{2012}{7}$ m^2

15. The dimensions of a field are 12 m × 10 m. A pit 5 m long, 4 m wide and 2 m deep is dug in one corner of the field and the earth removed has been evenly spread over the remaining area of the field. The level of the field is raised by
(a) 30 cm (b) 35 cm (c) 38 cm (d) 40 cm

16. A plate of metal 1 cm thick, 9 cm broad, 81 cm long is melted into a cube. The difference in the surface area of two solids is
(a) 1152 cm^2 (b) 1150 cm^2
(c) 1052 cm^2 (d) 1050 cm^2

17. The sum of the radius of the base and the height of a cylinder is 37 m. If the total surface area of the solid cylinder is 1628 m^2. The circumference of base of cylinder is
(a) 11 m (b) 22 m (c) 33 m (d) 44 m

18. The volume of a metallic cylindrical pipe is 770 cm^3. Its length is 14 cm and its external radius is 9 cm. Then, its thickness is
(a) 1 cm (b) 1.5 cm
(c) 2 cm (d) 2.5 cm

19. A 20 m deep well with diameter 14 m is dug up and the earth from digging is spread evenly to form a platform 22m ×14 m. The height of platform is
(a) 10 m (b) 15 m
(c) 20 m (d) 25 m

20. The circumference of the base of a 9 m high wooden solid cone is 44 m. The slant height of the cone is
(a) $\sqrt{120}$ m (b) $\sqrt{130}$ m
(c) $\sqrt{150}$ m (d) $7\sqrt{5}$ m

21. How many metres of cloth 50 m wide will be required to make a conical tent, the radius of whose base is 7 m and whose height is 24 m ?
(a) 9 m (b) 11 m (c) 12 m (d) 13 m

22. It is required to make a hollow cone 24 cm high whose base radius is 7 cm. The area of sheet required including the base is
(a) 700 cm^2 (b) 704 cm^2
(c) 708 cm^2 (d) 710 cm^2

23. The radius of a sphere whose surface area is 154 cm^2, is
(a) 3.5 cm (b) 3.6 cm
(c) 3.7 cm (d) None of these

24. A hemispherical bowl made of brass has inner diameter 10.5 cm. The cost of tin plating it on the inside at the rate of ₹ 16 per 100 cm^2 is
(a) ₹ 28 (b) ₹ 27.72
(c) ₹29.27 (d) ₹28.52

25. If the volume of a sphere is double that of the other sphere, then the ratio of their radii is
(a) $2\sqrt{2}:1$ (b) $\sqrt[3]{2}:1$
(c) $1:\sqrt[3]{2}$ (d) 2:1

26. The internal and external diameters of a hollow hemispherical vessel are 24 cm and 25 cm respectively. The total area to be painted is

(a) $\frac{13211}{7}$ cm^2 (b) $\frac{26961}{14}$ cm^2
(c) $\frac{6961}{14}$ cm^2 (d) $\frac{16951}{14}$ cm^2

27. Three cubes each of side 10 cm are joined end to end. The surface area of the resultant figure is

(a) 1400 cm^2 (b) 1500 cm^2
(c) 1450 cm^2 (d) 1550 cm^2

28. Height of a solid cylinder is 10 cm and diameter 8 cm. Two equal conical holes have been made from its both ends. If the diameter of the hole is 6 cm and height 4 cm. The volume of remaining portion is

(a) 24π cm^3 (b) 36π cm^3
(c) 72π cm^3 (d) 136π cm^3

29. The length, breadth and height of a room are in the ratio of 3 : 2 : 1. If its volume be 1296 m^3, its breadth is

(a) 12 m (b) 18 m
(c) 16 m (d) 24 m

30. The diameters of two cones are equal. If their slant height be in the ratio 5 : 7, the ratio of their curved surface areas is

(a) 25 : 7 (b) 25 : 49
(c) 5 : 49 (d) 5 : 7

31. A rectangular paper 11 cm by 8 cm can be exactly wrapped to cover the curved surface of a cylinder of height 8 cm. The volume of the cylinder is

(a) 66 cm^3 (b) 77 cm^3
(c) 88 cm^3 (d) 12 cm^3

32. A cylindrical tube open at both ends is made of metal. The internal diameter of the tube is 11.2 cm and its length is 21 cm. The metal everywhere is 0.8 cm thick. The volume of the metal is

(a) 316 cm^3 (b) 310 cm^3
(c) 306.24 cm^3 (d) 280.52 cm^3

33. A solid right circular cylinder of radius 8 cm and height 2 cm is melted into a right circular cone with radius of the base 8 cm. Its height is

(a) 5 cm (b) 6 cm
(c) 5.75 cm (d) 6.25 cm

34. A hemispherical bowl is made from a metal sheet having thickness 0.3 cm. The inner radius of the bowl is 24.7 cm. The cost of polishing its outer surface at the rate of ₹ 4 per 100cm^2 is (take $\pi = 3.14$)

(a) ₹ 159 (b) ₹ 157
(c) ₹ 160 (d) ₹ 165

35. If the radius of a cylinder is increased from 7 m to 10 m and the surface area of it kept same. If its height is 4 m, then new height will be

(a) 2.8 m (b) 3.1 m
(c) 3.6 m (d) 3.3 m

36. Find the volume of the cone which is exactly fit in the cube of side 8 cm.

(a) 133 cm^3 (b) 134 cm^3
(c) 135 cm^3 (d) None of these

Answers

1	(a)	2	(a)	3	(a)	4	(b)	5	(c)	6	(b)	7	(d)	8	(c)	9	(a)	10	(c)
11	(a)	12	(a)	13	(b)	14	(c)	15	(d)	16	(a)	17	(d)	18	(a)	19	(a)	20	(b)
21	(b)	22	(b)	23	(a)	24	(b)	25	(b)	26	(b)	27	(a)	28	(d)	29	(a)	30	(d)
31	(b)	32	(c)	33	(b)	34	(b)	35	(a)	36	(c)								

Hints and Solutions

1. Height $= \dfrac{\text{Volume of the cuboid}}{\text{Area of its base}}$

$$= \frac{440}{88} = 5\,\text{cm}$$

2. Let edge of cube be a.

Surface area of the cube $= 6a^2$

$\therefore \quad 6a^2 = 486$

$\Rightarrow \quad a^2 = 81$

$\Rightarrow \quad a = 9$

$\therefore$ Volume of the cube $= (\text{edge})^3$

$$= (9)^3 = 729\ \text{m}^3$$

3. Sand needed to fill the tank

$$= \left(5 \times 2 \times \frac{10}{100}\right) = 1\ \text{m}^3$$

4. We know that, the volume of a cylinder having base radius r and height h is $V = \pi r^2 h$

Now, if new height is $\frac{1}{4}$th of the original height and the radius is doubled, i.e.

$h' = \frac{1}{4}h$ and $r' = 2r$, then

New volume, $V' = \pi(2r)^2 \times \frac{1}{4}h = 4\pi r^2 \times \frac{1}{4}h$

$$= \pi r^2 h = V$$

Hence, the new volume of cylinder is same as the original volume.

5. Now, surface area of cube $= 4(\text{side})^2$

$$= 4 \times (5)^2 = 100\,\text{cm}^2$$

6. Length of longest pencil

$= \text{diagonal of the box}$

$= \sqrt{8^2 + 6^2 + 2^2} = \sqrt{104} = 2\sqrt{26}$ cm

7. Given, $\quad l + b + h = 20\,\text{cm}$

and $\quad \sqrt{l^2 + b^2 + h^2} = 4\sqrt{5}$

$\therefore$ Surface area $= 2\,(lb + bh + hl)$

$= (l + b + h)^2 - (l^2 + b^2 + h^2)$

$= (20)^2 - (4\sqrt{5})^2$

$= 400 - 80 = 320\ \text{cm}^2$

8. Volume of cube $= 6 \times 6 \times 6\ \text{m}^3$

Volume of cuboid $= 18 \times 15 \times 8\ \text{m}^3$

$\therefore$ Required number of cube

$$= \frac{\text{Volume of cuboid}}{\text{Volume of cube}}$$

$$= \frac{18 \times 15 \times 8}{6 \times 6 \times 6} = 10$$

9. Let r_1, r_2 be radii of two cylinders and h_1, h_2 be their heights.

Then, $\quad \dfrac{r_1}{r_2} = \dfrac{1}{2}$ and $\dfrac{h_1}{h_2} = \dfrac{2}{3}$

$$\therefore \quad \frac{V_1}{V_2} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2} = \left(\frac{1}{2}\right)^2 \times \frac{2}{3}$$

$$= \frac{1}{4} \times \frac{2}{3} = \frac{1}{6} = 1:6 = \frac{1}{4} \times \frac{2}{3} = \frac{1}{6} = 1:6$$

10. Let a and b be the edges of the two cubes, respectively.

Then, according to the question,

$a^3 : b^3 = 1 : 64 \quad [\because \text{volume of cube} = (\text{edge})^3]$

$\Rightarrow \quad \dfrac{a^3}{b^3} = \dfrac{1}{64}$

$\Rightarrow \quad \left(\dfrac{a}{b}\right)^3 = \left(\dfrac{1}{4}\right)^3$

$\Rightarrow \quad \dfrac{a}{b} = \dfrac{1}{4} \quad$ [taking cube roots on both sides]

Now, ratio of areas, $\left(\dfrac{a}{b}\right)^2 = \left(\dfrac{1}{4}\right)^2$

$[\because \text{surface area of cube} = 6 \times (\text{edge})^2]$

$\Rightarrow \quad \dfrac{a^2}{b^2} = \dfrac{1}{16}$

$\therefore \quad a^2 : b^2 = 1:16$

11. Let the edges of cubes be x and y, then volumes are x^3 and y^3 respectively.

$\therefore \quad \dfrac{x^3}{y^3} = \dfrac{8}{1}$

$\Rightarrow \quad \dfrac{x}{y} = \dfrac{2}{1}$

12. Total surface area of right circular cylinder

$$= 2\pi r\,(h + r) = 2 \times \frac{22}{7} \times 7\,(15 + 7)$$
$$= 2 \times 22 \times 22 = 968 \text{ cm}^2$$

13. External volume of the box

$$= 10 \times 8 \times 7 = 560 \text{ cm}^3$$

Internal dimensions as thickness of wood

$= 1$ cm

Internal length $= 10 - 2 = 8$ cm,

breadth $= 8 - 2 = 6$ cm, height $= 7 - 2 = 5$ cm

$\therefore$ Internal volume $= 8 \times 6 \times 5 = 240 \text{ cm}^2$

$\Rightarrow$ Volume of wood

= External volume – Internal volume

$= 560 - 240 = 320 \text{ cm}^3$

$\therefore$ Total cost of wood required to make the box

$= 320 \times 2 =$ ₹ 640

14. Total surface area $= \pi r\,(l + r)$

$$= \frac{22}{7} \times 6 \times (10 + 6) = \frac{2112}{7} \text{ m}^2$$

15. Area of the field = Length × Breadth

$= 12 \times 10 = 120 \text{ m}^2$

Area of the pit's surface $= 5 \times 4 = 20 \text{ m}^2$

Area on which the earth is to be spread

$= 120 - 20 = 100 \text{ cm}^2$

Volume of earth dug out $= 5 \times 4 \times 2 = 40 \text{ m}^3$

$\therefore$ Level of field raised $= \dfrac{40}{100} = \dfrac{2}{5}$ m

$$= \frac{2}{5} \times 100 = 40 \text{ cm}$$

16. Let the edge of the cube be 'x'.

Then, volume of the cube is

$$x^3 = 9 \times 81 \times 1 \text{ cm}^3 = 729 \text{ cm}^3$$

$\therefore$ $x = \sqrt[3]{729} = 9$ cm

Surface area of metal plate

$= 2\,(81 \times 9 + 9 \times 1 + 1 \times 81) = 2 \times (819)$

$= 1638 \text{ cm}^2$

Total surface area of the cube

$= 6\,(\text{edge})^2 = 6\,(9)^2 = 486 \text{ cm}^2$

$\therefore$ Difference of surface area of two solids

$$= 1638 - 486 = 1152 \text{ cm}^2$$

17. Given, $r + h = 37$ m

and total surface area $= 1628 \text{ m}^2$

$= 2\pi r\,(h + r) = 1628 \text{ m}^2 \Rightarrow 2\pi r\,(37) = 1628$

$\Rightarrow$ $r = \dfrac{1628 \times 7}{2 \times 22 \times 37} = 7$

$\therefore$ Circumference of its base $= 2\pi r$

$$= 2 \times \frac{22}{7} \times 7 = 44 \text{ m}$$

18. External radius, $R = 9$ cm

Internal radius, r cm.

Length of pipe $= 14$ cm

Since, volume of pipe $= 770 \text{ cm}^3$

$\Rightarrow$ Volume of hollow cylinder $= 770$

$\therefore$ $r(R^2 - r^2)h = 770$

$\Rightarrow$ $81 - r^2 = \dfrac{770 \times 7}{22 \times 14} = 17$

$\Rightarrow$ $r^2 = 64 \Rightarrow r = 8$ cm

$\therefore$ Thickness $= R - r = 9 - 8 = 1$ cm

19. Volume of earth dug out from the well

$$\pi r^2 h = \frac{22}{7} \times \left(\frac{14}{2}\right)^2 \times 20$$
$$= 22 \times 7 \times 20 \text{ m}^3$$

Let height of platform be h m.

$\therefore$ Volume of platform $= 22 \times 14 \times h$

$\Rightarrow$ $22 \times 14 \times h = 22 \times 7 \times 20$

$\Rightarrow$ $h = \dfrac{22 \times 7 \times 20}{22 \times 14} = 10$ m

20. Since, circumference of cone $= 44$ m

$\Rightarrow$ $2\pi r = 44 \Rightarrow r = \dfrac{44}{2\pi} = 7$m

$\therefore$ Slant height $= \sqrt{r^2 + h^2} = \sqrt{49 + 81}$

$$= \sqrt{130} \text{ m}^3$$

21. Slant height $= \sqrt{r^2 + h^2} = \sqrt{24^2 + 7^2}$

$$= \sqrt{576 + 49}$$
$$= \sqrt{625} = 25$$

Curved surface area $= \pi rl = \frac{22}{7} \times 7 \times 25 = 550\ \text{m}^2$

Since, width of cloth = 50 m

$\therefore$ Length of required cloth $= \frac{550}{50} = 11$ m

22. Given, $h = 24$ cm, $r = 7$ cm

Now, slant height

$$l = \sqrt{r^2 + h^2}$$
$$= \sqrt{24^2 + 7^2} = \sqrt{576 + 49}$$
$$= \sqrt{625} = 25 \text{ cm}$$

$\therefore$ Area of metal sheet required

= Total surface area of cone

$$= \pi r (l + r) = \frac{22}{7} \times 7 (7 + 25)$$
$$= 22 (32) = 704\ \text{cm}^2$$

23. Let the radius of the sphere be r cm.

Surface area of the sphere = 154 cm²

$\therefore \quad 4\pi r^2 = 154$

[$\because$ surface area of a sphere $= 4\pi r^2$]

$\Rightarrow \quad 4 \times \frac{22}{7} \times r^2 = 154$

$\Rightarrow \quad r^2 = \frac{154 \times 7}{22 \times 4} = 12.25$

$\Rightarrow \quad r = \sqrt{12.25} = 3.5$ cm

Hence, the radius of the sphere is 3.5 cm.

24. We have, inner diameter = 10.5 cm

$\therefore$ Inner radius $(r) = \frac{10.5}{2} = 5.25$ cm

Curved surface area of hemispherical bowl of inner side $= 2\pi r^2 = 2 \times \frac{22}{7} \times (5.25)^2$

$$= 2 \times \frac{22}{7} \times 5.25 \times 5.25$$
$$= 173.25\ \text{cm}^2$$

$\because$ Cost of tin plating on inside for 100 cm²

= ₹16

$\therefore$ Cost of tin plating on the inside for 173.25 cm²

$$= \frac{16 \times 173.25}{100} = ₹\ 27.72$$

25. Let r_1 and r_2 be the radii and V_1, V_2 be its volume of spheres respectively.

Given, $V_1 = 2V_2 \Rightarrow \frac{V_1}{V_2} = 2$

$$\therefore \quad \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{2}{1} \Rightarrow \frac{r_1^3}{r_2^3} = \frac{2}{1}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{\sqrt[3]{2}}{1}$$

26. Internal radius $(r) = 12$ cm

and external radius $(R) = \frac{25}{2}$ cm

$\therefore$ Area to be painted

= Internal area + External area + Area of edge

$2\pi r^2 + 2\pi R^2 + \pi (R^2 - r^2)$

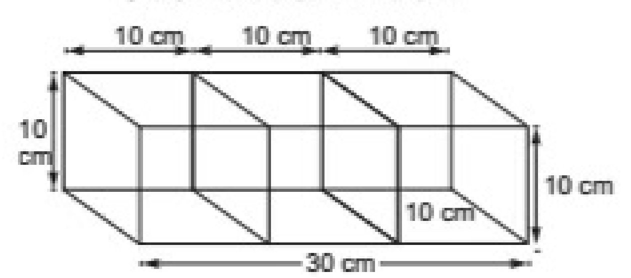
$$= 2 \times \frac{22}{7} \times 12 \times 12 + 2 \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2} + \frac{22}{7}\left(\frac{25}{2} \times \frac{25}{2} - 12 \times 12\right)$$

$$= \frac{6336}{7} + \frac{6875}{7} + \frac{539}{14}$$

$$= \frac{26422}{14} + \frac{539}{14} = \frac{26961}{14}\ \text{cm}^2$$

27. If three cubes each of side 10 cm are joined, then a cuboid will be formed of dimensions

30 cm × 10 cm × 10 cm



$\therefore$ Surface area of the cuboid $= 2[lb + bh + hl]$

$$= 2[30 \times 10 + 10 \times 10 + 30 \times 10]$$
$$= 2[300 + 100 + 300] = 2[700] = 1400\ \text{cm}^2$$

28. Volume of cylinder $= \pi(4)^2 \times 10 = 160\pi \text{ cm}^3$

Volume of one cone $= \frac{1}{3} \times \pi \times 3^2 \times 4$

$= 12\pi \text{ cm}^3$

$\therefore$ Volume of both cones $= 24\pi \text{ cm}^3$

$\therefore$ Volume of remaining portion

$= 160\pi - 24\pi = 136\pi \text{ cm}^3$

29. Let the sides be $3x, 2x$ and $1x$.

$\therefore$ Volume $= l \times b \times h$

$\Rightarrow \quad 1296 = 3x \times 2x \times x$

$\Rightarrow \quad 6x^3 = 1296$

$\Rightarrow \quad x^3 = 216 \Rightarrow x = 6$

$\therefore$ Breadth $= 2 \times 6 = 12$ m

30. Ratios of two curved surface area

$= C_1 : C_2 = \pi r l_1 : \pi r l_2$

$= l_1 : l_2 = 5 : 7$

31. $\therefore$ Surface area of cylinder = Area of paper

$\Rightarrow \quad 2\pi rh = l \times b$

$\Rightarrow \quad 2\pi r \times 8 = 11 \times 8$

$\Rightarrow \quad 2\pi r = 11 \Rightarrow r = \frac{11 \times 7}{2 \times 22} = \frac{7}{4}$

$\therefore$ Volume $= \pi \left(\frac{7}{4}\right)^2 \times 8 = 77 \text{cm}^3$

32. Volume of metal $= \pi [R^2 - r^2] h$

$= \frac{22}{7} [6^2 - (5.6)^2] \times 21$

$= 66 \times 4.64$

$= 306.24 \text{ cm}^3$

33. Volume of circular cylinder $= \pi(8)^2(2) = 128\pi$ cm

Volume of right circular cone $= \frac{1}{3}\pi r^2 h$

$\therefore \quad \frac{1}{3}\pi r^2 h = 128\pi$

$\Rightarrow \quad \frac{1}{3} \times \pi \times (8)^2 \times h = 128\pi$

$\Rightarrow \quad h = 6$ cm

34. Given, inner radius of the hemispherical bowl

$= 24.7$ cm

Thickness of metal sheet $= 0.3$ cm

Now, outer radius of the hemispherical bowl

$= 24.7 + 0.3 = 25$ cm

$\therefore$ Outer surface area of the hemispherical bowl

$= 2\pi r^2$

$= 2 \times 3.14 \times (25)^2 = 157 \times 25 = 3925 \text{ cm}^2$

Now, cost of polishing $100 \text{ cm}^2 = ₹ 4$

$\therefore$ Cost of polishing 3925 cm^2

$= \frac{4 \times 3925}{100} = ₹157$

35. When radius $r = 7$ m and height $h = 4$ m,

then surface area of cylinder, $S_1 = 2\pi rh$

$= 2\pi \times 7 \times 4 = 56\pi \text{m}^2$

When radius $r_1 = 10$ m and height $= h_1$ m,

then surface area of new cylinder,

$S_2 = 2\pi r_1 h_1 = 2\pi \times 10 \times h_1$

$= 20\pi h_1$

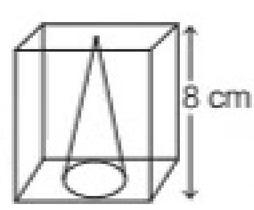
According to the given condition,

$S_1 = S_2$

$\Rightarrow \quad 56\pi = 20\pi \times h_1$

$\Rightarrow \quad h_1 = \frac{56}{20} = 2.8$ m

36. Here, height of cone $h = 8$ cm



and radius of cone $r = \frac{8}{2} = 4$ cm

Now, volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (4)^2 \times 8$

$= \frac{2816}{21} = 135 \text{ cm}^3$

CHAPTER

14

DATA HANDLING

In this chapter, we study the data in frequency distribution table, representation of there data in various types of graph such as Bar graph Pie chart, Line graph etc. and also study probability of event.

Data

A collection of numerical facts regarding a particular type of information is called data.

(i) **Primary Data** The data collected actually in the process of investigation by the investigator is known as primary data.

(ii) **Secondary Data** Data which is already collected by other persons is called secondary data.

e.g. As investigator collects data related to industries through the government publications.

Organising Data

The initial form of data is in unorganised form, i.e. raw data form. To organise the given data in systematic manner, we use a term frequency.

Frequency

The number of times a particular observation occurs is called frequency. e.g. The frequency distribution table is shown below.

Subjects	Tally marks	Number of students
Art	𝍸 II	7
Mathematics	𝍸	5
Science	𝍸 I	6
English	IIII	4

Here we see that, the number of students corresponding to each subject are the frequencies.

Grouping Data

Sometimes, the given data is very large, so we make a group. The data arranged in each group is known as class-interval (or class).

Each of group 0-10, 10-20, 20-30, etc, is called class-interval. The upper value of a class-interval is called its upper class limit and the lower value of the class-interval is called its lower class limit.

Here, in the class interval 10-20,
the upper class limit =20

and the lower class limit =10.

Some more related terms of grouping data are given below.

(i) **Width** The difference between the upper class limit and lower class limit of a class is called the width or size of the class-interval.

(ii) **Tally marks** Frequency distribution table involves the tally marks for counting purpose. We use bar (i.e.| |) known as tally marks.

(iii) **Class Frequency** The frequency of a class in a continuous frequency distribution is called as class frequency.

(iv) **Class Marks** It is the mid-value of the class interval.

Class mark

$$= \frac{\text{Lower limit of class} + \text{Upper limit of class}}{2}$$

(v) **Range** It is the difference between the highest and the lowest values of the given observation on data.

Example 1 The first and second class interval of a grouped data are 6-12, 12-18. Its fifth class interval is

(a) 20-26 (b) 30-36 (c) 24-30 (d) 31-37

Sol. *(b)* Given, class intervals are 6-12 and 12-18

Here, width of the class is $12 - 6$ i.e. 6

∴ Then, next class intervals will be 18-24, 24-30, 30-36 etc.

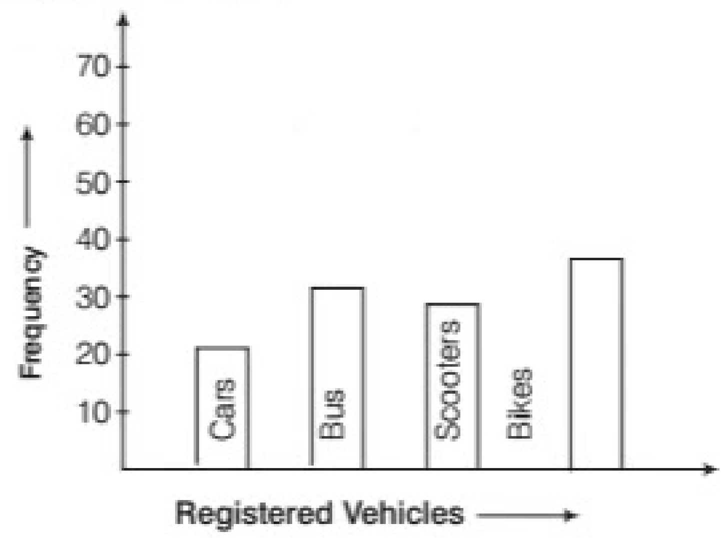
Hence, fifth class interval is 30-36.

Types of Graphical Data

The following types of groups are given below

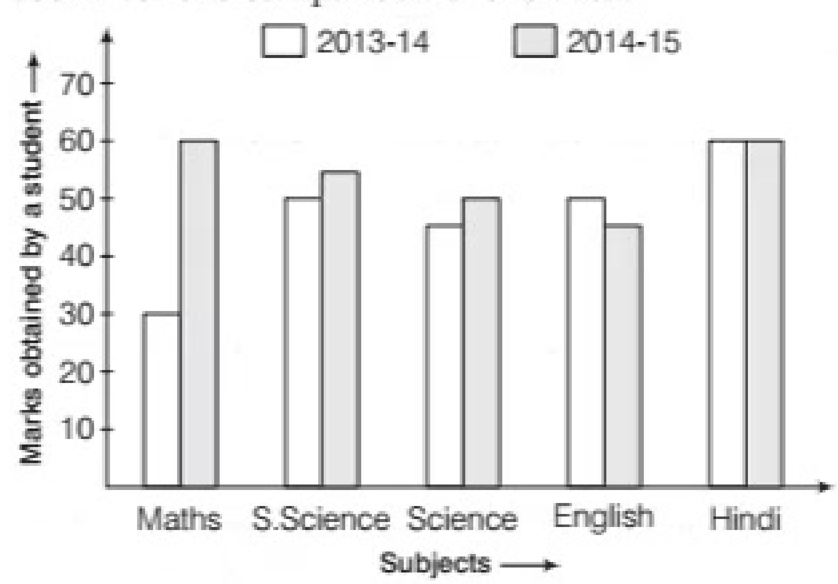
1. Bar Graph

A pictorial representation of numerical data in the form of rectangles (or bars) of uniform width and various heights is called a bar graph, where the commodity taken on the horizontal axis and the heights of the bars (rectangles) show the frequency of the commodity. e.g. The registration of vehicles versus number of vehicles shown through the bar graph.



2. Double Bar Graph

A bar graph showing two sets of data simultaneously is called a double bar graph. It is useful for the comparison of the data.

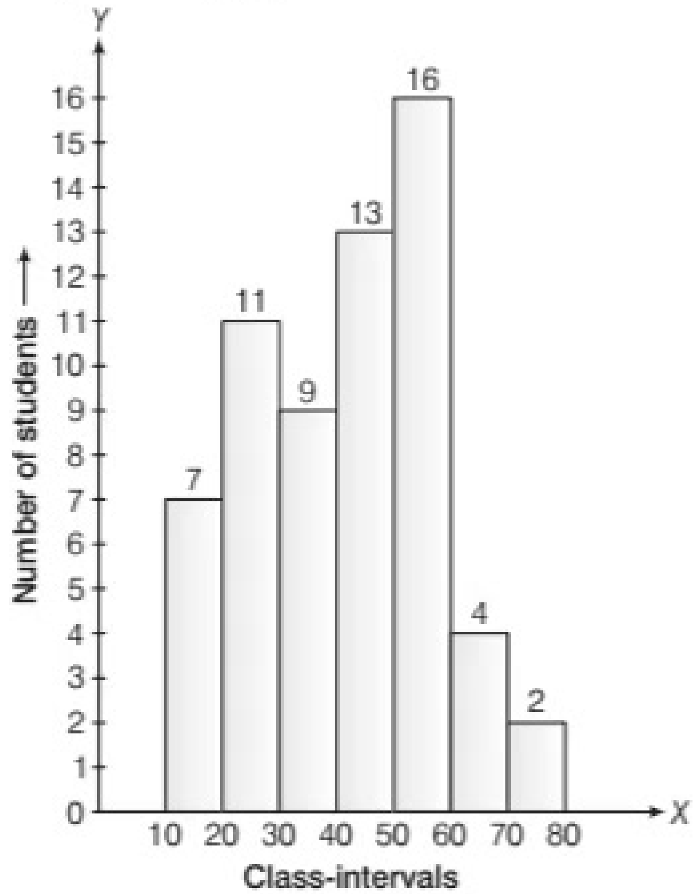


3. Histogram Graph

A histogram is the graphical representation of a grouped frequency distribution in exclusive form with continuous classes in the form of rectangles with class-intervals as bases and the corresponding frequencies as heights.

There is no gap between any two consecutive rectangles.

The shape of the graph is shown below.



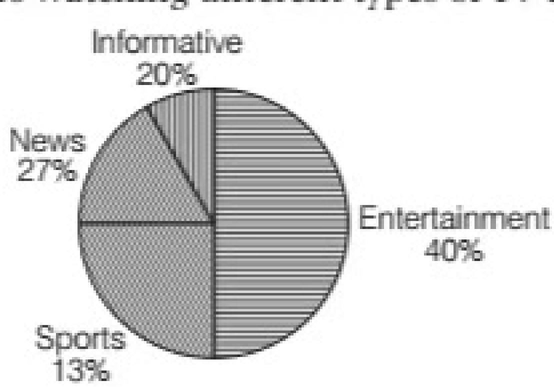
4. Pie Chart

In a pie chart, the various observations or components are represented by the sectors of a circle and the whole circle represents the sum of the values of all components.

The central angle for a component is given by Central angle for a component

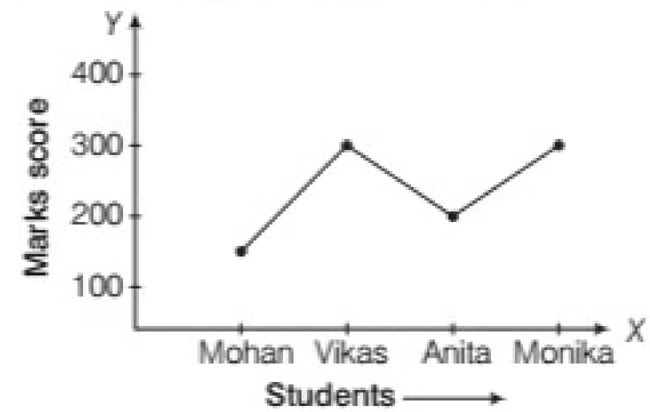
$$=\left(\frac{\text{Value of the component}}{\text{Sum of the values of all components}}\times 360^\circ\right)$$

e.g. The following pie graph shows the percentage of viewers watching different types of TV channels.

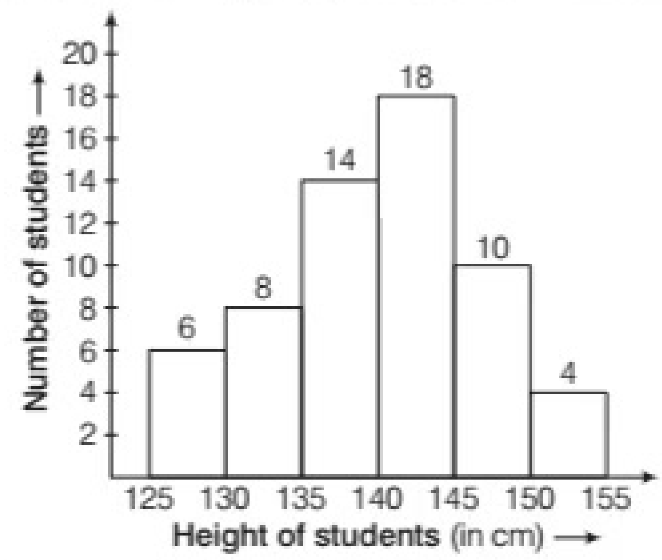


5. Line Graph

A line graph is a graph, which is used to display data that changes continuously over period of time. e.g. The marks score by the students shown through the line graph as given below.



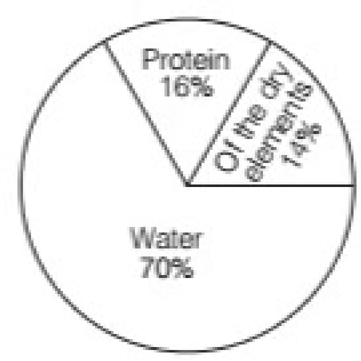
Example 2 In a histogram, the number of students having height less than 140 cm is



(a) 32 (b) 28 (c) 40 (d) 34

Sol. *(b)* Number of students who have height less than 140 cm $= 6 + 8 + 14 = 28$.

Example 3 The following pie chart gives the distribution of constituents in the human body. The central angle of the sector showing the distribution of protein and other constituents is



(a) 108° (b) 54°
(c) 30° (d) 216°

Sol. *(a)* Distribution of protein and other constituents in human body $= 16 + 14 = 30\%$

Central angle of the sector showing the distribution of protein and other constituents

$$= \frac{30}{100} \times 360° = 108°$$

Experiment

An operation which can produce some well-defined outcomes is called an experiment.

Random Experiment

An experiment in which the outcomes cannot be predicted exactly in advance and all possible outcomes are known, is called random experiment. e.g. On tossing a coin, head or tail are the two outcomes, but you cannot predict, which outcome you will get.

Event

Each outcome of an experiment or a collection of outcomes make an event.

e.g.

(i) Throwing a die and getting each of the outcomes 1, 2, 3, 4, 5 or 6 is an event.

(ii) Tossing a coin and getting a head or a tail is an event.

Negation of an event Negation of an event A is 'not A', which occurs only when A does not occur. It is denoted by '$\overline{A}$'.

Probability of an Event

Let E be an event. Then, the probability of occurrence of E is defined as

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

Note The probability of any event lies between 0 to 1.

Important Points

(i) In tossing a coin, all possible outcomes are $\{H, T\}$.

(ii) In tossing 2 coins, all possible outcomes are $\{HH, HT, TH, TT\}$.

(iii) On rolling a dice, all possible outcomes are $\{1, 2, 3, 4, 5, 6\}$.

(iv) In drawing a card from a well-shuffled deck of 52 cards, it has 4 suits (spades, clubs, hearts and diamonds), each having 13 cards.

In out of 52 cards, 26 cards are black and 26 cards are red.

(a) Cards of spades and clubs are black cards.

(b) Cards of hearts and diamonds are red cards.

(c) Kings, queens and jacks (or knaves) are known as face cards.

Thus, there are in 12 face cards.

Example 4 The two coins are tossed together. The probability of getting two heads together is?

(a) $\frac{1}{4}$ (b) $\frac{2}{4}$ (c) $\frac{1}{2}$ (d) 1

Sol. *(a)* When two coins are tossed together, all possible outcomes are – $\{HH, HT, TH, TT\}$

$\therefore$ Number of all possible outcomes = 4

Possibility of getting two heads together $= \frac{1}{4}$

Example 5 A dice is thrown. What is the probability of getting a prime number?

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

Sol. *(b)* In throwing a die, all possible outcomes are 1, 2, 3, 4, 5, 6.

$\therefore$ Number of all possible outcomes = 6

Prime numbers are 2, 3, 5.

Number of prime numbers = 3

$\therefore P(\text{getting a prime number}) = \frac{3}{6} = \frac{1}{2}$

Example 6 From a well-shuffled deck of 52 cards, one card is drawn at random. What is the probability that the card drawn is an ace?

(a) $\frac{2}{13}$ (b) $\frac{3}{13}$ (c) $\frac{1}{13}$ (d) $\frac{5}{13}$

Sol. *(c)* Total number of cards = 52

Total number of possible outcomes = 52.

Number of all aces = 4.

$\therefore \quad P(\text{getting an ace}) = \frac{4}{52} = \frac{1}{13}$

PRACTICE EXERCISE

1. A graph that displays data that changes continuously over periods of time is
(a) bar graph (b) pie chart
(c) histogram (d) line graph

2. The range of the data 30, 61, 55, 56, 60, 20, 26, 46, 28, 56 is
(a) 26 (b) 30
(c) 41 (d) 61

3. Which of the following is not a random experiment?
(a) Tossing a coin
(b) Rolling a die
(c) Choosing a card from a deck of 52 cards
(d) Throwing a stone to the roof of a building

4. Tally marks are used to find
(a) class intervals
(b) range
(c) frequency
(d) upper limit

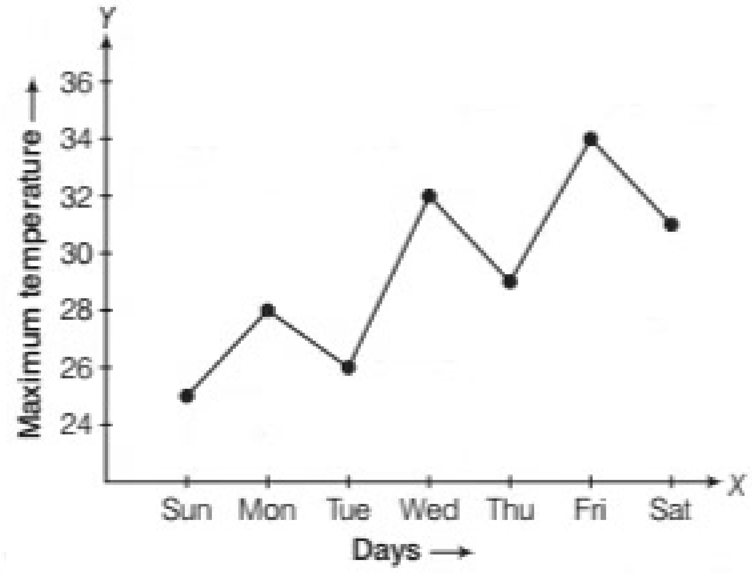
5. In a frequency distribution with classes 0-10,10-20 etc., the size of the class intervals is 10. The lower limit of fourth class is
(a) 40 (b) 50
(c) 20 (d) 30

6. Monthly salary of a person is ₹ 15000. The central angle of the sector representing his expenses on food and house rent on a pie chart is 60°. The amount he spends on food and house rent, is
(a) ₹ 5000 (b) ₹ 2500
(c) ₹ 6000 (d) ₹ 9000

7. The first and second class interval of a grouped data are 5-15, 15-25, 25-35. Its sixth class interval is
(a) 65-70 (b) 44-55
(c) 45-50 (d) 55-65

8. The class mark of the interval 13.5-18.5 is
(a) 15 (b) 16
(c) 14 (d) 13

Directions (Q. Nos. 9-11) *Study the graph and answer the questions that follow.*

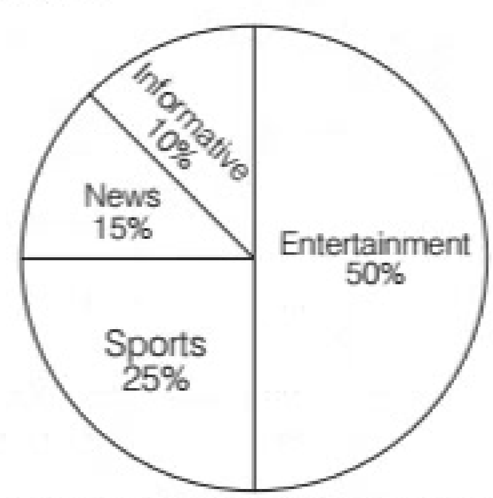


9. On which day the temperature was least?
(a) Sunday
(b) Monday
(c) Friday
(d) None of the above

10. On which day the temperature was 31°C?
(a) Monday (b) Saturday
(c) Tuesday (d) Friday

11. Which was the hottest day?
(a) Monday (b) Friday
(c) Tuesday (d) Saturday

Directions (Q.Nos. 12 and 13) *A pie chart is given below.*



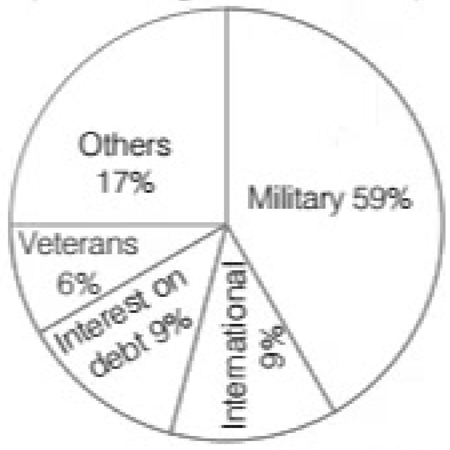
Viewers watching different types of channels on TV

12. Which type of programmes are viewed the most?
(a) Entertainment viewers
(b) Informative viewers
(c) Sports viewers
(d) None of the above

13. Which two types of programmes have number of viewers equal to those watching sports channels?
(a) News and Sports
(b) News and Informative
(c) News and Entertainment
(d) None of the above

Directions (Q. Nos. 14 and 15) *Study the following pie chart and answer the questions that follow.*

National budget expenditure in the year 2012
(Percentage distribution)

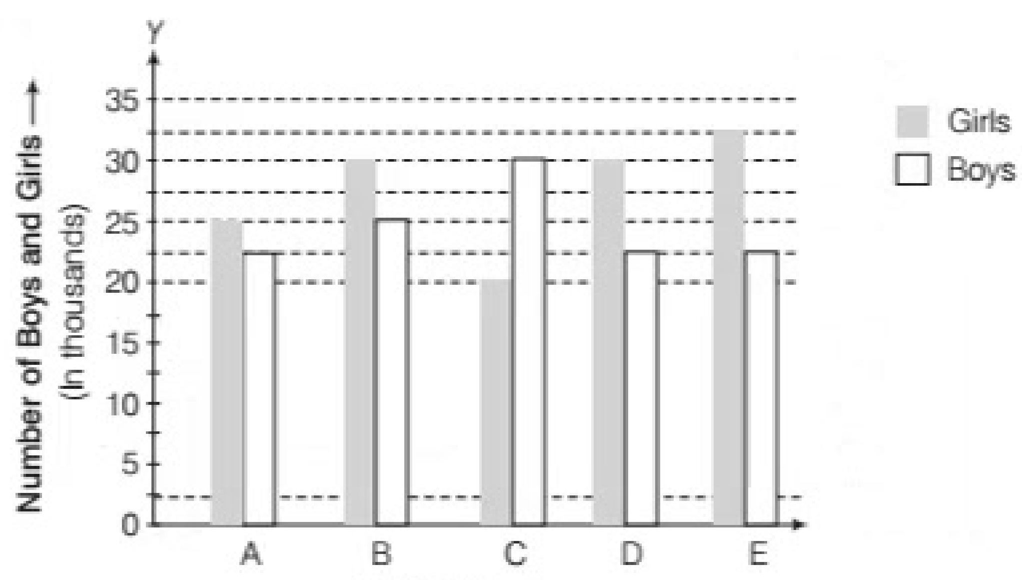


14. In year 2012, if India had a total expenditure of ₹ 120 billion, then how many billions did it spend on interest on debt?
(a) ₹ 10.8 billion
(b) ₹ 11.2 billion
(c) ₹ 11.50 billion
(d) ₹ 11.70 billion

15. If ₹ 9 billion were spent in year 2012 for veterans, then what would have been the total expenditure for that year (in billions)?
(a) ₹ 160 billion
(b) ₹ 165 billion
(c) ₹ 150 billion
(d) ₹ 170 billion

Directions (Q. Nos. 16 and 17) *Study the following graph carefully and answer the questions given below.*

Total Number of Boys and Girls in Various Colleges (Number in Thousands)



16. What is the average number of girls from all the colleges together?

(a) 25000 (b) 27500
(c) 27000 (d) 25500

17. What is the difference between the total number of girls and the total number of boys from all the colleges together?

(a) 13500 (b) 14000
(c) 15000 (d) 16000

18. A dice is throw, find the probability of getting not even number?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{5}$ (d) $\frac{1}{3}$

19. What is the probability of choosing a vowel from the alphabets?

(a) $\frac{21}{26}$ (b) $\frac{5}{26}$ (c) $\frac{1}{26}$ (d) $\frac{3}{26}$

20. A coin is tossed 12 times and the outcomes are observed as shown below

The chance of occurrence of head is

(a) $\frac{1}{2}$ (b) $\frac{5}{12}$ (c) $\frac{7}{12}$ (d) $\frac{5}{7}$

21. Ram puts some buttons on the table. There were 4 blue, 7 red, 3 black and 6 white buttons in all. All of a sudden, a cat jumped on the table and knocked out one button on the floor. What is the probability that the button on the floor is blue?

(a) $\frac{7}{20}$ (b) $\frac{3}{5}$ (c) $\frac{1}{5}$ (d) $\frac{1}{4}$

22. Two coins are tossed simultaneously. What is the probability of getting one head and one tail?

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{2}{3}$

23. A bag contains 3 white and 2 red balls. One ball is drawn at random. What is the probability that the ball drawn is red?

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

24. A dice is thrown. What is the probability of getting 6?

(a) 1 (b) $\frac{1}{6}$
(c) $\frac{5}{6}$ (d) None of these

25. A dice is thrown. What is the probability of getting an even number?

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{5}{6}$ (d) $\frac{1}{6}$

26. From a well-shuffled deck of 52 cards, one card is drawn at random. What is the probability that the drawn card is a queen?

(a) $\frac{1}{4}$ (b) $\frac{1}{52}$ (c) $\frac{1}{13}$ (d) $\frac{1}{26}$

27. From a well-shuffled deck of 52 cards, one card is drawn at random. What is the probability that the drawn card is a black 6?

(a) $\frac{3}{26}$ (b) $\frac{1}{26}$ (c) $\frac{1}{13}$ (d) $\frac{1}{52}$

28. Numbers 1 to 10 are written on ten separate slips (one slip of one number), kept in a box and mixed well. One slip is chosen from the box without looking into it. What is the probability of getting a number less than 6?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

Answers

1	(d)	2	(c)	3	(d)	4	(c)	5	(d)	6	(b)	7	(d)	8	(b)	9	(a)	10	(b)
11	(b)	12	(a)	13	(b)	14	(a)	15	(c)	16	(b)	17	(c)	18	(b)	19	(b)	20	(b)
21	(c)	22	(b)	23	(d)	24	(b)	25	(a)	26	(c)	27	(b)	28	(a)				

Hints and Solutions

1. Line graph is an important way to represent and compare the data which varies continuously.

   A line graph displays the relation between two varying quantities.

   In a line graph, we connect all the points by a line segment while in bar graph and histogram, we use rectangles of uniform width.

2. Range of data

   = Maximum value – Minimum value

   $= 61 - 20 = 41$

3. Tossing a coin, rolling a die and choosing a card from a deck of 52 cards are the random experiments, as we don't have an idea about the output of these experiments. But if we throw a stone to the roof of a building, we know that output, it will fall on the ground.

4. Tally marks are used to find the frequency of the observations.

5. Given classes are 0-10 and 10-20.

   As, the class of given classes is 10, so the next classes will be 20-30 and 30-40.

   As, the fourth class is 30-40.

   Hence, the lower limit of fourth class is 30.

6. Central angle of the sector representing his expenses on food and house rent on a pie chart

   $= 60^\circ$

   Part of the monthly salary he is expending on food and house rent $= \frac{60^\circ}{360^\circ} = \frac{1}{6}$

   Hence, the amount he spends on food and house rent $= \frac{1}{6} \times$ Monthly salary

   $= \frac{1}{6} \times 15000 = ₹\ 2500$

7. Given class intervals are 5-15, 15-25 and 25-35

   Here, width of the class is 15-5 i.e. 10.

   ∴ The next class intervals will be

   35-45, 45-55, 55-65 etc

   Hence, sixth class interval is 55-65.

8. Class mark $= \frac{\text{Lower limit} + \text{Upper limit}}{2}$

   $= \frac{13.5+18.5}{2} = \frac{32.0}{2} = 16$

9. On Sunday, the temperature was 25°C. So, it is least temperature in the week.

10. On Saturday, the temperature was 31°C.

11. On Friday, the temperature was maximum i.e. 34°C. Hence, it is the hottest day of the week.

12. From the given pie chart, we have the following table :

Types of viewers	Percentage
Sports viewers	25
News viewers	15
Informative viewers	10
Entertainment viewers	50

   Since, percentage of entertainment is highest, so entertainment programme are viewed the most.

13. Here, the number of viewers watching news = 15%

   Number of viewers watching informative = 10%

   ∴ Sum of number of viewers watching news and informative = (15 + 10)% = 25%

   = Number of viewers watching sports

   Hence, news and informative programmes have number of viewers equal to those watching sports channel.

14. Total expenditure = ₹ 120 billion

   ∴ Expenditure of interest on debt = 9% of 120

   $= \frac{9}{100} \times 120 = ₹\ 10.8$ billion

15. ₹ 9 billion were spent for veterans. 6% of the total expenditure was spent on veterans in the year 2012.

   Hence, the total expenditure

   $= \frac{9}{6} \times 100 = ₹\ 150$ billion

16. Total number of girls

   $= (25 + 30 + 20 + 30 + 32.5)$ thousands

   $= 137.5 \times 1000 = 137500$

   Average number of girls $= \frac{137500}{5} = 27500.$

17. Total number of boys

$=(22.5+25+30+22.5+22.5)$ thousands

$=(122.5\times1000)=122500$

Total number of girls $=137500$

Required difference $=(137500-122500)=15000$.

18. The probability of getting not even number

$=$Probability of getting odd number

$=\frac{3}{6}=\frac{1}{2}$ [$\because$ Odd numbers in a die are 1, 3, 5]

19. Total number of alphabets = 26

Total number of vowels = 5

$\therefore$ Probability of choosing a vowel from the alphabets $=\frac{\text{Total number of vowels}}{\text{Total number of alphabets}}=\frac{5}{26}$

20. Total number of times a coin is tossed = 12

Total number of occurrence of head = 5

$\therefore$ The chance of occurrence of head

$=\frac{\text{Number of times head appeared}}{\text{Number of times a coin is tossed}}=\frac{5}{12}$

21. Total number of buttons $=4+7+3+6=20$

$\therefore$ Probability that button on the floor is blue

$=\frac{\text{Number of blue buttons}}{\text{Total number of buttons}}=\frac{4}{20}=\frac{1}{5}$

22. Total number of outcomes $=2\times2=4$

Number of favourable outcomes

$=2$ [i.e. (H, T), (T, H)]

$\therefore$ Probability of getting one head and one tail

$=\frac{2}{4}=\frac{1}{2}$

23. Total number of balls $=3+2=5$ balls

Number of favourable outcomes = Number of red balls = 2

$\therefore$ Probability of getting a red ball $=\frac{2}{5}$

24. Total number of outcomes in a die = 6

Number of favourable outcomes = 1

$\therefore$ P (getting 6) $=\frac{1}{6}$

25. Total number of outcomes in a die = 6

Number of favourable outcomes = 3

[$\because$ even numbers are 2, 4, 6]

$\therefore P$ (getting even number) $=\frac{3}{6}=\frac{1}{2}$

26. Total number of outcomes in a deck of cards

$=52$

Number of favourable outcomes (queen) = 4

$\therefore P$ (getting queen) $=\frac{4}{52}=\frac{1}{13}$

27. Total number of outcomes = 52

Number of favourable outcomes = 2

$\therefore P$ (getting card of black 6) $=\frac{2}{52}=\frac{1}{26}$

28. Total number of outcomes $=10$

Number of favourable outcomes = 5

$\therefore P$ (getting number less than 6) $=\frac{5}{10}=\frac{1}{2}$