BASIC ELECTRICAL ENGINEERING

DIGITAL NOTES

Prepared by

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MALLA REDDY COLLEGE OF ENGINEERING & TECHNOLOGY
(Autonomous Institution – UGC, Govt. of India)
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MALLA REDDY COLLEGE OF ENGINEERING AND TECHNOLOGY

I Year B.Tech II Sem – EEE/ECE/CSE/IT

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(R18A0201) BASIC ELECTRICAL ENGINEERING

Objectives:

- This course introduces the basic concepts of electrical circuits & networks and their analysis which is the foundation for all the subjects in the electrical engineering discipline.
- The emphasis is laid on the basic elements in electrical circuits.
- Analysis of Circuits which includes Network Analysis & Network Theorems.
- Analysis of Single Phase AC Circuits and Basic Treatment of Single Phase Transformers and DC Machines is introduced.

UNIT –I:

Introduction to Electrical Circuits: Concept of Circuit and Network, Types of elements, R-L-C Parameters, Independent and Dependent sources, Source transformation and Kirchhoff's Laws

UNIT –II:

Network Analysis: Network Reduction Techniques- Series and parallel connections of resistive networks, Star–to-Delta and Delta-to-Star Transformations for Resistive Networks, Mesh Analysis, and Nodal Analysis,

Network Theorems**:** Thevenin's theorem, Norton's theorem, Maximum Power Transfer theorem and Superposition theorem, Illustrative Problems.

UNIT-III:

Single Phase A.C. Circuits: Average value, R.M.S. value, form factor and peak factor for sinusoidal wave form, Complex and Polar forms of representation. Steady State Analysis of series R-L-C circuits. Concept of Reactance, Impedance, Susceptance, Admittance, Phase and Phase difference, Concept of Power Factor, Real, Reactive and Complex power, Illustrative Problems.

UNIT –IV:

Electrical Machines (elementary treatment only):

Single phase transformers: principle of operation, constructional features and emf equation.

DC.Generator: principle of operation, constructional features, emf equation. DC Motor: principle of operation, Back emf, torque equation.

UNIT –V:

Electrical Installations:

Components of LT Switchgear: Switch Fuse Unit (SFU), MCB, ELCB, MCCB, Types of Wires and Cables, Earthing. Types of Batteries, Important Characteristics for Batteries. Elementary calculations for energy consumption and battery backup.

TEXT BOOKS:

1. Engineering Circuit Analysis - William Hayt, Jack E. Kemmerly, S M Durbin, Mc Graw Hill Companies.
2. Electric Circuits - A. Chakrabarhty, Dhanipat Rai & Sons.
3. Electrical Machines – P.S.Bimbra, Khanna Publishers.

REFERENCE BOOKS:

1. Network analysis by M.E Van Valkenburg, PHI learning publications.
2. Network analysis - N.C Jagan and C. Lakhminarayana, BS publications.
3. Electrical Circuits by A. Sudhakar, Shyammohan and S Palli, Mc Graw Hill Companies.
4. Electrical Machines by I.J. Nagrath & D. P. Kothari, Tata Mc Graw-Hill Publishers.

Outcomes:

At the end of this course the student would get

- A thorough knowledge of the basic RLC circuit elements.
- Understanding of the basic concepts of networks and circuits with RLC.
- Concepts of single phase AC circuits.
- Network theorems and their application to solve problems in Network analysis.
- Fundamentals Of Constructional Details And Principle Of Operation Of DC Machines And Transformers.

UNIT-I

INTRODUCTION TO ELECTRICAL CIRCUITS

- **Concept of Circuit and Network**
- **Types of elements**
- **R-L-C Parameters**
- **Independent and Dependent sources**
- **Source transformation Technique**
- **Kirchhoff's Laws**
- **Simple Problems**

INTRODUCTION TO ELECTRICAL CIRCUITS

Network theory is the study of solving the problems of electric circuits or electric networks. In this introductory chapter, let us first discuss the basic terminology of electric circuits and the types of network elements.

Basic Terminology

In Network Theory, we will frequently come across the following terms −

- Electric Circuit
- Electric Network
- Current
- Voltage
- Power

So, it is imperative that we gather some basic knowledge on these terms before proceeding further. Let's start with Electric Circuit.

Electric Circuit

An electric circuit contains a closed path for providing a flow of electrons from a voltage source or current source. The elements present in an electric circuit will be in series connection, parallel connection, or in any combination of series and parallel connections.

Electric Network

An electric network need not contain a closed path for providing a flow of electrons from a voltage source or current source. Hence, we can conclude that "all electric circuits are electric networks" but the converse need not be true.

Current

The current **"I"** flowing through a conductor is nothing but the time rate of flow of charge. Mathematically, it can be written as

$$I = \frac{dQ}{dt}$$

Where,

- Q is the charge and its unit is Coloumb.
- t is the time and its unit is second.

As an analogy, electric current can be thought of as the flow of water through a pipe. Current is measured in terms of Ampere. In general, Electron current flows from negative terminal of source to positive terminal, whereas, Conventional current flows from positive terminal of source to negative terminal.

Electron current is obtained due to the movement of free electrons, whereas, Conventional current is obtained due to the movement of free positive charges. Both of these are called as electric current.

Voltage

The voltage "V" is nothing but an electromotive force that causes the charge (electrons) to flow. Mathematically, it can be written as

$$V = \frac{dW}{dQ}$$

Where,

- W is the potential energy and its unit is Joule.
- Q is the charge and its unit is Coloumb.

As an analogy, Voltage can be thought of as the pressure of water that causes the water to flow through a pipe. It is measured in terms of Volt.

Power

The power "P" is nothing but the time rate of flow of electrical energy. Mathematically, it can be written as

$$P = \frac{dW}{dt}$$

Where,

- W is the electrical energy and it is measured in terms of Joule.
- t is the time and it is measured in seconds.

We can re-write the above equation a

$$P = \frac{dW}{dt} = \frac{dW}{dQ} \times \frac{dQ}{dt} = VI$$

Therefore, power is nothing but the product of voltage V and current I. Its unit is Watt.

Types of Network Elements

We can classify the Network elements into various types based on some parameters. Following are the types of Network elements −

- Active Elements and Passive Elements
- Linear Elements and Non-linear Elements
- Bilateral Elements and Unilateral Elements
- Lumped Elements and Distributed Elements

Active Elements and Passive Elements

We can classify the Network elements into either active or passive based on the ability of delivering power.

- Active Elements deliver power to other elements, which are present in an electric circuit. Sometimes, they may absorb the power like passive elements. That means active elements have the capability of both delivering and absorbing power.

  Examples: Voltage sources and current sources.
- Passive Elements can't deliver power (energy) to other elements, however they can absorb power. That means these elements either dissipate power in the form of heat or store energy in the form of either magnetic field or electric field.

  Examples: Resistors, Inductors, and capacitors.

Linear Elements and Non-Linear Elements

We can classify the network elements as linear or non-linear based on their characteristic to obey the property of linearity.

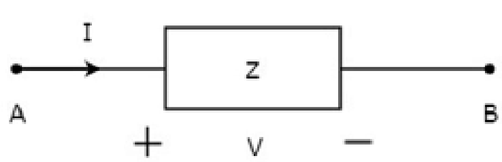
- Linear Elements are the elements that show a linear relationship between voltage and current. Examples: Resistors, Inductors, and capacitors.
- Non-Linear Elements are those that do not show a linear relation between voltage and current. Examples: Voltage sources and current sources.

Bilateral Elements and Unilateral Elements

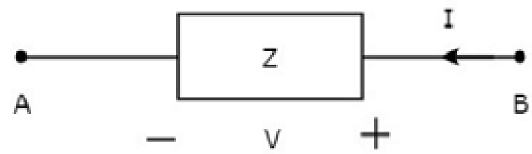
Network elements can also be classified as either bilateral or unilateral based on the direction of current flows through the network elements.

Bilateral Elements are the elements that allow the current in both directions and offer the same impedance in either direction of current flow. Examples: Resistors, Inductors and capacitors.

The concept of Bilateral elements is illustrated in the following figures.



In the above figure, the current (I) is flowing from terminals A to B through a passive element having impedance of Z Ω. It is the ratio of voltage (V) across that element between terminals A & B and current (I).



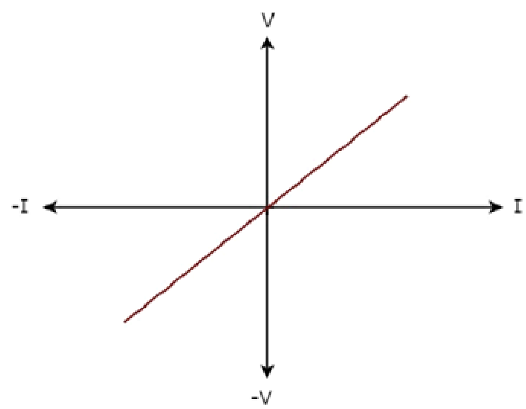
In the above figure, the current (I) is flowing from terminals B to A through a passive element having impedance of $Z\ \Omega$. That means the current (–I) is flowing from terminals A to B. In this case too, we will get the same impedance value, since both the current and voltage having negative signs with respect to terminals A & B.

Unilateral Elements are those that allow the current in only one direction. Hence, they offer different impedances in both directions.

We discussed the types of network elements in the previous chapter. Now, let us identify the nature of network elements from the V-I characteristics given in the following examples.

Example 1

The V-I characteristics of a network element is shown below.



Step 1 − Verifying the network element as linear or non-linear.

From the above figure, the V-I characteristics of a network element is a straight line passing through the origin. Hence, it is linear element.

Step 2 − Verifying the network element as active or passive.

The given V-I characteristics of a network element lies in the first and third quadrants.

- In the first quadrant, the values of both voltage (V) and current (I) are positive. So, the ratios of voltage (V) and current (I) gives positive impedance values.

- Similarly, in the third quadrant, the values of both voltage (V) and current (I) have negative values. So, the ratios of voltage (V) and current (I) produce positive impedance values.

Since, the given V-I characteristics offer positive impedance values, the network element is a Passive element.

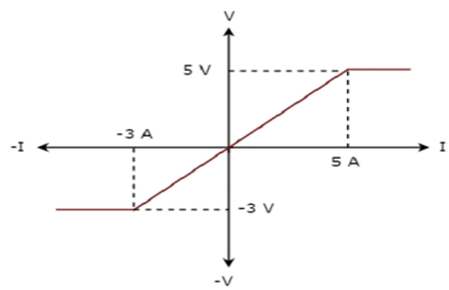
Step 3 − Verifying the network element as bilateral or unilateral.

For every point (I, V) on the characteristics, there exists a corresponding point (-I, -V) on the given characteristics. Hence, the network element is a Bilateral element.

Therefore, the given V-I characteristics show that the network element is a Linear, Passive, and Bilateral element.

Example 2

The V-I characteristics of a network element is shown below.



Step 1 − Verifying the network element as linear or non-linear.

From the above figure, the V-I characteristics of a network element is a straight line only between the points (-3A, -3V) and (5A, 5V). Beyond these points, the V-I characteristics are not following the linear relation. Hence, it is a Non-linear element.

Step 2 − Verifying the network element as active or passive.

The given V-I characteristics of a network element lies in the first and third quadrants. In these two quadrants, the ratios of voltage (V) and current (I) produce positive impedance values. Hence, the network element is a Passive element.

Step 3 − Verifying the network element as bilateral or unilateral.

Consider the point (5A, 5V) on the characteristics. The corresponding point (-5A, -3V) exists on the given characteristics instead of (-5A, -5V). Hence, the network element is a Unilateral element.

Therefore, the given V-I characteristics show that the network element is a Non-linear, Passive, and Unilateral element. The circuits containing them are called unilateral circuits.

Lumped and Distributed Elements

Lumped elements are those elements which are very small in size & in which simultaneous actions takes place. Typical lumped elements are capacitors, resistors, inductors.

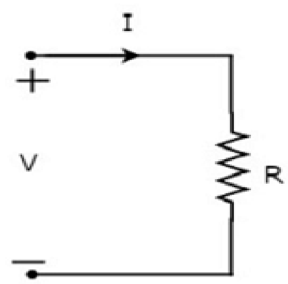
Distributed elements are those which are not electrically separable for analytical purposes.

For example a transmission line has distributed parameters along its length and may extend for hundreds of miles.

R-L-C Parameters

Resistor

The main functionality of Resistor is either opposes or restricts the flow of electric current. Hence, the resistors are used in order to limit the amount of current flow and / or dividing (sharing) voltage. Let the current flowing through the resistor is I amperes and the voltage across it is V volts. The symbol of resistor along with current, I and voltage, V are shown in the following figure.



According to Ohm's law, the voltage across resistor is the product of current flowing through it and the resistance of that resistor. Mathematically, it can be represented as

$$V = IR \qquad \text{Equation 1}$$

$$\Rightarrow I = \frac{V}{R} \qquad \text{Equation 2}$$

Where, **R** is the resistance of a resistor.

From Equation 2, we can conclude that the current flowing through the resistor is directly proportional to the applied voltage across resistor and inversely proportional to the resistance of resistor.

Power in an electric circuit element can be represented as

$$P = VI \qquad \text{Equation 3}$$

Substitute, Equation 1 in Equation 3.

$$P = (IR)I$$

$$\Rightarrow P = I^2R \qquad \text{Equation 4}$$

Substitute, Equation 2 in Equation 3.

$$P = V\left(\frac{V}{R}\right)$$

$$\Rightarrow P = \frac{V^2}{R} \qquad \text{Equation 5}$$

So, we can calculate the amount of power dissipated in the resistor by using one of the formulae mentioned in Equations 3 to 5.

Inductor

In general, inductors will have number of turns. Hence, they produce magnetic flux when current flows through it. So, the amount of total magnetic flux produced by an inductor depends on the current, I flowing through it and they have linear relationship.

Mathematically, it can be written as

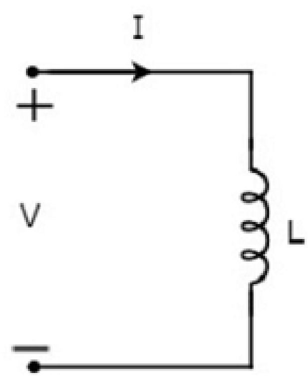
$$\Psi \,\alpha\, I$$

$$\Rightarrow \Psi = LI$$

Where,

- Ψ is the total magnetic flux
- L is the inductance of an inductor

Let the current flowing through the inductor is I amperes and the voltage across it is V volts. The symbol of inductor along with current I and voltage V are shown in the following figure.



According to Faraday's law, the voltage across the inductor can be written as

$$V = \frac{d\Psi}{dt}$$

Substitute $\Psi = LI$ in the above equation.

$$V = \frac{d(LI)}{dt}$$

$$\Rightarrow V = L\frac{dI}{dt}$$

$$\Rightarrow I = \frac{1}{L}\int V dt$$

From the above equations, we can conclude that there exists a linear relationship between voltage across inductor and current flowing through it.

We know that power in an electric circuit element can be represented as

$$P = VI$$

Substitute $V = L\frac{dI}{dt}$ in the above equation.

$$P = (L\frac{dI}{dt})I$$

$$\Rightarrow P = LI\frac{dI}{dt}$$

By integrating the above equation, we will get the energy stored in an inductor as

$$W = \frac{1}{2}LI^2$$

So, the inductor stores the energy in the form of magnetic field.

Capacitor

In general, a capacitor has two conducting plates, separated by a dielectric medium. If positive voltage is applied across the capacitor, then it stores positive charge. Similarly, if negative voltage is applied across the capacitor, then it stores negative charge.

So, the amount of charge stored in the capacitor depends on the applied voltage **V** across it and they have linear relationship. Mathematically, it can be written as

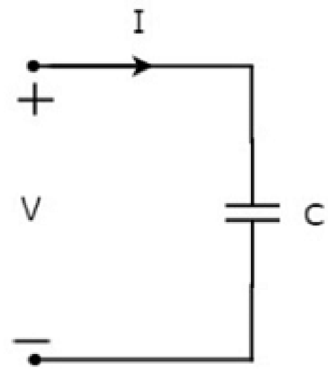
$$Q \, \alpha \, V$$

$$\Rightarrow Q = CV$$

Where,

- ***Q*** is the charge stored in the capacitor.
- ***C*** is the capacitance of a capacitor.

Let the current flowing through the capacitor is *I* amperes and the voltage across it is *V* volts. The symbol of capacitor along with current *I* and voltage *V* are shown in the following figure.



We know that the **current** is nothing but the **time rate of flow of charge**. Mathematically, it can be represented as

$$I = \frac{dQ}{dt}$$

Substitute $Q = CV$ in the above equation.

$$I = \frac{d(CV)}{dt}$$

$$\Rightarrow I = C\frac{dV}{dt}$$

$$\Rightarrow V = \frac{1}{C}\int I dt$$

From the above equations, we can conclude that there exists a linear relationship between voltage across capacitor and current flowing through it.

We know that power in an electric circuit element can be represented as

$$P = VI$$

Substitute $I = C\frac{dV}{dt}$ in the above equation.

$$P = V(C\frac{dV}{dt})$$

$$\Rightarrow P = CV\frac{dV}{dt}$$

By integrating the above equation, we will get the **energy** stored in the capacitor as

$$W = \frac{1}{2}CV^2$$

So, the capacitor stores the energy in the form of electric field.

Types of Sources

Active Elements are the network elements that deliver power to other elements present in an electric circuit. So, active elements are also called as sources of voltage or current type. We can classify these sources into the following two categories −

- Independent Sources
- Dependent Sources

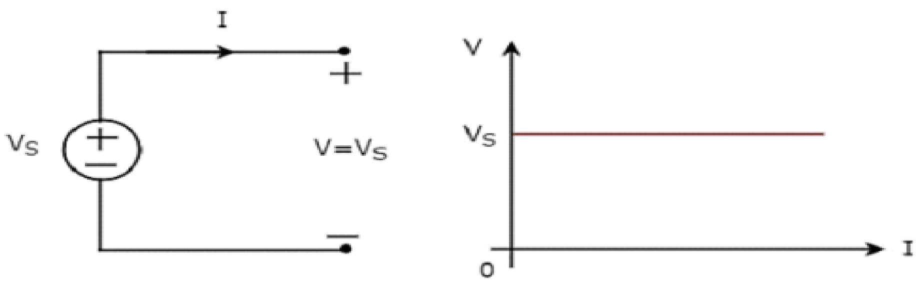
Independent Sources

As the name suggests, independent sources produce fixed values of voltage or current and these are not dependent on any other parameter. Independent sources can be further divided into the following two categories −

- Independent Voltage Sources
- Independent Current Sources

Independent Voltage Sources

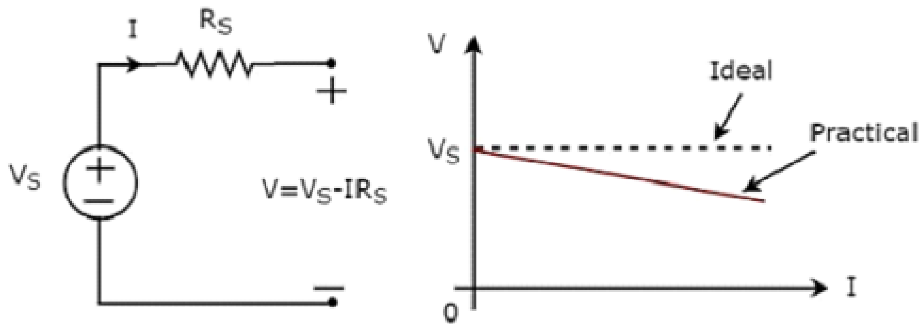
An independent voltage source produces a constant voltage across its two terminals. This voltage is independent of the amount of current that is flowing through the two terminals of voltage source. Independent ideal voltage source and its V-I characteristics are shown in the following figure.



The V-I characteristics of an independent ideal voltage source is a constant line, which is always equal to the source voltage (VS) irrespective of the current value (I). So, the internal resistance of an independent ideal voltage source is zero Ohms.

Hence, the independent ideal voltage sources do not exist practically, because there will be some internal resistance.

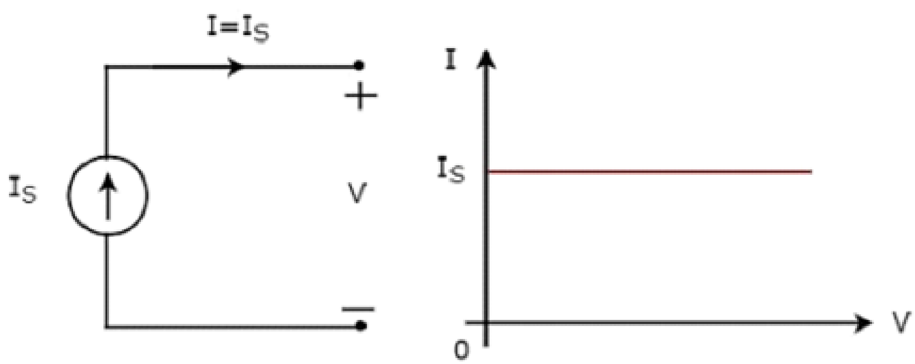
Independent practical voltage source and its V-I characteristics are shown in the following figure.



There is a deviation in the V-I characteristics of an independent practical voltage source from the V-I characteristics of an independent ideal voltage source. This is due to the voltage drop across the internal resistance (R_S) of an independent practical voltage source.

Independent Current Sources

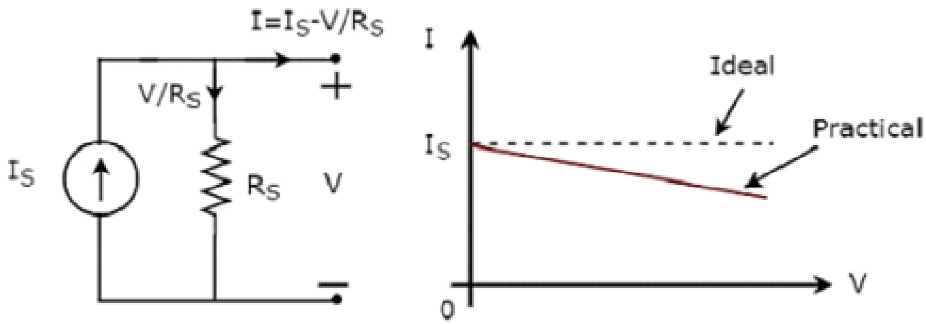
An independent current source produces a constant current. This current is independent of the voltage across its two terminals. Independent ideal current source and its V-I characteristics are shown in the following figure.



The V-I characteristics of an independent ideal current source is a constant line, which is always equal to the source current (I_S) irrespective of the voltage value (V). So, the internal resistance of an independent ideal current source is infinite ohms.

Hence, the independent ideal current sources do not exist practically, because there will be some internal resistance.

Independent practical current source and its V-I characteristics are shown in the following figure.



There is a deviation in the V-I characteristics of an independent practical current source from the V-I characteristics of an independent ideal current source. This is due to the amount of current flows through the internal shunt resistance (R_S) of an independent practical current source.

Dependent Sources

As the name suggests, dependent sources produce the amount of voltage or current that is dependent on some other voltage or current. Dependent sources are also called as controlled sources. Dependent sources can be further divided into the following two categories −

- Dependent Voltage Sources
- Dependent Current Sources

Dependent Voltage Sources

A dependent voltage source produces a voltage across its two terminals. The amount of this voltage is dependent on some other voltage or current. Hence, dependent voltage sources can be further classified into the following two categories −

- Voltage Dependent Voltage Source (VDVS)
- Current Dependent Voltage Source (CDVS)

Dependent voltage sources are represented with the signs '+' and '-' inside a diamond shape. The magnitude of the voltage source can be represented outside the diamond shape.

Dependent Current Sources

A dependent current source produces a current. The amount of this current is dependent on some other voltage or current. Hence, dependent current sources can be further classified into the following two categories −

- Voltage Dependent Current Source (VDCS)
- Current Dependent Current Source (CDCS)

Dependent current sources are represented with an arrow inside a diamond shape. The magnitude of the current source can be represented outside the diamond shape. We can observe these dependent or controlled sources in equivalent models of transistors.

Source Transformation Technique

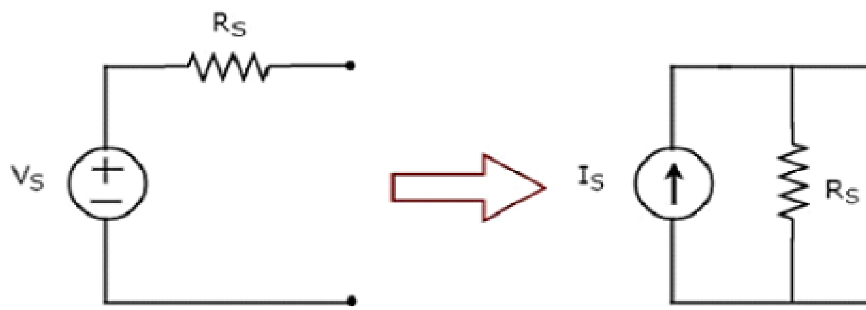
We know that there are two practical sources, namely, voltage source and current source. We can transform (convert) one source into the other based on the requirement, while solving network problems.

The technique of transforming one source into the other is called as source transformation technique. Following are the two possible source transformations −

- Practical voltage source into a practical current source
- Practical current source into a practical voltage source

Practical voltage source into a practical current source

The transformation of practical voltage source into a practical current source is shown in the following figure



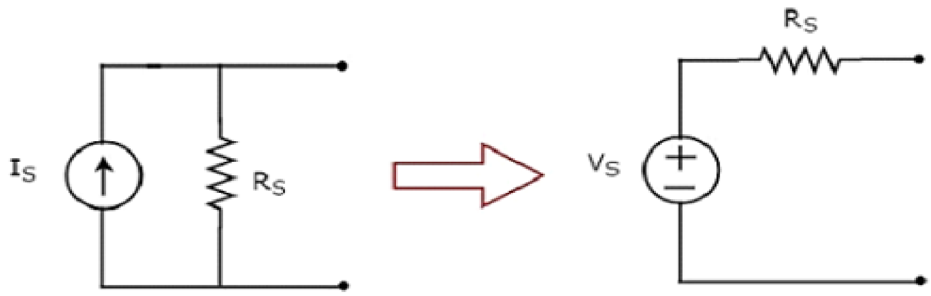
Practical voltage source consists of a voltage source (V_S) in series with a resistor (R_S). This can be converted into a practical current source as shown in the figure. It consists of a current source (I_S) in parallel with a resistor (R_S).

The value of IS will be equal to the ratio of V_S and R_S. Mathematically, it can be represented as

$$I_S = \frac{V_S}{R_S}$$

Practical current source into a practical voltage source

The transformation of practical current source into a practical voltage source is shown in the following figure.



Practical current source consists of a current source (I_S) in parallel with a resistor (R_S). This can be converted into a practical voltage source as shown in the figure. It consists of a voltage source (V_S) in series with a resistor (R_S).

The value of V_S will be equal to the product of I_S and R_S. Mathematically, it can be represented as

$$V_S = I_S R_S$$

In this chapter, we will discuss in detail about the passive elements such as Resistor, Inductor, and Capacitor. Let us start with Resistors.

Kirchhoff's Laws

Network elements can be either of active or passive type. Any electrical circuit or network contains one of these two types of network elements or a combination of both.

Now, let us discuss about the following two laws, which are popularly known as Kirchhoff's laws.

- Kirchhoff's Current Law
- Kirchhoff's Voltage Law

Kirchhoff's Current Law

Kirchhoff's Current Law (KCL) states that the algebraic sum of currents leaving (or entering) a node is equal to zero.

A Node is a point where two or more circuit elements are connected to it. If only two circuit elements are connected to a node, then it is said to be simple node. If three or more circuit elements are connected to a node, then it is said to be Principal Node.

Mathematically, KCL can be represented as

$$\sum_{m=1}^{M} I_m = 0$$

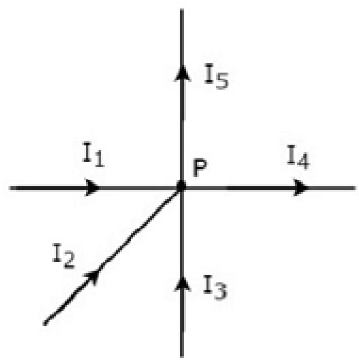
Where,

- I_m is the m^th^ branch current leaving the node.
- M is the number of branches that are connected to a node.

The above statement of KCL can also be expressed as "the algebraic sum of currents entering a node is equal to the algebraic sum of currents leaving a node". Let us verify this statement through the following example.

Example

Write KCL equation at node P of the following figure.



- In the above figure, the branch currents I_1, I_2 and I_3 areentering at node P. So, consider negative signs for these three currents.
- In the above figure, the branch currents I_4 and I_5 areleaving from node P. So, consider positive signs for these two currents.

The KCL equation at node P will be

$$-I_1 - I_2 - I_3 + I_4 + I_5 = 0$$

$$\Rightarrow I_1 + I_2 + I_3 = I_4 + I_5$$

In the above equation, the left-hand side represents the sum of entering currents, whereas the right-hand side represents the sum of leaving currents.

In this tutorial, we will consider positive sign when the current leaves a node and negative sign when it enters a node. Similarly, you can consider negative sign when the current leaves a node and positive sign when it enters a node. In both cases, the result will be same.

Note − KCL is independent of the nature of network elements that are connected to a node.

Kirchhoff's Voltage Law

Kirchhoff's Voltage Law (KVL) states that the algebraic sum of voltages around a loop or mesh is equal to zero.

A Loop is a path that terminates at the same node where it started from. In contrast, a Mesh is a loop that doesn't contain any other loops inside it.

Mathematically, KVL can be represented as

$$\sum_{n=1}^{N} V_n = 0$$

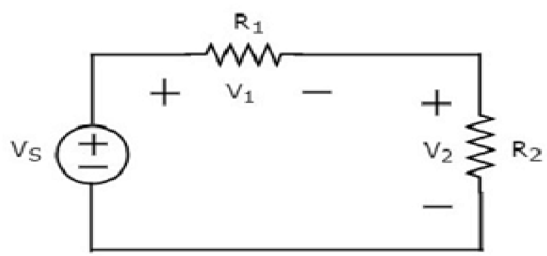
Where,

- $\mathbf{V_n}$ is the n^{th} element's voltage in a loop (mesh).
- **N** is the number of network elements in the loop (mesh).

The above statement of KVL can also be expressed as "the algebraic sum of voltage sources is equal to the algebraic sum of voltage drops that are present in a loop." Let us verify this statement with the help of the following example.

Example

Write KVL equation around the loop of the following circuit.



The above circuit diagram consists of a voltage source, V_S in series with two resistors R_1 and R_2. The voltage drops across the resistors R_1 and R_2 are V_1 and V_2 respectively.

Apply KVL around the loop.

$$V_S - V_1 - V_2 = 0$$

$$\Rightarrow V_S = V_1 + V_2$$

In the above equation, the left-hand side term represents single voltage source VS. Whereas, the right-hand side represents the sum of voltage drops. In this example, we considered only one voltage source. That's why the left-hand side contains only one term. If we consider multiple voltage sources, then the left side contains sum of voltage sources.

In this tutorial, we consider the sign of each element's voltage as the polarity of the second terminal that is present while travelling around the loop. Similarly, you can consider the sign of each voltage as the polarity of the first terminal that is present while travelling around the loop. In both cases, the result will be same.

Note − KVL is independent of the nature of network elements that are present in a loop.

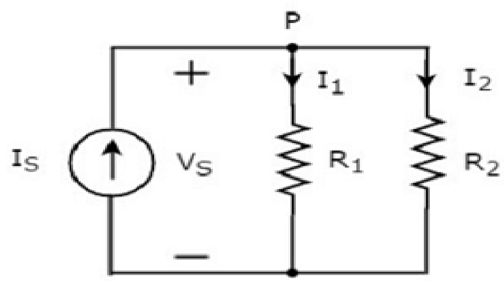
In this chapter, let us discuss about the following two division principles of electrical quantities.

- Current Division Principle
- Voltage Division Principle

Current Division Principle

When two or more passive elements are connected in parallel, the amount of current that flows through each element gets divided(shared) among themselves from the current that is entering the node.

Consider the following circuit diagram.



The above circuit diagram consists of an input current source $\mathbf{I_S}$ in parallel with two resistors $\mathbf{R_1}$ and $\mathbf{R_2}$. The voltage across each element is $\mathbf{V_S}$. The currents flowing through the resistors $\mathbf{R_1}$ and$\mathbf{R_2}$ are $\mathbf{I_1}$ and $\mathbf{I_2}$ respectively.

The KCL equation at node P will be

$$I_S = I_1 + I_2$$

- Substitute $I_1 = \frac{V_S}{R_1}$ and $I_2 = \frac{V_S}{R_2}$ in the above equation.

$$I_S = \frac{V_S}{R_1} + \frac{V_S}{R_2} = V_S\left(\frac{R_2 + R_1}{R_1 R_2}\right)$$

$$\Rightarrow V_S = I_S\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

- Substitute the value of V_S in $I_1 = \frac{V_S}{R_1}$.

$$I_1 = \frac{I_S}{R_1}\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

$$\Rightarrow I_1 = I_S\left(\frac{R_2}{R_1 + R_2}\right)$$

- Substitute the value of V_S in $I_2 = \frac{V_S}{R_2}$.

$$I_2 = \frac{I_S}{R_2}\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

$$\Rightarrow I_2 = I_S\left(\frac{R_1}{R_1 + R_2}\right)$$

From equations of I_1 and I_2, we can generalize that the current flowing through any passive element can be found by using the following formula.

$$I_N = I_S\left(\frac{Z_1 || Z_2 || \dots || Z_{N-1}}{Z_1 + Z_2 + \dots + Z_N}\right)$$

This is known as current division principle and it is applicable, when two or more passive elements are connected in parallel and only one current enters the node.

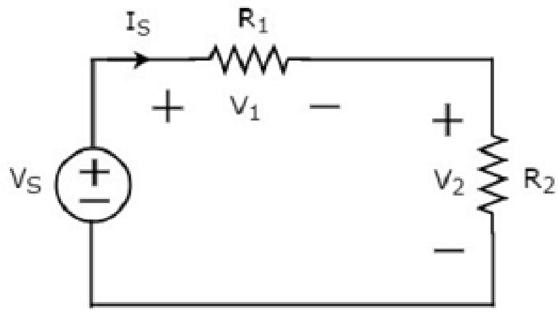
Where,

- I_N is the current flowing through the passive element of Nth branch.
- I_S is the input current, which enters the node.
  - $Z_1, Z_2, \dots, Z_N$ are the impedances of 1st branch, 2ndbranch, …, Nth branch respectively.

Voltage Division Principle

When two or more passive elements are connected in series, the amount of voltage present across each element gets divided (shared) among themselves from the voltage that is available across that entire combination.

Consider the following circuit diagram.



The above circuit diagram consists of a voltage source, V_S in series with two resistors R_1 and R_2. The current flowing through these elements is I_S. The voltage drops across the resistors R_1 and R_2 are V_1 and V_2 respectively.

The KVL equation around the loop will be

$$V_S = V_1 + V_2$$

- Substitute $V_1 = I_S R_1$ and $V_2 = I_S R_2$ in the above equation

$$V_S = I_S R_1 + I_S R_2 = I_S (R_1 + R_2)$$

$$I_S = \frac{V_S}{R_1 + R_2}$$

- Substitute the value of I_S in $V_1 = I_S R_1$.

$$V_1 = \left(\frac{V_S}{R_1 + R_2}\right) R_1$$

$$\Rightarrow V_1 = V_S \left(\frac{R_1}{R_1 + R_2}\right)$$

- Substitute the value of I_S in $V_2 = I_S R_2$.

$$V_2 = \left(\frac{V_S}{R_1 + R_2}\right) R_2$$

$$\Rightarrow V_2 = V_S \left(\frac{R_2}{R_1 + R_2}\right)$$

From equations of V_1 and V_2, we can generalize that the voltage across any passive element can be found by using the following formula.

$$V_N = V_S \left(\frac{Z_N}{Z_1 + Z_2 + \ldots + Z_N}\right)$$

This is known as voltage division principle and it is applicable, when two or more passive elements are connected in series and only one voltage available across the entire combination.

Where,

- V_N is the voltage across Nth passive element.
- V_S is the input voltage, which is present across the entire combination of series passive elements.
- Z_1,Z_2, …,Z_3 are the impedances of 1st passive element, 2nd passive element, …, Nth passive element respectively.

UNIT-II
NETWORK ANALYSIS

- **Network Reduction Techniques**
- **Series and Parallel connection of Resistive Networks**
- **Star–to-Delta and Delta-to-Star Transformations for Resistive Networks**
- **Mesh Analysis**
- **Nodal Analysis**
- **Network Theorems: Thevenin's Theorem**
- **Norton's Theorem**
- **Maximum Power Transfer Theorem**
- **Superposition Theorem**
- **Problems**

Network Reduction Techniques:

There are two basic methods that are used for solving any electrical network: Nodal analysis and Mesh analysis. In this chapter, let us discuss about the Mesh analysis method.

Series and parallel connections of resistive networks:

If a circuit consists of two or more similar passive elements and are connected in exclusively of series type or parallel type, then we can replace them with a single equivalent passive element. Hence, this circuit is called as an equivalent circuit.

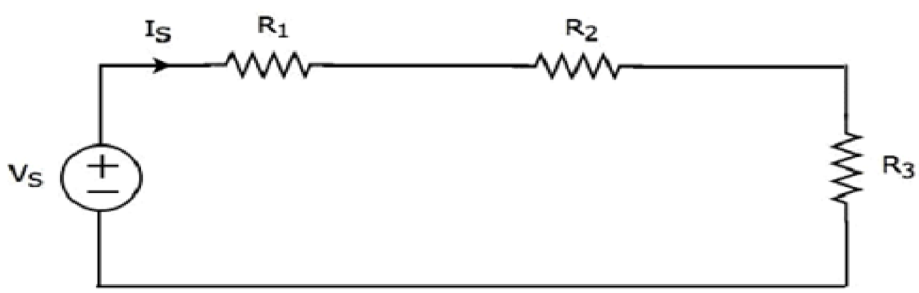
In this chapter, let us discuss about the following two equivalent circuits.

- Series Equivalent Circuit
- Parallel Equivalent Circuit

Series Equivalent Circuit

If similar passive elements are connected in series, then the same current will flow through all these elements. But, the voltage gets divided across each element.

Consider the following circuit diagram.



It has a single voltage source (V_S) and three resistors having resistances of R_1, R_2 and R_3. All these elements are connected in series. The current IS flows through all these elements.

The above circuit has only one mesh. The KVL equation around this mesh is

$$V_S = V_1 + V_2 + V_3$$

Substitute $V_1 = I_S R_1$, $V_2 = I_S R_2$ and $V_3 = I_S R_3$ in the above equation.

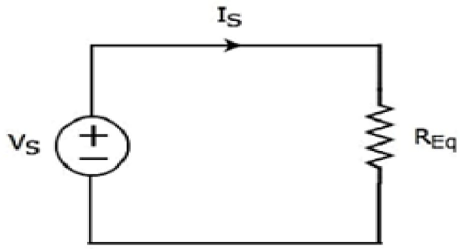
$$V_S = I_S R_1 + I_S R_2 + I_S R_3$$

$$\Rightarrow V_S = I_S (R_1 + R_2 + R_3)$$

The above equation is in the form of $V_S = I_S R_{Eq}$ where,

$$R_{Eq} = R_1 + R_2 + R_3$$

The equivalent circuit diagram of the given circuit is shown in the following figure.



That means, if multiple resistors are connected in series, then we can replace them with an equivalent resistor. The resistance of this equivalent resistor is equal to sum of the resistances of all those multiple resistors.

Note 1 − If 'N' inductors having inductances of L_1, L_2, ..., L_N are connected in series, then the equivalent inductance will be

$$L_{Eq} = L_1 + L_2 + \ldots + L_N$$

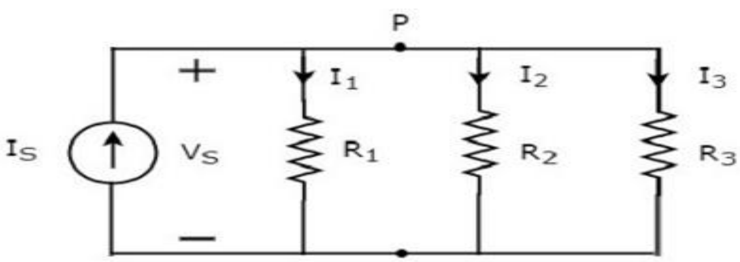
Note 2 − If 'N' capacitors having capacitances of C_1, C_2, ..., $C_N$are connected in series, then the equivalent capacitance will be

$$\frac{1}{C_{Eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_N}$$

Parallel Equivalent Circuit

If similar passive elements are connected in parallel, then the same voltage will be maintained across each element. But, the current flowing through each element gets divided.

Consider the following circuit diagram.



It has a single current source (I_S) and three resistors having resistances of R_1, R_2, and R_3. All these elements are connected in parallel. The voltage (V_S) is available across all these elements.

The above circuit has only one principal node (P) except the Ground node. The KCL equation at this principal node (P) is

$$I_S = I_1 + I_2 + I_3$$

Substitute $I_1 = \frac{V_S}{R_1}$, $I_2 = \frac{V_S}{R_2}$ and $I_3 = \frac{V_S}{R_3}$ in the above equation.

$$I_S = \frac{V_S}{R_1} + \frac{V_S}{R_2} + \frac{V_S}{R_3}$$

$$\Rightarrow I_S = V_S\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

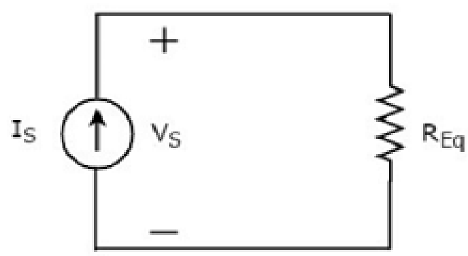
$$\Rightarrow V_S = I_S\left[\frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}\right]$$

The above equation is in the form of $V_S = I_S R_{Eq}$ where,

$$R_{Eq} = \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

$$\frac{1}{R_{Eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

The equivalent circuit diagram of the given circuit is shown in the following figure.



That means, if multiple resistors are connected in parallel, then we can replace them with an equivalent resistor. The resistance of this equivalent resistor is equal to the reciprocal of sum of reciprocal of each resistance of all those multiple resistors.

Note 1 − If 'N' inductors having inductances of L_1, L_2, ..., L_N are connected in parallel, then the equivalent inductance will be

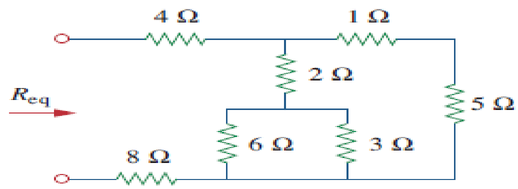
$$\frac{1}{L_{Eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_N}$$

Note 2 − If 'N' capacitors having capacitances of C_1, C_2, ..., $C_N$are connected in parallel, then the equivalent capacitance will be

$$C_{Eq} = C_1 + C_2 + \ldots + C_N$$

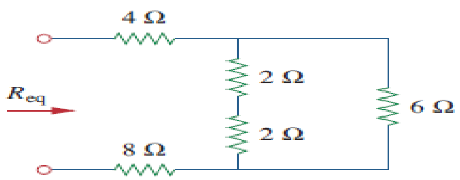
Example Problems:

1) Find the Req for the circuit shown in below figure.

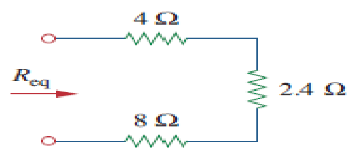


fig(a)

Solution:



fig(b)



fig(c)

To get Req we combine resistors in series and in parallel. The 6 ohms and 3 ohms resistors are in parallel, so their equivalent resistance is

$$6\ \Omega \parallel 3\Omega = \frac{6 \times 3}{6 + 3} = 2\ \Omega$$

Also, the 1 ohm and 5ohms resistors are in series; hence their equivalent resistance is

$$1\ \Omega + 5\ \Omega = 6\ \Omega$$

Thus the circuit in Fig.(b) is reduced to that in Fig. (c). In Fig. (b), we notice that the two 2 ohms resistors are in series, so the equivalent resistance is

$$2\ \Omega + 2\ \Omega = 4\ \Omega$$

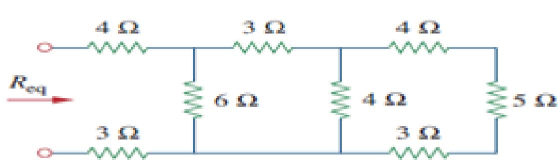
This 4 ohms resistor is now in parallel with the 6 ohms resistor in Fig.(b); their equivalent resistance is

$$4\ \Omega \parallel 6\ \Omega = \frac{4 \times 6}{4 + 6} = 2.4\ \Omega$$

The circuit in Fig.(b) is now replaced with that in Fig.(c). In Fig.(c), the three resistors are in series. Hence, the equivalent resistance for the circuit is

$$R_{eq} = 4\ \Omega + 2.4\ \Omega + 8\ \Omega = 14.4\ \Omega$$

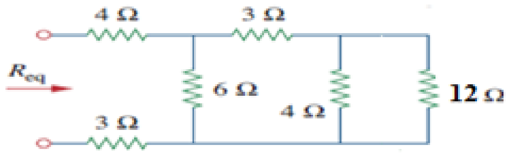
2) Find the Req for the circuit shown in below figure.



fig(a)

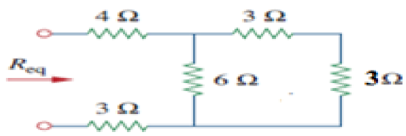
Solution:

In the given network 4 ohms, 5 ohms and 3 ohms comes in series then equivalent resistance is 4+5 + 3 = 12 ohms



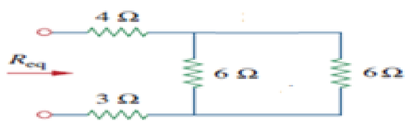
fig(b)

From fig(b), 4 ohms and 12 ohms are in parallel, equivalent is 3 ohms



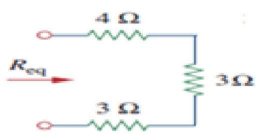
fig(c)

From fig(c), 3 ohms and 3 ohms are in series, equivalent resistance is 6 ohms



fig(d)

From fig(d), 6 ohms and 6 ohms are in parallel, equivalent resistance is 3 ohms



fig(e)

From fig(e), 4 ohms, 3 ohms and 3 ohms are in series .Hence Req = 4+ 3+ 3 =10 ohms

Star–to-Delta and Delta-to-Star Transformations for Resistive Networks:

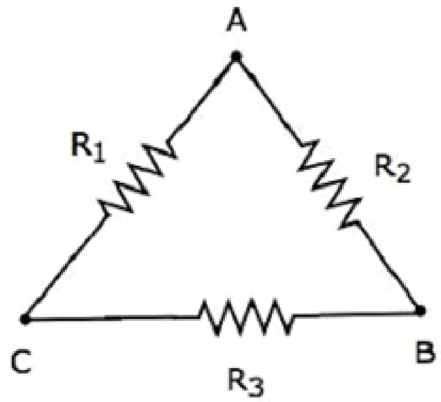
Delta to Star Transformation

In the previous chapter, we discussed an example problem related equivalent resistance. There, we calculated the equivalent resistance between the terminals A & B of the given electrical network easily. Because, in every step, we got the combination of resistors that are connected in either series form or parallel form.

However, in some situations, it is difficult to simplify the network by following the previous approach. For example, the resistors connected in either delta (δ) form or star form. In such situations, we have to convert the network of one form to the other in order to simplify it further by using series combination or parallel combination. In this chapter, let us discuss about the Delta to Star Conversion.

Delta Network

Consider the following delta network as shown in the following figure.



The following equations represent the equivalent resistance between two terminals of delta network, when the third terminal is kept open.

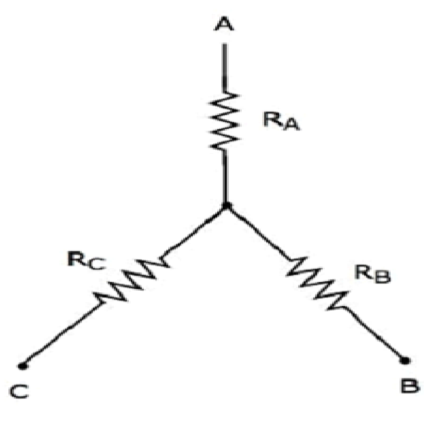
$$R_{AB} = \frac{(R_1 + R_3)R_2}{R_1 + R_2 + R_3}$$

$$R_{BC} = \frac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3}$$

$$R_{CA} = \frac{(R_2 + R_3)R_1}{R_1 + R_2 + R_3}$$

Star Network

The following figure shows the equivalent star network corresponding to the above delta network.



The following equations represent the equivalent resistance between two terminals of star network, when the third terminal is kept open.

$$R_{AB} = R_A + R_B$$

$$R_{BC} = R_B + R_C$$

$$R_{CA} = R_C + R_A$$

Star Network Resistances in terms of Delta Network Resistances

We will get the following equations by equating the right-hand side terms of the above equations for which the left-hand side terms are same.

$$R_A + R_B = \frac{(R_1+R_3)R_2}{R_1+R_2+R_3} \qquad \text{Equation 1}$$

$$R_B + R_C = \frac{(R_1+R_2)R_3}{R_1+R_2+R_3} \qquad \text{Equation 2}$$

$$R_C + R_A = \frac{(R_2+R_3)R_1}{R_1+R_2+R_3} \qquad \text{Equation 3}$$

By adding the above three equations, we will get

$$2(R_A + R_B + R_C) = \frac{2(R_1R_2 + R_2R_3 + R_3R_1)}{R_1 + R_2 + R_3}$$

$$\Rightarrow R_A + R_B + R_C = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1+R_2+R_3} \qquad \text{Equation 4}$$

Subtract Equation 2 from Equation 4.

$$R_A + R_B + R_C - (R_B + R_C) = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1+R_2+R_3} - \frac{(R_1+R_2)R_3}{R_1+R_2+R_3}$$

$$R_A = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

By subtracting Equation 3 from Equation 4, we will get

$$R_B = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

By subtracting Equation 1 from Equation 4, we will get

$$R_C = \frac{R_3 R_1}{R_1 + R_2 + R_3}$$

By using the above relations, we can find the resistances of star network from the resistances of delta network. In this way, we can convert a delta network into a star network.

Star to Delta Transformation

In the previous chapter, we discussed about the conversion of delta network into an equivalent star network. Now, let us discuss about the conversion of star network into an equivalent delta network. This conversion is called as Star to Delta Conversion.

In the previous chapter, we got the resistances of star network from delta network as

$$R_A = \frac{R_1 R_2}{R_1+R_2+R_3} \qquad \text{Equation 1}$$

$$R_B = \frac{R_2 R_3}{R_1+R_2+R_3} \qquad \text{Equation 2}$$

$$R_C = \frac{R_3 R_1}{R_1+R_2+R_3} \qquad \text{Equation 3}$$

Delta Network Resistances in terms of Star Network Resistances

Let us manipulate the above equations in order to get the resistances of delta network in terms of resistances of star network.

- Multiply each set of two equations and then add.

$$R_A R_B + R_B R_C + R_C R_A = \frac{R_1 R_2^2 R_3 + R_2 R_3^2 R_1 + R_3 R_1^2 R_2}{(R_1 + R_2 + R_3)^2}$$

$$\Rightarrow R_A R_B + R_B R_C + R_C R_A = \frac{R_1 R_2 R_3 (R_1 + R_2 + R_3)}{(R_1 + R_2 + R_3)^2}$$

$$\Rightarrow R_A R_B + R_B R_C + R_C R_A = \frac{R_1 R_2 R_3}{R_1 + R_2 + R_3}$$

Equation 4

- By dividing Equation 4 with Equation 2, we will get

$$\frac{R_A R_B + R_B R_C + R_C R_A}{R_B} = R_1$$

$$\Rightarrow R_1 = R_C + R_A + \frac{R_C R_A}{R_B}$$

- By dividing Equation 4 with Equation 3, we will get

$$R_2 = R_A + R_B + \frac{R_A R_B}{R_C}$$

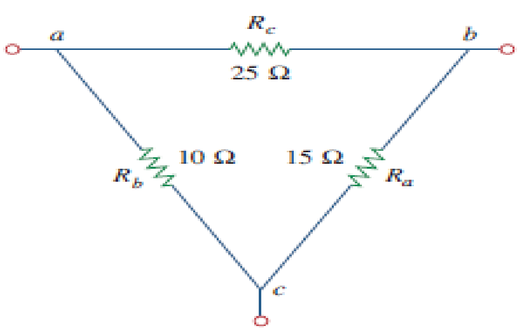
- By dividing Equation 4 with Equation 1, we will get

$$R_3 = R_B + R_C + \frac{R_B R_C}{R_A}$$

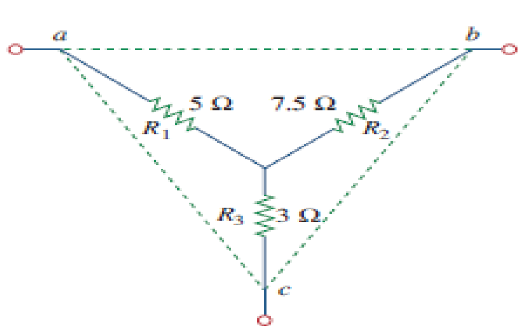
By using the above relations, we can find the resistances of delta network from the resistances of star network. In this way, we can convert star network into delta network.

Example problems:

1) Convert the Delta network in Fig.(a) to an equivalent star network
Solution:



(a)



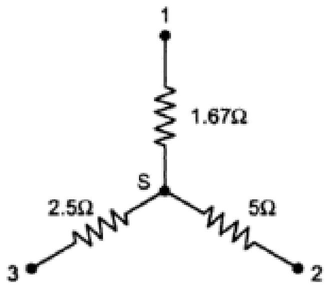
(b)

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = \frac{250}{50} = 5\ \Omega$$

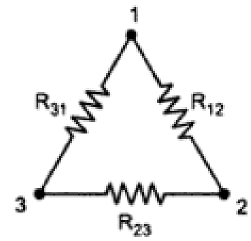
$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\ \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\ \Omega$$

2) Convert the star network in fig(a) to delta network



fig(a)



fig(b)

Solution: The equivalent delta for the given star is shown in fig(b), where

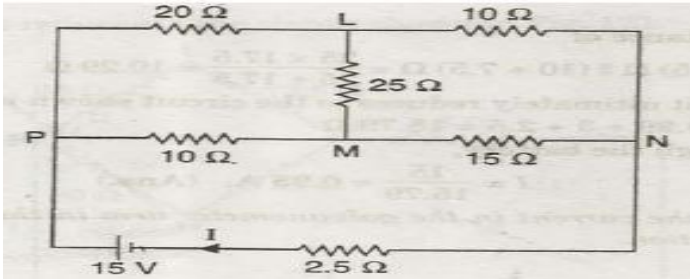
$$R_{12} = 1.67 + 5 + \frac{1.67 \times 5}{2.5} = 1.67 + 5 + 3.33 = \mathbf{10\ \Omega}$$

$$R_{23} = 5 + 2.5 + \frac{5 \times 2.5}{1.67} = 5 + 2.5 + 7.5 = \mathbf{15\ \Omega}$$

$$R_{31} = 2.5 + 1.67 + \frac{2.5 \times 1.67}{5} = 2.5 + 1.67 + 0.833$$

$$= \mathbf{5\ \Omega}$$

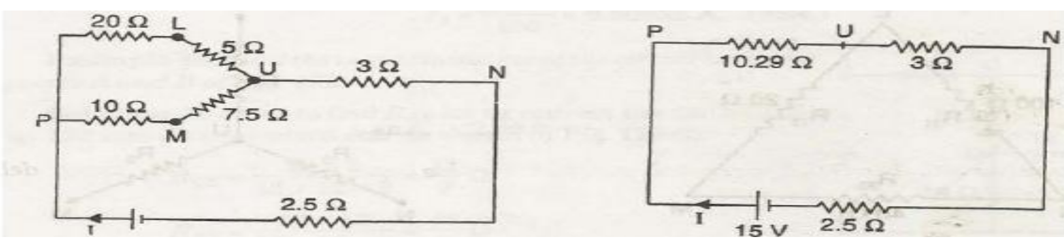
3) Determine the total current I in the given circuit.



Solution: Delta connected resistors 25 ohms, 10 ohms and 15 ohms are converted in to star as shown in given figure.

$R_1 = R_{12}\, R_{31} / R_{12} + R_{23} + R_{31} = 10 \times 25 / 10 + 15 + 25 = 5$ ohms
$R_2 = R_{23}\, R_{12} / R_{12} + R_{23} + R_{31} = 15 \times 10 / 10 + 15 + 25 = 3$ ohms
$R_3 = R_{31}\, R_{23} / R_{12} + R_{23} + R_{31} = 25 \times 15 / 10 + 15 + 25 = 7.5$ ohms

The given circuit thus reduces to the circuit shown in below fig.



The equivalent resistance of
$(20 + 5)$ ohms || $(10 + 7.5)$ ohms $= 25 \times 17.5 / 25 + 17.5 = 10.29$ ohms
Total resistance $= 10.29 + 3 + 2.5 = 15.79$ ohms
Hence the total current through the battery,
$I = 15 / 15.79 = 0.95$ A

Mesh Analysis:

Mesh analysis provides general procedure for analyzing circuits using mesh currents as the circuit variables. Mesh Analysis is applicable only for planar networks. It is preferably useful for the circuits that have many loops .This analysis is done by using KVL and Ohm's law.

In Mesh analysis, we will consider the currents flowing through each mesh. Hence, Mesh analysis is also called as Mesh-current method.

A branch is a path that joins two nodes and it contains a circuit element. If a branch belongs to only one mesh, then the branch current will be equal to mesh current.

If a branch is common to two meshes, then the branch current will be equal to the sum (or difference) of two mesh currents, when they are in same (or opposite) direction.

Procedure of Mesh Analysis

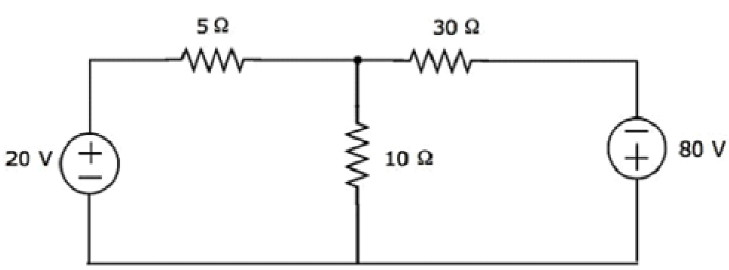
Follow these steps while solving any electrical network or circuit using Mesh analysis.

- **Step 1** − Identify the meshes and label the mesh currents in either clockwise or anti-clockwise direction.
- **Step 2** − Observe the amount of current that flows through each element in terms of mesh currents.
- **Step 3** − Write mesh equations to all meshes. Mesh equation is obtained by applying KVL first and then Ohm’s law.
- **Step 4** − Solve the mesh equations obtained in Step 3 in order to get the mesh currents.

Now, we can find the current flowing through any element and the voltage across any element that is present in the given network by using mesh currents.

Example

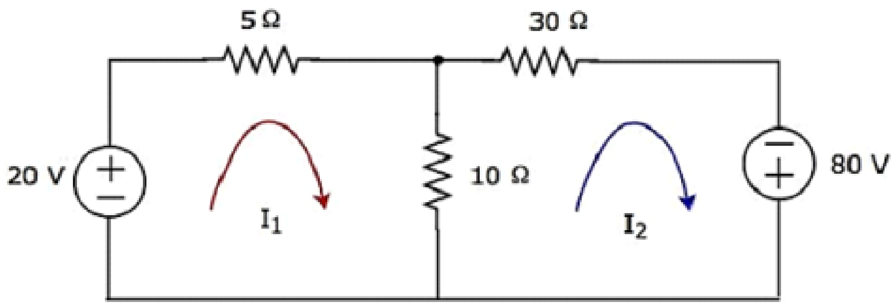
Find the voltage across 30 Ω resistor using Mesh analysis.



Step 1 − There are two meshes in the above circuit. The mesh currents I_1 and I_2 are considered in clockwise direction. These mesh currents are shown in the following figure.

Step 2 − The mesh current I_1 flows through 20 V voltage source and 5 Ω resistor. Similarly, the mesh current I_2 flows through 30 Ω resistor and -80 V voltage source. But, the difference of two mesh currents, I_1 and I_2, flows through 10 Ω resistor, since it is the common branch of two meshes.

Step 3 − In this case, we will get two mesh equations since there are two meshes in the given circuit. When we write the mesh equations, assume the mesh current of that particular mesh as greater than all other mesh currents of the circuit. The mesh equation of first mesh is



$$20 - 5I_1 - 10(I_1 - I_2) = 0$$

$$\Rightarrow 20 - 15I_1 + 10I_2 = 0$$

$$\Rightarrow 10I_2 = 15I_1 - 20$$

Divide the above equation with 5.

$$2I_2 = 3I_1 - 4$$

Multiply the above equation with 2.

$$4I_2 = 6I_1 - 8 \quad \text{Equation 1}$$

The **mesh equation** of second mesh is

$$-10(I_2 - I_1) - 30I_2 + 80 = 0$$

Divide the above equation with 10.

$$-(I_2 - I_1) - 3I_2 + 8 = 0$$

$$\Rightarrow -4I_2 + I_1 + 8 = 0$$

$$4I_2 = I_1 + 8 \quad \text{Equation 2}$$

Step 4 − Finding mesh currents I_1 and I_2 by solving Equation 1 and Equation 2.

The left-hand side terms of Equation 1 and Equation 2 are the same. Hence, equate the right-hand side terms of Equation 1 and Equation 2 in order find the value of I_1.

$$6I_1 - 8 = I_1 + 8$$

$$\Rightarrow 5I_1 = 16$$

$$\Rightarrow I_1 = \frac{16}{5} A$$

Substitute I_1 value in Equation 2.

$$4I_2 = \frac{16}{5} + 8$$

$$\Rightarrow 4I_2 = \frac{56}{5}$$

$$\Rightarrow I_2 = \frac{14}{5} A$$

So, we got the mesh currents I_1 and I_2 as $\frac{16}{5}$ A and $\frac{14}{5}$ A respectively.

Step 5 – The current flowing through 30 Ω resistor is nothing but the mesh current I_2 and it is equal to $\frac{14}{5}$ A. Now, we can find the voltage across 30 Ω resistor by using Ohm's law.

$$V_{30\Omega} = I_2 R$$

Substitute the values of I_2 and R in the above equation.

$$V_{30\Omega} = \left(\frac{14}{5}\right)30$$

$$\Rightarrow V_{30\Omega} = 84V$$

Therefore, the voltage across 30 Ω resistor of the given circuit is84 V.

Note 1 − From the above example, we can conclude that we have to solve 'm' mesh equations, if the electric circuit is having 'm' meshes. That's why we can choose Mesh analysis when the number of meshes is less than the number of principal nodes (except the reference node) of any electrical circuit.

Note 2 − We can choose either Nodal analysis or Mesh analysis, when the number of meshes is equal to the number of principal nodes (except the reference node) in any electric circuit.

Nodal analysis:

Nodal analysis provides another general procedure for analyzing circuits nodal voltages as the circuit variables. It is preferably useful for the circuits that have many no. of nodes. It is applicable for the both planar and non planar circuits. This analysis is done by using KCL and Ohm's law.

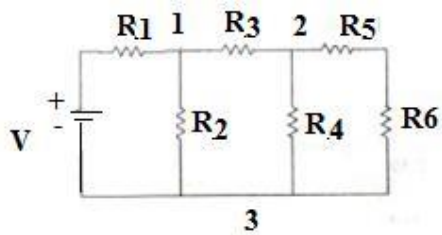
Node: It is a junction at which two or more branches are interconnected.

Simple Node: Node at which only two branches are interconnected.

Principal Node: Node at which more than two branches are interconnected.

Nodal analysis with example:

Determination of node voltages:

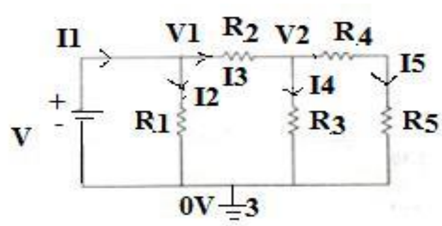


Procedure:

Step (1): Identify the no. nodes, simple nodes and principal nodes in the given circuit. Among all the nodes one node is taken as reference node. Generally bottom is taken as reference node. The potential at the reference node is 0v.

In the given circuit there are 3 principal nodes in which node (3) is the reference node.

Step (2): Assign node voltages to the all the principal nodes except reference node and assign branch currents to all branches.



Step (3): Apply KCL to those principal nodes for nodal equations and by using ohm's law express the node voltages in terms of branch current.

Applying KCL to node (1) ----**I1=I2+I3**

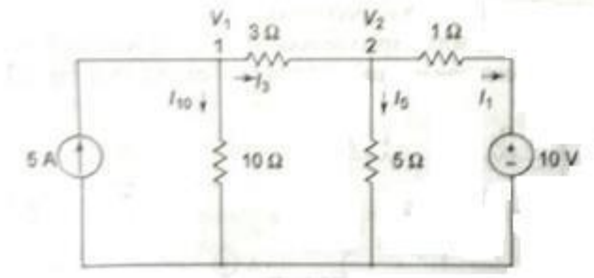
Using ohm's law, we get **(V-V1)/R1 =(V2-0)/R2 +(V1-V2)/R3.......... (1)**

Applying KCL to node (2) ----**I3=I4 +I5**

Using ohm's law, we get **(V1-V2)/R3 =(V4-0)/R4 +(V5-0)/R5 .... (2)**

Step(4): Solve the above nodal equations to get the node voltages.

Example: Write the node voltage equations and find out the currents in each branch of the circuit shown in the figure below.



Solution:

The node voltages and the directions of the branch currents are assigned as shown in given figure. Applying KCL to node 1, we get: **5 = I10+ I3**

5= (V1-0)/10 +(V1-V2)/3

V1(13/30) -V2(1/3) = 5 ....(1)

Applying KCL to node 2, we get: **I3= I5 + I1**

(V1-V2)/3 = (V2 -0)/5 + (V2-10) /1

V1(1/3)-V2(23/15) = -10................ (2)

Solving the these two equations for V1 and V2 we get :

V1 = 19.85 V and V2 = 10.9 V and the currents are :
I10= V1/10 = 1.985A
I3 = (V1-V2)/3 = (19.85-10.9)/3 = 2.98A
I5 = V2/5 = 10.9/5 =2.18A
I1 = (V2-10) = (10.9-10)/1 = 0.9A

Network Theorems:

Introduction:

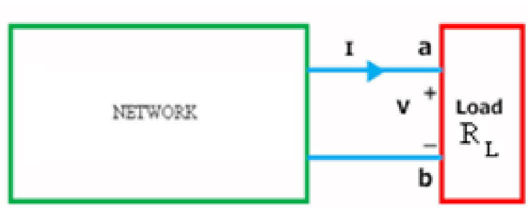
Any complicated network i.e. several sources, multiple resistors are present if the single element response is desired then use the network theorems. Network theorems are also can be termed as network reduction techniques. Each and every theorem got its importance of solving network. Let us see some important theorems with DC and AC excitation with detailed procedures.

Thevenin's Theorem and Norton's theorem (Introduction) :

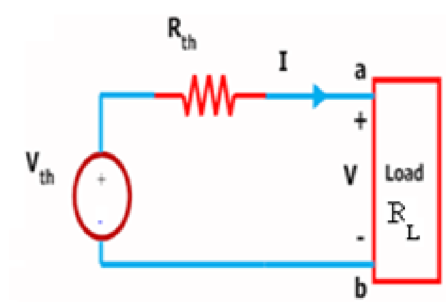
Thevenin's Theorem and Norton's theorem are two important theorems in solving Network problems having many active and passive elements. Using these theorems the networks can be reduced to simple equivalent circuits with one active source and one element. In circuit analysis many a times the current through a branch is required to be found when it's value is changed with all other element values remaining same. In such cases finding out every time the branch current using the conventional mesh and node analysis methods is quite awkward and time consuming. But with the simple equivalent circuits (with one active source and one element) obtained using these two theorems the calculations become very simple. Thevenin's and Norton's theorems are dual theorems.

Thevenin's Theorem Statement:

Any linear, bilateral two terminal network consisting of sources and resistors(Impedance),can be replaced by an equivalent circuit consisting of a voltage source in series with a resistance (Impedance).The equivalent voltage source V_{Th} is the open circuit voltage looking into the terminals(with concerned branch element removed) and the equivalent resistance R_{Th} while all sources are replaced by their internal resistors at ideal condition i.e. voltage source is short circuit and current source is open circuit.



(a)



(b)

Figure (a) shows a simple block representation of a network with several active / passive elements with the load resistance R_L connected across the terminals 'a & b' and figure (b) shows the Thevenin's equivalent circuit with V_{Th} connected across R_{Th} & R_L .

Main steps to find out V_{Th} and R_{Th} :

1. The terminals of the branch/element through which the current is to be found out are marked as say a & b after removing the concerned branch/element

2. Open circuit voltage V_{OC} across these two terminals is found out using the conventional network mesh/node analysis methods and this would be V_{Th} .

3. Thevenin's resistance R_{Th} is found out by the method depending upon whether the network contains dependent sources or not.

   a. With dependent sources: $R_{Th} = V_{oc} / I_{sc}$

   b. Without dependent sources : R_{Th} = Equivalent resistance looking into the concerned terminals with all voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited)

4. Replace the network with V_{Th} in series with R_{Th} and the concerned branch resistance (or) load resistance across the load terminals (A&B) as shown in below fig.

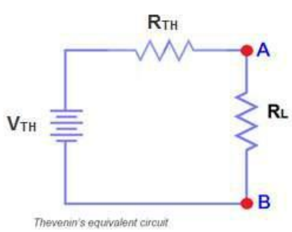
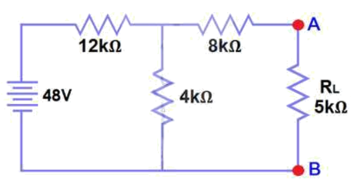


Fig.(a)

Example: Find V_{TH}, R_{TH} and the load current and load voltage flowing through R_L resistor as shown in fig. by using Thevenin's Theorem?



Solution:

The resistance R_L is removed and the terminals of the resistance R_L are marked as A & B as shown in the fig. (1)

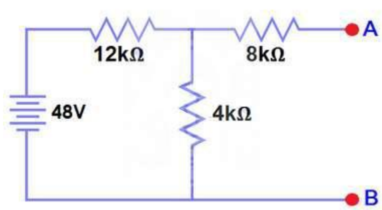


Fig.(1)

Calculate / measure the Open Circuit Voltage. This is the Thevenin Voltage (V_{TH}). We have already removed the load resistor from fig.(a), so the circuit became an open circuit as shown in fig (1). Now we have to calculate the Thevenin's Voltage. Since 3mA Current flows in both 12kΩ and 4kΩ resistors as this is a series circuit because current will not flow in the 8kΩ resistor as it is open. So 12V (3mA x 4kΩ) will appear across the 4kΩ resistor. We also know that current is not flowing through the 8kΩ resistor as it is open circuit, but the 8kΩ resistor is in parallel with 4k resistor. So the same voltage (i.e. 12V) will appear across the 8kΩ resistor as 4kΩ resistor. Therefore 12V will appear across the AB terminals.

So, V_{TH} = 12V

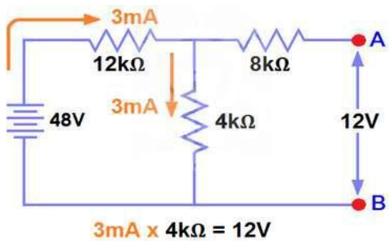
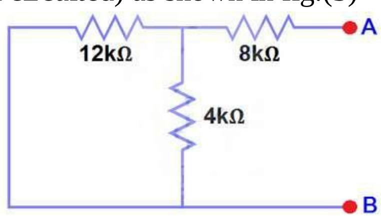


Fig (2)

All voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited) as shown in fig.(3)



Fig(3)

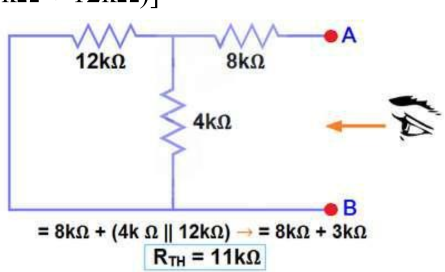
Calculate /measure the Open Circuit Resistance. This is the Thevenin's Resistance (R_{TH})We have Reduced the 48V DC source to zero is equivalent to replace it with a short circuit as shown in figure (3) We can see that 8kΩ resistor is in series with a parallel connection of 4kΩ resistor and 12k Ω resistor. i.e.:

8kΩ + (4k Ω || 12kΩ) ….. (|| = in parallel with)

R_{TH} = 8kΩ + [(4kΩ x 12kΩ) / (4kΩ + 12kΩ)]

R_{TH} = 8kΩ + 3kΩ

R_{TH} = 11kΩ



Fig(4)

Connect the R_{TH} in series with Voltage Source V_{TH} and re-connect the load resistor across the load terminals(A&B) as shown in fig (5) i.e. Thevenin's circuit with load resistor. This is the Thevenin's equivalent circuit.

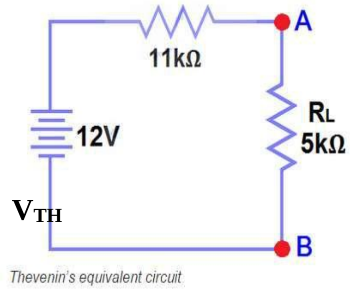


Fig (5)

Now apply Ohm's law and calculate the load current from fig 5.

$I_L = V_{TH}/ (R_{TH} + R_L) = 12V / (11k\Omega + 5k\Omega) = 12/16k\Omega$

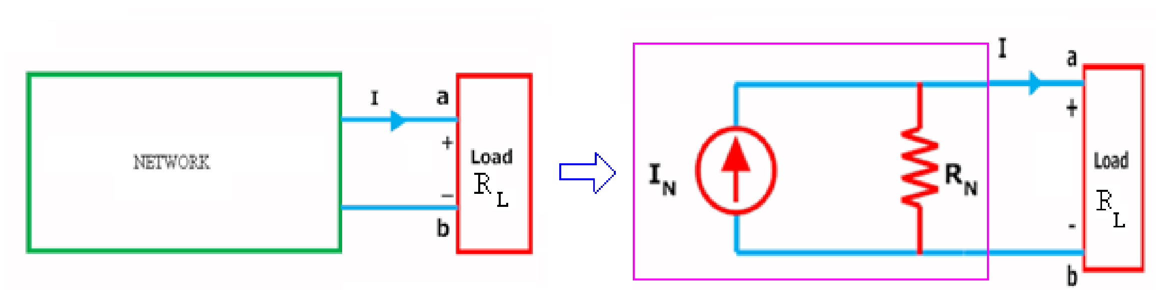
$I_L = 0.75mA$

And $V_L = I_L x R_L = 0.75mA \times 5k\Omega$

$V_L = 3.75V$

Norton's Theorem Statement:

Any linear, bilateral two terminal network consisting of sources and resistors(Impedance),can be replaced by an equivalent circuit consisting of a current source in parallel with a resistance (Impedance),the current source being the short circuited current across the load terminals and the resistance being the internal resistance of the source network looking through the open circuited load terminals.



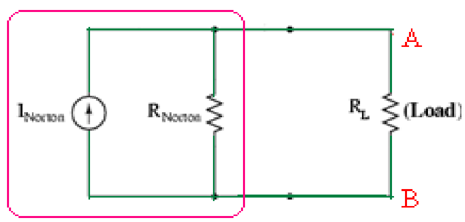
(a) (b)

Figure (a) shows a simple block representation of a network with several active / passive elements with the load resistance $\mathbf{R_L}$ connected across the terminals **'a & b'** and figure (b) shows the **Norton equivalent circuit** with $\mathbf{I_N}$ connected across $\mathbf{R_N}$ & $\mathbf{R_L}$ **.**

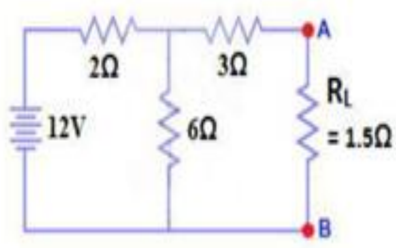
Main steps to find out I_N and R_N:

- The terminals of the branch/element through which the current is to be found out are marked as say **a & b** after removing the concerned branch/element.

- Open circuit voltage $\mathbf{V_{OC}}$ across these two terminals and $\mathbf{I_{SC}}$ through these two terminals are found out using the conventional network mesh/node analysis methods and they are same as what we obtained in Thevenin's equivalent circuit.
- Next **Norton resistance** $\mathbf{R_N}$ is found out depending upon whether the network contains dependent sources or not.
  a) With dependent sources: $\mathbf{R_N = V_{oc} / I_{sc}}$
  b) Without dependent sources : $\mathbf{R_N}$ **=** Equivalent resistance looking into the concerned terminals with all voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited)
- Replace the network with $\mathbf{I_N}$ in parallel with $\mathbf{R_N}$ and the concerned branch resistance across the load terminals(A&B) as shown in below fig

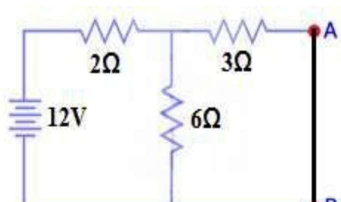


Example: Find the current through the resistance $\mathbf{R_L}$ (1.5 Ω) of the circuit shown in the figure (a) below using Norton's equivalent circuit.



Fig(a)

Solution: To find out the Norton's equivalent ckt we have to find out $\mathbf{I_N = I_{sc}}$ **,**$\mathbf{R_N = V_{oc} / I_{sc}}$**.** Short the 1.5Ω load resistor as shown in (Fig 2), and Calculate / measure the Short Circuit Current. This is the Norton Current (I_N).



Fig(2)

We have shorted the AB terminals to determine the Norton current, I_N. The 6Ω and 3Ω are then in parallel and this parallel combination of 6Ω and 3Ω are then in series with 2Ω.So the Total Resistance of the circuit to the Source is:-

2Ω + (6Ω || 3Ω) ….. (|| = in parallel with)

R_T = 2Ω + [(3Ω x 6Ω) / (3Ω + 6Ω)]

R_T = 2Ω + 2Ω

R_T = 4Ω

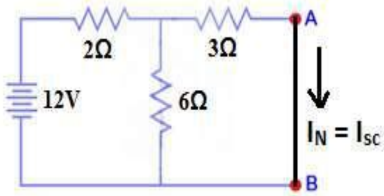
I_T = V / R_T

I_T = 12V / 4Ω= 3A..

Now we have to find I_{SC} = I_N… Apply CDR… (Current Divider Rule)…

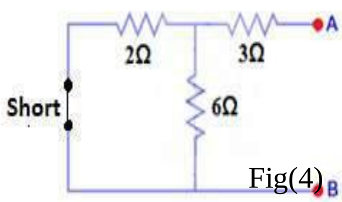
I_{SC} = I_N = 3A x [(6Ω / (3Ω + 6Ω)] = 2A.

I_{SC}= I_N = 2A.



Fig(3)

All voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited) and Open Load Resistor. as shown in fig.(4)



Fig(4)

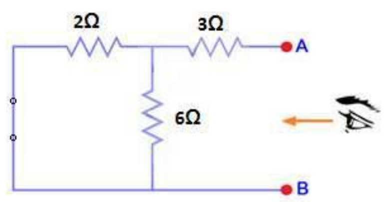
Calculate /measure the Open Circuit Resistance. This is the Norton Resistance (R_N) We have Reduced the 12V DC source to zero is equivalent to replace it with a short circuit as shown in fig(4), We can see that 3Ω resistor is in series with a parallel combination of 6Ω resistor and 2Ω resistor. i.e.:

3Ω + (6Ω || 2Ω) ….. (|| = in parallel with)

R_N = 3Ω + [(6Ω x 2Ω) / (6Ω + 2Ω)]

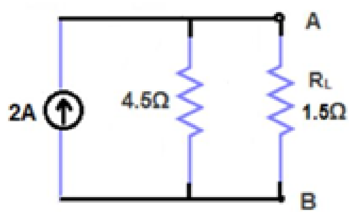
R_N = 3Ω + 1.5Ω

R_N = 4.5Ω



Fig(5)

Connect the R_N in Parallel with Current Source I_N and re-connect the load resistor. This is shown in fig (6) i.e. Norton Equivalent circuit with load resistor.



Fig(6)

Now apply the Ohm's Law and calculate the load current through Load resistance across the terminals A&B. Load Current through Load Resistor is

$I_L = I_N$ x $[R_N / (R_N + R_L)]$

$I_{L=}$ 2A x (4.5Ω /4.5Ω +1.5kΩ)

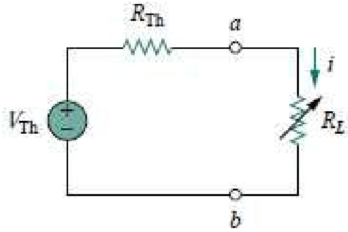
I_L = 1.5A **I_L = 1. 5A**

Maximum Power Transfer Theorem:

In many practical situations, a circuit is designed to provide power to a load. While for electric utilities, minimizing power losses in the process of transmission and distribution is critical for Efficiency and economic reasons, there are other applications in areas such as communications where it is desirable to maximize the power delivered to a load. electrical applications with electrical loads such as Loud speakers, antennas, motors etc. it would be required to find out the condition under which maximum power would be transferred from the circuit to the load.

Maximum Power Transfer Theorem Statement:

Any linear, bilateral two terminal network consisting of a resistance load, being connected to a dc network, receives maximum power when the load resistance is equal to the internal resistance (Thevenin's equivalent resistance) of the source network as seen from the load terminals.



According to Maximum Power Transfer Theorem, for maximum power transfer from the network to the load resistance , $\mathbf{R_L}$ must be equal to the source resistance i.e. Network's Thevenin equivalent resistance $\mathbf{R_{Th}}$ **. i.e.** $\mathbf{R_L = R_{Th}}$

The load current **I** in the circuit shown above is given by,

$$I = \frac{V_{TH}}{R_{TH}+R_L}$$

The power delivered by the circuit to the load:

$$P = I^2R = \frac{V_{TH}^2}{(R_{TH}+R_L)^2} R_L$$

The condition for maximum power transfer can be obtained by differentiating the above expression for power delivered with respect to the load resistance (Since we want to find out the value of $\mathbf{R_L}$ for maximum power transfer) and equating it to zero as :

$$\frac{\partial P}{\partial R_L} = 0 = \frac{V_{TH}^2}{(R_{TH}+R_L)^2} - \frac{2V_{TH}^2}{(R_{TH}+R_L)^3} R_L = 0$$

Simplifying the above equation, we get:

$$(R_{TH} + R_L) - 2R_L = 0 \Longrightarrow R_L = R_{TH}$$

Under the condition of maximum power transfer, the power delivered to the load is given by :

$$P_{MAX} = \frac{V_{TH}^2}{(R_L+R_L)^2} \times R_L = \frac{V_{TH}^2}{4R_L}$$

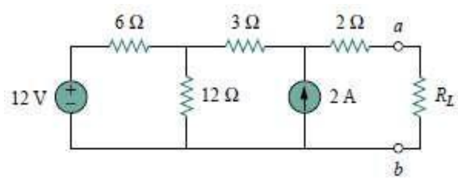
Under the condition of maximum power transfer, the efficiency $\boldsymbol{\eta}$ of the network is then given by:

$$P_{LOSS} = \frac{V_{TH}^2}{(R_L+R_L)^2} \times R_{TH} = \frac{V_{TH}^2}{4R_L}$$

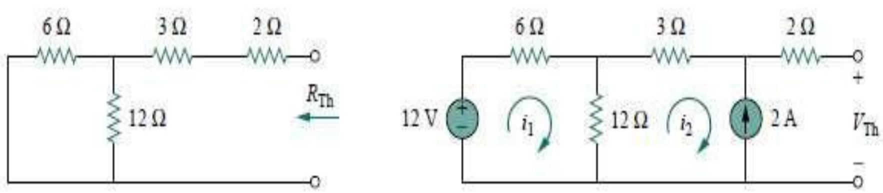
$$\boldsymbol{\eta} = \frac{\text{output}}{\text{input}} = \frac{\frac{V_{TH}^2}{4R_L}}{\left(\frac{V_{TH}^2}{4R_L} + \frac{V_{TH}^2}{4R_L}\right)} = 0.50$$

For maximum power transfer the load resistance should be equal to the Thevenin's equivalent resistance (or Norton equivalent resistance) of the network to which it is connected . Under the condition of maximum power transfer the efficiency of the system is 50 %.

Example: Find the value of $\mathbf{R_L}$ for maximum power transfer in the circuit of Fig. Find the maximum power.?



Solution: We need to find the Thevenin's resistance R_{Th} and the Thevenin's voltage V_{Th} across the terminals *a-b*. To get R_{Th}, we use the circuit in Fig. (a)



(a) (b)

R_{Th}= 2 + 3 + (6 // 12)=5+($\frac{6\times12}{6+12}$)=5+4=9Ω

To get V_{Th}, we consider the circuit in Fig.(b).Applying mesh analysis,

$-12 + 18i_1 - 12i_2 = 0$,

$i_2 = -2$ A,

Solving for i1, we get $i_1 = -2/3$.

Applying KVL around the outer loop to get V_{Th} across terminals *a*-*b*, we obtain,

$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0$

V_{Th}= 22 V

For maximum power transfer, $R_L = R_{Th}$ = 9Ω and the maximum power is,

$$P_{MAX} = \frac{V_{TH}^2}{4R_L} = \frac{22\times22}{4\times9} = 13.44\text{W}$$

Superposition Theorem:

The principle of superposition helps us to analyze a linear circuit with more than one current or voltage sources sometimes it is easier to find out the voltage across or current in a branch of the circuit by considering the effect of one source at a time by replacing the other sources with their ideal internal resistances.

Superposition Theorem Statement:

Any linear, bilateral two terminal network consisting of more than one sources, The total current or voltage in any part of a network is equal to the algebraic sum of the currents or voltages in the required branch with each source acting individually while other sources are replaced by their ideal internal resistances. (i.e. Voltage sources by a short circuit and current sources by open circuit)

Steps to Apply Super position Principle:

1. Replace all independent sources with their internal resistances except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Example: By Using the superposition theorem find I in the circuit shown in figure?

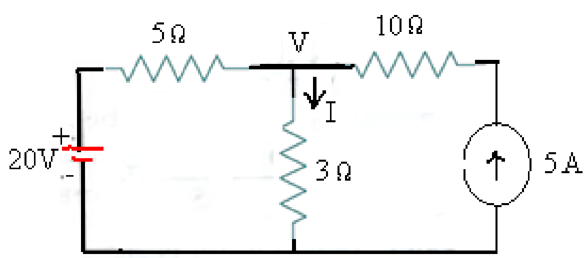


Fig.(a)

Solution: Applying the superposition theorem, the current $\mathbf{I_2}$ in the resistance of 3 Ω due to the voltage source of 20V alone, with current source of 5A open circuited [as shown in the figure.1 below] is given by :

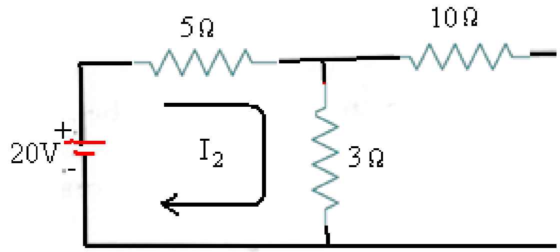


Fig.1

$I_2 = 20/(5+3) = 2.5A$

Similarly the current I_5 in the resistance of 3 Ω due to the current source of 5A alone with voltage source of 20V short circuited [as shown in the figure.2 below] is given by :

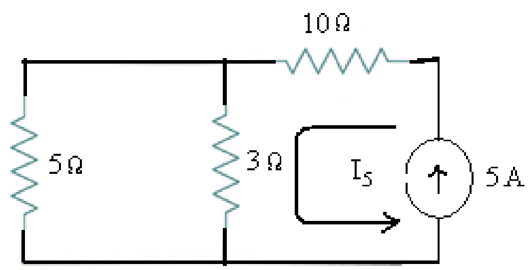


Fig.2

$I_5 = 5 \times 5/(3+5) = 3.125$ A

The total current passing through the resistance of 3Ω is then = $I_2 + I_5 = 2.5 + 3.125 =$ **5.625 A**

Let us verify the solution using the basic nodal analysis referring to the node marked with V in fig.(a).Then we get :

$$\frac{V-20}{5} + \frac{V}{3} = 5$$

3V-60+5V=15× 5

8V-60=75

8V=135

V=16.875

The current I passing through the resistance of 3Ω =V/3 = 16.875/3 = **5.625 A.**

UNIT-III
SINGLE PHASE A.C. CIRCUITS

- **Average value, R.M.S. value, form factor and peak factor for sinusoidal wave form.**
- **Complex and Polar forms of representation.**
- **Steady State Analysis of series R-L-C circuits.**
- **Concept of Reactance, Impedance, Susceptance, Admittance, Phase and Phase difference.**
- **Concept of Power Factor, Real, Reactive and Complex power.**
- **Illustrative Problems.**

2.1 ALTERNATING QUANTITY

An alternating quantity is that which acts in alternate directions and whose magnitude undergoes a definite cycle of changes in definite intervals of time. When a simple loop revolves in a magnetic field, an alternating emf is induced in the loop. If the loop revolves with an uniform angular velocity the induced alternating emf is sinusoidal in nature. The alternating quantity may have various other wave forms like triangular, semicircular, stepped, distorted, etc. as shown in Fig. 2.1(a), (b), (c) and (d), respectively. The graph repeats after regular intervals. One complete set of positive and negative values of an alternating quantity is called a cycle. The important alternating quantities, $f(t)$ that will be discussed in the chapter are current and voltage.

2.2 ALTERNATING VOLTAGE

Alternating voltage may be generated by

(A) By rotating a coil in a stationary magnetic field.

(B) By rotating a magnetic field within a stationary coil.

The value of the voltage generated in each case depends on:

(i) The number of turns in the coils.

(ii) The strength of the field.

(iii) The speed at which the coil or magnetic field rotates.

- (a) Maximum flux links with the coil when its plane is in vertical position (perpendicular) to the direction of flux between the poles.
- (b) When the plane of a coil is horizontal no flux links with the coil.

2.5 CYCLE

A cycle may be defined as one complete set of positive and negative values of an alternating quantity repeating at equal intervals. Each complete cycle is spread over 360° electrical as shown in Fig. 2.5.

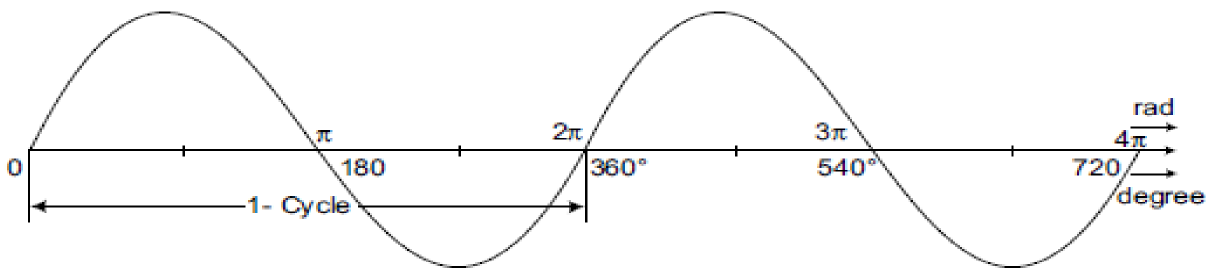


Fig. 2.5

2.6 PERIODIC TIME

The time taken by an alternating quantity in seconds to trace one complete cycle is called periodic time or time-period. It is usually denoted by symbol T.

2.7 FREQUENCY

The number of cycles per second is called frequency and is denoted by symbol f.

Thus, $$f = \frac{1}{T}$$

or, $$T = \frac{1}{f}$$

If the angular velocity w is expressed in radians per second, then

$$\omega = \frac{2\pi}{T} = 2\pi f$$

2.8' PHASE DIFFERENCE

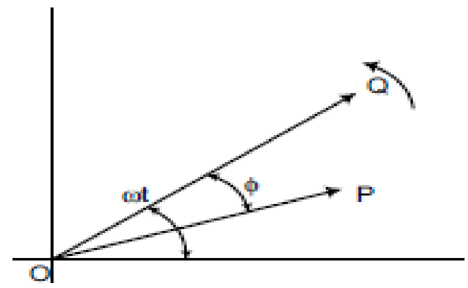


Fig. 2.6

Let OP and OQ be the two vectors (more preferred to be called phasors) representing two alternating quantities of the same frequency at any instant. The angle ϕ between them is called the phase angle.

The direction of rotation in counter clock-wise direction is usually taken as positive. If OQ and OP represent voltage and current vectors, then

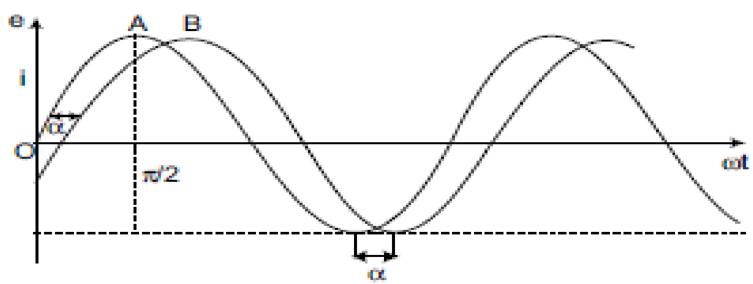
$$e = OQ \sin \omega t$$

and,
$$i = OP \sin (\omega t - \phi)$$

where, ϕ is called the phase difference. In above phasor OQ is said to lead the phasor OP.

The 'phase' of an AC wave may be defined as its position with respect to a reference axis or reference wave.

Phase angle as the angle of lead or lag with respect to reference axis or with respect to another wave.



→ Phase shift = α degrees
→ *A* is ahead of *B* because *A* attains its maxima or minima before *B*
→ *A* leads *B*
→ *B* lags *A*

Fig. 2.7

- A is α degree ahead of B.
- A attains its maxima α degrees before B or $\dfrac{\alpha}{2\pi}T$ second or

$$\alpha = \omega t \left\{ t = \frac{\alpha}{\omega} \right\} \text{ sec before } B.$$

$$\begin{cases} \alpha = \omega t \\ t = \dfrac{\alpha}{\omega} = \dfrac{\alpha}{2\pi} T \text{ sec} \end{cases}$$

2.9 PHASOR NOTATION

Sinusoidal quantities can be represented by a function.

$$f(t) = V_m e^{j\omega t} = V_m\, e^{j\theta} = V_m \angle\theta$$

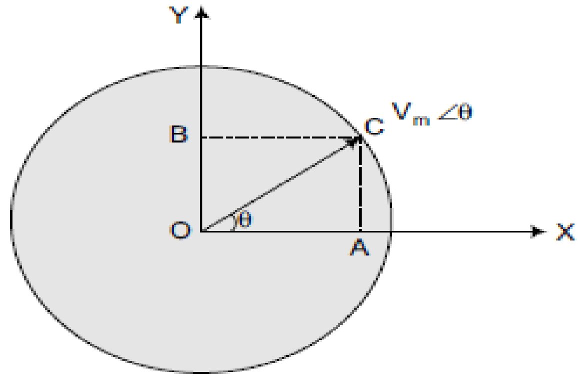


Fig. 2.8

This function has constant magnitude V_m and as ωt moves through 0 ot 2π radians.

$$OA = V_m \cos\theta,\ \ OB = V_m \sin\theta$$

$$OC = (OA) + j(OB),\ \ \theta = \tan^{-1}\frac{OB}{OA}$$

by Euler theorem $e^{j\theta} = \cos\theta + j\sin\theta$

$$\boxed{V = V_m \angle\theta = V_m(\cos\theta + j\sin\theta)}$$

In rectangular form

$$\overline{OC} = \overline{OA} + j\,\overline{OB}$$

$$|OC| = \overline{x} + j\,\overline{y} \quad \text{where } \theta = \tan^{-1} y/x$$

$$|OC| = \sqrt{x^2 + y^2}$$

$$V_1 = V_{m_1}\angle\theta_1, \quad V_2 = V_{m_2}\angle\theta_2$$

Then $V_1 V_2 = V_{m_1} V_{m_2} \angle\theta_1 + \theta_2 = V_m[\cos(\theta_1 + \theta_2) + j\sin(\theta_1 + \theta_2)]$

$$V_1/V_2 = \frac{V_{m_1}}{V_{m_2}}\angle\theta_1 - \theta_2 = V_m[\cos(\theta_1 - \theta_2) + j\sin(\theta_1 - \theta_2)]$$

$$V_1 + V_2 = V_{m_1}\angle\theta_1 + V_{m_2}\angle\theta_2$$

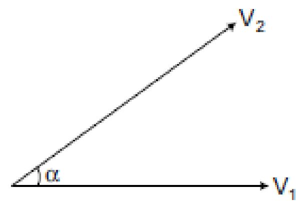
$$= V_{m_1}(\cos\theta_1 + j\sin\theta_1) + V_{m_2}(\cos\theta_2 + j\sin\theta_2)$$

$$\boxed{V_1 + V_2 = \left(V_{m_1}\cos\theta_1 + V_{m_2}\cos\theta_2\right) + j\left(V_{m_1}\sin\theta_1 + V_{m_2}\sin\theta_2\right)}$$

Phasor diagram:

Let $V_1 = V_{m_1} \sin\theta = V_{m_1} \angle O \qquad \theta = \omega t$

$V_2 = V_{m_2} \sin(\omega t + \alpha) = V_{m_2} \angle\alpha$ (means V_2 leads, V_1 by angle $\alpha°$)

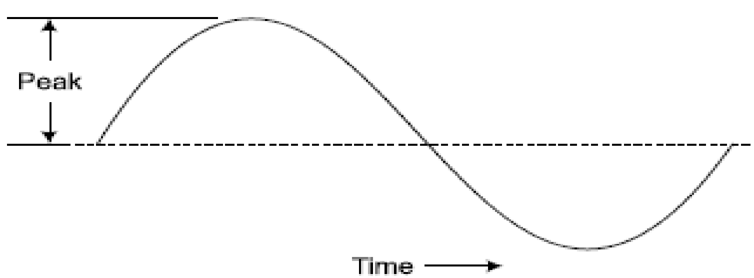


2.10 MEASUREMENTS OF AC MAGNITUDE

So far we know that AC voltage alternates in polarity and AC current alternates in direction. We also know that AC can alternate in a variety of different ways, and by tracing the alternation over time we can plot it as a "waveform". We can measure the rate of alternation by measuring the time it takes for a wave to evolve before it repeats itself (the "period"), and express this as cycles per unit time, of "frequency". In music, frequency is the same as pitch, which is the essential property distinguishing one note from another.

However, we encounter a measurement problem if we try to express how large or small an AC quantity is. With DC, where quantities of voltage and current are generally stable, we have little trouble expressing how much voltage or current we have in any part of a circuit. But how do you grant a single measurement of magnitude to something that is constantly changing?

One way to express the intensity, or magnitude (also called the amplitude), of an AC quantity is to measure its peak height on a waveform graph. This is known as the peak or crest value of an AC waveform:



Another way is to measure the total height between opposite peaks. This is known as the peak-to-peak (P-P) value of an AC waveform.

Unfortunately, either one of these expressions of waveform amplitude can be misleading when comparing two different types of waves. For example, a square wave peaking at 10 volts is obviously a greater amount of voltage for a greater amount of time than a triangle wave peaking at 10 volts. The effects of these two AC voltages powering a load would be quite different.

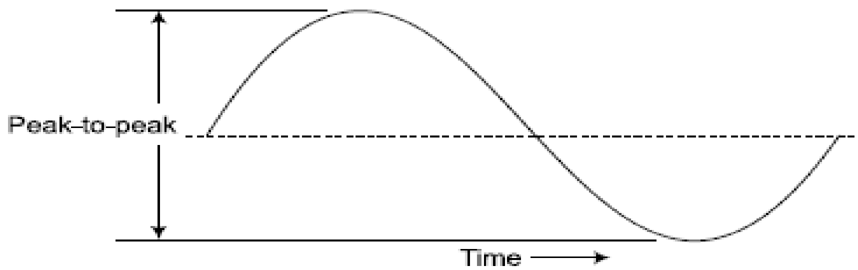
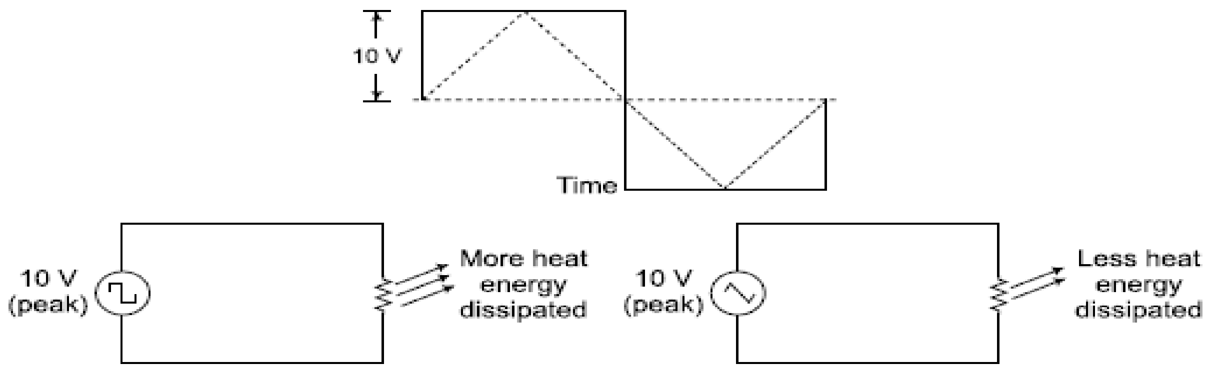
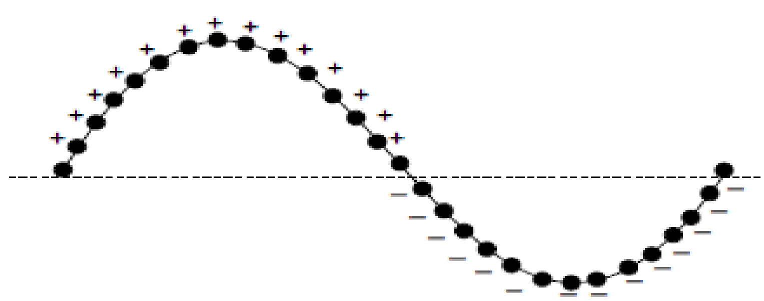


Fig. 2.11



(Same load resistance)

One way of expressing the amplitude of different wave-shapes in a more equivalent fashion is to mathematically average the values of all the points on the graph of a waveform to a single, aggregate number. This amplitude measure is known as the average value of the waveform. If we average all the points on the waveform algebraically (that is, to consider their sign, either positive or negative), the average



True average value of all points (considering their signs) is zero.

Fig. 2.13

value for most waveforms is technically zero, because all the positive points cancel all the negative points over a full cycle.

This, of course, will be true for any waveform having equal-area portions above and below the "zero" line of a plot. However, as a practical measure of a waveform's aggregate value, "average" is usually defined as the mathematical mean of all the points' absolute values over a cycle. In other words, we calculate the practical average value of the waveform by considering all points on the wave as positive quantities as if the waveform looked like this:

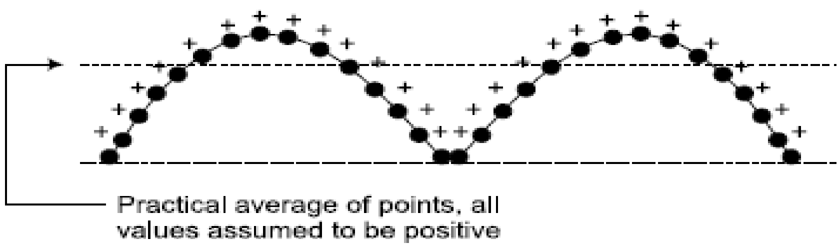


Fig. 2.14

Polarity-insensitive mechanical meter movements (meters designed to respond equally to the positive and negative half-cycles of an alternating voltage or current) register in proportion to the waveform's (practical) average value, because the inertia of the pointer against the tension of the spring naturally averages the force produced by the varying voltage/current values over time. Conversely polarity-sensitive meter movements vibrate uselessly if exposed to AC voltage or current, their needles oscillating rapidly about the zero mark, indicating the true (algebraic) average value of zero for a symmetrical waveform. When the "average" value of a waveform is referenced in this text, it will be assumed that the "practical" definition of average is intended unless otherwise specified.

Another method of deriving an aggregate value for waveform amplitude is based on the waveform's ability to do useful work when applied to a load resistance. Unfortunately, an AC measurement based on work performed by a waveform is not the same as that waveform's average value, because the power dissipated by a given load (work performed per unit time) is not directly proportional to the magnitude of either the voltage or current impressed upon it. Rather, power is proportional to the square of the voltage or current applied to a resistance ($P = E^2/R$, and $P = I^2R$). Although the mathematics of such an amplitude measurement might not be straightforward, the utility of it, is.

Current would produce the same amount of heat energy dissipation through an equal resistance:

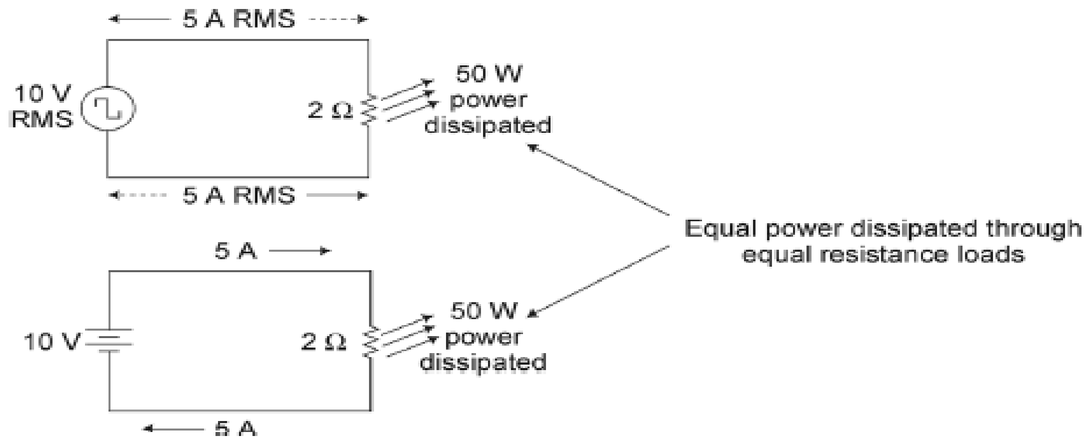


Fig. 2.15

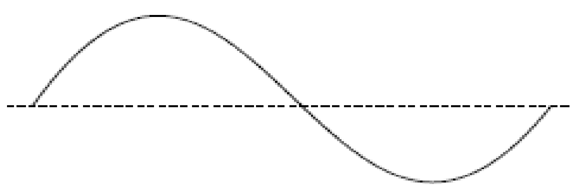
In the two circuits above, we have the same amount of load resistance (2 Ω) dissipating the same amount of power in the form of heat (50 watts), one powered by AC and the other by DC. Because the AC voltage source pictured above is equivalent (in terms of power delivered to a load) to a 10 volt DC battery, we would call this a "10 volt" AC source. More specifically, we would denote its voltage value as being 10 volts RMS. The qualifier "RMS" stands for Root Mean Square, the algorithm used to obtain the DC equivalent value from point on a graph (essentially, the procedure consists of squaring all the positive and negative points on a waveform graph, averaging those squared values, then taking the square root of the average to obtain the final answer). Sometimes the alternative terms equivalent or DC equivalent are used instead of "RMS", but the quantity and principle are both the same.

RMS amplitude measurement is the best way to relate AC quantities to DC quantities, or other AC quantities of differing waveform shapes, when dealing with measurements of electric power. For other considerations, peak or peak-to-peak measurements may be the best to employ. For instance, when determining the proper size of wire (ampacity) to conduct electric power from a source to a load, RMS current measurement is the best to use, because the principal concern with current is overheating of the wire, which is a function of power dissipation caused by current through the resistance of the wire. However, when rating insulators for service in high-voltage AC applications, peak voltage measurements are the most appropriate, because the principal concern here is insulator "flashover" caused by brief spikes of voltage, irrespective of time.

Peak and peak-to-peak measurements are best performed with an oscilloscope, which can capture the crests of the waveform with a high degree of accuracy due to the fast action of the cathode-ray-tube in response to changes in voltage. For RMS measurements, analog meter movements (D' Arsonval, Weston, iron vane, electrodynamometer) will work so long as they have been calibrated in RMS figures. Because the mechanical inertia and dampening effects of an electromechanical meter movement makes the deflection of the needle naturally proportional to the average value of the AC, not the true RMS value, analog meters must be specifically calibrated (or mis-calibrated depending on how you look at it) to indicate voltage or current in RMS units. The accuracy of this calibration depends on an assumed waveshape, usually a sine wave.

Electronic meters specifically designed for RMS measurement are best for the task. Some instrument manufacturers have designed ingenious methods for determining the RMS value of any waveform. One such manufacturer produces "True-RMS" meters with a tiny resistive heating element powered by a voltage proportional to that being measured. The heating effect of that resistance element is measured thermally to give a true RMS value with no mathematical calculations whatsoever, just the laws of physics in action in fulfilment of the definition of RMS. The accuracy of this type of RMS measurement is independent of waveshape.

For "pure" waveforms, simple conversion coefficients exist for equating peak, peak-to-peak, average (practical, not algebraic), and RMS measurements to one another:



$$\left.\begin{array}{l}\text{RMS} = 0.707 \text{ (Peak)} \\ \text{AVG} = 0.637 \text{ (Peak)} \\ \text{P-P} = 2 \text{ (Peak)}\end{array}\right\}\text{Sinusoidal wave}$$

$$I_{\text{rms}} = \left[\frac{1}{T}\int_0^T i^2 dt\right]^{1/2}$$

- For a sinusoidal wave

$$i = I_m \sin \omega t$$

$$I_{\text{rms}} = \left[\frac{1}{T}\int_0^T (i)^2 dt\right]^{1/2}$$

$$= \left[\frac{1}{T}\int_0^T I_m^2 \sin^2 \omega t . dt\right]^{1/2}$$

$$= \left[\frac{1}{T}\int_0^T I_m^2 \frac{(1-\cos 2\omega t)}{2} dt\right]^{1/2}$$

$$= \left[\frac{I_m^2}{T}\left(\frac{t}{2} - \frac{\sin 2\omega t}{4\omega}\right)_0^T\right]^{1/2}$$

$$= \left[\frac{I_m^2}{T}\left(\frac{T}{2} - 0\right)\right]^{1/2} \quad \left\{\sin 2\omega t = \sin 2.2\pi f t = \sin 2.\frac{2\pi}{T} t\right.$$

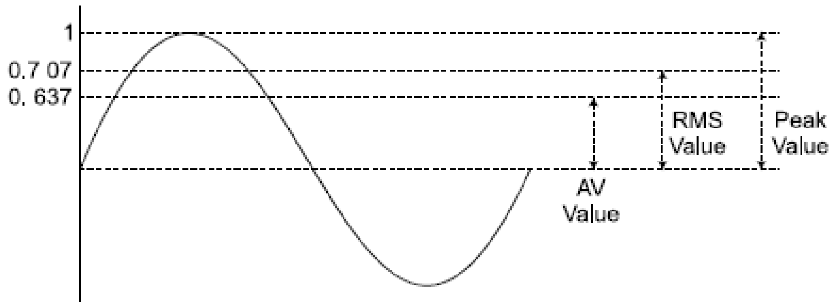
when $t = T$ $\{\sin 2\omega t = \sin 4\pi = 0$

$$= \frac{I_m}{\sqrt{2}}$$

- Ratio of *maximum* value to the RMS value is known as *crest or peak factor* or amplitude factor. Peak factor $= \dfrac{\text{Maximum Value}}{\text{RMS value}}$

- Ratio of *effective value* to *average value* is known as *form factor*

$$\text{form factor} = \frac{\text{RMS value}}{\text{Average value}}$$



2.13 FORM FACTOR

The form factor is defined as the ratio of the effective value to the average value of an alternating quantity.

$$= \frac{1}{4} V_m^2$$

$$V_{\text{rms}} = \frac{1}{2}\ V_m$$

$$\text{From factor} = \frac{V_{\text{rms}}}{V_{\text{av}}} = \frac{0.5V_m}{0.318V_m} = 1.572$$

Example 3: Find the average and rms value of the full wave rectified sine wave shown in Fig. 2.22

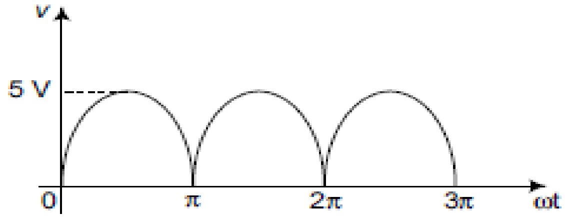


Fig. 2.22

Solution: Average value $V_{av} = \frac{1}{\pi}\int_0^{\pi} 5 \sin \omega t \, d(\omega t)$

$$= 3.185$$

Effective value or rms value $= \sqrt{\frac{1}{\pi}\int_0^{\pi} (5\sin\omega t)^2 \, d(\omega t)}$

$$= \sqrt{\frac{25}{2}} = 3.54$$

Example 4: The full wave rectified sine wave shown in Fig. 2.23 has a delay angle of 60°. Calculate V_{av} and V_{rms}.

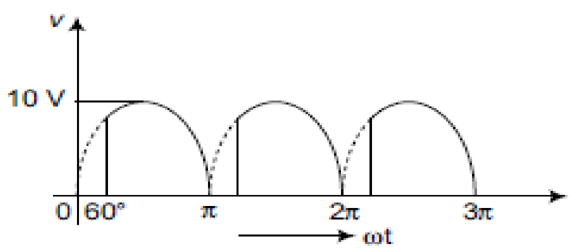


Fig. 2.23

Solution: Average value $V_{av} = \frac{1}{\pi}\int_0^{\pi} 10 \sin (\omega t) \, d(\omega t)$

$$= \frac{1}{\pi}\int_{60^\circ}^{\pi} 10 \sin \omega t \, d(\omega t)$$

$$V_{av} = \frac{10}{\pi}(-\cos\omega t)_{60}^{\pi} = 4.78$$

Effective value
$$V_{rms} = \sqrt{\frac{1}{\pi}\int_{60^\circ}^{\pi}(10\sin\omega t)^2\, d(\omega t)}$$

$$= \sqrt{\frac{100}{\pi}\int_{60^\circ}^{\pi}\left(\frac{1-\cos 2\omega t}{2}\right) d(\omega t)}$$

$$= 6.33$$

2.15 OPERATOR *j*

- An alternating voltage or current is a phasor quantity, but since the instantaneous values are changing continuously, it must be represented by a rotating vector phasor.
- A phasor is a vector rotating at a constant angular velocity.
- j is defined as an operator which turns a phasor by 90° counter-clockwise (CCW) without changing the magnitude of phasor

$$j = 1\angle 90^\circ, \quad jr = r\angle 90^\circ$$

2.16 CIRCUIT WITH PURE RESISTANCE ONLY

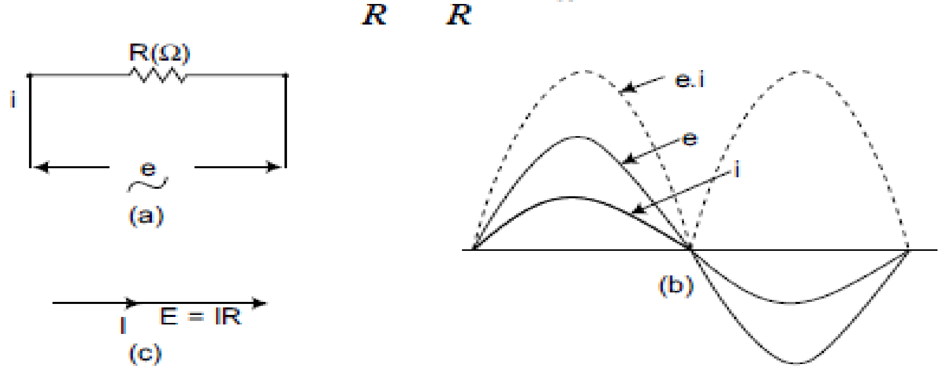
A pure resistance is that in which there is ohmic voltage drop only. Consider a circuit having a pure resistance R as shown is Fig. 2.24 below.

Let the instantaneous value of the alternating voltage applied be,

$$e = E_m \sin\omega t$$

The instantaneous value of current,

$$i = \frac{e}{R} = \frac{E_m}{R}\sin\omega t$$



2.17 CIRCUIT WITH PURE INDUCTANCE ONLY

A pure inductive circuit possesses only inductance and no resistance or capacitance as shown in Fig. 2.25. When an alternating voltage is applied to it, a back emf of self inductance is induced in it. As there is no ohmic resistance drop, the applied voltage has to oppose the self induced emf only. So the applied voltage is equal and opposite to the back emf at all instants.

Let the applied voltage

$$e = E_m \sin \omega t \quad (1)$$

instantaneous value of self induced emf is e'

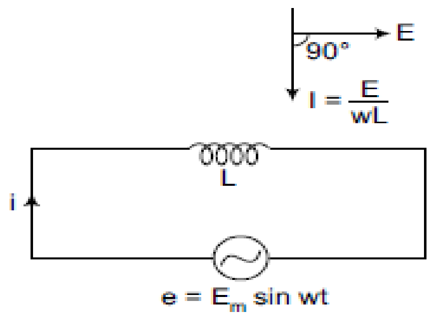


Fig. 2.25

$$e' = -L \frac{di}{dt} = -e$$

$$di = \frac{1}{L} e \, dt$$

integrating both side, we get

$$\int di = \frac{1}{L} \int E_m \sin \omega t . dt$$

$$i = \frac{E_m}{\omega L} (-\cos \omega t)$$

$$i = \frac{E_m}{\omega L} \sin \left(\omega t - \frac{\pi}{2} \right) \qquad \left\{ \begin{array}{l} \text{integration constant will} \\ \text{cancel out from both side} \end{array} \right.$$

$$i = I_m \sin \left(\omega t - \frac{\pi}{2} \right) \qquad (2) \quad \left\{ I_m = \frac{E_m}{\omega L} \right.$$

observing (1) and (2) we find that the current lags the applied voltage by 90° or $\frac{\pi}{2}$ radian.

$\Rightarrow$ impedance
$$Z = \frac{E}{I} = \frac{\frac{E_m}{\sqrt{2}} \angle 0^\circ}{\frac{E_m}{\sqrt{2}} \angle -\frac{\pi}{2}}$$

$$Z = \frac{E_m}{I_m} \angle \frac{\pi}{2} = \omega L \angle \frac{\pi}{2}$$

The quantity ωL is called inductive reactance and is usually devoted by symbol X_L and units is ohm.

$$X_L = \omega L \text{ ohms}$$

where, L is in henry and ω is in rad/sec.

Wave diagram and Phasor diagram for Pure inductance

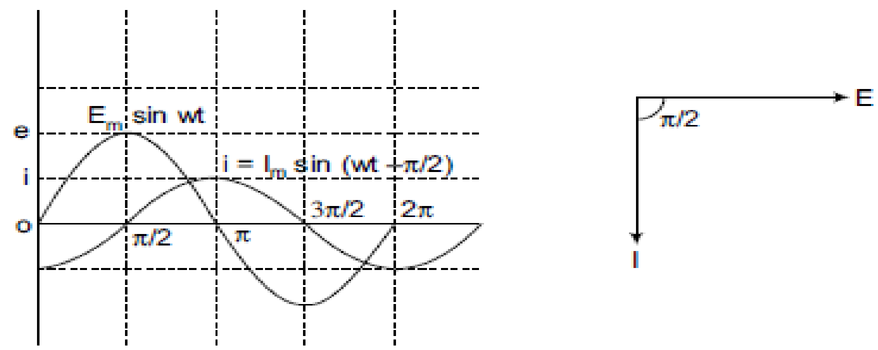


Fig. 2.26

Average Power→

$$P = \frac{1}{2\pi}\int_0^{2\pi} e\,i\,d(wt)$$

$$= \frac{1}{2\pi}\int_0^{2\pi} E_m \sin wt . I_m \sin\left(wt - \frac{\pi}{2}\right) d(wt)$$

$$= \frac{1}{2\pi}\int_0^{2\pi} -E_m\, I_m \sin wt . \cos wt . d(wt)$$

$$= -\frac{V_m I_m}{2\pi}\int_0^{2\pi} \frac{\sin 2wt}{2} . d(wt)$$

$$= 0$$

This shows power consumed in purely inductive circuit is zero.

Hence, the average power consumption in an inductive circuit is zero and is periodic with twice the supply frequency as expressed by equation (1). The stored energy in the inductive circuit in one quarter of a cycle is released in the next quarter.

$$= C\omega\, E_m \sin\left(\omega t + \frac{\pi}{2}\right) = \frac{E_m}{1/\omega C}.\sin\left(\omega t + \frac{\pi}{2}\right)$$

Comparing equations, we see that the current leads the voltage vector by 90° as shown in Fig. 2.28.

Maximum value of current is given by,

$$I_m = \frac{E_m}{1/C\omega}$$

The quantity $1/C\omega$ is called inductive capacitance and is usually denoted by X_c. Its unit is ohm.

$$\therefore \qquad X_c = \frac{1}{C\omega} \text{ ohms}$$

where, C = Capacity in farads

ω = angular velocity in rad/sec

Impedance $$Z = \frac{E}{I} = \frac{E_m \angle 0/\sqrt{2}}{I_m \angle 90/\sqrt{2}}$$

$$Z = \frac{E_m}{I_m} \angle -90^\circ$$

Since $$\frac{E_m}{I_m} = X_C = \frac{1}{\omega C}$$

$$\therefore \qquad Z = X_C \angle -90^\circ$$

$$= -j\, X_C\, \Omega$$

Average Power→

instantaneous power $P = vi$

$$P = V_m \sin \omega t . I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= V_m I_m \sin \omega t . \cos \omega t$$

$$= \frac{V_m I_m}{2} \sin 2\omega t \qquad (1)$$

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} P\, d(\omega t)$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\omega t . d(\omega t)$$

$$= 0$$

This shows that the power consumed in purely capacitive circuit is zero.

A capacitor receives energy during the first quarter cycle of voltage and returns the same during the next quarter cycle.

2.19 CIRCUIT WITH RESISTANCE AND INDUCTANCE IN SERIES

Consider circuit of Fig. 2.29.

Let R = Resistance in ohms in the circuit.

L = Inductance in henries

X_L = Inductive reactance

$= \omega L$

E = Effective value of applied emf

I = Effective value of current in circuit.

Voltage drop across resistance,

$E_R = R.I$ in phase with current vector as shown in vector diagram of Fig. 2.30.

Voltage across reactance,

$E_L = I.\omega L = IX_L$, 90° ahead of vector I

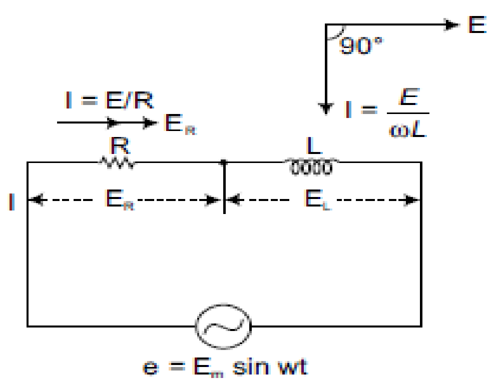


Fig. 2.29

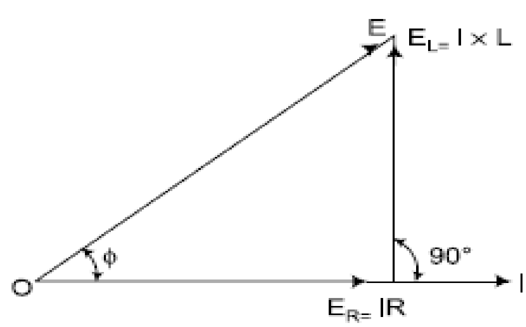


Fig. 2.30

$$Z = R + jX_L$$

$$= \sqrt{R^2 + X_L^2} \angle \tan^{-1} \frac{X_L}{R}$$

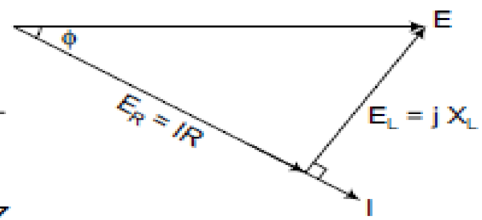
here $$\phi = \tan^{-1} \frac{X_L}{R}$$

and
$$|Z| = \sqrt{R^2 + X_L^2}$$
$$Z = |Z| \angle\phi$$
$$I = \frac{E}{Z} = \frac{E \angle\phi}{|Z| \angle\phi}$$
$$\boxed{I = \frac{E}{Z} \angle -\phi}$$

instantaneous value of current is, $i = I_m \sin(\omega t - \phi)$, where $I_m = \frac{E}{Z}$

hence in R-L circuit current lags the applied voltage by angle $\phi = \tan^{-1} \frac{X_L}{R}$

The applied voltage is therefore given by,



$$\therefore \quad E = \sqrt{E_r^2 + E_L^2}$$
$$= \sqrt{(IR)^2 + (IX_L)^2}$$
$$\therefore \quad E = I\sqrt{R^2 + X_L^2} = IZ$$
$$\phi = \tan^{-1} \frac{X_L}{R} = \tan^{-1} \frac{\omega L}{R}$$

or,
$$I = \frac{E}{\sqrt{R^2 + X_L^2}}$$

The quantity $\sqrt{R^2 + X_L^2}$ is called impedance.

Since, the power is consumed by the resistance only, so the power in the circuit is given by,
$$P = I^2R = I.IR$$
$$= \frac{E}{\sqrt{R^2 + X_L^2}} . IR$$

or,
$$P = E.I\frac{R}{\sqrt{R^2 + X_L^2}}$$

If ϕ is the angle between E and I, then
$$\cos\phi = \frac{E_R}{E} = \frac{IR}{I\sqrt{R^2 + X_L^2}} = \frac{R}{Z}$$

$\therefore \qquad P = E.I \cos \phi$

Cos ϕ is called the power factor of the circuit. Obviously the power factor is lagging in an inductive circuit. So instantaneous current across R-L is

$$i = I_m \sin(\omega t - \phi).$$

2.20 CIRCUIT WITH RESISTANCE AND CAPACITANCE IN SERIES

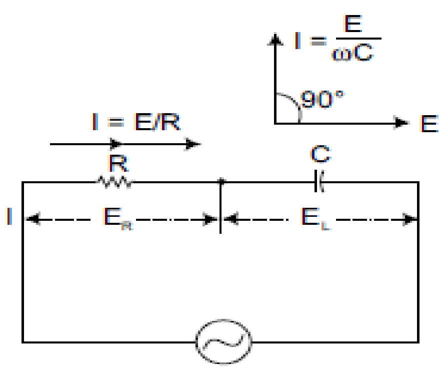
Consider circuit of Fig. 2.31.

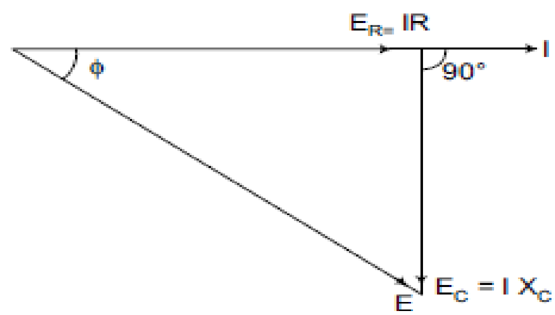
Voltage drop across resistance,

$E_R = IR$ in phase with I as shown in vector diagram of a Fig. 2.32.

$$E_C = I.\frac{1}{C\omega} = IX_C,\ 90^\circ \text{ lagging}$$

with respect to the current vector.





The applied voltage is, therefore, given by,

$$E = \sqrt{E_R^2 + E_C^2}$$

$$= I\sqrt{R^2 + X_C^2} = IZ$$

Thus, ohm's law is applicable to AC circuit also after replacing the term resistance by impedance.

$$\text{Power} = EI \cos \phi$$

$$\text{Cos } \phi = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{R}{Z} \qquad \text{...(2.53)}$$

$$Z = R - j X_C$$

$$Z = \sqrt{R^2 + X_C^2} \quad \tan^{-1}\left(\frac{-X_C}{R}\right)$$

$$Z = |Z| \angle -\phi \quad \text{here} \quad \phi = \tan^{-1}\left(\frac{-X_C}{R}\right)$$

$$I = \frac{E}{Z} = \frac{E}{|Z| \angle -\phi}$$

$$I = \left|\frac{E}{Z}\right| \angle\phi = I_m \angle\phi$$

instantaneous value of current throw R-C is

$$i = I_m \sin(\omega t + \phi) \text{ where } I_m = \left|\frac{E}{Z}\right|$$

hence current leads the voltages by aug $\phi = \tan^{-1}\left(\frac{-X_C}{R}\right)$.

2.21 SERIES R-L-C CIRCUIT

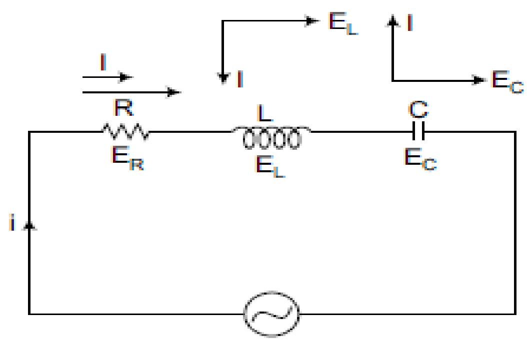


Fig. 2.33

Problems on alternating current circuits can be attempted easily by using j-notation.

$\therefore$ Voltage across inductance $= +jIX_L = E_L$

Voltage across capacitance $= -jIX_C = E_C$

Net voltage across them $= +jI(X_L - X_C) = j(E_L - E_C)$

Resistance drop $= IR = E_R$.

$\therefore$ Applied voltage in j-notation is represented by,

$$E = IR + jI(X_L - X_C)$$

or, $$E = I\sqrt{R^2 + (X_L - X_C)^2}$$

Impedance in j-notation may be written as,

$$Z = R + j(X_L - X_C)$$

or, $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$I = \frac{E}{Z} \qquad E = \frac{E_m}{\sqrt{2}} \angle 0°$$

$$Z = R + j(X_L - X_C)$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \angle\phi = |Z| \angle\phi$$

where $$\phi = \tan^{-1} \frac{X_L - X_C}{R}$$

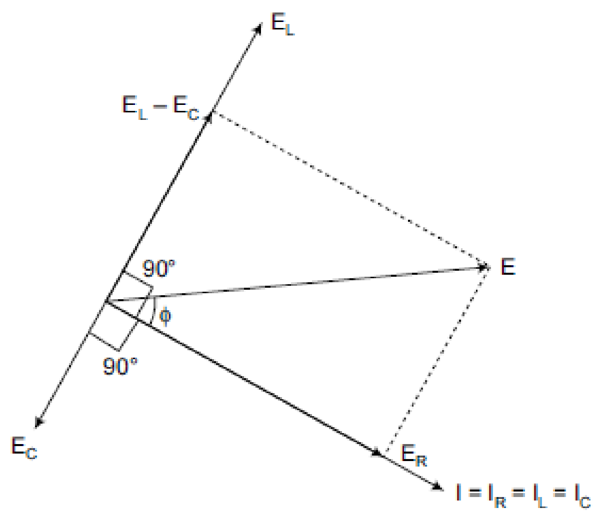
$\Rightarrow$ if $X_L > X_C$ then ϕ is +ve

if $X_L < X_C$ then ϕ is –ve

$$I = \frac{E_m}{\sqrt{2}} \angle 0° \Big/ |Z| \angle\phi$$

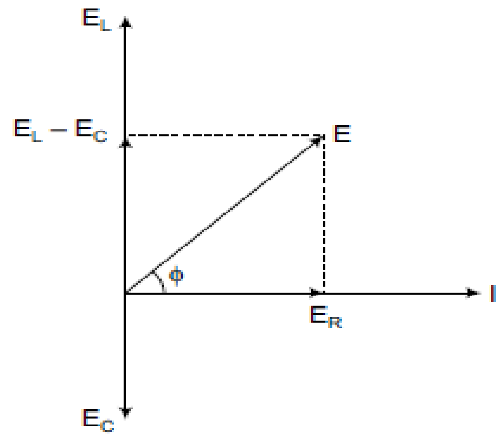
hence if $X_L > X_C$ then current lags the applied volt.

$$I = I' \angle -\phi$$



Phasor diagram of series R–L–C Circuit

R-L-C Ckt taking *E* as a refrence phasor when $X_L > X_C$



Phasor diagram of a series R-L-C Ckt taking current *I* as a refrence phasor.

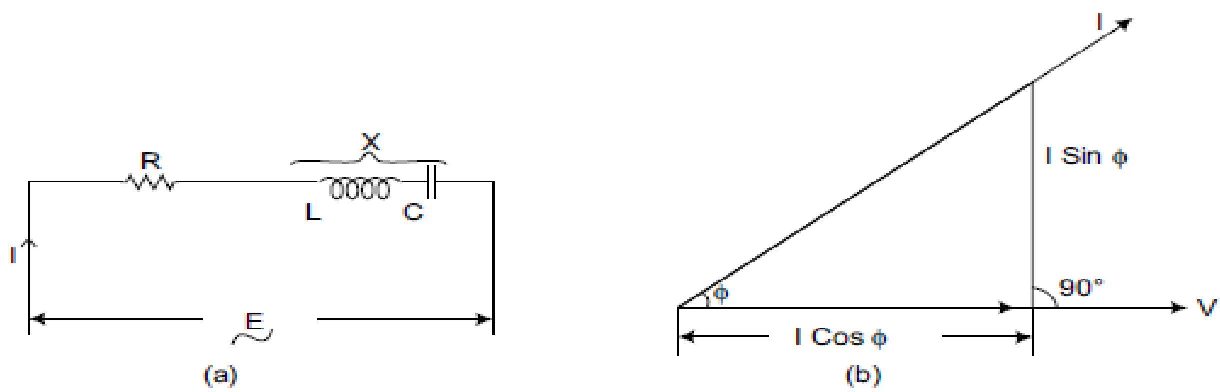
2.22 POWER IN AC CIRCUITS

- When the current is out of phase with the voltage the power indicated by the *product of the applied* voltage and the total current gives only what is known as *apparent power* and measured in volt-ampere.
- Power that is returned to the source by the reactive components in the circuit is called *reactive power* and is measured in VAR.
- Power that actually used in the circuit (dissipated in resistance) is true or active power and is measured in watts or kW.

2.22.1 Active and Reactive Power and Apparent Power

Form Fig. given below

Impedance $$Z = R \pm jX = |Z| \angle\phi = |Z| \cos\phi + j|Z| \sin\phi$$



Magnitude or amplitude of impedance,

$$|Z| = \sqrt{R^2 + X^2} \qquad \Rightarrow \qquad R = |Z| \cos \phi, \quad X = |Z| \sin \phi$$

Power factor of the circuit,

$$\cos \phi = \frac{R}{Z}.$$

Current in the circuit $I = \dfrac{E}{Z}$.

This current has two components $I \cos \phi$ and $I \sin \phi$. The component $I \cos \phi$ is called in phase or **wattfull** component and $I \sin \phi$ is perpendicular to E and is called **wattless** component, as shown in Fig. 2.30(b). Then

Active (Real) Power = Voltage × Current × cos ϕ watts

Since, the angle between the voltage and the wattless component of current is 90°, hence the power absorbed by this component is zero. The power is only absorbed by the wattful component.

The total power EI in volt amperes supplied to a circuit consists of two components:

(a) Active power = $EI \cos \phi$ watts

(b) Reactive power = $EI \sin \phi$ volt amperes reactive or simply VAR.

The above components can be shown in vector from in Fig. 2.34.

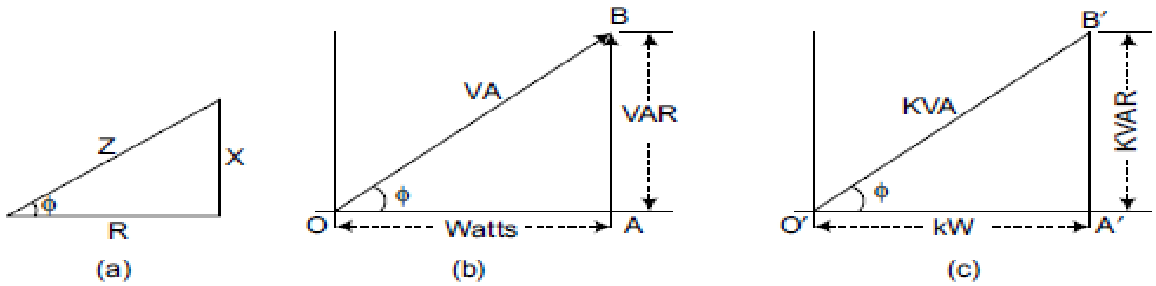


Fig. 2.34

From Fig. 2.31(b)

OA = Active power = $EI \cos \phi$ presented by watts

AB = Reactive power = $EI \sin \phi$ expressed by VAR

OB = Total power = EI expressed by VA

Obviously $\qquad \text{VA} = \sqrt{\text{Watts}^2 + \text{VAR}^2} \qquad$ ...(2.4)

$$\cos\theta_2 = \frac{a_2}{\sqrt{a_2^2+b_2^2}},\ \sin\theta_2 = \frac{b_2}{\sqrt{a_2^2+b_2^2}}$$

Active power:

$$= OA.OB\cos AOB = \sqrt{a_1^2+b_1^2}\cdot\sqrt{a_2^2+b_2^2}\ .\cos(\theta_2-\theta_1)$$

$$= \sqrt{a_1^2+b_1^2}\cdot\sqrt{a_2^2+b_2^2}\ [\cos\theta_2.\cos\theta_1+\sin\theta_2.\sin\theta_1]$$

$$= a_1^2+b_1^2\sqrt{a_2^2+b_2^2}\left\{\frac{a_2}{\sqrt{a_2^2+b_2^2}}\cdot\frac{a_1}{\sqrt{a_1^2+b_1^2}}+\frac{b_2}{\sqrt{a_2^2+b_2^2}}\cdot\frac{b_1}{\sqrt{a_1^2+b_1^2}}\right\}$$

$$= a_1a_2+b_1b_2$$

Reactive power:

$$= OA.OB\sin AOB = \sqrt{a_1^2+b_1^2}\cdot\sqrt{a_2^2+b_2^2}\ .\sin(\theta_2-\theta_1)$$

$$= \sqrt{a_1^2+b_1^2}\cdot\sqrt{a_2^2+b_2^2}\ [\sin\theta_2.\cos\theta_1-\cos\theta_2.\sin\theta_1]$$

$$= \sqrt{a_1^2+b_1^2}\cdot\sqrt{a_2^2+b_2^2}\left\{\frac{b_2}{\sqrt{a_2^2+b_2^2}}\cdot\frac{a_1}{\sqrt{a_1^2+b_1^2}}-\frac{a_2}{\sqrt{a_2^2+b_2^2}}\cdot\frac{b_1}{\sqrt{a_1^2+b_1^2}}\right\}$$

$$= a_1b_2-a_1b_2$$

Note: $\quad$ $V.I. = (a_1+jb_1)(a_2+jb_2)$

$$= (a_1a_2-b_1b_1)+j(a_2b_1+a_1b_2)$$

If we write $V\times$ Conjugate of I

$$= (a_1+jb_1)(a_2-jb_2) = (a_1a_2+b_1b_2)+j(a_1b_2-b_1a_2)$$

$$= \underset{\text{(Active power)}}{a_1a_2+b_1b_2} + \underset{\text{(Reactive power)}}{j(a_1b_2-b_1a_2)}$$

Note 1: Hence, the active and reactive powers would be given by the real and *j* ports of the vector product of voltage with the conjugate of the current vector.

Note 2: Active power can also be expressed by the sum of the algebraic product of the real parts of the current and the voltage and the algebraic product of the *j* parts of the current and voltage.

Alternate approach→

Let *E* and *I* are the phasors given by

$$\overline{E} = E\angle\theta_1$$

and $\bar{I} = I \angle \pm\theta_2$ $\begin{cases} + \text{ sign for leading current} \\ - \text{ sign for lagging current} \end{cases}$

there complex power is given by S

$$\begin{aligned} S &= \bar{E} \times \bar{I}^* \\ &= E \angle \theta_1 . I \angle \mp \theta_2 \\ &= EI \angle \theta_1 \mp \theta_2 \end{aligned} \qquad \begin{cases} \text{if } \bar{I} = I \angle \pm\theta_2 \\ \quad \bar{I}^* = I \angle \mp\theta_2 \end{cases}$$

$$S = EI \cos(\theta_1 \mp \theta_2) + j\, EI \sin(\theta_1 \mp \theta_2)$$

$$\boxed{S = P + jQ}$$

if V is the refrence phasor $\theta_1 = 0$

$$S = EI \cos\theta_2 + j\, EI \sin(\mp\theta_2)$$
$$S = EI \cos\theta_2 \mp j\, EI \sin\theta_2$$

$$\boxed{S = P \mp jQ} \qquad \begin{cases} -\text{ve for leading P.f load} \\ S = P - jQ \\ +\text{ve for lagging P.f load} \\ S = P + jQ \end{cases}$$

P = active power
Q = reactive power

2.24 POWER FACTOR

The power factor of an alternating-current device or circuit or electric power system is defined as the ratio of real or true power to the apparent power (VA) and is between 0 to 1.

Real power is the capacity of the circuit for performing work in a particular time, and apparent power is the product of current and voltage of a system. Reactive power is the power that magnetic equipment (transformer, motor and relays) needs to produce the magnetizing flux.

- In a single-phase circuit the power factor is also a measure of the phase angle θ between the phase voltage (V_{ph}) and phase current (I_{ph})

$$\text{Power factor} = \frac{P}{S} = \frac{\text{real power}}{\text{apparent power}} = \frac{V_{ph}\, I_{ph} \cos\theta}{V_{ph}\, I_{ph}}$$

$$\boxed{\text{Power factor} = \cos\theta}$$

- Power factor is said to be lagging if the current lags behind voltage and leading if the current leads the voltage.
- The significance of power factor lies in the fact that utility companies supply customers with volt-amperes, but bill them for watts.

2.24.1 Problems of Low Power Factor

(1) Power factor below 1.0 requires a utility to generate more than the minimum volt-amperes necessary to supply the real power (watts). This increases generation and transmission cost.

(2) If the load power factor were as low as 0.7, the apparent power would be $\left(\frac{\text{real power}}{0.7}\right)$ 1.4 times the real power used by the load. Line current in the circuit would also be 1.4 times the current required at unity power factor, so the losses in the circuit would be doubled (proportional to square of current) result in all components of the system such as generator, conductors, transformers and switchgear would be increased in size (cost) to carry the extra current.

(3) Higher current produces larger voltage drop in cables and other apparatus. This results in poor voltage regulation. In practice, power factor is rarely corrected to unity because the cost of equipment required to improve the power is usually greater than the saving on tariff.

2.24.2 Causes of Low Power Factor

Many alternating-current machines (transformer, induction motors) absorb reactive power to produce their magnetic fields, this decreases the power factor. Reactive power (kVAr) required by inductive loads increases the amount of apparent power (kVAr) in our, distribution system (Fig. 2.36). This increase in ***reactive*** and ***apparent power*** results in a larger angle (measured between **kW** and **kVA**). Recall that, as θ increases, cosine θ (or ***power factor***) decreases.

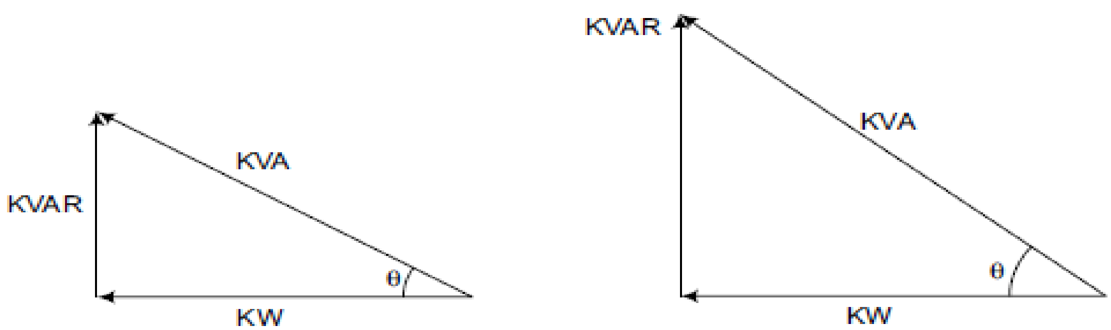


Fig. 2.36

So ***inductive loads*** (with large **kVAr**) results in low ***power factor***.

(3) The more increased voltage level in the electrical system and cooler, the more efficient motors will be.

As mentioned above, uncorrected power factor causes power system losses in the distribution system. As power losses increase, we may experience voltage drops. Excessive voltage drops can cause overheating and premature failure of motors and other inductive equipment.

So, by raising the power factor, these voltage drops can be minimized along feeder cables and avoid related problems. The motor will run cooler and more efficiently, with a slight increase in capacity and starting torque.

2.24.4 Power Factor Correction

Power factor correction is the process of adjusting the characteristics of elective loads in order to improve power factor closer to unity. A high power factor is generally desirable in a transmission system to reduce transmission losses and improve voltage regulation at the load.

- **The presence of reactive power causes the real power to be less than the apparent power, and so the electrical load has a power factor less than unity.**
- **The reactive power increases the current flowing between the power source and the load, which increase the power losses though transmission and distribution lines.**

(1) Power factor correction can be done by supplying reactive power of opposite sign adding capacitors or inductors which act to cancel the inductive or capacitive effects of the load, respectively.

For example, the inductive effect of motor loads may be offset by locally connected capacitors, sometimes when the power factor is leading due to capacitive loading, inductors are used to correct the power factor.

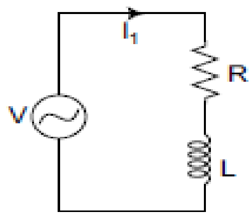
(2) Minimizing operation of idling or lightly loaded motors because low power factor is caused by running induction motor lightly loaded.

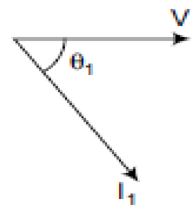
(3) Avoiding operation of equipment above its rated voltage.

(4) Replacing standard motors as they burn out with energy-efficient motors.

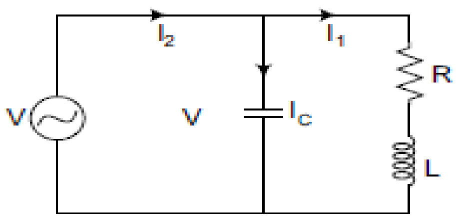
(5) By using synchronous motor or synchronous condenser.

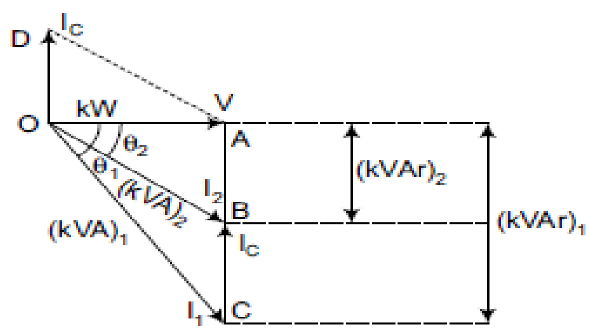
Power Factor Correction by Static Capacitor: Consider an inducting load consisting of resistor R and an inductor L connected to an ac supply. Current I_1 lags the voltage V by angle θ_1 so PF is $\cos\theta_1$.





Let us now for improving the power factor connect a capacitor parallel to a load. This capacitor takes a leading current from the supply. The capacitor produces a reactive power in the opposite direction hence net reactive power decreases.





from the phasor diagram

$$OA = I_1 \cos\theta_1 = I_2 \cos\theta_2$$

$$I_2 = \frac{I_1 \cos\theta_1}{\cos\theta_2}$$

$\Rightarrow$ $\theta_2 > \theta_1$ so $\cos\theta_2 > \cos\theta_1$

Since $\cos\theta_2 > \cos\theta_1$, $I_2 < I_1$

Hence, current drawn from the supply is less than the load current I_1. Hence if power factor reduces then apparent power (VI) from the supply will also reduce.

$\Rightarrow$ $I_2 \cos\theta_2 = I_1 \cos\theta_1$

$\Rightarrow$ $VI_2 \cos\theta_2 = VI_1 \cos\theta_1 = \text{Real power}$

The above relation shows that active or true power taken from supply has not altered,

CONCEPT OF REACTANCE, IMPEDANCE, SUSCEPTANCE AND ADMITTANCE:

Reactance is essentially inertia against the motion of electrons. It is present anywhere electric or magnetic fields are developed in proportion to applied voltage or current, respectively; but most notably in capacitors and inductors. When alternating current goes through a pure reactance, a voltage drop is produced that is 90^O out of phase with the current. Reactance is mathematically symbolized by the letter "X" and is measured in the unit of ohms (Ω).

Impedance is a comprehensive expression of any and all forms of opposition to electron flow, including both resistance and reactance. It is present in all circuits, and in all components. When alternating current goes through an impedance, a voltage drop is produced that is somewhere between 0^O and 90^O out of phase with the current. Impedance is mathematically symbolized by the letter "Z" and is measured in the unit of ohms (Ω), in complex form

Admittance is also a complex number as impedance which is having a real part, Conductance (G) and imaginary part, Susceptance (B).

$$Y = G + jB$$
$$Y \rightarrow Admittance\ in\ Siemens$$
$$G \rightarrow Conductance\ in\ Siemens = \frac{R}{R^2 + X^2}$$
$$B \rightarrow\ Susceptance\ in\ Siemens = -\frac{X}{R^2 + X^2}$$

(it is negative for capacitive susceptance and positive for inductive susceptance)

$$j^2 = -1$$
$$|Y| = \sqrt{G^2 + B^2} = \frac{1}{\sqrt{R^2 + X^2}}$$
$$\angle Y = arctan\left(\frac{B}{G}\right) = arctan\left(-\frac{X}{R}\right)$$

Susceptance (symbolized *B*) is an expression of the ease with which alternating current (AC) passes through a capacitance or inductance

UNIT-IV
ELECTRICAL MACHINES

Dc Generator

- **Principle Of Operation**
- **Constructional Features**
- **EMF Equation**

Dc Motor

- **Principle Of Operation**
- **Back EMF**
- **Torque Equation**

Single Phase Transformer

- **Principle Of Operation**
- **Constructional Features**
- **EMF Equation**
- **Simple Problems**

DC GENERATOR

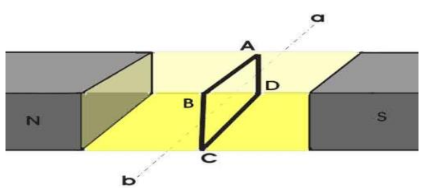
Principle of DC Generator

There are two types of generators, one is ac generator and other is DC generator. Whatever may be the types of generators, it always converts mechanical power to electrical power. An AC generator produces alternating power. A DC generator produces direct power. Both of these generators produce electrical power, based on same fundamental principle of Faraday's law of electromagnetic induction. According to this law, when a conductor moves in a magnetic field it cuts magnetic lines of force, due to which an emf is induced in the conductor. The magnitude of this induced emf depends upon the rate of change of flux (magnetic line force) linkage with the conductor. This emf will cause a current to flow if the conductor circuit is closed.
Hence the most basic tow essential parts of a generator are

1. a magnetic field
2. conductors which move inside that magnetic field.

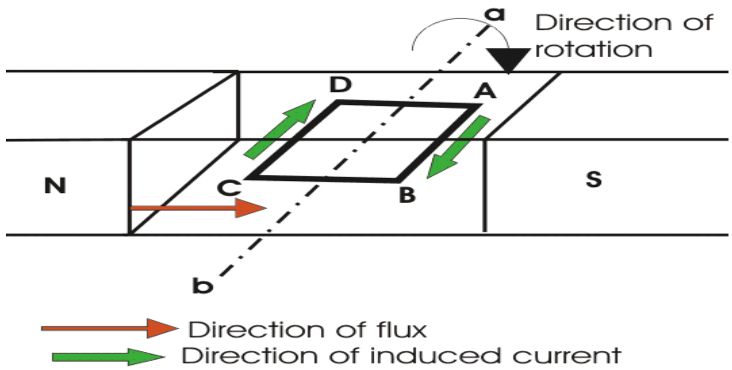
Now we will go through working principle of DC generator. As, the working principle of ac generator is not in scope of our discussion in this section.

Single Loop DC Generator

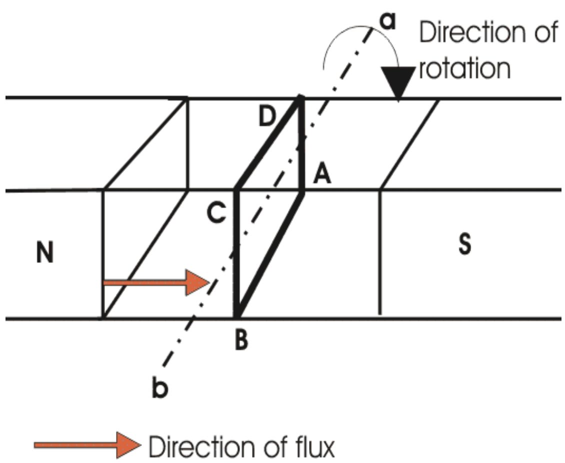


In the figure above, a single loop of conductor of rectangular shape is placed between two opposite poles of magnet.

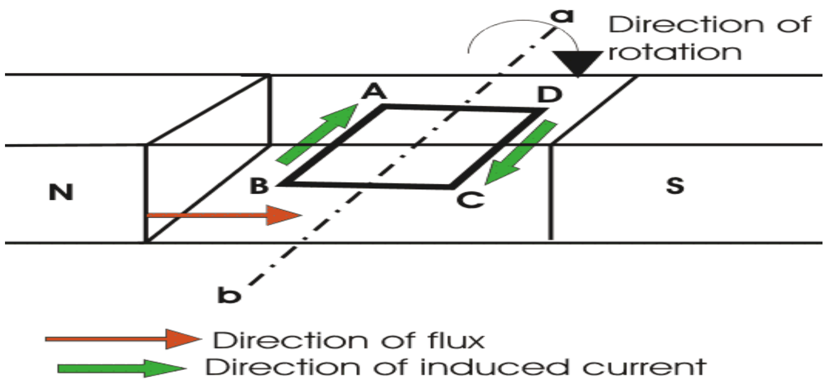
Let's us consider, the rectangular loop of conductor is ABCD which rotates inside the magnetic field about its own axis ab. When the loop rotates from its vertical position to its horizontal position, it cuts the flux lines of the field. As during this movement two sides, i.e. AB and CD of the loop cut the flux lines there will be an emf induced in these both of the sides (AB and BC) of the loop.



As the loop is closed there will be a current circulating through the loop. The direction of the current can be determined by Flemming's right hand Rule. This rule says that if you stretch thumb, index finger and middle finger of your right hand perpendicular to each other, then thumbs indicates the direction of motion of the conductor, index finger indicates the direction of magnetic field i.e. N - pole to S - pole, and middle finger indicates the direction of flow of current through the conductor. Now if we apply this right hand rule, we will see at this horizontal position of the loop, current will flow from point A to B and on the other side of the loop current will flow from point C to D.



Now if we allow the loop to move further, it will come again to its vertical position, but now upper side of the loop will be CD and lower side will be AB (just opposite of the previous vertical position). At this position the tangential motion of the sides of the loop is parallel to the flux lines of the field. Hence there will be no question of flux cutting and consequently there will be no current in the loop. If the loop rotates further, it comes to again in horizontal position. But now, said AB side of the loop comes in front of N pole and CD comes in front of S pole, i.e. just opposite to the previous horizontal position as shown in the figure beside.

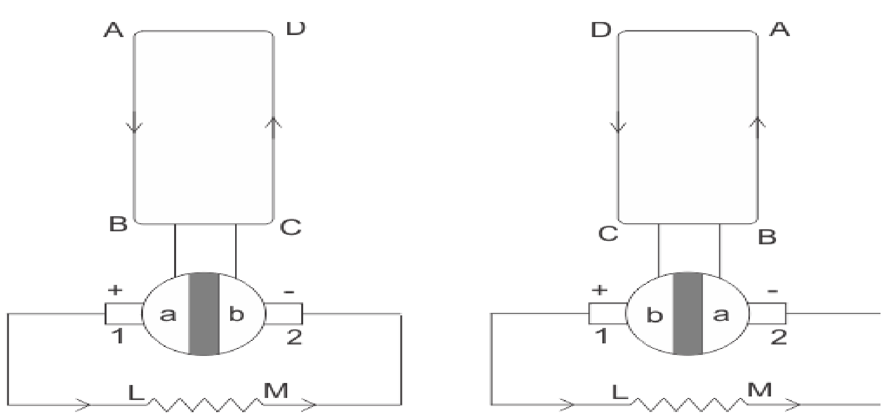


Here the tangential motion of the side of the loop is perpendicular to the flux lines, hence rate of flux cutting is maximum here and according to Fleming's right hand rule, at this position current flows from B to A and on other side from D to C.Now if the loop is continued to rotate about its axis, every time the side AB comes in front of S pole, the current flows from A to B and when it comes in front of N pole, the current flows from B to A. Similarly, every time the side CD comes in front of S pole the current flows from C to D and when it comes in front of N pole the current flows from D to C.

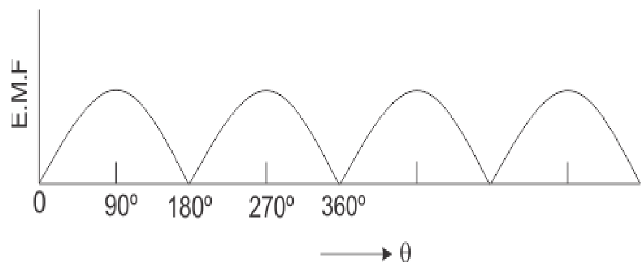
If we observe this phenomena in different way, it can be concluded, that each side of the loop comes in front of N pole, the current will flow through that side in same direction i.e. downward to the reference plane and similarly each side of the loop comes in front of S pole, current through it flows in same direction i.e. upwards from reference plane. From this, we will come to the topic of principle of DC generator.

Now the loop is opened and connected it with a split ring as shown in the figure below. Split ring are made out of a conducting cylinder which cuts into two halves or segments insulated from each other. The external load terminals are connected with two carbon brushes which are rest on these split slip ring segments.

Working Principle of DC Generator



It is seen that in the first half of the revolution current flows always along ABLMCD i.e. brush no 1 in contact with segment a. In the next half revolution, in the figure the direction of the induced current in the coil is reversed. But at the same time the position of the segments a and b are also reversed which results that brush no 1 comes in touch with the segment b. Hence, the current in the load resistance again flows from L to M. The wave from of the current through the load circuit is as shown in the figure. This current is unidirectional.



This is basic working principle of DC generator, explained by single loop generator model. The position of the brushes of DC generator is so arranged that the change over of the segments a and b from one brush to other takes place when the plane of rotating coil is at right angle to the plane of the lines of force. It is so become in that position, the induced emf in the coil is zero.

Construction of DC Generator

During explaining working principle of DC Generator, we have used a single loop DC generator. But now we will discuss about practical construction of DC Generator. A DC generator has the following parts

1. Yoke
2. Pole of generator
3. Field winding
4. Armature of DC generator
5. Brushes of generator and Commentator
6. Bearing

Yoke of DC Generator

Yoke or the outer frame of DC generator serves two purposes,

1. It holds the magnetic pole cores of the generator and acts as cover of the generator.
2. It carries the magnetic field flux.

In small generator, yoke are made of cast iron. Cast iron is cheaper in cost but heavier than steel. But for large construction of DC generator, where weight of the machine is concerned, lighter cast steel or rolled steel is preferable for constructing yoke of DC generator. Normally larger yokes are formed by rounding a rectangular steel slab and the edges are welded together at the bottom. Then feet, terminal box and hangers are welded to the outer periphery of the yoke frame.

Pole Cores and Pole Shoes

Let's first discuss about pole core of DC generator. There are mainly two types of construction available.

One: Solid pole core, where it is made of a solid single piece of cast iron or cast steel.

Two: Laminated pole core, where it made of numbers of thin, limitations of annealed steel which are riveted together. The thickness of the lamination is in the range of 0.04" to 0.01". The pole core is fixed to the inner periphery of the yoke by means of bolts through the yoke and into the pole body. Since the poles project inwards they are called salient poles. The pole shoes are so typically shaped, that, they spread out the magnetic flux in the air gap and reduce the reluctance of the magnetic path. Due to their larger cross-section they hold the pole coil at its position.

Pole Coils: The field coils or pole coils are wound around the pole core. These are a simple coil of insulated copper wire or strip, which placed on the pole which placed between yoke and pole shoes as shown.

Armature Core

The purpose of armature core is to hold the armature winding and provide low reluctance path for the flux through the armature from N pole to S pole. Although a DC generator provides direct current but induced current in the armature is alternating in nature. That is why, cylindrical or drum shaped armature core is build up of circular laminated sheet. In every circular lamination, slots are either die - cut or punched on the outer periphery and the key way is located on the inner periphery as shown. Air ducts are also punched of cut on each lamination for circulation of air through the core for providing better cooling. Up to diameter of 40", the circular stampings are cut out in one piece of lamination sheet. But above 40", diameter, number of suitable sections of a circle is cut. A complete circle of lamination is formed by four or six or even eight such segment.

Armature Winding

Armature winding are generally formed wound. These are first wound in the form of flat rectangular coils and are then pulled into their proper shape in a coil puller. Various conductors of the coils are insulated from each other. The conductors are placed in the armature slots, which are lined with tough insulating material. This slot insulation is folded over above the armature conductors placed in it and secured in place by special hard wooden or fiber wedges. Two types of armature windings are used – Lap winding and Wave winding.

Commutator

The commentator plays a vital role in DC generator. It collects current from armature and sends it to the load as direct current. It actually takes alternating current from armature and converts it to direct current and then send it to external load. It is cylindrical structured and is build up of wedge-shaped segments of high conductivity, hard drawn or drop forged copper. Each segment is insulated from the shaft by means of insulated commutator segment shown below. Each commentator segment is connected with corresponding armature conductor through segment riser or lug.

Brushes

The brushes are made of carbon. These are rectangular block shaped. The only function of these carbon brushes of DC generator is to collect current from commutator segments. The brushes are housed in the rectangular box shaped brush holder or brush box. As shown in figure, the brush face is placed on the commutator segment which is attached to the brush holder.

Bearing

For small machine, ball bearing is used and for heavy duty DC generator, roller bearing is used. The bearing must always be lubricated properly for smooth operation and long life of generator.

Armature winding

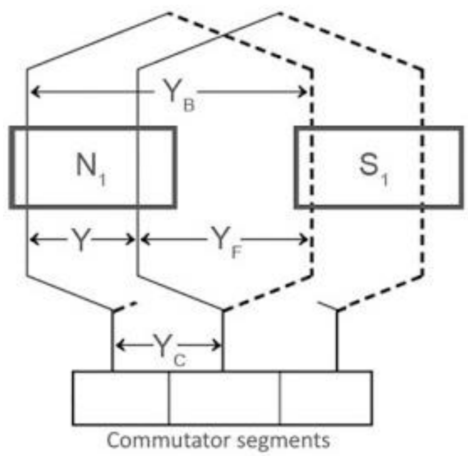
Basically armature winding of a DC machine is wound by one of the two methods, lap winding or wave winding. The difference between these two is merely due to the end connections and commutator connections of the conductor. To know how armature winding is done, it is essential to know the following terminologies -

1. Pole pitch: It is defined as number of armature slots per pole. For example, if there are 36 conductors and 4 poles, then the pole pitch is 36/4=9.
2. Coil span or coil pitch (Ys): It is the distance between the two sides of a coil measured in terms of armature slots.
3. Front pitch (Yf): It is the distance, in terms of armature conductors, between the second conductor of one coil and the first conductor of the next coil. OR it is the distance between two coil sides that are connected to the same commutator segment.
4. Back pitch (Yb): The distance by which a coil advances on the back of the armature is called as back pitch of the coil. It is measured in terms of armature conductors.
5. Resultant pitch (Yr): The distance, in terms of armature conductor, between the beginning of one coil and the beginning of the next coil is called as resultant pitch of the coil.

Armature winding can be done as single layer or double layer. It may be simplex, duplex or multiplex, and this multiplicity increases the number of parallel paths.

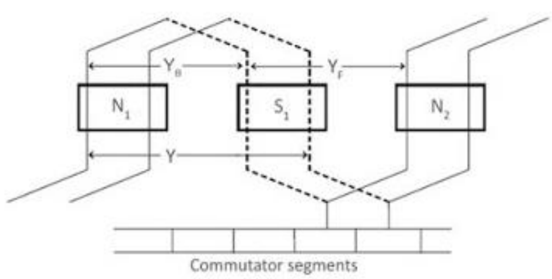
Lap Winding and Wave Winding

In lap winding, the successive coils overlap each other. In a simplex lap winding, the two ends of a coil are connected to adjacent commutator segments. The winding may be progressive or retrogressive. A progressive winding progresses in the direction in which the coil is wound. The opposite way is retrogressive. The following image shows progressive simplex lap winding.



Lap Winding

In wave winding, a conductor under one pole is connected at the back to a conductor which occupies an almost corresponding position under the next pole which is of opposite polarity. In other words, all the coils which carry e.m.f in the same direction are connected in series. The following diagram shows a part of simplex wave winding.



Wave winding

Basis For Comparison	**Lap Winding**	**Wave Winding**
Definition	The coil is lap back to the succeeding coil.	The coil of the winding form the wave shape.
Connection	The end of the armature coil is connected to an adjacent segment on the commutators.	The end of the armature coil is connected to commutator segments some distance apart.
Parallel Path	The numbers of parallel path are equal to the total of number poles.	The number of parallel paths is equal to two.
Other Name	Parallel Winding or Multiple Winding	Two-circuit or Series Winding.
EMF	Less	More
Number of Brushes	Equal to the number of parallel paths.	Two
Types	Simplex and Duplex lap winding.	Progressive and Retrogressive wave winding
Efficiency	Less	High
Additional Coil	Equalizer Ring	Dummy coil
Winding Cost	High (because more conductor is required)	Low
Uses	In low voltage, high current machines.	In high voltage, low current machines.

EMF Equation of a DC Generator

Consider a DC generator with the following parameters,

P = number of field poles

Ø = flux produced per pole in Wb (weber)

Z = total no. of armature conductors

A = no. of parallel paths in armature

N = rotational speed of armature in revolutions per min. (rpm)

Now,

- Average emf generated per conductor is given by dΦ/dt (Volts) ... eq. 1
- Flux cut by one conductor in one revolution = dΦ = PΦ ….(Weber),
- Number of revolutions per second (speed in RPS) = N/60
- Therefore, time for one revolution = dt = 60/N (Seconds)
- From eq. 1, emf generated per conductor = dΦ/dt = PΦN/60 (Volts) …..(eq. 2)

Above equation-2 gives the emf generated in one conductor of the generator. The conductors are connected in series per parallel path, and the emf across the generator terminals is equal to the generated emf across any parallel path.

Therefore, Eg = PΦNZ / 60A

For simplex lap winding, number of parallel paths is equal to the number of poles (i.e. A=P),

Therefore, for simplex lap wound dc generator, Eg = PΦNZ / 60P

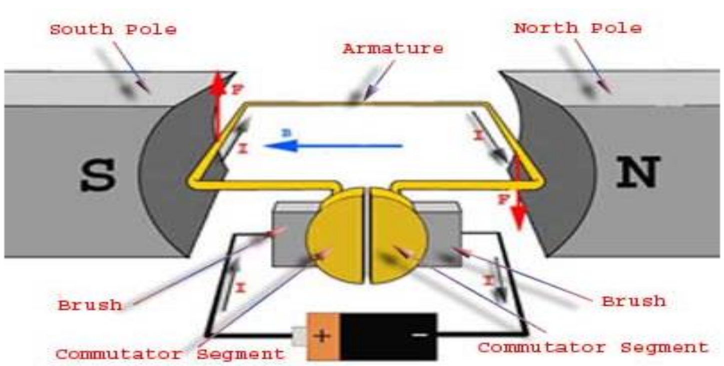
For simplex wave winding, number of parallel paths is equal to 2 (i.e P=2),

Therefore, for simplex wave wound dc generator, Eg = PΦNZ / 120

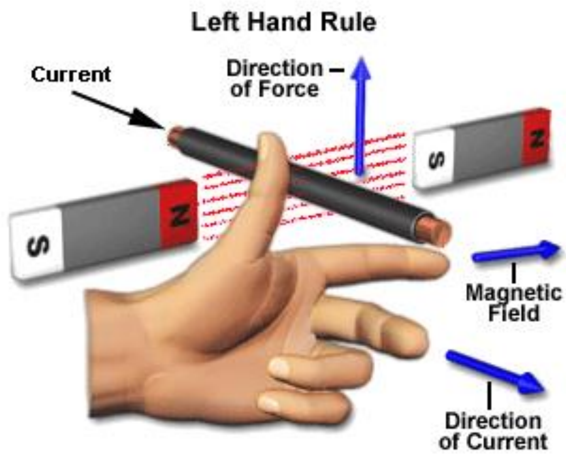
DC MOTOR

Working or Operating Principle of DC Motor

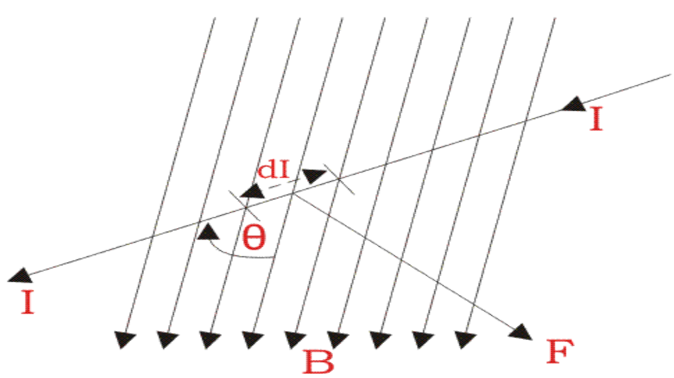
A DC motor in simple words is a device that converts electrical energy (direct current system) into mechanical energy. It is of vital importance for the industry today, and is equally important for engineers to look into the working principle of DC motor in details that has been discussed in this article. In order to understand the operating principle of DC motor we need to first look into its constructional feature. The very basic construction of a DC motor contains a current carrying armature which is connected to the supply end through commutator segments and brushes. The armature is placed in between north south poles of a permanent or an electromagnet as shown in the diagram above.



As soon as we supply direct current in the armature, a mechanical force acts on it due to electromagnetic effect of the magnet. Now to go into the details of the operating principle of DC motor its important that we have a clear understanding of Fleming's left hand rule to determine the direction of force acting on the armature conductors of DC motor.



If a current carrying conductor is placed in a magnetic field perpendicularly, then the conductor experiences a force in the direction mutually perpendicular to both the direction of field and the current carrying conductor. Fleming's left hand rule says that if we extend the index finger, middle finger and thumb of our left hand perpendicular to each other, in such a way that the middle finger is along the direction of current in the conductor, and index finger is along the direction of magnetic field i.e. north to south pole, then thumb indicates the direction of created mechanical force. For clear understanding the principle of DC motor we have to determine the magnitude of the force, by considering the diagram below.



We know that when an infinitely small charge dq is made to flow at a velocity 'v' under the influence of an electric field E, and a magnetic field B, then the Lorentz Force dF experienced by the charge is given by:-

$$dF = dq(E + vB)$$

For the **operation of DC motor**, considering E = 0

$$dF = dq \times v \times B$$

i.e. it's the cross product of dq v and magnetic field B.

$$dF = dq\frac{dL}{dt} \times B \qquad \left[V = \frac{dL}{dt}\right]$$

Where, dL is the length of the conductor carrying charge q.

$$dF = dq\frac{dL}{dt} \times B$$

$$or,\ dF = IdL \times B \qquad \left[Since,\ current\ I = \frac{dq}{dt}\right]$$

$$or,\ F = IL \times B = ILB\sin\theta$$

$$or,\ F = BIL\sin\theta$$

From the 1st diagram we can see that the construction of a DC motor is such that the direction of current through the armature conductor at all instance is perpendicular to the field. Hence the force acts on the armature conductor in the direction perpendicular to the both uniform field and current is constant.

$$i.e.\ \theta = 90^\circ$$

So if we take the current in the left hand side of the armature conductor to be I, and current at right hand side of the armature conductor to be -I, because they are flowing in the opposite direction with respect to each other.

Then the force on the left hand side armature conductor,

$$F_i = BIL\sin 90^\circ = BIL$$

Similarly force on the right hand side conductor

$$F_r = B(-I)L\sin 90^\circ = -BIL$$

Therefore, we can see that at that position the force on either side is equal in magnitude but opposite in direction. And since the two conductors are separated by some distance w = width of the armature turn, the two opposite forces produces a rotational force or a torque that results in the rotation of the armature conductor.

Now let's examine the expression of torque when the armature turn crate an angle of α (alpha) with its initial position.

The torque produced is given by,

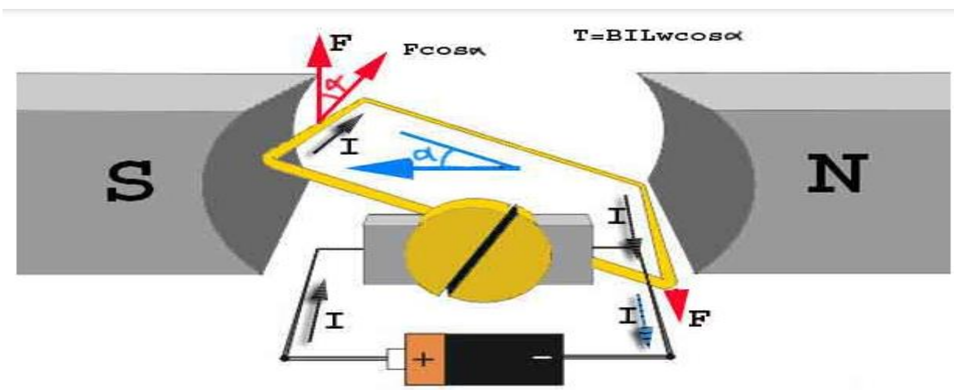
$$Torque = (force,\ tangential\ to\ the\ direction\ of\ armature\ rotation) \times (distance)$$

$$or,\ \tau = F\cos\alpha \times w$$

$$or,\ \tau = BILw\cos\alpha$$

Where, α (alpha) is the angle between the plane of the armature turn and the plane of reference or the initial position of the armature which is here along the direction of magnetic field.

The presence of the term cosα in the torque equation very well signifies that unlike force the torque at all position is not the same. It in fact varies with the variation of the angle α (alpha). To explain the variation of torque and the principle behind rotation of the motor let us do a step wise analysis.

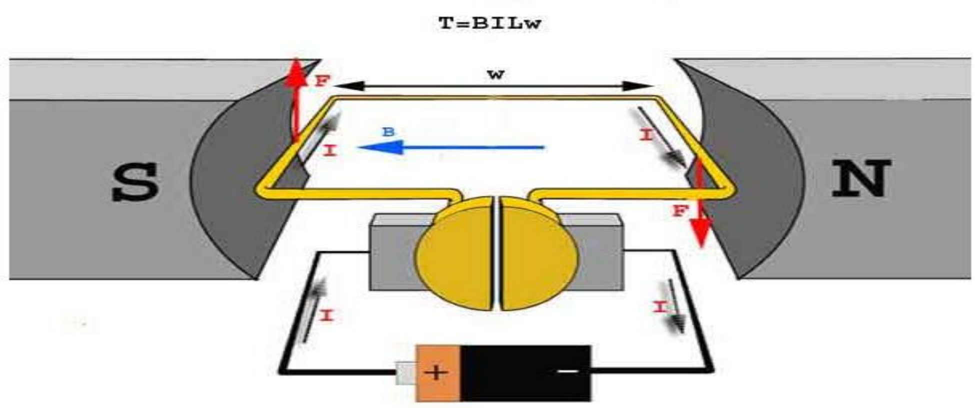


Step 1:

Initially considering the armature is in its starting point or reference position where the angle α = 0.

$$\therefore\ \tau = BILw \times \cos 0^\circ = BILw$$

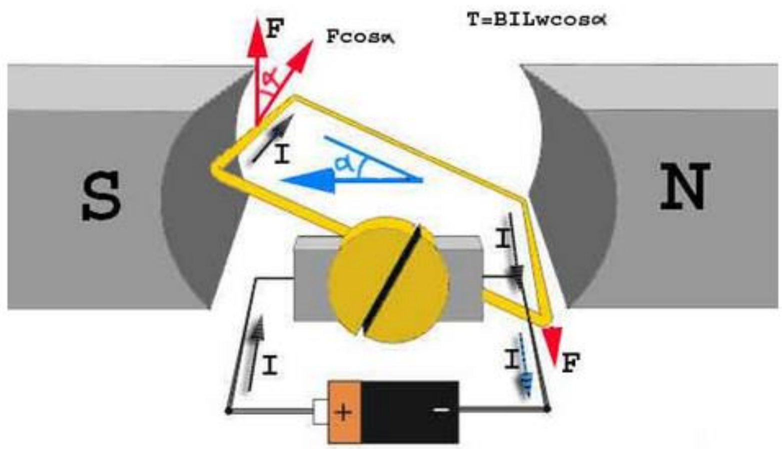
Since, α = 0, the term cos α = 1, or the maximum value, hence torque at this position is maximum given by τ = BILw. This high starting torque helps in overcoming the initial inertia of rest of the armature and sets it into rotation.



Step 2:

Once the armature is set in motion, the angle α between the actual position of the armature and its reference initial position goes on increasing in the path of its rotation until it becomes 90° from its initial position. Consequently the term cosα decreases and also the value of torque.

The torque in this case is given by τ = BILwcosα which is less than BIL w when α is greater than 0°.

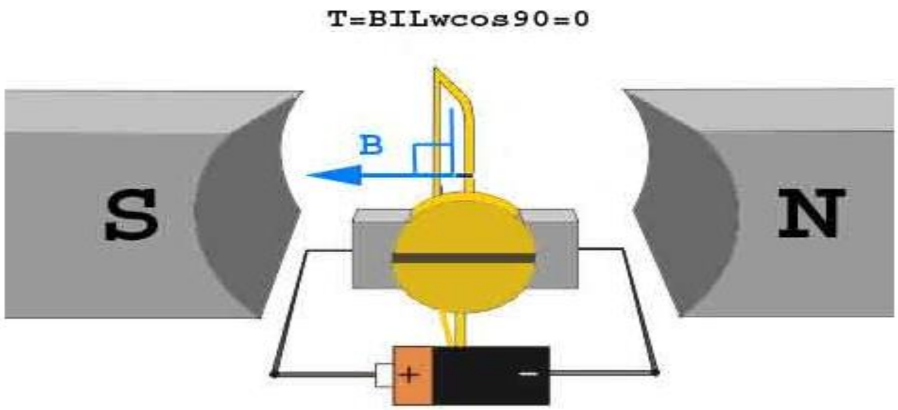


Step 3:

In the path of the rotation of the armature a point is reached where the actual position of the rotor is exactly perpendicular to its initial position, i.e. α = 90°, and as a result the term cosα = 0.

The torque acting on the conductor at this position is given by,

$$\therefore \tau = BIL\omega \times \cos 90^o = 0$$



i.e. virtually no rotating torque acts on the armature at this instance. But still the armature does not come to a standstill, this is because of the fact that the operation of DC motor has been engineered in such a way that the inertia of motion at this point is just enough to overcome this point of null torque. Once the rotor crosses over this position the angle between the actual position of the armature and the initial plane again decreases and torque starts acting on it again.

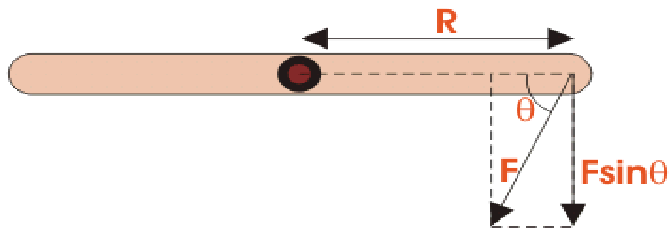
Torque Equation of DC Motor

When a DC machine is loaded either as a motor or as a generator, the rotor conductors carry current. These conductors lie in the magnetic field of the air gap. Thus each conductor experiences a force. The conductors lie near the surface of the rotor at a common radius from its center. Hence torque is produced at the circumference of the rotor and rotor starts rotating. The term torque as best explained by Dr. Huge d Young is the quantitative measure of the tendency of a force to cause a rotational motion, or to bring about a change in rotational motion. It is in fact the moment of a force that produces or changes a rotational motion.

The equation of torque is given by,

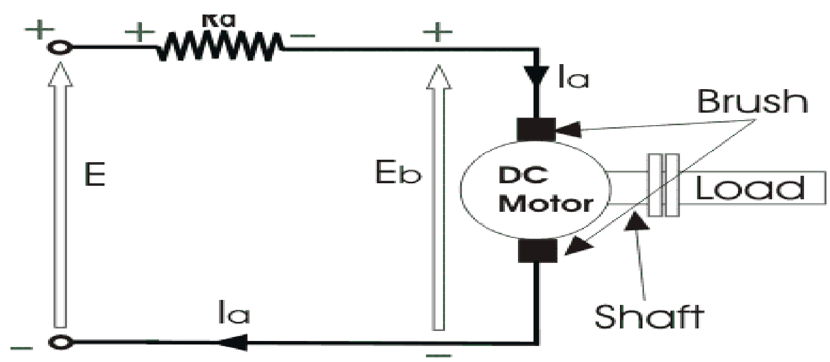
$$\tau = FR\sin\theta \text{..................} (1)$$

Where, F is force in linear direction.
R is radius of the object being rotated,
and θ is the angle, the force F is making with R vector



The DC motor as we all know is a rotational machine, and torque of DC motor is a very important parameter in this concern, and it's of utmost importance to understand the torque equation of DC motor for establishing its running characteristics.

To establish the torque equation, let us first consider the basic circuit diagram of a DC motor, and its voltage equation.



Referring to the diagram beside, we can see, that if E is the supply voltage, E_b is the back emf produced and I_a, R_a are the armature current and armature resistance respectively then the voltage equation is given by,

$$E = E_b + I_a R_a \text{ .... } (2)$$

But keeping in mind that our purpose is to derive the torque equation of DC motor we multiply both sides of equation (2) by I_a.

$$Therefore,\ EI_a = E_b I_a + I_a^2 R_a \text{ .... } (3)$$

Now $I_a^2.R_a$ is the power loss due to heating of the armature coil, and the true effective mechanical power that is required to produce the desired torque of DC machine is given by,

$P_m = E_b I_a$ .... (4)

The mechanical power P_m is related to the electromagnetic torque T_g as,

$P_m = T_g \omega$ .... (5)

Where ω is speed in rad/sec.

Now equating equation (4) and (5) we get,

$$E_b I_a = T_g \omega$$

Now for simplifying the torque equation of DC motor we substitute.

$$E_b = \frac{P\varphi ZN}{60A} \text{ .... (6)}$$

Where, P is no of poles,

φ is flux per pole,

Z is no. of conductors,

A is no. of parallel paths,

and N is the speed of the DC motor.

$$Hence,\ w = \frac{2\pi N}{60} \text{ .... (7)}$$

Substituting equation (6) and (7) in equation (4), we get:

$$T_g = \frac{P.Z.\varphi.I_a}{2\pi A}$$

The torque we so obtain, is known as the electromagnetic torque of DC motor, and subtracting the mechanical and rotational losses from it we get the mechanical torque. Therefore,

$$T_m = T_g - mechanical\ losses$$

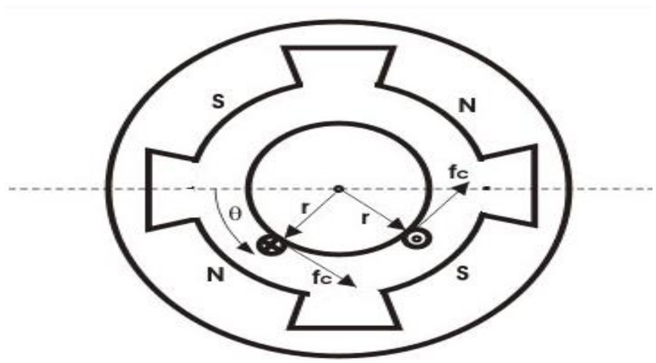
\This is the torque equation of DC motor. It can be further simplified as:

$$T_g = k_a \phi I_A$$

$$Where,\ k_a = \frac{P.Z}{2\pi A}$$

Which is constant for a particular machine and therefore the torque of DC motor varies with only flux φ and armature current I_a.

The Torque equation of a DC motor can also be explained considering the figure below.



$$\text{Here we can see Area per pole } A_r = \frac{2\pi.r.L}{P}$$

$$B = \frac{\varphi}{A_r}$$

$$\text{Here we can see Area per pole } A_r = \frac{2\pi.r.L}{P}$$

$$B = \frac{\varphi}{A_r}$$

$$B = \frac{P.\varphi}{2\pi r L}$$

Current/conductor $I_C = I_a$ A
Therefore, force per conductor = $f_C = BLI_a/A$
Now torque $T_C = f_C.\ r = BLI_a.r/A$

$$\therefore\ T_c = \frac{\varphi P.I_a}{2\pi A}$$

Hence, the total torque developed of a DC machine is,

$$T_g = \frac{P.Z.\varphi.I_a}{2\pi.A}$$

This torque equation of DC motor can be further simplified as:

$$T_g = k_a \phi I_a$$

$$Where,\ k_a = \frac{P.Z}{2\pi.A}$$

Which is constant for a particular machine and therefore the torque of DC motor varies with only flux φ and armature current I_a.

TRANSFORMER

Introduction

The transformer is a device that transfers electrical energy from one electrical circuit to another electrical circuit. The two circuits may be operating at different voltage levels but always work at the same frequency. Basically transformer is an electro-magnetic energy conversion device. It is commonly used in electrical power system and distribution systems. It can change the magnitude of alternating voltage or current from one value to another. This useful property of transformer is mainly responsible for the widespread use of alternating currents rather than direct currents i.e., electric power is generated, transmitted and distributed in the form of alternating current. Transformers have no moving parts, rugged and durable in construction, thus requiring very little attention. They also have a very high efficiency as high as 99%.

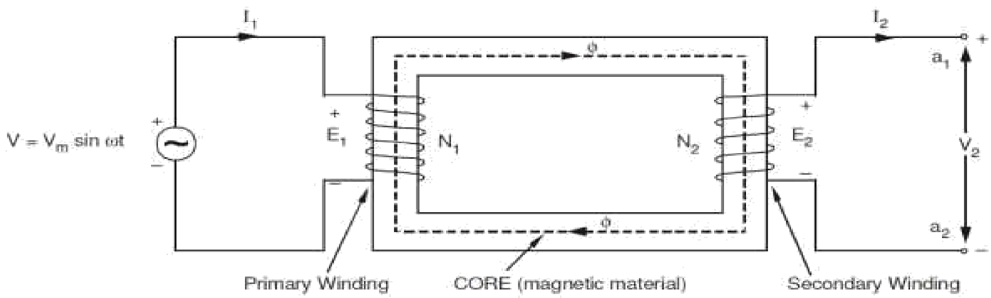
Single Phase Transformer

A transformer is a static device of equipment used either for raising or lowering the voltage of an a.c. supply with a corresponding decrease or increase in current. It essentially consists of two windings, the primary and secondary, wound on a common laminated magnetic core as shown in Fig 1. The winding connected to the a.c. source is called primary winding (or primary) and the one connected to load is called secondary winding (or secondary). The alternating voltage V1 whose magnitude is to be changed is applied to the primary.

Depending upon the number of turns of the primary (N1) and secondary (N2), an alternating e.m.f. E2 is induced in the secondary. This induced e.m.f. E2 in the secondary causes a secondary current I2. Consequently, terminal voltage V2 will appear across the load.

If V2 > V1, it is called a step up-transformer.

If V2 < V1, it is called a step-down transformer.



Constructional Details

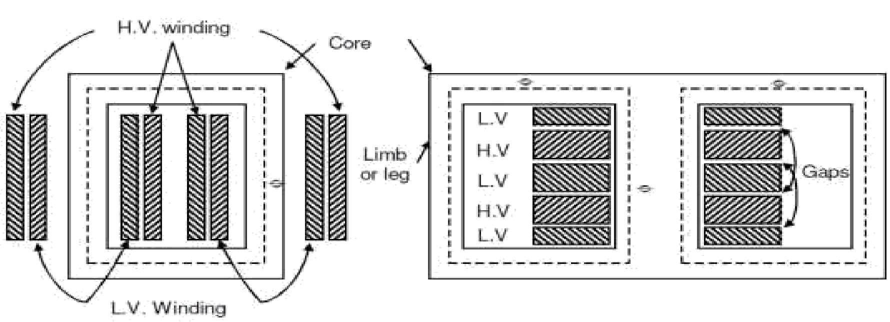
Depending upon the manner in which the primary and secondary windings are placed on the core, and the shape of the core, there are two types of transformers, called

(a) Core type

(b) Shell type.

Core-type and Shell-type Construction

In core type transformers, the windings are placed in the form of concentric cylindrical coils placed around the vertical limbs of the core. The low-voltage (LV) as well as the high- voltage (HV) winding are made in two halves, and placed on the two limbs of core. The LV winding is placed next to the core for economy in insulation cost. Figure 2.1(a) shows the cross- section of the arrangement. In the shell type transformer, the primary and secondary windings are wound over the central limb of a three-limb core as shown in Figure 2.1(b). The HV and LV windings are split into a number of sections, and the sections are interleaved or sandwiched i.e. the sections of the HV and LV windings are placed alternately.



(a) Core Type (b) Shell Type

Core

The core is built-up of thin steel laminations insulated from each other. This helps in reducing the eddy current losses in the core, and also helps in construction of the transformer. The steel used for core is of high silicon content, sometimes heat treated to produce a high permeability and low hysteresis loss. The material commonly used for core is CRGO (Cold Rolled Grain Oriented) steel. Conductor material used for windings is mostly copper. However, for small distribution transformer aluminum is also sometimes used. The conductors, core and whole windings are insulated using various insulating materials depending upon the voltage.

Insulating Oil

In oil-immersed transformer, the iron core together with windings is immersed in insulating oil. The insulating oil provides better insulation, protects insulation from moisture and transfers the heat produced in core and windings to the atmosphere.

The transformer oil should possess the following qualities:

(a)High dielectric strength,

(b)Low viscosity and high purity,

(c)High flash point, and

(d)Free from sludge.

Transformer oil is generally a mineral oil obtained by fractional distillation of crude oil.

Tank and Conservator

The transformer tank contains core wound with windings and the insulating oil. In large transformers small expansion tank is also connected with main tank is known as conservator. Conservator provides space when insulating oil expands due to heating. The transformer tank is provided with tubes on the outside, to permits circulation of oil, which aides in cooling. Some additional devices like breather and Buchholz relay are connected with main tank. Buchholz relay is placed between main tank and conservator. It protect the transformer under extreme heating of transformer winding. Breather protects the insulating oil from moisture when the cool transformer sucks air inside. The silica gel filled breather absorbs moisture when air enters the tank. Some other necessary parts are connected with main tank like, Bushings, Cable Boxes, Temperature gauge, Oil gauge, Tapings, etc.

Principle of Operation

When an alternating voltage V1 is applied to the primary, an alternating flux ϕ is set up in the core. This alternating flux links both the windings and induces e.m.f.s E1 and E2 in them according to Faraday's laws of electromagnetic induction. The e.m.f. E1 is termed as primary e.m.f. and E2 is termed as secondary e.m.f.

Clearly, $E_1 = -N_1 \frac{d\phi}{dt}$

and $E_2 = -N_2 \frac{d\phi}{dt}$

$$\therefore \quad \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

Note that magnitudes of E2 and E1 depend upon the number of turns on the secondary and primary respectively.

If N2 > N1, then E2 > E1 (or V2 > V1) and we get a step-up transformer.

If N2 < N1, then E2 < E1(or V2< V1) and we get a step-down transformer.

If load is connected across the secondary winding, the secondary e.m.f. E2 will cause a current I2 to flow through the load. Thus, a transformer enables us to transfer a.c. power from one circuit to another with a change in voltage level.

The following points may be noted carefully

(a)The transformer action is based on the laws of electromagnetic induction.

(b)There is no electrical connection between the primary and secondary.

(c)The a.c. power is transferred from primary to secondary through magnetic flux.

(d)There is no change in frequency i.e., output power has the same frequency as the input power.

(e)The losses that occur in a transformer are:

(a)core losses—eddy current and hysteresis losses

(b) copper losses—in the resistance of the windings

In practice, these losses are very small so that output power is nearly equal to the input primary power. In other words, a transformer has very high efficiency

E.M.F. Equation of a Transformer

Consider that an alternating voltage V1 of frequency f is applied to the primary as shown in Fig.2.3. The sinusoidal flux ϕ produced by the primary can be represented as:

$$\phi = \phi m \sin\omega t$$

When the primary winding is excited by an alternating voltage V1, it is circulating alternating current, producing an alternating flux ϕ.

ϕ - Flux

ϕm - maximum value of flux ,

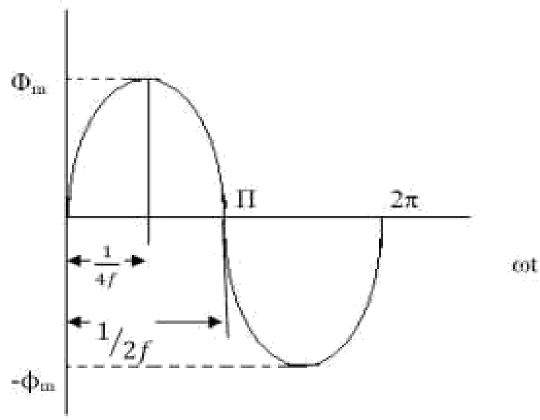
N1 - Number of primary turns ,

N2 - Number of secondary turns

f - Frequency of the supply voltage

E1 - R.M.S. value of the primary induced e.m.f ,E2 - R.M.S. value of the secondary induced e.m.f

The instantaneous e.m.f. e1 induced in the primary is –



The flux increases from zero value to maximum value ϕm in 1/4f of the time period that is in 1/4f seconds.

The change of flux that takes place in 1/4f seconds = $\phi m - 0 = \phi m$ webers

Voltage Ratio

$$\frac{d\phi}{dt} = \frac{dt}{1/4f} = 4f\phi_m \ w_b/\text{sec.}$$

Since flux ϕ varies sinusoidally, the R.m.s value of the induced e.m.f is obtained by multiplying the average value with the form factor

$$\text{Form factor of a sinwave} = \frac{\text{R.m.s value}}{\text{Average value}} = 1.11$$

R.M.S Value of e.m.f induced in one turns = $4\phi_m f \times 1.11$ Volts.

$= 4.44\phi_m f$ Volts.

R.M.S Value of e.m.f induced in primary winding = $4.44\phi_m f$ N1Volts.

R.M.S Value of e.m.f induced in secondary winding = $4.44\phi_m f$ N2Volts.

The expression of E_1 and E_2 are called e.m.f equation of a transformer

$V_1 = E_1 = 4.44\phi_m f N_1$ Volts.
$V_2 = E_2 = 4.44\phi_m f N_2$ Volts.

$$\frac{E2}{E1} = \frac{4.44\phi mf\,N2}{4.44\phi mf\,N1}$$

$$\boxed{\frac{E2}{E1} = \frac{N2}{N1} = K}$$

Voltage transformation ratio is the ratio of e.m.f induced in the secondary winding to the e.m.f induced in the primary winding.

This ratio of secondary induced e.m.f to primary induced e.m.f is known as voltage transformation ratio

1. If N2>N1 i.e. K>1 we get E2>E1 then the transformer is called step up transformer.
2. If N2< N1 i.e. K<1 we get E2< E2 then the transformer is called step down transformer.
3. If N2= N1 i.e. K=1 we get E2= E2 then the transformer is called isolation transformer or 1:1 Transformer.

$$E_2 = KE_1 \qquad \text{where } K = \frac{N2}{N1}$$

Current Ratio

Current ratio is the ratio of current flow through the primary winding (I1) to the current flowing through the secondary winding (I2). In an ideal transformer -

Apparent input power = Apparent output power.

V1I1 = V2I2

$$\frac{I1}{I2} = \frac{V2}{V1} = \frac{N2}{N1} = K$$

Volt-Ampere Rating

i)The transformer rating is specified as the products of voltage and current (VA rating).

ii)On both sides, primary and secondary VA rating remains same. This rating is generally expressed in KVA (Kilo Volts Amperes rating)

$$\frac{V1}{V2} = \frac{I2}{I1} = K$$

$$V_1I_1 = V_2I_2$$

$$\text{KVA Rating of a transformer} = \frac{V1I1}{1000} = \frac{V2I2}{1000} \quad \text{(1000 is to convert KVA to VA)}$$

V_1 and V_2 are the V_t of primary and secondary by using KVA rating we can calculate I_1 and I_2 Full load current and it is safe maximum current.

$$I_1 \text{ Full load current} = \frac{KVA\ Rating\ X\ 1000}{V1}$$

$$I_1 \text{ Full load current} = \frac{KVA\ Rating\ X\ 1000}{V2}$$

UNIT –V

ELECTRICAL INSTALLATIONS

- **Components of LT Switchgear: Switch Fuse Unit (SFU), MCB, ELCB, MCCB.**
- **Types of Wires and Cables.**
- **Earthing.**
- **Types of Batteries.**
- **Important Characteristics for Batteries.**
- **Elementary calculations for energy consumption and battery backup.**

Wire and cable:

The use of Conductors and their insulation is regulated by Indian Electricity (IE) regulation and Indian Standard (IS) Code Of Practice. Wires and cables are the most common forms of conductors. They carry electric current through all types of circuits and systems. A conductor is a wire or cable or any other form of mental, suitable for carrying current from generating station the point where it is used.

Difference Between Wire and Cable:

According to Bureau of Indian Standards (BIS), wire and cable can be defined as follows :

Bare Conductors : They have no covering. The best example is overhead transmission and distribution lines.

Wire: If bare conductors are provided with Insulation, then it is known as a wire. The insulation separates the conductor electrically from other conductors.

Cable: It consists of two or more conductors covered with suitable insulation and surrounded by a protecting cover. The necessary requirements of a cable are that it should conduct electricity efficiently, cheaply, and safely. This should neither be so small that it has a large internal voltage drop nor be too large so that it costs too much. Its insulation should be such that it prevents leakage of current in unwanted direction to minimize risk of fire and shock.

The cable essentially consists of three parts :

(i) Conductor or core- the metal wire, or strand of wires, carrying the current

(II) insulation of dielectric- a covering of insulating material to avoid leakage of current from the conductor and

(iii) protective covering for protection of insulation from mechanical damage

Basically, there is no difference between a cable and a wire. It is a relative term. The term cable is used for all heavy section insulated conductors, whereas a wire means a thin (i.e., smaller) section insulated conductor used for carrying current from one point to another point.

Classifications of Wire / Cables:

The wires/ cables used for domestic or industrial wiring are classified into different groups as follows :

(i) According to the conductor material used

(a) Copper conductor cables

(b) Aluminium conductor cable

(ii) According to number of cores

(a) Singles core cable (SCC)

(b) Double core or twin core cables (DCC)

(c) Three core cables

(d) four core cables

(e) Two core with earth continuity conductor cables

(iii) According to type of insulation

(a) Vulcanized Indian rubber (VIR) insulated wires/cables

(b) Tough rubber sheathed (TRS) or cable tyre sheathed (CTS) cables

(c) Polyvinyl chloride (PVC) cables

(b) Lead sheathed cables

(e) Weather proof cables

(f) Flexible cords and cables

(g) XLPE cables

(IV) According to the voltage at which they are manufactured

(a) Low tension (LT) cables – up to 1000V

(b) High tension (HT) cables – up to 11kV

(c) Super tension (ST) cables – from 22-33kV

(d) Extra high tension (EHT) cables – from 33-66kV

(e) Extra super voltage cables – beyond 132 kV

Specifications of Cables:

Cables are specified by providing

(i) Size of the cable in metric system (e.g., 19/2.24, 7/1.70, 7/2.24, 7/2.50 etc) giving the Number of strands used and diameter of each strand, or giving the area of cross- section of conductor used.

(ii) Type of conductor used in cables (copper or aluminium)

(iii) Number of cores that cable consists of e.g. single core, twin core, three core, four core etc.

(iv) Voltage grade (240/415V or 650/1100V grade)

(v) Type of cable with clear description regarding insulation, shielding, armouring, bedding etc.

A few specifications of a cable are given below:

(i) 7/20, VIR, aluminium conductor, twin core,650/1100 grade.

in this case, the numerator 7 indicates the number of stands in cable and denominator 20 represents the gauge number of each strand. The cable has two cores made with Aluminium, With VIR insulation and is used for 650/1100 voltage

(ii) 19/1.12, aluminium conductor,3 ½ core, 1100V, PVC cable, PVC sheathed.

in this case, the cable consists of 19 strands, each strand has a diameter of 1.12mm. The conductor is made with aluminium, insulation is made with PVC, is covered with PVC sheathing, and is used for 1100Vsupply system.

Earthing of Grounding:

The process of connecting the metallic frame (i.e., non- current carrying part) of electrical equipment or some electrical part of the system (e.g., neutral point in a star-connected system, one conductor of the secondary of a transformer, etc.) to the earth (i.e., soil) is called grounding or Earthing. The potential of the earth is to be considered zero for all practical purposes. Earthing is to connect any electrical equipment to earth with a very low resistance wire, making it to attain earth's potential, This ensures safe discharge of electrical energy due to failure of the insulation line coming in contact with the casing, etc. Earthing brings the potential of the body of the equipment to zero i.e., to the earth's potential, thus ptotecting the operating personnel against electrical shock.

The earth resistance is affected by the following factors :

(a) Material properties of the earth, wire and the electrode

(b) Temperature and moisture content of the soil

(c) Depth of the pit

(d) Quantity of the charcoal used

Necessity of Earthing:

The requirement for provision of earthing can be listed as follows :

(1) To protect the operating personnel from the danger of shock.

(2) To maintain the line voltage constant, under unbalanced load condition.

(3) To avoid risk of fire due to earth leakage current through unwanted path.

(4) Protection of the equipments.

(5) Protection of large buildings and all machines fed from overhead lines against lighting.

Methods of Earthing:

The various methods of earthing in common use are

(i) Plate earthing

(ii) Pipe earthing

(iii) Rod earthing

(iv) Strip or wire earthing

(i) Plate earthing:

In this method either a copper plate of 60cm × 60cm ×3.18 or GI plate of 60cm × 60cm × 6.35 is used for earthing. The plate is buried into the ground not less than 3m from the ground level. The earth plate is embedded in alternate layers of coal and salt for a thickness of 15cm as shown in figure (12.4). In addition, water is poured for keeping the earth's electrode resistance value below a maximum of 5Ω. The earth wire is securely bolted to the earth plate.

A cement masonry chamber is built with a cast iron cover for easy regular maintenance

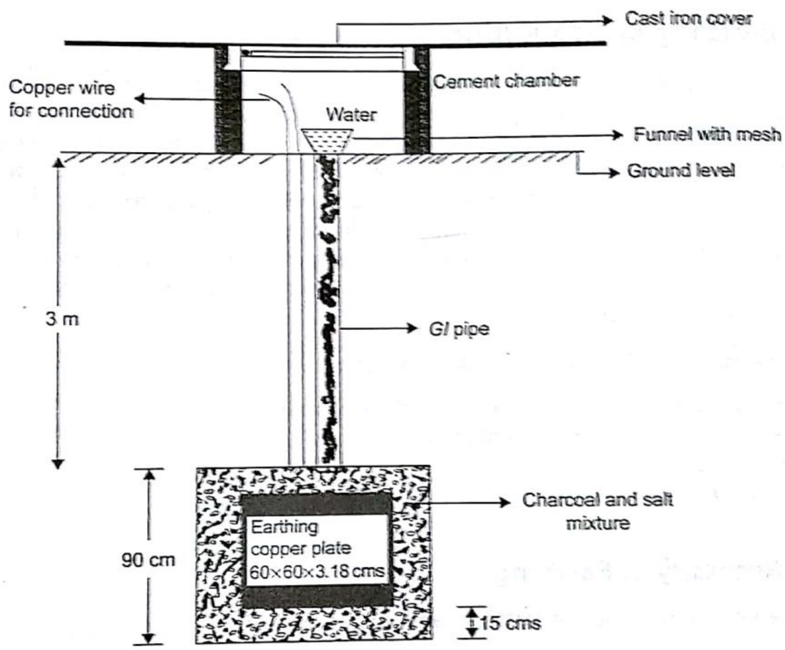


Figure (12.4): *Plate earthing*

(ii) Pipe earthing:

Earth electrode made of a GI (galvanized iron) pipe of 38mm in diameter and length of 2m (depending on the current) with 12mm holes on the surface is placed upright at a depth of 4.75cm in a permanently wet ground. To keep the value of the earth resistance at the desired level, the area (15 cm) surrounding the GI pipe is filled with a mixture of salt and coal. The efficiency of the earthing system is improved by pouring water through the funnel periodically. The GI earth wires of sufficient cross-sectional area are run through a 12.7mm diameter pipe (at 60cm below) from the 19mm diameter pipe and secured tightly at the top as shown in

figure (12.5).

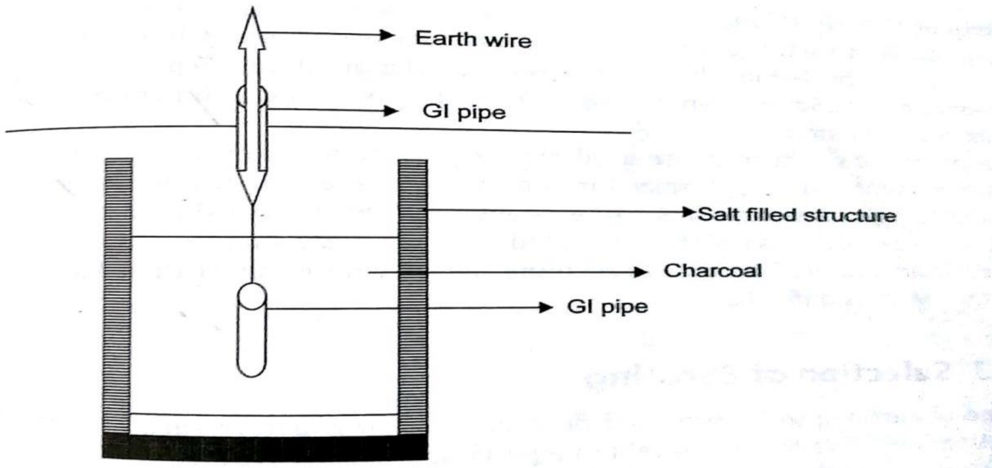


Figure (12.5): *Pipe earthing*

When compared to the plate earth system the pipe earth system can carry larger leakage currents due to larger surface area is in contact with the soil for given electrode size. This system also enables easy maintenance as the earth wire connection is housed at the ground levels.

(iii) Rod earthing:

It is the same method as pipe earthing, A copper rod of 12.5cm (1/2 inch) diameter or 16mm (0.6in) diameter of galvanized steel or hollow section 25mm (1 inch) of GI pipe of length above 2.5m (8.2 ft) are buried upright in the earth manually or with the help of a pneumatic hammer. The length of embedded electrodes in the soil reduces earth resistance to a desired value.

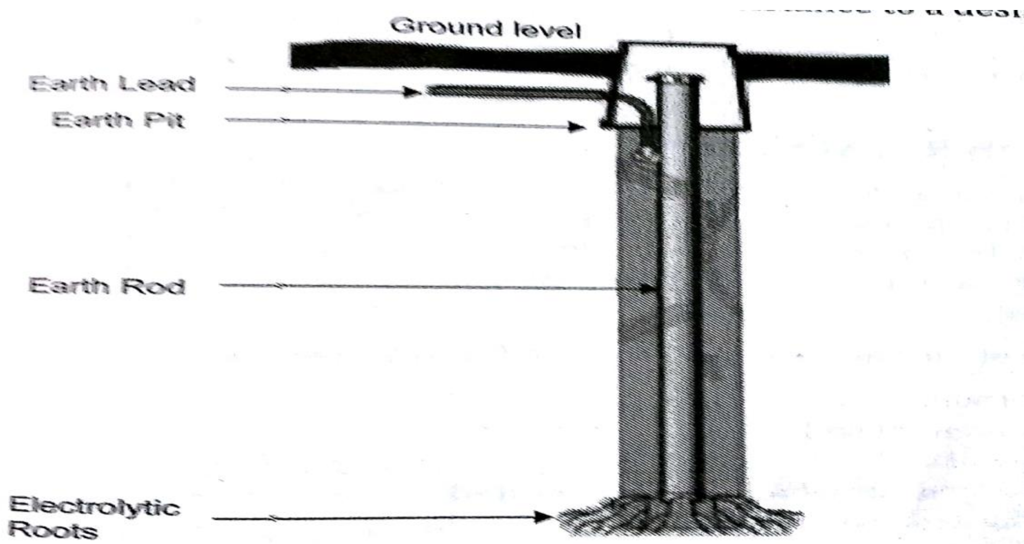


Figure (12.6): *Rod earthing*

(iv) Strip or wire earthing:

In this method of earthing strip electrodes of cross- section not less than 25mm × 1.6mm (1 in × 0.06in) is buried in a horizontal trenches of a minimum depth of 0.5m. If copper with a cross-section of 25mm × 4mm (1in × 0.15in) is used and a dimension of 3.0 mm^2 if it's a galvanized iron or steel.

If at all round conductors are used, their cross-section area should not be too small, say less than 6.0 mm^2 if it's a galvanized iron or steel. The length of the conductor buried in the ground would give a sufficient earth resistance and this length should not be less than 15m. The electrodes shall be as widely distributed as possible in a single straight or circular trenches radiating from a point. This type of earthing is used where the earth bed has a rocky soil and excavation work is difficult.

Selection of Earthing:

The type of earthing to be provided depends on many factors such as type of soil, type of installation, etc.. The following table helps in selecting a type of earthing for a particular application

S.No	Type of Earthing	Application
01	Plate earthing	Large installations such as transmission towers, all sub-stations generating stations
02	Pipe earthing	• For domestic installations such as heaters, coolers, refrigerators, geysers, electric iron, etc. • For 11kV/400V distribution transformers • For induction motors rating upto 100HP • For conduit pipe in a wall, all wall brackets
03	Rod earthing	In areas where the soil is loose or sandy
04	Strip of wire earthing	In rocky ares

Earth Resistance:

The earth resistance should be kept as low as possible so that the neutral of any electrical system, which is earthed, is maintained almost at the earth potential. The earth resistance for copper wire is 1Ω and that of GI wire less than 3Ω. The typical value of the earth resistance at large power stations is 0.5Ω , major sub-stations is 1Ω, small sub-stations is 2 Ω and in all other cases 5 Ω.

The resistance of the earth depends on the following factors

Condition of soil.

ii. Moisture content of soil.

iii. Temperature of soil.

iv. Depth of electrode at which it is embedded.

v. Size, material and spacing of earth electrode.

vi. Quality and quantity of coal and salt in the earth pit.

Difference Between Earth Wire and Neutral Wire:

Neutral Wire :

(i) In a 3-phase 4-wire system, the fourth wire is a neutral wire.

(ii) IT acts a return path for 3-phase currents when the load is not balanced.

(iii) IN domestic single phase AC circuit, the neutral wire acts as return path for the line current.

Earth Wire :

(i) Earth wire is actually connected to the general mass of the earth and metallic body of the equipment

(ii) It is provided to transfer any leakage current from the metallic body to the earth.

Fuse:

The electrical equipment are designed to carry a particular rated value of current under normal conditions. Under abnormal conditions such as short circuits, overload, or any fault; the current rises above this value, damaging the equipment and sometimes resulting in fire hazard. Fuses come into operation under fault conditions.

A fuse is short piece of metal, inserted in the circuit, which melts when excessive current flows through it and thus breaks the circuits. Under normal operating conditions it designed to carry the full load current. If the current increases beyond this designed value due to any of the reasons mentioned above, the fuse melts, isolating the power supply from the load.

(a) Desirable characteristics of a Fuse Element:

The material used foe fuse wires must have the following characteristics:

i. Low melting point e.g., tin, lead.

ii. High conductivity e.g., copper.

iii. Free from deterioration due oxidation e.g., silver.

iv. Low cost e.g., tin, copper.

(b) Materials:

Material used are tin lead or silver having low melting points. Use of copper or iron is dangerous, though tinned copper may be used.

(c) Types of Fuses:

Fuses are classified into following types

(i) Re-wireable or kit-Kat fuse and

(ii) High rupturing capacity (H.R.C) cartridge fuse

Re-wireable or Kit-Kat Fuse:

Re-wireable fuse is used where low values of fault current are to be interrupted. These fuses are simple in construction, cheap and available up to a current rating of 200A.They are erratic in operation and their performance deteriorates with time. An image of re-wireable fuse is as shown in figure (12.7)

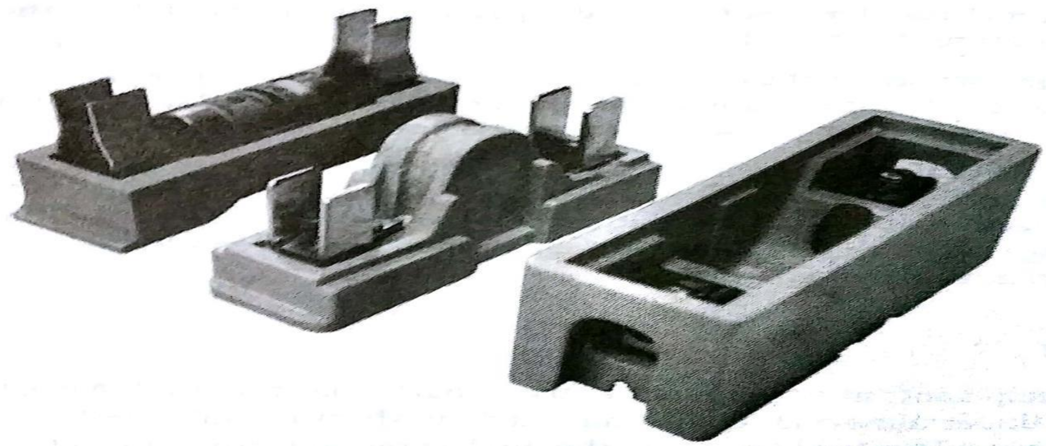


Figure (12.7): *Re-wireable or kit-kat fuse*

High Rupturing Capacity (HRC) Cartridge Fuse:

Figure (12.8) shown an image of HRC cartridge fuse and figure (12.9) shown the essential parts of a typical HRC cartridge fuse. It consists of a heat resisting ceramic body having metal end-caps to which a silver current-carrying element is welded. The space within the body surrounding the elements is completely packed with a filling powder. The filling material my be chalk, plaster of Paris, quartz or marble dust and acts as an arc quenching and cooling medium. Therefore, it carries the normal current without overheating

Under normal loading conditions, the fuse element is at a temperature below its melting point. When a fault occurs, the current increases and the fuse element melts before the fault current reaches its first peak. The heat produced in the process vaporizes the melted silver element. The chemical reaction between the silver vapors and the filling powder results in the formation of a high resistance substance which helps in quenching the arc.

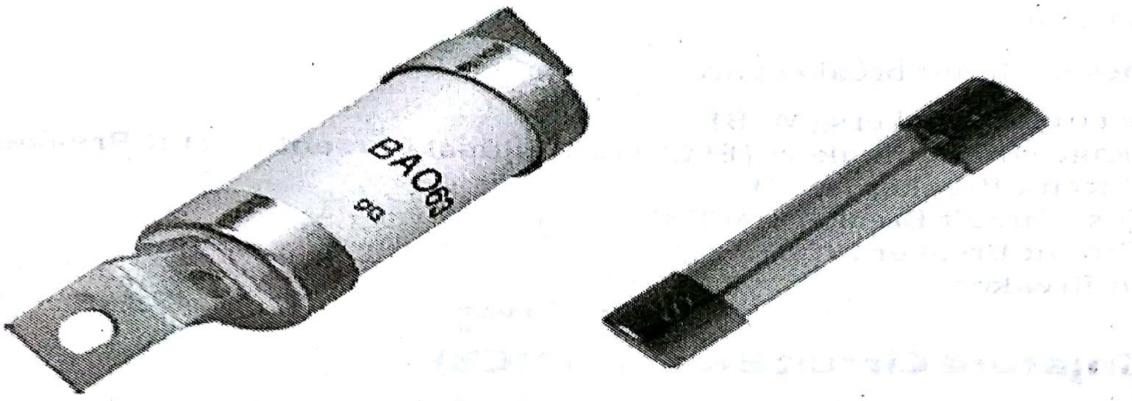


Figure (12.8): *HRC cartridge fuse*

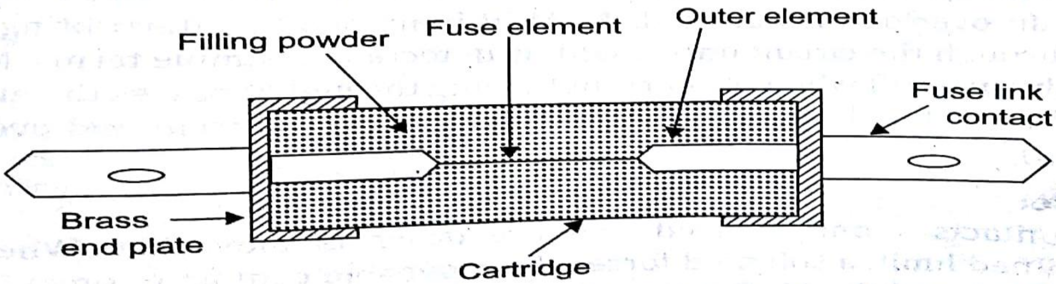


Figure (12.9): *Cross-section of HRC cartridge fuse*

Circuit Breaker:

Electrical circuits breaker is a switching device which can be operated manually and automatically for the controlling and protection of electrical power system, respectively. The modern power system deals with a huge power network and huge numbers of associated electrical equipments. During shirt circuits fault or any other type of electrical fault, these equipments, as well as the power network, suffer a high stress of fault current, which in turn damage the equipment and networks permanently. For saving these equipment and the power networks, the fault current should be cleared from the system as quickly as possible. Again after the cleared, the system must come to its normal working condition as soon is possible for supplying reliable quality power to the receiving ends. The circuits breaker is the special device all the required switching operations during current carrying condition.

A circuits breaker essentially consists of fixed and moving contacts, called electrodes. Under normal operating conditions, these contacts remain closed and will not open automatically until and unless the system becomes faulty. The contacts can be opened manually or by remote control whenever desired. When a fault occurs in any part of the system, the trip coils of the breaker get energized and the moving contacts are pulled apart by some mechanism, thus opening the circuits.

The main types of circuits breakers are

i. Miniature circuits breakers (MCB)

ii. Earth leakage circuits breakers (ELCB) or Residual Current Breaker (RCCB)

iii. Air blast Circuits Breaker (ACB)

iv. Molded Case Circuits Breakers (MCCB)

v. Vacuum Circuits Breaker (VCB)

vi. SF_6 Circuits Breaker

Miniature Circuit Breaker (MCB):

Minimum circuits breakers are electromechanical devices which protect an electrical circuits from over currents. Over currents in an electrical circuits may results from short circuits overload, or faulty design. An MCB is better alternative than fuse, since it does not require replacement once an overload is detected. An MCB functions by interrupting the continuity of electrical flow through the circuits once a fault is detected. In simple terms, MCB is a switch which automatically turns off when the current flowing through it passes the maximum allowable limit. Generally MCB is designed to protect against over current and over temperature faults (over heating).

Working Principle:

There are two contact - one is fixed and the other is moveable. When the current exceeds the predefined limit, a solenoid forces the moveable contact to open (i.e., disconnect from the fixed contact) and the MCB turns off, thereby stopping the current from flowing in the circuits.

Operation :

An image of MCB is shown in figure (12.10) and internal parts of an MCB are shown in figure (12.11). It mainly consists of one bi- metallic strip, one trip coil and one hand operated on-off lever. Electric current carrying path of a MCB is as follows - first left hand side power terminal-then bimetallic strip - then current coil - then moving contact - then fixed contact and - lastly
right hand side power terminal, and all are arranged in series.

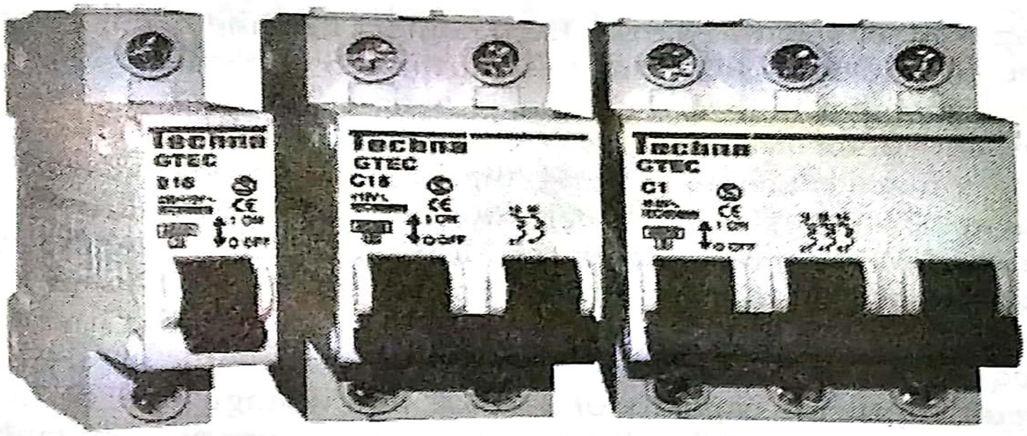


Figure (12.10): *Miniature circuit breaker*

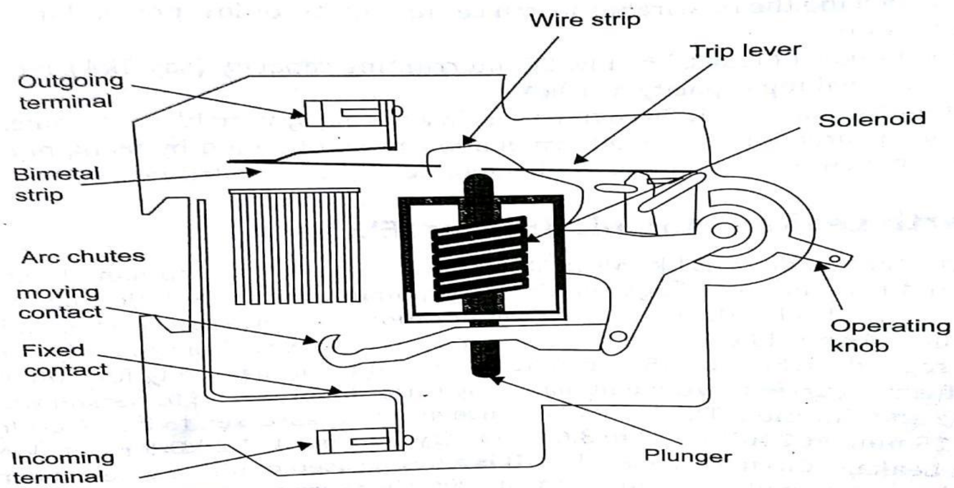


Figure (12.11): *Cross-section of MCB*

if circuits is overload for a long time, the bi -metallic strip becomes over heated and deformed. This deformation of bi-metallic strip causes displacement of latch point. The moving contact of the MCB is so arranged by means of spring, with this latch point, that a little displacement of latch causes releases of spring and makes the moving contact to move for opening the MCB. The current coil or trip coil placed in such a manner that during SC faults, the MMF of that coil causes its plunger to hit the same latch point and force the latch to be displaced. Hence, the MCB will open in the same manner. Again when operating lever of the MCB is operated by hand, that means when we make the MCB at off position manually, the same latch point is displaced as a result moving contact separated from fixed contact in same manner. So, whatever may be the operating mechanism, i.e., may be due to deformation of bi-metallic strip or may be due to increased MMF of trip coil or may be due to manual operation - actually the same latch point is displaced and the deformed spring is released, which is ultimately responsible for movement of the moving contact. When the moving contacts is separated from fixed contact, there may be a high chance of arc. This are then goes up thorough the arc runner and enters into arc splitters and is finally quenched. When we switch on the MCB, we actually reset the displaced operating latch to its previous on position and make the MCB ready for another switch off or trip operation.

These are available in single pole, double pole, triple pole, and four pole versions with neutral poles, if required. The normal current ratings are available from 0.5-63 A with a sym-metrical short circuits rupturing capacity of 3-10kA, at a voltage level of 230/440v. MCBs are generally designed to trip within 2.5 millisecond when an over current fault arises. In case of temperature rise or over heating it may take 2 seconds to 2 min. For the MCB to trip.

Advantages :

i. MCBs are replacing the re-wireable switch i.e., fuse units for low power domestic and industrial applications.

ii. The disadvantages of fuses, like low SC interrupting capacity (say 3kA), Etc. Are overcome with high SC breaking capacity of 10kA.

iii. MCB is combination of all three functions in a wiring system like switching, overload and short circuits protection. Overload protection can be obtained by using bi-metallic strips where as shorts circuits protection can be obtained by using solenoid

Earth Leakage Circuits Breaker (ELCB):

None of the protection devices like MCB, MCCB, etc. Can protect the human life against electric shocks or avoid fire due to leakage current. The human resistance noticeably drops with an increase in voltage. It also depends upon the duration of impressed voltage and drops with increase in time. As per IS code, a contact potential of 65V is within tolerable limit of human body for 10 seconds, where as 250V can be withstood by human body for 100 milliseconds. The actual effect of current thorough human body varies from person to person with reference to magnitude and duration. The body resistance at 10V is assessed to be 19 kΩ for 1 second and 8kΩ for 15 min. At 240V, 3 to 3.6 kΩ for dry skin and 1-1.2 kΩ for wet skin.

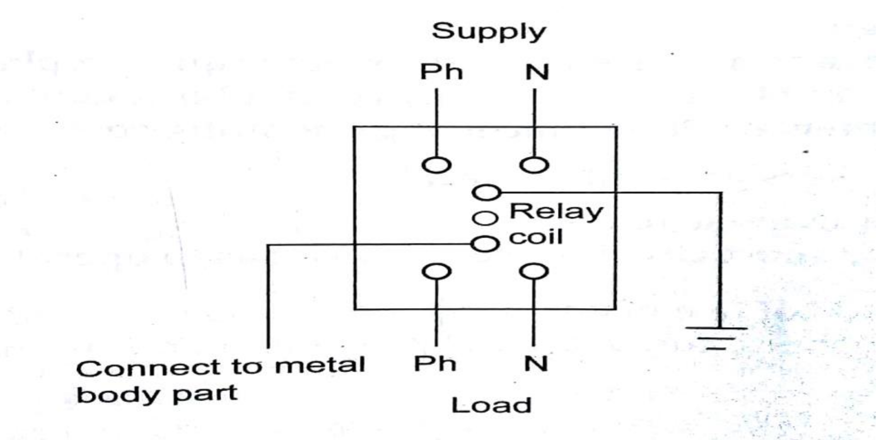
An Earth Leakage Circuits Breakers (ELCB) is a device used to directly detect currents leaking to earth from an installation and cut the power. There are two types of ELCBs :

(i) Voltage Earth Leakage Circuits Breaker (voltage -ELCB)

(ii) Current Earth Leakage Circuits Breaker (Current -ELCB)

(i) Voltage Earth Leakage Circuits Breaker (voltage -ELCB):

Voltage –ELCB is a voltage operated circuits breakers. The device will function when the current passes thorough the ELCB. Voltage-ELCB contains relay coil and one end of the coil is connected to metallic load body and the other end is connected to ground wire as shown in figure(12.12). If the voltage of the equipment body rises (by touching phase to metal part or insulation failure of equipment), which could cause the difference between earth and load body voltage and the danger of electric shock will occur. This voltage difference will produce an electric current from the load metallic body and phase through the loop to the Earth. When voltage on the equipment metallic body rises to danger level i.e., which exceed to 50V, the flowing current through relay loop could move the relay contact by disconnecting the supply current avoid from any danger electric shock. The ELCB detects fault currents from line to the earth (ground) wire within the installation it protects. If sufficient voltage appears across the ELCB's sensing coil, it will switch off the power, and remain off until manually reset. A voltage –sensing ELCB does not sense fault current from line to any other earthed body.



(ii) Current Earth Leakage Circuits Breaker (Current -ELCB):

Current –ELCB is a current operated circuits breaker which is a commonly used ELCB. Current-ELCB consists of a 3- winding transformer, which has two primary windings and 1 secondary winding as shown in figure (12.13). Neutral and line wires act as the two primary windings. A wire wound coil is the secondary winding. The current thorough the secondary winding is zero at the balanced condition. In the balanced condition, the flux due to current through the phase wire will be neutralized by the current through the neutral wire, since the current which flows from the phase will be returned back to the neutral. When a fault occurs, a small current will flow to the ground also. This makes an unbalanced between line and neutral currents and creates an unbalanced magnetic filed. This induces a current through the secondary winding, which is connected to the sensing circuits. This will sense the leakage and send a signal to the tripping system and trips the contact.

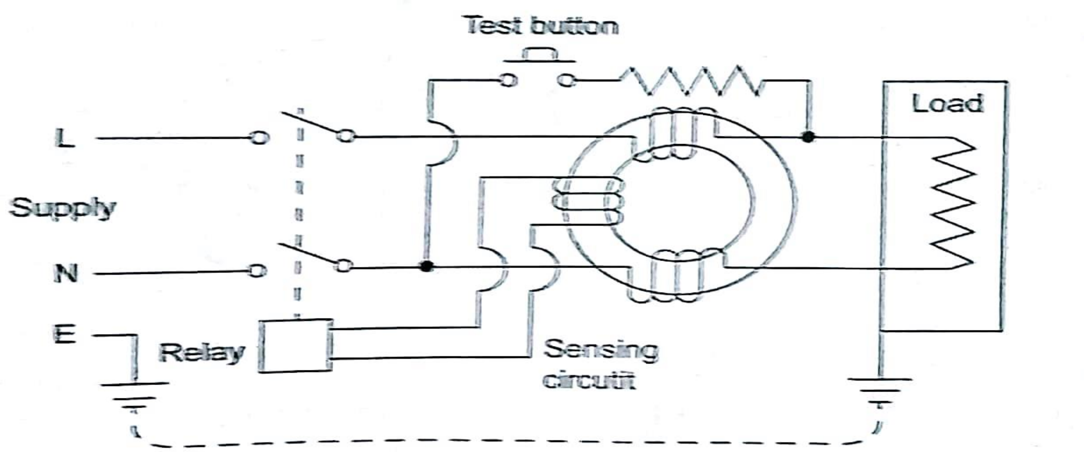


Figure (12.13): *Current earth leakage circuit breaker*

Molded Case Circuits Breaker(MCCB):

Molded case circuits breakers are electromechanical devices which protect a circuits from over current and short circuits. They provide over current and short circuits protection for circuits ranging from 63A up to 3000 A. Their primary function are to provide a means to manually open a circuits and automatically open a circuits under overload or short circuits conditions respectively. The over current, in an electrical circuits, may result from short circuits, overload of faulty design.

MCCB is an alternative to a fuse, since it does not require replacement once an overload is detected. Unlike a fuse, an MCCB can be easily reset after a fault and offer improved operational safety and convenience without incurring operating cost.

Molded case circuits breakers generally have a

i. Thermal element for over current and

ii. Magnetic element for short circuits release which has to operate faster.

The MCCBs are comprised of five major components such as molded case or frame operating mechanism, arc extinguishers, contacts and trip components as shown in figure (12.14)

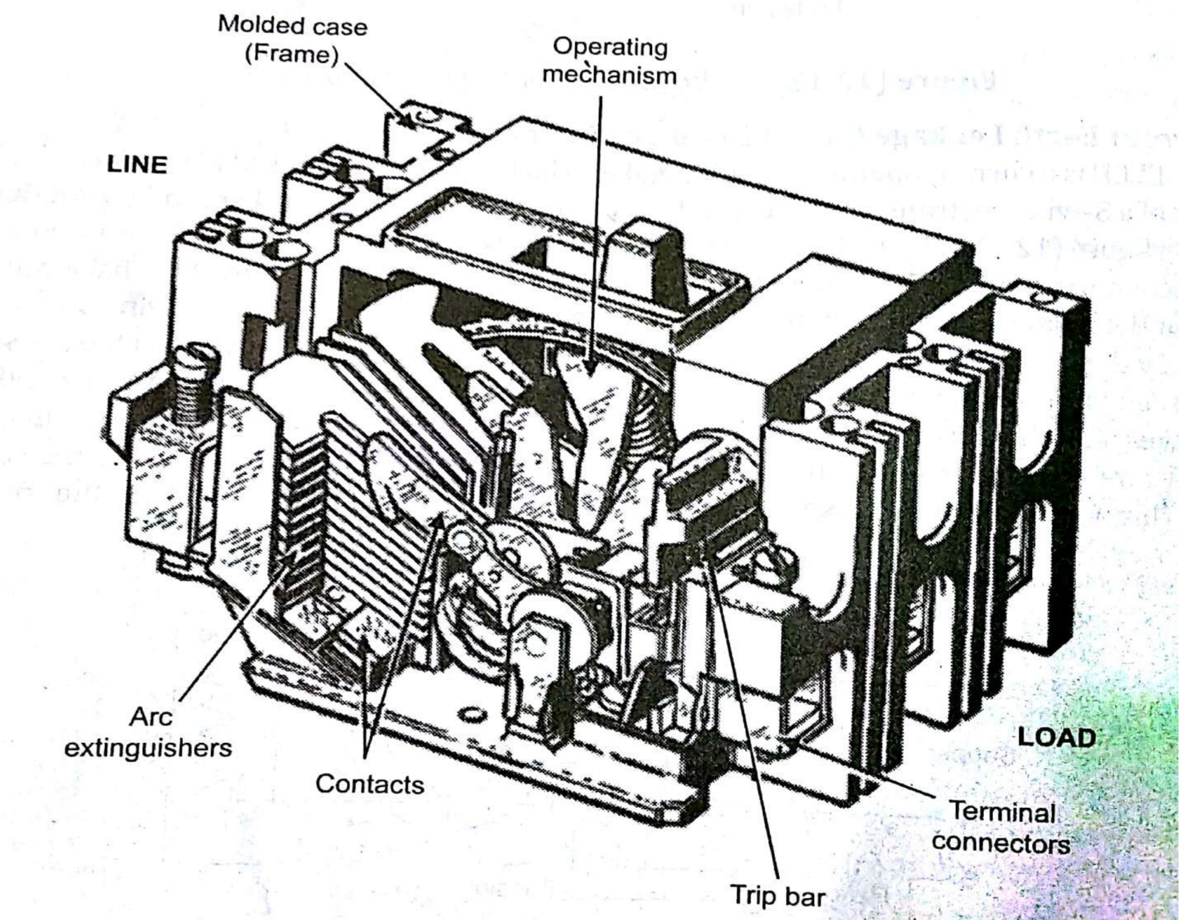


Figure (12.14): *Molded case circuit breaker*

MCCB are manufactured such that the end user will not have access to internal workings of the over-current protection device. Generally constructed of two pieces of heavy-duty electrically insulated plastic, these halves are riveted together to form the whole. Inside the plastic shell is series of thermal elements and a spring-loaded trigger. When the thermal element gets too warm, from an over current situation, the spring trips, which in turn will shut off the electrical circuits.

Operating mechanism :

At its core, the protection mechanism employed by MCCBs is based on the same physical principles used by all type of thermal – magnetic circuits breakers.

Overload protection is accomplished by means of a thermal mechanism. MCCBs have a bimetallic contact that expands and contacts in response to changes on temperature. Under normal operating conditions, the contact allows electric current through the MCCB. However as soon as the current exceeds the adjusted trip value, the contact will start to heat and expend until the circuits the circuit is interrupted. The thermal protection against overload is designed with a time delay to allow short duration over current, which is a normal part of operation for many devices. However, any over current conditions that last more than what is normally expected represent an overload, and the MCCB is tripped to protect the equipment and personnel.

On the other hand, fault protection is accomplished with electromagnetic induction, and the response is instant. Fault currents should be interrupted immediately, no matter if their duration is short or long. Whenever a fault occurs, the extremely high current induces a magnetic field in a solenoid coil located inside the breaker – this magnetic induction trips a contact and current it interrupted. As a complement to the magnetic protection mechanism, MCCBs have internal arc dissipation measure to facilitate interruption.

As with all types of circuits breakers, the MCCB includes a disconnection switch which is used to trip the breaker manually. It is used whenever the electric supply must be disconnected to carry out field work such as maintenance or equipment upgrades.

Applications :

Molded case circuits breakers can have very high current ratings, which allows them to be used in heavy duty applications such as main electric feeder protection, capacitor bank protection, generator protection, welding applications, low current application that require adjustable trip setting and motor protection.

Safety precautions in Handling Electrical Appliance:

It is essentially important to take precautions when we are working with electricity and using electrical appliances. Here, some of the basic precautions are mentioned for safe usage of electrical appliance :

(i) Follow the manufacture's instructions :

Always read the manufacture's instructions carefully before using a new appliance.

(ii) Replace or repair damaged power cords :

Exposed wiring is a danger that cannot be ignored. If you see the protective coating on a wire is stripped away, be sure to replace it or cover it with electrical tape as soon as possible.

(iii) Keep electrical equipment or outlets away from water :

Avoid water at all times when working with electricity. Never touch or repairing any electrical equipment or circuits with wet hands. It increases the conductivity of electrical current. Keep all electrical appliance away from water such as sinks, bathtubs, pools or overhead vents that may drip.

(iv) use insulated tools while working :

Always use appropriate insulated rubber gloves, goggles, protective clothes and shoes with insulated soles while working on any branch circuits or any other electrical circuits. Use only tools and equipment with non-conducting handles when working on electrical devices. Never use metallic pencils or rulers or wear rings or metal watchbands when working with electrical equipment as they cause a strong electric shock.|

(v) Don't overload your outlets :

Every outlet in your home is designed to deliver a certain amount of electricity; by plugging too many devices into it at once, you could cause a small explosion or a fire. If you have a lot of things to plug in, use a power strip that can safely accommodate your needs.

(vi) Shut-off the power supply :

Always make sure that the power source should be shut-off before performing any work related to electricity. For example; inspecting, installing, maintaining or repairing.

(vii) Avoid extension cords as much as possible :

Running extension cords through the house can trip up residence; this can cause injury and damage to the wire or outlet if it cause the cord to be ripped out of the wall. If you find yourself using extension cords very often, consider having an electrician install new outlets throughout your home.

(viii) When to repair :

Everyone want to have the safe electrical environment. Equipment producing “tingle” sound should be disconnected and reported promptly for repair.

(ix) Avoid the usage of flammable liquids :

Never use highly flammable liquids near electrical equipment. Never touch another person's equipment or electrical control devices unless instructed to do so.

(x) Use electric tester :

Never try repairing energized equipment. Always check that it is de-energized first by using a tester. When an electric tester touches a live or hot wire, the bulb inside the tester lights up showing that an electrical current is flowing through the respective wire. Check all the wires, the outer metallic covering of the service panel any other hanging wires with an electrical tester before proceeding with your work.

(xi) In case of electric shock :

If an individual comes in contact with a live electrical conductor, do not touch the equipment, cord person. Disconnect the power source from the circuits breaker or pull out the plug using a leather belt. By enclosing all electric conductors and contacts can save people from getting the electric shock. Use three-pin plugs, which have earth wire connection which prevents electrical shock.

(xii) Display danger board :

Danger board should be displayed at the work place. We should not allow any unauthorized person to enter in the working place and we should not put any new equipment into the service without necessary testing by the concern authority.

(xiii) Usage of proper ladder :

Never use an aluminium or steel ladder if you are working on any receptacle at height in your home. An electrical surge will ground you and the whole electric current will pass through your body. Use a bamboo, wooden or a fibreglass ladder instead.

(xiv) Usage of circuits breaker or fuse :

Always use a circuits breaker or fuse with the appropriate current rating. Circuits breakers and fuses are protection devices that automatically disconnect the live wire a condition of short circuits or over current occurs. The selection of the appropriate fuse or circuit breaker is essential. Normally for protection against short circuits a fuse rated of 150% of the normal circuit current is selected. In the case of a circuit with 10 amperes of current, a 15 ampere fuse will against direct short circuits a 9.5 amperes fuse will blow out.

(xv) Use ceiling on live wire :

Always put a cap on the hot/live wire while working on an electric board or service panel as you could end up short circuiting the bare ends of the live wire with the neutral. The cap insulates the copper ends of the cable thus preventing any kind of shock even if touched mistakenly.

(xvi) Precaution during soldering :

Always take care while soldering your circuits boards. Wear goggles and keep yourself away from the fumes. Keep the solder iron in its stand when not in use; it can get extremely hot and can easily cause burns.

(xvii) Things to remember :

The circuits is bad, electricity appliances are not working well, and lights are fluctuating. It means you need an electrical inspection or repair. In this case, either you'll call an electrician or do it yourself. So if you are trying to repair, always remember that your hands are well dry, you have essential tools, rubber gloves & shoe are good, As all these acts as an insulator. Do not wear loose clothing or tied near electrical equipment.

(xviii) Keep heaters away from bedclothes, clothing and curtains to avoid risk fire. Be extra careful when using electrical appliances attached to power outlets near kitchen or bathroom sinks, tubs, swimming pools, and other wet areas. Don't cover an electric heater with clothing or other items.

Batteries:

A battery is a devices which converts chemical energy into electrical energy and is made up of a number of cells. Batteries consists of two or more voltaic cell that are connected in series to provide a steady DC voltage at the battery's output terminals. The voltage is produced by a chemical reaction inside the cell. Electrodes are immersed in an electrolyte, which forces the electric charge to separate in the from of ions and free electrons. A battery's voltage output and current rating are determined by the elements used for the electrodes, the size of the electrodes, and the type of electrolyte used. Whether a battery may be recharged or not depends on the cells used to make up the battery.

Batteries are classified into two types such as primary batteries and secondary batteries.

(i) Primary Batteries

As the name indicates, these batteries are meats for single usage. Once these batteries are used they cannot be recharged as the devices are not easily reversible and active material may not return to their original forms. Other name for these batteries is disposable batteries. Some of the example for the disposable batteries are the normal AA, AAA batteries which we use in wall clocks, television remote, etc.

(ii) Secondary Batteries

Secondary batteries are also known as rechargeable batteries. These batteries can be used and charged simultaneously. A secondary battery or storage battery can be recharged because its chemical reaction is reversible. Rechargeable batteries are (re) charged by applying electric current, which reverses the chemical reactions that occur during discharge/use. Some of the examples for rechargeable batteries are the batteries used in mobile phones MP3 players, etc.

Types of primary Cells/Batteries:

There are several types of primary cells in use today, such as

(i) Carbon- zinc dry cell

(ii) Alkaline cell

(iii) Zinc chloride cell

(iv) Mercury cell

(v) Silver oxide cell

(vi) Lithium cell

(i) Carbon- zinc dry cell

(a) This is one the most popular primary cells (often used for type AAA, AA, D)

(b) The negative electrode is made of zinc

(c) The positive electrode is made of carbon.

(d) The output voltage of a single cell is about 1.5 V.

(e) Performance of the cell is better with intermittent operation.

(ii) Alkaline cell

(i) The alkaline cell is another popular type also used for type AA, C, D, etc.

(ii) It has the same 1.5V output as carbon- zinc cells, but they are longer -lasting.

(iii) It consists of a zinc anode and manganese dioxide cathode in an alkaline electrolyte (potassium hydroxide)

(iv) It works with high efficiency even with continuous use, due to low internal resistance.

(iii) Zinc chloride cell

(i) This cell is also referred to as a “heavy-duty” type battery.

(ii) It is modified zinc-carbon cell.

(iii) It has little chance of liquid leakage because the cell consumes water along with the chemically active materials. The cell is usually dry at the end of its useful life.

(iv) Mercury cell

(i) This cell consists of a zinc anode, mercury compound cathode, and potassium or sodium hydroxide electrolyte.

(ii) It is becoming obsolete due to the hazards associated with proper disposal of mercury.

(v) Silver oxide cell

(i) This cell consists of a zinc anode, silver oxide cathode, and potassium of sodium hydroxide electrolyte.

(ii)It is typically available as 1.5V, miniature button form.

(iii) Applications include hearing aids, cameras, and watches

(vi) Lithium cell

(i) This cell offers high output voltage, long shelf life, low weight, and small volume.

(ii) It comes in two forms of 3V output in widespread use:

(a) Lithium-sulfur dioxide($LiSO_2$).

(b) Lithium- thionyl chloride.

(iii) $LiSO_2$ type batteries contain methyl cyanide liquid solvent; if its container is punctured

(iv) Safe disposal of these cells is critical.

Types of Secondary Cells/Batteries:

There are several types of secondary cells in use today, such as

(i) Lead-acid cell

(ii) Nickel cadmium (NiCd) cell

(iii) Lithium-ion battery

(iv) Nickel-metal- hydride (NiMH) cell

(v) Nickel-iron (Edison) cell

(vi) Fuel cell

(vii) Solar cell

(i) Lead-acid cell:

(a) This cell is a widely applied type of secondary cell, used extensively in automobiles inverters, backup power system, etc. Requiring high values of load current.

(b) Anode: Porous lead

(c) Cathode: Lead-dioxide

(d) Electrolyte: Sulfuric acid, 6 molar H_2SO_4

(e) The output is about 2.1 V per cell.

(f) Cells are typically used in series combinations of 3 (6V battery) or 6 (12V battery)

(ii) Nickel cadmium (NiCd) cell:

(a) This type of cell delivers high current.

(b) It can be recharged many times.

(c) Anode: Nickel hydroxide $Ni(OH)_2$

(d) Cathode: Cadmium hydroxide, $Cd(OH)_2$

(e) Electrolyte : Cadmium hydroxide, KOH

(f) Maintain a steady voltage of 1.2V per cell until completely depleted

(g) It can be stored for long periods of times.

(h) Its specific gravity does not charge with the state of charge.

(I) Applications include portable power tools, alarm systems, portable radio and TV equipment.

(iii) Lithium-ion battery:

(i) Li-based cells are most compact ways of storing electrical energy.

(ii) Lower in energy density than lithium metal, lithium-ion is safe.

(iii) Anode: Graphite

(iv) Cathode: Lithium manganese dioxide

(v) Electrolyte : mixture of lithium salts

(vi) Energy density is twice of the standard nickel-cadmium.

(vii) No memory and no scheduled cycling is required to prolong battery life.

(iv) Nickel-metal- hydride (NiMH) cell:

(i) These cells are used in applications demanding long-running battery performance (e.g., high-end portable electrical or electronic products like power tools).

(ii) They offer 40% more capacity over a comparably-sized NiCb cell.

(iii) They contain the same components as a NiCd cell, expect for the negative electrode.

(iv) They are more expensive than NiCd cells, self-discharge more rapidly, and cannot be cycled as frequently as NiCd cells.

(v) Nickel-iron (Edison) cell:

(i) Anode: Nickel hydroxide, $Ni(OH)_2$

(ii) Cathode: iron

(iii) Electrolyte: potassium hydroxide

(iv) The specific gravity of electrolyte remains unaffected during the charging and discharging process.

(v) They are now almost obsolete due to lead-acid batteries.

(vi) These are used in emergency lamps in hospitals and at places where the rate of discharge and charge are rapid.

(vi) Fuel cell:

(i) A fuel cell is an electrochemical device that converts chemicals (such as hydrogen and oxygen)into water and produces electricity in the process.

(ii) As long as the reactants (H and O) are supplied to the fuel cell, it will continually produce electricity and never go dead, unlike conventional batteries.

(iii) Fuel cells are used extensively in the space program as sources of DC power.

(iv) They are very efficient, capable of providing hundreds of kilowatts of power.

(vii) Solar cell:

(i) Solar cells convert the sun's light energy into electric energy.

(ii) They are made of semiconductor materials.

(iii) They are arranged in modules that are assembled into a large solar array to produce the required power.

(iv) An applied voltage higher than the voltage of one cell can be obtained by connecting cells in series.

(v) The total voltage available across the battery of cells is equal to the sum of the individual values for each cell.

(vi) Parallel cells have the same voltage as one current capacity, To provide a higher output voltage and more current capacity, cell can be connected in series-parallel combinations.

Comparison of Primary and Secondary Cells:

S.No	**Primary Cell**	**Secondary Cell**
01	If discharged once cannot be charged again	If discharged once can be charged again
02	Light in weight	Heavy in weight
03	Used for intermittent use with low load current rating	Used for continuous rating with high load current rating
04	Short life	Long life
05	Low cost	High life
06	Low efficiency	High efficiency
07	Low power output	High power output
08	Less maintenance	More maintenance

Battery Characteristics:

There are many characteristics that can help to identify a battery that can help to identify a battery and we can distinguish the three main ones as; chemistry, battery capacity and voltage. However, if the battery is only a starter, it also delivers cold cranking amps (CCA), which permits to offer high current at cold temperatures.

(i) Chemistry

The main battery chemistries are lead, nickel and lithium. They all need a specific designated charger, this is why charging these batteries on a different charger from their own might cause an incorrect charge, despite it seeming to work at first. This happens because of the different regulatory requirement of each chemistry.

(ii) Battery Capacity

Battery capacity is a measure (typically in Amp-hr) of the charge stored by the battery, and is determined by the mass of active material contained in the battery. The battery capacity represent the maximum amount of energy that can be extracted from the battery under certain specified conditions. However, the actual energy storage capabilities of the battery can vary significantly from the "nominal" rated capacity, as the battery capacity depends strongly on the age and past history of the battery, the charging or discharging regimes of the battery and the temperature.

The energy stored in a battery, called the battery capacity, is measured in either watt-hours (Wh), kilowatt-hours (kWh), or ampere-hours (Ahr). The most common measure of battery capacity is Ah, defined as the number of hours for which a battery can provide a current equal to the discharge rate at the nominal voltage of the battery. The unit of Ah is commonly used when working with battery systems as the battery voltage will vary throughout the charging or discharging cycle.

(ii) Voltage

A battery feature a nominal voltage. Along with the amount of cells connected in series, chemistry provides the open circuits voltage (OCV), which is about 5-7% higher on a fully charged battery. It is important to check the correct nominal voltage of a battery before connecting it.

(iv) Cold Cranking Amps (CCA)

Every starter battery is marked with cold cranking amps, also abbreviated CCA. The number denotes the amount of amps that the battery is able to provide at -18^{0}C.

Energy Consumption Calculation:

Energy and power are closely related. Electrical energy can be measured only when electrical power is known. So first we understand the electrical power. Electrical power it the amount of electrical current that results from a certain amount of voltage or we can say that power is the rate which energy is delivered. It is measured in watts. Mathematically it is written as

Power = Voltage × Current

The measurement of electrical energy is completely dependent on power which is measured in watt, kilowatts, megawatts, gigawatts, and time which is measured in an hour. Joule is the smallest unit of energy. But for some bigger calculation, some better unit it required. So, the unit used for electrical energy is watt-hour.

Electrical energy is the product of electrical power and time, and it measured in joules. It is defined as “1 joule of energy is equal to 1 watt of power is consumed for 1 second”. I.e.,

Energy = Power × Time

1 Joule = 1 watt × 1 second

Watts are the basic unit of power in which electrical power is measured or we can say that rate at which electrical current is being used at a particular moment.

Watt-hour is the standard unit used for measurement of energy, describing the amount of watts used over a time. It shows how fast the power is consumed in the period of time.

Energy in watt hours = Power in watts × Time in hours

Kilowatt-hour is simply a bigger unit of energy when large appliance drawn power in kilowatts. It can be described as one kilowatt hour is the amount of energy drawn by the 1000 watts appliance when used for an hour.

Where, One kilowatt = 1000 watts

Energy in kilowatt hours = Power in kilowatts × Time in hours

The electrical supply companies take electric energy charges from their consumer per kilowatt hour unit basis.

This kilowatt hour is board of trade (BOT) unit.

Illustration for Energy Consumption:

A consumer uses a 10 kW geezer, a 6 kW electric furnace and five 100 W bulbs for 15 hours. How many units (kWh) of electrical energy have been used?

Explanation : Given that

Load – 1 = 10 kW geezer

Load – 2 = 6 kW electric furnace

Load – 3 = 500 watt (five 100 watt bulbs)

Total load = 10kW + 6kW + 0.5kW = 16.5kW

Time taken = 155 hours

Energy consumed = Power in kW × Time in hours

= 16.5 × 15 = 247.5 kWh

For above electrical energy consumption, the tariff can be calculated as follows :

1 unit = 1kWh

So, the total energy consumption = 247.5 units

If the cost per unit is 2.5, then the total cost of energy consumption

$= 247.5 \times 2.5 = 618.75$/-