Problem Statement:

Perform an analysis to understand the impact of the optical beam's output power on inter-satellite communication. Assume the transmitter and receiver parameters as needed. Implement your analysis in Python 3. Submit a GitHub repository containing:

1. Source code (Python 3 compatible).

2. A PDF summarising your methodology, assumptions, equations used, graphs, and key results. Upload the PDF to the same GitHub repository.

Tasks

The objective is to analyze the impact of optical beam output power on inter-satellite communication in outer space. This involves characterizing the optical beam, modeling the space channel, analyzing power transfer, and evaluating system performance using BER for OOK modulation, as outlined in the assignment instructions.

Assumptions:

- The optical communication link operates in a vacuum (outer space), so only geometric spreading is considered—no atmospheric attenuation.
- The transmitter uses a laser source with a typical wavelength (e.g., 1550 nm) and a defined beam waist (e.g., 5 cm).
- The receiver has a circular aperture (e.g., 10 cm diameter).

1. Characterise Optical Beam in Outer Space

Answer:

The beam is modelled as a Gaussian beam, characterized by its divergence angle, beam spread, and intensity profile. Beam spread is essentially how much a beam diverges or spreads out from its initial size as it travels.

A. **Beam Divergence Angle (θ)**

It is defined as the angular spread of the beam as it propagates in space.

$$\theta = \frac{\lambda}{\pi w_0}$$

Where: λ: Wavelength (e.g., 1550 nm)

w_0: Beam waist radius at the transmitter

B. **Beam Radius at Distance z**

The beam radius at given distance z is

$$w(z) = w_0 * \sqrt{1 + (\frac{\lambda z}{\pi {w_0}^2})^2}$$

Where z is the propagation distance. The beam spread is then quantified by how $w(z)$ increases with z.

C. **Intensity Profile**

The spatial distribution of optical power across the beam is:

$$I(r,z) = I_0 * \exp(-2\frac{r^2}{w(z)^2})$$

Where r is the radial distance from the beam center.

D. **Rayleigh Range (zR)**

It is the distance over which the beam waist grows by $\sqrt{2}$ given by:

Formula:

$$zR = \frac{\pi {w_0}^2}{\lambda}$$

Assumptions:

Assume practical values based on space optical systems:

1. λ=1550 nm (common for space comms)
2. w_0=5 cm (beam waist)
3. Distances: z=200 km, 400 km,…,1000 km

Calculation has been done for z=1000 km:

A. **Divergence Angle**:

$$\theta = \frac{\lambda}{\pi w_0}$$

$$= \frac{1550 * 10^{-9}}{\pi * 0.05} = 9.87\ \mu rad$$

B. **Beam Radius**:

$$w(z) = w_0 * \sqrt{1 + \left(\frac{\lambda z}{\pi {w_0}^2}\right)^2}$$

$$= 0.05 * \sqrt{1 + \left(\frac{1550 * 10^{-9} * 10^6}{\pi * 0.05^2}\right)^2} = 0.05m$$

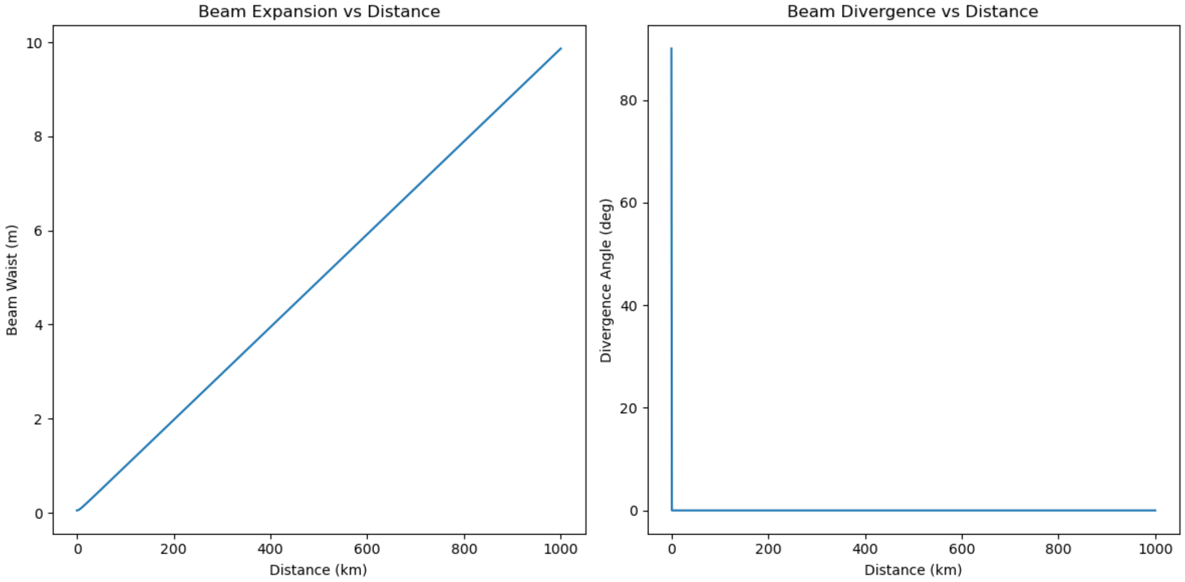
C. **Intensity at *r*=0** (peak intensity):

$$I(0, z) = \frac{P_{tx}}{\pi {w_z}^2}$$

D. Rayleigh Range (zR):

$$zR = \frac{\pi {w_0}^2}{\lambda}$$

$$= \frac{\pi(0.05)^2}{1550 * 10^{-9}} = 5.06\ km$$



2. **Characterise the Channel (Outer Space)**

Quantify the effect of vacuum on signal propagation (e.g., free-space path loss, no atmospheric attenuation). Quantify how the vacuum affects signal propagation, focusing on free-space path loss.

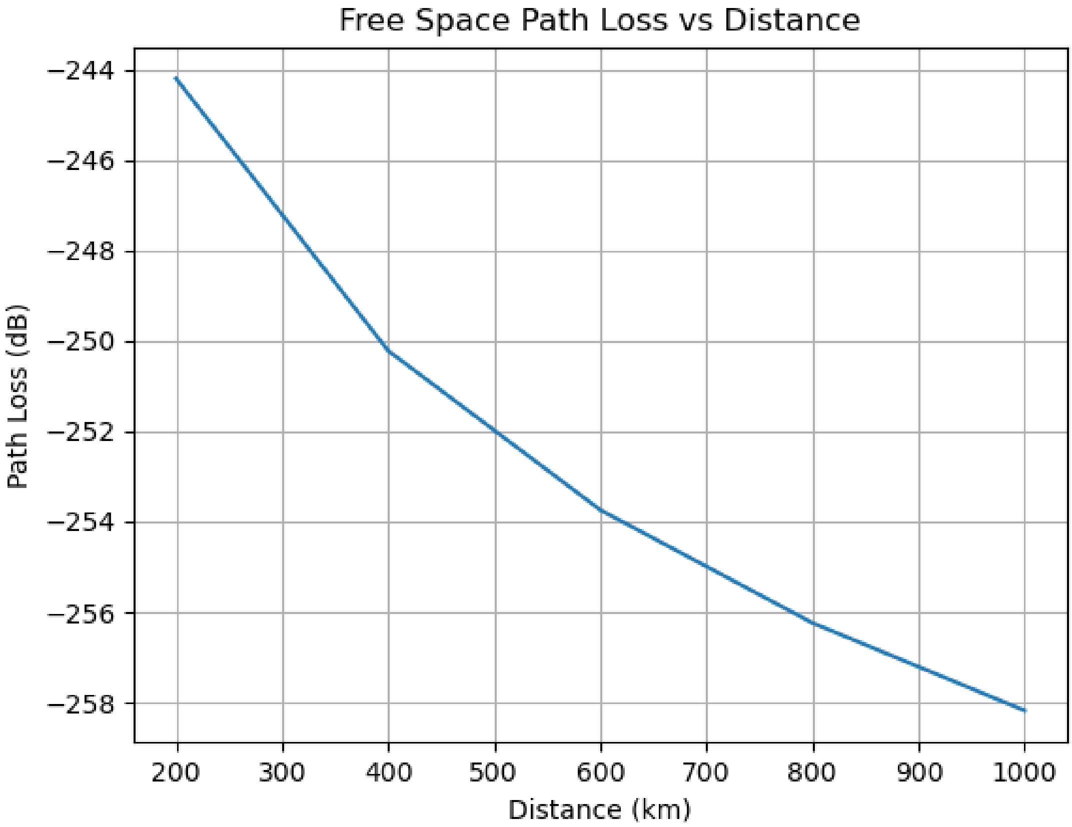
Answer:

The signal attenuation due to the spreading of the beam as it propagates through space. In vacuum, there's no atmospheric absorption, scattering, or turbulence. The primary factor affecting signal strength is geometric spreading (FSPL). The free-space path loss increases significantly with distance, following an inverse square law.

$$FSPL = (\frac{4\pi d}{\lambda})^2$$

$$FSPL\ (dB) = 20 * \log(\frac{4\pi d}{\lambda})$$

Where *d* is the distance between satellites.



3. Power Analysis

- Using the characteristics of the optical beam and the space channel, plot Transmitted Output Power vs Received Input Power for a fixed distance between transmitter and receiver.
- Tx Power Range: 100 mW to 1 W (in 10 mW steps).
- Fixed Distance: 200 km, 400 km, 600 km, 800 km, 1000 km

Answer:

For each distance (200 km, 400 km, 600 km, 800 km, 1000 km), and for transmitted powers from 100 mW to 1 W (in 10 mW steps), the received power is calculated using the formula given below:

Assumptions:

Tx Power Range: 100 mW to 1 W (in 10 mW steps)

Fixed Distances: 200 km, 400 km, 600 km, 800 km, 1000 km

Transmitter Aperture Diameter (D_{tx}): 0.1 m

Receiver Aperture Diameter (D_{rx}): 0.1 m

Transmitter Efficiency (η_{tx}): 0.8

Receiver Efficiency (η_{rx}): 0.7

$$P_{rx} = \frac{P_{tx} * G_t * G_r * (\lambda)^2}{(4*pi*d)^2}$$

$$G_t = \frac{4 * pi * A_{tx}}{(\lambda)^2}$$

$$G_r = \frac{4 * pi * A_{rx}}{(\lambda)^2}$$

Putting the values of G_t and G_r in P_{rx} and considering the efficiency of receiver and transmitter, we get:

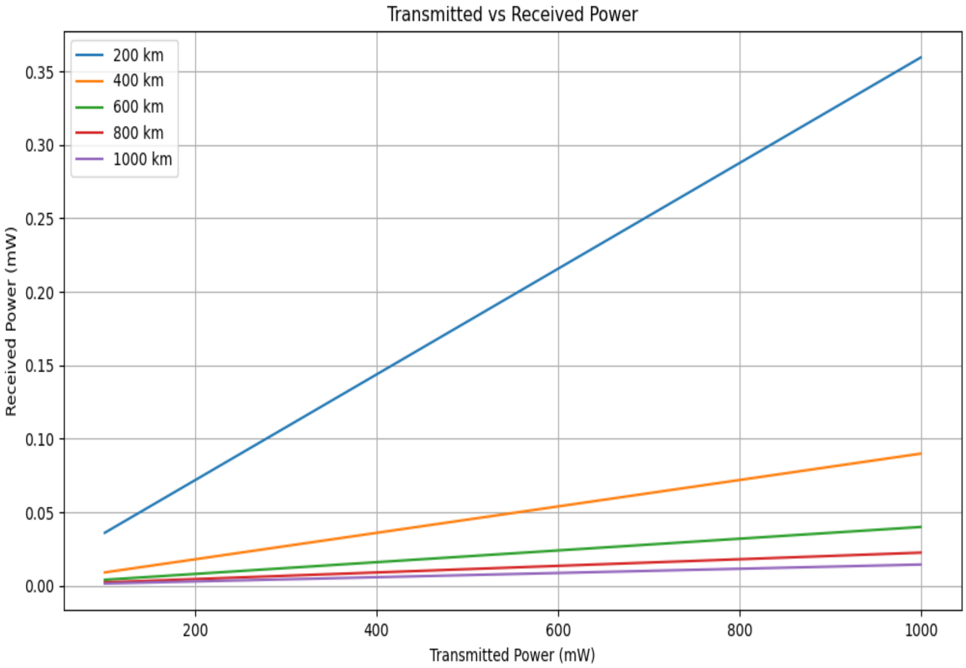
Received Power (P_{rx})

$$P_{rx} = \frac{P_{tx} * \eta_{tx} * \eta_{rx} * (A_{tx} * A_{rx})}{(\lambda\ d)^2}$$

Where:

$A_{tx} = \pi\ (D_{tx}/2)^2$ (Transmitter Aperture Area)

$A_{rx} = \pi\ (D_{rx}/2)^2$ (Receiver Aperture Area)



The received power decreases with the square of the distance. The relationship between transmitted and received power is linear for a fixed distance. The difference in received power between 200 km and 400 km is greater than the others because of the inverse square relationship between power and distance.

4. **BER vs SNR for OOK Modulation**

- Plot Bit Error Rate (BER) vs Signal-to-Noise Ratio (SNR) for On-Off Keying (OOK) modulation.
- Tx Power Range: 200 mW
- Fixed Distance: 200 km, 400 km, 600 km, 800 km, 1000 km
- SNR Range: -2 dB to 7 dB.

Answer:

The BER (Bit Error Rate) of an optical beam refers to the number of bits that are incorrectly received in a data transmission over an optical communication link. This is influenced by factors like detector noise, amplifier noise, and modulation scheme. A lower BER indicates a more reliable transmission. In optical communication systems, BER is often used as a key performance metric to evaluate the quality of the transmitted signal.

Assumptions:

Tx Power: 200 mW
Fixed Distances: 200 km, 400 km, 600 km, 800 km, 1000 km
SNR Range: -2 dB to 7 dB
Receiver Noise Figure (F): 3 dB
Receiver Responsivity (R): 0.65 A/W
Bandwidth (B): 1 GHz
Temperature (T): 300 K

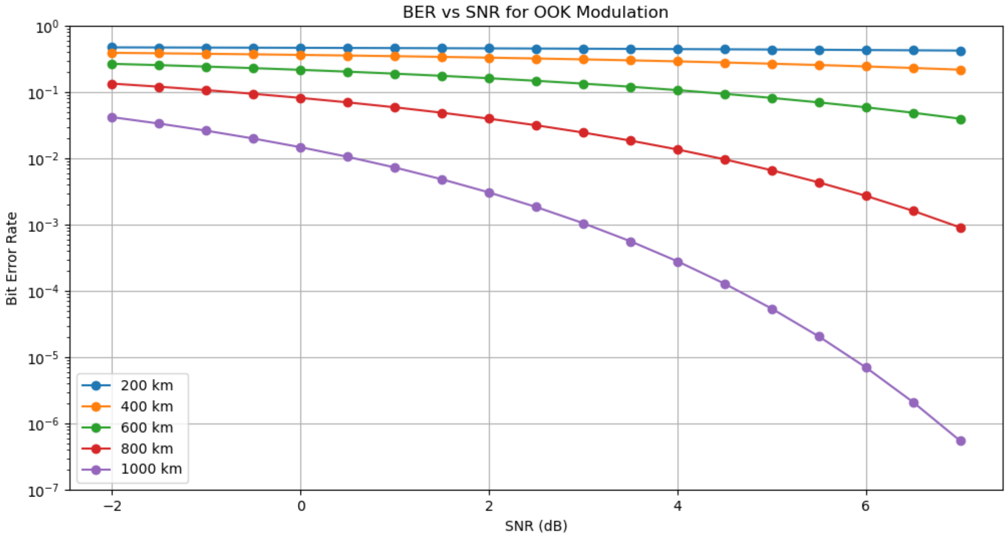
For an Additive White Gaussian Noise (AWGN) channel, the BER for OOK for coherent detection is given by:

$$SNR = \frac{(RP_{rx})^2}{2qRP_{rx}B + 4K_BTBF}$$

Where:
q is the electron charge (1.6 x 10^-19 C)
K_B is Boltzmann's constant (1.38 x 10^-23 J/K)

BER (Bit Error Rate) for OOK:

$$BER = \frac{1}{2} * erfc(\sqrt{\frac{SNR}{2}})$$



The Bit Error Rate (BER) decreases as the Signal-to-Noise Ratio (SNR) increases. At longer distances, the BER is higher for a given SNR. In this Prx is dependent on distance as received power is inversely proportional to distance. As distance increases, Prx reduces, further reducing SNR , leading to a higher BER for longer links.