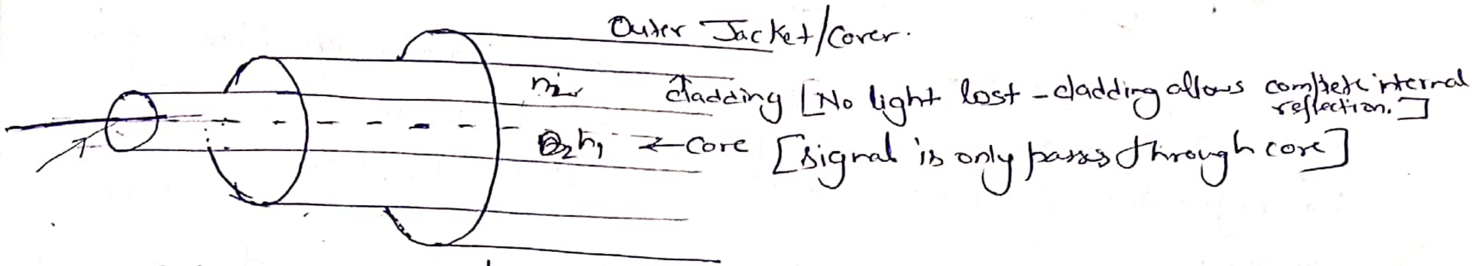
(F)

Optical fibres:-

* To transmit the information of telephone comm., computer data etc. which are in the form of coded light signals.
* To transmit the optical images (eg - Endoscopy).
* To act as a light source at the inaccessible places.

Structure of Optical fibre



n_2 → Refractive index of Cladding
n_1 → ———————— core

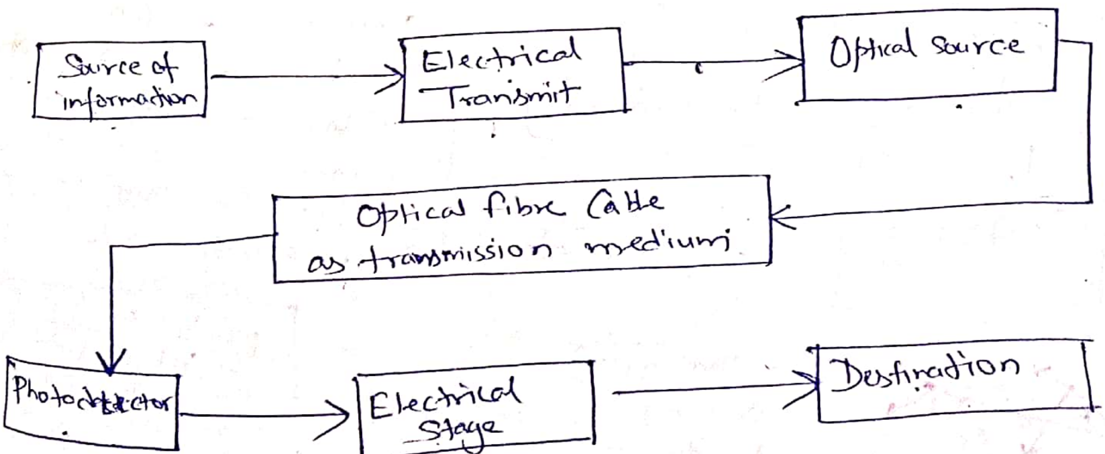
$\boxed{n_1 > n_2}$ ← {It is always remember that}

$\boxed{n_1 > n_2}$

~~Types of optical fibre~~ -

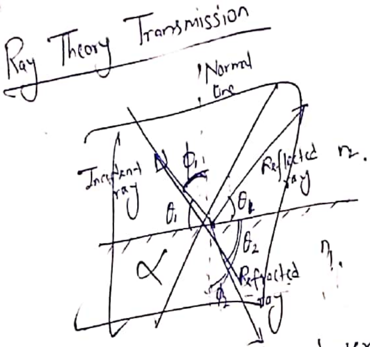
Advantages of OFC:-

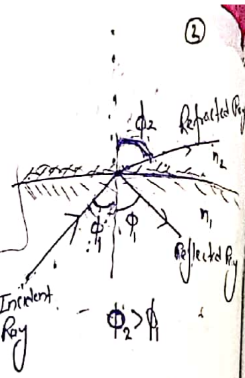
1) Large information capacity
2) Long dist. transmission
3) Small size & low weight
4) Enhanced safety
5) System reliability & easy maintenance., etc.



The Basic Block diagram of OFC system

Ray Theory Transmission





$\phi_2 > \phi_1$

The ray is propagating from denser medium of high refractive index n_1 to rarer medium of low refractive index n_2. There will be refraction of light and refracted ray will move away from the normal.

If the incident angle is ϕ_1 & $\angle$ of refraction is ϕ_2 than from

Snell's Law—

$n_1 \sin\phi_1 = n_2 \sin\phi_2$

$\Rightarrow \boxed{\dfrac{\sin\phi_1}{\sin\phi_2} = \dfrac{n_2}{n_1}}$ — (1)

where, $n_1 \to$ Refractive index of core
$n_2 \to$ " " " cladding
$\phi_1 \to \angle$ of incidence
$\phi_2 \to \angle$ of refraction

As we know that, n_1 is greater than n_2, the angle of refraction is always > angle of incidence.

When $\angle$ of refraction is $90°$ then the $\angle$ of incidence is known as critical angle ϕ_c.

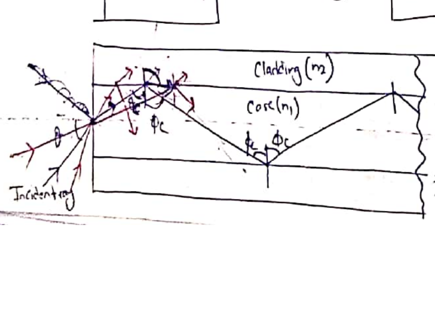
i.e $\dfrac{\sin\phi_1}{\sin 90°} = \dfrac{n_2}{n_1}$

$\Rightarrow \dfrac{\sin\phi_1}{1} = \dfrac{n_2}{n_1}$ $[\because \sin 90° = 1]$

$\Rightarrow \boxed{\sin\phi_c = \dfrac{n_2}{n_1}}$ — (2)

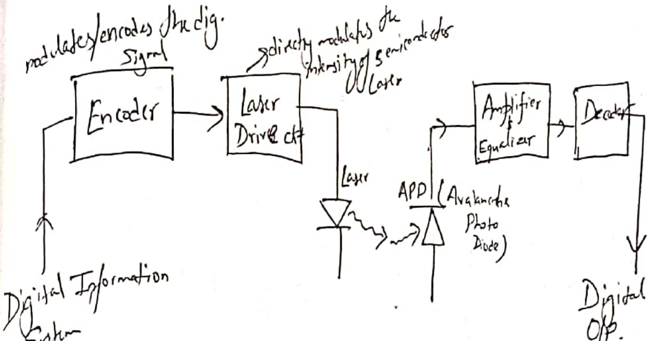
Material	Refractive Index
Air	$\to$ 1.0003
Vaccum	— 1.0
Water	— 1.33
Glass	— 1.5
Diamond	— 2.0
Silicon	— 3.4
Gallium arsenide	— 3.6

$\eta = \dfrac{\text{Vel. of Light in free sp}}{\text{Vel. of light in medi}}$

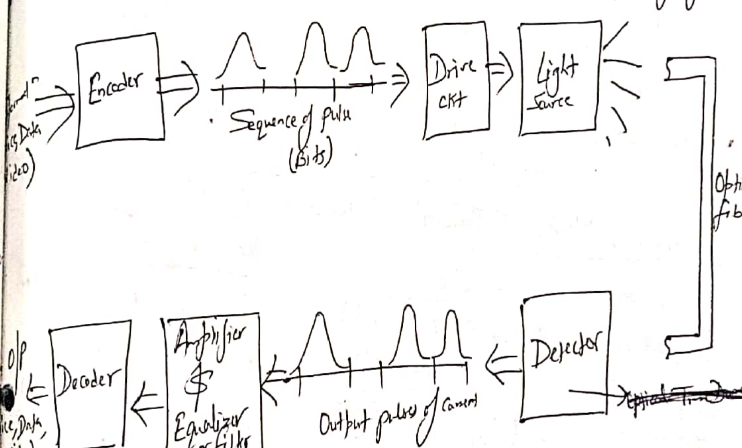


$\rightarrow$ c) Refraction of Light at air fibre Interface.

Basic Block diagram of Digital Optical Communication system



Information Transmission Sequence:-



Steps:-

1) Informatⁿ is suitably encoded into Electrical form.
2) Electrical Signals are converted into light signals by light source.
3) Light signals travel down the fibre
4) Detector changes the light signals into electrical form.
5) Amplifier or Equalizer or filter provide gain as well as linear signal processing & reduction in noise BW.
6) Finally, electrical signals are decoded to give the original information.

Prof. Sushma Kumari

Light :- Optics :- (Electromagnetic Wave)

Maxwell proved that EM waves propagate with the velocity of light

i.e $C = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ → permittivity of free space (8.85×10^{-12} Farad/m) ← [It represents the capability of a vacuum to permit electric fields]

μ_0 → Permeability of free space (Energy stored in a magnetic field) * $4\pi \times 10^{-7}$ N/A²

Different Bands of Electromagnetic Spectrum :-

~~Ultraviolet, visible~~

Band	freq. (Hz)	Major transmission Media Used
Ultraviolet	10^{15} to 10^{16}	Opt. fibre & Laser beams
Visible	10^{14} to 10^{15}	"
UHF	10^{8} to 10^{9}	Coaxial cable
UHF	10^{7} to 10^{8}	Coaxial cable
VHF	10^{6} to 10^{7}	Coaxial cable
HF	10^{5} to 10^{6}	" , twisted pair
MF	10^{4} to 10^{5}	twisted pair
LF	10^{3} to 10^{4}	Wire pairs
VLF		

LF, MF, HF, VHF, UHF, SHF, EHF

Very Low freq. Band

Comparative Merits & Demerits of Transmission Media :-

Microwave

* Fast installation (+)
* Point to multipoint (+) working
* Best for difficult (+) areas
* Less Bandwidth than fibre (−)
* Costly (−)
* Line of Sight limitation (+)

Copper

* High BW for coaxial (+)
* Easy to interconnect (+)
* Unaffected by Rain, fog (+)
* Heavy weight (−)
* Easy to tap (−)
* Affected by thermal Noise (−)
* point-to-point working (+)

Optical Fibre

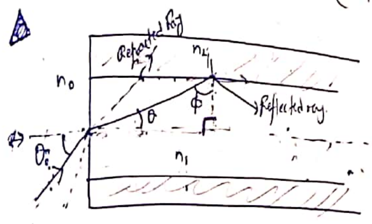
* Long Repeaterless distance
* Enormous BW (+)
* Unaffected by Rain, fog. (+)
* Light weight (+)
* Costly (−)
* Difficult to join/fuse (−)
* point-to-point working. (−)
* Highest signal security (+)

(θ_a)

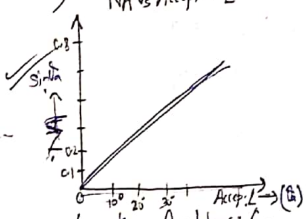
Acceptance angle and Numerical Aperture :-

* The maximum angle θ_{max} with which a ray of light can enter through the entrance end of the fibre and still be totally internally reflected is called Acceptance angle of fiber.

* Numerical Aperture (NA) of the fiber is the light collecting capability of the fiber & is the measure of the amount of light rays that can be accepted by the fiber. It is equal to the sine of Acceptance ∠. i.e ($NA = \sin\theta_a$).



NA vs Acceptance ∠



Let a incident light ray at an ∠θ_0 which is less than Acceptance ∠.

Then using Snell's Law,

$n_0 \sin\theta_0 = n_1 \sin\theta$

& $\theta = \frac{\pi}{2} - \phi$

So, $n_0 \sin\theta_0 = n_1 \sin\left[\frac{\pi}{2} - \phi\right]$

$\Rightarrow n_0 \sin\theta_0 = n_1 \cos\phi$

~~$\Rightarrow n_0 \sin\theta_0 = n_1\sqrt{1-\sin^2\phi}$~~ — (1)

$\Rightarrow \boxed{\sin\theta_0 = \frac{n_1}{n_0}\cos\phi}$ — (1)

As we know,

$\sin\phi_c = \frac{n_2}{n_1}$ → $[\sin^2\theta + \cos^2\theta = 1]$

$\cos\phi_c = \sqrt{1-\left(\frac{n_2}{n_1}\right)^2} = \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$ — (2)

Now, in eqn (1)

$\sin\theta_0 = \sin\theta_a = \frac{n_1}{n_0}\cos\phi_c$

$\Rightarrow \sin\theta_a = \frac{n_1}{n_0} \times \frac{\sqrt{n_1^2 - n_2^2}}{n_1}$

$\boxed{\sin\theta_a = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}}$ → [For free space $n_0 = 1$]

Consider the limiting case, ϕ becomes the critical angle for Core-cladding interface & θ_0 become the acceptance angle θ_a then –

$n_0 \sin\theta_a = \sqrt{n_1^2 - n_2^2}$ — (2)

Now, $NA = n_0 \sin\theta_a = \sqrt{n_1^2 - n_2^2}$

If consider surrounding medium as air then $n_0 = 1$

$\boxed{NA = \sin\theta_a ; \quad NA = n_1 (2\Delta)^{1/2}}$, or $\boxed{NA = n_1\sqrt{2\Delta}}$

where, $\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} = \frac{n_1 - n_2}{n_1}$.

↓

Relative Refractive index difference.

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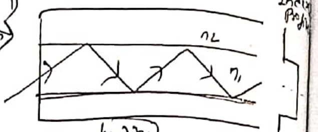
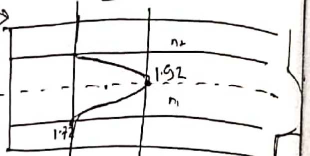
Types of optical fibre :- A) On the basis of Refractive Index :-

1) Step Index fibre → Single mode / Multi mode
(Ref. Index of core is Uniform)



$n_1 > n_2$

2) Graded index fibre → Multi mode
(Ref. Index of core n_1 is highest at axis & it will dec. as we move towards perimeter of core)



$n_1 > n_2$

B) On the basis of Propagation :-

1) Mono mode or Single mode fibre

2) Multi mode fibre.

* If the refractive index of the core in a fibre is uniform throughout & undergoes abrupt change (or step) at the cladding boundary, it is called Step Index Fibre.

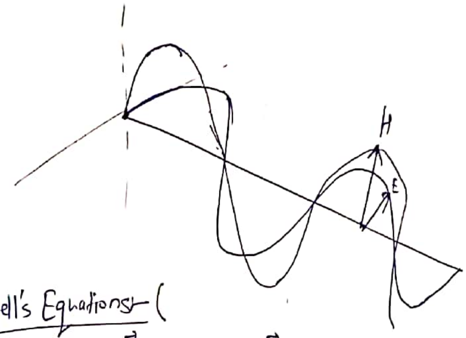
* If the refractive index of the core in fibre is made to vary as a function of the radial distance from the centre of the fibre, it is called graded index fibre.

Electromagnetic Theory :-

To obtain detail understanding of Propagation of light in an optical fibre.

Light as a variety of EM vibrations :-

E & H fields at right angle to each other & perpendicular to direction of propagation.



* Maxwell's Equations :- (

1) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ 3) $\nabla \cdot \vec{D} = 0$

2) $\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$ 4) $\nabla \cdot \vec{B} = 0$

where $\vec{D} = \epsilon \vec{E}$ & $\vec{B} = \mu \vec{H}$

* For a complete treatment of optical propagation in the fibre, it is necessary to consider both forms :-

i) For planar wave guides described by Cartesian coordinates (x, y, z) :-

$$\boxed{\nabla^2 \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}}$$

ii) For Circular fibres described by Cylindrical coordinates (r, ϕ, z)

$$\boxed{\nabla^2 \psi = \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \phi^2} + \frac{\partial^2 \psi}{\partial z^2}}$$

* Basic solution of wave equation is a sinusoidal wave, and most important form is a uniform plane or linearly polarized wave given by :-

$$\boxed{\psi = \psi_0 \exp j(\omega t - k.r)}$$

where, $\omega \rightarrow$ angular freq. of field
$t \rightarrow$ time
$k \rightarrow$ propagation vector (gives direction of propagation & rate of change of phase with distance)

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* **Concept of Modes:-**

A plane monochromatic wave propagating in direction of ray path within the guide of refractive index n_1 sandwiched b/w two regions of lower refractive index n_2.

Classifications of Optical fibres:-

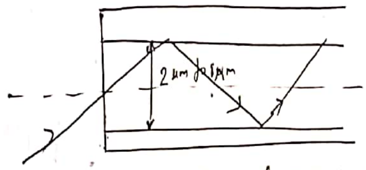
A) Depending on Material Used in Manufacturing of OF Cables:-

1) Glass fibres - [These consists of ~~core~~ glass as a core & also glass as a cladding] with impurities such as Ge, B, Ph, F)
2) Plastic Clad Silica → (By replacing the cladding with a plastic coating of Ref. index)
3) Plastic fibres — These fibres consist of both core & cladding of the plastic material.

B) Depending on Number of Modes:-

1) Mono mode or Single mode fibre

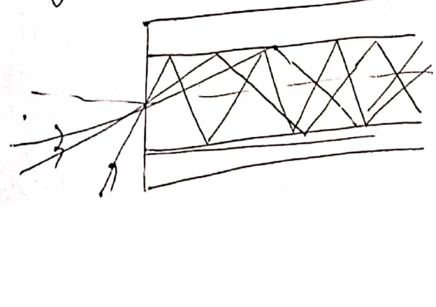
* Transmission of light or modes in only one mode. In this mode, diameter of core is in b/w 2 to 8 μm. Only one ray of light can enter the core & get guided by Total internal reflection.



* ~~Advantages~~ Major advantage of SMF is that it exhibits minimum dispersion loss & hence, highest transmission B.W. It is used for long distance applications.

2) Multimode fibre:-

* In this mode, fibre is capable of transmitting more than one mode;
So The core diameter is of order 50 μm. ~~& cladding~~
* Many diff. rays of light or modes, each entering the core at diff. angles
Since, diff. modes have diff. group velocities,
Hence, dispersion losses are more & B.W is smaller.
* Ease of splicing or jointing.



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c) Depending on the Index profiles or Refractive Index

A) Step Index Fibre
- 1) Single mode Step Index
- 2) Multi mode Step Index

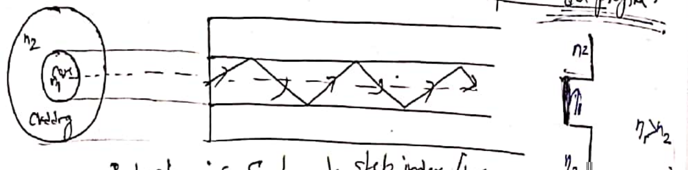
[Refractive Index of core is uniform in every where.]

B) Graded Index fibre → 1) Multimode Graded Index

(Ref. Index of core n_1 is highest at axis & it will decrease as we move towards perimeter of core)

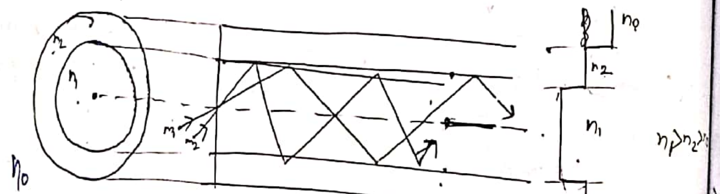
1) Single mode step Index fibre:

Ref. Index profile.

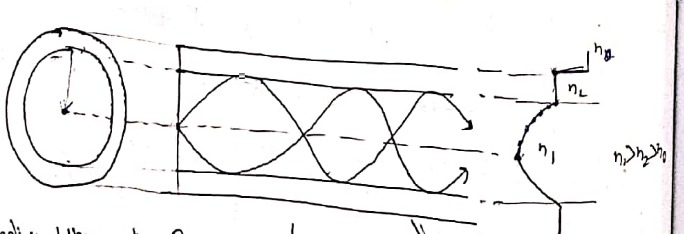


Propagation is Single mode step index fibre

2) Multimode Step Index fibre



3) Multimode Graded Index fiber



Various propagation mode.

D) Depending upon the Requirements & Usage
1) Active fibre
2) Passive fibre
3) L

Numericals

1) If the refractive index of an optical fibre core is 1.40 & the relative refractive index difference is 1%. Determine the refractive index of the cladding.

$\frac{1}{100} = 0.01 = \Delta$

As we know that, $\Delta = \frac{n_1^2 - n_2^2}{2n_1^2}$

or, $\Delta = \frac{n_1 - n_2}{n_1} = 1 - \frac{n_2}{n_1}$

or, $\frac{n_2}{1.40} = 1 - \frac{1}{100} \Rightarrow n_2 = 1.40(1 - 0.01) = \underline{1.386}$

2) Find the value of a critical angle of the core-cladding interface of a silica optical fibre having core refractive index of 1.50 & cladding refractive index of 1.47.

Given, $n_1 = 1.50$ & $n_2 = 1.47$

From Snell's Law, $\sin\phi_c = \frac{n_2}{n_1}$

$\Rightarrow \phi_c = \sin^{-1}\left(\frac{1.47}{1.50}\right) = \sin^{-1} 0.98$

$\boxed{\phi_c = 78.5^\circ}$

3) A typical relative refractive index difference for an optical fibre designed for long distance transmission is 1%. Find the numerical aperture of the fibre if the core refractive index is 1.50.

$NA = n_1\sqrt{2\Delta}$

$\Delta = 0.01$ & $n_1 = 1.50$ (given)

$NA = 1.50\sqrt{2 \times 0.01} = \underline{0.212}$

4) The velocity of light in the core of silica fibre is 2×10^8 m/s & the critical angle at the core-cladding interface is 60°. Determine the
a) The Refractive index of core & cladding.

Sol: As we know that, $n_1 = \frac{\text{Vel. of light in free-space}}{\text{Vel. of light in core}} = \frac{3 \times 10^8}{2 \times 10^8} = \underline{1.5}$

$\sin\phi_c = \frac{n_2}{n_1} \Rightarrow n_2 = n_1 \sin\phi_c = 1.5 \times \sin 60^\circ = \underline{1.3}$

$\frac{3}{2} \times \frac{\sqrt{3}}{2}$

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* Normalized Frequency & Cut-off Wavelength of a fibre:- (10)

Normalized freq. (or cut-off parameter or V number or fibre characteristics) or $\boxed{V = \frac{2\pi a}{\lambda}(n_1^2 - n_2^2)^{1/2}}$

Where $a \rightarrow$ Radius of core

$\lambda \rightarrow$ light wavelength

$n_1 \rightarrow$ Ref. Index of core

$n_2 \rightarrow$ " " " cladding.

~~Proof~~ As we know that, $NA = (n_1^2 - n_2^2)^{1/2}$ —— (1)

& $\Delta = \frac{n_1 - n_2}{n_1} = 1 - \frac{n_2}{n_1}$

$\Rightarrow \frac{n_2}{n_1} = 1 - \Delta$ —— (2)

Putting eqn (2) in (1), we get

$NA = \left\{n_1^2\left(1 - \frac{n_2^2}{n_1^2}\right)\right\}^{1/2}$

$= n_1\left[1 - (1-\Delta)^2\right]^{1/2}$

$= n_1\left[2\Delta - \Delta^2\right]^{1/2}$

Δ^2 is very small, hence can be neglected.

So, $\boxed{NA = n_1 (2\Delta)^{1/2}}$

and $\boxed{V = \frac{2\pi a}{\lambda} n_1 \sqrt{2\Delta}}$ or $\boxed{V = \frac{2\pi a}{\lambda} . NA}$

~~* Cut-off~~

* No. of modes $\boxed{N = \frac{V^2}{2}}$ for Step Index

* $\boxed{N = \frac{V^2}{4}}$ for graded Index

Normalized frequency depends only on the fibre characteristics and the wavelength of light being propagated.

And the normalized freq. determines the number of modes propagating along the waveguide.

In above eqn (4), find Numerical Aperture for the fibre. (11)

$NA = \sqrt{n_1^2 - n_2^2}$

$= \sqrt{1.5^2 - 1.3^2}$

$= \sqrt{2.8 \times 0.2}$

$= \sqrt{0.56}$

$= 0.748331$

A multimode step Index fibre with core diameter of 60μm & a Relative Ref. index difference is 1% is operating at a wavelength of 0.80 μm. If the Ref. index of core is 1.5, determine:

a) The Normalized freq. for the fibre.

b) The Approximate no. of modes it will support.

a) As we know,

$V = \frac{2\pi a}{\lambda} . NA$

$= \frac{2\pi a}{\lambda} . n_1 \sqrt{2\Delta} = \frac{2\pi \times \left(\frac{60 \times 10^{-6}}{2}\right) \times 1.5 \times \sqrt{2 \times 0.01}}{0.8 \times 10^{-6}} = 52.1$

b) No. of modes, $N = \frac{V^2}{2}$

$= \frac{(52.1)^2}{2} = 1353$

A ~~SIF~~ step index fibre with a large core diameter compared with the wavelength of the transmitted light has an acceptance angle in air of 22° & a relative Ref. index difference of 3%. Determine:

a) The numerical aperture of the fibre.

b) The critical angle at the core-cladding interface.

As we know that,

a) $NA = \sin \theta_a$

$= \sin 22° = 0.375$

b) $\Delta = 0.03$

$\Delta = 1 - \frac{n_2}{n_1} \Rightarrow \frac{n_2}{n_1} = 0.97$

$\theta_c = 75.93°$

Prof. Sudha Kumari

Q.8 Calculate the number of modes of an optical fibre having diameter of $50\mu m$, $n_1 = 1.48$, $n_2 = 1.46$ & wavelength 'λ' of $820nm$. — 5 marks

Soln. Given,

diameter of fibre, $d = 50\mu m$

$\therefore$ Radius, $a = \frac{d}{2} = 25\mu m$

No. of Modes, $N = ?$

$$N = \frac{V^2}{2} \quad \text{--- (1)}$$

& $V = \frac{2\pi a}{\lambda} NA = \frac{2\pi 25\times10^{-6}}{820\times10^{-9}} \sqrt{(1.48)^2 - (1.46)^2}$

$\boxed{V = 46.45}$

$\boxed{N = \frac{(46.45)^2}{2} = 1078.8}$

Q.9 Estimate the Maximum core diameter for an optical fibre with refractive index diff. of 1.6% & a core ref. index of 1.48 in order that it may be suitable for single mode operation for an operating wavelength of $0.9\mu m$. Further estimate the maximum core diameter for a Single Mode operation when the Relative refractive index diff. is reduced by a factor of 10. Assume V-number as 2.405.

Soln. Case-I → Core diameter, $d = ?$

Given, $\Delta = 1.6\% = 0.016$, $n_1 = 1.48$, $\lambda = 0.9\mu m$

and $V = 2.405$

As we know that, $V = \frac{2\pi a}{\lambda} NA$

$NA = n_1\sqrt{2\Delta} = 1.48\sqrt{2\times0.016} = 0.26475$

$\therefore$ Core radius, $a = \frac{V\cdot\lambda}{2\pi \times NA} = \frac{2.405\times0.9\times10^{-6}}{2\times3.14\times0.26475}$

$a = 1.3\times10^{-6}m$

$\boxed{a = 1.3\mu m}$

$\therefore$ Core diameter, $d = 2a = 2(1.3\mu m) = \underline{2.6\mu m}$

Case II :- (Q.9 soln. continue)

If Relative refractive index difference is reduced by 10.

i.e $\Delta = \frac{0.016}{10} = 0.0016$ (13)

Core diameter, $d = ?$

$\therefore NA = n_1\sqrt{2\Delta} = 1.48\sqrt{2\times0.0016} = 0.08372$

$\therefore$ Core radius, $a = \frac{V\cdot\lambda}{2\pi\times NA} = \frac{2.405\times0.9\times10^{-6}}{2\times3.14\times0.08372}$

$a = 4.115\times10^{-6}m$

$\therefore d = 2a = 8.23\times10^{-6}m$

$\boxed{d = 8.23\mu m}$

A step index multi-mode fibre with a NA of 0.2 supports approx 1000 modes at an 850nm wavelength. What is the diameter of its core? How many modes does the fibre supports at 1320nm & at 1550nm?

Given

i) $NA = 0.2$, $N = 1000$ at $850nm$, $d = ?$

ii) $N = ?$ at $1320nm$ &

$N = ?$ at $1550nm$

As we know that

$N = 1000$ at $\lambda = 850nm$

$\Rightarrow \frac{V^2}{2} = 1000 \Rightarrow V = \sqrt{2000} = \underline{44.72}$

$\therefore V = \frac{2\pi a}{\lambda} NA$

$a = \frac{V\lambda}{2\pi\times NA} = \frac{44.72\times850\times10^{-9}}{2\times3.14\times0.2} = \underline{30.25\mu m}$

$\boxed{d = 60.5\mu m}$

ii) No. of Guided modes at $\lambda = 1320nm$:-

As we know that, $N = \frac{V^2}{2}$ --- (1)

& $V = \frac{2\pi a}{\lambda} NA = \frac{2\pi\times30.25\times10^{-6}}{\lambda}\times0.2 \simeq 28.78$

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$\therefore N = \frac{V^2}{2} = \frac{(28.78)^2}{2} = 414.66 \simeq 415$

$\boxed{N = 415 \text{ modes}}$ at $\lambda = 1320\,nm$

iii) No. of Guided modes at $\lambda = 1550\,nm$.

$V = \frac{2\pi a}{\lambda} NA = \frac{2\pi \times 30.25 \times 10^{-6}}{1550 \times 10^{-9}} \times 0.2$

$\simeq 24.525$

$\therefore N = \frac{V^2}{2} = 300.73$

$\boxed{N \simeq 301}$ modes at $\lambda = 1550\,nm$

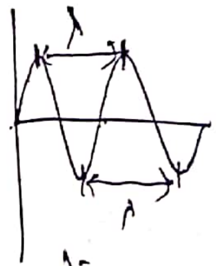
9) Normalized freq. & Cut-off Wavelength of a fiber →

Normalized freq. (or Cut-off parameter or V-number or fibre characteristics)

$$\boxed{V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}}$$

where, $a \rightarrow$ Radius of core

$\lambda \rightarrow$ Light wavelength



$\lambda =$

OFC

Formulae:-

① Snell's Law → $\boxed{\dfrac{\sin\phi_1}{\sin\phi_2} = \dfrac{n_2}{n_1}}$

② Critical Angle, ϕ_c

$\Rightarrow \sin\phi_c = \dfrac{n_2}{n_1} \Rightarrow \boxed{\phi_c = \sin^{-1}\left(\dfrac{n_2}{n_1}\right)}$

③ Acceptance Angle, θ_a:-

$\sin\theta_a = \sqrt{n_1^2 - n_2^2}$

$\Rightarrow \boxed{\theta_a = \sin^{-1}\left(\sqrt{n_1^2 - n_2^2}\right)}$

④ Numerical Aperture:- (NA)

$\boxed{NA = \sin\theta_a = \sqrt{n_1^2 - n_2^2}}$

$\boxed{NA = n_1\left(\sqrt{2\Delta}\right)}$ where $\Delta = \dfrac{n_1 - n_2}{n_1}$ → Relative Refractive Index difference.

⑤ Wave Equation:-

$\boxed{\psi = \psi_0 \exp j(\omega t - kr)}$

where, ψ → Wave Eqn. of sinusoidal wave

ω → angular freq. of field

t → time

k → propagation vector (gives direction of propagation)

r → coordination pt.

Optical fiber:-

A OF is a flexible & transparent fibre which is made by drawing glass (Silica) or Plastics.

& have diameter slightly thicker than human hair.

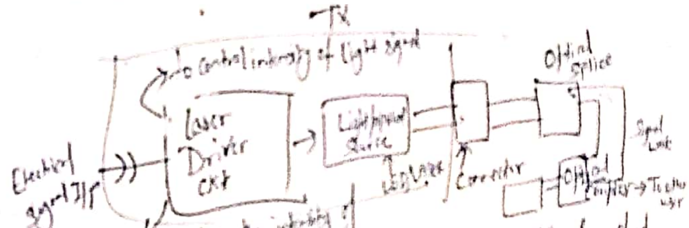
Used in communication have diameter of 0.25mm - 0.5mm

It core, cladding & cover.

Based on refractive index profile, there are two categories of OF

→ Step Index OF

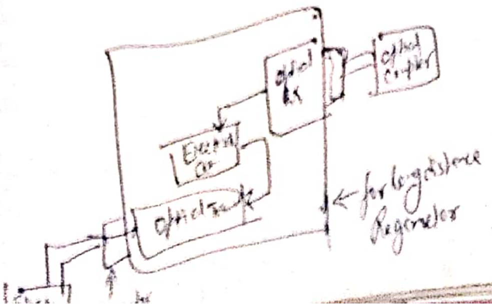
→ Graded Index OF



Semiconductor laser with the encoded dig. signal. Hence a digital optical signal is launched into the optfibre cable.

* At the Rx, we have to detect the dig. Optical Signal for which we are using another Avalanche Photo diode (APD) as detector.

APD detector is followed by a front end amplifier & equalizer or filter to provide gain as well as linear signal processing & Noise B/W reduction.



Block diag. of OFC & its Working