SOFT COMPUTING NOTES

For

7th Sem B.Tech (EE/ETC/CSE/CIVIL)

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Computing

$$X \longrightarrow \boxed{Y = f(X)} \longrightarrow Y$$

Antecedent — Consequent.

f → formal method / Algorithm / mapping func.

Algorithm → finite steps.

Features of Computing :—

1. Precise solution — computing always gives precise and accurate solution.
2. Unambiguous and Accurate —

Ex:- Adjust the temp. to suitable level.
↳ Ambiguous st. (Not precisely defined)

3. Mathematical model.

Computing :—

2 types. ① Hard computing
② Soft computing.

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Hard computing :—

1. Precise results — (No approximate results)
   → Proper mathematical model gives precise solution
2. Control actions (It manipulates the inputs inside our func^n or algorithm to give desired output).

   So control actions must be
   ① Unambiguous (1 step of alg. should have only 1 meaning)
   ② Formally defined (there must be the mathematical model / Algorithm)

Ex:— ① Numerical problems (Integration, Differentiation, Square root)

(ii) Searching and sorting — (Binary search to search and sort the element.

(iii) Computational Geometry problem

Soft Computing :—

1. Imprecision — It is not precise. (So we may not get precise solution)
2. Dynamic — Adaptive in nature. Must have handling capacity according to our requirement.
3. Uncertainty — Uncertain over wide range. Don't have certain output. Output changes each time.
4. Low solution cost — (hard computing results high solution cost)
5. Don't require mathematical model.

Paradigm / Fundamental tools of SC :—

1. Fuzzy logic
2. Neural Network
3. Evolutionary / Genetic computing — (decision based solution)

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Example of SC :—

Stock market predictions in business, computer aided diagnosis in medical hard writing recognision, biometric application in image processing etc.

Hard Computing	Soft Computing
→ Precise solution.	→ Imprecise solution.
→ Based on binary logic and numerical analysis (True/False) (Yes/No)	→ Based on fuzzy logic, Neural network, evolutionary computation
→ No approximation result.	→ Approximation
→ Deal with exact i/p data. (Unambiguous i/p data is free from unwanted data)	→ Deal with ambiguous & noisy data. (Can process with unwanted data)
→ Allow strictly sequential manner.	→ Allow parallel computation (Fast proc)
→ Certainty	→ Uncertainty.

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Fuzzy set :— In fuzzy set all the elements are in order pairs.

$G = \{g, \mu(g)\}$ U = Universe of discourse.

↓ membership function.

U = All students
G = Good students
S = Bad students

$G = \{(A, 0.9), (B, 0.7), (C, 0.3)\}$

↑ degree of goodness / membership value.

→ Membership value ranges from 0 to 1 including 0 & 1.

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Crisp set :—

Power set P(A)

$A = \{a, b, c\}$

$P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{c,a\}, \{a,b,c\}\}$.

$|P(A)| = 2^{|A|}$

Union :—

$A \cup B = \{x \mid x \in A \text{ OR } x \in B\}$

Intersection :—

$A \cap B = \{x \mid x \in A \text{ AND } x \in B\}$

Difference :—

$A - B = \{x \mid x \in A \text{ and } x \notin B\}$

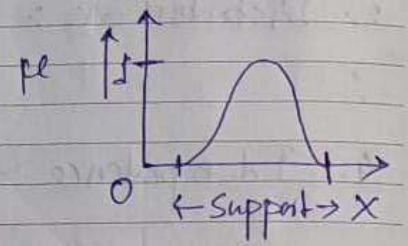
Properties :—

1. Commutative :— $A \cup B = B \cup A$
   $A \cap B = B \cap A$
2. Associative :— $(A \cup B) \cup C = A \cup (B \cup C)$
   $(A \cap B) \cap C = A \cap (B \cap C)$
3. Distributive :— $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
4. Idempotence :— $A \cup A = A$, $A \cap A = A$
5. Identity :— $A \cap \phi = \phi$ $A \cap E = A$
   $A \cup \phi = A$ $A \cup E = E$
6. Law of absorption :—
   $A \cup (A \cap B) = A$
   $A \cap (A \cup B) = A$
7. Involution :— $(A^c)^c = A$
8. Law of Contradiction :— $A \cap A^c = \phi$
9. Demorgan's Law :—
   $(A \cup B)^c = A^c \cap B^c$
   $(A \cap B)^c = A^c \cup B^c$

Features of Membership func^n :-

1. **Support** :- Set of all points such that $\{\mu_A(x) > 0\}$ then it belongs to support set.

Support $(A) = \{x \mid \mu_A(x) > 0\}$

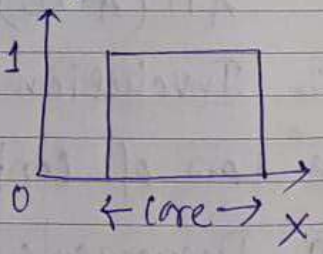


2. **Core** :- $\mu_A(x) = 1$.

If membership value equals to 1, then it belongs to core of A.

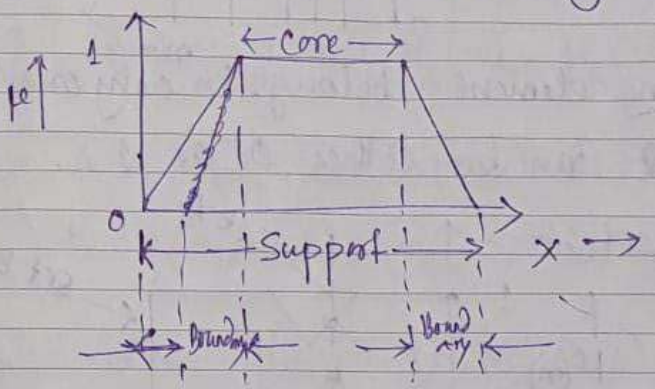
$core(A) = \{x \mid \mu_A(x) = 1\}$

Fully belongs to set 'A'.



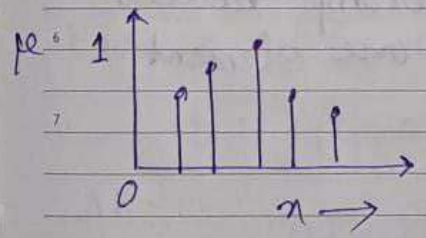
3. **Boundary** :- $1 > \mu_A(x) > 0$

Then it said to be boundary element.

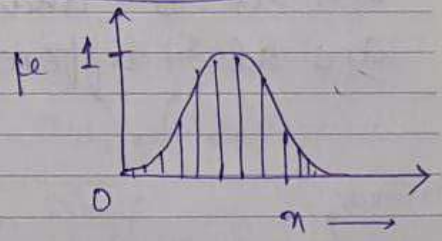


→ Those elements belongs core then doesn't come under boundary elements.

Discrete MF & **continuous MF**.



→ Discrete elements & discrete membership values.



→ Elements are cont. & Membership values are also cont.

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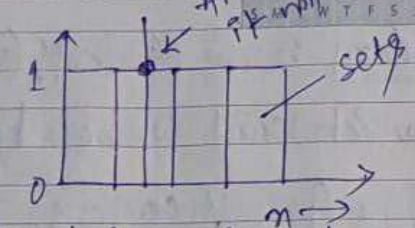
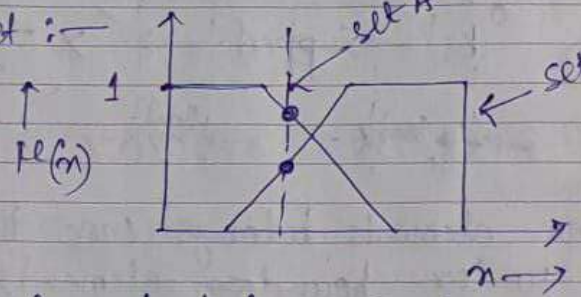
Crisp set :—



Here one element belongs to only one set.
Element can be either 0 or 1.

fuzzy set :—



Here one element belongs to both set A and set B. and membership values are also different for one element.

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Fuzzy Set Operations :—

Union :—

$$\mu_{A \cup B}(x) = \max\left(\mu_A(x), \mu_B(x)\right)$$

Ex :— $A = \{(x_1, 0.5), (x_2, 0.3), (x_3, 0.2)\}$

$B = \{(x_1, 0.4), (x_2, 0.2), (x_3, 0.1)\}$

$\mu_{A \cup B} = A \cup B = \{(x_1, 0.5), (x_2, 0.3), (x_3, 0.2)\}$

Intersection :—

$$\mu_{A \cap B}(x) = \min\left(\mu_A(x), \mu_B(x)\right)$$

Ex :— $A = \{(x_1, 0.3), (x_2, 0.2), (x_3, 0.1)\}$

$B = \{(x_1, 0.2), (x_2, 0.1), (x_3, 0.7)\}$

$A \cap B = \{(x_1, 0.2), (x_2, 0.1), (x_3, 0.1)\}$

Complement :—

$$\mu_{A^c}(x) = 1 - \mu_A(x)$$

Ex :— $A^c = \{(x_1, 0.7), (x_2, 0.8), (x_3, 0.9)\}$

Product :—

$$\mu_{A\cdot B}(x) = \mu_A(x) \cdot \mu_B(x)$$

Equality :— $\mu_A(x) = \mu_B(x)$ then $A = B$.

Ex :- $A = \{(x_1, 0.2), (x_2, 0.8)\}$

$B = \{(x_1, 0.6), (x_2, 0.8)\}$ $A \neq B$

$C = \{(x_1, 0.2), (x_2, 0.8), (x_3, 0)\}$ $A = C$

Power :—

$$\mu_{A^\alpha}(x) = \left(\mu_A(x)\right)^\alpha$$

If $\alpha = 2 \rightarrow$ Concentration

$\alpha = 1/2 \rightarrow$ Dilation

Difference :—

$$A - B = A \cap B^c$$

Disjunctive sum :—

$$A \oplus B = (A^c \cap B) \cup (A \cap B^c)$$

Properties :—

(1) $A \cup A^c \neq X$, (2) $A \cap A^c \neq \phi$

Crisp Relations :—

Cartesian Product :—

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$$

Crisp Relation :—

A binary relation $R(X, Y)$ where

$X = \{x_1, x_2, \ldots x_n\}$

$Y = \{y_1, y_2 \ldots y_n\}$

The relation matrix 'R' is a two dim. matrix.

where X = rows, Y = columns

$R(i, j) = 1$ if $(x_i, y_i) \in R$

$= 0$ if $(x_i, y_i) \notin R$.

Ex :- $X = \{1, 2, 3, 4\}$

$Y = \{1, 2, 3, 4\}$

$X \times Y = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)\}$

The relation is defined as

$R = \{(x,y) \mid y = x+1, \text{ where } x \in X, y \in Y\}$

$R = \{(1,2), (2,3), (3,4)\}$

$R(i,j) = 1$ if $(x_i, y_i) \in R$

0 if $(x_i, y_i) \notin R$

$$R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Operation on Relation :-

Let R & S two relations defined on X & Y.

Union :-

$RUS(x,y) = \max\{R(x,y), S(x,y)\}$

$$R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad S = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$RUS = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Intersection :-

$R \cap S(x,y) = \min\{R(x,y), S(x,y)\}$

Complement :-

$\bar{R}(x,y) = 1 - R(x,y)$

Composition of Relation / Max-min Relation

$R \circ S$

Let X, Y, Z are 3 sets.

Let R : relations from X and Y (x,y)

S : relations from Y and Z (y,z)

$$R \circ S = \{(x,z) \mid (x,z) \in X \times Z \text{ for some } y \in Y, (x,y) \in R \text{ and } (y,z) \in S\}$$

$$R \circ S = \max(\min(R(x,y), S(y,z)))$$

Fuzzy Cartesian Product :—

$$R = A \times B,$$

$$\mu_R(x,y) = \mu_{A\times B} = \min(\mu_A(x), \mu_B(y))$$

Ex :— $A = \{(x_1, 0.6), (x_2, 0.2), (x_3, 0.1)\}$

$B = \{(y_1, 0.7), (y_2, 0.3), (y_3, 0.7)\}$

$$A \times B = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 0.6 & 0.3 & 0.6 \\ 0.2 & 0.2 & 0.2 \\ 0.1 & 0.1 & 0.1 \end{bmatrix} \end{array}$$

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Union :— (Operations of fuzzy relations)

$$\mu_{R \cup S} = \max(\mu_R(x,y), \mu_S(x,y))$$

Intersection :—

$$\mu_{R \cap S} = \min(\mu_R(x,y), \mu_S(x,y))$$

Complement :—

$$\mu_{\bar{R}}(x,y) = 1 - \mu_R(x,y)$$

Composition / Max-min Relation :—

$R \rightarrow X \times Y$

$S \rightarrow Y \times Z$

$R \circ S \rightarrow X \times Z$

$$\mu_{R \circ S}(x,z) = \max(\min(\mu_R(x,y), \mu_S(y,z)))$$

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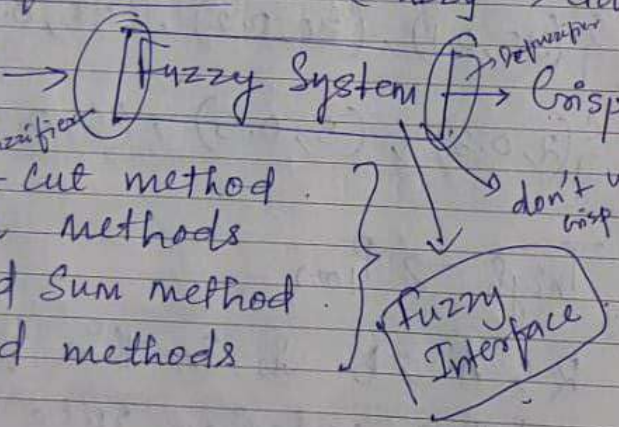
Machine level lang. → Low level.
high level lang. → (User understandable)

Crisp Logic Vs Fuzzy Logic

Crisp o/p → Yes / No.

Fuzzy o/p → May be
May not be
Absolutely
Partially.

Defuzzification :— (Fuzzy → Crisp)



1. Lamda-Cut method.
2. Maxima methods
3. Weighted Sum method.
4. Centroid methods

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Lamda-cut Method (Fuzzy sets)

Fuzzy set (A) ⟶ Crisp set A_λ $(0 \le \lambda \le 1)$

$$A_\lambda = \{x \mid \mu_A(x) \geq \lambda\}$$

Ex:- $A = \{(x_1, 0.2), (x_2, 0.4), (x_3, 0.6\}$

$\lambda = 0.3$

$A_{0.3} = \{(x_1, 0), (x_2, 1), (x_3, 1)\} = \{x_2, x_3\}$

$B = \{(y_1, 0.5), (y_2, 0.4), (y_3, 0.7)\}$

$B_{0.7} = \{(y_1, 0), (y_2, 0), (y_3, 1)\} = \{y_3\}$.

Element which having membership value '0' just eliminated that element.

Fuzzy Relation :— ⟶ Crisp relation.

$$R = \begin{bmatrix} 0.2 & 1 \\ 0.3 & 0.6 \end{bmatrix}$$

$\lambda = 1, 0.5, 0.1, 0$.

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Fuzzy relation

R_λ → Crisp relation

Ex:- $R = \begin{bmatrix} 0.2 & 1 \\ 0.3 & 0.6 \end{bmatrix}$ $\lambda = 1, 0.5, 0.1, 0$

$R_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $R_{0.5} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

$R_{0.1} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $R_0 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

→ λ changes then crisp relation also changes.

(2) Maxima Methods

(i) First of Maxima (FOM)

(ii) Last of Maxima (LOM)

(iii) Mean of Maxima (MOM)

Maxima → Maximum height of fuzzy sets.

highest peak of membership value.

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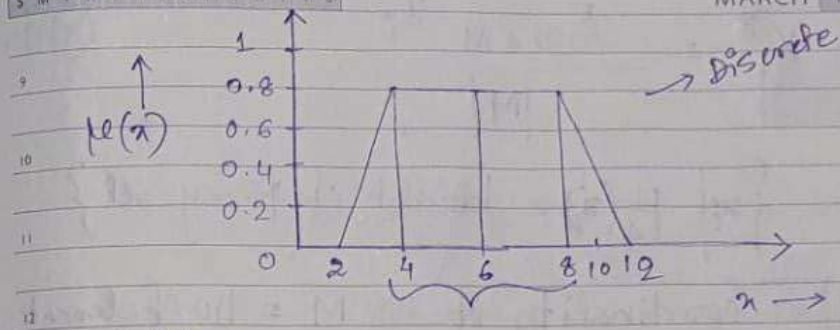


height = 0.8

Here 4, 6, 8 elements having highest membership value i.e 0.8.

① FOM → The 1st element which having the highest membership value.

Here $x^* = 4$.

Crisp value

② LOM → The last element which having highest membership value.

Here $x^* = 8$.

(III) $X^* = \frac{\sum_{x_i \in M} x_i}{|M|}$

$M = \{x_i \mid \mu_A(x_i) = \text{height of fuzzy set}\}$

$|M|$ = cardinality of set M = no of elements.

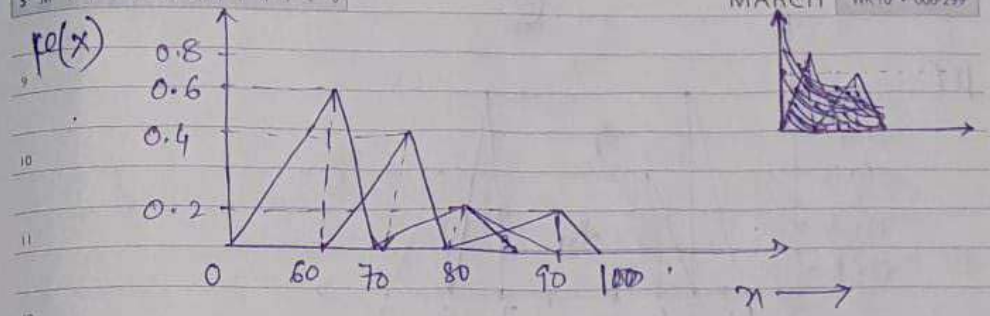
$x = 4, 6, 8 \rightarrow$ belongs to set M.

So, $x^* = \frac{4+6+8}{3} = \frac{18}{3} = 6$

(III) Weighted Average method :-

$x^* = \frac{\sum \mu(x) \cdot x_c}{\sum \mu(x_c)}$

x^* ↑ Crisp value.



$X^* = \frac{(60 \times 0.6 + 70 \times 0.4 + 80 \times 0.2 + 90 \times 0.2)}{(0.6 + 0.4 + 0.2 + 0.2)}$

$X^* = 70 \rightarrow$ Crisp value.

A crisp value for a given fuzzy set.

(IV) Centroid method :- (we cal. the sum of area of ind. fuzzy sets)

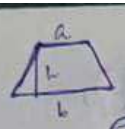
(a) Centre of Sum (COS) :-

$X^* = \frac{\sum A \cdot x_c}{\sum A}$ A = Area.

x_c = centre value = medium

→ Overlapping area twice.

→ COG / COA - overlapping area once.



$\frac{1}{2}(a+b) \times h$

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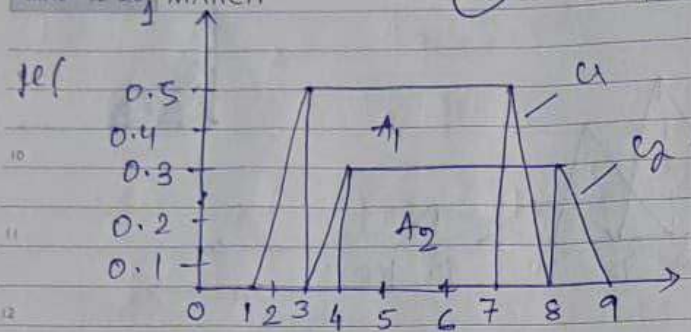
$\frac{8-1}{2} = \frac{7}{2} = 3.5$

3.5



Fuzzy sets C_1 & C_2

Area of 1 :- $A_1 = \frac{1}{2}\left[\underbrace{(8-1)+(7-3)}_{\text{base}}\right] \times \underbrace{0.5}_{\text{height}}$

$= 2.75$

Area of 2 :- $A_2 = \frac{1}{2}\left[(9-3)+(8-4)\right] \times 0.3$

$= 1.5$

$X_c \Rightarrow$ centre of set C.

Here we have to find centre of C_1 & C_2.

$X_{c_1} = 5$, $X_{c_2} = 6\left(\frac{8+4}{2}\right)$

$\frac{7+3}{2}$

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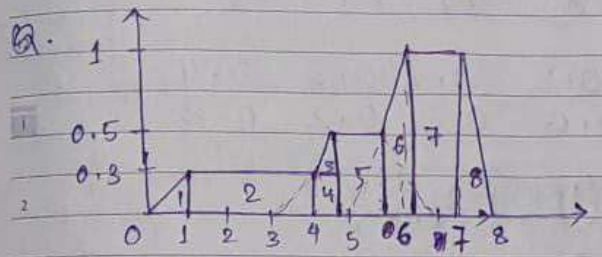
$$X^* = \frac{A_1 \cdot X_{c_1} + A_2 \cdot X_{c_2}}{A_1 + A_2}$$

$$= \frac{2.75 \times 5 + 1.5 \times 6}{2.75 + 1.5} = \frac{5.35}{\text{Crisp value}}$$

Q.



Centroid method

$X^* = ?$

$A_1 = \frac{1}{2}\{(4-1)+(5-0)\} \times 0.3 = \frac{1}{2}\times(3+5)\times 0.3 = 0.12$

$A_2 = \frac{1}{2}\{(6-4)+(7-3)\} \times 0.5 = \frac{1}{2}(2+4)\times 0.5 = 0.15$

$A_3 = \frac{1}{2}\{(9-6)+(8-5)\} \times 1 = \frac{1}{2}\times(1+3)\times 1 = 2$

$$X^* = \frac{0.12 \times \frac{5}{2} + 0.15 \times \left(\frac{7-3}{2}+3\right) + \left(\frac{8-5}{2}+5\right) \times 2}{0.12 + 0.15 + 2}$$

$$= \frac{0.3 + 0.75 + 13}{2.27} = 6.18$$

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λ -cut sets

① $(A \cup B)_\lambda = A_\lambda \cup B_\lambda$

2. $(A \cap B)_\lambda = A_\lambda \cap B_\lambda$

3. $(\bar{A})_\lambda \neq \bar{A}_\lambda$ except for value of $\lambda = 0.5$

Q.

$\mu(x)$	x_1	x_2	x_3	x_4	x_5
P	0.1	0.2	0.7	0.5	0.4
Q	0.9	0.6	0.3	0.2	0.8

(i) $(P \cup Q)_{0.6} = ?$ $(P \cup \bar{P})_{0.8} = ?$

(ii) $P_{0.2}, Q_{0.3} = ?$ $(P \cap Q)_{0.4} = ?$

$P \cup Q = 0.9, 0.6, 0.7, 0.5, 0.8$

$(P \cup Q)_{0.6} = \{(x_1, 1), (x_2, 1), (x_3, 1), (x_4, 0), (x_5, 0)\}$

$= \{x_1, x_2, x_3, x_5\}$

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$P_{0.6} = \{x_3\}$ $Q_{0.6} = \{x_1, x_2, x_5\}$

$P_{0.6} \cup Q_{0.6} = \{x_1, x_2, x_3, x_5\}$

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Fuzzy Inference system

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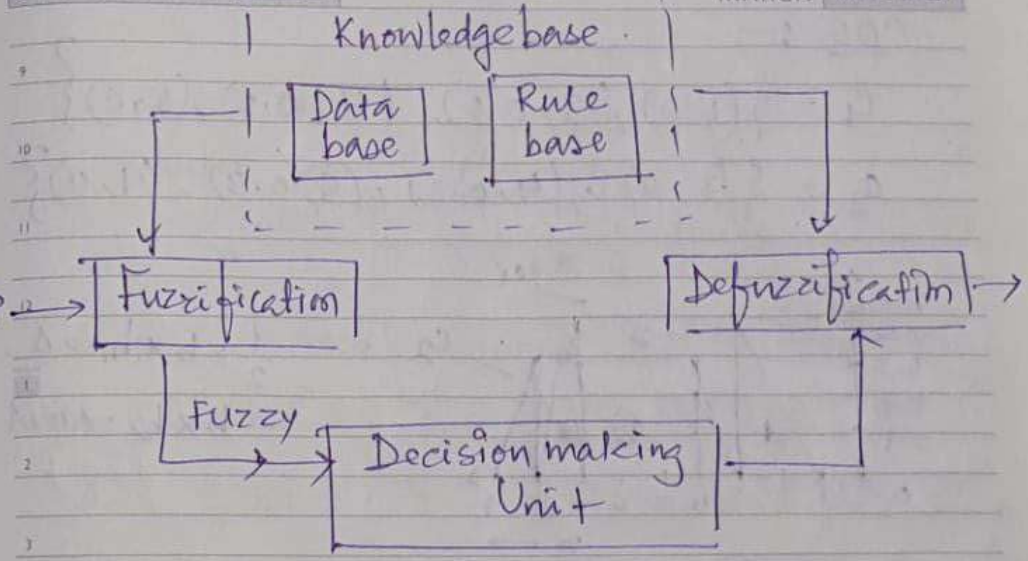


- FIS → uses fuzzy "IF ... THEN" rule along with OR and AND for drawing decision rules.

Rule base – It contains fuzzy IF-THEN rule

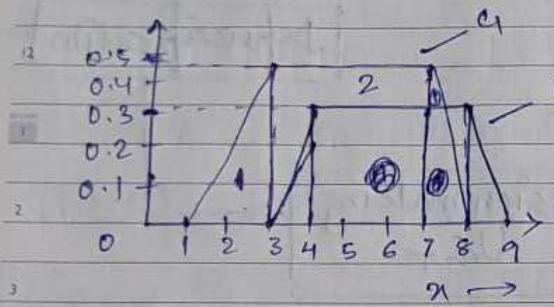
Data base – It defines membership functions of fuzzy sets used in fuzzy rule.

Decision making Unit – It performs the operation on rules.

COS :-

$C_1 = \{(1,0), (3,0.5), (7,0.5), (8,0)\}$

$C_2 = \{(3,0), (4,0.3), (8,0.3), (9,0)\}$



$\frac{1}{2} \times b \times h \rightarrow \Delta$.

$b \times h \rightarrow$ rect.

$$x^* = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$

$x_1 = \frac{7+3}{2}$ = centre of area.

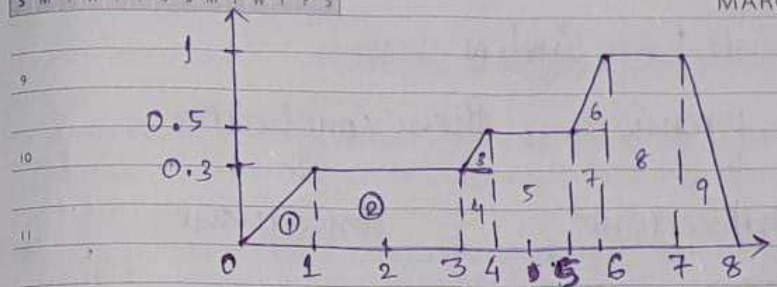
(avg. of max value of x)

$A_1 =$

COG :-

$$X^* = \frac{\sum_{i=1}^{n} x_i A_i}{\sum_{i=1}^{n} A_i}$$

x_i = centre of gravity of ith element.



$A_1 = \frac{1}{2} \cdot 1 \cdot 0.3 = 0.15$, $G_1 = \frac{0+1+1}{3} = 0.66$

$A_2 = 2 \times 0.3 = 0.6$, $G_2 = \frac{1+3}{2} = 2$

$A_3 = \frac{1}{2} \times 1 \times 0.2 = 0.1$, $G_3 = \frac{3+4+4}{3} = 3.66$

$A_4 = 1 \times 0.3 = 0.3$, $G_4 = \frac{3+4}{2} = 3.5$

$A_5 = 1 \times 0.5 = 0.5$, $G_5 = \frac{4+5}{2} = 4.5$

$A_6 = \frac{1}{2} \times 1 \times 0.5 = 0.25$, $G_6 = \frac{5+6+6}{3} = 5.67$

$A_7 = 1 \times 0.5 = 0.5$, $G_7 = \frac{5+6}{2} = 5.5$

$A_8 = 1 \times 1 = 1$, $G_8 = \frac{6+7}{2} = 6.5$

$A_9 = \frac{1}{2} \times 1 \times 1 = 0.5$ $= \frac{7+7+8}{3} = 7.33$

$$X^* = \frac{A_1 G_1 + A_2 G_2 + \cdots + A_9 G_9}{A_1 + A_2 + \cdots A_9} = 4.95$$

Fuzzy Rule Base System :—

If premise, then conclusion
↓ ↓
antecedent consequent

→ IF-THEN rule based form

Canonical forms of rule based System

Rule 1 :- If condition C^1, then restriction R^1

Rule 2 :- If condition C^2, then res. R^2

⋮ ⋮ ⋮

Rule r :- If cond. C^r, then res. R^r.

→ All of these are in If-then rule based form.

⇒ What is restriction ?

∴ Restrictions are generally modelled by fuzzy sets and relations. These restriction statements are connected by some linguistic connectives like AND, OR, ELSE.

Conjunction (∧) (and)
disjunction (∨) (or)

$R^1, R^2 \cdots R^r$ applied across O/P side of a statement.

So restrictions are the consequents of a particular rule.

→ Now we have no of conditions in a fuzzy rule base system and either they are connected by either conjuction or disjunction.

Hence we have multiple conjuctive antecedents for a particular rule.

Multiple Conjuctive Antecedents :—

If x is A^1 and A^2 and $A^3 \cdots A^L$, then y is B^S.
antecedent (they are connected by conjunctions like AND, OR)

x = I/P , y = O/P

$A^1, A^2, \cdots A^L$ and B^S are fuzzy sets of fuzzy nos.

Let's assume a new fuzzy subset A^S

$$A^S = A^1 \cap A^2 \cap A^3 \cdots A^L \quad \text{——} (1)$$

$$\mu_{A^S}(x) = \min\left[\mu_{A^1}(x), \mu_{A^2}(x), \cdots \mu_{A^L}(x)\right]$$

If x is A^S, then y is B^S.

Multiple Disjunctive Antecedents

If x is A^1 or A^2 or $A^3 \cdots A^L$ then y is B^S

Antecedent

Let's assume a new fuzzy subset A^S.

$$A^S = A^1 \cup A^2 \cup A^3 \cdots A^L \quad \text{— (1)}$$

$$\mu_{A^S}(x) = \max\left[\mu_{A^1}(x), \mu_{A^2}(x) \cdots \mu_{A^L}(x)\right]$$

If x is A^S, then y is B^S.

Aggregation of fuzzy rules :—

1. Conjuctive System of Rules :—

18 Sunday In the case of a system of rules that must be jointly satisfied, the rules are connected by 'AND' connectives.

In this case, the aggregated O/P, y is found by the fuzzy intersection of all the 2018 individual rule consequents, y^i where $i = 1, 2, 3 \cdots r$

$$y = y^1 \text{ and } y^2 \text{ and } \cdots \text{ and } y^r$$

or

$$y = y^1 \cap y^2 \cap \cdots \cap y^r$$

$$\mu_y(y) = \min\left[\mu_{y^1}(y), \mu_{y^2}(y) \cdots \mu_{y^r}(y)\right]$$

Disjunctive System of rules :—

In this rule system, we should satisfy atleast one rule. The rules are connected by the 'OR' connectives. The aggregated O/P, y is found by the fuzzy union of all the individual rule consequents y^i where $i = 1, 2, 3 \cdots r$

→ Here we are combining consequents of all the individual rules present in the rule system by the 'OR' connectives, in such a way that O/P 'y' is depended upon the satisfaction of atleast one rule.

$$y = y^1 \text{ or } y^2 \text{ or } \cdots \text{ or } y^r$$

or

$$y = y^1 \cup y^2 \cup y^3 \cdots y^r$$

$$\mu_y(y) = \max\left[\mu_{y^1}(y), \mu_{y^2}(y), \cdots \mu_{y^r}(y)\right]$$ 2018

Graphical Techniques of Inference

1. Mamdani Method :—

If x_1 is A_1 and x_2 is A_2 then y is B

Here we are having two antecedent one consequent

When we have multiple rules, then we can write,

If x_1 is A_1^k and x_2 is A_2^k, then y^k is B^k

where $k = 1, 2, \ldots n$

We will considering a 2-input mamdani system.

There are 2 cases for 2-input mamdani system.

1. Max - min Inference method
2. Max product Inference method.

Max-min Inference method :—

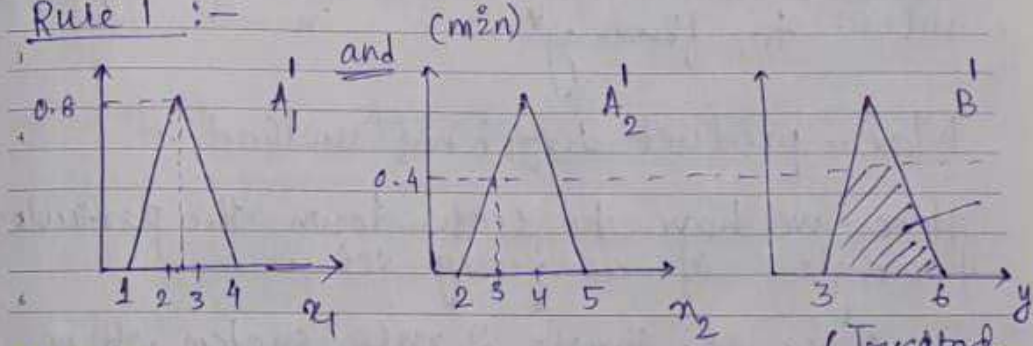
Consider a simple 2 rule system where each rule has 2 antecedents and one consequent as follows :

Rule 1 : If x_1 is A_1^1 and x_2 is A_2^1 then y^1 is B^1

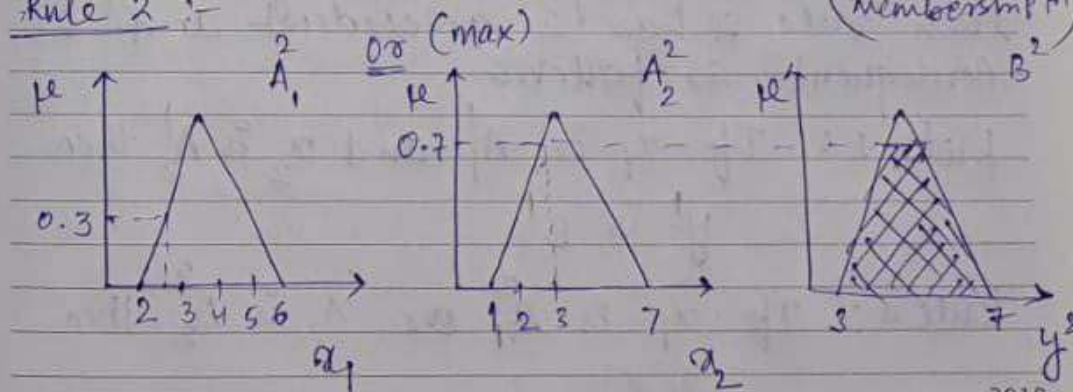
Rule 2 : If x_1 is A_1^2 or x_2 is A_2^2 then y^2 is B^2

$x_1 = 2.5$, $x_2 = 3$

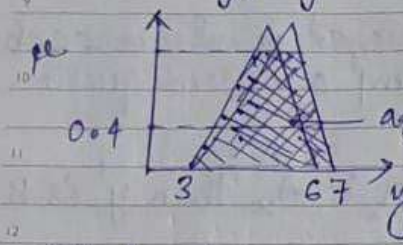
Rule 1 :—



Rule 2 :—



Now aggregate the output graph.



In rule 1, $y^1 = 0.4$

rule 2, $y^2 = 0.7$

It is in case of mamdani method.
Next step is to find out defuzzified value which we will obtain by defuzzification rules. to find y^*.

Max - product Inference method :-

Here we have to scale down the particular triangle of a fuzzy set.

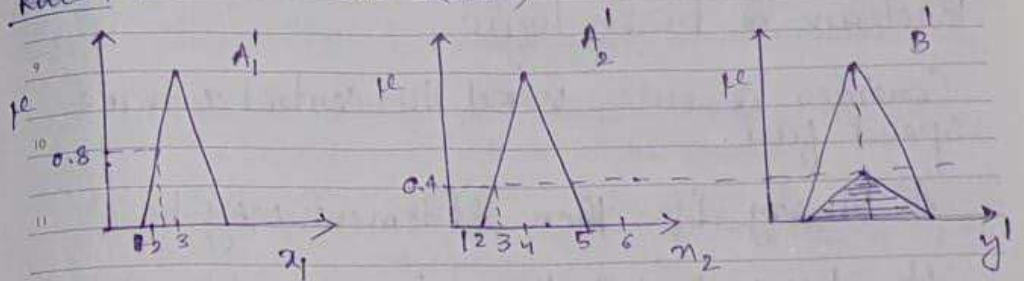
Consider a simple 2 rule system where each rule has 2 antecedent and 1 consequent as follows.

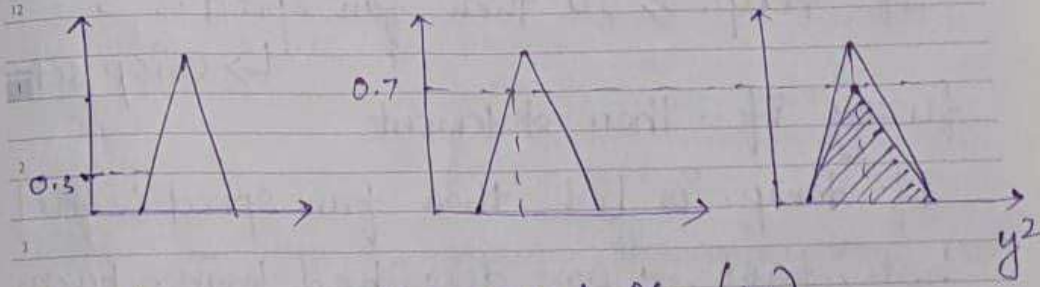
Rule 1 : If x_1 is A_1^1 and x_2 is A_2^1 then y^1 is B^1

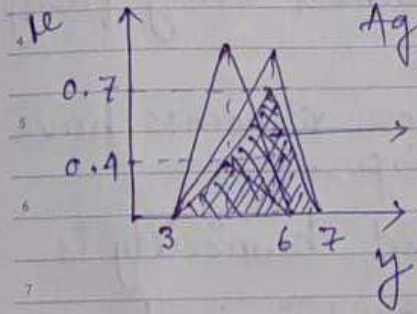
Rule 2 :- If x_1 is A_1^2 or x_2 is A_2^2 then y^2 is B^2

$x_1 = 2.5$, $x_2 = 3$

Rule 1 and (min)







Aggregated o/p (y)

→ Scaled membership function.

L. A Zadeh → father of sc

24 Saturday (40) MARCH - 2018

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Boolean Vs fuzzy logic

Consider a rule used to control a three speed fan.

A binary if-then statement may be

If temp. ≥ 30 then fan speed is 3
↳ crisp set.

fuzzy if-then statement

If temp. is hot then fan speed is fast.

hot, fast → are described using fuzzy sets.

fuzzy sets → A set whose elements have degree of membership.

Ex:- If temp. is hot and humidity is high then fan speed is fast.

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Sugeno method :—

In case of mamdani method, there are 2 antecedents and one consequent.

Mamdani systems : If x_1 is A_1 and x_2 is A_2, then y is B.

Here x_1, x_2 → inputs
y → outputs

A_1, A_2, B → Fuzzy sets.

But in Sugeno system, the antecedents are fuzzy sets or fuzzy numbers and the consequent is given as a function of the two inputs in the antecedent.

Sugeno Systems : If x_1 is A_1 and x_2 is A_2, then y is $y = f(x_1, x_2)$

where, $y = f(x_1, x_2)$ is a crisp function in the consequent.

So here o/p y is a crisp number. The overall aggregated o/p will be obtained through weighted average deffuzification method.

2018

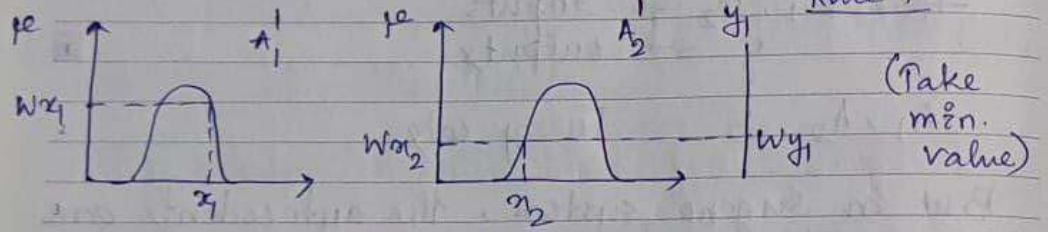
Rule 1 : If x_1 is A_1^1 and x_2 is A_2^1, then y_1 is

$$y_1 = p_1 x_1 + q_1 x_2 + r_1$$

Rule 2 : If x_1 is A_1^2 and x_2 is A_2^2, then y_2 is

$$y_2 = p_2 x_1 + q_2 x_2 + r_2$$

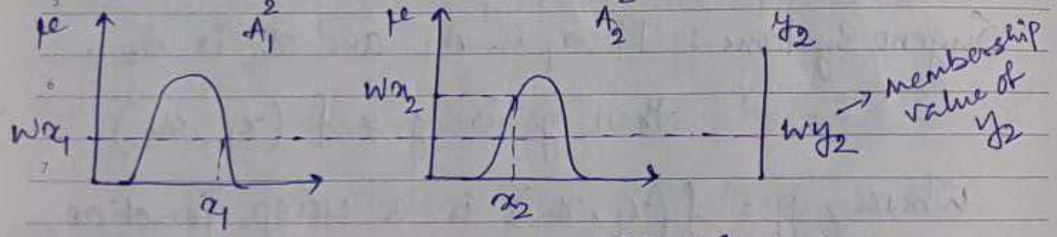
Rule 1



(Take min. value)

$$y_1 = p_1 x_1 + q_1 x_2 + r_1$$

Rule 2



$$y_2 = p_2 x_1 + q_2 x_2 + r_2$$

Now to obtain overall output now apply weighted Average method.

$$y^* = \frac{(w_{y_1} \cdot y_1) + (w_{y_2} \cdot y_2)}{w_{y_1} + w_{y_2}}$$

(y_1, y_2 values are given.)

Ex 2 :-

Consider a two input - one single output Sugeno model with 4 rules are :

Rule 1 : If x_1 is small and x_2 is small,

then $y_1 = -x_1 + x_2 + 1$

Rule 2 : If x_1 is small and x_2 is large,

then $y_2 = -x_2 + 3$

Rule 3 : If x_1 is large and x_2 is small,

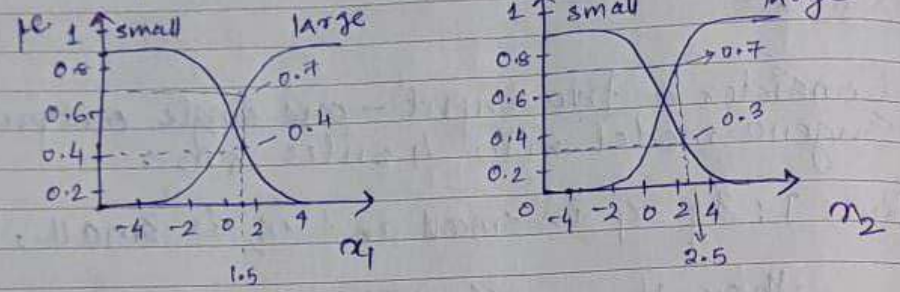
then $y_3 = -x_1 + 3$

Rule 4 : If x_1 is large and x_2 is large,

then $y_4 = -x_1 + x_2 + 2$

Find the output when $x_1 = 1.5$, $x_2 = 2.5$

The graphs for small and large fuzzy sets for x_1 and x_2 is given as :



Rule 1 : $x_1 = 1.5 \rightarrow 0.4$ (small)
$x_2 = 2.5 \rightarrow 0.3$ (small) $\Big\} wy_1 = \min(w_{x_1}, w_{x_2}) = 0.3$

Rule 2 : $x_1 = 1.5$ (small) $\rightarrow 0.4$
$x_2 = 2.5$ (large) $\rightarrow 0.7$ $\Big\} wy_2 = 0.4$

Rule 3 : $x_1 = 1.5$ (large) $\rightarrow 0.7$
$x_2 = 2.5$ (small) $\rightarrow 0.3$ $\Big\} wy_3 = 0.3$

Rule 4 : $x_1 = 1.5$ (large) $\rightarrow 0.7$
$x_2 = 2.5$ (large) $\rightarrow 0.7$ $\Big\} wy_4 = 0.7$

Crisp value

$y_1 = -x_1 + x_2 + 1 = -1.5 + 2.5 + 1 = 2$

$y_2 = ?$, $y_3 = ?$, $y_4 = ?$

Apply weighted Avg. method $y^* = \dfrac{wy_1 \cdot y_1 + wy_2 \cdot y_2 + \cdots}{wy_1 + wy_2 + wy_3 + wy_4}$

NOTE : Sugeno method is much more better than Mamdani since it avoids the time consuming methods of defuzzification that are needed for Mamdani method.