

① Given

Sequence of n numbers
and each number is distinct (or) occurs in duplicate (at
most 2 times).

Goal: No of permutation that are good (no duplicates
occur together).

Let a_1, a_2, \dots, a_n be the sequence
let the number of repetitive pairs be denoted
by " d ".

\Rightarrow If $1, 2, 2, 3$ is the sequence $d=1$

Let n be the total no of elements

Hence, # of unique numbers = $n-d$

Let us construct a table with rows i where

$0 \leq i \leq n$ and

columns j where $0 \leq j \leq d$

$c[i, 0] \Rightarrow$ represents good permutations of length i
 $c[i, j] \Rightarrow$ represents permutations of length i
with error " j ".

I am trying to populate 2×2 matrix using an example

Sequence : 1 2 2 3

$n=4$ and $d=1$

$i \downarrow j \rightarrow$	0	1	
0	0	0	$C\{0, j\} = 0$ since length = 0
1	3	0	
2	6	1 $\rightarrow 2C_0 \cdot 1!$	
3	8	4 $\rightarrow 2C_1 \cdot 2!$	
4	6	6 $\rightarrow 2C_2 \cdot 3!$	

This is the no of good permutations of length "n"
 $C\{n, 0\}$

So from above

$C\{i, 1\}$ is filled using the formula

$$\left[\begin{matrix} n-d \\ C_{i-d-1} \cdot (i-1)! \end{matrix} \right]$$

using the nCr formula = $nC_{r+1} = \left(\frac{n-r}{r+1} \right) nC_r$

Hence we can fill up the columns using value in the row above instead of computing the value every time

$$c[i+1, j] = \binom{n-(i-d-2)}{i-d-1} (i-1) \cdot c[i, j] \quad (3)$$

using this recursion formula, we calculate the values of that column.

We compute the value in $c[i, 0]$ by subtracting the total value for each row and $c[i, 1]$

$$\text{Total value of row } i \text{ is } = \frac{n-d-1}{i-d-1} (i-1)!$$