Programming Assignment - 2

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Problem statement:

$P = (x_1, y_1), (x_2, y_2) \ldots (x_n, y_n)$

we say that a point (x_i, y_i) dominates a point (x_j, y_j) if it is bigger in both coordinates, that is if $(x_i > x_j)$ and $(y_i > y_j)$. A maximal point is not dominated by any other. In this problem, we develop and analyze algorithm for finding the maximal points

Algorithm: Kirkpatrick Seidel (p)

if $P = 0$ then return 0
if $|p| = 1$ then return p
} Base cases

let x be the median x-coordinate of points in p

let p' be all the points with x-coordinate $\leq x$

let pr be all the points with x-coordinate $> x$

let q be the point in pr with maximum y-co-ordinate

Delete q from pr

Delete every point dominated by q in p'

Delete every point dominated by q in pr

S' = kirkpatrickseidel (p') ; Sr = kirkpatrick seidel (pr)

return S' U Sr U {q}

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Time complexity:

This is a divide and conquer algorithm with time $O(n \log h)$ sorting x-co-ordinates, and y coordinates. h is size of hull

Analysis

we divide the points in S into two sets S_L and S_R of approximately Equal size by using vertical separating line. we can compute upper supporting line before we compute convex hulls of S_L and S_R, since these are functions of the point set, and not convex hulls of S_L and S_R

we delete all points that are immediately below the bridge specifically, if the bridge joins points p and q. we can delete all the points with x co-ordinates between p and q. we recurse on the remaining points to find the upper chain of S_L' and S_R'. This will potentially reduce the number of points we recurse on.

if upper chain of S_R' has size h_R and upper chain of S_L' has size h_L, then we can have $h = 1 + h_R + h_L$

let running time be $T(n,h)$

$$T(n,h) = cn \quad \text{if } h=1$$

$$T(n,h) = cn + T(n/2, h_L) + T(n/2, h_R)$$

we need to argue that regardless of split of h between h_L and h_R, the algorithm runs in $O(n\log h)$ time

By induction.

$$T(m,h') \le cm\log h' \text{ for } m<n \text{ and } 1+h'<h$$

$$T(n,h) \le cn + c\frac{n}{2}\log h_L + c\frac{n}{2}\log h_R$$

Now use the fact that $\frac{1}{2}(\log h_L + \log h_R) \le \log\frac{h_L+h_R}{2}$

$$T(n,h) \le cn + c\frac{n}{2}\left(2\log\left(\frac{h_L+h_R}{2}\right)\right) \le cn + cn(\log h - 1)$$

$$\le cn\log h.$$