EXERCISE 3.2

(a) The coefficient of *EXPER* indicates that, on average, a technical artist's quality rating goes up by 0.076 for every additional year of experience.

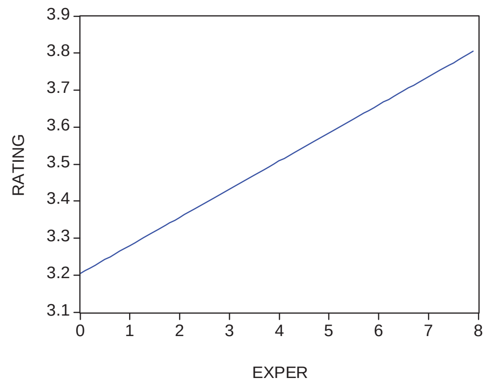

Figure xr3.2(a) Estimated regression function

(b) Using the value $t_c = t_{(0.975,22)} = 2.074$, the 95% confidence interval for β_2 is given by

$$b_2 \pm t_c \text{se}(b_2) = 0.076 \pm 2.074 \times 0.044 = (-0.015,\ 0.167)$$

We are 95% confident that the procedure we have used for constructing a confidence interval will yield an interval that includes the true parameter β_2.

(c) To test $H_0: \beta_2 = 0$ against $H_1: \beta_2 \neq 0$, we use the test statistic $t = b_2/\text{se}(b_2) = 0.076/0.044 = 1.727$. The t critical value for a two tail test with $N - 2 = 22$ degrees of freedom is 2.074. Since $-2.074 < 1.727 < 2.074$ we fail to reject the null hypothesis.

(d) To test $H_0: \beta_2 = 0$ against $H_1: \beta_2 > 0$, we use the *t*-value from part (c), namely $t = 1.727$, but the right-tail critical value $t_c = t_{(0.95,22)} = 1.717$. Since $1.727 > 1.717$, we reject H_0 and conclude that β_2 is positive. Experience has a positive effect on quality rating.

Exercise 3.2 (continued)

(e) The *p*-value of 0.0982 is given as the sum of the areas under the *t*-distribution to the left of −1.727 and to the right of 1.727. We do not reject H_0 because, for $\alpha = 0.05$, *p*-value > 0.05. We can reject, or fail to reject, the null hypothesis just based on an inspection of the *p*-value. Having the *p*-value > α is equivalent to having $|t| < t_c = 2.074$.

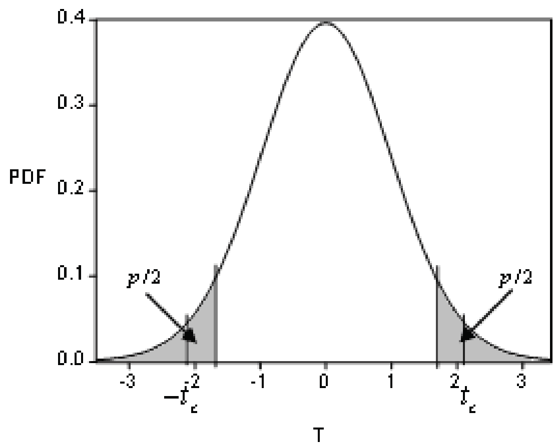

Figure xr3.2(e) *p*-value diagram

EXERCISE 3.4

(a) $b_1 = t \times \text{se}(b_1)$ = 1.257 × 2.174 = 2.733

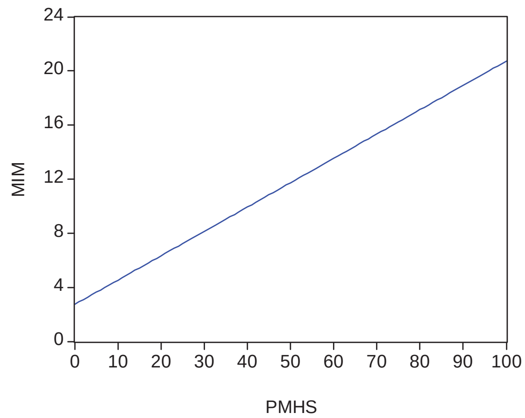

Figure xr3.4(a) Estimated regression function

(b) $\text{se}(b_2) = b_2/t = 0.180/5.754 = 0.0313$

(c) p-value = 2 × $\left(1 - P(t < 1.257)\right)$ = 2 × (1 – 0.8926) = 0.2147

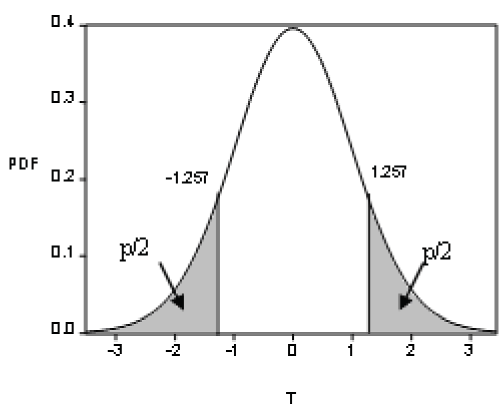

Figure xr3.4(c) *p*-value diagram

(d) The estimated slope $b_2 = 0.18$ indicates that a 1% increase in males 18 and older, who are high school graduates, increases average income of those males by $180. The positive sign is as expected; more education should lead to higher salaries.

(e) Using $t_c = t_{(0.995,49)} = 2.68$, a 99% confidence interval for the slope is given by

$$b_2 \pm t_c \text{se}(b_2) = 0.180 \pm 2.68 \times 0.0313 = (0.096, 0.264)$$

Exercise 3.4 (continued)

(f) For testing $H_0:\beta_2 = 0.2$ against $H_1:\beta_2 \neq 0.2,$ we calculate

$$t = \frac{0.180 - 0.2}{0.0313} = -0.639$$

The critical values for a two-tailed test with a 5% significance level and 49 degrees of freedom are $\pm t_c = \pm 2.01.$ Since t = –0.634 lies in the interval (–2.01, 2.01), we do not reject H_0. The null hypothesis suggests that a 1% increase in males 18 or older, who are high school graduates, leads to an increase in average income for those males of $200. Non-rejection of H_0 means that this claim is compatible with the sample of data.

EXERCISE 3.5

(a) The linear relationship between life insurance and income is estimated as

$$\widehat{INSURANCE} = 6.8550 + 3.8802 INCOME$$
$$(\text{se}) \qquad (7.3835)(0.1121)$$

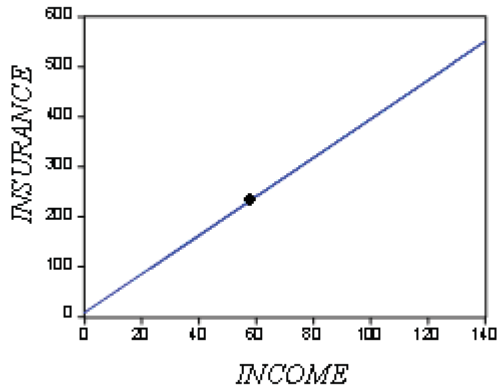

Figure xr3.5 Fitted regression line and mean

(b) The relationship in part (a) indicates that, as income increases, the amount of life insurance increases, as is expected. If taken literally, the value of b_1 = 6.8550 implies that if a family has no income, then they would purchase \$6855 worth of insurance. However, given the lack of data in the region where $INCOME = 0$, this value is not reliable.

(i) If income increases by \$1000, then an estimate of the resulting change in the amount of life insurance is \$3880.20.

(ii) The standard error of b_2 is 0.1121. To test a hypothesis about β_2 the test statistic is

$$\frac{b_2 - \beta_2}{\text{se}(b_2)} \sim t_{(N-2)}$$

An interval estimator for β_2 is $\left[b_2 - t_c\text{se}(b_2), b_2 + t_c\text{se}(b_2)\right]$, where t_c is the critical value for t with $(N-2)$ degrees of freedom at the α level of significance.

(c) To test the claim, the relevant hypotheses are H_0: $\beta_2 = 5$ versus H_1: $\beta_2 \neq 5$. The alternative $\beta_2 \neq 5$ has been chosen because, before we sample, we have no reason to suspect $\beta_2 > 5$ or $\beta_2 < 5$. The test statistic is that given in part (b) (ii) with β_2 set equal to 5. The rejection region (18 degrees of freedom) is $|\, t \,| > 2.101$. The value of the test statistic is

$$t = \frac{b_2 - 5}{\text{se}(b_2)} = \frac{3.8802 - 5}{0.1121} = -9.99$$

As $t = -9.99 < -2.101$, we reject the null hypothesis and conclude that the estimated relationship does not support the claim.

Exercise 3.5 (continued)

(d) To test the hypothesis that the slope of the relationship is one, we proceed as we did in part (c), using 1 instead of 5. Thus, our hypotheses are H_0: $\beta_2 = 1$ versus H_1: $\beta_2 \neq 1$. The rejection region is $|\, t \,| > 2.101$. The value of the test statistic is

$$t = \frac{3.8802 - 1}{0.1121} = 25.7$$

Since $t = 25.7 > t_c = 2.101$, we reject the null hypothesis. We conclude that the amount of life insurance does not increase at the same rate as income increases.

(e) Life insurance companies are interested in household characteristics that influence the amount of life insurance cover that is purchased by different households. One likely important determinant of life insurance cover is household income. To see if income is important, and to quantify its effect on insurance, we set up the model

$$INSURANCE_i = \beta_1 + \beta_2 INCOME_i + e_i$$

where $INSURANCE_i$ is life insurance cover by the i-th household, $INCOME_i$ is household income, β_1 and β_2 are unknown parameters that describe the relationship, and e_i is a random uncorrelated error that is assumed to have zero mean and constant variance σ^2.

To estimate our hypothesized relationship, we take a random sample of 20 households, collect observations on *INSURANCE* and *INCOME* and apply the least-squares estimation procedure. The estimated equation, with standard errors in parentheses, is

$$\begin{aligned}\widehat{INSURANCE} &= 6.8550 + 3.8802 INCOME \\ (\text{se}) \quad & \quad (7.3835)(0.1121)\end{aligned}$$

The point estimate for the response of life-insurance coverage to an income increase of \$1000 (the slope) is \$3880 and a 95% interval estimate for this quantity is (\$3645, \$4116). This interval is a relatively narrow one, suggesting we have reliable information about the response. The intercept estimate is not significantly different from zero, but this fact by itself is not a matter for concern; as mentioned in part (b), we do not give this value a direct economic interpretation.

The estimated equation could be used to assess likely requests for life insurance and what changes may occur as a result of income changes.

EXERCISE 3.7

(a) We set up the hypotheses $H_0:\beta_j = 1$ versus $H_1:\beta_j \neq 1$. The economic relevance of this test is to test whether the return on the firm's stock is risky relative to the market portfolio. Each beta measures the volatility of the stock relative to the market portfolio and volatility is often used to measure risk. A beta value of one indicates that the stock's volatility is the same as that of the market portfolio. The test statistic given H_0 is true, is

$$t = \frac{b_j - 1}{\text{se}(b_j)} \sim t_{(130)}$$

The rejection region is $t < -1.978$ and $t > 1.978$, where $t_{(0.975,130)} = 1.978$.

The results for each company are given in the following table:

Stock	t-value	Decision rule
Disney	$t = \frac{0.89794 - 1}{0.12363} = -0.826$	Since $-1.978 < t < 1.978$, fail to reject H_0
GE	$t = \frac{0.89926 - 1}{0.098782} = -1.020$	Since $-1.978 < t < 1.978$, fail to reject H_0
GM	$t = \frac{1.26141 - 1}{0.20222} = 1.293$	Since $-1.978 < t < 1.978$, fail to reject H_0
IBM	$t = \frac{1.18821 - 1}{0.126433} = 1.489$	Since $-1.978 < t < 1.978$, fail to reject H_0
Microsoft	$t = \frac{1.31895 - 1}{0.16079} = 1.984$	Since $t > 1.978$, reject H_0
Exxon-Mobil	$t = \frac{0.41397 - 1}{0.089713} = -6.532$	Since $t < -1.978$, reject H_0

For Disney, GE, GM and IBM we fail to reject the null hypothesis, indicating that the sample data are consistent with the conjecture that the Disney, GE, GM, and IBM stocks have the same volatility as the market portfolio. For Microsoft and Exxon-Mobil, we reject the null hypothesis, and conclude that these stocks do not have the same volatility as the market portfolio.

Exercise 3.7 (continued)

(b) We set up the hypotheses $H_0: \beta_j \geq 1$ versus $H_1: \beta_j < 1$ where j = Mobil-Exxon. The relevant test statistic, given H_0 is true, is

$$t = \frac{b_j - 1}{\text{se}\left(b_j\right)} \sim t_{(130)}$$

The rejection region is $t < -1.658$ where $t_c = t_{(0.05,130)} = -1.657$. The value of the test statistic is

$$t = \frac{0.41397 - 1}{0.089713} = -6.532$$

Since $t = -6.532 < t_c = -1.657$, we reject H_0 and conclude that Mobil-Exxon's beta is less than 1. A beta equal to 1 suggests a stock's variation is the same as the market variation. A beta less than 1 implies the stock is less volatile than the market; it is a defensive stock.

(c) We set up the hypotheses $H_0: \beta_j \leq 1$ versus $H_1: \beta_j > 1$ where j = Microsoft. The relevant test statistic, given H_0 is true, is

$$t = \frac{b_j - 1}{\text{se}\left(b_j\right)} \sim t_{(130)}$$

The rejection region is $t > 1.6567$ where $t_{(0.95,130)} = 1.6567$. The value of the test statistic is

$$t = \frac{1.31895 - 1}{0.16079} = 1.9836$$

Since $t = 1.9836 > t_c = 1.6567$, we reject H_0 and conclude that Microsoft's beta is greater than 1. A beta equal to 1 suggests a stock's variation is the same as the market variation. A beta greater than 1 implies the stock is more volatile than the market; it is an aggressive stock.

(d) A 95% interval estimator for Microsoft's beta is $b_j \pm t_{(0.975,130)} \times \text{se}(b_j)$. Using our sample of data the corresponding interval estimate is

$$1.3190 \pm 1.978 \times 0.16079 = (1.001, 1.637)$$

Thus we estimate, with 95% confidence, that Microsoft's beta falls in the interval 1.001 to 1.637. It is possible that Microsoft's beta falls outside this interval, but we would be surprised if it did, because the procedure we used to create the interval works 95% of the time. This result appears in line with our conclusion in both parts (a) and (c).

Exercise 3.7 (continued)

(e) The two hypotheses are $H_0\colon \alpha_j = 0$ versus $H_1\colon \alpha_j \neq 0$. The test statistic, given H_0 is true, is

$$t = \frac{a_j}{\operatorname{se}(a_j)} \sim t_{(130)}$$

The rejection region is $t < -1.978$ and $t > 1.978$, where $t_{(0.975,130)} = 1.978$.
The results for each company are given in the following table:

Stock	t-value	Decision rule
Disney	$t = \frac{-0.00115}{0.005956} = -0.193$	Since $-1.978 < t < 1.978$, fail to reject H_0
GE	$t = \frac{-0.001167}{0.004759} = -0.245$	Since $-1.978 < t < 1.978$, fail to reject H_0
GM	$t = \frac{-0.01155}{0.009743} = -1.185$	Since $-1.978 < t < 1.978$, fail to reject H_0
IBM	$t = \frac{0.005851}{0.006091} = 0.961$	Since $-1.978 < t < 1.978$, fail to reject H_0
Microsoft	$t = \frac{0.006098}{0.007747} = 0.787$	Since $-1.978 < t < 1.978$, fail to reject H_0
Mobil-Exxon	$t = \frac{0.00788}{0.004322} = 1.823$	Since $-1.978 < t < 1.978$, fail to reject H_0

We do not reject the null hypothesis for any of the stocks. This result indicates that the sample data is consistent with the conjecture from economic theory that the intercept term equals 0.

EXERCISE 3.9

(a) We set up the hypotheses H_0: $\beta_2 = 0$ versus H_1: $\beta_2 > 0$. The alternative $\beta_2 > 0$ is chosen because we assume that growth, if it does influence the vote, will do so in a positive way. The test statistic, given H_0 is true, is

$$t = \frac{b_2}{\text{se}(b_2)} \sim t_{(22)}$$

The rejection region is $t > 1.717 = t_{(0.95,22)}$. The estimated regression model is

$$\widehat{VOTE} = 50.8484 + 0.8859GROWTH$$
$$\text{(se)} \quad (1.0125) \quad (0.1819)$$

The value of the test statistic is

$$t = \frac{0.8859}{0.1819} = 4.870$$

Since t = 4.870 > 1.717, we reject the null hypothesis that $\beta_2 = 0$ and accept the alternative that $\beta_2 > 0$. We conclude that economic growth has a positive effect on the percentage vote earned by the incumbent party.

(b) A 95% interval estimate for β_2 from the regression in part (a) is:

$$b_2 \pm t_{(0.975,22)}\text{se}(b_2) = 0.8859 \pm 2.074 \times 0.1819 = (0.509,\ 1.263)$$

This interval estimate suggests that, with 95% confidence, the true value of β_2 is between 0.509 and 1.263. Since β_2 represents the change in percentage vote due to economic growth, we expect that an increase in the growth rate of 1% will increase the percentage vote by an amount between 0.509 and 1.263.

(c) We set up the hypotheses H_0: $\beta_2 = 0$ versus H_1: $\beta_2 < 0$. The alternative $\beta_2 < 0$ is chosen because we assume that inflation, if it does influence the vote, will do so in a negative way. The test statistic, given H_0 is true, is

$$t = \frac{b_2}{\text{se}(b_2)} \sim t_{(22)}$$

Selecting a 5% significance level, the rejection region is $t < -1.717 = t_{(0.05,22)}$. The estimated regression model is

$$\widehat{VOTE} = 53.4077 - 0.4443INFLATION$$
$$\text{(se)} \quad (2.2500) \quad (0.5999)$$

The value of the test statistic is

$$t = \frac{-0.4443}{0.5999} = -0.741$$

Since $-0.741 > -1.717$, we do not reject the null hypothesis. There is not enough evidence to suggest inflation has a negative effect on the vote.

Exercise 3.9 (continued)

(d) A 95% interval estimate for β_2 from the regression in part (c) is:

$$b_2 \pm t_{(0.975,22)}\text{se}(b_2) = -0.4443 \pm 2.074 \times 0.5999 = (-1.688, 0.800)$$

This interval estimate suggests that, with 95% confidence, the true value of β_2 is between -1.688 and 0.800. It suggests that an increase in the inflation rate of 1% could increase or decrease or have no effect on the percentage vote earned by the incumbent party.

(e) When $INFLATION = 0$, the expected vote in favor of the incumbent party is

$$E\left(VOTE \mid INFLATION = 0\right) = \beta_1 + \beta_2 \times 0 = \beta_1$$

Thus, we wish to test $H_0: \beta_1 \geq 50$ against the alternative $H_1: \beta_1 < 50$. The test statistic, assuming H_0 is true at the point $\beta_1 = 50$, is

$$t = \frac{b_1 - 50}{\text{se}(b_1)} \sim t_{(22)}$$

The rejection region is $t < -1.717 = t_{(0.05,22)}$. The value of the test statistic is

$$t = \frac{53.4077 - 50}{2.2500} = 1.515$$

Since $1.515 > -1.717$, we do not reject the null hypothesis. There is no evidence to suggest that the expected vote in favor of the incumbent party is less than 50% when there is no inflation.

(f) A point estimate of the expected vote in favor of the incumbent party when $INFLATION = 2$ is

$$\widehat{E\left(VOTE\right)} = b_1 + 2b_2 = 53.4077 + 2 \times (-0.44431) = 52.5191$$

The standard error of this estimate is the square root of

$$\begin{aligned}\widehat{\text{var}\left(b_1 + 2b_2\right)} &= \widehat{\text{var}\left(b_1\right)} + 2^2 \cdot \widehat{\text{var}\left(b_2\right)} + 2 \cdot 2 \cdot \widehat{\text{cov}\left(b_1, b_2\right)} \\ &= 5.0625 + 4\left(0.3599\right) + 4\left(-1.0592\right) \\ &= 2.2653\end{aligned}$$

The 95% interval estimate is therefore:

$$\begin{aligned}\left(b_1 + 2b_2\right) \pm t_{(0.975,22)}\text{se}\left(b_1 + 2b_2\right) &= 52.5191 \pm 2.074\sqrt{2.2653} \\ &= 52.5191 \pm 3.1216 \\ &= \left(49.40, 55.64\right)\end{aligned}$$

We estimate with 95% confidence that the expected vote in favor of the incumbent party when inflation is at 2% is between 49.40% and 55.64%. In repeated samples of elections with inflation at 2%, we expect the mean vote to lie within 95% of the interval estimates constructed from the repeated samples.