SVM-Example2

Question 1

The training data for an SVM consists of 5 points: $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4), (x_5, y_5)$, where: $y_1 = -1, y_2 = 1, y_3 = -1, y_4 = 1, y_5 = 1$. The values of the feature vectors $(x_1, \ldots, x_5)$ are not known explicitly but their Gram matrix is known:

$$G = \begin{pmatrix} 9 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -3 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & -2 \\ 0 & 0 & 0 & -2 & 1 \end{pmatrix}$$

(Element i, j of the Gram matrix is the dot product of x_i and x_j.) Let $\alpha_1, \ldots, \alpha_5$ be the Lagrange multipliers associated with this data. (α_i is associated with (x_i, y_i).)

Part A

a.

Using the linear kernel, what (dual) optimization problem needs to be solved in terms of the α_i in order to determine their values?

Answer

b.

Using the polynomial kernel of order 2 , and soft margins specified by the parameter $C = 10$, what (dual) optimization problem needs to be solved in terms of the α_i in order to determine their values?

Answer

Part B

Four optimization problems were solved by a numeric algorithm, and the corresponding Lagrange multiplies are shown below. You should take into consideration the fact that the numeric algorithm is not perfect, so that the results for the alphas are just an approximation.

Case	kernel	C	α_1	α_2	α_3	α_4	α_5
1	linear	∞	$6.6 \cdot 10^7$	$9.4 \cdot 10^7$	$2.0 \cdot 10^8$	$5.6 \cdot 10^7$	$1.1 \cdot 10^8$
2	2nd order polynomial	∞	0	0.666	0.666	0	0
3	linear	10	3.6	4.8	10	2.9	5.8
4	2nd order polynomial	10	0	0.666	0.666	0	0

a.

Select one of these cases and show that the SVM correctly classifies the entire training data. Show and explain your computations.

Answer

I am using Case ___.
The following values need to be calculated so that the SVM can be applied:

a1 The following computation needs to be carried out to classify (x_1, y_1):

.

a2 The following computation needs to be carried out to classify (x_2, y_2):

.

a3 The following computation needs to be carried out to classify (x_3, y_3):

.

a4 The following computation needs to be carried out to classify (x_4, y_4):

.

a5 The following computation needs to be carried out to classify (x_5, y_5):

.

b.

Select one of these cases and show that the SVM does not correctly classify the entire training data. Show and explain your computations.

Answer

I am using Case ___.
The following values need to be calculated so that the SVM can be applied:

b1 The following computation needs to be carried out to classify (x_1, y_1):

.

b2 The following computation needs to be carried out to classify (x_2, y_2):

.

b3 The following computation needs to be carried out to classify (x_3, y_3):

.

b4 The following computation needs to be carried out to classify (x_4, y_4):

.

b5 The following computation needs to be carried out to classify (x_5, y_5):

.