The exact (optimal) steepest descent algorithm

This algorithm finds a local minimum of $f(x) = f(x_1, \dots, x_n)$ when given ∇f.

Start with arbitrary x.

Repeat:

- $r = -\nabla f(x)$
- Compute a, where $t = a$ minimizes $\phi(t)$

$$\phi(t) = f(x + tr)$$

- $x = x + ar$

Terminate when $|r|$ is small enough so that x can be considered the local minimum point.

The ϵ-step steepest descent algorithm

This algorithm finds a local minimum of $f(x) = f(x_1, \dots, x_n)$ when given ∇f.

Start with arbitrary x.

Repeat:

- $r = -\nabla f(x)$
- choose a small number $\epsilon > 0$
- $x = x + \epsilon r$

Terminate when $|r|$ is small enough so that x can be considered the local minimum point.

Alternative formulation of the ϵ-step steepest descent algorithm

This algorithm finds a local minimum of $(x) = f(x_1, \dots, x_n)$ when given the derivatives $\frac{\partial f}{\partial x_i}$ for $i = 1, \dots, n$.

Start with arbitrary $x_1, \dots, x_n$.

Repeat:

- choose a small number $\epsilon > 0$
- $x_i = x_i - \epsilon \, \frac{\partial f}{\partial x_i}$ for $i = 1, \dots, n$.

Terminate when the norm of all the partial derivatives $|\frac{\partial f}{\partial x_i}|$ for $i = 1, \dots, n$ are small.