Integer Linear Programming

Introduction

In many network planning problems the variables can take only integer values, because they represent choices among a finite number of possibilities. Such a mathematical program is known as *integer program*.

Remark: Often the variables can only take two values, 0 and 1. Then we speak about 0-1 programming.

If the problem, apart from the restriction that the variables are integer valued, has the same formulation as a linear program, then it is called an *integer linear program (ILP)*.

Sometimes it happens that only a subset of the variables are restricted to be integer valued, others may vary continuously. Then we speak about a *mixed programming* task. If it is also linear, then it is a mixed ILP.

A general ILP is formulated as follows:

$$\min Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

Subject to

$$\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \\
\vdots \qquad\qquad & \quad \vdots \\
a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &= b_m \\
x_1, \ldots, x_n &\in \mathbf{Z}
\end{aligned}$$

where $\mathbf{Z}$ denotes the set of integers.

One can, of course, use other LP formulations, too, and then add the integrality constraint $x_1, \ldots, x_n \in \mathbf{Z}$ to obtain an ILP.

Comments:

The minimum can be replaced by maximum, this does not make any essential difference.

If $x_1, \ldots, x_n \in \mathbf{Z}$ is replaced by $x_1, \ldots, x_n \in \{0, 1\}$, then we obtain a 0-1 programming problem. Many of the tasks encountered in network design belong to this class, as will be seen later.

It is important to know that ILP is usually much more difficult to solve than LP. Often we have to apply heuristic or approximative approaches, because finding the exact global optimum for a large problem is rarely feasible.