10-3-2019

① Each process has an input value from $\{0, 1\}$

Allow a (small) probability of error in agreement

Run for ~~r~~ r rounds; r is chosen a priori

Communication pattern: $\gamma \in$ gamma

↓

$\{$ ~~(i, j, k)~~ . $\}$

triples .

$(i, j, k) \in \gamma$ (the communication pattern)

$\Rightarrow$ message sent by i to j in round k is successfully delivered (not lost)

from i to j

When a message is delivered in round k, process j "learns" about what i knows ~~during~~ at the time i sent that message.

Relation $\leq_\gamma$ ← gamma : reflexive partial ordering

① $(i, k) \leq_\gamma (i, k')$ $\forall i$, $\forall k, k' : 0 \leq k \leq k'$ (info flow within a process)

process id / round #

② if $(i, j, k) \in \gamma$ then $(i, k-1) \leq_\gamma (j, k)$ info flow from sender (i) to receiver (j)

③ if $(i, k) \leq_\gamma (i', k')$ and $(i', k') \leq_\gamma (i'', k'')$ then $(i, k) \leq_\gamma (i'', k'')$

② Information Level

① $k = 0$: $Level_q(i, 0) = 0$ $\quad$ $Level(i, 0) = 0$

Common pattern — process id

③ $k > 0$ and there exists $j \neq i$ such that $(j, 0) \not\leq_q (i, k)$ then $Level(i, k) = 0$

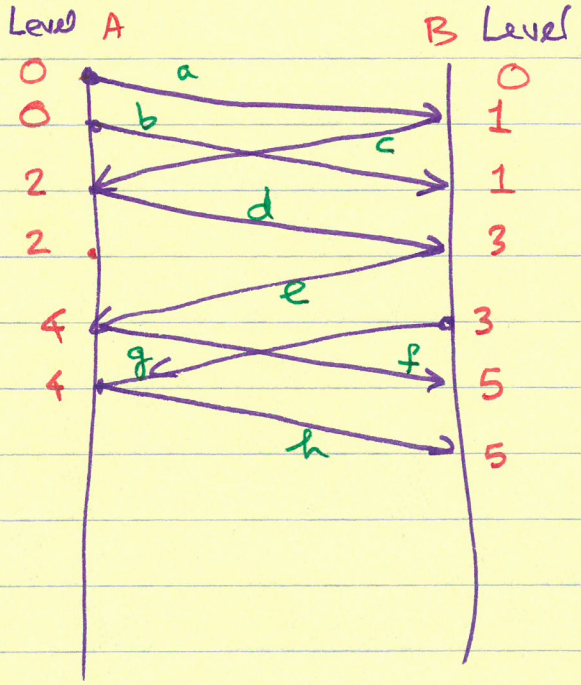
INFORMALLY, → { process i does not know input values of all processes in round k }

④ $k > 0$ and $(j, 0) \leq_q (i, k)$ for all $j \neq i$

let $l_j = \max \{ Level(j, k') : (j, k') \leq_q (i, k) \}$

$Level(i, k) = 1 + \min \{ l_j : j \neq i \}$

example



$a = (A, B, 1)$

$b = (A, B, 2)$

$c = (B, A, 2)$

$d = (A, B, 3)$

$e = (B, A, 4)$

$f = (A, B, 5)$

$g = (B, A, 5)$

$h = (A, B, 6)$

③ Lemma 5.3 : If q is the Complete Communication pattern (all triples $(i, j, k) \in V$) then $Level(i, k) = k \quad \forall i, \forall k$

Algorithm for Consensus:

All process have L : level vector
V . Value set (vector of input vals of all)

Uniformly chosen

Process 1 (special process) chooses a random (key) in range $1..r$ &

(key, L & V) are sent to all process at every round.

When a message is received update these 3 variables.

Run these steps for r rounds.

At the end.

Process i decides on 1 if all input values are 1 and $Level_i[r] \geq key$.

Process i decides on 0 otherwise.

Correctness:

① Termination ✓ at end of r rounds

② Validity : ✓

③ Agreement.

④ $\Pr$ [Some process decides on 0 and others decide on 1] $\leq \frac{1}{r}$.

disagreement:

$$|\text{level}_i(k) - \text{level}_j(k)| \leq 1 \quad \forall i, \forall j, \forall k \leq r$$

↓ happens when key = max $\{\text{level}_j(r)\}$

$$\Pr[\text{key} = \text{max level}] = \frac{1}{r}$$ ■

Lower round: Any r-round algorithm for randomized Consensus Problem has probability of disagreement $\geq \frac{1}{r+1}$

Chapter 6. Consensus Despite Process failures.

(fail stop, Completely Connected, Synchronous network, all links Connecting non-faulty process dont fail)

f = max # of processes that can fail.