

Mathematical Programming for Planning

What is a mathematical program?

An optimization task where we want to find a solution that minimizes or maximizes an *objective function* under *constraints*. Typically, it has many variables and constraints.

Historical note: the word “program” in the name had originally nothing to do with computer programming, even though the solving algorithm is naturally implemented by a computer program. The name “programming” was originally motivated by its meaning “preparing a schedule of activities,” before computers existed at all. It is still used, e.g., in oil refineries, where the refinery programmers prepare detailed schedules of how the various process units will be operated and the products blended.

Classification

- *Continuous — Discrete:* If all the variables can take continuous values (real numbers) then the task is called continuous. If the variables are allowed to take discrete values only (e.g., integers), then we speak about a discrete programming task. (Sometimes both cases occur within the same problem, then we speak about mixed programming).

- *Linear Programming (LP)*: A linear programming problem (see more later) is a continuous optimization task in which the objective function is linear *and* the constraints are expressed by linear inequalities or linear equations.
- *Nonlinear Programming*: If either the objective function or the constraints (or both) are nonlinear, then we speak about nonlinear programming.
- *Combinatorial Optimization*: Many optimization problems that occur in network design are associated with a combinatorial structure, typically a *graph*. This often imposes the constraint that the variables are 0-1 valued, i.e., can only take 2 possible values (0 or 1). Then we often speak about *combinatorial optimization*.

Our first objective is to show how to formulate network design related problems as mathematical programming tasks.

Why is it important? Because once the task is formulated as a standard mathematical programming problem, the solution can be found using known algorithms that are available as commercial software, or even freeware.

Thus, if we can ask the question properly, the answer is often available off-the-shelf. But, as we are going to see, it is not always easy to properly formulate the task in a given framework.