

The SoftMax

Forward propagation

Consider a network for selecting between k categories. It is designed such that at the last step there are k computed real values V_1, \dots, V_k . The SoftMax converts these values into probabilities as follows:

$$q_i = e^{V_i}, \quad i = 1, \dots, k \quad (1)$$

$$Z = \sum_{i=1}^k q_i \quad (2)$$

$$p_i = \frac{q_i}{Z} = \frac{e^{V_i}}{\sum_{j=1}^k e^{V_j}}, \quad i = 1, \dots, k \quad (3)$$

Error back propagation

Suppose the true category is j . The SoftMax error is defined as follows:

$$E = E_j = -\ln p_j = \ln \frac{1}{p_j}$$

Observe that the error is always nonnegative, in the range 0 to ∞ . Simple calculation shows:

$$\frac{\partial p_j}{\partial V_i} = p_j(1_{ij} - p_i), \quad \text{where } 1_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

Therefore:

$$\frac{\partial E_j}{\partial V_i} = -\frac{1}{p_j} \frac{\partial p_j}{\partial V_i} = p_i - 1_{ij}$$

We now assume that the top layer is linear: $V_i = g(h_i) = h_i$. Then:

$$\frac{\partial E}{\partial h_i} = \frac{\partial E_j}{\partial h_i} = \frac{\partial E_j}{\partial V_i} = p_i - 1_{ij}$$

From the BP proof theorem:

$$\delta_i = -\frac{1}{2} \frac{\partial E}{\partial h_i} = \frac{1}{2}(1_{ij} - p_i)$$

The rese of the BP algorithm is the same as before.