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Lower bound on Comparison based algorithms

Definitions: i) order equivalence of ~~of~~ sequence of ids

(ii) Corresponding States. (with respect to id sequences)

(iii) k-neighborhoods of a process.

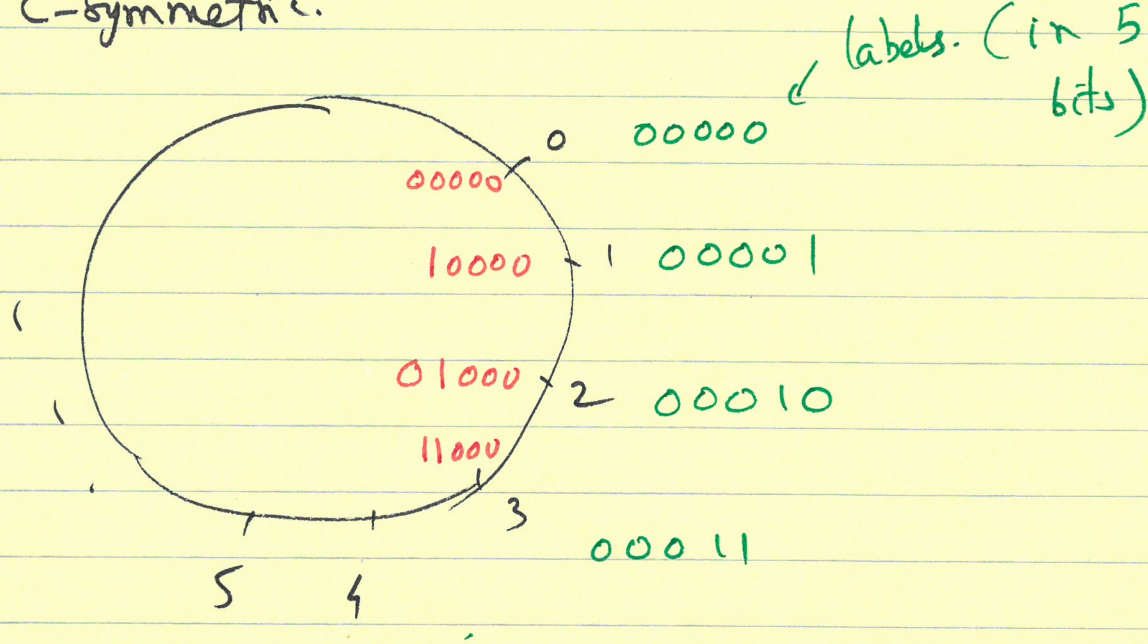
Lemma 3.5 A is a comparison based algorithm in a ring R of n processes. k is an int $0 \leq k < \lfloor \frac{n}{2} \rfloor$, i & j are processes and

their k-neighborhoods are order equivalent. After at most k rounds, i & j are in corresponding states.

~~Proof b~~ Proof by induction on k

Definition c is a constant; $0 \leq c \leq 1$. R is a ring of size n. R is said to be c-symmetric if for all l, ~~...~~ $\sqrt{n} \leq l \leq n$, and for all segments S of length l, there are at least $\lfloor \frac{cn}{l} \rfloor$ segments of R that ~~...~~ are order-equivalent.

for $c = 0.5$, every bit reversal ring is c-symmetric.

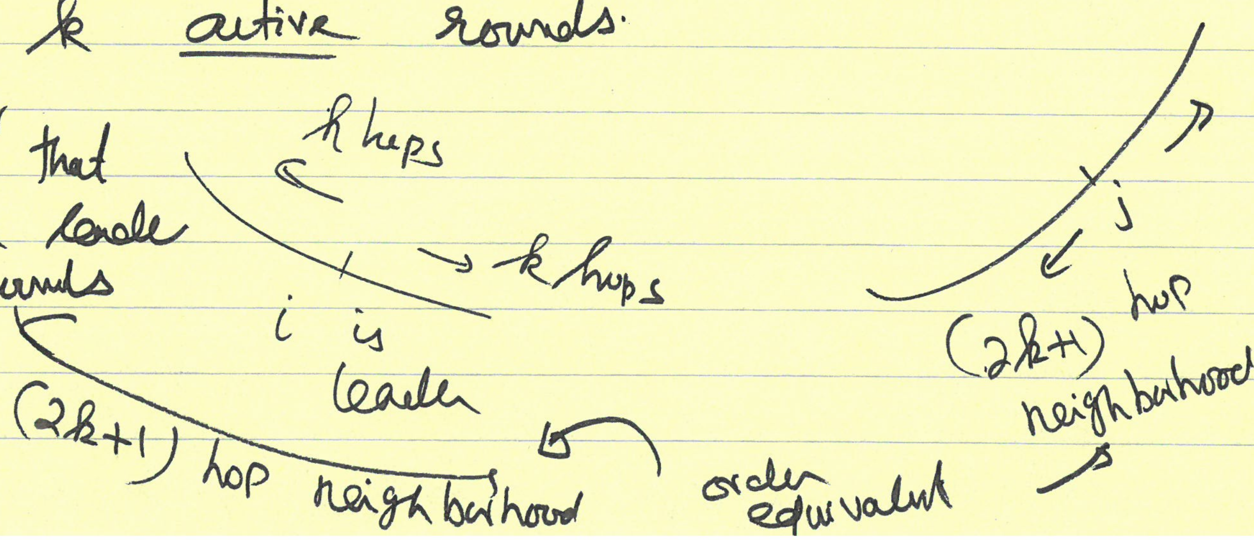


Red is id
Green is label } Red is bit reversal of Green.

Can be proved.

<u>Lemma 3.8</u> A is a comparison based algorithm in a c-symmetric ring. A elects a leader. Let k be an integer such that $\sqrt{n} \le 2k+1$ and $\left\lfloor \frac{cn}{2k+1} \right\rfloor \geq 2$. Then A has more than k <u>active</u> rounds.

Assume for contradiction that A elects a leader in k rounds



Theorem 3.9 A . . . . . . A needs $\Omega(n \log n)$ messages.

Proof Let c be a constant ($0 \le c \le 1$) & R be a c-symmetric ring.

Let $k = \left\lfloor \frac{cn-2}{4} \right\rfloor$. Then $\sqrt{n} \le 2k+1$ if n is sufficiently large. and

$$\frac{cn}{2k+1} \geq 2$$

By Lemma 3-8, A needs more than k active rounds (at least $k+1$ rounds)

Consider round r; $\sqrt{n}+1 \le r \le k+1$
This round r is active.
Some process i sends a message in round r
$S = (r-1)$ neighborhood of process i
Since ring is c-symmetric, there are $\left\lfloor \frac{cn}{2r-1} \right\rfloor$ order equivalent segments in R (these are all order equivalent to S).
The mid points of these send messages in round r.
Let $r_1 = \lceil \sqrt{n} \rceil + 1$; $r_2 = k+1 = \left\lfloor \frac{cn-2}{4} \right\rfloor + 1$

# of messages sent

$$\geq \sum_{r=r_1}^{r=r_2} \left\lfloor \frac{cn}{2r-1} \right\rfloor$$

$$\geq \sum_{r=r_1}^{r_2} \frac{cn}{2r-1} - r_2 \geq n \sum_{r=r_1}^{r=r_2} \frac{c}{2r-1}$$

$$= \Omega\, n \sum_{r=r_1}^{r=r_2} \frac{1}{r}$$

$$= \Omega(n \log n)$$

HS algorithm is asymptotically message optimal.

If no problem with time complexity, $O(n)$ message algorithm is possible (variable speeds)

If you want to be frugal in time, $\Omega(n \log n)$ messages are needed

Ch 4

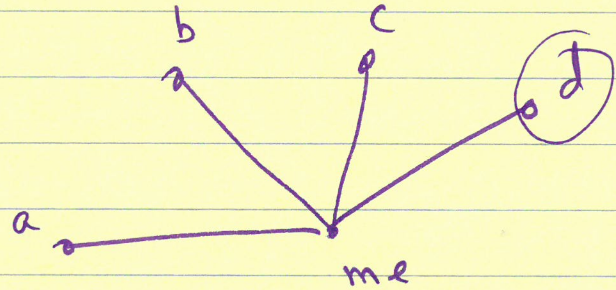
Synchronous ~~Symmetric~~ General networks

$G = (V, E)$ is the communication network

Leader election

Each process has a unique id.

Process with max id to be leader.



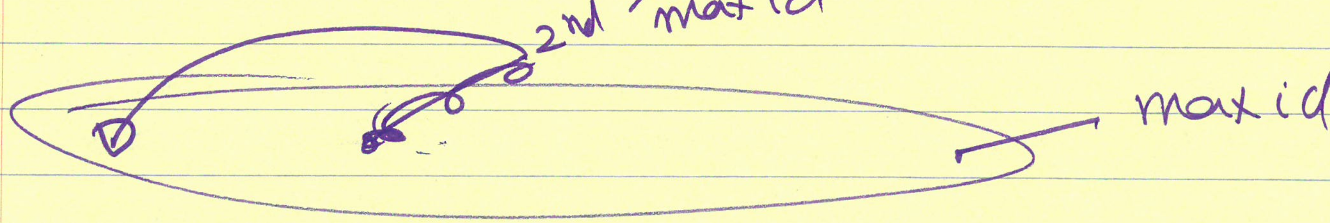
largest among $\{me, a, b, c, d\}$

FloodMax algorithm

forward largest id known to you until now to all

do this for diam rounds, then terminate

if upper bound on diam is known, we can terminate.



$a \longrightarrow b$

diam known.

$O(|E| \cdot \text{diam})$

# of messages $O(\text{diam}\ |E|)$ whether diam is known or unknown