MIS 8

+ set of nodes is independent if no two nodes are adjacent.
(one-hop away)

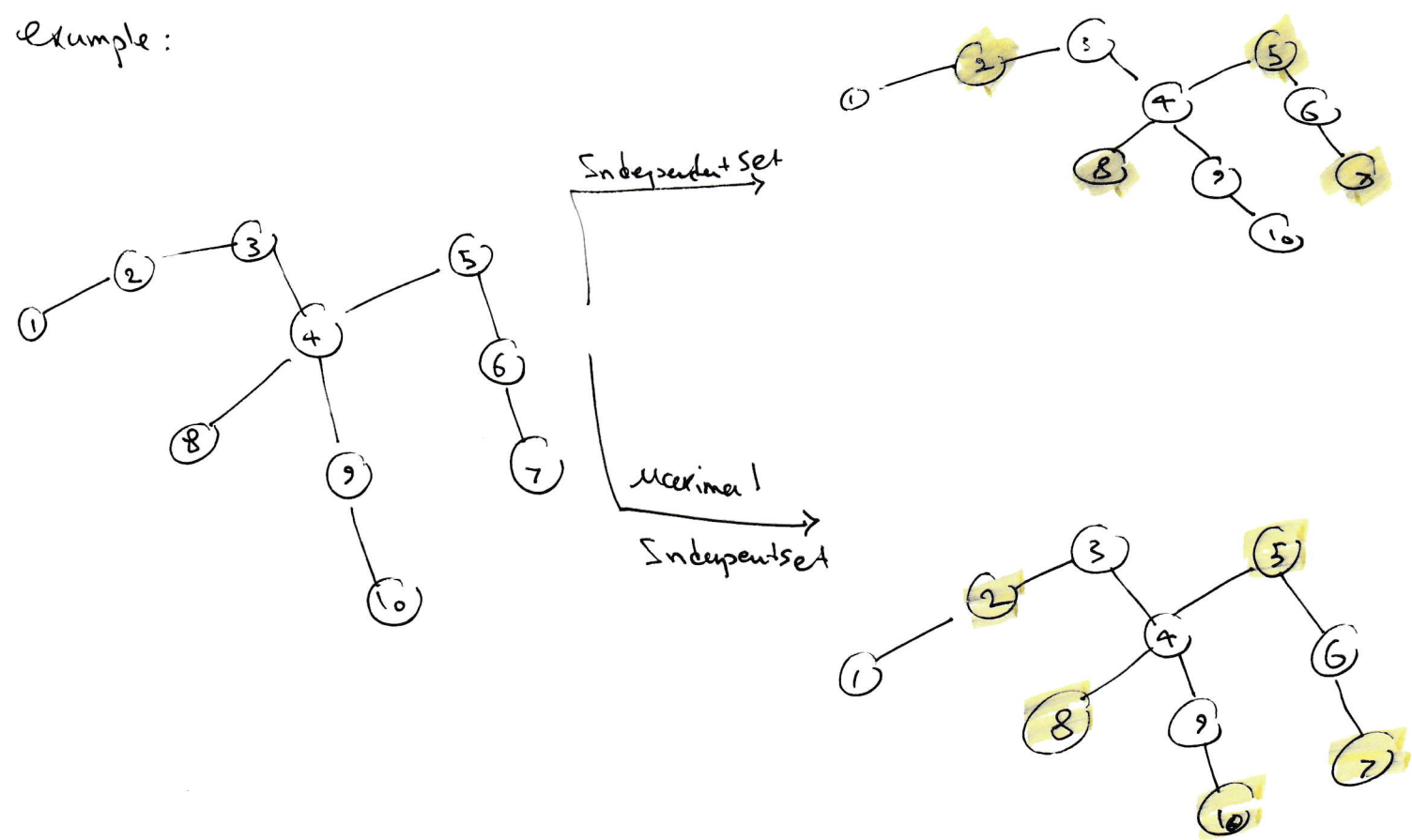
+ Independent set (IS) is maximal if it cannot be increased to form a larger set.

+ An independent set of maximum cardinality is called maximum.

+ We are interested in maximal not maximum.

+ Motivation: Broadcast — difficult problem.

Example:



↳ Sequential Algorithm

$G = (V, E)$

$I \leftarrow \phi$; Nodes $\leftarrow V$; Edges $\leftarrow E$;

while Nodes $\neq \phi$

{ choose a non-empty set $I' \subseteq$ Nodes

I' is an Independent Set;

$I \leftarrow I \cup I'$;

Nodes $\leftarrow$ Nodes $-$ $\{I'\}$ $-$ {neighbors of I'}

Edges $\leftarrow$ Edges $-$ {edges incident of I'} $-$

{edges incident of neighbors of I'}

↳ How to compute in distributed way?

↳ How to choose I'?

↳ Randomization is used.

each process i chooses val_i randomly in the range $\{1, \ldots, n^4\}$

sufficiently large to reduce possibility where 2 process choose same value.

$j \in I'$ if val_j is max among all its neighbors.

we define I' to consist of all nodes j that are local winners.

↳ It means, those nodes j s.t $val_j > val_i$ for all neighbor

I' might be empty set at some stage → (wasted stage)

The algorithm works in Stages, each consisting of 3 rounds.

one stage

Round 1: choose Vales locally and send to neighbors.

Round 2: winners notify neighbors.

By the end of this round, loser are known.

I' is known.

loser: the nodes that have neighbors in I'.

Round 3: each loser notifies it's neighbors.

winner & loser remove themselves.

↳ it means that, loser and winners discontinue participate after this stage.

loser's neighbors remove all edges.

↳ How many stages?

Depend on $|I'|$.

In the randomized algorithm, Result is correct ; # of Stage may vary.

Lemma 4.7: Expected # of edges removed from G in Single Stage is at least $\frac{|E|}{8}$.

↳ the algorithm ensures that every edge with at least one-endpoint in I is removed.

Lemma 4.8: with prob at least $\frac{1}{16}$, # number of edges removed from G in Single Stage is at least $\frac{|E|}{16}$

Th 4.9 # of Stages is $O(\log n)$ with high probability.