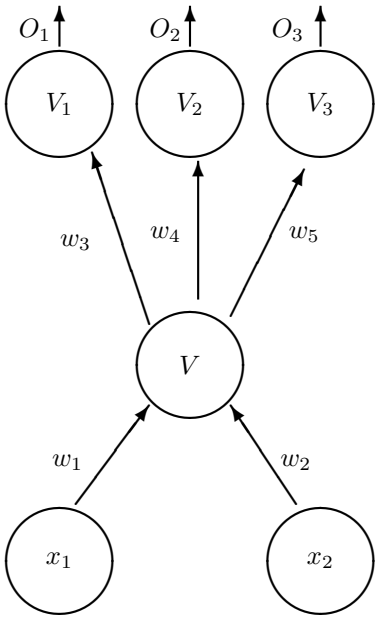
Back-Propagation-Example-Solutions

Question 1



The above neural network has two layers (one hidden layer), two inputs, and three outputs. (There are NO bias connections.) All nodes compute the sigmoid function with $\beta = 1$.

A.1 Give explicit expressions to the values of all nodes in forward propagation when the network is given the input $x_1 = 3, x_2 = 9$, with the desired output $y_1 = 1$, $y_2 = 0$,$y_3 = 1$. Your answer should be in terms of the old weights w_1, w_2, w_3, w_4, w_5. You may use the notation $S(.)$ instead of explicitly computing sigmoid values.

Answer

$$
\begin{aligned}
V &= S(3w_1 + 9w_2) \\
V_1 &= S(w_3 V) \\
V_2 &= S(w_4 V) \\
V_3 &= S(w_5 V) \\
O_1 &= V_1 \\
O_2 &= V_2 \\
O_3 &= V_3
\end{aligned}
$$

A.2 Give explicit expressions to how the weights change by back propagation when the network is given the same example as above. Use $\epsilon = 0.1$.

Your answer should be in terms of the old weights w_1, w_2, w_3, w_4, w_5 and the node values V, V_1, V_2, V_3, O_1, O_2, O_3, that were computed in A.1. You may use the notation $S(.)$ instead of explicitly computing sigmoid values.

You may use temporary variables in your answer, but make sure that they are defined in terms of the above variables.

Answer I am using the following temporary variables in my answer:

$$\begin{aligned}
\delta_1 &= 2\beta O_1(1-O_1)(y_1-O_1) = 2O_1(1-O_1)^2 \\
\delta_2 &= 2\beta O_2(1-O_2)(y_2-O_2) = -2O_2^2(1-O_2) \\
\delta_3 &= 2\beta O_3(1-O_3)(y_3-O_3) = 2O_3(1-O_3)^2 \\
\delta &= 2\beta V(1-V)(w_3\delta_1 + w_4\delta_2 + w_5\delta_3) = 2V(1-V)(w_3\delta_1 + w_4\delta_2 + w_5\delta_3)
\end{aligned}$$

Answer

$$\begin{aligned}
\text{new } w_1 &= w_1 + 0.3\delta \\
\text{new } w_2 &= w_2 + 0.9\delta \\
\text{new } w_3 &= w_3 + 0.1\delta_1 V \\
\text{new } w_4 &= w_4 + 0.1\delta_2 V \\
\text{new } w_5 &= w_5 + 0.1\delta_3 V
\end{aligned}$$