Basic Reliability Configurations

Assumptions on the reliability model:

- Each component has two possible states: operational or failed.
- The failure of each component is an independent event.
- Component i is functioning (operational) with probability p_i and is inoperational (failed) with probability $1 - p_i$. (These probabilities are usually known.)
- The reliability R of the system is some fuction of the component reliabilities:

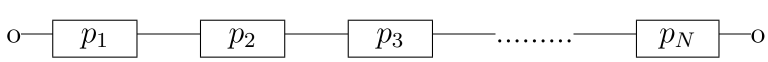
  $$R = f(p_1, p_2, \ldots, p_N)$$

  where N is the number of components.

The function $f(...)$ above depends on the *configuration*, which defines when the system is considered operational, given the states of the components. Basic examples are shown in the configurations discussed below.

Series Configuration

In the series configuration the system is operational if and only if *all* components are functioning. This can be schematically represented by the figure below, in which the system is considered operational if there is an operational path between the two endpoints, that is, all components are functioning:



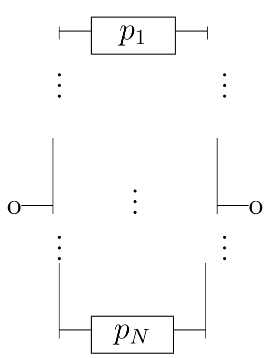
The reliability of the series configuration is computed simply as the product of the component reliabilities:

$$R_{series} = p_1 p_2 \dots p_N$$

Note: If many components are connected in series, then the reliability may be much lower than the individual reliabilities. For example, if $p = 0.98$ and $N = 10$, then $R_{series} = (0.98)^{10} = 0.82$, significantly lower than the individual reliabilities.

Parallel Configuration

The parallel configuration is defined operational if *at least* one of the components are functioning. This is schematically represented in the figure below:



The reliability can be computed as follows. The probability that component i fails is $1 - p_i$. The probability that *all* components fail is $(1 - p_1)(1 - p_2)\dots(1 - p_N)$. The complement of this is that *not* all component fails, that is, at least one of them works:

$$R_{parallel} = 1 - (1 - p_1)(1 - p_2)\dots(1 - p_N) = 1 - \prod_{i=1}^{N}(1 - p_i)$$

k out of N Configuration

In this configuration the system is considered functional if at least k components out of the total of N are functioning.

The probability that a *given* set of k components are functioning is

$$p^k(1-p)^{N-k}.$$

The probability that a *any* set of k components are functioning is

$$\binom{N}{k} p^k(1-p)^{N-k}$$

where

$$\binom{N}{k} = \frac{N!}{k!(N-k)!}$$

represents the number of ways one can choose a k-element set out of N.

Finally, since we need at least k operational componenets, we have to sum up the above for all possible acceptable values of k. This gives the reliability of the k out of N configuration:

$$R_{k/N} = \sum_{i=k}^{N} \binom{N}{i} p^i(1-p)^{N-i}$$