9-12-2019

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time: n-1 rounds
messages: $(n-1)\ |E|$

Minimum Spanning Tree

$G = (V, E)$ is a weighted undirected graph.

$e \in E$ has an edge weight.

Start with trivial spanning forest:
each node is a tree by itself

merge these trees till we have one tree.

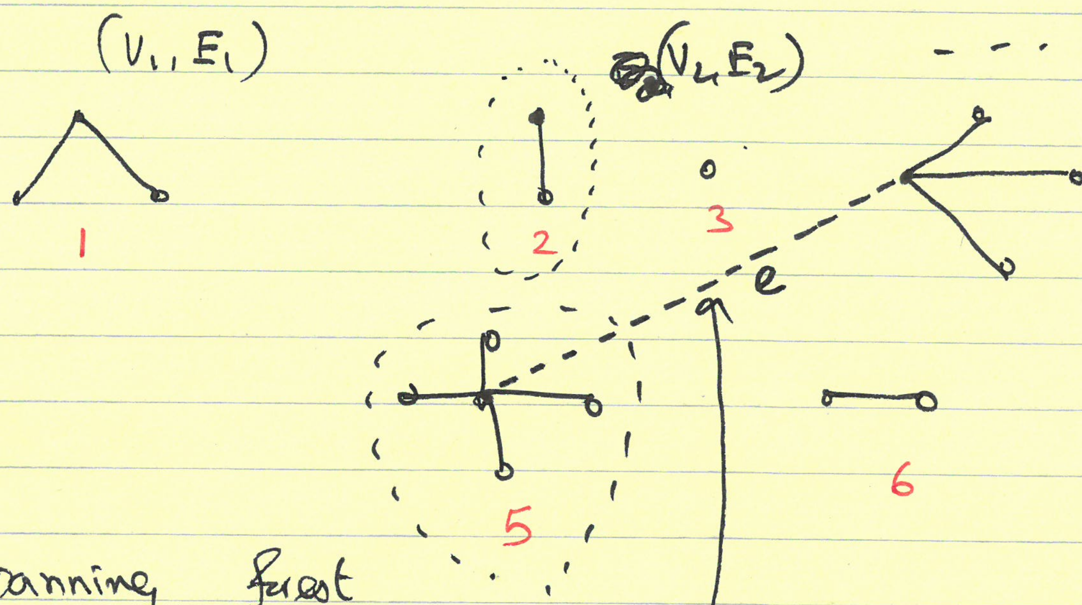
T T' are two trees
How to merge these 2. ?

Find an edge that connects T & T' and of smallest cost among all edges that connect T & T'

Lemma: $G = (V, E)$ is a weighted undirected graph.
$\{(V_i, E_i\}: 1 \le i \le k\}$ is a spanning forest of G and $k > 1$.
e = an edge of smallest cost (weight) such that e has one end point in V_i for some V_i

(2)

There exists a spanning tree that includes ($\bigcup E_j$ and e) and this spanning tree is of smallest cost among all ~~span~~ spanning trees ~~to~~ of G that include $\bigcup_j E_j$



Spanning forest
$i = \cancel{2}\,5$

Smallest cost among all edges where one end point is in $\{V_5, E_5\}$ and other end point is in a different subtree.

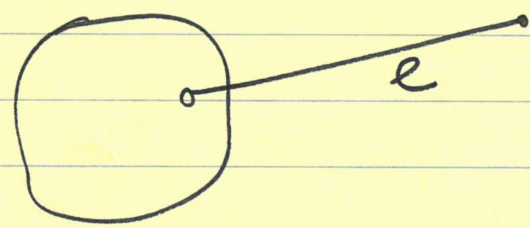
$E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5 \cup E_6 \cup \{e\}$ will be a subset of MST - that includes all edges of E_1, E_2, E_3, E_4, E_5 & E_6

Proof by Contradiction.

Assume $T = E_1 \cup E_2 \ldots E_k \cup \ldots$ and this is a spanning tree of smallest cost among all spanning trees that include $E_1 \cup \ldots E_k$ and T does not include e. (for contradiction)

one end of $e \in (V_i, E_i)$

add e to T. we have a cycle.

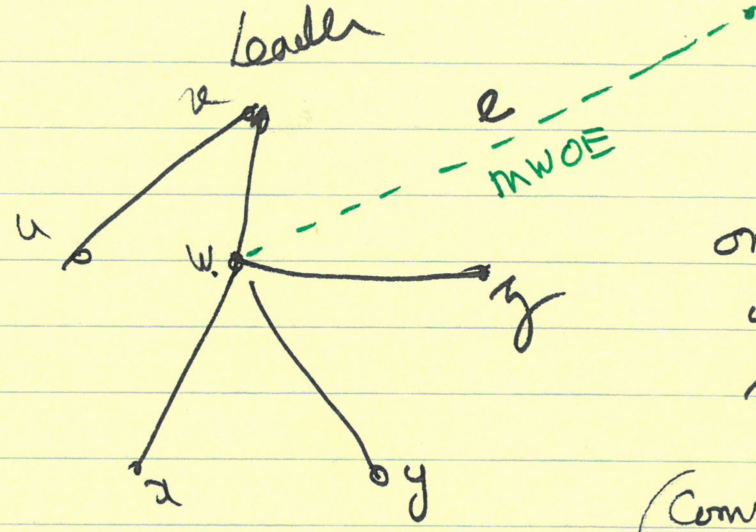


i^{th} subtree (V_i, E_i)

$\exists$ a cycle. ~~in tree~~

$\exists$ an edge e' such that one end of e' is in V_i ~~and ...~~ (other end is outside V_i) and e' has higher cost than e.

$T \cup \{e\} - \{e'\}$ has smaller cost than T, a contradiction. ■

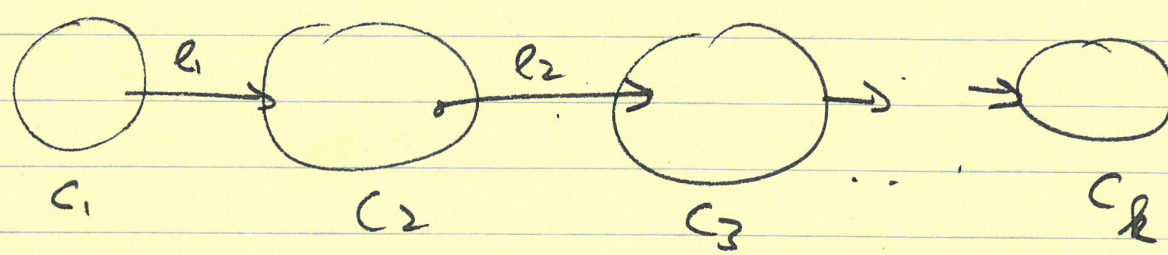


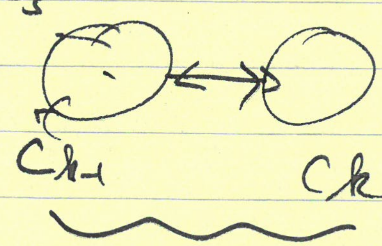
one subtree of the spanning forest (Component)

Find MWOE (min weight outgoing edge) for this Component

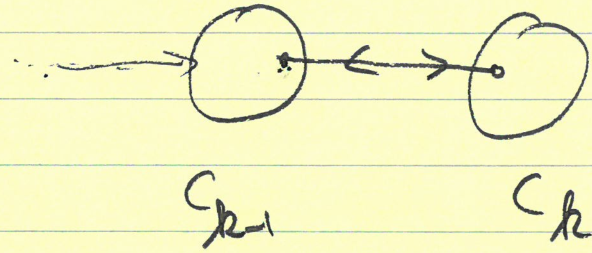
e = MWOE of this Component

1. Finding MWOE of a Component.
Leader broadcasts a message containing Component id.
Each process finds a local candidate for MWOE



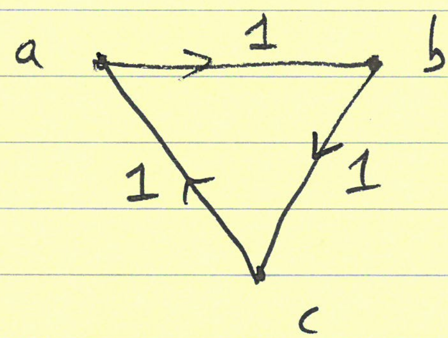


Share common MWOE



Common MWOE
or
Core edge

one of the two end points of the core edge becomes the leader. of the combined component.

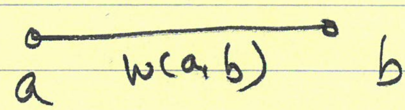


Cycle creation is possible.

Lemma. If all edge weights are distinct then there is a unique minimum sp-tree.

To be proved next week

Making edge weights distinct:



min id

max id

weight of this edge = (w(a,b), a, b)