# Homework 1

**1.**

i) Let $f(x, y) = x^2 + \ln(y) + xy + y^3$, where $y > 0$.

$$\frac{\partial f}{\partial x} = 2x + y$$
$$\frac{\partial f}{\partial y} = \frac{1}{y} + x + 3y^2$$

At $x = -10, y = -10$:

$$2x + y = 2(-10) + (-10) = 20 - 10 = 10$$
$$\frac{1}{y} + x + 3y^2 = \frac{1}{-10} + 10 + 3(-10)^2 = 309.9$$

Gradient $= (10, 309.9)$.

ii) Let $f(x, y, z) = \tanh\left(x^3 y^3\right) + \sin(z^2)$.

$$\frac{\partial f}{\partial x} = 3x^2 y^3 \text{sech}^2(x^3 y^3)$$
$$\frac{\partial f}{\partial y} = 3x^3 y^2 \text{sech}^2(x^3 y^3)$$
$$\frac{\partial f}{\partial z} = 2z \cos(z^2)$$

At $(-1, 0, \frac{\pi}{2})$:

$$\left(0, 0, 2\frac{\pi}{2} \cos\frac{\pi^2}{4}\right) = (0, 0, 3.1386)$$

**2.**

i) Given the matrix:

$$\begin{bmatrix} 10 \\ -5 \\ 2 \\ 8 \end{bmatrix} and \begin{bmatrix} 0 & 3 & 0 & 1 \end{bmatrix}$$

Multiplication:

$$\begin{bmatrix} 10*0 & 10*3 & 10*0 & 10*1 \\ -5*0 & -5*3 & -5*0 & -5*1 \\ 2*0 & 2*3 & 2*0 & 2*1 \\ 8*0 & 8*3 & 8*0 & 8*1 \end{bmatrix}$$

which is equal to:

$$\begin{bmatrix} 0 & 30 & 0 & 10 \\ 0 & -15 & 0 & -5 \\ 0 & 6 & 0 & 2 \\ 0 & 24 & 0 & 8 \end{bmatrix}$$

ii) For the matrix operation involving:

$$\begin{bmatrix} 1 & -1 & 6 & 7 \\ 9 & 0 & 8 & 1 \\ -8 & 1 & 2 & 3 \\ 10 & 4 & 0 & 1 \end{bmatrix} and \begin{bmatrix} 6 & 2 & 0 \\ 0 & -1 & 1 \\ -3 & 0 & 4 \\ 3 & 4 & 7 \end{bmatrix}$$

Multiplication:

$$\begin{bmatrix} 1*6+0+6*-3+7*3 & 1*2+-1*-1+0+7*4 & 0+-1*1+6*4+7*7 \\ 9*6++8*-3+1*3 & 9*2+0+8*-1+1*4 & 0+0+8*4+1*7 \\ -8*6+0+2*-3+3*3 & -8*2+1*-1+0+3*4 & 0+1*1+2*4+3*7 \\ 10*6+0+0+1*3 & 10*2+4*-1+0+1*4 & 0+4*1+0+7*1 \end{bmatrix}$$

which is equal to:

$$\begin{bmatrix} 6+0-18+21 & 2+1+0+28 & 0-1+24+49 \\ 54+0-24+3 & 18+0-8+4 & 0+032+7 \\ -48+0-6+9 & -16+2+0+12 & 0+1+8+21 \\ 60+0+0+3 & 20-4+0+4 & 0+4+0+7 \end{bmatrix}$$

i.e.

$$\begin{bmatrix} 9 & 31 & 72 \\ 33 & 22 & 39 \\ -45 & -5 & 30 \\ 63 & 20 & 11 \end{bmatrix}$$