DTU Wind and Energy Systems

Autonomous and sustainable optimisation of flexible energy assets for grid services

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DTU Wind & Energy Systems is a department of the Technical University of Denmark with a unique integration of research, education, innovation and public/private sector consulting in the field of wind energy. Our activities develop new opportunities and technology for the global and Danish exploitation of wind energy. Research focuses on key technical-scientific fields, which are central for the development, innovation and use of wind energy and provides the basis for advanced education at the education.

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Approval

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Abstract

With the significant rise of renewable energy installations and large-scale unscheduled consumer loads, the need for a higher degree of flexibility in grid-connected demand assets has become imperative for grid security. This, in turn, has increased the value derived from participation in the ancillary services market. A framework to evaluate and quantify the energy flexibility of a demand-side asset for participation in frequency regulation markets is presented and it is applied to an electric boiler. With the adoption of the pan-European PICASSO market, the secondary Automatic Frequency Restoration Reserve (aFRR) market is divided into the day-ahead capacity market and the real-time energy activation, creating new revenue opportunities for flexible and deferrable energy assets. However, as the energy activation of the committed power is completely dependent on grid imbalances, the energy balance of the asset could deviate from the predicted day-ahead trajectory, leading to unmet demand or unexploited profits. This thesis proposes a receding horizon algorithm as a secondary control interface between the electric asset and the electricity market. The model is designed for a boiler in West Denmark DK1 to enable optimal trading in the aFRR real-time energy market via PICASSO. The bidding strategy is evaluated using aFRR market price data and the boiler's operational schedule over a two week period in March, 2025. The results of the case study demonstrates the viability of concurrent participation in ancillary services while fulfilling the boiler's regular operational requirements, and assess the impact of aFRR participation on achieving the objective of minimizing operational costs.

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Nomenclature

$\mathcal{T}$	Prediction Horizon
$\mathcal{K}$	Control Cycle of PICASSO Activation Optimisation Function
P_t^{ref}	Day-ahead Schedule of the Boiler (MW)
$p_t^{\text{capacity bid}}$	Day-ahead aFRR Capacity Bid (MW)
p_t^{demand}	Forecasted Heat Demand (MW)
λ_t^{spot}	Hourly spot prices (€/MWh)
$\lambda_{t,k}^{\uparrow}$	Up aFRR energy price (€/MWh)
$\lambda_{t,k}^{\downarrow}$	Down aFRR energy price (€/MWh)
η_{P2H}	Power to Heat conversion efficiency (%)
$p_t^{\uparrow}$	Upward Bid Power (MW)
$p_t^{\downarrow}$	Downward Bid Power (MW)
p_t^{boiler}	Boiler Power (MW)
$e_t^{\uparrow}$	Upward Energy activated (MWh)
$e_t^{\downarrow}$	Downward Energy activated (MWh)
SoE_t	State of Energy of the Boiler (%)
u_t	Binary variable to indicate Bid Direction
β_t	Binary variable to indicate Bid Status

1 Introduction

The rising share of renewable energy in the generation mix, along with the gradual shift of fossil-fuel-based loads to electric systems, has significantly increased the stress on the power grid stability. Projections for the coming decade indicate a growing need for greater amount of ancillary reserves to balance disturbances in the grid [1]. Higher uncertainty of solar and wind generation, paired with large-scale unscheduled consumer loads like electric vehicles, heat pumps, and other motor-based systems increases the occurrence of supply-demand imbalances leading to frequency events [2, 3]. On the other hand, these electric assets can also flexibly change their power profile to support the grid during imbalances.

While the majority of balancing reserves procured currently are generation units and energy storage systems, demand-side assets offer cost-effective frequency regulation services at no standby costs using existing infrastructure [4]. However, only a limited portion of these demand-side electric assets currently utilize the system's flexibility to participate in the ancillary service markets. To offer frequency regulation, the asset must quantify the available flexibility in terms of electrical power and energy and meet the basic requirements of sufficient demand capacity and fast ramping capabilities. For demand-side assets, the estimation of available reserves depends on the consumer needs. The asset must maintain normal operation, in many cases essential and emergency services and can only offer the surplus flexibility for frequency regulation. Additionally, large upfront investments for aggregation of small individual capacities and their synchronized response to market signals creates challenges for large-scale adoption [5, 6].

The economic impact of demand-side asset participation in frequency regulation has been investigated in [7, 8], highlighting the technical barriers of committing large energy volumes to participate in different energy markets. The most common approach to market participation is to run a day-ahead optimization to determine the capacity bids for the following day of delivery [9, 10, 11]. This static day-ahead planning is generally conservative to account for possible changes throughout the day and is limited by uncertainty in the load baseline. The main drawback is that the energy activation of the committed power is purely determined by grid imbalances and the consumer's demand may also shift from the day-ahead forecasts as the day progresses. This means the real-time system flexibility (energy balance) could deviate from the predicted day-ahead trajectory, resulting in unmet demand or unexploited profits.

With this motivation, the objective of this work is to create a secondary control interface between the demand asset and the market that tracks the system's flexibility in real-time and trades the surplus on the market. A framework is proposed to evaluate and quantify the available flexibility of any demand-side asset and it is applied to an electric boiler in the case study. A case study of an electric boiler participating in the Automatic Frequency Restoration Reserve (aFRR) Market via PICASSO platform in West Denmark DK1 is studied. To address the limitation of day-ahead scheduling, a receding horizon optimisation algorithm is created that determines the optimal control inputs to the asset and the bid decisions on the aFRR energy market. Finally, the model is simulated using perfect prices and estimated price forecasts to determine the impact of market participation on the operational costs of the asset.

2 Operation of Power systems

The primary objective of power system operation is to maintain a reliable, secure, and efficient electricity supply that continuously meets real-time demand. This section provides an overview of the key operational services that support system reliability and security, with a focus on the current power market structure in the West Denmark synchronous area (DK1).

2.1 Ancillary Services

The stable and safe operation of power system is primarily dependent on the electric assets connected to the grid. These assets include various types of generation units like synchronous generators (SGs), renewable generation (wind and solar), energy storage plants (batteries, hydrogen systems), large industrial demands and distributed household demands. All assets connected to the grid maintain equilibrium of electricity demand and power supply reflected in key system parameters such as voltage and frequency. The power system operation planning, therefore, must account for both anticipated and unanticipated changes in grid connected asset composition to ensure uninterrupted power supply [12].

Expected changes in grid-connected assets are managed by the Transmission System Operators (TSOs) and Distribution System Operators (DSOs) through various energy market mechanisms. The operation of various generation units is scheduled in advance using Unit Commitment (UC), based on forecasts of future electricity demand [13]. However, these demand forecasts of the upcoming period have large uncertainties, especially as most demand assets, unlike generation units, are highly distributed and influenced by public usage patterns. In addition to forecast uncertainty, the power system may also encounter unexpected operational issues such as generator outages, transmission line failures, or sudden, large demand fluctuations caused by internal or external factors [14]. The SOs take into account such possible scenarios to design the power system to handle these events without interruption.

Ancillary Services are operations designed to support the power grid by ensuring system stability and flexibility. These are mainly electricity reserves of different capacities, response speeds and at crucial locations of the power grid. The TSOs are responsible for procuring the electricity reserves which serve as backup used to address imbalances between consumption and generation and ensure power system recovery to a stable state following any disturbance[15]. The primary parameters that indicate these disturbances are voltage and frequency. The frequency is same throughout the entire grid while the voltage is fixed to certain sections within the grid. This allows ancillary services to deliver different services at different levels of the grid [16].

The main ancillary services are Frequency regulation, Voltage control and Emergency services [17]. Frequency regulation is responsible for maintaining the balance of active power injected and consumed from the grid. This balance keeps the frequency constant at reference value throughout the system. Any positive deviation from the base frequency means there is more active power injected into the grid than consumed and vice-versa for negative deviation. Voltage Regulation is responsible for maintaining the reactive power in the grid which is injected or consumed by synchronous sources, capacitive and inductive grid connected elements. Emergency services include black-start capability in the event of system-wide blackouts and other control actions as spinning reserves, load scheduling and dispatch. In this project work, the study is focused on the frequency regulation.

With the significant rise of renewable energy penetration in the power system, the composition of traditional synchronous generator (SG) based units are largely replaced by inverter based renewable generation [18]. Historically, power systems comprised of large and centralized generation units, such as fossil fuel, nuclear and hydro power plants. These units are highly reliable in terms of scheduling, and easier to control the unidirectional power flow from the transmission network to distribution level [19]. Moreover, SGs are synchronously coupled with the grid's frequency. These have large rotational inertia that is necessary to damp system oscillations as a result of disturbances. As more of these conventional generators are replaced by solar and wind generators, which do not have any rotating mass and are coupled to the grid through inverters, the overall system inertia is reduced, thus shortening the response times to disturbances [20]. The complexity of control of grid connected assets has also increased as more centralised large units are replaced by small distributed renewable sources. In addition, the uncertainty of renewable production also raises the chances of unexpected disturbances like shortages in production.

On the demand side, with growing focus on sustainability and reducing fossil fuel use, large energy consumers are increasingly shifting toward electricity. Gas-powered boilers are being replaced by electric boilers and heat pumps, combustion-engine vehicles are being replaced by electric vehicles, and the addition of large battery storage systems etc. are significantly altering traditional consumption patterns. These distributed loads when operated in unison can cause significant stress on the grid stability and control. These makes the need for robust ancillary services more pronounced and complex. Close coordination between asset operators and TSO/DSOs is necessary to unlock higher system flexibility, improve system security and pave way for larger integration of renewable energy [21].

2.2 Electricity Markets

Electricity markets play a critical role in ensuring reliable and efficient operation of the power system. By facilitating the exchange of electrical energy between suppliers and consumers, these markets help maintain the real-time balance between supply and demand essential for system stability and security. Through competitive mechanisms, electricity markets promote cost-reflective pricing, encourage efficient resource allocation, and maximises social welfare [22]. Because large-scale storage of electric power is currently uneconomical, electricity trading is structured to align production closely with real-time demand. As a result, electricity markets are organized based on the timing of physical power delivery, with different market segments corresponding to how far in advance the electricity is traded[23]. The different types of electricity markets vary largely depending on the system preferences and needs of the local jurisdictions. Figure 1 shows the timelines of different electricity market w.r.t the time of delivery T and actors responsible for each segment. Market operators such as Nordpool manages the electricity market trading on their platforms up to the time of delivery while the System operators as Energinet, who is responsible for technical operation of the power system, oversee the real-time balancing market. The main market segments are discussed below.

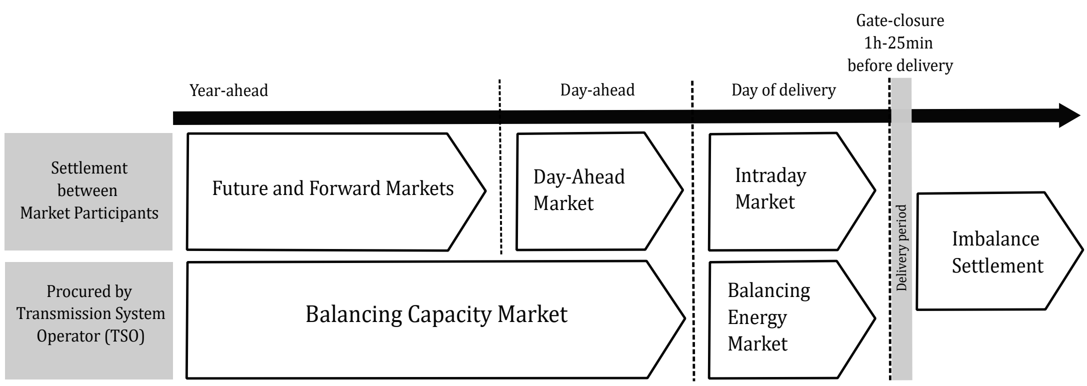


Figure 1: Structure and timeline of electricity markets.

Future and Forward Markets

The future and forward energy markets allows the trading of energy in longer time horizon, ranging from weeks, months and even years in advance. These markets provide price certainty and help hedge against future price volatility and market risks. They are particularly important for long term planning and investment decisions for both producers and large consumers. Forward contracts are traded over-the-counter between two parties with customized clauses and terms of agreement while future contracts are standardized agreements traded on the energy exchanges with predefined terms and conditions.

Day-Ahead Markets

The Day-Ahead (DA) market operates one day before the actual delivery of electricity, allowing market participants to buy or sell energy for each hour of the upcoming 24-hour period. The market is cleared through a closed auction taking into account both merit order and network constraints. This process determines the hourly electricity prices for the next day, commonly referred to as spot market price.

Intraday Markets

The intraday market is designed to help maintain the balance between supply and demand closer to the actual time of delivery. It allows market participants to trade surpluses or deficits in their power profiles caused by uncertainties in renewable generation or changes in demand that occur after their Day-Ahead (DA) commitments. The Intraday market operates continuously, allowing real-time adjustments, and typically remains open until one hour before the time of delivery. As of March 2025, the Nordic market has adopted a 15-minute trading granularity to better reflect short-term fluctuations and improve system responsiveness.

Ancillary Markets

The system operators, generally TSOs are responsible for the procurement and deployment of ancillary reserves which include both balancing energy and balancing capacity. These reserves can come from various sources, including spinning and non spinning reserves, demand response providers, and other entities capable of quick response to real-time grid imbalances. This market facilitates trading of both balancing capacity, which refers to reserved resources ready to be activated if needed, and balancing energy, which is the actual energy used to correct deviations between supply and demand [24].

Imbalance Settlement

The imbalance settlement process is carried after the time of delivery is completed and applied to the difference between scheduled and actual metered production and consumption. The direction of imbalance energy determines whether the balance responsible party (BRP) is penalized or incentivized.

2.3 Frequency Regulation Services

Frequency regulation ancillary services maintain active power balance in the grid by procuring capacity and energy reserves to be deployed in the event of an imbalance. To keep the system frequency at its nominal value of 50 Hz in Europe, the power injected by generation units must match the power consumed by demand assets. Any mismatch leads to frequency deviations: positive deviation indicating surplus power, while negative deviation indicates deficit of active power. In the event of a system outage, generation loss or large unexpected demand variation, the energy reserves are deployed in order of response speed and activation time [25]. The response speed is determined by how quickly an energy reserve can react to imbalance in the system and the activation time is determined by how long the production or consumption unit must stay active and support the grid. Therefore, the procurement of frequency reserves is divided into three categories: primary, secondary and tertiary reserves.

The primary reserves, known as Frequency Containment Reserves (FCR), are the first to respond to any imbalances in the grid by activating the reserves without any external signal from TSO. These energy assets have the fastest response time and are activated for short periods to halt frequency deviations from the reference value and find a new stable operating point following a disturbance. Once the frequency deviation is contained, secondary reserves are deployed to restore the frequency back to its nominal value. These reserves respond slower than FCR, but they offer a large capacity and have longer activation time. The secondary reserves are remotely activated by the TSOs and hence called Automatic Frequency Restoration Reserve (aFRR). The tertiary Manual Frequency Restoration Reserves (mFRR) are responsible for restoring the frequency if system imbalance persists after aFRR is deployed, thereby reinstating the primary and secondary frequency control reserves. The market design for procuring these reserves in terms of response time and sizing specifications is determined by the TSOs based on the region of operation and system requirements. The regulations discussed below is primarily applicable to West Denmark DK1 and certain common pan-European market products, on which the case study in this thesis is based.

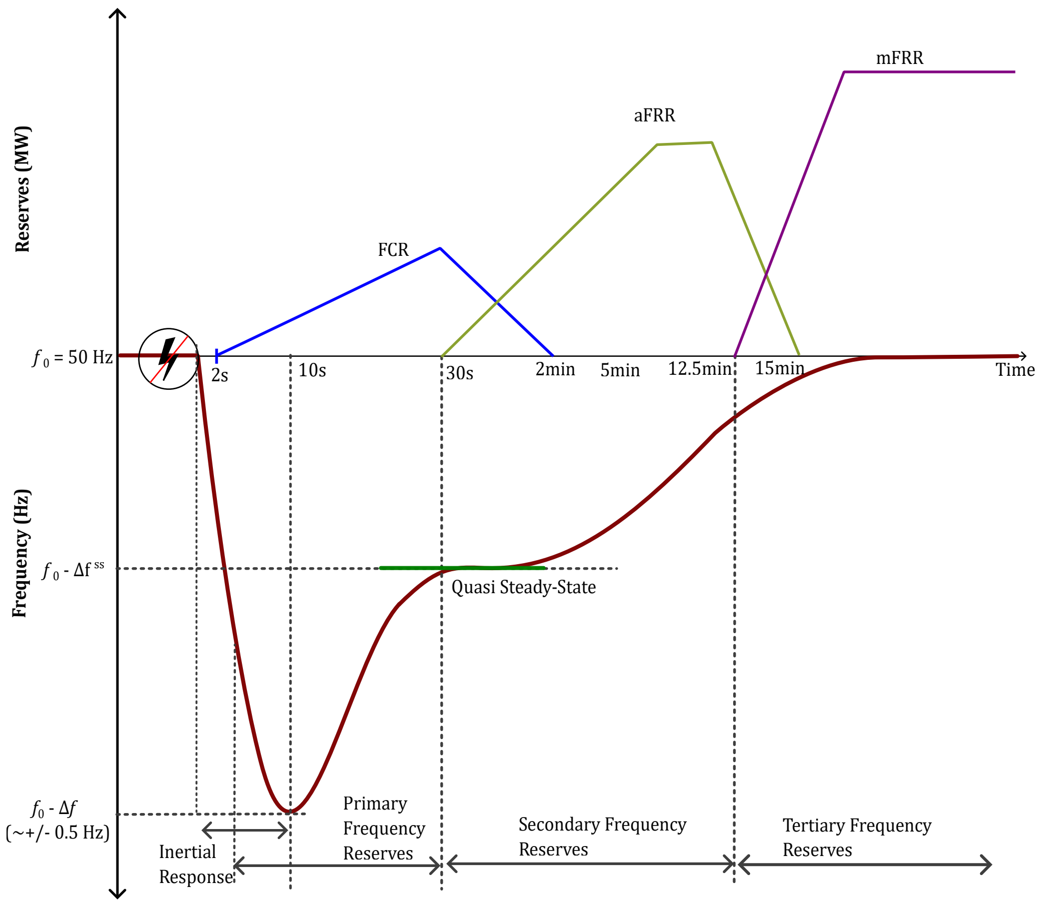


Figure 2: Frequency regulation services in West Denmark DK1 with response times.

Frequency Containment Reserve (FCR)

Frequency containment reserve stabilises the frequency close to the nominal value to reduce the number of frequency dips and jumps irrespective of the size of deviation [26]. The units providing FCR measure the frequency locally and continuously respond to grid frequency deviations by activating reserves, without the need for an external activation signal from the TSO. In DK1, FCR is a symmetrical product meaning assets must reserve power for both upward and downward regulation. The power supplied in response to frequency deviation is linear within a +/- 0.2 Hz band around the nominal 50 Hz frequency, with 50% of the total reserve capacity activated when frequency deviates by +/- 0.1 Hz. The full activation time (FAT) of FCR reserves is 30 seconds with more than 50% activation within first 15 seconds. The initial response after frequency deviates should be within 2 seconds. For limited energy reservoir (LER) assets, such as batteries which deplete in short periods, the regulation is stricter for qualification on the FCR market. The LER units must have a energy management system and can only participate with 75% of the pre-qualified volume in FCR.

Automatic Frequency Restoration Reserve (aFRR)

The procurement of aFRR in DK1 is done separately for day-ahead reserve capacity and real-time energy activation. From October 2024, both DK1 and DK2 joined the pan-European PICASSO platform for the aFRR energy market while capacity is procured in the local DK1 market and the Nordic market for DK2. PICASSO stands for Platform for the International Coordination of the Automatic frequency restoration process and Stable System Operation which integrates all the European aFRR energy markets via the TSOs. The role of the platform is to run an activation optimization function (AOF) to determine the optimal set of assets to be activated for aFRR energy regulation taking into account the common merit order and other constraints as network topology and grid congestion [27]. The aFRR capacity market is cleared in a day-ahead auction where the assets can reserve capacity in steps of 1 MW for both upward and downward regulation. This is a unsymmetrical product with a bid resolution of 15 minutes. The response time is 5 minutes to reach full activation, and it should be maintained till end of the bid validity if necessary. Within each 15 minute market time unit (MTU), the PICASSO algorithm clears the market every 4 second in a control cycle to select a new set of optimal bids and then delivers the results in real-time to TSOs. The TSO forwards the corresponding real-time aFRR activation signal to the balance responsible party (BRP) whose bids are accepted. The BRP responds to the TSO by activating the reserved energy of the asset. The current market structure mandates any power committed in the day-ahead capacity market must be met with the equivalent energy bid in PICASSO, however, assets can participate directly in the aFRR energy market via PICASSO without reserving any capacity in the day-ahead market. An overview of the aFRR market characteristics can be found in Table 1.

Table 1: Bid characteristics for the aFRR market.

Bid Currency	€
Maximum Bid Price	15,000 €/MWh
Minimum Bid Size	1 MW
Maximum Bid Size	9,999 MW
Bid Granularity	1 MW
Bid Time Resolution	15 min

Manual Frequency Restoration Reserves (mFRR)

The mFRR is the balancing reserves deployed manually after all other faster reserves are activated to restore system balance during extended period of frequency deviations. The mFRR procurement is done separately for capacity and energy market similar to aFRR, and market participants are remunerated for capacity committed in the day-ahead and energy activated in real time. The market is asymmetric, allowing assets to commit either in up or down regulation in 15 minute bid resolution. The response time to reach full activation is 12.5 minutes [28]. Currently, both DK1 and DK2 has a common mFRR market, allowing the demand of one area to be fulfilled by activation in the other area, via the Great Belt connection between the two synchronous areas [1]. The mFRR demand in DK1 is 284 MW and 600 MW in DK2. The mFRR market is planned to join the pan-European Manually Activated Reserves Initiative (MARI) energy activation platform from 2026. The activation of mFRR via MARI is automatic and with a bid time resolution of 15 minutes.

2.4 PICASSO Platform

The work in this thesis focuses on the integration of a demand asset in the aFRR energy market via the PICASSO platform. This section details the complete operational overview and bidding process on the platform. The procurement of energy reserves is carried out individually by the TSOs in coordination with Market Operators across all of Continental Europe [29]. The body European Network of Transmission System Operators for Electricity (ENTSOE) is an association of TSOs which oversees and manages the Continental Europe synchronous area facilitating interconnection and collaboration between TSOs. In order to optimally use the interconnection between separate load frequency control areas (LFCs) to navigate imbalances, the International Grid Control Cooperation (IGCC) platform for imbalance netting is implemented from February 2016 [30]. Imbalance netting is the process where two or more LFCs operated by different TSOs avoid simultaneous activation of frequency reserves in opposite direction, aimed at harmonising the grid by offsetting cross-border imbalances [31]. This decreases the total volume of activated reserves, reducing the total cost of aFRR procurement to maximise social welfare at the pan-European level. Following the advantages of the IGCC platform and successful integration of day-ahead and intra-day market in Europe, the Electricity Balancing Guideline is established allowing member countries to share resources procured by the TSO to maintain generation-demand equilibrium [32]. Under this guideline, PICASSO is implemented as a centralised platform to merge aFRR markets of different regions and enable common activation of balancing reserves. PICASSO is based on a TSO-TSO model, meaning respective TSOs are responsible for dimensioning, prequalification and procurement of aFRR and is the single point of contact for BRPs. The platform is responsible to gather all the available cross-zonal capacities and carry out imbalance netting and cross-border activations via the TSOs. This allows activation of energy reserves in one LFC area during imbalances in another, if the affected area has more expensive or insufficient reserves. A detailed study of the economic impact of the PICASSO is carried out in [33] comparing to IGCC.

The platform employs an economic optimisation model to select the optimal balancing reserves to activate in order to maximize social welfare [27]. The model, called as Activation Optimisation Function (AOF) runs every 4-seconds in a multi-step process given in Figure 3. It begins with a Common Merit Order (CMO) optimisation problem, then proceeds to an Imbalance Netting (IN) optimisation, followed by a second CMO optimisation. The system constantly balances out demand through netting process in both CMO and IN steps, however, the actual selection of energy bids from market participants for activation happens only during the CMO optimisations. After these three operational steps are completed, a separate Price Optimisation problem is solved to calculate the marginal price for each LFC area, which acts as the price signal for market participants and for financial settlements between grid operators. The aFRR demand in any LFC area is divided into inelastic demand, which the model must to satisfy and elastic demand, which the model tries to satisfy if there are bids below a certain price threshold. This essentially splits the demand into high and low priority during imbalances. Both CMO and IN are multi-objective problems with predefined priorities assigned to each objective.

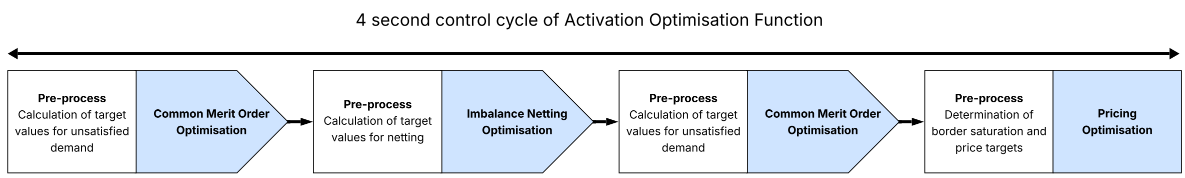


Figure 3: Sequence of optimisation processes in the PICASSO Activation Optimisation Function (AOF)

The objective of the CMO optimisation in order of priority is to satisfy the aFRR inelastic demand of individual LFC areas as much as possible, then minimizing activation volumes, maximising economic surplus taking into account elastic demand and minimizing cross border power flows. Before the CMO, a pre-process calculates target values of possible unsatisfied demand, ensuring that local areas have priority access to their own aFRR volumes. This means if a LFC area has enough of its own aFRR energy bids to cover its demand, the model assigns higher weight to prioritize these bids. Additionally, the model allows all possible balancing of opposite demands (netting), regardless of the bid prices. If one LFC area needs more power and another has a surplus, they will be offset against each other to the maximum extent possible before new bids are activated. Following netting, if demand is still unmet, the model distributes the unsatisfied demand fairly among the affected LFC areas in proportion to the initial needs. The final objective to minimize cross-border power flows is achieved by considering real-time grid congestions and flow restrictions as constraints in the model. The second step in the cycle, IN focuses on minimizing the deviations from the target value of netting. This netting target value is determined by a pre-process based on the overall aFRR demand across all participating LFC areas. This model also accounts for grid congestions, similar to CMO, but only for the LFC areas participating in IN. The two optimisations follow the sequence CMO-IN-CMO-PRICING in each 4-second control cycle of the AOF, linked such that the output of one step (remaining aFRR demand, available bids) feeds into the next step to form a continuous coordinated process. Finally, the Pricing Optimisation problem is solved to determine the Cross-Border Marginal Prices (CBMP) for each LFC area, which serve as price signals for market participants and are used for TSO-TSO settlement.

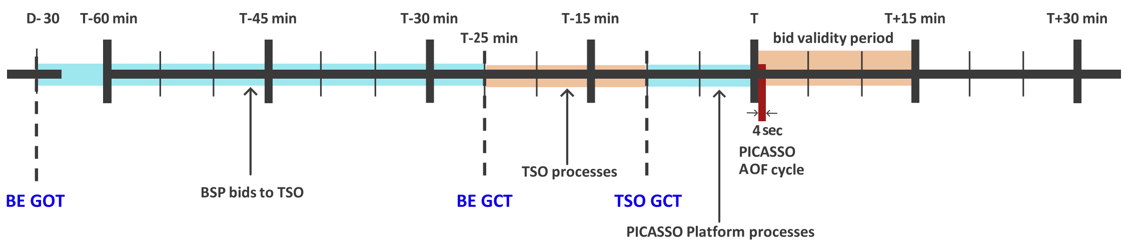


Figure 4: Bid Timeline of the aFRR energy market on PICASSO platform [34]

In order to submit energy bids to the platform, the BRP interacts only through the respective TSO of the LFC area [34]. The bid timeline on the aFRR market is shown in Figure 4. As discussed earlier, the aFRR energy market has a 15-minute bid resolution period for which the bid placement window opens 30 days before time of delivery T (D-30). The BRPs can send the energy bid for time T to the TSO once this gate opens until the balancing energy gate (BEGCT) closes 25 minutes before the delivery time T (T-25 min). Once the bid is submitted and the gate closes, the BRP is responsible for maintaining availability of the asset from T to T+15 minutes indicated as the bid validity period. The bids are submitted according to the ENTSOE-E CIM *ReserveBid_MarketDocument* to which TSO responds with a *Acknowledgement_MarketDocument* if the bid is placed successfully [34]. The TSO then submits the local merit order lists of aFRR energy bids to the PICASSO platform until the TSO energy bid submission gate closure time (TSOGCT) of 10 minutes. Once the bid validity period starts at T, the TSOs sends the aFRR demand to PICASSO AOF as a real-time-signal. The AOF, after each 4-second control cycle, determines the clearing prices and sends a real-time activation signal to the TSO which the TSO then forwards to the BRP. The BRP then activates the energy reserve and answers with a real-time current activation signal. The bid document contains the attributes as Bid Quantity, Bid Price, Bid Time Interval, Flow Direction (A01 for Upward and A02 for Downward regulation) and other regulatory parameters. Another mandatory attribute in the bid document is Geotags which refers to the 400kV/220kV/150kV(DK1)/132kV(DK2) substation used by the BRP. This parameter is used to handle the local grid congestions by the TSO.

3 Demand Response in Frequency regulation

As discussed in the previous section, the uncertainty from the changing composition of generation assets paired with the largely inelastic demand creates a precarious scenario where system operators are forced to procure more reserves. Instead of solely relying on the varying the supply to meet the demand, Demand response (DR) allows the consumers to change consumption in order to mitigate the disturbance in the grid [35]. Using demand response for system regulation is different in that the consumers are more distributed and hold larger autonomy as opposed to centralised generation units. As such, while in the event of large disturbance, the SO may implement load shedding to change demand, under regular operation, consumers are motivated to support the grid imbalances through market-based mechanism and remunerations.

Implementing demand response in frequency regulation is more cost effective than using energy storage and conventional generation as reserves because DR has no standby costs, lower carbon emissions and uses existing infrastructure [4]. However, control of demand assets to participate in ancillary services is more complicated than generation units as the determination of available reserve varies with consumer needs [36]. These assets must maintain their regular operation, in many cases emergency and essential services while they participate in demand response with the remaining flexibility. Flexibility, in context of demand assets, refer to the additional increase or decrease of consumption the asset can support in response to a disturbance without hampering the normal operation [37]. Therefore, in order to provide demand response, these assets need a more robust control mechanism to maintain stable operation. Another barrier in unlocking this flexibility is the relatively small individual capacities of the electric assets, such as electric vehicles, heat pumps etc. and the need for aggregation and synchronised response to grid imbalances in order to participate in the ancillary services market [5, 6].

In Demand Response, the asset offers to change active power consumed through upward and downward regulation. From a load's perspective, upward regulation is the ability to reduce the power consumption of the asset over a specific time period and downward regulation involves increasing the power consumption over the time period. Depending on the type of product, asymmetric markets allow assets to offer either upward or downward regulation services while symmetric markets such as FCR require asset to offer equal amount of both upward and downward capacity as reserve over the same time period.

3.1 Framework for DR in ancillary services

Based on the type of electric asset, the available flexibility depends largely on the normal operational limits and inherits the uncertainties of consumer's demand. In addition, the market regulators set certain limits such as response speed, continuous activation time that the asset must fulfil in order to qualify for the frequency regulation services. The regulating body such as TSOs define these technical requirements and perform tests to pre-qualify for the specific market. Here, a standard framework is proposed to evaluate and quantify the available flexibility of any demand-side asset. The framework does not cover the specific technical tests required by TSO for qualification, rather offers a general tool for pre-assessment of eligible markets. The framework is built upon the modular analysis presented in [7] to asses the barriers for entry of distributed energy resources (DERs) in ancillary markets as a guide to TSOs and market regulator for market redesign.

Capacity of the demand asset

The participation in all frequency market products requires the demand asset to satisfy the minimum bid size requirement which is the minimum amount of power the asset can offer to provide as reserve capacity usually between 0.1 MW or 1 MW. The BRP may be responsible for multiple assets as an load aggregator or a single asset. In both cases, the offered capacity as reserve must be equal to or higher than the minimum bid size for eligibility.

Operational baseline of the demand asset

The baseline of an electric asset is the operational reference it follows to satisfy the regular demand under standard conditions without any external control action. To qualify in the frequency markets, the TSO requires this baseline consumption characterization for the demand asset to prove that the available flexibility satisfies the minimum requirements to enter the market and guarantee certainty of availability at the time of delivery. As per Denmark's TSO Energinet's requirements, they allow only a small margin of error corresponding to 10th percentile of a probabilistic forecast of the baseline. The 10th percentile ensures that the predefined baseline is correct and the reserve capacity is available 90% of the time.

This baseline depends largely on the type of demand asset and whether it operates as a scheduled or unscheduled load. Generally, large-volume scheduled loads have a explicit fixed schedule for the upcoming period and procure energy in the day-ahead market which is the baseline. The schedule is decided based on demand forecasts and may be optimised over DA spot-prices to minimize cost of operation in the day of operation. For smaller unscheduled loads, the baseline corresponds to expected power consumption trajectory during the day of operation. This can be estimated using historical trends in power consumption and demand forecasts. Forecasting methodologies and data fitting models are implemented to predict the power consumption. Unlike day-ahead energy forecasts, which can be adjusted in the intraday market, reserve capacity forecasts must be highly accurate, as there is limited or no opportunity to correct them at time of delivery.

Flexibility estimation

Once the baseline is determined, the flexibility of a demand asset is the allowable deviation of the operating point from the baseline for a specific time period while respecting existing system constraints. The difference between the baseline and the actual power consumption is the capacity reserve the asset can offer on the market. Reserve activation will change the operating point of the system, hence, it is essential to ensure that this deviation is within a stable region of operation of the asset. This means the system physical parameters must be within limits and the normal operation of the unit must be satisfied. The type of frequency market product determines the bid time resolution, during which the asset may be required to maintain the new operating point for the entire bid validity period if the bid is accepted. Therefore, the smaller the bid resolution, the greater flexibility the asset can offer.

Identification of key limiting factors

Any deviation from the baseline of an asset is dependent on the stability of operation in the new operating point. The parameter representing the state of operation of the system must, therefore, be within the permissible operating limits. For a battery system, the internal resistance, thermal management and state of charge are parameters that define the system stability and for rotor system, parameters such as rotor speed, torque balance etc. determine system stability. This identification of the system stable region of operation is essential to ascertain the available flexibility that can be offered in the regulation markets.

Technical requirements of the market product

To find the suitable frequency market product for the demand asset, these following market requirements must be considered: response rate, bid validity period, proximity to real-time reservation, and product symmetry. The asset must be able to respond to the disturbances very quickly and satisfy all the pre-qualification tests such as sine response test, ramp test set by regulators. The required response speed varies by product and market region, as outlined in 2.3. The bid validity period when asset must maintain the committed reserve capacity, directly influences the amount of power it can offer to the market. Third, the time at which reserve bids are submitted relative to actual delivery impacts the decision-making process of the BRP, as the level of uncertainty differs between day-ahead capacity bids and those made closer to real-time energy delivery. When the reserves are submitted long before delivery, the BRP must account for larger uncertainty of demand forecast over long time horizons, and strategise on risk-based over or under-commitment. This also allows BRP to use real-time market products to course-correct this uncertainty from the day-ahead commitments to fulfil unmet demand or trade unutilized flexibility. Lastly, whether the market permits symmetrical or asymmetrical bids in the upward and downward directions affects the amount of reserves the BRP can provide. In symmetrical markets, the BRP may be constrained to offer reserves equal to the lesser of the available upward or downward reserves, potentially leaving flexibility in one direction unutilized. This consideration is particularly important for assets that typically operate at either maximum or minimum power levels, where available flexibility at any given time exists only in one direction, regardless of the total capacity.

3.2 Integration of Demand Assets to Ancillary Market

The integration of electric assets to the frequency regulation markets to provide demand response requires real-time online communication between them. The demand asset is responsible for tracking the state of the system to ensure the availability of committed power. This power reserve is submitted as a market bid to the respective market platform, some via the TSOs. Therefore, to operate a demand asset on various markets, a secondary control layer is needed to interface between the two components, as illustrated in Figure 5. As stated from the previous section, the volume of flexibility of an asset may vary in real-time according to the frequency market considered. Assets can exploit greater flexibility in faster frequency markets with small bid resolutions, as compared to longer aFRR and mFRR markets, given the asset qualifies for participation. Secondly, the revenue from market participation does not necessarily correlate with offered volume, higher revenue can be generated in one market despite offering less volume than another. This is mainly influenced by the market clearing prices and asset's internal operations which the controller optimizes to find the best market bid combinations.

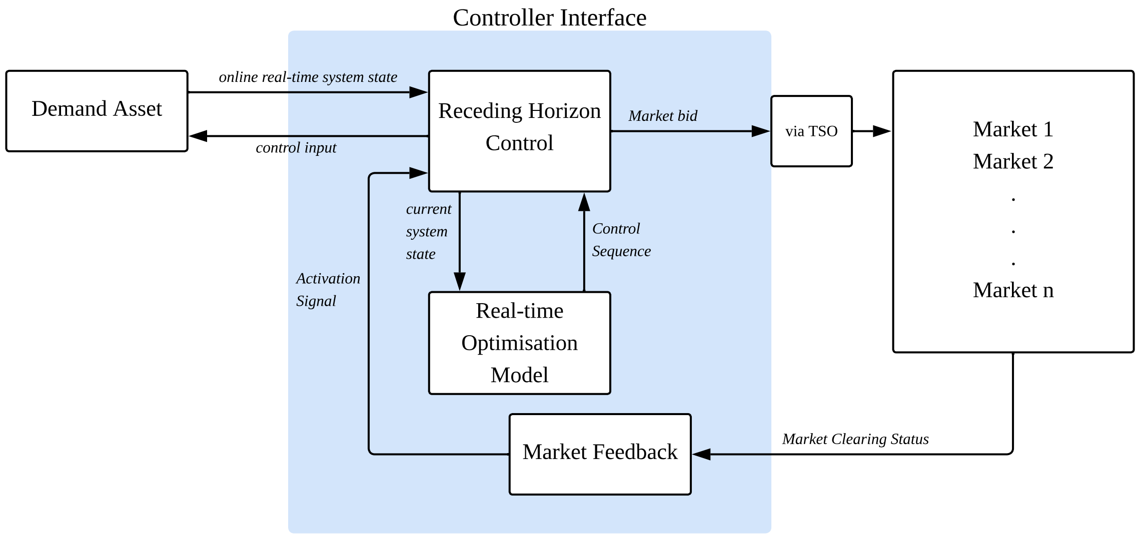


Figure 5: Block diagram of the Controller Interface between demand asset and ancillary market.

In this work, the study is simplified to interface a single demand asset to only one frequency market. The controller employs a receding horizon algorithm along with a real-time optimisation model. The demand asset shares its internal system state to the controller in real-time, with which the controller initialises the optimiser. The quantification of flexibility and bid decision making is done by the optimiser and returns the optimal control sequence to follow over a given prediction horizon. The controller then submits the market bid to the TSO before the bid gate closing time. In case of the aFRR market, the TSOs send the activation signal if the bids are accepted, while for some markets such as FCR, the asset is responsible for tracking the system frequency and activation in the event of a disturbance. At the start of the bid validity period, the controller sends the first step of the control sequence from optimiser to the demand asset and subsequently, forwards all activation signals received from the market. The market feedback process updates the bid clearing status and tracks the financial reconciliation over time.

4 System Description

4.1 Boiler Physical Description

The electric asset studied in this thesis is an electric boiler within a District Heating (DH) plant. DH plants typically consists of different technologies such as gas boiler, electric boiler and heat pumps. The selection of which asset to operate to meet the heat demand depends on minimizing operational costs, primarily influenced by gas and electricity prices. The heat demand of DH plants follow a consistent daily pattern with large seasonal variation between summer and winter months. Therefore, the operation of the DH plant is planned on the day-ahead with heat demand forecasts with relatively low uncertainty. This section details the working principle and physical operational set-up of an electric boiler.

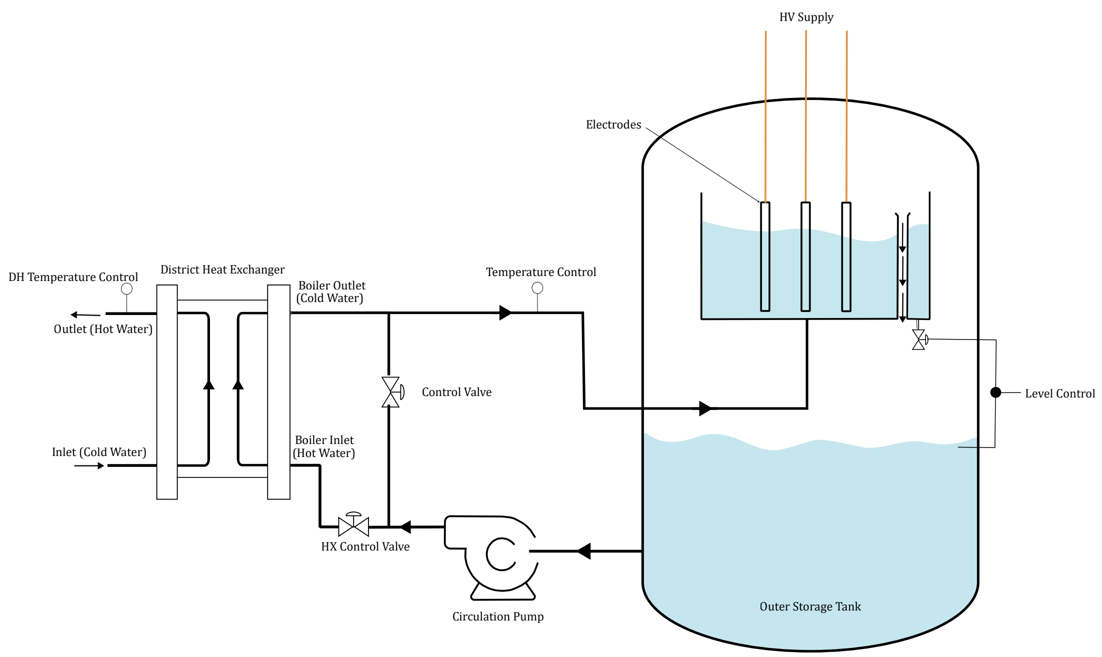


Figure 6: Physical set-up of an Electrode boiler hot water system to Heat Exchanger [38]

The type of electric boiler studied here is an electrode boiler where water is heated through electrical resistance Figure 6. The data used for the analysis in this work is from an electrode boiler of the manufacturer PARAT Halvorsen AS [38] and the operational description is summarised from [39]. The boiler consists of two concentric water tanks - the inner/upper tank and the outer/lower storage tank. Water is heated in the smaller inner tank and the hot water is stored in the larger lower tank. Two electrodes connected to medium voltage AC supply are submerged in the inner small tank, to which the water is flown through a valve level controller. The water in contact with the electrodes acts as electrical resistance, allowing the current flow to heat the water. The amount of heat produced is proportional to the water level the electrodes are immersed in, such that higher water level reduces the resistance and increases the current flow, thereby thermal power output. The valve allows heated water to drain into the lower tank from where it is pumped out of the boiler.

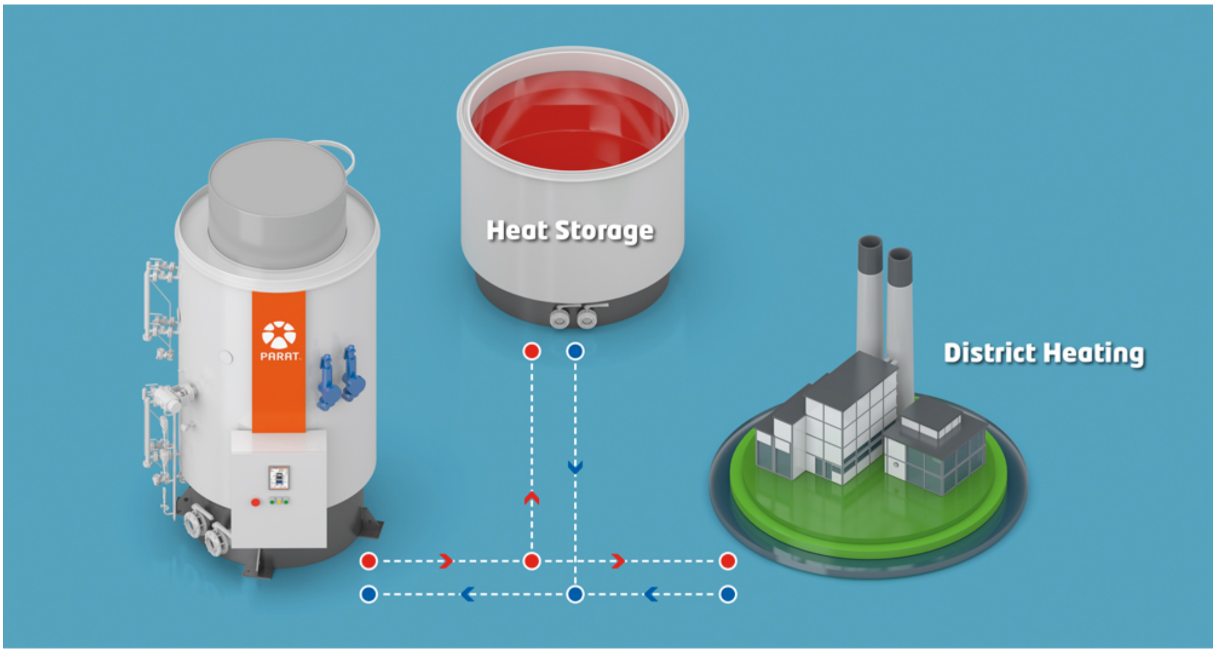


Figure 7: Electric boiler, Heat Storage and District Heating plant connection [38]

On the outside, the electrode boiler is connected to an external thermal storage tank, enabling it to receive and store large amounts of energy for longer hours illustrated in Figure 7. This stored energy is then released from the tank as needed to meet the current heat demand. A pump circulates the flow of water directly between the lower boiler tank to the DH heat exchanger (HX) or transfers the hot water to the external storage tank. The return cold water is flown into the boiler's upper tank from the HX for reheating. The HX is external element where the heat from the boiler's hot water is transferred to the district heating network, and the cold return water flows back to the boiler. The flow of water is controlled by the pumps and control valves while the sensors and actuators monitor the safety operation of the boiler such as maintaining permissible temperature limits, flow rates etc. The rate of heating is controlled by varying the water level in the upper small tank. The power dissipated is proportional to the square of the voltage and the water's electrical conductance, which in turn is proportional to the water level and conductivity. The boiler uses purified, conditioned water to manage the water conductivity. The plant operators, generally, are responsible for providing the power setpoint to maintain the heat output stored in the outer large tank to the primary inner control system of the boiler.

4.2 Modelling Electric Boiler as a VBESS

The operation of electric boiler is planned according to the expected heat forecast and the corresponding thermal energy to be stored in order to meet this demand. The rate of heating, as discussed above, is controlled by the power setpoint and is much faster than the rate of discharge to the HX. Therefore, the electric boiler is preferred to operate intermittently, heating water during short periods usually when the cost of electricity procurement is lowest. The stored hot water is then circulated from the thermal storage tank over extended periods to meet heat demand, even while the boiler is turned off. This allows the boiler to operate only for a few hours during the day. The self-discharge of heat due to thermal losses from the storage tank occurs very slowly (over several days) and is negligible for short-term operational modelling.

The heating of water is directly proportional to the boiler's power consumption (in MW), which is reflected as a change in the amount of thermal energy stored in the external storage tank. This stored thermal energy, referred to as the state of energy (SoE), depends on the hot water level in the tank and the water temperature. To simplify the model, the temperature is assumed constant, so changes in power consumption directly alter the hot water level in the tank. This stored thermal energy can be equivalently expressed in terms of electrical energy (in MWh).

This heating process, therefore, mimics the charging behaviour of a battery, where input electrical power increases the system's internal energy. The size of the thermal storage tank determines the system's energy capacity, analogous to a battery's capacity, measured in MWh. The state of the system corresponds to the state of charge (SoC) in a battery, where a 100% SoE indicates the tank is full and 0% SoE means it is empty and cannot discharge further. When hot water is drawn from the storage tank via the heat exchanger to meet the demand, the system releases the stored thermal energy, similar to battery discharging. However, unlike a battery, discharging in the boiler only reduces the water level, and does not inject power back into the grid.

Not all the electrical power supplied to the electrodes is completely converted into thermal energy due to factors such as convective heat losses and scaling build-up on the electrodes. The effective energy stored is reduced by a power to heat conversion efficiency factor, denoted as η_{P2H}. Since both the charging (heating) and discharging (hot water usage) processes are controllable and subject to physical and operational constraints, the external operational characteristics of the electric boiler such as power limits, storage capacity, efficiency losses, and energy level dynamics, can all be described using the similar parameters of conventional battery models. Thus, the external operation of the electric boiler can be simplified and represented as a virtual battery energy storage system (vBESS). The physical parameters of the electric boiler used in this study are summarized in Table Table 2.

Table 2: Physical parameters of the boiler model and their vBESS equivalents

Boiler Parameter	**vBESS Equivalent**	**Value**
Maximum Power	Maximum Charge Power	12 MW
Maximum Storage tank's Thermal Capacity	Energy Storage Capacity	60 MWh
State of Energy (SoE) (Water Level)	State of Charge (SoC)	0-100%
Ramp Up/Down Rate	Battery ramp rate	$\leq$ 1 min
Power to Heat Conversion Efficiency η_{P2H}	Charging/Discharging Efficiency	0.97

Using the demand response framework discussed in the previous section, the characteristics of the electric boiler are examined and outlined in Table 3.

Table 3: Examining the Boiler's Characteristics on the DR Framework

DR Framework Component	**Boiler Characteristics**
Capacity of the Electric Boiler	Rated power capacity: 12 MW, adjustable in steps of 0.1 MW or 1 MW depending on control resolution
Operational baseline of the Electric Boiler	Day-ahead Scheduled operation P^{ref} procured at Spot market prices
Flexibility estimation	Real-time modulation of active power p^{boiler} around the baseline P^{ref}, constrained within thermally stable and operationally feasible regions
Key limiting factor	Available thermal storage capacity, represented by the State of Energy (SoE)
Full activation response time	Less than 1 minute

5 Methodology and Mathematical Formulation

This section presents the trading algorithm for participating on the PICASSO market. As discussed earlier, the available flexibility of the boiler can be quantified from the DA operation schedule for energy procurement at spot prices. The BRP offers some this reserve energy as a power bid in the DA aFRR capacity market conservatively accounting for uncertainties in heat demand and system dynamics. Since the BRP is obligated to commit the equivalent amount of energy in the real-time market to fulfil the DA capacity bids, it must ensure that sufficient energy is available for those MTUs of the following day. However, the actual volume of energy activated during the bid period is unknown and determined by grid imbalance. For instance, the activation may last the whole MTU, thereby exhausting the entire energy reserve, or only a shorter duration, leaving part of the reserve unused and available for trading in the subsequent MTUs.

When these uncertainties materialize at the time of delivery, the actual system state (i.e. the energy level in the storage tank) may drift from the state predicted by the DA power profile and heat forecasts. This creates two scenarios. First, when the conservative DA capacity commitment results in unutilized energy flexibility, it can be traded on the day of delivery, closer to real-time. Second, if the volume of energy activated is less than what the BRP reserved for the MTU, this balance energy accumulates in subsequent MTUs. To trade this energy in real-time, BRP must constantly track the system state to recalculate the available flexibility after each bid period, and revise the operation schedule to maximise revenue from the energy market.

With this objective, a receding horizon optimisation model is designed which acts as a secondary control layer to the electric boiler providing the power setpoint in real-time to integrate with the market. The single-market model is compatible with PICASSO regulations and implemented in the sequence of steps given below:

- The receding horizon controller initializes a real-time optimisation (RTO) model with the current system state to predict the process output over a prediction horizon. It then applies the first step of output power to the boiler, and submits the energy bid for the first MTU on PICASSO. Following this, the controller shifts the prediction horizon ahead by the control horizon (15 minutes).
- A real-time optimisation (RTO) model takes the DA operation schedule, DA capacity bids, heat demand and current system state as inputs to find the optimal power setpoint of the boiler over the prediction horizon. The model uses forecast aFRR clearing prices for

energy bid decisions during the prediction horizon.

- The real-time feedback from the market is, then, obtained from the ENTSOE-E published clearing prices and the actual system state at the end of the MTU is calculated and fed into the controller.

The decision making process on the PICASSO platform is strictly bounded by the gate closing time of 25 minutes. The methodology formulated here makes certain assumptions to simplify the model and simulate the trading algorithm. The model does not take into account the BEGCT in the receding horizon control, allowing the decision of second bid after the market outcome of first bid is determined. This is not the case in reality, where for Bid 1, say at day D [00:00] hours, the BEGCT is at D-1 [23:35] hours, followed by Bid 2, at D [00:15] hours, the BEGCT is at D-1 [23:55] hours. The actual market outcome of Bid 1 is known only before the BEGCT of Bid 4 at D [00:25] hours. However, if online real-time system state measurement is available, the BRP can make subsequent bid decisions without determining the market outcome. Secondly, the RTO assumes linear change of state of energy (SoE) in the storage tank, without considering the delay in response to the TSO's activation signal.

5.1 Receding Horizon Control

For the optimal trading on the real-time energy market, the algorithm must track the dynamic state of the system and recalculate the system's flexibility after each bid period is completed. Receding Horizon Control (RHC) uses an explicit dynamic model to predict how future changes in the manipulated variables will affect the system output and the control signal is obtained by minimizing a cost function [40]. The main objective of the RHC is to steer the system towards the optimal steady-state computed in the optimization layer. However, in this case, the control objective also contains a economic cost term to be minimized and so, the final equilibrium state may not be the most economical. The close-loop control does not realise the predicted state trajectory, but instead tracks only the input profile over the prediction horizon. The state evolution is tracked implicitly by the system's dynamic constraints. This tracking methodology is summarised in the following steps [41], represented in Figure 8.

- The future states of the system for a determined horizon T, called the prediction horizon are predicted at each instant t using the process model. These predicted output states $\hat{x}(t+k|t)$ for $k \in \{1, ..., \text{T}\}$ depend on the known system input $u(t|t)$ and state $x(t|t)$ and on the future control inputs $u(t+k|t)$ for $k \in \{0, ..., \text{T-1}\}$.

- At the sampling instant *t*, the process model receives a state measurement of the current process which initializes the model. The sequence of future control inputs $u(t+k|t)$ is then computed over the prediction horizon corresponding to time $t \in \{t, t+\mathrm{T}\}$ in real-time. The process model optimizes a performance criterion to keep the process as close as possible to a given reference trajectory $w(t+k|t)$. The model tries to minimize the error between the predicted input signal and the reference trajectory as quadratic function and the cost function $l(x_s, u_s)$. To account for the process constraints, the solver uses an iterative optimisation method.

- From the set of control inputs, only the first control step $u(t|t)$ is sent to the system model to measure the system state $x(t+1)$ at the next sampling instant. Then, the prediction horizon T window is shifted by the control horizon *t* and the steps are repeated to obtain the next control step $u(t+1|t+1)$.

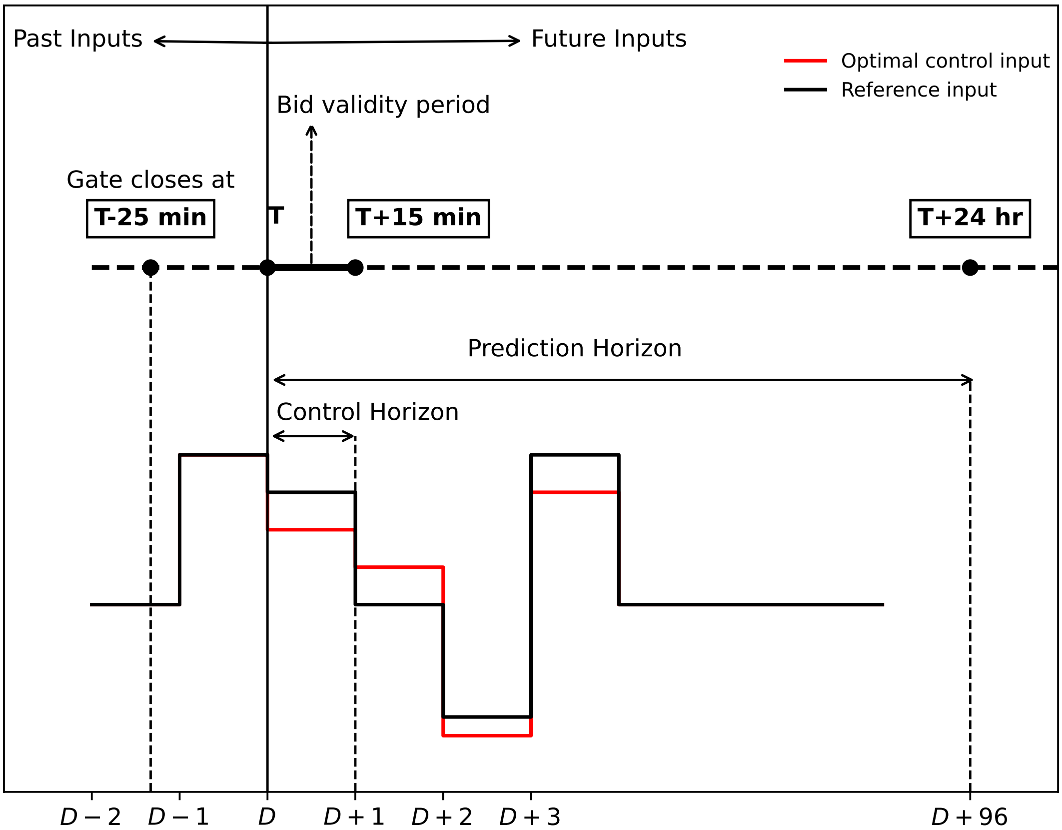


Figure 8: Implementation of Receding Horizon Control on the aFRR market.

The control horizon *t* in this problem is 15 minutes MTU and the prediction horizon can be any period in multiples of 15 minutes longer than 1 MTU. At the start of the control strategy at instant t, the past power inputs (MW) and system states (SoE) are known. The model has a reference power trajectory for all future inputs within the prediction horizon and the expected state trajectory predicted with the reference power, heat demand and DA committed energy bids. Here, the process model only minimises the error between the control input, not the states and a second term maximises revenue from the market. The optimization problem is formulated as follows:

$$
\begin{aligned}
\min_{u_t, x_t} \quad & \sum_{t=0}^{\mathcal{T}} \left\| u_t^{\text{ref}} - u_t \right\|_R^2 + l(x_s, u_s) \\
s.t. \quad & \\
& x_{t+1} = f(x_t, u_t), \\
& x_0 = x \\
& x_t \in \mathcal{X} \\
& u_t \in \mathcal{U}
\end{aligned}
$$

5.2 Real-time Energy Market Optimisation Model

The real-time optimisation (RTO) problem is formulated as a deterministic model for the electric boiler participating in the PICASSO platform. The boiler's DA scheduled operation is optimised over the spot market prices to meet the heat demand forecast of the day. This typically results in the boiler operating during off-peak hours, when spot prices are lowest. However, the DA schedule may also be influenced by power commitments made in the aFRR capacity market. As discussed earlier, real-time activations and conservative DA commitments can cause the actual state of energy (SoE) to deviate from the predicted trajectory. To account for the residual energy following each activation in a MTU, an optimization model is proposed.

The model is initialised with the boiler's current SoE and uses the DA operation schedule as power reference. The DA capacity bids are considered if the BRP has committed power. The model determines the optimal power input for each MTU over a fixed horizon to maximize aFRR revenue while meeting operational constraints. During DA commitment periods, the power set-point must ensure sufficient energy to meet capacity obligations. Outside these periods, the boiler may deviate from the reference schedule to offer its energy flexibility on the energy market, provided it meets current heat demand and maintains adequate SoE for future requirements.

The objectives of the RTO model are to minimize deviations from the reference schedule while maximizing aFRR revenue.

The prediction horizon, expressed in multiples of MTU (15 minutes), defines the time window over which the boiler operation is optimized in each RTO run [42]. A longer horizon, such as 24 hours, enables the boiler to better anticipate future energy needs, especially to maintain availability for mandatory DA bids. However, this comes at the cost of increased computational burden. In contrast, shorter horizon makes the model short-sighted, often leading to overcommitting available flexibility without considering future commitments. The prediction horizon must be longer than the control horizon, which is fixed at 15 minutes.

Within the prediction horizon, the clearing prices of the aFRR energy market are treated as known inputs, which may be forecasted clearing prices. Based on these prices, the RTO determines the optimal bid decisions: power quantity and bid direction (upward or downward). However, power bids are only selected when sufficient energy is available to fully support the bid for the entire MTU, even if the model estimates a lower activation volume. This conservative approach accounts for the fact that actual market outcomes may differ significantly from internal forecasts. By doing so, the algorithm ensures the stable operation of the boiler, regardless of the real-time activation signal. The output of the model includes the optimal power input sequence to the boiler and the corresponding bid decisions, such as bid size (MW) and direction. For simplicity, the RTO sets fixed bid price, since the aFRR market follows a pay-as-clear mechanism, which is discussed in detail later.

The optimisation model is created for the PICASSO platform and hence, the upward and downward prices are indicated by mathematical signs as per regulations. The sign, positive or negative, on the published energy clearing prices follow Table 4 indicating payment between BRP and TSO.

Table 4: Payment of Balancing Energy in aFRR market

	Positive Clearing Price	Negative Clearing Price
Upward Regulation	**Payment from TSO to BRP**	Payment from BRP to TSO
Downward Regulation	Payment from BRP to TSO	**Payment from TSO to BRP**

The prediction horizon in this case study is defined by the set $\mathcal{T} \in \{0, 1, \cdots, 96\}$ for each 15-minute MTU in a day, and the control cycle of the PICASSO AOF is defined by the set $\mathcal{K} \in \{0, 4, \cdots, 225\}$ for each 4-second interval in an MTU.

Decision Variables

The primary decision variables in the optimisation model are upward and downward power bid size $p_t^{\uparrow}, p_t^{\downarrow}$, and the boiler power input p_t^{boiler} in MW. The auxiliary variables are the upward and downward energy activated $e_{t,k}^{\uparrow}, e_{t,k}^{\downarrow}$ in MWh, the state of energy $\text{SoE}_{t,k}$. The model has two binary variables to determine either-or decisions related to energy bid in each 15-minute MTU. The binary variable u_t represents the direction of bid, with 1 indicating upward flexibility and 0 for downward flexibility. The second binary variable $\beta_{t,k}$ represents the predicted market outcome in control cycle k within MTU t, where $\beta_{t,k} = 1$ means the submitted bid is accepted. The decision variables $p_t^{\uparrow}, p_t^{\downarrow}, p_t^{\text{boiler}}, u_t$ vary in each subsequent MTU, and are fixed throughout the 15-minute interval. The time resolution of variables $e_{t,k}^{\uparrow}, e_{t,k}^{\downarrow}, \text{SoE}_{t,k}, \beta_{t,k}$ is 4-seconds inside a MTU.

Objective Function

The objective function in Eq.(1), framed as a cost minimisation term, minimizes the deviation of real-time boiler power from the power reference while maximizing revenue from energy activations in the aFRR energy market. The power reference to the boiler is the day-ahead (D-1) operational schedule of the boiler designed to fulfil the heat demand forecast of day D. In the first term, the weighting factor *w* determines the penalty of deviation from the reference power. The deviation is the L2 norm of the difference of actual boiler power p_t^{boiler} from power reference P_t^{ref}. The second term represents the revenues calculated as a product of the clearing price given by $\lambda_{t,k}^{\uparrow}, \lambda_{t,k}^{\downarrow}$ (€/MWh) and the total energy (MWh) activated during each control cycle. Revenue is earned from upward activation when $u_t = 1$ and from downward activation when $u_t = 0$. The negative sign on the $\lambda_{t,k}^{\downarrow}$ is to take into account the sign format from Table 4.

$$
\begin{aligned}
\min_{\substack{p_t^{\uparrow}, p_t^{\downarrow}, e_{t,k}^{\uparrow}, e_{t,k}^{\downarrow}, \\ p_t^{\text{boiler}}, \text{SoE}_{t,k}, u_t, \beta_{t,k}}} \quad & w \cdot \left\| P_t^{\text{ref}} - p_t^{\text{boiler}} \right\|_2^2 \\
& - \sum_{\mathcal{T}} \left[\left(\sum_{\mathcal{K}} \lambda_{t,k}^{\uparrow} \cdot e_{t,k}^{\uparrow} \right) u_t + \left(\sum_{\mathcal{K}} -\lambda_{t,k}^{\downarrow} \cdot e_{t,k}^{\downarrow} \right) (1 - u_t) \right]
\end{aligned}
\tag{1}
$$

System Constraints

These set of constraints in Eq.(2) applies the boiler's physical system limits on power and energy. The change in power of the boiler is limited to integer values in Eq.(2a) to satisfy the bid granularity of the aFRR market 1 and Eq.(2b) apply the boiler's rated capacity limits. The upward power bid cannot be higher that the current power consumption of the boiler given in Eq.(2c) and the downward power bid to increase consumption cannot be higher than available power under maximum capacity limit, given in Eq.(2d).

The permissible state of energy (SoE) is restricted in Eq.(2e), considered within 10%–90% bounds in the simulation. Eq.(2f) and (2g) ensures that the energy is reserved for the entire bid validity period corresponding to the power bid. This is because the final activation of the reserve is unknown, and the model should account for stable operation under worst case scenario when the reserve is activated for the complete MTU. The time interval denoted by $\Delta T = 0.25$ for a MTU.

$$p_t^{\uparrow}, p_t^{\downarrow}, p_t^{\text{boiler}} \in \mathbb{Z} \tag{2a}$$

$$P_{\text{min}} \leq p_t^{\text{boiler}} \leq P_{\text{max}} \tag{2b}$$

$$p_t^{\uparrow} \leq p_t^{\text{boiler}} \tag{2c}$$

$$p_t^{\downarrow} \leq P_{\text{max}} - p_t^{\text{boiler}} \tag{2d}$$

$$\text{SoE}_{\text{min}} \leq \text{SoE}_{t,k} \leq \text{SoE}_{\text{max}} \tag{2e}$$

$$\text{SoE}_{t-1,k-1} - \frac{\eta_{\text{P2H}} \cdot \Delta T \cdot p_t^{\uparrow}}{E_{\text{max}}} \geq \text{SoE}_{\text{min}} \tag{2f}$$

$$\text{SoE}_{t-1,k-1} + \frac{\eta_{\text{P2H}} \cdot \Delta T \cdot p_t^{\downarrow}}{E_{\text{max}}} \leq \text{SoE}_{\text{max}} \tag{2g}$$

Market Constraints

As the aFRR market follows a pay-as-clear remuneration mechanism, the bid price submitted is only to ensure the bid clears and any activation is paid at the clearing price. This means as long as there is system flexibility in a given direction, it is beneficial to submit the lowest possible bid price to stay below the clearing price. Power is procured in the DA market at spot prices for the hours the boiler is scheduled to operate. For any bid in the upward direction, the minimum bid price to breakeven the cost of energy procurement is the spot price. In the downward direction, the clearing prices are negative indicating the TSO pays the BRP. The maximum bid price is zero to ensure any activation (i.e. increase boiler consumption) is free of cost. This bid offering price strategy is adapted in this model for the electric boiler assuming the boiler operational costs is only influenced by the energy procurement costs.

The constraints in Eq.(3) enforce the binary variable $\beta_{t,k}$ in each 4-second control cycle, considering λ_t^{spot} is submitted for upward regulation bids and 0 for downward regulation bids. The model then compares this with the known upward clearing price $\lambda_{t,k}^{\uparrow}$ and downward clearing price $\lambda_{t,k}^{\downarrow}$. In subsequent discussions, the impact of perfect price knowledge and estimated prices on the final outcome of the model is studied in detail. A big M method is used to implement this pricing logic with M $= 1e4$.

$$\lambda_t^{\text{spot}} \leq \lambda_{t,k}^{\uparrow} + \mathrm{M} \cdot (1 - \beta_{t,k}) + \mathrm{M} \cdot (1 - u_t) \tag{3a}$$

$$-\lambda_{t,k}^{\downarrow} \geq \mathrm{M} \cdot (1 - \beta_{t,k}) + \mathrm{M} \cdot u_t \tag{3b}$$

As per market regulations, BRPs has to submit an equivalent power bid in the PICASSO platform corresponding to all DA capacity bids submitted. The model takes the DA capacity bids as an input $P_t^{\text{capacity bid}}$ where positive value indicate upward bid and negative value indicate downward capacity bid. The constraints in Eq.(4) forces the upward or downward power bids $p_t^{\uparrow}, p_t^{\downarrow}$ to match the capacity bid for those specific MTU.

$$p_t^{\uparrow}, p_t^{\downarrow} = \begin{cases} P_t^{\text{capacity bid}}, \quad 0 & \text{if } P_t^{\text{capacity bid}} > 0 \\ 0, \quad -P_t^{\text{capacity bid}} & \text{if } P_t^{\text{capacity bid}} < 0 \end{cases} \tag{4}$$

State Tracking Constraints

The set of constraints in Eq.(5) track the outcome of the power bids using $\beta_{t,k}$ to update the auxiliary variables $e_{t,k}^{\uparrow}, e_{t,k}^{\downarrow}$. The energy activated when a bid is cleared ($\beta_{t,k} = 1$) is equivalent to the power bid over the given interval times the power to heat conversion efficiency η_{P2H}. The direction of bid u_t and the outcome $\beta_{t,k}$ from aFRR clearing price determines whether the energy activated is up-regulation ($e_{t,k}^{\uparrow}$) or down-regulation ($e_{t,k}^{\downarrow}$). The time interval for 4-second control cycle given by $\Delta K = \frac{4}{3600}$.

For up activation,

$$e^{\uparrow}_{t,k} \leq \eta_{\text{P2H}} \cdot \Delta K \cdot p^{\uparrow}_{t} \cdot u_t \tag{5a}$$

$$e^{\uparrow}_{t,k} \geq \eta_{\text{P2H}} \cdot \Delta K \cdot p^{\uparrow}_{t} \cdot u_t - \text{M} \cdot (1 - \beta_{t,k}) \tag{5b}$$

$$e^{\uparrow}_{t,k} \leq \text{M} \cdot \beta_{t,k} \tag{5c}$$

For down activation,

$$e^{\downarrow}_{t,k} \leq \eta_{\text{P2H}} \cdot \Delta K \cdot p^{\downarrow}_{t} \cdot (1 - u_t) \tag{5d}$$

$$e^{\downarrow}_{t,k} \geq \eta_{\text{P2H}} \cdot \Delta K \cdot p^{\downarrow}_{t} \cdot (1 - u_t) - \text{M} \cdot (1 - \beta_{t,k}) \tag{5e}$$

$$e^{\downarrow}_{t,k} \leq \text{M} \cdot \beta_{t,k} \tag{5f}$$

The energy stored in the tank given by the State of Energy $\text{SoE}_{t,k}$ evolves linearly as the bids are cleared, the boiler is activated, and heat demand (p^{demand}_{t}) is supplied. The constraint Eq.(6) updates the SoE trajectory with 4-second resolution as the model progresses over the prediction horizon. The SoE is updated taking into account the current boiler power, the energy activated in upward or downward regulation and the heat demand supplied. The system state can be initialised at any value between 10%-90% bounds, and subsequent states are updated using the constraint.

$$\Delta\text{SoE}_{t,k} = \left(\frac{\eta_{\text{P2H}} \cdot \Delta K p^{\text{boiler}}_{t} + e^{\downarrow}_{t} - e^{\uparrow}_{t} - \Delta K P^{\text{demand}}_{t}}{E_{\text{max}}} \right) \tag{6}$$

5.3 Real-time feedback from the market

Once a bid is submitted by the BRP, in practice, the TSO sends an activation signal to the asset when the bid is accepted and in response, BRP activates the energy reserves. As the real-time activation signal is not available for simulation, the algorithm derives the energy activations using energy prices feedback from the market. Again, the clearing prices are not published in real-time, generally with a delay of approximately 30 minutes on the ENTSOE-E API. In actual application, the BRP will receive the energy activation signal in real-time and can measure the revenue with a delay when the prices are published. Here, the simulation is carried out on historical data and the need for energy activation signals is replaced with calculated activation from aFRR price data.

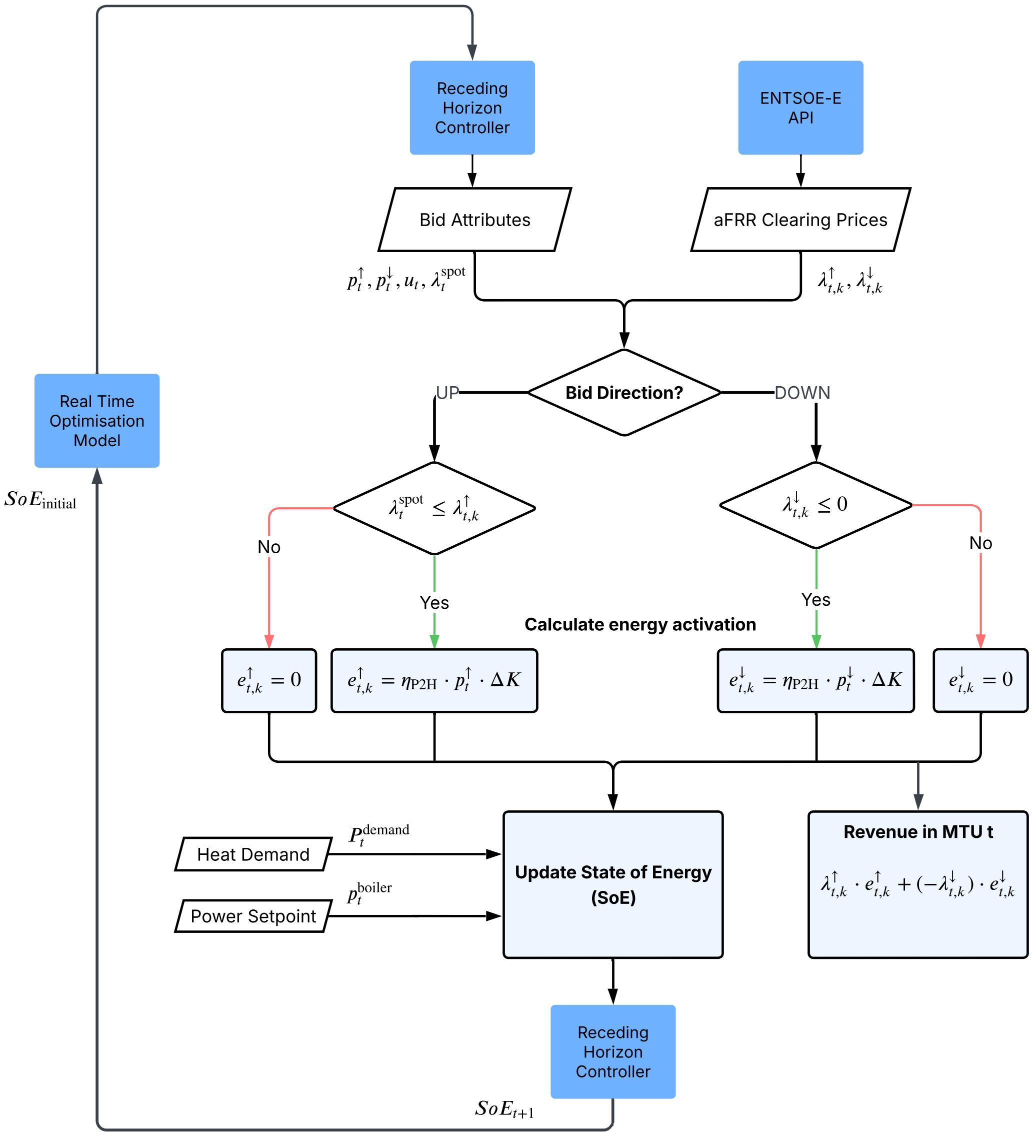


Figure 9: Tracking the real-time market feedback from PICASSO.

The process to track the real-time feedback and the information flow between the three components is outlined in Figure 9. The first step of the output signal from RTO is considered as the submitted bid attributes: Bid quantity (MW) and Bid direction. The bid price submitted is the hourly spot price for upward regulation and 0 for downward regulation. This feedback system compares the offered fix bid price with the aFRR clearing prices published by ENTSOE-E API for every 4-second control cycle of the MTU. The bids are accepted for upward activation when spot price is less than the upward energy price and for downward regulation, bids are accepted when downward energy prices are negative. The model then updates the state of energy (SoE) considering the volume of activated reserve, the heat demand supplied and the power setpoint during the MTU and sends the updated SoE_{t+1} to the controller. The revenue from the activation is calculated and stored. The controller re-runs the RTO with the new initial state SoE_{t+1} and repeats the process.

6 Data analysis

The simulation of the proposed algorithm is carried out using historical data from an electric boiler in West Denmark DK1 and the published market data from the Entsoe API [43] and Energinet API [44]. In this section, the data used is presented and analysed to find useful characteristics and underlying patterns.

6.1 Historical data of Boiler's operation schedule

The operational schedule data from the boiler contains the heat demand forecast of the following day, the DA planned operation schedule and the predicted state of energy (SoE) level in the storage tank. From Table 2, the boiler maximum rated power is 12 MW and the boiler can switch power levels in steps of 0.1 MW. The full capacity of the storage tank is 60 MWh corresponding to 100% SoE. Figure 10 shows the two-days snippet of the boiler data used. The heat forecast has a consistent daily pattern and all datapoints are available with 15-minute time resolution. The boiler is schedule mainly during afternoon hours and off during peak load periods around 18:00 following spot market prices and also, due to solar production at the district heating facility. This additional source is not considered in the simulation here.

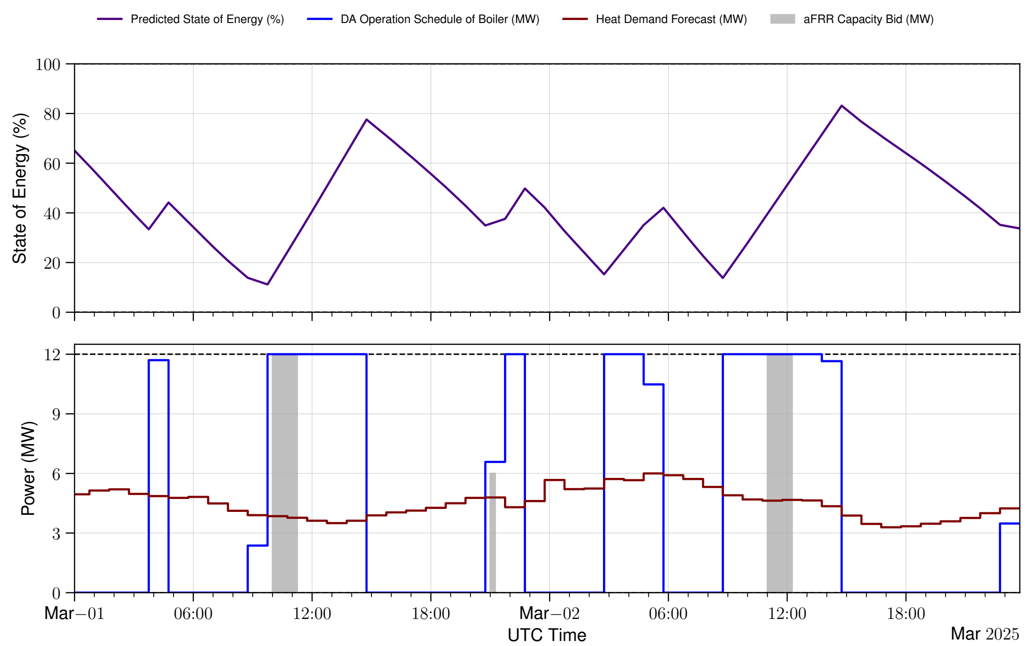


Figure 10: Operation Schedule of an Electric Boiler with heat demand forecasts, DA Capacity Bids and predicted State of Energy trajectory

In order to verify the model's effectiveness in assigning energy bids for the commitment in the DA aFRR Capacity market, an additional schedule is created to indicate the DA capacity bid for those MTUs.

6.2 Historical aFRR Energy Prices

The market data from the APIs consist of the upward and downward clearing prices from PICASSO aFRR energy market and the hourly spot market prices of the day. As observed, the clearing prices have very large volatility with large price spikes for few control cycles up to the upper limit of +/- 15,000 €/MWh. The cause of these problematic price fluctuations can be attributed to multiple reasons, such as cross-border grid congestions [45], artificial scarcity of energy volume, uncertainty in renewable generation [46], and unexpected bidding behaviour by market participants [47]. These characteristics and the state of system-wide imbalances are outside the observability of market participants. In addition to the unpredictable spikes, the energy prices have high frequency fluctuations due to short 4-second control cycles. Therefore, standard forecast models that rely on historical data as features cannot capture the small variations, resulting in poor results.

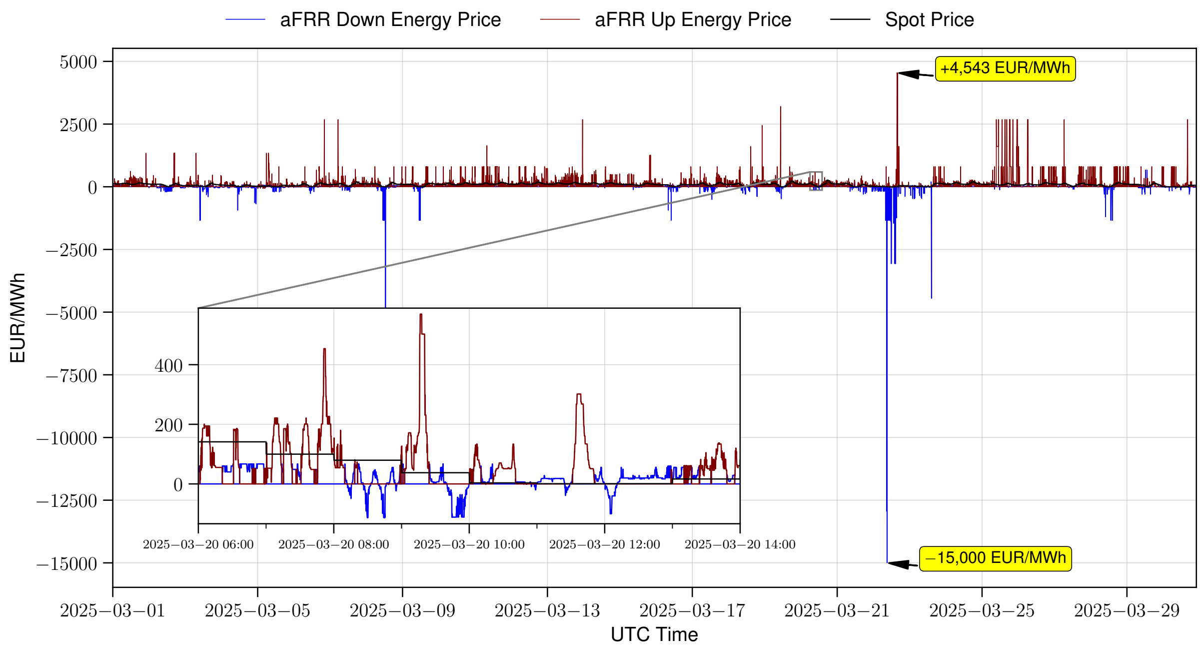


Figure 11: Historical aFRR energy prices in March 2025

The historical clearing prices for upward and downward regulation and hourly spot prices in

West Denmark DK1 for the month of March 2025 is presented in Figure 11. Periods with no aFRR activation appear as missing values in the dataset and were replaced with zero to maintain continuity. There is significant price jumps with two notable spikes on 22nd March 09:00 reaching -15,000 €/MWh for down regulation and at 15:30 with +4,543 €/MWh for up regulation. The case study in [46] provides a detailed analysis of causes leading to similar price spikes in DK1 and DK2.

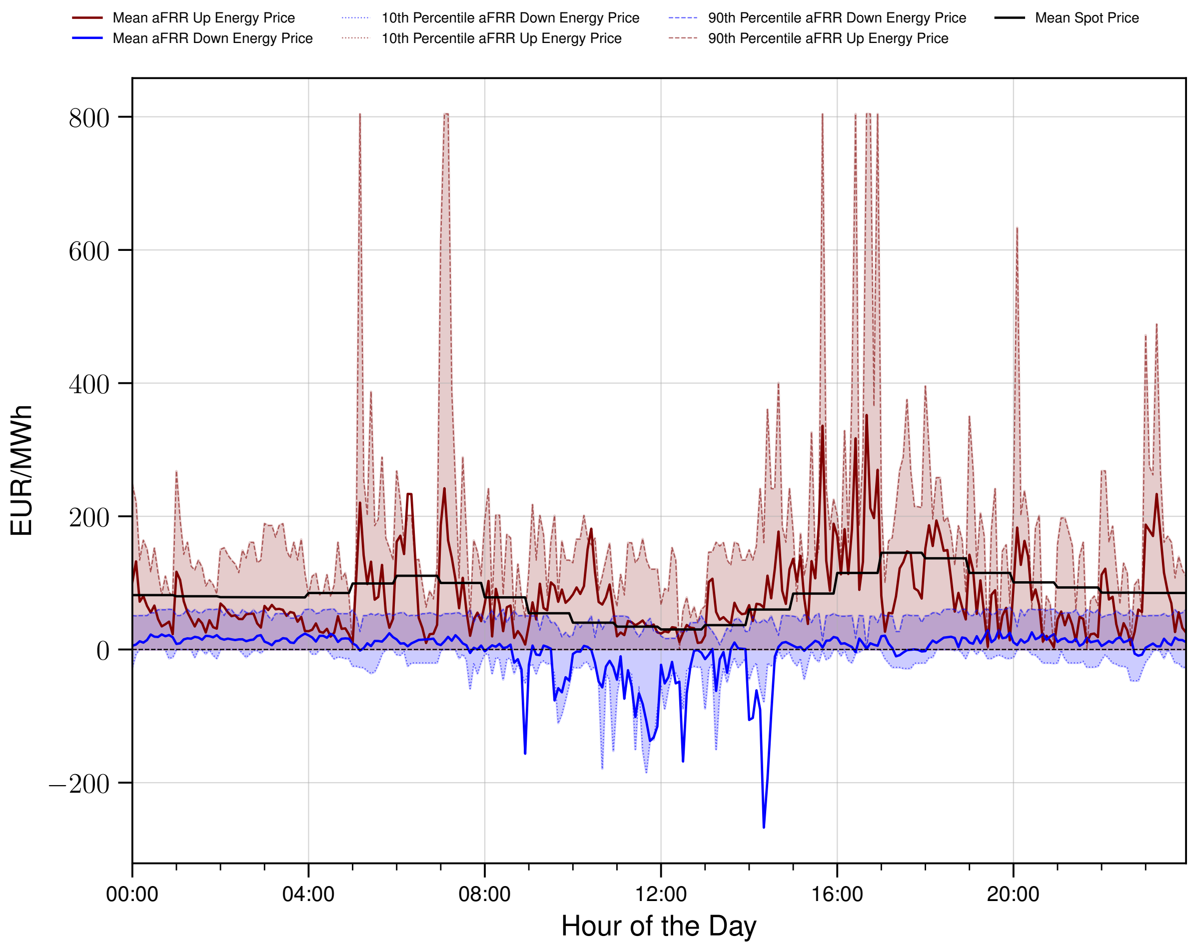


Figure 12: Daily trend of aFRR energy prices and spot prices in March 2025

Figure 12 shows the general daily trend of the aFRR clearing prices and spot price averaged over 5-minute intervals of a day. The upward clearing prices are consistently positive, with the 10th percentile at zero. The highest prices occur around peak hours loosely following the spot price profile, which is expected as the system is more stressed with shortage of positive balancing energy. The 90th percentile of the price distribution shows scenarios where extreme prices in the common merit order is reached. Upward prices also shows more volatility compared to downward prices, as seen in Figure 11, it has more consistent spikes albeit less severe prices. Next, as the bidding strategy of the boiler sets the upward bid at spot price, the chance of clearing the market is highest during the peak hours between 05:00-08:00 and 15:00-18:00. However, given that the boiler's operation is also optimized around spot prices, typically avoiding peak-hour usage, this may affect its ability to participate in upward regulation.

The mean downward clearing prices are positive throughout the day, except between 09:00 and 15:00, when they are negative. This negative balancing energy is typically required during mid-day when there is surplus from PV production during daytime and relatively low demand. The 90^{th} percentile of downward prices remains positive throughout the day, while the 10^{th} percentile is negative during the 09:00–15:00 window and close to zero at other times. Therefore, at least 90% of the time, outside of 09:00–15:00, downward clearing prices are positive, meaning the BRP pays the TSO for activation.

6.3 Estimation of aFRR Energy Prices

The optimisation model requires the aFRR clearing prices as known inputs over the prediction horizon in order to make the bidding decisions. As such, the model's performance improves with the accuracy of the provided prices compared to the actual clearing prices published by PICASSO. Since actual clearing prices are never available to the BRP in advance, using them in the model represents a best-case benchmark. In the following analysis, the boiler's operation in the aFRR energy market is simulated using estimated aFRR prices and compared against this best-case scenario.

Moving Average Price Estimates

The aFRR energy prices exhibits characteristics typical of high frequency time series data: a fine temporal resolution of 4 seconds, extreme short-term volatility and frequent price spikes at irregular time intervals. These fluctuations are large in magnitude and occurs frequently, indicating highly non-stationary data where both the mean and variance continuously shift over time [48]. The aFRR prices do not hover around a stable average or maintain consistent variability, instead shifting entirely based on market and system conditions. This behaviour of the price distribution complicates both visualization and statistical inference to detect any underlying seasonal or cyclical trends or observe patterns in the price changes. The time series can be considered noisy because extreme price spikes caused by rare imbalances in the system are outliers in the dataset that have to be filtered in order to observe the regular price variation patterns. To address this, simple moving average (MA) is used to capture a linear relationship between current prices to historical prices over a given past time window.

A moving average (MA) model is a linear filter to smooth out the short-term fluctuations considered as white noise to identify meaningful trends, and repeating patterns in the data that are not strictly seasonal [49]. The simple MA calculates the equally weighted mean of the past *k* data points in the time series p_t. Mathematically, for the look-back window size *k*, this is defined as:

$$\hat{p}_t = \frac{1}{k} \sum_{i=0}^{k-1} p_{t-i}$$

Here, the trailing MA window only takes past values, so the calculated average $\hat{p}_t$ lags behind the actual price signal p_t depending on the window size.

The MA window size is the main parameter that influences the outcome of the model. A larger window size considers more past values of prices, effectively filtering out short-term fluctuations and high frequency noise resulting in a smooth price signal. However, this also introduces more lag delaying the response to sudden price shifts and may ignore useful shorter trends. Conversely, a smaller window is more sensitive to recent price changes and responds quickly, but also picks up noise reducing the clarity of underlying trends. For example in the aFRR price data, a small 5-minute window will be more sensitive to a price outlier that persists for just a minute or two compared to a 1-hour window. Therefore, selection of the window size is depends on the optimal trade-off between sensitivity and smoothness of the resulting price signal. Some statistical criteria discussed below is used to evaluate the estimated prices and choose the optimal window size.

Residuals: The residual for each observation is the difference between the actual observed value p_t and the estimated value $\hat{p}_t$, given as:

$$r_i = p_t - \hat{p}_t$$

A negative residual means the predicted value is too high ($\hat{p}_t > p_t$) resulting in overestimation while a positive residual indicates that the predicted value is too low, leading to underestimation. The estimated prices $\hat{y}_t$ varies with different MA window size, consequently the distribution of residuals changes. The residuals of aFRR upward and downward clearing prices are calculated for the month of March, 2025 for window sizes from 15 minute to 24 hour in increasing order.

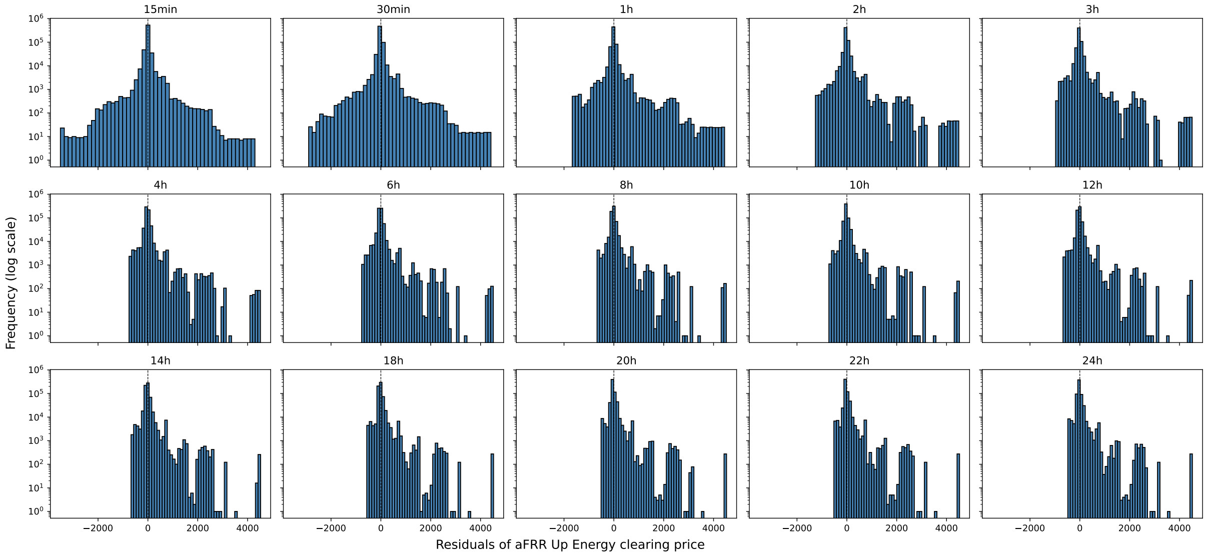


Figure 13: Histogram of residuals of aFRR upward energy prices for varying moving average window.

Figure 13 shows the histogram of residuals of aFRR upward energy prices for different windows, with frequency plotted on a logarithmic y-scale. The residuals approximate a normal distribution for shorter windows, with both positive and negative values indicating that the model reasonably tracks the price signal and captures sudden spikes. As the window length increases, the distribution becomes increasingly right-skewed, meaning the model is underestimating all the upward price spikes. This suggests that the model becomes less responsive to rapid price changes, as the moving average smooths out fluctuations and filters out price spikes, resulting in an overly damped price trajectory. Rare extreme outliers above 4,000 €/MWh are still present, as seen in Figure 11.

Figure 14 shows the histogram of residuals of aFRR downward energy clearing prices for different rolling windows. The frequency of residuals tightly clustered around zero is significantly higher than for upward residuals, necessitating a logarithmic scale for visualization. This indicates that down-regulation prices, while highly fluctuating, tend to oscillate close to their moving average, resulting in lower variance compared to upward prices which has more spread and frequent large deviations. The histogram also shows a left-skewed distribution, especially for longer window sizes, with a negative tail indicating overestimation by the moving average. Despite this general stability, extreme outliers persist, with downward prices spiking up to 15,000€/MWh, as shown in Figure 11.

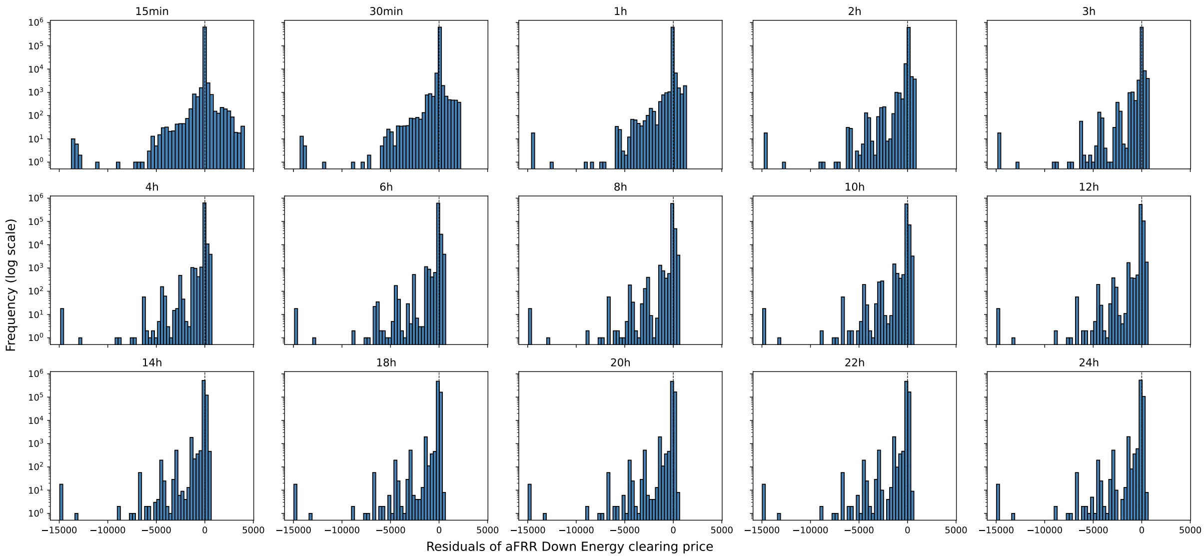


Figure 14: Histogram of residuals of aFRR downward energy prices for varying moving average window.

Error Metrics: The error between the actual price and the MA estimated price is calculated for both upward and downward energy prices using two metrics. The Mean Absolute Error (MAE) represents the average of the absolute differences between predicted and actual values. The MAE is defined as follows, where y_t represents the actual values, $\hat{y}_t$ denotes the predicted values, and n is the total number of observations:

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|$$

MAE assigns equal weights to all individual differences, so outliers do not have a disproportionately large impact on the error value.

The Root Mean Square Error (RMSE) metric measures the square root of the average squared differences between predicted and actual values, taking into account both the magnitude and variance of prediction errors. As the individual differences are squared, this penalizes large errors more severely, making it more sensitive to outliers. It offers insights into the overall goodness-of-fit of the model and is defined as follows:

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}$$

Table 5: Comparison of Error Metrics for aFRR Upward and Downward Energy Clearing Prices for different moving average window.

	MAE		RMSE	
Rolling Window	UP Price	Down Price	Up Price	Down Price
15 minute	59.23	26.37	177.63	150.33
1 hour	85.94	35.41	235.52	173.19
2 hour	93.47	38.11	242.01	177.44
6 hour	101.92	40.02	244.98	179.05
12 hour	103.62	40.88	247.69	182.36
24 hour	106.00	40.55	252.24	183.16

Table 5 shows the MAE and RMSE errors for aFRR upward and downward energy prices over different MA windows sizes. The error values plotted in Figure 15 are normalised relative to minimum-maximum observed errors.. As expected, the prediction errors increases significantly with window size for both price types before plateauing around 2-hour mark. In all case, the RMSE error is substantially higher than MAE, with the difference widening as the window increases. This indicates the presence of large outliers that disproportionately affect RMSE due to its squared error formulation, particularly at larger window sizes where price dynamics are smoothed out. Additionally, across all windows, errors for upward prices are significantly higher than for downward prices. This is expected, as downward prices tend to have lower variance and fluctuate more tightly around the moving average. Finally, the gap between RMSE and MAE is larger for upward prices, suggesting that consistent large deviations contribute to higher overall error, while downward prices experience infrequent but extreme spikes that affect RMSE more than MAE.

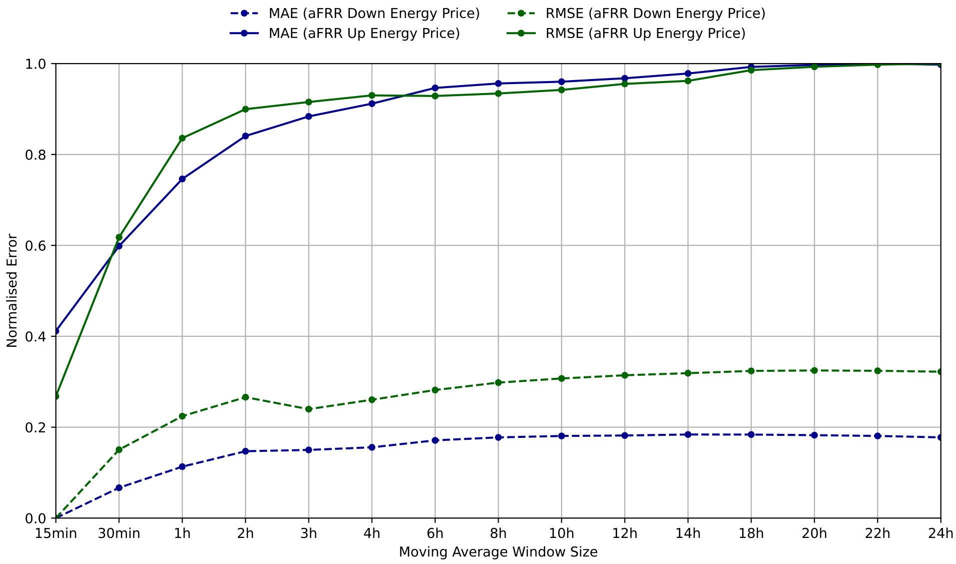


Figure 15: Normalised error of aFRR energy price estimation using different moving average window.

Based on the observed error and the normal distribution of residuals, the least error occurs with the shortest window. However, to evaluate the model's performance with subpar estimates and to prevent large bias in price estimation from the actual values, the simulations are carried out with the 2-hour window size. The window size retains the overall trend of the clearing prices without capturing any sudden price spikes. Since the accuracy of the MA price signal affects the model's outcome, the 2-hour window provides a deliberately smoothed yet robust estimate to test the algorithm's effectiveness under suboptimal price forecasts.

7 Results

In this section, the historical data from a 12 MW electric boiler and the aFRR market are employed to verify the effectiveness of the real-time optimisation algorithm outlined above. Key model parameters are analysed through a series of simulations to assess their impact on performance.

7.1 Impact of Weighting Factor

The selection of the weighting factor w in the objective function Eq. (1) determines the trade-off between the allowed deviation from the baseline and the market revenue. The baseline of the boiler limits the amount of energy it can offer on the market, especially when market conditions diverge from the planned operation. For example, downward regulation may be needed around midday in the aFRR market, but if the boiler is already operating at maximum power due to low spot prices, only upward regulation can be offered without deviating from the baseline. In such cases, the optimization must decide whether to maintain the DA schedule or allow deviations to capture market revenue.

In the objective function Eq. (1), the magnitude of the deviation term $\left\|P_t^{\text{ref}} - p_t^{\text{boiler}}\right\|_2^2$ is significantly smaller in absolute scale compared to the revenue term. The power deviation remains within a narrow range (under 12 MW), while the revenue component calculated based on market prices can span from hundreds to thousands of euros. To account for this disparity in scale and to ensure a meaningful trade-off between cost and revenue, the weight on the revenue term is fixed to 1, and the weighting factor w is varied to appropriately scale the penalty on deviations.

Higher weights indicate a larger penalty for any deviation from the DA operational schedule, which results in lower revenue from energy activation. A parametric sweep method is used to find the impact of different weighting factors on the resulting revenue and find the optimal value of w for the simulation. Figure 16 shows the trend of the normalized cost terms from the objective function, evaluated across daily simulations from 1$^{\text{st}}$ to 14$^{\text{th}}$ March 2025. For each day, the optimization is run independently for increasing values of w and the resulting deviation and revenue are plotted (faint lines). The median deviation and revenue across all days is highlighted in the plot. For a penalty factor of $w = 5$, the median deviation from the baseline schedule drops to only 5.66%, while the corresponding revenues reduces to just 31.65% of the maximum possible revenue. It is expected that the stricter adherence to the DA operational schedule reduces the available flexibility of the boiler and results in lower revenues from the energy market. The few outlier lines with large revenues deviate significantly from the median due to large price spikes on the days, between 7th and 9th March Figure 11. Here, the model accepts large deviation from the baseline even at higher weights.

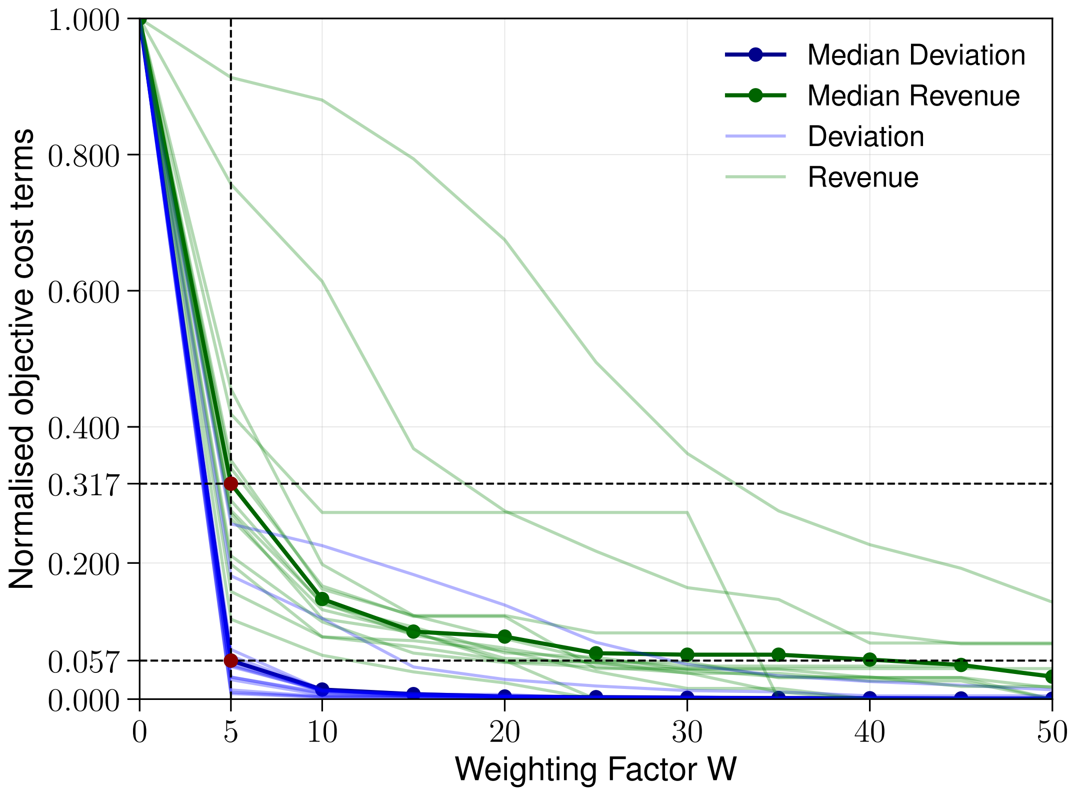


Figure 16: Trend of objective cost term: Deviation and Revenue with varying weighting factor w

From an operational point of view, it is desirable to minimize deviations from the baseline to ensure stable operation of the boiler and reduce imbalances which, in some cases can be penalized in imbalance settlement market. Based on the observed variation of revenue concerning w, a penalty factor greater than 10 effectively keeps deviations under 1% while achieving approximately 15–20% of the maximum possible revenue.

7.2 Selection of Prediction Horizon

The prediction horizon $\mathcal{T}$ can be any duration that extends beyond the control horizon in steps of 15 minutes. The RTO solves the problem over the entire prediction horizon using the price and demand forecasts to obtain the expected state trajectory. A longer horizon captures more of the market's future behaviour, enabling the model to optimise the boiler's operation for more profitable scenario in the future, albeit at a cost of increased computation time. Additionally, the uncertainty error of both price and demand forecast increases with longer horizon. Shorter prediction horizon, on the other hand, are computationally faster, but may be short sighted, exploiting all available flexibility without accounting for future scenarios.

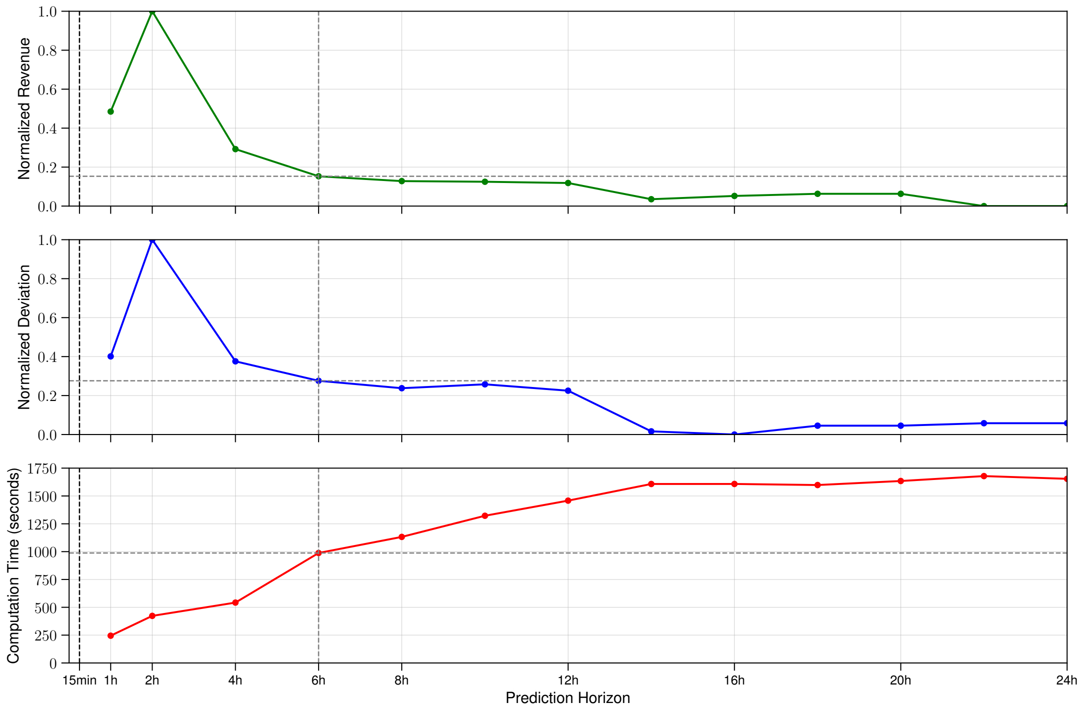


Figure 17: Trend of Revenue, Deviation and Computation time with varying prediction horizon $\mathcal{T}$

Figure 17 shows the normalised revenue, power deviation from baseline and computation time for prediction horizon ranging from 1-hour to 24-hours, based on a simulation of the boiler on the aFRR market. At very short horizon (1-hour), the simulation is fastest, but the realized revenue and deviation is low indicating the model fails to respond effectively to market dynamics. The highest revenue is observed at 2-hour prediction horizon but it also has the highest deviation which is undesirable. To select the optimal prediction horizon, there is a trade-off between maximising revenue and minimising deviation. The computation time rises steadily with longer horizons. Beyond the 12-hour mark, further increase in horizon have no significant impact on revenue, deviation or computation time. In particular, the prediction horizon between 6-hours to 12-hours have consistent values. It is important to note that certain trends, such as sharp peaks or drops in performance, are influenced by the specific characteristics of the input data on this simulation and may not generalize to all operational scenarios. Based on these observations, a 6-hour prediction horizon is chosen for the subsequent simulations offering a good compromise between computational efficiency, revenue performance, and operational stability.

7.3 Simulation Results

Simulation Using Actual aFRR Prices

A simulation is carried out from 1st March to 14th March, 2025, assuming the actual aFRR energy clearing prices for backtesting. The real-time optimisation is run over 6-hour prediction horizon with initial system state of energy at 90% and updated every 15 minutes using receding horizon control. The simulation is initially solved with a weighting factor of 30. However, if the system constraints do not permit a feasible solution while adhering to the baseline schedule, the weighting factor is progressively relaxed until feasibility is achieved. In actual market implementation, the bid window closing time (BEGCT) shifts the prediction horizon and control horizon by 25 minutes. However, this offset is not applied here, as the model runs on historical data with perfect knowledge.

Figure 18 shows a snapshot of the results from the simulation period. The first panel shows the direction of available flexibility in each 15 minute MTU for positive upward regulation and negative downward regulation. The energy activated (in MWh) as a result of the market outcome is plotted. In periods where there is available flexibility but no energy activation, the bid is placed but not accepted. The second panel shows the revenue (in €) calculated in 15-minute bid periods corresponding to energy activation and clearing prices from PICASSO. The third panel shows the DA operational schedule, DA capacity bids and real-time control input to the boiler. The bottom panel shows how the state of energy (SoE) in the boiler's storage tank changes in real time as the bids are activated.

The deviation of the input power to the boiler from the baseline primarily serves to either increase flexibility in a given direction or to satisfy the safety limits, such as the maximum and minimum SoE. As seen in period between 09:00 and 15:00 on 4th March, the boiler power is reduced in large amounts from the baseline in order to ensure power availability during the day-ahead committed capacity bids and maintain the SoE within its limits. Similarly, between 16:00 and 23:00 hours on 6th March, the boiler power deviates between 1-3 MW from 0 MW reference to create flexibility in the upward direction.

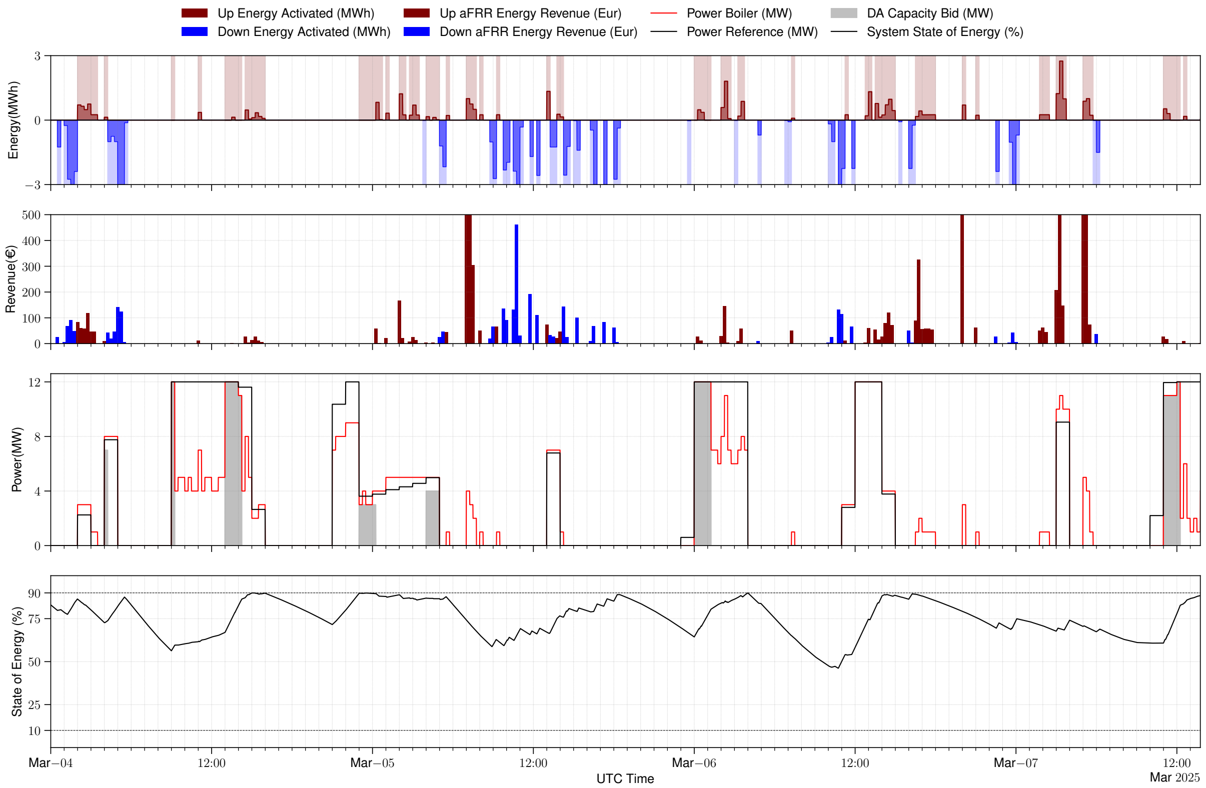


Figure 18: Simulation with Actual aFRR prices: Power input, SoE evolution and Revenue over 4 days in March 2025, based on 15-minute upward and downward aFRR bids and energy activations.

Table 6: Selected Balancing Energy Data and Revenue per MTU on 2025-03-05

Timestamp	Upward Energy activated (MWh)	Down Energy activated (MWh)	Total Revenue in the MTU (€)
2025-03-05 05:00:00	0	1.19	25.56
2025-03-05 05:15:00	0	2.16	46.66
2025-03-05 07:00:00	1	0	1291.77
2025-03-05 07:15:00	0.75	0	705.44

From the simulation results in Table 6, it is evident that the volume of energy activated does not directly correspond to the revenue earned. Although more downward energy was activated than upward energy during this period, the revenue, shown in the second panel of Figure 18, is significantly higher for the upward activations as the market cleared at a very high aFRR upward price. The model leverages this information to make optimal bid decisions, since it has full knowledge of the market prices in this scenario. This highlights that more accurate price forecasts can lead to better bidding strategies. The behaviour of the model and its bidding decisions when using the simple moving average price signal are discussed in the following section.

Simulation Using Estimated aFRR Prices

The simulation is run for the same period from 1st March to 14th March, 2025, using the moving average price signals for upward and downward regulation. These price inputs to the RTO model are shifted by 25 minutes to account for the bid gate closure time (BEGCT). The RTO solves the problem over a 6-hour prediction horizon, after which the real-time market feedback is obtained, and the receding horizon control updates the solution every 15 minutes. The weighting factor selected in this scenario is 10 using similar parametric sweep method where the revenue is approximately 30% of the maximum possible revenue.

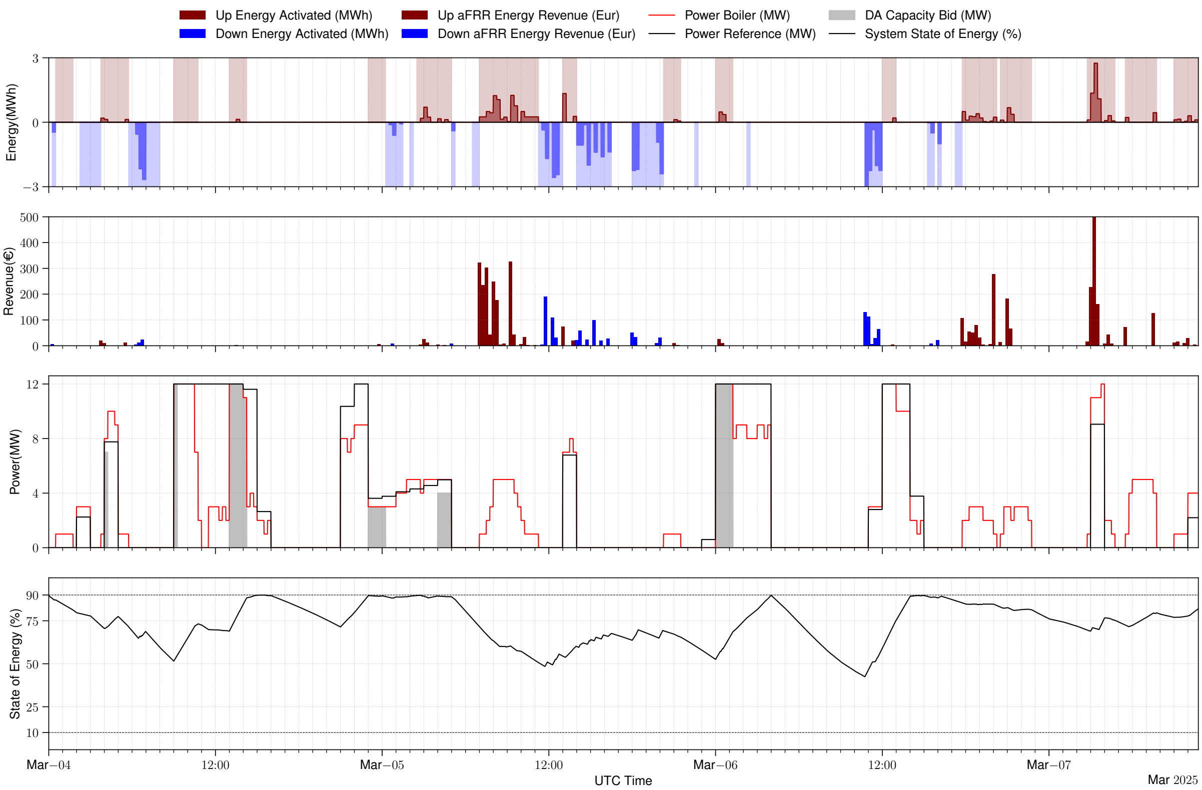


Figure 19: Simulation with Estimated aFRR prices: Power input, SoE evolution and Revenue over 4 days in March 2025, based on 15-minute upward and downward aFRR bids and energy activations.

Table 7 presents the average daily energy activated and the corresponding revenue from aFRR activations over the simulation period, comparing the Base Model (with actual market prices) to the Moving Average Model (with estimated prices). As expected, the revenue decreases by almost 50% with estimated prices. Since the RTO relies on the price input for bidding decisions, it cannot accurately track the price spikes, resulting in significantly lower revenues even when comparable amounts of energy are activated. While the Base Model yields much higher revenue, the increase in total energy activation is only slightly higher, indicating that price timing and spike prediction are critical to optimizing revenue in the aFRR market.

Table 7: Daily Traded energy volume and revenue from aFRR Activation in March 2025

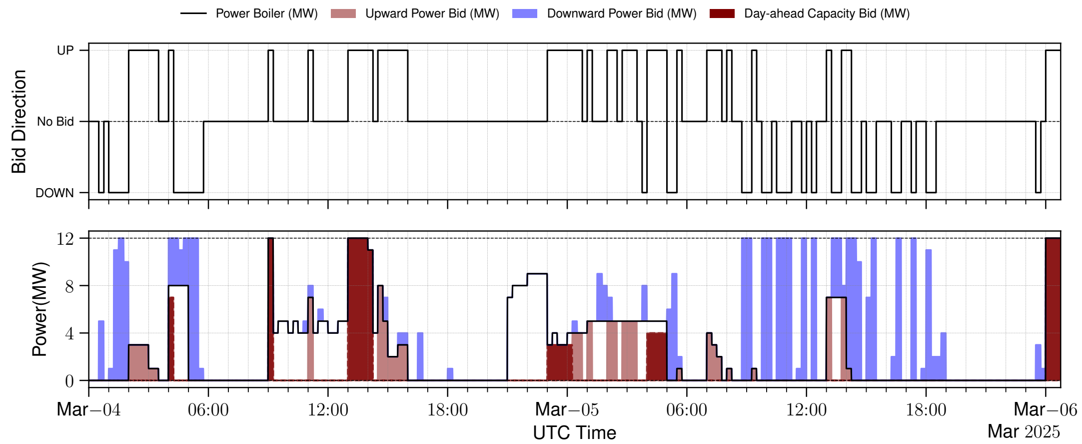
	Base Model	Moving Average Model
Upward Regulation		
Energy Activated (Mwh)	13.04	8.25
Total Revenue (€)	3522.46	1848.49
Downward Regulation		
Energy Activated (Mwh)	11.08	6.448
Total Revenue (€)	682.01	183.65
Total Revenue (€)	4204.47	2032.14

The revenue from upward activation is significantly higher than from downward activation, despite comparable energy volumes in both directions. Upward regulation earned over twice more revenue with €270.21/MWh compared to downward regulation with €61.55/MWh in the base model and in the MA model, the revenue from upward activation is €224/MWh and only €28.5/MWh for downregulation. This disparity reflects the inherent asymmetry in aFRR energy clearing prices: upward prices are generally cleared at high positive prices, with more frequent and extreme spikes, and thus offer greater revenue potential. In contrast, downward clearing prices are generally lower than upward clearing prices, and a substantial portion of them have positive prices in which case the BRP pays the TSO for activation. The Base Model leverages perfect foresight of these spikes, while the MA Model, though showing lower prediction error for downward prices, fails to capture the rare but high-magnitude downward price events primarily during mid-day due to smoothing.

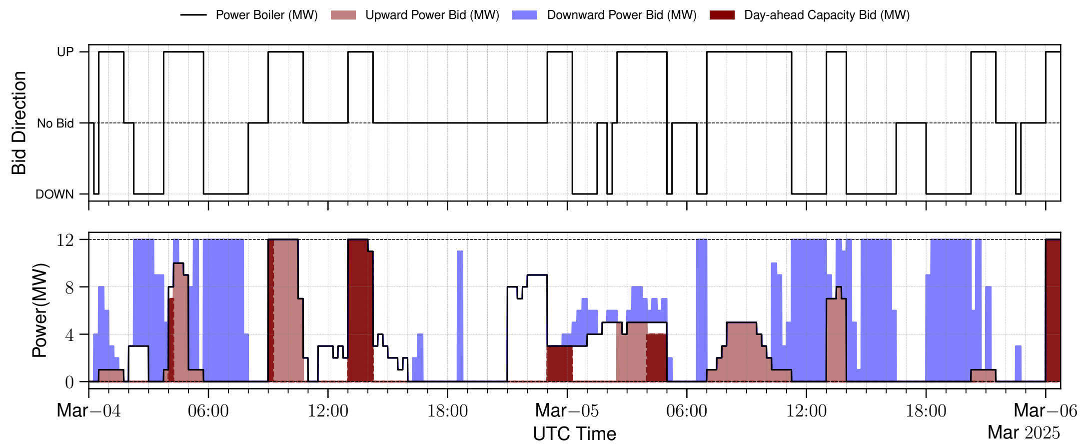
The overall revenue in both models is also partly influenced by the traditional day-ahead operational schedule of the boiler, which optimizes for spot market prices and typically runs the boiler at maximum power for short durations. The boiler has upward flexibility during period of operation thereby, limiting its participation in the upward market to these small durations. When stricter weights are applied to enforce the DA schedule, the boiler further loses the opportunity to offer greater upward flexibility and capture additional revenue.

Bid attributes

The output of the RTO includes the bid attributes: Bid direction (UP, DOWN) and Bid quantity (MW) plotted in Figure 20 for base and moving average model for 4th and 5th March. Upward bids ($p_t^{\uparrow}$) appear below the power curve, representing the amount of power the boiler can reduce if activated. Conversely, downward bids ($p_t^{\downarrow}$) lie above the power curve, reflecting the additional power the boiler can supply.



(a) Bid attribute decisions using actual aFRR energy prices.



(b) Bid attribute decisions using moving average aFRR prices.

Figure 20: Comparison of bid attributes using actual and moving average aFRR price models (March 4th–6th).

From the plots, the bid power is not always equal to the total available power flexibility. This is because the model chooses the bids conservatively, only if it can ensure energy delivery for the entire MTU. Additionally, the model calculates the feasible power bids in both directions for each MTU. For example on 4^{th} March at 05:00 in MA model, the downward power bid is greater than upward bid size, but the selected direction is UP, based on the expected revenue from the bid. Similarly, at 18:45 on the same day, the model chooses to not bid even though downward flexibility is available, because the cost of deviating from the schedule outweighs the expected revenue from energy activation.

Most bid decisions differ between the base and MA models for the same time periods. The base model, with access to actual aFRR energy prices, frequently switches bid directions to capture short-term price spikes, where small activations results in high revenues. In contrast, the MA model produces steadier bid patterns, reflecting the smooth nature of the moving average price signal, which does not have sharp spikes and evolves more gradually. A comparison of the actual and estimated price signals, along with the resulting market feedback, is discussed next.

Market Feedback on estimated prices

Figure 21 shows the aFRR market clearing prices, the moving average estimated price, and the spot price for the same period, 4^{th} and 5^{th} March. The market feedback logic employed in Figure 9 accepts the bid when spot price is lower than the aFRR upward energy price or when the aFRR downward energy price is negative. As seen from the plots, the moving average price signal is much smoother than the highly volatile market prices and does not capture any short term price fluctuations. Additionally, the peaks in the MA price signal are delayed relative to the actual price, due to the 2-hour MA window and the 25 minutes shift accounting for BEGCT. This means the model makes decisions on price assumptions that are quite outdated and does not reflect the real market conditions.

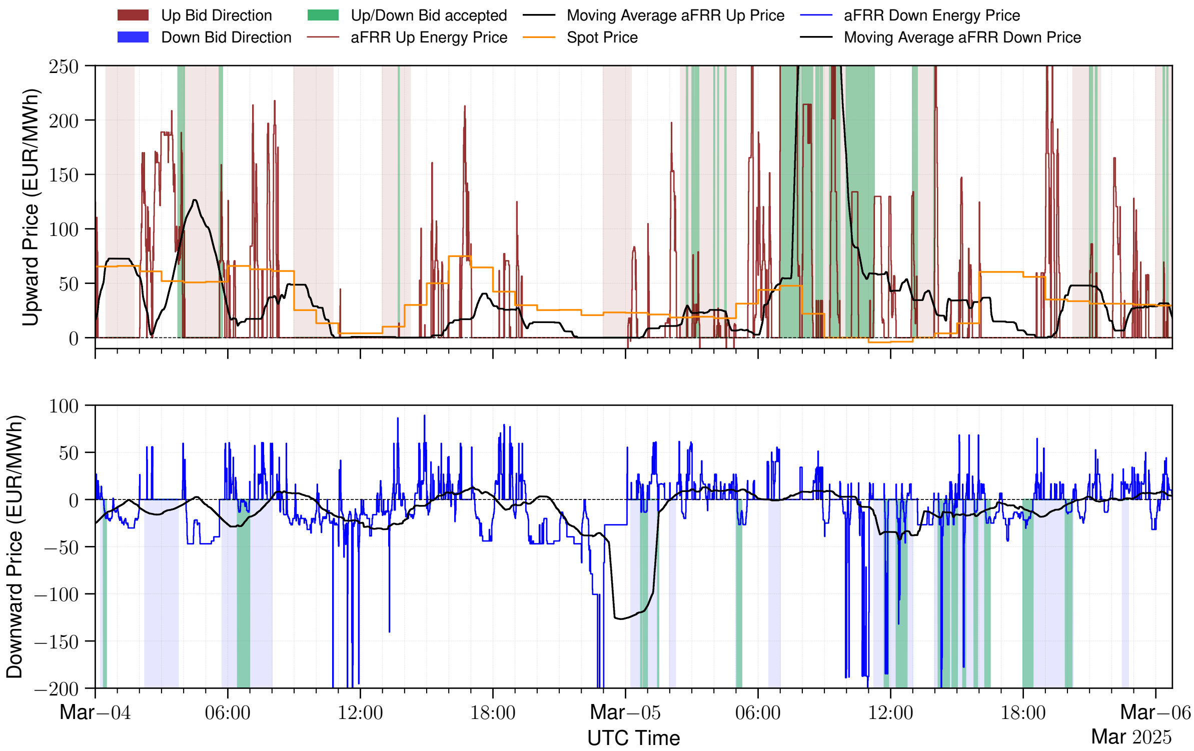


Figure 21: Market feedback on bid decision in the Moving Average model (March 4th–6th).

The bids submitted for upward and downward regulation are highlighted in the plot corresponding to the bid attributes in the MA model Figure 20b. The outcome of the market feedback process is also indicated, showing which bids were accepted. Over the full simulation period, the base model submits bids for a total of 517 MTUs and the market outcome shows 97.76% of bids are accepted when measured on the 4-second control cycle resolution. In comparison, the moving average model submits bids for 539 MTUs with 72.91% of them accepted.

This difference in bid acceptance highlights the impact of price signal quality, while MA models bids more frequently, many do not align with the real-time market conditions. For example, at 09:00 on 4th March, the model selects upward bids based on the delayed MA price signal, while the market has a higher need for downward regulation. Nevertheless, due to the current trend of high aFRR clearing prices, the MA model still achieves a significant volume of activations, resulting in positive revenues.

Impact on Boiler's operational cost

The revenue from the energy activations on the aFRR market contribute to reducing the cost of operation, however, the new power trajectory the boiler follows in order to participate in the aFRR market also may negatively contribute in rising the operation costs. This is important in scenarios when the model bids for upward regulation, so the boiler has to maintain the committed power level which accumulates costs irrespective of whether the bid is accepted or not. Figure 22 shows the 15-minute MTU average cost profile of the boiler with and without energy activations in the Moving Average model.

The DA scheduled cost, shown on the negative y-axis (where negative values indicate cost), inversely follows the spot price trend, which is plotted on the secondary y-axis. As observed, periods with lower scheduled cost typically coincide with higher spot prices, and vice versa, indicating that the DA schedule is effectively optimized to minimize costs during high-price hours. The DA schedule cost is calculated as:

$$\text{Scheduled Cost} = \lambda_t^{\text{spot}} \cdot P_t^{\text{ref}}$$

The effective cost represents the cost taking into account the average revenue from aFRR activations over the day. Practically, cost of operation of the boiler outside the DA scheduled hours is calculated at the intra-day prices but for simplicity, spot price is used here.

$$\text{Effective Cost} = \lambda_t^{\text{spot}} \cdot P_t^{\text{boiler}} - \text{Total Revenue}$$

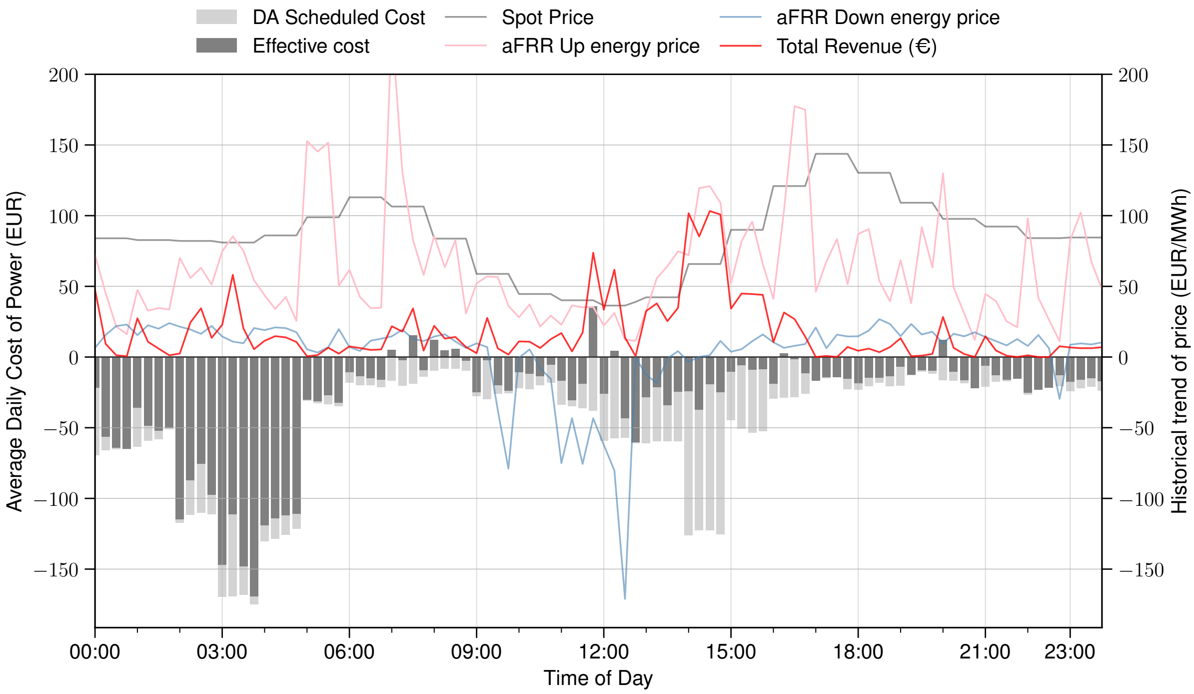


Figure 22: Impact on aFRR activation on the cost of operation of the boiler over 24 hours in the Moving Average model.

The comparison of operational cost in the 2-week period of simulation for both models is given in Table 8. The base model has much higher drop due to perfect price forecast. However, the MA model achieves a significant reduction, particularly due to downward regulation volumes, which, despite lower revenue, consistently offsets operational costs Table 7.

Table 8: Cost Comparison of operation of the boiler in two weeks of March between Base and Moving Average model.

Model	Total Scheduled Cost (€)	Total Effective Cost (€)	Relative Drop in Cost (%)
Base Model	63867.46	21962.60	65.61
MA Model	63867.46	43305.52	32.03

The significant revenue comes during the operational hours of the boiler from upward activation between 00:00 and 05:00 hours and from 12:00 and 15:00 hours when the boiler has upward flexibility. The average aFRR Up energy clearing price is much higher than the down clearing price during the simulation period. Downward prices, as expected, are negative only for a short period around midday, and most downward activations occur during this window. This is particularly interesting, because while upward activation brings more revenue, it also accumulates more costs to run the boiler at higher power levels. However, downward activations allows for an increase of consumption at no extra cost and extra revenue. So even at lower revenue, downward activation can be useful as it increases the state of energy without incurring additional costs.

Willingness to pay for Forecast

As shown in Figure 23, the revenue achieved using actual market prices reflects the oracle benchmark, while the revenue from the moving average (MA) forecast shows the performance under limited price foresight. The widening gap between the two revenue trajectories over the simulation period highlights the cumulative impact of suboptimal bidding decisions due to forecast inaccuracy. This revenue gap represents the marginal benefit a BRP could capture with improved price predictions. For a BRP operating in real-time on aFRR PICASSO, this determines the willingness to pay (WTP) for more accurate forecast models that better capture market dynamics and price volatility, ultimately enabling more profitable and risk-adjusted bidding strategies.

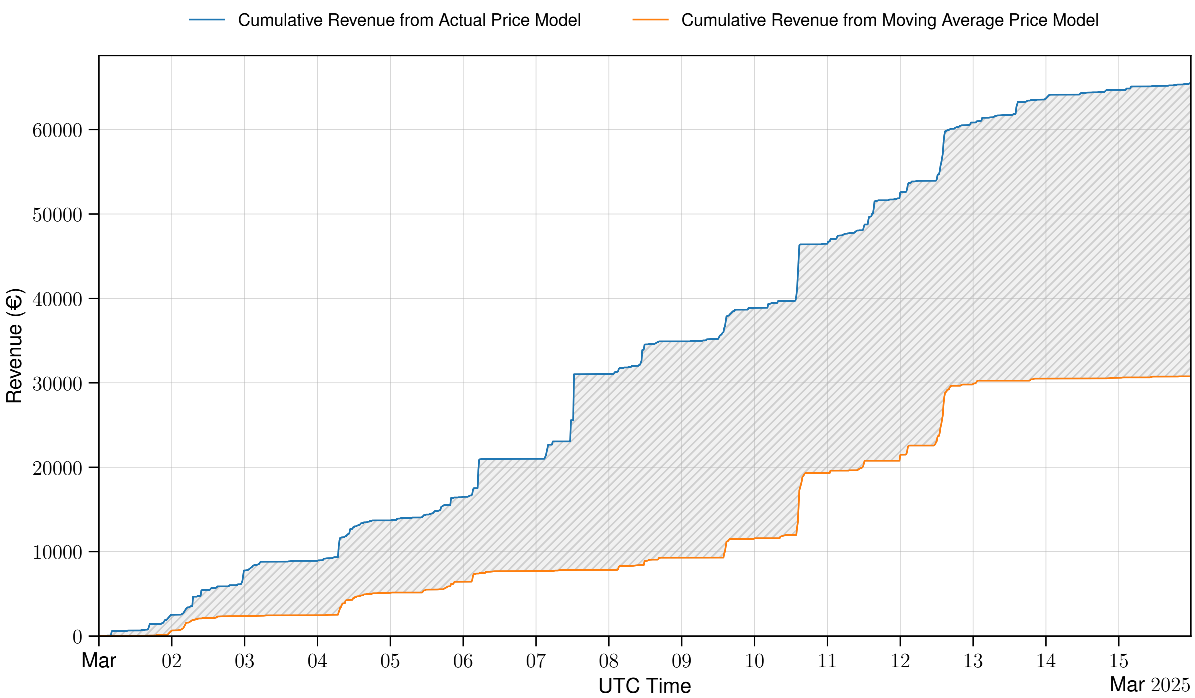


Figure 23: Cumulative revenue from aFRR activation in Base and Moving Average Model over simulation period.

8 Discussion and Limitations

Impact of price forecast on model's performance

As seen from the comparison of the base model to the moving average model, the drop in revenue is significant from both upward and downward regulation. The MA model bids more frequently, but the resulting outcome of the market where bids are accepted is much lower. Particularly the value for per MWh of energy activated is much lower. This disparity is due to the poor price estimation using moving average which does not capture price spikes and reacts to the market price changes with a delay. This means the RTO solves the problem with outdated market conditions to make the bid decisions. Better price forecasts which capture price spikes and overall market behaviour without delay will improve the bid decisions of the model yielding higher bid acceptance. The timing of the bids is crucial, as the aFRR market clears with a fine time resolution, meaning the market changes very quickly and the response of the model must be adequate for better results.

Forecasting the high frequency price signal with standard data-driven models is challenging, as price variations are driven by system-wide imbalances that are not directly observable to market participants. Since the launch of the aFRR market in DK1 in October 2024, prices have shown high volatility, although the magnitude of price spikes has declined in recent months. Both DK1 and DK2 are relatively small bidding zones with a high share of renewables generation mix, which contributed to similar price spikes observed in February and March [46]. These factors makes estimation of clearing prices using historical data as features more difficult for longer periods. The aFRR energy price forecasts will also accumulate errors with increasing length of the prediction horizon.

Even with perfect price forecasts, the revenue from upward regulation is significantly higher than that from downward regulation for similar energy volumes, due to the asymmetric clearing prices in the aFRR energy market. The boiler offers more downward flexibility, as it remains off for extended periods. Notably, negative downward regulation prices at midday coincide with low spot prices, both resulting from surplus power generation. So, it is a trade-off between operating the boiler at low spot prices or aiming for activation in downward regulation to increase consumption at no cost.

The DA operation schedule also impacts the amount of flexibility the boiler can offer throughout the day. Upward regulation is only possible when the boiler is operational, which under the current schedule occurs for short durations at maximum power. If these operational periods do not align with times of upward regulation demand, the boiler misses potential revenue. A revised DA schedule with lower power setpoint over a longer duration can increase the chances of upward activation. In addition, the planning of the operation schedule can be improved by considering the revenue from aFRR capacity market bids.

Overall, the control algorithm gives a net positive outcome by reducing the boiler's operational costs during the simulation period even under poor price forecasts. It increases boiler utilisation, and with the current high aFRR prices, the algorithm offers a viable strategy to optimally use all available flexibility of the boiler while satisfying the heat demand. To further improve the outcome of the model, the BRP has to use better price forecasts to improve accuracy of bids. This will allow the model to capitalise on the sudden price spikes, identifying periods of deficit in the system to gain large revenue for small energy activations. Better price forecasts also reduces the possibility of incurring additional costs due to the boiler's new power setpoint to follow bids submitted without any energy activation.

Effectiveness of fixed price strategy

The model uses a fixed bid price strategy offering upward regulation at the spot price and downward regulation at zero. This is based on the assumption that the boiler's operational costs consists solely of energy procurement cost. Since the aFRR market follows pay-as-clear remuneration, this fixed price approach simplifies the optimisation by reducing the number of decision variables. Moreover, it provides a baseline for analysing the behaviour of the aFRR market prices ahead of optimisation. As shown in the daily trend of aFRR prices in March Figure 12, this deterministic bid price strategy allows to identify periods of upward and downward regulation availability in the market, without relying on the optimisation model. The effectiveness of this approach is evident from the overall reduced operational cost of the boiler through market participation.

The boiler has a large flexibility to offer on the market, especially during summer months when heat demand is low. Since, they do not have strict restrictions on real-time system state as long as the heat demand is met and system limits are respected, the model should prioritise some bids over others to maximise market revenue. In situations where the system state approaches operational limits, the model could allocate its limited flexibility to periods with higher clearing prices. However, this level of strategic bidding is not possible with fixed price strategy, which does not reflect the opportunity costs or adjust to the value of flexibility.

Currently, downward energy activation (i.e. increase of power consumption) is only allowed when the clearing price is negative (TSO pays BRP). However, historical trends show that downward aFRR prices are predominantly positive, except briefly around midday. Here, an alternative bidding approach is to configure bid prices such that downward bids are accepted even when the clearing price is positive, but lower than the prevailing spot market price. In this case, the BRP pays the TSO for the activation, but the cost remains below the spot market rate, thereby reducing the total effective cost. This approach increases market participation, while currently the model has long periods during which no bids are placed. A key limitation is the risk of additional operational costs if the boiler is activated when it would otherwise have remained idle, thereby reducing the net benefit. The strategy also relies on accurate price forecasts to avoid unintended costs. In contrast, the current fixed price strategy is more conservative to account for incorrect price estimations.

Certain assumptions are made to simplify the formulation of the algorithm and so, despite the positive results, the model has these following limitations.

- The RTO model simplifies the boiler's state dynamics as a linear model with no delay in response time. This assumption is reasonable at the 15-minute time resolution, when the response delay is negligible. However, the PICASSO AOF operates at a 4-second resolution and the current model assumes instantaneous response to the activation signals which is not practical. The main limitation in capturing these system dynamics is the absence of the actual activation signal during simulation, instead it is only estimated from the market feedback.
- As seen from the simulation results, the boiler's control input signal has abrupt frequent jumps between power levels which can be undesirable from a operational point of view. This is due to the absence of ramp-rate limits or actuator constraints in the model. Adding penalties for rapid changes or physical constraints would result in smoother and more realistic control signals.
- The RTO is a deterministic model that does not consider demand uncertainty over the prediction horizon. This can lead to over-commitment in the aFRR energy market, potentially disrupting normal boiler operation if actual demand deviates from forecasts.

9 Conclusions and future scope of work

This project explores the main limitations of extracting demand-side flexibility for participation in the frequency regulation services. A standard framework is outlined to evaluate existing demand units to find suitable markets for participation and quantify the energy available to commit as reserve. With an electric boiler as a case study, the framework is applied to find the operational baseline and measure flexibility in real-time.

The objective to optimally utilise the asset to satisfy normal operation and maximise participation on the market is achieved with the receding horizon control scheme. The simulation of the electric boiler trading on the real-time PICASSO platform demonstrates the viability of real-time market integration. The operation costs of the boiler drops significantly with the market integration without any interruption to the heat demand supply.

While the results from the estimated price simulation are significantly lower than those from the perfect price run, the model still shows a net positive outcome. The lost revenues, due to the model's inability to capture market price spikes, can be mitigated with the use of more accurate forecasts. However, this depends on the BRP's willingness to invest in better forecasting tools and whether the potential increase in revenue justifies the additional cost.

Future works will focus on improvements to the model's limitations discussed earlier to consider demand forecast uncertainties, and realistic system dynamics. The deviation from the baseline also has a economic cost due to the intraday market and imbalance settlement. It would be interesting to optimise the RTO taking these costs into consideration, and the impact on the overall operation of the asset. Furthermore, as the boiler qualifies on multiple frequency markets, the goal is to create a multi-market trading algorithm that determines the optimal bid decisions for each specific market.

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