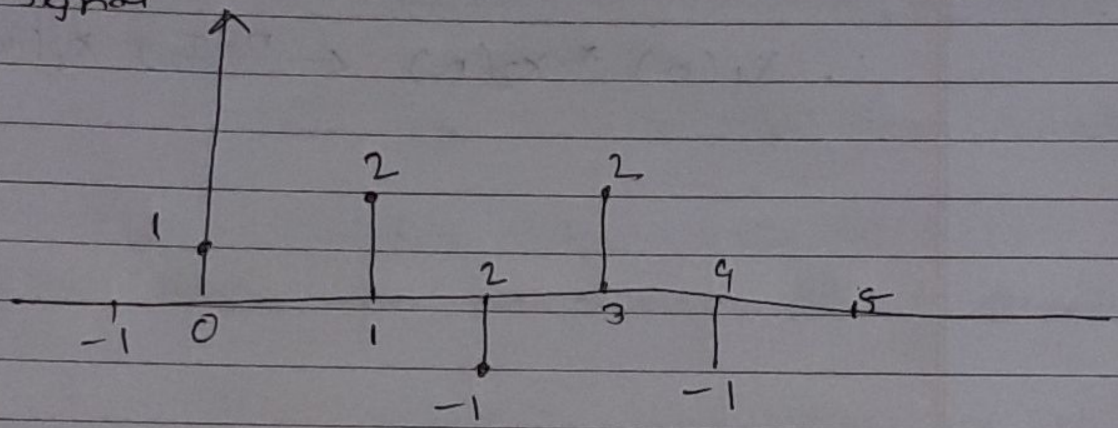
Q.1]
2]

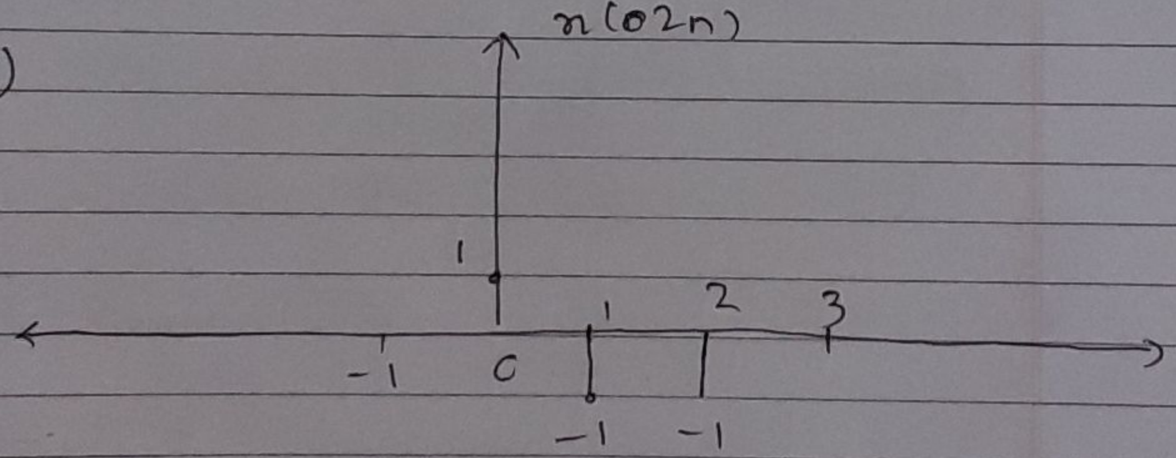
$x(n) = \{1, 2, -1, 2, -1\}$

$x(n) = \{\underset{\uparrow}{1}, 2, -1, 2, -1\}$

So original graphical representation of signal

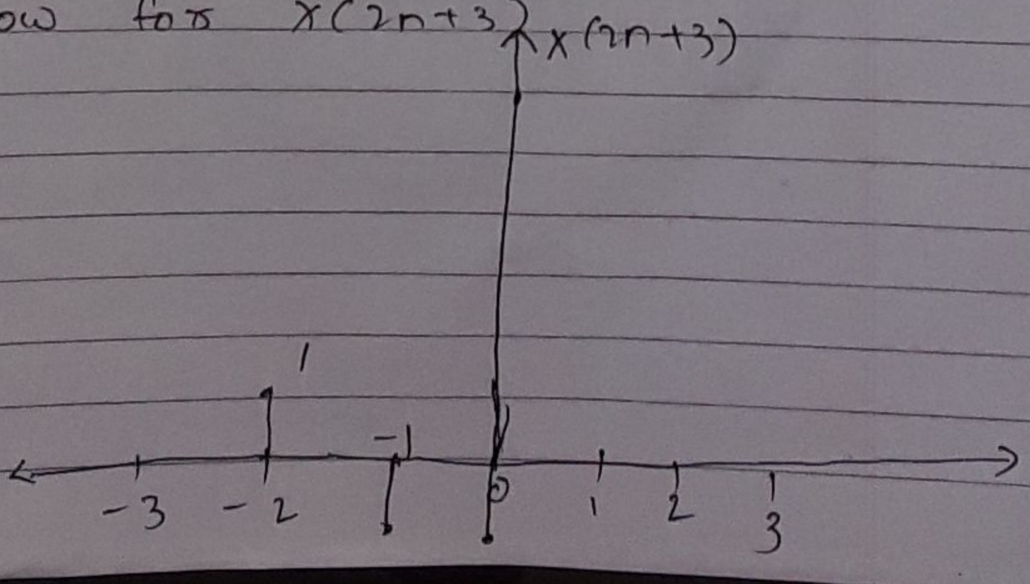


a)

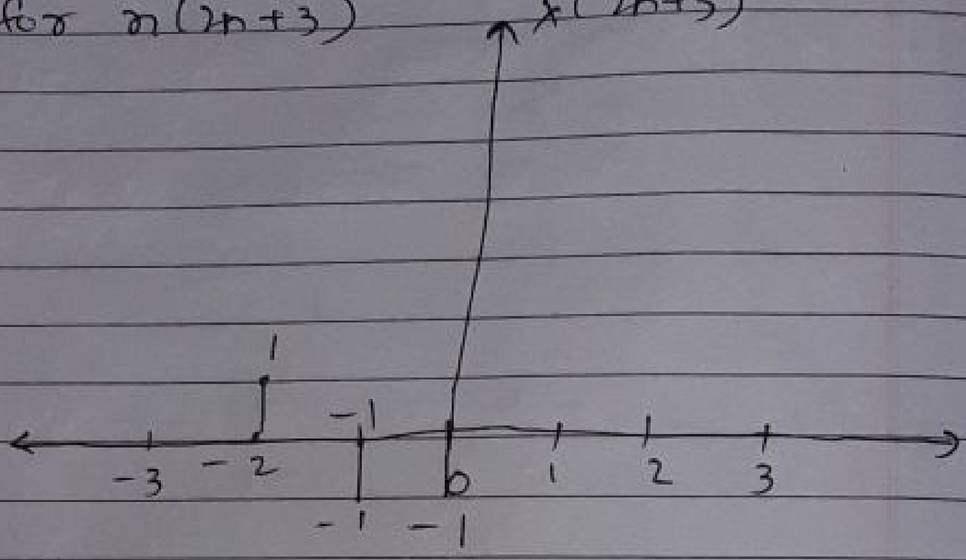


This is $x(2n)$

Now for $x(2n+3)$

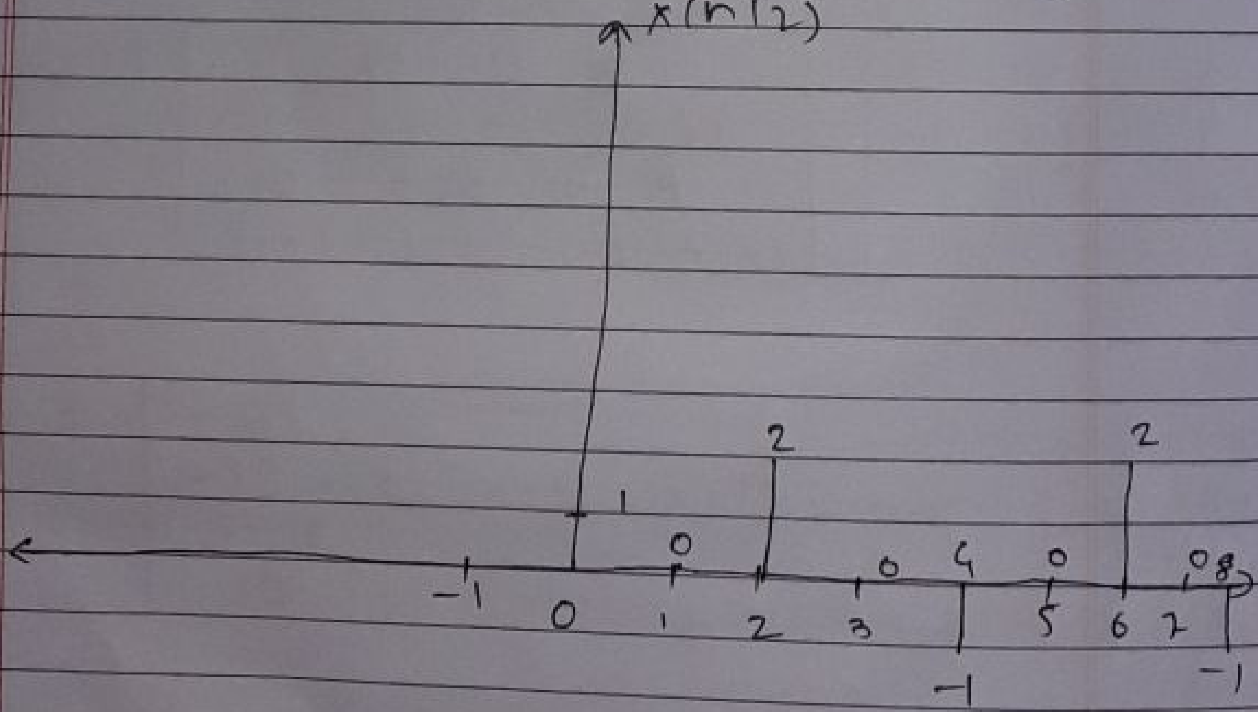


i) Now for $x(2n+3)$



So this is $x(2n+3)$

ii) $x(n/2) = \{1, 0, 2, 0, -1, 0, 2, 0, -1\}$



Q.1]

3) $y[n] = \sum_{i=-\infty}^{\infty} x[i]\,h[n-i]$

$x[n] = \{4, 1, -2, 1\}$ $h[n] = \{3, -1, 1\}$

$$\begin{array}{rrrrrrr}
 & 3 & -1 & 1 & & & \\
\times & & 4 & 1 & -2 & 1 & \\
\hline
 & 12 & -4 & 4 & & & \\
 & & 3 & -1 & 1 & & \\
 & & & -6 & 2 & -2 & \\
+ & & & & 3 & -1 & 1 \\
\hline
 & 12 & -1 & -3 & 6 & -3 & 1
\end{array}$$

$y[n] = \{12, -1, -3, 6, -3, 1\}$

Q.2)
1) DFT of $x(n) = \{2, 1, 1, -1\}$
length $N = 4$
$x(k) = [\omega_4]_{N \times N} \times x(n)$

$$x(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 + 1 + 1 - 1 \\ 2 - j - 1 - j \\ 2 - 1 + 1 + 1 \\ 2 + j - 1 + j \end{bmatrix} = \begin{bmatrix} 3 \\ 1 - 2j \\ 3 \\ 1 + 2j \end{bmatrix}$$

so, DFT of $x(n)$
$x(k) = \{3, 1-2j, 3, 1+2j\}$

for DFT $x(n-1)$
$x'(n) - x(n-1)$ i.e $x(n)$ is shifted by 1

$x(n) \xleftrightarrow{DFT} x(k)$
$x(n-p) \xleftrightarrow{DFT} \bar{e}^{j2\pi pk/N} x(k)$
$x(n-p) \xleftrightarrow{DFT} \omega_n^{pk} x(k)$

so,
$x'(k) = \omega^k x(k)$

$k=0$ $\omega^0 = 1$ so $x'(0) = 3$
$k=1$ $\omega^1 = 1$ $x'(1) = 3$
$k=2$ $\omega^2 = -1$ $x'(2) = -3$
$k=3$ $\omega^3 = j$ $x'(3) = -1$

$x'(n-1) \to x'(k) = \{3, 3, -3, -1\}$

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Convulation property of DFT

If $x_1(n) \xleftrightarrow{DFT} X_1(k)$
and $x_2(n) \xleftrightarrow{DFT} X_2(k)$
then

$$x_1(n) \odot x_2(n) \xleftrightarrow{DFT} X_1(k) . X_2(k)$$

$$X_3(k) = X_1(k)\, X_2(k)$$

Hence $(\odot)$ is symbol used for circular convulation

we find $x_3(n)$ for which DFT is $X_3(k)$.

~~also use Symbol~~ to

Proof:-

From the periodic convulation we know,

$$x_{3p}(n) = \sum_{m=0}^{N-1} x_{1p}(m)\, x_{2p}(n-m)$$

p stands for periodic

i.e

$$x_3((n))_N = \sum_{m=0}^{N-1} x_1((m))_N\, x_2((n-m))_N$$

i.e

$$x_3((n))_N = \sum_{m=0}^{N-1} x_1((m))_N\, x_2((n-m))_N$$

for $0 \le n \le N-1$ (one period)

$$x_3((n))_N = x_3(n),$$

$$x_1((m))_N = x_1(m)$$

$$x_3(n) = \sum_{m=0}^{N-1} x_1(m)\, x_2((n-m))_N$$

RHS represents circular convulation of $x_1(n)$ and $x_2(n)$ which is represented as (⊛)

$\therefore x_3(n) = x_1(n) \circledast x_2(n)$

$DFT(x_3(n)) = DFT\{x_1(n) \circledast x_2(n)]$

$= x_3(k) = X_1(k) \cdot X_2(k)$

$\therefore x_1(n) * x_2(n) \xleftrightarrow{DFT} X_1(k) \cdot X_2(k)$