CS 561 Assignment 2

Due on 13th April 5:30 PM

Exercise 1: The objective of this exercise is to understand the Metropolis-Hastings algorithm, a Markov chain Monte Carlo (MCMC) method for sampling.

Consider the following distribution

$$P(x) = \frac{\exp(-x^4)\ (2 + \sin(5x) + \sin(-2x^2))}{\int_{-\infty}^{\infty} \exp(-x'^4)\ \left(2 + \sin(5x') + \sin\left(-2x'^2\right)\right)dx'}$$

Assume that the integration is difficult to solve and you know that

$$P(x) \propto \exp(-x^4)\ (2 + \sin(5x) + \sin(-2x^2))$$

The distribution is shown in Figure 1. Generate samples from this distribution using Metropolis-Hastings algorithm with normal distribution as proposal distribution.

- Generate the candidate using normal distribution with the current state as mean of the distribution i.e. $x^*|x_n \sim Normal(x_n, \sigma^2)$.

- Set $x_0 = -1$, generate 1500 samples for three different values of σ $(low = .05,\ medium = 1,$ and $high = 50)$. Plot the histogram of the generated samples and compare with actual distribution for each of the σ values, and also plot the generated sample versus iteration (the actual Markov chain-the sequence of generated values) for each of the σ values.

- Submit your code and a report (hard copy not more than one page –both sides printed) that should have the plots and the conclusions drawn from the plots.

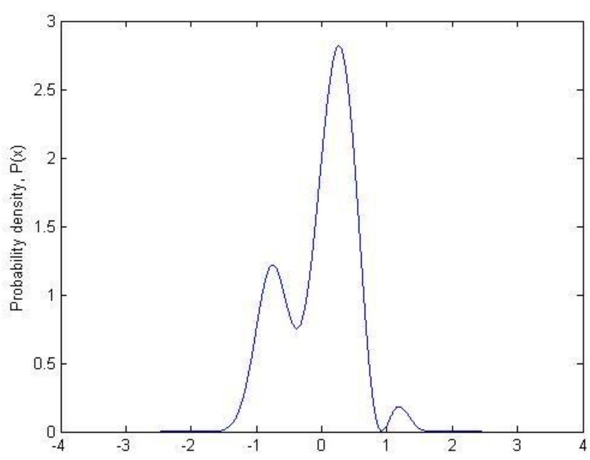

Figure 1. The distribution from which samples are required to be drawn

Exercise 2: The objective of this exercise is to apply Hidden Markov Models to localization problem. Consider a robot with the task of localization (inferring where it is from the available data) given a map of the world and a sequence of percepts and actions. The robot is placed in a maze-like environment as shown in Figure 2.

Figure 2. The robot environment

The robot is equipped with four sonar sensors that tell whether there is an obstacle- the outer wall or a shaded square in the figure – in each of the compass directions (NSEW). We assume that the robot has a correct map. The robot performs action $Move$ to move to one of the adjacent or neighbouring square.

X_t : state variable representing the location of the robot on the discrete grid.

$dom(X_t) = \{s_1, \dots, s_n\}$: domain of X_t the set of empty squares.

$NEIGHBOURS(s)$: the set of empty squares that are adjacent to s and let N(s) be the size of that set.

The transition model for $Move$ action is given as

$$P(X_{t+1} = j \mid X_t = i) = \boldsymbol{T}_{ij\,=} = \left(\frac{1}{N(i)} \;\; if \; \in NEIGHBOURS(I) \; else(0)\right)$$

Assume uniform distribution over all the squares; $P(X_0 = i) = 1/n.$

E_t: sensor variable that can have 16 possible values, each a four-bit sequence giving the presence or absence of an obstacle in a particular compass direction.

The observation model can be given as

$$P(E_t = e_t \mid X_t = i) = \boldsymbol{O}_{t_{ii}} = (1-\epsilon)^{4-d_{it}} \;\; \epsilon^{d_{it}}$$

ϵ is the sensor's error rate and errors occur independently for the four sensor directions, $(1-\epsilon)^4$ is the probability of getting all the four bits right and the probability of getting all of them wrong is $\epsilon^{d_{it}}$, and d_{it} is the discrepancy that is the number of bits that are different – between true values for square i and the actual reading e_t from the sensor.

(a) First the robot needs to estimate its current location and then determine the most likely path it has taken to get where it is now for a given time t.

(b) Repeat (a) and find out the localization error as a function of number of observations for various values of ϵ (0.00, 0.02, 0.05, 0.10, 0.20) and plot them. Localization error is Manhattan distance from the true location.

(c) Repeat (b) for path accuracy, which is defined as the fraction of correct states on the Viterbi path.

(You can find more details on Exercise 2 on page number 591-593, Chapter 15 , Russell and Norvig book , Third edition, Indian edition)

Submit your code and a report (hard copy not more than one page –both sides printed) that should have the plots and the conclusions drawn from the plots.