

# Lesson 6: Image Pyramids, Wavelets and Parametric Transformations

## Multi-Resolution Analysis and Global Transformations

AI 879: Machine Vision

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# Today's Agenda

- 1 Introduction & Motivation
- 2 Image Pyramids
  - Template Matching and Correlation
  - Gaussian Image Pyramids
  - Laplacian Image Pyramids
  - Application: Image Compression
- 3 Wavelet Transforms
- 4 Parametric Transformations
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# Introduction & Motivation

# Welcome to Lesson 6

## From Single-Scale to Multi-Scale

### Lesson 5 Recap:

- Point operators (single pixel); Linear filtering (neighborhood); Frequency analysis (global)
- All at **one scale**

### The Challenge

- Objects appear at different sizes
- Need scale-invariant detection
- Efficient multi-resolution analysis
- Image blending and compression

### Today's Topics

- Image Pyramids
- Wavelet Transforms
- Parametric Transformations
- Image Blending

# Learning Objectives

By the end of this lesson, you will be able to:

① **Define and construct** image pyramids:

- Gaussian pyramids
- Laplacian pyramids
- Steerable pyramids

② **Understand** wavelet transforms:

- Continuous vs. discrete wavelets
- Multi-resolution decomposition
- Denoising applications

③ **Apply** parametric transformations:

- Translation, rotation, scaling
- Affine transformations
- Matrix representations

④ **Implement** image blending algorithms

⑤ **Code** these methods in MATLAB

# The Scale Problem

## Motivating Example: Object Detection

**Problem:** Find sunflowers in a field

### Single-Scale Approach:

- Create a template ( $46 \times 44$  pixels)
- Use correlation to find matches
- **Problem:** Only finds sunflowers of similar size!

### Multi-Scale Solution:

- Create image pyramid
- Search at each scale
- Find sunflowers of all sizes

### Why Not Resize Template?

- More computationally expensive
- Template smaller than image
- Better to downsample large image

### Efficiency

Image:  $960 \times 594$  pixels

Template:  $46 \times 44$  pixels

Downsampling image is faster!

# Applications of Multi-Scale Analysis

## Where are image pyramids used?

### Computer Vision:

- Object detection
- Face recognition
- Feature matching
- Motion estimation

### Image Processing:

- Image blending
- Panorama stitching
- HDR imaging
- Texture synthesis

### Compression:

- JPEG 2000
- Progressive images
- Multi-resolution storage
- Efficient transmission

## Real-World Impact

Image pyramids are fundamental to modern computer vision systems, from smartphone cameras to autonomous vehicles!

# Image Pyramids



# What are Image Pyramids?

## Definition

**Image pyramids** are multi-resolution representations of images, where each level represents the image at a different scale.

## Structure:

- Base level: Original image (highest resolution)
- Each higher level: Reduced resolution (typically 50%)
- Pyramid gets smaller → fewer pixels at top

## Three Types:

- ① **Gaussian Pyramid:** Smoothed and downsampled versions
- ② **Laplacian Pyramid:** Difference images (bandpass filtered)
- ③ **Steerable Pyramid:** Adds orientation information

# The Sunflower Detection Example



**Key Observation:** Sunflowers at different distances require different scales for detection

# Normalized Cross-Correlation

**How do we find objects in an image?**

**Normalized Cross-Correlation:**

$$c(u, v) = \frac{\sum_{x,y} [I(x, y) - \bar{I}] \cdot [T(x - u, y - v) - \bar{T}]}{\sqrt{\sum_{x,y} [I(x, y) - \bar{I}]^2 \cdot \sum_{x,y} [T(x - u, y - v) - \bar{T}]^2}}$$

where:

- $I$  = input image
- $T$  = template
- $\bar{I}, \bar{T}$  = means
- $c(u, v)$  = correlation at position  $(u, v)$

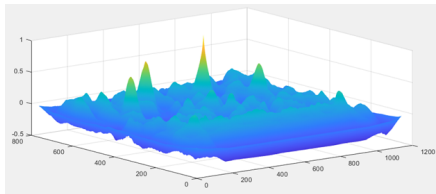
**Properties:**

- Values range:  $[-1, 1]$
- $c = 1$ : Perfect match
- $c \approx 0$ : No correlation
- Invariant to brightness changes

# Template Matching: MATLAB Code

```
1  I = im2gray(imread('L06 sunflower.png'));
2  T = im2gray(imread('L06 sunflower Template.png'));
3  c = normxcorr2(T, I); % Calculate normalized cross-correlation
4  % Visualize correlation surface
5  figure;
6  surf(c);
7  shading flat;
8  title('Correlation Surface');
9  % Find local maxima above threshold and display
10 threshold = 0.5;
11 [yvals, xvals] = find(c > threshold * max(c(:)) & ...
12 islocalmax(c, 1) & islocalmax(c, 2));
13 figure; imshow(I);
14 for i = 1:length(yvals)
15     yoffSet = yvals(i) - size(T,1);
16     xoffSet = xvals(i) - size(T,2);
17     rectangle('Position', [xoffSet, yoffSet, ...
18 size(T,2), size(T,1)], 'EdgeColor', [1 0 0]);
19 end
```

# Correlation Results



## Observation:

- 4 correlation peaks found (0.51-1.0); Each peak corresponds to a detected sunflower
- One false detection near the edge (proximity issue)

# Gaussian Pyramid Construction

## Algorithm:

- 1 Start with original image (Level 0)
- 2 For each subsequent level:
  - Apply Gaussian blur (smooth)
  - Downsample by factor of 2 (reduce size)
- 3 Repeat until desired number of levels

## Why Gaussian Blur?

- Prevents aliasing
- Removes high frequencies before downsampling
- Satisfies Nyquist-Shannon sampling theorem

## Key Properties:

- Each level is smoothed version of previous
- Default: downsample by factor of 2
- Pyramid height determined by image size

# Gaussian Pyramid: MATLAB Implementation

```
1  I = imread('L06_sunflower.png');
2  % Create Gaussian pyramid with 4 levels
3  level1 = I; % Original
4  level2 = impyramid(level1, 'reduce'); % 50% size
5  level3 = impyramid(level2, 'reduce'); % 25% size
6  level4 = impyramid(level3, 'reduce'); % 12.5% size
7  % Display pyramid
8  figure;
9  subplot(2,2,1); imshow(level1); title('Level 1 (Original)');
10 subplot(2,2,2); imshow(level2); title('Level 2 (50%)');
11 subplot(2,2,3); imshow(level3); title('Level 3 (25%)');
12 subplot(2,2,4); imshow(level4); title('Level 4 (12.5%)');
13 % Alternative: Use cell array
14 pyramid = cell(1, 4);
15 pyramid{1} = I;
16 for k = 2:4
17     pyramid{k} = impyramid(pyramid{k-1}, 'reduce');
18 end
19
```

# Gaussian Pyramid Visualization

Level 1 (Original)



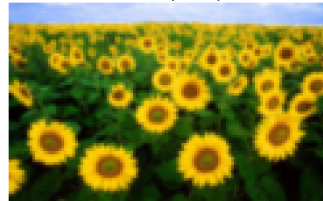
Level 2 (50%)



Level 3 (25%)



Level 4 (12.5%)





# The `impyramid` Function

## MATLAB's `impyramid`:

```
J = impyramid(I, direction)
```

### Parameters:

- `I`: Input image
- `direction`:
  - `'reduce'`: Smooth and downsample (default factor: 2)
  - `'expand'`: Upsample and smooth

### What it does internally:

- ① Applies Gaussian filter ( $5 \times 5$  kernel)
- ② For `'reduce'`: Downsamples by keeping every other pixel
- ③ For `'expand'`: Upsamples by inserting zeros, then filters

## Important

`impyramid` automatically handles the Gaussian smoothing. Don't blur before calling it!

# Multi-Scale Object Detection

```
1  I = im2gray(imread('L06_sunflower.png'));
2  T = im2gray(imread('L06_sunflower_Template.png'));
3  T = imgaussfilt(T, 2);
4  T = imresize(T, 0.5, 'nearest'); % Smaller template
5
6  figure; imshow(I);
7  for iter = 1:3 % 3 pyramid levels
8      if iter == 1
9          I_tmp = imgaussfilt(I, 2); % Smooth original
10         else
11             I_tmp = impyramid(I_tmp, 'reduce'); % Downsample
12         end
13
14         % Find matches at this scale
15         c = normxcorr2(T, I_tmp);
16         [yvals, xvals] = find(c > 0.5 & islocalmax(c,1) & islocalmax(c,2));
17
18         % Draw rectangles (scaled back to original size)
19         for x = 1:length(yvals)
20             yoffSet = iter * (yvals(x) - size(T,1));
21             xoffSet = iter * (xvals(x) - size(T,2));
22             rectangle('Position', [xoffSet, yoffSet, ...
23                 iter*size(T,2), iter*size(T,1)], ...
24                 'EdgeColor', [1 0 0], 'LineWidth', 2);
25         end
26     end
27
```

# Multi-Scale Detection Results



# From Gaussian to Laplacian

## Limitation of Gaussian Pyramids:

- Only store smoothed versions
- Lose high-frequency information
- Inefficient for reconstruction

## Laplacian Pyramid Solution:

- Store **differences** between levels
- Capture information lost during downsampling
- Enable perfect reconstruction
- Useful for compression and blending

## Construction:

$$L_i = G_i - \text{upsample}(G_{i+1})$$

where:

- $L_i$  = Laplacian at level  $i$
- $G_i$  = Gaussian at level  $i$
- Except top level:  $L_n = G_n$

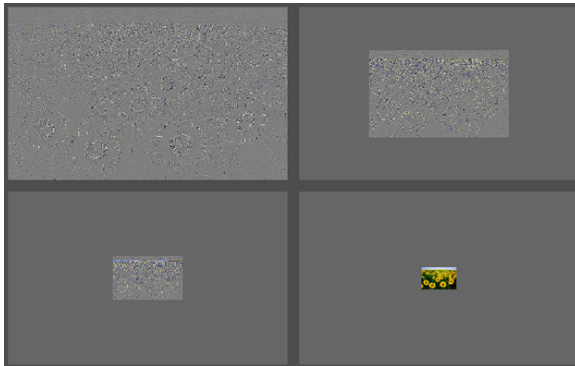
# Laplacian Pyramid: Construction Code

```
1 function lapp = laplacianPyramid(gaussian_pyramid)
2     num_levels = numel(gaussian_pyramid);
3     lapp = cell(1, num_levels);
4     % Process all levels except the last
5     for k = 1:(num_levels - 1)
6         A = gaussian_pyramid{k};
7         B = imresize(gaussian_pyramid{k+1}, 2, 'bilinear');
8         % Crop B to match A's size (handle size mismatch)
9         [M, N, ~] = size(A);
10        lapp{k} = A - B(1:M, 1:N, :);
11    end
12    % Top level is just the Gaussian
13    lapp{num_levels} = gaussian_pyramid{num_levels};
14 end
15
```

## Key Steps:

- Expand (upsample) next coarser level
- Subtract from current level
- Store difference (the Laplacian)

# Laplacian Pyramid Visualization



**Note:** Laplacian images appear mostly gray because differences are small

# Reconstruction from Laplacian Pyramid

## Perfect Reconstruction:

Starting from the top (coarsest level), work down:

$$G_i = L_i + \text{upsample}(G_{i+1})$$

```
1 function out = reconstructFromLaplacian(lapp)
2     num_levels = numel(lapp);
3     out = lapp{end}; % Start with top level
4
5     % Work backwards through pyramid
6     for k = (num_levels - 1) : -1 : 1
7         % Upsample current reconstruction
8         out = imresize(out, 2, 'bilinear');
9
10        % Add Laplacian at this level
11        g = lapp{k};
12        [M, N, ~] = size(g);
13        out = out(1:M, 1:N, :) + g;
14    end
15 end
16
```

**Result:** Exactly reconstructs the original image (within numerical precision)

# Laplacian Pyramids for Compression

**Idea:** Quantize Laplacian coefficients to reduce file size

**Why it works:**

- Most Laplacian values are small (near zero)
- Can round to fewer decimal places
- Loses some detail but maintains structure

**Simple Quantization:**

```
1 function lapp_quantized = roundPixelValues(lapp)
2     for k = 1:(numel(lapp) - 1)
3         % Round to 1 decimal place
4         lapp_quantized{k} = round(lapp{k}, 1);
5     end
6     % Keep top level unchanged
7     lapp_quantized{numel(lapp)} = lapp{end};
8 end
9
```



# Compression Results



**Key Result:** 36% size reduction with minimal visual quality loss!

# Complete Laplacian Compression Example

```
1 % Read image and create Gaussian pyramid
2 I = imread('L06_sunflower.png');
3 mrp = multiresolutionPyramid(I, 4);
4
5 % Create Laplacian pyramid
6 lapp = laplacianPyramid(mrp);
7
8 % Quantize (compress)
9 lapp_quantized = roundPixelValues(lapp);
10
11 % Reconstruct
12 out = reconstructFromLaplacianPyramid(lapp_quantized);
13
14 % Compare file sizes
15 imwrite(I, 'original.png');
16 imwrite(out, 'compressed.png');
17 info_orig = dir('original.png');
18 info_comp = dir('compressed.png');
19 reduction = (1 - info_comp.bytes/info_orig.bytes) * 100;
20 fprintf('Size reduction: %.1f%%\n', reduction);
21
22 % Display
23 subplot(2,1,1); imshow(I); title('Original');
24 subplot(2,1,2); imshow(out); title('Compressed & Reconstructed');
25
```

# Wavelet Transforms

# Why Wavelets?

## Limitations of Fourier Transform:

- Global frequency analysis
- No time/spatial localization
- Fixed resolution across frequencies

## Wavelets Advantages:

- Time-frequency localization
- Multi-resolution analysis
- Better for non-stationary signals
- Efficient for compression

## Analogy

**Fourier:** Decomposes signal into sine waves

**Wavelets:** Decomposes signal into scaled and shifted wavelets

# What is a Wavelet?

## Definition

A **wavelet** is a rapidly decaying, wave-like oscillation with zero mean.

## Properties:

- Localized in both time and frequency
- Different wavelets for different applications
- Orthogonal or biorthogonal

## Continuous Wavelet Transform (CWT):

$$W(a, b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} f(t) \psi^* \left( \frac{t - b}{a} \right) dt$$

where:

- $a$  = scale parameter
- $b$  = translation parameter
- $\psi$  = mother wavelet

# Discrete Wavelet Transform (DWT)

## For Images:

### 2D DWT Process:

- ① Apply 1D DWT to each row
- ② Apply 1D DWT to each column
- ③ Results in 4 subbands:
  - LL: Low-low (approximation)
  - LH: Low-high (horizontal details)
  - HL: High-low (vertical details)
  - HH: High-high (diagonal details)

### Multi-Level Decomposition:

- Recursively decompose LL subband
- Creates multi-resolution representation
- Similar to pyramid structure

# Wavelet Families

## Common Wavelets in MATLAB:

Family	Type	Examples
Daubechies	Orthogonal	db1 (Haar), db2, ..., db45
Symlets	Orthogonal	sym2, sym4, ..., sym45
Coiflets	Orthogonal	coif1, coif2, ..., coif5
BiorSplines	Biorthogonal	bior1.1, bior2.2, bior3.3
Fejér-Korovkin	Orthogonal	fk4, fk6, fk8, fk14, fk22

## Choosing a Wavelet:

- Daubechies: General purpose, good compression
- Symlets: More symmetric than Daubechies
- Biorthogonal: Perfect reconstruction, linear phase
- Application-specific: Experiment to find best

# Application: Image Denoising

## Wavelet Denoising Principle:

- ① **Decompose:** Apply DWT to noisy image
- ② **Threshold:** Remove small wavelet coefficients (noise)
- ③ **Reconstruct:** Apply inverse DWT

## Why it works:

- Signal energy concentrated in large coefficients
- Noise spread across all coefficients
- Thresholding removes noise while preserving signal

## MATLAB Function:

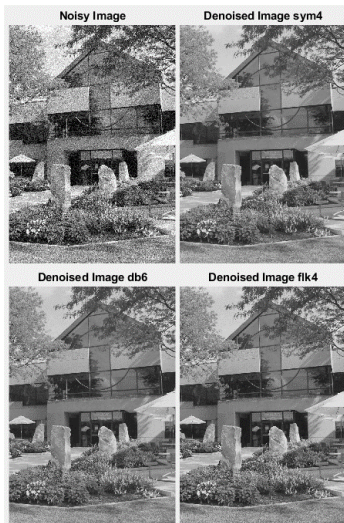
- `wdenoise2`: 2D wavelet denoising
- Automatic threshold selection
- Multiple wavelet options



# Wavelet Denoising: MATLAB Code

```
1  I = imread('L01 greatvalley.jpg');
2  J = imnoise(I, 'gaussian'); % Add Gaussian noise
3  % Denoise with different wavelets
4  out1 = wdenoise2(J); % Default: sym4, level 7
5  out2 = wdenoise2(J, 4, Wavelet="db6"); % Level 4, db6
6  out3 = wdenoise2(J, 3, Wavelet="fk14"); % Level 3, fk14
7  % Display results
8  figure;
9  subplot(2,2,1); imshow(J); title('Noisy Image');
10 subplot(2,2,2); imshow(out1, []); title('sym4 (default)');
11 subplot(2,2,3); imshow(out2, []); title('db6, level 4');
12 subplot(2,2,4); imshow(out3, []); title('fk14, level 3');
13 % Compare PSNR
14 psnr1 = psnr(out1, I);
15 psnr2 = psnr(out2, I);
16 psnr3 = psnr(out3, I);
17 fprintf('PSNR: sym4=%.2f, db6=%.2f, fk14=%.2f\n', ...
18 psnr1, psnr2, psnr3);
19
```

# Wavelet Denoising Results



# Decomposition Levels

## Choosing the Number of Levels:

### Maximum levels:

$$L_{\max} = \lfloor \log_2(\min(M, N)) \rfloor$$

where  $M \times N$  is image size

### For Great Valley image (960×594):

- Maximum:  $\lfloor \log_2(594) \rfloor = 9$
- Default: 7 (automatically chosen by MATLAB)
- Higher levels: More smoothing
- Lower levels: More detail preservation

### Trade-offs:

- More levels: Better denoising, possible oversmoothing
- Fewer levels: More details, possible residual noise
- Optimal level depends on noise level and image content

# Wavelets vs. Pyramids

Aspect	Pyramids	Wavelets
<b>Decomposition</b>	Isotropic (no direction)	Directional (H, V, D)
<b>Reconstruction</b>	Approximate (Gaussian) or exact (Laplacian)	Exact (orthogonal)
<b>Compression</b>	Good for images	Excellent (JPEG 2000)
<b>Computation</b>	Simple, fast	More complex
<b>Applications</b>	Blending, detection	Denoising, compression
<b>Flexibility</b>	Limited wavelet choice	Many wavelet families

## When to Use Which?

**Pyramids:** Real-time processing, object detection, blending

**Wavelets:** Compression, denoising, feature extraction

# Parametric Transformations

# What are Parametric Transformations?

## Definition

**Parametric transformations** apply global deformations to images, controlled by a small number of parameters.

## Key Characteristic:

- Apply to the **entire image**
- Defined by **matrix multiplication**
- Preserve certain properties (lines, parallelism, etc.)

## Common Types:

- Translation (2 parameters)
- Rotation (1 parameter)
- Scaling (2 parameters)
- Shear (2 parameters)
- Reflection (1 parameter)

# Homogeneous Coordinates

## Why $3 \times 3$ matrices for 2D transformations?

**Homogeneous coordinates:**

$$\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

## Advantages:

- Allows translation as matrix multiplication
- Can combine transformations by matrix multiplication
- Uniform representation for all transformations

## General Form:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & t_x \\ c & d & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

# Translation

Move image by  $(t_x, t_y)$ :

$$T = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

**Effect:**

- $x' = x + t_x$
- $y' = y + t_y$
- Shifts entire image

**Parameters:**

- $t_x$ : horizontal displacement
- $t_y$ : vertical displacement
- Can be positive or negative



# Scaling

**Scale by factors  $(s_x, s_y)$ :**

$$S = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**Effect:**

- $x' = s_x \cdot x$
- $y' = s_y \cdot y$
- Scales about origin

**Special Cases:**

- $s_x = s_y$ : Uniform scaling
- $s_x = s_y > 1$ : Enlargement
- $s_x = s_y < 1$ : Reduction
- $s_x \neq s_y$ : Non-uniform (aspect ratio change)

# Rotation

Rotate by angle  $\theta$  about origin:

$$R = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**Effect:**

- $x' = x \cos \theta - y \sin \theta$
- $y' = x \sin \theta + y \cos \theta$
- Counterclockwise rotation

**Properties:**

- Preserves distances
- Preserves angles
- Orthogonal matrix:  $R^T = R^{-1}$

# Shear

## Shear transformations:

### Horizontal shear:

$$\text{Shear}_x = \begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

### Vertical shear:

$$\text{Shear}_y = \begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

### Effect:

- Skews the image
- Preserves areas
- Creates parallelogram from rectangle

# Reflection

## Mirror transformations:

### Vertical reflection (flip up-down):

$$\text{Reflect}_v = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

### Horizontal reflection (flip left-right):

$$\text{Reflect}_h = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

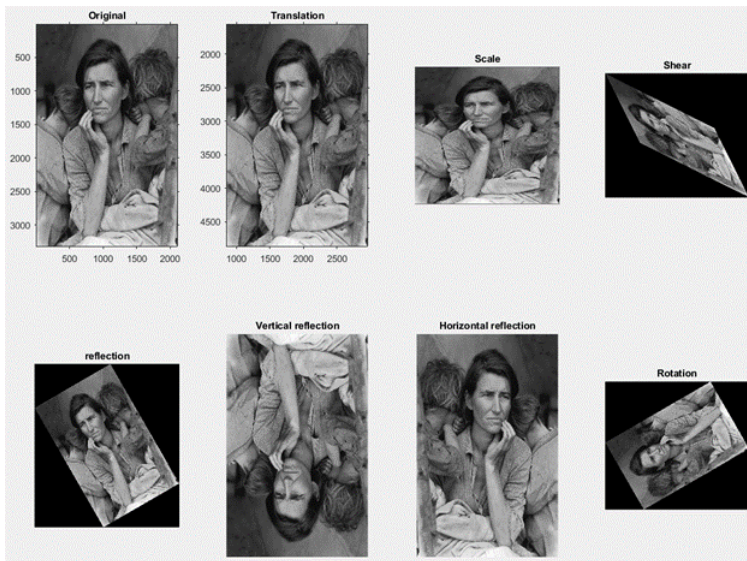
### Reflection about arbitrary line (angle $\phi$ ):

$$\text{Reflect}_\phi = \begin{bmatrix} \cos(2\phi) & \sin(2\phi) & 0 \\ \sin(2\phi) & -\cos(2\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

# MATLAB Implementation: affine2d

```
1 % Load image
2 I = imread('L01 Migrant Mother.png');
3 I = im2double(I);
4 % Translation
5 T = [1 0 0; 0 1 0; 850 1550 1];
6 tform = affine2d(T);
7 J1 = imwarp(I, tform);
8 % Scaling
9 T = [0.5 0 0; 0 0.3 0; 0 0 1];
10 tform = affine2d(T);
11 J2 = imwarp(I, tform);
12 % Rotation (30 degrees)
13 theta = 30;
14 T = [cosd(theta) -sind(theta) 0; sind(theta) cosd(theta) 0; 0 0 1];
15 tform = affine2d(T);
16 J3 = imwarp(I, tform);
17 % Shear
18 T = [1 2 0; 2 1 0; 0 0 1];
19 tform = affine2d(T);
20 J4 = imwarp(I, tform);
21 % Display all
22 subplot(2,3,1); imshow(I); title('Original');
23 subplot(2,3,2); imshow(J1); title('Translation');
24 subplot(2,3,3); imshow(J2); title('Scale');
25 subplot(2,3,4); imshow(J3); title('Rotation');
26 subplot(2,3,5); imshow(J4); title('Shear');
```

# Parametric Transformation Results



## Homework 6 Discussion

# Homework 6: Image Pyramids

**Objective:** Demonstrate understanding of Gaussian and Laplacian pyramids

## Task 1: Modify Gaussian Pyramid Code

Make the following modifications and discuss results:

- **A.** Increase input image size by 150%
- **B.** Keep template size at 50% of original
- **C.** Increase iterations to 5
- **D.** Replace `impyramid` with `imresize` at factor 0.8

## Task 2: Image Blending

- Implement blending algorithm (Szeliski 3.5.5)
- Choose any two images
- Create smooth composite



## Summary & Knowledge Check

# Lesson Summary

## Key Concepts Covered:

### ① Image Pyramids:

- Gaussian: Multi-resolution representation
- Laplacian: Bandpass decomposition
- Applications: Detection, compression, blending

### ② Wavelet Transforms:

- Time-frequency localization
- Multi-resolution analysis
- Applications: Denoising, compression

### ③ Parametric Transformations:

- Affine transformations (6 DOF)
- Matrix representations
- Combining transformations

### ④ Applications:

- Scale-invariant object detection
- Seamless image blending
- Image denoising

# Knowledge Check - Question 1

**Question:** What is the main advantage of using a Gaussian pyramid for object detection compared to a single-scale approach?

- ☐ A) It makes the image look better
- ☐ B) It allows detection of objects at multiple scales
- ☐ C) It reduces image noise
- ☐ D) It increases image resolution

# Knowledge Check - Question 1

**Question:** What is the main advantage of using a Gaussian pyramid for object detection compared to a single-scale approach?

- ☐ A) It makes the image look better
- ☐ B) It allows detection of objects at multiple scales
- ☐ C) It reduces image noise
- ☐ D) It increases image resolution

## Answer

**B) It allows detection of objects at multiple scales**

Gaussian pyramids create versions of the image at different resolutions, enabling detection of objects regardless of their size in the image. A single-scale template would only match objects of a specific size.

## Knowledge Check - Question 2

**Question:** In a Laplacian pyramid, what does each level (except the top) represent?

- ☐ A) A smoothed version of the image
- ☐ B) The difference between adjacent Gaussian pyramid levels
- ☐ C) The average of two pyramid levels
- ☐ D) Random noise

## Knowledge Check - Question 2

**Question:** In a Laplacian pyramid, what does each level (except the top) represent?

- A) A smoothed version of the image
- B) The difference between adjacent Gaussian pyramid levels
- C) The average of two pyramid levels
- D) Random noise

### Answer

**B) The difference between adjacent Gaussian pyramid levels**

Each Laplacian level stores the difference between a Gaussian level and an upsampled version of the next coarser level:  $L_i = G_i - \text{upsample}(G_{i+1})$ . This captures bandpass-filtered information.

## Knowledge Check - Question 3

**Question:** Why is it important to apply Gaussian blur before downsampling an image?

- A) To make the image prettier
- B) To prevent aliasing artifacts
- C) To increase computation speed
- D) To enhance edges

## Knowledge Check - Question 3

**Question:** Why is it important to apply Gaussian blur before downsampling an image?

- A) To make the image prettier
- B) To prevent aliasing artifacts
- C) To increase computation speed
- D) To enhance edges

### Answer

**B) To prevent aliasing artifacts**

Gaussian blurring acts as a low-pass filter, removing high frequencies before downsampling. This prevents aliasing (high frequencies appearing as low frequencies), following the Nyquist-Shannon sampling theorem.



## Knowledge Check - Question 4

**Question:** What is the key advantage of wavelets over Fourier transforms for image analysis?

- Ⓐ) Wavelets are always faster
- Ⓑ) Wavelets provide time-frequency localization
- Ⓒ) Wavelets only work in 1D
- Ⓓ) Wavelets require less memory

## Knowledge Check - Question 4

**Question:** What is the key advantage of wavelets over Fourier transforms for image analysis?

- ☐ A) Wavelets are always faster
- ☐ B) Wavelets provide time-frequency localization
- ☐ C) Wavelets only work in 1D
- ☐ D) Wavelets require less memory

### Answer

**B) Wavelets provide time-frequency localization**

Unlike Fourier transforms which provide only frequency information, wavelets decompose signals with both temporal/spatial and frequency localization. This makes them better for analyzing non-stationary signals and localized features.

# Knowledge Check - Question 5

**Question:** Which transformation preserves parallel lines?

- Ⓐ Only translation
- Ⓑ Only rotation
- Ⓒ All affine transformations
- Ⓓ No transformations preserve parallel lines

## Knowledge Check - Question 5

**Question:** Which transformation preserves parallel lines?

- ☐ A) Only translation
- ☐ B) Only rotation
- ☐ C) All affine transformations
- ☐ D) No transformations preserve parallel lines

### Answer

**C) All affine transformations**

Affine transformations (including translation, rotation, scaling, and shear) preserve parallelism. Lines that are parallel before an affine transformation remain parallel after. This is one of the defining properties of affine transformations.

## Knowledge Check - Question 6

**Question:** In the `impyramid('reduce')` function, by what factor is the image downsampled?

- ☐ A) 0.5 (half the size)
- ☐ B) 0.75 (three-quarters)
- ☐ C) 0.8 (80%)
- ☐ D) 0.25 (quarter size)

## Knowledge Check - Question 6

**Question:** In the `impyramid('reduce')` function, by what factor is the image downsampled?

- ☐ A) 0.5 (half the size)
- ☐ B) 0.75 (three-quarters)
- ☐ C) 0.8 (80%)
- ☐ D) 0.25 (quarter size)

### Answer

**A) 0.5 (half the size)**

The `impyramid('reduce')` function downsamples by a factor of 2 in each dimension, resulting in an image that is 50% the width and 50% the height (25% the total pixels) of the original. It also applies Gaussian smoothing automatically.

# Thank You!

Good Luck with Homework 6!

See You Next Time!

**Next Lesson:** Points and Matches