Sardar Patel Institute of Technology

Bhavan's Campus, Munshi Nagar, Andheri (W), Mumbai : 400058, India

(Autonomous College of Affiliated to University of Mumbai)

End Semester Examination
December 2022

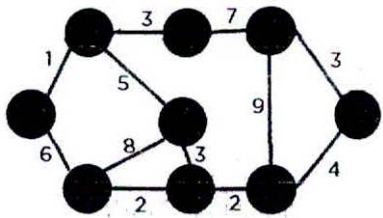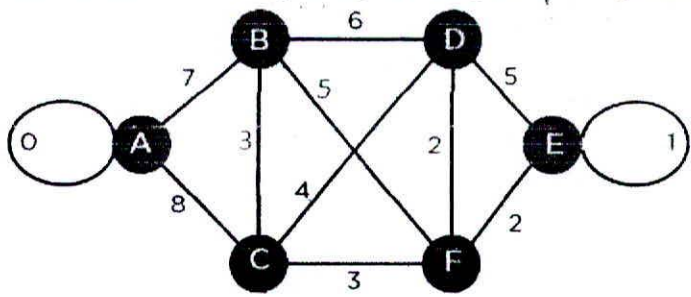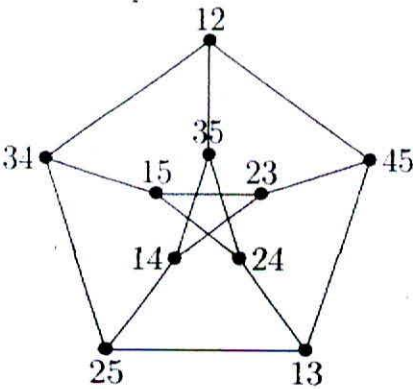|	
Max Marks : 100	Duration : 3 hours
Class : SE	Semester : III
Course code: CS201/DS201/AI201	Branch : SE COMP/CSE - AIML,DS
Name of the course : Discrete Structure And Graph Theory	

Q No		Max Marks	CO	BL
Q1	1. find the minimum cost spanning tree using prim's algorithm if the starting node is A. **OR** find the minimum cost spanning tree using kruskal's algorithm	5	CO3	3
	2. Write the following statement in English, using the predicate S(x, y) for "x shops in y", where x represents people and y represents stores: $\exists x1\ \exists y\ \forall x2\ [S(x1, y) \wedge (x1 \neq x2 \rightarrow \neg S(x2, y))]$	2	CO1	3
	3. Write the following statement using quantifiers and the predicates S(x, y): "x shops in y" T (x): "x is a student" where the universe for x consists of people and the universe for y consists of stores: "The only shoppers in some stores are students."	2	CO1	3
	4. Suppose you want to prove a theorem of the form $p \rightarrow (q \vee r)$. Prove that this is equivalent to showing that $(p \wedge \neg q) \rightarrow r$. Prove the statement without using the truth table.	6	CO1	3
	5. A total of 1232 students have taken a course in Spanish, 879 have taken a course in French, and 114 have taken a course in Russian. Further, 103 have	6	CO1	3

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	taken courses in both Spanish and French, 23 have taken courses in both Spanish and Russian, and 14 have taken courses in both French and Russian. If 2092 students have taken a course in at least one of Spanish, French and Russian, how many students have taken a course in all 3 languages. 6. Use the Principle of Mathematical Induction to prove that $$\sum_{i=1}^{n}(2i+3) = n(n+4) \quad \text{for all } n \geq 1.$$	4	CO1	3
Q2 A)	1.The duration of flight travel from Chennai to London through British Airlines is approximately 11 hours. The airplane begins its journey on Sunday at 23:30 hours. If the time at Chennai is four and half hours ahead of that of London's time, then find the time at London, when will the flight land at London Airport?	2	CO4	3
	2. Alice and Bob agree to use the prime number $p = 941$ and the $g = 627$. Use p=941 for mod n operation. Alice chooses the secret key a = 347 Similarly, Bob chooses the secret key b = 781. Find the shared secret key.	3	CO4	3
Q.2 B	1.What is the number of colors required to color a complete graph containing n vertices?	1	CO3	3
	2.What is the smallest number of colors that you can assign to its vertices such that any two adjacent vertices get different colors ? Justify your answer. 	2	CO3	3
	3. Let G = (V, E) be a loop-free connected 4-regular planar graph. If $|E| = 16$, how many regions are there in a planar depiction of G?	2	CO3	3
Q.2c)	Suppose that a new company has five employees: Zamora, Agraharani, Smith, Chou, and Macintyre. Each employee will assume one of six responsibilities: planning, publicity, sales, marketing, development, and industry relations. Each employee is capable of doing one or more of these jobs: Zamora could do planning, sales, marketing, or industry relations; Agraharam could do planning or development; Smith could do publicity, sales, or industry relations; Chou could do planning, sales, or industry relations; and Macintyre could do planning, publicity, sales, or industry relations. a) Model the capabilities of these employees using a bipartite graph.	5	CO3	3

	b) Find an assignment of responsibilities such that each employee is assigned one responsibility. c) Is the matching of responsibilities you found in part (b) a complete matching? Is it a maximum matching?			
Q.2d)	Use Ford Fulkerson algorithm to find max flow at t from s 	5	CO3	3
Q2 e)	Following graphs Hamiltonian or not? Justify your answer. **OR** Given graphs are planar or not. Justify your answer 	5	CO3	3
Q3A	Consider A={1,2,3,5,6,10,15,30} and ≤ be the partial order relation of divisibility on A. Let S = {a,b,c} be another poset with partial order relation ⊆. Show that (A,≤) and (S,⊆) are isomorphic.	10	CO2	3
Q3B	Consider a set of integers Z. Let aRb if $b = a^r$ for some positive integer r. Show that R is a partially ordered set. **OR**	05	CO2	3

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	Define a relation R on the set of integers Z by aRb if a-b is a non-negative even integer. Verify whether R is a partial order relation.			
Q3C	Let R be a relation on the set of positive integers such that R={(a,b)| (a-b) is an odd positive integer}. Is R an equivalence relation, a partial order relation?	05	CO2	3
Q3D	If f, g, h : R → R are defined as $f(x) = x+2$, $g(x) = 1/(x^2+1)$, $h(x) = 3$. Find		CO2	3
	(i) gohof(x)	2		
	(ii) hogof(x)	2		
	(iii) gof⁻¹of(x)	6		
Q4A	Draw the digraph of the poset and construct the Hasse diagram for the set A = {1,2,5,7,10,14,35,70}, with the relationship of divisibility: aRb iff a|b. Determine whether it is a lattice by finding GLB and LUB for every pair of elements.	10	CO2	3
Q4B	Find the generating function for the following sequences:	5	CO2	3
	(i) 0,0,0,1,2,3,4,5,...			
	OR			
	(ii) $a_k = 4^k$ where k=0,1,2,...∞			
Q4C	Solve the following recurrence relation: $a_{n+2} + 2a_{n+1} - 3a_n = 0$, given $a_0 = 1$, $a_1 = 2$	5	CO2	3
	OR			
	Solve the following recurrence relation: $a_n - 7a_{n-1} + 10a_{n-2} = 0$, given $a_0 = 1$, $a_1 = 6$.			