29/02/2024.

$x + iy \in \mathbb{C}$ - complex no.

$x, y \in \mathbb{R}$.

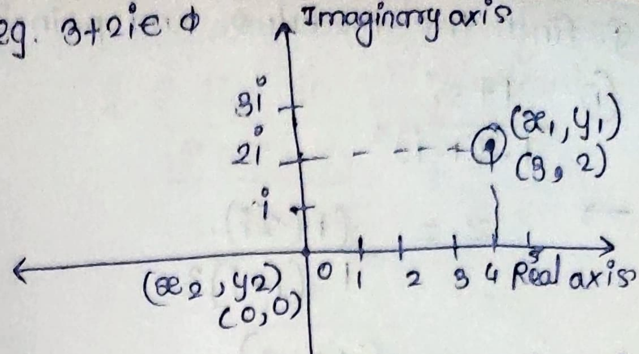
x = Real part.

y = Imaginary part

$i = \sqrt{-1}$

$i^2 = -1$.

eg. $3+2i \in \mathbb{C}$



This two dimensional plane is called Argand plane in complex.

one dimension = $d = (x - y)$.

distance between two points $= d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ or r.

eg. $r = \sqrt{(3-0)^2 + (2-0)^2}$

$r = \sqrt{13}$

[0 to 360° ← normal argument.
$-\pi$ to π ← principle value at argument] θ

for $z = x + iy \in \mathbb{C}$

(1) Distance from origin or modulus of complex no.

$|z| = \sqrt{(x-0)^2 + (y-0)^2}$

$|z| = r = \sqrt{x^2 + y^2}$

(2) Amplitude of z.

$\theta = \tan^{-1}|y/x|$ slop = tan..

(3) Conjugate. of conplex. no.

$\bar{z} = x - iy$.

If $z = yi$ then it is called pure imaginary complex no.

Q. find the modulus and principle argument of the complex no

① $\frac{1+2i}{1-(1-i)^2}$

→ $z = \frac{(1+2i)}{1-(1-i)^2}$

$= \frac{(1+2i)}{\cancel{1}-\cancel{(1}-2i+i^2)}$

$= \frac{(1+2i)}{+2i+i^2}$

put $i = -1$.

$= \frac{(1+2i)}{(2i+1)} = 1$.

$= 1+0i$.

$z = 1+0i$

$x = 1 + y = 0$

modulus $= r = |z| = \sqrt{1^2+0^2} = 1$.

Argument $= \tan^{-1}\left(\frac{y}{x}\right) = 0$.

~~$= 45°$~~.

since, $\theta \in (-\pi, \pi)$

$\therefore \theta = 0$ is the principle argument.

② $\frac{1+2i}{1-3i}$

→ $\frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i}$

$= \frac{(1+2i)(1+3i)}{1^2 \cancel{+} - 3i^2}$

$= \frac{(1+2i)(1+3i)}{1-9i^2}$.

put $i^2 = -1$

$\therefore$ we get.

$$= \frac{(1+2i)(1+3i)}{1-9}$$

$$= \frac{(1+2i)(1+3i)}{10}$$

$$= \frac{1(1+3i)+2i(1+3i)}{1-9}$$

$$= \frac{1+3i+2i+6i^2}{10}$$

$$= \frac{1+5i+6i^2}{10}$$

$$= \frac{1+5i-6}{10} = \frac{-5+5i}{10} = \frac{1}{2}(1+i)$$

$$\boxed{x = -\frac{1}{2}} \quad \boxed{y = +i/2}$$

Modulus. $|z| = \sqrt{(-1/2)^2 + (1/2)^2}$

$$= \sqrt{\frac{1}{2}}$$

$$r = \frac{1}{\sqrt{2}}.$$

Argument $= \theta = \tan^{-1}\left|y/x\right|.$

$$= \tan^{-1}\left|\frac{1/2}{-1/2}\right|.$$

$$= \tan^{-1}(1)$$

$$= \pi/4 -$$

By comparing the values of x, y θ lies in 2nd quadrant.

$$\boxed{\theta = \pi - \alpha.}$$

$$= \pi - \pi/4$$

$$= \frac{4\pi - \pi}{4} = 3\pi/4.$$

* Polar representation of complex no :-

$$z = x + iy.$$

$$= r\cos\theta + ir\sin\theta.$$

$$= r(\cos\theta + i\sin\theta).$$

$$z = re^{i\theta}.$$

Q.no 1) $\frac{2+i}{3-i}$

$z = \frac{2+i}{3-i}$

$\frac{2+i}{3-i} \times \frac{3+i}{3+i}$

$= \frac{(2+i)(3+i)}{3^2-i^2}$

$= \frac{2(3+i)+i(3+i)}{3^2-i^2}$

$= \frac{6+2i+3i+i^2}{9-i^2}$

$= \frac{6+5i+i^2}{9-i^2}$

$= \frac{6+5i-1}{9+1} = \frac{5+5i}{10}$

$x = \frac{1}{2}, y = i/2$

$r = \sqrt{\left(\frac{1}{2}\right)^2 + (1/2)^2}$

$= \cancel{\sqrt{}}\ \sqrt{2}/2$

$= 1/\sqrt{2}$

$\tan^{-1}(y/x) = \left(\frac{1/2}{1/2}\right)$

$= \tan^{-1}(1)$

$= \pi/4.$

$= \frac{1}{\sqrt{2}} e^{\pi/4 i}$ ← polar form.

formula $= re^{i\theta}$.

Q.no 1) $\frac{2+i}{3-i}$

$z = \frac{2+i}{3-i}$

$\frac{2+i}{3-i} \times \frac{3+i}{3+i}$

$= \frac{(2+i)(3+i)}{3^2-i^2}$

$= \frac{2(3+i)+i(3+i)}{3^2-i^2}$

$= \frac{6+2i+3i+i^2}{9-i^2}$

$= \frac{6+5i+i^2}{9-i^2}$

$= \frac{6+5i-1}{9+1} = \frac{5+5i}{10}$

$x = \frac{1}{2}, y = i/2$

$r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(1/2\right)^2}$

$= \sqrt{2}/2$

$= 1/\sqrt{2}$

$\tan^{-1}(y/x) = \left(\frac{1/2}{1/2}\right)$

$= \tan^{-1}(1)$

$= \pi/4.$

$= \frac{1}{\sqrt{2}} e^{\pi/4 i} \leftarrow$ polar form.

formula $= re^{i\theta}$.

* De-Moivre's Theorem :-

$$\boxed{(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)}$$

1] solve $x^4 - x^3 + x^2 - x + 1 = 0$

~~$x^n - x^{n-1} + \dots\dots x + 1 = 0$~~

~~$(x+1)($~~

formula =

$$\boxed{x^n - 1 = (x-1)(x^{n-1} + x^{n-2} + \dots + x + 1)}$$

$x^n + 1 = (x+1)(x^{n-1} - x^{n-2} + \dots\dots x + 1)$.

To solve :-

since $x^5 + 1 = (x+1)(x^4 - x^3 + x^2 - x + 1)$.

$\therefore$ if we want to solve ~~this~~ $(x^4 - x^3 + x^2 - x + 1)$ then we need to solve LHS $(x^5 + 1)$.

$x^5 + 1 = 0$.

$x^5 = -1 \rightarrow x = (-1)^{1/5}$.

$-1 = z = (-1) + 0.i$

$r = 1$.

$\theta = \pi$.

$-1 = r(\cos\theta + i\sin\theta)$

$-1 = (1)(\cos\pi + i\sin\pi)$.

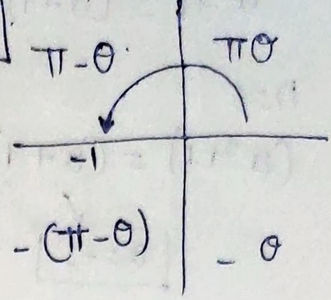
$\therefore (-1)^{1/5} = [\cos(\pi) + i\sin(\pi)]^{1/5}$

$= [\cos(\pi + 2n\pi) + i\sin(\pi + 2n\pi)]^{1/5}$

$x = \cos\left[\frac{\pi + 2n\pi}{5}\right] + i\sin\left[\frac{\pi + 2n\pi}{5}\right]$.

$n = 0, 1, 2, 3, 4$.

(-1).



Q.] $x^2 + 1 = 0$

$\rightarrow x^2 = -1$.

$x = (-1)^{1/2}$.

$-1 = (-1) + 0i = z$

$r = \sqrt{(-1)^2 + (0)^2} = 1$.

$\theta = \tan^{-1}\left|\frac{0}{-1}\right| = \tan^{-1}(0)$.

since (-1) is on $-ve$ x-axis.

$\pi - 0 = \pi$.

$$\boxed{\theta = \pi}$$

$z = re^{i\theta} = (1)e^{i\pi}$

$r(\cos\theta + i\sin\theta]$.

$z = -1 = (1)[\cos(\pi) + i\sin(\pi)]$.

$x = (-1)^{1/2} = [\cos\pi + i\sin\pi]^{1/2}$.

$= \cos\left[\frac{\pi}{2}\right] + i\sin\left[\frac{\pi}{2}\right]$.

$x = 0 + i(1)$

$$\boxed{x = i}$$

steps :- 1) find r, θ.

2) Express constant in terms of $\cos\theta + \sin\theta$.

3) $1/n$ power.

4) De-Moivre's theorem.

Q.1 Find the roots of $x^4 - x^3 + x^2 - x + 1 = 0$

$x^n + 1 = (x+1)(x^{n-1} - x^{n-2} + \dots - x + 1)$

$n = 5$

$(x^5 + 1) = (x+1)(x^4 - x^3 + x^2 - x + 1)$

$\boxed{x = -1}$ 4 roots

$x^5 + 1 = 0$

$\boxed{x^5 = -1 \longrightarrow x = (-1)^{1/5}}$

$-1 = z = (-1) + 0 \cdot i$

$r = \sqrt{(-1)^2 + 0^2} = 1$

$\theta = \tan^{-1}(0/1) = 0$

-1 Lies on $-$ve x-axis.

$\therefore \boxed{\theta = \pi}$

$\boxed{r = 1}$

$\therefore -1 = z = re^{i\theta}$

$-1 = r(\cos\theta + i\sin\theta)$.

$-1 = 1(\cos\pi + i\sin\pi)$

$\cos(2n\pi + \theta) = \cos\theta$

& $\sin(2n\pi + \theta) = \sin\theta$

$-1 = \cos(\pi + 2n\pi) + i\sin(\pi + 2n\pi)$

$-1 = \cos[\pi(2n+1)] + i\sin(\pi(2n+1)]$.

$(-1)^{1/5} = [\cos(2n+1)\pi + i\sin(2n+1)\pi]^{1/5}$

$-1 = \cos[\pi(2n+1)] + i\sin[\pi(2n+1)]$

$(-1)^{1/5} = [\cos(2n+1)\pi + i\sin(2n+1)\pi]^{1/5}$

$x = (-1)^{1/5} = \cos\left[\frac{2n+1\pi}{5}\right] + i\sin\left[\frac{(2n+1)\pi}{5}\right]$

$n = 0, 1, 2, 3, 4$.

$n = 0$

$x = \cos\left(\frac{\pi}{5}\right) + i\sin\left(\frac{\pi}{5}\right)$

$n = 1$

$x = \cos\left(\frac{3\pi}{5}\right) + i\sin\left(\frac{3\pi}{5}\right)$

$n = 2$

$x = \cos\left(\frac{5\pi}{5}\right) + i\sin\left(5\pi/5\right)$

$n = 3$

$x = \cos(7\pi/5) + i\sin(7\pi/5)$

$n = 4$

$x = \cos\left(\frac{9\pi}{5}\right) + i\sin(9\pi/5)$

Q.2] find the continued product of the five values of the expression $(1+i)^{1/5}$.

$z = 1+i$ (1st quadrant).

$r = \sqrt{1^2+1^2} = \sqrt{2}$.

$\theta = \tan^{-1}|1/1| = \tan^{-1}(1) = \pi/4$.

$z = 1+i = re^{i\theta}$

$= r(\cos\theta + i\sin\theta)$.

$(1+i) = \sqrt{2}(\cos \pi/4 + i\sin \pi/4)$.

$(1+i) = \sqrt{2}\left(\cos(\pi/4 + 2n\pi) + i\sin(\pi/4 + 2n\pi)\right]$

$(1+i)^{1/5} = \left\{\sqrt{2}\left[\cos[\pi/4 + 2n\pi) + i\sin(\pi/4 + 2n\pi]\right]\right\}^{1/5}$.

$= (\sqrt{2})^{1/5} \times \left[\cos(\pi/4 + 2n\pi) + i\sin(\pi/4 + 2n\pi)\right]^{1/5}$

$= 2^{1/10} \times \left[\cos\left(\frac{\pi/4 + 2n\pi}{5}\right) + i\sin\left[\frac{\pi/4 + 2n\pi}{5}\right]\right]$ By Demoivre's theorem.

$n = 0$

$x_1 = 2^{1/10} \cos(\pi/20) + i\sin(\pi/20)$.

$n = 1$

$x_1 = 2^{1/10} \cos\left[\frac{\pi/4 + 2\pi}{5}\right] + i\sin\left[\frac{\pi/4 + 2\pi}{5}\right] =$

$= 2^{1/10} \cos[9/20]\pi + i\sin\left[\frac{9}{20}\pi\right]$.

$n = 2$.

$x_2 = 2^{1/10}\left[\cos\left[\cancel{\cos}\ \frac{17\pi}{20} + \sin \frac{17\pi}{20}\right]\right.$

$n = 3$

$x_3 = 2^{1/10}\left[\cos\left[\frac{25\pi}{20}\right) + i\sin\left(\frac{25\pi}{20}\right)\right]$

$n = 4$.

$x_4 = 2^{1/10}\left[\cos\left[\frac{33\pi}{20}\right] + i\sin\left(\frac{33\pi}{20}\right]\right]$

$x_0 \cdot x_1 \cdot x_2 \cdot x_3 \cdot x_4 =$

$$\left[2^{1/10}\right]^5 \left[\cos\left(\frac{\pi}{20} + \frac{9\pi}{20} + \frac{17\pi}{20} + \frac{25\pi}{20} + \frac{33\pi}{20}\right) + i\sin\left[\frac{\pi}{20} + \frac{9\pi}{20} + \frac{17\pi}{20} + \frac{25\pi}{20} + \frac{33\pi}{20}\right]\right]$$

H.W. $(1+i)^{1/3}$.