

Linear Regression Algorithm

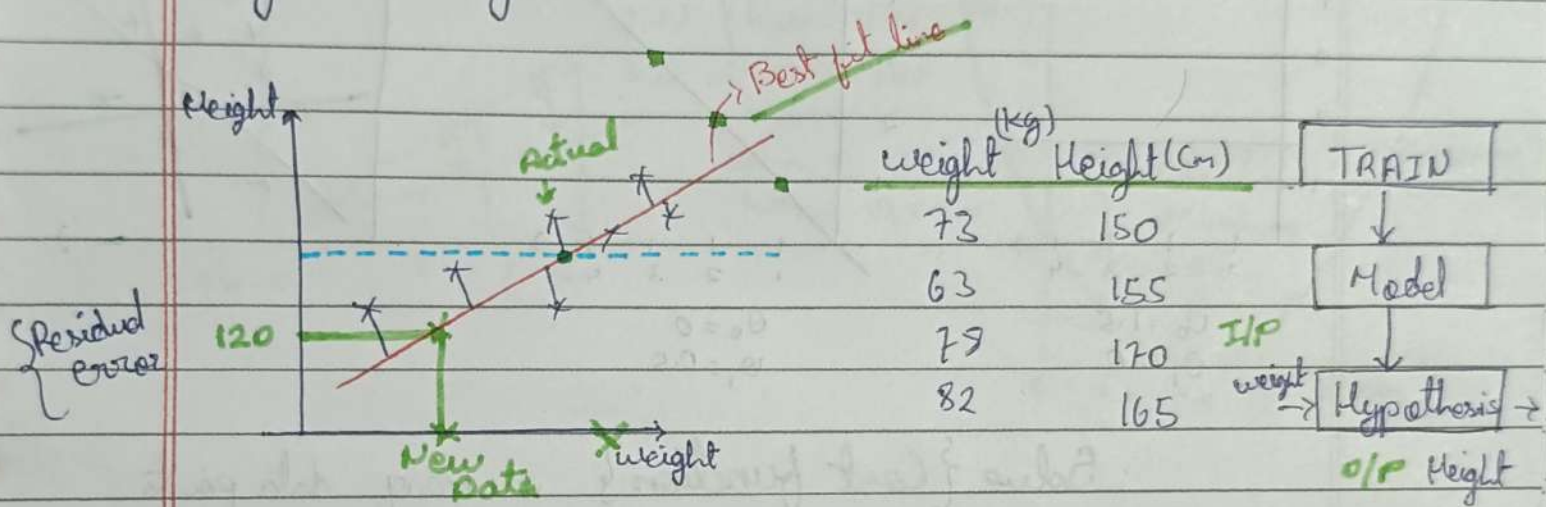
Lucky

Date / /

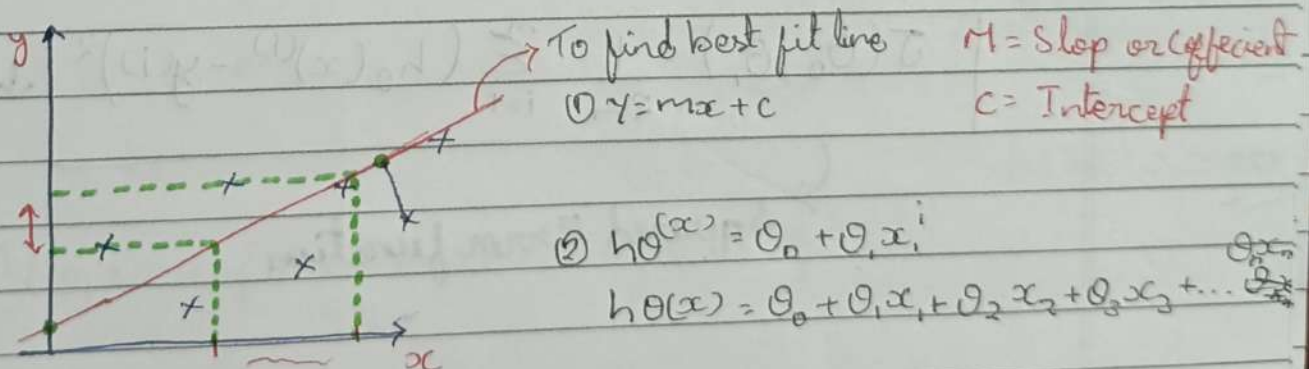
Page

Linear Regression is a way to predict a number by drawing the best straight line through data points.

It learns the relationship between input (X) and output (Y) using a straight line.



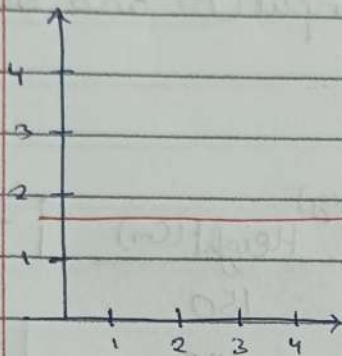
Residual Error is difference between the actual value and predicted value



Hypothesis $\{y \text{ is a linear function of } x\}$

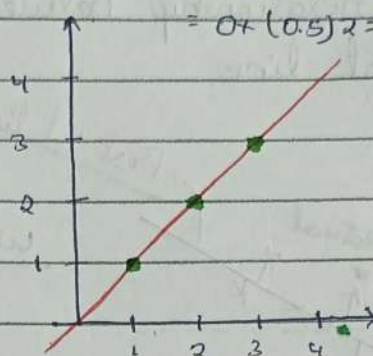
$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

↗ Coefficient
↘ Intercept



$$\theta_0 = 1.5$$

$$\theta_1 = 0$$

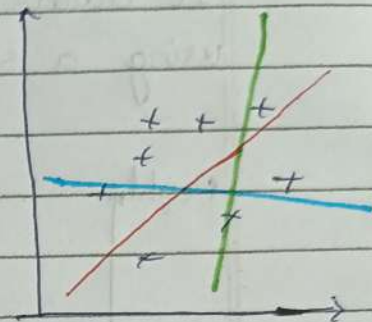


$$\theta_0 = 0$$

$$\theta_1 = 0.5$$

$$= 0 \times (0.5)1 = 0.5$$

$$= 0 + (0.5)2 = 1$$



Solve { Cost function } $m = \text{No. of data points}$

Minimize θ_0, θ_1

$$\sum_{i=1}^m \frac{1}{2m} (h_{\theta}(x^{(i)}) - y^{(i)})^2 \downarrow \downarrow$$

$$\frac{d(x^2)}{dx} = 2x^{2-1} = 2x$$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 \downarrow \downarrow \downarrow$$

↘ Squared Error function

Hypothesis

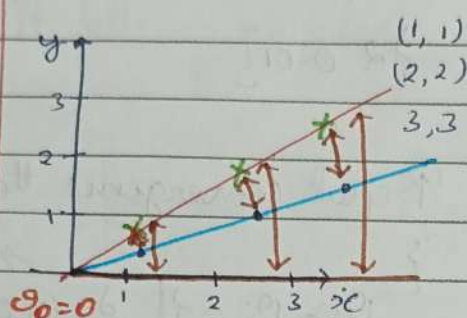
$$h_0(x) = \theta_0 + x,$$

where $\theta_1 = 1$

let $\theta_0 = 0$

Cost function

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_0(x^{(i)}) - y^{(i)})^2$$



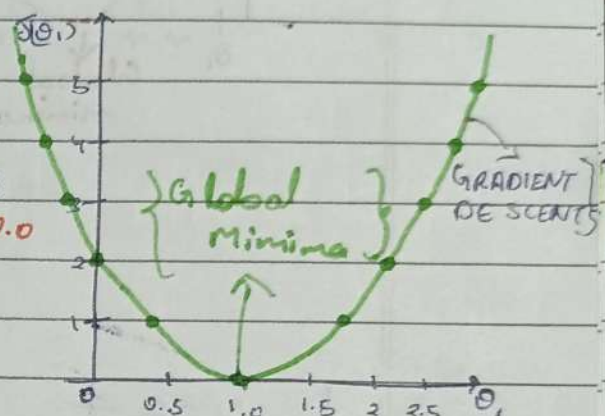
$$h_{\text{red}} = 1 + 1$$

$$h_0(x) = 1 + 2$$

$$h_0(x) = 1 + 3$$

when $\theta_1 = 0.5$

where $\theta_1 = 0.0$



$$J(\theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_0(x^{(i)}) - y^{(i)})^2 \text{ when } \theta_1 = 1$$

$$= \frac{1}{6} [(0)^2 + (0)^2 + (0)^2]$$

$$= 0$$

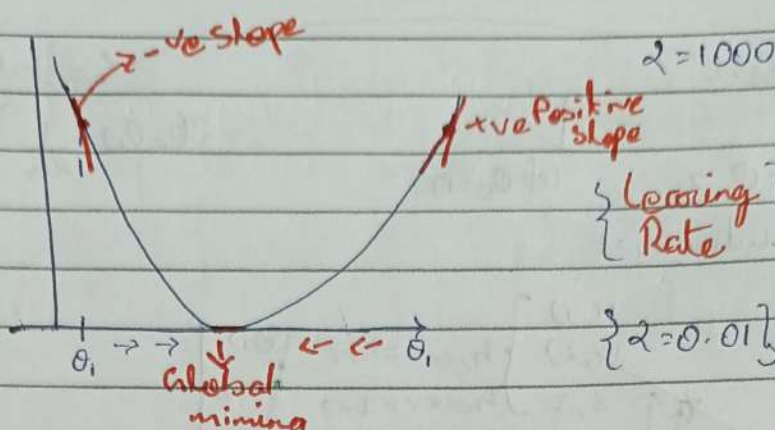
$$J(\theta_1) = \frac{1}{6} [(0.5-1)^2 + (1.2)^2 + (1.5-3)^2]$$

when $\theta = 0.5$

$$= \frac{1}{6} (3.5) = \approx 0.58$$

$$J(\theta_1) = \frac{1}{6} [(0-1)^2 + (0-2)^2 + (0-3)^2]$$

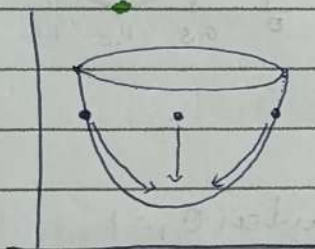
$$= \frac{14}{6} = \approx \underline{\underline{2.3}}$$



Repeat convergence theorem

$$\theta_j := \theta_j - \left\{ \frac{\partial}{\partial \theta_j} (J(\theta_0, \theta_1)) \right\}$$

$$\begin{aligned} \theta_1 &:= \theta_1 - \alpha (+ve) \quad \text{decrement} \\ \theta_0 &:= \theta_0 - \alpha (-ve) \quad \text{increment} \\ \theta_1 &:= \theta_1 + \end{aligned}$$



Outline

- ① Start with θ_0 & θ_1
- ② Keep changing θ_0, θ_1 to reduce $J(\theta_0, \theta_1)$ until we reach near global minima.

③ Convergence Theorem

$$\theta_j := \theta_j - \left\{ \frac{\partial}{\partial \theta_j} (J(\theta_0, \theta_1)) \right\}$$

$$J = 0 \text{ and } 1$$

}