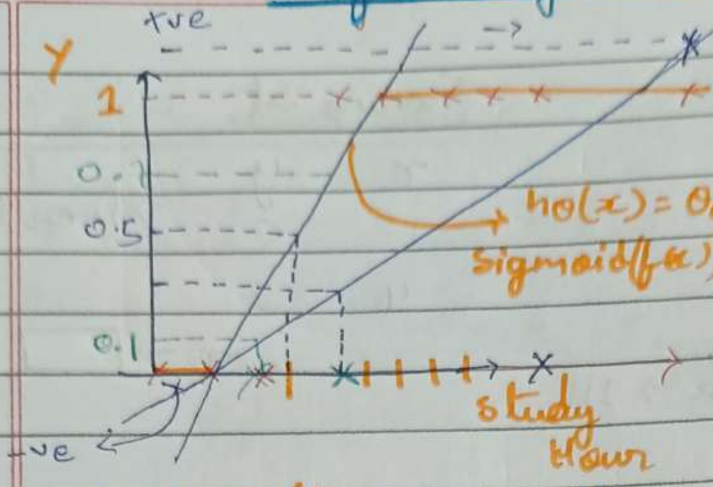
Logistic Regression → Classification



$h_\theta(x) = \theta_0 + \theta_1 x_1$

$\text{Sigmoid}(f(x)) = \frac{1}{1+e^{-z}}$

Study hour	Pass/Fail
4	Pass
2	Fail
0	Fail
1	Fail
2.5	Pass

{ ① Outlier
② Output will be >1 or <0 }

$$h_\theta(x) = g(\theta_0 + \theta_1 x_1)$$

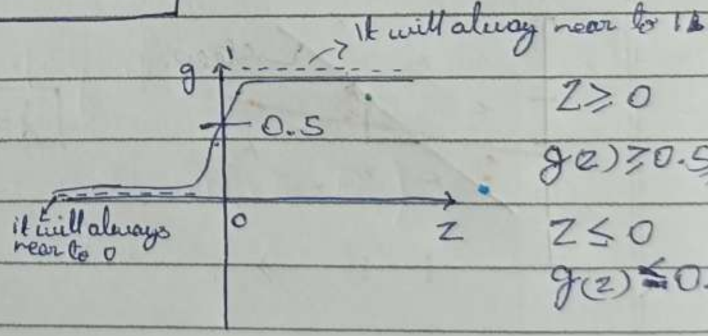
$$g = \frac{1}{1+e^{-z}} \qquad z = \theta_0 + \theta_1 x_1$$

$$g\, h_\theta(x) = \frac{1}{1+e^{-z}} \quad \text{Sigmoid} \quad \theta_0 + \theta_1 x_1$$

$$\boxed{h_\theta(x) = \frac{1}{1+e^{-(\theta_0+\theta_1 x_1)}}}$$

Sigmoid

$$g = \frac{1}{1+e^{-z}}$$



$Z \geq 0$
$g(z) \geq 0.5$

$Z \leq 0$
$g(z) \leq 0.5$

Trainset $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots (x^{n}, y^{n})\}$

$\in$ ⇒ belongs to

$$x \in \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad y \in \{0, 1\}$$

$$Z = \theta_0 + \theta_1 x_1$$
$$= \theta^T x$$

$$h_\theta(x) = \frac{1}{1+e^{-(\theta_0+\theta_1 x_1)}}$$

$$h_\theta(x) = \frac{1}{1+e^{-(\theta^T x)}}$$ ⇒ Hypothesis Test

Linear Regression	Logistic Regression
$J(\theta_0,\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x)^{(i)} - y^{(i)}\right)^2$	$J(\theta_0,\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(h_\theta(x)^{i} - y^{(i)}\right)^2$

$h_\theta(x) = \theta^T x$

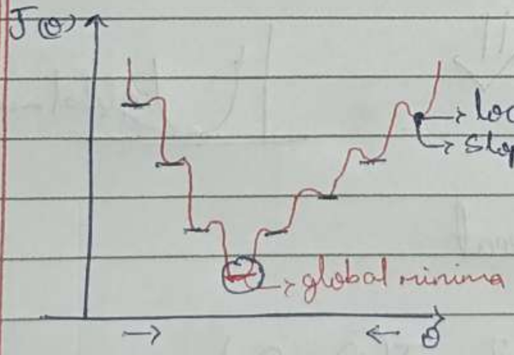
$= \theta_0 + \theta_1 x$ replace

$h_\theta(x) = \frac{1}{1+e^{-(\theta^T x)}}$ replace

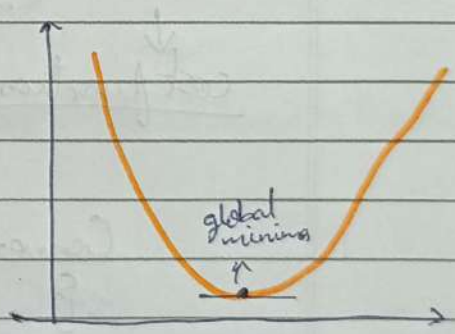
⇓ Non Convex function

it will always make a → Convex

Non Convex



Convex

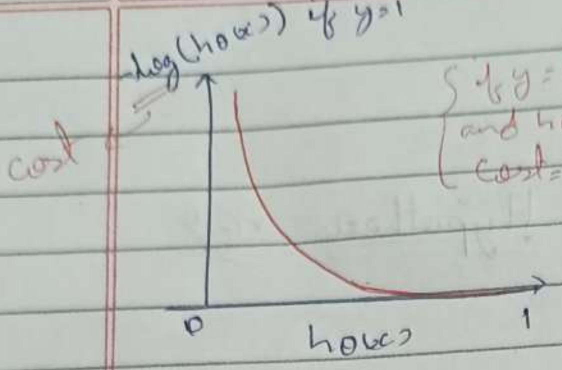


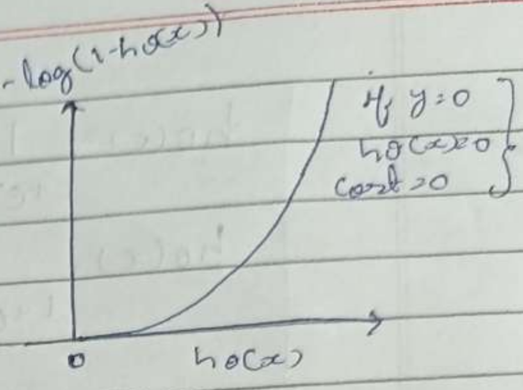
$$J(\theta_0,\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}\underbrace{\left(h_\theta(x)^{(i)} - y^{(i)}\right)^2}_{\Downarrow}$$

$\text{Cost}(h_\theta(x^{(i)}), y)$

$h_\theta(x)^{(i)} = \frac{1}{1+e^{-z}}$ $\quad z = \theta_0 + \theta_1 x_1$

$$\text{Cost}(h_\theta(x^{(i)}), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y=1 \\ -\log(1-h_\theta(x)) & \text{if } y=0 \end{cases}$$ ⇒ Cost function





$$\left\{ Cost(h_\theta(x), y) = -y\log(h_\theta(x)) - (1-y)\log(1-h_\theta(x)) \right\}$$

if $y=1$

$$= -\log(h_\theta(x))$$

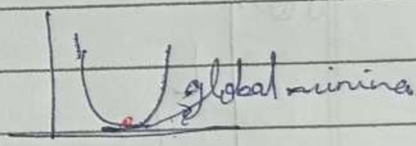
if $y=0$

$$= -\log(1-h_\theta(x))$$

$$J(\theta_0, \theta_1) = \frac{-1}{2m} \sum_{i=1}^{m} \left(y^{(i)} \log h_\theta(x)^{(i)} + (1-y^{(i)}) \log(1-h_\theta(x)^{(i)})\right)$$

↓

cost function



Convergence Repeat

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$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$