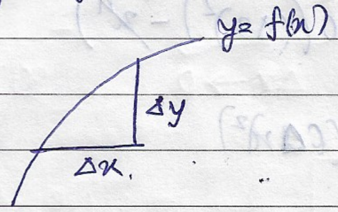
LECTURE 2

What is a Derivative? (Continued)

Rate of Change



$\frac{\Delta y}{\Delta x}$ "Another interpretation - average change"

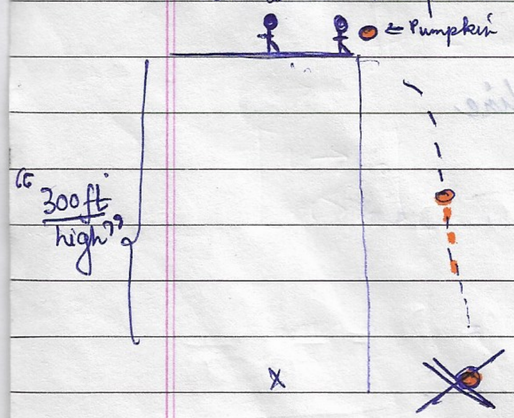
$\frac{\Delta y}{\Delta x} \longrightarrow \frac{dy}{dx}$ } Instantaneous rate of change.

Examples from physics

1. q = charge, $\frac{dq}{dt}$ = current.

2. s = distance, $\frac{ds}{dt}$ = speed.

"Halloween Pumpkin drop"



"for sake of convienience let the height of 80 conviently."

$h = 80 - 5t^2$ meters.

$t = 0$, $h = 80$; $t = 4$, $h = 0$

Average speed : $\frac{\Delta h}{\Delta t} = \frac{0-80}{4-0} = -20\, m/sec.$

instaneous speed at bottom = $\frac{d h}{dt} = 0 - 10t$

$$\left(\frac{d}{dt} 80 = 0, \frac{d}{dt} t^2 = 2t\right)$$

$t = 4$, $h' = -40\ m/s$. ("90 miles per hour")

3. T = temperature

$\frac{dT}{dx}$ = temperature gradient.

4. Sensitivity of measurements.

Problem set 1. GPS

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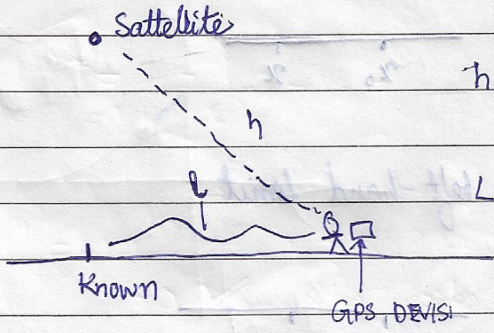
Thursday.



h measured by radiowaves and clock

L is deduced from h

Error in h : Δh

"The main uncertainity on GPS is from the ionosphere. but lots of corrections of all kinds also if inside a buildings its a problem to measure it.

We then get

ΔL estimated by $\Delta L / \Delta h \approx \frac{dL}{dh}$ "easy"

"Its used to land airplanes"

Limits + Continuity

1. Easy limits

$$\lim_{x \to 4} \frac{x+3}{x^2+1} = \frac{4+3}{(4)^2+1} = \frac{7}{17}$$

"With this kind of limit All I have to do is plug in $x=4$.

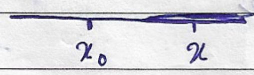
2. Derivatives are always harder:

$$\lim_{x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{x - x_0}$$ $x = x_0$ gives $\frac{0}{0}$

needs cancellation to make any sense.

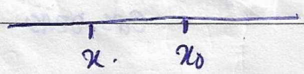
$\lim_{x \to x^+} f(x)$ = right-hand limit

$\begin{bmatrix} x \to x_0 \\ x > x_0 \end{bmatrix}$



$\lim_{x \to x^-} f(x)$ = left-hand limit

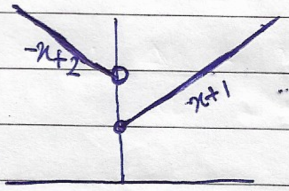
$\begin{bmatrix} x \to x_0 \\ x < x_0 \end{bmatrix}$



, Comma
– hypen

Example

$$f(x) = \begin{cases} x+1, & x \geq 0 \\ -x+2, & x < 0 \end{cases}$$



o – open does not include value

• – closed does include value.

On evaluating the limits

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0} x+1 = 1$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0} (-x+2) = 2$$

"Did not need $x=0$ value".

Definition of Continutly: f is continuous at x_0 means.

$$\lim_{x \to x_0} f(x) = f(x_0) \longrightarrow \text{Easy limits}$$

$\uparrow_{x \neq x_0}$

continuous at x_0:

1. $\lim_{x \to x_0} f(x)$ exists. (both from $L + R$, $L = R$)
2. $f(x_0)$ is defined
3. they are equal.

"Left and Right hand side limits are evaluated without using $x = x_0$"

ZOO of Discontinuity

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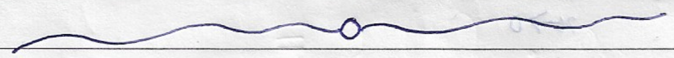
Thursday

1. Jump Discontinuity.

lim from left and right exist but are not equal (as in example).

2. Removable Discontinuity

lim from left and right are equal but not defined at x_0



removable discontinuity at $x=0$

Examples $g(x) = \frac{\sin x}{x}$, $h(x) = \frac{1-\cos x}{x}$

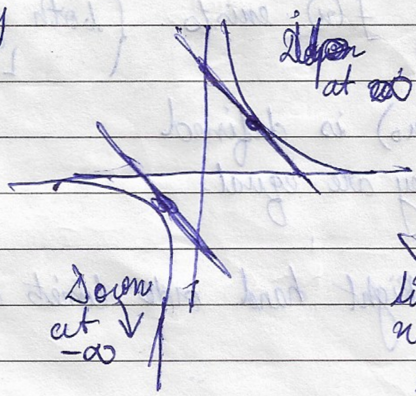
$g(0) = ??$ "removable singularity"

$$\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1}$$

$$\boxed{\lim_{x\to 0} \frac{1-\cos x}{x} = 0}$$

3. Infinite Discontinuity

$y = \frac{1}{x}$.



"odd function"

$\lim_{x\to 0^+} \frac{1}{x} = \infty$

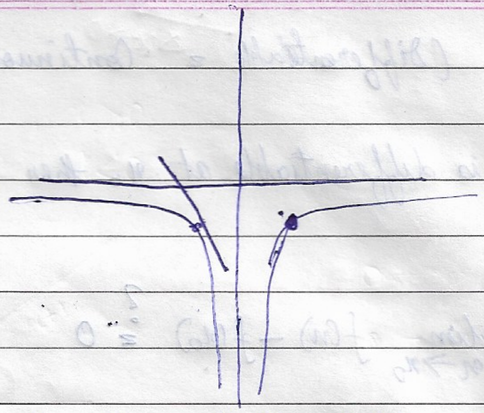
$\lim_{x\to 0^-} \frac{1}{x} = -\infty$

$\cancel{\lim_{x\to 0} \frac{1}{x} = \infty}$

Sloppy and wrong.

Solved earlier

$y' = \frac{-1}{x^2}$



"even function"

"Derivative of a function looks nothing like the function. That intuition is wrong.

$\lim_{x \to 0} \frac{-1}{x^2} = -\infty$

$x \to 0^+$ and $x \to 0^-$ both

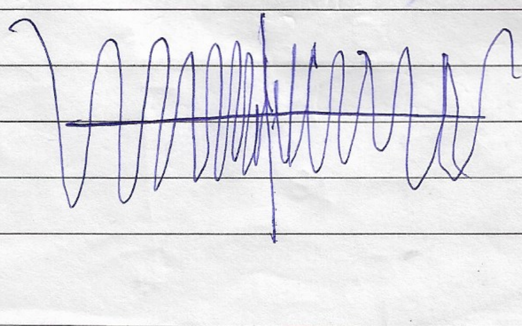
"The derivative of an odd function you always get an even function"

"That closely related to $\frac{1}{x}$ is an odd power

and $\frac{1}{x^2}$ is a even power"

4. Other Ugly discontinuities.

$y = \sin\frac{1}{x}$ as $x \to 0$



"no left or right limit".

Theorem (Differentiable = Continuous)

If f is differentiable at x_0 then f is continuous at x_0

Proof

$$\lim_{x \to x_0} f(x) - f(x_0) \overset{?}{=} 0$$

. we rewrite the limit

$$\lim_{x \to x_0} \frac{f(x) - f(x_0)}{(x - x_0)} (x - x_0)$$

$$= \underbrace{f'(x_0)}_{\text{exists by our assumpti}} \cdot \underbrace{0}_{\text{clearly zero}}$$

$= 0$ Thats it the answer is 0 what we want