LECTURE - 4.

Example $\frac{d}{dt}(cu) = c\frac{du}{dt}$, c is a constant

or $(cu)' = cu'$, c is a constant

- $\frac{d}{dt}(u+v) = \frac{du}{dt}v + u\frac{dv}{dt}$

or $(u+v)' = u'v + uv'$

Product Rule :

$(uv)' = u'v + uv'$.

"Little bit funny differentiating a product gives you a sum"

Example. $\frac{d}{dx}(x^n \sin(x)) = nx^{n-1}\sin(x) + x^n\cos(x)$.

"Obviously you can ~~do~~ differentiate it for ~~bigger~~ longer product taking one at a time"

Proof. $\Delta(uv)$ ← "looking at the change in the function you are differentiating".

$= u(x+\Delta x)\,v(x+\Delta x) - u(x)\,v(x)$

"New value" "old Value"

$= \big(u(x+\Delta x) - u(x)\big)\,v(x+\Delta x) + u(x)\big(v(x+\Delta x) - v(x)\big)$

"Multiply out and cancel out middle terms"

$= (\Delta u) v(x+\Delta x) + u(x)\Delta v.$

$$\frac{\Delta(uv)}{\Delta x} = \frac{\Delta u}{\Delta x} v(x+\Delta x) + u\frac{\Delta v}{\Delta x}$$

$\Delta x \to 0 \downarrow$

$$\frac{d(uv)}{dx} = \frac{du}{dx} v + u\frac{dv}{dx}.$$

"$v(x+\Delta x) \longrightarrow v(x)$ by continuity"

Quotient Rule

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \quad , v \neq 0$$

Proof: $\Delta\left(\frac{u}{v}\right) = \frac{u+\Delta u}{v+\Delta v} - \frac{u}{v}$

$$= \frac{uv + (\Delta u)v - uv - u\Delta v}{(v+\Delta v)v}$$

$$= \frac{(\Delta u)v - u\Delta v}{(v+\Delta v)v}$$

$$\frac{\Delta\left(\frac{u}{v}\right)}{\Delta x} = \frac{\frac{\Delta u}{\Delta x}v - u\frac{\Delta v}{\Delta x}}{(v+\Delta v)v}$$

$\downarrow \Delta x \to 0$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{du}{dx} \cdot v - u\frac{dv}{dx}}{v \cdot v}$$

Example $u = 1$,

$$\frac{d}{dx}\left(\frac{1}{v}\right) = \frac{-1 \cdot v'}{v^2} = -v^{-2} v'$$

Subexample $u = 1$, $v = x^n$

$$\frac{d}{dx}(x^{-n}) \quad \frac{d}{dx}\left(\frac{1}{x^n}\right) = -x^{-2n} \cdot n x^{n-1}$$

$$= (-n) x^{(-n)-1}$$

$\boxed{\frac{d}{dx} x^n = n x^{n-1}}$ "That comes out of Quotient Rule"

Also can be written as $\frac{d}{dx}(x^{-n})$

Chain Rule -
Composition Rule

Example $y = (\sin t)^{10} =$,

Method: use new variable name

$x = \sin(t)$

$y = x^{10}$

$y = \sin^{10}(t)$

proof $\frac{\Delta y}{\Delta t} = \frac{\Delta y}{\Delta x} \cdot \frac{\Delta x}{\Delta t}$ "Solved by basic algebra"

$\downarrow$

$\Delta t \to 0$

$$\boxed{\frac{dy}{dt} = \frac{dy}{dx} \frac{dx}{dt}}$$

"Chain Rule"

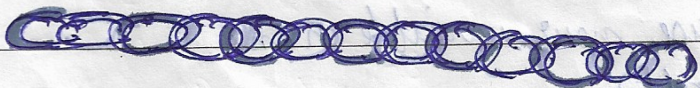
because delta x goes to 0
as delta t goes to 0

Principle "Chain Rule" - "Differentiation of a composition is a product"

Example. $\frac{d}{dt}(\sin t)^{10}$

Inside function, $x = \sin(t)$
Outside function, $y = x^{10}$

$= 10x^9 \cdot \cos(t)$
$= 10(\sin(t))^9 \cdot \cos(t)$



"Chain Rule lets you burst free"

Example. $\frac{d}{dt} \sin(10t)$

Inside function $x = 10t$
Outside function $y = \sin(x)$

$$\frac{dy}{dt} = \cos(x) \cdot 10$$
$$= 10 \cos(10t)$$

or briefly "After getting used to it"

$$\frac{d}{dt} \sin(10)t$$
$$= \cos(10t) \cdot 10.$$

Higher Derivatives "Just means differentiate over and over again."

$u = u(x)$, u' first Derivative ↓ Differentiated once, $u'' = (u')'$ second Derivative ↓ Differentiated again, $u''' = (u'')'$ third Derivative,

Example $u = \sin x$, $u' = \cos(x)$, $u'' = -\sin(x)$, $u''' = -\cos(x)$

(cont)

$u^{(4)} = u^{(111)'}$ fourth derivative

$u^{(4)} = \sin(x)$ "Returned to same value"

Other Notations $\frac{du}{dx} = \left(\frac{d}{dx}\right)u = Du$ where $D = \frac{d}{dx}$

"operator to be applied to a function"

$$u'' = \frac{d}{dx}\cdot\frac{du}{dx} = \frac{d}{dx}\cdot\frac{d}{dx}u$$

$$= \left(\frac{d}{dx}\right)^2 u$$

$$= \frac{d^2}{(dx)^2}u \qquad \text{"Abusive Notation"}$$

$$= \frac{d^2u}{dx^2} \qquad \text{"This is not } d(x^2)\text{"}$$

$$= D^2u.$$

$$u''' = \frac{d^3u}{dx^3} \quad \text{etc.}$$

$$= D^3u$$

Example. $D^n x^n = ?$

$Dx^n = nx^{n-1}$

$D^2x^n = n(n-1)x^{n-2}.$

$D^3x^n = n(n-1)(n-2)x^{n-3}$

$D^{n-1}x = (n(n-1)\ldots\ldots)x^1$

$D^n x^n = (n(n-1)\ldots\ldots 2.1)\ 1$

n factorial

$n!$

Constant 1

"We've just used Mathematical InDoctrination"

$\boxed{D^n x^n = n!}$. constant = 1

$D^{n+1} x^n = 0$ "Its the derivative of a constant so it is zero".