$\sin(a+b) = \sin a \cos b + \cos a \sin b$

LECTURE-3

Derivative Formulaes.

1. Specific $f(x)$ $\left(f(x) = x^n, \ 1/x \right)$
2. General eg $(u+v)' = u'+v'$
   $(cu)' = cu'$ where c is constant

July 10, 2020
Friday

"Both these kinds are needed".

Example - need both kinds for polynomials.

Specific x in radians

- $\frac{d}{dx} \sin x = \cos x$ "The derivative of the sine is a cosine".
- $\frac{d}{dx} \cos x = -\sin x$ "The derivative of the cosine is negative sine".

- $\frac{\sin(x+\Delta x) - \sin x}{\Delta x}$ - Difference quotient $\xrightarrow{\Delta x \to 0} \cos x$

"Any tricky limit is $\frac{0}{0}$ when we set $\Delta x \to 0$"

$= \frac{\sin x \cos \Delta x + \cos x \sin \Delta x - \sin x}{\Delta x}$

$= \sin x \underbrace{\left(\frac{\cos \Delta x - 1}{\Delta x}\right)}_{\to 0} + \cos x \underbrace{\left(\frac{\sin \Delta x}{\Delta x}\right)}_{\to 1}$

"We group the terms so that a zero stays over zero, otherwise one over zero we get something meaningless"

$\boxed{\Delta x \to 0}$

$= \cos x.$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$$\frac{\cos(x+\Delta x) - \cos x}{\Delta x} \xrightarrow[\Delta x \to 0]{} -\sin x$$

"Applying sum rule of cosine"

$$= \frac{\cos x \cos \Delta x - \sin x \sin \Delta x - \cos x}{\Delta x}$$

$$= \cos x \underbrace{\left(\frac{\cos \Delta x - 1}{\Delta x}\right)}_{\downarrow \text{ A} \atop 0} + (-\sin x) \underbrace{\left(\frac{\sin \Delta x}{\Delta x}\right)}_{\downarrow \text{ B} \atop 1}$$

$$\boxed{\Delta x \to 0}$$

$= -\sin x$

July 12, 2020
Sunday

Remarks

$$\left.\frac{d}{dx}\cos x\right|_{x=0} = \lim_{\Delta x \to 0} \frac{\cos(\Delta x) - 1}{\Delta x} = 0$$

property A

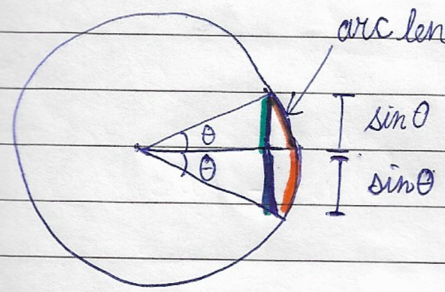
$$\left.\frac{d}{dx}\sin x\right|_{x=0} = \lim_{\Delta x \to 0} \frac{\sin(\Delta x) - 0}{\Delta x} = 1$$

Property B

July 13, 2020
Monday

Derivatives of sine and cosine at $x=0$ give all values of $\frac{d}{dx}\sin x$, $\frac{d}{dx}\cos x$

(B) Geometric Proof : $\Delta x \rightsquigarrow \theta$, here θ is in radians



bowstring

$$\frac{2\sin\theta}{2\theta} = \frac{\sin\theta}{\theta} \longrightarrow 1$$

bow

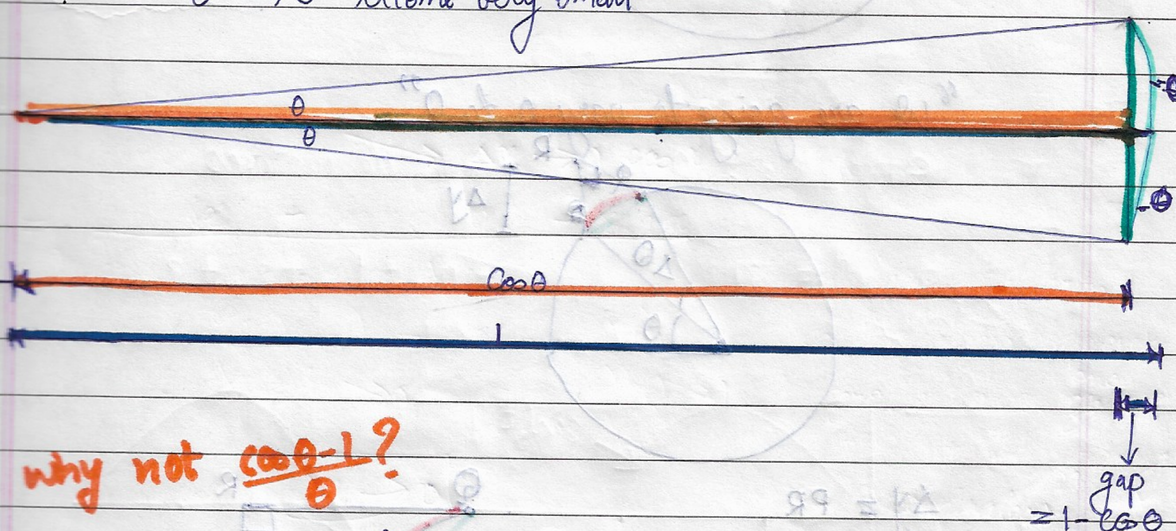
$\theta \to 0$

Principle : short pieces of curves are nearly straight

(A) $\frac{1-\cos\theta}{\theta}$, here θ is in radians

why bow length is θ?

$\theta \longrightarrow 0$ "become very small"



why not $\frac{\cos\theta - 1}{\theta}$?

$$\frac{\cos\theta - 1}{\theta} = -\frac{(1-\cos\theta)}{\theta} \longrightarrow 0$$

same thing

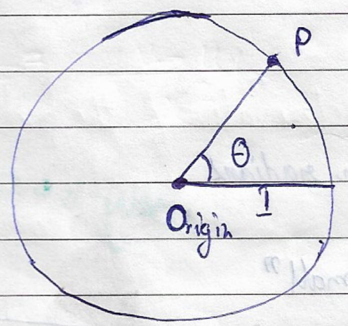
July 14, 2020
Tuesday

Geometric Proof

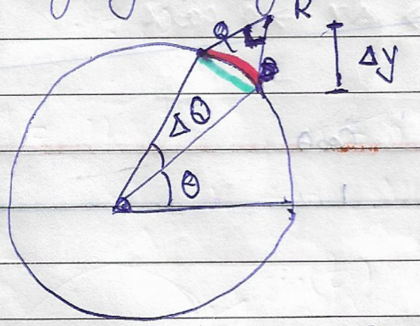
$$\frac{d}{d\theta}\sin\theta = \cos\theta$$

(for all θ)

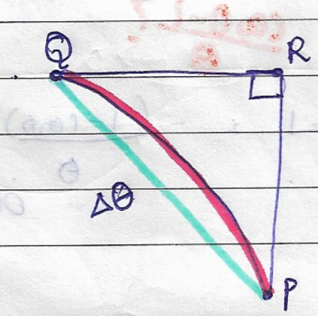
$y = \sin\theta$
vertical position of circular motion.



"We are going to vary θ to Q"



$\Delta y = PR$



"It is the straight line approximation to the curve that we are interested in"

Principle: "Short pieces of curves are nearly straight"

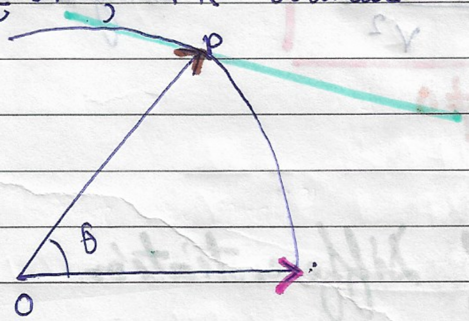
$$\overset{\frown}{PQ} \approx \overline{PQ}$$

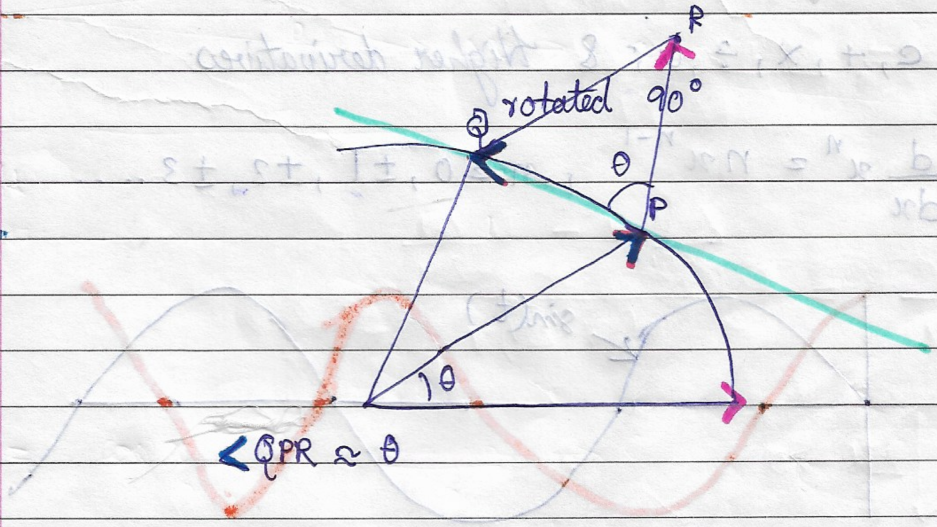
$PQ \approx \Delta\theta$

a) What is $\angle QPR$? And b) $PR \approx$?

The line $\overline{PQ}$ is almost perpendicular to the circle.

$PQ \underset{\sim}{\perp} OP$; PR vertical





$\angle QPR \approx \theta$

$$\therefore \frac{\Delta y}{\Delta \theta} \approx \cos\theta$$

$$\lim_{\Delta\theta \to 0} \frac{\Delta y}{\Delta \theta} = \cos\theta$$

General Rules Derivatives

1. Product Rule
$$(uv)' = u'v + uv'$$
"Change one at a time"

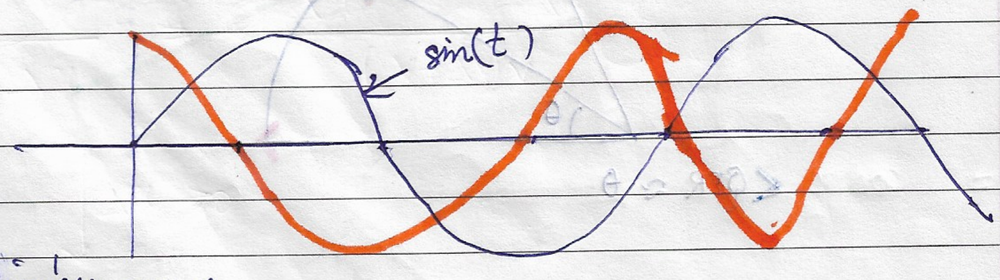
2. Quotient Rule
$$(u/v)' = \frac{u'v - uv'}{v^2}$$
"Everytime hope you get it right"

$v \neq 0$

Next Lecture.
Rules of Differentiation.

c, +, x, ÷, 0 ; & Higher derivatives

$$\frac{d}{dx} x^n = n x^{n-1}, \quad n = 0, \pm 1, \pm 2, \pm 3 \ldots$$



$\sin'(t) = \cos$