ANTIDOTE OF FEAR, ANGER AND JEALOUSY IS KARUNA

Date July 3, 2020

LECTURE 1

Unit 1 Differentiation

A What is a Derivative?

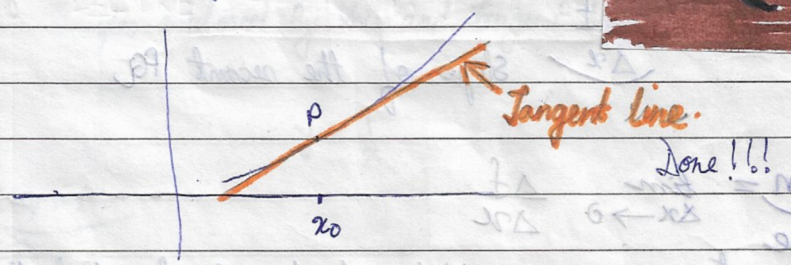
- geometrical interpretation.
- physical interpretation
- importance of derivatives to all measurements (science, engineering, economics, politics etc)

B. How to differentiate any function you know.

Geometric Interpretation

Problem: Find the Tangent line to $y = f(x)$ at $P = (x_0, y_0)$.

Solution



Done!!!

x x x

"The task that we have now is to figure out analytically, do it in a way that a machine can do it just as well as done above by me"

$y - y_0 = m(x - x_0)$ Equation of the line

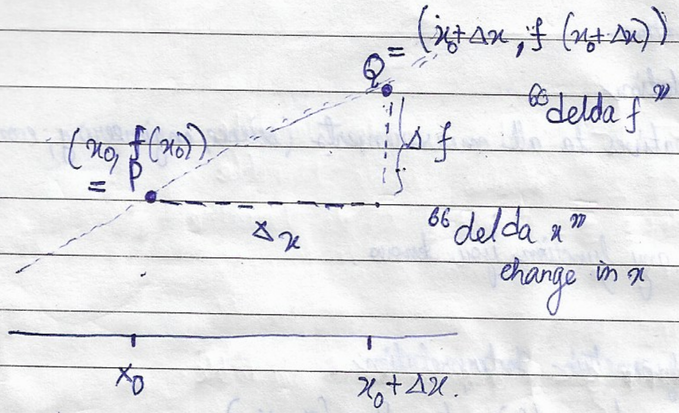
(i) point $y_0 = f(x_0)$

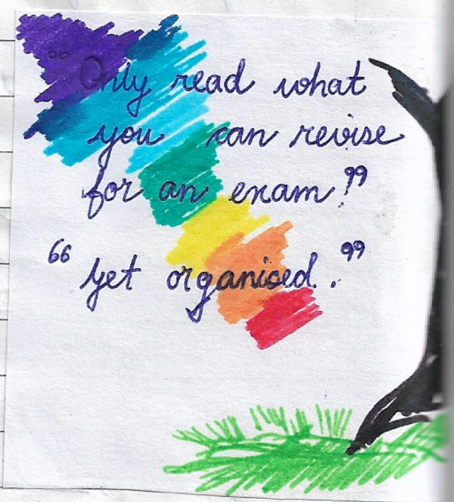
(ii) slope $m = f'(x_0)$ namely f prime of x_0 or derivative of x_0

Definition $f'(x_0)$, the derivative of f at x_0, is the slope of the tangent line to $y = f(x)$ at P.

Remember geometrical interpretation

Tangent line = Limit of Secant lines PQ as $Q \to P$ (P fixed).





$\frac{\Delta f}{\Delta x}$ Slope of the secant PQ

$$\underbrace{m}_{\text{slope of tangent}} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$$

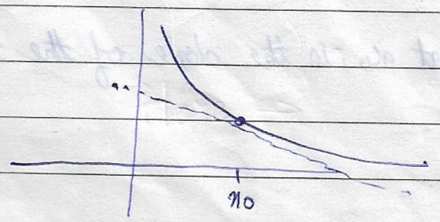
Different formula for derivative → Difference Quotient

$$\boxed{f'(x_0) = m = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}}$$

"By far the most important formula with which we derive pretty much everything else"

Example 1 $f(x) = \frac{1}{x}$ → hyperbola.

Solution - Geometrically



Algebraically

$$\frac{\Delta f}{\Delta x} = \frac{\frac{1}{x_0+\Delta x} - \frac{1}{x_0}}{\Delta x}$$

Cont.

$$= \frac{1}{\Delta x}\left(\frac{x_0 - (x_0+\Delta x)}{(x_0+\Delta x)\,x_0}\right) = \frac{1}{\Delta x}\left(\frac{-\Delta x}{(x_0+\Delta x)\,x_0}\right)$$

$$= \frac{-1}{(x_0+\Delta x)\,x_0} \xrightarrow[\Delta x \to 0]{} \boxed{\frac{-1}{x_0^2}}$$

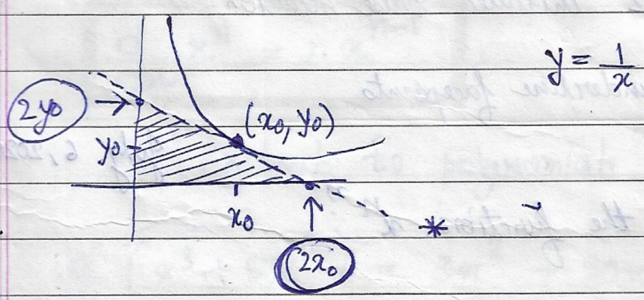
$$\therefore \boxed{f'(x_0) = \frac{-1}{x_0^2}}\ , <0 \text{ negative slope}$$

$x_0 \to \infty$ less & less steep.

July 4, 2020 Saturday.

Problem: Find the area of triangles enclosed By Axes and Tangent to $y = 1/x$

Solution:



$$* \quad y - y_0 = \frac{-1}{x_0^2}(x - x_0)$$

"Only bit of Calculus"

Find x intercept $(y=0)$: $\quad 0 - y_0 = \frac{-1}{x_0^2}(x - x_0)$

$$\Rightarrow 0 - \frac{1}{x} = \frac{-1}{x_0^2}(x - x_0) = \frac{-x}{x_0^2} + \frac{1}{x_0}$$

$$\Rightarrow x/x_0^2 = 2/x_0 \Rightarrow \boxed{x = 2x_0}$$

Shortcut to y-intercept (use symmetry). $\quad y = 2y_0$

exchange of $(x, y) \rightsquigarrow (y, x)$

Symmetry Explanation

$y = \frac{1}{x} \Leftrightarrow xy = 1 \Leftrightarrow x = \frac{1}{y}$

Can also get y-intercept by plugging $x = 0$ into (x)

Δ Area of triangle $= \frac{1}{2} x y = \frac{1}{2}(2x_0)(2y_0) = x_0 \times 2 \times \frac{1}{x_0} = 2$

More Notations

$y = f(x)$, $\Delta y = \Delta f$

$f' =$ Newton's $\underbrace{\frac{df}{dx} = \frac{dy}{dx} = \frac{d}{dx} f = \frac{d}{dx} y}$

Leibniz initiated these notation.

(omits x_0)

↳ underline facepoints

July 6, 2020

"Calculation of the derivative of the function x^n"

Example 2: $f(x) = x^n$, $n = 1, 2, 3, \ldots$

$$\frac{d}{dx} x^n = ?$$

Solution. $\frac{\Delta f}{\Delta x} = \frac{(x+\Delta x)^n - x^n}{\Delta x}$.

(binomial theorem) • $(x+\Delta x)^n = (x+\Delta x) \ldots\ldots (x+\Delta x)$

Here → $= x^n + n x^{n-1} \Delta x + \text{junk}$

of $(\Delta x)^2, (\Delta x)^3 \ldots$ junk $\Rightarrow O((\Delta x)^2)$ terms

$$\frac{\Delta f}{\Delta x} = \frac{1}{\Delta x}\left((x+\Delta x)^n - x^n \right)$$

$$= \frac{1}{\Delta x}\left(\cancel{x^n} + nx^{n-1}\Delta x + O((\Delta x)^2) - \cancel{x^n} \right)$$

$$= \frac{1}{\Delta x}\left(nx^{n-1}\Delta x + O((\Delta x)^2) \right)$$

$$= nx^{n-1} + O(\Delta x)$$

in limit $\Delta x \to 0$ $= 0$

$$\xrightarrow[\Delta x \to 0]{} nx^{n-1}$$

$$\boxed{\frac{d}{dx} x^n = nx^{n-1}}$$

"Extends to polynomials"

$$\frac{d}{dx}(x^3 + 5x^{10}) = 3x^2 + 50x^9$$

Recall

Derivative = Slope of Tangent line

$$\frac{d}{dx}\frac{1}{x} = \frac{-1}{x^2}$$

$$\frac{d}{dx}x^n = nx^{n-1}, \quad n = 1, 2, 3 \ldots$$

$n = 0$ ✓ Also works.