Applications of Derivatives

1. Given $P(x) = x^4 + ax^3 + bx^2 + cx + d$ such that $x = 0$ is the only real root of $P'(x) = 0$. If $P(-1) < P(1)$, then in the interval $[-1, 1]$ **[AIEEE-2009]**

   (1) $P(-1)$ is not minimum but $P(1)$ is the maximum of P

   (2) $P(-1)$ is minimum but $P(1)$ is not the maximum of P

   (3) Neither $P(-1)$ is the minimum nor $P(1)$ is the maximum of P

   (4) $P(-1)$ is the minimum and $P(1)$ is the maximum of P

2. The shortest distance between the line $y - x = 1$ and the curve $x = y^2$ is **[AIEEE-2009]**

   (1) $\frac{2\sqrt{3}}{8}$ (2) $\frac{3\sqrt{2}}{5}$

   (3) $\frac{\sqrt{3}}{4}$ (4) $\frac{3\sqrt{2}}{8}$

3. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis, is **[AIEEE-2010]**

   (1) $y = 0$ (2) $y = 1$

   (3) $y = 2$ (4) $y = 3$

4. Let $f : \boldsymbol{R} \to \boldsymbol{R}$ be defined by

   $$f(x) = \begin{cases} k - 2x, & \text{if} \quad x \le -1 \\ 2x + 3, & \text{if} \quad x > -1 \end{cases}$$

   If f has a local minimum at $x = -1$, then a possible value of k is **[AIEEE-2010]**

   (1) 1 (2) 0

   (3) $-\frac{1}{2}$ (4) -1

5. The curve that passes through the point (2, 3) and has the property that the segment of any tangent to it lying between the coordinate axes bisected by the point of contact is given by **[AIEEE-2011]**

   (1) $x^2 + y^2 = 13$ (2) $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 2$

   (3) $2y - 3x = 0$ (4) $y = \frac{6}{x}$

6. Let f be a function defined by

   $$f(x) = \begin{cases} \frac{\tan x}{x}, & x \ne 0 \\ 1, & x = 0 \end{cases}$$

   Statement-1: $x = 0$ is point of minima of f.

   Statement-2: $f'(0) = 0$. **[AIEEE-2011]**

   (1) Statement-1 is true, statement-2 is false.

   (2) Statement-1 is false, statement-2, is true.

   (3) Statement-1 is true, statement-2 is true, statement-2 is a correct explanation for statement-1

   (4) Statement-1 is true, statement-2 is true; statement-2 is ***0*** a correct explanation for statement-1

7. A spherical balloon is filled with 4500π cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π cubic meters per minute, then the rate (in meters per unit) at which the radius of the balloon decreases 49 minutes after the leakage began is **[AIEEE-2012]**

   (1) 7/9 (2) 2/9

   (3) 9/2 (4) 9/7

8. Let $a, b \in R$ be such that the function f given be $f(x) = \ln|x| + bx^2 + ax$, $x \ne 0$ has extreme values at $x = -1$ and $x = 2$.

   Statement-1: f has local maximum at $x = -1$ and at $x = 2$.

   Statement-2: $a = \frac{1}{2}$ and $b = \frac{-1}{4}$. **[AIEEE-2012]**

(1) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.

(2) Statement-1 is true, statement-2 is true, statement-2 is **not** a correct explanation for statement-1.

(3) Statement-1 is true, statement-2 is false.

(4) Statement-1 is false, statement-2 is true.

9. If $x = -1$ and $x = 2$ are extreme points of $f(x) = \alpha \log|x| + \beta x^2 + x$ then **[JEE (Main)-2014]**

(1) $\alpha = 2,\ \beta = -\frac{1}{2}$ (2) $\alpha = 2,\ \beta = \frac{1}{2}$

(3) $\alpha = -6,\ \beta = \frac{1}{2}$ (4) $\alpha = -6,\ \beta = -\frac{1}{2}$

10. The normal to the curve, $x^2 + 2xy - 3y^2 = 0$ at (1,1) **[JEE (Main)-2015]**

(1) Does not meet the curve again

(2) Meets the curve again in the second quadrant

(3) Meets the curve again in the third quadrant

(4) Meets the curve again in the fourth quadrant

11. Let $f(x)$ be a polynomial of degree four having extreme values at $x = 1$ and $x = 2$. If $\lim\limits_{x\to 0}\left[1+\frac{f(x)}{x^2}\right] = 3$, then $f(2)$ is equal to

[JEE (Main)-2015]

(1) –8 (2) –4

(3) 0 (4) 4

12. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side = x units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then **[JEE (Main)-2016]**

(1) $(4-\pi)x = \pi r$ (2) $x = 2r$

(3) $2x = r$ (4) $2x = (\pi + 4)r$

13. The normal to the curve $y(x-2)(x-3) = x + 6$ at the point where the curve intersects the y-axis passes through the point **[JEE (Main)-2017]**

(1) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (2) $\left(\frac{1}{2}, -\frac{1}{3}\right)$

(3) $\left(\frac{1}{2}, \frac{1}{3}\right)$ (4) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

14. Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is

[JEE (Main)-2017]

(1) 10 (2) 25

(3) 30 (4) 12.5

15. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is

[JEE (Main)-2018]

(1) 6 (2) $\frac{7}{2}$

(3) 4 (4) $\frac{9}{2}$

16. Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}$, $x \in R - \{-1, 0, 1\}$. If $h(x) = \frac{f(x)}{g(x)}$, then the local minimum value of $h(x)$ is **[JEE (Main)-2018]**

(1) 3 (2) –3

(3) $-2\sqrt{2}$ (4) $2\sqrt{2}$

17. If θ denotes the acute angle between the curves, $y = 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then $|\tan\theta|$ is equal to

[JEE (Main)-2019]

(1) $\frac{8}{15}$ (2) $\frac{7}{17}$

(3) $\frac{8}{17}$ (4) $\frac{4}{9}$

18. The maximum volume (in cu. m) of the right circular cone having slant height 3 m is

[JEE (Main)-2019]

(1) $\frac{4}{3}\pi$ (2) $2\sqrt{3}\,\pi$

(3) $3\sqrt{3}\,\pi$ (4) $6\,\pi$

19. The shortest distance between the point $\left(\frac{3}{2}, 0\right)$ and the curve $y = \sqrt{x}, (x > 0)$, is

[JEE (Main)-2019]

(1) $\frac{3}{2}$ (2) $\frac{5}{4}$

(3) $\frac{\sqrt{3}}{2}$ (4) $\frac{\sqrt{5}}{2}$

20. The tangent to the curve $y = xe^{x^2}$ passing through the point $(1, e)$ also passes through the point **[JEE (Main)-2019]**

(1) $(2, 3e)$ (2) $\left(\frac{4}{3}, 2e\right)$

(3) $(3, 6e)$ (4) $\left(\frac{5}{3}, 2e\right)$

21. A helicopter is flying along the curve given by $y - x^{3/2} = 7$, $(x \geq 0)$. A soldier positioned at the point $\left(\frac{1}{2}, 7\right)$ wants to shoot down the helicopter when it is nearest to him. Then this nearest distance is **[JEE (Main)-2019]**

(1) $\frac{1}{6}\sqrt{\frac{7}{3}}$ (2) $\frac{\sqrt{5}}{6}$

(3) $\frac{1}{2}$ (4) $\frac{1}{3}\sqrt{\frac{7}{3}}$

22. The maximum value of the function $f(x) = 3x^3 - 18x^2 + 27x - 40$ on the set $S = \{x \in R: x^2 + 30 \leq 11x\}$ is **[JEE (Main)-2019]**

(1) 122 (2) –122

(3) 222 (4) –222

23. Let x, y be positive real numbers and m, n positive integers. The maximum value of the expression $\frac{x^m y^n}{(1+x^{2m})(1+y^{2n})}$ is **[JEE (Main)-2019]**

(1) $\frac{1}{2}$ (2) $\frac{m+n}{6mn}$

(3) 1 (4) $\frac{1}{4}$

24. Let $f(x) = \frac{x}{\sqrt{a^2 + x^2}} - \frac{(d-x)}{\sqrt{b^2 + (d-x)^2}}, x \in R$ where a, b and d are non-zero real constants. Then **[JEE (Main)-2019]**

(1) f is an increasing function of x

(2) f is a decreasing function of x

(3) f is neither increasing nor decreasing function of x

(4) f' is not a continuous function of x

25. The tangent to the curve $y = x^2 - 5x + 5$, parallel to the line $2y = 4x + 1$, also passes through the point **[JEE (Main)-2019]**

(1) $\left(\frac{1}{4}, \frac{7}{2}\right)$ (2) $\left(\frac{1}{8}, -7\right)$

(3) $\left(\frac{7}{2}, \frac{1}{4}\right)$ (4) $\left(-\frac{1}{8}, 7\right)$

26. If a curve passes through the point $(1, -2)$ and has slope of the tangent at any point (x, y) on it as $\frac{x^2 - 2y}{x}$, then the curve also passes through the point **[JEE (Main)-2019]**

(1) $(-1, 2)$ (2) $\left(\sqrt{3}, 0\right)$

(3) $(3, 0)$ (4) $\left(-\sqrt{2}, 1\right)$

27. If the function f given by $f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$, for some $a \in R$ is increasing in $(0, 1]$ and decreasing in $[1, 5)$, then a root of the equation, $\frac{f(x) - 14}{(x-1)^2} = 0 \ (x \neq 1)$ is **[JEE (Main)-2019]**

(1) –7 (2) 5

(3) 6 (4) 7

28. The equation of a tangent to the parabola, $x^2 = 8y$, which makes an angle θ with the positive direction of x-axis, is **[JEE (Main)-2019]**

(1) $x = y\cot\theta - 2\tan\theta$

(2) $y = x\tan\theta + 2\cot\theta$

(3) $x = y\cot\theta + 2\tan\theta$

(4) $y = x\tan\theta - 2\cot\theta$

29. If a straight line passing through the point $P(-3, 4)$ is such that its intercepted portion between the coordinate axes is bisected at P, then its equation is **[JEE (Main)-2019]**

(1) $3x - 4y + 25 = 0$ (2) $4x - 3y + 24 = 0$

(3) $x - y + 7 = 0$ (4) $4x + 3y = 0$

30. The shortest distance between the line $y = x$ and the curve $y^2 = x - 2$ is **[JEE (Main)-2019]**

(1) $\frac{11}{4\sqrt{2}}$ (2) $\frac{7}{8}$

(3) 2 (4) $\frac{7}{4\sqrt{2}}$

31. If S_1 and S_2 are respectively the sets of local minimum and local maximum points of the function, $f(x) = 9x^4 + 12x^3 - 36x^2 + 25$, $x \in R$, then **[JEE (Main)-2019]**

(1) $S_1 = \{-2\}; S_2 = \{0, 1\}$

(2) $S_1 = \{-2, 1\}; S_2 = \{0\}$

(3) $S_1 = \{-1\}; S_2 = \{0, 2\}$

(4) $S_1 = \{-2, 0\}; S_2 = \{1\}$

32. Let $f : [0, 2] \to R$ be a twice differentiable function such that $f''(x) > 0$, for all $x \in (0, 2)$. If $\phi(x) = f(x) + f(2 - x)$, then ϕ is **[JEE (Main)-2019]**

(1) Decreasing on (0, 2)

(2) Increasing on (0, 2)

(3) Decreasing on (0, 1) and increasing on (1, 2)

(4) Increasing on (0, 1) and decreasing on (1, 2)

33. The height of a right circular cylinder of maximum volume inscribed in a sphere of radius 3 is **[JEE (Main)-2019]**

(1) $\frac{2}{3}\sqrt{3}$ (2) $2\sqrt{3}$

(3) $\sqrt{3}$ (4) $\sqrt{6}$

34. If $f(x)$ is a non-zero polynomial of degree four, having local extreme points at $x = -1, 0, 1$; then the set

$S = \{x \in R : f(x) = f(0)\}$

contains exactly **[JEE (Main)-2019]**

(1) Four irrational numbers

(2) Four rational numbers

(3) Two irrational and one rational number

(4) Two irrational and two rational numbers

35. Let S be the set of all values of x for which the tangent to the curve $y = f(x) = x^3 - x^2 - 2x$ at (x, y) is parallel to the line segment joining the points $(1, f(1))$ and $(-1, f(-1))$, then S is equal to **[JEE (Main)-2019]**

(1) $\left\{\frac{1}{3}, -1\right\}$ (2) $\left\{-\frac{1}{3}, 1\right\}$

(3) $\left\{-\frac{1}{3}, -1\right\}$ (4) $\left\{\frac{1}{3}, 1\right\}$

36. If the tangent to the curve, $y = x^3 + ax - b$ at the point $(1, -5)$ is perpendicular to the line, $-x + y + 4 = 0$, then which one of the following points lies on curve? **[JEE (Main)-2019]**

(1) $(-2, 1)$ (2) $(2, -2)$

(3) $(2, -1)$ (4) $(-2, 2)$

37. A water tank has the shape of an inverted right circular cone, whose semi-vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at a constant rate of 5 cubic metre per minute. Then the rate (in m/min.), at which the level of water is rising at the instant when the depth of water in the tank is 10 m; is **[JEE (Main)-2019]**

(1) $\frac{2}{\pi}$ (2) $\frac{1}{15\pi}$

(3) $\frac{1}{10\pi}$ (4) $\frac{1}{5\pi}$

38. Let $f(x) = e^x - x$ and $g(x) = x^2 - x$, $\forall\, x \in R$. Then the set of all $x \in R$, where the function $h(x) = (fog)(x)$ is increasing, is **[JEE (Main)-2019]**

(1) $\left[-1, \frac{-1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$ (2) $[0, \infty)$

(3) $\left[0, \frac{1}{2}\right] \cup [1, \infty)$ (4) $\left[\frac{-1}{2}, 0\right] \cup [1, \infty)$

39. If the tangent to the curve $y = \frac{x}{x^2 - 3}$, $x \in R$, $(x \neq \pm\sqrt{3})$, at a point $(\alpha, \beta) \neq (0, 0)$ on it is parallel to the line $2x + 6y - 11 = 0$, then

[JEE (Main)-2019]

(1) $|6\alpha + 2\beta| = 19$ (2) $|2\alpha + 6\beta| = 19$

(3) $|6\alpha + 2\beta| = 9$ (4) $|2\alpha + 6\beta| = 11$

40. A spherical iron ball of radius 10 cm is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of the ice is 5 cm, then the rate at which the thickness (in cm/min) of the ice decreases, is

[JEE (Main)-2019]

(1) $\frac{5}{6\pi}$ (2) $\frac{1}{36\pi}$

(3) $\frac{1}{9\pi}$ (4) $\frac{1}{18\pi}$

41. If m is the minimum value of k for which the function $f(x) = x\sqrt{kx - x^2}$ is increasing in the interval [0, 3] and M is the maximum value of f in [0, 3] when $k = m$, then the ordered pair (m, M) is equal to **[JEE (Main)-2019]**

(1) $(4, 3\sqrt{2})$ (2) $(3, 3\sqrt{3})$

(3) $(5, 3\sqrt{6})$ (4) $(4, 3\sqrt{3})$

42. If the angle of intersection at a point where the two circles with radii 5 cm and 12 cm intersect is 90°, then the length (in cm) of their common chord is **[JEE (Main)-2019]**

(1) $\frac{120}{13}$ (2) $\frac{13}{2}$

(3) $\frac{13}{5}$ (4) $\frac{60}{13}$

43. The tangents to the curve $y = (x - 2)^2 - 1$ at its points of intersection with the line $x - y = 3$, intersect at the point **[JEE (Main)-2019]**

(1) $\left(\frac{5}{2}, 1\right)$ (2) $\left(-\frac{5}{2}, 1\right)$

(3) $\left(\frac{5}{2}, -1\right)$ (4) $\left(-\frac{5}{2}, -1\right)$

44. Let $f(x)$ be a polynomial of degree 5 such that $x = \pm 1$ are its critical points. If $\lim\limits_{x \to 0}\left(2 + \frac{f(x)}{x^3}\right) = 4$, then which one of the following is not true? **[JEE (Main)-2020]**

(1) f is an odd function

(2) $x = 1$ is a point of minima and $x = -1$ is a point of maxima of f

(3) $f(1) - 4f(-1) = 4$

(4) $x = 1$ is a point of maxima and $x = -1$ is a point of minimum of f

45. Let $f(x) = x\cos^{-1}(-\sin|x|)$, $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, then which of the following is true? **[JEE (Main)-2020]**

(1) $f'(0) = -\frac{\pi}{2}$

(2) f' is decreasing in $\left(-\frac{\pi}{2}, 0\right)$ and increasing in $\left(0, \frac{\pi}{2}\right)$

(3) f is not differentiable at $x = 0$

(4) f' is increasing in $\left(-\frac{\pi}{2}, 0\right)$ and decreasing in $\left(0, \frac{\pi}{2}\right)$

46. Let $f : (1, 3) \to R$ be a function defined by $f(x) = \frac{x[x]}{1 + x^2}$, where $[x]$ denotes the greatest integer $\leq x$. Then the range of f is **[JEE (Main)-2020]**

(1) $\left(\frac{2}{5}, \frac{3}{5}\right] \cup \left(\frac{3}{4}, \frac{4}{5}\right)$ (2) $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$

(3) $\left(\frac{2}{5}, \frac{4}{5}\right]$ (4) $\left(\frac{3}{5}, \frac{4}{5}\right)$

47. A spherical iron ball of 10 cm radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate (in cm/min.) at which of the thickness of ice decreases, is **[JEE (Main)-2020]**

(1) $\frac{1}{36\pi}$ (2) $\frac{1}{18\pi}$

(3) $\frac{1}{54\pi}$ (4) $\frac{5}{6\pi}$

48. Let a function $f : [0, 5] \to R$ be continuous, $f(1) = 3$ and F be defined as

$$F(x) = \int_1^x t^2 g(t)dt, \text{ where } g(t) = \int_1^t f(u)du.$$

Then for the function F, the point $x = 1$ is **[JEE (Main)-2020]**

(1) A point of inflection.

(2) Not a critical point.

(3) A point of local minima.

(4) A point of local maxima.

49. If $p(x)$ be a polynomial of degree three that has a local maximum value 8 at $x = 1$ and a local minimum value 4 at $x = 2$; then $p(0)$ is equal to **[JEE (Main)-2020]**

(1) 12 (2) 6

(3) –24 (4) –12

50. Let $P(h, k)$ be a point on the curve $y = x^2 + 7x + 2$, nearest to the line, $y = 3x - 3$. Then the equation of the normal to the curve at P is **[JEE (Main)-2020]**

(1) $x - 3y - 11 = 0$ (2) $x - 3y + 22 = 0$

(3) $x + 3y - 62 = 0$ (4) $x + 3y + 26 = 0$

51. If the tangent to the curve $y = x + \sin y$ at a point (a, b) is parallel to the line joining $\left(0, \frac{3}{2}\right)$ and $\left(\frac{1}{2}, 2\right)$, then **[JEE (Main)-2020]**

(1) $b = a$ (2) $b = \frac{\pi}{2} + a$

(3) $|a + b| = 1$ (4) $|b - a| = 1$

52. The equation of the normal to the curve $y = (1 + x)^{2y} + \cos^2(\sin^{-1}x)$ at $x = 0$ is **[JEE (Main)-2020]**

(1) $y = 4x + 2$ (2) $y + 4x = 2$

(3) $x + 4y = 8$ (4) $2y + x = 4$

53. Let $f : (-1, \infty) \longrightarrow R$ be defined by $f(0) = 1$ and $f(x) = \frac{1}{x}\log_e(1 + x),\ x \neq 0$. Then the function f **[JEE (Main)-2020]**

(1) Increases in $(-1, \infty)$

(2) Increases in $(-1, 0)$ and decreases in $(0, \infty)$

(3) Decreases in $(-1, 0)$ and increases in $(0, \infty)$

(4) Decreases in $(-1, \infty)$

54. The function, $f(x) = (3x - 7)x^{2/3}$, $x \in R$, is increasing for all x lying in : **[JEE (Main)-2020]**

(1) $(-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$

(2) $(-\infty, 0) \cup \left(\frac{3}{7}, \infty\right)$

(3) $\left(-\infty, -\frac{14}{15}\right) \cup (0, \infty)$

(4) $\left(-\infty, \frac{14}{15}\right)$

55. Suppose $f(x)$ is a polynomial of degree four, having critical points at $-1, 0, 1$. If $T = \{x \in R \mid f(x) = f(0)\}$, then the sum of squares of all the elements of T is **[JEE (Main)-2020]**

(1) 4 (2) 2

(3) 6 (4) 8

56. If the surface area of a cube is increasing at a rate of 3.6 cm^2/sec, retaining its shape; then the rate of change of its volume (in cm^3/sec), when the length of a side of the cube is 10 cm, is: **[JEE (Main)-2020]**

(1) 9 (2) 20

(3) 10 (4) 18

57. Let f be a twice differentiable function on $(1, 6)$. If $f(2) = 8$, $f'(2) = 5$, $f'(x) \geq 1$ and $f''(x) \geq 4$, for all $x \in (1, 6)$, then **[JEE (Main)-2020]**

(1) $f(5) + f'(5) \geq 28$ (2) $f'(5) + f''(5) \leq 20$

(3) $f(5) \leq 10$ (4) $f(5) + f'(5) \leq 26$

58. The area (in sq. units) of the largest rectangle $ABCD$ whose vertices A and B lie on the x-axis and vertices C and D lie on the parabola, $y = x^2 - 1$ below the x-axis, is **[JEE (Main)-2020]**

(1) $\frac{4}{3\sqrt{3}}$ (2) $\frac{1}{3\sqrt{3}}$

(3) $\frac{2}{3\sqrt{3}}$ (4) $\frac{4}{3}$

59. The minimum value of $2^{\sin x} + 2^{\cos x}$ is **[JEE (Main)-2020]**

(1) $2^{-1+\sqrt{2}}$ (2) $2^{1-\sqrt{2}}$

(3) $2^{1-\frac{1}{\sqrt{2}}}$ (4) $2^{-1+\frac{1}{\sqrt{2}}}$

60. If the minimum and the maximum values of the function $f : \left[\frac{\pi}{4}, \frac{\pi}{2}\right] \to R$, defined by

$$f(\theta) = \begin{vmatrix} -\sin^2\theta & -1-\sin^2\theta & 1 \\ -\cos^2\theta & -1-\cos^2\theta & 1 \\ 12 & 10 & -2 \end{vmatrix}$$

are m and M respectively, then the ordered pair (m, M) is equal to **[JEE (Main)-2020]**

(1) $(0, 2\sqrt{2})$ (2) $(0, 4)$

(3) $(-4, 4)$ (4) $(-4, 0)$

61. Which of the following points lies on the tangent to the curve $x^4e^y + 2\sqrt{y+1} = 3$ at the point $(1, 0)$? **[JEE (Main)-2020]**

(1) $(2, 2)$ (2) $(-2, 4)$

(3) $(2, 6)$ (4) $(-2, 6)$

62. If $x = 1$ is a critical point of the function $f(x) = (3x^2 + ax - 2 - a)e^x$, then **[JEE (Main)-2020]**

(1) $x = 1$ is a local maxima and $x = -\frac{2}{3}$ is a local minima of f

(2) $x = 1$ and $x = -\frac{2}{3}$ are local maxima of f

(3) $x = 1$ and $x = -\frac{2}{3}$ are local minima of f

(4) $x = 1$ is a local minima and $x = -\frac{2}{3}$ is a local maxima of f

63. Let m and M be respectively the minimum and maximum values of

$$\begin{vmatrix} \cos^2 x & 1+\sin^2 x & \sin 2x \\ 1+\cos^2 x & \sin^2 x & \sin 2x \\ \cos^2 x & \sin^2 x & 1+\sin 2x \end{vmatrix}$$

Then the ordered pair (m, M) is equal to **[JEE (Main)-2020]**

(1) $(1, 3)$ (2) $(-3, -1)$

(3) $(-4, -1)$ (4) $(-3, 3)$

64. The position of a moving car at time t is given by $f(t) = at^2 + bt + c$, $t > 0$, where a, b and c are real numbers greater than 1. Then the average speed of the car over the time interval $[t_1, t_2]$ is attained at the point **[JEE (Main)-2020]**

(1) $2a(t_1 + t_2) + b$

(2) $(t_2 - t_1)/2$

(3) $a(t_2 - t_1) + b$

(4) $(t_1 + t_2)/2$

65. The set of all real values of λ for which the function $f(x) = (1 - \cos^2x)\cdot(\lambda + \sin x)$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, has exactly one maxima and exactly one minima, is **[JEE (Main)-2020]**

(1) $\left(-\frac{3}{2}, \frac{3}{2}\right)$ (2) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

(3) $\left(-\frac{1}{2}, \frac{1}{2}\right) - \{0\}$ (4) $\left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$

66. For all twice differentiable functions $f : R \to R$, with $f(0) = f(1) = f'(0) = 0$, **[JEE (Main)-2020]**

(1) $f''(0) = 0$

(2) $f''(x) = 0$, for some $x \in (0, 1)$

(3) $f''(x) = 0$, at every point $x \in (0, 1)$

(4) $f''(x) \neq 0$, at every point $x \in (0, 1)$

67. If the tangent to the curve, $y = f(x) = x\log_e x$, $(x > 0)$ at a point $(c, f(c))$ is parallel to the line-segment joining the points $(1, 0)$ and (e, e), then c is equal to **[JEE (Main)-2020]**

(1) $\frac{1}{e-1}$ (2) $\frac{e-1}{e}$

(3) $e^{\left(\frac{1}{1-e}\right)}$ (4) $e^{\left(\frac{1}{e-1}\right)}$

68. Let $f(x)$ be a polynomial of degree 3 such that $f(-1) = 10$, $f(1) = -6$, $f(x)$ has a critical point at $x = -1$, and $f'(x)$ has a critical point $x = 1$. Then $f(x)$ has a local minima at $x =$ _________. **[JEE (Main)-2020]**

69. If the tangent to the curve, $y = e^x$ at a point (c, e^c) and the normal to the parabola, $y^2 = 4x$ at the point $(1, 2)$ intersect at the same point on the x-axis, then the value of c is _________ . **[JEE (Main)-2020]**

70. If the lines $x + y = a$ and $x - y = b$ touch the curve $y = x^2 - 3x + 2$ at the points where the curve intersects the x-axis, then $\frac{a}{b}$ is equal to _______. **[JEE (Main)-2020]**

71. Let AD and BC be two vertical poles at A and B respectively on a horizontal ground. If $AD = 8$ m, $BC = 11$ m and $AB = 10$ m; then the distance (in meters) of a point M on AB from the point A such that $MD^2 + MC^2$ is minimum is _________. **[JEE (Main)-2020]**

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