10 Arithmetic Progression

10.1 Introduction :

A group of numbers, which are arranged in a definite order following a certain rule, is called a **sequence**.

For example :

1. 2, 4, 6, 8, ... is a sequence in which each number **is 2 more than** its preceding number.
2. 2, 4, 16, 256, ... is a sequence in which each number **is square of** its preceding number.
3. 30, 27, 24, ... is a sequence in which each number (term) is **3 less than** its preceding term.
4. 5, 8, 6, 15, ... is not a sequence as numbers of this group do not have any definite relation with their preceding numbers.

The numbers used in a sequence are called its **elements** or **terms.**

When the numbers (terms) in a sequence are connected to each other by positive (plus) sign or negative (minus) sign, the **sequence becomes series**.

Thus, (i) $2 + 4 + 6 + 8 +$ .... is a series.

(ii) $-2 - 4 - 6 - 8$ .... is a series.

(iii) $2 + 4 + 16 + 256 +$ .... is a series and so on.

Sequence and series are used in the same sense.

When the members (elements) of a series (or, sequence) are written in a definite order, following certain rule, we get a **progression**.

For example :

3, 7, 11, 15 .... is a progression and successive

term of 15 is $15 + 4 = 19$, successive term of 19 is $19 + 4 = 23$ and so on.

Similarly, $5 + 15 + 45 + 135 +$ .... is progression.

In a progression, its terms (elements) are in such a pattern which can suggest the successor of every element of it.

10.2 Arithmetic Progression (A.P.) :

An **arithmetic progression** is a sequence (series) of numbers in which each term can be obtained by adding a certain quantity to its preceding term.

For example, the sequence 3, 8, 13, 18, 23, ... is an arithmetic progression in which every term (other than the first term) can be obtained by adding 5 to its preceding term *i.e.*

$3 + 5 = 8, \quad 8 + 5 = 13, \quad 13 + 5 = 18$ and so on.

In other words, we can say that the difference between any two consecutive terms of the sequence, given above, is 5.

In an A.P., the difference between two consecutive terms is called its **common difference** and is denoted by letter **'*d*'**.

Thus, if $t_1, t_2, t_3, t_4,$ .... are terms (numbers) in A.P., its **first term** is t_1 and is denoted by $\boldsymbol{a}$.

So, $a = t_1$

$d = t_2 - t_1 = t_3 - t_2 = t_4 - t_3 =$ .... and so on.

Each of the following series is an arithmetic progression.

1. 4, 6, 8, 10, ...
   with first term $a = 4$ and common difference $d = 6 - 4 = 2$.
2. $4 + 8 + 12 + 16 +$ ....
   with $a = 4$ and common difference $d = 8 - 4 = 4$.
3. $-4 - 6 - 8 - 10$ .... has $a = -4$ and $d = -6 - (-4) = -2$.
4. 28, 25, 22, 19, ... has $a = 28$ and $d = 25 - 28 = -3$.

Conversely, if first term (a) of an A.P. is 8 and its common difference (d) is 3, the A.P. is :

$8, 8 + 3, 8 + 3 + 3, 8 + 3 + 3 + 3,$ ....

$= 8, 11, 14, 17,$ ....

More examples :

	First term (a)	Common difference (d)	A.P.
1.	5	2	5, 7, 9, 11, 13, ...
2.	18	−2	18, 16, 14, 12, ...
3.	a	d	$a, a + d, a + 2d, a + 3d,$ ....
4.	−20	−4	−20, −24, −28, ...
5.	0	−3	0, −3, −6, −9, ...

1 Is the sequence 12, 8, 4, 0, ... an A.P. ?
If yes; state its first term and common difference.

Solution :

Since, $8 - 12 = -4, \quad 4 - 8 = -4$ and $0 - 4 = -4$

⇒ Difference between consecutive terms is the same.

⇒ **The given sequence is an A.P.** **Ans.**

Clearly, **first term = 12** and **common difference = −4** **Ans.**

2 For the A.P. 7, 15, 23, 31, ..., write the first term, common difference and next two terms.

Solution :

First term of the given A.P. = **7.** Its common difference = 15 – 7 = **8**

Next two terms are 31 + d = 31 + 8 = **39** and 39 + 8 = **47** **Ans.**

10.3 General Term of An Arithmetic Progression :

Let the first term of an A.P. be 'a' and its common difference be 'd', the terms of the A.P. can be taken as :

$a, a + d, a + 2d, a + 3d,$ ....

or, $a + (a + d) + (a + 2d) + (a + 3d) +$ ....

First term $= a = a + (1 - 1)d = a +$ (no. of term $- 1)d$

Second term $= a + d = a + (2 - 1)d = a +$ (no. of term $- 1)d$

Third term $= a + 2d = a + (3 - 1)d = a +$ (no. of term $- 1)d$

On proceeding in the similar manner, we find :

10^{th} term $= a + (10 - 1)d = a + 9d$

24^{th} term $= a + (24 - 1)d = a + 23d$

5th term $= a + 4d$, 12^{th} term $= a + 11d$ and so on.

On combining all the discussions made in this article, we find :

n^{th} term of an A.P. $= a + (n - 1)d$ *i.e.* $\boldsymbol{t_n = a + (n - 1)d}$

Here, $\boldsymbol{t_n = a + (n - 1)d}$ is called the general term of the A.P. in which by putting $n = 1, 2, 3$, etc., we can get first term, second term, third term, respectively of the A.P. under consideration.

If an A.P. has only n terms, its n^{th} term (last term) is denoted by letter l.

Thus for an A.P. with first term $= a$, common difference $= d$ and number of terms = n, we have $\boxed{l = a + (n - 1)d.}$

For an A.P., $t_n = a + (n - 1)d$

$\Rightarrow$ if $d > 0$, the A.P. is increasing,

if $d < 0$, the A.P. is decreasing and

if $d = 1$, all the terms of the A.P. are same.

Thus, for A.P. of n terms,

A.P. $= a + (a + d) + (a + 2d) +$ ....

$= a + (a + d) + (a + 2d) +$ .... $+ \{a + (n - 1)d\}$

$= a + (a + d) + (a + 2d) +$ .... $+ l$

$= a + (a + d) + (a + 2d) +$ .... $+ (l - 2d) + (l - d) + l.$

3 Find the 20^{th} term of the sequence : 9, 5, 1, –3, ...

Solution :

Since, $5 - 9 = -4$, $1 - 5 = -4$, $-3 - 1 = -4$, etc. the given sequence is an A.P. with first term $a = 9$ and common difference $d = -4$.

$$t_n = a + (n-1)d \Rightarrow t_{20} = 9 + (20-1) \times -4$$
$$= 9 + 19 \times -4 = 9 - 76 = \mathbf{-67} \qquad \textbf{Ans.}$$

4 Find the A.P. whose second term is 12 and 7th term exceeds the 4^{th} by 15.

Solution :

Second term $= 12 \Rightarrow a + d = 12$

7^{th} term $- 4^{th}$ term $= 15 \Rightarrow (a + 6d) - (a + 3d) = 15$

i.e. $3d = 15 \Rightarrow d = 5$ and $a + d = 12 \Rightarrow a = 7$

$\therefore$ **The required A.P.** $= a, a + d, a + 2d, a + 3d, \ldots\ldots$

$= 7, 7 + 5, 7 + 2 \times 5, 7 + 3 \times 5, \ldots\ldots$

$= \mathbf{7, 12, 17, 22, \ldots\ldots}$ **Ans.**

5 Find the A.P. whose 6th term = 5 and 10th term = 9.

Solution :

Given $a + 5d = 5$ and $a + 9d = 9$

On solving, we get : $a = 0$ and $d = 1$,

$\therefore$ **The required A.P.** $= a, a + d, a + 2d, a + 3d, \ldots\ldots$

$= \mathbf{0, 1, 2, 3, 4, 5, \ldots\ldots}$ **Ans.**

6 Which term of the A.P. 4·2, 4·7, 5·2, 5·7, ... is 8·7 ?

Solution :

Given $a = 4{\cdot}2$, $d = 4{\cdot}7 - 4{\cdot}2 = 0{\cdot}5$ and $t_n = 8{\cdot}7$

To find : number of terms *i.e.* n.

$$t_n = a + (n-1)d \Rightarrow 8{\cdot}7 = 4{\cdot}2 + (n-1) \times 0{\cdot}5$$

i.e. $8{\cdot}7 - 4{\cdot}2 = (n-1) \times 0{\cdot}5 \Rightarrow \dfrac{4{\cdot}5}{0{\cdot}5} = (n-1)$

i.e. $9 = n - 1$ and $\mathbf{n = 10}$ **Ans.**

7 Find the 12th term from the end in A.P. 13, 18, 23, ..., 158.

Solution :

For an A.P. $a, a + d, a + 2d, \ldots\ldots, l$ [The common difference $= d$]

its reverse A.P. $= l, l - d, l - 2d, \ldots\ldots, a$ [The common difference $= -d$]

Clearly, if the common difference of given A.P. is positive, the common difference of its reverse A.P. is negative and vice-versa.

The given A.P. is 13, 18, 23, ..., 158.

Its first term = 13, common difference = 5 and last term = 158

The reverse A.P. = 158, 153, 148, ..., 18, 13

For the reverse A.P., first term = 158 and common difference = –5.

$\therefore$ **12^{th} term from the end of given A.P.**

= 12^{th} term from the beginning of its reverse A.P.

= 12^{th} term of the A.P. 158, 153, ..., 18, 13

= $a + (n - 1)d$

= $158 + (12 - 1) \times -5 =$ **103** **Ans.**

Alternative Method :

Let the given A.P. 13, 18, 23, ..., 158 has n terms

$\therefore \quad 158 = 13 + (n - 1) \times 5$ $\quad [\because tn = a + (n - 1)d]$

$\Rightarrow \quad n = 30$

12^{th} term from the end

= $(30 - 12 + 1)^{th}$ term from the beginning

= 19^{th} term from the beginning

= $a + 18d = 13 + 18 \times 5 =$ **103** **Ans.**

In general, if a sequence has n terms, its r^{th} term from the end
= Its $(n - r + 1)^{th}$ term from the beginning

EXERCISE 10(A)

1. Which of the following sequences are in arithmetic progression ?
   (i) 2, 6, 10, 14, ...
   (ii) 15, 12, 9, 6, ...
   (iii) 5, 9, 12, 18, ...
   (iv) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$, ...
2. The n^{th} term of a sequence is $(2n - 3)$, find its fifteenth term.
3. If the p^{th} term of an A.P. is $(2p + 3)$; find the A.P.
4. Find the 24^{th} term of the sequence :
   12, 10, 8, 6, ...
5. Find the 30^{th} term of the sequence :
   $\frac{1}{2}, 1, \frac{3}{2}$, ...
6. Find the 100^{th} term of the sequence :
   $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}$, ...
7. Find the 50^{th} term of the sequence :
   $\frac{1}{n}, \frac{n+1}{n}, \frac{2n+1}{n}$, ...
8. Is 402 a term of the sequence :
   8, 13, 18, 23, ... ?
9. Find the common difference and 99^{th} term of the arithmetic progression :
   $7\frac{3}{4}, 9\frac{1}{2}, 11\frac{1}{4}$, ...
10. How many terms are there in the series :
    (i) 4, 7, 10, 13, ..., 148 ?
    (ii) 0·5, 0·53, 0·56, ..., 1·1 ?
    (iii) $\frac{3}{4}, 1, 1\frac{1}{4}$, ..., 3 ?
11. Which term of the A.P. 1 + 4 + 7 + 10 + ... is 52 ?
12. If 5^{th} and 6^{th} terms of an A.P. are respectively 6 and 5. find the 11^{th} term of the A.P.
13. If t_n represents n^{th} term of an A.P., $t_2 + t_5 - t_3 = 10$ and $t_2 + t_9 = 17$, find its first term and its common difference.

If first term of an A.P. is a and its common difference is d.

$$t_2 + t_5 - t_3 = 10$$

$$\Rightarrow (a + d) + (a + 4d) - (a + 2d) = 10$$

i.e. $a + 3d = 10$ I

Also, $t_2 + t_9 = 17$

$\Rightarrow (a + d) + (a + 8d) = 17$

i.e. $2a + 9d = 17$ II

Solve the two equations to get the values of a and d

14. Find the 10^{th} term from the end of the A.P. 4, 9, 14, ..., 254.

15. Determine the arithmetic progression whose 3^{rd} term is 5 and 7^{th} term is 9.

16. Find the 31^{st} term of an A.P. whose 10^{th} term is 38 and 16^{th} term is 74.

17. Which term of the series :

    21, 18, 15, ... is –81 ?

    Can any term of this series be zero ?

    If yes, find the number of term.

8 If 8 times the eighth term of an A.P. is equal to 15 times its fifteenth term, find its 23^{rd} term.

Solution :

Given $8 \times t_8 = 15 \times t_{15}$

$\Rightarrow 8 \times [a + (8 - 1)d] = 15 \times [a + (15 - 1)d]$

$\Rightarrow 8a + 56d = 15a + 210d$

$\Rightarrow -154d = 7a$ i.e. $a = -22d$

Now, **23^{rd} term** $= a + 22d$

$= -22d + 22d = \mathbf{0}$ **Ans.**

Remember : For an A.P., if :

(i) $2 \times t_2 = 8 \times t_8 \Rightarrow t_{10} = 0$ [2 + 8 = 10]

(ii) $20 \times t_{20} = 30 \times t_{30} \Rightarrow t_{50} = 0$ [20 + 30 = 50]

In general, for an A.P., if :

m times the m^{th} term = n times the n^{th} term

$\Rightarrow$ its $(m + n)^{th}$ term = 0.

9 Find the number of all natural numbers between 20 and 80, which are divisible by 3.

Solution :

The natural numbers between 20 and 80 which are divisible by 3

= 21, 24, 27, ..., 78

This is an A.P. with first term $a = 21$, common difference $d = 3$ and last term $l = 78$

$l = a + (n - 1)d \Rightarrow 78 = 21 + (n - 1) \times 3$

$\Rightarrow 57 = 3n - 3$ i.e $n = 20$

$\Rightarrow$. **Required number of terms = 20** **Ans.**

10 How many whole numbers, each divisible by 7, lie between 200 and 500 ?

Solution :

$\because \frac{200}{7} = 28\frac{4}{7}$ and $\frac{500}{7} = 71\frac{3}{7}$

The numbers between 200 and 500 and divisible by 7 are :

$29 \times 7, 30 \times 7, 31 \times 7, \ldots\ldots, 71 \times 7$

$= 203, 210, 217, \ldots\ldots, 497$

It is an A.P. with first term $a = 203$, common difference $d = 7$ and last term $l = 497$

$l = a + (n - 1)d \Rightarrow 497 = 203 + (n - 1) \times 7$

i.e. $294 = (n - 1) \times 7 \Rightarrow n - 1 = \dfrac{294}{7} = 42$ and $n = 43$

$\therefore$ **There are 43 numbers between 200 and 500 which are divisible by 7. Ans.**

11 Which term of the A.P. 4, 11, 18, 25, ... is 42 more than its 25th term ?

Solution :

For given A.P.; $a = 4, d = 11 - 4 = 7$ and

$$t_{25} = a + 24d = 4 + 24 \times 7 = 172$$

Let the required term be n^{th} term

$\therefore \quad t_n = t_{25} + 42 \Rightarrow a + (n - 1)d = t_{25} + 42 \qquad [\because t_n = a + (n - 1)d]$

$\Rightarrow 4 + (n - 1) \times 7 = 172 + 42$

i.e. $4 + 7n - 7 = 214 \Rightarrow 7n = 217$ and $n = 31$

$\therefore$ **Required term = 31st term** **Ans.**

EXERCISE 10(B)

1. In an A.P., ten times of its tenth term is equal to thirty times of its 30th term. Find its 40th term.
2. How many two-digit numbers are divisible by 3 ?
3. Which term of A.P. 5, 15, 25, ... will be 130 more than its 31st term ?
4. Find the value of p, if x, $2x + p$ and $3x + 6$ are in A.P.
5. If the 3rd and the 9th terms of an arithmetic progression are 4 and –8 respectively, which term of it is zero ?
6. How many three-digit numbers are divisible by 87 ?
7. For what value of n, the n^{th} term of A.P. 63, 65, 67, ... and n^{th} term of A.P. 3, 10, 17, ... are equal to each other ?
8. Determine the A.P. whose 3rd term is 16 and the 7th term exceeds the 5th term by 12.
9. If numbers $n - 2$, $4n - 1$ and $5n + 2$ are in A.P., find the value of n and its next two terms.
10. Determine the value of k for which $k^2 + 4k + 8$, $2k^2 + 3k + 6$ and $3k^2 + 4k + 4$ are in A.P.
11. If a, b and c are in A.P. show that :
    (i) $4a$, $4b$ and $4c$ are in A.P.
    (ii) $a + 4$, $b + 4$ and $c + 4$ are in A.P.
12. An A.P. consists of 57 terms of which 7th term is 13 and the last term is 108. Find the 45th term of this A.P.
13. 4th term of an A.P. is equal to 3 times its first term and 7th term exceeds twice the 3rd term by 1. Find the first term and the common difference.
14. The sum of the 2nd term and the 7th term of an A.P. is 30. If its 15th term is 1 less than twice of its 8th term, find the A.P.
15. In an A.P., if m^{th} term is n and n^{th} term is m, show that its r^{th} term is $(m + n - r)$.
16. Which term of the A.P. 3, 10, 17, ... will be 84 more than its 13th term ?

10.4 Sum of n terms of an A.P. :

Let for the given A.P., first term = a, common difference = d, number of terms = n and last term = l

If the sum of terms of the A.P. be denoted by S,

$$S = a + (a + d) + (a + 2d) + (a + 3d) + \dots\dots + (l - 2d) + (l - d) + l$$

On writing the terms of the above A.P. in reverse order, we get :

$$S = l + (l - d) + (l - 2d) + \dots\dots + (a + 2d) + (a + d) + a$$

Now adding the two equations, we get :

$$\begin{aligned} 2S &= (a + l) + (a + d + l - d) + (a + 2d + l - 2d) + \dots\dots \\ &\qquad + (l - 2d + a + 2d) + (l - d + a + d) + (l + a) \\ &= (a + l) + (a + l) + (a + l) + (a + l) + \dots\dots\ n \text{ times} \\ &= n(a + l) \end{aligned}$$

$$\Rightarrow \quad S = \frac{n}{2}(a + l)$$

Since, $l = a + (n - 1)d$

$$\begin{aligned} \therefore \quad \mathbf{S} &= \frac{n}{2}(a + l) \\ &= \frac{n}{2}[a + a + (n - 1)d] = \frac{n}{2}[2a + (n - 1)d] \end{aligned}$$

For an A.P. when

(i) a, n and l are known, take its sum $\mathbf{S} = \frac{n}{2}(a + l)$

(ii) a, n and d are known, take $\mathbf{S} = \frac{n}{2}[2a + (n - 1)d]$

12 Find the sum of the first 20 terms of the A.P. : 5, 8, 11, 14, ...

Solution :

Clearly, $a = 5$, $d = 8 - 5 = 3$ and $n = 20$

$$\begin{aligned} \therefore \quad \mathbf{S} &= \frac{n}{2}[2a + (n - 1)d] \\ &= \frac{20}{2}[2 \times 5 + (20 - 1) \times 3] \\ &= 10(10 + 19 \times 3) = 10 \times 67 = 670 \end{aligned}$$

Ans.

13 Find the sum of first ten terms of the A.P. : 8, 4, 0, –4, –8, ...

Solution :

Clearly, $a = 8$, $d = 4 - 8 = -4$ and $n = 10$

$$\therefore \quad \mathbf{S} = \frac{n}{2}[2a + (n - 1)d]$$

$$= \frac{10}{2}[2 \times 8 + (10 - 1) \times -4]$$

$$= 5(16 - 36) = 5 \times -20 = -100$$ **Ans.**

14 Find the sum of the first 40 terms of the A.P. whose 4^{th} term is 8 and 6^{th} term is 14.

Solution :

Given, $a + 3d = 8$ and $a + 5d = 14$ $[\because t_4 = a + 3d$ and $t_6 = a + 5d]$

On solving these two equations, we get :

$d = 3$ and $a = -1$

Now, for finding the sum of first 40 term :

$$\therefore \quad S = \frac{n}{2}[2a + (n - 1)d]$$

$$= \frac{40}{2}[2 \times -1 + (40 - 1) \times 3]$$

$$= 20(-2 + 117) = 20 \times 115 = 2300$$ **Ans.**

$a = -1$,
$d = 3$ and
$n = 40$

15 For the A.P. : 10, 15, 20, ..., 195; find :

(i) the number of terms in the above A.P.

(ii) the sum of all its terms.

Solution :

Given : $a = 10$, $d = 5$ and last term $l = 195$

(i) If the given A.P. has n terms :

$$l = a + (n - 1)d \Rightarrow 195 = 10 + (n - 1) \times 5$$

$$\Rightarrow 185 = (n - 1) \times 5$$

$$\Rightarrow 37 = (n - 1) \text{ i.e. } n = 38$$ **Ans.**

(ii) **Sum of all the terms**

$$= \frac{n}{2}[2a + (n - 1)d]$$

$$= \frac{38}{2}[2 \times 10 + (38 - 1) \times 5] = 3895$$ **Ans.**

Alternative method :

$\because \quad a = 10$, $l = 195$ and $n = 38$

$$\therefore \quad S = \frac{n}{2}[a + l]$$

$$= \frac{38}{2}[10 + 195] = 19 \times 205 = 3895$$ **Ans.**

16 Find the sum of first 16 terms of a sequence whose n^{th} term is given by $t_n = 5n - 3$, where n is a natural number.

Solution :

$\because$ n^{th} term of the sequence is $t_n = 5n - 3$ and n is a natural number

$\therefore$ Its first term $t_1 = 5 \times 1 - 3 = 2$,

second term $t_2 = 5 \times 2 - 3 = 7$,

third term $t_3 = 5 \times 3 - 3 = 12$ and so on.

Clearly, the sequence is 2, 7, 12, 17, ...; which is an A.P. with first term $a = 2$ and common difference $d = 5$

Now, we have to find the sum of first 16 terms of the sequence (A.P.)

And, the sum of n terms of an A.P. $= \frac{n}{2}[2a + (n-1)d]$

$\therefore$ **Sum of 16 terms** $= \frac{16}{2}[2 \times 2 + (16 - 1) \times 5]$

$= 8(4 + 75) = \mathbf{632}$ **Ans.**

17 The sum of n terms of an A.P. $= n^2 + 3n$. Find its 14^{th} term.

Solution :

$\mathbf{S_n = n^2 + 3n} \Rightarrow S_1 = 1^2 + 3 \times 1 = 4$ *i.e.* $t_1 = 4$

Also, $\mathbf{S_2 = 2^2 + 3 \times 2 = 10}$

i.e. $t_1 + t_2 = 10 \Rightarrow 4 + t_2 = 10$ and $t_2 = 6$

$\mathbf{S_3 = 3^2 + 3 \times 3 = 18}$

i.e. $t_1 + t_2 + t_3 = 18$ and $10 + t_3 = 18 \Rightarrow t_3 = 18 - 10 = 8$

$\therefore$ The given A.P. is 4, 6, 8, ...

Clearly, its first term $(a) = 4$ and common difference $(d) = 6 - 4 = 2$

$\therefore$ $\mathbf{t_n = a + (n-1)d} \Rightarrow t_{14} = 4 + (14 - 1) \times 2 = \mathbf{30}$ **Ans.**

Alternative method :

$\because$ Sum of n terms, $S_n = n^2 + 3n$

$\therefore$ Sum of $(n - 1)$ terms, $S_{n-1} = (n-1)^2 + 3(n-1)$

$= n^2 - 2n + 1 + 3n - 3 = n^2 + n - 2$

$\therefore$ n^{th} term, $t_n = S_n - S_{n-1}$

$= (n^2 + 3n) - (n^2 + n - 2)$

$= n^2 + 3n - n^2 - n + 2 = 2n + 2$

$\Rightarrow$ **14^{th} term** $= 2 \times 14 + 2 = \mathbf{30}$ **Ans.**

n^{th} term of an A.P. = Sum of n terms – Sum of $(n - 1)$ terms

$\Rightarrow$ (i) 10^{th} term of A.P. = Sum of 10 terms – Sum of 9 terms

i.e. $t_{10} = S_{10} - S_9$

(ii) 25^{th} term of A.P. = Sum of 25 terms – Sum of 24 terms

i.e. $t_{25} = S_{25} - S_{24}$ and so on.

18 How many terms of the A.P. 43, 39, 35, ... be taken so that their sum is 252 ?

Solution :

Let the number of terms taken be n.

$\because a = 43$ and $d = 39 - 43 = -4$

$\therefore$ $\quad$ **$S = \frac{n}{2}[2a + (n-1)d]$** $\quad \Rightarrow 252 = \frac{n}{2}[2 \times 43 + (n-1) \times -4]$

$\Rightarrow 252 \times 2 = n(86 - 4n + 4)$

$\Rightarrow 504 = 90n - 4n^2$

$\Rightarrow 2n^2 - 45n + 252 = 0$

$\Rightarrow (n - 12)(2n - 21) = 0$

$\Rightarrow n = 12$ or $n = \frac{21}{2}$

$\Rightarrow n = 12 \qquad \left[\because n \neq \frac{21}{2}\right]$

$\therefore$ **The required number of terms = 12** **Ans.**

19 How many terms of the A.P. $20 + 19\frac{1}{3} + 18\frac{2}{3} + \ldots\ldots\ldots\ldots$ must be taken so that their sum is 300 ?

Solution :

Let the number of terms taken be n.

$\because a = 20$ and $d = 19\frac{1}{3} - 20 = -\frac{2}{3}$

$\therefore$ **$S_n = \frac{n}{2}[2a + (n-1)d]$** $\quad \Rightarrow 300 = \frac{n}{2}[2 \times 20 + (n-1) \times -\frac{2}{3}]$

$\Rightarrow 600 = n[40 - \frac{2}{3}n + \frac{2}{3}]$

$\Rightarrow 1800 = 120n - 2n^2 + 2n$

$\Rightarrow 2n^2 - 122n + 1800 = 0$

$\Rightarrow n^2 - 61n + 900 = 0$

$\Rightarrow n = 25$ or $n = 36$

$\therefore$ **Required number of terms = 25 or 36** **Ans.**

EXERCISE 10(C)

1. Find the sum of the first 22 terms of the A.P. : $\quad 8, 3, -2, \ldots\ldots\ldots\ldots$.
2. How many terms of the A.P. : 24, 21, 18, ... must be taken so that their sum is 78 ?
3. Find the sum of 28 terms of an A.P. whose n^{th} term is $8n - 5$.
4. Find the sum of :
   (i) all odd natural numbers less than 50.
   (ii) first 12 natural numbers each of which is a multiple of 7.
5. Find the sum of first 51 terms of an A.P. whose 2^{nd} and 3^{rd} terms are 14 and 18 respectively.
6. The sum of first 7 terms of an A.P. is 49 and that of first 17 terms of it is 289. Find the sum of first n terms.
7. The first term of an A.P. is 5, the last term is 45 and the sum of its terms is 1000. Find the number of terms and the common difference of the A.P.
8. Find the sum of all natural numbers between 250 and 1000 which are divisible by 9.

9. The first and the last terms of an A.P. are 34 and 700 respectively. If the common difference is 18, how many terms are there and what is their sum ?

10. In an A.P., the first term is 25, n^{th} term is -17 and the sum of n terms is 132. Find n and the common difference.

11. If the 8^{th} term of an A.P. is 37 and the 15^{th} term is 15 more than the 12^{th} term, find the A.P. Also, find the sum of first 20 terms of this A.P..

12. Find the sum of all multiples of 7 lying between 300 and 700.

13. The sum of n natural numbers is $5n^2 + 4n$. Find its 8^{th} term.

14. The fourth term of an A.P. is 11 and the eighth term exceeds twice the fourth term by 5. Find the A.P. and the sum of first 50 terms.

10.5 Three or more terms in A.P. :

1. When the sum of three consecutive terms of an A.P. is given, we take the terms as : $(a - d)$, a and $(a + d)$.
2. When the sum of four consecutive terms of an A.P. is given, we take the terms as : $a - 3d$, $a - d$, $a + d$ and $a + 3d$.

   Similarly,
3. **For consecutive five terms in A.P. :** take terms as $(a - 2d)$, $(a - d)$, a, $(a + d)$ and $(a + 2d)$.
4. **For consecutive six terms in A.P. :** take terms as $(a - 5d)$, $(a - 3d)$, $(a - d)$, $(a + d)$, $(a + 3d)$ and $(a + 5d)$.

20 The sum of three terms in A.P. is 33 and their product is 1155. Find the terms.

Solution :

Let the terms be $a - d$, a and $a + d$.

$\therefore$ $(a - d) + a + (a + d) = 33$ and $(a - d) \times a \times (a + d) = 1155$

$\Rightarrow$ $3a = 33$ and $a(a^2 - d^2) = 1155$

$\Rightarrow$ $a = 11$ and $11(11^2 - d^2) = 1155$

$\Rightarrow$ $a = 11$ and $121 - d^2 = 105$

i.e. $d^2 = 16$ and $d = \pm 4$ *i.e.* $d = 4$ or -4

Now, $a = 11$ and $d = 4$

$\Rightarrow$ **terms are** $= a - d$, a and $a + d$

$= 11 - 4$, 11 and $11 + 4 =$ **7, 11 and 15** **Ans.**

Also, $a = 11$ and $d = -4$

$\Rightarrow$ **terms are** $= a - d$, a and $a + d$

$= 11 + 4$, 11, $11 - 4 =$ **15, 11 and 7** **Ans.**

10.6 Arithmetic mean :

If three numbers a, A and b are in arithmetic progression, then A is called **arithmetic mean** (A.M.) between a and b.

Since; a, A and b are in A.P.

$\Rightarrow \quad A - a = b - A$

$\Rightarrow \quad 2A = a + b$ and $A = \dfrac{a+b}{2}$

$\therefore$ Arithmetic mean between a and $b = \dfrac{a+b}{2}$.

21 Insert two arithmetic means between 5 and 11.

Solution :

Let the required arithmetic means (A.M.s) between 5 and 11 be A_1 and A_2

$\Rightarrow$ 5, A_1, A_2 and 11 are in A.P.

$\Rightarrow$ 11 = 4^{th} term of this A.P.

$\Rightarrow$ $11 = 5 + 3d$ *i.e.* $d = 2$ $\quad [\because l = a + (n-1)d]$

$\therefore$ $A_1 = 5 + d = 5 + 2 = 7$ and $A_2 = 7 + 2 = 9$

$\Rightarrow$ **Required A.M.s between 5 and 11 = 7 and 9** **Ans.**

22 Insert five arithmetic means between 8 and 26.

Solution :

Let the required arithmetic means be A_1, A_2, A_3, A_4 and A_5.

$\Rightarrow$ 8, A_1, A_2, A_3, A_4, A_5 and 26 are in A.P.

$\Rightarrow$ 26 = 7^{th} term of this A.P.

$\Rightarrow$ $26 = 8 + 6d$ *i.e.* $d = 3$

$\Rightarrow$ **Required A.M.s** = A_1, A_2, A_3, A_4 and A_5

= $a + d$, $a + 2d$, $a + 3d$, $a + 4d$ and $a + 5d$

= $8 + 3$, $8 + 2 \times 3$, $8 + 3 \times 3$, $8 + 4 \times 3$ and $8 + 5 \times 3$

= **11, 14, 17, 20 and 23** **Ans.**

EXERCISE 10(D)

1. Find three numbers in A.P. whose sum is 24 and whose product is 440.
2. The sum of three consecutive terms of an A.P. is 21 and the sum of their squares is 165. Find these terms.
3. The angles of a quadrilateral are in A.P. with common difference 20°. Find its angles.
4. Divide 96 into four parts which are in A.P. and the ratio between product of their means to product of their extremes is 15 : 7.
5. Find five numbers in A.P. whose sum is $12\frac{1}{2}$ and the ratio of the first to the last terms is 2 : 3.
6. Split 207 into three parts such that these parts are in A.P. and the product of the two smaller parts is 4623.
7. The sum of three numbers in A.P. is 15 and the sum of the squares of the extreme terms is 58. Find the numbers.
8. Find four numbers in A.P. whose sum is 20 and the sum of whose squares is 120.
9. Insert one arithmetic mean between 3 and 13.
10. Insert two arithmetic means between 15 and 21.
11. Insert three arithmetic means between 15 and 27.
12. Insert four A.M.s between 14 and –1.
13. Insert five A.M.s between –12 and 8.
14. Insert six A.M.s between 15 and –15.

23 If p^{th}, q^{th} and r^{th} terms of an A.P. are x, y and z respectively, find the value of : $x(q - r) + y(r - p) + z(p - q)$.

Solution :

Given : $a + (p - 1)d = x$ I

$a + (q - 1)d = y$ II

and, $a + (r - 1)d = z$ III

Eq. II – Eq. III $\Rightarrow$ $(q - 1 - r + 1)d = y - z$

$\Rightarrow$ $(q - r) = \dfrac{y}{d} - \dfrac{z}{d}$

Eq. III – Eq. I $\Rightarrow$ $(r - 1 - p + 1)d = z - x$

$\Rightarrow$ $(r - p) = \dfrac{z}{d} - \dfrac{x}{d}$

Eq. I – Eq. II $\Rightarrow$ $(p - 1 - q + 1)d = x - y$

$\Rightarrow$ $p - q = \dfrac{x}{d} - \dfrac{y}{d}$

$\therefore$ $x(q - r) + y(r - p) + z(p - q)$

$$= x\left(\frac{y}{d} - \frac{z}{d}\right) + y\left(\frac{z}{d} - \frac{x}{d}\right) + z\left(\frac{x}{d} - \frac{y}{d}\right)$$

$$= \frac{xy}{d} - \frac{zx}{d} + \frac{yz}{d} - \frac{xy}{d} + \frac{zx}{d} - \frac{yz}{d} = \mathbf{0}$$ **Ans.**

24 Solve the equation : $1 + 4 + 7 + 10 + \ldots\ldots\ldots\ldots + x = 287$.

Solution :

Let the given equation has n terms.

Since, the given equation is an A.P., with $a = 1$ and $d = 4 - 1 = 3$

$\therefore$ $\dfrac{n}{2}[2a + (n - 1)d] = 287$ $\Rightarrow$ $n[2 \times 1 + (n - 1) \times 3] = 287 \times 2$

$\Rightarrow$ $2n + 3n^2 - 3n = 574$

i.e. $3n^2 - n - 574 = 0$

and $n = \dfrac{1 \pm \sqrt{1 - 4 \times 3 \times -574}}{2 \times 3} = \dfrac{1 \pm \sqrt{6889}}{6}$

$= \dfrac{1 \pm 83}{6} = \dfrac{1 + 83}{6}$, or $\dfrac{1 - 83}{6} = 14$, or $-\dfrac{41}{3}$

$\Rightarrow$ $n = 14$ $\left[\text{Rejecting } n = -\dfrac{41}{3}\right]$

$\therefore$ $x = n^{th}$ term

$= 14^{th}$ term

$= 1 + (14 - 1) \times 3 = \mathbf{40}$ **Ans.**

25 Solve the equation : $(x + 1) + (x + 4) + (x + 7) + \ldots\ldots\ldots + (x + 28) = 165$.

Solution :

Since, $(x + 4) - (x + 1) = x + 4 - x - 1 = 3$

and, $(x + 7) - (x + 4) = x + 7 - x - 4 = 3$

$\Rightarrow\ (x + 1) + (x + 4) + (x + 7) + \ldots\ldots\ldots\ldots + (x + 28)$ are in A.P.

$\Rightarrow\ 1 + 4 + 7 + \ldots\ldots\ldots\ldots + 28$ are in A.P. [On subtracting x from each term]

If the sequence has n terms; $t_n = 28$

i.e. $1 + (n - 1) \times 3 = 28 \Rightarrow n = 10$

Now $(x + 1) + (x + 4) + (x + 7) + \ldots\ldots\ldots\ldots\ldots + (x + 28) = 165$

$$\Rightarrow \quad \frac{n}{2}[a + l] = 165$$

$$\Rightarrow \quad \frac{10}{2}[x + 1 + x + 28] = 165 \Rightarrow x = 2$$

Ans.

> **26** If the sum of first m terms of an A.P. be n and the sum of first n terms of the same A.P. be m, show that the sum of its $(m + n)$ terms is $-(m + n)$.

Solution :

Sum of m terms = n $\Rightarrow \frac{m}{2}[2a + (m - 1)d] = n$

$\Rightarrow m[2a + (m - 1)d] = 2n$

$\Rightarrow 2am + m^2d - md = 2n$ I

Sum of n terms = m $\Rightarrow \frac{n}{2}[2a + (n - 1)d] = m$

$\Rightarrow 2an + n^2d - nd = 2m$ II

Eq. I – Eq. II $\Rightarrow 2a(m - n) + (m^2 - m - n^2 + n)d = 2n - 2m$

i.e. $2a(m - n) + (m^2 - n^2)d - (m - n)d = -2(m - n)$

$\Rightarrow \quad 2a + (m + n)d - d = -2$ [Dividing each term by $(m - n)$]

$\Rightarrow \quad 2a + (m + n - 1)d = -2$

On multiplying both the sides by $\frac{m+n}{2}$, we get :

$$\Rightarrow \quad \left(\frac{m+n}{2}\right)[2a + (m + n - 1)d] = -2\left(\frac{m+n}{2}\right)$$

$$\Rightarrow \quad \frac{m+n}{2}[2a + (m + n - 1)d] = -(m + n)$$

$\Rightarrow$ **Sum of $(m + n)$ terms = $-(m + n)$** **Hence Proved.**

10.7 Properties of an A.P. :

Property 1 :

If same fixed non-zero number is added or subtracted from each term of an A.P., the resulting sequence is also an A.P.

e.g., (i) 5, 8, 11, 14, ... are in A.P.

$\Rightarrow$ 5 + 7, 8 + 7, 11 + 7, 14 + 7, ... are in A.P. [Adding 7 to each term]

and, 5 – 7, 8 – 7, 11–7, 14 – 7, ... are also in A.P. [Subtracting 7 from each term]

(ii) 15, 13, 11, 9, ... are in A.P.

$\Rightarrow$ 15 – 10, 13 – 10, 11 – 10, 9 – 10, ... are in A.P. [Subtracting 10 from each term]

and, 15 + 8, 13 + 8, 11 + 8, 9 + 8, ... are also in A.P. [Adding 8 to each term]

Property 2 :

If each term of a given A.P. is multiplied or divided by a given non-zero fixed number, the resulting sequence is an A.P.

e.g., (i) 5, 8, 11, 14, ... are in A.P.

$\Rightarrow$ $5 \times 8, 8 \times 8, 11 \times 8, 14 \times 8$, ... are also in A.P. [Multiplying each term by 8]

(ii) 27, 25, 23, 21, ... are in A.P.

$\Rightarrow$ $\frac{27}{4}, \frac{25}{4}, \frac{23}{4}, \frac{21}{4}$ .... are also in A.P. [Dividing each term by 4]

27 If a, b and c are in A.P., show that : $(b + c)$, $(c + a)$ and $(a + b)$ are also in A.P.

Solution :

$\because$ a, b and c are in A.P.

$\Rightarrow$ $a - (a + b + c)$, $b - (a + b + c)$ and $c - (a + b + c)$ are in A.P.

[Subtracting same quantity $(a + b + c)$ from each term of the A.P.]

$\Rightarrow$ $-(b + c)$, $-(c + a)$ and $-(a + b)$ are in A.P.

$\Rightarrow$ $(b + c)$, $(c + a)$ and $(a + b)$ are in A.P. [Dividing each term by –1]

Hence Proved.

Similarly,

$$\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c} \text{ are in A.P.}$$

$$\Rightarrow \frac{b+c-a}{a} + 2, \frac{c+a-b}{b} + 2, \frac{a+b-c}{c} + 2 \text{ are in A.P.}$$

[Adding 2 to each term of the given A.P.]

$$\Rightarrow \frac{b+c-a+2a}{a}, \frac{c+a-b+2b}{b}, \frac{a+b-c+2c}{c} \text{ are in A.P.}$$

$$\Rightarrow \frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c} \text{ are in A.P.}$$

$$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \text{ are in A.P.}$$

[Dividing each term by $(a + b + c)$]

EXERCISE 10(E)

1. The angles of a polygon are in A.P. with common difference 5°. If the smallest angle is 120°, find the number of sides of the polygon.

Let the number of sides be n

$\therefore \quad \frac{n}{2}[2 \times 120° + (n-1) \times 5°]$
$= (2n - 4) \times 90°$

2. Solve : $25 + 22 + 19 + \text{.........} + x = 115$.

3. $\frac{1}{a}$, $\frac{1}{b}$ and $\frac{1}{c}$ are in A.P.
Show that : bc, ca and ab are also in A.P.

4. a, b and c are in A.P. Show that :
$(b + c - a)$, $(c + a - b)$ and $(a + b - c)$ are also in A.P.

5. $\frac{b+c}{a}$, $\frac{c+a}{b}$ and $\frac{a+b}{c}$ are in A.P.
Show that :
(i) $\frac{1}{a}$, $\frac{1}{b}$ and $\frac{1}{c}$ are in A.P.
(ii) bc, ca and ab are in A.P.

6. $\frac{1}{a}$, $\frac{1}{b}$ and $\frac{1}{c}$ are in A.P. Show that :
$\frac{b+c}{a}$, $\frac{c+a}{b}$ and $\frac{a+b}{c}$ are also in A.P.

7. The p^{th} term of an A.P. is 20 and its q^{th} term is 10, show that the sum of its first $(p + q)$ terms is $\frac{p+q}{2}\left\{30+\frac{10}{p-q}\right\}$.

10.8 Word problems :

28 A sum of ₹ 8,000 is invested at 10% simple interest per annum. Calculate the interest at the end of each year.

Does the sequence of interests at the end of consecutive years form an A.P. ?
If yes, write its first term and the common difference.

Solution :

Interest at the end of 1st year $= \frac{₹ 8{,}000 \times 10 \times 1}{100}$ $\left[\because I = \frac{P \times R \times T}{100}\right]$

$= ₹\ 800$

Interest at the end of 2nd year $= \frac{₹ 8{,}000 \times 10 \times 2}{100}$

$= ₹\ 1{,}600$

Interest at the end of 3rd year $= \frac{₹ 8{,}000 \times 10 \times 3}{100}$

$= ₹\ 2{,}400$

$\therefore$ Sequence formed by interests of consecutive years

$= ₹\ 800,\ ₹\ 1{,}600,\ ₹\ 2{,}400,\ \text{..........}$

Yes, it is an A.P. with first term = ₹ 800

and, **common difference = ₹ 800** **Ans.**

29 In a school, students stand in rows. If 30 students stand in the first row, twenty-seven in the second row, twenty four in the third row and six in the last row; find how many rows are there and what is the total number of students ?

Solution :

Sequence formed by the number of students standing in different rows

$= 30, 27, 24, \ldots\ldots\ldots\ldots, 6$

It is an A.P. with $a = 30$, $d = -3$ and $l = 6$

Since, $a + (n-1)d = l$

$\Rightarrow \quad 30 + (n-1) \times -3 = 6$ i.e. $n = 9$

$\therefore$ **There are 9 rows** **Ans.**

Also, total number of students

$= $ Sum of students in different rows

$= \frac{9}{2}[2 \times 30 + (9-1) \times -3] \qquad \left[\because S_n = \frac{n}{2}\{2a + (n-1)d\}\right]$

$= \frac{9}{2}[60 - 24] = \frac{9}{2} \times 36 = 162$ **Ans.**

EXERCISE 10(F)

1. Two cars start together in the same direction from the same place. The first car goes at uniform speed of 10 km h^{-1}. The second car goes at a speed of 8 km h^{-1} in the first hour and thereafter increasing the speed by 0·5 km h^{-1} each succeeding hour. After how many hours will the two cars meet ?

Let the two cars meet after n hours, then

$10 \times n = \frac{n}{2}[2 \times 8 + (n-1) \times 0{\cdot}5]$

2. A sum of ₹ 700 is to be paid to give seven cash prizes to the students of a school for their overall academic performance. If the cost of each prize is ₹ 20 less than its preceding prize; find the value of each of the prizes.

3. An article can be bought by paying ₹ 28,000 at once or by making 12 monthly instalments. If the first instalment paid is ₹ 3,000 and every other instalment is ₹ 100 less than the previous one, find :
   (i) amount of instalment paid in the 9th month
   (ii) total amount paid in the instalment scheme.

4. A manufacturer of TV sets produces 600 units in the third year and 700 units in the 7th year. Assuming that the production increases uniformly by a fixed number every year, find:
   (i) the production in the first year.
   (ii) the production in the 10th year.
   (iii) the total production in 7 years.

5. Mrs. Gupta repays her total loan of ₹ 1,18,000 by paying instalments every month. If the instalment for the first month is ₹ 1,000 and it increases by ₹ 100 every month, what amount will she pay as the 30th instalment of loan ? What amount of loan she still has to pay after the 30th instalment ?

6. In a school, students decided to plant trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be five times of the class to which the respective section belongs. If there are 1 to 10 classes in the school and each class has three sections, find how many trees were planted by the students ?

EXERCISE 10(G)

1. The n^{th} term of an arithmetic progression is $15 - 7n$. Find its common difference.

2. The angles of a triangle form an arithmetic progression. If the largest angle is twice the least, find all the angles of the triangle.

3. Find the 11^{th} term from the last of the sequence 10, 7, 4, ..., – 62.

4. The 15^{th} term of an A.P. exceeds its 8^{th} term by 7. Find its common difference.

5. How many multiples of 4 lie between 10 and 250 ?

6. The sum of the 4^{th} and the 8^{th} terms of an A.P. is 24 and the sum of the sixth term and the tenth term is 44. Find the first three terms of the A.P.

7. In an A.P., show that :

$$(m + n)^{th} \text{ term} + (m - n)^{th} \text{ term} = 2 \times m^{th} \text{ term}.$$

8. If the m^{th} term of an A.P. is $\frac{1}{n}$ and the n^{th} term of it is $\frac{1}{m}$.

Show that : $(mn)^{th}$ term of this A.P. is 1.

9. Find a and b, if 12, $a + b$, $2a$ *and* b are in A.P.

> 12, $a + b$ and $2a$ are in A.P.
>
> $\Rightarrow a + b = \frac{12+2a}{2}$.
>
> $a + b$, $2a$ and b are in A.P. $\Rightarrow 2a = \frac{a+b+b}{2}$.
>
> Solve the two equations simultaneously to get $a = 4$ and $b = 6$.

10. Find the 31^{st} term of an A.P. whose 11^{th} term is 38 and the 16^{th} term is 73.

11. The sum of first n terms of an A.P. is $5n^2 - 8n$. Find the A.P. and hence find its 15^{th} term.

12. In an A.P., the sum of first ten terms is –80 and the sum of next ten terms is –280. Find the A.P. :

> Clearly, sum of first 10 terms = –80
>
> $\Rightarrow \frac{10}{2}[2a + (10 - 1)d] = -80$
>
> and sum of first 20 terms = –80 + (–280)
>
> = –360
>
> $\Rightarrow \frac{20}{2}[2a + (20 - 1)d] = -360$

13. In an A.P., the first term is –4 and the last term is 29. If the sum of all its terms is 150, find the common difference.

14. Find the sum of all three digit numbers which leave the remainder 3 when divided by 5.

15. The sum of how many terms of the A.P. 17, 15, 13, ... is 72 ?

16. The sum of first 15 terms of an A.P. is 0. If its 4th term is 12, find its 12^{th} term.

17. Find the sum of all
    (i) odd numbers between 50 and 150.
    (ii) even numbers between 100 and 200.

18. The sum of n terms of a sequence is $an^2 + bn$, show that the sequence is an A.P.

19. Two arithmetic progressions are such that the common difference of both is 8. If the difference between their 50^{th} terms is 50; what will be the difference between their 80^{th} terms ?

> Difference between 80^{th} terms
>
> = difference between 50^{th} terms.

20. The sum of first n, $2n$ and $3n$ terms of an A.P. are S_1, S_2 and S_3 respectively.

Prove that : $S_3 = 3(S_2 - S_1)$.