Quadratic Equations

1. If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$ is **[AIEEE-2009]**

   (1) Less than $4ab$ (2) Greater than $-4ab$

   (3) Less than $-4ab$ (4) Greater than $4ab$

2. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$ **[AIEEE-2010]**

   (1) -2 (2) -1

   (3) 1 (4) 2

3. Let for $a \neq a_1 \neq 0$,

   $f(x) = ax^2 + bx + c$, $g(x) = a_1x^2 + b_1x + c_1$ and $p(x) = f(x) - g(x)$.

   If $p(x) = 0$ only for $x = -1$ and $p(-2) = 2$, then the value of $p(2)$ is **[AIEEE-2011]**

   (1) 6 (2) 18

   (3) 3 (4) 9

4. Sachin and Rahul attempted to solve a quadratic equation. Sachin made a mistake in writing down the constant term and ended up in roots (4, 3). Rahul made a mistake in writing down coefficient of x to get roots (3, 2). The correct roots of equation are: **[AIEEE-2011]**

   (1) $-6, -1$ (2) $-4, -3$

   (3) 6, 1 (4) 4, 3

5. The real number k for which the equation $2x^3 + 3x + k = 0$ has two distinct real roots in [0, 1] **[JEE (Main)-2013]**

   (1) Lies between 1 and 2

   (2) Lies between 2 and 3

   (3) Lies between -1 and 0

   (4) Does not exist

6. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a, b, c \in R$, have a common root, then $a : b : c$ is **[JEE (Main)-2013]**

   (1) 1 : 2 : 3 (2) 3 : 2 : 1

   (3) 1 : 3 : 2 (4) 3 : 1 : 2

7. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to **[JEE (Main)-2015]**

   (1) 6 (2) -6

   (3) 3 (4) -3

8. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2+4x-60} = 1$ is **[JEE (Main)-2016]**

   (1) -4 (2) 6

   (3) 5 (4) 3

9. If, for a positive integer n, the quadratic equation, $x(x+1) + (x+1)(x+2) + \ldots + (x+\overline{n-1})(x+n) = 10n$ has two consecutive integral solutions, then n is equal to **[JEE (Main)-2017]**

   (1) 9 (2) 10

   (3) 11 (4) 12

10. Let $S = \{x \in R : x \geq 0$ and $2\left|\sqrt{x} - 3\right| + \sqrt{x}\left(\sqrt{x} - 6\right) + 6 = 0\}$. Then S **[JEE (Main)-2018]**

    (1) Is an empty set

    (2) Contains exactly one element

    (3) Contains exactly two elements

    (4) Contains exactly four elements

11. If $\alpha, \beta \in C$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to **[JEE (Main)-2018]**

    (1) -1 (2) 0

    (3) 1 (4) 2

12. Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to **[JEE (Main)-2019]**

    (1) -512 (2) 512

    (3) 256 (4) -256

13. If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval [1, 5] then m lies in the interval **[JEE (Main)-2019]**

(1) (–5, –4) (2) (3, 4)

(3) (4, 5) (4) (5, 6)

14. The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2 - 11x + \alpha = 0$ are rational numbers is **[JEE (Main)-2019]**

(1) 4 (2) 5

(3) 2 (4) 3

15. Consider the quadratic equation $(c - 5)x^2 - 2cx + (c - 4) = 0$, $c \neq 5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval (0, 2) and its other root lies in the interval (2, 3). Then the number of elements in S is **[JEE (Main)-2019]**

(1) 11 (2) 18

(3) 12 (4) 10

16. The value of λ such that sum of the squares of the roots of the quadratic equation, $x^2 + (3 - \lambda)x + 2 = \lambda$ has the least value is **[JEE (Main)-2019]**

(1) 2 (2) 1

(3) $\frac{15}{8}$ (4) $\frac{4}{9}$

17. If one real root of the quadratic equation $81x^2 + kx + 256 = 0$ is cube of the other root, then a value of k is **[JEE (Main)-2019]**

(1) –300 (2) 144

(3) –81 (4) 100

18. Let α and β the roots of the quadratic equation

$x^2 \sin\theta - x(\sin\theta\cos\theta + 1) + \cos\theta = 0$

$(0 < \theta < 45°)$, and $\alpha < \beta$. Then

$\sum_{n=0}^{\infty}\left(\alpha^n + \frac{(-1)^n}{\beta^n}\right)$ is equal to **[JEE (Main)-2019]**

(1) $\frac{1}{1+\cos\theta} - \frac{1}{1-\sin\theta}$ (2) $\frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$

(3) $\frac{1}{1-\cos\theta} - \frac{1}{1+\sin\theta}$ (4) $\frac{1}{1+\cos\theta} + \frac{1}{1-\sin\theta}$

19. If λ be the ratio of the roots of the quadratic equation in x, $3m^2x^2 + m(m - 4)x + 2 = 0$, then the least value of m for which $\lambda + \frac{1}{\lambda} = 1$, is **[JEE (Main)-2019]**

(1) $4 - 2\sqrt{3}$ (2) $4 - 3\sqrt{2}$

(3) $2 - \sqrt{3}$ (4) $-2 + \sqrt{2}$

20. The number of integral values of m for which the quadratic expression, $(1 + 2m)x^2 - 2(1 + 3m)x + 4(1 + m)$, $x \in R$, is always positive, is **[JEE (Main)-2019]**

(1) 8 (2) 3

(3) 6 (4) 7

21. If α and β be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n = 1$ is **[JEE (Main)-2019]**

(1) 4 (2) 5

(3) 3 (4) 2

22. The sum of the solutions of the equation $\left|\sqrt{x} - 2\right| + \sqrt{x}\left(\sqrt{x} - 4\right) + 2 = 0, (x > 0)$ is equal to **[JEE (Main)-2019]**

(1) 4 (2) 10

(3) 9 (4) 12

23. If three distinct numbers a, b, c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct? **[JEE (Main)-2019]**

(1) d, e, f are in A.P.

(2) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

(3) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

(4) d, e, f are in G.P.

24. The number of integral values of m for which the equation $(1 + m^2)x^2 - 2(1 + 3m)x + (1 + 8m) = 0$ has no real root is : **[JEE (Main)-2019]**

(1) Infinitely many (2) 3

(3) 2 (4) 1

25. Let $p, q \in R$. If $2-\sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then **[JEE (Main)-2019]**

(1) $q^2 - 4p - 16 = 0$ (2) $p^2 - 4q + 12 = 0$

(3) $p^2 - 4q - 12 = 0$ (4) $q^2 + 4p + 14 = 0$

26. If m is chosen in the quadratic equation $(m^2 + 1)x^2 - 3x + (m^2 + 1)^2 = 0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is **[JEE (Main)-2019]**

(1) $8\sqrt{3}$ (2) $10\sqrt{5}$

(3) $4\sqrt{3}$ (4) $8\sqrt{5}$

27. If α and β are the roots of the equation $375x^2 - 25x - 2 = 0$, then $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\alpha^r + \lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\beta^r$ is equal to **[JEE (Main)-2019]**

(1) $\frac{21}{346}$ (2) $\frac{7}{116}$

(3) $\frac{29}{358}$ (4) $\frac{1}{12}$

28. If α, β and γ are three consecutive terms of a non-constant G.P. such that the equations $\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root, then $\alpha(\beta + \gamma)$ is equal to **[JEE (Main)-2019]**

(1) 0 (2) $\alpha\gamma$

(3) $\beta\gamma$ (4) $\alpha\beta$

29. Let α and β be two real roots of the equation $(k+1)\tan^2 x - \sqrt{2}\cdot\lambda\tan x = (1-k)$, where $k(\neq -1)$ and λ are real numbers. If $\tan^2(\alpha + \beta) = 50$, then a value of λ is **[JEE (Main)-2020]**

(1) 10 (2) $10\sqrt{2}$

(3) 5 (4) $5\sqrt{2}$

30. Let α and β be the roots of the equation $x^2 - x - 1 = 0$. If $p_k = (\alpha)^k + (\beta)^k$, $k \geq 1$, then which one of the following statements is not true? **[JEE (Main)-2020]**

(1) $p_3 = p_5 - p_4$

(2) $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$

(3) $p_5 = 11$

(4) $p_5 = p_2 \cdot p_3$

31. Let S be the set of all real roots of the equation, $3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$. Then S **[JEE (Main)-2020]**

(1) Contains at least four elements

(2) Is a singleton

(3) Contains exactly two elements

(4) Is an empty set

32. The number of real roots of the equation, $e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$ is **[JEE (Main)-2020]**

(1) 4 (2) 2

(3) 3 (4) 1

33. Let a, $b \in R$, $a \neq 0$ be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to **[JEE (Main)-2020]**

(1) 25 (2) 24

(3) 26 (4) 28

34. Let α and β be the roots of the equation, $5x^2 + 6x - 2 = 0$. If $S_n = \alpha^n + \beta^n$, $n = 1, 2, 3, \ldots$, then **[JEE (Main)-2020]**

(1) $5S_6 + 6S_5 = 2S_4$ (2) $6S_6 + 5S_5 + 2S_4 = 0$

(3) $6S_6 + 5S_5 = 2S_4$ (4) $5S_6 + 6S_5 + 2S_4 = 0$

35. Let $f(x)$ be a quadratic polynomial such that $f(-1) + f(2) = 0$. If one of the roots of $f(x) = 0$ is 3, then its other root lies in **[JEE (Main)-2020]**

(1) (–1, 0) (2) (–3, –1)

(3) (0, 1) (4) (1, 3)

36. If α and β are the roots of the equation $x^2 + px + 2 = 0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2 + 2qx + 1 = 0$, then $\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$ is equal to **[JEE (Main)-2020]**

(1) $\frac{9}{4}(9 - q^2)$ (2) $\frac{9}{4}(9 + p^2)$

(3) $\frac{9}{4}(9 + q^2)$ (4) $\frac{9}{4}(9 - p^2)$

37. The set of all real values of λ for which the quadratic equations, $(\lambda^2 + 1)\,x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval (0, 1) is **[JEE (Main)-2020]**

(1) (–3, –1) (2) (2, 4]

(3) (0, 2) (4) (1, 3]

38. Let $\lambda \neq 0$ be in R. If α and β are the roots of the equation, $x^2 - x + 2\lambda = 0$ and α and γ are the roots of the equation, $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to **[JEE (Main)-2020]**

(1) 18 (2) 9

(3) 27 (4) 36

39. The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$, is **[JEE (Main)-2020]**

(1) $\frac{25}{9}$ (2) $\frac{25}{81}$

(3) $\frac{5}{9}$ (4) $\frac{5}{27}$

40. If α and β are the roots of the equation, $7x^2 - 3x - 2 = 0$, then the value of $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$ is equal to **[JEE (Main)-2020]**

(1) $\frac{1}{24}$ (2) $\frac{27}{32}$

(3) $\frac{3}{8}$ (4) $\frac{27}{16}$

41. If α and β be two roots of the equation $x^2 - 64x + 256 = 0$. Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8}$ is **[JEE (Main)-2020]**

(1) 3 (2) 2

(3) 4 (4) 1

42. If α and β are the roots of the equation $2x(2x + 1) = 1$, then β is equal to **[JEE (Main)-2020]**

(1) $2\alpha^2$

(2) $-2\alpha(\alpha + 1)$

(3) $2\alpha(\alpha - 1)$

(4) $2\alpha(\alpha + 1)$

43. The least positive value of 'a' for which the equation, $2x^2 + (a - 10)x + \frac{33}{2} = 2a$ has real roots is _____. **[JEE (Main)-2020]**

❑ ❑ ❑