Trigonometric Functions

1. Let A and B denote the statements :

   A : $\cos\alpha + \cos\beta + \cos\gamma = 0$

   B : $\sin\alpha + \sin\beta + \sin\gamma = 0$

   If $\cos(\beta-\gamma) + \cos(\gamma-\alpha) + \cos(\alpha-\beta) = -\frac{3}{2}$, then **[AIEEE-2009]**

   (1) A is false and B is true

   (2) Both A and B are true

   (3) Both A and B are false

   (4) A is true and B is false

2. Let $\cos(\alpha+\beta) = \frac{4}{5}$ and let $\sin(\alpha-\beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then tan 2$\alpha$ = **[AIEEE-2010]**

   (1) $\frac{25}{16}$ (2) $\frac{56}{33}$

   (3) $\frac{19}{12}$ (4) $\frac{20}{7}$

3. For a regular polygon, let *r* and *R* be the radii of the inscribed and the circumscribed circles. A ***false*** statement among the following is **[AIEEE-2010]**

   (1) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$

   (2) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$

   (3) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$

   (4) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

4. The possible values of $\theta \in (0, \pi)$ such that $\sin(\theta) + \sin(4\theta) + \sin(7\theta) = 0$ are **[AIEEE-2011]**

   (1) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

   (2) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}$

   (3) $\frac{\pi}{4}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

   (4) $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{35\pi}{36}$

5. In a ΔPQR, if 3 sin *P* + 4 cos *Q* = 6 and 4 sin *Q* + 3 cos *P* = 1, then the angle *R* is equal to **[AIEEE-2012]**

   (1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$

   (3) $\frac{3\pi}{4}$ (4) $\frac{5\pi}{6}$

6. *ABCD* is a trapezium such that *AB* and *CD* are parallel and $BC \perp CD$. If $\angle ADB = \theta$, *BC* = *p* and *CD* = *q*, then *AB* is equal to **[JEE (Main)-2013]**

   (1) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta + q\sin\theta}$ (2) $\frac{p^2+q^2\cos\theta}{p\cos\theta+q\sin\theta}$

   (3) $\frac{p^2+q^2}{p^2\cos\theta+q^2\sin\theta}$ (4) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta+q\sin\theta)^2}$

7. The expression $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ can be written as **[JEE (Main)-2013]**

   (1) sin*A* cos*A* + 1

   (2) sec*A* cosec*A* + 1

   (3) tan*A* + cot*A*

   (4) sec*A* + cosec*A*

8. Let $f_k(x)=\frac{1}{k}(\sin^k x+\cos^k x)$ where $x\in R$ and $k\geq 1$. Then $f_4(x) - f_6(x)$ equals **[JEE (Main)-2014]**

(1) $\frac{1}{4}$ (2) $\frac{1}{12}$

(3) $\frac{1}{6}$ (4) $\frac{1}{3}$

9. A bird is sitting on the top of a vertical pole 20 m high and its elevation from a point *O* on the ground is 45°. It flies off horizontally straight away from the point *O*. After one second, the elevation of the bird from *O* is reduced to 30°. Then the speed (in m/s) of the bird is **[JEE (Main)-2014]**

(1) $20\sqrt{2}$ (2) $20\left(\sqrt{3}-1\right)$

(3) $40\left(\sqrt{2}-1\right)$ (4) $40\left(\sqrt{3}-\sqrt{2}\right)$

10. If the angles of elevation of the top of a tower from three collinear points *A*, *B* and *C*, on a line leading to the foot of the tower, are 30º, 45º and 60º respectively, then the ratio, *AB* : *BC*, is **[JEE (Main)-2015]**

(1) $\sqrt{3}:1$ (2) $\sqrt{3}:\sqrt{2}$

(3) $1:\sqrt{3}$ (4) 2 : 3

11. If $0\leq x<2\pi$, then the number of real values of *x*, which satisfy the equation $\cos x+\cos 2x+\cos 3x+\cos 4x=0$, is **[JEE (Main)-2016]**

(1) 5 (2) 7

(3) 9 (4) 3

12. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point *A* on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 minutes from *A* in the same direction, at a point *B*, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from *B* to reach the pillar, is **[JEE (Main)-2016]**

(1) 10 (2) 20

(3) 5 (4) 6

13. If $5(\tan^2 x-\cos^2 x)=2\cos 2x+9$, then the value of $\cos 4x$ is **[JEE (Main)-2017]**

(1) $\frac{1}{3}$ (2) $\frac{2}{9}$

(3) $-\frac{7}{9}$ (4) $-\frac{3}{5}$

14. Let a vertical tower *AB* have its end *A* on the level ground. Let *C* be the mid-point of *AB* and *P* be a point on the ground such that *AP* = 2*AB*. If $\angle BPC=\beta$ then tan β is **[JEE (Main)-2017]**

(1) $\frac{1}{4}$ (2) $\frac{2}{9}$

(3) $\frac{4}{9}$ (4) $\frac{6}{7}$

15. If sum of all the solutions of the equation $8\cos x\cdot\left(\cos\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$ in [0, π] is *k*π, then *k* is equal to : **[JEE (Main)-2018]**

(1) $\frac{2}{3}$ (2) $\frac{13}{9}$

(3) $\frac{8}{9}$ (4) $\frac{20}{9}$

16. *PQR* is a triangular park with *PQ* = *PR* = 200 m. A T.V. tower stands at the mid-point of *QR*. If the angles of elevation of the top of the tower at *P*, *Q* and *R* are respectively 45º, 30º and 30º, then the height of the tower (in m) is **[JEE (Main)-2018]**

(1) 100 (2) 50

(3) $100\sqrt{3}$ (4) $50\sqrt{2}$

17. For any $\theta\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)$, the expression $3(\sin\theta-\cos\theta)^4+6(\sin\theta+\cos\theta)^2+4\sin^6\theta$ equals **[JEE (Main)-2019]**

(1) $13-4\cos^2\theta+6\sin^2\theta\cos^2\theta$

(2) $13-4\cos^2\theta+6\cos^4\theta$

(3) $13-4\cos^6\theta$

(4) $13-4\cos^4\theta+2\sin^2\theta\cos^2\theta$

18. If $0\leq x<\frac{\pi}{2}$, then the number of values of *x* for which $\sin x-\sin 2x+\sin 3x=0$, is **[JEE (Main)-2019]**

(1) 2 (2) 3

(3) 1 (4) 4

19. Consider a triangular plot *ABC* with sides *AB* = 7 m, *BC* = 5 m and *CA* = 6 m. A vertical lamp-post at the mid point *D* of *AC* subtends an angle 30° at *B*. The height (in m) of the lamp-post is **[JEE (Main)-2019]**

(1) $2\sqrt{21}$ (2) $7\sqrt{3}$

(3) $\frac{2}{3}\sqrt{21}$ (4) $\frac{3}{2}\sqrt{21}$

20. The sum of all values of $\theta \in \left(0, \frac{\pi}{2}\right)$ satisfying $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$ is **[JEE (Main)-2019]**

(1) $\frac{5\pi}{4}$ (2) $\frac{\pi}{2}$

(3) $\frac{3\pi}{8}$ (4) π

21. If 5, $5r$, $5r^2$ are the lengths of the sides of a triangle, then r **cannot** be equal to **[JEE (Main)-2019]**

(1) $\frac{3}{2}$ (2) $\frac{7}{4}$

(3) $\frac{3}{4}$ (4) $\frac{5}{4}$

22. With the usual notation, in ΔABC, if $\angle A + \angle B = 120°$, $a = \sqrt{3}+1$ and $b = \sqrt{3}-1$, then the ratio $\angle A : \angle B$, is **[JEE (Main)-2019]**

(1) 7 : 1 (2) 3 : 1

(3) 9 : 7 (4) 5 : 3

23. The value of

$$\cos\frac{\pi}{2^2}.\cos\frac{\pi}{2^3}.\ldots.\cos\frac{\pi}{2^{10}}.\sin\frac{\pi}{2^{10}}$$ is **[JEE (Main)-2019]**

(1) $\frac{1}{512}$ (2) $\frac{1}{256}$

(3) $\frac{1}{2}$ (4) $\frac{1}{1024}$

24. Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for k = 1, 2, 3, ...
Then for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to **[JEE (Main)-2019]**

(1) $\frac{-1}{12}$ (2) $\frac{1}{12}$

(3) $\frac{5}{12}$ (4) $\frac{1}{4}$

25. Given $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ for ΔABC with usual notation. If $\frac{\cos A}{\alpha} = \frac{\cos B}{\beta} = \frac{\cos C}{\gamma}$, then the ordered triplet (α, β, γ) has a value **[JEE (Main)-2019]**

(1) (3, 4, 5) (2) (7, 19, 25)

(3) (19, 7, 25) (4) (5, 12, 13)

26. The maximum value of $3\cos\theta + 5\sin\left(\theta - \frac{\pi}{6}\right)$ for any real value of θ is **[JEE (Main)-2019]**

(1) $\sqrt{34}$ (2) $\sqrt{19}$

(3) $\frac{\sqrt{79}}{2}$ (4) $\sqrt{31}$

27. If the angle of elevation of a cloud from a point P which is 25 m above a lake be 30° and the angle of depression of reflection of the cloud in the lake from P be 60°, then the height of the cloud (in meters) from the surface of the lake is **[JEE (Main)-2019]**

(1) 45 (2) 50

(3) 42 (4) 60

28. If $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$; $\alpha, \beta \in [0, \pi]$, then $\cos(\alpha + \beta) - \cos(\alpha - \beta)$ is equal to **[JEE (Main)-2019]**

(1) $\sqrt{2}$ (2) $-\sqrt{2}$

(3) -1 (4) 0

29. If $\cos(\alpha+\beta) = \frac{3}{5}, \sin(\alpha-\beta) = \frac{5}{13}$ and $0 < \alpha, \beta < \frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to **[JEE (Main)-2019]**

(1) $\frac{21}{16}$ (2) $\frac{63}{52}$

(3) $\frac{33}{52}$ (4) $\frac{63}{16}$

30. Two vertical poles of heights, 20 m and 80 m stand apart on a horizontal plane. The height (in meters) of the point of intersection of the lines joining the top of each pole to the foot of the other, from this horizontal plane is **[JEE (Main)-2019]**

(1) 16 (2) 18

(3) 15 (4) 12

31. If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is **[JEE (Main)-2019]**

(1) 4 : 5 : 6 (2) 3 : 4 : 5

(3) 5 : 9 : 13 (4) 5 : 6 : 7

32. Let $S = \{\theta \in [-2\pi, 2\pi] ; 2\cos^2\theta + 3\sin\theta = 0\}$. Then the sum of the elements of S is **[JEE (Main)-2019]**

(1) π (2) 2π

(3) $\frac{13\pi}{6}$ (4) $\frac{5\pi}{3}$

33. The value of $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$ is **[JEE (Main)-2019]**

(1) $\frac{3}{4}$ (2) $\frac{3}{2}(1+\cos 20°)$

(3) $\frac{3}{2}$ (4) $\frac{3}{4}+\cos 20°$

34. Two poles standing on a horizontal ground are of heights 5 m and 10 m respectively. The line joining their tops makes an angle of 15° with the ground. Then the distance (in m) between the poles, is **[JEE (Main)-2019]**

(1) $5\left(2+\sqrt{3}\right)$ (2) $10\left(\sqrt{3}-1\right)$

(3) $5\left(\sqrt{3}+1\right)$ (4) $\frac{5}{2}\left(2+\sqrt{3}\right)$

35. The value of $\sin 10° \sin 30° \sin 50° \sin 70°$ is **[JEE (Main)-2019]**

(1) $\frac{1}{18}$ (2) $\frac{1}{32}$

(3) $\frac{1}{16}$ (4) $\frac{1}{36}$

36. All the pairs (x, y) that satisfy the inequality $2^{\sqrt{\sin^2 x - 2\sin x + 5}} \cdot \frac{1}{4^{\sin^2 y}} \le 1$ also satisfy the equation **[JEE (Main)-2019]**

(1) $\sin x = |\sin y|$ (2) $\sin x = 2 \sin y$

(3) $2 \sin x = \sin y$ (4) $2|\sin x| = 3 \sin y$

37. ABC is a triangular park with $AB = AC = 100$ metres. A vertical tower is situated at the mid-point of BC. If the angles of elevation of the top of the tower at A and B are $\cot^{-1}(3\sqrt{2})$ and $\text{cosec}^{-1}(2\sqrt{2})$ respectively, then the height of the tower (in metres) is : **[JEE (Main)-2019]**

(1) 20 (2) $10\sqrt{5}$

(3) 25 (4) $\frac{100}{3\sqrt{3}}$

38. The angles A, B and C of a triangle ABC are in A.P. and $a : b = 1 : \sqrt{3}$. If $c = 4$ cm, then the area (in sq.cm) of this triangle is : **[JEE (Main)-2019]**

(1) $\frac{2}{\sqrt{3}}$ (2) $4\sqrt{3}$

(3) $2\sqrt{3}$ (4) $\frac{4}{\sqrt{3}}$

39. A 2 m ladder leans against a vertical wall. If the top of the ladder begins to slide down the wall at the rate 25 cm/sec., then the rate (in cm/sec.) at which the bottom of the ladder slides away from the wall on the horizontal ground when the top of the ladder is 1 m above the ground is **[JEE (Main)-2019]**

(1) $\frac{25}{3}$ (2) $25\sqrt{3}$

(3) $\frac{25}{\sqrt{3}}$ (4) 25

40. The number of solutions of the equation $1 + \sin^4 x = \cos^2 3x$, $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$ is **[JEE (Main)-2019]**

(1) 3 (2) 5

(3) 4 (4) 7

41. The angle of elevation of the top of a vertical tower standing on a horizontal plane is observed to be 45° from a point A on the plane. Let B be the point 30 m vertically above the point A. If the angle of elevation of the top of the tower from B be 30°, then the distance (in m) of the foot of the tower from the point A is **[JEE (Main)-2019]**

(1) $15\,(3+\sqrt{3})$

(2) $15\,(1+\sqrt{3})$

(3) $15\,(3-\sqrt{3})$

(4) $15\,(5-\sqrt{3})$

42. Let S be the set of all $\alpha \in R$ such that the equation, $\cos 2x + \alpha \sin x = 2\alpha - 7$ has a solution. Then S is equal to **[JEE (Main)-2019]**

(1) [1, 4] (2) R

(3) [2, 6] (4) [3, 7]

43. The value of

$$\cos^3\left(\frac{\pi}{8}\right).\cos\left(\frac{3\pi}{8}\right)+\sin^3\left(\frac{\pi}{8}\right).\sin\left(\frac{3\pi}{8}\right) \text{ is}$$

[JEE (Main)-2020]

(1) $\frac{1}{4}$ (2) $\frac{1}{2\sqrt{2}}$

(3) $\frac{1}{2}$ (4) $\frac{1}{\sqrt{2}}$

44. If the equation $\cos^4\theta + \sin^4\theta + \lambda = 0$ has real solutions for θ, then λ lies in the interval

[JEE (Main)-2020]

(1) $\left[-1,-\frac{1}{2}\right]$ (2) $\left[-\frac{3}{2},-\frac{5}{4}\right]$

(3) $\left(-\frac{1}{2},-\frac{1}{4}\right]$ (4) $\left(-\frac{5}{4},-1\right)$

45. Two vertical poles *AB* = 15 m and *CD* = 10 m are standing apart on a horizontal ground with points *A* and *C* on the ground. If *P* is the point of intersection of *BC* and *AD*, then the height of *P* (in m) above the line *AC* is

[JEE (Main)-2020]

(1) 6 (2) 20/3

(3) 10/3 (4) 5

46. If $L=\sin^2\left(\frac{\pi}{16}\right)-\sin^2\left(\frac{\pi}{8}\right)$ and

$M=\cos^2\left(\frac{\pi}{16}\right)-\sin^2\left(\frac{\pi}{8}\right)$, then

[JEE (Main)-2020]

(1) $L=-\frac{1}{2\sqrt{2}}+\frac{1}{2}\cos\frac{\pi}{8}$

(2) $M=\frac{1}{2\sqrt{2}}+\frac{1}{2}\cos\frac{\pi}{8}$

(3) $M=\frac{1}{4\sqrt{2}}+\frac{1}{4}\cos\frac{\pi}{8}$

(4) $L=\frac{1}{4\sqrt{2}}-\frac{1}{4}\cos\frac{\pi}{8}$

47. The angle of elevation of a cloud *C* from a point *P*, 200 m above a still lake is 30°. If the angle of depression of the image of *C* in the lake from the point *P* is 60°, then *PC* (in m) is equal to

[JEE (Main)-2020]

(1) 400

(2) $400\sqrt{3}$

(3) 100

(4) $200\sqrt{3}$

48. The angle of elevation of the summit of a mountain from a point on the ground is 45°. After climbing up one km towards the summit at an inclination of 30° from the ground, the angle of elevation of the summit is found to be 60°. Then the height (in km) of the summit from the ground is **[JEE (Main)-2020]**

(1) $\frac{1}{\sqrt{3}+1}$ (2) $\frac{1}{\sqrt{3}-1}$

(3) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (4) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

49. If $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}}=\frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}}=\frac{1}{\sqrt{10}}$, $\alpha, \beta \in \left(0,\frac{\pi}{2}\right)$, then $\tan(\alpha + 2\beta)$ is equal to _____. **[JEE (Main)-2020]**

50. The number of distinct solutions of the equation, $\log_{\frac{1}{2}}|\sin x| = 2-\log_{\frac{1}{2}}|\cos x|$ in the interval $[0, 2\pi]$, is _____. **[JEE (Main)-2020]**

51. The angle of elevation of the top of a hill from a point on the horizontal plane passing through the foot of the hill is found to be 45°. After walking a distance of 80 meters towards the top, up a slope inclined at an angle of 30° to the horizontal plane, the angle of elevation of the top of the hill becomes 75°. Then the height of the hill (in meters) is _____. **[JEE (Main)-2020]**

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