

Definite Integrals

1. $\int_0^{\pi} [\cot x] dx$, where $[.]$ denotes the greatest integer function, is equal to [AIEEE-2009]
- (1) 1 (2) -1
 (3) $-\frac{\pi}{2}$ (4) $\frac{\pi}{2}$
2. Let $p(x)$ be a function defined on R such that $\lim_{x \rightarrow \infty} \frac{f(3x)}{f(x)} = 1$, $p'(x) = p'(1-x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x) dx$ equals [AIEEE-2010]
- (1) $\sqrt{41}$ (2) 21
 (3) 41 (4) 42
3. Let $[.]$ denote the greatest integer function, then the value of $\int_0^{1.5} x[x^2] dx$ is [AIEEE-2011]
- (1) $\frac{3}{4}$ (2) $\frac{5}{4}$
 (3) 0 (4) $\frac{3}{2}$
4. If $g(x) = \int_0^x \cos 4t dt$, then $g(x + \pi)$ equals [AIEEE-2012]
- (1) $g(x) + g(\pi)$ (2) $g(x) - g(\pi)$
 (3) $g(x) \cdot g(\pi)$ (4) $\frac{g(x)}{g(\pi)}$
5. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is [JEE (Main)-2013]
- (1) 2500 (2) 3000
 (3) 3500 (4) 45000
6. Statement - I : The value of the integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \sqrt{\tan x}}$ is equal to $\frac{\pi}{6}$
- Statement - II : $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$. [JEE (Main)-2013]
- (1) Statement - I is true; Statement - II is true; Statement - II is a correct explanation for Statement - I.
 (2) Statement - I is true; Statement - II is true; Statement - II is not a correct explanation for Statement - I.
 (3) Statement - I is true; Statement - II is false.
 (4) Statement - I is false; Statement - II is true.
7. The intercepts on x -axis made by tangents to the curve, $y = \int_0^x |t| dt$, $x \in R$, which are parallel to the line $y = 2x$, are equal to [JEE (Main)-2013]
- (1) ± 1 (2) ± 2
 (3) ± 3 (4) ± 4
8. The integral $\int_0^{\pi} \sqrt{1 + 4 \sin^2 \frac{x}{2} - 4 \sin \frac{x}{2}} dx$ equals [JEE (Main)-2014]
- (1) $4\sqrt{3} - 4$ (2) $4\sqrt{3} - 4 - \frac{\pi}{3}$
 (3) $\pi - 4$ (4) $\frac{2\pi}{3} - 4 - 4\sqrt{3}$
9. The integral $\int_2^4 \frac{\log x^2}{\log x^2 + \log(36 - 12x + x^2)} dx$ is equal to [JEE (Main)-2015]
- (1) 2 (2) 4
 (3) 1 (4) 6

10. The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to [JEE (Main)-2017]
- (1) 2 (2) 4
 (3) -1 (4) -2
- (1) $\frac{1}{12}(7\pi - 5)$ (2) $\frac{3}{10}(4\pi - 3)$
 (3) $\frac{3}{20}(4\pi - 3)$ (4) $\frac{1}{12}(7\pi + 5)$

11. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$ is [JEE (Main)-2018]
- (1) $\frac{\pi}{8}$ (2) $\frac{\pi}{2}$
 (3) 4π (4) $\frac{\pi}{4}$

12. The value of $\int_0^{\pi} |\cos x|^3 dx$ is [JEE (Main)-2019]
- (1) 0 (2) $\frac{2}{3}$
 (3) $-\frac{4}{3}$ (4) $\frac{4}{3}$

13. If $\int_0^{\pi/3} \frac{\tan \theta}{\sqrt{2k \sec \theta}} d\theta = 1 - \frac{1}{\sqrt{2}}$, ($k > 0$), then the value of k is [JEE (Main)-2019]
- (1) 4 (2) 2
 (3) 1 (4) $\frac{1}{2}$

14. Let $I = \int_a^b (x^4 - 2x^2) dx$. If I is minimum then the ordered pair (a, b) is [JEE (Main)-2019]
- (1) $(-\sqrt{2}, 0)$ (2) $(0, \sqrt{2})$
 (3) $(\sqrt{2}, -\sqrt{2})$ (4) $(-\sqrt{2}, \sqrt{2})$

15. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{dx}{[x] + [\sin x] + 4}$, where $[t]$ denotes the greatest integer less than or equal to t , is [JEE (Main)-2019]

16. If $\int_0^x f(t) dt = x^2 + \int_x^1 t^2 f(t) dt$, then [JEE (Main)-2019]
- (1) $\frac{6}{25}$ (2) $\frac{24}{25}$
 (3) $\frac{4}{5}$ (4) $\frac{18}{25}$

17. The value of the integral $\int_{-2}^2 \frac{\sin^2 x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} dx$ (where $[x]$ denotes the greatest integer less than or equal to x) is [JEE (Main)-2019]

- (1) $\sin 4$ (2) $4 - \sin 4$
 (3) 0 (4) 4

18. The integral $\int_{\pi/6}^{\pi/4} \frac{dx}{\sin 2x (\tan^5 x + \cot^5 x)}$ equals [JEE (Main)-2019]

- (1) $\frac{1}{10} \left(\frac{\pi}{4} - \tan^{-1} \left(\frac{1}{9\sqrt{3}} \right) \right)$
 (2) $\frac{1}{20} \tan^{-1} \left(\frac{1}{9\sqrt{3}} \right)$
 (3) $\frac{\pi}{40}$
 (4) $\frac{1}{5} \left(\frac{\pi}{4} - \tan^{-1} \left(\frac{1}{3\sqrt{3}} \right) \right)$

19. Let f and g be continuous functions on $[0, a]$ such that $f(x) = f(a - x)$ and $g(x) + g(a - x) = 4$, then

- $\int_0^a f(x)g(x) dx$ is equal to [JEE (Main)-2019]
- (1) $\int_0^a f(x) dx$ (2) $4 \int_0^a f(x) dx$
 (3) $-3 \int_0^a f(x) dx$ (4) $2 \int_0^a f(x) dx$

20. The integral $\int_1^e \left\{ \left(\frac{x}{e} \right)^{2x} - \left(\frac{e}{x} \right)^x \right\} \log_e x dx$ is equal to
[JEE (Main)-2019]

- (1) $\frac{3}{2} - e - \frac{1}{2e^2}$ (2) $-\frac{1}{2} + \frac{1}{e} - \frac{1}{2e^2}$
 (3) $\frac{1}{2} - e - \frac{1}{e^2}$ (4) $\frac{3}{2} - \frac{1}{e} - \frac{1}{2e^2}$

21. $\lim_{n \rightarrow \infty} \left(\frac{n}{n^2 + 1^2} + \frac{n}{n^2 + 2^2} + \frac{n}{n^2 + 3^2} + \dots + \frac{1}{5n} \right)$ is equal to
[JEE (Main)-2019]
 (1) $\pi/4$ (2) $\tan^{-1}(3)$
 (3) $\tan^{-1}(2)$ (4) $\pi/2$

22. If $f(x) = \frac{2 - x \cos x}{2 + x \cos x}$ and $g(x) = \log_e x$, ($x > 0$) then

- the value of the integral $\int_{-\pi/4}^{\pi/4} g(f(x)) dx$ is :
[JEE (Main)-2019]
 (1) $\log_e 1$ (2) $\log_e 3$
 (3) $\log_e 2$ (4) $\log_e e$

23. Let $f(x) = \int_0^x g(t) dt$, where g is a non-zero even function. If $f(x+5) = g(x)$, then $\int_0^x f(t) dt$, equals:
[JEE (Main)-2019]

- (1) $\int_{x+5}^5 g(t) dt$ (2) $2 \int_5^{x+5} g(t) dt$
 (3) $\int_5^{x+5} g(t) dt$ (4) $5 \int_{x+5}^5 g(t) dt$

24. The value of $\int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx$ is
[JEE (Main)-2019]

- (1) $\frac{\pi - 2}{4}$ (2) $\frac{\pi - 2}{8}$
 (3) $\frac{\pi - 1}{4}$ (4) $\frac{\pi - 1}{2}$

25. If $\cos x \frac{dy}{dx} - y \sin x = 6x$, $\left(0 < x < \frac{\pi}{2} \right)$ and $y\left(\frac{\pi}{3}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to
[JEE (Main)-2019]

- (1) $-\frac{\pi^2}{2}$ (2) $-\frac{\pi^2}{4\sqrt{3}}$
 (3) $\frac{\pi^2}{2\sqrt{3}}$ (4) $-\frac{\pi^2}{2\sqrt{3}}$

26. The value of the integral $\int_0^1 x \cot^{-1}(1 - x^2 + x^4) dx$ is
[JEE (Main)-2019]

- (1) $\frac{\pi}{4} - \log_e 2$ (2) $\frac{\pi}{2} - \log_e 2$
 (3) $\frac{\pi}{2} - \frac{1}{2} \log_e 2$ (4) $\frac{\pi}{4} - \frac{1}{2} \log_e 2$

27. If $f: R \rightarrow R$ is a differentiable function and $f(2) = 6$, then $\lim_{x \rightarrow 2} \int_6^{f(x)} \frac{2t dt}{(x-2)}$ is
[JEE (Main)-2019]

- (1) 0 (2) $2f'(2)$
 (3) $12f'(2)$ (4) $24f'(2)$

28. The value of $\int_0^{2\pi} [\sin 2x (1 + \cos 3x)] dx$, where $[t]$ denotes the greatest integer function, is
[JEE (Main)-2019]

- (1) π (2) $-\pi$
 (3) -2π (4) 2π

29. $\lim_{n \rightarrow \infty} \left(\frac{(n+1)^{\frac{1}{3}}}{n^{\frac{4}{3}}} + \frac{(n+2)^{\frac{1}{3}}}{n^{\frac{4}{3}}} + \dots + \frac{(2n)^{\frac{1}{3}}}{n^{\frac{4}{3}}} \right)$ is equal to
[JEE (Main)-2019]

- (1) $\frac{4}{3} (2)^{\frac{4}{3}}$ (2) $\frac{3}{4} (2)^{\frac{4}{3}} - \frac{3}{4}$
 (3) $\frac{4}{3} (2)^{\frac{3}{4}}$ (4) $\frac{3}{4} (2)^{\frac{4}{3}} - \frac{4}{3}$

30. The integral $\int_{\pi/6}^{\pi/3} \sec^{\frac{2}{3}} x \cosec^{\frac{4}{3}} x dx$ is equal to
[JEE (Main)-2019]

- (1) $3^{\frac{7}{6}} - 3^{\frac{5}{6}}$ (2) $3^{\frac{5}{3}} - 3^{\frac{1}{3}}$
 (3) $3^{\frac{5}{6}} - 3^{\frac{2}{3}}$ (4) $3^{\frac{4}{3}} - 3^{\frac{1}{3}}$

42. Let $f(x) = |x - 2|$ and $g(x) = f(f(x))$, $x \in [0, 4]$.

Then $\int_0^3 (g(x) - f(x)) dx$ is equal to

(1) 0

(2) $\frac{3}{2}$

(3) $\frac{1}{2}$

(4) 1

43. The integral

$$\int_{\pi/6}^{\pi/3} \tan^3 x \cdot \sin^2 3x (2\sec^2 x \cdot \sin^2 3x + 3\tan x \cdot \sin 6x) dx$$

is equal to

(1) $\frac{7}{18}$

(2) $\frac{9}{2}$

(3) $-\frac{1}{18}$

(4) $-\frac{1}{9}$

44. The value of $\int_{-\pi/2}^{\pi/2} \frac{1}{1 + e^{\sin x}} dx$ is

(1) $\frac{\pi}{4}$

(2) π

(3) $\frac{3\pi}{2}$

(4) $\frac{\pi}{2}$

[JEE (Main)-2020]

45. If $I_1 = \int_0^1 (1-x^{50})^{100} dx$ and $I_2 = \int_0^1 (1-x^{50})^{101} dx$ such that $I_2 = \alpha I_1$ then α equals to

[JEE (Main)-2020]

(1) $\frac{5051}{5050}$

(2) $\frac{5050}{5051}$

(3) $\frac{5050}{5049}$

(4) $\frac{5049}{5050}$

46. The integral $\int_1^2 e^x \cdot x^x (2 + \log_e x) dx$ equals

[JEE (Main)-2020]

(1) $e(2e - 1)$

(2) $e(4e - 1)$

(3) $4e^2 - 1$

(4) $e(4e + 1)$

47. The integral $\int_0^2 |x-1| - x | dx$ is equal to _____.

[JEE (Main)-2020]

48. Let $[t]$ denote the greatest integer less than or equal to t . Then the value of $\int_1^2 |2x - [3x]| dx$ is _____.

[JEE (Main)-2020]

49. Let $\{x\}$ and $[x]$ denote the fractional part of x and the greatest integer $\leq x$ respectively of a real number x . If $\int_0^n \{x\} dx$, $\int_0^n [x] dx$ and $10(n^2 - n)$, ($n \in N$, $n > 1$) are three consecutive terms of a G.P., then n is equal to _____.

[JEE (Main)-2020]

[JEE (Main)-2020]

