Conic Sections

(Parabola, Ellipse and Hyperbola)

1. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0). Then the equation of the ellipse is **[AIEEE-2009]**

   (1) $x^2 + 12y^2 = 16$ (2) $4x^2 + 48y^2 = 48$

   (3) $4x^2 + 64y^2 = 48$ (4) $x^2 + 16y^2 = 16$

2. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of p is **[AIEEE-2010]**

   (1) $x = 1$ (2) $2x + 1 = 0$

   (3) $x = -1$ (4) $2x - 1 = 0$

3. The equation of the hyperbola whose foci are (–2, 0) and (2, 0) and eccentricity is 2 is given by: **[AIEEE-2011]**

   (1) $-x^2 + 3y^2 = 3$ (2) $-3x^2 + y^2 = 3$

   (3) $x^2 - 3y^2 = 3$ (4) $3x^2 - y^2 = 3$

4. **Statement 1 :** An equation of a common tangent to the hyperbola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$. **[AIEEE-2012]**

   Statement 2 : If the line $y = mx + \frac{4\sqrt{3}}{m}$, $(m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$.

   (1) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1

   (2) Statement 1 is true, Statement 2 is true, Statement 2 is **not** a correct explanation for Statement 1

   (3) Statement 1 is true, Statement 2 is false

   (4) Statement 1 is false, Statement 2 is true

5. An ellipse is drawn by taking a diameter of the circle $(x - 1)^2 + y^2 = 1$ as its semi-minor axis and a diameter of the circle $x^2 + (y - 2)^2 = 4$ as its semi-major axis. If the centre of the ellipse is at the original and its axes are the coordinate axes, then the equation of the ellipse is **[AIEEE-2012]**

   (1) $x^2 + 4y^2 = 8$ (2) $4x^2 + y^2 = 8$

   (3) $x^2 + 4y^2 = 16$ (4) $4x^2 + y^2 = 4$

6. Given : A circle, $2x^2 + 2y^2 = 5$ and a parabola, $y^2 = 4\sqrt{5}\,x$

   Statement-I : An equation of a common tangent to these curves is $y = x + \sqrt{5}$.

   Statement-II : If the line, $y = mx + \frac{\sqrt{5}}{m}$ $(m \neq 0)$ is their common tangent, then m satisfies $m^4 - 3m^2 + 2 = 0$. **[JEE (Main)-2013]**

   (1) Statement-I is true; statement-II is true; statement-II is a **correct** explanation for statement-I

   (2) Statement-I is true; statement-II is true; statement-II is **not** a correct explanation for statement-I

   (3) Statement-I is true; statement-II is false

   (4) Statement-I is false; statement-II, is true

7. The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is **[JEE (Main)-2014]**

   (1) $(x^2 + y^2)^2 = 6x^2 + 2y^2$

   (2) $(x^2 + y^2)^2 = 6x^2 - 2y^2$

   (3) $(x^2 - y^2)^2 = 6x^2 + 2y^2$

   (4) $(x^2 - y^2)^2 = 6x^2 - 2y^2$

8. The slope of the line touching both the parabolas $y^2 = 4x$ and $x^2 = -32y$ is **[JEE (Main)-2014]**

   (1) $\frac{1}{8}$ (2) $\frac{2}{3}$

   (3) $\frac{1}{2}$ (4) $\frac{3}{2}$

9. The area (in sq. units) of the quadrilateral formed by the tangents at the end points of the latera recta to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$, is **[JEE (Main)-2015]**

(1) $\frac{27}{4}$ (2) 18

(3) $\frac{27}{2}$ (4) 27

10. Let O be the vertex and Q be any point on the parabola, $x^2 = 8y$. If the point P divides the line segment OQ internally in the ratio 1 : 3, then the locus of P is **[JEE (Main)-2015]**

(1) $x^2 = y$ (2) $y^2 = x$

(3) $y^2 = 2x$ (4) $x^2 = 2y$

11. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y + 6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is **[JEE (Main)-2016]**

(1) $x^2 + y^2 - x + 4y - 12 = 0$

(2) $x^2+y^2-\frac{x}{4}+2y-24=0$

(3) $x^2 + y^2 - 4x + 9y + 18 = 0$

(4) $x^2 + y^2 - 4x + 8y + 12 = 0$

12. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is **[JEE (Main)-2016]**

(1) $\frac{4}{\sqrt{3}}$ (2) $\frac{2}{\sqrt{3}}$

(3) $\sqrt{3}$ (4) $\frac{4}{3}$

13. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directrices is $x = -4$, then the equation of the normal to it at $\left(1, \frac{3}{2}\right)$ is **[JEE (Main)-2017]**

(1) $4x - 2y = 1$ (2) $4x + 2y = 7$

(3) $x + 2y = 4$ (4) $2y - x = 2$

14. A hyperbola passes through the point $P(\sqrt{2}, \sqrt{3})$ and has foci at (±2, 0). Then the tangent to this hyperbola at P also passes through the point **[JEE (Main)-2017]**

(1) $(2\sqrt{2}, 3\sqrt{3})$

(2) $(\sqrt{3}, \sqrt{2})$

(3) $(-\sqrt{2}, -\sqrt{3})$

(4) $(3\sqrt{2}, 2\sqrt{3})$

15. Two sets A and B are as under :

$A = \{(a, b) \in R \times R : |a - 5| < 1 \text{ and } |b - 5| < 1\}$

$B = \{(a, b) \in R \times R : 4(a - 6)^2 + 9(b - 5)^2 \le 36\}$, then **[JEE (Main)-2018]**

(1) $B \subset A$

(2) $A \subset B$

(3) $A \cap B = \phi$ (an empty set)

(4) Neither $A \subset B$ nor $B \subset A$

16. Tangent and normal are drawn at $P(16, 16)$ on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is **[JEE (Main)-2018]**

(1) $\frac{1}{2}$ (2) 2

(3) 3 (4) $\frac{4}{3}$

17. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point $T(0, 3)$ then the area (in sq. units) of ΔPTQ is **[JEE (Main)-2018]**

(1) $45\sqrt{5}$ (2) $54\sqrt{3}$

(3) $60\sqrt{3}$ (4) $36\sqrt{5}$

18. Let $0<\theta<\frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2\theta}-\frac{y^2}{\sin^2\theta}=1$ is greater than 2, then the length of its latus rectum lies in the interval **[JEE (Main)-2019]**

(1) (2, 3] (2) (3/2, 2]

(3) (1, 3/2] (4) (3, ∞)

19. Axis of a parabola lies along x-axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it?

[JEE (Main)-2019]

(1) (4, –4) (2) $\left(5, 2\sqrt{6}\right)$

(3) $\left(6, 4\sqrt{2}\right)$ (4) (8, 6)

20. Let $A(4, -4)$ and $B(9, 6)$ be points on the parabola, $y^2 = 4x$. Let C be chosen on the arc AOB of the parabola, where O is the origin, such that the area of ΔACB is maximum. Then, the area (in sq. units) of ΔACB, is **[JEE (Main)-2019]**

(1) 32 (2) $31\frac{3}{4}$

(3) $31\frac{1}{4}$ (4) $30\frac{1}{2}$

21. A hyperbola has its centre at the origin, passes through the point (4, 2) and has transverse axis of length 4 along the x-axis. Then the eccentricity of the hyperbola is **[JEE (Main)-2019]**

(1) $\frac{3}{2}$ (2) $\sqrt{3}$

(3) $\frac{2}{\sqrt{3}}$ (4) 2

22. If the parabolas $y^2 = 4b(x - c)$ and $y^2 = 8ax$ have a common normal, then which one of the following is a valid choice for the ordered triad (a, b, c)?

[JEE (Main)-2019]

(1) $\left(\frac{1}{2}, 2, 0\right)$ (2) $\left(\frac{1}{2}, 2, 3\right)$

(3) (1, 1, 0) (4) (1, 1, 3)

23. The equation of a tangent to the hyperbola $4x^2 - 5y^2 = 20$ parallel to the line $x - y = 2$ is

[JEE (Main)-2019]

(1) $x - y + 7 = 0$ (2) $x - y + 1 = 0$

(3) $x - y - 3 = 0$ (4) $x - y + 9 = 0$

24. The length of the chord of the parabola $x^2 = 4y$ having equation $x - \sqrt{2}y + 4\sqrt{2} = 0$ is

[JEE (Main)-2019]

(1) $3\sqrt{2}$ (2) $6\sqrt{3}$

(3) $2\sqrt{11}$ (4) $8\sqrt{2}$

25. Let $S = \left\{(x, y) \in R^2 : \frac{y^2}{1+r} - \frac{x^2}{1-r} = 1\right\}$, where $r \neq \pm 1$. Then S represents

[JEE (Main)-2019]

(1) An ellipse whose eccentricity is $\frac{1}{\sqrt{r+1}}$, when $r > 1$.

(2) An ellipse whose eccentricity is $\sqrt{\frac{2}{r+1}}$, when $r > 1$.

(3) A hyperbola whose eccentricity is $\frac{2}{\sqrt{r+1}}$, when $0 < r < 1$.

(4) A hyperbola whose eccentricity is $\frac{2}{\sqrt{1-r}}$, when $0 < r < 1$.

26. Equation of a common tangent to the parabola $y^2 = 4x$ and the hyperbola $xy = 2$ is

[JEE (Main)-2019]

(1) $4x + 2y + 1 = 0$

(2) $x + 2y + 4 = 0$

(3) $x - 2y + 4 = 0$

(4) $x + y + 1 = 0$

27. If tangents are drawn to the ellipse $x^2 + 2y^2 = 2$ at all points on the ellipse other than its four vertices then the mid points of the tangents intercepted between the coordinate axes lie on the curve **[JEE (Main)-2019]**

(1) $\frac{1}{4x^2} + \frac{1}{2y^2} = 1$ (2) $\frac{x^2}{2} + \frac{y^2}{4} = 1$

(3) $\frac{x^2}{4} + \frac{y^2}{2} = 1$ (4) $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$

28. If the area of the triangle whose one vertex is at the vertex of the parabola, $y^2 + 4(x - a^2) = 0$ and the other two vertices are the points of intersection of the parabola and y-axis, is 250 sq. units, then a value of 'a' is **[JEE (Main)-2019]**

(1) $5\sqrt{5}$

(2) $(10)^{2/3}$

(3) $5(2^{1/3})$

(4) 5

29. If a hyperbola has length of its conjugate axis equal to 5 and the distance between its foci is 13, then the eccentricity of the hyperbola is **[JEE (Main)-2019]**

(1) 2 (2) $\frac{13}{8}$

(3) $\frac{13}{6}$ (4) $\frac{13}{12}$

30. Let the length of the latus rectum of an ellipse with its major axis along x-axis and centre at the origin, be 8. If the distance between the foci of this ellipse is equal to the length of its minor axis, then which one of the following points lies on it? **[JEE (Main)-2019]**

(1) $\left(4\sqrt{3}, 2\sqrt{3}\right)$ (2) $\left(4\sqrt{3}, 2\sqrt{2}\right)$

(3) $\left(4\sqrt{2}, 2\sqrt{2}\right)$ (4) $\left(4\sqrt{2}, 2\sqrt{3}\right)$

31. Let $P(4, -4)$ and $Q(9, 6)$ be two points on the parabola, $y^2 = 4x$ and let X be any point on the arc POQ of this parabola, where O is the vertex of this parabola, such that the area of ΔPXQ is maximum. Then this maximum area (in sq. units) is **[JEE (Main)-2019]**

(1) $\frac{75}{2}$ (2) $\frac{125}{4}$

(3) $\frac{625}{4}$ (4) $\frac{125}{2}$

32. Let C_1 and C_2 be the centres of the circles $x^2 + y^2 - 2x - 2y - 2 = 0$ and $x^2 + y^2 - 6x - 6y + 14 = 0$ respectively. If P and Q are the points of intersection of these circles, then the area (in sq. units) of the quadrilateral PC_1QC_2 is **[JEE (Main)-2019]**

(1) 4 (2) 9

(3) 6 (4) 8

33. The maximum area (in sq. units) of a rectangle having its base on the x-axis and its other two vertices on the parabola, $y = 12 - x^2$ such that the rectangle lies inside the parabola, is **[JEE (Main)-2019]**

(1) 32 (2) 36

(3) $20\sqrt{2}$ (4) $18\sqrt{3}$

34. If the vertices of a hyperbola be at $(-2, 0)$ and $(2, 0)$ and one of its foci be at $(-3, 0)$, then which one of the following points does not lie on this hyperbola? **[JEE (Main)-2019]**

(1) $\left(4, \sqrt{15}\right)$ (2) $\left(6, 5\sqrt{2}\right)$

(3) $\left(2\sqrt{6}, 5\right)$ (4) $\left(-6, 2\sqrt{10}\right)$

35. Let S and S' be the foci of an ellipse and B be any one of the extremities of its minor axis. If $\Delta S'BS$ is a right angled triangle with right angle at B and area $(\Delta S'BS) = 8$ sq. units, then the length of a latus rectum of the ellipse is **[JEE (Main)-2019]**

(1) $4\sqrt{2}$ (2) 4

(3) $2\sqrt{2}$ (4) 2

36. If the tangents on the ellipse $4x^2 + y^2 = 8$ at the points $(1, 2)$ and (a, b) are perpendicular to each other, then a^2 is equal to **[JEE (Main)-2019]**

(1) $\frac{64}{17}$ (2) $\frac{2}{17}$

(3) $\frac{4}{17}$ (4) $\frac{128}{17}$

37. Let $O(0, 0)$ and $A(0, 1)$ be two fixed points. Then the locus of a point P such that the perimeter of ΔAOP is 4, is **[JEE (Main)-2019]**

(1) $8x^2 - 9y^2 + 9y = 18$

(2) $9x^2 + 8y^2 - 8y = 16$

(3) $9x^2 - 8y^2 + 8y = 16$

(4) $8x^2 + 9y^2 - 9y = 18$

38. In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $\left(0, 5\sqrt{3}\right)$, then the length of its latus rectum is **[JEE (Main)-2019]**

(1) 5 (2) 6

(3) 8 (4) 10

39. The tangent to the parabola $y^2 = 4x$ at the point where it intersects the circle $x^2 + y^2 = 5$ in the first quadrant, passes through the point **[JEE (Main)-2019]**

(1) $\left(\frac{3}{4}, \frac{7}{4}\right)$ (2) $\left(-\frac{1}{3}, \frac{4}{3}\right)$

(3) $\left(\frac{1}{4}, \frac{3}{4}\right)$ (4) $\left(-\frac{1}{4}, \frac{1}{2}\right)$

40. If the eccentricity of the standard hyperbola passing through the point (4, 6) is 2, then the equation of the tangent to the hyperbola at (4, 6) is **[JEE (Main)-2019]**

(1) $2x - 3y + 10 = 0$

(2) $x - 2y + 8 = 0$

(3) $3x - 2y = 0$

(4) $2x - y - 2 = 0$

41. If one end of a focal chord of the parabola, $y^2 = 16x$ is at (1, 4), then the length of this focal chord is **[JEE (Main)-2019]**

(1) 24 (2) 20

(3) 22 (4) 25

42. If the line $y = mx + 7\sqrt{3}$ is normal to the hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$, then a value of m is **[JEE (Main)-2019]**

(1) $\frac{2}{\sqrt{5}}$ (2) $\frac{3}{\sqrt{5}}$

(3) $\frac{\sqrt{15}}{2}$ (4) $\frac{\sqrt{5}}{2}$

43. The area (in sq. units) of the smaller of the two circles that touch the parabola, $y^2 = 4x$ at the point (1, 2) and the x-axis is **[JEE (Main)-2019]**

(1) $4\pi\left(3+\sqrt{2}\right)$ (2) $8\pi\left(2-\sqrt{2}\right)$

(3) $8\pi\left(3-2\sqrt{2}\right)$ (4) $4\pi\left(2-\sqrt{2}\right)$

44. If the tangent to the parabola $y^2 = x$ at a point (α, β), $(\beta > 0)$ is also a tangent to the ellipse, $x^2 + 2y^2 = 1$, then α is equal to **[JEE (Main)-2019]**

(1) $\sqrt{2} - 1$ (2) $\sqrt{2} + 1$

(3) $2\sqrt{2} + 1$ (4) $2\sqrt{2} - 1$

45. If a directrix of a hyperbola centred at the origin and passing through the point $\left(4, -2\sqrt{3}\right)$ is $5x = 4\sqrt{5}$ and its eccentricity is e, then : **[JEE (Main)-2019]**

(1) $4e^4 + 8e^2 - 35 = 0$

(2) $4e^4 - 24e^2 + 35 = 0$

(3) $4e^4 - 12e^2 - 27 = 0$

(4) $4e^4 - 24e^2 + 27 = 0$

46. If the line $x - 2y = 12$ is tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $\left(3, \frac{-9}{2}\right)$, then the length of the latus rectum of the ellipse is **[JEE (Main)-2019]**

(1) 5 (2) $8\sqrt{3}$

(3) $12\sqrt{2}$ (4) 9

47. If $5x + 9 = 0$ is the directrix of the hyperbola $16x^2 - 9y^2 = 144$, then its corresponding focus is **[JEE (Main)-2019]**

(1) $\left(\frac{5}{3}, 0\right)$ (2) $\left(-\frac{5}{3}, 0\right)$

(3) (–5, 0) (4) (5, 0)

48. If the line $ax + y = c$, touches both the curves $x^2 + y^2 = 1$ and $y^2 = 4\sqrt{2}x$, then $|c|$ is equal to **[JEE (Main)-2019]**

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2}$

(3) 2 (4) $\sqrt{2}$

49. The tangent and normal to the ellipse $3x^2 + 5y^2 = 32$ at the point $P(2, 2)$ meet the x-axis at Q and R, respectively. Then the area (in sq. units) of the triangle PQR is **[JEE (Main)-2019]**

(1) $\frac{16}{3}$ (2) $\frac{14}{3}$

(3) $\frac{34}{15}$ (4) $\frac{68}{15}$

50. If the normal to the ellipse $3x^2 + 4y^2 = 12$ at a point P on it is parallel to the line, $2x + y = 4$ and the tangent to the ellipse at P passes through $Q(4, 4)$ then PQ is equal to **[JEE (Main)-2019]**

(1) $\frac{\sqrt{61}}{2}$ (2) $\frac{5\sqrt{5}}{2}$

(3) $\frac{\sqrt{157}}{2}$ (4) $\frac{\sqrt{221}}{2}$

51. Let P be the point of intersection of the common tangents to the parabola $y^2 = 12x$ and the hyperbola $8x^2 - y^2 = 8$. If S and S' denote the foci of the hyperbola where S lies on the positive x-axis then P divides SS' in a ratio **[JEE (Main)-2019]**

(1) 13 : 11 (2) 14 : 13

(3) 5 : 4 (4) 2 : 1

52. An ellipse, with foci at (0, 2) and (0, –2) and minor axis of length 4, passes through which of the following points? **[JEE (Main)-2019]**

(1) $\left(\sqrt{2}, 2\right)$ (2) $\left(2, 2\sqrt{2}\right)$

(3) $\left(1, 2\sqrt{2}\right)$ (4) $\left(2, \sqrt{2}\right)$

53. The equation of a common tangent to the curves, $y^2 = 16x$ and $xy = -4$, is **[JEE (Main)-2019]**

(1) $x + y + 4 = 0$

(2) $2x - y + 2 = 0$

(3) $x - 2y + 16 = 0$

(4) $x - y + 4 = 0$

54. If $y = mx + 4$ is a tangent to both the parabolas, $y^2 = 4x$ and $x^2 = 2by$, then b is equal to **[JEE (Main)-2020]**

(1) –64 (2) 128

(3) –32 (4) –128

55. If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is **[JEE (Main)-2020]**

(1) $\frac{3}{\sqrt{2}}$ (2) $\sqrt{3}$

(3) $3\sqrt{2}$ (4) $2\sqrt{3}$

56. If $3x + 4y = 12\sqrt{2}$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ for some $a \in R$, then the distance between the foci of the ellipse is **[JEE (Main)-2020]**

(1) $2\sqrt{5}$ (2) $2\sqrt{7}$

(3) 4 (4) $2\sqrt{2}$

57. The locus of a point which divides the line segment joining the point (0, –1) and a point on the parabola, $x^2 = 4y$, internally in the ratio 1 : 2, is **[JEE (Main)-2020]**

(1) $9x^2 - 12y = 8$

(2) $4x^2 - 3y = 2$

(3) $x^2 - 3y = 2$

(4) $9x^2 - 3y = 2$

58. Let the line $y = mx$ and the ellipse $2x^2 + y^2 = 1$ intersect at a point P in the first quadrant. If the normal to this ellipse at P meets the co-ordinate axes at $\left(-\frac{1}{3\sqrt{2}}, 0\right)$ and $(0, \beta)$, then β is equal to **[JEE (Main)-2020]**

(1) $\frac{\sqrt{2}}{3}$ (2) $\frac{2}{3}$

(3) $\frac{2}{\sqrt{3}}$ (4) $\frac{2\sqrt{2}}{3}$

59. The length of the perpendicular from the origin, on the normal to the curve, $x^2 + 2xy - 3y^2 = 0$ at the point (2, 2) is **[JEE (Main)-2020]**

(1) $2\sqrt{2}$ (2) $\sqrt{2}$

(3) $4\sqrt{2}$ (4) 2

60. If a hyperbola passes through the point $P(10, 16)$ and it has vertices at $(\pm 6, 0)$, then the equation of the normal to it at P is **[JEE (Main)-2020]**

(1) $x + 2y = 42$ (2) $2x + 5y = 100$

(3) $x + 3y = 58$ (4) $3x + 4y = 94$

61. If e_1 and e_2 are the eccentricities of the ellipse, $\frac{x^2}{18} + \frac{y^2}{4} = 1$ and the hyperbola, $\frac{x^2}{9} - \frac{y^2}{4} = 1$ respectively and (e_1, e_2) is a point on the ellipse, $15x^2 + 3y^2 = k$, then k is equal to **[JEE (Main)-2020]**

(1) 14 (2) 15

(3) 17 (4) 16

62. The length of the minor axis (along y-axis) of an ellipse in the standard form is $\frac{4}{\sqrt{3}}$. If this ellipse touches the line, $x + 6y = 8$; then its eccentricity is **[JEE (Main)-2020]**

(1) $\frac{1}{3}\sqrt{\frac{11}{3}}$ (2) $\frac{1}{2}\sqrt{\frac{5}{3}}$

(3) $\sqrt{\frac{5}{6}}$ (4) $\frac{1}{2}\sqrt{\frac{11}{3}}$

63. If one end of a focal chord AB of the parabola $y^2 = 8x$ is at $A\left(\frac{1}{2},-2\right)$, then the equation of the tangent to it at B is **[JEE (Main)-2020]**

(1) $x - 2y + 8 = 0$ (2) $x + 2y + 8 = 0$

(3) $2x - y - 24 = 0$ (4) $2x + y - 24 = 0$

64. A line parallel to the straight line $2x - y = 0$ is tangent to the hyperbola $\frac{x^2}{4}-\frac{y^2}{2}=1$ at the point (x_1, y_1). Then $x_1^2+5y_1^2$ is equal to **[JEE (Main)-2020]**

(1) 8 (2) 6

(3) 10 (4) 5

65. For some $\theta\in\left(0,\frac{\pi}{2}\right)$, if the eccentricity of the hyperbola, $x^2 - y^2\sec^2\theta = 10$ is $\sqrt{5}$ times the eccentricity of the ellipse, $x^2\sec^2\theta + y^2 = 5$, then the length of the latus rectum of the ellipse, is **[JEE (Main)-2020]**

(1) $2\sqrt{6}$ (2) $\frac{2\sqrt{5}}{3}$

(3) $\frac{4\sqrt{5}}{3}$ (4) $\sqrt{30}$

66. The area (in sq. units) of an equilateral triangle inscribed in the parabola $y^2 = 8x$, with one of its vertices on the vertex of this parabola, is **[JEE (Main)-2020]**

(1) $64\sqrt{3}$ (2) $256\sqrt{3}$

(3) $128\sqrt{3}$ (4) $192\sqrt{3}$

67. Let P be a point on the parabola, $y^2 = 12x$ and N be the foot of the perpendicular drawn from P on the axis of the parabola. A line is now drawn through the mid-point M of PN, parallel to its axis which meets the parabola at Q. If the y-intercept of the line NQ is $\frac{4}{3}$, then **[JEE (Main)-2020]**

(1) $MQ=\frac{1}{4}$ (2) $PN = 3$

(3) $PN = 4$ (4) $MQ=\frac{1}{3}$

68. A hyperbola having the transverse axis of length $\sqrt{2}$ has the same foci as that of the ellipse of $3x^2 + 4y^2 = 12$, then this hyperbola does not pass through which of the following points? **[JEE (Main)-2020]**

(1) $\left(-\sqrt{\frac{3}{2}},1\right)$ (2) $\left(\sqrt{\frac{3}{2}},\frac{1}{\sqrt{2}}\right)$

(3) $\left(\frac{1}{\sqrt{2}},0\right)$ (4) $\left(1,-\frac{1}{\sqrt{2}}\right)$

69. Let e_1 and e_2 be the eccentricities of the ellipse, $\frac{x^2}{25}+\frac{y^2}{b^2}=1\ (b<5)$ and the hyperbola, $\frac{x^2}{16}-\frac{y^2}{b^2}=1$ respectively satisfying $e_1e_2 = 1$. If α and β are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair (α, β) is equal to **[JEE (Main)-2020]**

(1) (8, 10) (2) $\left(\frac{24}{5},10\right)$

(3) $\left(\frac{20}{3},12\right)$ (4) (8, 12)

70. Let $P(3, 3)$ be a point on the hyperbola, $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If the normal to it at P intersects the x-axis at (9, 0) and e is its eccentricity, then the ordered pair (a^2, e^2) is equal to **[JEE (Main)-2020]**

(1) (9, 3) (2) $\left(\frac{9}{2},3\right)$

(3) $\left(\frac{3}{2},2\right)$ (4) $\left(\frac{9}{2},2\right)$

71. Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b)$ be a given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function, $\phi(t)=\frac{5}{12}+t-t^2$, then $a^2 + b^2$ is equal to **[JEE (Main)-2020]**

(1) 135 (2) 116

(3) 126 (4) 145

72. Let $x = 4$ be a directrix to an ellipse whose centre is at the origin and its eccentricity is $\frac{1}{2}$. If $P(1, \beta)$, $\beta > 0$ is a point on this ellipse, then the equation of the normal to it at P is **[JEE (Main)-2020]**

(1) $7x - 4y = 1$ (2) $4x - 2y = 1$

(3) $4x - 3y = 2$ (4) $8x - 2y = 5$

73. If the common tangent to the parabolas, $y^2 = 4x$ and $x^2 = 4y$ also touches the circle, $x^2 + y^2 = c^2$, then c is equal to **[JEE (Main)-2020]**

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$

(3) $\frac{1}{2\sqrt{2}}$ (4) $\frac{1}{\sqrt{2}}$

74. If the point P on the curve, $4x^2 + 5y^2 = 20$ is farthest from the point $Q(0, -4)$, then PQ^2 is equal to **[JEE (Main)-2020]**

(1) 29 (2) 48

(3) 21 (4) 36

75. If the line $y = mx + c$ is a common tangent to the hyperbola $\frac{x^2}{100} - \frac{y^2}{64} = 1$ and the circle $x^2 + y^2 = 36$, then which one of the following is true ? **[JEE (Main)-2020]**

(1) $5m = 4$ (2) $8m + 5 = 0$

(3) $c^2 = 369$ (4) $4c^2 = 369$

76. Let L_1 be a tangent to the parabola $y^2 = 4(x + 1)$ and L_2 be a tangent to the parabola $y^2 = 8(x + 2)$ such that L_1 and L_2 intersect at right angles. Then L_1 and L_2 meet on the straight line **[JEE (Main)-2020]**

(1) $2x + 1 = 0$ (2) $x + 3 = 0$

(3) $x + 2y = 0$ (4) $x + 2 = 0$

77. Which of the following points lies on the locus of the foot of perpendicular drawn upon any tangent to the ellipse, $\frac{x^2}{4} + \frac{y^2}{2} = 1$ from any of its foci? **[JEE (Main)-2020]**

(1) (1, 2)

(2) $\left(-2, \sqrt{3}\right)$

(3) $\left(-1, \sqrt{3}\right)$

(4) $\left(-1, \sqrt{2}\right)$

78. If the normal at an end of a latus rectum of an ellipse passes through an extremity of the minor axis, then the eccentricity e of the ellipse satisfies **[JEE (Main)-2020]**

(1) $e^2 + 2e - 1 = 0$

(2) $e^2 + e - 1 = 0$

(3) $e^4 + 2e^2 - 1 = 0$

(4) $e^4 + e^2 - 1 = 0$

79. Let the normal at a point P on the curve $y^2 - 3x^2 + y + 10 = 0$ intersect the y-axis at $\left(0, \frac{3}{2}\right)$. If m is the slope of the tangent at P to the curve, then $|m|$ is equal to _____. **[JEE (Main)-2020]**

80. Let a line $y = mx$ ($m > 0$) intersect the parabola, $y^2 = x$ at a point P, other than the origin. Let the tangent to it at P meet the x-axis at the point Q. If area (ΔOPQ) = 4 sq. units, then m is equal to_____. **[JEE (Main)-2020]**

81. If the curves, $x^2 - 6x + y^2 + 8 = 0$ and $x^2 - 8y + y^2 + 16 - k = 0$, ($k > 0$) touch each other at a point, then the largest value of k is _____. **[JEE (Main)-2020]**

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