Sequences and Series

1. The sum to infinity of the series $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+.....$ is **[AIEEE-2009]**

(1) 3 (2) 4

(3) 6 (4) 2

2. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11}, ...$ are in an AP with common difference –2, then the time taken by him to count all notes is **[AIEEE-2010]**

(1) 24 minutes (2) 34 minutes

(3) 125 minutes (4) 135 minutes

3. Let a_n be the n^{th} term of an A.P.

If $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{100} a_{2r-1} = \beta$, then the common difference of the A.P. is **[AIEEE-2011]**

(1) $\beta - \alpha$ (2) $\frac{\alpha-\beta}{200}$

(3) $\alpha - \beta$ (4) $\frac{\alpha-\beta}{100}$

4. **Statement-1:** The sum of the series 1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) +…+ (361 + 380 + 400) is 8000.

Statement-2: $\sum_{k=1}^{n}(k^3-(k-1)^3) = n^3$ for any natural number n. **[AIEEE-2012]**

(1) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.

(2) Statement-1 is true, statement-2 is true, statement-2 is **not** a correct explanation for statement-1.

(3) Statement-1 is true, statement-2 is false.

(4) Statement-1 is false, statement-2 is true.

5. If 100 times the 100^{th} term of an AP with non-zero common difference equals the 50 times its 50^{th} term, then the 150^{th} term of this AP is **[AIEEE-2012]**

(1) 150 times its 50^{th} term

(2) 150

(3) Zero

(4) –150

6. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, ..., is **[JEE (Main)-2013]**

(1) $\frac{7}{81}(179-10^{-20})$ (2) $\frac{7}{9}(99-10^{-20})$

(3) $\frac{7}{81}(179+10^{-20})$ (4) $\frac{7}{9}(99+10^{-20})$

7. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q, r are in A.P. and $\frac{1}{\alpha}+\frac{1}{\beta}=4$, then the value of $|\alpha - \beta|$ is **[JEE (Main)-2014]**

(1) $\frac{\sqrt{34}}{9}$ (2) $\frac{2\sqrt{13}}{9}$

(3) $\frac{\sqrt{61}}{9}$ (4) $\frac{2\sqrt{17}}{9}$

8. If $(10)^9 + 2(11)^1 (10)^8 + 3(11)^2 (10)^7 + ... + 10(11)^9 = k(10)^9$, then k is equal to **[JEE (Main)-2014]**

(1) 100 (2) 110

(3) $\frac{121}{10}$ (4) $\frac{441}{100}$

9. Three positive numbers form an increasing G.P. If the middle term in this G.P. is doubled, the new numbers are in A.P. Then the common ratio of the G.P. is **[JEE (Main)-2014]**

(1) $2-\sqrt{3}$ (2) $2+\sqrt{3}$

(3) $\sqrt{2}+\sqrt{3}$ (4) $3+\sqrt{2}$

10. If m is the A.M. of two distinct real numbers l and n $(l, n > 1)$ and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals. **[JEE (Main)-2015]**

(1) $4\, l^2mn$ (2) $4\, lm^2n$

(3) $4\, lmn^2$ (4) $4\, l^2m^2n^2$

11. The sum of first 9 terms of the series

$$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\text{........ is}$$

[JEE (Main)-2015]

(1) 71 (2) 96

(3) 142 (4) 192

12. If the 2nd, 5th and 9th terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is **[JEE (Main)-2016]**

(1) $\frac{4}{3}$ (2) 1

(3) $\frac{7}{4}$ (4) $\frac{8}{5}$

13. If the sum of the first ten terms of the series

$$\left(1\frac{3}{5}\right)^2+\left(2\frac{2}{5}\right)^2+\left(3\frac{1}{5}\right)^2+4^2+\left(4\frac{4}{5}\right)^2+\text{.....,}$$

is $\frac{16}{5}m$, then m is equal to **[JEE (Main)-2016]**

(1) 101 (2) 100

(3) 99 (4) 102

14. For any three positive real numbers a, b and c,

$$9(25a^2+b^2)+25(c^2-3ac)=15b(3a+c).$$

Then **[JEE (Main)-2017]**

(1) b, c and a are in A.P.

(2) a, b and c are in A.P.

(3) a, b and c are in G.P.

(4) b, c and a are in G.P.

15. Let $a, b, c \in R$. If $f(x) = ax^2 + bx + c$ is such that $a + b + c = 3$ and

$$f(x+y)=f(x)+f(y)+xy,\ \forall\ x, y \in R,$$

then $\sum_{n=1}^{10} f(n)$ is equal to **[JEE (Main)-2017]**

(1) 165 (2) 190

(3) 255 (4) 330

16. Let $a_1, a_2, a_3, \ldots, a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$.

If $a_1^2 + a_2^2 + \ldots + a_{17}^2 = 140m$, then m is equal to **[JEE (Main)-2018]**

(1) 66 (2) 68

(3) 34 (4) 33

17. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series

$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots..$

If $B - 2A = 100\lambda$, then λ is equal to

[JEE (Main)-2018]

(1) 232 (2) 248

(3) 464 (4) 496

18. Let $a_1, a_2 \ldots a_{30}$ be an A.P., $S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{(2i-1)}$. If $a_5 = 27$ and $S - 2T = 75$, then a_{10} is equal to **[JEE (Main)-2019]**

(1) 47 (2) 57

(3) 52 (4) 42

19. If a, b and c be three distinct real numbers in G.P. and $a + b + c = xb$, then x cannot be

[JEE (Main)-2019]

(1) 2 (2) –3

(3) –2 (4) 4

20. The sum of the following series

$$1+6+\frac{9(1^2+2^2+3^2)}{7}+\frac{12(1^2+2^2+3^2+4^2)}{9}+\frac{15(1^2+2^2+\ldots+5^2)}{11}+\ldots$$

up to 15 terms, is **[JEE (Main)-2019]**

(1) 7830 (2) 7820

(3) 7520 (4) 7510

21. Let a, b and c be the 7th, 11th and 13th terms respectively of a non-constant A.P. If these are also the three consecutive terms of a G.P., then $\frac{a}{c}$ is equal to **[JEE (Main)-2019]**

(1) $\frac{1}{2}$ (2) 4

(3) $\frac{7}{13}$ (4) 2

22. Let a_1, a_2, a_3 ... , a_{10} in G.P with $a_i > 0$ for i = 1, 2, ... , 10 and S be the set of pairs (r, k), r, $k \in N$ (the set of natural numbers for which

$$\begin{vmatrix} \log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k \\ \log_e a_4^r a^k & \log_e a_5^r a_6^k & \log_e a_6^r a_7^k \\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k \end{vmatrix} = 0$$

[JEE (Main)-2019]

(1) 2 (2) 10

(3) 4 (4) Infinitely many

23. Let a_1, a_2, ..., a_{10} be a G.P. If $\frac{a_3}{a_1}$ = 25, then $\frac{a_9}{a_5}$ equals **[JEE (Main)-2019]**

(1) 5^3

(2) 5^4

(3) $2(5^2)$

(4) $4(5^2)$

24. The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of this series is **[JEE (Main)-2019]**

(1) $\frac{1}{3}$ (2) $\frac{2}{9}$

(3) $\frac{2}{3}$ (4) $\frac{4}{9}$

25. If 19th term of a non-zero A.P. is zero, then its (49th term) : (29th term) is **[JEE (Main)-2019]**

(1) 2 : 1 (2) 1 : 3

(3) 4 : 1 (4) 3 : 1

26. Let $S_n = 1 + q + q^2 + \ldots + q^n$ and

$$T_n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + \ldots + \left(\frac{q+1}{2}\right)^n$$

where q is a real number and $q \neq 1$.

If ${}^{101}C_1 + {}^{101}C_2 \cdot S_1 + \ldots + {}^{101}C_{101} \cdot S_{100} = \alpha\, T_{100}$.

[JEE (Main)-2019]

(1) 200 (2) 202

(3) 2^{99} (4) 2^{100}

27. The product of three consecutive terms of a G.P. is 512. If 4 is added to each of the first and the second of these terms, the three terms now form an A.P. Then the sum of the original three terms of the given G.P. is **[JEE (Main)-2019]**

(1) 36 (2) 32

(3) 24 (4) 28

28. Let $S_k = \frac{1+2+3+\ldots+k}{k}$. If

$S_1^2 + S_2^2 + \ldots + S_{10}^2 = \frac{5}{12}A$, then A is equal to

[JEE (Main)-2019]

(1) 303 (2) 156

(3) 301 (4) 283

29. If the sum of the first 15 terms of the series $\left(\frac{3}{4}\right)^3 + \left(1\frac{1}{2}\right)^3 + \left(2\frac{1}{4}\right)^3 + 3^3 + \left(3\frac{3}{4}\right)^3 + \ldots$ is equal to 225 k, then k is equal to **[JEE (Main)-2019]**

(1) 108 (2) 27

(3) 9 (4) 54

30. The sum of all natural numbers 'n' such that 100 < n < 200 and H.C.F. (91, n) > 1 is :

[JEE (Main)-2019]

(1) 3303 (2) 3121

(3) 3203 (4) 3221

31. The sum $\sum_{k=1}^{20} k \frac{1}{2^k}$ is equal to **[JEE (Main)-2019]**

(1) $2 - \frac{3}{2^{17}}$ (2) $1 - \frac{11}{2^{20}}$

(3) $2 - \frac{21}{2^{20}}$ (4) $2 - \frac{11}{2^{19}}$

32. Let the sum of the first n terms of a non-constant A.P., a_1, a_2, a_3, ... be $50n + \frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to

[JEE (Main)-2019]

(1) (50, 50 + 46A) (2) (A, 50 + 45A)

(3) (A, 50 + 46A) (4) (50, 50 + 45A)

33. The sum of the series 1 + 2 × 3 + 3 × 5 + 4 × 7 + ... upto 11th term is **[JEE (Main)-2019]**

(1) 916 (2) 946

(3) 945 (4) 915

34. If the sum and product of the first three terms in an A.P. are 33 and 1155, respectively, then a value of its 11th term is **[JEE (Main)-2019]**

(1) –36 (2) 25

(3) –25 (4) –35

35. If a_1, a_2, a_3, ..., a_n are in A.P. and $a_1 + a_4 + a_7 + ... + a_{16} = 114$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to **[JEE (Main)-2019]**

(1) 98 (2) 38

(3) 64 (4) 76

36. The sum

$$\frac{3\times1^3}{1^2}+\frac{5\times(1^3+2^3)}{1^2+2^2}+\frac{7\times(1^3+2^3+3^3)}{1^2+2^2+3^2}+....$$

upto 10th term, is **[JEE (Main)-2019]**

(1) 620 (2) 600

(3) 680 (4) 660

37. The sum $1+\frac{1^3+2^3}{1+2}+\frac{1^3+2^3+3^3}{1+2+3}+...$

$$+\frac{1^3+2^3+3^3+...+15^3}{1+2+3+...+15}-\frac{1}{2}(1+2+3+...+15)$$

is equal to **[JEE (Main)-2019]**

(1) 1860 (2) 620

(3) 660 (4) 1240

38. Let a, b and c be in G.P. with common ratio r, where $a \neq 0$ and $0 < r \le \frac{1}{2}$. If $3a$, $7b$ and $15c$ are the first three terms of an A.P., then the 4th term of this A.P. is **[JEE (Main)-2019]**

(1) $\frac{2}{3}a$ (2) a

(3) $\frac{7}{3}a$ (4) $5a$

39. Let a_1, a_2, a_3, ... be an A.P. with $a_6 = 2$. Then the common difference of this A.P., which maximises the product $a_1 a_4 a_5$, is **[JEE (Main)-2019]**

(1) $\frac{2}{3}$ (2) $\frac{8}{5}$

(3) $\frac{3}{2}$ (4) $\frac{6}{5}$

40. Let S_n denote the sum of the first n terms of an A.P. If $S_4 = 16$ and $S_6 = -48$, then S_{10} is equal to **[JEE (Main)-2019]**

(1) –260 (2) –380

(3) –320 (4) –410

41. If a_1, a_2, a_3, ... are in A.P. such that $a_1 + a_7 + a_{16} = 40$, then the sum of the first 15 terms of this A.P. is **[JEE (Main)-2019]**

(1) 150 (2) 280

(3) 200 (4) 120

42. The greatest positive integer k, for which $49^k + 1$ is a factor of the sum

$49^{125} + 49^{124} + ... + 49^2 + 49 + 1$, is **[JEE (Main)-2020]**

(1) 65 (2) 60

(3) 32 (4) 63

43. Five numbers are in A.P., whose sum is 25 and product is 2520. If one of these five numbers is $-\frac{1}{2}$, then the greatest number amongst them is **[JEE (Main)-2020]**

(1) $\frac{21}{2}$ (2) 7

(3) 27 (4) 16

44. Let a_1, a_2, a_3 ... be a G.P. such that $a_1 < 0$, $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^{9} a_i = 4\lambda$, then λ is equal to **[JEE (Main)-2020]**

(1) –513 (2) –171

(3) $\frac{511}{3}$ (4) 171

45. If the sum of the first 40 terms of the series, $3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + ...$ is (102)m, then m is equal to **[JEE (Main)-2020]**

(1) 5 (2) 20

(3) 25 (4) 10

46. Let $f : R \to R$ be such that for all $x \in R$ $(2^{1+x} + 2^{1-x})$, $f(x)$ and $(3^x + 3^{-x})$ are in A.P., then the minimum value of $f(x)$ is **[JEE (Main)-2020]**

(1) 2 (2) 0

(3) 3 (4) 4

47. If the 10^{th} term of an A.P. is $\frac{1}{20}$ and its 20^{th} term is $\frac{1}{10}$, then the sum of its first 200 terms is **[JEE (Main)-2020]**

(1) $50\frac{1}{4}$ (2) 50

(3) 100 (4) $100\frac{1}{2}$

48. The product

$2^{\frac{1}{4}}.4^{\frac{1}{16}}.8^{\frac{1}{48}}.16^{\frac{1}{128}}.\ ...$ to ∞ is equal to **[JEE (Main)-2020]**

(1) $2^{\frac{1}{2}}$ (2) $2^{\frac{1}{4}}$

(3) 2 (4) 1

49. Let a_n be the n^{th} term of a G.P. of positive terms. If $\sum_{n=1}^{100} a_{2n+1} = 200$ and $\sum_{n=1}^{100} a_{2n} = 100$, then $\sum_{n=1}^{200} a_n$ is equal to **[JEE (Main)-2020]**

(1) 300 (2) 150

(3) 175 (4) 225

50. If $|x| < 1$, $|y| < 1$ and $x \neq y$, then the sum to infinity of the following series $(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + ...$ is **[JEE (Main)-2020]**

(1) $\frac{x+y+xy}{(1+x)(1+y)}$ (2) $\frac{x+y-xy}{(1-x)(1-y)}$

(3) $\frac{x+y-xy}{(1+x)(1+y)}$ (4) $\frac{x+y+xy}{(1-x)(1-y)}$

51. The sum of the first three terms of a G.P. is S and their product is 27. Then all such S lie in **[JEE (Main)-2020]**

(1) $(-\infty, -9] \cup [3, \infty)$

(2) $[-3, \infty)$

(3) $(-\infty, -3] \cup [9, \infty)$

(4) $(-\infty, 9]$

52. If the sum of first 11 terms of an A.P., a_1, a_2, a_3, ... is 0 ($a_1 \neq 0$), then the sum of the A.P., a_1, a_3, a_5, ..., a_{23} is ka_1, where k is equal to **[JEE (Main)-2020]**

(1) $-\frac{121}{10}$ (2) $-\frac{72}{5}$

(3) $\frac{72}{5}$ (4) $\frac{121}{10}$

53. Let S be the sum of the first 9 terms of the series : $\{x + ka\} + \{x^2 + (k + 2)a\} + \{x^3 + (k + 4)a\} + \{x^4 + (k + 6)a\} + ...$ where $a \neq 0$ and $x \neq 1$. If $S = \frac{x^{10} - x + 45a(x-1)}{x-1}$, then k is equal to **[JEE (Main)-2020]**

(1) –3 (2) 1

(3) –5 (4) 3

54. If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is **[JEE (Main)-2020]**

(1) $\frac{1}{6}$ (2) $\frac{1}{4}$

(3) $\frac{1}{7}$ (4) $\frac{1}{5}$

55. If the sum of the series

$20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + ...$ upto n^{th} term is 488 and the n^{th} term is negative, then **[JEE (Main)-2020]**

(1) $n = 41$ (2) n^{th} term is $-4\frac{2}{5}$

(3) $n = 60$ (4) n^{th} term is –4

56. Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x^2 - 6x + q = 0$. If $\alpha, \beta, \gamma, \delta$ form a geometric progression. Then ratio $(2q + p) : (2q - p)$ is **[JEE (Main)-2020]**

(1) 3 : 1 (2) 5 : 3

(3) 9 : 7 (4) 33 : 31

57. If $1 + (1 - 2^2\cdot1) + (1 - 4^2\cdot3) + (1 - 6^2\cdot5) + ... + (1 - 20^2\cdot19) = \alpha - 220\beta$, then an ordered pair (α, β) is equal to **[JEE (Main)-2020]**

(1) (10, 103)

(2) (10, 97)

(3) (11, 97)

(4) (11, 103)

58. Let $a_1, a_2, ...a_n$ be a given A.P. whose common difference is an integer and $S_n = a_1 + a_2 + ... + a_n$. If $a_1 = 1$, $a_n = 300$ and $15 \le n \le 50$, then the ordered pair (S_{n-4}, a_{n-4}) is equal to

[JEE (Main)-2020]

(1) (2490, 249) (2) (2480, 249)

(3) (2490, 248) (4) (2480, 248)

59. If $3^{2\sin 2\alpha - 1}$, 14 and $3^{4-2\sin 2\alpha}$ are the first three terms of an A.P. for some α, then the sixth term of this A.P is **[JEE (Main)-2020]**

(1) 65 (2) 78

(3) 81 (4) 66

60. If $2^{10} + 2^9 \cdot 3^1 + 2^8 \cdot 3^2 + ... + 2 \cdot 3^9 + 3^{10} = S - 2^{11}$, then S is equal to **[JEE (Main)-2020]**

(1) $2 \cdot 3^{11}$ (2) $3^{11} - 2^{12}$

(3) $\dfrac{3^{11}}{2} + 2^{10}$ (4) 3^{11}

61. If the sum of the second, third and fourth terms of a positive term G.P. is 3 and the sum of its sixth, seventh and eighth terms is 243, then the sum of the first 50 terms of this G.P. is

[JEE (Main)-2020]

(1) $\dfrac{2}{13}\left(3^{50} - 1\right)$ (2) $\dfrac{1}{13}\left(3^{50} - 1\right)$

(3) $\dfrac{1}{26}\left(3^{49} - 1\right)$ (4) $\dfrac{1}{26}\left(3^{50} - 1\right)$

62. If the sum of the first 20 terms of the series $\log_{(7^{1/2})} x + \log_{(7^{1/3})} x + \log_{(7^{1/4})} x + ...$ is 460, then x is equal to **[JEE (Main)-2020]**

(1) 7^2 (2) e^2

(3) $7^{1/2}$ (4) $7^{46/21}$

63. Let a, b, c, d and p be any non zero distinct real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) = 0$. Then

[JEE (Main)-2020]

(1) a, c, p are in G.P.

(2) a, b, c, d are in A.P.

(3) a, c, p are in A.P.

(4) a, b, c, d are in G.P.

64. The common difference of the A.P. $b_1, b_2, \ldots, b_m$ is 2 more than the common difference of A.P. $a_1, a_2, \ldots, a_n$. If $a_{40} = -159$, $a_{100} = -399$ and $b_{100} = a_{70}$, then b_1 is equal to **[JEE (Main)-2020]**

(1) –127 (2) –81

(3) 127 (4) 81

65. The sum $\sum_{k=1}^{20}(1 + 2 + 3 + ... + k)$ is ___________.

[JEE (Main)-2020]

66. The sum, $\sum_{n=1}^{7}\dfrac{n(n+1)(2n+1)}{4}$ is equal to

[JEE (Main)-2020]

67. The number of terms common to the two A.P.'s 3, 7, 11, ..., 407 and 2, 9, 16, ..., 709 is ________.

[JEE (Main)-2020]

68. The value of $(0.16)^{\log_{2.5}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots \text{ to } \infty\right)}$ is equal to _________. **[JEE (Main)-2020]**

69. If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4th A.M. is equal to 2nd G.M., then m is equal to _________ . **[JEE (Main)-2020]**

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