

# Three Dimensional Geometry

1. Let the line  $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$  lie in the plane  $x + 3y - \alpha z + \beta = 0$ . Then  $(\alpha, \beta)$  equals [AIEEE-2009]
- (1)  $(-6, 7)$       (2)  $(5, -15)$   
 (3)  $(-5, 5)$       (4)  $(6, -17)$
2. A line  $AB$  in three-dimensional space makes angles  $45^\circ$  and  $120^\circ$  with the positive  $x$ -axis and the positive  $y$ -axis respectively. If  $AB$  makes an acute angle  $\theta$  with the positive  $z$ -axis, then  $\theta$  equals [AIEEE-2010]
- (1)  $30^\circ$       (2)  $45^\circ$   
 (3)  $60^\circ$       (4)  $75^\circ$
3. **Statement-1 :** The point  $A(3, 1, 6)$  is the mirror image of the point  $B(1, 3, 4)$  in the plane  $x - y + z = 5$ .
- Statement-2 :** The plane  $x - y + z = 5$  bisects the line segment joining  $A(3, 1, 6)$  and  $B(1, 3, 4)$ . [AIEEE-2010]
- (1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1  
 (2) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1  
 (3) Statement-1 is true, Statement-2 is false  
 (4) Statement-1 is false, Statement-2 is true
4. There are 10 points in a plane, out of these 6 are collinear. If  $N$  is the number of triangles formed by joining these points, then [AIEEE-2011]
- (1)  $140 < N \leq 190$       (2)  $N > 190$   
 (3)  $N \leq 100$       (4)  $100 < N \leq 140$
5. The length of the perpendicular drawn from the point  $(3, -1, 11)$  to the line  $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  is [AIEEE-2011]
- (1)  $\sqrt{53}$       (2)  $\sqrt{66}$   
 (3)  $\sqrt{29}$       (4)  $\sqrt{33}$
6. The distance of the point  $(1, -5, 9)$  from the plane  $x - y + z = 5$  measured along a straight line  $x = y = z$  is [AIEEE-2011]
- (1)  $3\sqrt{10}$       (2)  $3\sqrt{5}$   
 (3)  $10\sqrt{3}$       (4)  $5\sqrt{3}$
7. An equation of a plane parallel to the plane  $x - 2y + 2z - 5 = 0$  and at a unit distance from the origin is [AIEEE-2012]
- (1)  $x - 2y + 2z + 1 = 0$   
 (2)  $x - 2y + 2z - 1 = 0$   
 (3)  $x - 2y + 2z + 5 = 0$   
 (4)  $x - 2y + 2z - 3 = 0$
8. If the lines  $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$  and  $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$  intersect, then  $k$  is equal to [AIEEE-2012]
- (1)  $\frac{2}{9}$       (2)  $\frac{9}{2}$   
 (3) 0      (4)  $-1$
9. Distance between two parallel planes  $2x + y + 2z = 8$  and  $4x + 2y + 4z + 5 = 0$  is [JEE (Main)-2013]
- (1)  $\frac{3}{2}$       (2)  $\frac{5}{2}$   
 (3)  $\frac{7}{2}$       (4)  $\frac{9}{2}$
10. If the lines  $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$  and  $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$  are coplanar, then  $k$  can have [JEE (Main)-2013]
- (1) Any value  
 (2) Exactly one value  
 (3) Exactly two values  
 (4) Exactly three values

11. The image of the line

$\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5}$  in the plane  $2x - y + z + 3 = 0$   
is the line

[JEE (Main)-2014]

(1)  $\frac{x-3}{3} = \frac{y+5}{1} = \frac{z-2}{-5}$

(2)  $\frac{x-3}{-3} = \frac{y+5}{-1} = \frac{z-2}{5}$

(3)  $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$

(4)  $\frac{x+3}{-3} = \frac{y-5}{-1} = \frac{z+2}{5}$

12. The distance of the point  $(1, 0, 2)$  from the point

of intersection of the line  $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$  and  
the plane  $x - y + z = 16$ , is

[JEE (Main)-2015]

(1)  $2\sqrt{14}$  (2) 8

(3)  $3\sqrt{21}$  (4) 13

13. The equation of the plane containing the line  $2x - 5y + z = 3$ ;  $x + y + 4z = 5$ , and parallel to the plane,  $x + 3y + 6z = 1$ , is

[JEE (Main)-2015]

(1)  $2x + 6y + 12z = 13$

(2)  $x + 3y + 6z = -7$

(3)  $x + 3y + 6z = 7$

(4)  $2x + 6y + 12z = -13$

14. If the line,  $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$  lies in the plane,

$|lx + my - z| = 9$ , then  $l^2 + m^2$  is equal to

[JEE (Main)-2016]

(1) 18 (2) 5

(3) 2 (4) 26

15. The distance of the point  $(1, -5, 9)$  from the plane  $x - y + z = 5$  measured along the line  $x = y = z$  is

[JEE (Main)-2016]

(1)  $10\sqrt{3}$  (2)  $\frac{10}{\sqrt{3}}$

(3)  $\frac{20}{3}$  (4)  $3\sqrt{10}$

16. The distance of the point  $(1, 3, -7)$  from the plane passing through the point  $(1, -1, -1)$ , having normal perpendicular to both the lines

$\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-4}{3}$  and  $\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z+7}{-1}$ , is

[JEE (Main)-2017]

(1)  $\frac{10}{\sqrt{83}}$

(2)  $\frac{5}{\sqrt{83}}$

(3)  $\frac{10}{\sqrt{74}}$

(4)  $\frac{20}{\sqrt{74}}$

17. If the image of the point  $P(1, -2, 3)$  in the plane,  $2x + 3y - 4z + 22 = 0$  measured parallel to the

line,  $\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$  is  $Q$ , then  $PQ$  is equal to

[JEE (Main)-2017]

(1)  $2\sqrt{42}$

(2)  $\sqrt{42}$

(3)  $6\sqrt{5}$

(4)  $3\sqrt{5}$

18. If  $L_1$  is the line of intersection of the planes  $2x - 2y + 3z - 2 = 0$ ,  $x - y + z + 1 = 0$  and  $L_2$  is the line of intersection of the planes  $x + 2y - z - 3 = 0$ ,  $3x - y + 2z - 1 = 0$ , then the distance of the origin from the plane containing the lines  $L_1$  and  $L_2$ , is

[JEE (Main)-2018]

(1)  $\frac{1}{4\sqrt{2}}$

(2)  $\frac{1}{3\sqrt{2}}$

(3)  $\frac{1}{2\sqrt{2}}$

(4)  $\frac{1}{\sqrt{2}}$

19. The length of the projection of the line segment joining the points  $(5, -1, 4)$  and  $(4, -1, 3)$  on the plane,  $x + y + z = 7$  is:

[JEE (Main)-2018]

(1)  $\frac{2}{\sqrt{3}}$

(2)  $\frac{2}{3}$

(3)  $\frac{1}{3}$

(4)  $\frac{\sqrt{2}}{3}$

20. The plane through the intersection of the planes  $x + y + z = 1$  and  $2x + 3y - z + 4 = 0$  and parallel to  $y$ -axis also passes through the point

[JEE (Main)-2019]

(1)  $(3, 2, 1)$

(2)  $(3, 3, -1)$

(3)  $(-3, 0, -1)$

(4)  $(-3, 1, 1)$





42. Let  $P$  be the plane, which contains the line of intersection of the planes,  $x + y + z - 6 = 0$  and  $2x + 3y + z + 5 = 0$  and it is perpendicular to the  $xy$ -plane. Then the distance of the point  $(0, 0, 256)$  from  $P$  is equal to [JEE (Main)-2019]

(1) $63\sqrt{5}$	(2) $\frac{17}{\sqrt{5}}$
(3) $205\sqrt{5}$	(4) $\frac{11}{\sqrt{5}}$

43. If the length of the perpendicular from the point  $(\beta, 0, \beta)$  ( $\beta \neq 0$ ) to the line,  $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1}$  is  $\frac{\sqrt{3}}{2}$ , then  $\beta$  is equal to [JEE (Main)-2019]

(1) -1	(2) -2
(3) 1	(4) 2

44. Let  $A(3, 0, -1)$ ,  $B(2, 10, 6)$  and  $C(1, 2, 1)$  be the vertices of a triangle and  $M$  be the mid point of  $AC$ . If  $G$  divides  $BM$  in the ratio,  $2 : 1$  then  $\cos(\angle GOA)$  ( $O$  being the origin) is equal to [JEE (Main)-2019]

(1) $\frac{1}{6\sqrt{10}}$	(2) $\frac{1}{\sqrt{30}}$
(3) $\frac{1}{2\sqrt{15}}$	(4) $\frac{1}{\sqrt{15}}$

45. If  $Q(0, -1, -3)$  is the image of the point  $P$  in the plane  $3x - y + 4z = 2$  and  $R$  is the point  $(3, -1, -2)$ , then the area (in sq. units) of  $\triangle PQR$  is :

[JEE (Main)-2019]

(1) $\frac{\sqrt{65}}{2}$	(2) $\frac{\sqrt{91}}{4}$
(3) $2\sqrt{13}$	(4) $\frac{\sqrt{91}}{2}$

46. A perpendicular is drawn from a point on the line  $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{1}$  to the plane  $x + y + z = 3$  such that the foot of the perpendicular  $Q$  also lies on the plane  $x - y + z = 3$ . Then the co-ordinates of  $Q$  are [JEE (Main)-2019]

(1) $(1, 0, 2)$	(2) $(2, 0, 1)$
(3) $(4, 0, -1)$	(4) $(-1, 0, 4)$

47. If the plane  $2x - y + 2z + 3 = 0$  has the distances  $\frac{1}{3}$  and  $\frac{2}{3}$  units from the planes  $4x - 2y + 4z + \lambda = 0$  and  $2x - y + 2z + \mu = 0$ , respectively, then the maximum value of  $\lambda + \mu$  is equal to [JEE (Main)-2019]

(1) 13	(2) 15
(3) 5	(4) 9

48. If the line  $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$  intersects the plane  $2x + 3y - z + 13 = 0$  at a point  $P$  and the plane  $3x + y + 4z = 16$  at a point  $Q$ , then  $PQ$  is equal to [JEE (Main)-2019]

(1) $2\sqrt{14}$	(2) 14
(3) $2\sqrt{7}$	(4) $\sqrt{14}$

49. The length of the perpendicular drawn from the point  $(2, 1, 4)$  to the plane containing the lines  $\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$  and

$$\vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + \hat{j} - 2\hat{k})$$

(1) $\frac{1}{3}$	(2) 3
(3) $\frac{1}{\sqrt{3}}$	(4) $\sqrt{3}$

50. Let  $P$  be a plane through the points  $(2, 1, 0)$ ,  $(4, 1, 1)$  and  $(5, 0, 1)$  and  $R$  be any point  $(2, 1, 6)$ . Then the image of  $R$  in the plane  $P$  is

[JEE (Main)-2020]

(1) $(6, 5, 2)$	(2) $(4, 3, 2)$
(3) $(3, 4, -2)$	(4) $(6, 5, -2)$

51. The shortest distance between the lines

$$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1} \text{ and } \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{2}$$

[JEE (Main)-2020]

(1) $2\sqrt{30}$	(2) $\frac{7}{2}\sqrt{30}$
(3) $3\sqrt{30}$	(4) 3

52. The mirror image of the point  $(1, 2, 3)$  in a plane is  $\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$ . Which of the following points lies on this plane? [JEE (Main)-2020]

(1) $(-1, -1, -1)$	(2) $(1, 1, 1)$
(3) $(-1, -1, 1)$	(4) $(1, -1, 1)$

53. The plane passing through the points  $(1, 2, 1)$ ,  $(2, 1, 2)$  and parallel to the line,  $2x = 3y, z = 1$  also passes through the point [JEE (Main)-2020]  
 (1)  $(0, 6, -2)$       (2)  $(-2, 0, 1)$   
 (3)  $(0, -6, 2)$       (4)  $(2, 0, -1)$
54. A plane passing through the point  $(3, 1, 1)$  contains two lines whose direction ratios are  $1, -2, 2$  and  $2, 3, -1$  respectively. If this plane also passes through the point  $(\alpha, -3, 5)$ , then  $\alpha$  is equal to [JEE (Main)-2020]  
 (1) 5      (2) 10  
 (3) -10      (4) -5
55. The foot of the perpendicular drawn from the point  $(4, 2, 3)$  to the line joining the points  $(1, -2, 3)$  and  $(1, 1, 0)$  lies on the plane [JEE (Main)-2020]  
 (1)  $x - 2y + z = 1$       (2)  $x + 2y - z = 1$   
 (3)  $x - y - 2z = 1$       (4)  $2x + y - z = 1$
56. The lines  $\vec{r} = (\hat{i} - \hat{j}) + l(2\hat{i} + \hat{k})$  and  $\vec{r} = (2\hat{i} - \hat{j}) + m(\hat{i} + \hat{j} - \hat{k})$  [JEE (Main)-2020]  
 (1) do not intersect for any values of  $l$  and  $m$   
 (2) intersect for all values of  $l$  and  $m$   
 (3) intersect when  $l = 2$  and  $m = \frac{1}{2}$   
 (4) intersect when  $l = 1$  and  $m = 2$
57. The plane which bisects the line joining the points  $(4, -2, 3)$  and  $(2, 4, -1)$  at right angles also passes through the point [JEE (Main)-2020]  
 (1)  $(4, 0, 1)$       (2)  $(0, -1, 1)$   
 (3)  $(0, 1, -1)$       (4)  $(4, 0, -1)$
58. The distance of the point  $(1, -2, 3)$  from the plane  $x - y + z = 5$  measured parallel to the line  $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$  is [JEE (Main)-2020]  
 (1) 7      (2)  $\frac{7}{5}$   
 (3) 1      (4)  $\frac{1}{7}$
59. If  $(a, b, c)$  is the image of the point  $(1, 2, -3)$  in the line,  $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$ , then  $a + b + c$  is equal to [JEE (Main)-2020]  
 (1) 2      (2) 1  
 (3) 3      (4) -1
60. If for some  $\alpha \in R$ , the lines  $L_1 : \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$  and  $L_2 : \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$  are coplanar, then the line  $L_2$  passes through the point [JEE (Main)-2020]  
 (1)  $(10, 2, 2)$       (2)  $(2, -10, -2)$   
 (3)  $(10, -2, -2)$       (4)  $(-2, 10, 2)$
61. The shortest distance between the lines  $\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$  and  $x + y + z + 1 = 0, 2x - y + z + 3 = 0$  is [JEE (Main)-2020]  
 (1)  $\frac{1}{\sqrt{2}}$       (2) 1  
 (3)  $\frac{1}{\sqrt{3}}$       (4)  $\frac{1}{2}$
62. A plane  $P$  meets the coordinate axes at  $A, B$  and  $C$  respectively. The centroid of  $\Delta ABC$  is given to be  $(1, 1, 2)$ . Then the equation of the line through this centroid and perpendicular to the plane  $P$  is [JEE (Main)-2020]  
 (1)  $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$   
 (2)  $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-2}{2}$   
 (3)  $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$   
 (4)  $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-2}{1}$
63. If the foot of the perpendicular drawn from the point  $(1, 0, 3)$  on a line passing through  $(\alpha, 7, 1)$  is  $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$ , then  $\alpha$  is equal to [JEE (Main)-2020]

64. The projection of the line segment joining the points  $(1, -1, 3)$  and  $(2, -4, 11)$  on the line joining the points  $(-1, 2, 3)$  and  $(3, -2, 10)$  is \_\_\_\_\_.
- [JEE (Main)-2020]

65. If the distance between the plane,  $23x - 10y - 2z + 48 = 0$  and the plane containing the lines

$$\frac{x+1}{2} = \frac{y-3}{4} = \frac{z+1}{3} \text{ and}$$

$$\frac{x+3}{2} = \frac{y+2}{6} = \frac{z-1}{\lambda} (\lambda \in R)$$

is equal to  $\frac{k}{\sqrt{633}}$ , then  $k$  is equal to

[JEE (Main)-2020]

66. Let a plane  $P$  contain two lines

$$\vec{r} = \hat{i} + \lambda(\hat{i} + \hat{j}), \lambda \in R \text{ and}$$

$$\vec{r} = -\hat{j} + \mu(\hat{j} - \hat{k}), \mu \in R$$

If  $Q(\alpha, \beta, \gamma)$  is the foot of the perpendicular drawn from the point  $M(1, 0, 1)$  to  $P$ , then  $3(\alpha + \beta + \gamma)$  equals \_\_\_\_\_.

[JEE (Main)-2020]

67. If the equation of a plane  $P$ , passing through the intersection of the planes  $x + 4y - z + 7 = 0$  and  $3x + y + 5z = 8$  is  $ax + by + 6z = 15$  for some  $a, b \in R$ , then the distance of the point  $(3, 2, -1)$  from the plane  $P$  is \_\_\_\_\_.
- [JEE (Main)-2020]

