

15

Similarity

(With Applications to Maps and Models)

15.1 Introduction :

1. Similarity of Figures : Two figures are said to be similar, if they have the *same shape* but may *differ in size*.

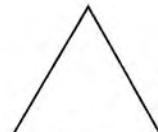
Examples :



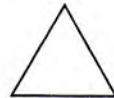
and



(ii)



and



- (a) *Same shape means* : Angles of one figure are equal to corresponding angles of other figure each to each.
- (b) *Same size means* : Sides of one figure are equal to corresponding sides of the other figure each to each.

2. Congruency of Figures : Two figures are said to be *congruent*, if they have the *same shape* and the *same size*.

Congruent figures are always *similar*, whereas *similar figures* are not necessarily *congruent*.

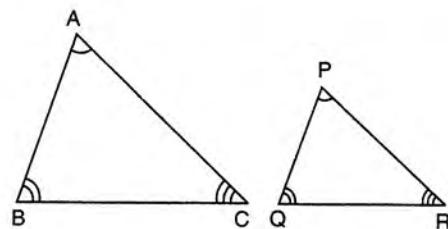
15.2 Similar Triangles :

Two triangles are said to be *similar*, if their corresponding angles are *equal* and corresponding sides are *proportional* (*i.e.* the ratios between the lengths of corresponding sides are equal).

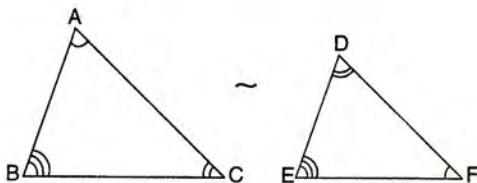
e.g. Triangles ABC and PQR are similar, if :

- (a) their corresponding angles are equal
i.e. $\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$
- (b) their corresponding sides are in proportion

$$\text{i.e. } \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}.$$



Symbolically, we write : $\Delta ABC \sim \Delta PQR$; where symbol \sim is read as, “is similar to”. In the same way;

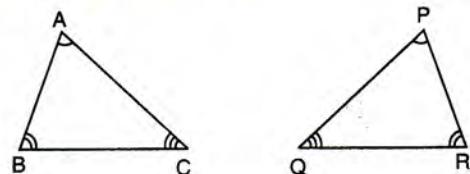


$$\Leftrightarrow \begin{cases} \angle A = \angle D, \angle B = \angle E \text{ and } \angle C = \angle F. \\ \text{Also, } \frac{AB}{DE} = \frac{BC}{DF} = \frac{AC}{DF} \end{cases}$$

15.3 Corresponding sides and corresponding angles :

1. In similar triangles, the *sides opposite to equal angles* are said to be the *corresponding sides*.

e.g. the adjoining figure shows, $\Delta ABC \sim \Delta PRQ$ in which $\angle A = \angle P$, $\angle B = \angle R$ and $\angle C = \angle Q$.



- \therefore (i) $\angle A = \angle P$ \Rightarrow Sides opposite to $\angle A$ and $\angle P$ are the corresponding sides.
 \Rightarrow Sides BC and QR are the corresponding sides.
- (ii) $\angle B = \angle R$ \Rightarrow Sides opposite to $\angle B$ and $\angle R$ are the corresponding sides
 \Rightarrow Sides AC and PQ are the corresponding sides.

Similarly, (iii) $\angle C = \angle Q \Rightarrow$ Sides AB and PR are the corresponding sides.

As, the triangles ABC and PRQ are similar; we have :

$$\frac{BC}{QR} = \frac{AC}{PQ} = \frac{AB}{PR} \quad [\text{Corresponding sides of similar triangles are proportional}]$$

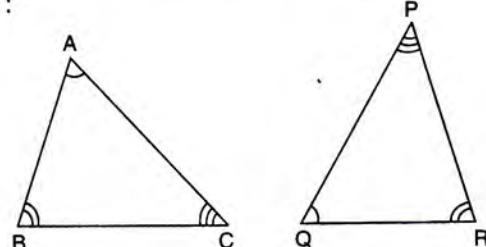
In the same way, if in ΔABC and ΔPQR :

$$\angle A = \angle Q, \angle B = \angle R \text{ and } \angle C = \angle P;$$

the two triangles are similar

i.e. ΔABC is similar to ΔQRP

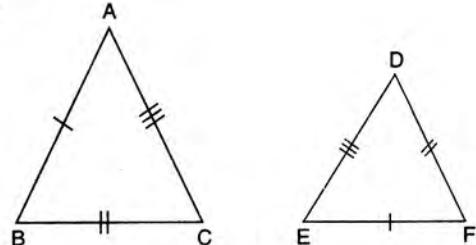
$$\begin{aligned} \text{And, } \frac{\text{side opp. to } \angle A}{\text{side opp. to } \angle Q} &= \frac{\text{side opp. to } \angle B}{\text{side opp. to } \angle R} \\ &= \frac{\text{side opp. to } \angle C}{\text{side opp. to } \angle P} \Rightarrow \frac{BC}{PR} = \frac{AC}{PQ} = \frac{AB}{QR} \end{aligned}$$



2. In similar triangles, the *angles opposite to proportional sides* are the corresponding angles and so, they are equal.

e.g. in similar triangles ABC and EFD,

$$\text{if } \frac{AB}{EF} = \frac{BC}{DF} = \frac{AC}{DE}, \text{ then}$$



- (i) ratio $\frac{AB}{EF} \Rightarrow$ angle opposite to side AB = angle opposite to side EF
i.e. $\angle C = \angle D$,

- (ii) ratio $\frac{BC}{DF} \Rightarrow$ angle opposite to side BC = angle opposite to side DF
i.e. $\angle A = \angle E$ and

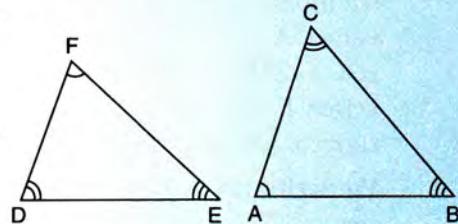
- (iii) ratio $\frac{AC}{DE} \Rightarrow$ angle opposite to side AC = angle opposite to side DE
i.e. $\angle B = \angle F$.

1. Please note that *in congruent triangles* the *corresponding sides* are equal, whereas *in similar triangles*, the *corresponding sides* are in *proportion*.

e.g. in the adjoining figures; if $\triangle ABC$ and $\triangle FED$ are congruent, then $BC = DE$, $AB = EF$ and $AC = DF$.

But, if $\triangle ABC$ and $\triangle FED$ are similar, then

$$\frac{BC}{DE} = \frac{AB}{EF} = \frac{AC}{DF}$$



2. Triangles, which are similar to the same triangle, are similar to each other also.

15.4 Condition of Similar Triangle : (SAS, AA or AAA and SSS)

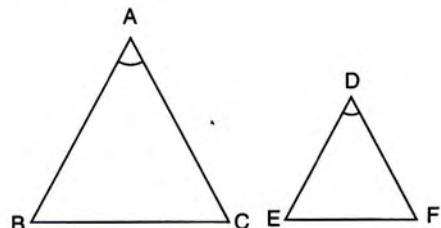
1. If one angle of a triangle is equal to any angle of the other triangle and in both the triangles, the sides including the equal angles are in proportion; then the triangles are similar. (SAS postulate)

Example :

If in $\triangle ABC$ and $\triangle DEF$,

$$\angle A = \angle D \text{ and } \frac{AB}{DE} = \frac{AC}{DF}$$

then by SAS, $\triangle ABC \sim \triangle DEF$.



Similarly, if $\angle B = \angle E$ and $\frac{AB}{DE} = \frac{BC}{EF}$, then also $\triangle ABC \sim \triangle DEF$ and so on.

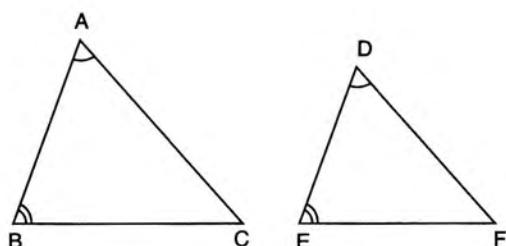
2. If two triangles have atleast two pairs of corresponding angles equal; the triangles are similar. (AA or AAA postulate)

Example :

If in $\triangle ABC$ and $\triangle DEF$,

$$\angle A = \angle D \text{ and } \angle B = \angle E, \triangle ABC \sim \triangle DEF.$$

Since, the sum of the angles of a triangle is 180° , therefore if two angles of one triangle are equal to two angles of another triangle each to each, their third angles are also equal.



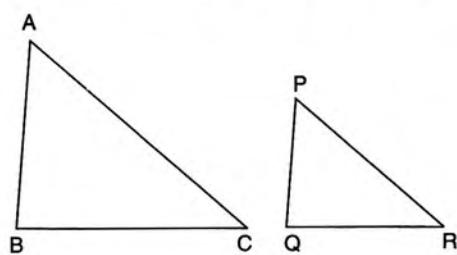
3. If two triangles have their three pairs of corresponding sides proportional, then the triangles are similar. (SSS postulate)

Example :

If in $\triangle ABC$ and $\triangle PQR$,

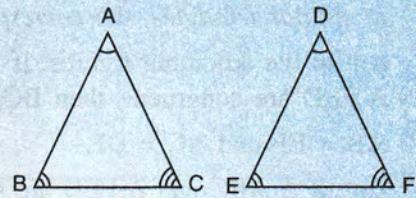
$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

$\triangle ABC \sim \triangle PQR$.



Important :

- The given figure shows two similar triangles ABC and DEF such that vertex A corresponds to vertex D (as, $\angle A = \angle D$); vertex B corresponds to vertex E (as, $\angle B = \angle E$) and, vertex C corresponds to vertex F (as, $\angle C = \angle F$).



We write : $\Delta ABC \sim \Delta DEF$ and not $\Delta ABC \sim \Delta DFE$ or $\Delta BAC \sim \Delta DEF$, etc.

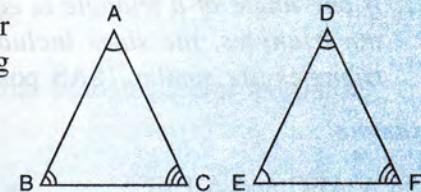
In fact, the order of vertices of two similar triangles must be written in such a way that the corresponding vertices occupy the same position.

Thus, triangle ABC is similar to triangle DEF

$$\Rightarrow \Delta ABC \sim \Delta DEF \quad [A \leftrightarrow D, B \leftrightarrow E \text{ and } C \leftrightarrow F]$$

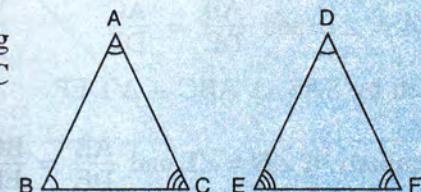
- The adjoining figure shows two similar triangles such that, the corresponding vertices are:
 $A \leftrightarrow E$; $B \leftrightarrow D$ and $C \leftrightarrow F$.

$$\therefore \Delta ABC \sim \Delta EDF$$



- In the adjoining figure, the corresponding vertices are : A and F, B and D and C and E.

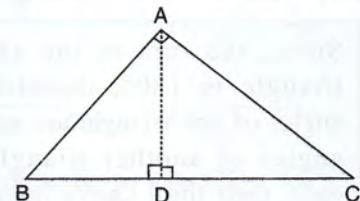
$$\therefore \Delta ABC \sim \Delta FDE$$



- 1 A perpendicular drawn from the vertex of the right angle of a right-angled triangle divides the triangle into two triangles similar to each other and also to the original triangle. Prove it.

Solution :

Let ABC be a right angled triangle, right angled at vertex A and AD is perpendicular drawn to the hypotenuse BC.



To Prove : $\Delta DBA \sim \Delta DAC \sim \Delta ABC$

Proof :

Statement :

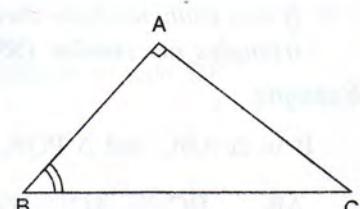
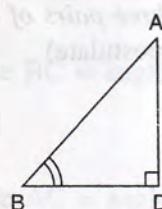
- In ΔDBA and ΔABC :

$$(i) \angle ABD = \angle ABC \quad [\text{Common}]$$

$$(ii) \angle ADB = \angle BAC \quad [\text{Each } 90^\circ]$$

$$\therefore \Delta DBA \sim \Delta ABC \quad [\text{AA or AAA postulate}]$$

Reason :

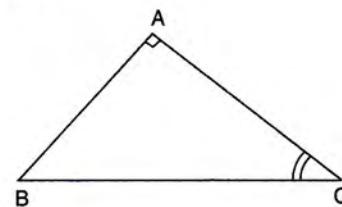
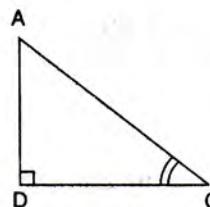


2. In $\triangle DAC$ and $\triangle ABC$:

(i) $\angle ACD = \angle ACB$ [Common]

(ii) $\angle ADC = \angle BAC$ [Each 90°]

$\therefore \triangle DAC \sim \triangle ABC$ [AA or AAA postulate]



3. $\therefore \triangle DBA \sim \triangle DAC \sim \triangle ABC$

[From 1 and 2]

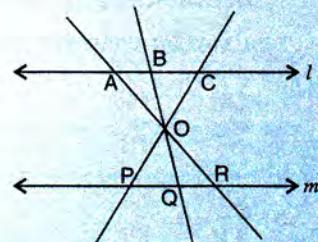
Hence Proved.

2

In the given figure, lines l and m are parallel.

Three concurrent lines through point O meet line l at points A, B and C ; and line m at points P, Q and R as shown.

$$\text{Prove that : } \frac{AB}{BC} = \frac{QR}{PQ}.$$



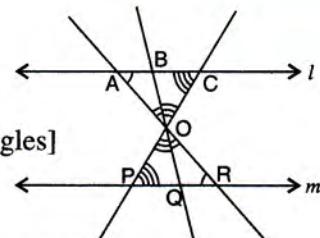
Solution :

Since, line l is parallel to line m and AR is transversal

$$\therefore \angle OAB = \angle ORQ \quad [\text{Alternate angles}]$$

$$\text{Also, } \angle AOB = \angle QOR \quad [\text{Vertically opposite angles}]$$

$$\therefore \triangle OAB \sim \triangle ORQ \quad [\text{By A.A. postulate}]$$



$$\Rightarrow \frac{AB}{QR} = \frac{OB}{OQ} \dots \text{I} \quad [\text{Corresponding sides of similar triangles are in proportion}]$$

$$\text{Similarly, } \angle OCB = \angle OPQ \quad [\text{Alternate angles}]$$

$$\text{and, } \angle BOC = \angle POQ \quad [\text{Vertically opposite angles}]$$

$$\therefore \triangle BOC \sim \triangle QOP \quad [\text{By A.A. postulate}]$$

$$\Rightarrow \frac{BC}{PQ} = \frac{OB}{OQ} \dots \text{II} \quad [\text{Corresponding sides of similar } \Delta\text{s}]$$

From equations I and II, we get :

$$\frac{AB}{QR} = \frac{BC}{PQ}$$

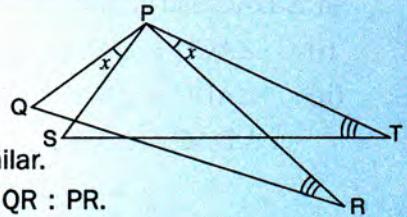
$$\Rightarrow \frac{AB}{BC} = \frac{QR}{PQ}$$

Hence proved.

- 3** In the figure, given alongside,

$\angle QPS = \angle RPT$
and $\angle PRQ = \angle PTS$.

- (i) Prove that triangles PQR and PST are similar.
(ii) If $PT : ST = 3 : 4$; find the ratio between $QR : PR$.



Solution :

- (i) In $\triangle PQR$ and $\triangle PST$,

1. $\angle QPR = \angle SPT$ [Each equal to $\angle x + \angle SPR$]
 2. $\angle PRQ = \angle PTS$ [Given]
- $\therefore \triangle PQR \sim \triangle PST$ [AA postulate]

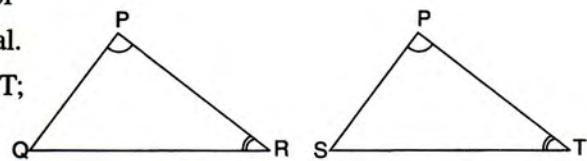
- (ii) Since the corresponding sides of similar triangles are proportional.

\therefore In similar triangles PQR and PST;

$$\frac{QR}{ST} = \frac{PR}{PT}$$

$$\Rightarrow \frac{QR}{PR} = \frac{ST}{PT} = \frac{4}{3}$$

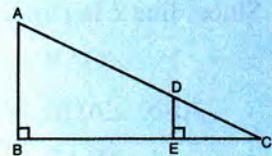
$$\Rightarrow QR : PR = 4 : 3$$



$$[\text{Given, } \frac{PT}{ST} = \frac{3}{4} \Rightarrow \frac{ST}{PT} = \frac{4}{3}]$$

Ans.

- 4** In the given figure, AB and DE are perpendiculars to BC. If $AB = 9\text{ cm}$, $DE = 3\text{ cm}$ and $AC = 24\text{ cm}$, calculate AD. [2005]



Solution :

In $\triangle ABC$ and $\triangle DEC$:

- (i) $\angle ABC = \angle DEC$ [Each 90°]

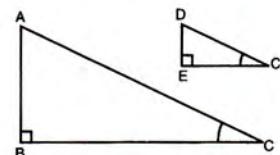
- (ii) $\angle C$ is common

$\therefore \triangle ABC \cong \triangle DEC$ [By A.A.]

$$\Rightarrow \frac{AC}{DC} = \frac{AB}{DE} \quad i.e. \quad \frac{24\text{ cm}}{DC} = \frac{9\text{ cm}}{3\text{ cm}}$$

$$\Rightarrow DC = \frac{24 \times 3}{9} \text{ cm} = 8 \text{ cm}$$

$$\therefore AD = AC - DC = 24 \text{ cm} - 8 \text{ cm} = 16 \text{ cm}$$

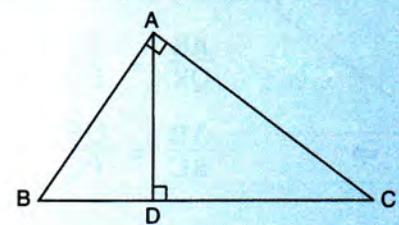


Ans.

- 5** In the adjoining figure, ABC is a triangle right-angled at vertex A and AD is altitude.

- (i) Prove that : $\triangle ABD$ is similar to $\triangle CAD$.

- (ii) If $BD = 3.6\text{ cm}$ and $CD = 6.4\text{ cm}$; find the length of AD.



Solution :

(i) In $\triangle CAD$, let $\angle C = x$

$$\Rightarrow \angle CAD = 90^\circ - x$$

In $\triangle ABC$,

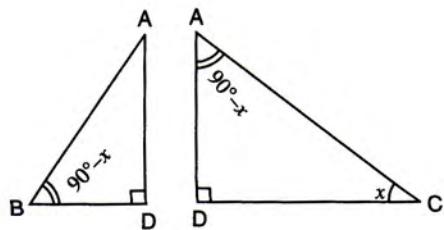
$$\angle C = x \Rightarrow \angle B = 90^\circ - x$$

Now in $\triangle ABD$ and $\triangle CAD$,

$$1. \angle ADB = \angle CDA \quad [\text{Each is } 90^\circ]$$

$$2. \angle ABD = \angle CAD \quad [\text{Each is } 90^\circ - x]$$

$\therefore \triangle ABD \sim \triangle CAD$ [By AA]



Hence Proved.

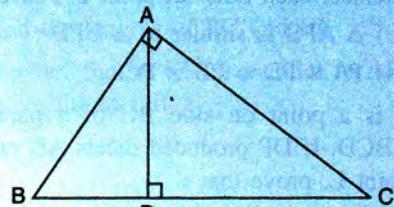
(ii) Since $\triangle ABD \sim \triangle CAD \Rightarrow \frac{AD}{CD} = \frac{BD}{AD}$

$$\text{i.e. } AD^2 = BD \times CD = 3.6 \times 6.4$$

$$\Rightarrow \text{AD} = 4.8 \text{ cm}$$

Ans.

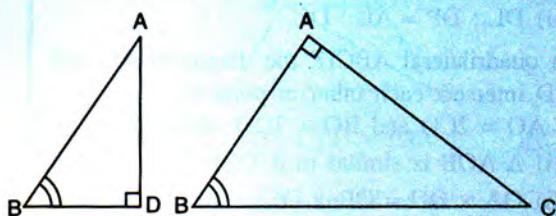
In $\triangle ABC$, with $\angle A = 90^\circ$ and $AD \perp BC$, we get three pairs of similar triangles.



1. $\triangle ABD \sim \triangle CBA$ [By AA]

$$\Rightarrow \frac{AB}{BC} = \frac{BD}{AB}$$

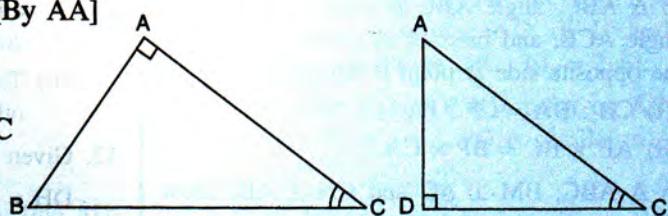
$$\Rightarrow AB^2 = BD \times BC$$



2. $\triangle ABC \sim \triangle DAC$ [By AA]

$$\Rightarrow \frac{AC}{DC} = \frac{BC}{AC}$$

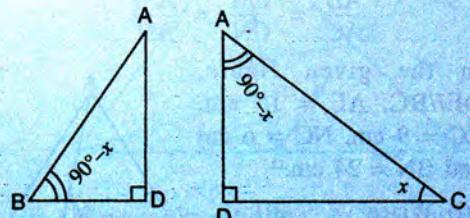
$$\Rightarrow AC^2 = DC \times BC$$



3. $\triangle BAD \sim \triangle ACD$ [By AA]

$$\Rightarrow \frac{AD}{DC} = \frac{BD}{AD}$$

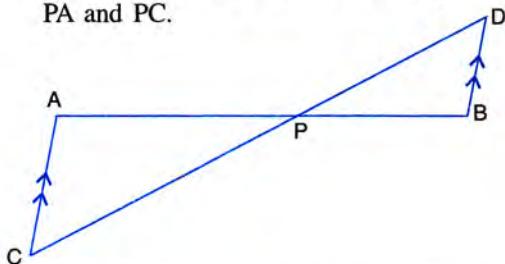
$$\Rightarrow AD^2 = BD \times DC$$



EXERCISE 15(A)

1. In the figure, given below, straight lines AB and CD intersect at P; and AC // BD. Prove that :

- (i) $\triangle APC$ and $\triangle BPD$ are similar.
- (ii) If $BD = 2.4$ cm, $AC = 3.6$ cm, $PD = 4.0$ cm and $PB = 3.2$ cm; find the lengths of PA and PC.



2. In a trapezium ABCD, side AB is parallel to side DC; and the diagonals AC and BD intersect each other at point P. Prove that :

- (i) $\triangle APB$ is similar to $\triangle CPD$.
- (ii) $PA \times PD = PB \times PC$.

3. P is a point on side BC of a parallelogram ABCD. If DP produced meets AB produced at point L, prove that :

- (i) $DP : PL = DC : BL$.
- (ii) $DL : DP = AL : DC$.

4. In quadrilateral ABCD, the diagonals AC and BD intersect each other at point O.

If $AO = 2CO$ and $BO = 2DO$; show that :

- (i) $\triangle AOB$ is similar to $\triangle COD$.
- (ii) $OA \times OD = OB \times OC$.

5. In $\triangle ABC$, angle ABC is equal to twice the angle ACB, and bisector of angle ABC meets the opposite side at point P. Show that :

- (i) $CB : BA = CP : PA$
- (ii) $AB \times BC = BP \times CA$

6. In $\triangle ABC$; $BM \perp AC$ and $CN \perp AB$; show that :

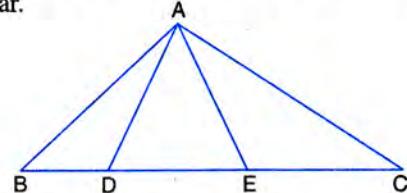
$$\frac{AB}{AC} = \frac{BM}{CN} = \frac{AM}{AN}$$

7. In the given figure, $DE \parallel BC$, $AE = 15$ cm, $EC = 9$ cm, $NC = 6$ cm and $BN = 24$ cm.

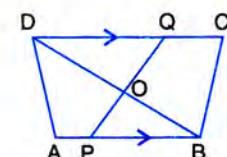
- (i) Write all possible pairs of similar triangles.
- (ii) Find lengths of ME and DM.

8. In the given figure, $AD = AE$ and $AD^2 = BD \times EC$.

Prove that : triangles ABD and CAE are similar.



9. In the given figure, $AB \parallel DC$, $BO = 6$ cm and $DQ = 8$ cm; find: $BP \times DO$.



10. Angle BAC of triangle ABC is obtuse and $AB = AC$. P is a point in BC such that $PC = 12$ cm. PQ and PR are perpendiculars to sides AB and AC respectively. If $PQ = 15$ cm and $PR = 9$ cm; find the length of PB.

11. State, true or false :

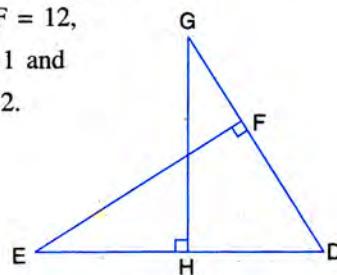
- (i) Two similar polygons are necessarily congruent.
- (ii) Two congruent polygons are necessarily similar.
- (iii) All equiangular triangles are similar.
- (iv) All isosceles triangles are similar.
- (v) Two isosceles-right triangles are similar.
- (vi) Two isosceles triangles are similar, if an angle of one is congruent to the corresponding angle of the other.
- (vii) The diagonals of a trapezium divide each other into proportional segments.

12. Given : $\angle GHE = \angle DFE = 90^\circ$,

$$DH = 8, DF = 12,$$

$$DG = 3x - 1 \text{ and}$$

$$DE = 4x + 2.$$

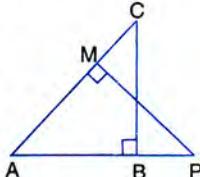


Find : the lengths of segments DG and DE.

13. D is a point on the side BC of triangle ABC such that angle ADC is equal to angle BAC. Prove that : $CA^2 = CB \times CD$.

14. In the given figure, $\triangle ABC$ and $\triangle AMP$ are right angled at B and M respectively.

Given AC = 10 cm, AP = 15 cm and PM = 12 cm.

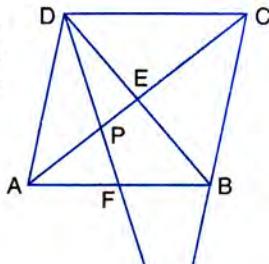


- (i) Prove that : $\triangle ABC \sim \triangle AMP$
(ii) Find : AB and BC. [2012]

15. Given : RS and PT are altitudes of $\triangle PQR$.
Prove that :

- (i) $\triangle PQT \sim \triangle QRS$,
(ii) $PQ \times QS = RQ \times QT$.

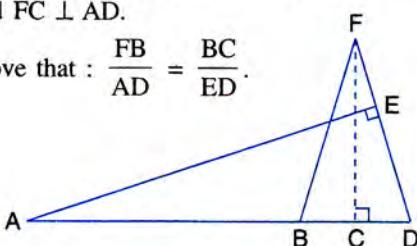
16. Given : ABCD is a rhombus, DPR and CBR are straight lines.



Prove that :
 $DP \times CR = DC \times PR$.

17. Given : $FB = FD$, $AE \perp FD$ and $FC \perp AD$.

Prove that : $\frac{FB}{AD} = \frac{BC}{ED}$.



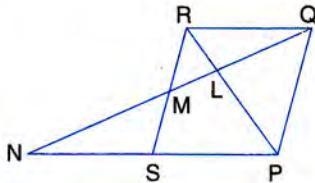
18. In $\triangle PQR$, $\angle Q = 90^\circ$ and QM is perpendicular to PR. Prove that :

- (i) $PQ^2 = PM \times PR$
(ii) $QR^2 = PR \times MR$
(iii) $PQ^2 + QR^2 = PR^2$

19. In $\triangle ABC$, $\angle B = 90^\circ$ and $BD \perp AC$.

- (i) If $CD = 10$ cm and $BD = 8$ cm; find AD .
(ii) If $AC = 18$ cm and $AD = 6$ cm; find BD .
(iii) If $AC = 9$ cm and $AB = 7$ cm; find AD .

20. In the figure, PQRS is a parallelogram with $PQ = 16$ cm and $QR = 10$ cm. L is a point



on PR such that $RL : LP = 2 : 3$. QL produced meets RS at M and PS produced at N.

Find the lengths of PN and RM.

21. In quadrilateral ABCD, diagonals AC and BD intersect at point E such that

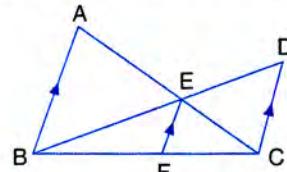
$$AE : EC = BE : ED.$$

Show that : ABCD is a trapezium.

22. In triangle ABC, AD is perpendicular to side BC and $AD^2 = BD \times DC$.

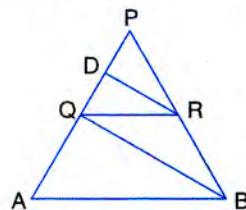
Show that angle BAC = 90° .

23. In the given figure, $AB \parallel EF \parallel DC$; $AB = 67.5$ cm, $DC = 40.5$ cm and $AE = 52.5$ cm.



- (i) Name the three pairs of similar triangles.
(ii) Find the lengths of EC and EF.

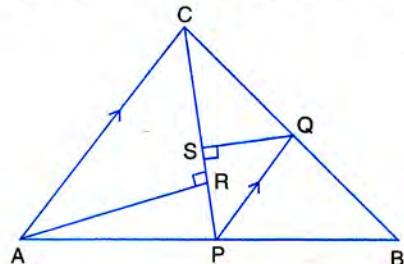
24. In the given figure, QR is parallel to AB and DR is parallel to QB.



Prove that : $PQ^2 = PD \times PA$.

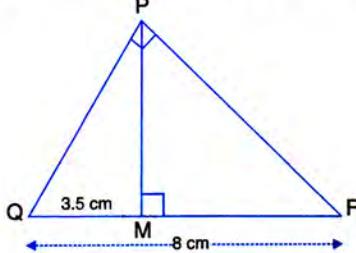
25. Through the mid-point M of the side CD of a parallelogram ABCD, the line BM is drawn intersecting diagonal AC in L and AD produced in E. Prove that : $EL = 2 BL$.

26. In the given figure, P is a point on AB such that $AP : PB = 4 : 3$. PQ is parallel to AC.



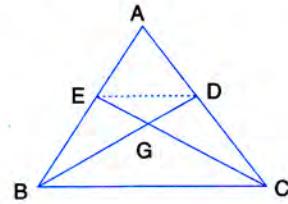
- (i) Calculate the ratio $PQ : AC$, giving reason for your answer.
(ii) In triangle ARC, $\angle ARC = 90^\circ$ and in triangle PQS, $\angle PSQ = 90^\circ$. Given $QS = 6$ cm, calculate the length of AR.

27. In the right-angled triangle QPR, PM is an altitude.



Given that $QR = 8 \text{ cm}$ and $MQ = 3.5 \text{ cm}$, calculate the value of PR . [2000]

28. In the figure, given below, the medians BD and CE of a triangle ABC meet at G. Prove that :



- (i) $\triangle EGD \sim \triangle CGB$ and
(ii) $BG = 2 GD$ from (i) above.

15.5

Basic Proportionality Theorem With Applications :

Theorem 1

A line drawn parallel to one side of a triangle divides the other two sides proportionally.

Given : In $\triangle ABC$; line DE is drawn parallel to side BC which meets AB at D and AC at E.

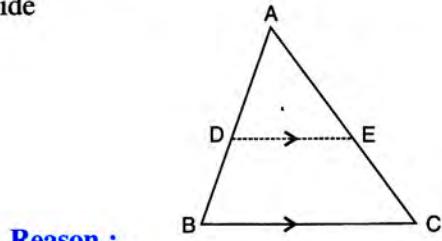
To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Proof :

Statement :

In $\triangle ABC$ and $\triangle ADE$,

$$1. \quad \angle ABC = \angle ADE$$



Reason :

[Corresponding angles]

$$2. \quad \angle ACB = \angle AED$$

[Corresponding angles]

$$3. \quad \angle BAC = \angle DAE$$

[Common]

$$\therefore \triangle ABC \sim \triangle ADE$$

[AAA postulate]

$$\therefore \frac{AB}{AD} = \frac{AC}{AE}$$

[Corresponding sides of similar triangles are proportional]

$$\Rightarrow \frac{AD + DB}{AD} = \frac{AE + EC}{AE}$$

$$\Rightarrow 1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$$

$[\frac{AD}{AD} = 1 \text{ and } \frac{AE}{AE} = 1]$

$$\Rightarrow \frac{DB}{AD} = \frac{EC}{AE}$$

[Cancelling 1 from both the sides]

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

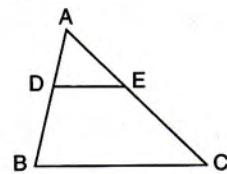
[Taking the reciprocal]

Hence Proved.

Conversely : If a line divides any two sides of a triangle proportionally, the line is parallel to the third side.

Note : In $\triangle ABC$, D is a point in AB and E is a point AC

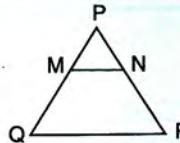
such that $\frac{AD}{BD} = \frac{AE}{CE}$, then $DE \parallel BC$.



1. M and N are points on sides PQ and PR respectively of $\triangle PQR$; then :

$$(i) \text{ if } MN//QR \Rightarrow \frac{PM}{MQ} = \frac{PN}{NR}$$

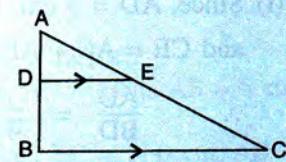
and, (ii) if $\frac{PM}{MO} = \frac{PN}{NR} \Rightarrow MN \parallel QR$



2. In $\triangle ABC$, DE is parallel to BC

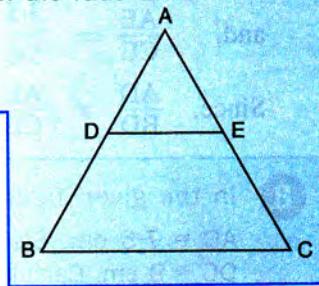
$$\Rightarrow \text{(i)} \frac{AD}{BD} = \frac{AE}{CE}$$

$$\text{and, (ii)} \quad \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC} \quad (\text{As, } \triangle ADE \sim \triangle ABC)$$



- 6** In the adjoining figure; $DE \parallel BC$ and D divides AB in the ratio $2 : 3$. Find :

(i) $\frac{AE}{EC}$ (ii) $\frac{AE}{AC}$ (iii) DE, if BC = 7.5 cm.



Solution :

- (i) Since, a line drawn parallel to one side of a triangle, divides the other two sides proportionally;

Also, given that D divides AB in the ratio $2 : 3$ i.e. $\frac{AD}{DB} = \frac{2}{3}$ II

From I and II, we get : $\frac{AE}{EC} = \frac{2}{3}$

Ans.

$$(ii) \quad \frac{AE}{AC} = \frac{AE}{AE+EC} = \frac{2}{2+3} = \frac{2}{5}$$

Ans.

$$(iii) \quad \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC} \quad [\text{As, } \Delta ADE \sim \Delta ABC]$$

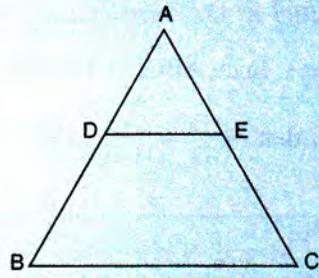
$$\Rightarrow \frac{DE}{7.5} = \frac{2}{5} \quad [\text{From (ii), } \frac{AE}{AC} = \frac{2}{5}]$$

$$\Rightarrow \text{DE} = \frac{2}{5} \times 7.5 \text{ cm} = 3 \text{ cm}$$

Ans.

- 7 In $\triangle ABC$, D and E are points on the sides AB and AC respectively. Find whether $DE \parallel BC$; if :

- (i) $AD = 3 \text{ cm}$, $BD = 4.5 \text{ cm}$,
 $AE = 4 \text{ cm}$ and $AC = 10 \text{ cm}$
- (ii) $AB = 7 \text{ cm}$, $BD = 4.5 \text{ cm}$,
 $AE = 3.5 \text{ cm}$ and $CE = 5.6 \text{ cm}$



Solution :

DE will be parallel to BC, only when $\frac{AD}{BD} = \frac{AE}{CE}$

- (i) Since, $AD = 3 \text{ cm}$, $BD = 4.5 \text{ cm}$; $AE = 4 \text{ cm}$
and $CE = AC - AE = 10 \text{ cm} - 4 \text{ cm} = 6 \text{ cm}$.

$$\therefore \frac{AD}{BD} = \frac{3}{4.5} = \frac{2}{3} \text{ and, } \frac{AE}{CE} = \frac{4}{6} = \frac{2}{3}$$

$$\Rightarrow \frac{AD}{BD} = \frac{AE}{CE} \therefore DE \parallel BC$$

$$(ii) \frac{AD}{BD} = \frac{AB - BD}{BD} = \frac{7 - 4.5}{4.5} = \frac{2.5}{4.5} = \frac{5}{9}$$

$$\text{and, } \frac{AE}{CE} = \frac{3.5}{5.6} = \frac{5}{8}$$

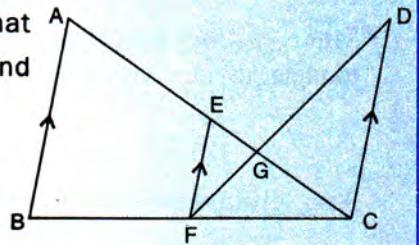
Since, $\frac{AD}{BD} \neq \frac{AE}{CE} \Rightarrow DE \text{ is not parallel to BC.}$

Ans.

Ans.

- 8 In the given figure; $AB \parallel EF \parallel CD$. Given that

$AB = 7.5 \text{ cm}$, $EG = 2.5 \text{ cm}$, $GC = 5 \text{ cm}$ and
 $DC = 9 \text{ cm}$. Calculate : (i) EF (ii) AC.



Solution :

- (i) In $\triangle EGF$ and $\triangle CGD$,

$$\angle EGF = \angle CGD \quad [\text{Vertically opposite angles}]$$

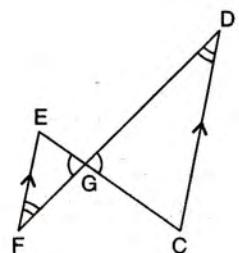
$$\text{and, } \angle EFG = \angle D \quad [\text{Alternate angles}]$$

$$\therefore \triangle EGF \sim \triangle CGD \quad [\text{By A.A. postulate}]$$

$$\Rightarrow \frac{EF}{DC} = \frac{EG}{GC} \quad [\text{Corresponding parts of similar triangles are proportional}]$$

$$\Rightarrow \frac{EF}{9 \text{ cm}} = \frac{2.5 \text{ cm}}{5 \text{ cm}}$$

$$\Rightarrow EF = \frac{2.5}{5} \times 9 \text{ cm} = 4.5 \text{ cm}$$



Ans.

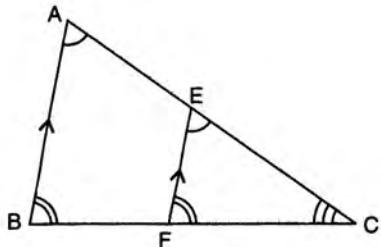
(ii) $EC = EG + GC$
 $= 2.5 \text{ cm} + 5 \text{ cm} = 7.5 \text{ cm}$

$\Delta ABC \sim \Delta EFC$ [By A.A.A.]

$$\Rightarrow \frac{AC}{EC} = \frac{AB}{EF}$$

$$\Rightarrow \frac{AC}{7.5 \text{ cm}} = \frac{7.5 \text{ cm}}{4.5 \text{ cm}}$$

$$\Rightarrow AC = \frac{7.5 \times 7.5}{4.5} \text{ cm} = 12.5 \text{ cm}$$



Ans.

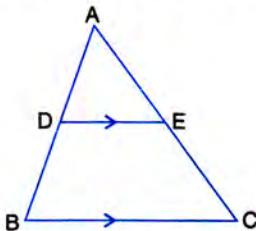
EXERCISE 15(B)

1. In the following figure, point D divides AB in the ratio 3 : 5. Find :

(i) $\frac{AE}{EC}$ (ii) $\frac{AD}{AB}$ (iii) $\frac{AE}{AC}$

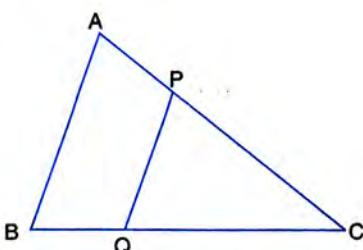
Also, if:

- (iv) $DE = 2.4 \text{ cm}$, find the length of BC.
(v) $BC = 4.8 \text{ cm}$, find the length of DE.



2. In the given figure, $PQ \parallel AB$; $CQ = 4.8 \text{ cm}$, $QB = 3.6 \text{ cm}$ and $AB = 6.3 \text{ cm}$. Find :

- (i) $\frac{CP}{PA}$ (ii) PQ
(iii) If $AP = x$, then the value of AC in terms of x .



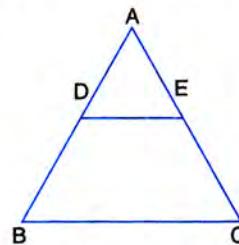
3. A line PQ is drawn parallel to the side BC of ΔABC which cuts side AB at P and side AC at Q. If $AB = 9.0 \text{ cm}$, $CA = 6.0 \text{ cm}$ and $AQ = 4.2 \text{ cm}$, find the length of AP.

4. In ΔABC , D and E are the points on sides AB and AC respectively.

Find whether $DE \parallel BC$, if :

- (i) $AB = 9 \text{ cm}$, $AD = 4 \text{ cm}$, $AE = 6 \text{ cm}$ and $EC = 7.5 \text{ cm}$.
(ii) $AB = 6.3 \text{ cm}$, $EC = 11.0 \text{ cm}$, $AD = 0.8 \text{ cm}$ and $AE = 1.6 \text{ cm}$.

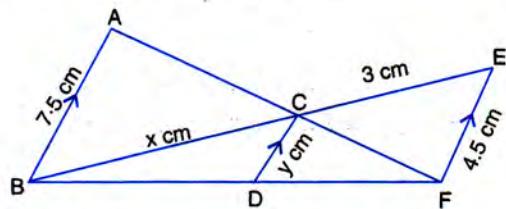
5. In the given figure, $\Delta ABC \sim \Delta ADE$. If $AE : EC = 4 : 7$ and $DE = 6.6 \text{ cm}$, find BC. If 'x' be the length of the perpendicular from



A to DE, find the length of perpendicular from A to BC in terms of 'x'.

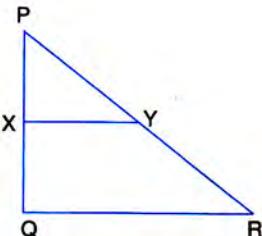
6. A line segment DE is drawn parallel to base BC of ΔABC which cuts AB at point D and AC at point E. If $AB = 5$ BD and $EC = 3.2 \text{ cm}$, find the length of AE.

7. In the figure, given below, AB, CD and EF are parallel lines. Given $AB = 7.5 \text{ cm}$, $DC = y \text{ cm}$, $EF = 4.5 \text{ cm}$, $BC = x \text{ cm}$ and $CE = 3 \text{ cm}$, calculate the values of x and y .

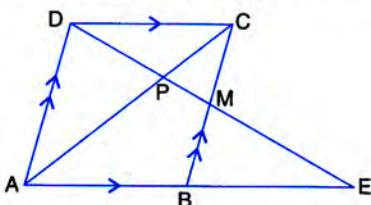


8. In the figure, given below, PQR is a right-angle triangle right angled at Q. XY is parallel

to QR, PQ = 6 cm, PY = 4 cm and PX : XQ = 1 : 2. Calculate the lengths of PR and QR.

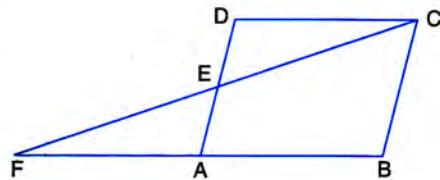


9. In the following figure, M is mid-point of BC of a parallelogram ABCD. DM intersects the



diagonal AC at P and AB produced at E. Prove that : PE = 2 PD

10. The given figure shows a parallelogram ABCD. E is a point in AD and CE produced meets BA produced at point F. If AE = 4 cm, AF = 8 cm and AB = 12 cm, find the perimeter of the parallelogram ABCD.



15.6 Relation between the areas of two triangles :

Theorem 2

The areas of two similar triangles are proportional to the squares on their corresponding sides.

Given : $\triangle ABC \sim \triangle DEF$

such that $\angle BAC = \angle EDF$,
 $\angle B = \angle E$ and $\angle C = \angle F$.

To Prove :

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$

Construction :

Draw AM \perp BC and DN \perp EF.

Proof :

Statement

Reason

1. Area of $\triangle ABC$ = $\frac{1}{2} BC \times AM$	Area of Δ = $\frac{1}{2}$ base \times altitude
Area of $\triangle DEF$ = $\frac{1}{2} EF \times DN$	Area of Δ = $\frac{1}{2}$ base \times altitude
$\Rightarrow \frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \frac{\frac{1}{2} BC \times AM}{\frac{1}{2} EF \times DN}$	
$= \frac{BC}{EF} \times \frac{AM}{DN}$I

2. In ΔABM and ΔDEN :

$$(i) \angle B = \angle E$$

[Given]

$$(ii) \angle AMB = \angle DNE$$

[Each angle being 90°]

$$\therefore \Delta ABM \sim \Delta DEN$$

[By AA postulate]

$$\Rightarrow \frac{AM}{DN} = \frac{AB}{DE}$$

.....II

[Corresponding sides of similar triangles are in proportion]

3. Since, $\Delta ABC \sim \Delta DEF$

[Given]

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

.....III

[Corresponding sides of similar triangles are in proportion]

$$\therefore \frac{AM}{DN} = \frac{BC}{EF}$$

[From II and III]

Substituting $\frac{AM}{DN} = \frac{BC}{EF}$ in equation I, we get :

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{BC}{EF} \times \frac{BC}{EF} = \frac{BC^2}{EF^2}$$

.....IV

Now combining eq. III and eq. IV, we get :

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$

Hence Proved.

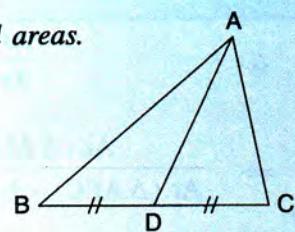
Remember :

1. Median divides the triangle into two triangles of equal areas.

In the given figure, AD is median

$$\Rightarrow \text{Area of } \Delta ABD = \text{Area of } \Delta ACD$$

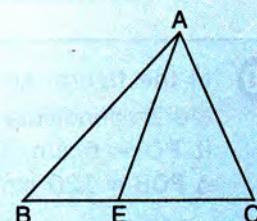
$$= \frac{1}{2} \times \text{Area of } \Delta ABC.$$



2. If two or many triangles have the common vertex and their bases are along the same straight line, the ratio between their areas is equal to the ratio between the lengths of their bases.

In the given figure, triangles ABE and ACE have the common vertex at point A and their bases are along the same straight line BC.

$$\Rightarrow \frac{\text{Area of } \Delta ABE}{\text{Area of } \Delta ACE} = \frac{BE}{CE}.$$



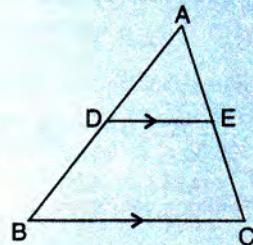
9 In the given figure, $DE \parallel BC$.

(i) Prove that ΔADE and ΔABC are similar.

(ii) Given that $AD = \frac{1}{2} BD$, calculate DE , if $BC = 4.5$ cm.

[2004]

Also, find $\frac{\text{Ar.}(\Delta ADE)}{\text{Ar.}(\Delta ABC)}$ and $\frac{\text{Ar.}(\Delta ADE)}{\text{Ar.}(\text{trapezium BCED})}$



Solution :

(i) $DE \parallel BC$

$$\Rightarrow \angle ADE = \angle ABC$$

[Corresponding angles]

$$\text{and, } \angle AED = \angle ACB$$

[Corresponding angles]

$$\therefore \Delta ADE \sim \Delta ABC$$

[By A.A.]

Hence Proved.

(ii) Since the corresponding sides of the similar triangles are in proportion :

$$\therefore \frac{DE}{BC} = \frac{AD}{AB}$$

$$\text{i.e. } \frac{DE}{4.5 \text{ cm}} = \frac{1}{3}$$

$$\Rightarrow DE = \frac{1}{3} \times 4.5 \text{ cm} = 1.5 \text{ cm} \quad \text{Ans.}$$

$$\text{Given, } AD = \frac{1}{2} BD$$

$$\text{Since, } \frac{AD}{BD} = \frac{1}{2}$$

$$\Rightarrow \frac{AD}{AD + BD} = \frac{1}{1+2}$$

$$\Rightarrow \frac{AD}{AB} = \frac{1}{3}$$

We know the ratio between the areas of two similar triangles

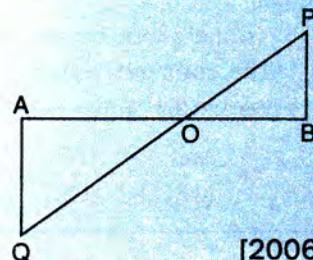
= ratio between the squares of its corresponding sides.

$$\therefore \frac{\text{Ar.}(\Delta ADE)}{\text{Ar.}(\Delta ABC)} = \frac{DE^2}{BC^2} = \frac{(1.5)^2}{(4.5)^2} = \frac{1}{9} \quad \text{Ans.}$$

$$\Rightarrow \frac{\text{Ar.}(\Delta ADE)}{\text{Ar.}(\Delta ABC) - \text{Ar.}(\Delta ADE)} = \frac{1}{9-1}$$

$$\Rightarrow \frac{\text{Ar.}(\Delta ADE)}{\text{Ar.}(\text{trapezium BCED})} = \frac{1}{8} \quad \text{Ans.}$$

10 In the figure, given alongside, PB and QA are perpendiculars to the line segment AB . If $PO = 6$ cm, $QO = 9$ cm and area of $\Delta POB = 120 \text{ cm}^2$, find the area of ΔQOA .



[2006]

Solution :

In $\triangle POB$ and $\triangle QOA$:

$$1. \quad \angle PBO = \angle QAO \quad [\text{Each } 90^\circ]$$

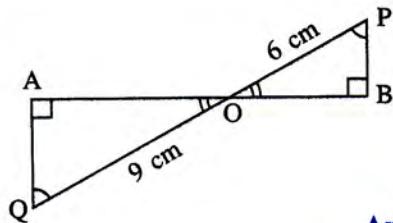
$$2. \quad \angle POB = \angle QOA \quad [\text{Vertically opposite angles}]$$

$$\Rightarrow \quad \triangle POB \sim \triangle QOA \quad [\text{By A.A.}]$$

$$\Rightarrow \quad \frac{\text{Ar.}(\Delta POB)}{\text{Ar.}(\Delta QOA)} = \frac{PO^2}{QO^2} \quad [\text{Ratio between the areas of two similar triangles is equal to the ratio between the squares of the corresponding sides}]$$

$$\Rightarrow \quad \frac{120}{\text{Ar.}(\Delta QOA)} = \frac{6^2}{9^2}$$

$$\Rightarrow \quad \text{Ar.}(\Delta QOA) = \frac{120 \times 81}{36} \text{ cm}^2 \\ = 270 \text{ cm}^2$$

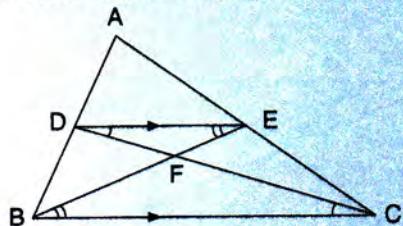


Ans.

- 11** In the given figure, DE is parallel to the base BC of triangle ABC and $AD : DB = 5 : 3$. Find the ratio :

(i) $\frac{AD}{AB}$ and then $\frac{DE}{BC}$.

(ii) $\frac{\text{Area of } \triangle DEF}{\text{Area of } \triangle DEC}$.



Solution :

$$(i) \quad \frac{AD}{DB} = \frac{5}{3} \quad \Rightarrow \quad \frac{AD}{AD + DB} = \frac{5}{5+3}$$

$$\Rightarrow \quad \frac{AD}{AB} = \frac{5}{8} \quad \text{Ans.}$$

Since, $\triangle ADE \sim \triangle ABC$ [By AA postulate]

$$\Rightarrow \quad \frac{DE}{BC} = \frac{AD}{AB} = \frac{5}{8} \quad \text{Ans.}$$

- (ii) Since, $\triangle DEF$ and $\triangle DEC$ have common vertex at E and their bases DF and DC are along the same straight line

$$\therefore \quad \frac{\text{Area of } \triangle DEF}{\text{Area of } \triangle DEC} = \frac{DF}{DC} \quad \dots\dots(I)$$

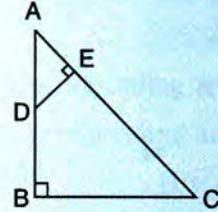
Now, show that $\Delta DFE \sim \Delta CFB$

$$\Rightarrow \frac{DF}{FC} = \frac{DE}{BC} = \frac{5}{8}$$

$$\Rightarrow \frac{DF}{DF + FC} = \frac{5}{5 + 8} \Rightarrow \frac{DF}{DC} = \frac{5}{13} \quad \dots\dots(\text{II})$$

From I and II, we get : $\frac{\text{Area of } \Delta DEF}{\text{Area of } \Delta DEC} = \frac{5}{13}$ Ans.

- 12 In ΔABC , $\angle B = 90^\circ$, $AB = 12 \text{ cm}$ and $AC = 15 \text{ cm}$.
D and E are points on AB and AC respectively such that $\angle AED = 90^\circ$ and $DE = 3 \text{ cm}$. Calculate the area of ΔABC and then the area of ΔADE .



Solution :

In right-angled triangle ABC,

$$AB^2 + BC^2 = AC^2 \quad [\text{Pythagoras theorem}]$$

$$\begin{aligned} \Rightarrow BC^2 &= 15^2 - 12^2 \\ &= 225 - 144 = 81 \\ BC &= 9 \text{ cm.} \end{aligned}$$

$$\begin{aligned} \therefore \text{Area of } \Delta ABC &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times BC \times AB \\ &= \frac{1}{2} \times 9 \times 12 \text{ cm}^2 = 54 \text{ cm}^2 \quad \text{Ans.} \end{aligned}$$

In ΔADE and ΔABC ,

$$\angle ABC = \angle AED = 90^\circ$$

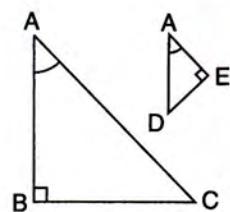
and $\angle A$ is common to both.

\therefore By A.A. axiom, $\Delta ADE \sim \Delta ABC$

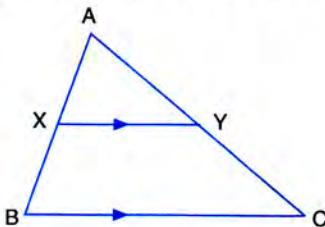
$$\Rightarrow \frac{\text{Ar. of } \Delta ADE}{\text{Ar. of } \Delta ABC} = \frac{DE^2}{BC^2}$$

$$\Rightarrow \frac{\text{Ar. of } \Delta ADE}{54 \text{ cm}^2} = \frac{3^2}{9^2}$$

$$\begin{aligned} \Rightarrow \text{Ar. of } \Delta ADE &= \frac{9}{81} \times 54 \text{ cm}^2 \\ &= 6 \text{ cm}^2 \quad \text{Ans.} \end{aligned}$$

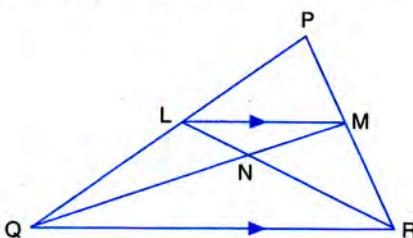


- (i) The ratio between the corresponding sides of two similar triangles is 2 is to 5. Find the ratio between the areas of these triangles.
 (ii) Areas of two similar triangles are 98 sq. cm and 128 sq. cm. Find the ratio between the lengths of their corresponding sides.
- A line PQ is drawn parallel to the base BC of $\triangle ABC$ which meets sides AB and AC at points P and Q respectively. If $AP = \frac{1}{3}PB$; find the value of :
 (i) $\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle APQ}$
 (ii) $\frac{\text{Area of } \triangle APQ}{\text{Area of trapezium PBCQ}}$
- The perimeters of two similar triangles are 30 cm and 24 cm. If one side of the first triangle is 12 cm, determine the corresponding side of the second triangle.
- In the given figure, $AX : XB = 3 : 5$



Find :

- the length of BC, if the length of XY is 18 cm.
 - the ratio between the areas of trapezium XBCY and triangle ABC.
- ABC is a triangle. PQ is a line segment intersecting AB in P and AC in Q such that $PQ \parallel BC$ and divides triangle ABC into two parts equal in area. Find the value of ratio $BP : AB$.
 - In the given triangle PQR, LM is parallel to QR and $PM : MR = 3 : 4$.



Calculate the value of ratio :

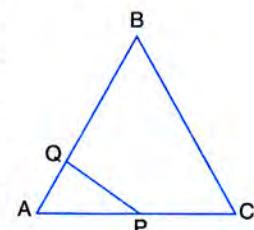
(i) $\frac{PL}{PQ}$ and then $\frac{LM}{QR}$

(ii) $\frac{\text{Area of } \triangle LMN}{\text{Area of } \triangle MNR}$ (iii) $\frac{\text{Area of } \triangle LQM}{\text{Area of } \triangle LQN}$

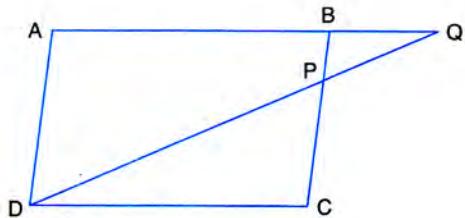
- The given diagram shows two isosceles triangles which are similar. In the given diagram, PQ and BC are not parallel; $PC = 4$, $AQ = 3$, $QB = 12$, $BC = 15$ and $AP = PQ$.

Calculate :

- the length of AP,
- the ratio of the areas of triangle APQ and triangle ABC.



- In the figure, given below, ABCD is a parallelogram. P is a point on BC such that $BP : PC = 1 : 2$. DP produced meets AB produced at Q. Given the area of triangle CPQ = 20 cm^2 .

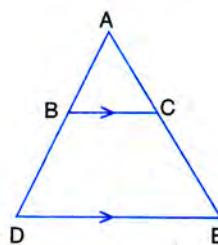


Calculate :

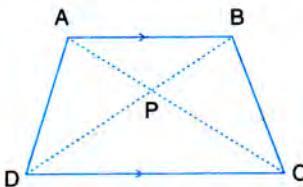
- area of triangle CDP,
- area of parallelogram ABCD.

- ABC is a triangle. PQ is a line segment intersecting AB in P and AC in Q such that $PQ \parallel BC$ and divides triangle ABC into two parts equal in area. Calculate the length of BC.

Also, find the area of triangle BCD.



10. The given figure shows a trapezium in which AB is parallel to DC and diagonals AC and BD intersect at point P. If $AP : CP = 3 : 5$,



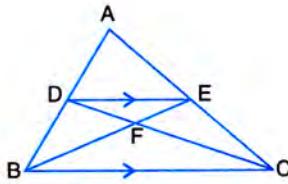
Find :

- (i) $\Delta APB : \Delta CPB$
- (ii) $\Delta DPC : \Delta APB$
- (iii) $\Delta ADP : \Delta APB$
- (iv) $\Delta APB : \Delta ADB$

11. In the given figure, ABC is a triangle. DE is parallel to BC and $\frac{AD}{DB} = \frac{3}{2}$.

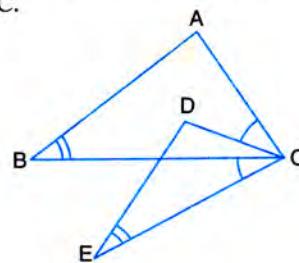
- (i) Determine the ratios $\frac{AD}{AB}$ and $\frac{DE}{BC}$.
- (ii) Prove that ΔDEF is similar to ΔCBF .

Hence, find $\frac{EF}{FB}$.



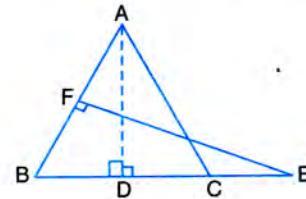
- (iii) What is the ratio of the areas of ΔDEF and ΔBFC ? [2007]

12. In the given figure, $\angle B = \angle E$, $\angle ACD = \angle BCE$, $AB = 10.4$ cm and $DE = 7.8$ cm. Find the ratio between areas of the ΔABC and ΔDEC .



13. Triangle ABC is an isosceles triangle in which $AB = AC = 13$ cm and $BC = 10$ cm. AD is perpendicular to BC. If $CE = 8$ cm and $EF \perp AB$, find :

- (i) $\frac{\text{area of } \Delta ADC}{\text{area of } \Delta FEB}$
- (ii) $\frac{\text{area of } \Delta FEB}{\text{area of } \Delta ABC}$



15.7 Similarity as a Size Transformation :

Draw a triangle ABC and mark a point P outside it.

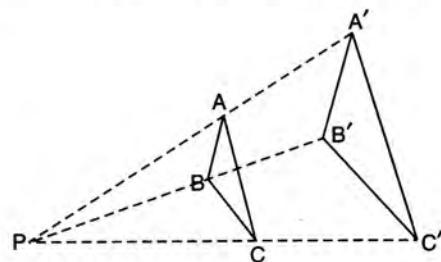
Join P and A. Produce PA to point A' such that $PA' = 2PA$. Similarly, get $PB' = 2PB$ and $PC' = 2PC$.

On joining A' , B' and C' ; we get another triangle $A'B'C'$ which is similar to the original triangle ABC. If the sides of the triangle $A'B'C'$ be measured and compared with the corresponding sides of the original triangle ABC, we find :

$$A'B' = 2AB, B'C' = 2BC \text{ and } C'A' = 2CA.$$

Since, each side of the resulting triangle $A'B'C'$ is twice the corresponding side of the original triangle ABC; $\Delta A'B'C'$ is similar to ABC and we say, that the triangle ABC has been **enlarged by a scale factor 2 about the centre of enlargement P**.

Similarly, if for the same triangle ABC and a point P outside it, we take a point A' in PA such that $PA' = \frac{1}{2}PA$; a point B' in PB such that $PB' = \frac{1}{2}PB$ and a point C' in



PC such that $PC' = \frac{1}{2} PC$. On joining A', B' and C' we again get a triangle A' B' C' which is similar to the original triangle ABC.

On comparing the resulting triangle A' B' C' and the original triangle ABC, it will be observed that $A' B' = \frac{1}{2} AB$, $B' C' = \frac{1}{2} BC$ and $C' A' = \frac{1}{2} CA$.

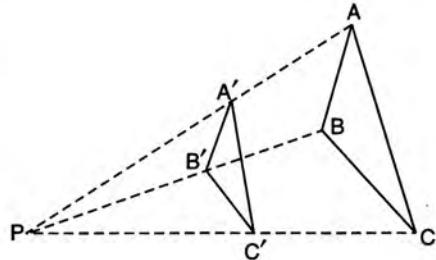
Since, each side of the resulting triangle A' B' C' is **half** of the corresponding side of the given triangle ABC; we say, that the triangle ABC has been **reduced** by a scale factor $\frac{1}{2}$ about the **centre of reduction P**.

The type of **enlargement or reduction**, as discussed above, is called **size transformation**.

Thus in a size transformation, a given figure is enlarged or reduced by a scale factor k, such that the resulting figure (the image) is similar to the given figure (the object or the pre-image).

In a size transformation :

- (i) the image of a line is a line,
- (ii) the image of a triangle is a triangle,
- (iii) the image of a quadrilateral is a quadrilateral and so on.



15.8 Applications to Maps and Models :

Maps and Models : Students use different maps of India and of Asia in Geography. Consider the map of India in which the positions of its major cities are shown. Measure the distance between any two cities marked in the map and compare it with the actual distance between those two cities to get the ratio between the two distances. In the same way, choose two more cities marked in the map and compare their distance (on the map) with the actual distance between them to get the ratio between the two distances; we find that the ratio of distances in both the cases is the same.

In fact, this ratio is already marked (written) on every map and is known as **scale factor** which is denoted by letter, ' k '.

For Example :

If the scale of a map is $1 : 20,000$, this implies that, a distance of one cm on the map is equal to an actual distance of 20,000 cm (0.2 km) on the ground. And, also scale factor $k = \frac{1}{20,000}$.

The same principle is applicable to models. In the case of models,

$$\begin{aligned}\frac{\text{Height of the model}}{\text{Height of the object}} &= \frac{\text{Length of the model}}{\text{Length of the object}} \\ &= \frac{\text{Width of the model}}{\text{Width of the object}} \\ &= \text{Scale factor (}k\text{)}\end{aligned}$$

If the scale factor is k , then :

- (i) each side of the resulting figure (the image) is k times the corresponding side of the given figure (the object or the pre-image).
- (ii) the area of the resulting figure is k^2 times the area of the given figure.
- (iii) in the case of solids, the volume of the resulting figure is k^3 times the volume of the given figure.
- (iv) $k > 1 \Rightarrow$ the transformation is an enlargement,
 $k < 1 \Rightarrow$ the transformation is a reduction and
 $k = 1 \Rightarrow$ the transformation is an identity transformation.

- 13** A model of a ship is made to a scale of $1 : 200$. If the length of the model is 4 m ; calculate the length of the ship.

Solution :

$$\text{Clearly, the scale factor } k = \frac{1}{200}$$

And, the length of model = k times the length of the ship.

$$\Rightarrow 4\text{ m} = \frac{1}{200} \times \text{the length of the ship}$$

$$\Rightarrow \text{The length of the ship} = 800\text{ m}$$

Ans.

- 14** The scale of map is $1 : 50,000$. In the map, a triangular plot ABC of land has the following dimensions :
 $AB = 2\text{ cm}$, $BC = 3.5\text{ cm}$ and angle $ABC = 90^\circ$.
Calculate : (i) the actual length of side BC, in km, of the land.
(ii) the area of the plot in sq. km.

Solution :

$$\text{Clearly, scale factor } k = \frac{1}{50,000}$$

- (i) Length of side BC in the map = k times the actual length of side BC in the land.

$$\Rightarrow 3.5\text{ cm} = \frac{1}{50,000} \times \text{actual length of BC}$$

$$\text{(ii) Since, the area of triangle ABC in the map} = \frac{1}{2} \times 2 \text{ cm} \times 3.5 \text{ cm} \\ = 3.5 \text{ sq. cm}$$

And, the area of Δ ABC in the map = k^2 times the actual area of triangular plot ABC.

$$\Rightarrow 3.5 \text{ cm}^2 = \left(\frac{1}{50,000} \right)^2 \times \text{actual area of the plot}$$

$$\Rightarrow \text{Actual area of the plot} = 3.5 \times 50,000 \times 50,000 \text{ sq. cm} \\ = 0.875 \text{ sq. km}$$

Ans.

- 15** A rectangular tank has length = 4 m, width = 3 m and capacity = 30 m^3 . A small model of the tank is made with capacity 240 cm^3 . Find :
- (i) the dimensions of the model.
 - (ii) the ratio between the total surface area of the tank and its model.

Solution :**For the tank :**

$$\text{Its length} \times \text{breadth} \times \text{height} = \text{Volume}$$

$$\Rightarrow 4 \text{ m} \times 3 \text{ m} \times \text{height} = 30 \text{ m}^3$$

$$\Rightarrow \text{Height} = \frac{30}{4 \times 3} \text{ m} = 2.5 \text{ m.}$$

Let the scale factor for reduction = k

$$\therefore \text{Volume of the model} = k^3 \times \text{volume of the tank}$$

$$\Rightarrow 240 \text{ cm}^3 = k^3 \times 30 \text{ m}^3$$

$$\Rightarrow 240 \text{ cm}^3 = k^3 \times 30 \times 100 \times 100 \times 100 \text{ cm}^3$$

$$\Rightarrow k^3 = \frac{240}{30 \times 100 \times 100 \times 100} \\ = \frac{1}{125000}$$

$$\Rightarrow \text{Scale factor, } k = \frac{1}{50}$$

$$\text{(i) } \therefore \text{Length of the model} = k \times \text{length of the tank} \\ = \frac{1}{50} \times 4 \text{ m} = 8 \text{ cm,}$$

$$\text{breadth of the model} = k \times \text{breadth of the tank} \\ = \frac{1}{50} \times 3 \text{ m} = 6 \text{ cm}$$

and, height of the model = $\frac{1}{50} \times 2.5 \text{ m} = 5 \text{ cm}$

∴

Dimensions of the model = 8 cm × 6 cm × 5 cm

Ans.

(ii) Since, total surface area of the model = $k^2 \times$ total S.A. of the tank

$$\Rightarrow \frac{\text{Total S.A. of the tank}}{\text{Total S.A. of the model}} = \frac{1}{k^2} = (50)^2 \\ = 2500 = \mathbf{2500 : 1}$$

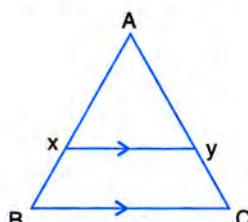
Ans.

EXERCISE 15(D)

1. A triangle ABC has been enlarged by scale factor $m = 2.5$ to the triangle $A' B' C'$. Calculate :
 - (i) the length of AB, if $A' B' = 6 \text{ cm}$.
 - (ii) the length of $C' A'$ if $CA = 4 \text{ cm}$.
2. A triangle LMN has been reduced by scale factor 0.8 to the triangle $L' M' N'$. Calculate:
 - (i) the length of $M' N'$, if $MN = 8 \text{ cm}$.
 - (ii) the length of LM, if $L' M' = 5.4 \text{ cm}$.
3. A triangle ABC is enlarged, about the point O as centre of enlargement, and the scale factor is 3. Find :
 - (i) $A' B'$, if $AB = 4 \text{ cm}$.
 - (ii) BC, if $B' C' = 15 \text{ cm}$.
 - (iii) OA, if $OA' = 6 \text{ cm}$.
 - (iv) OC' , if $OC = 21 \text{ cm}$.
- Also, state the value of :
 - (a) $\frac{OB'}{OB}$
 - (b) $\frac{C'A'}{CA}$
4. A model of an aeroplane is made to a scale of 1 : 400. Calculate :
 - (i) the length, in cm, of the model; if the length of the aeroplane is 40 m.
 - (ii) the length, in m, of the aeroplane, if length of its model is 16 cm.
5. The dimensions of the model of a multistorey building are $1.2 \text{ m} \times 75 \text{ cm} \times 2 \text{ m}$. If the scale factor is 1 : 30; find the actual dimensions of the building.
6. On a map drawn to a scale of 1 : 2,50,000; a triangular plot of land has the following measurements : $AB = 3 \text{ cm}$, $BC = 4 \text{ cm}$ and angle $ABC = 90^\circ$. Calculate :
 - (i) the actual lengths of AB and BC in km.
 - (ii) the area of the plot in sq. km.
7. A model of a ship is made to a scale of 1 : 200.
 - (i) The length of the model is 4 m; calculate the length of the ship.
 - (ii) The area of the deck of the ship is 160000 m^2 ; find the area of the deck of the model.
 - (iii) The volume of the model is 200 litres; calculate the volume of the ship in m^3 .
8. An aeroplane is 30 m long and its model is 15 cm long. If the total outer surface area of the model is 150 cm^2 , find the cost of painting the outer surface of the aeroplane at the rate of ₹ 120 per sq. m. Given that 50 sq. m of the surface of the aeroplane is left for windows.

EXERCISE 15(E)

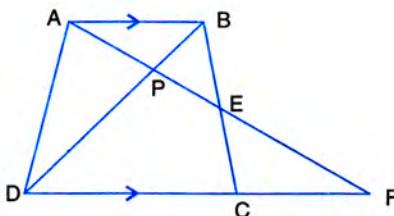
1. In the following figure, XY is parallel to BC, $AX = 9 \text{ cm}$, $XB = 4.5 \text{ cm}$ and $BC = 18 \text{ cm}$.



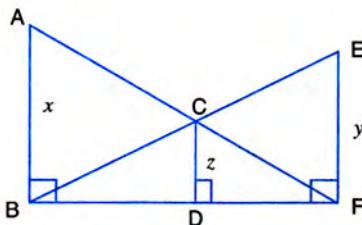
Find :

$$(i) \frac{AY}{YC} \quad (ii) \frac{YC}{AC} \quad (iii) XY$$

2. In the following figure, ABCD is a trapezium with $AB // DC$. If $AB = 9 \text{ cm}$, $DC = 18 \text{ cm}$, $CF = 13.5 \text{ cm}$, $AP = 6 \text{ cm}$ and $BE = 15 \text{ cm}$. Calculate :
 - (i) EC
 - (ii) AF
 - (iii) PE



3. In the following figure, AB, CD and EF are perpendicular to the straight line BDF.



If $AB = x$ and, $CD = z$ unit and $EF = y$ unit,
prove that : $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$

Let $BD = a$ and $DF = b$. In $\triangle ABF$, CD is parallel to AB

$$\Rightarrow \frac{CD}{AB} = \frac{DF}{BF} \quad \text{i.e., } \frac{z}{x} = \frac{b}{a+b}.$$

And, in $\triangle BEF$, CD is parallel to EF

$$\Rightarrow \frac{CD}{EF} = \frac{BD}{BF} \quad \text{i.e. } \frac{z}{y} = \frac{a}{a+b}.$$

$$\therefore \frac{z}{x} + \frac{z}{y} = \frac{b}{a+b} + \frac{a}{a+b} = 1$$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$

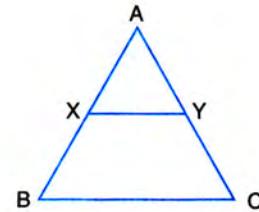
4. Triangle ABC is similar to triangle PQR. If AD and PM are corresponding medians of the two triangles, prove that : $\frac{AB}{PQ} = \frac{AD}{PM}$.

5. Triangle ABC is similar to triangle PQR. If AD and PM are altitudes of the two triangles, prove that : $\frac{AB}{PQ} = \frac{AD}{PM}$.

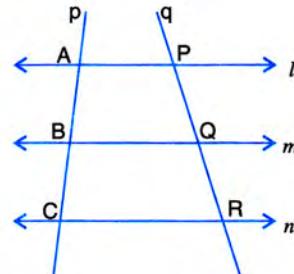
6. Triangle ABC is similar to triangle PQR. If bisector of angle BAC meets BC at point D and bisector of angle QPR meets QR at point M, prove that : $\frac{AB}{PQ} = \frac{AD}{PM}$.

7. In the following figure, $\angle AXB = \angle AYX$.

If $\frac{BX}{AX} = \frac{CY}{AY}$, show that triangle ABC is isosceles.



8. In the following diagram, lines l , m and n are parallel to each other. Two transversals p and q intersect the parallel lines at points A, B, C and P, Q, R as shown.



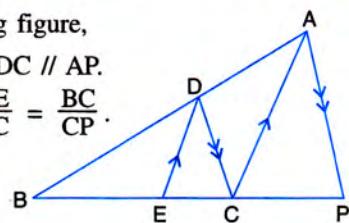
$$\text{Prove that : } \frac{AB}{BC} = \frac{PQ}{QR}$$

Join A and R. Let AR meets BQ at point D.

9. In the following figure,

$DE \parallel AC$ and $DC \parallel AP$.

$$\text{Prove that : } \frac{BE}{EC} = \frac{BC}{CP}.$$



In $\triangle ACB$, $DE \parallel AC$

$$\Rightarrow \frac{BD}{DA} = \frac{BE}{EC} \quad \dots \text{I}$$

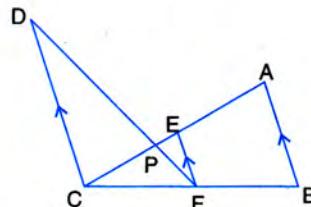
In $\triangle BAP$, $DC \parallel AP$

$$\Rightarrow \frac{BD}{DA} = \frac{BC}{CP} \quad \dots \text{II}$$

From I and II, we get : $\frac{BE}{EC} = \frac{BC}{CP}$

10. In the figure given below, $AB \parallel EF \parallel CD$. If $AB = 22.5$ cm, $EP = 7.5$ cm, $PC = 15$ cm and $DC = 27$ cm.

Calculate : (i) EF (ii) AC



11. In quadrilateral ABCD, its diagonals AC and BD intersect at point O such that

$$\frac{OC}{OA} = \frac{OD}{OB} = \frac{1}{3}.$$

Prove that :

- (i) $\triangle OAB \sim \triangle OCD$
- (ii) ABCD is a trapezium.

Further if $CD = 4.5$ cm; find the length of AB.

12. In triangle ABC, angle A is obtuse and $AB = AC$. P is any point in side BC. $PM \perp AB$ and $PN \perp AC$.

Prove that : $PM \times PC = PN \times PB$

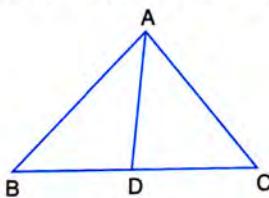
13. In triangle ABC, $AB = AC = 8$ cm, $BC = 4$ cm and P is a point in side AC such that $AP = 6$ cm. Prove that $\triangle BPC$ is similar to $\triangle ABC$. Also, find the length of BP.

In $\triangle ABC$, $\frac{AB}{BC} = \frac{8}{4} = 2$, in $\triangle BPC$, $\frac{BC}{CP} = \frac{4}{2} = 2$ and $\angle ABC = \angle C$. Therefore, by SAS, $\triangle ABC$ is similar to $\triangle BPC$.

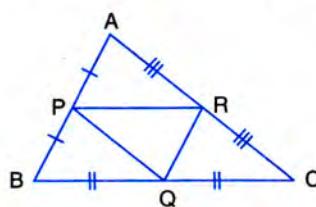
14. In $\triangle ABC$, $\angle ABC = \angle DAC$, $AB = 8$ cm, $AC = 4$ cm and $AD = 5$ cm.

- (i) Prove that $\triangle ACD$ is similar to $\triangle ABC$.
- (ii) Find BC and CD.
- (iii) Find area of $\triangle ACD$: area of $\triangle ABC$.

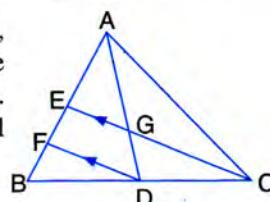
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15. In the given triangle P, Q and R are the midpoints of sides AB, BC and AC respectively. Prove that triangle PQR is similar to triangle ABC.



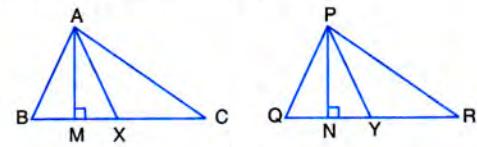
16. In the following figure, AD and CE are medians of $\triangle ABC$. DF is drawn parallel to CE.



Prove that :

- (i) $EF = FB$,
- (ii) $AG : GD = 2 : 1$.

17. In the given figure, triangle ABC is similar to triangle PQR. AM and PN are altitudes whereas AX and PY are medians.



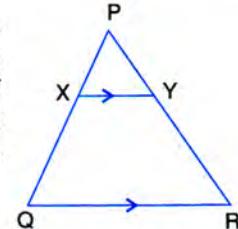
Prove that : $\frac{AM}{PN} = \frac{AX}{PY}$

18. The two similar triangles are equal in area. Prove that the triangles are congruent.

19. The ratio between the altitudes of two similar triangles is 3 : 5; write the ratio between their :
- (i) medians.
 - (ii) perimeters.
 - (iii) areas.

20. The ratio between the areas of two similar triangles is 16 : 25. Find the ratio between their :
- (i) perimeters.
 - (ii) altitudes.
 - (iii) medians.

21. The given figure shows a triangle PQR in which XY is parallel to QR. If $PX : XQ = 1 : 3$ and $QR = 9$ cm, find the length of XY.



Further, if the area of $\triangle PXY = x$ cm^2 ; find, in terms of x , the area of :

- (i) triangle PQR.
- (ii) trapezium XQRY.

22. On a map, drawn to a scale of 1 : 20000, a rectangular plot of land ABCD has $AB = 24$ cm and $BC = 32$ cm. Calculate :

- (i) the diagonal distance of the plot in kilometre.
- (ii) the area of the plot in sq. km.

23. The dimensions of the model of a multistoreyed building are 1 m by 60 cm by 1.20 m. If the scale factor is 1 : 50, find the actual dimensions of the building.

Also, find :

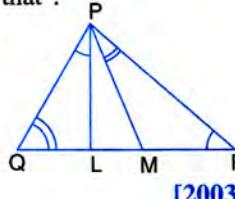
- (i) the floor area of a room of the building, if the floor area of the corresponding room in the model is 50 sq. cm.
- (ii) the space (volume) inside a room of the model, if the space inside the corresponding room of the building is 90 m^3 .

24. In a triangle PQR, L and M are two points on the base QR, such that $\angle LPQ = \angle QRP$ and $\angle RPM = \angle RQP$. Prove that :

(i) $\Delta PQL \sim \Delta RPM$

(ii) $QL \times RM = PL \times PM$

(iii) $PQ^2 = QR \times QL$



[2003]

25. In ΔABC , $\angle ACB = 90^\circ$ and $CD \perp AB$.

Prove that : $\frac{BC^2}{AC^2} = \frac{BD}{AD}$.

26. A triangle ABC with $AB = 3$ cm, $BC = 6$ cm and $AC = 4$ cm is enlarged to ΔDEF such that the longest side of $\Delta DEF = 9$ cm. Find the scale factor and hence, the lengths of the other sides of ΔDEF .

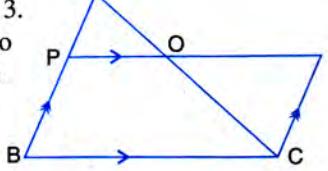
27. Two isosceles triangles have equal vertical angles. Show that the triangles are similar.

If the ratio between the areas of these two triangles is $16 : 25$, find the ratio between their corresponding altitudes.

28. In ΔABC ,

$AP : PB = 2 : 3$.

PO is parallel to BC and is extended to Q so that CQ is parallel to BA.

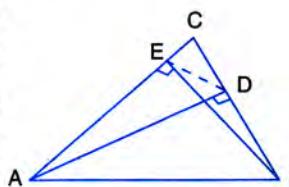


Find :

(i) area ΔAPO : area ΔABC .

(ii) area ΔAPO : area ΔCQO .

29. The following figure shows a triangle ABC in which AD and BE are perpendiculars to BC and AC respectively.



Show that :

(i) $\Delta ADC \sim \Delta BEC$

(ii) $CA \times CE = CB \times CD$

(iii) $\Delta ABC \sim \Delta DEC$

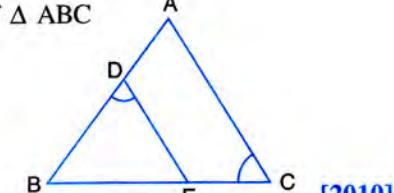
(iv) $CD \times AB = CA \times DE$

30. In the given figure, ABC is a triangle with $\angle EDB = \angle ACB$. Prove that $\Delta ABC \sim \Delta EBD$.

If $BE = 6$ cm, $EC = 4$ cm, $BD = 5$ cm and area of $\Delta BED = 9$ cm². Calculate the :

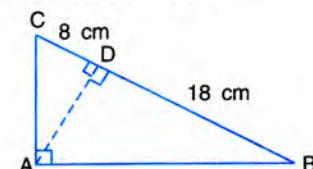
(i) length of AB

- (ii) area of ΔABC



[2010]

31. In the given figure, ABC is a right angled triangle with $\angle BAC = 90^\circ$.



(i) Prove that : $\Delta ADB \sim \Delta CDA$.

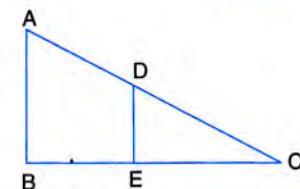
(ii) If $BD = 18$ cm and $CD = 8$ cm, find AD.

(iii) Find the ratio of the area of ΔADB to area of ΔCDA .

[2011]

32. In the given figure,

AB and DE are perpendiculars to BC.



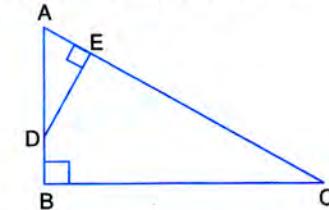
(i) Prove that : $\Delta ABC \sim \Delta DEC$

(ii) If $AB = 6$ cm, $DE = 4$ cm and $AC = 15$ cm. Calculate CD.

(iii) Find the ratio of the area of ΔABC : area of ΔDEC .

[2013]

33. ABC is a right angled triangle with $\angle ABC = 90^\circ$. D is any point on AB and DE is perpendicular to AC. Prove that :



(i) $\Delta ADE \sim \Delta ACB$.

(ii) If $AC = 13$ cm, $BC = 5$ cm and $AE = 4$ cm. Find DE and AD.

(iii) Find, area of ΔADE : area of quadrilateral BCED.

[2015]

34. Given : $AB \parallel DE$ and $BC \parallel EF$. Prove that :

(i) $\frac{AD}{DG} = \frac{CF}{FG}$

(ii) $\Delta DFG \sim \Delta ACG$.

