8 Remainder and Factor Theorems

8.1 A Basic Concept :

1. In equation $f(x) = 2x^2 - 5x - 7$, $f(x)$ is said to be a function of variable x as the value of $f(x)$ depends on the value of x. The following examples prove this statement.

$f(x) = 2x^2 - 5x - 7 \Rightarrow$

(i) if $x = 3$, $\quad f(3) = 2 \times (3)^2 - 5 \times 3 - 7 = 18 - 22 = -4$

(ii) if $x = -3$, $\quad f(-3) = 2(-3)^2 - 5(-3) - 7 = 18 + 15 - 7 = 26$ and so on.

In the same way :

in $f(y) = 2y^3 - 3y + 1$, $f(y)$ is a function of variable y as the value of $f(y)$ depends on the value of y, e.g.,

$f(y) = 2y^3 - 3y + 1 \Rightarrow$

(i) if $y = 5$, $\quad f(5) = 2 \times 5^3 - 3 \times 5 + 1 = 250 - 15 + 1 = 236$.

(ii) if $y = -2$, $\quad f(-2) = 2 \times (-2)^3 - 3 \times -2 + 1 = -16 + 6 + 1 = -9$ and so on.

2. Find the remainder obtained on dividing $f(x) = x^2 - 5x + 8$ by $x - 2$.

$$
\begin{array}{r|l}
 & x - 3 \\
\hline
x - 2 & x^2 - 5x + 8 \\
 & x^2 - 2x \\
 & - \quad + \\
\hline
 & -3x + 8 \\
 & -3x + 6 \\
 & + \quad - \\
\hline
 & 2
\end{array}
$$

The remainder = 2

Now, find $f(2)$

$f(x) = x^2 - 5x + 8$

$\Rightarrow f(2) = (2)^2 - 5 \times 2 + 8$

$= 4 - 10 + 8 = 2$

= **The remainder when $x^2 - 5x + 8$ is divided by $x - 2$.**

3. Again, find the remainder obtained on dividing $f(x) = 6x^3 - 3x^2 + 8x - 5$ by $x + 3$

$$
\begin{array}{r|l}
 & 6x^2 - 21x + 71 \\
\hline
x + 3 & 6x^3 - 3x^2 + 8x - 5 \\
 & 6x^3 + 18x^2 \\
 & - \quad - \\
\hline
 & -21x^2 + 8x \\
 & -21x^2 - 63x \\
 & + \quad + \\
\hline
 & 71x - 5 \\
 & 71x + 213 \\
 & - \quad - \\
\hline
 & -218
\end{array}
$$

The remainder = -218

Now, find $f(-3)$

$f(x) = 6x^3 - 3x^2 + 8x - 5$

$\Rightarrow f(-3) = 6(-3)^3 - 3(-3)^2 + 8(-3) - 5$

$= -162 - 27 - 24 - 5$

$= -218$

= **The remainder when $6x^3 - 3x^2 + 8x - 5$ is divided by $x + 3$**

It is clear from the examples, given above, that :

1. when $f(x)$ is divided by $x - 2$, the remainder = the value of $f(2)$.
2. when $f(x)$ is divided by $x + 3$, the remainder = the value of $f(-3)$

The method of finding the remainder without actually performing the process of division is called **Remainder Theorem.**

8.2 Remainder Theorem :

If $f(x)$, a polynomial in x, is divided by $(x - a)$, the remainder = $f(a)$

e.g. If $f(x)$ is divided by $(x - 3)$, the remainder is $f(3)$.

For finding the remainder, using Remainder Theorem :

Step 1 : Put the divisor equal to zero and solve the equation obtained to get the value of its variable.

Step 2 : Substitute the value of the variable, obtained in step 1, in the given polynomial and simplify it to get the required remainder.

1 Find the remainder when $x^2 - 8x + 4$ is divided by $2x + 1$.

Solution :

Step 1 : $2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$

Step 2 : Required remainder = Value of given polynomial $x^2 - 8x + 4$ at $x = -\frac{1}{2}$

$\therefore$ **Remainder** $= \left(-\frac{1}{2}\right)^2 - 8\left(-\frac{1}{2}\right) + 4$

$= \frac{1}{4} + 4 + 4 = 8\frac{1}{4}$ **Ans.**

2 Find the value of 'a' if the division of $ax^3 + 9x^2 + 4x - 10$ by $x + 3$ leaves a remainder of 5.

Solution :

$x + 3 = 0 \Rightarrow x = -3$

Given, remainder is 5; therefore :

The value of $ax^3 + 9x^2 + 4x - 10$ at $x = -3$ is 5

$\Rightarrow a(-3)^3 + 9(-3)^2 + 4(-3) - 10 = 5$

$\Rightarrow -27a + 81 - 12 - 10 = 5$ or $a = 2$ **Ans.**

3 When the polynomial $2x^3 - kx^2 + (5k - 3)x - 8$ is divided by $x - 2$, the remainder is 14. Find the value of 'k'.

Solution :

$x - 2 = 0 \Rightarrow x = 2$

Given, remainder is 14, therefore :

$$\Rightarrow \quad 2(2)^3 - k(2)^2 + (5k - 3) \times 2 - 8 = 14$$

$$\Rightarrow \quad 16 - 4k + 10k - 6 - 8 = 14$$

$$\Rightarrow \quad 6k = 12 \text{ and } \mathbf{k = 2}$$ **Ans.**

4 The polynomials $3x^3 - ax^2 + 5x - 13$ and $(a + 1)x^2 - 7x + 5$ leave the same remainder when divided by $x - 3$. Find the value of 'a'.

Solution :

$x - 3 = 0 \Rightarrow x = 3$

Since, the given polynomials leave the same remainder when divided by $x - 3$

Value of polynomial $3x^3 - ax^2 + 5x - 13$ at $x = 3$ is the same as the value of polynomial $(a + 1)\,x^2 - 7x + 5$ at $x = 3$

$$\Rightarrow \quad 3(3)^3 - a(3)^2 + 5 \times 3 - 13 = (a + 1)\,(3)^2 - 7 \times 3 + 5$$

$$\Rightarrow \quad 81 - 9a + 15 - 13 = 9a + 9 - 21 + 5$$

$$\Rightarrow \quad 18a = 90 \text{ and } \mathbf{a = 5}$$ **Ans.**

5 When $f(x) = x^3 + ax^2 - bx - 8$ is divided by $x - 2$, the remainder is zero and when divided by $x + 1$, the remainder is -30. Find the values of 'a' and 'b'.

Solution :

Since, $x - 2 = 0 \Rightarrow x = 2$

And, given that on dividing $f(x) = x^3 + ax^2 - bx - 8$ by $x - 2$, the remainder $= 0$

$$\Rightarrow \quad f(2) = 0$$

$$\Rightarrow \quad (2)^3 + a(2)^2 - b(2) - 8 = 0 \quad i.e. \quad 8 + 4a - 2b - 8 = 0$$

$$\Rightarrow \quad 4a - 2b = 0 \quad i.e. \quad 2a - b = 0 \quad \text{....I}$$

Again, given that on dividing $f(x) = x^3 + ax^2 - bx - 8$ by $x + 1$, the remainder $= -30$

$$\Rightarrow \quad f(-1) = -30 \qquad [x + 1 = 0 \Rightarrow x = -1]$$

$$\Rightarrow \quad (-1)^3 + a(-1)^2 - b(-1) - 8 = -30 \quad i.e. \quad -1 + a + b - 8 = -30$$

$$\Rightarrow \quad a + b = -21 \quad \text{....II}$$

On solving equations I and II, we get : $\mathbf{a = -7}$ **and** $\mathbf{b = -14}$ **Ans.**

6 What number should be added to $2x^3 - 3x^2 + x$ so that when the resulting polynomial is divided by $x - 2$, the remainder is 3 ?

Solution :

Let the number added be k so the resulting polynomial is

$$2x^3 - 3x^2 + x + k$$

Given, when this polynomial is divided by $x - 2$, the remainder $= 3$

$$\Rightarrow \quad 2(2)^3 - 3(2)^2 + 2 + k = 3 \qquad [x - 2 = 0 \Rightarrow x = 2]$$

$\Rightarrow \quad 16 - 12 + 2 + k = 3$ *i.e.,* $k = -3$

$\therefore$ **The required number to be added = –3** **Ans.**

8.3 Factor Theorem :

When a polynomial $f(x)$ is divided by $x - a$, the remainder = $f(a)$. And, if remainder $f(a) = 0$; $x - a$ is a factor of the polynomial $f(x)$.

For example :

Let $f(x) = x^2 - 5x + 6$ be divided by $x - 3$;

$$\text{then remainder} = f(3) = (3)^2 - 5 \times 3 + 6 = 0$$

$\therefore$ Remainder = 0

$\Rightarrow \quad x - 3$ is a factor of $f(x) = x^2 - 5x + 6$

7 Determine whether $x - 1$ is a factor of $x^6 - x^5 + x^4 + x^3 - x^2 - x + 1$ or not ?

Solution :

$x - 1 = 0 \quad \Rightarrow \quad x = 1$

$\therefore$ When given polynomial is divided by $x - 1$, the remainder

$$= (1)^6 - (1)^5 + (1)^4 + (1)^3 - (1)^2 - (1) + 1$$
$$= 1 - 1 + 1 + 1 - 1 - 1 + 1$$
$= 4 - 3 = 1$, *which is not equal to zero.*

$\therefore$ $x - 1$ is **not a factor** of the given polynomial. **Ans.**

8 If $x - 2$ is a factor of $x^2 - 7x + 2a$, find the value of a.

Solution :

$x - 2 = 0 \quad \Rightarrow \quad x = 2$

Since, $x - 2$ is a factor of polynomial $x^2 - 7x + 2a$

$\Rightarrow$ Remainder $= 0 \Rightarrow (2)^2 - 7(2) + 2a = 0 \Rightarrow$ $\boldsymbol{a = 5}$ **Ans.**

9 Find the value of 'k' if $(x - 2)$ is a factor of $x^3 + 2x^2 - kx + 10$. Hence, determine whether $(x + 5)$ is also a factor. **[2011]**

Solution :

$x - 2$ is a factor and $x - 2 = 0 \Rightarrow x = 2$

$\therefore$ The value of given expression $x^3 + 2x^2 - kx + 10$ is zero at $x = 2$

i.e. remainder = 0

$\Rightarrow \quad (2)^3 + 2(2)^2 - k \times 2 + 10 = 0$

$\Rightarrow \quad 8 + 8 - 2k + 10 = 0$

$\Rightarrow \quad$ $\boldsymbol{k = 13}$ **Ans.**

On substituting $k = 13$, the given expression becomes $x^3 + 2x^2 - 13x + 10$.

Now to check whether $(x + 5)$ is also a factor or not,

find the value of the given expression for $x = -5$. $[\because x + 5 = 0 \Rightarrow x = -5]$

$\therefore$ $x^3 + 2x^2 - 13x + 10$ (at $x = -5$)

$= (-5)^3 + 2(-5)^2 - 13(-5) + 10$

$= -125 + 50 + 65 + 10 = -125 + 125 = 0$

Since, the remainder is 0 $\Rightarrow$ **$(x + 5)$ is a factor** **Ans.**

10 Given that $x + 2$ and $x - 3$ are factors of $x^3 + ax + b$; calculate the values of a and b.

Solution :

Given, $x + 2$ is a factor of $x^3 + ax + b$;

$\Rightarrow$ $(-2)^3 + a(-2) + b = 0$ $[x + 2 = 0 \Rightarrow x = -2]$

$\Rightarrow$ $-2a + b = 8$ I

Again, given that :

$x - 3$ is a factor of $x^3 + ax + b$;

$\Rightarrow$ $(3)^3 + a(3) + b = 0$ $[x - 3 = 0 \Rightarrow x = 3]$

$\Rightarrow$ $3a + b = -27$ II

On solving equations I and II, we get $a = -7$ and $b = -6$ **Ans.**

11 Polynomial $x^3 - ax^2 + bx - 6$ leaves remainder $- 8$ when divided by $x - 1$ and $x - 2$ is a factor of it. Find the values of 'a' and 'b'.

Solution :

On dividing by $x - 1$, the polynomial $x^3 - bx^2 + bx - 6$ leaves remainder $- 8$

$\Rightarrow$ $(1)^3 - a(1)^2 + b(1) - 6 = - 8$ $[x - 1 = 0 \Rightarrow x = 1]$

$\Rightarrow$ $- a + b = - 3$

i.e. $a - b = 3$ I

$(x - 2)$ is a factor of polynomial $x^3 - bx^2 + bx - 6$

$\Rightarrow$ $(2)^3 - a(2)^2 + b(2) - 6 = 0$ $[x - 2 = 0 \Rightarrow x = 2]$

$\Rightarrow$ $8 - 4a + 2b - 6 = 0$

i.e. $2a - b = 1$ II

On solving equations I and II, we get :

$a = - 2$ and $b = - 5$ **Ans.**

EXERCISE 8(A)

1. Find, in each case, the remainder when :
   (i) $x^4 - 3x^2 + 2x + 1$ is divided by $x - 1$.
   (ii) $x^3 + 3x^2 - 12x + 4$ is divided by $x - 2$.
   (iii) $x^4 + 1$ is divided by $x + 1$.
2. Show that :
   (i) $x - 2$ is a factor of $5x^2 + 15x - 50$.
   (ii) $3x + 2$ is a factor of $3x^2 - x - 2$.
3. Use the Remainder Theorem to find which of the following is a factor of $2x^3 + 3x^2 - 5x - 6$.
   (i) $x + 1$ (ii) $2x - 1$
   (iii) $x + 2$
4. (i) If $2x + 1$ is a factor of $2x^2 + ax - 3$, find the value of a.
   (ii) Find the value of k, if $3x - 4$ is a factor of expression $3x^2 + 2x - k$.
5. Find the values of constants a and b when $x - 2$ and $x + 3$ both are the factors of expression $x^3 + ax^2 + bx - 12$.
6. Find the value of k, if $2x + 1$ is a factor of $(3k + 2)\, x^3 + (k - 1)$.
7. Find the value of a, if $x - 2$ is a factor of $2x^5 - 6x^4 - 2ax^3 + 6ax^2 + 4ax + 8$.
8. Find the values of m and n so that $x - 1$ and $x + 2$ both are factors of
   $$x^3 + (3m + 1)\, x^2 + nx - 18.$$
9. When $x^3 + 2x^2 - kx + 4$ is divided by $x - 2$, the remainder is k. Find the value of constant k.
10. Find the value of a, if the division of $ax^3 + 9x^2 + 4x - 10$ by $x + 3$ leaves a remainder 5.
11. If $x^3 + ax^2 + bx + 6$ has $x - 2$ as a factor and leaves a remainder 3 when divided by $x - 3$, find the values of a and b. **[2005]**
12. The expression $2x^3 + ax^2 + bx - 2$ leaves remainder 7 and 0 when divided by $2x - 3$ and $x + 2$ respectively. Calculate the values of a and b.
13. What number should be added to $3x^3 - 5x^2 + 6x$ so that when resulting polynomial is divided by $x - 3$, the remainder is 8 ?
14. What number should be subtracted from $x^3 + 3x^2 - 8x + 14$ so that on dividing it by $x - 2$, the remainder is 10 ?
15. The polynomials $2x^3 - 7x^2 + ax - 6$ and $x^3 - 8x^2 + (2a + 1)x - 16$ leave the same remainder when divided by $x - 2$. Find the value of 'a'.
16. If $(x - 2)$ is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when the expression is divided by $(x - 3)$, it leaves a remainder 52, find the values of a and b. **[2013]**
17. Find 'a' if the two polynomials $ax^3 + 3x^2 - 9$ and $2x^3 + 4x + a$, leave the same remainder when divided by $x + 3$. **[2015]**

8.4 Using the Factor Theorem to factorise the given polynomial :

[Factorising a polynomial completely after obtaining one factor by factor theorem]

When expression $f(x)$ is divided by $x - a$, the remainder $= f(a)$.

If the remainder $f(a) = 0$.

$\Rightarrow$ $x - a$ is a factor of expression $f(x)$.

Conversely, if for the expression $f(x)$, $f(a) = 0$; $\Rightarrow$ $(x - a)$ is a factor.

For example :

(i) Let $f(x) = x^2 - 7x + 10$; then

$$f(2) = (2)^2 - 7 \times 2 + 10 = 0$$

$\Rightarrow$ $x - 2$ is a factor of $f(x) = x^2 - 7x + 10$

(ii) Let $f(x) = 2x^2 - x - 3$; then $f(-1) = 2(-1)^2 - (-1) - 3 = 0$

$\Rightarrow$ $x + 1$ is a factor of $f(x) = 2x^2 - x - 3$ and so on.

12 Using the Factor Theorem, show that $(x - 2)$ is a factor of $3x^2 - 5x - 2$. Hence, factorise the given expression.

Solution :

$\because\ x - 2 = 0 \Rightarrow x = 2$

$\therefore$ Remainder $=$ The value of $3x^2 - 5x - 2$ at $x = 2$

$= 3(2)^2 - 5(2) - 2$

$= 12 - 10 - 2 = 0$

$\Rightarrow$ **$(x - 2)$ is a factor of $3x^2 - 5x - 2$**

$$\begin{array}{r|l} & 3x + 1 \\ \hline x - 2 & 3x^2 - 5x - 2 \\ & 3x^2 - 6x \\ & - \quad + \\ \hline & \qquad x - 2 \\ & \qquad x - 2 \\ & \qquad - \quad + \\ \hline & \qquad\quad \times \end{array}$$

Now, dividing $(3x^2 - 5x - 2)$ by $(x - 2)$, we get quotient $= 3x + 1$

$\therefore$ **$3x^2 - 5x - 2 = (x - 2)(3x + 1)$** **Ans.**

13 Show that $2x + 7$ is a factor of $2x^3 + 5x^2 - 11x - 14$. Hence, factorise the given expression completely, using the factor theorem. [2006]

Solution :

$2x + 7 = 0 \Rightarrow x = -\frac{7}{2}$

Remainder $=$ Value of $2x^3 + 5x^2 - 11x - 14$ at $x = -\frac{7}{2}$

$= 2\left(-\frac{7}{2}\right)^3 + 5\left(-\frac{7}{2}\right)^2 - 11\left(-\frac{7}{2}\right) - 14$

$= -\frac{343}{4} + \frac{245}{4} + \frac{77}{2} - 14$

$= \frac{-343 + 245 + 154 - 56}{4} = 0$

$$\begin{array}{r|l} & x^2 - x - 2 \\ \hline 2x + 7 & 2x^3 + 5x^2 - 11x - 14 \\ & 2x^3 + 7x^2 \\ & - \quad - \\ \hline & \quad - 2x^2 - 11x - 14 \\ & \quad - 2x^2 - 7x \\ & \quad + \quad + \\ \hline & \qquad - 4x - 14 \\ & \qquad - 4x - 14 \\ & \qquad + \quad + \\ \hline & \qquad\quad \times \end{array}$$

$\Rightarrow$ $(2x + 7)$ is a factor of $2x^3 + 5x^2 - 11x - 14$.

$\therefore$ **$2x^3 + 5x^2 - 11x - 14$** $= (2x + 7)(x^2 - x - 2)$

$= (2x + 7)(x^2 - 2x + x - 2)$

$= (2x + 7)[x(x - 2) + 1(x - 2)]$

$=$ **$(2x + 7)(x - 2)(x + 1)$** **Ans.**

14 Using the Remainder Theorem, factorise the expression $2x^3 + x^2 - 2x - 1$ completely.

Solution :

First Step : For $x = 1$, the value of given expression

$= 2(1)^3 + (1)^2 - 2(1) - 1.$

$= 2 + 1 - 2 - 1 = 0$

$\Rightarrow$ $x - 1$ is a factor of $2x^3 + x^2 - 2x - 1$

$$\begin{array}{r|l} & 2x^2 + 3x + 1 \\ \hline x - 1 & 2x^3 + x^2 - 2x - 1 \\ & 2x^3 - 2x^2 \\ & - \quad + \\ \hline & \quad 3x^2 - 2x - 1 \\ & \quad 3x^2 - 3x \\ & \quad - \quad + \\ \hline & \qquad\quad x - 1 \\ & \qquad\quad x - 1 \\ & \qquad\quad - \quad + \\ \hline & \qquad\qquad \times \end{array}$$

Second Step :

$$2x^3 + x^2 - 2x - 1 = (x-1)(2x^2 + 3x + 1)$$
$$= (x-1)(2x^2 + 2x + x + 1)$$
$$= (x-1)[2x(x+1) + 1(x+1)]$$
$$= (x-1)(x+1)(2x+1) \qquad \textbf{Ans.}$$

15 Find the values of 'a' and 'b' so that the polynomial $x^3 + ax^2 + bx - 45$ has $(x-1)$ and $(x+5)$ as its factors.
For the values of 'a' and 'b', as obtained above, factorise the given polynomial completely.

Solution :

$(x - 1)$ is a factor of given polynomial $x^3 + ax^2 + bx - 45$

$\Rightarrow \quad (1)^3 + a(1)^2 + b(1) - 45 = 0 \qquad [x - 1 = 0 \Rightarrow x = 1]$

i.e. $\quad a + b = 44 \qquad$ ...I

$(x + 5)$ is a factor of given polynomial

$\Rightarrow \quad (-5)^3 + a(-5)^2 + b(-5) - 45 = 0 \qquad [x + 5 = 0 \Rightarrow x = -5]$

$\Rightarrow \quad -125 + 25a - 5b - 45 = 0$

i.e. $\quad 5a - b = 34 \qquad$ II

On solving equations I and II, we get :

$a = 13$ and $b = 31$ **Ans.**

$\therefore$ The given polynomial $x^3 + ax^2 + bx - 45$
$= x^3 + 13x^2 + 31x - 45$

Now divide this polynomial by $(x - 1)$ as shown alongside :

$$\begin{array}{r|l} & x^2 + 14x + 45 \\ \hline x - 1 & x^3 + 13x^2 + 31x - 45 \\ & x^3 - x^2 \\ & - \quad + \\ \hline & \quad 14x^2 + 31x - 45 \\ & \quad 14x^2 - 14x \\ & \quad - \quad + \\ \hline & \qquad 45x - 45 \\ & \qquad 45x - 45 \\ & \qquad - \quad + \\ \hline & \qquad \quad \times \end{array}$$

$\therefore \quad x^3 + 13x^2 + 31x - 45$
$$= (x-1)(x^2 + 14x + 45)$$
$$= (x-1)(x^2 + 9x + 5x + 45)$$
$$= (x-1)[x(x+9) + 5(x+9)]$$
$$= (x-1)(x+9)(x+5) \qquad \textbf{Ans.}$$

16 If $(x - 2)$ is a factor of $2x^3 - x^2 - px - 2$
(i) find the value of p.
(ii) with the value of p, factorise the above expression completely. **[2008]**

Solution :

(i) $\quad x - 2 = 0 \quad \Rightarrow \quad x = 2$

Since, $(x - 2)$ is a factor of given expression

$\therefore \quad$ Remainder $= 0$

$\Rightarrow\ 2(2)^3 - (2)^2 - p \times 2 - 2 = 0$

$\Rightarrow\ 10 - 2p = 0$ and $p = 5$ **Ans.**

(ii) $\therefore\ 2x^3 - x^2 - px - 2 = 2x^3 - x^2 - 5x - 2$

On dividing $2x^3 - x^2 - 5x - 2$ by $x - 2$, we get :

$$\begin{array}{r|l} & 2x^2 + 3x + 1 \\ \hline x - 2 & 2x^3 - x^2 - 5x - 2 \\ & 2x^3 - 4x^2 \\ & - \quad + \\ \hline & \quad 3x^2 - 5x - 2 \\ & \quad 3x^2 - 6x \\ & \quad - \quad + \\ \hline & \qquad x - 2 \\ & \qquad x - 2 \\ & \qquad - \quad + \\ \hline & \qquad \times \end{array}$$

quotient $= 2x^2 + 3x + 1$

$\therefore\ \mathbf{2x^3 - x^2 - 5x - 2}$

$= (x - 2)(2x^2 + 3x + 1)$

$= (x - 2)(2x^2 + 2x + x + 1)$

$= (x - 2)[2x(x + 1) + 1(x + 1)]$

$= \mathbf{(x - 2)(x + 1)(2x + 1)}$ **Ans.**

EXERCISE 8(B)

1. Using the Factor Theorem, show that :
   (i) $(x - 2)$ is a factor of $x^3 - 2x^2 - 9x + 18$. Hence, factorise the expression $x^3 - 2x^2 - 9x + 18$ completely.
   (ii) $(x + 5)$ is a factor of $2x^3 + 5x^2 - 28x - 15$. Hence, factorise the expression $2x^3 + 5x^2 - 28x - 15$ completely.
   (iii) $(3x + 2)$ is a factor of $3x^3 + 2x^2 - 3x - 2$. Hence, factorise the expression $3x^3 + 2x^2 - 3x - 2$ completely.

2. Using the Remainder Theorem, factorise each of the following completely :
   (i) $3x^3 + 2x^2 - 19x + 6$ **[2012]**
   (ii) $2x^3 + x^2 - 13x + 6$
   (iii) $3x^3 + 2x^2 - 23x - 30$
   (iv) $4x^3 + 7x^2 - 36x - 63$
   (v) $x^3 + x^2 - 4x - 4$. **[2004]**

3. Using the Remainder Theorem, factorise the expression $3x^3 + 10x^2 + x - 6$. Hence, solve the equation $3x^3 + 10x^2 + x - 6 = 0$

4. Factorise the expression
   $f(x) = 2x^3 - 7x^2 - 3x + 18$.
   Hence, find all possible values of x for which $f(x) = 0$.

5. Given that $x - 2$ and $x + 1$ are factors of $f(x) = x^3 + 3x^2 + ax + b$; calculate the values of a and b. Hence, find all the factors of $f(x)$.

6. The expression $4x^3 - bx^2 + x - c$ leaves remainders 0 and 30 when divided by $x + 1$ and $2x - 3$ respectively. Calculate the values of b and c. Hence, factorise the expression completely.

7. If $x + a$ is a common factor of expressions $f(x) = x^2 + px + q$ and $g(x) = x^2 + mx + n$; show that : $a = \dfrac{n - q}{m - p}$

8. The polynomials $ax^3 + 3x^2 - 3$ and $2x^3 - 5x + a$, when divided by $x - 4$, leave the same remainder in each case. Find the value of a.

9. Find the value of 'a', if $(x - a)$ is a factor of $x^3 - ax^2 + x + 2$. **[2003]**

10. Find the number that must be subtracted from the polynomial $3y^3 + y^2 - 22y + 15$, so that the resulting polynomial is completely divisible by $y + 3$.

EXERCISE 8(C)

1. Show that $(x - 1)$ is a factor of
$$x^3 - 7x^2 + 14x - 8.$$
Hence, completely factorise the given expression.

2. Using Remainder Theorem, factorise :
$x^3 + 10x^2 - 37x + 26$ completely. **[2014]**

3. When $x^3 + 3x^2 - mx + 4$ is divided by $x - 2$, the remainder is $m + 3$. Find the value of m.

4. What should be subtracted from $3x^3 - 8x^2 + 4x - 3$, so that the resulting expression has $x + 2$ as a factor ?

> The number to be subtracted
> = Remainder obtained on dividing
> $3x^3 - 8x^2 + 4x - 3$ by $x + 2$.

5. If $(x + 1)$ and $(x - 2)$ are factors of $x^3 + (a + 1)x^2 - (b - 2)x - 6$, find the values of a and b. And then, factorise the given expression completely.

6. If $x - 2$ is a factor of $x^2 + ax + b$ and $a + b = 1$, find the values of a and b.

7. Factorise $x^3 + 6x^2 + 11x + 6$ completely using factor theorem.

8. Find the value of 'm', if $mx^3 + 2x^2 - 3$ and $x^2 - mx + 4$ leave the same remainder when each is divided by $x - 2$.

9. The polynomial $px^3 + 4x^2 - 3x + q$ is completely divisible by $x^2 - 1$; find the values of p and q. Also, for these values of p and q factorize the given polynomial completely.

10. Find the number which should be added to $x^2 + x + 3$ so that the resulting polynomial is completely divisible by $(x + 3)$.

11. When the polynomial $x^3 + 2x^2 - 5ax - 7$ is divided by $(x - 1)$, the remainder is A and when the polynomial $x^3 + ax^2 - 12x + 16$ is divided by $(x + 2)$, the remainder is B. Find the value of 'a' if $2A + B = 0$.

12. $(3x + 5)$ is a factor of the polynomial $(a - 1)x^3 + (a + 1)x^2 - (2a + 1)x - 15$. Find the value of '$a$'. For this value of '$a$', factorise the given polynomial completely.

13. When divided by $x - 3$ the polynomials $x^3 - px^2 + x + 6$ and $2x^3 - x^2 - (p + 3)x - 6$ leave the same remainder. Find the value of 'p'. **[2010]**

14. Use the Remainder Theorem to factorise the following expression :
$2x^3 + x^2 - 13x + 6$ **[2010]**