Determinants

1. Let A be a 2 × 2 matrix

   Statement-1 : adj (adj A) = A

   Statement-2 : |adj A| = |A| **[AIEEE-2009]**

   (1) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

   (2) Statement-1 is true, Statement-2 is false

   (3) Statement-1 is false, Statement-2 is true

   (4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

2. Let a, b, c be such that $b(a + c) \neq 0$. If

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0,$$

   then the value of n is **[AIEEE-2009]**

   (1) Any even integer (2) Any odd integer

   (3) Any integer (4) Zero

3. Consider the system of linear equations:

   $x_1 + 2x_2 + x_3 = 3$

   $2x_1 + 3x_2 + x_3 = 3$

   $3x_1 + 5x_2 + 2x_3 = 1$

   The system has **[AIEEE-2010]**

   (1) Infinite number of solutions

   (2) Exactly 3 solutions

   (3) A unique solutions

   (4) No solution

4. If the trival solution is the only solution of the system of equations

   $x - ky + z = 0$

   $kx + 3y - kz = 0$

   $3x + y - z = 0$

   then the set of all values of k is **[AIEEE-2011]**

   (1) $R - \{-3\}$ (2) $\{2, -3\}$

   (3) $R - \{2, -3\}$ (4) $R - \{2\}$

5. Let P and Q be 3 × 3 matrices with $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to **[AIEEE-2012]**

   (1) 1 (2) 0

   (3) –1 (4) –2

6. The number of values of k, for which the system of equations

   $(k + 1)x + 8y = 4k$

   $kx + (k + 3)y = 3k - 1$

   has no solution, is **[JEE (Main)-2013]**

   (1) Infinite (2) 1

   (3) 2 (4) 3

7. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3 × 3 matrix A and $|A| = 4$, then α is equal to **[JEE (Main)-2013]**

   (1) 4 (2) 11

   (3) 5 (4) 0

8. If $\alpha, \beta \neq 0$, and $f(n) = \alpha^n + \beta^n$ and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = K(1-\alpha)^2 (1-\beta)^2 (\alpha - \beta)^2,$$

   then K is equal to **[JEE (Main)-2014]**

   (1) 1 (2) –1

   (3) $\alpha\beta$ (4) $\frac{1}{\alpha\beta}$

9. The set of all values of λ for which the system of linear equations

   $2x_1 - 2x_2 + x_3 = \lambda x_1$

   $2x_1 - 3x_2 + 2x_3 = \lambda x_2$

   $-x_1 + 2x_2 = \lambda x_3$

   has a non-trivial solution **[JEE (Main)-2015]**

(1) Is an empty set

(2) Is a singleton

(3) Contains two elements

(4) Contains more than two elements

10. The system of linear equations

$x + \lambda y - z = 0$

$\lambda x - y - z = 0$

$x + y - \lambda z = 0$

has a non-trivial solution for **[JEE (Main)-2016]**

(1) Exactly one value of λ

(2) Exactly two values of λ

(3) Exactly three values of λ

(4) Infinitely many values of λ

11. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and A.adj $A = A\,A^T$, then $5a + b$ is equal to : **[JEE (Main)-2016]**

(1) 5 (2) 4

(3) 13 (4) –1

12. Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$. If

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k,$$ then k is equal to **[JEE (Main)-2017]**

(1) z (2) –1

(3) 1 (4) $-z$

13. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to **[JEE (Main)-2017]**

(1) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$ (2) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$

(3) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$ (4) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$

14. If S is the set of distinct values of b for which the following system of linear equations

$x + y + z = 1$

$x + ay + z = 1$

$ax + by + z = 0$

has no solution, then S is **[JEE (Main)-2017]**

(1) An infinite set

(2) A finite set containing two or more elements

(3) A singleton

(4) An empty set

15. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$, then the ordered pair (A, B) is equal to **[JEE (Main)-2018]**

(1) (–4, –5) (2) (–4, 3)

(3) (–4, 5) (4) (4, 5)

16. If the system of linear equations

$x + ky + 3z = 0$

$3x + ky - 2z = 0$

$2x + 4y - 3z = 0$

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to **[JEE (Main)-2018]**

(1) –10 (2) 10

(3) –30 (4) 30

17. The system of linear equations

$x + y + z = 2$

$2x + 3y + 2z = 5$

$2x + 3y + (a^2 - 1)z = a + 1$ **[JEE (Main)-2019]**

(1) has infinitely many solutions for $a = 4$

(2) is inconsistent when $|a| = \sqrt{3}$

(3) has a unique solution for $|a| = \sqrt{3}$

(4) is inconsistent when $a = 4$

18. If the system of linear equations

$x - 4y + 7z = g$

$3y - 5z = h$

$-2x + 5y - 9z = k$

is consistent, then **[JEE (Main)-2019]**

(1) $g + h + k = 0$ (2) $g + 2h + k = 0$

(3) $g + h + 2k = 0$ (4) $2g + h + k = 0$

19. Let $d \in R$, and

$$A = \begin{bmatrix} -2 & 4+d & (\sin\theta) - 2 \\ 1 & (\sin\theta) + 2 & d \\ 5 & (2\sin\theta) - d & (-\sin\theta) + 2 + 2d \end{bmatrix},$$

$\theta \in [0, 2\pi]$. If the minimum value of det(A) is 8, then a value of d is **[JEE (Main)-2019]**

(1) –5 (2) $2(\sqrt{2}+1)$

(3) –7 (4) $2(\sqrt{2}+2)$

20. If the system of equations

$x + y + z = 5$

$x + 2y + 3z = 9$

$x + 3y + \alpha z = \beta$

has infinitely many solutions, then $\beta - \alpha$ equals **[JEE (Main)-2019]**

(1) 18 (2) 21

(3) 8 (4) 5

21. The number of values of $\theta \in (0, \pi)$ for which the system of linear equations

$x + 3y + 7z = 0$

$-x + 4y + 7z = 0$

$(\sin 3\theta)x + (\cos 2\theta)y + 2z = 0$

has a non-trivial solution, is **[JEE (Main)-2019]**

(1) Four (2) One

(3) Three (4) Two

22. If the system of linear equations

$2x + 2y + 3z = a$

$3x - y + 5z = b$

$x - 3y + 2z = c$

where a, b, c are non-zero real numbers, has more than one solution, then **[JEE (Main)-2019]**

(1) $b - c + a = 0$

(2) $b + c - a = 0$

(3) $a + b + c = 0$

(4) $b - c - a = 0$

23. If $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a + b + c)(x + a + b + c)^2$, $x \neq 0$ and $a + b + c \neq 0$, then x is equal to **[JEE (Main)-2019]**

(1) $2(a + b + c)$ (2) $-(a + b + c)$

(3) abc (4) $-2(a + b + c)$

24. An ordered pair (α, β) for which the system of linear equations

$(1 + \alpha)x + \beta y + z = 2$

$\alpha x + (1 + \beta)y + z = 3$

$\alpha x + \beta y + 2z = 2$

has a unique solution, is **[JEE (Main)-2019]**

(1) (1, –3) (2) (2, 4)

(3) (–3, 1) (4) (–4, 2)

25. If $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$; then for all $\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$, det ($A$) lies in the interval **[JEE (Main)-2019]**

(1) $\left(1, \frac{5}{2}\right]$ (2) $\left(0, \frac{3}{2}\right]$

(3) $\left[\frac{5}{2}, 4\right)$ (4) $\left(\frac{3}{2}, 3\right]$

26. The set of all values of λ for which the system of linear equations

$x - 2y - 2z = \lambda x$

$x + 2y + z = \lambda y$

$-x - y = \lambda z$ **[JEE (Main)-2019]**

(has a non-trivial solution)

(1) Contains exactly two elements

(2) Contains more than two elements

(3) Is a singleton

(4) Is an empty set

27. The greatest value of $c \in R$ for which the system of linear equations

$x - cy - cz = 0$

$cx - y + cz = 0$

$cx + cy - z = 0$

has a non-trivial solution, is **[JEE (Main)-2019]**

(1) –1 (2) 0

(3) 2 (4) $\frac{1}{2}$

28. If the system of linear equations

$x - 2y + kz = 1$

$2x + y + z = 2$

$3x - y - kz = 3$

has a solution (x, y, z), $z \neq 0$, then (x, y) lies on the straight line whose equation is **[JEE (Main)-2019]**

(1) $3x - 4y - 4 = 0$ (2) $3x - 4y - 1 = 0$

(3) $4x - 3y - 1 = 0$ (4) $4x - 3y - 4 = 0$

29. Let α and β be the roots of the equation $x^2 + x + 1 = 0$. Then for $y \neq 0$ in R,

$$\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$$ is equal to

(1) $y(y^2 - 1)$ (2) $y^3 - 1$

(3) $y(y^2 - 3)$ (4) y^3

30. If the system of equations $2x + 3y - z = 0$, $x + ky - 2z = 0$ and $2x - y + z = 0$ has a non-trivial solution (x, y, z), then $\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + k$ is equal to **[JEE (Main)-2019]**

(1) $\frac{1}{2}$ (2) -4

(3) $\frac{3}{4}$ (4) $-\frac{1}{4}$

31. If $\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ and

$\Delta_2 = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}$, $x \neq 0$; then for all $\theta \in \left(0, \frac{\pi}{2}\right)$ **[JEE (Main)-2019]**

(1) $\Delta_1 + \Delta_2 = -2x^3$

(2) $\Delta_1 - \Delta_2 = -2x^3$

(3) $\Delta_1 + \Delta_2 = -2(x^3 + x - 1)$

(4) $\Delta_1 - \Delta_2 = x(\cos 2\theta - \cos 4\theta)$

32. If the system of linear equations

$x + y + z = 5$

$x + 2y + 2z = 6$

$x + 3y + \lambda z = \mu$, $(\lambda, \mu \in R)$, has infinitely many solutions, then the value of $\lambda + \mu$ is **[JEE (Main)-2019]**

(1) 10 (2) 12

(3) 7 (4) 9

33. The sum of the real roots of the equation

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0,$$ is equal to **[JEE (Main)-2019]**

(1) 6 (2) 0

(3) -4 (4) 1

34. Let λ be a real number for which the system of linear equations

$x + y + z = 6$

$4x + \lambda y - \lambda z = \lambda - 2$

$3x + 2y - 4z = -5$

has infinitely many solutions. Then λ is a root of the quadratic equation **[JEE (Main)-2019]**

(1) $\lambda^2 + 3\lambda - 4 = 0$ (2) $\lambda^2 - \lambda - 6 = 0$

(3) $\lambda^2 + \lambda - 6 = 0$ (4) $\lambda^2 - 3\lambda - 4 = 0$

35. If $[x]$ denotes the greatest integer $\leq x$, then the system of linear equations $[\sin\theta]x + [-\cos\theta]y = 0$

$[\cot\theta]x + y = 0$ **[JEE (Main)-2019]**

(1) Has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and have infinitely many solutions if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$.

(2) Has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$.

(3) Have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and has a unique solution if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$.

(4) Have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$.

36. A value of $\theta \in (0, \pi/3)$, for which

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0,$$ is **[JEE (Main)-2019]**

(1) $\frac{\pi}{18}$ (2) $\frac{7\pi}{36}$

(3) $\frac{7\pi}{24}$ (4) $\frac{\pi}{9}$

37. If the system of linear equations

$2x + 2ay + az = 0$

$2x + 3by + bz = 0$

$2x + 4cy + cz = 0$,

where $a, b, c \in R$ are non-zero and distinct;

has a non-zero solution, then **[JEE (Main)-2020]**

(1) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

(2) a, b, c are in A.P.

(3) $a + b + c = 0$

(4) a, b, c are in G.P.

38. For which of the following ordered pairs (μ, δ), the system of linear equations

$x + 2y + 3z = 1$

$3x + 4y + 5z = \mu$

$4x + 4y + 4z = \delta$

is inconsistent? **[JEE (Main)-2020]**

(1) (4, 3)

(2) (4, 6)

(3) (3, 4)

(4) (1, 0)

39. The system of linear equations

$\lambda x + 2y + 2z = 5$

$2\lambda x + 3y + 5z = 8$

$4x + \lambda y + 6z = 10$ has **[JEE (Main)-2020]**

(1) Infinitely many solutions when $\lambda = \mathbf{2}$

(2) No solution when $\lambda = \mathbf{8}$

(3) A unique solution when $\lambda = -\mathbf{8}$

(4) No solution when $\lambda = \mathbf{2}$

40. If for some α and β in R, the intersection of the following three planes

$x + 4y - 2z = 1$

$x + 7y - 5z = \beta$

$x + 5y + \alpha z = 5$

is a line in R^3, then $\alpha + \beta$ is equal to **[JEE (Main)-2020]**

(1) –10 (2) 0

(3) 2 (4) 10

41. If $x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n}\theta$ and $y = \sum_{n=0}^{\infty} \cos^{2n}\theta$,

for $0 < \theta < \frac{\pi}{4}$, then **[JEE (Main)-2020]**

(1) $x(1 - y) = 1$ (2) $y(1 + x) = 1$

(3) $y(1 - x) = 1$ (4) $x(1 + y) = 1$

42. The following system of linear equations

$7x + 6y - 2z = 0$

$3x + 4y + 2z = 0$

$x - 2y - 6z = 0$, has **[JEE (Main)-2020]**

(1) Infinitely many solutions, (x, y, z) satisfying $x = 2z$

(2) No solution

(3) Only the trivial solution

(4) Infinitely many solutions, (x, y, z) satisfying $y = 2z$

43. Let S be the set of all $\lambda \in R$ for which the system of linear equations

$2x - y + 2z = 2$

$x - 2y + \lambda z = -4$

$x + \lambda y + z = 4$

has no solution. Then the set S **[JEE (Main)-2020]**

(1) Is a singleton.

(2) Contains more than two elements.

(3) Is an empty set.

(4) Contains exactly two elements.

44. If $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} =$

$Ax^3 + Bx^2 + Cx + D$, then $B + C$ is equal to **[JEE (Main)-2020]**

(1) 9 (2) –1

(3) 1 (4) –3

45. If the system of equations

$x + y + z = 2$

$2x + 4y - z = 6$

$3x + 2y + \lambda z = \mu$

has infinitely many solutions, then **[JEE (Main)-2020]**

(1) $2\lambda - \mu = 5$

(2) $\lambda - 2\mu = -5$

(3) $\lambda + 2\mu = 14$

(4) $2\lambda + \mu = 14$

46. Suppose the vectors x_1, x_2 and x_3 are the solutions of the system of linear equations, $Ax = b$ when the vector b on the right side is equal to b_1, b_2 and b_3 respectively. If

$x_1 = \begin{bmatrix}1\\1\\1\end{bmatrix}, x_2 = \begin{bmatrix}0\\2\\1\end{bmatrix}, x_3 = \begin{bmatrix}0\\0\\1\end{bmatrix}, b_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}, b_2 = \begin{bmatrix}0\\2\\0\end{bmatrix}$ and

$b_3 = \begin{bmatrix}0\\0\\2\end{bmatrix}$, then the determinant of A is equal to

[JEE (Main)-2020]

(1) 4 (2) $\frac{1}{2}$

(3) 2 (4) $\frac{3}{2}$

47. Let $\lambda \in R$. The system of linear equations

$2x_1 - 4x_2 + \lambda x_3 = 1$

$x_1 - 6x_2 + x_3 = 2$

$\lambda x_1 - 10x_2 + 4x_3 = 3$

is inconsistent for **[JEE (Main)-2020]**

(1) exactly two values of λ

(2) exactly one positive value of λ

(3) every value of λ

(4) exactly one negative value of λ

48. If $a + x = b + y = c + z + 1$, where a, b, c, x, y, z are non-zero distinct real numbers, then

$\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$ is equal to **[JEE (Main)-2020]**

(1) $y(b - a)$ (2) $y(a - b)$

(3) $y(a - c)$ (4) 0

49. If the system of linear equations

$x + y + 3z = 0$

$x + 3y + k^2z = 0$

$3x + y + 3z = 0$

has a non-zero solution (x, y, z) for some $k \in R$, then $x + \left(\frac{y}{z}\right)$ is equal to **[JEE (Main)-2020]**

(1) 9 (2) 3

(3) –9 (4) –4

50. The values of λ and μ for which the system of linear equations

$x + y + z = 2$

$x + 2y + 3z = 5$

$x + 3y + \lambda z = \mu$

has infinitely many solutions are, respectively

[JEE (Main)-2020]

(1) 5 and 7 (2) 6 and 8

(3) 4 and 9 (4) 5 and 8

51. If the system of linear equations,

$x + y + z = 6$

$x + 2y + 3z = 10$

$3x + 2y + \lambda z = \mu$

has more than two solutions, then $\mu - \lambda^2$ is equal to _____. **[JEE (Main)-2020]**

52. Let S be the set of all integer solutions, (x, y, z), of the system of equations

$x - 2y + 5z = 0$

$-2x + 4y + z = 0$

$-7x + 14y + 9z = 0$

such that $15 \le x^2 + y^2 + z^2 \le 150$. Then, the number of elements in the set S is equal to _____ **[JEE (Main)-2020]**

53. If the system of equations

$x - 2y + 3z = 9$

$2x + y + z = b$

$x - 7y + az = 24$, has infinitely many solutions, then $a - b$ is equal to _____. **[JEE (Main)-2020]**

54. The sum of distinct values of λ for which the system of equations

$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$

$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$

$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0,$

has non-zero solutions, is _____ .

[JEE (Main)-2020]

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