

4***Linear Inequations***
(In one variable)**4.1 Introduction :**

If x and y are two quantities; then both of these quantities will satisfy any one of the following four conditions (relations) :

i.e. either (i) $x > y$ (ii) $x \geq y$ (iii) $x < y$ or (iv) $x \leq y$

Each of the four conditions, given above, is an **inequation**.

In the same way, each of the following also represents an inequation :

$x < 8$, $x \geq 5$, $-x + 4 \leq 3$, $x + 8 > 4$, etc.

4.2 Linear Inequations In One Variable :

If a , b and c are real numbers, then each of the following is called a **linear inequation** in one variable :

- (i) $ax + b > c$. Read as : $ax + b$ is greater than c .
- (ii) $ax + b < c$. Read as : $ax + b$ is less than c .
- (iii) $ax + b \geq c$. Read as : $ax + b$ is greater than or equal to c .
- (iv) $ax + b \leq c$. Read as : $ax + b$ is less than or equal to c .

In an inequation, the signs ' $>$ ', ' $<$ ', ' \geq ' and ' \leq ' are called signs of inequality.

4.3 Solving a Linear Inequation Algebraically :

To solve a given linear inequation means to find the value or values of the variable used in it.

Thus; (i) to solve the inequation $3x + 5 > 8$ means to find the variable x .

(ii) to solve the inequation $8 - 5y \leq 3$ means to find the variable y and so on

The following working rules must be adopted for solving a given linear inequation :

Rule 1 : On transferring a positive term from one side of an inequation to its other side, the sign of the term becomes negative.

$$\begin{aligned} \text{e.g. } 2x + 3 &> 7 \Rightarrow 2x > 7 - 3, \quad 5x + 4 \leq 15 \Rightarrow 5x \leq 15 - 4, \\ 23 &\geq 4x + 15 \Rightarrow 23 - 15 \geq 4x \quad \text{and so on.} \end{aligned}$$

Rule 2 : On transferring a negative term from one side of an inequation to its other side, the sign of the term becomes positive.

$$\begin{aligned} \text{e.g. } 2x - 3 &> 7 \Rightarrow 2x > 7 + 3, \quad 5x - 4 \leq 15 \Rightarrow 5x \leq 15 + 4, \\ 23 &\geq 4x - 15 \Rightarrow 23 + 15 \geq 4x \quad \text{and so on.} \end{aligned}$$

Rule 3 : If each term of an inequation be multiplied or divided by the same positive number, the sign of inequality remains the same.

That is, if p is positive

$$(i) \quad x < y \Rightarrow px < py \quad \text{and} \quad \left(\frac{x}{p}\right) < \left(\frac{y}{p}\right),$$

$$(ii) \quad x > y \Rightarrow px > py \quad \text{and} \quad \left(\frac{x}{p}\right) > \left(\frac{y}{p}\right),$$

$$(iii) \quad x \leq y \Rightarrow px \leq py \quad \text{and} \quad \left(\frac{x}{p}\right) \leq \left(\frac{y}{p}\right)$$

$$\text{and, (iv)} \quad x \geq y \Rightarrow px \geq py \quad \text{and} \quad \left(\frac{x}{p}\right) \geq \left(\frac{y}{p}\right).$$

$$\text{Thus, } x \leq 6 \Rightarrow 4x \leq 4 \times 6,$$

$$x \geq 5 \Rightarrow 3x \geq 3 \times 5,$$

$$x \leq 2 \Rightarrow \frac{x}{10} \leq \frac{2}{10} \quad \text{and so on.}$$

Rule 4 : If each term of an inequation be multiplied or divided by the same negative number, the sign of inequality reverses.

That is, if p is negative

$$(i) \quad x < y \Rightarrow px > py \quad \text{and} \quad \left(\frac{x}{p}\right) > \left(\frac{y}{p}\right)$$

$$(ii) \quad x \geq y \Rightarrow px \leq py \quad \text{and} \quad \left(\frac{x}{p}\right) \leq \left(\frac{y}{p}\right)$$

$$\text{Thus, } x \leq 6 \Rightarrow -4x \geq -4 \times 6,$$

$$x > 5 \Rightarrow -3x < -3 \times 5$$

$$x \geq y \Rightarrow \frac{x}{-2} \leq \frac{y}{-2} \quad \text{and so on.}$$

Rule 5 : If sign of each term on both the sides of an inequation is changed, the sign of inequality gets reversed.

$$\text{i.e. (i)} \quad -x > 5 \Leftrightarrow x < -5 \quad \text{(ii)} \quad 3y \leq 15 \Leftrightarrow -3y \geq -15$$

$$\text{(iii)} \quad -2y < -7 \Leftrightarrow 2y > 7 \quad \text{and so on.}$$

Rule 6 : If both the sides of an inequation are positive or both are negative, then on taking their reciprocals, the sign of inequality reverses.

i.e. if x and y both are either positive or both are negative, then

$$(i) \quad x > y \Leftrightarrow \frac{1}{x} < \frac{1}{y} \quad \text{(ii)} \quad x \leq y \Leftrightarrow \frac{1}{x} \geq \frac{1}{y}$$

$$\text{(iii)} \quad x \geq y \Leftrightarrow \frac{1}{x} \leq \frac{1}{y} \quad \text{and so on.}$$

4.4 Replacement Set and Solution Set :

The set, from which the value of the variable x is to be chosen, is called **replacement set** and its subset, whose elements satisfy the given inequation, is called **solution set**.

e.g. Let the given inequation be $x < 3$, if :

- (i) the replacement set = N, the set of natural numbers;
the solution set = {1, 2}
- (ii) the replacement set = W, the set of whole numbers;
the solution set = {0, 1, 2}
- (iii) the replacement set = Z or I, the set of integers;
the solution set = {....., -2, -1, 0, 1, 2}

But, if the replacement set is the set of real numbers, the solution set can only be described in set-builder form, i.e. $\{x : x \in \mathbb{R} \text{ and } x < 3\}$.

1 If the replacement set is the set of natural numbers (N), find the solution set of : (i) $3x + 4 < 16$ (ii) $8 - x \leq 4x - 2$.

Solution :

$$\begin{aligned} \text{(i) } 3x + 4 &< 16 & \Rightarrow & 3x < 16 - 4 & [\text{Using rule 1}] \\ & & \Rightarrow & 3x < 12 & \\ & & \Rightarrow & \frac{3x}{3} < \frac{12}{3} & [\text{Using rule 3}] \\ & & \text{i.e.} & x < 4 & \end{aligned}$$

Since, the replacement set = N (set of natural numbers)

\therefore Solution set = {1, 2, 3} Ans.

$$\begin{aligned} \text{(ii) } 8 - x &\leq 4x - 2 & \Rightarrow & -x - 4x \leq -2 - 8 & [\text{Using rule 1}] \\ & & \Rightarrow & -5x \leq -10 & \\ & & \Rightarrow & \frac{-5x}{-5} \geq \frac{-10}{-5} & [\text{Using rule 4}] \\ & & \text{i.e.} & x \geq 2 & \end{aligned}$$

Since, the replacement set = N

\therefore Solution set = {2, 3, 4, 5, 6,} Ans.

Alternative method :

$$\begin{aligned} 8 - x &\leq 4x - 2 & \Rightarrow & 4x - 2 \geq 8 - x & [x \leq y \text{ and } y \geq x \text{ mean the same}] \\ & & \Rightarrow & 4x + x \geq 8 + 2 & [\text{Using rule 2}] \\ & & \Rightarrow & 5x \geq 10 & \\ & & \Rightarrow & x \geq 2 \text{ and } x \in \mathbb{N} & \end{aligned}$$

\therefore Solution set = {2, 3, 4, 5, 6,} Ans.

- 2** If the replacement set is the set of whole numbers (W), find the solution set of : (i) $5x + 4 \leq 24$ (ii) $4x - 2 < 2x + 10$.

Solution :

$$\begin{aligned} \text{(i)} \quad 5x + 4 \leq 24 &\Rightarrow 5x \leq 24 - 4 \\ &\Rightarrow 5x \leq 20 \quad \text{and} \quad x \leq \frac{20}{5} \quad \text{i.e. } x \leq 4 \end{aligned}$$

Since, the replacement set is the set of whole numbers

$$\therefore \text{Solution set} = \{0, 1, 2, 3, 4\} \quad \text{Ans.}$$

$$\begin{aligned} \text{(ii)} \quad 4x - 2 < 2x + 10 &\Rightarrow 4x - 2x < 10 + 2 \\ &\Rightarrow 2x < 12 \quad \text{and} \quad x < 6 \end{aligned}$$

Since, the replacement set = W(whole numbers)

$$\therefore \text{Solution set} = \{0, 1, 2, 3, 4, 5\} \quad \text{Ans.}$$

- 3** If the replacement set is the set of integers, (I or Z), between -6 and 8 , find the solution set of : (i) $6x - 1 \geq 9 + x$ (ii) $15 - 3x > x - 3$.

Solution :

$$\begin{aligned} \text{(i)} \quad 6x - 1 \geq 9 + x &\Rightarrow 6x - x \geq 9 + 1 \\ &\Rightarrow 5x \geq 10 \quad \text{and} \quad x \geq 2 \end{aligned}$$

Since, the replacement set is the set of integers between -6 and 8 .

$$\therefore \text{Solution set} = \{2, 3, 4, 5, 6, 7\} \quad \text{Ans.}$$

$$\begin{aligned} \text{(ii)} \quad 15 - 3x > x - 3 &\Rightarrow -3x - x > -3 - 15 \\ &\Rightarrow -4x > -18 \\ &\Rightarrow \frac{-4x}{-4} < \frac{-18}{-4} \quad [\text{Using rule 4}] \\ &\Rightarrow x < 4.5 \end{aligned}$$

Since, the replacement set is the set of integers between -6 and 8

$$\therefore \text{Solution set} = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4\} \quad \text{Ans.}$$

Alternative method :

$$\begin{aligned} 15 - 3x > x - 3 &\Rightarrow x - 3 < 15 - 3x \\ &\Rightarrow x + 3x < 15 + 3 \\ &\Rightarrow 4x < 18 \quad \text{and} \quad x < 4.5 \end{aligned}$$

$$\therefore \text{Solution set} = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4\} \quad \text{Ans.}$$

- 4** If the replacement set is the set of real numbers (R), find the solution set of : (i) $5 - 3x < 11$ (ii) $8 + 3x \geq 28 - 2x$.

Solution :

$$\text{(i)} \quad 5 - 3x < 11 \Rightarrow -3x < 11 - 5$$

$$\Rightarrow \frac{-3x}{-3} > \frac{6}{-3}$$

$$\Rightarrow x > -2$$

[Using rule 4]

Since, the replacement set is the set of real numbers (R)

$$\therefore \text{Solution set} = \{x : x > -2 \text{ and } x \in \mathbb{R}\}$$

Ans.

$$(ii) 8 + 3x \geq 28 - 2x \Rightarrow 3x + 2x \geq 28 - 8$$

$$\Rightarrow 5x \geq 20 \text{ and } x \geq 4$$

Since, the replacement set is the set of real numbers

$$\therefore \text{Solution set} = \{x : x \geq 4 \text{ and } x \in \mathbb{R}\}$$

Ans.

5 Solve : $\frac{x}{2} - 5 \leq \frac{x}{3} - 4$, where x is a positive odd integer.

Solution :

$$\frac{x}{2} - 5 \leq \frac{x}{3} - 4 \Rightarrow \frac{x}{2} - \frac{x}{3} \leq -4 + 5$$

$$\Rightarrow \frac{3x - 2x}{6} \leq 1 \Rightarrow x \leq 6$$

Since, x is a positive odd integer

$$\therefore \text{Solution set} = \{1, 3, 5\}$$

Ans.

6 Solve the following inequation : $2y - 3 < y + 1 \leq 4y + 7$; if :

- (i) $y \in \{\text{Integers}\}$
- (ii) $y \in \mathbb{R}$ (real numbers)

Solution :

$$2y - 3 < y + 1 \leq 4y + 7 \Rightarrow 2y - 3 < y + 1 \text{ and } y + 1 \leq 4y + 7$$

$$\Rightarrow y < 4 \text{ and } -6 \leq 3y$$

$$\Rightarrow y < 4 \text{ and } y \geq -2$$

$$\Rightarrow -2 \leq y < 4$$

- (i) **When $y \in \{\text{Integers}\}$**

$$\therefore \text{Solution set} = \{-2, -1, 0, 1, 2, 3\}$$

Ans.

- (ii) **When $y \in \mathbb{R}$ (real numbers)**

$$\therefore \text{Solution set} = \{y : -2 \leq y < 4 \text{ and } y \in \mathbb{R}\}$$

Ans.

EXERCISE 4(A)

1. State, true or false :

- (i) $x < -y \Rightarrow -x > y$
- (ii) $-5x \geq 15 \Rightarrow x \geq -3$
- (iii) $2x \leq -7 \Rightarrow \frac{2x}{-4} \geq \frac{-7}{-4}$
- (iv) $7 > 5 \Rightarrow \frac{1}{7} < \frac{1}{5}$

2. State, whether the following statements are true or false.

- (i) If $a < b$, then $a - c < b - c$
- (ii) If $a > b$, then $a + c > b + c$
- (iii) If $a < b$, then $ac > bc$
- (iv) If $a > b$, then $\frac{a}{c} < \frac{b}{c}$
- (v) If $a - c > b - d$; then $a + d > b + c$
- (vi) If $a < b$, and $c > 0$, then $a - c > b - c$
where a , b , c and d are real numbers and $c \neq 0$.

3. If $x \in \mathbb{N}$, find the solution set of inequations.

- (i) $5x + 3 \leq 2x + 18$
- (ii) $3x - 2 < 19 - 4x$

4. If the replacement set is the set of whole numbers, solve :

- (i) $x + 7 \leq 11$
- (ii) $3x - 1 > 8$
- (iii) $8 - x > 5$
- (iv) $7 - 3x \geq -\frac{1}{2}$

$$(v) x - \frac{3}{2} < \frac{3}{2} - x \quad (vi) 18 \leq 3x - 2$$

5. Solve the inequation :

$$3 - 2x \geq x - 12 \text{ given that } x \in \mathbb{N}.$$

6. If $25 - 4x \leq 16$, find :

- (i) the smallest value of x , when x is a real number,
- (ii) the smallest value of x , when x is an integer.

7. If the replacement set is the set of real numbers, solve :

$$\begin{array}{ll} (i) -4x \geq -16 & (ii) 8 - 3x \leq 20 \\ (iii) 5 + \frac{x}{4} > \frac{x}{5} + 9 & (iv) \frac{x+3}{8} < \frac{x-3}{5} \end{array}$$

8. Find the smallest value of x for which $5 - 2x < 5\frac{1}{2} - \frac{5}{3}x$, where x is an integer.

9. Find the largest value of x for which $2(x - 1) \leq 9 - x$ and $x \in \mathbb{W}$.

10. Solve the inequation :

$$12 + 1\frac{5}{6}x \leq 5 + 3x \text{ and } x \in \mathbb{R}.$$

11. Given $x \in \{\text{integers}\}$, find the solution set of : $-5 \leq 2x - 3 < x + 2$.

12. Given $x \in \{\text{whole numbers}\}$, find the solution set of : $-1 \leq 3 + 4x < 23$.

4.5 Representation of the solution on the number line :

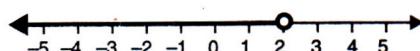
A real number line can be used to represent the solution set of an inequation.

The convention is that \circ (a hollow circle) marks the end of a range with a *strict inequality* (i.e. $<$ or $>$) and \bullet (a darkened circle) marks the end of a range involving an equality as well (i.e. \leq or \geq).

For Example :

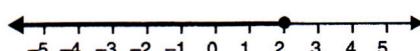
The adjacent figure shows

$$x < 2 \text{ and } x \in \mathbb{R}.$$

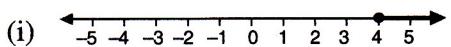


The number 2 is encircled and the circle is not darkened to show that 2 is not included in the graph.

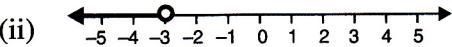
If 2 is also included i.e. $x \leq 2$, then the circle will be darkened and the graph will be as shown alongside:



Similarly, study each of the following carefully :



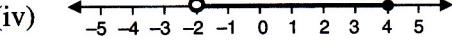
$$\{x : x \geq 4 \text{ and } x \in \mathbb{R}\}$$



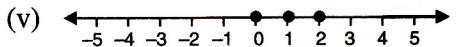
$$\{x : x < -3 \text{ and } x \in \mathbb{R}\}$$



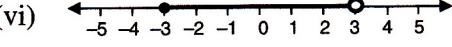
$$\{x : -2 < x \leq 4 \text{ and } x \in \mathbb{Z}\}$$



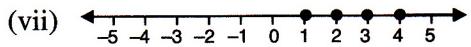
$$\{x : -2 < x \leq 4 \text{ and } x \in \mathbb{R}\}$$



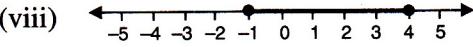
$$\{x : -3 \leq x < 3 \wedge x \in \mathbb{W}\}$$



$$\{x : -3 \leq x < 3 \text{ and } x \in \mathbb{R}\}$$



$$\{x : -1 \leq x \leq 4 \wedge x \in \mathbb{N}\}$$



$$\{x : -1 \leq x \leq 4 \text{ and } x \in \mathbb{R}\}$$

[Note: The symbol ‘ \wedge ’ stands for ‘and’].

7 Given that $x \in \mathbb{R}$, solve the following inequality and graph the solution on the number line : $-1 \leq 3 + 4x < 23$ [2006]

Solution :

$$\text{Given : } -1 \leq 3 + 4x < 23; x \in \mathbb{R}$$

$$\Rightarrow -1 \leq 3 + 4x \quad \text{and} \quad 3 + 4x < 23$$

$$\Rightarrow -4 \leq 4x \quad \text{and} \quad 4x < 20$$

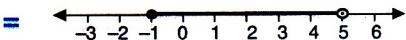
$$\Rightarrow -1 \leq x \quad \text{and} \quad x < 5$$

$$\Rightarrow -1 \leq x < 5; x \in \mathbb{R}$$

$$\therefore \text{Solution} = \{-1 \leq x < 5 : x \in \mathbb{R}\}$$

Ans.

Solution on the number line is :



Ans.

8 Simplify : $-\frac{1}{3} \leq \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6} : x \in \mathbb{R}$.

Graph the values of x on the real number line.

Solution :

The given inequation has two parts :

$$-\frac{1}{3} \leq \frac{x}{2} - 1\frac{1}{3} \quad \text{and} \quad \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}$$

$$\Rightarrow -\frac{1}{3} + 1\frac{1}{3} \leq \frac{x}{2} \quad \mid \quad \Rightarrow \frac{x}{2} < \frac{1}{6} + \frac{4}{3}$$

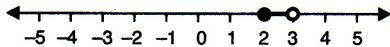
$$\Rightarrow 1 \leq \frac{x}{2} \quad \mid \quad \Rightarrow \frac{x}{2} < \frac{9}{6}$$

$$\Rightarrow 2 \leq x \quad \mid \quad \Rightarrow x < \frac{9}{6} \times 2$$

$$\Rightarrow \boxed{x < 3}$$

∴ On simplifying, the given inequation reduces to $2 \leq x < 3$ and the required graph on number line is :

Ans.



- 9 List the solution set of $50 - 3(2x - 5) < 25$, given that $x \in \mathbb{W}$.

Also, represent the solution set obtained on a number line.

Solution :

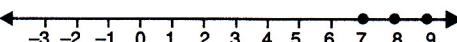
$$\begin{aligned}50 - 3(2x - 5) &< 25 \Rightarrow 50 - 6x + 15 < 25 \\&\Rightarrow -6x < 25 - 65 \\&\Rightarrow -6x < -40 \\&\Rightarrow \frac{-6x}{-6} > \frac{-40}{-6} \\&\Rightarrow x > \frac{20}{3}\end{aligned}$$

Division by a negative number
reverses the sign of inequality.

Required solution set = {7, 8, 9, }

Ans.

And, the required number line is :



Ans.

4.6 Combining Inequations :

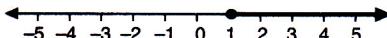
- 10 Solve and graph the solution set of $3x + 6 \geq 9$ and $-5x > -15$; where $x \in \mathbb{R}$.

Solution :

$$\begin{aligned}3x + 6 \geq 9 &\Rightarrow 3x \geq 9 - 6 \\&\Rightarrow 3x \geq 3 \quad \Rightarrow x \geq 1\end{aligned}$$

$$\text{And, } -5x > -15 \Rightarrow \frac{-5x}{-5} < \frac{-15}{-5} \Rightarrow x < 3$$

Graph for $x \geq 1$:

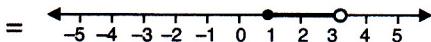


Graph for $x < 3$:



∴ Graph of solution set of $x \geq 1$ and $x < 3$

= Graph of points common to both $x \geq 1$ and $x < 3$

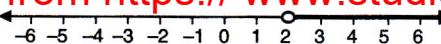


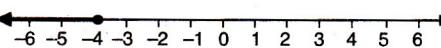
Ans.

- 11 Solve and graph the solution set of $-2 < 2x - 6$ or $-2x + 5 \geq 13$; where $x \in \mathbb{R}$.

Solution :

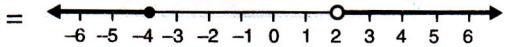
$$\begin{aligned}-2 < 2x - 6 &\Rightarrow 2x - 6 > -2 \\&\Rightarrow 2x > 4 \quad \Rightarrow x > 2 \\-2x + 5 &\geq 13 \Rightarrow -2x \geq 8 \\&\Rightarrow x \leq -4\end{aligned}$$

Graph for $x > 2$: 

Graph for $x \leq -4$: 

\therefore Graph of solution set of $x > 2$ or $x \leq -4$

= Graph of points which belong to $x > 2$ or $x \leq -4$ or both



Ans.

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Given : $P = \{x : 5 < 2x - 1 \leq 11, x \in R\}$

$Q = \{x : -1 \leq 3 + 4x < 23, x \in I\}$

where $R = \{\text{real numbers}\}$ and $I = \{\text{integers}\}$.

Represent P and Q on two different number lines. Write down the elements of $P \cap Q$.

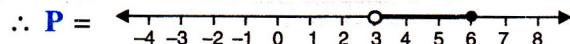
Solution :

For P : $5 < 2x - 1 \leq 11$, where $x \in R$

$$\Rightarrow 5 < 2x - 1 \quad \text{and} \quad 2x - 1 \leq 11$$

$$\Rightarrow 3 < x \quad \text{and} \quad x \leq 6$$

i.e. $3 < x \leq 6; x \in R$

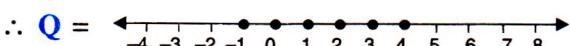


For Q : $-1 \leq 3 + 4x < 23, x \in I$

$$\Rightarrow -1 \leq 3 + 4x \quad \text{and} \quad 3 + 4x < 23$$

$$\Rightarrow -1 \leq x \quad \text{and} \quad x < 5$$

i.e. $-1 \leq x < 5; \text{ where } x \in I$



Hence, $P \cap Q = \{\text{elements common to both P and Q}\} = \{4\}$.

Ans.

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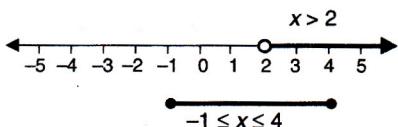
Write down the range of values of x ($x \in R$) for which both the inequations $x > 2$ and $-1 \leq x \leq 4$ are true.

Solution :

Both the given inequations are true in the range; where their graphs on the real number lines overlap.

The graphs for the given inequalities are drawn alongside :

It is clear from both the graphs that their common range is $2 < x \leq 4$.



\therefore Required range is $2 < x \leq 4$

Ans.

For any two solution sets A and B :

(i) $A \text{ and } B = \text{Intersection of sets A and B}$

= Set of elements common to set A and to set B

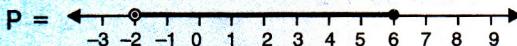
= $A \cap B$

(ii) $A \text{ or } B = \text{Union of sets A and B}$

= Set of elements which belong either set A and to set B

= $A \cup B$

- 14** The diagram, given below, represents two inequations P and Q on real number lines :



(i) Write down P and Q in set builder notation.

(ii) Represent each of the following sets on different number lines :

(a) $P \cup Q$

(b) $P \cap Q$

(c) $P - Q$

(d) $Q - P$

(e) $P \cap Q'$

(f) $P' \cap Q$

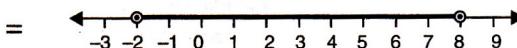
Solution :

(i) $P = \{x : -2 < x \leq 6 \text{ and } x \in \mathbb{R}\}$ and,

$Q = \{x : 2 \leq x < 8 \text{ and } x \in \mathbb{R}\}$

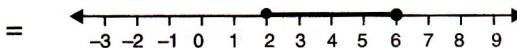
Ans.

(ii) (a) $P \cup Q$ = Numbers which belong to P or to Q or to both



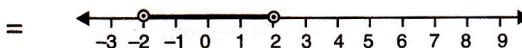
Ans.

(b) $P \cap Q$ = Numbers common to both P and Q



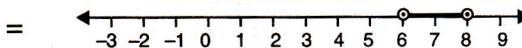
Ans.

(c) $P - Q$ = Numbers which belong to P but do not belong to Q



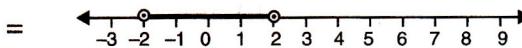
Ans.

(d) $Q - P$ = Numbers which belong to Q but do not belong to P



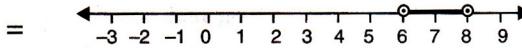
Ans.

(e) $P \cap Q'$ = Numbers which belong to P but do not belong to Q = $P - Q$



Ans.

(f) $P' \cap Q$ = Numbers which do not belong to P but belong to Q = $Q - P$



Ans.

- 15** Find three smallest consecutive whole numbers such that the difference between one-fourth of the largest and one-fifth of the smallest is at least 3.

Solution :

Let the required whole numbers be x , $x + 1$ and $x + 2$.

According to the given statement :

$$\frac{x+2}{4} - \frac{x}{5} \geq 3 \Rightarrow \frac{5x+10-4x}{20} \geq 3$$

$$\Rightarrow x + 10 \geq 60 \text{ i.e., } x \geq 50$$

Since, the smallest value of $x = 50$ that satisfies the inequation $x \geq 50$.

∴ Required smallest consecutive whole numbers are :

$$= x, x + 1 \text{ and } x + 2$$

$$= 50, 50 + 1 \text{ and } 50 + 2$$

$$= \mathbf{50, 51 \text{ and } 52}$$

Ans.

EXERCISE 4(B)

1. Represent the following inequalities on real number lines :

$$(i) 2x - 1 < 5$$

$$(ii) 3x + 1 \geq -5$$

$$(iii) 2(2x - 3) \leq 6$$

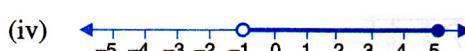
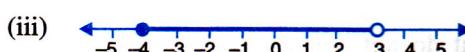
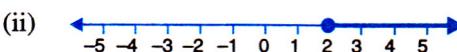
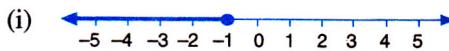
$$(iv) -4 < x < 4$$

$$(v) -2 \leq x < 5$$

$$(vi) 8 \geq x > -3$$

$$(vii) -5 < x \leq -1$$

2. For each graph given alongside, write an inequation taking x as the variable :



3. For the following inequations, graph the solution set on the real number line :

$$(i) -4 \leq 3x - 1 < 8$$

$$(ii) x - 1 < 3 - x \leq 5$$

4. Represent the solution of each of the following inequalities on the real number line:

$$(i) 4x - 1 > x + 11$$

$$(ii) 7 - x \leq 2 - 6x$$

$$(iii) x + 3 \leq 2x + 9$$

$$(iv) 2 - 3x > 7 - 5x$$

$$(v) 1 + x \geq 5x - 11$$

$$(vi) \frac{2x+5}{3} > 3x - 3$$

5. $x \in \{\text{real numbers}\}$ and $-1 < 3 - 2x \leq 7$, evaluate x and represent it on a number line.

6. List the elements of the solution set of the inequation $-3 < x - 2 \leq 9 - 2x$; $x \in \mathbb{N}$.

7. Find the range of values of x which satisfies

$$-2\frac{2}{3} \leq x + \frac{1}{3} < 3\frac{1}{3}, x \in \mathbb{R}$$

Graph these values of x on the number line.

[2007]

8. Find the values of x , which satisfy the inequation:

$$-2 \leq \frac{1}{2} - \frac{2x}{3} \leq 1\frac{5}{6}, x \in \mathbb{N}$$

Graph the solution on the number line.

9. Given $x \in \{\text{real numbers}\}$, find the range of values of x for which $-5 \leq 2x - 3 < x + 2$ and represent it on a real number line.

10. If $5x - 3 \leq 5 + 3x \leq 4x + 2$, express it as $a \leq x \leq b$ and then state the values of a and b .

11. Solve the following inequation and graph the solution set on the number line :

$$2x - 3 < x + 2 \leq 3x + 5; x \in \mathbb{R}$$

12. Solve and graph the solution set of :

$$(i) 2x - 9 < 7 \text{ and } 3x + 9 \leq 25; x \in \mathbb{R}$$

$$(ii) 2x - 9 \leq 7 \text{ and } 3x + 9 > 25; x \in \mathbb{I}$$

$$(iii) x + 5 \geq 4(x - 1) \text{ and } 3 - 2x < -7; x \in \mathbb{R}$$

13. Solve and graph the solution set of :

$$(i) 3x - 2 > 19 \text{ or } 3 - 2x \geq -7; x \in \mathbb{R}$$

$$(ii) 5 > p - 1 > 2 \text{ or } 7 \leq 2p - 1 \leq 17; p \in \mathbb{R}$$

14. The diagram represents two inequations A and B on real number lines :



- (i) Write down A and B in set builder notation.

- (ii) Represent $A \cap B$ and $A \cap B'$ on two different number lines.

15. Use real number line to find the range of values of x for which :

$$(i) x > 3 \text{ and } 0 < x < 6$$

$$(ii) x < 0 \text{ and } -3 \leq x < 1$$

$$(iii) -1 < x \leq 6 \text{ and } -2 \leq x \leq 3$$

16. Illustrate the set $\{x : -3 \leq x < 0 \text{ or } x > 2; x \in \mathbb{R}\}$ on a real number line.

17. Given $A = \{x : -1 < x \leq 5, x \in \mathbb{R}\}$ and $B = \{x : -4 \leq x < 3, x \in \mathbb{R}\}$

Represent on different number lines :

(i) $A \cap B$ (ii) $A' \cap B$ (iii) $A - B$

18. P is the solution set of $7x - 2 > 4x + 1$ and Q is the solution set of $9x - 45 \geq 5(x - 5)$; where $x \in \mathbb{R}$. Represent :

(i) $P \cap Q$

(ii) $P - Q$

(iii) $P \cap Q'$ on different number lines.

19. If $P = \{x : 7x - 4 > 5x + 2, x \in \mathbb{R}\}$ and $Q = \{x : x - 19 \geq 1 - 3x, x \in \mathbb{R}\}$; find the range of set $P \cap Q$ and represent it on a number line.

20. Find the range of values of x , which satisfy:

$$-\frac{1}{3} \leq \frac{x}{2} + 1\frac{2}{3} < 5\frac{1}{6}$$

Graph, in each of the following cases, the values of x on the different real number lines:

(i) $x \in \mathbb{W}$ (ii) $x \in \mathbb{Z}$ (iii) $x \in \mathbb{R}$.

21. Given : $A = \{x : -8 < 5x + 2 \leq 17, x \in \mathbb{I}\}$

$$B = \{x : -2 \leq 7 + 3x < 17, x \in \mathbb{R}\}$$

Where $\mathbb{R} = \{\text{real numbers}\}$ and $\mathbb{I} = \{\text{integers}\}$.

Represent A and B on two different number lines. Write down the elements of $A \cap B$.

22. Solve the following inequation and represent the solution set on the number line $2x - 5 \leq 5x + 4 < 11$, where $x \in \mathbb{I}$. [2011]

23. Given that $x \in \mathbb{I}$, solve the inequation and graph the solution on the number line :

$$3 \geq \frac{x-4}{2} + \frac{x}{3} \geq 2. \quad [2004]$$

24. Given :

$$A = \{x : 11x - 5 > 7x + 3, x \in \mathbb{R}\} \text{ and}$$

$$B = \{x : 18x - 9 \geq 15 + 12x, x \in \mathbb{R}\}.$$

Find the range of set $A \cap B$ and represent it on a number line. [2005]

25. Find the set of values of x , satisfying :

$$7x + 3 \geq 3x - 5 \text{ and } \frac{x}{4} - 5 \leq \frac{5}{4} - x,$$

where $x \in \mathbb{N}$.

26. Solve :

(i) $\frac{x}{2} + 5 \leq \frac{x}{3} + 6$, where x is a positive odd integer

(ii) $\frac{2x+3}{3} \geq \frac{3x-1}{4}$, where x is a positive even integer

27. Solve the inequation :

$$-2\frac{1}{2} + 2x \leq \frac{4x}{5} \leq \frac{4}{3} + 2x, x \in \mathbb{W}.$$

Graph the solution set on the number line.

28. Find three consecutive largest positive integers such that the sum of one-third of first, one-fourth of second and one-fifth of third is at most 20.

29. Solve the given inequation and graph the solution on the number line.

$$2y - 3 < y + 1 \leq 4y + 7, y \in \mathbb{R} \quad [2008]$$

30. Solve the inequation :

$$3z - 5 \leq z + 3 < 5z - 9; z \in \mathbb{R}.$$

Graph the solution set on the number line.

31. Solve the following inequation and represent the solution set on the number line.

$$-3 < -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}, x \in \mathbb{R}. \quad [2010]$$

32. Solve the following inequation and represent the solution set on the number line :

$$4x - 19 < \frac{3x}{5} - 2 \leq \frac{-2}{5} + x, x \in \mathbb{R} \quad [2012]$$

33. Solve the following inequation, write the solution set and represent it on the number line:

$$-\frac{x}{3} \leq \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}, x \in \mathbb{R} \quad [2013]$$

34. Find the values of x , which satisfy the inequation

$$-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \leq 2, x \in \mathbb{W}. \text{ Graph the solution set on the number line.} \quad [2014]$$

35. Solve the following inequation and write the solution set :

$$13x - 5 < 15x + 4 < 7x + 12, x \in \mathbb{R}$$

Represent the solution on a real number line.

[2015]

4.7 Important :

1. If the product of two numbers is less than 0 (zero) i.e. product is negative; one of the two numbers is positive and the other is negative.
2. If the product of two numbers is greater than 0 (zero) i.e. positive; both the numbers are positive or both are negative.

1. For $(x - 3)(x + 5) < 0$

If $(x - 3)$ is positive, then $x + 5$ is negative

and if $x - 3$ is negative, then $x + 5$ is positive

Case 1 : When $(x - 3)$ is positive and $(x + 5)$ is negative

$$\text{i.e. } x - 3 > 0 \text{ and } x + 5 < 0$$

$$\Rightarrow x > 3 \text{ and } x < -5$$

It is not possible, as number greater than 3 cannot be smaller than -5 .

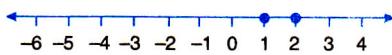
Case 2 : When $(x - 3)$ is negative and $(x + 5)$ is positive

$$\text{i.e. } x - 3 < 0 \text{ and } x + 5 > 0$$

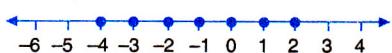
$$\Rightarrow x < 3 \text{ and } x > -5$$

$$\Rightarrow -5 < x < 3, \text{ which is possible.}$$

If $x \in N$, then : $-5 < x < 3$ on the number line is :



If $x \in Z$, then : $-5 < x < 3$ on the number line is :



If $x \in R$, then : $-5 < x < 3$ on the number line is :



2. For $(x - 3)(x + 5) > 0$

If $(x - 3)$ is negative, then $(x + 5)$ is also negative

and, if $(x - 3)$ is positive, then $(x + 5)$ is also positive

Case 1 : $x - 3$ is negative and $x + 5$ is negative

$$\text{i.e. } x - 3 < 0 \text{ and } x + 5 < 0$$

$$\Rightarrow x < 3 \text{ and } x < -5$$

$$\Rightarrow x < -5 \text{ as the number smaller than } -5 \text{ is less than } 3 \text{ also.}$$

Case 2 : $x - 3$ is positive and $x + 5$ is positive

$$\text{i.e. } x - 3 > 0 \text{ and } x + 5 > 0$$

$$\Rightarrow x > 3 \text{ and } x > -5$$

$$\Rightarrow x > 3 \text{ as the number greater than } 3 \text{ is also greater than } -5.$$

If $x \in R$, then for $(x - 3)(x + 5) > 0$ the number line is :

