24 Measures of Central Tendency

(Mean, Median, Quartiles and Mode)

24.1 Introduction :

The numerical expressions which represent the characteristics of a group (a large collection of numerical data) are called **Measures of Central Tendency** (or, Averages).

An average which is used to represent a whole series should neither have the lowest value nor the highest value in the group, but a value somewhere between two limits, possibly in the centre, where most of the items of the group cluster.

There are many types of statistical averages, out of them the following averages will be studied in this chapter.

1. Arithmetic Average or Mean 2. Median 3. Mode

24.2 Arithmetic Mean :

The ***arithmetic mean*** (or, simply, mean) of a set of numbers is obtained by dividing the sum of numbers of the set by the number of numbers.

For example :

The mean of n numbers $x_1, x_2, x_3,$ ----------, x_n is

$$= \frac{x_1 + x_2 + x_3 + \text{-----} + x_n}{n} = \frac{\sum x}{n}$$

The Greek letter Σ (called sigma) represents the sum of numbers.

1 The weights (in kilogram) of 5 persons are 67, 65, 71, 57 and 45. Find the arithmetic mean of their weights.

Solution :

According to the definition :

$$\textbf{Arithmetic mean} = \frac{\sum x}{n} = \frac{67 + 65 + 71 + 57 + 45}{5} \text{ kg}$$

$$= \frac{305}{5} \text{ kg} = \mathbf{61\ kg.}$$ **Ans.**

24.3 Arithmetic Mean of Tabulated Data :

For a given discrete frequency distribution, the arithmetic mean can be obtained by using any one of the following three methods :

1. Direct method. 2. Short-cut method. 3. Step-deviation method.

1. Direct method :

Steps :

1. Prepare a frequency table with three columns :
   (a) In the first column from the left, write the values of the variate (x).
   (b) In the second column from the left, write the corresponding frequency (f) of each variate in column (a).
   (c) In the third column, write the product of each x with its frequency (f) *i.e.* write each value of fx.
2. Add all the entries in the second column to get Σf (sum of all the frequencies).
3. Add all the entries in the third column to get Σfx.
4. Then required mean = $\dfrac{\Sigma fx}{\Sigma f}$, by using direct method.

2 Find the mean of :

x	5	6	7	8	9
f	4	5	3	6	2

Solution :

x	f	fx
5	4	20
6	5	30
7	3	21
8	6	48
9	2	18
	$\Sigma f = 20$	$\Sigma fx = 137$

$\because$ $\Sigma f = 20$ and,

$\Sigma fx = 137$

$\therefore$ **Mean** $= \dfrac{\Sigma fx}{\Sigma f}$

$= \dfrac{137}{20} =$ **6·85** **Ans.**

3 Using direct method, find the mean of following frequency distribution :

x	5	15	25	35	44·5
f	14	16	20	30	20

Solution :

x	f	fx
5	14	70
15	16	240
25	20	500
35	30	1050
44·5	20	890
	$\Sigma f = 100$	$\Sigma fx = 2750$

$\because$ $\Sigma f = n = 100$ and $\Sigma fx = 2750$

$\therefore$ **Mean** $(\bar{x}) = \dfrac{\Sigma fx}{n}$

$= \dfrac{2750}{100} =$ **27·50** **Ans.**

In general, mean is denoted by $\bar{x}$.

2. Short-cut method :

Steps :

1. Prepare a frequency table with four columns.
   (a) In the first column from the left, write the values of the variate (x).
   (b) In the second column from the left, write the frequency (f) of each variate in column (a).
2. Take a number 'A' (preferably from given values of variate 'x' in the first column). Here, the number 'A' is called **assumed mean**.

   From each value of variate 'x' in first column, subtract assumed mean 'A' to get deviation 'd'. $\therefore$ Deviation (d) = x – A.

   Write down the values of all the deviations (d = x – A) in the third column and against their corresponding frequencies.
3. Multiply the frequency (f) in the second column with the corresponding deviation (d) in third column to get the values of fd.

   Write the values of fd obtained in the fourth column and against the corresponding values of deviations 'd'.
4. Find Σfd, the sum of all the values of fd in fourth column.

   Also, find $\Sigma f = n$, the sum of all values of frequency 'f'.
5. The following formula gives the required mean by using **short-cut method** :

$$\text{Mean} = \text{A} + \frac{\Sigma fd}{\Sigma f}$$

4 The weights of 25 students of a class are given in the following table :

Weight (in kg)	65	66	67	68	69
Number of students	8	6	4	4	3

Using short-cut method, find the mean weight.

Solution :

Let the assumed mean A = 67.

Therefore :

Weight (x) (in kg)	Number of students (f)	$d = x - \text{A}$ $= x - 67$	fd
65	8	65 – 67 = –2	–16
66	6	66 – 67 = –1	–6
A = 67	4	67 – 67 = 0	0
68	4	68 – 67 = 1	4
69	3	69 – 67 = 2	6
	$\Sigma f = 25$		$\Sigma fd = -12$

$$\therefore \quad \textbf{Mean} = \text{A} + \frac{\Sigma fd}{\Sigma f} = 67 + \frac{-12}{25} = 67 - 0{\cdot}48 = \textbf{66{\cdot}52}$$ **Ans.**

3. Step-deviation method :

Steps :

1. Construct a frequency table with five columns.
   (a) In the first column from left, write the values of the variate (x).
   (b) In the second column from the left, write the corresponding frequency (f) of each variate in column (a).
2. Take a number 'A' (preferably from given values of variate 'x' in the first column). Here, the number 'A' is called **assumed mean.**

   From each value of variate 'x' in first column, subtract assumed mean 'A' to get deviation 'd'. $\therefore$ Deviation $(d) = x - A$.

   Write down the values of all the deviations ($d = x - A$) in the third column and against their corresponding frequencies.
3. Choose the largest number i, which may divide each value of d in third column. Divide each value of d by i to get $\frac{d}{i} = \frac{x-A}{i}$. Denote values so obtained by t and write in the fourth column.
4. Multiply the frequencies in second column with the corresponding values of t in the fourth column to get the values of ft and write each value of ft in the fifth column.
5. Add all the numbers in fifth column to get the value of Σft. Also, add all the numbers in second column to get the value Σf.
6. The following formula gives the required mean by using **step-deviation method**:

$$\text{Mean} = A + \frac{\Sigma ft}{\Sigma f} \times i$$

5 Using step-deviation method, find the mean of following frequency distribution:

x	10	30	50	70	90	110
f	135	187	240	273	124	151

Solution :

Let the assumed mean A = 70.

x	f	$d = x - A$	$t = \frac{x-A}{i} = \frac{x-70}{20}$	ft
10	135	– 60	– 3	– 405
30	187	– 40	– 2	– 374
50	240	– 20	– 1	– 240
A = 70	273	0	0	0
90	124	20	1	124
110	151	40	2	302
	$\Sigma f = 1110$			$\Sigma ft = -593$

$\therefore$ **Mean** $= A + \frac{\Sigma ft}{\Sigma f} \times i = 70 + \frac{-593}{1110} \times 20 =$ **59·32** **Ans.**

6 If the mean of the following distribution is 7·5, find the missing frequency 'f' :

Variable :	5	6	7	8	9	10	11	12
Frequency :	20	17	f	10	8	6	7	6

[2005]

Solution :

Variable x	Frequency f	fx
5	20	100
6	17	102
7	f	$7f$
8	10	80
9	8	72
10	6	60
11	7	77
12	6	72
	$\Sigma f = 74 + f$	$\Sigma fx = 563 + 7f$

$\because \quad \Sigma f = 74 + f$

and $\quad \Sigma fx = 563 + 7f$

$\therefore \quad$ **Mean** $= \dfrac{\Sigma fx}{\Sigma f}$

$\Rightarrow \quad 7{\cdot}5 = \dfrac{563 + 7f}{74 + f}$

$\Rightarrow \quad 555 + 7{\cdot}5f = 563 + 7f$

i.e. $\quad 0{\cdot}5f = 8$

and, $\quad f = \dfrac{8}{0{\cdot}5} = \mathbf{16}$ **Ans.**

7 Find the value of p, if the mean of following distribution is 20.

x	15	17	19	$20+p$	23
f	6	9	12	$15p$	18

Solution :

x	f	fx
15	6	90
17	9	153
19	12	228
$20 + p$	$15p$	$300p + 15p^2$
23	18	414
	$\Sigma f = 45+15p$	$\Sigma fx = 15p^2 + 300p + 885$

$\therefore \quad$ **Mean** $= \dfrac{\Sigma fx}{\Sigma f}$

$\Rightarrow \quad 20 = \dfrac{15p^2 + 300p + 885}{45 + 15p}$

$\Rightarrow \quad 900 + 300p = 15p^2 + 300p + 885$

$\Rightarrow \quad 15p^2 = 15$ and $p = \pm 1$

But, frequency 15p cannot be negative

$\therefore \quad p = \mathbf{1}$ **Ans.**

EXERCISE 24(A)

1. Find the mean of following set of numbers :

   (i) 6, 9, 11, 12 and 7 (ii) 11, 14, 23, 26, 10, 12, 18 and 6

2. Marks obtained (in mathematics) by 9 students are given below :

   60, 67, 52, 76, 50, 51, 74, 45 and 56

   (a) Find the arithmetic mean.

   (b) If marks of each student be increased by 4; what will be the new value of arithmetic mean ?

> 1. If each given number is increased or decreased by the same quantity (say, p), the value of mean is also increased or decreased by quantity p.
> 2. Similarly, if each given number is multiplied or divided by the same quantity (say, n), the value of mean is also multiplied or divided by quantity n.

3. Find the mean of natural numbers from 3 to 12.

4. (a) Find the mean of 7, 11, 6, 5 and 6.

   (b) If each number given in (a) is diminished by 2; find the new value of mean.

5. If the mean of 6, 4, 7, a and 10 is 8. Find the value of 'a'.

6. The mean of the number 6, y, 7, x and 14 is 8. Express y in terms of x.

7. The ages of 40 students are given in the following table :

Age (in years)	12	13	14	15	16	17	18
Frequency	2	4	6	9	8	7	4

   Find the arithmetic mean.

8. If 69·5 is the mean of 72, 70, x, 62, 50, 71, 90, 64, 58 and 82 : find the value of x.

9. The following table gives the heights of plants in centimetre. If the mean height of plants is 60·95 cm; find the value of 'f'.

Height (cm)	50	55	58	60	65	70	71
No. of plants	2	4	10	f	5	4	3

10. From the data, given below, calculate the mean wage, correct to the nearest rupee.

Category :	A	B	C	D	E	F
Wages in ₹/day :	50	60	70	80	90	100
No. of workers	2	4	8	12	10	6

   (i) If the number of workers in each category is doubled, what would be the new mean wage ?

   (ii) If the wages per day in each category are increased by 60%; what is the new mean wage ?

   (iii) If the number of workers in each category is doubled and the wages per day per worker are reduced by 40%; what would be the new mean wage ?

11. The contents of 100 match boxes were checked to determine the number of matches they contained.

No. of matches :	35	36	37	38	39	40	41
No. of boxes :	6	10	18	25	21	12	8

(i) Calculate, correct to one decimal place, the mean number of matches per box.

(ii) Determine, how many extra matches would have to be added to the total contents of the 100 boxes to bring the mean up to exactly 39 matches ?

12. If the mean of the following distribution is 3, find the value of p.

x	1	2	3	5	$p + 4$
f	9	6	9	3	6

13. In the following table, $\Sigma f = 200$ and mean = 73. Find the missing frequencies f_1, and f_2.

x	0	50	100	150	200	250
f	46	f_1	f_2	25	10	5

14. Find the arithmetic mean (correct to the nearest whole-number) by using step-deviation method.

x	5	10	15	20	25	30	35	40	45	50
y	20	43	75	67	72	45	39	9	8	6

15. Find the mean (correct to one place of decimal) by using short-cut method.

x	40	41	43	45	46	49	50
f	14	28	38	50	40	20	10

24.4 To Find Mean for Grouped Data (both continuous and discontinuous) :

1. Direct Method :

Step 1 :

Find the mean (mid value) of each class interval.

The mean value of a class interval is obtained by dividing the sum of its lower and upper limits by 2. For example, the mean value of class interval 10 – 20 is $\frac{10 + 20}{2} = 15$.

Step 2 :

Represent the mean value by x and find the arithmetic mean using the same method as is used in example 2.

8 Find the mean of :

Class interval	0–10	10–20	20–30	30–40	40–50
Frequency	10	6	8	12	5

[2007]

Solution :

Class interval	Frequency (f)	Mean value (x)	$f \times x$
0 – 10	10	$\frac{0+10}{2} = 5$	50
10 – 20	6	15	90
20 – 30	8	25	200
30 – 40	12	35	420
40 – 50	5	45	225
	$\Sigma f = 41$		$\Sigma fx = 985$

$\therefore$ **Mean** $= \frac{\Sigma fx}{\Sigma f} = \frac{985}{41} = 24\frac{1}{41}$ **Ans.**

2. Short-Cut Method :

By using this method, big quantities get converted into small quantities which make the process of multiplication and division easier.

In this method, an assumed mean (A) is taken according to the convenience and then arithmetic mean is obtained by using the following formula :

Mean = A + $\frac{\Sigma fd}{\Sigma f}$ [see example 4]

Here, 'A' is the assumed mean and 'd' is the deviation of 'x' from assumed mean 'A'.

9 Find mean of the following distribution using short-cut method :

C.I.	35-40	40-45	45-50	50-55	55-60
f	7	6	9	5	3

Solution :

C.I.	f	Mid-value x	A = 47.5 $d = x - A$	$f \times d$
35 – 40	7	37·5	–10	–70
40 – 45	6	42·5	–5	–30
45 – 50	9	47·5	0	0
50 – 55	5	52·5	5	25
55 – 60	3	57·5	10	30
	$\Sigma f = 30$			$\Sigma fd = -45$

$$\therefore \textbf{Mean} = A + \frac{\sum fd}{\sum f} = 47{\cdot}5 + \frac{-45}{30} = \mathbf{46}$$ **Ans.**

Note : In the above method, any number can be taken as assumed mean. But to make the calculations simpler, the assumed mean should be taken from the middle of the values of x.

3. Step-Deviation Method :

According to this method;

$$\text{Mean} = A + \frac{\sum ft}{\sum f} \times i \quad \text{Where, A = assumed mean, } t = \frac{x - A}{i} \text{ and}$$

i = class-size (*i.e.* upper class limit – lower class limit).

10 The weights of 50 apples were recorded as given below. Calculate the mean weight, to the nearest gram, by the *Step Deviation Method.* **[2008]**

Weight in grams	80-85	85-90	90-95	95-100	100-105	105-110	110-115
No. of apples	5	8	10	12	8	4	3

Solution :

Weight	f	Mid-value x	Let A = 97·5 ∴ $d = x - A$	$t = \frac{x - A}{i}$	$f \times t$
80 – 85	5	82·5	–15	–3	–15
85 – 90	8	87·5	–10	–2	–16
90 – 95	10	92·5	–5	–1	–10
95 – 100	12	97·5	0	0	0
100 – 105	8	102·5	5	1	8
105 – 110	4	107·5	10	2	8
110 – 115	3	112·5	15	3	9
	$\Sigma f = 50$				$\Sigma ft = -16$

$$\therefore \quad \textbf{Mean} = A + \frac{\Sigma ft}{\Sigma f} \times i = 97{\cdot}5 + \frac{-16}{50} \times 5 = 95{\cdot}9$$

$= \mathbf{96\ g}$ (correct to the nearest gram) **Ans.**

Whichever method of finding the mean is used, the mean in each case will be the same.

11 Find the mean of the following distribution :

Class interval	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
Frequency	10	6	8	12	5	9

[2006]

Solution :

(i) Direct Method :

C.I.	f	*Class mark (x)*	$f.x$
20–30	10	25	250
30–40	6	35	210
40–50	8	45	360
50–60	12	55	660
60–70	5	65	325
70–80	9	75	675
	50		2480

$\because \quad n = \Sigma f = 50$

and, $\Sigma fx = 2480$

$\therefore \quad$ **Mean** $= \dfrac{\Sigma fx}{n}$

$= \dfrac{2480}{50}$

$=$ **49·6** **Ans.**

(ii) Short cut method (method of assumed mean) **:**

C.I.	f	x	Assumed mean A = 45 $\therefore d = x - A$	$f.d.$	
20–30	10	25	–20	–200	–260
30–40	6	35	–10	–60	
40–50	8	45	0	0	
50–60	12	55	10	120	490
60–70	5	65	20	100	
70–80	9	75	30	270	
	50			230	

$\because \quad n = \Sigma f = 50$, $\Sigma fd = 230$ and assumed mean, A = 45

$\therefore \quad$ Mean $= A + \dfrac{\Sigma fd}{n}$

$\Rightarrow \quad$ **Mean** $= 45 + \dfrac{230}{50} = 45 + 4{\cdot}6 =$ **49·6** **Ans.**

(iii) Step deviation method : **[2013]**

C.I.	f	x	If A = 45, $t = \dfrac{x - A}{i}$	$f.t.$
20–30	10	25	–2	–20
30–40	6	35	–1	–6
40–50	8	45	0	0
50–60	12	55	1	12
60–70	5	65	2	10
70–80	9	75	3	27
	50			23

$\because$ i = class-size = 10, $n = \Sigma f = 50$, assumed mean (A) = 45 and $\Sigma ft = 23$

$\therefore$ $$\text{Mean} = A + \frac{\Sigma ft}{n} \times i$$

$\Rightarrow$ $$\textbf{Mean} = 45 + \frac{23}{50} \times 10$$

$$= 45 + 4{\cdot}6 = \mathbf{49{\cdot}6}$$ **Ans.**

12 The total number of observations in the following distribution table is 120 and their mean is 50. Find the values of missing frequencies f_1 and f_2.

Class :	0-20	20-40	40-60	60-80	80-100
Frequency :	17	f_1	32	f_2	19

Solution :

C.I.	f	Mid-value (x)	$f \times x$
0-20	17	10	170
20-40	f_1	30	$30 f_1$
40-60	32	50	1600
60-80	f_2	70	$70 f_2$
80-100	19	90	1710

$$n = \Sigma f = 17 + f_1 + 32 + f_2 + 19 = 68 + f_1 + f_2$$

$$\Sigma fx = 170 + 30f_1 + 1600 + 70f_2 + 1710 = 3480 + 30 f_1 + 70 f_2$$

Given : Number of observations = 120 *i.e.* $n = 120$

$\Rightarrow$ $68 + f_1 + f_2 = 120$ *i.e.* $f_1 + f_2 = 52$ I

Also, Mean = 50 $\Rightarrow$ $$\frac{\Sigma fx}{n} = 50$$

$$\Rightarrow \frac{3480 + 30 f_1 + 70 f_2}{120} = 50$$

$$\Rightarrow 3480 + 30f_1 + 70f_2 = 6000$$

$$\Rightarrow 3f_1 + 7f_2 = 252 \quad \text{..... II}$$

On solving equations I and II, we get :

$\mathbf{f_1 = 28}$ and $\mathbf{f_2 = 24}$ **Ans.**

EXERCISE 24(B)

1. The following table gives the ages of 50 students of a class. Find the arithmetic mean of their ages.

Age (years)	16 – 18	18 – 20	20 – 22	22 – 24	24 – 26
No. of students	2	7	21	17	3

2. The following table gives the weekly wages of workers in a factory.

Weekly Wages (₹)	50 – 55	55 – 60	60 – 65	65 – 70	70 – 75	75 – 80	80 – 85	85 – 90
No. of workers	5	20	10	10	9	6	12	8

Calculate the mean, by using :
(i) Direct Method (ii) Short-Cut Method.

3. The following are the marks obtained by 70 boys in a class test.

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
No. of boys	10	12	14	12	9	7	6

Calculate the mean by :
(i) Short-cut Method (ii) Step-deviation Method.

4. Find mean by 'step-deviation method' :

C.I.	63 – 70	70 – 77	77 – 84	84 – 91	91 – 98	98 – 105	105 – 112
Frequency	9	13	27	38	32	16	15

5. The mean of following frequency distribution is $21\frac{1}{7}$. Find the value of 'f'.

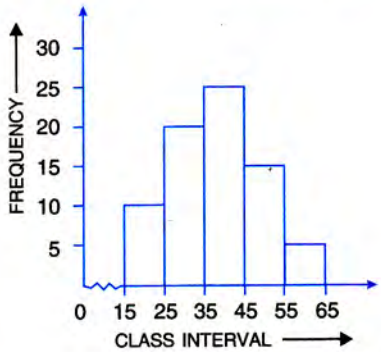
Class interval	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	8	22	31	f	2

6. Using step-deviation method, calculate the mean marks of the following distribution.

Class interval	50-55	55-60	60-65	65-70	70-75	75-80	80-85	85-90
Frequency	5	20	10	10	9	6	12	8

[2011]

7. Using the information given in the adjoining histogram; calculate the mean.



8. If the mean of the following observations is 54, find the value of p.

Class :	0-20	20-40	40-60	60-80	80-100
Frequency :	7	p	10	9	13

9. The mean of the following distribution is 62·8 and the sum of all the frequencies is 50. Find the missing frequencies f_1 and f_2.

Class :	0-20	20-40	40-60	60-80	80-100	100-120
Frequency :	5	f_1	10	f_2	7	8

10. Calculate the mean of the distribution, given below, using the short cut method :

Marks	11-20	21-30	31-40	41-50	51-60	61-70	71-80
No. of students	2	6	10	12	9	7	4

[2014]

11. Calculate the mean of the following distribution :

Class interval	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	8	5	12	35	24	16

[2015]

24.5 Median :

Median is the *value of middle term* of a set of variables *when the variables of the set are arranged in ascending or descending order.*

24.6 Median for Raw Data :

13 Find the median of 7, 8, 4, 3 and 10.

Solution :

First of all arrange the given terms in ascending or descending order of their magnitude.

On arranging the given terms in ascending order, we get : 3, 4, 7, 8, 10

Clearly, the middle term is 7 $\Rightarrow$ **Median = 7** **Ans.**

Let there be n terms in a given set of variables and all these terms are expressed in ascending or descending order. Then :

(i) *if n is odd*; the median = $\left(\frac{n+1}{2}\right)^{th}$ term

In example 13, given above, $n = 5$

$\therefore$ Median = $\left(\frac{5+1}{2}\right)^{th}$ term = 3rd term = **7** **Ans.**

(ii) *if n is even*; there are two middle terms *i.e.* $\left(\frac{n}{2}\right)^{th}$ term and $\left(\frac{n}{2}+1\right)^{th}$ term.

In this case, *the arithmetic mean of these two terms is the median.*

i.e. Median = $\dfrac{\left(\frac{n}{2}\right)^{th} \text{ term} + \left(\frac{n}{2}+1\right)^{th} \text{ term}}{2}$

14 Find the median of : 7, 12, 15, 6, 20, 8, 4 and 10

Solution :

On arranging the given terms in ascending order of magnitude, we get :

4, 6, 7, 8, 10, 12, 15, 20

Since no. of terms $n = 8$ (even)

$$\therefore \text{ Median} = \frac{\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2}+1\right)^{th} \text{term}}{2}$$

$$= \frac{4^{th} \text{ term} + 5^{th} \text{ term}}{2} = \frac{8+10}{2} = 9$$ **Ans.**

Note : Whether the given terms are arranged in ascending or descending order, the value of median always remains the same.

15 The following numbers are written in descending order of their values :
68, 60, 52, $x - 3$, $x - 8$, $x - 11$, 30, 25, 22 and 20.
If their median is 39, find the value of x.

Solution :

$\because$ n = No. of terms = 10 (even)

$$\therefore \quad \text{Median} = \frac{1}{2}\left[\left(\frac{10}{2}\right)^{th} \text{term} + \left(\frac{10}{2}+1\right)^{th} \text{term}\right]$$

$\Rightarrow$ $39 = \frac{1}{2}[5^{th} \text{ term} + 6^{th} \text{ term}]$

$\Rightarrow$ $78 = x - 8 + x - 11$

$\Rightarrow$ $2x = 97$ and $x = 48{\cdot}5$ **Ans.**

24.7 Median for Tabulated Data :

16 The weights of 45 children in a class were recorded, to the nearest kg, as follows :

Wt. (in nearest kg)	46	48	50	52	53	54	55
No. of children	7	5	8	12	10	2	1

Calculate the median weight.

Solution :

Construct the cumulative frequency table as given below :

Weight (x)	No. of students (f)	Cumulative freq. ($c.f.$)
46	7	7
48	5	12
50	8	20
52	12	32
53	10	42
54	2	44
55	1	45
	45	

Clearly, total number of children = 45
i.e. $n = 45$, *which is odd*

Since total number of children = 45

i.e $n = 45$; *which is odd*

$$\therefore \quad \textbf{Median} = \left(\frac{n+1}{2}\right)^{th} \text{ term} = \left(\frac{45+1}{2}\right)^{th} \text{ term} = 23^{rd} \text{ term}$$

$$= \text{weight of } 23^{rd} \text{ child.}$$

According to the table obtained above, it can be observed that the weight of each child from 21st child to 32nd child is 52 kg.

$\therefore$ Weight of 23rd child = 52 kg

$\Rightarrow$ **Median weight = 52 kg** **Ans.**

24.8 Median for Grouped Data (both continuous and discontinuous) :

1. Draw a cumulative frequency curve (Ogive).
2. If there be n terms in the given distribution; then use the ogive to find the value of $\left(\frac{n}{2}\right)^{th}$ or $\left(\frac{n+1}{2}\right)^{th}$ term which is the median of given distribution.

17 Find the median for the following distribution :

C.I.	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	5	7	10	8	5

Solution :

Using the given data construct a cumulative frequency table as shown alongside. On a graph paper, mark class-intervals (C.I.) along x-axis and cumulative frequencies along y-axis.

On this graph mark points (10, 5), (20, 12), (30, 22), (40, 30) and (50, 35).

C.I.	Frequency	Cumulative Frequency
0 – 10	5	5
10 – 20	7	12
20 – 30	10	22
30 – 40	8	30
40 – 50	5	35
	$n = \Sigma f = 35$	

Then draw a free-hand curve passing through the points marked, starting from the lower limit of first class and terminating at upper limit of the last class. The curve (graph) so obtained is ogive show below.

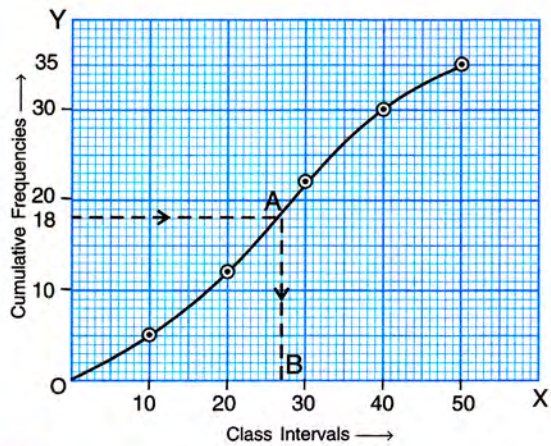
Since, the no. of terms, $n = 35$

$$\therefore \text{ Median} = \left(\frac{n+1}{2}\right)^{th} \text{ term}$$

$$= \left(\frac{35+1}{2}\right)^{th} \text{ term}$$

$$= 18^{th} \text{ term}$$

Through mark 18 on y-axis, draw a horizontal line which meets the curve at point A.



Through point A, on the curve, draw a vertical line which meets x-axis at point B. The value of point B on x-axis is median.

It is clear from the ogive drawn that the **median is 27.** **Ans.**

The class interval, to which the value of median for the given data belongs, is called the ***median class***. For the example, given above, the median class = 20 – 30.

18 The daily wages of 160 workers in a building project are given below :

Wages in ₹	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of Workers	12	20	30	38	24	16	12	8

Using a graph paper, draw an Ogive for the above distribution.

Use your Ogive to estimate :

(i) the median wage of the workers

(ii) the percentage of workers who earn more than ₹ 45 a day ? **[2006]**

Solution :

Wages in ₹	No. of workers (f)	Cumulative frequency ($c.f.$)
0 – 10	12	12
10 – 20	20	32
20 – 30	30	62
30 – 40	38	100
40 – 50	24	124
50 – 60	16	140
60 – 70	12	152
70 – 80	8	160

Plot the points (10, 12), (20, 32), (30, 62), (40, 100), (50, 124), (60, 140), (70, 152) and (80, 160) on a properly marked graph paper. Then draw a free hand curve passing through the points marked, starting from the lower limit of first class and terminating at upper limit of last class. The ogive so drawn is shown on the next page. Using this graph, we get :

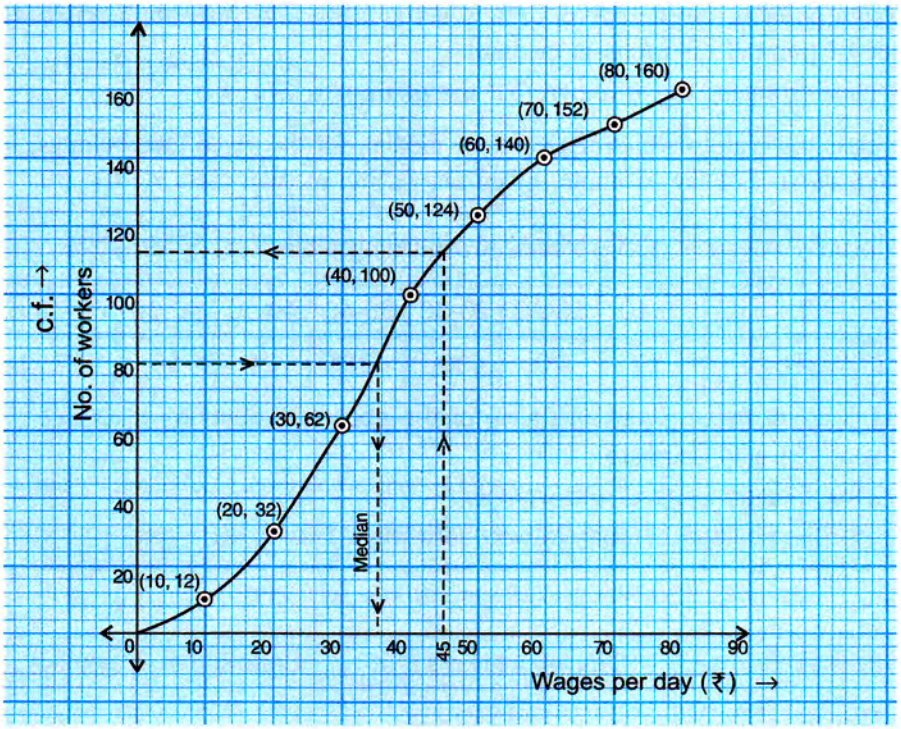
(i) **Median** $= \left(\frac{n}{2}\right)^{\text{th}}$ term $= \left(\frac{160}{2}\right)^{\text{th}}$ term

$=$ 80th term $=$ **35 (approximately)** **Ans.**

(ii) The number of workers who earn upto ₹ 45 per day = 112

$\Rightarrow$ The number of workers who earn more than ₹ 45 per day = 160 – 112 = 48

$\therefore$ **Required percentage** $= \frac{48}{160} \times 100\% =$ **30%** **Ans.**



19 The marks obtained by 200 students in an examination are given below :

Marks	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
No. of students	05	10	11	20	27	38	40	29	14	06

Using a graph paper, draw an Ogive for the above distribution. Use your Ogive to estimate:

(i) the median;

(ii) the number of students who obtained more than 80% marks in the examination and

(iii) the number of students who did not pass, if the pass percentage was 35.

Use the scale as 2 cm = 10 marks on one axis and

2 cm = 20 students on the other axis. **[2004]**

Solution :

Marks	*f*	*c.f.*
0 – 10	05	05
10 – 20	10	15
20 – 30	11	26
30 – 40	20	46
40 – 50	27	73
50 – 60	38	111
60 – 70	40	151
70 – 80	29	180
80 – 90	14	194
90–100	06	200

Using the graph drawn on the next page, we get :

(i) **Median** $= \left(\frac{n}{2}\right)^{th}$ term, where $n = 200$

$= 100^{th}$ term = **57** **Ans.**

(ii) No. of students who score marks upto 80%

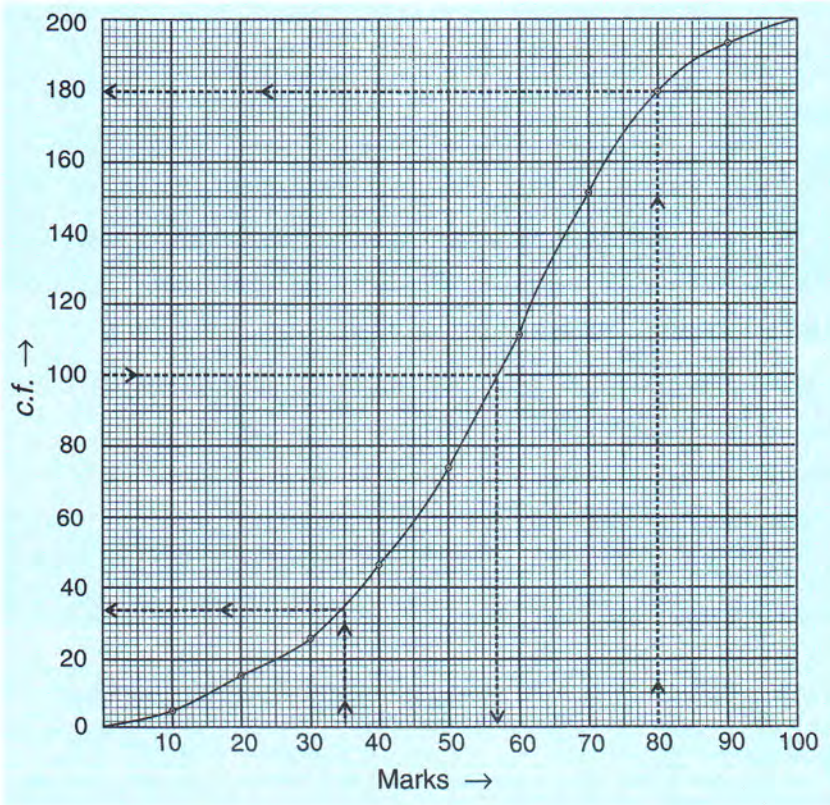
= 180

⇒ No. of students who score marks more than 80%

= 200 – 180 = **20** **Ans.**

(iii) The no. of students who did not pass

= **34** **Ans.**

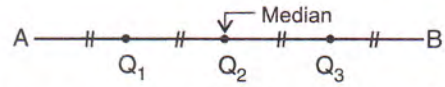


24.9 Quartiles :

Quartiles are those values of the variate which divide the total set of data in four equal parts.

> For finding quartiles, the given terms (data) are always arranged in ascending order of their magnitudes.

Let a given set of data, in ascending order, be arranged along a line segment AB. Mark Q_1, Q_2 and Q_3 on the line AB dividing the line into four equal parts. Then, Q_2 is called the ***middle quartile*** and is same as median for the same set of data whereas Q_1 and Q_3 are called ***lower quartile*** and ***upper quartile*** respectively.



1. Lower quartile (Q_1)

When the lower half, before the median, is divided into two equal parts, the value of the dividing variate is called ***lower quartile***.

Let there be n terms arranged in ascending order, then :

Lower quartile $(Q_1) = \left(\frac{n}{4}\right)^{th}$ term or $\left(\frac{n+1}{4}\right)^{th}$ term, depending whether n is even or odd.

2. Upper quartile (Q_3)

When the upper half, after the median, is divided into two equal parts, the value of the dividing variate is called ***upper quartile***.

$\therefore$ Upper quartile $(Q_3) = \left(\frac{3n}{4}\right)^{th}$ term or $\left(\frac{3(n+1)}{4}\right)^{th}$ term.

1. **If n is odd**, lower quartile $(Q_1) = \left(\frac{n+1}{4}\right)^{th}$ term and

upper quartile $(Q_3) = \frac{3(n+1)}{4}$ th term.

2. **If n is even**, lower quartile $= \left(\frac{n}{4}\right)^{th}$ term and upper quartile $= \left(\frac{3n}{4}\right)^{th}$ term.

24.10 Inter-quartile Range :

For a given data, the difference between the upper quartile (Q_3) and the lower quartile (Q_1) is called **inter-quartile range**.

i.e. Inter-quartile range $= Q_3 - Q_1$, which is always positive as $Q_3 > Q_1$.

20 Find the lower quartile, upper quartile and inter quartile range for the data : 9, 11, 15, 19, 17, 13, 7.

Solution :

On arranging the given data in ascending order of their magnitudes, we get :

7, 9, 11, 13, 15, 17, 19

Clearly, $n = 7$, *which is an odd number.*

$\therefore$ **Lower quartile (Q_1)** $= \left(\frac{n+1}{4}\right)^{th}$ term $= \left(\frac{7+1}{4}\right)^{th}$ term $= 2^{nd}$ term $=$ **9** **Ans.**

Upper quartile (Q_3) $= \left(\frac{3(n+1)}{4}\right)^{th}$ term

$= \left(\frac{3(7+1)}{4}\right)^{th}$ term $= 6^{th}$ term $=$ **17** **Ans.**

Inter-quartile range $= Q_3 - Q_1 = 17 - 9 =$ **8** **Ans.**

21 From the following frequency distribution table, find :

(i) Lower quartile (ii) Upper quartile (iii) Inter-quartile range

C.I.	5-10	10-15	15-20	20-25	25-30	30-35
Frequency	3	4	6	9	7	1

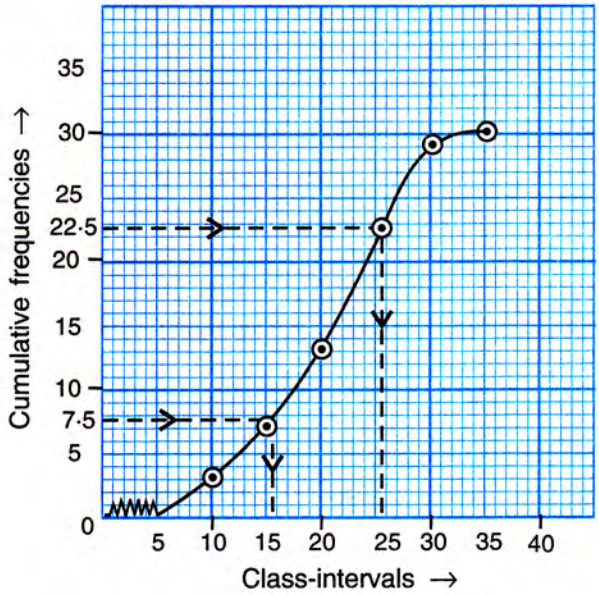
Solution :

First of all, construct a cumulative frequency table for the given distribution.

On a graph paper, plot the points (10, 3), (15, 7), (20, 13), (25, 22), (30, 29) and (35, 30).

C.I.	Frequency	Cumulative Frequency
5 – 10	3	3
10 – 15	4	7
15 – 20	6	13
20 – 25	9	22
25 – 30	7	29
30 – 35	1	30

Draw an ogive as shown below :



(i) **Lower quartile (Q_1)** = $\left(\frac{30}{4}\right)^{th}$ term = $7{\cdot}5^{th}$ term = **15·5** **Ans.**

(ii) **Upper quartile (Q_3)** = $\left(\frac{3\times30}{4}\right)^{th}$ term = $22{\cdot}5^{th}$ term = **25·5** **Ans.**

(iii) **Inter-quartile range** = $Q_3 - Q_1 = 25{\cdot}5 - 15{\cdot}5 =$ **10** **Ans.**

22 The table below shows the distribution of the scores obtained by 120 shooters in a shooting competition. Using a graph sheet, draw an ogive for the distribution.

Score obtained	Number of shooters
0 - 10	5
10 - 20	9
20 - 30	16
30 - 40	22
40 - 50	26
50 - 60	18
60 - 70	11
70 - 80	6
80 - 90	4
90 - 100	3

Use your ogive to estimate :

(i) The median.

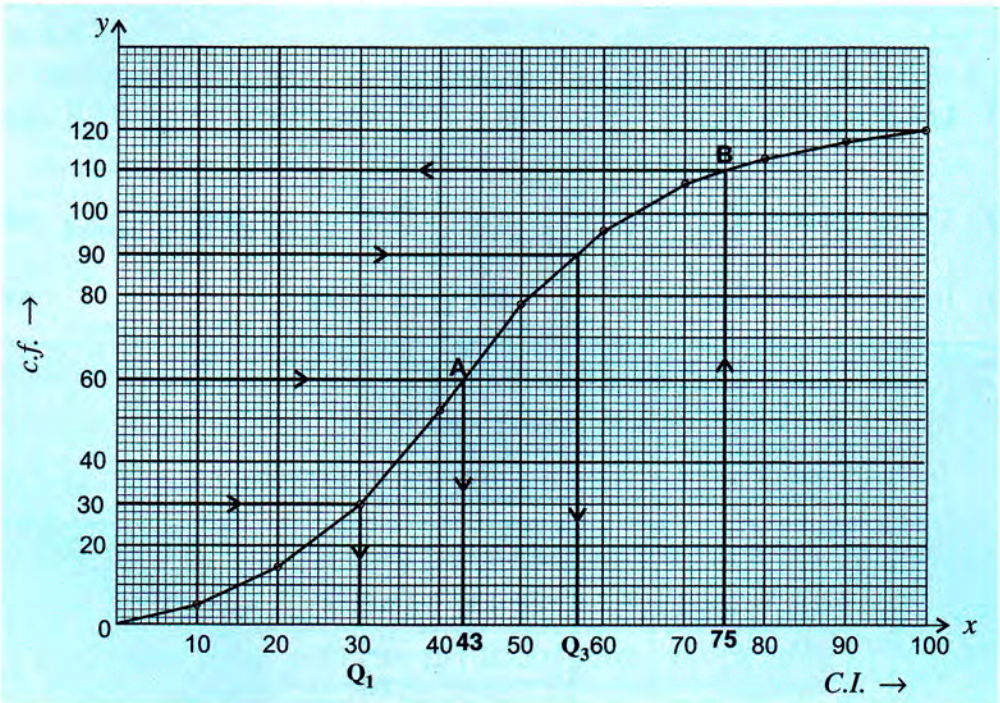
(ii) The interquartile range.

(iii) The number of shooters who obtained more than 75% scores. [2007]

Solution :

Scores	*f*	*c.f.*
0 – 10	5	5
10 – 20	9	14
20 – 30	16	30
30 – 40	22	52
40 – 50	26	78
50 – 60	18	96
60 – 70	11	107
70 – 80	6	113
80 – 90	4	117
90 – 100	3	120
	N = 120	

Taking scores (class intervals along) *x*-axis and cumulative frequency (*c.f.*) along *y*-axis, draw an ogive as shown below :



Scale : *x*-axis : 2 units = 10 scores

y-axis : 2 units = 20 shooters

(i) $$\text{Median} = \left(\frac{N}{2}\right)^{th} \text{term} = \left(\frac{120}{2}\right)^{th} \text{term} = 60^{th} \text{ term}$$

Through mark for 60 on *y*-axis, draw a horizontal line which meets the ogive drawn at point A. Through point A, draw a vertical line which meets the *x*-axis at the mark of 43.

∴ **Median = 43** **Ans.**

(ii) Since, number of terms = 120

$$\text{Lower quartile } (Q_1) = \left(\frac{120}{4}\right)^{th} \text{term} = 30^{th} \text{ term} = 30$$

$$\text{Upper quartile } (Q_3) = \left(\frac{3\times120}{4}\right)^{th} \text{ term} = 90^{th} \text{ term} = 57$$

$\therefore$ **Inter-quartile range** $= Q_3 - Q_1$

$= 57 - 30 =$ **27** **Ans.**

(iii) Since, 75% scores = 75% of 100 = 75

Through mark for 75 on x-axis, draw a vertical line which meets the ogive drawn at point B. Through the point B, draw a horizontal line which meets the y-axis at the mark of 110.

$\therefore$ The number of shooters who obtained more than 75% scores

$= 120 - 110 =$ **10** **Ans.**

23 Using a graph paper, draw an *ogive* for the following distribution which shows the marks obtained in the General Knowledge paper by 100 students.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of students	5	10	20	25	15	12	9	4

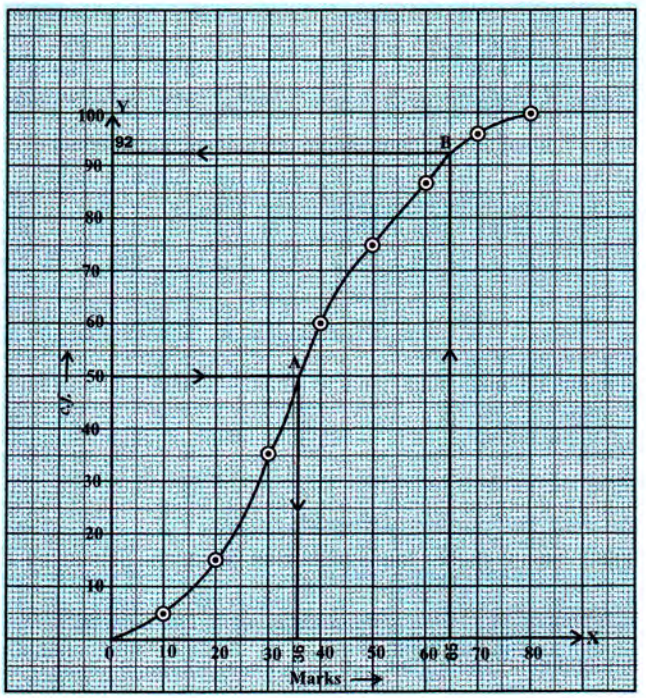
Use the ogive to estimate :

(i) the median (ii) the number of students who score marks above 65 ? **[2008]**

Solution :

Marks	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
f	5	10	20	25	15	12	9	4
$c.f.$	5	15	35	60	75	87	96	100

On a graph paper, draw an ogive as shown alongside :



(i) Since, $n = 100$,

$$\text{Median} = \left(\frac{n}{2}\right)^{th} \text{ term}$$

$$= \left(\frac{100}{2}\right)^{th} \text{ term} = 50^{th} \text{ term}$$

Through mark 50 on y-axis, draw a horizontal line which meets the graph at point A. Then through point A, draw a vertical line which meets x-axis at the mark 36.

$\therefore$ **Median = 36** **Ans.**

(ii) Through mark 65 on x-axis, draw a vertical line which meets the graph at point B. Then through point B, draw a horizontal line which meets y-axis at the mark of 92.

$\therefore$ **No. of students scoring marks above 65** $= 100 - 92 =$ **8** **Ans.**

EXERCISE 24(C)

1. A student got the following marks in 9 questions of a question paper.
   3, 5, 7, 3, 8, 0, 1, 4 and 6. Find the median of these marks.
2. The weights (in kg) of 10 students of a class are given below :
   21, 28·5, 20·5, 24, 25·5, 22, 27·5, 28, 21 and 24. Find the median of their weights.
3. The marks obtained by 19 students of a class are given below :
   27, 36, 22, 31, 25, 26, 33, 24, 37, 32, 29, 28, 36, 35, 27, 26, 32, 35 and 28. Find :
   (i) Median (ii) Lower quartile (iii) Upper quartile (iv) Inter-quartile range
4. From the following data; find : (i) Median (ii) Upper quartile (iii) Inter-quartile range.
   25, 10, 40, 88, 45, 60, 77, 36, 18, 95, 56, 65, 7, 0, 38 and 83
5. The ages of 37 students in a class are given in the following table :

Age (in years)	11	12	13	14	15	16
Frequency	2	4	6	10	8	7

   Find the median.
6. The weights of 60 boys are given in the following distribution table :

Weight (kg)	37	38	39	40	41
No. of boys	10	14	18	12	6

   Find : (i) Median (ii) Lower quartile (iii) Upper quartile (iv) Inter-quartile range.
7. Estimate the median for the given data by drawing ogive :

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	4	9	15	14	8

8. By drawing an ogive; estimate the median for the following frequency distribution :

Weight (kg)	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35
No. of boys	11	25	12	5	2

9. From the following cumulative frequency table draw ogive and then use it to find :
   (i) Median (ii) Lower quartile (iii) Upper quartile.

Marks (less than)	10	20	30	40	50	60	70	80	90	100
Cumulative Frequency	5	24	37	40	42	48	70	77	79	80

10. In a school, 100 pupil have heights as tabulated below :

Height (in cm)	121 – 130	131 – 140	141 – 150	151 – 160	161 – 170	171 – 180
No. of pupils	12	16	30	20	14	8

   Find the median height by drawing an ogive.
11. Attempt this question on graph paper.
   Table below shows the distribution of marks gained by a group of 400 students in an examination :

Marks less than :	10	20	30	40	50	60	70	80	90	100
No. of students :	5	10	30	60	105	180	270	355	390	400

   Using a scale of 2 cm to represent 10 marks and 2 cm to represent 50 students, plot these points and draw a smooth curve through the points.
   Estimate from the graph : (i) Median marks (ii) quartile marks.

12. Attempt this question on graph paper.

Age (yrs.)	5 – 15	15 – 25	25 – 35	35 – 45	45 – 55	55 – 65	65 – 75
No. of casualties	6	10	15	13	24	8	7

(i) Construct the 'less than' cumulative frequency curve for the above data, using 2 cm = 10 years, on one axis and 2 cm = 10 casualties on the other.

(ii) From your graph determine : (a) Median (b) Lower quartile.

24.11 Mode :

Mode is the value which occurs most frequently in a set of observations. It is *the point of maximum frequency.*

1. Mode for raw data :

24 Find the mode of the data : 4, 7, 4, 3, 2, 7, 7, 6, 4, 7 and 8.

Solution :

In the given data; 7 occurs most frequently.

∴ **Mode = 7** **Ans.**

2. Mode for tabulated data :

25 Find the mode from the following frequency distribution :

Number	8	9	10	11	12	13	14	15	16
Frequency	3	8	12	15	14	17	12	8	6

Solution :

Since, the frequency of number 13 is maximum.

∴ **Mode = 13** **Ans.**

To find the mode of a group frequency distribution (using a histogram) :

3. Mode for grouped data :

26 Find the mode of the following frequency distribution :

Class	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
Frequency	4	7	9	11	6	2

Solution :

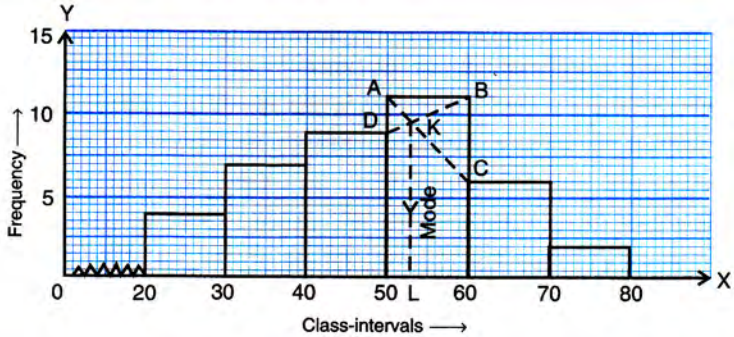
Steps :

1. Draw a histogram of the given distribution.
2. Inside the highest rectangle, which represents the maximum frequency (or modal class), draw two lines AC and BD diagonally from the upper corners C and D of adjacent rectangles.
3. Through the point K (the point of intersection of diagonals AC and BD), draw KL perpendicular to the horizontal axis.
4. The value of point L on the horizontal axis represents the value of mode.

∴ **Mode = 53** **Ans.**

The class-interval to which the value of mode for the given data belongs, is called the **modal-class**.

In this example, the modal-class = 50 – 60.



EXERCISE 24(D)

1. Find the mode of the following data :

   (i) 7, 9, 8, 7, 7, 6, 8, 10, 7 and 6 (ii) 9, 11, 8, 11, 16, 9, 11, 5, 3, 11, 17 and 8

2. The following table shows the frequency distribution of heights of 50 boys :

Height (cm)	120	121	122	123	124
Frequency	5	8	18	10	9

   Find the mode of heights.

3. Find the mode of following data, using a histogram :

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	5	12	20	9	4

4. The following table shows the expenditure of 60 boys on books. Find the mode of their expenditure :

Expenditure (₹)	20 – 25	25 – 30	30 – 35	35 – 40	40 – 45	45 – 50
No. of students	4	7	23	18	6	2

5. Find the median and the mode for the set of numbers : 2, 2, 3, 5, 5, 5, 6, 8 and 9.

6. A boy scored the following marks in various class tests during a term, each test being marked out of 20.

   15, 17, 16, 7, 10, 12, 14, 16, 19, 12 and 16.

   (i) What are his modal marks ? (ii) What are his median marks ?

   (iii) What are his total marks ? (iv) What are his mean marks ?

7. Find the mean, median and mode of the following marks obtained by 16 students in a class test marked out of 10 marks :

   0, 0, 2, 2, 3, 3, 3, 4, 5, 5, 5, 5, 6, 6, 7 and 8.

8. At a shooting competition the scores of a competitor were as given below :

Score	0	1	2	3	4	5
No. of shots	0	3	6	4	7	5

   (i) What was his modal score ? (ii) What was his median score ?

   (iii) What was his total score ? (iv) What was his mean score ?

EXERCISE 24(E)

1. The following distribution represents the height of 160 students of a school.

Height (in cm)	No. of Students
140 – 145	12
145 – 150	20
150 – 155	30
155 – 160	38
160 – 165	24
165 – 170	16
170 – 175	12
175 – 180	8

Draw an ogive for the given distribution taking 2 cm = 5 cm of height on one axis and 2 cm = 20 students on the other axis. Using the graph, determine :

(i) the median height.

(ii) the inter-quartile range.

(iii) the number of students whose height is above 172 cm ? **[2012]**

2. The following table gives the weekly wages of workers in a factory :

Weekly wages (in ₹)	No. of workers
50 – 55	5
55 – 60	20
60 – 65	10
65 – 70	10
70 – 75	9
75 – 80	6
80 – 85	12
85 – 90	8

Calculate :

(i) the mean,

(ii) the modal class,

(iii) the number of workers getting weekly wages below ₹ 80 and

(iv) the number of workers getting ₹ 65 or more but less than ₹ 85 as weekly wages.

[2002]

3. Draw an ogive for the data given below and from the graph determine :

(i) the median marks,

(ii) the number of students who obtained more than 75% marks ?

Marks	0–9	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–99
No. of students	5	9	16	22	26	18	11	6	4	3

4. The mean of 1, 7, 5, 3, 4 and 4 is m. The numbers 3, 2, 4, 2, 3, 3 and p have mean $m - 1$ and median q. Find p and q.

5. The marks of 200 students in a test were recorded as follows :

Marks %	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59	60 – 69	70 – 79	80 – 89
No. of students	7	11	20	46	57	37	15	7

Construct the cumulative frequency table.

Draw an ogive and use it to find :

(i) the median and

(ii) the number of students who score more than 35% marks ?

6. In a malaria epidemic, the number of cases diagnosed were as follows :

Date (July)	1	2	3	4	5	6	7	8	9	10	11	12
Number	5	12	20	27	46	30	31	18	11	5	0	1

On what days does the mode, the upper and the lower quartiles occur ?

7. The incomes of the parents of 100 students in a class in a certain university are tabulated below.

Income (in thousand ₹)	0 – 8	8 – 16	16 – 24	24 – 32	32 – 40
No. of students	8	35	35	14	8

(i) Draw a cumulative frequency curve to estimate the median income.

(ii) If 15% of the students are given freeships on the basis of the income of their parents, find the annual income of parents, below which the freeships will be awarded.

(iii) Calculate the Arithmetic mean.

8. The marks of 20 students in a test were as follows :

2, 6, 8, 9, 10, 11, 11, 12, 13, 13, 14, 14, 15, 15, 15, 16, 16, 18, 19 and 20.

Calculate : (i) the mean (ii) the median (iii) the mode. **[2002]**

9. The marks obtained by 120 students in a Mathematics test are given below :

Marks	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
No. of students	5	9	16	22	26	18	11	6	4	3

Draw an ogive for the given distribution on a graph sheet. Use a suitable scale for your ogive. Use your ogive to estimate :

(i) the median

(ii) the number of students who obtained more than 75% marks in a test ?

(iii) the number of students who did not pass in the test if the pass percentage was 40 ? **[2002, 2013]**

(iv) the lower quartile.

10. Find the mean for the following frequency distribution :

Class-interval	0 – 50	50 – 100	100 – 150	150 – 200	200 – 250	250 – 300
Frequency	4	8	16	13	6	3

[2003]

11. Draw a histogram and hence estimate the mode for the following frequency distribution :

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Frequency	2	8	10	5	4	3

[2003, 2013]

12. For the following set of data, find the median :

10, 75, 3, 81, 17, 27, 4, 48, 12, 47, 9 and 15. **[2004]**

13. For the following frequency distribution, draw a histogram. Hence, calculate the mode.

Class :	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30
Frequency	7	18	10	8	5

[2004]

14. Using a graph paper, draw an Ogive for the following distribution which shows a record of the weight in kilograms of 200 students.

Weight	40–45	45–50	50–55	55–60	60–65	65–70	70–75	75–80
Frequency	5	17	22	45	51	31	20	9

Use your Ogive to estimate the following :

(i) The percentage of students weighing 55 kg or more,

(ii) The weight above which the heaviest 30% of the students fall,

(iii) The number of students who are (a) under-weight and (b) over-weight, if 55·70 kg is considered as standard weight ? **[2005]**

15. The daily wages of 80 workers in a building project are given below :

Wages in ₹	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100	100 – 110
No. of workers	6	10	15	19	12	8	6	4

Using graph paper, draw an Ogive for the above distribution.

Use your Ogive to estimate :

(i) the median wages of the workers.

(ii) the percentage of workers who earn more than ₹ 75 day.

(iii) the upper quartile wage of the workers.

(iv) the lower quartile wage of the workers.

(v) Inter quartile range.

16. The distribution, given below, shows the marks obtained by 25 students in an aptitude test. Find the mean, median and mode of the distribution.

Marks obtained	5	6	7	8	9	10
No. of students	3	9	6	4	2	1

[2010]

17. The mean of the following distribution is 52 and the frequency of class interval 30-40 is 'f'. Find 'f'.

Class Interval	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	5	3	f	7	2	6	13

[2010]

18. The monthly income of a group of 320 employees in a company is given below :

Monthly Income [in thousands]	6-7	7-8	8-9	9-10	10-11	11-12	12-13
No. of Employees	20	45	65	95	60	30	5

[2010]

Draw an ogive of the given distribution on a graph sheet taking 2 cm = ₹ 1000 on one axis and 2 cm = 50 employees on the other axis. From the graph determine :

(i) the median wage.

(ii) the number of employees whose income is below ₹ 8,500.

(iii) if the salary of a senior employee is above ₹ 11,500, find the number of senior employees in the company.

(iv) the upper quartile. **[2010]**

19. A Mathematics aptitude test of 50 students was recorded as follows :

Marks	50 - 60	60 - 70	70 - 80	80 - 90	90 - 100
No. of Students	4	8	14	19	5

Draw a histogram for the above data using a graph paper and locate the mode. **[2011]**

20. Marks obtained by 200 students in an examination are given below :

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of students	5	11	10	20	28	37	40	29	14	6

Draw an ogive for the given distribution taking 2 cm = 10 marks on one axis and 2 cm = 20 students on the other axis. Using the graph, determine :

(i) The median marks.

(ii) The number of students who failed if minimum marks required to pass is 40 ?

(iii) If scoring 85 and more marks is considered as grade one, find the number of students who secured grade one in the examination ? **[2011]**

21. Marks obtained by 40 students in a short assessment is given below, where a and b are two missing data.

Marks	5	6	7	8	9
Number of Students	6	a	16	13	b

If the mean of the distribution is 7·2, find a and b. **[2012]**

22. Find the mode and the median of the following frequency distribution :

x	10	11	12	13	14	15
f	1	4	7	5	9	3

[2012]

23. The median of the observations 11, 12, 14, $(x - 2)$, $(x + 4)$, $(x + 9)$, 32, 38, 47 arranged in ascending order is 24. Find the value of x and hence find the mean. **[2013]**

24. The numbers 6, 8, 10, 12, 13 and x are arranged in an ascending order. If the mean of the observations is equal to the median, find the value of x. **[2015]**

25. (Use a graph paper for this question). The daily pocket expenses of 200 students in a school are given below :

Pocket expenses (in ₹)	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
No. of students (frequency)	10	14	28	42	50	30	14	12

Draw a histogram representing the above distribution and estimate the mode from the graph. **[2014]**

26. The marks obtained by 100 students in a mathematics test are given below :

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of students	3	7	12	17	23	14	9	6	5	4

Draw an ogive for the given distribution on a graph sheet.

Use a scale of 2 cm = 10 units on both the axes.

Use the ogive to estimate :

(i) median. (ii) lower quartile.

(iii) number of students who obtained more than 85% marks in the test.

(iv) number of students failed, if the pass percentage was 35. **[2014]**

27. The marks obtained by 30 students in a class assessment of 5 marks is given below :

Marks	0	1	2	3	4	5
No. of students	1	3	6	10	5	5

Calculate the mean, median and mode of the above distribution. **[2015]**

28. The weight of 50 workers is given below :

Weight in Kg	50-60	60-70	70-80	80-90	90-100	100-110	110-120
No. of workers	4	7	11	14	6	5	3

Draw an ogive of the given distribution using a graph sheet. Take 2 cm = 10 kg on one axis and 2 cm = 5 workers along the other axis. Use the ogive drawn to estimate the following :

(i) the upper and lower quartiles.

(ii) if weighing 95 Kg and above is considered overweight, find the number of workers who are overweight. **[2015]**