Integrals

1. If the integral

$$\int \frac{5\tan x}{\tan x - 2}dx = x + a\ln\left|\sin x - 2\cos x\right| + k,$$

then a is equal to **[AIEEE-2012]**

(1) –2 (2) 1

(3) 2 (4) –1

2. If $\int f(x)dx = \psi(x)$, then $\int x^5 f(x^3)dx$ is equal to **[JEE (Main)-2013]**

(1) $\frac{1}{3}\left[x^3\psi(x^3) - \int x^2\psi(x^3)dx\right] + C$

(2) $\frac{1}{3}x^3\psi\left(x^3\right) - 3\int x^3\psi(x^3)dx + C$

(3) $\frac{1}{3}x^3\psi\left(x^3\right) - \int x^2\psi(x^3)dx + C$

(4) $\frac{1}{3}\left[x^3\psi\left(x^3\right) - \int x^3\psi(x^3)dx\right] + C$

3. Let the population of rabbits surviving at a time t be governed by the differential equation

$\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$. If $p(0) = 100$, then $p(t)$ equals **[JEE (Main)-2014]**

(1) $600 - 500\ e^{t/2}$ (2) $400 - 300\ e^{-t/2}$

(3) $400 - 300\ e^{t/2}$ (4) $300 - 200\ e^{-t/2}$

4. The integral $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ equals **[JEE (Main)-2015]**

(1) $\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$ (2) $(x^4+1)^{\frac{1}{4}} + c$

(3) $-(x^4+1)^{\frac{1}{4}} + c$ (4) $-\left(\frac{x^4+1}{x^4}\right)^{\frac{1}{4}} + c$

5. The integral $\int \frac{2x^{12} + 5x^9}{\left(x^5 + x^3 + 1\right)^3}$ is equal to **[JEE (Main)-2016]**

(1) $\frac{x^{10}}{2\left(x^5 + x^3 + 1\right)^2} + C$

(2) $\frac{x^5}{2\left(x^5 + x^3 + 1\right)^2} + C$

(3) $\frac{-x^{10}}{2\left(x^5 + x^3 + 1\right)^2} + C$

(4) $\frac{-x^5}{\left(x^5 + x^3 + 1\right)^2} + C$

where C is an arbitrary constant.

6. Let $I_n = \int \tan^n x dx, (n > 1)$.

If $I_4 + I_6 = a\ \tan^5 x + bx^5 + C$, where C is a constant of integration, then the ordered pair (a, b) is equal to **[JEE (Main)-2017]**

(1) $\left(\frac{1}{5}, 0\right)$ (2) $\left(\frac{1}{5}, -1\right)$

(3) $\left(-\frac{1}{5}, 0\right)$ (4) $\left(-\frac{1}{5}, 1\right)$

7. The integral

$$\int \frac{\sin^2 x\cos^2 x}{(\sin^5 x + \cos^3 x\sin^2 x + \sin^3 x\cos^2 x + \cos^5 x)^2}dx$$

is equal to **[JEE (Main)-2018]**

(1) $\frac{1}{3(1+\tan^3 x)} + C$ (2) $\frac{-1}{3(1+\tan^3 x)} + C$

(3) $\frac{1}{1+\cot^3 x} + C$ (4) $\frac{-1}{1+\cot^3 x} + C$

(where C is a constant of integration)

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8. For $x^2 \neq n\pi + 1$, $n \in N$ (the set of natural numbers), the integral $\int x\sqrt{\frac{2\sin\left(x^2-1\right)-\sin 2\left(x^2-1\right)}{2\sin\left(x^2-1\right)+\sin 2\left(x^2-1\right)}}\,dx$ is equal to

(where c is a constant of integration)

[JEE (Main)-2019]

(1) $\frac{1}{2}\log_e\left|\sec\left(x^2-1\right)\right|+c$

(2) $\frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$

(3) $\log_e\left|\cos\left(\frac{x^2-1}{2}\right)\right|+c$

(4) $\log_e\left|\frac{1}{2}\sec^2\left(x^2-1\right)\right|+c$

9. If $f(x) = \int\frac{5x^8+7x^6}{\left(x^2+1+2x^7\right)^2}dx, (x \geq 0)$, and $f(0) = 0$, then the value of $f(1)$ is **[JEE (Main)-2019]**

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$

(3) $-\frac{1}{2}$ (4) $-\frac{1}{4}$

10. Let $n \geq 2$ be a nutural number and $0 < \theta < \pi/2$. Then $\int\frac{(\sin^n\theta-\sin\theta)^{\frac{1}{n}}\cos\theta}{\sin^{n+1}\theta}d\theta$ is equal to

(where C is a constant of integration)

[JEE (Main)-2019]

(1) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n+1}\theta}\right)^{\frac{n+1}{n}}+C$

(2) $\frac{n}{n^2+1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

(3) $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

(4) $\frac{n}{n^2-1}\left(1+\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

11. If $\int x^5e^{-4x^3}dx = \frac{1}{48}e^{-4x^3}f(x)+C$, where C is a constant of integration, then $f(x)$ is equal to

[JEE (Main)-2019]

(1) $4x^3 + 1$ (2) $-4x^3 - 1$

(3) $-2x^3 + 1$ (4) $-2x^3 - 1$

12. If $\int\frac{\sqrt{1-x^2}}{x^4}dx = A(x)\left(\sqrt{1-x^2}\right)^m+C$, for a suitable chosen integer m and a function $A(x)$, where C is a constant of integration, then $(A(x))^m$ equals **[JEE (Main)-2019]**

(1) $\frac{-1}{3x^3}$ (2) $\frac{1}{27x^6}$

(3) $\frac{1}{9x^4}$ (4) $\frac{-1}{27x^9}$

13. If $\int\frac{x+1}{\sqrt{2x-1}}dx = f(x)\sqrt{2x-1}+C$, where C is a constant of integration, then $f(x)$ is equal to **[JEE (Main)-2019]**

(1) $\frac{1}{3}(x+1)$ (2) $\frac{1}{3}(x+4)$

(3) $\frac{2}{3}(x-4)$ (4) $\frac{2}{3}(x+2)$

14. The integral $\int\cos\left(\log_e x\right)dx$ is equal to (where C is a constant of integration) **[JEE (Main)-2019]**

(1) $x\left[\cos\left(\log_e x\right)-\sin\left(\log_e x\right)\right]+C$

(2) $\frac{x}{2}\left[\cos\left(\log_e x\right)+\sin\left(\log_e x\right)\right]+C$

(3) $\frac{x}{2}\left[\sin\left(\log_e x\right)-\cos\left(\log_e x\right)\right]+C$

(4) $x\left[\cos\left(\log_e x\right)+\sin\left(\log_e x\right)\right]+C$

15. The integral $\int\frac{3x^{13}+2x^{11}}{(2x^4+3x^2+1)^4}dx$ is equal to

(where C is a constant of integration)

[JEE (Main)-2019]

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(1) $\frac{x^4}{6(2x^4+3x^2+1)^3}+C$

(2) $\frac{x^{12}}{(2x^4+3x^2+1)^3}+C$

(3) $\frac{x^{12}}{6(2x^4+3x^2+1)^3}+C$

(4) $\frac{x^4}{(2x^4+3x^2+1)^3}+C$

16. $\int \frac{\sin\frac{5x}{2}}{\sin\frac{x}{2}}dx$ is equal to

(where c is a constant of integration)

[JEE (Main)-2019]

(1) $2x + \sin x + 2\sin 2x + c$

(2) $x + 2\sin x + 2\sin 2x + c$

(3) $x + 2\sin x + \sin 2x + c$

(4) $2x + \sin x + \sin 2x + c$

17. If $\int \frac{dx}{x^3(1+x^6)^{2/3}} = xf(x)(1+x^6)^{\frac{1}{3}}+C$ where C is a constant of integration, then the function $f(x)$ is equal to: **[JEE (Main)-2019]**

(1) $-\frac{1}{2x^3}$ (2) $\frac{3}{x^2}$

(3) $-\frac{1}{6x^3}$ (4) $-\frac{1}{2x^2}$

18. The integral $\int \sec^{2/3}x \operatorname{cosec}^{4/3}x\,dx$ is equal to (Here C is a constant of integration)

[JEE (Main)-2019]

(1) $-3\cot^{-1/3}x + C$ (2) $-3\tan^{-1/3}x + C$

(3) $-\frac{3}{4}\tan^{-4/3}x + C$ (4) $3\tan^{-1/3}x + C$

19. If $\int e^{\sec x}\left(\sec x\tan x f(x)+\sec x\tan x+\sec^2 x\right)dx = e^{\sec x}f(x)+C$, then a possible choice of $f(x)$ is

[JEE (Main)-2019]

(1) $\sec x-\tan x-\frac{1}{2}$ (2) $\sec x+\tan x+\frac{1}{2}$

(3) $\sec x+x\tan x-\frac{1}{2}$ (4) $x\sec x+t\text{an}\,x+\frac{1}{2}$

20. If $\int \frac{dx}{(x^2-2x+10)^2}$

$= A\left(\tan^{-1}\left(\frac{x-1}{3}\right)+\frac{f(x)}{x^2-2x+10}\right)+C$

where C is a constant of integration, then

[JEE (Main)-2019]

(1) $A=\frac{1}{81}$ and $f(x)=3(x-1)$

(2) $A=\frac{1}{54}$ and $f(x)=3(x-1)$

(3) $A=\frac{1}{27}$ and $f(x)=9(x-1)$

(4) $A=\frac{1}{54}$ and $f(x)=9(x-1)^2$

21. If $\int x^5e^{-x^2}dx = g(x)e^{-x^2}+c$, where c is a constant of integration, then $g(-1)$ is equal to

[JEE (Main)-2019]

(1) -1 (2) $-\frac{1}{2}$

(3) 1 (4) $-\frac{5}{2}$

22. The integral $\int \frac{2x^3-1}{x^4+x}dx$ is equal to

(Here C is a constant of integration)

[JEE (Main)-2019]

(1) $\log_e\left|\frac{x^3+1}{x}\right|+C$ (2) $\frac{1}{2}\log_e\frac{(x^3+1)^2}{|x^3|}+C$

(3) $\frac{1}{2}\log_e\frac{|x^3+1|}{x^2}+C$ (4) $\log_e\frac{|x^3+1|}{x^2}+C$

23. Let $\alpha \in (0, \pi/2)$ be fixed. If the integral $\int \frac{\tan x+\tan\alpha}{\tan x-\tan\alpha}dx =$

$A(x)\cos 2\alpha + B(x)\sin 2\alpha + C$, where C is a constant of integration, then the functions $A(x)$ and $B(x)$ are respectively **[JEE (Main)-2019]**

(1) $x-\alpha$ and $\log_e|\cos(x-\alpha)|$

(2) $x+\alpha$ and $\log_e|\sin(x-\alpha)|$

(3) $x-\alpha$ and $\log_e|\sin(x-\alpha)|$

(4) $x+\alpha$ and $\log_e|\sin(x+\alpha)|$

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24. If θ_1 and θ_2 be respectively the smallest and the largest values of θ in $(0, 2\pi) - \{\pi\}$ which satisfy the equation, $2\cot^2\theta - \frac{5}{\sin\theta} + 4 = 0$, then $\int_{\theta_1}^{\theta_2} \cos^2 3\theta\, d\theta$, is equal to **[JEE (Main)-2020]**

(1) $\frac{\pi}{3} + \frac{1}{6}$ (2) $\frac{\pi}{3}$

(3) $\frac{2\pi}{3}$ (4) $\frac{\pi}{9}$

25. If $\int \frac{\cos x dx}{\sin^3 x(1+\sin^6 x)^{2/3}} = f(x)(1+\sin^6 x)^{1/\lambda} + c$

where c is a constant of integration, then

$\lambda f\left(\frac{\pi}{3}\right)$ is equal to **[JEE (Main)-2020]**

(1) $\frac{9}{8}$ (2) $-\frac{9}{8}$

(3) –2 (4) 2

26. The integral $\int \frac{dx}{(x+4)^{\frac{8}{7}}(x-3)^{\frac{6}{7}}}$ is equal to

(where C is a constant of integration) **[JEE (Main)-2020]**

(1) $\frac{1}{2}\left(\frac{x-3}{x+4}\right)^{3/7} + C$ (2) $-\frac{1}{13}\left(\frac{x-3}{x+4}\right)^{-13/7} + C$

(3) $\left(\frac{x-3}{x+4}\right)^{1/7} + C$ (4) $-\left(\frac{x-3}{x+4}\right)^{-1/7} + C$

27. If $\int \frac{d\theta}{\cos^2\theta(\tan 2\theta + \sec 2\theta)} =$

$\lambda\tan\theta + 2\log_e|f(\theta)| + C$ where C is a constant of integration, then the ordered pair $(\lambda, f(\theta))$ is equal to **[JEE (Main)-2020]**

(1) $(1, 1 - \tan\theta)$ (2) $(-1, 1 + \tan\theta)$

(3) $(-1, 1 - \tan\theta)$ (4) $(1, 1 + \tan\theta)$

28. If $\int \sin^{-1}\left(\sqrt{\frac{x}{1+x}}\right)dx = A(x)\tan^{-1}\left(\sqrt{x}\right) + B(x) + C$,

where C is a constant of integration, then the ordered pair $(A(x), B(x))$ can be **[JEE (Main)-2020]**

(1) $\left(x+1, -\sqrt{x}\right)$ (2) $\left(x+1, \sqrt{x}\right)$

(3) $\left(x-1, -\sqrt{x}\right)$ (4) $\left(x-1, \sqrt{x}\right)$

29. The integral $\int\left(\frac{x}{x\sin x + \cos x}\right)^2 dx$ is equal to (where C is a constant of integration) **[JEE (Main)-2020]**

(1) $\tan x - \frac{x\sec x}{x\sin x + \cos x} + C$

(2) $\sec x - \frac{x\tan x}{x\sin x + \cos x} + C$

(3) $\tan x + \frac{x\sec x}{x\sin x + \cos x} + C$

(4) $\sec x + \frac{x\tan x}{x\sin x + \cos x} + C$

30. Let $f(x) = \int \frac{\sqrt{x}}{(1+x)^2} dx$ $(x \geq 0)$. Then $f(3) - f(1)$ is equal to **[JEE (Main)-2020]**

(1) $\frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$ (2) $-\frac{\pi}{6} + \frac{1}{2} + \frac{\sqrt{3}}{4}$

(3) $-\frac{\pi}{12} + \frac{1}{2} + \frac{\sqrt{3}}{4}$ (4) $\frac{\pi}{6} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

31. If $\int \left(e^{2x} + 2e^x - e^{-x} - 1\right)e^{(e^x + e^{-x})}dx$

$= g(x)e^{(e^x+e^{-x})} + c$, where c is a constant of integration, then $g(0)$ is equal to **[JEE (Main)-2020]**

(1) e^2 (2) 1

(3) 2 (4) e

32. If $\int \frac{\cos\theta}{5 + 7\sin\theta - 2\cos^2\theta} d\theta = A\log_e|B(\theta)| + C$,

where C is a constant of integration, then $\frac{B(\theta)}{A}$ can be **[JEE (Main)-2020]**

(1) $\frac{2\sin\theta + 1}{5(\sin\theta + 3)}$ (2) $\frac{2\sin\theta + 1}{\sin\theta + 3}$

(3) $\frac{5(2\sin\theta + 1)}{\sin\theta + 3}$ (4) $\frac{5(\sin\theta + 3)}{2\sin\theta + 1}$

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