21

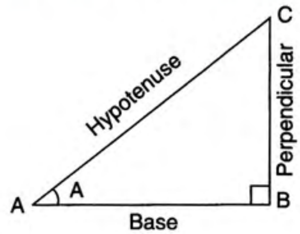
Trigonometrical Identities

(Including Trigonometrical Ratios of Complementary Angles and Use of Four Figure Trigonometrical Tables)

21.1 Trigonometry :

Trigonometry means; *the science which deals with the **measurements of triangles**.*

21.2 Trigonometrical Ratios :



There are six trigonometrical ratios relating to the three sides of a right-angled triangle (this has already been done by students in Class IX).

For an acute angle of a right-angled triangle :

(1) **sine** (sin) $= \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} \Rightarrow \sin A = \dfrac{BC}{AC}$

(2) **cosine** (cos) $= \dfrac{\text{Base}}{\text{Hypotenuse}} \Rightarrow \cos A = \dfrac{AB}{AC}$

(3) **tangent** (tan) $= \dfrac{\text{Perpendicular}}{\text{Base}} \Rightarrow \tan A = \dfrac{BC}{AB}$

(4) **cotangent** (cot) $= \dfrac{\text{Base}}{\text{Perpendicular}} \Rightarrow \cot A = \dfrac{AB}{BC}$

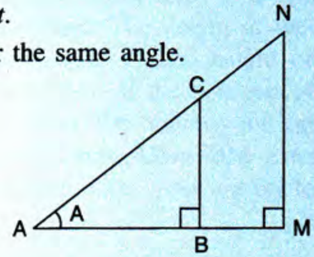
(5) **secant** (sec) $= \dfrac{\text{Hypotenuse}}{\text{Base}} \Rightarrow \sec A = \dfrac{AC}{AB}$

(6) **cosecant** (cosec) $= \dfrac{\text{Hypotenuse}}{\text{Perpendicular}} \Rightarrow \text{cosec } A = \dfrac{AC}{BC}$

Remember :

1. Each *trigonometrical ratio* is a *real number* and has *no unit*.
2. The values of trigonometrical ratios are always the same for the same angle.

For Example :



In right triangle ABC, $\sin A = \dfrac{BC}{AC}$

and in right triangle AMN, $\sin A = \dfrac{MN}{AN}$

Since the angle A is same for both the triangles; we have $\sin A = \dfrac{BC}{AC} = \dfrac{MN}{AN}$

For the same reason : $\cos A = \dfrac{AB}{AC} = \dfrac{AM}{AN}$, $\tan A = \dfrac{BC}{AB} = \dfrac{MN}{AM}$ and so on.

21.3 Relations Between Different Trigonometrical Ratios :

1. Reciprocal relations :

Since $\sin A = \dfrac{\text{perpendicular}}{\text{hypotenuse}}$ and $\operatorname{cosec} A = \dfrac{\text{hypotenuse}}{\text{perpendicular}}$

$\Rightarrow$ sin A and cosec A are reciprocals of each other

i.e. $\mathbf{\sin A = \dfrac{1}{\operatorname{cosec} A}}$ and $\mathbf{\operatorname{cosec} A = \dfrac{1}{\sin A}}$

Similarly, (i) cos A and sec A are reciprocals of each other

i.e. $\mathbf{\cos A = \dfrac{1}{\sec A}}$ and $\mathbf{\sec A = \dfrac{1}{\cos A}}$

(ii) tan A and cot A are reciprocals of each other

i.e. $\mathbf{\tan A = \dfrac{1}{\cot A}}$ and $\mathbf{\cot A = \dfrac{1}{\tan A}}$.

2. Quotient relations :

Since $\sin A = \dfrac{\text{perpendicular}}{\text{hypotenuse}}$ and $\cos A = \dfrac{\text{base}}{\text{hypotenuse}}$

$$\therefore \quad \frac{\sin A}{\cos A} = \frac{\text{perpendicular}}{\text{hypotenuse}} \times \frac{\text{hypotenuse}}{\text{base}}$$

$$= \frac{\text{perpendicular}}{\text{base}} = \tan A$$

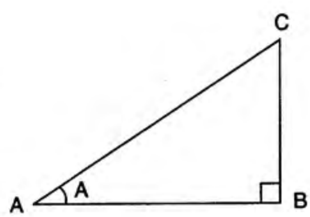
Similarly, $\dfrac{\cos A}{\sin A} = \cot A$

Hence, $\mathbf{\tan A = \dfrac{\sin A}{\cos A}}$ and $\mathbf{\cot A = \dfrac{\cos A}{\sin A}}$

3. Square relations :

In right-angled triangle ABC, with angle B = 90°;

$\sin A = \dfrac{BC}{AC}$ and $\cos A = \dfrac{AB}{AC}$



$$\Rightarrow \quad \mathbf{\sin^2 A + \cos^2 A} = \left(\frac{BC}{AC}\right)^2 + \left(\frac{AB}{AC}\right)^2$$

$$= \frac{BC^2 + AB^2}{AC^2}$$

$$= \frac{AC^2}{AC^2} = 1 \qquad [\text{As, } AB^2 + BC^2 = AC^2]$$

$$\boxed{\therefore \sin^2 A + \cos^2 A = 1}$$

Similarly,

(i) $1 + \tan^2 A = 1 + \left(\frac{BC}{AB}\right)^2$

$$= \frac{AB^2 + BC^2}{AB^2} = \frac{AC^2}{AB^2} \qquad [\because AB^2 + BC^2 = AC^2]$$

$$= \left(\frac{AC}{AB}\right)^2 = \sec^2 A \qquad [\because \sec A = \frac{AC}{AB}]$$

(ii) $1 + \cot^2 A = 1 + \left(\frac{AB}{BC}\right)^2$

$$= \frac{BC^2 + AB^2}{BC^2} = \frac{AC^2}{BC^2}$$

$$= \left(\frac{AC}{BC}\right)^2 = \text{cosec}^2 A \qquad [\because \text{cosec } A = \frac{AC}{BC}]$$

Hence,

$$\mathbf{\sin^2 A + \cos^2 A = 1; \quad 1 + \tan^2 A = \sec^2 A \quad \text{and} \quad 1 + \cot^2 A = \text{cosec}^2 A.}$$

Remember :

(i) $\sin^2 A + \cos^2 A = 1 \Rightarrow \sin^2 A = 1 - \cos^2 A$ and $\cos^2 A = 1 - \sin^2 A$

(ii) $1 + \tan^2 A = \sec^2 A \Rightarrow \sec^2 A - \tan^2 A = 1$ and $\sec^2 A - 1 = \tan^2 A$

(iii) $1 + \cot^2 A = \text{cosec}^2 A \Rightarrow \text{cosec}^2 A - \cot^2 A = 1$ and $\text{cosec}^2 A - 1 = \cot^2 A$

21.4 Trigonometric Identities :

When an equation, involving trigonometrical ratios of an angle A, is true for all values of A; the equation is called a ***trigonometrical identity***.

Each of the relations given above; viz. reciprocal relations, quotient relations and square relations; is a trigonometrical identity.

1 Prove the identity : tan A + cot A = sec A . cosec A

Solution :

L.H.S. $= \tan A + \cot A$

$$= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} = \frac{\sin^2 A + \cos^2 A}{\cos A \cdot \sin A}$$

$$= \frac{1}{\cos A \sin A} \qquad [\because \sin^2 A + \cos^2 A = 1]$$

$= \sec A \cdot \text{cosec } A =$ **R.H.S.** $\qquad [\because \sec A = \frac{1}{\cos A}$ and $\text{cosec } A = \frac{1}{\sin A}]$

Important :

In order to prove a trigonometrical identity : start with any side **left-hand-side** (L.H.S.) or **right-hand-side** (R.H.S.) and by applying trigonometrical relations reach to the other side, *i.e.*, if you start with L.H.S.; reach to R.H.S. and if you start with R.H.S. reach to L.H.S.

In general, *start with the more complicated side.*

Sometimes both the sides are complicated. In this situation, both the sides may be taken and reduced independently to the same result.

2 Prove that : (i) $\cos^4 A - \sin^4 A = 2\cos^2 A - 1$

(ii) $(1 + \cot A)^2 + (1 - \cot A)^2 = 2\,\text{cosec}^2 A$

(iii) $\tan^4 A + \tan^2 A = \sec^4 A - \sec^2 A$

Solution :

(i) **L.H.S.** $= (\cos^2 A)^2 - (\sin^2 A)^2$

$= (\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)$

$= \cos^2 A - \sin^2 A$ [As, $\cos^2 A + \sin^2 A = 1$]

$= \cos^2 A - (1 - \cos^2 A)$ [As, $\sin^2 A = 1 - \cos^2 A$]

$= 2\cos^2 A - 1 =$ **R.H.S.**

(ii) **L.H.S.** $= 1 + \cot^2 A + 2\cot A + 1 + \cot^2 A - 2\cot A$

$= 2 + 2\cot^2 A$

$= 2(1 + \cot^2 A)$

$= 2\,\text{cosec}^2 A$ [As, $1 + \cot^2 A = \text{cosec}^2 A$]

$=$ **R.H.S.**

(iii) **L.H.S.** $= \tan^2 A \,.\, (\tan^2 A + 1)$

$= (\sec^2 A - 1)\,.\,\sec^2 A$ [As, $\sec^2 A = 1 + \tan^2 A$]

$= \sec^4 A - \sec^2 A =$ **R.H.S.**

3 Prove that :

(i) $\dfrac{\sin A}{1+\cos A} + \dfrac{1+\cos A}{\sin A} = 2\,\text{cosec}\,A$ **[2004, 2009]**

(ii) $\dfrac{1+\cos A}{1-\cos A} = (\text{cosec}\,A + \cot A)^2$ (iii) $\dfrac{\cot A+\tan B}{\cot B+\tan A} = \cot A \tan B.$

(iv) $\dfrac{\cos A \cot A}{1-\sin A} = 1 + \text{cosec}\,A$ **[2006 type]**

Solution :

(i) **L.H.S.** $= \dfrac{\sin^2 A+(1+\cos A)^2}{(1+\cos A)\sin A}$

$= \dfrac{\sin^2 A+1+\cos^2 A+2\cos A}{(1+\cos A)\sin A}$

$= \dfrac{1+1+2\cos A}{(1+\cos A)\sin A}$ [$\because \sin^2 A + \cos^2 A = 1$]

$= \dfrac{2(1+\cos A)}{(1+\cos A)\sin A} = \dfrac{2}{\sin A} = 2\,\text{cosec}\,A =$ **R.H.S.**

(ii) **R.H.S.** $= \left(\dfrac{1}{\sin A} + \dfrac{\cos A}{\sin A}\right)^2$ [Starting with the complicated side'

$= \dfrac{(1+\cos A)^2}{\sin^2 A} = \dfrac{(1+\cos A)^2}{1-\cos^2 A}$ [$\because \sin^2 A = 1 - \cos^2 A$]

$= \dfrac{(1+\cos A)(1+\cos A)}{(1+\cos A)(1-\cos A)} = \dfrac{1+\cos A}{1-\cos A}$ = **L.H.S.**

Alternative method :

L.H.S. $= \dfrac{1+\cos A}{1-\cos A} \times \dfrac{1+\cos A}{1+\cos A}$ [Multiplying and dividing by (1 + cos A)]

$= \dfrac{(1+\cos A)^2}{1-\cos^2 A}$

$= \dfrac{(1+\cos A)^2}{\sin^2 A} = \left(\dfrac{1+\cos A}{\sin A}\right)^2 = \left(\dfrac{1}{\sin A} + \dfrac{\cos A}{\sin A}\right)^2$

$= (\text{cosec } A + \cot A)^2$ = **R.H.S.**

(iii) **L.H.S.** $= \dfrac{\dfrac{\cos A}{\sin A} + \dfrac{\sin B}{\cos B}}{\dfrac{\cos B}{\sin B} + \dfrac{\sin A}{\cos A}} = \dfrac{\dfrac{\cos A \cos B + \sin A \sin B}{\sin A \cos B}}{\dfrac{\cos A \cos B + \sin A \sin B}{\sin B . \cos A}} = \dfrac{\sin B \cos A}{\sin A \cos B}$

$= \left(\dfrac{\cos A}{\sin A}\right) \cdot \left(\dfrac{\sin B}{\cos B}\right) = \cot A \cdot \tan B$ = **R.H.S.**

Alternative method :

L.H.S. $= \dfrac{\cot A + \tan B}{\dfrac{1}{\tan B} + \dfrac{1}{\cot A}}$

$= \dfrac{\cot A + \tan B}{\dfrac{\cot A + \tan B}{\tan B \cot A}} = \dfrac{(\cot A + \tan B)\cot A \tan B}{\cot A + \tan B} = \cot A \tan B$ = **R.H.S.**

(iv) **L.H.S.** $= \dfrac{\cos A \cot A}{1-\sin A} = \dfrac{\cos A \times \dfrac{\cos A}{\sin A}}{1-\sin A} = \dfrac{\cos^2 A}{\sin A(1-\sin A)}$

$= \dfrac{(1-\sin A)(1+\sin A)}{\sin A(1-\sin A)}$ [$\because \cos^2 A = 1 - \sin^2 A = (1 - \sin A)(1 + \sin A)$]

$= \dfrac{1+\sin A}{\sin A} = \dfrac{1}{\sin A} + \dfrac{\sin A}{\sin A} = \text{cosec } A + 1$ = **R.H.S.**

4 Prove that : $\dfrac{\sec A - \tan A}{\text{cosec } A + \cot A} = \dfrac{\text{cosec } A - \cot A}{\sec A + \tan A}$

Solution :

Since $\sec^2 A - \tan^2 A = 1$ and $\text{cosec}^2 A - \cot^2 A = 1$.

$$\therefore \quad \sec^2 A - \tan^2 A = \text{cosec}^2 A - \cot^2 A$$

$$\Rightarrow \quad (\sec A - \tan A)(\sec A + \tan A) = (\text{cosec } A - \cot A)(\text{cosec } A + \cot A)$$

$$\Rightarrow \quad \frac{\sec A - \tan A}{\text{cosec } A + \cot A} = \frac{\text{cosec } A - \cot A}{\sec A + \tan A}$$

Hence Proved.

Alternative method :

$$\text{L.H.S.} = \frac{\sec A - \tan A}{\text{cosec } A + \cot A}$$

$$= \frac{\sec A - \tan A}{\text{cosec } A + \cot A} \times \frac{\text{cosec } A - \cot A}{\text{cosec } A - \cot A} \times \frac{\sec A + \tan A}{\sec A + \tan A}$$

$$= \frac{(\sec^2 A - \tan^2 A)(\text{cosec } A - \cot A)}{(\text{cosec}^2 A - \cot^2 A)(\sec A + \tan A)}$$

$$= \frac{1 \times (\text{cosec } A - \cot A)}{1 \times (\sec A + \tan A)} = \frac{\text{cosec } A - \cot A}{\sec A + \tan A} = \text{R.H.S}$$

5 Prove that : (i) $\sqrt{\dfrac{1-\sin A}{1+\sin A}} = \sec A - \tan A$

(ii) $\dfrac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \dfrac{1+\sin A}{\cos A}$

Solution :

(i) $\text{L.H.S.} = \sqrt{\dfrac{1-\sin A}{1+\sin A}} \times \dfrac{\sqrt{1-\sin A}}{\sqrt{1-\sin A}}$ [Multiplying and dividing by $\sqrt{1-\sin A}$]

$$= \frac{1-\sin A}{\sqrt{1-\sin^2 A}} = \frac{1-\sin A}{\cos A} \qquad [\because 1 - \sin^2 A = \cos^2 A]$$

$$= \frac{1}{\cos A} - \frac{\sin A}{\cos A} = \sec A - \tan A = \text{R.H.S.}$$

(ii) $\text{L.H.S.} = \dfrac{\tan A + \sec A - (\sec^2 A - \tan^2 A)}{\tan A - \sec A + 1}$ $[\because \sec^2 A - \tan^2 A = 1]$

$$= \frac{(\tan A + \sec A) - (\sec A + \tan A)(\sec A - \tan A)}{\tan A - \sec A + 1}$$

$$= \frac{(\tan A + \sec A)(1 - \sec A + \tan A)}{\tan A - \sec A + 1}$$

$$= \tan A + \sec A = \frac{\sin A}{\cos A} + \frac{1}{\cos A} = \frac{\sin A + 1}{\cos A} = \text{R.H.S.}$$

Alternative method :

$$\text{L.H.S.} = \frac{\dfrac{\sin A}{\cos A} + \dfrac{1}{\cos A} - 1}{\dfrac{\sin A}{\cos A} - \dfrac{1}{\cos A} + 1} = \frac{\dfrac{\sin A + 1 - \cos A}{\cos A}}{\dfrac{\sin A - 1 + \cos A}{\cos A}}$$

$$= \frac{\sin A + 1 - \cos A}{\sin A - 1 + \cos A}$$

$$= \frac{\sin A + 1 - \cos A}{\sin A - 1 + \cos A} \times \frac{1 + \sin A}{1 + \sin A}$$ [Multiplying and dividing by 1 + sin A]

$$= \frac{(\sin A + 1 - \cos A)(1 + \sin A)}{\sin A - 1 + \cos A + \sin^2 A - \sin A + \sin A \cos A}$$

$$= \frac{(\sin A + 1 - \cos A)(1 + \sin A)}{-1 + \cos A + (1 - \cos^2 A) + \sin A \cos A}$$ [$\because \sin^2 A = 1 - \cos^2 A$]

$$= \frac{(\sin A + 1 - \cos A)(1 + \sin A)}{\cos A - \cos^2 A + \sin A \cos A}$$

$$= \frac{(\sin A + 1 - \cos A)(1 + \sin A)}{\cos A (1 - \cos A + \sin A)} = \frac{1 + \sin A}{\cos A} =$$ **R.H.S.**

EXERCISE 21(A)

Prove the following identities :

1. $\frac{\sec A - 1}{\sec A + 1} = \frac{1 - \cos A}{1 + \cos A}$ **[2007]**
2. $\frac{1 + \sin A}{1 - \sin A} = \frac{\text{cosec } A + 1}{\text{cosec } A - 1}$
3. $\frac{1}{\tan A + \cot A} = \cos A \sin A$
4. $\tan A - \cot A = \frac{1 - 2\cos^2 A}{\sin A \cos A}$
5. $\sin^4 A - \cos^4 A = 2\sin^2 A - 1$
6. $(1 - \tan A)^2 + (1 + \tan A)^2 = 2\sec^2 A$ **[2005]**
7. $\text{cosec}^4 A - \text{cosec}^2 A = \cot^4 A + \cot^2 A$
8. $\sec A (1 - \sin A)(\sec A + \tan A) = 1$
9. $\text{cosec } A (1 + \cos A)(\text{cosec } A - \cot A) = 1$
10. $\sec^2 A + \text{cosec}^2 A = \sec^2 A \cdot \text{cosec}^2 A$
11. $\frac{(1 + \tan^2 A)\cot A}{\text{cosec}^2 A} = \tan A$
12. $\tan^2 A - \sin^2 A = \tan^2 A \cdot \sin^2 A$
13. $\cot^2 A - \cos^2 A = \cos^2 A \cdot \cot^2 A$
14. $(\text{cosec } A + \sin A)(\text{cosec } A - \sin A)$
$= \cot^2 A + \cos^2 A$
15. $(\sec A - \cos A)(\sec A + \cos A)$
$= \sin^2 A + \tan^2 A$
16. $(\cos A + \sin A)^2 + (\cos A - \sin A)^2 = 2$
17. $(\text{cosec } A - \sin A)(\sec A - \cos A)(\tan A + \cot A)$
$= 1$
18. $\frac{1}{\sec A + \tan A} = \sec A - \tan A$
19. $\text{cosec } A + \cot A = \frac{1}{\text{cosec } A - \cot A}$
20. $\frac{\sec A - \tan A}{\sec A + \tan A} = 1 - 2\sec A \tan A + 2\tan^2 A$
21. $(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2$
$= 7 + \tan^2 A + \cot^2 A$
22. $\sec^2 A \cdot \text{cosec}^2 A = \tan^2 A + \cot^2 A + 2$
23. $\frac{1}{1 + \cos A} + \frac{1}{1 - \cos A} = 2\,\text{cosec}^2 A$
24. $\frac{1}{1 - \sin A} + \frac{1}{1 + \sin A} = 2\sec^2 A$
25. $\frac{\text{cosec } A}{\text{cosec } A - 1} + \frac{\text{cosec } A}{\text{cosec } A + 1} = 2\sec^2 A$
26. $\frac{\sec A}{\sec A + 1} + \frac{\sec A}{\sec A - 1} = 2\,\text{cosec}^2 A$
27. $\frac{1 + \cos A}{1 - \cos A} = \frac{\tan^2 A}{(\sec A - 1)^2}$ **[2012]**
28. $\frac{\cot^2 A}{(\text{cosec } A + 1)^2} = \frac{1 - \sin A}{1 + \sin A}$
29. $\frac{1 + \sin A}{\cos A} + \frac{\cos A}{1 + \sin A} = 2\sec A$

30. $\dfrac{1-\sin A}{1+\sin A} = (\sec A - \tan A)^2$

31. $(\cot A - \operatorname{cosec} A)^2 = \dfrac{1-\cos A}{1+\cos A}$

32. $\dfrac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} = \left(\dfrac{\cos A}{1+\sin A}\right)^2$

33. $\tan^2 A - \tan^2 B = \dfrac{\sin^2 A - \sin^2 B}{\cos^2 A . \cos^2 B}$

34. $\dfrac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$

35. $\dfrac{\sin A}{1+\cos A} = \operatorname{cosec} A - \cot A$ **[2008]**

36. $\dfrac{\cos A}{1-\sin A} = \sec A + \tan A$

37. $\dfrac{\sin A \tan A}{1-\cos A} = 1 + \sec A$

38. $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$

39. $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

40. $\sqrt{\dfrac{1-\cos A}{1+\cos A}} = \operatorname{cosec} A - \cot A$ **[2000]**

41. $\sqrt{\dfrac{1-\cos A}{1+\cos A}} = \dfrac{\sin A}{1+\cos A}$ **[2000, 2013]**

42. $\sqrt{\dfrac{1-\sin A}{1+\sin A}} = \dfrac{\cos A}{1+\sin A}$

43. $1 - \dfrac{\cos^2 A}{1+\sin A} = \sin A$ **[2001]**

44. $\dfrac{1}{\sin A + \cos A} + \dfrac{1}{\sin A - \cos A} = \dfrac{2\sin A}{1 - 2\cos^2 A}$ **[2002]**

45. $\dfrac{\sin A + \cos A}{\sin A - \cos A} + \dfrac{\sin A - \cos A}{\sin A + \cos A} = \dfrac{2}{2\sin^2 A - 1}$

46. $\dfrac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1} = \dfrac{1+\cos A}{\sin A}$

47. $\dfrac{\sin\theta \tan\theta}{1-\cos\theta} = 1 + \sec\theta$ **[2006]**

48. $\dfrac{\cos\theta \cot\theta}{1+\sin\theta} = \operatorname{cosec}\theta - 1$

6 Prove that : (i) $\dfrac{\cos A}{1-\tan A} + \dfrac{\sin A}{1-\cot A} = \cos A + \sin A$ **[2015]**

(ii) $(1 + \tan^2 A) + \left(1 + \dfrac{1}{\tan^2 A}\right) = \dfrac{1}{\sin^2 A - \sin^4 A}$

Solution :

(i) **L.H.S.** $= \dfrac{\cos A}{1-\tan A} + \dfrac{\sin A}{1-\cot A}$

$$= \frac{\cos A}{1-\frac{\sin A}{\cos A}} + \frac{\sin A}{1-\frac{\cos A}{\sin A}}$$

$$= \frac{\cos A}{\frac{\cos A - \sin A}{\cos A}} + \frac{\sin A}{\frac{\sin A - \cos A}{\sin A}}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A}$$

$$= \frac{(\cos A - \sin A)(\cos A + \sin A)}{\cos A - \sin A} = \cos A + \sin A = \textbf{R.H.S.}$$

(ii) **L.H.S.** $= (1 + \tan^2 A) + (1 + \frac{1}{\tan^2 A})$

$= \sec^2 A + (1 + \cot^2 A)$

$= \sec^2 A + \text{cosec}^2 A$

$= \frac{1}{\cos^2 A} + \frac{1}{\sin^2 A} = \frac{\sin^2 A + \cos^2 A}{\cos^2 A \sin^2 A} = \frac{1}{\cos^2 A \sin^2 A}$

$= \frac{1}{(1 - \sin^2 A)\sin^2 A} = \frac{1}{\sin^2 A - \sin^4 A}$ = **R.H.S.**

7 If $\tan A + \sin A = m$ and $\tan A - \sin A = n$;

prove that : $m^2 - n^2 = 4\sqrt{mn}$.

Solution :

$$m^2 - n^2 = (m + n)(m - n)$$

$$= (\tan A + \sin A + \tan A - \sin A)(\tan A + \sin A - \tan A + \sin A)$$

$$= (2 \tan A)(2 \sin A)$$

$$= 4 \tan A \sin A \qquad ...(I)$$

$$4\sqrt{mn} = 4\sqrt{(\tan A + \sin A)(\tan A - \sin A)}$$

$$= 4\sqrt{\tan^2 A - \sin^2 A}$$

$$= 4\sqrt{\frac{\sin^2 A}{\cos^2 A} - \sin^2 A}$$

$$= 4 \sin A \sqrt{\sec^2 A - 1} \qquad [\because \frac{1}{\cos^2 A} = \sec^2 A]$$

$$= 4 \sin A \, . \, \tan A \qquad ...(II) \qquad [\because \sec^2 A - 1 = \tan^2 A]$$

$\therefore \quad m^2 - n^2 = 4\sqrt{mn}$ [From I and II]

Hence Proved.

8 If $x = a \sec A \cos B$, $y = b \sec A \sin B$ and $z = c \tan A$; show that :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

Solution :

L.H.S. $= \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$

$= \frac{(a \sec A \cos B)^2}{a^2} + \frac{(b \sec A \sin B)^2}{b^2} - \frac{(c \tan A)^2}{c^2}$

$$= \frac{a^2 \sec^2 A \cos^2 B}{a^2} + \frac{b^2 \sec^2 A \sin^2 B}{b^2} - \frac{c^2 \tan^2 A}{c^2}$$

$= \sec^2 A \cos^2 B + \sec^2 A \sin^2 B - \tan^2 A$

$= \sec^2 A (\cos^2 B + \sin^2 B) - \tan^2 A$

$= \sec^2 A - \tan^2 A$ $\quad [\because \cos^2 B + \sin^2 B = 1]$

$= 1 =$ **R.H.S.** $\quad [\because \sec^2 A - \tan^2 A = 1]$

EXERCISE 21(B)

1. Prove that :

(i) $\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A$

[2003]

(ii) $\frac{\cos^3 A + \sin^3 A}{\cos A + \sin A} + \frac{\cos^3 A - \sin^3 A}{\cos A - \sin A} = 2$

(iii) $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A} = \sec A \operatorname{cosec} A + 1$

(iv) $\left(\tan A + \frac{1}{\cos A}\right)^2 + \left(\tan A - \frac{1}{\cos A}\right)^2 = 2\left(\frac{1 + \sin^2 A}{1 - \sin^2 A}\right)$

(v) $2 \sin^2 A + \cos^4 A = 1 + \sin^4 A$

(vi) $\frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B} = 0$

(vii) $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

(viii) $(1 + \tan A . \tan B)^2 + (\tan A - \tan B)^2 = \sec^2 A \sec^2 B$

(ix) $\frac{1}{\cos A + \sin A - 1} + \frac{1}{\cos A + \sin A + 1} = \operatorname{cosec} A + \sec A$

2. If $x \cos A + y \sin A = m$ and $x \sin A - y \cos A = n$, then prove that :
$x^2 + y^2 = m^2 + n^2$

3. If $m = a \sec A + b \tan A$ and $n = a \tan A + b \sec A$, then prove that :
$m^2 - n^2 = a^2 - b^2$

4. If $x = r \sin A \cos B$, $y = r \sin A \sin B$ and $z = r \cos A$, then prove that :
$x^2 + y^2 + z^2 = r^2$

5. If $\sin A + \cos A = m$ and $\sec A + \operatorname{cosec} A = n$, show that :
$n (m^2 - 1) = 2 m$

6. If $x = r \cos A \cos B$, $y = r \cos A \sin B$ and $z = r \sin A$, show that :
$x^2 + y^2 + z^2 = r^2$

7. If $\frac{\cos A}{\cos B} = m$ and $\frac{\cos A}{\sin B} = n$, show that :
$(m^2 + n^2) \cos^2 B = n^2$.

21.5 Trigonometrical Ratios of Complementary Angles :

For an acute angle A,

(i) $\sin (90° - A) = \cos A$, (ii) $\cos (90° - A) = \sin A$,

(iii) $\tan (90° - A) = \cot A$, (iv) $\cot (90° - A) = \tan A$,

(v) $\sec (90° - A) = \operatorname{cosec} A$ and (vi) $\operatorname{cosec} (90° - A) = \sec A$.

9 Find the value of x, if :
$\cos x = \cos 60° \cos 30° + \sin 60° \sin 30°$.

Solution :

$$\cos x = \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}$$
$$= \frac{\sqrt{3}+\sqrt{3}}{4} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} = \cos 30° \quad \therefore \; x = 30°$$ **Ans.**

10 Given $\cos 38° \sec (90° - 2A) = 1$; find the value of angle A.

Solution :

$$\cos 38° \sec (90° - 2A) = 1 \Rightarrow \cos 38° \operatorname{cosec} 2A = 1 \quad [\because \sec (90° - \theta) = \operatorname{cosec} \theta]$$
$$\Rightarrow \cos 38° \times \frac{1}{\sin 2A} = 1 \quad [\because \operatorname{cosec} \theta = \frac{1}{\sin \theta}]$$
$$\Rightarrow \sin 2A = \cos 38°$$
$$= \cos (90° - 52°)$$
$$\Rightarrow \sin 2A = \sin 52° \quad [\because \cos (90° - \theta) = \sin \theta]$$
$\therefore$ $2A = 52°$ and $A = 26°$ **Ans.**

EXERCISE 21(C)

1. Show that :
   (i) $\tan 10° \tan 15° \tan 75° \tan 80° = 1$
   (ii) $\sin 42° \sec 48° + \cos 42° \operatorname{cosec} 48° = 2$
   (iii) $\frac{\sin 26°}{\sec 64°} + \frac{\cos 26°}{\operatorname{cosec} 64°} = 1$
2. Express each of the following in terms of angles between 0° and 45° :
   (i) $\sin 59° + \tan 63°$
   (ii) $\operatorname{cosec} 68° + \cot 72°$
   (iii) $\cos 74° + \sec 67°$
3. Show that :
   (i) $\frac{\sin A}{\sin (90° - A)} + \frac{\cos A}{\cos (90° - A)} = \sec A \operatorname{cosec} A$
   (ii) $\sin A \cos A - \frac{\sin A \cos (90° - A) \cos A}{\sec (90° - A)} - \frac{\cos A \sin (90° - A) \sin A}{\operatorname{cosec} (90° - A)} = 0$
4. For triangle ABC, show that :
   (i) $\sin \frac{A+B}{2} = \cos \frac{C}{2}$
   (ii) $\tan \frac{B+C}{2} = \cot \frac{A}{2}$
5. Evaluate :
   (i) $3 \frac{\sin 72°}{\cos 18°} - \frac{\sec 32°}{\operatorname{cosec} 58°}$
   (ii) $3 \cos 80° \operatorname{cosec} 10° + 2 \sin 59° \sec 31°$. **[2013]**
   (iii) $\frac{\sin 80°}{\cos 10°} + \sin 59° \sec 31°$ **[2007]**
   (iv) $\tan (55° - A) - \cot (35° + A)$
   (v) $\operatorname{cosec} (65° + A) - \sec (25° - A)$
   (vi) $2 \frac{\tan 57°}{\cot 33°} - \frac{\cot 70°}{\tan 20°} - \sqrt{2} \cos 45°$
   (vii) $\frac{\cot^2 41°}{\tan^2 49°} - 2 \frac{\sin^2 75°}{\cos^2 15°}$

(viii) $\dfrac{\cos 70^\circ}{\sin 20^\circ} + \dfrac{\cos 59^\circ}{\sin 31^\circ} - 8 \sin^2 30^\circ$

(ix) $14 \sin 30^\circ + 6 \cos 60^\circ - 5 \tan 45^\circ$. **[2004]**

6. A triangle ABC is right angled at B; find the value of $\dfrac{\sec A . \operatorname{cosec} C - \tan A . \cot C}{\sin B}$

7. Find (in each case, given below) the value of x, if :

(i) $\sin x = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

(ii) $\sin x = \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

(iii) $\cos x = \cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$

(iv) $\tan x = \dfrac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ}$

(v) $\sin 2x = 2 \sin 45^\circ \cos 45^\circ$

(vi) $\sin 3x = 2 \sin 30^\circ \cos 30^\circ$

(vii) $\cos (2x - 6^\circ) = \cos^2 30^\circ - \cos^2 60^\circ$

8. In each case, given below, find the value of angle A, where $0^\circ \leq A \leq 90^\circ$.

(i) $\sin (90^\circ - 3A) \cdot \operatorname{cosec} 42^\circ = 1$

(ii) $\cos (90^\circ - A) \cdot \sec 77^\circ = 1$

9. Prove that :

(i) $\dfrac{\cos (90^\circ - \theta) \cos \theta}{\cot \theta} = 1 - \cos^2 \theta$

(ii) $\dfrac{\sin \theta \cdot \sin (90^\circ - \theta)}{\cot (90^\circ - \theta)} = 1 - \sin^2 \theta$

10. Evaluate :

$\dfrac{\sin 35^\circ \cos 55^\circ + \cos 35^\circ \sin 55^\circ}{\operatorname{cosec}^2 10^\circ - \tan^2 80^\circ}$ **[2010]**

11. Evaluate :

$\sin^2 34^\circ + \sin^2 56^\circ + 2 \tan 18^\circ \tan 72^\circ - \cot^2 30^\circ$ **[2014]**

21.6 Using the Trigonometrical Tables :

(*i.e.*, to find the trigonometrical ratios of acute angles other than 0°, 30°, 45° and 60°)

The *trigonometrical tables* give the values of *natural sines, cosines* and *tangents* to **four decimal places.** A trigonometrical table consists of three parts :

(i) *a column* on the extreme left which contains degrees from 0° to 89°.

(ii) *ten columns* headed by 0′, 6′, 12′, 18′, 24′, 30′, 36′ 42′, 48′ and 54′ respectively.

(iii) *five columns* headed by 1′, 2′, 3′, 4′ and 5′ respectively.

Note : *When one degree* (1°) is divided into *sixty* equal parts, each part is called **one minute** (1′).

∴ One degree = 60 minute *i.e.* 1° = 60′.

11 Find : sin 36° 51′.

Solution :

See the table given for natural sines :

x°	0′	6′ 12′ 18′	24′ 30′ 36′	42′ 48′ 54′	1′ 2′ 3′ 4′ 5′ Difference to add
36	0·5878			5990	7

Since sin 36° 51′ = sin (36° 48′ + 3′)

From table,

$\sin 36° 48' = 0{\cdot}5990$ [See the number in the row against 36° & in the column headed 48′]

diff for 3′ $= 0{\cdot}0007$ (To add) [See the number in the same row and under 3′]

$\therefore$ **$\sin 36° 51' = 0{\cdot}5997$** **Ans.**

12 Find : tan 53° 38′.

Solution :

See the table for natural tangents :

$x°$	0′	6′ 12′ 18′	24′ 30′ 36′	42′ 48′ 54′	1′ 2′ 3′ 4′ 5′ Difference to add
53	1·3270		3564		16

Since $\tan 53° 38' = \tan (53° 36' + 2')$

$\therefore$ $\tan 53° 36' = 1{\cdot}3564$ [From table]

diff for 2′ $= 0{\cdot}0016$ [To add]

$\therefore$ **$\tan 53° 38' = 1{\cdot}3580$** **Ans.**

13 Find : cos 62° 27′.

Solution :

See the table for natural cosines :

$x°$	0′	6′ 12′ 18′	24′ 30′ 36′	42′ 48′ 54′	1′ 2′ 3′ 4′ 5′ Difference to subtract
62	0·4695		4633		8

Since $\cos 62° 27' = \cos (62° 24' + 3')$

$\therefore$ $\cos 62° 24' = 0{\cdot}4633$ [From table]

diff for 3′ $= 0{\cdot}0008$ [To subtract]

$\therefore$ **$\cos 62° 27' = 0{\cdot}4625$** **Ans.**

Note : The trigonometrical tables can also be used to find an acute angle.

14 Find θ; if sin θ = 0·5798.

Solution :

From the table of natural sines find the angle whose sine is just smaller than 0.5798.

$x°$	0′	6′ 12′ 18′	24′ 30′ 36′	42′ 48′ 54′	1′ 2′ 3′ 4′ 5′ Difference to add
35	0·5736		5793		5

From the table, it is clear that;

$$\sin 35° 24' = 0{\cdot}5793$$

$\sin\theta - \sin 35° 24' = 0{\cdot}5798 - 0{\cdot}5793 = 0{\cdot}0005$

From the table; diff of $2' = 0{\cdot}0005$

$\therefore \quad \theta = 35° 24' + 2' = \mathbf{35° 26'}$ **Ans.**

15 Use tables to find, θ if : (i) $\cos\theta = 0{\cdot}4457$ (ii) $\tan\theta = 0{\cdot}8516$.

Solution :

(i) See the table for natural cosines.

$x°$	0′	6′ 12′ 18′	24′ 30′ 36′	42′ 48′ 54′	1′ 2′ 3′ 4′ 5′ Difference to subtract
63	0·4540		4446		10 13

Given, $\cos\theta = 0{\cdot}4457$

$\cos 63° 36' = 0{\cdot}4446$ [From table]

diff. in values $= 0{\cdot}0011$ [$0{\cdot}4457 - 0{\cdot}4446 = 0{\cdot}0011$]

From table, diff of $4' = 0{\cdot}0011$ [To subtract]

$\therefore \quad \theta = 63° 36' - 4' = \mathbf{63° 32'}$ **(Ans)** [Greater is the value of $\cos\theta$, lesser is θ]

(ii) Similarly, given that $\tan\theta = 0{\cdot}8516$

And, from the table of natural tangents, we observe :

$\tan 40° 24' = 0{\cdot}8511$

diff. in values $= 0{\cdot}0005$ [$0{\cdot}8516 - 0{\cdot}8511 = 0{\cdot}0005$]

Since, diff. for $1' = 0{\cdot}0005$ [From table]

$\therefore \quad \theta = 40° 24' + 1' = \mathbf{40° 25'}$ **Ans.**

EXERCISE 21(D)

1. Use tables to find sine of :
   (i) 21° (ii) 34° 42′
   (iii) 47° 32′ (iv) 62° 57′
   (v) 10° 20′ + 20° 45′
2. Use tables to find cosine of :
   (i) 2° 4′ (ii) 8° 12′
   (iii) 26° 32′ (iv) 65° 41′
   (v) 9° 23′ + 15° 54′
3. Use trigonometrical tables to find tangent of :
   (i) 37° (ii) 42° 18′
   (iii) 17° 27′
4. Use tables to find the acute angle θ, if the value of sin θ is :
   (i) 0·4848 (ii) 0·3827
   (iii) 0·6525
5. Use tables to find the acute angle θ, if the value of cos θ is :
   (i) 0·9848 (ii) 0·9574
   (iii) 0·6885
6. Use tables to find the acute angle θ, if the value of tan θ is :
   (i) 0·2419 (ii) 0·4741
   (iii) 0·7391

1. Prove the following identities :

(i) $\dfrac{1}{\cos A+\sin A}+\dfrac{1}{\cos A-\sin A}$
$= \dfrac{2\cos A}{2\cos^2 A-1}$

(ii) $\operatorname{cosec} A - \cot A = \dfrac{\sin A}{1+\cos A}$

(iii) $1 - \dfrac{\sin^2 A}{1+\cos A} = \cos A$

(iv) $\dfrac{1-\cos A}{\sin A}+\dfrac{\sin A}{1-\cos A} = 2 \operatorname{cosec} A$

(v) $\dfrac{\cot A}{1-\tan A}+\dfrac{\tan A}{1-\cot A} = 1 + \tan A + \cot A$

(vi) $\dfrac{\cos A}{1+\sin A} + \tan A = \sec A$

(vii) $\dfrac{\sin A}{1-\cos A} - \cot A = \operatorname{cosec} A$

(viii) $\dfrac{\sin A - \cos A+1}{\sin A + \cos A-1} = \dfrac{\cos A}{1-\sin A}$

(ix) $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \dfrac{\cos A}{1-\sin A}$

(x) $\sqrt{\dfrac{1-\cos A}{1+\cos A}} = \dfrac{\sin A}{1+\cos A}$

(xi) $\dfrac{1+(\sec A-\tan A)^2}{\operatorname{cosec} A\,(\sec A-\tan A)} = 2 \tan A$

(xii) $\dfrac{(\operatorname{cosec} A-\cot A)^2+1}{\sec A\,(\operatorname{cosec} A-\cot A)} = 2 \cot A$

(xiii) $\cot^2 A \left(\dfrac{\sec A-1}{1+\sin A}\right) + \sec^2 A \left(\dfrac{\sin A-1}{1+\sec A}\right) = 0$

(xiv) $\dfrac{(1-2\sin^2 A)^2}{\cos^4 A-\sin^4 A} = 2 \cos^2 A - 1$

(xv) $\sec^4 A\,(1 - \sin^4 A) - 2 \tan^2 A = 1$

(xvi) $\operatorname{cosec}^4 A\,(1 - \cos^4 A) - 2 \cot^2 A = 1$

(xvii) $(1 + \tan A + \sec A)(1 + \cot A - \operatorname{cosec} A) = 2$

2. If $\sin A + \cos A = p$ and $\sec A + \operatorname{cosec} A = q$, then prove that : $q(p^2 - 1) = 2p$.

3. If $x = a \cos \theta$ and $y = b \cot \theta$, show that :
$$\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$$

4. If $\sec A + \tan A = p$, show that :
$$\sin A = \frac{p^2 - 1}{p^2 + 1}$$

5. If $\tan A = n \tan B$ and $\sin A = m \sin B$, prove that :
$$\cos^2 A = \frac{m^2 - 1}{n^2 - 1}.$$

6. (i) If $2 \sin A - 1 = 0$, show that : $\sin 3A = 3 \sin A - 4 \sin^3 A$ **[2001]**

(ii) If $4 \cos^2 A - 3 = 0$, show that : $\cos 3A = 4 \cos^3 A - 3 \cos A$

7. Evaluate :

(i) $2\left(\dfrac{\tan 35^\circ}{\cot 55^\circ}\right)^2+\left(\dfrac{\cot 55^\circ}{\tan 35^\circ}\right)^2-3\left(\dfrac{\sec 40^\circ}{\operatorname{cosec} 50^\circ}\right)$ **[2011]**

(ii) $\sec 26^\circ \sin 64^\circ + \dfrac{\operatorname{cosec} 33^\circ}{\sec 57^\circ}$

(iii) $\dfrac{5\sin 66^\circ}{\cos 24^\circ} - \dfrac{2\cot 85^\circ}{\tan 5^\circ}$

(iv) $\cos 40^\circ \operatorname{cosec} 50^\circ + \sin 50^\circ \sec 40^\circ$

(v) $\sin 27^\circ \sin 63^\circ - \cos 63^\circ \cos 27^\circ$

(vi) $\dfrac{3\sin 72^\circ}{\cos 18^\circ} - \dfrac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$ **[2000]**

(vii) $3 \cos 80^\circ \operatorname{cosec} 10^\circ + 2 \cos 59^\circ \operatorname{cosec} 31^\circ$ **[2002]**

(viii) $\dfrac{\cos 75^\circ}{\sin 15^\circ} + \dfrac{\sin 12^\circ}{\cos 78^\circ} - \dfrac{\cos 18^\circ}{\sin 72^\circ}$ **[2003]**

8. Prove that :

(i) $\tan (55^\circ + x) = \cot (35^\circ - x)$

(ii) $\sec (70^\circ - \theta) = \operatorname{cosec} (20^\circ + \theta)$

(iii) $\sin (28^\circ + A) = \cos (62^\circ - A)$

(iv) $\dfrac{1}{1+\cos (90^\circ-A)}+\dfrac{1}{1-\cos (90^\circ-A)}$
$= 2 \operatorname{cosec}^2 (90^\circ - A)$

(v) $\dfrac{1}{1+\sin (90^\circ-A)}+\dfrac{1}{1-\sin (90^\circ-A)}$
$= 2 \sec^2 (90^\circ - A)$

9. If A and B are complementary angles, prove that :

(i) $\cot B + \cos B = \sec A \cos B (1 + \sin B)$

(ii) $\cot A \cot B - \sin A \cos B - \cos A \sin B = 0$

(iii) $\operatorname{cosec}^2 A + \operatorname{cosec}^2 B = \operatorname{cosec}^2 A \operatorname{cosec}^2 B$

(iv) $\dfrac{\sin A + \sin B}{\sin A - \sin B} + \dfrac{\cos B - \cos A}{\cos B + \cos A} = \dfrac{2}{2\sin^2 A - 1}$

10. Prove that :

(i) $\dfrac{1}{\sin A - \cos A} - \dfrac{1}{\sin A + \cos A} = \dfrac{2\cos A}{2\sin^2 A - 1}$

(ii) $\dfrac{\cot^2 A}{\operatorname{cosec} A - 1} - 1 = \operatorname{cosec} A$

(iii) $\dfrac{\cos A}{1 + \sin A} = \sec A - \tan A$

(iv) $\cos A (1 + \cot A) + \sin A (1 + \tan A)$
$= \sec A + \operatorname{cosec} A$

(v) $(\sin A - \cos A)(1 + \tan A + \cot A)$
$= \dfrac{\sec A}{\operatorname{cosec}^2 A} - \dfrac{\operatorname{cosec} A}{\sec^2 A}$

(vi) $\sqrt{\sec^2 A + \operatorname{cosec}^2 A} = \tan A + \cot A$

(vii) $(\sin A + \cos A)(\sec A + \operatorname{cosec} A)$
$= 2 + \sec A \operatorname{cosec} A$

(viii) $(\tan A + \cot A)(\operatorname{cosec} A - \sin A)$
$(\sec A - \cos A) = 1$

(ix) $\cot^2 A - \cot^2 B = \dfrac{\cos^2 A - \cos^2 B}{\sin^2 A \sin^2 B}$
$= \operatorname{cosec}^2 A - \operatorname{cosec}^2 B$

11. If $4 \cos^2 A - 3 = 0$ and $0° \le A \le 90°$, then prove that :

(i) $\sin 3A = 3 \sin A - 4 \sin^3 A$

(ii) $\cos 3A = 4 \cos^3 A - 3 \cos A$

$4 \cos^2 A - 3 = 0 \Rightarrow \cos^2 A = \dfrac{3}{4}$

and $\cos A = \dfrac{\sqrt{3}}{2} \Rightarrow A = 30°$

(i) **$\sin 3A$** $= \sin 90° =$ **1**

and, **$3 \sin A - 4 \sin^3 A$**

$= 3 \sin 30° - 4 \sin^3 30°$

$= 3 \times \dfrac{1}{2} - 4 \times \left(\dfrac{1}{2}\right)^3$

$= \dfrac{3}{2} - \dfrac{1}{2} = \mathbf{1}$

$\therefore \sin 3A = 3 \sin A - 4 \sin^3 A.$

12. Find A, if $0° \le A \le 90°$ and :

(i) $2 \cos^2 A - 1 = 0$

(ii) $\sin 3A - 1 = 0$

(iii) $4 \sin^2 A - 3 = 0$

(iv) $\cos^2 A - \cos A = 0$

(v) $2 \cos^2 A + \cos A - 1 = 0$

13. If $0° < A < 90°$; find A, if :

(i) $\dfrac{\cos A}{1 - \sin A} + \dfrac{\cos A}{1 + \sin A} = 4$

(ii) $\dfrac{\sin A}{\sec A - 1} + \dfrac{\sin A}{\sec A + 1} = 2$

14. Prove that :

$(\operatorname{cosec} A - \sin A)(\sec A - \cos A) \sec^2 A = \tan A$

[2011]

15. Prove the identity $(\sin\theta + \cos\theta)(\tan\theta + \cot\theta)$

$= \sec\theta + \operatorname{cosec}\theta.$ **[2014]**