Binomial Theorem and Principle of Mathematical Induction

1. In a binomial distribution $B\left(n, p=\frac{1}{4}\right)$, if the probability of at least one success is greater than or equal to $\frac{9}{10}$, then n is greater than **[AIEEE-2009]**

(1) $\frac{1}{\log_{10} 4+\log_{10} 3}$ (2) $\frac{9}{\log_{10} 4-\log_{10} 3}$

(3) $\frac{4}{\log_{10} 4-\log_{10} 3}$ (4) $\frac{1}{\log_{10} 4-\log_{10} 3}$

2. The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9 is **[AIEEE-2009]**

(1) 2 (2) 7

(3) 8 (4) 0

3. Let $S_1=\sum_{j=1}^{10} j(j-1)^{10}C_j, S_2=\sum_{j=1}^{10} j^{10}C_j$ and $S_3=\sum_{j=1}^{10} j^2\ {}^{10}C_j$

Statement-1 : $S_3 = 55 \times 2^9$

Statement-2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$ **[AIEEE-2010]**

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is false, Statement-2 is true

4. **Statement-1:** For each natural number n, $(n + 1)^7 - n^7 - 1$ is divisible by 7.

Statement-2: For each natural number n, $n^7 - n$ is divisible by 7. **[AIEEE-2011]**

(1) Statement-1 is true, statement-2 is false.

(2) Statement-1 is false, statement-2, is true.

(3) Statement-1 is true, statement-2 is true, statement-2 is a correct explanation for statement-1

(4) Statement-1 is true, statement-2 is true; statement-2 is ***not*** a correct explanation for statement-1

5. If n is a positive integer, then $(\sqrt{3}+1)^{2n}-(\sqrt{3}-1)^{2n}$ is **[AIEEE-2012]**

(1) An odd positive integer

(2) An even positive integer

(3) A rational number other than positive integer

(4) An irrational number

6. Let A and B be two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is **[JEE (Main)-2013]**

(1) 256 (2) 220

(3) 219 (4) 211

7. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$ is **[JEE (Main)-2013]**

(1) 4 (2) 120

(3) 210 (4) 310

8. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n - 1) : n \in N\}$, where N is the set of natural numbers, then $X \cup Y$ is equal to **[JEE (Main)-2014]**

(1) X (2) Y

(3) N (4) $Y - X$

9. If the coefficients of x^3 and x^4 in the expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$ in powers of x are both zero, then (a, b) is equal to **[JEE (Main)-2014]**

(1) $\left(14, \frac{272}{3}\right)$ (2) $\left(16, \frac{272}{3}\right)$

(3) $\left(16, \frac{251}{3}\right)$ (4) $\left(14, \frac{251}{3}\right)$

10. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set $A \times B$, each having at least three elements is **[JEE (Main)-2015]**

(1) 219 (2) 256

(3) 275 (4) 510

11. The sum of coefficients of integral powers of x in the binomial expansion of $\left(1-2\sqrt{x}\right)^{50}$ is **[JEE (Main)-2015]**

(1) $\frac{1}{2}(3^{50}+1)$ (2) $\frac{1}{2}(3^{50})$

(3) $\frac{1}{2}(3^{50}-1)$ (4) $\frac{1}{2}(2^{50}+1)$

12. If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n$, $x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is **[JEE (Main)-2016]**

(1) 2187 (2) 243

(3) 729 (4) 64

13. The value of

$({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + \ldots + ({}^{21}C_{10} - {}^{10}C_{10})$ is **[JEE (Main)-2017]**

(1) $2^{21} - 2^{10}$ (2) $2^{20} - 2^9$

(3) $2^{20} - 2^{10}$ (4) $2^{21} - 2^{11}$

14. The sum of the co-efficients of all odd degree terms in the expansion of

$\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$, $(x > 1)$ is **[JEE (Main)-2018]**

(1) –1 (2) 0

(3) 1 (4) 2

15. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to **[JEE (Main)-2019]**

(1) 8 (2) 4

(3) 6 (4) 14

16. The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ is **[JEE (Main)-2019]**

(1) 15 (2) 14

(3) 12 (4) 10

17. If $\sum_{i=1}^{20}\left(\frac{{}^{20}C_{i-1}}{{}^{20}C_i + {}^{20}C_{i-1}}\right)^3 = \frac{k}{21}$, then k equals **[JEE (Main)-2019]**

(1) 400 (2) 100

(3) 200 (4) 50

18. The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is **[JEE (Main)-2019]**

(1) 1465 (2) 1356

(3) 1365 (4) 1256

19. If the third term in the binomial expansion of $(1+x^{\log_2 x})^5$ equals 2560, then a possible value of x is **[JEE (Main)-2019]**

(1) $2\sqrt{2}$ (2) $\frac{1}{4}$

(3) $4\sqrt{2}$ (4) $\frac{1}{8}$

20. Consider the statement : "$P(n)$: $n^2 - n + 41$" is prime." Then which one of the following is true? **[JEE (Main)-2019]**

(1) $P(5)$ is false but $P(3)$ is true

(2) $P(3)$ is false but $P(5)$ is true

(3) Both $P(3)$ and $P(5)$ are false

(4) Both $P(3)$ and $P(5)$ are true

21. The positive value of λ for which the co-efficient of x^2 in the expression $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$ is 720, is **[JEE (Main)-2019]**

(1) 3 (2) 4

(3) $\sqrt{5}$ (4) $2\sqrt{2}$

22. The sum of the real values of x for which the middle term in the binomial expansion of $\left(\frac{x^3}{3}+\frac{3}{x}\right)^8$ equals 5670 is **[JEE (Main)-2019]**

(1) 4 (2) 8

(3) 0 (4) 6

23. Let $(x + 10)^{50} + (x - 10)^{50}$ $= a_0 + a_1x + a_2x^2 + ... + a_{50}x^{50}$, for all $x \in R$; then $\frac{a_2}{a_0}$ is equal to **[JEE (Main)-2019]**

(1) 12.25 (2) 12.75

(3) 12.00 (4) 12.50

24. A ratio of the 5th term from the beginning to the 5th term from the end in the binomial expansion of $\left(2^{\frac{1}{3}}+\frac{1}{2(3)^{\frac{1}{3}}}\right)^{10}$ is **[JEE (Main)-2019]**

(1) $1:4(16)^{\frac{1}{3}}$ (2) $1:2(6)^{\frac{1}{3}}$

(3) $2(36)^{\frac{1}{3}}:1$ (4) $4(36)^{\frac{1}{3}}:1$

25. The total number of irrational terms in the binomial expansion of $\left(7^{1/5}-3^{1/10}\right)^{60}$ is **[JEE (Main)-2019]**

(1) 48 (2) 49

(3) 54 (4) 55

26. The sum of the co-efficients of all even degree terms in x in the expansion of $(x+\sqrt{x^3-1})^6 + (x-\sqrt{x^3-1})^6, (x>1)$ is equal to : **[JEE (Main)-2019]**

(1) 24 (2) 32

(3) 26 (4) 29

27. If the fourth term in the binomial expansion of $\left(\sqrt{\frac{1}{x^{1+\log_{10}x}}}+x^{\frac{1}{12}}\right)^6$ is equal to 200, and $x > 1$, then the value of x is : **[JEE (Main)-2019]**

(1) 10 (2) 10^3

(3) 100 (4) 10^4

28. If the fourth term in the Binomial expansion of $\left(\frac{2}{x}+x^{\log_8 x}\right)^6$ $(x > 0)$ is 20×8^7, then a value of x is **[JEE (Main)-2019]**

(1) 8^3 (2) 8

(3) 8^{-2} (4) 8^2

29. If some three consecutive coefficients in the binomial expansion of $(x + 1)^n$ in powers of x are in the ratio 2 : 15 : 70, then the average of these three coefficients is **[JEE (Main)-2019]**

(1) 625 (2) 964

(3) 232 (4) 227

30. If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $(1 + ax + bx^2)(1 - 3x)^{15}$ in powers of x, then the ordered pair (a, b) is equal to : **[JEE (Main)-2019]**

(1) (– 54, 315) (2) (28, 861)

(3) (–21, 714) (4) 28, 315

31. If α and β are the roots of the quadratic equation, $x^2 + x \sin\theta - 2\sin\theta = 0$, $\theta \in \left(0, \frac{\pi}{2}\right)$, then $\frac{\alpha^{12}+\beta^{12}}{(\alpha^{-12}+\beta^{-12})\cdot(\alpha-\beta)^{24}}$ is equal to **[JEE (Main)-2019]**

(1) $\frac{2^{12}}{(\sin\theta-8)^6}$ (2) $\frac{2^{12}}{(\sin\theta-4)^{12}}$

(3) $\frac{2^6}{(\sin\theta+8)^{12}}$ (4) $\frac{2^{12}}{(\sin\theta+8)^{12}}$

32. The smallest natural number n, such that the coefficient of x in the expansion of $\left(x^2+\frac{1}{x^3}\right)^n$ is ${}^nC_{23}$, is **[JEE (Main)-2019]**

(1) 58 (2) 35

(3) 38 (4) 23

33. The coefficient of x^{18} in the product $(1 + x)(1 - x)^{10}(1 + x + x^2)^9$ is **[JEE (Main)-2019]**

(1) 84 (2) –126

(3) –84 (4) 126

34. If ${}^{20}C_1 + (2^2)\,{}^{20}C_2 + (3^2)\,{}^{20}C_3 + ... + (20^2)\,{}^{20}C_{20} = A(2^\beta)$, then the ordered pair (A, β) is equal to **[JEE (Main)-2019]**

(1) (420, 19) (2) (380, 19)

(3) (420, 18) (4) (380, 18)

35. The term independent of x in the expansion of $\left(\frac{1}{60}-\frac{x^8}{81}\right)\cdot\left(2x^2-\frac{3}{x^2}\right)^6$ is equal to **[JEE (Main)-2019]**

(1) –108 (2) –36

(3) –72 (4) 36

36. The number of ordered pairs (r, k) for which $6\cdot{}^{35}C_r = (k^2-3)\cdot{}^{36}C_{r+1}$, where k is an integer, is **[JEE (Main)-2020]**

(1) 3 (2) 6

(3) 2 (4) 4

37. The coefficient of x^7 in the expression $(1+x)^{10} + x(1+x)^9 + x^2(1+x)^8 + \ldots + x^{10}$ is **[JEE (Main)-2020]**

(1) 120 (2) 330

(3) 420 (4) 210

38. If α and β be the coefficients of x^4 and x^2 respectively in the expansion of $\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6$, then **[JEE (Main)-2020]**

(1) $\alpha-\beta=60$ (2) $\alpha+\beta=60$

(3) $\alpha-\beta=-132$ (4) $\alpha+\beta=-30$

39. In the expansion of $\left(\frac{x}{\cos\theta}+\frac{1}{x\sin\theta}\right)^{16}$, if I_1 is the least value of the term independent of x when $\frac{\pi}{8}\le\theta\le\frac{\pi}{4}$ and I_2 is the least value of the term independent of x when $\frac{\pi}{16}\le\theta\le\frac{\pi}{8}$, then the ratio $I_2 : I_1$ is equal to **[JEE (Main)-2020]**

(1) 1 : 16 (2) 16 : 1

(3) 1 : 8 (4) 8 : 1

40. Let $\alpha > 0$, $\beta > 0$ be such that $\alpha^3+\beta^2=4$. If the maximum value of the term independent of x in the binomial expansion of $\left(\alpha x^{\frac{1}{9}}+\beta x^{-\frac{1}{6}}\right)^{10}$ is $10k$, then k is equal to **[JEE (Main)-2020]**

(1) 84 (2) 176

(3) 336 (4) 352

41. If the number of integral terms in the expansion of $\left(3^{\frac{1}{2}}+5^{\frac{1}{8}}\right)^n$ is exactly 33, then the least value of n is **[JEE (Main)-2020]**

(1) 264 (2) 128

(3) 256 (4) 248

42. If the term independent of x in the expansion of $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$ is k, then $18k$ is equal to **[JEE (Main)-2020]**

(1) 9 (2) 11

(3) 5 (4) 7

43. The value of $\sum_{r=0}^{20}{}^{50-r}C_6$ is equal to **[JEE (Main)-2020]**

(1) ${}^{51}C_7-{}^{30}C_7$ (2) ${}^{50}C_7-{}^{30}C_7$

(3) ${}^{51}C_7+{}^{30}C_7$ (4) ${}^{50}C_6-{}^{30}C_6$

44. If for some positive integer n, the coefficients of three consecutive terms in the binomial expansion of $(1+x)^{n+5}$ are in the ratio 5 : 10 : 14, then the largest coefficient in this expansion is **[JEE (Main)-2020]**

(1) 252 (2) 462

(3) 792 (4) 330

45. If $\{p\}$ denotes the fractional part of the number p, then $\left\{\frac{3^{200}}{8}\right\}$, is equal to **[JEE (Main)-2020]**

(1) $\frac{5}{8}$ (2) $\frac{1}{8}$

(3) $\frac{7}{8}$ (4) $\frac{3}{8}$

46. If the constant term in the binomial expansion of $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$ is 405, then $|k|$ equals **[JEE (Main)-2020]**

(1) 2 (2) 3

(3) 9 (4) 1

47. If a, b and c are the greatest values of ${}^{19}C_p$, ${}^{20}C_q$ and ${}^{21}C_r$ respectively, then

[JEE (Main)-2020]

(1) $\frac{a}{10}=\frac{b}{11}=\frac{c}{42}$ (2) $\frac{a}{11}=\frac{b}{22}=\frac{c}{42}$

(3) $\frac{a}{10}=\frac{b}{11}=\frac{c}{21}$ (4) $\frac{a}{11}=\frac{b}{22}=\frac{c}{21}$

48. If the sum of the coefficients of all even powers of x in the product

$\left(1+x+x^2+...+x^{2n}\right)\left(1-x+x^2-x^3+...+x^{2n}\right)$

is 61, then n is equal to _____.

[JEE (Main)-2020]

49. The coefficient of x^4 in the expansion of $(1 + x + x^2)^{10}$ is _____. **[JEE (Main)-2020]**

50. If $C_r \equiv {}^{25}C_r$ and $C_0 + 5.C_1 + 9.C_2 + ... + (101).C_{25} = 2^{25}.k$, then k is equal to _____.

[JEE (Main)-2020]

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51. For a positive integer n, $\left(1+\frac{1}{x}\right)^n$ is expanded in increasing powers of x. If three consecutive coefficients in this expansion are in the ratio, 2 : 5 : 12, then n is equal to ____.

[JEE (Main)-2020]

52. Let $\left(2x^2+3x+4\right)^{10}=\sum_{r=0}^{20} a_r x^r$. Then $\frac{a_7}{a_{13}}$ is equal to _____. **[JEE (Main)-2020]**

53. The natural number m, for which the coefficient of x in the binomial expansion of $\left(x^m+\frac{1}{x^2}\right)^{22}$ is 1540, is _____. **[JEE (Main)-2020]**

54. The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^6$ in powers of x, is _____.

[JEE (Main)-2020]