Differential Equations

1. The differential equation which represents the family of curves $y = c_1 e^{c_2 x}$, where c_1 and c_2 are arbitrary constants, is **[AIEEE-2009]**

   (1) $y'' = y' y$

   (2) $yy'' = y'$

   (3) $yy'' = (y')^2$

   (4) $y' = y^2$

2. Solution of the differential equation

   $\cos x dy = y(\sin x - y)dx, 0 < x < \frac{\pi}{2}$ is **[AIEEE-2010]**

   (1) $\sec x = (\tan x + c)y$

   (2) $y \sec x = \tan x + c$

   (3) $y \tan x = \sec x + c$

   (4) $\tan x = (\sec x + c)y$

3. Consider the differential equation

   $y^2 dx + \left(x - \frac{1}{y}\right)dy = 0$. If $y(1) = 1$, then x is given by **[AIEEE-2011]**

   (1) $1 + \frac{1}{y} - \frac{e^{\frac{1}{y}}}{e}$ (2) $1 - \frac{1}{y} + \frac{e^{\frac{1}{y}}}{e}$

   (3) $4 - \frac{2}{y} - \frac{e^{\frac{1}{y}}}{e}$ (4) $3 - \frac{1}{y} + \frac{e^{\frac{1}{y}}}{e}$

4. The population $p(t)$ at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5p(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is **[AIEEE-2012]**

   (1) ln 9 (2) $\frac{1}{2}$ ln 18

   (3) ln 18 (4) 2 ln 18

5. Let $y(x)$ be the solution of the differential equation

   $(x \log x)\frac{dy}{dx} + y = 2x \log x, (x \geq 1)$.

   Then $y(e)$ is equal to **[JEE (Main)-2015]**

   (1) e (2) 0

   (3) 2 (4) $2e$

6. If a curve $y = f(x)$ passes through the point $(1, -1)$ and satisfies the differential equation, $y(1 + xy)\,dx = x\,dy$, then $f\left(-\frac{1}{2}\right)$ is equal to **[JEE (Main)-2016]**

   (1) $-\frac{4}{5}$ (2) $\frac{2}{5}$

   (3) $\frac{4}{5}$ (4) $-\frac{2}{5}$

7. If $(2 + \sin x)\frac{dy}{dx} + (y + 1)\cos x = 0$ and $y(0) = 1$, then $y\left(\frac{\pi}{2}\right)$ is equal to **[JEE (Main)-2017]**

   (1) $-\frac{2}{3}$ (2) $-\frac{1}{3}$

   (3) $\frac{4}{3}$ (4) $\frac{1}{3}$

8. let $y = y(x)$ be the solution of the differential equation $\sin x \frac{dy}{dx} + y \cos x = 4x$, $x \in (0, \pi)$. If $y = \left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to **[JEE (Main)-2018]**

   (1) $\frac{4}{9\sqrt{3}}\pi^2$ (2) $\frac{-8}{9\sqrt{3}}\pi^2$

   (3) $-\frac{8}{9}\pi^2$ (4) $-\frac{4}{9}\pi^2$

9. If $y = y(x)$ is the solution of the differential equation, $x\frac{dy}{dx}+2y=x^2$ satisfying $y(1) = 1$, then $y\left(\frac{1}{2}\right)$ is equal to **[JEE (Main)-2019]**

(1) $\frac{13}{16}$ (2) $\frac{7}{64}$

(3) $\frac{1}{4}$ (4) $\frac{49}{16}$

10. Let $f : [0, 1] \to R$ be such that $f(xy) = f(x).f(y)$, for all $x, y \in [0, 1]$, and $f(0) \neq 0$. If $y = y(x)$ satisfies the differential equation, $\frac{dy}{dx}=f(x)$ with $y(0) = 1$, then $y\left(\frac{1}{4}\right)+y\left(\frac{3}{4}\right)$ is equal to **[JEE (Main)-2019]**

(1) 4 (2) 3

(3) 2 (4) 5

11. If $\frac{dy}{dx}+\frac{3}{\cos^2 x}y=\frac{1}{\cos^2 x}, x\in\left(\frac{-\pi}{3},\frac{\pi}{3}\right)$ and $y\left(\frac{\pi}{4}\right)=\frac{4}{3}$, then $y\left(-\frac{\pi}{4}\right)$ equals **[JEE (Main)-2019]**

(1) $\frac{1}{3}+e^6$ (2) $\frac{1}{3}+e^3$

(3) $\frac{1}{3}$ (4) $-\frac{4}{3}$

12. The curve amongst the family of curves represented by the differential equation, $(x^2 – y^2)dx + 2xydy = 0$ which passes through (1, 1) is **[JEE (Main)-2019]**

(1) A hyperbola with transverse axis along the x-axis.

(2) A circle with centre on the y-axis.

(3) An ellipse with major axis along the y-axis.

(4) A circle with centre on the x-axis.

13. If $y(x)$ is the solution of the differential equation $\frac{dy}{dx}+\left(\frac{2x+1}{x}\right)y=e^{-2x}, x>0$, where $y(1)=\frac{1}{2}e^{-2}$, then **[JEE (Main)-2019]**

(1) $y(x)$ is decreasing in $\left(\frac{1}{2},1\right)$

(2) $y(\log_e 2)=\frac{\log_e 2}{4}$

(3) $y(\log_e 2)=\log_e 4$

(4) $y(x)$ is decreasing in (0, 1)

14. The solution of the differential equation, $\frac{dy}{dx}=(x-y)^2$, when $y(1) = 1$, is **[JEE (Main)-2019]**

(1) $\log_e\left|\frac{2-y}{2-x}\right|=2(y-1)$

(2) $-\log_e\left|\frac{1+x-y}{1-x+y}\right|=x+y-2$

(3) $-\log_e\left|\frac{1-x+y}{1+x-y}\right|=2(x-1)$

(4) $\log_e\left|\frac{2-x}{2-y}\right|=x-y$

15. Let $y = y(x)$ be the solution of the differential equation, $x\frac{dy}{dx}+y=x\log_e x,(x>1)$. If $2y(2) = \log_e 4 – 1$, then $y(e)$ is equal to **[JEE (Main)-2019]**

(1) $\frac{e^2}{4}$ (2) $-\frac{e}{2}$

(3) $-\frac{e^2}{2}$ (4) $\frac{e}{4}$

16. Let $y = y(x)$ be the solution of the differential equation, $(x^2+1)^2\frac{dy}{dx}+2x(x^2+1)y=1$ such that $y(0) = 0$. If $\sqrt{a}\,y(1)=\frac{\pi}{32}$, then the value of '$a$' is **[JEE (Main)-2019]**

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$

(3) 1 (4) $\frac{1}{16}$

17. Given that the slope of the tangent to a curve $y = y(x)$ at any point (x,y) is $\frac{2y}{x^2}$. If the curve passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, then its equation is **[JEE (Main)-2019]**

(1) $x \log_e |y| = -2(x - 1)$

(2) $x \log_e |y| = x - 1$

(3) $x \log_e |y| = 2(x - 1)$

(4) $x^2 \log_e |y| = -2(x - 1)$

18. The solution of the differential equation $x\frac{dy}{dx} + 2y = x^2 (x \neq 0)$ with $y(1) = 1$, is **[JEE (Main)-2019]**

(1) $y = \frac{4}{5}x^3 + \frac{1}{5x^2}$ (2) $y = \frac{x^3}{5} + \frac{1}{5x^2}$

(3) $y = \frac{x^2}{4} + \frac{3}{4x^2}$ (4) $y = \frac{3}{4}x^2 + \frac{1}{4x^2}$

19. If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} = (\tan x - y)\sec^2 x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that $y(0) = 0$, then $y\left(-\frac{\pi}{4}\right)$ is equal to **[JEE (Main)-2019]**

(1) $\frac{1}{e} - 2$ (2) $\frac{1}{2} - e$

(3) $e - 2$ (4) $2 + \frac{1}{e}$

20. Let $y = y(x)$ be the solution of the differential equation, $\frac{dy}{dx} + y\tan x = 2x + x^2 \tan x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that $y(0) = 1$.Then **[JEE (Main)-2019]**

(1) $y\left(\frac{\pi}{4}\right) + y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{2} + 2$

(2) $y\left(\frac{\pi}{4}\right) - y\left(-\frac{\pi}{4}\right) = \sqrt{2}$

(3) $y'\left(\frac{\pi}{4}\right) + y'\left(-\frac{\pi}{4}\right) = -\sqrt{2}$

(4) $y'\left(\frac{\pi}{4}\right) - y'\left(-\frac{\pi}{4}\right) = \pi - \sqrt{2}$

21. If $e^y + xy = e$, the ordered pair $\left(\frac{dy}{dx}, \frac{d^2y}{dx^2}\right)$ at $x = 0$ is equal to **[JEE (Main)-2019]**

(1) $\left(\frac{1}{e}, -\frac{1}{e^2}\right)$ (2) $\left(-\frac{1}{e}, -\frac{1}{e^2}\right)$

(3) $\left(-\frac{1}{e}, \frac{1}{e^2}\right)$ (4) $\left(\frac{1}{e}, \frac{1}{e^2}\right)$

22. Consider the differential equation, $y^2 dx + \left(x - \frac{1}{y}\right)dy = 0$. If value of y is 1 when $x = 1$, then the value of x for which $y = 2$, is **[JEE (Main)-2019]**

(1) $\frac{5}{2} + \frac{1}{\sqrt{e}}$ (2) $\frac{3}{2} - \sqrt{e}$

(3) $\frac{3}{2} - \frac{1}{\sqrt{e}}$ (4) $\frac{1}{2} + \frac{1}{\sqrt{e}}$

23. The general solution of the differential equation $(y^2 - x^3)\,dx - xydy = 0$ $(x \neq 0)$ is (where c is a constant of integration) **[JEE (Main)-2019]**

(1) $y^2 + 2x^3 + cx^2 = 0$ (2) $y^2 - 2x^2 + cx^3 = 0$

(3) $y^2 - 2x^3 + cx^2 = 0$ (4) $y^2 + 2x^2 + cx^3 = 0$

24. Let $x^k + y^k = a^k$, $(a, k > 0)$ and $\frac{dy}{dx} + \left(\frac{y}{x}\right)^{\frac{1}{3}} = 0$, then k is **[JEE (Main)-2020]**

(1) $\frac{3}{2}$ (2) $\frac{1}{3}$

(3) $\frac{4}{3}$ (4) $\frac{2}{3}$

25. If $y = y(x)$ is the solution of the differential equation, $e^y\left(\frac{dy}{dx} - 1\right) = e^x$ such that $y(0) = 0$, then $y(1)$ is equal to **[JEE (Main)-2020]**

(1) $2e$

(2) $\log_e 2$

(3) $2 + \log_e 2$

(4) $1 + \log_e 2$

26. Let $y = y(x)$ be the solution curve of the differential equation, $\left(y^2 - x\right)\frac{dy}{dx} = 1$, satisfying $y(0) = 1$. This curve intersects the x-axis at a point whose abscissa is **[JEE (Main)-2020]**

(1) $2 - e$

(2) $2 + e$

(3) $-e$

(4) 2

27. Let $y = y(x)$ be a function of x satisfying $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$ where k is a constant and $y\left(\frac{1}{2}\right) = -\frac{1}{4}$. Then $\frac{dy}{dx}$ at $x = \frac{1}{2}$, is equal to **[JEE (Main)-2020]**

(1) $-\frac{\sqrt{5}}{2}$ (2) $\frac{2}{\sqrt{5}}$

(3) $\frac{\sqrt{5}}{2}$ (4) $-\frac{\sqrt{5}}{4}$

28. Let $f(x) = (\sin(\tan^{-1}x) + \sin(\cot^{-1}x))^2 - 1$, $|x| > 1$. If $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}\left(\sin^{-1}\left(f(x)\right)\right)$ and $y\left(\sqrt{3}\right) = \frac{\pi}{6}$, then $y\left(-\sqrt{3}\right)$ is equal to **[JEE (Main)-2020]**

(1) $\frac{\pi}{3}$ (2) $-\frac{\pi}{6}$

(3) $\frac{2\pi}{3}$ (4) $\frac{5\pi}{6}$

29. Let $y = y(x)$ be a solution of the differential equation, $\sqrt{1-x^2}\frac{dy}{dx} + \sqrt{1-y^2} = 0, |x| < 1$.

If $y\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$, then $y\left(\frac{-1}{\sqrt{2}}\right)$ is equal to **[JEE (Main)-2020]**

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1}{\sqrt{2}}$

(3) $-\frac{\sqrt{3}}{2}$ (4) $-\frac{1}{\sqrt{2}}$

30. The differential equation of the family of curves, $x^2 = 4b(y + b)$, $b \in R$, is **[JEE (Main)-2020]**

(1) $x(y')^2 = x - 2yy'$

(2) $x(y')^2 = 2yy' - x$

(3) $x(y')^2 = x + 2yy'$

(4) $xy'' = y'$

31. If $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$; $y(1) = 1$; then a value of x satisfying $y(x) = e$ is **[JEE (Main)-2020]**

(1) $\frac{e}{\sqrt{2}}$ (2) $\sqrt{3}e$

(3) $\sqrt{2}e$ (4) $\frac{1}{2}\sqrt{3}e$

32. Let y = y(x) be the solution of the differential equation,

$\frac{2 + \sin x}{y + 1} \cdot \frac{dy}{dx} = -\cos x$, $y > 0$, $y(0) = 1$. If $y(\pi) = a$ and $\frac{dy}{dx}$ at $x = \pi$ is b, then the ordered pair (a, b) is equal to **[JEE (Main)-2020]**

(1) (2, 1)

(2) (1, –1)

(3) $\left(2, \frac{3}{2}\right)$

(4) (1, 1)

33. If a curve $y = f(x)$, passing through the point (1, 2), is the solution of the differential equation, $2x^2dy = (2xy + y^2)dx$, then $f\left(\frac{1}{2}\right)$ is equal to **[JEE (Main)-2020]**

(1) $\frac{1}{1 + \log_e 2}$ (2) $\frac{-1}{1 + \log_e 2}$

(3) $1 + \log_e 2$ (4) $\frac{1}{1 - \log_e 2}$

34. The solution curve of the differential equation, $\left(1 + e^{-x}\right)\left(1 + y^2\right)\frac{dy}{dx} = y^2$, which passes through the point (0, 1), is **[JEE (Main)-2020]**

(1) $y^2 + 1 = y\left(\log_e\left(\frac{1+e^{-x}}{2}\right) + 2\right)$

(2) $y^2 + 1 = y\left(\log_e\left(\frac{1+e^{x}}{2}\right) + 2\right)$

(3) $y^2 = 1 + y\log_e\left(\frac{1+e^{-x}}{2}\right)$

(4) $y^2 = 1 + y\log_e\left(\frac{1+e^{x}}{2}\right)$

35. If $y^2 + \log_e(\cos^2 x) = y$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then **[JEE (Main)-2020]**

(1) $|y'(0)| + |y''(0)| = 3$

(2) $|y'(0)| + |y''(0)| = 1$

(3) $y''(0) = 0$

(4) $|y''(0)| = 2$

36. If $x^3dy + xy\,dx = x^2dy + 2y\,dx$; $y(2) = e$ and $x > 1$, then $y(4)$ is equal to **[JEE (Main)-2020]**

(1) $\frac{\sqrt{e}}{2}$ (2) $\frac{1}{2} + \sqrt{e}$

(3) $\frac{3}{2} + \sqrt{e}$ (4) $\frac{3}{2}\sqrt{e}$

37. Let $y = y(x)$ be the solution of the differential equation,

$xy' - y = x^2(x\cos x + \sin x)$, $x > 0$. If $y(\pi) = \pi$, then $y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right)$ is equal to **[JEE (Main)-2020]**

(1) $1 + \frac{\pi}{2}$ (2) $1 + \frac{\pi}{2} + \frac{x^2}{4}$

(3) $2 + \frac{\pi}{2}$ (4) $2 + \frac{\pi}{2} + \frac{\pi^2}{4}$

38. The solution of the differential equation $\frac{dy}{dx} - \frac{y+3x}{\log_e(y+3x)} + 3 = 0$ is

(where C is a constant of integration.)

[JEE (Main)-2020]

(1) $x - \frac{1}{2}(\log_e(y+3x))^2 = C$

(2) $y + 3x - \frac{1}{2}(\log_e x)^2 = C$

(3) $x - 2\log_e(y+3x) = C$

(4) $x - \log_e(y+3x) = C$

39. If $y = y(x)$ is the solution of the differential equation $\frac{5+e^x}{2+y} \cdot \frac{dy}{dx} + e^x = 0$ satisfying $y(0) = 1$, then a value of $y(\log_e 13)$ is **[JEE (Main)-2020]**

(1) –1 (2) 2

(3) 0 (4) 1

40. Let $y = y(x)$ be the solution of the differential equation $\cos x\frac{dy}{dx} + 2y\sin x = \sin 2x$, $x \in \left(0, \frac{\pi}{2}\right)$. If $y(\pi/3) = 0$, then $y(\pi/4)$ is equal to **[JEE (Main)-2020]**

(1) $\frac{1}{\sqrt{2}} - 1$ (2) $\sqrt{2} - 2$

(3) $2 - \sqrt{2}$ (4) $2 + \sqrt{2}$

41. The general solution of the differential equation $\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0$ is

(where C is a constant of integration)

[JEE (Main)-2020]

(1) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$

(2) $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + C$

(3) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + C$

(4) $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$

42. If $y = \left(\frac{2}{\pi}x - 1\right)$ cosec x is the solution of the differential equation, $\frac{dy}{dx} + p(x)y = \frac{2}{\pi}$ cosec x, $0 < x < \frac{\pi}{2}$, then the function $p(x)$ is equal to

[JEE (Main)-2020]

(1) tan x (2) cot x

(3) cosec x (4) sec x

43. If for $x \geq 0$, $y = y(x)$ is the solution of the differential equation,

$(x + 1)dy = ((x + 1)^2 + y - 3)dx$, $y(2) = 0$,

then $y(3)$ is equal to _____. **[JEE (Main)-2020]**

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