12 Reflection

UNIT 3 : Co-ordinate Geometry

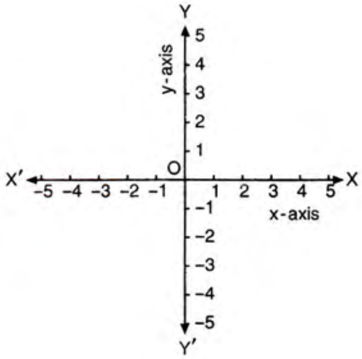
(In x-axis, y-axis, x = a, y = a and the origin; Invariant Points)

12.1 Introduction :

Co-ordinate geometry is the branch of geometry in which two numbers, called ***co-ordinates***, are used to locate the position of a point in a plane.

12.2 Co-ordinate Axes :

The two mutually perpendicular **number lines** intersecting each other at their zeroes, are called ***rectangular axes*** *or* ***co-ordinate axes*** *or* ***axes of reference.***



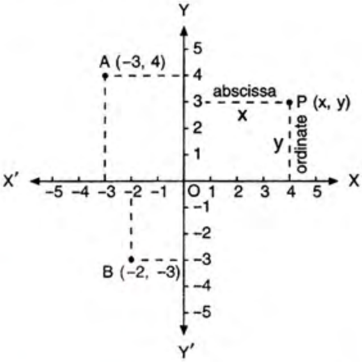
As shown in the adjacent figure, the *horizontal number line* XOX′ is called the ***x-axis;*** the *vertical number line* YOY′ is called the ***y-axis*** and their *point of intersection*, O is called the ***origin.***

12.3 Co-ordinates :

The position of a point in a plane is expressed by a pair of two numbers (one concerning x-axis and the other concerning y-axis) called ***co-ordinates***.

Consider a point P (x, y)

Here (x, y) is a pair of two numbers, which gives the co-ordinates of point P.



The first number x of the pair (x, y) is the distance of the point P from y-axis and is called ***x-co-ordinate*** or ***abscissa.***

The second number y of the pair (x, y) is the distance of the point P from x-axis and is called the ***y-co-ordinate*** or ***ordinate.***

Suppose the co-ordinates of point A are (–3, 4), then its abscissa = –3 and ordinate = 4.

And, if for a point B, abscissa = –2 and ordinate = –3, then its co-ordinates are (–2, –3).

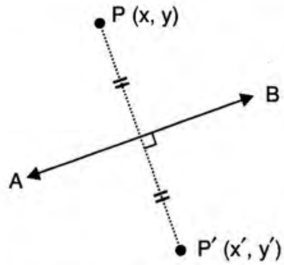
Remember :

1. In stating the co-ordinates of a point the abscissa precedes the ordinate. The two co-ordinates are separated by a comma and are enclosed in a bracket.
   Thus, a point with abscissa x and ordinate y is denoted by (x, y).
2. Co-ordinates of origin O = (0, 0).
3. Co-ordinates of a point on the x-axis = $(x, 0)$ and
4. Co-ordinates of a point on the y-axis = $(0, y)$.

12.4 Reflection :

When an object is placed before a plane mirror, the image formed is *at the same distance behind the mirror as the object is in front of it.*

Therefore, to find the image of a point P in a line AB, consider AB as the *plane mirror* and point P as the *object.* Now, find a point P′ on the other side of AB, such that P′ is at the same distance from AB as P is from it.



Thus, point P′ is the *image of point* P in line AB and line AB, which is also the perpendicular bisector of PP′, is said to be the **mirror line** or **mediator** of segment PP′.

The transformation which maps a point P to P′ is called ***reflection***.

The *reflection* can be denoted in several ways, but here it will be denoted by M_l, where M denotes *reflection* and l is the line or point in which the reflection takes place.

Thus, M_x represents *reflection in the x-axis*;

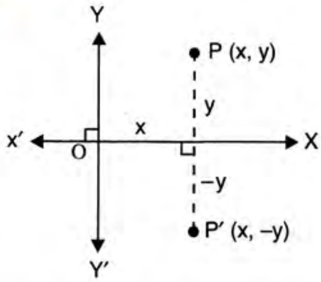
M_y represents *reflection in the y-axis*;

and M_o represents *reflection in the origin*.

12.5 Reflection in the line $y = 0$ *i.e.* in the x-axis :

The line $y = 0$ means the x-axis

The adjoining figure shows the reflection of point P(x, y) in the x-axis. It is clear from the figure that **P′ is the image of P** in the x-axis such that P′ = (x, –y).



Symbolically, $\mathbf{M_x\ (x, y) = (x, -y)}$.

Therefore, when a *point is reflected in the x-axis*, the *sign of its ordinate changes.*

For Example :

Reflection of point (2, 3) in the x-axis = (2, –3) *i.e.* M_x (2, 3) = (2, –3)

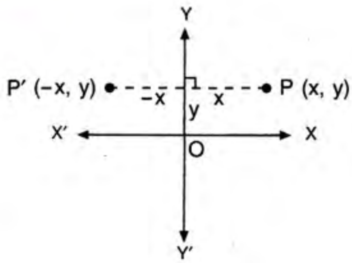
Reflection of point (2, –3) in the x-axis = (2, 3) *i.e.* M_x (2, –3) = (2, 3)

Similarly, M_x (–5, 7) = (–5, –7); M_x (–a, –b) = (–a, b) and so on.

12.6 Reflection in the line $x = 0$ *i.e.* in the y-axis :

The line $x = 0$ means the y-axis

As is clear from the adjoining figure, the reflection of point P (x, y) in the y-axis is P′ such that P′ = (–x, y).



Symbolically, $\mathbf{M_y\ (x, y) = (-x, y)}$

Therefore, when a *point is reflected in the y-axis*, the *sign of its abscissa changes.*

For Example :

Reflection of point (2, 3) in the y-axis = (–2, 3) *i.e.* M_y (2, 3) = (–2, 3)

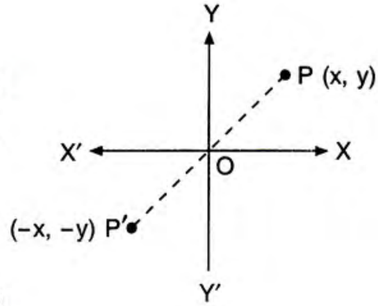
Reflection of point (2, –3) in the y-axis = (–2, –3) *i.e.* M_y (2, –3) = (–2, –3)

Similarly, M_y (–5, 7) = (5, 7), M_y (–a, –b) = (a, –b) and so on.

12.7 Reflection in the origin :

When a point P (x, y) is reflected in the origin, the *signs of its abscissa and ordinate both change i.e.* if P′ is the image of P (x, y) in the origin, then P′ = $(-x, -y)$

Symbolically, $\mathbf{M_o (x, y) = (-x, -y)}$



For Example :

Reflection of point (2, 3) in the origin = (–2, –3)

i.e. M_o (2, 3) = (–2, –3)

Reflection of point (2, –3) in the origin = (–2, 3)

i.e. M_o (2, –3) = (–2, 3)

Similarly, M_o (–5, 7) = (5, –7); M_o $(-a, -b) = (a, b)$ and so on.

1 The triangle A(1, 2), B(4, 4) and C(3, 7) is first reflected in the line $y = 0$ onto triangle A′B′C′ and then triangle A′B′C′ is reflected in the origin onto triangle A″B″C″. Write down the co-ordinates of :

(i) A′, B′ and C′ (ii) A″, B″ and C″.

Solution :

Reflection in $y = 0$ means reflection in x-axis.

(i) Since, reflection in the x-axis is given by $M_x (x, y) = (x, -y)$

∴ A′ = reflection of A (1, 2) in the x-axis = (1, –2)

Similarly, B′ = **(4, –4)** and C′ = **(3, –7)** **Ans.**

(ii) Since, reflection in the origin is given by $M_o (x, y) = (-x, -y)$

∴ A″ = reflection of A′ (1, –2) in the origin = (–1, 2)

Similarly, B″ = **(–4, 4)** and C″ = **(–3, 7)** **Ans.**

2 A point P is reflected in the x-axis. Co-ordinates of its image are (8, –6).

(i) Find the co-ordinates of P.

(ii) Find the co-ordinates of the image of P under reflection in the y-axis.

Solution :

(i) **P = (8, 6)** $\boxed{\text{Since, } M_x (8, 6) = (8, -6)}$ **Ans.**

(ii) Co-ordinates of the image of P(8, 6) under reflection in the y-axis

= **(–8, 6)** **Ans.**

3 Perform the operations $M_x \,.\, M_y$ and $M_y \,.\, M_x$ on the point (3, –4).

State whether $M_x \,.\, M_y = M_y \,.\, M_x$.

If 'yes'; then state whether it is always true.

Solution :

$$\mathbf{M_x \,.\, M_y (3, -4)} = M_x [M_y (3, -4)]$$

$$= M_x (-3, -4) = \mathbf{(-3, 4)}$$ **Ans.**

$$M_y \cdot M_x (3, -4) = M_y [M_x (3, -4)]$$
$$= M_y (3, 4) = (-3, 4) \quad \textbf{Ans.}$$
$$\therefore M_x \cdot M_y = M_y \cdot M_x \quad \textbf{Ans.}$$
'Yes', it is always true. **Ans.**

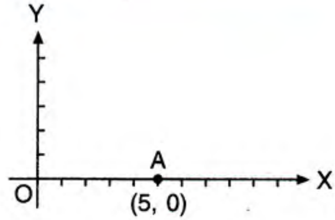
> ***Remember :***
> The combination of the reflections is always commutative, *i.e.*
> (i) $M_x \cdot M_y = M_y \cdot M_x = M_o$ (ii) $M_o \cdot M_x = M_x \cdot M_o = M_y$
> (iii) $M_o \cdot M_y = M_y \cdot M_o = M_x$ and so on.

12.8 Invariant Point :

Any point that remains *unaltered* under a given transformation is called an **invariant.**

e.g. when the point A(5, 0) is reflected in the *x-axis,* the co-ordinates of its image are also (5, 0) *i.e.,* the co-ordinates remain unchanged.

$\therefore$ Point A (5, 0) is said to be invariant under reflection in the *x-axis.*



The same is with :

(i) B (0, 5) in invariant under reflection in the *y-axis.*

(ii) O (0, 0) in invariant under reflection in the *x-axis*, *y-axis* and origin.

(iii) C (–3, 0) in invariant under reflection in the *x-axis* and so on.

> ***Remember :***
> In case of an invariant point, the point is its own image *i.e.* reflection of the point is the point itself. Such transformation (reflection) is called invariant transformation.
> Similarly, reflection of any point is invariant under reflection in a line, if the point lies in the same line.

EXERCISE 12(A)

1. Complete the following table :

	Point	*Transformation*	*Image*
(a)	(5, –7)	-------------------	(–5, 7)
(b)	(4, 2)	Reflection in *x*-axis	-------
(c)	-------	Reflection in *y*-axis	(0, 6)
(d)	(6, –6)	-------------------	(–6, 6)
(e)	(4, –8)	-------------------	(–4, –8)

2. A point P is its own image under the reflection in a line *l*. Describe the position of the point P with respect to the line *l*.

3. State the co-ordinates of the following points under reflection in *x*-axis :
   (i) (3, 2) (ii) (–5, 4) (iii) (0, 0)

4. State the co-ordinates of the following points under reflection in *y*-axis :
   (i) (6, –3) (ii) (–1, 0) (iii) (–8, –2)

5. State the co-ordinates of the following points under reflection in origin :
   (i) (–2, –4) (ii) (–2, 7) (iii) (0, 0)

6. State the co-ordinates of the following points under reflection in the line $x = 0$:
   (i) (–6, 4) (ii) (0, 5) (iii) (3, –4)

7. State the co-ordinates of the following points under reflection in the line $y = 0$;
   (i) (–3, 0) (ii) (8, –5) (iii) (–1, –3)

8. A point P is reflected in the *x*-axis. Co-ordinates of its image are (–4, 5).
   (i) Find the co-ordinates of P.

(ii) Find the co-ordinates of the image of P under reflection in the y-axis.

9. A point P is reflected in the origin. Co-ordinates of its image are (–2, 7).

   (i) Find the co-ordinate of P.

   (ii) Find the co-ordinates of the image of P under reflection in the x-axis.

10. The point P(a, b) is first reflected in the origin and then reflected in the y-axis to P′. If P′ has co-ordinates (4, 6); evaluate a and b.

11. The point P(x, y) is first reflected in the x-axis and then reflected in the origin to P′. If P′ has co-ordinates (–8, 5); evaluate x and y.

12. The point A(–3, 2) is reflected in the x-axis to the point A′. Point A′ is then reflected in the origin to point A″.

    (i) Write down the co-ordinates of A″.

    (ii) Write down a single transformation that maps A onto A″.

13. The point A(4, 6) is first reflected in the origin to point A′. Point A′ is then reflected in the y-axis to point A″.

    (i) Write down the co-ordinates of A″.

    (ii) Write down a single transformation that maps A onto A″.

14. The triangle ABC, where A is (2, 6), B is (–3, 5) and C is (4, 7), is reflected in the y-axis to triangle A′B′C′. Triangle A′B′C′ is then reflected in the origin to triangle A″B″C″.

    (i) Write down the co-ordinates of A″, B″ and C″.

    (ii) Write down a single transformation that maps triangle ABC onto triangle A″B″C″.

15. P and Q have co-ordinates (–2, 3) and (5, 4) respectively. Reflect P in the x-axis to P′ and Q in the y-axis to Q′. State the co-ordinates of P′ and Q′.

16. On a graph paper, plot the triangle ABC, whose vertices are at the points A (3, 1), B (5, 0) and C (7, 4).

    On the same diagram, draw the image of the triangle ABC under reflection in the origin O (0, 0).

17. Find the image of point (4, –6) under the following operations :

    (i) $M_x \cdot M_y$ (ii) $M_y \cdot M_x$

    (iii) $M_o \cdot M_x$ (iv) $M_x \cdot M_o$

    (v) $M_o \cdot M_y$ (vi) $M_y \cdot M_o$

    Write down a single transformation equivalent to each operation given above. State whether :

    (a) $M_o \cdot M_x = M_x \cdot M_o$

    (b) $M_y \cdot M_o = M_o \cdot M_y$

18. Point A (4, –1) is reflected as A′ in the y-axis. Point B on reflection in the x-axis is mapped as B′ (–2, 5). Write the co-ordinates of A′ and B.

19. The point (–5, 0) on reflection in a line is mapped as (5, 0) and the point (–2, –6) on reflection in the same line is mapped as (2, –6).

    (a) Name the line of reflection.

    (b) Write the co-ordinates of the image of (5, –8) in the line obtained in (a).

4 Points (–5, 0) and (4, 0) are invariant points under reflection in the line L_1; points (0, – 6) and (0, 5) are invariant on reflection in the line L_2.

(a) Name or write equations for the lines L_1 and L_2.

(b) Write down the images of P(2, 6) and Q (–8, –3) on reflection in L_1. Name the images as P′ and Q′ respectively.

(c) Write down the images of P and Q on reflection in L_2. Name the images as P″ and Q″ respectively.

(d) State or describe a single transformation that maps Q′ onto Q″.

Solution :

(a) We know that every point in a line is invariant under the reflection in the same line.

Since, points (–5, 0) and (4, 0) lie on the x-axis

⇒ Points (–5, 0) and (4, 0) are invariant under reflection in x-axis.

Given that the points (–5, 0) and (4, 0) are invariant on reflection in line L_1.

$\therefore$ **The line L_1 is x-axis,** whose **equation is $y = 0$** **Ans.**

Similarly, the given points (0, – 6) and (0, 5) lie on the y-axis and are invariant on reflection in line L_2.

$\therefore$ **The line L_2 is y-axis, whose equation is $x = 0$** **Ans.**

(b) **P′** = The image of P(2, 6) in L_1
= The image of P(2, 6) in x-axis = **(2, –6)** **Ans.**

And, **Q′** = The image of Q(–8, –3) in L_1
= The image of Q(–8, –3) in x-axis = **(–8, 3)** **Ans.**

(c) **P″** = The image of P(2, 6) in L_2
= The image of P(2, 6) in y-axis = **(–2, 6)** **Ans.**

Q″ = The image of Q(–8, –3) in L_2
= The image of Q(–8, –3) in y-axis = **(8, –3)** **Ans.**

(d) Since, Q′ = (–8, 3) and Q″ = (8, –3)
and we know $M_0(-x, y) = (x, -y)$

$\therefore$ **The single transformation that maps Q′ onto Q″ = Reflection in origin** **Ans.**

12.9 Using Graph Paper :

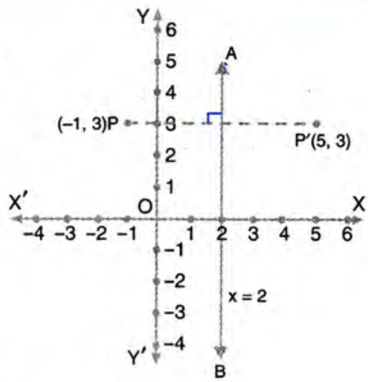
Reflection of a point in the lines $x = a$ and $y = a$.

$x = a$ is a line parallel to y-axis and at a distance of a unit from it.

Similarly, $y = a$ is a line parallel to x-axis and at a distance of a unit from it.

5 (i) Find the reflection of the point P(–1, 3) in the line $x = 2$.
(ii) Find the reflection of the point Q(2, 1) in the line $y + 3 = 0$.

Solution :

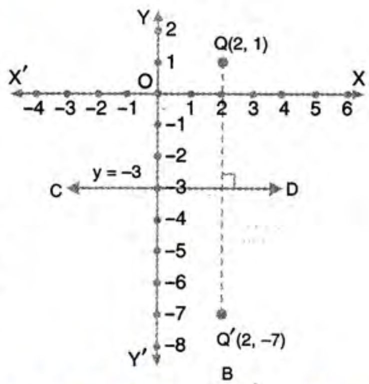


(i) Since, $x = 2$ is a straight line parallel to y-axis and at a distance of 2 unit from it, therefore, in the adjoining figure straight line AB represents $x = 2$.

Mark P(–1, 3) on the same graph. From the point P, draw a straight line perpendicular to AB and produce. On this line mark a point P′ which is at the same distance behind AB as P(–1, 3) is before it.

The co-ordinates of P′ = (5, 3),

$\therefore$ **P′(5, 3) is the reflection of P(–1, 3) in the line $x = 2$.** **Ans.**



(ii) $y + 3 = 0 \Rightarrow y = -3$

Which is the line CD parallel to x-axis and at a distance of –3 unit from it.

Mark the point Q(2, 1) on the same graph.

From the point Q, draw a straight line perpendicular to line CD and produce. On this line mark a point Q′ which is at the same distance below CD as Q(2, 1) is above it.

The co-ordinates of Q′ = (2, –7).

$\therefore$ **Q′(2, –7) is the reflection of Q(2, 1) in the line $y + 3 = 0$.** **Ans.**

6 The points P(5, 1) and Q(–2, –2) are reflected in line $x = 2$. Use graph paper to find the images P′ and Q′ of points P and Q respectively in line $x = 2$. Take 2 cm equal to 2 units.

Solution :

The graph of line $x = 2$ is the straight line AB, as shown below, which is parallel to y-axis and is at a distance of 2 units from it.

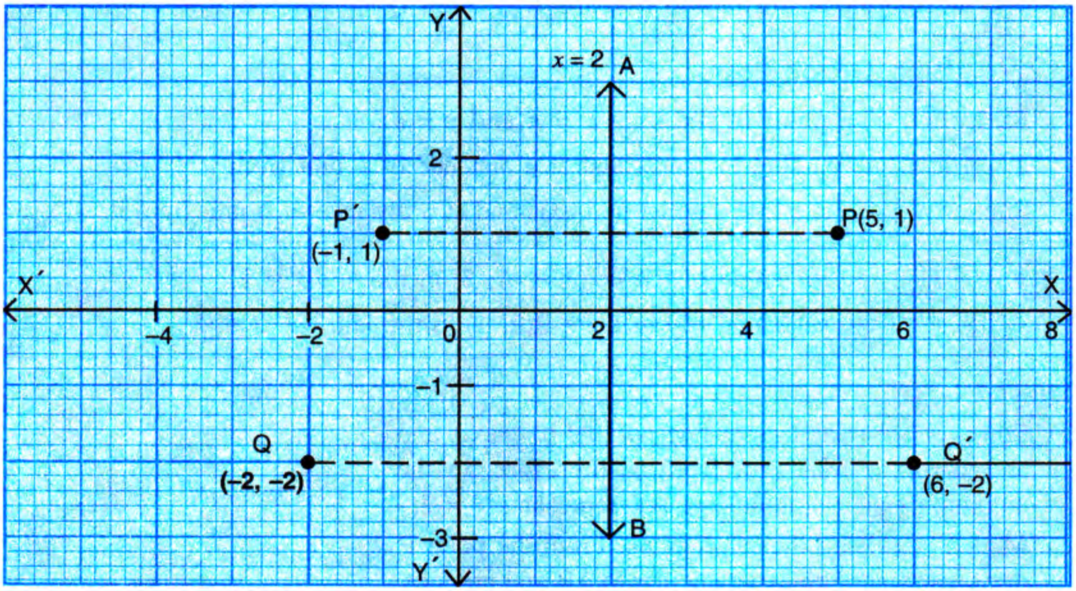
Mark P(5, 1) and Q (–2, –2) on the same graph paper.

To find P′, the image of P :

Mark P′ at the same distance behind AB as P is before it. Since, P is 3 units before AB, its image P′ will be 3 units behind AB.

Clearly, the co-ordinates of P′ = **(–1, 1)** **Ans.**

In the same way, since Q(–2, –2) is 4 units before AB, its image Q′ will be 4 units behind AB.



On marking position of Q′, we find : Q′ = **(6, –2)** **Ans.**

7 Use a graph paper for this question. (Take two divisions = 1 unit on both the axes).

Plot the points P (3, 2) and Q (–3, –2). From P and Q, draw perpendiculars PM and QN on the x-axis.

(a) Write the co-ordinates of points M and N.

(b) Name the image of P on reflection in the origin.

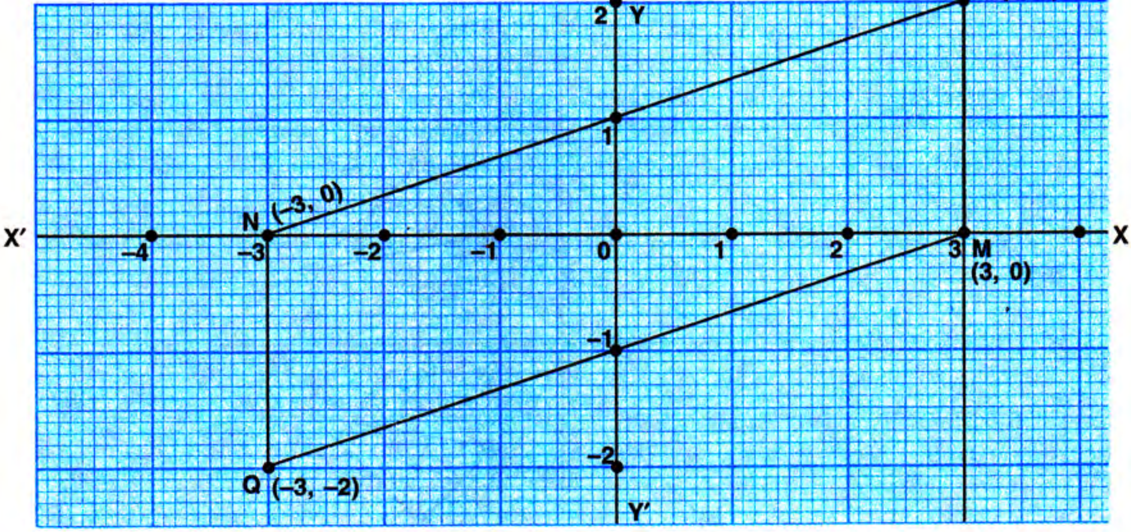
(c) Assign the special name to geometrical figure PMQN and find its area.

(d) Write the co-ordinates of the point to which M is mapped on reflection in :
(i) x-axis, (ii) y-axis, (iii) origin. [2003]

Solution :

(a) Co-ordinates of M = **(3, 0)** and

Co-ordinates of N = **(–3, 0)** **Ans.**



(b) **Image of P(3, 2) in origin = (–3, –2) = Q** **Ans.**

(c) **PMQN is a parallelogram**

Area of PMQN = 2 (Area of Δ PMN) $= 2\left(\frac{1}{2} \times 6 \times 2\right)$ sq. units

= **12 sq. units** **Ans.**

(d) (i) M (3, 0) reflected in x-axis gives **(3, 0)** **Ans.**

(ii) M (3, 0) reflected in y-axis gives **(–3, 0)** **Ans.**

(iii) M (3, 0) reflected in origin gives **(–3, 0)** **Ans.**

8 Use graph paper for this question.

The points A(2, 3), B(4, 5) and C(7, 2) are the vertices of Δ ABC.

(i) Write down the co-ordinates of A′, B′, C′ if Δ A′ B′ C′ is the image of Δ ABC, when reflected in the origin.

(ii) Write down the co-ordinates of A″, B″, C″ if Δ A″ B″ C″ is the image of Δ ABC, when reflected in the x-axis.

(iii) Mention the special name of the quadrilateral BCC″B″ and find its area. [2006]

Solution :

See the graph on the next page

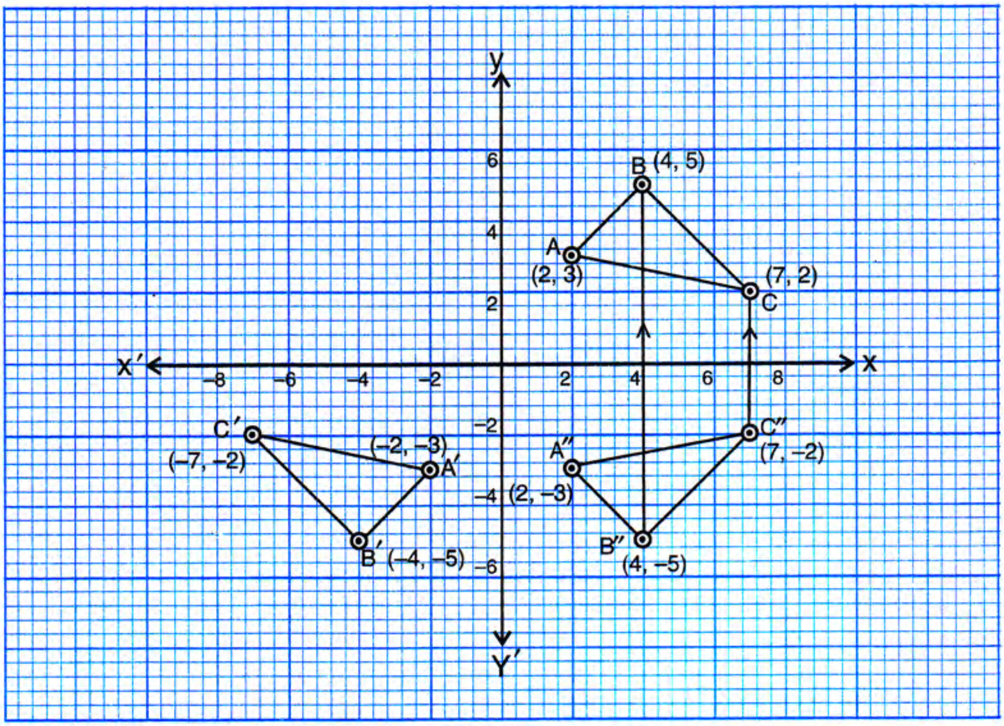
(i) **A′ = (–2, –3), B′ = (–4, –5)** and **C′ = (–7, –2)** **Ans.**

(ii) **A″ = (2, –3), B″ = (4, –5)** and **C″ = (7, –2)** **Ans.**

(iii) **BCC″ B″ is an isosceles trapezium** as BB″ is parallel to CC″ and BC = B″ C″. **Ans.**

Area of quadrilateral (trapezium) **BCC″ B″**

$= \frac{1}{2}$ (BB″ + CC″) × 3 $= \frac{1}{2}$ × (10 + 4) × 3 sq. unit = **21 sq. unit** **Ans.**



EXERCISE 12(B)

1. Attempt this question on graph paper.
   (a) Plot A (3, 2) and B (5, 4) on graph paper. Take 2 cm = 1 unit on both the axes.
   (b) Reflect A and B in the x-axis to A′ and B′ respectively. Plot these points also on the same graph paper.
   (c) Write down :
      (i) the geometrical name of the figure ABB′A′;
      (ii) the measure of angle ABB′;
      (iii) the image A″ of A, when A is reflected in the origin.
      (iv) the single transformation that maps A′ to A″.
2. Points (3, 0) and (–1, 0) are invariant points under reflection in the line L_1; points (0, –3) and (0, 1) are invariant points on reflection in line L_2.
   (i) Name or write equations for the lines L_1 and L_2.
   (ii) Write down the images of points P (3, 4) and Q (–5, –2) on reflection in L_1. Name the images as P′ and Q′ respectively.
   (iii) Write down the images of P and Q on reflection in L_2. Name the images as P″ and Q″ respectively.
   (iv) State or describe a single transformation that maps P′ onto P″.
3. (i) Point P (a, b) is reflected in the x-axis to P′ (5, –2). Write down the values of a and b.
   (ii) P″ is the image of P when reflected in the y-axis. Write down the co-ordinates of P″.
   (iii) Name a single transformation that maps P′ to P″.
4. The point (–2, 0) on reflection in a line is mapped to (2, 0) and the point (5, –6) on reflection in the same line is mapped to (–5, –6).
   (i) State the name of the mirror line and write its equation.
   (ii) State the co-ordinates of the image of (–8, –5) in the mirror line.
5. The points P (4, 1) and Q (–2, 4) are reflected in line y = 3. Find the co-ordinates of P′, the image of P and Q′, the image of Q.
6. A point P (–2, 3) is reflected in line x = 2 to point P′. Find the co-ordinates of P′.

7. A point P (a, b) is reflected in the x-axis to P′ (2, –3). Write down the values of a and b. P″ is the image of P, reflected in the y-axis. Write down the co-ordinates of P″. Find the co-ordinates of P‴, when P is reflected in the line, parallel to y-axis, such that $x = 4$.

8. Points A and B have co-ordinates (3, 4) and (0, 2) respectively. Find the image :
   (a) A′ of A under reflection in the x-axis.
   (b) B′ of B under reflection in the line AA′.
   (c) A″ of A under reflection in the y-axis.
   (d) B″ of B under reflection in the line AA″.

9. (i) Plot the points A (3, 5) and B(–2, –4). Use 1 cm = 1 unit on both the axes.
   (ii) A′ is the image of A when reflected in the x-axis. Write down the co-ordinates of A′ and plot it on the graph paper.
   (iii) B′ is the image of B when reflected in the y-axis, followed by reflection in the origin. Write down the co-ordinates of B′ and plot it on the graph paper.
   (iv) Write down the geometrical name of the figure AA′BB′.
   (v) Name two invariant points under reflection in the x-axis.

10. The point P (5, 3) was reflected in the origin to get the image P′.
   (a) Write down the co-ordinates of P′.
   (b) If M is the foot of the perpendicular from P to the x-axis, find the co-ordinates of M.
   (c) If N is the foot of the perpendicular from P′ to the x-axis, find the co-ordinates of N.
   (d) Name the figure PMP′N.
   (e) Find the area of the figure PMP′N. **[2001]**

11. The point P(3, 4) is reflected to P′ in the x-axis; and O′ is the image of O (the origin) when reflected in the line PP′. Write :
   (i) the co-ordinates of P′ and O′,
   (ii) the length of the segments PP′ and OO′,
   (iii) the perimeter of the quadrilateral POP′O′,
   (iv) the geometrical name of the figure POP′O′. **[2002]**

12. A (1, 1), B (5, 1), C (4, 2) and D (2, 2) are vertices of a quadrilateral. Name the quadrilateral ABCD. A, B, C, and D are reflected in the origin on to A′, B′, C′ and D′ respectively. Locate A′, B′, C′ and D′ on the graph sheet and write their co-ordinates. Are D, A, A′ and D′ collinear ? **[2004]**

13. P and Q have co-ordinates (0, 5) and (–2, 4).
   (a) P is invariant when reflected in an axis. Name the axis.
   (b) Find the image of Q on reflection in the axis found in (a).
   (c) (0, k) on reflection in the origin is invariant. Write the value of k.
   (d) Write the co-ordinates of the image of Q, obtained by reflecting it in the origin followed by reflection in x-axis. **[2005]**

14. The points P(1, 2), Q(3, 4) and R(6, 1) are the vertices of Δ PQR.
   (a) Write down the co-ordinates of P′, Q′ and R′, if Δ P′Q′R′ is the image of Δ PQR, when reflected in the origin.
   (b) Write down the co-ordinates of P″, Q″ and R″, if Δ P″Q″R″ is the image of Δ PQR, when reflected in the x-axis.
   (c) Mention the special name of the quadrilateral QRR″Q″ and find its area.

15. (a) The point P (2, –4) is reflected about the line $x = 0$ to get the image Q. Find the co-ordinates of Q.
   (b) The point Q is reflected about the line $y = 0$ to get the image R. Find the co-ordinates of R.
   (c) Name the figure PQR.
   (d) Find the area of figure PQR. **[2007]**

16. A′ and B′ are images of A(–3, 5) and B(–5, 3) respectively on reflection in y-axis. Find :
   (a) the co-ordinates of A′ and B′.
   (b) Assign special name of quadrilateral AA′B′B.
   (c) Are AB′ and BA′ equal in length ?

17. Using a graph paper, plot the points A(6, 4) and B(0, 4).
   (a) Reflect A and B in the origin to get the images A′ and B′.
   (b) Write the co-ordinates of A′ and B′.
   (c) State the geometrical name for the figure ABA′B′.
   (d) Find its perimeter. **[2013]**