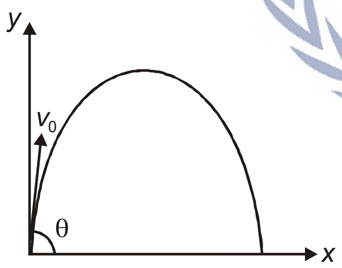
System of Particles and Rotational Motion

1. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of **[AIEEE-2009]**

   (1) $\frac{1}{6}\frac{l\omega}{g}$ (2) $\frac{1}{2}\frac{l^2\omega^2}{g}$

   (3) $\frac{1}{6}\frac{l^2\omega^2}{g}$ (4) $\frac{1}{3}\frac{l^2\omega^2}{g}$

2. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is **[AIEEE-2010]**



   (1) $\frac{1}{2}mg v_0 t^2 \cos\theta\, \hat{i}$ (2) $-mg v_0 t^2 \cos\theta\, \hat{j}$

   (3) $mg v_0 t \cos\theta\, \hat{k}$ (4) $-\frac{1}{2}mg v_0 t^2 \cos\theta\, \hat{k}$

   where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively

3. A particle of mass m is projected with a velocity v making an angle of 30° with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height h is **[AIEEE-2011]**

   (1) $\frac{\sqrt{3}}{16}\frac{mv^3}{g}$ (2) $\frac{\sqrt{3}}{2}\frac{mv^2}{g}$

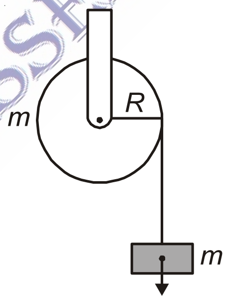
   (3) Zero (4) $\frac{mv^3}{\sqrt{2}g}$

4. A ring of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip? **[JEE (Main)-2013]**

   (1) $\frac{r\omega_0}{4}$ (2) $\frac{r\omega_0}{3}$

   (3) $\frac{r\omega_0}{2}$ (4) $r\omega_0$

5. A mass m is supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall on release? **[JEE (Main)-2014]**



   (1) $\frac{2g}{3}$ (2) $\frac{g}{2}$

   (3) $\frac{5g}{6}$ (4) g

6. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical. About the point of suspension **[JEE (Main)-2014]**

   (1) Angular momentum is conserved

   (2) Angular momentum changes in magnitude but not in direction

   (3) Angular momentum changes in direction but not in magnitude

   (4) Angular momentum changes both in direction and magnitude

7. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to **[JEE (Main)-2015]**

(1) $\frac{h^2}{4R}$ (2) $\frac{3h}{4}$

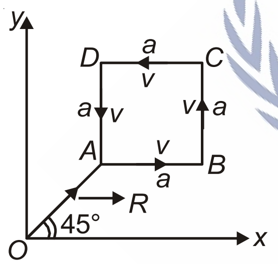
(3) $\frac{5h}{8}$ (4) $\frac{3h^2}{8R}$

8. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its center and perpendicular to one of its faces is **[JEE (Main)-2015]**

(1) $\frac{MR^2}{32\sqrt{2}\pi}$ (2) $\frac{MR^2}{16\sqrt{2}\pi}$

(3) $\frac{4MR^2}{9\sqrt{3}\pi}$ (4) $\frac{4MR^2}{3\sqrt{3}\pi}$

9. A particle of mass m is moving along the side of a square of side a, with a uniform speed v in the x-y plane as shown in the figure



Which of the following statements is false for the angular momentum $\vec{L}$ about the origin? **[JEE (Main)-2016]**

(1) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from C to D

(2) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from B to C

(3) $\vec{L} = \frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from D to A

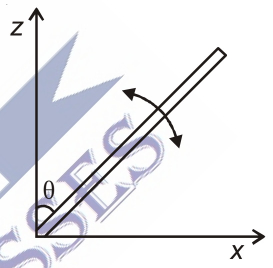
(4) $\vec{L} = -\frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from A to B

10. The moment of inertia of a uniform cylinder of length ℓ and radius R about its perpendicular bisector is I. What is the ratio $\frac{\ell}{R}$ such that the moment of inertia is minimum? **[JEE (Main)-2017]**

(1) $\sqrt{\frac{3}{2}}$ (2) $\frac{\sqrt{3}}{2}$

(3) 1 (4) $\frac{3}{\sqrt{2}}$

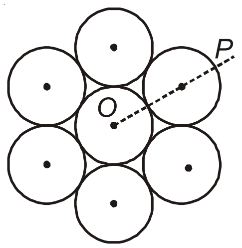
11. A slender uniform rod of mass M and length l is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is **[JEE (Main)-2017]**



(1) $\frac{3g}{2\ell}\sin\theta$ (2) $\frac{2g}{3\ell}\sin\theta$

(3) $\frac{3g}{2\ell}\cos\theta$ (4) $\frac{2g}{3\ell}\cos\theta$

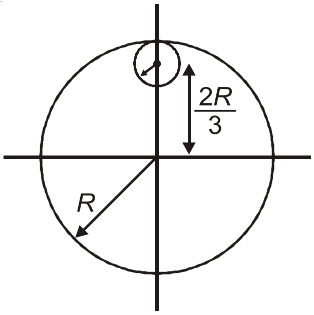
12. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is **[JEE (Main)-2018]**



(1) $\frac{19}{2}MR^2$ (2) $\frac{55}{2}MR^2$

(3) $\frac{73}{2}MR^2$ (4) $\frac{181}{2}MR^2$

13. From a uniform circular disc of radius R and mass $9M$, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is **[JEE (Main)-2018]**

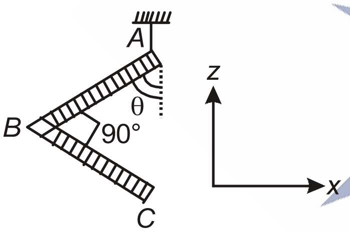


(1) $4MR^2$ (2) $\frac{40}{9}MR^2$

(3) $10MR^2$ (4) $\frac{37}{9}MR^2$

14. An L- shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If $AB = BC$, and the angle made by AB with downward vertical is θ, then

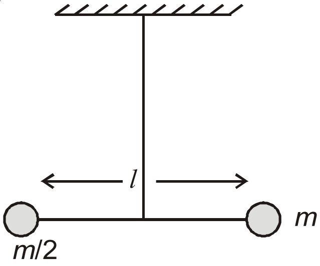
[JEE (Main)-2019]



(1) $\tan\theta = \frac{1}{2}$ (2) $\tan\theta = \frac{2}{\sqrt{3}}$

(3) $\tan\theta = \frac{1}{3}$ (4) $\tan\theta = \frac{1}{2\sqrt{3}}$

15. Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length l. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system (see figure). Because of torsional constant k, the restoring torque is $\tau = k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be **[JEE (Main)-2019]**

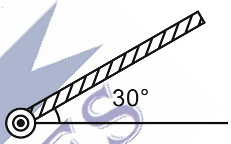


(1) $\frac{k\theta_0^2}{2l}$ (2) $\frac{k\theta_0^2}{l}$

(3) $\frac{2k\theta_0^2}{l}$ (4) $\frac{3k\theta_0^2}{l}$

16. A rod of length 50 cm is pivoted at one end. It is raised such that it makes an angle of 30° from the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rad s^{-1}) will be (g = 10 ms^{-2})

[JEE (Main)-2019]



(1) $\frac{\sqrt{20}}{3}$ (2) $\sqrt{30}$

(3) $\sqrt{\frac{30}{2}}$ (4) $\frac{\sqrt{30}}{2}$

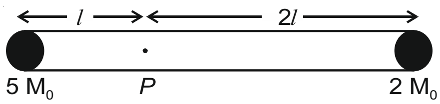
17. A homogeneous solid cylindrical roller of radius R and mass M is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is

[JEE (Main)-2019]

(1) $\frac{F}{2mR}$ (2) $\frac{2F}{3mR}$

(3) $\frac{F}{3mR}$ (4) $\frac{3F}{2mR}$

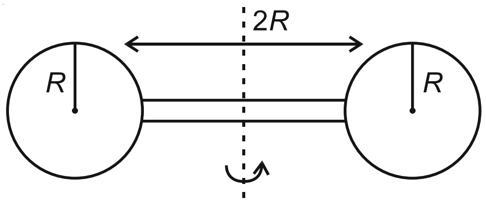
18. A rigid massless rod of length $3l$ has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be **[JEE (Main)-2019]**



(1) $\dfrac{g}{2\ell}$ (2) $\dfrac{g}{3\ell}$

(3) $\dfrac{g}{13\ell}$ (4) $\dfrac{7g}{3\ell}$

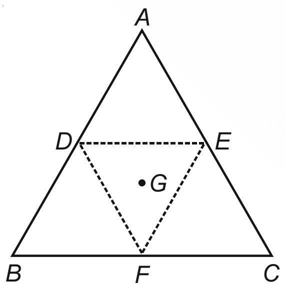
19. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length $2R$ and mass M (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is **[JEE (Main)-2019]**



(1) $\dfrac{152}{15}MR^2$ (2) $\dfrac{17}{15}MR^2$

(3) $\dfrac{209}{15}MR^2$ (4) $\dfrac{137}{15}MR^2$

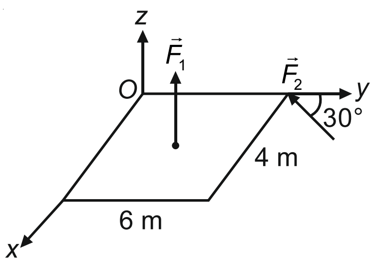
20. An equilateral triangle ABC is cut from a thin solid sheet of wood. (See figure) D, E and F are the mid-points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0. If the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I. Then **[JEE (Main)-2019]**



(1) $I = \dfrac{3}{4}I_0$ (2) $I = \dfrac{15}{16}I_0$

(3) $I = \dfrac{I_0}{4}$ (4) $I = \dfrac{9}{16}I_0$

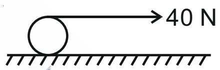
21. A slab is subjected to two forces $\vec{F_1}$ and $\vec{F_2}$ of same magnitude F as shown in the figure. Force $\vec{F_2}$ is in XY-plane while force F_1 acts along z-axis at the point $(2\vec{i}+3\vec{j})$. The moment of these forces about point O will be **[JEE (Main)-2019]**



(1) $(3\hat{i}-2\hat{j}+3\hat{k})F$ (2) $(3\hat{i}+2\hat{j}-3\hat{k})F$

(3) $(3\hat{i}+2\hat{j}+3\hat{k})F$ (4) $(3\hat{i}-2\hat{j}-3\hat{k})F$

22. A string is wound around a hollow cylinder of mass 5 kg and radius 0.5 m. If the string is now pulled with a horizontal force of 40 N, and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string) : **[JEE (Main)-2019]**



(1) 16 rad/s^2 (2) 20 rad/s^2

(3) 12 rad/s^2 (4) 10 rad/s^2

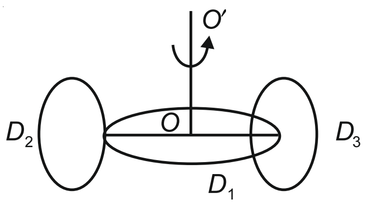
23. The magnitude of torque on a particle of mass 1 kg is 2.5 Nm about the origin. If the force acting on it is 1 N, and the distance of the particle from the origin is 5 m, the angle between the force and the position vector is (in radians): **[JEE (Main)-2019]**

(1) $\dfrac{\pi}{8}$ (2) $\dfrac{\pi}{6}$

(3) $\dfrac{\pi}{3}$ (4) $\dfrac{\pi}{4}$

24. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly at its opposite ends (see figure). The moment of inertia of the system about the axis OO', passing through the centre of D_1 as shown in the figure, will be

[JEE (Main)-2019]



(1) $3MR^2$ (2) $\dfrac{4}{5}MR^2$

(3) MR^2 (4) $\dfrac{2}{3}MR^2$

25. Let the moment of inertia of a hollow cylinder of length 30 cm (inner radius 10 cm and outer radius 20 cm), about its axis be I. The radius of a thin cylinder of the same mass such that its moment of inertia about its axis is also I, is

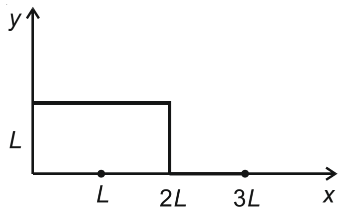
[JEE (Main)-2019]

(1) 14 cm (2) 12 cm

(3) 16 cm (4) 18 cm

26. The position vector of the centre of mass $\vec{r}_{cm}$ of an asymmetric uniform bar of negligible area of cross-section as shown in figure is

[JEE (Main)-2019]



(1) $\vec{r}_{cm} = \frac{5}{8}L\hat{x} + \frac{13}{8}L\hat{y}$

(2) $\vec{r}_{cm} = \frac{3}{8}L\hat{x} + \frac{11}{8}L\hat{y}$

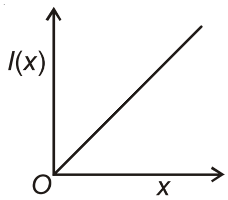
(3) $\vec{r}_{cm} = \frac{13}{8}L\hat{x} + \frac{5}{8}L\hat{y}$

(4) $\vec{r}_{cm} = \frac{11}{8}L\hat{x} + \frac{3}{8}L\hat{y}$

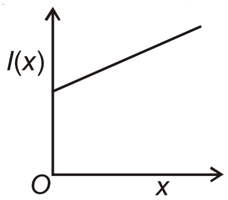
27. The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of x from it, is '$I(x)$'. Which one of the graphs represents the variation of $I(x)$ with x correctly?

[JEE (Main)-2019]

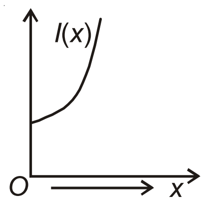
(1)



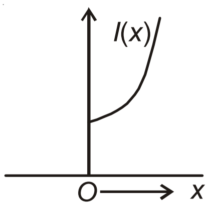
(2)



(3)

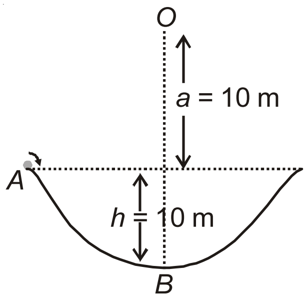


(4)



28. A particle of mass 20 g is released with an initial velocity 5 m/s along the curve from the point A, as shown in the figure. The point A is a height h from point B. The particle slides along the frictionless surface. When the particle reaches point B, its angular momentum about O will be

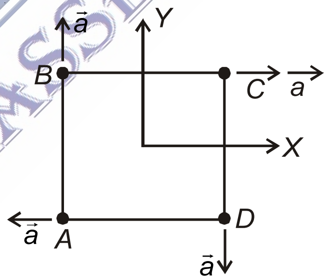
(Take g = 10 m/s^2) **[JEE (Main)-2019]**



(1) 2 kg m^2/s (2) 3 kg m^2/s

(3) 8 kg m^2/s (4) 6 kg m^2/s

29. Four particles A, B, C and D with masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is **[JEE (Main)-2019]**



(1) Zero (2) $a(\hat{i} + \hat{j})$

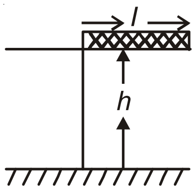
(3) $\frac{a}{5}(\hat{i} + \hat{j})$ (4) $\frac{a}{5}(\hat{i} - \hat{j})$

30. A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with ρ_0 as constant and r is the distance from its center. The moment of inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I = a MR^2$. The value of the coefficient a is **[JEE (Main)-2019]**

(1) $\frac{3}{2}$ (2) $\frac{1}{2}$

(3) $\frac{3}{5}$ (4) $\frac{8}{5}$

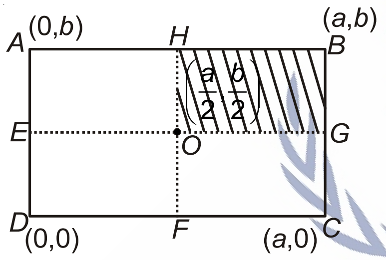
31. A rectangular solid box of length 0.3 m is held horizontally, with one of its sides on the edge of a platform of height 5 m. When released, it slips off the table in a very short time τ = 0.01 s, remaining essentially horizontal. The angle by which it would rotate when it hits the ground will be (in radians) close to: **[JEE (Main)-2019]**



(1) 0.3 (2) 0.02

(3) 0.28 (4) 0.5

32. A uniform rectangular thin sheet *ABCD* of mass *M* has length a and breadth *b*, as shown in the figure. If the shaded portion *HBGO* is cut-off, the coordinates of the centre of mass of the remaining portion will be: **[JEE (Main)-2019]**

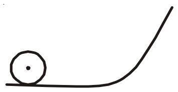


(1) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$ (2) $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

(3) $\left(\frac{3a}{4}, \frac{3b}{4}\right)$ (4) $\left(\frac{5a}{3}, \frac{5b}{3}\right)$

33. A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights h_{sph} and h_{cyl} on the incline. The ratio $\frac{h_{sph}}{h_{cyl}}$ is given by

[JEE (Main)-2019]



(1) $\frac{4}{5}$ (2) $\frac{2}{\sqrt{5}}$

(3) 1 (4) $\frac{14}{15}$

34. A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which it has rotated, is given as $k\theta^2$. If its moment of inertia is *I*, then the angular acceleration of the disc is **[JEE (Main)-2019]**

(1) $\frac{2k}{I}\theta$ (2) $\frac{k}{I}\theta$

(3) $\frac{k}{2I}\theta$ (4) $\frac{k}{4I}\theta$

35. The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane : (i) a ring of radius *R*, (ii) a solid cylinder of radius $\frac{R}{2}$ and (iii) a solid sphere of radius $\frac{R}{4}$. If, in each case, the speed of the center of mass at the bottom of the incline is same, the ratio of the maximum heights they climb is **[JEE (Main)-2019]**

(1) 14 : 15 : 20 (2) 10 : 15 : 7

(3) 4 : 3 : 2 (4) 20 : 15 : 14

36. A thin smooth rod of length *L* and mass *M* is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its center. Two beads of mass *m* and negligible size are at the center of the rod initially. The beads are free to slide along the rod. The angular speed of the system, when the beads reach the opposite ends of the rod, will be **[JEE (Main)-2019]**

(1) $\frac{M\omega_0}{M+3m}$ (2) $\frac{M\omega_0}{M+m}$

(3) $\frac{M\omega_0}{M+6m}$ (4) $\frac{M\omega_0}{M+2m}$

37. Moment of inertia of a body about a given axis is 1.5 kg m^2. Initially the body is at rest. In order to produce a rotational kinetic energy of 1200 J, the angular acceleration of 20 rad/s^2 must be applied about the axis for a duration of **[JEE (Main)-2019]**

(1) 2.5 s (2) 2 s

(3) 5 s (4) 3 s

38. Two coaxial discs, having moments of inertia I_1 and $\frac{I_1}{2}$, are rotating with respective angular velocities ω_1 and $\frac{\omega_1}{2}$ about their common axis. They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then $(E_f - E_i)$ is : **[JEE (Main)-2019]**

(1) $-\frac{I_1\omega_1^2}{12}$ (2) $\frac{3}{8}I_1\omega_1^2$

(3) $\frac{I_1\omega_1^2}{6}$ (4) $-\frac{I_1\omega_1^2}{24}$

39. A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$ where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is **[JEE (Main)-2019]**

(1) $\frac{MR^2}{3}$ (2) $\frac{MR^2}{6}$

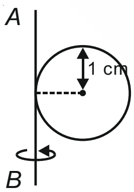
(3) $\frac{MR^2}{2}$ (4) $\frac{2MR^2}{3}$

40. The time dependence of the position of a particle of mass $m = 2$ is given by $\vec{r}(t) = 2t\hat{i} - 3t^2\hat{j}$. Its angular momentum, with respect to the origin, at time $t = 2$ is **[JEE (Main)-2019]**

(1) $-34\left(\hat{k}-\hat{i}\right)$ (2) $48\left(\hat{i}+\hat{j}\right)$

(3) $36\,\hat{k}$ (4) $-48\,\hat{k}$

41. A metal coin of mass 5 g and radius 1 cm is fixed to a thin stick AB of negligible mass as shown in the figure. The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in 5 s, is closed to **[JEE (Main)-2019]**



(1) 4.0×10^{-6} Nm (2) 7.9×10^{-6} Nm

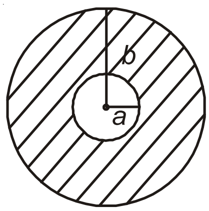
(3) 2.0×10^{-5} Nm (4) 1.6×10^{-5} Nm

42. A solid sphere of mass M and radius R is divided into two unequal parts. The first part has a mass of $\frac{7M}{8}$ and is converted into a uniform disc of radius $2R$. The second part is converted into a uniform solid sphere. Let I_1 be the moment of inertia of the disc about its axis and I_2 be the moment of inertia of the new sphere about its axis. The ratio I_1/I_2 is given by **[JEE (Main)-2019]**

(1) 140 (2) 185

(3) 285 (4) 65

43. A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\frac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is **[JEE (Main)-2019]**



(1) $\frac{a+b}{2}$ (2) $\sqrt{\frac{a^2+b^2+ab}{2}}$

(3) $\frac{a+b}{3}$ (4) $\sqrt{\frac{a^2+b^2+ab}{3}}$

44. A man (mass = 50 kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son so that he starts moving at a speed of 0.70 ms^{-1} with respect to the man. The speed of the man with respect to the surface is **[JEE (Main)-2019]**

(1) 0.20 ms^{-1} (2) 0.14 ms^{-1}

(3) 0.47 ms^{-1} (4) 0.28 ms^{-1}

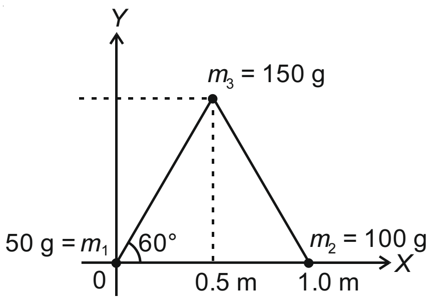
45. A person of mass M is, sitting on a swing of length L and swinging with an angular amplitude θ_0. If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his centre of mass moves by a distance l (l<<L), is close to **[JEE (Main)-2019]**

(1) Mgl (2) $Mgl\left(1+\theta_0^2\right)$

(3) $Mgl\left(1+\frac{\theta_0^2}{2}\right)$ (4) $Mgl\left(1-\theta_0^2\right)$

46. Three particles of masses, 50 g, 100 g and 150 g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be

[JEE (Main)-2019]



(1) $\left(\frac{\sqrt{3}}{8}\text{ m}, \frac{7}{12}\text{ m}\right)$ (2) $\left(\frac{\sqrt{3}}{4}\text{ m}, \frac{5}{12}\text{ m}\right)$

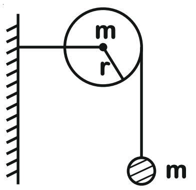
(3) $\left(\frac{7}{12}\text{ m}, \frac{\sqrt{3}}{8}\text{ m}\right)$ (4) $\left(\frac{7}{12}\text{ m}, \frac{\sqrt{3}}{4}\text{ m}\right)$

47. The radius of gyration of a uniform rod of length l, about an axis passing through a point $\frac{l}{4}$ away from the centre of the rod, and perpendicular to it, is **[JEE (Main)-2020]**

(1) $\sqrt{\frac{7}{48}}\,l$ (2) $\frac{1}{8}l$

(3) $\frac{1}{4}l$ (4) $\sqrt{\frac{3}{8}}\,l$

48.



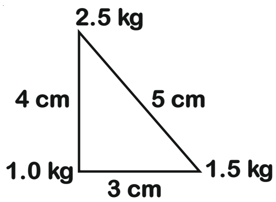
As shown in the figure, a bob of mass m is tied by a massless string whose other end portion is wound on a fly wheel (disc) of radius r and mass m. When released from rest the bob starts falling vertically. When it has covered a distance of h, the angular speed of the wheel will be

[JEE (Main)-2020]

(1) $r\sqrt{\frac{3}{2gh}}$ (2) $r\sqrt{\frac{3}{4gh}}$

(3) $\frac{1}{r}\sqrt{\frac{4gh}{3}}$ (4) $\frac{1}{r}\sqrt{\frac{2gh}{3}}$

49. Three point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The center of mass of system is at a point **[JEE (Main)-2020]**



(1) 1.5 cm right and 1.2 cm above 1 kg mass

(2) 2.0 cm right and 0.9 cm above 1 kg mass

(3) 0.9 cm right and 2.0 cm above 1 kg mass

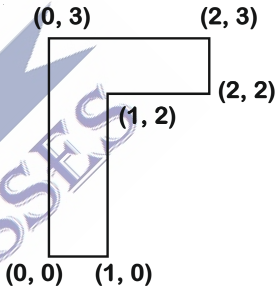
(4) 0.6 cm right and 2.0 cm above 1 kg mass

50. Mass per unit area of a circular disc of radius a depends on the distance r from its centre as $\sigma(r) = A + Br$. The moment of inertia of the disc about the axis, perpendicular to the plane and passing through its centre is **[JEE (Main)-2020]**

(1) $2\pi a^4\left(\frac{A}{4}+\frac{B}{5}\right)$ (2) $\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$

(3) $2\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$ (4) $2\pi a^4\left(\frac{aA}{4}+\frac{B}{5}\right)$

51. The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plate) of mass 4 kg. (The coordinates of the same are shown in figure) are **[JEE (Main)-2020]**



(1) (0.75 m, 1.75 m) (2) (1.25 m, 1.50 m)

(3) (1 m, 1.75 m) (4) (0.75 m, 0.75 m)

52. Consider a uniform rod of mass M = 4 m and length l pivoted about its centre. A mass m moving with velocity v making angle $\theta = \frac{\pi}{4}$ to the rod's long axis collides with one end of the rod and sticks to it. The angular speed of the rod-mass system just after the collision is

[JEE (Main)-2020]

(1) $\frac{4}{7}\frac{v}{l}$ (2) $\frac{3}{7}\frac{v}{l}$

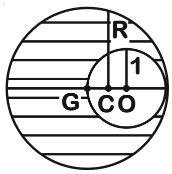
(3) $\frac{3}{7\sqrt{2}}\frac{v}{l}$ (4) $\frac{3\sqrt{2}}{7}\frac{v}{l}$

53. A uniform sphere of mass 500 g rolls without slipping on a plane horizontal surface with its centre moving at a speed of 5.00 cm/s. Its kinetic energy is **[JEE (Main)-2020]**

(1) 6.25×10^{-4} J (2) 1.13×10^{-3} J

(3) 8.75×10^{-4} J (4) 8.75×10^{-3} J

54. As shown in figure when a spherical cavity (centred at O) of radius 1 is cut out of a uniform sphere of radius R (centred at C), the centre of mass of remaining (shaded) part of sphere is at G, i.e., on the surface of the cavity. R can be determined by the equation **[JEE (Main)-2020]**



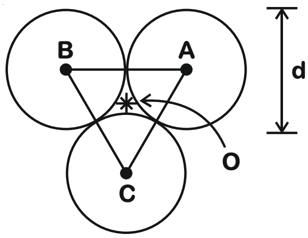
(1) $(R^2 + R + 1)(2 - R) = 1$

(2) $(R^2 - R + 1)(2 - R) = 1$

(3) $(R^2 - R - 1)(2 - R) = 1$

(4) $(R^2 + R - 1)(2 - R) = 1$

55.



Three solid spheres each of mass m and diameter d are stuck together such that the lines connecting the centres form an equilateral triangle of side of length d. The ratio I_0/I_A of moment of inertia I_0 of the system about an axis passing the centroid and about center of any of the spheres I_A and perpendicular to the plane of the triangle is **[JEE (Main)-2020]**

(1) $\frac{23}{13}$ (2) $\frac{15}{13}$

(3) $\frac{13}{15}$ (4) $\frac{13}{23}$

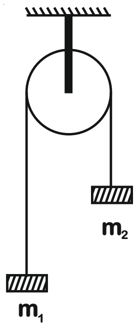
56. A rod of length L has non-uniform linear mass density given by $\rho(x) = a + b\left(\frac{x}{L}\right)^2$, where a and b are constants and $0 \le x \le L$. The value of x for the centre of mass of the rod is at **[JEE (Main)-2020]**

(1) $\frac{3}{2}\left(\frac{a+b}{2a+b}\right)L$ (2) $\frac{4}{3}\left(\frac{a+b}{2a+3b}\right)L$

(3) $\frac{3}{2}\left(\frac{2a+b}{3a+b}\right)L$ (4) $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$

57. A uniformly thick wheel with moment of inertia I and radius R is free to rotate about its centre of mass (see fig.), A massless string is warapped over its rim and two blocks of masses m_1 and m_2 ($m_1 > m_2$) are attached to the ends of the string. The system is released from rest. The angular speed of the wheel when m_1 descents by a distance h is

[JEE (Main)-2020]



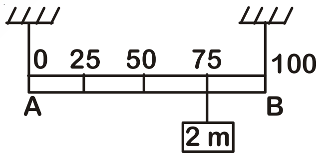
(1) $\left[\frac{(m_1 - m_2)}{(m_1 + m_2)R^2 + I}\right]^{\frac{1}{2}} gh$

(2) $\left[\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + I}\right]^{\frac{1}{2}}$

(3) $\left[\frac{m_1 + m_2}{(m_1 + m_2)R^2 + I}\right]^{\frac{1}{2}} gh$

(4) $\left[\frac{2(m_1 + m_2)gh}{(m_1 + m_2)R^2 + I}\right]^{\frac{1}{2}}$

58.

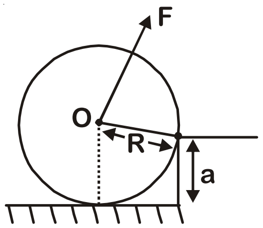


Shown in the figure is rigid and uniform one meter long rod AB held in horizontal position by two strings tied to its ends and attached to the ceiling. The rod is of mass 'm' and has another weight of mass 2 m hung at a distance of 75 cm from A. The tension in the string at A is **[JEE (Main)-2020]**

(1) 0.5 mg (2) 2 mg

(3) 0.75 mg (4) 1 mg

59. A uniform cylinder of mass M and radius R is to be pulled over a step of height a ($a < R$) by applying a force F at its centre 'O' perpendicular to the plane through the axes of the cylinder on the edge of the step (see figure). The minimum value of F required is **[JEE (Main)-2020]**



(1) $Mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$ (2) $Mg\sqrt{\left(\frac{R}{R-a}\right)^2-1}$

(3) $Mg\sqrt{1-\frac{a^2}{R^2}}$ (4) $Mg\frac{a}{R}$

60. Two uniform circular discs are rotating independently in the same direction around their common axis passing through their centres. The moment of inertia and angular velocity of the first disc are 0.1 kg-m^2 and 10 rad s^{-1} respectively while those for the second one are 0.2 kg-m^2 and 5 rad s^{-1} respectively. At some instant they get stuck together and start rotating as a single system about their common axis with some angular speed. The kinetic energy of the combined system is **[JEE (Main)-2020]**

(1) $\frac{20}{3}$ J (2) $\frac{5}{3}$ J

(3) $\frac{10}{3}$ J (4) $\frac{2}{3}$ J

61. Moment of inertia of a cylinder of mass M, length L and radius R about an axis passing through its centre and perpendicular to the axis of the cylinder is $I = M\left(\frac{R^2}{4}+\frac{L^2}{12}\right)$. If such a cylinder is to be made for a given mass of a material, the ratio L/R for it to have minimum possible I is

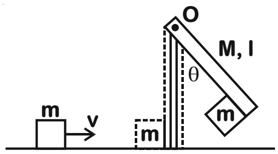
[JEE (Main)-2020]

(1) $\sqrt{\frac{2}{3}}$ (2) $\frac{2}{3}$

(3) $\frac{3}{2}$ (4) $\sqrt{\frac{3}{2}}$

62. A block of mass m = 1 kg slides with velocity v = 6 m/s on a frictionless horizontal surface and collides with a uniform vertical rod and sticks to it as shown. The rod is pivoted about O and swings as a result of the collision making angle θ before momentarily coming to rest. If the rod has mass M = 2 kg and length l = 1 m, the value of θ is approximately (take g = 10 m/s^2)

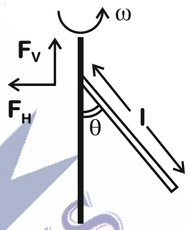
[JEE (Main)-2020]



(1) 49° (2) 55°

(3) 63° (4) 69°

63. A uniform rod of length 'l' is pivoted at one of its ends on a vertical shaft of negligible radius. When the shaft rotates at angular speed ω the rod makes an angle θ with it (see figure). To find θ equate the rate of change of angular momentum (direction going into the paper) $\frac{ml^2}{12}\omega^2\sin\theta\cos\theta$ about the centre of mass (CM) to the torque provided by the horizontal and vertical forces F_H and F_V about the CM. The value of θ is then such that

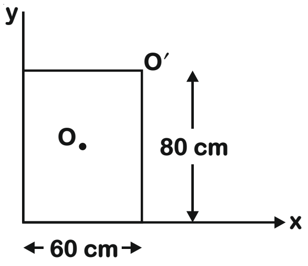


[JEE (Main)-2020]

(1) $\cos\theta = \frac{g}{l\omega^2}$ (2) $\cos\theta = \frac{2g}{3l\omega^2}$

(3) $\cos\theta = \frac{g}{2l\omega^2}$ (4) $\cos\theta = \frac{3g}{2l\omega^2}$

64. For a uniform rectangular sheet shown in the figure, the ratio of moments of inertia about the axes perpendicular to the sheet and passing through O (the centre of mass) and O' (corner point) is



[JEE (Main)-2020]

(1) 1/2 (2) 1/4

(3) 1/8 (4) 2/3

65. A circular coil has moment of inertia 0.8 kg m^2 around any diameter and is carrying current to produce a magnetic moment of 20 Am^2. The coil is kept initially in a vertical position and it can rotate freely around a horizontal diameter. When a uniform magnetic field of 4 T is applied along the

vertical, it starts rotating, around its horizontal diameter. The angular speed the coil acquires after rotating by 60° will be **[JEE (Main)-2020]**

(1) 10 rad s^{-1} (2) 10π rad s^{-1}

(3) 20 rad s^{-1} (4) 20π rad s^{-1}

66. Consider two uniform discs of the same thickness and different radii $R_1 = R$ and $R_2 = \alpha R$ made of the same material. If the ratio of their moments of inertia I_1 and I_2, respectively, about their axes is $I_1 : I_2 = 1 : 16$ then the value of α is **[JEE (Main)-2020]**

(1) 2 (2) 4

(3) $2\sqrt{2}$ (4) $\sqrt{2}$

67. A wheel is rotating freely with an angular speed ω on a shaft. The moment of inertia of the wheel is I and the moment of inertia of the shaft is negligible. Another wheel of moment of inertia $3I$ initially at rest is suddenly coupled to the same shaft. The resultant fractional loss in the kinetic energy of the system is **[JEE (Main)-2020]**

(1) $\frac{5}{6}$ (2) $\frac{1}{4}$

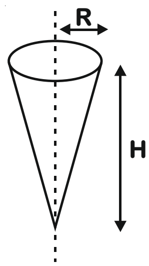
(3) 0 (4) $\frac{3}{4}$

68. A spaceship in space sweeps stationary interplanetary dust. As a result, its mass increases at a rate $\frac{dM(t)}{dt} = bv^2(t)$, where $v(t)$ is its instantaneous velocity. The instantaneous acceleration of the satellite is **[JEE (Main)-2020]**

(1) $-bv^3(t)$ (2) $-\frac{2bv^3}{M(t)}$

(3) $-\frac{bv^3}{M(t)}$ (4) $-\frac{bv^3}{2M(t)}$

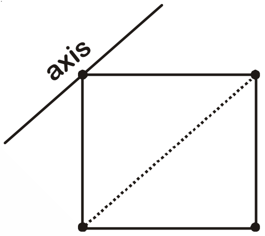
69. Shown in the figure is a hollow icecream cone (it open at the top). If its mass is M, radius of its top, R and height, H, then its moment of inertia about its axis is **[JEE (Main)-2020]**



(1) $\frac{M(R^2 + H^2)}{4}$ (2) $\frac{MR^2}{2}$

(3) $\frac{MR^2}{3}$ (4) $\frac{MH^2}{3}$

70. Four point masses, each of mass m, are fixed at the corners of a square of side ℓ. The square is rotating with angular frequency ω, about an axis passing through one of the corners of the square and parallel to its diagonal, as shown in the figure. The angular momentum of the square about this axis is **[JEE (Main)-2020]**



(1) $3\ m\ell^2\omega$ (2) $4\ m\ell^2\omega$

(3) $m\ell^2\omega$ (4) $2\ m\ell^2\omega$

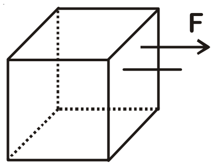
71. The linear mass density of a thin rod AB of length L varies from A to B as

$\lambda(x) = \lambda_0\left(1+\frac{x}{L}\right)$, where x is the distance from A.

If M is the mass of the rod then its moment of inertia about an axis passing through A and perpendicular to the rod is **[JEE (Main)-2020]**

(1) $\frac{2}{5}ML^2$ (2) $\frac{5}{12}ML^2$

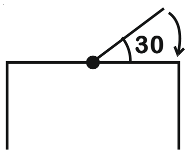
(3) $\frac{3}{7}ML^2$ (4) $\frac{7}{18}ML^2$

72. 

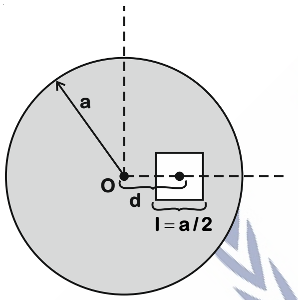
Consider a uniform cubical box of side a on a rough floor that is to be moved by applying minimum possible force F at a point b above its centre of mass (see figure). If the coefficient of friction is μ = 0.4, the maximum possible value of $100\times\frac{b}{a}$ for box not to topple before moving is ________.

[JEE (Main)-2020]

73. One end of a straight uniform 1 m long bar is pivoted on horizontal table. It is released from rest when it makes an angle 30° from the horizontal (see figure). Its angular speed when it hits the table is given as $\sqrt{n}\ s^{-1}$, where n is an integer. The value of n is ______ **[JEE (Main)-2020]**

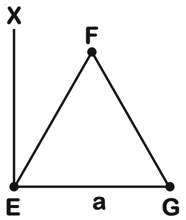


74. A square shaped hole of side $l = \frac{a}{2}$ is carved out at a distance $d = \frac{a}{2}$ from the centre 'O' of a uniform circular disk of radius a. If the distance of the centre of mass of the remaining portion from O is $-\frac{a}{X}$, value of X (to the nearest integer) is _________. **[JEE (Main)-2020]**

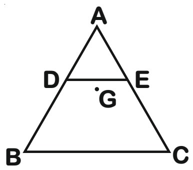


75. A person of 80 kg mass is standing on the rim of a circular platform of mass 200 kg rotating about its axis at 5 revolutions per minute (rpm). The person now starts moving towards the centre of the platform. What will be the rotational speed (in rpm) of the platform when the person reaches its centre _________. **[JEE (Main)-2020]**

76. An massless equilateral triangle EFG of side 'a' (As shown in figure) has three particles of mass m situated at its vertices. The moment of inertia of the system about the line EX perpendicular to EG in the plane of EFG is $\frac{N}{20}ma^2$ where N is an integer. The value of N is _________. **[JEE (Main)-2020]**



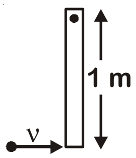
77. ABC is a plane lamina of the shape of an equilateral triangle. D, E are mid points of AB, AC and G is the centroid of the lamina. Moment of inertia of the lamina about an axis passing through G and perpendicular to the plane ABC is I_0. If part ADE is removed, the moment of inertia of the remaining part about the same axis is $\frac{NI_0}{16}$ where N is an integer. Value of N is ___________. **[JEE (Main)-2020]**



78. A circular disc of mass M and radius R is rotating about its axis with angular speed ω_1. If another stationary disc having radius $\frac{R}{2}$ and same mass M is dropped co-axially on to the rotating disc. Gradually both discs attain constant angular speed ω_2. The energy lost in the process is p% of the initial energy. Value of p is _____. **[JEE (Main)-2020]**

79. A force $\vec{F} = (\hat{i} + 2\hat{j} + 3\hat{k})$ N acts at a point $(4\hat{i} + 3\hat{j} - \hat{k})$ m. Then the magnitude of torque about the point $(\hat{i} + 2\hat{j} + \hat{k})$ m will be $\sqrt{x}$ N-m. The value of x is ____ . **[JEE (Main)-2020]**

80. A thin rod of mass 0.9 kg and length 1 m is suspended, at rest, from one end so that it can freely oscillate in the vertical plane. A particle of move 0.1 kg moving in a straight line with velocity 80 m/s hits the rod at its bottom most point and sticks to it (see figure). The angular speed (in rad/s) of the rod immediately after the collision will be ____ . **[JEE (Main)-2020]**



81. The centre of mass of a solid hemisphere of radius 8 cm is x cm from the centre of the flat surface. Then value of x is _________. **[JEE (Main)-2020]**

❑ ❑ ❑