

# 6

## Solving (simple) Problems (Based On Quadratic Equations)

### 6.1 Introduction :

In this chapter, we shall be dealing with word problems based on quadratic equations. (The method of solving a quadratic equation has already been done in previous classes and in the previous chapter).

**For solving a word problem based on Quadratic Equation adopt the following steps :**

1. Represent the unknown quantity of the problem by variable 'x'.
2. Translate the given statement to form an equation in terms of 'x'.
3. Solve the equation.

### 6.2 Problems Based on Numbers :

- 1** Find two natural numbers which differ by 3 and the sum of whose squares is 117.

**Solution :**

Let the natural numbers be  $x$  and  $x + 3$ .

$$\therefore x^2 + (x + 3)^2 = 117$$

$$x^2 + x^2 + 6x + 9 = 117$$

$$\Rightarrow 2x^2 + 6x - 108 = 0$$

$$\Rightarrow x^2 + 3x - 54 = 0$$

[Dividing each term by 2]

$$\Rightarrow (x + 9)(x - 6) = 0$$

[Factorising]

$$\Rightarrow x + 9 = 0, \text{ or } x - 6 = 0$$

$$\Rightarrow x = -9, \text{ or } x = 6$$

$$\therefore \text{One number} = 6$$

[Since,  $-9$  is not a natural number]

$$\text{and other number} = 6 + 3 = 9.$$

$\therefore$  Numbers are 6 and 9

**Ans.**

- 2** Five times a certain whole number is equal to three less than twice the square of the number. Find the number.

**Solution :**

Let the number be  $x$ .

Given, five times the number = 3 less than twice the square of the number.

$$\therefore 5x = 2x^2 - 3$$

$$\Rightarrow 2x^2 - 5x - 3 = 0$$

$$\Rightarrow (x - 3)(2x + 1) = 0$$

[Factorising]

$$\Rightarrow x - 3 = 0, \text{ or } 2x + 1 = 0$$

$$\Rightarrow x = 3, \text{ or } x = -\frac{1}{2} \quad \therefore \text{Required whole number is 3} \quad \text{Ans.}$$

- 3** Sum of two natural numbers is 8 and the difference of their reciprocals is  $\frac{2}{15}$ .  
Find the numbers. [2015]

**Solution :**

Let the natural numbers be  $x$  and  $8 - x$

$$\Rightarrow \frac{1}{x} - \frac{1}{8-x} = \frac{2}{15} \quad \text{i.e.} \quad \frac{8-x-x}{x(8-x)} = \frac{2}{15}$$

$$\Rightarrow 2(8x - x^2) = 15(8 - 2x) \quad \text{i.e.} \quad 16x - 2x^2 = 120 - 30x$$

$$\Rightarrow 2x^2 - 46x + 120 = 0 \quad \text{i.e.} \quad x^2 - 23x + 60 = 0$$

$$\Rightarrow x^2 - 20x - 3x + 60 = 0 \quad \text{i.e.} \quad x(x - 20) - 3(x - 20) = 0$$

$$\Rightarrow (x - 20)(x - 3) = 0 \quad \text{i.e.} \quad x - 20 = 0 \text{ or } x - 3 = 0$$

$$\Rightarrow x = 20 \text{ or } x = 3$$

Reject  $x = 20$  as the sum of natural numbers is 8.

$$\therefore x = 3 \text{ and } 8 - x = 8 - 3 = 5$$

$\Rightarrow$  Required natural numbers are 3 and 5. Ans.

### 6.3 Problems Based on Time and Work :

- 4** For the same amount of work, A takes 6 hours less than B. If together they complete the work in 13 hours 20 minutes; find how much time will B alone take to complete the work ?

**Solution :**

If B alone takes  $x$  hours then A alone takes  $(x - 6)$  hours for the same work.

$$\therefore \frac{1}{x-6} + \frac{1}{x} = \frac{3}{40} \quad [\because 13 \text{ hrs. } 20 \text{ min.} = \left(13 + \frac{20}{60}\right) \text{ hrs.} = \frac{40}{3} \text{ hrs.}]$$

$$\Rightarrow \frac{x+x-6}{(x-6)x} = \frac{3}{40}$$

$$\Rightarrow 3x^2 - 18x = 80x - 240 \quad \text{i.e.} \quad 3x^2 - 98x + 240 = 0$$

$$\Rightarrow 3x^2 - 90x - 8x + 240 = 0 \quad \text{i.e.} \quad (x - 30)(3x - 8) = 0$$

$$\Rightarrow x = 30, \text{ or } x = \frac{8}{3} \quad \text{i.e.} \quad x = 30$$

$\therefore$  B alone will take 30 hrs. to complete the work. Ans.



- The product of two consecutive integers is 56. Find the integers.
- The sum of the squares of two consecutive natural numbers is 41. Find the numbers.
- Find the two natural numbers which differ by 5 and the sum of whose squares is 97.
- The sum of a number and its reciprocal is 4.25. Find the number.
- Two natural numbers differ by 3. Find the numbers, if the sum of their reciprocals is  $\frac{7}{10}$ .
- Divide 15 into two parts such that the sum of their reciprocals is  $\frac{3}{10}$ .
- The sum of the squares of two positive integers is 208. If the square of the larger number is 18 times the smaller number, find the numbers.
- The sum of the squares of two consecutive positive even numbers is 52. Find the numbers.
- Find two consecutive positive odd numbers, the sum of whose squares is 74.
- The denominator of a positive fraction is one more than twice the numerator. If the sum of the fraction and its reciprocal is 2.9; find the fraction.
- Three positive numbers are in the ratio  $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$ . Find the numbers if the sum of their squares is 244.
- Divide 20 into two parts such that three times the square of one part exceeds the other part by 10.
- Three consecutive natural numbers are such that the square of the middle number exceeds the difference of the squares of the other two by 60.  
Assume the middle number to be  $x$  and form a quadratic equation satisfying the above statement. Hence; find the three numbers.
- Out of three consecutive positive integers, the middle number is  $p$ . If three times the square of the largest is greater than the sum of the squares of the other two numbers by 67; calculate the value of  $p$ .
- A can do a piece of work in ' $x$ ' days and B can do the same work in  $(x + 16)$  days. If both working together can do it in 15 days; calculate ' $x$ '.
- One pipe can fill a cistern in 3 hours less than the other. The two pipes together can fill the cistern in 6 hours 40 minutes. Find the time that each pipe will take to fill the cistern.
- A positive number is divided into two parts such that the sum of the squares of the two parts is 20. The square of the larger part is 8 times the smaller part. Taking  $x$  as the smaller part of the two parts, find the number.

[2010]

## 6.4 Problems Based on Geometrical Figures :

- 5** The hypotenuse of a right triangle is 13 cm and the difference between the other two sides is 7 cm.

Taking ' $x$ ' as the length of the shorter of the two sides, write an equation in ' $x$ ' that represents the above statement and also solve the equation to find the two unknown sides of the triangle.

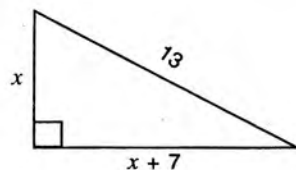
**Solution :**

Since, the shorter side =  $x$  cm.

$\therefore$  Longer side =  $(x + 7)$  cm.

Using Pythagoras Theorem, we get :

$$x^2 + (x + 7)^2 = 13^2$$



$$\Rightarrow x^2 + x^2 + 14x + 49 = 169$$

$$\Rightarrow 2x^2 + 14x - 120 = 0$$

$$\Rightarrow x^2 + 7x - 60 = 0$$

[Dividing each term by 2]

On solving, it gives

$$x = -12, \text{ or } x = 5$$

Since, the side of triangle cannot be negative, therefore,  $x = 5$ .

$$\therefore \text{One side of the triangle} = x \text{ cm} = 5 \text{ cm.}$$

$$\text{and other side of the triangle} = (x + 7) \text{ cm} = (5 + 7) \text{ cm} = 12 \text{ cm.}$$

Ans.

**6** The length of a verandah is 3 m more than its breadth. The numerical value of its area is equal to the numerical value of its perimeter.

- Taking 'x' as the breadth of the verandah, write an equation in 'x' that represents the above statement.
- Solve the equation obtained in (i) above and hence find the dimensions of the verandah.

**Solution :**

Since breadth =  $x$  m  $\therefore$  Length =  $(x + 3)$  m

(i) Given : Area of verandah = its perimeter

[Numerically]

$$\text{i.e. length} \times \text{breadth} = 2 (\text{length} + \text{breadth})$$

$$\Rightarrow (x + 3) \cdot x = 2(x + 3 + x)$$

$$\Rightarrow x^2 + 3x = 4x + 6$$

$$\Rightarrow x^2 - x - 6 = 0$$

Ans.

$$(ii) \quad x^2 - x - 6 = 0$$

$$\Rightarrow (x - 3)(x + 2) = 0$$

[On factorising]

$$\Rightarrow x = 3, \text{ or } x = -2$$

Since, the breadth cannot be negative,  $\therefore x = 3$

$$\text{Hence, blue length of verandah} = (x + 3) \text{ m} = (3 + 3) \text{ m} = 6 \text{ m}$$

$$\text{and, its breadth} = x \text{ m} = 3 \text{ m}$$

Ans.

### EXERCISE 6(B)

- The sides of a right-angled triangle containing the right angle are  $4x$  cm and  $(2x - 1)$  cm. If the area of the triangle is  $30 \text{ cm}^2$ ; calculate the lengths of its sides.
- The hypotenuse of a right-angled triangle is 26 cm and the sum of other two sides is 34 cm. Find the lengths of its sides.
- The sides of a right-angled triangle are  $(x - 1)$  cm,  $3x$  cm and  $(3x + 1)$  cm. Find :
  - the value of  $x$ ,

- the lengths of its sides,
- its area.

- The hypotenuse of a right-angled triangle exceeds one side by 1 cm and the other side by 18 cm; find the lengths of the sides of the triangle.
- The diagonal of a rectangle is 60 m more than its shorter side and the larger side is 30 m more than the shorter side. Find the sides of the rectangle.



6. The perimeter of a rectangle is 104 m and its area is  $640 \text{ m}^2$ . Find its length and breadth.
7. A footpath of uniform width runs round the inside of a rectangular field 32 m long and 24 m wide. If the path occupies  $208 \text{ m}^2$ , find the width of the footpath.
8. Two squares have sides  $x \text{ cm}$  and  $(x + 4) \text{ cm}$ . The sum of their areas is  $656 \text{ sq. cm}$ . Express this as an algebraic equation in  $x$  and solve the equation to find the sides of the squares.
9. The dimensions of a rectangular field are 50 m by 40 m. A flower bed is prepared inside this field leaving a gravel path of uniform width all around the flower bed. The total cost of laying the flower bed and gravelling the path at ₹ 30 and ₹ 20 per square metre, respectively, is ₹ 52,000. Find the width of the gravel path.
10. An area is paved with square tiles of a certain size and the number required is 128. If the tiles had been 2 cm smaller each way, 200 tiles would have been needed to pave the same area. Find the size of the larger tiles.
11. A farmer has 70 m of fencing, with which he encloses three sides of a rectangular sheep pen; the fourth side being a wall. If the area of the pen is  $600 \text{ sq. m}$ , find the length of its shorter side.
12. A square lawn is bounded on three sides by a path 4 m wide. If the area of the path is  $\frac{7}{8}$  that of the lawn, find the dimensions of the lawn.
13. The area of a big rectangular room is  $300 \text{ m}^2$ . If the length were decreased by 5 m and the breadth increased by 5 m; the area would be unaltered. Find the length of the room.

### 6.5 Problems Based on Distance, Speed and Time :

- 7** By increasing the speed of a car by 10 km/hr, the time of journey for a distance of 72 km is reduced by 36 minutes. Find the original speed of the car. [2005]

**Solution :**

Let the original speed of the car =  $x \text{ km/hr}$

$$\therefore \text{Time taken by it to cover 72 km} = \frac{72}{x} \text{ hrs} \quad \left[ \text{Time} = \frac{\text{Distance}}{\text{Speed}} \right]$$

New speed of the car =  $(x + 10) \text{ km/hr}$

$$\therefore \text{New time taken by the car to cover 72 km} = \frac{72}{x+10} \text{ hrs}$$

**Given :** Time is reduced by 36 minutes :

$$\Rightarrow \frac{72}{x} - \frac{72}{x+10} = \frac{36}{60}$$

On solving, we get :  $x = -40$  or  $x = 30$

Since, speed cannot be negative hence the value of  $x = 30$

i.e.

**The original speed of the car = 30 km/hr.**

**Ans.**

- 8** Car A travels  $x \text{ km}$  for every litre of petrol, while car B travels  $(x + 5) \text{ km}$  for every litre of petrol.

- (i) Write down the number of litres of petrol used by car A and car B in covering a distance of 400 km.
- (ii) If car A uses 4 litres of petrol more than car B in covering the 400 km, write down an equation in  $x$  and solve it to determine the number of litres of petrol used by car B for the journey.

**Solution :**

(i) No. of litres of petrol used by car A =  $\frac{400}{x}$  litre Ans.

No. of litres of petrol used by car B =  $\frac{400}{x+5}$  litre Ans.

(ii) Given :  $\frac{400}{x} - \frac{400}{x+5} = 4$  i.e.  $\frac{400x + 2000 - 400x}{x(x+5)} = 4$   
 $\Rightarrow 4(x^2 + 5x) = 2000$  i.e.  $x^2 + 5x - 500 = 0$   
 $\Rightarrow x = -25, \text{ or } x = 20$  [On solving]  
 $\Rightarrow x = 20$  [ $\because$  Distance cannot be negative]

$\therefore$  No. of litres of petrol used by car B =  $\frac{400}{x+5}$  litres  
 $= \frac{400}{20+5}$  litres = **16 litres** Ans.

## 6.6 Problems on C.P. and S.P. :

- 9** By selling an article for ₹ 24, a trader loses as much percent as the cost price of the article. Calculate the cost price.

**Solution :**

Let C.P. of the article be ₹  $x$ .

$$\therefore \text{Loss} = x\% \text{ of C.P.} = \frac{x}{100} \times ₹ x = ₹ \frac{x^2}{100}$$

$$\therefore x - \frac{x^2}{100} = 24 \quad [\text{C.P.} - \text{Loss} = \text{S.P.}]$$

$$\Rightarrow 100x - x^2 = 2400 \quad \text{i.e. } x^2 - 100x + 2400 = 0$$

On solving, we get :  $x = 60$  and  $x = 40$

$\therefore$  C.P. of the article is ₹ 60 or ₹ 40. Ans.

### EXERCISE 6(C)

- The speed of an ordinary train is  $x$  km per hr and that of an express train is  $(x + 25)$  km per hr.
  - Find the time taken by each train to cover 300 km.
  - If the ordinary train takes 2 hrs more than the express train; calculate speed of the express train.
- If the speed of a car is increased by 10 km per hr, it takes 18 minutes less to cover a distance of 36 km. Find the speed of the car.
- If the speed of an aeroplane is reduced by 40 km per hr, it takes 20 minutes more to cover 1200 km. Find the speed of the aeroplane.
- A car covers a distance of 400 km at a certain speed. Had the speed been 12 km/h more, the time taken for the journey would have been 1 hour 40 minutes less. Find the original speed of the car. [2012]
- A girl goes to her friend's house, which is at a distance of 12 km. She covers half of the distance at a speed of  $x$  km/hr and



the remaining distance at a speed of  $(x + 2)$  km/hr. If she takes 2 hrs 30 minutes to cover the whole distance, find 'x'.

6. A car made a run of 390 km in 'x' hours. If the speed had been 4 km/hour more, it would have taken 2 hours less for the journey. Find 'x'.
7. A goods train leaves a station at 6 p.m., followed by an express train which leaves at 8 p.m. and travels 20 km/hour faster than the goods train. The express train arrives at a station, 1040 km away, 36 minutes before the goods train. Assuming that the speeds of both

the trains remain constant between the two stations; calculate their speeds.

8. A man bought an article for ₹  $x$  and sold it for ₹ 16. If his loss was  $x$  percent, find the cost price of the article.
9. A trader bought an article for ₹  $x$  and sold it for ₹ 52, thereby making a profit of  $(x - 10)$  percent on his outlay. Calculate the cost price.
10. By selling a chair for ₹ 75, Mohan gained as much percent as its cost. Calculate the cost of the chair.

## 6.7 Miscellaneous Problems :

- 10** The sum  $S$  of first  $n$  natural numbers is given by the relation :  $S = \frac{1}{2} n (n + 1)$ . Find  $n$ , if the sum is 276.

**Solution :**

$$\text{Given : } S = 276$$

$$\Rightarrow \frac{1}{2} n (n + 1) = 276 \quad \text{i.e.} \quad n^2 + n - 552 = 0$$

$$\Rightarrow (n + 24) (n - 23) = 0 \quad [\text{On factorising}]$$

$$\Rightarrow n = -24, \text{ or } n = 23 \quad [\text{Zero product rule}]$$

Since,  $n$  is a natural number, reject  $n = -24$ .

$$\therefore n = 23$$

**Ans.**

- 11** A two-digit number is such that the product of its digits is 6. When 9 is added to this number; the digits interchange their places. Find the number. [2014]

**Solution :**

Let the required two digit number be  $10x + y$

$$\text{Given : } xy = 6 \text{ and } 10x + y + 9 = 10y + x$$

$$10x + y + 9 = 10y + x \Rightarrow 9y = 9x + 9 \text{ i.e. } y = x + 1$$

$$\text{Now, } xy = 6 \Rightarrow x(x + 1) = 6$$

$$\Rightarrow x^2 + x - 6 = 0$$

$$\Rightarrow x = -3, \text{ or } x = 2 \quad [\text{On solving}]$$

Since,  $-3$  is not a digit,  $x = 2 \Rightarrow y = x + 1 = 2 + 1 = 3$

$$\therefore \text{The required two digit number} = 10x + y$$

$$= 10 \times 2 + 3 = 23 \quad \text{Ans.}$$

- 12** Five years ago, a woman's age was the square of her son's age. Ten years hence her age will be twice that of her son's age. Find :

- (i) the age of the son five years ago.  
(ii) the present age of the woman.

[2007]

**Solution :**

Let the age of the son 5 years ago =  $x$  years.

$\therefore$  The woman's age 5 years ago =  $x^2$  years.

$\Rightarrow$  The present age of the woman =  $(x^2 + 5)$  years

and, the present age of her son =  $(x + 5)$  years

**10 years hence :**

The woman's age will be =  $(x^2 + 5) + 10$  years =  $(x^2 + 15)$  years

and, her son's age will be =  $(x + 5) + 10$  years =  $(x + 15)$  years.

**According to the given statement :**

10 years hence, woman's age = twice her son's age

$\Rightarrow x^2 + 15 = 2(x + 15)$

$\Rightarrow x^2 - 2x - 15 = 0$

On solving, we get :  $x = 5$  or  $x = -3$

Since, age cannot be negative, reject  $x = -3$ .

$\therefore x = 5$

(i) **The age of the son 5 years ago** =  $x$  years = **5 years** **Ans.**

(ii) **The present age of the woman** =  $(x^2 + 5)$  years =  $(5^2 + 5)$  years  
= **30 years** **Ans.**

- 13** A motor-boat, whose speed is 9 km/h in still water, goes 12 km downstream and comes back in a total time of 3 hours. Find the speed of the stream.

**Solution :**

Let the speed of the stream =  $x$  km/hr

$\therefore$  Speed of the boat downstream =  $(9 + x)$  km/hr

and speed of the boat upstream =  $(9 - x)$  km/hr

Also, time taken to go 12 km downstream =  $\frac{12}{9+x}$  hr  $\left[ \because \text{Time} = \frac{\text{Distance}}{\text{Speed}} \right]$

And, time taken to come back =  $\frac{12}{9-x}$  hr

Given :  $\frac{12}{9+x} + \frac{12}{9-x} = 3$  i.e.  $\frac{108 - 12x + 108 + 12x}{(9+x)(9-x)} = 3$

$\Rightarrow 3(81 - x^2) = 216$

$\Rightarrow 81 - x^2 = 72$  and  $x^2 = 9$  i.e.  $x = \pm 3$

Since, speed can not be negative (i.e.  $x \neq -3$ ) therefore  $x = 3$

Hence, **the speed of the stream** = **3 km/hr**

**Ans.**



- 14** A piece of cloth costs ₹ 200. If the piece was 5 m longer and each metre of cloth costs ₹ 2 less; the cost of the piece would have remained unchanged. How long is the piece and what is the original rate per metre ?

**Solution :**

Let the length of the piece be  $x$  metre

Since, the cost of  $x$  metre cloth = ₹ 200

$$\Rightarrow \text{Cost of each metre of cloth} = ₹ \frac{200}{x}$$

New length of cloth =  $(x + 5)$  m

$$\Rightarrow \text{New cost of each metre of cloth} = ₹ \frac{200}{x+5}$$

$$\text{Given : } \frac{200}{x} - \frac{200}{x+5} = 2 \text{ i.e. } \frac{200x + 1000 - 200x}{x(x+5)} = 2$$

$$\text{i.e. } 2(x^2 + 5x) = 1000$$

$$\Rightarrow x^2 + 5x = 500$$

$$\text{i.e. } x^2 + 5x - 500 = 0$$

$$\Rightarrow x = -25, \text{ or } x = 20 \quad [\text{On solving}]$$

$$\therefore x = 20 \Rightarrow \text{The length of the piece} = 20 \text{ m}$$

$$\text{And, the original rate per metre} = ₹ \frac{200}{20} = ₹ 10 \quad \text{Ans.}$$

**Alternative method :**

$$\text{Let the length} = x \text{ m} \Rightarrow \text{Original cost per m} = ₹ \frac{200}{x}$$

$$\text{Given : If length} = (x + 5) \text{ m, cost per m} = ₹ \left( \frac{200}{x} - 2 \right) \text{ and total cost} = ₹ 200$$

$$\therefore (x + 5) \left( \frac{200}{x} - 2 \right) = 200 \quad [\text{Length} \times \text{Rate} = \text{Cost}]$$

$$\Rightarrow (x + 5) \left( \frac{200 - 2x}{x} \right) = 200$$

$$\Rightarrow 200x - 2x^2 + 1000 - 10x = 200x$$

$$\Rightarrow x^2 + 5x - 500 = 0$$

$$\Rightarrow x = -25 \text{ or } 20.$$

$$\therefore \text{The length of the cloth} = 20 \text{ m and original rate per m} = ₹ \frac{200}{20} = ₹ 10 \quad \text{Ans.}$$

- 15** A shopkeeper buys a certain number of books for ₹ 960. If the cost per book was ₹ 8 less, the number of books that could be bought for ₹ 960 would be 4 more. Taking the original cost of each book to be ₹  $x$ , write an equation in  $x$  and solve it. [2013]

**Solution :**

$$\therefore \text{Original cost of each book} = ₹ x$$

$$\therefore \text{No. of books bought for ₹ 960} = \frac{960}{x}$$

**In 2nd case :**

$$\text{The cost of each book} = ₹ (x - 8)$$

$$\therefore \text{No. of books bought for ₹ 960} = \frac{960}{x-8}$$

$$\text{Given : } \frac{960}{x-8} - \frac{960}{x} = 4 \quad \text{i.e.} \quad \frac{960x - 960x + 7680}{x(x-8)} = 4$$

$$\Rightarrow 7680 = 4(x^2 - 8x)$$

$$\Rightarrow x^2 - 8x = 1920$$

$$\Rightarrow x^2 - 8x - 1920 = 0$$

**Ans.**

$$\text{Now } x^2 - 8x - 1920 = 0$$

$$\Rightarrow x^2 - 48x + 40x - 1920 = 0$$

$$\Rightarrow x(x - 48) + 40(x - 48) = 0$$

$$\Rightarrow (x - 48)(x + 40) = 0$$

$$\Rightarrow x = 48 \text{ or } x = -40$$

$$\text{Rejecting } x = -40, \text{ we get : } x = 48$$

**Ans.**

**Alternative method :**

$$\therefore \text{Cost of each book} = ₹ x$$

$$\Rightarrow \text{No. of books bought for ₹ 960} = \frac{960}{x}$$

**In 2nd case :**

$$\text{Cost of each book} = ₹ (x - 8)$$

$$\text{and, no. of books bought} = \left( \frac{960}{x} + 4 \right)$$

$$\therefore \text{Cost of each book} \times \text{no. of books} = \text{Cost of all the books}$$

$$\Rightarrow (x - 8) \left( \frac{960}{x} + 4 \right) = 960$$

$$\Rightarrow (x - 8) \left( \frac{960 + 4x}{x} \right) = 960$$

$$\Rightarrow 960x + 4x^2 - 7680 - 32x = 960x$$

$$\Rightarrow 4x^2 - 32x - 7680 = 0$$

$$\Rightarrow x^2 - 8x - 1920 = 0$$

$$\Rightarrow (x - 48)(x + 40) = 0$$

$$\Rightarrow x = 48 \text{ or } x = -40$$

$$\Rightarrow x = 48$$

**Ans.**

- 16** Some students planned a picnic. The budget for the food was ₹ 480. As eight of them failed to join the party, the cost of the food for each member increased by ₹ 10. Find, how many students went for the picnic ? [2008]

**Solution :**

Let the number of students who planned the picnic =  $x$



Since, the budget for food for all = ₹ 480

$$\therefore \text{Share of each in it} = ₹ \frac{480}{x}$$

Given eight of them failed to join the party

$$\Rightarrow (x - 8) \text{ students went for the picnic}$$

So, the share of each will be = ₹  $\frac{480}{x-8}$

Since, the cost of food for each member (student) is increased by ₹ 10

$$\therefore \frac{480}{x-8} - \frac{480}{x} = 10 \quad \text{i.e.} \quad \frac{480x - 480x + 3840}{x(x-8)} = 10$$

$$\Rightarrow 3840 = 10(x^2 - 8x) \quad \text{i.e.} \quad x^2 - 8x - 384 = 0$$

On solving, it gives  $x = 24$  and  $x = -16$

But the number of students cannot be negative,  $\therefore x = 24$

$\therefore$  The number of students who went for picnic =  $x - 8 = 24 - 8 = 16$  Ans.

**Alternative method :**

Let  $x$  students went for picnic

$$\Rightarrow \text{Each student paid} = ₹ \frac{480}{x}$$

Clearly, no. of students who planned for picnic =  $x + 8$

$$\Rightarrow \text{Each was to pay} = ₹ \frac{480}{x+8}$$

$$\text{Given : } \frac{480}{x} - \frac{480}{x+8} = 10 \quad \text{i.e.} \quad \frac{480x + 3840 - 480x}{x(x+8)} = 10$$

$$\Rightarrow 3840 = 10(x^2 + 8x) \text{ and } x^2 + 8x - 384 = 0$$

On solving it gives  $x = -24$  and  $x = 16$

Since, the number of students cannot be negative,  $\therefore x = 16$

$\Rightarrow$  The number of students who went for picnic =  $x = 16$

Ans.

### EXERCISE 6(D)

1. The sum  $S$  of  $n$  successive odd numbers starting from 3 is given by the relation :  $S = n(n + 2)$ . Determine  $n$ , if the sum is 168.
2. A stone is thrown vertically downwards and the formula  $d = 16t^2 + 4t$  gives the distance,  $d$  metres, that it falls in  $t$  seconds. How long does it take to fall 420 metres ?
3. The product of the digits of a two digit number is 24. If its unit's digit exceeds twice its ten's digit by 2; find the number.
4. The ages of two sisters are 11 years and 14 years. In how many years time will the product of their ages be 304 ?
5. One year ago, a man was 8 times as old as his son. Now his age is equal to the square of his son's age. Find their present ages.
6. The age of a father is twice the square of the age of his son. Eight years hence, the age of the father will be 4 years more than three times the age of the son. Find their present ages.
7. The speed of a boat in still water is 15 km/hr. It can go 30 km upstream and return downstream to the original point in 4 hours 30 minutes. Find the speed of the stream.
8. Mr. Mehra sends his servant to the market to buy oranges worth ₹. 15. The servant having

eaten three oranges on the way, Mr. Mehra pays 25 paise per orange more than the market price.

Taking  $x$  to be the number of oranges which Mr. Mehra receives, form a quadratic equation in  $x$ . Hence, find the value of  $x$ .

9. ₹ 250 is divided equally among a certain number of children. If there were 25 children more, each would have received 50 paise less. Find the number of children.
10. An employer finds that if he increases the weekly wages of each worker by ₹ 5 and employs five workers less, he increases his weekly wage bill from ₹ 3,150 to ₹ 3,250. Taking the original weekly wage of each worker as ₹  $x$ ; obtain an

equation in  $x$  and then solve it to find the weekly wages of each worker.

11. A trader bought a number of articles for ₹ 1,200. Ten were damaged and he sold each of the remaining articles at ₹ 2 more than what he paid for it, thus getting a profit of ₹ 60 on the whole transaction ?  
Taking the number of articles he bought as  $x$ , form an equation in  $x$  and solve it.
12. The total cost price of a certain number of identical articles is ₹ 4,800. By selling the articles at ₹ 100 each, a profit equal to the cost price of 15 articles is made. Find the number of articles bought.

### EXERCISE 6(E)

1. The distance by road between two towns A and B is 216 km, and by rail it is 208 km. A car travels at a speed of  $x$  km/hr and the train travels at a speed which is 16 km/hr faster than the car. Calculate :
  - (i) the time taken by the car to reach town B from A, in terms of  $x$ ;
  - (ii) the time taken by the train, to reach town B from A, in terms of  $x$ .
  - (iii) If the train takes 2 hours less than the car, to reach town B, obtain an equation in  $x$ , and solve it.
  - (iv) Hence, find the speed of the train.
2. A trader buys  $x$  articles for a total cost of ₹ 600.
  - (i) Write down the cost of one article in terms of  $x$ .  
If the cost per article were ₹ 5 more, the number of articles that can be bought for ₹ 600 would be four less.
  - (ii) Write down the equation in  $x$  for the above situation and solve it for  $x$ .
3. A hotel bill for a number of people for overnight stay is ₹ 4,800. If there were 4 people more, the bill each person had to pay, would have reduced by ₹ 200. Find the number of people staying overnight.
4. An aeroplane travelled a distance of 400 km at an average speed of  $x$  km/hr. On the return journey, the speed was increased by 40 km/hr. Write down an expression for the time taken for :

- (i) the onward journey;
- (ii) the return journey.

If the return journey took 30 minutes less than the onward journey, write down an equation in  $x$  and find its value. [2002]

5. ₹ 6,500 was divided equally among a certain number of persons. Had there been 15 persons more, each would have got ₹ 30 less. Find the original number of persons.
6. A plane left 30 minutes later than the scheduled time and in order to reach its destination 1500 km away in time, it has to increase its speed by 250 km/hr from its usual speed. Find its usual speed.
7. Two trains leave a railway station at the same time. The first train travels due west and the second train due north. The first train travels 5 km/hr faster than the second train. If after 2 hours, they are 50 km apart, find the speed of each train.
8. The sum  $S$  of first  $n$  even natural numbers is given by the relation  $S = n(n + 1)$ . Find  $n$ , if the sum is 420.
9. The sum of the ages of a father and his son is 45 years. Five years ago, the product of their ages (in years) was 124. Determine their present ages.
10. In an auditorium, seats were arranged in rows and columns. The number of rows was equal to the number of seats in each row. When the number of rows was doubled and the number of seats in each row was reduced by 10, the total number of seats increased by 300. Find :



- (i) the number of rows in the original arrangement.
- (ii) the number of seats in the auditorium after re-arrangement. [2003]
11. Mohan takes 16 days less than Manoj to do a piece of work. If both working together can do it in 15 days, in how many days will Mohan alone complete the work ?
12. Two years ago, a man's age was three times the square of his son's age. In three years time, his age will be four times his son's age. Find their present ages.
13. In a certain positive fraction, the denominator is greater than the numerator by 3. If 1 is subtracted from the numerator and the denominator both, the fraction reduces by  $\frac{1}{14}$ . Find the fraction.
14. In a two digit number, the ten's digit is bigger. The product of the digits is 27 and the difference between two digits is 6. Find the number.
15. Some school children went on an excursion by a bus to a picnic spot at a distance of 300 km. While returning, it was raining and the bus had to reduce its speed by 5 km/hr and it took two hours longer for returning. Find the time taken to return.
16. ₹ 480 is divided equally among 'x' children. If the number of children were 20 more then each would have got ₹ 12 less. Find 'x'. [2011]