

# Functions

1. Let  $f(x) = (x + 1)^2 - 1$ ,  $x \geq -1$ .

**Statement-1 :** The set  $\{x : f(x) = f^{-1}(x)\} = \{0, -1\}$ .

**Statement-2 :**  $f$  is a bijection. [AIEEE-2009]

- (1) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1
- (2) Statement-1 is true, Statement-2 is false
- (3) Statement-1 is false, Statement-2 is true
- (4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

2. For real  $x$ , let  $f(x) = x^3 + 5x + 1$ , then

[AIEEE-2009]

- (1)  $f$  is onto  $\mathbf{R}$  but not one-one
- (2)  $f$  is one-one and onto  $\mathbf{R}$
- (3)  $f$  is neither one-one nor onto  $\mathbf{R}$
- (4)  $f$  is one-one but not onto  $\mathbf{R}$

3. Let  $y$  be an implicit function of  $x$  defined by  $x^{2x} - 2x^x \cot y - 1 = 0$ . Then  $y'(1)$  equals

[AIEEE-2009]

- (1) 1
- (2)  $\log 2$
- (3)  $-\log 2$
- (4) -1

4. Let  $f$  be a function defined by

$$f(x) = (x - 1)^2 + 1, (x \geq 1).$$

**Statement - 1 :** The set  $\{x : f(x) = f^{-1}(x)\} = \{1, 2\}$ .

**Statement - 2 :**  $f$  is a bijection and  $f^{-1}(x) = 1 + \sqrt{x - 1}$ ,  $x \geq 1$ . [AIEEE-2011]

- (1) Statement-1 is true, Statement-2 is false
- (2) Statement-1 is false, Statement-2 is true
- (3) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation for Statement-1
- (4) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1

5. The equation  $e^{\sin x} - e^{-\sin x} - 4 = 0$  has

[AIEEE-2012]

- (1) No real roots

- (2) Exactly one real root

- (3) Exactly four real roots

- (4) Infinite number of real roots

6. If  $a \in \mathbf{R}$  and the equation

$$-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$$

(where  $[x]$  denotes the greatest integer  $\leq x$ ) has no integral solution, then all possible values of  $a$  lie in the interval

[JEE (Main)-2014]

- (1)  $(-2, -1)$

- (2)  $(-\infty, -2) \cup (2, \infty)$

- (3)  $(-1, 0) \cup (0, 1)$

- (4)  $(1, 2)$

7. If  $g$  is the inverse of a function  $f$  and

$$f'(x) = \frac{1}{1+x^5}, \text{ then } g'(x) \text{ is equal to}$$

[JEE (Main)-2014]

- (1)  $\frac{1}{1+\{g(x)\}^5}$
- (2)  $1 + \{g(x)\}^5$

- (3)  $1 + x^5$
- (4)  $5x^4$

8. If  $f(x) + 2f\left(\frac{1}{x}\right) = 3x$ ,  $x \neq 0$ , and

$$S = \{x \in \mathbf{R} : f(x) = f(-x)\}; \text{ then } S$$

[JEE (Main)-2016]

- (1) Contains exactly one element

- (2) Contains exactly two elements

- (3) Contains more than two elements

- (4) Is an empty set

9. The function  $f : \mathbf{R} \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$  defined as

$$f(x) = \frac{x}{1+x^2}, \text{ is}$$

[JEE (Main)-2017]

- (1) Injective but not surjective  
 (2) Surjective but not injective  
 (3) Neither injective nor surjective  
 (4) Invertible
10. For  $x \in R - \{0,1\}$ , let  $f_1(x) = \frac{1}{x}, f_2(x) = 1-x$  and  $f_3(x) = \frac{1}{1-x}$  be three given functions. If a function,  $J(x)$  satisfies  $(f_2 \circ J \circ f_1)(x) = f_3(x)$  then  $J(x)$  is equal to [JEE (Main)-2019]
- (1)  $f_1(x)$   
 (2)  $\frac{1}{x} f_3(x)$   
 (3)  $f_2(x)$   
 (4)  $f_3(x)$
11. Let  $A = \{x \in R : x \text{ is not a positive integer}\}$ . Define a function  $f : A \rightarrow R$  as  $f(x) = \frac{2x}{x-1}$ , then  $f$  is [JEE (Main)-2019]
- (1) Injective but not surjective  
 (2) Neither injective nor surjective  
 (3) Surjective but not injective  
 (4) Not injective
12. Let  $f : R \rightarrow R$  be a function such that  $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$ ,  $x \in R$ . Then  $f(2)$  equals [JEE (Main)-2019]
- (1) 8  
 (2) -4  
 (3) -2  
 (4) 30
13. Let  $N$  be the set of natural numbers and two functions  $f$  and  $g$  be defined as  $f, g : N \rightarrow N$  such that
- $$f(n) = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$$
- and  $g(n) = n - (-1)^n$ . Then  $fog$  is [JEE (Main)-2019]
- (1) One-one but not onto.  
 (2) Onto but not one-one.  
 (3) Neither one-one nor onto.  
 (4) Both one-one and onto.
14. Let  $f : R \rightarrow R$  be defined by  $f(x) = \frac{x}{1+x^2}$ ,  $x \in R$ . Then the range of  $f$  is [JEE (Main)-2019]
- (1)  $R - \left[-\frac{1}{2}, \frac{1}{2}\right]$   
 (2)  $\left[-\frac{1}{2}, \frac{1}{2}\right]$   
 (3)  $(-1, 1) - \{0\}$   
 (4)  $R - [-1, 1]$
15. Let a function  $f : (0, \infty) \rightarrow [0, \infty)$  be defined by  $f(x) = \left|1 - \frac{1}{x}\right|$ . Then  $f$  is [JEE (Main)-2019]
- (1) Injective only  
 (2) Both injective as well as surjective  
 (3) Not injective but it is surjective  
 (4) Neither injective nor surjective
16. If  $f(x) = \log_e \left( \frac{1-x}{1+x} \right)$ ,  $|x| < 1$ , then  $f\left(\frac{2x}{1+x^2}\right)$  is equal to : [JEE (Main)-2019]
- (1)  $2f(x)$   
 (2)  $2f(x^2)$   
 (3)  $-2f(x)$   
 (4)  $(f(x))^2$
17. Let  $f : [-1, 3] \rightarrow R$  be defined as
- $$f(x) = \begin{cases} |x| + [x], & -1 \leq x < 1 \\ x + |x|, & 1 \leq x < 2 \\ x + [x], & 2 \leq x \leq 3, \end{cases}$$
- where  $[t]$  denotes the greatest integer less than or equal to  $t$ . Then,  $f$  is discontinuous at : [JEE (Main)-2019]
- (1) Only one point  
 (2) Only two points  
 (3) Only three points  
 (4) Four or more points
18. If  $f(1) = 1, f'(1) = 3$ , then the derivative of  $f(f(f(x))) + (f(x))^2$  at  $x = 1$  is [JEE (Main)-2019]
- (1) 33  
 (2) 12  
 (3) 9  
 (4) 15
19. Let  $f(x) = a^x$  ( $a > 0$ ) be written as  $f(x) = f_1(x) + f_2(x)$ , where  $f_1(x)$  is an even function and  $f_2(x)$  is an odd function. Then  $f_1(x+y) + f_1(x-y)$  equals [JEE (Main)-2019]
- (1)  $2f_1(x)f_1(y)$   
 (2)  $2f_1(x+y)f_1(x-y)$   
 (3)  $2f_1(x+y)f_2(x-y)$   
 (4)  $2f_1(x)f_2(y)$

20. Let  $\sum_{k=1}^{10} f(a+k) = 16(2^{10} - 1)$ , where the function  $f$  satisfies  $f(x+y) = f(x)f(y)$  for all natural numbers  $x, y$  and  $f(1) = 2$ . Then the natural number  $a$  is [JEE (Main)-2019]

(1) 2      (2) 3  
 (3) 16      (4) 4

21. Let  $f(x) = 15 - |x - 10|$ ;  $x \in R$ . Then the set of all values of  $x$ , at which the function,  $g(x) = f(f(x))$  is not differentiable, is [JEE (Main)-2019]

(1)  $(10, 15)$       (2)  $\{5, 10, 15, 20\}$   
 (3)  $\{10\}$       (4)  $\{5, 10, 15\}$

22. If the function  $f : R - \{1, -1\} \rightarrow A$  defined by  $f(x) = \frac{x^2}{1-x^2}$ , is surjective, then  $A$  is equal to [JEE (Main)-2019]

(1)  $[0, \infty)$       (2)  $R - \{-1\}$   
 (3)  $R - (-1, 0)$       (4)  $R - [-1, 0)$

23. The domain of the definition of the function  $f(x) = \frac{1}{4-x^2} + \log_{10}(x^3 - x)$  is [JEE (Main)-2019]

(1)  $(-1, 0) \cup (1, 2) \cup (3, \infty)$   
 (2)  $(-1, 0) \cup (1, 2) \cup (2, \infty)$   
 (3)  $(1, 2) \cup (2, \infty)$   
 (4)  $(-2, -1) \cup (-1, 0) \cup (2, \infty)$

24. Let  $f(x) = x^2$ ,  $x \in R$ . For any  $A \subseteq R$ , define  $g(A) = \{x \in R : f(x) \in A\}$ . If  $S = [0, 4]$ , then which one of the following statements is not true ? [JEE (Main)-2019]

(1)  $f(g(S)) = S$       (2)  $g(f(S)) = g(S)$   
 (3)  $g(f(S)) \neq S$       (4)  $f(g(S)) \neq f(S)$

25. Let  $f(x) = \log_e(\sin x)$ ,  $(0 < x < \pi)$  and  $g(x) = \sin^{-1}(e^{-x})$ ,  $(x \geq 0)$ . If  $\alpha$  is a positive real number such that  $a = (fog)'(\alpha)$  and  $b = (fog)(\alpha)$ , then [JEE (Main)-2019]

(1)  $a\alpha^2 - b\alpha - a = 1$       (2)  $a\alpha^2 + b\alpha + a = 0$   
 (3)  $a\alpha^2 - b\alpha - a = 0$       (4)  $a\alpha^2 + b\alpha - a = -2\alpha^2$

26. The number of real roots of the equation  $5 + |2^x - 1| = 2^x(2^x - 2)$  is [JEE (Main)-2019]

(1) 4      (2) 2  
 (3) 1      (4) 3

27. For  $x \in R$ , let  $[x]$  denote the greatest integer  $\leq x$ , then the sum of the series [JEE (Main)-2019]

$$\left[ -\frac{1}{3} \right] + \left[ -\frac{1}{3} - \frac{1}{100} \right] + \left[ -\frac{1}{3} - \frac{2}{100} \right] + \dots + \left[ -\frac{1}{3} - \frac{99}{100} \right]$$
 is: [JEE (Main)-2019]

(1) -135      (2) -153  
 (3) -133      (4) -131

28. For  $x \in (0, \frac{3}{2})$ , let  $f(x) = \sqrt{x}$ ,  $g(x) = \tan x$  and  $h(x) = \frac{1-x^2}{1+x^2}$ . If  $\phi(x) = ((hof)og)(x)$ , then  $\phi\left(\frac{\pi}{3}\right)$  is equal to [JEE (Main)-2019]

(1)  $\tan \frac{5\pi}{12}$       (2)  $\tan \frac{\pi}{12}$   
 (3)  $\tan \frac{11\pi}{12}$       (4)  $\tan \frac{7\pi}{12}$

29. If  $g(x) = x^2 + x - 1$  and  $(gof)(x) = 4x^2 - 10x + 5$ , then  $f\left(\frac{5}{4}\right)$  is equal to [JEE (Main)-2020]

(1)  $-\frac{1}{2}$       (2)  $\frac{3}{2}$   
 (3)  $\frac{1}{2}$       (4)  $-\frac{3}{2}$

30. The inverse function of  $f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$ ,  $x \in (-1, 1)$ , is [JEE (Main)-2020]

(1)  $\frac{1}{4} \log_e \left( \frac{1-x}{1+x} \right)$   
 (2)  $\frac{1}{4} (\log_8 e) \log_e \left( \frac{1+x}{1-x} \right)$   
 (3)  $\frac{1}{4} \log_e \left( \frac{1+x}{1-x} \right)$   
 (4)  $\frac{1}{4} (\log_8 e) \log_e \left( \frac{1-x}{1+x} \right)$

