Matrices

1. The number of 3 × 3 non-singular matrices, with four entries as 1 and all other entries as 0, is **[AIEEE-2010]**

   (1) Less than 4 (2) 5

   (3) 6 (4) At least 7

2. Let A be a 2 × 2 matrix with non-zero entries and let $A^2 = I$, where I is 2 × 2 identity matrix. Define

   Tr(A) = sum of diagonal elements of A and $|A|$ = determinant of matrix A.

   Statement-1 : Tr(A) = 0.

   Statement-2 : $|A|$ = **1.** **[AIEEE-2010]**

   (1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

   (2) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

   (3) Statement-1 is true, Statement-2 is false

   (4) Statement-1 is false, Statement-2 is true

3. Consider the following relation R on the set of real square matrices of order 3.

   $R = \{(A, B) | A = P^{-1} BP$ for some invertible matrix $P\}$.

   Statement-1 : R is an equivalence relation.

   Statement-2 : For any two invertible 3 × 3 matrices M and N, $(MN)^{-1} = N^{-1}M^{-1}$.

   [AIEEE-2011]

   (1) Statement-1 is true, statement-2 is false

   (2) Statement-1 is false, statement-2 is true

   (3) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1

   (4) Statement-1 is true, statement-2 is true; statement-2 is ***not*** a correct explanation for statement-1

4. **Statement-1 :** Determinant of a skew-symmetric matrix of order 3 is zero.

   Statement-2 : For any matrix A, $\det(A^T) = \det(A)$ and $\det(-A) = -\det(A)$.

   Where det(B) denotes the determinant of matrix B. Then **[AIEEE-2011]**

   (1) Statement-1 is false and statement-2 is true

   (2) Statement-1 is true and statement-2 is false

   (3) Both statements are true

   (4) Both statements are false

5. If $\omega \neq 1$ is the complex cube root of unity and matrix $H = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$, then H^{70} is equal to **[AIEEE-2011]**

   (1) H^2 (2) H

   (3) 0 (4) – H

6. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$ if u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to **[AIEEE-2012]**

   (1) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ (2) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$

   (3) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ (4) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

7. If A is an 3 × 3 non-singular matrix such that $AA' = A'A$ and $B = A^{-1}A'$, then BB' equals **[JEE (Main)-2014]**

   (1) B^{-1} (2) $(B^{-1})'$

   (3) $I + B$ (4) I

8. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3×3 identity matrix, then the ordered pair (a, b) is equal to **[JEE (Main)-2015]**

(1) (2, –1) (2) (–2, 1)

(3) (2, 1) (4) (–2, –1)

9. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \frac{\pi}{12}$, is equal to **[JEE (Main)-2019]**

(1) $\begin{bmatrix} \frac{1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$ (2) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

(3) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ (4) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

10. If

$$A = \begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix},$$

then A is **[JEE (Main)-2019]**

(1) Invertible only if $t = \pi$

(2) Invertible for all $t \in$ R.

(3) Invertible only if $t = \frac{\pi}{2}$

(4) Not invertible for any $t \in$ R.

11. Let $A = \begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix}$ where $b > 0$. Then **[JEE (Main)-2019]**

(1) $-\sqrt{3}$ (2) $\sqrt{3}$

(3) $2\sqrt{3}$ (4) $-2\sqrt{3}$

12. Let $A = \begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$. If $AA^T = I_3$, then $|p|$ is **[JEE (Main)-2019]**

(1) $\frac{1}{\sqrt{3}}$ (2) $\frac{1}{\sqrt{6}}$

(3) $\frac{1}{\sqrt{5}}$ (4) $\frac{1}{\sqrt{2}}$

13. Let A and B be two invertible matrices of order 3×3. If $det(ABA^T) = 8$ and $det(AB^{-1}) = 8$, then $det(BA^{-1}B^T)$ is equal to **[JEE (Main)-2019]**

(1) 1 (2) 16

(3) $\frac{1}{16}$ (4) $\frac{1}{4}$

14. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ and $Q = [q_{ij}]$ be two 3×3 matrices such that $Q - P^5 = I_3$. Then $\frac{q_{21} + q_{31}}{q_{32}}$ is equal to **[JEE (Main)-2019]**

(1) 10 (2) 135

(3) 9 (4) 15

15. Let $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}, (\alpha \in R)$ such that $A^{32} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Then a value of α is **[JEE (Main)-2019]**

(1) $\frac{\pi}{32}$ (2) $\frac{\pi}{64}$

(3) 0 (4) $\frac{\pi}{16}$

16. Let the numbers 2, b, c be in an A.P. and $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$. If det($A$) $\in$ [2, 16], then c lies in the interval **[JEE (Main)-2019]**

(1) [2, 3) (2) $(2 + 2^{3/4}, 4)$

(3) $[3, 2 + 2^{3/4}]$ (4) [4, 6]

17. If $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\cdots\cdot\begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$,

then the inverse of $\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$ is **[JEE (Main)-2019]**

(1) $\begin{bmatrix}1 & 0\\13 & 1\end{bmatrix}$ (2) $\begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}$

(3) $\begin{bmatrix}1 & -12\\0 & 1\end{bmatrix}$ (4) $\begin{bmatrix}1 & 0\\12 & 1\end{bmatrix}$

18. The total number of matrices

$$A=\begin{pmatrix}0 & 2y & 1\\2x & y & -1\\2x & -y & 1\end{pmatrix},\ (x,\ y \in R,\ x \neq y)\text{ for}$$

which $A^TA = 3I_3$ is **[JEE (Main)-2019]**

(1) 6 (2) 3

(3) 4 (4) 2

19. If A is a symmetric matrix and B is a skew-symmetric matrix such that $A+B=\begin{bmatrix}2 & 3\\5 & -1\end{bmatrix}$, then

AB is equal to **[JEE (Main)-2019]**

(1) $\begin{bmatrix}4 & -2\\1 & -4\end{bmatrix}$ (2) $\begin{bmatrix}4 & -2\\-1 & -4\end{bmatrix}$

(3) $\begin{bmatrix}-4 & -2\\-1 & 4\end{bmatrix}$ (4) $\begin{bmatrix}-4 & 2\\1 & 4\end{bmatrix}$

20. If $B=\begin{bmatrix}5 & 2\alpha & 1\\0 & 2 & 1\\\alpha & 3 & -1\end{bmatrix}$ is the inverse of a 3 × 3 matrix A, then the sum of all value of α for which det (A) + 1 = 0, is **[JEE (Main)-2019]**

(1) –1 (2) 2

(3) 0 (4) 1

21. Let α be a root of the equation $x^2 + x + 1 = 0$ and the matrix $A=\frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\\1 & \alpha & \alpha^2\\1 & \alpha^2 & \alpha^4\end{bmatrix}$, then the matrix

A^{31} is equal to **[JEE (Main)-2020]**

(1) A^2 (2) A

(3) I_3 (4) A^3

22. Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two 3 × 3 real matrices such that $b_{ij} = (3)^{(i+j-2)}a_{ji}$, where i, j = 1, 2, 3. If the determinant of B is 81, then the determinant of A is **[JEE (Main)-2020]**

(1) 1/9 (2) 1/81

(3) 3 (4) 1/3

23. If $A=\begin{pmatrix}2 & 2\\9 & 4\end{pmatrix}$ and $I=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$, then $10A^{-1}$ is equal to **[JEE (Main)-2020]**

(1) $6I - A$ (2) $4I - A$

(3) $A - 4I$ (4) $A - 6I$

24. If the matrices $A=\begin{bmatrix}1 & 1 & 2\\1 & 3 & 4\\1 & -1 & 3\end{bmatrix}$, B = adj A and $C = 3A$, then $\dfrac{|\text{adj } B|}{|C|}$ is equal to

[JEE (Main)-2020]

(1) 16 (2) 2

(3) 72 (4) 8

25. Let $a - 2b + c = 1$.

If $f(x)=\begin{vmatrix}x+a & x+2 & x+1\\x+b & x+3 & x+2\\x+c & x+4 & x+3\end{vmatrix}$, then

[JEE (Main)-2020]

(1) $f(50) = 1$ (2) $f(-50) = 501$

(3) $f(-50) = -1$ (4) $f(50) = -501$

26. Let A be a 2 × 2 real matrix with entries from {0, 1} and $|A| \neq 0$. Consider the following two statements **[JEE (Main)-2020]**

(P) If $A \neq I_2$, then $|A| = -1$

(Q) If $|A| = 1$, then tr(A) = 2,

where I_2 denotes 2 × 2 identity matrix and tr(A) denotes the sum of the diagonal entries of A. Then

[JEE (Main)-2020]

(1) (P) is true and (Q) is false

(2) Both (P) and (Q) are false

(3) Both (P) and (Q) are true

(4) (P) is false and (Q) is true

27. Let a, b, $c \in R$ be all non-zero and satisfy $a^3 + b^3 + c^3 = 2$. If the matrix

$$A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$$

satisfies $A^TA = I$, then a value of abc can be **[JEE (Main)-2020]**

(1) 3 (2) $\frac{1}{3}$

(3) $-\frac{1}{3}$ (4) $\frac{2}{3}$

28. Let $A = \{X = (x, y, z)^T : PX = 0 \text{ and } x^2 + y^2 + z^2 = 1\}$,

where $P = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix}$, then the set A **[JEE (Main)-2020]**

(1) Is an empty set.

(2) Contains more than two elements.

(3) Contains exactly two elements.

(4) Is a singleton.

29. Let A be a 3 × 3 matrix such that

$$\text{adj } A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix}$$ and $B = \text{adj}(\text{adj } A)$.

If $|A| = \lambda$ and $\left|(B^{-1})^T\right| = \mu$, then the ordered pair, $(|\lambda|, \mu)$ is equal to **[JEE (Main)-2020]**

(1) (3, 81) (2) $\left(9, \frac{1}{9}\right)$

(3) $\left(3, \frac{1}{81}\right)$ (4) $\left(9, \frac{1}{81}\right)$

30. If $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}, \left(\theta = \frac{\pi}{24}\right)$ and $A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $i = \sqrt{-1}$, then which one of the following is not true? **[JEE (Main)-2020]**

(1) $a^2 - b^2 = \frac{1}{2}$ (2) $a^2 - c^2 = 1$

(3) $a^2 - d^2 = 0$ (4) $0 \le a^2 + b^2 \le 1$

31. Let $\theta = \frac{\pi}{5}$ and $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$. If $B = A + A^4$, then det (B) **[JEE (Main)-2020]**

(1) Lies in (2, 3)

(2) Is zero.

(3) Is one

(4) Lies in (1, 2)

32. The number of all 3 × 3 matrices A, with enteries from the set {–1, 0, 1} such that the sum of the diagonal elements of AA^T is 3, is _____________.

[JEE (Main)-2020]

33. Let $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}, x \in R$ and $A^4 = [a_{ij}]$. If $a_{11} = 109$, then a_{22} is equal to _________.

[JEE (Main)-2020]

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