

Limits

1. Let $f: R \rightarrow R$ be a positive increasing function with

$$\lim_{x \rightarrow \infty} \frac{f(3x)}{f(x)} = 1. \text{ Then } \lim_{x \rightarrow \infty} \frac{f(2x)}{f(x)} = \quad [\text{AIEEE-2010}]$$

(1) 1

(2) $\frac{2}{3}$

(3) $\frac{3}{2}$

(4) 3

2. Let $f: R \rightarrow [0, \infty)$ be such that $\lim_{x \rightarrow 5} f(x)$ exists

$$\text{and } \lim_{x \rightarrow 5} \frac{(f(x))^2 - 9}{\sqrt{|x-5|}} = 0. \text{ Then } \lim_{x \rightarrow 5} f(x) \text{ equals}$$

[AIEEE-2011]

(1) 2

(2) 3

(3) 0

(4) 1

3. $\lim_{x \rightarrow 0} \frac{(1-\cos 2x)(3+\cos x)}{x \tan 4x}$ is equal to

[JEE (Main)-2013]

(1) $-\frac{1}{4}$

(2) $\frac{1}{2}$

(3) 1

(4) 2

4. $\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2 x)}{x^2}$ is equal to [JEE (Main)-2014]

(1) $-\pi$

(2) π

(3) $\frac{\pi}{2}$

(4) 1

5. $\lim_{x \rightarrow 0} \frac{(1-\cos 2x)(3+\cos x)}{x \tan 4x}$ is equal to

[JEE (Main)-2015]

(1) 4

(2) 3

(3) 2

(4) $\frac{1}{2}$

6. Let $p = \lim_{x \rightarrow 0+} (1 + \tan^2 \sqrt{x})^{\frac{1}{2x}}$ then $\log p$ is equal to
[JEE (Main)-2016]

(1) 1

(2) $\frac{1}{2}$

(3) $\frac{1}{4}$

(4) 2

7. $\lim_{n \rightarrow \infty} \left(\frac{(n+1)(n+2)\dots 3n}{n^{2n}} \right)^{\frac{1}{n}}$ is equal to

[JEE (Main)-2016]

(1) $\frac{27}{e^2}$

(2) $\frac{9}{e^2}$

(3) $3 \log 3 - 2$

(4) $\frac{18}{e^4}$

8. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ equals

[JEE (Main)-2017]

(1) $\frac{1}{16}$

(2) $\frac{1}{8}$

(3) $\frac{1}{4}$

(4) $\frac{1}{24}$

9. For each $t \in R$, let $[t]$ be the greatest integer less than or equal to t . Then

[JEE (Main)-2018]

$$\lim_{x \rightarrow 0^+} x \left(\left[\frac{1}{x} \right] + \left[\frac{2}{x} \right] + \dots + \left[\frac{15}{x} \right] \right)$$

(1) Is equal to 0

(2) Is equal to 15

(3) Is equal to 120

(4) Does not exist (in R)

10. $\lim_{y \rightarrow 0} \frac{\sqrt{1+\sqrt{1+y^4}} - \sqrt{2}}{y^4}$

[JEE (Main)-2019]

(1) Exists and equals $\frac{1}{2\sqrt{2}(\sqrt{2}+1)}$

(2) Does not exist

(3) Exists and equals $\frac{1}{4\sqrt{2}}$

(4) Exists and equals $\frac{1}{2\sqrt{2}}$

11. For each $x \in R$, let $[x]$ be the greatest integer less than or equal to x . Then $\lim_{x \rightarrow 0} \frac{x([x]+|x|)\sin[x]}{|x|}$ is equal to [JEE (Main)-2019]

(1) $-\sin 1$ (2) 1
(3) $\sin 1$ (4) 0

12. For each $t \in R$, let $[t]$ be the greatest integer less than or equal to t . Then,

$$\lim_{x \rightarrow 1^+} \frac{(1-|x| + \sin |1-x|)\sin\left(\frac{x}{2}[1-x]\right)}{|1-x|[1-x]}$$

[JEE (Main)-2019]

(1) Equals 0 (2) Equals 1
(3) Equals -1 (4) Does not exist

13. Let $[x]$ denote the greatest integer less than or equal to x . Then

$$\lim_{x \rightarrow 0} \frac{\tan(\pi \sin^2 x) + (|x| - \sin(x[x])))^2}{x^2}$$

[JEE (Main)-2019]

(1) Equals 0 (2) Equals $\pi + 1$
(3) Equals π (4) Does not exist

14. $\lim_{x \rightarrow 0} \frac{x \cot(4x)}{\sin^2 x \cot^2(2x)}$ is equal to

[JEE (Main)-2019]

(1) 2 (2) 4
(3) 1 (4) 0

15. $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cot^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$ is [JEE (Main)-2019]

(1) $8\sqrt{2}$ (2) 4
(3) $4\sqrt{2}$ (4) 8

16. $\lim_{x \rightarrow 1} \frac{\sqrt{\pi} - \sqrt{2\sin^{-1} x}}{\sqrt{1-x}}$ is equal to [JEE (Main)-2019]

(1) $\sqrt{\frac{2}{\pi}}$

(2) $\sqrt{\frac{\pi}{2}}$

(3) $\sqrt{\pi}$

(4) $\frac{1}{\sqrt{2\pi}}$

17. $\lim_{x \rightarrow 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}}$ equals [JEE (Main)-2019]

(1) $\sqrt{2}$

(2) $2\sqrt{2}$

(3) 4

(4) $4\sqrt{2}$

18. Let $f: R \rightarrow R$ be a differentiable function satisfying

$f'(3) + f'(2) = 0$. Then $\lim_{x \rightarrow 0} \left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)} \right)^{\frac{1}{x}}$ is

equal to

[JEE (Main)-2019]

(1) e

(2) 1

(3) e^2

(4) e^{-1}

19. If $f(x) = [x] - \left[\frac{x}{4}\right]$, $x \in R$, where $[x]$ denotes the greatest integer function, then [JEE (Main)-2019]

(1) $\lim_{x \rightarrow 4^+} f(x)$ exists but $\lim_{x \rightarrow 4^-} f(x)$ does not exist

(2) f is continuous at $x = 4$

(3) $\lim_{x \rightarrow 4^-} f(x)$ exists but $\lim_{x \rightarrow 4^+} f(x)$ does not exist

(4) Both $\lim_{x \rightarrow 4^-} f(x)$ and $\lim_{x \rightarrow 4^+} f(x)$ exist but are not equal

20. If $\lim_{x \rightarrow 1} \frac{x^4 - 1}{x - 1} = \lim_{x \rightarrow k} \frac{x^3 - k^3}{x^2 - k^2}$, then k is

[JEE (Main)-2019]

(1) $\frac{4}{3}$

(2) $\frac{3}{2}$

(3) $\frac{8}{3}$

(4) $\frac{3}{8}$

21. If $\lim_{x \rightarrow 1} \frac{x^2 - ax + b}{x - 1} = 5$, then $a + b$ is equal to

[JEE (Main)-2019]

(1) 5

(2) -4

(3) 1

(4) -7

22. $\lim_{x \rightarrow 0} \frac{x + 2 \sin x}{\sqrt{x^2 + 2 \sin x + 1} - \sqrt{\sin^2 x - x + 1}}$ is [JEE (Main)-2019]
 (1) 3 (2) 6
 (3) 1 (4) 2
23. Let $f(x) = 5 - |x - 2|$ and $g(x) = |x + 1|$, $x \in R$. If $f(x)$ attains maximum value at α and $g(x)$ attains minimum value at β , then
 $\lim_{x \rightarrow \alpha^+} \frac{(x - 1)(x^2 - 5x + 6)}{x^2 - 6x + 8}$ is equal to [JEE (Main)-2019]
 (1) $1/2$ (2) $-1/2$
 (3) $-3/2$ (4) $3/2$
24. $\lim_{x \rightarrow 0} \left(\frac{3x^2 + 2}{7x^2 + 2} \right)^{\frac{1}{x^2}}$ is equal to [JEE (Main)-2020]
 (1) e (2) $\frac{1}{e}$
 (3) $\frac{1}{e^2}$ (4) e^2
25. $\lim_{x \rightarrow 0} \left(\tan \left(\frac{\pi}{4} + x \right) \right)^{\frac{1}{x}}$ is equal to
 (1) e^2 (2) 1
 (3) e (4) 2
26. Let $[t]$ denote the greatest integer $\leq t$. If for some $\lambda \in R - \{0, 1\}$, $\lim_{x \rightarrow 0} \left| \frac{1-x+|x|}{\lambda-x+[x]} \right| = L$, then L is equal to [JEE (Main)-2020]
 (1) 2 (2) $\frac{1}{2}$
 (3) 0 (4) 1
27. $\lim_{x \rightarrow a} \frac{(a+2x)^{\frac{1}{3}} - (3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}} - (4x)^{\frac{1}{3}}} (a \neq 0)$ is equal to [JEE (Main)-2020]
 (1) $\left(\frac{2}{9} \right) \left(\frac{2}{3} \right)^{\frac{1}{3}}$ (2) $\left(\frac{2}{3} \right) \left(\frac{2}{9} \right)^{\frac{1}{3}}$
 (3) $\left(\frac{2}{3} \right)^{\frac{4}{3}}$ (4) $\left(\frac{2}{9} \right)^{\frac{4}{3}}$

28. Let $f : (0, \infty) \rightarrow (0, \infty)$ be a differentiable function such that $f(1) = e$ and
 $\lim_{t \rightarrow x} \frac{t^2 f^2(x) - x^2 f^2(t)}{t - x} = 0$
 If $f(x) = 1$, then x is equal to [JEE (Main)-2020]
 (1) $2e$ (2) e
 (3) $\frac{1}{2e}$ (4) $\frac{1}{e}$
29. If α is the positive root of the equation, $p(x) = x^2 - x - 2 = 0$, then $\lim_{x \rightarrow \alpha^+} \frac{\sqrt{1 - \cos(p(x))}}{x + \alpha - 4}$ is equal to [JEE (Main)-2020]
 (1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2}$
 (3) $\frac{3}{\sqrt{2}}$ (4) $\frac{3}{2}$
30. $\lim_{x \rightarrow 0} \frac{x \left(e^{(\sqrt{1+x^2+x^4}-1)/x} - 1 \right)}{\sqrt{1+x^2+x^4}-1}$ [JEE (Main)-2020]
 (1) Is equal to 0 (2) Is equal to \sqrt{e}
 (3) Is equal to 1 (4) Does not exist
31. $\lim_{x \rightarrow 1} \left(\frac{\int_0^{(x-1)^2} t \cos(t^2) dt}{(x-1) \sin(x-1)} \right)$ [JEE (Main)-2020]
 (1) Does not exist (2) Is equal to $-\frac{1}{2}$
 (3) Is equal to 1 (4) Is equal to $\frac{1}{2}$
32. $\lim_{x \rightarrow 2} \frac{3^x + 3^{3-x} - 12}{3^{-\frac{x}{2}} - 3^{1-x}}$ is equal to _____ [JEE (Main)-2020]
33. If $\lim_{x \rightarrow 1} \frac{x + x^2 + x^3 + \dots + x^n - n}{x-1} = 820$, ($n \in N$) then the value of n is equal to _____ [JEE (Main)-2020]
34. If $\lim_{x \rightarrow 0} \left\{ \frac{1}{x^8} \left(1 - \cos \frac{x^2}{2} - \cos \frac{x^2}{4} + \cos \frac{x^2}{2} \cos \frac{x^2}{4} \right) \right\} = 2^{-k}$, then the value of k is _____ [JEE (Main)-2020]