Complex Numbers

1. If $\left|Z-\frac{4}{Z}\right|=2$, then the maximum value of $|Z|$ is equal to **[AIEEE-2009]**

   (1) $\sqrt{5}+1$ (2) 2

   (3) $2+\sqrt{2}$ (4) $\sqrt{3}+1$

2. The number of complex numbers z such that $|z-1| = |z+1| = |z-i|$ equals **[AIEEE-2010]**

   (1) 0 (2) 1

   (3) 2 (4) ∞

3. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies **[AIEEE-2012]**

   (1) On a circle with centre at the origin.

   (2) Either on the real axis or on a circle not passing through the origin

   (3) On the imaginary axis

   (4) Either on the real axis or on a circle passing through the origin

4. If z is a complex number of unit modulus and argument θ, then arg $\left(\frac{1+z}{1+\bar{z}}\right)$ equals **[JEE (Main)-2013]**

   (1) $-\theta$ (2) $\frac{\pi}{2}-\theta$

   (3) θ (4) $\pi-\theta$

5. If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z+\frac{1}{2}\right|$ **[JEE (Main)-2014]**

   (1) Is strictly greater than $\frac{5}{2}$

   (2) Is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

   (3) Is equal to $\frac{5}{2}$

   (4) Lies in the interval (1, 2)

6. A complex number z is said to be unimodular if $|z| = 1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1-2z_2}{2-z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a **[JEE (Main)-2015]**

   (1) Straight line parallel to x-axis

   (2) Straight line parallel to y-axis

   (3) Circle of radius 2

   (4) Circle of radius $\sqrt{2}$

7. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is **[JEE (Main)-2016]**

   (1) $\frac{\pi}{6}$ (2) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

   (3) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (4) $\frac{\pi}{3}$

8. Let $A=\left\{\theta\in\left(-\frac{\pi}{2},\pi\right):\frac{3+2i\sin\theta}{1-2i\sin\theta}\text{ is purely imaginary}\right\}$.

   Then the sum of the elements in A is **[JEE (Main)-2019]**

   (1) $\frac{5\pi}{6}$ (2) π

   (3) $\frac{3\pi}{4}$ (4) $\frac{2\pi}{3}$

9. Let z_0 be a root of the quadratic equation, $x^2 + x + 1 = 0$. If $z = 3 + 6iz_0^{81} - 3iz_0^{93}$, then arg z is equal to **[JEE (Main)-2019]**

   (1) 0 (2) $\frac{\pi}{3}$

   (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

10. Let z_1 and z_2 be any two non-zero complex numbers such that $3|z_1| = 4|z_2|$. If $z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$ then **[JEE (Main)-2019]**

(1) $\text{Im}(z) = 0$ (2) $\frac{3}{2} \le |z| \le \frac{5}{2}$

(3) $|z| = \frac{1}{2}\sqrt{\frac{17}{2}}$ (4) $\text{Re}(z) = 0$

11. Let $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$. If $R(z)$ and $I(z)$ respectively denote the real and imaginary parts of z, then **[JEE (Main)-2019]**

(1) $I(z) = 0$

(2) $R(z) > 0$ and $I(z) > 0$

(3) $R(z) < 0$ and $I(z) > 0$

(4) $R(z) = -3$

12. Let $\left(-2 - \frac{1}{3}i\right)^3 = \frac{x + iy}{27}\ (i = \sqrt{-1})$, where x and y are real numbers, then $y - x$ equals **[JEE (Main)-2019]**

(1) –85 (2) –91

(3) 85 (4) 91

13. Let z be a complex number such that $|z| + z = 3 + i$ (where $i = \sqrt{-1}$). Then $|z|$ is equal to **[JEE (Main)-2019]**

(1) $\frac{\sqrt{41}}{4}$ (2) $\frac{5}{4}$

(3) $\frac{5}{3}$ (4) $\frac{\sqrt{34}}{3}$

14. If $\frac{z - \alpha}{z + \alpha}\ (\alpha \in R)$ is a purely imaginary number and $|z| = 2$, then a value of α is **[JEE (Main)-2019]**

(1) $\sqrt{2}$ (2) 2

(3) $\frac{1}{2}$ (4) 1

15. Let z_1 and z_2 be two complex numbers satisfying $|z_1| = 9$ and $|z_2 - 3 - 4i| = 4$. Then the minimum value of $|z_1 - z_2|$ is **[JEE (Main)-2019]**

(1) 0 (2) $\sqrt{2}$

(3) 1 (4) 2

16. If $z = \frac{\sqrt{3}}{2} + \frac{i}{2}\left(i = \sqrt{-1}\right)$, then **[JEE (Main)-2019]**

(1) 0 (2) $(-1 + 2i)^9$

(3) -1 (4) 1

17. All the points in the set

$$S = \left\{\frac{\alpha + i}{\alpha - i} : \alpha \in R\right\} (i = \sqrt{-1})$$

lie on a **[JEE (Main)-2019]**

(1) Straight line whose slope is 1

(2) Circle whose radius is $\sqrt{2}$

(3) Circle whose radius is 1

(4) Straight line whose slope is –1

18. Let $z \in C$ be such that $|z| < 1$. If $\omega = \frac{5 + 3z}{5(1 - z)}$, then **[JEE (Main)-2019]**

(1) $5\,\text{Re}(\omega) > 4$ (2) $5\,\text{Re}(\omega) > 1$

(3) $4\,\text{Im}(\omega) > 5$ (4) $5\,\text{Im}(\omega) < 1$

19. If $a > 0$ and $z = \frac{(1+i)^2}{a - i}$, has magnitude $\sqrt{\frac{2}{5}}$, then $\bar{z}$ is equal to : **[JEE (Main)-2019]**

(1) $-\frac{1}{5} + \frac{3}{5}i$ (2) $-\frac{3}{5} - \frac{1}{5}i$

(3) $\frac{1}{5} - \frac{3}{5}i$ (4) $-\frac{1}{5} - \frac{3}{5}i$

20. If z and w are two complex numbers such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$, then : **[JEE (Main)-2019]**

(1) $z\bar{w} = \frac{1 - i}{\sqrt{2}}$ (2) $\bar{z}w = i$

(3) $z\bar{w} = \frac{-1 + i}{\sqrt{2}}$ (4) $\bar{z}w = -i$

21. The equation $|z - i| = |z - 1|$, $i = \sqrt{-1}$, represents : **[JEE (Main)-2019]**

(1) The line through the origin with slope –1

(2) A circle of radius $\frac{1}{2}$

(3) A circle of radius 1

(4) The line through the origin with slope 1

22. Let $z \in C$ with $\text{Im}(z) = 10$ and it satisfies $\dfrac{2z - n}{2z + n} = 2i - 1$ for some natural number n. Then

[JEE (Main)-2019]

(1) $n = 20$ and $Re(z) = 10$

(2) $n = 20$ and $Re(z) = -10$

(3) $n = 40$ and $Re(z) = -10$

(4) $n = 40$ and $Re(z) = 10$

23. If $\text{Re}\left(\dfrac{z-1}{2z+i}\right) = 1$, where $z = x + iy$, then the point (x, y) lies on a **[JEE (Main)-2020]**

(1) Circle whose centre is at $\left(-\dfrac{1}{2}, -\dfrac{3}{2}\right)$

(2) Straight line whose slope is $-\dfrac{2}{3}$

(3) Circle whose diameter is $\dfrac{\sqrt{5}}{2}$

(4) Straight line whose slope is $\dfrac{3}{2}$

24. If $\dfrac{3 + i\sin\theta}{4 - i\cos\theta}$, $\theta \in [0, 2\pi]$, is a real number, then an argument of $\sin\theta + i\cos\theta$ is

[JEE (Main)-2020]

(1) $\pi - \tan^{-1}\left(\dfrac{3}{4}\right)$ (2) $\pi - \tan^{-1}\left(\dfrac{4}{3}\right)$

(3) $-\tan^{-1}\left(\dfrac{3}{4}\right)$ (4) $\tan^{-1}\left(\dfrac{4}{3}\right)$

25. If the equation, $x^2 + bx + 45 = 0$ $(b \in R)$ has conjugate complex roots and they satisfy $|z+1| = 2\sqrt{10}$, then **[JEE (Main)-2020]**

(1) $b^2 - b = 42$ (2) $b^2 - b = 30$

(3) $b^2 + b = 12$ (4) $b^2 + b = 72$

26. Let $\alpha = \dfrac{-1 + i\sqrt{3}}{2}$. If $a = (1+\alpha)\sum_{k=0}^{100} \alpha^{2k}$ and $b = \sum_{k=0}^{100} \alpha^{3k}$, then a and b are the roots of the quadratic equation **[JEE (Main)-2020]**

(1) $x^2 - 101x + 100 = 0$

(2) $x^2 - 102x + 101 = 0$

(3) $x^2 + 101x + 100 = 0$

(4) $x^2 + 102x + 101 = 0$

27. Let z be a complex number such that $\left|\dfrac{z-i}{z+2i}\right| = 1$ and $|z| = \dfrac{5}{2}$. Then the value of $|z + 3i|$ is

[JEE (Main)-2020]

(1) $2\sqrt{3}$ (2) $\dfrac{7}{2}$

(3) $\sqrt{10}$ (4) $\dfrac{15}{4}$

28. If z be a complex number satisfying $|\text{Re}(z)| + |\text{Im}(z)| = 4$, then $|z|$ cannot be

[JEE (Main)-2020]

(1) $\sqrt{10}$ (2) $\sqrt{8}$

(3) $\sqrt{\dfrac{17}{2}}$ (4) $\sqrt{7}$

29. The value of $\left(\dfrac{1 + \sin\dfrac{2\pi}{9} + i\cos\dfrac{2\pi}{9}}{1 + \sin\dfrac{2\pi}{9} - i\cos\dfrac{2\pi}{9}}\right)^3$ is

[JEE (Main)-2020]

(1) $-\dfrac{1}{2}(1 - i\sqrt{3})$ (2) $\dfrac{1}{2}(1 - i\sqrt{3})$

(3) $\dfrac{1}{2}(\sqrt{3} - i)$ (4) $-\dfrac{1}{2}(\sqrt{3} - i)$

30. The imaginary part of

$\left(3 + 2\sqrt{-54}\right)^{1/2} - \left(3 - 2\sqrt{-54}\right)^{1/2}$ can be

[JEE (Main)-2020]

(1) $\sqrt{6}$ (2) $-\sqrt{6}$

(3) $-2\sqrt{6}$ (4) 6

31. If z_1, z_2 are complex numbers such that $\text{Re}(z_1) = |z_1 - 1|$, $\text{Re}(z_2) = |z_2 - 1|$, and $\arg(z_1 - z_2) = \dfrac{\pi}{6}$, then $\text{Im}(z_1 + z_2)$ is equal to

[JEE (Main)-2020]

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1}{\sqrt{3}}$

(3) $\frac{2}{\sqrt{3}}$ (4) $2\sqrt{3}$

32. Let $u = \frac{2z+i}{z-ki}$, $z = x+iy$ and $k > 0$. If the curve represented by Re(u) + Im(u) = 1 intersects the y-axis at the point P and Q where PQ = 5, then the value of K is **[JEE (Main)-2020]**

(1) 1/2 (2) 3/2

(3) 2 (4) 4

33. If a and b are real numbers such that $(2+\alpha)^4 = a + b\alpha$, where $\alpha = \frac{-1+i\sqrt{3}}{2}$, then $a + b$ is equal to **[JEE (Main)-2020]**

(1) 33 (2) 9

(3) 24 (4) 57

34. If the four complex numbers z, $\bar{z}$, $\bar{z} - 2\text{Re}(\bar{z})$ and $z - 2\text{Re}(z)$ represent the vertices of a square of side 4 units in the Argand plane, then $|z|$ is equal to **[JEE (Main)-2020]**

(1) $4\sqrt{2}$ (2) 2

(3) $2\sqrt{2}$ (4) 4

35. The value of $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30}$ is **[JEE (Main)-2020]**

(1) $-2^{15}i$ (2) -2^{15}

(3) $2^{15}i$ (4) 65

36. The region represented by $\{z = x + iy \in C : |z| - \text{Re}(z) \le 1\}$ is also given by the inequality **[JEE (Main)-2020]**

(1) $y^2 \le x + \frac{1}{2}$ (2) $y^2 \le 2\left(x + \frac{1}{2}\right)$

(3) $y^2 \ge x + 1$ (4) $y^2 \ge 2(x+1)$

37. Let $z = x + iy$ be a non-zero complex number such that $z^2 = i|z|^2$, where $i = \sqrt{-1}$, then z lies on the **[JEE (Main)-2020]**

(1) Line, $y = x$ (2) Imaginary axis

(3) Real axis (4) Line, $y = -x$

38. If $\left(\frac{1+i}{1-i}\right)^{m/2} = \left(\frac{1+i}{i-1}\right)^{n/3} = 1$, $(m, n \in N)$ then the greatest common divisor of the least values of m and n is ________. **[JEE (Main)-2020]**

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