Continuity and Differentiability

1. Let $f(x) = x|x|$ and $g(x) = \sin x$.

Statement-1 : gof is differentiable at $x = 0$ and its derivative is continuous at that point.

Statement-2 : gof is twice differentiable at $x = 0$.

[AIEEE-2009]

(1) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is false

(3) Statement-1 is false, Statement-2 is true

(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

2. Let $f: (-1, 1) \to R$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $g(x) = [f(2f(x) + 2)]^2$. Then $g'(0) =$ **[AIEEE-2010]**

(1) 4 (2) –4

(3) 0 (4) –2

3. Let $f: \boldsymbol{R} \to \boldsymbol{R}$ be a continuous function defined by

$$f(x) = \frac{1}{e^x + 2e^{-x}}$$

Statement-1 : $f(c) = \frac{1}{3}$, for some $c \in \boldsymbol{R}$.

Statement-2 : $0 < f(x) \le \frac{1}{2\sqrt{2}}$, for all $x \in \boldsymbol{R}$.

[AIEEE-2010]

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is false, Statement-2 is true

4. Define $F(x)$ as the product of two real functions

$$f_1(x) = x, x \in \mathbb{R}, \text{ and } f_2(x) = \begin{cases} \sin\frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

as follows:

$$F(x) = \begin{cases} f_1(x).f_2(x) & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

Statement - 1 : $F(x)$ is continuous on $\mathbb{R}$.

Statement - 2 : $\boldsymbol{f_1(x)}$ and $\boldsymbol{f_2(x)}$ are continuous on R.

[AIEEE-2011]

(1) Statement-1 is true, Statement-2 is false

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation of Statement-1

(4) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation of Statement-1

5. If function $f(x)$ is differentiable at $x = a$, then

$$\lim_{x \to a} \frac{x^2 f(a) - a^2 f(x)}{x - a} \text{ is}$$ **[AIEEE-2011]**

(1) $2a\, f(a) - a^2 f'(a)$

(2) $2a\, f(a) + a^2 f'(a)$

(3) $-a^2 f'(a)$

(4) $a\, f(a) - a^2 f'(a)$

6. If $f : R \to R$ is a function defined by $f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi$, where $[x]$ denotes the greatest integer function, then f is **[AIEEE-2012]**

(1) Discontinuous only at $x = 0$

(2) Discontinuous only at non-zero integral values of x

(3) Continuous only at $x = 0$

(4) Continuous for every real x

7. **Consider the function,**

$f(x) = |x-2| + |x-5|, x \in R$

Statement-1 : $f'(4) = 0$

Statement-2 : f is continuous in [2, 5], differentiable in (2, 5) and $f(2) = f(5)$.

[AIEEE-2012]

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is false, Statement-2 is true

8. If f and g are differentiable functions in [0, 1] satisfying $f(0) = 2 = g(1)$, $g(0) = 0$ and $f(1) = 6$, then for some $c \in]0, 1[$ **[JEE (Main)-2014]**

(1) $f'(c) = g'(c)$ (2) $f'(c) = 2g'(c)$

(3) $2f'(c) = g'(c)$ (4) $2f'(c) = 3g'(c)$

9. If the function.

$$g(x) = \begin{cases} k\sqrt{x+1} & , \quad 0 \le x \le 3 \\ mx+2 & , \quad 3 < x \le 5 \end{cases}$$

is differentiable, the value of $k + m$ is **[JEE (Main)-2015]**

(1) 2 (2) $\frac{16}{5}$

(3) $\frac{10}{3}$ (4) 4

10. For $x \in R$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then **[JEE (Main)-2016]**

(1) $g'(0) = \cos(\log 2)$

(2) $g'(0) = -\cos(\log 2)$

(3) g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$

(4) g is not differentiable at $x = 0$

11. If for $x \in \left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x} \cdot g(x)$, then $g(x)$ equals **[JEE (Main)-2017]**

(1) $\frac{3x\sqrt{x}}{1-9x^3}$ (2) $\frac{3x}{1-9x^3}$

(3) $\frac{3}{1+9x^3}$ (4) $\frac{9}{1+9x^3}$

12. Let $S = \{t \in R : f(x) = |x-\pi| \cdot (e^{|x|}-1)\sin|x|$ is not differentiable at $t\}$. Then the set S is equal to **[JEE (Main)-2018]**

(1) ϕ (an empty set) (2) $\{0\}$

(3) $\{\pi\}$ (4) $\{0, \pi\}$

13. Let $f : R \to R$ be a function defined as

$$f(x) = \begin{cases} 5, & \text{if} \quad x \le 1 \\ a+bx, & \text{if} \quad 1 < x < 3 \\ b+5x, & \text{if} \quad 3 \le x < 5 \\ 30, & \text{if} \quad x \ge 5 \end{cases}$$

Then, f is **[JEE (Main)-2019]**

(1) Continuous if $a = -5$ and $b = 10$

(2) Continuous if $a = 5$ and $b = 5$

(3) Continuous if $a = 0$ and $b = 5$

(4) Not continuous for any values of a and b

14. Let f be a differentiable function from R to R such that $|f(x)-f(y)| \le 2|x-y|^{\frac{3}{2}}$, for all $x, y \in$ R. If $f(0) = 1$ then $\int_0^1 f^2(x)dx$ equal to **[JEE (Main)-2019]**

(1) 1 (2) 0

(3) $\frac{1}{2}$ (4) 2

15. If $x = 3\tan t$ and $y = 3\sec t$, then the value of $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$, is **[JEE (Main)-2019]**

(1) $\frac{1}{6\sqrt{2}}$ (2) $\frac{1}{3\sqrt{2}}$

(3) $\frac{3}{2\sqrt{2}}$ (4) $\frac{1}{6}$

16. Let $f(x)=\begin{cases}\max\{|x|,x^2\}, & |x|\le 2\\ 8-2|x|, & 2<|x|\le 4\end{cases}$

Let S be the set of points in the interval (–4, 4) at which f is not differentiable. Then S

[JEE (Main)-2019]

(1) Equals {–2, –1, 0, 1, 2}

(2) Equals {–2, 2}

(3) Is an empty set

(4) Equals {–2, –1, 1, 2}

17. Let $f:(-1, 1)\to R$ be a function defined by $f(x)=\max\left\{-|x|,-\sqrt{1-x^2}\right\}$. If K be the set of all points at which f is not differentiable, then K has exactly **[JEE (Main)-2019]**

(1) Three elements (2) Two elements

(3) One element (4) Five elements

18. Let f be a differentiable function such that $f'(x)=7-\frac{3}{4}\frac{f(x)}{x}$, $(x>0)$ and $f(1)\ne 4$. Then $\lim\limits_{x\to 0^+} xf\left(\frac{1}{x}\right)$ **[JEE (Main)-2019]**

(1) Exist and equals 4

(2) Does not exist

(3) Exists and equals $\frac{4}{7}$

(4) Exists and equals 0

19. Let $f(x)=\begin{cases}-1, & -2\le x<0\\ x^2-1, & 0\le x\le 2\end{cases}$ and $g(x) = |f(x)|+f(|x|)$. Then, in the interval (–2, 2), g is **[JEE (Main)-2019]**

(1) Not differentiable at two points

(2) Not differentiable at one point

(3) Not continuous

(4) Differentiable at all points

20. If $x\log_e(\log_e x)-x^2+y^2=4$ $(y>0)$, then $\frac{dy}{dx}$ at $x = e$ is equal to **[JEE (Main)-2019]**

(1) $\frac{(2e-1)}{2\sqrt{4+e^2}}$ (2) $\frac{(1+2e)}{2\sqrt{4+e^2}}$

(3) $\frac{(1+2e)}{\sqrt{4+e^2}}$ (4) $\frac{e}{\sqrt{4+e^2}}$

21. Let K be the set of all real values of x where the function $f(x) = \sin|x| - |x| + 2(x-\pi)$ is not differentiable. Then the set K is equal to

[JEE (Main)-2019]

(1) $\{\pi\}$

(2) ϕ (an empty set)

(3) {0}

(4) $\{0, \pi\}$

22. For $x > 1$, if $(2x)^{2y} = 4e^{2x-2y}$, then $\left(1+\log_e 2x\right)^2\frac{dy}{dx}$ is equal to **[JEE (Main)-2019]**

(1) $\log_e 2x$

(2) $x\log_e 2x$

(3) $\frac{x\log_e 2x+\log_e 2}{x}$

(4) $\frac{x\log_e 2x-\log_e 2}{x}$

23. Let S be the set of all points in $(-\pi, \pi)$ at which the function, $f(x) = \min\{\sin x, \cos x\}$ is not differentiable. Then S is a subset of which of the following? **[JEE (Main)-2019]**

(1) $\left\{-\frac{3\pi}{4},-\frac{\pi}{4},\frac{3\pi}{4},\frac{\pi}{4}\right\}$

(2) $\left\{-\frac{3\pi}{4},-\frac{\pi}{2},\frac{\pi}{2},\frac{3\pi}{4}\right\}$

(3) $\left\{-\frac{\pi}{2},-\frac{\pi}{4},\frac{\pi}{4},\frac{\pi}{2}\right\}$

(4) $\left\{-\frac{\pi}{4},0,\frac{\pi}{4}\right\}$

24. Let f be a differentiable function such that $f(1) = 2$ and $f'(x) = f(x)$ for all $x\in R$. If $h(x) = f(f(x))$, then $h'(1)$ is equal to **[JEE (Main)-2019]**

(1) $2e$ (2) $2e^2$

(3) $4e$ (4) $4e^2$

25. If $2y=\left(\cot^{-1}\left(\frac{\sqrt{3}\cos x+\sin x}{\cos x-\sqrt{3}\sin x}\right)\right)^2, x\in\left(0,\frac{\pi}{2}\right)$ then $\frac{dy}{dx}$ is equal to **[JEE (Main)-2019]**

(1) $2x-\frac{\pi}{3}$ (2) $x-\frac{\pi}{6}$

(3) $\frac{\pi}{3}-x$ (4) $\frac{\pi}{6}-x$

26. If the function f defined on $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ by

$$f(x)=\begin{cases}\dfrac{\sqrt{2}\cos x-1}{\cot x-1}, & x\neq\dfrac{\pi}{4}\\ k, & x=\dfrac{\pi}{4}\end{cases}$$

is continuous, then k is equal to

[JEE (Main)-2019]

(1) 1 (2) $\frac{1}{2}$

(3) $\frac{1}{\sqrt{2}}$ (4) 2

27. If the function $f(x)=\begin{cases}a|\pi-x|+1, & x\leq 5\\ b|x-\pi|+3, & x>5\end{cases}$ is continuous at $x = 5$, then the value of $a - b$ is

[JEE (Main)-2019]

(1) $\frac{2}{\pi-5}$ (2) $\frac{-2}{\pi+5}$

(3) $\frac{2}{\pi+5}$ (4) $\frac{2}{5-\pi}$

28. If $f(x)=\begin{cases}\dfrac{\sin(p+1)x+\sin x}{x}, & x<0\\ q, & x=0\\ \dfrac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}}, & x>0\end{cases}$

is continuous at $x = 0$, then the ordered pair (p, q) is equal to : **[JEE (Main)-2019]**

(1) $\left(\frac{5}{2}, \frac{1}{2}\right)$ (2) $\left(-\frac{3}{2}, \frac{1}{2}\right)$

(3) $\left(-\frac{3}{2}, -\frac{1}{2}\right)$ (4) $\left(-\frac{1}{2}, \frac{3}{2}\right)$

29. Let $f : R \to R$ be differentiable at $c \in R$ and $f(c) = 0$. If $g(x) = |f(x)|$, then at $x = c$, g is :

[JEE (Main)-2019]

(1) Not differentiable if $f'(c) = 0$

(2) Differentiable if $f'(c) = 0$

(3) Not differentiable

(4) Differentiable if $f'(c) \neq 0$

30. The derivative of $\tan^{-1}\left(\frac{\sin x-\cos x}{\sin x+\cos x}\right)$, with respect to $\frac{x}{2}$, where $\left(x\in\left(0,\frac{\pi}{2}\right)\right)$ is

[JEE (Main)-2019]

(1) $\frac{1}{2}$ (2) $\frac{2}{3}$

(3) 2 (4) 1

31. Let the function, $f : [-7, 0] \to R$ be continuous on $[-7, 0]$ and differentiable on $(-7, 0)$. If $f(-7) = -3$ and $f'(x) \leq 2$, for all $x \in (-7, 0)$, then for all such functions f, $f(-1) + f(0)$ lies in the interval

[JEE (Main)-2020]

(1) $[-3, 11]$ (2) $(-\infty, 20]$

(3) $(-\infty, 11]$ (4) $[-6, 20]$

32. If $y(\alpha)=\sqrt{2\left(\frac{\tan\alpha+\cot\alpha}{1+\tan^2\alpha}\right)+\frac{1}{\sin^2\alpha}}, \alpha\in\left(\frac{3\pi}{4},\pi\right)$, then $\frac{dy}{d\alpha}$ at $\alpha=\frac{5\pi}{6}$ is **[JEE (Main)-2020]**

(1) 4 (2) $\frac{4}{3}$

(3) $-\frac{1}{4}$ (4) -4

33. The value of c in the Lagrange's mean value theorem for the function $f(x) = x^3 - 4x^2 + 8x + 11$, when $x \in [0,1]$ is **[JEE (Main)-2020]**

(1) $\frac{4-\sqrt{7}}{3}$ (2) $\frac{4-\sqrt{5}}{3}$

(3) $\frac{2}{3}$ (4) $\frac{\sqrt{7}-2}{3}$

34. If c is a point at which Rolle's theorem holds for the function, $f(x)=\log_e\left(\frac{x^2+\alpha}{7x}\right)$ in the interval $[3, 4]$, where $\alpha \in R$, then $f''(c)$ is equal to

[JEE (Main)-2020]

(1) $-\frac{1}{24}$ (2) $\frac{1}{12}$

(3) $\frac{\sqrt{3}}{7}$ (4) $-\frac{1}{12}$

35. Let S be the set of all functions $f:[0,1]\to R$, which are continuous on [0, 1] and differentiable on (0, 1). Then for every f in S, there exists a $c \in (0, 1)$, depending on f, such that

[JEE (Main)-2020]

(1) $|f(c) - f(1)| < |f'(c)|$

(2) $\dfrac{f(1)-f(c)}{1-c}f'(c)$

(3) $|f(c) - f(1)| < (1 - c)|f'(c)|$

(4) $|f(c) + f(1)| < (1 + c)|f'(c)|$

36. Let f be any function continuous on $[a, b]$ and twice differentiable on (a, b). If for all $x \in (a, b)$, $f'(x) > 0$ and $f''(x) < 0$, then for any $c \in (a, b)$, $\dfrac{f(c)-f(a)}{f(b)-f(c)}$ is greater than

[JEE (Main)-2020]

(1) 1

(2) $\dfrac{b+a}{b-a}$

(3) $\dfrac{c-a}{b-c}$

(4) $\dfrac{b-c}{c-a}$

37. If $f(x)=\begin{cases} \dfrac{\sin(a+2)x+\sin x}{x} & ; \ x<0 \\ b & ; \ x=0 \\ \dfrac{(x+3x^2)^{1/3}-x^{1/3}}{x^{4/3}} & ; \ x>0 \end{cases}$

is continuous at $x = 0$, then $a + 2b$ is equal to

[JEE (Main)-2020]

(1) –1

(2) 1

(3) 0

(4) –2

38. Let f and g be differentiable functions on R such that fog is the identity function. If for some a, $b \in R$, $g'(a) = 5$ and $g(a) = b$, then $f'(b)$ is equal to **[JEE (Main)-2020]**

(1) 1

(2) 5

(3) $\dfrac{1}{5}$

(4) $\dfrac{2}{5}$

39. If $x = 2\sin\theta - \sin 2\theta$ and $y = 2\cos\theta - \cos 2\theta$, $\theta \in [0, 2\pi]$, then $\dfrac{d^2y}{dx^2}$ at $\theta = \pi$ is

[JEE (Main)-2020]

(1) $\dfrac{3}{4}$

(2) $\dfrac{3}{2}$

(3) $-\dfrac{3}{8}$

(4) $-\dfrac{3}{4}$

40. Let $[t]$ denote the greatest integer $\le t$ and $\lim\limits_{x\to 0} x\left[\dfrac{4}{x}\right] = A$. Then the function, $f(x) = [x^2]\sin(\pi x)$ is discontinuous, when x is equal to

[JEE (Main)-2020]

(1) $\sqrt{A+21}$

(2) $\sqrt{A}$

(3) $\sqrt{A+1}$

(4) $\sqrt{A+5}$

41. If a function $f(x)$ defined by **[JEE (Main)-2020]**

$$f(x)=\begin{cases} ae^x+be^{-x}, & -1 \le x<1 \\ cx^2 , & 1 \le x \le 3 \\ ax^2+2cx , & 3<x \le 4 \end{cases}$$

be continuous for some $a, b, c \in R$ and $f'(0) + f'(2) = e$, then the value of a is

[JEE (Main)-2020]

(1) $\dfrac{e}{e^2+3e+13}$

(2) $\dfrac{e}{e^2-3e+13}$

(3) $\dfrac{e}{e^2-3e-13}$

(4) $\dfrac{1}{e^2-3e+13}$

42. If $(a+\sqrt{2}b\cos x)(a-\sqrt{2}b\cos y)=a^2-b^2$, where $a > b > 0$, then $\dfrac{dx}{dy}$ at $\left(\dfrac{\pi}{4},\dfrac{\pi}{4}\right)$ is

[JEE (Main)-2020]

(1) $\dfrac{2a+b}{2a-b}$

(2) $\dfrac{a-b}{a+b}$

(3) $\dfrac{a+b}{a-b}$

(4) $\dfrac{a-2b}{a+2b}$

43. The function $f(x)=\begin{cases} \dfrac{\pi}{4}+\tan^{-1}x, & |x|\le 1 \\ \dfrac{1}{2}(|x|-1), & |x|>1 \end{cases}$ is

[JEE (Main)-2020]

(1) both continuous and differentiable on $R - \{-1\}$

(2) both continuous and differentiable on $R - \{1\}$

(3) continuous on $R - \{-1\}$ and differentiable on $R - \{-1, 1\}$

(4) continuous on $R - \{1\}$ and differentiable on $R - \{-1, 1\}$.

44. If the function

$$f(x) = \begin{cases} k_1(x-\pi)^2 - 1, & x \le \pi \\ k_2 \cos x, & x > \pi \end{cases}$$ is twice

differentiable, then the ordered pair (k_1, k_2) is equal to **[JEE (Main)-2020]**

(1) $(1, 0)$ (2) $\left(\frac{1}{2}, 1\right)$

(3) $(1, 1)$ (4) $\left(\frac{1}{2}, -1\right)$

45. Let $f : R \to R$ be defined as

$$f(x) = \begin{cases} x^5 \sin\left(\frac{1}{x}\right) + 5x^2, & x < 0 \\ 0, & x = 0 \\ x^5 \cos\left(\frac{1}{x}\right) + \lambda x^2, & x > 0 \end{cases}$$

The value of λ for which $f''(0)$ exists, is_______.

[JEE (Main)-2020]

46. Let $f : R \to R$ be a function defined by $f(x) = \max\{x, x^2\}$. Let S denote the set of all points in R, where f is not differentiable. Then

[JEE (Main)-2020]

(1) ϕ(an empty set)

(2) $\{0, 1\}$

(3) $\{1\}$

(4) $\{0\}$

47. If the function f defined on $\left(-\frac{1}{3}, \frac{1}{3}\right)$ by

$$f(x) = \begin{cases} \frac{1}{x}\log_e\left(\frac{1+3x}{1-2x}\right), & \text{when } x \ne 0 \\ k, & \text{when } x = 0 \end{cases}$$

is continuous, then k is equal to _________.

[JEE (Main)-2020]

48. Suppose a differentiable function $f(x)$ satisfies the identity $f(x + y) = f(x) + f(y) + xy^2 + x^2y$, for all real x and y. If $\lim\limits_{x\to 0}\frac{f(x)}{x} = 1$, then $f'(3)$ is equal to _______. **[JEE (Main)-2020]**

49. Let $f(x) = x \cdot \left[\frac{x}{2}\right]$, for $-10 < x < 10$, where $[t]$ denotes the greatest integer function. Then the number of points of discontinuity of f is equal to ______. **[JEE (Main)-2020]**

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