CS 5350/6350: Machine Learining Fall 2018

Homework 1

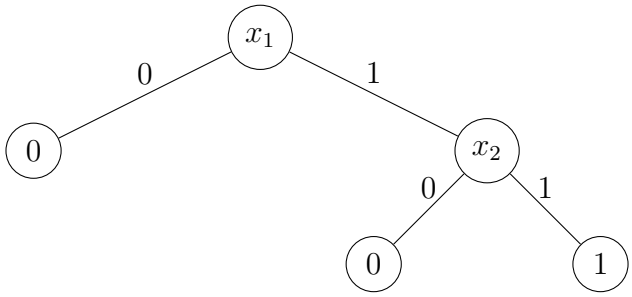
Handed out: 28 August, 2018
Due date: 11 September, 2018

General Instructions

- You are welcome to talk to other members of the class about the homework. I am more concerned that you understand the underlying concepts. However, you should write down your own solution. Please keep the class collaboration policy in mind.
- Feel free discuss the homework with the instructor or the TAs.
- Your written solutions should be brief and clear. You need to show your work, not just the final answer, but you do *not* need to write it in gory detail. Your assignment should be **no more than 10 pages**. Every extra page will cost a point.
- Handwritten solutions will not be accepted.
- The homework is due by midnight of the due date. Please submit the homework on Canvas.
- Some questions are marked **For 6350 students**. Students who are registered for CS 6350 should do these questions. Of course, if you are registered for CS 5350, you are welcome to do the question too, but you will not get any credit for it.

1 Decision Trees

1. [6 points] In this warm-up question, you will be drawing decision trees for boolean functions. For example, the following decision tree represents the function $x_1 \wedge x_2$.



Represent the following boolean functions as decision trees. You can use the tikz code for the tree above as reference to create your trees. (See the latex source of the homework for the code.) If you are more comfortable with GUI, you can use a tool such as www.draw.io to create your tree and export it as a png.

(a) $(x_1 \wedge x_2) \vee \neg x_3$

(b) x_1 xor $(x_2 \wedge x_3)$

(c) $x_1 \wedge (x_2 \vee (x_3 \wedge x_4))$

2. [24 points] Mark loves mangoes. Unfortunately he is lousy at picking ripe mangoes at the grocery. He needs your help. You need to build a decision tree that will help Mark decide if a mango is ripe or not. You need to make the decision based on four features described below:

   (a) **Variety** (*Alphonso, Keitt or Haden*): Describes the variety of the mango.

   (b) **Color** (*Red, Yellow or Green*): Describes the color of the mango.

   (c) **Smell** (*Sweet or None*): Describes the smell of the mango.

   (d) **Time** (*One or Two*): Number of weeks since the mango was plucked.

   You are given the following dataset which contains data for 8 different mangoes. For each mango, the values of the above four features are listed. The label of whether the mango was ripe or not is also provided.

Variety	Color	Smell	Time	Ripe?
Alphonso	Red	None	Two	False
Keitt	Red	None	One	True
Alphonso	Yellow	Sweet	Two	True
Keitt	Green	None	Two	False
Haden	Green	Sweet	One	True
Alphonso	Yellow	None	Two	False
Keitt	Yellow	Sweet	One	False
Alphonso	Red	Sweet	Two	True

Table 1: Training data for the mango prediction problem.

   (a) [5 points] How many possible functions are there to map these four features to a boolean decision? How many functions are consistent with the given training dataset?

   (b) [3 points] What is the entropy of the labels in this data? When calculating entropy, the base of the logarithm should be base 2.

   (c) [4 points] Compute the information gain of each feature and enter it into Table 2. Specify upto 3 decimal places.

   (d) [1 points] Which attribute will you use to construct the root of the tree using the information gain heuristic of the ID3 algorithm?

   (e) [8 points] Using the root that you selected in the previous question, construct a decision tree that represents the data. You do not have to use the ID3 algorithm here, you can show any tree with the chosen root.

Feature	Information Gain
Variety	
Color	
Smell	
Time	

Table 2: Information gain for each feature.

(f) [3 points] Suppose you are given three more examples, listed in Table 3. Use your decision tree to predict the label for each example. Also report the accuracy of the classifier that you have learned.

Variety	Color	Smell	Time	Ripe?
Alphonso	Green	Sweet	Two	True
Keitt	Red	Sweet	One	False
Haden	Yellow	None	Two	True

Table 3: Test data for mango prediction problem

3. [10 points] Recall that in the ID3 algorithm, we want to identify the best attribute that splits the examples that are relatively pure in one label. Aside from entropy, which we saw in class and you used in the previous question, there are other methods to measure impurity.

We will now develop a variant of the ID3 algorithm that does not use entropy. If, at some node, we stopped growing the tree and assign the most common label of the remaining examples at that node, then the empirical error on the training set at that node will be

$$MajorityError = 1 - \max_i p_i$$

where, p_i is the fraction of examples that are labeled with the i^{th} label.

(a) [2 points] Notice that $MajorityError$ can be thought of as a measure of impurity just like entropy. Just like we used entropy to define information gain, we can define a new version of information gain that uses $MajorityError$ in place of entropy. Write down an expression that defines a new version of information gain that uses $MajorityError$ in place of entropy.

(b) [6 points] Calculate the value of your newly defined information gain from the previous question for the four features in the mango dataset from 1. Use 3 significant digits. Enter the information gain into Table 4.

(c) [2 points] According to your results in the last question, which attribute should be the root for the decision tree? Do these two measures (entropy and majority error) lead to the same tree?

Feature	Information Gain (using majority error)
Variety	
Color	
Smell	
Time	

Table 4: Information gain for each feature.

2 Linear Classifier

In the questions in this section, we have four features x_1, x_2, x_3 and x_4 and the label is represented by o.

1. [5 points] Find a linear classifier that correctly classifies the given dataset. You dont need to run any learning algorithm here. Recall that a linear classifier is described by a weight vector $\mathbf{w}$ and a bias b. The classifier predicts 1 if $\mathbf{w}^T\mathbf{x}+b \geq 0$ and -1 otherwise. For this problem, the dataset has 4 features, so the weight vector has 4 components w_1, w_2, w_3 and w_4. So the linear classifier will predict 1 if $w_1x_1+w_2x_2+w_3x_3+w_4x_4+b \geq 0$ and -1 otherwise. Specify the values of w_1, w_2, w_3, w_4 and b that correctly predicts the label for the dataset below.

x_1	x_2	x_3	x_4	o
1	1	1	1	1
0	1	1	1	1
1	1	0	0	-1
1	0	0	0	-1

2. [6 points] Suppose the dataset below is an extension of the above dataset. Check if your classifier from the previous question correctly classifies the dataset. Report its accuracy.

x1	x2	x3	x4	o
0	0	0	0	-1
0	0	0	1	-1
0	0	1	0	-1
0	0	1	1	-1
1	0	1	1	1
1	1	0	1	1

3. [9 points] Given the remaining missing data points of the above dataset in the table below, find a linear classifier that correctly classifies the whole dataset (all three tables together). Specify the values of w_1, w_2, w_3, w_4 and b for the classifier.

x1	x2	x3	x4	o
0	1	0	0	-1
0	1	0	1	-1
0	1	1	0	-1
1	0	0	1	-1
1	0	1	0	-1
1	1	1	0	1

3 Experiments

This problem uses a modified version of the famous the Mushroom Dataset from the UCI machine learning repository. Each data point has 21 features indicating different characteristics of a mushroom. You can find definitions of each feature in information.txt file. The labels are either 'p' or 'e' which stand for poisonous and edible, respectively. Your task is to implement the ID3 algorithm and build a decision tree using the training data `data/train.csv`. The goal of the decision tree is to learn how to distinguish between poisonous and edible mushrooms. We also provide the test set `data/test.csv` to evaluate how your decision tree classifier is performing.

You may use any programming language for your implementation. However, the graders should be able to execute your code on the CADE machines without having to install any libraries.

In the folder, you will also find the file `data.py`, which is code that will help you with the data loading and manipulation if you choose to use Python. We have also included a Jupyter notebook `datademo.ipynb` that demonstrates how to use the file and what functionality it provides. Using the code is not required, although we think it should help you with the implementation of the algorithm.

Cross-Validation

The depth of the tree is a *hyper-parameter* to the decision tree algorithm that helps reduce overfitting. By depth, we refer to the maximum path length from the root to any leaf. That is, a tree with just a single node has depth 0, a tree with a root attribute A leading to labels in one step has depth 1 and so on. You will see later in the semester that many machine learning algorithm (SVM, logistic-regression etc) have some hyper-parameters as their input. One way to determine a proper value for the hyper-parameter is to use a technique called cross-validation.

As usual we have a training set and a test set. Our goal is to discover good hyper-parameters using the training set *only*. To do so, you can put aside some of the training data aside, and when training is finished, you can test the resulting classifier on the held out data. This allows you to get an idea of how well the particular choice of hyper-parameters does. However, since you did not train on your whole dataset you may have introduced a statistical bias in the classifier. To correct for this, you will need to train many classifiers with different subsets of the training data removed and average out the accuracy across these trials.

For problems with small data sets, a popular method is the leave-one-out approach. For each example, a classifier is trained on the rest of the data and the chosen example is then evaluated. The performance of the classifier is the average accuracy on all the examples. The downside to this method is for a data set with n examples you must train n different classifiers. Of course, this is not practical for the data set you will use in this problem, so you will hold out subsets of the data many times instead.

Specifically, for this problem, you should implement k-fold cross validation. The general approach for k-fold cross validation is the following: Suppose you want to evaluate how good a particular hyper-parameter is. You randomly split the training data into k equal sized parts. Now, you will train your model on all but one part with the chosen hyper-parameter and evaluate the trained model on the remaining part. You should repeat this k times, choosing a different part for evaluation each time. This will give you k values of accuracy. Their average cross-validation accuracy gives you an idea of how good this choice of the hyper-parameter is. To find the best value of the hyper-parameter, you will need to repeat this procedure for different choices of the hyper-parameter. Once you find the best value of the hyper-parameter, use the value to retrain you classifier using the entire training set.

1. [20 points] **Implementation**

   For this problem, your will be using the data in **data** folder. This folder contains two files: **train.csv** and **test.csv**. You will train your algorithm on the training file. Remember that you should not look at or use your testing file until your training is complete.

   (a) [15 points] Implement the decision tree data structure and the ID3 algorithm for your decision tree (Remember that the decision tree need not be a binary tree!). For debugging your implementation, you can use the previous toy examples like the mango data from Table 1. Discuss what approaches and design choices you had to make for your implementation and what data structures you used.

   (b) [1 points] Report the error of your decision tree on the **data/train.csv** file.

   (c) [1 points] Report the error of your decision tree on the **data/test.csv** file.

   (d) [3 points] Report the maximum depth of your decision tree.

2. [20 points] **Limiting Depth**

   In this section, you will be using 5-fold cross-validation in order to limit the depth of your decision tree, effectively pruning the tree to avoid overfitting. You will be using the 5 cross-validation files for this section, titled data/CVfolds/foldX.csv where X is a number between 1 and 5 (inclusive)

   (a) [10 points] Run 5-fold cross-validation using the specified files. Experiment with depths in the set $\{1, 2, 3, 4, 5, 10, 15\}$, reporting the average cross-validation accuracy and standard deviation for each depth. Explicitly specify which depth should be chosen as the best, and explain why.

   (b) [5 points] Using the depth with the greatest cross-validation accuracy from your experiments: train your decision tree on the **data/train.csv** file. Report the accuracy of your decision tree on the **data/test.csv** file.

(c) [5 points] Discuss the performance of the depth limited tree as compared to the full decision tree. Do you think limiting depth is a good idea? Why?

Experiment Submission Guidelines

1. The report should detail your experiments. For each step, explain in no more than a paragraph or so how your implementation works. You may provide the results for the final step as a table or a graph.

2. *Your code should run on the CADE machines.* You should include a shell script, `run.sh`, that will execute your code in the CADE environment. Your code should produce similar output to what you include in your report.

   You are responsible for ensuring that the grader can execute the code using only the included script. If you are using an esoteric programming language, you should make sure that its runtime is available on CADE.

3. Please do not hand in binary files! We will *not* grade binary submissions.

4 CS 6350 only: Decision Trees with Attribute Costs

[10 points] Sometimes, we may encounter situations where the features in our learning problem are associated with costs. For example, if we are building a classifier in a medical scenario, features may correspond to the results of different tests that are performed on a patient. Some tests may be inexpensive (or inflict no harm), such as measuring the patient's body temperature or weight. Some other tests may be expensive (or may cause discomfort to the patient), such as blood tests or radiographs.

In this question, we will explore the problem of learning decision trees in such a scenario. We prefer decision trees that use features associated with low costs at the top of the tree and only use higher cost features if needed at the bottom of the trees. In order to impose this preference, we can modify the information gain heuristic that selects attributes at the root of a tree to penalize costly attributes.

In this question, we will explore different such variations. Suppose we denote $Gain(S, A)$ as the information gain of an attribute A for a dataset S (using the original version of information from ID3). Let $Cost(A)$ denote the cost of the attribute A. We can define two cost-sensitive information gain criteria for attributes as:

1. $Gain_T(S, A) = \frac{Gain(S,A)^2}{Cost(A)}$

2. $Gain_N(S, A) = \frac{2^{Gain(S,A)}-1}{\sqrt{Cost(A)+1}}$

In both cases, note that attributes with higher costs are penalized and so will get chosen only if the information gain is really high.

To evaluate these two methods for root selection, we will use the following training set:

Shape	Color	Size	Material	Label
square	red	big	metal	+
square	blue	small	plastic	+
triangle	yellow	medium	metal	+
triangle	pink	big	leather	-
square	pink	medium	leather	-
circle	red	small	plastic	-
circle	blue	small	metal	-
ellipse	yellow	small	plastic	-
ellipse	blue	big	leather	+
ellipse	pink	medium	wood	+
circle	blue	big	wood	+
triangle	blue	medium	plastic	+

Suppose we know the following costs of the attributes:

Attribute	Cost
Shape	10
Color	30
Size	50
Material	100

1. [8 points] Compute the modified gains $Gain_T$ and $Gain_S$ for each attribute using these costs. Fill in your results in the table below. (upto 3 decimal places)

Attribute	$Gain_T$	$Gain_S$
Shape		
Color		
Size		
Material		

2. [2 points] For each variant of gain, which feature would you choose as the root?

(You may modify your code from the experiments to calculate these values instead of doing so by hand.)