Manish Semwal

204101032

Ans 1: No.

Counter example:

Let us consider an instance of 3 men and 3 women represented by m1, m2, m3 and w1, w2, w3 resp.

Men's initial preference list in decreasing order of preference:

	1st Preference	2nd Preference	3rd Preference
m1	w1	w2	w3
m2	w2	w3	w1
m3	w3	w1	w2

Women's initial preference list in decreasing order of preference:

	1st Preference	2nd Preference	3rd Preference
w1	m1	m2	m3
w2	m1	m2	m3
w3	m1	m2	m3

Stable matching S: m1-w1, m2-w2, m3-w3

Reversed preference list of all women which is their new preference list, in decreasing order of preference is given by:

	1st Preference	2nd Preference	3rd Preference
w1	m3	m2	m1
w2	m3	m2	m1
w3	m3	m2	m1

Still, there is no unstable pair in S as,

- m1 neither prefers w2 nor w3 over w1
- m2 neither prefers w1 nor w3 over w2
- m3 neither prefers w1 nor w2 over w3

Hence, "a stable matching becomes unstable if we reverse the preference list of all the women" is not necessarily true.

Ans 2: No.

Counter example:

Let us consider an instance of 3 men and 3 women represented by m1, m2, m3 and w1, w2, w3 resp.

Men's initial preference list in decreasing order of preference:

	1st Preference	2nd Preference	3rd Preference
m1	w1	w2	w3
m2	w1	w2	w3
m3	w1	w2	w3

Women's initial preference list in decreasing order of preference:

	1st Preference	2nd Preference	3rd Preference
w1	m1	m2	m3
w2	m1	m2	m3
w3	m1	m2	m3

Stable matching S: m1-w1, m2-w2, m3-w3

Reversed preference list of all men which is their new preference list, in decreasing order of preference is given by:

	1st Preference	2nd Preference	3rd Preference
m1	w3	w2	w1
m2	w3	w2	w1
m3	w3	w2	w1

Reversed preference list of all women which is their new preference list, in decreasing order of preference is given by:

	1st Preference	2nd Preference	3rd Preference
w1	m3	m2	m1
w2	m3	m2	m1
w3	m3	m2	m1

Still, there is no unstable pair in S as,

- m1 prefers w2 over w1 but w2 doesn't prefer m1 over m2 & m1 prefers w3 over w1 but w3 doesn't prefer m1 over m3
- m2 doesn't prefers w1 over w2 & m2 prefers w3 over w2 but w3 doesn't prefer m2 over m3
- m3 neither prefers w1 nor w2 over w3

Hence, “a stable matching becomes unstable if we reverse the preference list of all the women and all the men” is not necessarily true.

Ans 3: No.

Conter example:

Let us consider an instance of 3 men and 3 women represented by m1, m2, m3 and w1, w2, w3 resp.

Men’s preference list in decreasing order of preference:

	1st Preference	2nd Preference	3rd Preference
m1	w3	w2	w1
m2	w3	w1	w2
m3	w1	w2	w3

Women’s preference list in decreasing order of preference:

	1st Preference	2nd Preference	3rd Preference
w1	m1	m2	m3
w2	m1	m2	m3
w3	m1	m2	m3

6 Perfect matchings:

1. m1-w1, m2-w2, m3-w3: regret=3+3+3+1+2+3=15
2. m1-w1, m2-w3, m3-w2: regret=3+1+2+1+2+3=12
3. m1-w2, m2-w1, m3-w3: regret=2+2+3+1+2+3=13
4. m1-w2, m2-w3, m3-w1: regret=2+1+1+1+2+3=10
5. m1-w3, m2-w1, m3-w2: regret=1+2+2+1+2+3=11
6. m1-w3, m2-w2, m3-w1: regret=1+3+1+1+2+3=11

Minimum regret = 10 for 4th perfect matching which is m1-w2, m2-w3, m3-w1.

Here,

- m1-w3 is an unstable pair, since m1 prefers w3 over w2 and w3 prefers m1 over m2.

Hence, for an instance a perfect matching that have minimum regret is **not necessarily stable**.