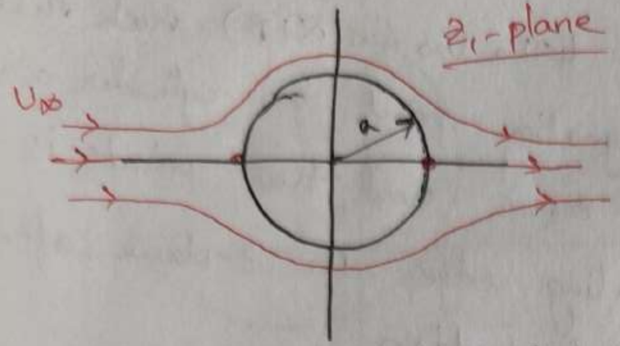
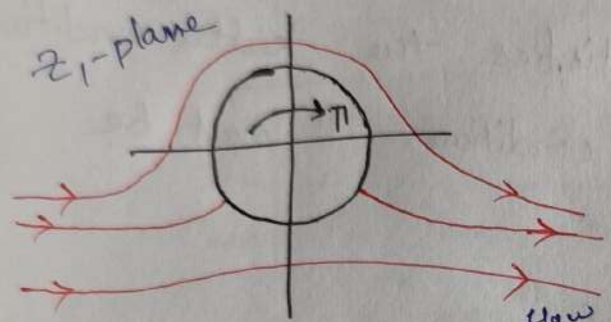
Q1 Let start with a non-lifting over a circular cylinder of radius 'a' in z_1-plane



Complex potential for this flow

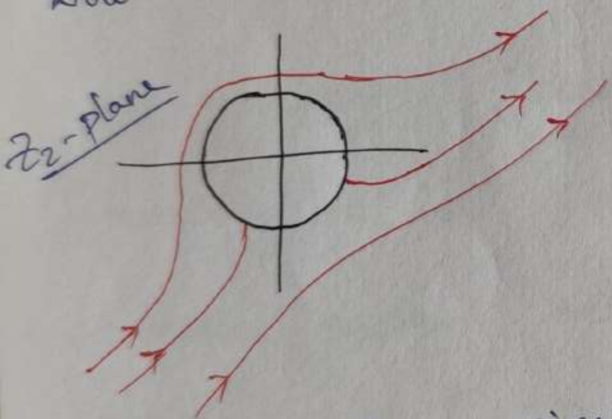
$$W(z_1) = U_\infty z_1 + \frac{U_\infty a^2}{z_1} \quad -(1.1)$$

add circulation (Γ) to get lifting flow



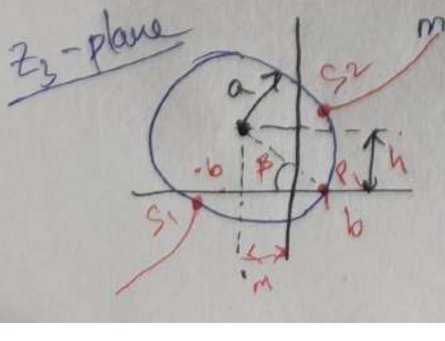
$$w(z_1) = U_\infty z_1 + \frac{U_\infty a^2}{z_1} + \frac{i\Gamma}{2\pi}\ln\frac{z_1}{a} \quad -(1.2)$$

Now rotate the ~~[illegible]~~ flow by angle α anticlockwise, $z_2 = z_1 e^{i\alpha}$



$$z_2 = z_1 e^{i\alpha} \quad -(1.3)$$

Lets shift the circle in the both real and imaginary axes. Such that the circle intersects the real axes twice. Also one of the intersecting point should be at a distance 'b' which is the joukowski constant. So that we get a sharp corner. ~~[illegible]~~



$m + b = a\cos\beta$

$\Rightarrow m = a\cos\beta - b \quad -(1.4)$

$h = a\sin\beta$

$m << a, b.$

In the z_3-plane, $z_3 = z_2 - m + ia\sin\beta = z_2 - (a\cos\beta - b) + ia\sin\beta$. (1.5)

applying the jokowski transformation to get cambered airfoil in z-plane. - (1.6)

$$\frac{dw}{dz} = \frac{dw}{dz_1}\,\frac{dz_1}{dz_2}\cdot\frac{dz_2}{dz_3}\cdot\frac{dz_3}{dz}. \leftarrow \text{complex velocity.}$$

joukowski transformation $\quad z = z_3 + \dfrac{b^2}{z_3}$. - (1.7)

using the above equations.

$$\frac{dw}{dz} = \left(U_\infty\left(1-\frac{a^2}{z_1^2}\right) + \frac{i\Gamma}{2\pi z_1}\right)e^{-i\alpha}\,\frac{1}{1-\frac{b^2}{z_3^2}}. \quad - (1.8)$$

To satisfy the kutta condition, S_2 point must be at point P_1 $z_3 = b$. This condition helps us to find value of the circulation (Γ).

from eq^n (1.5) $\quad z_2 = \cancel{b} + a\cos\beta - \cancel{b} - ia\sin\beta$

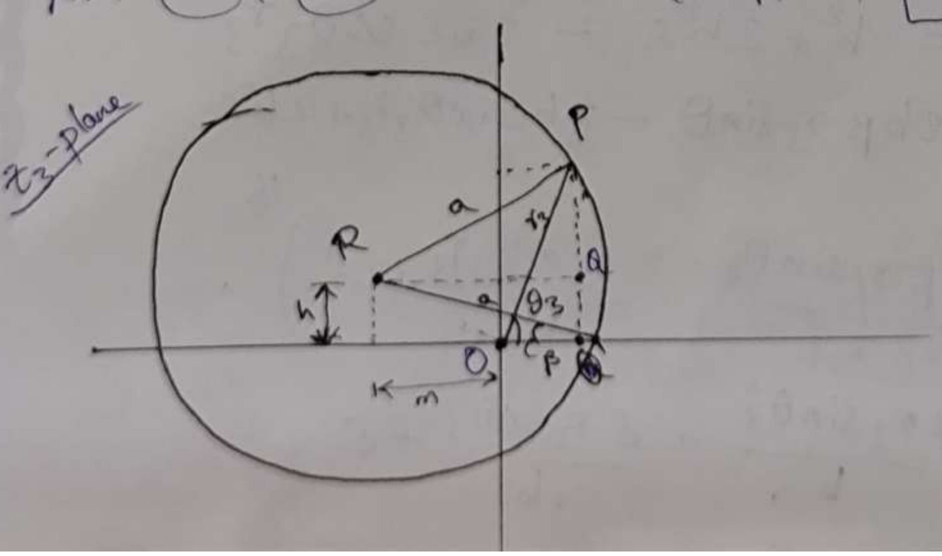
$\Rightarrow \quad z_2 = a\cos\beta - ia\sin\beta. = ae^{-i\beta}$ — (1.9)

from eq^n (1.3) $\quad z_1 = ae^{-i\beta}e^{-i\alpha} = ae^{-i(\alpha+\beta)}$. - (1.10).

Now in z_1-plane we know a relation of circulation and angular location of a stagnation point.

$$\sin\theta_s = -\frac{\Gamma}{4\pi U_\infty a}. \quad - (1.11).$$

from (1.10) & (1.11) $\quad \theta_s = -(\alpha+\beta) \Rightarrow \boxed{\Gamma = 4\pi U_\infty a\sin(\alpha+\beta)}$ -(1.12)



from the above figure

$$PQ = r_3 \sin\theta_3 - h = r_3 \sin\theta_3 - a\sin\beta$$

also $PQ = \sqrt{a^2 - (m + r_3\cos\theta_3)^2}$

$$\Rightarrow \quad (r_3 \sin\theta_3 - a\sin\beta)^2 = a^2 - (m + r_3\cos\theta_3)^2 \quad -(1.13)$$

$$\Rightarrow \quad r_3^2 \sin^2\theta_3 + a^2\sin^2\beta - 2ar_3\sin\beta\sin\theta_3$$

$$= a^2 - m^2 - r_3^2\cos^2\theta_3 - 2mr_3\cos\theta_3$$

also $\quad h = a\sin\beta \approx a\beta . \quad -(1.14)$

$$a\cos\beta = m + b \Rightarrow a \approx m + b = b(\varepsilon+1) \quad \because \varepsilon = \frac{m}{b} .$$

$$-(1.15)$$

$$\Rightarrow \quad a = b(\varepsilon+1)$$

$$r_3^2\sin^2\theta_3 + b^2(\varepsilon+1)^2\beta^2 - 2b(\varepsilon+1)r_3\beta\sin\theta_3$$

$$= b^2(\varepsilon+1)^2 - b^2\varepsilon^2 - r_3^2\cos^2\theta_3 - 2b\varepsilon\cos\theta_3 . r_3$$

retain 1st order terms.

$$r_3^2\sin^2\theta_3 - 2b\beta(\varepsilon+1)r_3\sin\theta_3 = b^2(\overset{\approx 0}{\cancel{\varepsilon^2}} + 1 + 2\varepsilon)$$

$$- r_3^2\cos^2\theta_3 - 2b\varepsilon\cos\theta_3 r_3$$

$$r_3^2(\sin^2\theta_3 + \cos^2\theta_3) - 2b\overset{\approx 0}{\cancel{\beta\varepsilon}}r_3\sin\theta_3 - 2b\beta r_3\sin\theta_3$$

$$= b^2 + 2b^2\varepsilon - 2b\varepsilon\cos\theta_3 r_3$$

$$r_3^2 = b^2 + 2b\beta r_3\sin\theta - 2b\varepsilon\cos\theta_3 r_3 + 2b^2\varepsilon .$$

$$\frac{r_3}{b} = \left(1 + \frac{2\beta r_3\sin\theta_3}{b} - \frac{2\varepsilon\cos\theta_3 r_3}{b} + 2\varepsilon\right)^{1/2}$$

$$\frac{r_3}{b} = 1 + \frac{\beta r_3\sin\theta_3}{b} - \frac{\varepsilon r_3\cos\theta_3}{b} + \varepsilon$$

$$\frac{r_3}{b} = \frac{r_3}{b}\left(\beta \sin\theta_3 - \varepsilon\cos\theta_3\right) + (1+\varepsilon)$$

$$\Rightarrow \frac{r_3}{b} = \frac{(1+\varepsilon)}{(1-\beta\sin\theta_3 + \varepsilon\cos\theta_3)}$$

again $\frac{r_3}{b} = (1+\varepsilon)(1+\beta\sin\theta_3 + \varepsilon\cos\theta_3)$

$$\Rightarrow \frac{r_3}{b} = 1+\beta\sin\theta_3 + \varepsilon\cos\theta_3 + \varepsilon + \cancel{\varepsilon\beta}^{\approx 0}\sin\theta_3 + \cancel{\varepsilon^2}^{=0}\cos\theta_3$$

retaining only first order terms, $\varepsilon\beta \approx 0$, $\varepsilon^2 \approx 0$

$$\Rightarrow \boxed{\frac{r_3}{b} = 1+\varepsilon + \beta\sin\theta_3 + \varepsilon\cos\theta_3} \quad - (1.16).$$

"Airfoil equation in z_3-plane."

To get the airfoil equation in x, y co-ordinates, we do the following.

from eq^n 1.7, $z = z_3 + \frac{b^2}{z_3}$.

$$\Rightarrow x+iy = r_3 e^{i\theta_3} + \frac{b^2}{r_3}e^{-i\theta_3} = \left(r_3 + \frac{b^2}{r_3}\right)\cos\theta_3 + i\left(r_3 - \frac{b^2}{r_3}\right)\sin\theta_3.$$

$$\Rightarrow \frac{x}{\cos\theta_3} = r_3 + \frac{b^2}{r_3}, \quad \frac{y}{\sin\theta_3} = r_3 - \frac{b^2}{r_3}. \quad - (1.17)$$

$\Rightarrow$ from eq^n 1.16, $\frac{b}{r_3} = \frac{1}{1+\varepsilon+\beta\sin\theta_3 - \varepsilon\cos\theta_3}$

$$\Rightarrow \frac{b^2}{r_3} = b\left(1-\varepsilon+\varepsilon\cos\theta_3 - \beta\sin\theta_3\right) \quad - (1.18)$$

$$r_3 + \frac{b^2}{r_3} = b\left[1+\varepsilon+\beta\sin\theta_3 - \varepsilon\cos\theta_3 + 1-\varepsilon+\varepsilon\cos\theta_3 - \beta\sin\theta_3\right] \quad - (1.19)$$

$$= 2b.$$

$$r_3 - \frac{b^2}{r_3} = 2b\left(\varepsilon - \varepsilon\cos\theta_3 + \beta\sin\theta_3\right) \quad - (1.20).$$

from eqn's 1.17, 1.19, 1.20,

$$x = 2b\cos\theta_3 \Rightarrow \cos\theta_3 = \frac{x}{2b} \Rightarrow \sin\theta_3 = \sqrt{1-\left(\frac{x}{2b}\right)^2}.$$

Substitute $\cos\theta_3$, $\sin\theta_3$ into equation 1.17.

$$\frac{y}{\sin\theta_3} = r_3 - \frac{b^2}{r_3} = 2b\left(\varepsilon - \varepsilon\cos\theta_3 + \beta\sin\theta_3\right)$$

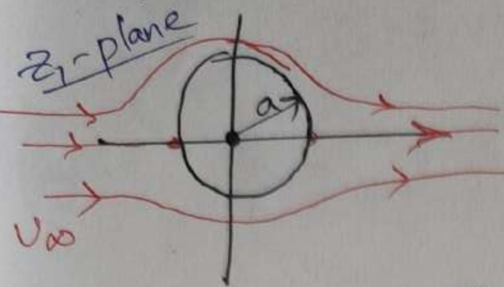
$$y = 2b.\sin\theta_3\left(\varepsilon - \varepsilon\cos\theta_3 + \beta\sin\theta_3\right)$$

$$\Rightarrow y = 2b\sqrt{1-\left(\frac{x}{2b}\right)^2}\left(\varepsilon - \varepsilon.\frac{x}{2b} + \beta\sqrt{1-\left(\frac{x}{2b}\right)^2}\right).$$

$$\Rightarrow \boxed{y = 2b\sqrt{1-\left(\frac{x}{2b}\right)^2}\left[\varepsilon\left(1-\frac{x}{2b}\right) + \beta\sqrt{1-\left(\frac{x}{2b}\right)^2}\right]}$$

"Airfoil equation" — 1.21 —

Q2 Lets start with non-lifting flow over a circular cylinder of radius a & center at origin.

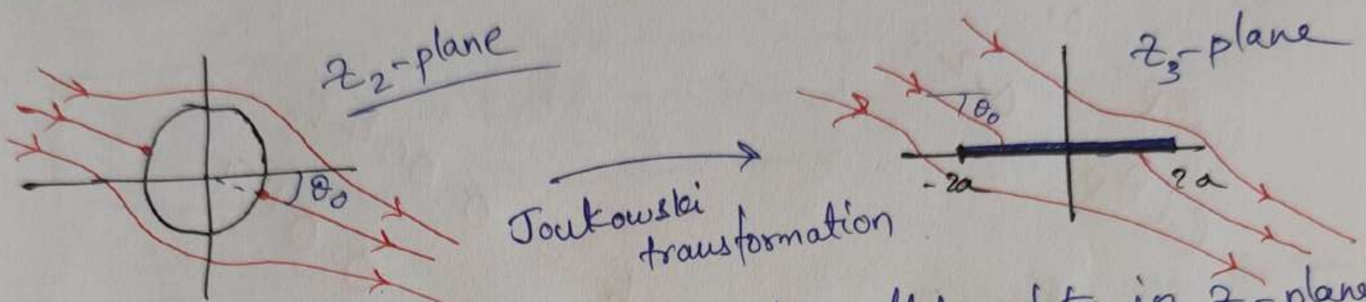


complex potential

$$w(z_1) = U_\infty z_1 + \frac{U_\infty a^2}{z_1} \quad - (2.1)$$

Now rotate the ~~axes~~ flow by an angle θ_0 ~~anti~~- clockwise direction.

$$z_2 = z_1 e^{-i\theta_0} \quad - (2.2)$$



Now transform the cylinder to flate plate in z_3-plane by choosing joukowski constant $b = a$.

$$z_3 = z_2 + \frac{a^2}{z_2} \quad - (2.3)$$

Now eq^n (2.1) & (2.2)

$$\frac{w}{U_\infty} = z_1 + \frac{a^2}{z_1} = z_2 e^{i\theta_0} + \frac{a^2}{z_2} e^{-i\theta_0}. \quad - (2.4)$$

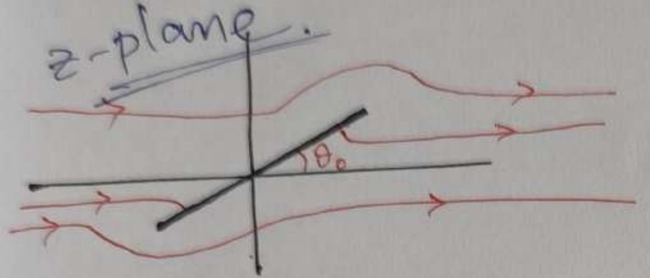
from eq^n (2.3) $\quad z_2 = \dfrac{z \pm \sqrt{z^2 - 4a^2}}{2}$.

only $z_2 = \dfrac{z + \sqrt{z^2-4a^2}}{2}$ ~~only~~ satisfies far field flow condition.

Note, $L = 4a$

$$\Rightarrow \boxed{w(z) = \left(\frac{z + \sqrt{z^2-4a^2}}{2}\right) U_\infty e^{i\theta_0} + \frac{U_\infty a^2 \, e^{-i\theta_0}}{\left(\frac{z + \sqrt{z^2-4a^2}}{2}\right)}} \quad - (2.5).$$

Now rotate the ~~axes~~ flow by θ_0 angle in anti clockwise rotation so that flow (oncoming) is parellel to real axis as given in the problem. $z = z_3 e^{i\theta_0}$ — (2.4).



Now from eq^n (2.1) & eq^n (2.2)

$$\omega = U_\infty z_2 e^{i\theta_0} + \frac{U_\infty a^2}{z_2} e^{-i\theta_0} \quad -(2.5)$$

from eq^n (2.3). $$z_2 = \frac{z_3 \pm \sqrt{z_3^2 - 4a^2}}{2}$$

only $z_2 = \dfrac{z_3 \mp \sqrt{z_3^2 - 4a^2}}{2}$. satisfies farfield flow condition. — (2.6)

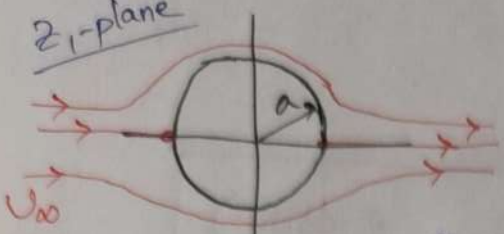
$$\frac{\omega}{U_\infty} = \left(\frac{z_3 \pm \sqrt{z_3^2 - 4a^2}}{2}\right) e^{i\theta_0} + \frac{a^2 e^{-i\theta_0}}{\left(\dfrac{z_3 + \sqrt{z_3^2 - 4a^2}}{2}\right)} \quad -(2.7)$$

finally use eq^n (2.4).

$$\boxed{\omega(z) = U_\infty \left(\frac{z e^{-i\theta_0} \pm \sqrt{z^2 e^{-2i\theta_0} - 4a^2}}{2}\right) e^{i\theta_0} + \frac{U_\infty a^2 e^{-i\theta_0}}{\left(\dfrac{z e^{-i\theta_0} + \sqrt{z^2 e^{-2i\theta_0} - 4a^2}}{2}\right)}} \quad -(2.8)$$

$\boxed{\text{Note: } L = 4a}$

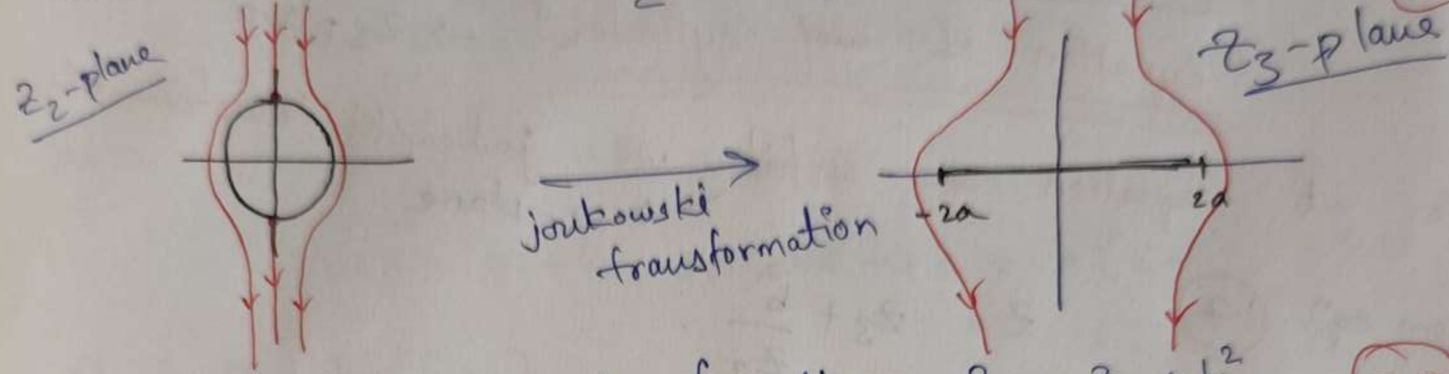
Q3 Let start again with a non-lifting flow over a cylinder with radius a and center at origin.



complex potential

$$w(z_1) = U_\infty z_1 + \frac{U_\infty a^2}{z_1} \quad - (3.1)$$

Now rotate ~~axes~~ flow $\frac{\pi}{2}$ in anti-clockwise direction, $z_2 = z_1 e^{-i\pi/2}$ $- (3.2)$

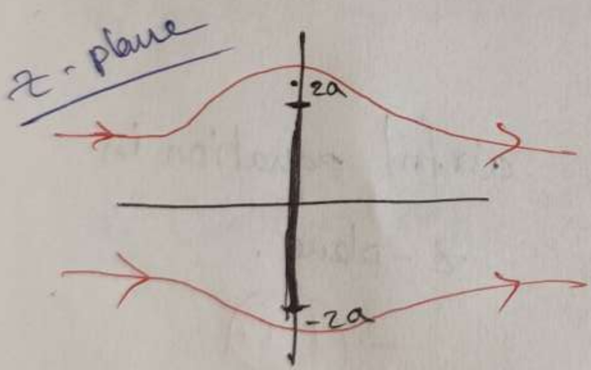


applying joukowski transformation $z_3 = z_2 + \frac{b^2}{z_2}$ $- (3.3)$

when $b = a$, then cylinder turns / transforms to ~~pl~~ flat plate of length $- 4a$.

Now rotate the ~~axes~~ flow $\frac{\pi}{2}$ in anti-clockwise direction.

$$z = z_3 e^{i\pi/2} \quad - (3.4)$$



use equations 3.2, 3.3, 3.4 & (3.1)

$$\frac{w}{U_\infty} = z_1 + \frac{a^2}{z_1} = z_2 e^{i\pi/2} + \frac{a^2}{z_2} e^{-i\pi/2}$$

$$\Rightarrow \frac{w}{U_\infty} = z_2 i + \frac{a^2}{z^2}(-i) = \left(z_2 - \frac{a^2}{z_2}\right) i$$

$\Rightarrow \dfrac{w}{U_\infty} = -\left(z_2 - \dfrac{a^2}{z_2}\right) i$ — (3.5).

NOW from eq^n (3.4). $z = z_3 i = \left(z_2 + \dfrac{a^2}{z_2}\right) i$ — (3.6).

Square & subtract 3.5 & 3.6.

$$\frac{w^2}{U_\infty^2} - z^2 = -\left(z_2 - \frac{a^2}{z_2}\right)^2 + \left(z_2 + \frac{a^2}{z_2}\right)^2$$

$$= -\cancel{z_2^2} - \cancel{\frac{a^4}{z_2^2}} + 2a^2 + \cancel{z_2^2} + \cancel{\frac{a^4}{z_2^2}} + 2a^2.$$

$$\frac{w^2}{U_\infty^2} - z^2 = 4a^2 \Rightarrow w^2 = U_\infty^2 (z^2 + 4a^2)$$

$$\Rightarrow \boxed{w(z) = U_\infty \sqrt{z^2 + 4a^2}} \quad - (3.7).$$

"complex potential."

Now consider $L \to \infty$, where $L = 4a$

$$\Rightarrow \frac{w}{U_\infty} = \sqrt{z^2 + \frac{(4a)^2}{4}} = \sqrt{z^2 + \frac{L^2}{4}} = \frac{L}{2}\left(1 + \left(\frac{2z}{L}\right)^2\right)^{1/2}$$

$\Rightarrow L \to \infty$, $\dfrac{z}{L} \to$ small number (near the plate).

$$\Rightarrow \frac{w}{U_\infty} \approx \frac{L}{2}\left(1 + \frac{1}{2}\cdot\left(\frac{2z}{L}\right)^2\right)$$

$$\Rightarrow \frac{w}{U_\infty} \approx \frac{L}{2} + \frac{\cancel{L}}{\cancel{2}}\cdot\frac{1}{\cancel{2}}\cdot\frac{\cancel{4}z^2}{L^{\cancel{2}}} \approx \frac{L}{2} + \frac{z^2}{L}$$

$$\Rightarrow \boxed{w = U_\infty \frac{L}{2} + \frac{U_\infty}{L} z^2} \quad - (3.8)$$

we know complex potential of stagnation point flow. $\boxed{W(z) = Az^2}$ – (3.9)

There eqn (3.8) that we got $w(z) = \frac{U_\infty L}{2} + \frac{U_\infty}{L} z^2$

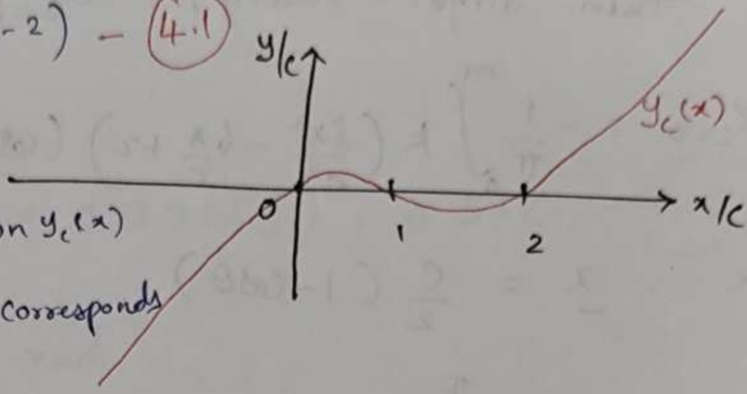
we recovered the stagnation point flow potential.

$\boxed{w(z) \sim \frac{U_\infty}{L} z^2}$ & $\boxed{W(z) = Az^2}$

– (3.10).

Q4 Given that, a thin camber airfoil has a camber line

$$\frac{y_c}{c} = k\,\frac{x}{c}\left(\frac{x}{c}-1\right)\left(\frac{x}{c}-2\right) \quad - \text{(4.1)}$$



we know that given relation $y_c(x)$ has a domain $[0,1]$ which corresponds to the camber line.

we have to show $k = 0.052$, if maximum camber is 2%

first lets find location of maximum camber $\left(\frac{x_{c\,max}}{c}\right)$ which occurs at $\frac{dy_c}{dx} = 0$ — (4.2)

$\Rightarrow \frac{dy_c}{dx} = k\left(\frac{x}{c}-1\right)\left(\frac{x}{c}-2\right) + kx\left(\frac{1}{c}\right)\left(\frac{x}{c}-2\right) + kx\left(\frac{1}{c}\right)\left(\frac{x}{c}-1\right)$.

$$= k\left[\frac{x^2}{c^2} - \frac{x}{c} - \frac{2x}{c} + 2 + \frac{x^2}{c^2} - \frac{2x}{c} + \frac{x^2}{c^2} - \frac{x}{c}\right]$$

$\Rightarrow \frac{dy_c}{dx} = k\left[3\frac{x^2}{c^2} - 6\frac{x}{c} + 2\right]$ — (4.3)

$\Rightarrow$ for $\frac{dy_c}{dx} = 0 \Rightarrow \cancel{x_{max}} = k\left[\frac{3x^2}{c^2} - 6\frac{x}{c} + 2\right] = 0$

$\Rightarrow \frac{x_{max}}{c} = \frac{6 \pm \sqrt{36 - 4\cdot 3\cdot 2}}{6} = \frac{6 \pm \sqrt{36-24}}{6} = 1 \pm 0.577$

$\Rightarrow \frac{x_{max}}{c} = 1.577$ or 0.423. — (4.4)

only $\boxed{\frac{x_{max}}{c} = 0.423}$ is in domain $\frac{x_c}{c} \in [0,1]$ — (4.5)

max camber's location.

$\Rightarrow$ if max camber $\frac{y_{c\,max}}{c} = 0.02$ at $\frac{x_{max}}{c} = 0.423$.

Substitute $y_{c\,max}$ & x_{max} in eq^n (4.1) to get $\boxed{k = 0.052}$ — (4.6)

Zero lift angle of attack: ($\alpha_{L=0}$)

from thin airfoil theory, $\alpha_{L=0} = -\frac{1}{\pi}\int_0^\pi \frac{dy_c}{dx}(\cos\theta - 1)\,d\theta.$

$$\Rightarrow \alpha_{L=0} = -\frac{1}{\pi}\int_0^\pi k\left(\frac{3x^2}{c^2} - 6\frac{x}{c} + 2\right)(\cos\theta - 1)\,d\theta$$

use $\quad x = \frac{c}{2}(1-\cos\theta)$

$$\Rightarrow \alpha_{L=0} = -\frac{k}{\pi}\int_0^\pi \left(3\cdot\frac{1}{4}(1-\cos\theta)^2 - 6\cdot\frac{1}{2}(1-\cos\theta) + 2\right)(\cos\theta - 1)\,d\theta$$

$$= -\frac{k}{\pi}\int_0^\pi \left[\frac{3}{4}(1+\cos^2\theta - 2\cos\theta) - 3(1-\cos\theta) + 2\right](\cos\theta - 1)\,d\theta.$$

$$= -\frac{k}{\pi}\int_0^\pi \left(\frac{3}{4} + \frac{3}{4}\cos^2\theta - \frac{3}{2}\cos\theta) - 3 + 3\cos\theta + 2\right)(\cos\theta - 1)\,d\theta.$$

$$= -\frac{k}{\pi}\int_0^\pi \left(\frac{3}{4}\cos^2\theta + \frac{3}{2}\cos\theta - \frac{1}{4}\right)(\cos\theta - 1)\,d\theta$$

$$= -\frac{k}{\pi}\int_0^\pi \left[\frac{3}{4}\cos^3\theta + \frac{3}{2}\cos^2\theta - \frac{1}{4}\cos\theta - \frac{3}{4}\cos^2\theta - \frac{3}{2}\cos\theta + \frac{1}{4}\right]d\theta.$$

$$= -\frac{k}{\pi}\int_0^\pi \left[\frac{3}{4}\cos^3\theta + 0.75\cos^2\theta - 1.75\cos\theta + \frac{1}{4}\right]d\theta.$$

$$= -\frac{k}{\pi}\left[\frac{3}{4}\left(\sin\theta - \frac{\sin^3\theta}{3}\right) + 0.75\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right) - 1.75\sin\theta + \frac{\theta}{4}\right]_0^\pi$$

$\alpha_{L=0} = -0.625k \Rightarrow \boxed{\alpha_{L=0} = -0.0325\,\text{rad} = -1.86^\circ}$ - (4.7)

Determine C_L at 3° & 6° angle of attack.

we know that $C_L = 2\pi(\alpha - \alpha_{L=0})$ - 4.8

for $\alpha = 3^\circ = 0.0523$ rad.

$C_L = 2\pi\,(0.0523 - (-0.0325)) = 0.532$ - 4.9

for $\alpha = 6^\circ = 0.1047$ rad.

$C_L = 2\pi\,(0.1047 - (-0.0325)) = 0.862$ - 4.10

$\therefore$ $\boxed{C_{L_{\alpha=3^\circ}} = 0.532 \quad\Big|\quad C_{L_{\alpha=6^\circ}} = 0.862}$

Q5 Given that NACA 2412.

maximum camber $= \frac{2}{100} = 0.02$

location of max camber $= \frac{4}{10} = 0.4$ — (5.1).

thickness $= \frac{12}{100} = 0.12$

equation $\cancel{y_c = \frac{0.02}{0.4}}$

$$y_c = \begin{cases} \frac{m}{p^2}(2px - x^2), & 0 \leq x \leq p \\ \frac{m}{(1-p)^2}\left((1-2p) + 2px - x^2\right), & p \leq x \leq 1. \end{cases}$$

here x,y are fractions of chord.

here $m = \frac{\text{maximum camber}}{\text{chord}} = 0.02$

$p = \text{Location } \frac{x_{max}}{c} = 0.4$

Let $\xi = \frac{x}{c}$.

2) $\cancel{y_c = \frac{m}{p^2}(2p\xi c - \xi^2 c^2}$

In Houghton & Carpenter $x = \xi$.

$$y_c = \begin{cases} \frac{0.02}{0.4^2}(2 \cdot 0.4\xi - \xi^2), & \xi \leq p \\ \frac{0.02}{(1-0.4)^2}\left((1-2 \cdot 0.4) + 2 \cdot 0.4\xi - \xi^2\right), & p \leq \xi. \end{cases}$$

$$\Rightarrow \quad y_c = \begin{cases} 0.125\,(0.8\xi - \xi^2) & , \ \xi \le P \\ 0.056\,(0.2 + 0.8\xi - \xi^2) & , \ P \le \xi \end{cases}$$

$$\Rightarrow \quad \boxed{y_c = \begin{cases} 0.1\xi - 0.125\xi^2 & , \ \xi \le P \\ 0.011 + 0.045\xi - 0.056\xi^2 & , \ \xi \ge P \end{cases}} \quad - (5.2)$$

Let's find C_L & $C_{mc/4}$, from thin airfoil theory,

$$C_L = 2\pi(\alpha - \alpha_{L=0}) \quad , \quad C_{mc/4} = \frac{\pi}{4}(A_2 - A_1)$$

Determine $\alpha_{L=0}$, A_1, A_2.

$$\alpha_{L=0} = -\frac{1}{\pi}\int_0^{\pi} \frac{dy_c}{d\xi}(\cos\theta - 1)\,d\theta.$$

first find θ at which max camber present. $\xi = \dfrac{1-\cos\theta}{2}$

when $\xi = 0.4 \Rightarrow \quad 0.8 = 1 - \cos\theta \Rightarrow \boxed{\cos\theta = 0.2}$

at $\theta = 78.46^\circ = 0.436\pi$.

$\Rightarrow$ max camber $\xi = 0.4$ at $\theta = 0.436\pi$.

$\Rightarrow$ find the derivative $\dfrac{dy_c}{d\xi}$

$$\frac{dy_c}{d\xi} = \begin{cases} 0.1 - 0.25\xi & , \ \xi \le 0.4 \\ 0.045 - 0.112\xi & , \ \xi \ge 0.4 \end{cases}$$

$$\Rightarrow \quad \alpha_{L=0} = -\frac{1}{\pi}\int_0^{\pi} \frac{dy_c}{d\xi}(\cos\theta - 1)\,d\theta$$

$$= -\frac{1}{\pi}\int_0^{0.436\pi}(0.1 - 0.25\xi)(\cos\theta - 1)\,d\theta - \frac{1}{\pi}\int_{0.436\pi}^{\pi}(0.045 - 0.112\xi)(\cos\theta - 1)\,d\theta$$

$$\Rightarrow \alpha_{L=0} = \frac{-1}{\pi}\int_0^{0.436\pi} \left(0.1 - 0.25\left(\frac{1-\cos\theta}{2}\right)\right)(\cos\theta - 1)\,d\theta$$
$$-\frac{1}{\pi}\int_{0.436\pi}^{\pi} \left(0.045 - 0.112\left(\frac{1-\cos\theta}{2}\right)\right)(\cos\theta - 1)\,d\theta$$

$$= -\frac{1}{\pi}\int_0^{0.436\pi} (0.1 - 0.125 + 0.125\cos\theta)(\cos\theta - 1)\,d\theta$$
$$-\frac{1}{\pi}\int_{0.436\pi}^{\pi} (0.045 - 0.056 + 0.056\cos\theta)(\cos\theta - 1)\,d\theta$$

$$= -\frac{1}{\pi}\int_0^{0.436\pi} (0.125\cos\theta - 0.025)(\cos\theta - 1)\,d\theta - \frac{1}{\pi}\int_{0.436\pi}^{\pi} (0.056\cos\theta - 0.011)(\cos\theta - 1)\,d\theta$$

$$= -\frac{1}{\pi}\int_0^{0.436\pi} (0.125\cos^2\theta - 0.125\cos\theta - 0.025\cos\theta + 0.025)\,d\theta.$$
$$-\frac{1}{\pi}\int_{0.436\pi}^{\pi} (0.056\cos^2\theta - 0.011\cos\theta - 0.056\cos\theta + 0.011)\,d\theta$$

$$= -\frac{1}{\pi}\int_0^{0.436\pi} (0.125\cos^2\theta - 0.15\cos\theta + 0.025)\,d\theta - \frac{1}{\pi}\int_{0.436\pi}^{\pi} (0.056\cos^2\theta - 0.067\cos\theta + 0.011)\,d\theta$$

$$\Rightarrow \alpha_{L=0} = \frac{-1}{\pi}\left[0.125\left(\frac{1}{2}\theta + \frac{\sin 2\theta}{4}\right) - 0.15\sin\theta + 0.025\theta\right]_0^{0.436\pi}$$
$$-\frac{1}{\pi}\left[0.056\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right) - 0.067\sin\theta + 0.011\theta\right]_{0.436\pi}^{\pi}.$$

$$= -\frac{1}{\pi}\left[0.125\left(\frac{0.436\pi}{2} + \frac{\sin(2\times 0.436\pi)}{4}\right) - 0.15\sin(0.436\pi) + 0.025(0.436\pi)\right]$$
$$-\frac{1}{\pi}\left[0.056\left(\frac{\pi - 0.436\pi}{2} + \frac{0 - \sin(2\times 0.436\pi)}{4}\right) + 0.067\sin(0.436\pi) + 0.011(\pi - 0.436\pi)\right].$$

$$= -\frac{1}{\pi}\left[0.125(0.685 + 0.098) - 0.147 + 0.034\right]$$
$$-\frac{1}{\pi}\left[0.056(0.885 - 0.098) + 0.066 + 0.019\right]$$

$\Rightarrow \alpha_{L=0} = -\frac{1}{\pi}\left[-0.015 + 0.129\right] = -0.036$ rad.

$\Rightarrow$ $\boxed{\alpha_{L=0} = -0.036 \text{ rad} = -2.081^\circ}$

$$A_1 = \frac{2}{\pi}\int_0^{\pi} \frac{dz}{d\xi}\cos\theta\, d\theta.$$

$$= \frac{2}{\pi}\int_0^{0.436\pi} (0.1 - 0.25\xi)\cos\theta\, d\theta + \frac{2}{\pi}\int_{0.436\pi}^{\pi} (0.045 - 0.112\xi)\cos\theta\, d\theta$$

$$A_1 = \frac{2}{\pi}\int_0^{0.436\pi} \left(0.1 - 0.25\left(\frac{1-\cos\theta}{2}\right)\right)\cos\theta\, d\theta + \frac{2}{\pi}\int_{0.436\pi}^{\pi} \left(0.045 - 0.112\left(\frac{1-\cos\theta}{2}\right)\right)\cos\theta\, d\theta.$$

$$\frac{\pi A_1}{2} = \int_0^{0.436\pi} (0.1 - 0.125 + 0.125\cos\theta)\cos\theta\, d\theta + \int_{0.436\pi}^{\pi} (0.045 - 0.056 + 0.056\cos\theta)\cos\theta\, d\theta$$

$$\frac{\pi A_1}{2} = \int_0^{0.436\pi} (-0.025 + 0.125\cos\theta)\cos\theta\, d\theta + \int_{0.436\pi}^{\pi} (-0.011 + 0.056\cos\theta)\cos\theta\, d\theta.$$

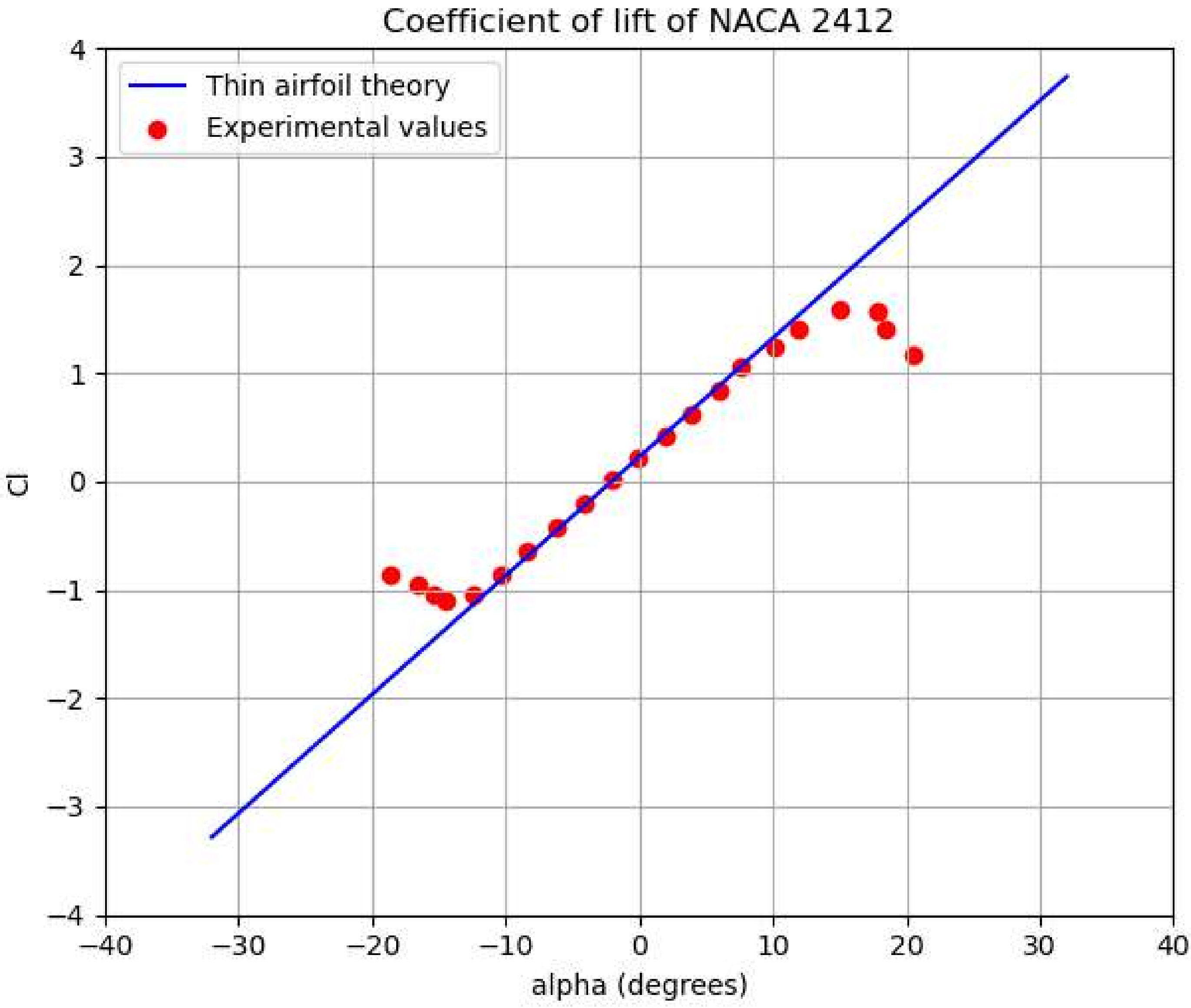
$\frac{\pi A_1}{2} = 0.128 \Rightarrow$ $\boxed{A_1 = 0.082}$

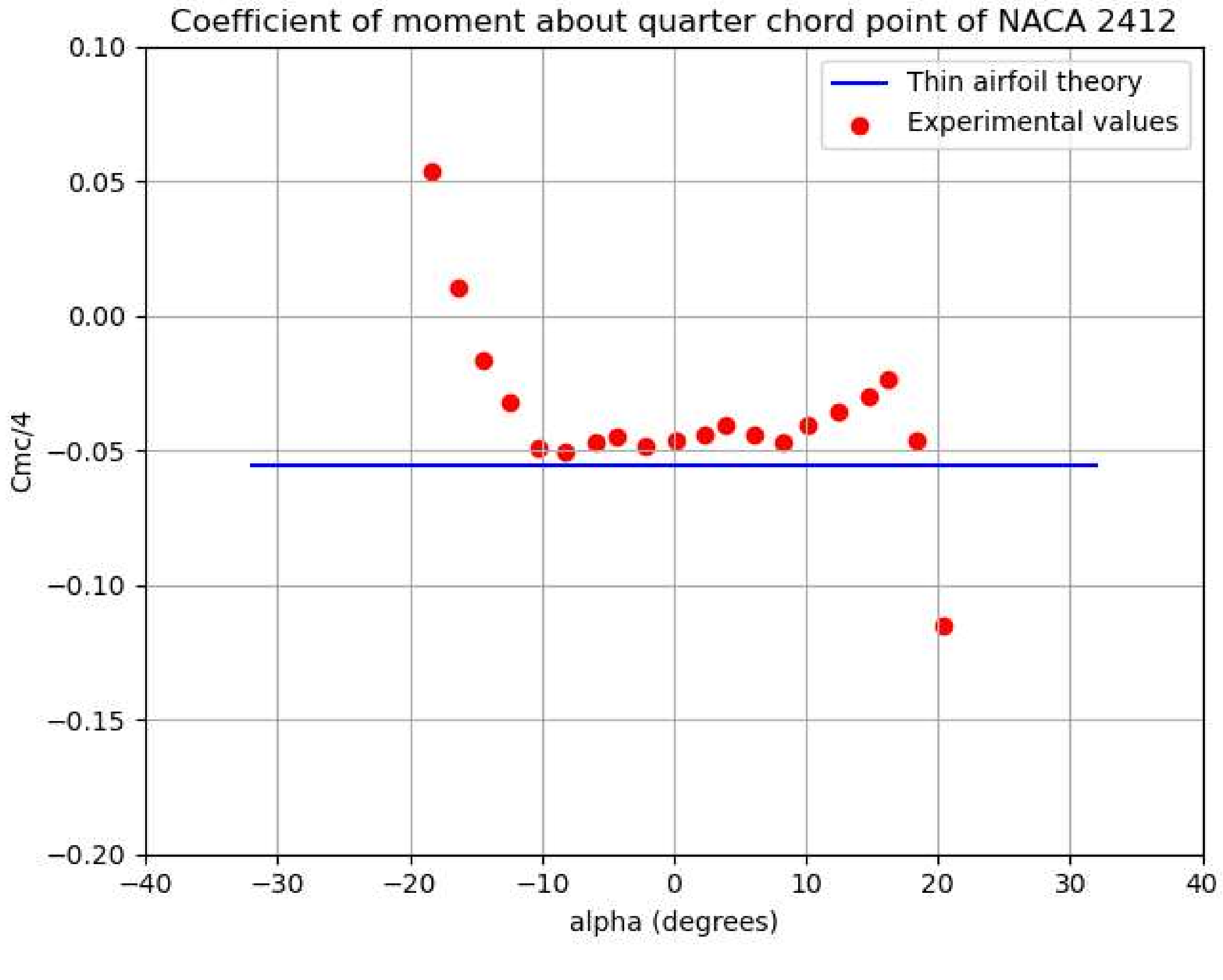
Similarly A_2

$$\frac{\pi A_2}{2} = \int_0^{0.436\pi} (-0.025 + 0.125\cos\theta)\cos 2\theta\, d\theta + \int_{0.436\pi}^{\pi} (-0.011 + 0.056\cos\theta)\cos 2\theta\, d\theta$$

$\cancel{\frac{\pi A_2}{2} = 0.022} \Rightarrow$ $\boxed{A_2 = 0.014}$

$\Rightarrow$ $\boxed{\begin{array}{l} C_L = 2\pi(\alpha - \alpha_{L=0}) = 2\pi(\alpha + 0.036) \\ C_{m_{c/4}} = \frac{\pi}{4}(A_2 - A_1) = -0.054 \\ \qquad = -0.054. \end{array}}$


Coefficient of lift of NACA 2412
Thin airfoil theory
Experimental values
Cl
alpha (degrees)
4
3
2
1
0
−1
−2
−3
−4
−40
−30
−20
−10
0
10
20
30
40


Coefficient of moment about quarter chord point of NACA 2412
Thin airfoil theory
Experimental values
Cmc/4
alpha (degrees)
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
−40
−30
−20
−10
0
10
20
30
40