Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) When a function $f(x)$ is said to be continuous at a point. A function is defined as, $f(x) = \begin{cases} 2x + 1 \ when \ x < 1 \\ 3 \ when \ x = 1 \\ x^2 + 2 \ when \ x > 1 \end{cases}$. Show that $f(x)$ is continuous and differentiable at the given point.   **8**

   OR

   State Leibnitz theorem for successive derivative of the product of two functions. If $y = \sin^{-1}x$, show that
   (i) $(1-x^2)y_2 - xy_1 = 0$
   (ii) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$.
   (iii) $(y_{n+2})_0 = (n^2 y_n)_0$

   b) State and prove Lagrange's mean value theorem. Verify Langrange's Mean Value Theorem for the function $f(x) = x(x-1)(x-2)$ in $[0, \frac{1}{2}]$.   **7**

2. a) Define indeterminate forms. State L'Hospital rule and hence evaluate $\lim\limits_{x \to 0} \dfrac{(1+x)^{1/x} - e}{x}$ **7**

   b) Define asymptotes and its types. Find the asymptotes of the curve $(x^2-y^2)^2 - 8(x^2+y^2) + 8x - 16 = 0$.   **8**

   OR

   Find the altitude of the right circular cone of maximum value that can be inscribed in a sphere of radius a.

3. Integrate: (any three)   **3×5**

   i) $\int \dfrac{1}{1+3e^x + 2e^{2x}} dx$

   ii) $\int_0^a \dfrac{dx}{x + \sqrt{a^2 - x^2}}$

   iii) $\int \dfrac{dx}{2 + \cos x + \sin x}$

   iv) $\int_0^1 \sqrt{x} \ dx$ (by summation method)

4. a) Find the area bounded by the curves $x + y^2 = 0$ and $x + 3y^2 = 0$.   **7**

   OR

   Find the volume of the solid generated by revolving the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x- axis.

   b) Use Trapezoidal and Simpson's rule with n = 6 to find the approximate area between the curve y = sinx ordinates x = 0, x = π and x – axis and compare the result with exact value.   **8**

5. a) Define conic section and classify them with respect to eccentricity. Find center, foci, vertices, equation of directrix of the conic section $4x^2 + y^2 - 16x + 4y + 16 = 0$.   **7**

   b) Find the equation of plane through (1, 0 , −1) and (−1, 2 , 1) and parallel to the line of intersection of the planes $3x + y - 2z = 0$ and $4x - y + 3z = 0$.   **8**

6. a) Find the equation of the plane through the points (2,4,5) and perpendicular to the line $\dfrac{x-5}{1} = \dfrac{y-1}{3} = \dfrac{z}{4}$ (by vector method).   **7**

   b) Define vector product of three vectors. Show that the vectors $\vec{a} \times (\vec{b} \times \vec{c})$, $\vec{b} \times (\vec{c} \times \vec{a})$, and $\vec{c} \times (\vec{a} \times \vec{b})$, are coplanar.   **8**

7. Attempt all the questions: **4×2.5**
   a) Find radius of curvature $y^2 = 4ax$ at (x,y)
   b) Find the domain and range of function $f(x) = (\sqrt{x})^2$
   c) Find the value of p, when the vectors $2\vec{i} - p\vec{j} + \vec{k}$, $5\vec{i} + \vec{j} + 2\vec{k}$ and $\vec{i} + 6\vec{k}$ are coplanar.
   d) Evaluate improper integral $\int_0^\infty \dfrac{1}{x^2 + 9} dx$.