POKHARA UNIVERSITY

Level: Bachelor Semester: Spring Year : 2021
Programme: BE Full Marks: 100
Course: Engineering Mathematics I Pass Marks: 45
Time . 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Define Continuity and Differentiability of a function f(x) at x = a.. Show that the function f(x) defined by 8

$$f(x)=\begin{cases} x & when\ 1 \le x < 2 \\ \frac{x^3}{4} & when\ 2 \le x < 3 \end{cases}$$

is continuous at x = 2 but not differentiable at x = 2.

OR

State & prove Leibnitz's theorem. If $y=\sin^{-1}x$ show that: i) $(1-x^2)y_2 - xy_1=0$,

ii) $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$

b) State "Rolle's theorem". Write its geometrical meaning. Verify Rolle's theorem for the function $f(x) = (x - a)^m (x - b)^n$, where m and n are positive integers in [a, b]. 7

2. a) Define Indeterminate form state L Hospital rule and using it, Evaluate the liming value of $\lim_{x \to 0}\left(\frac{1}{x^2}\right)^{\tan x}$ 7

b) Find the asymptotes of given curve $x^2(x-y)^2-a^2(x^2+y^2)=0$ 8

OR

Find the total surface area of the right circular cylinder of greatest surface that can be inscribed in a given sphere of radius r.

3. Attempt any three:

a) Evaluate: $\int \frac{x+\sin x}{1+\cos x}dx$. 5

b) Evaluate: $\int \frac{dx}{4\sin x+3\cos x+13}$. 5

c) $\int_0^1 \frac{\log x dx}{\sqrt{1-x^2}}$ 5

d) $\int_a^b e^{-x}\,dx$ (Using summation Method.) 5

4. a) Find the area between the curves $x=y^2$ and $2y^2=-x+3$ 7

b) Using Trapezoidal & Simpson's rule. Estimate the integral $\int_0^4 \frac{dx}{x^2+1}$ with n=4 subintervals and compare with the exact value. 8

5. a) A tangent to the parabola $y^2 = 8x$ makes an angle of 45° with the straight line y = 3x + 5. Find its equation. Find the equation of the hyperbola whose focus, directrix and eccentricity respectively are (6, 0), $4x - 3y = 6$ and $\frac{5}{4}$. 8

b) What is conic section? What is the eccentricity of an ellipse? Derive the equation of ellipse in standard form. 7

6. a) Find the equation of straight line through (2, -9, 5) and parallel to $2\vec{i} + 5\vec{j} + 6\vec{k}$. 7

b) Find the equation of plane through the points (2,4,5), (1,5,7), (-1,6,8) (using vector method) 8

7. Attempt all questions:

a) Find the domain and range of $y = \frac{1}{x-3}$. 2.5

b) Find the arc length of the curve $y=x^{\frac{3}{2}}$ from x= 0 to x=2. 2.5

c) Find the vector projection of $\vec{a}$ on $\vec{b}$ if $\vec{a} = 3\vec{i} - \vec{j} + \vec{k}$, and $\vec{b} = 2\vec{i} + \vec{j} - 2\vec{k}$. 2.5

d) Find the radius of curvature of the curve y=4sinx-sin2x at $x = \frac{\pi}{2}$ 2.5