Some Important Question of Multiple Integral

A. Evaluate the following integral: a. $\int_0^2\int_{y^2}^4 y\cos(x^2)dxdy$ b. $\int_0^1\int_x^{\sqrt{2-x^2}} \frac{xdydx}{\sqrt{x^2+y^2}}$ c. $\int_0^{\pi}\int_0^x x\sin y dydx$

d. $\int_0^2\int_1^{e^x} dydx$ e. $\int_0^{\pi}\int_x^{\pi} \frac{\sin y}{y} dydx$ f. $\int_0^1\int_y^1 x^2 e^{xy} dxdy$

g. $\int_0^2\int_{y^2}^4 y\cos(x^2)dxdy$ h. $\int_0^1\int_{4y}^4 e^{x^2} dxdy$ i. $\int_0^4\int_y^4 \frac{x}{x^2+y^2} dxdy$ j. $\int_0^{\infty}\int_x^{\infty} \frac{e^{-y}}{y} dydx$

B. Evaluate the following Integral by using Polar form:

$a.\int_0^1\int_x^{\sqrt{2-x^2}} \frac{xdydx}{\sqrt{x^2+y^2}}$ $b.\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} \frac{xdydx}{\sqrt{x^2+y^2}}$ $c.\int_{-a}^{a}\int_0^{\sqrt{a^2-x^2}} e^{-(x^2+y^2)}dydx$

$d.\int_0^2\int_0^{\sqrt{4-y^2}} \cos(x^2+y^2)dxdy$ $d.\int_0^a\int_y^{\sqrt{a^2-y^2}} (x^2+y^2)dxdy$ $e.\int_0^{\frac{a}{\sqrt{2}}}\int_y^{\sqrt{a^2-y^2}} \log(x^2+y^2)dxdy$

$f.\int_0^{4a}\int_{y}^{\frac{y^2}{4a}} \left(\frac{x^2-y^2}{x^2+y^2}\right)dxdy$

C. Evaluate the following

$a.\int_{-1}^{1}\int_{-2}^{2}\int_{-3}^{3} dxdydz$ $b.\int_1^3\int_2^3\int_1^2 (x-y-z)dxdydz$ $c.\int_0^{\log 2}\int_0^{x}\int_0^{x+\log y} e^{x+y+z}dxdydz$

d. Define the Dirichlet's Theorem. Compute $\iiint_V x^2yzdxdydz$ over the volume of the tetrahedron bounded by x=0,y=0,z=0 and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

e. Evaluate $\iiint_V (x^2+y^2+z^2)dxdydz$ taken over the volume of the sphere $x^2+y^2+z^2=1$.

f. Evaluate the integral $\iiint_V xdxdydz$ over the region V in the first octant bounded $x^{\frac{2}{3}}+y^{\frac{2}{3}}+z^{\frac{2}{3}}=1$.

D. Attempt the following Question:

a. Find the volume in the first octant bounded by the co-ordinate planes, the cylinder

$x^2 + y^2 = 4$ and the plane $z + y = 3$.

b. Find the volume of the solid whose base in the region in xy-plane that is bounded by the parabola $y = 4 - x^2$ and the line $y = 3x$, while the top of the solid is bounded by the plane $z = x + 4$.

c. Find the volume bounded by the XY-plane, the paraboloid $2z = x^2 + y^2$ and the cylinder $x^2 + y^2 = 4$.

d. Find the volume bounded by the XY-plane, the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 3$.

Some Important question of Series Solution of Differential Equations and Special Functions

A Applying the power series method, Solve the following differential equations.

a. $y' - 2xy=0$. b. $y' +2xy=0$. C. $x y' - 3 y=k$. d. $y'' +9y=0$.

e. $y'' +4 y=0$. f. $(1+x) y' - y=0$. g. $y'' - 4xy'+ (4x^2 - 2) y=0$.

h. $y'' + (1-x^2)y =0$. i. $y'=3y$.

B. Attempt the following question.

1. a. Define the Legendre's Equation of order n. Write the formula of $P_n(x)$ and sketch the graph of $p_2(x)$.

   b. Express $x^3 +2x^2 -x -3$ in terms of Legendre Polynomials.

   c. If $P_n(x) = \dfrac{1}{2^n n!}\dfrac{d^n[(x^2-1)}{dx^n}$ then show that: $P_n(1)=1$ and $P_n(-1)=(-1)^n$.

   d. Define Legendre's equation and Legendre's Polynomial.

   e. Solve Legendre's Equation $(1-x^2)y''-2xy' +n(n+1)y=0$.

2. a. Define the Bessel's equation of order n. Also show that $2J'_v(x) = J_{v-1}(x) - J_{v+1}(x)$.

   b. Show that $J_{\frac{1}{2}}(x) = \sqrt{\dfrac{2}{\pi x}}\sin x$.

   c. Solve the Bessel's equation: $x^2y'' + xy'+ (x^2 - n^2) y=0$.By using Frobenius method.

   d. Prove that $\dfrac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x)$.

   e. Show that $J'_0(x) = -J_1(x)$.

   f.. Show that $J_{-n}(x) = (-1)^n J_n(x)$.

Some Important Question of Laplace Transform.

A. Solve the Following initial value problems using the Laplace transform

a. y'' +4y' +3y = e^{-t} . y (0)=y'(0)=1.

b. x'' +2x' +5x = e^{-t} sint. x (0)=0, x'(0)=1.

C. y'' +2y' +2y = 0. y (0)= 0, y'(0)=1.

d. y'' -2y' +10y = 0. y (0)= 3, y'(0)=3.

e. 4y'' +8y' +2y = 0. y (0)= 0, y'(0)=1.

f. y'' - 2y' +y =e^{t} . Y (0) = 2, y' (0) =-1.

g. 9y'' -6 y' +y = 0. y(0)= 3, y'(0)=1.

h. y'' +2y' +2y = e^{-t} . y(0)=-1, y'(0)=1.

i. y'' +2y' -3y = 6e^{-2t} . y(0)=2, y'(0)=-14.

j. y'' +4y' +4y = sint . y(0)=1, y'(0)=3.

k. y'' +π^2 y =0 . y(0)=2, y'(0)=0.

B

a. State and prove the first shifting theorem of Laplace Transform.

b. State and prove the Second shifting theorem of Laplace Transform.

c. Define Convolution of two functions. State and prove Convolution Theorem.

d. Define Unit function. Laplace Transform.

C Find the Laplace Transform of the following:

a. tsinat b. tcoshat c. t^2e^t d. t^2e^{-3t} e. $t^2\sin 2t$

$f.$ $t^2\cos wt$ g. $t^2\cos 3t$ h. t^2e^{2t} i. t^2e^t j. t^ne^{at}

k. $te^t\cos t$ l. $te^{2t}\sin t$ m. $e^{2t}\sin n\pi t$ n. $\dfrac{\sin t}{t}$

o. sin2t.u$(t-\pi)$

D Find the Inverse Laplace Transform of the following:

a. $\dfrac{1}{(s^2+w^2)s^2}$ $b.\dfrac{se^{-2s}}{s^2+9}$ $c.\dfrac{s^2}{(s^2+a^2)(s^2+b^2)}$

$d.\dfrac{s+4}{(s^2-4)}$ $e.\dfrac{\pi}{(s+2)^2+\pi^2}$ $e.\dfrac{s}{(s^2+w^2)^2}$

$f.\dfrac{e^{-2a}s}{(s^2+1)}$ $g.\log\dfrac{(s+1)}{s^2(s^2+1)}$ $i.\log\dfrac{s(s+1)}{s^2+4}$

$j.\log\left(1+\dfrac{w^2}{s^2}\right)$ $k.\dfrac{1}{s^2(s^2+w^2)}$ $l.\dfrac{1}{4s+s^2}$

$m.\dfrac{1}{(s^2+36)}$

The end

Some Important Question of vector Calculus (near about* 35 *marks)

1 If $\vec{V} = \dfrac{x\vec{i} + y\vec{j} + z\vec{k}}{\sqrt{x^2+y^2+z^2}}$ Show that: $\nabla.\vec{V} = \dfrac{2}{\sqrt{x^2+y^2+z^2}}$ and $\nabla \times \vec{V} = 0$.

2. If $\phi = \log(x^2+y^2+z^2)$ then find div(grad ϕ) and curl(grad ϕ) .

3. Prove that: $\vec{F} = r^2\,\vec{r}$, Show that $\vec{F}$ is a conservative vector field and scalar potential is

$$\phi = \frac{r^4}{4} + \text{Constant.}$$

4. Define directional derivative of the function in the direction a. Find the directional Derivative of F $=xy^2+yz^3$ at (2, -1, 1) along the direction of the normal to the surface

S : $x \log z - y^2 + 4 = 0$ at (-1,2 ,1).

5. Find the directional derivate of F at p in the direction $\vec{a}$ where at P (3,0,4) ; $\vec{a} = \vec{i} + \vec{j} + \vec{k}$

$$F = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

6. Calculate $\oint_c \vec{F}.\vec{dr}$ where $\vec{F}$ = (cosh x, sinh y, e^z) and C be a path given by

$\vec{r}$ = $(t, \quad t^2, \quad t^3)$ From (0,0, 0) to (2,4, 8).

7. Prove that if $\vec{F} = (2xz^3 + 6y \;\; , 6x - 2yz \;\; , 3x^2z^2 - y^2)$ $\vec{F}$ is a conservative vector field

Also find the scalar potential.

8. A particle moves along the curve $x= t^3+1$, $y= t^2$, $z= 2t+5$.Find the component of velocity and acceleration at t= 1 in the direction $\vec{i} + \vec{j} + \vec{k}$.

9. Calculate $\oint_c \vec{F}.\vec{dr}$ where $\vec{F}$ = $(e^x, \; e^{-y}, \; e^z)$ and C be a path given by

$\vec{r}$ = $(t, \quad t^2, \quad t)$ From (0,0, 0) to (1,1, 1).

10. Find the work done in moving a particle in the force field $\vec{F}$ = $(3x^2, \; 2xz - y, \; z)$ along the curve defined by $x^2=4y$, $3x^3=8z$ from x=0 to x=2.

11. The necessary and sufficient condition for the vector value function $\vec{a}$ of the scalar variable t to have a constant direction is $\vec{a} \times \dfrac{d\vec{a}}{dt} = 0$.

12. The necessary and sufficient condition for the vector value function $\vec{a}$ of the scalar variable t to have a constant magnitude is $\vec{a} . \dfrac{d\vec{a}}{dt} = 0$.

13. Evaluate: a. $\int_{(0,0,0)}^{(4,1,2)} [3ydx + 3xdy + 2zdz]$

b. $\int_{(0,1)}^{(2,3)} [(2x + y^3)dx + (3xy^2 + 4)dy]$

c. $\int_{\left(0,1,\frac{1}{2}\right)}^{\left(\frac{\pi}{2},3,2\right)} [y^2 \cos x dx + (2y \sin x + e^{2z})dy + 2ye^{2z}dz]$

B Green's Theorem

1. Evaluate by using Green's Theorem of $\oint_c [(y - \sin x)dx + \cos x dy]$ where C is the triangle with vertices (0,0), $\left(\frac{\pi}{2},0\right)$ and $\left(\frac{\pi}{2},1\right)$.

2. Evaluate by using Green's Theorem of $\oint_c (\sqrt{y}dx + \sqrt{x}dy)$ where C is the triangle with vertices (1,1), $(2,2)$ and $(3,1)$.

3. Evaluate by using Green's Theorem of $\oint_c (5xydx + x^3dy)$ where C is the closed curve consisting of the graph of $y = x^2$ and y = 2x between the points (0,0) and (2,4).

4. Evaluate by using Green's Theorem of $\oint_c (x^2 + y^2)\vec{i} - 2xy\vec{j}).d\vec{r}$ along the rectangle bounded by y=0, y=b, x=0, x=a.

C Surface Integral

1. Find $\iint_s (\vec{F}.\vec{n})ds$, for $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$, $\vec{r} = (u\cos v, \quad u\sin v, 3v); 0 \le u \le 1, 0 \le v \le 2\pi.$

2. Define the surface integral of $\vec{F}$, on the surface S. Evaluate $\iint_s (\vec{F}.\vec{n})ds$, where $\vec{F} = x^2\vec{i} + e^x\vec{j} + \vec{k}$, Where S is the surface, x+ y+ z=1, $x \ge 0, y \ge 0, z \ge 0$.

3. Find $\iint_s (\vec{F}.\vec{n})ds$, for $\vec{F} = 4x\vec{i} + x^2y\vec{j} - x^2z\vec{k}$, ; S is the surface of the tetrahedron with vertices (0,0,0) , (1,0,0), (0,1,0) ,(0,0,1).

4. Find $\iint_s (\vec{F}.\vec{n})dA$, for $\vec{F} = (x^2, e^y, 1)$; S is the portion of the plane x+ y+ z=1 lying in the first Octant.

5. Find the flux integral of $\vec{F} = (x, y, z)$ through the surface S, Where S is the portion of the plane 2x+3y+z =6 in first octant.

6. Find the flux integral of $\vec{F} = (yz, zx, xy)$ through the surface S, Where S is the portion of the sphere, $x^2+y^2+z^2=1$ in first octant.

7. Find the flux integral of $\vec{F} = (3x, 3y, z)$ through the surface S, Where S is the part of the graph $z=9-x^2-y^2$.

D Stoke's Theorem

1. Evaluate $\oint_c \vec{F}.\vec{dr}$ by using Stoke's Theorem where $\vec{F} = y\,\vec{i} + xz^3\,\vec{j} - zy^3\,\vec{k}$, and

   C: $x^2+y^2 = 4$, $z = -3$.

2. Evaluate $\oint_c \vec{F}.\vec{dr}$ by using Stoke's Theorem where $\vec{F} = -3y\,\vec{i} + 3x\,\vec{j} + z\,\vec{k}$, and

   C: $x^2+y^2 = 4$, $z = 1$.

3. Evaluate $\oint_c \vec{F}.\vec{dr}$ by using Stoke's Theorem where $\vec{F} = y^3\,\vec{i} + x^3\,\vec{k}$, and C is the boundary of the triangle with vertices (1,0,0), (0,1,0), (0,0,1).

4. Evaluate $\oint_c \vec{F}.\vec{dr}$ by using Stoke's Theorem where $\vec{F} = (2x-y)\,\vec{i} - yz^2\,\vec{j} - y^2z\,\vec{k}$, and S is the upper half surface of $x^2+y^2+z^2=1$, bounded by its projection on xy –plane.

5. Evaluate $\oint_c \vec{F}.\vec{dr}$ by using Stoke's Theorem where $\vec{F} = (z^2, 5x, 0)$ and S is the square

   $0 \le x \le 1,\ 0 \le y \le 1,\quad z = 1$

6. Evaluate $\oint_c \vec{F}.\vec{dr}$ by using Stoke's Theorem where $\vec{F} = (y^2, z^2, x^2)$ and S is the first portion of the plane x+ y+ z =1 .

7. Evaluate $\oint_c \vec{F}.\vec{dr}$ by using Stoke's Theorem where $\vec{F} = (y^2, 2xy + \sin x, 0)$ where c is the boundary of the of $0 \le x \le \frac{\pi}{2}$, $0 \le y \le 2$.

Gauss Divergence Theorem

1. Using the divergence theorems to find $\iint_s (\vec{F}.\vec{n})ds$, where $\vec{F} = e^x\,\vec{i} + \vec{j} + e^z\,\vec{k}$ and

   S: $0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le 1.$

2. Using the divergence theorems to find $\iint_s (\vec{F}.\vec{n})ds$, where $\vec{F} = y^2e^z\,\vec{i} - xy\,\vec{j} + x\tan^{-1}y\,\vec{k}$ and S is the surface of the region bounded by the coordinate planes and the plane x+y+z=1.

3. State Gauss divergence Theorem. Use it to evaluate $\iint_S (\vec{F}.\vec{n})dA,$ where $\vec{F} = (4x, -2y^2, z^2)$, S is the surface bounding the region $x^2 + y^2 = 4$, $z = 3$, $z = 0$ ·

4. Using the divergence theorems to find $\iint_S (\vec{F}.\vec{n})dA,$ where $\vec{F} = y^3\,\vec{i} + x^3\,\vec{j} + z^3\,\vec{k}$ and

   S: $x^2 + 4y^2 = 1, \quad x \geq 0, y \geq 0, 0 \leq z \leq h$,

5. Using the divergence theorems to find $\iint_S (\vec{F}.\vec{n})dA,$ where $\vec{F} = 4x\,\vec{i} + 2y^2\,\vec{j} + z^2\,\vec{k}$ and

   S is the surface of the cube: $|x| \leq 1, |y| \leq 1, |z| \leq 1$.

6. Using the divergence theorems to find $\iint_S (\vec{F}.\vec{n})ds,$ where $\vec{F} = x^2\,\vec{i} + e^y\,\vec{j} + 1\vec{k}$ and

   S: $x + y + z = 1$, $x \geq 0$, $y \geq 0$ $z \geq 0$.

Some Important Question of Fourier series

1. Find the Fourier Series of the following function

   a. $f(x) = \frac{x^2}{2} \quad for -\pi < x < \pi.$

   Show that: *i.* $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \ldots\ldots\ldots\ldots = \frac{\pi^2}{6}$

   ii. $1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} + \ldots\ldots\ldots\ldots = \frac{\pi^2}{12}$

   b. $f(x) = |x| \quad for -\pi < x < \pi.$

   c. $f(x) = x + |x| \quad for -\pi < x < \pi.$

   d. $f(x) = \begin{cases} -1 & for \quad -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 1 & for \quad \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}.$

   e. $f(x) = \begin{cases} x & for \quad -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \pi - x & for \quad \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}.$

   f. $f(x) = \begin{cases} 1 & for \quad 0 < x < \pi \\ 0 & for \quad \pi < x < 2\pi \end{cases}.$

2. Find the Fourier Series of the following function

   a. $f(x) = |x| \quad for -2 < x < 2.$

   b. $f(x) = \begin{cases} -x & for \quad -2 < x < 0 \\ x & for \quad 0 < x < 2 \end{cases}.$

   c. $f(x) = \begin{cases} x & for \quad 0 < x < 1 \\ 1 - x & for \quad 1 < x < 2 \end{cases}.$

   d. $f(x) = \begin{cases} 0 & for \quad -1 < x < 0 \\ -2x & for \quad 0 \le x < 1 \end{cases}.$

3. Find the Fourier cosine and sine Series of the following function in half range.

   a. $f(x) = x^2 \quad for \quad 0 < x < L.$

   b. $f(x) = x \quad for \quad 0 < x < L.$

   c. $f(x) = e^x \quad for \quad 0 < x < \pi.$

   e. $f(x) = \pi - x \quad for \quad 0 < x < \pi.$

   f. $f(x) = \sin x \quad for \quad 0 < x < \pi.$

4. Define the following function.

   a. Periodic function
   b. Fourier function
   c. Odd and Even function
   d. Fourier cosine and sine function