POKHARA UNIVERSITY

Level: Bachelor | Semester: Fall | Year : 2021
Programme: BE | Full Marks: 100
Course: Engineering Mathematics I | Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) State Leibnitz's theorem for successive derivative of product of two functions y = u.v If $y=\left(x+\sqrt{1+x^2}\right)^m$, show that $\left(1+x^2\right)y_{n+2}+\left(2n+1\right)xy_{n+1}+\left(n^2-m^2\right)y_n=0$ 8

OR

Show that the function $f(x)=\begin{cases} x & for \quad x<1 \\ 2-x & for \quad 1\le x\le 2. \\ x-\frac{x^2}{2} & for \quad x>2 \end{cases}$

is continuous at x = 1& x = 2. Does f'(x) exists at these points.

b) State and prove Cauchy's mean value theorem. How does it differ from Lagrange's mean value theorem? Verify Cauchy mean value theorem for the functions f(x) = x, g(x) = x^2 in [-2,0] 7

2. a) State L'Hospital rule. Prove that: $\lim_{x\to 0}(cotx)^{sin2x}=1$ 7

b) Find the asymptotes of the curve 8

$x^3+3x^2y-xy^2-3y^3+x^2-2xy+3y^2+4x+5=0$

OR

Find the total surface area of the right circular cylinder of greatest surface that can be inscribed in a given sphere of radius r.

3. Integrate the following (**Any three**) 3×5 =15

i) $\int \frac{1}{4+5\sin x}dx$ ii) $\int_0^1 \frac{\log x dx}{\sqrt{1-x^2}}$

iii) $\int_0^2 x^2 dx$. (by summation) iv) $\int_0^{\frac{\pi}{2}} sin^4 x cos^2 x dx$

4. a) Find the volume of the solid generated by revolving the region in the first quadrant bounded above by the parabola $y = x^2$ below by the X-axis,and on the right by the line x=2 about y-axis. 7

b) Find the approximate area using Simpson's and Trapezoidal rule for the area bounded by the curve y = sinx, the x- axis and the lines x = $\pi/2$ and x= 2π (using n = 6) and compare these results with exact value. 8

5. a) Define eccentricity of a conic section, and derive the equation of hyperbola in its standard form. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ 8

b) Find the equation of tangent to the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$, which is parallel to the line x = y+5. 7

6. a) Explain the scalar triple product. Write any three properties. If vectors $\vec{a}, \vec{b}$ *and* $\vec{c}$. 8

Show that: $\left[\vec{a} \times \vec{b} \quad \vec{b} \times \vec{c} \quad \vec{c} \times \vec{a}\right] = 0$

b) Find the condition that the line ax+by+c = 0 may be tangent to the parabola $y^2 = 4ax$. 7

7. **Attempt all question** 4×2.5

a. Find the domain and range for $y = \sqrt{4 - x^2}$

b. If $\vec{a} = i - 2\vec{j} + \vec{k}$ and $\vec{b} = \vec{i} + 2\vec{j} - \vec{k}$ find the projection of $\vec{a}$ on $\vec{b}$

c. Integrate: $\int_0^\infty xe^{-x^2} dx$

d. Find the center, vertices and foci of the ellipse

$$x^2 + 10x + 25y^2 = 0$$