3.13 SIMULATION OF SINGLE-SERVER QUEUEING SYSTEM

Queue represents a certain number of customers waiting for service. People form queues to be served in banks, post offices, ships, and railway ticket counters.

Problem of Queuing System

A queueing problem is essentially a problem of balancing the cost of waiting against the cost of idle time for the services facilities in the system. This balancing requires an analysis of the queueing system, like idle time, average waiting time, queue length, etc. The problem arises due to the stochastic nature of the times between the arrivals of customers as well as the time it takes to serve each customer. We can solve this problem by the help of simulation.

Elements of Queuing Systems

There are following three elements of queueing systems:

- Arrivals of customers
- Service mechanism
- Queueing Discipline

Figure. 3.3 shows the elements of single server queueing system.

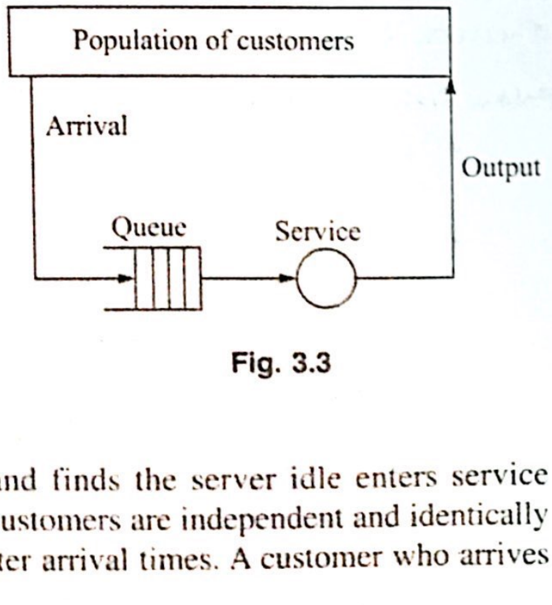

Fig. 3.3

(*i*) **Arrivals of Customers:** It defines the way customers enter into the system. Mostly the arrivals are random with random intervals between two adjacent arrivals. Typically the arrivals is described by a random distribution of intervals also called *Arrival Pattern*.

(*ii*) **Service Mechanism:** A customer who arrives and finds the server idle enters service immediately, and the service time s_1, s_2 .... of the successive customers are independent and identically distributed (IID) random variables that are independent of inter arrival times. A customer who arrives and finds the server busy joins the end of a single queue.

(*iii*) **Queueing Discipline:** It represents the way the queue is organised or it is the order in which service is provided such as:

- FIFO – (First in First out) are also called FCFS.
- LIFO – (Last in First out) are also called LCFS.
- Priority Queue – It may be viewed as a no. of queues for various priorities.
- SIRO – Serve in Random order.

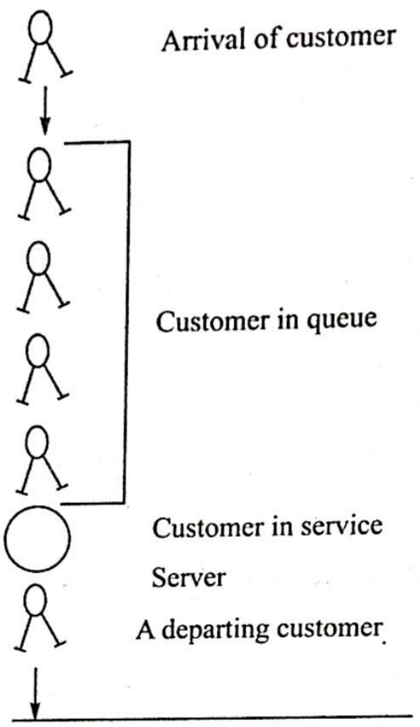

Fig. 3.4. A single server queueing system.

Example 1: Calculate different parameter for the single server queueing system when inter-arrival time and service times are given by the Tables 3.2 and 3.3.

Table 3.2. Inter-arrival and clock times

Customer	Inter-arrival time	Arrival time on clock
1	–	0
2	4	4
3	1	5
4	4	9
5	2	11
6	4	15

Table 3.3. Service Time of Each Customer

Customer	Service time
1	2
2	3
3	1
4	4
5	2
6	3

Solution: Formula used in simulation Table 3.4

1. Time since last arrival for $(i + 1)$ customer = Arrival time of i^{th} customer – Arrival time of $(i - 1)^{th}$ customer
2. Time customer wait in queue for i^{th} customer = Arrival time of i^{th} customer – Time service begin of i^{th} customer
3. Time service ends of i^{th} customer = Time service begin of i^{th} customer (min) + Service Time of i^{th} customer
4. Time customer spends in system = Time service ends of i^{th} customer – Arrival time of i^{th} customer
5. Idle time of server for i^{th} customer = Time service begins of i^{th} customer – Time service ends of $(i - 1)^{th}$ customer
6. Time service Begins: For first customer it is zero (always) for rest of customer it is a maximum between arrival of i^{th} customer and time service ends of $(i - 1)^{th}$ customer, *i.e.* Max. (Arrival of (i^{th}) customer, Time service ends of $(i - 1)^{th}$ customer)

Example: For second customer. Arrival time of customer 2 is 4 while service ends of first customer is 2 then

$$\text{Max } (4, 2)$$

Time service begin of 2^{nd} customer = 4

Now calculate the following parameters for short time simulation for this example:

1. Average waiting time $= \dfrac{\text{Total customer wait in queue}}{\text{Total no. of customer}}$

Table 3.4: Simulation for above queueing problem

Customer	Time service last arrival	Arrival time	Service time	Time service begin	Waiting in queue	Time service ends	Time customer spends in system	Idle time of queue
1	–	0	2	0	(0 – 0) = 0	(0 + 2) = 2	(2 – 0) = 2	0 – 0 = 0
2	4	4	3	4	(4 – 4) = 0	(4 + 3) = 7	(7 – 4) = 3	4 – 2 = 2
3	1	5	1	7	(7 – 5) = 2	(7 + 1) = 8	(8 – 7) + 2 = 3 or 8 – 5 = 3	7 – 7 = 0
4	4	9	4	9	(9 – 9) = 0	(9 + 4) = 13	(13 – 9) = 4	9 – 8 = 1
5	2	11	2	13	(13 – 11) = 2	(13 + 2) = 15	(15 – 13) + 2 = 4 or 15 – 11 = 4	13 – 13 = 0
6	4	15	3	15	(15 – 15) = 0	(15 + 3) = 18	(18 – 15) = 3	15 – 15 = 0
	15		15		4		19	3

$$= \frac{4}{6} = 0.667 \text{ min}$$

2. Probability that a customer has to wait in the queue is

$$\text{Probability (wait)} = \frac{\text{No. of customer who has to wait}}{\text{Total no. of customer}}$$

$$= \frac{2}{6} = 0.33$$

3. Probability of idle server $= \dfrac{\text{Total run time of idle server (min)}}{\text{Total run time of simulation}}$

$$= \frac{3}{19} = 0.157$$

Probability of busy server is $= 1 - 0.157$
$= 0.84$

4. Average service time $= \dfrac{\text{Total service time (min)}}{\text{Total no. of customer}}$

$$= \frac{15}{6} = 2.5$$

5. Average time between arrivals (min)

$$= \frac{\text{Sum of all time between arrival}}{\text{No. of arrivals} - 1}$$

$$= \frac{15}{6-1} = 3$$

6. Average waiting time of those who wait

$$= \frac{\text{Sum of total waiting time in queue}}{\text{No. of customer who has to wait}}$$

$$= \frac{4}{2} = 2 \text{ min}$$

7. Average time customers spends in the system (min)

$$= \frac{\text{Total time customer spends in system}}{\text{Total no. of customer}}$$

$$= \frac{19}{6} = 3.16 \text{ minutes}$$

or by second alternative formula

= Average time customer spends waiting in the queue (min) + Average time customer spends in service (min) Average time customer

$= 0.667 + 2.5 = 3.167$

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We can make following decision:

- Less customer has to wait with 2 minute average waiting time.
- Server is not often free, it is busy appropriate 84% of time.

Note: Simulation requiring variation of the arrival and service distribution as well as implementation in a spreadsheet for analysis.

Example 2. Calculate different parameter for a single server queueing system when arrival time and service times are given by the Table 3.5 and analysis the system.

Table 3.5 For Arrival time and service time

Customer	Arrival time	Service time
1	0	4
2	8	1
3	14	4
4	15	3
5	23	2
6	26	4
7	34	5
8	41	4
9	43	5
10	46	3

Solution: Consider following parameter for single server queueing system.

1. The average waiting time for a customer, is determined in the following manner:

$$\text{Average waiting time} = \frac{\text{Total time customers wait in queue}}{\text{Total no. of customer}}$$

$$= \frac{9}{10} = 0.9 \text{ minutes}$$

2. The probability that a customer has to wait in the queue, is determined in the following manner

$$\text{Probability (wait)} = \frac{\text{No. of customers who wait}}{\text{Total no. of customers}}$$

$$= \frac{3}{10} = 0.3 \text{ minutes}$$

3. The fraction of idle-time of the server is determined in the following manner:

$$\text{Probability of idle} = \frac{\text{Total idle time of server (minutes)}}{\text{Total run time of simulation (minutes)}}$$

$$= \frac{18}{44} = .409 \approx .41$$

The probability of server being busy is

$$= 1 - .41$$

$$= .59$$

4. The average service time is determined as follows:

$$\text{Average service time (minutes)} = \frac{\text{Total service time (minutes)}}{\text{Total number of customers}}$$

$$= \frac{35}{10} = 3.5 \text{ minutes}$$

5. The average time between arrivals is determined in following manner:

Average time between arrivals (minutes)

$$= \frac{\text{Sum of all times between arrivals (min)}}{\text{Number of arrivals} - 1}$$

$$= \frac{46}{10-1} = \frac{46}{9} = 5.1 \text{ minutes}$$

6. The average waiting time of those who wait is determined in the following manner:

Average waiting time of those who wait (minutes)

$$= \frac{\text{Total time customers wait is queue (minutes)}}{\text{Total number of customers who wait}}$$

$$= \frac{9}{3} = 3 \text{ minutes}$$

7. The average time a customer spends in the system is determined in two ways

(*i*) Average time customer spends in system

$$= \frac{\text{Total time customers spend in the system (minutes)}}{\text{Total number of customers}}$$

$$= \frac{44}{10} = 4.4 \text{ minutes}$$

(*ii*) Average time customer spends in the system (minutes)

$$= \begin{array}{l}\text{Avg. time customer}\\ \text{spends waiting}\\ \text{in the queue (min)}\end{array} + \begin{array}{l}\text{Average time}\\ \text{customer spends}\\ \text{in service (min)}\end{array}$$

$$= 0.9 + 3.5$$

$$= 4.4$$

Now on the basis of above parameters. We can analysis following points:

- less customer has to wait
- server is quite idle, almost 41% it is idle
- average waiting time is not excessive.

Table 3.6: Simulation for Queueing Problem

Customer	Time since last arrival (minutes)	Arrival time	Service time (minutes)	Time service begins	Time customer wait in queue	Time service ends	Time customer spends in system (minutes)	Idle time of server (minutes)
1	–	0	4	0	0	4	4	0
2	8	8	1	8	0	9	1	4
3	6	14	4	14	0	18	4	5
4	1	15	3	18	3	21	6	0
5	8	23	2	23	0	25	2	2
6	3	26	4	26	0	30	4	1
7	8	34	5	34	0	39	5	4
8	7	41	4	41	0	45	4	2
9	2	43	5	45	2	50	7	0
10	3	46	3	50	4	53	7	0
			35		9		44	18