POKHARA UNIVERSITY

Level: Bachelor | Semester: Spring | Year : 2025
Programme: BE | Full Marks : 100
Course: Applied Mathematics | Pass Marks : 45
Time : 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Define analytic function. Show that the necessary condition for a function $f(z) = u + iv$ to be analytic is $u_x = v_y$ and $u_y = -v_x$. 7

   b) Define harmonic function. Check $u = \sin x \cosh y$ is harmonic or not? If yes, find a corresponding harmonic conjugate v of u and corresponding analytic function $f(z)$. 8

2. a) Find the Laurent's series expansion of $f(z) = \frac{4z+3}{z^3-z^2-6z}$ in the following regions 7

   i. $0 < |z| < 2$

   ii. $2 < |z| < 3$

   iii. $|z| > 3$

   OR

   Define bilinear transformation. Find the bilinear transformation which maps the points $z = 0, -1, i$ onto the points $w = i, 0, \infty$. Also, find the image of unit circle $|z| = 1$.

   b) State and prove Cauchy integral formula. Evaluate: $\oint_C \frac{e^{2z}}{(z-1)^4} dz$, where C is the circle $|z| = 2$ in counter clockwise direction. 8

3. a) Find:

   i. $Z^{-1}\left[\frac{z}{(z^2-5z+6)}\right]$ 3

   ii. $Z^{-1}\left[\frac{z}{(z+2)(z-1)^2}\right]$ 4

   b) Define Z-transform of a function. Prove that 8

   $Z[a^n f(t)] = F\left(\frac{z}{a}\right)$, where $Z[f(t)] = F(z)$.

   Using it, find $Z(a^n e^{ibt})$ and hence deduce the values of $Z(a^n \cos bt)$ and $Z(a^n \sin bt)$.

4. a) Using Z-transform, solve the difference equation: 7

$y_{n+2} + 6y_{n+1} + 9y_n = 2^n$, $y_0 = 0, y_1 = 0$

b) Derive one dimensional wave equation with required assumptions. 8

OR

Express the Laplacian $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ in polar co-ordinates.

5. a) A homogeneous rod of conducting material of length 100 cm has its ends kept at zero temperature and the temperature initially is 7

$$f(x) = \begin{cases} x & \text{if } 0 \le x \le 50 \\ (100 - x) & \text{if } 50 < x \le 100 \end{cases}$$

Find the temperature $u(x,t)$.

b) Find the solution of one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ assuming the appropriate initial and boundary conditions. 8

6. a) Using Fourier integral representation, show that 7

$$\int_0^{\infty} \frac{cosxw + wsinxw}{1 + w^2}\, dw = \begin{cases} 0 & \text{if } x < 0 \\ \frac{\pi}{2} & \text{if } x = 0. \\ \pi e^{-x} & \text{if } x > 0 \end{cases}$$

b) Find the Fourier cosine transform of $f(x) = e^{-x}$ and by using Parseval's identity, show that $\int_0^{\infty} \frac{dx}{(1+x^2)^2} = \frac{\pi}{4}$. 8

7. Attempt all the questions 4×2.5

a) Evaluate: $\oint_C \frac{2z}{(z-1)(z+3)} dz$, where C is the circle $C: |z| = 2$ in counter clockwise direction by using Cauchy residue theorem.

b) Check whether the function $u(x,y) = z\bar{z}$ is analytic or not.

c) Find $Z(n^2)$.

d) Solve $u_x + u_y = 0$ by using separation of variables.