$-\omega$ $-h$ diff con

POKHARA UNIVERSITY

Level: Bachelor
Programme: BE
Course: Applied Mathematics

Semester: Fall

Year : 2024
Full Marks : 100
Pass Marks : 45
Time : 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Define analyticity of a function f(z). Show that the necessary condition for the function $f(z) = u(x,y) + iv(x,y)$ to be analytic on a domain D is $u_x = v_y$ and $u_y = -v_x$ at each point (x, y) of D. 7

   b) State and prove Cauchy integral formula. Evaluate the integral $\oint_C \frac{e^{5z}}{(z+i)^4} dz$, where c: $|z| = 2$. 8

2. a) State Cauchy Residue Theorem. By applying Cauchy Residue Theorem, evaluate $\oint_C \frac{dz}{(z^2+4)^3}$ where C : $|z - i| = 2$. 8

   b) Find the linear transformation that maps $z = 0$, $z = 1$, $z = \infty$ into the points $w = -3$, $w = -1$, $w = 1$ and find the fixed point of the transformation. 7

3. a) State and prove first shifting theorem on Z-transform. Find Z-transform of $e^{\frac{in\pi}{2}}$ and then find $Z(\cos\frac{n\pi}{2})$ and $Z(\sin\frac{n\pi}{2})$. 7

   b) Solve the difference equation $y_{n+2} - 7y_{n+1} + 12y_n = 2n$, $y_0 = 0$, $y_1 = 0$ by using Z-transform. 8

4. a) Show that $Z[nf(t)] = -z\frac{d}{dz}[F(z),]$ where $F(z) = Z[f(t)]$. 7

   Find $Z^{-1}\left[\frac{z}{(z+1)^2(z-1)}\right]$.

   b) Derive one dimensional heat equation. 8

5. a) Change the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ into polar form $r^2 u_{rr} + r u_r + u_{\theta\theta} = 0$. 8

   b) The diameter of a semi-circular plate of radius 'a' is kept at 0°C and the temperature at the semi-circular boundary is T°C. Find the steady state temperature of the plate. 7

OR

Find the temperature in a laterally insulated bar of length L whose ends are kept at temperature 0, assuming that the initial temperature

$$\text{is } f(x) = \begin{cases} x & \text{if } 0 < x < \dfrac{L}{2} \\ L - x & \text{if } \dfrac{L}{2} < x < L \end{cases}.$$

6. a) Find the Fourier cosine transform of $f(x) = e^{-mx}$ for $m > 0$, and then show that $\int_0^\infty \frac{\cos kx}{1+x^2}\,dx = \frac{\pi}{2}e^{-k}$. 7

OR

Find the Fourier transform of $f(x) = xe^{-x^2}$.

b) Show that $\int_0^\infty \frac{\cos wx + w \sin wx}{1+w^2}\,dw = \begin{cases} 0 & \text{if } x < 0 \\ \frac{\pi}{2} & \text{if } x = 0. \\ \pi e^{-x} & \text{if } x > 0 \end{cases}$ 8

7. Attempt any two questions. 2×5

a) Find Laurent series of the function $f(z) = \dfrac{z+3}{z(z+1)(z-2)}$ in the region $1 < |z| < 2$.

b) Find the solution of the differential equation, $y^2u_x - x^2u_y = 0$, by using separating of variables.

c) Show that the function $u(x, y) = e^{2x}(x \cos 2y - y \sin 2y)$ is a harmonic function. Find its harmonic conjugate.