POKHARA UNIVERSITY

Level: Bachelor Semester: Fall Year : 2018
Programme: BE Full Marks: 100
Course: Engineering Mathematics I Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Prove that the differentiability of a function at a point implies the continuity of the function at that point. Give an example to show that the converse may not be true. 8

OR

If $y = a\cos(\log x) + b\sin(\log x)$ show that

i. $x^2y_2 + xy_1 + y = 0$ and

ii. $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$.

b) State and prove Lagrange's Mean value theorem. 7

Show that $\frac{b-a}{b} < \log(\frac{b}{a}) < \frac{b-a}{a}$ by using Lagrange's mean value theorem.

2. a) Evaluate $\lim_{x\to 0}(\cos x)^{\cot^2 x}$ 7

b) Find the asymptotes of the curve $x^2(x-y)^2 - a^2(x^2+y^2) = 0$. 8

OR

A square piece of tin of side 18 cm is to be made into a box without lid, cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the value of box is maximum possible?

3. Evaluate the following integrals (Any three) 5×3

a) $\int \frac{x^3}{(x-2)(x-3)}dx$

b) $\int \frac{1}{2+\cos x+\sin x}dx$

c) $\int_a^b e^{-x}\,dx$ by summation method

d) $\int_0^{\frac{\pi}{2}} \frac{x\,dx}{\sin x+\cos x} = \frac{\pi}{2\sqrt{2}}\log(\sqrt{2}+1)$.

4. a) Find the volume of the solid in the region in the first quadrants bounded by the parabola $x = \sqrt{y}$ and the line y=x is revolved about y-axis. 7

b) Find approximate value of $\int_0^3 (x^2+1)\,dx$ by Simpson's and Trapezoidal Rule with n = 6. Compare the result with exact value. 8

5. a) Find the condition that the line $lx + my + n = 0$ may be a tangent to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. 7

b) Define conic section and derive the standard equation of Ellipse. 8

6. a) Find the equation of the plane through the points (2,4,5) and perpendicular to the line $\frac{x-5}{1} = \frac{y-1}{3} = \frac{z}{4}$ by vector method. 7

b) Define vector triple product. If 8

$\vec{a} = \vec{i} - 2\vec{j} - 3\vec{k},\ \vec{b} = 2\vec{i} + \vec{j} - \vec{k}$ and $\vec{c} = \vec{i} + 3\vec{j} - 2\vec{k}$

Also verify that $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$.

7. Attempt all questions. 2.5×4

a) Find the radius of curvature at any point (r, θ) for the curve $r = ae^{\theta\cot\alpha}$.

b) Find the center, vertices and foci of the ellipse

$x^2 + 10x + 25y^2 = 0$

c) Evaluate $\int \frac{x}{(x-3)(x+1)}dx$

d) Find the value of p so that the vectors $\vec{a} = 2\vec{i} - \vec{j} + \vec{k},\ \vec{b} = \vec{i} + 2\vec{j} + 3\vec{k}$ and $\vec{c} = 3\vec{i} + p\vec{j} + 5\vec{k}$ are coplanar.