

POKHARA UNIVERSITY

Level: Bachelor Semester: Spring Year : 2019
 Programme: BE Full Marks: 100
 Course: Engineering Mathematics I Pass Marks: 45
 Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Define continuity and differentiability of a function. Show that the function $f(x)$ defined by $f(x) = \begin{cases} x^2 + 1 & \text{when } 0 \leq x < 1 \\ x + 1 & \text{when } 1 \leq x < 2 \end{cases}$ 8

Is continuous of $x=1$ but not differentiable at $x=1$.

OR

If $y = \ln(x + \sqrt{a^2 + x^2})$, show that

(i) $(a^2 + x^2)y_2 + xy_1 = 0$

(ii) $(a^2 + x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2y_n = 0$

- b) State Lagrange's mean value theorem. Verify Lagrange's Mean Value Theorem for the function $f(x) = x(x-1)(x-2)$ in $[0, \frac{11}{2}]$. 7

2. a) Show that the limit: $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \cot^2 x \right) = \frac{2}{3}$ 7
 b) What are the types of asymptotes? Find the asymptotes of the curve: $y^3 - x^2y + 2xy^2 + 4y + x = 0$ 8

OR

Find the total surface area of the right circular cylinder of greatest surface that can be inscribed in a given sphere of radius r .

3. Integrate (Any Three) 3×5

a) $\int \frac{dx}{(x+3)\sqrt{x^2+6x+10}}$

b) $\int_0^1 \frac{\sin^{-1}x}{\sqrt{1-x^2}} dx$

c) $\int \frac{x + \sin x}{1 + \cos x} dx$

d) $\int_a^b e^{-x} dx$ (Using summation Method.)

4. a) Find the area between the curves $x=y^2$ and $2y^2 = -x+3$. 7

OR

Find the volume of the solid in the region in the first quadrant evolved by the parabola $y = x^2$ the y -axis and the line $y = 1$ revolved about the line $x = \frac{3}{2}$.

- b) Find the approximate area using Simpson's and Trapezoidal rule for the region bounded by the curves $y = x^2 + 2$, the x -axis with $a = 1$, $b = 5$ and $n = 6$ and compare with exact value. 8

5. a) Find the condition that the line $lx + my + n = 0$ is tangent to the parabola $y^2 = 4ax$. Also find the point of contact. 7

- b) Define eccentricity of a conic section, and derive the equation of hyperbola in its standard form. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ 8

6. a) Define scalar and vector product. Show that: $[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2[\vec{a}, \vec{b}, \vec{c}]$ 7

- b) Find the equation for the plane through the points $(-1, 4, 3)$, and perpendicular to the line $\frac{x-3}{2} = \frac{y+1}{5} = \frac{z-4}{1}$ (by vector method). 8

7. Attempt all the questions. 4×2 = 8

- a) Transform the equation $x^2 - 3y^2 + 4x + 6y = 0$ by transferring the origin to the point $(-2, 1)$ the co-ordinate axes remaining parallel. 5

- b) Find the arc length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 2$.

- c) Evaluate: $\lim_{x \rightarrow 0} \frac{x^n}{e^x}$

- d) Find the radius of curvature of the curve $y = 4\sin x - \sin 2x$ at $x = \frac{\pi}{2}$