Level: Bachelor   Semester: Fall   Year   : 2022
Programme: BE   Full Marks: 100
Course: Probability and Statistics   Pass Marks: 45
                                   Time   : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) From the following distribution of 500 students of a college, find the marks if only 20% students had failed and also find the minimum marks obtained by the top 25% of the students.   7

Marks	0–20	20–40	40–50	50–60	60–80	80–100
No. of students	50	100	150	90	60	50

    Also represent the data by histogram and locate the mode.

   b) The scores of two golfers for 10 rounds each are: 8

      A: 58   59   60   54   65   66   52   75   69   52

      B: 84   56   92   65   86   78   44   54   78   68

    Which may be regarded as the more consistent player?

2. a) Define the mathematical definition of probability. From a group of 3 engineers, 4 economists and 5 statisticians, a sub – group of 4 persons are selected at random from the group. Find the probability that the sub – group will consist of   8
     i.   2 engineers and 2 economists
     ii.   1 engineer, 1 economist and 2 statistician
     iii   All statisticians.

   b) Customers who purchase a certain make of car can order an engine in any of three sizes. Of all cars sold, 45% have the smallest engine, 35% have the medium-sized one, and 20% have the largest. Of cars with the smallest engine, 10% fail an emissions test within two years of purchase, while 12% of those with the medium size and 15% of those with the largest engine fail. What is the probability that a randomly chosen car will fail an emissions test within two years?   7

3. a) a random variable X has the following probability function,   8

X:	2	3	4	5	6	7
F(X):	0.1	K	0.2	2K	0.3	K

    Find the value of K and find mean and variance.

   b) In a certain factory turning out optical lenses, there is a small chance 1/500 for any lens to be defective. The lenses are supplied in a packet of 10. Use Poisson distribution to calculate the approximate number of packets containing no defective, one defective, two defective and three defective lenses respectively in a consignment of 10,000 packets.   7

4. a) In a class quiz, 8% of the students obtained marks below 25 and 90% of the students got below 85 marks. Assuming that the marks are normally distributed, find the mean and standard deviation of the distribution.   7

   b) Let X and Y be two continuous random variables with joint probability density function   8

$$f(x,y) = Axy; \quad 0 < x < 1 \ and \ 0 < y < x$$

     i.   Find the constant A.
     ii.   Find the marginal density functions of X and Y.
     iii.   Find the conditional density function of Y given X = x.
     iv.   Check the independency of X and Y.

5. a) A random sample of 8 envelops is taken from letter box of post office and their weights in gms are found to be 12.1, 11.9, 12.3, 11.9, 12.1, 12.4, 12.1, and 11.5. Find 99% fiducial limits for the mean weight of envelops received at the post office.   8

   b) What do you mean by p-value? In a factory 2% fans were found to be defectives in a lot of 3000 fans and in other factory 3% fans were found to be defective in a lot of 2500 fans. Do you find that the fans in secondary factory are significantly inferior compared to the fans in the first factory? (Use $\alpha=5\%$)   7

6. a) Resistors labeled as hundred *ohm* are purchased from two different vendors. The specification for this type of resistor is that its actual resistance be within 5% of its labeled resistance. In a sample of 180 resistors from vendor A, 150 of them met the specification. In a sample of 270 resistors purchased from vendor B, 233 of them met the specification. Vendor A is the current supplier, but if the data demonstrate convincingly that a greater proportion of the resistors from vendor B meet the specification, a change will be made.   7
     i.   State null and alternative hypothesis.

ii. Calculate test statistic and draw a conclusion that a change should be made? (Use α=5%)

b) The following table presents shear strengths (in kN/mm) and weld diameters (in mm) for a sample of spot welds. 8

Diameter	4.2	4.4	4.6	4.8	5.0	5.2	5.4
Strength	51	54	69	81	75	79	89

i. Compute the least-squares line for predicting strength from diameter.

ii. Predict the strength for a diameter of 5.5 mm.

7. Write short notes on: (**Any two**) 2×5

a) Error of hypothesis testing

b) Confidence Interval

c) Probability Sampling