Basic Electrical Engineering

Chapter-5: Electric Machines

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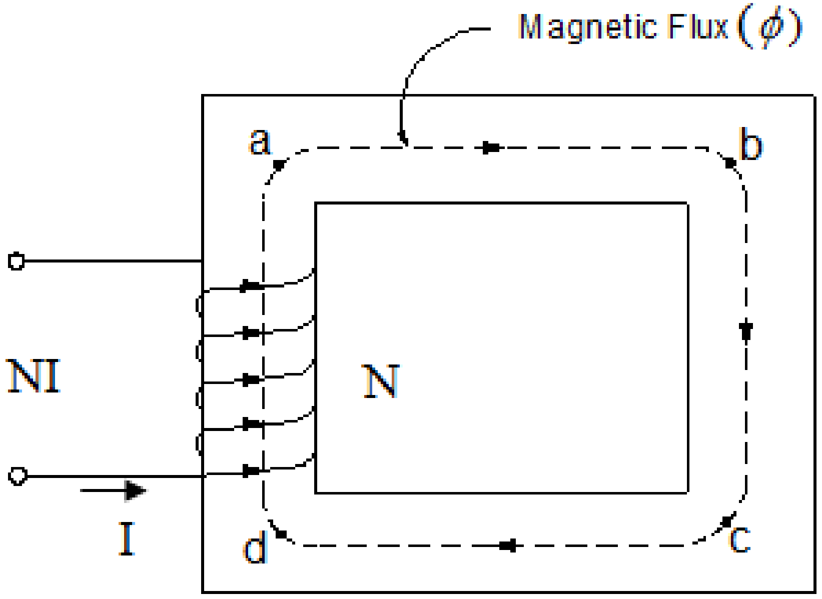
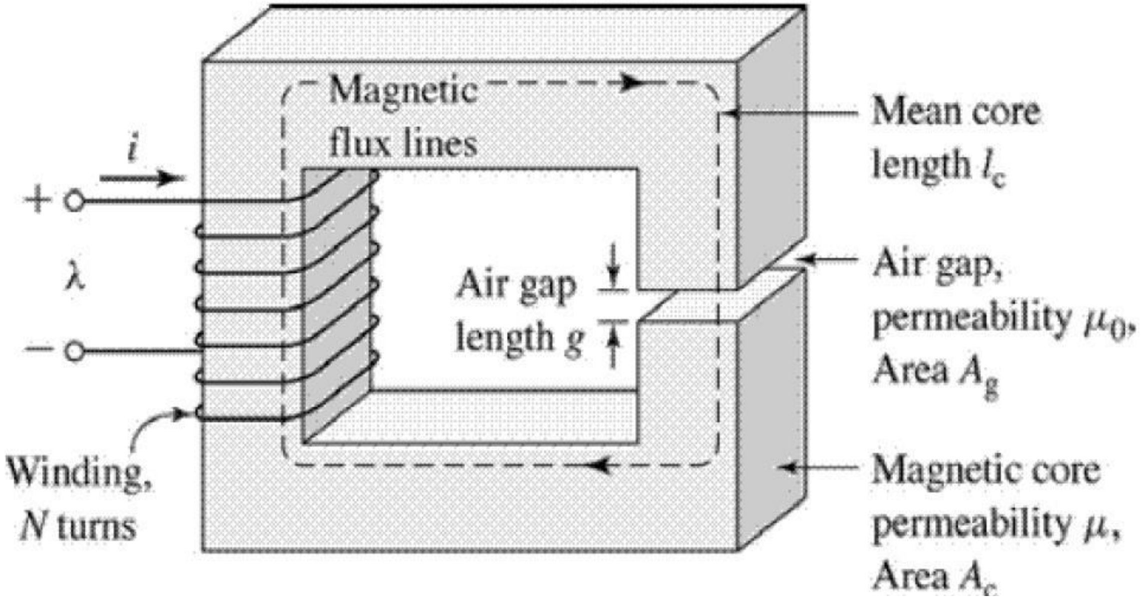
Magnetic circuit

The path followed by the magnetic flux is known as magnetic circuit. The path a-b-c-d-a shown in Fig. is a magnetic circuit consisting of iron core and winding.





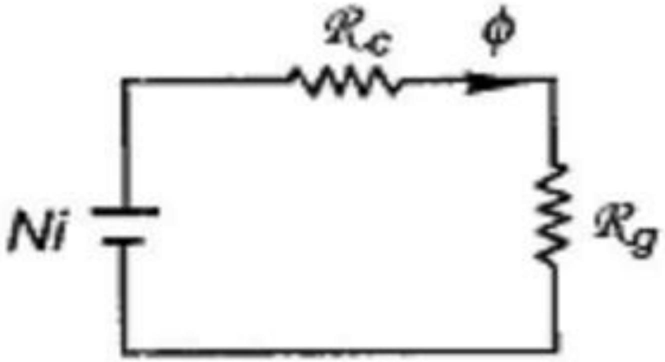
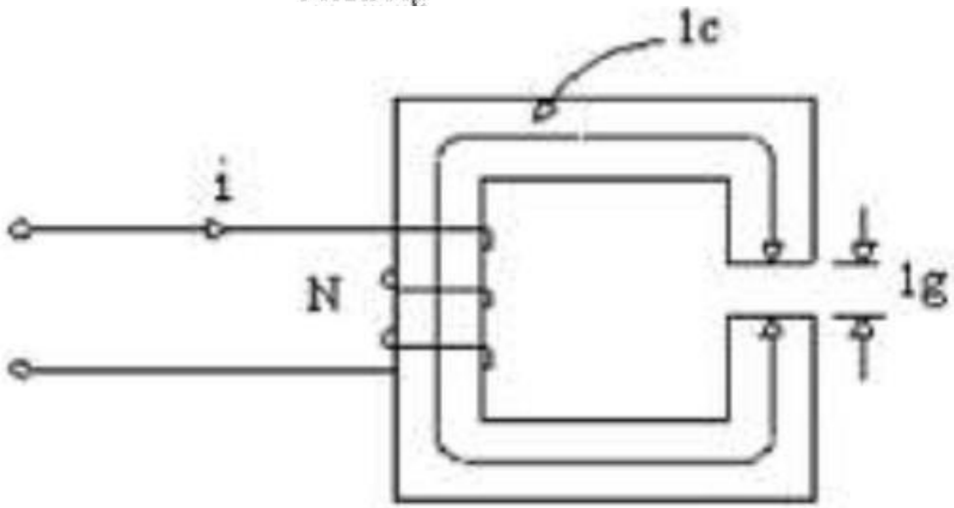
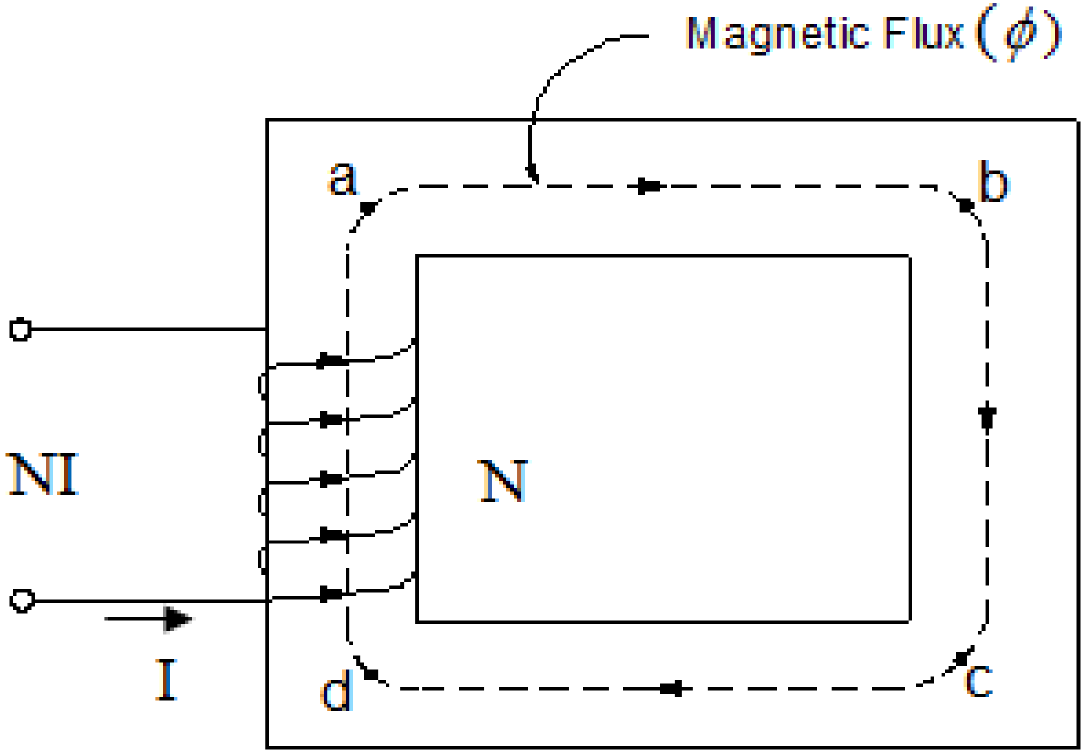


Fig. Magnetic circuit



Magnetic circuit



Let I = Current through the exciting winding (Amp)
N = Number of turns in exciting winding
ϕ = Magnitude of magnetic flux (wb)
A = Cross-sectional area of the iron core (m^2)
L = Mean length of the magnetic flux path (a-b-c-d-a)

Magnetic flux density in the core is given by: $B = \dfrac{\phi}{A}$

And, magnetic field intensity in side the core is given by: $H = \dfrac{N.I}{L}$

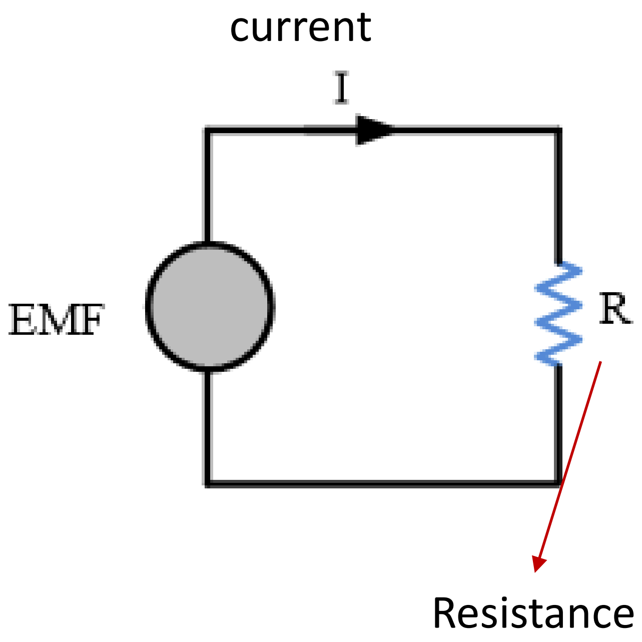
For linear part of the magnetization curve: $\dfrac{B}{H} = \mu$

Or B = μ.H OR $\dfrac{\phi}{A} = \dfrac{\mu.\text{N.I}}{\text{L}}$ OR $\phi = \dfrac{\text{N.I}}{L/\mu.A} = \dfrac{\text{mmf}}{\text{Reluctance}}$ (ohm's Law)

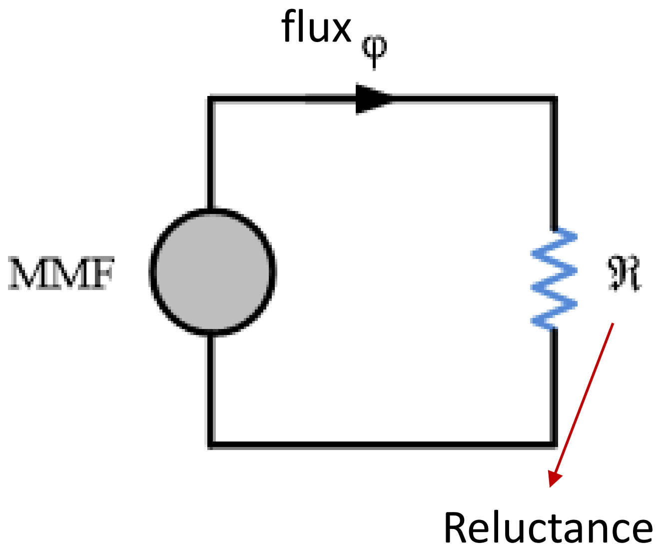
Where, **N.I= mmf = magnetomotive force**, which push the magnetic flux in the magnetic circuit.

$$\text{Rel} = \frac{\text{L}}{\mu.A} = \text{Reluctance of magnetic circuit}$$

Analogy between Electric Circuit and Magnetic circuit



Electric Circuit	Magnetic Circuit
Path traced by the current is known as electric current.	Path traced by the magnetic flux is called as magnetic circuit.
EMF is the driving force in the electric circuit. The unit is Volts.	MMF is the driving force in the magnetic circuit. The unit is ampere turns.
There is a current I in the electric circuit which is measured in amperes.	There is flux φ in the magnetic circuit which is measured in the weber.
The flow of electrons decides the current in conductor.	The number of magnetic lines of force decides the flux.
Resistance (R) oppose the flow of the current. The unit is Ohm	Reluctance (S) is opposed by magnetic path to the flux. The Unit is ampere turn/weber.
$R = \rho . l/a$. Directly proportional to l. Inversely proportional to a. Depends on nature of material.	$S = l/(\mu_0\mu_r a)$. Directly proportional to l. Inversely proportional to $\mu = \mu_0\mu_r$. Inversely proportional to a
The current I = EMF/ Resistance	The Flux = MMF/ Reluctance
The current density	The flux density
Kirchhoff current law and voltage law is applicable to the electric circuit.	Kirchhoff mmf law and flux law is applicable to the magnetic flux.



Series Magnetic circuit

Series magnetic circuit is such circuit, where the same magnetic flux passes through the all sections of the magnetic circuit as shown in Fig.

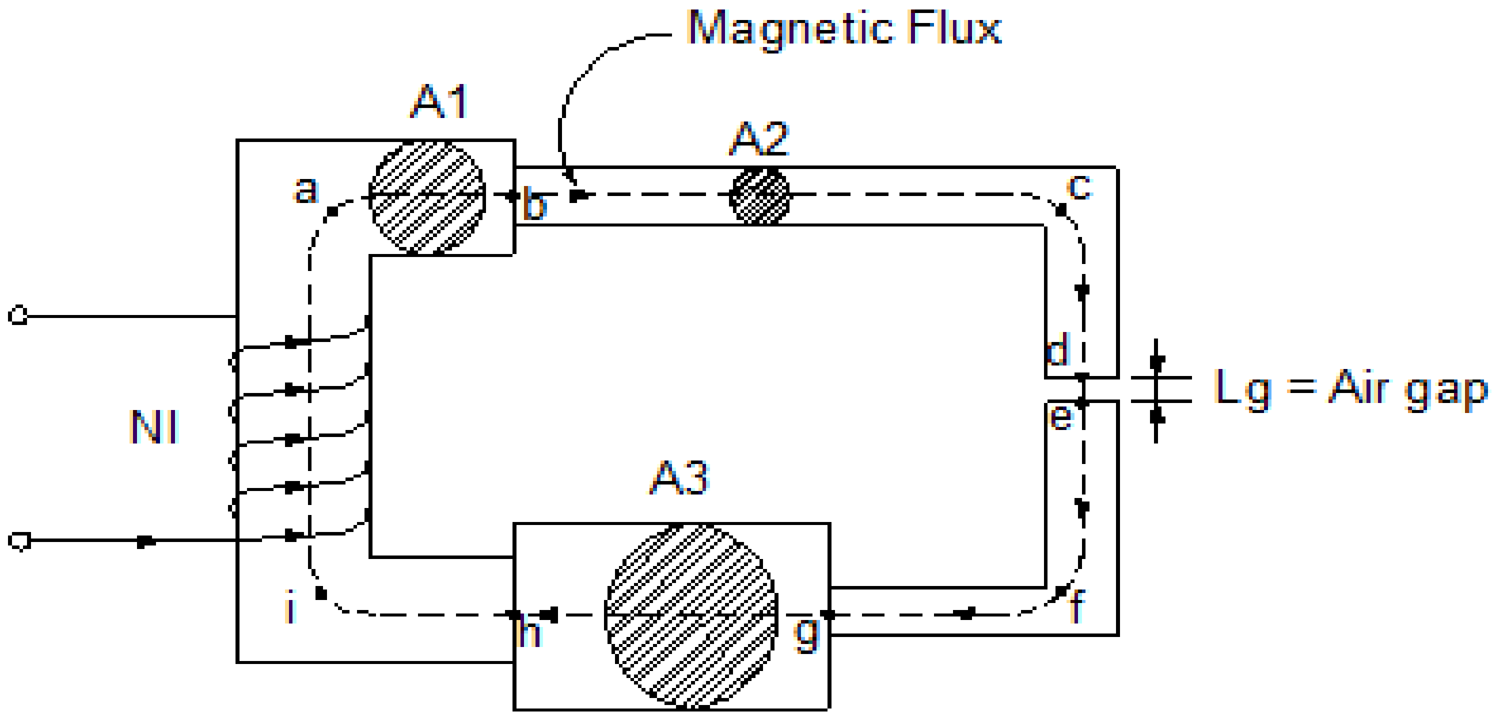


Fig. Series magnetic circuit

The magnetic circuit has four different sections with different length and cross-sectional area as follow:

Section-1 : L_1 = ba + ai + ih, Area = A_1, Permeability = μ_1
Section-2 : L_2 = bc + cd + ef + fg , Area = A_2 Permeability = μ_2
Section-3 : L_3 = gh, Area = A_3 Permeability = μ_3
Section-4 : L_g = de, Area = A_g=A_2 Permeability = μ_0

Parallel Magnetic circuit

If the magnetic flux divides into two or more parallel paths, such magnetic circuit is known as parallel magnetic circuit.

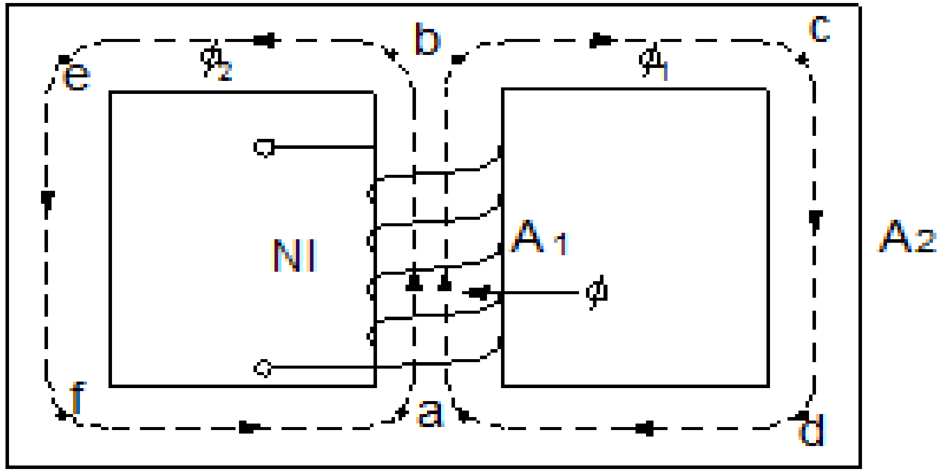


Fig.1.18(a) Parallel magnetic circuit

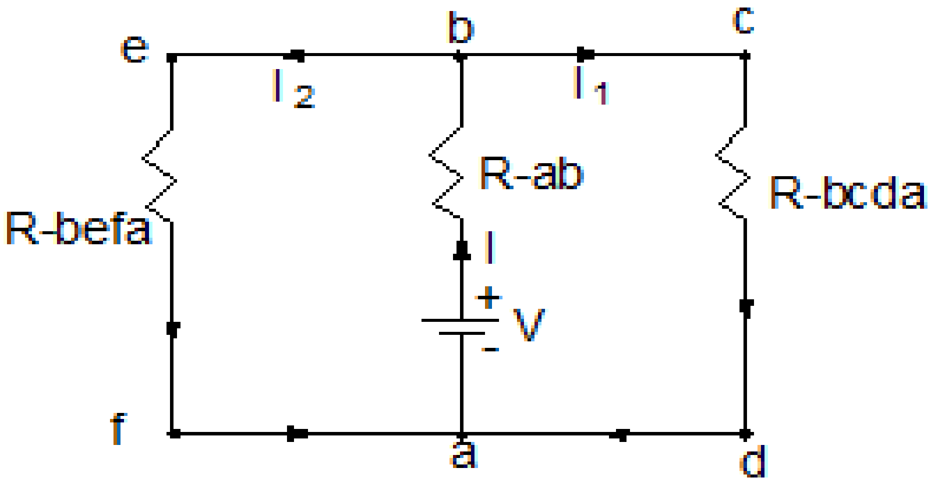


Fig.1.18(b) Corresponding electric circuit

For electric circuit, $I = I_1 + I_2$ If $R_{bcda} = R_{befa}$, Then $I_1 = I_2$ Or $I_1 = I/2$

Writing KVL for loop a-b-c-d: $V = I \times R_{ab} + I_1 \times R_{bcda}$ OR $V = I \times R_{ab} + 0.5I \times R_{bcda}$

$$\boxed{\text{OR } I = \frac{V}{(R_{ab} + 0.5\,R_{bcda})}}$$

Similarly, for magnetic circuit:

$\phi = \phi 1 + \phi 2$ If Reluctance of path-1 = Reluctance of path-2,

then $\phi 1 = \phi 2 = 0.5\,\phi$

Therefore, $N.I = \phi \times Rel_{(ab)} + \phi_1 \times Rel_{(bcda)}$

OR $N.I = \phi \times Rel_{(ab)} + 0.5\,\phi \times Rel_{(bcda)}$

$$\boxed{\phi = \frac{N.I}{(Rel_{(ab)} + 0.5\,Rel_{(bcda)})}}$$

TRANSFORMERS

Introduction:

- Transformer is a **static machine** which **transfers electrical power from one circuit to another circuit.**

- The two circuits **are electrically isolated** from each other, but they are linked by **common magnetic flux.**

- While transferring the electrical power from one circuit to another circuit, the voltage level of the second circuit may be different from that of the first circuit, but the **frequency of both circuits remains same**.

- Fig.2.1 represents the block diagram of a transformer.

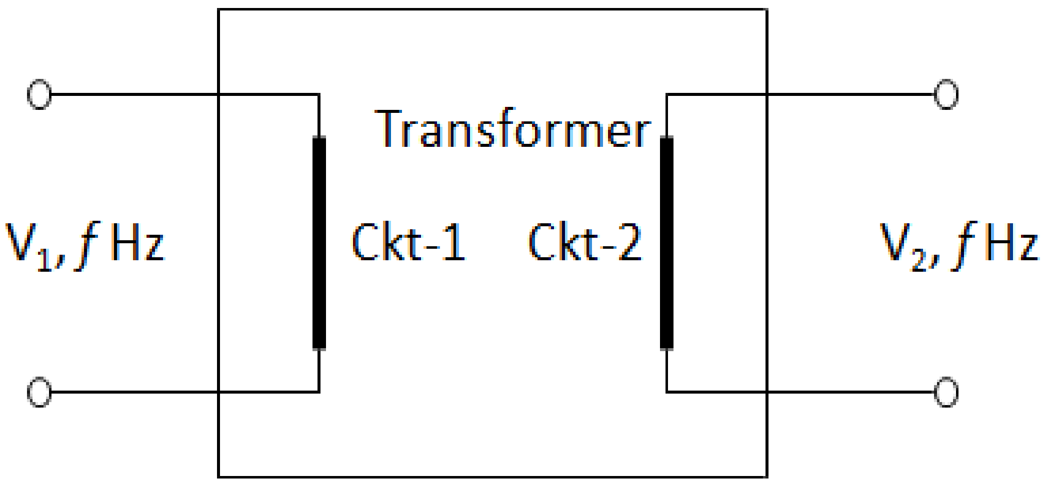


Fig.2.1 Block diagram representation of transformer

Basic construction and operating principle:

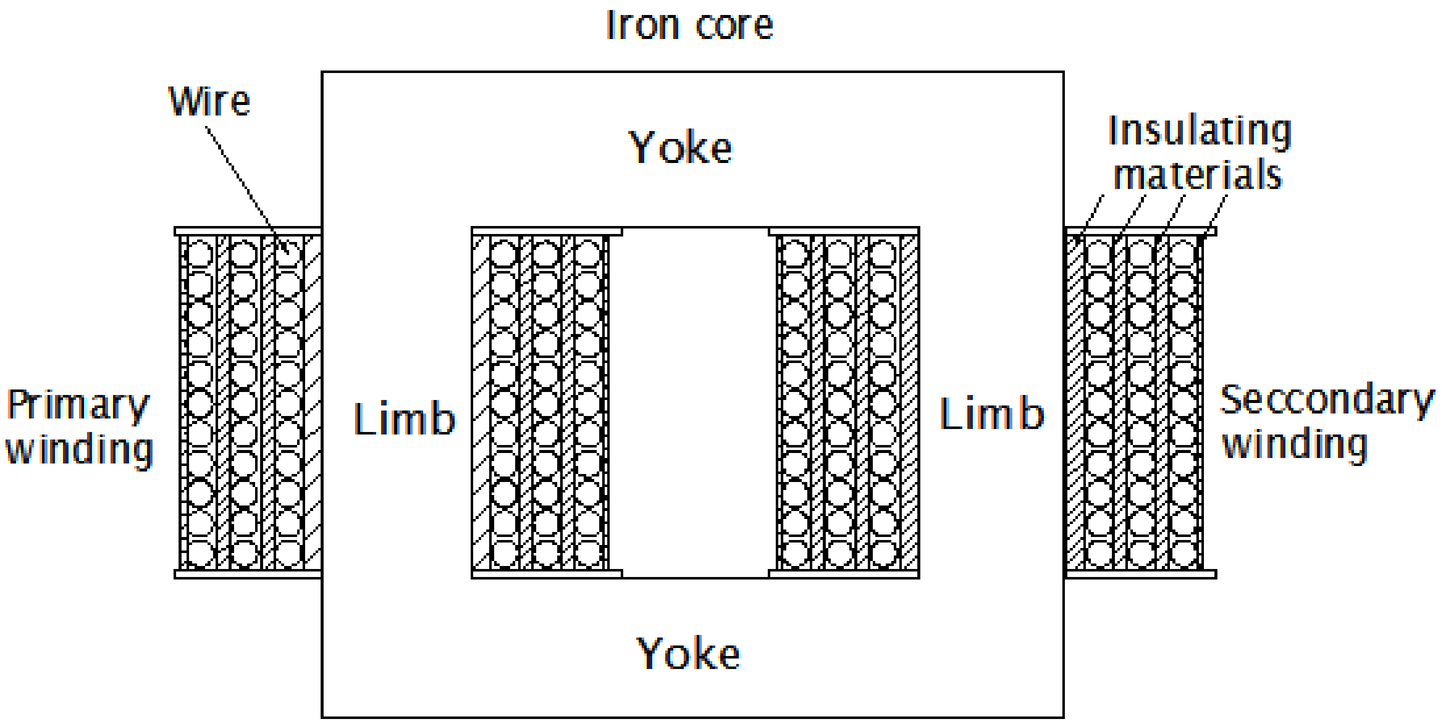


Fig.2.2 Basic construction of transformer

- Basically, a transformer consists of a rectangular window shaped iron core as shown in Fig.2.2. The horizontal part of the core is known as ‘**yoke**’ and the vertical part of the core is known as **‘limb**’.

- The core is made of **laminated silicon steel**. Two separate coils (windings) are wound on the two separate limbs of the core. The coils are made of **enamel insulated copper wire**.

Operating principle:

When one of the winding (say a-a’) is excited by ac voltage source ‘V_1’, then the winding will draw some current (say I_0). If the winding is assumed to be purely inductive with zero resistance, the current I_0 lags the supply V_1 by 90^0 as shown in Fig.2.4.

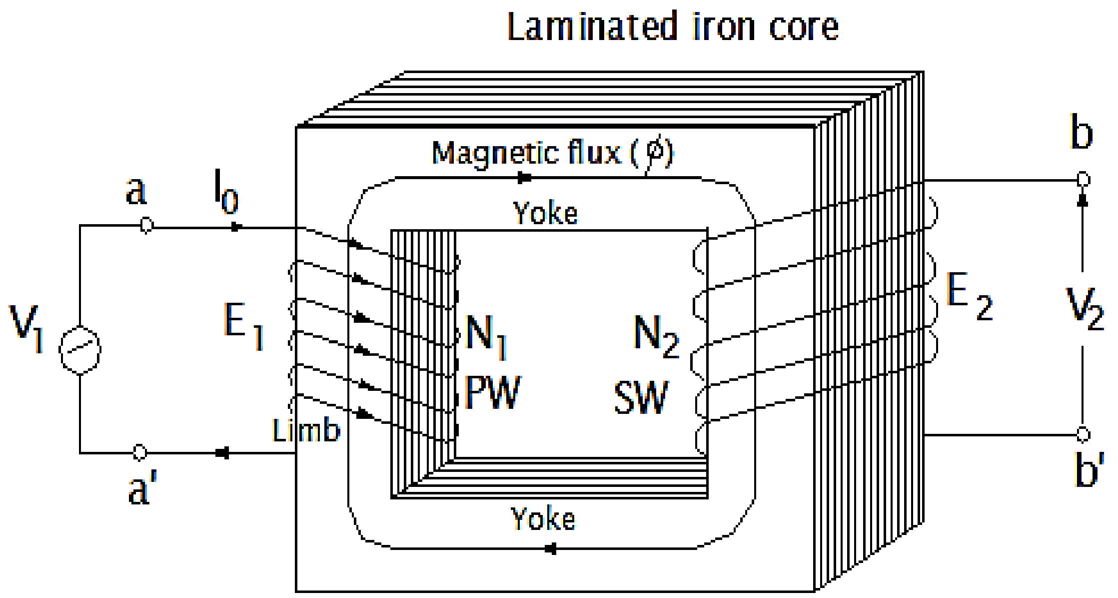


Fig.2.3 Circuit representation of transformer

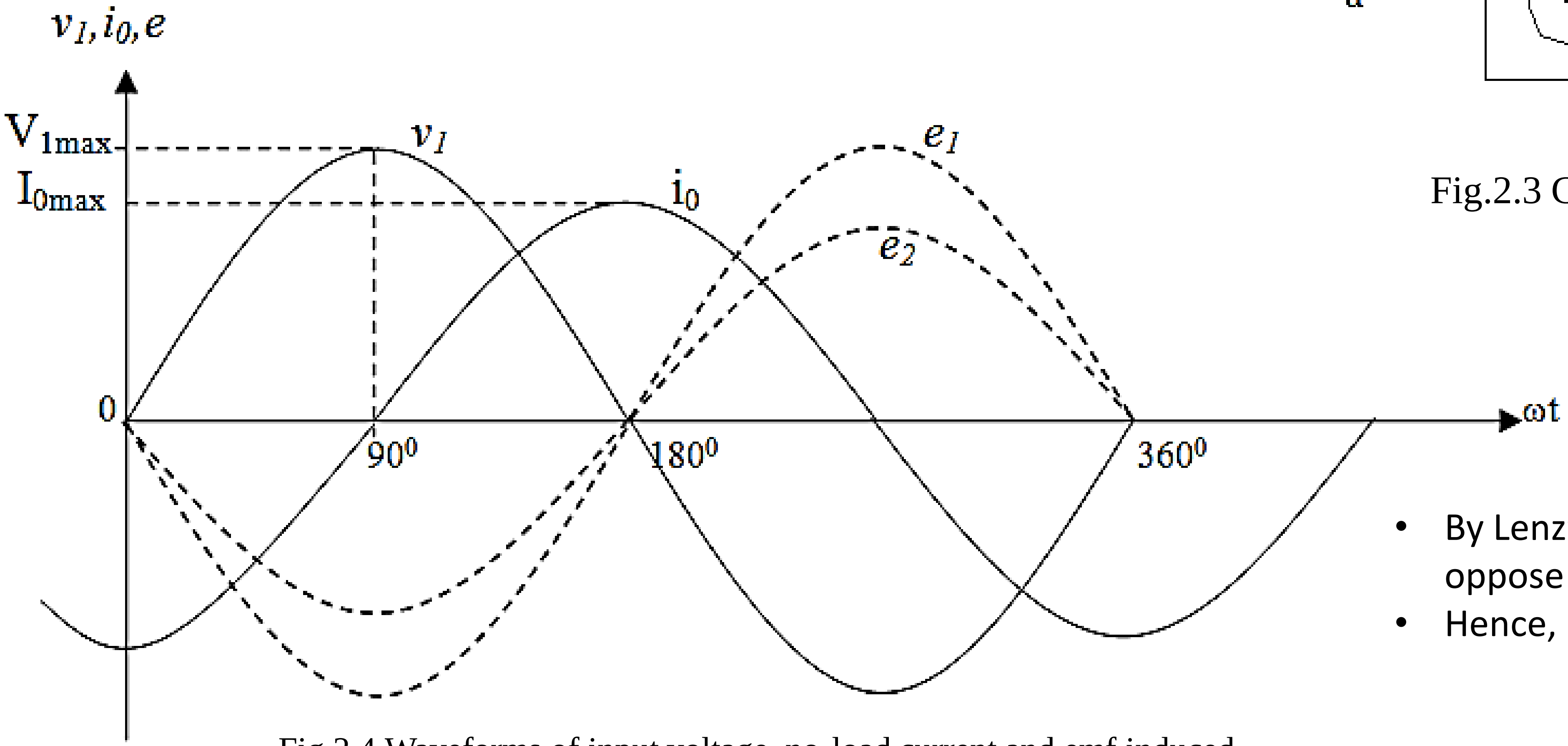
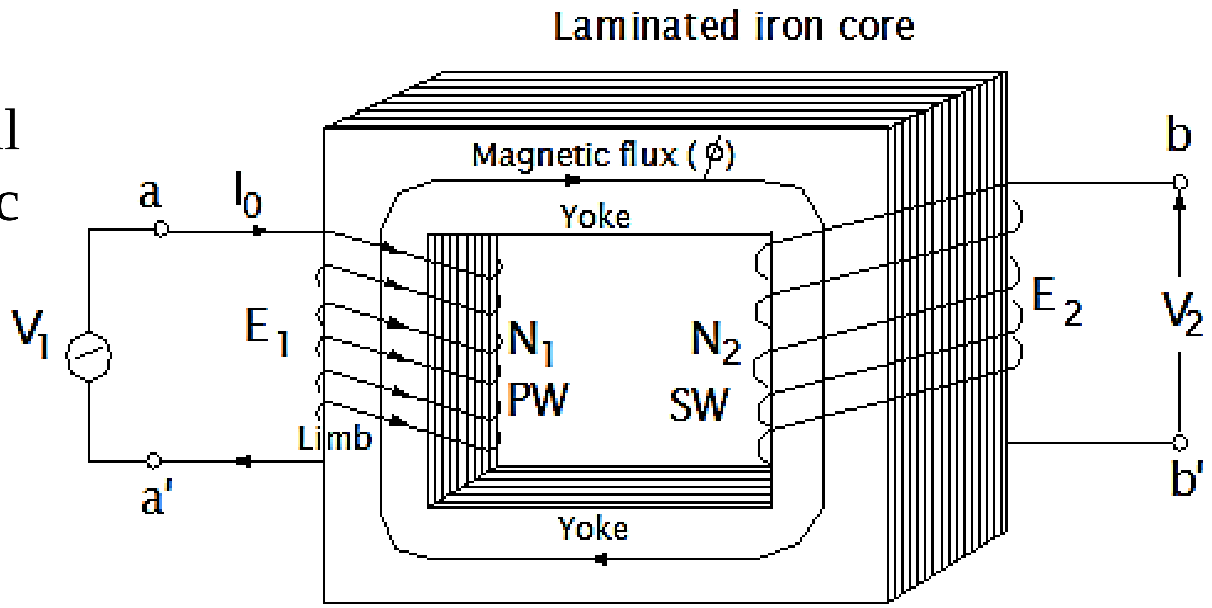


Fig.2.4.Waveforms of input voltage, no-load current and emf induced

- By Lenz’s law, emf induced always tries to oppose the supply voltage
- Hence, emf induced is out of phase.

Operating principle:

The iron core gets magnetized and magnetic flux will circulate through the iron core. The magnitude of magnetic flux is given by:



$$\phi = \frac{N_1 . i_0}{\mathrm{Re}\, l} \qquad (2.1)$$

Where, N_1 = Number of turns in the coil a-a’
i_0 = Instantaneous value of current through the coil a-a’
Rel = Reluctance of the core.

Since the applied voltage v_1 is alternating in nature, the current i_0 also will be alternating in nature. Hence, the **magnetic flux (ϕ) also will be alternating in nature and in phase with i_0** as shown in Fig.2.5.

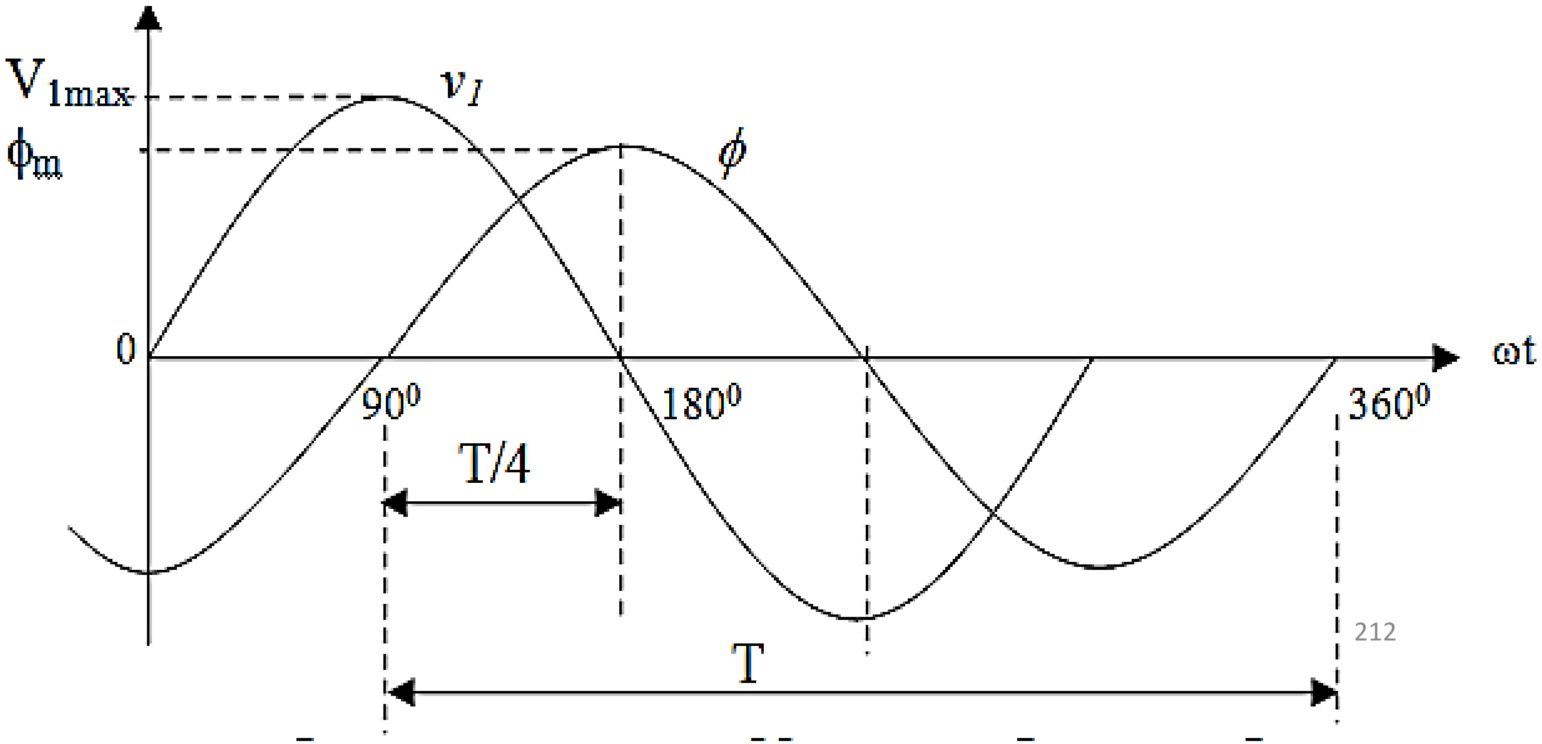


Fig.2.5 Waveforms of input voltage and magnetic flux

Operating principle:

- The magnitude of magnetic flux in the core is changing with respect to time and it is linking with the second coil on the another limb.

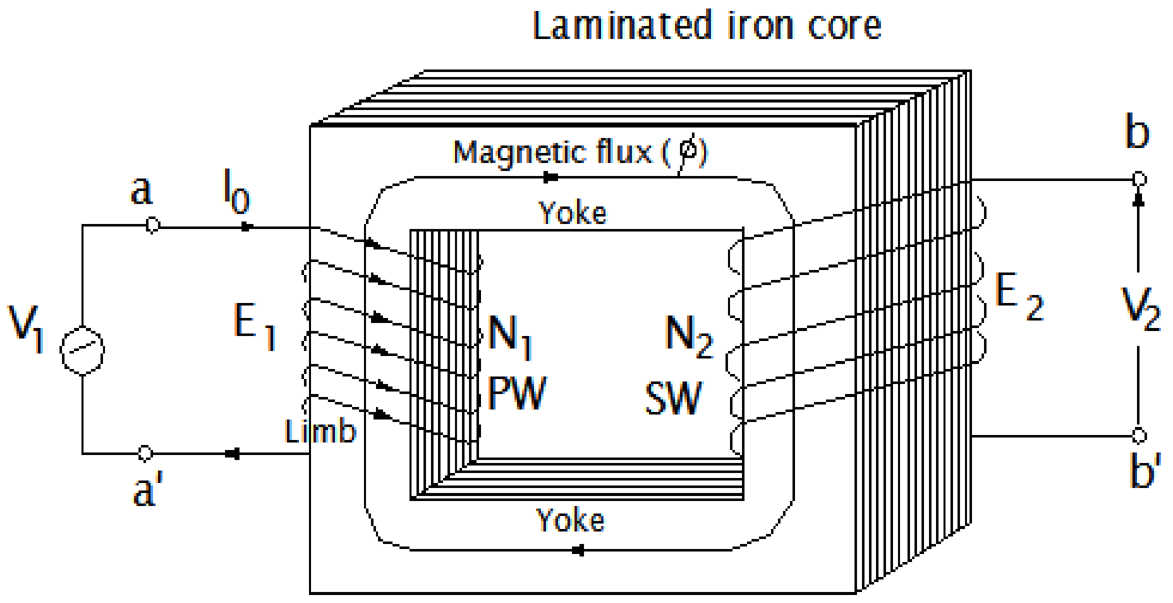
- Hence, according to **Faraday's law of electromagnetic induction, emf (e_2)** will induce in the second coil b-b'.

- If the load is connected across the second coil, electric current will circulate through the load thus by transferring the electrical power from coil a-a' to coil b-b'.

- This is the operating principle of transformer

- The coil, on which the supply voltage is applied, is known as **primary winding (PW)**

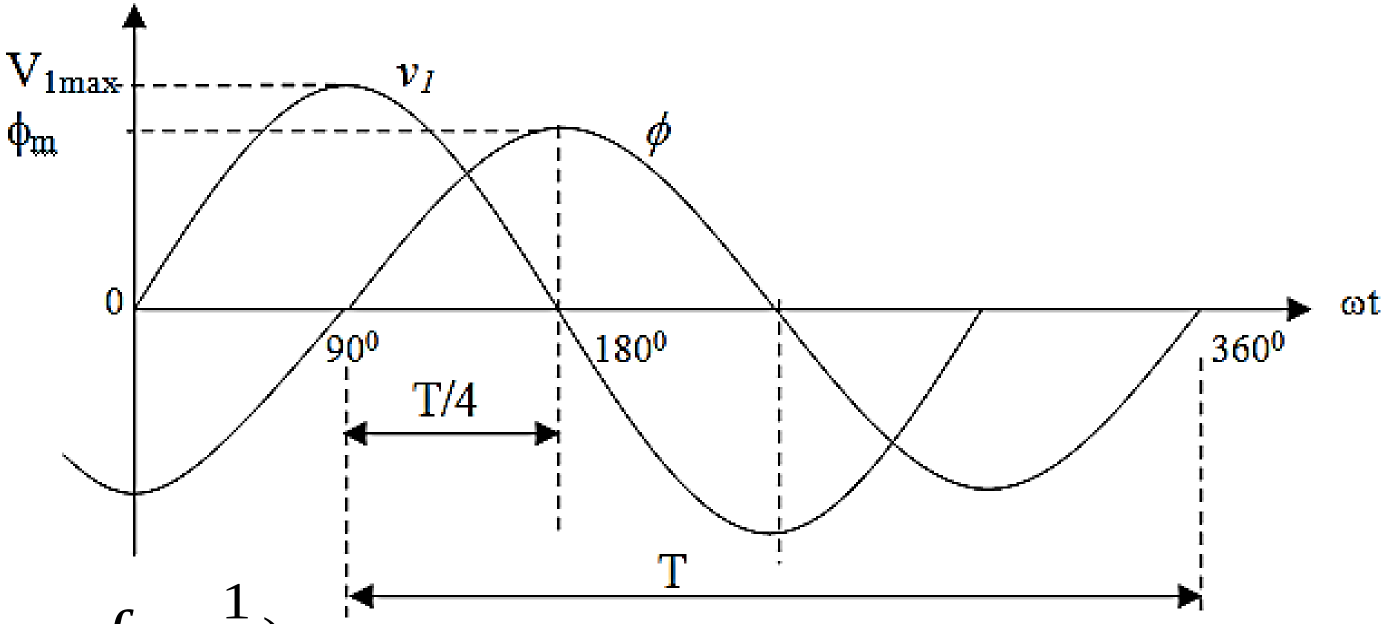
- The second coil, on which the **emf (e_2) is induced**, is known as **secondary winding (SW).**



Calculation of induced emf:

According to 2nd law of Faraday's law of electromagnetic induction, the average value of emf induced (e_2) is given by:

$$\boxed{E_{2(avg)} = N_2 \frac{d\phi}{dt}} \qquad (2.2)$$



Where, N_2 = Number of turns in secondary winding

$$\frac{d\phi}{dt} = \text{Average rate of change of magetic flux}$$

Magnetic flux changes from 0 to ϕ_m in T/4 sec.

Hence, $$\frac{d\phi}{dt} = \frac{\phi_m - 0}{T/4} = \frac{4.\phi m}{T} = 4\ f\ \phi m \quad (\text{Because, } f = \frac{1}{T})$$

$$\therefore\ E_{2(avg)} = N_2 \frac{d\phi}{dt} = 4\,N_2\ f\ \phi m$$

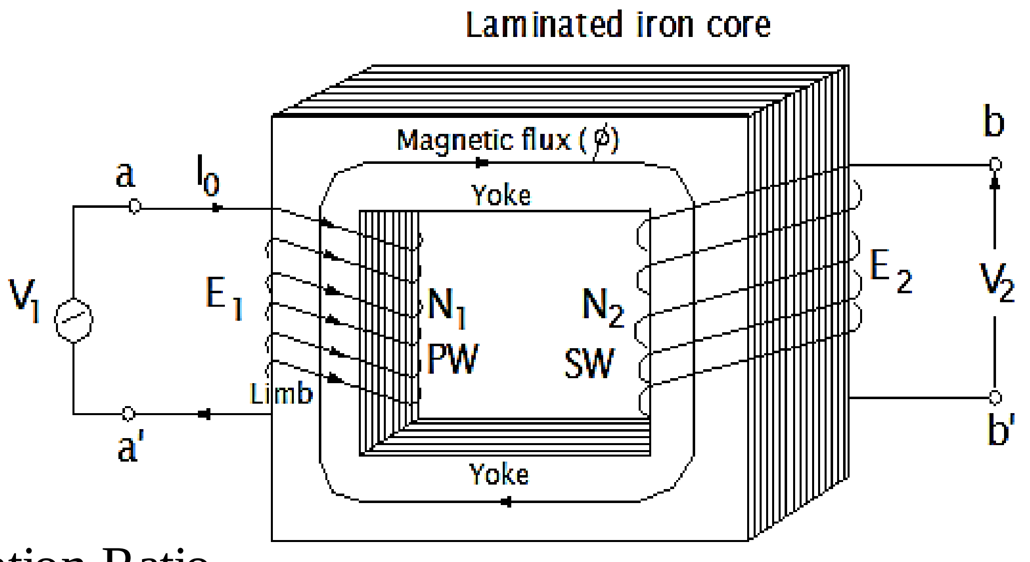
$$\text{For sine - wave, form factor} = \frac{\text{RMS Value}}{\text{Average Value}} = 1.11$$

$\therefore$ RMS value of emf induced in the secondary winding is given by:

$$\boxed{E_2 = 4.44\ N_2\ f\ \phi_m} \text{ Volt} \qquad (2.3)$$

- Similarly, the magnitude of emf induced in the primary winding is given by:

$$E_1 = 4.44\, N_1\, f\, \phi_m \qquad (2.4)$$



- Dividing eqn (2.3) by eqn (2.4) gives:

$$\frac{E_2}{E_1} = \frac{4.44\, N_2\, f\, \phi_m}{4.44\, N_1\, f\, \phi_m} = \frac{N_2}{N_1} = \frac{V_2}{V_1}$$

$$V_2 = \frac{N_2}{N_1} V_1 = K.V_1 \qquad (2.7) \qquad \text{Where, } K = \frac{N_2}{N_1} = \text{Transformation Ratio}$$

The magnitude of emf induced in the secondary winding depends upon the transformation ratio.

Case-I : If $\mathbf{N_2 > N_1}$, i.e. $K > 1$, then $\mathbf{V_2 > V_1}$

Such a transformer is known as **step up transformer**

Case-II : If $\mathbf{N_2 < N_1}$, i.e. $K < 1$, then $\mathbf{V_2 < V_1}$

Such a transformer is known as **step down transformer**

Case-III : If $\mathbf{N_2 = N_1}$, i.e. $K = 1$, then $\mathbf{V_2 = V_1}$

Such a transformer is known as **isolation transformer**

Ideal Transformer and Practical Transformer

- An ideal transformer is that which has **purely inductive winding** without any resistance, **without any magnetic leakage flux** and which is **100% efficient** without any power loss within the transformer.

- This is just the mathematical realization and such transformer can not be constructed in real practice.

- The operating principle of the transformer so far explained was based on the assumption of ideal transformer.

- Now, operation of **real transformer** shall be described in the following sections.

No-Load Operation of Real Transformer

- **No - load current I_0 has two components** as shown in the Fig.2.7. supplying active power loss and the reactive power loss.
- That means the **no-load current (I_0)** of a real transformer does not lag by 90^0 with V_1 as explained in the operating principal of ideal transformer. It **lags by an angle ϕ_0 which is less than 90^0.**

I_w = Component of I_0 in phase with V_1,
$I_w = I_0 \cos\phi_0$ = Loss component of I_0

I_μ = Component of I_0 which lags V_1, by 90^0 ,
$I_\mu = I_0 \sin\phi_0$ = Magnetizing component of I_0

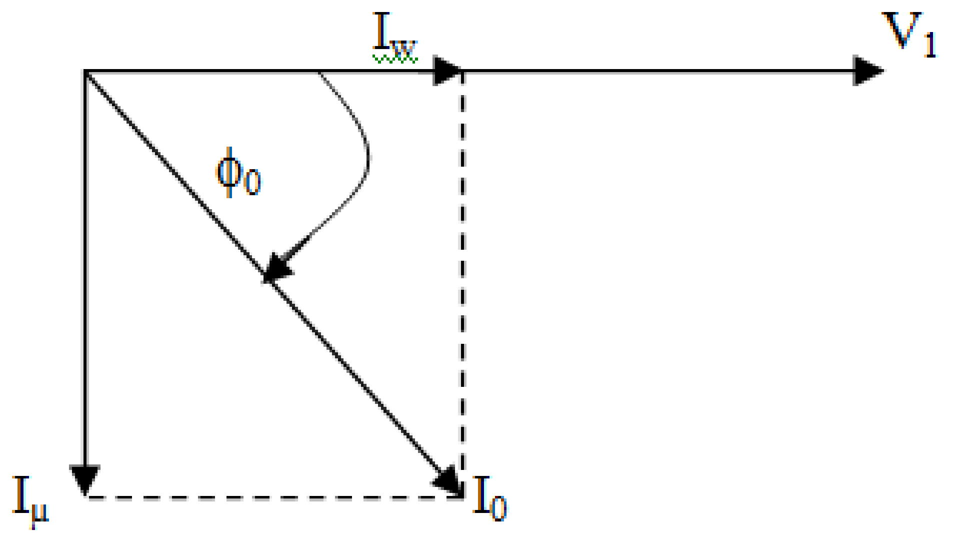


Fig.2.7 Phasor diagram for no-load operation

From the phasor diagram , it can be written as:

$$I_0 = \sqrt{I_w^2 + I_\mu^2}$$

No-Load Operation of Real Transformer

The active power consumed by transformer at no-load is given by:

$$W_0 = V_1 I_0 \cos\phi_0 = V_1 I_w \text{ Watts} \qquad (2.8)$$

- This power will be lost within the transformer in **heating the iron core**. Hence, $\mathbf{W_0}$ is known as no-load power loss or iron loss of the transformer.
- Therefore, the component $\mathbf{I_w}$ is responsible for producing heat loss in the iron core.
- $\mathbf{Cos\phi_0}$ is known as **no-load power factor** of the transformer.

The reactive power consumed by transformer at no-load is given by:

$$W_0 = V_1 I_0 \sin\phi_0 = V_1 I_\mu \text{ VARs} \qquad (2.9)$$

- The component I_μ is responsible for reactive power which is used in maintaining magnetic flux in the iron core.
- Here it shall be noted that only reactive power can establish magnetic flux in magnetic circuit excited by AC voltage.

Equivalent Circuit of Real Transformer at no load:

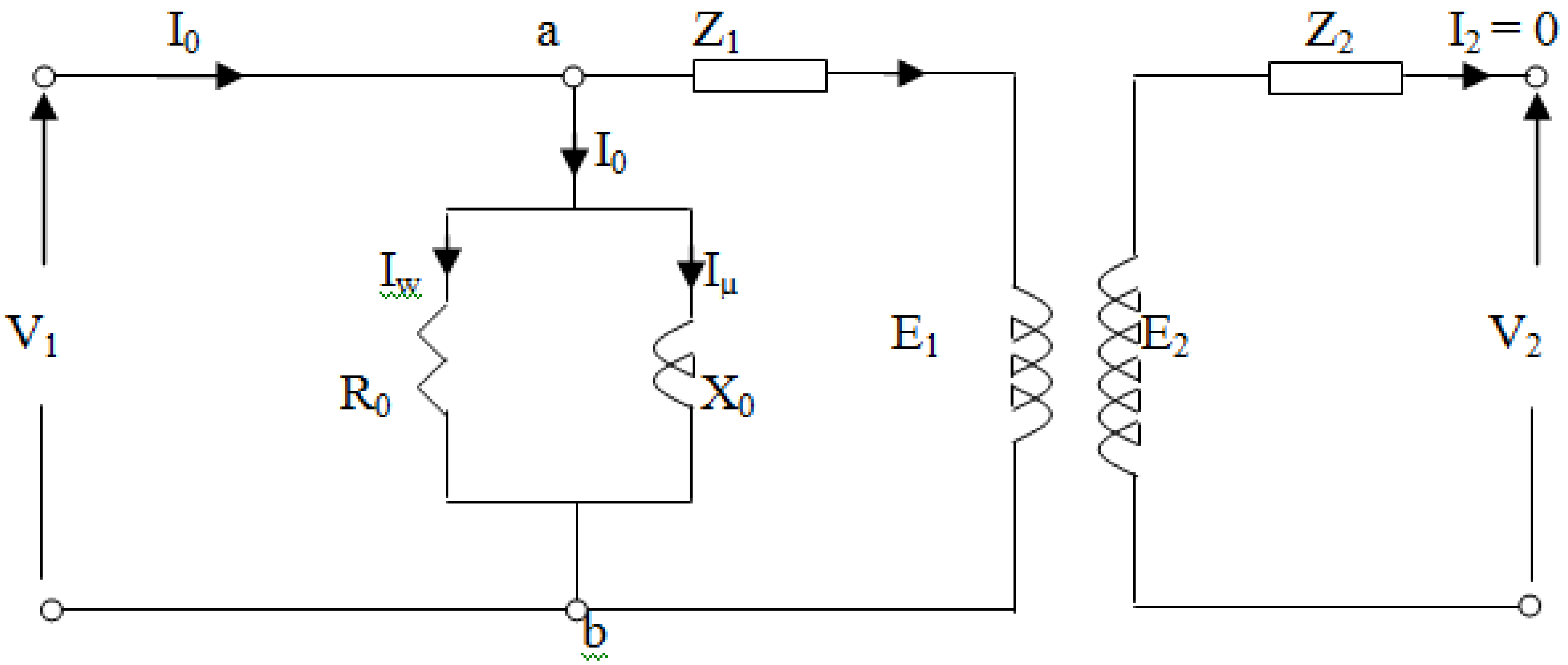


Fig.2.9 Equivalent circuit of transformer at no-load

V_1 = Input voltage
E_1 = Emf induced across the P.W.
E_2 = Emf induced across the S.W.
V_2 = Load terminal voltage (< E_2)
I_0 = No-load primary current (remains constant)
I_2 = Load current = S.W. current (Varies with load)
R_1 = Resistance of P.W.
X_1 = Leakage reactance of P.W.
R_2 = Resistance of S.W.
X_2 = Leakage reactance of S.W.
R_0 = Shunt branch core loss resistance
X_0 = Shunt branch magnetizing reactance
$I_w = V_1/ R_0$ = In phase component of I_0
$I_\mu = V_1/ X_0$ = 90^0 lagging component of I_0
$I_w^{\ 2} R_0$ = Iron loss (core loss)
$I_\mu^{\ 2} X_0$ =Reactive power consumes by transformer to produce magnetic flux in the core

Equivalent Circuit of Real Transformer at load:

$Z_1 = (R_1 + jX_1)$ = Series Impedance of primary winding $\quad Z_2 = (R_2 + jX_2)$ = Series Impedance of secondary winding

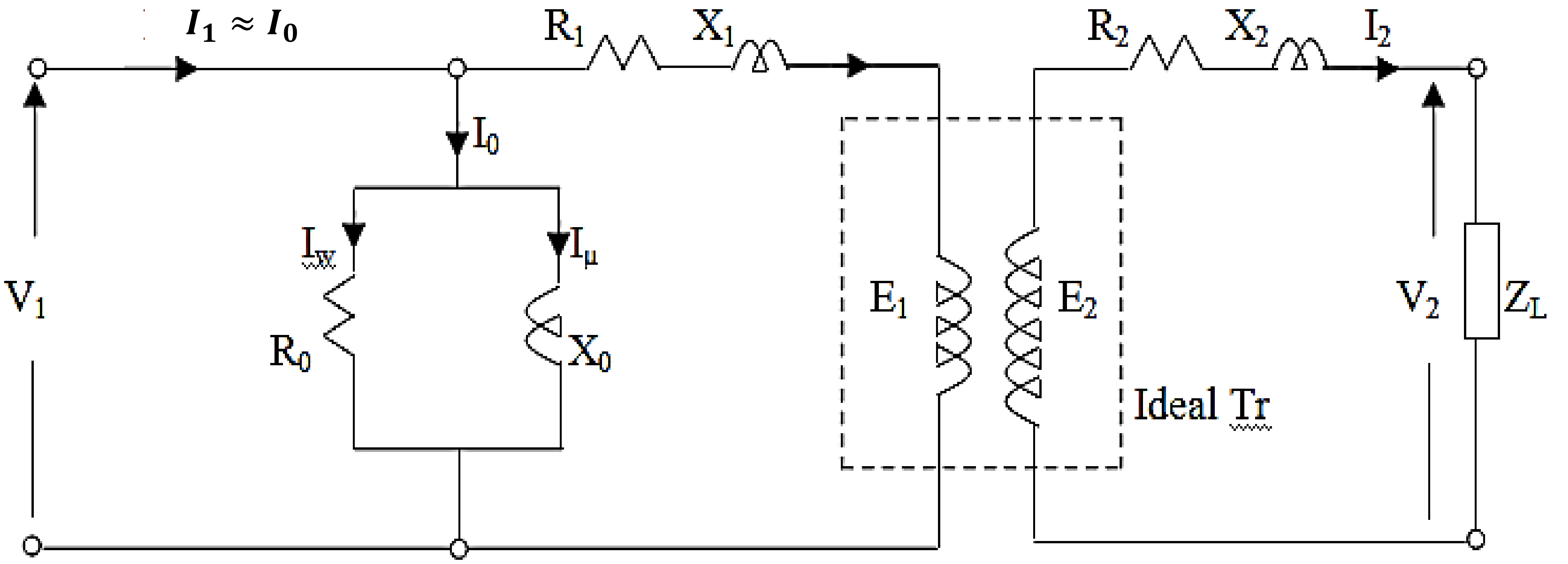


Fig.2.10 Detail Equivalent circuit of transformer

$$\textit{Therefore} \quad \tilde{V}_1 - \tilde{I}_1 .(R_1 + jX_1) = \tilde{E}_1 \text{ (approximately)} \qquad (2.11)$$

$$\text{And} \quad \tilde{V}_2 = \tilde{E}_2 - \tilde{I}_2 .(R_2 + jX_2) \qquad (2.12)$$

Transformation of Impedance :

The equivalent circuit shown in Fig.2.10 can be simplified ***by transferring the resistance and leakage reactance of the secondary winding to the primary side*** as shown in Fig.2.13

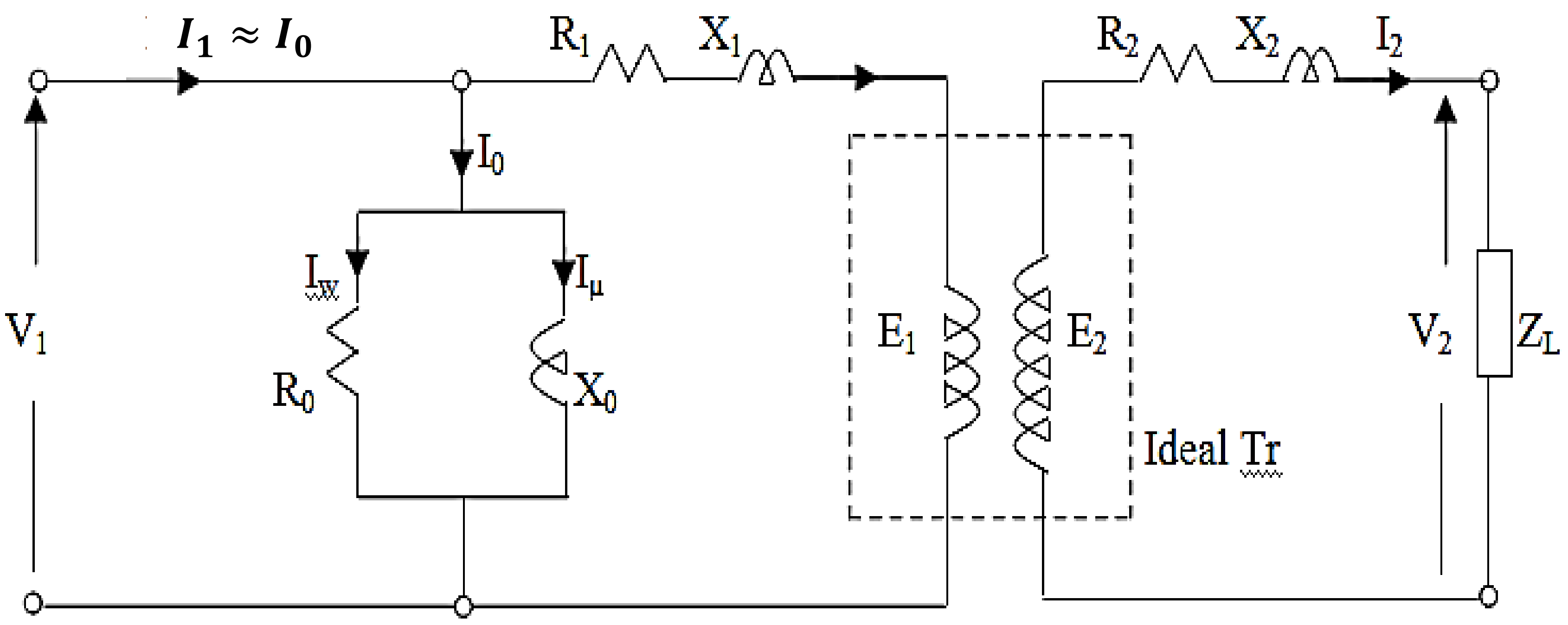


Fig.2.10 Detail Equivalent circuit of transformer

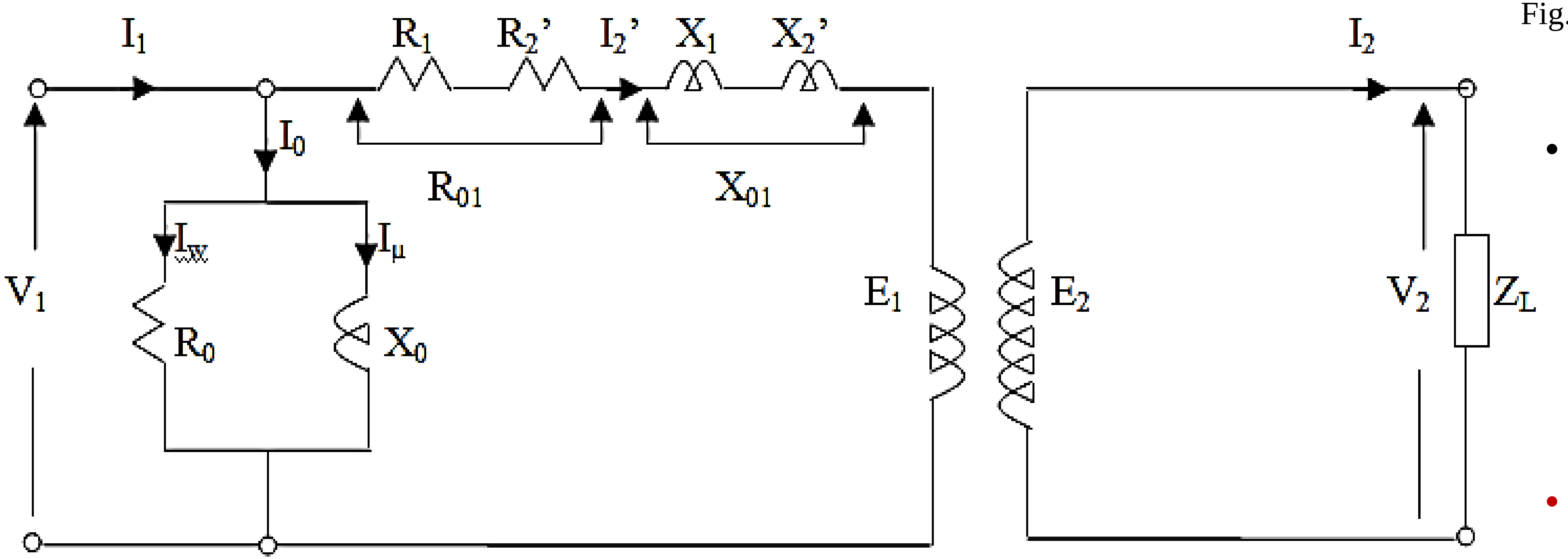


Fig.2.13 Equivalent circuit of transformer referred to primary side

- R_2 is transferred to primary side with a new value R_2' in such a way that **R_2' produces same amount of power loss in primary side as it produces in the secondary side**.
- **R_2' is known as equivalent of R_2 referred to primary side.**

Transformation of Impedance :

Equating power loss in primary and secondary side, it gives:

$(I_2')^2 R_2' = (I_2)^2 R_2$

(Assuming $I_1 \approx I_2'$)

$(I_1)^2 R_2' = (I_2)^2 R_2$

OR $R_2' = \left(\dfrac{I_2}{I_1}\right)^2 .R_2$

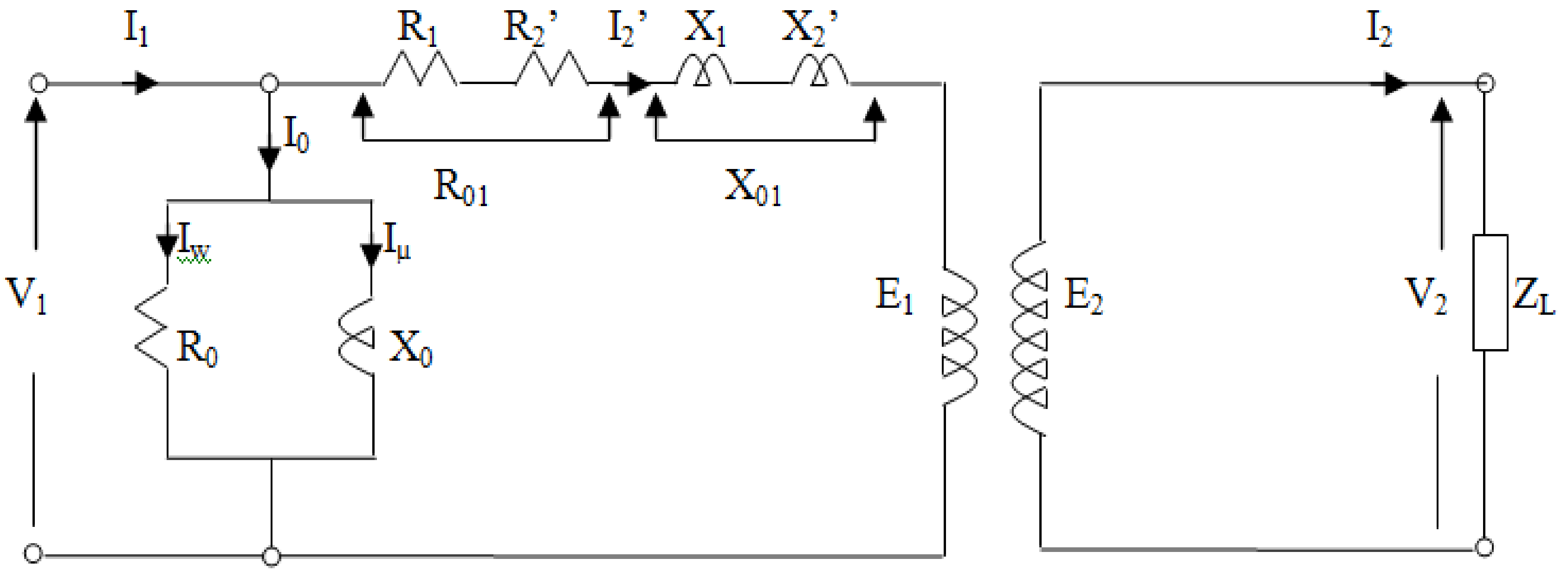


Fig.2.13 Equivalent circuit of transformer referred to primary side

But $\dfrac{I_2}{I_1} = \dfrac{N_1}{N_2} = \dfrac{1}{K}$

$$\therefore \quad R_2' = \frac{1}{K^2}.R_2 \qquad (2.16)$$

Similarly $X_2' = \dfrac{1}{K^2}.X_2 \qquad (2.17)$

Final equivalent circuit of the transformer referred to primary side

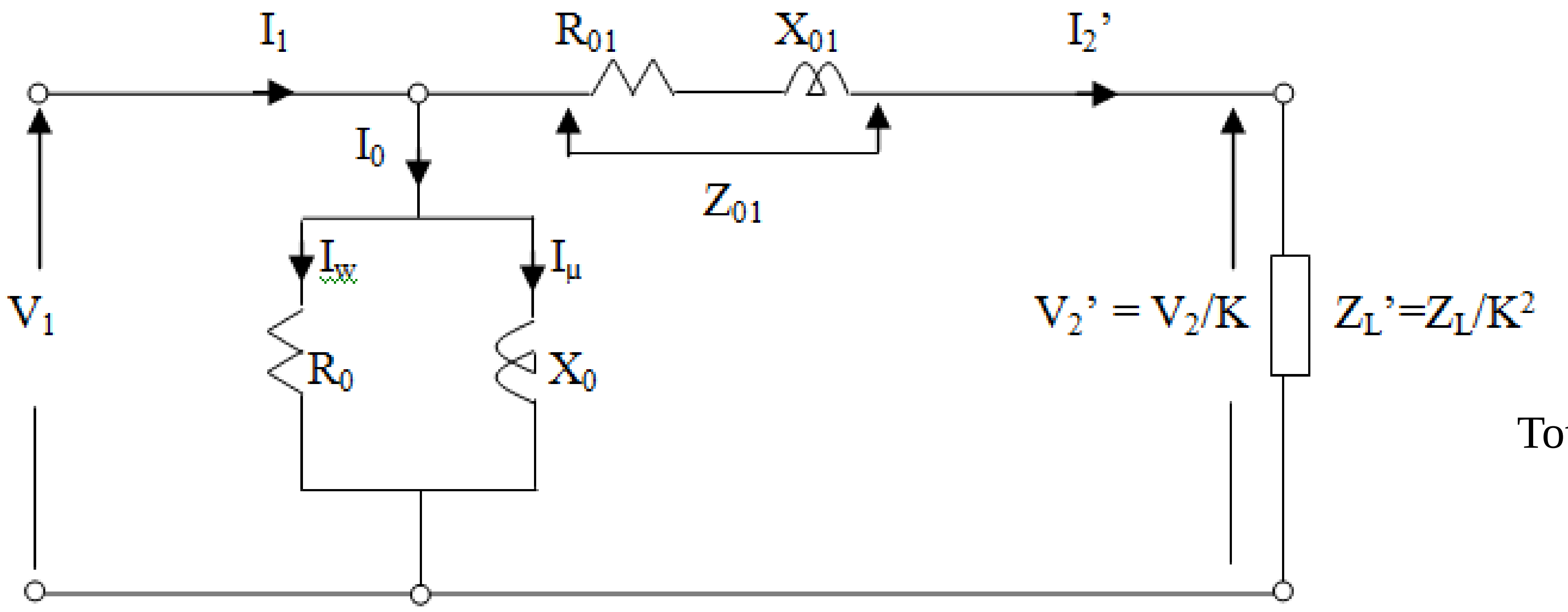


Fig.2.14 Final equivalent circuit of the transformer referred to primary side

Total series Impedance of the transformer referred to primary side

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$$

Total series resistance and the reactance of the transformer referred to primary side

$$\mathbf{R_{01} = R_1 + R_2'}$$

$$\mathbf{X_{01} = X_1 + X_2'}$$

Here, Load impedance and load voltage also has been transferred to primary side.

$$Z_L' = \frac{Z_L}{K^2} = \text{ Equivalent of load impedance refer to primary side}$$

$$V_2' = \frac{V_2}{K} = \text{ Equivalent of load voltage refer to primary side}$$

Transformation of Impedance :

The equivalent circuit shown in Fig.2.10 can be simplified ***by transferring the resistance and leakage reactance of the primary winding to the secondary side*** as shown in Fig.2.13

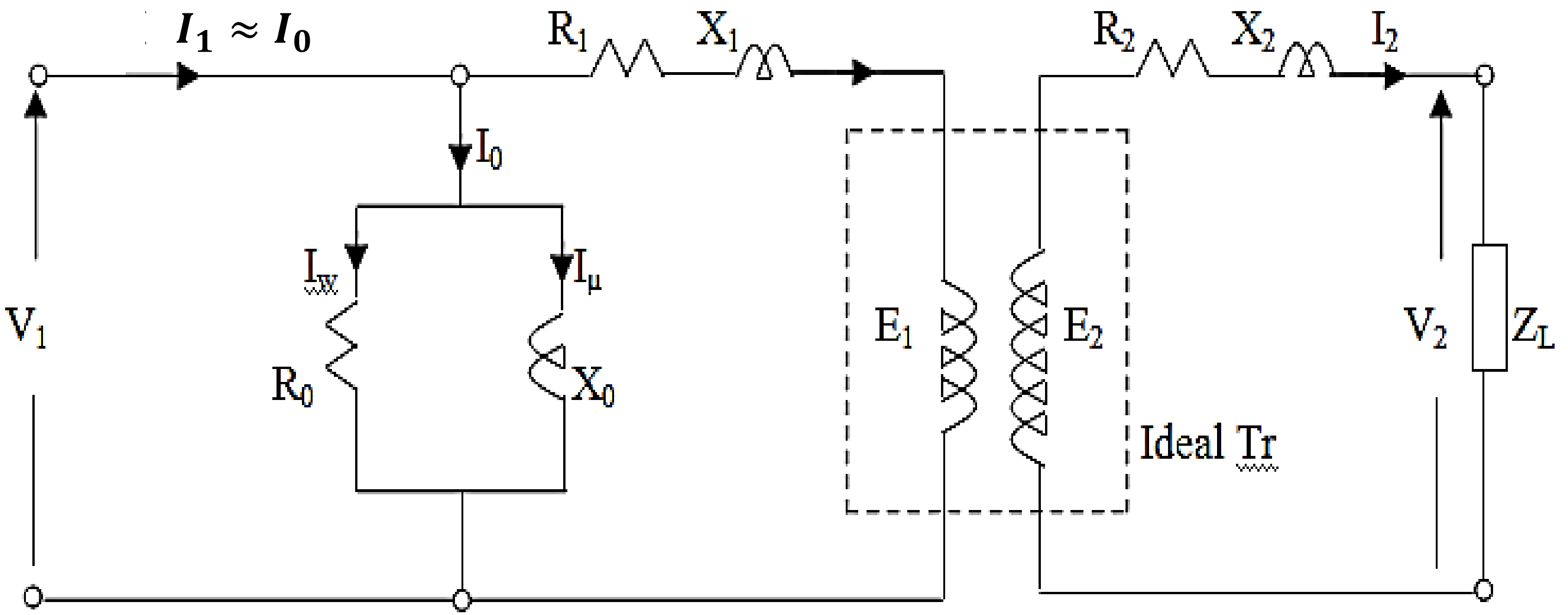


Fig.2.10 Detail Equivalent circuit of transformer

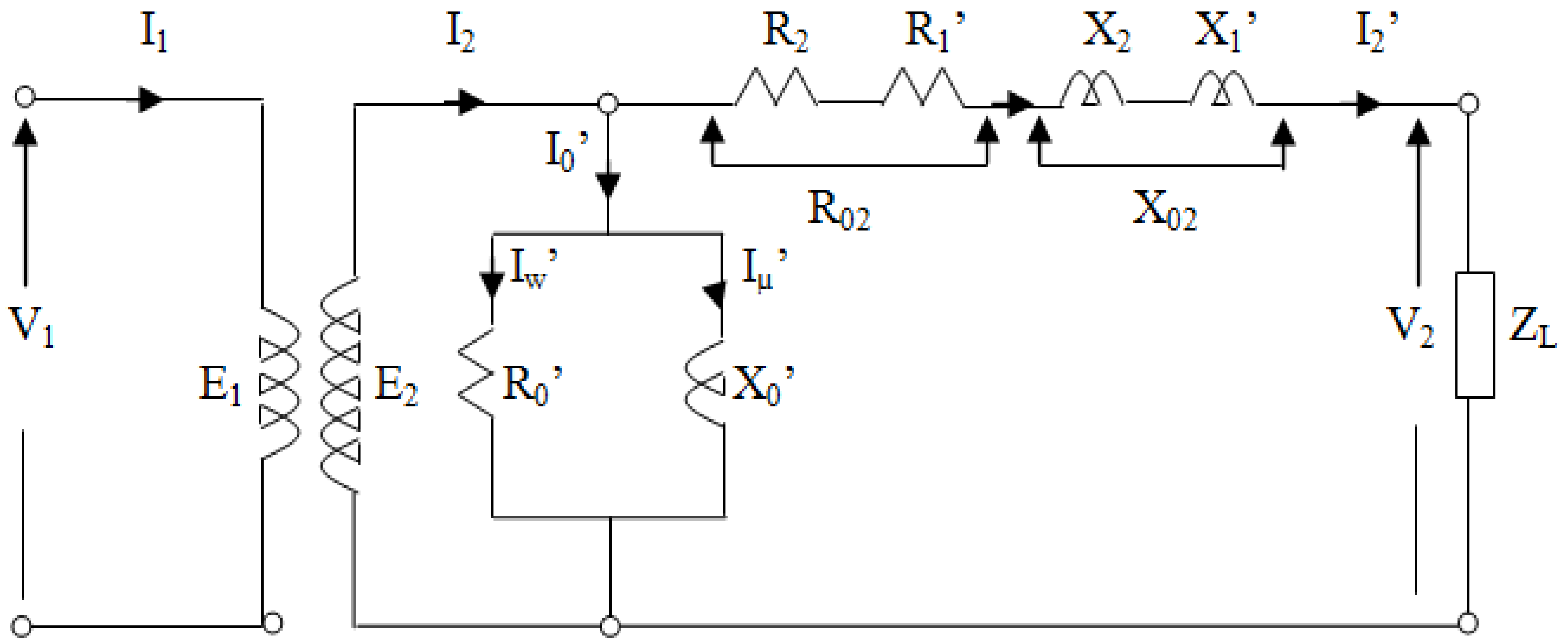


Fig.2.15 Equivalent circuit of transformer referred to secondary side

- R_1 is transferred to secondary side with a new value R_1' in such a way that **R_1' produces same amount of power loss in secondary side as it produces in the primary side**.

- **R_2' is known as equivalent of R_1 referred to primary side.**

Final equivalent circuit of the transformer referred to secondary side

Equating power loss in secondary and primary side, it gives:

$(I_2)^2 R_1' = (I_2')^2 R_1$

(Assuming $I_2' \approx I_1$)

OR $R_1' = \left(\dfrac{I_2'}{I_2}\right)^2 . R_1 = \left(\dfrac{I_1}{I_2}\right)^2 R_1$

But $\dfrac{I_1}{I_2} = \dfrac{N_2}{N_1} = K \quad \therefore \quad R_1' = R_1 . K^2$

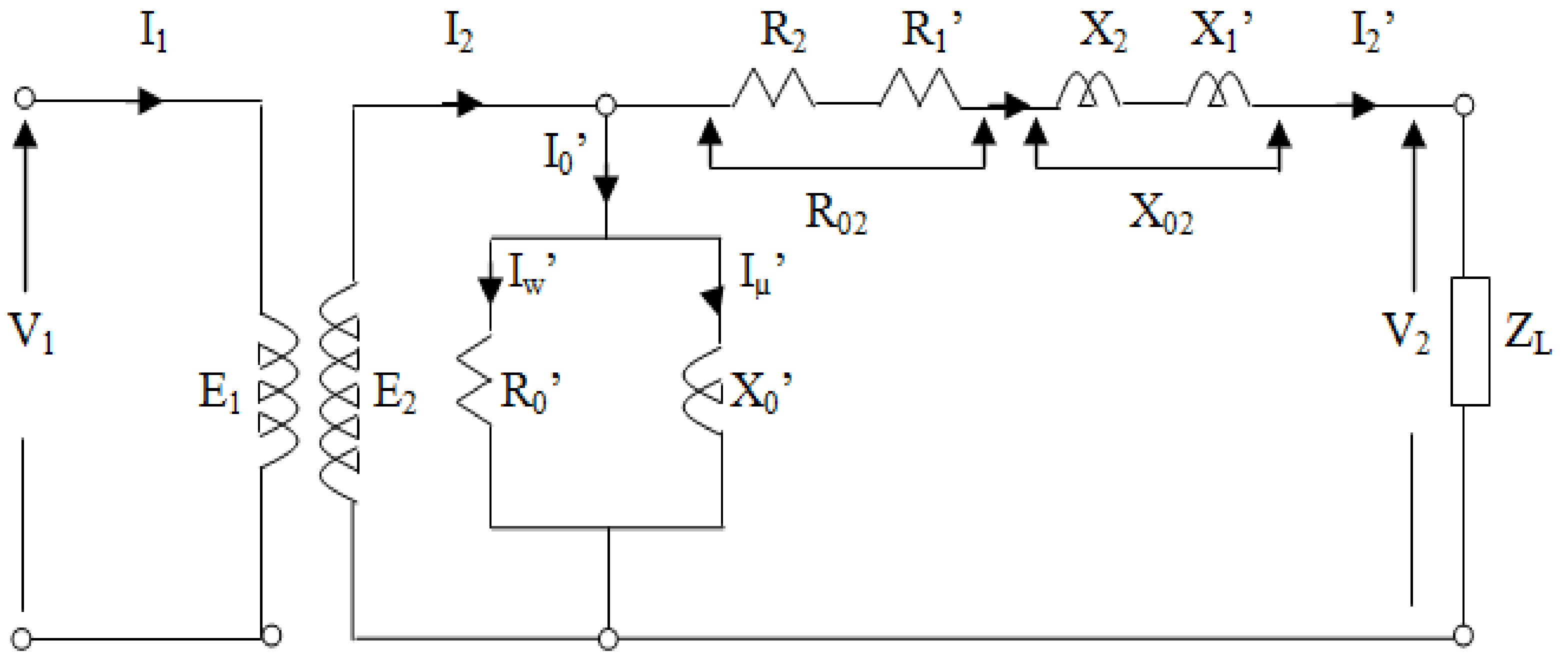


Fig.2.15 Equivalent circuit of transformer referred to secondary side

Similarly, $X_1' = X_1 . K^2$, $\quad R_0' = R_0 . K^2$ And $X_0' = X_0 . K^2$

Final equivalent circuit of the transformer referred to secondary side

Total series Impedance of the transformer referred to secondary side

$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2}$$

Total series resistance and the reactance of the transformer referred to secondary side

$\mathbf{R_{02} = R_2 + R_1'}$

$\mathbf{X_{02} = X_2 + X_1'}$

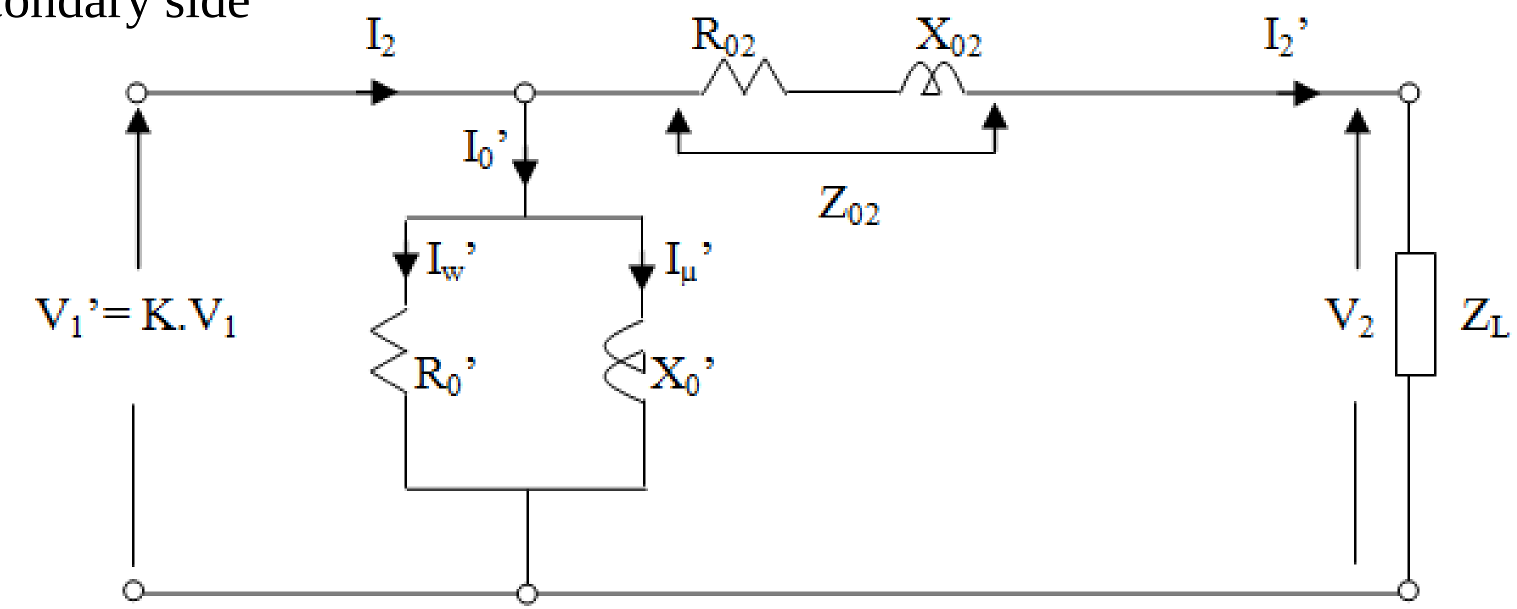


Fig.2.16 Complete equivalent circuit of transformer referred to secondary side

Here, input voltage V_1, I_0 and I_μ also has been transferred to secondary side.

$$V_1' = K.V_1 = \text{Equivalent of } V_1 \text{ refer to seconary side}$$

$$I_\mu' = \frac{I_\mu}{K} = \text{Equivalent of } I_\mu \text{ refer to secondary side}$$

$$I_0' = \frac{I_0}{K} = \text{Equivalent of } I_0 \text{ refer to secondary side}$$

$$I_w' = \frac{I_w}{K} = \text{Equivalent of } I_w \text{ refer to secondary side}$$

Power Losses of a Transformer:

- The output power ($V_2I_2Cos\phi_2$) of a transformer is always less than the input power ($V_1I_1Cos\phi_1$), because there are some power losses within the transformer.
- There are mainly two types of power losses in the transformer: **i) Iron loss and ii) Copper loss.**

i) Iron loss:

- power loss due to heating of iron core of the transformer which is equal to the no-load power loss and remains constant at any load.
- Therefore, iron loss is also known as constant power loss.
- The power loss in the iron core take place due to **eddy current loss and hysteresis loss**.

ii) Copper loss:

- When the transformer is loaded, current flows through primary winding as well as secondary winding.
- The internal resistance of the primary winding and the secondary winding produces heat due to current flowing through them. The power loss due to the heat so produced is known as copper loss.
- The magnitude of copper loss depends upon the square of current and thus it is known as variable loss.
- It can be calculated as follows:

$$\textbf{Total copper loss} = \textbf{Copper loss in PW} + \textbf{Copper loss in SW} = I_1^2 R_1 + I_2^2 R_2 \qquad (2.21)$$

$$\textbf{Or, Total copper loss} = I_1^2 R_{01} = I_2^2 R_{02} \text{ (watts)}$$

Efficiency of a Transformer:

Input power is given by: $P_{in} = V_1 I_1 \text{Cos}\phi_1$

Output power: $P_{out} = P_{in} - \text{Iron loss} - \text{Copper loss} = V_1 I_1 \text{Cos}\phi_1 - W_i - I_1^2 \mathbf{R_{01}}$

$$\boxed{\text{Efficiency of transformer } \eta = \frac{P_{out}}{P_{in}} \text{ pu.} \quad \text{Or} \quad \eta = \frac{P_{out}}{P_{in}} \times 100\,\%}$$

$$\text{OR} \quad \eta = \frac{V_1 . I_1 \text{ Cos}\phi_1 - W_i - I_1^2 R_{01}}{V_1 . I_1 \text{ Cos}\phi_1} \qquad (2.22)$$

Voltage Regulation of a Transformer:

- If the magnitude of output voltage of a transformer remains constant from no-load to full-load, it would be a very good transformer.

- However, a real transformer can not give such performance. There will be some voltage drop in the series impedance of primary winding and secondary winding. Therefore, **the output voltage at full-load will be less than that at no-load.**

- If a transformer has a minimum voltage drop, the transformer is said to be a good transformer from voltage drop point of view.

- The quality of a transformer from voltage drop point of view is measured in terms of voltage regulation.

- The voltage regulation is defined as the "**change in the magnitude of output voltage from full-load to no-load, expressed as percentage of full load voltage**".

Voltage Regulation of Transformer:

Let $_fV_2$ = Full load terminal voltage

$_0V_2$ = No- load terminal voltage

$$\therefore \quad \text{Voltage Regulation, } V_{Reg} = \frac{_0V_2 - _fV_2}{_fV_2} \quad \text{(in pu)}$$

If the load power factor is lagging,

$$V_{Reg} = \frac{I_2R_{02}\,Cos\varphi_2 + I_2X_{02}\,Sin\varphi_2}{_fV_2}$$

$$\text{or, } V_{Reg} = (R_{pu})\,Cos\varphi_2 + (X_{pu})Sin\varphi_2 \qquad (2.6)$$

If the load power factor is leading,

$$V_{Reg} = \frac{I_2R_{02}\,Cos\varphi_2 - I_2X_{02}\,Sin\varphi_2}{_fV_2}$$

$$\text{or, } V_{Reg} = (R_{pu})\,Cos\varphi_2 - (X_{pu})Sin\varphi_2 \qquad (2.7)$$

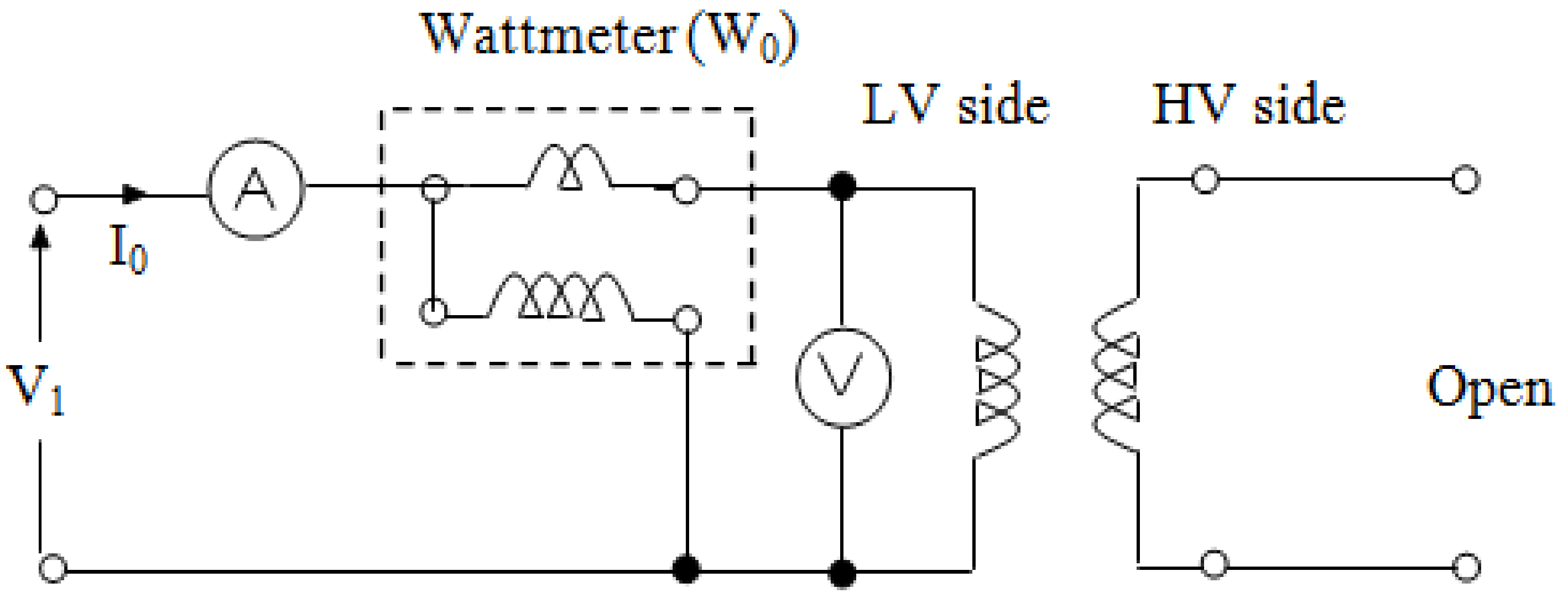
Where, $R_{pu} = \frac{I_2R_{02}}{_fV_2}$ = Per Unit resistance of transformer

And $X_{pu} = \frac{I_2X_{02}}{_fV_2}$ = Per Unit reactance of transformer

Then $Z_{pu} = \sqrt{(R_{pu})^2 + (X_{pu})^2}$ = Per unit Impedance of the transformer

No-load Test (Open Circuit Test)

- The purpose of this test is to evaluate the **shunt branch parameters** of the equivalent circuit, **iron loss** of the transformer**, no-load current** and **no-load power factor**.
- In this test, the **high voltage winding is kept open and the low voltage winding is supplied by rated voltage**



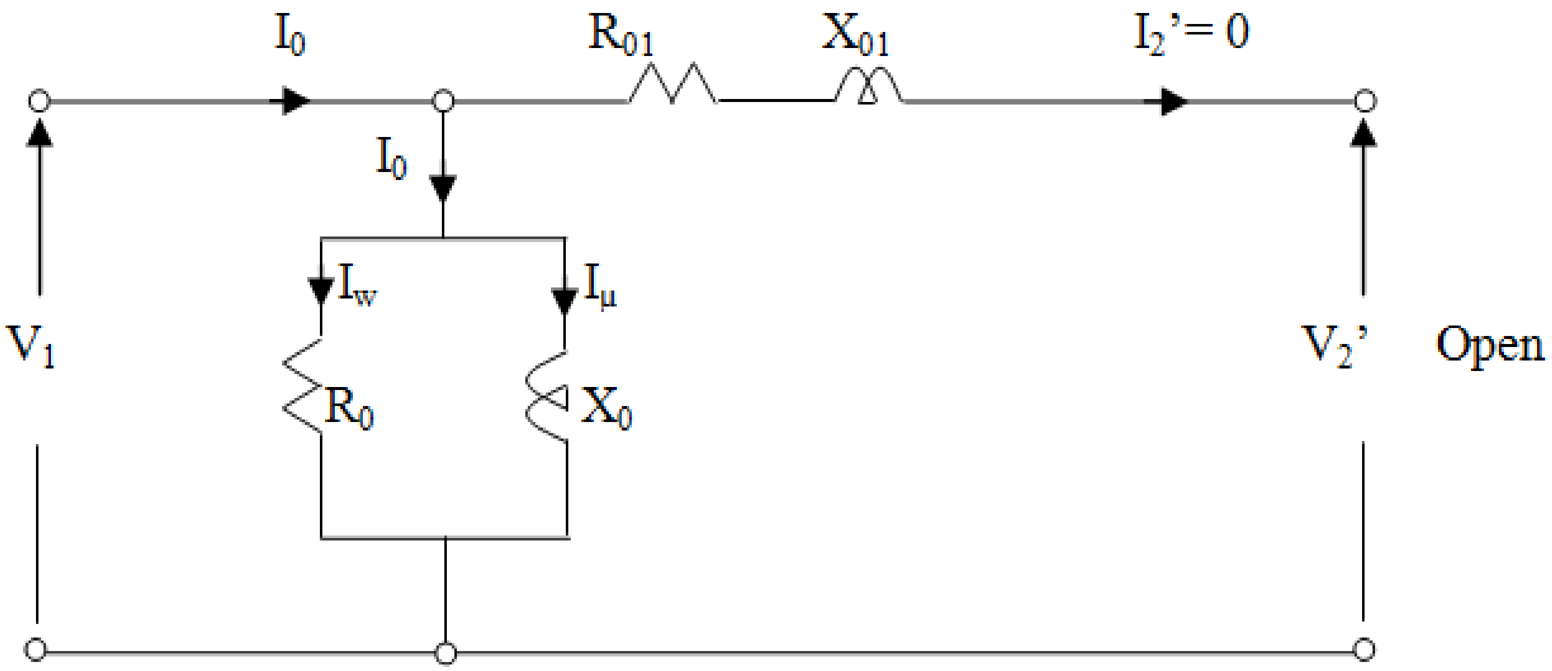
V_1 = Voltmeter reading
I_0 = Ammeter reading
W_0 = Wattmeter reading

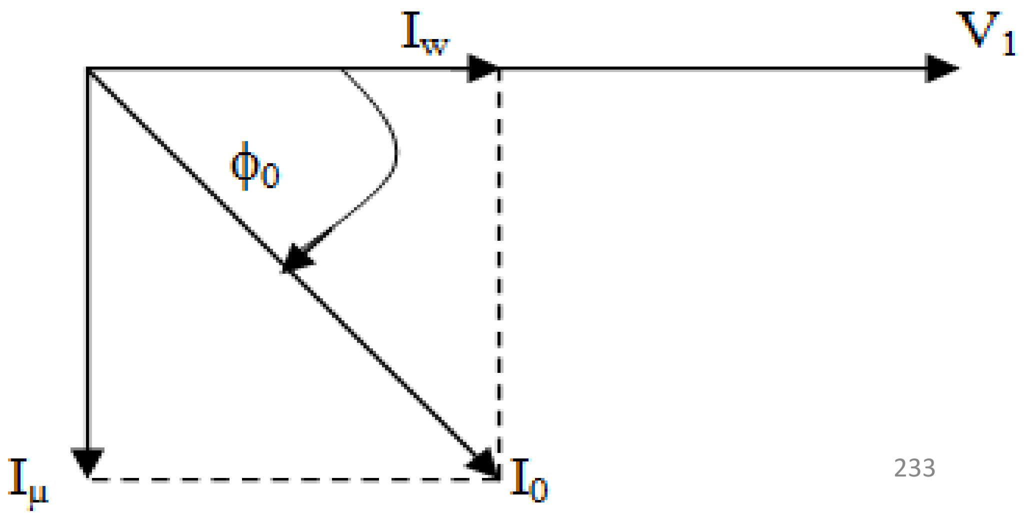
No-load Test (Open Circuit Test)

- As the no-load current is very small, **copper loss at no-load can be neglected.**
- Hence, the wattmeter reading is equal to the **no-load power loss or iron loss of the transformer.**
- The wattmeter reading is equal to the power consumed by the transformer at no-load and it is given by:

$$\mathbf{W_0 = V_1\, I_0\, Cos\phi_0,} \quad \text{where, } Cos\phi_0 = \text{no-load power factor.}$$

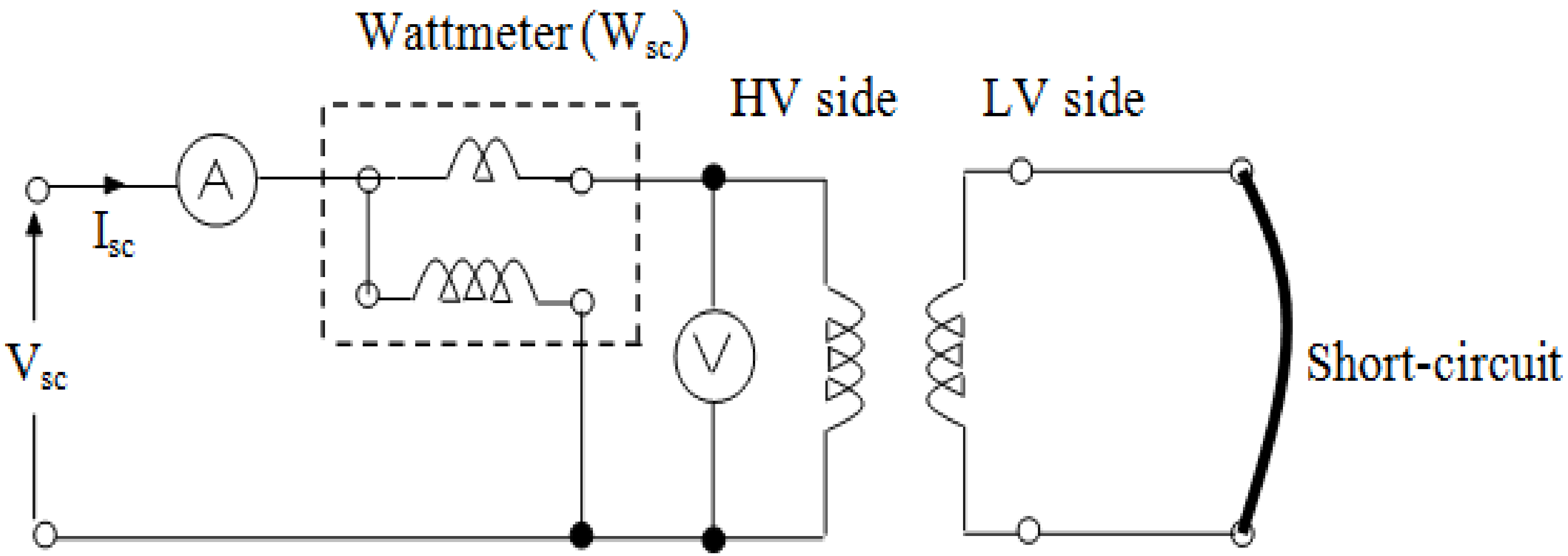
- Then I_w and I_μ can be calculated as: $\mathbf{I_w = I_0\, Cos\phi_0}$ **and** $\mathbf{I_\mu = I_0\, Sin\phi_0}$





Short - Circuit Test

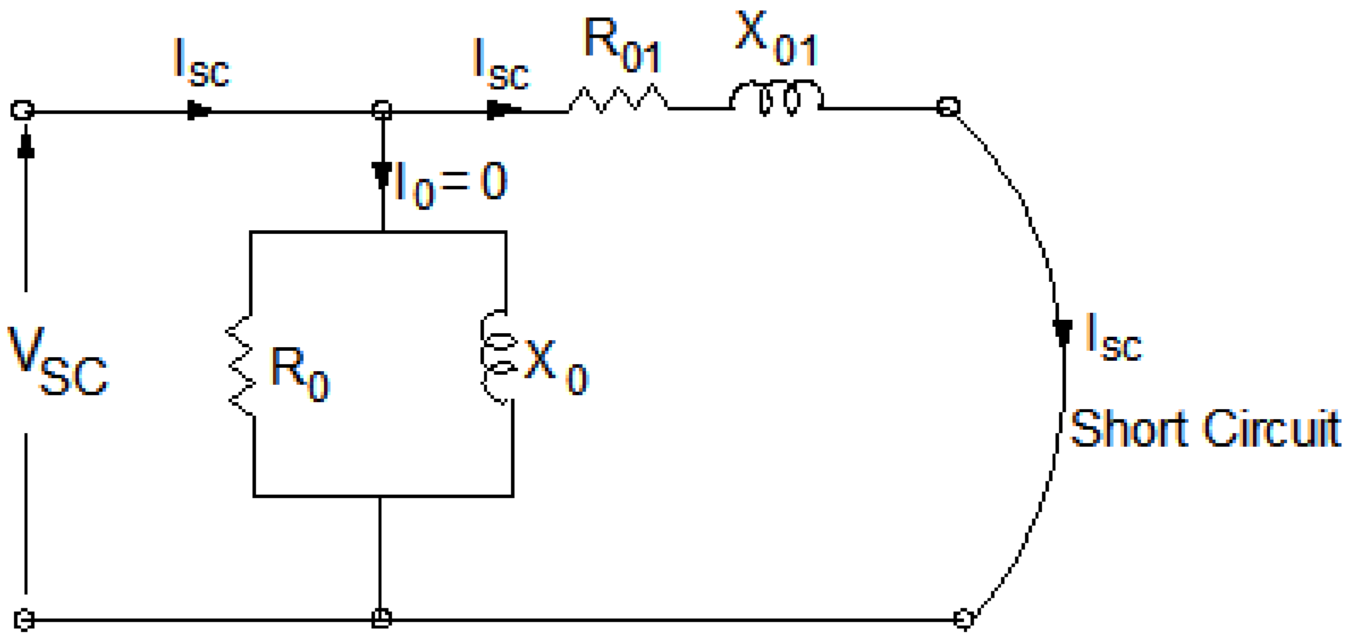
- the purpose of this test is to evaluate the **series resistance and reactance of the transformer and copper loss at full load.**
- In this test, the **low voltage side is short circuited by a thick wire**
- the **high voltage side is supplied by reduced low voltage** of such a value, which is **just sufficient to circulate full load currents at primary and secondary windings**



Let V_{sc} = Voltmeter reading
I_{sc} = Ammeter reading
W_{sc} = Wattmeter reading

Short - Circuit Test

- Since the magnitude of applied voltage during the short circuit test is very small, **I_0 and magnetic flux in the core will be very small** with compare to that in case of normal operation and the **iron core will not be saturated.**
- Therefore, **the eddy current loss and hysteresis loss during short circuit test will be very small** with compare to copper loss in the series resistance.
- the **wattmeter reading** during short circuit test will be **equal to the copper loss at full load**



Short - Circuit Test

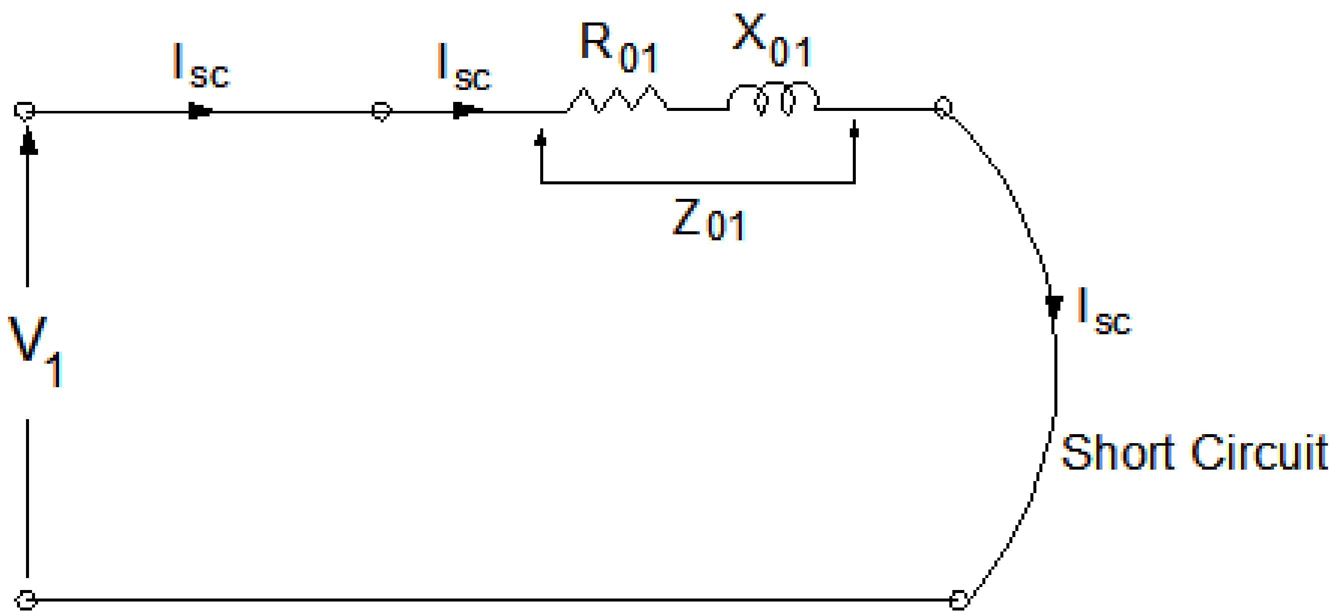
- Since wattmeter reads the total copper loss of the transformer at full load, it can be written as:

$$W_{sc} = I_{sc}^{2} R_{01}$$

Hence, R_{01} can be calculated as: $R_{01} = \dfrac{W_{sc}}{I_{sc}^{2}}$ (2.30)

- The equivalent series impedance Z_{01} can be calculated as: $Z_{01} = \dfrac{V_{sc}}{I_{sc}}$ (2.31)

Then $X_{01} = \sqrt{Z_{01}^{2} - R_{01}^{2}}$



Illustrative example 2.5:

A 40 kVA, 6600V/250V single phase transformer has R_1 =8 ohm, X_1 = 15 ohm, R_2 = 0.02 ohm, X_2 = 0.05 ohm. Calculate the voltage regulation at full load (a) with 0.8 lagging power factor and (b) with 0.8 leading power factor.

Solution:

$$V_{\mathrm{Re}\,g} = \frac{\mathrm{I_2 R_{02}\ Cos}\phi_2 + \mathrm{I_2 X_{02}\ Sin}\phi_2}{{}_fV_2}$$

Transformation ratio $\mathrm{K} = \frac{N_2}{N_1} = \frac{V_2}{V_1} = 0.0378$

R_1 and X_1 can be transferred to the secondary side as follow:

$$R_1^{'} = K^2 \times R_1 = (0.0378)^2 \times 8 = 0.0114\,\Omega \qquad X_1^{'} = K^2 \times X_1 = (0.0378)^2 \times 15 = 0.0214\,\Omega$$

Then, total series resistance and reactance of the transformer referred to secondary side is given by:

$R_{02} = R_2 + R_1' = 0.02 + 0.0114 = 0.0314\ \Omega$

$X_{02} = X_2 + X_1' = 0.05 + 0.0214 = 0.0714\Omega$

Case-a: Fully loaded with 0.8 lagging power factor

$$\text{Full load current } I_2 = \frac{\text{Capacity (S)}}{V_2} = \frac{40{,}000}{250} = 160\,Amp$$

Here, $\text{Cos}\phi_2 = 0.8$ and $\phi_2 = \text{Cos}^{-1}(0.8) = 36.86^0$ $\text{Sin}\phi_2 = 0.6$

$$\therefore V_{\text{Re}\,g} = \frac{\text{I}_2\text{R}_{02}\,\text{Cos}\phi_2 + \text{I}_2\text{X}_{02}\,\text{Sin}\phi_2}{{}_fV_2} = \frac{160\times 0.0314\times 0.8 + 160\times 0.0714\times 0.6}{250}$$

Or, $V_{reg} = 0.0436$ Or $4.36\ \%$

Case-b: Fully loaded with 0.8 leading power factor

$$V_{\text{Re}\,g} = \frac{\text{I}_2\text{R}_{02}\,\text{Cos}\phi_2 - \text{I}_2\text{X}_{02}\,\text{Sin}\phi_2}{{}_fV_2} = \frac{160\times 0.0314\times 0.8 - 160\times 0.0714\times 0.6}{250} = \text{-}\,0.011\text{pu} = \text{-}1.1\%$$

Illustrative example :

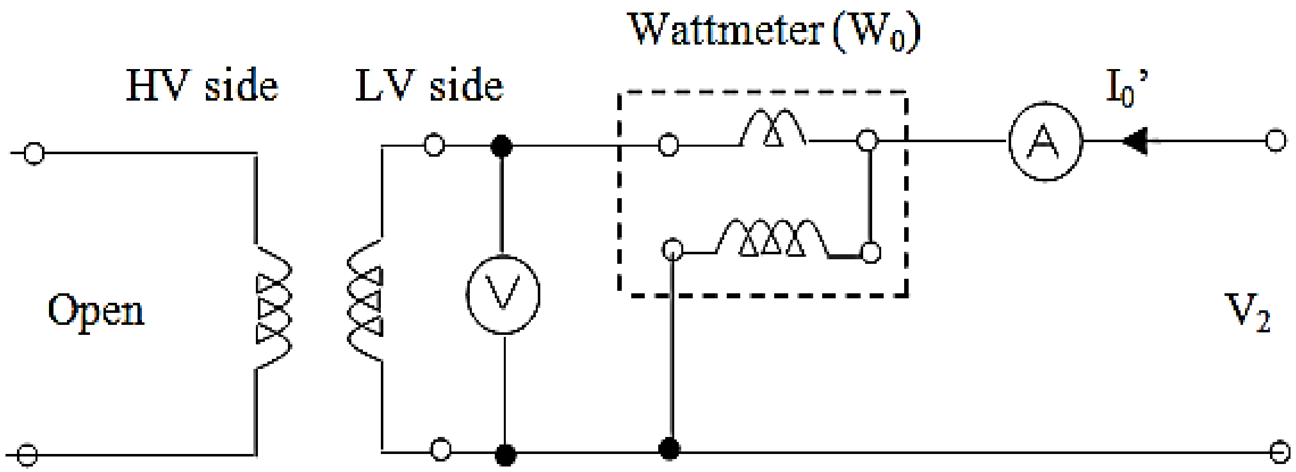
A 200 kVA, 2000V/440V, 50Hz single phase transformer gave the following test results:

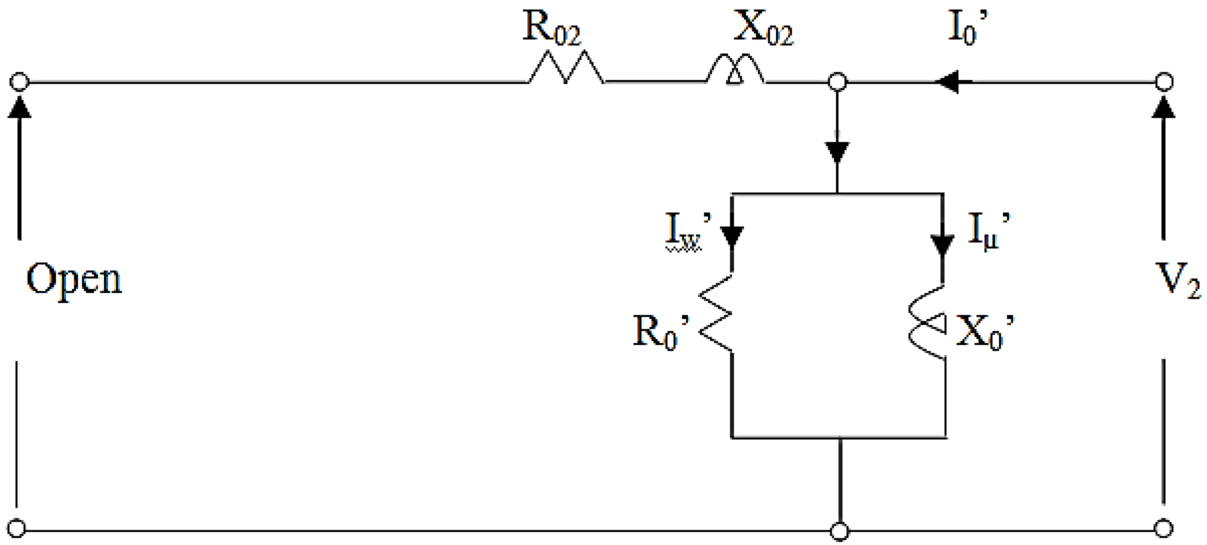
No-load test (with HV side open) : 440V 1500 W 8 A

Short circuit test (LV side S/C) : 30V 2000 W 300 A

a) Calculate the equivalent circuit parameter referred to primary side

b) Calculate efficiency at full load with 0.8 lagging power factor

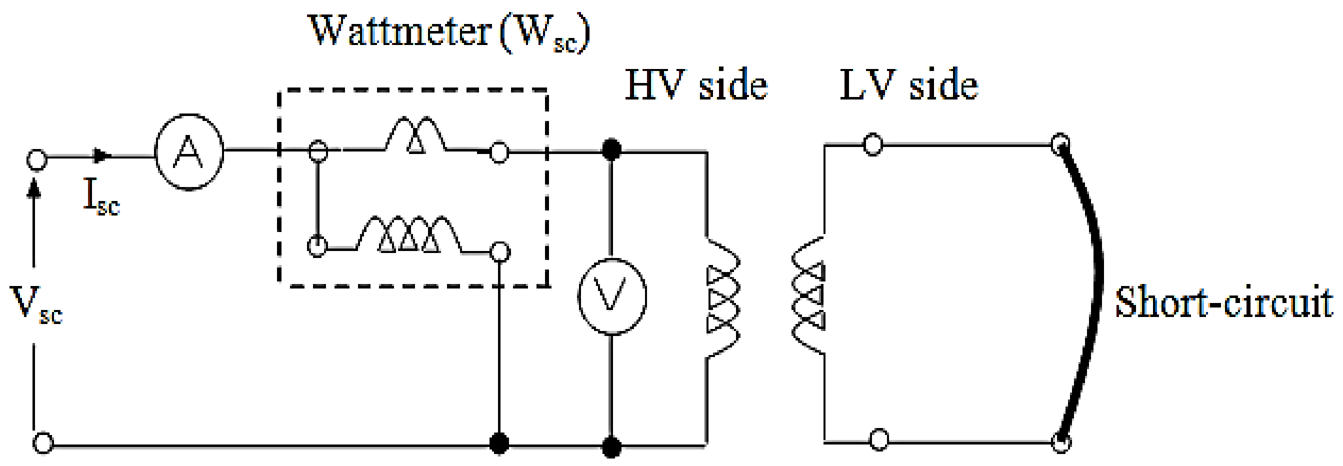


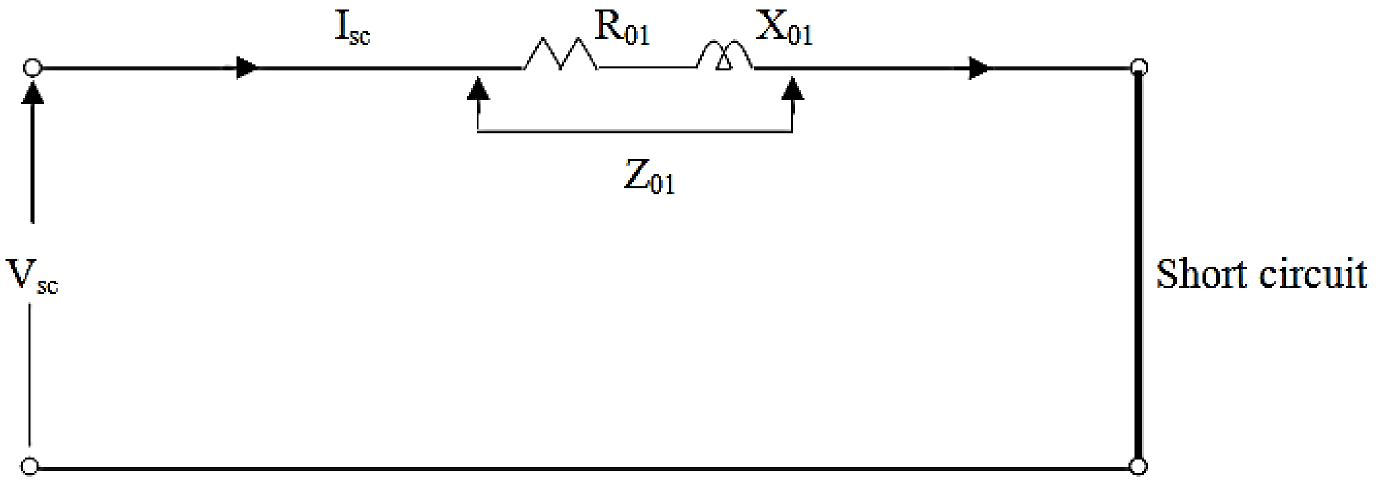


$\mathbf{W_0 = V_1\, I_0\, Cos\phi_0,}$ where, $Cos\phi_0$ = no-load power factor.

$\mathbf{I_w = I_0\, Cos\phi_0}$ **and** $\mathbf{I_\mu = I_0\, Sin\phi_0}$

$$R_0 = \frac{V_1}{I_w} \qquad (2.28) \qquad \text{and} \qquad X_0 = \frac{V_1}{I_\mu} \qquad (2.29)$$

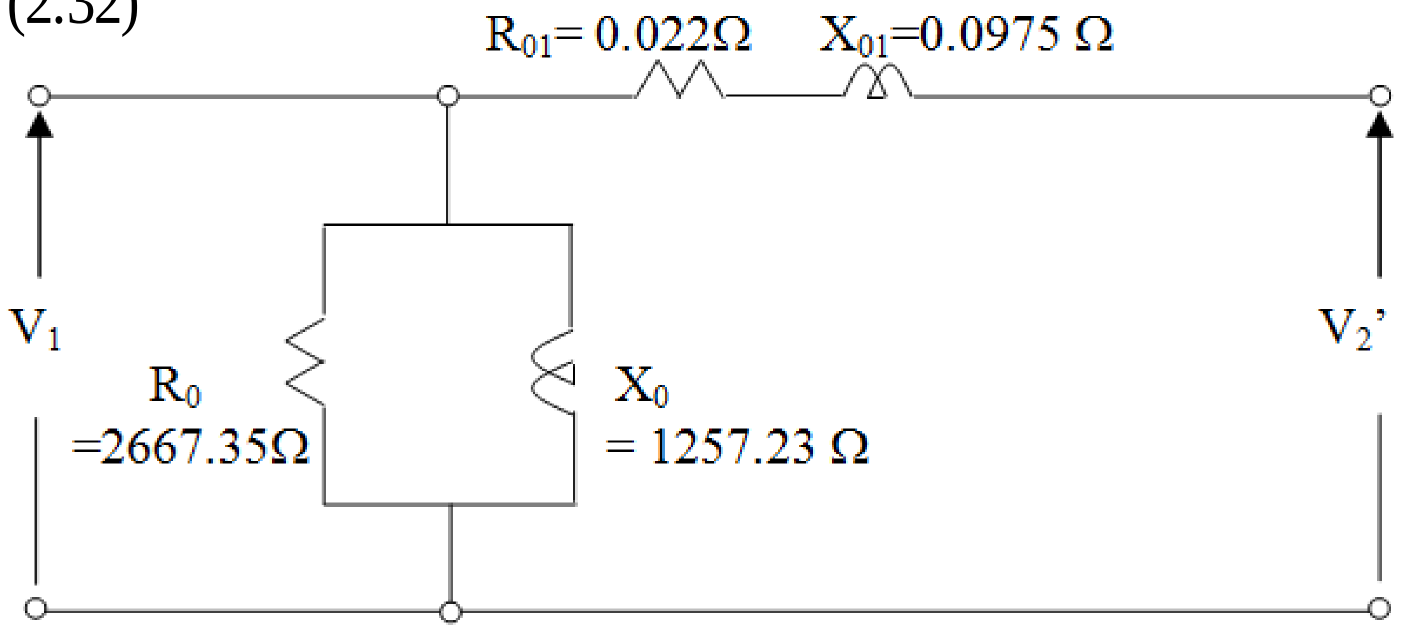




$\mathbf{W_{sc} = I_{sc}^{\,2}\, R_{01}}$ Hence, R_{01} can be calculated as: $R_{01} = \dfrac{W_{sc}}{I_{sc}^2}$ (2.30)

Then $X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$ (2.32)

$$Z_{01} = \frac{V_{sc}}{I_{sc}} \qquad (2.31)$$



6) A 200/400 V, 50 Hz, single-phase transformer gave the following test results :

O.C. Test (L.V. Side) : 200 V, 0.7 A, 70 W

S.C. Test (H.V. Side) : 15 V, 10A, 110 W

(i) Calculate the parameters of the equivalent circuit referred to the L.V. side.

(ii) Calculate the total transformer drop as referred to secondary.

Solution

From O.C. test,

$\mathbf{V_1 I_0 \cos\phi_0 = W_0}$

or, $200 \times 0.7 \times \cos\phi_0 = 70$

$\therefore$ $\mathbf{\cos\phi_0 = 0.5}$ and $\mathbf{\sin\phi_0 = 0.866}$

$\mathbf{I_W = I_0 \cos\phi_0} = 0.7 \times 0.5 = 0.35$ A

$\mathbf{I_\mu = I_0 \sin\phi_0} = 0.7 \times 0.866 = 0.606$ A

$\mathbf{R_0 = V_1/I_W} = 200/0.35 = 571.4\ \Omega$

$\mathbf{X_0 = V_1/I_\mu} = 200/0.666 = 330\ \Omega$

From S.C. test,

Here, the secondary is HV side. So primary side (LV winding) is short circuited and measurements are made on the secondary side.

So, $\mathbf{Z_{02} = V_{SC}/I_2} = 15/10 = 1.5\ \Omega$

$\mathbf{K} = 400/200 = 2$ $\quad\therefore \mathbf{Z_{01} = Z_{02}/K^2} = 1.5/4 = 0.375\ \Omega$

Also, $\mathbf{I_2^2 R_{02} = W}$ $\quad\therefore \mathbf{R_{02}} = 85/100 = 0.85\ \Omega$

$\mathbf{R_{01} = R_{02}/K^2} = 0.85/4 = 0.21\ \Omega$

$\mathbf{X_{01} = \sqrt{(Z_{01}^2 - R_{01}^2)}} = \sqrt{(0.375^2 - 0.21^2)} = 0.31\ \Omega$

Now,

$\mathbf{Z_{02}} = 1.5\ \Omega$ and $\mathbf{R_{02}} = 0.85\ \Omega$

Then, $\mathbf{X_{02} = \sqrt{(Z_{02}^2 - R_{02}^2)}} = \sqrt{(1.5^2 - 0.85^2)} = 1.24\ \Omega$

The total transformer drop as referred to secondary $= \mathbf{I_2(R_{02}\cos\phi_2 + X_{02}\sin\phi_2)}$

$= 15.6\ (0.85 \times 0.8 + 1.24 \times 0.6)$

$= 22.2$ V