POKHARA UNIVERSITY

Level: Bachelor Semester: Spring Year : 2018
Programme: BE Full Marks: 100
Course: Engineering Mathematics I Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Show that the function $f(x)$ defined by 8

$$f(x) = \begin{cases} -x \text{ when } x \le o \\ x \text{ when } 0 < x < 1 \\ 2 - x \text{ when } x \ge 1 \end{cases}$$

is continuous at $x = 0$ *and* $x = 1$, but is not differentiable at $x = 1$.

OR

If $y=\sqrt{\frac{1+x}{1-x}}$ prove that

(i) $(1-x)y^2 = 1+x$

(ii) $(1-x^2)y_n - \{2(n-1)x+1\}y_{n-1} - (n-1)y_{n-2} = 0$

b) State and prove that Cauchy's Mean Value theorem. Is the theorem applicable to the functions f(x)= x and g(x)= x^2-2x in the interval [0,2]? Why? 7

2. a) Evaluate: $\lim_{x \to 0} \left(\frac{1}{x^2}\right)^{tanx}$ 7

b) A cone is inscribed in a sphere of radius r, prove that it's volume as well as its curved surface is greatest when the altitude is $\frac{4r}{3}$ 8

OR

Find the asymptote to the curve $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$

3. Integrate (Any Three): 3×5

a. $\int \frac{(x+2)}{\sqrt{4x-x^2}} dx$

b. $\int \frac{1}{1-\cos x+\sin x} dx$

c. Prove: $\int_0^1 \cot^{-1}(1-x-x^2)dx = \frac{\pi}{2} - \log 2$

d. $\int_0^1 \sqrt{x}dx$ by summation method.

4. a) Find the volume of the solid generated by revolving the region bounded by y = $\sqrt{x}$ and the lines y=1,x=4 about the line y=1. 7

b) Approximate the area by using Trapezoidal and Simpson's rule to the integral $\int_1^4 \frac{1}{1+x} dx, n = 6$. Also compare with exact. 8

5. a) Define hyperbola. Derive the standard equation of hyperbola. 7

b) Find the condition that the line $y = mx + c$ may touch the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Also find the point of contact. 8

6. a) Define scalar and vector product of three vectors. Prove that the scalar triple product of three vectors represent the volume of parallelepiped. What conclusion can be drawn if $\vec{a}.(\vec{b} \times \vec{c}) = 0$? 7

b) Show that the vectors $\vec{a} \times (\vec{b} \times \vec{c}), \vec{b} \times (\vec{c} \times \vec{a})$ *and* $\vec{c} \times (\vec{a} \times \vec{b})$ are coplanar. 8

7. Attempt all the questions: 2.5×4

a) Find the radius of curvature at (s, ψ) *for the curve* $s = 8a\sin^2\frac{\psi}{6}$

b) Find centre, vertices and foci of the ellipse: $x^2+10x+25y^2=0$

c) Find the volume of a parallelepiped whose concurrent edges are represented by $\vec{i} + \vec{j} + \vec{k}$, $2\vec{i} + \vec{j} - 2\vec{k}$ *and* $3\vec{i} + 2\vec{j} - \vec{k}$.

d) $\int x^3 \log x \, dx$.