POKHARA UNIVERSITY

Level: Bachelor   Semester: Fall   Year : 2017
Programme: BE   Full Marks: 100
Course: Engineering Mathematics I   Pass Marks: 45
                                        Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1.

a) Examine the continuity and derivability at $x = 0$ *and* $x = \dfrac{\pi}{2}$ of the function   **8**

$$f(x) = \begin{cases} 1 & when \;\; (-\infty, 0) \\ 1 + \sin x & when \;\; x \in [0, \pi/2) \\ 2 + (x - \pi/2)^2 & when \;\; x \in [\pi/2, \infty) \end{cases}$$

OR

State Leibnitz theorem for successive derivative of the product of two functions. If $y = \sin^{-1} x$ then show that

   i. $(1 - x^2) y_2 - x y_1 = 0$

   ii. $(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} - n^2 y_n = 0.$

b) State and prove Rolle's Theorem. Verify the theorem for the function   **7**
$f(x) = \log\left(\dfrac{x^2 + ab}{(a+b)x}\right)$ in $[a, b]$ where $0 < a < b$.

2. a) A cone is inscribed in a sphere of radius r, prove that it's volume as   **8** well as its curved surface is greatest when the altitude is $\dfrac{4r}{3}$.

OR

Find the asymptote to the curve $y^2x^2 - 3yx^2 - 5x\,y^2 + 2x^2 + 6\,y^2 - x - 3y + 2 = 0.$

b) Show that $\lim\limits_{x \to \infty} \left(x - x^2 \ln\left(1 + \dfrac{1}{x}\right)\right) = \dfrac{1}{2}.$ **7**

3. Evaluate *Any Three*   **3×5**

a) $\displaystyle\int \frac{\cos x\,dx}{\sqrt{2\sin^2 x + 3\sin x + 4}}$

b) $\displaystyle\int \frac{2\sin x + 3\cos x}{3\sin x + 4c\cos x}dx$

c) $\displaystyle\int_0^1 \frac{\log(1 + x)\,dx}{1 + x^2}$

d) Evaluate $\displaystyle\int_a^b e^{-x}dx$ by summation method.

4. a) Find the volume of the solid generated by revolving the region in the   **7** first quadrant bounded by the parabola $y = x^2$, below by the x-axis and on the right by the line x=2 about y-axis.

b) Find approximate values of $\int_2^5 (x^2 + 1)dx$ using Simpson's and   **8** Trapezoidal rules with $n = 6$. Also compare the results with exact value.

5. a) Find the plane through A(1,1,1)and perpendicular to the line of   **8** intersection of the planes 2x+y+3z = 5 and 3x+2y+z=7.

b) Prove that the four points having position vectors   **7**
$-\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}, 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}, 6\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $-12\mathbf{i} - \mathbf{j} - 3\mathbf{k}$ are coplanar.

6. a) Define conic section by their eccentricity and classify them. Derive   **8** standard equation of parabola $y^2 = 4ax$.

b) Find the condition that the line $y = mx + c$ may be tangent to the   **7** ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

7. Attempt all   **4×2.5**

a) Find the vertical asymptotes to the curve $x^2 + xy + 4y + 3 = 0$

b) Find the radius of curvature at the origin of the curve $x^3 + y^3 = 3axy$

c) Evaluate $\int x^5 e^x dx$

d) Transform to parallel axis through the point $(3, -4)$ the equation
$x^2 - y^2 + 2x - 3y = 0.$