Realization of IIR filter [Lattice structure]

Given: $H(z) = \dfrac{1}{1+0.9z^{-1} - 0.8z^{-2} + 0.5z^{-3}}$. Determine the lattice coefficient/ draw the lattice structure of an all pole IIR filter

Soln, you should know the structure of IIR filter in Lattice form

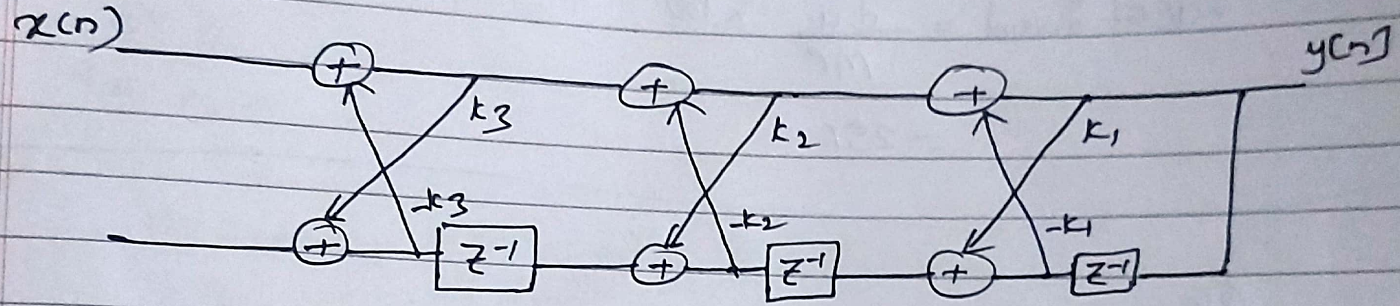

fig. for IIR filter

No. of delays = highest power of the denominator polynomial

Steps: Find k_1, k_2, k_3

formula to remember

$$A_{m-1}(z) = \frac{A_m(z) - k_m B_m(z)}{1 - k_m^2}$$

where $B_m(z) = z^{-m}(A_m(z^{-1}))$

from question

$$A_3(z) = 1 + 0.9z^{-1} - 0.8z^{-2} + 0.5z^{-3}$$

$$B_3(z) = z^{-3}(1 + 0.9z - 0.8z^2 + 0.5z^3)$$

$$= z^{-3} + 0.9z^{-2} - 0.8z^{-1} + 0.5$$

$$= 0.5 - 0.8z^{-1} + 0.9z^{-2} + z^{-3}$$

[Note: The coefficients of $B_3(z)$ are reversed from $A_3(z)$]

$k_3 = 0.5$ (No need to calculate, you got it from question ie the coefficient of the highest power)

[Note: From $H(z)$, $A_m(z)$ ($= A_3(z)$) is obtained directly
ie $A_3(z) = 1 + 0.9z^{-1} - 0.8z^{-2} + 0.5z^{-3}$ and
coefficient of max. z^{-1} is $k_3 = 0.5$]

put $m=3$, we will get A_2 and from A_2, we'll get k_2

ie $$A_2(z) = \frac{A_3(z) - k_3 B_3(z)}{1 - k_3^2}$$

$$= \frac{1 + 0.9z^{-1} - 0.8z^{-2} + 0.5z^{-3} - 0.5[z^{-3} + 0.9z^{-2} - 0.8z^{-1} + 0.5]}{1 - (0.5)^2}$$

$$= 1 + 1.73z^{-1} - 1.67z^{-2}$$

$\therefore k_2 = -1.67$

Now, $A_1(z) = A_2(z) - k_2 B_2(z)$
where $B_2(z) = -1.67 + 1.73z^{-1} + z^{-2}$

$$= \frac{1 + 1.73z^{-1} - 1.67z^{-2} + 1.67[1.67 + 1.73z^{-1} - z^{-2}]}{1 - (1.67)^2}$$

$\therefore A_1(z) = 1 - 1.67z^{-1}$

$\therefore k_1 = 1.67$

Substituting this value of k_1, k_2 and k_3 in the lattice structure for IIR filter, we get

Consider a three-stage FIR lattice structure having the coefficients $k_1 = 0.65$, $k_2 = 0.5$ and $k_3 = 0.9$. Find its impulse response and direct form structure.

Soln: formulae to remember

$\alpha_m(0) = 1$ — (I)

$\alpha_m(m) = k_m$ — (II)

$\alpha_m(k) = \alpha_{m-1}(k) + k_m \alpha_{m-1}(m-k)$ — (III)

$m = 3$

$\alpha_m(0) = 1$

$\alpha_1(1) = k_1 = 0.65$

$\alpha_2(2) = k_2 = 0.5$

$\alpha_3(3) = k_3 = 0.9$

To realize direct form structure, we need to find $\alpha_3(1)$, $\alpha_3(2)$, $\alpha_3(3)$

put $m = 3$, $k = 1$ in equation (III)

$\alpha_3(1) = \alpha_2(1) + k_3 \alpha_2(2)$ — (A)

Now Calculate $\alpha_2(1)$

put $m = 2$, $k = 1$ in (III)

$\alpha_2(1) = \alpha_1(1) + k_2 \alpha_1(1)$

$= 0.65 + 0.5 \times 0.65$

$= 0.975$

Substitute $\alpha_2(1)$ in equation (A)

$\therefore \alpha_3(1) = 0.975 + 0.9 \times 0.5$

$= 1.425$

Now, Sustitute $m_3 =$, $k_2 = 2$ in eqtn III

$\alpha_3(2) = \alpha_2(1) + k_3 \times \alpha_2(1)$

$= 0.5 + 0.9 \times 0.975$

$= 1.3775$

$\alpha_3(3) = 0.9$

we know that

$$H(z) = 1 + \sum_{k=1}^{m} \alpha_m(k) z^{-k}$$

$$= 1 + \sum_{k=1}^{3} \alpha_m(k) z^{-k}$$

$$= 1 + \alpha_3(1) z^{-1} + \alpha_3(2) z^{-2} + \alpha_3(3) z^{-3}$$

$$= 1 + 1.425 z^{-1} + 1.3775 z^{-2} + 0.9 z^{-3}$$

$$\frac{Y(z)}{X(z)} = 1 + 1.425 z^{-1} + 1.3775 z^{-2} + 0.9 z^{-3}$$

$$y(n) = x(n) + 1.425\, x(n-1) + 1.3775\, x(n-2) + 0.9\, x(n-3)$$

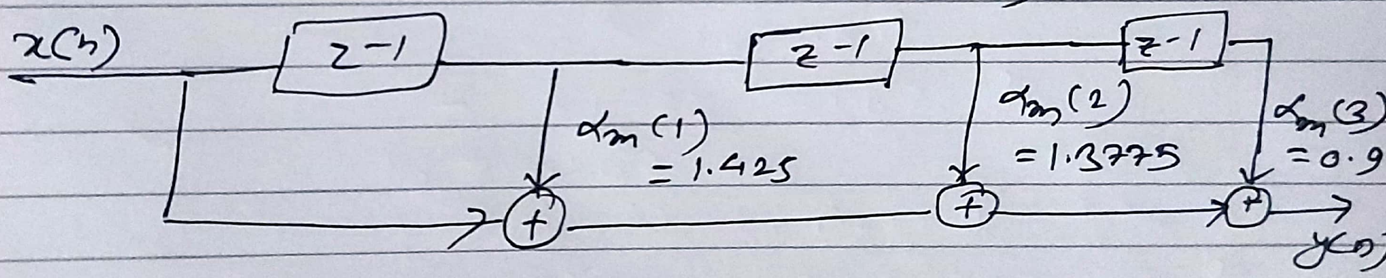

fig. Direct form structure