POKHARA UNIVERSITY

Level: Bachelor Semester: Fall Year : 2019
Programme: BE Full Marks: 100
Course: Engineering Mathematics I Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all the questions.

1. a) State Leibnitz's theorem for successive derivative of product of two functions y = u.v If $y=\left(x+\sqrt{1+x^2}\right)^m$, show that $\left(1+x^2\right)y_{n+2}+\left(2n+1\right)xy_{n+1}+\left(n^2-m^2\right)y_n=0$ 8

OR

Show that the function $f(x)=\begin{cases} x & for \quad x<1 \\ 2-x & for \quad 1\le x\le 2. \\ x-\frac{x^2}{2} & for \quad x>2 \end{cases}$

is continuous at x =1 & x=2. Does f'(x) exists at these points.

b) State Rolle's theorem with its geometrical meaning. Also verify the theorem for the function $f(x)=(x-a)^m(x-b)^n$ 7

2. a) State L'Hospital rule. Prove that: $\lim_{x\to 0}(cotx)^{sin2x}=1$. 7

b) Find the asymptotes of given curve $x^2(x-y)^2-a^2(x^2+y^2)=0$ 8

OR

Find the total surface area of the right circular cylinder of greatest surface that can be inscribed in a given sphere of radius r.

3. Integrate (Any Three) 15

a) $\int \frac{dx}{4+5\sin x}$

b) $\int_0^a \frac{dx}{x+\sqrt{a^2-x^2}}$

c.) $\int_0^1 \frac{dx}{(1-x^6)^{\frac{1}{6}}}$ d) $\int_0^2 x^2 dx$. (by summation)

4. a) Find the area between the curves $x = y^2$ and $2y^2 = -x+3$ 7

OR

Show that the volume of sphere of radius r is $\frac{4}{3}\pi r^3$

b) Find the approximate area using Simpson's and Trapezoidal rule for the area bounded by the curve $y = \sin x$, the x- axis and the lines $x = \pi/2$ and $x = 2\pi$ (using $n = 6$) and compare these results with exact value. 8

OR

Obtain the reduction formula for $\int_{\pi/4}^{\pi/2} \cot^n x dx$ and evaluate $\int_{\pi/4}^{\pi/2} \cot^7 x dx$

5. a) Define eccentricity of a conic section, and derive the equation of a hyperbola in its standard form. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ 8

b) Find the equation of tangent to the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$, which is parallel to the line $x=y+5$. 7

6. a) Define scalar and vector product of three vectors. Prove that the scalar triple product of three vectors represent the volume of parallelepiped. What conclusion can you draw about these vectors if $\vec{a}.(\vec{b} \times \vec{c}) = 0$. 8

b) Find the equation of the plane through (1, 0, -1) and (-1, 2, 1) and parallel to the line of intersection of the planes $3x + y - 2z = 0$ and $4x - y + 3z = 0$ 7

7. Attempt All questions 10

a) Find the domain and range for $y = \sqrt{4 - x^2}$

b) Find the equation of parabola having focus (-3,0) and directrix $x+5=0$

c) If $\vec{a} = i - 2\vec{j} + \vec{k}$ and $\vec{b} = \vec{i} + 2\vec{j} - \vec{k}$ find the projection of $\vec{a}$ on $\vec{b}$

d) Evaluate $\int \tan^{-1} x$