Important questions:

Q.1 Define Analytic function. Show that $\sinh z$ is an analytic function.

Q.2 State and prove the necessary condition of C-R equation. Check the analyticity of $f(z) = z^2$.

Q.3 Show that the function $f(z) = z^n$, where n is a positive integer is an analytic function.

Q.4 Define harmonic function. Check $u = \sin x \cosh y$ is harmonic or not? If yes, find corresponding analytic function $f(z) = u + iv$.

Q.5 Show that the function $u(x,y) = 3x^2y + x^2 - y^3 - y^2$ is a harmonic function. Find a function $v(x,y)$ such that $u+iv$ is an analytic function.

Q.6 Given that $f(z) = u+iv$ is an analytic function and $u+v = e^x(\cos y + \sin y)$. Find $f(z)$.

Q.7 If $v = \arg z$ is harmonic? If yes find a corresponding harmonic conjugate.

Q.8 If $v = (x^2-y^2)^2$ is harmonic? If yes find its harmonic conjugate and analytic function $f(z) = u+iv$.

Q.9 If $u = ax^3 + bxy$, evaluate a and b such that the given function is harmonic and find its harmonic conjugate.

Q.10 Determine value of a, when $u = \cos ax \cosh 2y$, is harmonic and find its harmonic conjugate.

Q.11. Show that $v = 2xy - \dfrac{y}{x^2+y^2}$ is a harmonic function. Find harmonic conjugate u of v.

Q.12. Given that $u = x^2 - y^2$ and $v = \left[\dfrac{-y}{x^2+y^2}\right]$ prove that both u and v are harmonic functions but $u+iv$ is not analytic function of z.

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Q.13 Determine whether the following functions are harmonic. If your answer is yes, find a corresponding analytic function $f(z) = u+iv$ (a) $u = \frac{x}{x^2+y^2}$ b. $v = x^3 - 3xy^2$

Q.14. Determine a and b such that the given functions are harmonics and find a harmonic conjugate.

(a) $u = ax^3 + by^3$ (b) $u = e^{ax} \cdot \cos 5y$.

Q.15 Show that the function $u = \cos x . \cosh y$ is harmonic and find its harmonic conjugate.

Q.16 Prove that $u = y^3 - 3x^2y$ is harmonic function. Determine its harmonic conjugate and find the corresponding analytic function $f(z)$ interms of z.

Q.17 If $u = x^3 - 3xy^2$. Show that there exist a function $v(x,y)$ such that $w = u+iv$ is analytic in a finite region.

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Q.18. Define Bilinear transformation (linear fractional transformation). prove that a bilinear transformation maps circles into circles.

Q.19. Find a bilinear transformation which maps the points $i, -i, 1$ of the z-plane into $0, 1, \infty$ of the w-plane respectively.

1.2 Integrals in complex plane:
Integrals in complex-plane

Q.20 State cauchy's integral formula and use it to evaluate $\int_c \frac{z^2+1}{z^2-1}\, dz$ where c is the circle of radius 1 with centre at the point (i) $z=1$ (ii) $z=-1$

Q.21. State and prove cauchy's integral formula and hence evaluate by using this

(I) $\int_c \frac{2z^2+3}{(z-2)^3}\, dz$ $\quad c: |z-1| = 2$

(II) $\int_c \frac{4-\sin z}{z^2-2z} \cdot dz$

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(iii) $\oint_c \frac{1}{z^2+4} dz$, where c is the ellipse, $4x^2+(y-2)^2=4$

(iv) $\oint_c \frac{2z^2+4z}{z-2} dz$ $c: |z|=1$

(v) $\oint_c \frac{e^{5z}}{(z+i)^4} dz$ where c is the circle $|z|=3$ counter clockwise

(vi) $f(z) = \frac{z^3}{2z-i}$ around unit circle clockwise

(vii) $\oint_c \frac{\cot z}{(z-\pi/2)^2} dz$ where c is the ellipse $4x^2+9y^2=36$

(viii) $\oint_c \frac{\log(z-1)}{(z-6)} \cdot dz$ where c is the circle $|z-6|=4$

(ix) $\oint_c \left(\frac{z+1}{z^3-4z}\right) \cdot dz$ where c is the circle $|z+2| = \frac{3}{2}$ in anticlockwise direction.

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(x) evaluate: by using C.F.
$\oint_c \frac{z^2}{(z^2-1)} dz$, where c is a positively oriented circle $|z-1|=1$

(xi) $\oint_c \frac{z}{z-2} dz$ where $c: |z|=2$ counterclockwise

(xii) By using C.I.T. evaluate

$\int_c \frac{e^z}{z-2} dz$

(I) c is positively oriented circle. $|z|=1$

(II) c is positively oriented circle $|z-2|=2$

(III) $\int_c \frac{z \sin z}{2z-1} dz$, where c is the unit circle counterclockwise

(IV) $\oint_c \frac{\cosh 4z}{(z-4)^3}$ where c consists of $|z|=6$ (counterclockwise) and $|z-3|=2$ (clockwise)

(V) $\oint_c \frac{z+1}{z^3-2z^2} dz$, c is the unit circle.

Q.22 State and prove cauchy integral formula, evaluate the integral $\int \frac{\cos z}{(z-\pi i)^3} . dz$ where c is the unit-circle enclosing the point πi.

Short Note:

Q3. a) Evaluate $\int_c \frac{1}{z} dz$, where c is the unit-circle

b) Show that $\int_c \frac{dz}{z} = 2\pi i$, where c is the unit-circle, counter clockwise.

c) $\oint_c \frac{dz}{z-3i}$ where c is the circle $|z|=4$ counter clock-wise direction.

d) $\oint \frac{z^3 \sin z}{3z-1} dz$ along unit-circle.

e) $\oint_c \frac{dz}{z}$ where c is the unit disk $|z|=1$

(1.3.) Taylor and laurent Series for the complex variables.

Q.1 (I) State Taylor's and laurent Series
(II) Define singularities, Zero's and Residue.
(III) State cauchy Residue theorem.

Q.2 Expand: by using Maclaurin's series:

(a) $f(z) = \frac{1}{1-z^4}$

(b) $f(z) = \frac{2-z}{(1-z)^2}$

(c) $f(z) = \tan^{-1} z$

Q.3. Find Taylor's expansion of (a) $f(z) = z^5$ at $z = -1$

(b) $f(z) = \cos \pi z$ at $z = \frac{1}{2}$.

(c) $f(z) = \frac{1}{(z+i)^2}$ at $z = i$ by using Taylor's expansion.

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(d) $f(z) = \cos^2 z$ at $z = \frac{\pi}{2}$.

(e) $\frac{2z^3+1}{z(z+1)}$ about the point $z = -i$

(f) $\frac{z^3+2z^2}{z^2+2z+3}$ in $\left|z+\frac{1}{2}\right| < 1$

Q.4. Find the laurent series of

(a) $f(z) = \frac{e^z}{z(1-z)}$ which converges

$0 < |z-1| < R$

(b) $f(z) = \frac{z^2-4}{(z-1)^2}$ for $0 < |z-1| < R$

Q.5. Find the taylor or laurent series which represent the function

(i) $\frac{1}{(1+z^2)(z+2)}$

(a) when $|z| < 1$ (b) when $1 < |z| < 2$

(c) when $|z| > 2$

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(ii) $f(z) = \frac{z^2-1}{(z+2)(z+3)}$

(a) when $|z| < 2$ (b) when $2 < |z| < 3$

(c) when $|z| > 3$.

Q.6 Expand $f(z) = \frac{z+3}{z(z^2-z-2)}$ in power of z; where

(i) $|z| < 1$ (ii) $1 < |z| < 2$

(iii) $|z| > 2$.

Q.7 Represent the function $f(z) = \frac{4z+3}{z(z-3)(z+2)}$ in laurent series

(i) within $|z| = 1$

(ii) In annular region between $|z| = 2$ and $|z| = 3$.

(iii) exterior to $|z| = 3$ i.e. $|z| > 3$.

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Q.8 Expand in the series the function

$f(z) = \frac{1}{z^2-3z+2}$ in the regions

(i) $|z|<1$ (ii) $1<|z|<2$ (iii) $|z|>2$

Q.9 Find the laurent series for $f(z) = \frac{z^2-1}{z^2+5z+6}$ in the region $2<|z|<3$.

Q.10 Find the laurent expansion for $f(z) = \frac{1}{z^2(1-z)}$ in the region

(a) $0<|z|<1$

(b) $1<|z|<4$

Q.11 Find the expression of $f(z) = \frac{1}{(z-z^3)}$ in the region $1<|z-1|<2$

Q.12 Find the expansion for $\frac{7z-2}{(z+1)z(z-2)}$ in the regions given by

a. $0<|z+1|<1$ b. $1<|z+1|<3$

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c. $|z+1|>3$

Q.13 Give the laurent's series expansion for $f(z) = \frac{1}{(z+1)(z+3)}$ for the region

$0<|z+1|<2$

Q.14 Find the laurent's series expansion for the function $f(z)$ given below in the specified regions:

(a) $\frac{1}{z(1+z)^2}$ in $0<|z|<1$

(b) $\frac{z+3}{z(z^2-z-2)}$ in (i) $0<|z|<1$ (ii) $1<|z|<2$

Q.15 Find the laurent's series expansion for $f(z)$ in the region specified

(a) $\frac{z}{(z-1)(z-3)}$ in $0<|z-1|<2$

(b) $\frac{e^{2z}}{(z-1)^3}$ in $|z-1|>1$

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(c) $\frac{1}{z-z^3}$ in $1 < |z+1| < 2$.

Zeros and singularities of a function:

1. Find the residue of

(a) $\frac{z^2}{z^2+a^2}$ at $z = ia$

(b) $\frac{z^3}{(z-1)^4(z-2)(z-3)}$ at $z = 1$

(c) $\frac{1}{(z^2+a^2)^2}$ at $z = ia$

(d) Find the singular points and residues of the function

$$f(z) = \frac{(z+2)}{(z-2)(z^2+1)^2}$$

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2. Integrate $\oint_{C} \left(\frac{z-23}{z^2-4z-5}\right) dz$;

where $C: |z-2| = 4$

3. Evaluate: $\oint_c \left(\frac{z^2 \sin z}{4z^2-1}\right) dz$;

where $C: |z| = 2$

4. Integrate: $\oint_c \frac{e^z+z}{z^3-z} . dz$; where

$C: |z| = \frac{\pi}{2}$.

5. Evaluate: $\oint_c \frac{e^{2z}}{(z+1)^3} . dz$ where c is th[e]

ellipse $4x^2 + 9y^2 = 16$.

The fourier integral:

Q.1 Find the fourier integral representation of the function

$$f(x) = \begin{cases} 1 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$$

Q.2 Find fourier integral representation of

$$f(x) = \begin{cases} \frac{\pi}{2} & \text{if } 0 \le x < 1 \\ \frac{\pi}{4} & \text{if } x = 1 \\ 0 & \text{if } x > 1 \text{ and } x < 0 \end{cases}$$

Q.3 (a) Define fourier cosine and sine integrals.

(b) Find fourier cosine integral representation of the function,

$$f(x) = \begin{cases} x^2 & \text{if } 0 < x < 1 \\ 0 & \text{if } x > 1 \end{cases}$$

(b) $f(x) = \begin{cases} a^2 - x^2 & \text{if } 0 < x < a \\ 0 & \text{if } x > a \end{cases}$

Q.4 Find fourier cosine integral representation of $f(x) = e^{-x} + e^{-2x}$ for $x > 0$

Q.5 Find the fourier sine integral representation of the function

$$f(x) = \begin{cases} x & \text{if } 0 < x < a \\ 0 & \text{if } x > a \end{cases}$$

Q.6 Show that: $\int_0^\infty \frac{w^3 \sin xw}{w^4+4}\, dw = \frac{\pi}{2} e^{-x} \cos x$ if $x > 0$

Q.7. Find the fourier cosine and sine integrals of the function

$$f(x) = e^{-kx} \quad \text{for } x > 0,\ k > 0$$

Q.8 Show that

(a) $$\int_0^\infty \left[\frac{\cos xw + w \sin xw}{1+w^2}\right] dw = \begin{cases} 0 & \text{if } x < 0 \\ \frac{\pi}{2} & \text{if } x = 0 \\ \pi e^{-x} & \text{if } x > 0 \end{cases}$$

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(b) $\int_0^\infty \left(\frac{1-\cos \pi w}{w}\right) \sin xw \, dw = \begin{cases} \frac{\pi}{2} & \text{if } 0<x<\pi \\ 0 & \text{if } x>\pi \end{cases}$

(c) $\int_0^\infty \frac{\cos xw}{1+w^2} \, dw = \frac{\pi}{2} e^{-x}$ if $x>0$

(d) $\int_0^\infty \frac{\sin \pi w \cdot \sin xw}{1-w^2} \, dw = \begin{cases} \frac{\pi}{2}\sin x & \text{if } 0\le x\le\pi \\ 0 & \text{if } x>\pi \end{cases}$

(e) $\int_0^\infty \left(\frac{w \sin wx}{a^2+w^2}\right) dw = \frac{\pi}{2} e^{-ax}$ where $x>0$, $a<0$

(f) $\int_0^\infty \left(\frac{\cos xw}{k^2+w^2}\right) dw = \frac{\pi e^{-kx}}{2k}$ when $k>0$, $x>0$

Q.9 Define fourier cosine and sine transform.

(a) Find the fourier cosine transform of $f(x) = e^{-mx}$, where $m>0$.

(b) Find the fourier sine transform of

$f(x) = \begin{cases} 2x & \text{if } 0<x<1 \\ 2-x & \text{if } 1\le x<2 \\ 0 & \text{if } x\ge 2 \end{cases}$

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Q.10 Find the fourier sine transform of $e^{-|x|}$, hence evaluate, $\int_0^\infty \frac{w \sin wx}{1+w^2} \, dw$

Q.11 Find the fourier cosine transform of e^{-x^2}.

Q.12 Find the fourier cosine transform of $f(x) = e^{-ax}$ where $a>0$.

Q.13 (a) Define fourier transform and its inverse

(b) Find the fourier transform of

$f(x) = \begin{cases} e^{-kx} & \text{if } x>0,\ k>0 \\ 0 & \text{if } x<0 \end{cases}$

(c) $f(x) = \begin{cases} x^2 & \text{if } 0<x<1 \\ 0 & \text{otherwise} \end{cases}$

(d) $f(x) = \begin{cases} e^{x} & \text{if } x<0 \\ e^{-x} & \text{if } x>0 \end{cases}$

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(e) $f(x) = \begin{cases} |x| & \text{if } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

(f) $f(x) = \begin{cases} kx & \text{if } a < x < b \\ 0 & \text{otherwise.} \end{cases}$

(g) $f(x) = \begin{cases} e^{-ax^2} & \text{where } a > 0 \end{cases}$

Q.14 Find the fourier transform of

$f(x) = e^{-\frac{x^2}{2}}$.

Q.15 Find the fourier transform of

$f(x) = e^{-mx}$ $m > 0$ and then show that

$$\int_0^\infty \left(\frac{\cos kx}{1+x^2}\right) dx = \frac{\pi}{2} e^{-k}.$$

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Q.16 Find the fourier transform of the function, $f(x) = \begin{cases} 1-x^2 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1. \end{cases}$

Q.17. Starting from the fourier series obtain the fourier integral in complex form.

Q.18. Show that fourier transform is a linear operator.