| [II Sem Math]

Tutorial-I
Pokhara Engineering College

Level: Bachelor
Course: Algebra and Geometry

1) Find the intercepts made on the coordinates axes by the plane x+2y-2z=9. Find also the direction cosine of the normal to this plane
2) Find the equation of the plane containing the lines through the origin with direction Cosines proportional to 1,-2, 2 and 2, 3,-1.
3) Find the angle between the following pair of planes: 2x+3y+5z=0 and x-2y+z=20
4) Find the equation of the plane through (α,β,δ) parallel to ax+by+cz+d=0.
5) Find the equation of the plane containing the point (1,-1, 2) and perpendicular to the Planes 2x+3y-2z=5 and x+2y-3z=8.
6) Show that the points (4,5,1),(3,9,4),(-4,4,4) and (0,-1,-1) are coplanar.
7) Find the equation of the plane containing the point (6, 2, 1), (-1, 1,-1) and normal to the plane 2x+y+z=5
8) Find the equation of the plane through the intersection of the planes 2x+3y+10z=8, 2x-3y+7z=2 and normal to the plane 3x-2y+4z=5.
9) The plane x+3y+5z=7 is rotated through a right angle about its intersection with the plane x-2y-6z=8. Find the equation in its new position.
10) Show that the origin lies in the acute angle between the planes x+2y+2z-9=0 and 4x-3y+12z+13=0. Find the planes bisecting the angles between them and point out which bisects the acute angle.
11) Show $2x^2-6y^2-12z^2+18yz+2zx+xy=0$ represents the pair of planes. Find their equations and angle between them
12) Find the distance of the point (1,-3, 5) from the plane 3x-2y+6z=15 along a line with direction cosines proportional to (2, 1,-2).
13) Find the equation to the line through (-1, 3, 3) and perpendicular to the plane x+2y+2z=3. Also find the length of perpendicular and coordinates of foot.
14) Find the angle between the lines in which the plane x-y+z=5 is cut by the planes 2x+y-z=3 and 2x+2y+3z-1=0
15) Find the equation of the plane parallel to the line x-2= $\frac{y-1}{3}=\frac{z-3}{2}$ containing (1,0,-1) and (3, 2, 2).
16) Find the equation of a plane containing the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-3}{4}$ and is perpendicular to the plane x+2y+z=12.
17) Show that the lines $\frac{x+4}{3}=\frac{y+6}{5}=\frac{1-z}{2}$ and 3x-2y+z+5=0=2x+3y+4z are coplanar. Find their point of intersection and the plane in which they lie.
18) Show that the lines x-1=2y-4=3z, 3x-5=4y-9=3z meet in a point and the equation of the plane in which they lie is 3x-8y+3z+13=0.
19) Find the length and equation of the shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$; 2x-3y+27=0, 2y-z+20=0.
20) Find the shortest distance between the lines ax+by+cz+d=0=$a_1x+b_1x+c_1z+d$, and z-axis.

21) Prove that the shortest distance between the diagonal of a rectangular parallelepiped and the edges not meeting it are $\frac{bc}{\sqrt{b^2+c^2}}$, $\frac{c}{\sqrt{b^2+c^2}}$, $\frac{ab}{\sqrt{a^2+b^2}}$, where a, b, c are the lengths of the edges.

22) Find the condition that the two lines $\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$ are coplanar. Also find the equation of plane containing the lines.

23) Find the magnitude and equation of shortest distance between the two lines $\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$

24) Define sphere. Find the centre and radius of the sphere $x^2+y^2+z^2$-2x+4y-4z-7=0
Ans: (1,-2, 2); 4

25) Find the equation of the sphere which passes through the origin and the points (0,1,-1), (-1,2,0) and (1,2,3). Ans: $7(x^2+y^2+z^2)$-15x-25y-11z=0

26) Obtain the equation of the sphere which passes through the three points(1,0,0),(0,1,0),(0,0,1) and has its radius as small as possible
Ans: $3(x^2+y^2+z^2)$-2(x+y+z)-1=0

27) A plane passes through a fixed point (a,b,c) and cuts the axes in A,B,C.Prove that the locus of the Centre of the sphere OABC is $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$.

28) A sphere of radius k passes through (0,0,0) and meets the axes in A,B,C. Prove that the centroid of the triangle ABC lies on the sphere $9(x^2+y^2+z^2)=4k^2$

29) Find the equation of the sphere on of whose great circle is $x^2+y^2+z^2$=4, $x^2+y^2+z^2$-2x+4y-6z=11. Ans: $4(x^2+y^2+z^2)$ +2x-4y+6z-9=0

30) Show that the equation of the sphere through the circle $x^2+y^2+z^2$-2x-3y+4z+8=0, $x^2+y^2+z^2$+4x+5y-6z+2=0 and having its centre on the plane 4x-5y-z=3 is $x^2+y^2+z^2$+7x+9y-11z-1=0.

31) Find the equation of the sphere through the circle $x^2+y^2=4, z=0$ and is cut by the plane x+2y+2z=0 in a circle of radius 3. Ans: $x^2+y^2+z^2$-6z-4=0.

32) Find the centre and radius of the circle $x^2+y^2+z^2$+x+y+z=4,x+y+z=0. Ans: (0,0,0),2..

33) Show that the plane 2x-2y+z+12=0 touches the sphere $x^2+y^2+z^2$-2x-4y+2z-3=0 and find the point of contact. Ans: (-1,4,-2)

34) Find the equation of the sphere which passes through the circle $x^2+y^2+z^2$=5,x+2y+3z=3 and touch the plane 4x+3y=15.

35) Find the equation to the tangent planes to the sphere $x^2+y^2+z^2$-2x+4y-6z+10=0 which pass through $\frac{x+3}{14}=\frac{y+1}{-3}=\frac{z-5}{1}$.

36) Find the value of c for which the plane x+y+z=c$\sqrt{3}$ touches the sphere $x^2+y^2+z^2$-2x-2y-2z-6=0. Ans ; $\sqrt{3}\pm3$.

Tutorial-II

Level: Bachelor

Course: Algebra and Geometry

1) Define conic section and derive the standard equation of parabola, ellipse and Hyperbola.
2) Derive the equation of parabola with vertex (h, k) and focus (h + a, k).
3) Find the equation of tangent and normal at p(x_1, y_1) on the parabola $y^2 = 4ax$, ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
4) Find the condition that the line y= mx + c is tangent to the parabola $y^2 = 4ax$, ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and hyperbola. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ .Also find point of contact.
5) Find the condition that the line lx+my+n=0 is tangent to the parabola $y^2 = 4ax$, ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Also find point of contact.
6) Obtain the vertex, focus, directrix, line of symmetry and length of latus rectum of the following parabola and sketch. i) $x^2 - 4y + 2x - 3 = 0$. ii) $4y^2 + 3x - 8y - 2 = 0$
7) Obtain the Centre, vertices, coordinates of foci, directrix, eccentricity and length of latus rectum of the following ellipse $9x^2 + 5y^2 - 30y = 0$.
8) Obtain the centre, vertices, coordinates of foci, directrix, eccentricity and length of latus rectum of the following hyperbola i) $4(y+3)^2 - 9(x-2)^2 = 1$ ii) $9x^2 + 18x - 16y^2 + 32y = 151$
9) Find the equation of parabola having focus and directrix. Focus (1, 2) directrix x+2y+4=0.
10) Find the equation of ellipse having focus, directrix and eccentricity F (-1, 1), 4x-3y=0 and e= $\frac{5}{6}$
11) Find the equation of hyperbola having focus, directrix and eccentricity are (6, 0), 4x-3y=6, e= $\frac{5}{4}$
12) A tangent to $y^2 = 16x$ makes an angle 60^0 with x-axis. Find the equation of tangent and its point of contact.

Tutorial-III

Level: Bachelor

Course Algebra and Geometry

1) Define scalar and vector product of two, three and four vectors. Give geometrical interpretation of scalar product of two and three vectors and vector product of two and three vectors.

2) (i) If $\vec{a}=4\vec{i}+3\vec{j}+\vec{k}$, $\vec{b}=2\vec{i}-\vec{j}+2\vec{k}$ then find the cosine and sine angle between $\vec{a}$ and $\vec{b}$. Also find the unit vector $\hat{n}$ perpendicular to the vectors $\vec{a}$ and $\vec{b}$.

(ii) Find the area of the parallelogram having the adjacent sides as $2\vec{i}+5\vec{j}+6\vec{k}$ and $4\vec{i}+3\vec{j}+\vec{k}$.

3) (i) Find the volume of a parallelepiped whose concurrent edges are represented by the vectors. $\vec{i}+\vec{j}+\vec{k}$; $\vec{i}-\vec{j}+\vec{k}$ and $\vec{i}+2\vec{j}-\vec{k}$

(ii) Prove that the following four points are coplanar.
$2\vec{i}+3\vec{j}-\vec{k}$; $\vec{i}-2\vec{j}+3\vec{k}$, $3\vec{i}+4\vec{j}-2\vec{k}$ and $\vec{i}-6\vec{j}+6\vec{k}$.

(iii) The position vectors of the points A, B, C and D are $3\vec{i}-2\vec{j}-\vec{k}$; $2\vec{i}+3\vec{j}-4\vec{k}$, $-\vec{i}+\vec{j}+2\vec{k}$ and $4\vec{i}+5\vec{j}+\lambda\vec{k}$ .If the points A, B, C and D are coplanar, find the value of λ.

4) Show that the vectors $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})+(\vec{a}\times\vec{c})\times(\vec{d}\times\vec{b})+(\vec{a}\times\vec{d})\times(\vec{b}\times\vec{c})$ is parallel to the vector $\vec{a}$.

5) Prove, $[\vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\ \ \vec{a}\times\vec{b}]=[\vec{a}\vec{b}\vec{c}]^2$

6) Prove distributive law of vector product of two vectors using scalar triple product.

7) Find the set of reciprocal vector of $\vec{a}=2\vec{i}+3\vec{j}-\vec{k}$, $\vec{b}=\vec{i}-\vec{j}-2\vec{k}$ and $\vec{c}=-\vec{i}+2\vec{j}+2\vec{k}$.

8) Show that $(\vec{a}\times\vec{b})\times\vec{c}=\vec{a}\times(\vec{b}\times\vec{c})$ iff $(\vec{a}\times\vec{c})\times\vec{b}=0$.

9) Show that $(\vec{b}\times\vec{c}).(\vec{a}\times\vec{d})+(\vec{c}\times\vec{a}).(\vec{b}\times\vec{d})+(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d})=0$

10) Show that the vectors $\vec{a}\times(\vec{b}\times\vec{c})$, $\vec{b}\times(\vec{c}\times\vec{a})$, $\vec{c}\times(\vec{a}\times\vec{b})$ are coplanar.

Tutorial-IV

Level: Bachelor

Course Algebra and Geometry

1) State and prove Hyper-Harmonic series.

2) State and prove Leibnitz's theorem for convergence of an infinite series.

3) State Cauchy Root test, D'Alembert's Ratio test, Limit comparison test for convergence of infinite series.

4) Test the convergence and divergence of the following series by using limit comparison test

i) $\frac{2}{3^2}+\frac{3}{4^2}+\frac{4}{5^2}+.....$ ii) $\frac{1}{1+\sqrt{2}}+\frac{2}{1+2\sqrt{3}}+\frac{3}{1+3\sqrt{4}}+.....$

iii) $\sum \sqrt{n^4+1}-\sqrt{n^4-1}$ iv) $\sum u_n=\sum \frac{\sqrt{n}}{n^2+1}$

5) Test the convergence and divergence of the following series by using D'Alembert ratio test

i) $\frac{1}{2}+\frac{4}{2^2}+\frac{9}{2^3}+.....$ ii) $\sum u_n=\sum \sqrt{\frac{2^n-1}{3^n-1}}$ iii) $\sum u_n=\sum \frac{n(n+1)^2}{n!}$

6) Test the convergence and divergence of the following series by using Cauchy root test

i) $1+\frac{1}{2^2}+\frac{1}{3^3}+\frac{1}{4^4}+.....$ ii) $\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}+.....$ iii) $\sum\{\left(\frac{n+1}{n}\right)^{n+1}-\frac{n+1}{n}\}^{-n}$

7) Define Alternating series. Test the convergence and divergence of the following series by using Leibnitz's test

i) $1-\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}-\frac{1}{4\sqrt{4}}+.....$ ii) $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+.....$

iii) $\frac{1}{\sqrt{2}+1}-\frac{1}{\sqrt{3}+1}+\frac{1}{\sqrt{4}+1}+.....$ iv) $\sum u_n=\sum(-1)^{n+1}\frac{1+n}{n^2}$

8) Define power series, center and interval of convergence of series.

Find the interval of convergence, center of convergence and radius of convergence of the series

i) $\sum u_n=\sum_1^{\alpha}\frac{(x-5)^n}{n5^n}$ ii) $\sum u_n=\sum_1^{\alpha}\frac{(x)^{2n+1}}{(-4)^n}$

iii) $x-\frac{x^2}{2^2}+\frac{x^3}{3^3}-\frac{x^4}{4^4}+.....$ iv) $\sum u_n=\sum_1^{\alpha}\frac{(x)^n}{n^2}$

9) Using simplex method, solve the following LPP

i)Maximize $Z=3x_1+x_2$ subject to constraints $2x_1+x_2\le 6,\ x_1+3x_2\le 9\quad x_1,x_2\ge 0$

ii) Minimize $Z=5x_1-20x_2$

subject to constraints $-2x_1+10x_2\le 5,\ 2x_1+5x_2\le 10\quad x_1,x_2\ge 0$

iii) Maximize $Z=4x_1-10x_2-20x_3$ subject to constraints $3x_1+4x_2+5x_3\le 60,$

$2x_1+x_2\le 20$, $2x_1+3x_3\le 30\ x_1,x_2,x_3\ge 0$

10) Construct the dual problem and solve by simplex method.

Minimize $Z=8x_1+9x_2$ subject to constraints $x_1+x_2\ge 6,\ 3x_1+x_2\ge 21\quad x_1,x_2\ge 0$

11) Solve the following LPP by Big-M method.

Maximize $Z=5x_1-2x_2$ subject to constraints $3x_1-4x_2\le 2,\ x_1+2x_2\ge 4$, $x_2\le 4,$

$x_1,x_2\ge 0$

Tutorial-V

Level: Bachelor
Course Algebra and Geometry

1) Define the term Rank of matrix Find Rank of following matrix by using Echelon form.

a) $\begin{pmatrix} 2 & 1 & 3 \\ 4 & 7 & 13 \\ 4 & -3 & -1 \end{pmatrix}$ b) $\begin{pmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{pmatrix}$

2) Check the given system of linear equations is consistent or not, if consistent then solve.

a) x+2y-3z=9 2x-y+2z=-8 3x-y-4z=3

b) 2x-y+3z=8 -x+2y+z=4 3x+y-4z=0

3) Determine the value of p and q for which the system of linear equations

x+y+z=6 x+2y+5z=10 2x+3y+pz=q has

a)unique solution b) no solution c) infinitely many solutions.

3) Define linearly dependent and independent vectors.

Determine the following vectors are linearly dependent or independent.

a)(1,1,1), (2,1,0), (1,-1,2) b) (1,1,1), (1,2,3), (-1,1,3)

4) Define basis of vectors. Check the following vectors form the basis of R^3

a)(1,2,2), (2,5,4), (2,7,4) b) (1,0,0), (0,1,0), (0,0,1)

5) Define Linear transformation and determine the following transformation is linear or not.

a) $T: R^3 \to R^2$ define by T(x,y,z)=(x+2y-z,0)

a) $T: R^3 \to R^3$ define by T(x,y,z)=(x-2y,y+2z,x+3z)

6) Find Eigen value and corresponding Eigen vectors of the matrix.

a) $A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$ b) $A = \begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$

7) State Cayley Hamilton theorem.Verify Cayley Hamilton theorem and find inverse of A

a) a) $A = \begin{pmatrix} 2 & 3 \\ 1 & 5 \end{pmatrix}$ b) $A = \begin{pmatrix} 6 & 2 & 3 \\ 3 & 1 & 1 \\ 10 & 3 & 4 \end{pmatrix}$

8) Define Diagonalization of matrices and diagonalize the matrix $A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$.