POKHARA UNIVERSITY

Level: Bachelor
Programme: BE
Course: Calculus II

Semester: Fall

Year : 2023
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1\.

a) Evaluate $\int_0^2 \int_0^{\sqrt{4-y^2}} \cos(x^2+y^2)dxdy$ by changing into polar integration. 5

b) Evaluate: $\int_0^1 \int_0^1 \int_0^1 (x^2+y^2+z^2)dzdydx$ 5

c) Find the volume in the first octant bounded by the coordinate planes, the cylinder $x^2+y^2=4$ and $z+y=3$. 5

2\. a) Solve the differential equation $y''+(1-x^2)y=0$ by using power series method. 7

b) Define Legendre's equation. Also derive the solution of Legendre's equation. 8

OR

If $J_n(x)$ represents the Bessel's function of order n then show that:

i. $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$.

ii. $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$,

3\. a) State First Shifting Theorem of Laplace transforms. 8

i. Find the Laplace transform of $te^{-t}\cosh t$

ii. Find the inverse Laplace transform of $\frac{1}{s^2-5s+6}$

b) Solve the initial value problem:
$y''+4y'+3y=e^{-t}, y(0)=y'(0)=1$ by using Laplace transform. 7

4\. a) *If* $\emptyset = \log(x^2+y^2+z^2)$ then find div (gradΦ). 5

b) Find the directional derivative of $f = x^2yz + 4xz^2$ at (1, -2,1) in the direction $2\vec{i} - \vec{j} - 2\vec{k}$. 5

c) Calculate: $\oint_c \vec{F}.\vec{dr}$ if $\vec{F} = (y, z, x)$, C; $\vec{r} = (t, t^2, t^3)$, from (0,0,0) and (2,4,8). 5

5. a) Define Green's Theorem. Evaluate $\oint_c \vec{F}.\vec{dr}$, where $\vec{F} = (\sin y, \cos x)$ and c is the triangle with vertices $(0,0), (\pi, 0), (\pi, 1)$, by using Green's theorem. 8

b) Evaluate $\oint_c \vec{F}.\vec{dr}$ by using Stoke's theorem, where $\vec{F} = (y^2, z^2, x^2)$ and c is the boundary of the surface $S: x + y + z = 1$ in the first octant. 7

OR

Evaluate the surface integral $\iint_s \vec{F}.\vec{n}dA$ where $\vec{F} = (x^2, y^2, z^2)$ and s: $\vec{r}$=(ucosv,usinv,3v), $0 \le u \le 1,\ 0 \le v \le 2\pi$

6. a) Find the fourier series of $f(x) = x + |x|$ for $-\pi < x < \pi$. 7

b) Expand the function $f(x) = x^2$ in the interval $0 < x \le \pi$ in half range Fourier cosine series and show that $\frac{\pi^2}{6} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$ 8

7. Solve Any Two: 2×5

a) Find the general solution of $2u_x + 2u_y - u = 0$.

b) Derive the one-dimensional traffic flow model using conservation law.

c) Show that the value under integral sign $\int_{(4,0,3)}^{(-1,1,2)} [(yz + 1)dx + (xz + 1)dy + (xy + 1)dz]$ is exact and evaluate the integral.