POKHARA UNIVERSITY

Level: Bachelor | Semester: Fall | Year : 2023
Programme: BE | | Full Marks: 100
Course: Engineering Mathematics IV | | Pass Marks: 45
| | Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Define harmonic function. Check $u = x^3 - 3xy^2$ is harmonic or not? If yes, find corresponding harmonic conjugate v of u. 7

   b) State Cauchy Integral formula for derivative. Evaluate $\oint_c \frac{z^6}{(2z-1)^6} dz$, where c is the unit circle $|z| = 1$, counterclockwise. 8

a) Find the image of triangular region of the Z-plane bounded by the lines $x = 0, y = 0, x + y = 1$ under the transformation of $w = z\,e^{\frac{i\pi}{4}}$ and show the sketch in the diagram. 8

OR

Find the Laurent series for $f(z) = \frac{z+3}{z(z^2-z-2)}$ in the region

(i) $0 < |z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$

b) State Cauchy Residue Theorem. By applying Cauchy Residue Theorem, evaluate $\int_c \left(\frac{4-3z}{z(z-1)(z-2)}\right) dz$ where C: $|z| = \frac{3}{2}$. 7

a) State and prove first shifting theorem of Z transform. Using it evaluate the Z transform of $a^n \cos bt$ and $a^n \sin bt$. 7

b) Solve the difference equation by using Z-transform: 8

$y_{n+2} - 4y_{n+1} + 4y_n = 2^n$ with $y_0 = 0, y_1 = 1$

Show that $\int_0^\infty \frac{\sin \pi w \sin xw}{1-w^2} dw = \begin{cases} \frac{\pi}{2}\sin x & if \ 0 \le x \le \pi \\ 0 & if \ x > \pi \end{cases}$ 7

Find Fourier sine transform of $f(x) = e^{-x}$ for $x > 0$. Then prove that $\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m}$ for $m > 0$. 8

5. a) Derive one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ with necessary assumptions. 7

   b) Find u(x, t) from one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, with boundary condition. $u(0,t) = 0 = u(L,t)$, initial deflection $f(x)$ and initial velocity $\left.\frac{\partial u}{\partial t}\right|_{t=0} = g(x)$. 8

6. a) A homogeneous rod of conducting material of length 100cm has its ends kept at zero temperature and the temperature initially is $f(x) = \begin{cases} x, & 0 \le x \le 50 \\ 100-x, & 50 \le x \le 100 \end{cases}$ find the temperature distribution on the rod at any time. 7

   b) Express the Laplacian $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ in polar co-ordinates. 8

   OR

   Derive Helmholtz equation $F_{xx} + F_{yy} + FV^2 = 0$ and find its solution under boundary condition.

7. Attempt all the questions: 4×2.5

   a) Find tangent vector on the curve $\vec{r} = \cos t\vec{i} + 2\sin t\vec{j}$, at $P(\frac{1}{2}, \sqrt{3}, 0)$

   b) Find z–transform of $z(a^n)$

   c) Check analyticity of: $f(z) = z^3$

   d) Solve the partial differential equation $u_{yy} = u$.

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The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Define harmonic function. Check $u = x^3 - 3xy^2$ is harmonic or not? If yes, find corresponding harmonic conjugate v of u. 7

   b) State Cauchy Integral formula for derivative. Evaluate $\oint_c \frac{z^6}{(2z-1)^6} dz$, where c is the unit circle $|z| = 1$, counterclockwise. 8

2. a) Find the image of triangular region of the Z-plane bounded by the lines $x = 0, y = 0, x + y = 1$ under the transformation of $w = z\, e^{\frac{i\pi}{4}}$ and show the sketch in the diagram. 8

OR

$A = -\frac{3}{2}$

$B = \frac{2}{3}$

$C = \frac{5}{6}$

Find the Laurent series for $f(z) = \frac{z+3}{z(z^2-z-2)}$ in the region

(i) $0 < |z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$

   b) State Cauchy Residue Theorem. By applying Cauchy Residue Theorem, evaluate $\oint_c \left(\frac{4-3z}{z(z-1)(z-2)}\right) dz$ where C: $|z| = \frac{3}{2}$. 7

3. a) State and prove first shifting theorem of Z transform. Using it evaluate the Z transform of $a^n \cos bt$ and $a^n \sin bt$. 7

   b) Solve the difference equation by using Z-transform: 8

   $y_{n+2} - 4y_{n+1} + 4y_n = 2^n$ *with* $y_0 = 0, y_1 = 1$

4. 7

   a) Show that $\int_0^{\infty} \frac{\sin \pi w \sin xw}{1-w^2} dw = \begin{cases} \frac{\pi}{2} \sin x & \text{if } 0 \le x \le \pi \\ 0 & \text{if } x > \pi \end{cases}$

   b) Find Fourier sine transform of $f(x) = e^{-x}$ for $x > 0$. Then prove that $\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m}$ for $m > 0$. 8

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5\. a) Derive one dimensional heat equation $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$ with necessary assumptions.

b) Find u(x, t) from one dimensional wave equation $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$, with boundary condition. $u(0,t)=0=u(L,t)$, initial deflection $f(x)$ and initial velocity $\left.\frac{\partial u}{\partial t}\right|_{t=0}$ = g(x). 8

6\. a) A homogeneous rod of conducting material of length 100cm has its ends kept at zero temperature and the temperature initially is $f(x)=\begin{cases} x, & 0\le x\le 50 \\ 100-x, & 50\le x\le 100\end{cases}$ find the temperature distribution on the rod at any time. 7

b) Express the Laplacian $\nabla^2 u=\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}$ in polar co-ordinates. 8

OR

Derive Helmholtz equation $F_{xx}+F_{yy}+FV^2=0$ and find its solution under boundary condition.

7\. Attempt all the questions: 4×2.5

a) Find tangent vector on the curve $\vec{r}=\cos t\vec{i}+2\sin t\vec{j}$, at $P(\frac{1}{2},\sqrt{3},0)$

b) Find z–transform of $z(a^n)$

c) Check analyticity of: $f(z)=z^3$

d) Solve the partial differential equation $u_{yy}=u$.

Gandaki College of Engineering and Science
Lamachaur, Pokhara
Assessment Exam
Semester: Fall

Level: Bachelor
Program: B.E. Computer
Subject: Mathematics IV

Year: 2023
Full Marks: 100
Pass Marks: 45
Time: 3 hrs.

1. a) What do you mean by analyticity of function f(z). State Cauchy Riemann equation and hence show that it is the necessary condition for the function to be analytic. [8]

   b) State Cauchy integral formula.

   Evaluate $\int \frac{1}{z^2+4} dz$; where integration

   is along the ellipse $4x^2 + (y-2)^2 = 4$ [7]

2. a) Integrate $f(z) = \frac{e^z+z}{z^3-z} dz$ around a unit circle: $|z| = \frac{\pi}{2}$ using Cauchy Residue theorem. [8]

   b) Define bilinear transformation find the bilinear transformation which maps $z_1=0$, $z_2=1$, $z_3=\infty$ into $w_1=i$, $w_2=-1$, $w_3=-i$ [7]

3. a) Define fourier transform. Find the fourier transform of $f(x) = e^{\frac{-x^2}{2}}$ [8]

   b) Choosing a suitable function show that [7]

$$\int_0^{\infty} \frac{(\cos xw + w \sin xw)}{1+w^2} dw = \begin{cases} 0 & if \quad x < 0 \\ \frac{\pi}{2} & if \quad x = 0 \\ \pi e^{-x} & if \quad x > 0 \end{cases}$$

4. a) State and prove first shifting theorem of z-transform. using it evaluate the z-transform of $a^n \cos bt$ and $a^n \sin bt$. [8]

b) Solve the difference equation by using z-transform [7]

$y_{n+2} - 3y_{n+1} + 2y_n = 4^n$ with $y_0 = y_1 = 1$

5. a) State one dimensional heat equation and then obtain its solution with necessary assumptions. [8]

b) A tightly stretched string with fixed ends at $x = 0$ and $x = L$ is initially at rest in its equilibrium position. Find $u(x,t)$ if it is set vibrating by giving to each of its points a velocity $3(Lx-x^2)$ [7]

6. a) Express the Laplacian

$\nabla^2 u = \frac{d^2u}{dx^2} + \frac{d^2u}{dy^2}$ in polar co-ordinates. [8]

b) Find the temperature in a laterally insulated bar of length L whose ends are kept at temperature O, assuming that the initial temperature is

$$f(x) = \begin{cases} x & \text{if} \quad 0 < x < \frac{L}{2} \\ L - x & \text{if} \quad \frac{L}{2} < x < L \end{cases}$$ [7]

7. Attempt all [4x2.5=10]

a) Write the equation of hyperboloid of one sheet and then sketch.

b) Find the unit tangent vector to the curve:

$\vec{r}(t) = \cosh t \ \vec{i} + \sinh t \ \vec{j}$ at P $(\frac{5}{3}, \frac{4}{3}, 0)$

c) Evaluate z inverse of $f(z) = \frac{z}{(z-1)(z-2)}$

d) Verify the given function to satisfy two dimensional Laplace equation:

$u = \tan^{-1}(\frac{y}{x})$.

POKHARA UNIVERSITY

Level: Bachelor Semester: Spring Year : 2021
Programme: BE Full Marks: 100
Course: Engineering Mathematics IV Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Define harmonic function. Prove that the function $v = \arg z$ is harmonic. Also, find its conjugate and the corresponding analytic function. 8

   b) State Cauchy's integral formula and using it integrate $\oint_c \frac{z^2}{(z^4-1)} dz$ where c is the circle |z+i|=1 in counter clockwise 7

2. a) Find the image of triangular region of the z-plane bounded by the lines x = 0, y = 0 and $\sqrt{3}\,x + y = 1$ under the transformation of w = $e^{i\pi/3}$z and show the sketch in the diagram. 7

   b) Define Singularities of a function f(z). Find the residues of $f(z) = \frac{z+2}{(z+1)(z^2+1)^2}$. 8

OR

Find Laurent series of the function $f(z) = \frac{z^2-1}{z^2+5z+6}$ in the region i) when $|z| < 2$ ii) when $2 < |z| < 3$ and iii) when $|z| > 3$.

3. a) State and prove second shifting theorem of z-transform. Find z-transform of e^{-iat} and hence find Z(cos at) 7

   b) Use z-transform to solve the difference equation: $y_{n+2} - 3y_{n+1} + 2y_n = 4''$, where $y_0 = 0$ and $y_1 = 1$. 8

4. a) Using Fourier cosine integral, show that 7

$$\int_0^{\infty} \frac{\sin\omega \cos\omega x \, d\omega}{\omega} = \begin{cases} \pi/2 & if\ 0 \le x < 1 \\ \pi/4 & if\ x = 1 \\ 0 & if\ x > 1 \end{cases}$$

b) Find Fourier cosine transform of $f(x) = e^{-mx}$ for m > 0. Then prove that $\int_0^{\infty} \frac{\cos kx}{1+x^2}\, dx = \frac{\pi}{2} e^{-k}$ 8

5. a) A tightly stretched string of length L, fixed at its ends, is initially in a position given by $u(x,0) = u_0 \sin^3\left(\frac{\pi x}{L}\right)$. If it is released from the rest from this position, find the displacement at any point x at time t. 8

b) Find the temperature in a laterally insulated bar of length L=10cm whose ends are kept at a zero temperature, assuming that the initial temperature is $f(x) = \begin{cases} x & if\ 0 < x < \frac{L}{2} \\ L - x & if\ \frac{L}{2} < x < L \end{cases}$ 7

6. a) Find the solution of differential equation $y^2 u_x - x^2 u_y = 0$ using separating of variables. 7

b) Find the solution of one-dimensional wave equation by D'Alembert's method. 8

OR

Express the Laplacian $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ into polar co-ordinates.

7. Attempt all questions:

a) Find a tangent vector and the corresponding unit tangent vector $\vec{u(t)}$ at a given point $r(t) = 2\cos t.\vec{i} + \sin t.\vec{j}$ at P $(\sqrt{2}, \sqrt{2}, 0)$ 2.5

b) Check analyticity of $f(z) = z^2$ 2.5

c) Find the poles of the function $f(z) = \frac{\sinh z}{(z - i\pi)}$. 2.5

d) Find z–transform of $Z(n^2)$ 2.5

POKHARA UNIVERSITY

Level: Bachelor | Semester: Spring | Year : 2023
Programme: BE | Full Marks: 100
Course: Engineering Mathematics IV | Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Define harmonic function. Is a function $v = 2xy - \frac{y}{x^2+y^2}$ harmonic? If yes, find a corresponding harmonic conjugate and the analytic function. 8

   b) State Cauchy integral formula for derivative. Evaluate $\oint_c \frac{z^6}{(2z-1)^6} dz$, where c is the unit circle $|z| = 1$, counterclockwise. 7

2. a) State Laurent's theorem. Find Laurent's series for 7

$$f(z) = \frac{1}{(z - z^3)}$$ in the region $1 < |z+1| < 2$.

OR

Find the image of infinite strip $\frac{1}{4} < y < \frac{1}{2}$ under the transformation $\omega = \frac{1}{z}$

   b) Define singularity, zeros, and poles of a function. Evaluate $\oint_c f(z)dz$ where $f(z) = \frac{e^{2z}}{(z+1)^3}$ where c is the ellipse $4x^2+9y^2=16$. 8

3. a) State and prove initial theorem and find the inverse z-transform of 8

$$F(z) = \frac{z^2 - 3z}{(z-5)(z+2)}$$

   b) Use z-transform to solve 7

$$y_{n+2} + 6y_{n+1} + 9y_n = 2^n \text{ given } y_0 = y_1 = 0$$

4. a) Define Fourier integral. By choosing a suitable function, show that 7

$$\int_0^\infty \left(\frac{\cos xw + w\sin xw}{1+w^2}\right) dw = \begin{cases} 0 & if\ x < 0 \\ \frac{\pi}{2} & if\ x = 0 \\ \pi e^{-x} & if\ x > 0 \end{cases}$$

   b) Find the Fourier transform of the function $f(x) = e^{\frac{-x^2}{2}}$ 8

5. a) Derive the one dimensional wave equation with required assumptions. 7

OR

Define partial differential equation. By separating variables solve

i. $u_{xy} - u = 0$

ii. $xu_{xy} + 2yu = 0$

   b) Find the temperature distribution in a laterally insulated thin copper bar ($c^2 = 1.158\ cm^2/sec$) 100 cm long and of constant cross-section whose end points at $x = 0$ and $x = 100$ are kept at $0°C$ and its initial temperature is $f(x) = \sin(0.01)\pi x$. 8

6. a) Find the temperature in a laterally insulated bar of length π whose ends are kept at a zero temperature, assuming that the initial 8

temperature is $f(x) = \begin{cases} x & if\ 0 < x < \frac{\pi}{2} \\ \pi - x & if\ \frac{\pi}{2} < x < \pi \end{cases}$

   b) Express the Laplacian $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ in polar coordinates. 7

7. Write short notes on: (**Any two**) 2.5×4=10

   a) Show that $z\bar{z}$ is not an analytic function.

   b) Find Z- transform of $\sin(\frac{n\pi}{2})$ *and* $\cos(\frac{n\pi}{2})$

   c) solve $u_{xx} - u_{yy} = 0$

   d) Write equation of an ellipsoid. Sketch it with centre and axis of symmetry.

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Course: Engineering Mathematics IV | Pass Marks: 45
Time : 3hrs.

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The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Define Laplace equation and harmonic function. Determine a and b such that $u = ax^3 + by^3$ is harmonic and also find the harmonic conjugate. 7

   b) State and prove Cauchy Integral Formula. Evaluate the integral $\oint_c \left(\frac{e^{5z}}{(z+i)^4}\right) dz$, where c: $|z| = 2$ 8

2. a) Show that the transformation $w = \frac{2z+3}{z-4}$ maps the circle $x^2 + y^2 - 4x = 0$ on to the straight line $4u + 3 = 0$ in w plane. 7

   b) Find the series expansion of the function $f(z) = \frac{7z-2}{(z+1)z(z-2)}$ in the regions by using Laurentz series. 8

   i) $0 < |z+1| < 1$ *ii)* $1 < |z+1| < 3$.

3. a) State and prove first shifting theorem of Z-transform. Using it find $Z[te^{bt}]$ 7

   b) Solve the difference equation by using Z transform: 8

   $$y_{n+2} - 4y_{n+1} + 4y_n = 2^n, \text{where } y_0 = 0, y_1 = 1.$$

4. a) Show that $\int_0^\infty \frac{w \sin xw}{a^2 + w^2} dw = \frac{\pi}{2} e^{-ax}$ *where* $x > 0, a > 0$ 7

   b) Find Fourier cosine transform of $f(x) = e^{-mx}$ for m> 0. Then prove that $\int_0^\infty \frac{\cos kx}{1+x^2} dx = \frac{\pi}{2} e^{-k}$ 8

5. a) Find the solution of the differential equation $y^2 u_x - x^2 u_y = 0$ by using separating of variables. 7

b) Find the temperature in a laterally insulated bar of length L = 20cm whose ends are kept at a zero temperature, assuming that the initial temperature is $f(x) = \begin{cases} x & \text{if } 0 < x < \frac{L}{2} \\ L - x & \text{if } \frac{L}{2} < x < L \end{cases}$ 8

6. a) Express the Laplacian $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ in polar co-ordinates. 7

b) Derive the solution of one dimensional wave equation for a vibrating sting.by using D Alembert's method. 8

7. Attempt all questions 2.5×4

a) Check analyticity of $f(z) = z^2$

b) Show that the Z transform is linear operator.

c) Solve the partial differential equation $u_{xx}+9u=0$.

d) Find the unit tangent vector to the curve

$$\vec{r}(t) = 2\cos t\vec{i} + \sin t\vec{j} \quad at \quad (\sqrt{2}, \sqrt{2}, 0).$$

POKHARA UNIVERSITY

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Course: Engineering Mathematics IV | | Pass Marks: 45
| | Time : 3hrs.

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The figures in the margin indicate full marks.

Attempt all the questions.

1.

a) Define harmonic function. Is a function $v = 2xy - \dfrac{y}{x^2+y^2}$ is harmonic? If yes, find a corresponding harmonic conjugate and the analytic function. 8

b) State Cauchy Integral formula for derivative. Evaluate $\oint_c \dfrac{z^6}{(2z-1)^6}$ dz, where c is the unit circle $|z|=1$, counterclockwise 7

2.

a) Integrate f(z) = $\dfrac{e^z+z}{z^3-z}dz$ around a unit circle : $|z| = \dfrac{\pi}{2}$ using Cauchy's Residue theorem. 7

b) Define a bilinear transformation. Find the bilinear transformation which maps the points $z = 0, -1, i$ onto the points $w = i, 0, \infty$. Also find image of the unit circle $|z| = 1$. 8

3. a) Define Fourier integral. Choosing a suitable function, show that 7

$$\int_0^\infty \frac{\sin \pi\omega}{\omega} \sin wx\, dw = \begin{cases} \dfrac{\pi \sin \pi x}{2} & if \quad 0 \le x \ge \pi \\ 0 & if\ x > \pi \end{cases}.$$

b) Find the Fourier Transform of the function $f(x) = e^{\frac{-x^2}{2}}$ 8

4. a) State and prove first shifting theorem of Z transform. Using it evaluate the Z transform of $a^n \cos bt$ and $a^n \sin bt$. 7

b) Solve the difference equation by using Z-transform: 8

$y_{n+2} - 3y_{n+1} + 2y_n = 4^n \; with \; y_0 = y_1 = 1$

5. a) Derive one dimensional wave equation with solution. 8

b) A tightly stretched string of length L, fixed at its ends, is initially 7

in a position given by $u(x,0) = u_0 \sin^3\left(\frac{\pi x}{L}\right)$. If it is released from

the rest from this position, find the displacement.

6. a) A homogeneous rod of conducting material of length 100cm has 7
its ends kept at zero temperature and the temperature initially is

$f(x) = \begin{cases} x, \; 0 \le x \le 50 \\ 100 - x, \; 50 \le x \le 100 \end{cases}$ find the temperature distribution

on the rod at any time.

b) Express the Laplacian $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ in polar co-ordinates. 8

7. Attempt all questions: 10

a) Find tangent vector on the curve $\vec{r} = \cos t\vec{i} + 2\sin t\vec{j}$, at $P(\frac{1}{2}, \sqrt{3}, 0)$

b) Verify that $u = x^2 + t^2$ is a solution of one dimensional wave equation.

c) Express f(z) = sinhz in terms of u+iv.

d) Solve $u_{xx} - u = 0$ by using separation of variables

POKHARA UNIVERSITY

Level: Bachelor Semester: Fall Year : 2020
Programme: BE Full Marks: 100
Course: Engineering Mathematic IV Pass Marks: 45
Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Check u = sinx coshy is harmonic or not? If yes, find corresponding harmonic conjugate v of u 8

   b) Evaluate $\oint_C \frac{\cot z}{\left(z-\frac{\pi}{2}\right)^2} dz$, where C is the ellipse $4x^2 + 9y^2 = 36$. 7

2. a) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent series valid for 7

   $(i)\ 1 < |z| < 3\ (ii)\ |z| > 3\ (iii)\ |z| < 1\ (iv)\ 0 < |z+1| < 2$

   b) State and prove Cauchy residue theorem. Using it evaluate $\int_C \left(\frac{z^2 \sin z}{4z^2 - 1}\right) dz$ where C is the circle $|z| = 2$. 8

3. a) Find the Z transform of (i) $r^n \cos n\theta$ (ii) $\frac{1}{n+2}$. 7

   b) Solve the differential equation $y_{k+2} + 2y_{k+1} + y_k = k$ where $y_0 = 0$, $y_1 = 0$ using Z-transform. 8

4. a) Find the solution of the differential equation, $y^2u_x - x^2u_y = 0$, by using separating of variables. 7

   b) Find the solution of one dimensional equation with boundary $u(0, t) = 0 = u(l, t)$ and initial condition $u(x, 0) = \left(\frac{100x}{l}\right)$. 8

5.

a) Derive one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ with necessary assumptions. 7

b) What is Helmboltz's equation on F(x, y) and solve it subjected to F(0, y) = 0 = F(x, 0) = F(x, b). 8

"OR"

Find the deflection u(x, y, t) of the square membrane with a= b = 1 and c = 1, if the initial velocity is zero and the initial deflection is (0, 1). Sin3 π xsin4 π y.

6.

a) Find the Fourier transform of $f(x) = \begin{cases} |x| & \text{if } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$ 7

b) Define Fourier cosine integral. Hence, show that 8

$$\int_0^\infty \frac{\sin\omega \cos\omega x}{\omega} d\omega = \begin{cases} \pi/2 & \text{if } 0 \le x < 1 \\ \pi/4 & \text{if } x = 1 \\ 0 & \text{if } x > 1 \end{cases}$$

7. Write short notes on: 10

a) Write down Laplacian in cylindrical coordinate systems.

b) Prove Z(a^n) = $\frac{z}{z-a}$

c) Show that the transformation w = e^z is conformal

d) Show that the fourier cosine transform satisfied linearity property.