

POKHARA UNIVERSITY

Level: Bachelor
 Programme: BE
 Course: Numerical Methods (Old)

Semester: Fall

Year : 2023
 Full Marks: 100
 Pass Marks: 45
 Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Find where the graph of $y = x - 3$ and $y = \ln(x)$ intersect using bisection method. Get the intersection value correct to four decimal places. 8
 - b) Find value of $\sqrt{18}$ using Newton Raphson method. 7
 2. a) Find the root of the equation $3x + \sin x - e^x = 0$ corrected up to 3 decimal places using Secant Method. 7
 - b) Given populations of a city in different year as shown in table below. 8
- | | | | | | |
|---------------------------------------|------|------|------|------|------|
| x (in year) | 1891 | 1901 | 1911 | 1921 | 1931 |
| f(x)(population of that year in 0000) | 46 | 66 | 81 | 93 | 101 |
- Observe the data carefully and answer the following questions:
- i. Which method is applicable for finding population in the year 1860 and why? Also find the population of that year.
 - ii. If you are asked to find the change in population between the year 1925 to 1930, how will you find? (mention formula and steps)
3. a) Fit a power function model of the form $y = ax^b$ from the given data points. 8
- | | | | | | |
|---|-----|---|-----|---|------|
| x | 1 | 2 | 3 | 4 | 5 |
| y | 0.5 | 2 | 4.5 | 8 | 12.5 |
- b) Use Romberg method to evaluate: 7
- $$\int_0^{3\pi/2} e^x \sin x \, dx$$
4. a) Solve the following system of equations using partial pivoting method. 8
- $$x + y + z = 4, \quad x + 4y + 3z = 8, \quad x + 6y + 2z = 6$$

- b) Obtain the solution of following system using Gauss-Seidel iteration method. 7

$$2x_1 + x_2 + x_3 = 5$$

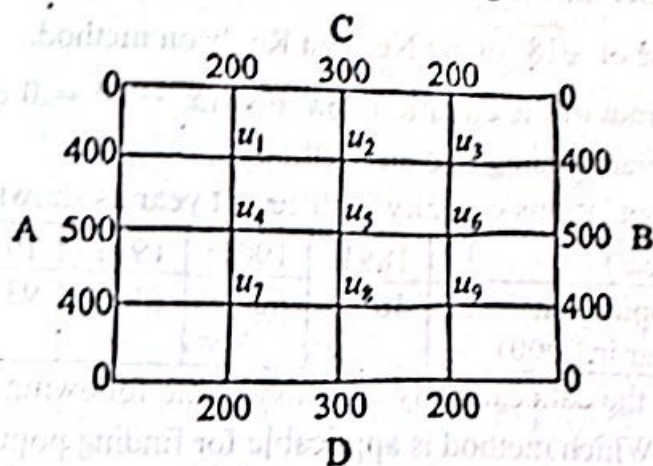
$$3x_1 + 5x_2 + 2x_3 = 15$$

$$2x_1 + x_2 + 4x_3 = 8$$

5. a) Determine the largest eigenvalue and the corresponding eigenvector of the matrix: 7

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix} \text{ using the power method.}$$

- b) Solve the Laplace equation $U_{xx} + U_{yy} = 0$ for the square mesh with boundary values as shown in the figure. 8



6. a) Solve the following equation for $y(0.2)$ using shooting method: 9

$$\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} - 3y = 6x \text{ Given } y(0) = 0, y'(0) = 1.$$

- b) Use Picard's method to approximate the value of y when $x=0.1, x=0.2$ and $x=0.4$, given that $y=1$ at $x=0$ and $dy/dx = 1+xy$ correct to three decimal places (Use upto second approximations). 6

7. Write short notes on: (Any two) 2x5

- Newton's backward differentiation formulas
- Partial differential equation and their examples
- Algorithm of fixed point iteration method

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Explain in brief the errors in numerical calculations. 5
b) Find a root of $3x + \sin x - e^x = 0$ using 10

- i. One of the bracketing methods and
ii. One of the non-bracketing methods

2. a) From the data given below, find the number of students whose weight is between 60 to 70. 8

Weight in lbs	0-40	40-60	60-80	80-100	100-120
No. of students	250	120	100	70	50

- b) Using the method of least squares, fit the curves $ax^2 + \frac{b}{x}$ to the following data. 7

x	1	2	3	4
y	-1.52	0.96	8.88	7.66

3. a) Use Romberg's method, to compute $\int_0^2 \frac{e^x + \sin x}{1+x^2} dx$ correct up to two decimal places. 8
b) Estimate approximate derivative of $f(x) = x^2$ at $x=1$ for $h=0.1, 0.2, 0.05, 0.01$. Use first order difference method and find the respective error. 7

4. a) Apply the factorization method to solve the equation $3x+2y+7z=4$; $2x+3y+z=5$; $3x+4y+z=7$ 8
b) Using SOR method, solve the following system of 7
 $4x+y+2z=4$; $3x+5y+z=7$; $x+y+3z=3$.
5. a) Find the largest eigen value and the corresponding eigen vector of the matrix 8
 $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ using power method
b) Using the R-K 1st order method, find an approximate value of y corresponding to $x=1$, given that $\frac{dy}{dx} = \frac{y-x}{y+x}$ and $y=1$. When $x=0$, and $h=0.02$. 7
6. a) Using the R-K method of fourth order, solve for y at $x=1.2, 1.4$, from $\frac{dy}{dx} = \frac{2xy+e^x}{x^2+xe^x}$ given $x_0=1, y_0=0$. 8
b) Solve the elliptic equation $U_{xx} + U_{yy} = 0$ over a square mesh of side four units satisfying the following boundary conditions; $u(0, y) = 0$ for $0 \leq y \leq 4$; $u(4, y) = 12 + y$ for $0 \leq y \leq 4$; $u(x, 0) = 3x$ for $0 \leq x \leq 4$; $u(x, 4) = x^2$ for $0 \leq x \leq 4$ 7
7. Write short notes on: (Any two) 2x5
a) Shooting Method
b) Algorithm of Gauss Jordan method
c) Algorithm of fixed point iteration method

POKHARA UNIVERSITY

Level: Bachelor

Semester: Fall

Year : 2022

Programme: BE

Full Marks: 100

Course: Numerical Methods

Pass Marks: 45

Time : 3hrs.

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The figures in the margin indicate full marks.

Attempt all the questions.

1. a) Find the root of the equation $f(x) = x^2 - 4x - 10$ correct to three decimal places by using False Position method. 7

- b) Estimate the root of the equation $f(x) = xe^x - \cos x$ using Newton Raphson method correct to three decimal places. 8

2. a) From the following table estimate the number of student who obtained marks between 40 and 45. 7

Marks	30-40	40-50	50-60	60-70
No of Students	31	42	51	35

- b) From the following data given in the table below evaluate $f'(2.5)$ by using Lagrange method. 8

x	1	2	4	5	7
f(x)	1	1.414	1.732	2.00	2.6

3. a) Evaluate $\int_1^5 \frac{1}{x} dx$ by using Gaussian Integration formula for $n=3$ and compare the value with exact solution. 7

- b) Use the Romberg integration to find the solution correct upto three decimal places. 8

$$I = \int_0^1 \frac{1}{1+x^2} dx$$

4. a) Find the solution of the given simultaneous linear equation using Gauss Seidel method. 7

$$6x_1 - 2x_2 + x_3 = 11$$

$$-2x_1 + 7x_2 + 2x_3 = 5$$

$$x_1 + 2x_2 - 5x_3 = -1$$

- b) Solve the following system of equations using Crout method.
 $x + y + z = 4, x + 4y + 3z = 8, x + 6y + 2z = 6$
5. a) Using the Euler's (R-K 1st order method) find an approximate value of y corresponding to $x=1$, given that $dy/dx = X+Y$ and $y=1$ when $x=0, h=0.1$.
- b) Apply Euler's method to approximate value of $y(0.3)$ for the differential equation:
 $\frac{dy}{dx} = y + x, y(0) = 1$.
6. a) Torsion on a square bar of size $15\text{cm} \times 15\text{cm}$. If two of the sides are held at 100°C and the other two sides are held at 0°C . Calculate the steady state temperature at interior points. Assume a grid size of $5\text{cm} \times 5\text{cm}$.
- b) Solve the Poisson equation $\nabla^2 f = 2x^2 + y$, over the square domain $1 \leq x \leq 4, 1 \leq y \leq 4$, with $f=0$ on the boundary. Take step size in x and $y, h=k=1$.
7. Write short notes on: (Any two)
- Ill-conditioned and Well-conditioned systems
 - Error in Numerical method
 - Cubic Spline.

POKHARA UNIVERSITY

Level: Bachelor
Programme: BE
Course: Numerical Methods

Semester: Spring

Year : 2021
Full Marks: 100
Pass Marks: 45
Time : 3 hrs.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all the questions.

1. a) What is the difference between the bisection method and false position even though both are bracketing methods? Find the real root of the given non-linear equation correct up to three decimal place using Newton Raphson method. 7

- b) Define error and write its different types in numerical methods with examples. If $x = 1.350253$ is rounded off to four significant digits, find the absolute and relative errors. 8

2. a) By using Least square method find the straight line that best fit the following data: 7

x:	1	2	3	4	5
y:	14	27	40	55	68

- b) Find the cubic spline interpolation formula for the following data: 8

x	1	2	3	4	5
f(x)	1	0	1	0	1

3. a) Evaluate $\int_4^{5.2} \log x \, dx$ from the following data 8

x	4.0	4.2	4.4	4.6	4.8	5.0	5.2
y	1.3863	1.4351	1.4816	1.5261	1.5686	1.6094	1.6487

by using

- Trapezoidal Method
- Simpson 1/3 Method
- Simpson 3/8 Method

- b) Evaluate $\int_0^2 \frac{x^2+2x+1}{1+(x+1)^4} dx$ by using Gaussian Integration formula for $n=2, n=3$ and compare their values with exact solution. 7
4. a) Solve the following system of equations by using relaxation method correct to two decimal places. 8
- $$\begin{aligned} 9x - y + 2z &= 9 \\ x + 10y - 2z &= 15 \\ 2x - 2y - 13z &= -17 \end{aligned}$$
- b) Using Dollittle LU decomposition method, solve the following system equations: 7
- $$\begin{aligned} 3x + 2y + z &= 10 \\ 2x + 3y + 2z &= 14 \\ x + 2y + 3z &= 14 \end{aligned}$$
5. a) Use Runge-kutta of order four to find the solution of the given differential equation at $x=1.5$ taking a step size of $h=0.25$. 8
- $$\frac{dy}{dx} + 2y = x^2, y(1) = 5$$
- b) Find the solution of the given ordinary differential equation at $x=0.5$ using the step size of $h=0.25$ using Heun's method. 7
- $$\frac{dy}{dx} + 0.4y = 3e^{-x}, y(0) = 5$$
6. a) Determine the steady-state heat distribution in a thin square metal plate with dimensions 0.5 m by 0.5 m using $n = m = 4$. Two adjacent boundaries are held at 0°C , and the heat on the other boundaries increases linearly from 0°C at one corner to 100°C where the sides meet. 7
- b) The following table gives the corresponding values of pressure and specific volume of superheated steam. 8
- | | | | | |
|---|-----|-------|------|------|
| V | 2 | 4 | 6 | 8 |
| P | 105 | 42.07 | 25.3 | 16.7 |
- (i) Find the rate of change of pressure with respect to volume when $V=2$.
(ii) Find the rate of change of volume with respect to pressure when $P=105$.
7. Write short notes on: (Any two) 2x5
- Ill-conditioned systems
 - Laplacian equation
 - Classification of Second Order Partial Differential Equation

POKHARA UNIVERSITY

Level: Bachelor
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Course: Numerical Methods

Semester: Fall

Year : 2021
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

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Attempt all the questions.

1. a) Solve $x^3 + x^2 - 3x - 3 = 0$ by secant method up to 8th iteration. Assume that the error should be less than 10^{-4} . 7
- b) Find the root of the equation $\log x - \cos x = 0$ correct to three decimal places by using N-R method. 8

2. a) Define interpolation. From the following table, estimate the number of students who passed marks between 40 and 45: 8

Marks	: 30-40	40-50	50-60	60-70	70-80
No. of students	: 30	40	50	38	31

- b) Fit cubic polynomial equations to the given data set and find the value of $f(3.7)$ and $f'(7.5)$. 7

X	2	4	7	9
f(X)	1	2	1	2

3. a) Integrate the following function by using Trapezoidal Rule, Simpson's 8

$\frac{1}{3}$ rule and Simpson's $\frac{3}{8}$ rule. Take $n = 6$. $\int_0^{\frac{\pi}{2}} \sin x \, dx$

- b) Integrate the given integral $\int_0^{\pi/2} \frac{\cos x}{\sqrt{1 + \sin x}} \, dx$ 7

Using Gauss quadrature Formula for $n=2$ and $n=3$

4. a) Find the inverse of the matrix, using Gauss Jordan method.

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 3 & -5 \\ -2 & -4 & -4 \end{bmatrix}$$

- b) Find the largest Eigen-value and the corresponding Eigen-vector of the following square matrix using Power method.

$$\begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & 4 \end{bmatrix}$$

5. a) Solve the following set of equations by using LU Crout method
- $$\begin{aligned} 3x + 2y + z &= 10 \\ 2x + 3y + 2z &= 14 \\ x + 2y + 3z &= 14 \end{aligned}$$
- b) Apply R-K-4 method to solve $y(0.2)$ for the given equation
 $\frac{d^2y}{dx^2} + x \frac{dy}{dx} - y$ given that $y=1$ and $\frac{dy}{dx} = 0$ when $x=0$.
6. a) In a square bar with dimension of 3 inch \times 3 inch, torsion function, ϕ , can be obtained from the following P.D.E: $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = -2$ where $\phi = 0$ on the outer boundary of the bar's cross-section. Subdivide the region into nine equal squares to form a mesh and find the values of ϕ in the interior nodes.
- b) Consider second order initial value problem $y'' - 4y' + 2y = e^t \sin(t)$ with $y(0) = 0.4$ and $y'(0) = -0.6$, using Heun's find value of $y(0.2)$ and $y'(0.2)$.
7. Write short notes on: (Any two)
- Taylor's series for solving ODE
 - Ill-Conditioned System
 - Classify the partial differential equation $U_{xx} + 2U_{xy} + U_{yy} = 0$

POKHARA UNIVERSITY

Level: Bachelor
 Programme: BE
 Course: Numerical Methods

Semester: Fall

Year : 2020
 Full Marks: 100
 Pass Marks: 45
 Time : 3hrs.

Candidates are required to give their answers in their own words as far as practicable.

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Attempt all the questions.

1. a) Find the positive real root of the equation $\cos x + e^x + x^2 = 3$ using False position method, correct to 3 decimal places. 7
 b) Discuss the drawbacks of Newton – Raphson method. Find the real root of the equation $x \sin x - \cos x = 0$ using Newton – Raphson method, correct to 3 decimal places. 8
2. a) From following experimental data, it is known that the relation connects v and t as $v = at^b$. Find the possible values of a and b . 8

V	350	400	500	600
T	61	26	7	2.6

 b) The following table gives the viscosity of oil as the function of temperature. Use Lagrange's interpolation formula to find the viscosity of oil at a temperature of 140 degree Celsius. 7

T(degCelsius)	110	130	160	190
Viscosity	10.8	8.1	5.5	4.8

3. a) Integrate the given integral using Romberg integration. 8

$$\int_1^2 \frac{1}{1+x^3} dx$$

10/14

- b) Compute the integral using Gaussian 3 - point formula.

$$\int_2^5 \frac{e^x + \sin x}{1 + x^2} dx$$

4. a) Solve the equation by Relaxation method:

$$9x - y + 2z = 9, x + 2y - 2z = 15, 2x - 2y - 13z = -17$$

- b) Determine the largest eigenvalue and the corresponding eigenvector of

the matrix: $A = \begin{bmatrix} 1 & 4 & 4 \\ 4 & 1 & 8 \\ 4 & 8 & 1 \end{bmatrix}$ using the power method

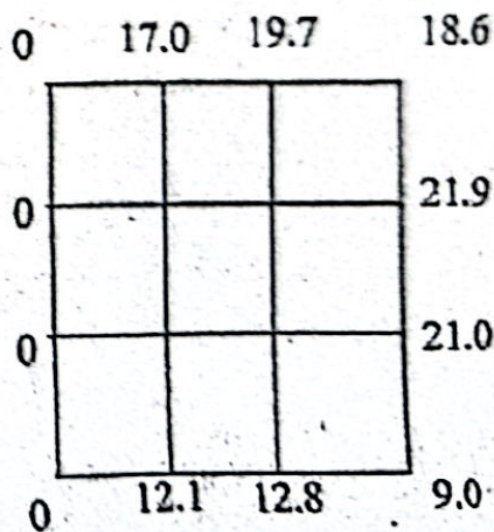
5. a) Apply Runge - Kutta fourth order method to find an approximate value of y when $x = 0.3$ give that: $y' = 2.5y + e^{0.3x}$; with an initial $y(0) = 1$, taking $h = 0.3$

- b) Solve the Boundary Value Problem (BVP) using Shooting method by dividing into four sub-interval employing Euler's method.

$$y'' + 2y' - y = x$$

subject to boundary condition $y(1) = 2$ and $y(2) = 4$

6. a) Solve the Poisson equation $\nabla^2 f = 4x^2y + 3xy^2$, over the square domain $x \leq 3, 1 \leq y \leq 3$, with f on the boundary is given in figure below. Take $h = k = 1$



POKHARA UNIVERSITY

Level: Bachelor
Programme: BE
Course: Numerical Methods

Semester: Spring

Year : 2019
Full Marks: 100
Pass Marks: 45
Time : 3hrs.

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Attempt all the questions.

1. a) Using Secant method, find the zero of function $f(x) = 2x - \log_{10} x - 7$ correct up to three decimal places. 8
- b) Find the root of the equation $\log x - \cos x = 0$ correct to three decimal placed by using N-R method. 7
2. a) The voltage v across a capacitor at time t seconds is given in following table. 8

Time t (sec)	0	2	4	6	8	4
voltage v	150	63	28	12	5.6	124

If the relationship between voltage v and time t is of the form $v = ae^{kt}$. Using least-square approximation estimate the temperature at $t=2.6$ minute.

- b) From the following table, estimate the number of students who obtained marks between 40 and 45. 7

Marks:	30-40	40-50	50-60	60-70	70-80
No. of students	31	42	51	35	31

3. a) The following table gives the velocity of a vehicle at various points of time. 7

Time, t (seconds)	1	2	4	5
Velocity, v (m/sec)	0.25	1	2.2	4

Find the acceleration of the vehicle at $t = 1.1$ second and $t = 2.5$ second using any suitable differential formula.

- b) Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin u}{u} du$ by using Trapezoidal, Simpson's 1/3 and 3/8 rule with $n=6$ 8

4. a) Determine the largest Eigen value and corresponding Eigenvector for the matrix using power method correct up-to 3 decimal places. 7

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

- b) Obtain the solution of the following system using the Dolittle LU decomposition method. 8

$$3x_1 + 2x_2 + x_3 = 10$$

$$2x_1 + 3x_2 + 2x_3 = 14$$

$$x_1 + 2x_2 + 3x_3 = 14$$

5. a) Solve the given differential equation by Heun's method $y'' - y' - 2y = 3e^{2x}$ with initial condition $y(0) = 0, y'(0) = -2$, for $y(0.2)$ taking $h = 0.1$ 8

- b) Solve: $y' = y + e^x, y(0) = 0$ for $y(0.2)$ and $y(0.4)$ by RK-4th order method. 7

6. a) Solve the poisson's equation $U_{xx} + U_{yy} = 243(x^2 + y^2)$ over a square domain $0 \leq x \leq 1, 0 \leq y \leq 1$ with step size $h = \frac{1}{3}$ with $u = 100$ on the boundary. 8

- b) Use Gauss-Legendre 2-point and 3-point formula to evaluate $\int_{0.5}^{1.5} e^{x^2} dx$ 7

7. Write short notes on: (Any two)

- a) Ill conditioned Method
b) Algorithm of bisection method
c) Cubic splines

2x5

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Semester: Fall

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Attempt all the questions.

1. a) Find where the graph of $y = x-3$ and $y = \ln(x)$ intersect using bisection method. Get the intersection value correct to four decimal places. 8
- b) Find value of $\sqrt{18}$ using Newton Raphson method. 7
2. a) The function $y = f(x)$ is given at the points (7, 3), (8, 1), (9, 1) and (10, 9) Find the value of y for $x = 9.5$ using Lagrange Interpolation formula. 7
- b) The following table shows pressure and specific volume of dry saturated steam. 8

V	38.4	20	8.51	4.44	3.03
P	10	20	50	100	150

Fit a curve of the form: $PV^a = \beta$ by using least square method.

3. a) Evaluate $\int_{-2}^2 \frac{x}{x + 2e^x} dx$ by using Trapezoidal, Simpson's 1/3 and 3/8 rule with $n = 6$. 7
- b) Using three-point Gaussian Quadrature formula, evaluate $\int_0^1 \frac{dx}{(1+x)}$ 8
4. a) Find inverse of the matrix, using Gauss Jordan method. 8

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

- b) Determine the largest eigenvalue and the corresponding eigenvector 7

of the matrix: $A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$ using the power method.

5. a) Use Picard's method to approximate the value of y when $x = 0.1$, $x = 0.2$ and $x = 0.4$, given that $y = 1$ at $x = 0$ and $dy/dx = 1 + xy$ correct to three decimal places (Use upto second approximations). 7
- b) Using Runge- Kutta method of second order (RK-2), obtain a solution of the equation $y'' = y + xy'$ with initial condition $y(0) = 1$, $y'(0) = 0$ to find $y(0.2)$ and $y'(0.2)$, taking $h = 0.1$ 8
6. a) Use Relaxation method to solve the given systems of equations: 8
- $$\begin{aligned} 20x + y - 2z &= 17 \\ 3x + 20y - z &= 18 \\ 2x - 3y + 20z &= 25 \end{aligned}$$
- b) Given the Poisson's equation: $\Delta^2 f = 4x^2y^2$ over the square domain: $0 \leq x \leq 3$ and $0 \leq y \leq 3$ with Dirichlet boundary condition of $f(x, y) = 100$ and $h = k = 1$. Calculate the steady state temperatures at the interior nodes by using Gauss-Seidel method. Iterate until the successive values at any point is correct to two decimal places. 7
7. Write short notes on: (Any two) 2x5
- a) Errors in numerical computations
- b) Obtain divided difference table for the given data set:
- | | | | | |
|---|----|---|---|----|
| X | -1 | 2 | 5 | 7 |
| Y | -8 | 3 | 1 | 12 |
- c) Write an algorithm for Romberg's integration method