Attack of the Clones

ESC101: Fundamentals of Computing

Purushottam Kar

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Proof by induction is basically a *recurs*

We used the proof for the case n-1 to prove the case n

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- **Proof**: *Base case*: for n = 1 true by inspection
- *Inductive case*: (1 + … + n) = (1 + … + n-1) + n = (n-1)n/2 + n = n(n+1)/2 ☺

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- Fundamental result in computer science: Church-Turing thesis
- <u>Disadvantage</u>: loop solutions sometimes very difficult to write and read
- <u>Advantage</u>: loop solutions can be much faster than recursion solution

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Will see several examples of this in ESC101. More examples in advanced courses e.g. ESO207, CS345

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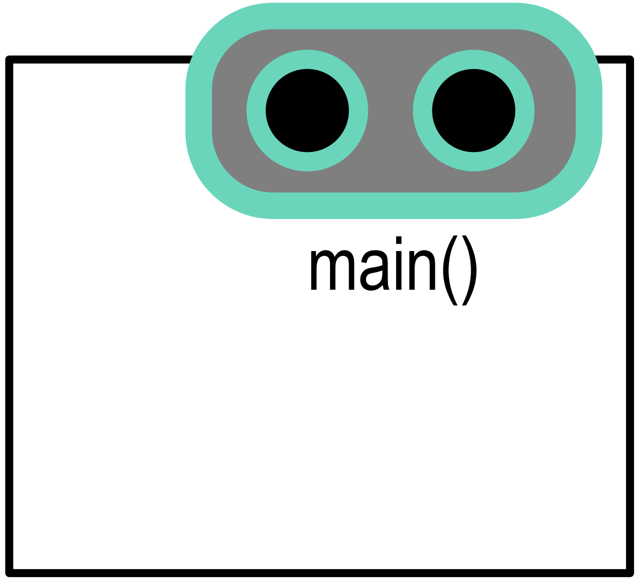

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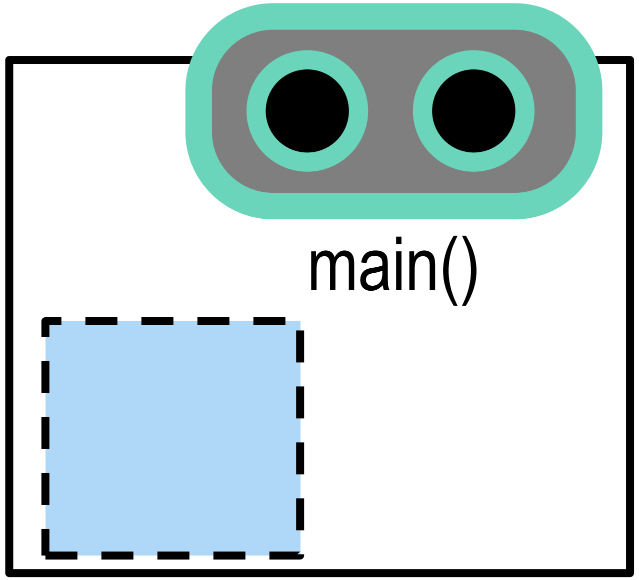

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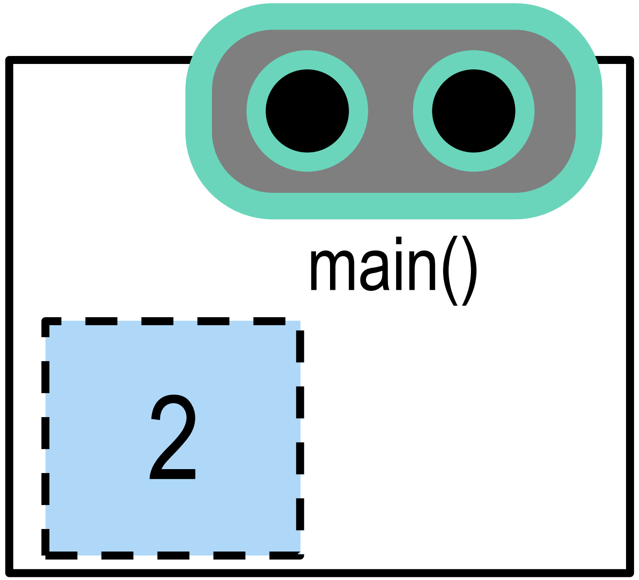

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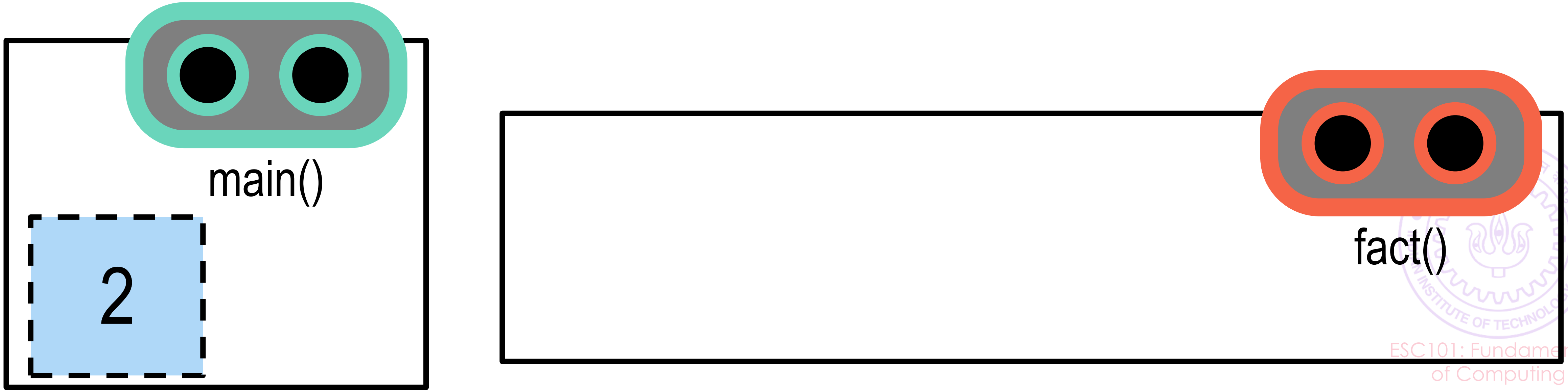

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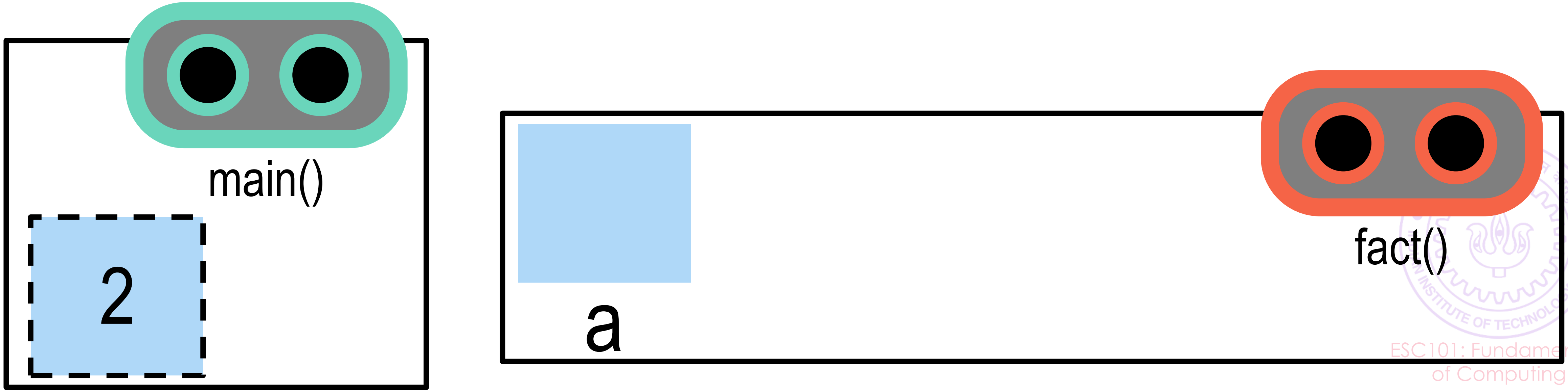

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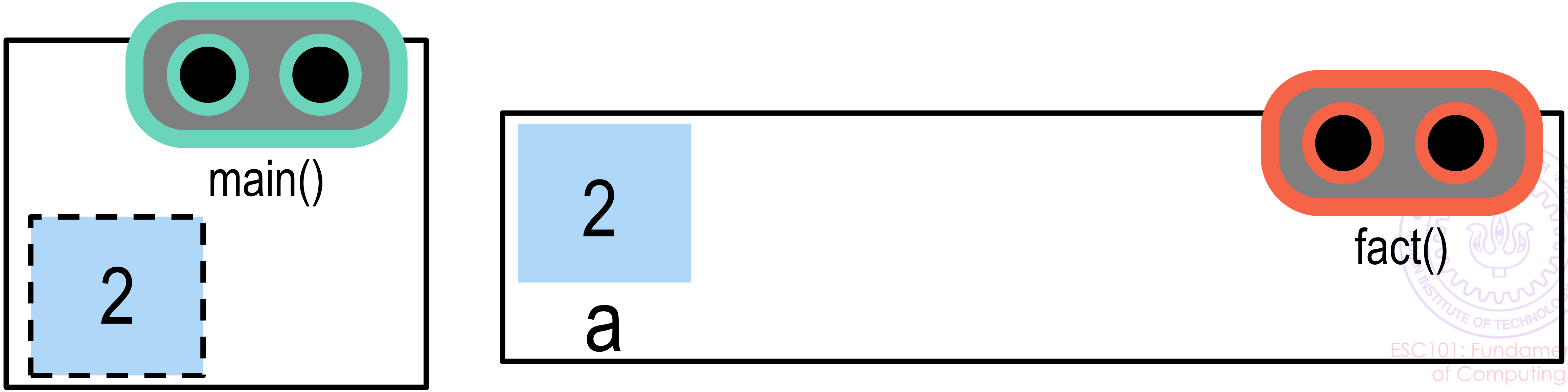

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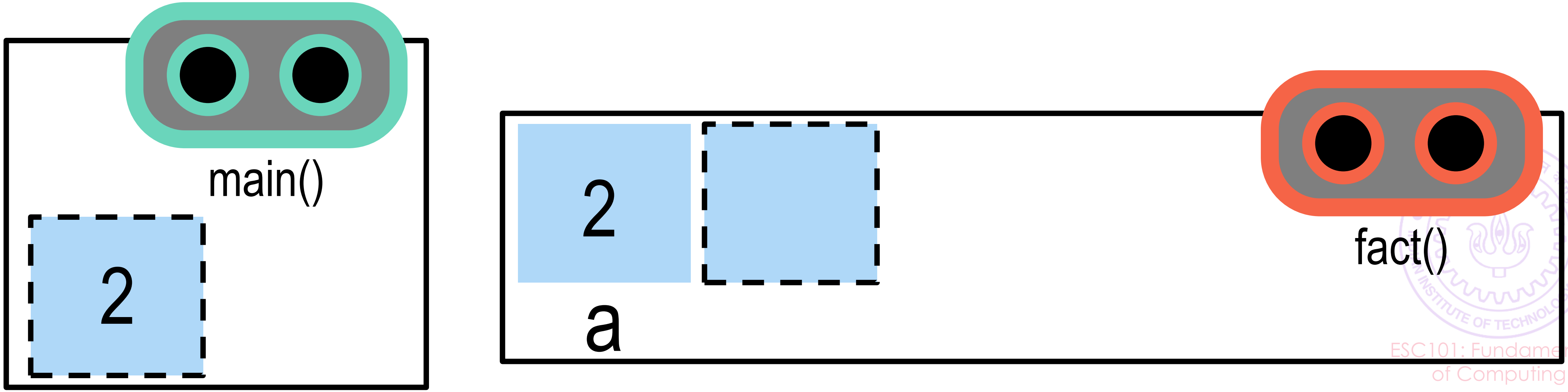

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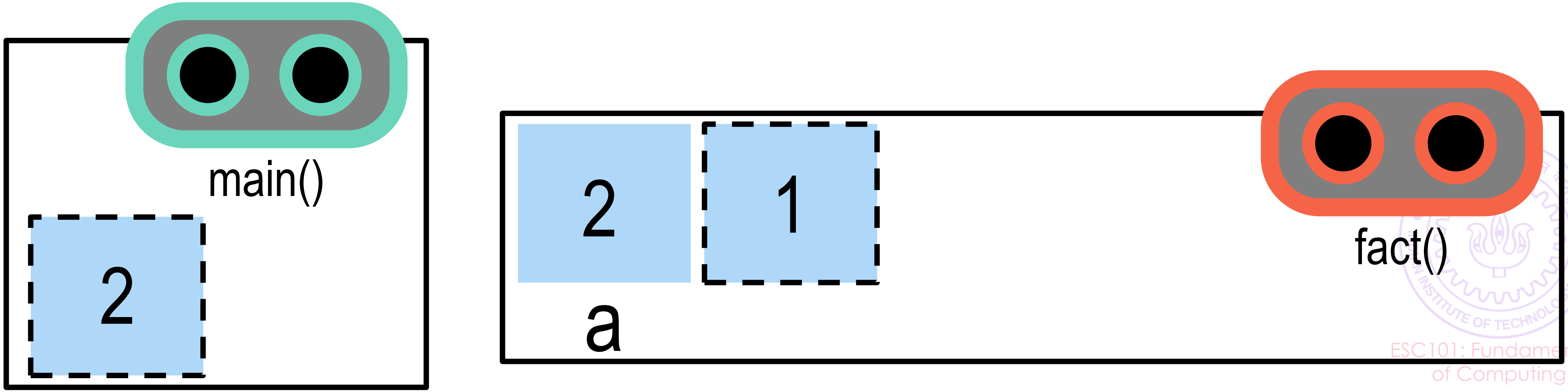

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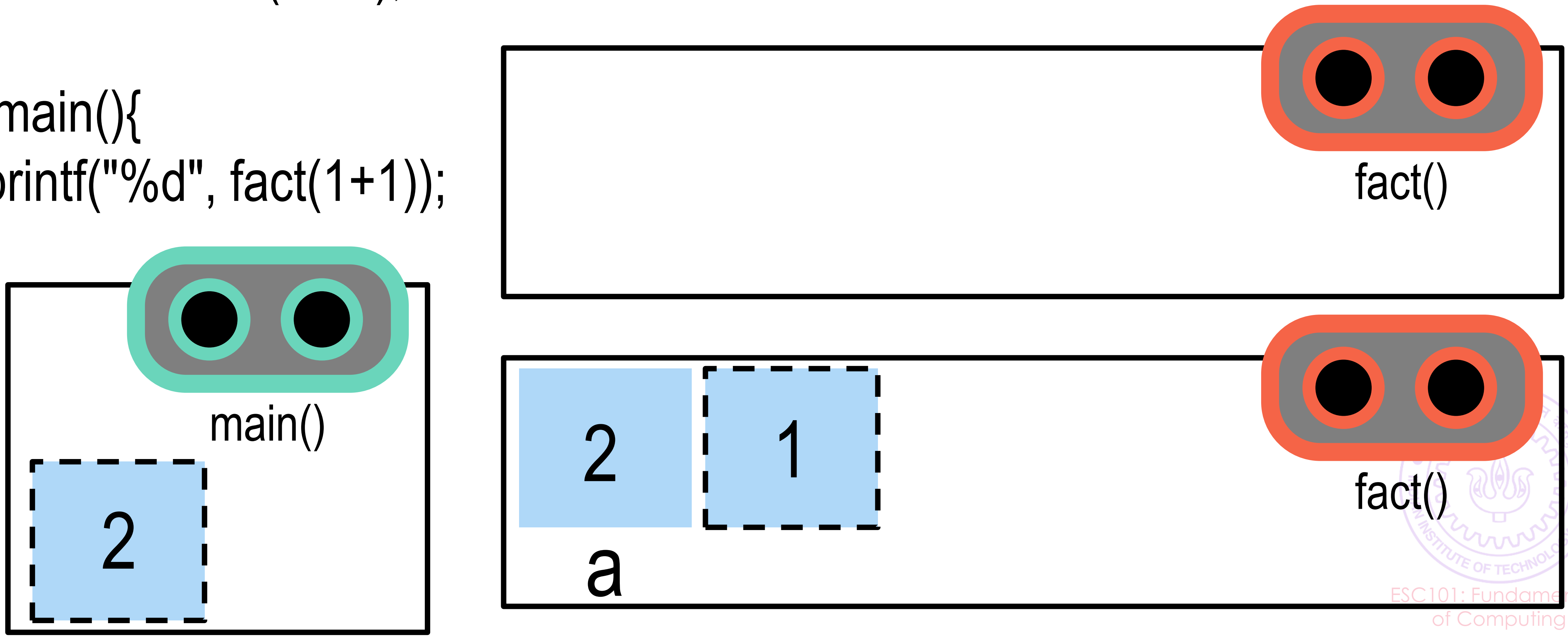

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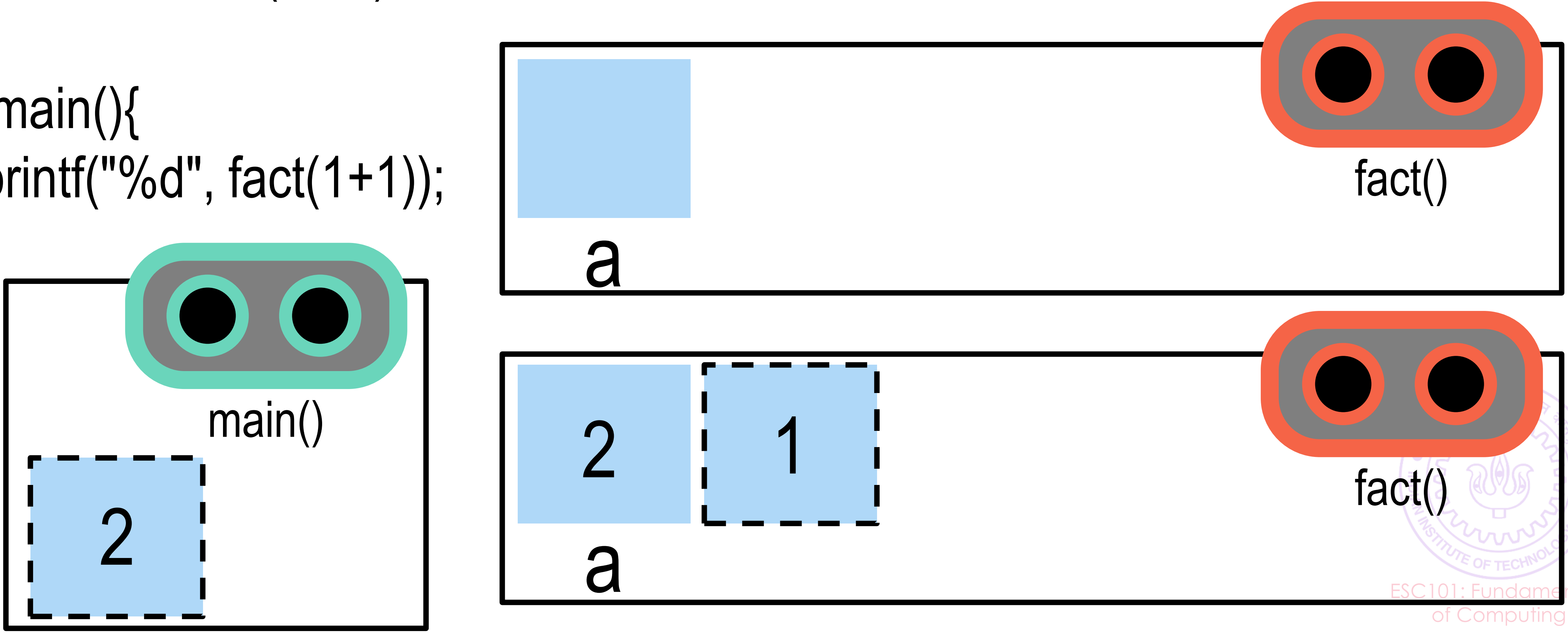

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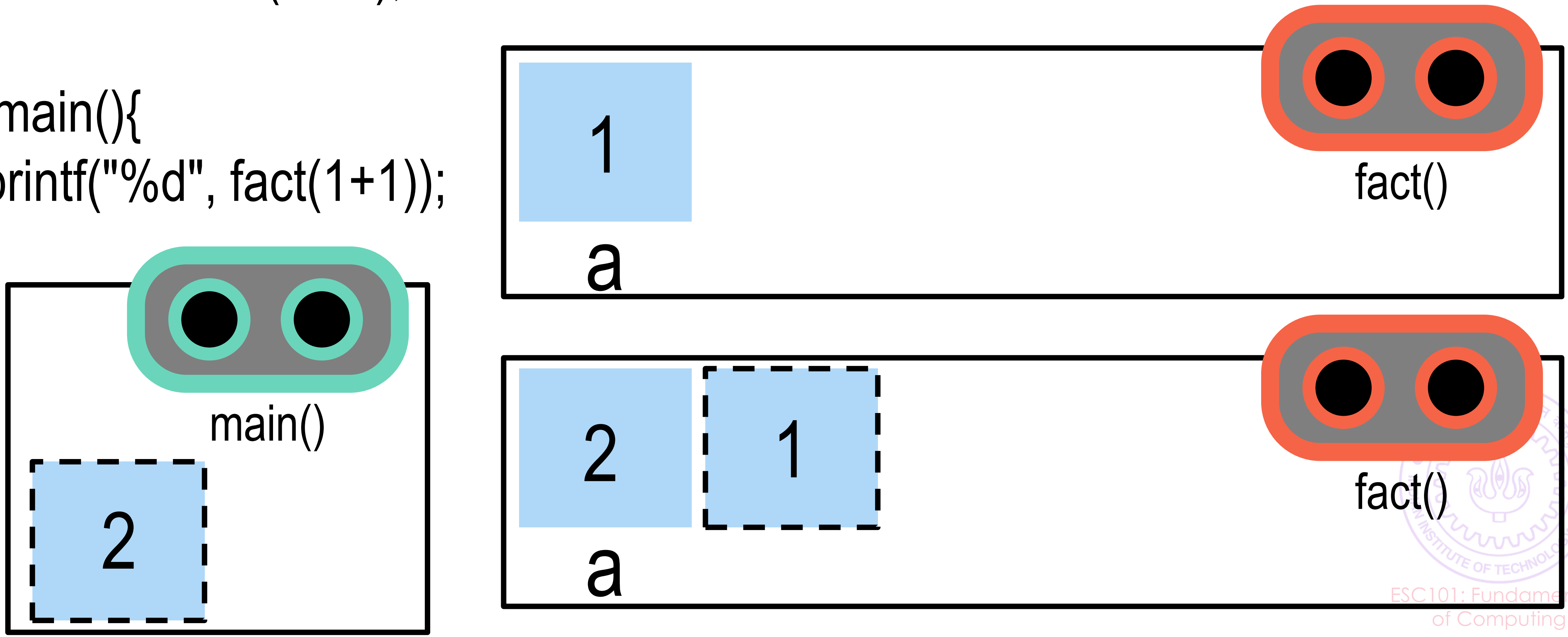

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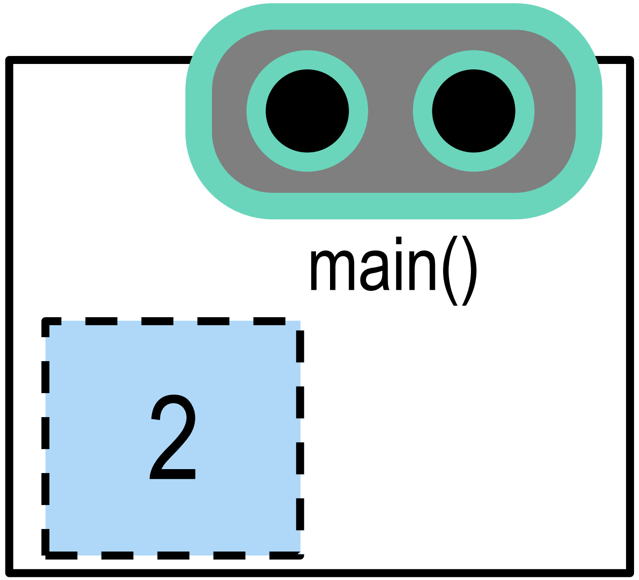

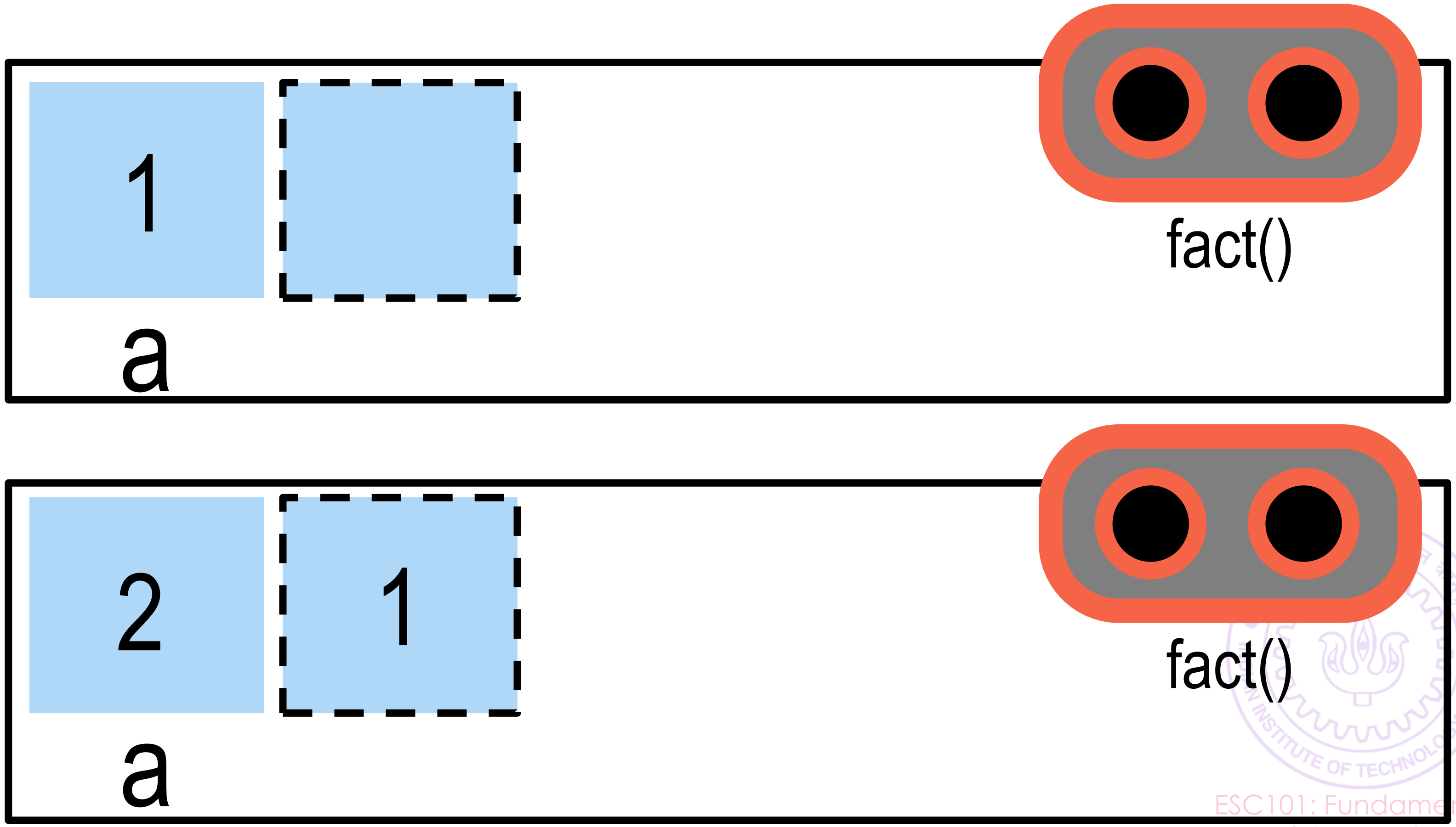

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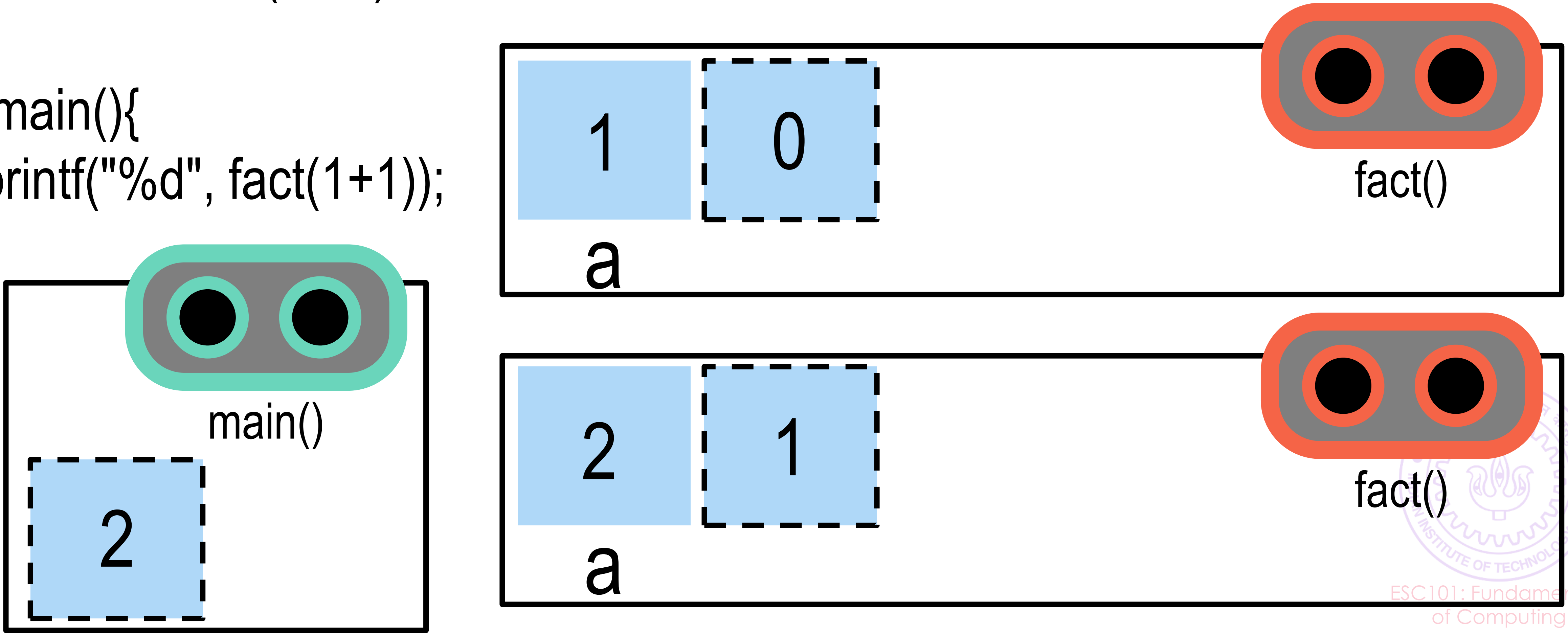

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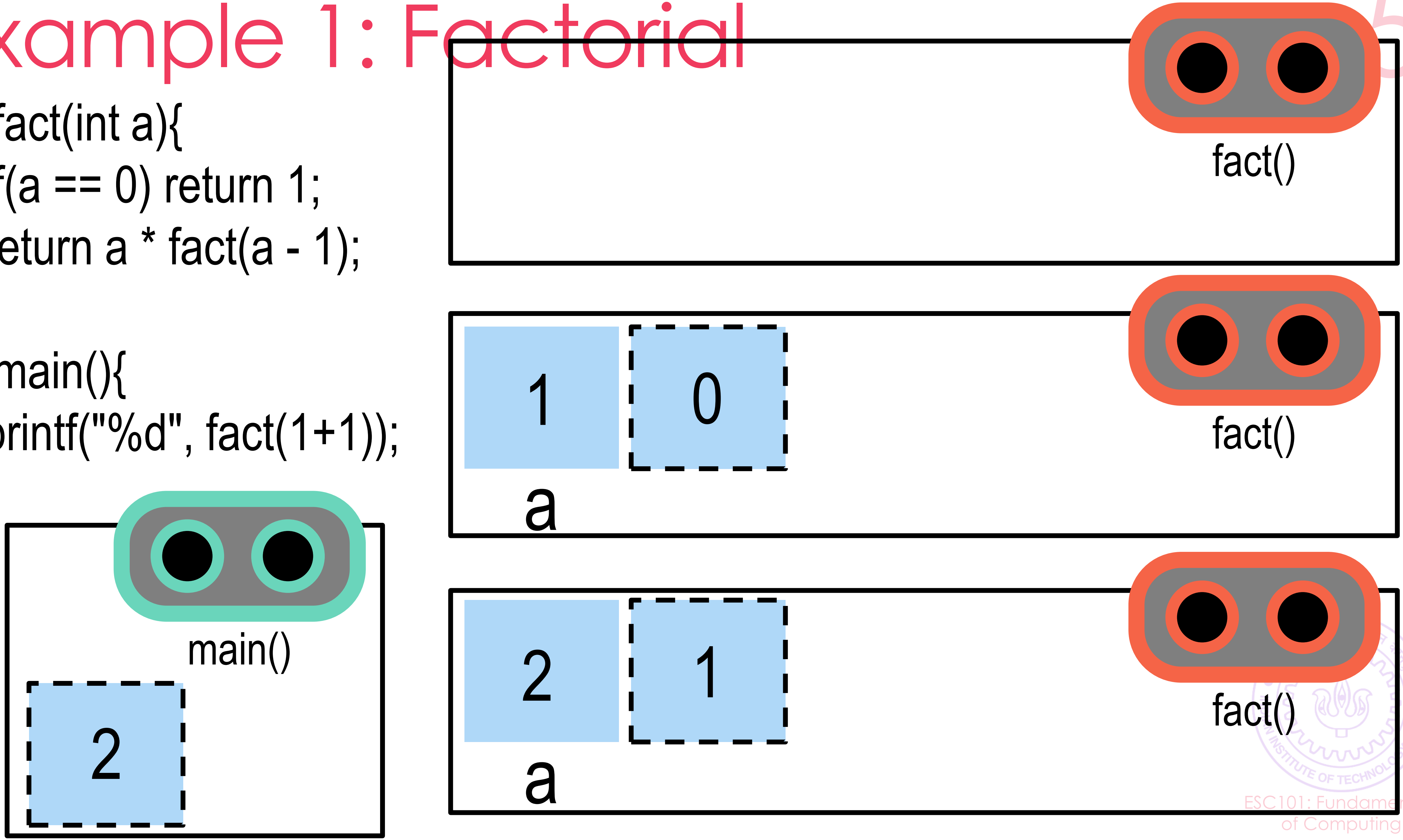

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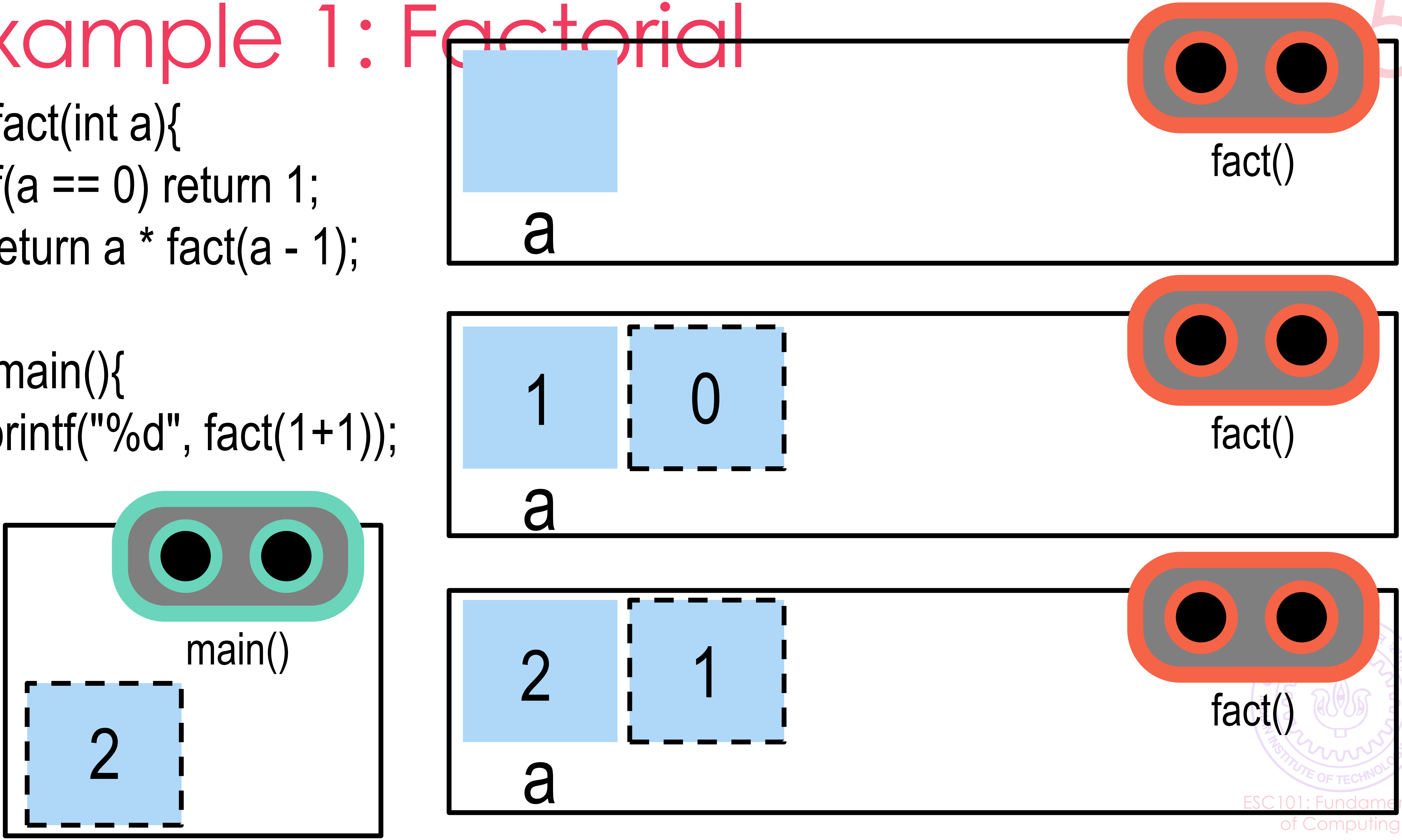

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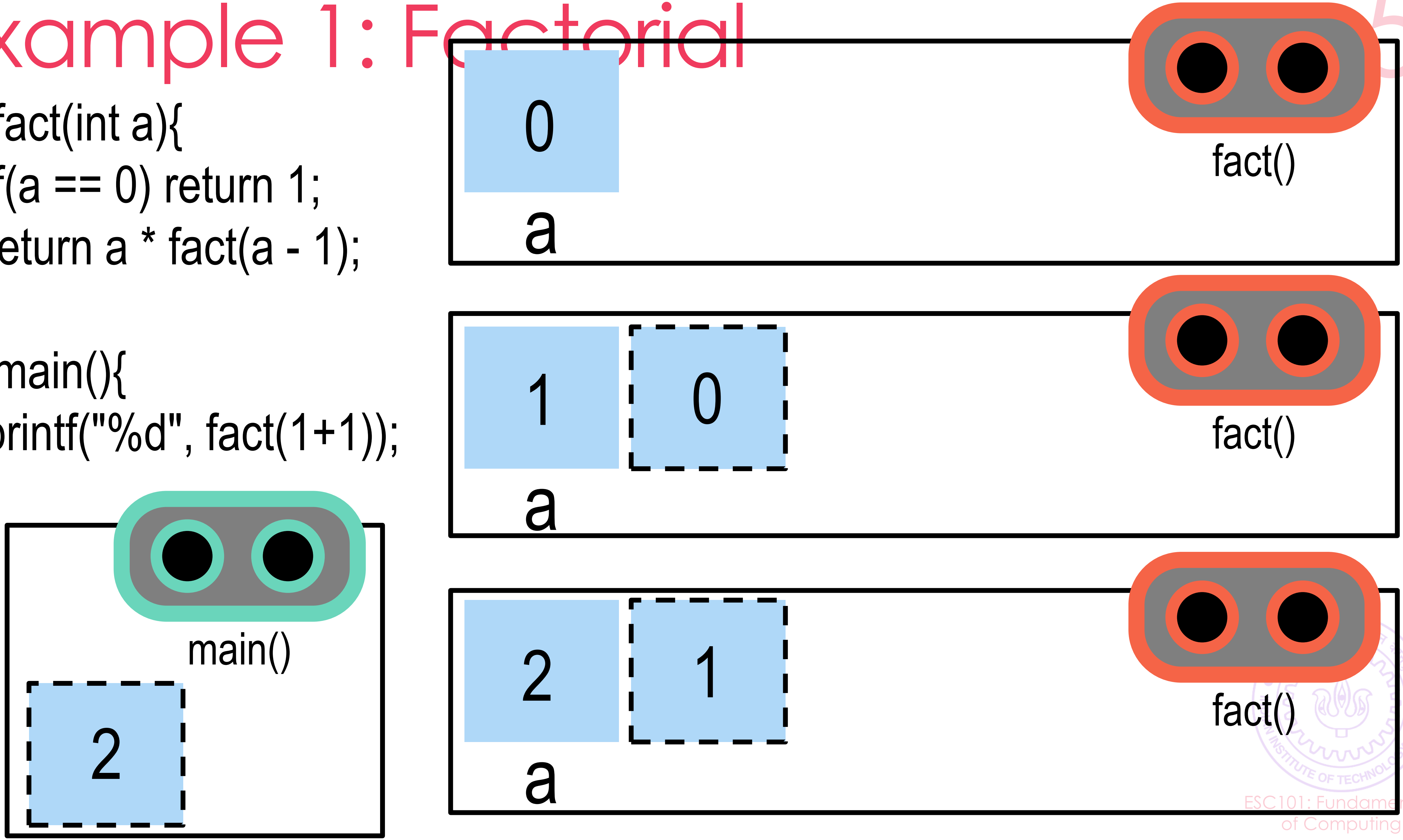

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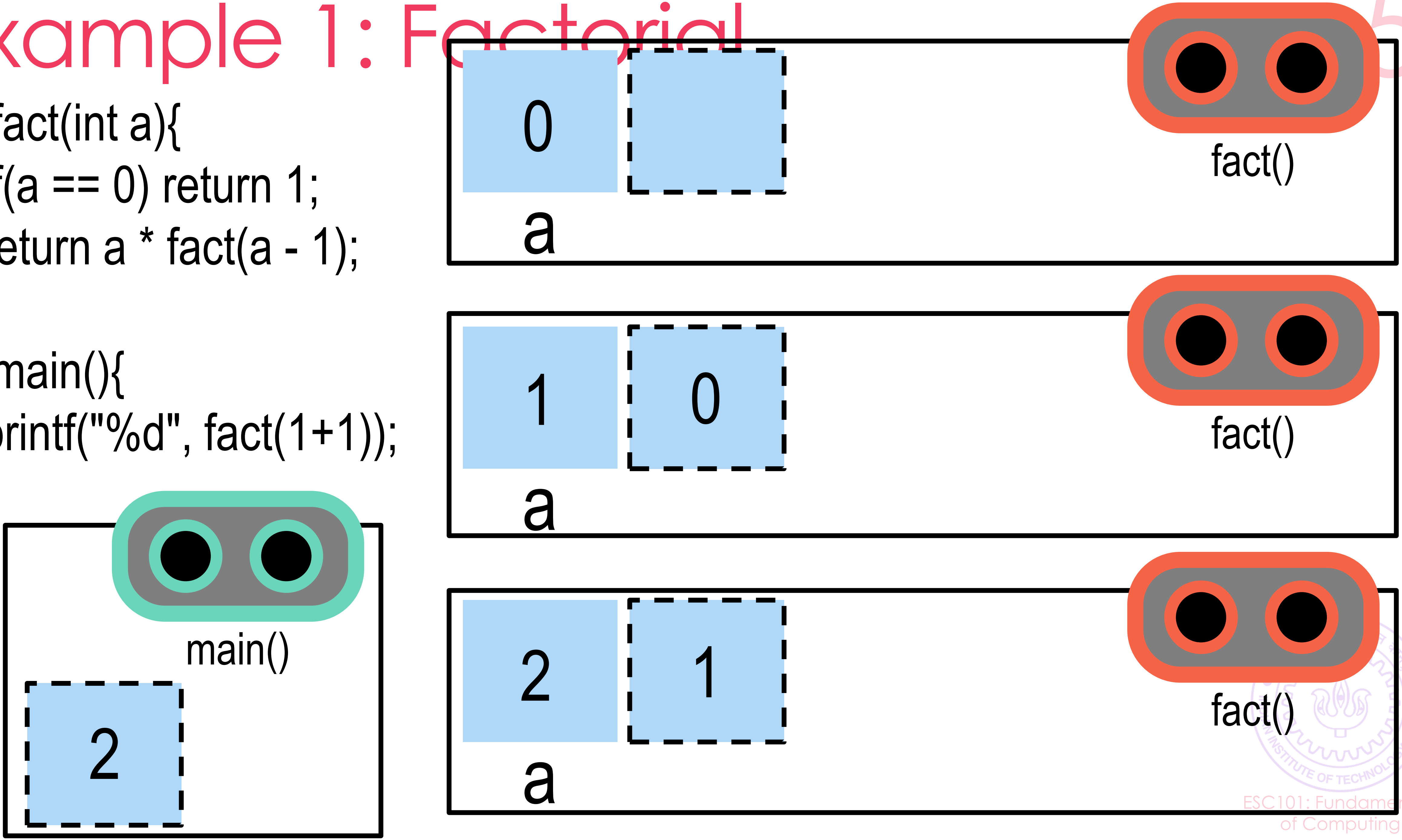

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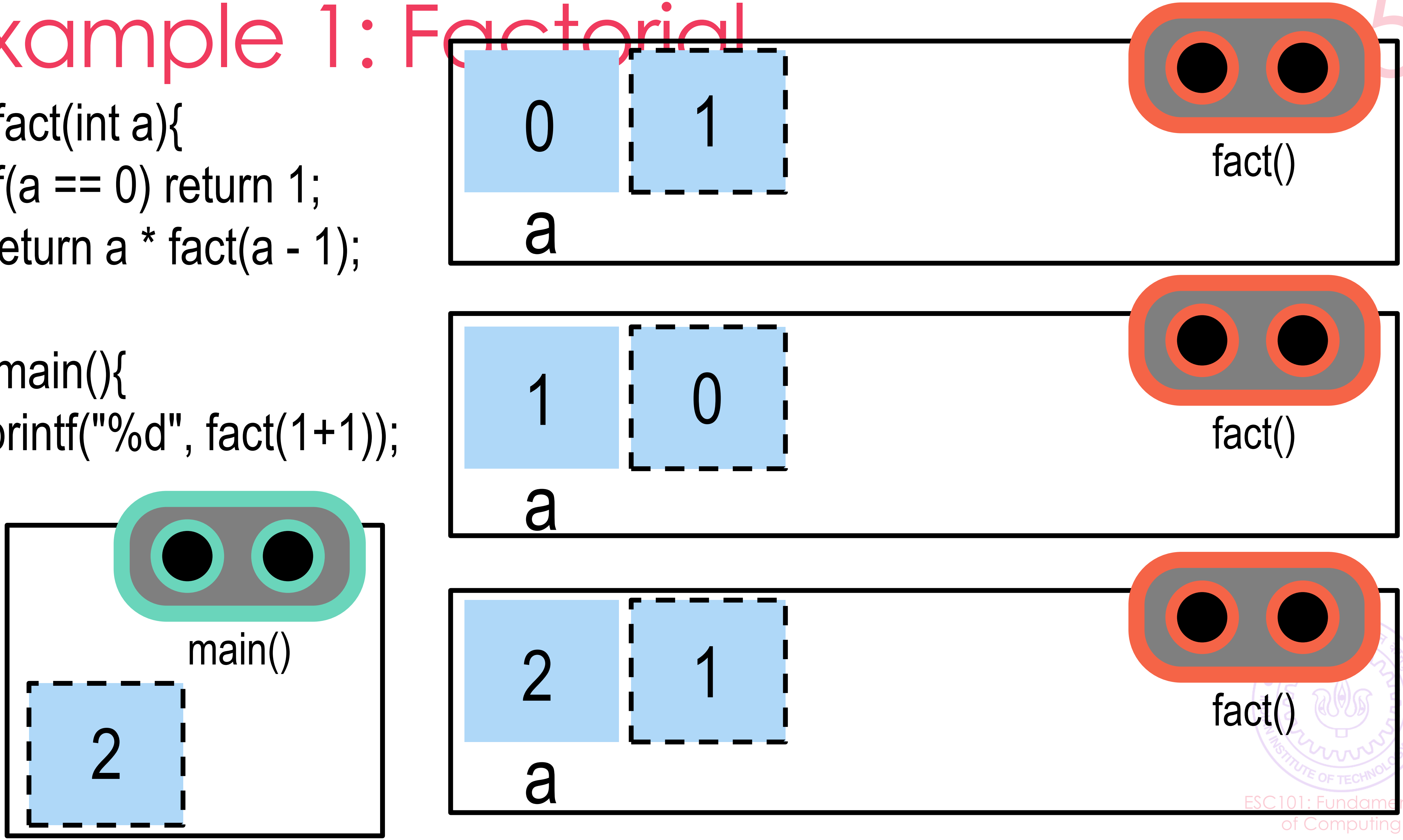

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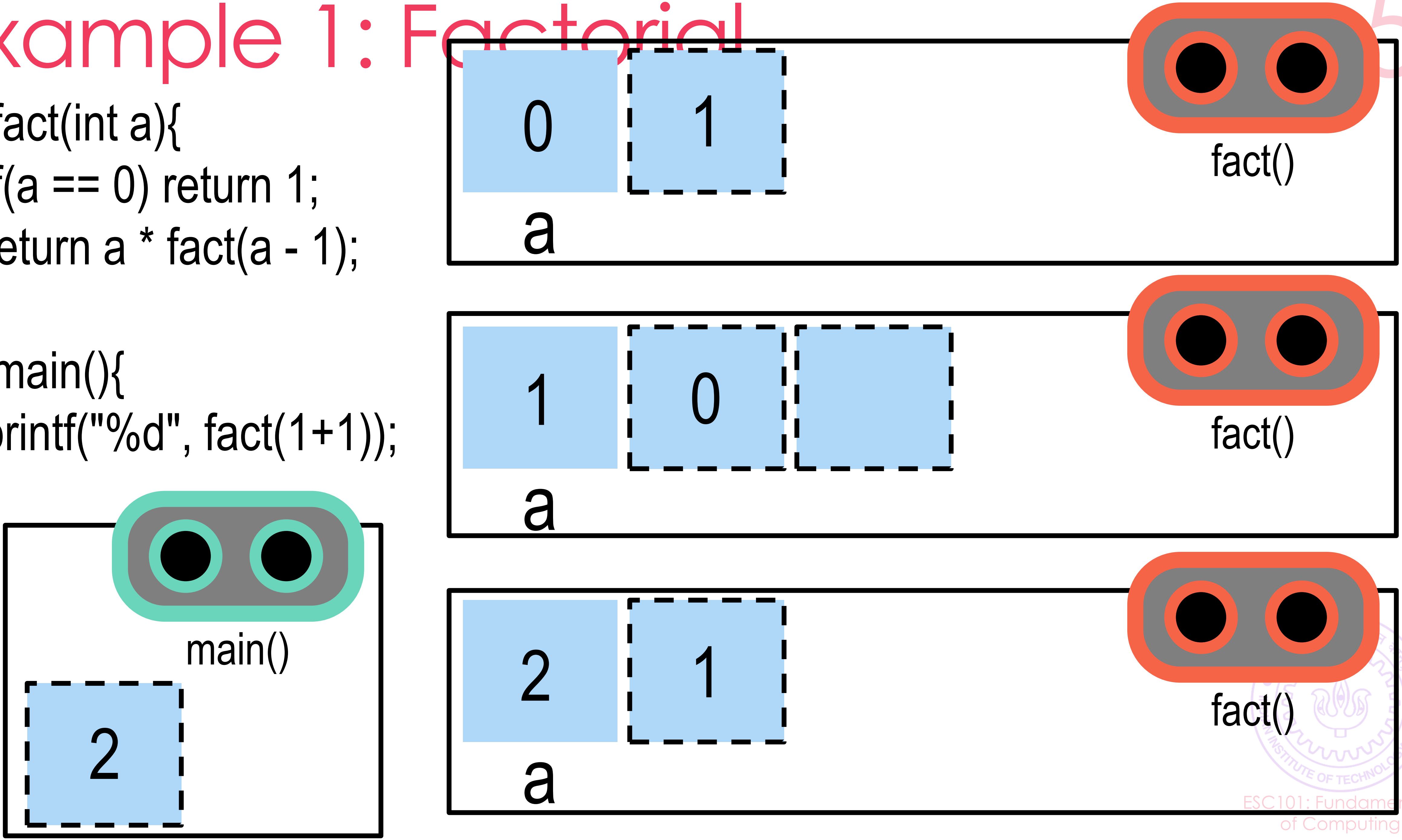

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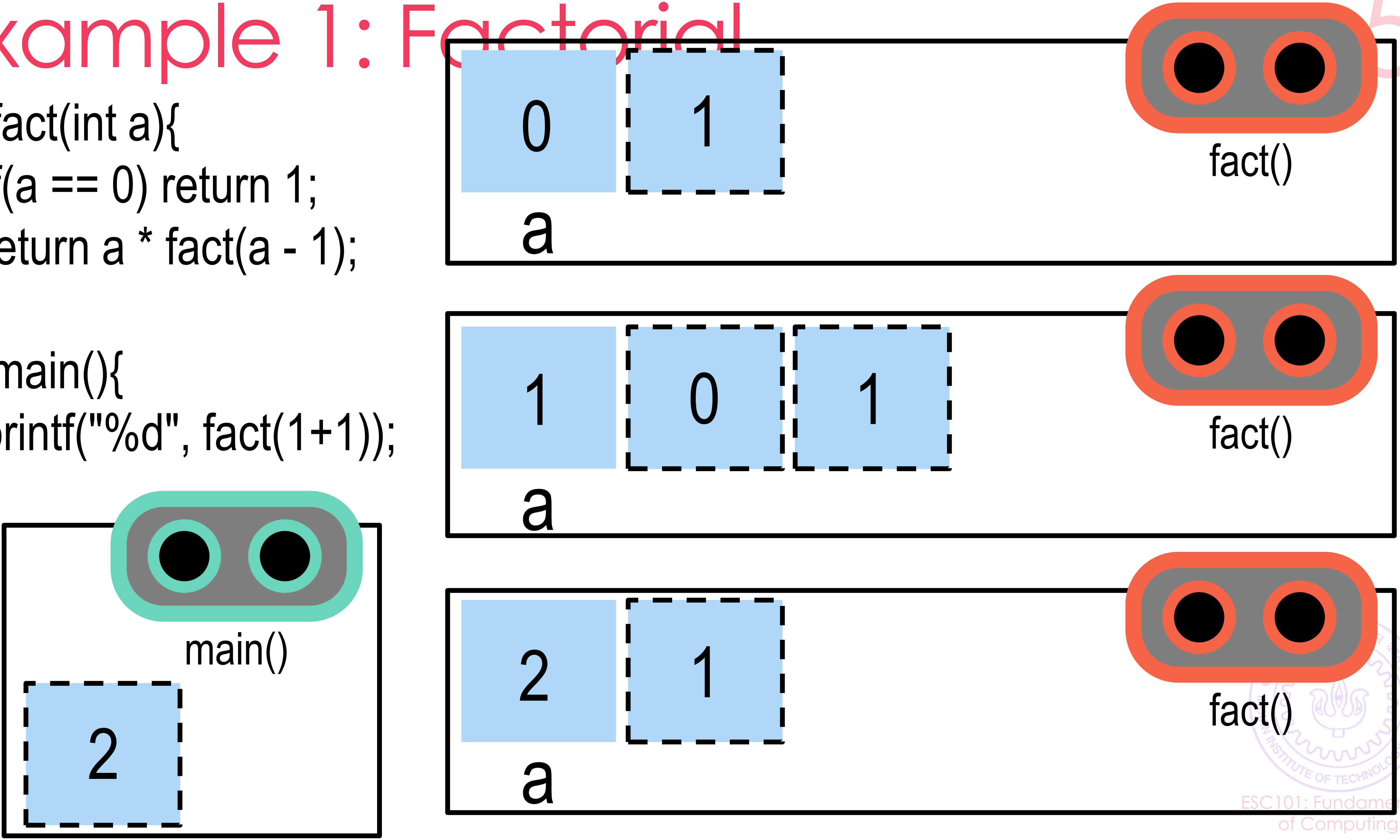

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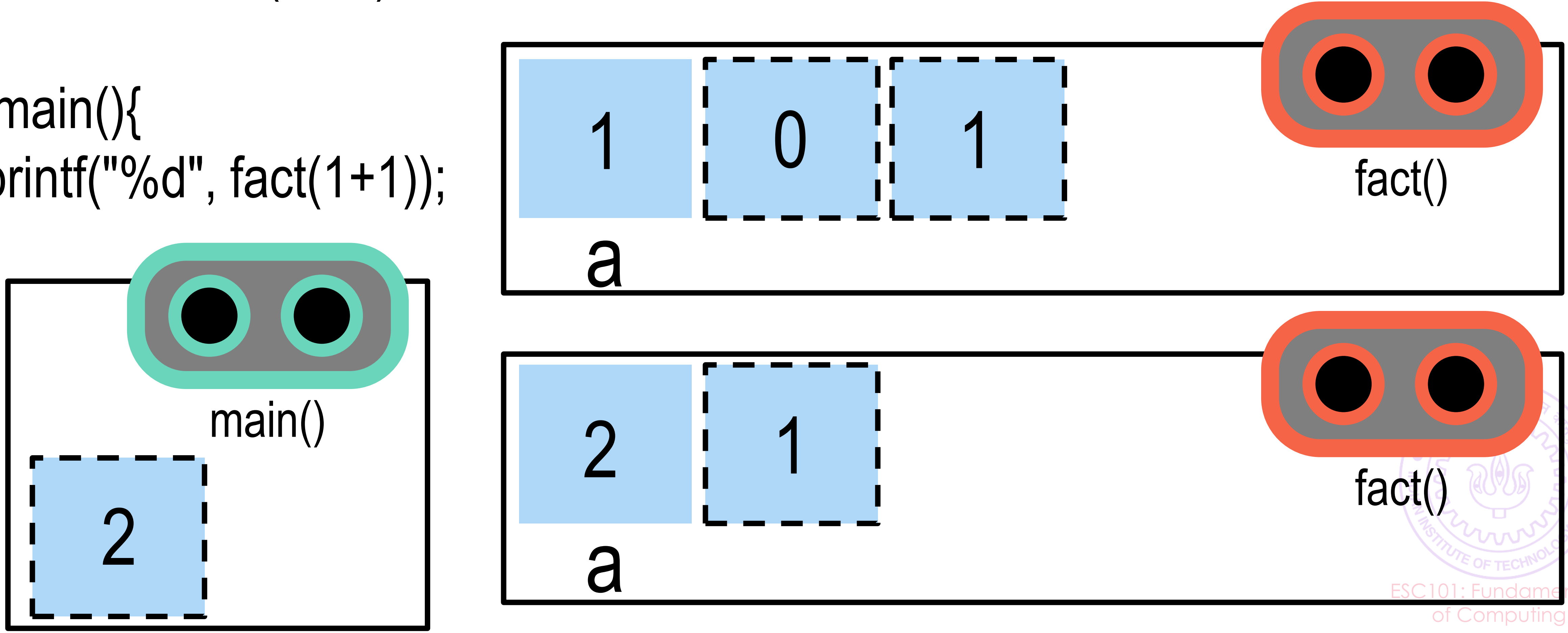

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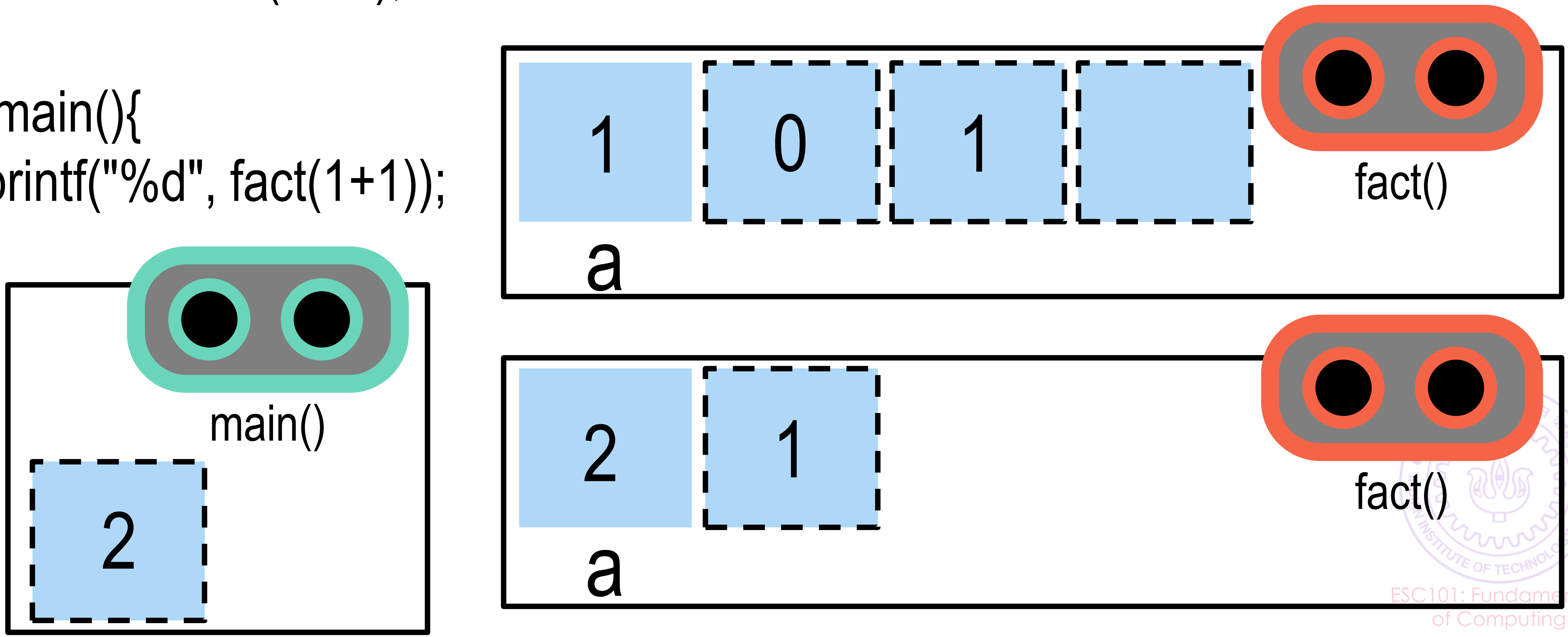

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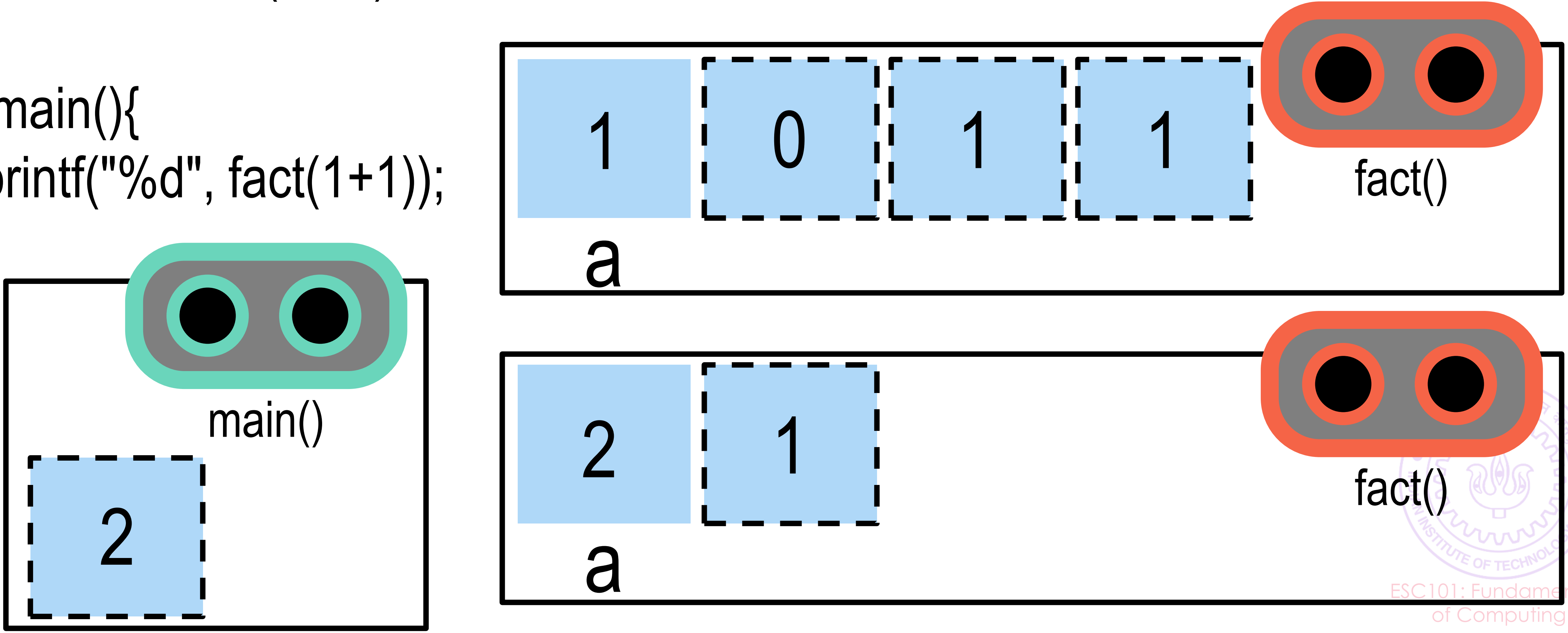

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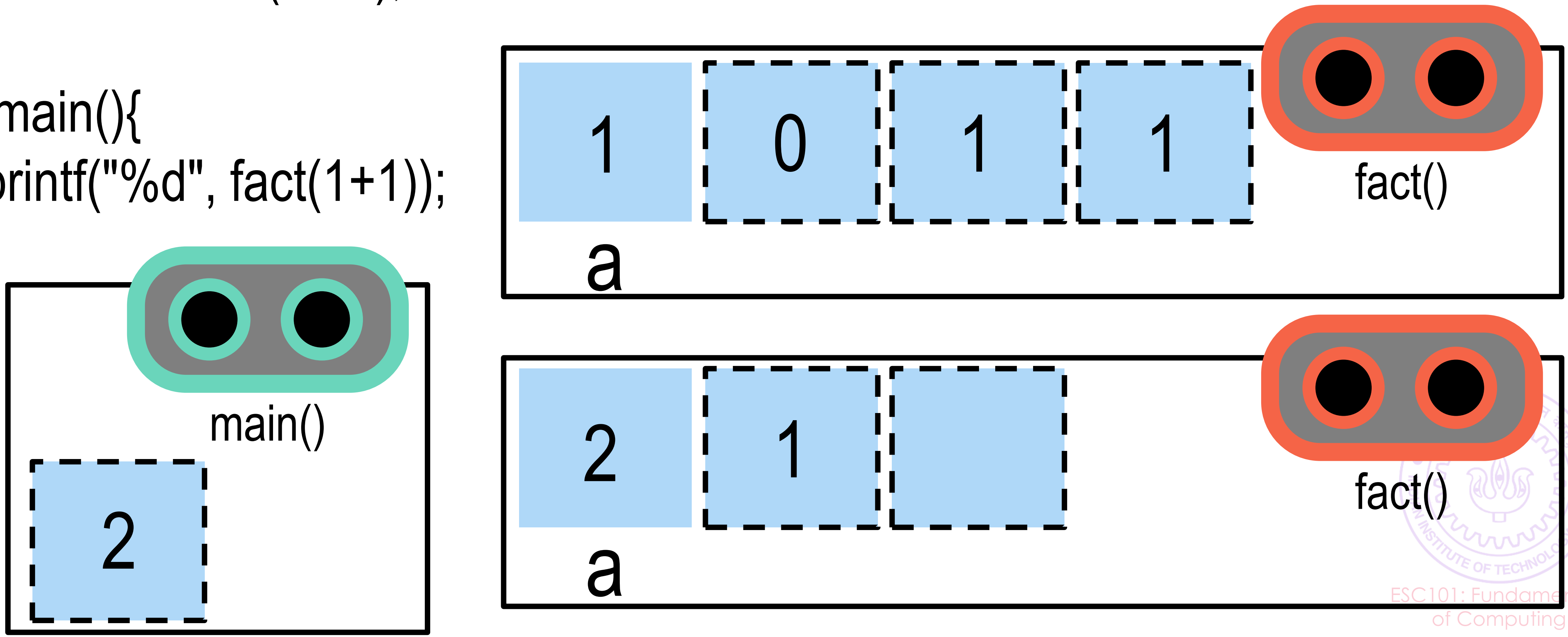

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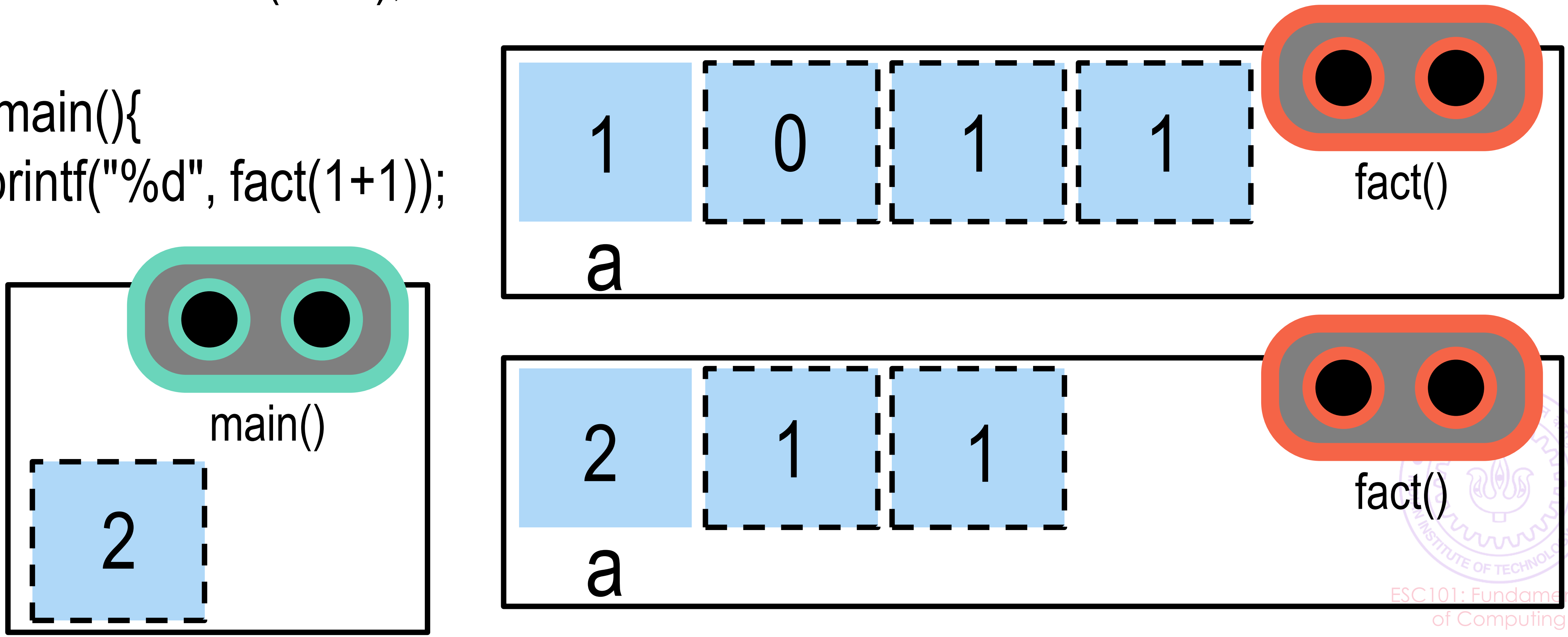

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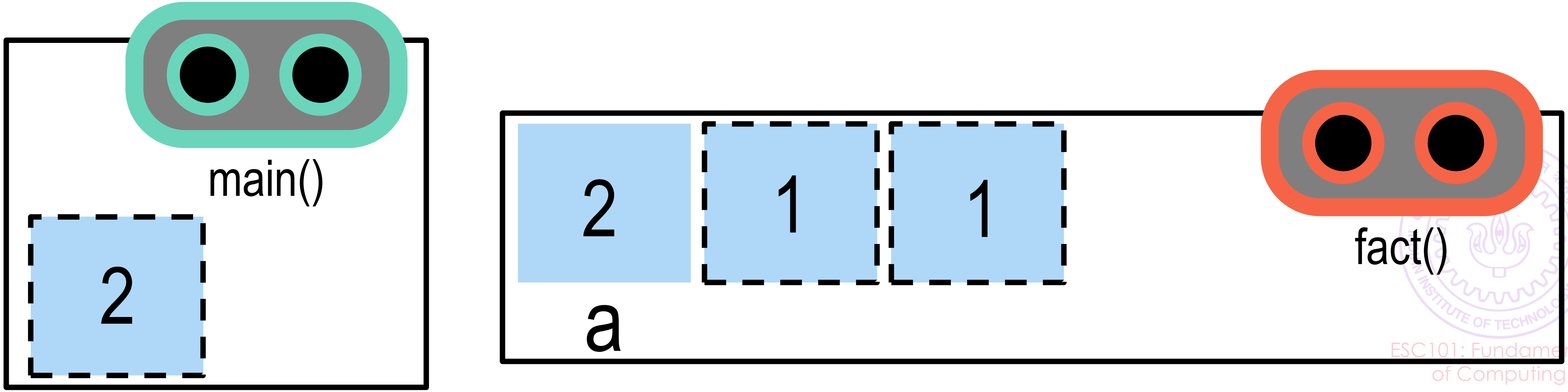

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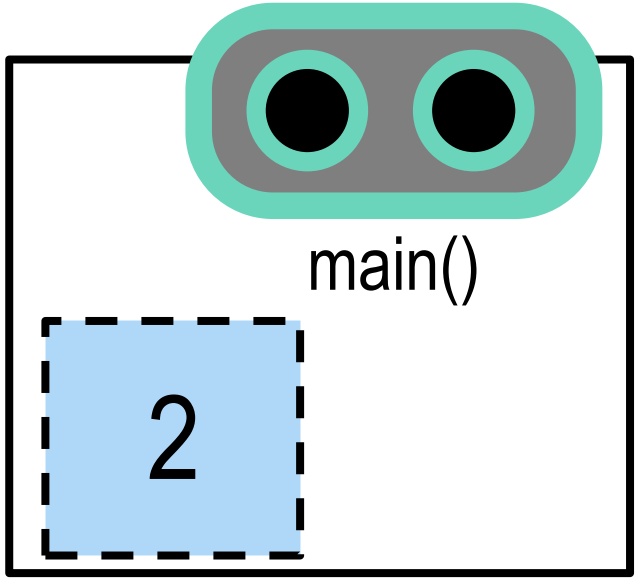

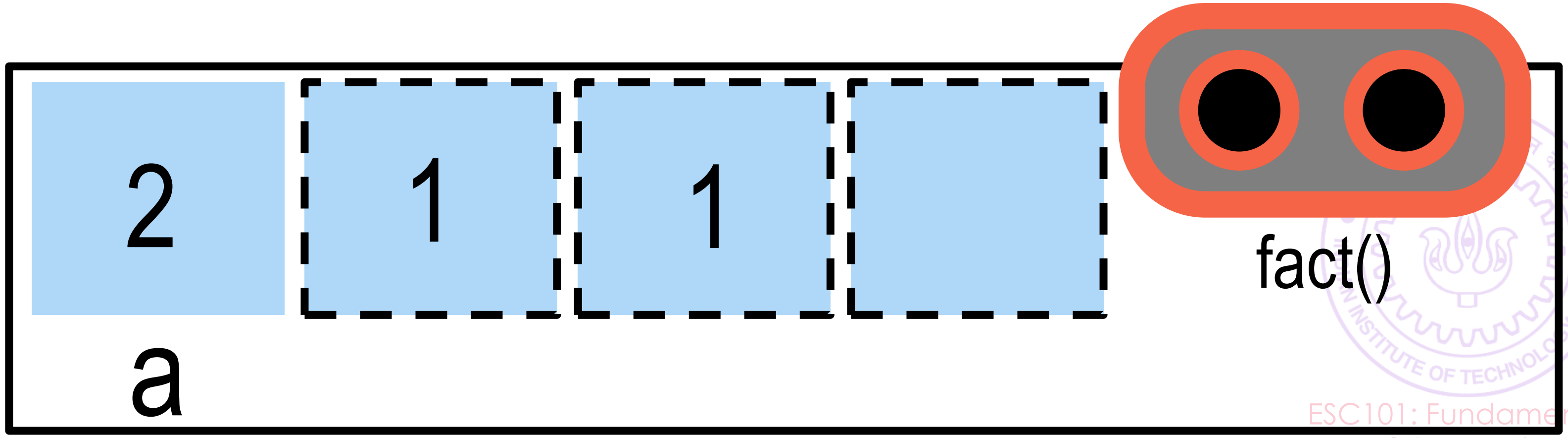

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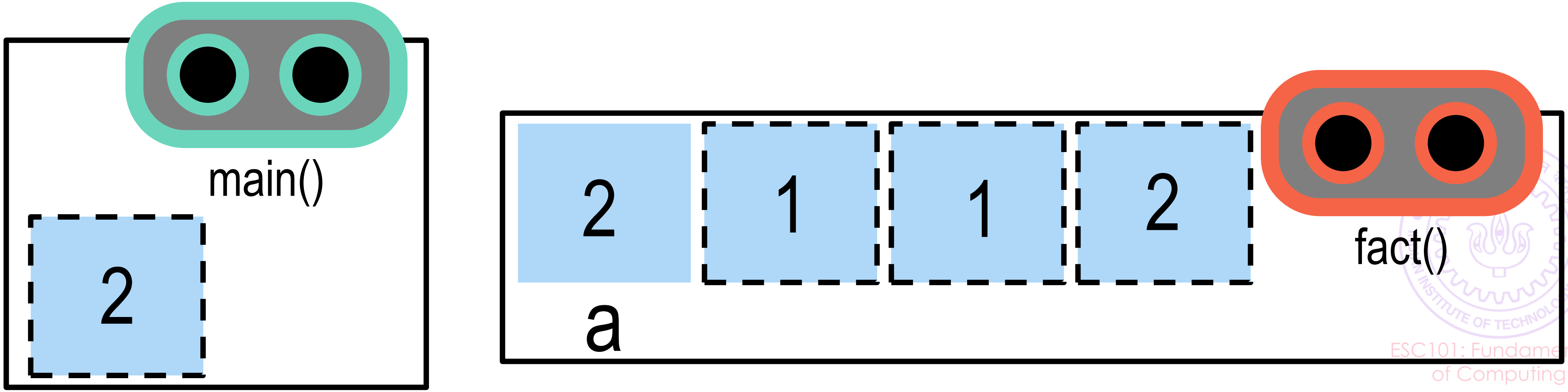
int fact(int a){
    if(a == 0) return 1;
    return a * fact(a - 1);
}
int main(){
    printf("%d", fact(1+1));
}
```



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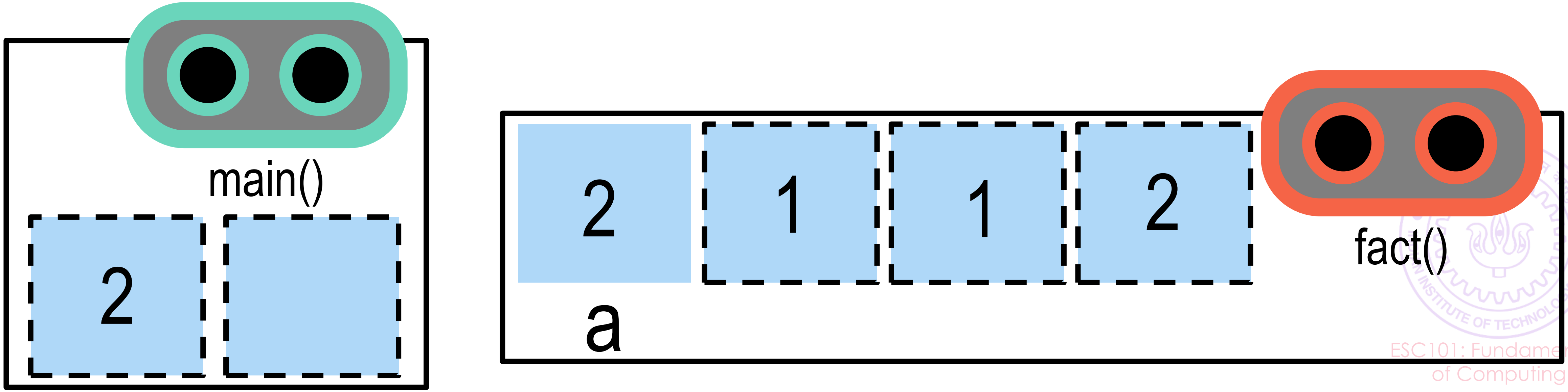

Example 1: Factorial


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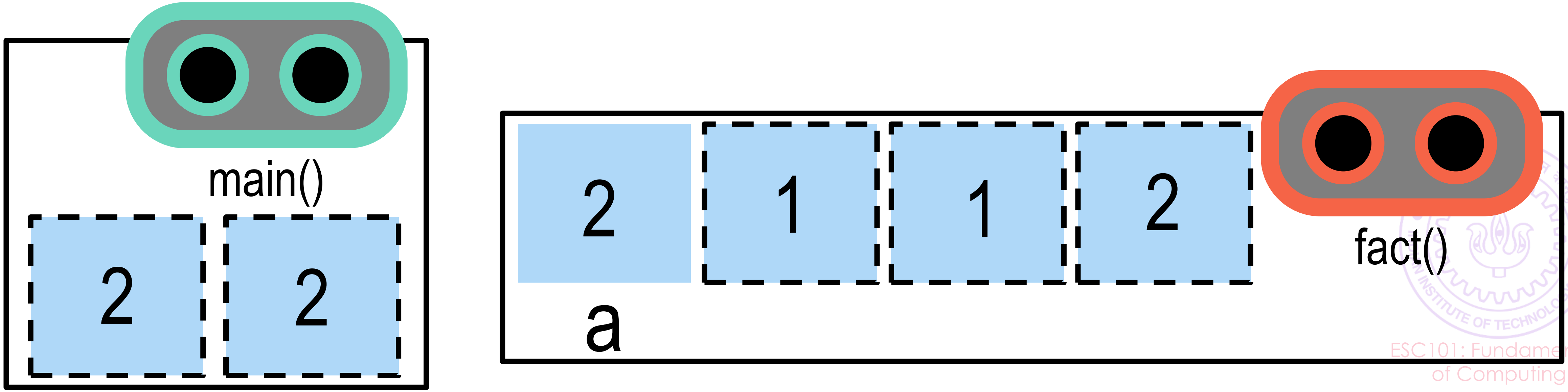

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int main(){
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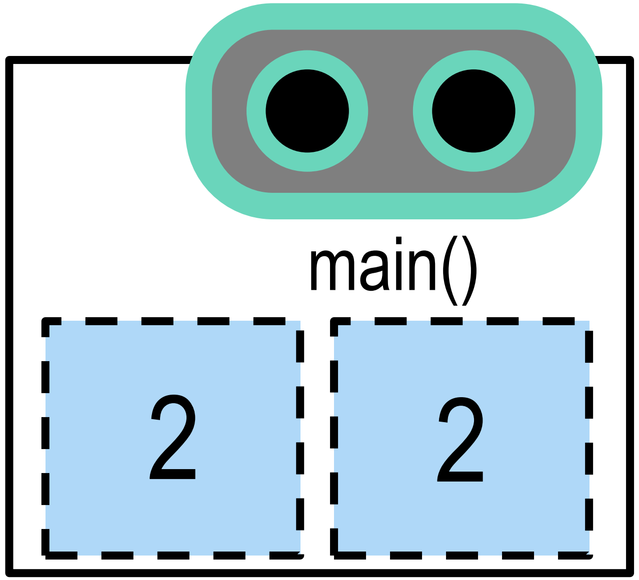

Example 1: Factorial


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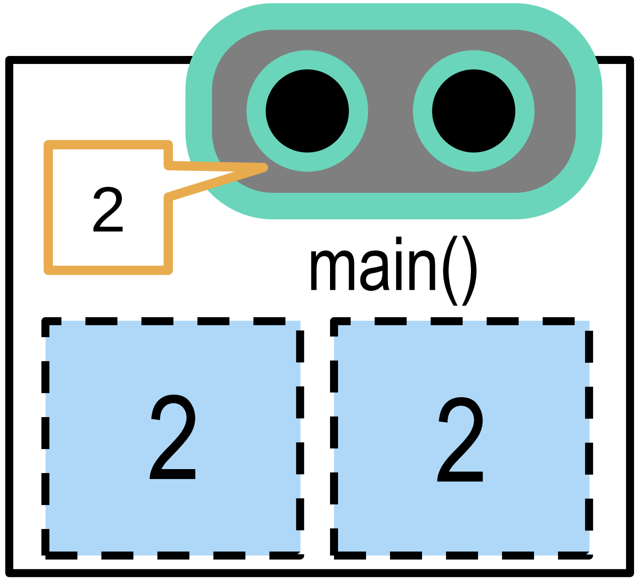

Example 1: Factorial

Example 1: Factorial

```
int fact(int a){
    if(a == 0) return 1;
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    printf("%d", fact(2*3));
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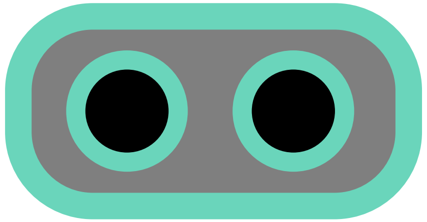

main()

Example 1: Factorial

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int main(){
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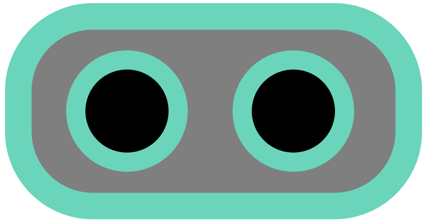
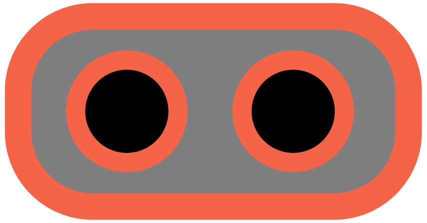


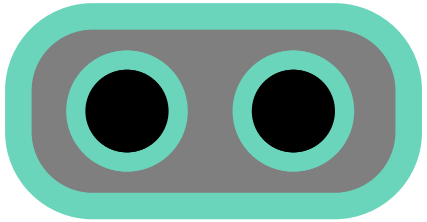
main()



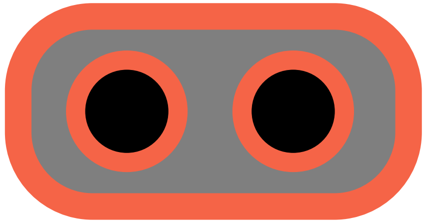
fact(6)

Example 1: Factorial

```
int fact(int a){
    if(a == 0) return 1;
    return a * fact(a - 1);
}
int main(){
    printf("%d", fact(2*3));
}
```



main()



fact(6)
= 6 * fact(5)

Example 1: Factorial

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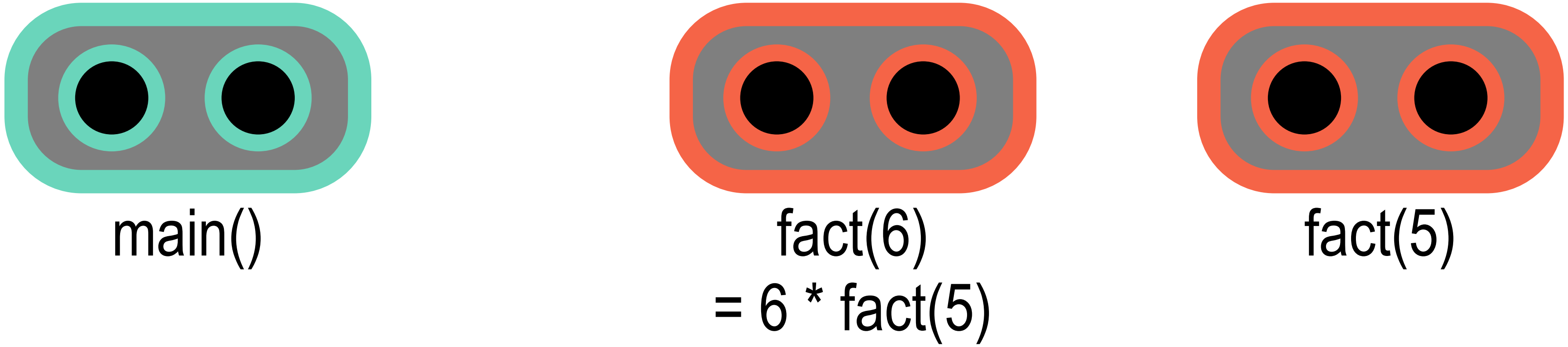

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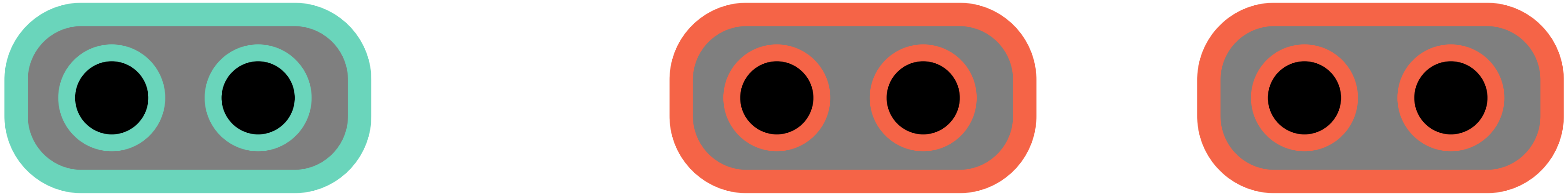

main()

fact(6)
= 6 * fact(5)

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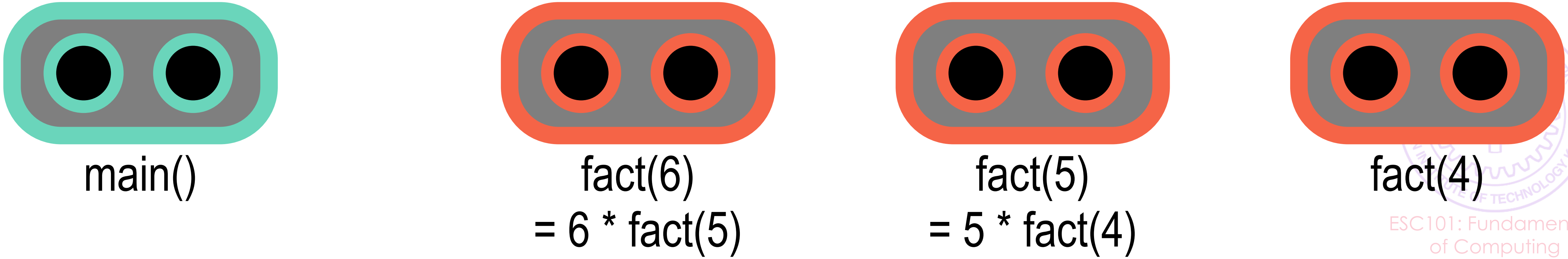

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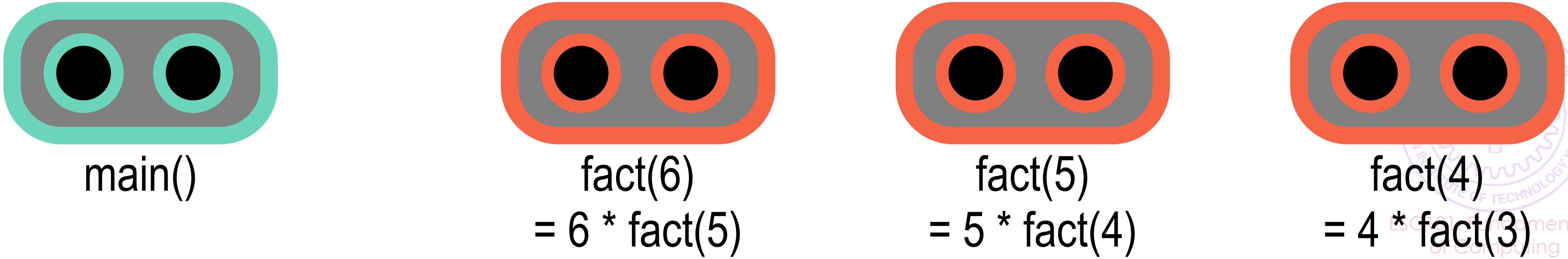

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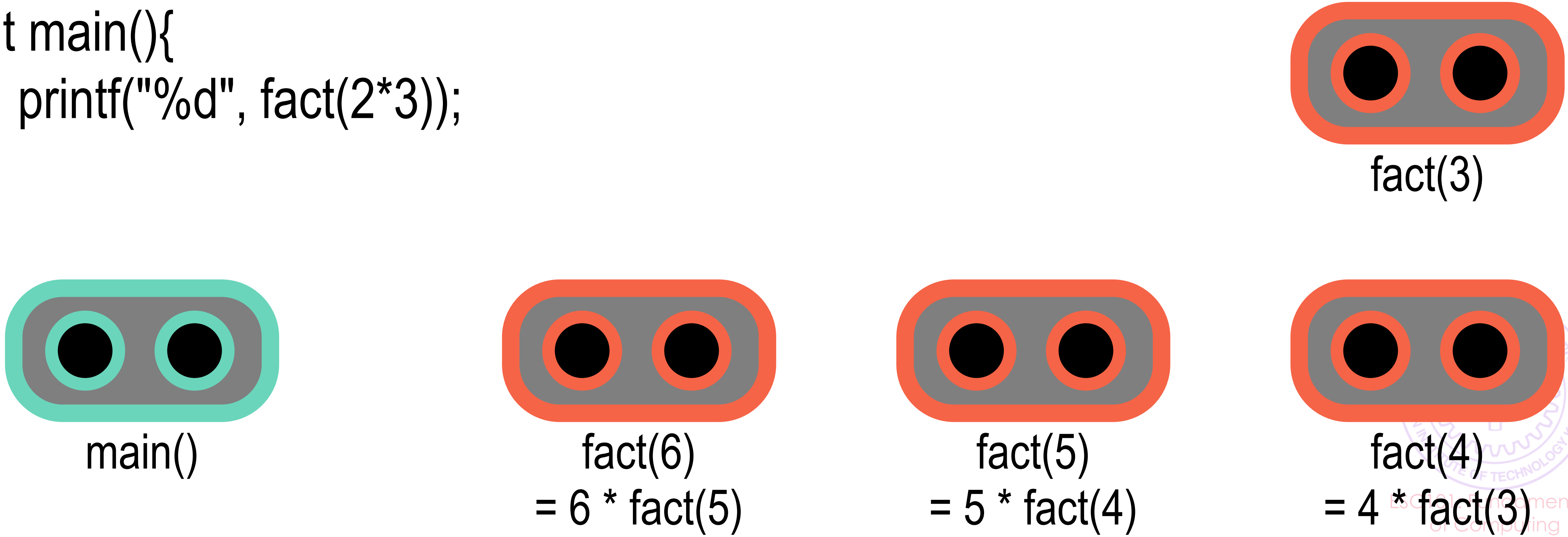

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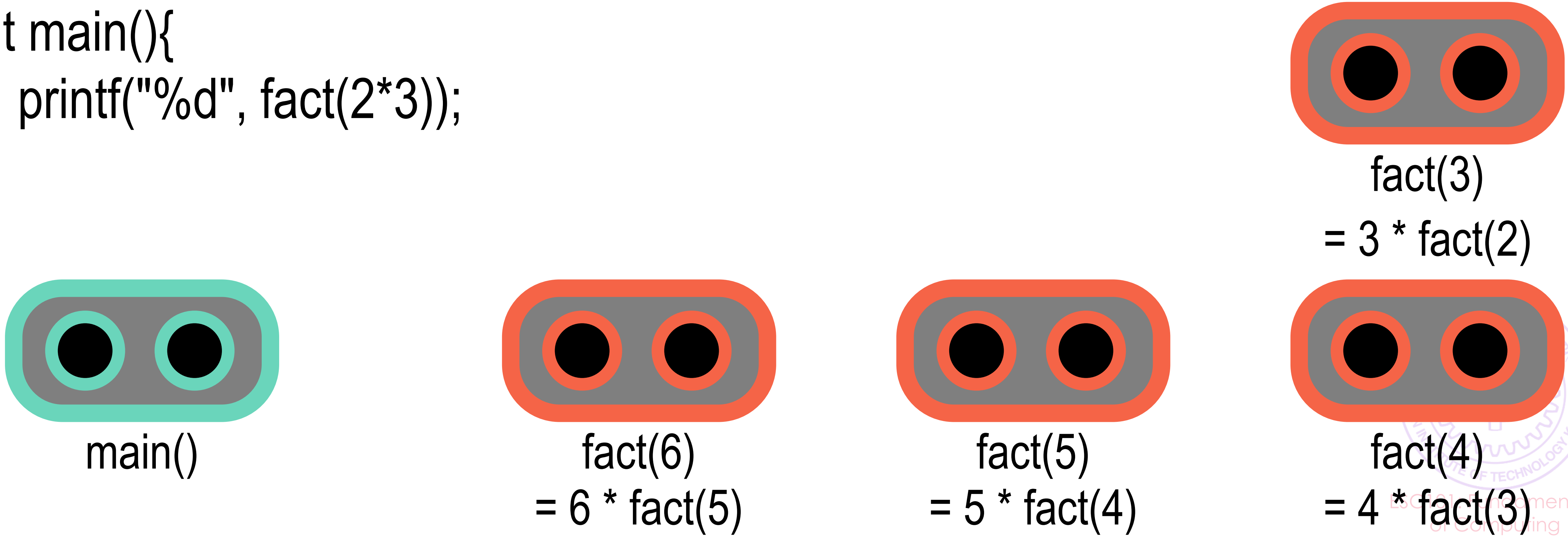

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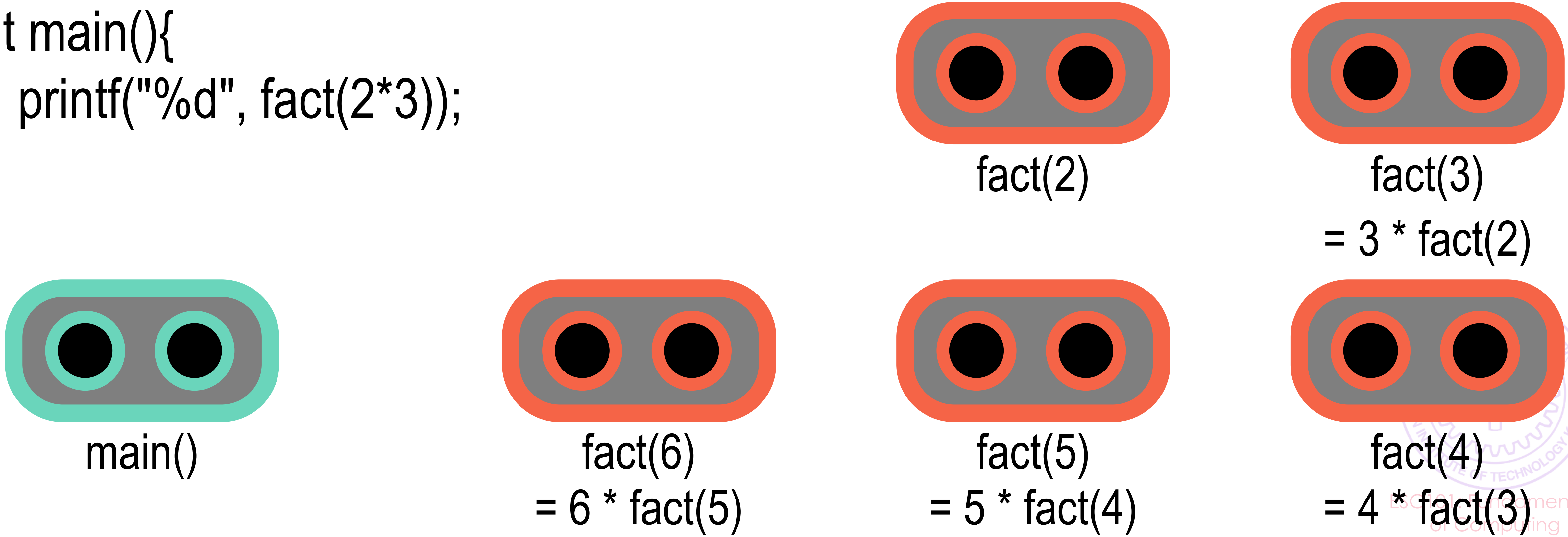

Example 1: Factorial

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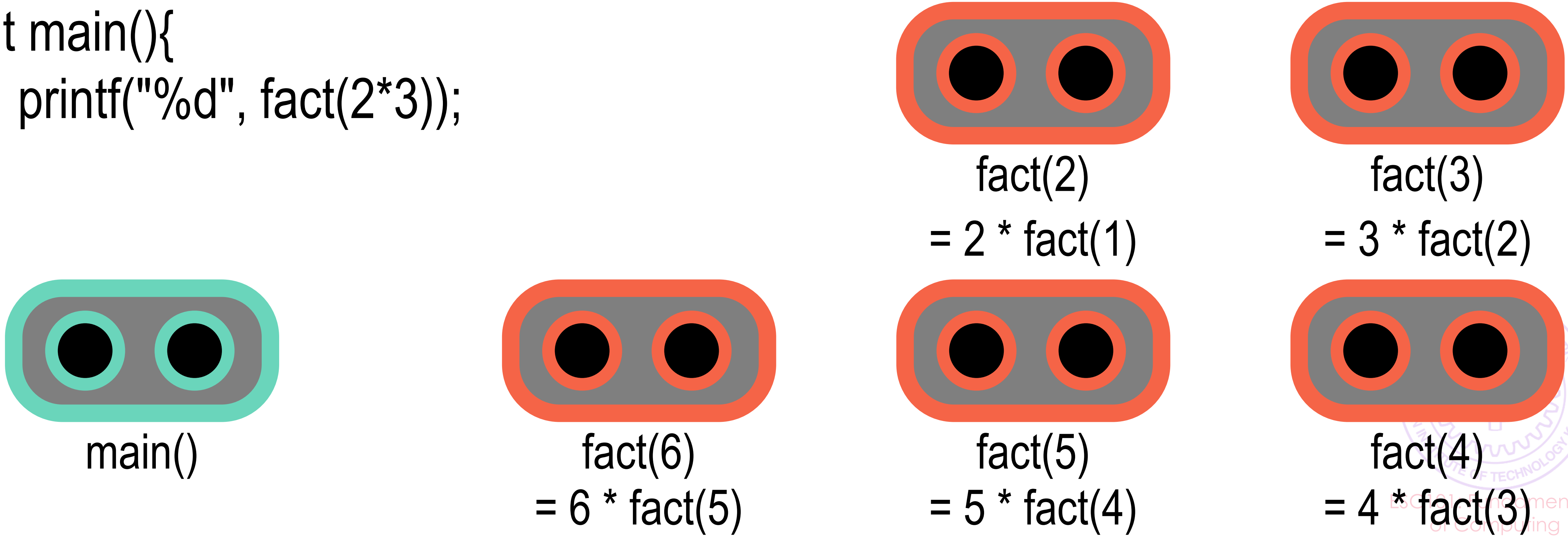

Example 1: Factorial

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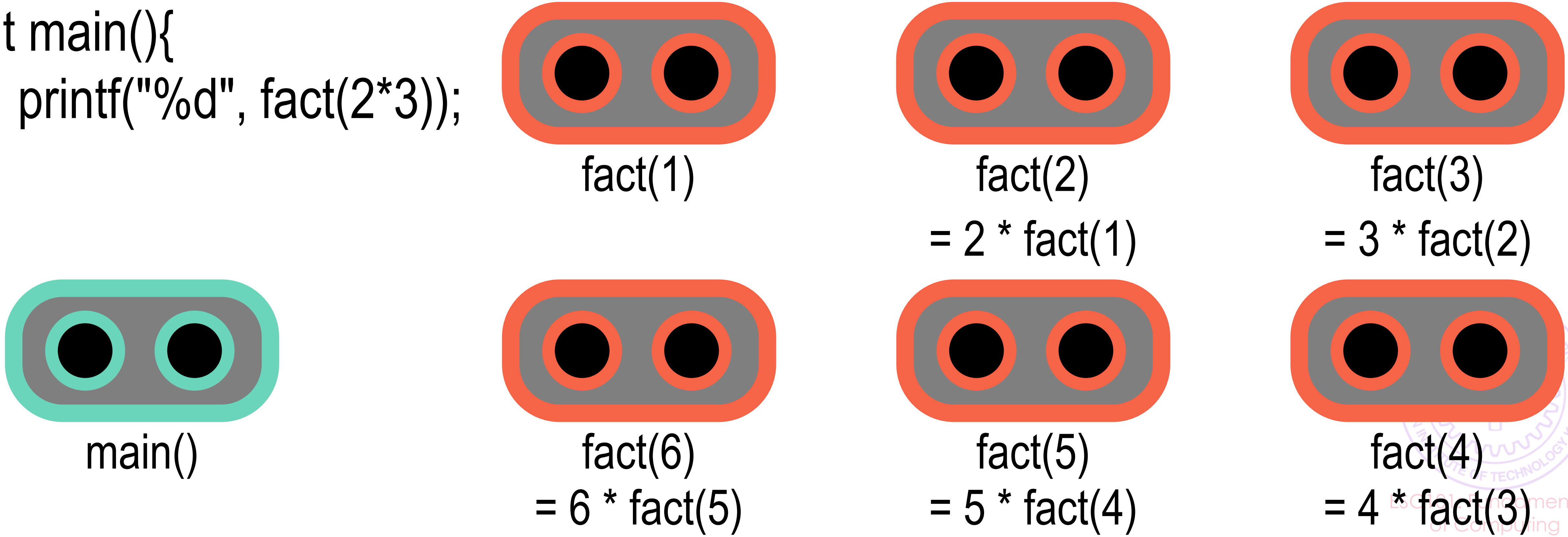

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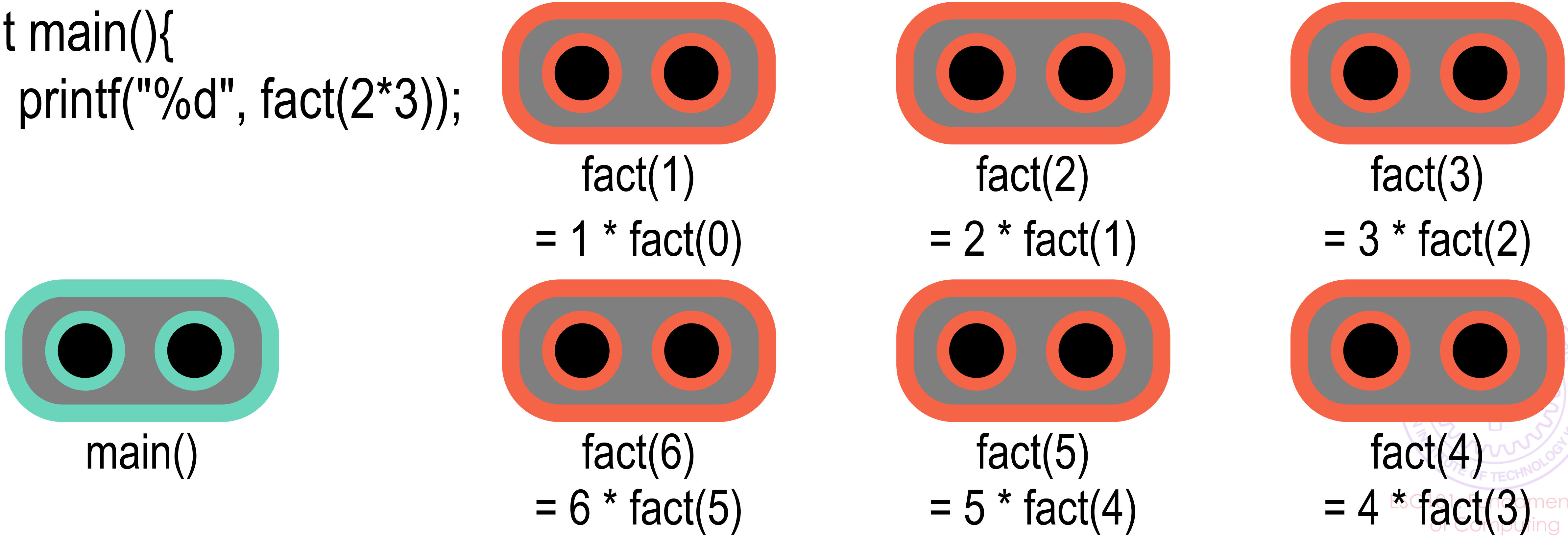

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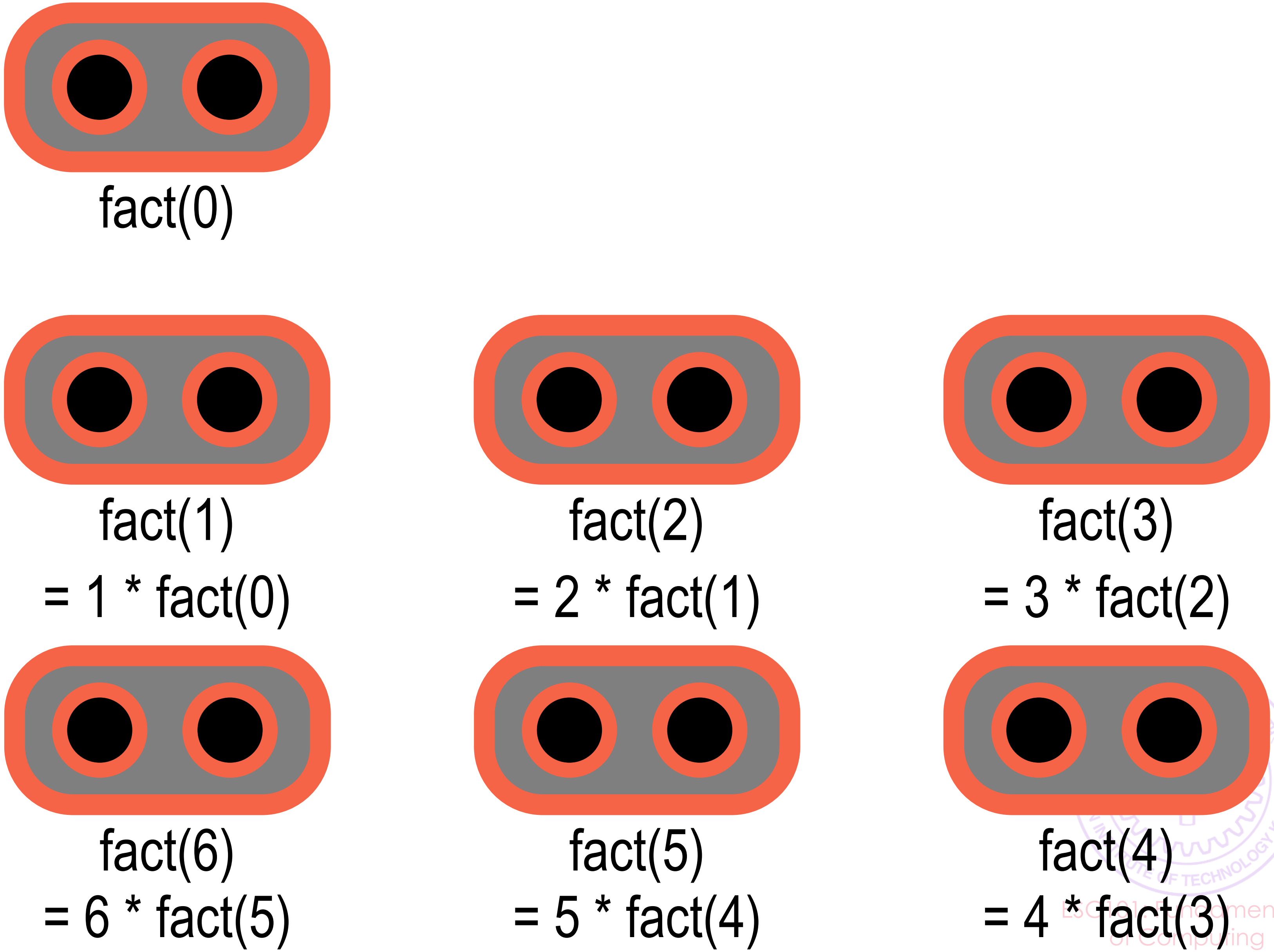

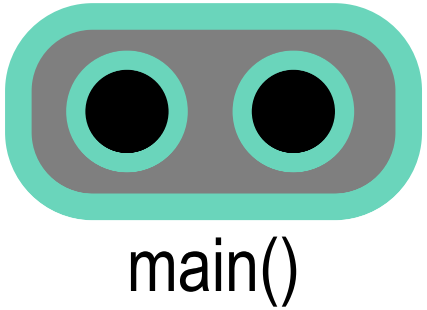

Example 1: Factorial

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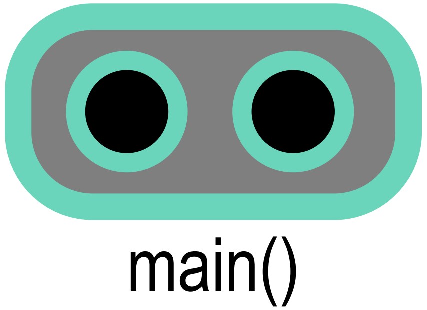

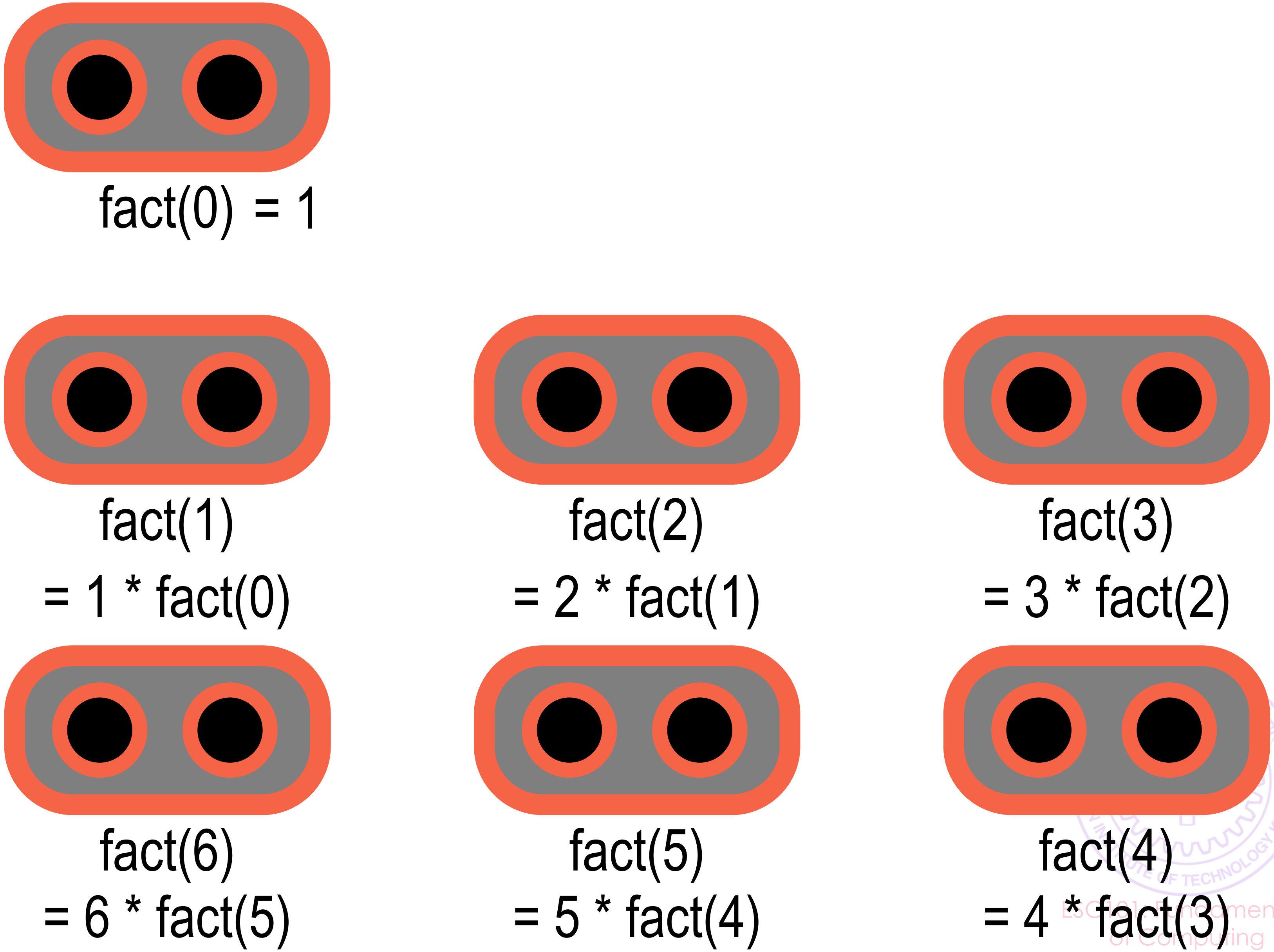

Example 1: Factorial

```
int fact(int a){
    if(a == 0) return 1;
    return a * fact(a - 1);
}
int main(){
    printf("%d", fact(2*3));
}
```

fact(0) = 1

fact(1)
= 1 * fact(0)

fact(2)
= 2 * fact(1)

fact(3)
= 3 * fact(2)

main()

fact(6)
= 6 * fact(5)

fact(5)
= 5 * fact(4)

fact(4)
= 4 * fact(3)

Example 1: Factorial

See how many clones got created!

```
int fact(int a){
    if(a == 0) return 1;
    return a * fact(a - 1);
}
int main(){
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Example 1: Factorial

See how many clones got created!

Creating each clone takes time and memory

```
int fact(int a){
    if(a == 0) return 1;
    return a * fact(a - 1);
}
int main(){
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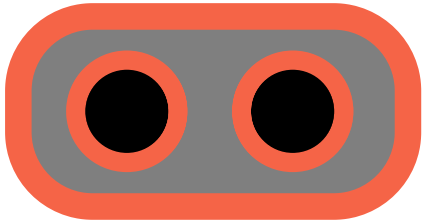
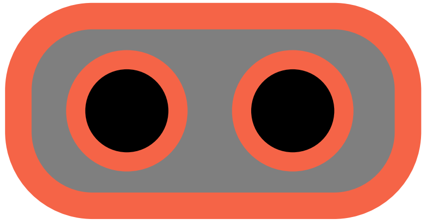
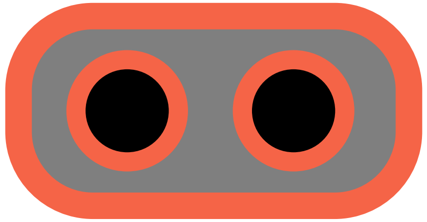
```



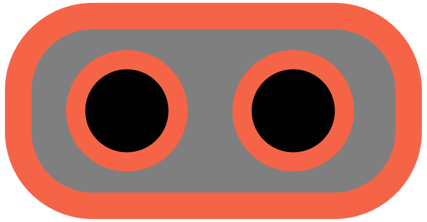
fact(0) = 1



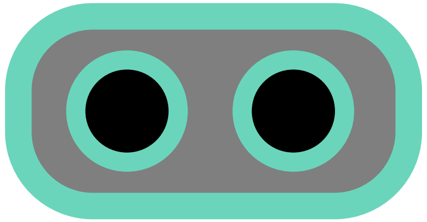
fact(1)
= 1 * fact(0)



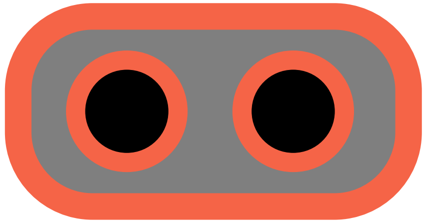
fact(2)
= 2 * fact(1)



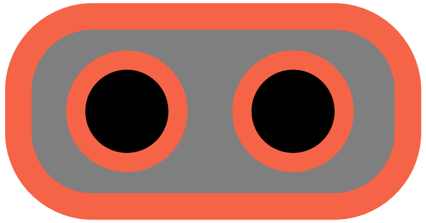
fact(3)
= 3 * fact(2)



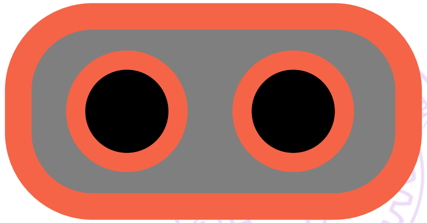
main()



fact(6)
= 6 * fact(5)



fact(5)
= 5 * fact(4)



fact(4)
= 4 * fact(3)

Example 1: Factorial

```
int fact(int a){
    if(a == 0) return 1;
    return a * fact(a - 1);
}
int main(){
    printf("%d", fact(2*3));
}
```

See how many clones got created!

Creating each clone takes time and memory

This is why recursive programs can be a bit slower – be careful

fact(0) = 1

fact(1) = 1 * fact(0)

fact(2) = 2 * fact(1)

fact(3) = 3 * fact(2)

main()

fact(6) = 6 * fact(5)

fact(5) = 5 * fact(4)

fact(4) = 4 * fact(3)

Example 1: Factorial

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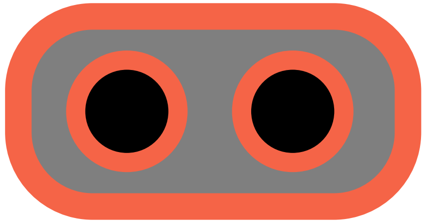
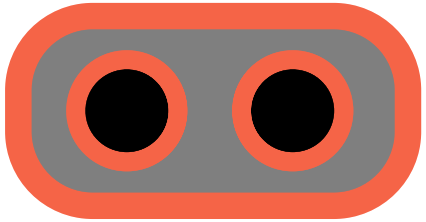
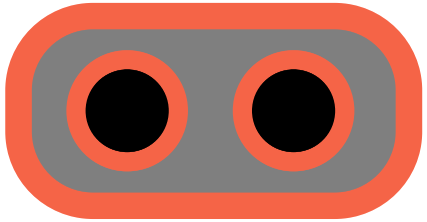


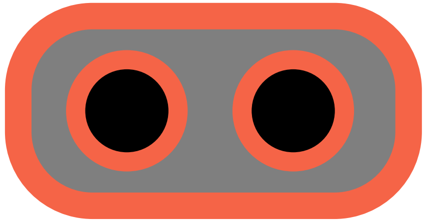
fact(0) = 1



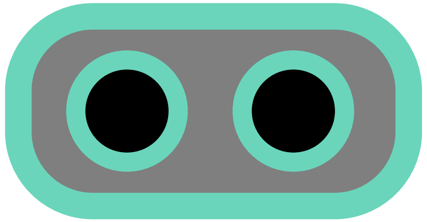
fact(1)
= 1 * 1 = 1



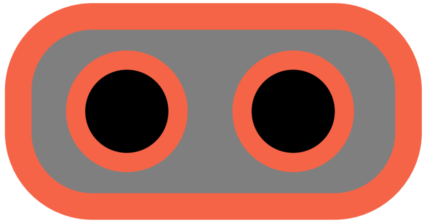
fact(2)
= 2 * fact(1)



fact(3)
= 3 * fact(2)



main()



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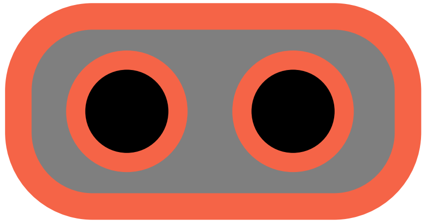

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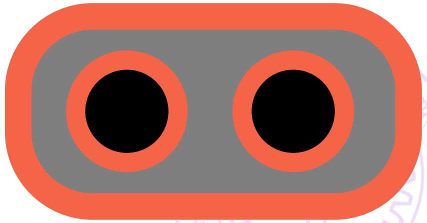

fact(4)
= 4 * fact(3)

See how many clones got created!

Creating each clone takes time and memory

This is why recursive programs can be a bit slower – be careful

Example 1: Factorial

```
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    if(a == 0) return 1;
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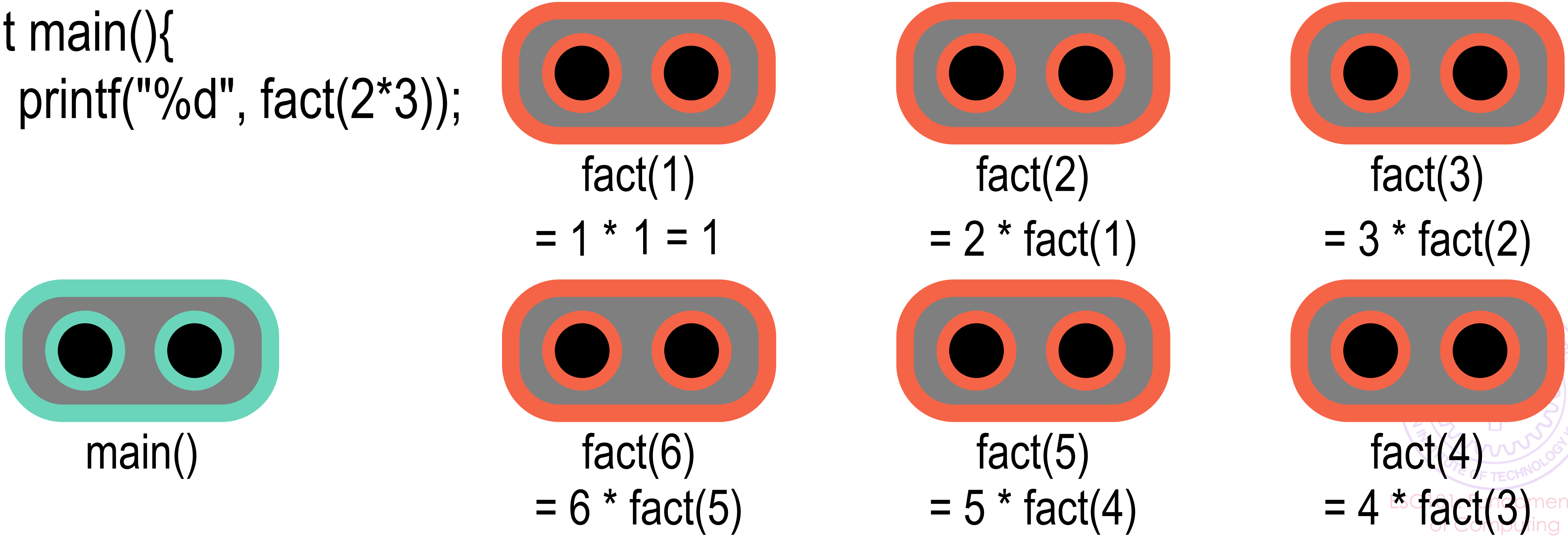

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fact(1) = 1 * 1 = 1

fact(2) = 2 * 1 = 2

fact(3) = 3 * fact(2)

main()

fact(6) = 6 * fact(5)

fact(5) = 5 * fact(4)

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Example 1: Factorial

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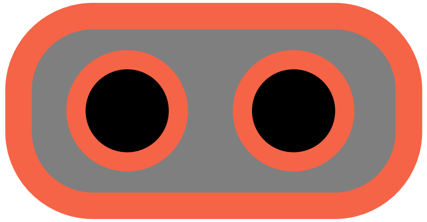
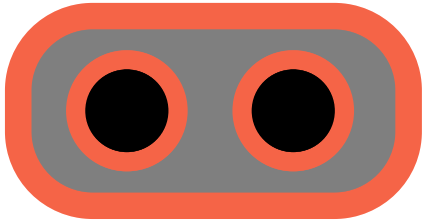
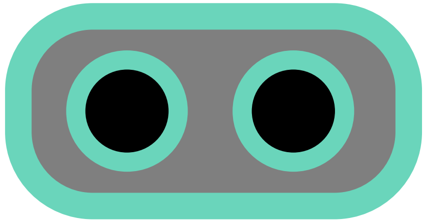


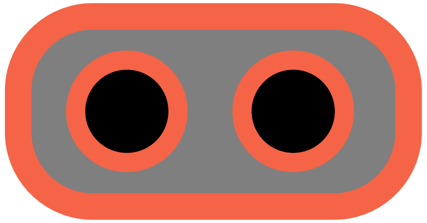
fact(2)
= 2 * 1 = 2



fact(3)
= 3 * fact(2)



main()



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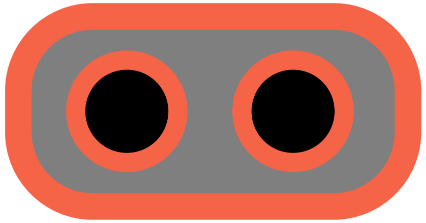

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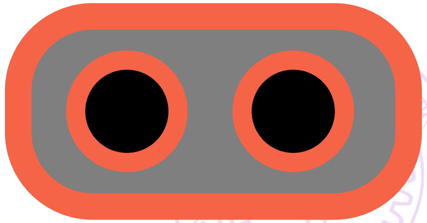

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Example 1: Factorial

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    if(a == 0) return 1;
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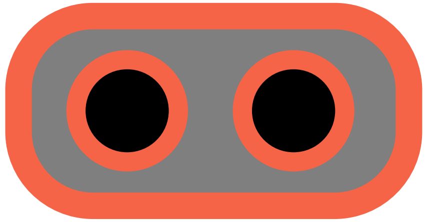
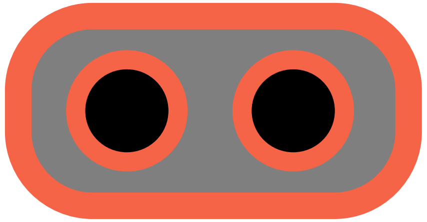
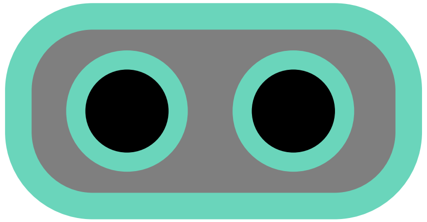


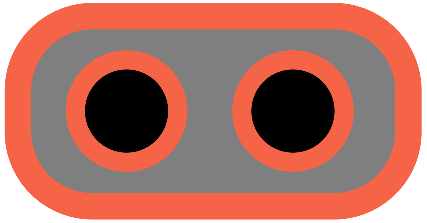
fact(2)
= 2 * 1 = 2



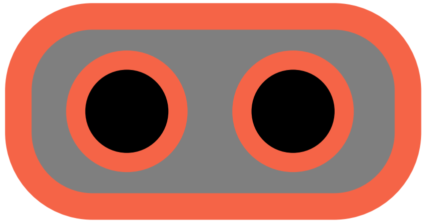
fact(3)
= 3 * 2 = 6



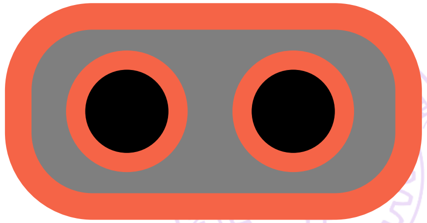
main()



fact(6)
= 6 * fact(5)



fact(5)
= 5 * fact(4)



fact(4)
= 4 * fact(3)

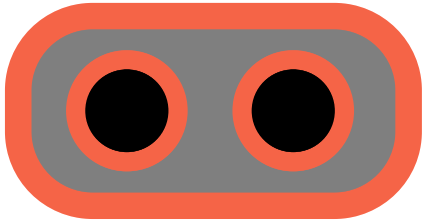
Example 1: Factorial

```
int fact(int a){
    if(a == 0) return 1;
    return a * fact(a - 1);
}
int main(){
    printf("%d", fact(2*3));
}
```

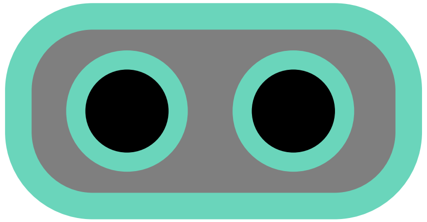
See how many clones got created!

Creating each clone takes time and memory

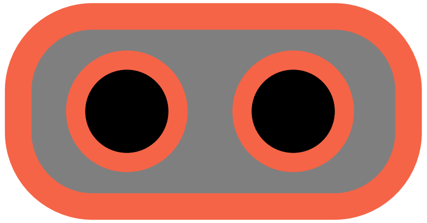
This is why recursive programs can be a bit slower – be careful



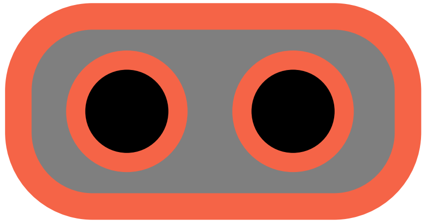
fact(3)
= 3 * 2 = 6



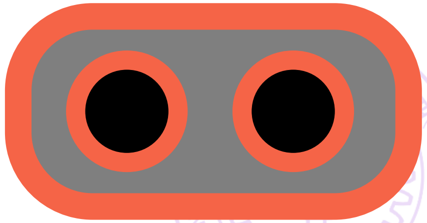
main()



fact(6)
= 6 * fact(5)



fact(5)
= 5 * fact(4)



fact(4)
= 4 * fact(3)

Example 1: Factorial

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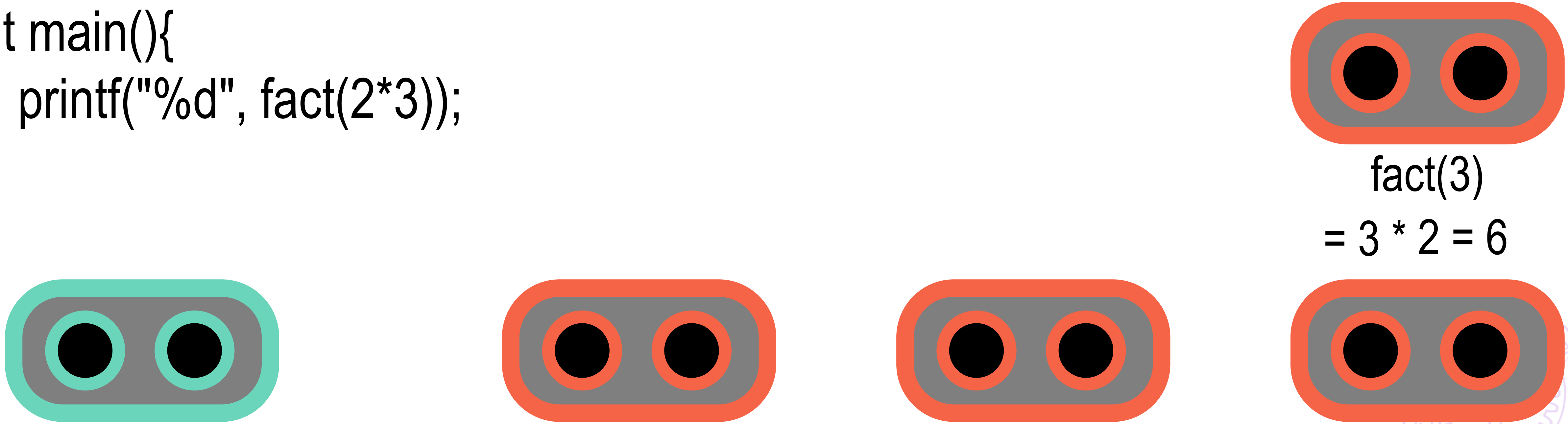

fact(3)
= 3 * 2 = 6

main()

fact(6)
= 6 * fact(5)

fact(5)
= 5 * fact(4)

fact(4)
= 4 * 6 = 24

Example 1: Factorial

```
int fact(int a){
    if(a == 0) return 1;
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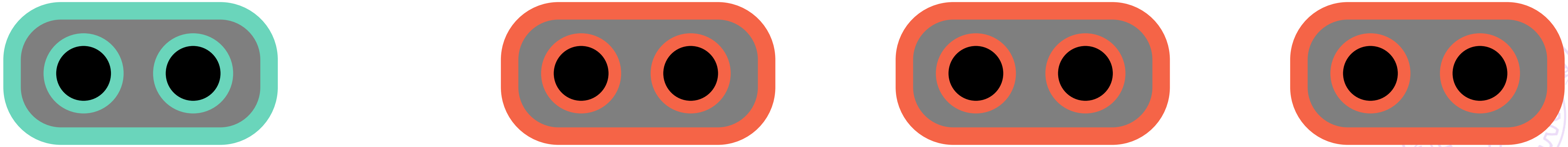

main()

fact(6)
= 6 * fact(5)

fact(5)
= 5 * fact(4)

fact(4)
= 4 * 6 = 24

Example 1: Factorial

```
int fact(int a){
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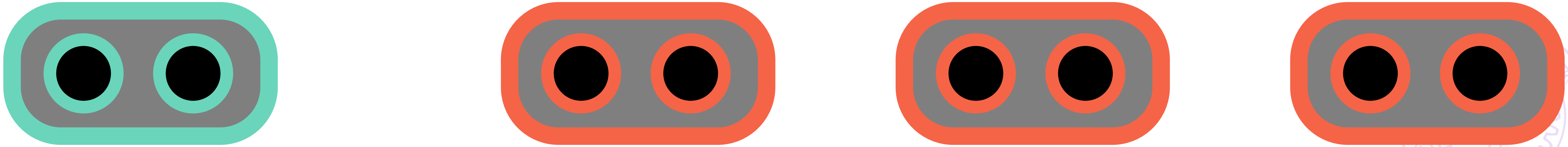

main()

fact(6)
= 6 * fact(5)

fact(5)
= 5 * 24 = 120

fact(4)
= 4 * 6 = 24

Example 1: Factorial

```
int fact(int a){
    if(a == 0) return 1;
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}
int main(){
    printf("%d", fact(2*3));
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```

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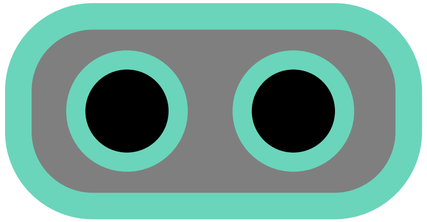
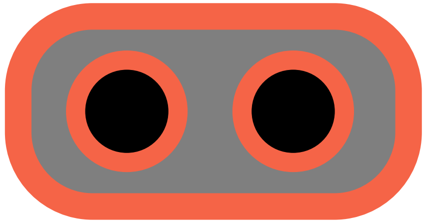
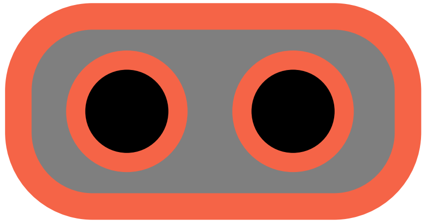


main()



fact(6)
= 6 * fact(5)



fact(5)
= 5 * 24 = 120

Example 1: Factorial

```
int fact(int a){
    if(a == 0) return 1;
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}
int main(){
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```

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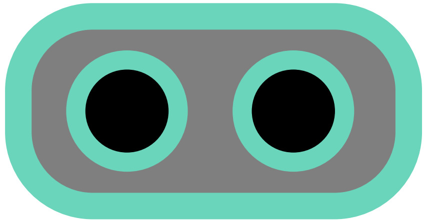

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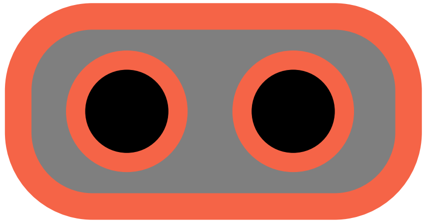
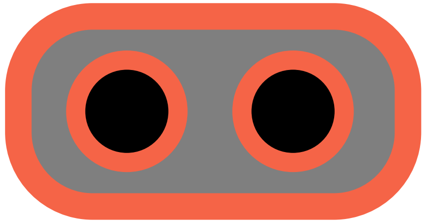


fact(6)
= 6 * 120 = 720



fact(5)
= 5 * 24 = 120

Example 1: Factorial

```
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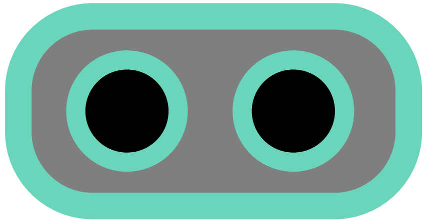
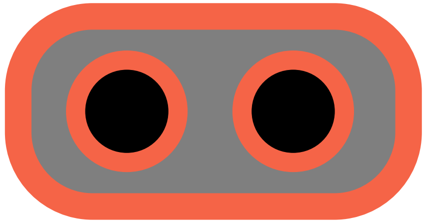


main()



fact(6)
= 6 * 120 = 720

Example 1: Factorial

```
int fact(int a){
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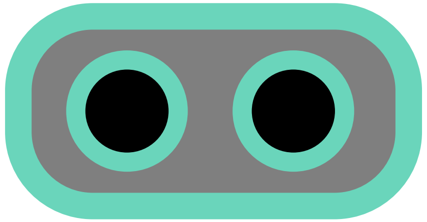

main()

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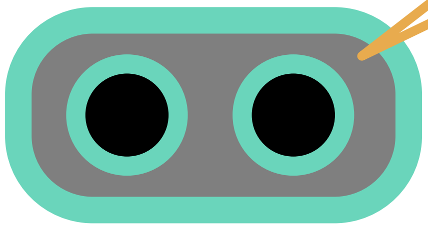
720



main()

Example 2: Fibonacci Numbers

Example 2: Fibonacci Numbers

There are two base cases for Fibonacci numbers

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- The first Fibonacci number is defined to be 0

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Recursive case: for n > 2, n-th Fibonacci number is the sum of the previous two Fibonacci numbers

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int fib(int n){
    if(n == 1) return 0;
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int main(){
    printf("%d", fib(5));
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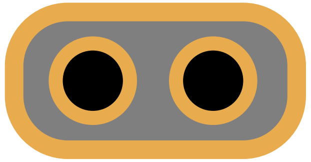

fib(5)

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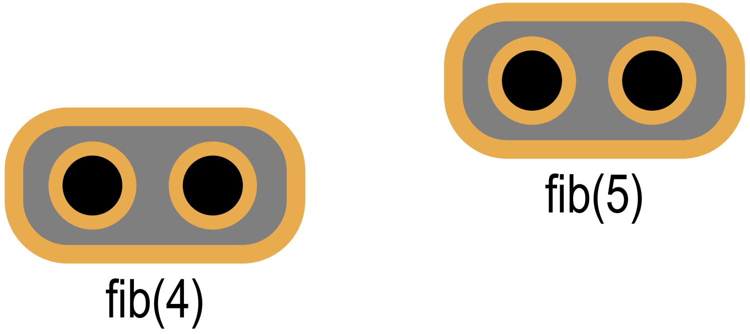

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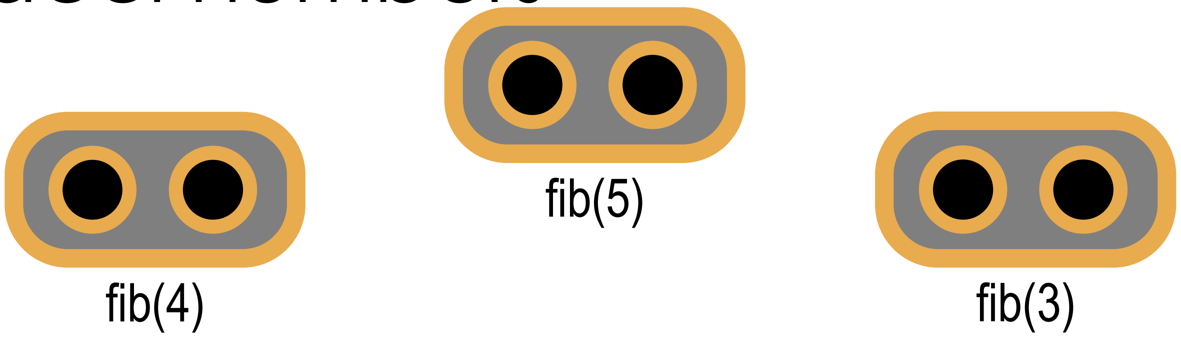

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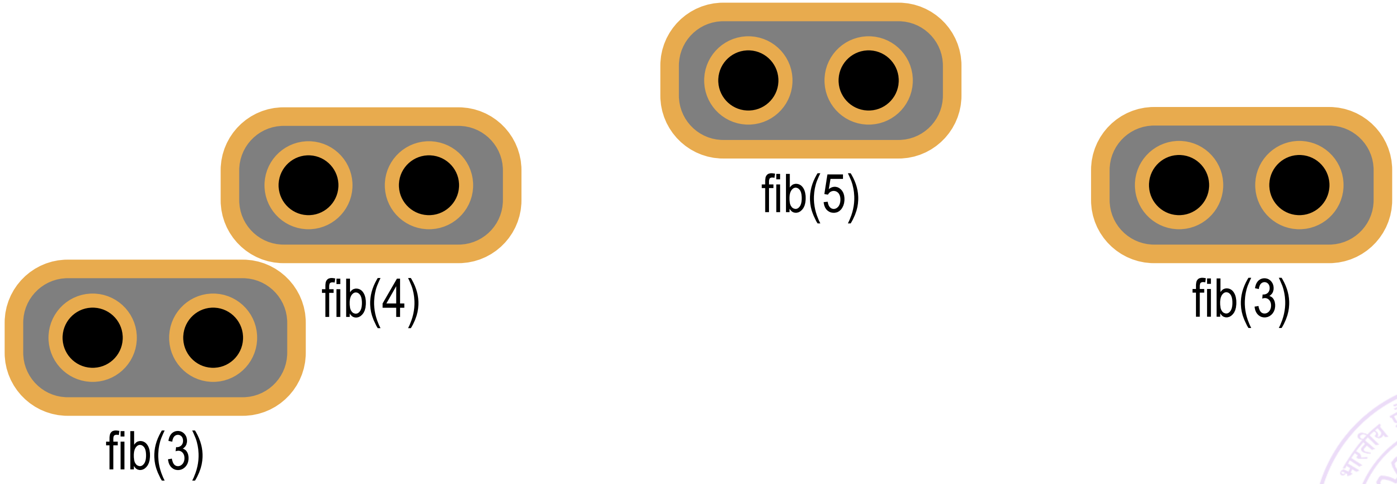

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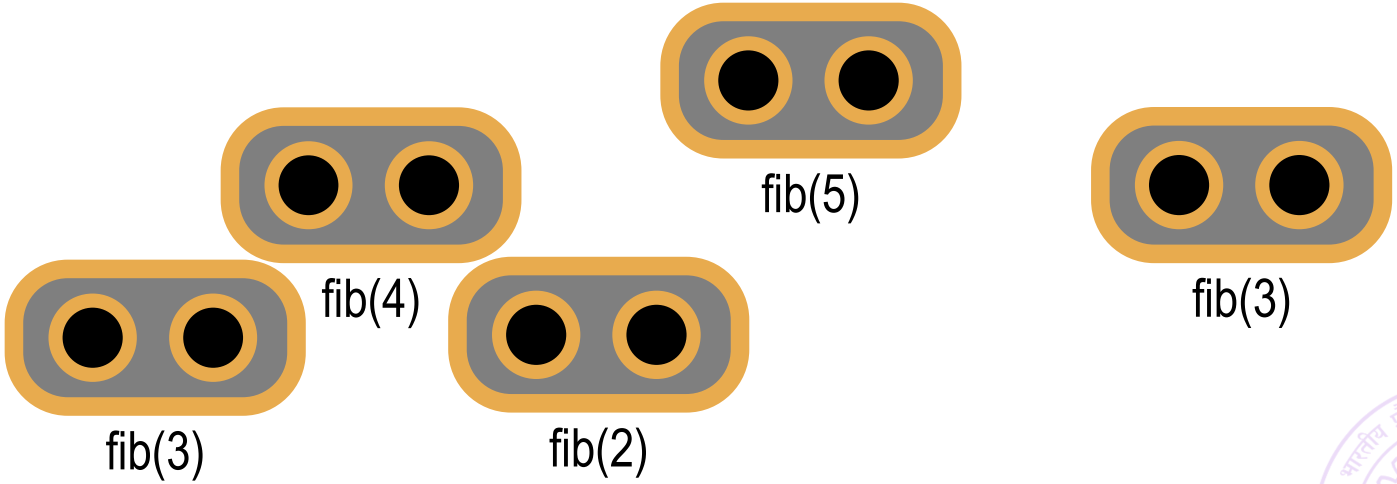

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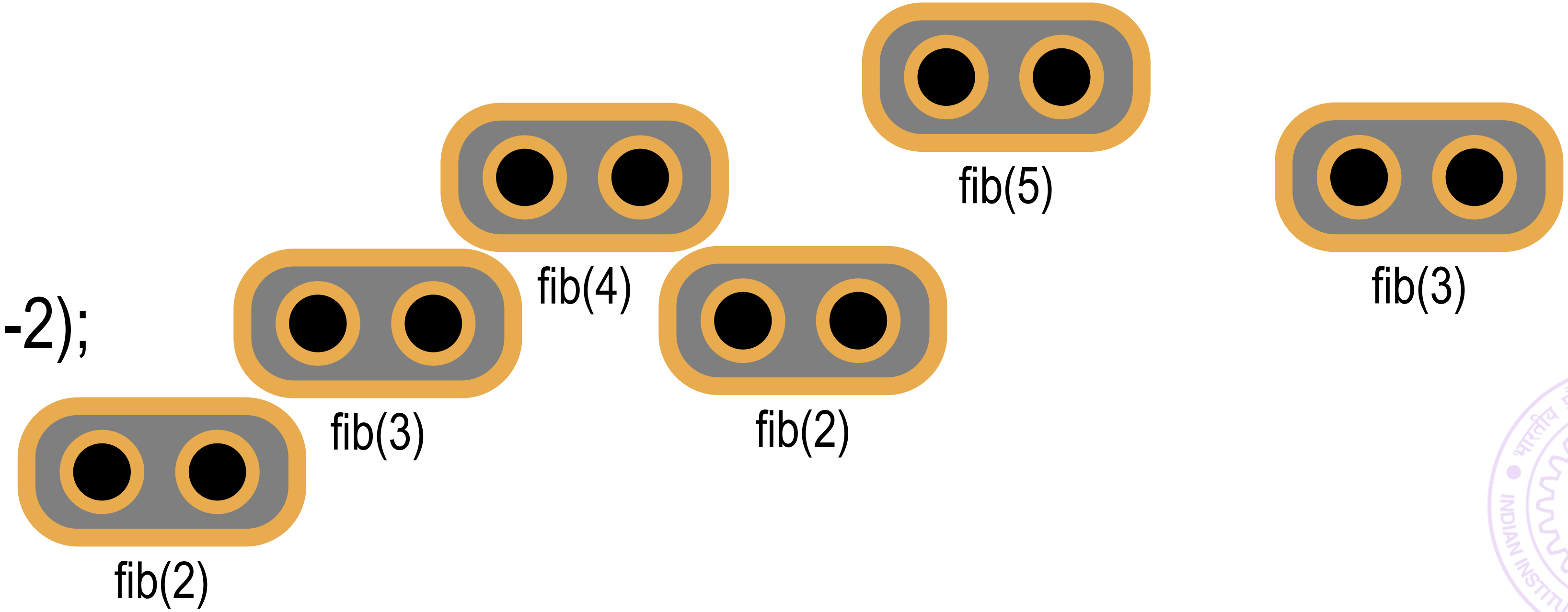

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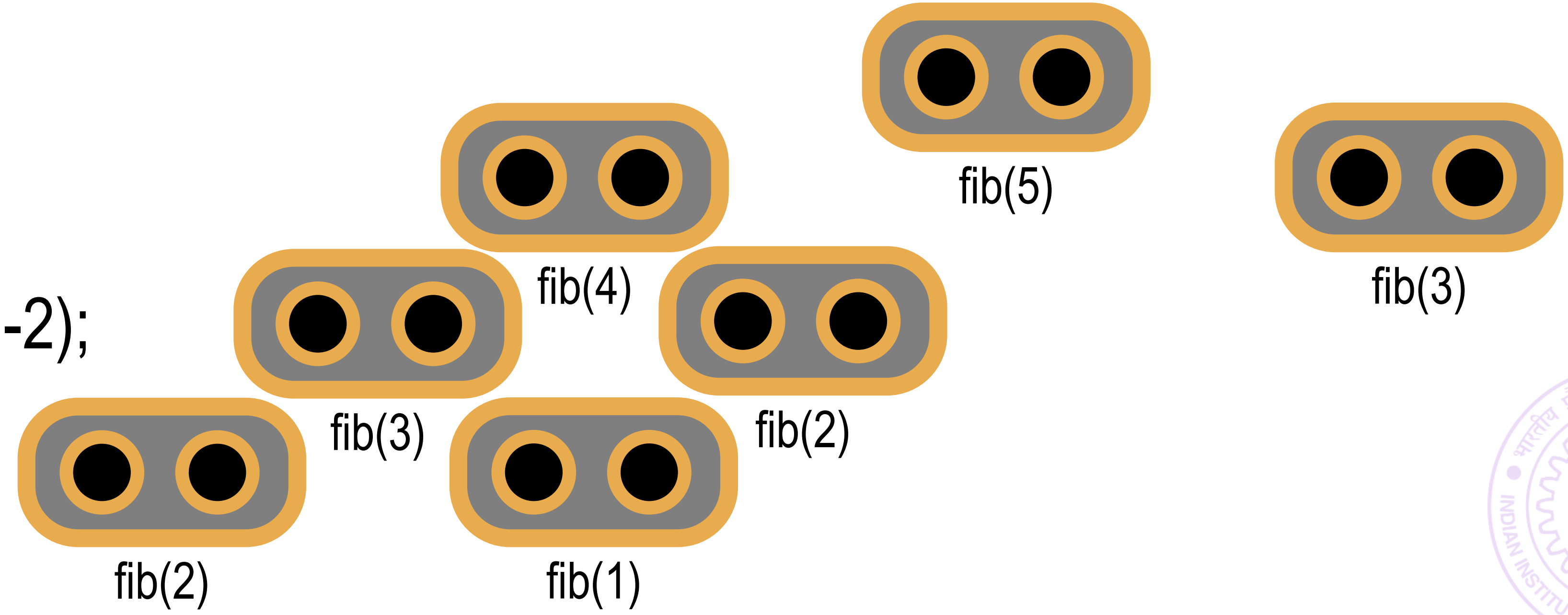

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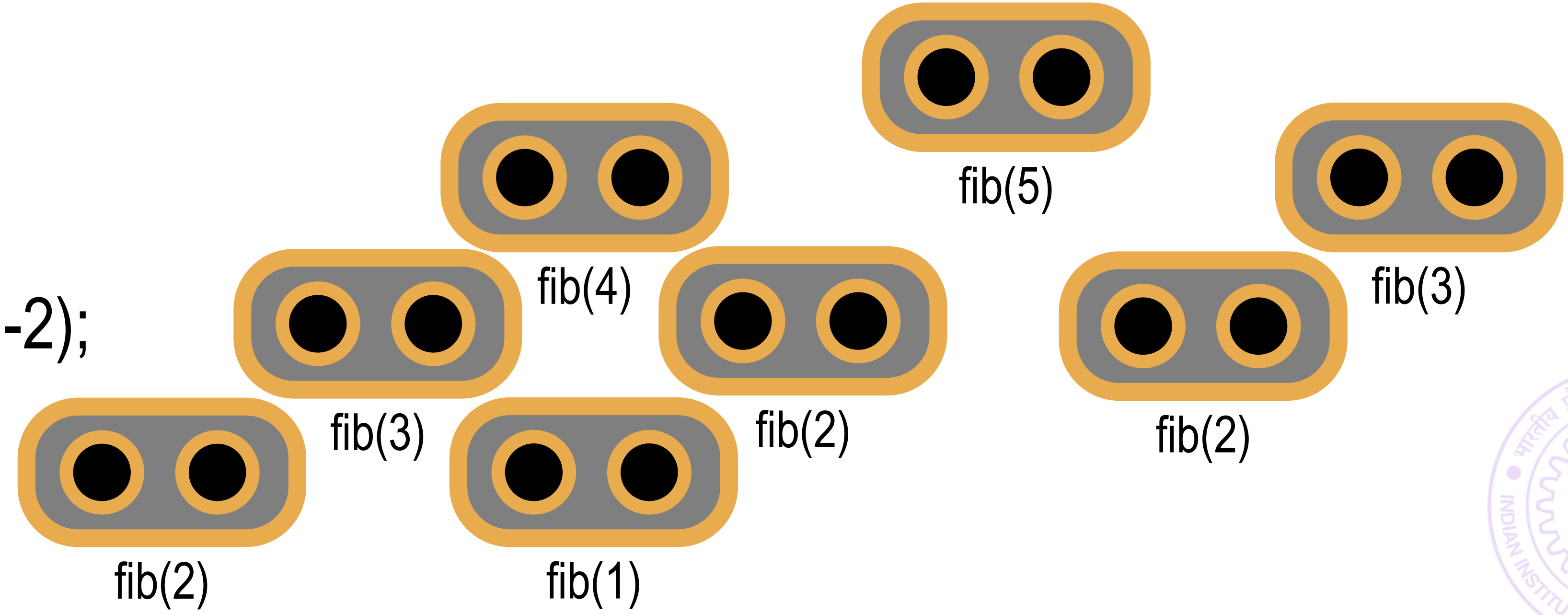

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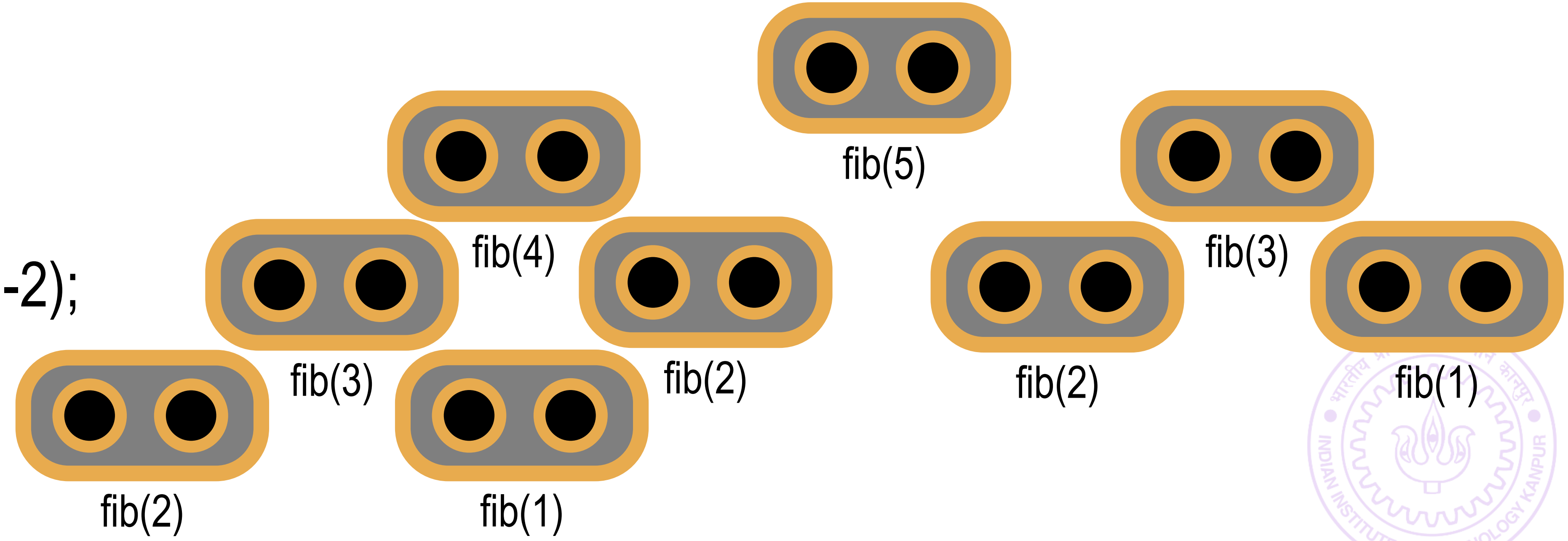

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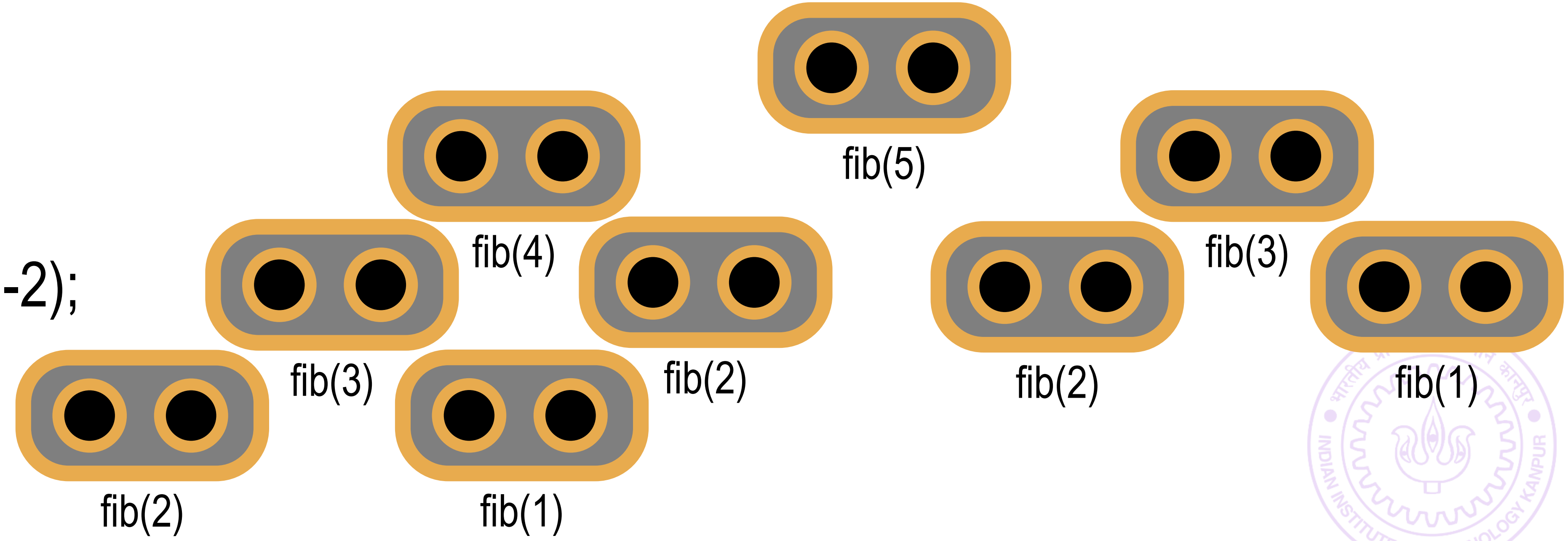

Example 2: Fibonacci Numb

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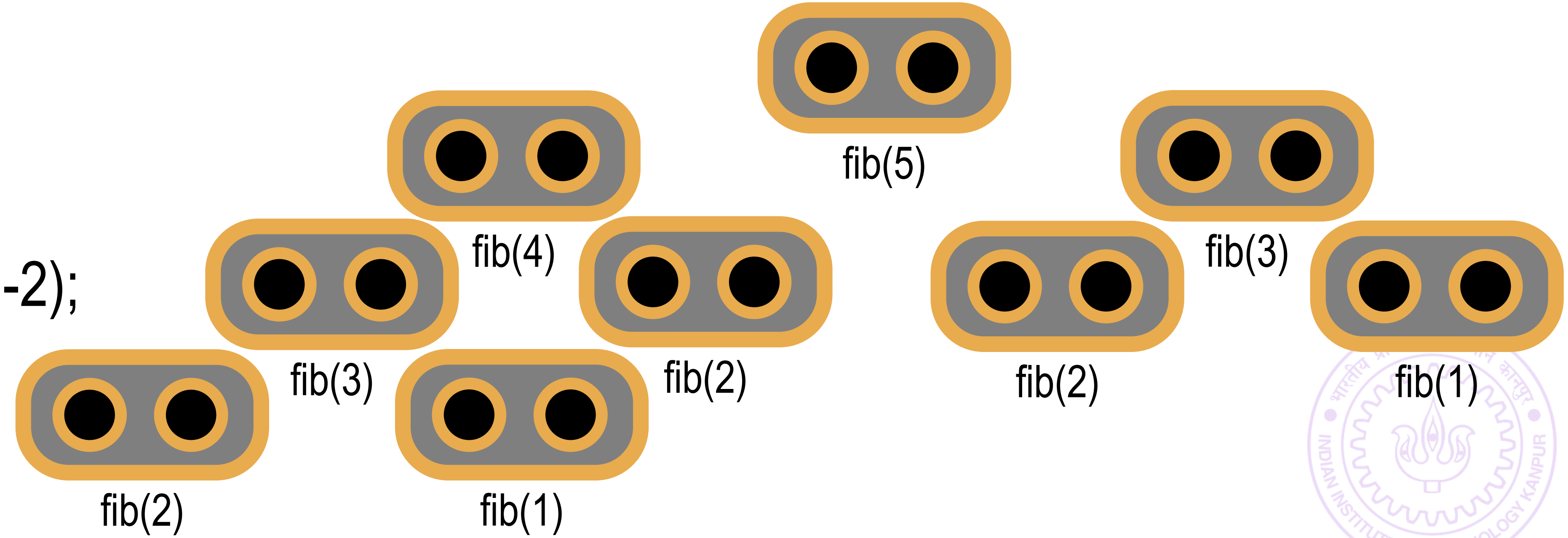
```



Example 2: Fibo

Wasted effort too!
fib(1) calculated twice
fib(2) calculated 3 times
fib(3) calculated twice

Explosion of clones!

There are two base cas umbers
- The first Fibonacci number is defined to be 0
- The second Fibonacci number is defined to be 1

Recursive case: for n > 2, n-th Fibonacci number is the sum of the previous two Fibonacci numbers

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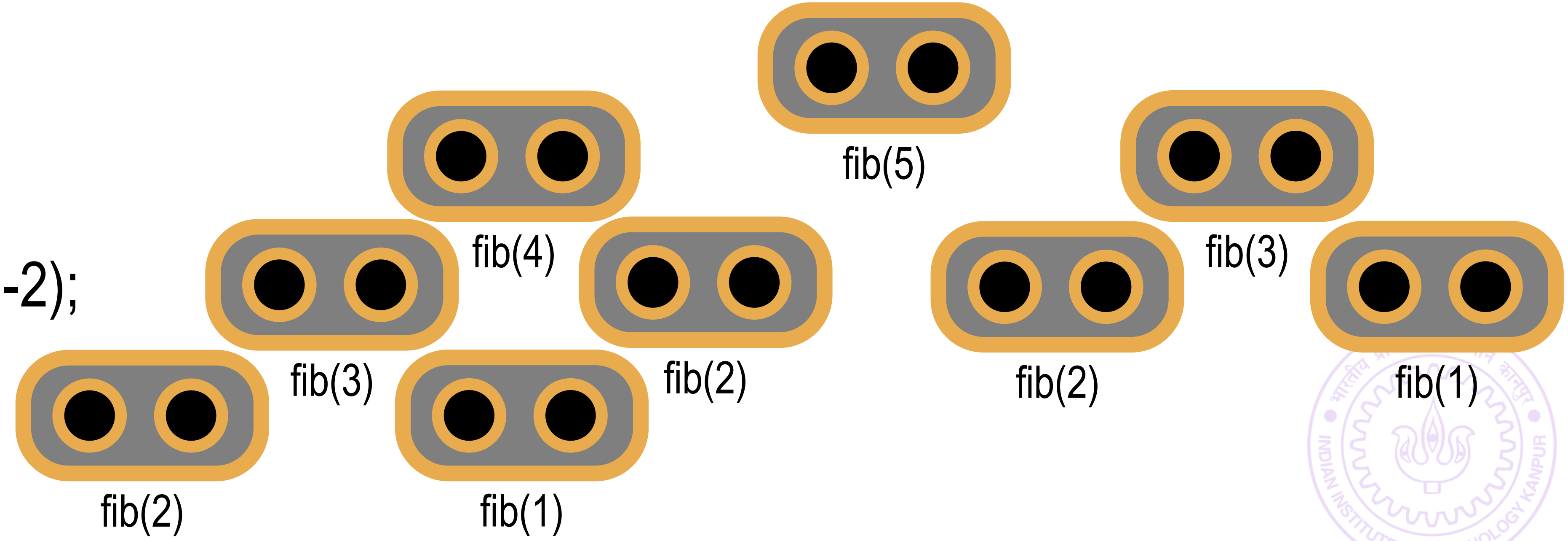

Imagine the number of clones to calculate fib(100). **Challenge**: don't imagine – find out ☺

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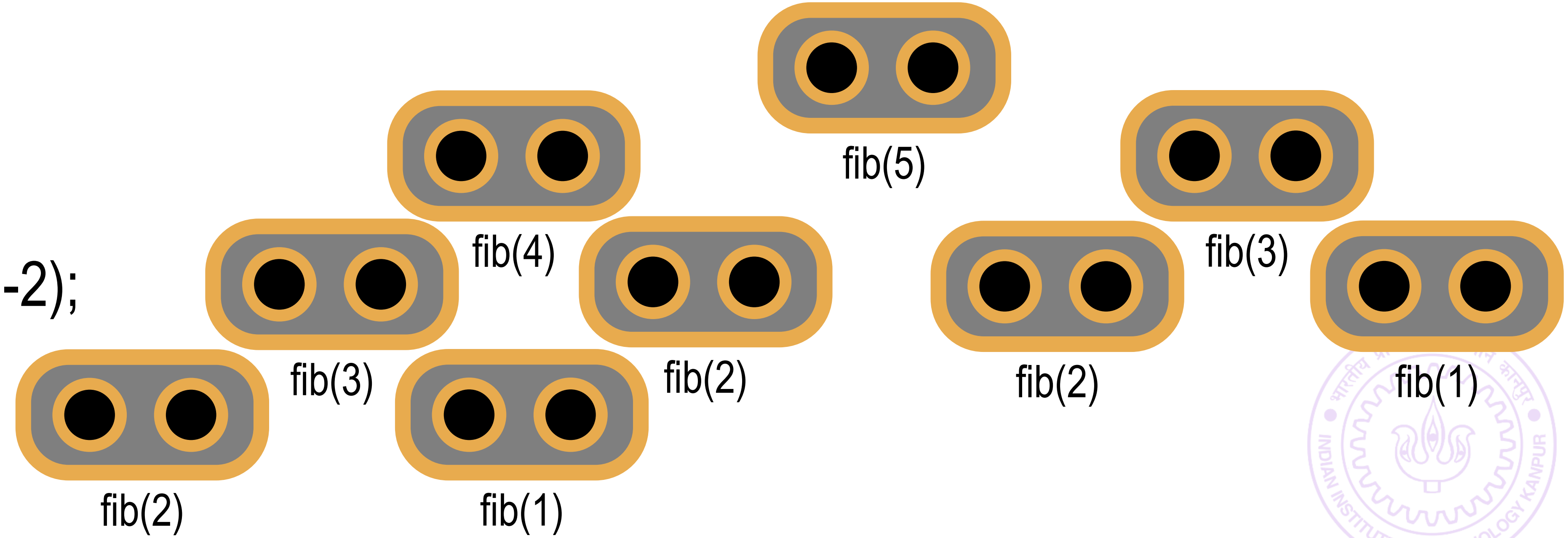

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- ...econd Fibonacci number is defined to be 1

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```
... n){
    if(n == 1) return 0;
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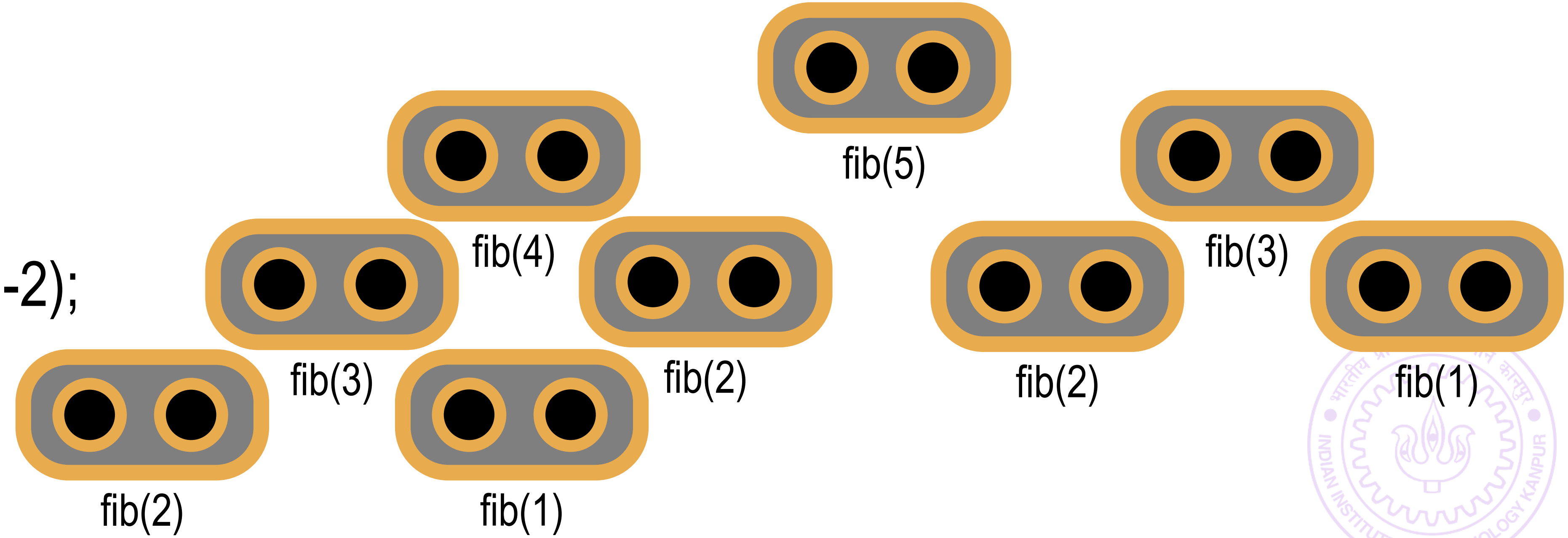

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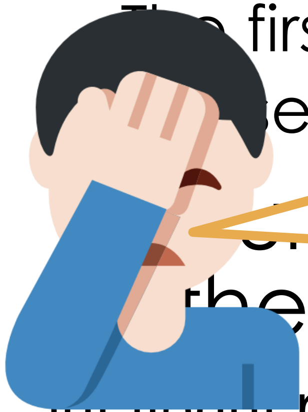
...seco... fined to be 1

...h Fibonacci number is the sum ...the previous two Fibonacci numbers



I could have easily solved this problem using a for loop – much faster and no clones

```
int fib(int n){
    if(n == 1) return 0;
    if(n == 2) return 1;
    return fib(n-1) + fib(n-2);
}
int main(){
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```

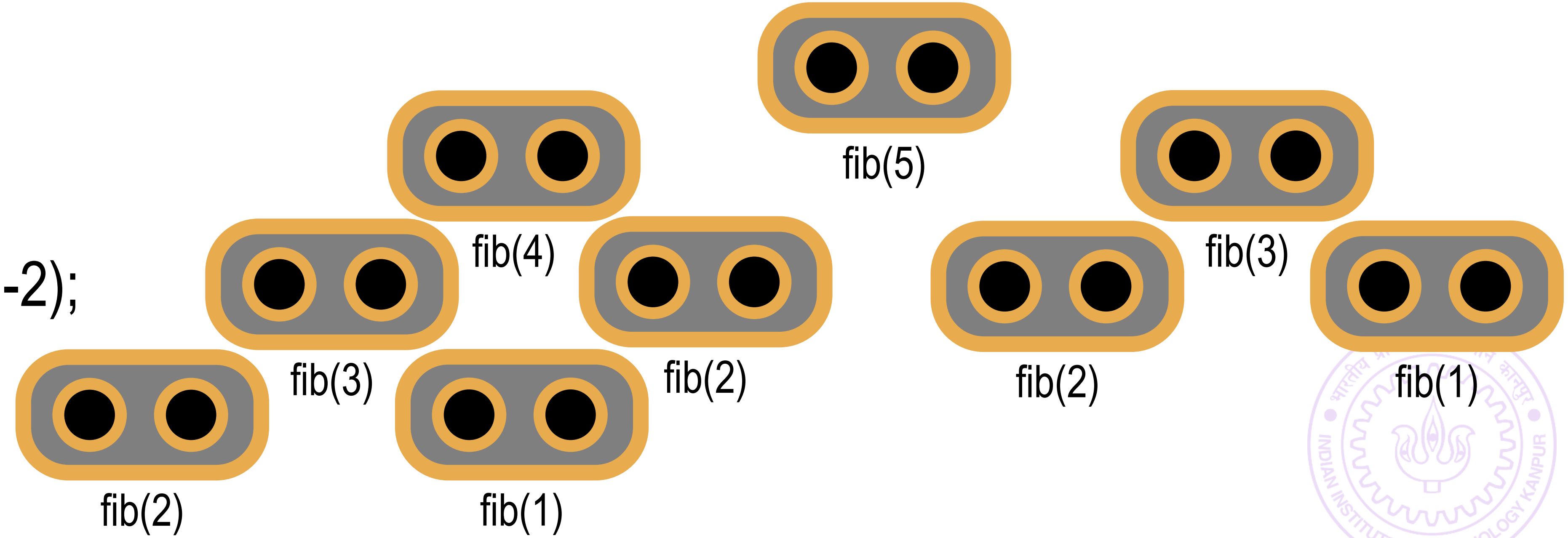

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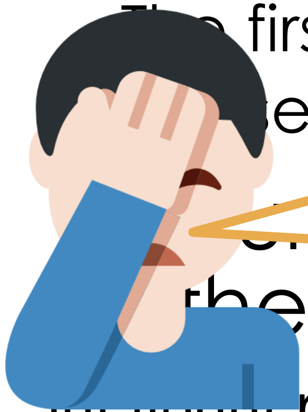
Explosion of clones!

There are two base cas... umbers

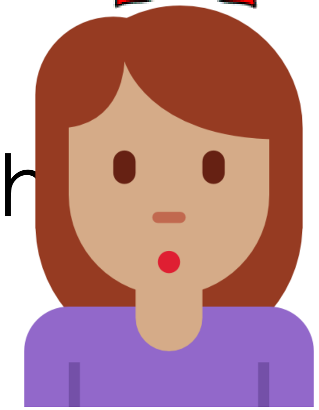
The first Fibonacci number is defined to be 0



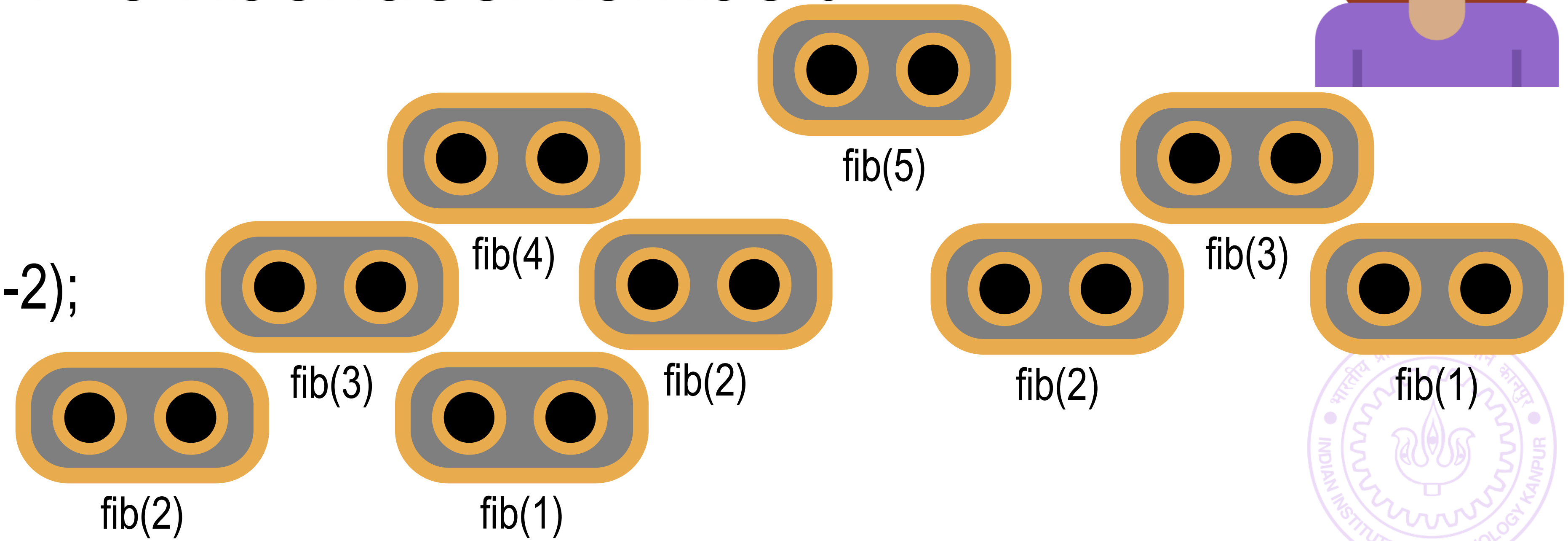
I could have easily solved this problem using a for loop – much faster and no clones

...seco... fined to be 1

...h Fibonacci number is th... the previous two Fibonacci numbers



```
int fib(int n){
    if(n == 1) return 0;
    if(n == 2) return 1;
    return fib(n-1) + fib(n-2);
}
int main(){
    printf("%d", fib(5));
}
```

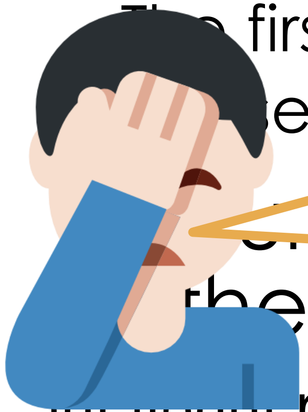


Imagine the number of clones to calculate fib(100). **Challenge**: don't imagine – find out ☺

Wasted effort too!
fib(1) calculated twice
fib(2) calculated 3 times
fib(3) calculated twice

Explosion of clones!

the previous two Fibonacci numbers



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That's why we were warned. Recursion allows neat code but sometimes can be slower

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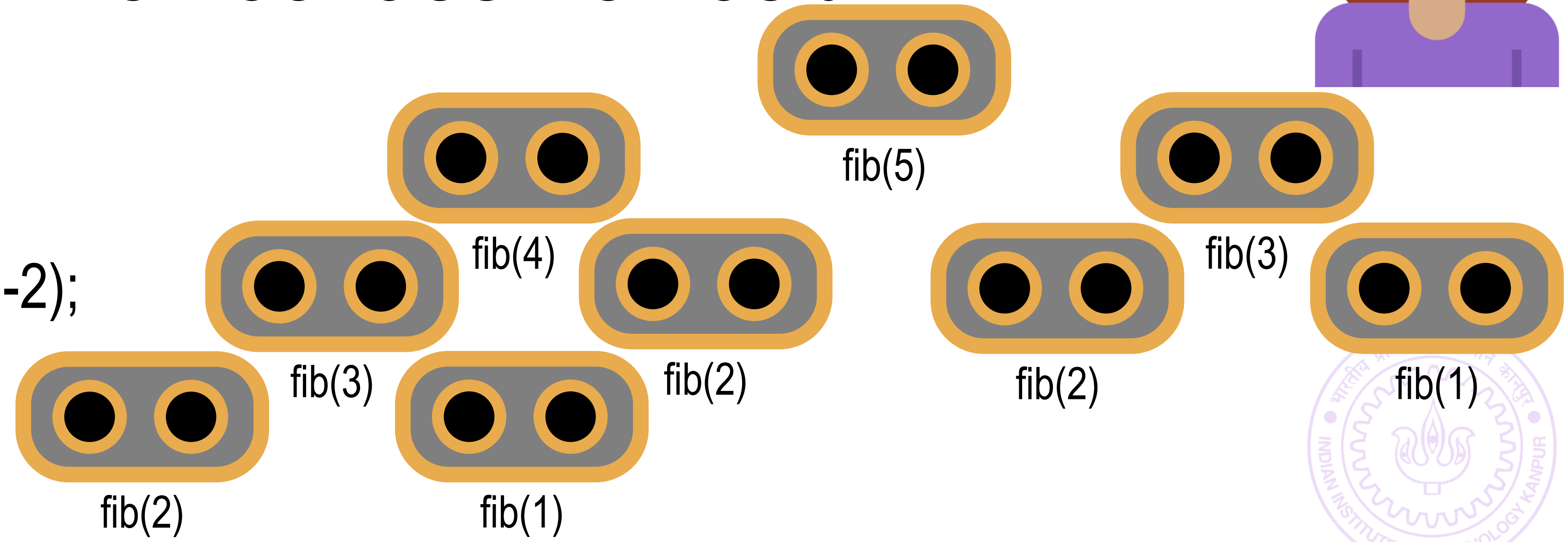

Attack of the Clones

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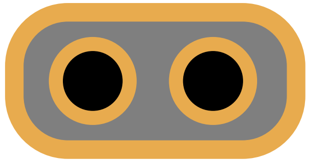
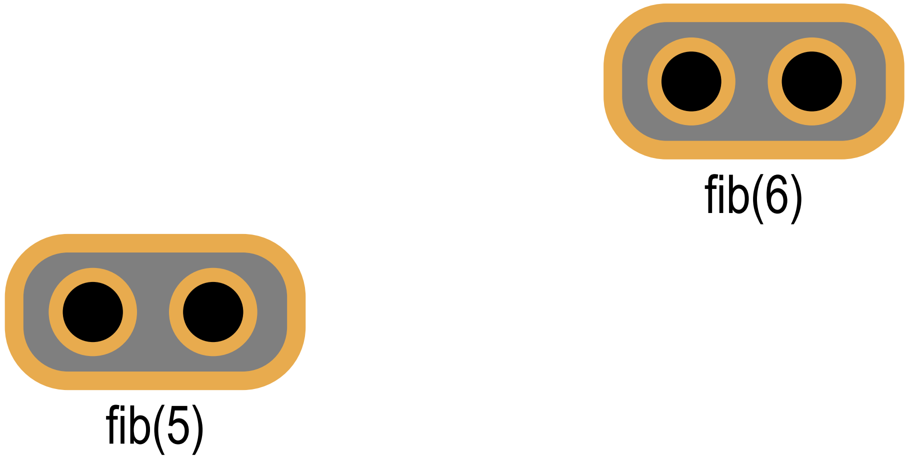


fib(6)

Attack of the Clones



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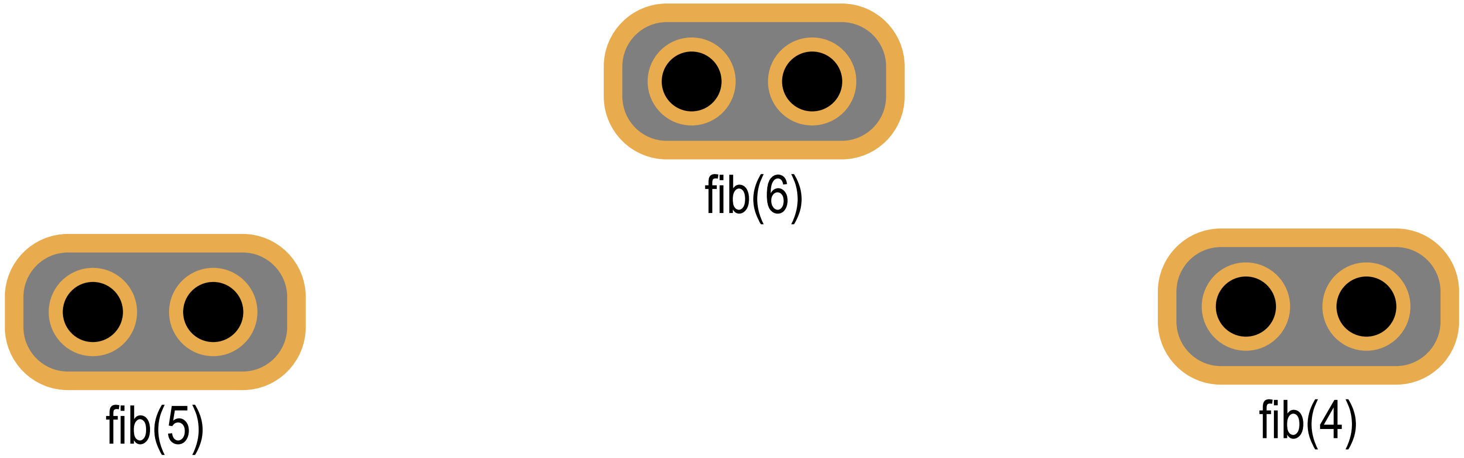

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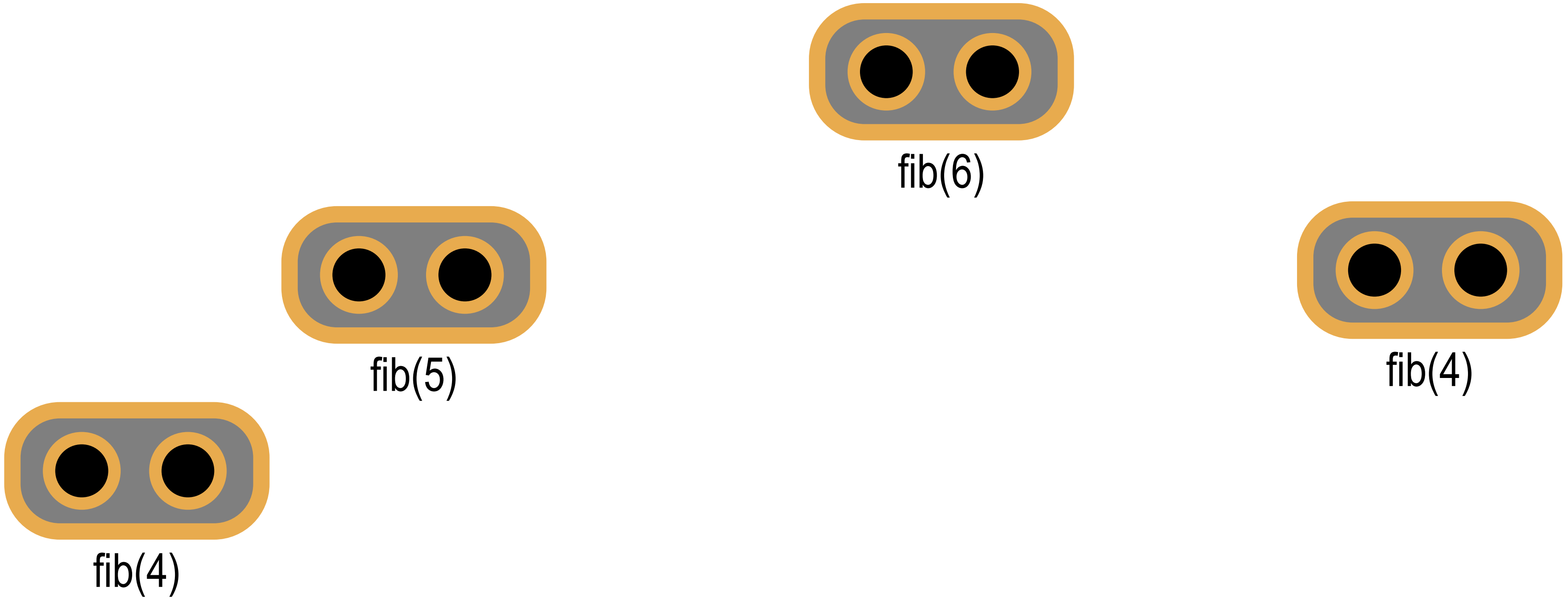

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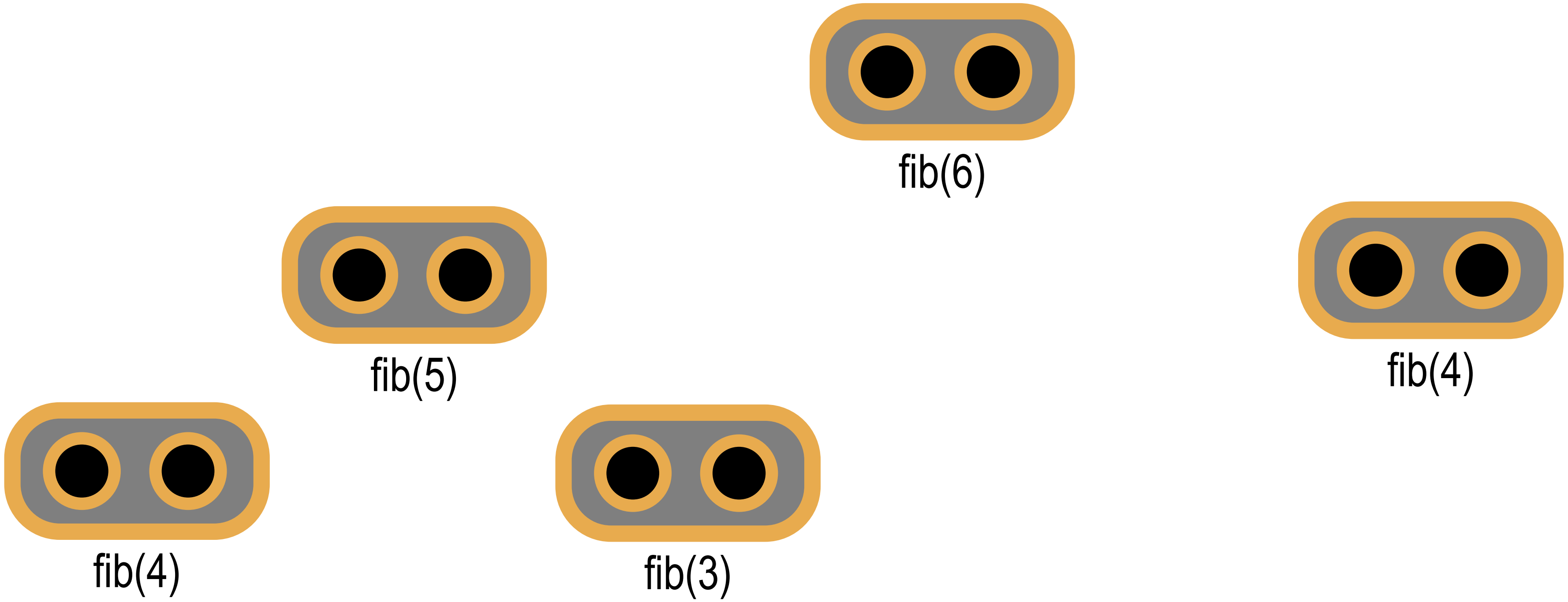

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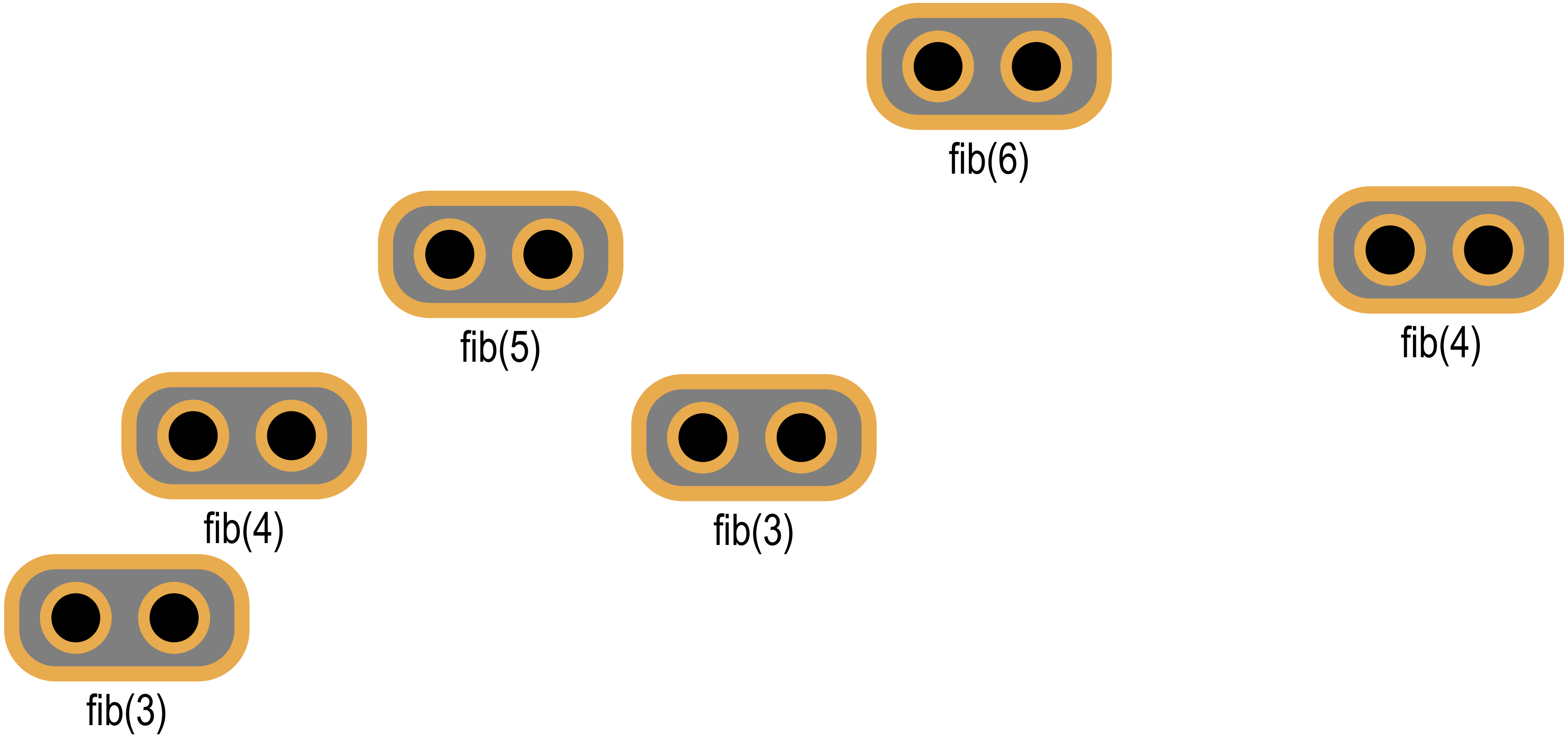

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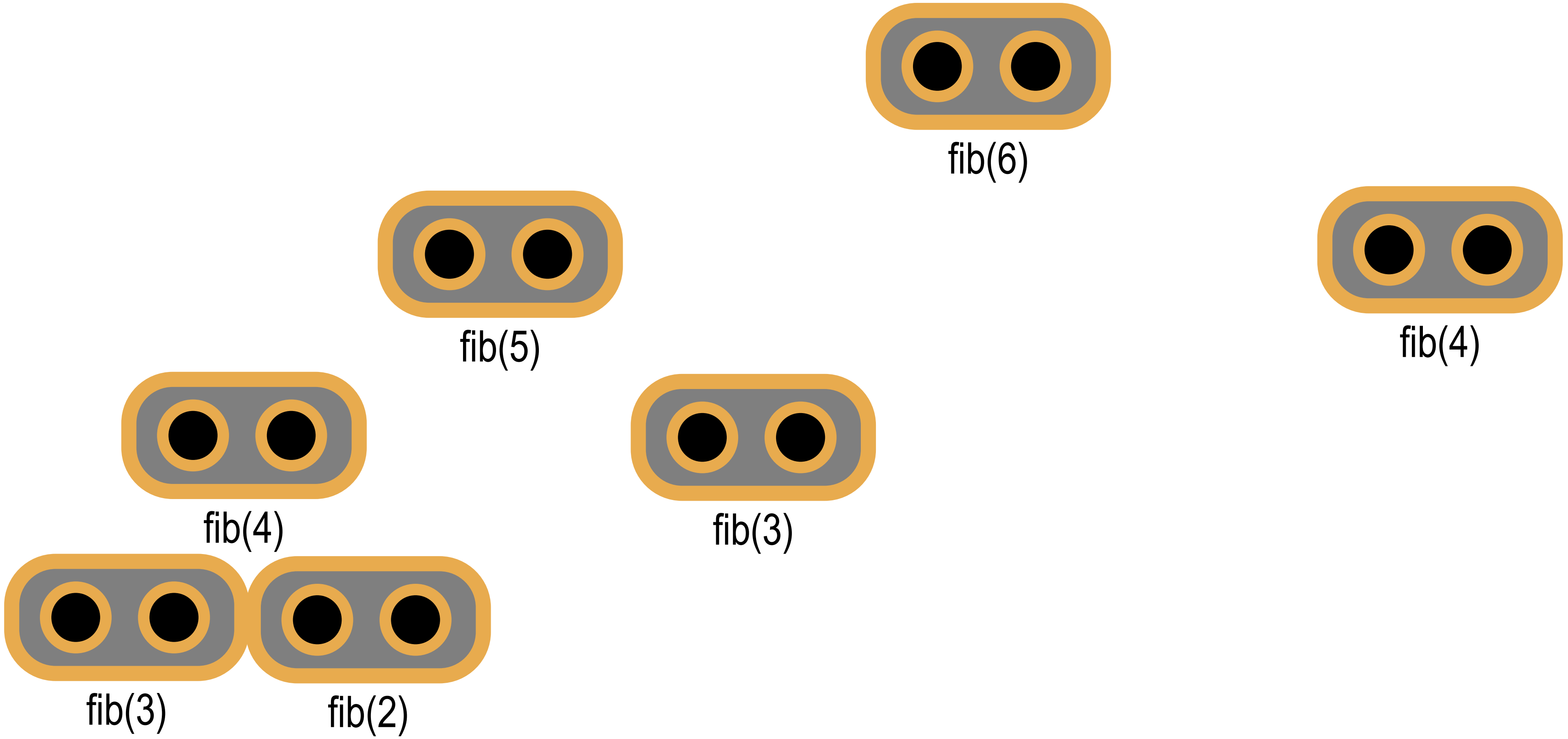

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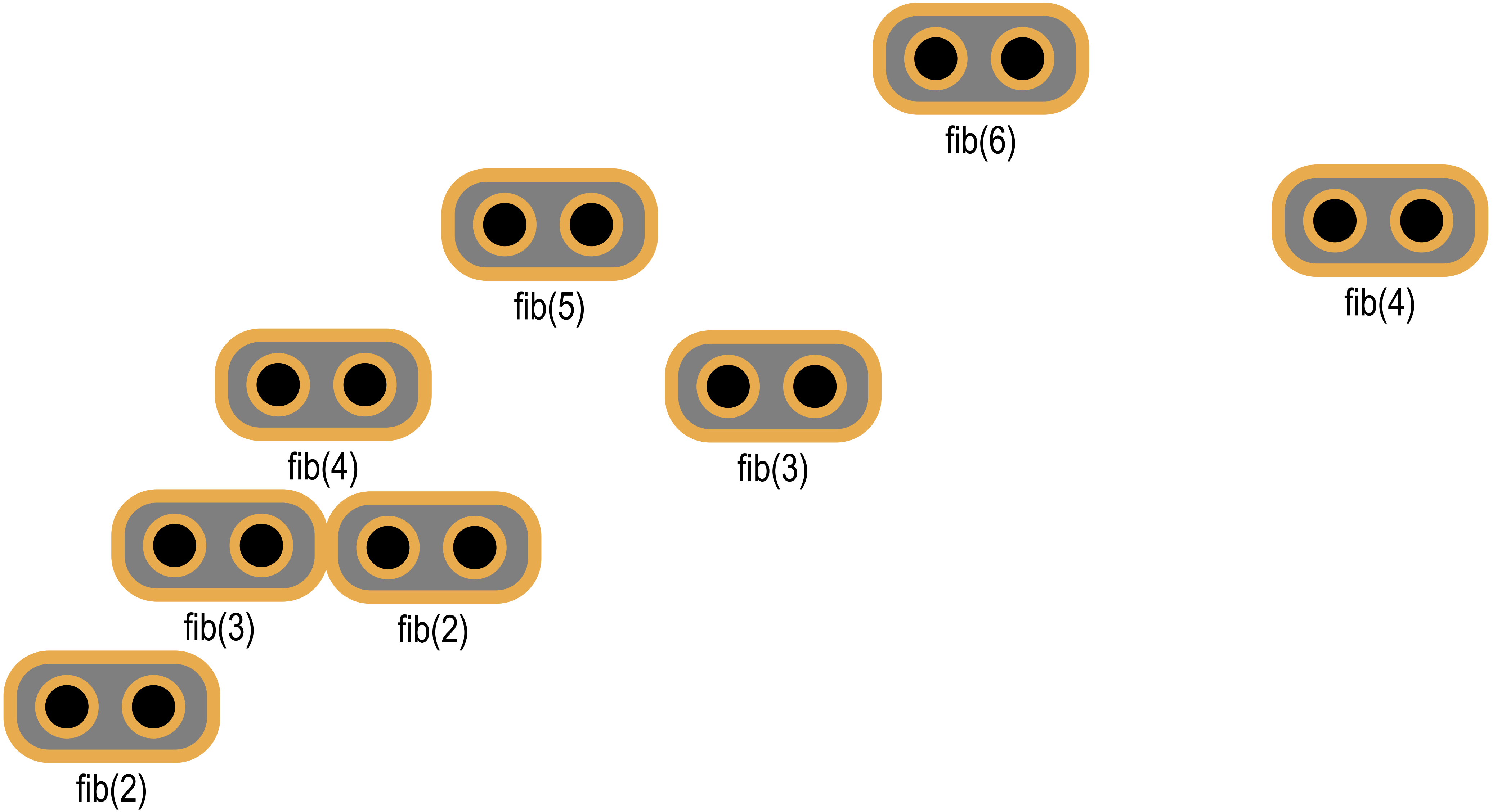

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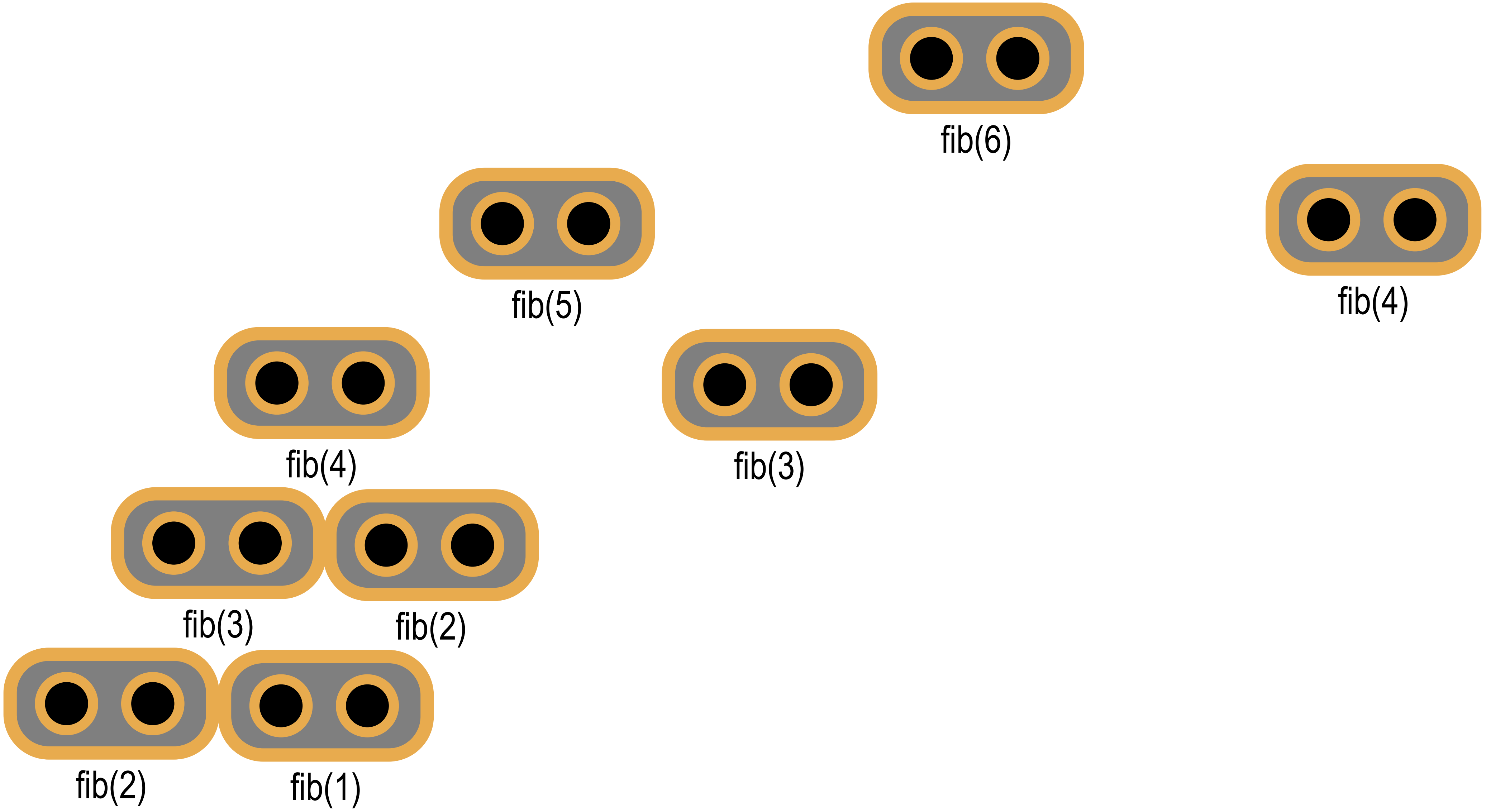

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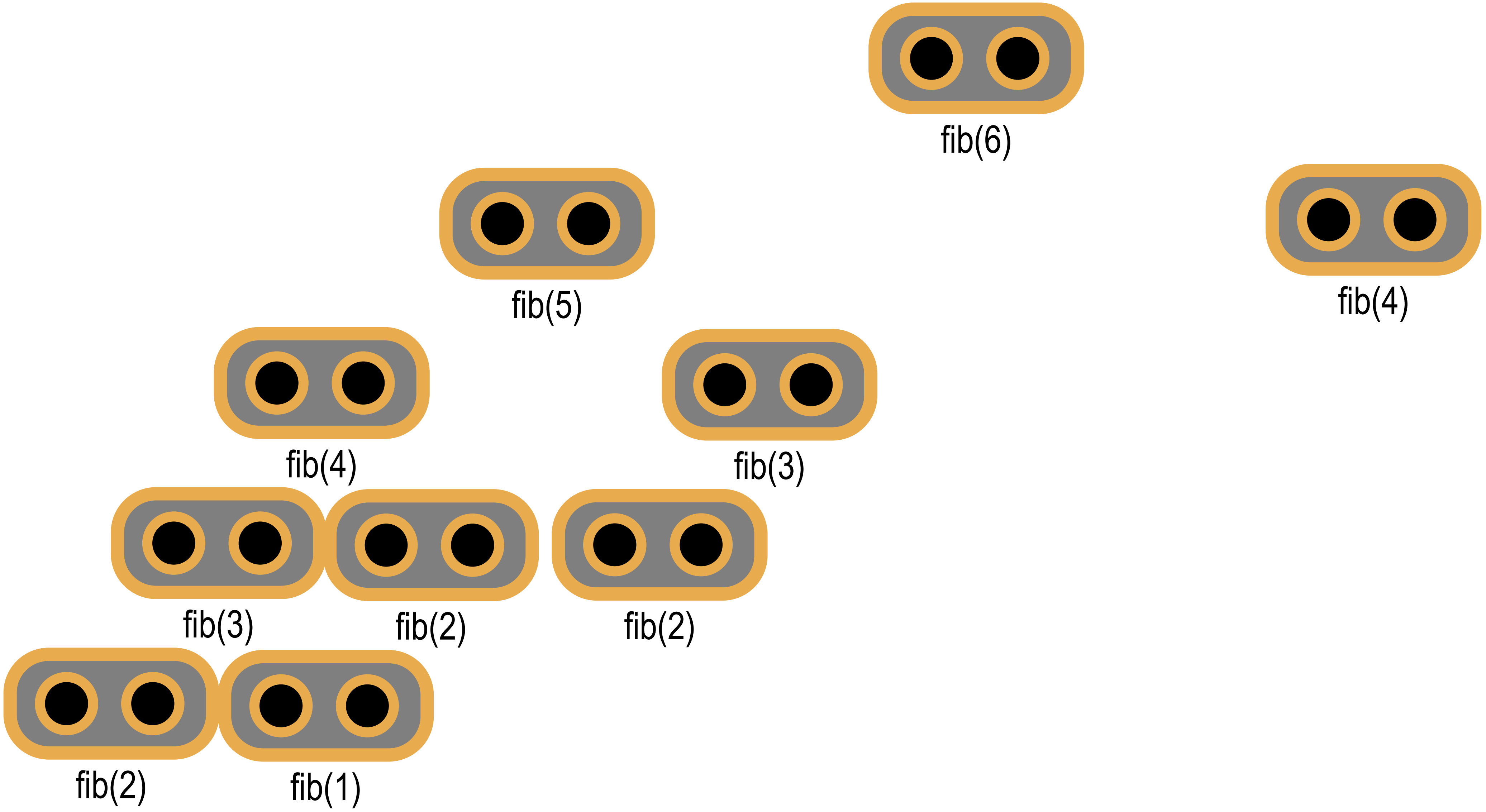

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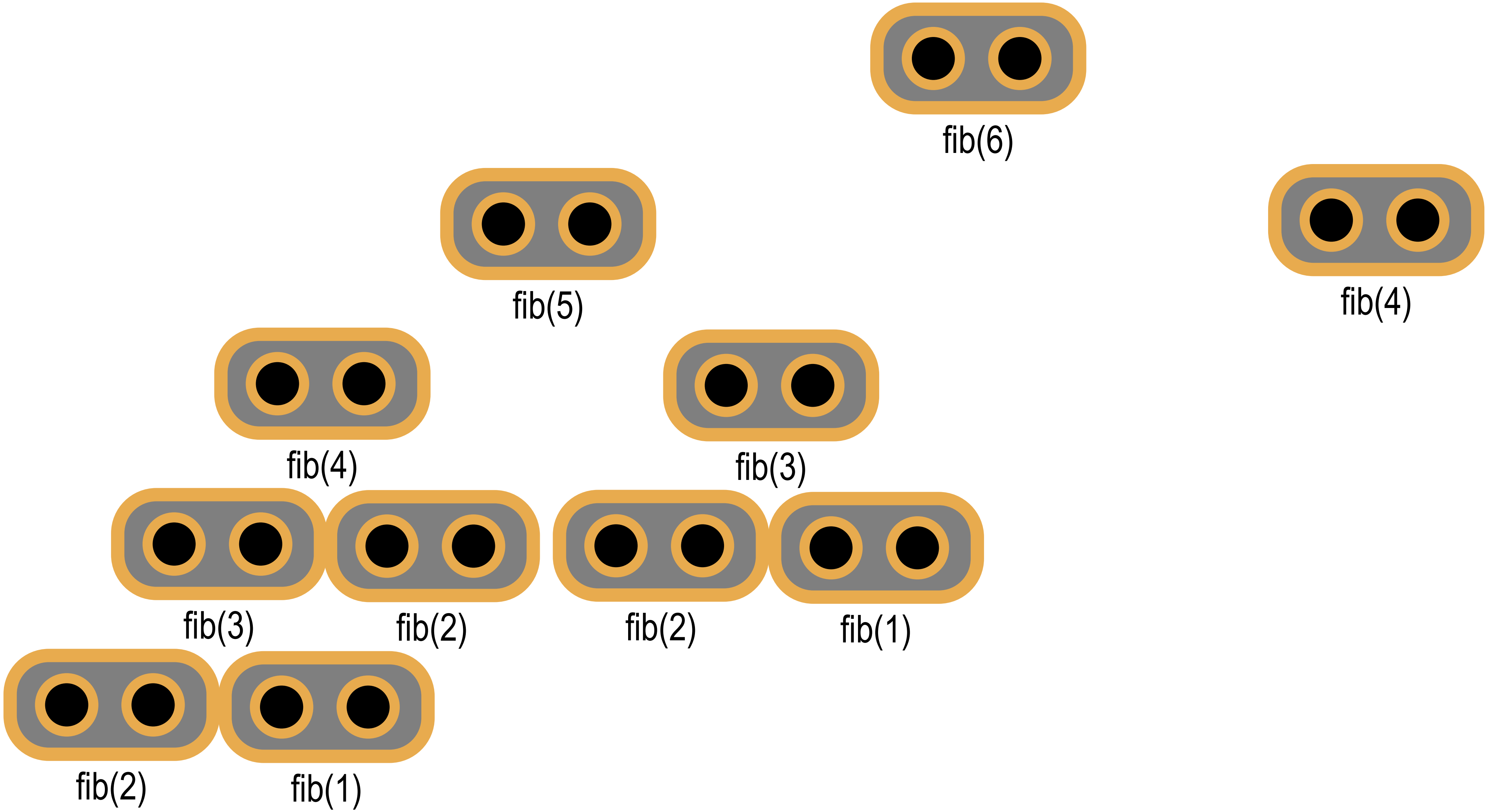

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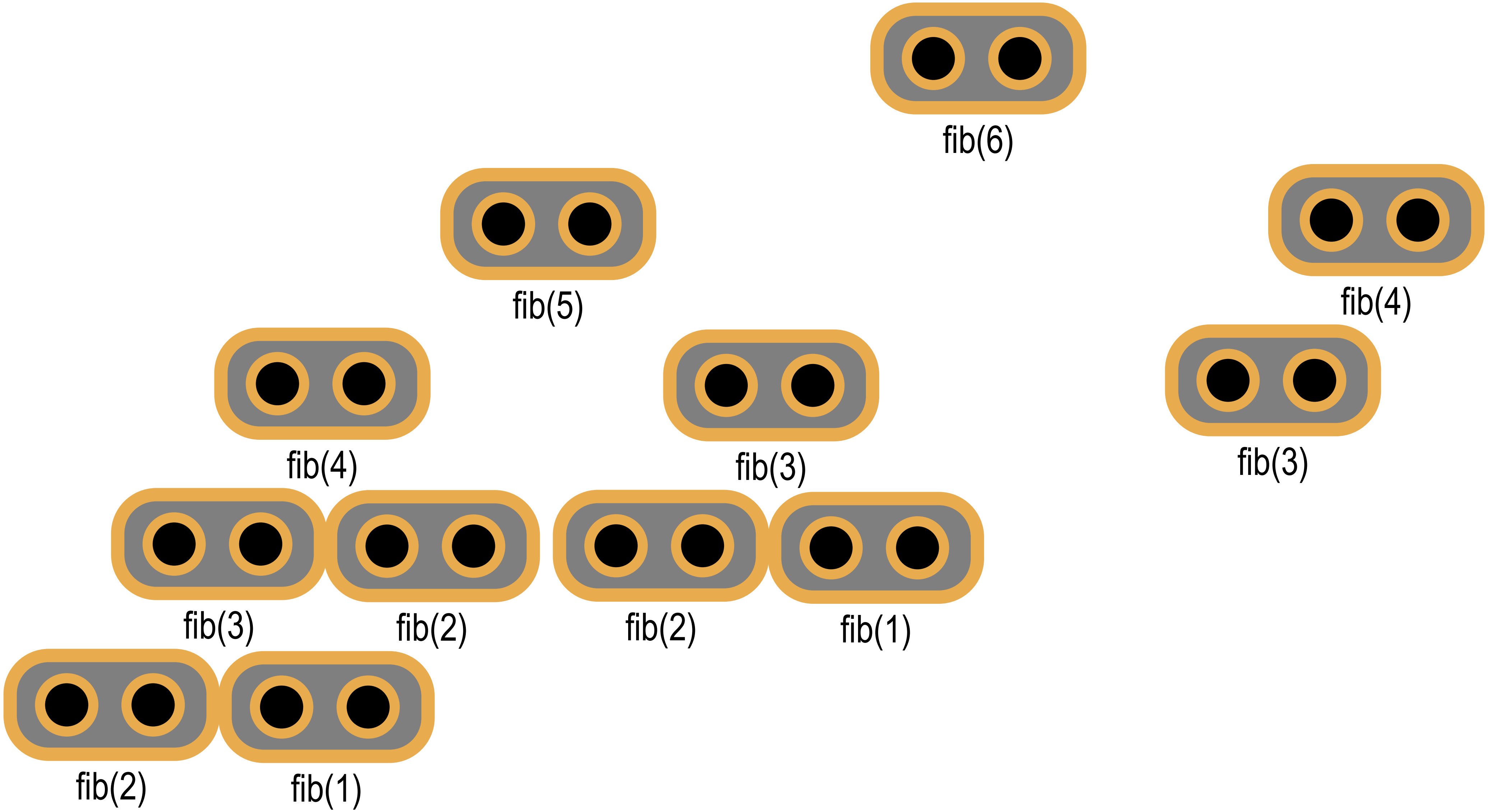

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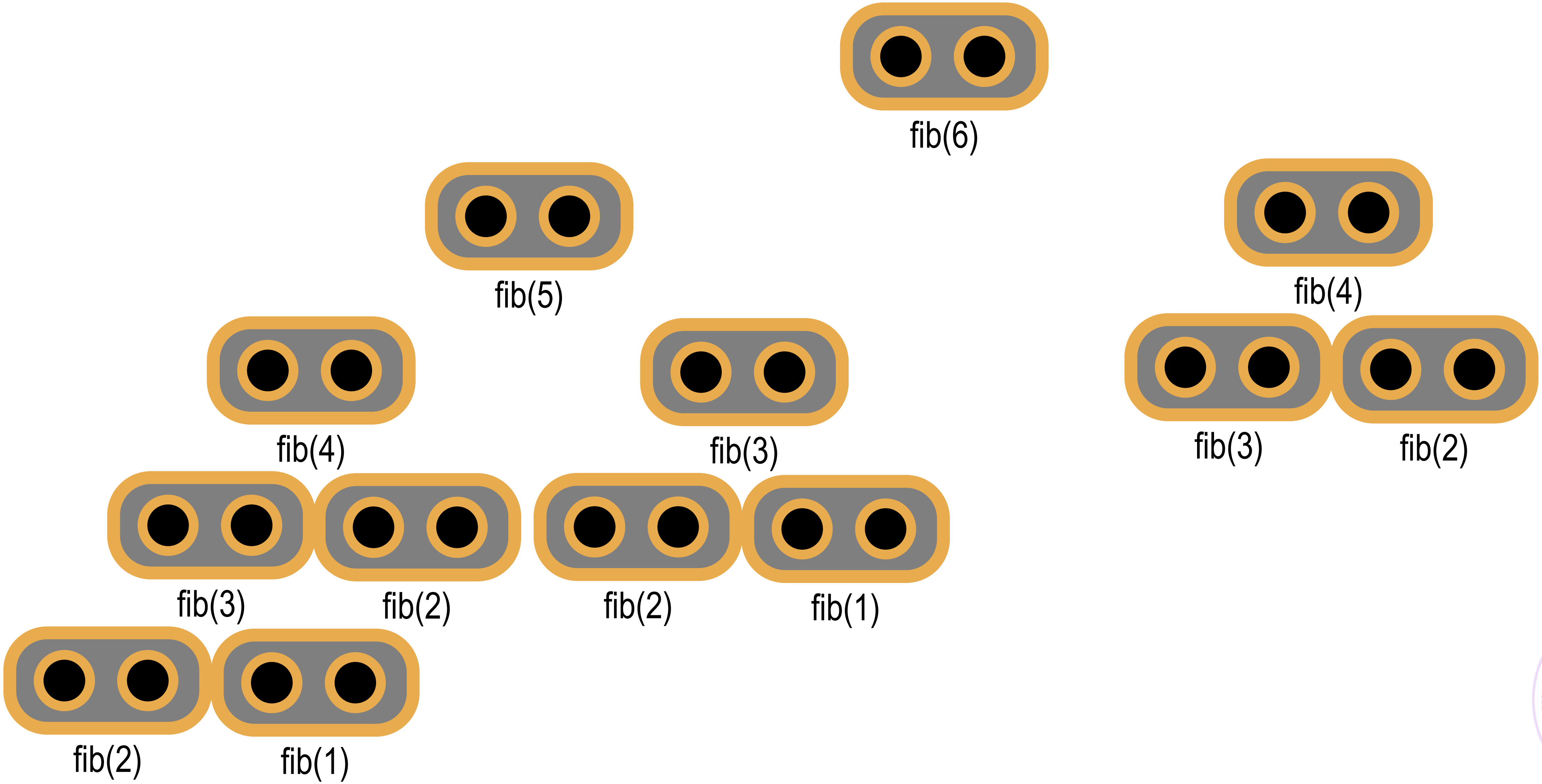

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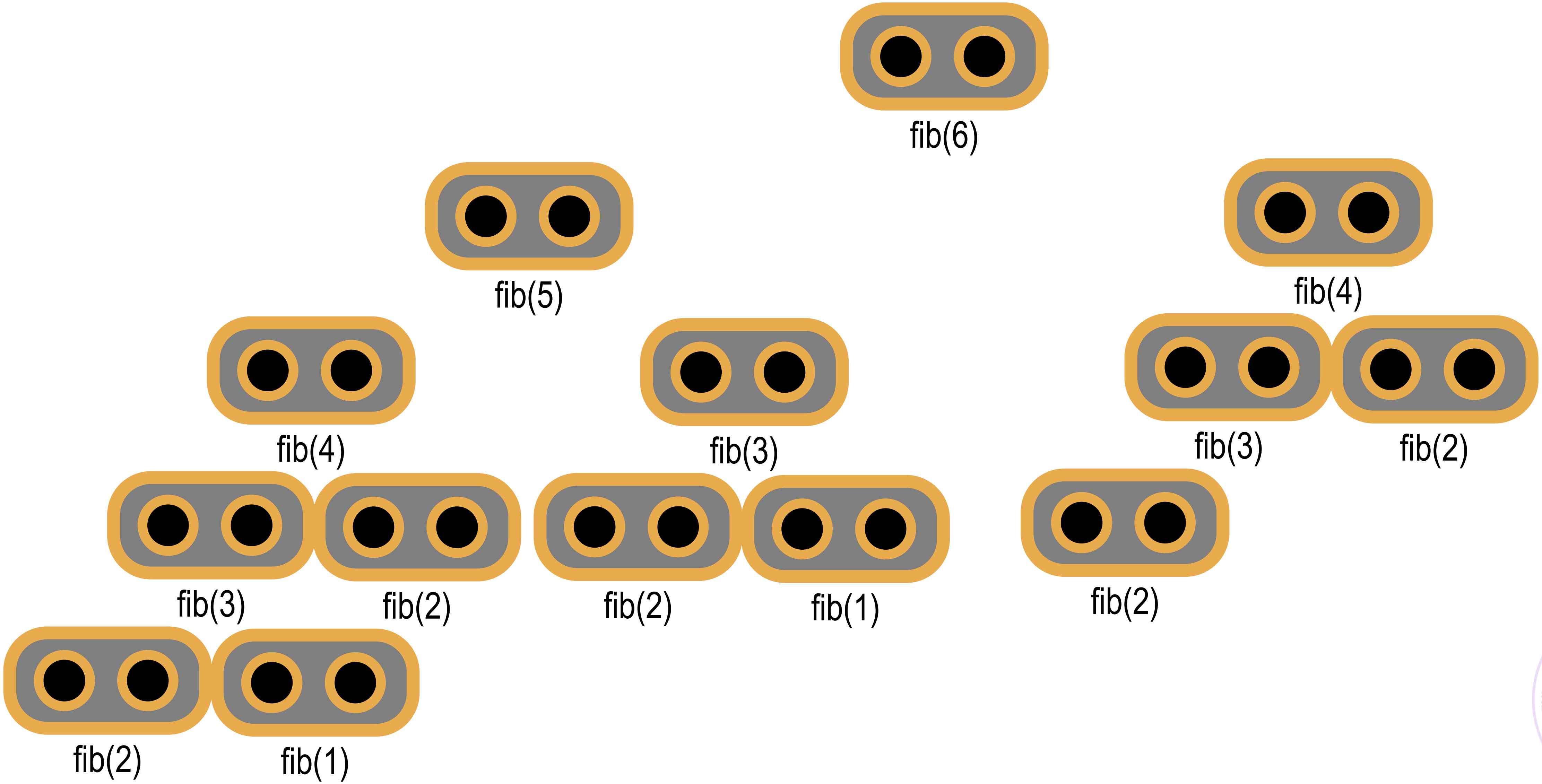

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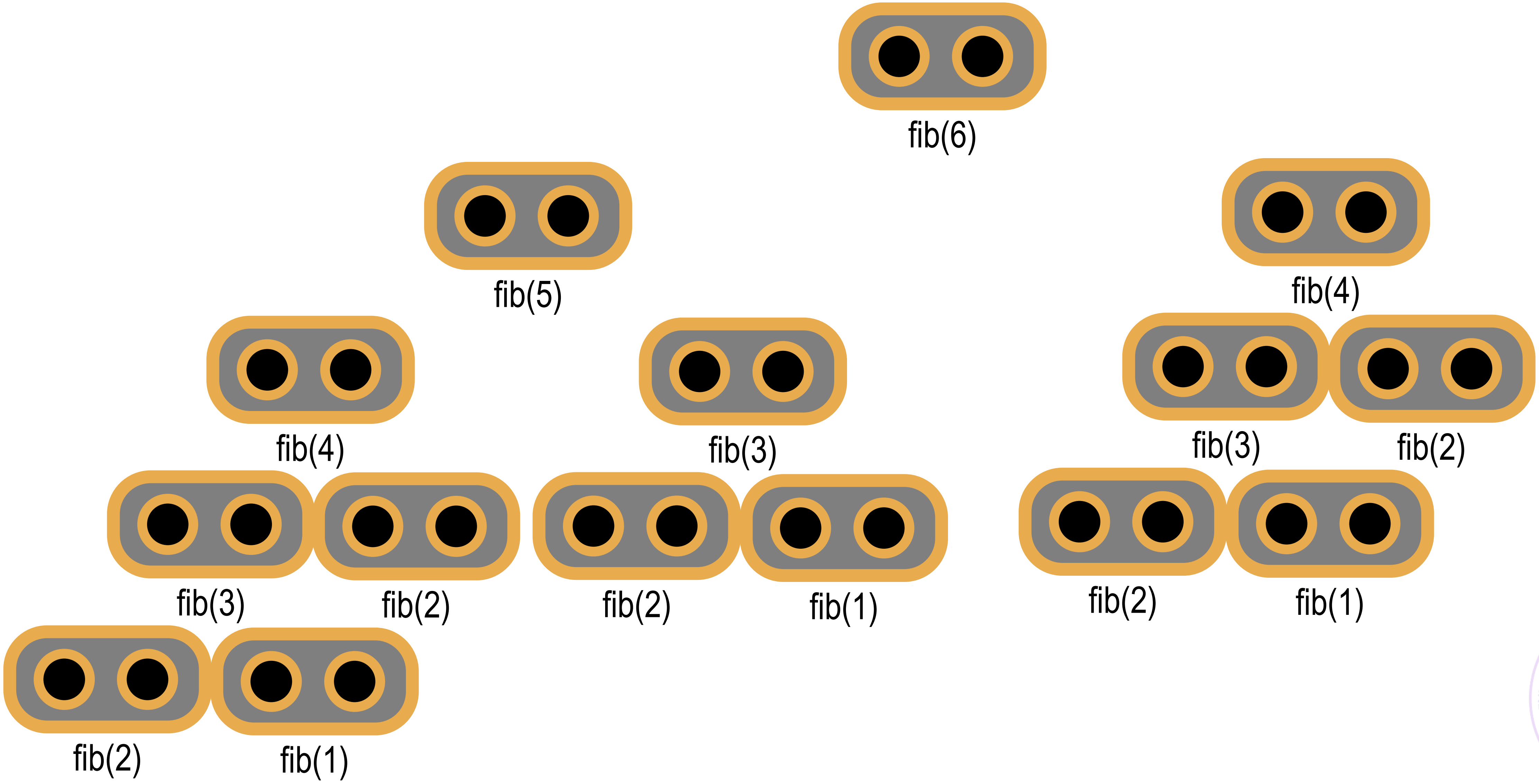

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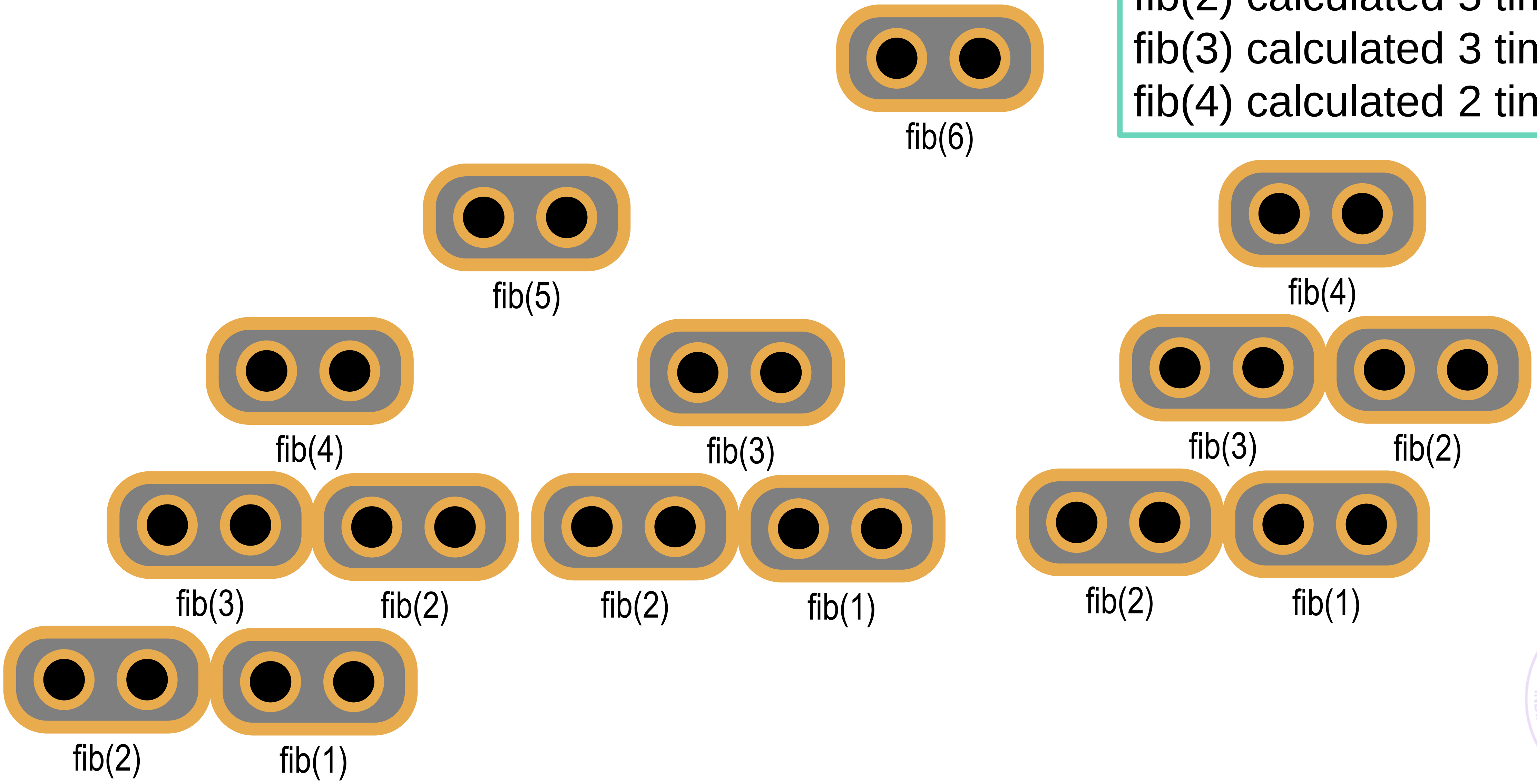

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We will prove this theorem in the next slide

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The proof of Lemma 2 is left as an easy exercise.

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This proves Theorem 1. QED (quod erat demonstrandum)

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The proof is compete after noticing that a – b = (m – n) * g and that (m – n) and n must be coprime which means g must be gcd of (m – n)* g and n * g i.e. gcd of (a-b) and b

Partitions

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Easy way of ensuring this – make sure that numbers are writing in increasing order so that 3 + 1 is disqualified

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