Let us give Mr C a Complement

ESC101: Fundamentals of Computing

Purushottam Kar

The Binary Number System

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Just two digits – 0 and 1

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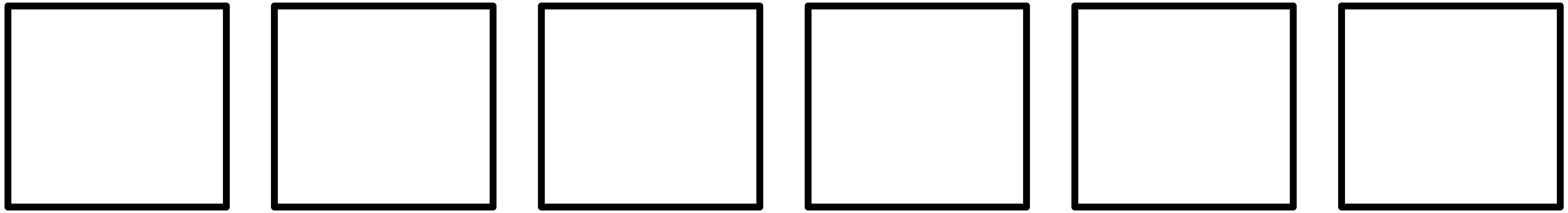

The Binary Number System

2

Just two digits – 0 and 1

1					

The Binary Number System

Just two digits – 0 and 1

1	0				

The Binary Number System

Just two digits – 0 and 1

1	0	1			

The Binary Number System

Just two digits – 0 and 1

1	0	1	1		

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1	0	1	1	0	

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Just two digits – 0 and 1

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The Binary Number System

Just two digits – 0 and 1

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$= 1 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 1 \times 2^3 + 0 \times 2^4 + 1 \times 2^5$

The Binary Number System

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$= 1 + 0 + 4 + 8 + 0 + 32$

The Binary Number System

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$= 45$

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$= 1 + 0 + 4 + 8 + 0 + 32$

$= 45$

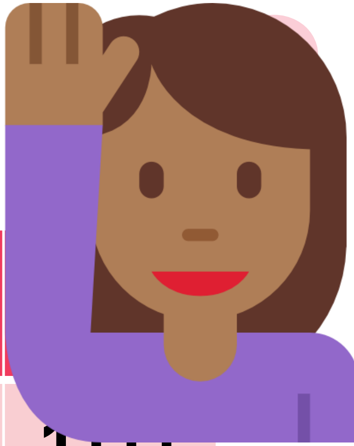
We will today see how the binary system is used to represent negative and fractional numbers

Bitwise Operators

Bitwise Operators

Operation	C Code	a	b	c	d	e	f
BITWISE AND	c = a & b	0000	1111	0000	1111	1111	1111
BITWISE OR	d = a | b	0101	1100	0100	1101	1001	1010
BITWISE XOR	e = a ^ b	1010	1110	1010	1110	0100	0101
BITWISE COMPLEMENT	f = ~a	1001	0111	0001	1111	1110	0110

Bitwise Operators



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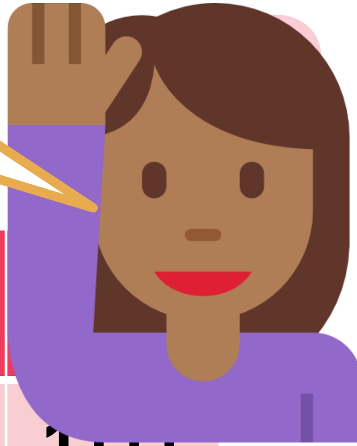
Bitwise Operato

Note: the word is complement and not compliment ☺



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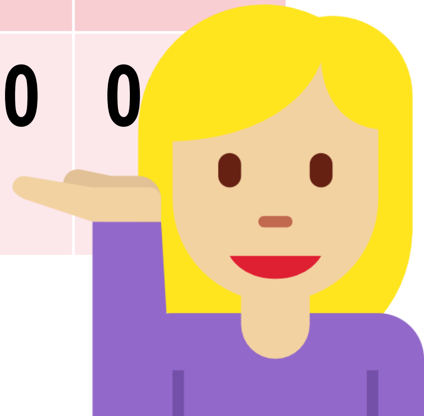
BITWISE COMPLEMENT	f = ~a	1001	0111	0001	1111	1110	0



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BITWISE OR	d = a | b	0101	1100	0100	1101	1001	1010
BITWISE XOR	e = a ^ b	1010	1110	1010	1110	0100	0101
BITWISE COMPLEMENT	f = ~a	1					0

The word compliment means to praise someone, complement means to enhance something

Bitwise And Operator &

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The output of bitwise AND is 1 if the corresponding bits of two operands is 1. If either bit of an operand is 0, the result of corresponding bit is evaluated to 0

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In C Programming, bitwise AND operator is denoted by &

```
12 = 00001100 (In Binary)
25 = 00011001 (In Binary)
Bitwise AND of 12 and 25
  0000 1100
& 0001 1001
  _________
  0000 1000  = 8 (In decimal)
```

Bitwise And Operator &

The output of bitwise AND is 1 if the corresponding bits of two operands is 1. If either bit of an operand is 0, the result of corresponding bit is evaluated to 0

In C Programming, bitwise AND operator is denoted by &

```
12 = 00001100 (In Binary)
25 = 00011001 (In Binary)
Bitwise AND of 12 and 25
   0000 1100
& 0001 1001
  ___________
   0000 1000  = 8 (In decimal)
```

```c
#include <stdio.h>
int main(){
    int a = 12, b = 25;
    printf("Output = %d", a & b);
    return 0;
}
```

Bitwise OR Operator |

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The output of bitwise OR is 1 if at least one corresponding bit of two operands is 1

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```
12 = 00001100 (In Binary)
25 = 00011001 (In Binary)
Bitwise OR of 12 and 25
   0000 1100
|  0001 1001
   _________
   00011101  = 29 (In decimal)
```

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In C Programming, bitwise OR operator is denoted by |

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12 = 00001100 (In Binary)
25 = 00011001 (In Binary)
Bitwise OR of 12 and 25
   0000 1100
|  0001 1001
   ________
   00011101  = 29 (In decimal)
```

```c
#include <stdio.h>
int main(){
    int a = 12, b = 25;
    printf("Output = %d", a | b);
    return 0;
}
```

Bitwise XOR Operator ^

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The result of bitwise XOR operator is 1 if the corresponding bits of two operands are opposite i.e. one is 1 and the other is 0

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```
12 = 00001100 (In Binary)
25 = 00011001 (In Binary)
Bitwise XOR of 12 and 25
   00001100
^ 00011001

  ________
  00010101  = 21 (In decimal)
```

Bitwise XOR Operator ^

The result of bitwise XOR operator is 1 if the corresponding bits of two operands are opposite i.e. one is 1 and the other is 0

In C Programming, bitwise XOR operator is denoted by ^

```
12 = 00001100 (In Binary)
25 = 00011001 (In Binary)
Bitwise XOR of 12 and 25
   00001100
^ 00011001
  ________
  00010101  = 21 (In decimal)
```

```c
#include <stdio.h>
int main(){
    int a = 12, b = 25;
    printf("Output = %d", a^b);
    return 0;
}
```

Bitwise Complement Operator ~

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A unary operator that simply flips each bit of the input

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```
12 = 0000 0000  0000 0000  0000 0000  0000 1100
Bitwise complement of 12 00001100
~ 0000 0000  0000 0000  0000 0000  0000 1100
  ________________________________________
  1111 1111   1111  1111    1111 1111   1111  0011
= -13 (decimal)
```

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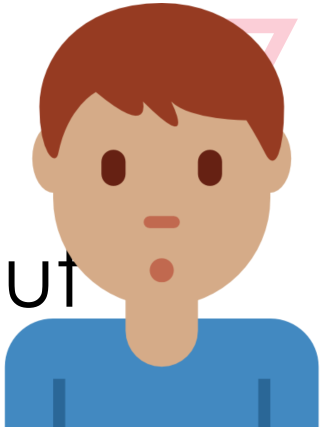
12 = 0000 0000  0000 0000  0000 0000  0000 1100
Bitwise complement of 12 00001100
~ 0000 0000  0000 0000  0000 0000  0000 1100
  ___________________________________________
  1111 1111   1111  1111    1111 1111   1111  0011
= -13 (decimal)
```

```
#include <stdio.h>
int main(){
    int a = 12;
    printf("Output = %d", ~a);
    return 0;
}
```

Bitwise Complement Operator ~



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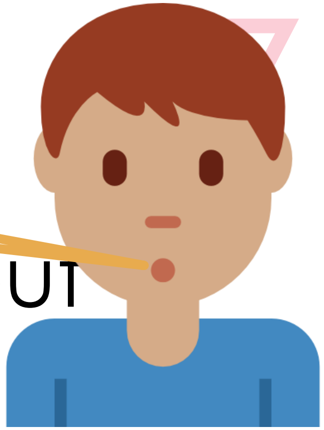

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In C Programming, bitwise complement operator is denoted by ~

12 = 0000 0000 0000 0000 0000 0000 0000 1100
Bitwise complement of 12 00001100
~ 0000 0000 0000 0000 0000 0000 0000 1100

1111 1111   1111 1111   1111 1111   1111 0011
= -13 (decimal)

```
#include <stdio.h>
int main(){
    int a = 12;
    printf("Output = %d", ~a);
    return 0;
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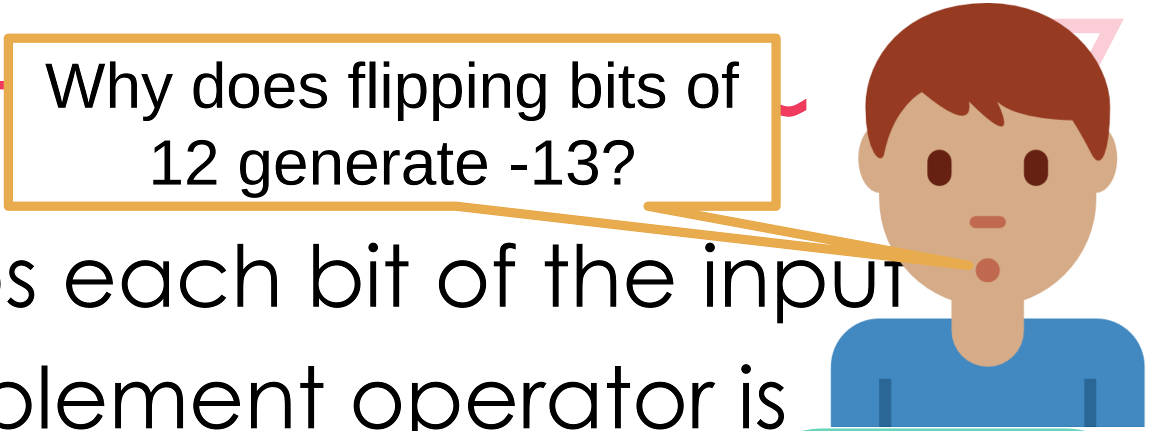
```

Bitwise Complement



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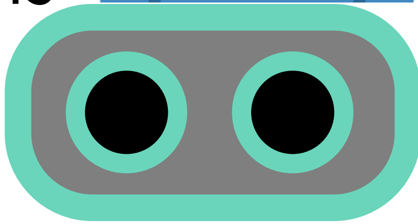
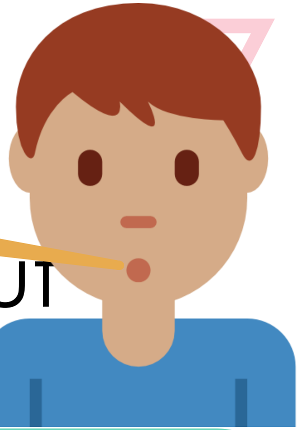


```
12 = 0000 0000  0000 0000  0000 0000  0000 1100
Bitwise complement of 12 00001100
~ 0000 0000  0000 0000  0000 0000  0000 1100
  ____________________________________________
  1111 1111   1111  1111    1111 1111   1111  0011
= -13 (decimal)
```

```c
#include <stdio.h>
int main(){
    int a = 12;
    printf("Output = %d", ~a);
    return 0;
}
```

Bitwise Complement



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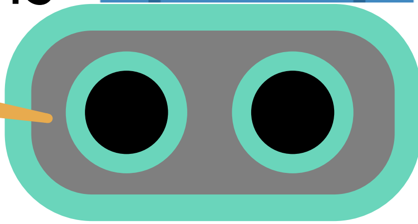

12 = 0000 0000 0000 0000 0000 0000 0000 1100
Bitwise complement of 12 00001100
~ 0000 0000 0000 0000 0000 0000 0000 1100

1111 1111 1111 1111 1111 1111 1111 0011
= -13 (decimal)

```
#include <stdio.h>
int main(){
    int a = 12;
    printf("Output = %d", ~a);
    return 0;
}
```

Right Shift Operator >>

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Right shift operator shifts all bits towards right by a certain number of locations

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Bits that “fall off” from the right most end are lost

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212 = 0000 0000   0000 0000   0000 0000   1101 0100

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212 = 0000 0000 0000 0000 0000 0000 1101 0100

212 >> 0 = 0000 0000 0000 0000 0000 0000 1101 0100

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212 = 0000 0000 0000 0000 0000 0000 1101 0100

212 >> 0 = 0000 0000 0000 0000 0000 0000 1101 0100

212 >> 4 = 0000 0000 0000 0000 0000 0000 0000 1101

Right Shift Operator >>

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212 = 0000 0000 0000 0000 0000 0000 1101 0100

212 >> 0 = 0000 0000 0000 0000 0000 0000 1101 0100

212 >> 4 = 0000 0000 0000 0000 0000 0000 0000 1101

212 >> 6 = 0000 0000 0000 0000 0000 0000 0000 0011

Right Shift Operator >>

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212 = 0000 0000   0000 0000   0000 0000   1101 0100

212 >> 0 = 0000 0000   0000 0000   0000 0000   1101 0100

212 >> 4 = 0000 0000   0000 0000   0000 0000   0000 1101

212 >> 6 = 0000 0000   0000 0000   0000 0000   0000 0011

212 >> 3 = 0000 0000   0000 0000   0000 0000   0001 1010

Right Shift Operator >>

Right shift operator shifts all bits towards right by a certain number of locations

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Blank spaces in the leftmost positions are filled with 0s

212 = 0000 0000 0000 0000 0000 0000 1101 0100

212 >> 0 = 0000 0000 0000 0000 0000 0000 1101 0100

212 >> 4 = 0000 0000 0000 0000 0000 0000 0000 1101

212 >> 6 = 0000 0000 0000 0000 0000 0000 0000 0011

212 >> 3 = 0000 0000 0000 0000 0000 0000 0001 1010

Right shift by k is equivalent to integer division with 2^k ☺

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212 = 0000 0000   0000 0000   0000 0000   1101 0100

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Left shift operator shifts all bits towards left by a certain number of locations

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212 = 0000 0000   0000 0000   0000 0000   1101 0100

212 << 0 = 0000 0000   0000 0000   0000 0000   1101 0100

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212 = 0000 0000 0000 0000 0000 0000 1101 0100

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212 << 4 = 0000 0000 0000 0000 0000 1101 0100 0000

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212 << 6 = 0000 0000 0000 0000 0011 0101 0000 0000

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212 = 0000 0000 0000 0000 0000 0000 1101 0100

212 << 0 = 0000 0000 0000 0000 0000 0000 1101 0100

212 << 4 = 0000 0000 0000 0000 0000 1101 0100 0000

212 << 6 = 0000 0000 0000 0000 0011 0101 0000 0000

212 << 28 = 0100 0000 0000 0000 0000 0000 0000 0000

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212 = 0000 0000   0000 0000   0000 0000   1101 0100

212 << 0 = 0000 0000   0000 0000   0000 0000   1101 0100

212 << 4 = 0000 0000   0000 0000   0000 1101   0100 0000

212 << 6 = 0000 0000   0000 0000   0011 0101   0000 0000

212 << 28 = 0100 0000   0000 0000   0000 0000   0000 0000

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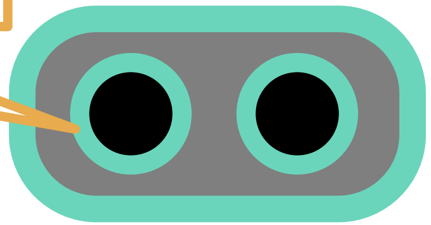
212 << 6 = 0000 0000 0000 0000 0011 0101 0000 0000

212 << 28 = 0100 0000 0000 0000 0000 0000 0000 0000

Left shift by k is equivalent to integer multiplication with 2^k

Left Shift Op

However, be warned. If you left shift too much, multiplication with 2^k may give you a number too large to be represented by an int. You may get garbage results.



Left shift operator … certain number of locations

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212 << 0 = 0000 0000 0000 0000 0000 0000 1101 0100

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212 << 6 = 0000 0000 0000 0000 0011 0101 0000 0000

212 << 28 = 0100 0000 0000 0000 0000 0000 0000 0000

Left shift by k is equivalent to integer multiplication with 2^k

Example use of bitwise operators

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Create a mask with only last bits set to 1 and take & with a

```
int a = 427;
int p = 1;
int q = p << 6;
int m = q – 1;
int r = a & m;
printf("%d", r); // 43
```

Example use of bitwise operators

Can use "masks" to extract certain bits of a number

Suppose I want to look at the last 6 bits of a number a

Create a mask with only last bits set to 1 and take & with a

```
a = 0000 0000  0000 0000  0000 0001  1010 1011
p = 0000 0000  0000 0000  0000 0000  0000 0001
q = 0000 0000  0000 0000  0000 0000  0100 0000
m = 0000 0000  0000 0000  0000 0000  0011 1111
r =  0000 0000  0000 0000  0000 0000  0010 1011
```

```
int a = 427;
int p = 1;
int q = p << 6;
int m = q – 1;
int r = a & m;
printf("%d", r); // 43
```

BODMAS table has more members

Operator Name	Symbol/Sign	Associativity
Brackets (array subscript), Post increment/decrement, structure field selection (dot and arrow)	(), [] ++, --, ., ->	Left
Unary negation, Pre-increment/decrement, NOT, (de)reference, sizeof, bitwise complement	-, ++, --, !, *, &, sizeof, ~	Right
Multiplication/division/remainder	*, /, %	Left
Addition/subtraction	+, -	Left
Bitwise left shift and right shift	<<, >>	
Relational	<, <=, >, >=	Left
Relational	==, !=	Left
Bitwise AND	&	Left
Bitwise XOR	^	Left
Bitwise OR	|	Left
Logical AND	&&	Left
Logical OR	||	Left
Ternary Conditional	? :	Right
Assignment, Compound assignment	=, +=, -=, *=, /=, %=, &=, ^=, |=, <<=, >>=	Right

Operator Name	Symbol/Sign	Associativity
Brackets (array subscript), Post increment/decrement, structure field selection (dot and arrow)	(), [] ++, --, ., ->	Left
Unary negation, Pre-increment/decrement, NOT, (de)reference, sizeof, bitwise complement	-, ++, --, !, *, &, sizeof, ~	Right
Multiplication/division/remainder	*, /, %	Left
Addition/subtraction	+, -	Left
Bitwise left shift and right shift	<<, >>	
Relational	<, <=, >, >=	Left
Relational	==, !=	Left
Bitwise AND	&	Left
Bitwise XOR	^	Left
Bitwise OR	|	Left
Logical AND	&&	Left
Logical OR	| |	Left
Ternary Conditional	? :	Right
Assignment, Compound assignment	=, +=, -=, *=, /=, %=, &=, ^=, |=, <<=, >>=	Right

HIGH

LOW

One's Compliment

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The one’s compliment of a binary number) is simply the bitwise compliment of that binary number

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35 as a 4 byte int is represented as
0000 0000   0000 0000   0000 0000   0010 0011

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0000 0000   0000 0000   0000 0000   0010 0011

So, in those old computers, -35 used to be represented as
1111 1111   1111 1111   1111 1111   1101 1100

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0000 0000 0000 0000 0000 0000 0010 0011

So, in those old computers, -35 used to be represented as
1111 1111 1111 1111 1111 1111 1101 1100

Note that b + ~b = 11111111 11111111 11111111 11111111

One’s Compliment

The[illegible] bit[illegible]

If we have n bits, then using one’s compliment, we can represent numbers between $-(2^{n-1} - 1)$ and $+(2^{n-1} - 1)$

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One's Compliment

Th
bit

If we have n bits, then using one's compliment, we can represent numbers between $-(2^{n-1} - 1)$ and $+(2^{n-1} - 1)$

The first bit acts as a sign bit – if the first bit is 1, it is treated as a negative number, if the first bit is 0, it is treated as a positive number

0000 0000 0000 0000 0000 0000 0010 0011

So, in those old computers, -35 used to be represented as
1111 1111 1111 1111 1111 1111 1101 1100

Note that b + ~b = 11111111 11111111 11111111 11111111

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The first bit acts as a sign bit – if the first bit is 1, it is treated as a negative number, if the first bit is 0, it is treated as a positive number

Largest positive number is 01111111 11111111 11111111 11111111
Smallest negative number is 1000000 00000000 00000000 00000000
Weird thing – negative 0 ☺
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One’s Compliment

If we have n bits, then using one’s compliment, we can represent numbers between $-(2^{n-1} - 1)$ and $+(2^{n-1} - 1)$

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Used no more. These days, computers use two’s compliment to represent negative numbers

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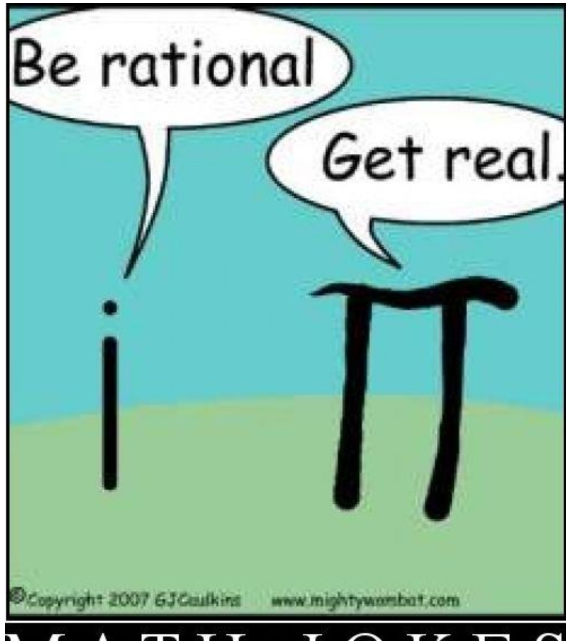
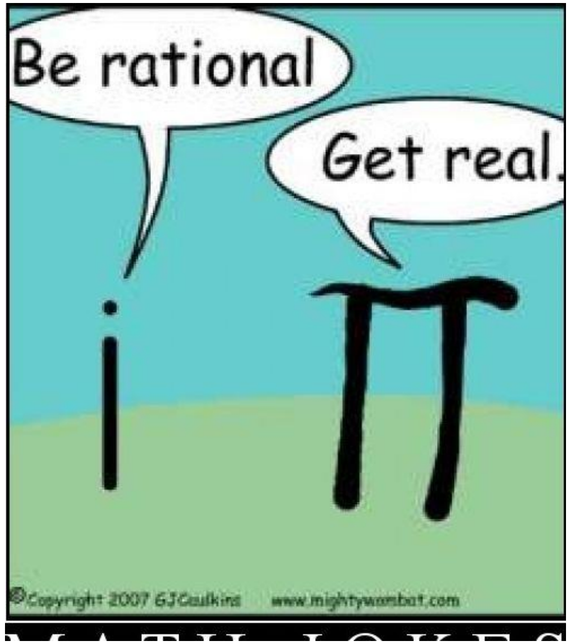
Floating Point Representation



Floating Point Representation

Have to represent three things

- Sign: uses 1 bit
- Mantissa: uses 23 bits
- Exponent: uses 8 bits



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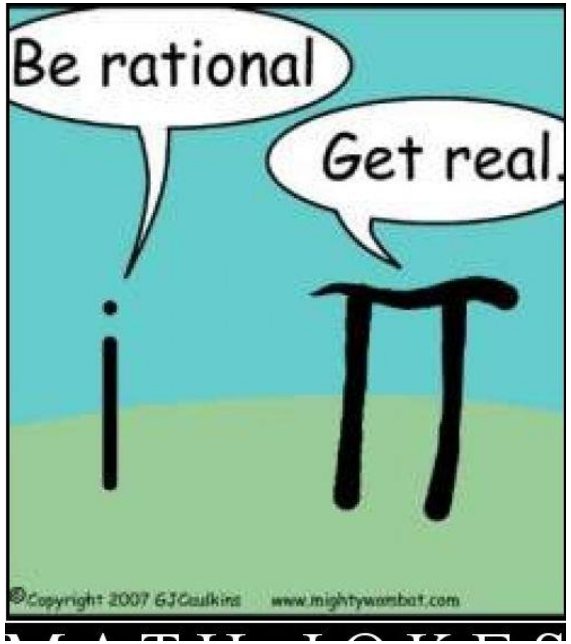

Base 2: d_i 's are binary digits 0 or 1.

$$\pm d_0.d_1d_2\cdots d_{p-1}\times 2^e$$

Its value is

$$\pm(d_0+d_1 2^{-1}+d_2{}^{-2}+\cdots+d_{p-1}2^{-(p-1)})2^e$$

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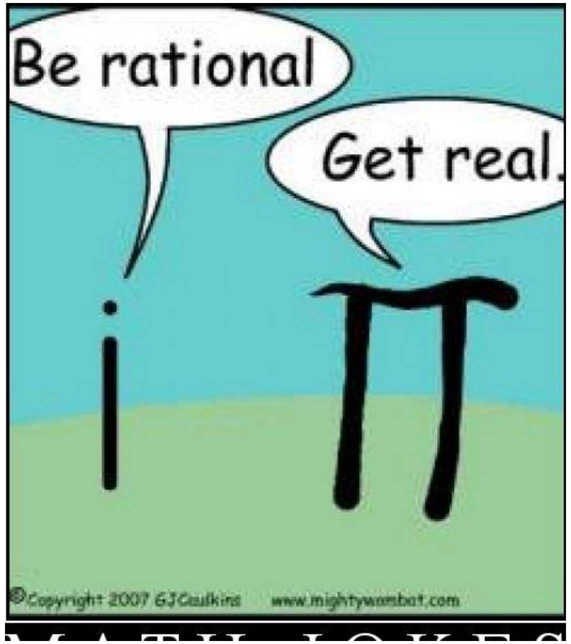

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1. p is called the precision of the representation.
2. Here base of the representation is 2. In general, the base could be different from 2. e.g., scientific calculators typically use base 10.

IEEE 754 Floating Point Representation

IEEE 754 Floating Point Standard

s	e=exponent	m=mantissa
1 bit	8 bits	23 bits

$$\text{number} = (-1)^{s} * (1.m) * 2^{e-127}$$

Range of Representation

- Base 2: d_i 's are binary digits 0 or 1.

$$\pm d_0.d_1d_2 \cdots d_{p-1} \times 2^e$$

- E.g., for number $(0.1)_{10}$ (i.e., 0.1 in decimal), p =24, the representation is

  $(1.10011001100110011001100) \text{ X } 2^{-4}$

1/16 + 1/32 + 1/256 + 1/512 = 51/512, ….

$(0.1)_{10}$ is not "exactly" represented in binary.

Real numbers cannot be exactly represented with finite precision and limited exponent.

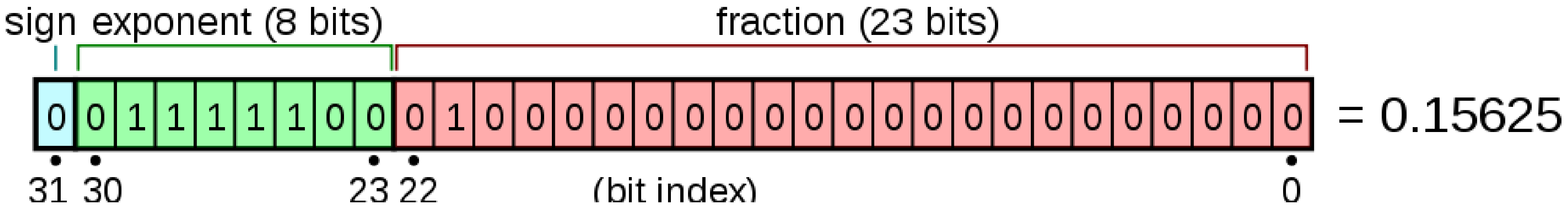
Single-precision (float)

0	0110 1000	101 0101 0100 0011 0100 0010

- Sign: 0 => positive
- Exponent:
  - $0110\ 1000_{two} = 104_{ten}$
  - Bias adjustment: 104 - 127 = -23
- Significand:
  - $1 + 1x2^{-1} + 0x2^{-2} + 1x2^{-3} + 0x2^{-4} + 1x2^{-5} + \ldots$
    $=1+2^{-1}+2^{-3}+2^{-5}+2^{-7}+2^{-9}+2^{-14}+2^{-15}+2^{-17}+2^{-22}$
    $= 1.0 + 0.666115$
- Represents: $1.666115{*}2^{-23} \sim 1.986{*}10^{-7}$

This is what you're using when you are invoking *float*

Imprecise Representation



Next number: $(1 + \frac{1}{4} + 2^{-23})2^{124-127} = 0.15625+2^{-26}$

0	0111100	01000000000000000000001

Given number: 0.15625

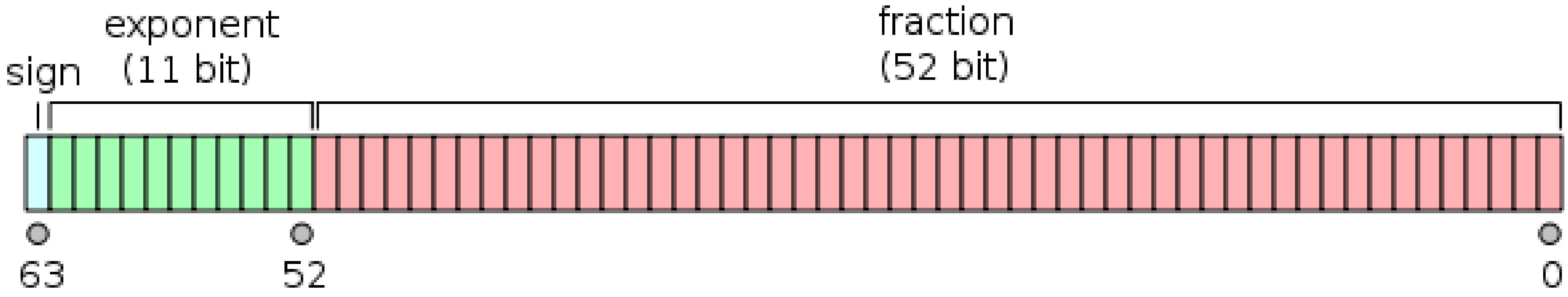
Previous number: $(1+1/8+1/16+\ldots 2^{-23})2^{-3}$
$= (1+1/4-2^{-23})2^{-3} = 0.15625-2^{-26}$

0	0111100	00111111111111111111111

Real numbers in-between 0.15625 and $0.15625 + 2^{-26}$ may be all represented as 0.15625.
Discrete representation: leads to approximation errors.

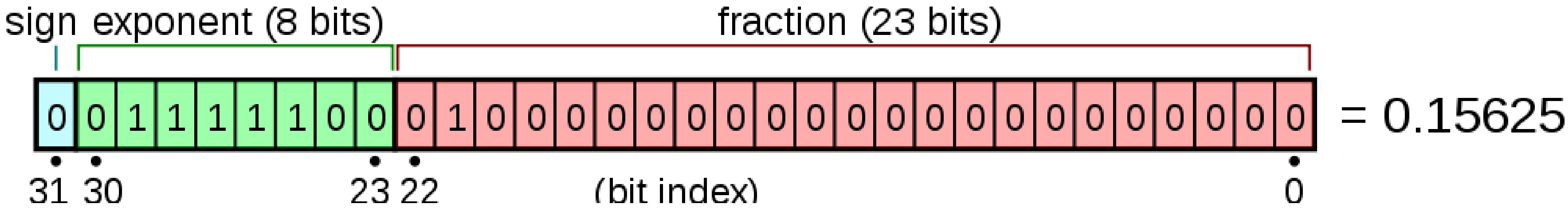
Double Precision (double)

Same logic as single precision, but with 64 bits of memory



Underflow and Overflow

Consider IEEE single precision.



Numbers smaller than 2^{-128} are indistinguishable from 0.

Such numbers occurring in calculations are said to be in underflow and are treated as 0.

Numbers that are larger than 2^{128} cannot be represented.

Such numbers occurring in calculations are said to have overflowed.

Warning: IEEE representation allows for +0.00 and –0.00. Be careful.