Mr C has to Sort things out

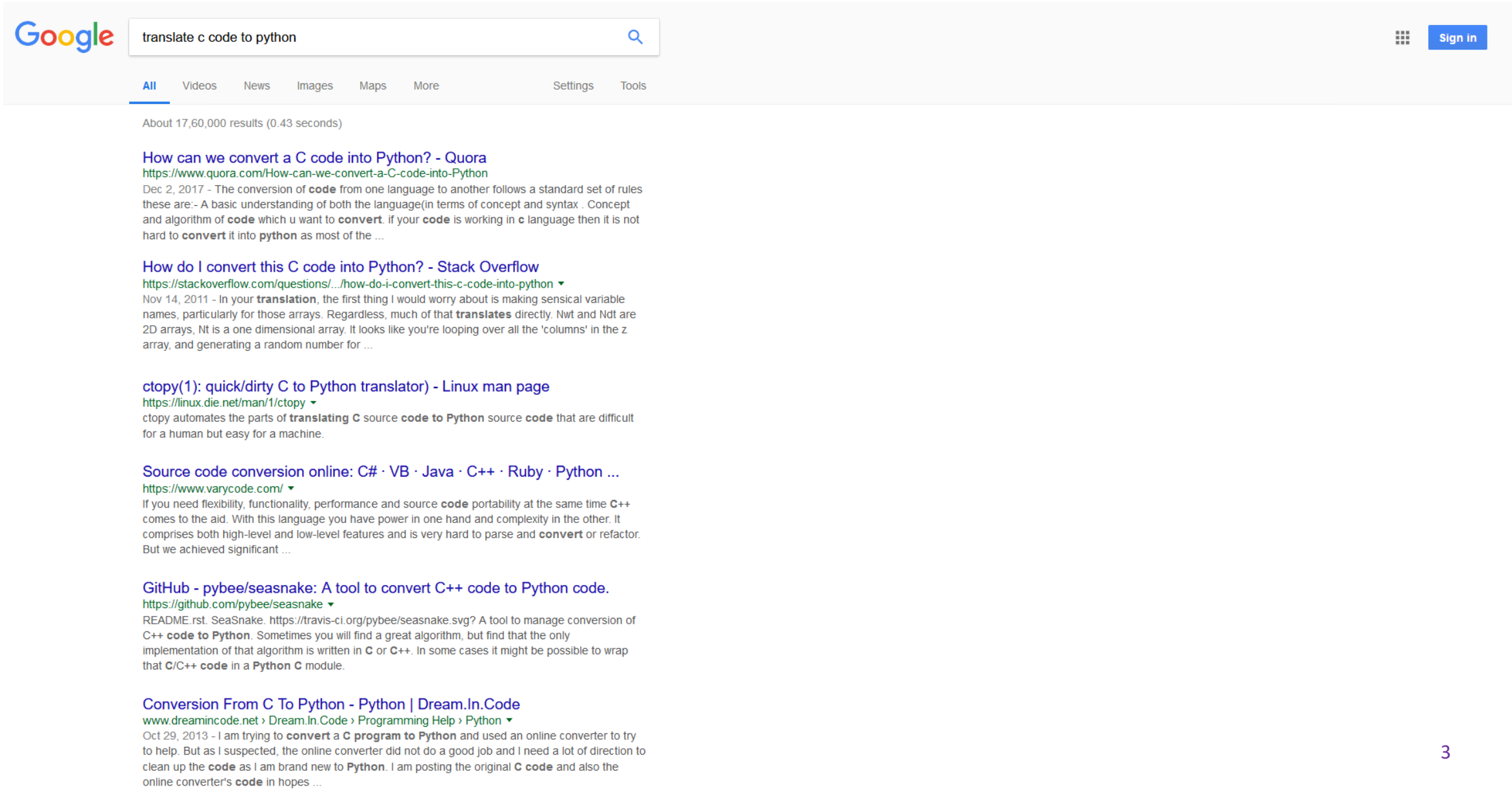
ESC101: Fundamentals of Computing

Purushottam Kar

What is Sorting?

> Sorting is the process of arranging items systematically, ordered by some criterion

Internet Searches



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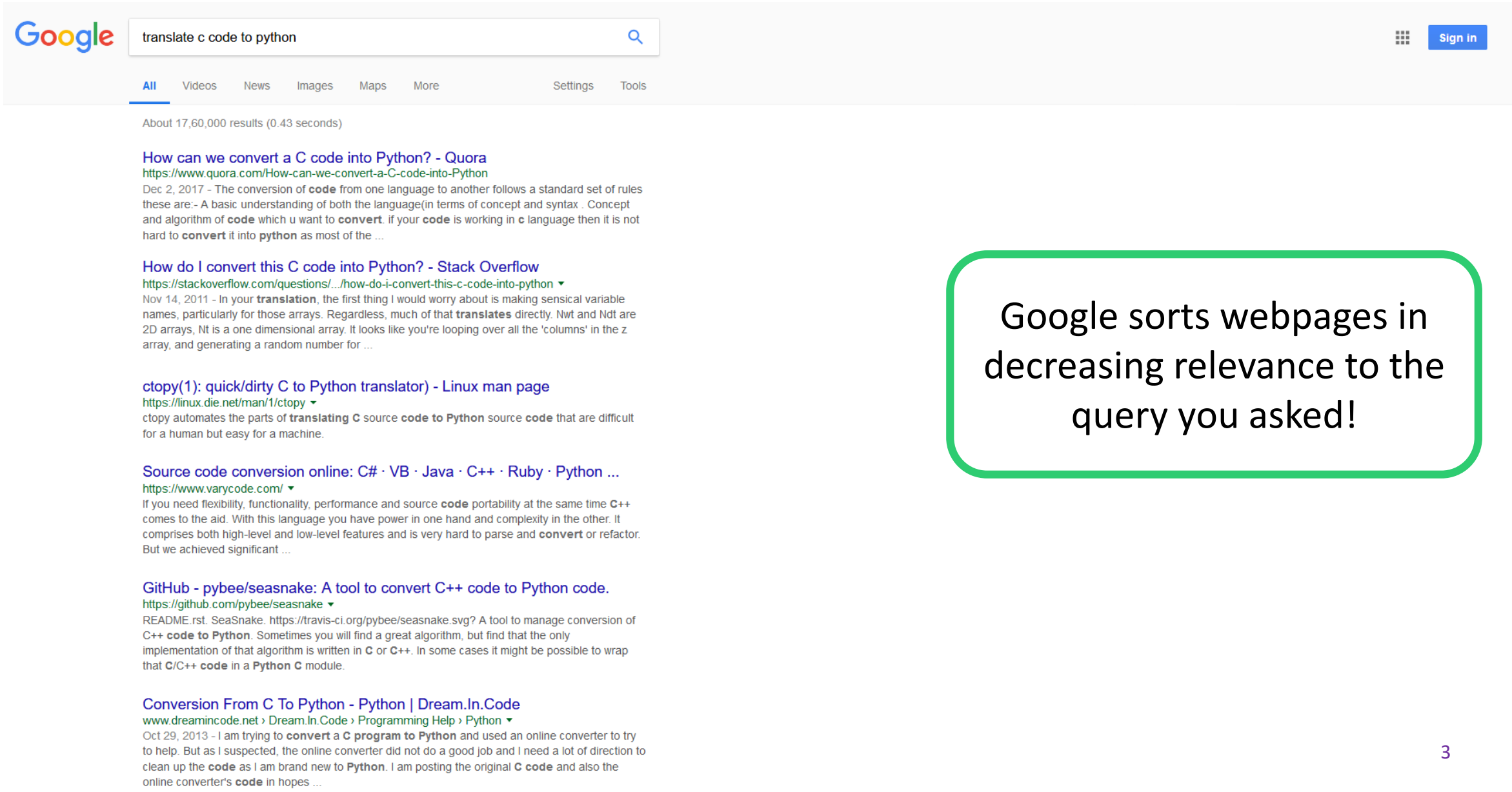
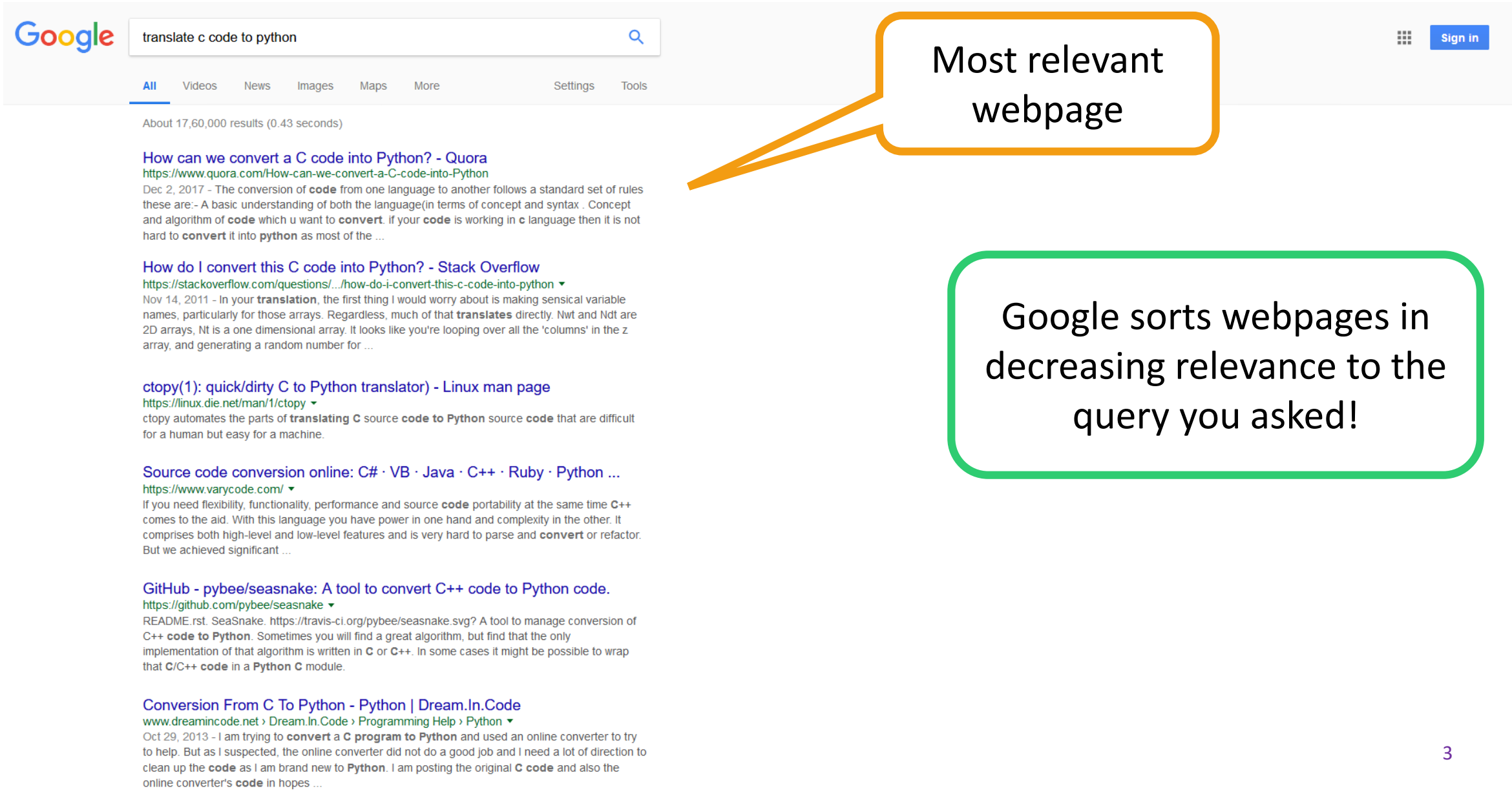


Google sorts webpages in decreasing relevance to the query you asked!

Internet Searches



Most relevant webpage

Google sorts webpages in decreasing relevance to the query you asked!

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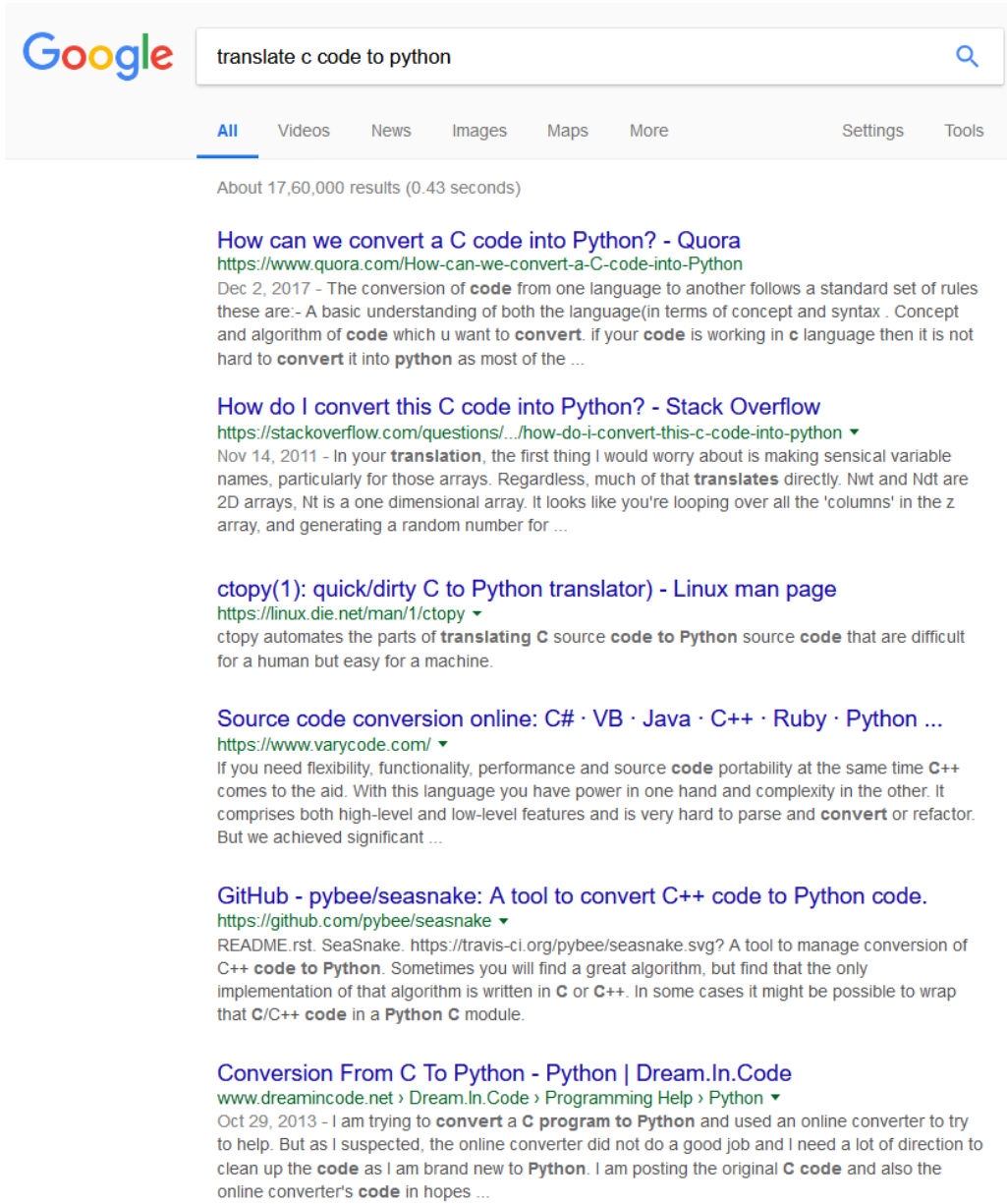

Most relevant webpage

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Less relevant webpage

Email Clients

Folders
Last Refresh:
Tue, 12:33 am
(Check mail)

- **INBOX** (5161/9964)
  - Drafts
  - Sent
  - Trash (Purge)
  - Draft

Folder Sizes

Current Folder: **INBOX** — Sign Out

Compose | Addresses | Folders | Options | Search | Help | Calendar | Auto Response — Help (LAN)

Previous | Next | 1 2 3 4 5 ... 14 15 16 17 18 19 20 21 22 23 24 ... 196 197 198 199 200 | Show All | Toggle All — Viewing Messages: **901** to **950** (9964 total)

Move Selected To: INBOX | Move | Forward — Transform Selected Messages: Flag | Unflag | Read | Unread | Delete

Thread View

From	Date	Subject	Size
Nitin Gupta	Aug 27, 2018	[acadstaff] Re: M.Tech. defense: Surbhit Wagle (BS...	7.9 k
Sameer Khandekar	Aug 27, 2018	[acadstaff] Notice: Ph. D. Oral Examination - Mr. ...	20 k
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Sounak Choudhury	Aug 27, 2018	[acadstaff] M.Tech. Thesis Defense in Mech. Engg. ...	5 k
DOAA	Aug 27, 2018	[All] [Fwd: Best student paper award]	6.6 k
SANJEEV GARG	Aug 27, 2018	[acadstaff] PhD Oral Examination (10102073)	78 k
eccc@weizmann.ac.il	Aug 27, 2018	[ECCC] New Paper published	5.6 k
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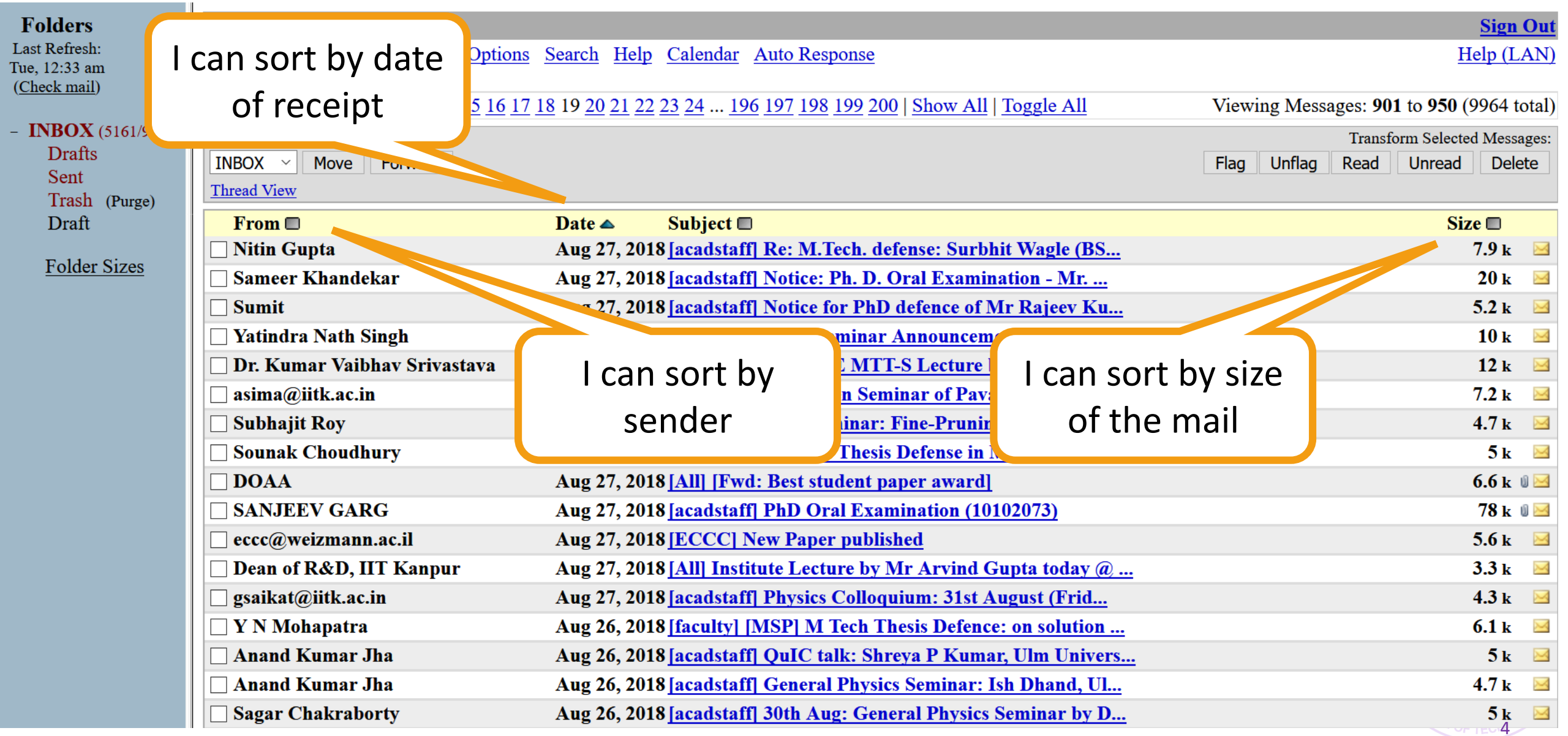

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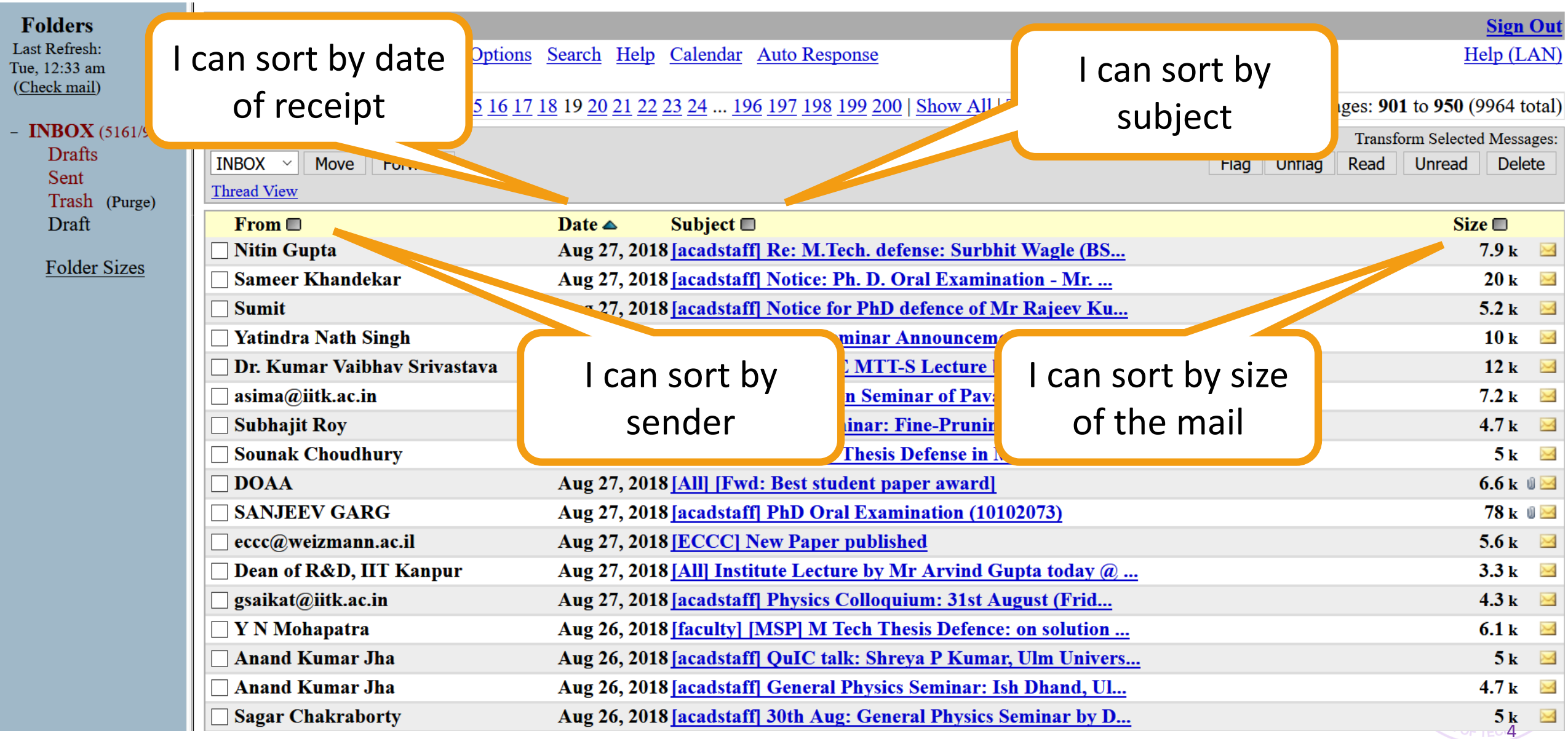

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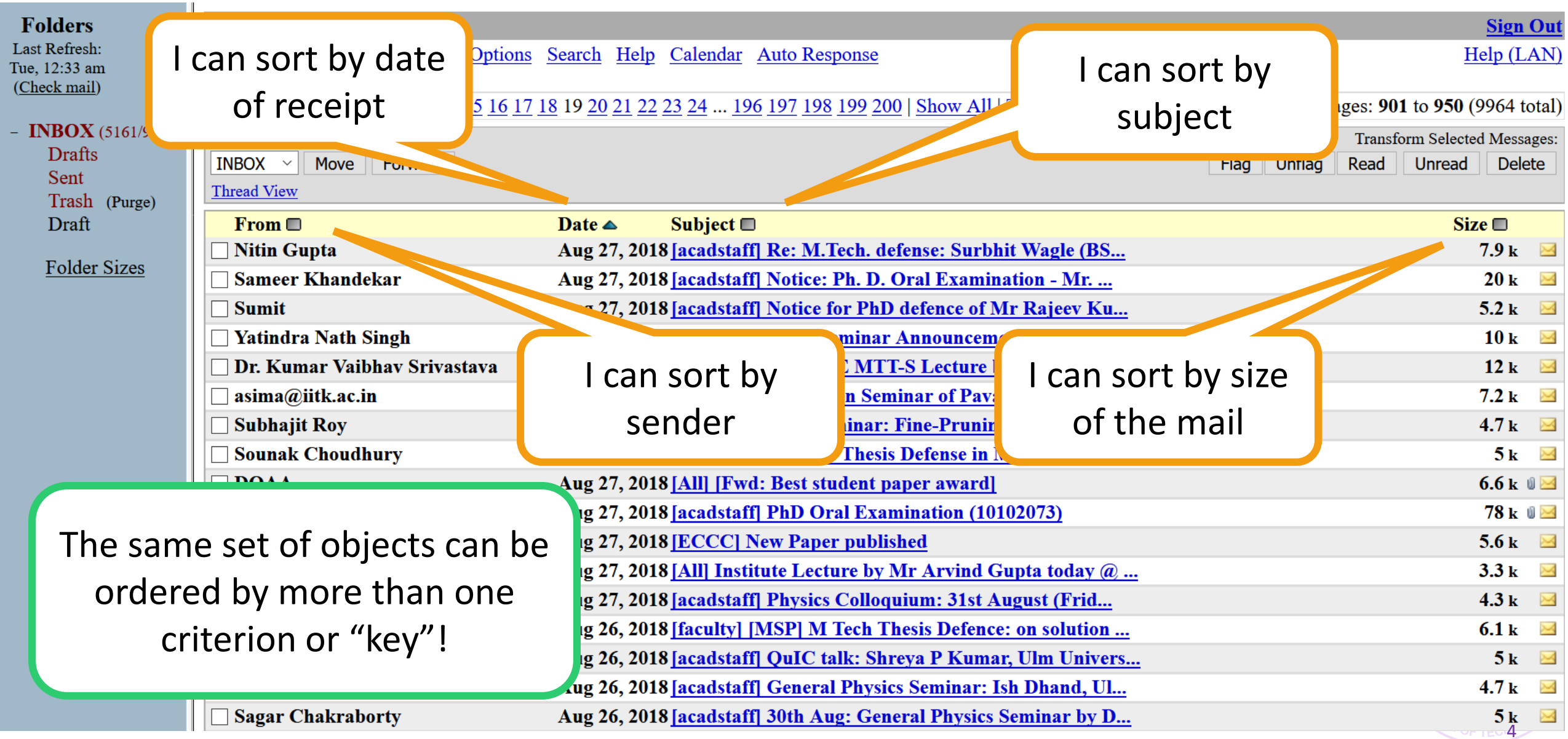

I can sort by date of receipt

I can sort by subject

I can sort by sender

I can sort by size of the mail

The same set of objects can be ordered by more than one criterion or “key”!

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The same set of objects can be ordered by more than one criterion or “key”!

Sort criterion can be numeric (e.g. size, date, relevance) or lexicographic (e.g. alphabetic)

Applications of Sorting

- The ability to order objects (webpages, emails, movies, songs) according to varied and user-dictated criteria can be lucrative
- A subfield of machine learning called *ranking* is dedicated to this and has immense applications – recommendation systems (Amazon, Flipkart), internet search, personalized medicine
- Profs especially love sorting since it helps them assign grades!
- Sorting can also make other (less fancy) operations very simple
- Agenda for the next couple of lectures
  - Look at a few applications of sorting
  - Look at a few efficient techniques of sorting
  - Get introduced to the divide and conquer technique

Search in Sorted Arrays

Brute Force Search

- Is the element 4 present in the array?
  - Can search the array from left to right or right to left
  - `for(i=0;i<11;i++) if(a[i]==4) return i; return -1;`
  - `for(i=10;i>=0;i--) if(a[i]==4) return i; return -1;`
  - Searching from left seems faster for the query 4
- Is the element 3 present in the array?
- Is the element 5 present in the array?
- If there are N elements in the array we have to do at least N operations (to verify absence) - can we do any better?

Binary Search

1	1	1	2	3	4	4	6	8	8	9

- The above array is sorted in ascending order $a[i] \leq a[i+1]$
- Can sort arrays in descending order too i.e. $a[i] \geq a[i+1]$
- Now lets try searching again by exploiting sortedness
- Crucial insight: if we are searching for the element K and if we know $a[j] < K$ then we also know $a[i] < K$ for all $i < j$
  - Proof: $a[i] < a[j]$ since $i < j$ and array is sorted and we know $a[j] < K$
  - Similarly, if $a[j] > K$ then we also know $a[i] > K$ for all $i > j$
- We will use the above to eliminate vast swathes of the array

Binary Search

Binary Search

- Suppose we check a[6] == K? Three possible outcomes

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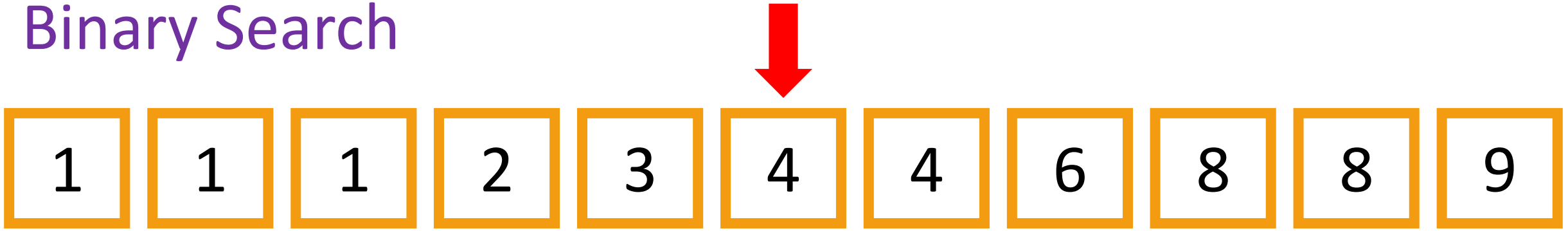

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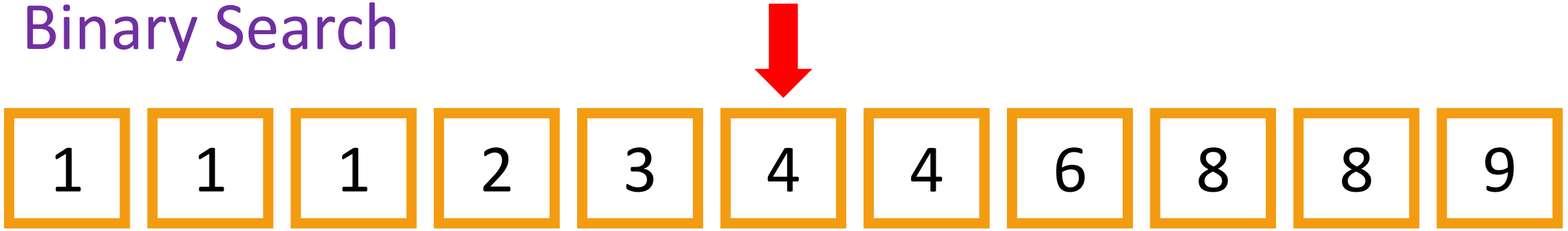

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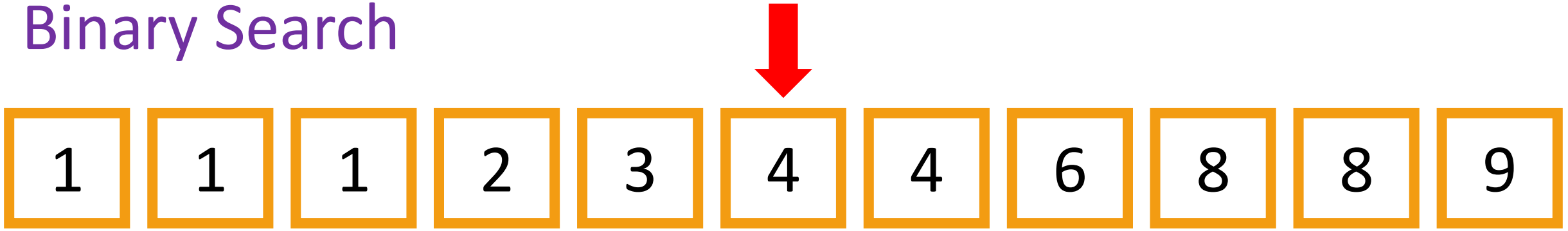

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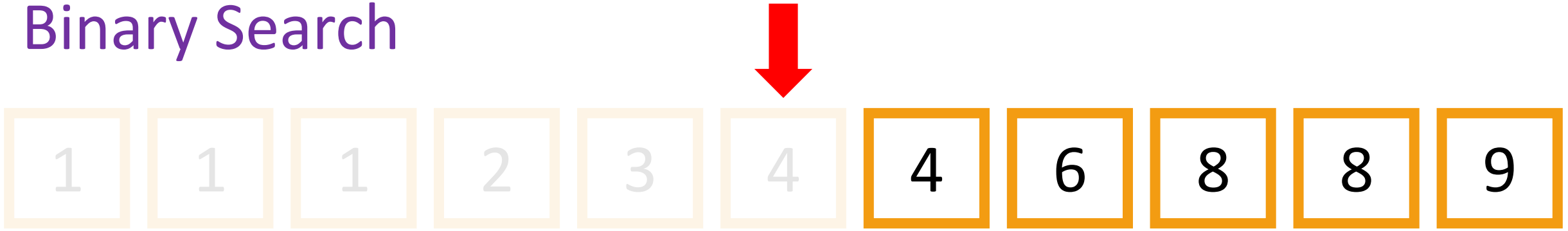

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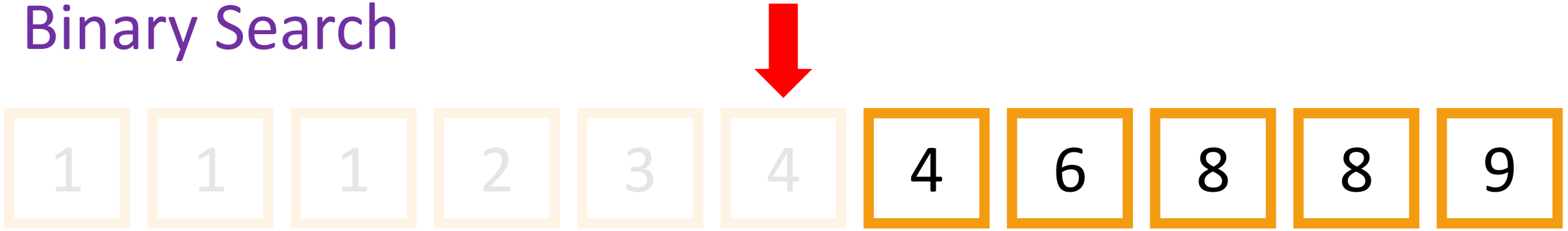

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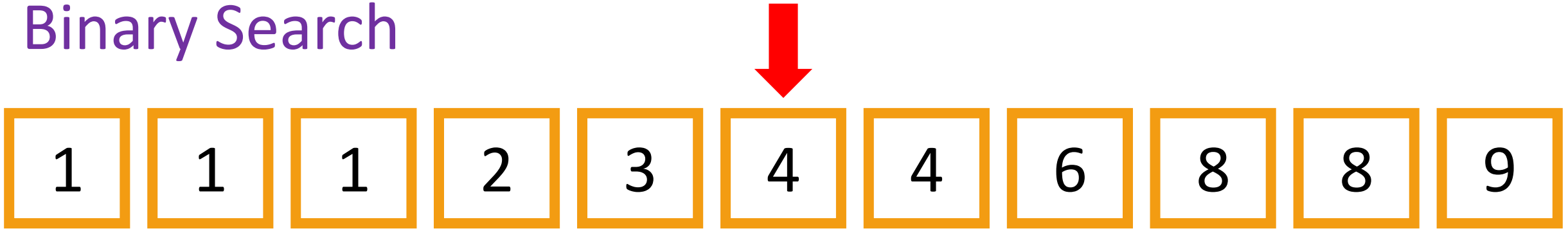

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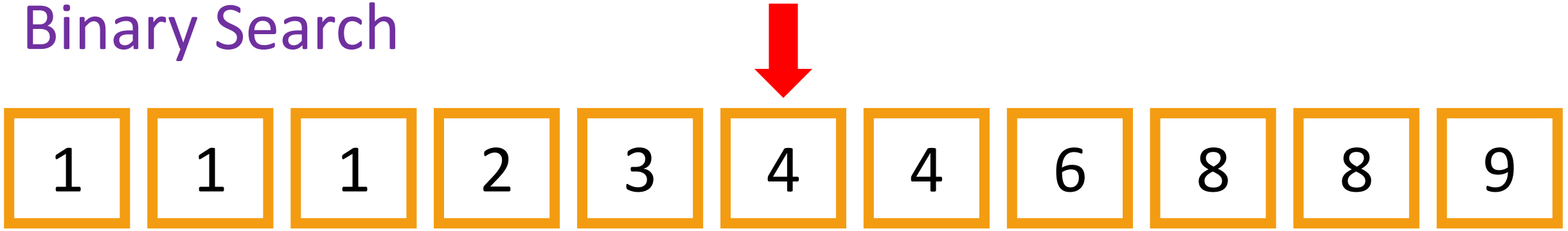

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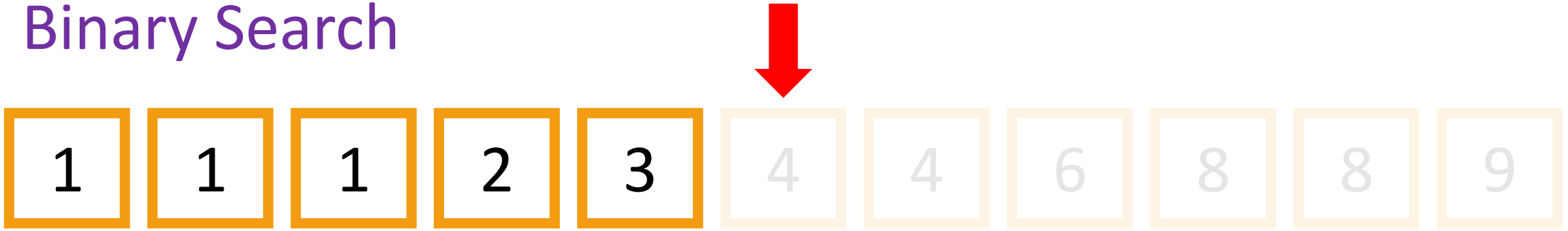

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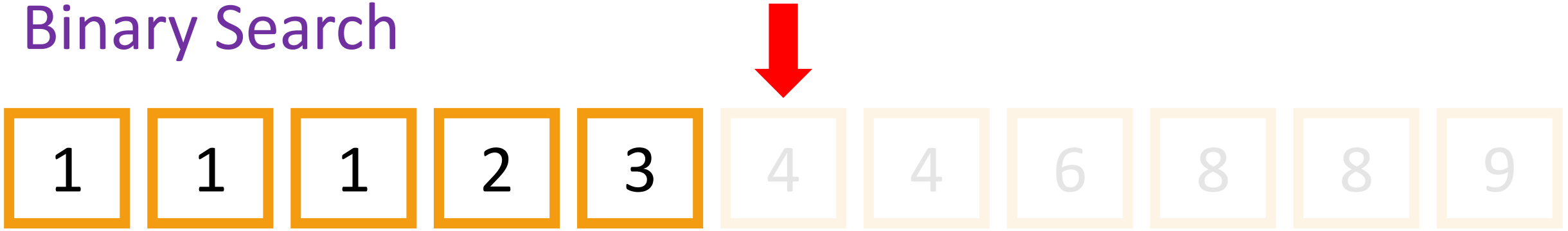

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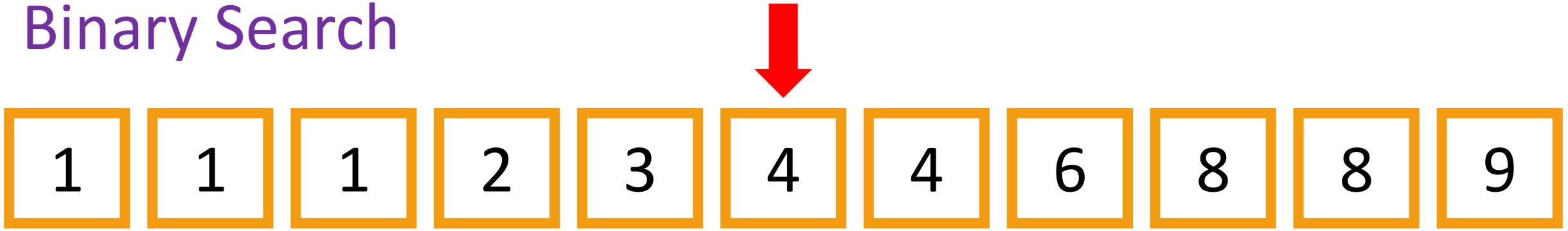

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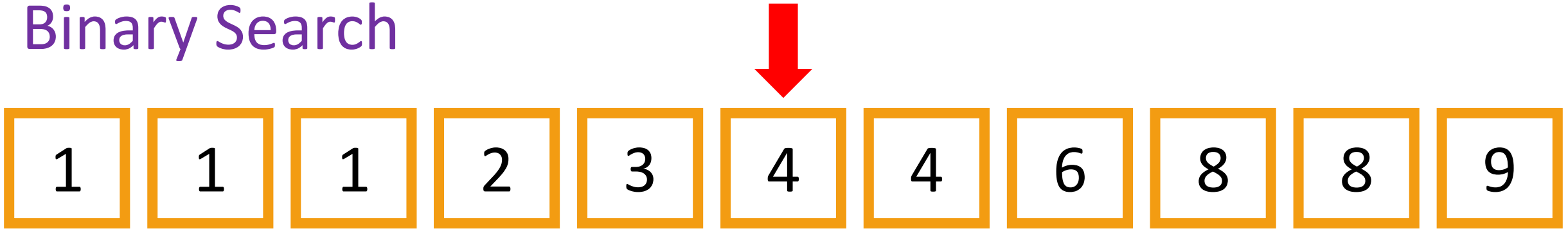

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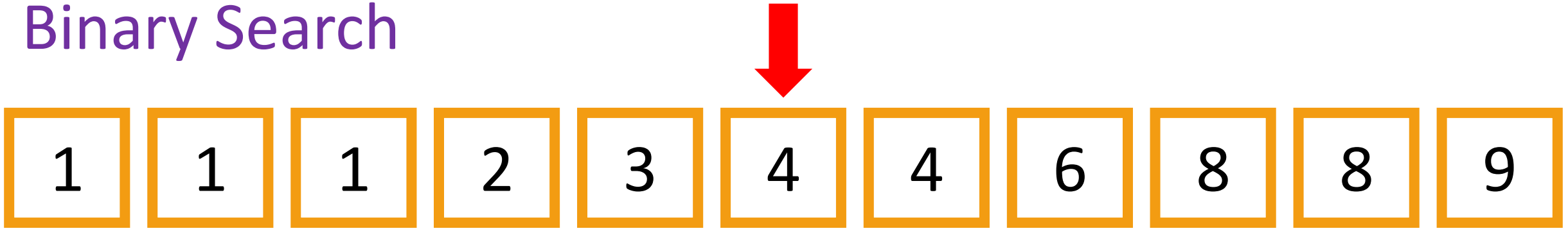

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- So a win-win situation – we either find the element or else reduce the search space to only half of the array
- Example of the *Divide and Conquer* technique – divide original problem into smaller instances of the same problem

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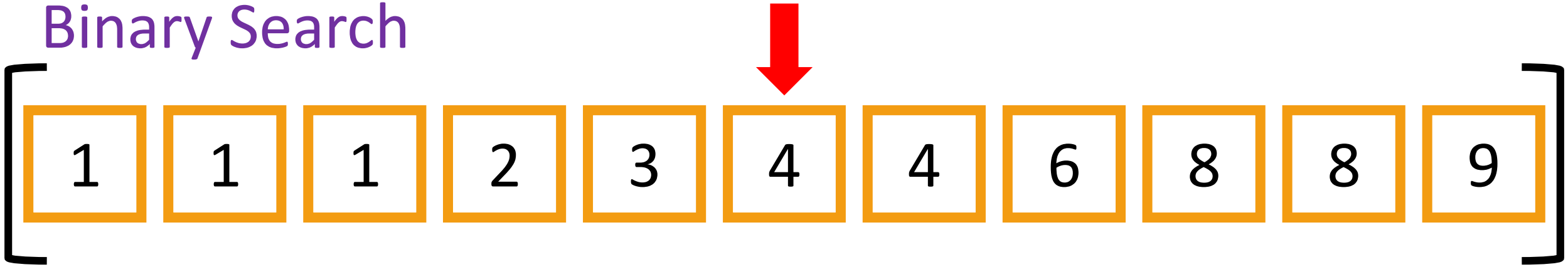

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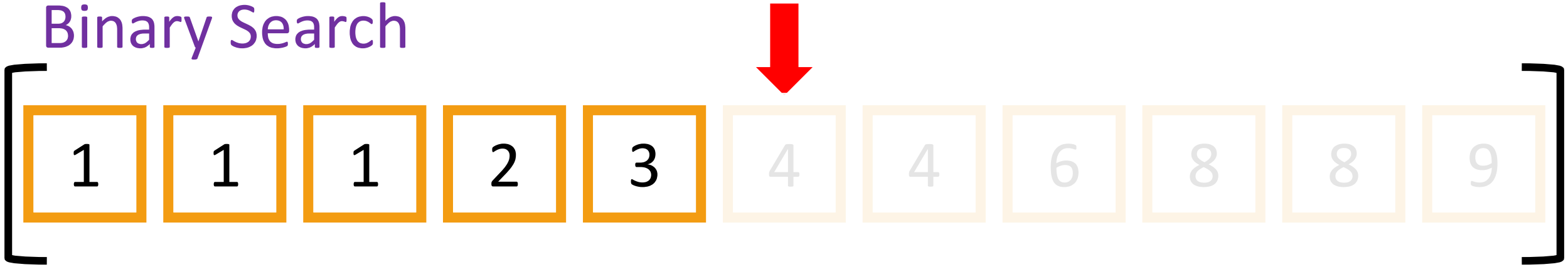

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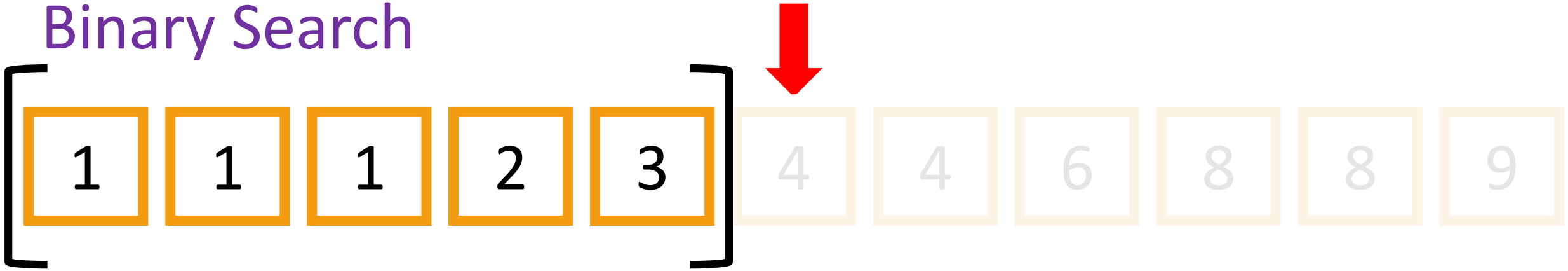

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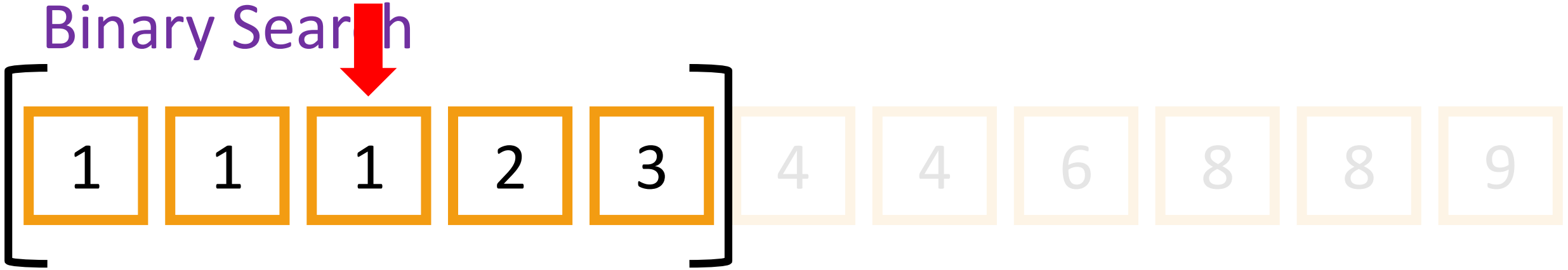

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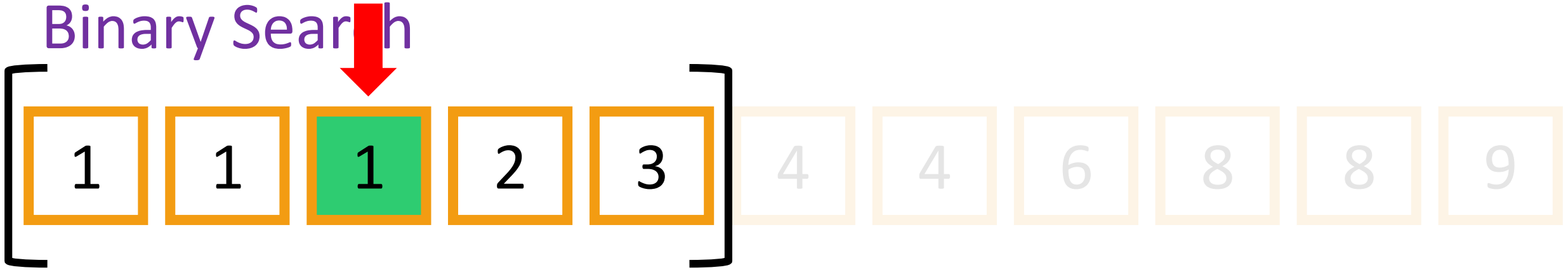

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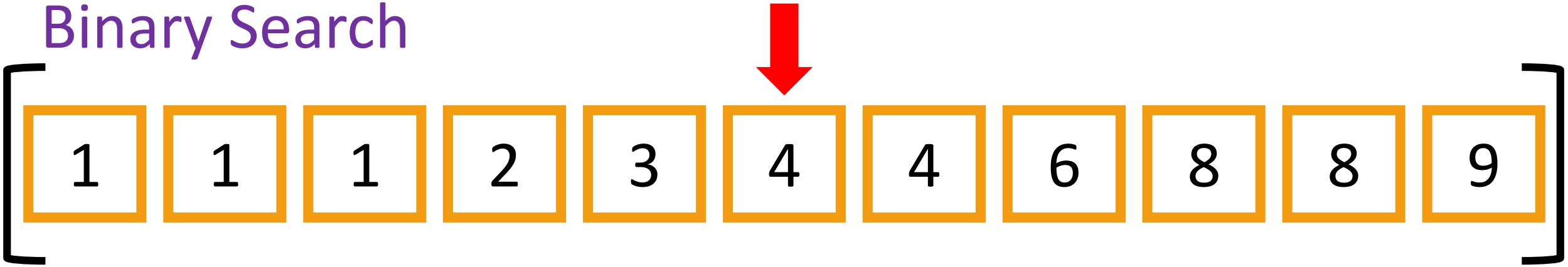

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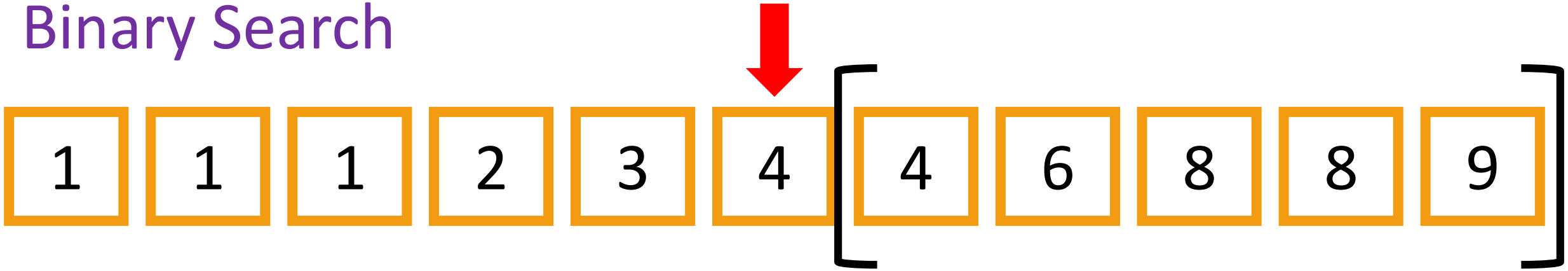

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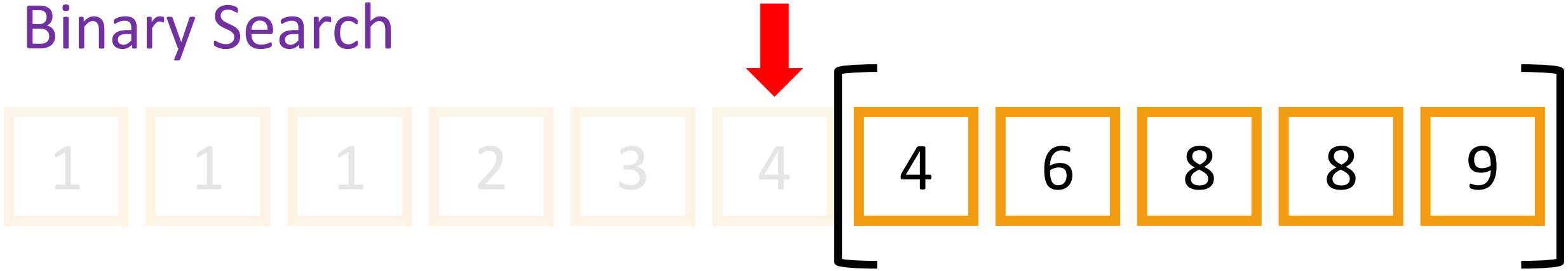

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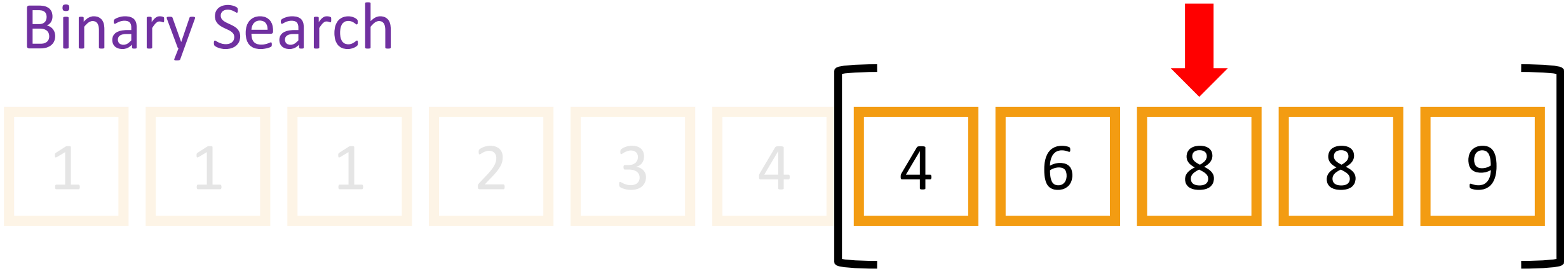

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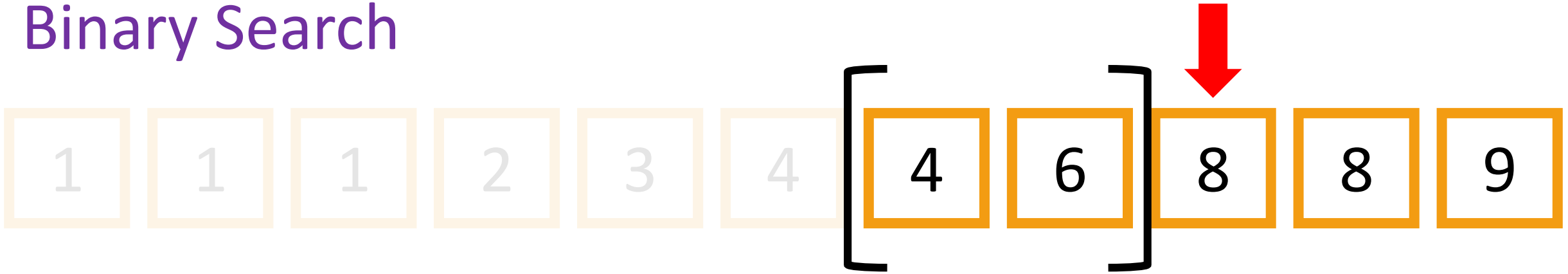

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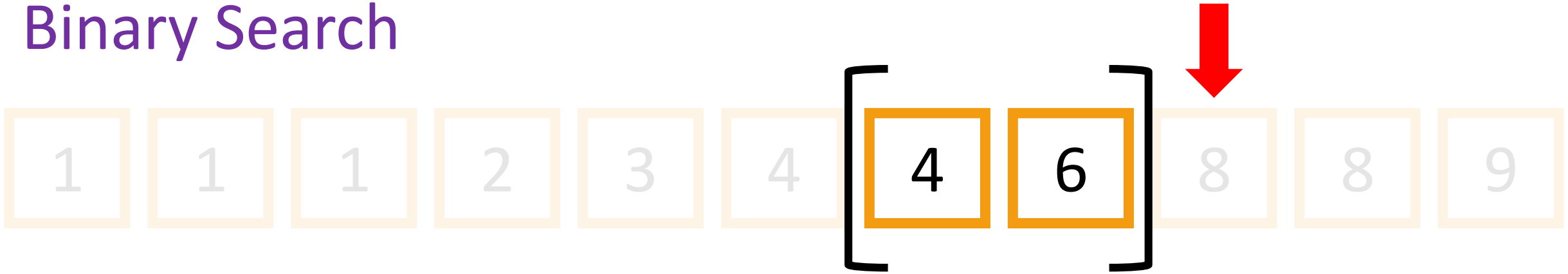

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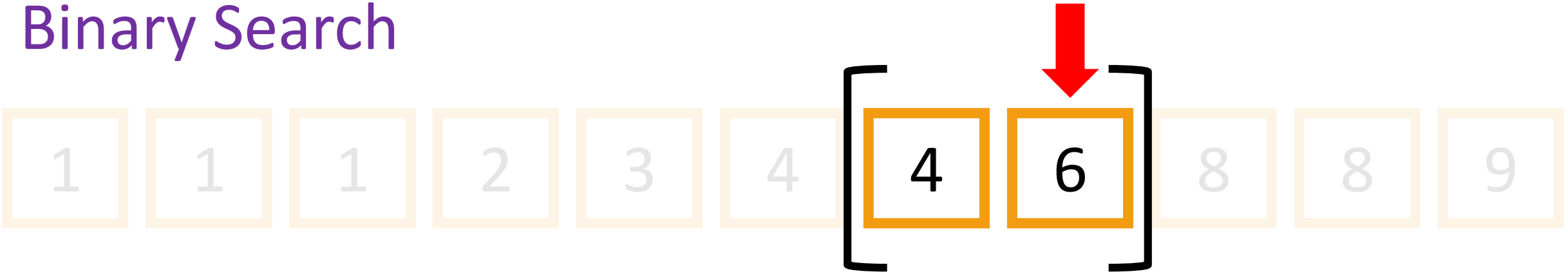

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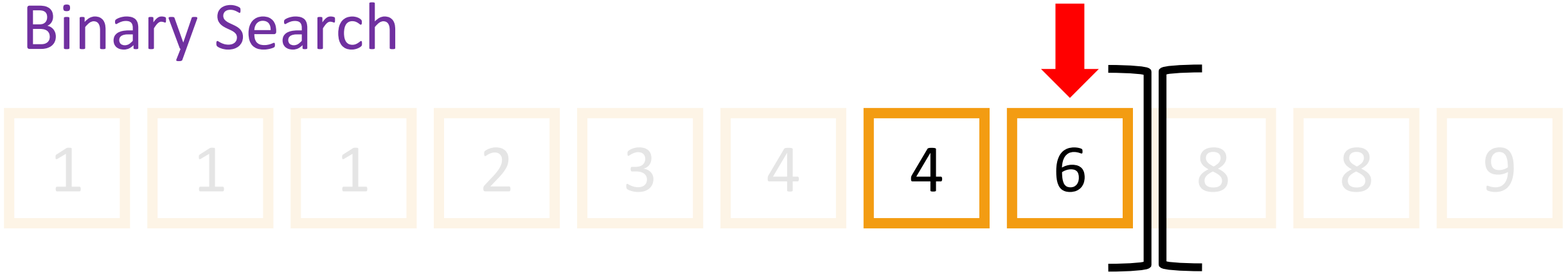

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2. Let $L \leftarrow 0$ and $R \leftarrow N-1$ *//Initial active range is full array*
3. While $L \leq R$
   1. Let $M \leftarrow \text{ceil}((L+R)/2)$
   2. If $a[M] == K$, return M *//Found key, return location*
   3. If $a[M] > K$, set $R \leftarrow M-1$ *//Right portion can't host K*
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Exercise: convert this to proper C code

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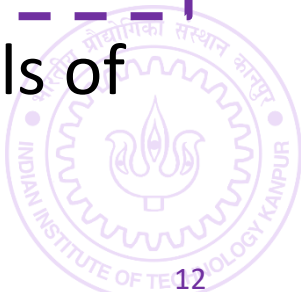

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Exercise: write a recursive version

Exercise: convert this to proper C code

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Asymptotic Time Complexity

- An effort to quantify the *speed* of algorithms in a manner that is independent of the computer on which they are executed
- Arguably binary search seems “faster” than brute force search
- We saw that in the worse case, brute force search on an unsorted array must check all N elements before answering
- Can binary search on sorted arrays also be forced to do so?
- Let $T(N)$ denote the time taken by binary search to search for a key in a sorted array with N elements
- We know that at every iteration of the while loop, binary search either discovers the element being searched or else reduces the length of the active range by a factor of 2

Asymptotic Time Complexity

- Thus, we must have $T(N) \leq c + T(N/2)$
  - c is the time taken to compare the middle element and update L, R
  - Note that c does not depend on N at all. Also note that $T(1) \leq c$
  - The above is a *recurrence relation*. It expresses T in terms of itself
- Applying the above to $T(N/2)$ gives us $T(N/2) \leq c + T(N/4)$ i.e. $T(N) \leq 2c + T(N/4) = 2c + T(N/2^2)$
- Repeating this gives us $T(N) < cm + T(N/2^m)$ for any $m > 0$
- However for $m \geq \text{ceil}(\log_2 N)$ we have $2^m \geq N$
- This means that $T(N) \leq c \cdot \text{ceil}(\log N) + T(1) \leq c \cdot \text{ceil}(\log N) + c$
- For all $N \geq 4$ we have $\text{ceil}(\log N) + 1 \leq 2 \log N$ which gives us

$$T(N) \leq 2c \cdot \log N$$

Big-Oh Notation

- Suppose we have two functions $f, g: \mathbb{R}_+ \to \mathbb{R}_+$ such that there exists a constant $c > 0$ so that for all "large" values of $x \in \mathbb{R}_+$ i.e. for all $x \geq M$ for some $M > 0$, we have

$$f(x) \leq c \cdot g(x)$$

  Then we say that $f(x) = \mathcal{O}\big(g(x)\big)$
- Be careful that c must not depend on x for the above statement
- The above discussion shows that the runtime complexity of Binary search is $\mathrm{T(N)} = \mathcal{O}(\log N)$ since for some constant c that doesn't depend on N we have $T(N) \leq 2c \cdot \log N$ for all $N \geq 4$
- Exercise: show that the runtime of brute force search is $O(N)$

Exercises

- Given a array `int a[N];` sorted in ascending order
  - Find the number of occurrences of a given number K in the array
    - Generalizes the search problem we just studied
  - Find the predecessor of a given number K in the array
    - Largest number in the array that is strictly smaller than K. Return NULL if none.
    - Be careful, the key K may itself occur say $N/2$ times in the array
  - Given a positive integer $M \leq N$, find the element of the array which is greater than or equal to exactly M elements of the array
    - $M = 1$ gives the smallest element, $M = N$ the largest element, $M = N/2$ the median
    - If you are interested, look up the term *quantile* on the internet for more info
- Make sure your algorithms take no more than $O(\log N)$ steps!
- Can you do the above operations as fast if the array is not sorted?

Exercises

- Given a (non-sorted) array `int a[N];` count the number of swaps A swap is a pair $0 \leq i \neq j < N$ such that $i < j$ but `a[i] > a[j]`
  - This problem is related to a ranking metric known as *area under the ROC curve*. Check it out if interested
- We have two arrays of N numbers `int P[N],F[N];` containing 12th marks of N students each who cleared and did not clear JEE
  - Find out the number of students who did not clear JEE but had 12th std marks more than at least 50% of students who did clear JEE
- Solve these problems faster than $O(N^2)$ time (Hint may involve sorting). Assume you have a routine that can sort N elements in $O(N \log N)$ time – will see such methods soon.

Sorting Algorithms

Selection Sort

Selection Sort

- One of the many (many) algorithms for sorting – very simple
- Like binary search, maintains *active range* $a[0:R]$ with $0 \leq R < N$
  - Initially the active range is entire array i.e. $R = N - 1$
- Invariants: we will ensure two things
  - At all points of time, the non-active portion will be sorted in ascending order i.e. for all $R \leq i < j$ we will ensure $a[i] \leq a[j]$
  - The non-active elements will never be smaller than the elements in the active range i.e. if $i \leq R < j$ then $a[i] \leq a[j]$
- The active region will shrink by one element at each step

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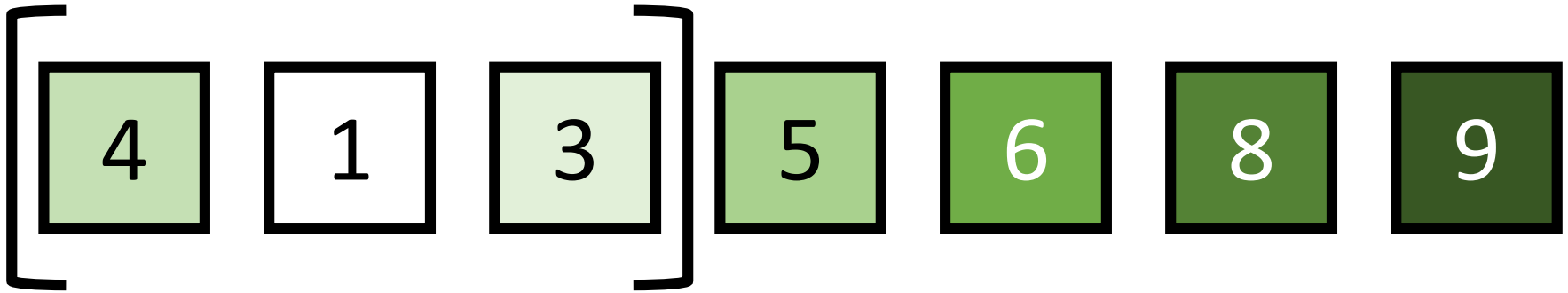

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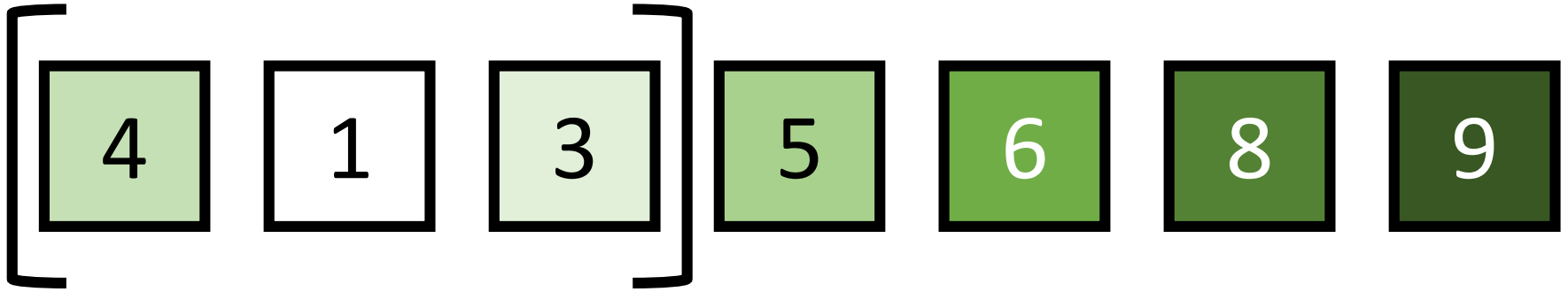

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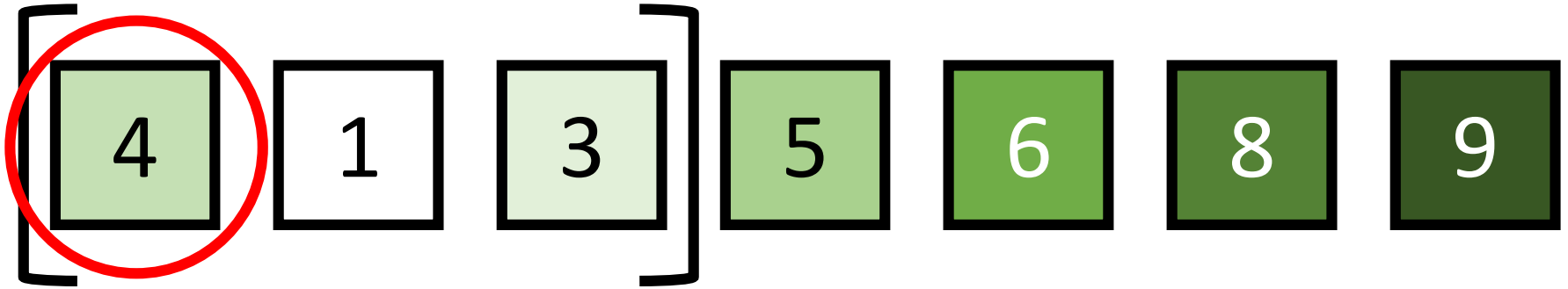

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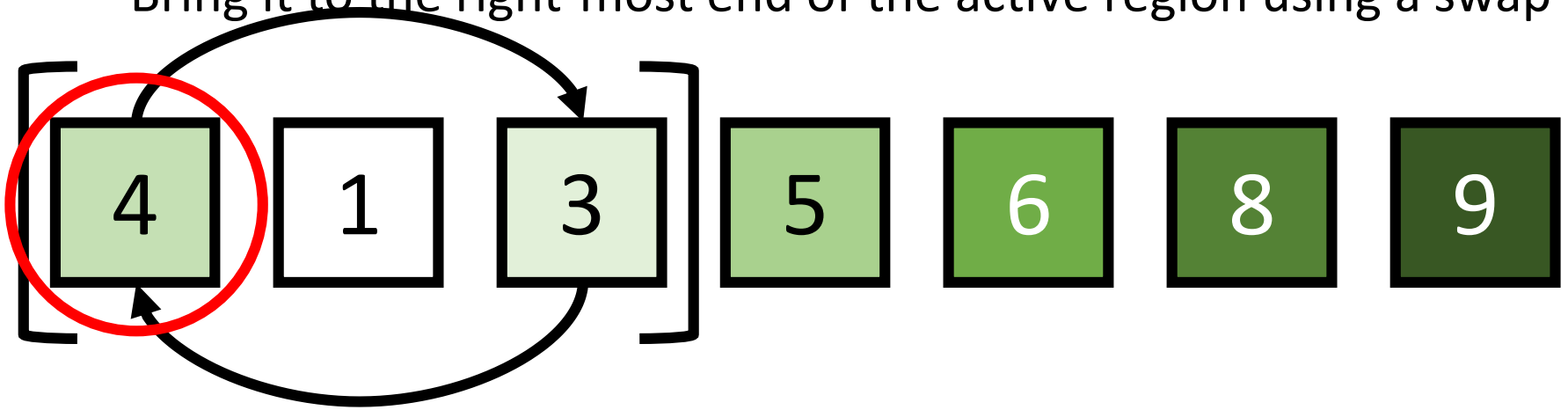

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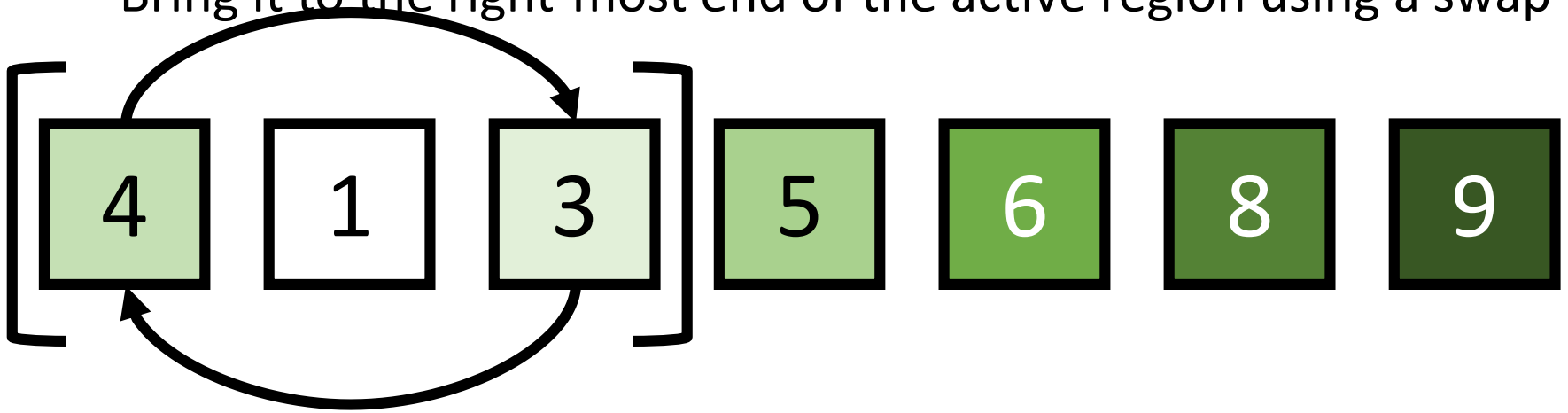

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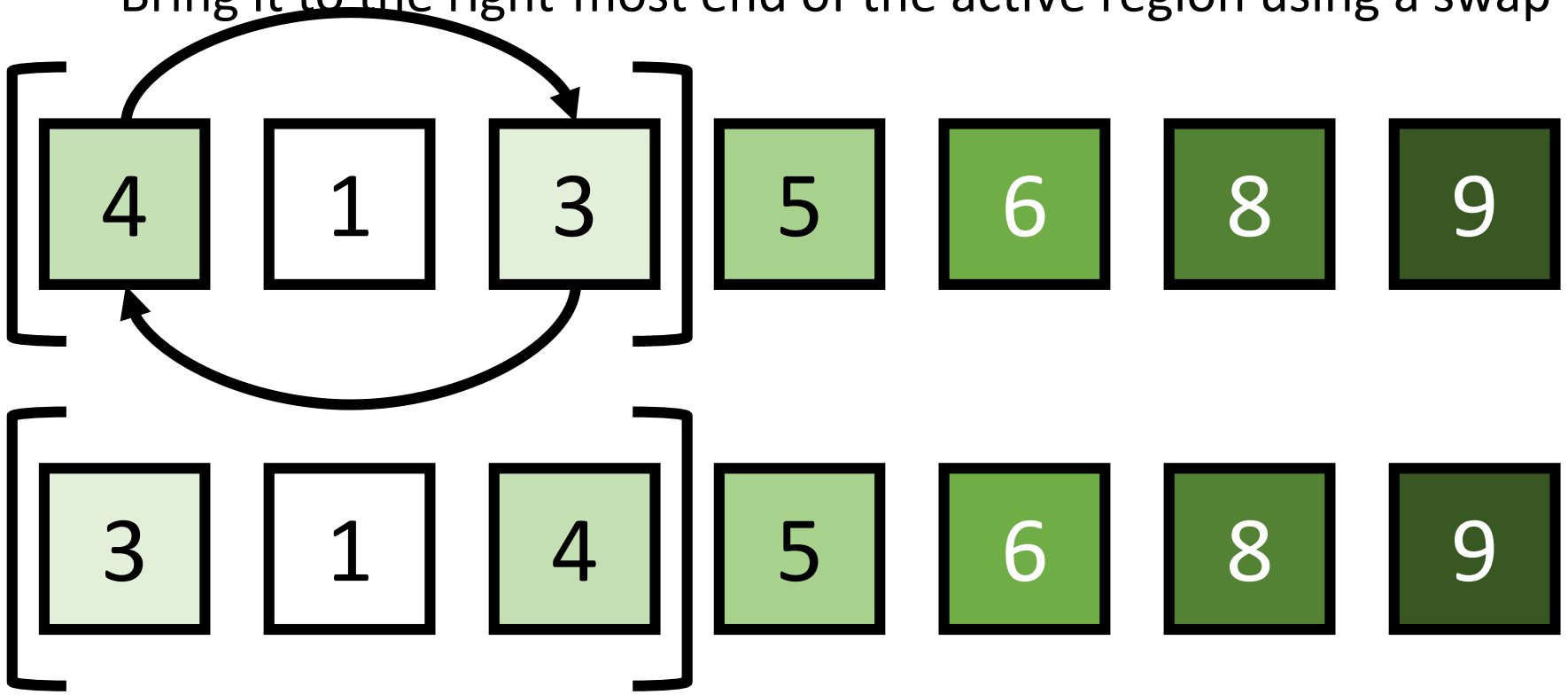

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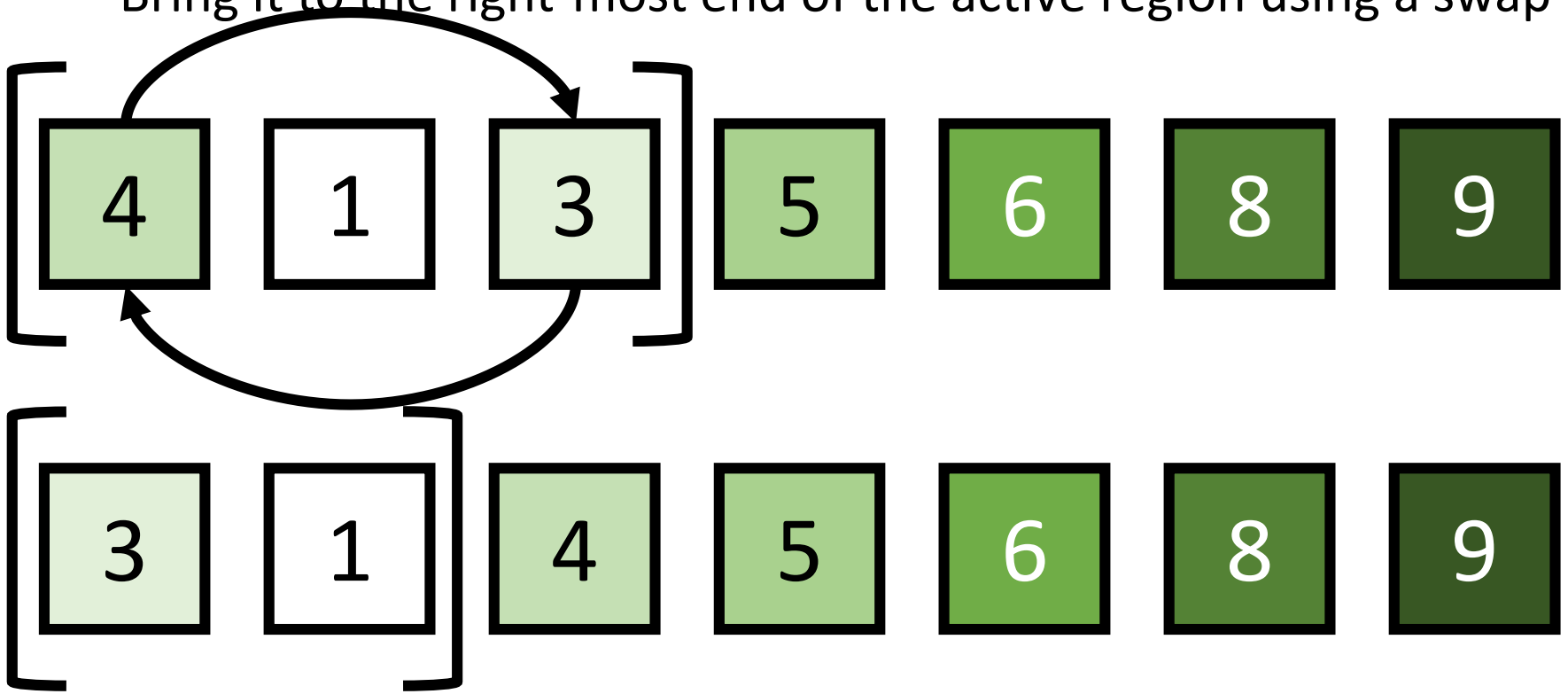

Selection Sort

- Notice that we never have to touch the non-active region ☺
- To maintain the invariant and still shrink the active region
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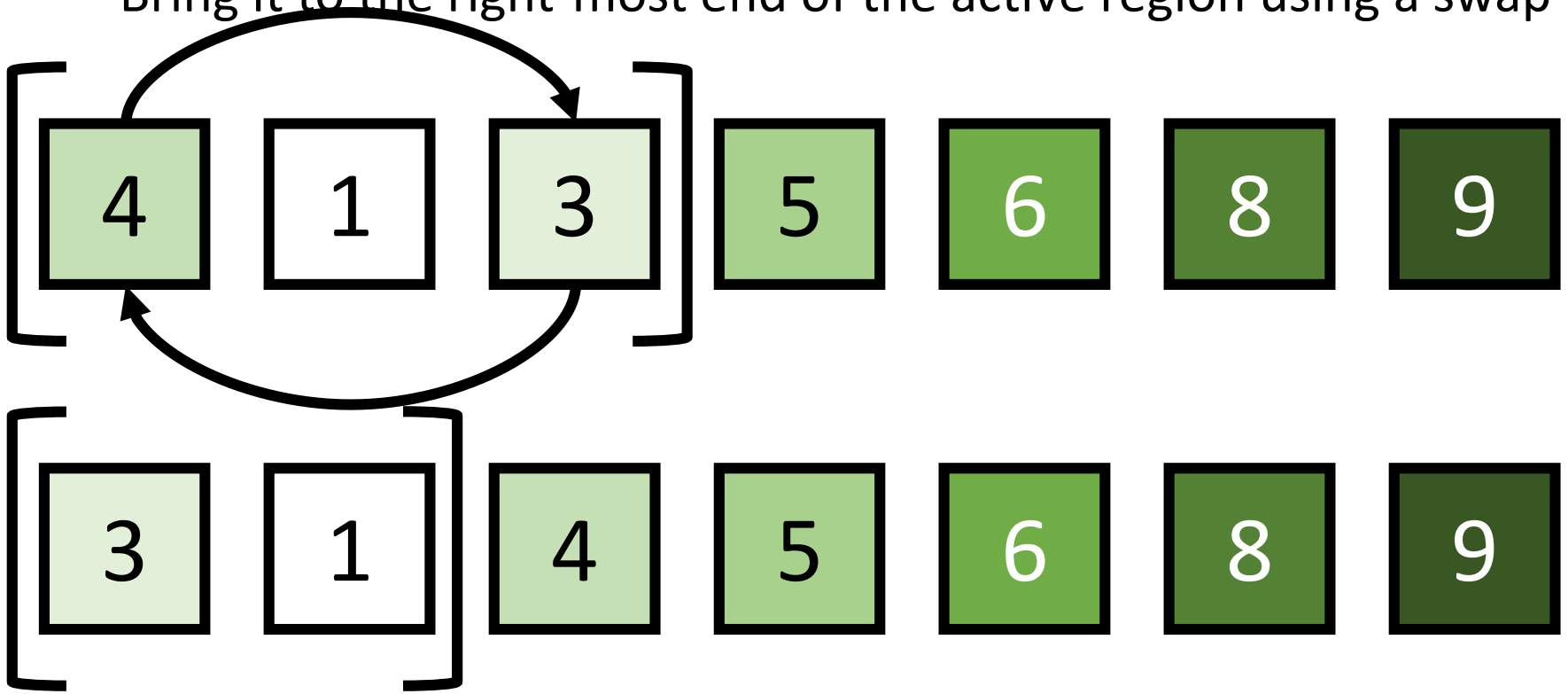

Verify that all promises of the invariant still hold

Selection Sort

SELECTION SORT

1. Given: Array a with N elements
2. For $R = N - 1; R > 0; R - -$ *//Initial active range is full array*
   1. $i \leftarrow \text{FINDMAX}(a, 0, R)$ *//Location of largest element in* $a[0, R]$
   2. $\text{SWAP}(a, i, R)$ *//Bring largest element to the end*

SWAP

1. Given: Array a, location i, j
2. Let $tmp \leftarrow a[i]$
3. Let $a[i] \leftarrow a[j]$
4. Let $a[j] \leftarrow tmp$

FINDMAX

1. Given: Array a, locations i, j
2. Let $k \leftarrow i, \max = a[k]$
3. For $l = i; i \leq j; l + +$
   1. If $a[l] > \max, \max = a[l], k = l$
4. Return k

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Exercise: convert this to proper C code

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Selection Sort

Exercise: write a recursive version

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Time Complexity

- Let $T(N)$ be the time taken for selection sort to sort N elements
- Let $M(N)$ be the time taken to find location of max of N elements
- At any time step when active region is $[0:R]$, we do two things
  - Find the largest element within the active region – takes time $M(R+1)$
  - Swap the largest element with the element at $a[R]$ - takes time c (const)
- Thus, we have $T(N) \leq M(N) + c + T(N-1)$
- It is easy to show that $M(N) \leq d \cdot N$ for all N for some constant d
- Exercise: expand the recurrence as before and show that

$$T(N) \leq \mathcal{O}(N^2)$$

  Assume $T(1) \leq c$
- Notice that selection sort doesn't need any extra memory (except a few tmp variables to store one integer each) – *in-place sorting*

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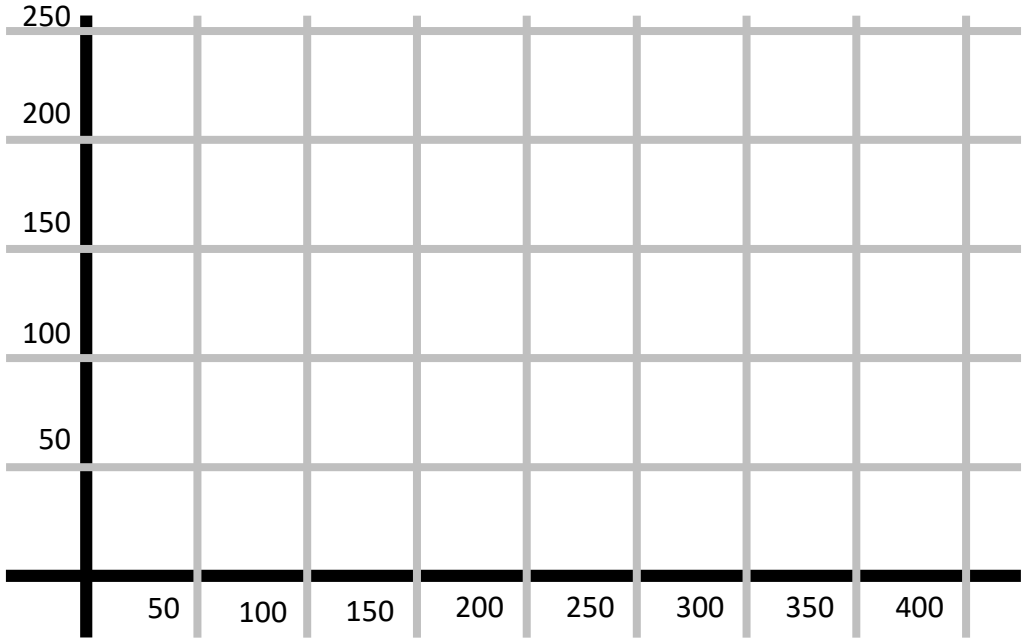

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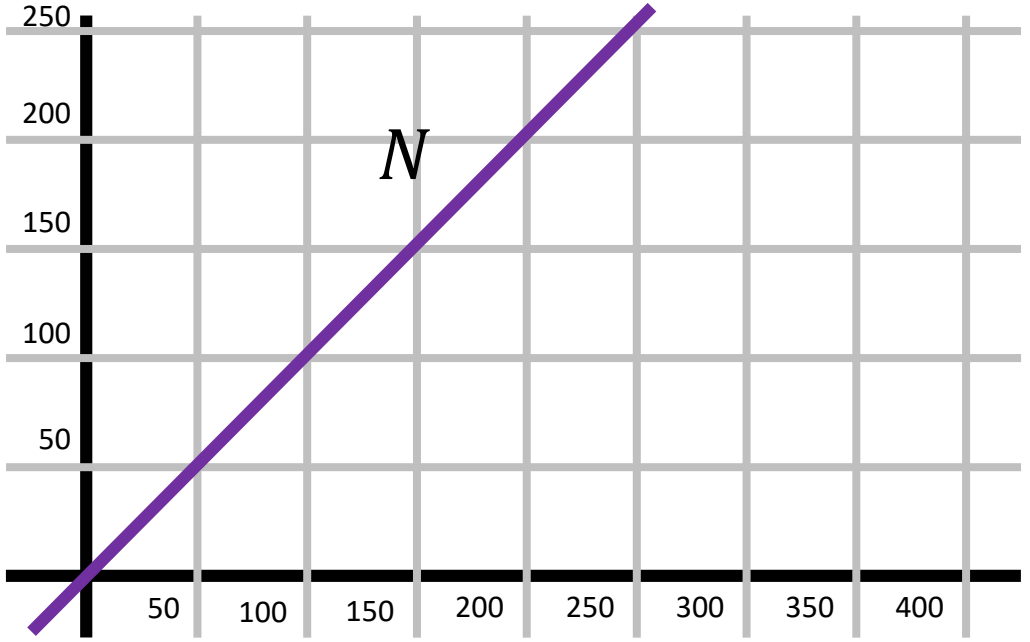

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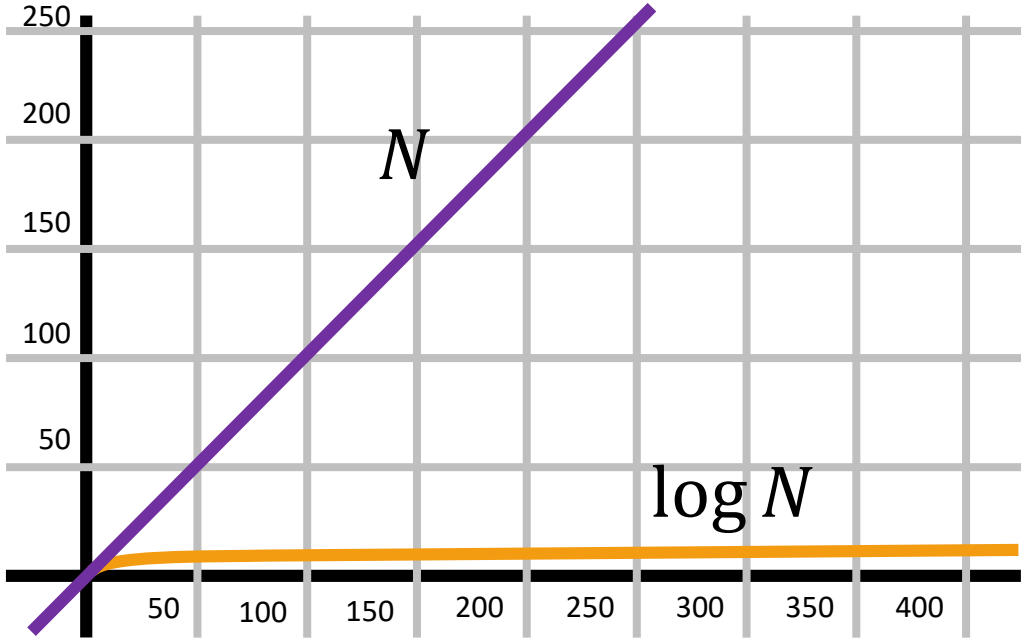

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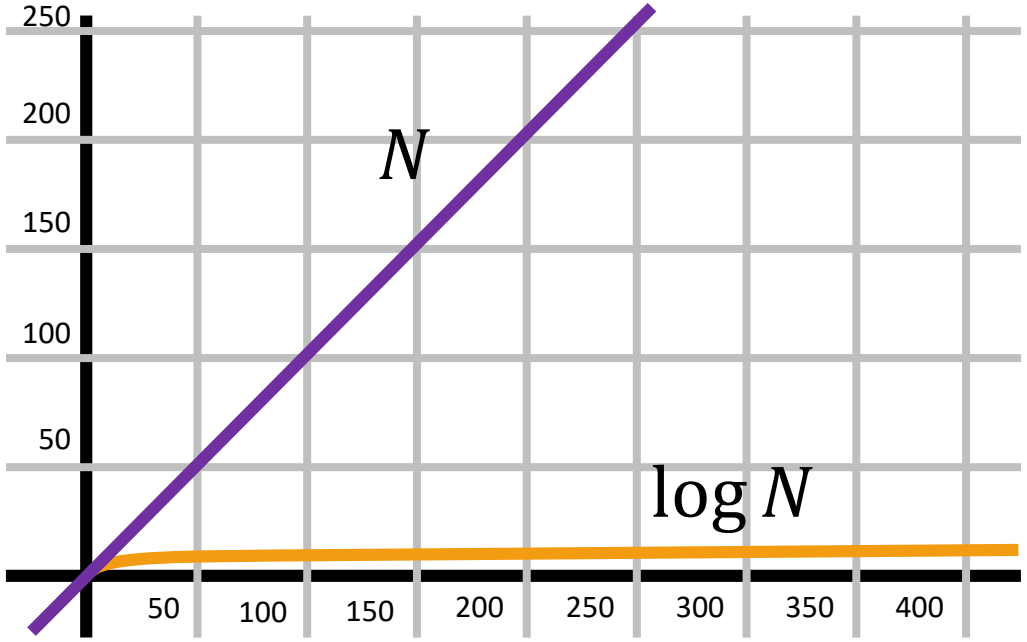

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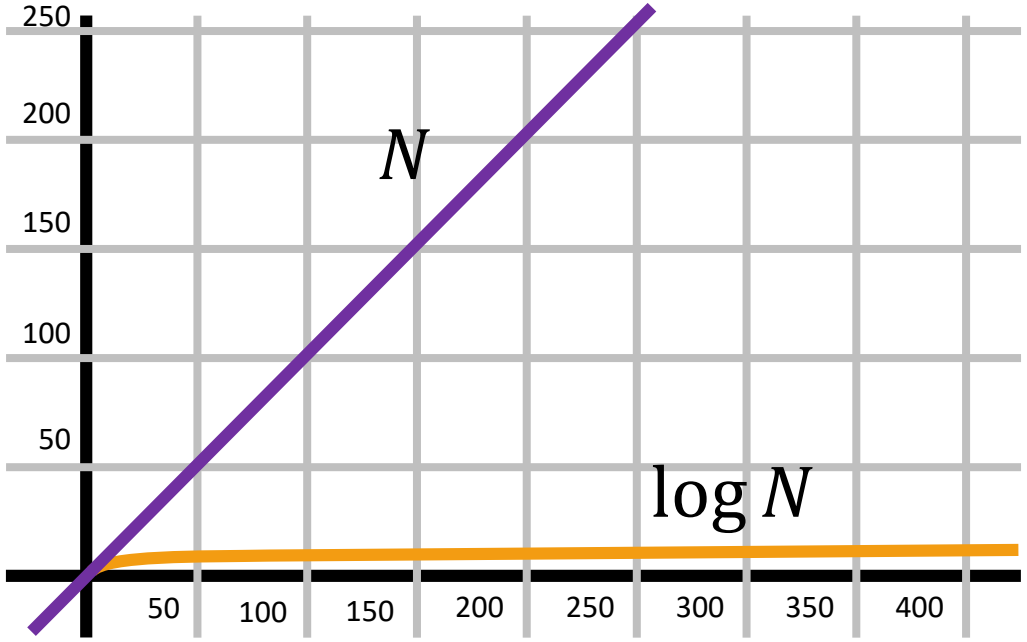

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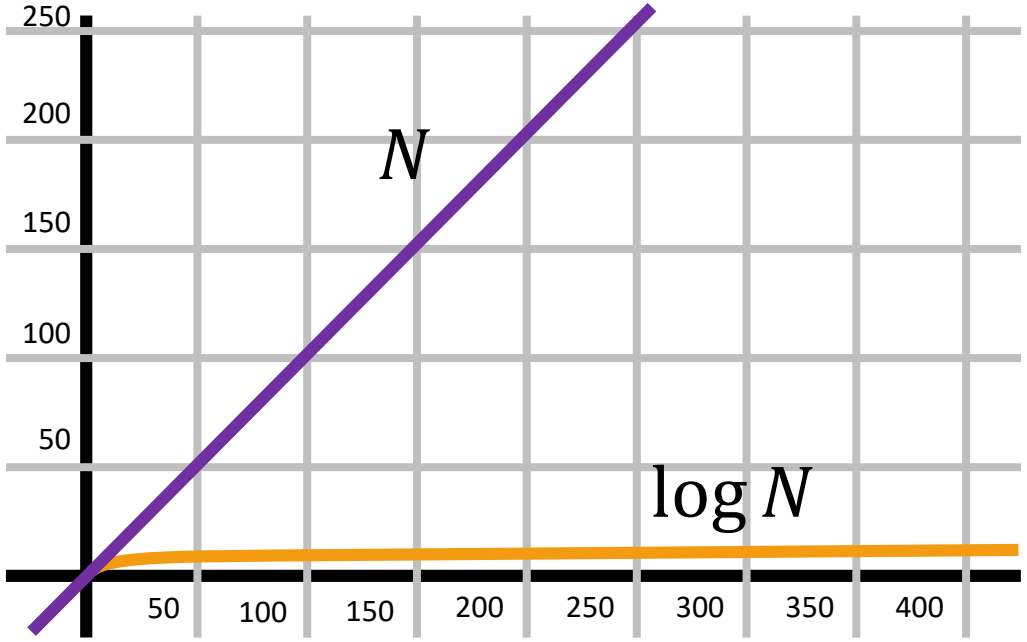

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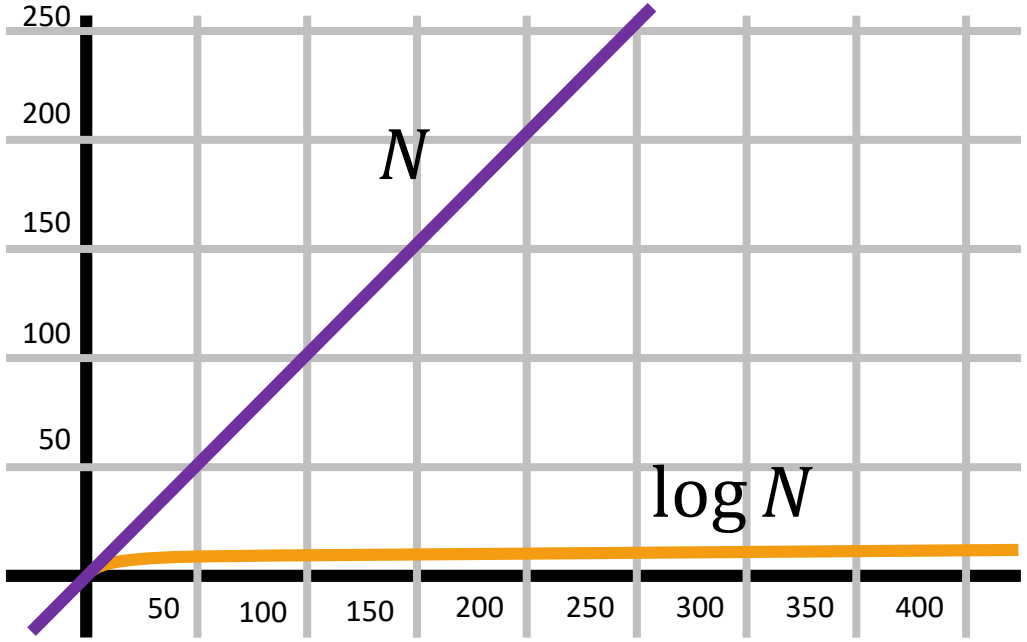

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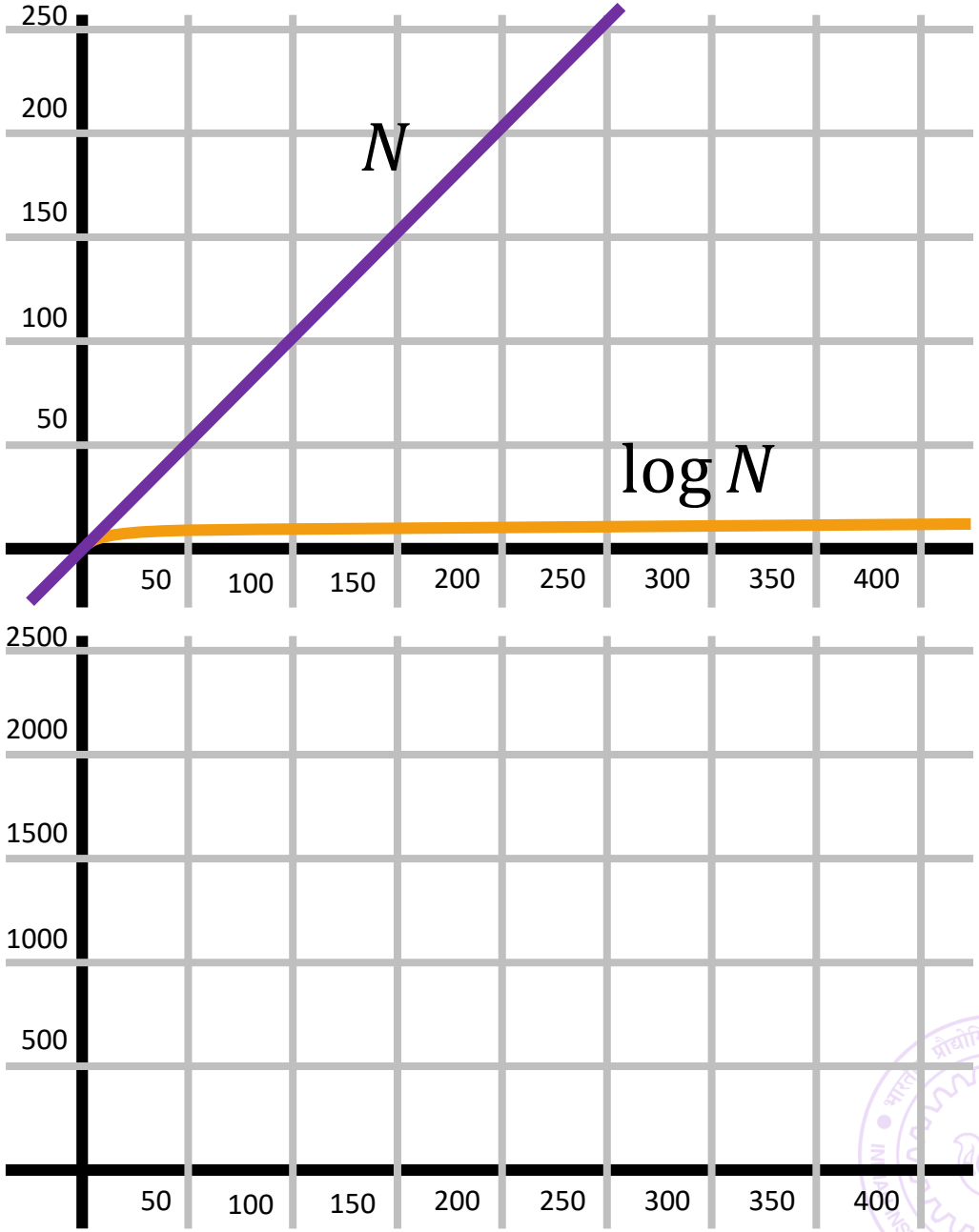

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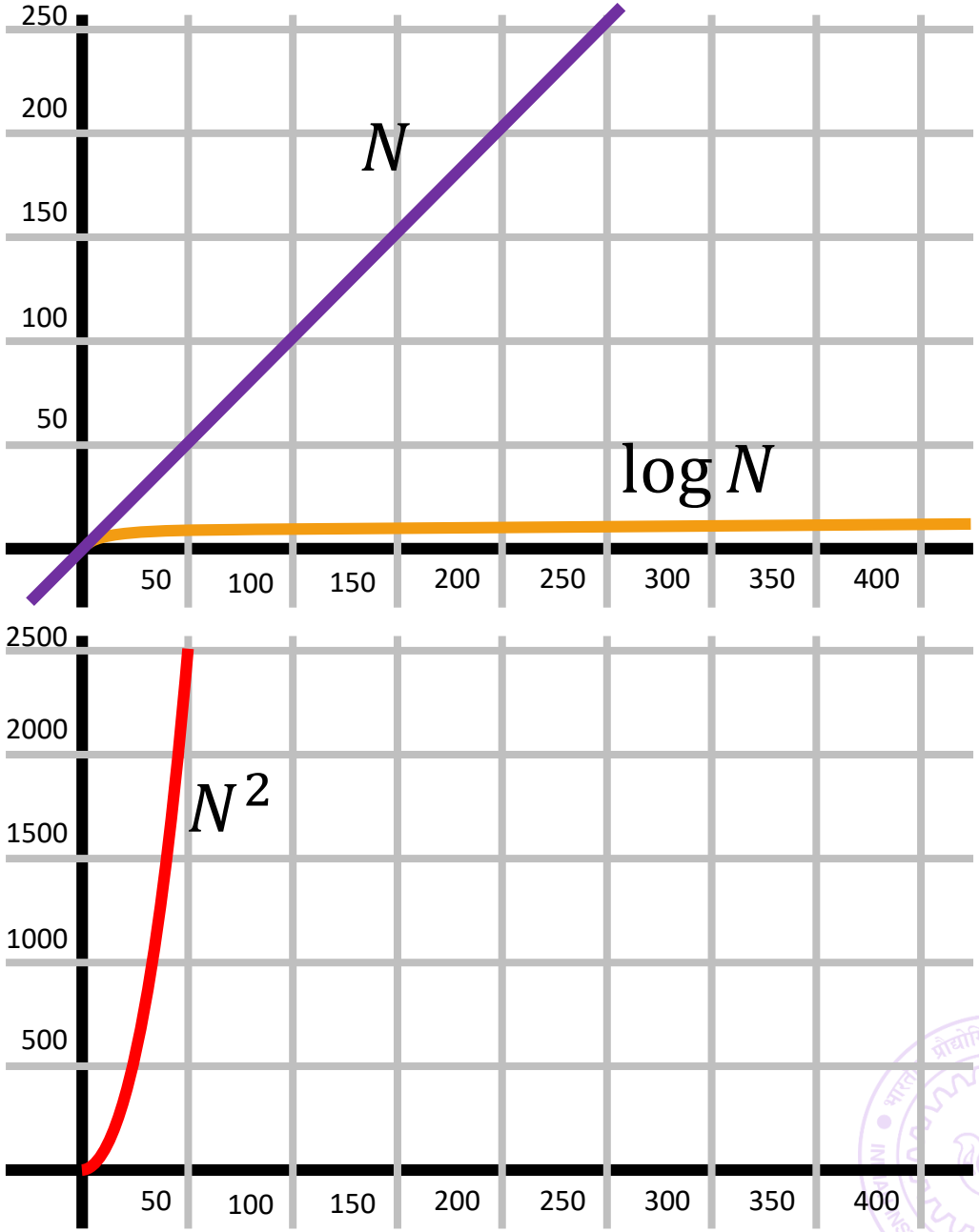

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