NCERT MISCELLANEOUS EXERCISE

Question 1:

Prove that: $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$

Ans:

L.H.S.

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}$$

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\left(\frac{\frac{3\pi}{13}+\frac{5\pi}{13}}{2}\right)\cos\left(\frac{\frac{3\pi}{13}-\frac{5\pi}{13}}{2}\right)\quad\left[\cos x+\cos y=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)\right]$$

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\frac{4\pi}{13}\cos\left(\frac{-\pi}{13}\right)$$

$$=2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+2\cos\frac{4\pi}{13}\cos\frac{\pi}{13}$$

$$=2\cos\frac{\pi}{13}\left[\cos\frac{9\pi}{13}+\cos\frac{4\pi}{13}\right]$$

$$=2\cos\frac{\pi}{13}\left[2\cos\left(\frac{\frac{9\pi}{13}+\frac{4\pi}{13}}{2}\right)\cos\left(\frac{\frac{9\pi}{13}-\frac{4\pi}{13}}{2}\right)\right]$$

$$=2\cos\frac{\pi}{13}\left[2\cos\frac{\pi}{2}\cos\frac{5\pi}{26}\right]$$

$$=2\cos\frac{\pi}{13}\times2\times0\times\cos\frac{5\pi}{26}$$

$$=0=\text{R.H.S}$$

Question 2:

Prove that: $(\sin 3x + \sin x)\sin x + (\cos 3x - \cos x)\cos x = 0$

Ans:

L.H.S.

$$=(\sin 3x+\sin x)\sin x+(\cos 3x-\cos x)\cos x$$

$$=\sin 3x\sin x+\sin^2 x+\cos 3x\cos x-\cos^2 x$$

$$=\cos 3x\cos x+\sin 3x\sin x-\left(\cos^2 x-\sin^2 x\right)$$

$$=\cos(3x-x)-\cos 2x\qquad\left[\cos(A-B)=\cos A\cos B+\sin A\sin B\right]$$

$$=\cos 2x-\cos 2x$$

$$=0$$

$$=\text{RH.S.}$$

Question 3:

Prove that: $(\cos x+\cos y)^2+(\sin x-\sin y)^2=4\cos^2\frac{x+y}{2}$

Ans:

$$\text{L.H.S.} = (\cos x + \cos y)^2 + (\sin x - \sin y)^2$$

$$= \cos^2 x + \cos^2 y + 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y$$
$$= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) + 2(\cos x \cos y - \sin x \sin y)$$
$$= 1 + 1 + 2\cos(x+y) \qquad [\cos(A+B) = (\cos A \cos B - \sin A \sin B)]$$
$$= 2 + 2\cos(x+y)$$
$$= 2[1 + \cos(x+y)]$$
$$= 2\left[1 + 2\cos^2\left(\frac{x+y}{2}\right) - 1\right] \qquad [\cos 2A = 2\cos^2 A - 1]$$
$$= 4\cos^2\left(\frac{x+y}{2}\right) = \text{R.H.S.}$$

Question 4:

Prove that: $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2\frac{x-y}{2}$

Ans:

$$\text{L.H.S.} = (\cos x - \cos y)^2 + (\sin x - \sin y)^2$$

$$= \cos^2 x + \cos^2 y - 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y$$
$$= (\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) - 2[\cos x \cos y + \sin x \sin y]$$
$$= 1 + 1 - 2[\cos(x-y)] \qquad [\cos(A-B) = \cos A \cos B + \sin A \sin B]$$
$$= 2[1 - \cos(x-y)]$$
$$= 2\left[1 - \left\{1 - 2\sin^2\left(\frac{x-y}{2}\right)\right\}\right] \qquad [\cos 2A = 1 - 2\sin^2 A]$$
$$= 4\sin^2\left(\frac{x-y}{2}\right) = \text{R.H.S.}$$

Question 5:

Prove that: $\sin x + \sin 3x + \sin 5x + \sin 7x = 4\cos x \cos 2x \sin 4x$

Ans:

It is known that $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$.

$$\therefore \text{L.H.S.} = \sin x + \sin 3x + \sin 5x + \sin 7x$$

$$= (\sin x + \sin 5x) + (\sin 3x + \sin 7x)$$
$$= 2\sin\left(\frac{x+5x}{2}\right)\cdot\cos\left(\frac{x-5x}{2}\right) + 2\sin\left(\frac{3x+7x}{2}\right)\cos\left(\frac{3x-7x}{2}\right)$$
$$= 2\sin 3x \cos(-2x) + 2\sin 5x \cos(-2x)$$
$$= 2\sin 3x \cos 2x + 2\sin 5x \cos 2x$$
$$= 2\cos 2x[\sin 3x + \sin 5x]$$
$$= 2\cos 2x\left[2\sin\left(\frac{3x+5x}{2}\right)\cdot\cos\left(\frac{3x-5x}{2}\right)\right]$$
$$= 2\cos 2x[2\sin 4x \cdot \cos(-x)]$$
$$= 4\cos 2x \sin 4x \cos x = \text{R.H.S.}$$

Question 6:

Prove that: $\dfrac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x$

Ans:

It is known that

$$\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right),\ \cos A+\cos B=2\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$$

$$\text{L.H.S.}=\frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}$$

$$=\frac{\left[2\sin\left(\frac{7x+5x}{2}\right)\cdot\cos\left(\frac{7x-5x}{2}\right)\right]+\left[2\sin\left(\frac{9x+3x}{2}\right)\cdot\cos\left(\frac{9x-3x}{2}\right)\right]}{\left[2\cos\left(\frac{7x+5x}{2}\right)\cdot\cos\left(\frac{7x-5x}{2}\right)\right]+\left[2\cos\left(\frac{9x+3x}{2}\right)\cdot\cos\left(\frac{9x-3x}{2}\right)\right]}$$

$$=\frac{[2\sin 6x\cdot\cos x]+[2\sin 6x\cdot\cos 3x]}{[2\cos 6x\cdot\cos x]+[2\cos 6x\cdot\cos 3x]}$$

$$=\frac{2\sin 6x[\cos x+\cos 3x]}{2\cos 6x[\cos x+\cos 3x]}$$

$$=\tan 6x$$

$$=\text{R.H.S.}$$

Question 7:

Prove that: $\sin 3x+\sin 2x-\sin x=4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}$

Ans:

$$\text{L.H.S.}=\sin 3x+\sin 2x-\sin x$$

$$=\sin 3x+(\sin 2x-\sin x)$$

$$=\sin 3x+\left[2\cos\left(\frac{2x+x}{2}\right)\sin\left(\frac{2x-x}{2}\right)\right]\qquad\left[\sin A-\sin B=2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$=\sin 3x+\left[2\cos\left(\frac{3x}{2}\right)\sin\left(\frac{x}{2}\right)\right]$$

$$=\sin 3x+2\cos\frac{3x}{2}\sin\frac{x}{2}$$

$$=2\sin\frac{3x}{2}\cdot\cos\frac{3x}{2}+2\cos\frac{3x}{2}\sin\frac{x}{2}\qquad[\sin 2A=2\sin A\cdot\cos B]$$

$$=2\cos\left(\frac{3x}{2}\right)\left[\sin\left(\frac{3x}{2}\right)+\sin\left(\frac{x}{2}\right)\right]$$

$$=2\cos\left(\frac{3x}{2}\right)\left[2\sin\left\{\frac{\left(\frac{3x}{2}\right)+\left(\frac{x}{2}\right)}{2}\right\}\cos\left\{\frac{\left(\frac{3x}{2}\right)-\left(\frac{x}{2}\right)}{2}\right\}\right]\quad\left[\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$=2\cos\left(\frac{3x}{2}\right).2\sin x\cos\left(\frac{x}{2}\right)$$

$$=4\sin x\cos\left(\frac{x}{2}\right)\cos\left(\frac{3x}{2}\right)=\text{R.H.S.}$$

Question 8:

Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$, if

$\tan x=-\frac{4}{3}$, x in quadrant II

Ans:

Here, x is in quadrant II.

i.e., $\frac{\pi}{2} < x < \pi$

$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$

Therefore, $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are all positive.

It is given that $\tan x = -\frac{4}{3}$.

$\sec^2 x = 1 + \tan^2 x = 1 + \left(\frac{-4}{3}\right)^2 = 1 + \frac{16}{9} = \frac{25}{9}$

$\therefore \cos^2 x = \frac{9}{25}$

$\Rightarrow \cos x = \pm\frac{3}{5}$

As x is in quadrant II, cos$x$ is negative.

$\therefore \cos x = \frac{-3}{5}$

Now, $\cos x = 2\cos^2\frac{x}{2} - 1$

$\Rightarrow \frac{-3}{5} = 2\cos^2\frac{x}{2} - 1$

$\Rightarrow 2\cos^2\frac{x}{2} = 1 - \frac{3}{5}$

$\Rightarrow 2\cos^2\frac{x}{2} = \frac{2}{5}$

$\Rightarrow \cos^2\frac{x}{2} = \frac{1}{5}$

$\Rightarrow \cos\frac{x}{2} = \frac{1}{\sqrt{5}}$ $\left[\because \cos\frac{x}{2} \text{ is positive}\right]$

$\therefore \cos\frac{x}{2} = \frac{\sqrt{5}}{5}$

$$\sin^2\frac{x}{2}+\cos^2\frac{x}{2}=1$$

$$\Rightarrow \sin^2\frac{x}{2}+\left(\frac{1}{\sqrt{5}}\right)^2=1$$

$$\Rightarrow \sin^2\frac{x}{2}=1-\frac{1}{5}=\frac{4}{5}$$

$$\Rightarrow \sin\frac{x}{2}=\frac{2}{\sqrt{5}} \qquad \left[\because \sin\frac{x}{2} \text{ is positive}\right]$$

$$\therefore \sin\frac{x}{2}=\frac{2\sqrt{5}}{5}$$

$$\tan\frac{x}{2}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}=\frac{\left(\frac{2}{\sqrt{5}}\right)}{\left(\frac{1}{\sqrt{5}}\right)}=2$$

Thus, the respective values of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are $\frac{2\sqrt{5}}{5}$, $\frac{\sqrt{5}}{5}$, and 2.

Question 9:

Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ for $\cos x=-\frac{1}{3}$, x in quadrant III

Ans:

Here, x is in quadrant III.

i.e., $\pi < x < \frac{3\pi}{2}$

$$\Rightarrow \frac{\pi}{2}<\frac{x}{2}<\frac{3\pi}{4}$$

Therefore, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are negative, whereas $\sin\frac{x}{2}$ is positive.

It is given that $\cos x=-\frac{1}{3}$.

$$\cos x=1-2\sin^2\frac{x}{2}$$

$$\Rightarrow \sin^2\frac{x}{2}=\frac{1-\cos x}{2}$$

$$\Rightarrow \sin^2\frac{x}{2}=\frac{1-\left(-\frac{1}{3}\right)}{2}=\frac{\left(1+\frac{1}{3}\right)}{2}=\frac{\frac{4}{3}}{2}=\frac{2}{3}$$

$$\Rightarrow \sin\frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}} \qquad \left[\because \sin\frac{x}{2} \text{ is positive}\right]$$

$$\therefore \sin\frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

Now, $\cos x = 2\cos^2\frac{x}{2} - 1$

$$\Rightarrow \cos^2\frac{x}{2} = \frac{1+\cos x}{2} = \frac{1+\left(-\frac{1}{3}\right)}{2} = \frac{\left(\frac{3-1}{3}\right)}{2} = \frac{\left(\frac{2}{3}\right)}{2} = \frac{1}{3}$$

$$\Rightarrow \cos\frac{x}{2} = -\frac{1}{\sqrt{3}} \qquad \left[\because \cos\frac{x}{2} \text{ is negative}\right]$$

$$\therefore \cos\frac{x}{2} = -\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$$

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\left(\frac{\sqrt{2}}{\sqrt{3}}\right)}{\left(\frac{-1}{\sqrt{3}}\right)} = -\sqrt{2}$$

Thus, the respective values of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are $\frac{\sqrt{6}}{3}$, $\frac{-\sqrt{3}}{3}$, and $-\sqrt{2}$.

Question 10:

Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ for $\sin x = \frac{1}{4}$, x in quadrant II

Ans:

Here, x is in quadrant II.

i.e., $\frac{\pi}{2} < x < \pi$

$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

Therefore, $\sin\frac{x}{2}$, $\cos\frac{x}{2}$, and $\tan\frac{x}{2}$ are all positive.

It is given that $\sin x = \frac{1}{4}$.

$$\cos^2 x = 1 - \sin^2 x = 1 - \left(\frac{1}{4}\right)^2 = 1 - \frac{1}{16} = \frac{15}{16}$$

$$\Rightarrow \cos x = -\frac{\sqrt{15}}{4} \quad [\cos x \text{ is negative in quadrant II}]$$

$$\sin^2\frac{x}{2}=\frac{1-\cos x}{2}=\frac{1-\left(-\frac{\sqrt{15}}{4}\right)}{2}=\frac{4+\sqrt{15}}{8}$$

$$\Rightarrow \sin\frac{x}{2}=\sqrt{\frac{4+\sqrt{15}}{8}} \qquad \left[\because \ \sin\frac{x}{2} \text{ is positive}\right]$$

$$=\sqrt{\frac{4+\sqrt{15}}{8}\times\frac{2}{2}}$$

$$=\sqrt{\frac{8+2\sqrt{15}}{16}}$$

$$=\frac{\sqrt{8+2\sqrt{15}}}{4}$$

$$\cos^2\frac{x}{2}=\frac{1+\cos x}{2}=\frac{1+\left(-\frac{\sqrt{15}}{4}\right)}{2}=\frac{4-\sqrt{15}}{8}$$

$$\Rightarrow \cos\frac{x}{2}=\sqrt{\frac{4-\sqrt{15}}{8}} \qquad \left[\because \ \cos\frac{x}{2} \text{ is positive}\right]$$

$$=\sqrt{\frac{4-\sqrt{15}}{8}\times\frac{2}{2}}$$

$$=\sqrt{\frac{8-2\sqrt{15}}{16}}$$

$$=\frac{\sqrt{8-2\sqrt{15}}}{4}$$

$$\tan\frac{x}{2}=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}=\frac{\left(\frac{\sqrt{8+2\sqrt{15}}}{4}\right)}{\left(\frac{\sqrt{8-2\sqrt{15}}}{4}\right)}=\frac{\sqrt{8+2\sqrt{15}}}{\sqrt{8-2\sqrt{15}}}$$

$$=\sqrt{\frac{8+2\sqrt{15}}{8-2\sqrt{15}}\times\frac{8+2\sqrt{15}}{8+2\sqrt{15}}}$$

$$=\sqrt{\frac{\left(8+2\sqrt{15}\right)^2}{64-60}}=\frac{8+2\sqrt{15}}{2}=4+\sqrt{15}$$

Thus, the respective values of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are $\frac{\sqrt{8+2\sqrt{15}}}{4}$, $\frac{\sqrt{8-2\sqrt{15}}}{4}$, and $4+\sqrt{15}$

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