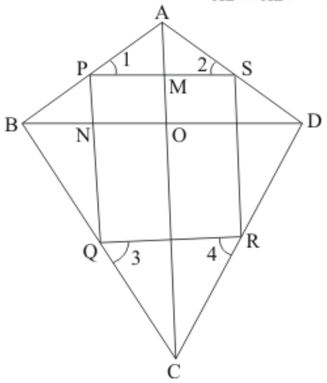
Quadrilaterals Ex 14.4 Q12

Answer :

ABCD is a kite such that $AB = AD$ and $BC = BD$



Quadrilateral *PQRS* is formed by joining the mid-points *P,Q,R* and *S* of sides *AB,BC,CD* and *AD* respectively.

We need to prove that Quadrilateral *PQRS* is a rectangle.

In ΔABC, *P* and *Q* are the mid-points of *AB* and *BC* respectively.

Therefore,

$PQ \parallel AC$ and $PQ = \frac{1}{2}AC$

Similarly, we have

$RS \parallel AC$ and $RS = \frac{1}{2}AC$

Thus,

$PQ \parallel RS$ and $PQ = RS$

Therefore, *PQRS* is a parallelogram.

Now,

$$AB = AD$$

$$\frac{1}{2}AB = \frac{1}{2}AD$$

But, *P* and *S* are the mid-points of *AB* and *AD*

$AP = AS$ ...... (I)

In ΔABD: *P* and *S* are the mid-point of side *AB* and *AD*

By mid-point Theorem, we get:

$PS \parallel BD$

Or,

$PM \parallel BO$

In ΔABO, *P* is the mid-point of side *AB* and $PM \parallel BO$

By Using the converse of mid-point theorem, we get:

M is the mid-point of *AO*

Thus,

$PM = MS$ ...... (II)

In ΔAPM and ΔASP, we have:

$AM = AM$ (Common)

$AP = AS$ [From (I)]

$PM = MS$ [From (II)]

By SSS Congruence theorem, we get:

$\Delta APM \cong \Delta ASP$

By corresponding parts of congruent triangles property, we get:

$\angle AMP = \angle AMS$

But,

$$\angle AMP + \angle AMS = 180^0$$
$$\angle AMS + \angle AMS = 180^0$$
$$2\angle AMS = 180^0$$
$$\angle AMS = 90^0$$

and $\angle AMP = 90^0$

Therefore,

$\angle AON = 90^0$ ($PM \| BO$, Corresponding angles should be equal)

Or, $\angle MON = 90^0$

We have proved that $PS \| BD$

Similarly, $PQ \| AC$.

Then we can say that $PM \| NO$ and $PN \| MO$

Therefore, $PMNO$ is a parallelogram with $\angle MON = 90^0$

Or, we can say that $PMNO$ is a rectangle.

$\angle MPN = 90^0$

We get:

$\angle SPQ = 90^0$

Also, *PQRS* is a parallelogram.

Therefore, *PQRS* is a rectangle.

Hence proved.

*********** END **********