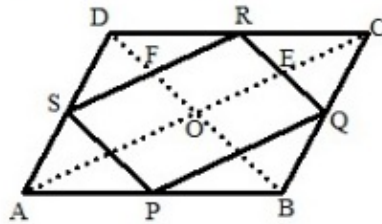




### Exercise 9C

Question 10:

Given : ABCD is a rhombus in which P,Q,R and S are the mid –points of AB, BC, CD and DA respectively.



To Prove : PQRS is a rectangle.

Construction : Join AC and BD.

Proof :

Midpoint Theorem: The line segment joining the midpoints of any two sides of a triangle is parallel to the third side and equal to half of it.

In  $\triangle ABC$

P and Q are the mid points of AB and BC.

So by Mid point Theorem,

$$PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC$$

Similarly, from  $\triangle ADC$ ,

$$RS \parallel AC \text{ and } RS = \frac{1}{2} AC$$

$$\Rightarrow PQ \parallel RS \text{ and } PQ = RS = \frac{1}{2} AC \dots\dots(1)$$

Now, in  $\triangle BAD$ ,

P and S are the mid –points of AB and AD.

So by Mid – point Theorem, we have

$$PS \parallel BD \text{ and } PS = \frac{1}{2} DB$$

Similarly, from  $\triangle BCD$ ,

$$RQ \parallel BD \text{ and } RQ = \frac{1}{2} DB$$

$$\Rightarrow PS \parallel RQ \text{ and } PS = RQ = \frac{1}{2} DB \dots\dots(2)$$

From (1) and (2), we have

PQRS is a parallelogram as its opposite sides are parallel.

We know, that in a rhombus, diagonals intersect at right angles.

$$\therefore \angle EOF = 90^\circ$$

Now,  $RQ \parallel DB$

$$\Rightarrow RE \parallel FO$$

Also,  $SR \parallel AC$

$$\Rightarrow FR \parallel OE$$

$\therefore$  OERF is a parallelogram

In a parallelogram, opposite angles are equal.

$$\text{So, } \angle FRE = \angle EOF = 90^\circ$$

Thus, PQRS is a parallelogram with  $\angle R = 90^\circ$

Hence, PQRS is a rectangle.

\*\*\*\*\* END \*\*\*\*\*