Indefinite Integrals Ex 19.22 Q5

Let $I = \int \frac{1}{1+3\sin^2 x}dx$

Diving numerator and denominator by $\cos^2 x$

$$I = \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\cos^2 x}+\frac{3\sin^2 x}{\cos^2 x}}dx$$

$$= \int \frac{\sec^2 x}{\sec^2 x + 3\tan^2 x}dx$$

$$= \int \frac{\sec^2 x}{1+\tan^2 x + 3\tan^2 x}dx$$

$$= \int \frac{\sec^2 x}{1+4\tan^2 x}dx$$

$$= \int \frac{\sec^2 x}{1+(2\tan x)^2}dx$$

Let $2\tan x = t$

$$2\sec^2 x\,dx = dt$$

$$I = \frac{1}{2}\int \frac{dt}{1+t^2}$$

$$= \frac{1}{2}\tan^{-1} t + c$$

$$I = \frac{1}{2}\tan^{-1}(2\tan x) + c$$

Indefinite Integrals Ex 19.22 Q6

Let $I = \int \frac{1}{3+2\cos^2 x}dx$

Diving numerator and denominator by $\cos^2 x$

$$I = \int \frac{\frac{1}{\cos^2 x}}{\frac{3}{\cos^2 x}+\frac{2\cos^2 x}{\cos^2 x}}dx$$

$$= \int \frac{\sec^2 x}{3\sec^2 x+2}dx$$

$$= \int \frac{\sec^2 x}{3\left(1+\tan^2 x\right)+2}dx$$

$$= \int \frac{\sec^2 x}{3+3\tan^2 x+2}dx$$

$$= \int \frac{\sec^2 x}{5+3\tan^2 x}dx$$

Let $\sqrt{3}\tan x = t$

$$\sqrt{3}\sec^2 x\,dx = dt$$

$$I = \frac{1}{\sqrt{3}}\int \frac{dt}{\left(\sqrt{5}\right)^2+t^2}$$

$$= \frac{1}{\sqrt{3}\times\sqrt{5}}\tan^{-1}\left(\frac{t}{\sqrt{5}}\right)+c$$

$$I = \frac{1}{\sqrt{15}}\tan^{-1}\left(\frac{\sqrt{3}\tan x}{\sqrt{5}}\right)+c$$

Indefinite Integrals Ex 19.22 Q7

Let $I = \int \frac{1}{(\sin x - 2\cos x)(2\sin x + \cos x)} dx$

$= \int \frac{1}{2\sin^2 x + \sin x \cos x - 4\sin x \cos x - 2\cos^2 x} dx$

$= \int \frac{1}{2\sin^2 x - 3\sin x \cos x - 2\cos^2 x} dx$

Diving numerator and denominator by $\cos^2 x$

$I = \int \frac{\sec^2 x}{2\tan^2 x - 3\tan x - 2} dx$

Let $\tan x = t$

$\sec^2 x \, dx = dt$

$I = \int \frac{dt}{2t^2 - 3t - 2}$

$= \frac{1}{2} \int \frac{dt}{t^2 - \frac{3}{2}t - 1}$

$= \frac{1}{2} \int \frac{dt}{t^2 - 2t\left(\frac{3}{4}\right) + \left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2 - 1}$

$I = \frac{1}{2} \int \frac{dt}{\left(t - \frac{3}{4}\right)^2 - \left(\frac{5}{4}\right)^2}$

$= \frac{1}{2} \times \frac{1}{2\left(\frac{5}{4}\right)} \log \left| \frac{t - \frac{3}{4} - \frac{5}{4}}{t - \frac{3}{4} + \frac{5}{4}} \right| + c$

$= \frac{1}{5} \log \left| \frac{t - 2}{2t + 1} \right| + c$

$I = \frac{1}{5} \log \left| \frac{\tan x - 2}{2\tan x + 1} \right| + c$

Indefinite Integrals Ex 19.22 Q8

Let $I = \int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$

Diving numerator and denominator by $\cos^4 x$

$I = \int \frac{2\tan x \sec^2 x}{\tan^4 x + 1} dx$

Let $\tan^2 x = t$

$2\tan x \sec^2 x \, dx = dt$

$I = \int \frac{dt}{t^2 + 1}$

$= \tan^{-1} t + c$

$I = \tan^{-1}\left(\tan^2 x\right) + c$

********** END **********