Question 21. Find the dimensions of the following quantities
(i) Acceleration
(ii) Angle
(iii) Density
(iv) Kinetic energy
(v) Gravitational constant
(vi) Permeability
Answer:

(i) $\text{Acceleration} = \dfrac{\text{Velocity}}{\text{Time}} \quad \therefore \quad [\text{Acceleration}] = \dfrac{[\text{Velocity}]}{[\text{Time}]} = \dfrac{LT^{-1}}{T} = LT^{-2}$

(ii) $\text{Angle} = \dfrac{\text{Distance}}{\text{Distance}} \quad \therefore$ angle is dimensionless.

(iii) $\text{Density} = \dfrac{\text{Mass}}{\text{Volume}} \quad \therefore \quad [\text{Density}] = \dfrac{[\text{Mass}]}{[\text{Volume}]} = \dfrac{M}{L^3} = ML^{-3}$

(iv) $\text{Kinetic energy} = \dfrac{1}{2}\text{Mass} \times \text{Velocity}^2$

$\therefore$ [Kinetic energy] = [Mass] × [Velocity]^2 = ML^2T^{-2}

(v) Constant of gravitation occurs in Newton's law of gravitation

$$F = G\frac{m_1 m_2}{d^2}$$

$$\therefore \quad [G] = \frac{[F][d^2]}{[m_1][m_2]} = \frac{MLT^{-2}L^2}{MM} = M^{-1}L^3T^{-2}$$

(vi) Permeability occurs in Ampere's law of force

$$\Delta F = \mu \frac{(i_1 \Delta l_1)(i_2 \Delta l_2)\sin\theta}{r^2}$$

$$\therefore \quad [\mu] = \frac{[\Delta F][r^2]}{[i_1 \Delta l_1][i_2 \Delta l_2]} = \frac{MLT^{-2}}{AL.AL} = MLT^{-2}A^{-2}$$

Question 22. The length, breadth and thickness of a block of metal were measured with the help of a Vernier Callipers. The measurements are l = (5.250 ± 0.001) cm, b = (3.450 ± 0.001) cm, t = (1.740 ± 0.001) cm.Find the percentage error in volume of the block.
Answer:
Volume of the block is given by V = l b t
Relative error in the volume of block

$$\frac{\Delta V}{V} = \frac{\Delta l}{l} + \frac{\Delta b}{b} + \frac{\Delta t}{t}$$

Here, $\Delta l = 0.001$ cm, $l = 5.250$ cm,
$\Delta b = 0.001$ cm, $b = 3.450$ cm,
$\Delta t = 0.001$ cm, $t = 1.740$ cm.

$$\therefore \quad \frac{\Delta V}{V} = \frac{0.001}{5.250} + \frac{0.001}{3.450} + \frac{0.001}{1.740}$$
$$= 0.0019 + 0.00289 + 0.00575$$
$$= 0.00954$$

$$\therefore \quad \%\ \text{Error} = \frac{\Delta V}{V} \times 100\%$$
$$= 0.00954 \times 100\%$$
$$= 0.9\% \simeq 1\%$$

Question 23. Find the value of 60 W on a system having 100 g, 20 cm and 1 minute as the fundamental units.
Answer: Here n_1 = 60 W. Obviously, the physical quantity is power whose dimensional formula is $[M^1 L^2 T^{-3}]$.
The first system, in which unit of power is 1 watt, is SI system in which M_1 = 1 kg, L_1 = 1 m and T_1= Is in second system, M_2 = 100 g, L_2 = 20 cm and T_2 = 1 min = 60 s.

$$\therefore \quad n_2 = n_1\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^1\left[\frac{T_1}{T_2}\right]^{-3}$$

$$= 60\left[\frac{1\text{ kg}}{100\text{ g}}\right]^1\left[\frac{1\text{ m}}{20\text{ cm}}\right]^1\left[\frac{1\text{ s}}{1\text{ min}}\right]^{-3} = 60\left[\frac{1000\text{ g}}{100\text{ g}}\right]^1\left[\frac{100\text{ cm}}{20\text{ cm}}\right]^2\left[\frac{1\text{ s}}{60\text{ s}}\right]^{-3}$$

$$= 60\times\frac{1000}{100}\times\frac{100}{20}\times\frac{100}{20}\times 60\times 60\times 60$$

$$= 3.24 \times 10^9 \text{ units.}$$

Question 24. By using the method of dimension, check the accuracy of the following formula: T =rh ρg/2cos θ , where T is the surface tension, h is the height of the liquid in a capillary tube, p is the density of the liquid, g is the acceleration due to gravity, 6 is the angle of contact, and r is the radius of the capillary tube.
Answer: In order to find out the accuracy of the given equation we shall compare the dimensions of T and rh ρg/2cos θ

The dimensions of surface tension, $T = \frac{\text{force}}{\text{length}} = \frac{[MLT^{-2}]}{[L]} = [MT^{-2}]$

The dimensions of $\frac{rh\rho g}{2\cos\theta} = [L]\,[L]\,[ML^{-3}]\,[LT^{-2}] = [MT^{-2}]$ (2 cos θ is dimensionless)

The dimensions of both the sides are the same and hence the equation is correct.

III. Long Answer Type Questions
Question 1. P.A.M. Dirac, a great physicist of 20th century found that from the following basic constants, a number having dimensions of time can be constructed:
(i) charge on electron (e),
(ii) permittivity of free space (ε_0),
(iii) mass of electron (me),
(iv) mass of proton (me)
(v) speed of light (c),
(vi) universal gravitational constant (G).
Obtain Dirac's number, given that the desired number is proportional to mp^{-1} and me^{-2}. What is the significance of this number?
Answer:

Let X be the desired number, then

$$X = k\, e^u\, \varepsilon_o^x\, m_e^{-2}\, m_p^{-1}\, c^y\, G^z \quad ...(i)$$

Here k is a dimensionless constant and x, y, z and u are unknowns, whose value is to be obtained from the principle of homogeneity of dimensions. Now

$$[X] = M^0 L^0 T^1 Q^0$$
$$[e] = M^0 L^0 T^0 Q^1$$
$$[\varepsilon_0] = M^{-1} L^{-3} T^2 Q$$
$$[C] = M^0 L T^{-1}$$
$$[G] = M^{-1} L^3 T^{-2}$$

Substituting dimensions of parameters involved in equation (*i*), we get

$$M^0 L^0 T^1 Q^0 = Q^u [M^{-1} L^{-3} T^2 Q^2]^x M^{-2} M^{-1} (LT^{-1})^y (M^{-1} L^3 T^{-2})^z$$
$$= M^{-x-3-z} L^{-3x+y+3z} T^{2x-y+2z} Q^{u+2x}$$

From the principle of homogeneity of dimensions

$$-x - 3 - z = 0 \quad ...(ii)$$
$$-3x + y + 3z = 0 \quad ...(iii)$$
$$2x - y - 2z = 1 \quad ...(iv)$$
$$u + 2x = 0 \quad ...(v)$$

Solving eqns. (*ii*), (*iii*), (*iv*) and (*v*), we get

$$u = +4, \quad x = -2, \quad y = -3, \quad z = -1$$

$$\therefore \quad x = ke^4\, \varepsilon_0^{-2}\, m_e^{-2}\, m_p^{-1}\, c^{-3}\, G^{-1}$$

or,
$$x = \frac{ke^4}{\varepsilon_0^2\, m_e^2\, m_p\, c^3\, G}$$

Experiments show that

$$k = \frac{1}{16\pi^2}$$

Hence,
$$x = \frac{e^4}{16\pi^2\, \varepsilon_0^2\, m_e^2\, m_p\, c^3\, G}$$

Substituting values of all known parameters we find that the value of x is nearly 15 billion years, which is approximately equal to the present estimate of the age of the universe.

Question 2. To determine acceleration due to gravity, the time of 20 oscillations of a simple pendulum of length 100 cm was observed to be 40 s. Calculate the value of g and maximum percentage error in the measured value of g.

Answer:

Here
$$T = 2\pi\sqrt{\frac{l}{g}}$$

or
$$T^2 = 4\pi^2 \frac{l}{g}$$

or
$$g = 4\pi^2 \frac{l}{T^2}$$

Given
$$l = 100 \text{ cm}, \quad T = \frac{40\text{s}}{20} = 2\text{s}$$

$$\therefore \quad g = 4 \times (3.14)^2 \times \frac{1000 \text{ cm}}{(2\text{s})^2} = \frac{4 \times 9.8596 \times 100}{4} \text{cms}^{-2}$$
$$= \mathbf{985.9\ cms^{-2}}$$

Let us now calculate the maximum error

$$g = 4\pi^2 \times \frac{l}{T^2} = 4\pi^2 \frac{1}{\left(\frac{t}{20}\right)^2} \left(\text{taking } T = \frac{t}{20}\right)$$

or
$$g = \frac{4\pi^2 l \times (20)^2}{t^2}$$

Taking log on both sides, we get

$$\log g = \log 4 + 2 \log \pi + \log l + 2 \log 20 - 2 \log t$$

[differentiating both the sides,]

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} - 2\frac{\Delta t}{t}$$

Given
$$l = 100 \text{ cm}$$
$$\Delta l = 0.1 \text{ cm (least count of the metre scale)}$$
$$t = 40 \text{ s}, \quad \Delta t = 0.1 \text{ s (least count of a stop watch)}$$

$$\therefore \text{Maximum error in } g = \frac{0.1}{100} + 2 \times \frac{0.1}{40}$$
$$= 0.001 + 0.005 = 0.006$$
$$= 0.006 \times 100\% = 0.6\%$$

Here 0.1% is the error in the measurement of length, and 0.5% is the error in the measurement of time. Therefore, time needs more careful measurement.

Question 3. It is known that the period T of a magnet of magnetic moment M vibrating in a uniform magnetic field of intensity B depends upon M, B and I where I is the moment of inertia of the magnet about its axis of oscillations. Show that

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

Answer: We first note that the dimension of I are [ML^2]. Also the magnetic moment has the units Am2 so that its dimensions can be written as [AL^2] where A stands for the dimensions of the electric current. Finally the magnetic field vector B has the units newton (per ampere metre) so that its dimensions can be written as

$$[B] = \frac{[MLT^{-2}]}{[A][L]} = [MT^{-2}A^{-1}]$$

We now assume that

$$T = k\, I^a M^b B^c$$

Substituting dimensions of all the quantities involved, we have

$$[T] = [ML^2]^a [AL^2]^b [MT^{-2}A^{-1}]^c$$
$$= M^{a+c} L^{2a+2b} T^{-2c} A^{b-c}$$

Equating powers of M, L, T and A on both sides, we have $a + c = 0$, $2(a + b) = 0$, $-2c = 1$, $b - c = 0$. From the first three equations, we get $c = \frac{-1}{2}$, $a = \frac{1}{2}$ and $b = \frac{-1}{2}$.

These values are consistent with the fourth equation. Thus

$$a = \frac{1}{2},\quad b = \frac{-1}{2} \quad \text{and} \quad c = \frac{-1}{2}$$

$$\therefore \quad T = k\, I^{1/2} M^{-1/2} B^{-1/2} = k\sqrt{\frac{I}{MB}}$$

Experiments show that $k = 2\pi$. Therefore

$$T = 2\pi\sqrt{\frac{I}{MB}}.$$

Question 4. Briefly explain how you will estimate the molecular diameter of oleic acid.
Answer: To determine the molecular diameter of oleic acid, we first of all dissolve 1 mL of oleic acid in 20 mL of alcohol. Then redissolve 1 mL of this solution in 20 mL of alcohol. Hence, the concentration of final solution is 1/20 x 1/20 =1/400 th part of oleic acid in alcohol. Now take a large sized trough filled with water. Lightly sprinkle lycopodium powder on water surface. Using a dropper of fine bore gently put few drops (say n) of the solution prepared on to water. The solution drops spread into a thin, large and roughly circular film of molecular thickness on water surface. Quickly measure the diameter of thin circular film and calculate its surface area S.
If volume of each drop of solution be V, then volume of n drops = n V
Volume of oleic acid in this volume of solution = nV/400
It t be the thickness of oleic acid film formed over water surface then the volume of oleic acid film = St
St = nV/400
⇒ t = nV/400S
As the film is extremely thin, this thickness t may be considered to be the size of one molecule of oleic acid i.e., t is the molecular diameter of oleic acid.
Experimentally, molecular diameter of oleic acid is found to be of the order of 10^{-9} m.

Question 5. Obtain a relation between the distance travelled by a body in time t, if its initial velocity be u and accelerationf.
Answer: Let the distance covered is S,

$$[L] = [LT^{-1}]^a [LT^{-2}]^b [T]^c$$

or $$[L] = [L^{a+b} T^{-a-2b+c}]$$

Comparing powers on both sides, we get

$$1 = a + b$$

and $$0 = -a - 2b + c$$

We have only two equations with three unknowns, therefore, we split the problem into two parts.

(*a*) Let the body have no acceleration,

then $$S = k_1 u^a t^b$$

or $$[L] = [LT^{-1}]^a [T]^b$$

$$= [L^a T^{-a+b}]$$

or $$a = 1$$

$$-a + b = 0 \quad \text{or} \quad b = 1$$

$$S = k_1 ut \qquad ...(i)$$

(*b*) Suppose the body has no initial velocity

then $$S = k_2 f^a t^b$$

$$[L] = k_2 [LT^{-2}]^a [T]^b$$

or $$[L] = [L^a T^{-2a+b}] \quad \text{or} \quad a = 1$$

$$-2a + b = 0 \quad \text{or} \quad b = 2a = 2$$

$\therefore$ $$S = k_2 ft^2 \qquad ...(ii)$$

If the body has both the initial velocity and acceleration comparing (*i*) and (*ii*), we get,

$$S = k_1 ut + k_2 ft^2$$

This is the required equation.

If we put $$k_1 = 1, \quad k_2 = \frac{1}{2}, \quad \text{we get}$$

$$\boxed{S = ut + \frac{1}{2} ft^2}$$

*********** END **********