Exercise Miscellaneous : Solutions of Questions on Page Number : **352**

Q1 : $\frac{1}{x-x^3}$

Answer :

$$\frac{1}{x-x^3}=\frac{1}{x\left(1-x^2\right)}=\frac{1}{x(1-x)(1+x)}$$

$$\text{Let } \frac{1}{x(1-x)(1+x)}=\frac{A}{x}+\frac{B}{(1-x)}+\frac{C}{1+x} \qquad \ldots(1)$$

$$\Rightarrow 1=A\left(1-x^2\right)+Bx(1+x)+Cx(1-x)$$

$$\Rightarrow 1=A-Ax^2+Bx+Bx^2+Cx-Cx^2$$

Equating the coefficients of x^2, x, and constant term, we obtain

$-A+B-C=0$

$B+C=0$

$A=1$

On solving these equations, we obtain

$$A=1, B=\frac{1}{2}, \text{ and } C=-\frac{1}{2}$$

From equation (1), we obtain

$$\frac{1}{x(1-x)(1+x)}=\frac{1}{x}+\frac{1}{2(1-x)}-\frac{1}{2(1+x)}$$

$$\Rightarrow \int\frac{1}{x(1-x)(1+x)}dx=\int\frac{1}{x}dx+\frac{1}{2}\int\frac{1}{1-x}dx-\frac{1}{2}\int\frac{1}{1+x}dx$$

$$=\log|x|-\frac{1}{2}\log|(1-x)|-\frac{1}{2}\log|(1+x)|$$

$$=\log|x|-\log\left|(1-x)^{\frac{1}{2}}\right|-\log\left|(1+x)^{\frac{1}{2}}\right|$$

$$=\log\left|\frac{x}{(1-x)^{\frac{1}{2}}(1+x)^{\frac{1}{2}}}\right|+C$$

$$=\log\left|\left(\frac{x^2}{1-x^2}\right)^{\frac{1}{2}}\right|+C$$

$$=\frac{1}{2}\log\left|\frac{x^2}{1-x^2}\right|+C$$

Answer needs Correction? **Click Here**

Q2 : $\frac{1}{\sqrt{x+a}+\sqrt{(x+b)}}$

Answer :

$$\frac{1}{\sqrt{x+a}+\sqrt{x+b}}=\frac{1}{\sqrt{x+a}+\sqrt{x+b}}\times\frac{\sqrt{x+a}-\sqrt{x+b}}{\sqrt{x+a}-\sqrt{x+b}}$$

$$=\frac{\sqrt{x+a}-\sqrt{x+b}}{(x+a)-(x+b)}$$

$$=\frac{\left(\sqrt{x+a}-\sqrt{x+b}\right)}{a-b}$$

$$\Rightarrow \int\frac{1}{\sqrt{x+a}-\sqrt{x+b}}dx=\frac{1}{a-b}\int\left(\sqrt{x+a}-\sqrt{x+b}\right)dx$$

$$=\frac{1}{(a-b)}\left[\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}}-\frac{(x+b)^{\frac{3}{2}}}{\frac{3}{2}}\right]$$

$$=\frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}}-(x+b)^{\frac{3}{2}}\right]+C$$

Answer needs Correction? **Click Here**

Q3 : $\frac{1}{x\sqrt{ax-x^2}}$ [Hint: Put $x=\frac{a}{t}$]

Answer :

$$\frac{1}{x\sqrt{ax-x^2}}$$

Let $x=\frac{a}{t} \Rightarrow dx=-\frac{a}{t^2}dt$

$$\Rightarrow \int \frac{1}{x\sqrt{ax-x^2}}dx=\int \frac{1}{\frac{a}{t}\sqrt{a\cdot\frac{a}{t}-\left(\frac{a}{t}\right)^2}}\left(-\frac{a}{t^2}dt\right)$$

$$=-\int \frac{1}{at}\cdot\frac{1}{\sqrt{\frac{1}{t}-\frac{1}{t^2}}}dt$$

$$=-\frac{1}{a}\int \frac{1}{\sqrt{\frac{t^2}{t}-\frac{t^2}{t^2}}}dt$$

$$=-\frac{1}{a}\int \frac{1}{\sqrt{t-1}}dt$$

$$=-\frac{1}{a}\left[2\sqrt{t-1}\right]+\mathrm{C}$$

$$=-\frac{1}{a}\left[2\sqrt{\frac{a}{x}-1}\right]+\mathrm{C}$$

$$=-\frac{2}{a}\left(\frac{\sqrt{a-x}}{\sqrt{x}}\right)+\mathrm{C}$$

$$=-\frac{2}{a}\left(\sqrt{\frac{a-x}{x}}\right)+\mathrm{C}$$

Answer needs Correction? Click Here

Q4 : $\frac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}$

Answer :

$$\frac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}$$

Multiplying and dividing by x^{-3}, we obtain

$$\frac{x^{-3}}{x^2\cdot x^{-3}\left(x^4+1\right)^{\frac{3}{4}}}=\frac{x^{-3}\left(x^4+1\right)^{\frac{-3}{4}}}{x^2\cdot x^{-3}}$$

$$=\frac{\left(x^4+1\right)^{\frac{-3}{4}}}{x^5\cdot\left(x^4\right)^{\frac{-3}{4}}}$$

$$=\frac{1}{x^5}\left(\frac{x^4+1}{x^4}\right)^{-\frac{3}{4}}$$

$$=\frac{1}{x^5}\left(1+\frac{1}{x^4}\right)^{-\frac{3}{4}}$$

Let $\frac{1}{x^4}=t \Rightarrow -\frac{4}{x^5}dx=dt \Rightarrow \frac{1}{x^5}dx=-\frac{dt}{4}$

$$\therefore \int \frac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}dx=\int \frac{1}{x^5}\left(1+\frac{1}{x^4}\right)^{-\frac{3}{4}}dx$$

$$=-\frac{1}{4}\int(1+t)^{-\frac{3}{4}}dt$$

$$=-\frac{1}{4}\left[\frac{(1+t)^{\frac{1}{4}}}{\frac{1}{4}}\right]+\mathrm{C}$$

$$=-\frac{1}{4}\frac{\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}}{\frac{1}{4}}+\mathrm{C}$$

$$=-\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}+\mathrm{C}$$

Answer needs Correction? Click Here

Q5 : $\frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}$ $\left[\text{Hint: }\frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}=\frac{1}{x^{\frac{1}{3}}\left(1+x^{\frac{1}{6}}\right)}\text{ Put } x=t^6\right]$

Answer :

$$\frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}=\frac{1}{x^{\frac{1}{3}}\left(1+x^{\frac{1}{6}}\right)}$$

Let $x=t^6 \Rightarrow dx=6t^5dt$

$$\therefore \int \frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}dx = \int \frac{1}{x^{\frac{1}{3}}\left(1+x^{\frac{1}{6}}\right)}dx$$
$$= \int \frac{6t^5}{t^2(1+t)}dt$$
$$= 6\int \frac{t^3}{(1+t)}dt$$

On dividing, we obtain

$$\int \frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}dx = 6\int\left\{\left(t^2-t+1\right)-\frac{1}{1+t}\right\}dt$$
$$= 6\left[\left(\frac{t^3}{3}\right)-\left(\frac{t^2}{2}\right)+t-\log|1+t|\right]$$
$$= 2x^{\frac{1}{2}}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\log\left(1+x^{\frac{1}{6}}\right)+C$$
$$= 2\sqrt{x}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\log\left(1+x^{\frac{1}{6}}\right)+C$$

Answer needs Correction? **Click Here**

Q6 : $\frac{5x}{(x+1)(x^2+9)}$

Answer :

$$\text{Let } \frac{5x}{(x+1)(x^2+9)} = \frac{A}{(x+1)}+\frac{Bx+C}{(x^2+9)} \qquad ...(1)$$
$$\Rightarrow 5x = A\left(x^2+9\right)+(Bx+C)(x+1)$$
$$\Rightarrow 5x = Ax^2+9A+Bx^2+Bx+Cx+C$$

Equating the coefficients of x^2, x, and constant term, we obtain

$A + B = 0$

$B + C = 5$

$9A + C = 0$

On solving these equations, we obtain

$$A = -\frac{1}{2}, B = \frac{1}{2}, \text{ and } C = \frac{9}{2}$$

From equation (1), we obtain

$$\frac{5x}{(x+1)(x^2+9)} = \frac{-1}{2(x+1)}+\frac{\frac{x}{2}+\frac{9}{2}}{(x^2+9)}$$
$$\int \frac{5x}{(x+1)(x^2+9)}dx = \int\left\{\frac{-1}{2(x+1)}+\frac{(x+9)}{2(x^2+9)}\right\}dx$$
$$= -\frac{1}{2}\log|x+1|+\frac{1}{2}\int\frac{x}{x^2+9}dx+\frac{9}{2}\int\frac{1}{x^2+9}dx$$
$$= -\frac{1}{2}\log|x+1|+\frac{1}{4}\int\frac{2x}{x^2+9}dx+\frac{9}{2}\int\frac{1}{x^2+9}dx$$
$$= -\frac{1}{2}\log|x+1|+\frac{1}{4}\log\left|x^2+9\right|+\frac{9}{2}\cdot\frac{1}{3}\tan^{-1}\frac{x}{3}$$
$$= -\frac{1}{2}\log|x+1|+\frac{1}{4}\log\left(x^2+9\right)+\frac{3}{2}\tan^{-1}\frac{x}{3}+C$$

Answer needs Correction? **Click Here**

Q7 : $\frac{\sin x}{\sin(x-a)}$

Answer :

$$\frac{\sin x}{\sin(x-a)}$$

Let $x - a = t \Rightarrow dx = dt$

$$\int\frac{\sin x}{\sin(x-a)}dx = \int\frac{\sin(t+a)}{\sin t}dt$$
$$= \int\frac{\sin t\cos a+\cos t\sin a}{\sin t}dt$$
$$= \int(\cos a+\cot t\sin a)dt$$
$$= t\cos a+\sin a\log|\sin t|+C_1$$
$$= (x-a)\cos a+\sin a\log|\sin(x-a)|+C_1$$
$$= x\cos a+\sin a\log|\sin(x-a)|-a\cos a+C_1$$
$$= \sin a\log|\sin(x-a)|+x\cos a+C$$

Answer needs Correction? **Click Here**

Q8 : $\frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}}$

Answer :

$$\frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}}=\frac{e^{4\log x}\left(e^{\log x}-1\right)}{e^{2\log x}\left(e^{\log x}-1\right)}$$
$$=e^{2\log x}$$
$$=e^{\log x^2}$$
$$=x^2$$
$$\therefore \int\frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}}dx=\int x^2dx=\frac{x^3}{3}+C$$

Answer needs Correction? **Click Here**

Q9 : $\frac{\cos x}{\sqrt{4-\sin^2 x}}$

Answer :

$$\frac{\cos x}{\sqrt{4-\sin^2 x}}$$

Let sin *x* = *t* ⇒ cos *x dx* = *dt*

$$\Rightarrow \int\frac{\cos x}{\sqrt{4-\sin^2 x}}dx=\int\frac{dt}{\sqrt{(2)^2-(t)^2}}$$
$$=\sin^{-1}\left(\frac{t}{2}\right)+C$$
$$=\sin^{-1}\left(\frac{\sin x}{2}\right)+C$$

Answer needs Correction? **Click Here**

Q10 : $\frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}$

Answer :

$$\frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}=\frac{\left(\sin^4 x+\cos^4 x\right)\left(\sin^4 x-\cos^4 x\right)}{\sin^2 x+\cos^2 x-\sin^2 x\cos^2 x-\sin^2 x\cos^2 x}$$
$$=\frac{\left(\sin^4 x+\cos^4 x\right)\left(\sin^2 x+\cos^2 x\right)\left(\sin^2 x-\cos^2 x\right)}{\left(\sin^2 x-\sin^2 x\cos^2 x\right)+\left(\cos^2 x-\sin^2 x\cos^2 x\right)}$$
$$=\frac{\left(\sin^4 x+\cos^4 x\right)\left(\sin^2 x-\cos^2 x\right)}{\sin^2 x\left(1-\cos^2 x\right)+\cos^2 x\left(1-\sin^2 x\right)}$$
$$=\frac{-\left(\sin^4 x+\cos^4 x\right)\left(\cos^2 x-\sin^2 x\right)}{\left(\sin^4 x+\cos^4 x\right)}$$
$$=-\cos 2x$$
$$\therefore \int\frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}dx=\int-\cos 2x\,dx=-\frac{\sin 2x}{2}+C$$

Answer needs Correction? **Click Here**

Q11 : $\frac{1}{\cos(x+a)\cos(x+b)}$

Answer :

$$\frac{1}{\cos(x+a)\cos(x+b)}$$

Multiplying and dividing by $\sin(a-b)$, we obtain

$$\frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x+a)\cos(x+b)}\right]$$
$$=\frac{1}{\sin(a-b)}\left[\frac{\sin\left[(x+a)-(x+b)\right]}{\cos(x+a)\cos(x+b)}\right]$$
$$=\frac{1}{\sin(a-b)}\left[\frac{\sin(x+a)\cdot\cos(x+b)-\cos(x+a)\sin(x+b)}{\cos(x+a)\cos(x+b)}\right]$$
$$=\frac{1}{\sin(a-b)}\left[\frac{\sin(x+a)}{\cos(x+a)}-\frac{\sin(x+b)}{\cos(x+b)}\right]$$
$$=\frac{1}{\sin(a-b)}\left[\tan(x+a)-\tan(x+b)\right]$$

$$\int\frac{1}{\cos(x+a)\cos(x+b)}dx=\frac{1}{\sin(a-b)}\int\left[\tan(x+a)-\tan(x+b)\right]dx$$
$$=\frac{1}{\sin(a-b)}\left[-\log\left|\cos(x+a)\right|+\log\left|\cos(x+b)\right|\right]+C$$
$$=\frac{1}{\sin(a-b)}\log\left|\frac{\cos(x+b)}{\cos(x+a)}\right|+C$$

Answer needs Correction? **Click Here**

Q12 : $\frac{x^3}{\sqrt{1-x^8}}$

Answer :

$\frac{x^3}{\sqrt{1-x^8}}$

Let $x^4 = t \Rightarrow 4x^3\, dx = dt$

$$\Rightarrow \int \frac{x^3}{\sqrt{1-x^8}}dx = \frac{1}{4}\int \frac{dt}{\sqrt{1-t^2}}$$
$$= \frac{1}{4}\sin^{-1} t + C$$
$$= \frac{1}{4}\sin^{-1}\left(x^4\right) + C$$

Answer needs Correction? **Click Here**

Q13 : $\frac{e^x}{\left(1+e^x\right)\left(2+e^x\right)}$

Answer :

$\frac{e^x}{\left(1+e^x\right)\left(2+e^x\right)}$

Let $e^x = t \Rightarrow e^x\, dx = dt$

$$\Rightarrow \int \frac{e^x}{\left(1+e^x\right)\left(2+e^x\right)}dx = \int \frac{dt}{(t+1)(t+2)}$$
$$= \int\left[\frac{1}{(t+1)} - \frac{1}{(t+2)}\right]dt$$
$$= \log|t+1| - \log|t+2| + C$$
$$= \log\left|\frac{t+1}{t+2}\right| + C$$
$$= \log\left|\frac{1+e^x}{2+e^x}\right| + C$$

Answer needs Correction? **Click Here**

Q14 : $\frac{1}{\left(x^2+1\right)\left(x^2+4\right)}$

Answer :

$$\therefore \frac{1}{\left(x^2+1\right)\left(x^2+4\right)} = \frac{Ax+B}{\left(x^2+1\right)} + \frac{Cx+D}{\left(x^2+4\right)}$$
$$\Rightarrow 1 = (Ax+B)\left(x^2+4\right) + (Cx+D)\left(x^2+1\right)$$
$$\Rightarrow 1 = Ax^3 + 4Ax + Bx^2 + 4B + Cx^3 + Cx + Dx^2 + D$$

Equating the coefficients of x^3, x^2, x, and constant term, we obtain

$A + C = 0$

$B + D = 0$

$4A + C = 0$

$4B + D = 1$

On solving these equations, we obtain

$A = 0,\ B = \frac{1}{3},\ C = 0, \text{and } D = -\frac{1}{3}$

From equation (1), we obtain

$$\frac{1}{\left(x^2+1\right)\left(x^2+4\right)} = \frac{1}{3\left(x^2+1\right)} - \frac{1}{3\left(x^2+4\right)}$$
$$\int \frac{1}{\left(x^2+1\right)\left(x^2+4\right)}dx = \frac{1}{3}\int \frac{1}{x^2+1}dx - \frac{1}{3}\int \frac{1}{x^2+4}dx$$
$$= \frac{1}{3}\tan^{-1} x - \frac{1}{3}\cdot\frac{1}{2}\tan^{-1}\frac{x}{2} + C$$
$$= \frac{1}{3}\tan^{-1} x - \frac{1}{6}\tan^{-1}\frac{x}{2} + C$$

Answer needs Correction? **Click Here**

Q15 : $\cos^3 x e^{\log \sin x}$

Answer :

$\cos^3 x e^{\log \sin x} = \cos^3 x \times \sin x$

Let $\cos x = t \Rightarrow -\sin x\, dx = dt$

$$\Rightarrow \int \cos^3 x\, e^{\log \sin x} dx = \int \cos^3 x \sin x dx$$
$$= -\int t \cdot dt$$
$$= -\frac{t^4}{4} + C$$
$$= -\frac{\cos^4 x}{4} + C$$

Answer needs Correction? **Click Here**

Q16 : $e^{3\log x}\left(x^4+1\right)^{-1}$

Answer :

$$e^{3\log x}\left(x^4+1\right)^{-1} = e^{\log x^3}\left(x^4+1\right)^{-1} = \frac{x^3}{\left(x^4+1\right)}$$

Let $x^4+1=t \Rightarrow 4x^3\, dx = dt$

$$\Rightarrow \int e^{3\log x}\left(x^4+1\right)^{-1} dx = \int \frac{x^3}{\left(x^4+1\right)} dx$$
$$= \frac{1}{4}\int \frac{dt}{t}$$
$$= \frac{1}{4}\log|t| + C$$
$$= \frac{1}{4}\log\left|x^4+1\right| + C$$
$$= \frac{1}{4}\log\left(x^4+1\right) + C$$

Answer needs Correction? **Click Here**

Q17 : $f'(ax+b)\left[f(ax+b)\right]^n$

Answer :

$f'(ax+b)\left[f(ax+b)\right]^n$

Let $f(ax+b) = t \Rightarrow af'(ax+b)dx = dt$

$$\Rightarrow \int f'(ax+b)\left[f(ax+b)\right]^n dx = \frac{1}{a}\int t^n dt$$
$$= \frac{1}{a}\left[\frac{t^{n+1}}{n+1}\right]$$
$$= \frac{1}{a(n+1)}\left(f(ax+b)\right)^{n+1} + C$$

Answer needs Correction? **Click Here**

Q18 : $\dfrac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}}$

Answer :

$$\frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}} = \frac{1}{\sqrt{\sin^3 x(\sin x\cos\alpha + \cos x \sin\alpha)}}$$
$$= \frac{1}{\sqrt{\sin^4 x\cos\alpha + \sin^3 x\cos x\sin\alpha}}$$
$$= \frac{1}{\sin^2 x\sqrt{\cos\alpha + \cot x\sin\alpha}}$$
$$= \frac{\operatorname{cosec}^2 x}{\sqrt{\cos\alpha + \cot x\sin\alpha}}$$

Let $\cos\alpha + \cot x\sin\alpha = t \Rightarrow -\operatorname{cosec}^2 x\sin\alpha\, dx = dt$

$$\therefore \int \frac{1}{\sin^3 x\sin(x+\alpha)} dx = \int \frac{\operatorname{cosec}^2 x}{\sqrt{\cos\alpha + \cot x\sin\alpha}} dx$$
$$= \frac{-1}{\sin\alpha}\int \frac{dt}{\sqrt{t}}$$
$$= \frac{-1}{\sin\alpha}\left[2\sqrt{t}\right] + C$$
$$= \frac{-1}{\sin\alpha}\left[2\sqrt{\cos\alpha + \cot x\sin\alpha}\right] + C$$
$$= \frac{-2}{\sin\alpha}\sqrt{\cos\alpha + \frac{\cos x\sin\alpha}{\sin x}} + C$$
$$= \frac{-2}{\sin\alpha}\sqrt{\frac{\sin x\cos\alpha + \cos x\sin\alpha}{\sin x}} + C$$
$$= -\frac{2}{\sin\alpha}\sqrt{\frac{\sin(x+\alpha)}{\sin x}} + C$$

Answer needs Correction? **Click Here**

Q19 : $\dfrac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}, x \in [0,1]$

Answer :

Answer needs Correction? **Click Here**

Q20 : $\sqrt{\dfrac{1-\sqrt{x}}{1+\sqrt{x}}}$

Answer :

$$I=\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}dx$$

$$\text{Let } x=\cos^2\theta \Rightarrow dx=-2\sin\theta\cos\theta\,d\theta$$

$$I=\int\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}(-2\sin\theta\cos\theta)d\theta$$

$$=-\int\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}}\sin 2\theta\,d\theta$$

$$=-\int\tan\frac{\theta}{2}\cdot 2\sin\theta\cos\theta\,d\theta$$

$$=-2\int\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)\cos\theta\,d\theta$$

$$=-4\int\sin^2\frac{\theta}{2}\cos\theta\,d\theta$$

$$=-4\int\sin^2\frac{\theta}{2}\cdot\left(2\cos^2\frac{\theta}{2}-1\right)d\theta$$

$$=-4\int\left(2\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}\right)d\theta$$

$$=-8\int\sin^2\frac{\theta}{2}\cdot\cos^2\frac{\theta}{2}d\theta+4\int\sin^2\frac{\theta}{2}d\theta$$

$$=-2\int\sin^2\theta\,d\theta+4\int\sin^2\frac{\theta}{2}d\theta$$

$$=-2\int\left(\frac{1-\cos 2\theta}{2}\right)d\theta+4\int\frac{1-\cos\theta}{2}d\theta$$

$$=-2\left[\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right]+4\left[\frac{\theta}{2}-\frac{\sin\theta}{2}\right]+C$$

$$=-\theta+\frac{\sin 2\theta}{2}+2\theta-2\sin\theta+C$$

$$=\theta+\frac{\sin 2\theta}{2}-2\sin\theta+C$$

$$=\theta+\frac{2\sin\theta\cos\theta}{2}-2\sin\theta+C$$

$$=\theta+\sqrt{1-\cos^2\theta}\cdot\cos\theta-2\sqrt{1-\cos^2\theta}+C$$

$$=\cos^{-1}\sqrt{x}+\sqrt{1-x}\cdot\sqrt{x}-2\sqrt{1-x}+C$$

$$=-2\sqrt{1-x}+\cos^{-1}\sqrt{x}+\sqrt{x(1-x)}+C$$

$$=-2\sqrt{1-x}+\cos^{-1}\sqrt{x}+\sqrt{x-x^2}+C$$

*********** END **********