



Trigonometric Ratios Ex 5.1 Q30

Answer :

Given: $3 \cos \theta - 4 \sin \theta = 2 \cos \theta + \sin \theta$

To find: $\tan \theta$

Now consider the given expression

$$3 \cos \theta - 4 \sin \theta = 2 \cos \theta + \sin \theta$$

Now by dividing both sides of the above expression by $\cos \theta$

We get,

$$\frac{3 \cos \theta - 4 \sin \theta}{\cos \theta} = \frac{2 \cos \theta + \sin \theta}{\cos \theta}$$

Now by separating the denominator for each terms

We get,

$$\frac{3 \cos \theta}{\cos \theta} - \frac{4 \sin \theta}{\cos \theta} = \frac{2 \cos \theta}{\cos \theta} + \frac{\sin \theta}{\cos \theta}$$

Now in the above expression $\cos \theta$ present in both numerator and denominator gets cancelled

Therefore,

$$3 - \frac{4 \sin \theta}{\cos \theta} = 2 + \frac{\sin \theta}{\cos \theta} \dots\dots (1)$$

Now we know that,

$$\frac{\sin \theta}{\cos \theta} = \tan \theta$$

Therefore by substituting $\frac{\sin \theta}{\cos \theta} = \tan \theta$ in equation (1)

We get,

$$3 - 4 \tan \theta = 2 + \tan \theta$$

Now by taking $\tan \theta$ on L.H.S

We get,

$$- \tan \theta - 4 \tan \theta = 2 - 3$$

Therefore,

$$-5 \tan \theta = -1$$

$$5 \tan \theta = 1$$

$$\tan \theta = \frac{1}{5}$$

$$\text{Hence } \tan \theta = \frac{1}{5}$$

Answer :

Given:

$$\tan \theta = \frac{20}{21} \dots\dots (1)$$

To show that:

$$\frac{1 - \sin \theta + \cos \theta}{1 + \sin \theta + \cos \theta} = \frac{3}{7}$$

Now we know $\tan \theta$ is defined as follows

$$\tan \theta = \frac{\text{Perpendicular side opposite to } \angle \theta}{\text{Base side adjacent to } \angle \theta} \dots\dots (2)$$

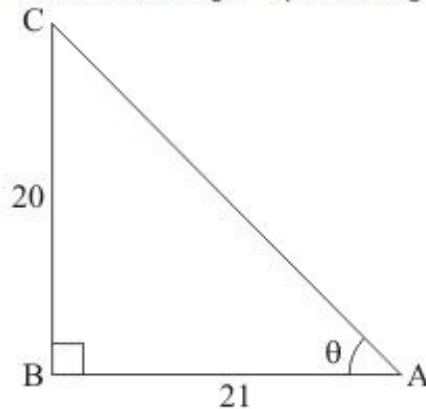
Now by comparing equation (1) and (2)

We get

Perpendicular side opposite to $\angle \theta = 20$

Base side adjacent to $\angle \theta = 21$

Therefore triangle representing angle θ is as shown below



Side AC is unknown and can be found using Pythagoras theorem

Therefore,

$$AC^2 = AB^2 + BC^2$$

Now by substituting the value of known sides from figure (a)

We get,

$$\begin{aligned} AC^2 &= 21^2 + 20^2 \\ &= 441 + 400 \\ &= 841 \end{aligned}$$

Now by taking square root on both sides

We get,

$$\begin{aligned} AC &= \sqrt{841} \\ &= 29 \end{aligned}$$

Therefore Hypotenuse side AC = 29 (3)

Now we know, $\sin \theta$ is defined as follows

$$\sin \theta = \frac{\text{Perpendicular side opposite to } \angle \theta}{\text{Hypotenuse}}$$

Therefore from figure (a) and equation (3)

We get,

$$\begin{aligned} \sin \theta &= \frac{BC}{AC} \\ &= \frac{20}{29} \end{aligned}$$

$$\sin \theta = \frac{20}{29} \dots\dots (4)$$

Now we know, $\cos \theta$ is defined as follows

$$\cos \theta = \frac{\text{Base side adjacent to } \angle \theta}{\text{Hypotenuse}}$$

Therefore from figure (a) and equation (3)

We get,

$$\begin{aligned} \cos \theta &= \frac{AB}{AC} \\ &= \frac{21}{29} \end{aligned}$$

$$\cos \theta = \frac{21}{29} \dots\dots (5)$$

Now we need to find the value of expression $\frac{1 - \sin \theta + \cos \theta}{1 + \sin \theta + \cos \theta}$

Therefore by substituting the value of $\sin \theta$ and $\cos \theta$ from equation (4) and (5) respectively, we get,

$$\frac{1 - \sin \theta + \cos \theta}{1 + \sin \theta + \cos \theta} = \frac{1 - \frac{20}{29} + \frac{21}{29}}{1 + \frac{20}{29} + \frac{21}{29}}$$

Now by taking L.C.M on R.H.S of above equation

We get

$$\begin{aligned} \frac{1 - \sin \theta + \cos \theta}{1 + \sin \theta + \cos \theta} &= \frac{29 - 20 + 21}{29 + 20 + 21} \\ &= \frac{29 + 1}{70} \\ &= \frac{30}{70} \\ &= \frac{30}{70} \times \frac{29}{29} \\ &= \frac{30}{70} \\ &= \frac{3 \times 10}{7 \times 10} \end{aligned}$$

Now as 10 is present in numerator as well as denominator of R.H.S of above equation, it gets cancelled and we get

$$\frac{1 - \sin \theta + \cos \theta}{1 + \sin \theta + \cos \theta} = \frac{3}{7}$$

$$\text{Hence } \frac{1 - \sin \theta + \cos \theta}{1 + \sin \theta + \cos \theta} = \frac{3}{7}$$

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