Exercise 4D

Question 17:
Given: D is the midpoint of side BC, AE ⊥ BC, BC = a, AC = b, AB = c, ED = x, AD = p and AE = h

In ΔAEC, $\angle AEC = 90^0$

$AD^2 = 2AE^2 + ED^2$ (by Pythagoras theorem)

$\Rightarrow p^2 = h^2 + x^2$

(i) In ΔAEC, $\angle AEC = 90^0$

$$b^2 = h^2 + \left(x + \frac{a}{2}\right)^2 = \left(h^2 + x^2\right) + ax + \frac{a^2}{4}$$

$$\therefore b^2 = p^2 + ax + \frac{a^2}{4} \text{ ----(1)}$$

(ii) In ΔABE, $\angle ABE = 90^0$

$AB^2 = AE^2 + BE^2$ (by pythagoras theorem)

$$\Rightarrow c^2 = h^2 + \left(\frac{a}{2} - x\right)^2 \text{ ----(2)}$$

$$= \left(h^2 + x^2\right) - ax + \frac{a^2}{4}$$

$$= p^2 - ax + \frac{a^2}{4}$$

Hence, $c^2 = p^2 - ax + \frac{a^2}{4}$

(iii) Adding (1) and (2), we get

$$b^2 + c^2 = p^2 + ax + \frac{a^2}{4} + p^2 - ax + \frac{a^2}{4}$$

$$\left(b^2 + c^2\right) = 2p^2 + \frac{1}{2}a^2$$

(iv) Subtracting (2) from (1), we get

$$b^2 - c^2 = p^2 + ax + \frac{a^2}{4} - \left(p^2 - ax + \frac{a^2}{4}\right)$$

$$= p^2 + ax + \frac{a^2}{4} - p^2 + ax - \frac{a^2}{4}$$

$$\left(b^2 - c^2\right) = 2ax$$

********** END **********