Higher Order Derivatives Ex 12.1 Q45

$x = a\left(\cos t + t\sin t\right)$

$\dfrac{dx}{dt} = -a\sin t + at\cos t + a\sin t$

$\quad = at\cos t$

$\dfrac{d^2x}{dt^2} = -at\sin t + a\cos t$

$y = a\left(\sin t - t\cos t\right)$

$\dfrac{dy}{dt} = a\cos t - a\cos t + at\sin t$

$\quad = at\sin t$

$\dfrac{d^2y}{dt^2} = at\cos t + a\sin t$

$\dfrac{d^2y}{dx^2} = \dfrac{\dfrac{dx}{dt}\dfrac{d^2y}{dt^2} - \dfrac{dy}{dt}\dfrac{d^2x}{dt^2}}{\left(\dfrac{dx}{dt}\right)^3}$

$= \dfrac{at\cos t\left(at\cos t + a\sin t\right) - at\sin t\left(-at\sin t + a\cos t\right)}{\left(at\cos t\right)^3}$

$= \dfrac{a^2t^2\cos^2 t + a^2t\cos t\sin t + a^2t^2\sin^2 t - a^2t\sin t\cos t}{\left(at\cos t\right)^3}$

$= \dfrac{a^2t^2}{a^3t^3\cos^3 t} = \dfrac{1}{at\cos^3 t}$

$\left.\dfrac{d^2y}{dx^2}\right|_{t-\frac{\pi}{4}} = \dfrac{1}{a\times\dfrac{\pi}{4}\cos^3\dfrac{\pi}{4}} = \dfrac{8\sqrt{2}}{\pi a}$

Higher Order Derivatives Ex 12.1 Q46

$$x = a\left(\cos t + \log \tan \frac{t}{2}\right) \text{ and } y = a \sin t$$

$$\frac{dx}{dt} = -a \sin t + a \frac{1}{\tan \frac{t}{2}} \times \sec^2 \frac{t}{2} \times \frac{1}{2}$$

$$= -a \sin t + a \frac{1}{2 \sin \frac{t}{2} \cos \frac{t}{2}}$$

$$= -a \sin t + a \operatorname{cosec} t$$

$$\frac{d^2x}{dt^2} = -a \cos t - a \operatorname{cosec} t \cot t$$

$$\frac{dy}{dt} = a \cos t$$

$$\frac{d^2y}{dt^2} = -a \sin t$$

$$\frac{d^2y}{dx^2} = \frac{\dfrac{dx}{dt} \dfrac{d^2y}{dt^2} - \dfrac{dy}{dt} \dfrac{d^2x}{dt^2}}{\left(\dfrac{dx}{dt}\right)^3}$$

$$= \frac{(-a \sin t + a \operatorname{cosec} t)(-a \sin t) - (a \cos t)(-a \cos t - a \operatorname{cosec} t \cot t)}{(-a \sin t + a \operatorname{cosec} t)^3}$$

$$= \frac{a^2 \sin^2 t + a^2 \cos^2 t - a^2 + a^2 \cot^2 t}{\left(-a \sin t + \dfrac{a}{\sin t}\right)^3}$$

$$= \frac{a^2 \cot^2 t}{a^3 \cos^6 t} \times \sin^3 t = \frac{1}{a} \times \frac{\sin t}{\cos^4 t}$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=\frac{\pi}{3}} = \frac{1}{a} \times \frac{\sin \frac{\pi}{3}}{\cos^4 \frac{\pi}{3}} = \frac{8\sqrt{3}}{a}$$

Higher Order Derivatives Ex 12.1 Q47

$$x = a\left(\cos 2t + 2t\sin 2t\right)$$

$$\frac{dx}{dt} = -2a\sin 2t + 2a\sin 2t + 4at\cos 2t = 4at\cos 2t$$

$$y = a\left(\sin 2t - 2t\cos 2t\right)$$

$$\frac{dy}{dt} = 2a\cos 2t - 2a\cos 2t + 4at\sin 2t = 4at\sin 2t$$

$$\frac{dy}{dx} = \tan 2t$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\tan 2t\right)$$

$$\frac{d^2y}{dx^2} = \sec^2 2t\,\frac{d}{dx}\left(2t\right)$$

$$\frac{d^2y}{dx^2} = 2\sec^2 2t\,\frac{d}{dx}\left(t\right)$$

$$\frac{d^2y}{dx^2} = 2\sec^2 2t \times \frac{1}{4at\cos 2t}$$

$$\frac{d^2y}{dx^2} = \frac{1}{2a}\sec^3 2t$$

********** END *********