Trigonometric Ratios Ex 5.3 Q2

Answer :

(i) We have to find: $\left(\frac{\sin 49^\circ}{\cos 41^\circ}\right)^2+\left(\frac{\cos 41^\circ}{\sin 49^\circ}\right)^2$

Since $\frac{\sec 70^\circ}{\operatorname{cosec} 20^\circ}+\frac{\sin 59^\circ}{\cos 31^\circ}$ $\sin\left(90^\circ-\theta\right)=\cos\theta$ and $\cos\left(90^\circ-\theta\right)=\sin\theta$

So

$$\left(\frac{\sin\left(90^\circ-41^\circ\right)}{\cos 41^\circ}\right)^2+\left(\frac{\cos\left(90^\circ-49^\circ\right)}{\sin 49^\circ}\right)^2=\left(\frac{\cos 41^\circ}{\cos 41^\circ}\right)^2+\left(\frac{\sin 49^\circ}{\sin 49^\circ}\right)^2$$
$$=1+1$$
$$=2$$

So value of $\left(\frac{\sin 49^\circ}{\cos 41^\circ}\right)^2+\left(\frac{\cos 41^\circ}{\sin 49^\circ}\right)^2$ is $\boxed{2}$

(ii) We have to find: $\cos 48^\circ-\sin 42^\circ$

Since $\cos\left(90^\circ-\theta\right)=\sin\theta$. So

$$\cos 48^\circ-\sin 42^\circ=\cos\left(90^\circ-42^\circ\right)-\sin 42^\circ$$
$$=\sin 42^\circ-\sin 42^\circ$$
$$=0$$

So value of $\cos 48^\circ-\sin 42^\circ$ is $\boxed{0}$

(iii) We have to find:

$$\frac{\cot 40^\circ}{\tan 50^\circ}-\frac{1}{2}\left(\frac{\cos 35^\circ}{\sin 55^\circ}\right)$$

Since $\cot\left(90^\circ-\theta\right)=\tan\theta$ and $\cos\left(90^\circ-\theta\right)=\sin\theta$

$$\frac{\cot 40^\circ}{\tan 50^\circ}-\frac{1}{2}\left(\frac{\cos 35^\circ}{\sin 55^\circ}\right)=\frac{\cot\left(90^\circ-50^\circ\right)}{\tan 50^\circ}-\frac{1}{2}\left(\frac{\cos\left(90^\circ-55^\circ\right)}{\sin 55^\circ}\right)$$
$$=\frac{\tan 50^\circ}{\tan 50^\circ}-\frac{1}{2}\left(\frac{\sin 55^\circ}{\sin 55^\circ}\right)$$
$$=1-\frac{1}{2}$$
$$=\frac{1}{2}$$

So value of $\frac{\cot 40^\circ}{\tan 50^\circ}-\frac{1}{2}\left(\frac{\cos 35^\circ}{\sin 55^\circ}\right)$ is $\boxed{\frac{1}{2}}$

(iv) We have to find: $\left(\frac{\sin 27^\circ}{\cos 63^\circ}\right)^2-\left(\frac{\cos 63^\circ}{\sin 27^\circ}\right)^2$

Since $\sin\left(90^\circ-\theta\right)=\cos\theta$ and $\cos\left(90^\circ-\theta\right)=\sin\theta$

$$\left(\frac{\sin 27^\circ}{\cos 63^\circ}\right)^2-\left(\frac{\cos 63^\circ}{\sin 27^\circ}\right)^2=\left(\frac{\sin\left(90^\circ-63^\circ\right)}{\cos 63^\circ}\right)^2-\left(\frac{\cos\left(90^\circ-27^\circ\right)}{\sin 27^\circ}\right)^2$$

$$=\left(\frac{\cos 63^\circ}{\cos 63^\circ}\right)^2-\left(\frac{\sin 27^\circ}{\sin 27^\circ}\right)^2$$
$$=1-1$$
$$=0$$

So value of $\left(\frac{\sin 27^\circ}{\cos 63^\circ}\right)^2-\left(\frac{\cos 63^\circ}{\sin 27^\circ}\right)^2$ is $\boxed{0}$

(v) We have to find:

$$\frac{\tan 35^\circ}{\cot 55^\circ}+\frac{\cot 78^\circ}{\tan 12^\circ}-1$$

Since $\tan\left(90^\circ-\theta\right)=\cot\theta$ and $\cot\left(90^\circ-\theta\right)=\tan\theta$

$$=1$$

So value of $\frac{\tan 35^\circ}{\cot 55^\circ}+\frac{\cot 78^\circ}{\tan 12^\circ}$ is $\boxed{1}$

(vi) We have to find: $\frac{\sec 70^\circ}{\operatorname{cosec} 20^\circ}+\frac{\sin 59^\circ}{\cos 31^\circ}$

Since $\frac{\sec 70^\circ}{\operatorname{cosec} 20^\circ}+\frac{\sin 59^\circ}{\cos 31^\circ}$ and $\sec\left(90^\circ-\theta\right)=\operatorname{cosec}\theta$

So

$$\frac{\sec 70^\circ}{\operatorname{cosec} 20^\circ}+\frac{\sin 59^\circ}{\cos 31^\circ}=\frac{\sec\left(90^\circ-20^\circ\right)}{\operatorname{cosec} 20^\circ}+\frac{\sin\left(90^\circ-31^\circ\right)}{\cos 31^\circ}$$
$$=\frac{\operatorname{cosec} 20^\circ}{\operatorname{cosec} 20^\circ}+\frac{\cos 31^\circ}{\cos 31^\circ}$$
$$=1+1$$
$$=2$$

So value of $\frac{\sec 70^\circ}{\operatorname{cosec} 20^\circ}+\frac{\sin 59^\circ}{\cos 31^\circ}$ is $\boxed{2}$

(vii) We have to find: $\operatorname{cosec} 31^\circ-\sec 59^\circ$

Since $\operatorname{cosec}\left(90^\circ-\theta\right)=\sec\theta$. So

$$=\operatorname{cosec} 31^\circ-\sec 59^\circ$$
$$=\operatorname{cosec}\left(90^\circ-59^\circ\right)-\sec 59^\circ$$
$$=\sec^\circ 59^\circ-\sec 59^\circ$$
$$=0$$

So value of $\operatorname{cosec} 31^\circ-\sec 59^\circ$ is $\boxed{0}$

(viii) We have to find: $\left(\sin 72^\circ+\cos 18^\circ\right)\left(\sin 72^\circ-\cos 18^\circ\right)$

Since $\sin\left(90^\circ-\theta\right)=\cos\theta$. So

$$\left(\sin 72^\circ+\cos 18^\circ\right)\left(\sin 72^\circ-\cos 18^\circ\right)=\left(\sin 72^\circ\right)^2-\left(\cos 18^\circ\right)^2$$
$$=\left[\sin\left(90^\circ-18^\circ\right)\right]^2-\left(\cos 18^\circ\right)^2$$
$$=\left(\cos 18^\circ\right)^2-\left(\cos 18^\circ\right)^2$$
$$=\cos^2 18^\circ-\cos^2 18^\circ$$
$$=0$$

So value of $\left(\sin 72^\circ + \cos 18^\circ\right)\left(\sin 72^\circ - \cos 18^\circ\right)$ is $\boxed{0}$

(ix) We find: $\sin 35^\circ \sin 55^\circ - \cos 35^\circ \cos 55^\circ$

Since $\sin\left(90^\circ - \theta\right) = \cos\theta$ and $\cos\left(90^\circ - \theta\right) = \sin\theta$

$$\begin{aligned}\sin 35^\circ \sin 55^\circ - \cos 35^\circ \cos 55^\circ &= \sin\left(90^\circ - 55^\circ\right)\sin 55^\circ - \cos\left(90^\circ - 55^\circ\right)\cos 55^\circ \\ &= \cos 55^\circ \sin 55^\circ - \sin 55^\circ \cos 55^\circ \\ &= 1 - 1 \\ &= 0\end{aligned}$$

So value of $\sin 35^\circ \sin 55^\circ - \cos 35^\circ \cos 55^\circ$ is $\boxed{0}$

(x) We have to find $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$

Since $\tan\left(90^\circ - \theta\right) = \cot\theta$. So

$$\begin{aligned}\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ &= \tan\left(90^\circ - 42^\circ\right)\tan\left(90^\circ - 67^\circ\right)\tan 42^\circ \tan 67^\circ - \\ &= \cot 42^\circ \cot 67^\circ \tan 42^\circ \tan 67^\circ \\ &= \left(\tan 67^\circ \cot 67^\circ\right)\left(\tan 42^\circ \cot 42^\circ\right) \\ &= 1 \times 1 \\ &= 1\end{aligned}$$

So value of $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$ is $\boxed{1}$

(xi) We find to find $\sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ$

Since $\cos\left(90^\circ - \theta\right) = \sin\theta$, $\sec\left(90^\circ - \theta\right) = \operatorname{cosec}\theta$ and $\sin\theta.\operatorname{cosec}\theta = 1$. So

$$\begin{aligned}\sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ &= \sec\left(90^\circ - 40^\circ\right)\sin 40^\circ + \cos\left(90^\circ - 50^\circ\right)\operatorname{cosec} 50^\circ \\ &= \operatorname{cosec} 40^\circ \sin 40^\circ + \sin 50^\circ \operatorname{cosec} 50^\circ \\ &= 1 + 1 \\ &= 2\end{aligned}$$

So value of $\sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ$ is $\boxed{2}$

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