Question 11.10:

Light of frequency 7.21 × 10^{14} Hz is incident on a metal surface. Electrons with a maximum speed of 6.0 × 10^{5} m/s are ejected from the surface. What is the threshold frequency for photoemission of electrons?

Answer

Frequency of the incident photon, $\nu = 488 \text{ nm} = 488 \times 10^{-9} \text{ m}$

Maximum speed of the electrons, v = 6.0 × 10^{5} m/s

Planck's constant, h = 6.626 × 10^{-34} Js

Mass of an electron, m = 9.1 × 10^{-31} kg

For threshold frequency ν_0, the relation for kinetic energy is written as:

$$\frac{1}{2}mv^2 = h(\nu - \nu_0)$$

$$\nu_0 = \nu - \frac{mv^2}{2h}$$

$$= 7.21 \times 10^{14} - \frac{(9.1 \times 10^{-31}) \times (6 \times 10^{5})^2}{2 \times (6.626 \times 10^{-34})}$$

$$= 7.21 \times 10^{14} - 2.472 \times 10^{14}$$

$$= 4.738 \times 10^{14} \text{ Hz}$$

Therefore, the threshold frequency for the photoemission of electrons is 4.738×10^{14} Hz.

Question 11.11:

Light of wavelength 488 nm is produced by an argon laser which is used in the photoelectric effect. When light from this spectral line is incident on the emitter, the stopping (cut-off) potential of photoelectrons is 0.38 V. Find the work function of the material from which the emitter is made.

Answer

Wavelength of light produced by the argon laser, λ = 488 nm

= 488 × 10^{-9} m

Stopping potential of the photoelectrons, V_0 = 0.38 V

1eV = 1.6 × 10^{-19} J

$\therefore V_0 = \frac{0.38}{1.6 \times 10^{-19}}$ eV

Planck's constant, $h = 6.6 \times 10^{-34}$ Js

Charge on an electron, $e = 1.6 \times 10^{-19}$ C

Speed of light, $c = 3 \times 10$ m/s

From Einstein's photoelectric effect, we have the relation involving the work function Φ_0 of the material of the emitter as:

$$eV_0 = \frac{hc}{\lambda} - \phi_0$$

$$\phi_0 = \frac{hc}{\lambda} - eV_0$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 488 \times 10^{-9}} - \frac{1.6 \times 10^{-19} \times 0.38}{1.6 \times 10^{-19}}$$

$$= 2.54 - 0.38 = 2.16 \text{ eV}$$

Therefore, the material with which the emitter is made has the work function of 2.16 eV.

Question 11.12:

Calculate the

(a) momentum, and

(b) de Broglie wavelength of the electrons accelerated through a potential difference of 56 V.

Answer

Potential difference, $V = 56$ V

Planck's constant, $h = 6.6 \times 10^{-34}$ Js

Mass of an electron, $m = 9.1 \times 10^{-31}$ kg

Charge on an electron, $e = 1.6 \times 10^{-19}$ C

(a) At equilibrium, the kinetic energy of each electron is equal to the accelerating potential, i.e., we can write the relation for velocity (v) of each electron as:

$$\frac{1}{2}mv^2 = eV$$

$$v^2 = \frac{2eV}{m}$$

$$\therefore v = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 56}{9.1 \times 10^{-31}}}$$

$$= \sqrt{19.69 \times 10^{12}} = 4.44 \times 10^6 \text{ m/s}$$

The momentum of each accelerated electron is given as:

$p = mv$

$= 9.1 \times 10^{-31} \times 4.44 \times 10^6$

$= 4.04 \times 10^{-24}$ kg m s^{-1}

Therefore, the momentum of each electron is 4.04×10^{-24} kg m s^{-1}.

(b) De Broglie wavelength of an electron accelerating through a potential V, is given by the relation:

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ Å}$$

$$= \frac{12.27}{\sqrt{56}} \times 10^{-10} \text{ m}$$

$$= 0.1639 \text{ nm}$$

Therefore, the de Broglie wavelength of each electron is 0.1639 nm.

Question 11.13:

What is the

(a) momentum,

(b) speed, and

(c) de Broglie wavelength of an electron with kinetic energy of 120 eV.

Answer

Kinetic energy of the electron, $E_k = 120$ eV

Planck's constant, $h = 6.6 \times 10^{-34}$ Js

Mass of an electron, $m = 9.1 \times 10^{-31}$ kg

Charge on an electron, $e = 1.6 \times 10^{-19}$ C

(a) For the electron, we can write the relation for kinetic energy as:

$$E_k = \frac{1}{2}mv^2$$

Where,

v = Speed of the electron

$$\therefore v^2 = \sqrt{\frac{2eE_k}{m}}$$

$$= \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 120}{9.1 \times 10^{-31}}}$$

$$= \sqrt{42.198 \times 10^{12}} = 6.496 \times 10^6 \text{ m/s}$$

Momentum of the electron, $p = mv$

$= 9.1 \times 10^{-31} \times 6.496 \times 10^6$

$= 5.91 \times 10^{-24}$ kg m s^{-1}

Therefore, the momentum of the electron is 5.91×10^{-24} kg m s^{-1}.

(b) Speed of the electron, $v = 6.496 \times 10^6$ m/s

(c) De Broglie wavelength of an electron having a momentum p, is given as:

$$\lambda = \frac{h}{p}$$

$$= \frac{6.6 \times 10^{-34}}{5.91 \times 10^{-24}} = 1.116 \times 10^{-10} \text{ m}$$

$$= 0.112 \text{ nm}$$

Therefore, the de Broglie wavelength of the electron is 0.112 nm.

Question 11.14:

The wavelength of light from the spectral emission line of sodium is 589 nm. Find the kinetic energy at which

(a) an electron, and

(b) a neutron, would have the same de Broglie wavelength.

Answer

Wavelength of light of a sodium line, $\lambda = 589$ nm $= 589 \times 10^{-9}$ m

Mass of an electron, $m_e = 9.1 \times 10^{-31}$ kg

Mass of a neutron, $m_n = 1.66 \times 10^{-27}$ kg

Planck's constant, $h = 6.6 \times 10^{-34}$ Js

(a) For the kinetic energy K, of an electron accelerating with a velocity v, we have the relation:

$$K = \frac{1}{2}m_e v^2 \qquad \dots(1)$$

We have the relation for de Broglie wavelength as:

$$\lambda = \frac{h}{m_e v}$$

$$\therefore v^2 = \frac{h^2}{\lambda^2 m_e^2} \qquad \dots(2)$$

Substituting equation (2) in equation (1), we get the relation:

$$K = \frac{1}{2}\frac{m_e h^2}{\lambda^2 m_e^2} = \frac{h^2}{2\lambda^2 m_e} \qquad \dots(3)$$

$$= \frac{\left(6.6 \times 10^{-34}\right)^2}{2 \times \left(589 \times 10^{-9}\right)^2 \times 9.1 \times 10^{-31}}$$

$$\approx 6.9 \times 10^{-25} \text{ J}$$

$$= \frac{6.9 \times 10^{-25}}{1.6 \times 10^{-19}} = 4.31 \times 10^{-6} \text{ eV} = 4.31 \mu\text{eV}$$

Hence, the kinetic energy of the electron is 6.9×10^{-25} J or 4.31 μeV.

(b) Using equation (3), we can write the relation for the kinetic energy of the neutron as:

$$\frac{h^2}{2\lambda^2 m_n}$$

$$= \frac{\left(6.6 \times 10^{-34}\right)^2}{2 \times \left(589 \times 10^{-9}\right)^2 \times 1.66 \times 10^{-27}}$$

$$= 3.78 \times 10^{-28} \text{ J}$$

$$= \frac{3.78 \times 10^{-28}}{1.6 \times 10^{-19}} = 2.36 \times 10^{-9} \text{ eV} = 2.36 \text{ neV}$$

Hence, the kinetic energy of the neutron is 3.78×10^{-28} J or 2.36 neV.

Question 11.15:

What is the de Broglie wavelength of

(a) a bullet of mass 0.040 kg travelling at the speed of 1.0 km/s,

(b) a ball of mass 0.060 kg moving at a speed of 1.0 m/s, and

(c) a dust particle of mass 1.0×10^{-9} kg drifting with a speed of 2.2 m/s?

Answer

(a) Mass of the bullet, m = 0.040 kg

Speed of the bullet, v = 1.0 km/s = 1000 m/s

Planck's constant, $h = 6.6 \times 10^{-34}$ Js

De Broglie wavelength of the bullet is given by the relation:

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.6 \times 10^{-34}}{0.040 \times 1000} = 1.65 \times 10^{-35} \text{ m}$$

(b) Mass of the ball, m = 0.060 kg

Speed of the ball, v = 1.0 m/s

De Broglie wavelength of the ball is given by the relation:

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.6 \times 10^{-34}}{0.060 \times 1} = 1.1 \times 10^{-32} \text{ m}$$

(c) Mass of the dust particle, $m = 1 \times 10^{-9}$ kg

Speed of the dust particle, $v = 2.2$ m/s

De Broglie wavelength of the dust particle is given by the relation:

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.6 \times 10^{-34}}{2.2 \times 1 \times 10^{-9}} = 3.0 \times 10^{-25} \text{ m}$$

Question 11.16:

An electron and a photon each have a wavelength of 1.00 nm. Find

(a) their momenta,

(b) the energy of the photon, and

(c) the kinetic energy of electron.

Answer

Wavelength of an electron (λ_e) and a photon (λ_p), $\lambda_e = \lambda_p = \lambda = 1$ nm

$= 1 \times 10^{-9}$ m

Planck's constant, $h = 6.63 \times 10^{-34}$ Js

(a) The momentum of an elementary particle is given by de Broglie relation:

$$\lambda = \frac{h}{p}$$

$$p = \frac{h}{\lambda}$$

It is clear that momentum depends only on the wavelength of the particle. Since the wavelengths of an electron and a photon are equal, both have an equal momentum.

$$\therefore p = \frac{6.63 \times 10^{-34}}{1 \times 10^{-9}} = 6.63 \times 10^{-25} \text{ kg m s}^{-1}$$

(b) The energy of a photon is given by the relation:

$$E = \frac{hc}{\lambda}$$

Where,

Speed of light, $c = 3 \times 10^8$ m/s

$$\therefore E = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1 \times 10^{-9} \times 1.6 \times 10^{-19}}$$

$$= 1243.1 \text{ eV} = 1.243 \text{ keV}$$

Therefore, the energy of the photon is 1.243 keV.

(c) The kinetic energy (K) of an electron having momentum p,is given by the relation:

$$K = \frac{1}{2}\frac{p^2}{m}$$

Where,

m = Mass of the electron = 9.1×10^{-31} kg

$p = 6.63 \times 10^{-25}$ kg m s^{-1}

$$\therefore K = \frac{1}{2} \times \frac{\left(6.63 \times 10^{-25}\right)^2}{9.1 \times 10^{-31}} = 2.415 \times 10^{-19} \text{ J}$$

$$= \frac{2.415 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.51 \text{ eV}$$

Hence, the kinetic energy of the electron is 1.51 eV.

Question 11.17:

(a) For what kinetic energy of a neutron will the associated de Broglie wavelength be 1.40×10^{-10} m?

(b) Also find the de Broglie wavelength of a neutron, in thermal equilibrium with matte having an average kinetic energy of (3/2) kT at 300 K.

Answer

(a) De Broglie wavelength of the neutron, $\lambda = 1.40 \times 10^{-10}$ m

Mass of a neutron, $m_n = 1.66 \times 10^{-27}$ kg

Planck's constant, $h = 6.6 \times 10^{-34}$ Js

Kinetic energy (K) and velocity (v) are related as:

$$K = \frac{1}{2} m_n v^2 \quad \dots (1)$$

De Broglie wavelength (λ) and velocity (v) are related as:

$$\lambda = \frac{h}{m_n v} \quad \dots (2)$$

Using equation (2) in equation (1), we get:

$$K = \frac{1}{2} \frac{m_n h^2}{\lambda^2 m_n^{\,2}} = \frac{h^2}{2\lambda^2 m_n}$$

$$= \frac{\left(6.63 \times 10^{-34}\right)^2}{2 \times \left(1.40 \times 10^{-10}\right)^2 \times 1.66 \times 10^{-27}} = 6.75 \times 10^{-21} \text{ J}$$

$$= \frac{6.75 \times 10^{-21}}{1.6 \times 10^{-19}} = 4.219 \times 10^{-2} \text{ eV}$$

Hence, the kinetic energy of the neutron is 6.75×10^{-21} J or 4.219×10^{-2} eV.

(b) Temperature of the neutron, T = 300 K

Boltzmann constant, $k = 1.38 \times 10^{-23}$ kg m^2 s^{-2} K^{-1}

Average kinetic energy of the neutron:

$$K' = \frac{3}{2} kT$$

$$= \frac{3}{2} \times 1.38 \times 10^{-23} \times 300 = 6.21 \times 10^{-21} \text{ J}$$

The relation for the de Broglie wavelength is given as:

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