Transformation Formulae Ex 8.2 Q 16.

We have,

$$y \sin\phi = x \sin(2\theta+\phi)$$

$$\Rightarrow \quad \frac{\sin\phi}{\sin(2\theta+\phi)} = \frac{x}{y} \qquad \text{---(i)}$$

Now,

$$\frac{\sin\phi}{\sin(2\theta+\phi)} = \frac{x}{y}$$

$$\Rightarrow \quad \frac{\sin\phi}{\sin(2\theta+\phi)} + 1 = \frac{x}{y} + 1$$

$$\Rightarrow \quad \frac{\sin\phi + \sin(2\theta+\phi)}{\sin(2\theta+\phi)} = \frac{x+y}{y} \qquad \text{---(ii)}$$

Again,

$$\frac{\sin\phi}{\sin(2\theta+\phi)} = \frac{x}{y} \qquad [\text{By equation (i)}]$$

$$\Rightarrow \quad \frac{\sin\phi}{\sin(2\theta+\phi)} - 1 = \frac{x}{y} - 1$$

$$\Rightarrow \quad \frac{\sin\phi - \sin(2\theta+\phi)}{\sin(2\theta+\phi)} = \frac{x-y}{y} \qquad \text{---(iii)}$$

Dividing equation (ii) by equation (iii), we get

$$\frac{\sin\phi + \sin(2\theta+\phi)}{\sin\phi - \sin(2\theta+\phi)} = \frac{x+y}{x-y}$$

$$\Rightarrow \quad \frac{2\sin\left(\frac{\phi+2\theta+\phi}{2}\right)\cos\left(\frac{\phi-2\theta-\phi}{2}\right)}{2\sin\left(\frac{\phi-2\theta-\phi}{2}\right)\cos\left(\frac{\phi+2\theta+\phi}{2}\right)} = \frac{x+y}{x-y}$$

$$\Rightarrow \quad \frac{\sin(\theta+\phi)\cos(\theta-\phi)}{\sin(-\theta)\cos(\theta+\phi)} = \frac{x+y}{x-y}$$

$$\Rightarrow \quad \frac{\sin(\theta+\phi)\cos(\theta)}{\cos(\theta+\phi)[-\sin(\theta)]} = \frac{x+y}{x-y}$$

$$\Rightarrow \quad \frac{-\cot(\theta)}{\cot(\theta+\phi)} = \frac{x+y}{x-y}$$

$$\Rightarrow \quad -(x-y)\cot\theta = (x+y)\cot(\theta+\phi)$$

$$\Rightarrow \quad (y-x)\cot\theta = (x+y)\cot(\theta+\phi)$$

$$\Rightarrow \quad (x+y)\cot(\theta+\phi) = (y-x)\cot\theta$$

Hence proved.

Transformation Formulae Ex 8.2 Q 17.

We have,

$$\cos(A+B)\sin(C-D)=\cos(A-B)\sin(C+D)$$

$$\Rightarrow \quad \frac{\cos(A+B)}{\cos(A-B)}=\frac{\sin(C+D)}{\sin(C-D)} \qquad \text{---(i)}$$

Now,

$$\frac{\cos(A+B)}{\cos(A-B)}=\frac{\sin(C+D)}{\sin(C-D)}$$

$$\Rightarrow \quad \frac{\cos(A+B)}{\cos(A-B)}+1=\frac{\sin(C+D)}{\sin(C-D)}+1$$

$$\Rightarrow \quad \frac{\cos(A+B)+\cos(A-B)}{\cos(A-B)}=\frac{\sin(C+D)+\sin(C-D)}{\sin(C-D)} \qquad \text{---(ii)}$$

Again,

$$\frac{\cos(A+B)}{\cos(A-B)}=\frac{\sin(C+D)}{\sin(C-D)} \qquad \text{[By equation (i)]}$$

$$\Rightarrow \quad \frac{\cos(A+B)}{\cos(A-B)}-1=\frac{\sin(C+D)}{\sin(C-D)}-1$$

$$\Rightarrow \quad \frac{\cos(A+B)-\cos(A-B)}{\cos(A-B)}=\frac{\sin(C+D)-\sin(C-D)}{\sin(C-D)} \qquad \text{---(iii)}$$

Dividing equation (ii) by equation (iii), we get

$$\frac{\cos(A+B)+\cos(A-B)}{\cos(A+B)-\cos(A-B)}=\frac{\sin(C+D)+\sin(C-D)}{\sin(C+D)-\sin(C-D)}$$

$$\Rightarrow \quad \frac{2\cos\left\{\frac{A+B+A-B}{2}\right\}\cos\left\{\frac{A+B-A+B}{2}\right\}}{-2\sin\left\{\frac{A+B+A-B}{2}\right\}\sin\left\{\frac{A+B-A+B}{2}\right\}}=\frac{2\sin\left\{\frac{C+D+C-D}{2}\right\}\cos\left\{\frac{C+D-C+D}{2}\right\}}{2\sin\left\{\frac{C+D-C+D}{2}\right\}\cos\left\{\frac{C+D+C-D}{2}\right\}}$$

$$\Rightarrow \quad \frac{\cos A\cos B}{-\sin A\sin B}=\frac{\sin C\cos D}{\sin D\cos C}$$

$$\Rightarrow \quad \frac{1}{-\tan A\tan B}=\frac{\sin C\cos D}{\cos C\sin D}$$

$$\Rightarrow \quad \frac{-1}{\tan A\tan B}=\frac{\tan C}{\tan D}$$

$$\Rightarrow \quad -\tan D=\tan A\tan B\tan C$$

$$\Rightarrow \quad \tan A\tan B\tan C=-\tan D$$

$$\Rightarrow \quad \tan A\tan B\tan C+\tan D=0 \qquad \text{Hence proved.}$$

Transformation Formulae Ex 8.2 Q 18.

Given $x\cos\theta=y\cos\left(\theta+\frac{2\pi}{3}\right)=z\cos\left(\theta+\frac{4\pi}{3}\right)=k(say)$

$$x=\frac{k}{\cos\theta}$$

$$y=\frac{k}{\cos\left(\theta+\frac{2\pi}{3}\right)}$$

$$z=\frac{k}{\cos\left(\theta+\frac{4\pi}{3}\right)}$$

$$xy+yz+zx=k^2\left[\frac{1}{\cos\theta\cos\left(\theta+\frac{2\pi}{3}\right)}+\frac{1}{\cos\left(\theta+\frac{2\pi}{3}\right)\cos\left(\theta+\frac{4\pi}{3}\right)}+\frac{1}{\cos\left(\theta+\frac{4\pi}{3}\right)\cos\theta}\right]$$

$$=k^2\left[\frac{\cos\left(\theta+\frac{4\pi}{3}\right)+\cos\theta+\cos\left(\theta+\frac{2\pi}{3}\right)}{\cos\theta\cos\left(\theta+\frac{2\pi}{3}\right)\cos\left(\theta+\frac{4\pi}{3}\right)}\right]$$

$$=k^2\left[\frac{\cos\theta\cos\frac{4\pi}{3}-\sin\theta\sin\frac{4\pi}{3}+\cos\theta+\cos\theta\cos\frac{2\pi}{3}-\sin\theta\sin\frac{2\pi}{3}}{\cos\theta\cos\left(\theta+\frac{2\pi}{3}\right)\cos\left(\theta+\frac{4\pi}{3}\right)}\right]$$

$$=k^2\left[\frac{\cos\theta\left(\frac{-1}{2}\right)-\sin\theta\left(\frac{-\sqrt{3}}{2}\right)+\cos\theta+\cos\theta\left(\frac{-1}{2}\right)-\sin\theta\left(\frac{\sqrt{3}}{2}\right)}{\cos\theta\cos\left(\theta+\frac{2\pi}{3}\right)\cos\left(\theta+\frac{4\pi}{3}\right)}\right]$$

$$=k^2\left[\frac{-\cos\theta+\sin\theta\left(\frac{\sqrt{3}}{2}\right)+\cos\theta+-\sin\theta\left(\frac{\sqrt{3}}{2}\right)}{\cos\theta\cos\left(\theta+\frac{2\pi}{3}\right)\cos\left(\theta+\frac{4\pi}{3}\right)}\right]$$

$$=0$$

Hence Proved

Transformation Formulae Ex 8.2 Q19

Given that $m\sin\theta = n\sin(\theta + 2a)$,

We need to prove that $\tan(\theta + a) = \dfrac{m+n}{m-n}\tan a$

$m\sin\theta = n\sin(\theta + 2a)$

$$\Rightarrow \frac{\sin(\theta + 2a)}{\sin\theta} = \frac{m}{n}$$

Using Componendo − Dividendo, we have,

$$\Rightarrow \frac{\sin(\theta + 2a) + \sin\theta}{\sin(\theta + 2a) - \sin\theta} = \frac{m+n}{m-n} \text{....(1)}$$

We know that,

$$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$$

and

$$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$$

Applying the above formulae in equation (1), we have,

$$\frac{2\sin\frac{\theta+2a+\theta}{2}\cos\frac{\theta+2a-\theta}{2}}{2\cos\frac{\theta+2a+\theta}{2}\sin\frac{\theta+2a-\theta}{2}} = \frac{m+n}{m-n}$$

$$\Rightarrow \frac{2\sin(\theta + a)\cos a}{2\cos(\theta + a)\sin a} = \frac{m+n}{m-n}$$

$$\Rightarrow \frac{\tan(\theta + a)}{\tan a} = \frac{m+n}{m-n}$$

$$\Rightarrow \tan(\theta + a) = \frac{m+n}{m-n} \times \tan a$$

Hence proved.

*********** END **********