Differentiation Ex 11.8 Q7(i)

Let $\quad u = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Put $\quad x = \sin\theta$, so

$$u = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$$
$$= \sin^{-1}(2\sin\theta\cos\theta)$$
$$u = \sin^{-1}(\sin 2\theta) \qquad \text{---(i)}$$

And,

Let $\quad v = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

$$= \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right)$$
$$= \sec^{-1}\left(\frac{1}{\cos\theta}\right)$$
$$= \sec^{-1}(\sec\theta)$$
$$= \cos^{-1}\left(\frac{1}{\frac{1}{\cos\theta}}\right) \qquad \left[\text{Since, } \sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right)\right]$$
$$v = \cos^{-1}(\cos\theta) \qquad \text{---(ii)}$$

Here,

$$x \in \left(0, \frac{1}{\sqrt{2}}\right)$$
$$\Rightarrow \quad \sin\theta \in \left(0, \frac{1}{\sqrt{2}}\right)$$
$$\Rightarrow \quad \theta \in \left(0, \frac{\pi}{4}\right)$$

So, from equation (i),

$$u = 2\theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

Let $\quad u = 2\sin^{-1}x \qquad [\text{Since, } x = \sin\theta]$

Differentiating it with respect to x,

$$\frac{du}{dx} = 2\left(\frac{1}{\sqrt{1-x^2}}\right)$$
$$\frac{du}{dx} = \frac{2}{\sqrt{1-x^2}} \qquad \text{---(iii)}$$

And, from equation (ii),

$$v = \theta \qquad \left[\text{Since, } \cos^{-1}(\cos\theta) = \theta, \text{ if } \theta \in [0, \pi]\right]$$
$$v = \sin^{-1}x \qquad [\text{Since, } x = \sin\theta]$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}} \qquad \text{---(iv)}$$

Dividing equation (iii) by (iv),3

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{1}$$

$$\frac{du}{dv} = 2$$

Differentiation Ex 11.8 Q7(ii)

Let $\quad u = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Put $\quad x = \sin\theta$, so

$$u = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$$
$$= \sin^{-1}\left(2\sin\theta\cos\theta\right)$$
$$u = \sin^{-1}\left(\sin 2\theta\right) \qquad \text{---(i)}$$

And,

Let $\quad v = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

$$= \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right)$$
$$= \sec^{-1}\left(\frac{1}{\cos\theta}\right)$$
$$= \sec^{-1}\left(\sec\theta\right)$$
$$= \cos^{-1}\left(\frac{1}{\sec\theta}\right) \qquad \left[\text{Since, } \sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right)\right]$$
$$v = \cos^{-1}\left(\cos\theta\right) \qquad \text{---(ii)}$$

Here,

$$x \in \left(\frac{1}{\sqrt{2}}, 1\right)$$
$$\Rightarrow \quad \sin\theta \in \left(\frac{1}{\sqrt{2}}, 1\right)$$
$$\Rightarrow \quad \theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

So, from equation (i),

$$u = 2\theta \qquad \left[\text{Since, } \sin^{-1}\left(\sin\theta\right) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$
$$u = 2\sin^{-1}x \qquad \left[\text{Since, } x = \sin\theta\right]$$

Differentiating it with respect to x,

$$\frac{du}{dx} = 2\left(\frac{1}{\sqrt{1-x^2}}\right) \qquad \text{---(iv)}$$

From equation (ii)

$$v = \theta \qquad \left[\text{Since, } \cos^{-1}\left(\cos\theta\right) = \theta, \text{ if } \theta \in [0, \pi]\right]$$
$$v = \sin^{-1}x$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}} \qquad \text{---(v)}$$

Dividing equation (iv) by (v),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{1}$$

$$\frac{du}{dv} = 2$$

Differentiation Ex 11.8 Q8

Let $\quad u = (\cos x)^{\sin x}$

Taking on both the sides,

$$\log u = \log(\cos x)^{\sin x}$$
$$\log u = \sin x \log(\cos x)$$

Differentiating it with respect to x using product and chain rule,

$$\frac{1}{u}\frac{du}{dx} = \sin x \frac{d}{dx}(\log\cos x) + \log\cos x \frac{d}{dx}(\sin x)$$
$$\frac{1}{u}\frac{du}{dx} = \sin x \left(\frac{1}{\cos x}\right)\frac{d}{dx}(\cos x) + \log\cos x (\cos x)$$
$$\frac{du}{dx} = 4\left[(\tan x)\times(-\sin x) + \log\log x \times (\cos x)\right]$$
$$\frac{du}{dx} = (\cos x)^{\sin x}\left[\cos x \log\cos x - \sin x \tan x\right] \qquad \text{---(i)}$$

Let $\quad v = (\sin x)^{\cos x}$

Taking log on both the sides,

$$\log v = \log(\sin x)^{\cos x}$$
$$\log v = \cos x \log(\sin x)$$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{1}{v}\frac{dv}{dx} = \cos x \frac{d}{dx}(\log\sin x) + \log\sin x \frac{d}{dx}(\cos x)$$
$$\frac{1}{v}\frac{dv}{dx} = \cos x \left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x) + \log\sin x (-\sin x)$$
$$\frac{dv}{dx} = v\left[\cot x (\cos x) - \sin x \log\sin x\right]$$
$$\frac{dv}{dx} = (\sin x)^{\cos x}\left[\cot x (\cos x) - \sin x \log\sin x\right] \qquad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\frac{du}{dv} = \frac{(\cos x)^{\sin x}\left[\cos x \log\cos x - \sin x \tan x\right]}{(\sin x)^{\cos x}\left[\cot x (\cos x) - \sin x \log\sin x\right]}$$

*********** END **********