Indefinite Integrals Ex 19.19 Q10

Let $I = \int \dfrac{x+2}{2x^2+6x+5}dx$

Let $x+2 = \lambda \dfrac{d}{dx}\left(2x^2+6x+5\right) + \mu$

$\qquad = \lambda\left(4x+6\right) + \mu$

$\qquad x+2 = \left(4\lambda\right)x + \left(6\lambda + \mu\right)$

Comparing the coefficients of like powers of x,

$\qquad 4\lambda = 1 \qquad\qquad \Rightarrow \qquad \lambda = \dfrac{1}{4}$

$\qquad 6\lambda + \mu = 2 \qquad \Rightarrow \qquad 6\left(\dfrac{1}{4}\right) + \mu = 2$

$\qquad\qquad\qquad\qquad\qquad\qquad \mu = \dfrac{1}{2}$

so, $\quad I = \int \dfrac{\dfrac{1}{4}\left(4x+6\right) + \dfrac{1}{2}}{2x^2+6x+5}dx$

$I = \dfrac{1}{4}\int \dfrac{4x+6}{2x^2+6x+5}dx + \dfrac{1}{2}\int \dfrac{1}{2x^2+6x+5}dx$

$I = \dfrac{1}{4}\int \dfrac{4x+6}{2x^2+6x+5}dx + \dfrac{1}{4}\int \dfrac{1}{x^2+3x+\dfrac{5}{2}}dx$

$I = \dfrac{1}{4}\int \dfrac{4x+6}{2x^2+6x+5}dx + \dfrac{1}{4}\int \dfrac{1}{x^2+2x\left(\dfrac{3}{2}\right)+\left(\dfrac{3}{2}\right)^2-\left(\dfrac{3}{2}\right)^3+\dfrac{5}{2}}dx$

$\quad = \dfrac{1}{4}\int \dfrac{4x+6}{2x^2+6x+5}dx + \dfrac{1}{4}\int \dfrac{1}{\left(x+\dfrac{3}{2}\right)^2+\dfrac{1}{4}}dx$

$I = \dfrac{1}{4}\int \dfrac{4x+6}{2x^2+6x+5}dx + \dfrac{1}{4}\int \dfrac{1}{\left(x+\dfrac{3}{2}\right)^2+\left(\dfrac{1}{2}\right)^2}dx$

$I = \dfrac{1}{4}\int \dfrac{4x+6}{2x^2+6x+5}dx + \dfrac{1}{4}\times \dfrac{1}{\dfrac{1}{2}} \tan^{-1}\left(\dfrac{x+\dfrac{3}{2}}{\dfrac{1}{2}}\right)+ c\,dx \qquad \left[\text{since, } \int \dfrac{1}{x^2+a^2}dx = \dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)+c\right]$

$I = \dfrac{1}{4}\log\left|2x^2+6x+5\right| + \dfrac{1}{2}\tan^{-1}\left(2x+3\right)+c$

Indefinite Integrals Ex 19.19 Q11

Let $I = \int \dfrac{(3\sin x - 2)\cos x}{5 - \cos^2 x - 4\sin x} dx$

$\therefore I = \int \dfrac{(3\sin x - 2)\cos x}{5 - (1 - \sin^2 x) - 4\sin x} dx$

$\Rightarrow I = \int \dfrac{(3\sin x - 2)\cos x}{5 - 1 + \sin^2 x - 4\sin x} dx$

Substitute $\sin x = t$

$\Rightarrow \cos x\, dx = dt$

Thus,

$I = \int \dfrac{(3t - 2)}{4 + t^2 - 4t} dt$

$\Rightarrow I = \int \dfrac{(3t - 2)}{t^2 - 4t + 4} dt$

$\Rightarrow I = \int \dfrac{(3t - 2)}{(t - 2)^2} dt$

Now let us separate the integrand into
the simplest form using partial fractions.

$\dfrac{(3t - 2)}{(t - 2)^2} = \dfrac{A}{(t - 2)} + \dfrac{B}{(t - 2)^2}$

$\qquad = \dfrac{A(t - 2) + B}{(t - 2)^2}$

$\qquad = \dfrac{At - 2A + B}{(t - 2)^2}$

$\Rightarrow 3t - 2 = At - 2A + B$

Comparing the coefficients, we have,

A=3

and

$-2A + B = -2$

Susbtituting the value of A=3 in the above equation, we have,

$\Rightarrow -2 \times 3 + B = -2$

$\Rightarrow -6 + B = -2$

$\Rightarrow B = 6 - 2$

$\Rightarrow B = 4$

Thus, $I = \int \dfrac{(3t - 2)}{(t - 2)^2} dt$ becomes,

$I = \int \dfrac{3}{(t - 2)} dt + \int \dfrac{4}{(t - 2)^2} dt$

$= 3\log|t - 2| - 4\left(\dfrac{1}{t - 2}\right) + C$

$= 3\log|2 - t| + 4\left(\dfrac{1}{2 - t}\right) + C$

Now substituting t=sinx, we have,

$I = 3\log|2 - \sin x| + 4\left(\dfrac{1}{2 - \sin x}\right) + C$

Indefinite Integrals Ex 19.19 Q12

Let $I = \int \dfrac{5x-2}{1+2x+3x^2}dx$

Rewriting the numerator we have,

$5x - 2 = A\dfrac{d}{dx}\left(1+2x+3x^2\right)+B$

$\Rightarrow 5x - 2 = A(2+6x)+B$

$\Rightarrow 5x - 2 = 6xA+2A+B$

Comparing the coefficients, we have,

$6A=5$ and $2A+B=-2$

$\Rightarrow A = \dfrac{5}{6}$

Substituting the value of A in $2A+B=-2, we have,$

$2\times\dfrac{5}{6}+B=-2$

$\Rightarrow \dfrac{10}{6}+B=-2$

$\Rightarrow B=-2-\dfrac{10}{6}$

$\Rightarrow B=\dfrac{-12-10}{6}$

$\Rightarrow B=\dfrac{-22}{6}$

$\Rightarrow B=\dfrac{-11}{3}$

$5x-2=\dfrac{5}{6}(2+6x)-\dfrac{11}{3}$

Thus, $I = \int\dfrac{5x-2}{1+2x+3x^2}dx$ becomes,

$I = \int \dfrac{\left[\dfrac{5}{6}(2+6x)-\dfrac{11}{3}\right]}{3x^2+2x+1}dx$

$= \dfrac{5}{6}\int\dfrac{(2+6x)}{3x^2+2x+1}dx - \dfrac{11}{3}\int\dfrac{dx}{3x^2+2x+1}$

$= \dfrac{5}{6}\log\left(3x^2+2x+1\right)-\dfrac{11}{3\times3}\int\dfrac{dx}{x^2+\dfrac{2}{3}x+\dfrac{1}{3}}+C$

$= \dfrac{5}{6}\log\left(3x^2+2x+1\right)-\dfrac{11}{9}\int\dfrac{dx}{x^2+\dfrac{2}{3}x+\left(\dfrac{4}{3}\right)^2+\dfrac{1}{3}-\left(\dfrac{4}{3}\right)^2}+C$

$= \dfrac{5}{6}\log\left(3x^2+2x+1\right)-\dfrac{11}{9}\int\dfrac{dx}{\left(x+\dfrac{1}{3}\right)^2+\left(\dfrac{\sqrt{2}}{3}\right)^2}+C$

$= \dfrac{5}{6}\log\left(3x^2+2x+1\right)-\dfrac{11}{9}\times\dfrac{1}{\dfrac{\sqrt{2}}{3}}\tan^{-1}\left[\dfrac{\left(x+\dfrac{1}{3}\right)}{\dfrac{\sqrt{2}}{3}}\right]+C$

$= \dfrac{5}{6}\log\left(3x^2+2x+1\right)-\dfrac{11}{9}\times\dfrac{3}{\sqrt{2}}\tan^{-1}\left[\dfrac{\left(\dfrac{3x+1}{3}\right)}{\dfrac{\sqrt{2}}{3}}\right]+C$

$= \dfrac{5}{6}\log\left(3x^2+2x+1\right)-\dfrac{11}{3\sqrt{2}}\tan^{-1}\left[\dfrac{3x+1}{\sqrt{2}}\right]+C$

*********** END **********

*********** END **********