$$V=\frac{1}{4\pi\in_0}\left(\frac{q}{z-a}\right)+\frac{1}{4\pi\in_0}\left(-\frac{q}{z+a}\right)$$
$$=\frac{q(z+a-z+a)}{4\pi\in_0\left(z^2-a^2\right)}$$
$$=\frac{2qa}{4\pi\in_0\left(z^2-a^2\right)}=\frac{p}{4\pi\in_0\left(z^2-a^2\right)}$$

Where,

$\in_0$ = Permittivity of free space

p = Dipole moment of the system of two charges = 2*qa*

(b) Distance *r* is much greater than half of the distance between the two charges.

Hence, the potential (*V*) at a distance *r* is inversely proportional to square of the distance

i.e., $V \propto \frac{1}{r^2}$

(c) Zero

The answer does not change if the path of the test is not along the *x*-axis.

A test charge is moved from point (5, 0, 0) to point (−7, 0, 0) along the *x*-axis.

Electrostatic potential (V_1) at point (5, 0, 0) is given by,

$$V_1=\frac{-q}{4\pi\in_0}\frac{1}{\sqrt{(5-0)^2+(-a)^2}}+\frac{q}{4\pi\in_0}\frac{1}{(5-0)^2+a^2}$$
$$=\frac{-q}{4\pi\in_0\sqrt{25^2+a^2}}+\frac{q}{4\pi\in_0\sqrt{25+a^2}}$$
$$=0$$

Electrostatic potential, V_2, at point (− 7, 0, 0) is given by,

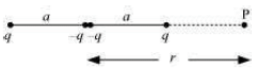
$$V_2=\frac{-q}{4\pi\in_0}\frac{1}{\sqrt{(-7)^2+(-a)^2}}+\frac{q}{4\pi\in_0}\frac{1}{\sqrt{(-7)^2+(a)^2}}$$
$$=\frac{-q}{4\pi\in_0\sqrt{49+a^2}}+\frac{q}{4\pi\in_0}\frac{1}{\sqrt{49+a^2}}$$
$$=0$$

Hence, no work is done in moving a small test charge from point (5, 0, 0) to point (−7, 0, 0) along the *x*-axis.

The answer does not change because work done by the electrostatic field in moving a test charge between the two points is independent of the path connecting the two points.

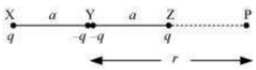
Question 2.22:

Figure 2.34 shows a charge array known as an *electric quadrupole*. For a point on the axis of the quadrupole, obtain the dependence of potential on *r* for *r*/*a* >> 1, and contrast your results with that due to an electric dipole, and an electric monopole (i.e., a single charge).



Answer

Four charges of same magnitude are placed at points X, Y, Y, and Z respectively, as shown in the following figure.



A point is located at P, which is *r* distance away from point Y.

The system of charges forms an electric quadrupole.

It can be considered that the system of the electric quadrupole has three charges.

Charge +*q* placed at point X

Charge −2*q* placed at point Y

Charge +*q* placed at point Z

XY = YZ = *a*

YP = *r*

PX = *r* + *a*

PZ = *r* − *a*

Electrostatic potential caused by the system of three charges at point P is given by,

$$V=\frac{1}{4\pi\in_0}\left[\frac{q}{\mathrm{XP}}-\frac{2q}{\mathrm{YP}}+\frac{q}{\mathrm{ZP}}\right]$$
$$=\frac{1}{4\pi\in_0}\left[\frac{q}{r+a}-\frac{2q}{r}+\frac{q}{r-a}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{r(r-a)-2(r+a)(r-a)+r(r+a)}{r(r+a)(r-a)}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{r^2-ra-2r^2+2a^2+r^2+ra}{r(r^2-a^2)}\right] = \frac{q}{4\pi\epsilon_0}\left[\frac{2a^2}{r(r^2-a^2)}\right]$$

$$= \frac{2qa^2}{4\pi\epsilon_0 r^3\left(1-\frac{a^2}{r^2}\right)}$$

Since $\frac{r}{a} >> 1$,

$\therefore \frac{a}{r} << 1$

$\frac{a^2}{r^2}$ is taken as negligible.

$\therefore V = \frac{2qa^2}{4\pi\epsilon_0 r^3}$

It can be inferred that potential, $V \propto \frac{1}{r^3}$

However, it is known that for a dipole, $V \propto \frac{1}{r^2}$

And, for a monopole, $V \propto \frac{1}{r}$

Question 2.23:

An electrical technician requires a capacitance of 2 μF in a circuit across a potential difference of 1 kV. A large number of 1 μF capacitors are available to him each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires the minimum number of capacitors.

Answer

Total required capacitance, C = 2 μF

Potential difference, V = 1 kV = 1000 V

Capacitance of each capacitor, C_1 = 1μF

Each capacitor can withstand a potential difference, V_1 = 400 V

Suppose a number of capacitors are connected in series and these series circuits are connected in parallel (row) to each other. The potential difference across each row must be 1000 V and potential difference across each capacitor must be 400 V. Hence, number of capacitors in each row is given as

$\frac{1000}{400} = 2.5$

Hence, there are three capacitors in each row.

Capacitance of each row $= \frac{1}{1+1+1} = \frac{1}{3}\ \mu\text{F}$

Let there are n rows, each having three capacitors, which are connected in parallel. Hence, equivalent capacitance of the circuit is given as

$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+..................n \text{ terms}$

$= \frac{n}{3}$

However, capacitance of the ciruit is given as 2 μF.

$\therefore \frac{n}{3} = 2$

$n = 6$

Hence, 6 rows of three capacitors are present in the circuit. A minimum of 6 × 3 i.e., 18 capacitors are required for the given arrangement.

Question 2.24:

What is the area of the plates of a 2 F parallel plate capacitor, given that the separation between the plates is 0.5 cm? [You will realize from your answer why ordinary capacitors are in the range of μF or less. However, electrolytic capacitors do have a much larger capacitance (0.1 F) because of very minute separation between the conductors.]

Answer

Capacitance of a parallel capacitor, V = 2 F

Distance between the two plates, d = 0.5 cm = 0.5 × 10^{-2} m

Capacitance of a parallel plate capacitor is given by the relation,

$C = \frac{\epsilon_0 A}{d}$

$A = \frac{Cd}{\epsilon_0}$

Where,

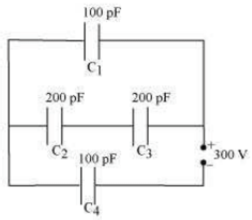
ϵ_0 = Permittivity of free space = 8.85 × 10^{-12} $C^2\ N^{-1}\ m^{-2}$

$\therefore A = \frac{2\times0.5\times10^{-2}}{8.85\times10^{-12}} = 1130\ \text{km}^2$

Hence, the area of the plates is too large. To avoid this situation, the capacitance is taken in the range of μF.

Question 2.25:

Obtain the equivalent capacitance of the network in Fig. 2.35. For a 300 V supply, determine the charge and voltage across each capacitor.



Answer

Capacitance of capacitor C_1 is 100 pF.

Capacitance of capacitor C_2 is 200 pF.

Capacitance of capacitor C_3 is 200 pF.

Capacitance of capacitor C_4 is 100 pF.

Supply potential, V = 300 V

Capacitors C_2 and C_3 are connected in series. Let their equivalent capacitance be C'.

$$\therefore \frac{1}{C'} = \frac{1}{200} + \frac{1}{200} = \frac{2}{200}$$

$$C' = 100 \text{ pF}$$

Capacitors C_1 and C' are in parallel. Let their equivalent capacitance be C''.

$$\therefore C'' = C' + C_1$$

$$= 100 + 100 = 200 \text{ pF}$$

C'' and C_4 are connected in series. Let their equivalent capacitance be C.

$$\therefore \frac{1}{C} = \frac{1}{C''} + \frac{1}{C_4}$$

$$= \frac{1}{200} + \frac{1}{100} = \frac{2+1}{200}$$

$$C = \frac{200}{3} \text{ pF}$$

Hence, the equivalent capacitance of the circuit is $\frac{200}{3}$ pF.

Potential difference across C'' = V''

Potential difference across C_4 = V_4

$$\therefore V'' + V_4 = V = 300 \text{ V}$$

Charge on C_4 is given by,

$Q_4 = CV$

$$= \frac{200}{3} \times 10^{-12} \times 300$$

$$= 2 \times 10^{-8} \text{ C}$$

$$\therefore V_4 = \frac{Q_4}{C_4}$$

$$= \frac{2 \times 10^{-8}}{100 \times 10^{-12}} = 200 \text{ V}$$

$\therefore$ Voltage across C_1 is given by,

$$V_1 = V - V_4$$

$$= 300 - 200 = 100 \text{ V}$$

Hence, potential difference, V_1, across C_1 is 100 V.

Charge on C_1 is given by,

$$Q_1 = C_1 V_1$$

$$= 100 \times 10^{-12} \times 100$$

$$= 10^{-8} \text{ C}$$

C_2 and C_3 having same capacitances have a potential difference of 100 V together. Since C_2 and C_3 are in series, the potential difference across C_2 and C_3 is given by,

$V_2 = V_3$ = 50 V

Therefore, charge on C_2 is given by,

$$Q_2 = C_2 V_2$$

$$= 200 \times 10^{-12} \times 50$$

$$= 10^{-8} \text{C}$$

And charge on C_3 is given by,

$$Q_3 = C_3 V_3$$

$$= 200 \times 10^{-12} \times 50$$

$$= 10^{-8} \text{C}$$

Hence, the equivalent capacitance of the given circuit is $\frac{200}{3}$ pF with,

$$Q_1 = 10^{-8} \text{ C}, \qquad V_1 = 100 \text{ V}$$

$$Q_2 = 10^{-8} \text{ C}, \qquad V_2 = 50 \text{ V}$$

$$Q_3 = 10^{-8} \text{ C}, \qquad V_3 = 50 \text{ V}$$

$Q_4 = 2\times10^{-8}\,\text{C}, \qquad V_4 = 200\,\text{V}$

Question 2.26:

The plates of a parallel plate capacitor have an area of 90 cm^2 each and are separated by 2.5 mm. The capacitor is charged by connecting it to a 400 V supply.

(a) How much electrostatic energy is stored by the capacitor?

(b) View this energy as stored in the electrostatic field between the plates, and obtain the energy per unit volume *u*. Hence arrive at a relation between *u* and the magnitude of electric field *E* between the plates.

Answer

Area of the plates of a parallel plate capacitor, A = 90 cm^2 = 90 × 10^{-4} m^2

Distance between the plates, d = 2.5 mm = 2.5 × 10^{-3} m

Potential difference across the plates, V = 400 V

(a) Capacitance of the capacitor is given by the relation,

$$C = \frac{\epsilon_0 A}{d}$$

Electrostatic energy stored in the capacitor is given by the relation, $E_1 = \frac{1}{2}CV^2$

$$= \frac{1}{2}\frac{\epsilon_0 A}{d}V^2$$

Where,

ϵ_0 = Permittivity of free space = 8.85 × 10^{-12} C^2 N^{-1} m^{-2}

$$\therefore E_1 = \frac{1\times8.85\times10^{-12}\times90\times10^{-4}\times(400)^2}{2\times2.5\times10^{-3}} = 2.55\times10^{-6}\ \text{J}$$

Hence, the electrostatic energy stored by the capacitor is 2.55×10^{-6} J.

(b) Volume of the given capacitor,

$$V' = A\times d$$
$$= 90\times10^{-4}\times25\times10^{-3}$$
$$= 2.25\times10^{-4}\ \text{m}^3$$

Energy stored in the capacitor per unit volume is given by,

$$u = \frac{E_1}{V'}$$
$$= \frac{2.55\times10^{-6}}{2.25\times10^{-4}} = 0.113\ \text{J m}^{-3}$$

$$\text{Again,}\ u = \frac{E_1}{V'}$$
$$= \frac{\frac{1}{2}CV^2}{Ad} = \frac{\frac{\epsilon_0 A}{2d}V^2}{Ad} = \frac{1}{2}\epsilon_0\left(\frac{V}{d}\right)^2$$

Where,

$\frac{V}{d}$ = Electric intensity = E

$$\therefore u = \frac{1}{2}\epsilon_0 E^2$$

Question 2.27:

A 4 μF capacitor is charged by a 200 V supply. It is then disconnected from the supply, and is connected to another uncharged 2 μF capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation?

Answer

Capacitance of a charged capacitor, $C_1 = 4\mu\text{F} = 4\times10^{-6}\,\text{F}$

Supply voltage, V_1 = 200 V

Electrostatic energy stored in C_1 is given by,

$$E_1 = \frac{1}{2}C_1V_1^2$$
$$= \frac{1}{2}\times4\times10^{-6}\times(200)^2$$
$$= 8\times10^{-2}\ \text{J}$$

Capacitance of an uncharged capacitor, $C_2 = 2\mu\text{F} = 2\times10^{-6}\,\text{F}$

When C_2 is connected to the circuit, the potential acquired by it is V_2.

According to the conservation of charge, initial charge on capacitor C_1 is equal to the final charge on capacitors, C_1 and C_2.

$$\therefore V_2(C_1+C_2) = C_1V_1$$
$$V_2\times(4+2)\times10^{-6} = 4\times10^{-6}\times200$$
$$V_2 = \frac{400}{3}\ \text{V}$$

Electrostatic energy for the combination of two capacitors is given by,

$$E_2 = \frac{1}{2}(C_1+C_2)V_2^2$$
$$= \frac{1}{2}(2+4)\times10^{-6}\times\left(\frac{400}{3}\right)^2$$
$$= 5.33\times10^{-2}\ \text{J}$$

Hence, amount of electrostatic energy lost by capacitor C_1

$= E_1 - E_2$

$= 0.08 - 0.0533 = 0.0267$

$= 2.67 \times 10^{-2}$ J

Question 2.28:

Show that the force on each plate of a parallel plate capacitor has a magnitude equal to (½) *QE*, where *Q* is the charge on the capacitor, and *E* is the magnitude of electric field between the plates. Explain the origin of the factor ½.

Answer

Let *F* be the force applied to separate the plates of a parallel plate capacitor by a distance of Δx. Hence, work done by the force to do so $= F\Delta x$

As a result, the potential energy of the capacitor increases by an amount given as $uA\Delta x$.

Where,

u = Energy density

A = Area of each plate

d = Distance between the plates

V = Potential difference across the plates

The work done will be equal to the increase in the potential energy i.e.,

$$F\Delta x = uA\Delta x$$

$$F = uA = \left(\frac{1}{2}\epsilon_0 E^2\right)A$$

Electric intensity is given by,

$$E = \frac{V}{d}$$

$$\therefore F = \frac{1}{2}\epsilon_0\left(\frac{V}{d}\right)EA = \frac{1}{2}\left(\epsilon_0 A\frac{V}{d}\right)E$$

However, capacitance, $C = \frac{\epsilon_0 A}{d}$

$$\therefore F = \frac{1}{2}(CV)E$$

Charge on the capacitor is given by,

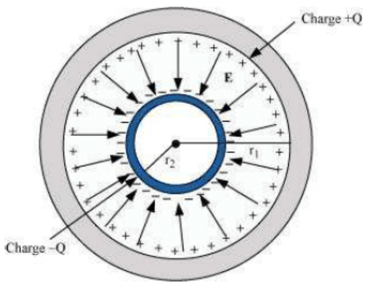
$Q = CV$

$$\therefore F = \frac{1}{2}QE$$

The physical origin of the factor, $\frac{1}{2}$, in the force formula lies in the fact that just outside the conductor, field is *E* and inside it is zero. Hence, it is the average value, $\frac{E}{2}$, of the field that contributes to the force.

Question 2.29:

A spherical capacitor consists of two concentric spherical conductors, held in position by suitable insulating supports (Fig. 2.36). Show



that the capacitance of a spherical capacitor is given by

$$C = \frac{4\pi\epsilon_0 r_1 r_2}{r_1 - r_2}$$

where r_1 and r_2 are the radii of outer and inner spheres, respectively.

Answer

Radius of the outer shell $= r_1$

Radius of the inner shell $= r_2$

The inner surface of the outer shell has charge $+Q$.

The outer surface of the inner shell has induced charge $-Q$.

Potential difference between the two shells is given by,

$$V = \frac{Q}{4\pi\epsilon_0 r_2} - \frac{Q}{4\pi\epsilon_0 r_1}$$

Where,

ϵ_0 = Permittivity of free space

$$V = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$= \frac{Q(r_1 - r_2)}{4\pi\epsilon_0 r_1 r_2}$$

Capacitance of the given system is given by,

$$C=\frac{\text{Charge}(Q)}{\text{Potential difference}(V)}$$

$$=\frac{4\pi\in_0 r_1r_2}{r_1-r_2}$$

Hence, proved.

Question 2.30:

A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of 2.5 µC. The space between the concentric spheres is filled with a liquid of dielectric constant 32.

(a) Determine the capacitance of the capacitor.

(b) What is the potential of the inner sphere?

(c) Compare the capacitance of this capacitor with that of an isolated sphere of radius 12 cm. Explain why the latter is much smaller.

Answer

Radius of the inner sphere, r_2 = 12 cm = 0.12 m

Radius of the outer sphere, r_1 = 13 cm = 0.13 m

Charge on the inner sphere, $q=2.5\mu\text{C}=2.5\times10^{-6}\text{C}$

Dielectric constant of a liquid, $\in_r=32$

(a) Capacitance of the capacitor is given by the relation,

$$C=\frac{4\pi\in_0\in_r r_1r_2}{r_1-r_2}$$

Where,

$\in_0$ = Permittivity of free space = $8.85\times10^{-12}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}$

$$\frac{1}{4\pi\in_0}=9\times10^9\ \text{N}\,\text{m}^2\,\text{C}^{-2}$$

$$\therefore C=\frac{32\times0.12\times0.13}{9\times10^9\times(0.13-0.12)}$$

$$\approx5.5\times10^{-9}\ \text{F}$$

Hence, the capacitance of the capacitor is approximately 5.5×10^{-9} F.

(b) Potential of the inner sphere is given by,

$$V=\frac{q}{C}$$

$$=\frac{2.5\times10^{-6}}{5.5\times10^{-9}}=4.5\times10^2\ \text{V}$$

Hence, the potential of the inner sphere is 4.5×10^2 V.

(c) Radius of an isolated sphere, r = 12 × 10^{-2} m

Capacitance of the sphere is given by the relation,

$$C'=4\pi\in_0 r$$

$$=4\pi\times8.85\times10^{-12}\times12\times10^{-12}$$

$$=1.33\times10^{-11}\ \text{F}$$

The capacitance of the isolated sphere is less in comparison to the concentric spheres. This is because the outer sphere of the concentric spheres is earthed. Hence, the potential difference is less and the capacitance is more than the isolated sphere.

Question 2.31:

Answer carefully:

(a) Two large conducting spheres carrying charges Q_1 and Q_2 are brought close to each other. Is the magnitude of electrostatic force between them exactly given by $Q_1 Q_2/4\pi\in_0 r^2$, where r is the distance between their centres?

(b) If Coulomb's law involved $1/r^3$ dependence (instead of $1/r^2$), would Gauss's law be still true?

(c) A small test charge is released at rest at a point in an electrostatic field configuration. Will it travel along the field line passing through that point?

(d) What is the work done by the field of a nucleus in a complete circular orbit of the electron? What if the orbit is elliptical?

(e) We know that electric field is discontinuous across the surface of a charged conductor. Is electric potential also discontinuous there?

(f) What meaning would you give to the capacitance of a single conductor?

(g) Guess a possible reason why water has a much greater dielectric constant (= 80) than say, mica (= 6).

Answer

(a) The force between two conducting spheres is not exactly given by the expression, $Q_1 Q_2/4\pi\in_0 r^2$, because there is a non-uniform charge distribution on the spheres.

(b) Gauss's law will not be true, if Coulomb's law involved $1/r^3$ dependence, instead of $1/r^2$, on r.

(c) Yes,

If a small test charge is released at rest at a point in an electrostatic field configuration, then it will travel along the field lines passing through the point, only if the field lines are straight. This is because the field lines give the direction of acceleration and not of straight. This is because the field lines give the direction of acceleration and not of velocity.

(d) Whenever the electron completes an orbit, either circular or elliptical, the work done by the field of a nucleus is zero.

(e) No

Electric field is discontinuous across the surface of a charged conductor. However, electric potential is continuous.

(f) The capacitance of a single conductor is considered as a parallel plate capacitor with one of its two plates at infinity.

(g) Water has an unsymmetrical space as compared to mica. Since it has a permanent dipole moment, it has a greater dielectric constant than mica.

Question 2.32:

A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (i.e., bending of field lines at the ends).

Answer

*********** END **********