Trigonometric Ratios Ex 5.1 Q21

Answer :

Given: $\cos\theta = \frac{3}{5}$ .....(1)

To find the value of $\frac{\sin\theta - \frac{1}{\tan\theta}}{2\tan\theta}$

Now, we know the following trigonometric identity

$\cos^2\theta + \sin^2\theta = 1$

Therefore, by substituting the value of $\cos\theta$ from equation (1) ,

We get,

$\left(\frac{3}{5}\right)^2 + \sin^2\theta = 1$

Therefore,

$$\sin^2\theta = 1 - \left(\frac{3}{5}\right)^2$$

$$= 1 - \frac{(3)^2}{(5)^2}$$

$$= 1 - \frac{9}{25}$$

$$\sin^2\theta = \frac{25-9}{25}$$

$$= \frac{16}{25}$$

Therefore by taking square root on both sides

We get,

$$\sin\theta = \sqrt{\frac{16}{25}}$$

$$= \frac{\sqrt{16}}{\sqrt{25}}$$

$$= \frac{4}{5}$$

Therefore,

$\sin\theta = \frac{4}{5}$ ..... (2)

Now, we know that

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

Therefore by substituting the value of $\sin\theta$ and $\cos\theta$ from equation (2) and (1) respectively

We get,

$\tan\theta = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$ ..... (4)

Now, by substituting the value of $\sin\theta$ and $\tan\theta$ from equation (2) and (4) respectively in the expression below

$$\frac{\sin\theta-\frac{1}{\tan\theta}}{2\tan\theta}$$

We get,

$$\frac{\sin\theta-\frac{1}{\tan\theta}}{2\tan\theta}=\frac{\frac{4}{5}-\frac{1}{\frac{4}{3}}}{2\times\frac{4}{3}}$$

$$=\frac{\frac{4}{5}-\frac{3}{4}}{\frac{2\times4}{3}}$$

$$=\frac{\frac{4\times4}{5\times4}-\frac{3\times5}{4\times5}}{\frac{8}{3}}$$

Therefore,

$$\frac{\sin\theta-\frac{1}{\tan\theta}}{2\tan\theta}=\frac{\frac{16}{20}-\frac{15}{20}}{\frac{8}{3}}$$

$$=\frac{\frac{1}{20}}{\frac{8}{3}}$$

$$=\frac{3}{160}$$

Therefore, $\dfrac{\sin\theta-\dfrac{1}{\tan\theta}}{2\tan\theta}=\dfrac{3}{160}$

********** END **********