Combinations Ex 17.1 Q18

${}^{n}C_4, {}^{n}C_5,$ and ${}^{n}C_6$ are in A.P

$$\therefore \quad {}^{n}C_5 - {}^{n}C_4 = {}^{n}C_6 - {}^{n}C_5$$

$$\frac{n!}{5!(n-5)!} - \frac{n!}{4!(n-4)!} = \frac{n!}{6!(n-6)!} - \frac{n!}{6!(n-5)!}$$

$$\Rightarrow \quad \frac{n!}{4!(n-5)!}\left[\frac{1}{5} - \frac{1}{n-4}\right] = \frac{n!}{5!(n-6)!}\left[\frac{1}{6} - \frac{1}{n-5}\right]$$

$$\Rightarrow \quad \frac{1}{n-5}\left[\frac{n-4-5}{5(n-4)}\right] = \frac{1}{5}\left[\frac{n-5-6}{6(n-5)}\right]$$

$$\Rightarrow \quad \frac{n-9}{n-4} = \frac{n-11}{6}$$

$$\Rightarrow \quad 6n - 54 = n^2 - 15n + 44$$

$$\Rightarrow \quad n^2 - 21n + 98 = 0$$

$$n = 7, 14$$

n is 7 or 14

Combinations Ex 17.1 Q19

We have $\alpha = {}^{m}C_2 = \frac{m(m-1)}{2} \left({}^{n}C_r = \frac{n!}{r!(n-r)!}\right)$

Now ${}^{\alpha}C_2 = \frac{\alpha(\alpha-1)}{2}$

$$= \frac{\left(\frac{m(m-1)}{2}\right)\left(\frac{m(m-1)}{2} - 1\right)}{2}$$

$$= \frac{m(m-1)(m^2-m-2)}{2 \times 2 \times 2} = \frac{m(m-1)(m+1)(m-2)}{8}$$

$$= \frac{m(m-1)(m+1)(m-2)}{4 \times 2}$$

multiplying with 3, numerator and denominator to make 4:

Or $= \frac{m(m+1)m(m-1)(m-2)}{4.3.2.1}$

$$= \frac{3(m+1)m(m-1)(m-2)}{4!}$$

$$= 3.\ {}^{m+1}C_4 \quad \left(\because {}^{n}C_r = \frac{n!}{r!(n-r)!}\right)$$

Combinations Ex 17.1 Q20(i)

$$^nC_r = \frac{n!}{r!(n-r)!}$$

$$^nC_{r-1} = \frac{n!}{(r-1)!(n-r+1)!}$$

$$\frac{^nC_r}{^nC_{r-1}} = \frac{n!(r-1)!(n-r+1)!}{r!(n-r)!\ n!}$$

$$= \frac{(r-1)!(n-r+1)\times(n-r)!}{r_2\times(r-1)!(n-r)!}$$

$$= \frac{n-r+1}{r}$$

Hence Proved

*********** END **********