$\Rightarrow a=-7k,\ b=8k,\ c=-3k$

Substituting the values of *a*, *b*, and *c* in equation (1), we obtain

$$-7k(x+1)+8k(y-3)-3k(z-2)=0$$
$$\Rightarrow(-7x-7)+(8y-24)-3z+6=0$$
$$\Rightarrow -7x+8y-3z-25=0$$
$$\Rightarrow 7x-8y+3z+25=0$$

This is the required equation of the plane.

Question 14:

If the points (1, 1, *p*) and (−3, 0, 1) be equidistant from the plane $\vec{r}\cdot\left(3\hat{i}+4\hat{j}-12\hat{k}\right)+13=0$, then find the value of *p*.

Answer

The position vector through the point (1, 1, *p*) is $\vec{a}_1=\hat{i}+\hat{j}+p\hat{k}$

Similarly, the position vector through the point (−3, 0, 1) is

$\vec{a}_2=-4\hat{i}+\hat{k}$

The equation of the given plane is $\vec{r}\cdot\left(3\hat{i}+4\hat{j}-12\hat{k}\right)+13=0$

It is known that the perpendicular distance between a point whose position vector is $\vec{a}$ and the plane, $\vec{r}\cdot\vec{N}=d$, is given by, $D=\dfrac{\left|\vec{a}\cdot\vec{N}-d\right|}{\left|\vec{N}\right|}$

Here, $\vec{N}=3\hat{i}+4\hat{j}-12\hat{k}$ and $d=-13$

Therefore, the distance between the point (1, 1, *p*) and the given plane is

$$D_1=\frac{\left|\left(\hat{i}+\hat{j}+p\hat{k}\right)\cdot\left(3\hat{i}+4\hat{j}-12\hat{k}\right)+13\right|}{\left|3\hat{i}+4\hat{j}-12\hat{k}\right|}$$
$$\Rightarrow D_1=\frac{\left|3+4-12p+13\right|}{\sqrt{3^2+4^2+(-12)^2}}$$
$$\Rightarrow D_1=\frac{\left|20-12p\right|}{13}\qquad ...(1)$$

Similarly, the distance between the point (−3, 0, 1) and the given plane is

$$D_2=\frac{\left|\left(-3\hat{i}+\hat{k}\right)\cdot\left(3\hat{i}+4\hat{j}-12\hat{k}\right)+13\right|}{\left|3\hat{i}+4\hat{j}-12\hat{k}\right|}$$
$$\Rightarrow D_2=\frac{\left|-9-12+13\right|}{\sqrt{3^2+4^2+(-12)^2}}$$
$$\Rightarrow D_2=\frac{8}{13}\qquad ...(2)$$

It is given that the distance between the required plane and the points, (1, 1, *p*) and (−3, 0, 1), is equal.

$\therefore D_1 = D_2$

$$\Rightarrow\frac{\left|20-12p\right|}{13}=\frac{8}{13}$$
$$\Rightarrow 20-12p=8 \text{ or } -(20-12p)=8$$
$$\Rightarrow 12p=12 \text{ or } 12p=28$$
$$\Rightarrow p=1 \text{ or } p=\frac{7}{3}$$

Question 15:

Find the equation of the plane passing through the line of intersection of the planes $\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=1$ and $\vec{r}\cdot\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0$ and parallel to *x*-axis.

Answer

The given planes are

$$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=1$$

$$\Rightarrow \vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1=0$$

$$\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4=0$$

The equation of any plane passing through the line of intersection of these planes is

$$\left[\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-1\right]+\lambda\left[\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4\right]=0$$

$$\vec{r}\cdot\left[(2\lambda+1)\hat{i}+(3\lambda+1)\hat{j}+(1-\lambda)\hat{k}\right]+(4\lambda+1)=0 \qquad ...(1)$$

Its direction ratios are $(2\lambda + 1)$, $(3\lambda + 1)$, and $(1 - \lambda)$.

The required plane is parallel to x-axis. Therefore, its normal is perpendicular to x-axis.

The direction ratios of x-axis are 1, 0, and 0.

$$\therefore 1.(2\lambda+1)+0(3\lambda+1)+0(1-\lambda)=0$$

$$\Rightarrow 2\lambda+1=0$$

$$\Rightarrow \lambda=-\frac{1}{2}$$

Substituting $\lambda=-\frac{1}{2}$ in equation (1), we obtain

$$\Rightarrow \vec{r}\cdot\left[-\frac{1}{2}\hat{j}+\frac{3}{2}\hat{k}\right]+(-3)=0$$

$$\Rightarrow \vec{r}(\hat{j}-3\hat{k})+6=0$$

Therefore, its Cartesian equation is $y - 3z + 6 = 0$

This is the equation of the required plane.

Question 16:

If O be the origin and the coordinates of P be (1, 2, −3), then find the equation of the plane passing through P and perpendicular to OP.

Answer

The coordinates of the points, O and P, are (0, 0, 0) and (1, 2, −3) respectively.

Therefore, the direction ratios of OP are (1 − 0) = 1, (2 − 0) = 2, and (−3 − 0) = −3

It is known that the equation of the plane passing through the point $(x_1, y_1\ z_1)$ is

$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$ where, a, b, and c are the direction ratios of normal.

Here, the direction ratios of normal are 1, 2, and −3 and the point P is (1, 2, −3).

Thus, the equation of the required plane is

$$1(x-1)+2(y-2)-3(z+3)=0$$

$$\Rightarrow x+2y-3z-14=0$$

Question 17:

Find the equation of the plane which contains the line of intersection of the planes

$\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})-4=0$, $\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})+5=0$ and which is perpendicular to the plane

$\vec{r}\cdot(5\hat{i}+3\hat{j}-6\hat{k})+8=0$.

Answer

The equations of the given planes are

$$\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})-4=0 \qquad ...(1)$$

$$\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})+5=0 \qquad ...(2)$$

The equation of the plane passing through the line intersection of the plane given in equation (1) and equation (2) is

$$\left[\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})-4\right]+\lambda\left[\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})+5\right]=0$$

$$\vec{r}\cdot\left[(2\lambda+1)\hat{i}+(\lambda+2)\hat{j}+(3-\lambda)\hat{k}\right]+(5\lambda-4)=0 \qquad ...(3)$$

The plane in equation (3) is perpendicular to the plane, $\vec{r}\cdot(5\hat{i}+3\hat{j}-6\hat{k})+8=0$

$$\therefore 5(2\lambda+1)+3(\lambda+2)-6(3-\lambda)=0$$

$$\Rightarrow 19\lambda-7=0$$

$$\Rightarrow \lambda=\frac{7}{19}$$

Substituting $\lambda=\frac{7}{19}$ in equation (3), we obtain

$$\Rightarrow \vec{r}\cdot\left[\frac{33}{19}\hat{i}+\frac{45}{19}\hat{j}+\frac{50}{19}\hat{k}\right]\frac{-41}{19}=0$$

$$\Rightarrow \vec{r}\cdot(33\hat{i}+45\hat{j}+50\hat{k})-41=0 \qquad ...(4)$$

This is the vector equation of the required plane.

The Cartesian equation of this plane can be obtained by substituting $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ in

The Cartesian equation of this plane can be obtained by substituting [illegible] in equation (3).

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\cdot\left(33\hat{i}+45\hat{j}+50\hat{k}\right)-41=0$$

$$\Rightarrow 33x+45y+50z-41=0$$

Question 18:

Find the distance of the point (−1, −5, −10) from the point of intersection of the line $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$ and the plane $\vec{r}.\left(\hat{i}-\hat{j}+\hat{k}\right)=5$.

Answer

The equation of the given line is

$$\vec{r}\cdot=2\hat{i}-\hat{j}+2\hat{k}+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right) \qquad ...(1)$$

The equation of the given plane is

$$\vec{r}.\left(\hat{i}-\hat{j}+\hat{k}\right)=5 \qquad ...(2)$$

Substituting the value of $\vec{r}$ from equation (1) in equation (2), we obtain

$$\left[2\hat{i}-\hat{j}+2\hat{k}+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)\right]\cdot\left(\hat{i}-\hat{j}+\hat{k}\right)=5$$

$$\Rightarrow\left[(3\lambda+2)\hat{i}+(4\lambda-1)\hat{j}+(2\lambda+2)\hat{k}\right]\cdot\left(\hat{i}-\hat{j}+\hat{k}\right)=5$$

$$\Rightarrow(3\lambda+2)-(4\lambda-1)+(2\lambda+2)=5$$

$$\Rightarrow\lambda=0$$

Substituting this value in equation (1), we obtain the equation of the line as

$$\vec{r}=2\hat{i}-\hat{j}+2\hat{k}$$

This means that the position vector of the point of intersection of the line and the plane is $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}$

This shows that the point of intersection of the given line and plane is given by the

*********** END **********