Chapter 10 Differentiability Ex 10.1 Q1

$$f(x) = |x-3|$$

$$= \begin{cases} -(x-3), & \text{if } x < 3 \\ x-3, & \text{if } x \geq 3 \end{cases}$$

$$f(3) = 3 - 3 = 0$$

$$\text{LHL} = \lim_{x \to 3^-} f(x)$$

$$= \lim_{h \to 0} f(3-h)$$

$$= \lim_{h \to 0} 3 - (3-h)$$

$$= \lim_{h \to 0} 0$$

$$\text{RHL} = \lim_{x \to 3^+} f(x)$$

$$= \lim_{h \to 0} f(3+h)$$

$$= \lim_{h \to 0} 3 + h - 3$$

$$= 0$$

$$\text{LHL} = f(3) = \text{RHL}$$

$\therefore$ $f(x)$ is continuous at $x = 3$

$$(\text{LHD at } x = 3) = \lim_{x \to 3^-} \frac{f(x) - f(3)}{x-3}$$

$$= \lim_{h \to 0} \frac{f(3-h) - f(3)}{3-h-3}$$

$$= \lim_{h \to 0} \frac{3-(3-h)-0}{-h}$$

$$= \lim_{h \to 0} \frac{h}{-h}$$

$$= -1$$

$$(\text{RHD at } x = 3) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x-3}$$

$$= \lim_{h \to 0} \frac{f(3+h) - f(3)}{3+h-3}$$

$$= \lim_{h \to 0} \frac{3+h-3-0}{h}$$

$$= \lim_{h \to 0} \frac{h}{h}$$

$$= 1$$

$$(\text{LHD at } x = 3) \neq (\text{RHD at } x = 3)$$

$\therefore$ $f(x)$ is continuous but not differentiable at $x = 3$.

Chapter 10 Differentiability Ex 10.1 Q2

$$f(x) = x^{\frac{1}{3}}$$

$$(\text{LHD at } x = 0) = \lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{h \to 0} \frac{f(0-h) - f(0)}{0 - h - 0}$$

$$= \lim_{h \to 0} \frac{(-h)^{\frac{1}{3}} - 0}{-h}$$

$$= \lim_{h \to 0} \frac{(-h)^{\frac{1}{3}}}{-h}$$

$$= \lim_{h \to 0} \frac{(-1)^{\frac{1}{3}} h^{\frac{1}{3}}}{(-1) h}$$

$$= \lim_{h \to 0} (-1)^{\frac{-2}{3}} h^{\frac{-2}{3}}$$

$$= \text{Not defined}$$

$$(\text{RHD at } x = 0) = \lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{h \to 0} \frac{f(0+h) - f(0)}{0 + h - 0}$$

$$= \lim_{h \to 0} \frac{h^{\frac{1}{3}} - 0}{h}$$

$$= \lim_{h \to 0} h^{\frac{-2}{3}}$$

$$= \text{Not defined}$$

Since,

LHD and RHD does not exists at $x = 0$

$\therefore$ $f(x)$ is not differentiable at $x = 0$

Chapter 10 Differentiability Ex 10.1 Q3

$$f(x) = \begin{cases} 12x - 13, & \text{if } x \le 3 \\ 2x^2 + 5, & \text{if } x > 3 \end{cases}$$

$$(\text{LHD at } x = 3) = \lim_{x \to 3^-} \frac{f(x) - f(3)}{x - 3}$$

$$= \lim_{h \to 0} \frac{f(3-h) - f(3)}{3 - h - 3}$$

$$= \lim_{h \to 0} \frac{\left[12(3-h) - 13\right] - \left[12(3) - 13\right]}{-h}$$

$$= \lim_{h \to 0} \frac{36 - 12h - 13 - 36 + 13}{-h}$$

$$= \lim_{h \to 0} \frac{-12h}{-h}$$

$$= 12$$

$$(\text{RHD at } x = 3) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x - 3}$$

$$= \lim_{h \to 0} \frac{f(3+h) - f(3)}{x - 3}$$

$$= \lim_{h \to 0} \frac{\left[2\left(3 + h^2\right) + 5\right] - \left[12(3) - 13\right]}{3 + h - 3}$$

$$= \lim_{h \to 0} \frac{18 + 12h + 2h^2 + 5 - 36 + 13}{h}$$

$$= \lim_{h \to 0} \frac{2h^2 + 12h}{h}$$

$$= \lim_{h \to 0} \frac{h(2h + 12)}{h}$$

$$= 12$$

Now,

$(\text{LHD at } x = 3) = (\text{RHD at } x = 3)$

$\therefore$ $f(x)$ is differentiable at $x = 3$

$f'(x) = 12$

Chapter 10 Differentiability Ex 10.1 Q4

$$f(x) = \begin{cases} 3x - 2 & , \quad 0 < x \le 1 \\ 2x^2 - x & , \quad 1 < x \le 2 \\ 5x - 4 & , \quad x > 2 \end{cases}$$

$$f(2) = 2(2)^2 - 2$$

$$= 8 - 2 = 6$$

$$\text{LHL} = \lim_{x \to 2^-} f(x)$$

$$= \lim_{h \to 0} f(2 - h)$$

$$= \lim_{h \to 0} \left[2(2-h)^2 - (2-h)\right]$$

$$= 8 - 2$$

$$= 6$$

$$\begin{aligned} \text{RHL} &= \lim_{x\to 2^+} f(x) \\ &= \lim_{h\to 0} f(2+h) \\ &= \lim_{h\to 0} 5(2+h) - 4 \\ &= 6 \end{aligned}$$

$$\text{LHL} = f(2) = \text{RHL}$$

$f(x)$ is continuous at $x = 2$

$$\begin{aligned} (\text{LHD at } x = 2) &= \lim_{x\to 2^-} \frac{f(x) - f(2)}{x - 2} \\ &= \lim_{h\to 0} \frac{f(2-h) - f(2)}{2 - h - 2} \\ &= \lim_{h\to 0} \frac{\left[2(2-h)^2 - (2-h)\right] - [8-2]}{-h} \\ &= \lim_{h\to 0} \frac{8 - 8h + 2h^2 - h - 6}{-h} \\ &= \lim_{h\to 0} \frac{2h^2 - 6h}{-h} \\ &= \lim_{h\to 0} \frac{h(2h-6)}{-h} \\ &= \lim_{h\to 0} (6 - 2h) \\ &= 6 \end{aligned}$$

$$\begin{aligned} (\text{RHD at } x = 2) &= \lim_{x\to 2^+} \frac{f(x) - f(2)}{x - 2} \\ &= \lim_{h\to 0} \frac{f(2+h) - f(2)}{2 + h - 2} \\ &= \lim_{h\to 0} \frac{\left[5(2+h) - 4\right] - [8-2]}{h} \\ &= \lim_{h\to 0} \frac{10 + 5h - 4 - 6}{h} \\ &= 5 \end{aligned}$$

*********** END **********