$$r_1 = \frac{\left(\frac{h}{2\pi}\right)^2}{Gm_p m_e^{\ 2}}$$

$$= \frac{\left(\frac{6.63\times10^{-34}}{2\times3.14}\right)^2}{6.67\times10^{-11}\times1.67\times10^{-27}\times\left(9.1\times10^{-31}\right)^2} \approx 1.21\times10^{29}\ m$$

It is known that the universe is 156 billion light years wide or 1.5 × 10^{27} m wide. Hence, we can conclude that the radius of the first Bohr orbit is much greater than the estimated size of the whole universe.

Question 12.13:

Obtain an expression for the frequency of radiation emitted when a hydrogen atom de-excites from level n to level (n−1). For large n, show that this frequency equals the classical frequency of revolution of the electron in the orbit.

Answer

It is given that a hydrogen atom de-excites from an upper level (n) to a lower level (n−1).

We have the relation for energy (E_1) of radiation at level n as:

$$E_1 = h\nu_1 = \frac{hme^4}{(4\pi)^3 \in_0^2 \left(\frac{h}{2\pi}\right)^3}\times\left(\frac{1}{n^2}\right) \qquad \text{... (i)}$$

Where,

ν_1 = Frequency of radiation at level n

h = Planck's constant

m = Mass of hydrogen atom

e = Charge on an electron

$\in_0$ = Permittivity of free space

Now, the relation for energy (E_2) of radiation at level ($n - 1$) is givenas:

$$E_2 = h\nu_2 = \frac{hme^4}{(4\pi)^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3} \times \frac{1}{(n-1)^2} \quad \text{... (ii)}$$

Where,

ν_2 = Frequency of radiation at level $(n-1)$

Energy (E) released as a result of de-excitation:

$E = E_2 - E_1$

$h\nu = E_2 - E_1$ ... (iii)

Where,

ν = Frequency of radiation emitted

Putting values from equations (i) and (ii) in equation (iii), we get:

$$\nu = \frac{me^4}{(4\pi)^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$= \frac{me^4 (2n-1)}{(4\pi)^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3 n^2 (n-1)^2}$$

For large n, we can write $(2n-1) \simeq 2n$ and $(n-1) \simeq n$.

$$\therefore \nu = \frac{me^4}{32\pi^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3 n^3} \quad \text{... (iv)}$$

Classical relation of frequency of revolution of an electron is given as:

$$\nu_c = \frac{v}{2\pi r} \quad \text{... (v)}$$

Where,

Velocity of the electron in the n^{th} orbit is given as:

$$v = \frac{e^2}{4\pi \epsilon_0 \left(\frac{h}{2\pi}\right) n} \quad \text{... (vi)}$$

And, radius of the n^{th} orbit is given as:

$$r = \frac{4\pi \epsilon_0 \left(\frac{h}{2\pi}\right)^2}{me^2} n^2 \quad \text{... (vii)}$$

Putting the values of equations (vi) and (vii) in equation (v), we get:

$$\nu_c = \frac{me^4}{32\pi^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3 n^3} \quad \text{... (viii)}$$

Hence, the frequency of radiation emitted by the hydrogen atom is equal to its classical orbital frequency.

Question 12.14:

Classically, an electron can be in any orbit around the nucleus of an atom. Then what determines the typical atomic size? Why is an atom not, say, thousand times bigger than its typical size? The question had greatly puzzled Bohr before he arrived at his famous model of the atom that you have learnt in the text. To simulate what he might well have done before his discovery, let us play as follows with the basic constants of nature and see if we can get a quantity with the dimensions of length that is roughly equal to the known size of an atom ($\sim 10^{-10}$ m).

(a) Construct a quantity with the dimensions of length from the fundamental constants e, m_e, and c. Determine its numerical value.

(b) You will find that the length obtained in (a) is many orders of magnitude smaller than the atomic dimensions. Further, it involves c. But energies of atoms are mostly in non-relativistic domain where c is not expected to play any role. This is what may have suggested Bohr to discard c and look for 'something else' to get the right atomic size. Now, the Planck's constant h had already made its appearance elsewhere. Bohr's great insight lay in recognising that h, m_e, and e will yield the right atomic size. Construct a quantity with the dimension of length from h, m_e, and e and confirm that its numerical value has indeed the correct order of magnitude.

Answer

(a) Charge on an electron, $e = 1.6 \times 10^{-19}$ C

Mass of an electron, $m_e = 9.1 \times 10^{-31}$ kg

Speed of light, $c = 3 \times 10^8$ m/s

Let us take a quantity involving the given quantities as $\left(\frac{e^2}{4\pi \in_0 m_e c^2}\right)$.

Where,

$\in_0$ = Permittivity of free space

And, $\frac{1}{4\pi \in_0} = 9\times10^9 \text{ N m}^2 \text{ C}^{-2}$

The numerical value of the taken quantity will be:

$$\frac{1}{4\pi \in_0}\times\frac{e^2}{m_e c^2}$$

$$= 9\times10^9 \times \frac{\left(1.6\times10^{-19}\right)^2}{9.1\times10^{-31}\times\left(3\times10^8\right)^2}$$

$$= 2.81\times10^{-15} \text{ m}$$

Hence, the numerical value of the taken quantity is much smaller than the typical size of an atom.

(b) Charge on an electron, $e = 1.6 \times 10^{-19}$ C

Mass of an electron, $m_e = 9.1 \times 10^{-31}$ kg

Planck's constant, $h = 6.63 \times 10^{-34}$ Js

Let us take a quantity involving the given quantities as $\frac{4\pi \in_0 \left(\frac{h}{2\pi}\right)^2}{m_e e^2}$.

Where,

$\in_0$ = Permittivity of free space

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