Sine and Cosine Formulae and their Applications Ex-10.1 Q21

$$\frac{b\sec B+c\sec C}{\tan B+\tan C}=\frac{c\sec C+a\sec A}{\tan C+\tan A}=\frac{a\sec A+b\sec B}{\tan A+\tan B}$$

$$\frac{b\sec B+c\sec C}{\tan B+\tan C}$$

$$=\frac{k\sin B\sec B+k\sin C\sec C}{\tan B+\tan C}$$

$$=\frac{k\sin B\dfrac{1}{\cos B}+k\sin C\dfrac{1}{\cos C}}{\tan B+\tan C}$$

$$=\frac{k\tan B+k\tan C}{\tan B+\tan C}=\frac{k(\tan B+\tan C)}{\tan B+\tan C}=k$$

$$Similarly,\frac{c\sec C+a\sec A}{\tan C+\tan A}=k$$

$$Similarly,\frac{a\sec A+b\sec B}{\tan A+\tan B}=k$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q22

$a\cos A + b\cos B + c\cos C = 2b\sin A\sin C = 2c\sin A\sin B$

LHS

$a\cos A + b\cos B + c\cos C$

$= k\sin A\cos A + k\sin B\cos B + k\sin C\cos C$

$= \frac{k}{2}(\sin 2A + \sin 2B + \sin 2C)$

$= \frac{k}{2}(2\sin(A+B).\cos(A-B) + 2\sin C.\cos C)$

$= \frac{2k}{2}(\sin(\pi - C).\cos(A-B) + \sin C.\cos C)$

$= k(\sin C.\cos(A-B) + \sin C.\cos C)$

$= k\sin C(\cos(A-B) + \cos C)$

$= k\sin C.2\cos(\frac{A-B+C}{2}).\cos(\frac{A-B-C}{2})$

$= k\sin C.2\cos(\frac{\pi-2B}{2}).\cos(\frac{A-\pi+A}{2})$

$= k\sin C.2\sin B.\cos(\frac{2A-\pi}{2})$

$= k\sin C.2\sin B.\cos(\frac{\pi-2A}{2})$

$= k\sin C.2\sin B.\sin A$

$= 2\sin B\sin C(k\sin A) = 2a\sin B\sin C$

$= RHS$

Similarly, $a\cos A + b\cos B + c\cos C = 2c\sin A\sin B$

Sine and Cosine Formulae and their Applications Ex-10.1 Q23

$a(\cos B\cos C + \cos A) = b(\cos A\cos C + \cos B) = c(\cos A\cos B + \cos C)$

$a(\cos B\cos C - \cos(\pi - (B+C)))$

$= a(\cos B\cos C - \cos(B+C))$

$= a(\cos B\cos C - \cos B.\cos C + \sin B\sin C)$

$= a\sin B\sin C$

$= k\sin A\sin B\sin C$

Similarly, $b(\cos A\cos C + \cos B) = k\sin A\sin B\sin C$

Similarly, $c(\cos A\cos B + \cos C) = k\sin A\sin B\sin C$

Sine and Cosine Formulae and their Applications Ex-10.1 Q24

Let $a = k\sin A$

$$a(\cos C - \cos B) = 2(b-c)\ \cos^2\frac{A}{2}$$

LHS

$$= a(\cos C - \cos B)$$

$$= a2.\sin\frac{C+B}{2}.\sin\frac{B-C}{2}$$

$$= 2k\sin A\sin\frac{\pi - A}{2}.\sin\frac{B-C}{2}$$

$$= 2k2\sin\frac{A}{2}.\cos\frac{A}{2}.\cos\frac{A}{2}.\sin\frac{B-C}{2}$$

$$= 2k\cos^2\frac{A}{2}\left(2\sin\frac{B-C}{2}.\sin\frac{A}{2}\right)$$

$$= 2k\cos^2\frac{A}{2}\left(2\sin\frac{B-C}{2}.\sin\frac{\pi-(B+C)}{2}\right)$$

$$= 2k\cos^2\frac{A}{2}\left(2\sin\frac{B-C}{2}.\cos\frac{B+C}{2}\right)$$

$$= 2k\cos^2\frac{A}{2}(\sin B - \sin C)$$

$$= 2\cos^2\frac{A}{2}(k\sin B - k\sin C)$$

$$= 2\cos^2\frac{A}{2}(b-c) = RHS$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q25

$b\cos\theta = c\cos(A-\theta) \; + a\cos(C+\theta)$

Let $a\sin C = c\sin A$[Using sine rule]

RHS

$= c\cos(A-\theta) + a\cos(C+\theta)$

$= c\cos A\cos\theta + c\sin A\cos\theta + a\cos C.\cos\theta - a\sin C\sin\theta$

$= k\sin C\cos A.\cos\theta + k\sin C\sin A\cos\theta + k\sin A\cos C.\cos\theta$

$-k\sin A\sin C\sin\theta$

$= k\sin C\cos A.\cos\theta + k\sin A\cos C.\cos\theta$

$= k\cos\theta(\sin C\cos A + \sin A\cos C)$

$= k\cos\theta\sin(C+A)$

$= k\cos\theta\sin(\pi - B)$

$= k\cos\theta\sin B$

$= k\sin B.\cos\theta = b\cos\theta = LHS$

*********** END **********