Exercise 1.1

Q4 :Use Euclid's division lemma to show that the square of any positive integer is either of form $3m$ or $3m + 1$ for some integer m.

[Hint: Let x be any positive integer then it is of the form $3q$, $3q + 1$ or $3q + 2$. Now square each of these and show that they can be rewritten in the form $3m$ or $3m + 1$.]

Answer :

Let a be any positive integer and $b = 3$.

Then $a = 3q + r$ for some integer $q \geq 0$

And $r = 0, 1, 2$ because $0 \leq r < 3$

Therefore, $a = 3q$ or $3q + 1$ or $3q + 2$

Or,

$$a^2 = (3q)^2 \text{ or } (3q+1)^2 \text{ or } (3q+2)^2$$

$$a^2 = (9q^2) \text{ or } 9q^2 + 6q + 1 \text{ or } 9q^2 + 12q + 4$$

$$= 3 \times (3q^2) \text{ or } 3(3q^2 + 2q) + 1 \text{ or } 3(3q^2 + 4q + 1) + 1$$

$$= 3k_1 \quad \text{or} \quad 3k_2 + 1 \quad \text{or} \quad 3k_3 + 1$$

Where k_1, k_2, and k_3 are some positive integers

Hence, it can be said that the square of any positive integer is either of the form $3m$ or $3m + 1$.

Q5 :Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$.

Answer :

Let a be any positive integer and $b = 3$

$a = 3q + r$, where $q \geq 0$ and $0 \leq r < 3$

$\therefore a = 3q$ or $3q + 1$ or $3q + 2$

Therefore, every number can be represented as these three forms. There are three cases.

Case 1: When $a = 3q$,

$$a^3 = (3q)^3 = 27q^3 = 9\left(3q^3\right) = 9m,$$

Where m is an integer such that $m = 3q^3$

Case 2: When $a = 3q + 1$,

$a^3 = (3q + 1)^3$

$a^3 = 27q^3 + 27q^2 + 9q + 1$

$a^3 = 9(3q^3 + 3q^2 + q) + 1$

$a^3 = 9m + 1$

Where m is an integer such that $m = (3q^3 + 3q^2 + q)$

Case 3: When $a = 3q + 2$,

$a^3 = (3q + 2)^3$

$a^3 = 27q^3 + 54q^2 + 36q + 8$

$a^3 = 9(3q^3 + 6q^2 + 4q) + 8$

$a^3 = 9m + 8$

Where m is an integer such that $m = (3q^3 + 6q^2 + 4q)$

Therefore, the cube of any positive integer is of the form $9m$, $9m + 1$,
or $9m + 8$.

*********** END **********