Continuity Ex 9.1 Q23

We have given that the function is continuous at $x = 2$

$\text{LHL} = \text{RHL} = f(2) \ldots\ldots(1)$

Now,

$$\text{LHL} = \lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h) = \lim_{h \to 0} a(2-h) + 5 = 2a + 5$$

$$f(2) = 2a + 5$$

$$\text{RHL} = \lim_{x \to 2^+} f(x) = \lim_{h \to 0} f(2+h) = \lim_{h \to 0} 2 + h - 1 = 1$$

$\therefore$ Using (1),

$2a + 5 = 1 \Rightarrow a = -2$

Continuity Ex 9.1 Q24

We have, at $x = 0$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{-h}{|-h| + 2(-h)^2} = \lim_{h \to 0} \frac{-h}{h + 2h^2} = \lim_{h \to 0} \frac{-1}{1 + 2h} = -1$$

$$f(0) = k$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{h}{|h| + 2h^2} = \lim_{h \to 0} \frac{1}{1 + 2h} = 1$$

Since, LHL $\neq$ RHL, function will remain discontinuous at $x = 0$, regardless the choice of k.

Continuity Ex 9.1 Q25

Since $f(x)$ is continuous at $x = \frac{\pi}{2}$, L.H.Limit = R.H.Limit.

$$\Rightarrow \lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{x \to \frac{\pi}{2}^+} f(x) = \lim_{x \to \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \lim_{x \to \frac{\pi}{2}^-} \frac{k\cos x}{\pi - 2x} = 3$$

$$\Rightarrow k \lim_{x \to \frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right)}{2\left(\frac{\pi}{2} - x\right)} = 3$$

$$\Rightarrow \frac{k}{2} \lim_{x \to \frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\left(\frac{\pi}{2} - x\right)} = 3$$

$$\Rightarrow \frac{k}{2} = 3$$

$$\Rightarrow k = 6$$

Continuity Ex 9.1 Q26

We have given that the function is continuous at $x = 0$

LHL = RHL = $f(0) \ldots (1)$

$f(0) = c$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{\sin(a+1)(-h) + \sin(-h)}{-h} = \lim_{h \to 0} \frac{-\sin(ah+h) - \sinh}{-h}$$

$$= \lim_{h \to 0} \frac{\sin(a+h)h}{h} + \lim_{h \to 0} \frac{\sinh}{h}$$

$$= a + 1 + 1 = a + 2$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{\sqrt{h + bh^2} - \sqrt{h}}{bh^{\frac{3}{2}}}$$

$$= \lim_{h \to 0} \frac{\sqrt{h + bh^2} - \sqrt{h}}{bh^{\frac{3}{2}}} \times \frac{\sqrt{h + bh^2} + \sqrt{h}}{\sqrt{h + bh^2} + \sqrt{h}}$$

$$= \lim_{h \to 0} \frac{h + bh^2 - h}{bh^{\frac{3}{2}}\left(\sqrt{h + bh^2} + \sqrt{h}\right)} = \lim_{h \to 0} \frac{bh^2}{bh^2\left(\sqrt{1 + bh} + 1\right)} = \frac{1}{2}$$

$\therefore$ from (1),

$$a + 2 = \frac{1}{2} \Rightarrow a = \frac{-3}{2}$$

$c = \frac{1}{2}$ and

$b \in R - \{0\}$

Hence, $a = \frac{-3}{2}$, $b \in R - \{0\}$, $c = \frac{1}{2}$

*********** END **********