Trigonometric Identities Ex 6.1 Q78

Answer :

Given:

$$x = a\cos^3\theta$$

$$\Rightarrow \frac{x}{a} = \cos^3\theta,$$

$$x = b\sin^3\theta$$

$$\Rightarrow \frac{y}{b} = \sin^3\theta$$

We have to prove $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$

We know that $\sin^2\theta + \cos^2\theta = 1$

So, we have

$$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = \left(\cos^3\theta\right)^{2/3} + \left(\sin^3\theta\right)^{2/3}$$

$$\Rightarrow \quad \left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = \cos^2\theta + \sin^2\theta$$

$$\Rightarrow \quad \left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$$

Hence proved.

Trigonometric Identities Ex 6.1 Q79

Answer :

Given: $3\sin\theta+5\cos\theta=5$

We have to prove that $5\sin\theta-3\cos\theta=\pm3$.

We know that, $\sin^2\theta+\cos^2\theta=1$

Squaring the given equation, we have

$$(3\sin\theta+5\cos\theta)^2=(5)^2$$

$$\Rightarrow 9\sin^2\theta+2\times3\sin\theta\times5\cos\theta+25\cos^2\theta=25$$

$$\Rightarrow 9(1-\cos^2\theta)+2\times3\sin\theta\times5\cos\theta+25(1-\sin^2\theta)=25$$

$$\Rightarrow 9-9\cos^2\theta+2\times3\sin\theta\times5\cos\theta+25-25\sin^2\theta=25$$

$$\Rightarrow 34-(9\cos^2\theta-2\times3\sin\theta\times5\cos\theta+25\sin^2\theta)=25$$

$$\Rightarrow -(25\sin^2\theta-2\times5\sin\theta\times3\cos\theta+9\cos^2\theta)=-9$$

$$\Rightarrow (25\sin^2\theta-2\times5\sin\theta\times3\cos\theta+9\cos^2\theta)=9$$

$$\Rightarrow (5\sin\theta-3\cos\theta)^2=9$$

$$\Rightarrow 5\sin\theta-3\cos\theta=\pm3$$

Hence proved.

Trigonometric Identities Ex 6.1 Q80

Answer :

Given:

$a\cos\theta+b\sin\theta=m,$

$a\sin\theta-b\cos\theta=n$

We have to prove $a^2+b^2=m^2+n^2$

We know that, $\sin^2\theta+\cos^2\theta=1$

Now, squaring and adding the two equations, we get

$$(a\cos\theta+b\sin\theta)^2+(a\sin\theta-b\cos\theta)^2=m^2+n^2$$

$$\Rightarrow (a^2\cos^2\theta+2ab\sin\theta\cos\theta+b^2\sin^2\theta)+(a^2\sin^2\theta-2ab\sin\theta\cos\theta+b^2\cos^2\theta)=2$$

$$\Rightarrow a^2(\cos^2\theta+\sin^2\theta)+b^2(\sin^2\theta+\cos^2\theta)=m^2+n^2$$

$$\Rightarrow a^2+b^2=m^2+n^2$$

Hence proved.

********** END **********