



Trigonometric Identities Ex 6.1 Q40

Answer :

We need to prove $\frac{1 - \cos A}{1 + \cos A} = (\cot A - \operatorname{cosec} A)^2$

Now, rationalising the L.H.S, we get

$$\begin{aligned}\frac{1 - \cos A}{1 + \cos A} &= \left(\frac{1 - \cos A}{1 + \cos A} \right) \left(\frac{1 - \cos A}{1 - \cos A} \right) \\ &= \frac{(1 - \cos A)^2}{1 - \cos^2 A} && \text{(Using } a^2 - b^2 = (a+b)(a-b) \text{)} \\ &= \frac{1 + \cos^2 A - 2 \cos A}{\sin^2 A} && \text{(Using } \sin^2 \theta = 1 - \cos^2 \theta \text{)} \\ &= \frac{1}{\sin^2 A} + \frac{\cos^2 A}{\sin^2 A} - \frac{2 \cos A}{\sin^2 A}\end{aligned}$$

Using $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ and $\cot \theta = \frac{\cos \theta}{\sin \theta}$, we get

$$\begin{aligned}\frac{1}{\sin^2 A} + \frac{\cos^2 A}{\sin^2 A} - \frac{2 \cos A}{\sin^2 A} &= \operatorname{cosec}^2 A + \cot^2 A - 2 \cot A \operatorname{cosec} A \\ &= (\cot A - \operatorname{cosec} A)^2 && \text{(Using } (a+b)^2 = a^2 + b^2 + 2ab \text{)}\end{aligned}$$

Hence proved.

Trigonometric Identities Ex 6.1 Q41

Answer :

We need to prove $\frac{1}{\sec A - 1} + \frac{1}{\sec A + 1} = 2 \operatorname{cosec} A \cot A$

Solving the L.H.S, we get

$$\begin{aligned}\frac{1}{\sec A - 1} + \frac{1}{\sec A + 1} &= \frac{\sec A + 1 + \sec A - 1}{(\sec A - 1)(\sec A + 1)} \\ &= \frac{2 \sec A}{\sec^2 A - 1}\end{aligned}$$

Further using the property $1 + \tan^2 \theta = \sec^2 \theta$, we get

So,

$$\begin{aligned}\frac{2 \sec A}{\sec^2 A - 1} &= \frac{2 \sec A}{\tan^2 A} \\ &= \frac{2 \left(\frac{1}{\cos A} \right)}{\frac{\sin^2 A}{\cos^2 A}} \\ &= 2 \frac{1}{\cos A} \times \frac{\cos^2 A}{\sin^2 A} \\ &= 2 \left(\frac{\cos A}{\sin A} \right) \times \frac{1}{\sin A} \\ &= 2 \operatorname{cosec} A \cot A\end{aligned}$$

Hence proved.

Trigonometric Identities Ex 6.1 Q42

Answer :

We need to prove $\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A$

Solving the L.H.S, we get

$$\begin{aligned}\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} &= \frac{\cos A}{1 - \frac{\sin A}{\cos A}} + \frac{\sin A}{1 - \frac{\cos A}{\sin A}} \\&= \frac{\cos A}{\frac{\cos A - \sin A}{\cos A}} + \frac{\sin A}{\frac{\sin A - \cos A}{\sin A}} \\&= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A} \\&= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A} \\&= \frac{(\cos A + \sin A)(\cos A - \sin A)}{\cos A - \sin A} \left[\text{using } a^2 - b^2 = (a+b)(a-b) \right] \\&= \cos A + \sin A \\&= \text{RHS}\end{aligned}$$

Hence proved.

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