Indefinite Integrals Ex 19.25 Q29

Let $\quad I = \int \operatorname{cosec}^3 dx$

$\quad = \int \operatorname{cosec} x - \operatorname{cosec}^2 x dx$

Using integration by parts,

$$= \operatorname{cosec} x \times \int \operatorname{cosec}^2 x dx + \int \left(\operatorname{cosec} x \cot x \int \operatorname{cosec}^2 x dx\right) dx$$

$$= \operatorname{cosec} x \times \left(-\cot x\right) + \int \operatorname{cosec} x \cot x \left(-\cot x\right) dx$$

$$= -\operatorname{cosec} x \cot x - \int \operatorname{cosec} x \cot^2 x dx$$

$$= -\operatorname{cosec} x \cot x - \int \operatorname{cosec} x \left(\operatorname{cosec}^2 x - 1\right) dx$$

$$= -\operatorname{cosec} x \cot x - \int \operatorname{cosec}^3 x dx + \int \operatorname{cosec} x dx$$

$$I = -\operatorname{cosec} x \cot x - I + \log\left|\tan \frac{x}{2}\right| + c_1$$

$$2I = -\operatorname{cosec} x \cot x + \log\left|\tan \frac{x}{2}\right| + c_1$$

$$I = -\frac{1}{2} \operatorname{cosec} x \cot x + \frac{1}{2} \log\left|\tan \frac{x}{2}\right| + c$$

Indefinite Integrals Ex 19.25 Q30

Let $\quad I = \int \sec^{-1} \sqrt{x}\, dx$

Let $\quad \sqrt{x} = t$

$$x = t^2$$

$$dx = 2t\,dt$$

$$I = \int 2t \sec^{-1} t\, dt$$

$$= 2\left[\sec^{-1} t \int t\,dt - \int \left(\frac{1}{t\sqrt{t^2 - 1}} \int t\,dt\right) dt\right]$$

$$= 2\left[\frac{t^2}{2} \sec^{-1} - \int \left(\frac{t}{2t\sqrt{t^2 - 1}}\right) dt\right]$$

$$= t^2 \sec^{-1} t - \int \frac{t}{\sqrt{t^2 - 1}} dt$$

$$= t^2 \sec^{-1} t - \frac{1}{2} \int \frac{2t}{\sqrt{t^2 - 1}} dt$$

$$= t^2 \sec^{-1} t - \frac{1}{2} \times 2\sqrt{t^2 - 1} + c$$

$$I = x \sec^{-1} \sqrt{x} - \sqrt{x - 1} + c$$

Indefinite Integrals Ex 19.25 Q31

$$\int \sin^{-1}\sqrt{x}\ dx =$$
let $x = t^2 \rightarrow dx = 2tdt$
$$\int \sin^{-1}\sqrt{x}\ dx = \int \sin^{-1}\sqrt{t^2}\ 2tdt = \int \sin^{-1}t\ 2tdt$$
$$= \sin^{-1}t\int 2tdt - \left(\int \frac{d\sin^{-1}t}{dt}\left(\int 2tdt\right)dt\right)$$
$$= \sin^{-1}t\left(t^2\right) - \int \frac{1}{\sqrt{1-t^2}}\left(t^2\right)dt$$
Lets solve $\int \frac{1}{\sqrt{1-t^2}}\left(t^2\right)dt$
$$\int \frac{1}{\sqrt{1-t^2}}\left(t^2\right)dt = \int \frac{t^2-1+1}{\sqrt{1-t^2}}dt = \int \frac{t^2-1}{\sqrt{1-t^2}}dt + \int \frac{1}{\sqrt{1-t^2}}dt$$
we know that, value of $\int \frac{1}{\sqrt{1-t^2}}dt = \sin^{-1}t$

Remaining integral to evaluate is $\int \frac{t^2-1}{\sqrt{1-t^2}}dt = \int -\sqrt{1-t^2}dt$

sub t=sin u, dt= cos$u$ du
$$\int -\sqrt{1-t^2}dt = \int -\cos^2 udu = -\int\left[\frac{1+\cos 2u}{2}\right]du$$
$$= -\frac{u}{2} - \frac{\sin 2u}{4}$$
Substitute back u=sin^{-1}t and $t = \sqrt{x}$
$$= -\frac{\sin^{-1}\sqrt{x}}{2} - \frac{\sin(2\sin^{-1}\sqrt{x})}{4}$$
$$\int \sin^{-1}\sqrt{x}\ dx = x\sin^{-1}\sqrt{x} - \frac{\sin^{-1}\sqrt{x}}{2} - \frac{\sin(2\sin^{-1}\sqrt{x})}{4}$$
$$\sin(2\sin^{-1}\sqrt{x}) = 2\sqrt{x}\sqrt{1-x}$$
$$\int \sin^{-1}\sqrt{x}\ dx = x\sin^{-1}\sqrt{x} - \frac{\sin^{-1}\sqrt{x}}{2} - \frac{\sqrt{x(1-x)}}{2}$$

Indefinite Integrals Ex 19.25 Q32

Let $I = \int x \tan^2 x\, dx$
$$= \int x\left(\sec^2 x - 1\right)dx$$
$$= \int x \sec^2 x\, dx - \int x\, dx$$
$$= \left[x\int \sec^2 x\, dx - \int\left(1\int \sec^2 x\, dx\right)dx\right] - \frac{x^2}{2}$$
$$= x\tan x - \int \tan x\, dx - \frac{x^2}{2}$$
$$I = x\tan x - \log\sec x - \frac{x^2}{2} + c$$

Indefinite Integrals Ex 19.25 Q33

Let $I = \int x \left(\frac{\sec 2x - 1}{\sec 2x + 1}\right) dx$

$= \int x \left(\frac{1 - \cos 2x}{1 + \cos 2x}\right) dx$

$= \int x \left(\frac{\sec^2 x}{\cos^2 x}\right) dx$

$= \int x \tan^2 x dx$

$= \int x \left(\sec^2 x - 1\right) dx$

$= \int x \sec^2 x dx - \int dx$

$= \left[x \int \sec^2 x \, dx - \int \left(1 \int \sec^2 x \, dx\right) dx\right] - \frac{x^2}{2}$

$= x \tan x - \int \tan x \, dx - \frac{x^2}{2}$

$I = x \tan x - \log \sec x - \frac{x^2}{2} + c$

Indefinite Integrals Ex 19.25 Q34

Let $I = \int \left(x + 1\right) e^x \log\left(xe^x\right) dx$

Let $xe^x = t$

$\left(1 \times e^x + xe^x\right) dx = dt$

$\left(x + 1\right) e^x dx = dt$

$I = \int \log t \, dt$

$= \int 1 \times \log t \, dt$

$= \log t \int dt - \int \left(\frac{1}{t} \int dt\right) dt$

$= t \log t - \int \left(\frac{1}{t} t\right) dt$

$= t \log t - \int dt$

$= t \log t - t + c$

$= t \left(\log t - 1\right) + c$

$I = xe^x \left(\log xe^x - 1\right) + c$

*********** END **********