Indefinite Integrals Ex 19.25 Q35

Let $I = \int \sin^{-1}\left(3x - 4x^3\right)dx$

Let $x = \sin\theta$

$dx = \cos\theta\, d\theta$

$= \int \sin^{-1}\left(3\sin\theta - 4\sin^3\theta\right)\cos\theta\, d\theta$

$= \int \sin^{-1}\left(\sin 3\theta\right)\cos\theta\, d\theta$

$= \int 3\theta \cos\theta\, d\theta$

$= 3\left[\theta\int \cos\theta\, d\theta - \int\left(1\int\cos\theta\, d\theta\right)d\theta\right]$

$= 3\left[\theta\sin\theta - \int\sin\theta\, d\theta\right]$

$= 3\left[\theta\sin\theta + \cos\theta\right] + c$

$I = 3\left[x\sin^{-1}x + \sqrt{1-x^2}\right] + c$

Indefinite Integrals Ex 19.25 Q36

Let $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

$\therefore \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) = \sin^{-1}\left(\dfrac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}\left(\sin 2\theta\right) = 2\theta$

$\Rightarrow \int \sin^{-1}\left(\dfrac{2x}{1+x^2}\right)dx = \int 2\theta\cdot\sec^2\theta\, d\theta = 2\int\theta\cdot\sec^2\theta\, d\theta$

Integrating by parts, we obtain

$2\left[\theta\cdot\int\sec^2\theta d\theta - \int\left\{\left(\dfrac{d}{d\theta}\theta\right)\int\sec^2\theta d\theta\right\}d\theta\right]$

$= 2\left[\theta\cdot\tan\theta - \int\tan\theta d\theta\right]$

$= 2\left[\theta\tan\theta + \log|\cos\theta|\right] + C$

$= 2\left[x\tan^{-1}x + \log\left|\dfrac{1}{\sqrt{1+x^2}}\right|\right] + C$

$= 2x\tan^{-1}x + 2\log\left(1+x^2\right)^{-\frac{1}{2}} + C$

$= 2x\tan^{-1}x + 2\left[-\dfrac{1}{2}\log\left(1+x^2\right)\right] + C$

$= 2x\tan^{-1}x - \log\left(1+x^2\right) + C$

Indefinite Integrals Ex 19.25 Q37

Let $\quad I = \int \tan^{-1}\left(\dfrac{3x - x^3}{1 - 3x^2}\right) dx$

Let $\quad x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$I = \int \tan^{-1}\left(\dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2 x}\right)\sec^2\theta\, d\theta$

$= \int \tan^{-1}(\tan 3\theta)\sec^2\theta\, d\theta$

$= \int 3\theta \sec^2\theta\, d\theta$

$= 3\left[\theta \int \sec^2\theta\, d\theta - \int\left(1\int \sec^2\theta\, d\theta\right) d\theta\right]$

$= 3\left[\theta \tan\theta - \int \tan\theta\, d\theta\right]$

$= 3\left[\theta \tan\theta + \log\sec\theta\right] + c$

$= 3\left[x \tan^{-1} x - \log\sqrt{1+x^2}\right] + c$

$$I = 3x \tan^{-1} x - \dfrac{3}{2}\log\left|1 + x^2\right| + c$$

Indefinite Integrals Ex 19.25 Q38

Let $\quad I = \int x^2 \sin^{-1} x\, dx$

$I = \sin^{-1} x \int x^2\, dx - \int\left(\dfrac{1}{\sqrt{1-x^2}}\int x^2\, dx\right) dx$

$= \dfrac{x^3}{3}\sin^{-1} x - \int \dfrac{x^3}{3\sqrt{1-x^2}}\, dx$

$I = \dfrac{x^3}{3}\sin^{-1} x - \dfrac{1}{3}I_1 + c_1 \;------(1)$

$I_1 = \int \dfrac{x^3}{\sqrt{1-x^2}}\, dx$

Let $\quad 1 - x^2 = t^2$

$-2x\, dx = 2t\, dt$

$-x\, dx = t\, dt$

$I_1 = -\int \dfrac{\left(1 - t^2\right)t\, dt}{t}$

$= \int \left(t^2 - 1\right) dt$

$= \dfrac{t^3}{3} - t + c_2$

$= \dfrac{\left(1 - x^2\right)^{\frac{3}{2}}}{3} - \left(1 - x^2\right)^{\frac{1}{2}} + c_2$

Now,

$$I = \dfrac{x^3}{3}\sin^{-1} x - \dfrac{1}{9}\left(1 - x^2\right)^{\frac{3}{2}} + \dfrac{1}{3}\left(1 - x^2\right)^{\frac{1}{2}} + c$$

Indefinite Integrals Ex 19.25 Q39

Let $I = \int \dfrac{\sin^{-1} x}{x^2} dx$

$= \int \left(\dfrac{1}{x^2}\right)\left(\sin^{-1} x\right) dx$

$I = \left[\sin^{-1} x \int \dfrac{1}{x^2} dx - \int\left(\dfrac{1}{\sqrt{1-x^2}} \int \dfrac{1}{x^2} dx\right) dx\right]$

$= \sin^{-1} x \left(-\dfrac{1}{x}\right) - \int \dfrac{1}{\sqrt{1-x^2}}\left(-\dfrac{1}{x}\right) dx$

$I = -\dfrac{1}{x}\sin^{-1} x + \int \dfrac{1}{x\sqrt{1-x^2}} dx$

$I = -\dfrac{1}{x}\sin^{-1} x + I_1 - - - - - -(1)$

Where,

$I_1 = \int \dfrac{1}{x\sqrt{1-x^2}} dx$

Let $1 - x^2 = t^2$

$-2x \, dx = 2t \, dt$

$I_1 = \int \dfrac{x}{x^2\sqrt{1-x^2}} dx$

$= -\int \dfrac{t \, dt}{\left(1-t^2\right)\sqrt{t}}$

$= -\int \dfrac{dt}{\left(1-t^2\right)}$

$= \int \dfrac{1}{t^2 - 1} dt$

$= \dfrac{1}{2}\log\left|\dfrac{t-1}{t+1}\right|$

$= \dfrac{1}{2}\log\left|\dfrac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1}\right| + c_1$

Now,

$I = -\dfrac{\sin^{-1} x}{x} + \dfrac{1}{2}\log\left|\left(\dfrac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1}\right)\left(\dfrac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}-1}\right)\right| + c$

$= -\dfrac{\sin^{-1} x}{x} + \dfrac{1}{2}\log\left|\dfrac{\left(\sqrt{1-x^2}-1\right)^2}{1-x^2-1}\right| + c$

$= -\dfrac{\sin^{-1} x}{x} + \dfrac{1}{2}\log\left|\dfrac{\left(\sqrt{1-x^2}-1\right)^2}{-x^2}\right| + c$

$= -\dfrac{\sin^{-1} x}{x} + \log\left|\dfrac{\sqrt{1-x^2}-1}{-x}\right| + c$

$I = -\dfrac{\sin^{-1} x}{x} + \log\left|\dfrac{1-\sqrt{1-x^2}}{x}\right| + c$

********** END **********