Trigonometric Ratios of Compound Angles Ex 7.2 Q2

Let $f(\theta) = \sqrt{3}\sin\theta - \cos\theta$

Multiplying and dividing by $\sqrt{(\sqrt{3})^2 + (-1)^2}$, we get

$$f(\theta) = \sqrt{(\sqrt{3})^2 + (-1)^2}\left[\frac{\sqrt{3}\sin\theta}{\sqrt{(\sqrt{3})^2 + (-1)^2}} - \frac{\cos\theta}{\sqrt{(\sqrt{3})^2 + (-1)^2}}\right]$$

$$= \sqrt{3+1}\left[\frac{\sqrt{3}\sin\theta}{\sqrt{3+1}} - \frac{\cos\theta}{\sqrt{3+1}}\right]$$

$\Rightarrow \quad f(\theta) = 2\left[\frac{\sqrt{3}\sin\theta}{2} - \frac{\cos\theta}{2}\right] \qquad ---(i)$

$\Rightarrow \quad f(\theta) = 2\left[\frac{\sqrt{3}}{2}\times\sin\theta - \frac{1}{2}\times\cos\theta\right]$

$$= 2\left[\cos\frac{\pi}{6}\times\sin\theta - \sin\frac{\pi}{6}\times\cos\theta\right]$$

$$= 2\left[\sin\theta\times\cos\frac{\pi}{6} - \cos\theta\times\sin\frac{\pi}{6}\right]$$

$$= 2\sin\left(\theta - \frac{\pi}{6}\right) \qquad \left[\because \sin(A-B) = \sin A\cos B - \cos A\sin B\right]$$

$\Rightarrow \quad f(\theta) = 2\sin\left(\theta - \frac{\pi}{6}\right)$

Again,

$$f(\theta) = 2\left[\frac{\sqrt{3}}{2}\sin\theta - \frac{\cos\theta}{2}\right]$$

$$= -2\left[\frac{1}{2}\times\cos\theta - \frac{\sqrt{3}}{2}\times\sin\theta\right]$$

$$= -2\left[\cos\frac{\pi}{3}\times\cos\theta - \sin\frac{\pi}{3}\times\sin\theta\right]$$

$$= -2\cos\left(\frac{\pi}{3} + \theta\right)$$

Let $f(\theta) = \cos\theta - \sin\theta$

Multiplying and dividing by $\sqrt{1^2 + 1^2}$, we get

$$f(\theta) = \sqrt{1^2 + 1^2}\left[\frac{\cos\theta}{\sqrt{1^2 + 1^2}} - \frac{\sin\theta}{\sqrt{1^2 + 1^2}}\right]$$

$\Rightarrow \qquad f(\theta) = \sqrt{2}\left[\frac{\cos\theta}{\sqrt{2}} - \frac{\sin\theta}{\sqrt{2}}\right] \qquad\qquad ---(i)$

Now, $\quad f(\theta) = \sqrt{2}\left[\frac{1}{\sqrt{2}} \times \cos\theta - \frac{1}{\sqrt{2}} \times \sin\theta\right]$

$\qquad\qquad = \sqrt{2}\left[\sin\frac{\pi}{4} \times \cos\theta - \cos\frac{\pi}{4} \times \sin\theta\right]$

$\qquad\qquad = \sqrt{2}\sin\left(\frac{\pi}{4} - \theta\right) \qquad\qquad [\because \sin(A - B) = \sin A \cos B - \cos A \sin B]$

$\Rightarrow \qquad f(\theta) = \sqrt{2}\sin\left(\frac{\pi}{4} - \theta\right)$

Again,

$\qquad f(\theta) = \sqrt{2}\left[\frac{1}{\sqrt{2}} \times \cos\theta - \frac{1}{\sqrt{2}} \times \sin\theta\right]$

$\qquad\qquad = \sqrt{2}\left[\cos\frac{\pi}{4} \times \cos\theta - \sin\frac{\pi}{4} \times \sin\theta\right]$

$\qquad\qquad = \sqrt{2}\cos\left(\frac{\pi}{4} + \theta\right) \qquad\qquad [\because \cos(A + B) = \cos A \cos B - \sin A \sin B]$

$\Rightarrow \qquad f(\theta) = \sqrt{2}\cos\left(\frac{\pi}{4} + \theta\right)$

Let $f(\theta) = 24\cos\theta + 7\sin\theta$

Multiplying and dividing by $\sqrt{(24)^2 + (7)^2}$, we get

$$f(\theta) = \sqrt{(24)^2 + 7^2}\left[\frac{24\cos\theta}{\sqrt{24^2 + 7^2}} + \frac{7\sin\theta}{\sqrt{24^2 + 7^2}}\right]$$

$\qquad\qquad = \sqrt{576 + 49}\left[\frac{24\cos\theta}{\sqrt{576 + 49}} + \frac{7\sin\theta}{\sqrt{576 + 49}}\right]$

$\qquad\qquad = \sqrt{625}\left[\frac{24\cos\theta}{\sqrt{625}} + \frac{7\sin\theta}{\sqrt{625}}\right]$

$\qquad\qquad = 25\left[\frac{24}{25} \times \cos\theta + \frac{7}{25} \times \sin\theta\right]$

$\Rightarrow \qquad f(\theta) = 25\left[\frac{24}{25} \times \cos\theta + \frac{7}{25} \times \sin\theta\right] \qquad\qquad ---(i)$

Now, $\quad f(\theta) = 25\left[\frac{24}{25} \times \cos\theta + \frac{7}{25} \times \sin\theta\right]$

$\qquad\qquad = 25\left[\sin\alpha \times \cos\theta + \cos\alpha \times \sin\theta\right]$

$\qquad\qquad\qquad\qquad$ where $\sin\alpha = \frac{24}{25}$ and $\cos\alpha = \frac{7}{25}$

$\Rightarrow \qquad f(\theta) = 25\sin(\alpha + \theta)$, where $\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{24}{7}$

Again,

$\qquad f(\theta) = 25\left[\frac{24}{25} \times \cos\theta + \frac{7}{25} \times \sin\theta\right]$

$\qquad\qquad = 25\left[\cos\alpha \times \cos\theta + \sin\alpha \times \sin\theta\right]$, where $\cos\alpha = \frac{24}{25}$ and $\sin\alpha = \frac{7}{25}$

$\Rightarrow \qquad f(\theta) = 25\cos(\alpha - \theta)$, where $\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{7}{24}$

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