Answer

It is given that $\vec{a}\cdot\vec{a}=0$ and $\vec{a}\cdot\vec{b}=0$.

Now,

$$\vec{a}\cdot\vec{a}=0\Rightarrow|\vec{a}|^2=0\Rightarrow|\vec{a}|=0$$

$\therefore \vec{a}$ is a zero vector.

Hence, vector $\vec{b}$ satisfying $\vec{a}\cdot\vec{b}=0$ can be any vector.

Question 14:

If either vector $\vec{a}=\vec{0}$ or $\vec{b}=\vec{0}$, then $\vec{a}\cdot\vec{b}=0$. But the converse need not be true. Justify your answer with an example.

Answer

Consider $\vec{a}=2\hat{i}+4\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}+3\hat{j}-6\hat{k}$.

Then,

$$\vec{a}\cdot\vec{b}=2.3+4.3+3(-6)=6+12-18=0$$

We now observe that:

$$|\vec{a}|=\sqrt{2^2+4^2+3^2}=\sqrt{29}$$

$$\therefore \vec{a}\neq\vec{0}$$

$$|\vec{b}|=\sqrt{3^2+3^2+(-6)^2}=\sqrt{54}$$

$$\therefore \vec{b}\neq\vec{0}$$

Hence, the converse of the given statement need not be true.

Question 15:

If the vertices A, B, C of a triangle ABC are (1, 2, 3), (−1, 0, 0), (0, 1, 2), respectively, then find □ABC. [□ABC is the angle between the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$]

Answer

The vertices of ΔABC are given as A (1, 2, 3), B (−1, 0, 0), and C (0, 1, 2).

Also, it is given that □ABC is the angle between the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$.

$$\overrightarrow{BA}=\{1-(-1)\}\hat{i}+(2-0)\hat{j}+(3-0)\hat{k}=2\hat{i}+2\hat{j}+3\hat{k}$$

$$\overrightarrow{BC}=\{0-(-1)\}\hat{i}+(1-0)\hat{j}+(2-0)\hat{k}=\hat{i}+\hat{j}+2\hat{k}$$

$$\therefore \overrightarrow{BA}\cdot\overrightarrow{BC}=\left(2\hat{i}+2\hat{j}+3\hat{k}\right)\cdot\left(\hat{i}+\hat{j}+2\hat{k}\right)=2\times1+2\times1+3\times2=2+2+6=10$$

$$\left|\overrightarrow{BA}\right|=\sqrt{2^2+2^2+3^2}=\sqrt{4+4+9}=\sqrt{17}$$

$$\left|\overrightarrow{BC}\right|=\sqrt{1+1+2^2}=\sqrt{6}$$

Now, it is known that:

$$\overrightarrow{BA}\cdot\overrightarrow{BC}=\left|\overrightarrow{BA}\right|\left|\overrightarrow{BC}\right|\cos(\angle ABC).$$

$$\therefore 10=\sqrt{17}\times\sqrt{6}\cos(\angle ABC)$$

$$\Rightarrow \cos(\angle ABC)=\frac{10}{\sqrt{17}\times\sqrt{6}}$$

$$\Rightarrow \angle ABC=\cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$$

Question 16:

Show that the points A (1, 2, 7), B (2, 6, 3) and C (3, 10, −1) are collinear.

Answer

The given points are A (1, 2, 7), B (2, 6, 3), and C (3, 10, −1).

$$\therefore \overrightarrow{AB}=(2-1)\hat{i}+(6-2)\hat{j}+(3-7)\hat{k}=\hat{i}+4\hat{j}-4\hat{k}$$

$$\overrightarrow{BC}=(3-2)\hat{i}+(10-6)\hat{j}+(-1-3)\hat{k}=\hat{i}+4\hat{j}-4\hat{k}$$

$$\overrightarrow{AC}=(3-1)\hat{i}+(10-2)\hat{j}+(-1-7)\hat{k}=2\hat{i}+8\hat{j}-8\hat{k}$$

$$\left|\overrightarrow{AB}\right|=\sqrt{1^2+4^2+(-4)^2}=\sqrt{1+16+16}=\sqrt{33}$$

$$\left|\overrightarrow{BC}\right|=\sqrt{1^2+4^2+(-4)^2}=\sqrt{1+16+16}=\sqrt{33}$$

$$\left|\overrightarrow{AC}\right|=\sqrt{2^2+8^2+8^2}=\sqrt{4+64+64}=\sqrt{132}=2\sqrt{33}$$

$\therefore \left|\overrightarrow{AC}\right| = \left|\overrightarrow{AB}\right| + \left|\overrightarrow{BC}\right|$

Hence, the given points A, B, and C are collinear.

Question 17:

Show that the vectors $2\hat{i} - \hat{j} + \hat{k},\ \hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the vertices of a right angled triangle.

Answer

Let vectors $2\hat{i} - \hat{j} + \hat{k},\ \hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ be position vectors of points A, B, and C respectively.

i.e., $\overrightarrow{OA} = 2\hat{i} - \hat{j} + \hat{k},\ \overrightarrow{OB} = \hat{i} - 3\hat{j} - 5\hat{k}$ and $\overrightarrow{OC} = 3\hat{i} - 4\hat{j} - 4\hat{k}$

Now, vectors $\overrightarrow{AB}, \overrightarrow{BC}$, and $\overrightarrow{AC}$ represent the sides of ΔABC.

i.e., $\overrightarrow{OA} = 2\hat{i} - \hat{j} + \hat{k},\ \overrightarrow{OB} = \hat{i} - 3\hat{j} - 5\hat{k}$, and $\overrightarrow{OC} = 3\hat{i} - 4\hat{j} - 4\hat{k}$

$$\therefore \overrightarrow{AB} = (1-2)\hat{i} + (-3+1)\hat{j} + (-5-1)\hat{k} = -\hat{i} - 2\hat{j} - 6\hat{k}$$

$$\overrightarrow{BC} = (3-1)\hat{i} + (-4+3)\hat{j} + (-4+5)\hat{k} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\overrightarrow{AC} = (2-3)\hat{i} + (-1+4)\hat{j} + (1+4)\hat{k} = -\hat{i} + 3\hat{j} + 5\hat{k}$$

$$\left|\overrightarrow{AB}\right| = \sqrt{(-1)^2 + (-2)^2 + (-6)^2} = \sqrt{1+4+36} = \sqrt{41}$$

$$\left|\overrightarrow{BC}\right| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{4+1+1} = \sqrt{6}$$

$$\left|\overrightarrow{AC}\right| = \sqrt{(-1)^2 + 3^2 + 5^2} = \sqrt{1+9+25} = \sqrt{35}$$

$$\therefore \left|\overrightarrow{BC}\right|^2 + \left|\overrightarrow{AC}\right|^2 = 6 + 35 = 41 = \left|\overrightarrow{AB}\right|^2$$

Hence, ΔABC is a right-angled triangle.

Question 18:

If $\vec{a}$ is a nonzero vector of magnitude '*a*' and λ a nonzero scalar, then $\lambda\vec{a}$ is unit vector if

(A) λ = 1 (B) λ = −1 (C) $a = |\lambda|$

(D) $a = \dfrac{1}{|\lambda|}$

Answer

Vector $\lambda\vec{a}$ is a unit vector if $|\lambda\vec{a}| = 1$.

Now,

$$|\lambda\vec{a}| = 1$$

$$\Rightarrow |\lambda||\vec{a}| = 1$$

$$\Rightarrow |\vec{a}| = \frac{1}{|\lambda|} \qquad [\lambda \neq 0]$$

$$\Rightarrow a = \frac{1}{|\lambda|} \qquad [|\vec{a}| = a]$$

Hence, vector $\lambda\vec{a}$ is a unit vector if $a = \dfrac{1}{|\lambda|}$.

The correct answer is D.

Exercise 10.4

Question 1:

Find $\left|\vec{a} \times \vec{b}\right|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

Answer

We have,

$\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$$

$$= \hat{i}(-14+14) - \hat{j}(2-21) + \hat{k}(-2+21) = 19\hat{j} + 19\hat{k}$$

$$\therefore \left|\vec{a} \times \vec{b}\right| = \sqrt{(19)^2 + (19)^2} = \sqrt{2 \times (19)^2} = 19\sqrt{2}$$

Question 2:

Find a unit vector perpendicular to each of the vector $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, where $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$.

Answer

We have,

$\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

$$\therefore \vec{a} + \vec{b} = 4\hat{i} + 4\hat{j},\ \vec{a} - \vec{b} = 2\hat{i} + 4\hat{k}$$

$$(a+b)\times(a-b)=\begin{vmatrix} 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix}=i(16)-j(16)+k(-8)=16i-16j-8k$$

$$\therefore \left|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})\right|=\sqrt{16^2+(-16)^2+(-8)^2}$$

$$=\sqrt{2^2\times 8^2+2^2\times 8^2+8^2}$$

$$=8\sqrt{2^2+2^2+1}=8\sqrt{9}=8\times 3=24$$

Hence, the unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ is given by the relation,

$$=\pm\frac{(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})}{\left|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})\right|}=\pm\frac{16\hat{i}-16\hat{j}-8\hat{k}}{24}$$

$$=\pm\frac{2\hat{i}-2\hat{j}-\hat{k}}{3}=\pm\frac{2}{3}\hat{i}\mp\frac{2}{3}\hat{j}\mp\frac{1}{3}\hat{k}$$

Question 3:

If a unit vector $\vec{a}$ makes an angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find θ and hence, the compounds of $\vec{a}$.

Answer

Let unit vector $\vec{a}$ have (a_1, a_2, a_3) components.

□ $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

Since $\vec{a}$ is a unit vector, $|\vec{a}|=1$.

Also, it is given that $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$, and an acute angle θ with $\hat{k}$.

Then, we have:

$$\cos\frac{\pi}{3}=\frac{a_1}{|\vec{a}|}$$

$$\Rightarrow \frac{1}{2}=a_1 \qquad \left[|\vec{a}|=1\right]$$

$$\cos\frac{\pi}{4}=\frac{a_2}{|\vec{a}|}$$

$$\Rightarrow \frac{1}{\sqrt{2}}=a_2 \qquad \left[|\vec{a}|=1\right]$$

$$\text{Also, } \cos\theta=\frac{a_3}{|\vec{a}|}.$$

$$\Rightarrow a_3=\cos\theta$$

Now,

$$|a|=1$$

$$\Rightarrow \sqrt{a_1^2+a_2^2+a_3^2}=1$$

$$\Rightarrow \left(\frac{1}{2}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2+\cos^2\theta=1$$

$$\Rightarrow \frac{1}{4}+\frac{1}{2}+\cos^2\theta=1$$

$$\Rightarrow \frac{3}{4}+\cos^2\theta=1$$

$$\Rightarrow \cos^2\theta=1-\frac{3}{4}=\frac{1}{4}$$

$$\Rightarrow \cos\theta=\frac{1}{2}\Rightarrow\theta=\frac{\pi}{3}$$

$$\therefore a_3=\cos\frac{\pi}{3}=\frac{1}{2}$$

*********** END **********