

Mathematical Induction Ex 12.2 Q27

Let $P(n): 11^{n+2} + 12^{2n+1}$ is divisible by 133

For n = 1 $11^3 + 12^3$ = 1331 + 1728

= 3059

it is divisible of 133

$$\Rightarrow$$
 $P(n)$ is true for $n = 1$
Let $P(n)$ is true for $n = k$, so

$$11^{k+2} + 12^{2k+1}$$
 is divisible by 133
$$11^{k+2} + 12^{2k+1} = 133\lambda \qquad \qquad ---(1)$$

We have to show that,

$$11^{k+3} + 12^{2k+3}$$
 is divisible by 133

Now,

$$\begin{aligned} &11^{k+2}.11+12^{2k+1}.12^2\\ &=\left(133\lambda-12^{2k+1}\right)11+12^{2k+1}.144\\ &=11.133\lambda-11.12^{2k+1}+144.12^{2k+1}\\ &=11.133\lambda+133.12^{2k+1}\\ &=133\left(11\lambda+12^{2k+1}\right) \end{aligned}$$

 $= 133 \mu$

$$\Rightarrow$$
 P(n) is true for $n = k + 1$

$$\Rightarrow$$
 $P(n)$ is true for all $n \in N$ by PMI

Mathematical Induction Ex 12.2 Q28

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Consider equation
1 \times 1! + 2 \times 2! + 3 \times 3! + \dots + n \times n!
Lets take (n+1)!-n! = n! (n+1-1)=n \times n!
Now substitue n=1,2,3,4,...n in above equation we get
2!-1!=1\times 1!
31-21=2\times 21
4!-3! = 3 \times 3!
(n+1)!-n!=n\times n!
Adding all the above terms gives
1 \times 1! + 2 \times 2! + 3 \times 3! + \dots + n! = 2! - 1! + 3! - 2! + 4! - 3! + \dots + (n+1)! - n!
1 \times 1! + 2 \times 2! + 3 \times 3! + \dots + n! = (n+1)! - 1
Mathematical Induction Ex 12.2 Q29
Let P(n) be the statement given by
P(n): n^3 - 7n + 3 is divisible by 3.
Step I:
P(1): 1^3 - 7(1) + 3 is divisible by 3
\therefore 1 - 7 + 3 = -3 is divisible by 3
∴ P(1) is true.
Step II:
Let P(m) is true. Then,
m<sup>3</sup> - 7m + 3 is divisible by 3
\Rightarrow m<sup>3</sup> - 7m + 3 = 3\lambda for some \lambda \in N ....(i)
We have to prove that P(m+1) is true.
(m+1)^3 - 7(m+1) + 3 = m^3 + 3m^2 + 3m + 1 - 7m - 7 + 3
                       = m^3 - 7m + 3 + 3m^2 + 3m + 1 - 7
                       =(m^3-7m+3)+3(m^2+m-2)
                       = 3\lambda + 3(m^2 + m - 2)...[Using (i)]
                       = 3[\lambda + (m^2 + m - 2)] which is divisible by 3
\Rightarrow P(m+1) is true.
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Hence by the principle of mathematical induction, the given result is true for all $n \in \mathbb{N}$.

********* END ********