Solutions Of Geometric Progressions Ex 20.1 Q 11

$5, 10, 20, \ldots n$ term
$1280, 640, 320, \ldots, n$ terms.

Let t_n be the general term if first G.P and t_n' be general term of record G.P whose nth terms are equal.

a for first G.P = 5

a for second G.P = 1280

r for first G.P = $\frac{10}{5} = 2$

r for second G.P = $\frac{640}{1280} = \frac{1}{2}$

$t_n = ar^{n-1}$

Applying and equating for both G.P_n'

$$(5)(2)^{n-1} = 1280\left(\frac{1}{2}\right)^{n-1}$$

$$(2)^{n-1} = \frac{1280}{5}\left(\frac{1}{2}\right)^{n-1}$$

$$= 256\left(\frac{1}{2}\right)^{n-1}$$

$$= 2^8\left(\frac{1}{2}\right)^{n-1}$$

$$\frac{(2)^{n-1}}{28} = \left(\frac{1}{2}\right)^{n-1} = 2^{n-1} = 2^{-n+1}$$

$\Rightarrow$ $2n = 10$

$n = 5$

Solutions Of Geometric Progressions Ex 20.1 Q 12

We have

$$\left(a^2+b^2+c^2\right)p^2 - 2\left(ab+bc+cd\right)p + \left(b^2+c^2+d^2\right) \le 0$$

$$\left(a^2p^2 - 2abp + b^2\right) + \left(b^2p^2 - 2bcp + c^2\right) + \left(c^2p^2 - 2cdp + d^2\right) \le 0$$

$$\left(ap-b\right)^2 + \left(bp-c\right)^2 + \left(cp-d\right)^2 \le 0$$

This is only possible when

$$ap - b = 0 \Rightarrow p = \frac{b}{a}$$

$$bp - c = 0 \Rightarrow p = \frac{c}{b}$$

$$cp - d = 0 \Rightarrow p = \frac{d}{c}$$

Thus

$$\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

Hence a, b, c and d are in G.P

Solutions Of Geometric Progressions Ex 20.1 Q 13

$\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$, two show that a, b, c, d are in G.P

$\Rightarrow$ to show $\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$ ---(i)

Now,

$$\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx} \text{ and } \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$$

Cross multiplying

$$(a+bx)(b-cx) = (b+cx)(a-bx)$$
$$ab - acx + b^2x - bcx^2 = ab - b^2x + acx - bcx^2$$

Cancelling ab and $-bcx^2$ on both sides

$$-acx + b^2x = -b^2x + acx$$
$$x\left(b^2 - ac\right) = -x\left(b^2 - ac\right)$$
$$2b^2x = 2acx$$
$$2b^2 = 2ac = b^2 = ac$$

From (i) $b^2 = ac$

Also,

$$\frac{cx+b}{b-cx} = \frac{c+dx}{c-dx}, \text{ cross multiplying}$$

$$c^2x - cdx^2 + bc - bdx = bc + bdx - c^2x - cdx^2$$
$$2c^2x = 2bdx$$

From (i) $c^2 = bd$

Hence, a, b, c, d are in G.P.

Solutions Of Geometric Progressions Ex 20.1 Q 14

We have

$a_5 = p$
$a_8 = q$
$a_{11} = s$

We have to show that

$$q^2 = ps$$
$$\Rightarrow \frac{q}{p} = \frac{s}{q}$$

Now, $q = ar^7$
$p = ar^4$
$s = ar^{10}$

$$\therefore \frac{q}{p} = \frac{s}{q}$$

$$\Rightarrow \frac{ar^7}{ar^4} = \frac{ar^{10}}{ar^7}$$
$$\Rightarrow r^3 = r^3$$

Hence proved.

*********** END **********