Indefinite Integrals Ex 19.9 Q15

Let $\quad I = \int \dfrac{1}{\sqrt{\tan^{-1} x}\ \left(1 + x^2\right)}\, dx\ -----\,(i)$

Let $\quad \tan^{-1} x = t \quad$ then,

$\qquad d\left(\tan^{-1} x\right) = dt$

$\Rightarrow \qquad \dfrac{1}{1 + x^2}\, dx = dt$

Putting $\quad \tan^{-1} x = t \quad$ and $\quad \dfrac{1}{1 + x^2}\, dx = dt \quad$ in equation (i), we get

$\qquad I = \int \dfrac{1}{\sqrt{t}}\, dt$

$\qquad = \int t^{-\frac{1}{2}}\, dt$

$\qquad = 2t^{\frac{1}{2}} + c$

$\qquad = 2\sqrt{\tan^{-1} x} + c$

$\therefore \qquad I = 2\sqrt{\tan^{-1} x} + c$

Indefinite Integrals Ex 19.9 Q16

Let $I = \displaystyle\int \dfrac{\sqrt{\tan x}}{\sin x \cos x}\, dx$

$\qquad = \displaystyle\int \dfrac{\sqrt{\tan x} \times \cos x}{\sin x \cos x \times \cos x}\, dx$

$\qquad = \displaystyle\int \dfrac{\sqrt{\tan x}}{\tan x \cos^2 x}\, dx$

$\qquad = \displaystyle\int \dfrac{\sec^2 x\, dx}{\sqrt{\tan x}}$

Let $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$\therefore I = \displaystyle\int \dfrac{dt}{\sqrt{t}}$

$\qquad = 2\sqrt{t} + C$

$\qquad = 2\sqrt{\tan x} + C$

Indefinite Integrals Ex 19.9 Q17

Let $\quad I = \int \dfrac{1}{x} \left(\log x\right)^2 dx \; ----- \; (i)$

Let $\quad \log x = t \qquad$ then,

$\qquad d\left(\log x\right) = dt$

$\Rightarrow \qquad \dfrac{1}{x} dx = dt$

Putting $\log x = t$ and $\dfrac{1}{x} dx = dt$ in equation (i), we get

$\qquad I = \int t^2 dt$

$\qquad = \dfrac{t^3}{3} + c$

$\qquad = \dfrac{\left(\log x\right)^3}{3} + c$

$\therefore \qquad I = \dfrac{1}{3}\left(\log x\right)^3 + c$

Indefinite Integrals Ex 19.9 Q18

Let $\quad I = \int \sin^5 x \cos x \, dx \; ----- \; (i)$

Let $\quad \sin x = t \qquad$ then,

$\qquad d\left(\sin x\right) = dt$

$\Rightarrow \qquad \cos x \, dx = dt$

Putting $\sin x = t$ and $\cos x \, dx = dt$ in equation (i), we get

$\qquad I = \int t^5 dt$

$\qquad = \dfrac{t^6}{6} + c$

$\qquad = \dfrac{\sin^6 x}{6} + c$

$\therefore \qquad I = \dfrac{1}{6}\sin^6 x + c$

Indefinite Integrals Ex 19.9 Q19

Let $\quad I = \int \tan^{\frac{3}{2}} x \sec^2 x \, dx$ ----- (i)

Let $\quad \tan x = t \quad$ then,

$\qquad d(\tan x) = dt$

$\Rightarrow \quad \sec^2 x \, dx = dt$

Putting $\tan x = t$ and $\sec^2 x \, dx = dt$ in equation (i), we get

$$I = \int t^{\frac{3}{2}} dt$$

$$= \frac{2}{5} t^{\frac{5}{2}} + c$$

$$= \frac{2}{5} (\tan x)^{\frac{5}{2}} + c$$

$\therefore \qquad I = \frac{2}{5} \tan^{\frac{5}{2}} x + c$

********** END *********