Indefinite Integrals Ex 19.23 Q1

Let $\quad I = \int \frac{1}{5 + 4\cos x} dx$

Put $\quad \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$I = \int \frac{1}{5 + 4\left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right)} dx$$

$$= \int \frac{1 + \tan^2 \frac{x}{2}}{5\left(1 + \tan^2 \frac{x}{2}\right) + 4\left(1 - \tan^2 \frac{x}{2}\right)} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{5 + 5\tan^2 \frac{x}{2} + 4 - 4\tan^2 \frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{9 + \tan^2 \frac{x}{2}} dx$$

Let $\quad \tan \frac{x}{2} = t$

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

$$= \int \frac{2dt}{(3)^2 + t^2}$$

$$= 2 \times \frac{1}{3} \tan^{-1}(t) + c$$

$$I = \frac{2}{3} \tan^{-1}\left(\frac{\tan \frac{x}{2}}{3}\right) + c$$

Indefinite Integrals Ex 19.23 Q2

Let $\quad I = \int \frac{1}{5-4\sin x}dx$

Put $\quad \sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$= \int \frac{1}{5-4\left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx$$

$$= \int \frac{1+\tan^2\frac{x}{2}}{5\left(1+\tan^2\frac{x}{2}\right)-4\left(2\tan\frac{x}{2}\right)}dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{5+5\tan^2\frac{x}{2}-8\tan\frac{x}{2}}dx$$

Let $\quad \tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2}dx = dt$$

$$= \int \frac{2dt}{5t^2-8t+5}$$

$$= \frac{2}{5}\int \frac{dt}{t^2-\frac{8}{5}t+1}$$

$$= \frac{2}{5}\int \frac{dt}{t^2-2t\left(\frac{4}{5}\right)+\left(\frac{4}{5}\right)^2-\left(\frac{4}{5}\right)^2+1}$$

$$I = \frac{2}{5}\int \frac{dt}{\left(t-\frac{4}{5}\right)^2+\left(\frac{3}{5}\right)^2}$$

$$= \frac{2}{5}\times\frac{1}{\frac{3}{5}}\tan^{-1}\left(\frac{t-\frac{4}{5}}{\frac{3}{5}}\right)+c$$

$$= \frac{2}{3}\tan^{-1}\left(\frac{5t-4}{3}\right)+c$$

$$I = \frac{2}{3}\tan^{-1}\left(\frac{5\tan\frac{x}{2}-4}{3}\right)+c$$

Indefinite Integrals Ex 19.23 Q3

Let $\quad I = \int \frac{1}{1-2\sin x}dx$

Put $\quad \sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$I = \int \frac{1}{1-2\left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx$$

$$= \int \frac{1+\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}-4\tan\frac{x}{2}}dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{\tan^2\frac{x}{2}-4\tan\frac{x}{2}+1}dx$$

Let $\quad \tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2}dx = dt$$

$$I = \int \frac{2dt}{t^2-4t+1}$$

$$= \int \frac{2dt}{t^2-2t(2)+(2)^2-(2)^2+1}$$

$$I = 2\int \frac{dt}{(t-2)^2+3}$$

$$= 2\int \frac{dt}{(t-2)^2+\left(\sqrt{3}\right)^2}$$

$$= 2\times\frac{1}{2\sqrt{3}}\log\left|\frac{t-2-\sqrt{3}}{t-2+\sqrt{3}}\right|+c$$

$$I = \frac{1}{\sqrt{3}}\log\left|\frac{\tan\frac{x}{2}-2-\sqrt{3}}{\tan\frac{x}{2}-2+\sqrt{3}}\right|+c$$

Indefinite Integrals Ex 19.23 Q4

Let $I = \int \frac{1}{4\cos x - 1}\,dx$

Put $\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$I = \int \frac{1}{4\left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right) - 1}\,dx$$

$$= \int \frac{1 + \tan^2 \frac{x}{2}}{4\left(1 - \tan^2 \frac{x}{2}\right) - \left(1 + \tan^2 \frac{x}{2}\right)}\,dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{4 - 4\tan^2 \frac{x}{2} - 1 - \tan^2 \frac{x}{2}}\,dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{3 - 5\tan^2 \frac{x}{2}}\,dx$$

Let $\sqrt{5}\tan\frac{x}{2} = t$

$$\frac{\sqrt{5}}{2}\sec^2 \frac{x}{2}\,dt = dt$$

$$I = \int \frac{dt}{\left(\sqrt{3}\right)^2 - t^2}$$

$$I = \frac{1}{\sqrt{15}}\log\left|\frac{\sqrt{3} + \sqrt{5}\tan\frac{x}{2}}{\sqrt{3} - \sqrt{5}\tan\frac{x}{2}}\right| + c$$

*********** END **********