Polynomials Ex 2.1 Q11

Answer :

Since α and β are the zeros of the quadratic polynomial $f(x) = x^2 - px + q$

$$\alpha + \beta = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$= \frac{-(-p)}{1}$$

$$= p$$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$= \frac{q}{1}$$

$$= q$$

We have,

$$\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{\alpha^2 \times \alpha^2}{\beta^2 \times \alpha^2} + \frac{\beta^2 \times \beta^2}{\alpha^2 \times \beta^2}$$

$$\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{\alpha^4}{\beta^2\alpha^2} + \frac{\beta^4}{\alpha^2\beta^2}$$

$$\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{\alpha^4 + \beta^4}{\alpha^2\beta^2}$$

$$\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{\left(\alpha^2 + \beta^2\right)^2 - 2\alpha^2\beta^2}{\alpha^2\beta^2}$$

$$\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}=\frac{\left[(\alpha+\beta)^2-2\alpha\beta\right]^2-2(\alpha\beta)^2}{(\alpha\beta)^2}$$

$$\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}=\frac{\left[(p)^2-2q\right]^2-2(q)^2}{q^2}$$

$$\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}=\frac{\left[p^2-2q\right]^2-2q^2}{q^2}$$

$$\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}=\frac{\left[p^2\times p^2-2\times p^2\times 2q+2q\times 2q\right]-2q^2}{q^2}$$

$$\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}=\frac{\left[p^4-4p^2q+4q^2\right]-2q^2}{q^2}$$

$$\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}=\frac{p^4-4p^2q+4q^2-2q^2}{q^2}$$

$$\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}=\frac{p^4-4p^2q+2q^2}{q^2}$$

$$\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}=\frac{p^4}{q^2}-\frac{4p^2q}{q^2}+\frac{2q^2}{q^2}$$

$$\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}=\frac{p^4}{q^2}-\frac{4p^2\cancel{q}}{q^{\cancel{2}}{}_1}+\frac{2\cancel{q^2}}{\cancel{q^2}}$$

$$\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}=\frac{p^4}{q^2}-\frac{4p^2}{q}+2$$

Hence, it is proved that $\frac{\alpha^2}{\beta^2}+\frac{\beta^2}{\alpha^2}$ is equal to $\boxed{\frac{p^4}{q^2}-\frac{4p^2}{q}+2}$.

*********** END **********