



$$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} - 5\hat{k})$$

**Question 8:**

Find the equation of the plane passing through  $(a, b, c)$  and parallel to the plane

$$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$$

**Answer**

Any plane parallel to the plane,  $\vec{r}_1 \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$ , is of the form

$$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = \lambda \quad \dots(1)$$

The plane passes through the point  $(a, b, c)$ . Therefore, the position vector  $\vec{r}$  of this point is  $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$

Therefore, equation (1) becomes

$$(a\hat{i} + b\hat{j} + c\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = \lambda$$

$$\Rightarrow a + b + c = \lambda$$

Substituting  $\lambda = a + b + c$  in equation (1), we obtain

$$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = a + b + c \quad \dots(2)$$

This is the vector equation of the required plane.

Substituting  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$  in equation (2), we obtain

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = a + b + c$$

$$\Rightarrow x + y + z = a + b + c$$

**Question 9:**

Find the shortest distance between lines  $\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$

and  $\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$ .

**Answer**

The given lines are

$$\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k}) \quad \dots(1)$$

$$\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k}) \quad \dots(2)$$

It is known that the shortest distance between two lines,  $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$ , is given by

$$d = \frac{|\left(\vec{b}_1 \times \vec{b}_2\right) \cdot (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}_1 \times \vec{b}_2|} \quad \dots(3)$$

Comparing  $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$  to equations (1) and (2), we obtain

$$\vec{a}_1 = 6\hat{i} + 2\hat{j} + 2\hat{k}$$

$$\vec{b}_1 = \hat{i} - 2\hat{j} + 2\hat{k}$$

$$\vec{a}_2 = -4\hat{i} - \hat{k}$$

$$\vec{b}_2 = 3\hat{i} - 2\hat{j} - 2\hat{k}$$

$$\Rightarrow \vec{a}_2 - \vec{a}_1 = (-4\hat{i} - \hat{k}) - (6\hat{i} + 2\hat{j} + 2\hat{k}) = -10\hat{i} - 2\hat{j} - 3\hat{k}$$

$$\Rightarrow \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix} = (4+4)\hat{i} - (-2-6)\hat{j} + (-2+6)\hat{k} = 8\hat{i} + 8\hat{j} + 4\hat{k}$$

$$\therefore |\vec{b}_1 \times \vec{b}_2| = \sqrt{(8)^2 + (8)^2 + (4)^2} = 12$$

$$(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = (8\hat{i} + 8\hat{j} + 4\hat{k}) \cdot (-10\hat{i} - 2\hat{j} - 3\hat{k}) = -80 - 16 - 12 = -108$$

Substituting all the values in equation (3), we obtain

$$d = \frac{|-108|}{12} = 9$$

Therefore, the shortest distance between the two given lines is 9 units.

Therefore, the shortest distance between the two given lines is 9 units.

**Question 10:**

Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the YZ-plane

Answer

It is known that the equation of the line passing through the points,  $(x_1, y_1, z_1)$  and  $(x_2,$

$y_2, z_2)$ , is  $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

The line passing through the points, (5, 1, 6) and (3, 4, 1), is given by,

$$\begin{aligned}\frac{x-5}{3-5} &= \frac{y-1}{4-1} = \frac{z-6}{1-6} \\ \Rightarrow \frac{x-5}{-2} &= \frac{y-1}{3} = \frac{z-6}{-5} = k \text{ (say)} \\ \Rightarrow x &= 5-2k, y=3k+1, z=6-5k\end{aligned}$$

Any point on the line is of the form  $(5-2k, 3k+1, 6-5k)$ .

The equation of YZ-plane is  $x=0$

Since the line passes through YZ-plane,

$$5-2k=0$$

$$\Rightarrow k = \frac{5}{2}$$

$$\Rightarrow 3k+1 = 3 \times \frac{5}{2} + 1 = \frac{17}{2}$$

$$6-5k = 6-5 \times \frac{5}{2} = \frac{-13}{2}$$

Therefore, the required point is  $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$ .

**Question 11:**

Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the ZX-plane.

Answer

It is known that the equation of the line passing through the points,  $(x_1, y_1, z_1)$  and  $(x_2,$

$y_2, z_2)$ , is  $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

The line passing through the points, (5, 1, 6) and (3, 4, 1), is given by,

$$\begin{aligned}\frac{x-5}{3-5} &= \frac{y-1}{4-1} = \frac{z-6}{1-6} \\ \Rightarrow \frac{x-5}{-2} &= \frac{y-1}{3} = \frac{z-6}{-5} = k \text{ (say)} \\ \Rightarrow x &= 5-2k, y=3k+1, z=6-5k\end{aligned}$$

Any point on the line is of the form  $(5-2k, 3k+1, 6-5k)$ .

Since the line passes through ZX-plane,

$$3k+1=0$$

$$\Rightarrow k = -\frac{1}{3}$$

$$\Rightarrow 5-2k = 5-2\left(-\frac{1}{3}\right) = \frac{17}{3}$$

$$6-5k = 6-5\left(-\frac{1}{3}\right) = \frac{23}{3}$$

Therefore, the required point is  $\left(\frac{17}{3}, 0, \frac{23}{3}\right)$ .

**Question 12:**

Find the coordinates of the point where the line through (3, -4, -5) and (2, -3, 1) crosses the plane  $2x+y+z=7$ .

Answer

It is known that the equation of the line through the points,  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$ , is

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

Since the line passes through the points, (3, -4, -5) and (2, -3, 1), its equation is given by,

$$\begin{aligned}\frac{x-3}{2-3} &= \frac{y+4}{-3+4} = \frac{z+5}{1+5} \\ \Rightarrow \frac{x-3}{-1} &= \frac{y+4}{1} = \frac{z+5}{6} = k \text{ (say)} \\ \Rightarrow x &= 3-k, y=k-4, z=6k-5\end{aligned}$$

Therefore, any point on the line is of the form  $(3-k, k-4, 6k-5)$ .

This point lies on the plane,  $2x+y+z=7$

$$\therefore 2(3-k) + (k-4) + (6k-5) = 7$$

$$\Rightarrow 5k-3=7$$

$$\Rightarrow k=2$$

Hence, the coordinates of the required point are  $(3 - 2, 2 - 4, 6 \times 2 - 5)$  i.e.,  $(1, -2, 7)$ .

**Question 13:**

Find the equation of the plane passing through the point  $(-1, 3, 2)$  and perpendicular to each of the planes  $x + 2y + 3z = 5$  and  $3x + 3y + z = 0$ .

Answer

The equation of the plane passing through the point  $(-1, 3, 2)$  is

$$a(x + 1) + b(y - 3) + c(z - 2) = 0 \dots (1)$$

where,  $a, b, c$  are the direction ratios of normal to the plane.

It is known that two planes,  $a_1x + b_1y + c_1z + d_1 = 0$  and  $a_2x + b_2y + c_2z + d_2 = 0$ , are

perpendicular, if  $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Plane (1) is perpendicular to the plane,  $x + 2y + 3z = 5$

$$\therefore a \cdot 1 + b \cdot 2 + c \cdot 3 = 0$$

$$\Rightarrow a + 2b + 3c = 0 \dots (2)$$

Also, plane (1) is perpendicular to the plane,  $3x + 3y + z = 0$

$$\therefore a \cdot 3 + b \cdot 3 + c \cdot 1 = 0$$

$$\Rightarrow 3a + 3b + c = 0 \dots (3)$$

From equations (2) and (3), we obtain

$$\frac{a}{2 \times 1 - 3 \times 3} = \frac{b}{3 \times 3 - 1 \times 1} = \frac{c}{1 \times 3 - 2 \times 3}$$

$$\Rightarrow \frac{a}{-7} = \frac{b}{8} = \frac{c}{-3} = k \text{ (say)}$$

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