



Maxima and Minima 18.3 Q1(v)

We have,

$$f(x) = xe^x$$

$$\therefore f'(x) = e^x + xe^x = e^x(x+1)$$

$$\begin{aligned} f''(x) &= e^x(x+1) + e^x \\ &= e^x(x+2) \end{aligned}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow e^x(x+1) = 0$$

$$\Rightarrow x = -1$$

Now,

$$f''(-1) = e^{-1} = \frac{1}{e} > 0$$

$\therefore x = -1$ is point of local minima

Hence,

$$\text{local min value} = f(-1) = \frac{-1}{e}.$$

Maxima and Minima 18.3 Q1(vi)

We have,

$$f(x) = \frac{x}{2} + \frac{2}{x}, \quad x > 0$$

$$\therefore f'(x) = \frac{1}{2} - \frac{2}{x^2}$$

$$\text{and, } f''(x) = \frac{4}{x^3}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \frac{1}{2} - \frac{2}{x^2} = 0$$

$$\Rightarrow \frac{x^2 - 4}{2x^2} = 0$$

$$\Rightarrow x = 2, -2$$

Now,

$$f''(2) = \frac{1}{2} > 0$$

$\therefore x = 2$ is point of minima

We will not consider $x = -2$ as $x > 0$

\therefore local min value = $f(2) = 2$.

We have,

$$f(x) = (x+1)(x+2)^{\frac{1}{3}}, \quad x \geq -2$$

$$\begin{aligned}\therefore f'(x) &= (x+2)^{\frac{1}{3}} + \frac{1}{3}(x+1)(x+2)^{\frac{-2}{3}} \\ &= (x+2)^{\frac{-2}{3}} \left(x+2 + \frac{1}{3}(x+1) \right) \\ &= \frac{1}{3}(x+2)^{\frac{-2}{3}} (4x+7)\end{aligned}$$

$$\text{and, } f''(x) = -\frac{2}{9}(x+2)^{\frac{-5}{3}}(4x+7) + \frac{1}{3}(x+2)^{\frac{-2}{3}} \times 4$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \frac{1}{3}(x+2)^{\frac{-2}{3}}(4x+7) = 0$$

$$\Rightarrow x = -\frac{7}{4}$$

Now,

$$f''\left(-\frac{7}{4}\right) = \frac{4}{3}\left(-\frac{7}{4}+2\right)^{\frac{-2}{3}}$$

$$\therefore x = -\frac{7}{4} \text{ is point of minima}$$

$$\therefore \text{local min value} = f\left(-\frac{7}{4}\right) = \frac{-3}{4^{\frac{2}{3}}}.$$

Maxima and Minima 18.3 Q1(viii)

We have,

$$f(x) = x\sqrt{32 - x^2}, -5 \leq x \leq 5$$

$$\begin{aligned}\therefore f'(x) &= \sqrt{32 - x^2} + \frac{x}{2\sqrt{32 - x^2}} \times (-2x) \\ &= \frac{2(32 - x^2) - 2x^2}{2\sqrt{32 - x^2}} \\ &= \frac{64 - 4x^2}{2\sqrt{32 - x^2}} \\ \text{and, } f''(x) &= \frac{2\sqrt{32 - x^2} \times (-8x) - \frac{-2(64 - 4x^2)}{2\sqrt{32 - x^2}} \times (-2x)}{4(32 - x^2)} \\ &= \frac{-4(32 - x^2) \times 8x + 4x(64 - x^2)}{8(32 - x^2)^{\frac{3}{2}}}\end{aligned}$$

For maxima and minima,

$$\begin{aligned}f'(x) &= 0 \\ \Rightarrow \frac{4(16 - x^2)}{2\sqrt{32 - x^2}} &= 0 \\ \Rightarrow x &= \pm 4\end{aligned}$$

Now,

$$f''(4) = \frac{4 \times 4(64 - 16 - 8 \times 32 + 8 \times 16)}{8(32 - 16)^{\frac{3}{2}}} < 0$$

$\therefore x = 4$ is point of maxima

***** END *****