Indefinite Integrals Ex 19.9 Q30

Let $\log \sin x = t$

$$\Rightarrow \frac{1}{\sin x} \cdot \cos x \; dx = dt$$
$$\therefore \cot x \; dx = dt$$

$$\Rightarrow \int \cot x \; \log \sin x \; dx = \int t \, dt$$
$$= \frac{t^2}{2} + C$$
$$= \frac{1}{2}(\log \sin x)^2 + C$$

Indefinite Integrals Ex 19.9 Q31

Let $I = \int \sec x \cdot \log(\sec x + \tan x) \, dx$ - - - - - (i)

Let $\log(\sec x + \tan x) = t$ then,
$d[\log(\sec x + \tan x)] = dt$

$\Rightarrow \quad \sec x \, dx = dt \qquad \left[\because \quad \frac{d}{dx}(\log(\sec x + \tan x)) = \sec x\right]$

Putting $\log(\sec x + \tan x) = t$ and $\sec x \, dx = dt$ in equation (i), we get

$$I = \int t \, dt$$
$$= \frac{t^2}{2} + c$$
$$= \frac{1}{2}[\log(\sec x + \tan x)]^2 + c$$

$\therefore \quad I = \frac{1}{2}[\log(\sec x + \tan x)]^2 + c$

Indefinite Integrals Ex 19.9 Q32

Let $I = \int \operatorname{cosec} x \log(\operatorname{cosec} x - \cot x)\, dx$ - - - - - (i)

Let $\log(\operatorname{cosec} x - \cot x) = t$ then,

$dx\left[\log(\operatorname{cosec} x - \cot x)\right] = dt$

$\Rightarrow \quad \operatorname{cosec} x\, dx = dt \qquad \left[\because \quad \frac{d}{dx}(\log(\operatorname{cosec} x - \cot x)) = \operatorname{cosec} x\right]$

Putting $\log(\operatorname{cosec} x - \cot x) = t$ and $\operatorname{cosec} x\, dx = dt$ in equation (i), we get

$$I = \int t\, dt$$
$$= \frac{t^2}{2} + c$$

$\therefore \quad I = \frac{1}{2}\left[\log(\operatorname{cosec} x - \cot x)\right]^2 + c$

Indefinite Integrals Ex 19.9 Q33

Let $I = \int x^3 \cos x^4 dx$ - - - - - (i)

Let $x^4 = t$ then,

$dx\left(x^4\right) = dt$

$\Rightarrow \quad 4x^3 dx = dt$

$\Rightarrow \quad x^3 = \frac{dt}{4}$

Putting $x^4 = t$ and $x^3 dx = \frac{dt}{4}$ in equation (i), we get

$$I = \int \cos t \frac{dt}{4}$$
$$= \frac{1}{4} \sin t + c$$

$\therefore \quad I = \frac{1}{4} \sin x^4 + c$

Indefinite Integrals Ex 19.9 Q34

Let $\quad I = \int x^3 \sin x^4 dx$ - - - - - (i)

Let $\quad x^4 = t$ then,

$d\left(x^4\right) = dt$

$\Rightarrow \quad 4x^3 dx = dt$

$\Rightarrow \quad x^3 = \dfrac{dt}{4}$

Putting $x^4 = t$ and $\quad x^3 dx = \dfrac{dt}{4}$ in equation (i), we get

$$I = \int \sin t \frac{dt}{4}$$

$$= \frac{1}{4}\int \sin t\, dt$$

$$= -\frac{1}{4}\cos t + c$$

$$= -\frac{1}{4}\cos x^4 + c$$

$\therefore \quad I = -\dfrac{1}{4}\cos x^4 + c$

********** END **********