



Functions Ex 2.5 Q15

We have given that

$f : \mathbb{R} \rightarrow (-1, 1)$ defined by

$$f(x) = \frac{10^x - 10^{-x}}{10^x + 10^{-x}} \text{ is invertible}$$

$$\text{let } f(x) = y$$

$$\Rightarrow \frac{10^x - 10^{-x}}{10^x + 10^{-x}} = y$$

$$\Rightarrow \frac{10^{2x} - 1}{10^{2x} + 1} = y$$

$$\Rightarrow 10^{2x} - 1 = y(10^{2x} + 1)$$

$$\Rightarrow 10^{2x} - 10^{2x}y = y + 1$$

$$\Rightarrow 10^{2x}(1 - y) = y + 1$$

$$\Rightarrow 10^{2x} = \frac{y+1}{1-y}$$

$$\Rightarrow 2x = \log_{10} \left(\frac{1+y}{1-y} \right)$$

$$x = \frac{1}{2} \log_{10} \left(\frac{1+y}{1-y} \right)$$

$$\therefore f^{-1}(x) = \frac{1}{2} \log_{10} \left(\frac{1+x}{1-x} \right)$$

Functions Ex 2.5 Q16

We have given that

$f : \mathbb{R} \rightarrow (0, 2)$ defined by

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} + 1 \text{ is invertible.}$$

$$\text{let } f(x) = y$$

$$\Rightarrow \frac{e^x - e^{-x}}{e^x + e^{-x}} + 1 = y$$

$$\Rightarrow \frac{2e^x}{e^x + e^{-x}} = y$$

$$\Rightarrow \frac{2e^{2x}}{e^{2x} + 1} = y$$

$$\Rightarrow 2e^{2x} = y(e^{2x} + 1)$$

$$\Rightarrow e^{2x}(2 - y) = y$$

$$\Rightarrow e^{2x} = \frac{y}{2 - y} \Rightarrow x = \frac{1}{2} \log_e \left(\frac{y}{2 - y} \right)$$

$$\Rightarrow f^{-1}(x) = \frac{1}{2} \log_e \left(\frac{x}{2 - x} \right)$$

Given: that

$f : [-1, \infty) \rightarrow [-1, \infty)$ is a function

given by $f(x) = (x+1)^2 - 1$

In order to show that f is invertible, we need to prove that f is bijective.

Injective: let $x, y \in [-1, \infty)$, Such that

$$f(x) = f(y)$$

$$\Rightarrow (x+1)^2 - 1 = (y+1)^2 - 1$$

$$\Rightarrow (x+1)^2 = (y+1)^2$$

$$\Rightarrow x+1 = y+1 \quad [x, y \in [-1, \infty)]$$

$$\Rightarrow x = y$$

$$\Rightarrow f \text{ is one-one}$$

Surjectivity: let $y \in [-1, \infty)$ be arbitrary

such that $f(x) = y$

$$\Rightarrow (x+1)^2 - 1 = y$$

$$\Rightarrow (x+1)^2 = y+1$$

$$\Rightarrow x+1 = \sqrt{y+1}$$

$$\Rightarrow x = \sqrt{y+1} - 1 \in [-1, \infty)$$

So, for each $y \in [-1, \infty)$ (co-domain) there exist $x = \sqrt{y+1} - 1 \in [-1, \infty)$ (domain)

$\therefore f$ is onto

Thus, f is bijective $\Rightarrow f$ is invertible.

Now,

$$f(x) = f^{-1}(x)$$

$$\Rightarrow (x+1)^2 - 1 = \sqrt{x+1} - 1$$

$$\Rightarrow (x+1)^2 - \sqrt{x+1} = 0$$

$$\Rightarrow \sqrt{x+1} \left((x+1)^{3/2} - 1 \right) = 0$$

$$\Rightarrow \sqrt{x+1} = 0 \text{ or } (x+1)^{3/2} - 1 = 0$$

$$\Rightarrow x = -1 \text{ or } x = 0$$

$$\therefore x = 0, -1$$

Hence, $S = \{0, -1\}$

Functions Ex 2.5 Q18

$A = \{x \in \mathbb{R} : -1 \leq x \leq 1\}$ and $f: A \rightarrow A$, $g: A \rightarrow A$ are two functions defined by $f(x) = x^2$ and $g(x) = \sin\left(\frac{\pi x}{2}\right)$

Here, $f: A \rightarrow A$ is defined by

$$f(x) = x^2$$

Clearly f is not injective, $\because f(1) = f(-1) = 1$

So, f is not bijective and hence not invertible.

Hence, f^{-1} does not exist

Now, $g: A \rightarrow A$ defined by

$$g(x) = \sin\left(\frac{\pi x}{2}\right)$$

Injectivity: Let $x_1 = x_2$

$$\begin{aligned} \Rightarrow \quad & \frac{\pi x_1}{2} = \frac{\pi x_2}{2} \\ \Rightarrow \quad & \sin\left(\frac{\pi x_1}{2}\right) = \sin\left(\frac{\pi x_2}{2}\right) \quad [\because -1 \leq x \leq 1] \\ \Rightarrow \quad & g(x_1) = g(x_2) \\ \Rightarrow \quad & g \text{ is one-one} \dots\dots\dots(i) \end{aligned}$$

Surjectivity: let y be arbitrary such that

$$\begin{aligned} & g(x) = y \\ \Rightarrow \quad & \sin\left(\frac{\pi x}{2}\right) = y \\ \Rightarrow \quad & \frac{\pi x}{2} = \sin^{-1} y \\ \Rightarrow \quad & x = \frac{2}{\pi} \sin^{-1} y = [-1, 1] \end{aligned}$$

Thus, for each y in codomain, there exists x in domain, such that

$$\begin{aligned} & g(x) = y \\ \Rightarrow \quad & g \text{ is surjective} \dots\dots\dots(ii) \end{aligned}$$

From (i) & (ii)

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