



Differentiation Ex 11.3 Q1

Let $y = \cos^{-1}\{2x\sqrt{1-x^2}\}$

Put $x = \cos \theta$

$$y = \cos^{-1}\{2 \cos \theta \sqrt{1 - \cos^2 \theta}\}$$

$$= \cos^{-1}\{2 \cos \theta \sin \theta\}$$

$$y = \cos^{-1}\{\sin 2\theta\}$$

$$[\text{Since } \sin 2\theta = 2 \sin \theta \cos \theta, \sin^2 \theta + \cos^2 \theta = 1]$$

$$y = \cos^{-1}\left[\cos\left(\frac{\pi}{2} - \theta\right)\right]$$

$$\text{---(i)}$$

Now,

$$\frac{1}{\sqrt{2}} < x < 1$$

$$\Rightarrow \frac{1}{\sqrt{2}} < \cos \theta < 1$$

$$\Rightarrow 0 < \theta < \frac{\pi}{4}$$

$$\Rightarrow 0 < 2\theta < \frac{\pi}{2}$$

$$\Rightarrow 0 > -2\theta > -\frac{\pi}{2}$$

$$\Rightarrow \frac{\pi}{2} > \left(\frac{\pi}{2} - 2\theta\right) > 0$$

Hence, from equation (i),

$$y = \frac{\pi}{2} - 2\theta$$

$$[\text{Since } \cos^{-1}(\cos \theta) = \theta, \text{ if } \theta \in [0, \pi]]$$

$$y = \frac{\pi}{2} - 2 \cos^{-1} x$$

$$[\text{Since } x = \cos \theta]$$

Differentiating it with respect to x ,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \left(\frac{\pi}{2} \right) - 2 \frac{d}{dx} (\cos^{-1} x) \\ &= 0 - 2 \left(\frac{-1}{\sqrt{1-x^2}} \right) \end{aligned}$$

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q2

$$\text{Let } y = \cos^{-1} \left\{ \sqrt{\frac{1+x}{2}} \right\}$$

$$\text{Put } x = \cos 2\theta$$

$$y = \cos^{-1} \left\{ \sqrt{\frac{1+\cos 2\theta}{2}} \right\}$$

$$= \cos^{-1} \left\{ \sqrt{\frac{2\cos^2 \theta}{2}} \right\}$$

$$y = \cos^{-1} \{ \cos \theta \} \quad \text{---(i)}$$

$$\text{Here, } -1 < x < 1$$

$$\Rightarrow -1 < \cos 2\theta < 1$$

$$\Rightarrow 0 < 2\theta < \pi$$

$$\Rightarrow 0 < \theta < \frac{\pi}{2}$$

So, from equation (i),

$$y = \theta$$

$$\left[\text{Since } \cos^{-1}(\cos \theta) = \theta \text{ if } \theta \in [0, \pi] \right]$$

$$y = \frac{1}{2} \cos^{-1} x$$

$$\left[\text{Since } x = \cos 2\theta \right]$$

Differentiating it with respect to x ,

$$\frac{dy}{dx} = -\frac{1}{2\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q3

$$\text{Let } y = \sin^{-1} \left\{ \sqrt{\frac{1-x}{2}} \right\}$$

$$\text{Let } x = \cos 2\theta$$

$$y = \sin^{-1} \left\{ \sqrt{\frac{1-\cos 2\theta}{2}} \right\}$$

$$= \sin^{-1} \left\{ \sqrt{\frac{2\sin^2 \theta}{2}} \right\}$$

$$y = \sin^{-1}(\sin \theta) \quad \text{---(i)}$$

$$\text{Here, } 0 < x < 1$$

$$\Rightarrow 0 < \cos 2\theta < 1$$

$$\Rightarrow 0 < 2\theta < \frac{\pi}{2}$$

$$\Rightarrow 0 < \theta < \frac{\pi}{4}$$

so, from equation (i),

$$y = \theta$$

$$\left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right]$$

$$y = \frac{1}{2} \cos^{-1} x$$

$$\left[\text{Since, } x = \cos 2\theta \right]$$

Differentiating it with respect to x ,

$$\frac{dy}{dx} = -\frac{1}{2\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q4

$$\text{Let } y = \sin^{-1} \left\{ \sqrt{1-x^2} \right\}$$

$$\text{Let } x = \cos \theta$$

$$y = \sin^{-1} \left\{ \sqrt{1-\cos^2 \theta} \right\}$$

$$y = \sin^{-1} (\sin \theta) \quad \text{---(i)}$$

$$\text{Here, } 0 < x < 1$$

$$\Rightarrow 0 < \cos \theta < 1$$

$$\Rightarrow 0 < \theta < \frac{\pi}{2}$$

From equatoin(i),

$$y = \theta$$

$$y = \cos^{-1} x$$

$$\left[\text{Since, } \sin^{-1} (\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right]$$

$$[\text{Since } x = \cos \theta]$$

Differentiating with respect to x ,

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q5

$$\text{Let } y = \tan^{-1} \left\{ \frac{x}{\sqrt{a^2-x^2}} \right\}$$

$$\text{Let } x = a \sin \theta$$

$$y = \tan^{-1} \left\{ \frac{a \sin \theta}{\sqrt{a^2 - a^2 \sin^2 \theta}} \right\}$$

$$y = \tan^{-1} \left\{ \frac{a \sin \theta}{\sqrt{a^2 (1 - \sin^2 \theta)}} \right\}$$

$$y = \tan^{-1} \left\{ \frac{a \sin \theta}{a \cos \theta} \right\}$$

$$y = \tan^{-1} (\tan \theta) \quad \text{---(i)}$$

$$\text{Here, } -a < x < a$$

$$\Rightarrow -1 < \frac{x}{a} < 1$$

$$\Rightarrow -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

From equatoin(i),

$$y = \theta$$

$$y = \sin^{-1} \left(\frac{x}{a} \right)$$

$$\left[\text{Since, } \tan^{-1} (\tan \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right]$$

$$[\text{Since } x = a \sin \theta]$$

Differentiating it with respect to x ,

Using chain rule,

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}} \frac{d}{dx} \left(\frac{x}{a} \right) \\ &= \frac{a}{\sqrt{a^2-x^2}} \times \left(\frac{1}{a} \right) \end{aligned}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{a^2-x^2}}.$$

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