

Functions Ex 2.5 Q12

Given that f(x) = 2x and g(x) = x + 2.

We need to prove that f and g are bijective maps.

Let $x,y \in Q$.

Consider f(x) = f(y)

$$\Rightarrow 2x = 2y$$

$$\Rightarrow \chi = y$$

$$\Rightarrow f$$
 is one – one.

Let y be an arbitrary element of Q such that f(x) = y

Then
$$f(x) = y = 2x \Rightarrow x = \frac{y}{2}$$

Thus, for any $y \in Q$, there exists $x = \frac{y}{2} \in Q$ such that,

$$f(x) = f\left(\frac{y}{2}\right) = 2\frac{y}{2} = y$$

So $f: Q \rightarrow Q$ is a bijection and hence invertible.

Let f^{-1} denote the inverse of f.

Thus,
$$f^{-1}(x) = \frac{x}{2}...(1)$$

Let $x,y \in Q$.

Consider g(x) = g(y)

$$\Rightarrow x + 2 = y + 2$$

$$\Rightarrow \chi = y$$

Let y be an arbitrary element of Q such that g(x) = y

Then
$$q(x) = y = x + 2 \Rightarrow x = y - 2$$

Thus, for any $y \in Q$, there exists x = y - 2, $y \in Q$ such that,

$$q(x) = q(y-2) = y-2+2=y$$

So $g: Q \rightarrow Q$ is a bijection and hence invertible.

Let g^{-1} denote the inverse of g.

Thus,
$$g^{-1}(x) = x - 2...(2)$$

Now consider
$$g \circ f = g[f(x)] = g(2x) = 2x + 2$$

Thus,
$$(g \circ f)^{-1} = \frac{x-2}{2}$$
...(3)

From (1) and (2), we have

$$f^{-1} \circ g^{-1} = f^{-1}[g^{-1}(x)] = f^{-1}[x-2] = \frac{x-2}{2}...(4)$$

From (3) and (4), it is clear that $(a \circ f)^{-1} = f^{-1} \circ a^{-1}$

Functions Ex 2.5 Q13

Given that
$$f(x) = \frac{x-2}{x-3}$$
;

Let
$$f(x) = y$$
;

$$\Rightarrow y = \frac{x-2}{x-3}$$

Interchange x and y;

$$\Rightarrow x = \frac{y-2}{y-3}$$

$$\Rightarrow (v-3)x=v-2$$

$$\Rightarrow xy - 3x = y - 2$$

$$\Rightarrow xy - y = 3x - 2$$

$$\Rightarrow y(x-1)=3x-2$$

$$\Rightarrow y = \frac{3x - 2}{x - 1}$$

$$\Rightarrow f^{-1}(x) = \frac{3x - 2}{x - 1}$$

Functions Ex 2.5 Q14

$$f: \mathbb{R}^+ \to [-9, \infty)$$
 given by $f(x) = 5x^2 + 6x - 9$

For any x, y
$$\in$$
 R⁺
 $f(x) = f(y)$
 $\Rightarrow 5x^2 + 6x - 9 = 5y^2 + 6y - 9$
 $\Rightarrow 5(x^2 - y^2) + 6(x - y) = 0$
 $\Rightarrow (x - y)[5(x + y) + 6] = 0$
 $\Rightarrow x - y = 0$ $[\because 5(x + y) + 6 \neq 0 \text{ as } x, y \in R^+]$
 $\Rightarrow x = y$

So, fis an injection.

Let y be an arbitrary element of $[-9, \infty)$.

$$f(x) = y$$

$$\Rightarrow 5x^{2} + 6x - 9 = y$$

$$\Rightarrow 25x^{2} + 30x - 45 = 5y$$

$$\Rightarrow 25x^{2} + 30x + 9 - 54 = 5y$$

$$\Rightarrow (5x + 3)^{2} = 5y + 54$$

$$\Rightarrow (5x + 3) = \sqrt{5y + 54} - 3$$

$$\Rightarrow x = \frac{\sqrt{5y + 54} - 3}{5}$$
Now, $y \in [-9, \infty)$

$$\Rightarrow y \ge -9$$

$$\Rightarrow 5y + 54 \ge 9$$

$$\Rightarrow \sqrt{5y + 54} \ge 3$$

$$\Rightarrow \sqrt{5y + 54} \ge 3$$

$$\Rightarrow \sqrt{5y + 54} = 3 \ge 0$$

$$\Rightarrow x \ge 0 \Rightarrow x \in \mathbb{R}^{+}$$
Thus, for every $y \in [-9, \infty)$ there exist $x = \frac{\sqrt{5y + 54} - 3}{5} \in \mathbb{R}^{+}$ such that $f(x) = y$.

So, $f: \mathbb{R}^+ \to [-9, \infty)$ is onto.

Thus, $f: \mathbb{R}^+ \to [-9, \infty)$ is a bijection and hence invertible.

Let f-1 denote the inverse of f.

Then,

$$(fof^{-1})(y) = y \text{ for all } y \in [-9, \infty)$$

$$f(f^{-1}(y)) = y \text{ for all } y \in [-9, \infty)$$

$$\Rightarrow 5\{f^{-1}(y)\}^2 + 6\{f^{-1}(y)\} - 9 = y \text{ for all } y \in [-9, \infty)$$

⇒
$$25\{f^{-1}(y)\}^2 + 30\{f^{-1}(y)\} - 45 = 5y \text{ for all } y \in [-9, \infty)$$

$$\Rightarrow 25\{f^{-1}(y)\}^2 + 30\{f^{-1}(y)\} + 9 = 5y + 54 \text{ for all } y \in [-9, \infty)$$

$$\Rightarrow \{5f^{-1}(y) + 3\}^2 = 5y + 54 \text{ for all } y \in [-9, \infty)$$

$$\Rightarrow$$
 5f⁻¹(y)+3 = $\sqrt{5y + 54}$ for all $y \in [-9, \infty)$

$$\Rightarrow f^{-1}\left(y\right)\frac{\sqrt{5y+54}-3}{5}$$

******* END ******