Indefinite Integrals Ex 19.9 Q20

Let $\quad I = \int \frac{x^3}{\left(x^2+1\right)^3} x\,dx \text{ - - - - - } (i)$

Let $\quad 1+x^2 = t \quad$ then,

$d\left(1+x^2\right) = dt$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad x\,dx = \frac{dt}{2}$

Putting $1+x^2 = t$ and $x\,dx = \frac{dt}{2}$ in equation (i), we get

$$
\begin{aligned}
I &= \int \frac{x^2}{t^3} \times \frac{dt}{2} \\
&= \frac{1}{2}\int \frac{(t-1)}{t^3} dt \qquad \left[\because \quad 1+x^2 = t\right] \\
&= \frac{1}{2}\int \left[\left(\frac{t}{t^3} - \frac{1}{t^3}\right) dt\right] \\
&= \frac{1}{2}\int \left(t^{-2} - t^{-3}\right) dt \\
&= \frac{1}{2}\left[-1t^{-1} - \frac{t^{-2}}{-2}\right] + c \\
&= \frac{1}{2}\left[-\frac{1}{t} + \frac{1}{2t^2}\right] + c \\
&= -\frac{1}{2t} + \frac{1}{4t^2} + c \\
&= -\frac{1}{2\left(1+x^2\right)} + \frac{1}{4\left(1+x^2\right)^2} + c \\
&= \frac{-2\left(1+x^2\right)+1}{4\left(1+x^2\right)^2} + c \\
&= \frac{-2-2x^2+1}{4\left(1+x^2\right)^2} + c \\
&= \frac{-2x^2-1}{4\left(1+x^2\right)^2} + c \\
&= -\frac{\left(1+2x^2\right)}{4\left(x^2+1\right)^2} + c
\end{aligned}
$$

$\therefore \quad I = \frac{-\left(1+2x^2\right)}{4\left(x^2+1\right)^2} + c$

Indefinite Integrals Ex 19.9 Q21

Let $x^2+x+1=t$

$(2x+1)dx = dt$

$$\int (4x+2)\sqrt{x^2+x+1}\ dx$$

$$= \int 2\sqrt{t}\ dt$$

$$= 2\int \sqrt{t}\ dt$$

$$= 2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$$

$$= \frac{4}{3}\left(x^2+x+1\right)^{\frac{3}{2}}+C$$

Indefinite Integrals Ex 19.9 Q22

Let $\quad I = \int \frac{4x+3}{\sqrt{2x^3+3x+1}}dx$ - - - - - (i)

Let $\quad 2x^3+3x+1 = t \qquad$ then,

$d\left(2x^3+3x+1\right) = dt$

$\Rightarrow \quad (4x+3)dx = dt$

Putting $2x^3+3x+1 = t$ and $(4x+3)dx = dt$ in equation (i), we get

$$I = \int \frac{dt}{\sqrt{t}}$$

$$= \int t^{\frac{-1}{2}}dt$$

$$= 2t^{\frac{1}{2}}+c$$

$$= 2\sqrt{t}+c$$

$\therefore \quad I = 2\sqrt{2x^3+3x+1}+c$

Indefinite Integrals Ex 19.9 Q23

Let $I = \int \frac{1}{1+\sqrt{x}}\,dx \text{-----} (i)$

Let $x = t^2$ then,

$dx = d\left(t^2\right)$

$\Rightarrow \quad dx = 2t\,dt$

Putting $x = t^2$ and $dx = 2t\,dt$ in equation (i), we get

$$
\begin{aligned}
I &= \int \frac{2t}{1+\sqrt{t^2}}\,dt \\
&= \int \frac{2t}{1+t}\,dt \\
&= 2\int \frac{t}{1+t}\,dt \\
&= 2\int \frac{1+t-1}{1+t}\,dt \\
&= 2\int \left[\frac{1+t}{1+t} - \frac{1}{1+t}\right]dt \\
&= 2\int dt - 2\int \frac{1}{1+t}\,dt \\
&= 2t - 2\log\left(1+t\right) + c \\
&= 2\sqrt{x} - 2\log\left(1+\sqrt{x}\right) + c
\end{aligned}
$$

$\therefore \quad I = 2\sqrt{x} - 2\log\left(1+\sqrt{x}\right) + c$

Indefinite Integrals Ex 19.9 Q24

Let $I = \int e^{\cos^2 x} \sin 2x\,dx \text{----} (i)$

Let $\cos^2 x = t$ then,

$d\left(\cos^2 x\right) = dt$

$\Rightarrow \quad -2\cos x \sin x\,dx = dt$

$\Rightarrow \quad -\sin 2x\,dx = dt$

$\Rightarrow \quad \sin 2x\,dx = -dt$

Putting $\cos^2 x = t$ and $\sin 2x\,dx = -dt$ in equation (i),
we get

$$
\begin{aligned}
I &= \int e^t\left(-dt\right) \\
&= -e^t + c \\
&= -e^{\cos^2 x} + c
\end{aligned}
$$

$\therefore \quad I = -e^{\cos^2 x} + c$

*********** END **********