Maxima and Minima 18.5 Q36

Let $S(x)$ be the selling price of x items and let $C(x)$ be the cost price of x items.

Then, we have $S(x) = \left(5 - \frac{x}{100}\right)x = 5x - \frac{x^2}{100}$

and $C(x) = \frac{x}{5} + 500$

Thus, the profit function $P(x)$ is given by

$$P(x) = S(x) - C(x) = 5x - \frac{x^2}{100} - \frac{x}{5} - 500 = \frac{24}{5}x - \frac{x^2}{100} - 500$$

$\therefore \quad P'(x) = \frac{24}{5} - \frac{x}{50}$

Now, $P'(x) = 0$

$\Rightarrow \frac{24}{5} - \frac{x}{50} = 0$

$\Rightarrow \quad x = \frac{24}{5} \times 50 = 240$

Also $P''(x) = -\frac{1}{50}$

So, $P''(240) = -\frac{1}{50} < 0$

Thus, $x = 240$ is a point of maxima.

Hence, the manufacturer can earn maximum profit, if he sells 240 items.

Maxima and Minima 18.5 Q37

Let l be the length of side of square base of the tank and h be the height of tank.
Then,

Volume of tank (v) = $l^2 h$

Total surface area (s) = $l^2 + 4lh$

Since the tank holds a given quantity of water the volume (v) is constant.

$\therefore \quad v = l^2 h \qquad$ ---(i)

Also, cost of lining with lead will be least if the total surface area is least.
So we need to minimise the surface area.

$\therefore \quad S = l^2 + 4lh \qquad$ ---(ii)

Now,

From (i) and (ii)

$$S = l^2 + \frac{4v}{l}$$

$$\therefore \quad \frac{ds}{dl} = 2l - \frac{4v}{l^2}$$

For maximum and minimum

$$\frac{ds}{dl} = 0$$

$$\Rightarrow \quad 2l - \frac{4v}{l^2} = 0$$

$$\Rightarrow \quad 2l^3 - 4v = 0$$

$$\Rightarrow \quad l^3 = 2v = 2t^2 h$$

$$\Rightarrow \quad l^2\left[l - 2h\right] = 0$$

$$\Rightarrow \quad l = 0 \text{ or } 2h$$

$l = 0$ is not possible.

$$\therefore \quad l = 2h$$

Now,

$$\frac{d^2 s}{dl^2} = 2 + \frac{8v}{l^3}$$

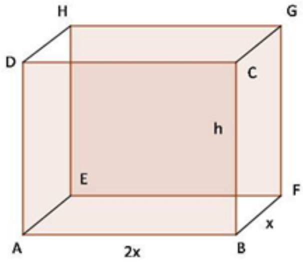
At $\quad l = 2h, \ \frac{d^2 s}{dl^2} > 0 \qquad$ for all h.

$\therefore \quad l = 2h$ is point of local minima

$\therefore \quad S$ is minimum when $l = 2h$

Maxima and Minima 18.5 Q38

Let *ABCDEFGH* be a box of constant volume *c*. We are given that the box is twice as long as its width.



$\therefore$ Let $BF = x$

$\Rightarrow$ $AB = 2x$

Cost of material of top and front side = $3 \times$ cost of material of the bottom of the box.

$\Rightarrow$ $2x \times x + xh + xh + 2xh + 2xh = 3 \times 2x^2$

$\Rightarrow$ $2x^2 + 2xh + 4xh = 6x^2$

$\Rightarrow$ $4x^2 - 6xh = 0$

$\Rightarrow$ $2x(2x - 3h) = 0$

$\Rightarrow$ $x = \frac{3h}{2}$ or $h = \frac{2x}{3}$

Volume of box = $2x \times x \times h$

$\Rightarrow$ $c = 2x^2h$

$\Rightarrow$ $h = \frac{c}{2x^2}$ ---(ii)

Now,

S = Surface area of box = $2\left(2x^2 + 2xh + xh\right)$

$\Rightarrow$ $S = 2\left(2x^2 + 3xh\right)$

From (i)

$$S = 2\left(2x^2 + \frac{3xc}{2x^2}\right)$$

$$\Rightarrow \quad S = 2\left(2x^2 + \frac{3}{2}\frac{c}{x}\right)$$

For maxima and minima,

$$\frac{dS}{dx} = 2\left(4x - \frac{3}{2}\frac{c}{x^2}\right) = 0$$

$$\Rightarrow \quad 8x^3 - 3c = 0$$

$$\Rightarrow \quad x = \left(\frac{3c}{8}\right)^{\frac{1}{3}}$$

Now,

$$\frac{d^2s}{dx^2} = 2\left(4 + 3\frac{c}{x^3}\right) > 0 \text{ as } x = \left(\frac{3c}{8}\right)^{\frac{1}{3}}$$

$x = \left(\frac{3c}{8}\right)^{\frac{1}{3}}$ is point of local minima

$\therefore$ Most economic dimension will be

x = width = $\left(\frac{3c}{8}\right)^{\frac{1}{3}}$

$2x$ = length = $2\left(\frac{3c}{8}\right)^{\frac{1}{3}}$

h = height = $\frac{2x}{3} = \frac{2}{3}\left(\frac{3c}{8}\right)^{\frac{1}{3}}$.

*********** END **********