Solution of Simultaneous Linear Equations Ex 8.1 Q9

Let the numbers are x, y, z.

$$x + y + z = 2 \qquad ---(1)$$

Also, $2y + (x + z) = 1$

$$x + 2y + z = 1 \qquad ---(2)$$

Again,

$$x + z + 5(x) = 6$$

$$5x + y + z = 6 \qquad ---(3)$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 5 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$$

or $\quad AX = B$

$$|A| = 1(1) - 1(-4) + 1(-9)$$
$$= 1 + 4 - 9 = -4 \neq 0$$

Hence, the unique solutions given by $x = A^{-1}B$

$$\begin{array}{lll} C_{11} = 1 & C_{21} = 0 & C_{31} = -1 \\ C_{12} = 4 & C_{22} = -4 & C_{32} = 0 \\ C_{13} = -9 & C_{23} = 4 & C_{33} = 1 \end{array}$$

or $\quad X = A^{-1}B = \dfrac{1}{|A|}(\text{adj } A) \times B = \dfrac{1}{-4}\begin{bmatrix} 1 & 0 & -1 \\ 4 & -4 & 0 \\ -9 & 4 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$

$$= \frac{-1}{4}\begin{bmatrix} 2-6 \\ 8-4 \\ -18+4+6 \end{bmatrix} = \frac{-1}{4}\begin{bmatrix} -4 \\ 4 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

Hence, $x = 1, y = -1, z = 2$

Solution of Simultaneous Linear Equations Ex 8.1 Q10

Let the three investments are x, y, z

$$x+y+z=10,000 \qquad \text{...... (1)}$$

Also

$$\frac{10}{100}x+\frac{12}{100}y+\frac{15}{100}z=1310$$
$$0.1x+0.12y+0.15z=1310 \qquad \text{...... (2)}$$

Also

$$\frac{10}{100}x+\frac{12}{100}y=\frac{15}{100}z-190$$
$$0.1x+0.12y-0.15z=-190 \qquad \text{...... (3)}$$

The above system can be written as

$$\begin{bmatrix} 1 & 1 & 1 \\ 0.1 & 0.12 & 0.15 \\ 0.1 & 0.12 & -0.15 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 10000 \\ 1310 \\ -190 \end{bmatrix}$$

Or $\quad AX = B$

$$|A|=1(-0.036)-1(-0.03)+1(0)=-0.006 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$C_{11}=-0.036 \qquad C_{21}=0.27 \qquad C_{31}=0.03$$
$$C_{12}=0.03 \qquad C_{22}=-0.25 \qquad C_{32}=-0.05$$
$$C_{13}=0 \qquad C_{23}=-0.02 \qquad C_{33}=0.02$$

$$\text{adj}A=\begin{bmatrix} -0.036 & 0.03 & 0 \\ 0.27 & -0.25 & -0.02 \\ 0.03 & -0.05 & 0.02 \end{bmatrix}^T=\begin{bmatrix} -0.036 & 0.27 & 0.03 \\ 0.03 & -0.25 & -0.05 \\ 0 & -0.02 & 0.02 \end{bmatrix}$$

Now,

$$X = A^{-1}B=\frac{1}{|A|}(\text{Adj}A)\times B$$
$$=\frac{1}{-0.006}\begin{bmatrix} -0.036 & 0.27 & 0.03 \\ 0.03 & -0.25 & -0.05 \\ 0 & -0.02 & 0.02 \end{bmatrix}\begin{bmatrix} 10000 \\ 1310 \\ -190 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\frac{1}{-0.006}\begin{bmatrix} -12 \\ -18 \\ -30 \end{bmatrix}=\begin{bmatrix} 2000 \\ 3000 \\ 5000 \end{bmatrix}$$

Hence, $x=\text{Rs}\,2000, y=\text{Rs}\,3000, z=\text{Rs}\,5000$

Solution of Simultaneous Linear Equations Ex 8.1 Q11

$$x + y + z = 45 \quad ---(1)$$
$$z = x + 8 \quad ---(2)$$
$$x + z = 2y \quad ---(3)$$

or $$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix}$$

$$|A| = 1(2) - 1(-2) + 1(2)$$
$$= 2 + 2 + 2 = 6 \neq 0$$

$$C_{11} = 2 \quad C_{21} = -3 \quad C_{31} = 1$$
$$C_{12} = 2 \quad C_{22} = 0 \quad C_{32} = -2$$
$$C_{13} = 2 \quad C_{23} = +3 \quad C_{33} = 1$$

$$X = A^{-1} \times B = \frac{1}{|A|}(\text{adj } A) \times B$$
$$= \frac{1}{6}\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}\begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix}$$
$$= \frac{1}{6}\begin{bmatrix} 90 - 24 \\ 90 \\ 90 + 24 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 66 \\ 90 \\ 114 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 15 \\ 19 \end{bmatrix}$$

Hence, $x = 11, y = 15, z = 19$

Solution of Simultaneous Linear Equations Ex 8.1 Q12

The given problem can be modelled using the following system of equations

$$3x+5y-4z=6000$$
$$2x-3y+z=5000$$
$$-x+4y+6z=13000$$

Which can write as $Ax=B$,

Where

$$A=\begin{bmatrix}3&5&-4\\2&-3&1\\-1&4&6\end{bmatrix},\quad x=\begin{bmatrix}x\\y\\z\end{bmatrix},\quad B=\begin{bmatrix}6000\\5000\\13000\end{bmatrix}$$

Now

$$|A|=3(-18-4)-2(30+16)-1(5-12)$$
$$=3(-22)-2(46)+7$$
$$=-66-92+7$$
$$=-151\neq 0$$

$\therefore$ A^{-1} exists.

Now $Ax=B \Rightarrow \quad x=A^{-1}B$

$$A^{-1}=\frac{adj(A)}{|A|}$$

Cofators of A are

$$C_{11}=-22 \qquad C_{21}=-13 \qquad C_{31}=5$$
$$C_{12}=-46 \qquad C_{22}=14 \qquad C_{32}=-17$$
$$C_{13}=-7 \qquad C_{23}=-11 \qquad C_{33}=-19$$

$$\therefore \quad adj(A)=\begin{bmatrix}-22&-46&-7\\-13&+14&-11\\5&-17&-19\end{bmatrix}$$

Hence,

$$X=\frac{1}{|A|}adj(A)(B)$$

$$=\frac{1}{-151}\begin{bmatrix}-22&-46&-7\\-13&+14&-11\\5&-17&-19\end{bmatrix}\begin{bmatrix}6000\\5000\\13000\end{bmatrix}$$

$$=\frac{1}{-151}\begin{bmatrix}-132000&-23000&-91000\\-78000&+70000&-143000\\-3000&-85000&-247000\end{bmatrix}$$

$$=\begin{bmatrix}3000\\1000\\2000\end{bmatrix}$$

$\therefore$ $x=3000, y=1000$ and $z=2000$.

Solution of Simultaneous Linear Equations Ex 8.1 Q13

From the given data, we get
the following three equations:

$x + y + z = 12$

$2x + 3y + 3z = 33$

$x - 2y + z = 0$

This system of equations can be written
in the matrix form as

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix} \ldots.(1)$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}$$

$$|A| = 1(9) - 1(-1) + 1(-7) = 3$$

$$cofA = \begin{bmatrix} 9 & 1 & -7 \\ -3 & 0 & 3 \\ 0 & -1 & 1 \end{bmatrix}$$

$$adjA = [cofA]^T = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{adjA}{|A|} = \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}\begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ 11 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 36 - 33 + 0 \\ 4 + 0 + 0 \\ -28 + 33 + 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

An award for organising different festivals in the colony
can be included by the management.

*********** END **********