Differentiation Ex 11.8 Q12

Let $\quad u = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Put $\quad x = \tan\theta$, so

$$u = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$$u = \tan^{-1}(\tan 2\theta) \qquad \text{---(i)}$$

Let $\quad v = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$$= \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$v = \cos^{-1}(\cos 2\theta) \qquad \text{---(ii)}$$

Here, $\quad 0 < x < 1$

$\Rightarrow \quad 0 < \tan\theta < 1$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{4}$

So, from equation (i),

$$u = 2\theta \qquad \left[\text{Since, } tab^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$u = 2\tan^{-1}x \qquad [\text{Since, } x = \tan\theta]$$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{2}{1+x^2} \qquad \text{---(iii)}$$

From equation (ii),

$$v = \theta \qquad \left[\text{Since, } \cos^{-1}(\cos\theta) = \theta, \text{ if } \theta \in [0, \pi]\right]$$

$$v = 2\tan^{-1}x \qquad [\text{Since, } x = \tan\theta]$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{2}{1+x^2} \qquad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{1+x^2} \times \frac{1+x^2}{2}$$

$$\frac{du}{dv} = 1$$

Differentiation Ex 11.8 Q13

Let $u = \tan^{-1}\left(\frac{x-1}{x+1}\right)$

Put $x = \tan\theta$, so

$$u = \tan^{-1}\left(\frac{\tan\theta - 1}{\tan\theta + 1}\right)$$

$$= \tan^{-1}\left(\frac{\tan\theta - \frac{\tan\pi}{4}}{1 + \tan\theta\frac{\tan\pi}{4}}\right)$$

$$u = \tan^{-1}\left(\tan\left(\theta - \frac{\pi}{4}\right)\right) \quad \text{---(i)}$$

Here, $-\frac{1}{2} < x < \frac{1}{2}$

$\Rightarrow \quad -\frac{1}{2} < \tan\theta < \frac{1}{2}$

$\Rightarrow \quad -\tan^{-1}\left(\frac{1}{2}\right) < \theta < \tan^{-1}\left(\frac{1}{2}\right)$

So,

$$u = \theta - \frac{\pi}{4} \qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$u = \tan^{-1}x - \frac{\pi}{4}$$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{1}{1+x^2} - 0$$

$$\frac{du}{dx} = \frac{1}{1+x^2} \quad \text{---(ii)}$$

And,

Let $v = \sin^{-1}\left(3x - 4x^3\right)$

Put $x = \sin\theta$, so

$$v = \sin^{-1}\left(3\sin\theta - 4\sin^3\theta\right)$$

$$v = \sin^{-1}(\sin 3\theta) \quad \text{---(iii)}$$

Now, $-\frac{1}{2} < x < \frac{1}{2}$

$\Rightarrow \quad -\frac{1}{2} < \sin\theta < \frac{1}{2}$

$\Rightarrow \quad -\frac{1}{6} < \theta < \frac{\pi}{6}$

So, from equation (iii),

$$v = 3\theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$v = 3\sin^{-1}x \qquad [\text{Since, } x = \sin\theta]$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{3}{\sqrt{1-x^2}} \quad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{1}{1+x^2} \times \frac{\sqrt{1-x^2}}{3}$$

$$\frac{du}{dv} = \frac{\sqrt{1-x^2}}{3\left(1+x^2\right)}$$

*********** END **********