Definite Integrals Ex 20.1 Q39

$$\int_0^2 \frac{dx}{x+4-x^2} = \int_0^2 \frac{dx}{-\left(x^2-x-4\right)}$$

$$= \int_0^2 \frac{dx}{-\left(x^2-x+\frac{1}{4}-\frac{1}{4}-4\right)}$$

$$= \int_0^2 \frac{dx}{-\left[\left(x-\frac{1}{2}\right)^2-\frac{17}{4}\right]}$$

$$= \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}$$

Let $x-\frac{1}{2}=t \Rightarrow dx = dt$

When $x=0,\ t=-\frac{1}{2}$ and when $x=2,\ t=\frac{3}{2}$

$$\therefore \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2} = \int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{dt}{\left(\frac{\sqrt{17}}{2}\right)^2-t^2}$$

$$= \left[\frac{1}{2\left(\frac{\sqrt{17}}{2}\right)} \log \frac{\frac{\sqrt{17}}{2}+t}{\frac{\sqrt{17}}{2}-t}\right]_{-\frac{1}{2}}^{\frac{3}{2}}$$

$$= \frac{1}{\ldots}\left[\log \frac{\frac{\sqrt{17}}{2}+\frac{3}{2}}{\ldots} - \log \frac{\frac{\sqrt{17}}{2}-\frac{1}{2}}{\ldots}\right]$$

$$= \frac{1}{\sqrt{17}}\left[\log \frac{\quad}{\frac{\sqrt{17}}{2}-\frac{3}{2}} - \frac{\quad}{\log \frac{\sqrt{17}}{2}+\frac{1}{2}}\right]$$

$$= \frac{1}{\sqrt{17}}\left[\log \frac{\sqrt{17}+3}{\sqrt{17}-3} - \log \frac{\sqrt{17}-1}{\sqrt{17}+1}\right]$$

$$= \frac{1}{\sqrt{17}}\log \frac{\sqrt{17}+3}{\sqrt{17}-3} \times \frac{\sqrt{17}+1}{\sqrt{17}-1}$$

$$= \frac{1}{\sqrt{17}}\log \left[\frac{17+3+4\sqrt{17}}{17+3-4\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}}\log \left[\frac{20+4\sqrt{17}}{20-4\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}}\log \left(\frac{5+\sqrt{17}}{5-\sqrt{17}}\right)$$

$$= \frac{1}{\sqrt{17}}\log \left[\frac{\left(5+\sqrt{17}\right)\left(5+\sqrt{17}\right)}{25-17}\right]$$

$$= \frac{1}{\sqrt{17}}\log \left[\frac{25+17+10\sqrt{17}}{8}\right]$$

$$= \frac{1}{\sqrt{17}}\log \left(\frac{42+10\sqrt{17}}{8}\right)$$

$$= \frac{1}{\sqrt{17}}\log \left(\frac{21+5\sqrt{17}}{4}\right)$$

Definite Integrals Ex 20.1 Q40

We have,

$$\int_0^1 \frac{1}{2x^2+x+1}dx$$

$$= \frac{1}{2}\int_0^1 \frac{1dx}{\left(x^2+\frac{1}{2}x+\frac{1}{2}\right)}$$

$$= \frac{1}{2}\int_0^1 \frac{dx}{\left(x+\frac{1}{4}\right)^2+\frac{1}{2}-\frac{1}{16}} \qquad \left[\text{Adding } \frac{1}{16} \text{ \& substracting} \frac{1}{16} \text{ in numerator}\right]$$

$$= \frac{1}{2}\int_0^1 \frac{dx}{\left(x+\frac{1}{4}\right)^2+\frac{7}{16}}$$

$$= \frac{1}{2}\int_0^1 \frac{dx}{\left(x+\frac{1}{4}\right)^2+\left(\frac{\sqrt{7}}{4}\right)^2}$$

$$= \frac{1}{2}\cdot\frac{4}{\sqrt{7}}\left[\tan^{-1}\left(\frac{x+\frac{1}{4}}{\frac{\sqrt{7}}{4}}\right)\right]_0^1$$

$$= \frac{2}{\sqrt{7}}\left\{\tan^{-1}\frac{5}{\sqrt{7}}-\tan^{-1}\left(\frac{1}{\sqrt{7}}\right)\right\}$$

$$\therefore \int_0^1 \frac{1}{2x^2+x+1}dx = \frac{2}{\sqrt{7}}\left\{\tan^{-1}\frac{5}{\sqrt{7}}-\tan^{-1}\left(\frac{1}{\sqrt{7}}\right)\right\}$$

Definite Integrals Ex 20.1 Q41

$$\text{Let } I = \int_0^1 \sqrt{x(1-x)}\,dx$$

$$\text{let } x = \sin^2\theta$$
$$\Rightarrow dx = 2\sin\theta.\cos\theta d\theta$$

Now,

$$x = 0 \Rightarrow \theta = 0$$
$$x = 1 \Rightarrow \theta = \frac{\pi}{2}$$

$$\therefore I = \int_0^{\frac{\pi}{2}} \sqrt{\sin^2\theta\left(1-\sin^2\theta\right)}.2\sin\theta.\cos\theta d\theta$$

$$= \int_0^{\frac{\pi}{2}} 2\sin^2\theta.\cos^2\theta d\theta$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} 4\sin^2\theta.\cos^2\theta d\theta$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \left(\sin^2 2\theta\right)d\theta$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}}\left(\frac{1-\cos 4\theta}{2}\right)d\theta$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}}(1-\cos 4\theta)\,d\theta$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} d\theta - \frac{1}{4}\int_0^{\frac{\pi}{2}} \cos 4\theta d\theta$$

$$= \frac{1}{4}\left[\theta\right]_0^{\frac{\pi}{2}} - \frac{1}{4}\left[\frac{\sin 4\theta}{4}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{4}\left[\frac{\pi}{2}-0\right] - \frac{1}{16}\left[\sin\pi - \sin 0\right]$$

$$= \frac{\pi}{8} - \frac{1}{16}\left[0-0\right]$$

$$= \frac{\pi}{8}$$

$$I = \frac{\pi}{8}$$

$$\therefore \int_0^1 \sqrt{x(1-x)}\,dx = \frac{\pi}{8}$$

Definite Integrals Ex 20.1 Q42

We have,

$$\int_0^2 \frac{dx}{\sqrt{3+2x-x^2}}$$

$$\int_0^2 \frac{dx}{\sqrt{3+1-(x^2-2x+1)}} \qquad \left[\text{Add and subtract 1 in denominator}\right]$$

$$= \int_0^2 \frac{dx}{\sqrt{(2)^2(x-1)^2}} \qquad \left[\because \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}\right]$$

$$= \left[\sin^{-1}\left(\frac{x-1}{2}\right)\right]_0^2$$

$$= \sin^{-1}\frac{1}{2} - \sin^{-1}\left(\frac{-1}{2}\right)$$

$$= \sin^{-1}\left(\sin\frac{\pi}{6}\right) - \sin^{-1}\left[\sin\left(\frac{-\pi}{6}\right)\right]$$

$$= \frac{\pi}{6} + \frac{\pi}{6}$$

$$= \frac{\pi}{3}$$

$$\therefore \int_0^2 \frac{dx}{\sqrt{3+2x-x^2}} = \frac{\pi}{3}$$

*********** END **********