Derivatives as a Rate Measurer Ex 13.2 Q28

Let edge of cube be x cm
Here,

$$\frac{dV}{dt} = 9 \ cm^3/sec$$

To find $\frac{dA}{dt}$ when $x = 10$ cm

We know that

$$V = x^3$$

$$\frac{dV}{dt} = 3x^2 \left(\frac{dx}{dt}\right)$$

$$9 = 3(10)^2 \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{3}{100} \ cm/sec$$

Now, $\quad A = 6x^2$

$$\frac{dA}{dt} = 12x \frac{dx}{dt}$$

$$= 12(10)\left(\frac{3}{100}\right)$$

$$\frac{dA}{dt} = 3.6 \ cm^2/sec.$$

Derivatives as a Rate Measurer Ex 13.2 Q29

Given, $\dfrac{dV}{dt} = 25$ cm^3/sec

To find $\dfrac{dA}{dt}$ when $r = 5$ cm

We know that,

$$V = \dfrac{4}{3}\pi r^3$$

$$\dfrac{dV}{dt} = \dfrac{4}{3}\pi\left(3r^2\right)\dfrac{dr}{dt}$$

$$25 = 4\pi\,(5)^2\,\dfrac{dr}{dt}$$

$$\dfrac{dr}{dt} = \dfrac{1}{4\pi}\ \text{cm/sec}$$

Now, $A = 4\pi r^2$

$$\dfrac{dA}{dt} = 8\pi r\dfrac{dr}{dt}$$

$$= 8\pi\,(5)\left(\dfrac{1}{4\pi}\right)$$

$$\dfrac{dA}{dt} = 10\ \text{cm}^2/\text{sec.}$$

Derivatives as a Rate Measurer Ex 13.2 Q30

Given,

$$\dfrac{dx}{dt} = -5\ \text{cm/min}$$

$$\dfrac{dy}{dt} = 4\ \text{cm/min}$$

(i) To find $\dfrac{dP}{dt}$ when $x = 8$ cm, $y = 6$ cm

$$P = 2\left(x + y\right)$$

$$\dfrac{dP}{dt} = 2\left(\dfrac{dx}{dt} + \dfrac{dy}{dt}\right)$$

$$= 2\left(-5 + 4\right)$$

$$\dfrac{dP}{dt} = -2\ \text{cm/min}$$

(ii) To find $\dfrac{dA}{dt}$ when $x = 8$ cm and $y = 6$ cm

$$A = xy$$

$$\dfrac{dA}{dt} = x\dfrac{dy}{dt} + y\dfrac{dx}{dt}$$

$$= (8)(4) + (6)(-5)$$

$$= 32 - 30$$

$$\dfrac{dA}{dt} = 2\ \text{cm}^2/\text{min.}$$

Derivatives as a Rate Measurer Ex 13.2 Q31

Let r be the radius of the given disc and A be its area.

Then, $A = \pi r^2$

$\therefore \quad \dfrac{dA}{dt} = 2\pi r \dfrac{dr}{dt}$ [by chain rule]

Now, the approximate increase of radius $= dr = \dfrac{dr}{dt}\Delta t = 0.05\,cm\,/sec$

$\therefore$ the approximate rate of increase in area is given by

$$dA = \dfrac{dA}{dt}(\Delta t) = 2\pi r\left(\dfrac{dr}{dt}\Delta t\right) = 2\pi\,(3.2)(0.05) = 0.320\pi\,cm^3\,/s$$

********** END *********