EXERCISE.8.1

Question-1

Expand the expression $(1-2x)^5$

Ans.

By using Binomial Theorem, the expression $(1-2x)^5$ can be expanded as

$$
\begin{aligned}
&(1-2x)^5\\
&= {}^5C_0(1)^5 - {}^5C_1(1)^4(2x) + {}^5C_2(1)^3(2x)^2 - {}^5C_3(1)^2(2x)^3 + {}^5C_4(1)^1(2x)^4 - {}^5C_5(2x)^5\\
&= 1 - 5(2x) + 10(4x^2) - 10(8x^3) + 5(16x^4) - (32x^5)\\
&= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5
\end{aligned}
$$

Question-2

Expand the expression $\left(\frac{2}{x}-\frac{x}{2}\right)^5$

Ans.

By using Binomial Theorem, the expression $\left(\frac{2}{x}-\frac{x}{2}\right)^5$ can be expanded as

$$
\begin{aligned}
\left(\frac{2}{x}-\frac{x}{2}\right)^5 &= {}^5C_0\left(\frac{2}{x}\right)^5 - {}^5C_1\left(\frac{2}{x}\right)^4\left(\frac{x}{2}\right) + {}^5C_2\left(\frac{2}{x}\right)^3\left(\frac{x}{2}\right)^2\\
&\quad - {}^5C_3\left(\frac{2}{x}\right)^2\left(\frac{x}{2}\right)^3 + {}^5C_4\left(\frac{2}{x}\right)\left(\frac{x}{2}\right)^4 - {}^5C_5\left(\frac{x}{2}\right)^5\\
&= \frac{32}{x^5} - 5\left(\frac{16}{x^4}\right)\left(\frac{x}{2}\right) + 10\left(\frac{8}{x^3}\right)\left(\frac{x^2}{4}\right) - 10\left(\frac{4}{x^2}\right)\left(\frac{x^3}{8}\right) + 5\left(\frac{2}{x}\right)\left(\frac{x^4}{16}\right) - \frac{x^5}{32}\\
&= \frac{32}{x^5} - \frac{40}{x^3} + \frac{20}{x} - 5x + \frac{5}{8}x^3 - \frac{x^5}{32}
\end{aligned}
$$

Question-3

Expand the expression $(2x - 3)^6$

Ans.

By using Binomial Theorem, the expression $(2x - 3)^6$ can be expanded as

$$\begin{aligned}(2x-3)^6 &= {}^6C_0(2x)^6 - {}^6C_1(2x)^5(3) + {}^6C_2(2x)^4(3)^2 - {}^6C_3(2x)^3(3)^3 \\ &\quad + {}^6C_4(2x)^2(3)^4 - {}^6C_5(2x)(3)^5 + {}^6C_6(3)^6 \\ &= 64x^6 - 6(32x^5)(3) + 15(16x^4)(9) - 20(8x^3)(27) \\ &\quad + 15(4x^2)(81) - 6(2x)(243) + 729 \\ &= 64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729\end{aligned}$$

Question-4

Expand the expression $\left(\frac{x}{3}+\frac{1}{x}\right)^5$

Ans.

By using Binomial Theorem, the expression $\left(\frac{x}{3}+\frac{1}{x}\right)^5$ can be expanded as

$$\begin{aligned}\left(\frac{x}{3}+\frac{1}{x}\right)^5 &= {}^5C_0\left(\frac{x}{3}\right)^5 + {}^5C_1\left(\frac{x}{3}\right)^4\left(\frac{1}{x}\right) + {}^5C_2\left(\frac{x}{3}\right)^3\left(\frac{1}{x}\right)^2 \\ &\quad + {}^5C_3\left(\frac{x}{3}\right)^2\left(\frac{1}{x}\right)^3 + {}^5C_4\left(\frac{x}{3}\right)\left(\frac{1}{x}\right)^4 + {}^5C_5\left(\frac{1}{x}\right)^5 \\ &= \frac{x^5}{243} + 5\left(\frac{x^4}{81}\right)\left(\frac{1}{x}\right) + 10\left(\frac{x^3}{27}\right)\left(\frac{1}{x^2}\right) + 10\left(\frac{x^2}{9}\right)\left(\frac{1}{x^3}\right) + 5\left(\frac{x}{3}\right)\left(\frac{1}{x^4}\right) + \frac{1}{x^5} \\ &= \frac{x^5}{243} + \frac{5x^3}{81} + \frac{10x}{27} + \frac{10}{9x} + \frac{5}{3x^3} + \frac{1}{x^5}\end{aligned}$$

Question-5

Expand $\left(x+\frac{1}{x}\right)^6$

Ans.

By using Binomial Theorem, the expression $\left(x+\frac{1}{x}\right)^6$ can be expanded as

$$\left(x+\frac{1}{x}\right)^6 = {}^6C_0(x)^6 + {}^6C_1(x)^5\left(\frac{1}{x}\right) + {}^6C_2(x)^4\left(\frac{1}{x}\right)^2$$
$$+ {}^6C_3(x)^3\left(\frac{1}{x}\right)^3 + {}^6C_4(x)^2\left(\frac{1}{x}\right)^4 + {}^6C_5(x)\left(\frac{1}{x}\right)^5 + {}^6C_6\left(\frac{1}{x}\right)^6$$
$$= x^6 + 6(x)^5\left(\frac{1}{x}\right) + 15(x)^4\left(\frac{1}{x^2}\right) + 20(x)^3\left(\frac{1}{x^3}\right) + 15(x)^2\left(\frac{1}{x^4}\right) + 6(x)\left(\frac{1}{x^5}\right) + \frac{1}{x^6}$$
$$= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$$

Question-6

Using Binomial Theorem, evaluate $(96)^3$

Ans.

96 can be expressed as the sum or difference of two numbers whose powers are easier to calculate and then, binomial theorem can be applied.

It can be written that, 96 = 100 – 4

$$\therefore (96)^3 = (100-4)^3$$
$$= {}^3C_0(100)^3 - {}^3C_1(100)^2(4) + {}^3C_2(100)(4)^2 - {}^3C_3(4)^3$$
$$= (100)^3 - 3(100)^2(4) + 3(100)(4)^2 - (4)^3$$
$$= 1000000 - 120000 + 4800 - 64$$
$$= 884736$$

Question-7

Using Binomial Theorem, evaluate $(102)^5$

Ans.

102 can be expressed as the sum or difference of two numbers whose powers are easier to calculate and then, Binomial Theorem can be applied.

It can be written that, 102 = 100 + 2

$$\therefore (102)^5 = (100+2)^5$$
$$= {}^5C_0(100)^5 + {}^5C_1(100)^4(2) + {}^5C_2(100)^3(2)^2 + {}^5C_3(100)^2(2)^3$$
$$+ {}^5C_4(100)(2)^4 + {}^5C_5(2)^5$$
$$= (100)^5 + 5(100)^4(2) + 10(100)^3(2)^2 + 10(100)^2(2)^3 + 5(100)(2)^4 + (2)^5$$
$$= 10000000000 + 1000000000 + 40000000 + 800000 + 8000 + 32$$
$$= 11040808032$$

Question-8

Using Binomial Theorem, evaluate $(101)^4$

Ans.

101 can be expressed as the sum or difference of two numbers whose powers are easier to calculate and then, Binomial Theorem can be applied.

It can be written that, 101 = 100 + 1

$$\begin{aligned}\therefore (101)^4 &= (100+1)^4\\ &= {}^4C_0(100)^4 + {}^4C_1(100)^3(1) + {}^4C_2(100)^2(1)^2 + {}^4C_3(100)(1)^3 + {}^4C_4(1)^4\\ &= (100)^4 + 4(100)^3 + 6(100)^2 + 4(100) + (1)^4\\ &= 100000000 + 4000000 + 60000 + 400 + 1\\ &= 104060401\end{aligned}$$

Question-9

Using Binomial Theorem, evaluate $(99)^5$

Ans.

99 can be written as the sum or difference of two numbers whose powers are easier to calculate and then, Binomial Theorem can be applied.

It can be written that, 99 = 100 − 1

$$\begin{aligned}\therefore (99)^5 &= (100-1)^5\\ &= {}^5C_0(100)^5 - {}^5C_1(100)^4(1) + {}^5C_2(100)^3(1)^2 - {}^5C_3(100)^2(1)^3\\ &\quad + {}^5C_4(100)(1)^4 - {}^5C_5(1)^5\\ &= (100)^5 - 5(100)^4 + 10(100)^3 - 10(100)^2 + 5(100) - 1\\ &= 10000000000 - 500000000 + 10000000 - 100000 + 500 - 1\\ &= 10010000500 - 500100001\\ &= 9509900499\end{aligned}$$

Question-10

Using Binomial Theorem, indicate which number is larger $(1.1)^{10000}$ or 1000.

Ans.

By splitting 1.1 and then applying Binomial Theorem, the first few terms of $(1.1)^{10000}$ can be obtained as

$$\begin{aligned}(1.1)^{10000} &= (1+0.1)^{10000}\\ &= {}^{10000}C_0 + {}^{10000}C_1(1.1) + \text{Other positive terms}\\ &= 1 + 10000\times 1.1 + \text{Other positive terms}\\ &= 1 + 11000 + \text{Other positive terms}\\ &> 1000\end{aligned}$$

Hence, $(1.1)^{10000} > 1000$

Question-11

Find $(a + b)^4 - (a - b)^4$. Hence, evaluate $\left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4$.

Ans.

Using Binomial Theorem, the expressions, $(a + b)^4$ and $(a - b)^4$, can be expanded as

$$(a+b)^4 = {}^4C_0a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_3ab^3 + {}^4C_4b^4$$
$$(a-b)^4 = {}^4C_0a^4 - {}^4C_1a^3b + {}^4C_2a^2b^2 - {}^4C_3ab^3 + {}^4C_4b^4$$
$$\therefore (a+b)^4 - (a-b)^4 = {}^4C_0a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_3ab^3 + {}^4C_4b^4$$
$$-\left[{}^4C_0a^4 - {}^4C_1a^3b + {}^4C_2a^2b^2 - {}^4C_3ab^3 + {}^4C_4b^4\right]$$
$$= 2\left({}^4C_1a^3b + {}^4C_3ab^3\right) = 2\left(4a^3b + 4ab^3\right)$$
$$= 8ab\left(a^2 + b^2\right)$$

By putting $a = \sqrt{3}$ and $b = \sqrt{2}$, we obtain

$$\left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4 = 8\left(\sqrt{3}\right)\left(\sqrt{2}\right)\left\{\left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2\right\}$$
$$= 8\left(\sqrt{6}\right)\{3+2\} = 40\sqrt{6}$$

Question-12

Find $(x + 1)^6 + (x - 1)^6$. Hence or otherwise evaluate $\left(\sqrt{2}+1\right)^6 + \left(\sqrt{2}-1\right)^6$.

Ans.

Using Binomial Theorem, the expressions, $(x + 1)^6$ and $(x - 1)^6$, can be expanded as

$$(x+1)^6 = {}^6C_0x^6 + {}^6C_1x^5 + {}^6C_2x^4 + {}^6C_3x^3 + {}^6C_4x^2 + {}^6C_5x + {}^6C_6$$
$$(x-1)^6 = {}^6C_0x^6 - {}^6C_1x^5 + {}^6C_2x^4 - {}^6C_3x^3 + {}^6C_4x^2 - {}^6C_5x + {}^6C_6$$
$$\therefore (x+1)^6 + (x-1)^6 = 2\left[{}^6C_0x^6 + {}^6C_2x^4 + {}^6C_4x^2 + {}^6C_6\right]$$
$$= 2\left[x^6 + 15x^4 + 15x^2 + 1\right]$$

By putting $x = \sqrt{2}$, we obtain

$$\left(\sqrt{2}+1\right)^6 + \left(\sqrt{2}-1\right)^6 = 2\left[\left(\sqrt{2}\right)^6 + 15\left(\sqrt{2}\right)^4 + 15\left(\sqrt{2}\right)^2 + 1\right]$$
$$= 2\left(8 + 15\times4 + 15\times2 + 1\right)$$
$$= 2\left(8 + 60 + 30 + 1\right)$$
$$= 2(99) = 198$$

Question-13

Show that $9^{n+1} - 8n - 9$ is divisible by 64, whenever n is a positive integer.

Ans.

In order to show that $9^{n+1} - 8n - 9$ is divisible by 64, it has to be proved that,

$9^{n+1} - 8n - 9 = 64k$, where k is some natural number

By Binomial Theorem,

$$(1+a)^m = {}^mC_0 + {}^mC_1a + {}^mC_2a^2 + \ldots + {}^mC_ma^m$$

For $a = 8$ and $m = n + 1$, we obtain

$$(1+8)^{n+1} = {}^{n+1}C_0 + {}^{n+1}C_1(8) + {}^{n+1}C_2(8)^2 + \ldots + {}^{n+1}C_{n+1}(8)^{n+1}$$
$$\Rightarrow 9^{n+1} = 1 + (n+1)(8) + 8^2\left[{}^{n+1}C_2 + {}^{n+1}C_3\times8 + \ldots + {}^{n+1}C_{n+1}(8)^{n-1}\right]$$
$$\Rightarrow 9^{n+1} = 9 + 8n + 64\left[{}^{n+1}C_2 + {}^{n+1}C_3\times8 + \ldots + {}^{n+1}C_{n+1}(8)^{n-1}\right]$$
$$\Rightarrow 9^{n+1} - 8n - 9 = 64k, \text{ where } k = {}^{n+1}C_2 + {}^{n+1}C_3\times8 + \ldots + {}^{n+1}C_{n+1}(8)^{n-1} \text{ is a natural number}$$

Thus, $9^{n+1} - 8n - 9$ is divisible by 64, whenever n is a positive integer.

Question-14

Prove that $\sum_{r=0}^{n} 3^r \, {}^nC_r = 4^n$.

Ans.

By Binomial Theorem,

$$\sum_{r=0}^{n} {}^nC_r \, a^{n-r} b^r = (a+b)^n$$

By putting b = 3 and a = 1 in the above equation, we obtain

$$\sum_{r=0}^{n} {}^nC_r \, (1)^{n-r} (3)^r = (1+3)^n$$

$$\Rightarrow \sum_{r=0}^{n} 3^r \, {}^nC_r = 4^n$$

Hence, proved.

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