Definite Integrals Ex 20.1 Q13

Let $I = \displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \, dx$

$\displaystyle\int \operatorname{cosec} x \, dx = \log|\operatorname{cosec} x - \cot x| = F(x)$

By second fundamental theorem of calculus, we obtain

$I = F\left(\dfrac{\pi}{4}\right) - F\left(\dfrac{\pi}{6}\right)$

$= \log\left|\operatorname{cosec} \dfrac{\pi}{4} - \cot \dfrac{\pi}{4}\right| - \log\left|\operatorname{cosec} \dfrac{\pi}{6} - \cot \dfrac{\pi}{6}\right|$

$= \log\left|\sqrt{2} - 1\right| - \log\left|2 - \sqrt{3}\right|$

$= \log\left(\dfrac{\sqrt{2} - 1}{2 - \sqrt{3}}\right)$

Definite Integrals Ex 20.1 Q14

We have,

$$\int_0^1 \frac{1-x}{1+x} dx$$

Let $x = \cos 2\theta \Rightarrow dx = -2\sin 2\theta d\theta$

Now,

$x = 0 \Rightarrow \theta = \dfrac{\pi}{4}$

$x = 1 \Rightarrow \theta = 0$

Now,

$$\int_0^1 \frac{1-x}{1+x} dx$$

$$= \int_{\frac{\pi}{4}}^0 \frac{1-\cos 2\theta}{1+\cos 2\theta} \times (-2\sin 2\theta) d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{2\sin^2 \theta}{2\cos^2 \theta} \times 2\sin 2\theta d\theta \qquad \left[\because -\int_a^b f(x) dx = \int_b^a f(x) dx\right]$$

$$= \int_0^{\frac{\pi}{4}} \frac{4\sin^3 \theta}{\cos \theta} d\theta$$

Let $\cos \theta = t$

$\Rightarrow -\sin \theta d\theta = dt$

Now,

$\theta = 0 \Rightarrow t = 1$

$\theta = \dfrac{\pi}{4} \Rightarrow t = \dfrac{1}{\sqrt{2}}$

$$\therefore \int_0^{\frac{\pi}{4}} \frac{4\sin^3 \theta}{\cos \theta} d\theta$$

$$= -4 \int_1^{\frac{1}{\sqrt{2}}} \frac{(1-t^2)}{t} dt$$

$$= -4 \left[\log t - \frac{t^2}{2}\right]_1^{\frac{1}{\sqrt{2}}}$$

$$= -4 \left[\log\left(\frac{1}{\sqrt{2}}\right) - \frac{1}{4} - 0 + \frac{1}{2}\right]$$

$$= -4 \left[-\log\sqrt{2} + \frac{1}{4}\right]$$

$$\therefore \int_0^1 \frac{1-x}{1+x} dx = 2\log 2 - 1$$

Definite Integrals Ex 20.1 Q15

$$I = \int_0^{\pi} \frac{1}{1+\sin x}\, dx$$

Multiplying Numerator and Denominator by $(1 - \sin x)$

$$I = \int_0^{\pi} \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x}\, dx$$

$$= \int_0^{\pi} \frac{(1-\sin x)}{\left(1^2 - \sin^2 x\right)}\, dx$$

$$= \int_0^{\pi} \frac{1-\sin x}{\left(\cos^2 x\right)}\, dx$$

$$= \int_0^{\pi} \frac{1}{\cos^2 x}\, dx - \int_0^{\pi} \frac{\sin x}{\cos^2 x}\, dx$$

$$= \int_0^{\pi} \sec^2 x\, dx - \int_0^{\pi} \tan x . \sec x\, dx$$

$$= \left[\tan x\right]_0^{\pi} - \left[\sec x\right]_0^{\pi}$$

$$= \left[\tan \pi - \tan 0\right] - \left[\sec \pi - \sec 0\right]$$

$$= \left[0 - 0\right] - \left[-1 - 1\right]$$

$$= 2$$

$$I = 2$$

$$\therefore \int_0^{\pi} \frac{1}{1+\sin x}\, dx = 2$$

********** END *********