Sine and Cosine Formulae and their Applications Ex-10.1 Q9

$$\sin\left(\frac{B-C}{2}\right)=\frac{b-c}{a}\cos\frac{A}{2}$$

Let $a=k\sin\ A, b=k\sin\ B, c=k\sin C$

RHS

$$\frac{b-c}{a}\cos\frac{A}{2}$$

$$=\frac{k\sin B-k\sin C}{k\sin A}.\cos\frac{A}{2}$$

$$\frac{\sin B-\sin C}{\sin A}.\cos\frac{A}{2}$$

$$=\frac{2\cos\frac{B+C}{2}.\sin\frac{B-C}{2}}{2\sin\frac{A}{2}.\cos\frac{A}{2}}\cos\frac{A}{2}$$

$$=\frac{\cos\frac{\pi-A}{2}\sin\frac{B-C}{2}}{\sin\frac{A}{2}}$$

$$=\frac{\sin\frac{A}{2}\sin\frac{B-C}{2}}{\sin\frac{A}{2}}=\sin\frac{B-C}{2}=RHS$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q10

$$\text{let } \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=k$$

LHS,

$$\frac{a^2-c^2}{b^2}$$

$$=\frac{k^2\sin^2 A-k^2\sin^2 C}{k^2\sin^2 B}$$

$$=\frac{k^2(\sin^2 A-\sin^2 C)}{k^2\sin^2 B}$$

$$=\frac{(\sin^2 A-\sin^2 C)}{\sin^2(\pi-(A+C))}$$

$$=\frac{\sin(A+C)\sin(A-C)}{\sin^2(A+C)}$$

$$=\frac{\sin(A-C)}{\sin(A+C)}=RHS$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q11

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}=k$$

RHS,

$$a\ \sin(B-C)$$

$$=a\sin B.\cos C-a\sin C.\cos B$$

$$=a(bk).\left(\frac{a^2+b^2-c^2}{2ab}\right)-a(ck).\left(\frac{a^2+c^2-b^2}{2ac}\right)$$

$$=k.\frac{(a^2+b^2-c^2)}{2}-k\frac{(a^2+c^2-b^2)}{2}$$

$$=2k.\frac{(b^2-c^2)}{2}$$

$$=b.(kb)-c(kc)$$

$$=b(\sin B)-c(\sin C)$$

LHS

Sine and Cosine Formulae and their Applications Ex-10.1 Q12

$$a^2 \sin(B-C) = (b^2-c^2)\sin A$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$$

$LHS,$

$$a^2 \sin(B-C)$$

$$= a^2\{\sin B.\cos C - \sin C.\cos B\}$$

$$= a^2 kb.\frac{a^2+b^2-c^2}{2ab} - a^2 ck.\frac{a^2+c^2-b^2}{2ac} \text{[Using cos rule and sine rule]}$$

$$= a^2 k.\frac{a^2+b^2-c^2}{2a} - a^2 k.\frac{a^2+c^2-b^2}{2a}$$

$$= a^2 k.\left(\frac{a^2+b^2-c^2-a^2-c^2+b^2}{2a}\right)$$

$$= a^2 k.\left(\frac{2b^2-2c^2}{2a}\right)$$

$$= ak.(b^2-c^2)$$

$$= \sin A(b^2-c^2) = RHS$$

Hence Proved

*********** END **********