Definite Integrals Ex 20.5 Q31

We have

$$\int_a^b f(x) = dx \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+\ldots+f(a+(n-1)h)\right]$$

Where $h=\dfrac{b-a}{n}$

Here

$$a=0, b=2 \text{ and } f(x)=x^2-x$$

Now

$$h=\frac{2}{n}$$
$$nh=2$$

Thus, we have

$$I=\int_0^2 \left(x^2-x\right)dx$$
$$=\lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+\ldots+f((n-1)h)\right]$$
$$=\lim_{h\to 0} h\left[\left\{(0)^2-(0)\right\}+\left\{(h)^2-(h)\right\}+\left\{(2h)^2-(2h)\right\}+\ldots\right]$$
$$=\lim_{h\to 0} h\left[\left((h)^2+(2h)^2+\ldots\right)-\left\{(h)+(2h)+\ldots\right\}\right]$$
$$=\lim_{h\to 0} h\left[h^2\left(1+2^2+3^3+\ldots+(n-1)^2\right)-h\{1+2+3+\ldots+(n-1)\}\right]$$
$$\because h=\frac{2}{n} \;\&\; \text{if } h\to 0 \Rightarrow n\to\infty$$
$$=\lim_{h\to\infty}\frac{2}{n}\left[\frac{9}{n^2}\frac{n(n-1)(2n-1)}{6}-\frac{9}{n}\frac{n(n-1)}{2}\right]$$
$$=\frac{2}{3}$$

Definite Integrals Ex 20.5 Q32

We have

$$\int_a^b f(x) = dx \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+\ldots+f(a+(n-1)h)\right]$$

Where $h=\dfrac{b-a}{n}$

Here

$$a=1, b=3 \text{ and } f(x)=2x^2+5x$$

Now

$$h=\frac{2}{n}$$
$$nh=2$$

Thus, we have

$$I=\int_1^3 \left(2x^2+5x\right)dx$$
$$=\lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+\ldots+f(1+(n-1)h)\right]$$
$$=\lim_{h\to 0} h\left[(2+5)+\left\{2(1+h)^2+5(1+h)\right\}+\left\{2(1+2h)^2+5(1+2h)\right\}+\ldots\right]$$
$$=\lim_{h\to 0} h\left[\left(7n+9h(1+2+3+\ldots)+2h^2\left(1+2^2+3^3+\ldots\right)\right)\right]$$
$$\because h=\frac{2}{n} \;\&\; \text{if } h\to 0 \Rightarrow n\to\infty$$
$$=\lim_{h\to\infty}\frac{2}{n}\left[7n+\frac{18}{n}\frac{n(n-1)}{2}+\frac{8}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$
$$=\frac{112}{3}$$

Definite Integrals Ex 20.5 Q33

Given,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a) + f(a+h) + f(a+2h) + \ldots + f(a+(n-1)h)\right],$$

where $h = \dfrac{b-a}{n}$

Here, $f(x) = 3x^2 + 1$, $a = 1$, $b = 3$. Therefore, $h = \dfrac{3-1}{n} = \dfrac{2}{n}$

$$\therefore I = \int_1^3 (3x^2 + 1)dx$$

$$\Rightarrow I = \lim_{h\to 0} h\left[f(1) + f(1+h) + f(1+2h) + \ldots + f(1+(n-1)h)\right]$$

$$\Rightarrow I = \lim_{h\to 0} h\left[3(1)^2 + 1 + 3(1+h)^2 + 1 + 3(1+2h)^2 + 1 + \ldots + 3(1+(n-1)h)^2 + 1\right]$$

$$\Rightarrow I = \lim_{h\to 0} h\left[3n + n + 6h(1+2+3+\ldots+(n-1)) + 3h^2\left(1^2 + 2^2 + \ldots + (n-1)^2\right)\right]$$

$$\Rightarrow I = \lim_{n\to\infty} \frac{2}{n}\left[4n + \frac{12}{n}(1+2+3+\ldots+(n-1)) + 3\times\frac{4}{n^2}\left(1^2 + 2^2 + \ldots + (n-1)^2\right)\right]$$

$$\Rightarrow I = \lim_{n\to\infty}\left[8 + \frac{24}{n^2}\times\frac{n(n-1)}{2} + \frac{24}{n^3}\times\frac{(n-1)(n)(2n-1)}{6}\right]$$

$$\Rightarrow I = \lim_{n\to\infty}\left[8 + 12\left(1-\frac{1}{n}\right) + 4\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)\right]$$

$$\Rightarrow I = 8 + 12 + 4\times 2 = 28$$

*********** END **********