Differentiation Ex 11.4 Q11

Given,

$$\sin xy + \cos(x+y) = 1$$

Differentiating with respect to x,

$$\frac{d}{dx}\sin xy + \frac{d}{dx}\cos(x+y) = \frac{d}{dx}(1)$$

$$\Rightarrow \quad \cos xy \frac{d}{dx}(xy) - \sin(x+y)\frac{d}{dx}(x+y) = 0 \qquad \text{[Using chain rule and product rule]}$$

$$\Rightarrow \quad \cos(xy)\left[x\frac{dy}{dx} + y\frac{d}{dx}(x)\right] - \sin(x+y)\left[1+\frac{dy}{dx}\right] = 0$$

$$\Rightarrow \quad \cos(xy)\left[x\frac{dy}{dx} + y(1)\right] - \sin(x+y) + \sin(x+y)\frac{dy}{dx} = 0$$

$$\Rightarrow \quad x\cos(xy)\frac{dy}{dx} + y\cos(xy) - \sin(x+y) + \sin(x+y)\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \left[x\cos(xy) + \sin(x+y)\right]\frac{dy}{dx} = \left[\sin(x+y) - y\cos xy\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = \left[\frac{\sin(x+y) - y\cos xy}{x\cos xy + \sin(x+y)}\right]$$

Differentiation Ex 11.4 Q12

Here,

$$\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$$

Let $\quad x = \sin A, y = \sin B,$ so

$$\Rightarrow \quad \sqrt{1-\sin^2 A} + \sqrt{1-\sin^2 B} = a(\sin A - \sin B)$$

$$\Rightarrow \quad \cos A + \cos B = a(\sin A - \sin B) \qquad \left[\text{Since } \left(1-\sin^2\theta\right) = \cos^2\theta\right]$$

$$\Rightarrow \quad a = \frac{\cos A + \cos B}{\sin A - \sin B}$$

$$\Rightarrow \quad a = \frac{2\cos\frac{A+B}{2}\times\cos\frac{A-B}{2}}{2\cos\frac{A+B}{2}\times\sin\frac{A-B}{2}} \qquad \left[\begin{array}{l}\text{Since, } \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\\ \qquad\quad \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\end{array}\right]$$

$$\Rightarrow \quad a = \cot\left(\frac{A-B}{2}\right)$$

$$\Rightarrow \quad \cot^{-1} a = \frac{A-B}{2}$$

$$\Rightarrow \quad 2\cot^{-1} a = A - B$$

$$\Rightarrow \quad 2\cot^{-1} a = \sin^{-1} x - \sin^{-1} y \qquad [\text{Since } x = \sin A, y = \sin B]$$

Differentiating with respect to x,

$$\frac{d}{dx}\left(2\cot^{-1} a\right) = \frac{d}{dx}\left(\sin^{-1} x\right) - \frac{d}{dx}\left(\sin^{-1} y\right)$$

$$\Rightarrow \quad 0 = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$

Differentiation Ex 11.4 Q13

Here,

$$y\sqrt{1-x^2}+x\sqrt{1-y^2}=1$$

Let $\quad x=\sin A, y=\sin B$

$\Rightarrow \quad \sin B\sqrt{1-\sin^2 A}+\sin A\sqrt{1-\sin^2 B}=1$

$\Rightarrow \quad \sin B\cos A+\sin A\cos B=1 \qquad \left[\begin{array}{l}\text{since } 1-\sin^2\theta=\cos^2\theta \text{ and}\\ \sin(x+y)=\sin x\cos y+\cos x\sin y\end{array}\right]$

$\Rightarrow \quad \sin(A+B)=1$

$\Rightarrow \quad A+B=\sin^{-1}(1)$

$\Rightarrow \quad \sin^{-1}x+\sin^{-1}y=\frac{\pi}{2} \qquad [\text{Since } x=\sin A, y=\sin B]$

Differentiating with respect to x,

$\Rightarrow \quad \frac{d}{dx}\left(\sin^{-1}x\right)+\frac{d}{dx}\left(\sin^{-1}y\right)=\frac{d}{dx}\left(\frac{\pi}{2}\right)$

$\Rightarrow \quad \frac{1}{\sqrt{1-x^2}}+\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=0$

$\Rightarrow \quad \frac{dy}{dx}=-\sqrt{\frac{1-y^2}{1-x^2}}$

Differentiation Ex 11.4 Q14

Here,

$$xy=1 \qquad ---(i)$$

Differentiating with respect to x,

$\quad \frac{d}{dx}(xy)=\frac{d}{dx}(1)$

$\Rightarrow \quad x\frac{dy}{dx}+y\frac{d}{dx}(x)=0 \qquad [\text{Using product rule}]$

$\Rightarrow \quad x\frac{dy}{dx}+y(1)=0$

$\Rightarrow \quad \frac{dy}{dx}=-\frac{y}{x} \qquad \left[\text{Put } x=\frac{1}{y} \text{ from equation (i)}\right]$

$\Rightarrow \quad \frac{dy}{dx}=-\frac{y}{\frac{1}{y}}$

$\Rightarrow \quad \frac{dy}{dx}=-y^2$

$\Rightarrow \quad \frac{dy}{dx}+y^2=0$

Differentiation Ex 11.4 Q15

Here,

$$xy^2 = 1$$

Differentiating with respect to x,

$$\frac{d}{dx}\left(xy^2\right) = \frac{d}{dx}(1)$$

$\Rightarrow \quad x\frac{d}{dx}\left(y^2\right) + y^2\frac{d}{dx}(x) = 0$ [Using product rule]

$$\Rightarrow \quad x(2y)\frac{dy}{dx} + y^2(1) = 0$$

$$\Rightarrow \quad 2xy\frac{dy}{dx} = -y^2$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y^2}{2xy}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y}{2x}$$

Put $x = \frac{1}{y^2}$ from equation (i)

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y}{2\left(\frac{1}{y^2}\right)}$$

$$\Rightarrow \quad 2\frac{dy}{dx} = -y^3$$

$$2\frac{dy}{dx} + y^3 = 0$$

*********** END **********