



**Question 4.18:**

Answer the following questions:

- (a)** A magnetic field that varies in magnitude from point to point but has a constant direction (east to west) is set up in a chamber. A charged particle enters the chamber and travels undeflected along a straight path with constant speed. What can you say about the initial velocity of the particle?
- (b)** A charged particle enters an environment of a strong and non-uniform magnetic field varying from point to point both in magnitude and direction, and comes out of it following a complicated trajectory. Would its final speed equal the initial speed if it suffered no collisions with the environment?
- (c)** An electron travelling west to east enters a chamber having a uniform electrostatic field in north to south direction. Specify the direction in which a uniform magnetic field should be set up to prevent the electron from deflecting from its straight line path.

Answer

- (a)** The initial velocity of the particle is either parallel or anti-parallel to the magnetic field. Hence, it travels along a straight path without suffering any deflection in the field.
- (b)** Yes, the final speed of the charged particle will be equal to its initial speed. This is because magnetic force can change the direction of velocity, but not its magnitude.
- (c)** An electron travelling from West to East enters a chamber having a uniform electrostatic field in the North-South direction. This moving electron can remain undeflected if the electric force acting on it is equal and opposite of magnetic field. Magnetic force is directed towards the South. According to Fleming's left hand rule, magnetic field should be applied in a vertically downward direction.

**Question 4.19:**

An electron emitted by a heated cathode and accelerated through a potential difference of 2.0 kV, enters a region with uniform magnetic field of 0.15 T. Determine the trajectory of the electron if the field (a) is transverse to its initial velocity, (b) makes an angle of  $30^\circ$  with the initial velocity.

Answer

Magnetic field strength,  $B = 0.15 \text{ T}$

Charge on the electron,  $e = 1.6 \times 10^{-19} \text{ C}$

Mass of the electron,  $m = 9.1 \times 10^{-31} \text{ kg}$

Potential difference,  $V = 2.0 \text{ kV} = 2 \times 10^3 \text{ V}$

Thus, kinetic energy of the electron  $= eV$

$$\Rightarrow eV = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2eV}{m}} \quad \dots (1)$$

Where,

$v$  = velocity of the electron

**(a)** Magnetic force on the electron provides the required centripetal force of the electron.

Hence, the electron traces a circular path of radius  $r$ .

Magnetic force on the electron is given by the relation,

$B ev$

$$\text{Centripetal force} = \frac{mv^2}{r}$$

$$\therefore Bev = \frac{mv^2}{r}$$

$$r = \frac{mv}{Be} \quad \dots (2)$$

From equations (1) and (2), we get

$$r = \frac{m}{Be} \left[ \frac{2eV}{m} \right]^{\frac{1}{2}}$$

$$= \frac{9.1 \times 10^{-31}}{0.15 \times 1.6 \times 10^{-19}} \times \left( \frac{2 \times 1.6 \times 10^{-19} \times 2 \times 10^3}{9.1 \times 10^{-31}} \right)^{\frac{1}{2}}$$

$$= 100.55 \times 10^{-5}$$

$$= 1.01 \times 10^{-3} \text{ m}$$

$$= 1 \text{ mm}$$

Hence, the electron has a circular trajectory of radius 1.0 mm normal to the magnetic field.

**(b)** When the field makes an angle  $\theta$  of  $30^\circ$  with initial velocity, the initial velocity will

de,

$$v_1 = v \sin \theta$$

From equation (2), we can write the expression for new radius as:

$$\begin{aligned} r_1 &= \frac{mv_1}{Be} \\ &= \frac{mv \sin \theta}{Be} \\ &= \frac{9.1 \times 10^{-31}}{0.15 \times 1.6 \times 10^{-19}} \times \left[ \frac{2 \times 1.6 \times 10^{-19} \times 2 \times 10^3}{9 \times 10^{-31}} \right]^{\frac{1}{2}} \times \sin 30^\circ \\ &= 0.5 \times 10^{-3} \text{ m} \\ &= 0.5 \text{ mm} \end{aligned}$$

Hence, the electron has a helical trajectory of radius 0.5 mm along the magnetic field direction.

#### Question 4.20:

A magnetic field set up using Helmholtz coils (described in Exercise 4.16) is uniform in a small region and has a magnitude of 0.75 T. In the same region, a uniform electrostatic field is maintained in a direction normal to the common axis of the coils. A narrow beam of (single species) charged particles all accelerated through 15 kV enters this region in a direction perpendicular to both the axis of the coils and the electrostatic field. If the beam remains undeflected when the electrostatic field is  $9.0 \times 10^5 \text{ V m}^{-1}$ , make a simple guess as to what the beam contains. Why is the answer not unique?

Answer

Magnetic field,  $B = 0.75 \text{ T}$

Accelerating voltage,  $V = 15 \text{ kV} = 15 \times 10^3 \text{ V}$

Electrostatic field,  $E = 9 \times 10^5 \text{ V m}^{-1}$

Mass of the electron =  $m$

Charge of the electron =  $e$

Velocity of the electron =  $v$

Kinetic energy of the electron =  $eV$

$$\Rightarrow \frac{1}{2}mv^2 = eV$$

$$\therefore \frac{e}{m} = \frac{v^2}{2V} \quad \dots (1)$$

Since the particle remains undeflected by electric and magnetic fields, we can infer that the electric field is balancing the magnetic field.

$$\therefore eE = evB$$

$$v = \frac{E}{B} \quad \dots (2)$$

Putting equation (2) in equation (1), we get

$$\begin{aligned} \frac{e}{m} &= \frac{1}{2} \frac{\left(\frac{E}{B}\right)^2}{V} = \frac{E^2}{2VB^2} \\ &= \frac{(9.0 \times 10^5)^2}{2 \times 15000 \times (0.75)^2} = 4.8 \times 10^7 \text{ C/kg} \end{aligned}$$

This value of specific charge  $e/m$  is equal to the value of deuteron or deuterium ions.

This is not a unique answer. Other possible answers are  $\text{He}^{++}$ ,  $\text{Li}^{++}$ , etc.

#### Question 4.21:

A straight horizontal conducting rod of length 0.45 m and mass 60 g is suspended by two vertical wires at its ends. A current of 5.0 A is set up in the rod through the wires.

(a) What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero?

(b) What will be the total tension in the wires if the direction of current is reversed keeping the magnetic field same as before? (Ignore the mass of the wires.)  $g = 9.8 \text{ m s}^{-2}$ .

Answer

Length of the rod,  $l = 0.45 \text{ m}$

Mass suspended by the wires,  $m = 60 \text{ g} = 60 \times 10^{-3} \text{ kg}$

Acceleration due to gravity,  $g = 9.8 \text{ m/s}^2$

Current in the rod flowing through the wire,  $I = 5 \text{ A}$

(a) Magnetic field ( $B$ ) is equal and opposite to the weight of the wire i.e.,

$$\begin{aligned} BIl &= mg \\ \therefore B &= \frac{mg}{Il} \\ &= \frac{60 \times 10^{-3} \times 9.8}{5 \times 0.45} = 0.26 \text{ T} \end{aligned}$$

A horizontal magnetic field of 0.26 T normal to the length of the conductor should be set  $= 0.26 \times 5 \times 0.45 + (60 \times 10^{-3}) \times 9.8$   
 $= 1.176 \text{ N}$

#### Question 4.22:

The wires which connect the battery of an automobile to its starting motor carry a current of 300 A (for a short time). What is the force per unit length between the wires if

current of 300 A (for a short time). What is the force per unit length between the wires if they are 70 cm long and 1.5 cm apart? Is the force attractive or repulsive?

Answer

Current in both wires,  $I = 300$  A

Distance between the wires,  $r = 1.5$  cm = 0.015 m

Length of the two wires,  $l = 70$  cm = 0.7 m

Force between the two wires is given by the relation,

$$F = \frac{\mu_0 I^2 l}{2\pi r}$$

Where,

$\mu_0$  = Permeability of free space =  $4\pi \times 10^{-7}$  T m A<sup>-1</sup>

$$\therefore F = \frac{4\pi \times 10^{-7} \times (300)^2}{2\pi \times 0.015}$$

$$= 1.2 \text{ N/m}$$

Since the direction of the current in the wires is opposite, a repulsive force exists between them.

#### Question 4.23:

A uniform magnetic field of 1.5 T exists in a cylindrical region of radius 10.0 cm, its direction parallel to the axis along east to west. A wire carrying current of 7.0 A in the north to south direction passes through this region. What is the magnitude and direction of the force on the wire if,

- (a) the wire intersects the axis,
- (b) the wire is turned from N-S to northeast-northwest direction,
- (c) the wire in the N-S direction is lowered from the axis by a distance of 6.0 cm?

Answer

Magnetic field strength,  $B = 1.5$  T

Radius of the cylindrical region,  $r = 10$  cm = 0.1 m

Current in the wire passing through the cylindrical region,  $I = 7$  A

(a) If the wire intersects the axis, then the length of the wire is the diameter of the cylindrical region.

Thus,  $l = 2r = 0.2$  m

Angle between magnetic field and current,  $\theta = 90^\circ$

Magnetic force acting on the wire is given by the relation,

$$F = BIl \sin \theta$$

$$= 1.5 \times 7 \times 0.2 \times \sin 90^\circ$$

$$= 2.1 \text{ N}$$

Hence, a force of 2.1 N acts on the wire in a vertically downward direction.

(b) New length of the wire after turning it to the Northeast-Northwest direction can be given as: :

$$l_1 = \frac{l}{\sin \theta}$$

Angle between magnetic field and current,  $\theta = 45^\circ$

Force on the wire,

$$F = BI l_1 \sin \theta$$

$$= BIl$$

$$= 1.5 \times 7 \times 0.2$$

$$= 2.1 \text{ N}$$

Hence, a force of 2.1 N acts vertically downward on the wire. This is independent of angle  $\theta$  because  $l \sin \theta$  is fixed.

(c) The wire is lowered from the axis by distance,  $d = 6.0$  cm

Let  $l_2$  be the new length of the wire.

$$\therefore \left(\frac{l_2}{2}\right)^2 = 4(d+r)^2$$

$$= 4(10+6)^2 = 4(16)$$

$$\therefore l_2 = 8 \times 2 = 16 \text{ cm} = 0.16 \text{ m}$$

Magnetic force exerted on the wire,

$$F_2 = BI l_2$$

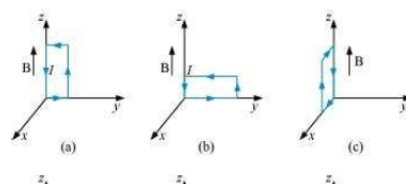
$$= 1.5 \times 7 \times 0.16$$

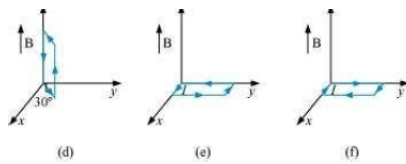
$$= 1.68 \text{ N}$$

Hence, a force of 1.68 N acts in a vertically downward direction on the wire.

#### Question 4.24:

A uniform magnetic field of 3000 G is established along the positive z-direction. A rectangular loop of sides 10 cm and 5 cm carries a current of 12 A. What is the torque on the loop in the different cases shown in Fig. 4.28? What is the force on each case? Which case corresponds to stable equilibrium?





Answer

Magnetic field strength,  $B = 3000 \text{ G} = 3000 \times 10^{-4} \text{ T} = 0.3 \text{ T}$

Length of the rectangular loop,  $l = 10 \text{ cm}$

Width of the rectangular loop,  $b = 5 \text{ cm}$

Area of the loop,

$$A = l \times b = 10 \times 5 = 50 \text{ cm}^2 = 50 \times 10^{-4} \text{ m}^2$$

Current in the loop,  $I = 12 \text{ A}$

Now, taking the anti-clockwise direction of the current as positive and vice-versa:

**(a)** Torque,  $\vec{\tau} = I \vec{A} \times \vec{B}$

From the given figure, it can be observed that  $A$  is normal to the  $y$ - $z$  plane and  $B$  is directed along the  $z$ -axis.

$$\begin{aligned} \therefore \tau &= 12 \times (50 \times 10^{-4}) \hat{i} \times 0.3 \hat{k} \\ &= -1.8 \times 10^{-2} \hat{j} \text{ N m} \end{aligned}$$

The torque is  $1.8 \times 10^{-2} \text{ N m}$  along the negative  $y$ -direction. The force on the loop is zero because the angle between  $A$  and  $B$  is zero.

**(b)** This case is similar to case (a). Hence, the answer is the same as (a).

**(c)** Torque  $\tau = I \vec{A} \times \vec{B}$

From the given figure, it can be observed that  $A$  is normal to the  $x$ - $z$  plane and  $B$  is directed along the  $z$ -axis.

$$\begin{aligned} \therefore \tau &= -12 \times (50 \times 10^{-4}) \hat{j} \times 0.3 \hat{k} \\ &= -1.8 \times 10^{-2} \hat{i} \text{ N m} \end{aligned}$$

The torque is  $1.8 \times 10^{-2} \text{ N m}$  along the negative  $x$  direction and the force is zero.

**(d)** Magnitude of torque is given as:

$$\begin{aligned} |\tau| &= IAB \\ &= 12 \times 50 \times 10^{-4} \times 0.3 \\ &= 1.8 \times 10^{-2} \text{ N m} \end{aligned}$$

Torque is  $1.8 \times 10^{-2} \text{ N m}$  at an angle of  $240^\circ$  with positive  $x$  direction. The force is zero.

**(e)** Torque  $\tau = I \vec{A} \times \vec{B}$

$$\begin{aligned} &= (50 \times 10^{-4} \times 12) \hat{k} \times 0.3 \hat{k} \\ &= 0 \end{aligned}$$

Hence, the torque is zero. The force is also zero.

**(f)** Torque  $\tau = I \vec{A} \times \vec{B}$

$$\begin{aligned} &= (50 \times 10^{-4} \times 12) \hat{k} \times 0.3 \hat{k} \\ &= 0 \end{aligned}$$

Hence, the torque is zero. The force is also zero.

In case (e), the direction of  $I \vec{A}$  and  $\vec{B}$  is the same and the angle between them is zero.

If displaced, they come back to an equilibrium. Hence, its equilibrium is stable.

Whereas, in case (f), the direction of  $I \vec{A}$  and  $\vec{B}$  is opposite. The angle between them is  $180^\circ$ . If disturbed, it does not come back to its original position. Hence, its equilibrium is unstable.

**Question 4.25:**

A circular coil of 20 turns and radius 10 cm is placed in a uniform magnetic field of 0.10 T normal to the plane of the coil. If the current in the coil is 5.0 A, what is the

**(a)** total torque on the coil,

**(b)** total force on the coil,

**(c)** average force on each electron in the coil due to the magnetic field?

(The coil is made of copper wire of cross-sectional area  $10^{-5} \text{ m}^2$ , and the free electron density in copper is given to be about  $10^{29} \text{ m}^{-3}$ .)

Answer

Number of turns on the circular coil,  $n = 20$

Radius of the coil,  $r = 10 \text{ cm} = 0.1 \text{ m}$

Magnetic field strength,  $B = 0.10 \text{ T}$

Current in the coil,  $I = 5.0 \text{ A}$

**(a)** The total torque on the coil is zero because the field is uniform.

**(b)** The total force on the coil is zero because the field is uniform.

**(c)** Cross-sectional area of copper coil,  $A = 10^{-5} \text{ m}^2$

Number of free electrons per cubic meter in copper,  $N = 10^{29} / \text{m}^3$

Charge on the electron,  $e = 1.6 \times 10^{-19} \text{ C}$

Magnetic force,  $F = Bev_d$

Where,

$v_d$  = Drift velocity of electrons

$$= \frac{I}{NeA}$$

$$\begin{aligned}\therefore F &= \frac{BeI}{NeA} \\ &= \frac{0.10 \times 5.0}{10^{29} \times 10^{-5}} = 5 \times 10^{-25} \text{ N}\end{aligned}$$

Hence, the average force on each electron is  $5 \times 10^{-25} \text{ N}$ .

**Question 4.26:**

A solenoid 60 cm long and of radius 4.0 cm has 3 layers of windings of 300 turns each. A 2.0 cm long wire of mass 2.5 g lies inside the solenoid (near its centre) normal to its axis; both the wire and the axis of the solenoid are in the horizontal plane. The wire is connected through two leads parallel to the axis of the solenoid to an external battery which supplies a current of 6.0 A in the wire. What value of current (with appropriate sense of circulation) in the windings of the solenoid can support the weight of the wire?  $g = 9.8 \text{ m s}^{-2}$

Answer

Length of the solenoid,  $L = 60 \text{ cm} = 0.6 \text{ m}$

Radius of the solenoid,  $r = 4.0 \text{ cm} = 0.04 \text{ m}$

It is given that there are 3 layers of windings of 300 turns each.

$\therefore$  Total number of turns,  $n = 3 \times 300 = 900$

Length of the wire,  $l = 2 \text{ cm} = 0.02 \text{ m}$

Mass of the wire,  $m = 2.5 \text{ g} = 2.5 \times 10^{-3} \text{ kg}$

Current flowing through the wire,  $i = 6 \text{ A}$

Acceleration due to gravity,  $g = 9.8 \text{ m/s}^2$

$$\text{Magnetic field produced inside the solenoid, } B = \frac{\mu_0 n I}{L}$$

Where,

$$\mu_0 = \text{Permeability of free space} = 4\pi \times 10^{-7} \text{ T m A}^{-1}$$

$I$  = Current flowing through the windings of the solenoid

Magnetic force is given by the relation,

$$\begin{aligned}F &= B i l \\ &= \frac{\mu_0 n I}{L} i l\end{aligned}$$

Also, the force on the wire is equal to the weight of the wire.

$$\begin{aligned}\therefore mg &= \frac{\mu_0 n I i l}{L} \\ I &= \frac{mgL}{\mu_0 n i l} \\ &= \frac{2.5 \times 10^{-3} \times 9.8 \times 0.6}{4\pi \times 10^{-7} \times 900 \times 0.02 \times 6} = 108 \text{ A}\end{aligned}$$

Hence, the current flowing through the solenoid is 108 A.

**Question 4.27:**

A galvanometer coil has a resistance of  $12 \Omega$  and the metre shows full scale deflection for a current of 3 mA. How will you convert the metre into a voltmeter of range 0 to 18 V?

Answer

Resistance of the galvanometer coil,  $G = 12 \Omega$

Current for which there is full scale deflection,  $I_g = 3 \text{ mA} = 3 \times 10^{-3} \text{ A}$

Range of the voltmeter is 0, which needs to be converted to 18 V.

$$\therefore V = 18 \text{ V}$$

Let a resistor of resistance  $R$  be connected in series with the galvanometer to convert it into a voltmeter. This resistance is given as:

$$\begin{aligned}R &= \frac{V}{I_g} - G \\ &= \frac{18}{3 \times 10^{-3}} - 12 = 6000 - 12 = 5988 \Omega\end{aligned}$$

Hence, a resistor of resistance  $5988 \Omega$  is to be connected in series with the galvanometer.

**Question 4.28:**

A galvanometer coil has a resistance of  $15 \Omega$  and the metre shows full scale deflection for a current of 4 mA. How will you convert the metre into an ammeter of range 0 to 6 A?

Answer

Resistance of the galvanometer coil,  $G = 15 \Omega$

Current for which the galvanometer shows full scale deflection,

$$I_g = 4 \text{ mA} = 4 \times 10^{-3} \text{ A}$$

Range of the ammeter is 0, which needs to be converted to 6 A.

$$\therefore \text{Current, } I = 6 \text{ A}$$

A shunt resistor of resistance  $S$  is to be connected in parallel with the galvanometer to convert it into an ammeter. The value of  $S$  is given as:

$$\begin{aligned}S &= \frac{I_g G}{I - I_g} \\ &= \frac{4 \times 10^{-3} \times 15}{6 - 4 \times 10^{-3}}\end{aligned}$$

$$S = \frac{6 - 4 \times 10^{-3}}{6 - 0.004} = \frac{0.06}{5.996}$$

$$\approx 0.01 \, \Omega = 10 \, \text{m}\Omega$$

Hence, a  $10 \, \text{m}\Omega$  shunt resistor is to be connected in parallel with the galvanometer.

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