Binomial Theorem Ex 18.2 Q34

We have,

$$(1+2a)^4(2-a)^5$$

Now,

$$(1+2a)^4 = {}^4C_0 + {}^4C_1 2a + {}^4C_2(2a)^2 + {}^4C_3(2a)^3 + {}^4C_4(2a)^4$$

and,

$$(2-5)^5 = {}^5C_0 \times 2^5 + {}^5C_2 \times 2^4(-a) + {}^5C_2 \times 2^3(-a)^3 + {}^5C_3 \times 2^2(-a)^3 + {}^5C_4 \times 2(-a)^4 + {}^5C_5(-a)^5$$

$$= {}^5C_0 \times 2^5 - {}^5C_1 \times 2^4 \times a + {}^5C_2 \times 2^3 \times a^2 - {}^5C_3 \times 2^2 \times a^3 + {}^5C_4 \times 2 \times a^4 - {}^5C_5 \times a^5$$

∴

$$(1+2a)^4(2-a)^5 = \left[{}^4C_0 + {}^4C_1 2a + {}^4C_2(2a)^2 + {}^4C_3(2a)^3 + {}^4C_4(2a)^4 \right] \begin{bmatrix} {}^5C_0 \times 2^5 - {}^5C_1 \times 2^4 \times a + {}^5C_2 \times 2^3 \times a^2 \\ - {}^5C_3 \times 2^2 \times a^3 + {}^5C_4 \times 2 \times a^4 - {}^5C_5 \times a^5 \end{bmatrix}$$

∴ Coefficients of $a^4 = 2\,{}^5C_4 - {}^4C_1 \times 2 \times {}^5C_3 \times 2^2 + {}^4C_2(2)^2 \times {}^5C_2 \times 2^3 - {}^4C_3(2)^3 \times {}^5C_1 \times 2^4 + {}^4C_4(2)^4 \times {}^5C_0 \times 2^5$

$$= 2 \times 5 - 8 \times 4 \times 10 + 32 \times 6 \times 10 - 128 \times 4 \times 5 + 512 \times 1 \times 1$$

$$= 10 - 320 + 1920 - 2560 + 512$$

$$= 2442 - 2880$$

$$= -438$$

∴ Coefficients of $a^4 = -438$.

Binomial Theorem Ex 18.2 Q35

$$\left(\sqrt{x} - \frac{k}{x^2} \right)^{10}$$

$$\binom{10}{2}\left(\sqrt{x} \right)^8 \left(-\frac{k}{x^2} \right)^2 = 405$$

$$45k^2 = 405$$

$$k^2 = 9$$

$$k = 3$$

Binomial Theorem Ex 18.2 Q36

$$\left(y^{1/2} + x^{1/3} \right)^n$$

$$\binom{n}{n-2}\left(y^{1/2} \right)^2 \left(x^{1/3} \right)^{n-2}$$

$$\binom{n}{n-2} = 45$$

$$n(n-1) = 90$$

$$n^2 - 10n + 9n - 90$$

$$n(n-10) + 9(n-10) = 0$$

$$n = -9 \: or \: 10$$

$n \: cannot \: be \: negative. \: So, n = 10$

$$6th \, term \binom{10}{5}\left(y^{1/2} \right)^5 \left(x^{1/3} \right)^5 = 252 y^{\frac{5}{3}} x^{\frac{5}{3}}$$

********** END *********