Differentiation Ex 11.2 Q16

Let $\quad y = \sqrt{\dfrac{1+\sin x}{1-\sin x}}$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1+\sin x}{1-\sin x}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(\frac{1+\sin x}{1-\sin x}\right)^{\frac{1}{2}-1}\frac{d}{dx}\left(\frac{1+\sin x}{1-\sin x}\right)$$

$$= \frac{1}{2}\left(\frac{1-\sin x}{1+\sin x}\right)^{\frac{1}{2}}\left[\frac{(1-\sin x)(\cos x)-(1+\sin x)(-\cos x)}{(1-\sin x)^2}\right]$$

$$= \frac{1}{2}\frac{(1-\sin x)^{\frac{1}{2}}}{(1+\sin x)^{\frac{1}{2}}}\left[\frac{\cos x - \cos x\sin x + \cos x + \sin x\cos x}{(1-\sin x)^2}\right]$$

$$= \frac{1}{2}\times\frac{2\cos x}{\sqrt{1+\sin x}\,(1-\sin x)^{\frac{3}{2}}}$$

$$= \frac{\cos x}{\sqrt{1+\sin x}\,(1-\sin x)^{\frac{3}{2}}}$$

$$= \frac{\cos x}{\sqrt{1+\sin x}\sqrt{1-\sin x}\,(1-\sin x)}$$

$$= \frac{\cos x}{\sqrt{1-\sin^2 x}\,(1-\sin x)}$$

$$= \frac{\cos x}{\cos x\,(1-\sin x)} \qquad \left[\text{Using } 1-\sin^2 x = \cos^2 x\right]$$

$$= \frac{1}{(1-\sin x)}\times\frac{(1+\sin x)}{(1+\sin x)}$$

$$= \frac{(1+\sin x)}{\left(1-\sin^2 x\right)}$$

$$= \frac{1+\sin x}{\cos^2 x}$$

$$\text{Thus, } \frac{dy}{dx} = \frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}$$

$$\Rightarrow \frac{dy}{dx} = \sec^2 x + \tan x\sec x$$

$$\Rightarrow \frac{dy}{dx} = \sec x[\tan x + \sec x]$$

Differentiation Ex 11.2 Q17

Let $y = \sqrt{\frac{1-x^2}{1+x^2}}$

$y = \left(\frac{1-x^2}{1+x^2}\right)^{\frac{1}{2}}$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(\frac{1-x^2}{1+x^2}\right)^{\frac{1}{2}-1} \frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right) \qquad \text{[Using chain rule]}$$

$$= \frac{1}{2}\left(\frac{1-x^2}{1+x^2}\right)^{\frac{-1}{2}}\left[\frac{\left(1+x^2\right)\frac{d}{dx}\left(1-x^2\right) - \left(1-x^2\right)\frac{d}{dx}\left(1+x^2\right)}{\left(1+x^2\right)^2}\right] \qquad \text{[Using quotient rule]}$$

$$= \frac{1}{2}\left(\frac{1+x^2}{1-x^2}\right)^{\frac{1}{2}}\left[\frac{\left(1+x^2\right)(-2x) - \left(1-x^2\right)(2x)}{\left(1+x^2\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{1+x^2}{1-x^2}\right)^{\frac{1}{2}}\left[\frac{-2x-2x^3-2x+2x^3}{\left(1+x^2\right)^2}\right]$$

$$= \frac{1}{2}\frac{-4x}{\sqrt{1-x^2}\left(1+x^2\right)^{\frac{3}{2}}}$$

So,

$$\frac{d}{dx}\left(\sqrt{\frac{1-x^2}{1+x^2}}\right) = \frac{-2x}{\sqrt{1-x^2}\left(1+x^2\right)^{\frac{3}{2}}}.$$

Differentiation Ex 11.2 Q18

Let $y = \left(\log\sin x\right)^2$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\log\sin x\right)^2$$

$$= 2\left(\log\sin x\right)\frac{d}{dx}\left(\log\sin x\right) \qquad \text{[Using chain rule]}$$

$$= 2\left(\log\sin x\right)\times\frac{1}{\sin x}\frac{d}{dx}\left(\log x\right)$$

$$= 2\left(\log\sin x\right)\times\frac{1}{\sin x}\times\frac{1}{x}$$

$$= \frac{2\log\sin x}{x\sin x}$$

So,

$$\frac{d}{dx}\left(\log\sin x\right)^2 = \frac{2\log\sin x}{x\sin x}$$

Differentiation Ex 11.2 Q19

Let $y = \sqrt{\dfrac{1+x}{1-x}}$

$\Rightarrow \quad y = \left(\dfrac{1+x}{1-x}\right)^{\frac{1}{2}}$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}-1}\frac{d}{dx}\left(\frac{1+x}{1-x}\right) \qquad \text{[Using chain rule]}$$

$$= \frac{1}{2}\left(\frac{1+x}{1-x}\right)^{\frac{-1}{2}}\left[\frac{(1-x)\frac{d}{dx}(1+x)-(1+x)\frac{d}{dx}(1-x)}{(1-x)^2}\right] \qquad \text{[Using chain rule]}$$

$$= \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{\frac{1}{2}}\left[\frac{(1-x)(1)-(1+x)(-1)}{(1-x)^2}\right]$$

$$= \frac{1}{2}\left(\frac{1-x}{1+x}\right)^{\frac{1}{2}}\left[\frac{1-x+1+x}{(1-x)^2}\right]$$

$$= \frac{1}{2}\frac{(1-x)^{\frac{1}{2}}}{(1+x)^{\frac{1}{2}}}\times\frac{2}{(1-x)^2}$$

$$= \frac{1}{\sqrt{1+x}\,(1-x)^{3/2}}$$

So,

$$\frac{d}{dx}\left(\sqrt{\frac{1+x}{1-x}}\right) = \frac{1}{\sqrt{1+x}\,(1-x)^{3/2}}$$

*********** END **********