



Tangents and Normals Ex 16.3 Q1(viii)

We know that angle of intersection of two curves is given by

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \quad \text{---(A)}$$

Where  $m_1$  and  $m_2$  are slopes of curves.

$$x^2 + y^2 = 2x \quad \text{---(i)}$$

$$y^2 = x \quad \text{---(ii)}$$

Solving (i) and (ii)

$$x^2 + x = 2x$$

$$\Rightarrow x^2 - x = 0$$

$$\Rightarrow x(x - 1) = 0$$

$$\Rightarrow x = 0, 1$$

$$\therefore y = 0 \text{ or } 1$$

$$\therefore \text{The points of intersection is } P = (0, 0), Q = (1, 1)$$

$\therefore$  Slope of (i)

$$2y \frac{dy}{dx} = 2 - 2x$$

$$\therefore \frac{dy}{dx} = \frac{2 - 2x}{2y} = \frac{1 - x}{y}$$

$$\therefore m_1 = 0$$

Slope of (ii)

$$m_2 = \frac{1}{2y} = \frac{1}{2}$$

From (A)

$$\tan \theta = \left| \frac{\frac{1}{2} - 0}{1 + \frac{1}{2} \times 0} \right| = \frac{1}{2}$$

$$\therefore \theta = \tan^{-1} \left( \frac{1}{2} \right)$$

Tangents and Normals Ex 16.3 Q1(ix)

$$y = 4 - x^2, \dots (i)$$

$$y = x^2, \dots (ii)$$

Substituting eq (ii) in (i) we get,

$$x^2 = 4 - x^2$$

$$\Rightarrow x^2 = 2$$

$$\Rightarrow x = \pm\sqrt{2}$$

From (i) when  $x = \sqrt{2}$ , we get  $y = 2$  and when  $x = -\sqrt{2}$ , we get  $y = 2$

Thus the two curves intersect at  $(\sqrt{2}, 2)$  and  $(-\sqrt{2}, 2)$ .

Differentiating (i) wrt  $x$ , we get

$$\frac{dy}{dx} = 0 - 2x = -2x$$

Differentiating (ii) wrt  $x$ , we get

$$\frac{dy}{dx} = 2x$$

Angle of intersection at  $(\sqrt{2}, 2)$

$$m_1 = \left( \frac{dy}{dx} \right)_{(\sqrt{2}, 2)} = -2\sqrt{2}$$

Angle of intersection at  $(-\sqrt{2}, 2)$

$$m_2 = \left( \frac{dy}{dx} \right)_{(-\sqrt{2}, 2)} = 2\sqrt{2}$$

Let  $\theta$  be the angle of intersection of the two curves.

$$\tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right| = \left| \frac{2\sqrt{2} + 2\sqrt{2}}{1 + (2\sqrt{2})(-2\sqrt{2})} \right| = \left| \frac{4\sqrt{2}}{-7} \right| = \frac{4\sqrt{2}}{7}$$

$$\Rightarrow \theta = \tan^{-1} \left( \frac{4\sqrt{2}}{7} \right)$$

\*\*\*\*\* END \*\*\*\*\*