Indefinite Integrals Ex 19.30 Q61

$$f(x)=\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$$

Now,

$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$$

$$=\frac{x^4+3x^2+2}{x^4+7x^2+12}$$

$$=\frac{(x^4+7x^2+12)-4x^2-10}{x^4+7x^2+12}$$

$$=1-\frac{4x^2+10}{x^4+7x^2+12}$$

Now,

$$\frac{4x^2+10}{x^4+7x^2+12}=\frac{4x^2+10}{(x^2+3)(x^2+4)}$$

Let $\quad \dfrac{4x^2+10}{(x^2+3)(x^2+4)}=\dfrac{Ax+B}{x^2+3}+\dfrac{Cx+D}{x^2+4}$

$\Rightarrow \quad 4x^2+10=(Ax+B)(x^2+4)+(Cx+D)(x^2+3)$

Let $x=0$, we get

$10=4B+3D \quad$ ---(i)

If $x=1$, we get

$14=5(A+B)+4(C+D)=5A+5B+4C+4D \quad$ ---(ii)

if $x=-1$, we get

$14=5(-A+B)+4(-C+D)=-5A+5B-4C+4D \quad$ ---(iii)

Applying (ii) and (iii), we get

$28=10B+8D$

$\Rightarrow \quad 14=5B+4D \quad$ ---(iv)

From (i), we get

$10=4B+3D \quad$ ---(i)

Multiplying equation (iv) by 3 and (i) by 4 and subtracting, we get

$42-40=15B-16B$

$\Rightarrow \quad 2=-B$

or $\quad B=-2 \quad$ ---(v)

Putting value of B in (i), we get

$10=4(-2)+3D$

$\dfrac{10+8}{3}=D$

$\Rightarrow \quad D=6 \quad$ ---(vi)

Comparing coefficients of x^3 in

$$4x^2+10=(Ax+B)\left(x^2+4\right)+(Cx+4)\left(x^2+3\right), \text{ we get,}$$

$$0=A+C \qquad ---(\text{vii})$$

Comparing coefficients of x, we get

$$0=4A+3C \qquad ---(\text{viii})$$

$$\Rightarrow \quad A=C=0$$

$$\therefore \quad f(x)=1-\frac{(-2)}{x^2+3}-\frac{6}{x^2+4}$$

$$=1+\frac{2}{x^2+3}-\frac{6}{x^2+4}$$

$$\therefore \quad \int f(x)\,dx=\int 1+\frac{2}{x^2+3}-\frac{6}{x^2+4}\,dx$$

$$=x+\frac{2}{\sqrt{3}}\tan^{-1}x\,\frac{x}{\sqrt{3}}-3\tan^{-1}\frac{x}{2}+c$$

Indefinite Integrals Ex 19.30 Q62

Let $x^2=y$

$$\therefore \quad \frac{4x^4+3}{\left(x^2+2\right)\left(x^2+3\right)\left(x^2+4\right)}=\frac{4y^2+3}{(y+2)(y+3)(y+4)}$$

Now,

Let $\dfrac{4y^2+3}{(y+2)(y+3)(y+4)}=\dfrac{A}{y+2}+\dfrac{B}{y+3}+\dfrac{C}{y+4}$

$$\Rightarrow \quad 4y^2+3=A(y+3)(y+4)+B=(y+2)(y+4)+C(y+2)(y+3)$$

$$=(A+B+C)y^2+(7A+6A+5C)y+12A+8B+6C$$

Equating similar terms,

$A+B+C=4,\ 7A+6A+5C=0,\ 12A+8B+6C=3$

Solving, we get

$A=\dfrac{19}{2},\ B=-39,\ C=\dfrac{67}{2}$

Thus,

$$I=\frac{19}{2}\int\frac{dx}{x^2+2}+(-39)\int\frac{dx}{x^2+3}+\frac{67}{2}\int\frac{dx}{x^2+4}$$

$$\Rightarrow \quad I=\frac{19}{2\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{39}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)+\frac{67}{4}\tan^{-1}\left(\frac{x}{2}\right)+c$$

Hence,

$$I=\frac{19}{2\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{39}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)+\frac{67}{4}\tan^{-1}\left(\frac{x}{2}\right)+c$$

Indefinite Integrals Ex 19.30 Q63

$$\frac{x^4}{(x-1)(x^2+1)} = \frac{x^4}{x^3-x^2+x-1}$$

$$= \frac{x(x^3-x^2+x-1)+1(x^3-x^2+x-1)+1}{(x^3-x^2+x-1)}$$

$$= x+1+\frac{1}{(x-1)(x^2+1)}$$

Now, suppose

$$\frac{1}{(x-1)(x^2+1)} = \frac{A}{x-1}+\frac{Bx+C}{x^2+1}$$

$\Rightarrow \quad 1 = A(x^2+1)+(Bx+C)(x-1)$

Put $x = 1$

$1 = 2A$

$\Rightarrow \quad A = \frac{1}{2}$

Put $x = 0$

$1 = A - C$

$\Rightarrow \quad C = A - 1 = -\frac{1}{2}$

Put $x = -1$

$1 = 2A + 2B - 2C = 2(A - C) + 2B$

$\Rightarrow \quad 1 = 2 + 2B$

$\Rightarrow \quad 2B = -1$

$\Rightarrow \quad B = -\frac{1}{2}$

$$\therefore \quad \int \frac{x^4}{(x-1)(x^2+1)}dx = \int x dx + \int 1 dx + \frac{1}{2}\int \frac{1}{x-1}dx - \frac{1}{2}\int \frac{x+1}{x^2+1}dx$$

$$= \frac{x^2}{2} + x + \frac{1}{2}\log|x-1| - \frac{1}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1}x + C$$

*********** END **********