Trigonometric Ratios Ex 5.1 Q7

Answer :

(i) Given: $\cot\theta = \frac{7}{8}$

To evaluate: $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ ...... (1)

We know the following formula

$(a+b)(a-b) = a^2 - b^2$

By applying the above formula in the numerator of equation (1) ,

We get,

$(1+\sin\theta)(1-\sin\theta) = 1^2 - \sin^2\theta$ ... (Where a = 1 and b = $\sin\theta$)

$(1+\sin\theta)(1-\sin\theta) = 1 - \sin^2\theta$ ...... (2)

Similarly,

By applying formula $(a+b)(a-b) = a^2 - b^2$ in the denominator of equation (1) ,

We get,

$(1+\cos\theta)(1-\cos\theta) = 1^2 - \cos^2\theta$ ... (Where a = 1 and b = $\cos\theta$)

$(1+\cos\theta)(1-\cos\theta) = 1 - \cos^2\theta$ ...... (3)

Substituting the value of numerator and denominator of equation (1) ,from equation (2) and(3)

Therefore,

$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta}$ ...... (4)

Since,

$\cos^2\theta + \sin^2\theta = 1$

Therefore,

$\cos^2\theta = 1 - \sin^2\theta$

Also, $\sin^2\theta = 1 - \cos^2\theta$

Putting the value of $1-\sin^2\theta$ and $1-\cos^2\theta$ in Equation (4)

We get,

$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{\cos^2\theta}{\sin^2\theta}$

$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \left(\frac{\cos\theta}{\sin\theta}\right)^2$

We know that, $\frac{\cos\theta}{\sin\theta} = \cot\theta$

$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = (\cot\theta)^2$

Since, It is given that $\cot\theta = \frac{7}{8}$

Therefore,

$$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}=\left(\frac{7}{8}\right)^2$$

$$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}=\frac{7^2}{8^2}$$

$$\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}=\frac{49}{64}$$

Answer: $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}=\frac{49}{64}$

(ii) Given: $\cot\theta = \frac{7}{8}$

To evaluate: $\cot^2\theta$

$$\cot\theta = \frac{7}{8}$$

Squaring on both sides,

We get,

$$(\cot\theta)^2 = \left(\frac{7}{8}\right)^2$$

$$\cot^2\theta = \frac{7^2}{8^2}$$

$$\cot^2\theta = \frac{49}{64}$$

Answer: $\cot^2\theta = \frac{49}{64}$

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