EXERCISE - 2.1

Question-1

If $\left(\frac{x}{3}+1,\ y-\frac{2}{3}\right)=\left(\frac{5}{3},\ \frac{1}{3}\right)$, find the values of x and y.

Ans.

It is given that $\left(\frac{x}{3}+1,\ y-\frac{2}{3}\right)=\left(\frac{5}{3},\ \frac{1}{3}\right)$.

Since the ordered pairs are equal, the corresponding elements will also be equal.

Therefore, $\frac{x}{3}+1=\frac{5}{3}$ and $y-\frac{2}{3}=\frac{1}{3}$.

$$\begin{array}{ll} \frac{x}{3}+1=\frac{5}{3} & \\ \Rightarrow \frac{x}{3}=\frac{5}{3}-1 & y-\frac{2}{3}=\frac{1}{3} \\ \Rightarrow \frac{x}{3}=\frac{2}{3} & \Rightarrow y=\frac{1}{3}+\frac{2}{3} \\ \Rightarrow x=2 & \Rightarrow y=1 \end{array}$$

$\therefore x=2$ and $y=1$

Question-2

If the set A has 3 elements and the set B = {3, 4, 5}, then find the number of elements in (A × B)?

Ans.

It is given that set A has 3 elements and the elements of set B are 3, 4, and 5.

$\Rightarrow$ Number of elements in set B = 3

Number of elements in (A × B)

= (Number of elements in A) × (Number of elements in B)

$= 3 \times 3 = 9$

Thus, the number of elements in (A × B) is 9.

Question-3

If $G = \{7, 8\}$ and $H = \{5, 4, 2\}$, find $G \times H$ and $H \times G$.

Ans.

$G = \{7, 8\}$ and $H = \{5, 4, 2\}$

We know that the Cartesian product $P \times Q$ of two non-empty sets P and Q is defined as

$P \times Q = \{(p, q): p \in P, q \in Q\}$

$\therefore G \times H = \{(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)\}$

$H \times G = \{(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)\}$

Question-4

State whether each of the following statement are true or false. If the statement is false, rewrite the given statement correctly.

(i) If $P = \{m, n\}$ and $Q = \{n, m\}$, then $P \times Q = \{(m, n), (n, m)\}$.

(ii) If A and B are non-empty sets, then $A \times B$ is a non-empty set of ordered pairs (x, y) such that $x \in A$ and $y \in B$.

(iii) If $A = \{1, 2\}$, $B = \{3, 4\}$, then $A \times (B \cap \Phi) = \Phi$.

Ans.

(i) False

If $P = \{m, n\}$ and $Q = \{n, m\}$, then

$P \times Q = \{(m, m), (m, n), (n, m), (n, n)\}$

(ii) True

(iii) True

Question-5

If $A = \{-1, 1\}$, find $A \times A \times A$.

Ans.

It is known that for any non-empty set A, $A \times A \times A$ is defined as

$A \times A \times A = \{(a, b, c): a, b, c \in A\}$

It is given that $A = \{-1, 1\}$

$\therefore A \times A \times A = \{(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1),$

$(1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)\}$

Question-6

If $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$. Find A and B.

Ans.

It is given that $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$

We know that the Cartesian product of two non-empty sets P and Q is defined as $P \times Q = \{(p, q): p \in P, q \in Q\}$

$\therefore$ A is the set of all first elements and B is the set of all second elements.

Thus, $A = \{a, b\}$ and $B = \{x, y\}$

Question-7

Let $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$. Verify that

(i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(ii) $A \times C$ is a subset of $B \times D$

Ans.

(i) To verify: $A \times (B \cap C) = (A \times B) \cap (A \times C)$

We have $B \cap C = \{1, 2, 3, 4\} \cap \{5, 6\} = \Phi$

$\therefore$ L.H.S. $= A \times (B \cap C) = A \times \Phi = \Phi$

$A \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)\}$

$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$

$\therefore$ R.H.S. $= (A \times B) \cap (A \times C) = \Phi$

$\therefore$ L.H.S. = R.H.S

Hence, $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(ii) To verify: $A \times C$ is a subset of $B \times D$

$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$

$B \times D = \{(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)\}$

We can observe that all the elements of set $A \times C$ are the elements of set $B \times D$.

Therefore, $A \times C$ is a subset of $B \times D$.

Question-8

Let $A = \{1, 2\}$ and $B = \{3, 4\}$. Write $A \times B$. How many subsets will $A \times B$ have? List them.

Ans.

$A = \{1, 2\}$ and $B = \{3, 4\}$

$\therefore A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$

$\Rightarrow n(A \times B) = 4$

We know that if C is a set with $n(C) = m$, then $n[P(C)] = 2^m$.

Therefore, the set $A \times B$ has $2^4 = 16$ subsets. These are

Φ, $\{(1, 3)\}$, $\{(1, 4)\}$, $\{(2, 3)\}$, $\{(2, 4)\}$, $\{(1, 3), (1, 4)\}$, $\{(1, 3), (2, 3)\}$,

$\{(1, 3), (2, 4)\}$, $\{(1, 4), (2, 3)\}$, $\{(1, 4), (2, 4)\}$, $\{(2, 3), (2, 4)\}$,

$\{(1, 3), (1, 4), (2, 3)\}$, $\{(1, 3), (1, 4), (2, 4)\}$, $\{(1, 3), (2, 3), (2, 4)\}$,

$\{(1, 4), (2, 3), (2, 4)\}$, $\{(1, 3), (1, 4), (2, 3), (2, 4)\}$

Question-9

Let A and B be two sets such that $n(A) = 3$ and $n(B) = 2$. If $(x, 1)$, $(y, 2)$, $(z, 1)$ are in $A \times B$, find A and B, where x, y and z are distinct elements.

Ans.

It is given that $n(A) = 3$ and $n(B) = 2$; and $(x, 1), (y, 2), (z, 1)$ are in $A \times B$.

We know that A = Set of first elements of the ordered pair elements of $A \times B$

B = Set of second elements of the ordered pair elements of $A \times B$.

$\therefore x, y,$ and z are the elements of A; and 1 and 2 are the elements of B.

Since $n(A) = 3$ and $n(B) = 2$, it is clear that $A = \{x, y, z\}$ and $B = \{1, 2\}$.

Question-10

The Cartesian product $A \times A$ has 9 elements among which are found $(-1, 0)$ and $(0, 1)$. Find the set A and the remaining elements of $A \times A$.

Ans.

We know that if $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.

$\therefore n(A \times A) = n(A) \times n(A)$

It is given that $n(A \times A) = 9$

$\therefore n(A) \times n(A) = 9$

$\Rightarrow n(A) = 3$

The ordered pairs $(-1, 0)$ and $(0, 1)$ are two of the nine elements of $A \times A$.

We know that $A \times A = \{(a, a): a \in A\}$. Therefore, $-1, 0,$ and 1 are elements of A.

Since $n(A) = 3$, it is clear that $A = \{-1, 0, 1\}$.

The remaining elements of set $A \times A$ are $(-1, -1), (-1, 1), (0, -1), (0, 0),$

$(1, -1), (1, 0),$ and $(1, 1)$

*********** END **********