



Algebra of Matrices Ex 5.3 Q65(i)

$$\text{Let, } A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$$

$$\begin{aligned} AB &= \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0+0 & ab+0 \\ 0+0 & 0+0 \end{bmatrix} \end{aligned}$$

$$AB = \begin{bmatrix} 0 & ab \\ 0 & 0 \end{bmatrix} \quad \text{---(i)}$$

$$\begin{aligned} BA &= \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} \end{aligned}$$

$$BA = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

From equation(i) and (ii)

$$AB \neq BA$$

$$\text{when } A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q65(ii)

$$\text{Let, } A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \neq 0$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix} \neq 0$$

$$\begin{aligned} AB &= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

Hence,

$$AB = 0$$

When,

$$A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \neq 0$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix} \neq 0$$

$$\text{Let, } A = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{aligned} AB &= \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix} \begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

$$AB = 0$$

$$\begin{aligned} BA &= \begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0+0 & ab+0 \\ 0+0 & 0+0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & ab \\ 0 & 0 \end{bmatrix} \end{aligned}$$

$$BA \neq 0$$

Hence,

for $AB = 0$ and $BA \neq 0$ we have,

$$A = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix}$$

Let, $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

Here,

$$A \neq 0, B \neq C$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\text{LHS} = \text{RHS}$$

So,

$$\text{for } A \neq 0, BC \neq 0 \text{ but } AB = AC$$

We have,

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

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