



Binary Operations Ex 3.4 Q1

Given,

$$a * b = a + b - 4 \text{ for all } a, b \in \mathbb{Z}$$

(i)

Commutative: Let $a, b \in \mathbb{Z}$, then

$$\Rightarrow a * b = a + b - 4 = b + a - 4 = b * a$$

$$\Rightarrow a * b = b * a$$

So, '*' is commutative on \mathbb{Z} .

Associativity: Let $a, b, c \in \mathbb{Z}$, then

$$\begin{aligned} (a * b) * c &= (a + b - 4) * c = a + b - 4 + c - 4 \\ &= a + b + c - 8 \end{aligned} \quad \text{--- (i)}$$

$$\text{and, } a * (b * c) = a * (b + c - 4) = a + b + c - 8 \quad \text{--- (ii)}$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

So, '*' is associative on \mathbb{Z} .

(ii)

Let $e \in \mathbb{Z}$ be the identity element with respect to *.

By identity property, we have

$$a * e = e * a = a \text{ for all } a \in \mathbb{Z}$$

$$\Rightarrow a + e - 4 = a$$

$$\Rightarrow e = 4$$

So, $e = 4$ will be the identity element with respect to *

(iii)

Let $b \in \mathbb{Z}$ be the inverse element of $a \in \mathbb{Z}$

$$\text{Then, } a * b = b * a = e$$

$$\Rightarrow a + b - 4 = e$$

$$\Rightarrow a + b - 4 = 4 \quad [\because e = 4]$$

$$\Rightarrow b = 8 - a$$

Thus, $b = 8 - a$ will be the inverse element of $a \in \mathbb{Z}$.

Binary Operations Ex 3.4 Q2

We have,

$$a * b = \frac{3ab}{5} \text{ for all } a, b \in Q_0$$

(i)

Commutative: Let $a, b \in Q_0$, then

$$a * b = \frac{3ab}{5} = \frac{3ba}{5} = b * a$$

$$\Rightarrow a * b = b * a$$

So, '*' is commutative on Q_0

Associativity: Let $a, b, c \in Q_0$, then

$$\begin{aligned} (a * b) * c &= \frac{3ab}{5} * c \\ &= \frac{9abc}{25} \end{aligned} \quad \text{--- (i)}$$

$$\begin{aligned} \text{and, } a * (b * c) &= a * \frac{3bc}{5} \\ &= \frac{9abc}{25} \end{aligned} \quad \text{--- (ii)}$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

So, '*' is associative on Q_0

(ii)

Let $e \in Q_0$ be the identity element with respect to *, then

$$a * e = e * a = a \text{ for all } a \in Q_0$$

$$\Rightarrow \frac{3ae}{5} = a$$

$$\Rightarrow e = \frac{5}{3}$$

will be the identity element with respect to *.

(iii)

Let $b \in Q_0$ be the inverse element of $a \in Q_0$, then

$$a * b = b * a = e$$

$$\Rightarrow \frac{3}{5}ab = e$$

$$\Rightarrow \frac{3}{5}ab = \frac{5}{3} \quad \left[\because e = \frac{5}{3} \right]$$

$$\Rightarrow b = \frac{25}{9a}$$

$\therefore \frac{25}{9a}$ is the inverse of $a \in Q_0$

.. $\frac{1}{9a}$ is the inverse of $a \in \mathbb{Q}_0$.

Binary Operations Ex 3.4 Q3

We have,

$$a * b = a + b + ab \text{ for all } a, b \in \mathbb{Q} - \{-1\}$$

(i)

Commutativity: Let $a, b \in \mathbb{Q} - \{-1\}$

$$\Rightarrow a * b = a + b + ab = b + a + ba = b * a$$

$$\Rightarrow a * b = b * a$$

$$\Rightarrow '*' \text{ is commutative on } \mathbb{Q} - \{-1\}$$

Associativity: Let $a, b, c \in \mathbb{Q} - \{-1\}$, then

$$\begin{aligned} \Rightarrow (a * b) * c &= (a + b + ab) * c \\ &= a + b + ab + c + ac + bc + abc \end{aligned} \quad \text{--- (i)}$$

$$\begin{aligned} \text{and, } a * (b * c) &= a * (b + c + bc) \\ &= a + b + c + bc + ab + ac + abc \end{aligned} \quad \text{--- (ii)}$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

$$\Rightarrow * \text{ is associative on } \mathbb{Q} - \{-1\}$$

(ii)

Let e be identity element with respect to $*$.

By identity property,

$$a * e = a = e * a \text{ for all } a \in \mathbb{Q} - \{-1\}$$

$$\Rightarrow a + e + ae = a$$

$$\Rightarrow e(1 + a) = 0 \Rightarrow e = 0 \quad \left[\because 1 + a \neq 0 \text{ as } a \neq -1 \right]$$

$\therefore e = 0$ is the identity element with respect to $*$

(iii)

Let b be the inverse of $a \in \mathbb{Q} - \{-1\}$

$$\text{Then, } a * b = b * a = e \quad [e \text{ is the identity element}]$$

$$\Rightarrow a + b + ab = e$$

$$\Rightarrow a + b + ab = 0$$

$$\Rightarrow b(1 + a) = -a$$

$$\Rightarrow b = \frac{-a}{1+a} \quad \left[\because \frac{-a}{1+a} \neq -1, \text{ because if } \frac{-a}{1+a} = -1 \right. \\ \left. \Rightarrow a = 1+a \Rightarrow 1 = 0 \text{ Not possible} \right]$$

$\therefore b = \frac{-a}{1+a}$ is the inverse of a with respect to $*$

Binary Operations Ex 3.4 Q4

We have,

$$(a, b) \odot (c, d) = (ac, bc + d) \text{ for all } (a, b), (c, d) \in R_0 \times R$$

(i)

Commutativity: Let $(a, b), (c, d) \in R_0 \times R$, then

$$\Rightarrow (a, b) \odot (c, d) = (ac, bc + d) \quad \text{--- (i)}$$

$$\text{and, } (c, d) \odot (a, b) = (ca, da + b) \quad \text{--- (ii)}$$

From (i) & (ii)

$$(a, b) \odot (c, d) \neq (c, d) \odot (a, b)$$

$$\Rightarrow ' \odot ' \text{ is not commutative on } R_0 \times R.$$

Associativity: Let $(a, b), (c, d), (e, f) \in R_0 \times R$, then

$$\begin{aligned} \Rightarrow ((a, b) \odot (c, d)) \odot (e, f) &= (ac, bc + d) \odot (e, f) \\ &= (ace, bce + de + f) \end{aligned} \quad \text{--- (i)}$$

$$\begin{aligned} \text{and, } (a, b) \odot (c, d \odot (e, f)) &= (a, b) \odot (ce, de + f) \\ &= (ace, bce + de + f) \end{aligned} \quad \text{--- (ii)}$$

$$\Rightarrow ((a, b) \odot (c, d)) \odot (e, f) = (a, b) \odot ((c, d) \odot (e, f))$$

$$\Rightarrow ' \odot ' \text{ is associative on } R_0 \times R.$$

(ii)

Let $(x, y) \in R_0 \times R$ be the identity element with respect to \odot , then

$$(a, b) \odot (x, y) = (x, y) \odot (a, b) = (a, b) \text{ for all } (a, b) \in R_0 \times R$$

$$\Rightarrow (ax, bx + y) = (a, b)$$

$$\Rightarrow ax = a \text{ and } bx + y = b$$

$$\Rightarrow x = 1, \text{ and } y = 0$$

$$\therefore (1, 0) \text{ will be the identity element with respect to } \odot.$$

(iii)

Let $(c, d) \in R_0 \times R$ be the inverse of $(a, b) \in R_0 \times R$, then

$$(a, b) \odot (c, d) = (c, d) \odot (a, b) = e$$

$$\Rightarrow (ac, bc + d) = (1, 0) \quad [\because e = (1, 0)]$$

$$\Rightarrow ac = 1 \text{ and } bc + d = 0$$

$$\Rightarrow c = \frac{1}{a} \text{ and } d = -\frac{b}{a}$$

$$\therefore \left(\frac{1}{a}, -\frac{b}{a} \right) \text{ will be the inverse of } (a, b).$$

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