



Complex Numbers Ex 13.2 Q7

$$\begin{aligned}
 \text{let } z &= \frac{1+i}{1-i} - \frac{1-i}{1+i} \\
 &= \frac{(1+i)^2 - (1-i)^2}{(1-i)(1+i)} \\
 &= \frac{1^2 + i^2 + 2 \times 1 \times i - (1^2 + i^2 - 2 \times 1 \times i)}{1^2 + 1^2} \\
 &= \frac{1 - 1 + 2i - (1 - 1 - 2i)}{2} \\
 &= \frac{2i + 2i}{2} \\
 &= \frac{4i}{2} \\
 \Rightarrow z &= 2i
 \end{aligned}$$

$$\begin{aligned}
 \therefore |z| &= |2i| \\
 &= 2|i| & (\because |z_1 z_2| &= |z_1| \times |z_2|) \\
 &= 2 \times 1 & (\because |i| &= 1) \\
 &= 2
 \end{aligned}$$

Complex Numbers Ex 13.2 Q8

$$x + iy = \frac{a+ib}{a-ib}$$

$$\Rightarrow \overline{(x+iy)} = \overline{\left(\frac{a+ib}{a-ib}\right)} \quad (\text{on taking conjugate both sides})$$

$$\begin{aligned}
 \Rightarrow x - iy &= \frac{\overline{(a+ib)}}{\overline{(a-ib)}} & \left( \because \overline{\left(\frac{z_1}{z_2}\right)} &= \frac{\overline{z_1}}{\overline{z_2}} \right) \\
 &= \frac{a-ib}{a+ib}
 \end{aligned}$$

$$\because (x+iy)(x-iy) = \frac{a+ib}{a-ib} \times \frac{a-ib}{a+ib}$$

$$\Rightarrow x^2 + y^2 = 1$$

proved

Complex Numbers Ex 13.2 Q9

For  $n = 1$ , we have,

$$\begin{aligned}\left(\frac{1+i}{1-i}\right)^1 &= \frac{1+i}{1-i} \\ &= \frac{1+i}{1-i} \times \frac{1+i}{1+i} \\ &= \frac{(1+i)^2}{1^2+1^2} \\ &= \frac{1^2+i^2+2 \times 1 \times i}{2} \\ &= \frac{2i}{2} \quad (\because i^2 = -1) \\ &= i, \text{ which is not real}\end{aligned}$$

For  $n = 2$ , we have

$$\begin{aligned}\left(\frac{1+i}{1-i}\right)^2 &= i^2 \quad \left(\because \frac{1+i}{1-i} = 1 \text{ from above}\right) \\ &= -1, \text{ which is real}\end{aligned}$$

Hence the least positive integral value of  $n$  is 2.

Complex Numbers Ex 13.2 Q10

$$\begin{aligned}\text{let } z &= \frac{1+i \cos \theta}{1-2i \cos \theta} \\ &= \frac{1+i \cos \theta}{1-2i \cos \theta} \times \frac{1+2i \cos \theta}{1+2i \cos \theta} \\ &= \frac{1+2i \cos \theta + i \cos \theta (1+2i \cos \theta)}{1^2 + (2 \cos \theta)^2} \\ &= \frac{1+2i \cos \theta + i \cos \theta - 2 \cos^2 \theta}{1+4 \cos^2 \theta} \\ &= \frac{1-2 \cos^2 \theta + 3i \cos \theta}{1+4 \cos^2 \theta} \\ &= \frac{1-2 \cos^2 \theta}{1+4 \cos^2 \theta} + \frac{3 \cos \theta}{1+4 \cos^2 \theta} i\end{aligned}$$

we know that  $z$  is purely real if and only if  $\text{Im } z = 0$

$$\begin{aligned}\therefore \frac{3 \cos \theta}{1+4 \cos^2 \theta} &= 0 \quad (\because z \text{ is given to be purely real}) \\ \Rightarrow 3 \cos \theta &= 0 \\ \Rightarrow \cos \theta &= 0 \\ \Rightarrow \cos \theta &= \cos \frac{\pi}{2}\end{aligned}$$

$\therefore$  The general solution is given by

$$\theta = 2n\pi \pm \frac{\pi}{2}, n \in \mathbb{Z}$$

\*\*\*\*\* END \*\*\*\*\*