

Functions Ex 2.1 Q5(xvi)

$$f: R \to R$$
 defined by  $f(x) = 1 + x^2$ 

Injective: let  $x, y \in R$  such that

$$f(x) = f(y)$$

$$\Rightarrow 1 + x^2 = 1 + y^2$$

$$\Rightarrow x^2 - y^2 = 0$$

$$\Rightarrow (x - y)(x + y) = 0$$

either x = y or x = -y or  $x \neq y$ 

∴ f is not one-one.

Surjective: let  $y \in R$  be arbitrary, then

$$f(x) = y$$

$$\Rightarrow 1 + x^2 = y$$

$$\Rightarrow x^2 + 1 - y = 0$$

$$\therefore \qquad X = \pm \sqrt{y-1} \notin R \text{ for } y < 1$$

f is not onto.

Functions Ex 2.1 Q6 Given,  $f: A \rightarrow B$  is injective such that range  $\{f\}$  =  $\{a\}$ 

- (, (,

We know that in injective map different elements have different images.  $\ \, ... \ \, \text{A has only one element}.$ 

Functions Ex 2.1 Q7

$$A = R - \{3\}, B = R - \{1\}$$

$$f: A \to B$$
 is defined as  $f(x) = \left(\frac{x-2}{x-3}\right)$ .

Let  $x, y \in A$  such that f(x) = f(y)

$$\Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$$

$$\Rightarrow$$
  $(x-2)(y-3)=(y-2)(x-3)$ 

$$\Rightarrow xy-3x-2y+6=xy-3y-2x+6$$

$$\Rightarrow$$
  $-3x-2y=-3y-2x$ 

$$\Rightarrow 3x-2x=3y-2y$$

$$\Rightarrow x = y$$

Therefore, f is one-one.

Let 
$$y \in B = \mathbf{R} - \{1\}.$$

Then,  $y \neq 1$ .

The function f is onto if there exists  $x \in A$  such that f(x) = y.

Now,

$$f(x) = y$$

$$\Rightarrow \frac{x-2}{x-3} = y$$

$$\Rightarrow x-2=xy-3y$$

$$\Rightarrow x(1-y) = -3y + 2$$

$$\Rightarrow x = \frac{2 - 3y}{1 - y} \in A \qquad [y \neq$$

Thus, for any  $y \in B$ , there exists  $\frac{2-3y}{1-y} \in A$  such that

$$f\left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right) - 2}{\left(\frac{2-3y}{1-y}\right) - 3} = \frac{2-3y-2+2y}{2-3y-3+3y} = \frac{-y}{-1} = y.$$

 $\therefore f$  is onto

Hence, function f is one-one and onto.

Functions Ex 2.1 Q8

We have  $f: R \to R$  given by f(x) = x - [x]Now,

check for injectivity:

$$\forall f(x) = x - [x] \Rightarrow f(x) = 0 \text{ for } x \in Z$$

∴ Range of  $f = [0,1] \neq R$ 

∴ f is not one-one, where as many-one

## Again, Range of $f = [0,1] \neq R$

## ∴ f is an into function

Functions Ex 2.1 Q9

Suppose  $f(n_1) = f(n_2)$ 

If  $n_1$  is odd and  $n_2$  is even, then we have

 $n_1 + 1 = n_2 - 1 \Rightarrow n_2 - n_1 = 2$ , not possible

If  $n_1$  is even and  $n_2$  is odd, then we have

 $n_1 - 1 = n_2 + 1 \Rightarrow n_1 - n_2 = 2$ , not possible

Therefore, both  $n_1$  and  $n_2$  must be either odd or even.

Suppose both n<sub>1</sub> and n<sub>2</sub> are odd.

Then,  $f(n_1) = f(n_2) \Rightarrow n_1 + 1 = n_2 + 1 \Rightarrow n_1 = n_2$ 

Suppose both n<sub>1</sub> and n<sub>2</sub> are even.

Then, 
$$f(n_1) = f(n_2) \Rightarrow n_1 - 1 = n_2 - 1 \Rightarrow n_1 = n_2$$

Thus, f is one - one.

Also, any odd number 2r+1 in the  $co-domain\ N$  will have an even number as image in domain N which is

$$f(n) = 2r + 1 \Rightarrow n - 1 = 2r + 1 \Rightarrow n = 2r + 2$$

any even number 2r in the co-domain N will have an odd number as image in domain N which is

$$f(n) = 2r \Rightarrow n+1 = 2r \Rightarrow n = 2r-1$$

Thus, f is onto.

Functions Ex 2.1 Q10

We have  $A = \{1, 2, 3\}$ 

All one-one functions from  $A = \{1, 2, 3\}$  to itself are obtained by re-arranging elements of A.

Thus all possible one-one functions are:

i) 
$$f(1) = 1$$
,  $f(2) = 2$ ,  $f(3) = 3$ 

ii) 
$$f(1) = 2$$
,  $f(2) = 3$ ,  $f(3) = 1$ 

iii) 
$$f(1) = 3$$
,  $f(2) = 1$ ,  $f(3) = 2$ 

$$|v| f(1) = 1, f(2) = 3, f(3) = 2$$

$$\forall f(1) = 3, f(2) = 2, f(3) = 1$$

$$\forall i \ f(1) = 2, \ f(2) = 1, \ f(3) = 3$$

\*\*\*\*\*\*\* END \*\*\*\*\*\*\*