Indefinite Integrals Ex 19.15 Q1

$$\text{Let } I = \int \frac{1}{4x^2 + 12x + 5} dx$$

$$= \frac{1}{4} \int \frac{1}{x^2 + 3x + \frac{5}{4}} dx$$

$$= \frac{1}{4} \int \frac{1}{x^2 + 2 \times x \times \left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + \frac{5}{4}} dx$$

$$I = \frac{1}{4} \int \frac{1}{\left(x + \frac{3}{2}\right)^2 - 1} dx$$

$$\text{Let } \left(x + \frac{3}{2}\right) = t \text{ - - - - - - - - (i)}$$

$$\Rightarrow \quad dx = dt$$

so,

$$I = \frac{1}{4} \int \frac{1}{t^2 - (1)^2} dt$$

$$I = \frac{1}{4} \times \frac{1}{2 \times (1)} \log \left| \frac{t-1}{t+1} \right| + c \qquad \left[\text{Since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c \right]$$

$$I = \frac{1}{8} \log \left| \frac{x + \frac{3}{2} - 1}{x + \frac{3}{2} + 1} \right| + c \qquad [\text{using (i)}]$$

$$I = \frac{1}{8} \log \left| \frac{2x+1}{2x+5} \right| + c$$

Indefinite Integrals Ex 19.15 Q2

$$\text{Let } I = \int \frac{1}{x^2 - 10x + 34} dx$$

$$= \int \frac{1}{x^2 - 2x \times 5 + (5)^2 - (5)^2 + 34} dx$$

$$= \int \frac{1}{(x-5)^2 + 9} dx$$

$$\text{Let } (x-1) = t \text{ - - - - - - - - (i)}$$

$$\Rightarrow \quad dx = dt$$

so,

$$I = \int \frac{1}{t^2 + (3)^2} dt$$

$$I = \frac{1}{3} \tan^{-1} \left(\frac{t}{3}\right) + c \qquad \left[\text{Since, } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{2}\right) + c \right]$$

$$I = \frac{1}{3} \tan^{-1} \left(\frac{x-5}{3}\right) + c \qquad [\text{using (i)}]$$

Indefinite Integrals Ex 19.15 Q3

$$\int \frac{1}{1+x-x^2}dx = \int \frac{1}{-\left(x^2-x-1\right)}dx$$

adding and subtracting $\frac{1}{4}$ in the denominator to make it a perfect square

$$= \int \frac{1}{-\left(x^2-x+\frac{1}{4}-1-\frac{1}{4}\right)}dx$$

$$= \int \frac{1}{-\left(\left[x^2-x+\frac{1}{4}\right]-1-\frac{1}{4}\right)}dx$$

$$= \int \frac{1}{-\left(\left[x-\frac{1}{2}\right]^2-1-\frac{1}{4}\right)}dx = \int \frac{1}{-\left(\left[x-\frac{1}{2}\right]^2-\frac{5}{4}\right)}dx$$

$$= \int \frac{1}{\left(\left(\frac{\sqrt{5}}{2}\right)^2-\left[x-\frac{1}{2}\right]^2\right)}d$$

$$= \frac{1}{2\left(\frac{\sqrt{5}}{2}\right)}\log\left|\frac{\frac{\sqrt{5}}{2}+\left(x-\frac{1}{2}\right)}{\frac{\sqrt{5}}{2}-\left(x-\frac{1}{2}\right)}\right|$$

$$= \frac{1}{\sqrt{5}}\log\left|\frac{\sqrt{5}+2x-1}{\sqrt{5}-2x+1}\right|$$

$$= \frac{1}{\sqrt{5}}\log\left|\frac{\sqrt{5}-1+2x}{\sqrt{5}+1-2x}\right|$$

Indefinite Integrals Ex 19.15 Q4

Let $I = \int \frac{1}{2x^2 - x - 1} dx$

$$= \frac{1}{2} \int \frac{1}{x^2 - \frac{x}{2} - \frac{1}{2}} dx$$

$$= \frac{1}{2} \int \frac{1}{x^2 - 2x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - \frac{1}{2}} dx$$

$$= \frac{1}{2} \int \frac{1}{\left(x - \frac{1}{4}\right)^2 - \frac{9}{16}} dx$$

Let $x - \frac{1}{4} = t$

$\Rightarrow \quad dx = dt$

$$I = \frac{1}{2} \int \frac{1}{t^2 - \left(\frac{3}{4}\right)^2} dt$$

$$I = \frac{1}{2} \times \frac{1}{2 \times \left(\frac{3}{4}\right)} \log \left| \frac{t - \frac{3}{4}}{t + \frac{3}{4}} \right| + c \quad \left[\text{Since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x - a}{x + a} \right| + c\right]$$

$$I = \frac{1}{3} \log \left| \frac{x - \frac{1}{4} - \frac{3}{4}}{x - \frac{1}{4} + \frac{3}{4}} \right| + c$$

$$I = \frac{1}{3} \log \left| \frac{x - 1}{2x + 1} \right| + c$$

Indefinite Integrals Ex 19.15 Q5

We have $x^2 + 6x + 13 = x^2 + 6x + 3^2 - 3^2 + 13 = (x + 3)^2 + 4$

Sol, $\int \frac{dx}{x^2 + 6x + 13} = \int \frac{1}{(x + 3)^2 + 2^2} dx$

Let $x + 3 = t$. Then $dx = dt$

Therefore, $\int \frac{dx}{x^2 + 6x + 13} = \int \frac{dt}{t^2 + 2^2} = \frac{1}{2} \tan^{-1} \frac{t}{2} + C$ [by 7.4 (3)]

$$= \frac{1}{2} \tan^{-1} \frac{x + 3}{2} + C$$

*********** END **********