Indefinite Integrals Ex 19.23 Q5

Let $\quad I = \int \dfrac{1}{1 - \sin x + \cos x}\, dx$

Put $\quad \sin x = \dfrac{2\tan\dfrac{x}{2}}{1 + \tan^2\dfrac{x}{2}}, \cos x = \dfrac{1 - \tan^2\dfrac{x}{2}}{1 + \tan^2\dfrac{x}{2}}$

$$I = \int \dfrac{1}{1 - \dfrac{2\tan\dfrac{x}{2}}{1 + \tan^2\dfrac{x}{2}} + \dfrac{1 - \tan^2\dfrac{x}{2}}{1 + \tan^2\dfrac{x}{2}}}\, dx$$

$$= \int \dfrac{1 + \tan^2\dfrac{x}{2}}{1 + \tan^2\dfrac{x}{2} - 2\tan\dfrac{x}{2} + 1 - \tan^2\dfrac{x}{2}}\, dx$$

$$= \int \dfrac{\sec^2\dfrac{x}{2}}{2 - 2\tan\dfrac{x}{2}}\, dx$$

Let $\quad \tan\dfrac{x}{2} = t$

$\dfrac{1}{2}\sec^2\dfrac{x}{2}\, dx = dt$

$= \dfrac{2}{2}\int \dfrac{dt}{1 - t}$

$= -\log|1 - t| + c$

$$I = -\log\left|1 - \tan\dfrac{x}{2}\right| + c$$

Indefinite Integrals Ex 19.23 Q6

Let $\quad I = \int \dfrac{1}{3 + 2\sin x + \cos x}\, dx$

Put $\quad \sin x = \dfrac{2\tan\dfrac{x}{2}}{1 + \tan^2\dfrac{x}{2}}, \quad \cos x = \dfrac{1 - \tan^2\dfrac{x}{2}}{1 + \tan^2\dfrac{x}{2}}$

$$I = \int \dfrac{1}{3 + 2\left(\dfrac{2\tan\dfrac{x}{2}}{1 + \tan^2\dfrac{x}{2}}\right) + \left(\dfrac{1 - \tan^2\dfrac{x}{2}}{1 + \tan^2\dfrac{x}{2}}\right)}\, dx$$

$$= \int \dfrac{\left(1 + \tan^2\dfrac{x}{2}\right)}{3\left(1 + \tan^2\dfrac{x}{2}\right) + 4\tan\dfrac{x}{2} + 1 - \tan^2\dfrac{x}{2}}\, dx$$

$$= \int \dfrac{\sec^2\dfrac{x}{2}}{3 + 3\tan^2\dfrac{x}{2} + 4\tan\dfrac{x}{2} + 1 - \tan^2\dfrac{x}{2}}\, dx$$

$$= \int \dfrac{\sec^2\dfrac{x}{2}}{2\tan^2\dfrac{x}{2} + 4\tan\dfrac{x}{2} + 4}\, dx$$

Let $\quad \tan\dfrac{x}{2} = t$

$$\dfrac{1}{2}\sec^2\dfrac{x}{2}\, dx = dt$$

$$I = \dfrac{2}{2}\int \dfrac{dt}{t^2 + 2t + 2}$$

$$= \int \dfrac{dt}{t^2 + 2t + 1 - 1 + 2}$$

$$I = \int \dfrac{dt}{(t + 1)^2 + (1)^2}$$

$$= \tan^{-1}(t + 1) + c$$

$$I = \tan^{-1}\left(\tan\dfrac{x}{2} + 1\right) + c$$

Indefinite Integrals Ex 19.23 Q7

Let $\quad I = \int \dfrac{1}{13 + 3\cos x + 4\sin x}\, dx$

Put $\quad \sin x = \dfrac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}, \qquad \cos x = \dfrac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$

$$I = \int \dfrac{1}{13 + 3\left(\dfrac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) + 4\left(\dfrac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right)}\, dx$$

$$= \int \dfrac{\left(1 + \tan^2\frac{x}{2}\right)}{13\left(1 + \tan^2\frac{x}{2}\right) + 3 - 3\tan^2\frac{x}{2} + 8\tan\frac{x}{2}}\, dx$$

$$= \int \dfrac{\sec^2\frac{x}{2}}{16 + 13\tan^2\frac{x}{2} - 3\tan^2\frac{x}{2} + 8\tan\frac{x}{2}}\, dx$$

Let $\quad \tan\dfrac{x}{2} = t$

$\dfrac{1}{2}\sec^2\dfrac{x}{2}\, dx = dt$

$I = \int \dfrac{2dt}{16 + 10t^2 + 8t}$

$= \dfrac{2}{10}\int \dfrac{dt}{t^2 + \dfrac{4}{5}t + \dfrac{8}{5}}$

$I = \dfrac{1}{5}\int \dfrac{dt}{t^2 + 2t\left(\dfrac{2}{5}\right) + \left(\dfrac{2}{5}\right)^2 - \left(\dfrac{2}{5}\right)^2 + \dfrac{8}{5}}$

$= \dfrac{1}{5}\int \dfrac{dt}{\left(t + \dfrac{2}{5}\right)^2 + \left(\dfrac{6}{5}\right)^2}$

$= \dfrac{1}{5} \times \dfrac{1}{\left(\dfrac{6}{5}\right)}\tan^{-1}\left(\dfrac{t + \dfrac{2}{5}}{\dfrac{6}{5}}\right) + c$

$= \dfrac{1}{6}\tan^{-1}\left(\dfrac{5t + 2}{6}\right) + c$

$$I = \dfrac{1}{6}\tan^{-1}\left(\dfrac{5\tan\frac{x}{2} + 2}{6}\right) + c$$

Indefinite Integrals Ex 19.23 Q8

Let $\quad I = \int \dfrac{1}{\cos x - \sin x} dx$

Put $\quad \cos x = \dfrac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \sin x = \dfrac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$I = \int \dfrac{1}{\left(\dfrac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right) - \left(\dfrac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right)} dx$$

$$= \int \dfrac{\left(1 + \tan^2 \frac{x}{2}\right)}{1 - \tan^2 \frac{x}{2} - 2 \tan \frac{x}{2}} dx$$

$$= \int \dfrac{\sec^2 \frac{x}{2}}{1 - \tan^2 \frac{x}{2} - 2 \tan \frac{x}{2}} dx$$

Let $\quad \tan \dfrac{x}{2} = t$

$$\dfrac{1}{2} \sec^2 \dfrac{x}{2} dx = dt$$

$$I = \int \dfrac{2dt}{1 - t^2 - 2t}$$

$$= -\int \dfrac{2dt}{t^2 + 2t - 1}$$

$$I = -\int \dfrac{2dt}{t^2 + 2t + 1 - 1 - 1}$$

$$I = -\int \dfrac{2dt}{(t + 1)^2 - (\sqrt{2})^2}$$

$$= \int \dfrac{2dt}{(\sqrt{2})^2 - (t + 1)^2}$$

$$= \dfrac{2}{2\sqrt{2}} \log \left| \dfrac{\sqrt{2} + t + 1}{\sqrt{2} - t - 1} \right| + c$$

$$I = \dfrac{1}{\sqrt{2}} \log \left| \dfrac{\sqrt{2} + \tan \frac{x}{2} + 1}{\sqrt{2} - \tan \frac{x}{2} - 1} \right| + c$$

*********** END *********