Indefinite Integrals Ex 19.24 Q1

Let $\quad I = \int \frac{1}{1 - \cot x} dx$

$= \int \frac{1}{1 - \frac{\cos x}{\sin x}} dx$

$= \int \frac{\sin x}{\sin x - \cos x} dx$

Let $\quad \sin x = \lambda \frac{d}{dx}(\sin x - \cos x) + \mu(\sin x - \cos x) + \nu$

$\sin x = \lambda \frac{d}{dx}(\cos x + \sin x) + \mu(\sin x - \cos x) + \nu$

$\sin x = \cos(\lambda - \mu) + \sin x\,(\lambda + \mu) + \nu$

Comparing the coefficients of $\sin x$ & $\cos x$ on the both the sides,

$\lambda + \mu = 1 \qquad \ldots(1)$

$\lambda - \mu = 1 \qquad \ldots(2)$

$\nu = 0 \qquad \ldots(3)$

Equation $(1),(2),(3)$ gives

$\lambda = \frac{1}{2}, \mu = \frac{1}{2}, \nu = 0$

$I = \int \frac{\frac{1}{2}(\cos x + \sin x) + \frac{1}{2}(\sin x - \cos x)}{(\sin x - \cos x)} dx$

$= \frac{1}{2} \int \frac{(\cos x + \sin x)}{(\sin x - \cos x)} dx + \frac{1}{2} \int dx$

$I = \frac{1}{2} \log|\sin x - \cos x| + \frac{1}{2} x + c$

Indefinite Integrals Ex 19.24 Q2

Let $\quad I = \int \frac{1}{1-\tan x} dx$

$= \int \frac{1}{1-\frac{\sin x}{\cos x}} dx$

$= \int \frac{\cos x}{\cos x - \sin x} dx$

Let $\quad \cos x = \lambda \frac{d}{dx}\left(\cos x - \sin x\right) + \mu\left(\cos x - \sin x\right) + \nu$

$= \lambda \frac{d}{dx}\left(-\sin x - \cos x\right) + \mu\left(\cos x - \sin x\right) + \nu$

$\cos x = \sin x\left(-\lambda - \mu\right) + \cos x\left(-\lambda + \mu\right) + \nu$

Camparing the coefficients of $\cos x$ & $\sin x$ on the both the sides,

$-\lambda - \mu = 0 ------ (1)$

$-\lambda + \mu = 1 ------ (2)$

$\nu = 0 --------- (3)$

Equation $(1), (2), (3)$ gives

$\lambda = -\frac{1}{2}, \mu = \frac{1}{2}, \nu = 0$

$I = \int \frac{-\frac{1}{2}\left(-\sin x - \cos x\right) + \frac{1}{2}\left(\cos x - \sin x\right)}{\left(\cos x - \sin x\right)} dx$

$= \frac{1}{2}\int \frac{\left(\sin x + \cos x\right)}{\left(\cos x - \sin x\right)} dx + \frac{1}{2}\int dx$

$= -\frac{1}{2}\log\left|\cos x - \sin x\right| + \frac{1}{2}x + c$

$I = \frac{1}{2}x - \frac{1}{2}\log\left|\cos x - \sin x\right| + c$

Indefinite Integrals Ex 19.24 Q3

Let $\quad I = \int \frac{3+2\cos x + 4\sin x}{2\sin x + \cos x + 3}dx$

Let $\quad 3+2\cos x + 4\sin x = \lambda \frac{d}{dx}(2\sin x + \cos x + 3) + \mu(2\sin x + \cos x + 3) + \nu$

$$3+2\cos x + 4\sin x = \lambda(2\cos x - \sin x) + \mu(2\sin x + \cos x + 3) + \nu$$

$$3+2\cos x + 4\sin x = (-\lambda + 2\mu)\sin x + (2\lambda + \mu)\cos x + 3\mu + \nu$$

Camparing the coefficients of $\sin x$ & $\cos x$ on the both the sides,

$$-\lambda + 2\mu = 4 ------(1)$$

$$2\lambda + \mu = 2 ------(2)$$

$$2\mu + \nu = 3 ---------(3)$$

Solving equation $(1),(2)$ and (3), we get

$$\lambda = 0, \mu = 2, \nu = -3$$

$$I = \int \frac{2(2\sin x + \cos x + 3) - 3}{(2\sin x + \cos x + 3)}dx$$

$$= 2\int dx - 3\int \frac{1}{2\sin x + \cos x + 3}dx$$

$$I = 2x - 3I_1 + C_1 -------(4)$$

Let $\quad I_1 = \int \frac{1}{2\sin x + \cos x + 3}dx$

Put $\quad \sin x = \frac{2\tan\frac{x}{2}}{\left(1+\tan^2\frac{x}{2}\right)}, \quad \cos x = \frac{1-\tan^2\frac{x}{2}}{\left(1+\tan^2\frac{x}{2}\right)}$

$$I_1 = \int \frac{1}{2\left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right) + \left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right) + 3}dx$$

$$= \int \frac{\left(1+\tan^2\frac{x}{2}\right)}{4\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2} + 3\left(1+\tan^2\frac{x}{2}\right)}dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{2\tan^2\frac{x}{2} + 4\tan\frac{x}{2} + 4}dx$$

Let $\quad \tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2} = dt$$

$$I_1 = \int \frac{2dt}{2t^2 + 4t + 4}$$

$$= \frac{2}{2}\int \frac{dt}{t^2 + 2t + 2}$$

$$= \int \frac{dt}{t^2 + 2t + 1 - 1 + 2}$$

$$= \int \frac{dt}{(t+1)^2 + 1}$$

$$= \tan^{-1}(t+1) + C_2$$

$$= \tan^{-1}\left(\tan\frac{x}{2} + 1\right) + C_2$$

Now, using equation (1),

$$I = 2x - 3\tan^{-1}\left(\tan\frac{x}{2} + 1\right) + C$$

*********** END **********