



Trigonometric Ratios of Compound Angles Ex 7.1 Q16

We have,

$$\begin{aligned}
 \text{LHS} &= \frac{\sin(A+B) + \sin(A-B)}{\cos(A+B) + \cos(A-B)} \\
 &= \frac{2 \sin A \cos B}{2 \cos A \cos B} \quad \left[\begin{array}{l} \because 2 \sin A \cos B = \sin(A+B) + \sin(A-B) \\ \text{and, } 2 \cos A \cos B = \cos(A+B) + \cos(A-B) \end{array} \right] \\
 &= \frac{\sin A}{\cos A} \\
 &= \tan A \\
 &= \text{RHS}
 \end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

$$\begin{aligned}
 \text{LHS} &= \frac{\sin(A-B)}{\cos A \cos B} + \frac{\sin(B-C)}{\cos B \cos C} + \frac{\sin(C-A)}{\cos C \cos A} \\
 &= \frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B} + \frac{\sin B \cos C - \cos B \sin C}{\cos B \cos C} + \frac{\sin C \cos A - \cos C \sin A}{\cos C \cos A} \\
 &= \frac{\sin A \cos B}{\cos A \cos B} - \frac{\cos A \sin B}{\cos A \cos B} + \frac{\sin B \cos C}{\cos B \cos C} - \frac{\cos B \sin C}{\cos B \cos C} + \frac{\sin C \cos A}{\cos C \cos A} \\
 &\quad - \frac{\cos C \sin A}{\cos C \cos A} \\
 &= \tan A - \tan B + \tan B - \tan C + \tan C - \tan A \\
 &= 0 \\
 &= \text{RHS}
 \end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

We have,

$$\begin{aligned}
 \text{LHS} &= \frac{\sin(A-B)}{\sin A \sin B} + \frac{\sin(B-C)}{\sin B \sin C} + \frac{\sin(C-A)}{\sin C \sin A} \\
 &= \frac{\sin A \cos B - \cos A \sin B}{\sin A \sin B} + \frac{\sin B \cos C - \cos B \sin C}{\sin B \sin C} + \frac{\sin C \cos A - \cos C \sin A}{\sin C \sin A} \\
 &= \frac{\sin A \cos B}{\sin A \sin B} - \frac{\cos A \sin B}{\sin A \sin B} + \frac{\sin B \cos C}{\sin B \sin C} - \frac{\cos B \sin C}{\sin B \sin C} + \frac{\sin C \cos A}{\sin C \sin A} \\
 &\quad - \frac{\cos C \sin A}{\sin C \sin A} \\
 &= \frac{\cos B}{\sin B} - \frac{\cos A}{\sin A} + \frac{\cos C}{\sin C} - \frac{\cos B}{\sin B} + \frac{\cos A}{\sin A} - \frac{\cos C}{\sin C} \\
 &= \cot B - \cot A + \cot C - \cot B + \cot A - \cot C \quad \left[\because \cot \theta = \frac{\cos \theta}{\sin \theta} \right] \\
 &= 0 \\
 &= \text{RHS}
 \end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

We have,

$$\begin{aligned}
 \text{RHS} &= \sin^2 A + \sin^2 (A - B) - 2 \sin A \cos B \sin (A - B) \\
 &= \sin^2 A + \sin (A - B) [\sin (A - B) - 2 \sin A \cos B] \\
 &= \sin^2 A + \sin (A - B) [\sin A \cos B - \cos A \sin B - 2 \sin A \cos B] \\
 &= \sin^2 A + \sin (A - B) [-\sin A \cos B - \cos A \sin B] \\
 &= \sin^2 A - \sin (A - B) (\sin A \cos B + \cos A \sin B) \\
 &= \sin^2 A - \sin (A - B) (\sin (A + B)) \\
 &= \sin^2 A - \sin (A - B) \sin (A + B) \\
 &= \sin^2 A - (\sin^2 A - \sin^2 B) \quad [\because \sin (A - B) \sin (A + B) = \sin^2 A - \sin^2 B] \\
 &= \sin^2 A - \sin^2 A + \sin^2 B \\
 &= \sin^2 B \\
 &= \text{LHS}
 \end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

$$\begin{aligned}
 \text{RHS} &= \cos^2 A + \cos^2 B - 2 \cos A \cos B \cos (A + B) \\
 &= \cos^2 A + (1 - \sin^2 B) - 2 \cos A \cos B \cos (A + B) \\
 &= [\cos^2 A - \sin^2 B] - 2 \cos A \cos B \cos (A + B) + 1 \\
 &= [\cos (A + B) \cos (A - B)] - 2 \cos A \cos B \cos (A + B) + 1 \\
 &= \cos (A + B) [\cos (A - B) - 2 \cos A \cos B] + 1 \\
 &= \cos (A + B) [\cos A \cos B + \sin A \sin B - 2 \cos A \cos B] + 1 \\
 &= \cos (A + B) [-\cos A \cos B + \sin A \sin B] + 1 \\
 &= -\cos (A + B) [\cos A \cos B - \sin A \sin B] + 1 \\
 &= -\cos (A + B) [\cos (A + B)] + 1 \\
 &= -\cos^2 (A + B) + 1 \\
 &= 1 - \cos^2 (A + B) \\
 &= \sin^2 (A + B) \quad [\sin^2 \theta = 1 - \cos^2 \theta] \\
 &= \text{RHS}
 \end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

We have,

$$\begin{aligned}\text{LHS} &= \frac{\tan(A+B)}{\cot(A-B)} \\&= \frac{\tan(A+B)}{\frac{1}{\tan(A-B)}} && \left[\because \cot \theta = \frac{1}{\tan \theta} \right] \\&= \tan(A+B) \tan(A-B) \\&= \left[\frac{\tan A + \tan B}{1 - \tan A \tan B} \right] \left[\frac{\tan A - \tan B}{1 + \tan A \tan B} \right] \\&= \frac{(\tan A + \tan B)(\tan A - \tan B)}{(1 - \tan A \tan B)(1 + \tan A \tan B)} \\&= \frac{\tan^2 A - \tan^2 B}{1 - (\tan A \tan B)^2} && \left[\because (a-b)(a+b) = a^2 - b^2 \right] \\&= \frac{\tan^2 A - \tan^2 B}{1 - \tan^2 A \tan^2 B} \\&= \text{RHS}\end{aligned}$$

\therefore LHS = RHS

Hence proved.

***** END *****