



Indefinite Integrals Ex 19.30 Q11

$$\text{Let } \int \frac{\sin 2x}{(1 + \sin x)(2 + \sin x)} dx = \frac{A}{1 + \sin x} + \frac{B}{2 + \sin x}$$

$$\Rightarrow \sin 2x = A(2 + \sin x) + B(1 + \sin x)$$

$$\Rightarrow 2 \sin x \cos x = (2A + B) + (A + B) \sin x$$

Equating similar terms, we get,

$$2A + B = 0 \quad \Rightarrow \quad B = -2A \text{ and}$$

$$A + B = 2 \cos x \Rightarrow -A = 2 \cos x$$

$$\Rightarrow A = -2 \cos x$$

$$\text{and } B = +4 \cos x$$

Thus,

$$\begin{aligned} I &= \int -\frac{2 \cos x}{1 + \sin x} dx + \int \frac{4 \cos x}{2 + \sin x} dx \\ &= -2 \log|1 + \sin x| + 4 \log|2 + \sin x| + c \end{aligned}$$

$$I = \log \left| \frac{(2 + \sin x)^4}{(1 + \sin x)^2} \right| + c$$

Indefinite Integrals Ex 19.30 Q12

$$\text{Let } \int \frac{2x}{(x^2+1)(x^2+3)} dx = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+3}$$

$$\begin{aligned} \Rightarrow 2x &= (Ax+B)(x^2+3) + (Cx+D)(x^2+1) \\ &= (A+C)x^3 + (B+D)x^2 + (3A+C)x + (3B+D) \end{aligned}$$

Equating similar terms, we get,

$$A+C=0, B+D=0, 3A+C=2 \text{ and } 3B+D=0$$

$$\Rightarrow A=-C, B=D=0 \quad 2A=2 \quad \Rightarrow A=1 \text{ \& } C=-1$$

Thus,

$$\begin{aligned} I &= \int \frac{x dx}{x^2+1} - \int \frac{x dx}{x^2+3} \\ &= \frac{1}{2} \log|x^2+1| - \frac{1}{2} \log|x^2+3| + c \end{aligned}$$

$$I = \frac{1}{2} \log \left| \frac{x^2+1}{x^2+3} \right| + c$$

Indefinite Integrals Ex 19.30 Q13

$$\text{Let } \int \frac{1}{x \log x (2 + \log x)} = \frac{A}{x \log x} + \frac{B}{x (2 + \log x)}$$

$$\Rightarrow 1 = A(2 + \log x) + B \log x$$

Put  $x = 1$

$$\Rightarrow 1 = 2A \Rightarrow A = \frac{1}{2}$$

Put  $x = 10^{-2}$

$$\Rightarrow 1 = -2B \Rightarrow B = -\frac{1}{2}$$

Thus,

$$\begin{aligned} I &= \frac{1}{2} \int \frac{dx}{x \log x} + \left(-\frac{1}{2}\right) \int \frac{dx}{x (2 + \log x)} \\ &= \frac{1}{2} \log|\log x| - \frac{1}{2} \log|2 + \log x| + c \end{aligned}$$

$$I = \frac{1}{2} \log \left| \frac{\log x}{2 + \log x} \right| + c$$

Indefinite Integrals Ex 19.30 Q15

$$\text{Let } \frac{ax^2+bx+c}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$$

$$\Rightarrow ax^2+bx+c = A(x-b)(x-c) + B(x-a)(x-c) + C(x-a)(x-b)$$

Put  $x = a$

$$\Rightarrow a^3+ba+c = (a-b)(a-c)A \Rightarrow A = \frac{a^3+ba+c}{(a-b)(a-c)}$$

Put  $x = b$

$$\Rightarrow ab^2+b^2+c = (b-a)(b-c)B \Rightarrow B = \frac{ab^2+b^2+c}{(b-a)(b-c)}$$

Put  $x = c$

$$\Rightarrow ac^2+bc+c = (c-a)(c-b)C \Rightarrow C = \frac{ac^2+bc+c}{(c-a)(c-b)}$$

Thus,

$$I = \frac{a^3+ba+c}{(a-b)(a-c)} \int \frac{dx}{x-a} + \frac{ab^2+b^2+c}{(b-a)(b-c)} \int \frac{dx}{x-b} + \frac{ac^2+bc+c}{(c-a)(c-b)} \int \frac{dx}{x-c}$$

Hence,

$$I = \frac{a^3+ba+c}{(a-b)(a-c)} \log|x-a| + \frac{ab^2+b^2+c}{(b-a)(b-c)} \log|x-b| + \frac{ac^2+bc+c}{(c-a)(c-b)} \log|x-c| + c$$

Indefinite Integrals Ex 19.30 Q16

Consider the integral

$$I = \int \frac{x}{(x^2 + 1)(x - 1)} dx$$

Now let us separate the fraction  $\frac{x}{(x^2 + 1)(x - 1)}$

through partial fractions.

$$\frac{x}{(x^2 + 1)(x - 1)} = \frac{A}{x - 1} + \frac{Bx + C}{x^2 + 1}$$

$$\Rightarrow \frac{x}{(x^2 + 1)(x - 1)} = \frac{A(x^2 + 1) + (Bx + C)(x - 1)}{(x^2 + 1)(x - 1)}$$

$$\Rightarrow x = A(x^2 + 1) + (Bx + C)(x - 1)$$

$$\Rightarrow x = Ax^2 + A + Bx^2 - Bx + Cx - C$$

Comparing the coefficients, we have,

$$A + B = 0, \quad -B + C = 1 \quad \text{and} \quad A - C = 0$$

Solving the equations, we get,

$$\Rightarrow A = \frac{1}{2}, \quad B = -\frac{1}{2} \quad \text{and} \quad C = \frac{1}{2}$$

$$\Rightarrow \frac{x}{(x^2 + 1)(x - 1)} = \frac{A}{x - 1} + \frac{Bx + C}{x^2 + 1}$$

$$\Rightarrow \frac{x}{(x^2 + 1)(x - 1)} = \frac{1}{2} \times \frac{1}{x - 1} - \frac{1}{2} \times \frac{x - 1}{x^2 + 1}$$

$$\Rightarrow \frac{x}{(x^2 + 1)(x - 1)} = \frac{1}{2(x - 1)} - \frac{x}{2(x^2 + 1)} + \frac{1}{2(x^2 + 1)}$$

Thus, we have,

$$I = \int \frac{x}{(x^2 + 1)(x - 1)} dx$$

$$= \int \left[ \frac{1}{2(x - 1)} - \frac{x}{2(x^2 + 1)} + \frac{1}{2(x^2 + 1)} \right] dx$$

$$= \int \frac{dx}{2(x - 1)} - \int \frac{xdx}{2(x^2 + 1)} + \int \frac{dx}{2(x^2 + 1)}$$

$$= \frac{1}{2} \int \frac{dx}{(x - 1)} - \frac{1}{2} \int \frac{xdx}{(x^2 + 1)} + \frac{1}{2} \int \frac{dx}{(x^2 + 1)}$$

$$= \frac{1}{2} \int \frac{dx}{(x - 1)} - \frac{1}{2} \times \frac{1}{2} \int \frac{2xdx}{(x^2 + 1)} + \frac{1}{2} \int \frac{dx}{(x^2 + 1)}$$

$$= \frac{1}{2} \log|x - 1| - \frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \tan^{-1} x + C$$

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