Transformation Formulae Ex 8.1 Q5(i)

$\cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^0 = \frac{3}{16}$

LHS $= \cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ$

$= \cos 30^\circ \cos 10^\circ \cos 50^\circ \cos 70^\circ$

$= \frac{\sqrt{3}}{2}(\cos 10^\circ \cos 50^\circ \cos 70^\circ)$

$= \frac{\sqrt{3}}{2}(\cos 10^\circ \cos 50^\circ)\cos 70^\circ$

$= \frac{\sqrt{3}}{4}(2\cos 10^\circ \cos 50^\circ)\cos 70^\circ$ [Multiiplying and dividing by 2]

Also,

$\Rightarrow \quad 2\cos A \cos B = \cos(A+B) + \cos(A-B)$ —(i)

$= \frac{\sqrt{3}}{4}\cos 70^\circ\left(\cos(50^\circ + 10^\circ) + \cos(10^\circ - 50^\circ)\right)$

$= \frac{\sqrt{3}}{4}\cos 70^\circ\left(\cos 60^\circ + \cos(-40^\circ)\right)$

Now,

$\cos(-\theta) = \cos\theta$

$= \frac{\sqrt{3}}{4}\cos 70^\circ\left(\frac{1}{2} + \cos 40^\circ\right)$ $\left[\because \cos 60^\circ = \frac{1}{2}\right]$

$= \frac{\sqrt{3}}{8}\cos 70^\circ + \frac{\sqrt{3}}{4}\cos 70^\circ \cos 40^\circ$

$= \frac{\sqrt{3}}{8}\cos 70^\circ + \frac{\sqrt{3}}{8}(2\cos 70^\circ \cos 40^\circ)$

$= \frac{\sqrt{3}}{8}\left[\cos 70^\circ + \cos(70^\circ + 40^\circ) + \cos(70^\circ - 40^\circ)\right]$ [from (i)]

$= \frac{\sqrt{3}}{8}\left[\cos 70^\circ + \cos 110^\circ + \cos 30^\circ\right]$

$= \frac{\sqrt{3}}{8}\left[\cos 70^\circ + \cos(180^\circ - 70^\circ) + \frac{\sqrt{3}}{2}\right]$

$= \frac{\sqrt{3}}{8}\left[\cos 70^\circ - \cos 70^\circ + \frac{\sqrt{3}}{2}\right]$ $\left[\because \cos(180^\circ - \theta) = -\cos\theta\right]$

$= \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2} = \frac{3}{16}$

= RHS

Transformation Formulae Ex 8.1 Q5(ii)

$$\cos 40^\circ \cos 80^\circ \cos 160^\circ = -\frac{1}{8}$$

$$\text{LHS} = \cos 40^\circ \cos 80^\circ \cos 160^\circ$$
$$= \cos 80^\circ \cos 40^\circ \cos 160^\circ$$

Multiplying and dividing by 2

$$= \frac{1}{2}\left(\cos 80^\circ \times \left(2\cos 40^\circ \cos 160^\circ\right)\right)$$

$$2\cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$= \frac{1}{2}\left(\cos 80^\circ \left(\cos(40^\circ + 160^\circ) + \cos(40^\circ - 160^\circ)\right)\right)$$

$$= \frac{1}{2}\left(\cos 80^\circ \left(\cos 200 + \cos(-120)\right)\right)$$

$$= \frac{1}{2}\cos 80^\circ \left(\cos(180^\circ + 20^\circ) + \cos(180^\circ - 60^\circ)\right)$$

$$= \frac{1}{2}\cos 80^\circ \left(\cos 20^\circ + \cos 60^\circ\right)$$

$$= \frac{1}{2}\cos 80^\circ \cos 20^\circ + \frac{1}{2}\cos 80^\circ + \cos 60^\circ$$

$$= -\frac{1}{2}\left(2\cos 80^\circ \cos 20^\circ\right) + \frac{1}{2}\cos 80^\circ \cos 60^\circ$$

$$= -\frac{1}{4}\left[2\cos 80^\circ \cos 20^\circ + \cos 80^\circ\right]$$

$$= -\frac{1}{4}\left[\cos(80^\circ + 20^\circ) + \cos(80^\circ - 20^\circ) + \cos 80^\circ\right]$$

$$= -\frac{1}{4}\left[\cos 100^\circ + \cos 60^\circ + \cos 80^\circ\right]$$

$$= -\frac{1}{4}\left[\cos(180^\circ - 80^\circ) + \cos 60^\circ + \cos 80^\circ\right]$$

$$= -\frac{1}{4}\left[-\cos 80^\circ + \cos 60^\circ + \cos 80^\circ\right]$$

$$= -\frac{1}{4}\cos 60^\circ$$

$$= -\frac{1}{4} \times \frac{1}{2}$$

$$= -\frac{1}{8} \quad \text{RHS}$$

Transformation Formulae Ex 8.1 Q5(iii)

$$\sin 20^\circ \sin 40^\circ \sin 80^\circ$$

$$= \frac{1}{2}\left(2\sin 20^\circ \sin 40^\circ\right)\sin 80^\circ$$

$$= \frac{1}{2}\left[\cos(40^\circ - 20^\circ) - \cos(40^\circ + 20^\circ)\right]\sin 80^\circ \qquad \left[\because 2\sin A \sin B = \cos(A-B) - \cos(A+B)\right]$$

$$= \frac{1}{2}\left[\cos 20^\circ - \cos 60^\circ\right]\sin 80^\circ$$

$$= \frac{1}{2}\left[\cos 20^\circ - \frac{1}{2}\right]\sin 80^\circ$$

$$= \frac{1}{2}\left[\cos 20^\circ \sin 80^\circ\right] - \frac{1}{4}\sin 80^\circ$$

$$= \frac{1}{4}\left[2\cos 20^\circ \sin 80^\circ - \sin 80^\circ\right]$$

$$= \frac{1}{4}\left[\sin(80^\circ + 20^\circ) + \sin(80^\circ - 20^\circ) - \sin 80^\circ\right] \qquad \left[\because 2\sin A \cos B = \sin(A+B) + \sin(A-B)\right]$$

$$= \frac{1}{4}\left[\sin 100^\circ + \sin 60^\circ - \sin 80^\circ\right]$$

$$= \frac{1}{4}\left[\sin(180^\circ - 80^\circ) + \frac{\sqrt{3}}{2} - \sin 80^\circ\right]$$

$$= \frac{1}{4}\left[\sin 80^\circ + \frac{\sqrt{3}}{2} - \sin 80^\circ\right]$$

$$= \frac{\sqrt{3}}{8} \quad = \text{RHS}$$

Transformation Formulae Ex 8.1 Q5(iv)

$\cos 20^\circ \cos 40^\circ \cos 80^\circ$

$= \frac{1}{2}(2\cos 20^\circ \cos 40^\circ)\cos 80^\circ$

$= \frac{1}{2}[\cos(40^\circ + 20^\circ) + \cos(40^\circ - 20^\circ)]\cos 80^\circ$ $\quad [\because 2\cos A \cos B = \cos(A+B) + \cos(A-B)]$

$= \frac{1}{2}[\cos 60^\circ + \cos 20^\circ]\cos 80^\circ$

$= \frac{1}{2}\left[\frac{1}{2} + \cos 20^\circ\right]\cos 80^\circ$

$= \frac{1}{2}[\cos 80^\circ + 2\cos 20^\circ \cos 80^\circ]$

$= \frac{1}{4}[\cos 80^\circ + \cos(80^\circ + 20^\circ) + \cos(20^\circ - 80^\circ)]$

$= \frac{1}{4}[\cos 80^\circ + \cos 100^\circ + \cos 60^\circ]$

$= \frac{1}{4}[\cos 80^\circ + \cos(180^\circ - 80^\circ) + \cos 60^\circ]$

$= \frac{1}{4}[\cos 80^\circ - \cos 80^\circ + \cos 60^\circ]$

$= \frac{1}{4}\left[\frac{1}{2}\right] = \frac{1}{8} = \text{RHS}$

Transformation Formulae Ex 8.1 Q5(V)

$\tan 20^\circ \tan 40^\circ \tan 60^\circ \tan 80^\circ$

$= (\tan 20^\circ \tan 40^\circ \tan 80^\circ)\sqrt{3}$ $\quad [\because \tan 60^\circ = \sqrt{3}]$

$= \left(\frac{\sin 20^\circ \sin 40^\circ \sin 80^\circ}{\cos 20^\circ \cos 40^\circ \cos 80^\circ}\right)\sqrt{3}$

$= \frac{(2\sin 20^\circ \sin 40^\circ)\sin 80^\circ \times \sqrt{3}}{(2\cos 20^\circ \cos 40^\circ)\cos 80^\circ}$

Applying

$\Rightarrow \quad 2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$2\cos A \cos B = \cos(A+B) + \cos(A-B)$

$= \frac{(\cos(40^\circ - 20^\circ) - \cos(40^\circ + 20^\circ))\sin 80^\circ \times \sqrt{3}}{(\cos(20^\circ + 40^\circ) + \cos(40^\circ - 20^\circ))\cos 80^\circ}$

$= \frac{(\cos 20^\circ - \cos 60^\circ)\sin 80^\circ \times \sqrt{3}}{(\cos 60^\circ + \cos 20^\circ)\cos 80^\circ}$

$= \frac{\left(\cos 20^\circ - \frac{1}{2}\right)\sin 80^\circ \times \sqrt{3}}{\left(\frac{1}{2} + \cos 20^\circ\right)\cos 80^\circ}$

$= \frac{(2\sin 80^\circ \cos 20^\circ - \sin 80^\circ)\sqrt{3}}{\cos 80^\circ + 2\cos 20^\circ \cos 80^\circ}$

$\Rightarrow \quad 2\sin A \cos B = \sin(A+B) + \sin(A-B)$

$= \frac{(\sin(80^\circ + 20^\circ) + \sin(80^\circ - 20^\circ) - \sin 80^\circ)\sqrt{3}}{\cos 80^\circ + (\cos(20^\circ + 80^\circ) + \cos(80^\circ - 20^\circ))}$

$= \frac{(\sin 100^\circ + \sin 60^\circ - \sin 80^\circ)\sqrt{3}}{\cos 80^\circ + \cos 100^\circ + \cos 60^\circ}$

$= \frac{\left(\sin(180^\circ - 80^\circ) + \frac{\sqrt{3}}{2} - \sin 80^\circ\right)\sqrt{3}}{\cos 80^\circ + \cos(180^\circ - 80^\circ) + \cos 60^\circ}$

$= \frac{\left(\sin 80^\circ + \frac{\sqrt{3}}{2} - \sin 80^\circ\right)\sqrt{3}}{\cos 80^\circ - \cos 80^\circ + \cos 60^\circ}$

$= \frac{\frac{3}{2}}{\frac{1}{2}} = 3 = \quad \text{RHS}$

Transformation Formulae Ex 8.1 Q5(vi)

$\tan 20^\circ \tan 30^\circ \tan 40^\circ \tan 80^\circ$

$$\frac{1}{\sqrt{3}}(\tan 20^\circ \tan 40^\circ \tan 80^\circ) \qquad \left[\because \tan 30^\circ = \frac{1}{\sqrt{3}}\right]$$

$$= \frac{(\sin 20^\circ \sin 40^\circ \sin 80^\circ)}{(\cos 20^\circ \cos 40^\circ \cos 80^\circ)\sqrt{3}}$$

$$= \frac{(2\sin 20^\circ \sin 40^\circ)\sin 80^\circ}{\sqrt{3}(2\cos 20^\circ \cos 40^\circ)\cos 80^\circ}$$

Applying

$\Rightarrow \quad 2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$$2\cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$= \frac{\{\cos(40^\circ - 20^\circ) - \cos(20^\circ + 40^\circ)\}\sin 80^\circ}{\cos(20^\circ + 40^\circ) + \cos(40^\circ - 20^\circ)\cos 80^\circ\sqrt{3}}$$

$$= \frac{(\cos 20^\circ - \cos 60^\circ)\sin 80^\circ}{\sqrt{3}(\cos 60^\circ + \cos 20^\circ)\cos 80^\circ}$$

$$= \frac{\left(\cos 20^\circ - \frac{1}{2}\right)\sin 80^\circ}{\sqrt{3}\left(\frac{1}{2} + \cos 20^\circ\right)\cos 80^\circ}$$

$$= \frac{2\sin 20^\circ \sin 80^\circ - \sin 80^\circ}{\sqrt{3}(\cos 80^\circ + 2\cos 20^\circ \cos 80^\circ)}$$

Now,

$\Rightarrow \quad 2\sin A \cos B = \sin(A+B) + \sin(A-B)$

$$= \frac{\sin(80^\circ + 20^\circ) + \sin(80^\circ - 20^\circ) - \sin 80^\circ}{\sqrt{3}\{\cos 80^\circ + \cos(20^\circ + 80^\circ) + \cos(80^\circ - 20^\circ)\}}$$

$$= \frac{\sin 100^\circ + \sin 60^\circ - \sin 80^\circ}{\sqrt{3}(\cos 80^\circ + \cos 100^\circ + \cos 60^\circ)}$$

$$= \frac{\sin 100^\circ + \sin 60^\circ - \sin(80^\circ - 100^\circ)}{\sqrt{3}\{\cos 80^\circ + \cos(1800^\circ - 80^\circ) + \sin 60^\circ\}}$$

$$= \frac{\sin 100^\circ + \frac{\sqrt{3}}{2} - \sin 100^\circ}{\sqrt{3}(\cos 80^\circ - \cos 80^\circ + \cos 60^\circ)}$$

$$= \frac{\frac{\sqrt{3}}{2}}{\sqrt{3}\left(\frac{1}{2}\right)} = 1 = \qquad \text{RHS}$$

Transformation Formulae Ex 8.1 Q5(vii)

$\sin 10^\circ \sin 50^\circ \sin 60^\circ \sin 70^\circ = \frac{\sqrt{3}}{16}$

LHS

$$\sin 10^\circ \sin 50^\circ \sin 70^\circ \frac{\sqrt{3}}{2} \qquad \left[\because \sin 60^\circ = \frac{\sqrt{3}}{2}\right]$$

$$= \sin(90^\circ - 80^\circ)\sin(90^\circ - 40^\circ)\sin(90^\circ - 20^\circ)\frac{\sqrt{3}}{2}$$

$$= \cos 80^\circ \cos 40^\circ \cos 20^\circ \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{2\times 2}(2\cos 40^\circ \cos 20^\circ)\cos 80^\circ \qquad [\because 2\cos A \cos B = \cos(A+B) + \cos(A-B)]$$

$$= \frac{\sqrt{3}}{2\times 2}[\cos(40^\circ + 20^\circ) + \cos(40^\circ - 20^\circ)]\cos 80^\circ$$

$$= \frac{\sqrt{3}}{2\times 2}[\cos 60^\circ + \cos 20^\circ]\cos 80^\circ$$

$$= \frac{\sqrt{3}}{2\times 2}\left[\frac{1}{2} + \cos 20^\circ\right]\cos 80^\circ$$

$$= \frac{\sqrt{3}}{4}\left[\frac{1}{2}\cos 80^\circ + \cos 20^\circ \cos 80^\circ\right]$$

$$= \frac{\sqrt{3}}{8}[\cos 80^\circ + 2\cos 20^\circ \cos 80^\circ]$$

$$= \frac{\sqrt{3}}{8}[\cos 80^\circ + \cos(80^\circ + 20^\circ) + \cos(80^\circ - 20^\circ)]$$

$$= \frac{\sqrt{3}}{8}[\cos 80^\circ + \cos 100^\circ + \cos 60^\circ]$$

$$= \frac{\sqrt{3}}{8}[\cos 80^\circ + \cos(180^\circ - 80^\circ) + \cos 60^\circ]$$

$$= \frac{\sqrt{3}}{8}[\cos 60^\circ] = \frac{\sqrt{3}}{16} = \text{RHS}$$

Transformation Formulae Ex 8.1 Q5(viii)

LHS $= \sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$

$= \sin 20^\circ \sin 40^\circ \sin 80^\circ \times \frac{\sqrt{3}}{2}$ $\left[\because \sin 60^\circ = \frac{\sqrt{3}}{2}\right]$

$= \frac{\sqrt{3}}{2} \times \frac{1}{2}(2 \sin 20^\circ \sin 40^\circ) \sin 80^\circ$

$= \frac{\sqrt{3}}{4}\left[\cos(40^\circ - 20^\circ) - \cos(40^\circ + 20^\circ)\right] \sin 80^\circ$

$= \frac{\sqrt{3}}{4}\left[\cos 20^\circ - \cos 60^\circ\right] \sin 80^\circ$

$= \frac{\sqrt{3}}{4}\left[\cos 20^\circ \sin 80^\circ - \frac{1}{2} \sin 80^\circ\right]$

$= \frac{\sqrt{3}}{8}\left[2 \cos 20^\circ \sin 80^\circ - \sin 80^\circ\right]$

$= \frac{\sqrt{3}}{8}\left[\sin(80^\circ + 20^\circ) + \sin(80^\circ - 20^\circ) - \sin 80^\circ\right]$

$= \frac{\sqrt{3}}{8}\left[\sin 100^\circ + \sin 60^\circ - \sin 80^\circ\right]$

$= \frac{\sqrt{3}}{8}\left[\sin 80^\circ + \sin 60^\circ - \sin 80^\circ\right]$

$= \frac{\sqrt{3}}{8} \times \sin 60^\circ = \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2}$

3 RHS

********** END **********