Consider a natural number (n) in co-domain **N**.

Case **I:** n is odd

$\therefore n = 2r + 1$ for some $r \in$ **N.** Then, there exists $4r + 1 \in$ **N** such that

$$f(4r+1) = \frac{4r+1+1}{2} = 2r+1.$$

Case **II:** n is even

$\therefore n = 2r$ for some $r \in$ **N.** Then,there exists $4r \in$ **N** such that $f(4r) = \frac{4r}{2} = 2r$.

$\therefore f$ is onto.

Hence, f is not a bijective function.

Question 10:

Let A = **R** − {3} and B = **R** − {1}. Consider the function f: A → B defined by

$f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

Answer

A = **R** − {3}, B = **R** − {1}

f: A → B is defined as $f(x) = \left(\frac{x-2}{x-3}\right)$.

Let $x, y \in$ A such that $f(x) = f(y)$.

$$\Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$$
$$\Rightarrow (x-2)(y-3) = (y-2)(x-3)$$
$$\Rightarrow xy - 3x - 2y + 6 = xy - 3y - 2x + 6$$
$$\Rightarrow -3x - 2y = -3y - 2x$$
$$\Rightarrow 3x - 2x = 3y - 2y$$
$$\Rightarrow x = y$$

$\therefore f$ is one-one.

Let $y \in$ B = **R** − {1}. Then, $y \neq 1$.

The function f is onto if there exists $x \in$ A such that $f(x) = y$.

Now,

$$f(x) = y$$
$$\Rightarrow \frac{x-2}{x-3} = y$$
$$\Rightarrow x - 2 = xy - 3y$$
$$\Rightarrow x(1-y) = -3y + 2$$
$$\Rightarrow x = \frac{2-3y}{1-y} \in \text{A} \qquad [y \neq 1]$$

Thus, for any $y \in$ B, there exists $\frac{2-3y}{1-y} \in \text{A}$ such that

$$f\left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right) - 2}{\left(\frac{2-3y}{1-y}\right) - 3} = \frac{2-3y-2+2y}{2-3y-3+3y} = \frac{-y}{-1} = y.$$

$\therefore f$ is onto.

Hence, function f is one-one and onto.

Question 11:

Let f: **R** → **R** be defined as $f(x) = x^4$. Choose the correct answer.

(A) f is one-one onto (B) f is many-one onto

(C) f is one-one but not onto (D) f is neither one-one nor onto

Answer

f: **R** → **R** is defined as $f(x) = x^4$.

Let $x, y \in$ **R** such that $f(x) = f(y)$.

$$\Rightarrow x^4 = y^4$$
$$\Rightarrow x = \pm y$$

$\therefore f(x_1) = f(x_2)$ does not imply that $x_1 = x_2$.

For instance,

$f(1)=f(-1)=1$

$\therefore$ f is not one-one.

Consider an element 2 in co-domain **R**. It is clear that there does not exist any x in domain **R** such that $f(x) = 2$.

$\therefore$ f is not onto.

Hence, function f is neither one-one nor onto.

The correct answer is D.

Question 12:

Let f: **R** → **R** be defined as $f(x) = 3x$. Choose the correct answer.

(A) f is one-one onto (B) f is many-one onto

(C) f is one-one but not onto (D) f is neither one-one nor onto

Answer

f: **R** → **R** is defined as $f(x) = 3x$.

Let $x, y \in$ **R** such that $f(x) = f(y)$.

$\Rightarrow 3x = 3y$

$\Rightarrow x = y$

$\therefore f$ is one-one.

Also, for any real number (y) in co-domain **R**, there exists $\frac{y}{3}$ in **R** such that

$f\left(\frac{y}{3}\right)=3\left(\frac{y}{3}\right)=y$.

$\therefore f$ is onto.

Hence, function f is one-one and onto.

The correct answer is A.

Exercise 1.3

Question 1:

Let f: {1, 3, 4} → {1, 2, 5} and g: {1, 2, 5} → {1, 3} be given by f = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}. Write down gof.

Answer

The functions f: {1, 3, 4} → {1, 2, 5} and g: {1, 2, 5} → {1, 3} are defined as f = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}.

$$gof(1)=g(f(1))=g(2)=3 \qquad [f(1)=2 \text{ and } g(2)=3]$$
$$gof(3)=g(f(3))=g(5)=1 \qquad [f(3)=5 \text{ and } g(5)=1]$$
$$gof(4)=g(f(4))=g(1)=3 \qquad [f(4)=1 \text{ and } g(1)=3]$$
$$\therefore gof=\{(1,3),(3,1),(4,3)\}$$

Question 2:

Let f, g and h be functions from **R** to **R**. Show that

$(f+g)oh=foh+goh$

$(f\cdot g)oh=(foh)\cdot(goh)$

Answer

To prove:

$(f+g)oh=foh+goh$

Consider:

$$((f+g)oh)(x)$$
$$=(f+g)(h(x))$$
$$=f(h(x))+g(h(x))$$
$$=(foh)(x)+(goh)(x)$$
$$=\{(foh)+(goh)\}(x)$$
$$\therefore ((f+g)oh)(x)=\{(foh)+(goh)\}(x) \qquad \forall x\in \mathbf{R}$$

Hence, $(f+g)oh=foh+goh$.

To prove:

$(f\cdot g)oh=(foh)\cdot(goh)$

Consider:

$$((f\cdot g)oh)(x)$$
$$=(f\cdot g)(h(x))$$
$$=f(h(x)).g(h(x))$$
$$=(foh)(x).(goh)(x)$$
$$=\{(foh).(goh)\}(x)$$
$$\therefore ((f\cdot g)oh)(x)=\{(foh).(goh)\}(x)\ \forall x\in \mathrm{R}$$

Hence, $(f\cdot g)oh=(foh)\cdot(goh)$.

Question 3:

Find gof and fog, if

(i) $f(x)=|x|$ and $g(x)=|5x-2|$

(ii) $f(x)=8x^3$ and $g(x)=x^{\frac{1}{3}}$

Answer

(i) $f(x)=|x|$ and $g(x)=|5x-2|$

$\therefore (gof)(x)=g(f(x))=g(|x|)=|5|x|-2|$

$(fog)(x)=f(g(x))=f(|5x-2|)=||5x-2||=|5x-2|$

(ii) $f(x)=8x^3$ and $g(x)=x^{\frac{1}{3}}$

$\therefore (gof)(x)=g(f(x))=g(8x^3)=(8x^3)^{\frac{1}{3}}=2x$

$(fog)(x)=f(g(x))=f\left(x^{\frac{1}{3}}\right)=8\left(x^{\frac{1}{3}}\right)^3=8x$

Question 4:

If $f(x)=\frac{(4x+3)}{(6x-4)}, x\neq\frac{2}{3}$, show that *f* o *f*(*x*) = *x*, for all $x\neq\frac{2}{3}$. What is the inverse of *f*?

Answer

It is given that $f(x)=\frac{(4x+3)}{(6x-4)}, x\neq\frac{2}{3}$.

$(fof)(x)=f(f(x))=f\left(\frac{4x+3}{6x-4}\right)$

$$=\frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4}=\frac{16x+12+18x-12}{24x+18-24x+16}=\frac{34x}{34}=x$$

Therefore, $fof(x)=x,$ for all $x\neq\frac{2}{3}.$

$\Rightarrow fof=\mathrm{I}$

Hence, the given function *f* is invertible and the inverse of *f* is *f* itself.

Question 5:

State with reason whether following functions have inverse

(i) *f*: {1, 2, 3, 4} → {10} with

f = {(1, 10), (2, 10), (3, 10), (4, 10)}

(ii) *g*: {5, 6, 7, 8} → {1, 2, 3, 4} with

g = {(5, 4), (6, 3), (7, 4), (8, 2)}

(iii) *h*: {2, 3, 4, 5} → {7, 9, 11, 13} with

h = {(2, 7), (3, 9), (4, 11), (5, 13)}

Answer

*********** END **********