



Indefinite Integrals Ex 19.30 Q42

$$\text{Let } \frac{x^2}{(x^2+1)(3x^2+4)} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(3x^2+4)}$$

$$\begin{aligned}\Rightarrow x^2 &= (Ax+B)(3x^2+4) + (Cx+D)(x^2+1) \\ &= (3A+C)x^3 + (3B+D)x^2 + (4A+C)x + 4B+D\end{aligned}$$

Equating similar terms, we get,

$$3A+C=0, 3B+D=1, 4A+C=0, 4B+D=0$$

$$\text{Solving, we get, } A=0, B=-1, C=0, D=4$$

Thus,

$$\begin{aligned}I &= \int \frac{-dx}{(x^2+1)} + \int \frac{4dx}{(3x^2+4)} \\ &= -\tan^{-1}x + \frac{4}{3} \int \frac{dx}{x^2 + \left(\frac{2}{\sqrt{3}}\right)^2} \\ &= -\tan^{-1}x + \frac{4}{3} \cdot \frac{\sqrt{3}}{2} \tan^{-1}\left(\frac{\sqrt{3}x}{2}\right) + c \\ I &= \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{\sqrt{3}x}{2}\right) - \tan^{-1}x + c\end{aligned}$$

Indefinite Integrals Ex 19.30 Q43

To evaluate the integral follow the steps:

$$\int \frac{3x+5}{x^3-x^2-x+1} dx$$

$$\text{Let } \frac{3x+5}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}$$

$$3x+5 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$$

$$\text{For } x=1 \quad B=4$$

$$\text{For } x=-1 \quad C=\frac{1}{2}$$

$$\text{For } x=0 \quad A=-\frac{1}{2},$$

Therefore

$$\begin{aligned} \int \frac{3x+5}{(x-1)^2(x+1)} dx &= -\frac{1}{2} \int \frac{dx}{x-1} + 4 \int \frac{dx}{(x-1)^2} + \frac{1}{2} \int \frac{dx}{x+1} \\ &= -\frac{1}{2} \ln |(x-1)| - \frac{4}{(x-1)} + \frac{1}{2} \ln |(x+1)| + c \\ &= \frac{1}{2} \ln \left| \frac{x+1}{x-1} \right| - \frac{4}{(x-1)} + c \end{aligned}$$

Indefinite Integrals Ex 19.30 Q44

$$\text{Let } I = \int \frac{x^3 - 1}{x^3 + x} dx$$

$$= \int 1 - \frac{(x+1)}{x^3 + x} dx$$

$$= \int dx - \int \frac{x+1}{x^3 + x} dx$$

$$\text{Let } \frac{x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$\Rightarrow \quad x+1 = A(x^2+1) + (Bx+C)(x) \\ = (A+B)x^2 + (B+C)x + A$$

Equating similar terms, we get,

$$A+B=0, \quad C=1, \quad A=1$$

Solving, we get, $A=1, \quad B=-1, \quad C=1$

Thus,

$$I = -\int \frac{dx}{x} - \int \frac{-x+1}{x^2+1} dx + \int dx$$

$$= -\log|x| + \int \frac{x dx}{x^2+1} - \int \frac{dx}{x^2+1} + \int dx$$

$$\therefore \quad I = x - \log|x| + \frac{1}{2} \log|x^2+1| - \tan^{-1}x + c$$

$$\therefore \quad I = x - \log|x| + \frac{1}{2} \log|x^2+1| - \tan^{-1}x + c$$

Indefinite Integrals Ex 19.30 Q45

To evaluate the integral follow the steps:

$$\int \frac{x^2+x+1}{(x+1)^2(x+2)} dx$$

$$\text{Let } \frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+2}$$

$$x^2+x+1 = A(x+1)(x+2) + B(x+2) + C(x+1)^2$$

$$\text{For } x=-1 \quad B=1$$

$$\text{For } x=-2 \quad C=3$$

$$\text{For } x=0 \quad A=-2,$$

Therefore

$$\int \frac{x^2+x+1}{(x+1)^2(x+2)} dx = -2 \int \frac{dx}{x+1} + \int \frac{dx}{(x+1)^2} + 3 \int \frac{dx}{x+2} \\ = -2 \ln|x+1| - \frac{1}{x+1} + 3 \ln|x+2| + c$$

Indefinite Integrals Ex 19.30 Q46

$$\text{Let } \frac{1}{x(x^4+1)} = \frac{A}{x} + \frac{Bx^3+Cx^2+Dx+E}{x^4+1}$$

$$\begin{aligned}\Rightarrow 1 &= A(x^4+1) + (Bx^3+Cx^2+Dx+E)x \\ &= (A+B)x^4 + Cx^3 + Dx^2 + Ex + A\end{aligned}$$

Equating similar terms, we get,

$$\begin{aligned}A+B &= 0, \quad C = 0, \quad D = 0, \quad E = 0, \quad A = 1 \\ \therefore B &= -1\end{aligned}$$

Thus,

$$\begin{aligned}I &= \int \frac{dx}{x} + \int -\frac{x^3 dx}{x^4+1} \\ &= \log|x| - \frac{1}{4} \log|x^4+1| + c\end{aligned}$$

$$I = \frac{1}{4} \log \left| \frac{x^4}{x^4+1} \right| + c$$

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