Differentiation Ex 11.6 Q4

Here,

$$y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \ldots \text{to } \infty}}}$$

$$y = \sqrt{\tan x + y}$$

Squaring both the sides,

$$y^2 = \tan x + y$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$$

$$\frac{dy}{dx}(2y - 1) = \sec^2 x$$

$$\frac{dy}{dx} = \frac{\sec^2 x}{2y - 1}$$

Differentiation Ex 11.6 Q5

Here,

$$y = (\sin x)^{(\sin x)^{(\sin x)^{\cdots\infty}}}$$

$$\Rightarrow \quad y = (\sin x)^{y}$$

Taking log on both the sides,

$$\log y = \log (\sin x)^{y}$$

$$\log y = y (\log \sin x)$$

Differentiating it with respect to x, using product rule,

$$\frac{1}{y}\frac{dy}{dx} = y\frac{d}{dx}(\log \sin x) + \log \sin x \frac{dy}{dx}$$

$$\frac{1}{y}\frac{dy}{dx} = y\frac{1}{\sin x}\frac{d}{dx}(\sin x) + \log \sin x \frac{dy}{dx}$$

$$\frac{dy}{dx}\left(\frac{1}{y} - \log \sin x\right) = \frac{y}{\sin x}(\cot x)$$

$$\frac{dy}{dx}\left(\frac{1 - y \log \sin x}{y}\right) = y \cot x$$

$$\frac{dy}{dx} = \frac{y^2 \cot x}{(1 - y \log \sin x)}$$

Differentiation Ex 11.6 Q6

Here,

$$y = (\tan x)^{(\tan x)^{(\tan x)^{\cdots\infty}}}$$

$$y = (\tan x)^{y}$$

Taking log on both the sides,

$$\log y = \log (\tan x)^{y}$$

$$\log y = y \log \tan x$$

Differentiating with respect to x using product rule and chain rule,

$$\frac{1}{y}\frac{dy}{dx} = y\frac{d}{dx}\log \tan x + \log \tan \frac{dy}{dx}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{y}{\tan x}\frac{d}{dx}(\tan x) + \log \tan x \frac{dy}{dx}$$

$$\frac{dy}{dx}\left(\frac{1}{y} - \log \tan x\right) = \frac{y}{\tan x}\sec^2 x$$

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = \frac{y \sec^2\left(\frac{\pi}{4}\right)}{\tan\left(\frac{\pi}{4}\right)} * \frac{y}{1 - y \log \tan\left(\frac{\pi}{4}\right)}$$

$$\left(\frac{dy}{dx}\right)_{\frac{\pi}{4}} = \frac{y^2\left(\sqrt{2}\right)^2}{1(1 - y \log \tan 1)}$$

$$= \frac{2(1)^2}{(1 - 0)}$$

$$\left(\frac{dy}{dx}\right)_{\frac{\pi}{4}} = 2$$

{since,

$$(y)_{\frac{\pi}{4}} = \left(\tan\frac{\pi}{4}\right)^{\left(\tan\frac{\pi}{4}\right)^{\left(\tan\frac{\pi}{4}\right)^{-\infty}}}$$

$$\Rightarrow y = (1)^{\infty}$$

$$\Rightarrow y = 1$$

}

********** END **********