Trigonometric Identities Ex 6.1 Q62

Answer :

We have to prove $(\sec A-\operatorname{cosec}A)(1+\tan A+\cot A)=\tan A\sec A-\cot A\operatorname{cosec}A$

We know that, $\sin^2 A+\cos^2 A=1$

So,

$$(\sec A-\operatorname{cosec}A)(1+\tan A+\cot A)=\left(\frac{1}{\cos A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}\right)$$

$$=\left(\frac{\sin A-\cos A}{\sin A\cos A}\right)\left(\frac{\sin A\cos A+\sin^2 A+\cos^2 A}{\sin A\cos A}\right)$$

$$=\left(\frac{\sin A-\cos A}{\sin A\cos A}\right)\left(\frac{\sin A\cos A+1}{\sin A\cos A}\right)$$

$$=\frac{(\sin A-\cos A)(\sin A\cos A+1)}{\sin^2 A\cos^2 A}$$

$$=\frac{\sin^2 A\cos A+\sin A-\cos^2 A\sin A-\cos A}{\sin^2 A\cos^2 A}$$

$$=\frac{(\sin^2 A\cos A-\cos A)+(\sin A-\cos^2 A\sin A)}{\sin^2 A\cos^2 A}$$

$$=\frac{\cos A(\sin^2 A-1)+\sin A(1-\cos^2 A)}{\sin^2 A\cos^2 A}$$

$$=\frac{\cos A(-\cos^2 A)+\sin A(\sin^2 A)}{\sin^2 A\cos^2 A}$$

$$=\frac{-\cos^3 A+\sin^3 A}{\sin^2 A\cos^2 A}$$

$$=\frac{\sin^3 A-\cos^3 A}{\sin^2 A\cos^2 A}$$

$$=\frac{\sin^3 A}{\sin^2 A\cos^2 A}-\frac{\cos^3 A}{\sin^2 A\cos^2 A}$$

$$=\frac{\sin A}{\cos^2 A}-\frac{\cos A}{\sin^2 A}$$

$$=\frac{\sin A}{\cos A}\frac{1}{\cos A}-\frac{\cos A}{\sin A}\frac{1}{\sin A}$$

$$=\tan A\sec A-\cot A\operatorname{cosec}A$$

Hence proved.

Trigonometric Identities Ex 6.1 Q63

Answer :

We have to prove $\frac{\cos A\operatorname{cosec}A-\sin A\sec A}{\cos A+\sin A}=\operatorname{cosec}A-\sec A$

So,

$$\begin{aligned}\frac{\cos A\operatorname{cosec}A-\sin A\sec A}{\cos A+\sin A}&=\frac{\cos A\frac{1}{\sin A}-\sin A\frac{1}{\cos A}}{\cos A+\sin A}\\&=\frac{\frac{\cos^2 A-\sin^2 A}{\sin A\cos A}}{\cos A+\sin A}\\&=\frac{\cos^2 A-\sin^2 A}{\sin A\cos A(\cos A+\sin A)}\\&=\frac{(\cos A-\sin A)(\cos A+\sin A)}{\sin A\cos A(\cos A+\sin A)}\\&=\frac{\cos A-\sin A}{\sin A\cos A}\\&=\frac{\cos A}{\sin A\cos A}-\frac{\sin A}{\sin A\cos A}\\&=\frac{1}{\sin A}-\frac{1}{\cos A}\\&=\operatorname{cosec}A-\sec A\end{aligned}$$

Hence proved.

********** END **********