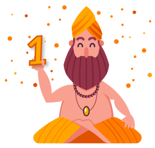

Question 13.22:

For the β^+ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the K−shell, is captured by the nucleus and a neutrino is emitted).

$$e^+ + {}_Z^A X \rightarrow {}_{Z-1}^{A} Y + \nu$$

Show that if β^+ emission is energetically allowed, electron capture is necessarily allowed but not vice−versa.

Answer

Let the amount of energy released during the electron capture process be Q_1. The nuclear reaction can be written as:

$$e^+ + {}_Z^A X \rightarrow {}_{Z-1}^{A} Y + \nu + Q_1 \qquad \dots (1)$$

Let the amount of energy released during the positron capture process be Q_2. The nuclear reaction can be written as:

$${}_Z^A X \rightarrow {}_{Z-1}^{A} Y + e^+ + \nu + Q_2 \qquad \dots (2)$$

$m_N\left({}_Z^A X\right)$ = Nuclear mass of ${}_Z^A X$

$m_N\left({}_{Z-1}^{A} Y\right)$ = Nuclear mass of ${}_{Z-1}^{A} Y$

$m\left({}_Z^A X\right)$ = Atomic mass of ${}_Z^A X$

$m\left({}_{Z-1}^{A} Y\right)$ = Atomic mass of ${}_{Z-1}^{A} Y$

m_e = Mass of an electron

c = Speed of light

Q-value of the electron capture reaction is given as:

$$Q_1 = \left[m_N\left({}_Z^A X\right) + m_e - m_N\left({}_{Z-1}^{A} Y\right)\right]c^2$$

$$= \left[m\left({}_Z^A X\right) - Zm_e + m_e - m\left({}_{Z-1}^{A} Y\right) + (Z-1)m_e\right]c^2$$

$$= \left[m\left({}_Z^A X\right) - m\left({}_{Z-1}^{A} Y\right)\right]c^2 \qquad \dots (3)$$

Q-value of the positron capture reaction is given as:

$$Q_2 = \left[m_N\left({}_Z^A X\right) - m_N\left({}_{Z-1}^{A} Y\right) - m_e\right]c^2$$

$$= \left[m\left({}_Z^A X\right) - Zm_e - m\left({}_{Z-1}^{A} Y\right) + (Z-1)m_e - m_e\right]c^2$$

$$= \left[m\left({}_Z^A X\right) - m\left({}_{Z-1}^{A} Y\right) - 2m_e\right]c^2 \qquad \dots (4)$$

It can be inferred that if $Q_2 > 0$, then $Q_1 > 0$; Also, if $Q_1 > 0$, it does not necessarily mean that $Q_2 > 0$.

Question 13.24:

The neutron separation energy is defined as the energy required to remove a neutron from the nucleus. Obtain the neutron separation energies of the nuclei ${}_{20}^{41}\mathrm{Ca}$ and ${}_{13}^{27}\mathrm{Al}$ from the following data:

$m\left({}_{20}^{40}\mathrm{Ca}\right)$ = 39.962591 u

$m\left({}_{20}^{41}\mathrm{Ca}\right)$) = 40.962278 u

$m\left({}_{13}^{26}\mathrm{Al}\right)$ = 25.986895 u

$m\left({}_{13}^{27}\mathrm{Al}\right)$) = 26.981541 u

Answer

For ${}_{20}^{41}\mathrm{Ca}$: Separation energy = 8.363007 MeV

For ${}_{13}^{27}\mathrm{Al}$: Separation energy = 13.059 MeV

A neutron $\left({}_0n\right)$ is removed from a ${}^{41}_{20}\mathrm{Ca}$ nucleus. The corresponding nuclear reaction can be written as:

$${}^{41}_{20}\mathrm{Ca} \longrightarrow {}^{40}_{20}\mathrm{Ca} + {}^{1}_{0}\mathrm{n}$$

It is given that:

Mass $m\left({}^{40}_{20}\mathrm{Ca}\right)$ = 39.962591 u

Mass $m\left({}^{41}_{20}\mathrm{Ca}\right)$) = 40.962278 u

Mass $m\left({}_0n^1\right)$ = 1.008665 u

The mass defect of this reaction is given as:

$$\Delta m = m\left({}^{40}_{20}\mathrm{Ca}\right) + \left({}^{1}_{0}\mathrm{n}\right) - m\left({}^{41}_{20}\mathrm{Ca}\right)$$

$$= 39.962591 + 1.008665 - 40.962278 = 0.008978\ \mathrm{u}$$

But $1\ \mathrm{u} = 931.5\ \mathrm{MeV}/c^2$

$\therefore \Delta m = 0.008978 \times 931.5\ \mathrm{MeV}/c^2$

Hence, the energy required for neutron removal is calculated as:

$$E = \Delta mc^2$$
$$= 0.008978 \times 931.5 = 8.363007\ \mathrm{MeV}$$

For ${}^{27}_{13}\mathrm{Al}$, the neutron removal reaction can be written as:

$${}^{27}_{13}\mathrm{Al} \longrightarrow {}^{26}_{13}\mathrm{Al} + {}^{1}_{0}\mathrm{n}$$

It is given that:

Mass $m\left({}^{27}_{13}\mathrm{Al}\right)$ = 26.981541 u

Mass $m\left({}^{26}_{13}\mathrm{Al}\right)$ = 25.986895 u

The mass defect of this reaction is given as:

$$\Delta m = m\left({}^{26}_{13}\mathrm{Al}\right) + m\left({}^{1}_{0}\mathrm{n}\right) - m\left({}^{27}_{13}\mathrm{Al}\right)$$

$$= 25.986895 + 1.008665 - 26.981541$$
$$= 0.014019\ \mathrm{u}$$
$$= 0.014019 \times 931.5\ \mathrm{MeV}/c^2$$

Hence, the energy required for neutron removal is calculated as:

$$E = \Delta mc^2$$
$$= 0.014019 \times 931.5 = 13.059\ \mathrm{MeV}$$

Question 13.25:

A source contains two phosphorous radio nuclides ${}^{32}_{15}\mathrm{P}$ ($T_{1/2}$ = 14.3d) and ${}^{33}_{15}\mathrm{P}$ ($T_{1/2}$ = 25.3d). Initially, 10% of the decays come from ${}^{33}_{15}\mathrm{P}$. How long one must wait until 90% do so?

Answer

Half life of ${}^{32}_{15}\mathrm{P}$, $T_{1/2}$ = 14.3 days

Half life of ${}^{33}_{15}\mathrm{P}$, $T'_{1/2}$ = 25.3 days

${}^{33}_{15}\mathrm{P}$ nucleus decay is 10% of the total amount of decay.

The source has initially 10% of ${}^{33}_{15}\mathrm{P}$ nucleus and 90% of ${}^{32}_{15}\mathrm{P}$ nucleus.

Suppose after t days, the source has 10% of ${}^{32}_{15}\mathrm{P}$ nucleus and 90% of ${}^{33}_{15}\mathrm{P}$ nucleus.

Initially:

Number of ${}^{33}_{15}\mathrm{P}$ nucleus = N

Number of ${}^{32}_{15}\mathrm{P}$ nucleus = 9 N

Finally:

Number of ${}^{33}_{15}\mathrm{P}$ nucleus $= 9\,N'$

Number of ${}^{32}_{15}\mathrm{P}$ nucleus $= N'$

For ${}^{32}_{15}\mathrm{P}$ nucleus, we can write the number ratio as:

$$\frac{N'}{9N} = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

$$N' = 9N(2)^{\frac{-t}{14.3}} \qquad \dots (1)$$

For ${}^{33}_{15}\mathrm{P}$, we can write the number ratio as:

For [illegible], we can write the number ratio as:

$$\frac{9N'}{N} = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

$$9N' = N(2)^{\frac{-t}{25.3}} \qquad \ldots (2)$$

On dividing equation (1) by equation (2), we get:

$$\frac{1}{9} = 9 \times 2^{\left(\frac{t}{25.3} - \frac{t}{14.3}\right)}$$

$$\frac{1}{81} = 2^{-\left(\frac{11t}{25.3 \times 14.3}\right)}$$

$$\log 1 - \log 81 = \frac{-11t}{25.3 \times 14.3} \log 2$$

$$\frac{-11t}{25.3 \times 14.3} = \frac{0 - 1.908}{0.301}$$

$$t = \frac{25.3 \times 14.3 \times 1.908}{11 \times 0.301} \approx 208.5 \text{ days}$$

Hence, it will take about 208.5 days for 90% decay of $_{15}P^{33}$.

Question 13.26:

Under certain circumstances, a nucleus can decay by emitting a particle more massive than an α-particle. Consider the following decay processes:

$$^{223}_{88}\text{Ra} \longrightarrow {}^{209}_{82}\text{Pb} + {}^{14}_{6}\text{C}$$

$$^{223}_{88}\text{Ra} \longrightarrow {}^{219}_{86}\text{Rn} + {}^{4}_{2}\text{He}$$

Calculate the *Q*-values for these decays and determine that both are energetically allowed.

Answer

Take a $^{14}_{6}C$ emission nuclear reaction:

$$^{223}_{88}\text{Ra} \longrightarrow {}^{209}_{82}\text{Pb} + {}^{14}_{6}\text{C}$$

We know that:

Mass of $^{223}_{88}Ra$, m_1 = 223.01850 u

Mass of $^{209}_{82}Pb$, m_2 = 208.98107 u

Mass of $^{14}_{6}C$, m_3 = 14.00324 u

Hence, the *Q*-value of the reaction is given as:

$Q = (m_1 - m_2 - m_3)\, c^2$

$= (223.01850 - 208.98107 - 14.00324)\, c^2$

$= (0.03419\, c^2)$ u

But 1 u = 931.5 MeV/c^2

$\therefore Q = 0.03419 \times 931.5$

= 31.848 MeV

Hence, the *Q*-value of the nuclear reaction is 31.848 MeV. Since the value is positive, the reaction is energetically allowed.

Now take a $^{4}_{2}He$ emission nuclear reaction:

$$^{223}_{88}\text{Ra} \longrightarrow {}^{229}_{86}\text{Rn} + {}^{4}_{2}\text{He}$$

We know that:

Mass of $^{223}_{88}Ra$, m_1 = 223.01850

Mass of $^{219}_{82}Rn$, m_2 = 219.00948

Mass of $^{4}_{2}He$, m_3 = 4.00260

*********** END **********