Question 8. 11. For the above problem, the direction of the gravitational intensity at an arbitrary point P is indicated by the arrow (i) d, (ii), e, (iii) f (iv) g.
Answer: Using the explanation given in the solution of the previous problem, the direction of the gravitational field intensity at P will be along e. So, option (ii) is correct.

Question 8. 12. A rocket is fired from the earth towards the sun. At what distance from the earth's centre is the gravitational force on the rocket zero? Mass of the sun = 2 x 10^{30} kg, mass of the earth = 6 x 10^{24} kg. Neglect the effect of other planets etc. (orbital radius = 1.5 x 10^{11} m).
Answer:
Mass of Sun, M = 2 x 10^{30} kg; Mass of Earth, m = 6 x 10^{24} kg Distance between Sim and Earth, r = 1.5 x 10^{11} m
Let at the point P, the gravitational force on the rocket due to Earth
= gravitational force on the rocket due to Sun

Let x = distance of the point P from the Earth

Then $$\frac{Gm}{x^2} = \frac{GM}{(r-x)^2}$$

$$\Rightarrow \quad \frac{(r-x)^2}{x^2} = \frac{M}{m} = \frac{2\times 10^{30}}{6\times 10^{24}} = \frac{10^6}{3}$$

or $$\frac{r-x}{x} = \frac{10^3}{\sqrt{3}} \Rightarrow \frac{r}{x} = \frac{10^3}{\sqrt{3}}+1 \simeq \frac{10^3}{\sqrt{3}}$$

or $$x = \frac{\sqrt{3}r}{10^3} = \frac{1.732\times 1.5\times 10^{11}}{10^3} = 2.6\times 10^8 \text{ m}.$$

Question 8. 13. How will you 'weigh the sun', that is, estimate its mass? The mean orbital radius of the earth around the sun is 1.5 x 10^8 km.
Answer:
The mean orbital radius of the Earth around the Sun
R = 1.5 x 10^8 km = 1.5 x 10^{11} m
Time period, T = 365.25 x 24 x 60 x 60 s
Let the mass of the Sun be M and that of Earth be m.
According to law of gravitation

$$F = G\frac{Mm}{R^2} \quad \text{...}(i)$$

Centripetal force,

$$F = \frac{mv^2}{R} = m.R.\omega^2 \quad \text{...}(ii)$$

From eqn. (i) and (ii), we have

$$\frac{GMm}{R^2} = m.R.\omega^2$$

$$= \frac{mR.4\pi^2}{T^2} \qquad \left[\because \omega = \frac{2\pi}{T}\right]$$

$$\therefore \quad M = \frac{4\pi^2 R^3}{G.T^2}$$

$$= \frac{4\times(3.14)^2\times(1.5\times 10^{11})^3}{6.67\times 10^{-11}\times(365.25\times 24\times 60\times 60)^2}$$

$$= 2.009\times 10^{30} \text{ kg} = 2.0\times 10^{30} \text{ kg.}$$

Question 8. 14. A Saturn year is 29.5 times the Earth year. How far is the Saturn from the Sun if the Earth is 1.50 x 10^8 km away from the Sun?
Answer:

We know that $T^2 \propto R^3$

$$\therefore \qquad \frac{T_s^2}{T_e^2} = \frac{R_s^3}{R_e^3}$$

where subscripts s and e refer to the Saturn and Earth respectively.

Now $\quad \frac{T_s}{T_e} = 29.5$ [given]; $\quad R_e = 1.50 \times 10^8$ km

$$\left(\frac{R_s}{R_e}\right)^3 = \left(\frac{T_s}{T_e}\right)^2$$

$$R_s = R_e \times [(29.5)^2]^{1/3} = 1.50 \times 10^8 \times (870.25)^{1/3} \text{ km}$$

$$= 1.43 \times 10^9 \text{ km} = 1.43 \times 10^{12} \text{ m}$$

$\therefore$ Distance of Saturn from Sun = **1.43×10^{12} m.**

Question 8. 15. A body weighs 63 N on the surface of the Earth. What is the gravitational force on it due to the Earth at a height equal to half the radius of the Earth?
Answer: Let g_h be the acceleration due to gravity at a height equal to half the radius of the Earth (h = R/2) and g its value on Earth's surface. Let the body have mass m.

We know that

$$\frac{g_h}{g} = \left(\frac{R}{R+h}\right)^2 \quad \text{or} \quad \frac{g_h}{g} = \left(\frac{R}{R+\frac{R}{2}}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Let W be the weight of body on the surface of Earth and W_h the weight of the body at height h.

Then, $\quad \frac{W_h}{W} = \frac{mg_h}{mg} = \frac{g_h}{g} = \frac{4}{9}$

or $\quad W_h = \frac{4}{9}W = \frac{4}{9} \times 63 \text{ N} = \mathbf{28\ N}.$

Question 8. 16. Assuming the earth to be a sphere of uniform mass density, how much would a body weigh half way down to the centre of the earth if it weighed 250 N on the surface?
Answer:

As $\quad g_d = g\left(1 - \frac{d}{R}\right) \Rightarrow mg_d = mg\left(1 - \frac{d}{R}\right)$

Here $\quad d = \frac{R}{2}$

$\therefore \quad mg_d = (250) \times \left(1 - \frac{R/2}{R}\right) = 250 \times \frac{1}{2} = 125 \text{ N}.$

Question 8. 17. A rocket is fired vertically with a speed of 5 km s^{-1} from the earth's surface. How far from the earth does the rocket go before returning to the earth? Mass of the earth = 6.0 x 10^{24} kg; mean radius of the earth = 6.4 x 10^6 m; G = 6.67 x 10^{-11} N m^2 kg^{-2}.
Answer:
Initial kinetic energy of rocket = 1/2 mv^2 = 1/2 x m x $(5000)^2$ = 1.25 x 10^7 mJ
At distance r from centre of earth, kinetic energy becomes zero
$\therefore$ Change in kinetic energy = 1.25 x 10^7 - 0 = 1.25 x 10^7 m J

This energy changes into potential energy.
Initial potential energy at the surface of earth = $GM_e m/r$

$$= \frac{-(6.67\times10^{-11})\times(6\times10^{24})m}{6.4\times10^{6}} = -6.25\ m \times 10^{7}\ \text{J}$$

Final potential energy at distance, $r = -\frac{GM_e m}{r}$

$$= \frac{-(6.67\times10^{-11})\times(6\times10^{24})m}{r} = -4\times10^{14}\frac{m}{r}\ \text{J}$$

$\therefore$ Change in potential energy $= 6.25 \times 10^{7}\ m - 4 \times 10^{14}\ \frac{m}{r}$

Using law of conservation of energy,

$$6.25\times10^{7}\ m - \frac{4\times10^{14}m}{r} = 1.25 \times 10^{7}\ \text{m}$$

i.e.,

$$r = \frac{4\times10^{14}}{5\times10^{7}}m = 8\times10^{16}m.$$

Question 8. 18. The escape speed of a projectile on the Earth's surface is 11.2 km s^{-1}. A body is projected out with thrice this speed. What is the speed of the body far away from the Earth? Ignore the presence of the Sun and other planets.
Answer:
Let v_{es} be the escape speed from surface of Earth having a value v_{es} = 11.2 kg s^{-1} = 11.2 × 10^{3} m s^{-1}. By definition

$$\frac{1}{2}mv_e^2 = \frac{GMm}{R^2} \quad ...(i)$$

When a body is projected with a speed $v_i = 3v_{es} = 3 \times 11.2 \times 10^{3}$ m/s, then it will have a final speed v_f such that

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 - \frac{GMm}{R^2} = \frac{1}{2}mv_i^2 - \frac{1}{2}mv_e^2$$

$$\Rightarrow \quad v_f = \sqrt{v_i^2 - v_e^2}$$

$$= \sqrt{(3\times11.2\times10^{3}) - (11.2\times10^{3})^2}$$

$$= 11.2\times10^{3}\times\sqrt{8}$$

$$= 31.7 \times 10^{3}\ \text{ms}^{-1} \text{ or } 31.7\ \text{km s}^{-1}.$$

Question 8. 19. A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of the earth's gravitational influence? Mass of the satellite = 200 kg; mass of the earth = 6.0 × 10^{24} kg; radius of the earth = 6.4 × 10^{6} m; G = 6.67 x 10^{-11} N m^2 kg^{-2}.
Answer:
Total energy of orbiting satellite at a height h

$$= -\frac{GMm}{(R+h)} + \frac{1}{2}mv^2 = -\frac{GMm}{(R+h)} + \frac{1}{2}m\frac{GM}{(R+h)}$$

$$= -\frac{GMm}{2(R+h)}$$

Energy expended to rocket the satellite out of the earth's gravitational field
= – (total energy of the orbiting satellite)

$$= \frac{GMm}{2(R+h)} = \frac{(6.67\times10^{-11})\times(6\times10^{24})\times200}{2\times(6.4\times10^{6}+4\times10^{5})}$$

$$= 5.9 \times 10^{9}\ \text{J}.$$

Question 8. 20. Two stars each of one solar mass (=2 x10^{30} kg) are approaching each other for a head on collision.When they are at a distance 10^{9} km, their speeds are negligible. What is the speed with which they collide? The radius of each star is 10^{4} km. Assume the stars to remain undistorted until they collide. (Use the known value of G).
Answer:
Here, mass of each star, M = 2 × 10^{30} kg

Initial potential between two stars, r = 10^9 km = 10^{12} m.
Initial potential energy of the system = -GMm/r'
Total K.E. of the stars = $1/2Mv^2 + 1/2Mv^2$
where v is the speed of stars with which they collide. When the stars are about to collide, the distance between their centres, r' = 2 R.
:. Final potential energy of two stars = -GMm/2R
Since gain in K.E. is at the cost of loss in P.E

$$\therefore \quad Mv^2 = -\frac{GMM}{r} - \left(-\frac{GMM}{2R}\right) = -\frac{GMM}{r} + \frac{GMM}{2R}$$

$$\text{or} \quad 2 \times 10^{30}\, v^2 = -\frac{6.67 \times 10^{-11} \times \left(2 \times 10^{30}\right)^2}{10^{12}} + \frac{6.67 \times 10^{-11} \times \left(2 \times 10^{30}\right)^2}{2 \times 10^{7}}$$

$$= -2.668 \times 10^{38} + 1.334 \times 10^{43}$$

$$= 1.334 \times 10^{43} \text{ J}$$

$$\therefore \quad v = \sqrt{\frac{1.334 \times 10^{43}}{2 \times 10^{30}}} = 2.583 \times 10^{6} \text{ ms}^{-1}.$$

*********** END **********