



## Factorisation of Polynomials Ex 6.4 Q22

**Answer :**

By division algorithm, when  $p(x) = 3x^3 + x^2 - 22x + 9$  is divided by  $3x^2 + 7x - 6$ , the remainder is a linear polynomial. So, let  $r(x) = ax + b$  be added to  $p(x)$  so that the result is divisible by  $q(x)$

Let

$$\begin{aligned} f(x) &= p(x) + r(x) \\ &= 3x^3 + x^2 - 22x + 9 + ax + b \\ &= 3x^3 + x^2 + (a - 22)x + 9 + b \end{aligned}$$

We have,

$$\begin{aligned} q(x) &= 3x^2 + 7x - 6 \\ &= 3x^2 + 9x - 2x - 6 \\ &= 3x(x + 3) - 2(x + 3) \\ &= (3x - 2)(x + 3) \end{aligned}$$

Clearly,  $(3x - 2)$  and  $(x + 3)$  are factors of  $q(x)$ .

Therefore,  $f(x)$  will be divisible by  $q(x)$  if  $(3x - 2)$  and  $(x + 3)$  are factors of  $f(x)$ , i.e.,

$$f\left(\frac{2}{3}\right) \text{ and } f(-3) \text{ are equal to zero.}$$

Now,

$$\begin{aligned} f\left(\frac{2}{3}\right) &= 0 \\ \Rightarrow 3\left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^2 + (a - 22)\left(\frac{2}{3}\right) + 9 + b &= 0 \\ \Rightarrow 3 \times \frac{8}{27} + \frac{4}{9} + \frac{2a}{3} - \frac{44}{3} + 9 + b &= 0 \\ \Rightarrow \frac{8}{9} + \frac{4}{9} - \frac{44}{3} + 9 + \frac{2a}{3} + b &= 0 \\ \Rightarrow \frac{8 + 4 - 132 + 81}{9} + \frac{2a}{3} + b &= 0 \\ \Rightarrow -\frac{39}{9} + \frac{2a}{3} + b &= 0 \\ \Rightarrow \frac{2a}{3} + b &= \frac{13}{3} \\ \Rightarrow 2a + 3b &= 13 \quad \dots\dots\dots (i) \end{aligned}$$

And

$$\begin{aligned} f(-3) &= 0 \\ \Rightarrow 3(-3)^3 + (-3)^2 + (a - 22)(-3) + 9 + b &= 0 \\ \Rightarrow -81 + 9 - 3a + 66 + 9 + b &= 0 \\ \Rightarrow -3a + b &= -3 \\ \Rightarrow b &= -3 + 3a \quad \dots\dots\dots (ii) \end{aligned}$$

Substituting the value of  $b$  from (ii) in (i), we get,

$$\begin{aligned} 2a + 3(3a - 3) &= 13 \\ \Rightarrow 2a + 9a - 9 &= 13 \\ \Rightarrow 11a &= 13 + 9 \\ \Rightarrow 11a &= 22 \\ \Rightarrow a &= 2 \end{aligned}$$

Now, from (ii), we get

$$b = -3 + 3(2) = -3 + 6 = 3$$

So, we have  $a = 2$  and  $b = 3$

Hence,  $p(x)$  is divisible by  $q(x)$ , if  $2x + 3$  is added to it.

## Factorisation of Polynomials Ex 6.4 Q23

**Answer :**(i) Let  $f(x) = x^3 - 2ax^2 + ax - 1$  be the given polynomial.By factor theorem, if  $(x - 2)$  is a factor of  $f(x)$ , then  $f(2) = 0$ 

Therefore,

$$\begin{aligned} f(2) &= (2)^3 - 2a(2)^2 + a(2) - 1 = 0 \\ 8 - 8a + 2a - 1 &= 0 \\ -6a + 7 &= 0 \\ a &= 7/6 \end{aligned}$$

Thus the value of  $a$  is  $7/6$ .(ii) Let  $f(x) = x^5 - 3x^4 - ax^3 + 3ax^2 + 2ax + 4$  be the given polynomial.By the factor theorem,  $(x - 2)$  is a factor of  $f(x)$ , if  $f(2) = 0$ 

Therefore,

$$\begin{aligned} f(2) &= (2)^5 - 3(2)^4 - a(2)^3 + 3a(2)^2 + 2a(2) + 4 = 0 \\ 32 - 48 - 8a + 12a + 4a + 4 &= 0 \\ -12 + 8a &= 0 \\ a &= 3/2 \end{aligned}$$

Thus, the value of  $a$  is  $\frac{3}{2}$ .

## Factorisation of Polynomials Ex 6.4 Q24

**Answer :**(i) Let  $f(x) = x^6 - ax^5 + x^4 - ax^3 + 3x - a + 2$  be the given polynomial.By factor theorem,  $(x - a)$  is a factor of the polynomial if  $f(a) = 0$ 

Therefore,

$$\begin{aligned} f(a) &= a^6 - a(a)^5 + (a)^4 - a(a)^3 + 3(a) - a + 2 = 0 \\ \cancel{a^6} - \cancel{a^6} + \cancel{a^4} - \cancel{a^4} + 2a + 2 &= 0 \\ 2a + 2 &= 0 \\ a &= -1 \end{aligned}$$

Thus, the value of  $a$  is  $-1$ .(ii) Let  $f(x) = x^5 - a^2x^3 + 2x + a + 1$  be the given polynomial.By factor theorem,  $(x - a)$  is a factor of  $f(x)$ , if  $f(a) = 0$ .

Therefore,

$$\begin{aligned} \Rightarrow f(a) &= (a)^5 - a^2(a)^3 + 2(a) + a + 1 = 0 \\ \cancel{a^5} - \cancel{a^5} + 2a + a + 1 &= 0 \\ 3a + 1 &= 0 \\ a &= -1/3 \end{aligned}$$

Thus, the value of  $a$  is  $-1/3$ .

## Factorisation of Polynomials Ex 6.4 Q25

**Answer :**

(i) Let  $f(x) = x^3 + ax^2 - 2x + a + 4$  be the given polynomial.

By the factor theorem,  $(x + a)$  is the factor of  $f(x)$ , if  $f(-a) = 0$ , i.e.,

$$f(-a) = (-a)^3 + a(-a)^2 - 2(-a) + a + 4 = 0$$

$$-a^3 + a^3 + 2a + a + 4 = 0$$

$$3a + 4 = 0$$

$$a = -4/3$$

Thus, the value of  $a$  is  $-4/3$ .

(ii) Let  $f(x) = x^4 - a^2x^2 + 3x - a$  be the polynomial. By factor theorem,  $(x + a)$  is a factor of the  $f(x)$ ,

if  $f(-a) = 0$ , i.e.,

$$f(-a) = (-a)^4 - a^2(-a)^2 + 3(-a) - a = 0$$

$$a^4 - a^4 - 3a - a = 0$$

$$-4a = 0$$

$$a = 0$$

Thus, the value of  $a$  is 0.

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