Increasing and Decreasing Functions Ex 17.2 Q1(xvii)

We have

$$f(x) = 2x^3 - 24x + 7$$
$$f'(x) = 6x^2 - 24$$

Critical points

$$f'(x) = 0$$
$$6x^2 - 24 = 0$$
$$6x^2 = 24$$
$$x^2 = 4$$
$$x = 2, -2$$

Clearly, $f'(x) > 0$ if $x > 2$ and $x < -2$

$$f'(x) < 0 \text{ if } -2 \le x \le 2$$

Thus, $f(x)$ is increasing in $(-\infty, -2) \cup (2, \infty)$, decreasing in $(-2, 2)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xviii)

We have $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$

$\therefore \quad f'(x) = \frac{3}{10}(4x^3) - \frac{4}{5}(3x^2) - 3(2x) + \frac{36}{5}$

$= \frac{6}{5}(x-1)(x+2)(x-3)$

Now $f'(x) = 0$

$\Rightarrow \frac{6}{5}(x-1)(x+2)(x-3) = 0$

$\Rightarrow x = 1, -2$ or 3

The points $x = 1, -2$ and 3 divide the number line into four disjoint intervals namely, $(-\infty, -2)$, $(-2, 1)$, $(1, 3)$ and $(3, \infty)$.

Consider the interval $(-\infty, -2)$, i.e $-\infty < x < -2$

In this case, we have $x - 1 < 0$, $x + 2 < 0$ and $x - 3 < 0$

$\therefore \quad f'(x) < 0$ when $-\infty < x < -2$

Thus, the function f is strictly decreasing in $(-\infty, -2)$

Consider the interval $(-2, 1)$, i.e $-2 < x < 1$

In this case, we have $x - 1 < 0$, $x + 2 > 0$ and $x - 3 < 0$

$\therefore \quad f'(x) > 0$ when $-2 < x < 1$

Thus, the function f is strictly increasing in $(-2, 1)$

Now, consider the interval $(1, 3)$, i.e $1 < x < 3$

In this case, we have $x - 1 > 0$, $x + 2 > 0$ and $x - 3 < 0$

$\therefore \quad f'(x) < 0$ when $1 < x < 3$

Thus, the function f is strictly decreasing in $(1, 3)$

Finally consider the interval $(3, \infty)$, i.e $3 < x < \infty$

In this case, we have $x - 1 > 0$, $x + 2 > 0$ and $x - 3 > 0$

$\therefore \quad f'(x) > 0$ when $x > 3$

Thus, the function f is strictly increasing in $(3, \infty)$

Increasing and Decreasing Functions Ex 17.2 Q1(xix)

We have,

$$f(x) = x^4 - 4x$$

$$\therefore \quad f'(x) = 4x^3 - 4$$

Critical points,

$$f'(x) = 0$$

$$\Rightarrow \quad 4\left(x^3 - 1\right) = 0$$

$$\Rightarrow \quad x = 1$$

Clearly, $f'(x) > 0$ if $x > 1$

$f'(x) < 0$ if $x < 1$

Thus, $f(x)$ increases in $(1, \infty)$, decreases in $(-\infty, 1)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xx)

$$f(x) = \frac{x^4}{4} + \frac{2}{3}x^3 - \frac{5}{2}x^2 - 6x + 7$$

$$\therefore \quad f'(x) = x^3 + 2x^2 - 5x - 6$$

Critical points

$$f'(x) = 0$$

$$\Rightarrow \quad x^3 + 2x^2 - 5x - 6 = 0$$

$$\Rightarrow \quad (x+1)(x+3)(x-2) = 0$$

$$\Rightarrow \quad x = -1, \ -3, \ 2$$

Clearly, $f'(x) > 0$ if $-3 < x < -1$ and $x > 2$

$f'(x) < 0$ if $x < -3$ and $-1 < x < 2$

Thus, $f(x)$ increases in $(-3, -1) \cup (2, \infty)$, decreases in $(-\infty, -3) \cup (-1, 2)$.

*********** END **********