(a) Maximum current is given as:

$$I_0 = \frac{V_0}{\left(\omega L - \frac{1}{\omega C}\right)}$$

$$= \frac{230\sqrt{3}}{\left(100\pi \times 80 \times 10^{-3} - \frac{1}{100\pi \times 60 \times 10^{-6}}\right)}$$

$$= \frac{230\sqrt{2}}{\left(8\pi - \frac{1000}{6\pi}\right)} = -11.63 \text{ A}$$

The negative sign appears because $\omega L < \frac{1}{\omega C}$.

Amplitude of maximum current, $|I_0| = 11.63 \text{ A}$

Hence, rms value of current, $I = \frac{I_0}{\sqrt{2}} = \frac{-11.63}{\sqrt{2}} = -8.22 \text{ A}$

(b) Potential difference across the inductor,

$V_L = I \times \omega L$

$= 8.22 \times 100\pi \times 80 \times 10^{-3}$

$= 206.61$ V

Potential difference across the capacitor,

$$V_c = I \times \frac{1}{\omega C}$$

$$= 8.22 \times \frac{1}{100\pi \times 60 \times 10^{-6}} = 436.3 \text{ V}$$

(c) Average power consumed by the inductor is zero as actual voltage leads the current by $\frac{\pi}{2}$.

(d) Average power consumed by the capacitor is zero as voltage lags current by $\frac{\pi}{2}$.

(e) The total power absorbed (averaged over one cycle) is zero.

Question 7.19:

Suppose the circuit in Exercise 7.18 has a resistance of 15 Ω. Obtain the average power transferred to each element of the circuit, and the total power absorbed.

Answer

Average power transferred to the resistor = 788.44 W

Average power transferred to the capacitor = 0 W

Total power absorbed by the circuit = 788.44 W

Inductance of inductor, L = 80 mH = 80×10^{-3} H

Capacitance of capacitor, C = 60 μF = 60×10^{-6} F

Resistance of resistor, R = 15 Ω

Potential of voltage supply, V = 230 V

Frequency of signal, ν = 50 Hz

Angular frequency of signal, $\omega = 2\pi\nu = 2\pi \times (50) = 100\pi$ rad/s

The elements are connected in series to each other. Hence, impedance of the circuit is given as:

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \sqrt{(15)^2 + \left(100\pi\left(80 \times 10^{-3}\right) - \frac{1}{\left(100\pi \times 60 \times 10^{-6}\right)}\right)^2}$$

$$= \sqrt{(15)^2 + (25.12 - 53.08)^2} = 31.728 \ \Omega$$

Current flowing in the circuit, $I = \frac{V}{Z} = \frac{230}{31.728} = 7.25 \text{ A}$

Average power transferred to resistance is given as:

$P_R = I^2 R$

$= (7.25)^2 \times 15 = 788.44$ W

Average power transferred to capacitor, P_C = Average power transferred to inductor, P_L

= 0

Total power absorbed by the circuit:

= $P_R + P_C + P_L$

= 788.44 + 0 + 0 = 788.44 W

Hence, the total power absorbed by the circuit is 788.44 W.

Question 7.20:

A series *LCR* circuit with L = 0.12 H, C = 480 nF, R = 23 Ω is connected to a 230 V variable frequency supply.

(a) What is the source frequency for which current amplitude is maximum. Obtain this maximum value.

(b) What is the source frequency for which average power absorbed by the circuit is maximum. Obtain the value of this maximum power.

(c) For which frequencies of the source is the power transferred to the circuit half the power at resonant frequency? What is the current amplitude at these frequencies?

(d) What is the *Q*-factor of the given circuit?

Answer

Inductance, L = 0.12 H

Capacitance, C = 480 nF = 480×10^{-9} F

Resistance, R = 23 Ω

Supply voltage, V = 230 V

Peak voltage is given as:

$V_0 = \sqrt{2} \times 230$ = 325.22 V

(a) Current flowing in the circuit is given by the relation, $I_0 = \dfrac{V_0}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$

Where,

I_0 = maximum at resonance

At resonance, we have

$$\omega_R L - \frac{1}{\omega_R C} = 0$$

Where,

ω_R = Resonance angular frequency

$$\therefore \omega_R = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{0.12 \times 480 \times 10^{-9}}} = 4166.67 \text{ rad/s}$$

∴Resonant frequency, $\nu_R = \dfrac{\omega_R}{2\pi} = \dfrac{4166.67}{2 \times 3.14} = 663.48 \text{ Hz}$

And, maximum current $(I_0)_{\text{Max}} = \dfrac{V_0}{R} = \dfrac{325.22}{23} = 14.14 \text{ A}$

(b) Maximum average power absorbed by the circuit is given as:

$$(P_{av})_{\text{Max}} = \frac{1}{2}(I_0)^2_{\text{Max}} R$$

$$= \frac{1}{2} \times (14.14)^2 \times 23 = 2299.3 \text{ W}$$

Hence, resonant frequency (ν_R) is 663.48 Hz.

(c) The power transferred to the circuit is half the power at resonant frequency.

Frequencies at which power transferred is half, = $\omega_R \pm \Delta\omega$

$= 2\pi(\nu_R \pm \Delta\nu)$

Where,

$$\Delta\omega = \frac{R}{2L}$$

$$= \frac{23}{2 \times 0.12} = 95.83 \text{ rad/s}$$

Hence, change in frequency, $\Delta\nu = \dfrac{1}{2\pi}\Delta\omega = \dfrac{95.83}{2\pi} = 15.26 \text{ Hz}$

$\therefore \nu_R + \Delta\nu = 663.48 + 15.26 = 678.74 \text{ Hz}$

And, $\nu_R - \Delta\nu = 663.48 - 15.26 = 648.22 \text{ Hz}$

Hence, at 648.22 Hz and 678.74 Hz frequencies, the power transferred is half.

At these frequencies, current amplitude can be given as:

$$I = \frac{1}{\sqrt{2}} \times (I_0)_{Max}$$

$$= \frac{14.14}{\sqrt{2}} = 10 \text{ A}$$

(d) Q-factor of the given circuit can be obtained using the relation, $Q = \frac{\omega_R L}{R}$

$$= \frac{4166.67 \times 0.12}{23} = 21.74$$

Hence, the Q-factor of the given circuit is 21.74.

Question 7.21:

Obtain the resonant frequency and Q-factor of a series LCR circuit with L = 3.0 H, C = 27 µF, and R = 7.4 Ω. It is desired to improve the sharpness of the resonance of the circuit by reducing its 'full width at half maximum' by a factor of 2. Suggest a suitable way.

Answer

Inductance, L = 3.0 H

Capacitance, C = 27 µF = 27 × 10^{-6} F

Resistance, R = 7.4 Ω

At resonance, angular frequency of the source for the given LCR series circuit is given as:

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{3 \times 27 \times 10^{-6}}} = \frac{10^3}{9} = 111.11 \text{ rad s}^{-1}$$

Q-factor of the series:

$$Q = \frac{\omega_r L}{R}$$

$$= \frac{111.11 \times 3}{7.4} = 45.0446$$

To improve the sharpness of the resonance by reducing its 'full width at half maximum' by a factor of 2 without changing ω_r, we need to reduce R to half i.e.,

Resistance = $\frac{R}{2} = \frac{7.4}{2} = 3.7\ \Omega$

Question 7.22:

Answer the following questions:

(a) In any ac circuit, is the applied instantaneous voltage equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit? Is the same true for rms voltage?

(b) A capacitor is used in the primary circuit of an induction coil.

(c) An applied voltage signal consists of a superposition of a dc voltage and an ac voltage of high frequency. The circuit consists of an inductor and a capacitor in series. Show that the dc signal will appear across C and the ac signal across L.

(d) A choke coil in series with a lamp is connected to a dc line. The lamp is seen to shine brightly. Insertion of an iron core in the choke causes no change in the lamp's brightness. Predict the corresponding observations if the connection is to an ac line.

(e) Why is choke coil needed in the use of fluorescent tubes with ac mains? Why can we not use an ordinary resistor instead of the choke coil?

Answer

(a) Yes; the statement is not true for rms voltage

It is true that in any ac circuit, the applied voltage is equal to the average sum of the instantaneous voltages across the series elements of the circuit. However, this is not true for rms voltage because voltages across different elements may not be in phase.

(b) High induced voltage is used to charge the capacitor.

A capacitor is used in the primary circuit of an induction coil. This is because when the circuit is broken, a high induced voltage is used to charge the capacitor to avoid sparks.

(c) The dc signal will appear across capacitor C because for dc signals, the impedance of an inductor (L) is negligible while the impedance of a capacitor (C) is very high (almost infinite). Hence, a dc signal appears across C. For an ac signal of high frequency, the impedance of L is high and that of C is very low. Hence, an ac signal of high frequency appears across L.

(d) If an iron core is inserted in the choke coil (which is in series with a lamp connected

*********** END **********