



### Relations Ex 1.2 Q5

We have,  $\mathbb{Z}$  be set of integers and

$R = \{(a, b) : a, b \in \mathbb{Z} \text{ and } a + b \text{ is even}\}$  be a relation on  $\mathbb{Z}$ .

We want to prove that  $R$  is an equivalence relation on  $\mathbb{Z}$ .

Now,

Reflexivity: Let  $a \in \mathbb{Z}$

$$\begin{aligned} \Rightarrow a + a & \text{ is even} && \begin{cases} \text{if } a \text{ is even} \Rightarrow a + a \text{ is even} \\ \text{if } a \text{ is odd} \Rightarrow a + a \text{ is even} \end{cases} \\ \Rightarrow (a, a) & \in R \\ \Rightarrow R & \text{ is reflexive} \end{aligned}$$

Symmetric: Let  $a, b \in \mathbb{Z}$  and  $(a, b) \in R$

$$\begin{aligned} \Rightarrow a + b & \text{ is even} \\ \Rightarrow b + a & \text{ is even} \\ \Rightarrow (b, a) & \in R, \\ \Rightarrow R & \text{ is symmetric} \end{aligned}$$

Transitivity: Let  $(a, b) \in R$  and  $(b, c) \in R$  For some  $a, b, c \in \mathbb{Z}$

$$\begin{aligned} \Rightarrow a + b & \text{ is even and } b + c \text{ is even} \\ \Rightarrow a + c & \text{ is even} && \begin{cases} \text{if } b \text{ is odd, then } a \text{ and } c \text{ must be odd} \Rightarrow a + c \text{ is even,} \\ \text{if } b \text{ is even, then } a \text{ and } c \text{ must be even} \Rightarrow a + c \text{ is even} \end{cases} \\ \Rightarrow (a, c) & \in R \\ \Rightarrow R & \text{ is transitive} \end{aligned}$$

Hence,  $R$  is an equivalence relation on  $\mathbb{Z}$

### Relations Ex 1.2 Q6

Let  $Z$  be set of integers

$R = \{(m, n) : m - n \text{ is divisible by } 13\}$  be a relation on  $Z$ .

Now,

Reflexivity: Let  $m \in Z$

- $\Rightarrow m - m = 0$
- $\Rightarrow m - m$  is divisible by 13
- $\Rightarrow (m, m) \in R,$
- $\Rightarrow R$  is reflexive

Symmetric: Let  $m, n \in Z$  and  $(m, n) \in R$

- $\Rightarrow m - n = 13p$  For some  $p \in Z$
- $\Rightarrow n - m = 13 \times (-p)$
- $\Rightarrow n - m$  is divisible by 13
- $\Rightarrow (n - m) \in R,$
- so
- $\Rightarrow R$  is symmetric

Transitivity: Let  $(m, n) \in R$  and  $(n, q) \in R$  For some  $m, n, q \in Z$

- $\Rightarrow m - n = 13p$  and  $n - q = 13s$  For some  $p, s \in Z$
- $\Rightarrow m - q = 13(p + s)$
- $\Rightarrow m - q$  is divisible by 13
- $\Rightarrow (m, q) \in R$
- $\Rightarrow R$  is transitive

Hence,  $R$  is an equivalence relation on  $Z$

Relations Ex 1.2 Q7

$(x, y) R (u, v) \Leftrightarrow xv = yu$

TPT Reflexive  $\therefore xy = yx$

$\therefore (x, y) R (x, y)$

TPT Symmetric Let  $(x, y) R (u, v)$

TPT  $(u, v) R (x, y)$

Given  $xv = yu$

$\Rightarrow yu = xv$

$\Rightarrow uy = vx$

$\therefore (u, v) R (x, y)$

Transitive Let  $(x, y) R (u, v)$  and  $(u, v) R (p, q)$  .....(i)

TPT  $(x, y) R (p, q)$

TPT  $xq = yp$

from (i)  $xv = yu$  &  $uq = vp$

$xvuq = yuvp$

$xq = yp$

$\therefore R$  is transitive

since  $R$  is reflexive symmetric & transitive all means it is an equivalence relation.]

Relations Ex 1.2 Q8

We have,  $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$  be a set and

$R = \{(a, b) : a = b\}$  be a relation on  $A$

Now,

Reflexivity: Let  $a \in A$

$$\Rightarrow a = a$$

$$\Rightarrow (a, a) \in R$$

$$\Rightarrow R \text{ is reflexive}$$

Symmetric: Let  $a, b \in A$  and  $(a, b) \in R$

$$\Rightarrow a = b$$

$$\Rightarrow b = a$$

$$\Rightarrow (b, a) \in R$$

$$\Rightarrow R \text{ is symmetric}$$

Transitive: Let  $a, b$  &  $c \in A$

and Let  $(a, b) \in R$  and  $(b, c) \in R$

$$\Rightarrow a = b \text{ and } b = c$$

$$\Rightarrow a = c$$

$$\Rightarrow (a, c) \in R$$

$$\Rightarrow R \text{ is transitive}$$

Since  $R$  is being reflexive, symmetric and transitive, so  $R$  is an equivalence relation.

Also, we need to find the set of all elements related to 1.

Since the relation is given by,  $R = \{(a, b) : a = b\}$ , and 1 is an element of  $A$ ,

$$R = \{(1, 1) : 1 = 1\}$$

Thus, the set of all elements related to 1 is 1.

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