Differentiation Ex 11.2 Q67

Given, $y = e^x \cos x$

Differentiating with respect to x,

$$\begin{aligned}
\frac{dy}{dx} &= \frac{d}{dx}\left(e^x \cos x\right) \\
&= e^x \frac{d}{dx}\cos x + \cos x \frac{d}{dx}e^x && \text{[Using product rule]} \\
&= e^x\left(-\sin x\right) + e^x \cos x \\
&= e^x\left(\cos x - \sin x\right) \\
&= \sqrt{2}e^x\left(\frac{\cos x}{\sqrt{2}} - \frac{\sin x}{\sqrt{2}}\right) && \left[\text{Multiplying and dividing by } \sqrt{2}\right] \\
&= \sqrt{2}e^x\left(\cos\frac{\pi}{4}\cos x - \sin\frac{\pi}{4}\sin x\right)
\end{aligned}$$

$$\frac{dy}{dx} = \sqrt{2}e^x \cos\left(x + \frac{\pi}{4}\right).$$

Differentiation Ex 11.2 Q68

Given, $y = \frac{1}{2}\log\left(\frac{1 - \cos 2x}{1 + \cos 2x}\right)$

$$\Rightarrow \quad y = \frac{1}{2}\log\left(\frac{2\sin^2 x}{2\cos^2 x}\right) \qquad \left[\begin{aligned}\text{Since, } 1 - \cos 2x &= 2\sin^2 x, \\ 1 + co2x &= 2\cos^2 x\end{aligned}\right]$$

$$\Rightarrow \quad y = \frac{1}{2}\log\left(\tan^2 x\right)$$

$$\Rightarrow \quad y = \frac{2}{2}\log\tan x \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

$$\Rightarrow \quad y = \log\tan x$$

Differentiate it with respect to x,

$$\begin{aligned}
\frac{dy}{dx} &= \left(\log\tan x\right) \\
&= \frac{1}{\tan x} \times \frac{d}{dx}\left(\tan x\right) && \text{[Using chain rule]} \\
&= \frac{\sec^2 x}{\tan x} \\
&= \frac{1}{\cos^2 x \times \frac{\sin x}{\cos x}} \\
&= \frac{1}{\sin x \cos x} \\
&= \frac{2}{2\sin x \cos x} \\
&= \frac{2}{\sin 2x} && \left[\text{Since, } \frac{1}{\sin x = \cos ecx}\right]
\end{aligned}$$

So,

$$\frac{dy}{dx} = 2\,\mathrm{cosec}\ 2x.$$

Differentiation Ex 11.2 Q69

Here, $y = x\sin^{-1}x + \sqrt{1-x^2}$

Differentiating with respect to x,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left[x\sin^{-1}x + \sqrt{1-x^2}\right]\\ &= \frac{d}{dx}\left(x\sin^{-1}x\right) + \frac{d}{dx}\left(\sqrt{1-x^2}\right)\\ &= \left[x\frac{d}{dx}\sin^{-1}x + \sin^{-1}x\frac{d}{dx}(x)\right] + \frac{1}{2\sqrt{1-x^2}}\frac{d}{dx}\left(1-x^2\right)\end{aligned}$$

[Using product rule and chain rule]

$$\begin{aligned}&= \left[\frac{x}{\sqrt{1-x^2}} + \sin^{-1}x\right] - \frac{2x}{2\sqrt{1-x^2}}\\ &= \frac{x}{\sqrt{1-x^2}} + \sin^{-1}x - \frac{x}{\sqrt{1-x^2}}\\ &= \sin^{-1}x\end{aligned}$$

So,

$$\frac{dy}{dx} = \sin^{-1}x.$$

Differentiation Ex 11.2 Q70

Here, $y = \sqrt{x^2+a^2}$

Differentiating with respect to x,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(\sqrt{x^2+a^2}\right)\\ &= \frac{1}{2\sqrt{x^2+a^2}}\frac{d}{dx}\left(x^2+a^2\right) && \text{[Using chain rule]}\\ &= \frac{1}{2\sqrt{x^2+a^2}}\times(2x)\\ &= \frac{x}{\sqrt{x^2+a^2}}\\ \frac{dy}{dx} &= \frac{x}{y} && \left[\text{Since } \sqrt{x^2+a^2} = y\right]\end{aligned}$$

$\Rightarrow \quad y\frac{dy}{dx} = x$

$\Rightarrow \quad y\frac{dy}{dx} - x = 0.$

*********** END **********