Trigonometric Ratios of Compound Angles Ex 7.1 Q31

We have,

$\tan\alpha = x + 1$ and $\tan\beta = x - 1$

Now, $2\cot(\alpha - \beta)$

$$= \frac{2}{\tan(\alpha-\beta)}$$

$$= \frac{2}{\frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}}$$

$$= \frac{2(1+\tan\alpha\tan\beta)}{\tan\alpha - \tan\beta}$$

$$= \frac{2[1+(x+1)(x-1)]}{x+1-(x-1)}$$

$$= \frac{2[1+x^2-1]}{x+1-x+1}$$

$$= \frac{2\times x^2}{2} = x^2$$

$\therefore \quad 2\cot(\alpha-\beta) = x^2$

Hence proved

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Let the two parts of the angle be θ and $\theta - \varnothing$.

$\tan(\theta - \varnothing) = \lambda\tan\varnothing$ [*According to question*]

$$\Rightarrow \frac{\tan(\theta-\varnothing)}{\tan\varnothing} = \frac{\lambda}{1}$$

$$\Rightarrow \frac{\tan(\theta-\varnothing)}{\tan\varnothing} = \frac{\lambda}{1}$$

$$\Rightarrow \frac{\tan(\theta-\varnothing)+\tan\varnothing}{\tan(\theta-\varnothing)-\tan\varnothing} = \frac{\lambda+1}{\lambda-1} \quad \text{[Using Componendo and Dividendo]}$$

$$\Rightarrow \frac{\frac{\tan\theta-\tan\varnothing}{1+\tan\theta.\tan\varnothing}+\tan\varnothing}{\frac{\tan\theta-\tan\varnothing}{1+\tan\theta.\tan\varnothing}-\tan\varnothing} = \frac{\lambda+1}{\lambda-1}$$

$$\Rightarrow \frac{\frac{\tan\theta-\tan\varnothing+\tan\varnothing(1+\tan\theta.\tan\varnothing)}{1+\tan\theta.\tan\varnothing}}{\frac{\tan\theta-\tan\varnothing-\tan\varnothing(1+\tan\theta.\tan\varnothing)}{1+\tan\theta.\tan\varnothing}} = \frac{\lambda+1}{\lambda-1}$$

$$\Rightarrow \frac{\tan\theta-\tan\varnothing+\tan\varnothing+\tan\theta.\tan^2\varnothing}{\tan\theta-\tan\varnothing-\tan\varnothing-\tan\theta.\tan^2\varnothing} = \frac{\lambda+1}{\lambda-1}$$

$$\Rightarrow \frac{\tan\theta+\tan\theta.\tan^2\varnothing}{\tan\theta-2\tan\varnothing-\tan\theta.\tan^2\varnothing} = \frac{\lambda+1}{\lambda-1}$$

$$\Rightarrow \frac{\tan}{}$$

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$$\tan\theta = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$$

$$\Rightarrow \tan\theta = \frac{\tan\alpha - 1}{\tan\alpha + 1} [\textit{Dividing both Numerator and Denominator by } \cos\alpha]$$

$$\Rightarrow \tan\theta = \frac{\tan\alpha - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{4}.\tan\alpha}$$

$$\Rightarrow \tan\theta = \tan\left(\alpha - \frac{\pi}{4}\right)$$

$$\Rightarrow \theta = \alpha - \frac{\pi}{4} \quad [\textit{Removing } \tan \textit{ from both sides}]$$

$$\Rightarrow \cos\theta = \cos\left(\alpha - \frac{\pi}{4}\right) [\textit{Taking } \cos \textit{ on both sides}]$$

$$\Rightarrow \cos\theta = \cos\alpha.\cos\frac{\pi}{4} + \sin\alpha.\sin\frac{\pi}{4}$$

$$\Rightarrow \cos\theta = \cos\alpha.\frac{1}{\sqrt{2}} + \sin\alpha.\frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos\theta = \frac{\cos\alpha + \sin\alpha}{\sqrt{2}}$$

$$\Rightarrow \sqrt{2}\cos\theta = \sin\alpha + \cos\alpha$$

Hence Proved

Trigonometric Ratios of Compound Angles Ex 7.1 Q34

RHS,

$$\frac{p+q}{1-pq}$$

$$= \frac{\tan(A+B) + \tan(A-B)}{1 - \tan(A+B).\tan(A-B)}$$

$$= \frac{\frac{\tan A + \tan B}{1 - \tan A.\tan B} + \frac{\tan A - \tan B}{1 + \tan A.\tan B}}{1 - \frac{\tan A + \tan B}{1 - \tan A.\tan B}.\frac{\tan A - \tan B}{1 + \tan A.\tan B}}$$

$$= \frac{\frac{(\tan A + \tan B)(1 + \tan A.\tan B) + (\tan A - \tan B)(1 - \tan A.\tan B)}{(1 - \tan A.\tan B)(1 + \tan A.\tan B)}}{\frac{(1 - \tan A.\tan B)(1 + \tan A.\tan B) - (\tan A + \tan B).(\tan A - \tan B)}{(1 - \tan A.\tan B)(1 + \tan A.\tan B)}}$$

$$= \frac{\tan A + \tan B + \tan^2 A.\tan B + \tan A.\tan^2 B + \tan A - \tan B - \tan^2 A.\tan B + \tan A.\tan^2 B}{1 - \tan^2 A.\tan^2 B - \tan^2 A + \tan^2 B}$$

$$= \frac{2\tan A + 2\tan A.\tan^2 B}{(1 - \tan^2 A)(1 + \tan^2 B)} = \frac{2\tan A(1 + \tan^2 B)}{(1 - \tan^2 A)(1 + \tan^2 B)} = \frac{2\tan A}{1 - \tan^2 A} = \tan 2A = LHS$$

Hence Proved

*********** END **********