Exercise 7B

Question 1:

$m = a\cos\theta + b\sin\theta$ and $n = a\sin\theta - b\cos\theta$

$$\therefore \text{LHS} = m^2 + n^2 = (a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2$$

$$= \left(a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta\right)$$

$$+ \left(a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta\right)$$

$$= a^2\left(\cos^2\theta + \sin^2\theta\right) + b^2\left(\sin^2\theta + \cos^2\theta\right)$$

$$= a^2 + b^2 = \text{RHS}$$

$\therefore$ LHS = RHS

Question 2:

$x = a\sec\theta + b\tan\theta$, and $y = a\tan\theta + b\sec\theta$

$$\text{LHS} = \left(x^2 - y^2\right) = (a\sec\theta + b\tan\theta)^2 - (a\tan\theta + b\sec\theta)^2$$

$$= \left(a^2\sec^2\theta + b^2\tan^2\theta + 2ab\sec\theta\tan\theta\right)$$

$$- \left(a^2\tan^2\theta + b^2\sec^2\theta + 2ab\tan\theta\sec\theta\right)$$

$$= a^2\left(\sec^2\theta - \tan^2\theta\right) - b^2\left(\sec^2\theta - \tan^2\theta\right)$$

$$= a^2 - b^2 = \text{RHS}$$

$\therefore$ LHS = RHS

Question 3:

$\left(\frac{x}{a}\sin\theta-\frac{y}{b}\cos\theta\right)=1$ and $\left(\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta\right)=1$

Now, $\left(\frac{x}{a}\sin\theta-\frac{y}{b}\cos\theta\right)=1$

(Squaring both sides, we get)

$$\frac{x^2}{a^2}\sin^2\theta+\frac{y^2}{b^2}\cos^2\theta-\frac{2xy}{ab}\sin\theta\cos\theta=1-------(1)$$

$\left(\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta\right)=1$

(Squaring both sides, we get)

$$\frac{x^2}{a^2}\cos^2\theta+\frac{y^2}{b^2}\sin^2\theta+\frac{2xy}{ab}\sin\theta\cos=1------(2)$$

Adding (1) & (2), we get

$$\frac{x^2}{a^2}\left(\sin^2\theta+\cos^2\theta\right)+\frac{y^2}{b^2}\left(\sin^2\theta+\cos^2\theta\right)=2$$

$$\frac{x^2}{a}+\frac{y^2}{b}=2(\text{proved})$$

********** END **********