



Question 19:

The area bounded by the y -axis, $y = \cos x$ and $y = \sin x$ when $0 \leq x \leq \frac{\pi}{2}$

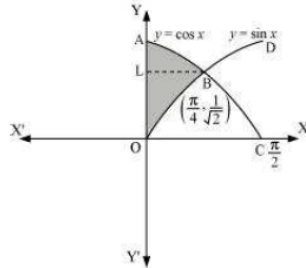
- A. $2(\sqrt{2}-1)$
 B. $\sqrt{2}-1$
 C. $\sqrt{2}+1$
 D. $\sqrt{2}$

Answer

The given equations are

$$y = \cos x \dots (1)$$

$$\text{And, } y = \sin x \dots (2)$$



Required area = Area (ABLA) + area (OBLO)

$$= \int_{\frac{1}{\sqrt{2}}}^1 x dy + \int_{\frac{1}{\sqrt{2}}}^1 x dy$$

$$= \int_{\frac{1}{\sqrt{2}}}^1 \cos^{-1} y dy + \int_{\frac{1}{\sqrt{2}}}^1 \sin^{-1} x dy$$

Integrating by parts, we obtain

$$= \left[y \cos^{-1} y - \sqrt{1-y^2} \right]_{\frac{1}{\sqrt{2}}}^1 + \left[x \sin^{-1} x + \sqrt{1-x^2} \right]_{\frac{1}{\sqrt{2}}}^1$$

$$= \left[\cos^{-1}(1) - \frac{1}{\sqrt{2}} \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{1-\frac{1}{2}} \right] + \left[\frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{1-\frac{1}{2}} - 1 \right]$$

$$= \frac{-\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} - 1$$

$$= \frac{2}{\sqrt{2}} - 1$$

$$= \sqrt{2} - 1 \text{ units}$$

Thus, the correct answer is B.

$$\text{Put } 2x = t \Rightarrow dx = \frac{dt}{2}$$

$$\text{When } x = \frac{3}{2}, t = 3 \text{ and when } x = \frac{1}{2}, t = 1$$

$$= \int_0^{\frac{1}{2}} 2\sqrt{x} dx + \frac{1}{4} \int_1^3 \sqrt{(3)^2 - (t)^2} dt$$

$$= 2 \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^{\frac{1}{2}} + \frac{1}{4} \left[\frac{t}{2} \sqrt{9-t^2} + \frac{9}{2} \sin^{-1}\left(\frac{t}{3}\right) \right]_1^3$$

$$= 2 \left[\frac{2}{3} \left(\frac{1}{2}\right)^{\frac{3}{2}} \right] + \frac{1}{4} \left[\left\{ \frac{3}{2} \sqrt{9-(3)^2} + \frac{9}{2} \sin^{-1}\left(\frac{3}{3}\right) \right\} - \left\{ \frac{1}{2} \sqrt{9-(1)^2} + \frac{9}{2} \sin^{-1}\left(\frac{1}{3}\right) \right\} \right]$$

$$= \frac{2}{3\sqrt{2}} + \frac{1}{4} \left[\left\{ 0 + \frac{9}{2} \sin^{-1}(1) \right\} - \left\{ \frac{1}{2} \sqrt{8} + \frac{9}{2} \sin^{-1}\left(\frac{1}{3}\right) \right\} \right]$$

$$= \frac{\sqrt{2}}{3} + \frac{1}{4} \left[\frac{9\pi}{4} - \sqrt{2} - \frac{9}{2} \sin^{-1}\left(\frac{1}{3}\right) \right]$$

$$= \frac{\sqrt{2}}{3} + \frac{9\pi}{16} - \frac{\sqrt{2}}{4} - \frac{9}{8} \sin^{-1}\left(\frac{1}{3}\right)$$

$$= \frac{9\pi}{16} - \frac{9}{8} \sin^{-1}\left(\frac{1}{3}\right) + \frac{\sqrt{2}}{12}$$

$$\text{Therefore, the required area is } \left[2 \times \left(\frac{9\pi}{16} - \frac{9}{8} \sin^{-1}\left(\frac{1}{3}\right) + \frac{\sqrt{2}}{12} \right) \right] = \frac{9\pi}{8} - \frac{9}{4} \sin^{-1}\left(\frac{1}{3}\right) + \frac{1}{3\sqrt{2}} \text{ units}$$

