Sine and Cosine Formulae and their Applications Ex-10.1 Q17

Let $a = k\sin A, b = k\sin B, c = k\sin C$

LHS

$b\cos B + c\cos C$

$= k\sin B\cos B + k\sin C\cos C$

$= \frac{k}{2}(2\sin B\cos B + 2\sin C\cos C)$

$= \frac{k}{2}(\sin 2B + \sin 2C)$

$= \frac{k}{2}2\sin(B+C)\cos(B-C)$

$= k\sin(\pi - A)\cos(B-C)$

$= k\sin A\cos(B-C)$

$= a\cos(B-C) = RHS$

Sine and Cosine Formulae and their Applications Ex-10.1 Q18

$\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2} = \frac{1}{a^2} - \frac{1}{b^2}$

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$

LHS

$= \frac{1-2\sin^2 A}{a^2} - \frac{1-2\sin^2 B}{b^2}$

$= \frac{1}{a^2} - \frac{1}{b^2} - 2\left(\frac{\sin^2 A}{a^2} - \frac{\sin^2 B}{b^2}\right)$

$= \frac{1}{a^2} - \frac{1}{b^2} - 2\left(k^2 - k^2\right)$[Using sine rule]

$= \frac{1}{a^2} - \frac{1}{b^2} = RHS$

hence Proved

Sine and Cosine Formulae and their Applications Ex-10.1 Q19

$$\frac{\cos^2 B-\cos^2 C}{b+c}+\frac{\cos^2 C-\cos^2 A}{c+a}+\frac{\cos^2 A-\cos^2 B}{a+b}=0$$

LHS

$$\frac{\cos^2 B-\cos^2 C}{b+c}+\frac{\cos^2 C-\cos^2 A}{c+a}+\frac{\cos^2 A-\cos^2 B}{a+b}$$

$$=\frac{\cos^2 B-\cos^2 C}{b+c}+\frac{\cos^2 C-\cos^2 A}{c+a}+\frac{\cos^2 A-\cos^2 B}{a+b}$$

$$=\frac{1-\sin^2 B-1+\sin^2 C}{b+c}+\frac{1-\sin^2 C-1+\sin^2 A}{c+a}+\frac{1-\sin^2 A-1+\sin^2 B}{a+b}$$

$$=\frac{\sin^2 C-\sin^2 B}{b+c}+\frac{\sin^2 A-\sin^2 C}{c+a}+\frac{\sin^2 B-\sin^2 A}{a+b}$$

$$=\frac{k^2c^2-k^2b^2}{b+c}+\frac{k^2a^2-k^2c^2}{c+a}+\frac{k^2b^2-k^2a^2}{a+b}$$

$$=k^2(\frac{c^2-b^2}{b+c}+\frac{a^2-c^2}{c+a}+\frac{b^2-a^2}{a+b})$$

$$=k^2(c-b+a-c+b-a)[\text{Using } b^2-a^2=(b-a)(b+a)]$$

$$=0=RHS$$

Hence Proved

Sine and Cosine Formulae and their Applications Ex-10.1 Q20

We know $a\sin B=b\sin A, c\sin B=b\sin C, a\sin C=c\sin B$

$$a\sin\frac{A}{2}\sin\left(\frac{B-C}{2}\right)+b\sin\frac{B}{2}\sin\left(\frac{C-A}{2}\right)+c\sin\frac{C}{2}\sin\left(\frac{A-B}{2}\right)=0$$

LHS

$$=a\sin\left(\frac{\pi-(B+C)}{2}\right)\sin\left(\frac{B-C}{2}\right)+b\sin\left(\frac{\pi-(C+A)}{2}\right)\sin\left(\frac{C-A}{2}\right)$$

$$+c\sin\left(\frac{\pi-(A+B)}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$=a\cos\left(\frac{B+C}{2}\right)\sin\left(\frac{B-C}{2}\right)+b\cos\left(\frac{C+A}{2}\right)\sin\left(\frac{C-A}{2}\right)$$

$$+c\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$$=a(\sin B-\sin C)+b(\sin C-\sin A)+c(\sin A-\sin B)$$

$$=a\sin B-a\sin C+b\sin C-b\sin A+c\sin A-c\sin B$$

$$=b\sin A-a\sin C+b\sin C-b\sin A+a\sin C-b\sin C$$

$$=0=RHS$$

*********** END **********