



Definite Integrals Ex 20.2 Q21

We have,

$$\int_0^{\pi} \frac{\sin x}{\sin x + \cos x} dx$$

$$\begin{aligned} \text{Let } \sin x &= K (\sin x + \cos x) + L \frac{d}{dx} (\sin x + \cos x) \\ &= K (\sin x + \cos x) + L (\cos x - \sin x) \\ &= \sin x (K - L) + \cos x (K + L) \end{aligned}$$

Equating similar terms

$$K - L = 1$$

$$K + L = 0$$

$$\Rightarrow K = \frac{1}{2} \text{ and } L = -\frac{1}{2}$$

$$\begin{aligned} \therefore \int_0^{\pi} \frac{\sin x}{\sin x + \cos x} dx &= \frac{1}{2} \int_0^{\pi} dx + \left(\frac{-1}{2} \right) \int_0^{\pi} \frac{\cos x - \sin x}{\sin x + \cos x} dx \\ &= \frac{1}{2} [x]_0^{\pi} - \frac{1}{2} (\log |\sin x + \cos x|)_0^{\pi} = \frac{\pi}{2} - \frac{1}{2} (0) = \frac{\pi}{2} \end{aligned}$$

$$\therefore \int_0^{\pi} \frac{\sin x}{\sin x + \cos x} dx = \frac{\pi}{2}$$

Definite Integrals Ex 20.2 Q22

We know,

$$\sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \quad \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$\begin{aligned} & \therefore \frac{1}{3 + 2 \sin x + \cos x} \\ &= \frac{1}{3 + 2 \left(\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right) + \left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right)} \\ &= \frac{\left(1 + \tan^2 \frac{x}{2} \right)}{3 \left(1 + \tan^2 \frac{x}{2} \right) + 4 \tan \frac{x}{2} + \left(1 - \tan^2 \frac{x}{2} \right)} \\ &= \frac{\sec^2 \frac{x}{2} dx}{3 + 3 \tan^2 \frac{x}{2} + 4 \tan \frac{x}{2} + 1 - \tan^2 \frac{x}{2}} \end{aligned}$$

$$\therefore \int_0^{\pi} \frac{1}{3 + 2 \sin x + \cos x} dx = \int_0^{\pi} \frac{\sec^2 \frac{x}{2} dx}{2 \tan^2 \frac{x}{2} + 4 \tan \frac{x}{2} + 4}$$

$$\text{Let } \tan \frac{x}{2} = t$$

Differentiating w.r.t. x , we get

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

Now,

$$x = 0 \Rightarrow t = 0$$

$$x = \pi \Rightarrow t = \infty$$

$$\begin{aligned}
& \therefore \int_0^{\frac{\pi}{2}} \frac{\sec^2 \frac{x}{2} dx}{2 \tan^2 \frac{x}{2} + 4 \tan \frac{x}{2} + 4} \\
&= \int_0^{\infty} \frac{dt}{t^2 + 2t + 2} \\
&= \int_0^{\infty} \frac{dt}{(t+1)^2 + 1} \\
&= \left[\tan^{-1}(t+1) \right]_0^{\infty} \\
&= \tan^{-1}(\infty) - \tan^{-1}(0+1) \\
&= \tan^{-1}(\infty) - \tan^{-1}(1) \\
&= \tan^{-1}\left(\tan \frac{\pi}{2}\right) - \tan^{-1}\left(\tan \frac{\pi}{4}\right) \\
&= \frac{\pi}{2} - \frac{\pi}{4} \\
&= \frac{2\pi - \pi}{4} \\
&= \frac{\pi}{4}
\end{aligned}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{1}{3 + 2 \sin x + \cos x} dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q23

We have,

$$\begin{aligned}
\int_0^1 1 \cdot \tan^{-1} x \, dx &= \tan^{-1} x \int_0^1 dx - \int_0^1 \left(\int dx \right) \frac{d}{dx} (\tan^{-1} x) dx \\
&= \left[x \tan^{-1} x \right]_0^1 - \int_0^1 \frac{x}{1+x^2} dx \\
&= \left[x \tan^{-1} x - \frac{1}{2} \log(1+x^2) \right]_0^1 \\
&= \frac{\pi}{4} - \frac{1}{2} (\log 2 - 0) \\
&= \frac{\pi}{4} - \frac{1}{2} \log 2
\end{aligned}$$

$$\therefore \int_0^1 \tan^{-1} x \, dx = \frac{\pi}{4} - \frac{1}{2} \log 2$$

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