Trigonometric Ratios Ex 5.1 Q11

Answer :

Given:

$3\cot\theta = 2$

Therefore,

$\cot\theta = \frac{2}{3}$ ..... (1)

Now, we know that $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Therefore equation (1) becomes

$\frac{\cos\theta}{\sin\theta} = \frac{2}{3}$ ..... (2)

Now, by applying Invertendo to equation (2)

We get,

$\frac{\sin\theta}{\cos\theta} = \frac{3}{2}$ ..... (3)

Now, multiplying by $\frac{4}{3}$ on both sides

We get,

$\frac{4}{3}\times\frac{\sin\theta}{\cos\theta} = \frac{4}{3}\times\frac{3}{2}$

Therefore, 3 cancels out on R.H.S and

We get,

$$\frac{4\sin\theta}{3\cos\theta}=\frac{2}{1}$$

Now by applying dividendo in above equation

We get,

$$\frac{4\sin\theta-3\cos\theta}{3\sin\theta}=\frac{2-1}{1}$$

$$\frac{4\sin\theta-3\cos\theta}{3\sin\theta}=\frac{1}{1} \quad \ldots\ldots (4)$$

Now, multiplying by $\frac{2}{6}$ on both sides of equation (3)

We get,

$$\frac{2}{6}\times\frac{\sin\theta}{\cos\theta}=\frac{2}{6}\times\frac{3}{2}$$

Therefore, 2 cancels out on R.H.S and

We get,

$$\frac{2\sin\theta}{6\cos\theta}=\frac{3}{6}$$

$$\frac{2\sin\theta}{6\cos\theta}=\frac{1}{2}$$

Now by applying componendo in above equation

We get,

$$\frac{2\cos\theta+6\sin\theta}{6\sin\theta}=\frac{1+2}{2}$$

$$\frac{2\cos\theta+6\sin\theta}{6\sin\theta}=\frac{3}{2}$$ ..... (5)

Now, by dividing equation (4) by equation (5)

We get,

$$\frac{\frac{4\sin\theta-3\cos\theta}{3\sin\theta}}{\frac{2\cos\theta+6\sin\theta}{6\sin\theta}}=\frac{\frac{1}{1}}{\frac{3}{2}}$$

Therefore,

$$\frac{4\sin\theta-3\cos\theta}{3\sin\theta}\times\frac{6\sin\theta}{2\cos\theta+6\sin\theta}=\frac{1}{1}\times\frac{2}{3}$$

$$\frac{4\sin\theta-3\cos\theta}{3\sin\theta}\times\frac{2\times(3\sin\theta)}{2\cos\theta+6\sin\theta}=\frac{1}{1}\times\frac{2}{3}$$

Therefore, on L.H.S $(3\sin\theta)$ cancels out and we get,

$$\frac{2\times(4\sin\theta-3\cos\theta)}{2\cos\theta+6\sin\theta}=\frac{2}{3}$$

Now, by taking 2 in the numerator of L.H.S on the R.H.S

We get,

$$\frac{4\sin\theta-3\cos\theta}{2\cos\theta+6\sin\theta}=\frac{2}{3\times2}$$

Therefore, 2 cancels out on R.H.S. and

We get,

$$\frac{4\sin\theta-3\cos\theta}{2\cos\theta+6\sin\theta}=\frac{1}{3}$$

Hence,

$$\frac{4\sin\theta-3\cos\theta}{2\cos\theta+6\sin\theta}=\frac{1}{3}$$

Answer :

Given:

$$\tan\theta = \frac{a}{b} \quad \ldots\ldots (1)$$

Now, we know that $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Therefore equation (1) becomes

$$\frac{\sin\theta}{\cos\theta} = \frac{a}{b} \quad \ldots\ldots (2)$$

Now, multiplying by $\frac{a}{b}$ on both sides of equation (2)

We get,

$$\frac{a}{b} \times \frac{\sin\theta}{\cos\theta} = \frac{a}{b} \times \frac{a}{b}$$

Therefore,

$$\frac{a\sin\theta}{b\cos\theta} = \frac{a^2}{b^2} \quad \ldots\ldots (3)$$

Now by applying dividendo in above equation (3)

We get,

$$\frac{a\sin\theta - b\cos\theta}{b\cos\theta} = \frac{a^2 - b^2}{b^2} \quad \ldots\ldots (4)$$

Now by applying componendo in equation (3)

We get,

$$\frac{a\sin\theta - b\cos\theta}{b\cos\theta} = \frac{a^2 - b^2}{b^2} \quad \ldots\ldots (4)$$

Now by applying componendo in equation (3)

We get,

$$\frac{a\sin\theta + b\cos\theta}{b\cos\theta} = \frac{a^2 + b^2}{b^2} \quad \ldots\ldots (5)$$

Now, by dividing equation (4) by equation (5)

We get,

$$\frac{\frac{a\sin\theta - b\cos\theta}{b\cos\theta}}{\frac{a\sin\theta + b\cos\theta}{b\cos\theta}} = \frac{\frac{a^2 - b^2}{b^2}}{\frac{a^2 + b^2}{b^2}}$$

Therefore,

$$\frac{a\sin\theta - b\cos\theta}{b\cos\theta} \times \frac{b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2 - b^2}{b^2} \times \frac{b^2}{a^2 + b^2}$$

Therefore, $b\cos\theta$ and b^2 cancels on L.H.S and R.H.S respectively and we get,

$$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2 - b^2}{a^2 + b^2}$$

Hence, it is proved that

$$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2 - b^2}{a^2 + b^2}$$

********** END **********