Question 4. 21.

A particle starts from the origin at t = 0 s with a velocity of 10.0 $\hat{j}$ m/s and moves in the x-y plane with a constant acceleration of $(8.0\hat{i} + 2.0\hat{j})\ m\ s^{-2}$.

(a) At what time is the x-coordinate of the particle 16 m? What is the y-coordinate of the particle at that time?

(b) What is the speed of the particle at the time?

Answer:

It is given that $\vec{r}_{(t=0s)} = 0, \vec{v}_{(0)} = 10.0\,\hat{j}$ m/s and $\vec{a}(t) = (8.0\,\hat{i} + 2.0\,\hat{j})\ \text{m s}^{-2}$

(*a*) It means $x_0 = 0$, $u_x = 0$, $a_x = 8.0$ m s^{-2} and $x = 16$ m

Using relation $\quad s = x - x_0 = u_x t + \frac{1}{2}a_x t^2$, we have

$$16 - 0 = 0 + \frac{1}{2} \times 8.0 \times t^2 \Rightarrow t = 2\text{ s}$$

$$\therefore \quad y = y_0 + u_y t + \frac{1}{2}a_y t^2 = 0 + 10.0 \times 2 + \frac{1}{2} \times 2.0 \times (2)^2$$

$$= 20 + 4 = 24\text{ m}$$

(*b*) Velocity of particle at $t = 2$ s along x-axis

$$v_x = u_x + a_x\,t = 0 + 8.0 \times 2 = 16.0\text{ m/s}$$

and along y-axis $\quad v_y = u_y + a_y\,t = 10.0 + 2.0 \times 2 = 14.0$ m/s

$\therefore$ Speed of particle at $t = 2$ s

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(16.0)^2 + (14.0)^2} = 21.26\text{ m s}^{-1}.$$

Question 4. 22.

$\hat{i}$ and $\hat{j}$ are unit vectors along x and y-axis respectively. What is the magnitude and direction of the vectors $\hat{i} + \hat{j}$, and $\hat{i} - \hat{j}$? What are the components of a vector $\vec{A} = 2\hat{i} + 3\hat{j}$ along the directions of $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$? [You may use graphical method]

Answer:

(*i*) $$\hat{i} + \hat{j} = \sqrt{(1)^2 + (1)^2 + 2 \times 1 \times 1 \times \cos 90^\circ} = \sqrt{2} = 1.414\text{ units}$$

$$\tan\theta = \frac{1}{1} = 1, \quad \therefore \quad \theta = 45^\circ$$

So the vector $\hat{i} + \hat{j}$ makes an angle of 45° with x-axis.

(*ii*) $$|\hat{i} - \hat{j}| = \sqrt{(1)^2 + (2)^2 - 2 \times 1 \times 1 \times \cos 90^\circ}$$

$$= \sqrt{2} = 1.414\text{ units}$$

The vector $\hat{i} - \hat{j}$ makes an angle of –45° with x-axis.

(*iii*) Let us now determine the component of $\vec{A} = 2\hat{i} + 3\hat{j}$ in the direction of $\hat{i} + \hat{j}$.

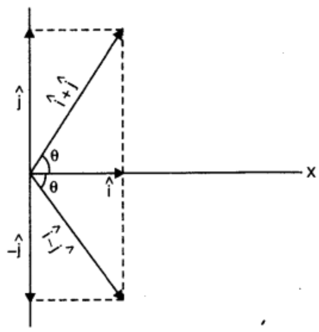
Let $\quad \vec{B} = \hat{i} + \hat{j}$

$$\vec{A}\cdot\vec{B} = AB\cos\theta = (A\cos\theta)\,B$$

So the component of $\vec{A}$ in the direction of $\vec{B} = \frac{\vec{A}\cdot\vec{B}}{B}$

$$= \frac{(2\hat{i} + 3\hat{j})\cdot(\hat{i} + \hat{j})}{\sqrt{(1)^2 + (1)^2}} = \frac{2\hat{i}\cdot\hat{i} + 2\hat{i}\cdot\hat{j} + 3\hat{j}\cdot\hat{i} + 3\hat{j}\cdot\hat{j}}{\sqrt{2}} = \frac{5}{\sqrt{2}}\text{ units}$$

(*iv*) Component of $\vec{A}$ in the direction of $\hat{i} - \hat{j} = \frac{(2\hat{i} + 3\hat{j})\cdot(\hat{i} - \hat{j})}{\sqrt{2}} = -\frac{1}{\sqrt{2}}$ units.



Question 4. 23. For any arbitrary motion in space, which of the following relations are true:

(a) $v_{average} = (1/2)\,[v\,(t_1) + v\,(t_2)]$

(b) $v_{average} = [r\,(t_2) - r\,(t_1)]/(t_2 - t_1)$

(c) $v\,(t) = v\,(0) + a\,t$

(d) $r\,(t) = r\,(0) + v\,(0)\,t + (1/2)\,a\,t^2$

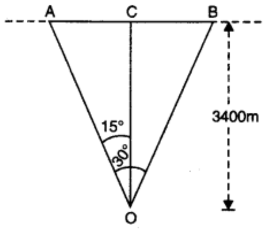
(e) $a_{average} = [v\,(t_2) - v\,(t_1)]/(t_2 - t_1)$

(The 'average' stands for average of the quantity over the time interval t_1 to t_2)

Answer: (b) and (e) are true; others are false because relations (a), (c) and (d) hold only for uniform acceleration.

Question 4. 24. Read each statement below carefully and state, with reasons and examples, if it is true or false: A scalar quantity is one that
(a) is conserved in a process
(b) can never take negative values
(c) must be dimensionless
(d) does not vary from one point to another in space
(e) has the same value for observers with different orientations of axes.
Answer:
(a) False, because kinetic energy is a scalar but does not remain conserved in an inelastic collision.
(b) False, because potential energy in a gravitational field may have negative values.
(c) False, because mass, length, time, speed, work etc., all have dimensions.
(d) False, because speed, energy etc., vary from point to point in space.
(e) True, because a scalar quantity will have the same value for observers with different orientations of axes since a scalar has no direction of its own.

Question 4. 25. An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10 s apart is 30°, what is the speed of the aircraft? Time taken by aircraft from A to B is 10 s.
Answer:



In Fig, O is the observation point at the ground. A and B are the positions of air craft for which $\angle AOB = 30°$. Draw a perpendicular OC on AB. Here $OC = 3400$ m and $\angle AOC = \angle COB = 15°$.

In ΔAOC, $AC = OC \tan 15° = 3400 \times 0.2679 = 910.86$ m.

$AB = AC + CB = AC + AC = 2\,AC = 2 \times 910.86$ m

Speed of the aircraft, $v = \dfrac{\text{distance } AB}{\text{time}} = \dfrac{2 \times 910.86}{10} = 182.17\ \text{ms}^{-1} = \mathbf{182.2\ ms^{-1}}$.

Question 4. 26. A vector has magnitude and direction.
(i) Does it have a location in the space?
(ii) Can it vary with time?
(iii) Will two equal vectors a and b at different locations in space necessarily have identical physical effects? Give examples in support of your answer.
Answer:
(i) Besides having magnitude and direction, each vector has also a location in space.
(ii) A vector can vary with time. As an example, velocity and acceleration vectors may vary with time.
(iii) Two equal vectors a and b having different locations may not have same physical effect. As an example, two balls thrown with the same force, one from earth and the other from moon will attain different 'maximum heights'.

Question 4. 27. A vector has both magnitude and direction. Does

that mean anything that has magnitude and direction is necessarily a vector? The rotation of a body can be specified by the direction of the axis of rotation and the angle of rotation about the axis. Does that make any rotation a vector?
Answer: No. Finite rotation of a body about an axis is not a vector because finite rotations do not obey the laws of vector addition.

Question 4. 28. Can you associate vectors with (a) the length of a wire bent into a loop (b) a plane area (c) a sphere? Explain.
Answer:
(a) We cannot associate a vector with the length of a wire bent into a loop. This is because the length of the loop does not have a definite direction.
(b) We can associate a vector with a plane area. Such a vector is called area vector and its direction is represented by a normal drawn outward to the area.
(c) The area of a sphere does not point in any difinite direction. However, we can associate a null vector with the area of the sphere. We cannot associate a vector with the volume of a sphere.

Question 4. 29. A bullet fired at an angle of 30° with the horizontal hits the ground 3 km away. By adjusting its angle of projection, can one hope to hit a target 5 km away? Assume the muzzle speed to the fixed, and neglect air resistance.
Answer:
Here $R = 3$ km $= 3000$ m, $\theta = 30°$, $g = 9.8$ m s^{-2}.

As $$R = \frac{u^2 \sin 2\theta}{g}$$

$$\Rightarrow \quad 3000 = \frac{u^2 \sin 2 \times 30°}{9.8} = \frac{u^2 \sin 60}{9.8}$$

$$\Rightarrow \quad u^2 = \frac{3000 \times 9.8}{\sqrt{3}/2} = 3464 \times 9.8$$

Also, $$R' = \frac{u^2 \sin 2\theta'}{g} \Rightarrow 5000 = \frac{3464 \times 9.8 \times \sin 2\theta}{9.8}$$

i.e., $$\sin 2\theta' = \frac{5000}{3464} = 1.44$$

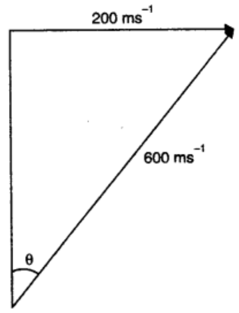
which is impossible because sine of an angle cannot be more than 1. Thus this target cannot be hoped to be hit.

Question 4. 30. A fighter plane flying horizontally at an altitude of 1.5 km with speed 720 km h^{-1} passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gun be fired for the shell with muzzle speed 600 ms^{-1} to hit the plane? At what minimum altitude should the pilot fly the plane to avoid being hit? (Take g = 10 ms^{-2})?
Answer: Velocity of plane, v_p=720 x 5/180 ms^{-1}=200 ms^{-1}

Velocity of shell = 600 ms^{-1};

$$\sin\theta = \frac{200}{600} = \frac{1}{3}$$

or $$\theta = \sin^{-1}\left(\frac{1}{3}\right) = 19.47°$$



This angle is with the vertical.
Let h be the required minimum height.
Using equation

$$v^2 - u^2 = 2as, \quad \text{we get}$$

$$(0)^2 - (600\cos\theta)^2 = -2 \times 10 \times h$$

or, $$h = \frac{600 \times 600\left(1 - \sin^2\theta\right)}{20}$$

$$= 30 \times 600\left(1 - \frac{1}{9}\right) = \frac{8}{9} \times 30 \times 600 \text{ m} = 16 \text{ km}.$$

Question 4. 31. A cyclist is riding with a speed of 27 km/h. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate of 0.50 m/s every second. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn?
Answer:

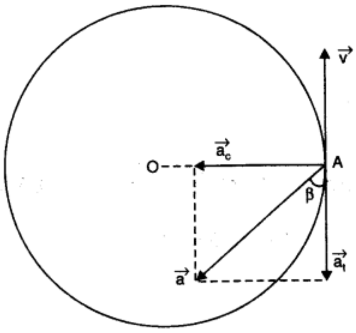
Here v = 27 km/h = $27 \times \frac{5}{18}$ m/s = 7.5 m/s, r = 80 m

and tangential acceleration a_t = – 0.50 m/s^2

$\therefore$ Centripetal acceleration $a_c = \frac{v^2}{r} = \frac{(7.5)^2}{80}$ = 0.70 ms^{-2} (radially inwards).

Thus, as shown in fig. above, two accelerations are acting in mutually perpendicular directions. If $\vec{a}$ be the resultant acceleration, then

$$\left|\vec{a}\right| = \sqrt{a_t^2 + a_c^2} = \sqrt{(0.5)^2 + (0.7)^2} = 0.86 \text{ ms}^{-2}$$



and $\tan\beta = \frac{a_c}{a_t} = \frac{0.7}{0.5} = 1.4$

$\Rightarrow$ $\beta = \tan^{-1}(1.4) = 54.5°$ from the direction of negative of the velocity.

Question 4.32. (a) Show that for a projectile the angle between the velocity and the x-axis as a function of time is given by

$$\theta(t) = \tan^{-1}\left(\frac{v_{oy} - gt}{v_{ox}}\right)$$

(b) Shows that the projection angle θ_0 for a projectile launched from the origin is given by

$$\theta_0 = \tan^{-1}\left(\frac{4h_m}{R}\right)$$

where the symbols have their usual meaning.

Answer:

(*a*) Let the projectile be fired at an angle θ with *x*-axis.
As θ depends on *t*, θ (*t*), at any instant

$$\tan\theta(t) = \frac{v_y}{v_x} = \frac{v_{oy} - gt}{v_{ox}} \qquad (\text{Since } v_y = v_{oy} - gt \text{ and } v_x = v_{ox})$$

$$\Rightarrow \quad \theta(t) = \tan^{-1}\left(\frac{v_{oy} - gt}{v_{ox}}\right)$$

(*b*) Since, $h_{max} = \frac{u^2 \sin^2\theta}{2g}$

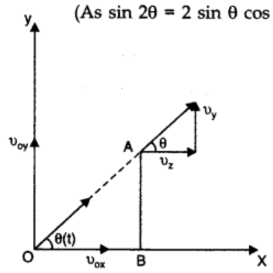
and $R = \frac{u^2 \sin 2\theta}{g}$

$$\Rightarrow \quad \frac{h_{max}}{R} = \frac{u^2\sin^2\theta/2g}{u^2\sin 2\theta/g} = \frac{\tan\theta}{4} \qquad (\text{As } \sin 2\theta = 2\sin\theta\cos\theta)$$

$$\Rightarrow \quad \frac{\tan\theta}{4} = \frac{h_{max}}{R}$$

or $\tan\theta = \frac{4\,h_{max}}{R}$

or $\theta = \tan^{-1}\left(\frac{4\,h_{max}}{R}\right)$.



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