Q12 : $\left(x^3-1\right)^{\frac{1}{3}} x^5$

Answer :

Let $x^3-1=t$

$\therefore\ 3x^2dx=dt$

$$\Rightarrow \int\left(x^3-1\right)^{\frac{1}{3}} x^5dx=\int\left(x^3-1\right)^{\frac{1}{3}} x^3\cdot x^2dx$$

$$=\int t^{\frac{1}{3}}(t+1)\frac{dt}{3}$$

$$=\frac{1}{3}\int\left(t^{\frac{4}{3}}+t^{\frac{1}{3}}\right)dt$$

$$=\frac{1}{3}\left[\frac{t^{\frac{7}{3}}}{\frac{7}{3}}+\frac{t^{\frac{4}{3}}}{\frac{4}{3}}\right]+\mathrm{C}$$

$$=\frac{1}{3}\left[\frac{3}{7}t^{\frac{7}{3}}+\frac{3}{4}t^{\frac{4}{3}}\right]+\mathrm{C}$$

$$=\frac{1}{7}\left(x^3-1\right)^{\frac{7}{3}}+\frac{1}{4}\left(x^3-1\right)^{\frac{4}{3}}+\mathrm{C}$$

Answer needs Correction? Click Here

Q13 : $\frac{x^2}{\left(2+3x^3\right)^3}$

Answer :

Let $2+3x^3=t$

$\therefore\ 9x^2\ dx = dt$

$$\Rightarrow \int\frac{x^2}{\left(2+3x^3\right)^3}dx=\frac{1}{9}\int\frac{dt}{(t)^3}$$

$$=\frac{1}{9}\left[\frac{t^{-2}}{-2}\right]+\mathrm{C}$$

$$=\frac{-1}{18}\left(\frac{1}{t^2}\right)+\mathrm{C}$$

$$=\frac{-1}{18\left(2+3x^3\right)^2}+\mathrm{C}$$

Answer needs Correction? Click Here

Q14 : $\frac{1}{x(\log x)^m}$, $x>0$

Answer :

Let log $x = t$

$\therefore\ \frac{1}{x}dx=dt$

$$\Rightarrow \int\frac{1}{x(\log x)^m}dx=\int\frac{dt}{(t)^m}$$

$$=\left(\frac{t^{-m+1}}{1-m}\right)+\mathrm{C}$$

$$=\frac{(\log x)^{1-m}}{(1-m)}+\mathrm{C}$$

Answer needs Correction? Click Here

Q15 : $\frac{x}{9-4x^2}$

Answer :

Let $9-4x^2=t$

$\therefore$ - 8$x\ dx = dt$

$$\Rightarrow \int\frac{x}{9-4x^2}dx=\frac{-1}{8}\int\frac{1}{t}dt$$

$$= \frac{-1}{8}\log|t| + C$$
$$= \frac{-1}{8}\log|9 - 4x^2| + C$$

Answer needs Correction? **Click Here**

Q16 : e^{2x+3}

Answer :

Let $2x + 3 = t$

$\therefore 2dx = dt$

$$\Rightarrow \int e^{2x+3}dx = \frac{1}{2}\int e^t dt$$
$$= \frac{1}{2}\left(e^t\right) + C$$
$$= \frac{1}{2}e^{(2x+3)} + C$$

Answer needs Correction? **Click Here**

Q17 : $\frac{x}{e^{x^2}}$

Answer :

Let $x^2 = t$

$\therefore 2xdx = dt$

$$\Rightarrow \int \frac{x}{e^{x^2}}dx = \frac{1}{2}\int \frac{1}{e^t}dt$$
$$= \frac{1}{2}\int e^{-t}dt$$
$$= \frac{1}{2}\left(\frac{e^{-t}}{-1}\right) + C$$
$$= -\frac{1}{2}e^{-x^2} + C$$
$$= \frac{-1}{2e^{x^2}} + C$$

Answer needs Correction? **Click Here**

Q18 : $\frac{e^{\tan^{-1}x}}{1+x^2}$

Answer :

Let $\tan^{-1}x = t$

$\therefore \frac{1}{1+x^2}dx = dt$

$$\Rightarrow \int \frac{e^{\tan^{-1}x}}{1+x^2}dx = \int e^t dt$$
$$= e^t + C$$
$$= e^{\tan^{-1}x} + C$$

Answer needs Correction? **Click Here**

Q19 : $\frac{e^{2x}-1}{e^{2x}+1}$

Answer :

$$\frac{e^{2x}-1}{e^{2x}+1}$$

Dividing numerator and denominator by e^x, we obtain

$$\frac{\frac{\left(e^{2x}-1\right)}{e^x}}{\frac{\left(e^{2x}+1\right)}{e^x}} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Let $e^x + e^{-x} = t$

$\therefore \left(e^x - e^{-x}\right)dx = dt$

$$\Rightarrow \int \frac{e^{2x}-1}{e^{2x}+1}dx = \int \frac{e^x - e^{-x}}{e^x + e^{-x}}dx$$
$$= \int \frac{dt}{t}$$
$$= \log|t| + C$$
$$= \log\left|e^x + e^{-x}\right| + C$$

Answer needs Correction? **Click Here**

Q20 : $e^{2x} - e^{-2x}$

$\frac{}{e^{2x}+e^{-2x}}$

Answer :

Let $e^{2x}+e^{-2x}=t$

$\therefore \left(2e^{2x}-2e^{-2x}\right)dx=dt$

$\Rightarrow 2\left(e^{2x}-e^{-2x}\right)dx=dt$

$$\Rightarrow \int\left(\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\right)dx=\int\frac{dt}{2t}$$
$$=\frac{1}{2}\int\frac{1}{t}dt$$
$$=\frac{1}{2}\log|t|+\mathrm{C}$$
$$=\frac{1}{2}\log\left|e^{2x}+e^{-2x}\right|+\mathrm{C}$$

Answer needs Correction? Click Here

Q21 : $\tan^2(2x-3)$

Answer :

$\tan^2(2x-3)=\sec^2(2x-3)-1$

Let $2x-3=t$

$\therefore 2dx=dt$

$$\Rightarrow \int\tan^2(2x-3)dx=\int\left[\left(\sec^2(2x-3)\right)-1\right]dx$$
$$=\frac{1}{2}\int\left(\sec^2 t\right)dt-\int 1dx$$
$$=\frac{1}{2}\int\sec^2 t\,dt-\int 1dx$$
$$=\frac{1}{2}\tan t-x+\mathrm{C}$$
$$=\frac{1}{2}\tan(2x-3)-x+\mathrm{C}$$

Answer needs Correction? Click Here

Q22 : $\sec^2(7-4x)$

Answer :

Let $7-4x=t$

$\therefore -4dx=dt$

$$\therefore \int\sec^2(7-4x)dx=\frac{-1}{4}\int\sec^2 t\,dt$$
$$=\frac{-1}{4}(\tan t)+\mathrm{C}$$
$$=\frac{-1}{4}\tan(7-4x)+\mathrm{C}$$

Answer needs Correction? Click Here

Q23 : $\frac{\sin^{-1}x}{\sqrt{1-x^2}}$

Answer :

Let $\sin^{-1}x=t$

$\therefore \frac{1}{\sqrt{1-x^2}}dx=dt$

$$\Rightarrow \int\frac{\sin^{-1}x}{\sqrt{1-x^2}}dx=\int t\,dt$$

$$=\frac{t^2}{2}+\mathrm{C}$$
$$=\frac{\left(\sin^{-1}x\right)^2}{2}+\mathrm{C}$$

Answer needs Correction? Click Here

Q24 : $\frac{2\cos x-3\sin x}{6\cos x+4\sin x}$

Answer :

$$\frac{2\cos x-3\sin x}{6\cos x+4\sin x}=\frac{2\cos x-3\sin x}{2(3\cos x+2\sin x)}$$

Let $3\cos x+2\sin x=t$

$\therefore \left(-3\sin x+2\cos x\right)dx=dt$

$$\int\frac{2\cos x-3\sin x}{6\cos x+4\sin x}dx=\int\frac{dt}{2t}$$

$$= \frac{1}{2}\int \frac{1}{t}dt$$
$$= \frac{1}{2}\log|t| + C$$
$$= \frac{1}{2}\log|2\sin x + 3\cos x| + C$$

Answer needs Correction? Click Here

Q25 : $\frac{1}{\cos^2 x(1-\tan x)^2}$

Answer :

$\frac{1}{\cos^2 x(1-\tan x)^2} = \frac{\sec^2 x}{(1-\tan x)^2}$

Let $(1-\tan x) = t$

$\therefore \; -\sec^2 x dx = dt$

$$\Rightarrow \int \frac{\sec^2 x}{(1-\tan x)^2}dx = \int \frac{-dt}{t^2}$$
$$= -\int t^{-2}dt$$
$$= +\frac{1}{t} + C$$
$$= \frac{1}{(1-\tan x)} + C$$

Answer needs Correction? Click Here

Q26 : $\frac{\cos\sqrt{x}}{\sqrt{x}}$

Answer :

Let $\sqrt{x} = t$

$\therefore \; \frac{1}{2\sqrt{x}}dx = dt$

$$\Rightarrow \int \frac{\cos\sqrt{x}}{\sqrt{x}}dx = 2\int \cos t \, dt$$
$$= 2\sin t + C$$
$$= 2\sin\sqrt{x} + C$$

Answer needs Correction? Click Here

Q27 : $\sqrt{\sin 2x}\cos 2x$

Answer :

Let $\sin 2x = t$

$\therefore \; 2\cos 2x \, dx = dt$

$$\Rightarrow \int \sqrt{\sin 2x}\;\cos 2x \, dx = \frac{1}{2}\int \sqrt{t}\, dt$$
$$= \frac{1}{2}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$
$$= \frac{1}{3}t^{\frac{3}{2}} + C$$
$$= \frac{1}{3}(\sin 2x)^{\frac{3}{2}} + C$$

Answer needs Correction? Click Here

Q28 : $\frac{\cos x}{\sqrt{1+\sin x}}$

Answer :

Let $1+\sin x = t$

$\therefore \cos x \, dx = dt$

$$\Rightarrow \int \frac{\cos x}{\sqrt{1+\sin x}}dx = \int \frac{dt}{\sqrt{t}}$$
$$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$$
$$= 2\sqrt{t} + C$$
$$= 2\sqrt{1+\sin x} + C$$

Answer needs Correction? Click Here

Q29 : cot x log sin x

Answer :

Let log sin $x = t$

$$\Rightarrow \frac{1}{\sin x} \cdot \cos x \, dx = dt$$
$$\therefore \cot x \, dx = dt$$

$$\Rightarrow \int \cot x \log \sin x \, dx = \int t \, dt$$
$$= \frac{t^2}{2} + C$$
$$= \frac{1}{2}(\log \sin x)^2 + C$$

Answer needs Correction? Click Here

Q30 : $\frac{\sin x}{1+\cos x}$

Answer :

Let $1 + \cos x = t$

$\therefore - \sin x \, dx = dt$

$$\Rightarrow \int \frac{\sin x}{1+\cos x} \, dx = \int -\frac{dt}{t}$$
$$= -\log|t| + C$$
$$= -\log|1+\cos x| + C$$

Answer needs Correction? Click Here

Q31 : $\frac{\sin x}{(1+\cos x)^2}$

Answer :

Let $1 + \cos x = t$

$\therefore - \sin x \, dx = dt$

$$\Rightarrow \int \frac{\sin x}{(1+\cos x)^2} \, dx = \int -\frac{dt}{t^2}$$
$$= -\int t^{-2} dt$$
$$= \frac{1}{t} + C$$
$$= \frac{1}{1+\cos x} + C$$

Answer needs Correction? Click Here

Q32 : $\frac{1}{1+\cot x}$

Answer :

$$\text{Let } I = \int \frac{1}{1+\cot x} dx$$
$$= \int \frac{1}{1+\frac{\cos x}{\sin x}} dx$$
$$= \int \frac{\sin x}{\sin x + \cos x} dx$$
$$= \frac{1}{2} \int \frac{2\sin x}{\sin x + \cos x} dx$$
$$= \frac{1}{2} \int \frac{(\sin x + \cos x) + (\sin x - \cos x)}{(\sin x + \cos x)} dx$$
$$= \frac{1}{2} \int 1 \, dx + \frac{1}{2} \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$
$$= \frac{1}{2}(x) + \frac{1}{2} \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$

Let $\sin x + \cos x = t \Rightarrow (\cos x - \sin x) \, dx = dt$

$$\therefore I = \frac{x}{2} + \frac{1}{2} \int \frac{-(dt)}{t}$$
$$= \frac{x}{2} - \frac{1}{2} \log|t| + C$$
$$= \frac{x}{2} - \frac{1}{2} \log|\sin x + \cos x| + C$$

Answer needs Correction? Click Here

Q33 : $\frac{1}{1-\tan x}$

Answer :

$$\text{Let } I = \int \frac{1}{1-\tan x} dx$$
$$= \int \frac{1}{1-\frac{\sin x}{\cos x}} dx$$

$$= \int \frac{\cos x}{\cos x - \sin x} dx$$
$$= \frac{1}{2} \int \frac{2\cos x}{\cos x - \sin x} dx$$
$$= \frac{1}{2} \int \frac{(\cos x - \sin x) + (\cos x + \sin x)}{(\cos x - \sin x)} dx$$
$$= \frac{1}{2} \int 1 dx + \frac{1}{2} \int \frac{\cos x + \sin x}{\cos x - \sin x} dx$$
$$= \frac{x}{2} + \frac{1}{2} \int \frac{\cos x + \sin x}{\cos x - \sin x} dx$$

Put $\cos x - \sin x = t \Rightarrow (-\sin x - \cos x)\, dx = dt$

$$\therefore I = \frac{x}{2} + \frac{1}{2} \int \frac{-(dt)}{t}$$
$$= \frac{x}{2} - \frac{1}{2} \log|t| + C$$
$$= \frac{x}{2} - \frac{1}{2} \log|\cos x - \sin x| + C$$

Answer needs Correction? Click Here

Q34 : $\frac{\sqrt{\tan x}}{\sin x \cos x}$

Answer :

$$\text{Let } I = \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$$
$$= \int \frac{\sqrt{\tan x} \times \cos x}{\sin x \cos x \times \cos x} dx$$
$$= \int \frac{\sqrt{\tan x}}{\tan x \cos^2 x} dx$$
$$= \int \frac{\sec^2 x\, dx}{\sqrt{\tan x}}$$

$$\text{Let } \tan x = t \Rightarrow \sec^2 x\, dx = dt$$

$$\therefore I = \int \frac{dt}{\sqrt{t}}$$
$$= 2\sqrt{t} + C$$
$$= 2\sqrt{\tan x} + C$$

Answer needs Correction? Click Here

Q35 : $\frac{(1+\log x)^2}{x}$

Answer :

Let $1 + \log x = t$

$$\therefore \frac{1}{x} dx = dt$$

$$\Rightarrow \int \frac{(1+\log x)^2}{x} dx = \int t^2 dt$$
$$= \frac{t^3}{3} + C$$
$$= \frac{(1+\log x)^3}{3} + C$$

Answer needs Correction? Click Here

Q36 : $\frac{(x+1)(x+\log x)^2}{x}$

Answer :

$$\frac{(x+1)(x+\log x)^2}{x} = \left(\frac{x+1}{x}\right)(x+\log x)^2 = \left(1+\frac{1}{x}\right)(x+\log x)^2$$

Let $(x+\log x) = t$

$$\therefore \left(1+\frac{1}{x}\right) dx = dt$$

$$\Rightarrow \int \left(1+\frac{1}{x}\right)(x+\log x)^2 dx = \int t^2 dt$$
$$= \frac{t^3}{3} + C$$
$$= \frac{1}{3}(x+\log x)^3 + C$$

Answer needs Correction? Click Here

Q37 : $\frac{x^3 \sin\left(\tan^{-1} x^4\right)}{1+x^8}$

Answer :

Let $x^4 = t$

$\therefore 4x^3 dx = dt$

$$\Rightarrow \int \frac{x^3 \sin\left(\tan^{-1} x^4\right)}{1+x^8} dx = \frac{1}{4}\int \frac{\sin\left(\tan^{-1} t\right)}{1+t^2} dt \quad ...(1)$$

Let $\tan^{-1} t = u$

$$\therefore \frac{1}{1+t^2} dt = du$$

From (1), we obtain

$$\int \frac{x^3 \sin\left(\tan^{-1} x^4\right) dx}{1+x^8} = \frac{1}{4}\int \sin u \, du$$

$$= \frac{1}{4}(-\cos u) + C$$

$$= \frac{-1}{4}\cos\left(\tan^{-1} t\right) + C$$

$$= \frac{-1}{4}\cos\left(\tan^{-1} x^4\right) + C$$

Answer needs Correction? Click Here

Q38 : $\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx$ equals

(A) $10^x - x^{10} + C$ (B) $10^x + x^{10} + C$

(C) $\left(10^x - x^{10}\right)^{-1} + C$ (D) $\log\left(10^x + x^{10}\right) + C$

Answer :

Let $x^{10} + 10^x = t$

$$\therefore \left(10x^9 + 10^x \log_e 10\right) dx = dt$$

$$\Rightarrow \int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx = \int \frac{dt}{t}$$

$$= \log t + C$$

$$= \log\left(10^x + x^{10}\right) + C$$

Hence, the correct answer is D.

Answer needs Correction? Click Here

Q39 : $\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx$ equals

(A) $10^x - x^{10} + C$ (B) $10^x + x^{10} + C$

(C) $\left(10^x - x^{10}\right)^{-1} + C$ (D) $\log\left(10^x + x^{10}\right) + C$

Answer :

Let $x^{10} + 10^x = t$

$$\therefore \left(10x^9 + 10^x \log_e 10\right) dx = dt$$

$$\Rightarrow \int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx = \int \frac{dt}{t}$$

$$= \log t + C$$

$$= \log\left(10^x + x^{10}\right) + C$$

Hence, the correct answer is D.

Answer needs Correction? Click Here

********** END **********