Co-Ordinate Geometry Ex 14.5 Q3

Answer :

GIVEN: The four vertices of quadrilateral are (1, 2), (−5, 6), (7, −4) and D (*k*, −2) taken in order. If the area of the quadrilateral is zero

TO FIND: value of *k*

PROOF: Let four vertices of quadrilateral are A (1, 2) and B (−5, 6) and C (7, −4) and D (*k*, −2)

We know area of triangle formed by three points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) is given by

$$\Delta = \frac{1}{2}\left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

$$\Delta = \frac{1}{2}\left| (x_1y_2 + x_2y_3 + x_3y_1) - (x_1y_3 + x_2y_1 + x_3y_2) \right|$$

Now Area of ΔABC

Taking three points when A (1, 2) and B (−5, 6) and C (7, −4)

$$\text{Area}(\Delta ABC)$$

$$= \frac{1}{2}\left|\{6 + 20 + 14\} - \{-10 + 42 - 4\}\right|$$

$$= \frac{1}{2}\left|\{40\} - \{28\}\right|$$

$$= \frac{1}{2}\left|\{12\}\right|$$

$$\boxed{\text{Area}(\Delta ABC) = 6 \text{ sq. units}}$$

Also,

Now Area of ΔACD

Taking three points when A (1, 2) and C (7, −4) and D (*k*, −2)

$$\text{Area}(\Delta ACD)$$

$$= \frac{1}{2}\left|\{-4 - 14 + 2k\} - \{14 - 4k - 2\}\right|$$

$$= \frac{1}{2}\left|\{2k - 18\} - \{12 - 4k\}\right|$$

$$= \frac{1}{2}\left|\{6k - 30\}\right|$$

$$= \left|\{3k - 15\}\right|$$

Hence

$$\text{Area}(ABCD) = \text{Area}(\Delta ABC) + \text{Area}(\Delta ACD)$$

$$0 = 6 + 3k - 15 (\text{substituting the values})$$

$$\boxed{k = 3}$$

Co-Ordinate Geometry Ex 14.5 Q4

Answer :

GIVEN: The vertices of triangle ABC are A (−2, 1) and B (5, 4) and C (2, −3)

TO FIND: The area of triangle ABC and length if the altitude through A

PROOF: We know area of triangle formed by three points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) is given by

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

Now Area of ΔABC

Taking three points A (−2, 1) and B (5, 4) and C(2, −3)

$$\begin{aligned}
\text{Area}(\Delta ABC) &= \frac{1}{2}\left|\{-8-15+2\}-\{5+8+6\}\right| \\
&= \frac{1}{2}\left|\{-21\}-\{19\}\right| \\
&= \frac{1}{2}\left|\{-40\}\right| \\
&= \frac{1}{2}(40) \\
&= 20
\end{aligned}$$

We have

$$BC = \sqrt{(5-2)^2 + (4+3)^2}$$

$$BC = \sqrt{(3)^2 + (7)^2}$$

$$BC = \sqrt{9+49}$$

$$BC = \sqrt{58}$$

Now,

$$\text{Area}(\Delta ABC) = \frac{1}{2} \times BC \times \text{length of altitude through A}$$

$$20 = \frac{1}{2} \times \sqrt{58} \times \text{length of altitude through A}$$

$$\boxed{\text{length of altitude through A} = \frac{40}{\sqrt{58}}}$$

Co-Ordinate Geometry Ex 14.5 Q5

Answer :

The formula for the area '*A*' encompassed by three points (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by the formula,

$$A = \frac{1}{2}\begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_2 - x_3 & y_2 - y_3 \end{vmatrix}$$

$$A = \frac{1}{2}\left\{(x_1 - x_2)(y_2 - y_3) - (x_2 - x_3)(y_1 - y_2)\right\}$$

If three points are collinear the area encompassed by them is equal to 0.

The three given points are *A*(2,5), *B*(4,6) and *C*(8,8). Substituting these values in the earlier mentioned formula we have,

$$\begin{aligned}
A &= \frac{1}{2}\begin{vmatrix} 2-4 & 5-6 \\ 4-8 & 6-8 \end{vmatrix} \\
&= \frac{1}{2}\begin{vmatrix} -2 & -1 \\ -4 & -2 \end{vmatrix} \\
&= \frac{1}{2}\left|(-2)(-2)-(-1)(-4)\right| \\
&= \frac{1}{2}\left|4-4\right| \\
A &= 0
\end{aligned}$$

Since the area enclosed by the three points is equal to 0, the three points need to be $\boxed{\text{collinear}}$.

The three given points are $A(1,-1)$, $B(2,1)$ and $C(4,5)$. Substituting these values in the earlier mentioned formula we have,

$$A = \frac{1}{2}\begin{vmatrix} 1-2 & -1-1 \\ 2-4 & 1-5 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} -1 & -2 \\ -2 & -4 \end{vmatrix}$$

$$= \frac{1}{2}\left|(-1)(-4)-(-2)(-2)\right|$$

$$= \frac{1}{2}\left|4-4\right|$$

$$A = 0$$

Since the area enclosed by the three points is equal to 0, the three points need to be $\boxed{\text{collinear}}$.

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