Indefinite Integrals Ex 19.29 Q1

$$\text{Let } I = \int (x+1)\sqrt{x^2 - x + 1}dx \qquad ---(1)$$

$$\text{Let } x+1 = \lambda \frac{d}{dx}\left(x^2 - x + 1\right) + \mu$$
$$= \lambda\left(2x - 1\right) + \mu$$

Equating similar terms, we get,

$$2\lambda = 1 \Rightarrow \qquad \lambda = \frac{1}{2}$$
$$-\lambda + \mu = 1$$

$$\Rightarrow \qquad \mu = 1 + \lambda = 1 + \frac{1}{2} = \frac{3}{2} \therefore \qquad \mu = \frac{3}{2}$$

So,

$$I = \int\left(\frac{1}{2}(2x-1) + \frac{3}{2}\right)\sqrt{x^2 - x + 1}dx$$
$$= \frac{1}{2}\int(2x-1)\sqrt{x^2 - x + 1}dx + \frac{3}{2}\int\sqrt{x^2 - x + 1}dx$$

$$\text{Let } x^2 - x + 1 = t$$
$$\Rightarrow \qquad (2x-1)dx = dt$$

$$= \frac{1}{2}\int\sqrt{t}dt + \frac{3}{2}\int\sqrt{\left(x - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}dx$$

$$= \frac{1}{2}\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{3}{2}\left\{\frac{\left(x - \frac{1}{2}\right)}{2}\sqrt{x^2 - x + 1} + \frac{3}{8}\log\left|\left(x - \frac{1}{2}\right) + \sqrt{x^2 - x + 1}\right|\right.$$

$$\Rightarrow \qquad I = \frac{1}{3}t^{\frac{3}{2}} + \frac{3}{8}(2x-1)\sqrt{x^2 - x + 1} + \frac{9}{16}.\log\left|\left(x - \frac{1}{2}\right) + \sqrt{x^2 - x + 1}\right| + c$$

Hence,

$$I = \frac{1}{3}\left(x^2 - x + 1\right)^{\frac{3}{2}} + \frac{3}{8}(2x-1)\sqrt{x^2 - x + 1} + \frac{9}{16}\log\left|\left(x - \frac{1}{2}\right) + \sqrt{x^2 - x + 1}\right| + c$$

Indefinite Integrals Ex 19.29 Q2

Let $I = \int (x+1)\sqrt{2x^2+3}dx$

Let $x + 1 = \lambda \frac{d}{dx}\left(2x^2+3\right) + \mu$
$= \lambda\left(4x\right) + \mu$

Equating similar terms, we get,

$4\lambda = 1 \Rightarrow \quad \lambda = \frac{1}{4}$
$\mu = 1$

$\therefore \quad I = \int \frac{1}{4}\left(4x\right)\sqrt{2x^2+3}dx + \int 1.\sqrt{2x^2+3}dx$

Let $2x^2 + 3 = t$
$\Rightarrow \quad 4xdx = dt$

$$I = \frac{1}{4}\int \sqrt{t}dt + \sqrt{2}\int \sqrt{x^2+\frac{3}{2}}dx$$
$$= \frac{1}{4}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \sqrt{2}\left\{\frac{x}{2}\sqrt{x^2+\frac{3}{2}} + \frac{3}{4}\log\left|x+\sqrt{x^2+\frac{3}{2}}\right| + c\right.$$

Hence,

$$I = \frac{1}{6}\left(2x^2+3\right)^{\frac{3}{2}} + \frac{x}{2}\sqrt{2x^2+3} + \frac{3}{2\sqrt{2}}\log\left|x+\sqrt{x^2+\frac{3}{2}}\right| + c$$

Indefinite Integrals Ex 19.29 Q3

Let $I = \int (2x-5)\sqrt{2+3x-x^2}\,dx$

Let $2x-5 = \lambda \dfrac{d}{dx}\left(2+3x-x^2\right) + \mu$

$= \lambda\left(3-2x\right) + \mu$

Equating similar terms, we get,

$-2\lambda = 2 \quad \Rightarrow \quad \lambda = -1$

$3\lambda + \mu = -5 \quad \Rightarrow \quad \mu = -5 - 3\lambda = -2$

$\therefore \quad \mu = -2$

So,

$I = \int \left(-1\left(3-2x\right) - 2\right)\sqrt{2+3x-x^2}\,dx$

$= -\int \left(3-2x\right)\sqrt{2+3x-x^2}\,dx - 2\int \sqrt{2+3x-x^2}\,dx$

Let $2+3x-x^2 = t$

$\Rightarrow \quad \left(3-2x\right)dx = dt$

$I = -\int \sqrt{t}\,dt - 2\int \sqrt{\dfrac{17}{4} - \left(\dfrac{9}{4} - 3x - x^2\right)}\,dx$

$= -\dfrac{t^{\frac{3}{2}}}{\frac{3}{2}} - 2\int \sqrt{\left(\dfrac{\sqrt{17}}{2}\right)^2 - \left(x - \dfrac{3}{2}\right)^2}\,dx$

$\Rightarrow \quad I = -\dfrac{2}{3}\left(2+3x-x^2\right)^{\frac{3}{2}} - 2\left\{\dfrac{\left(x-\frac{3}{2}\right)}{2}\sqrt{2+3x-x^2} + \dfrac{17}{8}\sin^{-1}\left(\dfrac{x-\frac{3}{2}}{\frac{\sqrt{17}}{2}}\right)\right. + c$

Hence,

$I = -\dfrac{2}{3}\left(2+3x-x^2\right)^{\frac{3}{2}} - \dfrac{\left(2x-3\right)}{2}\sqrt{2+3x-x^2} - \dfrac{17}{8}\sin^{-1}\left(\dfrac{2x-3}{\sqrt{17}}\right) + c$

Indefinite Integrals Ex 19.29 Q4

Let $I = \int (x+2)\sqrt{x^2+x+1}\,dx$

Let $x+2 = \lambda \frac{d}{dx}\left(x^2+x+1\right) + \mu$

$= \lambda\left(2x+1\right) + \mu$

Equating similar terms, we get,

$2\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$\lambda + \mu = 2 \quad \Rightarrow \quad \mu = 2 - \lambda = \frac{3}{2}$

$\therefore \quad \mu = \frac{3}{2}$

$\therefore \quad I = \int\left(\frac{1}{2}\left(2x+1\right) + \frac{3}{2}\right)\sqrt{x^2+x+1}\,dx$

$= \frac{1}{2}\int\left(2x+1\right)\sqrt{x^2+x+1} + \frac{3}{2}\int\sqrt{x^2+x+1}\,dx$

Let $x^2+x+1 = t$

$\left(2x+1\right)dx = dt$

$\therefore \quad I = \frac{1}{2}\int\sqrt{t}\,dt + \frac{3}{2}\int\sqrt{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\,dx$

$\Rightarrow \quad I = \frac{1}{2}\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{3}{2}\left\{\frac{\left(x+\frac{1}{2}\right)}{2}\sqrt{x^2+x+1} + \frac{3}{8}\log\left|\left(x+\frac{1}{2}\right) + \sqrt{x^2+x+1}\right| + c\right.$

Hence,

$I = \frac{1}{3}\left(x^2+x+1\right)^{\frac{3}{2}} + \frac{3}{8}\left(2x+1\right)\sqrt{x^2+x+1} + \frac{9}{16}\log\left|\left(x+\frac{1}{2}\right) + \sqrt{x^2+x+1}\right| + c$

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