Factorisation of Polynomials Ex 6.4 Q19

Answer :

Let $f(x) = ax^3 + x^2 - 2x + b$ be the given polynomial.

By factor theorem, if $(x+1)$ and $(x-1)$ both are factors of the polynomial f (x). if $f(-1)$ and $f(1)$ both are equal to zero.

Therefore,

$$f(-1) = a(-1)^3 + (-1)^2 - 2(-1) + b = 0$$
$$-a + 1 + 2 + b = 0$$
$$-a + b = -3 \qquad \text{...(i)}$$

And

$$f(1) = a(1)^3 + (1)^2 - 2(1) + b = 0$$
$$a + 1 - 2 + b = 0$$
$$a + b = 1 \qquad \text{...(ii)}$$

Adding (i) and (ii), we get

$2b = -2$

$b = -1$

And putting this value in equation (ii), we get,

$a = 2$

Hence, the value of a and b are 2 and -1 respectively.

Factorisation of Polynomials Ex 6.4 Q20

Answer :

Let $p(x) = x^3 - 3x^2 - 12x + 19$ and $q(x) = x^2 + x - 6$ be the given polynomial.

When $p(x)$ is divided by $q(x)$, the reminder is a linear polynomial in x.

So, let $r(x) = ax + b$ is added to $p(x)$, so that $p(x) + r(x)$ is divisible by $q(x)$.

Let $f(x) = p(x) + r(x)$

Then,

$$f(x) = x^3 - 3x^2 - 12x + 19 + ax + b$$
$$= x^3 - 3x^2 + (a-12)x + (19+b)$$

We have,

$$q(x) = x^2 + x - 6$$
$$= (x+3)(x-2)$$

Clearly, $q(x)$ is divisible by $(x+3)$ and $(x-2)$ i.e., $(x+3)$ and $(x-2)$ are the factors of $q(x)$.

Therefore, $f(x)$ is divisible by $q(x)$, if $(x+3)$ and $(x-2)$ are factors of $f(x)$, i.e.,

$f(-3)$ and $f(2) = 0$

Now, $f(-3) = 0$

$\Rightarrow f(-3) = (-3)^3 - 3(-3)^2 + (a-12)(-3) + 19 + b = 0$

$\Rightarrow -27 - 27 - 3a + 36 + 19 + b = 0$

$\Rightarrow$ -27 - 27 - 3a + 36 + 19 + b = 0

$\Rightarrow$ -54 - 3a + b + 55 = 0

$\Rightarrow$ -3a + b + 1 = 0 ---- (i)

And

$$f(2)=(2)^3-3(2)^2+(a-12)+19+b=0$$
$$8-12+2a-24+19+b=0$$
$$2a+b=9 \quad ...(ii)$$

Subtracting (i) from (ii), we get,

$$(2a+b)-(-3a+b)=10$$
$$5a=10$$
$$a=2$$

Putting this value in equation (ii), we get,

$$\Rightarrow 2\times 2+b=9$$
$$b=5$$

Hence, $p(x)$ is divisible by $q(x)$ if $(2x+5)$ added to it.

Factorisation of Polynomials Ex 6.4 Q21

Answer :

By divisible algorithm, when $p(x)=x^3-6x^2-15x+80$ is divided by $x^2+x-12,$ the reminder is a linear polynomial

Let $r(x)=a(x)+b$ be subtracted from $p(x)$ so that the result is divisible by $q(x)$.

Let

$$f(x)=p(x)-q(x)$$
$$=x^3-6x^2-15x+80-(ax+b)$$
$$=x^3-6x^2-(a+15)x+80-b$$

We have,

$$q(x)=x^2+x-12$$
$$=x^2+4x-3x-12$$
$$=(x+4)(x-3)$$

Clearly, $(x+4)$ and $(x-3)$ are factors of $q(x)$, therefore, $f(x)$ will be divisible by $q(x)$ if $(x+4)$ and $(x-3)$ are factors of $f(x)$, i.e. $f(-4)$ and $f(3)$ are equal to zero.

Therefore,

$$f(-4)=(-4)^3-6(-4)^2-(a+15)(-4)+80-b=0$$
$$-64-96+4a+60+80-b=0$$
$$-20+4a-b=0$$
$$+4a-b=20$$

and

$$f(3)=(3)^3-6(3)^2-(a+15)(3)+80-b=0$$
$$27-54-3a-45+80-b=0$$
$$-3a-b=8$$
$$3a+b=8$$

Adding (i) and (ii), we get,

$\boxed{a=4}$

Putting this value in equation (i), we get,

$b=-4$

Hence, $x^3-6x^2-15x+80$ will be divisible by $x^2+x-12,$ if $4x-4$ is subtracted from it

*********** END **********