



### Differentiation Ex 11.5 Q30

Here,

$$\begin{aligned}
 y &= (\tan x)^{\log x} + \cos^2 \left( \frac{\pi}{4} \right) \\
 y &= e^{\log(\tan x)^{\log x}} + \cos^2 \left( \frac{\pi}{4} \right) \\
 y &= e^{\log x \log \tan x} + \cos^2 \left( \frac{\pi}{4} \right) \quad \left[ \text{Since, } e^{\log a} = a \text{ and } \log a^b = b \log a \right]
 \end{aligned}$$

Differentiating it using chain rule and product rule,

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{d}{dx} \left( e^{\log x \log \tan x} \right) + \frac{d}{dx} \cos^2 \left( \frac{\pi}{4} \right) \\
 &= e^{\log x \log \tan x} \frac{d}{dx} (\log x \log \tan x) + 0 \\
 &= e^{\log(\tan x)^{\log x}} \left[ \log x \frac{d}{dx} (\log \tan x) + \log \tan x \frac{d}{dx} (\log x) \right] \\
 &= (\tan x)^{\log x} \left[ \log x \left( \frac{1}{\tan x} \right) \frac{d}{dx} (\tan x) + \log \tan x \left( \frac{1}{x} \right) \right] \\
 &= (\tan x)^{\log x} \left[ \log x \left( \frac{1}{\tan x} \right) (\sec^2 x) + \frac{\log \tan x}{x} \right] \\
 \frac{dy}{dx} &= (\tan x)^{\log x} \left[ \log x \left( \frac{\sec^2 x}{\tan x} \right) + \frac{\log \tan x}{x} \right]
 \end{aligned}$$

### Differentiation Ex 11.5 Q31

Here,

$$\begin{aligned}
 y &= x^x + x^{\frac{1}{x}} \\
 &= e^{\log x^x} + e^{\log x^{\frac{1}{x}}} \\
 y &= e^{x \log x} + e^{\left( \frac{1}{x} \log x \right)} \quad \left[ \text{Since, } e^{\log a} = a, \log a^b = b \log a \right]
 \end{aligned}$$

Differentiating it with respect to  $x$  using chain rule and product rule,

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{d}{dx} \left( e^{x \log x} \right) + \frac{d}{dx} \left( e^{\frac{1}{x} \log x} \right) \\
 &= e^{x \log x} + \frac{d}{dx} (x \log x) + e^{\frac{1}{x} \log x} \frac{d}{dx} \left( \frac{1}{x} \log x \right) \\
 &= e^{\log x^x} \left[ x \frac{d}{dx} (\log x) + \log x \frac{d}{dx} (x) \right] + e^{\log x^{\frac{1}{x}}} \left[ \frac{1}{x} \frac{d}{dx} (\log x) + \log x \frac{d}{dx} \left( \frac{1}{x} \right) \right] \\
 &= x^x \left[ x \left( \frac{1}{x} \right) + \log x (1) \right] + x^{\frac{1}{x}} \left[ \left( \frac{1}{x} \right) \left( \frac{1}{x} \right) + \log x \left( -\frac{1}{x^2} \right) \right] \\
 &= x^x [1 + \log x] + x^{\frac{1}{x}} \left( \frac{1}{x^2} - \frac{1}{x^2} \log x \right) \\
 \frac{dy}{dx} &= x^x [1 + \log x] + x^{\frac{1}{x}} \frac{(1 - \log x)}{x^2}
 \end{aligned}$$

### Differentiation Ex 11.5 Q32

$$\text{Let } y = (\log x)^x + x^{\log x}$$

$$\text{Also, let } u = (\log x)^x \text{ and } v = x^{\log x}$$

$$\therefore y = u + v$$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \dots(1)$$

$$u = (\log x)^x$$

$$\Rightarrow \log u = \log [(\log x)^x]$$

$$\Rightarrow \log u = x \log (\log x)$$

Differentiating both sides with respect to  $x$ , we obtain

$$\frac{1}{u} \frac{du}{dx} = \frac{d}{dx} (x \times \log (\log x)) + x \cdot \frac{d}{dx} [\log (\log x)]$$

$$\Rightarrow \frac{du}{dx} = u \left[ 1 \times \log (\log x) + x \cdot \frac{1}{\log x} \cdot \frac{d}{dx} (\log x) \right]$$

$$\Rightarrow \frac{du}{dx} = (\log x)^x \left[ \log (\log x) + \frac{x}{\log x} \cdot \frac{1}{x} \right]$$

$$\Rightarrow \frac{du}{dx} = (\log x)^x \left[ \log (\log x) + \frac{1}{\log x} \right]$$

$$\Rightarrow \frac{du}{dx} = (\log x)^x \left[ \frac{\log (\log x) \cdot \log x + 1}{\log x} \right]$$

$$\frac{1}{v} \cdot \frac{dv}{dx} = \frac{d}{dx} [(\log x)^2]$$

$$\Rightarrow \frac{1}{v} \cdot \frac{dv}{dx} = 2(\log x) \cdot \frac{d}{dx} (\log x)$$

$$\Rightarrow \frac{dv}{dx} = 2v(\log x) \cdot \frac{1}{x}$$

$$\Rightarrow \frac{dv}{dx} = 2x^{\log x} \frac{\log x}{x}$$

$$\Rightarrow \frac{dv}{dx} = 2x^{\log x - 1} \cdot \log x \quad \dots(3)$$

Therefore, from (1), (2), and (3), we obtain

$$\frac{dy}{dx} = (\log x)^{x-1} [1 + \log x \cdot \log (\log x)] + 2x^{\log x - 1} \cdot \log x$$

Here,

$$x^{13}y^7 = (x+y)^{20}$$

Taking log on both the sides,

$$\log(x^{13}y^7) = \log(x+y)^{20}$$

$$13\log x + 7\log y = 20\log(x+y)$$

$$[ \text{Since, } \log(AB) = \log A + \log B, \log a^b = b \log a ]$$

Differentiating it with respect to  $x$  using chain rule,

$$13 \frac{d}{dx}(\log x) + 7 \frac{d}{dx}(\log y) = 20 \frac{d}{dx} \log(x+y)$$

$$\frac{13}{x} + \frac{7}{y} \frac{dy}{dx} = \frac{20}{x+y} \frac{d}{dx}(x+y)$$

$$\frac{13}{x} + \frac{7}{y} \frac{dy}{dx} = \frac{20}{(x+y)} \left[ 1 + \frac{dy}{dx} \right]$$

$$\frac{7}{y} \frac{dy}{dx} - \frac{20}{(x+y)} = \frac{20}{(x+y)} - \frac{13}{x}$$

$$\frac{dy}{dx} \left[ \frac{7}{y} - \frac{20}{(x+y)} \right] = \frac{20}{(x+y)} - \frac{13}{x}$$

$$\frac{dy}{dx} \left[ \frac{7(x+y) - 20y}{y(x+y)} \right] = \left[ \frac{20x - 13(x+y)}{x(x+y)} \right]$$

$$\frac{dy}{dx} = \left[ \frac{20x - 13x - 13y}{x(x+y)} \right] \left( \frac{y(x+y)}{7x + 7y - 20y} \right)$$

$$= \frac{y}{x} \left( \frac{7x - 13y}{7x - 13y} \right)$$

$$\frac{dy}{dx} = \frac{y}{x}$$

\*\*\*\*\* END \*\*\*\*\*