



Binary Operations Ex 3.2 Q4(viii)

Commutativity: Let $a, b \in R$, then

$$\begin{aligned}a * b &= a + b - 7 \\&= b + a - 7 \\&= b * a\end{aligned}$$

$$\Rightarrow a * b = b * a$$

$$\Rightarrow * \text{ is commutative on } R$$

Associativity: Let $a, b, c \in Q$, then

$$\begin{aligned}(a * b) * c &= (a + b - 7) * c \\&= a + b - 7 + c - 7 \\&= a + b + c - 17\end{aligned}\quad \text{--- (i)}$$

$$\begin{aligned}\text{and, } a * (b * c) &= a * (b + c - 7) \\&= a + b + c - 7 - 7 \\&= a + b + c - 17\end{aligned}\quad \text{--- (ii)}$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

$$\Rightarrow * \text{ is associative on } R$$

Binary Operations Ex 3.2 Q4(ix)

Commutativity:

Let $a, b \in R - \{-1\}$, then

$$a * b = \frac{a}{b+1} \neq \frac{b}{a+1} = b * a$$

$$\Rightarrow a * b \neq b * a$$

$$\Rightarrow * \text{ is not commutative on } R - \{-1\}$$

Associativity:

Let $a, b, c \in R - \{-1\}$, then

$$\begin{aligned} (a * b) * c &= \left(\frac{a}{b+1} \right) * c \\ &= \frac{\frac{a}{b+1}}{c+1} = \frac{a}{(b+1)(c+1)} \end{aligned} \quad \text{--- (i)}$$

$$\begin{aligned} \& \quad a * (b * c) &= a * \left(\frac{b}{c+1} \right) \\ &= \frac{a}{\frac{b}{c+1} + 1} = \frac{a(c+1)}{b+c+1} \end{aligned} \quad \text{--- (ii)}$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$$\Rightarrow * \text{ is not associative on } R - \{-1\}$$

Binary Operations Ex 3.2 Q4(x)

Commutativity:

Let $a, b \in Q$, then

$$a * b = ab + 1 = ba + 1 = b * a$$

$$\Rightarrow a * b = b * a$$

$$\Rightarrow * \text{ is commutative on } Q$$

Associativity:

Let $a, b, c \in Q$, then

$$\begin{aligned} (a * b) * c &= (ab + 1) * c \\ &= abc + c + 1 \end{aligned} \quad \text{--- (i)}$$

$$\begin{aligned} a * (b * c) &= a * (bc + 1) \\ &= abc + a + 1 \end{aligned} \quad \text{--- (ii)}$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$$\Rightarrow * \text{ is not associative on } Q.$$

Binary Operations Ex 3.2 Q4(xi)

Commutativity:

Let $a, b \in N$, then

$$a * b = a^b \neq b^a = b * a$$

$$\Rightarrow a * b \neq b * a$$

$$\Rightarrow '*' \text{ is not commutative on } N$$

Associativity:

Let $a, b, c \in N$, then

$$(a * b) * c = a^b * c = (a^b)^c = a^{bc} \quad \text{--- (i)}$$

$$a * (b * c) = a * b^c = (a)^{b^c} \quad \text{--- (ii)}$$

From (i) and (ii)

$$a^{bc} \neq (a)^{b^c}$$

$$\Rightarrow (a * b) * c \neq a * (b * c)$$

$$\Rightarrow '*' \text{ is not associative on } N.$$

Binary Operations Ex 3.2 Q4(xii)

Commutativity:

Let $a, b \in N$, then

$$a * b = a^b \neq b^a = b * a$$

$$\Rightarrow a * b \neq b * a$$

$$\Rightarrow '*' \text{ is not commutative on } N$$

Associativity:

Let $a, b, c \in N$, then

$$(a * b) * c = a^b * c = (a^b)^c = a^{bc} \quad \text{--- (i)}$$

$$a * (b * c) = a * b^c = (a)^{b^c} \quad \text{--- (ii)}$$

From (i) and (ii)

$$a^{bc} \neq (a)^{b^c}$$

$$\Rightarrow (a * b) * c \neq a * (b * c)$$

$$\Rightarrow '*' \text{ is not associative on } N.$$

Binary Operations Ex 3.2 Q4(xiii)

Commutativity:

Let $a, b \in \mathbb{Z}$ then,

$$a * b = a - b \neq b - a = b * a$$

$$\Rightarrow a * b \neq b * a$$

$$\Rightarrow * \text{ is not commutative on } \mathbb{Z}$$

Associativity:

Let $a, b, c \in \mathbb{Z}$, then

$$(a * b) * c = (a - b) * c = (a - b - c) \quad \text{--- (i)}$$

$$\& \quad a * (b * c) = a * (b - c) = (a - b + c) \quad \text{--- (ii)}$$

From (i) & (ii)

$$(a * b) * c \neq a * (b * c)$$

$$\Rightarrow '*' \text{ is not associative on } \mathbb{Z}.$$

Binary Operations Ex 3.2 Q4(xiv)

Commutativity:

Let $a, b \in \mathbb{Q}$ then,

$$a * b = \frac{ab}{4} = \frac{ba}{4} = b * a$$

$$\Rightarrow a * b = b * a$$

$$\therefore * \text{ is commutative on } \mathbb{Q}$$

Associativity:

Let $a, b, c \in \mathbb{Q}$ then,

$$(a * b) * c = \frac{ab}{4} * c = \frac{abc}{16} \quad \text{--- (i)}$$

$$\text{and, } a * (b * c) = a * \frac{bc}{4} = \frac{abc}{16} \quad \text{--- (ii)}$$

From (i) and (ii)

$$(a * b) * c = a * (b * c)$$

$$\therefore '*' \text{ is associative on } \mathbb{Q}.$$

Binary Operations Ex 3.2 Q4(xv)

Commutativity:

Let $a, b \in Q$ then,

$$a * b = (a - b)^2 = (b - a)^2 = b * a$$

$$\Rightarrow a * b = b * a$$

\therefore '*' is commutative on Q .

Associativity:

Let $a, b, c \in Q$ then,

$$(a * b) * c = (a - b)^2 * c = \left[(a - b)^2 - c \right]^2 \quad \text{--- (i)}$$

$$\text{and, } a * (b * c) = a * (b - c)^2 = \left[a - (b - c)^2 \right]^2 \quad \text{--- (ii)}$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

\therefore '*' is not associative on Q .

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