Indefinite Integrals Ex 19.11 Q5

Let $I = \int \tan^5 x dx$. Then

$$I = \int \tan^2 x \tan^3 x dx$$
$$= \int \left(\sec^2 x - 1\right) \tan^3 x dx$$
$$= \int \sec^2 x \tan^3 x dx - \int \tan^3 x dx$$
$$= \int \sec^2 x \tan^3 x dx - \int \left(\sec^2 x - 1\right) \tan x dx$$
$$= \int \sec^2 x \tan^3 x dx - \int \sec^2 x \tan x dx + \int \tan x dx$$

Substituting $\tan x = t$ and $\sec^2 x dx = dt$ in first two integral, we get

$$I = \int t^3 dt - \int t dt + \int \tan x dx$$
$$= \frac{t^4}{4} - \frac{t^2}{2} + \log|\sec x| + c$$
$$= \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \log|\sec x| + c$$

$$\therefore \quad I = \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \log|\sec x| + c$$

Indefinite Integrals Ex 19.11 Q6

Let $I = \int \sqrt{\tan x} \sec^4 x dx$. Then

$$I = \int \sqrt{\tan x} \sec^2 x \sec^2 x dx$$
$$= \int \sqrt{\tan x} \left(1 + \tan^2 x\right) \sec^2 x dx$$
$$= \int \tan x^{\frac{1}{2}} \left(1 + \tan^2 x\right) \sec^2 x dx$$
$$\Rightarrow \quad I = \int \left(\tan x^{\frac{1}{2}} + \tan x^{\frac{5}{2}}\right) \sec^2 x dx$$

Substituting $\tan x = t$ and $\sec^2 x dx = dt$, we get

$$I = \int \left(t^{\frac{1}{2}} + t^{\frac{5}{2}}\right) dt$$
$$= \frac{2}{3} t^{\frac{3}{2}} + \frac{2}{7} t^{\frac{7}{2}} + c$$
$$= \frac{2}{3} (\tan x)^{\frac{3}{2}} + \frac{2}{7} (\tan x)^{\frac{7}{2}} + c$$

$$\therefore \quad I = \frac{2}{3} \tan^{\frac{3}{2}} x + \frac{2}{7} \tan^{\frac{7}{2}} x + c$$

Indefinite Integrals Ex 19.11 Q7

Let $I = \int \sec^4 2x dx$. Then

$$I = \int \sec^2 2x \sec^2 2x dx$$
$$= \int \left(1 + \tan^2 2x\right) \sec^2 2x dx$$
$$= \int \left(\sec^2 2x + \sec^2 2x \tan^2 2x\right) dx$$
$$\Rightarrow \quad I = \int \sec^2 2x dx + \int \sec^2 2x \tan^2 2x dx$$
$$\Rightarrow \quad I = \int \sec^2 2x \tan^2 2x dx + \int \sec^2 2x dx$$

Substituting $\tan 2x = t$ and $\sec^2 2x dx = \frac{dt}{2}$ in first integral, we get

$$I = \int t^2 \frac{dt}{2} + \int \sec^2 2x dx$$
$$= \frac{1}{2} \times \frac{t^3}{3} + \frac{1}{2} \tan 2x + c$$
$$\Rightarrow \quad I = \frac{1}{6} \tan^3 2x + \frac{1}{2} \tan 2x + c$$

$$\therefore \quad I = \frac{1}{2} \tan 2x + \frac{1}{6} \tan^3 2x + c$$

Indefinite Integrals Ex 19.11 Q8

Let $I = \int cosec^4 3x dx$. Then

$$I = \int cosec^2 3x \, cosec^2 3x dx$$
$$= \int \left(1 + \cot^2 3x\right) cosec^2 3x dx$$
$$= \int \left(cosec^2 3x + \cot^2 3x \, cosec^2 3x\right) dx$$
$$\Rightarrow \quad I = \int cosec^2 3x dx + \int \cot^2 3x \, cosec^2 3x dx$$

Substituting $\cot 3x = t$ and $cosec^2 3x dx = -dt$ in 2nd integral, we get

$$I = \int cosec^2 3x dx - \int t^2 \frac{dt}{3}$$
$$= \frac{-1}{3} \cot 3x - \frac{t^3}{9} + c$$
$$= \frac{-1}{3} \cot 3x - \frac{\cot^3 3x}{9} + c$$

$$\therefore \quad I = \frac{-1}{3} \cot 3x - \frac{1}{9} \cot^3 3x + c$$

*********** END **********