Binomial Theorem Ex 18.1 Q3

We have,

$$
\begin{aligned}
&(a+b)^4-(a-b)^4\\
&=\left[{}^4C_0a^4b^0+{}^4C_1a^3b^1+{}^4C_2a^2b^2+{}^4C_3a^1b^3+{}^4C_4a^0b^4\right]\\
&-\left[{}^4C_0a^4b^0-{}^4C_1a^3b^1+{}^4C_2a^2b^2-{}^4C_3a^1b^3+{}^4C_4a^0b^4\right]\\
&=\left[{}^4C_0a^4(-b)^0+{}^4C_1a^3(-b)^1+{}^4C_2a^2(-b)^2+{}^4C_3a^1(-b)^3+{}^4C_4a^0(-b)^4\right]\\
&\qquad-\left[{}^4C_0a^4(-b)^0+{}^4C_1a^3(-b)^1+{}^4C_2a^2(-b)^2+{}^4C_3a^1(-b)^3+{}^4C_4a^0(-b)^4\right]\\
&=\left[{}^4C_0a^4+{}^4C_1a^3b+{}^4C_2a^2b^2+{}^4C_3ab^3+{}^4C_4ab^4\right]-\left[{}^4C_0a^4-{}^4C_1a^3b+{}^4C_2a^2b^2-{}^4C_3ab^3+{}^4C_4b^4\right]\\
&={}^4C_0a^4+{}^4C_1a^3b+{}^4C_2a^2b^2+{}^4C_3ab^3+{}^4C_4ab^4-{}^4C_0a^4+{}^4C_1a^3b-{}^4C_2a^2b^2+{}^4C_3ab^3-{}^4C_4b^4\\
&=2\left[{}^4C_1a^3b+{}^4C_3ab^3\right]\\
&=2\left[4a^3b+4ab^3\right]\\
&=8\left[a^3b+ab^3\right]
\end{aligned}
$$

$\therefore \quad (a+b)^4-(a-b)^4=8\left(a^3b+ab^3\right) \qquad$ ---(i)

Putting $a=\sqrt{3}$ and $b=\sqrt{2}$ in equation (i), we get

$$
\begin{aligned}
\left(\sqrt{3}+\sqrt{2}\right)^4-\left(\sqrt{3}-\sqrt{2}\right)^4 &= 8\left[\left(\sqrt{3}\right)^3\times\sqrt{2}+\left(\sqrt{3}\right)\times\left(\sqrt{2}\right)^3\right]\\
&=8\left[3\sqrt{6}+2\sqrt{6}\right]\\
&=8\times5\sqrt{6}\\
&=40\sqrt{6}
\end{aligned}
$$

$\therefore \quad \left(\sqrt{3}+\sqrt{2}\right)^4-\left(\sqrt{3}-\sqrt{2}\right)^4=40\sqrt{6}.$

Binomial Theorem Ex 18.1 Q4

We have,

$$
\begin{aligned}
&(x+1)^6-(x-1)^6\\
&=\left[{}^6C_0x^6+{}^6C_1x^5+{}^6C_2x^4+{}^6C_3x^3+{}^6C_4x^2+{}^6C_5x^1+{}^6C_6x^0\right]\\
&+\left[{}^6C_0x^6(-1)^0+{}^6C_1x^5(-1)^1+{}^6C_2x^4(-1)^2+{}^6C_3x^3(-1)^3+{}^6C_4x^2(-1)^4+{}^6C_5x^1(-1)^5+{}^6C_6x^0(-1)^6\right]\\
&=\left[\begin{aligned}&{}^6C_0x^6+{}^6C_1x^5+{}^6C_2x^4+{}^6C_3x^3+{}^6C_4x^2+{}^6C_5x+{}^6C_6+{}^6C_0x^6-{}^6C_1x^5+{}^6C_2x^4-{}^6C_3x^3+{}^6C_4x^2\\&\qquad -{}^6C_5x+{}^6C_6\end{aligned}\right]\\
&=2\left[{}^6C_0x^6+{}^6C_2x^4+{}^6C_4x^2+{}^6C_6\right]\\
&=2\left[x^6+15x^4+15x^2+1\right]
\end{aligned}
$$

$\therefore \quad (x+1)^6+(x-1)^6=2\left[x^6+15x^4+15x^2+1\right] \qquad$ ---(i)

Putting $x=\sqrt{2}$ in equation (i), we get

$$
\begin{aligned}
(x+1)^6+(x-1)^6 &= 2\left[\left(\sqrt{2}\right)^6+15\left(\sqrt{2}\right)^4+15\left(\sqrt{2}\right)^2+1\right]\\
&=2[8+60+30+1]\\
&=2[99]\\
&=198
\end{aligned}
$$

$\therefore \quad (x+1)^6+(x-1)^6=198$

Binomial Theorem Ex 18.1 Q5(i)

We have,

$$
\begin{aligned}
(96)^3 &= (100-4)^3\\
&= {}^3C_0\times100^3+{}^3C_1\times100^2\times(-4)+{}^3C_2\times100\times(-4)^2+{}^3C_3\times(-4)^3\\
&=100^3-3\times100^2\times4+3\times100\times4^2-4^3\\
&=1000000-120000+4800-64\\
&=1004800-120064\\
&=884736
\end{aligned}
$$

$\therefore \quad (96)^3=884736$

*********** END **********