Complex Numbers Ex 13.2 Q15

$$a = \cos\theta + i\sin\theta$$

$$\frac{1+a}{1-a}$$

$$= \frac{1+\cos\theta+i\sin\theta}{1-\cos\theta-i\sin\theta}$$

$$= \frac{(1+\cos\theta+i\sin\theta)(1-\cos\theta+i\sin\theta)}{(1-\cos\theta-i\sin\theta)(1-\cos\theta+i\sin\theta)} \text{[Rationalizing the denominator]}$$

$$= \frac{(1+\cos\theta+i\sin\theta)(1-\cos\theta+i\sin\theta)}{(1-\cos\theta)^2-(i\sin\theta)^2}$$

$$= \frac{(1+i\sin\theta)^2-\cos^2\theta}{1-2\cos\theta+\cos^2\theta+\sin^2\theta}$$

$$= \frac{1+2i\sin\theta-\sin^2\theta-\cos^2\theta}{1-2\cos\theta+\cos^2\theta+\sin^2\theta}$$

$$= \frac{1+2i\sin\theta-1}{1-2\cos\theta+\cos^2\theta+\sin^2\theta} [\because \cos^2\theta+\sin^2\theta=1]$$

$$= \frac{2i\sin\theta}{1-2\cos\theta+1}$$

$$= \frac{2i\sin\theta}{2-2\cos\theta}$$

$$= \frac{i\sin\theta}{1-\cos\theta}$$

$$= \frac{i2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}$$

$$= \frac{i\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} = i\cot\frac{\theta}{2}$$

Complex Numbers Ex 13.2 Q16(i)

We have,

$$x = \frac{3-5i}{2}$$

$$\Rightarrow \quad 2x = 3-5i$$

$$\Rightarrow \quad 2x-3 = -5i$$

$$\Rightarrow (2x-3)^2 = (-5i)^2$$

$$\Rightarrow \quad 4x^2+9-12x = -25$$

$$\Rightarrow \quad 4x^2-12x+34 = 0$$

$$\Rightarrow 2\left(2x^2-6x+17\right) = 0$$

$$\Rightarrow \quad 2x^2-6x+17 = 0 \qquad \ldots\ldots\ldots\ldots(i)$$

$$\therefore 2x^3+2x^2-7x+72$$

$$= x\left(2x^2-6x+17\right)+6x^2-17x+2x^2-7x+72 \left(\text{adding and subtracting } 6x^2 \text{ and } 17x\right)$$

$$= x\times 0+8x^2-24x+72 \qquad (\text{using } (i))$$

$$= 4\left(2x^2-6x+17\right)+4$$

$$= 4\times 0+4 \qquad (\text{using } (i))$$

$$= 4$$

Complex Numbers Ex 13.2 Q16(ii)

We have,
$x = 3 + 2i$

$\Rightarrow x - 3 = 2i$
$\Rightarrow (x-3)^2 = (2i)^2$
$\Rightarrow x^2 + 3^2 - 2 \times 3 \times x = -4$
$\Rightarrow x^2 + 9 - 6x + 4 = 0$
$\Rightarrow x^2 - 6x + 13 = 0 \quad \ldots\ldots\ldots\ldots (i)$

Now,
$x^4 - 4x^3 + 4x^2 + 8x + 44$
$= x^2\left(x^2 - 6x + 13\right) + 6x^2 - 13x^2 - 4x^3 + 4x^2 + 8x + 44$ (adding and subtracting $6x^3$ and $13x^2$)
$= x^2 \times 0 + 2x^3 - 9x^2 + 8x + 44$ (using (i))
$= 2x\left(x^2 - 6x + 13\right) + 12x^2 - 26x - 9x^2 + 8x + 44$ (adding and subtracting $12x^2$ and $26x$)
$= 2x \times 0 + 3x^2 - 18x + 44$ (using (i))
$= 3\left(x^2 - 6x + 13\right) + 5$
$= 3 \times 0 + 5$ (using (i))
$= 5$

Complex Numbers Ex 13.2 Q16(iii)

We have,
$x = -1 + i\sqrt{2}$

$\Rightarrow x + 1 = i\sqrt{2}$
$\Rightarrow (x+1)^2 = \left(i\sqrt{2}\right)^2$ (squaring both sides)
$\Rightarrow x^2 + 1 + 2x = -2$
$\Rightarrow x^2 + 2x + 3 = 0 \ldots\ldots\ldots\ldots (i)$

Now,
$x^4 + 4x^3 + 6x^2 + 4x + 9$
$= x^2\left(x^2 + 2x + 3\right) + 2x^3 + 3x^2 + 4x + 9$
$= x^2 \times 0 + 2x\left(x^2 + 2x + 3\right) - x^2 - 2x + 9$ (using (i))
$= 2x \times 0 - \left(x^2 + 2x + 3\right) + 3 + 9$ (using (i) and adding and subtracting 3)
$= -0 + 3 + 9$ (using (i))
$= 12$

********** END **********