Exercise 3D

Question 21:

$x + (k + 1)y - 5 = 0$

$(k + 1)x + 9y - (8k - 1) = 0$

These are of the form

$$a_1x + b_1y + c_1 = 0,\ a_2x + b_2y + c_2 = 0$$

where $a_1 = 1, b_1 = (k + 1), c_1 = -5$

$a_2 = (k + 1), b_2 = 9, c_2 = -(8k - 1)$

For infinitely many solutions, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{1}{(k+1)} = \frac{(k+1)}{9} = \frac{-5}{-(8k-1)}$$

$$\Rightarrow \frac{1}{(k+1)} = \frac{(k+1)}{9} = \frac{5}{(8k-1)}$$

Case I : $\frac{1}{(k+1)} = \frac{(k+1)}{9}$ [Taking I and II]

$\Rightarrow (k + 1)^2 = 9 \Rightarrow (k + 1) = \pm 3$

$k + 1 = 3$ or $k + 1 = -3$

$k = 2$ or $k = -4$

Case II : $\frac{k+1}{9} = \frac{5}{8k-1}$ [Taking II and III]

$\Rightarrow (k + 1)(8k - 1) = 45$

$\Rightarrow 8k^2 + 7k - 46 = 0$

$8k^2 + 23k - 16k - 46 = 0$

$\Rightarrow k(8k + 23) - 23(8k + 23) = 0$

$\Rightarrow (k - 23)(8k + 23) = 0$

$\Rightarrow \quad k = \frac{-23}{8}$ or $k = 2$

Case III : $\frac{1}{(k+1)} = \frac{5}{(8k-1)}$ [Taking I and III]

$8k - 1 = 5(k + 1)$

$3k = 6 \Rightarrow k = 2$

Thus, $k = 2$ is the common value for which there are inifnitely many solutions

Question 22:

$(k-1)x - y - 5 = 0$

$(k+1)x + (1-k)y - (3k+1) = 0$

These are of the form

$$a_1x + b_1y + c_1 = 0,\ a_2x + b_2y + c_2 = 0$$

where, $a_1 = (k-1)$, $b_1 = -1$, $c_1 = -5$

$a_2 = (k+1)$, $b_2 = (1-k)$, $c_2 = -(3k+1)$

For infinitely many solution, we must now

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{k-1}{k+1} = \frac{-1}{-(k-1)} = \frac{-5}{-(3k+1)}$$

$$\frac{k-1}{k+1} = \frac{1}{(k-1)} = \frac{5}{(3k+1)}$$

Case I : $\frac{k-1}{k+1} = \frac{1}{(k-1)}$ [Taking I and II]

$$(k-1)^2 = k+1$$

$$\Rightarrow k^2 + 1 - 2k = k + 1$$

$$\Rightarrow k^2 + 1 - 1 - 2k - k = 0$$

$$\Rightarrow \quad k^2 = 3k \Rightarrow k = 3$$

case II : $\frac{1}{(k-1)} = \frac{5}{(3k+1)}$ [TakingII and III]

$$(3k+1) = 5(k-1) \Rightarrow 3k + 1 = 5k - 5$$

$$-2k = -6 \Rightarrow k = 3$$

Case III : $\frac{k-1}{k+1} = \frac{5}{(3k+1)}$ [Taking I and III]

$$(k-1)(3k+1) = 5(k+1)$$

$$3k^2 + k - 3k - 1 = 5k + 5$$

$$3k^2 - 2k - 5k - 1 - 5 = 0$$

$$3k^2 - 7k - 6 = 0$$

$$3k^2 - 9k + 2k - 6 = 0$$

$$3k(k-3) + 2(k-3) = 0$$

$$(3k+2)(k-3) = 0$$

$$(3k+2) = 0 \text{ or } (k-3) = 0$$

$$3k = -2 \text{ or } k = 3$$

$$k = \frac{-2}{3} \text{ or } k = 3$$

k = 3 is common value for which the number of solutions is infinitely many.

********** END **********