Question 5:

Find the vector and Cartesian equation of the planes

(a) that passes through the point (1, 0, −2) and the normal to the plane is $\hat{i}+\hat{j}-\hat{k}$.

(b) that passes through the point (1, 4, 6) and the normal vector to the plane is $\hat{i}-2\hat{j}+\hat{k}$.

Answer

(a) The position vector of point (1, 0, −2) is $\vec{a}=\hat{i}-2\hat{k}$

The normal vector $\vec{N}$ perpendicular to the plane is $\vec{N}=\hat{i}+\hat{j}-\hat{k}$

The vector equation of the plane is given by, $(\vec{r}-\vec{a}).\vec{N}=0$

$$\Rightarrow\left[\vec{r}-\left(\hat{i}-2\hat{k}\right)\right].\left(\hat{i}+\hat{j}-\hat{k}\right)=0 \qquad ...(1)$$

$\vec{r}$ is the position vector of any point P (x, y, z) in the plane.

$\therefore \vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

Therefore, equation (1) becomes

$$\left[\left(x\hat{i}+y\hat{j}+z\hat{k}\right)-\left(\hat{i}-2\hat{k}\right)\right].\left(\hat{i}+\hat{j}-\hat{k}\right)=0$$
$$\Rightarrow\left[(x-1)\hat{i}+y\hat{j}+(z+2)\hat{k}\right].\left(\hat{i}+\hat{j}-\hat{k}\right)=0$$
$$\Rightarrow(x-1)+y-(z+2)=0$$
$$\Rightarrow x+y-z-3=0$$
$$\Rightarrow x+y-z=3$$

This is the Cartesian equation of the required plane.

(b) The position vector of the point (1, 4, 6) is $\vec{a}=\hat{i}+4\hat{j}+6\hat{k}$

The normal vector $\vec{N}$ perpendicular to the plane is $\vec{N}=\hat{i}-2\hat{j}+\hat{k}$

The vector equation of the plane is given by, $(\vec{r}-\vec{a}).\vec{N}=0$

$$\Rightarrow\left[\vec{r}-\left(\hat{i}+4\hat{j}+6\hat{k}\right)\right].\left(\hat{i}-2\hat{j}+\hat{k}\right)=0 \qquad ...(1)$$

$\vec{r}$ is the position vector of any point P (x, y, z) in the plane.

$\therefore \vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

Therefore, equation (1) becomes

$$\left[\left(x\hat{i}+y\hat{j}+z\hat{k}\right)-\left(\hat{i}+4\hat{j}+6\hat{k}\right)\right].\left(\hat{i}-2\hat{j}+\hat{k}\right)=0$$
$$\Rightarrow\left[(x-1)\hat{i}+(y-4)\hat{j}+(z-6)\hat{k}\right]\cdot\left(\hat{i}-2\hat{j}+\hat{k}\right)=0$$
$$\Rightarrow(x-1)-2(y-4)+(z-6)=0$$
$$\Rightarrow x-2y+z+1=0$$

This is the Cartesian equation of the required plane.

Question 6:

Find the equations of the planes that passes through three points.

(a) (1, 1, −1), (6, 4, −5), (−4, −2, 3)

(b) (1, 1, 0), (1, 2, 1), (−2, 2, −1)

Answer

(a) The given points are A (1, 1, −1), B (6, 4, −5), and C (−4, −2, 3).

$$\begin{vmatrix}1 & 1 & -1\\ 6 & 4 & -5\\ -4 & -2 & 3\end{vmatrix}=(12-10)-(18-20)-(-12+16)$$
$$=2+2-4$$
$$=0$$

Since A, B, C are collinear points, there will be infinite number of planes passing through the given points.

(b) The given points are A (1, 1, 0), B (1, 2, 1), and C (−2, 2, −1).

$$\begin{vmatrix}1 & 1 & 0\\ 1 & 2 & 1\\ -2 & 2 & -1\end{vmatrix}=(-2-2)-(2+2)=-8\neq 0$$

Therefore, a plane will pass through the points A, B, and C.

It is known that the equation of the plane through the points, (x_1, y_1, z_1), (x_2, y_2, z_2), and (x_3, y_3, z_3), is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & y-1 & z \\ 0 & 1 & 1 \\ -3 & 1 & -1 \end{vmatrix} = 0$$

$$\Rightarrow (-2)(x-1) - 3(y-1) + 3z = 0$$
$$\Rightarrow -2x - 3y + 3z + 2 + 3 = 0$$
$$\Rightarrow -2x - 3y + 3z = -5$$
$$\Rightarrow 2x + 3y - 3z = 5$$

This is the Cartesian equation of the required plane.

Question 7:

Find the intercepts cut off by the plane $2x + y - z = 5$

Answer

$$2x + y - z = 5 \qquad ...(1)$$

Dividing both sides of equation (1) by 5, we obtain

$$\frac{2}{5}x + \frac{y}{5} - \frac{z}{5} = 1$$

$$\Rightarrow \frac{x}{\frac{5}{2}} + \frac{y}{5} + \frac{z}{-5} = 1 \qquad ...(2)$$

It is known that the equation of a plane in intercept form is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, where *a*, *b*, *c* are the intercepts cut off by the plane at *x*, *y*, and *z* axes respectively.

Therefore, for the given equation,

$$a = \frac{5}{2}, b = 5, \text{ and } c = -5$$

Thus, the intercepts cut off by the plane are $\frac{5}{2}$, 5, and -5.

Question 8:

Find the equation of the plane with intercept 3 on the *y*-axis and parallel to ZOX plane.

Answer

The equation of the plane ZOX is

y = 0

Any plane parallel to it is of the form, *y* = *a*

Since the *y*-intercept of the plane is 3,

∴ *a* = 3

Thus, the equation of the required plane is *y* = 3

Question 9:

Find the equation of the plane through the intersection of the planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and the point (2, 2, 1)

Answer

The equation of any plane through the intersection of the planes,

$3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$, is

$$(3x - y + 2z - 4) + \alpha(x + y + z - 2) = 0, \text{ where } \alpha \in \mathrm{R} \qquad ...(1)$$

The plane passes through the point (2, 2, 1). Therefore, this point will satisfy equation (1).

$$\therefore (3\times 2 - 2 + 2\times 1 - 4) + \alpha(2 + 2 + 1 - 2) = 0$$
$$\Rightarrow 2 + 3\alpha = 0$$
$$\Rightarrow \alpha = -\frac{2}{3}$$

Substituting $\alpha = -\frac{2}{3}$ in equation (1), we obtain

$$(3x - y + 2z - 4) - \frac{2}{3}(x + y + z - 2) = 0$$
$$\Rightarrow 3(3x - y + 2z - 4) - 2(x + y + z - 2) = 0$$
$$\Rightarrow (9x - 3y + 6z - 12) - 2(x + y + z - 2) = 0$$
$$\Rightarrow 7x - 5y + 4z - 8 = 0$$

This is the required equation of the plane.

Question 10:

Find the vector equation of the plane passing through the intersection of the planes $\vec{r}\cdot\left(2\hat{i}+2\hat{j}-3\hat{k}\right)=7,\ \vec{r}.\left(2\hat{i}+5\hat{j}+3\hat{k}\right)=9$ and through the point (2, 1, 3)

Answer

The equations of the planes are $\vec{r}\cdot\left(2\hat{i}+2\hat{j}-3\hat{k}\right)=7$ and $\vec{r}\cdot\left(2\hat{i}+5\hat{j}+3\hat{k}\right)=9$

$\Rightarrow \vec{r}\cdot\left(2\hat{i}+2\hat{j}-3\hat{k}\right)-7=0 \qquad ...(1)$

$\vec{r}\cdot\left(2\hat{i}+5\hat{j}+3\hat{k}\right)-9=0 \qquad ...(2)$

The equation of any plane through the intersection of the planes given in equations (1) and (2) is given by,

$\left[\vec{r}\cdot\left(2\hat{i}+2\hat{j}-3\hat{k}\right)-7\right]+\lambda\left[\vec{r}\cdot\left(2\hat{i}+5\hat{j}+3\hat{k}\right)-9\right]=0$, where $\lambda\in R$

$\vec{r}\cdot\left[\left(2\hat{i}+2\hat{j}-3\hat{k}\right)+\lambda\left(2\hat{i}+5\hat{j}+3\hat{k}\right)\right]=9\lambda+7$

$\vec{r}\cdot\left[(2+2\lambda)\hat{i}+(2+5\lambda)\hat{j}+(3\lambda-3)\hat{k}\right]=9\lambda+7 \qquad ...(3)$

The plane passes through the point (2, 1, 3). Therefore, its position vector is given by,

$\vec{r}=2\hat{i}+2\hat{j}+3\hat{k}$

Substituting in equation (3), we obtain

$\left(2\hat{i}+\hat{j}-3\hat{k}\right).\left[(2+2\lambda)\hat{i}+(2+5\lambda)\hat{j}+(3\lambda-3)\hat{k}\right]=9\lambda+7$

$\Rightarrow (2+2\lambda)+(2+5\lambda)+(3\lambda-3)=9\lambda+7$

$\Rightarrow 18\lambda-3=9\lambda+7$

$\Rightarrow 9\lambda=10$

*********** END **********