



Tangents and Normals Ex 16.3 Q1(v)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{---(i)}$$

$$x^2 + y^2 = ab \quad \text{---(ii)}$$

From (ii), we get

$$y^2 = ab - x^2$$

∴ From (i), we get

$$\frac{x^2}{a^2} + \frac{ab - x^2}{b^2} = 1$$

$$\Rightarrow b^2 x^2 + a^3 b - a^2 x^2 = a^2 b^2$$

$$\Rightarrow (b^2 - a^2) x^2 = a^2 b^2 - a^3 b$$

$$\begin{aligned} \Rightarrow x^2 &= \frac{a^2 b^2 - a^3 b}{b^2 - a^2} \\ &= \frac{a^2 b (b - a)}{(b - a)(b + a)} \\ &= \frac{a^2 b}{b + a} \end{aligned}$$

$$\therefore x = \pm \sqrt{\frac{a^2 b}{b + a}}$$

$$\begin{aligned} \therefore y^2 &= ab - x^2 = ab - \frac{a^2 b}{b + a} \\ &= \frac{a^2 b + ab^2 - a^2 b}{b + a} = \frac{ab^2}{b + a} \end{aligned}$$

Differentiating (i) and (ii) w.r.t x we get

$$\frac{2x}{a^2} + \frac{2y}{b^2} \times \left(\frac{dy}{dx} \right)_{C_1} = 0$$

$$\text{and } 2x + 2y \left(\frac{dy}{dx} \right)_{C_2} = 0$$

$$\begin{aligned} \therefore \left(\frac{dy}{dx} \right)_{C_1} &= \frac{-x}{a^2} \times \frac{b^2}{y} = \frac{-b^2 x}{a^2 y} \\ \left(\frac{dy}{dx} \right)_{C_2} &= \frac{-x}{y} \end{aligned}$$

$$\left(\frac{dy}{dx} \right)_{C_2} = y$$

At $\left(\pm \sqrt{\frac{a^2 b}{a+b}}, \pm \sqrt{\frac{ab^2}{a+b}} \right)$ we get

$$\left(\frac{dy}{dx} \right)_{C_1} = \frac{-b^2}{a^2} \sqrt{\frac{a}{b}} = \frac{-b^2 \sqrt{a}}{a^2 \sqrt{b}}$$

$$\left(\frac{dy}{dx} \right)_{C_2} = -\sqrt{\frac{a}{b}}$$

Tangents and Normals Ex 16.3 Q1(vi)

We know that angle of intersection of two curves is given by

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \quad \text{---(A)}$$

Where m_1 and m_2 are slopes of curves.

$$x^2 + 4y^2 = 8 \quad \text{---(i)}$$

$$x^2 - 2y^2 = 2 \quad \text{---(ii)}$$

Solving (i) and (ii)

$$6y^2 = 6 \Rightarrow y = \pm 1$$

$$\therefore x^2 = 2 + 2 \Rightarrow x = \pm 2$$

\therefore Point of intersection are

$$P = (2, 1) \text{ and } (-2, -1)$$

Now,

Slope m_1 for (i)

$$8y \frac{dy}{dx} = -2x \Rightarrow \frac{dy}{dx} = -\frac{x}{4y}$$

$$\therefore m_1 = \frac{1}{2}$$

Slope m_2 for (ii)

$$4y \frac{dy}{dx} = 2x \Rightarrow \frac{dy}{dx} = \frac{x}{2y}$$

$$\therefore m_2 = 1$$

From (A)

$$\tan \theta = \left| \frac{1 - \frac{1}{2}}{1 + 1 \times \frac{1}{2}} \right| = \frac{1}{3}$$

$$\therefore \theta = \tan^{-1} \left(\frac{1}{3} \right)$$

Tangents and Normals Ex 16.3 Q1(vii)

We know that angle of intersection of two curves is given by

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \quad \text{---(A)}$$

Where m_1 and m_2 are slopes of curves.

$$x^2 = 27y \quad \text{---(i)}$$

$$y^2 = 8x \quad \text{---(ii)}$$

Solving (i) and (ii) are

$$\frac{y^4}{64} = 27y$$

$$\Rightarrow y(y^3 - 27 \times 64) = 0$$

$$\Rightarrow y = 0 \text{ or } 12$$

$$\therefore x = 0 \text{ or } 18$$

\therefore Points of intersection is (0,0) and (18,12)

Now,

Slope of (i)

$$m_1 = \frac{2x}{27} = \frac{12}{9} = \frac{4}{3}$$

Slope of (ii)

$$m_2 = \frac{8}{2y} = \frac{8}{24} = \frac{1}{3}$$

From (A)

$$\tan \theta = \left| \frac{\frac{4}{3} - \frac{1}{3}}{1 + \frac{4}{3} \times \frac{1}{3}} \right| = \frac{9}{13}$$

$$\therefore \theta = \tan^{-1} \left(\frac{9}{13} \right)$$

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