Exercise 5.2 : Solutions of Questions on Page Number : **166**

Q1 : Differentiate the functions with respect to *x*.

$\sin\left(x^2+5\right)$

Answer :

Answer needs Correction? **Click Here**

Q2 : Differentiate the functions with respect to *x*.

$\cos(\sin x)$

Answer :

Let $f(x)=\cos(\sin x), u(x)=\sin x,$ and $v(t)=\cos t$

Then, $(vou)(x)=v(u(x))=v(\sin x)=\cos(\sin x)=f(x)$

Thus, *f* is a composite function of two functions.

Put *t* = *u* (*x*) = sin *x*

$$\therefore \frac{dv}{dt}=\frac{d}{dt}[\cos t]=-\sin t=-\sin(\sin x)$$

$$\frac{dt}{dx}=\frac{d}{dx}(\sin x)=\cos x$$

By chain rule, $\frac{df}{dx}=\frac{dv}{dt}\cdot\frac{dt}{dx}=-\sin(\sin x)\cdot\cos x=-\cos x\sin(\sin x)$

Alternate method

$$\frac{d}{dx}\left[\cos(\sin x)\right]=-\sin(\sin x)\cdot\frac{d}{dx}(\sin x)=-\sin(\sin x)\cdot\cos x=-\cos x\sin(\sin x)$$

Answer needs Correction? **Click Here**

Q3 : Differentiate the functions with respect to *x*.

$\sin(ax+b)$

Answer :

Let $f(x)=\sin(ax+b),\ u(x)=ax+b,$ and $v(t)=\sin t$

Then, $(vou)(x)=v(u(x))=v(ax+b)=\sin(ax+b)=f(x)$

Thus, *f* is a composite function of two functions, *u* and *v*.

Put *t* = *u* (*x*) = *ax* + *b*

Therefore,

$$\frac{dv}{dt}=\frac{d}{dt}(\sin t)=\cos t=\cos(ax+b)$$

$$\frac{dt}{dx}=\frac{d}{dx}(ax+b)=\frac{d}{dx}(ax)+\frac{d}{dx}(b)=a+0=a$$

Hence, by chain rule, we obtain

$$\frac{df}{dx}=\frac{dv}{dt}\cdot\frac{dt}{dx}=\cos(ax+b)\cdot a=a\cos(ax+b)$$

Alternate method

$$\begin{aligned}\frac{d}{dx}\left[\sin(ax+b)\right]&=\cos(ax+b).\frac{d}{dx}(ax+b)\\&=\cos(ax+b)\cdot\left[\frac{d}{dx}(ax)+\frac{d}{dx}(b)\right]\\&=\cos(ax+b).(a+0)\\&=a\cos(ax+b)\end{aligned}$$

Answer needs Correction? **Click Here**

Q4 : Differentiate the functions with respect to *x*.

$\sec\left(\tan\left(\sqrt{x}\right)\right)$

Answer :

Let $f(x)=\sec\left(\tan\sqrt{x}\right), u(x)=\sqrt{x}, v(t)=\tan t,$ and $w(s)=\sec s$

Then, $(wovou)(x)=w\left[v\left(u(x)\right)\right]=w\left[v\left(\sqrt{x}\right)\right]=w\left(\tan\sqrt{x}\right)=\sec\left(\tan\sqrt{x}\right)=f(x)$

Thus, *f* is a composite function of three functions, *u*, *v*, and *w*.

Put $s=v(t)=\tan t$ and $t=u(x)=\sqrt{x}$

$$\text{Then, } \frac{dw}{ds}=\frac{d}{ds}(\sec s)=\sec s\tan s=\sec(\tan t).\tan(\tan t) \quad [s=\tan t]$$
$$=\sec\left(\tan\sqrt{x}\right)\cdot\tan\left(\tan\sqrt{x}\right) \quad \left[t=\sqrt{x}\right]$$
$$\frac{ds}{dt}=\frac{d}{dt}(\tan t)=\sec^2 t=\sec^2\sqrt{x}$$
$$\frac{dt}{dx}=\frac{d}{dx}\left(\sqrt{x}\right)=\frac{d}{dx}\left(x^{\frac{1}{2}}\right)=\frac{1}{2}\cdot x^{\frac{1}{2}-1}=\frac{1}{2\sqrt{x}}$$

Hence, by chain rule, we obtain

$$\frac{dt}{dx}=\frac{dw}{ds}\cdot\frac{ds}{dt}\cdot\frac{dt}{dx}$$
$$=\sec\left(\tan\sqrt{x}\right)\cdot\tan\left(\tan\sqrt{x}\right)\times\sec^2\sqrt{x}\times\frac{1}{2\sqrt{x}}$$
$$=\frac{1}{2\sqrt{x}}\sec^2\sqrt{x}\sec\left(\tan\sqrt{x}\right)\tan\left(\tan\sqrt{x}\right)$$
$$=\frac{\sec^2\sqrt{x}\sec\left(\tan\sqrt{x}\right)\tan\left(\tan\sqrt{x}\right)}{2\sqrt{x}}$$

Alternate method

$$\frac{d}{dx}\left[\sec\left(\tan\sqrt{x}\right)\right]=\sec\left(\tan\sqrt{x}\right).\tan\left(\tan\sqrt{x}\right)\cdot\frac{d}{dx}\left(\tan\sqrt{x}\right)$$
$$=\sec\left(\tan\sqrt{x}\right)\cdot\tan\left(\tan\sqrt{x}\right)\cdot\sec^2\left(\sqrt{x}\right)\cdot\frac{d}{dx}\left(\sqrt{x}\right)$$
$$=\sec\left(\tan\sqrt{x}\right)\cdot\tan\left(\tan\sqrt{x}\right)\cdot\sec^2\left(\sqrt{x}\right)\cdot\frac{1}{2\sqrt{x}}$$
$$=\frac{\sec\left(\tan\sqrt{x}\right)\cdot\tan\left(\tan\sqrt{x}\right)\sec^2\left(\sqrt{x}\right)}{2\sqrt{x}}$$

Answer needs Correction? **Click Here**

Q5 : Differentiate the functions with respect to *x*.

$$\frac{\sin(ax+b)}{\cos(cx+d)}$$

Answer :

The given function is $f(x)=\frac{\sin(ax+b)}{\cos(cx+d)}=\frac{g(x)}{h(x)}$, where *g* (*x*) = sin (*ax* + *b*) and

h (*x*) = cos (*cx* + *d*)

$$\therefore f'=\frac{g'h-gh'}{h^2}$$

Consider $g(x)=\sin(ax+b)$

Let $u(x)=ax+b, v(t)=\sin t$

Then, $(vou)(x)=v(u(x))=v(ax+b)=\sin(ax+b)=g(x)$

∴ *g* is a composite function of two functions, *u* and *v*.

Put $t=u(x)=ax+b$

$$\frac{dv}{dt}=\frac{d}{dt}(\sin t)=\cos t=\cos(ax+b)$$
$$\frac{dt}{dx}=\frac{d}{dx}(ax+b)=\frac{d}{dx}(ax)+\frac{d}{dx}(b)=a+0=a$$

Therefore, by chain rule, we obtain

$$g'=\frac{dg}{dx}=\frac{dv}{dt}\cdot\frac{dt}{dx}=\cos(ax+b)\cdot a=a\cos(ax+b)$$

Consider $h(x)=\cos(cx+d)$

Let $p(x)=cx+d,\ q(y)=\cos y$

Then, $(qop)(x)=q(p(x))=q(cx+d)=\cos(cx+d)=h(x)$

∴ *h* is a composite function of two functions, *p* and *q*.

Put *y* = *p* (*x*) = *cx* + *d*

$$\frac{dq}{dy}=\frac{d}{dy}(\cos y)=-\sin y=-\sin(cx+d)$$
$$\frac{dy}{dx}=\frac{d}{dx}(cx+d)=\frac{d}{dx}(cx)+\frac{d}{dx}(d)=c$$

Therefore, by chain rule, we obtain

$$h'=\frac{dh}{dx}=\frac{dq}{dy}\cdot\frac{dy}{dx}=-\sin(cx+d)\times c=-c\sin(cx+d)$$

$$\therefore f'=\frac{a\cos(ax+b)\cdot\cos(cx+d)-\sin(ax+b)\{-c\sin(cx+d)\}}{[\cos(cx+d)]^2}$$
$$=\frac{a\cos(ax+b)}{\cos(cx+d)}+c\sin(ax+b)\cdot\frac{\sin(cx+d)}{\cos(cx+d)}\times\frac{1}{\cos(cx+d)}$$
$$=a\cos(ax+b)\sec(cx+d)+c\sin(ax+b)\tan(cx+d)\sec(cx+d)$$

Answer needs Correction? **Click Here**

Q6 : Differentiate the functions with respect to x.

Q6 : Differentiate the functions with respect to *x*.

$\cos x^3.\sin^2\left(x^5\right)$

Answer :

The given function is $\cos x^3.\sin^2\left(x^5\right)$.

$$\frac{d}{dx}\left[\cos x^3\cdot\sin^2\left(x^5\right)\right]=\sin^2\left(x^5\right)\times\frac{d}{dx}\left(\cos x^3\right)+\cos x^3\times\frac{d}{dx}\left[\sin^2\left(x^5\right)\right]$$
$$=\sin^2\left(x^5\right)\times\left(-\sin x^3\right)\times\frac{d}{dx}\left(x^3\right)+\cos x^3\times 2\sin\left(x^5\right)\cdot\frac{d}{dx}\left[\sin x^5\right]$$
$$=-\sin x^3\sin^2\left(x^5\right)\times 3x^2+2\sin x^5\cos x^3\cdot\cos x^5\times\frac{d}{dx}\left(x^5\right)$$
$$=-3x^2\sin x^3\cdot\sin^2\left(x^5\right)+2\sin x^5\cos x^5\cos x^3\cdot\times 5x^4$$
$$=10x^4\sin x^5\cos x^5\cos x^3-3x^2\sin x^3\sin^2\left(x^5\right)$$

Answer needs Correction? Click Here

Q7 : Differentiate the functions with respect to *x*.

$2\sqrt{\cot\left(x^2\right)}$

Answer :

$$\frac{d}{dx}\left[2\sqrt{\cot\left(x^2\right)}\right]$$
$$=2\cdot\frac{1}{2\sqrt{\cot\left(x^2\right)}}\times\frac{d}{dx}\left[\cot\left(x^2\right)\right]$$
$$=\sqrt{\frac{\sin\left(x^2\right)}{\cos\left(x^2\right)}}\times-\operatorname{cosec}^2\left(x^2\right)\times\frac{d}{dx}\left(x^2\right)$$
$$=-\sqrt{\frac{\sin\left(x^2\right)}{\cos\left(x^2\right)}}\times\frac{1}{\sin^2\left(x^2\right)}\times(2x)$$
$$=\frac{-2x}{\sqrt{\cos x^2}\sqrt{\sin x^2}\sin x^2}$$
$$=\frac{-2\sqrt{2}\,x}{\sqrt{2\sin x^2\cos x^2}\sin x^2}$$
$$=\frac{-2\sqrt{2}\,x}{\sin x^2\sqrt{\sin 2x^2}}$$

Answer needs Correction? Click Here

Q8 : Differentiate the functions with respect to *x*.

$\cos\left(\sqrt{x}\right)$

Answer :

Let $f(x)=\cos\left(\sqrt{x}\right)$

Also, let $u(x)=\sqrt{x}$

And, $v(t)=\cos t$

$$\begin{aligned}\text{Then, }(vou)(x)&=v(u(x))\\&=v\left(\sqrt{x}\right)\\&=\cos\sqrt{x}\\&=f(x)\end{aligned}$$

Clearly, *f* is a composite function of two functions, *u* and *v*, such that

$t=u(x)=\sqrt{x}$

$$\text{Then, }\frac{dt}{dx}=\frac{d}{dx}\left(\sqrt{x}\right)=\frac{d}{dx}\left(x^{\frac{1}{2}}\right)=\frac{1}{2}x^{-\frac{1}{2}}$$
$$=\frac{1}{2\sqrt{x}}$$

$$\text{And, }\frac{dv}{dt}=\frac{d}{dt}(\cos t)=-\sin t$$
$$=-\sin\left(\sqrt{x}\right)$$

By using chain rule, we obtain

$$\frac{dt}{dx}=\frac{dv}{dt}\cdot\frac{dt}{dx}$$
$$=-\sin\left(\sqrt{x}\right)\cdot\frac{1}{2\sqrt{x}}$$
$$=-\frac{1}{2\sqrt{x}}\sin\left(\sqrt{x}\right)$$
$$=-\frac{\sin\left(\sqrt{x}\right)}{2\sqrt{x}}$$

Alternate method

$$\frac{d}{dx}\left[\cos\left(\sqrt{x}\right)\right]=-\sin\left(\sqrt{x}\right)\cdot\frac{d}{dx}\left(\sqrt{x}\right)$$
$$=-\sin\left(\sqrt{x}\right)\times\frac{d}{dx}\left(x^{\frac{1}{2}}\right)$$
$$=-\sin\sqrt{x}\times\frac{1}{2}x^{-\frac{1}{2}}$$

$$= \frac{-\sin\sqrt{x}}{2\sqrt{x}}$$

Answer needs Correction? Click Here

Q9 : Prove that the function f given by

$f(x) = |x-1|,\ x \in \mathbf{R}$ is notdifferentiable at $x = 1$.

Answer :

The given function is $f(x) = |x-1|,\ x \in \mathbf{R}$

It is known that a function f is differentiable at a point $x = c$ in its domain if both

$\lim\limits_{h \to 0^-} \frac{f(c+h)-f(c)}{h}$ and $\lim\limits_{h \to 0^+} \frac{f(c+h)-f(c)}{h}$ are finite and equal.

To check the differentiability of the given function at $x = 1$,

consider the left hand limit of f at $x = 1$

$$\lim_{h \to 0^-} \frac{f(1+h)-f(1)}{h} = \lim_{h \to 0^-} \frac{|1+h-1|-|1-1|}{h}$$
$$= \lim_{h \to 0^-} \frac{|h|-0}{h} = \lim_{h \to 0^-} \frac{-h}{h} \qquad (h < 0 \Rightarrow |h| = -h)$$
$$= -1$$

Consider the right hand limit of f at $x = 1$

$$\lim_{h \to 0^+} \frac{f(1+h)-f(1)}{h} = \lim_{h \to 0^+} \frac{|1+h-1|-|1-1|}{h}$$
$$= \lim_{h \to 0^+} \frac{|h|-0}{h} = \lim_{h \to 0^+} \frac{h}{h} \qquad (h > 0 \Rightarrow |h| = h)$$
$$= 1$$

Since the left and right hand limits of f at $x = 1$ are not equal, f is not differentiable at $x = 1$

Answer needs Correction? Click Here

Q10 : Prove that the greatest integer function defined by $f(x) = [x], 0 < x < 3$ is not differentiable at $x = 1$ and $x = 2$.

Answer :

The given function f is $f(x) = [x], 0 < x < 3$

It is known that a function f is differentiable at a point $x = c$ in its domain if both

$\lim\limits_{h \to 0^-} \frac{f(c+h)-f(c)}{h}$ and $\lim\limits_{h \to 0^+} \frac{f(c+h)-f(c)}{h}$ are finite and equal.

To check the differentiability of the given function at $x = 1$, consider the left hand limit of f at $x = 1$

$$\lim_{h \to 0^-} \frac{f(1+h)-f(1)}{h} = \lim_{h \to 0^-} \frac{[1+h]-[1]}{h}$$
$$= \lim_{h \to 0^-} \frac{0-1}{h} = \lim_{h \to 0^-} \frac{-1}{h} = \infty$$

Consider the right hand limit of f at $x = 1$

$$\lim_{h \to 0^+} \frac{f(1+h)-f(1)}{h} = \lim_{h \to 0^+} \frac{[1+h]-[1]}{h}$$
$$= \lim_{h \to 0^+} \frac{1-1}{h} = \lim_{h \to 0^+} 0 = 0$$

Since the left and right hand limits of f at $x = 1$ are not equal, f is not differentiable at $x = 1$

To check the differentiability of the given function at $x = 2$, consider the left hand limit of f at $x = 2$

$$\lim_{h \to 0^-} \frac{f(2+h)-f(2)}{h} = \lim_{h \to 0^-} \frac{[2+h]-[2]}{h}$$
$$= \lim_{h \to 0^-} \frac{1-2}{h} = \lim_{h \to 0^-} \frac{-1}{h} = \infty$$

Consider the right hand limit of f at $x = 1$

$$\lim_{h \to 0^+} \frac{f(2+h)-f(2)}{h} = \lim_{h \to 0^+} \frac{[2+h]-[2]}{h}$$
$$= \lim_{h \to 0^+} \frac{2-2}{h} = \lim_{h \to 0^+} 0 = 0$$

Since the left and right hand limits of f at $x = 2$ are not equal, f is not differentiable at $x = 2$

Answer needs Correction? Click Here

<< Previous Chapter 4 : Determinants

Next Chapter 6 : Application of Derivatives >>

Exercise 5.3 : Solutions of Questions on Page Number : **169**

Q1 : Find $\frac{dy}{dx}$:

$2x + 3y = \sin x$

$2x+3y=\sin x$

Answer :

The given relationship is $2x+3y=\sin x$

Differentiating this relationship with respect to *x*, we obtain

$$\frac{d}{dx}(2x+3y)=\frac{d}{dx}(\sin x)$$
$$\Rightarrow \frac{d}{dx}(2x)+\frac{d}{dx}(3y)=\cos x$$
$$\Rightarrow 2+3\frac{dy}{dx}=\cos x$$
$$\Rightarrow 3\frac{dy}{dx}=\cos x-2$$
$$\therefore \frac{dy}{dx}=\frac{\cos x-2}{3}$$

Answer needs Correction? **Click Here**

Q2 : Find $\frac{dy}{dx}$:

$2x+3y=\sin y$

Answer :

The given relationship is $2x+3y=\sin y$

Differentiating this relationship with respect to *x*, we obtain

$$\frac{d}{dx}(2x)+\frac{d}{dx}(3y)=\frac{d}{dx}(\sin y)$$
$$\Rightarrow 2+3\frac{dy}{dx}=\cos y\frac{dy}{dx} \qquad \text{[By using chain rule]}$$
$$\Rightarrow 2=(\cos y-3)\frac{dy}{dx}$$
$$\therefore \frac{dy}{dx}=\frac{2}{\cos y-3}$$

Answer needs Correction? **Click Here**

Q3 : Find $\frac{dy}{dx}$:

$ax+by^2=\cos y$

Answer :

The given relationship is $ax+by^2=\cos y$

Differentiating this relationship with respect to *x*, we obtain

$$\frac{d}{dx}(ax)+\frac{d}{dx}(by^2)=\frac{d}{dx}(\cos y)$$
$$\Rightarrow a+b\frac{d}{dx}(y^2)=\frac{d}{dx}(\cos y) \qquad ...(1)$$

Using chain rule, we obtain $\frac{d}{dx}(y^2)=2y\frac{dy}{dx}$ and $\frac{d}{dx}(\cos y)=-\sin y\frac{dy}{dx}$ $\quad ...(2)$

From (1) and (2), we obtain

$$a+b\times 2y\frac{dy}{dx}=-\sin y\frac{dy}{dx}$$
$$\Rightarrow (2by+\sin y)\frac{dy}{dx}=-a$$
$$\therefore \frac{dy}{dx}=\frac{-a}{2by+\sin y}$$

Answer needs Correction? **Click Here**

Q4 : Find $\frac{dy}{dx}$:

$xy+y^2=\tan x+y$

Answer :

The given relationship is $xy+y^2=\tan x+y$

Differentiating this relationship with respect to *x*, we obtain

$$\frac{d}{dx}(xy+y^2)=\frac{d}{dx}(\tan x+y)$$
$$\Rightarrow \frac{d}{dx}(xy)+\frac{d}{dx}(y^2)=\frac{d}{dx}(\tan x)+\frac{dy}{dx}$$
$$\Rightarrow \left[y\cdot\frac{d}{dx}(x)+x.\frac{dy}{dx}\right]+2y\frac{dy}{dx}=\sec^2 x+\frac{dy}{dx} \qquad \text{[Using product rule and chain rule]}$$
$$\Rightarrow y\cdot 1+x.\frac{dy}{dx}+2y\frac{dy}{dx}=\sec^2 x+\frac{dy}{dx}$$
$$\Rightarrow (x+2y-1)\frac{dy}{dx}=\sec^2 x-y$$
$$\therefore \frac{dy}{dx}=\frac{\sec^2 x-y}{(x+2y-1)}$$

Answer needs Correction? **Click Here**

Q5 : Find $\frac{dy}{dx}$:

$x^2+xy+y^2=100$

Answer :

The given relationship is $x^2+xy+y^2=100$

Differentiating this relationship with respect to *x*, we obtain

$$\frac{d}{dx}\left(x^2+xy+y^2\right)=\frac{d}{dx}(100)$$
$$\Rightarrow \frac{d}{dx}\left(x^2\right)+\frac{d}{dx}(xy)+\frac{d}{dx}\left(y^2\right)=0 \text{ [Derivative of constant function is 0]}$$
$$\Rightarrow 2x+\left[y\cdot\frac{d}{dx}(x)+x\cdot\frac{dy}{dx}\right]+2y\frac{dy}{dx}=0 \qquad [\text{Using product rule and chain rule}]$$
$$\Rightarrow 2x+y\cdot 1+x\cdot\frac{dy}{dx}+2y\frac{dy}{dx}=0$$
$$\Rightarrow 2x+y+(x+2y)\frac{dy}{dx}=0$$
$$\therefore \frac{dy}{dx}=-\frac{2x+y}{x+2y}$$

Answer needs Correction? **Click Here**

Q6 : Find $\frac{dy}{dx}$:

$x^3+x^2y+xy^2+y^3=81$

Answer :

The given relationship is $x^3+x^2y+xy^2+y^3=81$

Differentiating this relationship with respect to *x*, we obtain

$$\frac{d}{dx}\left(x^3+x^2y+xy^2+y^3\right)=\frac{d}{dx}(81)$$
$$\Rightarrow \frac{d}{dx}\left(x^3\right)+\frac{d}{dx}\left(x^2y\right)+\frac{d}{dx}\left(xy^2\right)+\frac{d}{dx}\left(y^3\right)=0$$
$$\Rightarrow 3x^2+\left[y\frac{d}{dx}\left(x^2\right)+x^2\frac{dy}{dx}\right]+\left[y^2\frac{d}{dx}(x)+x\frac{d}{dx}\left(y^2\right)\right]+3y^2\frac{dy}{dx}=0$$
$$\Rightarrow 3x^2+\left[y\cdot 2x+x^2\frac{dy}{dx}\right]+\left[y^2\cdot 1+x\cdot 2y\cdot\frac{dy}{dx}\right]+3y^2\frac{dy}{dx}=0$$
$$\Rightarrow \left(x^2+2xy+3y^2\right)\frac{dy}{dx}+\left(3x^2+2xy+y^2\right)=0$$
$$\therefore \frac{dy}{dx}=\frac{-\left(3x^2+2xy+y^2\right)}{\left(x^2+2xy+3y^2\right)}$$

Answer needs Correction? **Click Here**

Q7 : Find $\frac{dy}{dx}$:

$\sin^2 y+\cos xy=\pi$

Answer :

The given relationship is $\sin^2 y+\cos xy=\pi$

Differentiating this relationship with respect to *x*, we obtain

$$\frac{d}{dx}\left(\sin^2 y+\cos xy\right)=\frac{d}{dx}(\pi)$$
$$\Rightarrow \frac{d}{dx}\left(\sin^2 y\right)+\frac{d}{dx}(\cos xy)=0 \qquad \ldots(1)$$

Using chain rule, we obtain

$$\frac{d}{dx}\left(\sin^2 y\right)=2\sin y\frac{d}{dx}(\sin y)=2\sin y\cos y\frac{dy}{dx} \qquad \ldots(2)$$
$$\frac{d}{dx}(\cos xy)=-\sin xy\frac{d}{dx}(xy)=-\sin xy\left[y\frac{d}{dx}(x)+x\frac{dy}{dx}\right]$$
$$=-\sin xy\left[y.1+x\frac{dy}{dx}\right]=-y\sin xy-x\sin xy\frac{dy}{dx} \qquad \ldots(3)$$

From (1), (2), and (3), we obtain

$$2\sin y\cos y\frac{dy}{dx}-y\sin xy-x\sin xy\frac{dy}{dx}=0$$
$$\Rightarrow (2\sin y\cos y-x\sin xy)\frac{dy}{dx}=y\sin xy$$
$$\Rightarrow (\sin 2y-x\sin xy)\frac{dy}{dx}=y\sin xy$$
$$\therefore \frac{dy}{dx}=\frac{y\sin xy}{\sin 2y-x\sin xy}$$

Answer needs Correction? **Click Here**

Q8 : Find $\frac{dy}{dx}$:

$\sin^2 x+\cos^2 y=1$

$\sin^2 x + \cos^2 y = 1$

Answer :

The given relationship is $\sin^2 x + \cos^2 y = 1$

Differentiating this relationship with respect to *x*, we obtain

$$\frac{d}{dx}\left(\sin^2 x + \cos^2 y\right) = \frac{d}{dx}(1)$$
$$\Rightarrow \frac{d}{dx}\left(\sin^2 x\right) + \frac{d}{dx}\left(\cos^2 y\right) = 0$$
$$\Rightarrow 2\sin x \cdot \frac{d}{dx}(\sin x) + 2\cos y \cdot \frac{d}{dx}(\cos y) = 0$$
$$\Rightarrow 2\sin x \cos x + 2\cos y(-\sin y) \cdot \frac{dy}{dx} = 0$$
$$\Rightarrow \sin 2x - \sin 2y \frac{dy}{dx} = 0$$
$$\therefore \frac{dy}{dx} = \frac{\sin 2x}{\sin 2y}$$

Answer needs Correction? **Click Here**

Q9 : Find $\frac{dy}{dx}$:

$$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

Answer :

The given relationship is $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$
$$\Rightarrow \sin y = \frac{2x}{1+x^2}$$

Differentiating this relationship with respect to *x*, we obtain

$$\frac{d}{dx}(\sin y) = \frac{d}{dx}\left(\frac{2x}{1+x^2}\right)$$
$$\Rightarrow \cos y \frac{dy}{dx} = \frac{d}{dx}\left(\frac{2x}{1+x^2}\right) \qquad ...(1)$$

The function, $\frac{2x}{1+x^2}$, is of the form of $\frac{u}{v}$.

Therefore, by quotient rule, we obtain

$$\frac{d}{dx}\left(\frac{2x}{1+x^2}\right) = \frac{\left(1+x^2\right).\frac{d}{dx}(2x) - 2x \cdot \frac{d}{dx}\left(1+x^2\right)}{\left(1+x^2\right)^2}$$
$$= \frac{\left(1+x^2\right).2 - 2x \cdot [0+2x]}{\left(1+x^2\right)^2} = \frac{2+2x^2-4x^2}{\left(1+x^2\right)^2} = \frac{2\left(1-x^2\right)}{\left(1+x^2\right)^2} \qquad ...(2)$$

Also, $\sin y = \frac{2x}{1+x^2}$

$$\Rightarrow \cos y = \sqrt{1-\sin^2 y} = \sqrt{1-\left(\frac{2x}{1+x^2}\right)^2} = \sqrt{\frac{\left(1+x^2\right)^2 - 4x^2}{\left(1+x^2\right)^2}}$$
$$= \sqrt{\frac{\left(1-x^2\right)^2}{\left(1+x^2\right)^2}} = \frac{1-x^2}{1+x^2} \qquad ...(3)$$

From (1), (2), and (3), we obtain

$$\frac{1-x^2}{1+x^2} \times \frac{dy}{dx} = \frac{2\left(1-x^2\right)}{\left(1+x^2\right)^2}$$
$$\Rightarrow \frac{dy}{dx} = \frac{2}{1+x^2}$$

Answer needs Correction? **Click Here**

Q10 : Find $\frac{dy}{dx}$:

$$y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$$

Answer :

The given relationship is $y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

$$y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$$
$$\Rightarrow \tan y = \frac{3x-x^3}{1-3x^2} \qquad ...(1)$$

It is known that, $\tan y = \frac{3\tan\frac{y}{3} - \tan^3\frac{y}{3}}{1-3\tan^2\frac{y}{3}} \qquad ...(2)$

Comparing equations (1) and (2), we obtain

Comparing equations (1) and (2), we obtain

$$x = \tan\frac{y}{3}$$

Differentiating this relationship with respect to *x*, we obtain

$$\frac{d}{dx}(x) = \frac{d}{dx}\left(\tan\frac{y}{3}\right)$$
$$\Rightarrow 1 = \sec^2\frac{y}{3}\cdot\frac{d}{dx}\left(\frac{y}{3}\right)$$
$$\Rightarrow 1 = \sec^2\frac{y}{3}\cdot\frac{1}{3}\cdot\frac{dy}{dx}$$
$$\Rightarrow \frac{dy}{dx} = \frac{3}{\sec^2\frac{y}{3}} = \frac{3}{1+\tan^2\frac{y}{3}}$$
$$\therefore \frac{dy}{dx} = \frac{3}{1+x^2}$$

Answer needs Correction? **Click Here**

Q11 : Find $\frac{dy}{dx}$:

$$y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$$

Answer :

The given relationship is,

$$y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$
$$\Rightarrow \cos y = \frac{1-x^2}{1+x^2}$$
$$\Rightarrow \frac{1-\tan^2\frac{y}{2}}{1+\tan^2\frac{y}{2}} = \frac{1-x^2}{1+x^2}$$

On comparing L.H.S. and R.H.S. of the above relationship, we obtain

$$\tan\frac{y}{2} = x$$

Differentiating this relationship with respect to *x*, we obtain

Answer needs Correction? **Click Here**

Q12 : Find $\frac{dy}{dx}$:

$$y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right),\ 0 < x < 1$$

Answer :

The given relationship is $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$$y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$
$$\Rightarrow \sin y = \frac{1-x^2}{1+x^2}$$

Differentiating this relationship with respect to *x*, we obtain

$$\frac{d}{dx}(\sin y) = \frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right) \qquad \text{...(1)}$$

Using chain rule, we obtain

$$\frac{d}{dx}(\sin y) = \cos y\cdot\frac{dy}{dx}$$
$$\cos y = \sqrt{1-\sin^2 y} = \sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}$$
$$= \sqrt{\frac{(1+x^2)^2-(1-x^2)^2}{(1+x^2)^2}} = \sqrt{\frac{4x^2}{(1+x^2)^2}} = \frac{2x}{1+x^2}$$
$$\therefore \frac{d}{dx}(\sin y) = \frac{2x}{1+x^2}\frac{dy}{dx} \qquad \text{...(2)}$$

$$\frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right) = \frac{(1+x^2)\cdot(1-x^2)' - (1-x^2)\cdot(1+x^2)'}{(1+x^2)^2} \qquad [\text{Using quotient rule}]$$

$$= \frac{(1+x^2)(-2x)-(1-x^2)\cdot(2x)}{(1+x^2)^2}$$
$$= \frac{-2x-2x^3-2x+2x^3}{(1+x^2)^2}$$
$$= \frac{-4x}{(1+x^2)^2} \qquad \text{...(3)}$$

From (1), (2), and (3), we obtain

$$\frac{2x}{1+x^2}\frac{dy}{dx}=\frac{-4x}{\left(1+x^2\right)^2}$$
$$\Rightarrow\frac{dy}{dx}=\frac{-2}{1+x^2}$$

Alternate method

$$y=\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$\Rightarrow\sin y=\frac{1-x^2}{1+x^2}$$

$$\Rightarrow\left(1+x^2\right)\sin y=1-x^2$$
$$\Rightarrow\left(1+\sin y\right)x^2=1-\sin y$$
$$\Rightarrow x^2=\frac{1-\sin y}{1+\sin y}$$

$$\Rightarrow x^2=\frac{\left(\cos\frac{y}{2}-\sin\frac{y}{2}\right)^2}{\left(\cos\frac{y}{2}+\sin\frac{y}{2}\right)^2}$$

$$\Rightarrow x=\frac{\cos\frac{y}{2}-\sin\frac{y}{2}}{\cos\frac{y}{2}+\sin\frac{y}{2}}$$

$$\Rightarrow x=\frac{1-\tan\frac{y}{2}}{1+\tan\frac{y}{2}}$$

$$\Rightarrow x=\tan\left(\frac{\pi}{4}-\frac{y}{2}\right)$$

Differentiating this relationship with respect to *x*, we obtain

$$\frac{d}{dx}(x)=\frac{d}{dx}\cdot\left[\tan\left(\frac{\pi}{4}-\frac{y}{2}\right)\right]$$
$$\Rightarrow 1=\sec^2\left(\frac{\pi}{4}-\frac{y}{2}\right)\cdot\frac{d}{dx}\left(\frac{\pi}{4}-\frac{y}{2}\right)$$
$$\Rightarrow 1=\left[1+\tan^2\left(\frac{\pi}{4}-\frac{y}{2}\right)\right]\cdot\left(-\frac{1}{2}\frac{dy}{dx}\right)$$
$$\Rightarrow 1=\left(1+x^2\right)\left(-\frac{1}{2}\frac{dy}{dx}\right)$$
$$\Rightarrow\frac{dy}{dx}=\frac{-2}{1+x^2}$$

Answer needs Correction? Click Here

Q13 : Find $\frac{dy}{dx}$:

$$y=\cos^{-1}\left(\frac{2x}{1+x^2}\right),-1<x<1$$

Answer :

The given relationship is $y=\cos^{-1}\left(\frac{2x}{1+x^2}\right)$

$$y=\cos^{-1}\left(\frac{2x}{1+x^2}\right)$$
$$\Rightarrow\cos y=\frac{2x}{1+x^2}$$

Differentiating this relationship with respect to *x*, we obtain

$$\frac{d}{dx}(\cos y)=\frac{d}{dx}\cdot\left(\frac{2x}{1+x^2}\right)$$
$$\Rightarrow-\sin y\cdot\frac{dy}{dx}=\frac{\left(1+x^2\right)\cdot\frac{d}{dx}(2x)-2x\cdot\frac{d}{dx}\left(1+x^2\right)}{\left(1+x^2\right)^2}$$

$$\Rightarrow-\sqrt{1-\cos^2 y}\,\frac{dy}{dx}=\frac{\left(1+x^2\right)\times 2-2x\cdot 2x}{\left(1+x^2\right)^2}$$
$$\Rightarrow\left[\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}\right]\frac{dy}{dx}=-\left[\frac{2\left(1-x^2\right)}{\left(1+x^2\right)^2}\right]$$
$$\Rightarrow\sqrt{\frac{\left(1+x^2\right)^2-4x^2}{\left(1+x^2\right)^2}}\,\frac{dy}{dx}=\frac{-2\left(1-x^2\right)}{\left(1+x^2\right)^2}$$
$$\Rightarrow\sqrt{\frac{\left(1-x^2\right)^2}{\left(1+x^2\right)^2}}\,\frac{dy}{dx}=\frac{-2\left(1-x^2\right)}{\left(1+x^2\right)^2}$$
$$\Rightarrow\frac{1-x^2}{1+x^2}\cdot\frac{dy}{dx}=\frac{-2\left(1-x^2\right)}{\left(1+x^2\right)^2}$$
$$\Rightarrow\frac{dy}{dx}=\frac{-2}{1+x^2}$$

Answer needs Correction? Click Here

Q14 : Find $\frac{dy}{dx}$:

$y=\sin^{-1}\left(2x\sqrt{1-x^2}\right),\ -\frac{1}{\sqrt{2}}<x<\frac{1}{\sqrt{2}}$

Answer :

The given relationship is $y=\sin^{-1}\left(2x\sqrt{1-x^2}\right)$

$$y=\sin^{-1}\left(2x\sqrt{1-x^2}\right)$$
$$\Rightarrow \sin y=2x\sqrt{1-x^2}$$

Differentiating this relationship with respect to *x*, we obtain

$$\cos y\frac{dy}{dx}=2\left[x\frac{d}{dx}\left(\sqrt{1-x^2}\right)+\sqrt{1-x^2}\frac{dx}{dx}\right]$$
$$\Rightarrow \sqrt{1-\sin^2 y}\frac{dy}{dx}=2\left[\frac{x}{2}\cdot\frac{-2x}{\sqrt{1-x^2}}+\sqrt{1-x^2}\right]$$
$$\Rightarrow \sqrt{1-\left(2x\sqrt{1-x^2}\right)^2}\frac{dy}{dx}=2\left[\frac{-x^2+1-x^2}{\sqrt{1-x^2}}\right]$$
$$\Rightarrow \sqrt{1-4x^2\left(1-x^2\right)}\frac{dy}{dx}=2\left[\frac{1-2x^2}{\sqrt{1-x^2}}\right]$$
$$\Rightarrow \sqrt{\left(1-2x^2\right)^2}\frac{dy}{dx}=2\left[\frac{1-2x^2}{\sqrt{1-x^2}}\right]$$
$$\Rightarrow \left(1-2x^2\right)\frac{dy}{dx}=2\left[\frac{1-2x^2}{\sqrt{1-x^2}}\right]$$
$$\Rightarrow \frac{dy}{dx}=\frac{2}{\sqrt{1-x^2}}$$

Answer needs Correction? **Click Here**

Q15 : Find $\frac{dy}{dx}$:

$y=\sec^{-1}\left(\frac{1}{2x^2-1}\right),0<x<\frac{1}{\sqrt{2}}$

Answer :

The given relationship is $y=\sec^{-1}\left(\frac{1}{2x^2-1}\right)$

$$y=\sec^{-1}\left(\frac{1}{2x^2-1}\right)$$
$$\Rightarrow \sec y=\frac{1}{2x^2-1}$$
$$\Rightarrow \cos y=2x^2-1$$
$$\Rightarrow 2x^2=1+\cos y$$
$$\Rightarrow 2x^2=2\cos^2\frac{y}{2}$$
$$\Rightarrow x=\cos\frac{y}{2}$$

Differentiating this relationship with respect to *x*, we obtain

$$\frac{d}{dx}(x)=\frac{d}{dx}\left(\cos\frac{y}{2}\right)$$
$$\Rightarrow 1=-\sin\frac{y}{2}\cdot\frac{d}{dx}\left(\frac{y}{2}\right)$$
$$\Rightarrow \frac{-1}{\sin\frac{y}{2}}=\frac{1}{2}\frac{dy}{dx}$$
$$\Rightarrow \frac{dy}{dx}=\frac{-2}{\sin\frac{y}{2}}=\frac{-2}{\sqrt{1-\cos^2\frac{y}{2}}}$$
$$\Rightarrow \frac{dy}{dx}=\frac{-2}{\sqrt{1-x^2}}$$

Answer needs Correction? **Click Here**

*********** END **********