Question 4.1:

A circular coil of wire consisting of 100 turns, each of radius 8.0 cm carries a current of 0.40 A. What is the magnitude of the magnetic field **B** at the centre of the coil?

Answer

Number of turns on the circular coil, n = 100

Radius of each turn, r = 8.0 cm = 0.08 m

Current flowing in the coil, I = 0.4 A

Magnitude of the magnetic field at the centre of the coil is given by the relation,

$$|\mathbf{B}| = \frac{\mu_0}{4\pi}\frac{2\pi nI}{r}$$

Where,

μ_0 = Permeability of free space

= $4\pi \times 10^{-7}$ T m A^{-1}

$$|\mathbf{B}| = \frac{4\pi\times10^{-7}}{4\pi}\times\frac{2\pi\times100\times0.4}{0.08}$$

$$= 3.14 \times 10^{-4}\ \text{T}$$

Hence, the magnitude of the magnetic field is 3.14×10^{-4} T.

Question 4.2:

A long straight wire carries a current of 35 A. What is the magnitude of the field **B** at a point 20 cm from the wire?

Answer

Current in the wire, I = 35 A

Distance of a point from the wire, r = 20 cm = 0.2 m

Magnitude of the magnetic field at this point is given as:

$$B = \frac{\mu_0}{4\pi}\frac{2I}{r}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}$ T m A^{-1}

$$B = \frac{4\pi\times10^{-7}\times2\times35}{4\pi\times0.2}$$

$$= 3.5 \times 10^{-5}\ \text{T}$$

Hence, the magnitude of the magnetic field at a point 20 cm from the wire is 3.5×10^{-5} T.

Question 4.3:

A long straight wire in the horizontal plane carries a current of 50 A in north to south direction. Give the magnitude and direction of **B** at a point 2.5 m east of the wire.

Answer

Current in the wire, I = 50 A

A point is 2.5 m away from the East of the wire.

∴ Magnitude of the distance of the point from the wire, r = 2.5 m.

Magnitude of the magnetic field at that point is given by the relation, $B = \frac{\mu_0 2I}{4\pi r}$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}$ T m A^{-1}

$$B = \frac{4\pi\times10^{-7}\times2\times50}{4\pi\times2.5}$$

$$= 4 \times 10^{-6}\ \text{T}$$

The point is located normal to the wire length at a distance of 2.5 m. The direction of the current in the wire is vertically downward. Hence, according to the Maxwell's right hand thumb rule, the direction of the magnetic field at the given point is vertically upward.

Question 4.4:

A horizontal overhead power line carries a current of 90 A in east to west direction. What is the magnitude and direction of the magnetic field due to the current 1.5 m below the line?

Answer

Current in the power line, I = 90 A

Point is located below the power line at distance, r = 1.5 m

Point is located below the power line at distance, $r = 1.5$ m

Hence, magnetic field at that point is given by the relation,

$$B = \frac{\mu_0 2I}{4\pi r}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}$ T m A^{-1}

$$B = \frac{4\pi \times 10^{-7} \times 2 \times 90}{4\pi \times 1.5} = 1.2 \times 10^{-5} \text{ T}$$

The current is flowing from East to West. The point is below the power line. Hence, according to Maxwell's right hand thumb rule, the direction of the magnetic field is towards the South.

Question 4.5:

What is the magnitude of magnetic force per unit length on a wire carrying a current of 8 A and making an angle of 30º with the direction of a uniform magnetic field of 0.15 T?

Answer

Current in the wire, I = 8 A

Magnitude of the uniform magnetic field, B = 0.15 T

Angle between the wire and magnetic field, θ = 30°.

Magnetic force per unit length on the wire is given as:

$f = BI \sin\theta$

$= 0.15 \times 8 \times 1 \times \sin 30°$

$= 0.6$ N m^{-1}

Hence, the magnetic force per unit length on the wire is 0.6 N m^{-1}.

Question 4.6:

A 3.0 cm wire carrying a current of 10 A is placed inside a solenoid perpendicular to its axis. The magnetic field inside the solenoid is given to be 0.27 T. What is the magnetic force on the wire?

Answer

Length of the wire, l = 3 cm = 0.03 m

Current flowing in the wire, I = 10 A

Magnetic field, B = 0.27 T

Angle between the current and magnetic field, θ = 90°

Magnetic force exerted on the wire is given as:

$F = BIl\sin\theta$

$= 0.27 \times 10 \times 0.03 \sin 90°$

$= 8.1 \times 10^{-2}$ N

Hence, the magnetic force on the wire is 8.1×10^{-2} N. The direction of the force can be obtained from Fleming's left hand rule.

Question 4.7:

Two long and parallel straight wires A and B carrying currents of 8.0 A and 5.0 A in the same direction are separated by a distance of 4.0 cm. Estimate the force on a 10 cm section of wire A.

Answer

Current flowing in wire A, I_A = 8.0 A

Current flowing in wire B, I_B = 5.0 A

Distance between the two wires, r = 4.0 cm = 0.04 m

Length of a section of wire A, l = 10 cm = 0.1 m

Force exerted on length l due to the magnetic field is given as:

$$B = \frac{\mu_0 2\, I_A I_B l}{4\pi r}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}$ T m A^{-1}

$$B = \frac{4\pi \times 10^{-7} \times 2 \times 8 \times 5 \times 0.1}{4\pi \times 0.04}$$

$$= 2 \times 10^{-5} \text{ N}$$

The magnitude of force is 2×10^{-5} N. This is an attractive force normal to A towards B because the direction of the currents in the wires is the same.

Question 4.8:

A closely wound solenoid 80 cm long has 5 layers of windings of 400 turns each. The diameter of the solenoid is 1.8 cm. If the current carried is 8.0 A, estimate the magnitude of **B** inside the solenoid near its centre.

Answer

Length of the solenoid, l = 80 cm = 0.8 m

There are five layers of windings of 400 turns each on the solenoid.

∴ Total number of turns on the solenoid, $N = 5 \times 400 = 2000$

Diameter of the solenoid, D = 1.8 cm = 0.018 m

Current carried by the solenoid, I = 8.0 A

Magnitude of the magnetic field inside the solenoid near its centre is given by the relation,

$$B = \frac{\mu_0 NI}{l}$$

l

Where,

μ_0 = Permeability of free space = 4π × 10^{-7} T m A^{-1}

$$B = \frac{4\pi \times 10^{-7} \times 2000 \times 8}{0.8}$$

$$= 8\pi \times 10^{-3} = 2.512 \times 10^{-2} \text{ T}$$

Hence, the magnitude of the magnetic field inside the solenoid near its centre is 2.512 × 10^{-2} T.

Question 4.9:

A square coil of side 10 cm consists of 20 turns and carries a current of 12 A. The coil is suspended vertically and the normal to the plane of the coil makes an angle of 30º with the direction of a uniform horizontal magnetic field of magnitude 0.80 T. What is the magnitude of torque experienced by the coil?

Answer

Length of a side of the square coil, l = 10 cm = 0.1 m

Current flowing in the coil, I = 12 A

Number of turns on the coil, n = 20

Angle made by the plane of the coil with magnetic field, θ = 30°

Strength of magnetic field, B = 0.80 T

Magnitude of the magnetic torque experienced by the coil in the magnetic field is given by the relation,

$\tau = n\,BIA \sin\theta$

Where,

A = Area of the square coil

$\Rightarrow l \times l = 0.1 \times 0.1 = 0.01\ \text{m}^2$

$\therefore \tau = 20 \times 0.8 \times 12 \times 0.01 \times \sin 30°$

= 0.96 N m

Hence, the magnitude of the torque experienced by the coil is 0.96 N m.

Question 4.10:

Two moving coil meters, M_1 and M_2 have the following particulars:

R_1 = 10 Ω, N_1 = 30,

A_1 = 3.6 × 10^{-3} m^2, B_1 = 0.25 T

R_2 = 14 Ω, N_2 = 42,

A_2 = 1.8 × 10^{-3} m^2, B_2 = 0.50 T

(The spring constants are identical for the two meters).

Determine the ratio of (a) current sensitivity and (b) voltage sensitivity of M_2 and M_1.

Answer

For moving coil meter M_1:

Resistance, R_1 = 10 Ω

Number of turns, N_1 = 30

Area of cross-section, A_1 = 3.6 × 10^{-3} m^2

Magnetic field strength, B_1 = 0.25 T

Spring constant $K_1 = K$

For moving coil meter M_2:

Resistance, R_2 = 14 Ω

Number of turns, N_2 = 42

Area of cross-section, A_2 = 1.8 × 10^{-3} m^2

Magnetic field strength, B_2 = 0.50 T

Spring constant, $K_2 = K$

(a) Current sensitivity of M_1 is given as:

$$I_{s1} = \frac{N_1\ B_1\ A_1}{K_1}$$

And, current sensitivity of M_2 is given as:

$$I_{s2} = \frac{N_2\ B_2\ A_2}{K_2}$$

$\therefore$ Ratio $\dfrac{I_{s2}}{I_{s1}} = \dfrac{N_2\ B_2\ A_2\ K_1}{K_2\ N_1\ B_1\ A_1}$

$$= \frac{42 \times 0.5 \times 1.8 \times 10^{-3} \times K}{K \times 30 \times 0.25 \times 3.6 \times 10^{-3}} = 1.4$$

Hence, the ratio of current sensitivity of M_2 to M_1 is 1.4.

(b) Voltage sensitivity for M_2 is given as:

$$V_{s2} = \frac{N_2\ B_2\ A_2}{K_2\ R_2}$$

And, voltage sensitivity for M_1 is given as:

$$V_{s1} = \frac{N_1\ B_1\ A_1}{K_1}$$

$\therefore$ Ratio $\dfrac{V_{s2}}{V_{s1}} = \dfrac{N_2\ B_2\ A_2\ K_1\ R_1}{K_2\ R_2\ N_1\ B_1\ A_1}$

∴ Ratio

$$= \frac{42\times0.5\times1.8\times10^{-3}\times10\times K}{K\times14\times30\times0.25\times3.6\times10^{-3}} = 1$$

Hence, the ratio of voltage sensitivity of M_2 to M_1 is 1.

Question 4.11:

In a chamber, a uniform magnetic field of 6.5 G (1 G = 10^{-4} T) is maintained. An electron is shot into the field with a speed of 4.8×10^6 m s^{-1} normal to the field. Explain why the path of the electron is a circle. Determine the radius of the circular orbit. (e = 1.6×10^{-19} C, m_e= 9.1×10^{-31} kg)

Answer

Magnetic field strength, B = 6.5 G = 6.5×10^{-4} T

Speed of the electron, v = 4.8×10^6 m/s

Charge on the electron, e = 1.6×10^{-19} C

Mass of the electron, m_e = 9.1×10^{-31} kg

Angle between the shot electron and magnetic field, θ = 90°

Magnetic force exerted on the electron in the magnetic field is given as:

$F = evB \sin\theta$

This force provides centripetal force to the moving electron. Hence, the electron starts moving in a circular path of radius r.

Hence, centripetal force exerted on the electron,

$$F_c = \frac{mv^2}{r}$$

In equilibrium, the centripetal force exerted on the electron is equal to the magnetic force i.e.,

$$F_c = F$$

$$\frac{mv^2}{r} = evB\sin\theta$$

$$r = \frac{mv}{Be\sin\theta}$$

$$= \frac{9.1\times10^{-31}\times4.8\times10^{6}}{6.5\times10^{-4}\times1.6\times10^{-19}\times\sin 90°}$$

$$= 4.2\times10^{-2}\text{ m } = 4.2\text{ cm}$$

Hence, the radius of the circular orbit of the electron is 4.2 cm.

Question 4.12:

In Exercise 4.11 obtain the frequency of revolution of the electron in its circular orbit. Does the answer depend on the speed of the electron? Explain.

Answer

Magnetic field strength, B = 6.5×10^{-4} T

Charge of the electron, e = 1.6×10^{-19} C

Mass of the electron, m_e = 9.1×10^{-31} kg

Velocity of the electron, v = 4.8×10^6 m/s

Radius of the orbit, r = 4.2 cm = 0.042 m

Frequency of revolution of the electron = ν

Angular frequency of the electron = ω = $2\pi\nu$

Velocity of the electron is related to the angular frequency as:

$v = r\omega$

In the circular orbit, the magnetic force on the electron is balanced by the centripetal force. Hence, we can write:

$$evB = \frac{mv^2}{r}$$

$$eB = \frac{m}{r}(r\omega) = \frac{m}{r}(r2\pi\nu)$$

$$\nu = \frac{Be}{2\pi m}$$

This expression for frequency is independent of the speed of the electron.

On substituting the known values in this expression, we get the frequency as:

$$\nu = \frac{6.5\times10^{-4}\times1.6\times10^{-19}}{2\times3.14\times9.1\times10^{-31}}$$

$$= 18.2 \times 10^6\text{ Hz}$$

$$\approx 18\text{ MHz}$$

Hence, the frequency of the electron is around 18 MHz and is independent of the speed of the electron.

Question 4.13:

(a) A circular coil of 30 turns and radius 8.0 cm carrying a current of 6.0 A is suspended vertically in a uniform horizontal magnetic field of magnitude 1.0 T. The field lines make an angle of 60º with the normal of the coil. Calculate the magnitude of the counter torque that must be applied to prevent the coil from turning.

(b) Would your answer change, if the circular coil in (a) were replaced by a planar coil of some irregular shape that encloses the same area? (All other particulars are also unaltered.)

Answer

(a) Number of turns on the circular coil, n = 30

Radius of the coil, r = 8.0 cm = 0.08 m

Radius of the coil, r = 8.0 cm = 0.08 m

Area of the coil $= \pi r^2 = \pi(0.08)^2 = 0.0201\ \text{m}^2$

Current flowing in the coil, I = 6.0 A

Magnetic field strength, B = 1 T

Angle between the field lines and normal with the coil surface,

$\theta = 60°$

The coil experiences a torque in the magnetic field. Hence, it turns. The counter torque applied to prevent the coil from turning is given by the relation,

$\tau = n\, IBA \sin\theta$ ... (*i*)

$= 30 \times 6 \times 1 \times 0.0201 \times \sin 60°$

= 3.133 N m

(b) It can be inferred from relation (*i*) that the magnitude of the applied torque is not dependent on the shape of the coil. It depends on the area of the coil. Hence, the answer would not change if the circular coil in the above case is replaced by a planar coil of some irregular shape that encloses the same area.

Question 4.14:

Two concentric circular coils X and Y of radii 16 cm and 10 cm, respectively, lie in the same vertical plane containing the north to south direction. Coil X has 20 turns and carries a current of 16 A; coil Y has 25 turns and carries a current of 18 A. The sense of the current in X is anticlockwise, and clockwise in Y, for an observer looking at the coils facing west. Give the magnitude and direction of the net magnetic field due to the coils at their centre.

Answer

Radius of coil X, r_1 = 16 cm = 0.16 m

Radius of coil Y, r_2 = 10 cm = 0.1 m

Number of turns of on coil X, n_1 = 20

Number of turns of on coil Y, n_2 = 25

Current in coil X, I_1 = 16 A

Current in coil Y, I_2 = 18 A

Magnetic field due to coil X at their centre is given by the relation,

$$B_1 = \frac{\mu_0 n_1 I_1}{2r_1}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}\ \text{T m A}^{-1}$

$$\therefore B_1 = \frac{4\pi \times 10^{-7} \times 20 \times 16}{2 \times 0.16}$$
$$= 4\pi \times 10^{-4}\ \text{T (towards East)}$$

Magnetic field due to coil Y at their centre is given by the relation,

$$B_2 = \frac{\mu_0 n_2 I_2}{2r_2}$$
$$= \frac{4\pi \times 10^{-7} \times 25 \times 18}{2 \times 0.10}$$
$$= 9\pi \times 10^{-4}\ \text{T (towards West)}$$

Hence, net magnetic field can be obtained as:

$$B = B_2 - B_1$$
$$= 9\pi \times 10^{-4} - 4\pi \times 10^{-4}$$
$$= 5\pi \times 10^{-4}\ \text{T}$$
$$= 1.57 \times 10^{-3}\ \text{T (towards West)}$$

Question 4.15:

A magnetic field of 100 G (1 G = 10^{-4} T) is required which is uniform in a region of linear dimension about 10 cm and area of cross-section about 10^{-3} m^2. The maximum current-carrying capacity of a given coil of wire is 15 A and the number of turns per unit length that can be wound round a core is at most 1000 turns m^{-1}. Suggest some appropriate design particulars of a solenoid for the required purpose. Assume the core is not ferromagnetic

Answer

Magnetic field strength, B = 100 G = 100 × 10^{-4} T

Number of turns per unit length, n = 1000 turns m^{-1}

Current flowing in the coil, I = 15 A

Permeability of free space, $\mu_0 = 4\pi \times 10^{-7}\ \text{T m A}^{-1}$

Magnetic field is given by the relation,

$$B = \mu_0 n I$$
$$\therefore nI = \frac{B}{\mu_0}$$
$$= \frac{100 \times 10^{-4}}{4\pi \times 10^{-7}} = 7957.74$$
$$\approx 8000\ \text{A/m}$$

If the length of the coil is taken as 50 cm, radius 4 cm, number of turns 400, and current 10 A, then these values are not unique for the given purpose. There is always a possibility of some adjustments with limits.

Question 4.16:

For a circular coil of radius *R* and *N* turns carrying current *I*, the magnitude of the magnetic field at a point on its axis at a distance *x* from its centre is given by,

$$B=\frac{\mu_0 IR^2N}{2\left(x^2+R^2\right)^{\frac{3}{2}}}$$

(a) Show that this reduces to the familiar result for field at the centre of the coil.

(b) Consider two parallel co-axial circular coils of equal radius *R*, and number of turns *N*, carrying equal currents in the same direction, and separated by a distance *R*. Show that the field on the axis around the mid-point between the coils is uniform over a distance that is small as compared to *R*, and is given by,

$B=0.72-\frac{\mu_0 BNI}{R}$, approximately.

[Such an arrangement to produce a nearly uniform magnetic field over a small region is known as *Helmholtz coils*.]

Answer

Radius of circular coil = *R*

Number of turns on the coil = *N*

Current in the coil = *I*

Magnetic field at a point on its axis at distance *x* is given by the relation,

$$B=\frac{\mu_0 IR^2N}{2\left(x^2+R^2\right)^{\frac{3}{2}}}$$

Where,

μ_0 = Permeability of free space

(a) If the magnetic field at the centre of the coil is considered, then *x* = 0.

$$\therefore B=\frac{\mu_0 IR^2N}{2R^3}=\frac{\mu_0 IN}{2R}$$

This is the familiar result for magnetic field at the centre of the coil.

(b) Radii of two parallel co-axial circular coils = *R*

Number of turns on each coil = *N*

Current in both coils = *I*

Distance between both the coils = *R*

Let us consider point Q at distance *d* from the centre.

Then, one coil is at a distance of $\frac{R}{2}+d$ from point Q.

∴ Magnetic field at point Q is given as:

$$B_1=\frac{\mu_0 NIR^2}{2\left[\left(\frac{R}{2}+d\right)^2+R^2\right]^{\frac{3}{2}}}$$

Also, the other coil is at a distance of $\frac{R}{2}-d$ from point Q.

∴ Magnetic field due to this coil is given as:

$$B_2=\frac{\mu_0 NIR^2}{2\left[\left(\frac{R}{2}-d\right)^2+R^2\right]^{\frac{3}{2}}}$$

Total magnetic field,

$$B=B_1+B_2$$

$$=\frac{\mu_0 IR^2}{2}\left[\left\{\left(\frac{R}{2}-d\right)^2+R^2\right\}^{-\frac{3}{2}}+\left\{\left(\frac{R}{2}+d\right)^2+R^2\right\}^{-\frac{3}{2}}\right]$$

$$=\frac{\mu_0 IR^2}{2}\left[\left(\frac{5R^2}{4}+d^2-Rd\right)^{-\frac{3}{2}}+\left(\frac{5R^2}{4}+d^2+Rd\right)^{-\frac{3}{2}}\right]$$

$$=\frac{\mu_0 IR^2}{2}\times\left(\frac{5R^2}{4}\right)^{-\frac{3}{2}}\left[\left(1+\frac{4}{5}\frac{d^2}{R^2}-\frac{4}{5}\frac{d}{R}\right)^{-\frac{3}{2}}+\left(1+\frac{4}{5}\frac{d^2}{R^2}+\frac{4}{5}\frac{d}{R}\right)^{-\frac{3}{2}}\right]$$

For $d \ll R$, neglecting the factor $\frac{d^2}{R^2}$, we get:

$$\approx\frac{\mu_0 IR^2}{2}\times\left(\frac{5R^2}{4}\right)^{-\frac{3}{2}}\times\left[\left(1-\frac{4d}{5R}\right)^{-\frac{3}{2}}+\left(1+\frac{4d}{5R}\right)^{-\frac{3}{2}}\right]$$

$$\approx\frac{\mu_0 IR^2N}{2R^3}\times\left(\frac{4}{5}\right)^{\frac{3}{2}}\left[1-\frac{6d}{5R}+1+\frac{6d}{5R}\right]$$

$$B=\left(\frac{4}{5}\right)^{\frac{3}{2}}\frac{\mu_0 IN}{R}=0.72\left(\frac{\mu_0 IN}{R}\right)$$

Hence, it is proved that the field on the axis around the mid-point between the coils is uniform.

Question 4.17:

A toroid has a core (non-ferromagnetic) of inner radius 25 cm and outer radius 26 cm, around which 3500 turns of a wire are wound. If the current in the wire is 11 A, what is the magnetic field (a) outside the toroid, (b) inside the core of the toroid, and (c) in the empty space surrounded by the toroid.

Answer

Inner radius of the toroid, r_1 = 25 cm = 0.25 m

Outer radius of the toroid, r_2 = 26 cm = 0.26 m

Number of turns on the coil, N = 3500

Current in the coil, I = 11 A

(a) Magnetic field outside a toroid is zero. It is non-zero only inside the core of a toroid.

(b) Magnetic field inside the core of a toroid is given by the relation,

$$B = \frac{\mu_0 N I}{l}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}\ \mathrm{T\,m\,A^{-1}}$

l = length of toroid

$$= 2\pi\left[\frac{r_1 + r_2}{2}\right]$$

$$= \pi(0.25 + 0.26)$$

$$= 0.51\pi$$

$$\therefore B = \frac{4\pi \times 10^{-7} \times 3500 \times 11}{0.51\pi}$$

$$\approx 3.0 \times 10^{-2}\ \mathrm{T}$$

(c) Magnetic field in the empty space surrounded by the toroid is zero.

*********** END **********