Functions Ex 2.3 Q11(ii)

We have, $f(x) = \sqrt{x-2}$

Clearly, $\text{Domain}(f) = [2,\infty)$ and $\text{Range}(f) = [0,\infty)$.

We observe that range(f) is not a subset of domain of f.

$$\begin{aligned}
\therefore \text{ Domain of } (fof) &= \{x : x \in \text{Domain } (f) \text{ and } f(x) \in \text{Domain } (f)\} \\
&= \{x : x \in [2,\infty) \text{ and } \sqrt{x-2} \in [2,\infty)\} \\
&= \{x : x \in [2,\infty) \text{ and } \sqrt{x-2} \geq 2\} \\
&= \{x : x \in [2,\infty) \text{ and } x-2 \geq 4\} \\
&= \{x : x \in [2,\infty) \text{ and } x \geq 6\} \\
&= [6,\infty)
\end{aligned}$$

Clearly, range of $f = [0,\infty) \not\subset$ Domain of (fof).

$$\begin{aligned}
\therefore \text{ Domain of } ((fof)of) &= \{x : x \in \text{Domain } (f) \text{ and } f(x) \in \text{Domain } (fof)\} \\
&= \{x : x \in [2,\infty) \text{ and } \sqrt{x-2} \in [6,\infty)\} \\
&= \{x : x \in [2,\infty) \text{ and } \sqrt{x-2} \geq 6\} \\
&= \{x : x \in [2,\infty) \text{ and } x-2 \geq 36\} \\
&= \{x : x \in [2,\infty) \text{ and } x \geq 38\} \\
&= [38,\infty)
\end{aligned}$$

Now,

$$(fof)(x) = f(f(x)) = f\left(\sqrt{x-2}\right) = \sqrt{\sqrt{x-2}-2}$$

$$(fofof)(x) = (fof)(f(x)) = (fof)\left(\sqrt{x-2}\right) = \sqrt{\sqrt{\sqrt{x-2}-2}-2}$$

$\therefore$ $fofof : [38,\infty) \to R$ defined as

$$(fofof)(x) = \sqrt{\sqrt{\sqrt{x-2}-2}-2}$$

Functions Ex 2.3 Q11(iii)

We have, $f(x) = \sqrt{x-2}$

Clearly, $\text{Domain}(f) = [2,\infty)$ and $\text{Range}(f) = [0,\infty)$.

We observe that range(f) is not a subset of domain of f.

$$\begin{aligned}
\therefore \text{Domain of } (fof) &= \{x : x \in \text{Domain } (f) \text{ and } f(x) \in \text{Domain } (f)\} \\
&= \{x : x \in [2,\infty) \text{ and } \sqrt{x-2} \in [2,\infty)\} \\
&= \{x : x \in [2,\infty) \text{ and } \sqrt{x-2} \geq 2\} \\
&= \{x : x \in [2,\infty) \text{ and } x-2 \geq 4\} \\
&= \{x : x \in [2,\infty) \text{ and } x \geq 6\} \\
&= [6,\infty)
\end{aligned}$$

Clearly, range of $f = [0,\infty) \not\subset$ Domain of (fof).

$$\begin{aligned}
\therefore \text{Domain of } ((fof)of) &= \{x : x \in \text{Domain } (f) \text{ and } f(x) \in \text{Domain } (fof)\} \\
&= \{x : x \in [2,\infty) \text{ and } \sqrt{x-2} \in [6,\infty)\} \\
&= \{x : x \in [2,\infty) \text{ and } \sqrt{x-2} \geq 6\} \\
&= \{x : x \in [2,\infty) \text{ and } x-2 \geq 36\} \\
&= \{x : x \in [2,\infty) \text{ and } x \geq 38\} \\
&= [38,\infty)
\end{aligned}$$

Now,

$$(fof)(x) = f(f(x)) = f\left(\sqrt{x-2}\right) = \sqrt{\sqrt{x-2}-2}$$

$$(fofof)(x) = (fof)(f(x)) = (fof)\left(\sqrt{x-2}\right) = \sqrt{\sqrt{\sqrt{x-2}-2}-2}$$

$\therefore$ fofof : $[38,\infty) \to R$ defined as

$$(fofof)(x) = \sqrt{\sqrt{\sqrt{x-2}-2}-2}$$

$$(fofof)(38) = \sqrt{\sqrt{\sqrt{38-2}-2}-2} = \sqrt{\sqrt{\sqrt{36}-2}-2} = \sqrt{\sqrt{6-2}-2} = \sqrt{\sqrt{4}-2} = \sqrt{2-2} = 0$$

Functions Ex 2.3 Q11(iv)

We have, $f(x) = \sqrt{x-2}$

Clearly, $\text{Domain}(f) = [2,\infty)$ and $\text{Range}(f) = [0,\infty)$.

We observe that range(f) is not a subset of domain of f.

$$\begin{aligned}
\therefore \text{Domain of } (fof) &= \{x : x \in \text{Domain } (f) \text{ and } f(x) \in \text{Domain } (f)\} \\
&= \{x : x \in [2,\infty) \text{ and } \sqrt{x-2} \in [2,\infty)\} \\
&= \{x : x \in [2,\infty) \text{ and } \sqrt{x-2} \geq 2\} \\
&= \{x : x \in [2,\infty) \text{ and } x-2 \geq 4\} \\
&= \{x : x \in [2,\infty) \text{ and } x \geq 6\} \\
&= [6,\infty)
\end{aligned}$$

Now,

$$(fof)(x) = f(f(x)) = f\left(\sqrt{x-2}\right) = \sqrt{\sqrt{x-2}-2}$$

$\therefore$ fof : $[6,\infty) \to R$ defined as

$$(fof)(x) = \sqrt{\sqrt{x-2}-2}$$

$$f^2(x) = [f(x)]^2 = \left[\sqrt{x-2}\right]^2 = x-2$$

$\therefore$ $f^2 : [2,\infty) \to R$ defined as

$$f^2(x) = x-2$$

$\therefore$ fof $\neq f^2$

Functions Ex 2.3 Q12

$$f(x)\begin{cases}1+x & 0 \le x \le 2 \\ 3-x & 2 \le x \le 3\end{cases}$$

$\therefore$ Range of $f = [0,3] \subseteq$ Domain of f.

$$\therefore \quad f \circ f(x) = f(f(x)) = f\begin{cases}1+x & 0 \le x \le 2 \\ 3-x & 2 < x \le 3\end{cases}$$

$$f \circ f(x) = \begin{cases}2+x & 0 \le x \le 1 \\ 2-x & 1 < x \le 2 \\ 4-x & 2 < x \le 3\end{cases}$$

*********** END **********