Chapter 5 Trigonometric Functions Ex 5.3 Q 4

$LHS = sin^2 \frac{\pi}{18} + sin^2 \frac{\pi}{9} + sin^2 \frac{7\pi}{18} + sin^2 \frac{4\pi}{9}$

$= sin^2 \frac{\pi}{18} + sin^2 \frac{4\pi}{9} + sin^2 \frac{\pi}{9} + sin^2 \frac{7\pi}{18}$

$= sin^2 \left(\frac{\pi}{2} - \frac{4\pi}{9} \right) + sin^2 \frac{4\pi}{9} + sin^2 \frac{\pi}{9} + sin^2 \left(\frac{\pi}{2} - \frac{\pi}{9} \right)$ $\left(\because \frac{\pi}{18} = \frac{\pi}{2} - \frac{4\pi}{9} \text{ and } \frac{7\pi}{18} = \frac{\pi}{2} - \frac{\pi}{9} \right)$

$= cos^2 \frac{4\pi}{9} + sin^2 \frac{4\pi}{9} + sin^2 \frac{\pi}{9} + cos^2 \frac{\pi}{9}$ $\left(\because sin \left(\frac{\pi}{2} - \theta \right) = cos\,\theta \right)$

$= 1 + 1$ $\left(\because sin^2 \theta + cos^2 \theta = 1 \right)$

$= 2$

$= RHS$

  Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 5

$LHS = sec \left(\frac{3\pi}{2} - \theta \right) sec \left(\theta - \frac{5\pi}{2} \right) + tan \left(\frac{5\pi}{2} + \theta \right) tan \left(\theta - \frac{3\pi}{2} \right)$

$= sec \left(\frac{3\pi}{2} - \theta \right) sec \left(-\left(\frac{5\pi}{2} - \theta \right) \right) + tan \left(\frac{5\pi}{2} + \theta \right) tan \left(-\left(\frac{3\pi}{2} - \theta \right) \right)$

$= -cosec\theta. sec \left(\frac{5\pi}{2} - \theta \right) - cot\,\theta \times (-) tan \left(\frac{3\pi}{2} - \theta \right)$

$\left[\because \left(sec \left(\frac{3\pi}{2} - \theta \right) \right) = -cosec\theta,\ sec (-\theta) = sec\,\theta, tan \left(\frac{5\pi}{2} + \theta \right) = -cot\,\theta \right.$

$\left. \& \ tan (-\theta) = -tan\,\theta \right]$

$= -cosec\,\theta \times cosec\,\theta - cot\,\theta \times (-1) \times cot\,\theta$ $\left[\left(\because sec \left(\frac{5\pi}{2} - \theta \right) \right) = cosec\theta \right.$

$\left. \& \ tan \left(\frac{3\pi}{2} - \theta \right) = cot\,\theta \right]$

$= -cosec^2\theta + cot^2 \theta$

$= -cosec^2\theta + cosec^2\theta - 1$ $\left(\because cosec^2\theta = 1 + cot^2 \theta \right)$

$= -1$

$= RHS$

  Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 6

We have $A + B + C = \pi$ $\left(\because \text{ sum of 3 angles of a triangle is } \pi = 180° \right)$

$\Rightarrow \qquad A + B = \pi - C$

$\Rightarrow \qquad cos (A + B) = cos (\pi - C)$

$\Rightarrow \qquad\qquad\quad = -cos\,C$ $\left(\because cos (\pi - \theta) = -cos\,\theta \right)$

$\Rightarrow \qquad cos (A + B) + cos\,C = 0$

   Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 6 ii

We have $A + B + C = \pi$ $\left(\because \text{ sum of 3 angles of a triangle is } \pi = 180° \right)$

$\Rightarrow \qquad A + B = \pi - C$

$\Rightarrow \qquad \frac{A + B}{2} = \frac{\pi - C}{2}$

$\Rightarrow \qquad \frac{A + B}{2} = \frac{\pi}{2} - \frac{C}{2}$

$\Rightarrow \qquad cos = \left(\frac{A + B}{2} \right) = cos \left(\frac{\pi}{2} - \frac{C}{2} \right)$

$\Rightarrow \qquad\qquad = sin \frac{C}{2}$ $\left(\because cos \left(\frac{\pi}{2} - \theta \right) = sin\,\theta \right)$

Hence $cos \left(\frac{A + B}{2} \right) = sin \frac{C}{2}$

   Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 6 iii

We have $A + B + C = \pi$ $\qquad \left(\because \text{ sum of 3 angles of a triangle is } \pi = 180° \right)$

$\Rightarrow \qquad A + B = \pi - C$

$\Rightarrow \qquad \dfrac{A + B}{2} = \dfrac{\pi - C}{2}$

$\Rightarrow \qquad \dfrac{A + B}{2} = \dfrac{\pi}{2} - \dfrac{C}{2}$

$\Rightarrow \qquad \tan\left(\dfrac{A + B}{2}\right) = \tan\left(\dfrac{\pi}{2} - \dfrac{C}{2}\right)$

$\qquad\qquad\qquad\quad = \cot\dfrac{C}{2} \qquad\qquad \left(\because \tan\left(\dfrac{\pi}{2} - \theta\right) = \cot\theta \right)$

Hence $\tan\left(\dfrac{A + B}{2}\right) = \cot\dfrac{C}{2}$

$\qquad\qquad\qquad$ Proved

*********** END *********