Higher Order Derivatives Ex 12.1 Q42

We know that, $\frac{d}{dx}\left(\operatorname{cosec}^{-1}x\right) = \frac{-1}{|x|\sqrt{x^2-1}}$

Let $y = \operatorname{cosec}^{-1}x$

$$\frac{dy}{dx} = \frac{-1}{|x|\sqrt{x^2-1}}$$

Since $x > 1$, $|x| = x$

Thus,

$$\frac{dy}{dx} = \frac{-1}{x\sqrt{x^2-1}} \ldots(1)$$

Differentiating the above function with respect to x, we have,

$$\frac{d^2y}{dx^2} = \frac{x\frac{2x}{2\sqrt{x^2-1}} + \sqrt{x^2-1}}{x^2\left(x^2-1\right)}$$

$$= \frac{\frac{x^2}{\sqrt{x^2-1}} + \sqrt{x^2-1}}{x^2\left(x^2-1\right)}$$

$$= \frac{x^2 + x^2 - 1}{x^2\left(x^2-1\right)^{\frac{3}{2}}}$$

$$= \frac{2x^2 - 1}{x^2\left(x^2-1\right)^{\frac{3}{2}}}$$

Thus, $x\left(x^2-1\right)\frac{d^2y}{dx^2} = \frac{2x^2-1}{x\sqrt{x^2-1}} \ldots(2)$

Similarly, from (1), we have

$$\left(2x^2-1\right)\frac{dy}{dx} = \frac{-2x^2+1}{x\sqrt{x^2-1}} \ldots(3)$$

Thus, from (2) and (3), we have,

$$x\left(x^2-1\right)\frac{d^2y}{dx^2} + \left(2x^2-1\right)\frac{dy}{dx} = \frac{2x^2-1}{x\sqrt{x^2-1}} + \left(\frac{-2x^2+1}{x\sqrt{x^2-1}}\right) = 0$$

Hence proved.

Higher Order Derivatives Ex 12.1 Q43

Given that, $x = \cos t + \log\tan\frac{t}{2}$, $y = \sin t$

Differentiating with respect to t, we have,

$$\frac{dx}{dt} = -\sin t + \frac{1}{\tan\frac{t}{2}} \times sec^2\frac{t}{2} \times \frac{1}{2}$$

$$= -\sin t + \frac{1}{\frac{\sin\frac{t}{2}}{\cos\frac{t}{2}}} \times \frac{1}{\cos^2\frac{t}{2}} \times \frac{1}{2}$$

$$= -\sin t + \frac{1}{\frac{\sin\frac{t}{2}}{\cos\frac{t}{2}}} \times \frac{1}{\cos^2\frac{t}{2}} \times \frac{1}{2}$$

$$= -\sin t + \frac{1}{2\sin\frac{t}{2}\cos\frac{t}{2}}$$

$$= -\sin t + \frac{1}{\sin t}$$

$$= \frac{1-\sin^2 t}{\sin t}$$

$$= \frac{\cos^2 t}{\sin t}$$

$$= \cos t \times \cot t$$

Now find the value of $\frac{dy}{dt}$:

$$\frac{dy}{dt} = \cos t$$

Thus, $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \cos t \times \frac{1}{\cos t \times \cot t}$

$$\Rightarrow \frac{dy}{dx} = \tan t$$

Since $\frac{dy}{dt} = \cos t$, *we have* $\frac{d^2y}{dt^2} = -\sin t$

At $t = \frac{\pi}{4}$, $\left(\frac{d^2y}{dt^2}\right)_{t=\frac{\pi}{4}} = -\sin\left(\frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}}$

$$\frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$$

$$=\frac{\frac{d}{dt}(\tan t)}{\cos t\times\cot t}$$

$$=\frac{sec^2t}{\cos t\times\cot t}$$

$$=\frac{sec^2t}{\cos t\times\frac{\cos t}{\sin t}}$$

$$=\frac{sec^2t}{\cos^2t}\times\sin t$$

$$=sec^4t\times\sin t$$

$$Thus,\left(\frac{d^2y}{dx^2}\right)_{t=\frac{\pi}{4}}=sec^4\left(\frac{\pi}{4}\right)\times\sin\frac{\pi}{4}=2$$

Higher Order Derivatives Ex 12.1 Q44

$$x=a\sin t\text{ and }y=a\left(\cos t+\log\tan\frac{t}{2}\right)$$

$$\frac{dx}{dt}=a\cos t$$

$$\frac{d^2x}{dt^2}=-a\sin t$$

$$\frac{dy}{dt}=-a\sin t+a\frac{1}{\tan\frac{t}{2}}\times\sec^2\frac{t}{2}\times\frac{1}{2}$$

$$=-a\sin t+a\frac{1}{2\sin\frac{t}{2}\cos\frac{t}{2}}$$

$$=-a\sin t+a\,cosect$$

$$\frac{d^2y}{dt^2}=-a\cos t-a\,cosect\cot t$$

$$\frac{d^2y}{dx^2}=\frac{\frac{dx}{dt}\frac{d^2y}{dt^2}-\frac{dy}{dt}\frac{d^2x}{dt^2}}{\left(\frac{dx}{dt}\right)^3}$$

$$=\frac{a\cos t(-a\cos t-a\,cosect\cot t)-(-a\sin t+a\,cosect)(-a\sin t)}{(a\cos t)^3}$$

$$=\frac{-a^2\cos^2t-a^2\cot^2t-a^2\sin^2t+a^2}{a^3\cos^3t}$$

$$=\frac{-a^2\cos^2t-a^2\sin^2t-a^2\cot^2t+a^2}{a^3\cos^3t}$$

$$=\frac{-a^2(\cos^2t+\sin^2t)-a^2\cot^2t+a^2}{a^3\cos^3t}$$

$$=-\frac{1}{a\sin^2t\cos t}$$

*********** END **********