Exercise 8.4

$$= \frac{(1-\cos\theta)(1-\cos\theta)}{1-\cos^2\theta}$$

$$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{1-\cos\theta}{1+\cos\theta} = \text{R.H.S.}$$

(ii) L.H.S. $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}$

$$= \frac{\cos^2\theta+1+\sin^2\theta+2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{\cos^2\theta+\sin^2\theta+1+2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{1+1+2\sin A}{(1+\sin A)\cos A} \left[\because \sin^2\theta+\cos^2\theta=1\right]$$

$$= \frac{2+2\sin A}{(1+\sin A)\cos A} = \frac{2(1+\sin A)}{(1+\sin A)\cos A}$$

$$= \frac{2}{\cos A} = 2\sec A = \text{R.H.S}$$

(iii) L.H.S. $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\sin\theta}{\cos\theta}\times\frac{\sin\theta}{\sin\theta-\cos\theta}+\frac{\cos\theta}{\sin\theta}\times\frac{\cos\theta}{\cos\theta-\sin\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$$

$$= \frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$$

$\left[\because a^3-b^3=(a-b)\left(a^2+b^2+ab\right)\right]$

$= \dfrac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}$ $\left[\because \sin^2\theta+\cos^2\theta=1\right]$

$= \dfrac{1}{\sin\theta\cos\theta}+1$ $= 1+\dfrac{1}{\sin\theta\cos\theta}$

$= 1+\sec\theta\cos ec\theta$

(iv) L.H.S. $\dfrac{1+\sec A}{\sec A} = \dfrac{1+\dfrac{1}{\cos A}}{\dfrac{1}{\cos A}}$

$= \dfrac{\cos A+1}{\cos A}\times\dfrac{\cos A}{1}$ $= 1+\cos A$

$= 1+\cos A\times\dfrac{1-\cos A}{1-\cos A}$

$= \dfrac{1-\cos^2 A}{1-\cos A}$

$= \dfrac{\sin^2 A}{1-\cos A}$ = R.H.S.

(v) L.H.S. $\dfrac{\cos A-\sin A+1}{\cos A+\sin A-1}$

Dividing all terms by $\sin A$,

$= \dfrac{\cot A-1+\cos ecA}{\cot A+1-\cos ecA}$ $= \dfrac{\cot A+\cos ecA-1}{\cot A-\cos ecA+1}$

$= \dfrac{(\cot A+\cos ecA)-\left(\cos ec^2A-\cot^2 A\right)}{(1+\cot A-\cos ecA)}$

$= \dfrac{(\cot A+\cos ecA)+\left(\cot^2 A-\cos ec^2A\right)}{(1+\cot A-\cos ecA)}$

$= \dfrac{(\cot A+\cos ecA)(1+\cot A-\cos ecA)}{(1+\cot A-\cos ecA)}$

$= \cot A+\cos ecA$ = R.H.S.

(vi) L.H.S. $\sqrt{\dfrac{1+\sin A}{1-\sin A}}$

$= \sqrt{\dfrac{1+\sin A}{1-\sin A}}\times\sqrt{\dfrac{1+\sin A}{1+\sin A}}$

$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$ $\left[\because (a+b)(a-b)=a^2-b^2\right]$

$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$ $\left[\because 1-\sin^2\theta=\cos^2\theta\right]$

$= \frac{1+\sin A}{\cos A}$ $= \frac{1}{\cos A}+\frac{\sin A}{\cos A}$

$= \sec A+\tan A$ = R.H.S.

(vii) L.H.S. $\frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}$

$= \frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(2\cos^2\theta-1\right)}$

$= \frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left[2\left(1-\sin^2\theta\right)-1\right]}$

$\left[\because 1-\sin^2\theta=\cos^2\theta\right]$

$= \frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(2-2\sin^2\theta-1\right)}$

$= \frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(1-2\sin^2\theta\right)}$ $= \frac{\sin\theta}{\cos\theta}$

$= \tan\theta$ = R.H.S

(viii) L.H.S. $(\sin A+\operatorname{cosec} A)^2+(\cos A+\sec A)^2$

$= \left(\sin A+\frac{1}{\sin A}\right)^2+\left(\cos A+\frac{1}{\cos A}\right)^2$

=

$\sin^2 A+\frac{1}{\sin^2 A}+2.\sin A.\frac{1}{\sin A}+\cos^2 A+\frac{1}{\cos^2 A}+2.\cos A.\frac{1}{\cos A}$

$= 2+2+\sin^2 A+\cos^2 A+\frac{1}{\sin^2 A}+\frac{1}{\cos^2 A}$

$= 4+1+\frac{1}{\sin^2 A}+\frac{1}{\cos^2 A}$

$= 5+\operatorname{cosec}^2 A+\sec^2 A$

$= 5+1+\cot^2 A+1+\tan^2 A$

$\left[\because \operatorname{cosec}^2\theta = 1+\cot^2\theta, \sec^2\theta = 1+\tan^2\theta\right]$

$= 7+\tan^2 A+\cot^2 A$

= R.H.S.

(ix) L.H.S. $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$= \left(\dfrac{1}{\sin A} - \sin A\right)\left(\dfrac{1}{\cos A} - \cos A\right)$

$= \left(\dfrac{1-\sin^2 A}{\sin A}\right)\left(\dfrac{1-\cos^2 A}{\cos A}\right)$

$= \dfrac{\cos^2 A}{\sin A} \times \dfrac{\sin^2 A}{\cos A} = \sin A.\cos A$

$= \dfrac{\sin A.\cos A}{\sin^2 A+\cos^2 A}$ $\left[\because \sin^2\theta+\cos^2\theta = 1\right]$

Dividing all the terms by $\sin A.\cos A$,

$$= \frac{\dfrac{\sin A.\cos A}{\sin A.\cos A}}{\dfrac{\sin^2 A}{\sin A.\cos A}+\dfrac{\cos^2 A}{\sin A.\cos A}}$$

$$= \frac{1}{\dfrac{\sin A}{\cos A}+\dfrac{\cos A}{\sin A}}$$

$= \dfrac{1}{\tan A+\cot A}$ = R.H.S.

(x) L.H.S. $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right) = \dfrac{\sec^2 A}{\operatorname{cosec}^2 A}$

$\left[\because 1+\tan^2\theta = \sec^2\theta, 1+\cot^2\theta = \operatorname{cosec}^2\theta\right]$

$= \dfrac{1}{\cos^2 A} \times \dfrac{\sin^2 A}{1} = \tan^2 A$ = R.H.S.

Now, Middle side = $\left(\dfrac{1-\tan A}{1-\cot A}\right)^2 = \left(\dfrac{1-\tan A}{1-\dfrac{1}{\tan A}}\right)^2$

$$= \left(\frac{1-\tan A}{\dfrac{\tan A-1}{}}\right)^2$$

$$\tan A$$

$$= \left(\frac{1-\tan A}{\frac{-(1-\tan A)}{\tan A}} \right) = (-\tan A)^2$$

$$= \tan^2 A = \text{R.H.S.}$$

********** END **********