

Q14: Sand is pouring from a pipe at the rate of 12 cm³/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

Answer:

The volume of a cone (V) with radius (r) and height (h) is given by,

$$V = \frac{1}{3}\pi r^2 h$$

It is given that,

$$h = \frac{1}{6}r \Rightarrow r = 6h$$

:.
$$V = \frac{1}{3}\pi (6h)^2 h = 12\pi h^3$$

The rate of change of volume with respect to time (*t)*is given by,

$$\frac{dV}{dt} = 12\pi \frac{d}{dh} (h^3) \cdot \frac{dh}{dt}$$
 [By chain rule]

$$=12\pi \left(3h^2\right)\frac{dh}{dt}$$

$$=36\pi h^2 \frac{dh}{dt}$$

It is also given that $\frac{dV}{dt} = 12 \text{ cm}^3/\text{s}$.

Therefore, when h=4 cm, we have:

$$12 = 36\pi \left(4\right)^2 \frac{dh}{dt}$$

$$12 = 36\pi (4)^2 \frac{dh}{dt}$$
$$\Rightarrow \frac{dh}{dt} = \frac{12}{36\pi (16)} = \frac{1}{48\pi}$$

Hence, when the height of the sand cone is 4 cm, its height is increasing at the rate of $\frac{1}{48\pi}$ cm/s.

Answer needs Correction? Click Here

Q15: The total cost C(x) in Rupees associated with the production of xunits of an item is given by

$$C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000$$

Find the marginal cost when 17 units are produced.

Marginal cost is the rate of change of total cost with respect to output.

:.Marginal cost (MC) =
$$\frac{dC}{dx}$$
 = 0.007(3 x^2) - 0.003(2 x) +15

$$= 0.021x^2 - 0.006x + 15$$

When x = 17, MC = 0.021 (17²) - 0.006 (17) + 15

= 6.069 - 0.102 + 15

Hence, when 17 units are produced, the marginal cost is Rs. 20.967.

Answer needs Correction? Click Here

Q16: The total revenue in Rupees received from the sale of x units of a product is given by

$$R(x) = 13x^2 + 26x + 15$$

Find the marginal revenue when x=7.

 $\label{thm:marginal} \mbox{Marginal revenue is the rate of change of total revenue with respect to the number of units sold.}$

∴ Marginal Revenue (MR) =
$$\frac{dR}{dx}$$
 = 13(2x) + 26 = 26x + 26

When x = 7,

Hence, the required marginal revenue is Rs 208.

Q17 : The rate of change of the area of a circle with respect to its radius rat r = 6 cm is (A) $10\tilde{A}$ a, $_{-}$ (B) $12\tilde{A}$ a, $_{-}$ (C) $8\tilde{A}$ a, $_{-}$ (D) $11\tilde{A}$ a, $_{-}$

Answer:

The area of a circle (A)with radius (r) is given by,

$$A = \pi r^2$$

Therefore, the rate of change of the area with respect to its radius *r*is

$$\frac{dA}{dr} = \frac{d}{dr} \left(\pi r^2 \right) = 2\pi r \,.$$

∴When
$$r = 6$$
 cm,

$$\frac{dA}{dr} = 2\pi \times 6 = 12\pi \,\mathrm{cm}^2/\mathrm{s}$$

Hence, the required rate of change of the area of a circle is $12\pi cm^2/s$.

The correct answer is B.

Answer needs Correction? Click Here

Q18: The total revenue in Rupees received from the sale of x units of a product is given by

$$R(x) = 3x^2 + 36x + 5$$
. The marginal revenue, when $x = 15$ is

(A) 116 (B) 96 (C) 90 (D) 126

Answer:

Marginal revenue is the rate of change of total revenue with respect to the number of units sold.

∴ Marginal Revenue (MR) =
$$\frac{dR}{dx}$$
 = 3(2x) + 36 = 6x + 36

∴When
$$x = 15$$
,

Hence, the required marginal revenue is Rs 126.

The correct answer is ${\sf D}.$

Answer needs Correction? Click Here

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