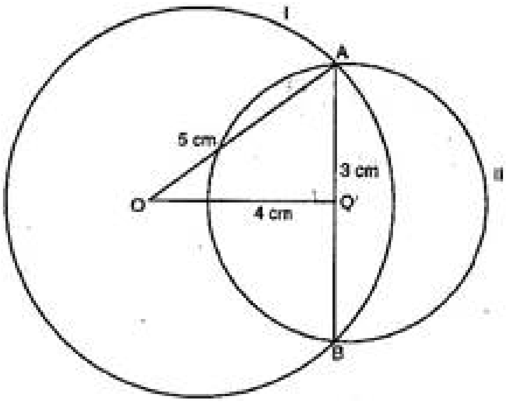
Q1. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centers is 4 cm. Find the length of the common chord.

Ans. Let two circles with centres O and O' intersect each other at points A and B. On joining A and B, AB is a common chord.



Radius OA = 5 cm, Radius O'A = 3 cm,

Distance between their centers OO' = 4 cm

In triangle AOO',

$5^2 = 4^2 + 3^2$

$\Rightarrow 25 = 16 + 9$

$\Rightarrow 25 = 25$

Hence AOO' is a right triangle, right angled at O'.

Since, perpendicular drawn from the center of the circle bisects the chord.

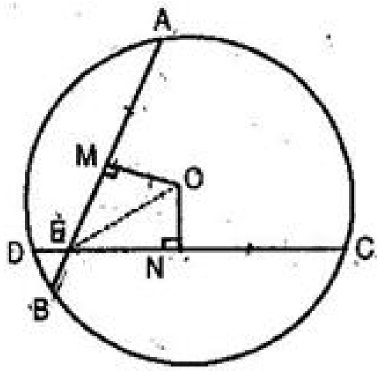
Hence O' is the mid-point of the chord AB. Also O' is the centre of the circle II.

Therefore length of chord AB = Diameter of circle II

$\therefore$ Length of chord AB = 2 x 3 = 6 cm.

Q2. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Ans. Given: Let AB and CD are two equal chords of a circle of centers O intersecting each other at point E within the circle.



To prove: (a) AE = CE

(b) BE = DE

Construction: Draw OM $\perp$ AB, ON $\perp$ CD. Also join OE.

Proof: In right triangles OME and ONE,

$\angle$ OME = $\angle$ ONE = 90°

OM = ON

[Equal chords are equidistance from the centre]

OE = OE [Common]

$\therefore$ ΔOME $\cong$ Δ ONE [RHS rule of congruency]

$\therefore$ ME = NE [By CPCT] ....(i)

Now, O is the centre of circle and OM $\perp$ AB

$\therefore$ AM = $\frac{1}{2}$ AB

[Perpendicular from the centre bisects the chord] .....(ii)

Similarly, NC = $\frac{1}{2}$ CD ....(iii)

But AB = CD [Given]

From eq. (ii) and (iii), AM = NC ....(iv)

Also MB = DN ....(v)

Adding (i) and (iv), we get,

AM + ME = NC + NE

$\Rightarrow$ AE = CE **[Proved part (a)]**

Now AB = CD [Given]

AE = CE [Proved]

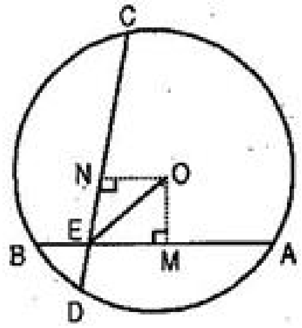
$\Rightarrow$ AB – AE = CD – CE

$\Rightarrow$ BE = DE **[Proved part (b)]**

Q3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chord.

Ans. Given: AB and CD be two equal chords of a circle with centre O intersecting each other

with in the circle at point E. OE is joined.



To prove: $\angle$ OEM = $\angle$ OEN

Construction: Draw OM $\perp$ AB and

ON $\perp$ CD.

Proof: In right angled triangles OME and ONE,

$\angle$ OME = $\angle$ ONE [Each 90°]

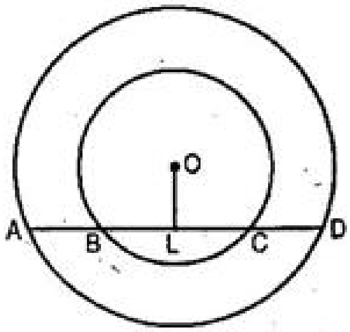
OM = ON [Equal chords are equidistant from the centre]

OE = OE [Common]

$\therefore$ ΔOME $\cong$ Δ ONE [RHS rule of congruency]

$\therefore$ $\angle$OEM = $\angle$ OEN [By CPCT]

Q4. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD. (See figure)



Ans. Given: Line l intersects two concentric circles with centre O at points A, B, C and D.

To prove: AB = CD

Construction: Draw OL $\perp l$

Proof: AD is a chord of outer circle and OL $\perp$ AD.

$\therefore$ AL = LD ....(i) [Perpendicular drawn from the centre bisects the chord]

Now, BC is a chord of inner circle and

OL $\perp$ BC

$\therefore$ BL = LC ...(ii) [Perpendicular drawn from the centre bisects the chord]

Subtracting (ii) from (i), we get,

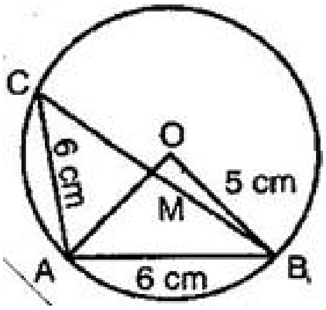
AL − BL = LD − LC

$\Rightarrow$ AB = CD

Q5. Three girls Reshma, Salma and Mandip are standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?

Ans. Let Reshma, Salma and Mandip takes the position C, A and B on the circle. Since AB = AC

The centre lies on the bisector of $\angle$ BAC.



Let M be the point of intersection of BC and OA.

Again, since AB = AC and AM bisects

$\angle$ CAB.

$\therefore$ AM $\perp$ CB and M is the mid-point of CB.

Let OM = x, then MA = $5-x$

From right angled triangle OMB,

$OB^2 = OM^2 + MB^2$

$\Rightarrow 5^2 = x^2 + MB^2$ ....(i)

Again, in right angled triangle AMB,

$AB^2 = AM^2 + MB^2$

$\Rightarrow 6^2 = (5-x)^2 + MB^2$ ....(ii)

Equating the value of MB^2 from eq. (i) and (ii),

$$5^2 - x^2 = 6^2 - (5-x)^2$$

$$\Rightarrow (5-x)^2 - x^2 = 6^2 - 5^2$$

$$\Rightarrow 25 - 10x + x^2 - x^2 = 36 - 25$$

$$\Rightarrow 10x = 25 - 11$$

$$\Rightarrow 10x = 14$$

$$\Rightarrow x = \frac{14}{10}$$

Hence, from eq. (i),

$$MB^2 = 5^2 - x^2 = 5^2 - \left(\frac{14}{10}\right)^2$$

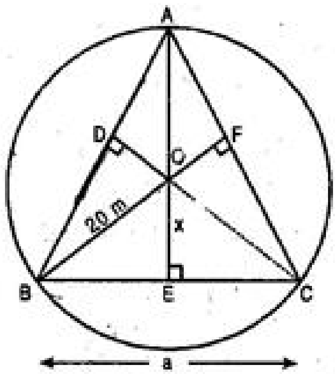
$$= \left(5 + \frac{4}{10}\right)\left(5 - \frac{14}{10}\right) = \frac{64}{10} \times \frac{36}{10}$$

$$\Rightarrow MB = \frac{8 \times 6}{10} = 4.8 \text{ cm}$$

$\therefore$ BC = 2MB = 2 x 4.8 = 9.6 cm

Q6. A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Ans. Let position of three boys Ankur, Syed and David are denoted by the points A, B and C respectively.



A = B = C = a [say]

Since equal sides of equilateral triangle are as equal chords and perpendicular distances of equal chords of a circle are equidistant from the centre.

$\therefore$ OD = OE = OF = x cm [say]

Join OA, OB and OC.

$\Rightarrow$ Area of ΔAOB

= Area of ΔBOC = Area of ΔAOC

And Area of ΔABC

= Area of ΔAOB + Area of ΔBOC + Area of Δ AOC

$\Rightarrow$ And Area of ΔABC = 3 x Area of ΔBOC

$\Rightarrow \frac{\sqrt{3}}{4}a^2 = 3\left(\frac{1}{2}\text{ BC x OE}\right)$

$\Rightarrow \frac{\sqrt{3}}{4}a^2 = 3\left(\frac{1}{2}\times a\times x\right)$

$\Rightarrow \frac{a^2}{a} = 3\times\frac{1}{2}\times\frac{4}{\sqrt{3}}\times x$

$\Rightarrow a = 2\sqrt{3}x$ ....(i)

Now, CE $\perp$ BC

$\therefore$ BE = EC = $\frac{1}{2}$ BC [$\because$ Perpendicular drawn from the centre bisects the chord]

$\Rightarrow$ BE = EC = $\frac{1}{2}a$

$\Rightarrow$ BE = EC = $\frac{1}{2}\left(2\sqrt{3}x\right)$ [Using eq. (i)]

$\Rightarrow$ BE = EC = $\sqrt{3}x$

Now in right angled triangle BEO,

$OE^2 + BE^2 = OB^2$ [Using Pythagoras theorem]

$\Rightarrow x^2 + \left(\sqrt{3}x\right)^2 = (20)^2$

$\Rightarrow x^2 + 3x^2 = 400$

$\Rightarrow 4x^2 = 400$

$\Rightarrow x^2 = 100$

$\Rightarrow x = 10$ m

And $a = 2\sqrt{3}x = 2\sqrt{3}\times 10 = 20\sqrt{3}$ m

Thus distance between any two boys is $20\sqrt{3}$ **m.**

*********** END **********