



Complex Numbers Ex 13.2 Q16(iv)

We have,

$$x = \frac{1+i}{\sqrt{2}}$$

$$\Rightarrow \sqrt{2}x = 1+i$$

$$\Rightarrow (\sqrt{2}x)^2 = (1+i)^2 \quad \text{(squaring both sides)}$$

$$\begin{aligned} \Rightarrow 2x^2 &= 1^2 + (i)^2 + 2 \times 1 \times i \\ &= 1 - 1 + 2i \end{aligned}$$

$$\Rightarrow 2x^2 = 2i$$

$$\Rightarrow x^2 = i$$

$$\Rightarrow (x^2)^2 = (i)^2 \quad \text{(squaring both sides)}$$

$$\Rightarrow x^4 = -1$$

$$\Rightarrow x^4 + 1 = 0 \quad \dots\dots\dots(i)$$

Now

$$x^6 + x^4 + x^2 + 1$$

$$= x^6 + x^2 + x^4 + 1$$

$$= x^2(x^4 + 1) + 1(x^4 + 1)$$

$$= x^2 \times 0 + 1 \times 0 \quad \text{(using (i))}$$

$$= 0$$

Complex Numbers Ex 13.2 Q16(v)

$$x = (-2 - \sqrt{3})$$

$$x^2 = (-2 - \sqrt{3})^2 = 4 + 4\sqrt{3} + 3 = 1 + 4\sqrt{3}$$

$$x^3 = (1 + 4\sqrt{3})(-2 - \sqrt{3}) = -2 - 8\sqrt{3} - \sqrt{3} - 12 = 10 - 9\sqrt{3}$$

$$x^4 = (1 + 4\sqrt{3})^2 = 1 + 8\sqrt{3} + 48 = -47 + 8\sqrt{3}$$

$$\begin{aligned} 2x^4 + 5x^3 + 7x^2 - x + 41 &= 2(-47 + 8\sqrt{3}) + 5(10 - 9\sqrt{3}) + 7(1 + 4\sqrt{3}) - (-2 - \sqrt{3}) + 41 \\ &= -94 + 16\sqrt{3} + 50 - 45\sqrt{3} + 7 + 28\sqrt{3} + 2 + \sqrt{3} + 41 \\ &= (-94 + 50 + 7 + 2 + 41) + (16\sqrt{3} - 45\sqrt{3} + 28\sqrt{3} + \sqrt{3}) \\ &= 6 + 0 \\ &= 6 \end{aligned}$$

Complex Numbers Ex 13.2 Q17

$$\begin{aligned}
& (1-i)^n \left(1 - \frac{1}{i}\right)^n \\
&= (1-i)^n \left(\frac{i-1}{i}\right)^n \\
&= \left\{ \frac{(1-i)(i-1)}{i} \right\}^n \\
&= \left\{ \frac{(1-i)(1-i)}{-i} \right\}^n \\
&= \left\{ \frac{(1-i)^2}{-i} \right\}^n \\
&= \left\{ \frac{1-2i-1}{-i} \right\}^n \\
&= \left\{ \frac{-2i}{-i} \right\}^n = 2^n
\end{aligned}$$

Complex Numbers Ex 13.2 Q18

$$\begin{aligned}
(1+i)z &= (1-i)\bar{z} \\
\Rightarrow z &= \frac{(1-i)\bar{z}}{(1+i)} \\
\Rightarrow z &= \frac{(1-i)(1-i)\bar{z}}{(1+i)(1-i)} \quad [\text{Rationalizing the denominator}] \\
\Rightarrow z &= \frac{(1-2i-1)\bar{z}}{(1+1)} \\
\Rightarrow z &= \frac{-2i}{2}\bar{z} \\
\Rightarrow z &= -i\bar{z}
\end{aligned}$$

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