Tangents and Normals Ex 16.2 Q1

The given equation of the curve is

$$\sqrt{x} + \sqrt{y} = a \qquad \text{---(i)}$$

Differentiating with respect to x, we get

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \times \frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$$

$$\therefore \quad m = \left(\frac{dy}{dx}\right)_{\left(\frac{a^2}{4}, \frac{a^2}{4}\right)} = -\frac{\frac{a}{2}}{\frac{a}{2}} = -1$$

Thus,

the equation of tangent is

$$y - y_1 = m\left(x - x_1\right)$$

$$\Rightarrow \quad y - \frac{a^2}{4} = (-1)\left(x - \frac{a^2}{4}\right)$$

$$\Rightarrow \quad x + y = \frac{a^2}{4} + \frac{a^2}{4}$$

$$\Rightarrow \quad x + y = \frac{a^2}{2}$$

Tangents and Normals Ex 16.2 Q2

The equation of the curve is

$$y = 2x^3 - x^2 + 3 \qquad \text{---(i)}$$

$$\text{Slope} = m = \frac{dy}{dx} = 6x^2 - 2x$$

$$\therefore \qquad m = \left(\frac{dy}{dx}\right)_{(1,4)} = 4$$

Now,

The equation of normal is (i) is

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$\Rightarrow \qquad (y - 4) = \frac{-1}{4}(x - 1)$$

$$\Rightarrow \qquad x + 4y = 16 + 1$$

$$\Rightarrow \qquad x + 4y = 17$$

Tangents and Normals Ex 16.2 Q3(i)

(i) The equation of the curve is $y = x^4 - 6x^3 + 13x^2 - 10x + 5$.

On differentiating with respect to x, we get:

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$

$$\left.\frac{dy}{dx}\right]_{(0,\,5)} = -10$$

Thus, the slope of the tangent at (0, 5) is −10. The equation of the tangent is given as:

$y - 5 = -10(x - 0)$

$\Rightarrow y - 5 = -10x$

$\Rightarrow 10x + y = 5$

The slope of the normal at (0, 5) is $\dfrac{-1}{\text{Slope of the tangent at }(0,5)} = \dfrac{1}{10}$.

Therefore, the equation of the normal at (0, 5) is given as:

$$y - 5 = \frac{1}{10}(x - 0)$$

$$\Rightarrow 10y - 50 = x$$

$$\Rightarrow x - 10y + 50 = 0$$

Tangents and Normals Ex 16.2 Q3(ii)

(ii) The equation of the curve is $y = x^4 - 6x^3 + 13x^2 - 10x + 5$.

On differentiating with respect to x, we get:

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$
$$\left.\frac{dy}{dx}\right]_{(1,\ 3)} = 4 - 18 + 26 - 10 = 2$$

Thus, the slope of the tangent at (1, 3) is 2. The equation of the tangent is given as:

$$y - 3 = 2(x - 1)$$
$$\Rightarrow y - 3 = 2x - 2$$
$$\Rightarrow y = 2x + 1$$

The slope of the normal at (1, 3) is $\dfrac{-1}{\text{Slope of the tangent at (1, 3)}} = \dfrac{-1}{2}$.

Therefore, the equation of the normal at (1, 3) is given as:

$$y - 3 = -\frac{1}{2}(x - 1)$$
$$\Rightarrow 2y - 6 = -x + 1$$
$$\Rightarrow x + 2y - 7 = 0$$

*********** END **********