Complex Numbers Ex 13.2 Q16(iv)

We have,

$$x = \frac{1+i}{\sqrt{2}}$$

$$\Rightarrow \sqrt{2}x = 1+i$$

$$\Rightarrow \left(\sqrt{2}x\right)^2 = (1+i)^2 \qquad \text{(squaring both sides)}$$

$$\Rightarrow 2x^2 = 1^2 + (i)^2 + 2\times 1\times i$$

$$= 1 - 1 + 2i$$

$$\Rightarrow 2x^2 = 2i$$

$$\Rightarrow x^2 = i$$

$$\Rightarrow \left(x^2\right)^2 = (i)^2 \qquad \text{(squaring both sides)}$$

$$\Rightarrow x^4 = -1$$

$$\Rightarrow x^4 + 1 = 0 \quad \ldots\ldots\ldots\ldots(i)$$

Now

$$x^6 + x^4 + x^2 + 1$$

$$= x^6 + x^2 + x^4 + 1$$

$$= x^2\left(x^4+1\right) + 1\left(x^4+1\right)$$

$$= x^2 \times 0 + 1 \times 0 \qquad \text{(using } (i)\text{)}$$

$$= 0$$

Complex Numbers Ex 13.2 Q16(v)

$$x = \left(-2-\sqrt{3}i\right)$$

$$x^2 = \left(-2-\sqrt{3}i\right)^2 = 4 + 4\sqrt{3}i + 3i^2 = 1 + 4\sqrt{3}i$$

$$x^3 = \left(1+4\sqrt{3}i\right)\left(-2-\sqrt{3}i\right) = -2 - 8\sqrt{3}i - \sqrt{3}i - 12i^2 = 10 - 9\sqrt{3}i$$

$$x^4 = \left(1+4\sqrt{3}i\right)^2 = 1 + 8\sqrt{3}i + 48i^2 = -47 + 8\sqrt{3}i$$

$$2x^4 + 5x^3 + 7x^2 - x + 41 = 2\left(-47+8\sqrt{3}i\right) + 5\left(10-9\sqrt{3}i\right) + 7\left(1+4\sqrt{3}i\right) - \left(-2-\sqrt{3}i\right) + 41$$

$$= -94 + 16\sqrt{3}i + 50 - 45\sqrt{3}i + 7 + 28\sqrt{3}i + 2 + \sqrt{3}i + 41$$

$$= \left(-94+50+7+2+41\right) + \left(16\sqrt{3}i - 45\sqrt{3}i + 28\sqrt{3}i + \sqrt{3}i\right)$$

$$= 6 + 0$$

$$= 6$$

Complex Numbers Ex 13.2 Q17

$$(1-i)^n\left(1-\frac{1}{i}\right)^n$$

$$=(1-i)^n\left(\frac{i-1}{i}\right)^n$$

$$=\left\{\frac{(1-i)(i-1)}{i}\right\}^n$$

$$=\left\{\frac{(1-i)(1-i)}{-i}\right\}^n$$

$$=\left\{\frac{(1-i)^2}{-i}\right\}^n$$

$$=\left\{\frac{1-2i-1}{-i}\right\}^n$$

$$=\left\{\frac{-2i}{-i}\right\}^n=2^n$$

Complex Numbers Ex 13.2 Q18

$$(1+i)z=(1-i)\bar{z}$$

$$\Rightarrow z=\frac{(1-i)}{(1+i)}\bar{z}$$

$$\Rightarrow z=\frac{(1-i)(1-i)}{(1+i)(1-i)}\bar{z} \quad \text{[Rationalizing the denominator]}$$

$$\Rightarrow z=\frac{(1-2i-1)}{(1+1)}\bar{z}$$

$$\Rightarrow z=\frac{-2i}{2}\bar{z}$$

$$\Rightarrow z=-i\bar{z}$$

*********** END **********