Sine and Cosine Formulae and their Applications Ex-10.1 Q5

$$(a-b)\cos\frac{C}{2} = c\sin\left(\frac{A-B}{2}\right)$$

Let $a = k\ \sin\ A, b = k\sin\ B, c = k\sin C$

LHS

$$(a-b)\cos\frac{C}{2}$$

$$= k(\sin A - \sin B).\cos\frac{C}{2}$$

$$= 2k\cos(\frac{A+B}{2})\sin(\frac{A-B}{2}).\cos\frac{C}{2}$$

$$= 2k\cos(\frac{\pi-C}{2})\sin(\frac{A-B}{2}).\cos\frac{C}{2}$$

$$= 2k\sin(\frac{C}{2}).\cos\frac{C}{2}.\sin(\frac{A-B}{2}) \qquad [\cos(\frac{\pi}{2}-\theta) = \sin\theta]$$

$$= k\sin C.\sin(\frac{A-B}{2})$$

$$= c.\sin(\frac{A-B}{2}) = RHS$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q6

$$\frac{c}{a-b} = \frac{\tan\left(\frac{A}{2}\right) + \tan\left(\frac{B}{2}\right)}{\tan\left(\frac{A}{2}\right) - \tan\left(\frac{B}{2}\right)}$$

LHS

$$\frac{c}{a-b}$$

$$= \frac{k\sin C}{k\sin A - k\sin B}$$

$$= \frac{\sin C}{\sin A - \sin B}$$

$$= \frac{2\sin\frac{C}{2}\cos\frac{C}{2}}{\sin A - \sin B}$$

$$= \frac{2\sin\frac{C}{2}\cos\frac{C}{2}}{2\cos(\frac{A+B}{2})\sin(\frac{A-B}{2})}$$

$$= \frac{\sin\frac{C}{2}\cos\frac{(\pi-(A+B))}{2}}{\cos(\frac{\pi-C}{2})\sin(\frac{A-B}{2})}$$

$$= \frac{\sin\frac{C}{2}\sin\frac{(A+B)}{2}}{\sin\frac{C}{2}\sin(\frac{A-B}{2})}$$

$$= \frac{\sin\frac{(A+B)}{2}}{\sin(\frac{A-B}{2})}$$

$$= \frac{\sin(\frac{A}{2}).\cos(\frac{B}{2}) + \sin(\frac{B}{2}).\cos(\frac{A}{2})}{\sin(\frac{A}{2}).\cos(\frac{B}{2}) - \sin(\frac{B}{2}).\cos(\frac{A}{2})}$$

$$= \frac{\tan(\frac{A}{2}) + \tan(\frac{B}{2})}{\tan(\frac{A}{2}). - \tan(\frac{B}{2}).} \text{[Dividing both Numerator and Denominator by } \cos(\frac{A}{2}).\cos(\frac{B}{2})\text{]}$$

$$= RHS$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q7

$$\frac{c}{a+b} = \frac{1-\tan\left(\frac{A}{2}\right)\tan\left(\frac{B}{2}\right)}{1+\tan\left(\frac{A}{2}\right)\tan\left(\frac{B}{2}\right)}$$

LHS

$$= \frac{c}{a+b}$$

$$= \frac{k\sin C}{k\sin A + k\sin B}$$

$$= \frac{2\sin\frac{C}{2}\cos\frac{C}{2}}{2\sin(\frac{A+B}{2}).\cos(\frac{A-B}{2})}$$

$$= \frac{\sin\frac{C}{2}\cos\frac{C}{2}}{\sin(\frac{\pi-C}{2}).\cos(\frac{A-B}{2})}$$

$$= \frac{\sin(\frac{\pi-(A+B)}{2})\cos\frac{C}{2}}{\cos(\frac{C}{2}).\cos(\frac{A-B}{2})}$$

$$= \frac{\cos(\frac{A+B}{2})}{\cos(\frac{A-B}{2})}$$

$$= \frac{\cos\frac{A}{2}.\cos\frac{B}{2} - \sin\frac{A}{2}\sin\frac{B}{2}}{\cos\frac{A}{2}.\cos\frac{B}{2} + \sin\frac{A}{2}.\sin\frac{B}{2}}$$

$$= \frac{1-\tan\frac{A}{2}\tan\frac{B}{2}}{1+\tan\frac{A}{2}.\tan\frac{B}{2}} \text{[Dividing both Numerator and Denominator by } \cos(\frac{A}{2}).\cos(\frac{B}{2})\text{]}$$

$$= RHS$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q8

$$\frac{a+b}{c}=\frac{\cos\left(\frac{A-B}{2}\right)}{\sin\frac{C}{2}}$$

Let $a=k\sin A, b=k\sin B, c=k\sin C$

LHS

$$\frac{k\sin A+k\sin B}{k\sin C}$$

$$=\frac{\sin A+\sin B}{\sin C}$$

$$=\frac{2\sin\frac{A+B}{2}.\cos\frac{A-B}{2}}{2\sin\frac{C}{2}.\cos\frac{C}{2}}$$

$$=\frac{\sin(\frac{\pi-C}{2}).\cos\frac{A-B}{2}}{\sin\frac{C}{2}.\cos\frac{C}{2}}$$

$$=\frac{\cos\frac{A-B}{2}}{\sin\frac{C}{2}}=RHS$$

*********** END **********