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Inverse Trigonometric Functions Ex 4.1 Q1.

Let $\tan^{-1}\left(-\sqrt{3}\right) = y$. Then, $\tan y = -\sqrt{3} = -\tan\frac{\pi}{3} = \tan\left(-\frac{\pi}{3}\right)$.

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{3}\right)$ is $-\sqrt{3}$.

Therefore, the principal value of $\tan^{-1}\left(\sqrt{3}\right)$ is $-\frac{\pi}{3}$.

Concept Insight:

The range for $\tan^{-1}$ is same as $\sin^{-1}$ except that it is an open interval, as $\tan(-\pi/2)$ and $\tan(\pi/2)$ are not defined. So the method of finding principal value is same as $\sin^{-1}$ given in the first problem. Also note that $\tan(-x) = -\tan x$.

Let $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = y$. Then, $\cos y = -\frac{1}{\sqrt{2}} = -\cos\left(\frac{\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right)$.

We know that the range of the principal value branch of $\cos^{-1}$ is $[0,\pi]$ and $\cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$.

Let $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right) = y$. Then, $\operatorname{cosec} y = -\sqrt{2} = -\operatorname{cosec}\left(\frac{\pi}{4}\right) = \operatorname{cosec}\left(-\frac{\pi}{4}\right)$.

We know that the range of the principal value branch of $\operatorname{cosec}^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ and $\operatorname{cosec}\left(-\frac{\pi}{4}\right) = -\sqrt{2}$.

Therefore, the principal value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$ is $-\frac{\pi}{4}$.

We know that for any $x \in [-1, 1]$, $\cos^{-1} x$ represents angle in $[0, \pi]$

$$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \text{an angle in } [0, \pi] \text{ whose cosine is } \left(-\frac{\sqrt{3}}{2}\right)$$

$$= \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$\therefore \quad \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$$

We know that, for any $x \in R$, $\tan^{-1} x$ represents an angle in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is x.

So,

$$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \text{An angle in } \left(\frac{-\pi}{2}, \frac{\pi}{2}\right) \text{ whose tangest is } \frac{1}{\sqrt{3}}$$

$$= \frac{\pi}{6}$$

$$\therefore \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}.$$

We know that, for $x \in R$, $sec^{-1}x$ represents an angle in $[0,\pi]-\left\{\frac{\pi}{2}\right\}$.

$sec^{-1}\left(-\sqrt{2}\right)$ = An angle in $[0,\pi]-\left\{\frac{\pi}{2}\right\}$ whose secant is $\left(-\sqrt{2}\right)$

$$= \pi - \frac{\pi}{4}$$

$$= \frac{3\pi}{4}$$

$sec^{-1}\left(-\sqrt{2}\right) = \frac{3\pi}{4}$.

We know that, for any $x \in R$, $cot^{-1}x$ represents an angle in $(0,\pi)$

$cot^{-1}\left(-\sqrt{3}\right)$ = An angle in $(0,\pi)$ whose contangent is $\left(-\sqrt{3}\right)$

$$= \pi - \frac{\pi}{6}$$

$$= \frac{5\pi}{6}$$

$\therefore cot^{-1}\left(-\sqrt{3}\right) = \frac{5\pi}{6}$.

We know that, for any $x \in R$, $\sec^{-1}x$ represents an angle in $[0,\pi]-\left\{\frac{\pi}{2}\right\}$.

$\sec^{-1}(2)$ = An angle is $[0,\pi]-\left\{\frac{\pi}{2}\right\}$ whose secant is 2

$$= \frac{\pi}{3}$$

$\therefore \sec^{-1}(2) = \frac{\pi}{3}$.

We know that, for any $x \in R$. $\operatorname{cosec}^{-1}x$ is an angle in $\left[\frac{-\pi}{2},0\right) \cup \left(0,\frac{\pi}{2}\right]$

$\operatorname{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$ = An angle is $\left[\frac{-\pi}{2},0\right) \cup \left(0,\frac{\pi}{2}\right]$ whose cosecant is $\left(\frac{2}{\sqrt{3}}\right)$

$$= \frac{\pi}{3}$$

$\therefore \operatorname{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{3}$.

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