Question 8. 21. Two heavy spheres each of mass 100 kg and radius 0.10 mare placed 1.0 m apart on ahorizontal table. What is the gravitational field and potential at the mid point of the line joining the centres of the spheres ? Is an object placed at that point in equilibrium ? If so, is the equilibrium stable or unstable?

Answer: Here G = 6.67 x 10^{-11} Nm^2 kg^{-2}; M = 100 kg; R = 0.1 m, distance between the two spheres, d = 1.0 m

Suppose that the distance of either sphere from the mid-point of the line joining their centre is r. Then r=d/2=0.5 m. The gravitational field at the mid-point due to two spheres will be equal and opposite.

Hence, the resultant gravitational field at the mid point = 0

The gravitational potential at the mid point = $\left(-\frac{GM}{r}\right)\times 2$

$$= -\frac{6.67\times 10^{-11}\times 100\times 2}{0.5} = -2.668\times 10^{-8}\ \text{J kg}^{-1}.$$

The object placed at that point would be in stable equilibrium.

Question 8. 22. As you have learnt in the text, a geostationary satellite orbits the Earth at a height of nearly 36,000 km from the surface of the Earth. What is the potential due to Earth's gravity at the site of this satellite? (Take the potential energy at infinity to be zero). Mass of the Earth = 6.0 x 10^{24} kg, radius = 6400 km.

Answer: Distance of satellite from the centre of earth = R = r + x

= 6400 + 36000 = 42400 km = 4.24 x 10^7 m

Using potential, $V = -\frac{GM}{R}$, we get

$$V = -\frac{\left(6.67\times 10^{-11}\right)\times\left(6\times 10^{24}\right)}{\left(4.24\times 10^{7}\right)} = -9.44\times 10^{6}\ \text{J kg}^{-1}$$

Question 8. 23. A star 2.5 times the mass of the sun and collapsed to a size of 12 km rotates with a speed of 1.2 rev. per second. (Extremely compact stars of this kind are known as neutron stars. Certain stellar objects called pulsars belong to this category). Will an object placed on its equator remain stuck to its surface due to gravity? (mass of the sun = 2 x 10^{30} kg).

Answer: Acceleration due to gravity of the star,g= GM/R^2 ....(i)

Here M is the mass and R is the radius of the star.

The outward centrifugal force acting on a body of mass m at the equator of the star =mv2/R =mR w_2-------(ii)

From equation (i), the acceleration due to the gravity of the star

$$= \frac{6.67\times 10^{-11}\times 2.5\times 2\times 10^{30}}{\left(12\times 10^{3}\right)^2} = 2.316\times 10^{12}\ \text{m/s}^2$$

$\therefore$ Inward force due to gravity on a body of mass m

$$= m\times 2.316\times 10^{12}\ \text{N}$$

From equation (*ii*), the outward centrifugal force = $mR\omega^2$

$$= m\times(12\times 10^{3})\times\left(\frac{2\pi\times 1.5}{-1}\right)^2$$

$$= m\times 1.06\times 10^{6}\ \text{N}$$

Since the inward force due to gravity on a body at the equator of the star is about 2.2 million times more than the outward centrifugal force, the body will remain stuck to the surface of the star.

Question 8. 24. A spaceship is stationed on Mars. How much energy must be expended on the spaceship to rocket it out of the solar system? Mass of the spaceship = 1000 kg, Mass of the Sun = 2 x 10^{30} kg. Mass of the Mars = 6.4 x 10^{23} kg, Radius of Mars = 3395 km. Radius of the orbit of Mars = 2.28 x 10^{11} m, G = 6.67 x 10^{-11} N m^2 kg^{-2}.
Answer: Let R be the radius of orbit of Mars and R' be the radius of the Mars. M be the mass of the Sun and M' be the mass of Mars. If m is the mass of the space-ship, then Potential energy of space-ship due to gravitational attraction of the Sun = - GM m/R Potential energy of space-ship due to gravitational attraction of Mars = - G M' m/R' Since the K.E. of space ship is zero, therefore,

$$\text{total energy of spaceship} = -\frac{GM\,m}{R} - \frac{GM'\,m}{R'} = -G\,m\left(\frac{M}{R} + \frac{M'}{R'}\right)$$

∴ Energy required to rocket out the spaceship from the solar system = – (total energy of spaceship)

$$= -\left[-G\,m\left(\frac{M}{R} + \frac{M'}{R'}\right)\right] = G\,m\left[\frac{M}{R} + \frac{M'}{R'}\right]$$

$$= 6.67 \times 10^{-11} \times 1000 \times \left[\frac{2 \times 10^{30}}{2.28 \times 10^{11}} + \frac{6.4 \times 10^{23}}{3395 \times 10^{3}}\right]$$

$$= 6.67 \times 10^{-8} \left[\frac{20}{2.28} + \frac{6.4}{33.95}\right] \times 10^{18}\text{ J} = 5.98 \times 10^{11}\text{ J.}$$

Question 8. 25. A rocket is fired 'vertically' from the surface of Mars with a speed of 2 km s-1. If 20% of its initial energy is lost due to Martian atmospheric resistance, how far will the rocket go from the surface of mars before returning to it? Mass of Mars = 6.4 x 10^{23} kg; radius of Mars = 3395 km; G = 6.67 x 10^{-11} N m^2 kg^{-2}
Answer:

Initial K.E. = $\frac{1}{2}mv^2$; Initial P.E. = $-\frac{GMm}{R}$

where m = Mass of rocket, M = Mass of Mars, R = Radius of Mars

∴ Total initial energy = $\frac{1}{2}mv^2 - \frac{GMm}{R}$

Since 20% of K.E. is lost, only 80% remains to reach the height.

∴ Total initial energy available

$$= \frac{4}{5} \times \frac{1}{2}mv^2 - \frac{GMm}{R}$$

$$= 0.4\,mv^2 - \frac{GMm}{R}$$

When the rocket reaches the highest point, at a height h above the surface, its K.E. is zero and P.E. = $-\frac{GMm}{R+h}$.

Using principle of conservation of energy.

$$0.4\text{ mv}^2 - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

or $$\frac{GMm}{R+h} = \frac{GMm}{R} - 0.4\,mv^2 \Rightarrow \frac{GM}{R+h} = \frac{GM}{R} - 0.4\,v^2$$

or $$\frac{GM}{R+h} = \frac{1}{R}\,[GM - 0.4\,Rv^2] \Rightarrow \frac{R+h}{R} = \frac{GM}{GM - 0.4\,Rv^2}$$

or $$\frac{h}{R} = \frac{GM}{GM - 0.4\,Rv^2} - 1$$

or $$\frac{h}{R} = \frac{0.4\,Rv^2}{GM - 0.4\,Rv^2} \Rightarrow h = \frac{0.4\,R^2v^2}{GM - 0.4\,Rv^2}$$

or $$h = \frac{0.4 \times (2 \times 10^3)^2 \times (3.395 \times 10^6)^2}{6.67 \times 10^{-11} \times 6.4 \times 10^{23} - 0.4 \times (2 \times 10^3)^2 \times 3.395 \times 10^6}\text{ m}$$

$$= 495\text{ km.}$$

*********** END **********