Definite Integrals Ex 20.4B Q29

$$I = \int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}\,dx$$

$$I = \int_{-\pi}^{\pi} \frac{2x}{1+\cos^2 x}\,dx + \int_{-\pi}^{\pi} \frac{2x\sin x}{1+\cos^2 x}\,dx$$

$$I = 0 + \int_{-\pi}^{\pi} \frac{2x\sin x}{1+\cos^2 x}\,dx \ldots\ldots\left[\because \frac{2x}{1+\cos^2 x} \text{ is an odd function}\right]$$

$$I = 2\int_{0}^{\pi} \frac{2x\sin x}{1+\cos^2 x}\,dx \ldots\ldots\left[\because \frac{2x\sin x}{1+\cos^2 x} \text{ is an even function}\right]$$

$$I = 4\int_{0}^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$$

$$I = 2\pi\int_{0}^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx \ldots\ldots\left[\because \int_{0}^{a} xf(x)\,dx = \frac{a}{2}\int_{0}^{a} f(x)\,dx\right]$$

Put $\cos x = t$ then $-\sin x\,dx = dt$

$$I = -2\pi\int_{1}^{-1} \frac{1}{1+t^2}\,dt$$

$$I = -2\pi\left[\tan^{-1} t\right]_{1}^{-1}$$

$$I = \pi^2$$

Definite Integrals Ex 20.4B Q30

$$I = \int_{-a}^{a} \log\left(\frac{a-\sin\theta}{a+\sin\theta}\right)d\theta$$

Let $f(\theta) = \log\left(\frac{a-\sin\theta}{a+\sin\theta}\right)$

$$f(-\theta) = \log\left(\frac{a-\sin(-\theta)}{a+\sin(-\theta)}\right) = -\log\left(\frac{a-\sin\theta}{a+\sin\theta}\right) = -f(\theta)$$

$\therefore f(\theta) = \log\left(\frac{a-\sin\theta}{a+\sin\theta}\right)$ is an odd function.

$$\therefore I = \int_{-a}^{a} \log\left(\frac{a-\sin\theta}{a+\sin\theta}\right)d\theta = 0$$

Definite Integrals Ex 20.4B Q31

$$I = \int_{-2}^{2} \frac{3x^3 + 2|x| + 1}{x^2 + |x| + 1}\, dx$$

$$I = \int_{-2}^{2} \frac{3x^3}{x^2 + |x| + 1}\, dx + \int_{-2}^{2} \frac{2|x| + 1}{x^2 + |x| + 1}\, dx$$

$$I = 0 + \int_{-2}^{2} \frac{2|x| + 1}{x^2 + |x| + 1}\, dx \ldots\ldots\ldots \left[\because \frac{3x^3}{x^2 + |x| + 1} \text{ is an odd function}\right]$$

$$I = 2\int_{0}^{2} \frac{2|x| + 1}{x^2 + |x| + 1}\, dx \ldots\ldots\ldots \left[\because \frac{2|x| + 1}{x^2 + |x| + 1} \text{ is an even function}\right]$$

$$I = 2\left[\log\left(x^2 + |x| + 1\right)\right]_0^2$$

$$I = 2\left[\log(4 + 2 + 1) - \log(1)\right]$$

$$I = 2\log_e(7)$$

Definite Integrals Ex 20.4B Q32

$$I = \int_{-3\pi/2}^{-\pi/2} \left\{\sin^2(3\pi + x) + (\pi + x)^3\right\}\, dx$$

Substitute $\pi + x = u$ then $dx = du$

$$I = \int_{-\pi/2}^{\pi/2} \left\{\sin^2(2\pi + u) + (u)^3\right\}\, du$$

$$I = \int_{-\pi/2}^{\pi/2} \left\{\sin^2(u) + (u)^3\right\}\, du$$

$$I = \left[\frac{1}{2}\left(u - \frac{1}{2}\sin(2u)\right) + \frac{u^4}{4}\right]_{-\pi/2}^{\pi/2}$$

$$I = \frac{\pi}{2}$$

********** END **********