Cubes and Cubes Roots Ex 4.4 Q5

Answer :

(i)

Let us consider the following rational number:

$$\frac{-125}{729}$$

Now

$$\sqrt[3]{\frac{-125}{729}}$$

$$= \frac{\sqrt[3]{-125}}{\sqrt[3]{729}} \qquad (\because \sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}})$$

$$= \frac{-\sqrt[3]{125}}{\sqrt[3]{729}} \qquad (\because \sqrt[3]{-a} = -\sqrt[3]{a})$$

$$= -\frac{5}{9} \qquad (\because 729 = 9 \times 9 \times 9 \text{ and } 125 = 5 \times 5 \times 5)$$

(ii)

Let us consider the following rational number:

$$\frac{10648}{12167}$$

Now

$$\sqrt[3]{\frac{10648}{12167}}$$
$$= \frac{\sqrt[3]{10648}}{\sqrt[3]{12167}} \qquad (\because \sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}})$$

Cube root by factors:

On factorising 10648 into prime factors, we get:
$10648 = 2 \times 2 \times 2 \times 11 \times 11 \times 11$

On grouping the factors in triples of equal factors, we get:
$10648 = \{2 \times 2 \times 2\} \times \{11 \times 11 \times 11\}$

Now, taking one factor from each triple, we get:
$\sqrt[3]{10648} = 2 \times 11 = 22$

Also

On factorising 12167 into prime factors, we get:
$12167 = 23 \times 23 \times 23$

On grouping the factors in triples of equal factors, we get:
$12167 = \{23 \times 23 \times 23\}$

Now, taking one factor from the triple, we get:
$\sqrt[3]{12167} = 23$

Now

$$\sqrt[3]{\frac{10648}{12167}}$$
$$= \frac{\sqrt[3]{10648}}{\sqrt[3]{12167}}$$
$$= \frac{22}{23}$$

(iii)
Let us consider the following rational number:

$$\frac{-19683}{24389}$$
Now,

$$\sqrt[3]{\frac{-19683}{24389}}$$
$$= \frac{\sqrt[3]{-19683}}{\sqrt[3]{24389}} \qquad (\because \sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}})$$
$$= \frac{-\sqrt[3]{19683}}{\sqrt[3]{24389}} \qquad (\because \sqrt[3]{-a} = -\sqrt[3]{a})$$

Cube root by factors:

On factorising 19683 into prime factors, we get:
$19683 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

On grouping the factors in triples of equal factors, we get:
$19683 = \{3 \times 3 \times 3\} \times \{3 \times 3 \times 3\} \times \{3 \times 3 \times 3\}$

Now, taking one factor from each triple, we get:
$\sqrt[3]{19683} = 3 \times 3 \times 3 = 27$

Also

On factorising 24389 into prime factors, we get:
$24389 = 29 \times 29 \times 29$

On grouping the factors in triples of equal factors, we get:
$24389 = \{29 \times 29 \times 29\}$

Now, taking one factor from each triple, we get:
$\sqrt[3]{24389} = 29$

Now

$\sqrt[3]{\frac{-19683}{24389}}$
$= \frac{\sqrt[3]{-19683}}{\sqrt[3]{24389}}$
$= \frac{-\sqrt[3]{19683}}{\sqrt[3]{24389}}$
$= \frac{-27}{29}$

(iv)
Let us consider the following rational number:

$\frac{686}{-3456}$
Now

$\sqrt[3]{\frac{686}{-3456}}$
$= -\sqrt[3]{\frac{2 \times 7^3}{2^7 \times 3^3}}$ (686 and 3456 are not perfect cubes; therefore, we simplify it as $\frac{686}{3456}$ by prime factorisation.)

$$= -\sqrt[3]{\frac{7^3}{2^3 \times 3^3}}$$

$$= \frac{-\sqrt[3]{7^3}}{\sqrt[3]{2^3 \times 3^3}} \qquad (\because \sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}})$$

$$= \frac{-7}{\sqrt[3]{2^3 \times 2^3 \times 3^3}}$$

$$= \frac{-7}{2 \times 2 \times 3}$$

$$= \frac{-7}{12}$$

(v)

Let us consider the following rational number:

$$\frac{-39304}{-42875}$$

Now

$$\sqrt[3]{\frac{-39304}{-42875}}$$

$$= \frac{\sqrt[3]{-39304}}{\sqrt[3]{-42875}} \qquad (\because \sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}})$$

$$= \frac{-\sqrt[3]{39304}}{-\sqrt[3]{42875}} \qquad (\because \sqrt[3]{-a} = -\sqrt[3]{a})$$

Cube root by factors:

On factorising 39304 into prime factors, we get:
$39304 = 2 \times 2 \times 2 \times 17 \times 17 \times 17$

On grouping the factors in triples of equal factors, we get:
$39304 = \{2 \times 2 \times 2\} \times \{17 \times 17 \times 17\}$

Now, taking one factor from each triple, we get:
$\sqrt[3]{39304} = 2 \times 17 = 34$

Also

On factorising 42875 into prime factors, we get:
$42875 = 5 \times 5 \times 5 \times 7 \times 7 \times 7$

On grouping the factors in triples of equal factors, we get:
$42875 = \{5 \times 5 \times 5\} \times \{7 \times 7 \times 7\}$

Now, taking one factor from each triple, we get:
$\sqrt[3]{42875} = 5 \times 7 = 35$

Now

$$\sqrt[3]{\frac{-39304}{-42875}}$$

$$= \frac{\sqrt[3]{-39304}}{\sqrt[3]{-42875}}$$

$$= \frac{-\sqrt[3]{39304}}{-\sqrt[3]{42875}}$$

$$= \frac{-34}{-35}$$

$$= \frac{34}{35}$$

********** END *********