Areas of Parallelograms and Triangles Ex 15.3 Q24

Answer :

Given: In ΔABC

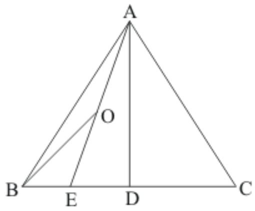
(1) D is the midpoint of the side BC

(2) E is the midpoint of the side BD

(3) O is the midpoint of the side AE

To prove: $\text{ar}(\Delta BOE)=\frac{1}{8}\text{ar}(\Delta ABC)$

Proof: We know that the median of a triangle divides the triangle into two triangles of equal area.



Since AD and AE are the medians of ΔABC and ΔABD respectively. And OB is the median of ΔABE

$$\text{ar}(\Delta ABD)=\frac{1}{2}\text{ar}(\Delta ABC) \quad \ldots\ldots (1)$$

$$\text{ar}(\Delta ABE)=\frac{1}{2}\text{ar}(\Delta ABD) \quad \ldots\ldots (2)$$

$$\text{ar}(\Delta BOE)=\frac{1}{2}\text{ar}(\Delta ABE) \quad \ldots\ldots (3)$$

Therefore

$$\text{ar}(\Delta BOE)=\frac{1}{2}\left(\frac{1}{2}\text{ar}(\Delta ABD)\right)(\text{from } 2)$$

$$\text{ar}(\Delta BOE)=\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\text{ar}(\Delta ABC)\right)\right)(\text{from } 1)$$

$$\text{ar}(\Delta BOE)=\frac{1}{8}\text{ar}(\Delta ABC)$$

Hence we have proved that

$$\boxed{\text{ar}(\Delta BOE)=\frac{1}{8}\text{ar}(\Delta ABC)}$$

Areas of Parallelograms and Triangles Ex 15.3 Q25

Answer :

Given:

(1) X and Y are the, midpoints of AC and AB respectively.

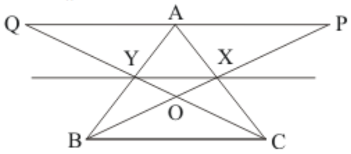
(2) QP|| BC

(3) CYQ and BXP are straight lines.

To prove: $\text{ar}(\Delta ABP) = \text{ar}(\Delta ACQ)$

Proof: Since X and Y are the, midpoints of AC and AB respectively.

So XY||BC



ΔBYC and ΔBXC are on the same base BC and between the same parallels XY and BC.

Therefore

$$\text{ar}(\Delta BYC) = \text{ar}(\Delta BXC)$$
$$\text{ar}(\Delta BYC) - \text{ar}(\Delta BOC) = \text{ar}(\Delta BXC) - \text{ar}(\Delta BOC)$$
$$\text{ar}(\Delta BOY) = \text{ar}(\Delta COX)$$
$$\text{ar}(\Delta BOY) + \text{ar}(\Delta XOY) = \text{ar}(\Delta COX) + \text{ar}(\Delta XOY)$$
$$\text{ar}(\Delta BXY) = \text{ar}(\Delta CXY) \quad \ldots\ldots (1)$$

Similarly the quadrilaterals XYAP and XYQA are on the same base XY and between the same parallels XY and PQ. Therefore

$$\text{ar}(\text{quad XYAP}) = \text{ar}(\text{quad XYQA}) \quad \ldots\ldots (2)$$

Adding equation 1 and 2 we get

$$\text{ar}(\Delta BXY) + \text{ar}(\text{quad XYAP}) = \text{ar}(\Delta CXY) + \text{ar}(\text{quad XYQA})$$
$$\Rightarrow \text{ar}(\Delta ABP) = \text{ar}(\Delta ACQ)$$

Hence we had proved that

$$\boxed{\text{ar}(\Delta ABP) = \text{ar}(\Delta ACQ)}$$

*********** END **********