Functions Ex 2.3 Q7

We are given that f is a real function and g is a function given by $g(x) = 2x$

To prove; $g \circ f = f + f$.

L.H.S

$g \circ f(x) = g(f(x)) = 2f(x)$

$\qquad = f(x) + f(x) = $ R.H.S

$\Rightarrow \qquad g \circ f = f + f$

Functions Ex 2.3 Q8

$$f(x) = \sqrt{1-x} \ , \qquad g(x) = \log_e^x$$

Domain of f and g are R.

$$\text{Range of } f = (-\infty, 1)$$

$$\text{Range of } g = (0, e)$$

Clearly Range $f \subset$ Domain $g \Rightarrow g \circ f$ exists

$\qquad$ Range $g \subset$ Domain $f \Rightarrow f \circ g$ exists

$$\therefore \ g \circ f(x) = g(f(x)) = g\left(\sqrt{1-x}\right)$$

$$g \circ f(x) = \log_e^{\sqrt{1-x}}$$

Again

$$f \circ g(x) = f(g(x)) = f\left(\log_e^x\right)$$

$$f \circ g(x) = \sqrt{1 - \log_e^x}$$

Functions Ex 2.3 Q9

$f : \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right) \to R$ and $g : [-1, 1] \to R$ defined as $f(x) = \tan x$ and $g(x) = \sqrt{1-x^2}$

Range of f : let $y = f(x) \Rightarrow y = \tan x$

$$\Rightarrow x = \tan^{-1} y$$

Since $x \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right), y \in (-\infty, \infty)$

∴ Range of $f \subset$ domain of $g = [-1, 1]$

∴ $g \circ f$ exists.

By similar argument $f \circ g$ exists.

Now,

$$f \circ g(x) = f(g(x)) = f\left(\sqrt{1-x^2}\right)$$

$$f \circ g(x) = \tan \sqrt{1-x^2}$$

Again

$$g \circ f(x) = g(f(x))$$
$$= g(\tan x)$$
$$g \circ f(x) = \sqrt{1 - \tan^2 x}$$

Functions Ex 2.3 Q10

$$f(x) = \sqrt{x+3} \text{ and } g(x) = x^2 + 1$$

Now,

$$\text{Range of } f = [-3, \infty] \text{ and}$$
$$\text{Range of } g = (1, \infty)$$

Then, Range of $f \subset$ Domain g and
Range of $g \subset$ Domain f

∴ $f \circ g$ and $g \circ f$ exist.

Now,

$$f \circ g(x) = f(g(x)) = f(x^2 + 1)$$
$$f \circ g(x) = \sqrt{x^2 + 4}$$

Again,

$$g \circ f(x) = g(f(x)) = g\left(\sqrt{x+3}\right)$$
$$= \left(\sqrt{x+3}\right)^2 + 1$$
$$g \circ f(x) = x + 4$$

Functions Ex 2.3 Q11(i)

We have, $f(x) = \sqrt{x-2}$

Clearly, Domain$(f) = [2, \infty)$ and Range$(f) = [0, \infty)$.

We observe that range(f) is not a subset of domain of f.

$\therefore$ Domain of $(fof) = \{x : x \in \text{Domain } (f) \text{ and } f(x) \in \text{Domain } (f)\}$

$\qquad\qquad\qquad = \{x : x \in [2, \infty) \text{ and } \sqrt{x-2} \in [2, \infty)\}$

$\qquad\qquad\qquad = \{x : x \in [2, \infty) \text{ and } \sqrt{x-2} \geq 2\}$

$\qquad\qquad\qquad = \{x : x \in [2, \infty) \text{ and } x-2 \geq 4\}$

$\qquad\qquad\qquad = \{x : x \in [2, \infty) \text{ and } x \geq 6\}$

$\qquad\qquad\qquad = [6, \infty)$

Now,

$(fof)(x) = f(f(x)) = f(\sqrt{x-2}) = \sqrt{\sqrt{x-2}-2}$

$\therefore$ fof $: [6, \infty) \to R$ defined as

$(fof)(x) = \sqrt{\sqrt{x-2}-2}$

********** END *********