



Differentiation Ex 11.7 Q10

Here,

$$x = e^{\theta} \left( \theta + \frac{1}{\theta} \right)$$

Differentiating it with respect to  $\theta$  using product rule,

$$\begin{aligned} \frac{dx}{d\theta} &= e^{\theta} \frac{d}{d\theta} \left( \theta + \frac{1}{\theta} \right) + \left( \theta + \frac{1}{\theta} \right) \frac{d}{d\theta} (e^{\theta}) \\ &= e^{\theta} \left( 1 - \frac{1}{\theta^2} \right) + \left( \frac{\theta^2 + 1}{\theta} \right) e^{\theta} \\ &= e^{\theta} \left( 1 - \frac{1}{\theta^2} + \frac{\theta^2 + 1}{\theta} \right) \\ &= e^{\theta} \left( \frac{\theta^2 - 1 + \theta^3 + \theta}{\theta^2} \right) \\ \frac{dx}{d\theta} &= \frac{e^{\theta} (\theta^3 + \theta^2 + \theta - 1)}{\theta^2} \quad \text{---(i)} \end{aligned}$$

And,  $y = e^{\theta} \left( \theta - \frac{1}{\theta} \right)$

Differentiating it with respect to  $\theta$  using product rule and chain rule,

$$\begin{aligned} \frac{dy}{d\theta} &= e^{-\theta} \frac{d}{d\theta} \left( \theta - \frac{1}{\theta} \right) + \left( \theta - \frac{1}{\theta} \right) \frac{d}{d\theta} (e^{-\theta}) \\ &= e^{-\theta} \left( 1 + \frac{1}{\theta^2} \right) + \left( \theta - \frac{1}{\theta} \right) e^{-\theta} \frac{d}{d\theta} (-\theta) \\ &= e^{-\theta} \left( 1 + \frac{1}{\theta^2} \right) + \left( \theta - \frac{1}{\theta} \right) e^{-\theta} (-1) \\ \frac{dy}{d\theta} &= e^{-\theta} \left[ 1 + \frac{1}{\theta^2} - \theta + \frac{1}{\theta} \right] \\ &= e^{-\theta} \left[ \frac{\theta^2 + 1 - \theta^3 + \theta}{\theta^2} \right] \\ \frac{dy}{d\theta} &= e^{-\theta} \left[ \frac{-\theta^3 + \theta^2 + \theta + 1}{\theta^2} \right] \quad \text{---(ii)} \end{aligned}$$

Differentiation Ex 11.7 Q11

Here,

$$x = \frac{2t}{1+t^2}$$

Differentiating it with respect to  $t$  using quotient rule,

$$\begin{aligned} \frac{dy}{dx} &= \left[ \frac{(1+t^2) \frac{d}{dt}(2t) - 2t \frac{d}{dt}(1+t^2)}{(1+t^2)^2} \right] \\ &= \left[ \frac{(1+t^2)(2) - 2t(2t)}{(1+t^2)^2} \right] \\ &= \left[ \frac{2+2t^2-4t^2}{(1+t^2)^2} \right] \\ &= \left[ \frac{2-2t^2}{(1+t^2)^2} \right] \\ \frac{dx}{dt} &= \frac{2(1-t^2)}{(1+t^2)^2} \quad \text{---(i)} \end{aligned}$$

And,  $y = \frac{1-t^2}{1+t^2}$

Differentiating it with respect to  $t$  using quotient rule,

$$\begin{aligned} \frac{dy}{dt} &= \left[ \frac{(1+t^2) \frac{d}{dt}(1-t^2) - (1-t^2) \frac{d}{dt}(1+t^2)}{(1+t^2)^2} \right] \\ &= \left[ \frac{(1+t^2)(-2t) - (1-t^2)(2t)}{(1+t^2)^2} \right] \\ &= \left[ \frac{-2t-2t^3-2t+2t^3}{(1+t^2)^2} \right] \\ \frac{dy}{dt} &= \left[ \frac{-4t}{(1+t^2)^2} \right] \quad \text{---(ii)} \end{aligned}$$

Dividing equation (ii) by (i),

$$\begin{aligned} \frac{\frac{dy}{dt}}{\frac{dx}{dt}} &= \frac{-4t}{(1+t^2)^2} \times \frac{(1+t^2)^2}{2(1-t^2)} \\ &= \frac{-2t}{1-t^2} \\ \frac{dy}{dx} &= -\frac{x}{y} \quad \left[ \text{Since, } \frac{x}{y} = \frac{2t}{1+t^2} \times \frac{1+t^2}{1-t^2} = \frac{2t}{1-t^2} \right] \end{aligned}$$

Differentiation Ex 11.7 Q12

Here,

$$x = \cos^{-1} \left( \frac{1}{\sqrt{1+t^2}} \right)$$

Differentiating it with respect to  $t$  using chain rule,

$$\frac{dx}{dt} = \frac{-1}{\sqrt{1-\left(\frac{1}{\sqrt{1+t^2}}\right)^2}} \frac{d}{dt} \left( \frac{1}{\sqrt{1+t^2}} \right)$$

$$\begin{aligned}
 & y = \sin^{-1} \left( \frac{1}{\sqrt{1+t^2}} \right) \\
 & = \frac{-1}{\sqrt{1 - \frac{1}{(1+t^2)}}} \left\{ \frac{-1}{2(1+t^2)^{\frac{3}{2}}} \right\} \frac{d}{dt} (1+t^2) \\
 & = \frac{(1+t^2)^{\frac{1}{2}}}{\sqrt{1+t^2} - 1} \times \frac{-1}{2(1+t^2)^{\frac{3}{2}}} (2t) \\
 & = \frac{-t}{\sqrt{t^2} \times (1+t^2)} \\
 \frac{dx}{dt} &= \frac{-1}{1+t^2} \quad \text{---(i)}
 \end{aligned}$$

Now,  $y = \sin^{-1} \left( \frac{1}{\sqrt{1+t^2}} \right)$

Differentiating it with respect to  $t$  using chain rule,

$$\begin{aligned}
 \frac{dy}{dt} &= \frac{1}{\sqrt{1 - \frac{1}{(\sqrt{1+t^2})^2}}} \times \frac{d}{dt} \left( \frac{1}{\sqrt{1+t^2}} \right) \\
 &= \frac{(1+t^2)^{\frac{1}{2}}}{\sqrt{1+t^2} - 1} \times \left( \frac{-1}{2(1+t^2)^{\frac{3}{2}}} \right) \frac{d}{dt} (1+t^2) \\
 &= \frac{-1}{2\sqrt{t^2} (1+t^2)} \times (2t) \\
 \frac{dy}{dt} &= \frac{-1}{(1+t^2)} \quad \text{---(ii)}
 \end{aligned}$$

Dividing equation (ii) by (i),

$$\begin{aligned}
 \frac{\frac{dy}{dt}}{\frac{dx}{dt}} &= -\frac{1}{(1+t^2)} \times \frac{(1+t^2)}{-1} \\
 \frac{dy}{dx} &= 1
 \end{aligned}$$

\*\*\*\*\* END \*\*\*\*\*