



Mean Value Theorems Ex 15.1 Q1(i)

$$f(x) = 3 + (x - 2)^{\frac{2}{3}} \text{ on } [1, 3]$$

Differentiating it with respect to x ,

$$f'(x) = \frac{2}{3} \times \frac{1}{(x - 2)^{\frac{1}{3}}}$$

$$\text{Clearly, } \lim_{x \rightarrow 2} = \frac{2}{3} \times \frac{1}{(x - 2)^{\frac{1}{3}}}$$

Thus, $f(x)$ is not differentiable at $x = 2 \in (1, 3)$

Hence, Rolle's theorem is not applicable for $f(x)$ in $x \in [1, 3]$.

Mean Value Theorems Ex 15.1 Q1(ii)

Here, $f(x) = [x]$ and $x \in [-1, 1]$, at $n = 1$

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow (1-h)} [x] \\ &= \lim_{h \rightarrow 0} [1 - h] \\ &= 0 \end{aligned}$$

$$\begin{aligned} \text{RHL} &= \lim_{x \rightarrow (1+h)} [x] \\ &= \lim_{h \rightarrow 0} [1 + h] \\ &= 1 \end{aligned}$$

$$\text{LHL} \neq \text{RHL}$$

So, $f(x)$ is not continuous at $1 \in [-1, 1]$

Hence, Rolle's theorem is not applicable on $f(x)$ in $[-1, 1]$.

Mean Value Theorems Ex 15.1 Q1(iii)

Here, $f(x) = \sin\left(\frac{1}{x}\right)$, $x \in [-1, 1]$, at $n = 0$

$$\begin{aligned}\text{LHS} &= \lim_{x \rightarrow (0-h)} \sin\left(\frac{1}{x}\right) \\ &= \lim_{h \rightarrow 0} \sin\left(\frac{1}{0-h}\right) \\ &= \lim_{h \rightarrow 0} \sin\left(\frac{-1}{h}\right) \\ &= -\lim_{h \rightarrow 0} \sin\left(\frac{1}{h}\right) \\ &= -k \quad \left[\text{Let } \lim_{h \rightarrow 0} \sin\left(\frac{1}{h}\right) = k \text{ as } k \in [-1, 1] \right]\end{aligned}$$

$$\begin{aligned}\text{RHS} &= \lim_{x \rightarrow (0+h)} \sin\left(\frac{1}{x}\right) \\ &= \lim_{h \rightarrow 0} \sin\left(\frac{1}{h}\right) \\ &= k\end{aligned}$$

$$\Rightarrow \text{LHS} \neq \text{RHS}$$

$$\Rightarrow f(x) \text{ is not continuous at } n = 0$$

So, Rolle's theorem is not applicable on $f(x)$ in $[-1, 1]$

Mean Value Theorems Ex 15.1 Q1(iv)

Here, $f(x) = 2x^2 - 5x + 3$ on $[1, 3]$

$f(x)$ is continuous in $[1, 3]$ and $f(x)$ is differentiable in $(1, 3)$ since it is a polynomial function.

Now,

$$f(x) = 2x^2 - 5x + 3$$

$$\begin{aligned}f(1) &= 2(1)^2 - 5(1) + 3 \\ &= 2 - 5 + 3\end{aligned}$$

$$f(1) = 0 \quad \text{---(i)}$$

$$\begin{aligned}f(3) &= 2(3)^2 - 5(3) + 3 \\ &= 18 - 15 + 3\end{aligned}$$

$$f(3) = 6 \quad \text{---(ii)}$$

From equation (i) and (ii),

$$f(1) \neq f(3)$$

So, Rolle's theorem is not applicable on $f(x)$ in $[1, 3]$.

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