Therefore, 100 kg of fertiliser F_1 and 80 kg of fertilizer F_2 should be used to minimize the cost. The minimum cost is Rs 1000.

Question 11:

The corner points of the feasible region determined by the following system of linear inequalities:

$2x+y\leq 10, x+3y\leq 15, xy\geq 0$ are $(0,0),(5,0),(3,4)$ and $(0,5)$ Let Z = $px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of Z occurs at both (3, 4) and (0, 5) is

(A) $p = q$ (B) $p = 2q$ (C) $p = 3q$ (D) $q = 3p$

Answer

The maximum value of Z is unique.

It is given that the maximum value of Z occurs at two points, (3, 4) and (0, 5).

∴ Value of Z at (3, 4) = Value of Z at (0, 5)

$\Rightarrow p(3) + q(4) = p(0) + q(5)$

$\Rightarrow 3p + 4q = 5q$

$\Rightarrow q = 3p$

Hence, the correct answer is D.

Miscellaneous Solutions

Question 1:

Refer to Example 9. How many packets of each food should be used to maximize the amount of vitamin A in the diet? What is the maximum amount of vitamin A in the diet?

Answer

Let the diet contain x and y packets of foods P and Q respectively. Therefore,

$x \geq 0$ and $y \geq 0$

The mathematical formulation of the given problem is as follows.

Maximize $z = 6x + 3y$ ... (1)

subject to the constraints,

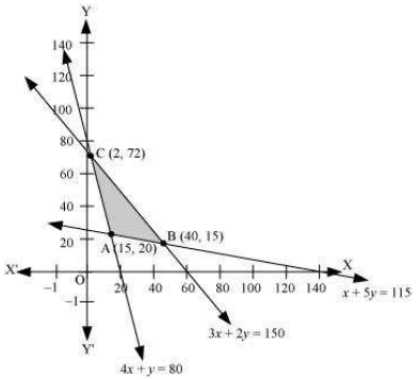
$4x+y\geq 80$ ...(2)

$x+5y\geq 115$ ...(3)

$3x+2y\leq 150$ ...(4)

$x, y\geq 0$ ...(5)

The feasible region determined by the system of constraints is as follows.



The corner points of the feasible region are A (15, 20), B (40, 15), and C (2, 72).

The values of z at these corner points are as follows.

Corner point	$z = 6x + 3y$	
A(15, 20)	150	
B(40, 15)	285	→ Maximum
C(2, 72)	228	

Thus, the maximum value of z is 285 at (40, 15).

Therefore, to maximize the amount of vitamin A in the diet, 40 packets of food P and 15 packets of food Q should be used. The maximum amount of vitamin A in the diet is 285 units.

Question 2:

Question 2:

A farmer mixes two brands P and Q of cattle feed. Brand P, costing Rs 250 per bag contains 3 units of nutritional element A, 2.5 units of element B and 2 units of element C. Brand Q costing Rs 200 per bag contains 1.5 units of nutritional elements A, 11.25 units of element B, and 3 units of element C. The minimum requirements of nutrients A, B and C are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag? What is the minimum cost of the mixture per bag?

Answer

Let the farmer mix x bags of brand P and y bags of brand Q.

The given information can be compiled in a table as follows.

	Vitamin A (units/kg)	Vitamin B (units/kg)	Cost (Rs/kg)
Food P	3	5	60
Food Q	4	2	80
Requirement (units/kg)	8	11	

The given problem can be formulated as follows.

Minimize $z = 250x + 200y$ ... (1)

subject to the constraints,

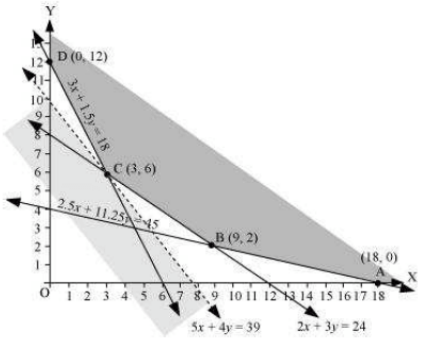
$3x+1.5y \geq 18$ ...(2)

$2.5x+11.25y \geq 45$ ...(3)

$2x+3y \geq 24$ ...(4)

$x, y \geq 0$ ...(5)

The feasible region determined by the system of constraints is as follows.



The corner points of the feasible region are A (18, 0), B (9, 2), C (3, 6), and D (0, 12).

The values of z at these corner points are as follows.

Corner point	z = 250x + 200y	
A (18, 0)	4500	
B (9, 2)	2650	
C (3, 6)	1950	→ Minimum
D (0, 12)	2400	

As the feasible region is unbounded, therefore, 1950 may or may not be the minimum value of z.

For this, we draw a graph of the inequality, $250x + 200y < 1950$ or $5x + 4y < 39$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $5x + 4y < 39$

Therefore, the minimum value of z is 2000 at (3, 6).

Thus, 3 bags of brand P and 6 bags of brand Q should be used in the mixture to minimize the cost to Rs 1950.

Question 3:

A dietician wishes to mix together two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin content of one kg food is given below:

Food	Vitamin A	Vitamin B	Vitamin C
X	1	2	3
Y	2	2	1

One kg of food X costs Rs 16 and one kg of food Y costs Rs 20. Find the least cost of the mixture which will produce the required diet?

Answer

Let the mixture contain x kg of food X and y kg of food Y.

The mathematical formulation of the given problem is as follows.

Minimize $z = 16x + 20y$ ... (1)

subject to the constraints,

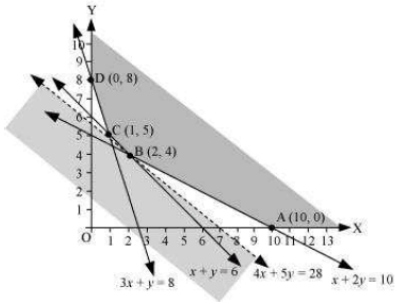
$x + 2y \geq 10$ ...(2)

$x + y \geq 6$ ...(3)

$3x + y \geq 8$ ...(4)

$x, y \geq 0$ ...(5)

The feasible region determined by the system of constraints is as follows.



The corner points of the feasible region are A (10, 0), B (2, 4), C (1, 5), and D (0, 8).

********** END **********