Indefinite Integrals Ex 19.12 Q10

Let $I = \int \frac{1}{\sin^4 x \cos^2 x} dx$ ---(i)

Then, $I = \int \sin^{-4} x \cos^{-2} x dx$

Since $-4-2=-6$, which is even integer. So, we divide both numerator and denominator by $\cos^6 x$.

$$\therefore \quad I = \int \frac{\frac{1}{\cos^6 x}}{\frac{\sin^4 x \cos^2 x}{\cos^6 x}} dx$$

$$= \int \frac{\sec^6 x}{\frac{\sin^4 x}{\cos^4 x}} dx$$

$$= \int \frac{\sec^6 x}{\tan^4 x} dx$$

$$= \int \frac{\sec^4 x \times \sec^2 x}{\tan^4 x} dx$$

$$= \int \frac{\left(\sec^2 x\right)^2 \times \sec^2 x}{\tan^4 x} dx$$

$$= \int \frac{\left(1+\tan^2 x\right)^2 \times \sec^2 x}{\tan^4 x} dx$$

$$\Rightarrow \quad I = \int \frac{\left(1+\tan^4 x + 2\tan^2 x\right) \times \sec^2 x}{\tan^4 x} dx \qquad \text{---(ii)}$$

Let $\tan x = t$. Then,

$$d(\tan x) = dt$$

$$\Rightarrow \quad \sec^2 x dx = dt$$

$$\Rightarrow \quad dx = \frac{dt}{\sec^2 x}$$

Putting $\tan x = t$ and $dx = \frac{dt}{\sec^2 x}$ in equation (i), we get

$$I = \int \frac{\left(1+t^4+2t^2\right)}{t^4} \times \sec^2 x \times \frac{dt}{\sec^2 x}$$

$$= \int \left(t^{-4}+1+2t^{-2}\right) dt$$

$$= -\frac{t^{-3}}{3} + t - 2t^{-1} + c$$

$$= -\frac{1}{3t^3} + t - \frac{2}{t} + c$$

$$= -\frac{1}{3\tan^3 x} + \tan x - \frac{2}{\tan x} + c$$

$$= -\frac{1}{3} \times \cot^3 x + \tan x - 2 \times \cot x + c$$

$$\therefore \quad I = \frac{-1}{3} \times \cot^3 x - 2\cot x + \tan x + c$$

Indefinite Integrals Ex 19.12 Q11

Let $I = \int \frac{1}{\sin^3 x \cos^5 x} dx$ ---(i)

Then, $I = \int \sin^{-3} x \cos^{-5} x dx$

Since $-3-5=-8$, which is even integer. So, we divide both numerator and denominator by $\cos^8 x$.

$$\therefore \quad I = \int \frac{\frac{1}{\cos^8 x}}{\frac{\sin^3 x \cos^5 x}{\cos^8 x}} dx$$

$$= \int \frac{\sec^8 x}{\tan^3 x} dx$$

$$= \int \frac{\sec^8 x}{\tan^3 x} \sec^2 x dx$$

$$= \int \frac{\left(\sec^2 x\right)^3}{\tan^3 x} \sec^2 x dx$$

$$= \int \frac{\left(1+\tan^2 x\right)^3}{\tan^3 x} \times \sec^2 x dx$$

$$\Rightarrow \quad I = \int \frac{\left(1+\tan^6 x + 3\tan^4 x + 3\tan^2 x\right) \times \sec^2 x}{\tan^3 x} dx \quad \text{---(ii)}$$

Let $t = \tan x$. Then,

$$d\left(\tan x\right) = dt$$

$$\Rightarrow \quad \sec^2 x dx = dt$$

$$\therefore \quad I = \int \frac{\left(1+t^6+3t^4+3t^2\right)}{t^3} dt$$

$$= \int \left(t^{-3}+t^3+3t+3t^{-1}\right) dt$$

$$= -\frac{t^{-2}}{2} + \frac{t^4}{4} + \frac{3}{2}t^2 + 3\log t + c$$

$$= -\frac{1}{2t^2} + \frac{t^4}{4} + \frac{3}{2}t^2 + 3\log t + c$$

$$= -\frac{1}{2} \times \frac{1}{\tan^2 x} + \frac{\tan^4 x}{4} + \frac{3}{2} \times \tan^2 x + 3\log\left|\tan x\right| + c$$

$$\therefore \quad I = \frac{-1}{2\tan^2 x} + 3\log\left|\tan x\right| + \frac{3}{2}\tan^2 x + \frac{1}{4} \times \tan^4 x + c$$

Indefinite Integrals Ex 19.12 Q12

Let $I = \int \frac{1}{\sin^3 x \cos x} dx$ ---(i)

Then, $I = \int \sin^{-3} x \cos^{-1} x dx$

Since $-3-1=-4$, which is even integer. So, we divide both numerator and denominator by $\cos^4 x$.

$$\therefore \quad I = \int \frac{\frac{1}{\cos^4 x}}{\frac{\sin^3 x \cos x}{\cos^4 x}} dx$$

$$= \int \frac{\sec^4 x}{\tan^3 x} dx$$

$$= \int \frac{\sec^4 x}{\tan^3 x} \sec^2 x dx$$

$$= \int \frac{\left(\sec^2 x\right)^3}{\tan^3 x} \sec^2 x dx$$

$$= \int \frac{\left(1+\tan^2 x\right)}{\tan^3 x} \times \sec^2 x dx$$

Putting $\tan x = t$ and $\sec^2 x dx = dt$, we get

$$I = \int \frac{1+t^2}{t^3} dt$$

$$= \int \left(t^{-3} + \frac{1}{t}\right) dt$$

$$= -\frac{t^{-2}}{2} + \log\left|t\right| + c$$

$$= -\frac{1}{2t^2} + \log\left|t\right| + c$$

$$= -\frac{1}{2\tan^2 x} + \log\left|\tan x\right| + c$$

Indefinite Integrals Ex 19.12 Q13

$$\frac{1}{\sin x\cos^3 x}=\frac{\sin^2 x+\cos^2 x}{\sin x\cos^3 x}$$

$$=\frac{\sin x}{\cos^3 x}+\frac{1}{\sin x\cos x}$$

$$=\tan x\sec^2 x+\frac{1\cos^2 x}{\frac{\sin x\cos x}{\cos^2 x}}$$

$$=\tan x\sec^2 x+\frac{\sec^2 x}{\tan x}$$

$$\therefore\ \int\frac{1}{\sin x\cos^3 x}dx=\int\tan x\sec^2 x\ dx+\int\frac{\sec^2 x}{\tan x}\ dx$$

Let $\tan x=t\Rightarrow\sec^2 x\ dx=dt$

$$\Rightarrow\int\frac{1}{\sin x\cos^3 x}dx=\int t\,dt+\int\frac{1}{t}dt$$

$$=\frac{t^2}{2}+\log|t|+\mathrm{C}$$

$$=\frac{1}{2}\tan^2 x+\log|\tan x|+\mathrm{C}$$

*********** END **********