



Exercise 2.3

Q 3. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

- (i) $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$
- (ii) $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$
- (iii) $x^3 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$

Answer :

(i) $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

$$t^2 - 3 = t^2 + 0.t - 3$$

$$\begin{array}{r}
 \overline{2t^2 + 3t + 4} \\
 t^2 + 0.t - 3 \overline{) 2t^4 + 3t^3 - 2t^2 - 9t - 12} \\
 \underline{2t^4 + 0.t^3 - 6t^2} \\
 - - + \\
 \hline
 3t^3 + 4t^2 - 9t - 12 \\
 3t^3 + 0.t^2 - 9t \\
 \underline{- - + } \\
 4t^2 + 0.t - 12 \\
 4t^2 + 0.t - 12 \\
 \underline{- - + } \\
 0
 \end{array}$$

Since the remainder is 0,

Hence, $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

$$(ii) \quad x^2 + 3x + 1, \quad 3x^4 + 5x^3 - 7x^2 + 2x + 2$$

$$\begin{array}{r}
 \overline{3x^2 - 4x + 2} \\
 x^2 + 3x + 1 \overline{) 3x^4 + 5x^3 - 7x^2 + 2x + 2} \\
 \underline{3x^4 + 9x^3 + 3x^2} \\
 -4x^3 - 10x^2 + 2x + 2 \\
 \underline{-4x^3 - 12x^2 - 4x} \\
 +2x^2 + 6x + 2 \\
 \underline{+2x^2 + 6x + 2} \\
 0
 \end{array}$$

Since the remainder is 0,

Hence, $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$.

(iii) $x^3 - 3x + 1$, $x^5 - 4x^3 + x^2 + 3x + 1$

$$\begin{array}{r}
 \overline{x^2 - 1} \\
 x^3 - 3x + 1 \overline{) x^5 - 4x^3 + x^2 + 3x + 1} \\
 \underline{x^5 - 3x^3 + x^2} \\
 - + - \\
 \underline{-x^3} + 3x + 1 \\
 \underline{-x^3} + 3x - 1 \\
 \underline{+} - + \\
 \underline{2} \\
 \underline{}
 \end{array}$$

Since the remainder $\neq 0$,

Hence, $x^3 - 3x + 1$ is not a factor of $x^5 - 4x^3 + x^2 + 3x + 1$.

Q 4. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, - 7, - 14 respectively.

Answer :

Let the polynomial be $ax^3 + bx^2 + cx + d$ and the zeroes be α, β , and γ .

It is given that

$$\alpha + \beta + \gamma = \frac{2}{1} = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{-7}{1} = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-14}{1} = \frac{-d}{a}$$

If $a = 1$, then $b = - 2$, $c = - 7$, $d = 14$

Hence, the polynomial is $x^3 - 2x^2 - 7x + 14$.

Q 5. Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Answer :

$$p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$$

Since the two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$,

$\therefore \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = \left(x^2 - \frac{5}{3}\right)$ is a factor of $3x^4 + 6x^3 - 2x^2 - 10x - 5$.

Therefore, we divide the given polynomial by $x^2 - \frac{5}{3}$.

$$\begin{array}{r} x^2 + 0x - \frac{5}{3} \overline{) 3x^4 + 6x^3 - 2x^2 - 10x - 5} \\ \underline{3x^4 + 0x^3 - 5x^2} \\ - x^3 + 3x^2 - 10x - 5 \\ \underline{6x^3 + 0x^2 - 10x} \\ - x^2 + 0x - 5 \\ \underline{3x^2 + 0x - 5} \\ - x^2 + 0x - 5 \\ \underline{3x^2 + 0x - 5} \\ 0 \end{array}$$

$$\begin{aligned} 3x^4 + 6x^3 - 2x^2 - 10x - 5 &= \left(x^2 - \frac{5}{3}\right)(3x^2 + 6x + 3) \\ &= 3\left(x^2 - \frac{5}{3}\right)(x^2 + 2x + 1) \end{aligned}$$

We factorize $x^2 + 2x + 1$

$$= (x+1)^2$$

Therefore, its zero is given by $x + 1 = 0$

$$x = -1$$

As it has the term $(x+1)^2$, therefore, there will be 2 zeroes at $x = -1$.

***** END *****