Mean Value Theorems Ex 15.1 Q3(xii)

Here,

$$f(x) = 2\sin x + \sin 2x \text{ on } [0, \pi]$$

We know that sine function is continuous and differentiable every where, so $f(x)$ is continuous is $[0, \pi]$ and differentiable is $(0, \pi)$.

Now,

$$f(0) = 2\sin 0 + \sin 0 = 0$$

$$f(\pi) = 2\sin \pi + \sin 2\pi = 0$$

$$\Rightarrow \quad f(0) = f(\pi)$$

So, Rolle's theorem is applicable, so there must exist a point $c \in (0, \pi)$ such that $f'(c) = 0$.

Now,

$$f(x) = 2\sin x + \sin 2x$$

$$f'(x) = 2\cos x + 2\cos 2x$$

Now,

$$f'(c) = 0$$

$$2\cos c + 2\cos 2c = 0$$

$$\Rightarrow \quad 2\left(\cos c + 2\cos^2 c - 1\right) = 0$$

$$\Rightarrow \quad \left(2\cos^2 + 2\cos c - \cos c - 1\right) = 0$$

$$\Rightarrow \quad (2\cos c - 1)(\cos c + 1) = 0$$

$$\Rightarrow \quad \cos c = \frac{1}{2}, \quad \cos c = -1$$

$$\Rightarrow \quad \tan c = 1$$

$$c = \frac{\pi}{3} \in (0, \pi), \quad c = \pi$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(xiii)

Here,

$$f(x) = \frac{x}{2} - \sin\frac{\pi x}{6} \text{ on } [-1, 0]$$

We know that sine function is continuous and differentiable every where, so $f(x)$ is continuous is $[-1, 0]$ and differentiable is $(-1, 0)$.

Now,

$$f(-1) = \frac{-1}{2} - \sin\left(-\frac{\pi}{6}\right)$$
$$= -\frac{1}{2} + \sin\frac{\pi}{6}$$
$$= -\frac{1}{2} + \frac{1}{2}$$
$$f(-1) = 0 \qquad \text{---(i)}$$

And $\quad f(0) = 0 - \sin 0$

$$f(0) = 0 \qquad \text{---(ii)}$$

From equation (i) and (ii),

$$f(-1) = f\binom{0}{}$$

So, Rolle's theorem is applicable, so there must exist a point $c \in (-1, 0)$ such that $f'(c) = 0$.

Now,

$$f(x) = \frac{x}{2} - \sin\left(\frac{\pi x}{6}\right)$$
$$f'(x) = \frac{1}{2} - \frac{\pi}{6}\cos\left(\frac{\pi x}{6}\right)$$

Now,

$$f'(c) = 0$$
$$\frac{1}{2} - \frac{\pi}{6}\cos\left(\frac{\pi c}{6}\right) = 0$$
$$\Rightarrow \quad -\frac{\pi}{6}\cos\left(\frac{\pi c}{6}\right) = -\frac{1}{2}$$
$$\Rightarrow \quad \cos\left(\frac{\pi c}{6}\right) = 3\pi$$
$$\Rightarrow \quad \frac{\pi c}{6} = \cos^{-1}\left(\frac{66}{7}\right)$$
$$\Rightarrow \quad c = \frac{6}{\pi}\cos^{-1}\left(\frac{66}{7}\right)$$
$$\Rightarrow \quad c = \frac{21}{11}\cos^{-1}\left(\frac{66}{7}\right)$$
$$\Rightarrow \quad c \in \left(-\frac{21}{11}, \frac{21}{11}\right) \qquad \left[\text{since, } \cos^{-1} x \in [-1, 1]\right]$$
$$\Rightarrow \quad c \in (-1.9, 1.9)$$
$$\Rightarrow \quad c \in (-1, 0)$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(xiv)

Here,

$$f(x) = \frac{6x}{\pi} - 4\sin^2 x \text{ on } \left[0, \frac{\pi}{6}\right]$$

We know that sine and its square function is continuous and differentiable every where, so $f(x)$ is continuous is $\left[0, \frac{\pi}{6}\right]$ and differentiable is $\left(0, \frac{\pi}{6}\right)$.

Now,

$$f(0) = 0 - 0 = 0$$

$$f\left(\frac{\pi}{6}\right) = 1 - 1 = 0$$

$$\Rightarrow \quad f(0) = f\left(\frac{\pi}{6}\right)$$

So, Rolle's theorem is applicable, so there must exist a point $c \in \left(0, \frac{\pi}{6}\right)$ such that $f'(c) = 0$.

Now,

$$f(x) = \frac{6x}{\pi} - 4\sin^2 x$$

$$f'(x) = \frac{6}{\pi} - 8\sin x \cos x$$

$$f'(x) = \frac{6}{\pi} - 4\sin 2x$$

Now,

$$f'(c) = 0$$

$$\frac{6}{\pi} - 4\sin 2c = 0$$

$$\Rightarrow \quad 4\sin 2c = \frac{6}{\pi}$$

$$\Rightarrow \quad \sin 2c = \frac{3}{2\pi}$$

$$\Rightarrow \quad 2c = \sin^{-1}\left(\frac{21}{44}\right)$$

$$\Rightarrow \quad c = \frac{1}{2}\sin^{-1}\left(\frac{21}{44}\right)$$

$$\Rightarrow \quad c \in \left(-\frac{1}{2}, \frac{1}{2}\right) \qquad \left[\text{since, } \sin^{-1} x \in [-1, 1]\right]$$

$$\Rightarrow \quad c \in \left(0, \frac{11}{21}\right)$$

$$\Rightarrow \quad c \in \left(0, \frac{\pi}{6}\right)$$

Hence, Rolle's theorem is verified.

*********** END **********