



Definite Integrals Ex 20.2 Q36

We have,

$$\int_0^{\frac{\pi}{2}} x^2 \sin x \, dx$$

Using by parts, we get

$$\begin{aligned} & x^2 \int \sin x \, dx - \int \left(\int \sin x \, dx \right) \frac{dx^2}{dx} \cdot dx \\ &= x^2 \cos x + \int \cos x \cdot 2x \, dx \end{aligned}$$

Again applying by parts

$$\begin{aligned} &= x^2 \cos x + 2 \left[x \int \cos x \, dx - \int \left(\int \cos x \, dx \right) \cdot \frac{dx}{dx} \cdot dx \right] \\ &= x^2 \cos x + 2 \left[x \sin x - \int \sin x \, dx \right] \\ &= \left[x^2 \cos x + 2x \sin x + 2 \cos x \right]_0^{\frac{\pi}{2}} \\ &= \pi + 0 - 0 - 0 - 2 \\ &= \pi - 2 \end{aligned}$$

$$\therefore \int_0^{\frac{\pi}{2}} x^2 \sin x \, dx = \pi - 2$$

Definite Integrals Ex 20.2 Q37

Let $x = \cos 2\theta$

Differentiating w.r.t. x , we get

$$dx = -2 \sin 2\theta d\theta$$

$$\text{Now, } x = 0 \Rightarrow \theta = \frac{\pi}{4}$$

$$x = 1 \Rightarrow \theta = 0$$

$$\therefore \int_0^1 \frac{\sqrt{1-x}}{\sqrt{1+x}} dx = \int_{\frac{\pi}{4}}^0 \frac{\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}} (-2 \sin 2\theta) d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}} (2 \sin 2\theta) d\theta \quad \left[\because \sin 2\theta = 2 \sin \theta \cos \theta; \text{ and } \sin^2 \theta = \frac{1-\cos 2\theta}{2} \right]$$

$$= 2 \int_0^{\frac{\pi}{4}} \frac{\sin \theta}{\cos \theta} \cdot \sin 2\theta d\theta$$

$$= 4 \int_0^{\frac{\pi}{4}} \sin^2 \theta d\theta$$

$$= 2 \int_0^{\frac{\pi}{4}} (1 - \cos 2\theta) d\theta$$

$$= 2 \left[\theta - \frac{\sin^2 \theta}{2} \right]_0^{\frac{\pi}{4}}$$

$$= 2 \left[\frac{\pi}{4} - \frac{1}{2} \right]$$

$$= \frac{\pi}{2} - 1$$

$$\therefore \int_0^1 \frac{\sqrt{1-x}}{\sqrt{1+x}} dx = \frac{\pi}{2} - 1$$

Definite Integrals Ex 20.2 Q38

We have,

$$\int_0^1 \frac{1-x^2}{(1+x^2)^2} dx = \int_0^1 \frac{-x^2 \left(1 - \frac{1}{x^2}\right) dx}{x^2 \left(x + \frac{1}{x}\right)^2} = - \int_0^1 \frac{\left(1 - \frac{1}{x^2}\right) dx}{\left(x + \frac{1}{x}\right)^2}$$

$$\text{Let } x + \frac{1}{x} = t \Rightarrow 1 - \frac{1}{x^2} dx = dt$$

$$\text{When } x = 0 \Rightarrow t = \infty$$

$$x = 1 \Rightarrow t = 2$$

$$\therefore \int_0^1 \frac{1-x^2}{(1+x^2)^2} dx = - \int_{\infty}^2 \frac{dt}{t^2} = \int_2^{\infty} \frac{dt}{t^2} = \left[-\frac{1}{t} \right]_2^{\infty} = \left(\frac{1}{2} - 0 \right) = \frac{1}{2}$$

Definite Integrals Ex 20.2 Q39

Put $t = x^5 + 1$, then $dt = 5x^4 dx$.

$$\text{Therefore, } \int_0^1 5x^4 \sqrt{x^5 + 1} dx = \int_1^2 \sqrt{t} dt = \frac{2}{3} t^{\frac{3}{2}} = \frac{2}{3} t^{\frac{3}{2}} = \frac{2}{3} (x^5 + 1)^{\frac{3}{2}}$$

$$\begin{aligned} \text{Hence, } \int_{-1}^1 5x^4 \sqrt{x^5 + 1} dx &= \frac{2}{3} \left[(x^5 + 1)^{\frac{3}{2}} \right]_{-1}^1 \\ &= \frac{2}{3} \left[(1^5 + 1)^{\frac{3}{2}} - ((-1)^5 + 1)^{\frac{3}{2}} \right] \\ &= \frac{2}{3} \left[2^{\frac{3}{2}} - 0^{\frac{3}{2}} \right] = \frac{2}{3} (2\sqrt{2}) = \frac{4\sqrt{2}}{3} \end{aligned}$$

Alternatively, first we transform the integral and then evaluate the transformed integral with new limits.

Let $t = x^5 + 1$. Then $dt = 5x^4 dx$.

Note that, when $x = -1$, $t = 0$ and when $x = 1$, $t = 2$.

Thus, as x varies from -1 to 1 , t varies from 0 to 2 .

$$\begin{aligned} \text{Therefore } \int_{-1}^1 5x^4 \sqrt{x^5 + 1} dx &= \int_0^2 \sqrt{t} dt \\ &= \frac{2}{3} \left[t^{\frac{3}{2}} \right]_0^2 = \frac{2}{3} \left[2^{\frac{3}{2}} - 0^{\frac{3}{2}} \right] = \frac{2}{3} (2\sqrt{2}) = \frac{4\sqrt{2}}{3} \end{aligned}$$

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