Polynomials Ex 2.1 Q2

Answer :

(i) Given α and β are the zeros of the quadratic polynomial $f(x) = ax^2 + bx + c$

$$\alpha + \beta = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$= \frac{-b}{a}$$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$= \frac{c}{a}$$

We have, $(\alpha - \beta)$

$$(\alpha-\beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$$

$$\alpha - \beta = \sqrt{\alpha^2 + \beta^2 - 2\alpha\beta}$$

$$\alpha - \beta = \sqrt{(\alpha+\beta)^2 - 2\alpha\beta - 2\alpha\beta}$$

$$\alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$$

Substituting $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$ then we get,

$$\alpha - \beta = \sqrt{\left(\frac{-b}{a}\right)^2 - 4\frac{c}{a}}$$

$$\alpha - \beta = \sqrt{\frac{b^2}{a^2} - \frac{4c}{a}}$$

$$\alpha - \beta = \sqrt{\frac{b^2}{a^2} - \frac{4c \times a}{a \times a}}$$

$$\alpha - \beta = \sqrt{\frac{b^2}{a^2} - \frac{4ac}{a^2}}$$

$$\alpha - \beta = \sqrt{\frac{b^2 - 4ac}{a^2}}$$

$$\alpha - \beta = \frac{\sqrt{b^2 - 4ac}}{a}$$

Hence, the value of $\alpha - \beta$ is $\boxed{\frac{\sqrt{b^2 - 4ac}}{a}}$.

(ii) Given α and β are the zeros of the quadratic polynomial $f(x) = ax^2 + bx + c$

$$\alpha + \beta = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$= \frac{-b}{a}$$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$= \frac{c}{a}$$

We have,

$$\frac{1}{\alpha}-\frac{1}{\beta}$$

$$\frac{1}{\alpha}-\frac{1}{\beta}=\sqrt{\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^2-4\frac{1}{\alpha\beta}}$$

$$\frac{1}{\alpha}-\frac{1}{\beta}=\sqrt{\left(\frac{\alpha+\beta}{\alpha\beta}\right)^2-4\frac{1}{\alpha\beta}}$$

Substituting $\alpha+\beta=\frac{-b}{a}$ and $\alpha\beta=\frac{c}{a}$ then we get,

$$\frac{1}{\alpha}-\frac{1}{\beta}=\sqrt{\left(\frac{\frac{-b}{a}}{\frac{c}{a}}\right)^2-\frac{4}{\frac{c}{a}}}$$

$$\frac{1}{\alpha}-\frac{1}{\beta}=\sqrt{\left(\frac{-b}{a}\times\frac{a}{c}\right)^2-4\times\frac{a}{c}}$$

$$\frac{1}{\alpha}-\frac{1}{\beta}=\sqrt{\left(\frac{-b}{\cancel{a}}\times\frac{\cancel{a}}{c}\right)^2-4\times\frac{a}{c}}$$

$$\frac{1}{\alpha}-\frac{1}{\beta}=\sqrt{\left(\frac{-b}{c}\right)^2-\frac{4a}{c}}$$

$$\frac{1}{\alpha}-\frac{1}{\beta}=\sqrt{\frac{b^2}{c^2}-\frac{4a}{c}}$$

By taking least common factor we get,

$$\frac{1}{\alpha}-\frac{1}{\beta}=\sqrt{\frac{b^2}{c^2}-\frac{4a\times c}{c\times c}}$$

$$\frac{1}{\alpha}-\frac{1}{\beta}=\sqrt{\frac{b^2}{c^2}-\frac{4ac}{c^2}}$$

$$\frac{1}{\alpha}-\frac{1}{\beta}=\sqrt{\frac{b^2-4ac}{c^2}}$$

$$\frac{1}{\alpha}-\frac{1}{\beta}=\frac{\sqrt{b^2-4ac}}{c}$$

Hence the value of $\frac{1}{\alpha}-\frac{1}{\beta}$ is $\boxed{\frac{\sqrt{b^2-4ac}}{c}}$.

(iii) Given α and β are the zeros of the quadratic polynomial $f(x)=ax^2+bx+c$

$$\alpha+\beta=\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$=\frac{-b}{a}$$

$$\alpha\beta=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$=\frac{c}{a}$$

We have, $\frac{1}{\alpha}+\frac{1}{\beta}-2\alpha\beta$

By cross multiplication we get,

$$\frac{1}{\alpha}+\frac{1}{\beta}-2\alpha\beta=\frac{\alpha+\beta}{\alpha\beta}-2\alpha\beta$$

By substituting $\alpha+\beta=\frac{-b}{a}$ and $\alpha\beta=\frac{c}{a}$ we get ,

$$\frac{1}{\alpha}+\frac{1}{\beta}-2\alpha\beta=\frac{\frac{-b}{a}}{\frac{c}{a}}-2\frac{c}{a}$$

$$\frac{1}{\alpha}+\frac{1}{\beta}-2\alpha\beta=\frac{-b}{a}\times\frac{a}{c}-\frac{2c}{a}$$

$$\frac{1}{\alpha}+\frac{1}{\beta}-2\alpha\beta=\frac{-b}{\cancel{a}}\times\frac{\cancel{a}}{c}-\frac{2c}{a}$$

$$\frac{1}{\alpha}+\frac{1}{\beta}-2\alpha\beta=\frac{-b}{c}-\frac{2c}{a}$$

$$\frac{1}{\alpha}+\frac{1}{\beta}-2\alpha\beta=-\left(\frac{b}{c}+\frac{2c}{a}\right)$$

Hence the value of $\frac{1}{\alpha}+\frac{1}{\beta}-2\alpha\beta$ is $\boxed{=-\left(\frac{b}{c}+\frac{2c}{a}\right)}$.

(iv) Given α and β are the zeros of the quadratic polynomial $f(x)=ax^2+bx+c$

$$\alpha+\beta=\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

*********** END **********