Areas of Parallelograms and Triangles Ex 15.3 Q19

Answer :

Given:

ABCD is a parallelogram

G is a point such that AG = 2GB

E is a point such that CE = 2DE

F is a point such that BF = 2FC

To prove:

(i) $\operatorname{ar}(\text{ADEG}) = \operatorname{ar}(\text{GBCE})$

(ii) $\operatorname{ar}(\Delta\text{EGB}) = \frac{1}{6}\operatorname{ar}(\text{ABCD})$

(iii) $\operatorname{ar}(\Delta\text{EFC}) = \frac{1}{2}\operatorname{ar}(\Delta\text{EBF})$

(iv) $\operatorname{ar}(\Delta\text{EBG}) = \frac{3}{2}\operatorname{ar}(\Delta\text{EFC})$

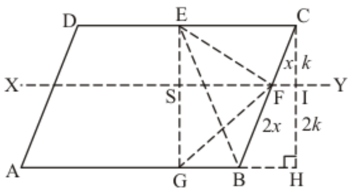
What portion of the area of parallelogram ABCD is the area of ΔEFG

Construction: draw a parallel line to AB through point F and a perpendicular line to AB through

PROOF:

(i) Since ABCD is a parallelogram,

So AB = CD and AD = BC



Consider the two trapeziums ADEG and GBCE:

Since AB = DC, EC = 2DE, AG = 2GB

$\Rightarrow \text{ED} = \frac{1}{3}\text{CD} = \frac{1}{3}\text{AB}$, and $\text{EC} = \frac{2}{3}\text{CD} = \frac{2}{3}\text{AB}$

$\Rightarrow \text{AG} = \frac{2}{3}\text{AB}$, and $\text{BG} = \frac{1}{3}\text{AB}$

So $\text{DE} + \text{AG} = \frac{1}{3}\text{AB} + \frac{2}{3}\text{AB} = \text{AB}$, and $\text{EC} + \text{BG} = \frac{2}{3}\text{AB} + \frac{1}{3}\text{AB} = \text{AB}$

Since the two trapeziums ADEG and GBCE have same height and their sum of two parallel sides are equal

Since $\text{Area of trapezium} = \frac{\text{sum of parallel sides}}{2} \times \text{height}$

So $\operatorname{ar}(\text{ADEG}) = \operatorname{ar}(\text{GBCE})$

Hence $\boxed{\operatorname{ar}(\text{ADEG}) = \operatorname{ar}(\text{GBCE})}$

(ii) Since we know from above that

$\text{BG} = \frac{1}{2}\text{AB}$. So

$$\begin{aligned}\text{ar}(\text{EGB}) &= \frac{1}{2}\times \text{GB}\times \text{Height}\\ &= \frac{1}{2}\times\frac{1}{3}\text{AB}\times\text{Height}\\ &= \frac{1}{6}\text{AB}\times\text{Height}\\ &= \frac{1}{6}\text{ar}(\text{ABCD})\end{aligned}$$

Hence $\boxed{\text{ar}(\Delta\text{EGB}) = \frac{1}{6}\text{ar}(\text{ABCD})}$

(iii) Since height of triangle EFC and triangle EBF are equal. So

$$\begin{aligned}\text{ar}(\text{EFC}) &= \frac{1}{2}\text{FC}\times\text{Height}\\ &= \frac{1}{2}\times\frac{1}{2}\times\text{FB}\times\text{Height}\\ &= \frac{1}{2}\text{ar}(\text{EBF})\end{aligned}$$

Hence $\boxed{\text{ar}(\Delta\text{EFC}) = \frac{1}{2}\text{ar}(\Delta\text{EBF})}$

(iv) Consider the trapezium in which

$$\text{ar}(\text{EGBC}) = \text{ar}(\Delta\text{EGB}) + \text{ar}(\Delta\text{EBF}) + \Delta(\text{EFC})$$

$$\Rightarrow \frac{1}{2}\text{ar}(\text{ABCD}) = \frac{1}{6}\text{ar}(\text{ABCD}) + 2\text{ar}(\Delta\text{EFC}) + \text{ar}(\Delta\text{EFC}) \text{ (From } (iii))$$

$$\Rightarrow \frac{1}{3}\text{ar}(\text{ABCD}) = 3\text{ar}(\Delta\text{EFC})$$

$$\Rightarrow \text{ar}(\Delta\text{EFC}) = \frac{1}{9}\text{ar}(\text{ABCD})$$

Now from (ii) part we have

$$\begin{aligned}\text{ar}(\Delta\text{EGB}) &= \frac{1}{6}\text{ar}(\Delta\text{EFC})\\ &= \frac{3}{2}\times\frac{1}{9}\text{ar}(\text{ABCD})\\ &= \frac{3}{2}\text{ar}(\Delta\text{EFC})\end{aligned}$$

$$\boxed{\Rightarrow \text{ar}(\Delta\text{EGB}) = \frac{3}{2}\text{ar}(\Delta\text{EFC})}$$

(v) In the figure it is given that FB = 2CF. Let CF = x and FB = $2x$

Now consider the tow triangles CFI and CBH which are similar triangles

So by the property of similar triangle CI = k and IH = $2k$

Now consider the triangle EGF in which

$$\begin{aligned}\text{ar}(\Delta\text{EFG}) &= \text{ar}(\Delta\text{ESF}) + \text{ar}(\Delta\text{SGF})\\ &= \frac{1}{2}\text{SF}\times\text{k} + \frac{1}{2}\text{SF}\times 2\text{k}\\ &= \frac{3}{2}\text{SF}\times\text{k} \qquad \ldots\ldots(\text{i})\end{aligned}$$

Now

$$\begin{aligned} ar(EGBC) &= ar(SGBF) + ar(ESFC) \\ &= \frac{1}{2}(SF + GB) \times 2k + \frac{1}{2}(SF + EC) \times k \\ &= \frac{3}{2}k \times SF + \left(GB + \frac{1}{2}EC\right) \times k \\ &= \frac{3}{2}k \times SF + \left(\frac{1}{3}AB + \frac{1}{2}\frac{2}{3}AB\right) \times k \end{aligned}$$

$$\frac{1}{2}ar(ABCD) = \frac{3}{2}k \times SF + \frac{2}{3}AB \times k$$

$$\Rightarrow ar(ABCD) = 3k \times SF + \frac{4}{3}AB \times k$$ (Multiply both sides by 2)

$$\Rightarrow ar(ABCD) = 3k \times SF + \frac{4}{9}ar(ABCD)$$

$$\Rightarrow k \times SF = \frac{5}{27}ar(ABCD) \quad \ldots\ldots (2)$$

From (1) and (2) we have

$$\begin{aligned} ar(\Delta EFG) &= \frac{3}{2}\frac{5}{27}ar(ABCD) \\ &= \frac{5}{18}ar(ABCD) \end{aligned}$$

$$\boxed{\Rightarrow ar(\Delta EFG) = \frac{5}{18}ar(ABCD)}$$

*********** END **********