



Trigonometric Ratios Ex 5.1 Q20

Answer :

Given: $\tan \theta = \frac{12}{13}$ (1)

To find the value of $\frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta}$

Now, we know the following trigonometric identity

$$\operatorname{cosec}^2 \theta = 1 + \tan^2 \theta$$

Therefore, by substituting the value of $\tan \theta$ from equation (1),

We get,

$$\begin{aligned} \operatorname{cosec}^2 \theta &= 1 + \left(\frac{12}{13}\right)^2 \\ &= 1 + \frac{(12)^2}{(13)^2} \\ &= 1 + \frac{144}{169} \end{aligned}$$

By taking L.C.M. on the R.H.S,

We get,

$$\begin{aligned} \operatorname{cosec}^2 \theta &= \frac{169 + 144}{169} \\ &= \frac{313}{169} \end{aligned}$$

Therefore

$$\begin{aligned} \operatorname{cosec} \theta &= \sqrt{\frac{313}{169}} \\ &= \frac{\sqrt{313}}{13} \end{aligned}$$

Therefore

$$\operatorname{cosec} \theta = \frac{\sqrt{313}}{13} \text{ (2)}$$

Now, we know that

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

Therefore,

$$\sin \theta = \frac{1}{\operatorname{cosec} \theta}$$

$$\begin{aligned} \sin \theta &= \frac{1}{\frac{\sqrt{313}}{13}} \\ &= \frac{13}{\sqrt{313}} \end{aligned}$$

Therefore

$$\sin \theta = \frac{13}{\sqrt{313}} \dots\dots (3)$$

Now, we know the following trigonometric identity

$$\cos^2 \theta + \sin^2 \theta = 1$$

Therefore,

$$\cos^2 \theta = 1 - \sin^2 \theta$$

Now by substituting the value of $\sin \theta$ from equation (3)

We get,

$$\begin{aligned}\cos^2 \theta &= 1 - \left(\frac{13}{\sqrt{313}} \right)^2 \\ &= 1 - \frac{(13)^2}{(\sqrt{313})^2} \\ &= 1 - \frac{169}{313}\end{aligned}$$

Therefore, by taking L.C.M on R.H.S

We get,

$$\begin{aligned}\cos^2 \theta &= \frac{313 - 169}{313} \\ &= \frac{144}{313}\end{aligned}$$

Now, by taking square root on both sides

We get,

$$\begin{aligned}\cos \theta &= \sqrt{\frac{144}{313}} \\ &= \frac{12}{\sqrt{313}}\end{aligned}$$

Therefore,

$$\cos \theta = \frac{12}{\sqrt{313}} \dots\dots (4)$$

Substituting the value of $\sin \theta$ and $\cos \theta$ from equation (3) and (4) respectively in the expression below

$$\frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta}$$

Therefore,

$$\begin{aligned}\frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta} &= \frac{2 \times \frac{13}{\sqrt{313}} \times \frac{12}{\sqrt{313}}}{\left(\frac{13}{\sqrt{313}} \right)^2 - \left(\frac{12}{\sqrt{313}} \right)^2} \\ &= \frac{\frac{2 \times 13 \times 12}{313}}{\frac{(13)^2}{313} - \frac{(12)^2}{313}} \\ &= \frac{\frac{312}{313}}{\frac{169}{313} - \frac{144}{313}}\end{aligned}$$

$$= \frac{\frac{312}{313}}{\frac{169-144}{313}}$$

$$= \frac{\frac{312}{313}}{\frac{25}{313}}$$

$$= \frac{312}{25}$$

Therefore, $\frac{2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta} = \frac{312}{25}$

***** END *****