Continuity Ex 9.1 Q15

We want to discuss the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} -(-h) = 0$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} h = 0$$

$$f(0) = 1$$

Thus, $\text{LHL} = \text{RHL} \neq f(0)$

Hence, the function is discontinuous at $x = 0$. And this is removable discontinuity.

Continuity Ex 9.1 Q16

We want to discuss the continuity of the function at $x = \frac{1}{2}$.

$$\text{LHL} = \lim_{x \to \frac{1}{2}^-} f(x) = \lim_{h \to 0} f\left(\frac{1}{2} - h\right) = \lim_{h \to 0} \frac{1}{2} - h = \frac{1}{2}$$

$$\text{RHL} = \lim_{x \to \left(\frac{1}{2}\right)^+} = \lim_{h \to 0} f\left(\frac{1}{2} + h\right) = \lim_{h \to 0} 1 - \left(\frac{1}{2} + h\right) = \frac{1}{2}$$

$$f\left(\frac{1}{2}\right) = \frac{1}{2}$$

Thus, $\text{LHL} = \text{RHL} = f\left(\frac{1}{2}\right) = \frac{1}{2}$

Hence, the function is continuous at $x = \frac{1}{2}$.

Continuity Ex 9.1 Q17

We want to check the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} 2(-h) - 1 = -1$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} 2h + 1 = 1$$

Thus, $\text{LHL} \neq \text{RHL}$

Hence, the function is discontinuous at $x = 0$. This is discontinuity of I^{st} kind.

Continuity Ex 9.1 Q18

We have given that the function is continuous at $x = 1$

$\text{LHL} = \text{RHL} = f(1) \ldots\ldots (1)$

Now, $\text{LHL} = \lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} \frac{(1-h)^2 - 1}{(1-h) - 1} = \lim_{h \to 0} \frac{h^2 - 2h}{-h} = 2$

$$f(1) = k$$

From (1), $\text{LHL} = f(1)$

$\therefore\ 2 = k$

*********** END **********