Complex Numbers Ex 13.4 Q4

$|z_1| = |z_2|$

Let $\arg(z_1)=\theta$

$\therefore \arg(z_2)=\pi\text{-}\theta$

In polar form, $z_1=|z_1|(\cos\theta+i\sin\theta)......(i)$

$z_2=|z_2|\{\cos(\pi-\theta)+i\sin(\pi-\theta)\}$

$=|z_2|(-\cos\theta+i\sin\theta)$

$=-|z_2|(\cos\theta-i\sin\theta)$

Finding conjugate of

$\overline{z_2}=-|z_2|(\cos\theta+i\sin\theta)......(ii)$

(i)/(ii) is equal to

$$\frac{z_1}{\overline{z_2}}=-\frac{|z_1|(\cos\theta+i\sin\theta)}{|z_2|(\cos\theta+i\sin\theta)}$$

$$\frac{z_1}{\overline{z_2}}=-\frac{|z_1|}{|z_1|} \qquad [\because |z_1|=|z_2|]$$

$$\frac{z_1}{\overline{z_2}}=-1$$

$z_1=-\overline{z_2}$

Hence Proved.

Complex Numbers Ex 13.4 Q5

z_1, z_2 are conjugates implies $z_2=\overline{z_1}$

z_3, z_4 are conjugates implies $z_4=\overline{z_3}$

Also we know that $\arg(z_1)+\arg(\overline{z_1})=0$

$$\arg\left(\frac{z_1}{z_4}\right)+\arg\left(\frac{z_2}{z_3}\right)$$

$$=\arg(z_1)-\arg(z_4)+\arg(z_2)-\arg(z_3) \qquad \left[\because \arg\left(\frac{z_1}{z_2}\right)=\arg(z_1)-\arg(z_2)\right]$$

$$=\arg(z_1)-\arg(\overline{z_3})+\arg(\overline{z_1})-\arg(z_3)$$

$$=\arg(z_1)+\arg(\overline{z_1})-\arg(\overline{z_3})-\arg(z_3)$$

$$=\arg(z_1)+\arg(\overline{z_1})-\left[\arg(\overline{z_3})+\arg(z_3)\right][\because \arg(z_1)+\arg(\overline{z_1})=0]$$

$$=0+0=0$$

Complex Numbers Ex 13.4 Q6

$$\sin\frac{\pi}{5} + i\left(1-\cos\frac{\pi}{5}\right)$$

$$= 2\sin\frac{\pi}{10}\cos\frac{\pi}{10} + i2\sin^2\frac{\pi}{10} [\text{Using } \sin 2\theta = 2\sin\theta\cos\theta \ \& \ 1-\cos 2\theta = 2\sin^2\theta]$$

$$= 2\sin\frac{\pi}{10}\left(\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right)$$

*********** END **********