

Chapter 6 Determinants Ex 6.4 Q19

$$D = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$$c_2 \to c_2 - c_1, c_3 \to c_3 - c_1$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ a & b - a & c - a \\ a^2 & b^2 - a^2 & c^2 - a^2 \end{vmatrix}$$

Now taking (b-a) from c_2 , and (c-a) c_3 common

$$= (b-a)(c-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & b+a & c+a \end{vmatrix}$$

Expanding along R₁

$$= (b-a)(c-a)[c+a-b-a]$$

$$= (b-a)(c-a)(c-b)$$

$$= (a-b)(b-c)(c-a)$$

Again
$$D_1 = -\begin{vmatrix} 1 & 1 & 1 \\ d & b & c \\ d^2 & b^2 & c^2 \end{vmatrix}$$

$$c_2 \to c_2 - c_1, c_3 \to c_3 - c_1$$

$$D_1 = - \begin{vmatrix} 1 & 0 & 0 \\ d & b - d & c - d \\ d^2 & b^2 - d^2 & c^2 - d^2 \end{vmatrix}$$

Taking (b-d) common from c_2 and (c-d) from c_3

$$= (b-d)(c-d)\begin{vmatrix} 1 & 0 & 0 \\ d & 1 & 1 \\ d^2 & b+d & c+d \end{vmatrix}$$

Expanding along R₁

$$= -(b-d)(c-d)[1(c+d-b-d)]$$

$$=-\left(b-d\right) \left(c-d\right) \left(c-b\right)$$

$$=-\left(b-c\right) \left(c-d\right) \left(d-b\right)$$

Again
$$D_2 = -\begin{vmatrix} 1 & 1 & 1 \\ a & d & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$$c_2 \to c_2 - c_1, c_3 \to c_3 - c_1$$

$$= -\begin{vmatrix} 1 & 0 & 0 \\ a & d - a & c - a \\ a^2 & d^2 - a^2 & c^2 - a^2 \end{vmatrix}$$

Taking (d - a) common from c_2 and (c - a) from c_3

$$= - (d-a)(c-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & d+a & c+a \end{vmatrix}$$

Expanding along R₁

$$= - (d-a)(c-a) \times 1[c+a-d-a]$$

$$= - (d-a)(c-a)(c-d)$$

$$= - (a-d)(d-c)(c-a)$$

Also
$$D_3 = -\begin{vmatrix} 1 & 1 & 1 \\ a & b & d \\ a^2 & b^2 & d^2 \end{vmatrix}$$

$$c_2 \to c_2 - c_1, c_3 \to c_3 - c_1$$

$$= -\begin{vmatrix} 1 & 0 & 0 \\ a & b - a & d - a \\ a^2 & b^2 - a^2 & d^2 - a^2 \end{vmatrix}$$

Now, taking (b-a) common from c_2 and (d-a) from c_3

$$= -(b-a)(d-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & b+a & d+a \end{vmatrix}$$

Expanding along R₁

$$= -(b-a)(d-a) \times 1[d+a-b-a]$$

$$= -(b-a)(d-a)(d-b)$$

$$= -(a-b)(b-d)(d-a)$$

Now
$$x = \frac{D_1}{D} = -\frac{(b-c)(c-d)(d-b)}{(a-b)(b-c)(c-a)}$$

 $y = \frac{D_2}{D} = -\frac{(a-d)(d-c)(c-a)}{(a-b)(b-c)(c-a)}$
 $z = \frac{D_3}{D} = -\frac{(a-b)(b-d)(d-a)}{(a-b)(b-c)(c-a)}$

Chapter 6 Determinants Ex 6.4 Q20

Here
$$D = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -2 & 2 & 2 \\ 2 & 1 & -2 & 2 \\ 3 & -1 & 3 & -3 \end{vmatrix}$$

$$D = \begin{vmatrix} 0 & 0 & 1 \\ -1 & -4 & 0 \\ -22 & 6 & -6 \end{vmatrix} \qquad \begin{bmatrix} C1 \to C1 + 3C3 \\ C2 \to C2 - C3 \end{bmatrix}$$
$$= 1(-6 - 88) = -94$$

$$D_1 = \begin{vmatrix} 2 & 1 & 1 & 1 \\ -6 & -2 & 2 & 2 \\ -5 & 1 & -2 & 2 \\ -3 & -1 & 3 & -3 \end{vmatrix} = 188$$

$$D_2 = \begin{vmatrix} 1 & 2 & 1 & 1 \\ 1 & -6 & 2 & 2 \\ 2 & -5 & -2 & 2 \\ 3 & -3 & 3 & -3 \end{vmatrix} = -282$$

$$D_3 = \begin{vmatrix} 1 & 1 & 2 & 1 \\ 1 & -2 & -6 & 2 \\ 2 & 1 & -5 & 2 \\ 3 & -1 & -3 & -3 \end{vmatrix} = -141$$

$$D_4 = \begin{vmatrix} 1 & 1 & 1 & 2 \\ 1 & -2 & 2 & -6 \\ 2 & 1 & -2 & -5 \end{vmatrix} = 47$$

Now
$$x = \frac{D_1}{D} = \frac{188}{-94} = -2$$

 $y = \frac{D_2}{D} = \frac{-282}{-94} = 3$
 $z = \frac{D_3}{D} = \frac{-141}{-94} = \frac{3}{2}$
 $w = \frac{D_4}{D} = \frac{47}{-94} = -\frac{1}{2}$

Hence
$$x = -2$$
, $y = 3$, $z = \frac{3}{2}$, $w = -\frac{1}{2}$

Chapter 6 Determinants Ex 6.4 Q21

Here
$$D = \begin{bmatrix} 2 & 0 & -3 & 1 \\ 1 & -1 & 0 & 2 \\ 0 & -3 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

$$D = -1 \begin{vmatrix} 1 & -6 & 1 \\ 4 & -3 & 2 \\ 0 & 0 & 1 \end{vmatrix}$$
$$= -1(-3 + 24) = -21$$
$$\begin{bmatrix} C1 \to C1 + 3C3 \\ C2 \to C2 - C3 \end{bmatrix}$$

$$D_1 = \begin{vmatrix} 1 & 0 & -3 & 1 \\ 1 & -1 & 0 & 2 \\ 1 & -3 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = -21$$

$$D_2 = \begin{vmatrix} 2 & 1 & -3 & 1 \\ 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = -6$$

$$D_3 = \begin{vmatrix} 2 & 0 & 1 & 1 \\ 1 & -1 & 1 & 2 \\ 0 & -3 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = -6$$

$$D_4 = \begin{vmatrix} 2 & 0 & -3 & 1 \\ 1 & -1 & 0 & 1 \\ 0 & -3 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{vmatrix} = 3$$

Now
$$x = \frac{D_1}{D} = \frac{-21}{-21} = 1$$

 $y = \frac{D_2}{D} = \frac{-6}{-21} = \frac{2}{7}$
 $z = \frac{D_3}{D} = \frac{-6}{-21} = \frac{2}{7}$
 $w = \frac{D_4}{D} = \frac{3}{-21} = -\frac{1}{7}$

Chapter 6 Determinants Ex 6.4 Q22

$$Let D = \begin{vmatrix} 2 & -1 \\ 4 & -2 \end{vmatrix}$$

Expanding along R_1

$$= -4 + 4 = 0$$

Also
$$D_1 = \begin{vmatrix} 5 & -1 \\ 7 & -2 \end{vmatrix} = -3$$

Also
$$D_2 = \begin{vmatrix} 2 & 5 \\ 4 & 7 \end{vmatrix} = -6$$

And since D = 0 and D_1 and D_2 are non-zero, hence the given system of equations is inconsistent.

Hence proved.

Chapter 6 Determinants Ex 6.4 Q23

$$D = \begin{vmatrix} 3 & 1 \\ -6 & -2 \end{vmatrix} = -6 + 6 = 0$$

$$D_1 = \begin{vmatrix} 5 & 1 \\ 9 & -2 \end{vmatrix} = -10 - 9 = -19 \neq 0$$

Since D = 0 but $D_1 \neq 0$

Hence the given system of equations is inconsistent.

********* END *******