



Definite Integrals Ex 20.1 Q16

We have,

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1 + \sin x} dx$$

We know,

$$\sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$\therefore \frac{1}{1 + \sin x} = \frac{1}{1 + \left(\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right)} = \frac{1 + \tan^2 \frac{x}{2}}{\left(1 + \tan \frac{x}{2} \right)^2} = \frac{\sec^2 \frac{x}{2}}{\left(1 + \tan \frac{x}{2} \right)^2}$$

$$\Rightarrow \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1 + \sin x} dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec^2 \frac{x}{2}}{\left(1 + \tan \frac{x}{2} \right)^2} dx$$

If $f(x)$ is an even function $\int_{-\frac{a}{2}}^{\frac{a}{2}} f(x) dx = 2 \int_0^{\frac{a}{2}} f(x) dx$

So,

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec^2 \frac{x}{2}}{\left(1 + \tan \frac{x}{2} \right)^2} dx = 2 \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec^2 \frac{x}{2}}{\left(1 + \tan \frac{x}{2} \right)^2} dx$$

$$\text{let } 1 + \tan \frac{x}{2} = t$$

$$\Rightarrow \frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

Now,

$$x = -\frac{\pi}{4} \Rightarrow t = 1 - \tan \frac{\pi}{8}$$

$$x = \frac{\pi}{4} \Rightarrow t = 1 + \tan \frac{\pi}{8}$$

$$\therefore 2 \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec^2 \frac{x}{2}}{\left(1 + \tan \frac{x}{2}\right)^2} dx = 2 \int_{1 - \tan \frac{\pi}{8}}^{1 + \tan \frac{\pi}{8}} \frac{8dt}{t^2}$$

$$= 2 \left[\frac{-1}{t} \right]_{1 - \tan \frac{\pi}{8}}^{1 + \tan \frac{\pi}{8}}$$

$$= 2 \left[\frac{1}{1 - \tan \frac{\pi}{8}} - \frac{1}{1 + \tan \frac{\pi}{8}} \right]$$

$$= 2 \left[\frac{2 \tan \frac{\pi}{8}}{1 - \tan^2 \frac{\pi}{8}} \right]$$

$$= 2 \tan \frac{\pi}{4} \quad \left[\because \tan 2A = \frac{2 \tan A}{1 - \tan^2 A} \right]$$

$$= 2$$

$$\therefore \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1 + \sin x} dx = 2$$

Definite Integrals Ex 20.1 Q17

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \cos^2 x \, dx$$

$$\int \cos^2 x \, dx = \int \left(\frac{1 + \cos 2x}{2} \right) dx = \frac{x}{2} + \frac{\sin 2x}{4} = \frac{1}{2} \left(x + \frac{\sin 2x}{2} \right) = F(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= \left[F\left(\frac{\pi}{2}\right) - F(0) \right] \\ &= \frac{1}{2} \left[\left(\frac{\pi}{2} + \frac{\sin \pi}{2} \right) - \left(0 + \frac{\sin 0}{2} \right) \right] \\ &= \frac{1}{2} \left[\frac{\pi}{2} + 0 - 0 - 0 \right] \\ &= \frac{\pi}{4} \end{aligned}$$

Definite Integrals Ex 20.1 Q18

We have,

$$\int_0^{\frac{\pi}{2}} \cos^3 x \, dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos 3x + 3 \cos x}{4} \, dx \quad \left[\because \cos 3x = 4 \cos^3 x - 3 \cos x \right]$$

$$= \frac{1}{4} \int_0^{\frac{\pi}{2}} (\cos 3x + 3 \cos x) \, dx$$

$$= \frac{1}{4} \left[\frac{\sin 3x}{3} + 3 \sin x \right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{4} \left[\left(\frac{\sin 3 \frac{\pi}{2}}{3} + 3 \sin \frac{\pi}{2} \right) - \left(\frac{\sin 0}{3} + 3 \sin 0 \right) \right]$$

$$= \frac{1}{4} \left[\left(\frac{-1}{3} + 3 \right) - (0 + 0) \right] = \frac{2}{3}$$

$$= \frac{1}{4} \left[\frac{8}{3} \right]$$

$$= \frac{2}{3}$$

$$\therefore \int_0^{\frac{\pi}{2}} \cos^3 x \, dx = \frac{2}{3}$$

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