Polynomials Ex 2.2 Q1

Answer :

We have,

$$f(x) = 2x^3 + 2x^2 - 5x + 2$$

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$$

$$f\left(\frac{1}{2}\right) = \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2$$

$$f\left(\frac{1}{2}\right) = 0$$

$$f(1) = 2(1)^3 + (1)^2 - 5(1) + 2$$

$$f(1) = 2 + 1 - 5 + 2$$

$$f(1) = 0$$

$$f(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$$

$$f(-2) = -16 + 4 + 10 + 2$$

$$f(-2) = 0$$

So, $\frac{1}{2}, 1$ and 2 are the zeros of polynomial p(x)

Let $\alpha = \frac{1}{2}, \beta = 1$ and $\gamma = -2$. Then

$$\alpha+\beta+\gamma=\frac{1}{2}+1-2$$

$$=\frac{1}{2}+\frac{1\times 2}{1\times 2}-\frac{2\times 2}{1\times 2}$$

$$=\frac{1+2-4}{2}$$

$$=\frac{-1}{2}$$

From $f(x)=2x^3+2x^2-5x+2$

$$\alpha+\beta+\gamma=\frac{-\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$$

$$\alpha+\beta+\gamma=-\left(\frac{1}{2}\right)$$

$$\alpha\beta+\beta\gamma+\gamma\alpha=\frac{1}{2}(1)+1(-2)-2\left(\frac{1}{2}\right)$$

$$=\frac{1}{2}-2-\frac{2}{2}$$

Taking least common factor we get,

$$\alpha\beta+\beta\gamma+\gamma\alpha=\frac{1}{2}-\frac{2\times 2}{1\times 2}-\frac{2}{2}$$

$$\alpha\beta+\beta\gamma+\gamma\alpha=\frac{1-4-2}{2}$$

$$\alpha\beta+\beta\gamma+\gamma\alpha=\frac{1-6}{2}$$

$\alpha\beta+\beta\gamma+\gamma\alpha=\frac{-5}{2}$

From $f(x)=2x^3+2x^2-5x+2$

$\alpha\beta+\beta\gamma+\gamma\alpha=\frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$

$\alpha\beta+\beta\gamma+\gamma\alpha=\frac{-5}{2}$

$\alpha\beta\gamma=\frac{1}{2}\times1\times(-2)$

$\alpha\beta\gamma=\frac{-\cancel{2}}{\cancel{2}}$

$\alpha\beta\gamma=-1$

From $f(x)=2x^3+2x^2-5x+2$

$\alpha\beta\gamma=\frac{-\text{Constant term}}{\text{Coefficient of } x^2}$

$\alpha\beta\gamma=-\left(\frac{\cancel{2}}{\cancel{2}}\right)$

$\alpha\beta\gamma=-1$

Hence, it is verified that the numbers given along side of the cubic polynomials are their zeros and also verified the relationship between the zeros and coefficients

(ii) We have,

$g(x)=x^3-4x^2+5x-2$

$g(2)=(2)^3-4(2)^2+5(2)-2$

$g(2)=8-16+10-2$

$g(2)=0$

$g(1)=(1)^3-4(1)^2+5(1)-2$

$=1-4+5-2$

$=0$

$g(1)=(1)^3-4(1)^2+5(1)-2$

$=1-4+5-2$

$=0$

So 2,1and 1 are the zeros of the polynomial g(x)

Let $\alpha=2,\beta=1$ and $\lambda=1$. Then,

$\alpha+\beta+\gamma=2+1+1$

$\alpha+\beta+\gamma=4$

From $g(x)=x^3-4x^2+5x-2$

$\alpha+\beta+\gamma=\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

$\alpha+\beta+\gamma=\frac{4}{1}$

$\alpha+\beta+\gamma=4$

$\alpha\beta+\beta\gamma+\gamma\alpha=2(1)+1(1)+1(2)$

$\alpha\beta+\beta\gamma+\gamma\alpha=2+1+2$

$\alpha\beta+\beta\gamma+\gamma\alpha=5$

From $g(x) = x^3 - 4x^2 + 5x - 2$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{5}{1}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 5$$

$$\alpha\beta\gamma = 2 \times 1 \times 1$$

$$\alpha\beta\gamma = 2$$

From $g(x) = x^3 - 4x^2 + 5x - 2$

$$\alpha\beta\gamma = \frac{-\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\alpha\beta\gamma = -\left(\frac{-2}{1}\right)$$

$$\alpha\beta\gamma = 2$$

Hence, it is verified that the numbers given along side of the cubic polynomials are their zeros and also verified the relationship between the zeros and coefficients.

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