Indefinite Integrals Ex 19.9 Q60

Let $\quad I = \int \frac{e^{2x}}{1+e^x}\,dx \text{ - - - -(i)}$

Let $\quad 1+e^x = t \qquad$ then,

$d\left(1+e^x\right) = dt$

$\Rightarrow \quad e^x\,dx = dt$

$\Rightarrow \quad dx = \frac{dt}{e^x}$

Putting $1+e^x = t$ and $dx = \frac{dt}{e^x}$ in equation (i),

we get

$$I = \int \frac{e^{2x}}{t} \times \frac{dt}{e^x}$$

$$= \int \frac{e^x}{t}\,dt$$

$$= \int \frac{t-1}{t}\,dt$$

$$= \int \left(\frac{t}{t} - \frac{1}{t}\right)dt$$

$$= t - \log|t| + c$$

$$= \left(1+e^x\right) - \log\left|1+e^x\right| + c$$

$\therefore \quad I = 1+e^x - \log\left|1+e^x\right| + c$

Indefinite Integrals Ex 19.9 Q61

Let $\quad I = \int \frac{\sec^2 \sqrt{x}}{\sqrt{x}} dx$ ----(i)

Let $\quad \sqrt{x} = t$ then,

$d\left(\sqrt{x}\right) = dt$

$\Rightarrow \quad \frac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow \quad dx = 2\sqrt{x}\, dt$

$\Rightarrow \quad dx = 2t\,dt \quad \left[\because \quad \sqrt{x} = t\right]$

Putting $\sqrt{x} = t$ and $dx = 2t\,dt$ in equation (i), we get

$$I = \int \frac{\sec^2 t}{t} \times 2t\,dt$$
$$= 2\int \sec^2 t\,dt$$
$$= 2\tan t + c$$
$$= 2\tan\sqrt{x} + c$$

$\therefore \quad I = 2\tan\sqrt{x} + c$

Indefinite Integrals Ex 19.9 Q62

$$\tan^3 2x \sec 2x = \tan^2 2x \tan 2x \sec 2x$$
$$= \left(\sec^2 2x - 1\right)\tan 2x \sec 2x$$
$$= \sec^2 2x \cdot \tan 2x \sec 2x - \tan 2x \sec 2x$$
$$\therefore \int \tan^3 2x \sec 2x\ dx = \int \sec^2 2x \tan 2x \sec 2x\ dx - \int \tan 2x \sec 2x\ dx$$
$$= \int \sec^2 2x \tan 2x \sec 2x\ dx - \frac{\sec 2x}{2} + \mathrm{C}$$

Let $\sec 2x = t$

$\therefore 2\sec 2x \tan 2x\ dx = dt$

$$\therefore \int \tan^3 2x \sec 2x\ dx = \frac{1}{2}\int t^2 dt - \frac{\sec 2x}{2} + \mathrm{C}$$
$$= \frac{t^3}{6} - \frac{\sec 2x}{2} + \mathrm{C}$$
$$= \frac{(\sec 2x)^3}{6} - \frac{\sec 2x}{2} + \mathrm{C}$$

Indefinite Integrals Ex 19.9 Q63

Let $\quad I = \int \frac{x + \sqrt{x+1}}{x+2} dx - - - - - (i)$

Let $\quad x + 1 = t^2 \quad$ then,

$d(x+1) = d(t^2)$

$\Rightarrow \quad dx = 2t\,dt$

Putting $x + 1 = t^2$ and $dx = 2t\,dt$ in equation (i), we get

$$I = \int \frac{x + \sqrt{t^2}}{x+2} 2t\,dt$$
$$= 2\int \frac{(t^2 - 1) + t}{(t^2 - 1) + 2} t\,dt \qquad [\because \quad x + 1 = t^2]$$
$$= 2\int \frac{t^2 + t - 1}{t^2 + 1} t\,dt$$
$$= 2\int \frac{t^3 + t^2 - t}{t^2 + 1} dt$$
$$= 2\left[\int \frac{t^3}{t^2+1} dt + \int \frac{t^2}{t^2+1} dt - \int \frac{t}{t^2+1} dt\right]$$

$\therefore \quad I = 2\left[\int \frac{t^3}{t^2+1} dt + \int \frac{t^2}{t^2+1} dt - \int \frac{t}{t^2+1} dt\right] - - - - (ii)$

Let $I_1 = \int \frac{t^3}{t^2+1} dt$

$I_2 = \int \frac{t^2}{t^2+1} dt$

and $\quad I_3 = \int \frac{t}{t^2+1} dt$

Now, $\quad I_1 = \int \frac{t^3}{t^2+1} dt$

$= \int \left(t - \frac{t}{t^2+1}\right) dt$

$= \frac{t^2}{2} - \frac{1}{2}\log(t^2+1)$

$\therefore \quad I = \frac{t^2}{2} - \frac{1}{2}\log(t^2+1) + c_1 - - - - (iii)$

Since, $I_2 = \int \frac{t^2}{t^2+1} dt$

$= \int \frac{t^2 + 1 - 1}{t^2+1} dt$

$= \int \frac{t^2+1}{t^2+1} dt - \int \frac{1}{t^2+1} dt$

$= \int dt - \int \frac{1}{t^2+1} dt$

$\Rightarrow \quad I_2 = t - \tan^{-1}(t^2) + c_2 - - - - (iv)$

and, $\quad I_3 = \int \frac{t}{t^2+1} dt$

$$= \frac{1}{2}\log\left(1+t^2\right) + c_3 \text{ - - - - - - } (v)$$

Using equations $(ii), (iii), (iv)$ and (v), we get

$$I = 2\left[\frac{t^2}{2} - \frac{1}{2}\log\left(t^2+1\right) + c_1 + t - \tan^{-1}\left(t^2\right) + c_2 - \frac{1}{2}\log\left(1+t^2\right) + c_3\right]$$

$$= 2\left[\frac{t^2}{2} + t - \tan^{-1}\left(t^2\right) - \log\left(1+t^2\right) + c_1 + c_2 + c_3\right]$$

$$= 2\left[\frac{t^2}{2} + t - \tan^{-1}\left(t^2\right) - \log\left(1+t^2\right) + c_4\right] \qquad \left[\text{Putting } c_1 + c_2 + c_3 = c_4\right]$$

$$= t^2 + 2t - 2\tan^{-1}\left(t^2\right) - 2\log\left(1+t^2\right) + 2c_4$$

$$= \left(x+1\right) + 2\sqrt{x+1} - 2\tan^{-1}\left(\sqrt{x+1}\right) - 2\log\left(1+x+1\right) + 2c_4$$

$$= \left(x+1\right) + 2\sqrt{x+1} - 2\tan^{-1}\left(\sqrt{x+1}\right) - 2\log\left(x+2\right) + c \qquad \left[\text{Putting } 2c_4 = c\right]$$

$$\therefore \quad I = \left(x+1\right) + 2\sqrt{x+1} - 2\tan^{-1}\left(\sqrt{x+1}\right) - 2\log\left(x+2\right) + c$$

*********** END **********