



Indefinite Integrals Ex 19.20 Q1

$$\begin{aligned}\text{Let } I &= \int \frac{x^2 + x + 1}{x^2 - x} dx \\ &= \int \left[1 + \frac{2x + 1}{x^2 - x} \right] dx \\ &= x + \int \frac{2x + 1}{x^2 - x} dx + c_1 \text{ ---- (i)} \\ I_1 &= \int \frac{2x + 1}{x^2 - x} dx\end{aligned}$$

$$\begin{aligned}\text{Let } 2x + 1 &= \lambda \frac{d}{dx}(x^2 - x) + \mu \\ &= \lambda(2x - 1) + \mu \\ 2x + 1 &= (2\lambda)x - \lambda + \mu\end{aligned}$$

Comparing the coefficients of like powers of x,

$$\begin{aligned}2 &= 2\lambda &\Rightarrow &\lambda = 1 \\ -\lambda + \mu &= 1 &\Rightarrow &\mu = 2\end{aligned}$$

$$\begin{aligned}\text{so, } I_1 &= \int \frac{(2x - 1) + 2}{x^2 - x} dx \\ I &= \int \frac{2x - 1}{x^2 - x} dx + 2 \int \frac{1}{x^2 - x} dx \\ I &= \int \frac{2x - 1}{x^2 - x} dx + 2 \int \frac{1}{x^2 - 2x\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx \\ &= \int \frac{2x - 1}{x^2 - x} dx + 2 \int \frac{1}{\left(x - \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx\end{aligned}$$

$$I = \log|x^2 - x| + 2 \times \frac{1}{2\left(\frac{1}{2}\right)} \log \left| \frac{x - \frac{1}{2} - \frac{1}{2}}{x - \frac{1}{2} + \frac{1}{2}} \right| + c_1 \left[\text{since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x - a}{x + a} \right| + c \right]$$

$$I_1 = \log|x^2 - x| + 2 \log \left| \frac{x - 1}{x} \right| + c_2 \text{ ---- (ii)}$$

Using equation (i) and (ii)

$$I = x + \log|x^2 - x| + 2 \log \left| \frac{x-1}{x} \right| + c$$

$$\begin{aligned} \text{Let } 2x + 1 &= \lambda \frac{d}{dx}(x^2 - x) + \mu \\ &= \lambda(2x - 1) + \mu \\ 2x + 1 &= (2\lambda)x - \lambda + \mu \end{aligned}$$

$$\begin{aligned} \text{Comparing the coefficients of like powers of } x, \\ 2 &= 2\lambda \quad \Rightarrow \quad \lambda = 1 \\ -\lambda + \mu &= 1 \quad \Rightarrow \quad \mu = 2 \end{aligned}$$

$$\begin{aligned} \text{so, } I_1 &= \int \frac{(2x-1)+2}{x^2-x} dx \\ I &= \int \frac{2x-1}{x^2-x} dx + 2 \int \frac{1}{x^2-x} dx \\ I &= \int \frac{2x-1}{x^2-x} dx + 2 \int \frac{1}{x^2 - 2x \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx \\ &= \int \frac{2x-1}{x^2-x} dx + 2 \int \frac{1}{\left(x - \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx \end{aligned}$$

$$I = \log|x^2 - x| + 2 \times \frac{1}{2 \left(\frac{1}{2}\right)} \log \left| \frac{x - \frac{1}{2} - \frac{1}{2}}{x - \frac{1}{2} + \frac{1}{2}} \right| + c_1 \left[\text{since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c \right]$$

$$I_1 = \log|x^2 - x| + 2 \log \left| \frac{x-1}{x} \right| + c_2 \text{ ---- (ii)}$$

Using equation (i) and (ii)

$$I = x + \log|x^2 - x| + 2 \log \left| \frac{x-1}{x} \right| + c$$

Indefinite Integrals Ex 19.20 Q2

$$\begin{aligned} \text{Let } I &= \int \frac{x^2 + x - 1}{x^2 + x - 6} dx \\ &= \int \left[1 + \frac{5}{x^2 + x - 6} \right] dx \\ I &= x + \int \frac{5}{x^2 + x - 6} dx + c_1 \text{ ---- (i)} \end{aligned}$$

$$\begin{aligned} \text{Let } I_1 &= 5 \int \frac{1}{x^2 + x - 6} dx \\ &= 5 \int \frac{1}{x^2 + 2x \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 6} dx \\ &= 5 \int \frac{1}{\left(x + \frac{1}{2}\right)^2 - \left(\frac{5}{2}\right)^2} dx \\ &= 5 \frac{1}{2 \left(\frac{5}{2}\right)} \log \left| \frac{x + \frac{1}{2} - \frac{5}{2}}{x + \frac{1}{2} + \frac{5}{2}} \right| + c_2 \quad \left[\text{since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c \right] \end{aligned}$$

$$I_1 = \log \left| \frac{x-2}{x+3} \right| + c_2 \text{ ---- (ii)}$$

Using equation (i) and (ii)

$$I = x + \log \left| \frac{x-2}{x+3} \right| + c$$

Indefinite Integrals Ex 19.20 Q3

$$\begin{aligned}
 \text{Let } I &= \int \frac{1-x^2}{x(1-2x)} dx \\
 &= \int \frac{1-x^2}{x-2x^2} dx \\
 &= \int \frac{x^2-1}{2x^2-x} dx \\
 &= \int \left[\frac{1}{2} + \frac{\frac{x}{2}-1}{2x^2-x} \right] dx \\
 I &= \frac{1}{2}x + \int \frac{\frac{x}{2}-1}{2x^2-x} dx + c_1 \text{ ---- (i)}
 \end{aligned}$$

$$\begin{aligned}
 \text{Let } I_1 &= \int \frac{\frac{x}{2}-1}{2x^2-x} dx \\
 \text{Let } \frac{x}{2}-1 &= \lambda \frac{d}{dx}(2x^2-x) + \mu \\
 &= \lambda(4x-1) + \mu \\
 \frac{x}{2}-1 &= (4\lambda)x - \lambda + \mu
 \end{aligned}$$

Comparing the coefficients of like powers of x,

$$\begin{aligned}
 \frac{1}{2} &= 4\lambda \quad \Rightarrow \quad \lambda = \frac{1}{8} \\
 -\lambda + \mu &= -1 \quad \Rightarrow \quad -\left(\frac{1}{8}\right) + \mu = -1 \\
 \mu &= -\frac{7}{8}
 \end{aligned}$$

$$\begin{aligned}
 \text{so, } I_1 &= \int \frac{\frac{1}{8}(4x-1) - \frac{7}{8}}{2x^2-x} dx \\
 I &= \frac{1}{8} \int \frac{4x-1}{2x^2-x} dx - \frac{7}{8} \int \frac{1}{2\left(x^2-\frac{x}{2}\right)} dx \\
 I &= \frac{1}{8} \int \frac{4x-1}{2x^2-x} dx - \frac{7}{16} \int \frac{1}{x^2-2x\left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2} dx \\
 &= \frac{1}{8} \int \frac{4x-1}{2x^2-x} dx - \frac{7}{16} \int \frac{1}{\left(x-\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2} dx
 \end{aligned}$$

$$I_1 = \frac{1}{8} \log|2x^2-x| - \frac{7}{16} \times \frac{1}{2\left(\frac{1}{4}\right)} \log \left| \frac{x-\frac{1}{4}-\frac{1}{4}}{x-\frac{1}{4}+\frac{1}{4}} \right| + c_2 \quad \left[\text{since, } \int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c \right]$$

$$\begin{aligned}
 I &= \frac{1}{8} \log|x| + \frac{1}{8} \log|2x-1| - \frac{7}{8} \log|1-2x| + \frac{7}{8} \log 2 + \frac{7}{8} \log|x| + c_2 \\
 I_1 &= \log|x| - \frac{3}{4} \log|1-2x| + c_3 \text{ ---- (ii)} \quad \left[\text{say, } c_3 = c_2 + \frac{7}{8} \log 2 \right]
 \end{aligned}$$

Using equation (i) and (ii)

$$I = \frac{1}{2}x + \log|x| - \frac{3}{4} \log|1-2x| + c$$

Indefinite Integrals Ex 19.20 Q4

Here the integrand $\frac{x^2+1}{x^2-5x+6}$ is not proper rational function, so we divide x^2+1 by x^2-5x+6 and find the

$$\frac{x^2+1}{x^2-5x+6} = 1 + \frac{5x-5}{x^2-5x+6} = 1 + \frac{5x-5}{(x-2)(x-3)}$$

$$\text{Let } \frac{5x-5}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$$

So that
Equating the coefficients of x and constant terms on both sides, we get A + B = 5 and 3A + 2B = 5. Solving the we get A = -5 and B = 10

$$\text{Thus, } \frac{x^2+1}{x^2-5x+6} = 1 - \frac{5}{x-2} + \frac{10}{x-3}$$

$$\begin{aligned}
 \text{Therefore, } \int \frac{x^2+1}{(x+1)^2(x+3)} dx &= \int dx - 5 \int \frac{1}{x-2} dx + 10 \int \frac{x}{x-3} \\
 &= x - 5 \log|x-2| + 10 \log|x-3| + C.
 \end{aligned}$$

Indefinite Integrals Ex 19.20 Q5

$$\begin{aligned}\text{Let } I &= \int \frac{x^2}{x^2 + 7x + 10} dx \\ &= \int \left\{ 1 - \frac{7x + 10}{x^2 + 7x + 10} \right\} dx \\ I &= x - \int \frac{7x + 10}{x^2 + 7x + 10} dx + c_1 \text{ --- (i)}\end{aligned}$$

$$\text{Let } I_1 = \int \frac{7x + 10}{x^2 + 7x + 10} dx$$

$$\begin{aligned}\text{Let } 7x + 10 &= \lambda \frac{d}{dx} (x^2 + 7x + 10) + \mu \\ &= \lambda (2x + 7) + \mu \\ 7x + 10 &= (2\lambda)x + 7\lambda + \mu\end{aligned}$$

Comparing the coefficients of like powers of x ,

$$\begin{aligned}7 = 2\lambda &\Rightarrow \lambda = \frac{7}{2} \\ 7\lambda + \mu = 10 &\Rightarrow 7\left(\frac{7}{2}\right) + \mu = 10 \\ \mu &= -\frac{29}{2}\end{aligned}$$

$$\begin{aligned}\text{so, } I_1 &= \int \frac{\frac{7}{2}(2x + 7) - \frac{29}{2}}{x^2 + 7x + 10} dx \\ &= \frac{7}{2} \int \frac{(2x + 7)}{x^2 + 7x + 10} dx - \frac{29}{2} \int \frac{1}{x^2 + 2x\left(\frac{7}{2}\right) + \left(\frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 + 10} dx \\ I_1 &= \frac{7}{2} \int \frac{2x + 7}{x^2 + 7x + 10} dx - \frac{29}{2} \int \frac{1}{\left(x + \frac{7}{2}\right)^2 - \left(\frac{3}{2}\right)^2} dx\end{aligned}$$

$$I_1 = \frac{7}{2} \log |x^2 + 7x + 10| - \frac{29}{2} \times \frac{1}{2\left(\frac{3}{2}\right)} \log \left| \frac{x + \frac{7}{2} - \frac{3}{2}}{x + \frac{7}{2} + \frac{3}{2}} \right| + c_2 \quad \left[\text{since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x - a}{x + a} \right| + c \right]$$

$$= \frac{7}{2} \log |x^2 + 7x + 10| - \frac{29}{6} \log \left| \frac{x + 2}{x + 5} \right| + c_2 \text{ --- (ii)}$$

Using equation (i) and (ii)

$$I = x - \frac{7}{2} \log |x^2 + 7x + 10| + \frac{29}{6} \log \left| \frac{x + 2}{x + 5} \right| + c$$

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