Co-Ordinate Geometry Ex 14.2 Q24

Answer :

The distance *d* between two points (x_1, y_1) and (x_2, y_2) is given by the formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

It is said that *P*(0,2) is equidistant from both *A*(3,*k*) and *B*(*k*,5).

So, using the distance formula for both these pairs of points we have

$$AP = \sqrt{(3)^2 + (k-2)^2}$$

$$BP = \sqrt{(k)^2 + (3)^2}$$

Now since both these distances are given to be the same, let us equate both.

$$AP = BP$$

$$\sqrt{(3)^2 + (k-2)^2} = \sqrt{(k)^2 + (3)^2}$$

Squaring on both sides we have,

$$(3)^2 + (k-2)^2 = (k)^2 + (3)^2$$

$$9 + k^2 + 4 - 4k = k^2 + 9$$

$$4k = 4$$

$$k = 1$$

Hence the value of '*k*' for which the point '*P*' is equidistant from the other two given points is $\boxed{k = 1}$.

Co-Ordinate Geometry Ex 14.2 Q25

Answer :

The distance *d* between two points (x_1, y_1) and (x_2, y_2) is given by the formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

In a square all the sides are of equal length. The diagonals are also equal to each other. Also in a square the diagonal is equal to $\sqrt{2}$ times the side of the square.

Here let the two points which are said to be the opposite vertices of a diagonal of a square be *A*(5,4) and *C*(1,−6).

Let us find the distance between them which is the length of the diagonal of the square.

$$AC = \sqrt{(5-1)^2 + (4+6)^2}$$

$$= \sqrt{(4)^2 + (10)^2}$$

$$= \sqrt{16 + 100}$$

$$AC = 2\sqrt{29}$$

Now we know that in a square,

$$\text{Side of the square} = \frac{\text{Diagonal of the square}}{\sqrt{2}}$$

Substituting the value of the diagonal we found out earlier in this equation we have,

$$\text{Side of the square} = \frac{2\sqrt{29}}{\sqrt{2}}$$

$$\text{Side of the square} = \sqrt{58}$$

Now, a vertex of a square has to be at equal distances from each of its adjacent vertices.

Let $P(x, y)$ represent another vertex of the same square adjacent to both 'A' and 'C'.

$$AP = \sqrt{(5-x)^2 + (4-y)^2}$$

$$CP = \sqrt{(1-x)^2 + (-6-y)^2}$$

But these two are nothing but the sides of the square and need to be equal to each other.

$$AP = CP$$

$$\sqrt{(5-x)^2 + (4-y)^2} = \sqrt{(1-x)^2 + (-6-y)^2}$$

Squaring on both sides we have,

$$(5-x)^2 + (4-y)^2 = (1-x)^2 + (-6-y)^2$$

$$25 + x^2 - 10x + 16 + y^2 - 8y = 1 + x^2 - 2x + 36 + y^2 + 12y$$

$$8x + 20y = 4$$

$$2x + 5y = 1$$

From this we have, $x = \frac{1-5y}{2}$

Substituting this value of 'x' and the length of the side in the equation for 'AP' we have,

$$AP = \sqrt{(5-x)^2 + (4-y)^2}$$

$$\sqrt{58} = \sqrt{(5-x)^2 + (4-y)^2}$$

Squaring on both sides,

$$58 = (5-x)^2 + (4-y)^2$$

$$58 = \left(5 - \left(\frac{1-5y}{2}\right)\right)^2 + (4-y)^2$$

$$58 = \left(\frac{9+5y}{2}\right)^2 + (4-y)^2$$

$$58 = \frac{81 + 25y^2 + 90y}{4} + 16 + y^2 - 8y$$

$$232 = 81 + 25y^2 + 90y + 64 + 4y^2 - 32y$$

$$87 = 29y^2 + 58y$$

We have a quadratic equation. Solving for the roots of the equation we have,

$$29y^2 + 58y - 87 = 0$$

$$29y^2 + 87y - 29y - 87 = 0$$

$$29y(y+3) - 29(y+3) = 0$$

$$(y+3)(29y - 29) = 0$$

$$(y+3)(y-1) = 0$$

The roots of this equation are −3 and 1.

Now we can find the respective values of 'x' by substituting the two values of 'y'

When $y = -3$

$$x = \frac{1-5(-3)}{2}$$

$$= \frac{1+15}{2}$$

$$x = 8$$

When $y = 1$

$$x = \frac{1-5(1)}{2}$$

$$= \frac{1-5}{2}$$

$$x = -2$$

Therefore the other two vertices of the square are $\boxed{(8,-3) \text{ and } (-2,1)}$.

*********** END **********