Trigonometric Identities Ex 6.1 Q13

Answer :

We have to prove $\frac{\sin\theta}{1-\cos\theta}=\operatorname{cosec}\theta+\cot\theta$.

We know that, $\sin^2\theta+\cos^2\theta=1$

Multiplying both numerator and denominator by $(1+\cos\theta)$, we have

$$\begin{aligned}\frac{\sin\theta}{1-\cos\theta}&=\frac{\sin\theta(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}\\&=\frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}\\&=\frac{\sin\theta(1+\cos\theta)}{\sin^2\theta}\\&=\frac{1+\cos\theta}{\sin\theta}\\&=\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}\\&=\operatorname{cosec}\theta+\cot\theta\end{aligned}$$

Trigonometric Identities Ex 6.1 Q14

Answer :

We have to prove $\frac{1-\sin\theta}{1+\sin\theta}=(\sec\theta-\tan\theta)^2$

We know that, $\sin^2\theta+\cos^2\theta=1$.

Multiplying both numerator and denominator by $(1-\sin\theta)$, we have

$$\begin{aligned}\frac{1-\sin\theta}{1+\sin\theta}&=\frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}\\&=\frac{(1-\sin\theta)^2}{1-\sin^2\theta}\\&=\left(\frac{1-\sin\theta}{\cos\theta}\right)^2\\&=\left(\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}\right)^2\\&=(\sec\theta-\tan\theta)^2\end{aligned}$$

Trigonometric Identities Ex 6.1 Q15

Answer :

We have to prove $(\text{cosec}\,\theta+\sin\theta)(\text{cosec}\,\theta-\sin\theta)=\cot^2\theta+\cos^2\theta$

We know that,

$\sin^2\theta+\cos^2\theta=1,$

$\text{cosec}^2\theta-\cot^2\theta=1$

So,

$$\begin{aligned}(\text{cosec}\,\theta+\sin\theta)(\text{cosec}\,\theta-\sin\theta)&=\cos ec^2\theta-\sin^2\theta\\&=(1+\cot^2\theta)-(1-\cos^2\theta)\\&=1+\cot^2\theta-1+\cos^2\theta\\&=\cot^2\theta+\cos^2\theta\end{aligned}$$

Trigonometric Identities Ex 6.1 Q16

Answer :

We have to prove $\dfrac{(1+\cot^2\theta)\tan\theta}{\sec^2\theta}=\cot\theta$

We know that, $\sec^2\theta-\tan^2\theta=1$

So,

$$\begin{aligned}\frac{(1+\cot^2\theta)\tan\theta}{\sec^2\theta}&=\frac{(1+\cot^2\theta)\tan\theta}{(1+\tan^2\theta)}\\&=\frac{\left(1+\dfrac{1}{\tan^2\theta}\right)\tan\theta}{(1+\tan^2\theta)}\\&=\frac{\left(\dfrac{\tan^2\theta+1}{\tan^2\theta}\right)\tan\theta}{(1+\tan^2\theta)}\\&=\frac{(1+\tan^2\theta)\tan\theta}{\tan^2\theta(1+\tan^2\theta)}\\&=\frac{1}{\tan\theta}\\&=\cot\theta\end{aligned}$$

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