Chapter 6 Determinants Ex 6.2 Q49

$$\Delta=\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

Applying $R_1 \to R_1+R_2+R_3$, we have:

$$\Delta=\begin{vmatrix} 2(a+b+c) & 2(a+b+c) & 2(a+b+c) \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

$$=2(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

Applying $C_2 \to C_2-C_1$ and $C_3 \to C_3-C_1$, we have:

$$\Delta=2(a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ c+a & b-c & b-a \\ a+b & c-a & c-b \end{vmatrix}$$

Expanding along R_1, we have:

$$\begin{aligned}\Delta&=2(a+b+c)(1)\left[(b-c)(c-b)-(b-a)(c-a)\right]\\&=2(a+b+c)\left[-b^2-c^2+2bc-bc+ba+ac-a^2\right]\\&=2(a+b+c)\left[ab+bc+ca-a^2-b^2-c^2\right]\end{aligned}$$

It is given that $\Delta=0$.

$$(a+b+c)\left[ab+bc+ca-a^2-b^2-c^2\right]=0$$

$\Rightarrow$ Either $a+b+c=0$, or $ab+bc+ca-a^2-b^2-c^2=0$.

Now,

$$\begin{aligned}&ab+bc+ca-a^2-b^2-c^2=0\\&\Rightarrow -2ab-2bc-2ca+2a^2+2b^2+2c^2=0\\&\Rightarrow (a-b)^2+(b-c)^2+(c-a)^2=0\\&\Rightarrow (a-b)^2=(b-c)^2=(c-a)^2=0 \qquad \left[(a-b)^2,(b-c)^2,(c-a)^2 \text{ are non-negative}\right]\\&\Rightarrow (a-b)=(b-c)=(c-a)=0\\&\Rightarrow a=b=c\end{aligned}$$

Hence, if $\Delta=0$, then either $a+b+c=0$ or $a=b=c$.

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$$\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} p-a & 0 & c-r \\ 0 & q-b & c-r \\ a & b & r \end{vmatrix} = 0 [R_1 = R_1 - R_3, R_2 = R_2 - R_3]$$

$$\Rightarrow (p-a)[r(q-b) - b(c-r)] + (c-r)[0 - a(q-b)] = 0$$

$$\Rightarrow (p-a)r(q-b) - (p-a)b(c-r) - (c-r)a(q-b) = 0$$

$$\Rightarrow \frac{(p-a)r(q-b)}{(p-a)(q-b)(r-c)} - \frac{(p-a)b(c-r)}{(p-a)(q-b)(r-c)} - \frac{(c-r)a(q-b)}{(p-a)(q-b)(r-c)} = 0$$

$$\Rightarrow \frac{r}{(r-c)} + \frac{b}{(q-b)} + \frac{a}{(p-a)} = 0$$

$$\Rightarrow \frac{r}{(r-c)} + \frac{b-q+q}{(q-b)} + \frac{a+p-p}{(p-a)} = 0$$

$$\Rightarrow \frac{r}{(r-c)} + \frac{q}{(q-b)} + \frac{(b-q)}{(q-b)} + \frac{(a-p)}{(p-a)} + \frac{p}{(p-a)} = 0$$

$$\Rightarrow \frac{r}{(r-c)} + \frac{q}{(q-b)} - 1 - 1 + \frac{p}{(p-a)} = 0$$

$$\Rightarrow \frac{r}{(r-c)} + \frac{q}{(q-b)} + \frac{p}{(p-a)} = 2$$

$$\therefore \frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$$

Chapter 6 Determinants Ex 6.2 Q51

Let us show that x = 2 is a root of the given equation:
Putting x = 2 in the LHS, we get

$$\begin{vmatrix} 2 & -6 & -1 \\ 2 & -6 & -1 \\ -3 & 4 & 4 \end{vmatrix} = 0$$

$\because R_1 = R_2$

Hence, x = 2 is a root of the given equation.

Now, we see if there are any other roots. For this we need to solve the equation:

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & -6 & -1 \\ x-1 & -3x & x-3 \\ x-1 & 2x & x+2 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)\begin{vmatrix} 1 & -6 & -1 \\ 1 & -3x & x-3 \\ 1 & 2x & x+2 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)\begin{vmatrix} 1 & -6 & -1 \\ 1 & -3x & x-3 \\ 1 & 2x & x+2 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)\begin{vmatrix} 1 & -6 & -1 \\ 0 & -3x+6 & x-3+1 \\ 0 & 2x+6 & x+2+1 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)\begin{vmatrix} 1 & -6 & -1 \\ 0 & -3(x-2) & x-2 \\ 0 & 2(x+3) & x+3 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)(x-2)(x+3)\begin{vmatrix} 1 & -6 & -1 \\ 0 & -3 & 1 \\ 0 & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)(x-2)(x+3) = 0$$

$$\Rightarrow (x-1) = 0 \quad (x-2) = 0 \quad (x+3) = 0$$

$$\Rightarrow x = 1 \quad x = 2 \quad x = -3$$

*********** END **********