Complex numbers Ex 13.1 Q3(iv)

$$\begin{aligned} i^5 + i^{10} + i^{15} &= i^{4\times1}\times i^1 + i^{4\times2}\times i^2 + i^{4\times3}\times i^3 \\ &= 1\times i + 1\times i^2 + 1\times i^3 \\ &= i - 1 - i \\ &= -1 \end{aligned}$$

$\therefore\ i^5 + i^{10} + i^{15} = -1$

Complex numbers Ex 13.1 Q3(v)

$$\begin{aligned} \frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}} &= \frac{i^{4\times148}+i^{147}\times i^2+i^{4\times147}+i^{4\times146}\times i^2+i^{4\times146}}{i^{4\times145}\times i^2+i^{4\times145}+i^{4\times144}\times i^2+i^{4\times144}+i^{4\times143}\times i^2} \\ &= \frac{1+1\times i^2+1+1\times i^2+1}{1\times i^2+1+1\times i^2+1+1\times i^2} \\ &= \frac{1-1+1-1+1}{-1+1-1+1-1} \\ &= \frac{1}{-1} \\ &= -1 \end{aligned}$$

$\therefore\ \frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}} = -1$

Complex numbers Ex 13.1 Q3(vi)

$$\begin{aligned} &1 + i^2 + i^4 + i^6 + i^8 + \ldots + i^{20} \\ &= 1 + i^2 + i^4 + i^{4\times1}\times i^2 + i^{4\times2} + i^{4\times2}\times i^2 + i^{4\times3} + i^{4\times3}\times i^2 + i^{4\times4} + i^{4\times4}\times i^2 + i^{4\times5} \\ &= 1 - 1 + 1 + 1\times i^2 + 1 + 1\times i^2 + 1 + 1\times i^2 + 1 + 1\times i^2 + 1 \\ &= 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + 1 \\ &= 1 \end{aligned}$$

Complex numbers Ex 13.1 Q3(vii)

$$\begin{aligned} (1+i)^6 + (1-i)^3 &= \left[(1+i)^2\right]^3 + (1-i)^3 \\ &= \left(1 + i^2 + 2i\right)^3 + \left(1 - 3i + 3i^2 - i^3\right) \\ &= \left(1 - 1 + 2i\right)^3 + \left(1 - 3i - 3 + i\right) \\ &= 8i^3 - 2 - 2i \\ &= -8i - 2 - 2i \\ &= -2 - 10i \end{aligned}$$

********** END **********