Exercise 3D

Question 17:

$2x + 3y - 7 = 0$

$(k - 1)x + (k + 2)y - 3k = 0$

These are of the form

$$a_1x + b_1y + c_1 = 0, \quad a_2x + b_2y + c_2 = 0$$

where, $a_1 = 2, b_1 = 3, c_1 = -7$

$$a_2 = (k-1), b_2 = (k+2), c_2 = -3k$$

For infinitely many solutions, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

This hold only when

$$\frac{2}{k-1} = \frac{3}{k+2} = \frac{-7}{-3k}$$

$$\frac{2}{k-1} = \frac{3}{k+2} = \frac{7}{3k}$$

Now the following cases arises

Case : I

$$\frac{2}{k-1} = \frac{3}{k+2}$$

$$\Rightarrow 2(k+2) = 3(k-1) \Rightarrow 2k + 4 = 3k - 3$$

$$\Rightarrow \quad k = 7$$

Case: II

$$\frac{3}{k+2} = \frac{7}{3k}$$

$$\Rightarrow 7(k+2) = 9k \Rightarrow 7k + 14 = 9k$$

$$\Rightarrow \quad k = 7$$

Case III

$$\frac{2}{k-1} = \frac{7}{3k}$$

$$\Rightarrow 7k - 7 = 6k$$

$$\Rightarrow k = 7$$

For k = 7, there are infinitely many solutions of the given system of equations.

Question 18:

$2x + (k - 2)y - k = 0$

$6x + (2k - 1)y - (2k + 5) = 0$

These are of the form

$$a_1x + b_1y + c_1 = 0,\ a_2x + b_2y + c_2 = 0$$

where $a_1 = 2,\ b_1 = (k - 2),\ c_1 = -k$

$$a_2 = 6,\ b_2 = (2k - 1),\ c_2 = -(2k + 5)$$

For infinite number of solutions, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

This hold only when

$$\frac{2}{6} = \frac{k-2}{2k-1} = \frac{-k}{-(2k+5)}$$

$$\frac{1}{3} = \frac{k-2}{2k-1} = \frac{k}{2k+5}$$

Case (1)

$$\frac{1}{3} = \frac{k-2}{2k-1}\ [\text{taking I and II}]$$

$$\Rightarrow 2k - 1 = 3k - 6 \Rightarrow k = 5$$

Case (2)

$$\frac{k-2}{2k-1} = \frac{k}{2k+5}\quad [\text{Taking II and III}]$$

$$(k-2)(2k+5) = k(2k-1)$$

$$\Rightarrow 2k^2 + 5k - 4k - 10 = 2k^2 - k$$

$$\Rightarrow\ k + k = 10 \Rightarrow 2k = 10$$

$$k = \frac{10}{2} = 5\ [\text{taking I and III}]$$

Case (3)

$$\frac{1}{3} = \frac{k}{2k+5}$$

$$2k + 5 = 3k \Rightarrow 3k - 2k = 5$$

$$k = 5$$

Thus, for k = 5 there are infinitely many solutions.

*********** END **********