Indefinite Integrals Ex 19.21 Q11

Let $I=\int\frac{x+1}{\sqrt{x^2+1}}dx$

Let $x+1=\lambda\frac{d}{dx}\left(x^2+1\right)+\mu$

$x+1=\lambda(2x)+\mu$

Comparing the coefficients of like powers of x,

$2\lambda=1 \quad \Rightarrow \quad \lambda=\frac{1}{2}$

$\Rightarrow \quad \mu=1$

so, $I=\int\frac{\frac{1}{2}(2x)+1}{\sqrt{x^2+1}}dx$

$=\frac{1}{2}\int\frac{(2x)}{\sqrt{x^2+1}}dx+\int\frac{1}{\sqrt{x^2+1}}dx$

$I=\frac{1}{2}\times2\sqrt{x^2+1}+\log\left|x+\sqrt{x^2+1}\right|+c \qquad \left[\text{since, }\int\frac{1}{\sqrt{x}}dx=2\sqrt{x}+c,\ \int\frac{1}{\sqrt{x^2+1}}dx=\log\left|x+\sqrt{x^2-a^2}\right|+c\right]$

$I=\sqrt{x^2+1}+\log\left|x+\sqrt{x^2+1}\right|+c$

Indefinite Integrals Ex 19.21 Q12

Let $I=\int\frac{2x+5}{\sqrt{x^2+2x+5}}dx$

Let $2x+5=\lambda\frac{d}{dx}\left(x^2+2x+5\right)+\mu$

$=\lambda(2x+2)+\mu$

$2x+5=(2\lambda)x+2\lambda+\mu$

Comparing the coefficients of like powers of x,

$2\lambda=2 \quad \Rightarrow \quad \lambda=1$

$2\lambda+\mu=5 \quad \Rightarrow \quad 2(1)+\mu=5$

$\Rightarrow \quad \mu=3$

so, $I=\int\frac{(2x+2)+3}{\sqrt{x^2+2x+5}}dx$

$=\int\frac{(2x+3)}{\sqrt{x^2+2x+5}}dx+3\int\frac{1}{\sqrt{x^2+2x+(1)^2-(1)^2+5}}dx$

$I=\int\frac{2x+3}{\sqrt{x^2+2x+5}}dx+3\int\frac{1}{\sqrt{(x+1)^2+(2)^2}}dx$

$I=2\sqrt{x^2+2x+5}+3\log\left|x+1+\sqrt{(x+1)^2+(2)^2}\right|+c \qquad \left[\text{since, }\int\frac{1}{\sqrt{x}}dx=2\sqrt{x}+c,\ \int\frac{1}{\sqrt{x^2+a^2}}dx=\log\left|x+\sqrt{x^2+a^2}\right|+c\right]$

$I=2\sqrt{x^2+2x+5}+3\log\left|x+1+\sqrt{x^2+2x+5}\right|+c$

Indefinite Integrals Ex 19.21 Q13

Let $I=\int\frac{3x+1}{\sqrt{5-2x-x^2}}dx$

Let $3x+1=\lambda\frac{d}{dx}\left(5-2x-x^2\right)+\mu$

$=\lambda(-2-2x)+\mu$

$3x+1=(-2\lambda)x-2\lambda+\mu$

Comparing the coefficients of like powers of x,

$-2\lambda=3 \quad \Rightarrow \quad \lambda=-\frac{3}{2}$

$-2\lambda+\mu=1 \quad \Rightarrow \quad -2\left(-\frac{3}{2}\right)+\mu=1$

$\Rightarrow \quad \mu=-2$

so, $I=\int\frac{-\frac{3}{2}(-2-2x)-2}{\sqrt{5-2x-x^2}}dx$

$=-\frac{3}{2}\int\frac{(-2-2x)}{\sqrt{5-2x-x^2}}dx-2\int\frac{1}{\sqrt{-\left[x^2+2x-5\right]}}dx$

$I=-\frac{3}{2}\int\frac{-2-2x}{\sqrt{5-2x-x^2}}dx-2\int\frac{1}{\sqrt{-\left[x^2+2x+(1)^2-(1)^2+5\right]}}dx$

$I=-\frac{3}{2}\int\frac{-2-2x}{\sqrt{5-2x-x^2}}dx-2\int\frac{1}{\sqrt{-\left[(x+1)^2-\left(\sqrt{6}\right)^2\right]}}dx$

$I=-\frac{3}{2}\int\frac{-2-2x}{\sqrt{5-2x-x^2}}dx-2\int\frac{1}{\sqrt{\left(\sqrt{6}\right)^2-(x+1)^2}}dx$

$I=-\frac{3}{2}\times2\sqrt{5-2x-x^2}-2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right)+c \qquad \left[\text{since, }\int\frac{1}{\sqrt{x}}dx=2\sqrt{x}+c,\ \int\frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\left(\frac{x}{a}\right)+c\right]$

$I=-3\sqrt{5-2x-x^2}-2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right)+c$

Indefinite Integrals Ex 19.21 Q14

Let $I = \int \sqrt{\frac{1-x}{1+x}}dx$

$= \int \sqrt{\frac{1-x}{1+x} \times \frac{1-x}{1-x}}dx$

$= \int \frac{1-x}{\sqrt{1-x^2}}dx$

Let $1-x = \lambda \frac{d}{dx}\left(1-x^2\right) + \mu$

$= \lambda\left(-2x\right) + \mu$

$1-x = \left(-2\lambda\right)x + \mu$

Comparing the coefficients of like powers of x,

$-2\lambda = -1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$\Rightarrow \quad \mu = 1$

so, $I = \int \frac{\frac{1}{2}\left(-2x\right) + 1}{\sqrt{1-x^2}}dx$

$= \frac{1}{2}\int \frac{\left(-2x\right)}{\sqrt{1-x^2}}dx + \int \frac{1}{\sqrt{1-x^2}}dx$

$I = \frac{1}{2} \times 2\sqrt{1-x^2} + \sin^{-1}x + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x} + c,\ \int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}\left(x\right) + c\right]$

$I = \sqrt{1-x^2} + \sin^{-1}x + c$

*********** END **********