Question 1:

Determine whether each of the following relations are reflexive, symmetric and transitive:

(i)Relation R in the set A = {1, 2, 3...13, 14} defined as

R = {(x, y): $3x - y = 0$}

(ii) Relation R in the set **N** of natural numbers defined as

R = {(x, y): $y = x + 5$ and $x < 4$}

(iii) Relation R in the set A = {1, 2, 3, 4, 5, 6} as

R = {(x, y): y is divisible by x}

(iv) Relation R in the set **Z** of all integers defined as

R = {(x, y): $x - y$ is as integer}

(v) Relation R in the set A of human beings in a town at a particular time given by

(a) R = {(x, y): x and y work at the same place}

(b) R = {(x, y): x and y live in the same locality}

(c) R = {(x, y): x is exactly 7 cm taller than y}

(d) R = {(x, y): x is wife of y}

(e) R = {(x, y): x is father of y}

Answer

(i) A = {1, 2, 3 ... 13, 14}

R = {(x, y): $3x - y = 0$}

∴R = {(1, 3), (2, 6), (3, 9), (4, 12)}

R is not reflexive since (1, 1), (2, 2) ... (14, 14) ∉ R.

Also, R is not symmetric as (1, 3) ∈R, but (3, 1) ∉ R. [$3(3) - 1 \neq 0$]

Also, R is not transitive as (1, 3), (3, 9) ∈R, but (1, 9) ∉ R.

[$3(1) - 9 \neq 0$]

Hence, R is neither reflexive, nor symmetric, nor transitive.

(ii) R = {(x, y): $y = x + 5$ and $x < 4$} = {(1, 6), (2, 7), (3, 8)}

It is seen that (1, 1) ∉ R.

∴R is not reflexive.

(1, 6) ∈R

But,

(1, 6) ∉ R.

∴R is not symmetric.

Now, since there is no pair in R such that (x, y) and (y, z) ∈R, then (x, z) cannot belong to R.

∴ R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

(iii) A = {1, 2, 3, 4, 5, 6}

R = {(x, y): y is divisible by x}

We know that any number (x) is divisible by itself.

⇒(x, x) ∈R

∴R is reflexive.

Now,

(2, 4) ∈R [as 4 is divisible by 2]

But,

(4, 2) ∉ R. [as 2 is not divisible by 4]

∴R is not symmetric.

Let (x, y), (y, z) ∈ R. Then, y is divisible by x and z is divisible by y.

∴z is divisible by x.

⇒ (x, z) ∈R

∴R is transitive.

Hence, R is reflexive and transitive but not symmetric.

(iv) R = {(x, y): $x - y$ is an integer}

Now, for every x ∈ **Z**, (x, x) ∈R as $x - x = 0$ is an integer.

∴R is reflexive.

Now, for every x, y ∈ **Z** if (x, y) ∈ R, then $x - y$ is an integer.

⇒ $-(x - y)$ is also an integer.

⇒ $(y - x)$ is an integer.

$\therefore (y, x) \in R$

$\therefore$R is symmetric.

Now,

Let (x, y) and $(y, z) \in$R, where $x, y, z \in \mathbf{Z}$.

$\Rightarrow (x - y)$ and $(y - z)$ are integers.

$\Rightarrow x - z = (x - y) + (y - z)$ is an integer.

$\therefore (x, z) \in$R

$\therefore$R is transitive.

Hence, R is reflexive, symmetric, and transitive.

(v) (a) R = $\{(x, y)$: x and y work at the same place$\}$

$\Rightarrow (x, x) \in R$

$\therefore$ R is reflexive.

If $(x, y) \in R$, then x and y work at the same place.

$\Rightarrow$ y and x work at the same place.

$\Rightarrow (y, x) \in R$.

$\therefore$R is symmetric.

Now, let $(x, y), (y, z) \in R$

$\Rightarrow$ x and y work at the same place and y and z work at the same place.

$\Rightarrow$ x and z work at the same place.

$\Rightarrow (x, z) \in$R

$\therefore$ R is transitive.

Hence, R is reflexive, symmetric, and transitive.

(b) R = $\{(x, y)$: x and y live in the same locality$\}$

Clearly $(x, x) \in R$ as x and x is the same human being.

$\therefore$ R is reflexive.

If $(x, y) \in$R, then x and y live in the same locality.

$\Rightarrow$ y and x live in the same locality.

$\Rightarrow (y, x) \in R$

$\therefore$R is symmetric.

Now, let $(x, y) \in R$ and $(y, z) \in R$.

$\Rightarrow$ x and y live in the same locality and y and z live in the same locality.

$\Rightarrow$ x and z live in the same locality.

$\Rightarrow (x, z) \in R$

$\therefore$ R is transitive.

Hence, R is reflexive, symmetric, and transitive.

(c) R = $\{(x, y)$: x is exactly 7 cm taller than $y\}$

Now,

$(x, x) \notin R$

Since human being x cannot be taller than himself.

$\therefore$R is not reflexive.

Now, let $(x, y) \in$R.

$\Rightarrow$ x is exactly 7 cm taller than y.

Then, y is not taller than x.

$\therefore (y, x) \notin$R

Indeed if x is exactly 7 cm taller than y, then y is exactly 7 cm shorter than x.

$\therefore$R is not symmetric.

Now,

Let $(x, y), (y, z) \in R$.

$\Rightarrow$ x is exactly 7 cm taller than y and y is exactly 7 cm taller than z.

$\Rightarrow$ x is exactly 14 cm taller than z .

$\therefore (x, z) \notin$R

$\therefore$ R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

(d) R = $\{(x, y)$: x is the wife of $y\}$

Now,

$(x, x) \notin R$

Since x cannot be the wife of herself.

$\therefore$R is not reflexive.

Now, let $(x, y) \in R$

$\Rightarrow$ x is the wife of y.

Clearly y is not the wife of x.

$\therefore (y, x) \notin R$

Indeed if x is the wife of y, then y is the husband of x.

$\therefore$ R is not transitive.

Let $(x, y), (y, z) \in R$

$\Rightarrow$ x is the wife of y and y is the wife of z.

This case is not possible. Also, this does not imply that x is the wife of z.

$\therefore (x, z) \notin R$

$\therefore$R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

(e) R = $\{(x, y)$: x is the father of $y\}$

$(x, x) \notin R$

As x cannot be the father of himself.

∴R is not reflexive.

Now, let $(x, y) \in R$.

⇒ x is the father of y.

⇒ y cannot be the father of y.

Indeed, y is the son or the daughter of y.

∴$(y, x) \notin R$

∴ R is not symmetric.

Now, let $(x, y) \in R$ and $(y, z) \in R$.

⇒ x is the father of y and y is the father of z.

⇒ x is not the father of z.

Indeed x is the grandfather of z.

∴ $(x, z) \notin R$

∴R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

Question 2:

Show that the relation R in the set **R** of real numbers, defined as

$R = \{(a, b): a \le b^2\}$ is neither reflexive nor symmetric nor transitive.

Answer

$R = \{(a, b): a \le b^2\}$

It can be observed that $\left(\frac{1}{2}, \frac{1}{2}\right) \notin \mathbf{R}$, since $\frac{1}{2} > \left(\frac{1}{2}\right)^2 = \frac{1}{4}$.

∴R is not reflexive.

Now, $(1, 4) \in R$ as $1 < 4^2$

But, 4 is not less than 1^2.

∴$(4, 1) \notin R$

∴R is not symmetric.

Now,

$(3, 2), (2, 1.5) \in R$

(as $3 < 2^2 = 4$ and $2 < (1.5)^2 = 2.25$)

But, $3 > (1.5)^2 = 2.25$

∴$(3, 1.5) \notin R$

∴ R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

Question 3:

Check whether the relation R defined in the set $\{1, 2, 3, 4, 5, 6\}$ as

$R = \{(a, b): b = a + 1\}$ is reflexive, symmetric or transitive.

Answer

Let $A = \{1, 2, 3, 4, 5, 6\}$.

A relation R is defined on set A as:

$R = \{(a, b): b = a + 1\}$

∴$R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$

We can find $(a, a) \notin R$, where $a \in A$.

For instance,

$(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) \notin R$

∴R is not reflexive.

It can be observed that $(1, 2) \in R$, but $(2, 1) \notin R$.

∴R is not symmetric.

Now, $(1, 2), (2, 3) \in \mathbf{R}$

But,

$(1, 3) \notin R$

∴R is not transitive

Hence, R is neither reflexive, nor symmetric, nor transitive.

Question 4:

Show that the relation R in **R** defined as $R = \{(a, b): a \le b\}$, is reflexive and transitive but not symmetric.

Answer

$R = \{(a, b); a \le b\}$

Clearly $(a, a) \in R$ as $a = a$.

∴R is reflexive.

Now,

$(2, 4) \in R$ (as $2 < 4$)

But, $(4, 2) \notin R$ as 4 is greater than 2.

∴ R is not symmetric.

Now, let $(a, b), (b, c) \in R$.

Then,

$a \leq b$ and $b \leq c$

$\Rightarrow a \leq c$

$\Rightarrow (a, c) \in R$

$\therefore$R is transitive.

Hence,R is reflexive and transitive but not symmetric.

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