

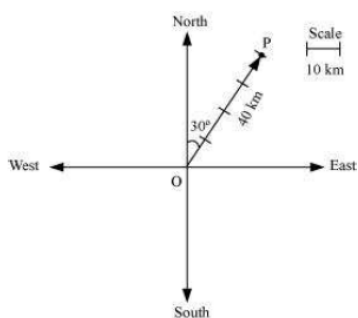


Exercise 10.1

Question 1:

Represent graphically a displacement of 40 km, 30° east of north.

Answer



Here, vector \overrightarrow{OP} represents the displacement of 40 km, 30° East of North.

Question 2:

Classify the following measures as scalars and vectors.

- (i) 10 kg (ii) 2 metres north-west (iii) 40°
(iv) 40 watt (v) 10^{-19} coulomb (vi) 20 m/s^2

Answer

- (i) 10 kg is a scalar quantity because it involves only magnitude.
(ii) 2 meters north-west is a vector quantity as it involves both magnitude and direction.
(iii) 40° is a scalar quantity as it involves only magnitude.
(iv) 40 watts is a scalar quantity as it involves only magnitude.
(v) 10^{-19} coulomb is a scalar quantity as it involves only magnitude.
(vi) 20 m/s^2 is a vector quantity as it involves magnitude as well as direction.

Question 3:

Classify the following as scalar and vector quantities.

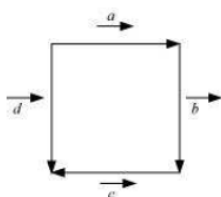
- (i) time period (ii) distance (iii) force
(iv) velocity (v) work done

Answer

- (i) Time period is a scalar quantity as it involves only magnitude.
(ii) Distance is a scalar quantity as it involves only magnitude.
(iii) Force is a vector quantity as it involves both magnitude and direction.
(iv) Velocity is a vector quantity as it involves both magnitude as well as direction.
(v) Work done is a scalar quantity as it involves only magnitude.

Question 4:

In Figure, identify the following vectors.



- (i) Coinitial (ii) Equal (iii) Collinear but not equal

Answer

- (i) Vectors \vec{a} and \vec{d} are coinitial because they have the same initial point.
(ii) Vectors \vec{b} and \vec{d} are equal because they have the same magnitude and direction.
(iii) Vectors \vec{a} and \vec{c} are collinear but not equal. This is because although they are parallel, their directions are not the same.

Question 5:

Answer the following as true or false.

- (i) \vec{a} and $-\vec{a}$ are collinear.
- (ii) Two collinear vectors are always equal in magnitude.
- (iii) Two vectors having same magnitude are collinear.
- (iv) Two collinear vectors having the same magnitude are equal.

Answer

(i) True.

Vectors \vec{a} and $-\vec{a}$ are parallel to the same line.

(ii) False.

Collinear vectors are those vectors that are parallel to the same line.

(iii) False.

Exercise 10.2**Question 1:**

Compute the magnitude of the following vectors:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}; \quad \vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}; \quad \vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$$

Answer

The given vectors are:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}; \quad \vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}; \quad \vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$$

$$|\vec{a}| = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3}$$

$$\begin{aligned} |\vec{b}| &= \sqrt{(2)^2 + (-7)^2 + (-3)^2} \\ &= \sqrt{4 + 49 + 9} \\ &= \sqrt{62} \end{aligned}$$

$$\begin{aligned} |\vec{c}| &= \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(-\frac{1}{\sqrt{3}}\right)^2} \\ &= \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = 1 \end{aligned}$$

Question 2:

Write two different vectors having same magnitude.

Answer

$$\text{Consider } \vec{a} = (\hat{i} - 2\hat{j} + 3\hat{k}) \text{ and } \vec{b} = (2\hat{i} + \hat{j} - 3\hat{k}).$$

$$\text{It can be observed that } |\vec{a}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14} \text{ and}$$

$$|\vec{b}| = \sqrt{2^2 + 1^2 + (-3)^2} = \sqrt{4 + 1 + 9} = \sqrt{14}.$$

Hence, \vec{a} and \vec{b} are two different vectors having the same magnitude. The vectors are different because they have different directions.

Question 3:

Write two different vectors having same direction.

Answer

$$\text{Consider } \vec{p} = (\hat{i} + \hat{j} + \hat{k}) \text{ and } \vec{q} = (2\hat{i} + 2\hat{j} + 2\hat{k}).$$

The direction cosines of \vec{p} are given by,

$$l = \frac{1}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}, \quad m = \frac{1}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}, \quad \text{and } n = \frac{1}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}.$$

The direction cosines of \vec{q} are given by

$$l = \frac{2}{\sqrt{2^2 + 2^2 + 2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}, \quad m = \frac{2}{\sqrt{2^2 + 2^2 + 2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}},$$

$$\text{and } n = \frac{2}{\sqrt{2^2 + 2^2 + 2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}.$$

The direction cosines of \vec{p} and \vec{q} are the same. Hence, the two vectors have the same direction.

Question 4:

Find the values of x and y so that the vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ are equal

Answer

The two vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ will be equal if their corresponding components are equal.

Hence, the required values of x and y are 2 and 3 respectively.

Question 5:

Find the scalar and vector components of the vector with initial point (2, 1) and terminal point (-5, 7).

Answer

The vector with the initial point P (2, 1) and terminal point Q (-5, 7) can be given by,

$$\overrightarrow{PQ} = (-5-2)\hat{i} + (7-1)\hat{j} \\ \Rightarrow \overrightarrow{PQ} = -7\hat{i} + 6\hat{j}$$

Hence, the required scalar components are -7 and 6 while the vector components are $-7\hat{i}$ and $6\hat{j}$.

Question 6:

Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.

Answer

The given vectors are $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.

$$\therefore \vec{a} + \vec{b} + \vec{c} = (1-2+1)\hat{i} + (-2+4-6)\hat{j} + (1+5-7)\hat{k} \\ = 0\hat{i} - 4\hat{j} - 1\hat{k} \\ = -4\hat{j} - \hat{k}$$

Question 7:

Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$.

Answer

The unit vector \hat{a} in the direction of vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ is given by $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$.

$$|\vec{a}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{1+1+4} = \sqrt{6} \\ \therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}} = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$$

Question 8:

Find the unit vector in the direction of vector \overrightarrow{PQ} , where P and Q are the points (1, 2, 3) and (4, 5, 6), respectively.

Answer

The given points are P (1, 2, 3) and Q (4, 5, 6).

$$\therefore \overrightarrow{PQ} = (4-1)\hat{i} + (5-2)\hat{j} + (6-3)\hat{k} = 3\hat{i} + 3\hat{j} + 3\hat{k} \\ |\overrightarrow{PQ}| = \sqrt{3^2 + 3^2 + 3^2} = \sqrt{9+9+9} = \sqrt{27} = 3\sqrt{3}$$

Hence, the unit vector in the direction of \overrightarrow{PQ} is

$$\frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|} = \frac{3\hat{i} + 3\hat{j} + 3\hat{k}}{3\sqrt{3}} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

Question 9:

For given vectors, $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$, find the unit vector in the direction of the vector $\vec{a} + \vec{b}$.

Answer

The given vectors are $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$.

$$\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k} \\ \vec{b} = -\hat{i} + \hat{j} - \hat{k} \\ \therefore \vec{a} + \vec{b} = (2-1)\hat{i} + (-1+1)\hat{j} + (2-1)\hat{k} = \hat{i} + 0\hat{j} + 1\hat{k} = \hat{i} + \hat{k} \\ |\vec{a} + \vec{b}| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

Hence, the unit vector in the direction of $(\vec{a} + \vec{b})$ is

$$\frac{(\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|} = \frac{\hat{i} + \hat{k}}{\sqrt{2}} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$$

Question 10:

Find a vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units.

Answer

Let $\vec{a} = 5\hat{i} - \hat{j} + 2\hat{k}$.

$$\therefore |\vec{a}| = \sqrt{5^2 + (-1)^2 + 2^2} = \sqrt{25+1+4} = \sqrt{30} \\ \therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}}$$

Hence, the vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units is given by,

$$\begin{aligned} 8\hat{a} &= 8 \left(\frac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}} \right) = \frac{40}{\sqrt{30}} \hat{i} - \frac{8}{\sqrt{30}} \hat{j} + \frac{16}{\sqrt{30}} \hat{k} \\ &= 8 \left(\frac{5\vec{i} - \vec{j} + 2\vec{k}}{\sqrt{30}} \right) \\ &= \frac{40}{\sqrt{30}} \vec{i} - \frac{8}{\sqrt{30}} \vec{j} + \frac{16}{\sqrt{30}} \vec{k} \end{aligned}$$

Question 11:

Show that the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.

Answer

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