



Indefinite Integrals Ex 19.31 Q1

$$\begin{aligned} I &= \int \frac{x^2 + 1}{x^4 + x^2 + 1} dx \\ &= \int \frac{1 + \frac{1}{x^2}}{x^2 + 1 + \frac{1}{x^2}} dx \end{aligned}$$

Dividing numerator and denominator by x^2

$$= \int \frac{\left(1 + \frac{1}{x^2}\right)}{\left(x - \frac{1}{x}\right)^2 + 3} dx$$

$$\text{Let } x - \frac{1}{x} = t \Rightarrow \left(1 + \frac{1}{x^2}\right) dx = dt$$

$$\begin{aligned} \therefore I &= \int \frac{dt}{t^2 + 3} \\ &= \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{t}{\sqrt{3}} \right) + c \end{aligned}$$

$$\therefore I = \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{3}x} \right) + c$$

Indefinite Integrals Ex 19.31 Q2

$$\int \sqrt{\cot \theta} d\theta$$

$$\text{Let } \cot \theta = x^2$$

$$\Rightarrow -\operatorname{cosec}^2 \theta d\theta = 2x dx$$

$$\begin{aligned}\Rightarrow d\theta &= \frac{-2x}{\operatorname{cosec}^2 \theta} dx \\ &= \frac{-2x}{1 + \cot^2 \theta} dx \\ &= \frac{-2x}{1 + x^4} dx\end{aligned}$$

$$\begin{aligned}\therefore I &= -\int \frac{2x^2}{1 + x^4} dx \\ &= -\int \frac{2}{\frac{1}{x^2} + x^2} dx\end{aligned}$$

Dividing numerator and denominator by x^2

$$\begin{aligned}&= -\int \frac{1 + \frac{1}{x^2} + 1 - \frac{1}{x^2}}{x^2 + \frac{1}{x^2}} dx \\ &= -\int \frac{\left(1 + \frac{1}{x^2}\right) dx}{\left(x - \frac{1}{x}\right)^2 + 2} - \int \frac{\left(1 - \frac{1}{x^2}\right) dx}{\left(x + \frac{1}{x}\right)^2 - 2}\end{aligned}$$

$$\text{Let } x - \frac{1}{x} = t \Rightarrow \left(1 + \frac{1}{x^2}\right) dx = dt$$

$$\text{and } x + \frac{1}{x} = z \Rightarrow \left(1 - \frac{1}{x^2}\right) dx = dz$$

$$\begin{aligned}\Rightarrow I &= -\int \frac{dt}{t^2 + 2} - \int \frac{dz}{z^2 - 2} \\ &= -\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{t}{\sqrt{2}} \right) - \frac{1}{2\sqrt{2}} \log \left| \frac{z - \sqrt{2}}{z + \sqrt{2}} \right| + c \\ &= -\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{2}x} \right) - \frac{1}{2\sqrt{2}} \log \left| \frac{x^2 + 1 - \sqrt{2}x}{x^2 + 1 + \sqrt{2}x} \right| + c \\ I &= -\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{\cot \theta - 1}{\sqrt{2} \cot \theta} \right) - \frac{1}{2\sqrt{2}} \log \left| \frac{\cot \theta + 1 - \sqrt{2} \cot \theta}{\cot \theta + 1 + \sqrt{2} \cot \theta} \right| + c\end{aligned}$$

Indefinite Integrals Ex 19.31 Q3

$$\text{Let } I = \int \frac{x^2 + 9}{x^4 + 81} dx$$

Dividing numerator and denominator by x^2

$$\begin{aligned}I &= \int \frac{1 + \frac{9}{x^2}}{x^2 + \frac{81}{x^2}} dx \\ &= \int \frac{1 + \frac{9}{x^2}}{\left(x - \frac{9}{x}\right)^2 + 18} dx\end{aligned}$$

$$\text{Let } \left(x - \frac{9}{x}\right) = t \Rightarrow \left(1 + \frac{9}{x^2}\right) dx = dt$$

$$\therefore I = \int \frac{dt}{t^2 + 18}$$

$$\Rightarrow I = \frac{1}{3\sqrt{2}} \tan^{-1} \left(\frac{t}{3\sqrt{2}} \right) + c$$

Thus,

$$I = \frac{1}{3\sqrt{2}} \tan^{-1} \left(\frac{x^2 - 9}{3\sqrt{2}x} \right) + c$$

Indefinite Integrals Ex 19.31 Q4

$$\text{Let } I = \int \frac{1}{x^4 + x^2 + 1} dx$$

Dividing numerator and denominator by x^2

$$\begin{aligned} I &= \int \frac{\frac{1}{x^2}}{x^2 + 1 + \frac{1}{x^2}} dx \\ &= \frac{1}{2} \int \frac{1 + \frac{1}{x^2} - 1 + \frac{1}{x^2}}{x^2 + 1 + \frac{1}{x^2}} dx \\ &= \frac{1}{2} \left\{ \int \frac{\left(1 + \frac{1}{x^2}\right) dx}{\left(x - \frac{1}{x}\right)^2 + 3} - \int \frac{\left(1 - \frac{1}{x^2}\right) dx}{\left(x + \frac{1}{x}\right)^2 - 1} \right\} \end{aligned}$$

$$\text{Let } \left(x - \frac{1}{x}\right) = t$$

$$\Rightarrow \left(1 + \frac{1}{x^2}\right) dx = dt$$

$$\text{and } \left(x + \frac{1}{x}\right) = z$$

$$\Rightarrow \left(1 - \frac{1}{x^2}\right) dx = dz$$

$$\therefore I = \frac{1}{2} \int \frac{dt}{t^2 + 3} - \frac{1}{2} \int \frac{dz}{z^2 - 1}$$

$$\Rightarrow I = \frac{1}{2\sqrt{3}} \tan^{-1} \left(\frac{t}{\sqrt{3}} \right) - \frac{1}{4} \log \left| \frac{z-1}{z+1} \right| + c$$

$$\Rightarrow I = \frac{1}{2\sqrt{3}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{3}x} \right) - \frac{1}{4} \log \left| \frac{x^2 + 1 - x}{x^2 + 1 + x} \right| + c$$

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