Differentiation Ex 11.7 Q17

Here,

$$x = a\left(t + \frac{1}{t}\right)$$

Differentiating it with respect to t,

$$\frac{dx}{dt} = a\frac{d}{dt}\left(t + \frac{1}{t}\right)$$

$$= a\left(1 - \frac{1}{t^2}\right)$$

$$\frac{dx}{dt} = a\left(\frac{t^2 - 1}{t^2}\right) \qquad \text{---(i)}$$

And, $y = a\left(t - \frac{1}{t}\right)$

Differentiating it with respect to t,

$$\frac{dy}{dt} = a\frac{d}{dt}\left(t - \frac{1}{t}\right)$$

$$= a\left(1 + \frac{1}{t^2}\right)$$

$$\frac{dy}{dt} = a\left(\frac{t^2 + 1}{t^2}\right) \qquad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = a\frac{\left(t^2 + 1\right)}{t^2} \times \frac{t^2}{a\left(t^2 - 1\right)}$$

$$\frac{dy}{dx} = \frac{t^2 + 1}{t^2 - 1}$$

$$\frac{dy}{dx} = \frac{x}{y} \qquad \left[\text{Since, } \frac{x}{y} = \frac{a\left(t^2 + 1\right)}{t} \times \frac{t}{a\left(t^2 - 1\right)} = \left(\frac{t^2 + 1}{t^2 - 1}\right)\right]$$

Differentiation Ex 11.7 Q18

Here,

$$x = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$$

Put $\quad t = \tan\theta$

$$x = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$= \sin^{\{1\}}(\sin 2\theta)$$

$$= 2\theta \qquad \left[\text{Since, } \sin 2x = \frac{2\tan x}{1+\tan^2 x}\right]$$

$$x = 2\left(\tan^{-1} t\right) \qquad \left[\text{Since, } t = \sin\theta\right]$$

Differentiating it with respect to t,

$$\frac{dx}{dt} = \frac{2}{1+t^2} \qquad \text{---(i)}$$

Now,

$$y = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$$

Put $\quad t = \tan\theta$

$$y = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$$= \tan^{-1}(\tan 2\theta) \qquad \left[\text{Sicne, } \tan 2x = \frac{2\tan x}{1-\tan^2 x}\right]$$

$$= 2\theta$$

$$y = 2\tan^{-1} t \qquad \left[\text{Since, } t = \tan\theta\right]$$

Differentiating it with respect to t,

$$\frac{dy}{dt} = \frac{2}{1+t^2} \qquad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2}{1+t^2} \times \frac{1+t^2}{2}$$

$$\frac{dy}{dx} = 1$$

Differentiation Ex 11.7 Q19

The given equations are $x = \dfrac{\sin^3 t}{\sqrt{\cos 2t}}$ and $y = \dfrac{\cos^3 t}{\sqrt{\cos 2t}}$

Then, $\dfrac{dx}{dt} = \dfrac{d}{dt}\left[\dfrac{\sin^3 t}{\sqrt{\cos 2t}}\right]$

$$= \frac{\sqrt{\cos 2t}\cdot\frac{d}{dt}\left(\sin^3 t\right) - \sin^3 t\cdot\frac{d}{dt}\sqrt{\cos 2t}}{\cos 2t}$$

$$= \frac{\sqrt{\cos 2t}\cdot 3\sin^2 t\cdot\frac{d}{dt}(\sin t) - \sin^3 t\times\frac{1}{2\sqrt{\cos 2t}}\cdot\frac{d}{dt}(\cos 2t)}{\cos 2t}$$

$$= \frac{3\sqrt{\cos 2t}\cdot\sin^2 t\cos t - \frac{\sin^3 t}{2\sqrt{\cos 2t}}\cdot(-2\sin 2t)}{\cos 2t}$$

$$= \frac{3\cos 2t\sin^2 t\cos t + \sin^3 t\sin 2t}{\cos 2t\sqrt{\cos 2t}}$$

$$\frac{dy}{dt}=\frac{d}{dt}\left[\frac{\cos^3 t}{\sqrt{\cos 2t}}\right]$$

$$=\frac{\sqrt{\cos 2t}.\frac{d}{dt}\left(\cos^3 t\right)-\cos^3 t\cdot\frac{d}{dt}\left(\sqrt{\cos 2t}\right)}{\cos 2t}$$

$$=\frac{\sqrt{\cos 2t}.3\cos^2 t\cdot\frac{d}{dt}\left(\cos t\right)-\cos^3 t.\frac{1}{2\sqrt{\cos 2t}}\cdot\frac{d}{dt}\left(\cos 2t\right)}{\cos 2t}$$

$$=\frac{3\sqrt{\cos 2t}.\cos^2 t\left(-\sin t\right)-\cos^3 t\cdot\frac{1}{2\sqrt{\cos 2t}}.\left(-2\sin 2t\right)}{\cos 2t}$$

$$=\frac{-3\cos 2t.\cos^2 t.\sin t+\cos^3 t\sin 2t}{\cos 2t\cdot\sqrt{\cos 2t}}$$

$$\therefore\frac{dy}{dx}=\frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}=\frac{-3\cos 2t.\cos^2 t.\sin t+\cos^3 t\sin 2t}{3\cos 2t\sin^2 t\cos t+\sin^3 t\sin 2t}$$

$$=\frac{-3\cos 2t.\cos^2 t.\sin t+\cos^3 t\left(2\sin t\cos t\right)}{3\cos 2t\sin^2 t\cos t+\sin^3 t\left(2\sin t\cos t\right)}$$

$$=\frac{\sin t\cos t\left[-3\cos 2t.\cos t+2\cos^3 t\right]}{\sin t\cos t\left[3\cos 2t\sin t+2\sin^3 t\right]}$$

$$=\frac{\left[-3\left(2\cos^2 t-1\right)\cos t+2\cos^3 t\right]}{\left[3\left(1-2\sin^2 t\right)\sin t+2\sin^3 t\right]}\qquad\begin{bmatrix}\cos 2t=\left(2\cos^2 t-1\right),\\ \cos 2t=\left(1-2\sin^2 t\right)\end{bmatrix}$$

$$=\frac{-4\cos^3 t+3\cos t}{3\sin t-4\sin^3 t}$$

$$=\frac{-\cos 3t}{\sin 3t}\qquad\begin{bmatrix}\cos 3t=4\cos^3 t-3\cos t,\\ \sin 3t=3\sin t-4\sin^3 t\end{bmatrix}$$

$$=-\cot 3t$$

*********** END **********