Maxima and Minima 18.5 Q5

Let r and h be the radius and height of the cylinder respectively.

Then, volume (V) of the cylinder is given by,

$$V = \pi r^2 h = 100 \qquad \text{(given)}$$

$$\therefore h = \frac{100}{\pi r^2}$$

Surface area (S) of the cylinder is given by,

$$S = 2\pi r^2 + 2\pi r h = 2\pi r^2 + \frac{200}{r}$$

$$\therefore \frac{dS}{dr} = 4\pi r - \frac{200}{r^2}, \quad \frac{d^2S}{dr^2} = 4\pi + \frac{400}{r^3}$$

$$\frac{dS}{dr} = 0 \ \Rightarrow \ 4\pi r = \frac{200}{r^2}$$

$$\Rightarrow r^3 = \frac{200}{4\pi} = \frac{50}{\pi}$$

$$\Rightarrow r = \left(\frac{50}{\pi}\right)^{\frac{1}{3}}$$

Now, it is observed that when $r = \left(\frac{50}{\pi}\right)^{\frac{1}{3}}$, $\frac{d^2S}{dr^2} > 0$.

$\therefore$By second derivative test, the surface area is the minimum when the radius of the cylinder is $\left(\frac{50}{\pi}\right)^{\frac{1}{3}}$ cm.

When $r = \left(\frac{50}{\pi}\right)^{\frac{1}{3}}$, $h = \dfrac{100}{\pi\left(\frac{50}{\pi}\right)^{\frac{2}{3}}} = \dfrac{2\times 50}{(50)^{\frac{2}{3}}(\pi)^{1-\frac{2}{3}}} = 2\left(\frac{50}{\pi}\right)^{\frac{1}{3}}$ cm.

Hence, the required dimensions of the can which has the minimum surface area is given by radius $= \left(\frac{50}{\pi}\right)^{\frac{1}{3}}$ cm and height $= 2\left(\frac{50}{\pi}\right)^{\frac{1}{3}}$ cm.

Maxima and Minima 18.5 Q6

We are given that the bending moment M at a distance x from one end of the beam is given by

(i) $M = \frac{WL}{2}x - \frac{W}{2}x^2$

$\therefore \quad \frac{dM}{dx} = \frac{WL}{2} - Wx$

For maxima and minima,

$$\frac{dM}{dx} = 0 \Rightarrow \quad \frac{WL}{2} - Wx = 0 \Rightarrow \quad x = \frac{L}{2}$$

Now,

$$\frac{d^2M}{dx^2} = -W < 0$$

$\therefore \quad x = \frac{L}{2}$ is point of local maxima.

(ii) $M = \frac{Wx}{3} - \frac{Wx^3}{3L^2}$

$\therefore \quad \frac{dM}{dx} = \frac{W}{3} - \frac{Wx^2}{L^2}$

For maxima and minima,

$$\frac{dM}{dx} = 0 \Rightarrow \quad \frac{W}{3} - \frac{Wx^2}{L^2} = 0 \Rightarrow \quad x = \frac{L}{\sqrt{3}}$$

Now,

$$\frac{d^2M}{dx^2} = -\frac{2xW}{L^2}$$

At $\quad x = \frac{L}{\sqrt{3}}, \ \frac{d^2M}{dx^2} = -\frac{2W}{\sqrt{3}L} < 0$

$\therefore \quad x = \frac{L}{\sqrt{3}}$ is point of local maxima

$$\Rightarrow \quad \frac{d^2s}{dx^2} = -\frac{\sqrt{2}r}{\frac{r^2}{2}}$$

$$= \frac{2\sqrt{2}}{r} < 0$$

$\therefore \quad x = \frac{r}{\sqrt{2}}$ is the point of local maxima

From (i)

$$y = \frac{r}{\sqrt{2}}$$

Hence, $x = \frac{r}{\sqrt{2}}, y = \frac{r}{\sqrt{2}}$ is the required number.

*********** END **********