Q1 : $x \sin x$

Answer :

Let $I = \int x \sin x \, dx$

Taking x as first function and $\sin x$ as second function and integrating by parts, we obtain

$$I = x \int \sin x \, dx - \int \left\{ \left(\frac{d}{dx} x \right) \int \sin x \, dx \right\} dx$$

$$= x(-\cos x) - \int 1 \cdot (-\cos x) \, dx$$

$$= -x \cos x + \sin x + C$$

Answer needs Correction? **Click Here**

Q2 : $x \sin 3x$

Answer :

Let $I = \int x \sin 3x \, dx$

Taking x as first function and $\sin 3x$ as second function and integrating by parts, we obtain

$$I = x \int \sin 3x \, dx - \int \left\{ \left(\frac{d}{dx} x \right) \int \sin 3x \, dx \right\}$$

$$= x \left(\frac{-\cos 3x}{3} \right) - \int 1 \cdot \left(\frac{-\cos 3x}{3} \right) dx$$

$$= \frac{-x \cos 3x}{3} + \frac{1}{3} \int \cos 3x \, dx$$

$$= \frac{-x \cos 3x}{3} + \frac{1}{9} \sin 3x + C$$

Answer needs Correction? **Click Here**

Q3 : $x^2 e^x$

Answer :

Let $I = \int x^2 e^x dx$

Taking x^2 as first function and e^x as second function and integrating by parts, we obtain

$$I = x^2 \int e^x \, dx - \int \left\{ \left(\frac{d}{dx} x^2 \right) \int e^x dx \right\} dx$$

$$= x^2 e^x - \int 2x \cdot e^x dx$$

$$= x^2 e^x - 2 \int x \cdot e^x dx$$

Again integrating by parts, we obtain

$$= x^2 e^x - 2 \left[x \cdot \int e^x dx - \int \left\{ \left(\frac{d}{dx} x \right) \cdot \int e^x dx \right\} dx \right]$$

$$= x^2 e^x - 2 \left[x e^x - \int e^x dx \right]$$

$$= x^2 e^x - 2 \left[x e^x - e^x \right]$$

$$= x^2 e^x - 2x e^x + 2e^x + C$$

$$= e^x \left(x^2 - 2x + 2 \right) + C$$

Answer needs Correction? **Click Here**

Q4 : $x \log x$

Answer :

Let $I = \int x \log x \, dx$

Taking $\log x$ as first function and x as second function and integrating by parts, we obtain

$$I = \log x \int x \, dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x \, dx \right\} dx$$

$$= \log x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2 \log x}{2} - \int \frac{x}{2} dx$$

$$= \frac{x^2 \log x}{2} - \frac{x^2}{} + C$$

Q5 : $x \log 2x$

Answer :

Let $I = \int x \log 2x\, dx$

Taking $\log 2x$ as first function and x as second function and integrating by parts, we obtain

$$I = \log 2x \int x\, dx - \int \left\{ \left(\frac{d}{dx} \log 2x \right) \int x\, dx \right\} dx$$

$$= \log 2x \cdot \frac{x^2}{2} - \int \frac{2}{2x} \cdot \frac{x^2}{2}\, dx$$

$$= \frac{x^2 \log 2x}{2} - \int \frac{x}{2}\, dx$$

$$= \frac{x^2 \log 2x}{2} - \frac{x^2}{4} + C$$

Q6 : $x^2 \log x$

Answer :

Let $I = \int x^2 \log x\, dx$

Taking $\log x$ as first function and x^2 as second function and integrating by parts, we obtain

$$I = \log x \int x^2\, dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x^2 dx \right\} dx$$

$$= \log x \left(\frac{x^3}{3} \right) - \int \frac{1}{x} \cdot \frac{x^3}{3}\, dx$$

$$= \frac{x^3 \log x}{3} - \int \frac{x^2}{3}\, dx$$

$$= \frac{x^3 \log x}{3} - \frac{x^3}{9} + C$$

Q7 : $x \sin^{-1} x$

Answer :

Let $I = \int x \sin^{-1} x\, dx$

Taking $\sin^{-1} x$ as first function and x as second function and integrating by parts, we obtain

$$I = \sin^{-1} x \int x\, dx - \int \left\{ \left(\frac{d}{dx} \sin^{-1} x \right) \int x\, dx \right\} dx$$

$$= \sin^{-1} x \left(\frac{x^2}{2} \right) - \int \frac{1}{\sqrt{1-x^2}} \cdot \frac{x^2}{2}\, dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \frac{-x^2}{\sqrt{1-x^2}}\, dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left\{ \frac{1-x^2}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} \right\} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left\{ \sqrt{1-x^2} - \frac{1}{\sqrt{1-x^2}} \right\} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left\{ \int \sqrt{1-x^2}\, dx - \int \frac{1}{\sqrt{1-x^2}}\, dx \right\}$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left\{ \frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x - \sin^{-1} x \right\} + C$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{x}{4} \sqrt{1-x^2} + \frac{1}{4} \sin^{-1} x - \frac{1}{2} \sin^{-1} x + C$$

$$= \frac{1}{4} \left(2x^2 - 1 \right) \sin^{-1} x + \frac{x}{4} \sqrt{1-x^2} + C$$

Q8 : $x \tan^{-1} x$

Answer :

Let $I = \int x \tan^{-1} x\, dx$

Taking $\tan^{-1} x$ as first function and x as second function and integrating by parts, we obtain

$$I = \tan^{-1} x \int x\, dx - \int \left\{ \left(\frac{d}{dx} \tan^{-1} x \right) \int x\, dx \right\} dx$$

$$= \tan^{-1} x \left(\frac{x^2}{2} \right) - \int \frac{1}{1+x^2} \cdot \frac{x^2}{2}\, dx$$

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \frac{x^2}{1+x^2}\, dx$$

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \left(\frac{x^2+1}{1+x^2} - \frac{1}{1+x^2} \right) dx$$

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \left(1 - \frac{1}{1+x^2} \right) dx$$

$$\frac{2}{2} \cdot \frac{2}{2} \int \left(\frac{}{1+x^2} \right)^{\cdots}$$

$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2}\left(x - \tan^{-1} x\right) + C$$

$$= \frac{x^2}{2}\tan^{-1} x - \frac{x}{2} + \frac{1}{2}\tan^{-1} x + C$$

Q9 : $x \cos^{-1} x$

Answer :

Let $I = \int x \cos^{-1} x\, dx$

Taking $\cos^{-1} x$ as first function and x as second function and integrating by parts, we obtain

$$I = \cos^{-1} x \int x\, dx - \int \left\{ \left(\frac{d}{dx}\cos^{-1} x \right) \int x\, dx \right\} dx$$

$$= \cos^{-1} x \cdot \frac{x^2}{2} - \int \frac{-1}{\sqrt{1-x^2}} \cdot \frac{x^2}{2}\, dx$$

$$= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2}\int \frac{1-x^2-1}{\sqrt{1-x^2}}\, dx$$

$$= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2}\int \left\{ \sqrt{1-x^2} + \left(\frac{-1}{\sqrt{1-x^2}} \right) \right\} dx$$

$$= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2}\int \sqrt{1-x^2}\, dx - \frac{1}{2}\int \left(\frac{-1}{\sqrt{1-x^2}} \right) dx$$

$$= \frac{x^2 \cos^{-1} x}{2} - \frac{1}{2}I_1 - \frac{1}{2}\cos^{-1} x \qquad \dots(1)$$

where, $I_1 = \int \sqrt{1-x^2}\, dx$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \int \frac{d}{dx}\sqrt{1-x^2} \int x\, dx$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \int \frac{-2x}{2\sqrt{1-x^2}} \cdot x\, dx$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \int \frac{-x^2}{\sqrt{1-x^2}}\, dx$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \int \frac{1-x^2-1}{\sqrt{1-x^2}}\, dx$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \left\{ \int \sqrt{1-x^2}\, dx + \int \frac{-dx}{\sqrt{1-x^2}} \right\}$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \left\{ I_1 + \cos^{-1} x \right\}$$

$$\Rightarrow 2I_1 = x\sqrt{1-x^2} - \cos^{-1} x$$

$$\therefore I_1 = \frac{x}{2}\sqrt{1-x^2} - \frac{1}{2}\cos^{-1} x$$

Substituting in (1), we obtain

$$I = \frac{x\cos^{-1} x}{2} - \frac{1}{2}\left(\frac{x}{2}\sqrt{1-x^2} - \frac{1}{2}\cos^{-1} x \right) - \frac{1}{2}\cos^{-1} x$$

$$= \frac{(2x^2-1)}{4}\cos^{-1} x - \frac{x}{4}\sqrt{1-x^2} + C$$

Q10 : $\left(\sin^{-1} x \right)^2$

Answer :

Let $I = \int \left(\sin^{-1} x \right)^2 \cdot 1\, dx$

Taking $\left(\sin^{-1} x \right)^2$ as first function and 1 as second function and integrating by parts, we obtain

$$I = \left(\sin^{-1} x \right)^2 \int 1\, dx - \int \left\{ \frac{d}{dx}\left(\sin^{-1} x \right)^2 \cdot \int 1 \cdot dx \right\} dx$$

$$= \left(\sin^{-1} x \right)^2 \cdot x - \int \frac{2\sin^{-1} x}{\sqrt{1-x^2}} \cdot x\, dx$$

$$= x\left(\sin^{-1} x \right)^2 + \int \sin^{-1} x \cdot \left(\frac{-2x}{\sqrt{1-x^2}} \right) dx$$

$$= x\left(\sin^{-1} x \right)^2 + \left[\sin^{-1} x \int \frac{-2x}{\sqrt{1-x^2}}\, dx - \int \left\{ \left(\frac{d}{dx}\sin^{-1} x \right) \int \frac{-2x}{\sqrt{1-x^2}}\, dx \right\} dx \right]$$

$$= x\left(\sin^{-1} x \right)^2 + \left[\sin^{-1} x \cdot 2\sqrt{1-x^2} - \int \frac{1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2}\, dx \right]$$

$$= x\left(\sin^{-1} x \right)^2 + 2\sqrt{1-x^2}\sin^{-1} x - \int 2\, dx$$

$$= x\left(\sin^{-1} x \right)^2 + 2\sqrt{1-x^2}\sin^{-1} x - 2x + C$$

Q11 : $\dfrac{x\cos^{-1} x}{\sqrt{1-x^2}}$

Answer :

Let $I = \int \frac{x\cos^{-1} x}{\sqrt{1-x^2}}\, dx$

$$I = \frac{-1}{2}\int\frac{-2x}{\sqrt{1-x^2}}\cdot\cos^{-1}x\,dx$$

Taking $\cos^{-1}x$ as first function and $\left(\dfrac{-2x}{\sqrt{1-x^2}}\right)$ as second function and integrating by parts, we obtain

$$I = \frac{-1}{2}\left[\cos^{-1}x\int\frac{-2x}{\sqrt{1-x^2}}\,dx - \int\left\{\left(\frac{d}{dx}\cos^{-1}x\right)\int\frac{-2x}{\sqrt{1-x^2}}\,dx\right\}dx\right]$$

$$= \frac{-1}{2}\left[\cos^{-1}x\cdot 2\sqrt{1-x^2} - \int\frac{-1}{\sqrt{1-x^2}}\cdot 2\sqrt{1-x^2}\,dx\right]$$

$$= \frac{-1}{2}\left[2\sqrt{1-x^2}\,\cos^{-1}x + \int 2\,dx\right]$$

$$= \frac{-1}{2}\left[2\sqrt{1-x^2}\,\cos^{-1}x + 2x\right] + C$$

$$= -\left[\sqrt{1-x^2}\,\cos^{-1}x + x\right] + C$$

Answer needs Correction? **Click Here**

Q12 : $x\sec^2 x$

Answer :

Let $I = \int x\sec^2 x\,dx$

Taking x as first function and $\sec^2 x$ as second function and integrating by parts, we obtain

$$I = x\int\sec^2 x\,dx - \int\left\{\left\{\frac{d}{dx}x\right\}\int\sec^2 x\,dx\right\}dx$$

$$= x\tan x - \int 1\cdot\tan x\,dx$$

$$= x\tan x + \log|\cos x| + C$$

Answer needs Correction? **Click Here**

Q13 : $\tan^{-1}x$

Answer :

Let $I = \int 1\cdot\tan^{-1}x\,dx$

Taking $\tan^{-1}x$ as first function and 1 as second function and integrating by parts, we obtain

$$I = \tan^{-1}x\int 1\,dx - \int\left\{\left(\frac{d}{dx}\tan^{-1}x\right)\int 1\cdot dx\right\}dx$$

$$= \tan^{-1}x\cdot x - \int\frac{1}{1+x^2}\cdot x\,dx$$

$$= x\tan^{-1}x - \frac{1}{2}\int\frac{2x}{1+x^2}\,dx$$

$$= x\tan^{-1}x - \frac{1}{2}\log|1+x^2| + C$$

$$= x\tan^{-1}x - \frac{1}{2}\log(1+x^2) + C$$

Answer needs Correction? **Click Here**

Q14 : $x(\log x)^2$

Answer :

$$I = \int x(\log x)^2\,dx$$

Taking $(\log x)^2$ as first function and x as second function and integrating by parts, we obtain

Answer needs Correction? **Click Here**

Q15 : $(x^2+1)\log x$

Answer :

Let $I = \int(x^2+1)\log x\,dx = \int x^2\log x\,dx + \int\log x\,dx$

Let $I = I_1 + I_2$ ... (1)

Where, $I_1 = \int x^2\log x\,dx$ and $I_2 = \int\log x\,dx$

$$I_1 = \int x^2\log x\,dx$$

Taking $\log x$ as first function and x^2 as second function and integrating by parts, we obtain

$$I_1 = \log x\int x^2 dx - \int\left\{\left(\frac{d}{dx}\log x\right)\int x^2 dx\right\}dx$$

$$= \log x\cdot\frac{x^3}{3} - \int\frac{1}{x}\cdot\frac{x^3}{3}\,dx$$

$$= \frac{x^3}{3}\log x - \frac{1}{3}\left(\int x^2 dx\right)$$

$$= \frac{x^3}{3}\log x - \frac{x^3}{9} + C_1 \qquad \dots(2)$$

$$I_2 = \int\log x\,dx$$

Taking log x as first function and 1 as second function and integrating by parts, we obtain

$$I_2 = \log x \int 1 \cdot dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int 1 \cdot dx \right\}$$

$$= \log x \cdot x - \int \frac{1}{x} \cdot x \, dx$$

$$= x \log x - \int 1 \, dx$$

$$= x \log x - x + C_2 \qquad \qquad \dots (3)$$

Using equations (2) and (3) in (1), we obtain

$$I = \frac{x^3}{3} \log x - \frac{x^3}{9} + C_1 + x \log x - x + C_2$$

$$= \frac{x^3}{3} \log x - \frac{x^3}{9} + x \log x - x + (C_1 + C_2)$$

$$= \left(\frac{x^3}{3} + x \right) \log x - \frac{x^3}{9} - x + C$$

Answer needs Correction? **Click Here**

Q16 : $e^x (\sin x + \cos x)$

Answer :

Let $I = \int e^x (\sin x + \cos x) \, dx$

Let $f(x) = \sin x$

$\Rightarrow f'(x) = \cos x$

$\therefore I = \int e^x \{ f(x) + f'(x) \} \, dx$

It is known that, $\int e^x \{ f(x) + f'(x) \} \, dx = e^x f(x) + C$

$\therefore I = e^x \sin x + C$

Answer needs Correction? **Click Here**

Q17 : $\dfrac{xe^x}{(1+x)^2}$

Answer :

Let $I = \int \dfrac{xe^x}{(1+x)^2} dx = \int e^x \left\{ \dfrac{x}{(1+x)^2} \right\} dx$

$$= \int e^x \left\{ \frac{1+x-1}{(1+x)^2} \right\} dx$$

$$= \int e^x \left\{ \frac{1}{1+x} - \frac{1}{(1+x)^2} \right\} dx$$

Let $f(x) = \dfrac{1}{1+x} \Rightarrow f'(x) = \dfrac{-1}{(1+x)^2}$

$$\Rightarrow \int \frac{xe^x}{(1+x)^2} dx = \int e^x \{ f(x) + f'(x) \} dx$$

It is known that, $\int e^x \{ f(x) + f'(x) \} dx = e^x f(x) + C$

$$\therefore \int \frac{xe^x}{(1+x)^2} dx = \frac{e^x}{1+x} + C$$

Answer needs Correction? **Click Here**

Q18 : $e^x \left(\dfrac{1 + \sin x}{1 + \cos x} \right)$

Answer :

$$e^x \left(\frac{1 + \sin x}{1 + \cos x} \right)$$

$$= e^x \left(\frac{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \right)$$

$$= \frac{e^x \left(\sin \frac{x}{2} + \cos \frac{x}{2} \right)^2}{2 \cos^2 \frac{x}{2}}$$

$$= \frac{1}{2} e^x \cdot \left(\frac{\sin \frac{x}{2} + \cos \frac{x}{2}}{\cos \frac{x}{2}} \right)^2$$

$$= \frac{1}{2} e^x \left[\tan \frac{x}{2} + 1 \right]^2$$

$$= \frac{1}{2} e^x \left(1 + \tan \frac{x}{2} \right)^2$$

$$= \frac{1}{2} e^x \left[1 + \tan^2 \frac{x}{2} + 2 \tan \frac{x}{2} \right]$$

$$= \frac{1}{2}e^x\left[\sec^2\frac{x}{2} + 2\tan\frac{x}{2}\right]$$

$$\frac{e^x(1+\sin x)dx}{(1+\cos x)} = e^x\left[\frac{1}{2}\sec^2\frac{x}{2} + \tan\frac{x}{2}\right] \qquad \text{...(1)}$$

Let $\tan\frac{x}{2} = f(x) \Rightarrow f'(x) = \frac{1}{2}\sec^2\frac{x}{2}$

It is known that, $\int e^x\{f(x) + f'(x)\}dx = e^x f(x) + C$

From equation (1), we obtain

$$\int \frac{e^x(1+\sin x)}{(1+\cos x)}dx = e^x\tan\frac{x}{2} + C$$

Answer needs Correction? **Click Here**

Q19 : $e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)$

Answer :

Let $I = \int e^x\left[\frac{1}{x} - \frac{1}{x^2}\right]dx$

Also, let $\frac{1}{x} = f(x) \Rightarrow f'(x) = \frac{-1}{x^2}$

It is known that, $\int e^x\{f(x) + f'(x)\}dx = e^x f(x) + C$

$\therefore I = \frac{e^x}{x} + C$

Answer needs Correction? **Click Here**

Q20 : $\frac{(x-3)e^x}{(x-1)^3}$

Answer :

$$\int e^x\left\{\frac{x-3}{(x-1)^3}\right\}dx = \int e^x\left\{\frac{x-1-2}{(x-1)^3}\right\}dx$$

$$= \int e^x\left\{\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3}\right\}dx$$

Let $f(x) = \frac{1}{(x-1)^2} \Rightarrow f'(x) = \frac{-2}{(x-1)^3}$

It is known that, $\int e^x\{f(x) + f'(x)\}dx = e^x f(x) + C$

$$\therefore \int e^x\left\{\frac{(x-3)}{(x-1)^2}\right\}dx = \frac{e^x}{(x-1)^2} + C$$

Answer needs Correction? **Click Here**

Q21 : $e^{2x}\sin x$

Answer :

Let $I = \int e^{2x}\sin x\, dx$ \qquad ...(1)

Integrating by parts, we obtain

$$I = \sin x\int e^{2x}dx - \int\left\{\left(\frac{d}{dx}\sin x\right)\int e^{2x}dx\right\}dx$$

$$\Rightarrow I = \sin x \cdot \frac{e^{2x}}{2} - \int\cos x \cdot \frac{e^{2x}}{2}dx$$

$$\Rightarrow I = \frac{e^{2x}\sin x}{2} - \frac{1}{2}\int e^{2x}\cos x\, dx$$

Again integrating by parts, we obtain

$$I = \frac{e^{2x} \cdot \sin x}{2} - \frac{1}{2}\left[\cos x\int e^{2x}dx - \int\left\{\left(\frac{d}{dx}\cos x\right)\int e^{2x}dx\right\}dx\right]$$

$$\Rightarrow I = \frac{e^{2x}\sin x}{2} - \frac{1}{2}\left[\cos x \cdot \frac{e^{2x}}{2} - \int(-\sin x)\frac{e^{2x}}{2}dx\right]$$

$$\Rightarrow I = \frac{e^{2x}\cdot\sin x}{2} - \frac{1}{2}\left[\frac{e^{2x}\cos x}{2} + \frac{1}{2}\int e^{2x}\sin x\, dx\right]$$

$$\Rightarrow I = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4} - \frac{1}{4}I \qquad \text{[From (1)]}$$

$$\Rightarrow I + \frac{1}{4}I = \frac{e^{2x}\cdot\sin x}{2} - \frac{e^{2x}\cos x}{4}$$

$$\Rightarrow \frac{5}{4}I = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4}$$

$$\Rightarrow I = \frac{4}{5}\left[\frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4}\right] + C$$

$$\Rightarrow I = \frac{e^{2x}}{5}[2\sin x - \cos x] + C$$

Answer needs Correction? **Click Here**

Q22 : $\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$

Answer :

Let $x = \tan\theta \Rightarrow dx = \sec^2\theta\,d\theta$

$$\therefore \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta$$

$$\Rightarrow \int\sin^{-1}\left(\frac{2x}{1+x^2}\right)dx = \int 2\theta\cdot\sec^2\theta\,d\theta = 2\int\theta\cdot\sec^2\theta\,d\theta$$

Integrating by parts, we obtain

$$2\left[\theta\cdot\int\sec^2\theta\,d\theta - \int\left\{\left(\frac{d}{d\theta}\theta\right)\int\sec^2\theta\,d\theta\right\}d\theta\right]$$

$$= 2\left[\theta\cdot\tan\theta - \int\tan\theta\,d\theta\right]$$

$$= 2\left[\theta\tan\theta + \log|\cos\theta|\right] + C$$

$$= 2\left[x\tan^{-1}x + \log\left|\frac{1}{\sqrt{1+x^2}}\right|\right] + C$$

$$= 2x\tan^{-1}x + 2\log\left(1+x^2\right)^{-\frac{1}{2}} + C$$

$$= 2x\tan^{-1}x + 2\left[-\frac{1}{2}\log\left(1+x^2\right)\right] + C$$

$$= 2x\tan^{-1}x - \log\left(1+x^2\right) + C$$

Q23 : $\int x^2 e^{x^3}\,dx$ **equals**

(A) $\dfrac{1}{3}e^{x^3} + C$ (B) $\dfrac{1}{3}e^{x^2} + C$

(C) $\dfrac{1}{2}e^{x^3} + C$ (D) $\dfrac{1}{3}e^{x^2} + C$

Answer :

Let $I = \int x^2 e^{x^3}\,dx$

Also, let $x^3 = t \Rightarrow 3x^2\,dx = dt$

$$\Rightarrow I = \frac{1}{3}\int e^t\,dt$$

$$= \frac{1}{3}\left(e^t\right) + C$$

$$= \frac{1}{3}e^{x^3} + C$$

Hence, the correct answer is A.

Q24 : $\int e^x \sec x(1+\tan x)\,dx$ **equals**

(A) $e^x \cos x + C$ (B) $e^x \sec x + C$

(C) $e^x \sin x + C$ (D) $e^x \tan x + C$

Answer :

$\int e^x \sec x(1+\tan x)\,dx$

Let $I = \int e^x \sec x(1+\tan x)\,dx = \int e^x(\sec x + \sec x\tan x)\,dx$

Also, let $\sec x = f(x) \Rightarrow \sec x\tan x = f'(x)$

It is known that, $\int e^x\{f(x) + f'(x)\}\,dx = e^x f(x) + C$

$\therefore I = e^x \sec x + C$

Hence, the correct answer is B.

********** END *********