

EXERCISE 5.2

Question 1:

Find the modulus and the argument of the complex number $z=-1-i\sqrt{3}$ Ans:

$$z = -1 - i\sqrt{3}$$

Let $r\cos\theta = -1$ and $r\sin\theta = -\sqrt{3}$

On squaring and adding, we obtain

$$(r\cos\theta)^2 + (r\sin\theta)^2 = (-1)^2 + (-\sqrt{3})^2$$

$$\Rightarrow r^2 \left(\cos^2 \theta + \sin^2 \theta\right) = 1 + 3$$

$$\Rightarrow$$
 r² = 4

$$\Rightarrow r^{2} = 4$$
 [cos² θ + sin² θ = 1]
$$\Rightarrow r = \sqrt{4} = 2$$
 [Conventionally, $r > 0$]

$$\therefore 2\cos\theta = -1 \text{ and } 2\sin\theta = -\sqrt{3}$$

$$\Rightarrow \cos \theta = \frac{-1}{2}$$
 and $\sin \theta = \frac{-\sqrt{3}}{2}$

Since both the values of $\sin\theta$ and $\cos\theta$ are negative and $\sin\theta$ and $\cos\theta$ are negative in III quadrant,

Argument =
$$-\left(\pi - \frac{\pi}{3}\right) = \frac{-2\pi}{3}$$

Thus, the modulus and argument of the complex number $-1-\sqrt{3}\,i$ are 2 and $\frac{-2\pi}{3}$ respectively.

Question 2:

Find the modulus and the argument of the complex number $z = -\sqrt{3} + i$ Ans:

$$z = -\sqrt{3} + i$$

Let $r \cos \theta = -\sqrt{3}$ and $r \sin \theta = 1$

On squaring and adding, we obtain

$$r^{2} \cos^{2} \theta + r^{2} \sin^{2} \theta = \left(-\sqrt{3}\right)^{2} + 1^{2}$$

$$\Rightarrow r^{2} = 3 + 1 = 4 \qquad \left[\cos^{2} \theta + \sin^{2} \theta = 1\right]$$

$$\Rightarrow r = \sqrt{4} = 2 \qquad \left[\text{Conventionally, } r > 0\right]$$

$$\therefore \text{ Modulus } = 2$$

$$\therefore 2 \cos \theta = -\sqrt{3} \text{ and } 2 \sin \theta = 1$$

$$\Rightarrow \cos \theta = \frac{-\sqrt{3}}{2} \text{ and } \sin \theta = \frac{1}{2}$$

Thus, the modulus and argument of the complex number $-\sqrt{3} + i$ are 2 and $\frac{5\pi}{6}$

[As θ lies in the II quadrant]

respectively.
Question 3:

 $\therefore \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

Convert the given complex number in polar form: 1 - i

Ans:

1-i

Let $r \cos \theta = 1$ and $r \sin \theta = -1$

On squaring and adding, we obtain

$$r^{2} \cos^{2} \theta + r^{2} \sin^{2} \theta = 1^{2} + (-1)^{2}$$

$$\Rightarrow r^{2} (\cos^{2} \theta + \sin^{2} \theta) = 1 + 1$$

$$\Rightarrow r^{2} = 2$$

$$\Rightarrow r = \sqrt{2} \qquad [Conventionally, r > 0]$$

$$\therefore \sqrt{2} \cos \theta = 1 \text{ and } \sqrt{2} \sin \theta = -1$$

$$\Rightarrow \cos \theta = \frac{1}{\sqrt{2}} \text{ and } \sin \theta = -\frac{1}{\sqrt{2}}$$

$$\therefore \theta = -\frac{\pi}{4} \qquad [As \theta \text{ lies in the IV quadrant}]$$

$$\therefore 1 - i = r\cos\theta + ir\sin\theta = \sqrt{2}\cos\left(-\frac{\pi}{4}\right) + i\sqrt{2}\sin\left(-\frac{\pi}{4}\right) = \sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right]$$

This is the required polar form.

Question 4:

Convert the given complex number in polar form: -1 + i Ans:

Let $r \cos \theta = -1$ and $r \sin \theta = 1$

On squaring and adding, we obtain

$$r^{2} \cos^{2} \theta + r^{2} \sin^{2} \theta = (-1)^{2} + 1^{2}$$

$$\Rightarrow r^{2} \left(\cos^{2} \theta + \sin^{2} \theta\right) = 1 + 1$$

$$\Rightarrow r^{2} = 2$$

$$\Rightarrow r = \sqrt{2} \qquad [Conventionally, r > 0]$$

$$\therefore \sqrt{2} \cos \theta = -1 \text{ and } \sqrt{2} \sin \theta = 1$$

$$\Rightarrow \cos \theta = -\frac{1}{\sqrt{2}} \text{ and } \sin \theta = \frac{1}{\sqrt{2}}$$

$$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \qquad [As \theta \text{ lies in the II quadrant}]$$

It can be written,

$$\therefore -1 + i = r\cos\theta + ir\sin\theta = \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

This is the required polar form.

Question 5:

Convert the given complex number in polar form: - 1 - i

Ans: -1-i

-1-1

Let $r \cos \theta = -1$ and $r \sin \theta = -1$

On squaring and adding, we obtain

$$r^{2} \cos^{2} \theta + r^{2} \sin^{2} \theta = (-1)^{2} + (-1)^{2}$$

$$\Rightarrow r^{2} \left(\cos^{2} \theta + \sin^{2} \theta\right) = 1 + 1$$

$$\Rightarrow r^{2} = 2$$

$$\Rightarrow r = \sqrt{2} \qquad [Conventionally, r > 0]$$

$$\therefore \sqrt{2} \cos \theta = -1 \text{ and } \sqrt{2} \sin \theta = -1$$

$$\Rightarrow \cos \theta = -\frac{1}{\sqrt{2}} \text{ and } \sin \theta = -\frac{1}{\sqrt{2}}$$

$$\therefore \theta = -\left(\pi - \frac{\pi}{4}\right) = -\frac{3\pi}{4} \qquad [As \ \theta \text{ lies in the III quadrant}]$$

$$\therefore -1 - i = r \cos \theta + i r \sin \theta = \sqrt{2} \cos \frac{-3\pi}{4} + i \sqrt{2} \sin \frac{-3\pi}{4} = \sqrt{2} \left(\cos \frac{-3\pi}{4} + i \sin \frac{-3\pi}{4}\right)$$

This is the required polar form.

Question 6:

Convert the given complex number in polar form: -3

Ans:

Let $r \cos \theta = -3$ and $r \sin \theta = 0$

On squaring and adding, we obtain

$$r^{2} \cos^{2} \theta + r^{2} \sin^{2} \theta = (-3)^{2}$$

$$\Rightarrow r^{2} \left(\cos^{2} \theta + \sin^{2} \theta\right) = 9$$

$$\Rightarrow r^{2} = 9$$

$$\Rightarrow r = \sqrt{9} = 3 \qquad \text{[Conventionally, } r > 0\text{]}$$

$$\therefore 3\cos \theta = -3 \text{ and } 3\sin \theta = 0$$

$$\Rightarrow \cos \theta = -1 \text{ and } \sin \theta = 0$$

$$\therefore \theta = \pi$$

$$\therefore -3 = r \cos \theta + ir \sin \theta = 3\cos \pi + \beta \sin \pi = 3\left(\cos \pi + i\sin \pi\right)$$

This is the required polar form.

Question 7:

Convert the given complex number in polar form: $\sqrt{3} + i$

$$\sqrt{3}+i$$

Let $r \cos \theta = \sqrt{3}$ and $r \sin \theta = 1$

On squaring and adding, we obtain

$$r^{2} \cos^{2} \theta + r^{2} \sin^{2} \theta = \left(\sqrt{3}\right)^{2} + 1^{2}$$

$$\Rightarrow r^{2} \left(\cos^{2} \theta + \sin^{2} \theta\right) = 3 + 1$$

$$\Rightarrow r^{2} = 4$$

$$\Rightarrow r = \sqrt{4} = 2 \qquad \text{[Conventionally, } r > 0\text{]}$$

$$\therefore 2 \cos \theta = \sqrt{3} \text{ and } 2 \sin \theta = 1$$

$$\Rightarrow \cos \theta = \frac{\sqrt{3}}{2} \text{ and } \sin \theta = \frac{1}{2}$$

$$\therefore \theta = \frac{\pi}{6} \qquad \text{[As } \theta \text{ lies in the I quadrant]}$$

$$\therefore \sqrt{3} + i = r\cos\theta + ir\sin\theta = 2\cos\frac{\pi}{6} + i2\sin\frac{\pi}{6} = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

This is the required polar form.

Question 8:

Convert the given complex number in polar form: *i* Ans:

Let $r \cos \theta = 0$ and $r \sin \theta = 1$

On squaring and adding, we obtain

$$r^{2} \cos^{2} \theta + r^{2} \sin^{2} \theta = 0^{2} + 1^{2}$$

$$\Rightarrow r^{2} \left(\cos^{2} \theta + \sin^{2} \theta\right) = 1$$

$$\Rightarrow r^{2} = 1$$

$$\Rightarrow r = \sqrt{1} = 1 \qquad [Conventionally, r > 0]$$

$$\therefore \cos \theta = 0 \text{ and } \sin \theta = 1$$

$$\therefore \theta = \frac{\pi}{2}$$

$$\therefore i = r\cos\theta + ir\sin\theta = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$$

This is the required polar form.

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