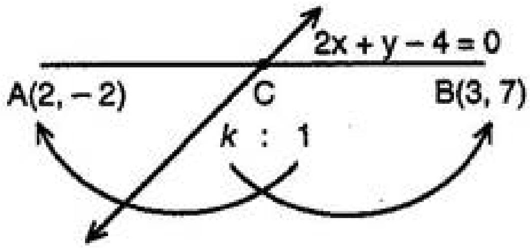
NCERT Solutions For Class 10 Chapter 7 Coordinate Geometry Exercise 7.4

1. Determine the ratio in which the line $2x+y-4=0$ divides the line segment joining the points A$(2,-2)$ and B$(3,7)$.

Ans. Let the line $2x+y-4=0$ divides the line segment joining A$(2,-2)$ and B (3, 7) in the ratio $k:1$ at point C. Then, the coordinates of C are $\left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$.



But C lies on $2x+y-4=0$, therefore

$$2\left(\frac{3k+2}{k+1}\right)+\left(\frac{7k-2}{k+1}\right)-4=0$$

$$\Rightarrow 6k+4+7k-2-4k-4=0$$

$$\Rightarrow 9k-2=0$$

$$\Rightarrow k=\frac{2}{9}$$

Hence, the required ratio if 2: 9 internally.

2. Find a relation between x and y if the points (x,y), $(1,2)$ and $(7,0)$ are collinear.

Ans. The points A(x,y), B (1, 2) and C (7, 0) will be collinear if

Area of triangle = 0

$$\Rightarrow \frac{1}{2}\left[x(2-0)+1(0-y)+7(y-2)\right]=0$$

$$\Rightarrow 2x-y+7y-14=0$$

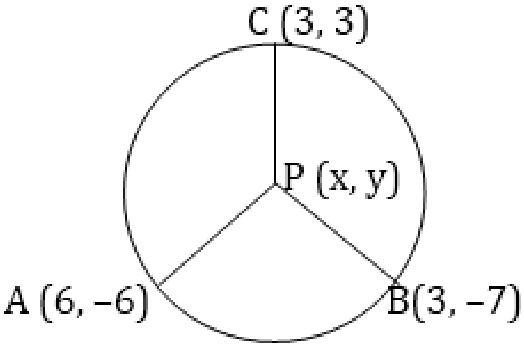
$$\Rightarrow 2x+6y-14=0$$

$$\Rightarrow x+3y-7=0$$

3. Find the centre of a circle passing through the points $(6,-6),(3,-7)$ and $(3,3)$.

Ans. Let P(x,y), be the centre of the circle passing through the points A$(6,-6)$, B$(3,-7)$ and C (3, 3). Then AP = BP = CP.

Taking AP = BP



$\Rightarrow\ AP^2 = BP^2$

$\Rightarrow\ (x-6)^2+(y+6)^2=(x-3)^2+(y+7)^2$

$\Rightarrow$

$x^2-12x+36+y^2+12y+36=x^2-6x+9+y^2+14y+49$

$\Rightarrow\ -12x+6x+12y-14y+72-58=0$

$\Rightarrow\ -6x-2y+14=0$

$\Rightarrow\ 3x+y-7=0$ ....(i)

Again, taking BP = CP

$\Rightarrow\ BP^2 = CP^2$

$\Rightarrow\ (x-3)^2+(y+7)^2=(x-3)^2+(y-3)^2$

$\Rightarrow$

$x^2-6x+9+y^2+14y+49=x^2-6x+9+y^2-6y+9$

$\Rightarrow\ -6x+6x+14y+6y+58-18=0$

$\Rightarrow\ 20y+40=0$

$\Rightarrow\ y=-2$

Putting the value of y in eq. (i),

$3x+y-7=0$

$\Rightarrow\ 3x=9$

$\Rightarrow\ x=3$

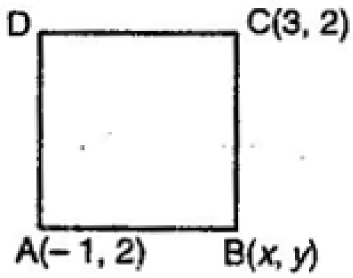
Hence, the centre of the circle is $(3,-2)$.

4. The two opposite vertices of a square are $(-1,2)$ and $(3,2)$. Find the coordinates of the other two vertices.

Ans. Let ABCD be a square and B(x,y) be the unknown vertex.

AB = BC

$\Rightarrow AB^2 = BC^2$



$\Rightarrow (x+1)^2+(y-2)^2=(x-3)^2+(y-2)^2$

$\Rightarrow 2x+1=-6x+9$

$\Rightarrow 8x=8$

$\Rightarrow x=1$ ....(i)

In ΔABC, $AB^2 + BC^2 = AC^2$

$\Rightarrow$

$(x+1)^2+(y-2)^2+(x-3)^2+(y-2)^2=(3+1)^2+(2-2)^2$

$\Rightarrow 2x^2+2y^2+2x-4y-6x-4y+1+4+9+4=16$

$\Rightarrow 2x^2+2y^2-4x-8y+2=0$

$\Rightarrow x^2+y^2-2x-4y+1=0$ ....(ii)

Putting the value of x in eq. (ii),

$1+y^2-2-4y+1=0$

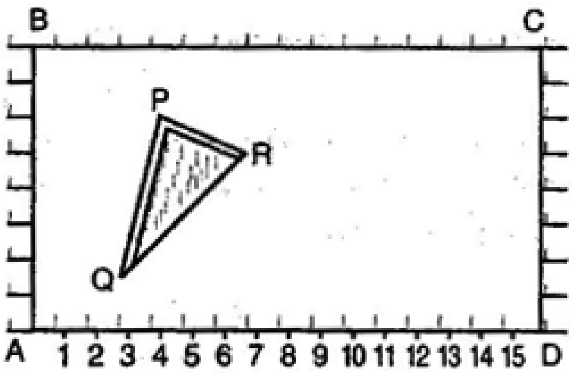
$\Rightarrow y^2-4y=0$

$\Rightarrow y(y-4)=0$

$\Rightarrow y$ = 0 or 4

Hence, the required vertices of the square are (1, 0) and (1, 4).

5. The class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Saplings of Gulmohar are planted on the boundary at a distance of 1 m from each other. There is a triangular grassy lawn in the plot as shown in the figure. The students are to sow seeds of flowering plants on the remaining area of the plot.



(i) Taking A as origin, find the coordinates of the vertices of the triangle.

(ii) What will be the coordinates of the vertices of ΔPQR if C is the origin? Also calculate the area of the triangle in these cases. What do you observe?

Ans. (i) Taking A as the origin, AD and AB as the coordinate axes. Clearly, the points P, Q and R are (4, 6), (3, 2) and (6, 5) respectively.

(ii) Taking C as the origin, CB and CD as the coordinate axes. Clearly, the points P, Q and R are given by (12, 2), (13, 6) and (10, 3) respectively.

We know that the area of the triangle =

$$\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$$

$\therefore$ Area of ΔPQR (First case) =

$$\frac{1}{2}\left[4(2-5)+3(5-6)+6(6-2)\right]$$

$$=\frac{1}{2}\left[4(-3)+3(-1)+6(4)\right]$$

$$=\frac{1}{2}[-12-3+24] = \frac{9}{2} \text{ sq. units}$$

And Area of ΔPQR (Second case) =

$$\frac{1}{2}\left[12(6-3)+13(3-2)+10(2-6)\right]$$

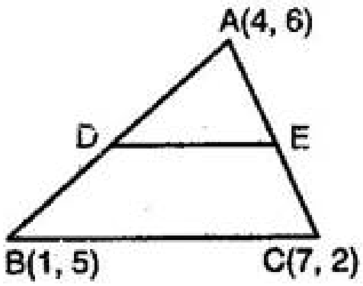
$= \frac{1}{2}[12(3)+13(1)+10(-4)]$

$= \frac{1}{2}[36+13-40] = \frac{9}{2}$ sq. units

Hence, the areas are same in both the cases.

6. The vertices of a ΔABC are A (4, 6), B (1, 5) and C (7, 2). A line is drawn to intersect sides AB and AC at D and E respectively such that $\frac{AD}{AB}=\frac{AE}{AC}=\frac{1}{4}$. Calculate the area of the ΔADE and compare it with the area of ΔABC.

Ans. Since, $\frac{AD}{AB}=\frac{AE}{AC}=\frac{1}{4}$



$\therefore$ DE $\parallel$ BC[By Thales theorem]

$\therefore$ $\Delta ADE \sim \Delta ABC$

$\therefore$ $\frac{\text{Area }(\Delta ADE)}{\text{Area }(\Delta ABC)} = \frac{AD^2}{AB^2}$

$= \left(\frac{AD}{AB}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$ ....(i)

Now, Area (ΔABC) =

$\frac{1}{2}[4(5-2)+1(2-6)+7(6-5)]$

$= \frac{1}{2}[12-4+7] = \frac{15}{2}$ sq. units..........(ii)

From eq. (i) and (ii),

Area (ΔADE) = $\frac{1}{16} \times$ Area (ΔABC) =

$\frac{1}{16} \times \frac{15}{2} = \frac{15}{32}$ sq. units

$\therefore$ Area (ΔADE): Area (ΔABC) = 1: 16

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