Exercise 10C

Question 9:

The given equation is $$x^2 - x + 2 = 0$$

This is of form $ax^2+bx+c=0$, when

$a = 1$, $b = -1$, $c = 2$

$$\therefore D = b^2 - 4ac = (-1)^2 - 4 \times 1 \times 2 = 1 - 8 = -7 < 0$$

So the equation has no real roots

Question 10:

The given equation is $$kx^2 + 2x + 1 = 0$$

This is of the form $ax^2+bx+c=0$, where $a = k$, $b = 2$, $c = 1$

$$\therefore D = b^2 - 4ac = (2)^2 - 4 \times k \times 1 = 4 - 4k$$

The given equation will have real and distinct roots if

Now,

$$D > 0 \Rightarrow 4 - 4k > 0 \Rightarrow 4k < k \Rightarrow k < 1$$

Question 11:

The given equation is $$2x^2 + px + 8 = 0$$

This is of the form $ax^2+bx+c=0$, where

$a = 2$, $b = p$, $c = 8$

$$\therefore D = b^2 - 4ac = \left[(p)^2 - 4 \times 2 \times 8\right] = p^2 - 64$$

The given equation will have real roots if $D \geq 0$

$$\text{Now, } D \geq 0 \Rightarrow p^2 - 64 \geq 0 \Rightarrow p^2 \geq 64$$

$$\Rightarrow p \geq 8 \text{ or } p \leq -8$$

Question 12:

The given equation is

$$(\alpha - 12)x^2 + 2(\alpha - 12)x + 2 = 0$$

This is of the form $ax^2 + bx + c = 0$,

where $a = (\alpha - 12)$, $b = 2(\alpha - 12)$, $c = 2$

$$\therefore D = 2^2 \times (\alpha - 12)^2 - 4 \times 2 \times (\alpha - 12)$$
$$= 4(\alpha - 12)^2 - 8(\alpha - 12)$$
$$= 4(\alpha - 12)\left[(\alpha - 12) - 2\right]$$
$$= 4((\alpha - 12)(\alpha - 14))$$

For real and equal roots, we must have D = 0
Now, D = 0
$4(\alpha - 12)(\alpha - 14) = 0$
$\Rightarrow (\alpha - 14) = 0, \quad (\alpha - 12) = 0$
But $\alpha \neq 12$ $\{\because$ the equation will not be a quadratic one if $\alpha = 12\}$
Hence $\alpha = 14$

Question 13:
The given equation is
$$\left(1 + m^2\right)x^2 + 2mcx + \left(c^2 - a^2\right) = 0$$

This is of the form $ax^2 + bx + c = 0$,

where $a = \left(1 + m^2\right)$, $b = 2mc$, $c = \left(c^2 - a^2\right)$

$$\therefore D = b^2 - 4ac = (2mc)^2 - 4\left(1 + m^2\right)\left(c^2 - a^2\right)$$
$$\Rightarrow \quad D = 4m^2c^2 - 4\left(c^2 - a^2 + m^2c^2 - m^2a^2\right)$$
$$\Rightarrow \quad D = 4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4m^2a^2$$
$$\Rightarrow \quad D = 4m^2a^2 + 4a^2 - 4c^2$$

For real and equal roots, we must have D = 0
$$\text{Now,} \quad D = 0 \Rightarrow 4\left(m^2a^2 + a^2 - c^2\right) = 0$$
$$\Rightarrow \quad m^2a^2 + a^2 - c^2 = 0$$
$$\Rightarrow \quad m^2a^2 + a^2 = c^2$$
$$\Rightarrow \quad a^2\left(m^2 + 1\right) = c^2$$
$$\Rightarrow \quad c^2 = a^2\left(m^2 + 1\right) \qquad \text{proved}$$

Question 14:
The given equation is
$$\left(c^2 - ab\right)x^2 - 2\left(a^2 - bc\right)x + \left(b^2 - ac\right) = 0$$

This is the form of $ax^2+bx+c=0$

$$a = (c^2 - ab),\ b = -2(a^2 - bc),\ c = (b^2 - ac)$$

$$\therefore D = B^2 - 4AC$$

$$= (-2)^2 \times (a^2 - bc)^2 - 4 \times (c^2 - ab) \times (b^2 - ac)$$

$$= 4 \times (a^4 + b^2c^2 - 2a^2bc) - 4 \times (c^2b^2 - ac^3 - ab^3 + a^2bc)$$

$$= 4a^4 + 4b^2c^2 - 8a^2bc - 4c^2b^2 + 4ac^3 + 4ab^3 - 4a^2bc$$

$$= 4a^4 - 12a^2bc + 4ac^3 + 4ab^3$$

$$= 4a(a^3 - 3abc + c^3 + b^3)$$

For real and equal roots, we must have D = 0

Now, $D = 0 \Rightarrow 4a(a^3 - 3abc + c^3 + b^3) = 0$

$\Rightarrow$ Either, $a^3 - 3abc + c^3 + b^3 = 0$ or $a = 0$

Either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$ proved

*********** END **********