Indefinite Integrals Ex 19.25 Q1

Let $I = \int x \cos x \, dx$

Using integration by parts,

$$I = x\int \cos x \, dx - \int \left(1 \times \int \cos x \, dx\right) dx + c$$

$$= x \sin x - \int \sin x \, dx + c$$

$$I = x \sin x + \cos x + c$$

Indefinite Integrals Ex 19.25 Q2

Let $I = \int \log(x+1) \, dx$

$$= \int 1 \times \log(x+1) \, dx$$

Using integration by parts,

$$I = \log(x+1)\int 1 \, dx - \int \left(\frac{1}{x+1} \times \int 1 \, dx\right) dx + c$$

$$= x \log(x+1) - \int \left(\frac{x}{x+1}\right) dx + c$$

$$= x \log(x+1) - \int \left(1 - \frac{1}{x+1}\right) dx + c$$

$$I = x \log(x+1) - x + \log(x+1) + c$$

Indefinite Integrals Ex 19.25 Q3

Let $I = \int x^3 \log x \, dx$

Using integration by parts,

$$I = \log x \int x^3 \, dx - \int \left(\frac{1}{x} \times \int x^3 \, dx\right) dx + c$$

$$= \frac{x^4}{4} \log x - \int \frac{x^4}{4x} dx + c$$

$$= \frac{x^4}{4} \log x - \frac{1}{4}\int x^3 \, dx + c$$

$$= \frac{x^4}{4} \log x - \frac{1}{4}\int \frac{x^4}{4} dx + c$$

$$I = \frac{x^4}{4} \log x - \frac{1}{16} x^4 + c$$

Indefinite Integrals Ex 19.25 Q4

Take first function as x and second function as e^x. The integral of the second function is e^x.
Therefore, $\int xe^x \, dx = xe^x - \int 1.\, e^x \, dx = xe^x - e^x + C.$

Indefinite Integrals Ex 19.25 Q5

Let $I = \int xe^{2x}dx$

Using integration by parts,

$$I = x\int e^{2x}\,dx - \int\left(1 \times \int e^{2x}\,dx\right)dx + c$$

$$= \frac{xe^{2x}}{2} - \int\left(\frac{e^{2x}}{2}\right)dx + c$$

$$= \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + c$$

$$I = \left(\frac{x}{2} - \frac{1}{4}\right)e^{2x} + c$$

Indefinite Integrals Ex 19.25 Q6

Let $I = \int x^2e^{-x}dx$

Using integration by parts,

$$I = x^2\int e^{-x}\,dx - \int\left(2x\int e^{-x}\,dx\right)$$

$$= -x^2e^{-x} - \int(2x)\left(-e^{-x}\right)$$

$$= -x^2e^{-x} + 2\int x\,e^{-x}dx$$

$$= -x^2e^{-x} + 2\left[x\int e^{-x}dx - \int\left(1 \times \int e^{-x}dx\right)dx\right]$$

$$= -x^2e^{-x} + 2\left[x\left(-e^{-x}\right) - \int\left(-e^{-x}\right)dx\right]$$

$$= -x^2e^{-x} - 2xe^{-x} + 2\int e^{-x}dx$$

$$I = -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + c$$

$$I = -e^{-x}\left(x^2 + 2x + 2\right) + c$$

Indefinite Integrals Ex 19.25 Q7

Let $I = \int x^2\cos x\,dx$

Using integration by parts,

$$I = x^2\int \cos x\,dx - \int\left(2x\int \cos x\,dx\right)dx$$

$$= x^2\sin x - 2\int(x)(\sin x)dx$$

$$= x^2\sin x - 2\left[x\int \sin x\,dx - \int\left(1 \times \int \sin x\,dx\right)dx\right]$$

$$= x^2\sin x - 2\left[x(-\cos x) - \int(-\cos x)dx\right]$$

$$= x^2\sin x + 2x\cos x - 2\int(\cos x)dx$$

$$I = x^2\sin x + 2x\cos x - 2\sin x + c$$

********** END **********