Geometric Progressions Ex 20.3 Q 11

$t_4 = \frac{1}{27}$, $t_7 = \frac{1}{729}$, $t_n = ar^{n-1}$

Where $t_n = n^{th}$ term, r = common difference, n = number of terms.

$$t_4 = ar^3 = \frac{1}{27} \qquad \text{---(i)}$$

$$t_7 = ar^6 = \frac{1}{729} \qquad \text{---(ii)}$$

Dividing (ii) by (i), we get

$$\frac{t_7}{t_4} = \frac{ar^6}{ar^3} = r^3 = \frac{27}{729} = \frac{1}{27},\ r = \frac{1}{3}$$

Sum of n term $= S_n = \frac{a\left(1-r^n\right)}{1-r}$ ---(i)

When, $r = 3$, $t_4 = ar^3 = \frac{1}{27}$

$$a\left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

$$a = 1$$

Substituting $a = 1$, $r = \frac{1}{3}$ in (i)

$$S_n = \frac{1\left[1-\left(\frac{1}{3}\right)^n\right]}{1-\frac{1}{3}}$$

$$= \frac{1-\left(\frac{1}{3}\right)^n}{\frac{2}{3}}$$

$$= \frac{3}{2}\left[1-\left(\frac{1}{3}\right)^n\right]$$

Geometric Progressions Ex 20.3 Q 12

$$\sum_{n=1}^{10}\left\{\left(\frac{1}{2}\right)^{n-1}+\left(\frac{1}{5}\right)^{n+1}\right\}$$

$$= \sum_{n=1}^{10}\left(\frac{1}{2}\right)^{n-1} + \sum_{n=1}^{10}\left(\frac{1}{5}\right)^{n+1}$$

$$= 1+\frac{1}{2}+\frac{1}{2^2}+\ldots\ldots+\frac{1}{5^2}+\frac{1}{5^3}+\frac{1}{5^4}+\ldots\ldots$$

$$= \frac{(1-\frac{1}{2^{10}})}{1-\frac{1}{2}} + \frac{\frac{1}{5}(1-\frac{1}{5^{10}})}{1-\frac{1}{5}}$$

$$= \frac{2^{10}-1}{2^9} + \frac{5^{10}-1}{5^{11}}$$

Geometric Progressions Ex 20.3 Q 13

Fifth term of series is

$ar^{5-1} = 81$................(1)

Second term of series is

$ar = 24$ ....(2)

Dividing (2) by (1) we get,

$$\frac{ar}{ar^4} = \frac{24}{81} = \frac{8}{27}$$

$$r^3 = \frac{27}{8}$$

$$r = \frac{3}{2}$$

Substituting r in (2), we get,

$$a = \frac{24 \times 2}{3} = 16$$

$$Sum = \frac{16\left[(\frac{3}{2})^8 - 1\right]}{\frac{3}{2} - 1}$$

$$= \frac{16\left[3^8 - 2^8\right]}{2^7}$$

$$= \frac{6305}{8}$$

Geometric Progressions Ex 20.3 Q14

S_1 = sum of n terms,

S_1 = sum of $2n$ terms,

S_1 = sum of $3n$ terms.

Then, $S_1^2 + S_2^2$

$$= (S_n)^2 + (S_{2n})^2$$

$$= \left(\frac{a(1 - r^n)}{1 - r}\right)^2 + \left(\frac{a(1 - r^{2n})}{1 - r}\right)^2$$

$$= \frac{a^2}{(1 - r)^2}\left[\left(1 - (r)^n\right)^2 + \left(1 - r^{2n}\right)^2\right]$$

$$= \frac{a^2}{(1 - r)^2}\left[1 + r^{2n} - 2r^n + 1 + r^{4n} - 2r^{2n}\right]$$

$$= \frac{a^2}{(1 - r)^2}\left[2 - r^{2n} - 2r^n + r^{4n}\right] \qquad \text{---(i)}$$

Also, $S_1(S_2 + S_3)$

$$= \frac{a(1 - r^n)}{1 - r}\left(\frac{a(1 - r^{2n})}{1 - r} + \frac{a(1 - r^{3n})}{1 - r}\right)$$

$$= \frac{a^2}{(1 - r)^2}\left[(1 - r)^n\left(1 - r^{2n}\right) + \left(1 - r^n\right)\left(1 - r^{3n}\right)\right]$$

$$= \frac{a^2}{(1 - r)^2}\left[1 - r^{2n} - r^n + r^{3n} - r^{3n} - r^n + 1 + r^{4n}\right]$$

$$= \frac{a^2}{(1 - r)^2}\left[2 - r^{2n} - 2r^n + r^{4n}\right] \qquad \text{---(ii)}$$

(i) = (ii) Hence, $S_1^2 + S_2^2 = S_1(S_2 + S_3)$

Geometric Progressions Ex 20.3 Q15

$S_1, S_2, \ldots, S_n$ are the sums of n terms of G.P. $a = 1,\ r = 1, 2, 3, \ldots, n$

Then, $S_1 + S_2 + 2S_3 + 3S_4 + \ldots + (n-1)S_n$

$$\frac{1\left(1^n - 1\right)}{1-1} + \frac{1\left(2^n - 1\right)}{2-1} + \frac{2\left(3^n - 1\right)}{3-1} + \ldots + (n-1)1\left(\frac{1^n - 1}{1-1}\right)$$
$$= 2^n - 1 + 23^n - 1 + 3.4^n - 1 + \ldots$$
$$= 2^n + 3^n + 4^n + \ldots + n^n$$

*********** END **********