Definite Integrals Ex 20.4B Q1

We have,

$$\frac{1}{1+\tan x} = \frac{1}{1+\frac{\sin x}{\cos x}} = \frac{\cos x}{\cos x + \sin x}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{dx}{1+\tan x} = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x}dx$$

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x}dx \qquad --(\mathrm{I})$$

So,

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2}-x\right)}{\cos\left(\frac{\pi}{2}-x\right)+\sin\left(\frac{\pi}{2}-x\right)}dx \qquad \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)\,dx\right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x}{\cos x + \sin x}dx \qquad --(\mathrm{II})$$

Hence, adding (I) & (II)

$$2I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x}dx + \int_0^{\frac{\pi}{2}} \frac{\sin x}{\cos x + \sin x}dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos x + \sin x}{\cos x + \sin x}dx$$

$$= \int_0^{\frac{\pi}{2}} dx$$

$$2I = \left[x\right]_0^{\frac{\pi}{2}}$$

$$2I = \left[\frac{\pi}{2}-0\right] \quad \Rightarrow \quad I = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q2

We have,

$$\frac{1}{1+\cot x}=\frac{1}{1+\frac{\cos x}{\sin x}}=\frac{\sin x}{\sin x+\cos x}$$

$$\therefore \int_0^{\frac{\pi}{2}}\frac{1}{1+\cot x}dx=\int_0^{\frac{\pi}{2}}\frac{\sin x}{\sin x+\cos x}dx$$

Let

$$I=\int_0^{\frac{\pi}{2}}\frac{\sin x}{\sin x+\cos x}dx \qquad --(\text{I})$$

So,

$$I=\int_0^{\frac{\pi}{2}}\frac{\sin\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}dx \qquad \left[\left[\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right]\right]$$

$$=\int_0^{\frac{\pi}{2}}\frac{\cos x}{\sin x+\cos x}dx \qquad --(\text{II})$$

Adding (I) & (II)

$$2I=\int_0^{\frac{\pi}{2}}\frac{\sin x}{\sin x+\cos x}dx+\int_0^{\frac{\pi}{2}}\frac{\cos x}{\sin x+\cos x}dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{\sin x+\cos x}{\sin x+\cos x}dx$$

$$2I=\int_0^{\frac{\pi}{2}}dx$$

$$=\left[x\right]_0^{\frac{\pi}{2}}$$

$$2I=\left[\frac{\pi}{2}-0\right]$$

$$I=\frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q3

We have,

$$\frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}}=\frac{\sqrt{\frac{\cos x}{\sin x}}}{\sqrt{\frac{\cos x}{\sin x}}+\sqrt{\frac{\sin x}{\cos x}}}=\frac{\sqrt{\frac{\cos x}{\sin x}}}{\frac{\cos x+\sin x}{\sqrt{\sin x}.\sqrt{\cos x}}}=\sqrt{\frac{\cos x}{\sin x}}\times\frac{\sqrt{\sin x}.\sqrt{\cos x}}{\cos x+\sin x}$$

$$=\frac{\cos x}{\cos x+\sin x}$$

$$\therefore \int_0^{\frac{\pi}{2}}\frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}}dx=\int_0^{\frac{\pi}{2}}\frac{\cos x}{\cos x+\sin x}dx$$

Let

$$I=\int_0^{\frac{\pi}{2}}\frac{\cos x}{\cos x+\sin x}dx \qquad --(\mathrm{I})$$

So,

B $$I=\int_0^{\frac{\pi}{2}}\frac{\cos\left(\frac{\pi}{2}-x\right)}{\cos\left(\frac{\pi}{2}-x\right)+\sin\left(\frac{\pi}{2}-x\right)}dx \qquad \left[\because\int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$$

$$=\int_0^{\frac{\pi}{2}}\frac{\sin x}{\cos x+\sin x}dx \qquad --(\mathrm{II})$$

Adding (I) & (II)

$$2I=\int_0^{\frac{\pi}{2}}\frac{\cos x}{\cos x+\sin x}dx+\int_0^{\frac{\pi}{2}}\frac{\sin x}{\cos x+\sin x}dx$$

$$2I=\int_0^{\frac{\pi}{2}}\frac{\cos x+\sin x}{\cos x+\sin x}dx$$

$$2I=\int_0^{\frac{\pi}{2}}dx$$

$$2I=[x]_0^{\frac{\pi}{2}}$$

$$2I=\left[\frac{\pi}{2}-0\right]$$

$$I=\frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q4

$$\text{Let } I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx \qquad \ldots(1)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)+\cos^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}dx \qquad \left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\cos^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx$$

$$\Rightarrow 2I=\int_0^{\frac{\pi}{2}}1\cdot dx$$

$$\Rightarrow 2I=[x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I=\frac{\pi}{2}$$

$$\Rightarrow I=\frac{\pi}{4}$$

********** END **********