Chapter 6 Determinants Ex 6.2 Q21

$$\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix} = -2$$

$$\text{LHS} = \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$$

Apply $R_3 \to R_3 - R_2$

$$= \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)2 & 1 & 0 \end{vmatrix}$$

Apply $R_2 \to R_2 - R_1$

$$= \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)2 & 1 & 0 \\ (a+3)2 & 1 & 0 \end{vmatrix}$$

$$= \left[(2a+4)(1) - (1)(2a+6)\right]$$

$$= -2$$

$$= \text{RHS}$$

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$$\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

$$\text{LHS} = \begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix}$$

Apply: $C_2 \to C_2 - 2C_1 - 2C_3$

$$= \begin{vmatrix} a^2 & a^2-(b-c)^2-2a^2-2bc & bc \\ b^2 & b^2-(c-a)^2-2b^2-2ca & ca \\ c^2 & c^2-(a-b)^2-2c^2-2ab & ab \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & -(b^2+c^2+a^2) & bc \\ b^2 & -(b^2+c^2+a^2) & ca \\ c^2 & -(b^2+c^2+a^2) & ab \end{vmatrix}$$

Take $-(a^2+b^2+c^2)$ common from C_2

$$= -(b^2+c^2+a^2)\begin{vmatrix} a^2 & 1 & bc \\ b^2 & 1 & ca \\ c^2 & 1 & ab \end{vmatrix}$$

$$= -(b^2+c^2+a^2)\begin{vmatrix} a^2 & 1 & bc \\ b^2-a^2 & 0 & ca-bc \\ c^2-a^2 & 0 & ab-bc \end{vmatrix}$$

$$= -(b^2+c^2+a^2)(a-b)(c-a)\begin{vmatrix} a^2 & 1 & bc \\ -(b+a) & 0 & c \\ c+a & 0 & -b \end{vmatrix}$$

$$= -(b^2+c^2+a^2)(a-b)(c-a)\left[(-(b+a))(-b)-(c)(c+a)\right]$$

$$= (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

$$= RHS$$

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$$\begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix} = -(a-b)(b-c)(c-a)\left(a^2+b^2+c^2\right)$$

$$\text{LHS} = \begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix}$$

Apply: $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1 & a^2+bc & a^3 \\ 0 & b^2+ca-a^2-bc & b^3-a^3 \\ 0 & c^2+abb^2+ca-a^2-bc & c^3-a^3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a^2+bc & a^3 \\ 0 & \left(b^2-a^2\right)-c(b-a) & b^3-a^3 \\ 0 & \left(c^2-a^2\right)-b(c-a) & c^3-a^3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a^2+bc & a^3 \\ 0 & (b-a)(b+a-c) & b^3-a^3 \\ 0 & (c-a)(c+a-b) & c^3-a^3 \end{vmatrix}$$

$$= (b-a)(c-a)\begin{vmatrix} 1 & a^2+bc & a^3 \\ 0 & (b+a-c) & b^2+a^2+ab \\ 0 & (c+a-b) & c^2+a^2+ac \end{vmatrix}$$

$$= (b-a)(c-a)\left[((b+a-c))\left(c^2+a^2+ac\right)-\left(b^2+a^2+ab\right)\left(c^2+a^2+ac\right)\right]$$

$$= -(a-b)(b-c)(c-a)\left(a^2+b^2+c^2\right)$$

$$= RHS$$

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We need to prove the following identity:

$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

Taking the term a,b,c *common from* C_1, C_2 *and* C_3, *respectively, we have,*

$$L.H.S = abc\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, *we have,*

$$L.H.S = abc\begin{vmatrix} 2a+2c & c & a+c \\ 2a+2b & b & a \\ 2b+2c & b+c & c \end{vmatrix}$$

$$\Rightarrow L.H.S = 2abc\begin{vmatrix} a+c & c & a+c \\ a+b & b & a \\ b+c & b+c & c \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$ *and* $C_3 \to C_3 - C_1$, *we have,*

$$L.H.S = 2abc\begin{vmatrix} a+c & -a & 0 \\ a+b & -a & -b \\ b+c & 0 & -b \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, *we have,*

$$\Rightarrow L.H.S = 2abc\begin{vmatrix} c & -a & 0 \\ 0 & -a & -b \\ c & 0 & -b \end{vmatrix}$$

Taking c, a, *and* b *from* C_1, C_2 *and* C_3 *respectively, we have,*

$$L.H.S = 2a^2b^2c^2\begin{vmatrix} 1 & -1 & 0 \\ 0 & -1 & -1 \\ 1 & 0 & -1 \end{vmatrix}$$

Applying $R_3 \to R_3 - R_1$, *we have*

$$L.H.S = 2a^2b^2c^2\begin{vmatrix} 1 & -1 & 0 \\ 0 & -1 & -1 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= 4a^2b^2c^2$$

*********** END **********