Complex numbers Ex 13.1 Q1(v)

We know that $i = \sqrt{-1}$

$$i^2 = -1$$
$$i^3 = -i$$
$$i^4 = 1$$

In order to find i^n where $n > 4$, we divide n by 4 to get quotient p and remainder q, so that

$$n = 4p + q, o \le q < 4$$

Then $i^n = i^{4p+q}$

$$= i^{4p} \times i^q$$
$$= \left(i^4\right)^p \times i^q$$
$$= 1^p \times i^q$$
$$= i^q \qquad \left[\because 1^{p=1}\right]$$

$Hence\, i^n = i^q, where\, o \le q < 4$

$$\left(i^{41} + \frac{1}{i^{257}}\right)^9 = \left(i^{4\times10} \times i^1 + \frac{1}{i^{4\times64} \times i^1}\right)^9$$
$$= \left(1 \times i + \frac{1}{1 \times i}\right)^9$$
$$= \left(i + \frac{1}{i}\right)^9$$
$$= \left(i + \frac{1}{i \times i} \times i\right)^9$$
$$= \left(i + \frac{i}{-1}\right)^9$$
$$= (i - i)^9$$
$$= 0$$

Complex numbers Ex 13.1 Q1(vi)

We know that $i = \sqrt{-1}$

$$i^2 = -1$$
$$i^3 = -i$$
$$i^4 = 1$$

In order to find i^n where $n > 4$, we divide n by 4 to get quotient p and remainder q, so that

$$n = 4p + q, o \le q < 4$$

Then $i^n = i^{4p+q}$

$$= i^{4p} \times i^q$$
$$= \left(i^4\right)^p \times i^q$$
$$= 1^p \times i^q$$
$$= i^q \qquad \left[\because 1^{p=1}\right]$$

$Hence\, i^n = i^q, where\, o \le q < 4$

$$\left(i^{77} + i^{70} + i^{87} + i^{414}\right)^3 = \left(i^{4\times19} \times i^1 + i^{4\times17} \times i^2 + i^{4\times21} \times i^3 + i^{4\times103} \times i^2\right)^3$$
$$= \left(1 \times i + 1 \times i^2 + 1 \times i^3 + 1 \times i^2\right)^3$$
$$= (i - 1 - i - 1)^3$$
$$= (-2)^3$$
$$= -8$$

$$\therefore \left(i^{77} + i^{70} + i^{87} + i^{414}\right)^3 = -8$$

Complex numbers Ex 13.1 Q1(vii)

We know that $i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

In order to find i^n where $n > 4$, we divide n by 4 to get quotient p and remainder q, so that

$$n = 4p + q, o \le q < 4$$

Then $i^n = i^{4p+q}$

$= i^{4p} \times i^q$

$= \left(i^4\right)^p \times i^q$

$= 1^p \times i^q$

$= i^q \qquad \left[\because 1^{p-1}\right]$

Hence $i^n = i^q$, *where* $o \le q < 4$

$\therefore\ i^{30} + i^{40} + i^{60} = i^{4\times7} \times i^2 + i^{4\times10} + i^{4\times15}$

$= 1 \times i^2 + 1 + 1$

$= -1 + 1 + 1$

$= 1$

$\therefore\ i^{30} + i^{40} + i^{60} = 1$

Complex numbers Ex 13.1 Q1(viii)

We know that $i = \sqrt{-1}$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

In order to find i^n where $n > 4$, we divide n by 4 to get quotient p and remainder q, so that

$$n = 4p + q, o \le q < 4$$

Then $i^n = i^{4p+q}$

$= i^{4p} \times i^q$

$= \left(i^4\right)^p \times i^q$

$= 1^p \times i^q$

$= i^q \qquad \left[\because 1^{p-1}\right]$

Hence $i^n = i^q$, *where* $o \le q < 4$

$i^{49} + i^{68} + i^{89} + i^{110} = i^{4\times12} \times i^1 + i^{4\times17} + i^{4\times22} \times i^1 + i^{4\times27} \times i^2$

$= 1 \times i + 1 + 1 \times i + 1 \times i^2$

$= i + 1 + i - 1$

$= 2i$

$\therefore\ i^{49} + i^{68} + i^{89} + i^{110} = 2i$

*********** END **********