1. Which term of the AP: 121, 117, 113, ... is its first negative term?

Ans. Given: 121, 117, 113, ...

Here $a = 121$, $d = 117 - 121 = 4$

Now, $a_n = a + (n-1)d$

$= 121 + (n-1)(-4)$

$= 121 - 4n + 4 = 125 - 4n$

For the first negative term, $a_n < 0$

$\Rightarrow 125 - 4n < 0$

$\Rightarrow 125 < 4n$

$\Rightarrow \frac{125}{4} < n$

$\Rightarrow 31\frac{1}{4} < n$

n is an integer and $n > 31\frac{1}{4}$.

Hence, the first negative term is 32^{nd} term.

2. The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of sixteen terms of the AP.

Ans. Let the AP be

$a-4d, a-3d, a-2d, a-d, a, a+d, a+2d, a+3d, \ldots$

Then, $a_3 = a-2d, a_7 = a+2d$

$\Rightarrow a_3 + a_7 = a-2d+a+2d = 6$

$\Rightarrow 2a = 6$

$\Rightarrow a = 3$ ....(i)

Also $(a-2d)(a+2d) = 8$

$\Rightarrow a^2 - 4d^2 = 8$

$\Rightarrow 4d^2 = a^2 - 8$

$\Rightarrow 4d^2 = 3^2 - 8$

$\Rightarrow 4d^2 = 1$

$\Rightarrow d^2 = \frac{1}{4} \Rightarrow d = \pm\frac{1}{2}$

Taking $d = \frac{1}{2}$,

$S_{16} = \frac{16}{2}\left[2\times(a-4d)+(16-1)d\right]$

$= 8\left[2\times\left(3-4\times\frac{1}{2}\right)+15\times\frac{1}{2}\right]$

$= 8\left[2+\frac{15}{2}\right] = 8\times\frac{19}{2} = 76$

Taking $d = \frac{-1}{2}$,

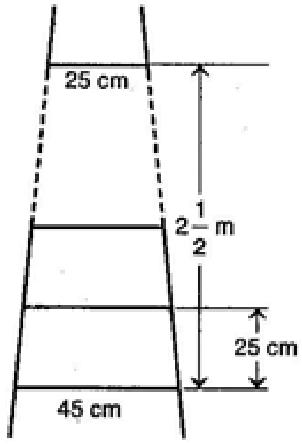
$S_{16} = \frac{16}{2}\left[2\times(a-4d)+(16-1)d\right]$

$= 8\left[2\times\left(3-4\times\frac{-1}{2}\right)+15\times\frac{-1}{2}\right]$

$= 8\left[\frac{20-15}{2}\right] = 8\times\frac{5}{2} = 20$

$\therefore S_{16} =$ 20 and 76

3. A ladder has rungs 25 cm apart (see figure). The rungs decrease uniformly in length from 45 cm, at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs?



Ans. Number of rungs $(n) = \dfrac{2\frac{1}{2}\times 100}{25}$ = 10

The length of the wood required for rungs = sum of 10 rungs

= $\dfrac{10}{2}[25+45]$ = 5×70 = 350 cm

4. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.

Ans. Here $a = 1$ and $d = 1$

$\therefore$ $S_{x-1} = \frac{x-1}{2}[2\times1+(x-1-1)\times1]$

$= \frac{x-1}{2}(2+x-2)$

$= \frac{(x-1)x}{2} = \frac{x^2-x}{2}$

$S_x = \frac{x}{2}[2\times1+(x-1)\times1]$

$= \frac{x}{2}(x+1) = \frac{x^2+x}{2}$

$S_{49} = \frac{49}{2}[2\times1+(49-1)\times1]$

$= \frac{49}{2}(2+48) = 49 \times 25$

According to question,

$S_{x-1} = S_{49} - S_x$

$\Rightarrow \frac{x^2-x}{2} = 49\times25 - \frac{x^2+x}{2}$

$\Rightarrow \frac{x^2-x}{2} + \frac{x^2+x}{2} = 49\times25$

$\Rightarrow \frac{x^2-x+x^2+x}{2} = 49\times25$

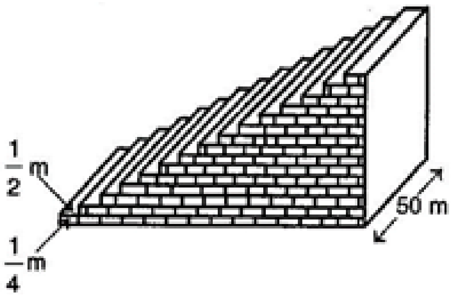
$\Rightarrow x^2 = 49\times25$

$\Rightarrow x = \pm35$

Since, x is a counting number, so negative value will be neglected.

$\therefore x = 35$

5. A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.



Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m (see figure). Calculate the total volume of concrete required to build the terrace.

Ans. Volume of concrete required to build the first step, second step, third step, ... (in m²) are

$$\frac{1}{4}\times\frac{1}{2}\times 50, \left(2\times\frac{1}{4}\right)\times\frac{1}{2}\times 50, \left(3\times\frac{1}{4}\right)\times\frac{1}{2}\times 50, \ldots.$$

$$\Rightarrow \frac{50}{8}, 2\times\frac{50}{8}, 3\times\frac{50}{8}, \ldots\ldots$$

$\therefore$ Total volume of concrete required =

$$\frac{50}{8}+2\times\frac{50}{8}+3\times\frac{50}{8}+\ldots\ldots$$

$$=\frac{50}{8}[1+2+3+\ldots\ldots]$$

$$=\frac{50}{8}\times\frac{15}{2}[2\times1+(15-1)\times1]\ [\because n=15]$$

$$=\frac{50}{8}\times\frac{15}{2}\times16$$

$$=750\ m^3$$

*********** END **********