Binary Operations Ex 3.3 Q1

The binary operator $*$ is defined on I^+ and is given by,

$$a * b = a + b \text{ for all } a, b \in I^+$$

Let $a \in I^+$ and $e \in I^+$ be the identity element with respect to $*$.

by identity property, we have,

$$a * e = e * a = a$$

$$\Rightarrow \quad a + e = a$$

$$\Rightarrow \quad e = 0$$

Thus the required identity element is 0.

Binary Operations Ex 3.3 Q2

Let $R - \{-1\}$ be the set and $*$ be a binary operator, given by

$$a * b = a + b + ab \text{ for all } a, b \in R - \{-1\}$$

Now,

Let $a \in R - \{-1\}$ and $e \in R - \{-1\}$ be the identity element with respect to $*$.

by identity property, we have,

$$a * e = e * a = a$$

$$\Rightarrow \quad a + e + ae = a$$

$$\Rightarrow \quad e(1 + a) = 0$$

$$\Rightarrow \quad e = 0 \qquad [\because 1 + a \neq 0 \text{ as } a \neq -1]$$

$\therefore$ The required identity element is 0.

Binary Operations Ex 3.3 Q3

We are given the binary operator $*$ defined on Z as

$$a * b = a + b - 5 \text{ for all } a, b \in Q.$$

Let e be the identity element with respect to $*$

Then, $a * e = e * a = a$ [By identity property]

$$\Rightarrow \quad a + e - 5 = a$$

$$\Rightarrow \quad e = 5$$

Hence, the required identity element with respect to $*$ is 5.

Binary Operations Ex 3.3 Q4

The binary operator $*$ is defined on Z, and is given by

$$a * b = a + b + 2 \text{ for all } a, b \in Z.$$

Let $a \in Z$ and $e \in Z$ be the identity element with respect to $*$, then

$$a * e = e * a = a \qquad \text{[By identity property]}$$

$$\Rightarrow \quad a + e + 2 = a$$

$$\Rightarrow \quad e = -2 \in Z$$

Hence, the identity element with respect to $*$ is -2.

********** END **********