Exercise 1E

Question 15:

For rationalising the denominator of a number, we multiply its numerator and denominator by its rationalising factor.

If a and b are integers, then $\left(a+\sqrt{b}\right)$ and $\left(a-\sqrt{b}\right)$ are rationalising factor of each other, as $\left(a+\sqrt{b}\right)\left(a-\sqrt{b}\right)=\left(a^2-b\right)$, which is rational.

Let us rationalise the denominator of the first term on the Left hand side.

We have,

$$\frac{4+\sqrt{5}}{4-\sqrt{5}}=\frac{4+\sqrt{5}}{4-\sqrt{5}}\times\frac{4+\sqrt{5}}{4+\sqrt{5}}$$
$$=\frac{\left(4+\sqrt{5}\right)^2}{(4)^2-\left(\sqrt{5}\right)^2}$$
$$=\frac{(4)^2+2(4)\left(\sqrt{5}\right)+\left(\sqrt{5}\right)^2}{16-5}$$
$$\frac{4+\sqrt{5}}{4-\sqrt{5}}=\frac{16+8\sqrt{5}+5}{11}=\frac{21+8\sqrt{5}}{11}\;.....(1)$$

Now consider the denominator of the second term on the left hand side:

$$\frac{4-\sqrt{5}}{4+\sqrt{5}}=\frac{4-\sqrt{5}}{4+\sqrt{5}}\times\frac{4-\sqrt{5}}{4-\sqrt{5}}$$
$$=\frac{\left(4-\sqrt{5}\right)^2}{(4)^2-\left(\sqrt{5}\right)^2}$$
$$=\frac{(4)^2-2(4)\left(\sqrt{5}\right)+\left(\sqrt{5}\right)^2}{16-5}$$
$$\frac{4-\sqrt{5}}{4+\sqrt{5}}=\frac{16-8\sqrt{5}+5}{11}=\frac{21-8\sqrt{5}}{11}\;......(2)$$

Adding equations (1) and (2), we have,

$$\therefore\frac{4+\sqrt{5}}{4-\sqrt{5}}+\frac{4-\sqrt{5}}{4+\sqrt{5}}=\frac{21+8\sqrt{5}}{11}+\frac{21-8\sqrt{5}}{11}$$
$$=\frac{21+8\sqrt{5}+21-8\sqrt{5}}{11}=\frac{42}{11}.$$

*********** END **********