



Differentiation Ex 11.5 Q21

Here,

$$y = \frac{(x^2 - 1)^3 (2x - 1)}{\sqrt{(x - 3)(4x - 1)}} \quad \text{---(i)}$$

$$y = \frac{(x^2 - 1)^3 (2x - 1)}{(x - 3)^{\frac{1}{2}} (4x - 1)^{\frac{1}{2}}}$$

Taking log on both the sides,

$$\log y = \log \left[\frac{(x^2 - 1)^3 (2x - 1)}{(x - 3)^{\frac{1}{2}} (4x - 1)^{\frac{1}{2}}} \right]$$

$$= \log(x^2 - 1)^3 + \log(2x - 1) - \log(x - 3)^{\frac{1}{2}} - \log(4x - 1)^{\frac{1}{2}}$$

$$\left[\text{Since, } \log(AB) = \log A + \log B, \log\left(\frac{A}{B}\right) = \log A - \log B \right]$$

$$= 3\log(x^2 - 1) + \log(2x - 1) - \frac{1}{2}\log(x - 3) - \frac{1}{2}\log(4x - 1)$$

Differentiating it with respect to x using chain rule,

$$\frac{1}{y} \frac{dy}{dx} = 3 \frac{d}{dx} \log(x^2 - 1) + \frac{d}{dx} \log(2x - 1) - \frac{1}{2} \frac{d}{dx} \log(x - 3) - \frac{1}{2} \frac{d}{dx} \log(4x - 1)$$

$$= 3 \left(\frac{1}{x^2 - 1} \right) \frac{d}{dx} (x^2 - 1) + \frac{1}{(2x - 1)} \frac{d}{dx} (2x - 1) - \frac{1}{2} \left(\frac{1}{x - 3} \right) \frac{d}{dx} (x - 3) - \frac{1}{2} \left(\frac{1}{4x - 1} \right) \frac{d}{dx} (4x - 1)$$

$$= 3 \left(\frac{1}{x^2 - 1} \right) (2x) + \frac{1}{(2x - 1)} (2) - \frac{1}{2} \left(\frac{1}{x - 3} \right) (1) - \frac{1}{2} \left(\frac{1}{4x - 1} \right) (4)$$

$$\frac{1}{y} \frac{dy}{dx} = \left[\frac{6x}{x^2 - 1} + \frac{2}{2x - 1} - \frac{1}{2(x - 3)} - \frac{2}{4x - 1} \right]$$

$$\frac{dy}{dx} = y \left[\frac{6x}{x^2 - 1} + \frac{2}{2x - 1} - \frac{1}{2(x - 3)} - \frac{2}{4x - 1} \right]$$

$$\frac{dy}{dx} = \frac{(x^2 - 1)^3 (2x - 1)}{\sqrt{(x - 3)(4x - 1)}} \left[\frac{6x}{x^2 - 1} + \frac{2}{2x - 1} - \frac{1}{2(x - 3)} - \frac{2}{4x - 1} \right] \quad \text{[Using equation (i)]}$$

Differentiation Ex 11.5 Q22

Here,

$$y = \frac{e^{ax} \sec^x \log x}{\sqrt{1 - 2x}} \quad \text{---(i)}$$

$$\Rightarrow y = \frac{e^{ax} \times \sec^x \times \log x}{(1 - 2x)^{\frac{1}{2}}}$$

Taking log on both the sides,

$$\log y = \log e^{ax} + \log \sec^x + \log \log x - \frac{1}{2} \log(1 - 2x) \quad \left[\text{Since, } \log\left(\frac{A}{B}\right) = \log A - \log B, \right.$$

$$\log y = ax + \log \sec^x + \log \log x - \frac{1}{2} \log(1 - 2x) \quad \left. \log(AB) = \log A + \log B \right]$$

$$\left[\text{Since, } \log a^b = b \log a \text{ and } \log_e e = 1 \right]$$

Differentiating it with respect to x using chain rule,

$$\frac{1}{y} \frac{dy}{dx} = \frac{d}{dx} (ax) + \frac{d}{dx} (\log \sec^x) + \frac{d}{dx} (\log \log x) - \frac{1}{2} \frac{d}{dx} \log(1 - 2x)$$

$$\frac{1}{y} \frac{dy}{dx} = a + \frac{1}{\sec^x} \frac{d}{dx} (\sec^x) + \frac{1}{\log x} \frac{d}{dx} (\log x) - \frac{1}{2} \left(\frac{1}{1 - 2x} \right) \frac{d}{dx} (1 - 2x)$$

$$\frac{1}{y} \frac{dy}{dx} = a + \frac{\sec^x \tan x}{\sec^x} + \frac{1}{(\log x)} \left(\frac{1}{x} \right) - \frac{1}{2} \left(\frac{1}{1 - 2x} \right) (-2)$$

$$\frac{dy}{dx} = y \left[a + \tan x + \frac{1}{x \log x} + \frac{1}{1 - 2x} \right]$$

$$\frac{dy}{dx} = \frac{e^{ax} \sec^x \log x}{\sqrt{1 - 2x}} \left[a + \tan x + \frac{1}{x \log x} + \frac{1}{1 - 2x} \right] \quad \text{[Using equation (i)]}$$

Differentiation Ex 11.5 Q23

Here,

$$y = e^{3x} \times \sin 4x \times 2^x \quad \text{--- (i)}$$

Taking log on both the sides,

$$\begin{aligned} \log y &= \log e^{3x} + \log \sin 4x + \log 2^x && [\text{Since, } \log(AB) = \log A + \log B] \\ \log y &= 3x \log e + \log \sin 4x + x \log 2 && [\text{Since, } \log_e e = 1, \log_a b = b \log a] \\ \log y &= 3x + \log \sin 4x + x \log 2 \end{aligned}$$

Differentiating it with respect to x ,

$$\begin{aligned} \frac{1}{y} \frac{dy}{dx} &= \frac{d}{dx} (3x) + \frac{d}{dx} (\log \sin 4x) + \frac{d}{dx} (x \log 2) \\ &= 3 + \frac{1}{\sin 4x} \frac{d}{dx} (\sin 4x) + \log 2 (1) \\ &= 3 + \frac{1}{\sin 4x} (\cos 4x) \frac{d}{dx} (4x) + \log 2 \\ &= 3 + \cot x (4) + \log 2 \\ \frac{1}{y} \frac{dy}{dx} &= 3 + 4 \cot 4x + \log 2 \\ \frac{dy}{dx} &= y [3 + 4 \cot 4x + \log 2] \end{aligned}$$

$$\frac{dy}{dx} = e^{3x} \times \sin 4x \times 2^x [3 + 4 \cot 4x + \log 2]$$

Differentiation Ex 11.5 Q24

Here,

$$y = \sin x \sin 2x \sin 3x \sin 4x \quad \text{--- (i)}$$

Taking log on both the sides,

$$\begin{aligned} \log y &= \log (\sin x \sin 2x \sin 3x \sin 4x) \\ \log y &= \log \sin x + \log \sin 2x + \log \sin 3x + \log \sin 4x \end{aligned}$$

Differentiating it with respect to x using chain rule,

$$\begin{aligned} \frac{1}{y} \frac{dy}{dx} &= \frac{d}{dx} \log \sin x + \frac{d}{dx} \log \sin 2x + \frac{d}{dx} \log \sin 3x + \frac{d}{dx} \log \sin 4x \\ &= \frac{1}{\sin x} \frac{d}{dx} (\sin x) + \frac{1}{\sin 2x} \frac{d}{dx} (\sin 2x) + \frac{1}{\sin 3x} \frac{d}{dx} (\sin 3x) + \frac{1}{\sin 4x} \frac{d}{dx} (\sin 4x) \\ &= \frac{1}{\sin x} (\cos x) + \frac{1}{\sin 2x} (\cos 2x) \frac{d}{dx} (2x) + \frac{1}{\sin 3x} (\cos 3x) \frac{d}{dx} (3x) + \frac{1}{\sin 4x} (\cos 4x) \frac{d}{dx} (4x) \\ \frac{1}{y} \frac{dy}{dx} &= [\cot x + \cot 2x (2) + \cot 3x (3) + \cot 4x (4)] \\ \frac{dy}{dx} &= y [\cot x + 2 \cot 2x + 3 \cot 3x + 4 \cot 4x] \\ \frac{dy}{dx} &= (\sin x \sin 2x \sin 3x \sin 4x) [\cot x + 2 \cot 2x + 3 \cot 3x + 4 \cot 4x] && [\text{Using equation (i)}] \end{aligned}$$

Differentiation Ex 11.5 Q25

$$\text{Let } y = x^{\sin x} + (\sin x)^x$$

$$\text{Also, let } u = x^{\sin x} \text{ and } v = (\sin x)^x$$

$$\therefore y = u + v$$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \dots(1)$$

$$u = x^{\sin x}$$

$$\Rightarrow \log u = \log(x^{\sin x})$$

$$\Rightarrow \log u = \sin x \log x$$

Differentiating both sides with respect to x , we obtain

$$\begin{aligned} \frac{1}{u} \frac{du}{dx} &= \frac{d}{dx}(\sin x) \cdot \log x + \sin x \cdot \frac{d}{dx}(\log x) \\ \Rightarrow \frac{du}{dx} &= u \left[\cos x \log x + \sin x \cdot \frac{1}{x} \right] \\ \Rightarrow \frac{du}{dx} &= x^{\sin x} \left[\cos x \log x + \frac{\sin x}{x} \right] \quad \dots(2) \end{aligned}$$

$$v = (\sin x)^x$$

$$\Rightarrow \log v = \log(\sin x)^x$$

$$\Rightarrow \log v = x \log(\sin x)$$

Differentiating both sides with respect to x , we obtain

$$\begin{aligned} \frac{1}{v} \frac{dv}{dx} &= \frac{d}{dx}(x) \times \log(\sin x) + x \times \frac{d}{dx}[\log(\sin x)] \\ \Rightarrow \frac{dv}{dx} &= v \left[\log(\sin x) + x \cdot \frac{1}{\sin x} \cdot \frac{d}{dx}(\sin x) \right] \\ \Rightarrow \frac{dv}{dx} &= (\sin x)^x \left[\log \sin x + \frac{x}{\sin x} \cos x \right] \\ \Rightarrow \frac{dv}{dx} &= (\sin x)^x \left[\log \sin x + x \cot x \right] \\ \Rightarrow \frac{dv}{dx} &= (\sin x)^{x-1} \left[\log \sin x + x \cot x \right] \quad \dots(3) \end{aligned}$$

From (1), (2), and (3), we obtain

$$\frac{dy}{dx} = x^{\sin x} \left(\cos x \log x + \frac{\sin x}{x} \right) + (\sin x)^{x-1} \left[\log \sin x + x \cot x \right]$$

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