It can be observed that the area ABCD is symmetrical about x-axis.

∴ Area ABCD = 2 × Area ABC

$$Area\ of\ ABC = \int_{\frac{a}{\sqrt{2}}}^{a} y\,dx$$

$$= \int_{\frac{a}{\sqrt{2}}}^{a} \sqrt{a^2 - x^2}\,dx$$

$$= \left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_{\frac{a}{\sqrt{2}}}^{a}$$

$$= \left[\frac{a^2}{2}\left(\frac{\pi}{2}\right) - \frac{a}{2\sqrt{2}}\sqrt{a^2 - \frac{a^2}{2}} - \frac{a^2}{2}\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]$$

$$= \frac{a^2\pi}{4} - \frac{a}{2\sqrt{2}} \cdot \frac{a}{\sqrt{2}} - \frac{a^2}{2}\left(\frac{\pi}{4}\right)$$

$$= \frac{a^2\pi}{4} - \frac{a^2}{4} - \frac{a^2\pi}{8}$$

$$= \frac{a^2}{4}\left[\pi - 1 - \frac{\pi}{2}\right]$$

$$= \frac{a^2}{4}\left[\frac{\pi}{2} - 1\right]$$

$$\Rightarrow Area\ ABCD = 2\left[\frac{a^2}{4}\left(\frac{\pi}{2} - 1\right)\right] = \frac{a^2}{2}\left(\frac{\pi}{2} - 1\right)$$

Therefore, the area of smaller part of the circle, $x^2 + y^2 = a^2$, cut off by the line, $x = \frac{a}{\sqrt{2}}$, is $\frac{a^2}{2}\left(\frac{\pi}{2} - 1\right)$ units.

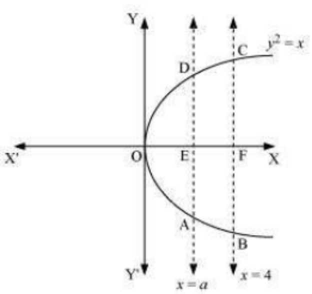
Question 8:

The area between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, find the value of a.

Answer

The line, $x = a$, divides the area bounded by the parabola and $x = 4$ into two equal parts.

∴ Area OAD = Area ABCD



It can be observed that the given area is symmetrical about x-axis.

⇒ Area OED = Area EFCD

$$Area\ OED = \int_0^a y\,dx$$

$$= \int_0^a \sqrt{x}\,dx$$

$$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^a$$

$$=\frac{2}{3}(a)^{\frac{3}{2}} \quad ...(1)$$

$$Area\ of\ EFCD=\int_a^4 \sqrt{x}dx$$

$$=\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_a^4$$

$$=\frac{2}{3}\left[8-a^{\frac{3}{2}}\right] \quad ...(2)$$

From (1) and (2), we obtain

$$\frac{2}{3}(a)^{\frac{3}{2}}=\frac{2}{3}\left[8-(a)^{\frac{3}{2}}\right]$$

$$\Rightarrow 2\cdot(a)^{\frac{3}{2}}=8$$

$$\Rightarrow (a)^{\frac{3}{2}}=4$$

$$\Rightarrow a=(4)^{\frac{2}{3}}$$

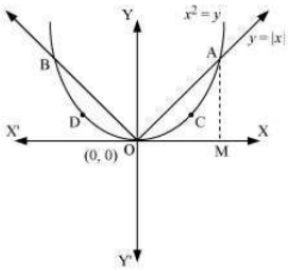
Therefore, the value of a is $(4)^{\frac{2}{3}}$.

Question 9:

Find the area of the region bounded by the parabola $y = x^2$ and $y=|x|$

Answer

The area bounded by the parabola, $x^2 = y$,and the line, $y=|x|$, can be represented as



The given area is symmetrical about y-axis.

∴ Area OACO = Area ODBO

The point of intersection of parabola, $x^2 = y$, and line, $y = x$, is A (1, 1).

Area of OACO = Area ΔOAB − Area OBACO

$$\therefore \text{Area of } \Delta\text{OAB}=\frac{1}{2}\times\text{OB}\times\text{AB}=\frac{1}{2}\times1\times1=\frac{1}{2}$$

$$\text{Area of OBACO}=\int_0^1 y\,dx=\int_0^1 x^2\,dx=\left[\frac{x^3}{3}\right]_0^1=\frac{1}{3}$$

⇒ Area of OACO = Area of ΔOAB − Area of OBACO

$$=\frac{1}{2}-\frac{1}{3}$$

$$=\frac{1}{6}$$

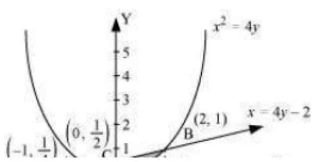
Therefore, required area = $2\left[\frac{1}{6}\right]=\frac{1}{3}$ units

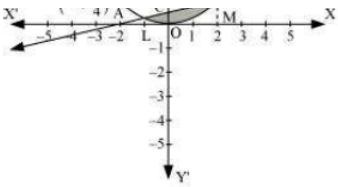
Question 10:

Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$

Answer

The area bounded by the curve, $x^2 = 4y$, and line, $x = 4y - 2$, is represented by the shaded area OBAO.





Let A and B be the points of intersection of the line and parabola.

Coordinates of point A are $\left(-1, \frac{1}{4}\right)$.

Coordinates of point B are (2, 1).

We draw AL and BM perpendicular to x-axis.

It can be observed that,

Area OBAO = Area OBCO + Area OACO ... (1)

Then, Area OBCO = Area OMBC − Area OMBO

$$= \int_0^2 \frac{x+2}{4} dx - \int_0^2 \frac{x^2}{4} dx$$

$$= \frac{1}{4}\left[\frac{x^2}{2} + 2x\right]_0^2 - \frac{1}{4}\left[\frac{x^3}{3}\right]_0^2$$

$$= \frac{1}{4}[2+4] - \frac{1}{4}\left[\frac{8}{3}\right]$$

$$= \frac{3}{2} - \frac{2}{3}$$

$$= \frac{5}{6}$$

Similarly, Area OACO = Area OLAC − Area OLAO

$$= \int_{-1}^0 \frac{x+2}{4} dx - \int_{-1}^0 \frac{x^2}{4} dx$$

$$= \frac{1}{4}\left[\frac{x^2}{2} + 2x\right]_{-1}^0 - \frac{1}{4}\left[\frac{x^3}{3}\right]_{-1}^0$$

$$= -\frac{1}{4}\left[\frac{(-1)^2}{2} + 2(-1)\right] - \left[-\frac{1}{4}\left(\frac{(-1)^3}{3}\right)\right]$$

$$= -\frac{1}{4}\left[\frac{1}{2} - 2\right] - \frac{1}{12}$$

$$= \frac{1}{2} - \frac{1}{8} - \frac{1}{12}$$

$$= \frac{7}{24}$$

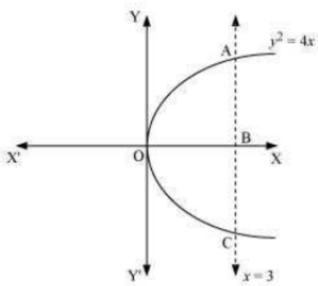
Therefore, required area = $\left(\frac{5}{6} + \frac{7}{24}\right) = \frac{9}{8}$ units

Question 11:

Find the area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$

Answer

The region bounded by the parabola, $y^2 = 4x$, and the line, $x = 3$, is the area OACO.



The area OACO is symmetrical about x-axis.

∴ Area of OACO = 2 (Area of OAB)

$$\text{Area OACO} = 2\left[\int_0^3 y\,dx\right]$$

$$= 2\int_0^3 2\sqrt{x}\,dx$$

$$= 4\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^3$$

$$= \frac{8}{3}\left[(3)^{\frac{3}{2}}\right]$$

$$= 8\sqrt{3}$$

Therefore, the required area is $8\sqrt{3}$ units.

Question 12:

Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is

*********** END **********