

Differentiation Ex 11.2 Q71

Here, $y = e^x + e^{-x}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx} \left(e^x + e^{-x} \right)$$

$$= \frac{d}{dx} e^x + \frac{d}{dx} e^{-x}$$

$$= e^x + e^{-x} \frac{d}{dx} (-x)$$

$$= e^x + e^{-x} (-1)$$

$$= \left(e^x - e^{-x} \right)$$

$$= \sqrt{\left(e^x + e^{-x} \right)^2 - 4e^x \times e^{-x}}$$

$$= \sqrt{y^2 - 4}$$

[Using chian rule]

$$\left[\text{Since } (a-b) = \sqrt{(a+b)^2 - 4ab}\right]$$

$$\left[\text{Since } e^x + e^{-x} = y\right]$$

$$\frac{dy}{dx} = \sqrt{y^2 - 4}.$$

Differentiation Ex 11.2 Q72 Given, $y = \sqrt{a^2 - x^2}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx} \left(\sqrt{a^2 - x^2} \right)$$

$$= \frac{1}{2\sqrt{a^2 - x^2}} \frac{d}{dx} \left(a^2 - x^2 \right)$$

$$= \frac{1}{2\sqrt{a^2 - x^2}} (-2x)$$

$$= \frac{-x}{\sqrt{a^2 - x^2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-x}{y}$$

$$\Rightarrow y \frac{dy}{dx} = -x$$

$$y \frac{dy}{dx} + x = 0$$
[Using chain rule]

[Since $\sqrt{a^2 - x^2} = y$]

Differentiation Ex 11.2 Q73

Here,
$$xy = 4$$

$$\Rightarrow y = \frac{4}{x}$$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{4}{x}\right)$$

$$= 4\frac{d}{dx} (x^{-1})$$

$$= 4\left(-1 \times x^{-1-1}\right)$$

$$= 4\left(-\frac{1}{x^2}\right)$$

$$= -\frac{4}{x^2}$$

$$= -\frac{4}{\left(\frac{x}{y}\right)^2}$$

$$= -\frac{4y^2}{16}$$

$$\frac{dy}{dx} = -\frac{y^2}{4}$$

$$\Rightarrow 4\frac{dy}{dx} = 3y^2 - 4y^2$$

$$\Rightarrow 4\frac{dy}{dx} + 4y^2 = 3y^2$$

$$\Rightarrow 4\left(\frac{dy}{dx} + y^2\right) = 3y^2$$

Dividing both the sides by x,

$$\Rightarrow \frac{4}{x} \left(\frac{dy}{dx} + y^2 \right) = \frac{3y^2}{x}$$

$$\Rightarrow y \left(\frac{dy}{dx} + y^2 \right) = \frac{3y^2}{x}$$

$$\Rightarrow x \left(\frac{dy}{dx} + y^2 \right) = \frac{3y^2}{y}$$

$$\Rightarrow x \left(\frac{dy}{dx} + y^2 \right) = 3y.$$
[Since $\frac{4}{x} = y$]

Differentiation Ex 11.2 Q74

$$\frac{d}{dx} \left\{ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right\} = \sqrt{a^2 - x^2}$$

$$\begin{aligned} \text{LHS} &&= \frac{d}{dx} \left\{ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right\} \\ &= \frac{d}{dx} \left(\frac{x}{2} \sqrt{a^2 - x^2} \right) + \frac{d}{dx} \left(\frac{a^2}{2} \sin^{-1} \frac{x}{a} \right) \\ &= \frac{1}{2} \left[x \frac{d}{dx} \sqrt{a^2 - x^2} + \sqrt{a^2 - x^2} \frac{d}{dx} (x) \right] + \frac{a^2}{2} \times \frac{1}{\sqrt{1 - \left(\frac{x}{a} \right)^2}} \times \frac{d}{dx} \left(\frac{x}{a} \right) \end{aligned}$$

[Using product rule, chain rule]

$$\begin{split} &= \frac{1}{2} \left[x \times \frac{1}{2\sqrt{a^2 - x^2}} \frac{d}{dx} \left(a^2 - x^2 \right) + \sqrt{a^2 - x^2} \right] + \left(\frac{a^2}{2} \right) \times \frac{1}{\sqrt{\frac{a^2 - x^2}{a^2}}} \times \left(\frac{1}{a} \right) \\ &= \frac{1}{2} \left[\frac{x \left(-2x \right)}{2\sqrt{a^2 - x^2}} + \sqrt{a^2 - x^2} \right] + \left(\frac{a^2}{2} \right) \frac{a}{\sqrt{a^2 - x^2}} \times \left(\frac{1}{a} \right) \\ &= \frac{1}{2} \left[\frac{-2x^2 + 2 \left(a^2 - x^2 \right)}{2\sqrt{a^2 - x^2}} \right] + \frac{a^2}{2\sqrt{a^2 - x^2}} \\ &= \frac{1}{2} \left[\frac{2 \left(a^2 - 2x^2 \right)}{2\sqrt{a^2 - x^2}} \right] + \frac{a^2}{2\sqrt{a^2 - x^2}} \\ &= \frac{a^2 - 2x^2}{2\sqrt{a^2 - x^2}} + \frac{a^2}{2\sqrt{a^2 - x^2}} \\ &= \frac{a^2 - 2x^2 + a^2}{2\sqrt{a^2 - x^2}} \\ &= \frac{2a^2 - 2x^2}{2\sqrt{a^2 - x^2}} \\ &= \frac{2 \left(a^2 - x^2 \right)}{2\sqrt{a^2 - x^2}} \\ &= \frac{2 \left(a^2 - x^2 \right)}{\sqrt{a^2 - x^2}} \\ &= \sqrt{a^2 - x^2} \\ &= \sqrt{a^2 - x^2} \\ &= RHS \end{split}$$

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