Higher Order Derivatives Ex 12.1 Q19

$x = \sin t; \quad y = \sin pt$

differentiating both w.r.t. t

$$\Rightarrow \quad \frac{dy}{dt} = \cos t \ \ldots\ldots (1); \quad \frac{dy}{dt} = p \cos pt \ \ldots\ldots (2)$$

dividing (2) by (1)

$$\Rightarrow \quad \frac{dy}{dx} = p \frac{\cos pt}{\cos t}$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{dt} = p\left\{\frac{p\cos t(-\sin pt) - (\cos pt)(-\sin t)}{\cos^2 t}\right\}$$

$$= p\left\{\frac{\sin t \cos pt - p\cos t \sin pt\,(-\sin t)}{\cos^2 t}\right\} \ldots\ldots (3)$$

$\Rightarrow$ dividing (3) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = p\left\{\frac{\sin t \cos pt - p\cos t \sin pt}{\cos^3 t}\right\} = \left\{\frac{\tan t \cos t - p \sin pt}{\cos^2 t}\right\}$$

$$\because \quad \sin^2 t + \cos^2 t = 1$$

$$\Rightarrow \quad 1 - \sin^2 t = \cos^2 t$$

$$\Rightarrow \quad 1 - x^2 = \cos^2 t$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = p\left\{\frac{\tan t \cos pt - p \sin pt}{1 - x^2}\right\}$$

$$\Rightarrow \quad \left(1 - x^2\right)\frac{d^2y}{dx^2} = p\frac{\sin t \cos pt}{\cos t} - p^2 \sin pt = \frac{dy}{dx} - p^2 y$$

$$\Rightarrow \quad \left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2 y = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q20

$y = e^{\tan^{-1} x}$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = e^{\tan^{-1} x}\left(\frac{1}{1+x^2}\right)$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{\left(1+x^2\right)\left(e^{\tan^{-1} x}\right) \times \frac{1}{1+x^2} - e^{\tan^{-1} x}\left(2x\right)}{\left(1+x^2\right)^2}$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} = \frac{e^{\tan^{-1} x} - 2xe^{\tan^{-1} x}}{1+x^2}$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} = \frac{e^{\tan^{-1} x}}{1+x^2}\left(1-2x\right) = \frac{dy}{dx}\left(1-2x\right)$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} + \left(2x-1\right)\frac{dy}{dx} = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q21

$y = e^{\tan^{-1} x}$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = e^{\tan^{-1} x}\left(\frac{1}{1+x^2}\right)$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{\left(1+x^2\right)\left(e^{\tan^{-1} x}\right) \times \frac{1}{1+x^2} - e^{\tan^{-1} x}\left(2x\right)}{\left(1+x^2\right)^2}$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} = \frac{e^{\tan^{-1} x} - 2xe^{\tan^{-1} x}}{1+x^2}$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} = \frac{e^{\tan^{-1} x}}{1+x^2}\left(1-2x\right) = \frac{dy}{dx}\left(1-2x\right)$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} + \left(2x-1\right)\frac{dy}{dx} = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q22

It is given that, $y = 3\cos(\log x) + 4\sin(\log x)$

Then,

$$y_1 = 3 \cdot \frac{d}{dx}\left[\cos(\log x)\right] + 4 \cdot \frac{d}{dx}\left[\sin(\log x)\right]$$
$$= 3 \cdot \left[-\sin(\log x) \cdot \frac{d}{dx}(\log x)\right] + 4 \cdot \left[\cos(\log x) \cdot \frac{d}{dx}(\log x)\right]$$
$$\therefore y_1 = \frac{-3\sin(\log x)}{x} + \frac{4\cos(\log x)}{x} = \frac{4\cos(\log x) - 3\sin(\log x)}{x}$$
$$\therefore y_2 = \frac{d}{dx}\left(\frac{4\cos(\log x) - 3\sin(\log x)}{x}\right)$$
$$= \frac{x\{4\cos(\log x) - 3\sin(\log x)\}' - \{4\cos(\log x) - 3\sin(\log x)\}(x)'}{x^2}$$
$$= \frac{x\left[4\{\cos(\log x)\}' - 3\{\sin(\log x)\}'\right] - \{4\cos(\log x) - 3\sin(\log x)\}.1}{x^2}$$
$$= \frac{x\left[-4\sin(\log x).(\log x)' - 3\cos(\log x).(\log x)'\right] - 4\cos(\log x) + 3\sin(\log x)}{x^2}$$
$$= \frac{x\left[-4\sin(\log x).\frac{1}{x} - 3\cos(\log x).\frac{1}{x}\right] - 4\cos(\log x) + 3\sin(\log x)}{x^2}$$
$$= \frac{-4\sin(\log x) - 3\cos(\log x) - 4\cos(\log x) + 3\sin(\log x)}{x^2}$$
$$= \frac{-\sin(\log x) - 7\cos(\log x)}{x^2}$$
$$\therefore x^2 y_2 + xy_1 + y$$
$$= x^2\left(\frac{-\sin(\log x) - 7\cos(\log x)}{x^2}\right) + x\left(\frac{4\cos(\log x) - 3\sin(\log x)}{x}\right) + 3\cos(\log x) + 4\sin(\log x)$$
$$= -\sin(\log x) - 7\cos(\log x) + 4\cos(\log x) - 3\sin(\log x) + 3\cos(\log x) + 4\sin(\log x)$$
$$= 0$$

Hence, proved.

*********** END **********