Indefinite Integrals Ex 19.10 Q5

Let $I = \int \left(2x^2 + 3\right)\sqrt{x+2}dx$

Substituting $x + 2 = t$ and $dx = dt$, we get

$$I = \int\left[2\left(t-2\right)^2 + 3\right]\sqrt{t}dt$$

$$= \int\left[2\left(t^2 + 4 - 4t\right) + 3\right]\sqrt{t}dt$$

$$= \int\left[2t^2 + 8 - 8t + 3\right]\sqrt{t}dt$$

$$= \int\left(2t^{\frac{5}{2}} + 11t^{-\frac{1}{2}} - 8t^{\frac{3}{2}}\right)dt$$

$$= \frac{4}{7}t^{\frac{7}{2}} + \frac{22}{3}t^{\frac{3}{2}} - \frac{16}{5}t^{\frac{5}{2}} + c$$

$$= \frac{4}{7}(x+2)^{\frac{7}{2}} - \frac{16}{5}(x+1)\frac{5}{2} + \frac{22}{3}(x+2)^{\frac{3}{2}} + c$$

$$\therefore \qquad I = \frac{4}{7}(x+2)^{\frac{7}{2}} - \frac{16}{5}(x+2)^{\frac{5}{2}} + \frac{22}{3}(x+2)^{\frac{3}{2}} + c$$

Indefinite Integrals Ex 19.10 Q6

Let $I = \int \frac{x^2 + 3x + 1}{\left(x+1\right)^2}dx$

Substituting $x + 1 = t$ and $dx = dt$, we get

$$I = \int \frac{(t-1)^2 + 3(t-1) + 1}{t^2}dt$$

$$= \int \frac{t^2 + 1 - 2t + 3t - 3 + 1}{t^2}dt$$

$$= \int \frac{t^2 + t - 1}{t^2}dt$$

$$= \int\left(\frac{t^2}{t^2} + \frac{t}{t^2} - \frac{1}{t^2}\right)dt$$

$$= \int\left(1 + \frac{1}{t} - r^{-2}\right)dt$$

$$= t + \log|t| + t^{-1} + c$$

$$= t + \log|t| + \frac{1}{t} + c$$

$$= \left(x+1\right) + \log|x+1| + \frac{1}{x+1} + c$$

Indefinite Integrals Ex 19.10 Q7

Let $I = \int \frac{x^2}{\sqrt{1-x}} dx$

Substituting $1 - x = t$ and $dx = -dt$, we get

$$I = \int \frac{(1-t)^2}{\sqrt{t}} \times -dt$$

$$= -\int \frac{1+t^2-2t}{\sqrt{t}} \times dt$$

$$= -\int \left(t^{\frac{-1}{2}} + t^{\frac{3}{2}} - 2t^{\frac{1}{2}} \right) dt$$

$$= -\left[2t^{\frac{1}{2}} + \frac{2}{5} t^{\frac{5}{2}} - \frac{4}{3} t^{\frac{3}{2}} \right] + c$$

$$= -\left[\frac{30t^{\frac{1}{2}} + 6t^{\frac{5}{2}} - 20t^{\frac{3}{2}}}{15} \right] + c$$

$$= -\frac{2t^{\frac{1}{2}}}{15} \left[15 + 3t^2 - 10t \right] + c$$

$$= -\frac{2}{15} \sqrt{1-x} \left[15 + 3(1-x)^2 - 10(1-x) \right] + c$$

$$= -\frac{2}{15} \sqrt{1-x} \left(15 + 3\left(1 + x^2 - 2x\right) - 10 + 10x \right) + c$$

$$= -\frac{2}{15} \sqrt{1-x} \left(5 + 3 + 3x^2 - 6x + 10x \right) + c$$

$$= -\frac{2}{15} \sqrt{1-x} \left(3x^2 + 4x + 8 \right) + c$$

$$= -\frac{2}{15} \left(3x^2 + 4x + 8 \right) \sqrt{1-x} + c$$

$$\therefore \quad I = -\frac{2}{15} \left(3x^2 + 4x + 8 \right) \sqrt{1-x} + c$$

Indefinite Integrals Ex 19.10 Q8

Let $I = \int x\left(1-x\right)^{23} dx$

Substituting $1-x = t$ and $dx = -dt$, we get

$$I = -\int \left(1-t\right) t^{23} dt$$

$$= -\int \left(t^{23} - t^{24}\right) dt$$

$$= -\int \left(\frac{t^{24}}{24} - \frac{t^{25}}{25}\right) + c$$

$$= \frac{t^{25}}{25} - \frac{t^{24}}{24} + c$$

$$= \frac{\left(1-x\right)^{25}}{25} - \frac{\left(1-x\right)^{24}}{24} + c$$

$$\therefore \quad I = \frac{\left(1-x\right)^{25}}{25} - \frac{\left(1-x\right)^{24}}{24} + c.$$

$$= \frac{1}{600}\left(1-x\right)^{24}\left[24\left(1-x\right) - 25\right] + c$$

$$= \frac{1}{600}\left(1-x\right)^{24}\left[24 - 24x - 25\right] + c$$

$$= \frac{1}{600}\left(1-x\right)^{24}\left[-1 - 24x\right] + c$$

$$= \frac{1}{600}\left(1-x\right)^{24} \times -\left[1 + 24x\right] + c$$

$$= -\frac{1}{600}\left(1-x\right)^{24}\left(1 + 24x\right) + c$$

*********** END **********