Indefinite Integrals Ex 19.5 Q1

Let $I = \int \frac{x+1}{\sqrt{2x+3}}dx$

Let $x+1 = \lambda(2x+3)+\mu$

On equating the coefficients of like powers of x on bothy sides, we get

$I = 2\lambda,\ 3\lambda+\mu = 1$

$\Rightarrow \quad \lambda = \frac{1}{2}$ and $3\times\frac{1}{2}+\mu = 1$

$\Rightarrow \quad \lambda = \frac{1}{2}$ and $\mu = \frac{-1}{2}$

Replacing $x+1$ by $\lambda(2x+3)+\mu$ in the given integral, we get

$$I = \int \frac{\lambda(2x+3)+\mu}{\sqrt{2x+3}}dx$$
$$= \int \frac{\lambda(2x+3)}{\sqrt{2x+3}}dx + \mu\int \frac{1}{\sqrt{2x+3}}dx$$
$$= \lambda\int (2x+3)^{\frac{1}{2}}dx + \mu\int (2x+3)^{\frac{-1}{2}}dx$$
$$= \lambda\frac{(2x+3)^{\frac{3}{2}}}{\frac{3}{2}\times 2} + \mu\frac{(2x+3)^{\frac{1}{2}}}{2\times\frac{1}{2}} + c$$
$$= \frac{1}{2}\times\frac{(2x+3)^{\frac{3}{2}}}{3} + \left(\frac{-1}{2}\right)\times(2x+3)^{\frac{1}{2}} + c \qquad \left[\because \lambda = \frac{1}{2}, \mu = \frac{-1}{2}\right]$$
$$= \frac{(2x+3)^{\frac{3}{2}}}{6} - \frac{(2x+3)^{\frac{1}{2}}}{2} + c$$

$\therefore \quad I = \frac{1}{6}\times(2x+3)^{\frac{3}{2}} - \frac{1}{2}(2x+3)^{\frac{1}{2}} + c.$

Indefinite Integrals Ex 19.5 Q2

Let $I = \int x\sqrt{x+2}dx$. Then,

$I = \int\{(x+2)-2\}x+2dx \qquad [\because x = (x+2)-2]$

$\Rightarrow \quad I = f\left\{(x+2)^{\frac{3}{2}} - 2(x+2)^{\frac{1}{2}}\right\}dx$

$\Rightarrow \quad I = \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + c$

Indefinite Integrals Ex 19.5 Q3

Let $I = \int \frac{x-1}{\sqrt{x+4}}dx$. Then,

$$I = \int \frac{x+4-4-1}{\sqrt{x+4}}$$
$$= \int \frac{x+4-5}{\sqrt{x+4}}dx$$
$$= \int \frac{x+4}{\sqrt{x+4}}dx - 5\int \frac{1}{\sqrt{x+4}}dx$$
$$= \int (x+4)^{\frac{1}{2}}dx - 5\int (x+4)^{\frac{-1}{2}}dx$$
$$= \frac{(x+4)^{\frac{3}{2}}}{\frac{3}{2}} - 5\frac{(x+4)^{\frac{1}{2}}}{\frac{1}{2}} + c$$
$$= \frac{2}{3}\times(x+4)^{\frac{3}{2}} - 10(x+4)^{\frac{1}{2}} + c$$

$$\therefore \quad I = \frac{2}{3}\times(x+4)^{\frac{3}{2}} - 10(x+4)^{\frac{1}{2}} + c$$

Indefinite Integrals Ex 19.5 Q4

Let $I = \int (x+2)\sqrt{3x+5}dx$

Let $x+2 = \lambda(3x+5) + \mu$ on equating the coefficients of like powers of x on both sides, we get

$$3\lambda = 1 \text{ and } 5\lambda + \mu = 2$$
$$\Rightarrow \quad \lambda = \frac{1}{3} \text{ and } 5\times\frac{1}{3} + \mu = 2$$
$$\Rightarrow \quad \lambda = \frac{1}{3} \text{ and } \mu = \frac{1}{3}$$

$$\therefore \quad I = \int \{\lambda(3x+5) + \mu\}\sqrt{3x+5}dx$$
$$= \lambda\int (3x+5)\sqrt{3x+5}dx + \mu\int \sqrt{3x+5}$$
$$= \lambda\int (3x+5)^{\frac{3}{2}}dx + \mu\int (3x+5)^{\frac{1}{2}}dx$$
$$= \lambda\times\frac{(3x+5)^{\frac{5}{2}}}{\frac{5}{2}\times 3} + \mu\frac{(3x+5)^{\frac{3}{2}}}{3\times\frac{3}{2}} + c$$
$$= \frac{1}{3}\times\frac{2}{15}\times(3x+5)^{\frac{5}{2}} + \frac{1}{3}\times\frac{2}{9}(3x+5)^{\frac{3}{2}} + c$$
$$= \frac{2}{45}\times(3x+5)^{\frac{5}{2}} + \frac{2}{27}\times(3x+5)^{\frac{3}{2}} + c$$
$$= \frac{2}{9}\times(3x+5)^{\frac{3}{2}}\left[\frac{1}{5}\times(3x+5)^{1} + \frac{1}{3}\right] + c$$
$$= \frac{2}{9}\times(3x+5)^{\frac{3}{2}}\left[\frac{3(3x+5)+5}{15}\right] + c$$
$$= \frac{2}{9}\times(3x+5)^{\frac{3}{2}}\frac{(9x+15+5)}{15} + c$$
$$= \frac{2}{135}\times(3x+5)^{\frac{3}{2}}(9x+20) + c$$

$$\therefore \quad I = \frac{2}{135}\times(9x+20)(3x+5)^{\frac{3}{2}} + c.$$

Indefinite Integrals Ex 19.5 Q5

Let $I = \int \frac{2x+1}{\sqrt{3x+2}}dx$

Let $2x+1 = \lambda(3x+2)+\mu$ on equating the coefficients of like powers of x on both sides, we get

$3\lambda = 2$ and $2\lambda + \mu = 1$

$\Rightarrow \quad \lambda = \frac{2}{3}$ and $2\times\frac{2}{3}+\mu = 1$

$\Rightarrow \quad \lambda = \frac{2}{3}$ and $\mu = \frac{-1}{3}$

$\therefore \quad I = \int \frac{\lambda(3x+2)+\mu}{\sqrt{3x+2}}dx$

$= \lambda\int \frac{3x+2}{\sqrt{3x+2}}dx + \mu\int \frac{1}{\sqrt{3x+2}}dx$

$= \lambda\int (3x+2)^{\frac{1}{2}}dx + \mu\int (3x+2)^{\frac{-1}{2}}dx$

$= \lambda\times\frac{(3x+2)^{\frac{3}{2}}}{\frac{3}{2}\times 3} + \mu\frac{(3x+2)^{\frac{1}{2}}}{\frac{1}{2}\times 3}+c$

$= \frac{2}{3}\times\frac{2}{9}\times(3x+2)^{\frac{3}{2}} - \frac{1}{3}\times\frac{2}{3}(3x+2)^{\frac{1}{2}}+c$

$= \frac{4}{27}\times(3x+2)^{\frac{3}{2}} - \frac{2}{9}\times(3x+2)^{\frac{1}{2}}+c$

$= \frac{2}{9}\times\sqrt{3x+2}\left[\frac{2}{3}\times(3x+2)-1\right]+c$

$= \frac{2}{9}\times\sqrt{3x+2}\left[\frac{6x+4-3}{3}\right]+c$

$= \frac{2}{27}\times\sqrt{3x+2}\,(6x+1)+c$

$\therefore \quad I = \frac{2}{27}\times(6x+1)\sqrt{3x+2}+c.$

********** END **********