Equation of line segment AB is

$$y - 0 = \frac{3-0}{1+1}(x+1)$$

$$y = \frac{3}{2}(x+1)$$

$$\therefore \text{Area}(ALBA) = \int_{-1}^{1} \frac{3}{2}(x+1)dx = \frac{3}{2}\left[\frac{x^2}{2} + x\right]_{-1}^{1} = \frac{3}{2}\left[\frac{1}{2} + 1 - \frac{1}{2} + 1\right] = 3 \text{ units}$$

Equation of line segment BC is

$$y - 3 = \frac{2-3}{3-1}(x-1)$$

$$y = \frac{1}{2}(-x+7)$$

$$\therefore \text{Area}(BLMCB) = \int_{1}^{3} \frac{1}{2}(-x+7)dx = \frac{1}{2}\left[-\frac{x^2}{2} + 7x\right]_{1}^{3} = \frac{1}{2}\left[-\frac{9}{2} + 21 + \frac{1}{2} - 7\right] = 5 \text{ units}$$

Equation of line segment AC is

$$y - 0 = \frac{2-0}{3+1}(x+1)$$

$$y = \frac{1}{2}(x+1)$$

$$\therefore \text{Area}(AMCA) = \frac{1}{2}\int_{-1}^{3}(x+1)dx = \frac{1}{2}\left[\frac{x^2}{2} + x\right]_{-1}^{3} = \frac{1}{2}\left[\frac{9}{2} + 3 - \frac{1}{2} + 1\right] = 4 \text{ units}$$

Therefore, from equation (1), we obtain
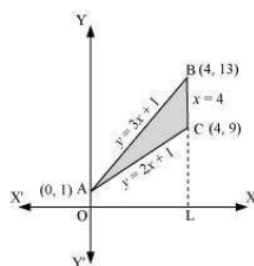Area (ΔABC) = (3 + 5 − 4) = 4 units

Question 5:
Using integration find the area of the triangular region whose sides have the equations *y* = 2*x* +1, *y* = 3*x* + 1 and *x* = 4.

Answer
The equations of sides of the triangle are *y* = 2*x* +1, *y* = 3*x* + 1, and *x* = 4.
On solving these equations, we obtain the vertices of triangle as A(0, 1), B(4, 13), and C (4, 9).

It can be observed that,
Area (ΔACB) = Area (OLBAO) −Area (OLCAO)

$$= \int_{0}^{4}(3x+1)dx - \int_{0}^{4}(2x+1)dx$$

$$= \left[\frac{3x^2}{2} + x\right]_{0}^{4} - \left[\frac{2x^2}{2} + x\right]_{0}^{4}$$
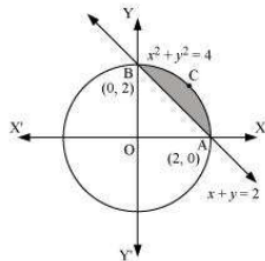
$$= (24+4) - (16+4)$$

$$= 28 - 20$$

= 8 units

Question 6:
Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ is

A. $2(\pi - 2)$

B. $\pi - 2$

C. $2\pi - 1$

D. $2(\pi + 2)$

Answer

The smaller area enclosed by the circle, $x^2 + y^2 = 4$, and the line, $x + y = 2$, is represented by the shaded area ACBA as

It can be observed that,

Area ACBA = Area OACBO – Area (ΔOAB)

$$= \int_0^2 \sqrt{4-x^2}\,dx - \int_0^2 (2-x)\,dx$$

$$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - \left[2x - \frac{x^2}{2}\right]_0^2$$

$$= \left[2 \cdot \frac{\pi}{2}\right] - [4-2]$$

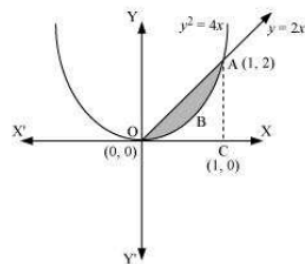$$= (\pi - 2) \text{ units}$$

Thus, the correct answer is B.

Question 7:
Area lying between the curve $y^2 = 4x$ and $y = 2x$ is

A. $\dfrac{2}{3}$

B. $\dfrac{1}{3}$

C. $\dfrac{1}{4}$

D. $\dfrac{3}{4}$

Answer

The area lying between the curve, $y^2 = 4x$ and $y = 2x$, is represented by the shaded area OBAO as

The points of intersection of these curves are O (0, 0) and A (1, 2).

We draw AC perpendicular to x-axis such that the coordinates of C are (1, 0).

∴ Area OBAO = Area (ΔOCA) – Area (OCABO)

$$= \int_0^1 2x\,dx - \int_0^1 2\sqrt{x}\,dx$$

$$= 2\left[\frac{x^2}{2}\right]_0^1 - 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1$$

$$= \left|1 - \frac{4}{3}\right|$$

$$= \left|-\frac{1}{3}\right|$$

$= \frac{1}{3}$ units
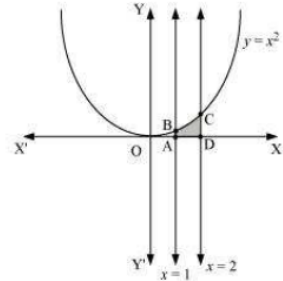
Thus, the correct answer is B.

Question 1:

Find the area under the given curves and given lines:

(i) $y = x^2$, $x = 1$, $x = 2$ and x-axis
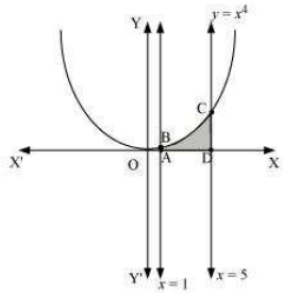
(ii) $y = x^4$, $x = 1$, $x = 5$ and x –axis

Answer

i.   The required area is represented by the shaded area ADCBA as

Area ADCBA $= \int_{1}^{2} y dx$

$= \int_{1}^{2} x^2 dx$

$= \left[\frac{x^3}{3} \right]_{1}^{2}$

$= \frac{8}{3} - \frac{1}{3}$

$= \frac{7}{3}$ units

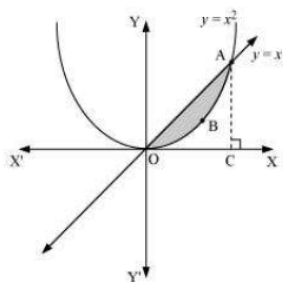ii.   The required area is represented by the shaded area ADCBA as

Area ADCBA $= \int_{1}^{5} x^4 dx$

$= \left[\frac{x^5}{5} \right]_{1}^{5}$

$= \frac{(5)^5}{5} - \frac{1}{5}$

$= (5)^4 - \frac{1}{5}$

$= 625 - \frac{1}{5}$

$= 624.8$ units

Question 2:

Find the area between the curves $y = x$ and $y = x^2$

Answer

The required area is represented by the shaded area OBAO as

The points of intersection of the curves, $y = x$ and $y = x^2$, is A (1, 1).

********** END **********