Indefinite Integrals Ex 19.8 Q16

Let $I = \int \frac{1}{x\left(3+\log x\right)} dx -----(i)$

Let $3+\log x = t$ then,

$d\left(3+\log x\right) = dt$

$\Rightarrow \quad \frac{1}{x} dx = dt$

$\Rightarrow \quad dx = x\,dt$

Putting $3+\log x = t$ and $dx = x\,dt$ in equation (i), we get,

$$I = \int \frac{1}{x \times t} \times x dt$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + c$$

$$= \log\left|\left(3+\log x\right)\right| + c$$

$$\therefore \quad I = \log\left|\left(3+\log x\right)\right| + c$$

Indefinite Integrals Ex 19.8 Q17

Let $I = \int \frac{e^x+1}{e^x+x} dx -----(i)$

Let $e^x + x = t$ then,

$d\left(e^x + x\right) = dt$

$\Rightarrow \quad \left(e^x + x\right) dx = dt$

$\Rightarrow \quad dx = \frac{dt}{e^x+1}$

Putting $e^x + x = t$ and $dx = \frac{dt}{e^x+1}$ in equation (i), we get,

$$I = \int \frac{e^x+1}{t} \times \frac{dt}{e^x+1}$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + c$$

$$= \log\left|e^x + x\right| + c$$

$$\therefore \quad I = \log\left|e^x + x\right| + c$$

Indefinite Integrals Ex 19.8 Q18

Let $I = \int \frac{1}{x \log x} dx$ - - - - - (i)

Let $\log x = t$ then,

$d(\log x) = dt$

$\Rightarrow \quad \frac{1}{x} dx = dt$

$\Rightarrow \quad dx = x\, dt$

Putting $\log x = t$ and $dx = x\, dt$ in equation (i), we get,

$$I = \int \frac{1}{x \times t} \times x\, dt$$
$$= \int \frac{dt}{t}$$
$$= \log|t| + c$$

$$= \log|(\log x)| + c$$

$\therefore \quad I = \log|(\log x)| + c$

Indefinite Integrals Ex 19.8 Q19

Let $I = \int \frac{\sin 2x}{a\cos^2 x + b\sin^2 x} dx$ - - - - - (i)

Let $a\cos^2 x + b\sin^2 x = t$ then,

$d(a\cos^2 x + b\sin^2 x) = dt$

$[a(2\cos x(-\sin x)) + b(2\sin x \cos x)]dx = dt$

$\Rightarrow \quad [-a(2\sin x \cos x) + b(2\sin x \cos x)]dx = dt$

$\Rightarrow \quad [-a\sin 2x + b\sin 2x]dx = dt$

$\Rightarrow \quad \sin 2x(b-a)dx = dt$

$\Rightarrow \quad dx = \frac{dt}{(b-a)\sin 2x}$

Putting $a\cos^2 x + b\sin^2 x = t$ and $dx = \frac{dt}{(b-a)\sin 2x}$ in equation (i), we get,

$$I = \int \frac{\sin 2x}{t} \times \frac{dt}{(b-a)\sin 2x}$$
$$= \frac{1}{b-a}\int \frac{dt}{t}$$
$$= \frac{1}{b-a}\log|t| + c$$

$$= \frac{1}{b-a}\log|a\cos^2 x + b\sin^2 x| + c$$

Indefinite Integrals Ex 19.8 Q20

Let $I = \int \frac{\cos x}{2 + 3\sin x} dx$ - - - - - (i)

Let $2 + 3\sin x = t$ then,

$d(2 + 3\sin x) = dt$

$\Rightarrow \quad 3\cos x\, dx = dt$

$\Rightarrow \quad dx = \frac{dt}{3\cos x}$

Putting $2 + 3\sin x = t$ and $dx = \frac{dt}{3\cos x}$ in equation (i), we get,

$$I = \int \frac{\cos x}{t} \times \frac{dt}{3\cos x}$$

$$= \frac{1}{3}\int \frac{dt}{t}$$

$$= \frac{1}{3}\log|t| + c$$

$$= \frac{1}{3}\log|2 + 3\sin x| + c$$

*********** END **********