Polynomials Ex 2.1 Q20

Answer :

If α and β are the zeros of the quadratic polynomial $f(x)=x^2+px+q$

$$\alpha+\beta=\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$=\frac{-p}{1}$$

$$\alpha\beta=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$=\frac{q}{1}$$

$$=q$$

Let S and P denote respectively the sums and product of the zeros of the polynomial whose zeros are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$. Then,

$$S=(\alpha+\beta)^2+(\alpha-\beta)^2$$

$$S=\alpha^2+\beta^2+\cancel{2\alpha\beta}+\alpha^2+\beta^2-\cancel{2\alpha\beta}$$

$$S=2\left[\alpha^2+\beta^2\right]$$

$$S=2\left[(\alpha+\beta)^2-2\alpha\beta\right]$$

$$S=2\left(p^2-2\times q\right)$$

$$S=2\left(p^2-2q\right)$$

$$S=2\left(p^2-2q\right)$$

$$P=(\alpha+\beta)^2(\alpha-\beta)^2$$

$$P=\left(\alpha^2+\beta^2+2\alpha\beta\right)\left(\alpha^2+\beta^2-2\alpha\beta\right)$$

$$P=\left((\alpha+\beta)^2-\cancel{2\alpha\beta}+\cancel{2\alpha\beta}\right)\left((\alpha+\beta)^2-2\alpha\beta-2\alpha\beta\right)$$

$$P=(p)^2\left((p)^2-4\times q\right)$$

$$P=p^2\left(p^2-4q\right)$$

The required polynomial of $f(x)=k\left(kx^2-sx+p\right)$ is given by

$$f(x)=k\left\{x^2-2\left(p^2-2q\right)x+p^2\left(p^2-4q\right)\right\}$$

$\boxed{f(x)=k\left\{x^2-2\left(p^2-2q\right)x+p^2\left(p^2-4q\right)\right\}}$, where k is any non-zero real number.

*********** END **********