Definite Integrals Ex 20.5 Q17

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+\ldots+f\{a+(n-1)h\}\right], \text{ where } h=\frac{b-a}{n}.$$

Since we have to find $\int_a^b \cos x dx$

We have, $f(x) = \cos x$

$\therefore \quad I = \int_a^b \cos x dx$

$\Rightarrow \quad I = \lim_{h\to 0} h\left[\cos a + \cos(a+h) + \cos(a+2h) + \ldots + \cos\{a+(n-1)h\}\right]$

$\Rightarrow \quad I = \lim_{h\to 0} h\left[\dfrac{\cos\left(a+(n-1)\frac{h}{2}\right)\sin\frac{nh}{2}}{\sin\frac{h}{2}}\right] = \lim_{h\to 0} h\left[\dfrac{\cos\left(a+\frac{nh}{2}-\frac{h}{2}\right)\sin\frac{nh}{2}}{\sin\frac{h}{2}}\right]$

$\Rightarrow \quad I = \lim_{h\to 0} h\left[\dfrac{\cos\left(a+\frac{b-a}{2}-\frac{h}{2}\right)\sin\left(\frac{b-a}{2}\right)}{\sin\frac{h}{2}}\right] \qquad [\because nh = b-a]$

$\Rightarrow \quad I = \lim_{h\to 0}\left[\dfrac{\frac{h}{2}}{\sin\frac{h}{2}} \times 2\cos\left(\frac{a+b}{2}-\frac{h}{2}\right)\sin\left(\frac{b-a}{2}\right)\right]$

$\Rightarrow \quad I = \lim_{h\to 0}\left(\dfrac{\frac{h}{2}}{\sin\frac{h}{2}}\right) \times \lim_{h\to 0} 2\cos\left(\frac{a+b}{2}-\frac{h}{2}\right)\sin\left(\frac{b-a}{2}\right) = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{b-a}{2}\right)$

$\Rightarrow \quad I = \sin b - \sin a \qquad [\because 2\cos A \sin B = \sin(A-B) - \sin(A+B)]$

Definite Integrals Ex 20.5 Q18

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 0,\ b = \frac{\pi}{2}$ and $f(x) = \sin x$

$$\therefore \quad h = \frac{\frac{\pi}{2}-0}{n} = \frac{\pi}{2n} \qquad nh = \frac{2}{\pi}$$

Thus, we have,

$$I = \int_0^{\frac{\pi}{2}} \sin x\,dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(0+h)+f(0+2h)+----f\{0+(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[\sin 0+\sin h+\sin 2h+----\sin(n-1)h\right]$$

$$= \lim_{h\to 0} h\left[\frac{\sin\left(\frac{nh}{2}-\frac{h}{2}\right)\times\sin\frac{nh}{2}}{\sin\frac{h}{2}}\right]$$

$$= \lim_{h\to 0} h\left[\frac{\sin\left(\frac{\pi}{4}-\frac{h}{2}\right)\times\sin\frac{\pi}{4}}{\sin\frac{h}{2}}\right]$$

$$\left[\therefore \lim_{h\to 0}\frac{\sin\theta}{\theta}=1\right] \qquad \therefore \lim_{h\to 0}\frac{h}{\sin\frac{h}{2}}\left[\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\right]$$

$$= 2\times\frac{1}{2} = 1$$

$$\therefore \int_0^{\frac{\pi}{2}} \sin x\,dx = 1$$

Definite Integrals Ex 20.5 Q19

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 0,\ b = \frac{\pi}{2}$ and $f(x) = \cos x$

$$\therefore \quad h = \frac{\frac{\pi}{2}-0}{n} = \frac{\pi}{2n} \qquad nh = \frac{2}{\pi}$$

Thus, we have,

$$I = \int_0^{\frac{\pi}{2}} \cos x\,dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(0+h)+f(0+2h)+----f\{0+(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[\cos 0+\cos h+\cos 2h+----\cos(n-1)h\right]$$

$$= \lim_{h\to 0} h\left[\frac{\cos\left(\frac{nh}{2}-\frac{h}{2}\right)\times\cos\frac{nh}{2}}{\cos\frac{h}{2}}\right]$$

$$= \lim_{h\to 0} h\left[\frac{\cos\left(\frac{\pi}{4}-\frac{h}{2}\right)\times\cos\frac{\pi}{4}}{\cos\frac{h}{2}}\right]$$

$$\left[\therefore \lim_{h\to 0}\frac{\cos\theta}{\theta} = 1\right] \qquad \therefore \lim_{h\to 0}\frac{h}{\cos\frac{h}{2}}\left[\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\right]$$

$$= 2\times\frac{1}{2} = 1$$

$$\therefore \int_0^{\frac{\pi}{2}} \cos x\,dx = 1$$

Definite Integrals Ex 20.5 Q20

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 1,\ b = 4$ and $f(x) = 3x^2+2x$

$$I = \lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(a+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[(3+2)+\{3(1+h)^2+2(1-+h)+\{3(1+2h)^2+2(1+2h)\}+----\right]$$

$$= \lim_{h\to 0} h\left[5+8h(1+2+3+--)+3h^2(1+2^2+3^2+---)\right]$$

$$\because h = \frac{3}{n} \ \&\ \text{if } h\to 0 \Rightarrow n\to\infty$$

$$= \lim_{n\to\infty}\frac{3}{n}\left[5n+\frac{24}{n}\frac{n(n-1)}{2}+\frac{27}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} 15+\frac{36}{n^2}n^2\left(1-\frac{1}{n}\right)+\frac{27}{2n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= 15+36+27 \qquad = 78$$

$$\therefore \int_1^4 \left(3x^2+2x\right)dx = 78$$

*********** END **********