Indefinite Integrals Ex 19.17 Q5

$$\text{Let } I = \int \frac{1}{\sqrt{(x-\alpha)(\beta-x)}}dx, (\text{as } \beta > \alpha)$$

$$= \int \frac{1}{\sqrt{-x^2 - x(\alpha+\beta) - \alpha\beta}}dx$$

$$= \int \frac{1}{\sqrt{-\left[x^2 - 2x\left(\frac{\alpha+\beta}{2}\right) + \left(\frac{\alpha+\beta}{2}\right)^2 - \left(\frac{\alpha+\beta}{2}\right)^2 + \alpha\beta\right]}}dx$$

$$= \int \frac{1}{\sqrt{-\left[\left(x - \frac{\alpha+\beta}{2}\right)^2 - \left(\frac{\alpha+\beta}{2}\right)^2\right]}}dx$$

$$= \int \frac{1}{\sqrt{\left[\left(\frac{\beta-\alpha}{2}\right)^2 - \left(x - \frac{\alpha+\beta}{2}\right)^2\right]}}dx, \qquad \left[\therefore \quad \beta > \alpha\right]$$

$$\text{Let } \left(x - \frac{\alpha+\beta}{2}\right) = t$$

$$\Rightarrow \quad dx = dt$$

$$I = \int \frac{1}{\sqrt{\left(\frac{\beta-\alpha}{2}\right)^2 - t^2}}dt$$

$$= \sin^{-1}\left(\frac{t}{\frac{\beta-\alpha}{2}}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \sin^{-1}\left(\frac{2\left(x - \frac{\alpha+\beta}{2}\right)}{\beta-\alpha}\right) + c$$

$$I = \sin^{-1}\left(\frac{2x-\alpha-\beta}{\beta-\alpha}\right) + c$$

Indefinite Integrals Ex 19.17 Q6

Let $I = \int \frac{1}{\sqrt{7-3x-2x^2}}dx$

$$= \int \frac{1}{\sqrt{-2\left[x^2+\frac{3}{2}x-\frac{7}{2}\right]}}dx$$

$$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{-\left[x^2+2x\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^2-\frac{7}{2}\right]}}dx$$

$$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{-\left[\left(x-\frac{3}{4}\right)^2-\frac{65}{16}\right]}}dx$$

$$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{\left(\frac{\sqrt{65}}{4}\right)^2-\left(x+\frac{3}{4}\right)^2}}dx$$

Let $\left(x+\frac{3}{4}\right) = t$

$\Rightarrow \quad dx = dt$

$$I = \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{\left(\frac{\sqrt{65}}{4}\right)^2-t^2}}dt$$

$$= \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{t}{\frac{\sqrt{41}}{4}}\right)+c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)+c\right]$$

$$I = \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{4\left(x+\frac{3}{4}\right)}{\sqrt{65}}\right)+c$$

$$I = \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{4x+3}{\sqrt{65}}\right)+c$$

Indefinite Integrals Ex 19.17 Q7

Let $I = \int \frac{1}{\sqrt{16-6x-x^2}}dx$

$$= \int \frac{1}{\sqrt{-\left[x^2+6x-16\right]}}dx$$

$$= \int \frac{1}{\sqrt{-\left[x^2+2x(3)+(3)^2-(3)^2-16\right]}}dx$$

$$= \int \frac{1}{\sqrt{-\left[(x+3)^2-25\right]}}dx$$

$$= \int \frac{1}{\sqrt{25-(x+3)^2}}dx$$

Let $(x+3) = t$

$\Rightarrow \quad dx = dt$

$$I = \int \frac{1}{\sqrt{5^2-t^2}}dt$$

$$= \sin^{-1}\left(\frac{t}{5}\right)+c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)+c\right]$$

$$I = \sin^{-1}\left(\frac{x+3}{5}\right)+c$$

Indefinite Integrals Ex 19.17 Q8

$7-6x-x^2$ can be written as $7-\left(x^2+6x+9-9\right)$.

Therefore,

$$7-\left(x^2+6x+9-9\right)$$
$$=16-\left(x^2+6x+9\right)$$
$$=16-\left(x+3\right)^2$$
$$=\left(4\right)^2-\left(x+3\right)^2$$

$$\therefore \int\frac{1}{\sqrt{7-6x-x^2}}dx=\int\frac{1}{\sqrt{\left(4\right)^2-\left(x+3\right)^2}}dx$$

Let $x+3=t$

$\Rightarrow dx=dt$

$$\Rightarrow \int\frac{1}{\sqrt{\left(4\right)^2-\left(x+3\right)^2}}dx=\int\frac{1}{\sqrt{\left(4\right)^2-\left(t\right)^2}}dt$$
$$=\sin^{-1}\left(\frac{t}{4}\right)+C$$
$$=\sin^{-1}\left(\frac{x+3}{4}\right)+C$$

Indefinite Integrals Ex 19.17 Q9

We have $\int\frac{dx}{\sqrt{5x^2-2x}}=\int\frac{dx}{\sqrt{5\left(x^2-\frac{2x}{5}\right)}}$

$$=\frac{1}{\sqrt{5}}\int\frac{dx}{\sqrt{\left(x-\frac{1}{5}\right)^2-\left(\frac{1}{5}\right)^2}}$$ (completing the square)

Put $x-\frac{1}{5}=t$. Then $dx=dt$.

Therefore, $\int\frac{dx}{\sqrt{5x^2-2x}}=\frac{1}{\sqrt{5}}\int\frac{dt}{\sqrt{t^2-\left(\frac{1}{5}\right)^2}}$

$$=\frac{1}{\sqrt{5}}\log\left|t+\sqrt{t^2-\left(\frac{1}{5}\right)^2}\right|+C$$ [by 7.4 (4)]

$$=\frac{1}{\sqrt{5}}\log\left|x-\frac{1}{5}+\sqrt{x^2-\frac{2x}{5}}\right|+C$$

*********** END **********