Indefinite Integrals Ex 19.30 Q37

Let $\frac{x}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$

$\Rightarrow \quad x = A(x^2+1) + (Bx+C)(x+1)$

Equating similar terms, we get,

$A+B = 0,\ B+C = 1,\ \ A+C = 0$

Solving, we get, $A = -\frac{1}{2},\ B = \frac{1}{2},\ C = \frac{1}{2}$

Thus,

$I = -\frac{1}{2}\int \frac{dx}{x+1} + \frac{1}{2}\int \frac{xdx}{x^2+1} + \frac{1}{2}\int \frac{dx}{x^2+1}$

$I = -\frac{1}{2}\log|x+1| + \frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1}x + c$

Indefinite Integrals Ex 19.30 Q38

Let $I = \int \frac{dx}{1+x+x^2+x^3}$

$\Rightarrow \quad I = \int \frac{dx}{(x^2+1)(x+1)}$

Now,

Let $\frac{1}{(x^2+1)(x+1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+1}$

$\Rightarrow \quad 1 = (Ax+B)(x+1) + C(x^2+1)$

Equating similar terms, we get,

$A + C = 0,\ A + B = 0,\ B + C = 1$

Solving, we get, $A = -\frac{1}{2},\ B = \frac{1}{2},\ C = \frac{1}{2}$

Thus,

$I = -\frac{1}{2}\int \frac{xdx}{x^2+1} + \frac{1}{2}\int \frac{dx}{x^2+1} + \frac{1}{2}\int \frac{dx}{x+1}$

$I = -\frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1}x + \frac{1}{2}\log|x+1| + c$

Indefinite Integrals Ex 19.30 Q39

Let $\frac{1}{(x+1)^2(x^2+1)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+1}$

$\Rightarrow \quad 1 = A(x+1)(x^2+1) + B(x^2+1) + (Cx+D)(x+1)^2$

$= (A+C)x^3 + (A+B+2C+D)x^2 + (A+C+2D)x + (A+B+D)$

Equating similar terms, we get,

$A + C = 0,\ A + B + 2C + D = 0,\ A + C + 2D = 0,\ A + B + D = 1$

Solving, we get, $A = \frac{1}{2},\ B = \frac{1}{2},\ C = -\frac{1}{2},\ D = 0$

Thus,

$I = \frac{1}{2}\int \frac{dx}{x+1} + \frac{1}{2}\int \frac{dx}{(x+1)^2} - \frac{1}{2}\int \frac{xdx}{x^2+1}$

$I = \frac{1}{2}\log|x+1| - \frac{1}{2(x+1)} - \frac{1}{4}\log|x^2+1| + c$

Indefinite Integrals Ex 19.30 Q40

Let $I = \int \frac{2x}{x^3 - 1} dx = \int \frac{2x}{(x-1)(x^2 + x + 1)} dx$

Now,

Let $\frac{2x}{(x-1)(x^2 + x + 1)} = \frac{A}{(x-1)} + \frac{Bx + C}{x^2 + x + 1}$

$\Rightarrow \quad 2x = A(x^2 + x + 1) + (Bx + C)(x - 1)$

$= (A + B)x^2 + (A - B + C)x + (A - C)$

Equating similar terms,

$A + B = 0,\ A - B + C = 2,\ A - C = 0,$

Solving, we get, $A = \frac{2}{3},\ B = -\frac{2}{3},\ C = \frac{2}{3}$

Thus,

$I = \frac{2}{3}\int \frac{dx}{x-1} - \frac{2}{3}\int \frac{(x-1)dx}{x^2 + x + 1}$

$= \frac{2}{3}\int \frac{dx}{x-1} - \frac{2}{3}\cdot\frac{1}{2}\int \frac{(2x-2)dx}{x^2 + x + 1}$

$\Rightarrow \quad I = \frac{2}{3}\int \frac{dx}{x-1} - \frac{1}{3}\int \frac{2x+1}{x^2 + x + 1} dx + \int \frac{dx}{x^2 + x + 1}$

$= \frac{2}{3}\int \frac{dx}{x-1} - \frac{1}{3}\int \frac{2x+1}{x^2 + x + 1} dx + \int \frac{dx}{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \frac{2}{3}\log|x-1| - \frac{1}{3}\log|x^2 + x + 1| + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x + \frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c$

Hence,

$I = \frac{2}{3}\log|x-1| - \frac{1}{3}\log|x^2 + x + 1| + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + c$

Indefinite Integrals Ex 19.30 Q41

Let $\frac{1}{(x^2+1)(x^2+4)} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{x^2+4}$

$\Rightarrow \quad 1 = (Ax+B)(x^2+4) + (Cx+D)(x^2+1)$

$= (A+C)x^3 + (B+D)x^2 + (4A+C)x + 4B + D$

Equating similar terms, we get,

$A + C = 0,\ B + D = 0,\ 4A + C = 0,\ 4B + D = 1$

Solving, we get, $A = 0,\ B = \frac{1}{3},\ C = 0,\ D = -\frac{1}{3}$

Thus,

$I = \int \frac{\frac{1}{3}dx}{(x^2+1)} - \int \frac{\frac{1}{3}dx}{(x^2+4)}$

$= \frac{1}{3}\tan^{-1}x - \frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right) + c \qquad \left[\because \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}\right]$

$\therefore \quad I = \frac{1}{3}\tan^{-1}x - \frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right) + c$

********** END **********