Tangents and Normals Ex 16.1 Q4

We have,

$$y = x^3 - 3x \qquad \text{---(i)}$$

$\therefore$ Slope of (i) is

$$\frac{dy}{dx} = 3x^2 - 3 \qquad \text{---(ii)}$$

Also,

The slope of the chord obtained by joining the points $(1, -2)$ and $(2, 2)$ is

$$\frac{2 - (-2)}{2 - 1} \qquad \left[\text{Slope } \frac{y_2 - y_1}{x_2 - x_1}\right]$$

$$= 4$$

According to the question slope of tangent to (i) and the chord are parallel

$\therefore$ $3x^2 - 3 = 4$

$\Rightarrow$ $3x^2 = 7$

$\Rightarrow$ $x = \pm\sqrt{\frac{7}{3}}$

From (i)

$$y = \pm\sqrt{\frac{7}{3}} \mp 3\sqrt{\frac{7}{3}}$$

$$= \mp\frac{2}{3}\sqrt{\frac{7}{3}}$$

Thus, the required point is

$$\pm\sqrt{\frac{7}{3}}, \ \mp\frac{2}{3}\sqrt{\frac{7}{3}}$$

Tangents and Normals Ex 16.1 Q5

The given equations are

$$y = x^3 - 2x^2 - 2x \qquad \text{---(i)}$$

$$y = 2x - 3 \qquad \text{---(ii)}$$

Slope to the tangents of (i) and (ii) are

$$\frac{dy}{dx} = 3x^2 - 4x - 2 \qquad \text{---(iii)}$$

and $$\frac{dy}{dx} = 2 \qquad \text{---(iv)}$$

According to the question slope to (i) and (ii) are parallel, so

$$3x^2 - 4x - 2 = 2$$

$$\Rightarrow \quad 3x^2 - 4x - 4 = 0$$

$$\Rightarrow \quad 3x^2 - 6x + 2x - 4 = 0$$

$$\Rightarrow \quad 3x(x-2) + 2(x-2) = 0$$

$$\Rightarrow \quad (3x+2)(x-2) = 0$$

$$\Rightarrow \quad x = \frac{-2}{3} \text{ or } 2$$

From (i)

$$y = \frac{4}{27} \text{ or } -4$$

Thus, the points are

$$\left(\frac{-2}{3}, \frac{4}{27}\right) \text{ and } (2, -4)$$

Tangents and Normals Ex 16.1 Q6

We have,

$$y^2 = 2x^3 \qquad \text{---(i)}$$

Differentiating (i) with respect to x, we get

$$2y\frac{dy}{dx} = 6x^2$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{3x^2}{y} \qquad \text{---(ii)}$$

According to the question

$$\frac{3x^2}{y} = 3$$

$$\Rightarrow \quad x^2 = y \qquad \text{---(iii)}$$

From (i) and (ii)

$$\left(x^2\right)^2 = 2x^3$$

$$\Rightarrow \quad x^4 - 2x^3 = 0$$

$$\Rightarrow \quad x^3\left(x - 2\right) = 0$$

$$\Rightarrow \quad x = 0 \text{ or } 2$$

If $x = 0$, then

$$\frac{dy}{dx} = \frac{3x^2}{y} \Rightarrow \frac{dy}{dx} = 0$$

Which is not possible.

$\therefore$ x = 2.

Putting x = 2 in the equation of the curve $y^2 = 2x^3$, we get y = 4.

Hence the required point is $(2, 4)$

Tangents and Normals Ex 16.1 Q7

We know that the slope to any curve is $\frac{dy}{dx} = \tan\theta$ where θ is the angle with possitive direction of x-axis.

Now,

The given curve is

$$xy + 4 = 0 \qquad \text{---(i)}$$

Differentiating with respect to x, we get

$$y + x\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y}{x} \qquad \text{---(ii)}$$

Also,

$$\frac{dy}{dx} = \tan 45° = 1 \qquad \text{---(iii)}$$

$\therefore$ From (ii) and (iii)

$$\frac{-y}{x} = 1$$

$$\Rightarrow \quad x = -y \qquad \text{---(iv)}$$

From (i) and (iv), we get

$$-y^2 + 4 = 0$$

$$\Rightarrow \quad y = \pm 2$$

$$\therefore \quad x = \mp 2$$

Thus, the points are

$(2, -2)$ and $(-2, 2)$

*********** END **********