Higher Order Derivatives Ex 12.1 Q38

$$y = \log\left(1+\cos x\right)$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{1+\cos x} \times -\sin x = \frac{-\sin x}{1+\cos x}$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\left[\frac{\left(1+\cos x\right)\cos x - \sin x\left(-\sin x\right)}{\left(1+\cos x\right)^2}\right]$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\left[\frac{\cos x + \cos^2 x + \sin^2 x}{\left(1+\cos x\right)^2}\right] = -\left[\frac{1+\cos x}{\left(1+\cos x\right)^2}\right] = \frac{-1}{1+\cos x}$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^3y}{dx^3} = -\left(\frac{+1}{\left(1+\cos x\right)^2} \times +\sin x\right) = -\left(\frac{-\sin x}{1+\cos x}\right) \times \left(\frac{-1}{1+\cos x}\right) = -\frac{dy}{dx}\cdot\frac{d^2y}{dx^2}$$

$$\Rightarrow \quad \frac{d^3y}{dx^3} + \frac{d^2y}{dx^2}\cdot\frac{dy}{dx} = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q39

$$y = \sin\left(\log x\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \cos\left(\log x\right) \times \frac{1}{x}$$

$$\Rightarrow \quad x\frac{dy}{dx} = \cos\left(\log x\right)$$

$$\Rightarrow \quad x\frac{d^2y}{dx^2} + \frac{dy}{dx} = -\sin\left(\log x\right) \times \frac{1}{x}$$

$$\Rightarrow \quad x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -y$$

$$\Rightarrow \quad x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q40

Given $y = 3e^{2x} + 2e^{3x}$

Then, $\frac{dy}{dx} = 6e^{2x} + 6e^{3x} = 6\left(e^{2x} + e^{3x}\right)$

$\therefore \quad \frac{d^2y}{dx^2} = 12e^{2x} + 18e^{3x} = 6\left(2e^{2x} + 3e^{3x}\right)$

Hence,

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 6\left(2e^{2x} + 3e^{3x}\right) - 30\left(e^{2x} + e^{3x}\right) + 6\left(3e^{2x} + 2e^{3x}\right)$$

$$= 0$$

Higher Order Derivatives Ex 12.1 Q41

$$y = \left(\cot^{-1} x\right)^2$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = y_1 = 2\cot^{-1} x \frac{-1}{1+x^2}$$

$$= \frac{-2\cot^{-1} x}{1+x^2} \text{(chain rule)}$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{dy}{dx} = -2\cot^{-1} x$$

differentiating w.r.t. x

$$\Rightarrow \quad \left(1+x^2\right)y_2 + 2xy_1 = +2\left(\frac{+1}{1+x^2}\right)$$

(multiplication rule on LHS)

$$\Rightarrow \quad \left(1+x^2\right)^2 y_2 + 2x\left(1+x^2\right)y_1 = 2$$

Hence proved!

*********** END **********