Exercise 9.2 : Solutions of Questions on Page Number : **385**

Q1 : $y = e^x + 1 \qquad : \; y'' - y' = 0$

Answer :

$y = e^x + 1$

Differentiating both sides of this equation with respect to *x*, we get:

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^x + 1\right)$$
$$\Rightarrow y' = e^x \qquad \ldots(1)$$

Now, differentiating equation (1) with respect to *x*, we get:

$$\frac{d}{dx}\left(y'\right) = \frac{d}{dx}\left(e^x\right)$$
$$\Rightarrow y'' = e^x$$

Substituting the values of y' and y'' in the given differential equation, we get the L.H.S. as:

$y'' - y' = e^x - e^x = 0 = \text{R.H.S.}$

Thus, the given function is the solution of the corresponding differential equation.

Answer needs Correction? **Click Here**

Q2 : $y = x^2 + 2x + C \qquad : \; y' - 2x - 2 = 0$

Answer :

$y = x^2 + 2x + C$

Differentiating both sides of this equation with respect to *x*, we get:

$$y' = \frac{d}{dx}\left(x^2 + 2x + C\right)$$
$$\Rightarrow y' = 2x + 2$$

Substituting the value of y' in the given differential equation, we get:

L.H.S. = $y' - 2x - 2 = 2x + 2 - 2x - 2 = 0$ = R.H.S.

Hence, the given function is the solution of the corresponding differential equation.

Answer needs Correction? **Click Here**

Q3 : $y = \cos x + C \qquad : \; y' + \sin x = 0$

Answer :

$y = \cos x + C$

Differentiating both sides of this equation with respect to *x*, we get:

$$y' = \frac{d}{dx}\left(\cos x + C\right)$$
$$\Rightarrow y' = -\sin x$$

Substituting the value of y' in the given differential equation, we get:

L.H.S. = $y' + \sin x = -\sin x + \sin x = 0$ = R.H.S.

Hence, the given function is the solution of the corresponding differential equation.

Answer needs Correction? **Click Here**

Q4 : $y = \sqrt{1 + x^2} \qquad : \; y' = \frac{xy}{1 + x^2}$

Answer :

$y = \sqrt{1 + x^2}$

Differentiating both sides of the equation with respect to *x*, we get:

$$y' = \frac{d}{dx}\left(\sqrt{1 + x^2}\right)$$
$$y' = \frac{1}{2\sqrt{1 + x^2}} \cdot \frac{d}{dx}\left(1 + x^2\right)$$
$$y' = \frac{2x}{2\sqrt{1 + x^2}}$$
$$y' = \frac{x}{\sqrt{1 + x^2}}$$

$\sqrt{1+x^2}$

$$\Rightarrow y' = \frac{x}{1+x^2} \times \sqrt{1+x^2}$$

$$\Rightarrow y' = \frac{x}{1+x^2} \cdot y$$

$$\Rightarrow y' = \frac{xy}{1+x^2}$$

$\therefore$ L.H.S. = R.H.S.

Hence, the given function is the solution of the corresponding differential equation.

Answer needs Correction? **Click Here**

Q5 : $y = \text{A}x \qquad : \; xy' = y \left(x \neq 0\right)$

Answer :

$y = \text{A}x$

Differentiating both sides with respect to *x*, we get:

$$y' = \frac{d}{dx}(\text{A}x)$$

$$\Rightarrow y' = \text{A}$$

Substituting the value of y' in the given differential equation, we get:

$\text{L.H.S.} = xy' = x \cdot \text{A} = \text{A}x = y = \text{R.H.S.}$

Hence, the given function is the solution of the corresponding differential equation.

Answer needs Correction? **Click Here**

Q6 : $y = x \sin x \qquad : \; xy' = y + x\sqrt{x^2 - y^2} \left(x \neq 0 \text{ and } x > y \text{ or } x < -y\right)$

Answer :

$y = x \sin x$

Differentiating both sides of this equation with respect to *x*, we get:

$$y' = \frac{d}{dx}(x \sin x)$$

$$\Rightarrow y' = \sin x \cdot \frac{d}{dx}(x) + x \cdot \frac{d}{dx}(\sin x)$$

$$\Rightarrow y' = \sin x + x \cos x$$

Substituting the value of y' in the given differential equation, we get:

$$\begin{aligned} \text{L.H.S.} = xy' &= x(\sin x + x \cos x) \\ &= x \sin x + x^2 \cos x \\ &= y + x^2 \cdot \sqrt{1 - \sin^2 x} \\ &= y + x^2 \sqrt{1 - \left(\frac{y}{x}\right)^2} \\ &= y + x\sqrt{y^2 - x^2} \\ &= \text{R.H.S.} \end{aligned}$$

Hence, the given function is the solution of the corresponding differential equation.

Answer needs Correction? **Click Here**

Q7 : $xy = \log y + \text{C} \qquad : \; y' = \frac{y^2}{1 - xy} \left(xy \neq 1\right)$

Answer :

$xy = \log y + \text{C}$

Differentiating both sides of this equation with respect to *x*, we get:

$$\frac{d}{dx}(xy) = \frac{d}{dx}(\log y)$$

$$\Rightarrow y \cdot \frac{d}{dx}(x) + x \cdot \frac{dy}{dx} = \frac{1}{y}\frac{dy}{dx}$$

$$\Rightarrow y + xy' = \frac{1}{y} y'$$

$$\Rightarrow y^2 + xy\, y' = y'$$

$$\Rightarrow (xy - 1) y' = -y^2$$

$$\Rightarrow y' = \frac{y^2}{1 - xy}$$

$\therefore$ L.H.S. = R.H.S.

Hence, the given function is the solution of the corresponding differential equation.

Answer needs Correction? **Click Here**

Q8 : $y - \cos y = x \qquad : \; (y \sin y + \cos y + x) y' = y$

Answer :

$y - \cos y = x \qquad \ldots(1)$

Differentiating both sides of the equation with respect to *x*, we get:

$$\frac{dy}{dx}-\frac{d}{dx}(\cos y)=\frac{d}{dx}(x)$$
$$\Rightarrow y'+\sin y\cdot y'=1$$
$$\Rightarrow y'(1+\sin y)=1$$
$$\Rightarrow y'=\frac{1}{1+\sin y}$$

Substituting the value of y' in equation (1), we get:

$$\begin{aligned}\text{L.H.S.}&=(y\sin y+\cos y+x)y'\\&=(y\sin y+\cos y+y-\cos y)\times\frac{1}{1+\sin y}\\&=y(1+\sin y)\cdot\frac{1}{1+\sin y}\\&=y\\&=\text{R.H.S.}\end{aligned}$$

Hence, the given function is the solution of the corresponding differential equation.

Answer needs Correction? Click Here

Q9 : $x+y=\tan^{-1}y \quad : \quad y^2y'+y^2+1=0$

Answer :

$x+y=\tan^{-1}y$

Differentiating both sides of this equation with respect to x, we get:

$$\frac{d}{dx}(x+y)=\frac{d}{dx}(\tan^{-1}y)$$
$$\Rightarrow 1+y'=\left[\frac{1}{1+y^2}\right]y'$$
$$\Rightarrow y'\left[\frac{1}{1+y^2}-1\right]=1$$
$$\Rightarrow y'\left[\frac{1-(1+y^2)}{1+y^2}\right]=1$$
$$\Rightarrow y'\left[\frac{-y^2}{1+y^2}\right]=1$$
$$\Rightarrow y'=\frac{-(1+y^2)}{y^2}$$

Substituting the value of y' in the given differential equation, we get:

$$\begin{aligned}\text{L.H.S.}=y^2y'+y^2+1=y^2\left[\frac{-(1+y^2)}{y^2}\right]+y^2+1\\&=-1-y^2+y^2+1\\&=0\\&=\text{R.H.S.}\end{aligned}$$

Hence, the given function is the solution of the corresponding differential equation.

Answer needs Correction? Click Here

Q10 : $y=\sqrt{a^2-x^2}x\in(-a,a) \quad : \quad x+y\frac{dy}{dx}=0(y\neq 0)$

Answer :

$y=\sqrt{a^2-x^2}$

Differentiating both sides of this equation with respect to x, we get:

$$\begin{aligned}\frac{dy}{dx}&=\frac{d}{dx}\left(\sqrt{a^2-x^2}\right)\\\Rightarrow\frac{dy}{dx}&=\frac{1}{2\sqrt{a^2-x^2}}\cdot\frac{d}{dx}(a^2-x^2)\\&=\frac{1}{2\sqrt{a^2-x^2}}(-2x)\\&=\frac{-x}{\sqrt{a^2-x^2}}\end{aligned}$$

Substituting the value of $\frac{dy}{dx}$ in the given differential equation, we get:

$$\begin{aligned}\text{L.H.S.}=x+y\frac{dy}{dx}=x+\sqrt{a^2-x^2}\times\frac{-x}{\sqrt{a^2-x^2}}\\&=x-x\\&=0\\&=\text{R.H.S.}\end{aligned}$$

Hence, the given function is the solution of the corresponding differential equation.

Answer needs Correction? Click Here

Q11 : The numbers of arbitrary constants in the general solution of a differential equation of fourth order are:

(A) 0 (B) 2 (C) 3 (D) 4

Answer :

We know that the number of constants in the general solution of a differential equation of order n is equal to its order.

Therefore, the number of constants in the general equation of fourth order differential equation is four.

Hence, the correct answer is D.

Q12 : The numbers of arbitrary constants in the particular solution of a differential equation of third order are:

(A) 3 (B) 2 (C) 1 (D) 0

Answer :

In a particular solution of a differential equation, there are no arbitrary constants.

Hence, the correct answer is D.

*********** END **********