



Continuity Ex 9.1 Q27

We have given that the function is continuous at $x = 0$

$$\therefore \text{LHL} = \text{RHL} = f(0) \dots (1)$$

$$f(0) = \frac{1}{2}$$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{1 - \cos k(-h)}{-h \sin(-h)} = \lim_{h \rightarrow 0} \frac{1 - \cos kh}{+h \sin h}$$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{2 \sin^2 \frac{kh}{2}}{h \cdot 2 \sin \frac{h}{2} \cdot \cos \frac{h}{2}} \\ &= \lim_{h \rightarrow 0} \left(\frac{\sin \frac{kh}{2}}{\frac{kh}{2}} \right)^2 \times \frac{\frac{k^2 h^2}{4}}{\frac{\sin \frac{h}{2}}{\frac{h}{2}} \times \frac{h}{2}} \cdot \frac{1}{h} \\ &= \lim_{h \rightarrow 0} \left(\frac{\sin \frac{kh}{2}}{\frac{kh}{2}} \right)^2 \cdot \frac{\frac{k^2}{4}}{\frac{\sin \frac{h}{2}}{\frac{h}{2}} \cdot \frac{1}{2}} \\ &= \frac{k^2}{2} \end{aligned}$$

\therefore Using (1) we get,

$$\frac{k^2}{2} = \frac{1}{2} \Rightarrow k = \pm 1$$

Continuity Ex 9.1 Q28

We have given that the function is continuous at $x = 4$

$$\therefore \text{LHL} = \text{RHL} = f(4) \dots (1)$$

$$f(4) = a + b \dots (A)$$

$$\text{LHL} = \lim_{x \rightarrow 4^-} f(x) = \lim_{h \rightarrow 0} f(4-h) = \lim_{h \rightarrow 0} \frac{(4-h)-4}{[(4-h)-4]} + a = \lim_{h \rightarrow 0} \frac{-h}{h} + a = a - 1 \dots (B)$$

$$\text{RHL} = \lim_{x \rightarrow 4^+} f(x) = \lim_{h \rightarrow 0} f(4+h) = \lim_{h \rightarrow 0} \frac{(4+h)-4}{[(4+h)-4]} + b = \lim_{h \rightarrow 0} \frac{h}{h} + b = b + 1 \dots (C)$$

\therefore from (1)

$$a - 1 = b + 1 \Rightarrow a - b = 2 \dots (D)$$

from (A) and (B)

$$a + b = a - 1 \Rightarrow b = -1$$

from (A) and (C)

$$a + b = b + 1 \Rightarrow a = 1$$

Thus, $a = 1$ and $b = -1$

Continuity Ex 9.1 Q29

We have given that the function is continuous at $x = 0$

$$\therefore \text{LHL} = \text{RHL} = f(0) \dots (1)$$

$$f(0) = k$$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} \frac{\sin 2(0 - h)}{-h} = \lim_{h \rightarrow 0} \frac{-\sin 2h}{-h} = 2$$

\therefore using (1), we get $k = 2$

Continuity Ex 9.1 Q30

We know that a function is continuous at $x = 0$ if,

$$\text{LHL} = \text{RHL} = f(0) \dots (1)$$

$$\begin{aligned} \text{LHL} = \lim_{x \rightarrow 0^-} f(x) &= \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} \frac{\log\left(1 - \frac{h}{a}\right) - \log\left(1 + \frac{h}{b}\right)}{(-h)} = \lim_{h \rightarrow 0} \frac{\log\left(1 + \left(\frac{-h}{a}\right)\right)}{\left(\frac{-h}{a}\right) \times a} + \frac{\log\left(1 + \frac{h}{b}\right)}{h} \\ &= \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} \end{aligned}$$

from (1),

$$f(0) = \frac{a+b}{ab}$$

***** END *****