Definite Integrals Ex 20.4B Q41

We have,

$$I = \int_0^{2a} f(x)dx = \int_0^{a} f(x)dx + \int_a^{2a} f(x)dx$$

$$I = \int_0^{a} f(x)dx + I_1$$

Let $2a - t = x$ then $dx = -dt$

$t = a, \ x = a$

$t = 2a \quad x = 0$

$$I_1 = \int_0^{2a} f(x) = \int_a^{0} f(2a - t)(-dt)$$

$$= -\int_a^{0} f(2a - t)dt$$

$$I_1 = \int_0^{a} f(2a - t)dt = \int_0^{a} f(2a - x)dx$$

$$I = \int_0^{a} f(x)dx + \int_0^{a} f(2a - x)dx$$

$$I = \int_0^{a} f(x)dx - \int_0^{a} f(x) \qquad [\because f(2a - x) = -f(x)]$$

$$I = 0$$

Hence,

$$\int_0^{2a} f(x)dx = 0$$

Definite Integrals Ex 20.4B Q42

(i) We have,

$$I = \int_{-a}^{a} f(x^2)dx$$

Clearly $f(x^2)$ is an even function.

So,

$$\int_{-a}^{a} f(t) = 2\int_0^{a} f(t)$$

$$I = 2\int_0^{a} f(x^2)dx$$

(ii) We have,

$$I = \int_{-a}^{a} xf(x^2)dx$$

Clearly, $xf(x^2)$ is odd function.

So, $I = 0$

$$\therefore \int_{-a}^{a} xf(x^2)dx = 0$$

Definite Integrals Ex 20.4B Q43

We have from LHS,

$$I = \int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_a^{2a} f(x)\,dx \qquad \ldots(i)$$

Let $x = 2a - t$, then $dx = -dt$

$x = a \Rightarrow t = a$, and $x = 2a \Rightarrow t = 0$

$$\therefore \int_0^{2a} f(x)\,dx = -\int_a^{0} f(2a-t)\,dt$$

$$\Rightarrow \int_0^{2a} f(x)\,dx = \int_0^{a} f(2a-t)\,dt$$

$$\Rightarrow \int_0^{2a} f(x)\,dx = \int_0^{a} f(2a-x)\,dx$$

Substituting $\int_0^{2a} f(x)\,dx = \int_0^{a} f(2a-x)\,dx$ in (i)

we get,

$$\int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a-x)\,dx$$

$$\Rightarrow \int_0^{2a} f(x)\,dx = \int_0^{a} \{f(x) + f(2a-x)\}\,dx$$

*********** END **********