Trigonometric Identities Ex 6.1 Q60

Answer :

We have to prove $(1+\cot A-\operatorname{cosec} A)(1+\tan A+\sec A)=2$

We know that, $\sin^2 A+\cos^2 A=1$.

So,

$$
\begin{aligned}
(1+\cot A-\operatorname{cosec} A)(1+\tan A+\sec A) &= \left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)\\
&= \left(\frac{\sin A+\cos A-1}{\sin A}\right)\left(\frac{\cos A+\sin A+1}{\cos A}\right)\\
&= \frac{(\sin A+\cos A-1)(\sin A+\cos A+1)}{\sin A\cos A}\\
&= \frac{\{(\sin A+\cos A)-1\}\{(\sin A+\cos A)+1\}}{\sin A\cos A}\\
&= \frac{(\sin A+\cos A)^2-1}{\sin A\cos A}\\
&= \frac{\sin^2 A+2\sin A\cos A+\cos^2 A-1}{\sin A\cos A}\\
&= \frac{(\sin^2 A+\cos^2 A)+2\sin A\cos A-1}{\sin A\cos A}\\
&= \frac{1+2\sin A\cos A-1}{\sin A\cos A}\\
&= \frac{2\sin A\cos A}{\sin A\cos A}\\
&= 2
\end{aligned}
$$

Hence proved.

Trigonometric Identities Ex 6.1 Q61

Answer :

We have to prove

$(\operatorname{cosec}\theta-\sec\theta)(\cot\theta-\tan\theta)=(\operatorname{cosec}\theta+\sec\theta)(\sec\theta\operatorname{cosec}\theta-2)$

We know that, $\sin^2\theta+\cos^2\theta=1$

Consider the LHS.

$$
\begin{aligned}
(\operatorname{cosec}\theta-\sec\theta)(\cot\theta-\tan\theta) &= \left(\frac{1}{\sin\theta}-\frac{1}{\cos\theta}\right)\left(\frac{\cos\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta}\right)\\
&= \left(\frac{\cos\theta-\sin\theta}{\sin\theta\cos\theta}\right)\left(\frac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta}\right)\\
&= \frac{(\cos\theta-\sin\theta)}{\sin\theta\cos\theta}\frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{\sin\theta\cos\theta}\\
&= \frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)^2}{\sin^2\theta\cos^2\theta}
\end{aligned}
$$

Now, consider the RHS.

$$\begin{aligned}(\operatorname{cosec}\theta+\sec\theta)(\sec\theta\operatorname{cosec}\theta-2) &= \left(\frac{1}{\sin\theta}+\frac{1}{\cos\theta}\right)\left(\frac{1}{\cos\theta}\frac{1}{\sin\theta}-2\right)\\ &= \left(\frac{\cos\theta+\sin\theta}{\sin\theta\cos\theta}\right)\left(\frac{1-2\sin\theta\cos\theta}{\sin\theta\cos\theta}\right)\\ &= \frac{(\cos\theta+\sin\theta)}{\sin\theta\cos\theta}\frac{(\cos^2\theta+\sin^2\theta-2\cos\theta\sin\theta)}{\sin\theta\cos\theta}\\ &= \frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)^2}{\sin^2\theta\cos^2\theta}\end{aligned}$$

∴ LHS = RHS

Hence proved.

*********** END **********