Definite Integrals Ex 20.2 Q28

We have,

$$\int_0^{\frac{\pi}{2}} \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx$$

Dividing numerator and denominator by $\cos^2 x$

$$\int_0^{\frac{\pi}{2}} \left(\frac{\frac{1}{\cos^2 x}}{a^2 \frac{\sin^2 x}{\cos^2 x} + b^2 \frac{\cos^2 x}{\cos^2 x}} \right) dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{\sec^2 x}{a^2 \tan^2 x + b^2} \right) dx$$

$$= \frac{1}{a^2} \int_0^{\frac{\pi}{2}} \left(\frac{\sec^2 x}{\tan^2 x + \left(\frac{b}{a}\right)^2} \right) dx$$

Let $\tan x = t$

Differentiating w.r.t. x, we get

$$\sec^2 x \, dx = dt$$

When $x = 0 \Rightarrow t = 0$

$$x = \frac{\pi}{2} \Rightarrow t = \infty$$

$$\therefore \frac{1}{a^2} \int_0^{\frac{\pi}{2}} \left(\frac{\sec^2 x}{\tan^2 x + \left(\frac{b}{a}\right)^2} \right) dx$$

$$= \frac{1}{a^2} \int_0^{\infty} \frac{dt}{\left(\frac{b}{a}\right)^2 + t^2}$$

$$= \frac{1}{a^2} \left[\frac{a}{b} \tan^{-1} \frac{at}{b} \right]_0^{\infty}$$

$$= \frac{1}{a^2} \frac{a}{b} \left[\tan^{-1} \infty - \tan^{-1} 0 \right]$$

$$= \frac{1}{ab} \left[\tan^{-1} \tan \frac{\pi}{2} \right] \quad = \frac{\pi}{2ab}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx = \frac{\pi}{2ab}$$

Definite Integrals Ex 20.2 Q29

$$I=\int_0^{\frac{\pi}{2}}\frac{x+\sin x}{1+\cos x}dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{x+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}dx$$

$$=\int_0^{\frac{\pi}{2}}\left(\frac{x\sec^2\frac{x}{2}}{2}+\tan\frac{x}{2}\right)dx$$

$$=\left[x\tan\left(\frac{x}{2}\right)-\int_0^{\frac{\pi}{2}}\tan\frac{x}{2}\,dx+\int_0^{\frac{\pi}{2}}\tan\frac{x}{2}\,dx\right]_0^{\frac{\pi}{2}}$$

$$=\frac{\pi}{2}$$

$$\therefore I=\int_0^{\frac{\pi}{2}}\frac{x+\sin x}{1+\cos x}dx=\frac{\pi}{2}$$

Definite Integrals Ex 20.2 Q30

$$I=\int_0^1\frac{\tan^{-1}x}{1+x^2}dx$$

Let $t=\tan^{-1}x$

$$dt=\frac{1}{1+x^2}dx$$

$$x=0,t=0$$

$$x=1,t=\frac{\pi}{4}$$

$$I=\int_0^{\frac{\pi}{4}}t\,dt$$

$$=\left[\frac{t^2}{2}\right]_0^{\frac{\pi}{4}}$$

$$=\frac{1}{2}\frac{\pi^2}{16}$$

$$=\frac{\pi^2}{32}$$

Definite Integrals Ex 20.2 Q31

$$I = \int_0^{\pi/4} \frac{\sin x + \cos x}{3 + \sin 2x}\, dx$$

$$I = \int_0^{\pi/4} \left(\frac{\sin x + \cos x}{3 + 1 - (\cos x - \sin x)^2} \right) dx$$

$$I = \int_0^{\pi/4} \left(\frac{\sin x + \cos x}{4 - (\cos x - \sin x)^2} \right) dx$$

$$I = \frac{1}{4}\left[\log\left| \frac{2 + \sin x - \cos x}{2 - \sin x + \cos x} \right| \right]_0^{\pi/4}$$

$$I = -\frac{1}{4}\log\left(\frac{1}{3}\right)$$

$$I = \frac{1}{4}\log_e 3$$

*********** END **********