Q1. Show that in a right angles triangle, the hypotenuse is the longest side.

Ans. Given: Let ABC be a right angled triangle, right angled at B.

To prove: Hypotenuse AC is the longest side.

Proof: In right angled triangle ABC,

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \angle A + 90^\circ + \angle C = 180^\circ$ $[\because \angle B = 90^\circ]$

$\Rightarrow \angle A + \angle C = 180^\circ - 90^\circ$

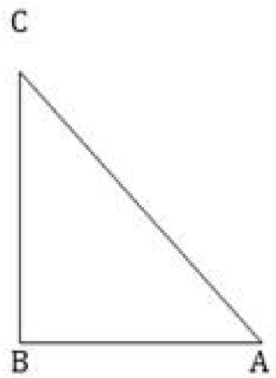
And $\angle B = 90^\circ$

$\Rightarrow \angle B > \angle C$ and $\angle B > \angle A$

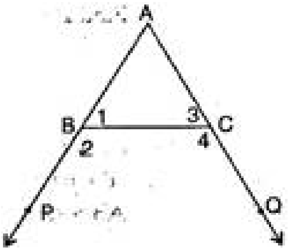
Since the greater angle has a longer side opposite to it.

$\Rightarrow AC > AB$ and $AC > AB$

Therefore $\angle B$ being the greatest angle has the longest opposite side AC, i.e. hypotenuse.



Q2. In figure, sides AB and AC of ΔABC are extended to points P and Q respectively. Also $\angle$ PBC < $\angle$ QCB. Show that AC > AB.



Ans. Given: In ΔABC, $\angle$PBC < $\angle$ QCB

To prove: AC > AB

Proof: In ΔABC,

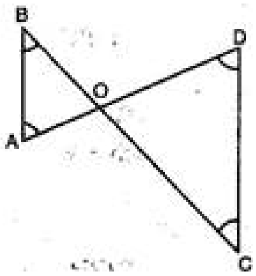
$\angle 4 > \angle 2$ [Given]

Now $\angle 1 + \angle 2 = \angle 3 + \angle 4 = 180^\circ$ [Linear pair]

$\therefore \angle 1 > \angle 3$ [$\because$ $\angle 4 > \angle 2$]

$\Rightarrow$ AC > AB [Side opposite to greater angle is longer]

Q3. In figure, $\angle$B < $\angle$A and $\angle$C < $\angle$D. Show that AD < BC.



Ans. In ΔAOB,

$\angle$B < $\angle$A [Given]

$\Rightarrow$ OA < OB ....(i) [Side opposite to greater angle is longer]

In ΔCOD,

$\angle$C < $\angle$D [Given]

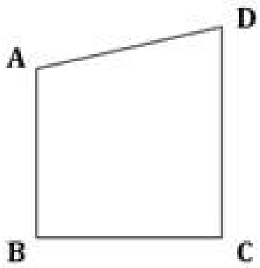
$\Rightarrow$ OD < OC ....(ii) [Side opposite to greater angle is longer]

Adding eq. (i) and (ii),

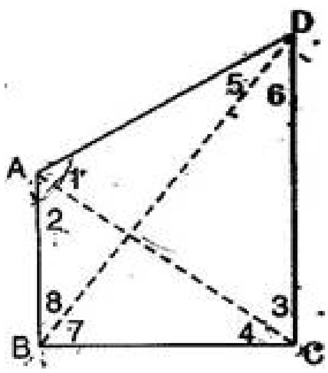
OA + OD < OB + OC

$\Rightarrow$ AD < BC

Q4. AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (See figure). Show that $\angle$A > $\angle$C and $\angle$B > $\angle$D.



Ans. Given: ABCD is a quadrilateral with AB as smallest and CD as longest side.



To prove: (i) $\angle$A > $\angle$C **(ii)** $\angle$B > $\angle$D

Construction: Join AC and BD.

Proof: **(i)** In Δ ABC, AB is the smallest side.

$\therefore \angle 4 < \angle 2$ ....(i)

[Angle opposite to smaller side is smaller]

In Δ ADC, DC is the longest side.

$\therefore \angle 3 < \angle 1$ ....(ii)

[Angle opposite to longer side is longer]

Adding eq. (i) and (ii),

$\angle 4 + \angle 3 < \angle 1 + \angle 2 \Rightarrow \angle C < \angle A$

(ii) In Δ ABD, AB is the smallest side.

$\therefore \angle 5 < \angle 8$ ....(iii)

[Angle opposite to smaller side is smaller]

In Δ BDC, DC is the longest side.

$\therefore \angle 6 < \angle 7$ ....(iv)

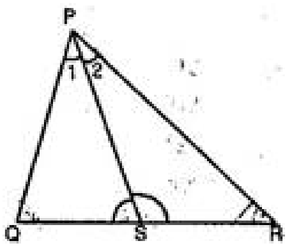
[Angle opposite to longer side is longer]

Adding eq. (iii) and (iv),

$\angle 5 + \angle 6 < \angle 7 + \angle 8$

$\Rightarrow \angle D < \angle B$

Q5. In figure, PR > PQ and PS bisects $\angle$ QPR. Prove that $\angle$ PSR > $\angle$ PSQ.



Ans. In Δ PQR, PR > PQ [Given]

$\therefore \angle$ PQR > $\angle$ PRQ .....(i) [Angle opposite to longer side is greater]

Again $\angle 1 = \angle 2$ .....(ii) [$\because$ PS is the bisector of $\angle$ P]

$\therefore \angle$ PQR + $\angle 1$ > $\angle$ PRQ + $\angle 2$ ....(iii)

But $\angle$ PQS + $\angle 1$ + $\angle$ PSQ = $\angle$ PRS + $\angle 2$ + $\angle$ PSR = 180° [Angle sum property]

$\Rightarrow \angle$ PQR + $\angle 1$ + $\angle$ PSQ = $\angle$ PRQ + $\angle 2$ + $\angle$ PSR ....(iv)

[$\angle$ PRS = $\angle$ PRQ and $\angle$ PQS = $\angle$ PQR]

From eq. (iii) and (iv),

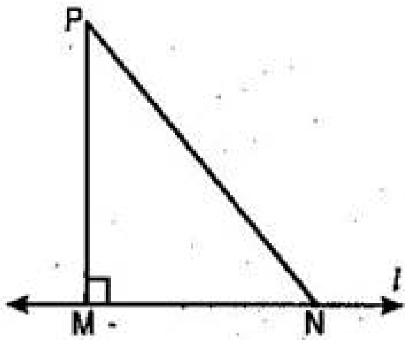
$\angle$ PSQ < $\angle$ PSR

Or $\angle$ PSR > $\angle$ PSQ

Q6. Show that all the line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

Ans. Given: l is a line and P is point not lying on l. PM $\perp l$

N is any point on l other than M.



To prove: PM < PN

Proof: In ΔPMN $\angle$ M is the right angle.

$\therefore$ N is an acute angle. (Angle sum property of Δ)

$\therefore \angle M > \angle N$

$\therefore$ PN > PM [Side opposite greater angle]

$\Rightarrow$ PM < PN

Hence of all line segments drawn from a given point not on it, the perpendicular is the shortest.

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