Indefinite Integrals Ex 19.3 Q11

$$\text{Let } I = \int \frac{1-\cos x}{1+\cos x} \times dx. \text{ Then,}$$

$$I = \int \frac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}} \times dx$$

$$= \int \frac{\sin^2 \frac{x}{2}}{\cos^2 \frac{x}{2}} \times dx$$

$$= \int \tan^2 \frac{x}{2} dx$$

$$= \int \left(\sec^2 \frac{x}{2} - 1 \right) dx$$

$$= \frac{\tan \frac{x}{2}}{\frac{1}{2}} - x + c$$

$$= 2\tan \frac{x}{2} - x + c$$

Indefinite Integrals Ex 19.3 Q12

Let $I = \int \frac{1}{1-\sin\frac{x}{2}}$. Then,

$$I = \int \frac{1}{1-\sin\frac{x}{2}} \times \frac{1+\sin\frac{x}{2}}{1+\sin\frac{x}{2}} dx$$

$$= \int \frac{1+\sin\frac{x}{2}}{1-\sin^2\frac{x}{2}} \times dx$$

$$= \int \frac{1+\sin\frac{x}{2}}{\cos^2\frac{x}{2}} \times dx$$

$$= \int \frac{1}{\cos^2\frac{x}{2}} dx + \int \frac{\sin\frac{x}{2}}{\cos^2\frac{x}{2}} dx$$

$$= \int \sec^2\frac{x}{2} dx + \int \sec\frac{x}{2}\tan\frac{x}{2} dx$$

$$= \frac{\tan\frac{x}{2}}{\frac{1}{2}2} + + \frac{\sec\frac{x}{2}}{\frac{1}{2}} + c$$

$$= 2\tan\frac{x}{2} + 2\sec\frac{x}{2} + c$$

$$\therefore \quad I = 2\left(\tan\frac{x}{2} + \sec\frac{x}{2}\right) + c$$

Indefinite Integrals Ex 19.3 Q13

Let $I = \int \frac{1}{1+\cos 3x} \times dx$. Then,

$$I = \int \frac{1}{1+\cos 3x} \times \frac{1-\cos 3x}{1-\cos 3x} \times dx$$

$$= \int \frac{1-\cos 3x}{1-\cos^2 3x} \times dx$$

$$= \int \frac{1-\cos 3x}{\sin^2 3x} \times dx$$

$$= \int \left(\frac{1}{\sin^2 3x} - \frac{\cos 3x}{\sin^2 3x}\right) dx$$

$$= \int \left(\operatorname{cosec}^2 3x - \operatorname{cosec} 3x \cot 3x\right) dx$$

$$= \frac{-\cot 3x}{3} + \frac{\operatorname{cosec} 3x}{3} + c$$

$$= \frac{-1}{3} \times \frac{\cos 3x}{\sin 3x} + \frac{1}{3} \times \frac{1}{\sin 3x} + c$$

$$= \frac{1-\cos 3x}{3\sin 3x} + c$$

$$\therefore \quad I = \frac{1-\cos 3x}{3\sin 3x} + c.$$

Indefinite Integrals Ex 19.3 Q14

Consider $I=\int (e^x+1)^2 e^x dx$

let $(e^x+1)=t \Rightarrow e^x dx = dt$

$$I=\int (e^x+1)^2 e^x dx$$
$$=\int (t)^2 dt$$
$$=\frac{t^3}{3}+C$$
$$=\frac{(e^x+1)^3}{3}+C$$

Indefinite Integrals Ex 19.3 Q15

Let $I=\int \left(e^x+\frac{1}{e^x}\right)^2 dx$. Then,

$$I=\int \left(e^x+\frac{1}{e^x}\right)^2 \times dx$$
$$=\int \left(e^{2x}+\frac{1}{e^{2x}}+2\right) dx$$
$$=\frac{e^{2x}}{2}-\frac{1}{2}e^{-2x}+2x+c$$

$\therefore \quad I=\frac{1}{2}\times e^{2x}+2x-\frac{1}{2}\times e^{-2x}+c$

********** END **********