



Trigonometric Identities Ex 6.1 Q68

Answer :

We have prove that

$$(1 + \cot A + \tan A)(\sin A - \cos A) = \frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A} = \sin A \tan A - \cot A \cos A$$

We know that, $\sin^2 A + \cos^2 A = 1$

So,

$$\begin{aligned} & (1 + \cot A + \tan A)(\sin A - \cos A) \\ &= \left(1 + \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A} \right) (\sin A - \cos A) \\ &= \left(\frac{\sin A \cos A + \cos^2 A + \sin^2 A}{\sin A \cos A} \right) (\sin A - \cos A) \\ &= \left(\frac{\sin A \cos A + 1}{\sin A \cos A} \right) (\sin A - \cos A) \\ &= \frac{(\sin A - \cos A)(\sin A \cos A + 1)}{\sin A \cos A} \\ &= \frac{\sin^2 A \cos A + \sin A - \cos^2 A \sin A - \cos A}{\sin A \cos A} \\ &= \frac{(\sin^2 A \cos A - \cos A) + (\sin A - \cos^2 A \sin A)}{\sin A \cos A} \end{aligned}$$

$$\begin{aligned}
&= \frac{\cos A(\sin^2 A - 1) + \sin A(1 - \cos^2 A)}{\sin A \cos A} \\
&= \frac{\cos A(-\cos^2 A) + \sin A(\sin^2 A)}{\sin A \cos A} \\
&= \frac{-\cos^3 A + \sin^3 A}{\sin A \cos A} \\
&= \frac{\sin^3 A - \cos^3 A}{\sin A \cos A} \\
&= \frac{\sin^3 A}{\sin A \cos A} - \frac{\cos^3 A}{\sin A \cos A} \\
&= \frac{\sin^2 A}{\cos A} - \frac{\cos^2 A}{\sin A} \\
&= \frac{\sin A}{\cos A} \sin A - \frac{\cos A}{\sin A} \cos A \\
&= \tan A \sin A - \cot A \cos A \\
&= \sin A \tan A - \cos A \cot A
\end{aligned}$$

Now,

$$\begin{aligned}
\frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A} &= \frac{\frac{1}{\cos A}}{\frac{1}{\sin^2 A}} - \frac{\frac{1}{\sin A}}{\frac{1}{\cos^2 A}} \\
&= \frac{\sin^2 A}{\cos A} - \frac{\cos^2 A}{\sin A} \\
&= \sin A \frac{\sin A}{\cos A} - \cos A \frac{\cos A}{\sin A} \\
&= \sin A \tan A - \cos A \cot A
\end{aligned}$$

Hence proved.

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