Definite Integrals Ex 20.4B Q37

$$I = \int_0^{\pi} \frac{x}{1+\cos\alpha \sin x}\,dx$$

Then,

$$I = \int_0^{\pi} \frac{(\pi - x)}{1+\cos\alpha \sin(\pi - x)}\,dx$$

$$I = \int_0^{\pi} \frac{(\pi - x)}{1+\cos\alpha \sin x}\,dx$$

$$2I = \pi\int_0^{\pi} \frac{1}{1+\cos\alpha \sin x}\,dx$$

$$2I = \pi\int_0^{\pi} \frac{1+\tan^2\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)+2\cos\alpha\tan\left(\frac{x}{2}\right)}\,dx$$

$$I = \frac{\pi}{2}\int_0^{\pi} \frac{\sec^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+2\cos\alpha\tan\left(\frac{x}{2}\right)+1}\,dx$$

Put $\tan\left(\frac{x}{2}\right) = t$ then $\sec^2\left(\frac{x}{2}\right)dx = 2dt$

$x = 0 \Rightarrow t = 0$ and $x = \pi \Rightarrow t = \infty$

$$I = \frac{\pi}{2}\int_0^{\infty} \frac{2}{t^2+2t\cos\alpha+1}\,dt$$

$$I = \pi\int_0^{\infty} \frac{1}{(t+\cos\alpha)^2+(1-\cos^2\alpha)}\,dt$$

$$I = \pi\int_0^{\infty} \frac{1}{(t+\cos\alpha)^2+\sin^2\alpha}\,dt$$

$$I = \frac{\pi}{\sin\alpha}\left[\tan^{-1}\left(\frac{t+\cos\alpha}{\sin\alpha}\right)\right]_0^{\infty}$$

$$I = \frac{\pi\alpha}{\sin\alpha}$$

Definite Integrals Ex 20.4B Q38

We know

$$\int_0^{2a} f(x)dx = \int_0^a f(x) + \int_0^a f(2a-x)dx$$

Also here

$$f(x) = f(2\Pi - x)$$

So

$$I = \int_0^{2\Pi} \sin^{100} x \cos^{101} x dx = 2\int_0^{\Pi} \sin^{100} x \cos^{101} x dx$$

$$I = 2\int_0^{\Pi} \sin^{100}(\Pi - x)\cos^{101}(\Pi - x)dx$$

$$I = -2\int_0^{\Pi} \sin^{100} x \cos^{101} x dx$$

Hence

$$2I = 0$$

$$I = 0$$

Definite Integrals Ex 20.4B Q39

$$I = \int_0^{\pi/2} \frac{a\sin x + b\cos x}{\sin x + \cos x} dx$$

Then,

$$I = \int_0^{\pi/2} \frac{a\sin\left(\frac{\pi}{2} - x\right) + b\cos\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$$I = \int_0^{\pi/2} \frac{a\cos x + b\sin x}{\cos x + \sin x} dx$$

$$2I = \int_0^{\pi/2} \frac{a\sin x + b\cos x}{\sin x + \cos x} dx + \int_0^{\pi/2} \frac{a\cos x + b\sin x}{\cos x + \sin x} dx$$

$$2I = (a+b)\int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$I = \frac{(a+b)}{2}\int_0^{\pi/2} 1 dx$$

$$I = \frac{(a+b)\pi}{4}$$

Definite Integrals Ex 20.4B Q40

We have,

$$I = \int_0^{2a} f(x)\,dx$$

Then

$$I = \int_0^{a} f(x)\,dx + \int_a^{2a} f(x)\,dx$$

$$I = \int_0^{a} f(x)\,dx + I_1$$

where, $I_1 = \int_a^{2a} f(x)\,dx$

Let $2a - t = x$ then $dx = -dt$

If $t = a \Rightarrow x = a$

If $t = 2a \Rightarrow x = 0$

$$I_1 = \int_0^{2a} f(x)\,dx = \int_a^{0} f(2a-t)(-dt) = -\int_a^{0} f(2a-t)\,dt$$

$$I_1 = \int_0^{a} f(2a-t)\,dt = \int_0^{a} f(2a-x)\,dx$$

$$\therefore\ I = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a-x)\,dx$$

$$I = \int_0^{a} f(x)\,dx + \int_0^{a} f(x)\,dx = 2\int_0^{a} f(x)\,dx \quad \left[f(2a-x) = f(x)\right]$$

Hence Proved.

*********** END **********