



Algebra of Matrices Ex 5.3 Q66

Given,

A and B are square matrices of same order

$$\begin{aligned}
 (A+B)^2 &= (A+B)(A+B) \\
 &= A(A+B) + B(A+B) && \text{(using distributive property)} \\
 &= A \times A + AB + BA + B^2 \\
 &= A^2 + AB + BA + B^2
 \end{aligned}$$

But,

$$(A+B)^2 = A^2 + 2AB + B^2 \text{ is possible only when } AB = BA$$

Here, we can not say that $AB = BA$

So,

$$(A+B)^2 = A^2 + 2AB + B^2 \text{ does not hold.}$$

Algebra of Matrices Ex 5.3 Q67

Given, A and B are square matrices of same order.

$$\begin{aligned}
 \text{(i) } (A+B)^2 &= (A+B)(A+B) \\
 &= A(A+B) + B(A+B) && \text{(using distributive property)} \\
 &= A \times A + AB + BA + B \times B \\
 &= A^2 + AB + BA + B^2 \\
 &\neq A^2 + 2AB + B^2
 \end{aligned}$$

Since, in general matrix multiplication is not commutative ($AB \neq BA$)

$$\text{So, } (A+B)^2 \neq A^2 + 2AB + B^2$$

$$\begin{aligned}
 \text{(ii) } (A-B)^2 &= (A-B)(A-B) \\
 &= A(A-B) - B(A-B) && \text{(using distributive property)} \\
 &= A \times A - AB - BA + B \times B \\
 &= A^2 - AB - BA + B^2 \\
 &\neq A^2 - 2AB + B^2
 \end{aligned}$$

Since, in general matrix multiplication is not commutative ($AB \neq BA$), so

$$\text{So, } (A-B)^2 \neq A^2 - 2AB + B^2$$

$$\begin{aligned}
 \text{(iii) } (A+B)(A-B) &= A(A-B) + B(A-B) && \text{(using distributive property)} \\
 &= A \times A - AB + BA - B \times B \\
 &= A^2 - AB + BA - B^2 \\
 &= A^2 - B^2
 \end{aligned}$$

Since, in general matrix multiplication is not commutative ($AB \neq BA$),

$$\text{So, } (A+B)(A-B) \neq A^2 - B^2$$

Algebra of Matrices Ex 5.3 Q68

The given equality is true only when we choose A and B to be a square matrix in such a way that $AB = BA$ else the result is not true in general.

Example: Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\begin{aligned} \text{Here } AB &= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 \times 0 + 0 \times 2 + 0 \times 0 & 1 \times 1 + 0 \times 1 + 0 \times 0 & 1 \times 0 + 0 \times 0 + 0 \times 1 \\ 1 \times 0 + 1 \times 2 + 0 \times 0 & 1 \times 1 + 1 \times 1 + 0 \times 0 & 1 \times 0 + 1 \times 0 + 0 \times 1 \\ 0 \times 0 + 0 \times 2 + 1 \times 0 & 0 \times 1 + 0 \times 1 + 1 \times 0 & 0 \times 0 + 0 \times 0 + 1 \times 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{and } BA &= \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 \times 1 + 1 \times 1 + 0 \times 0 & 0 \times 0 + 1 \times 1 + 0 \times 0 & 0 \times 0 + 1 \times 0 + 0 \times 1 \\ 2 \times 1 + 1 \times 1 + 0 \times 0 & 2 \times 0 + 1 \times 1 + 0 \times 0 & 2 \times 0 + 1 \times 0 + 0 \times 1 \\ 0 \times 1 + 0 \times 1 + 1 \times 0 & 0 \times 0 + 0 \times 1 + 1 \times 0 & 0 \times 0 + 0 \times 0 + 1 \times 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$AB \neq BA$$

$$\begin{aligned} \text{Now, } (AB)^2 &= \begin{bmatrix} 0 & 1 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 \times 0 + 1 \times 2 + 0 \times 0 & 0 \times 1 + 1 \times 2 + 0 \times 0 & 0 \times 0 + 1 \times 0 + 0 \times 1 \\ 2 \times 0 + 2 \times 2 + 0 \times 0 & 2 \times 1 + 2 \times 2 + 0 \times 0 & 2 \times 0 + 2 \times 0 + 0 \times 1 \\ 0 \times 0 + 0 \times 2 + 1 \times 0 & 0 \times 1 + 0 \times 2 + 1 \times 0 & 0 \times 0 + 0 \times 0 + 1 \times 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 2 & 0 \\ 4 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} A^2 &= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 \times 1 + 0 \times 1 + 0 \times 0 & 1 \times 0 + 0 \times 1 + 0 \times 0 & 1 \times 0 + 0 \times 0 + 0 \times 1 \\ 1 \times 1 + 1 \times 1 + 0 \times 0 & 1 \times 0 + 1 \times 1 + 0 \times 0 & 1 \times 0 + 1 \times 0 + 0 \times 1 \\ 0 \times 1 + 0 \times 1 + 1 \times 0 & 0 \times 0 + 0 \times 1 + 1 \times 0 & 0 \times 0 + 0 \times 0 + 1 \times 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} B^2 &= \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 \times 0 + 1 \times 2 + 0 \times 0 & 0 \times 1 + 1 \times 1 + 0 \times 0 & 0 \times 0 + 1 \times 0 + 0 \times 1 \\ 2 \times 0 + 1 \times 2 + 0 \times 0 & 2 \times 1 + 1 \times 1 + 0 \times 0 & 2 \times 0 + 1 \times 0 + 0 \times 1 \\ 0 \times 0 + 0 \times 2 + 1 \times 0 & 0 \times 1 + 0 \times 1 + 1 \times 0 & 0 \times 0 + 0 \times 0 + 1 \times 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 1 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned}
A^2B^2 &= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 \times 1 + 0 \times 1 + 0 \times 0 & 1 \times 1 + 0 \times 2 + 0 \times 0 & 1 \times 0 + 0 \times 0 + 0 \times 1 \\ 2 \times 1 + 1 \times 1 + 0 \times 0 & 2 \times 1 + 1 \times 2 + 0 \times 0 & 2 \times 0 + 1 \times 0 + 0 \times 1 \\ 0 \times 1 + 0 \times 1 + 1 \times 0 & 0 \times 1 + 0 \times 2 + 1 \times 0 & 0 \times 0 + 0 \times 0 + 1 \times 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 & 1 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\end{aligned}$$

We can see that if we have A and B two square matrices
with $AB \neq BA$ then $(AB)^2 \neq A^2B^2$

***** END *****