Trigonometric Identities Ex 6.1 Q73

Answer :

We have to prove $\tan^2 A \sec^2 B - \sec^2 A \tan^2 B = \tan^2 A - \tan^2 B$

We know that, $\sec^2 A - \tan^2 A = 1$

So,

$$\tan^2 A \sec^2 B - \sec^2 A \tan^2 B = \tan^2 A (1 + \tan^2 B) - (1 + \tan^2 A) \tan^2 B$$
$$= \tan^2 A + \tan^2 A \tan^2 B - \tan^2 B - \tan^2 A \tan^2 B$$
$$= \tan^2 A - \tan^2 B$$

Hence proved.

Trigonometric Identities Ex 6.1 Q74

Answer :

Given that,

$x = a\sec\theta + b\tan\theta,$

$y = a\tan\theta + b\sec\theta$

We have to prove $x^2 - y^2 = a^2 - b^2$

We know that, $\sec^2\theta - \tan^2\theta = 1$

So,

$x^2 - y^2$
$= (a\sec\theta + b\tan\theta)^2 - (a\tan\theta + b\sec\theta)^2$
$= (a^2 \sec^2\theta + 2ab\sec\theta\tan\theta + b^2\tan^2\theta) - (a^2\tan^2\theta + 2ab\sec\theta\tan\theta + b^2\sec^2\theta)$
$= a^2(\sec^2\theta - \tan^2\theta) - b^2(\sec^2\theta - \tan^2\theta)$
$= a^2 - b^2$

Hence proved.

Trigonometric Identities Ex 6.1 Q75

Answer :

Given that,

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1 \qquad \qquad ......(1)$$

$$\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta = 1 \qquad \qquad ......(2)$$

We have to prove $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 2$

We know that, $\sin^2\theta + \cos^2\theta = 1$

Squaring and then adding the above two equations, we have

$$\left(\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta\right)^2 + \left(\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta\right)^2 = 1 + 1$$

$$\Rightarrow \left(\frac{x^2}{a^2}\cos^2\theta + 2\frac{xy}{ab}\sin\theta\cos\theta + \frac{y^2}{b^2}\sin^2\theta\right) + \left(\frac{x^2}{a^2}\sin^2\theta - 2\frac{xy}{ab}\sin\theta\cos\theta + \frac{y^2}{b^2}\cos^2\theta\right) = 2$$

$$\Rightarrow \frac{x^2}{a^2}(\cos^2\theta + \sin^2\theta) + \frac{y^2}{b^2}(\sin^2\theta + \cos^2\theta) = 2$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$$

*********** END **********