



## Areas of Parallelograms and Triangles Ex 15.3 Q30

**Answer :**

**Given:**

- (1) ABCD is a right angled triangle at A
- (2) BCED, ACFG and ABMN are the squares on the sides of BC, CA and AB respectively.
- (3)  $AX \perp DE$ , meets BC at Y.

**To prove:**

- (i)  $\triangle MBC \cong \triangle ABD$
- (ii)  $\text{ar}(\text{BYXD}) = 2\text{ar}(\triangle MBC)$
- (iii)  $\text{ar}(\text{BYXD}) = \text{ar}(\text{ABMN})$
- (iv)  $\triangle FCB \cong \triangle ACE$
- (v)  $\text{ar}(\text{CYXE}) = 2\text{ar}(\triangle FCB)$
- (vi)  $\text{ar}(\text{CYXE}) = \text{ar}(\text{ACFG})$
- (vii)  $\text{ar}(\text{BCED}) = \text{ar}(\text{ABMN}) + \text{ar}(\text{ACFG})$

**Proof:**

- (i) In  $\triangle MBC$  and  $\triangle ABD$

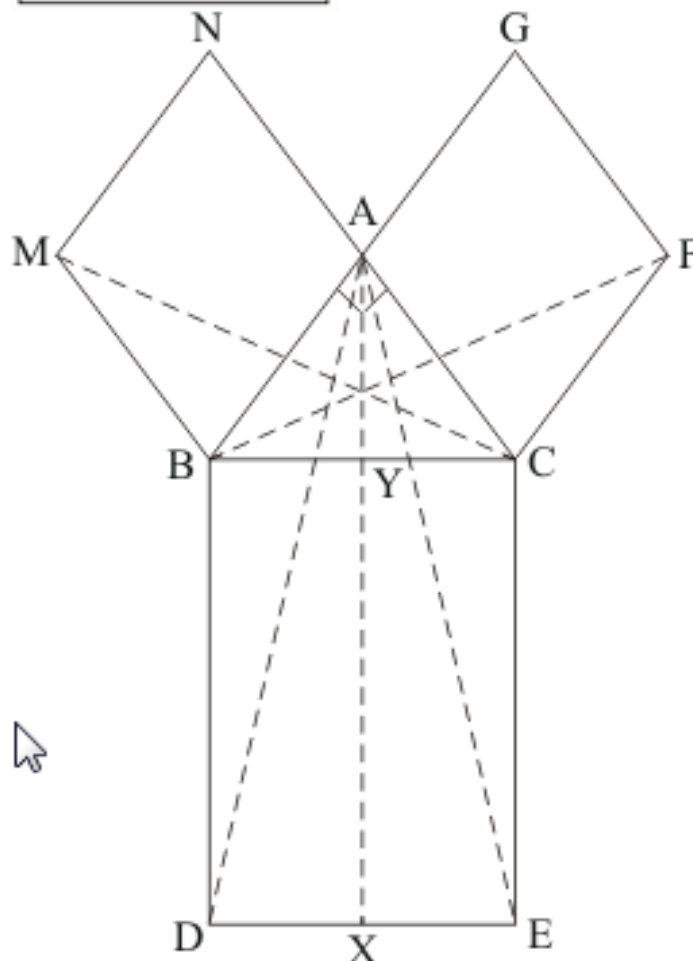
$$MB = AB$$

$$BC = BD$$

$$\angle MBC = \angle ABD (\angle MBC \text{ and } \angle ABD \text{ are obtained by adding } \angle ABC \text{ to a right angle})$$

$$\triangle MBC \cong \triangle ABD (\text{SAS criteria})$$

$$\boxed{\triangle MBC \cong \triangle ABD} \dots\dots (1)$$



(ii) Triangle ABD and rectangle BYXD are on the same base BD and between the same parallels AX and BD.

Therefore

$$\begin{aligned}\text{ar}(\triangle ABD) &= \frac{1}{2} \text{ar}(\text{BYXD}) \\ \Rightarrow 2\text{ar}(\triangle ABD) &= \text{ar}(\text{BYXD}) \\ \Rightarrow \boxed{\text{ar}(\text{BYXD}) = 2\text{ar}(\triangle ABC)} &\text{ (Using (1))} \dots\dots (2)\end{aligned}$$

(iii) Since  $\triangle MBC$  and square MBAN are on the same base MB and between the same parallels MB and NC.

$$\begin{aligned}\Rightarrow \text{ar}(\triangle MBC) &= \frac{1}{2} \text{ar}(\text{MBAN}) \\ \Rightarrow 2\text{ar}(\triangle MBC) &= \text{ar}(\text{MBAN}) \\ \Rightarrow \text{ar}(\text{MBAN}) &= 2\text{ar}(\triangle MBC) \dots\dots (3)\end{aligned}$$

From (2) and (3) we get

$$\boxed{\text{ar}(\text{MBAN}) = \text{ar}(\text{BYXD})}$$

(iv) In triangle FCB and ACE

$$BC = CE$$

$$AC = CF$$

$$\angle BCF = \angle ACE \text{ (}\angle BCF \text{ and } \angle ACE \text{ are obtained by adding } \angle ACB \text{ to a right angle)}$$

$$\triangle MBC \cong \triangle ABD \text{ (SAS criteria)}$$

$$\boxed{\triangle FCB = \triangle ACE} \dots\dots (4)$$

(v) Since  $\triangle ACE$  and rectangle CYXE are on the same base CE and between the same parallels CE and AX.

$$\begin{aligned}\Rightarrow \text{ar}(\triangle ACE) &= \frac{1}{2} \text{ar}(\text{CYXE}) \\ \Rightarrow 2\text{ar}(\triangle ACE) &= \text{ar}(\text{CYXE}) \\ \Rightarrow \text{ar}(\text{CYXE}) &= 2\text{ar}(\triangle ACE) \\ \Rightarrow \boxed{\text{ar}(\text{CYXE}) = 2\text{ar}(\triangle FCB)} &\dots\dots (5)\end{aligned}$$

(vi) Since  $\triangle FCB$  and rectangle FCAG are on the same base FC and between the same parallels FC and BG

$$\begin{aligned}\Rightarrow \text{ar}(\triangle FCB) &= \frac{1}{2} \text{ar}(\text{FCAG}) \\ \Rightarrow 2\text{ar}(\triangle FCB) &= \text{ar}(\text{FCAG}) \\ \Rightarrow \text{ar}(\text{FCAG}) &= 2\text{ar}(\triangle FCB) \dots\dots (6)\end{aligned}$$

From (5) and (6) we get

$$\boxed{\text{ar}(\text{CYXE}) = \text{ar}(\text{ACFG})}$$

(vii) Applying Pythagoras Theorem in  $\triangle ACB$ , WE get

$$BC^2 = AB^2 + AC^2$$

$$\Rightarrow BC \times BD = AB \times MB + AC \times FC$$

$$\boxed{\text{ar}(\text{BCED}) = \text{ar}(\text{ABMN}) + \text{ar}(\text{ACFG})}$$

\*\*\*\*\* END \*\*\*\*\*