Trigonometric Identities Ex 6.1 Q66

Answer :

We have to prove $\sec^4 A(1-\sin^4 A)-2\tan^2 A=1$

We know that, $\sin^2 A+\cos^2 A=1$

So,

$$\begin{aligned}
\sec^4 A(1-\sin^4 A)-2\tan^2 A &= \frac{1}{\cos^4 A}(1-\sin^4 A)-2\frac{\sin^2 A}{\cos^2 A}\\
&=\left(\frac{1}{\cos^4 A}-\frac{\sin^4 A}{\cos^4 A}\right)-2\frac{\sin^2 A}{\cos^2 A}\\
&=\left(\frac{1-\sin^4 A}{\cos^4 A}\right)-2\frac{\sin^2 A}{\cos^2 A}\\
&=\frac{(1-\sin^2 A)(1+\sin^2 A)}{\cos^4 A}-2\frac{\sin^2 A}{\cos^2 A}\\
&=\frac{\cos^2 A(1+\sin^2 A)}{\cos^4 A}-2\frac{\sin^2 A}{\cos^2 A}\\
&=\frac{1+\sin^2 A}{\cos^2 A}-2\frac{\sin^2 A}{\cos^2 A}\\
&=\frac{1+\sin^2 A-2\sin^2 A}{\cos^2 A}\\
&=\frac{1-\sin^2 A}{\cos^2 A}\\
&=\frac{\cos^2 A}{\cos^2 A}\\
&=1
\end{aligned}$$

Hence proved.

Trigonometric Identities Ex 6.1 Q67

Answer :

We have to prove $\frac{\cot^2 A(\sec A-1)}{1+\sin A}=\sec^2 A\left(\frac{1-\sin A}{1+\sec A}\right)$.

We know that, $\sin^2 A+\cos^2 A=1$

So,

$$\frac{\cot^2 A(\sec A-1)}{1+\sin A}=\sec^2 A\left(\frac{1-\sin A}{1+\sec A}\right)$$

$$=\frac{\frac{\cos^2 A}{\sin^2 A}\left(\frac{1}{\cos A}-1\right)}{1+\sin A}$$

$$=\frac{\frac{\cos^2 A}{\sin^2 A}\left(\frac{1-\cos A}{\cos A}\right)}{1+\sin A}$$

$$=\frac{\cos A(1-\cos A)}{\sin^2 A(1+\sin A)}$$

$$=\frac{\cos A(1-\cos A)}{(1-\cos^2 A)(1+\sin A)}$$

$$=\frac{\cos A(1-\cos A)}{(1-\cos A)(1+\cos A)(1+\sin A)}$$

$$=\frac{\cos A}{(1+\cos A)(1+\sin A)}$$

$$=\frac{\frac{1}{\sec A}}{\left(1+\frac{1}{\sec A}\right)(1+\sin A)}$$

$$=\frac{\frac{1}{\sec A}}{\left(\frac{\sec A+1}{\sec A}\right)(1+\sin A)}$$

$$=\frac{1}{(\sec A+1)(1+\sin A)}$$

Multiplying both the numerator and denominator by $(1-\sin A)$, we have

$$=\frac{(1-\sin A)}{(\sec A+1)(1+\sin A)(1-\sin A)}$$

$$=\frac{(1-\sin A)}{(\sec A+1)(1-\sin^2 A)}$$

$$=\frac{(1-\sin A)}{(\sec A+1)\cos^2 A}$$

$$=\sec^2 A\frac{(1-\sin A)}{(\sec A+1)}$$

$$=\sec^2 A\left(\frac{1-\sin A}{1+\sec A}\right)$$

Hence proved.

*********** END **********