



Question 14:

Using the method of integration find the area of the region bounded by lines:

$$2x + y = 4, 3x - 2y = 6 \text{ and } x - 3y + 5 = 0$$

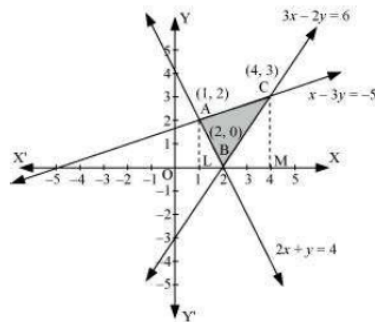
Answer

The given equations of lines are

$$2x + y = 4 \dots (1)$$

$$3x - 2y = 6 \dots (2)$$

$$\text{And, } x - 3y + 5 = 0 \dots (3)$$



The area of the region bounded by the lines is the area of ΔABC . AL and CM are the perpendiculars on x-axis.

$$\text{Area } (\Delta ABC) = \text{Area } (ALMCA) - \text{Area } (ALB) - \text{Area } (CMB)$$

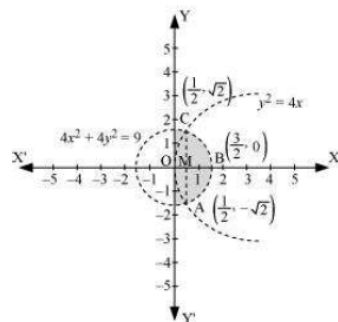
$$\begin{aligned} &= \int_1^4 \left(\frac{x+5}{3} \right) dx - \int_1^2 (4-2x) dx - \int_2^4 \left(\frac{3x-6}{2} \right) dx \\ &= \frac{1}{3} \left[\frac{x^2}{2} + 5x \right]_1^4 - \left[4x - x^2 \right]_1^2 - \frac{1}{2} \left[\frac{3x^2}{2} - 6x \right]_2^4 \\ &= \frac{1}{3} \left[8 + 20 - \frac{1}{2} - 5 \right] - [8 - 4 - 4 + 1] - \frac{1}{2} [24 - 24 - 6 + 12] \\ &= \left(\frac{1}{3} \times \frac{45}{2} \right) - (1) - \frac{1}{2} (6) \\ &= \frac{15}{2} - 1 - 3 \\ &= \frac{15}{2} - 4 = \frac{15-8}{2} = \frac{7}{2} \text{ units} \end{aligned}$$

Question 15:

Find the area of the region $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$

Answer

The area bounded by the curves, $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$, is represented as



The points of intersection of both the curves are $\left(\frac{1}{2}, \sqrt{2}\right)$ and $\left(\frac{1}{2}, -\sqrt{2}\right)$.

The required area is given by OABCO.

It can be observed that area OABCO is symmetrical about x-axis.

$$\therefore \text{Area OABCO} = 2 \times \text{Area OBC}$$

$$\text{Area OBCO} = \text{Area OMC} + \text{Area MBC}$$

$$= \int_0^1 2\sqrt{x} \, dx + \int_2^3 \frac{1}{2} \sqrt{9-4x^2} \, dx$$

$$= \int_0^1 2\sqrt{x} \, dx + \int_2^3 \frac{1}{2} \sqrt{(3)^2 - (2x)^2} \, dx$$

Question 16:

Area bounded by the curve $y = x^3$, the x-axis and the ordinates $x = -2$ and $x = 1$ is

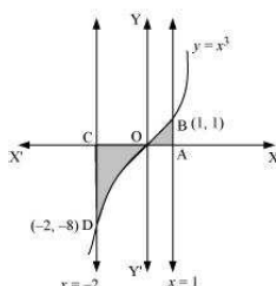
A. - 9

B. $-\frac{15}{4}$

C. $\frac{15}{4}$

D. $\frac{17}{4}$

Answer



$$\text{Required area} = \int_{-2}^1 y \, dx$$

$$= \int_{-2}^1 x^3 \, dx$$

$$= \left[\frac{x^4}{4} \right]_{-2}^1$$

$$= \left[\frac{1}{4} - \frac{(-2)^4}{4} \right]$$

$$= \left(\frac{1}{4} - 4 \right) = -\frac{15}{4} \text{ units}$$

Thus, the correct answer is B.

Question 17:

The area bounded by the curve $y = x|x|$, x-axis and the ordinates $x = -1$ and $x = 1$ is given by

[Hint: $y = x^2$ if $x > 0$ and $y = -x^2$ if $x < 0$]

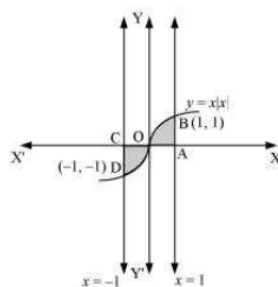
A. 0

B. $\frac{1}{3}$

C. $\frac{2}{3}$

D. $\frac{4}{3}$

Answer



$$\text{Required area} = \int_{-1}^1 y \, dx$$

$$= \int_{-1}^1 x|x| \, dx$$

$$= \int_{-1}^0 x^2 \, dx + \int_0^1 x^2 \, dx$$

$$\begin{aligned}
 &= \left[\frac{x^3}{3} \right]_{-1}^0 + \left[\frac{x^3}{3} \right]_0^1 \\
 &= -\left(-\frac{1}{3} \right) + \frac{1}{3} \\
 &= \frac{2}{3} \text{ units}
 \end{aligned}$$

Thus, the correct answer is C.

Question 18:

The area of the circle $x^2 + y^2 = 16$ exterior to the parabola $y^2 = 6x$ is

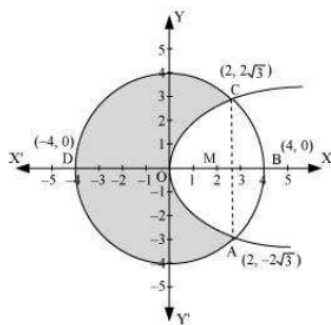
- A. $\frac{4}{3}(4\pi - \sqrt{3})$
 B. $\frac{4}{3}(4\pi + \sqrt{3})$
 C. $\frac{4}{3}(8\pi - \sqrt{3})$
 D. $\frac{4}{3}(4\pi + \sqrt{3})$

Answer

The given equations are

$$x^2 + y^2 = 16 \dots (1)$$

$$y^2 = 6x \dots (2)$$



Area bounded by the circle and parabola

$$\begin{aligned}
 &= 2 \left[\text{Area}(\text{OADO}) + \text{Area}(\text{ADBA}) \right] \\
 &= 2 \left[\int_0^2 \sqrt{16x} dx + \int_2^4 \sqrt{16-x^2} dx \right] \\
 &= 2 \left[\sqrt{6} \left\{ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right\}_0^2 \right] + 2 \left[\frac{x}{2} \sqrt{16-x^2} + \frac{16}{2} \sin^{-1} \frac{x}{4} \right]_2^4 \\
 &= 2\sqrt{6} \times \frac{2}{3} \left[x^{\frac{3}{2}} \right]_0^2 + 2 \left[8 \cdot \frac{\pi}{2} - \sqrt{16-4} - 8 \sin^{-1} \left(\frac{1}{2} \right) \right] \\
 &= \frac{4\sqrt{6}}{3} (2\sqrt{2}) + 2 \left[4\pi - \sqrt{12} - 8 \frac{\pi}{6} \right] \\
 &= \frac{16\sqrt{3}}{3} + 8\pi - 4\sqrt{3} - \frac{8}{3} \pi \\
 &= \frac{4}{3} [4\sqrt{3} + 6\pi - 3\sqrt{3} - 2\pi] \\
 &= \frac{4}{3} [\sqrt{3} + 4\pi] \\
 &= \frac{4}{3} [4\pi + \sqrt{3}] \text{ units}
 \end{aligned}$$

$$\text{Area of circle} = \pi (r)^2$$

$$= \pi (4)^2$$

$$= 16\pi \text{ units}$$

$$\begin{aligned}
 \therefore \text{Required area} &= 16\pi - \frac{4}{3} [4\pi + \sqrt{3}] \\
 &= \frac{4}{3} [4 \times 3\pi - 4\pi - \sqrt{3}] \\
 &= \frac{4}{3} (8\pi - \sqrt{3}) \text{ units}
 \end{aligned}$$

Thus, the correct answer is C.

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