Differentiation Ex 11.2 Q62

Given, $y = \sqrt{\frac{1+e^x}{1-e^x}}$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{\frac{1+e^x}{1-e^x}}\right)$$

$$= \frac{1}{2\sqrt{\left(\frac{1+e^x}{1-e^x}\right)}} \times \frac{d}{dx}\left(\frac{1+e^x}{1-e^x}\right) \qquad \text{[Using chain rule, quotient rule]}$$

$$= \frac{1}{2} \times \sqrt{\frac{1-e^x}{1+e^x}}\left[\frac{\left(1-e^x \frac{d}{dx}\left(1+e^x\right) - \left(1+e^x\right)\frac{d}{dx}\left(1-e^x\right)\right)}{\left(1-e^x\right)^2}\right]$$

$$= \frac{1}{2}\sqrt{\frac{1-e^x}{1+e^x}}\left[\frac{\left(1-e^x\right)e^x + \left(1+e^x\right)e^x}{\left(1-e^x\right)2}\right]$$

$$= \frac{1}{2}\sqrt{\frac{1-e^x}{1+e^x}} \times \left[\frac{2e^x}{\left(1-e^x\right)^2}\right]$$

$$= \frac{e^x}{\sqrt{\left(1+e^x\right)}\sqrt{\left(1-e^x\right)}} \frac{1}{\left(1-e^x\right)}$$

$$\frac{dy}{dx} = \frac{e^x}{\left(1-e^x\right)\sqrt{1-e^{2x}}}.$$

Differentiation Ex 11.2 Q63

Given, $y = \sqrt{x} + \frac{1}{\sqrt{x}}$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$$

$$= \frac{d}{dx}\left(\sqrt{x}\right) + \frac{d}{dx}\left(x^{-\frac{1}{2}}\right)$$

$$= \frac{1}{2\sqrt{x}} + \left(-\frac{1}{2} \times x^{-\frac{1}{2}-1}\right)$$

$$= \frac{1}{2\sqrt{x}} - \frac{1}{2x\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{x-1}{2x\sqrt{x}}$$

$$2x\frac{dy}{dx} = \frac{x-1}{\sqrt{x}}$$

$$\Rightarrow \quad 2x\frac{dy}{dx} = \frac{x}{\sqrt{x}} - \frac{1}{\sqrt{x}}$$

$$\Rightarrow \quad 2x\frac{dy}{dx} = \sqrt{x} - \frac{1}{\sqrt{x}}.$$

Differentiation Ex 11.2 Q64

Given, $y = \frac{x \sin^{-1} x}{\sqrt{1-x^2}}$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x \sin^{-1} x}{\sqrt{1-x^2}}\right)$$

$$= \left[\frac{\sqrt{1-x^2}\frac{d}{dx}\left(x \sin^{-1} x\right) - \left(x \sin^{-1} x\right)\frac{d}{dx}\left(\sqrt{1-x^2}\right)}{\left(\sqrt{1-x^2}\right)^2}\right]$$

[Using quotient rule, product rule, chain rule]

$$= \left[\frac{\sqrt{1-x^2}\left\{x\frac{d}{dx}\sin^{-1} x + \sin^{-1} x \frac{d}{dx}(x)\right\} - \left(x \sin^{-1} x\right)\frac{1}{2\sqrt{1-x^2}}\frac{d}{dx}\left(1-x^2\right)}{\left(1-x^2\right)}\right]$$

$$= \left[\frac{\sqrt{1-x^2}\left\{\frac{x}{\sqrt{1-x^2}} + \sin^{-1} x\right\} - \frac{x \sin^{-1} x\,(-2x)}{2\sqrt{1-x^2}}}{\left(1-x^2\right)}\right]$$

$$= \left[\frac{x + \sqrt{1-x^2}\sin^{-1} x + \frac{x^2 \sin^{-1} x}{\sqrt{1-x^2}}}{\left(1-x^2\right)}\right]$$

$$\left(1-x^2\right)\frac{dy}{dx} = x + \frac{\sqrt{1-x^2}\sin^{-1} x}{1} + \frac{x^2 \sin^{-1} x}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad \left(1-x^2\right)\frac{dy}{dx} = x + \left(\frac{\left(1-x^2\right)\sin^{-1} x + x^2 \sin^{-1} x}{\sqrt{1-x^2}}\right)$$

$$\Rightarrow \quad \left(1-x^2\right)\frac{dy}{dx} = x + \left(\frac{\sin^{-1} x - x^2 \sin^{-1} x + x^2 \sin^{-1} x}{\sqrt{1-x^2}}\right)$$

$$\Rightarrow \quad \left(1-x^2\right)\frac{dy}{dx} = x + \left(\frac{\sin^{-1} x}{\sqrt{1-x^2}}\right)$$

$$\left(1-x^2\right)\frac{dy}{dx} = x + \frac{y}{x} \qquad \left\{\text{Since, given } y = \frac{x \sin^{-1} x}{\sqrt{1-x^2}}\right\}.$$

Differentiation Ex 11.2 Q65

Given, $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)$$

$$= \left[\frac{\left(e^x + e^{-x}\right)\frac{d}{dx}\left(e^x - e^{-x}\right) - \left(e^x - e^{-x}\right)\frac{d}{dx}\left(e^x + e^{-x}\right)}{\left(e^x + e^{-x}\right)^2}\right]$$

[Using quotient rule and chain rule]

$$= \left[\frac{\left(e^x + e^{-x}\right)\left[e^x - e^{-x}\frac{d}{dx}(-x)\right) - \left(e^x - e^{-x}\right)\left(e^x + e^{-x}\frac{d}{dx}(-x)\right)\right]}{\left(e^x + e^{-x}\right)^2}\right]$$

$$= \left[\frac{\left(e^x + e^{-x}\right)\left(e^x + e^{-x}\right) - \left(e^x - e^{-x}\right)\left(e^x - e^{-x}\right)}{\left(e^x + e^{-x}\right)^2}\right]$$

$$= \left[\frac{e^{2x} + e^{-2x} + 2e^x \times e^{-x} - e^{2x} - e^{-2x} + 2e^x e^{-x}}{\left(e^x + e^{-x}\right)2}\right]$$

$$\frac{dy}{dx} = \left[\frac{4}{\left(e^x + e^{-x}\right)2}\right] \qquad \text{---(i)}$$

Now,

$$1 - y^2 = 1 - \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)^2$$

$$= 1 - \frac{\left(e^x - e^{-x}\right)^2}{\left(e^x + e^{-x}\right)^2}$$

$$= \frac{\left(e^x + e^{-x}\right)^2 - \left(e^x - e^{-x}\right)^2}{\left(e^x + e^{-x}\right)^2}$$

$$= \frac{4}{\left(e^x + e^{-x}\right)2}$$

Differentiation Ex 11.2 Q66

Given, $y = (x-1)\log(x-1) - (x+1)\log(x+1)$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[(x-1)\log(x-1) - (x+1)\log(x+1)\right]$$

$$= \left[(x-1)\frac{d}{dx}\log(x-1) + \log(x-1)\frac{d}{dx}(x-1)\right] -$$

$$\left[(x+1)\frac{d}{dx}\log(x+1) + \log(x+1)\frac{d}{dx}(x+1)\right]$$

[Using product rule, chain rule]

$$= \left[(x-1) \times \frac{1}{(x-1)}\frac{d}{dx}(x-1) + \log(x-1) \times (1)\right] -$$

$$\left[(x+1)\frac{1}{(x+1)} \times \frac{d}{dx}(x+1) + \log(x+1)(1)\right]$$

$$= \left[(1) + \log(x-1)\right] - \left[1 + \log(x+1)\right]$$

$$= \log(x-1) - \log(x+1)$$

$$\frac{dy}{dx} = \log\frac{(x-1)}{(x+1)} \qquad \left[\text{Since, } \log\left(\frac{a}{b}\right) = \log a - \log b\right].$$

*********** END **********