Definite Integrals Ex 20.2 Q24

Using Integration By parts

$$\int f'g = fg - \int fg'$$

$$f' = \frac{x}{\sqrt{1-x^2}}, g = \sin^{-1} x$$

$$f = -\sqrt{1-x^2}, g' = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx = -\sqrt{1-x^2} \sin^{-1} x - \int (-1) dx$$

$$\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx = -\sqrt{1-x^2} \sin^{-1} x + x$$

Hence

$$\int_0^{\frac{1}{2}} \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx = \left\{ x - \sqrt{1-x^2} \sin^{-1} x \right\}_0^{\frac{1}{2}}$$

$$\int_0^{\frac{1}{2}} \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx = \left\{ \frac{1}{2} - \sqrt{1-(\frac{1}{2})^2} \sin^{-1} \frac{1}{2} \right\}$$

$$\int_0^{\frac{1}{2}} \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx = \left\{ \frac{1}{2} - \frac{\sqrt{3}}{2} \frac{\Pi}{6} \right\}$$

Definite Integrals Ex 20.2 Q25

$$I = \int_0^{\pi/4} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$$

$$I = \int_0^{\pi/4} \left(\frac{\sqrt{\sin x}}{\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}}\right) dx$$

$$I = \int_0^{\pi/4} \left(\frac{\sin x + \cos x}{\sqrt{\sin x \cos x}}\right) dx$$

$$I = \sqrt{2}\int_0^{\pi/4} \left(\frac{\sin x + \cos x}{\sqrt{2 \sin x \cos x}}\right) dx$$

$$I = \sqrt{2}\int_0^{\pi/4} \left(\frac{\sin x + \cos x}{\sqrt{1-(\sin x - \cos x)^2}}\right) dx$$

Let $\sin x - \cos x = t$

$(\cos x + \sin x)dx = dt$

$x = 0 \Rightarrow t = -1$ and $x = \frac{\pi}{4} \Rightarrow t = 0$

$$I = \sqrt{2}\int_{-1}^{0} \left(\frac{1}{\sqrt{1-t^2}}\right) dt$$

$$I = \sqrt{2}\left[\sin^{-1} t\right]_{-1}^{0}$$

$$I = \sqrt{2}\left[\sin^{-1}(0) - \sin^{-1}(-1)\right]$$

$$I = \frac{\pi}{\sqrt{2}}$$

Definite Integrals Ex 20.2 Q26

We have,

$$\int_0^{\frac{\pi}{4}} \frac{\tan^3 x}{1+\cos 2x} dx = \int_0^{\frac{\pi}{4}} \frac{\tan^3 x}{2\cos^2 x} dx = \frac{1}{2}\int_0^{\frac{\pi}{4}} \tan^3 x \sec^2 x \, dx$$

Let $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

Now,

$x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{4} \Rightarrow t = 1$

$$\therefore \frac{1}{2}\int_0^{\frac{\pi}{4}} \sec^2 x \tan^3 x \, dx = \frac{1}{2}\int_0^1 t^3 \, dt = \frac{1}{2}\left[\frac{t^4}{4}\right]_0^1 = \frac{1}{8}$$

$$\therefore \int_0^{\frac{\pi}{4}} \frac{\tan^3 x}{1+\cos 2x} dx = \frac{1}{8}$$

Definite Integrals Ex 20.2 Q27

We know that,

$$\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$\frac{1}{5 + 3\cos x} = \frac{1}{5 + 3\left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right)} = \frac{1 + \tan^2 \frac{x}{2}}{5\left(1 + \tan^2 \frac{x}{2}\right) + 3\left(1 - \tan^2 \frac{x}{2}\right)} = \frac{\sec^2 \frac{x}{2} dx}{8 + 2\tan^2 \frac{x}{2}}$$

$$\therefore \int_0^{\pi} \frac{dx}{5 + 3\cos x} dx = \frac{1}{2}\int_0^{\pi} \frac{\sec^2 \frac{x}{2}}{2^2 + \tan^2 \frac{x}{2}} dx$$

Let $\tan \frac{x}{2} = t$

Differentiating w.r.t. x, we get

$$\frac{1}{2}\sec^2 \frac{x}{2} dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = \pi \Rightarrow t = \infty$

$$\therefore \frac{1}{2}\int_0^{\pi}\left(\frac{\sec^2 \frac{x}{2} dx}{2^2 + \tan^2 \frac{x}{2}}\right) dx$$

$$= \int_0^{\infty} \frac{dt}{2^2 + t^2}$$

$$= \left[\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)\right]_0^{\infty}$$

$$= \frac{1}{2}\left[\tan^{-1}(\infty) - \tan^{-1}(0)\right]$$

$$= \frac{1}{2}\left[\tan^{-1}\left(\tan\frac{\pi}{2}\right) - \tan^{-1}(\tan 0)\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{2} - 0\right]$$

$$= \frac{\pi}{4}$$

$$\therefore \int_0^{\pi} \frac{dx}{5 + 3\cos x} dx = \frac{\pi}{4}$$

*********** END **********