Exercise 7B

Question 8:

$$(\sin\theta + \cos\theta) = m \text{ and } (\sec\theta + \operatorname{cosec}\theta) = n$$

$$\text{LHS} = n(m^2 - 1) = (\sec\theta + \operatorname{cosec}\theta)\left[(\sin\theta + \cos\theta)^2 - 1\right]$$

$$= (\sec\theta + \operatorname{cosec}\theta) \times \left[\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1\right]$$

$$= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)(2\sin\theta\cos\theta)$$

$$= \left(\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}\right) \times 2\sin\theta\cos\theta$$

$$= 2(\sin\theta + \cos\theta) \; ----(1)$$

$$\text{RHS} = 2m = 2(\sin\theta + \cos\theta) \; ----(2)$$

$\therefore$ LHS = RHS

Question 9:

$$(\cot\theta + \tan\theta) = m \text{ and } (\sec\theta - \cos\theta) = n$$

$$\Rightarrow \left(\frac{1}{\tan\theta} + \tan\theta\right) = m \text{ and } \left(\frac{1}{\cos\theta} - \cos\theta\right) = n$$

$$\Rightarrow \left(\frac{1 + \tan^2\theta}{\tan\theta}\right) = m \text{ and } \frac{(1 - \cos^2\theta)}{\cos\theta} = n$$

$$\Rightarrow \left(\frac{\sec^2\theta}{\tan\theta}\right) = m \text{ and } \frac{(1 - \cos^2\theta)}{\cos\theta} = n$$

$$\Rightarrow \frac{1}{\cos^2\theta \times \frac{\sin\theta}{\cos\theta}} = m \text{ and } \frac{\sin^2\theta}{\cos\theta} = n$$

$$\Rightarrow \frac{1}{\cos\theta\sin\theta} = m \text{ and } \frac{\sin^2\theta}{\cos\theta} = n$$

$$\therefore (m^2 n)^{\frac{2}{3}} - (mn^2)^{\frac{2}{3}} = \left[\frac{1}{\cos^2\theta\sin^2\theta} \times \frac{\sin^2\theta}{\cos\theta}\right]^{\frac{2}{3}}$$

$$- \left[\frac{1}{\cos\theta\sin\theta} \times \frac{\sin^4\theta}{\cos^2\theta}\right]^{\frac{2}{3}}$$

$$= \left(\frac{1}{\cos^3\theta}\right)^{\frac{2}{3}} - \left(\frac{\sin^3\theta}{\cos^3\theta}\right)^{\frac{2}{3}} = \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \sec^2\theta - \tan^2\theta = 1 \quad \left[\because \sec^2\theta = 1 + \tan^2\theta\right]$$

$$\text{Hence, } (m^2 n)^{\frac{2}{3}} - (mn^2)^{\frac{2}{3}} = 1$$

*********** END **********