



Binary Operations Ex 3.2 Q4(i)

'\*' is a binary operator on  $Z$  defined by  $a * b = a + b + ab$  for all  $a, b \in Z$ .

Commutativity of '\*':

Let  $a, b \in Z$ , then

$$a * b = a + b + ab = b + a + ba = b * a$$

$$\therefore a * b = b * a$$

Associative of '\*':

Let  $a, b \in Z$ , then

$$\begin{aligned}(a * b) * c &= (a + b + ab) * c = a + b + ab + c + ac + bc + abc \\ &= a + b + c + ab + bc + ac + abc\end{aligned}\quad \text{--- (i)}$$

$$\text{Again, } a * (b * c) = a * (b + c + bc)$$

$$= a + b + c + bc + ab + ac + abc \quad \text{--- (ii)}$$

From (i) & (ii), we get

$$(a * b) * c = a * (b * c)$$

$\therefore$  \* is commutative and associative on  $Z$

Binary Operations Ex 3.2 Q4(ii)

Commutative:

Let  $a, b \in N$ , then

$$a * b = 2^{ab} = 2^{ba} = b * a$$

$$\therefore a * b = b * a$$

$\therefore$  \* is commutative on  $N$

Associative:

Let  $a, b, c \in N$ , then

$$(a * b) * c = 2^{ab} * c = 2^{2^{ab} \cdot c} \quad \text{--- (i)}$$

$$\text{and, } a * (b * c) = a * 2^{bc} = 2^{a \cdot 2^{bc}} \quad \text{--- (ii)}$$

From (i) & (ii), we get

$$(a * b) * c \neq a * (b * c)$$

$\therefore$  \* is not associative on  $N$

Binary Operations Ex 3.2 Q4(iii)

Commutativity:

Let  $a, b \in Q$ , then

$$a * b = a - b \neq b - a = b * a$$

$$\therefore a * b \neq b * a$$

$\Rightarrow$   $*$  is not commutative on  $Q$

Associative:

Let  $a, b, c \in Q$ , then

$$(a * b) * c = (a - b) * c = a - b - c \quad \text{--- (i)}$$

$$\text{and, } a * (b * c) = a * (b - c) = a - b + c \quad \text{--- (ii)}$$

From (i) & (ii), we get

$$(a * b) * c \neq a * (b * c)$$

$\therefore$   $*$  is not associative on  $Q$

Binary Operations Ex 3.2 Q4(iv)

Commutative:

Let  $a, b \in Q$ , then

$$a \circ b = a^2 + b^2 = b^2 + a^2 = b \circ a$$

$$\Rightarrow a \circ b = b \circ a$$

$\therefore$   $\circ$  is commutative on  $Q$ .

Associative:

Let  $a, b, c \in Q$ , then

$$(a \circ b) \circ c = (a^2 + b^2) \circ c = (a^2 + b^2)^2 + c^2 \quad \text{--- (i)}$$

$$\text{and, } a \circ (b \circ c) = a \circ (b^2 + c^2) = a^2 + (b^2 + c^2)^2 \quad \text{--- (ii)}$$

From (i) & (ii),

$$(a \circ b) \circ c \neq a \circ (b \circ c)$$

$\therefore$   $\circ$  is not associative on  $Q$ .

Binary Operations Ex 3.2 Q4(v)

Binary operation ' $\circ$ ' defined on  $Q$ , given by  $a \circ b = \frac{ab}{2}$  for all  $a, b \in Q$

Commutative:

Let  $a, b \in Q$ , then

$$a \circ b = \frac{ab}{2} = \frac{ba}{2} = b \circ a$$

$$\Rightarrow a \circ b = b \circ a$$

$\therefore \circ$  is commutative on  $Q$ .

Associativity:

Let  $a, b, c \in Q$ , then

$$(a \circ b) \circ c = \left( \frac{ab}{2} \right) \circ c = \frac{abc}{4} \quad \text{--- (i)}$$

$$a \circ (b \circ c) = a \circ \left( \frac{bc}{2} \right) = \frac{abc}{4} \quad \text{--- (ii)}$$

From (i) & (ii) we get

$$(a \circ b) \circ c = a \circ (b \circ c)$$

$\therefore \circ$  is associative on  $Q$ .

Binary Operations Ex 3.2 Q4(vi)

Commutative:

Let  $a, b \in Q$ , then

$$a * b = ab^2 \neq ba^2 = b * a$$

$$\Rightarrow a * b \neq b * a$$

$\therefore *$  is not commutative on  $Q$

Associativity:

Let  $a, b, c \in Q$ , then

$$(a * b) * c = ab^2 * c = ab^2c^2 \quad \text{--- (i)}$$

$$\& \quad a * (b * c) = a * bc^2 = a(bc^2)^2 \quad \text{--- (ii)}$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$\therefore *$  is not associative on  $Q$

Binary Operations Ex 3.2 Q4(vii)

Commutativity:

Let  $a, b \in Q$ , then

$$a * b = a + ab \quad \text{--- (i)}$$

$$b * a = b + ab \quad \text{--- (ii)}$$

From (i) & (ii)

$$a * b \neq b * a$$

$\Rightarrow$   $*$  is not commutative on  $Q$

Associativity:

Let  $a, b, c \in Q$ , then

$$(a * b) * c = (a + ab) * c = a + ab + ac + abc \quad \text{--- (i)}$$

$$\begin{aligned} a * (b * c) &= a * (b + bc) \\ &= a + ab + abc \end{aligned} \quad \text{--- (ii)}$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$\Rightarrow$   $*$  is not associative on  $Q$

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