Differentiation Ex 11.7 Q25

Consider the given functions,

$x = \cos t\left(3 - 2\cos^2 t\right)$

$x = 3\cos t - 2\cos^3 t$

$\frac{dx}{dt} = -3\sin t + 6\cos^2 t \sin t \ldots.(1)$

$y = \sin t \ \left(3 - 2\sin^2 t\right)$

$y = 3\sin t - 2\sin^3 t$

$\frac{dy}{dt} = 3\cos t - 6\sin^2 t \cos t \ldots.(2)$

$\frac{dy}{dx} = \left(\frac{dy}{dt}\right) / \left(\frac{dx}{dt}\right)$ ....[From equations (1) and (2)]

$= \frac{3\cos t - 6\sin^2 t \cos t}{-3\sin t + 6\cos^2 t \sin t}$

$= \frac{3\cos t\left(1 - 2\sin^2 t\right)}{3\sin t\left(2\cos^2 t - 1\right)}$

$= \cot t \frac{\left(1 - 2\left(1 - \cos^2 t\right)\right)}{\left(2\cos^2 t - 1\right)}$

$= \cot t$

$\left.\frac{dy}{dx}\right|_{\pi/4} = \cot\frac{\pi}{4} = 1$

Differentiation Ex 11.7 Q26

$x = \frac{1 + \log t}{t^2}, y = \frac{3 + 2\log t}{t}$

$\frac{dx}{dt} = \frac{t^2\left(\frac{1}{t}\right) - (1 + \log t)(2t)}{t^4} = \frac{t - 2t - 2t\log t}{t^4} = \frac{-2\log t - 1}{t^3}$

$\frac{dy}{dt} = \frac{t\left(\frac{2}{t}\right) - (3 + 2\log t)(1)}{t^2} = \frac{2 - 3 - 2\log t}{t^2} = \frac{-2\log t - 1}{t^2}$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{-2\log t - 1}{t^2}}{\frac{-2\log t - 1}{t^3}} = t$

Differentiation Ex 11.7 Q27

$x = 3\sin t - \sin 3t, y = 3\cos t - \cos 3t$

$\frac{dx}{dt} = 3\cos t - 3\cos 3t$

$\frac{dy}{dt} = -3\sin t + 3\sin 3t$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-3\sin t + 3\sin 3t}{3\cos t - 3\cos 3t}$

When $t = \frac{\pi}{3}$

$\frac{dy}{dx} = \frac{-3\sin\left(\frac{\pi}{3}\right) + 3\sin(\pi)}{3\cos\left(\frac{\pi}{3}\right) - 3\cos(\pi)} = \frac{-3\times\frac{\sqrt{3}}{2} + 0}{3\times\frac{1}{2} - 3(-1)} = -\frac{1}{\sqrt{3}}$

Differentiation Ex 11.7 Q28

$\sin x = \frac{2t}{1+t^2}, \tan y = \frac{2t}{1-t^2}$

$\Rightarrow x = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$ and $y = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$

$\frac{dx}{dt} = \frac{1}{\sqrt{1-\left(\frac{2t}{1+t^2}\right)^2}} \times \frac{2\left(1+t^2\right) - (2t)(2t)}{\left(1+t^2\right)^2}$

$\frac{dx}{dt} = \frac{2}{\left(1+t^2\right)}$

$\frac{dy}{dt} = \frac{1}{\left(\frac{2t}{1-t^2}\right)^2 + 1} \times \frac{2\left(1-t^2\right) - (2t)(-2t)}{\left(1-t^2\right)^2}$

$\frac{dy}{dt} = \frac{2}{\left(1+t^2\right)}$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{2}{\left(1+t^2\right)}}{\frac{2}{\left(1+t^2\right)}} = 1$

********** END **********