Exercise 10A

Question 44:

On putting $\left(\dfrac{2x-1}{x+3}\right) = y$, the given equation becomes

$$2y - \dfrac{3}{y} = 5 \Rightarrow 2y^2 - 3 = 5y$$

$$\Rightarrow 2y^2 - 5y - 3 = 0$$

$$\Rightarrow 2y^2 - 6y + y - 3 = 0$$

$$\Rightarrow 2(y-3) + 1(y-3) = 0$$

$$\Rightarrow (y-3)(2y+1) = 0$$

$$y = 3 \quad \text{or} \quad y = \dfrac{-1}{2}$$

Case I:

$$y = 3 \Rightarrow \dfrac{2x-1}{x+3} = 3$$

$$\Rightarrow 2x - 1 = 3x + 9$$

$$\Rightarrow x = -10$$

Case II:

$$y = \dfrac{-1}{2} \Rightarrow \dfrac{2x-1}{x+3} = \dfrac{-1}{2}$$

$$\Rightarrow 2(2x-1) = -1(x+3)$$

$$\Rightarrow 4x - 2 = -x - 3$$

$$\Rightarrow 5x = -1 \Rightarrow x = \dfrac{-1}{5}$$

Hence, -10 and $\dfrac{-1}{5}$ are the roots of the given equation

Question 45:

Putting $\left(\dfrac{4x-3}{2x+1}\right) = y$, the given equation becomes

$$y - \dfrac{10}{y} = 3 \Rightarrow y^2 - 10 = 3y$$

$$\Rightarrow y^2 - 3y - 10 = 0$$

$$\Rightarrow y^2 - 5y + 2y - 10 = 0$$

$$\Rightarrow y(y-5) + 2(y-5) = 0$$

$$\Rightarrow (y-5)(y+2) = 0$$

$$y - 5 = 0 \quad \text{or} \quad y + 2 = 0$$

$$y = 5 \quad \text{or} \quad y = -2$$

Case I

$$y = 5 \Rightarrow \dfrac{4x-3}{2x+1} = 5 \Rightarrow 4x - 3 = 10x + 5$$

$$-6x = 8 \Rightarrow x = \dfrac{-4}{3}$$

Case II

$$y = -2 \Rightarrow \dfrac{4x-3}{2x+1} = -2 \Rightarrow 4x - 3 = -4x - 2$$

$$8x = 1 \Rightarrow x = \dfrac{1}{8}$$

Hence, $\dfrac{-4}{3}$ and $\dfrac{1}{8}$ are the roots of given equation

Question 46:

The given equation

$$\left(\frac{a}{x-b} - 1\right) + \left(\frac{b}{x-a} - 1\right) = 0$$

$$\Rightarrow \frac{(a-x+b)}{(x-b)} + \frac{(b-x+a)}{(x-a)} = 0$$

$$\Rightarrow (a-x+b)\left[\frac{1}{(x-b)} + \frac{1}{(x-a)}\right] = 0$$

$$\Rightarrow (a-x+b)\left[\frac{2x-(a+b)}{(x-a)(x-b)}\right] = 0$$

$$\Rightarrow (a-x+b)\left[2x-(a+b)\right] = 0$$

$$\Rightarrow x = (a+b) \quad \text{or} \quad x = \frac{(a+b)}{2}$$

Hence, $(a+b)$ and $\dfrac{(a+b)}{2}$ is the roots of the given equation

Question 47:

$$\frac{a}{(ax-1)} + \frac{b}{(bx-1)} = (a+b), \qquad \left(x \neq \frac{1}{a}, \frac{1}{b}\right)$$

$$\Rightarrow \left[\frac{a}{(ax-1)} - b\right] + \left[\frac{b}{(bx-1)} - a\right] = 0$$

$$\Rightarrow \frac{(a-abx+b)}{(ax-1)} + \frac{(a-abx+b)}{(bx-1)} = 0$$

$$\Rightarrow (a-abx+b)\left[\frac{1}{ax-1} + \frac{1}{bx-1}\right] = 0$$

$$\Rightarrow (a-abx+b)\left[x(b+a)-2\right] = 0$$

$$\Rightarrow (a-abx+b) = 0 \quad \text{or} \quad x(b+a)-2 = 0$$

$$x = \frac{a+b}{ab} \quad \text{or} \quad x = \frac{2}{(b+a)}$$

Hence, $\dfrac{a+b}{ab}, \dfrac{2}{(a+b)}$ are the roots of the given equation

Question 48:

$$3^{x+2} + 3^{-x} = 10$$

$$3^x \cdot 3^2 + 3^{-x} = 10$$

$$\Rightarrow 9y + \frac{1}{y} = 10 \text{ where } 3^x = y$$

$$\Rightarrow 9y^2 - 10y + 1 = 0$$

$$\Rightarrow 9y^2 - 9y - y + 1 = 0$$

$$\Rightarrow 9y(y-1) - 1(y-1) = 0$$

$$\Rightarrow (9y - 1)(y - 1) = 0$$

$$\Rightarrow 9y - 1 = 0 \text{ or } y - 1 = 0$$

$$\Rightarrow y = \frac{1}{9} \text{ or } y = 1$$

$$\text{If } 3^x = \frac{1}{9} \Rightarrow 3^x = (3)^{-2} \Rightarrow x = -2$$

$$\text{If } 3^x = 1 = 3^0 \Rightarrow x = 0$$

Hence, -2,0 are the roots of the given equation

Question 49:

$4^{(x+1)} + 4^{(1-x)} = 10$

$4^x.4^1 + 4^1.4^{-x} = 10$

$4y + \dfrac{4}{y} = 10$ where $4^x = y$

$4y^2 - 10y + 4 = 0$

$\Rightarrow 4y^2 - 8y - 2y + 4 = 0$

$\Rightarrow 4y(y-2) - 2(y-2) = 0$

$\Rightarrow (y-2)(4y-2) = 0$

$y - 2 = 0$ or $4y - 2 = 0$

$y = 2$ and $y = \dfrac{2}{4} = \dfrac{1}{2}$

$y = 2$ or $y = \dfrac{1}{2}$

In caseI $4^x = 2 \Rightarrow (2)^{2x} = (2)^1 \Rightarrow 2x = 1$

$x = \dfrac{1}{2}$

In case II $4^x = \dfrac{1}{2} \Rightarrow (2)^{2x} = \left(\dfrac{1}{2}\right)^1 = (2)^{2x} = (2)^{-1}$

∴ $x = -\dfrac{1}{2}$

Hence, x=1/2 and x=1/2 are the roots of the given equation

Question 50:

$2^{2x} - 3.2^{(x+2)} + 32 = 0$

$2^{2x} - 3.2^x.2^2 + 32 = 0$

$y^2 - 12y + 32 = 0$ where $2^x = y$

$y^2 - 8y - 4y + 32 = 0$

$y(y-8) - 4(y-8) = 0$

$(y-8)(y-4) = 0$

$y - 8 = 0$ or $y - 4 = 0$

$y = 8$ or $y = 4$

$2^x = 8 \Rightarrow 2^x = (2)^3 \Rightarrow x = 3$

$2^x = 4 \Rightarrow 2^x = (2)^2 \Rightarrow x = 2$

Hence, 3 and 2 are the roots of the given equation.

********** END *********