



Continuity Ex 9.1 Q41

It is given that function is continuous at  $x = 0$ . then,

$$\text{LHL} = \text{RHL} = f(0) \dots (1)$$

Now,

$$f(0) = 2 \cdot 0 + k = k$$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} 2(-h)^2 + k = k$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0 + h) = \lim_{h \rightarrow 0} 2(h^2) + k = k$$

Thus, the function will be continuous for any  $k \in \mathbb{R}$ .

Continuity Ex 9.1 Q42

$$\text{The given function } f \text{ is } f(x) = \begin{cases} \lambda(x^2 - 2x), & \text{if } x \leq 0 \\ 4x + 1, & \text{if } x > 0 \end{cases}$$

If  $f$  is continuous at  $x = 0$ , then

$$\begin{aligned} \lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^+} f(x) = f(0) \\ \Rightarrow \lim_{x \rightarrow 0^-} \lambda(x^2 - 2x) &= \lim_{x \rightarrow 0^+} (4x + 1) = \lambda(0^2 - 2 \times 0) \\ \Rightarrow \lambda(0^2 - 2 \times 0) &= 4 \times 0 + 1 = 0 \\ \Rightarrow 0 &= 1 = 0, \text{ which is not possible} \end{aligned}$$

Therefore, there is no value of  $\lambda$  for which  $f$  is continuous at  $x = 0$

At  $x = 1$ ,

$$f(1) = 4x + 1 = 4 \times 1 + 1 = 5$$

$$\begin{aligned} \lim_{x \rightarrow 1} (4x + 1) &= 4 \times 1 + 1 = 5 \\ \therefore \lim_{x \rightarrow 1} f(x) &= f(1) \end{aligned}$$

Therefore, for any values of  $\lambda$ ,  $f$  is continuous at  $x = 1$

Continuity Ex 9.1 Q43

The function will be continuous at  $x = 2$

$$\text{if } \text{LHL} = \text{RHL} = f(2) \dots (1)$$

Now,

$$f(2) = k$$

$$\text{LHL} = \lim_{x \rightarrow 2^-} f(x) = \lim_{h \rightarrow 0} f(2-h) = \lim_{h \rightarrow 0} 2(2-h) + 1 = 5.$$

Thus, using (1) we get,

$$k = 5$$

Continuity Ex 9.1 Q44

It is given that the function is continuous at  $x = \frac{\pi}{2}$

$$\therefore \text{LHL} = \text{RHL} = f\left(\frac{\pi}{2}\right) \dots (1)$$

Now,

$$f\left(\frac{\pi}{2}\right) = a$$

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow \frac{\pi}{2}^-} f(x) = \lim_{h \rightarrow 0} f\left(\frac{\pi}{2} - h\right) = \lim_{h \rightarrow 0} \frac{1 - \sin^3\left(\frac{\pi}{2} - h\right)}{3 \cos^2\left(\frac{\pi}{2} - h\right)} = \lim_{h \rightarrow 0} \frac{1 - \cos^3 h}{3 \sin^2 h} \\ &= \lim_{h \rightarrow 0} \frac{(1 - \cosh)(1 + \cos^2 h + \cosh)}{3 \sin^2 h} \\ &= \lim_{h \rightarrow 0} \frac{2 \sin^2 \frac{h}{2} (1 + \cos^2 h + \cosh)}{3 \sin^2 h} \\ &= \lim_{h \rightarrow 0} \frac{2 \left(\frac{\sin \frac{h}{2}}{\frac{h}{2}}\right)^2 \times \frac{h^2}{4} \cdot (1 + \cos^2 h + \cosh)}{3 \left(\frac{\sinh}{h}\right)^2 \cdot h^2} \\ &= \lim_{h \rightarrow 0} \frac{2 \cdot \frac{1}{4} (1 + \cos^2 h + \cosh)}{3} = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{RHL} &= \lim_{x \rightarrow \frac{\pi}{2}^+} f(x) = \lim_{h \rightarrow 0} f\left(\frac{\pi}{2} + h\right) = \lim_{h \rightarrow 0} \frac{b \left(1 - \sin\left(\frac{\pi}{2} + h\right)\right)}{\left(\pi - 2\left(\frac{\pi}{2} + h\right)\right)^2} = \lim_{h \rightarrow 0} \frac{b(1 - \cosh)}{(\pi - \pi - 2h)^2} \\ &= \lim_{h \rightarrow 0} \frac{b \cdot 2 \sin^2 \frac{h}{2}}{(2h)^2} \\ &= \lim_{h \rightarrow 0} \frac{b}{2} \left(\frac{\sin \frac{h}{2}}{\frac{h}{2}}\right)^2 \times \frac{1}{4} \\ &= \lim_{h \rightarrow 0} \frac{b}{8} = \frac{b}{8} \end{aligned}$$

Thus, using (1) we get,

$$a = \frac{1}{2}$$

And

$$\frac{b}{8} = \frac{1}{2} \Rightarrow b = 4$$

Thus,  $a = \frac{1}{2}$  and  $b = 4$

\*\*\*\*\* END \*\*\*\*\*

