Definite Integrals Ex 20.1 Q35

We have,

$$\int_1^e \frac{\log x}{x}\,dx$$

Let $\log x = t$

$$\Rightarrow \frac{1}{x}dx = dt$$

Now,

$x = 1 \Rightarrow t = 0$

$x = e \Rightarrow t = 1$

$$\therefore \int_1^e \frac{\log x}{x}\,dx = \int_0^1 t\,dt$$

$$= \left[\frac{t^2}{2}\right]_0^1$$

$$= \frac{1}{2}$$

$$\therefore \int_1^e \frac{\log x}{x}\,dx = \frac{1}{2}$$

Definite Integrals Ex 20.1 Q36

We have,

$$\int_e^{e^2}\left\{\frac{1}{\log x}-\frac{1}{(\log x)^2}dx\right\}$$

$$I=\int\frac{1}{\log x}.1dx=\frac{1}{\log x}\int dx-\int\left(\int dx\right).\frac{d}{dx}\left(\frac{1}{\log x}\right)dx=\frac{x}{\log x}+\int\frac{1}{(\log x)^2}.x.\frac{1}{x}dx$$

$$=\frac{x}{\log x}+\int\frac{dx}{(\log x)^2}$$

$$\int_e^{e^2}\left\{\frac{1}{\log x}-\frac{1}{(\log x)^2}dx\right\}=\left[\frac{x}{\log x}\right]_e^{e^2}+\int_e^{e^2}\frac{dx}{(\log x)^2}-\int_e^{e^2}\frac{dx}{(\log x)^2}$$

$$=\left[\frac{x}{\log x}\right]_e^{e^2}$$

$$=\frac{e^2}{2}-e$$

Definite Integrals Ex 20.1 Q37

We have,

$$\int_1^2\frac{x+3}{x(x+2)}dx$$

$$=\int_1^2\frac{x}{x(x+2)}dx+\int_1^2\frac{3}{x(x+2)}dx$$

$$=\int_1^2\frac{dx}{(x+2)}+\int_1^2\frac{3}{x(x+2)}dx$$

$$=\left[\log(x+2)\right]_1^2+\frac{3}{2}\int_1^2\frac{1}{x}-\frac{1}{x+2}dx \qquad \left[\text{using partial fraction}\right]$$

$$=\left[\log(x+2)\right]_1^2+\left[\frac{3}{2}\log x-\frac{3}{2}\log(x+2)\right]_1^2$$

$$=\left[\frac{3}{2}\log x-\frac{1}{2}\log(x+2)\right]_1^2$$

$$=\frac{1}{2}\left[3\log 2-\log 4+\log 3\right]$$

$$=\frac{1}{2}\left[3\log 2-2\log 2+\log 3\right] \qquad \left[\because \log 4=2\log 2\right]$$

$$=\frac{1}{2}\left[\log 2+\log 3\right]$$

$$=\frac{1}{2}\left[\log 6\right]$$

$$=\frac{1}{2}\log 6$$

$$\therefore \int_1^2\frac{x+3}{x(x+2)}dx=\frac{1}{2}\log 6$$

Definite Integrals Ex 20.1 Q38

Let $I = \int_0^1 \frac{2x+3}{5x^2+1}dx$

$$\int \frac{2x+3}{5x^2+1}dx = \frac{1}{5}\int \frac{5(2x+3)}{5x^2+1}dx$$

$$= \frac{1}{5}\int \frac{10x+15}{5x^2+1}dx$$

$$= \frac{1}{5}\int \frac{10x}{5x^2+1}dx + 3\int \frac{1}{5x^2+1}dx$$

$$= \frac{1}{5}\int \frac{10x}{5x^2+1}dx + 3\int \frac{1}{5\left(x^2+\frac{1}{5}\right)}dx$$

$$= \frac{1}{5}\log\left(5x^2+1\right) + \frac{3}{5}\cdot\frac{1}{\frac{1}{\sqrt{5}}}\tan^{-1}\frac{x}{\frac{1}{\sqrt{5}}}$$

$$= \frac{1}{5}\log\left(5x^2+1\right) + \frac{3}{\sqrt{5}}\tan^{-1}\left(\sqrt{5}x\right)$$

$$= \mathrm{F}(x)$$

*********** END **********