Mathematical Induction Ex 12.2 Q41

$$\frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{2n}>\frac{13}{24},$$

Using induction we first show this is true for n=2:

$$\frac{1}{3}+\frac{1}{4}=\frac{7}{12}=\frac{14}{24}>\frac{13}{24}(True)$$

Now lets assume it is true for some n=k,

$$S_k=\frac{1}{k+1}+\frac{1}{k+2}+\ldots+\frac{1}{2k}>\frac{13}{24}$$

Finally we need to prove that this implies

it is also true for n=k+1:

$$S_{k+1}=\frac{1}{k+2}+\frac{1}{k+3}+\ldots+\frac{1}{2k+2}$$

$$=\frac{-1}{k+1}+\frac{1}{k+1}+\frac{1}{k+2}+\frac{1}{k+3}+\ldots\frac{1}{2k}+\frac{1}{2k+1}+\frac{1}{2k+2}$$

$$=\frac{-1}{k+1}+S_k+\frac{1}{2k+1}+\frac{1}{2k+2}$$

$$=S_k+\frac{1}{2(2k+1)(k+1)}$$

$$>S_k$$

$$\therefore S_{k+1}>\frac{13}{24}$$

Mathematical Induction Ex 12.2 Q42

$$a_1 = \frac{1}{2}\left(a_0 + \frac{A}{a_0}\right), A_2 = \frac{1}{2}\left(a_1 + \frac{A}{a_1}\right) \text{ and } a_{n+1} = \frac{1}{2}\left(a_n + \frac{A}{a_n}\right)$$

Let $\quad P(n): \dfrac{a_n - \sqrt{A}}{a_n + \sqrt{A}} = \left(\dfrac{a_1 - \sqrt{A}}{a_1 + \sqrt{A}}\right)^{2^{n-1}}$

For $n = 1$

$$\frac{a_1 - \sqrt{A}}{a_1 + \sqrt{A}} = \left(\frac{a_1 - \sqrt{A}}{a_1 + \sqrt{A}}\right)^{2^{n-1}}$$

$$\frac{a_1 - \sqrt{A}}{a_1 + \sqrt{A}} = \left(\frac{a_1 - \sqrt{A}}{a_1 + \sqrt{A}}\right)$$

$\Rightarrow \quad P(n)$ is true for $n = 1$

Let $P(n)$ is true for $n = k$

$$\frac{a_k - \sqrt{A}}{a_k + \sqrt{A}} = \left(\frac{a_1 - \sqrt{A}}{a_1 + \sqrt{A}}\right) \quad \text{---(i)}$$

We have to show that

$$\frac{a_{k+1} - \sqrt{A}}{a_{k+1} + \sqrt{A}} = \left(\frac{a_1 - \sqrt{A}}{a_1 + \sqrt{A}}\right)^{2^k}$$

$$\left(\frac{a_{k+1} - \sqrt{A}}{a_{k+1} + \sqrt{A}}\right)^{2^0}$$

$$= \left[\frac{\frac{1}{2}\left(a_k + \frac{A}{a_k}\right) - \sqrt{A}}{\frac{1}{2\left(a_k + \frac{A}{a_k}\right)} + \sqrt{A}}\right]^{2^0}$$

$$= \left[\frac{(a_k)^2 + A - 2ak\sqrt{A}}{(ak)^2 + A + 2a_k\sqrt{A}}\right]^{2^0}$$

$$= \frac{\left(a_k - \sqrt{A}\right)^2}{\left(a_k + \sqrt{A}\right)^2}$$

$$= \left[\frac{a_k - \sqrt{A}}{a_k + \sqrt{A}}\right]^{2^1}$$

$$= \left[\frac{a_1 - \sqrt{A}}{a_1 + \sqrt{A}}\right]^{2^k}$$

$\Rightarrow \quad P(n)$ is true for $n = k + 1$

$\Rightarrow \quad P(n)$ is true for all $n \in NE$ by PMI

Mathematical Induction Ex 12.2 Q43

$P(n): 2^n \geq 3n$

It is given that $P(r)$ is true, so

$2^r \geq 3r$ $\quad ---(1)$

Multiplying both the sides by 2,

$2^r.2 \geq 3r.2$
$2^{r+1} \geq 6r$
$2^{r+1} \geq 3r + 3r$
$2^{r+1} \geq 3 + 3r$ $\quad$ $[\text{Since } 3r \geq 3,\ 6r \geq 3 + 3r]$

$2^{r+1} \geq 3(r+1)$

So, $P(r+1)$ is true

But for $r = 1$
$2 \geq 3$

It is true, so

$P(n)$ is not true for all $n \in N$ by PMI

*********** END **********