Question 11.25:

Estimating the following two numbers should be interesting. The first number will tell you why radio engineers do not need to worry much about photons! The second number tells you why our eye can never 'count photons', even in barely detectable light.

(a) The number of photons emitted per second by a Medium wave transmitter of 10 kW power, emitting radiowaves of wavelength 500 m.

(b) The number of photons entering the pupil of our eye per second corresponding to

the minimum intensity of white light that we humans can perceive ($\sim 10^{-10}$ W m^{-2}). Take

the area of the pupil to be about 0.4 cm^2, and the average frequency of white light to be about 6×10^{14} Hz.

Answer

(a) Power of the medium wave transmitter, P = 10 kW = 10^4 W = 10^4 J/s

Hence, energy emitted by the transmitter per second, $E = 10^4$

Wavelength of the radio wave, λ = 500 m

The energy of the wave is given as:

$$E_1 = \frac{hc}{\lambda}$$

Where,

h = Planck's constant = 6.6×10^{-34} Js

c = Speed of light = 3×10^8 m/s

$$\therefore E_1 = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{500} = 3.96 \times 10^{-28} \text{ J}$$

Let n be the number of photons emitted by the transmitter.

$\therefore nE_1 = E$

$$n = \frac{E}{E_1}$$

$$= \frac{10^4}{3.96 \times 10^{-28}} = 2.525 \times 10^{31}$$

$$\approx 3 \times 10^{31}$$

The energy (E_1) of a radio photon is very less, but the number of photons (n) emitted per second in a radio wave is very large.

The existence of a minimum quantum of energy can be ignored and the total energy of a radio wave can be treated as being continuous.

(b) Intensity of light perceived by the human eye, $I = 10^{-10}$ W m^{-2}

Area of a pupil, $A = 0.4$ cm^2 = 0.4×10^{-4} m^2

Frequency of white light, $v = 6 \times 10^{14}$ Hz

The energy emitted by a photon is given as:

$E = hv$

Where,

h = Planck's constant = 6.6×10^{-34} Js

$\therefore E = 6.6 \times 10^{-34} \times 6 \times 10^{14}$

$= 3.96 \times 10^{-19}$ J

Let n be the total number of photons falling per second, per unit area of the pupil.

The total energy per unit for n falling photons is given as:

$E = n \times 3.96 \times 10^{-19}$ J s^{-1} m^{-2}

The energy per unit area per second is the intensity of light.

$\therefore E = I$

$n \times 3.96 \times 10^{-19} = 10^{-10}$

$$n = \frac{10^{-10}}{3.96 \times 10^{-19}}$$

$= 2.52 \times 10^8$ m^2 s^{-1}

The total number of photons entering the pupil per second is given as:

$n_A = n \times A$

$= 2.52 \times 10^8 \times 0.4 \times 10^{-4}$

$= 1.008 \times 10^4$ s^{-1}

This number is not as large as the one found in problem **(a)**, but it is large enough for the human eye to never see the individual photons.

Question 11.26:

Ultraviolet light of wavelength 2271 Å from a 100 W mercury source irradiates a photo-cell made of molybdenum metal. If the stopping potential is −1.3 V, estimate the work function of the metal. How would the photo-cell respond to a high intensity (~10^5 W m^{-2}) red light of wavelength 6328 Å produced by a He-Ne laser?

Answer

Wavelength of ultraviolet light, λ = 2271 Å = 2271×10^{-10} m

Stopping potential of the metal, V_0 = 1.3 V

Planck's constant, $h = 6.6 \times 10^{-34}$ J

Charge on an electron, $e = 1.6 \times 10^{-19}$ C

Work function of the metal = ϕ_0

Frequency of light = v

We have the photo-energy relation from the photoelectric effect as:

$\phi_0 = hv - eV_0$

$$= \frac{hc}{\lambda} - eV_0$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{2271 \times 10^{-10}} - 1.6 \times 10^{-19} \times 1.3$$

$$= 8.72 \times 10^{-19} - 2.08 \times 10^{-19}$$

$$= 6.64 \times 10^{-19} \text{ J}$$

$$= \frac{6.64 \times 10^{-19}}{1.6 \times 10^{-19}} = 4.15 \text{ eV}$$

Let v_0 be the threshold frequency of the metal.

$\therefore \phi_0 = hv_0$

$$v_0 = \frac{\phi_0}{h}$$

$$= \frac{6.64 \times 10^{-19}}{6.6 \times 10^{-34}} = 1.006 \times 10^{15} \text{ Hz}$$

Wavelength of red light, $\lambda_r = 6328 \text{ Å} = 6328 \times 10^{-10}$ m

$\therefore$Frequency of red light, $v_r = \frac{c}{\lambda_r}$

$$= \frac{3 \times 10^{8}}{6328 \times 10^{-10}} = 4.74 \times 10^{14} \text{ Hz}$$

Since $v_0 > v_r$, the photocell will not respond to the red light produced by the laser.

Question 11.27:

Monochromatic radiation of wavelength 640.2 nm (1nm = 10^{-9} m) from a neon lamp irradiates photosensitive material made of caesium on tungsten. The stopping voltage is measured to be 0.54 V. The source is replaced by an iron source and its 427.2 nm line irradiates the same photo-cell. Predict the new stopping voltage.

Answer

Wavelength of the monochromatic radiation, λ = 640.2 nm

= 640.2×10^{-9} m

Stopping potential of the neon lamp, V_0 = 0.54 V

Charge on an electron, $e = 1.6 \times 10^{-19}$ C

Planck's constant, $h = 6.6 \times 10^{-34}$ Js

Let ϕ_0 be the work function and v be the frequency of emitted light.

We have the photo-energy relation from the photoelectric effect as:

$eV_0 = hv - \phi_0$

$$\phi_0 = \frac{hc}{\lambda} - eV_0$$

$$= \frac{6.6\times 10^{-34} \times 3 \times 10^{8}}{640.2 \times 10^{-9}} - 1.6 \times 10^{-19} \times 0.54$$

$$= 3.093 \times 10^{-19} - 0.864 \times 10^{-19}$$

$$= 2.229 \times 10^{-19} \text{ J}$$

$$= \frac{2.229 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.39 \text{ eV}$$

Wavelength of the radiation emitted from an iron source, λ' = 427.2 nm

= 427.2×10^{-9} m

Let V_0' be the new stopping potential. Hence, photo-energy is given as:

$$eV_0' = \frac{hc}{\lambda'} - \phi_0$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{427.2 \times 10^{-9}} - 2.229 \times 10^{-19}$$

$$= 4.63 \times 10^{-19} - 2.229 \times 10^{-19}$$

$$= 2.401 \times 10^{-19} \text{ J}$$

$$= \frac{2.401 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.5 \text{ eV}$$

Hence, the new stopping potential is 1.50 eV.

Question 11.28:

A mercury lamp is a convenient source for studying frequency dependence of photoelectric emission, since it gives a number of spectral lines ranging from the UV to the red end of the visible spectrum. In our experiment with rubidium photo-cell, the following lines from a mercury source were used:

λ_1 = 3650 Å, λ_2= 4047 Å, λ_3= 4358 Å, λ_4= 5461 Å, λ_5= 6907 Å,

The stopping voltages, respectively, were measured to be:

V_{01} = 1.28 V, V_{02} = 0.95 V, V_{03} = 0.74 V, V_{04} = 0.16 V, V_{05} = 0 V

Determine the value of Planck's constant *h*, the threshold frequency and work function for the material.

[*Note:* You will notice that to get *h* from the data, you will need to know *e* (which you can take to be 1.6×10^{-19} C). Experiments of this kind on Na, Li, K, etc. were performed by Millikan, who, using his own value of *e* (from the oil-drop experiment) confirmed Einstein's photoelectric equation and at the same time gave an independent estimate of the value of *h*.]

Answer

Einstein's photoelectric equation is given as:

$eV_0 = h\nu - \phi_0$

$$V_0 = \frac{h}{e}\nu - \frac{\phi_0}{e} \qquad \ldots (1)$$

Where,

V_0 = Stopping potential

h = Planck's constant

e = Charge on an electron

ν = Frequency of radiation

ϕ_0 = Work function of a material

It can be concluded from equation (1) that potential V_0 is directly proportional to frequency ν.

Frequency is also given by the relation:

$$\nu = \frac{\text{Speed of light } (c)}{\text{Wavelength } (\lambda)}$$

This relation can be used to obtain the frequencies of the various lines of the given wavelengths.

$$\nu_1 = \frac{c}{\lambda_1} = \frac{3 \times 10^8}{3650 \times 10^{-10}} = 8.219 \times 10^{14} \text{ Hz}$$

$$\nu_2 = \frac{c}{\lambda_2} = \frac{3 \times 10^8}{4047 \times 10^{-10}} = 7.412 \times 10^{14} \text{ Hz}$$

$$\nu_3 = \frac{c}{\lambda_3} = \frac{3 \times 10^8}{4358 \times 10^{-10}} = 6.884 \times 10^{14} \text{ Hz}$$

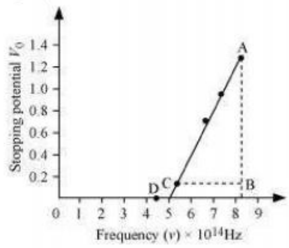
$$\nu_4 = \frac{c}{\lambda_4} = \frac{3 \times 10^8}{5461 \times 10^{-10}} = 5.493 \times 10^{14} \text{ Hz}$$

$$\nu_5 = \frac{c}{\lambda_5} = \frac{3 \times 10^8}{6907 \times 10^{-10}} = 4.343 \times 10^{14} \text{ Hz}$$

The given quantities can be listed in tabular form as:

Frequency × 10^{14} Hz	8.219	7.412	6.884	5.493	4.343
Stopping potential V_0	1.28	0.95	0.74	0.16	0

The following figure shows a graph between ν and V_0.



It can be observed that the obtained curve is a straight line. It intersects the ν-axis at 5 × 10^{14} Hz, which is the threshold frequency (ν_0) of the material. Point D corresponds to a frequency less than the threshold frequency. Hence, there is no photoelectric emission for the λ_5 line, and therefore, no stopping voltage is required to stop the current.

Slope of the straight line = $\frac{AB}{CB} = \frac{1.28 - 0.16}{(8.214 - 5.493) \times 10^{14}}$

From equation (1), the slope $\frac{h}{e}$ can be written as:

$$\frac{h}{e} = \frac{1.28 - 0.16}{(8.214 - 5.493) \times 10^{14}}$$

$$\therefore h = \frac{1.12 \times 1.6 \times 10^{-19}}{2.726 \times 10^{14}}$$

$$= 6.573 \times 10^{-34} \text{ Js}$$

The work function of the metal is given as:

$\phi_0 = h\nu_0$

$= 6.573 \times 10^{-34} \times 5 \times 10^{14}$

$= 3.286 \times 10^{-19}$ J

$= \frac{3.286 \times 10^{-19}}{1.6 \times 10^{-18}}$ = 2.054 eV

Question 11.29:

The work function for the following metals is given:

Na: 2.75 eV; K: 2.30 eV; Mo: 4.17 eV; Ni: 5.15 eV. Which of these metals will not give photoelectric emission for a radiation of wavelength 3300 Å from a He-Cd laser placed 1 m away from the photocell? What happens if the laser is brought nearer and placed 50 cm away?

Answer

Mo and Ni will not show photoelectric emission in both cases

Wavelength for a radiation, λ = 3300 Å = 3300×10^{-10} m

Speed of light, $c = 3 \times 10^8$ m/s

Planck's constant, $h = 6.6 \times 10^{-34}$ Js

The energy of incident radiation is given as:

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{3300 \times 10^{-10}} = 6 \times 10^{-19} \text{ J}$$

$$= \frac{6 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.158 \text{ eV}$$

It can be observed that the energy of the incident radiation is greater than the work function of Na and K only. It is less for Mo and Ni. Hence, Mo and Ni will not show photoelectric emission.

If the source of light is brought near the photocells and placed 50 cm away from them, then the intensity of radiation will increase. This does not affect the energy of the radiation. Hence, the result will be the same as before. However, the photoelectrons emitted from Na and K will increase in proportion to intensity.

Question 11.30:

Light of intensity 10^{-5} W m^{-2} falls on a sodium photo-cell of surface area 2 cm^2. Assuming that the top 5 layers of sodium absorb the incident energy, estimate time required for photoelectric emission in the wave-picture of radiation. The work function for the metal is given to be about 2 eV. What is the implication of your answer?

Answer

Intensity of incident light, $I = 10^{-5}$ W m^{-2}

Surface area of a sodium photocell, A = 2 cm^2 = 2×10^{-4} m^2

Incident power of the light, $P = I \times A$

$= 10^{-5} \times 2 \times 10^{-4}$

$= 2 \times 10^{-9}$ W

Work function of the metal, ϕ_0 = 2 eV

$= 2 \times 1.6 \times 10^{-19}$

$= 3.2 \times 10^{-19}$ J

Number of layers of sodium that absorbs the incident energy, n = 5

We know that the effective atomic area of a sodium atom, A_e is 10^{-20} m^2.

Hence, the number of conduction electrons in n layers is given as:

$$n' = n \times \frac{A}{A_e}$$

$$= 5 \times \frac{2 \times 10^{-4}}{10^{-20}} = 10^{17}$$

The incident power is uniformly absorbed by all the electrons continuously. Hence, the amount of energy absorbed per second per electron is:

$$E = \frac{P}{n'}$$

$$= \frac{2 \times 10^{-9}}{10^{17}} = 2 \times 10^{-26} \text{ J/s}$$

Time required for photoelectric emission:

$$t = \frac{\phi_0}{E}$$

$$= \frac{3.2 \times 10^{-19}}{2 \times 10^{-26}} = 1.6 \times 10^{7} \text{ s} \approx 0.507 \text{ years}$$

The time required for the photoelectric emission is nearly half a year, which is not practical. Hence, the wave picture is in disagreement with the given experiment.

Question 11.31:

Crystal diffraction experiments can be performed using X-rays, or electrons accelerated through appropriate voltage. Which probe has greater energy? (For quantitative comparison, take the wavelength of the probe equal to 1 Å, which is of the order of inter-atomic spacing in the lattice) (m_e= 9.11 × 10^{-31} kg).

Answer

An X-ray probe has a greater energy than an electron probe for the same wavelength.

Wavelength of light emitted from the probe, λ = 1 Å = 10^{-10} m

Mass of an electron, m_e = 9.11 × 10^{-31} kg

Planck's constant, h = 6.6 × 10^{-34} Js

Charge on an electron, e = 1.6 × 10^{-19} C

The kinetic energy of the electron is given as:

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