Exercise 7.4 : Solutions of Questions on Page Number : **315**

Q1 : $\dfrac{3x^2}{x^6+1}$

Answer :

Let $x^3 = t$

$\therefore 3x^2\,dx = dt$

$$\Rightarrow \int \frac{3x^2}{x^6+1}dx = \int \frac{dt}{t^2+1}$$
$$= \tan^{1} t + C$$
$$= \tan^{-1}\left(x^3\right) + C$$

Q2 : $\dfrac{1}{\sqrt{1+4x^2}}$

Answer :

Let $2x = t$

$\therefore 2dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{1+4x^2}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{1+t^2}}$$
$$= \frac{1}{2}\left[\log\left|t+\sqrt{t^2+1}\right|\right] + C \qquad \left[\int \frac{1}{\sqrt{x^2+a^2}}dt = \log\left|x+\sqrt{x^2+a^2}\right|\right]$$
$$= \frac{1}{2}\log\left|2x+\sqrt{4x^2+1}\right| + C$$

Q3 : $\dfrac{1}{\sqrt{(2-x)^2+1}}$

Answer :

Let $2 - x = t$

$\Rightarrow -dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{(2-x)^2+1}}dx = -\int \frac{1}{\sqrt{t^2+1}}dt$$
$$= -\log\left|t+\sqrt{t^2+1}\right| + C \qquad \left[\int \frac{1}{\sqrt{x^2+a^2}}dt = \log\left|x+\sqrt{x^2+a^2}\right|\right]$$
$$= -\log\left|2-x+\sqrt{(2-x)^2+1}\right| + C$$
$$= \log\left|\frac{1}{(2-x)+\sqrt{x^2-4x+5}}\right| + C$$

Q4 : $\dfrac{1}{\sqrt{9-25x^2}}$

Answer :

Let $5x = t$

$\therefore 5dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{9-25x^2}}\,dx = \frac{1}{5}\int \frac{1}{9-t^2}\,dt$$
$$= \frac{1}{5}\int \frac{1}{\sqrt{3^2-t^2}}\,dt$$
$$= \frac{1}{5}\sin^{-1}\left(\frac{t}{3}\right) + C$$
$$= \frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right) + C$$

Q5 : $\frac{3x}{1+2x^4}$

Answer :

Let $\sqrt{2}x^2 = t$

$\therefore\ 2\sqrt{2}x\ dx = dt$

$$\Rightarrow \int \frac{3x}{1+2x^4}dx = \frac{3}{2\sqrt{2}}\int \frac{dt}{1+t^2}$$
$$= \frac{3}{2\sqrt{2}}\left[\tan^{-1} t\right] + C$$
$$= \frac{3}{2\sqrt{2}}\tan^{-1}\left(\sqrt{2}x^2\right) + C$$

Answer needs Correction? **Click Here**

Q6 : $\frac{x^2}{1-x^6}$

Answer :

Let $x^3 = t$

$\therefore 3x^2\ dx = dt$

$$\Rightarrow \int \frac{x^2}{1-x^6}dx = \frac{1}{3}\int \frac{dt}{1-t^2}$$
$$= \frac{1}{3}\left[\frac{1}{2}\log\left|\frac{1+t}{1-t}\right|\right] + C$$
$$= \frac{1}{6}\log\left|\frac{1+x^3}{1-x^3}\right| + C$$

Answer needs Correction? **Click Here**

Q7 : $\frac{x-1}{\sqrt{x^2-1}}$

Answer :

$$\int \frac{x-1}{\sqrt{x^2-1}}\,dx = \int \frac{x}{\sqrt{x^2-1}}\,dx - \int \frac{1}{\sqrt{x^2-1}}\,dx \qquad ...(1)$$

For $\int \frac{x}{\sqrt{x^2-1}}dx$, let $x^2 - 1 = t \Rightarrow 2x\ dx = dt$

$$\therefore \int \frac{x}{\sqrt{x^2-1}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$
$$= \frac{1}{2}\int t^{-\frac{1}{2}}dt$$
$$= \frac{1}{2}\left[2t^{\frac{1}{2}}\right]$$
$$= \sqrt{t}$$
$$= \sqrt{x^2-1}$$

From (1), we obtain

$$\int \frac{x-1}{\sqrt{x^2-1}}dx = \int \frac{x}{\sqrt{x^2-1}}dx - \int \frac{1}{\sqrt{x^2-1}}dx \qquad \left[\int \frac{1}{\sqrt{x^2-a^2}}dt = \log\left|x+\sqrt{x^2-a^2}\right|\right]$$
$$= \sqrt{x^2-1} - \log\left|x+\sqrt{x^2-1}\right| + C$$

Answer needs Correction? **Click Here**

Q8 : $\frac{x^2}{\sqrt{x^6+a^6}}$

Answer :

Let $x^3 = t$

$\Rightarrow 3x^2\ dx = dt$

$$\therefore \int \frac{x^2}{\sqrt{x^6+a^6}}dx = \frac{1}{3}\int \frac{dt}{\sqrt{t^2+\left(a^3\right)^2}}$$
$$= \frac{1}{3}\log\left|t+\sqrt{t^2+a^6}\right| + C$$
$$= \frac{1}{3}\log\left|x^3+\sqrt{x^6+a^6}\right| + C$$

Answer needs Correction? **Click Here**

Q9 : $\frac{\sec^2 x}{\sqrt{\tan^2 x+4}}$

Answer :

Let $\tan x = t$

$\therefore \sec^2 x\ dx = dt$

$$\Rightarrow \int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx = \int \frac{dt}{\sqrt{t^2 + 2^2}}$$
$$= \log\left|t + \sqrt{t^2 + 4}\right| + C$$
$$= \log\left|\tan x + \sqrt{\tan^2 x + 4}\right| + C$$

Answer needs Correction? Click Here

Q10 : $\frac{1}{\sqrt{x^2 + 2x + 2}}$

Answer :

$$\int \frac{1}{\sqrt{x^2 + 2x + 2}} dx = \int \frac{1}{\sqrt{(x+1)^2 + (1)^2}} dx$$

Let $x + 1 = t$

$\therefore\ dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{x^2 + 2x + 2}} dx = \int \frac{1}{\sqrt{t^2 + 1}} dt$$
$$= \log\left|t + \sqrt{t^2 + 1}\right| + C$$
$$= \log\left|(x+1) + \sqrt{(x+1)^2 + 1}\right| + C$$
$$= \log\left|(x+1) + \sqrt{x^2 + 2x + 2}\right| + C$$

Answer needs Correction? Click Here

Q11 :

Answer :

Answer needs Correction? Click Here

Q12 : $\frac{1}{\sqrt{7 - 6x - x^2}}$

Answer :

$7 - 6x - x^2$ can be written as $7 - (x^2 + 6x + 9 - 9)$.

Therefore,

$7 - (x^2 + 6x + 9 - 9)$

$= 16 - (x^2 + 6x + 9)$

$= 16 - (x+3)^2$

$= (4)^2 - (x+3)^2$

$$\therefore\ \int \frac{1}{\sqrt{7 - 6x - x^2}} dx = \int \frac{1}{\sqrt{(4)^2 - (x+3)^2}} dx$$

Let $x + 3 = t$

$\Rightarrow\ dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{(4)^2 - (x+3)^2}} dx = \int \frac{1}{\sqrt{(4)^2 - (t)^2}} dt$$
$$= \sin^{-1}\left(\frac{t}{4}\right) + C$$
$$= \sin^{-1}\left(\frac{x+3}{4}\right) + C$$

Answer needs Correction? Click Here

Q13 : $\frac{1}{\sqrt{(x-1)(x-2)}}$

Answer :

$(x-1)(x-2)$ can be written as $x^2 - 3x + 2$.

Therefore,

$x^2 - 3x + 2$

$= x^2 - 3x + \frac{9}{4} - \frac{9}{4} + 2$

$= \left(x - \frac{3}{2}\right)^2 - \frac{1}{4}$

$= \left(x - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2$

$$\therefore \int \frac{1}{\sqrt{(x-1)(x-2)}} dx = \int \frac{1}{\sqrt{\left(x - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}} dx$$

Let $x - \frac{3}{2} = t$

$\therefore\ dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{\left(x - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}} dx = \int \frac{1}{\sqrt{t^2 - \left(\frac{1}{2}\right)^2}} dt$$

$$= \log\left|t+\sqrt{t^2-\left(\frac{1}{2}\right)^2}\right|+C$$
$$= \log\left|\left(x-\frac{3}{2}\right)+\sqrt{x^2-3x+2}\right|+C$$

Answer needs Correction? Click Here

Q14 : $\frac{1}{\sqrt{8+3x-x^2}}$

Answer :

$8+3x-x^2$ can be written as $8-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)$.

Therefore,

$$8-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)$$
$$=\frac{41}{4}-\left(x-\frac{3}{2}\right)^2$$
$$\Rightarrow \int\frac{1}{\sqrt{8+3x-x^2}}dx=\int\frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}}dx$$

Let $x-\frac{3}{2}=t$

$\therefore\ dx=dt$

$$\Rightarrow \int\frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}}dx=\int\frac{1}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2-t^2}}dt$$
$$=\sin^{-1}\left(\frac{t}{\frac{\sqrt{41}}{2}}\right)+C$$
$$=\sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{41}}{2}}\right)+C$$
$$=\sin^{-1}\left(\frac{2x-3}{\sqrt{41}}\right)+C$$

Answer needs Correction? Click Here

Q15 : $\frac{1}{\sqrt{(x-a)(x-b)}}$

Answer :

$(x-a)(x-b)$ can be written as $x^2-(a+b)x+ab$.

Therefore,

$$x^2-(a+b)x+ab$$
$$=x^2-(a+b)x+\frac{(a+b)^2}{4}-\frac{(a+b)^2}{4}+ab$$
$$=\left[x-\left(\frac{a+b}{2}\right)\right]^2-\frac{(a-b)^2}{4}$$
$$\Rightarrow \int\frac{1}{\sqrt{(x-a)(x-b)}}dx=\int\frac{1}{\sqrt{\left\{x-\left(\frac{a+b}{2}\right)\right\}^2-\left(\frac{a-b}{2}\right)^2}}dx$$

Let $x-\left(\frac{a+b}{2}\right)=t$

$\therefore\ dx=dt$

$$\Rightarrow \int\frac{1}{\sqrt{\left\{x-\left(\frac{a+b}{2}\right)\right\}^2-\left(\frac{a-b}{2}\right)^2}}dx=\int\frac{1}{\sqrt{t^2-\left(\frac{a-b}{2}\right)^2}}dt$$
$$=\log\left|t+\sqrt{t^2-\left(\frac{a-b}{2}\right)^2}\right|+C$$
$$=\log\left|\left\{x-\left(\frac{a+b}{2}\right)\right\}+\sqrt{(x-a)(x-b)}\right|+C$$

Answer needs Correction? Click Here

Q16 : $\frac{4x+1}{\sqrt{2x^2+x-3}}$

Answer :

Let $4x+1=A\frac{d}{dx}\left(2x^2+x-3\right)+B$

$\Rightarrow 4x+1=A(4x+1)+B$

$\Rightarrow 4x+1=4Ax+A+B$

Equating the coefficients of x and constant term on both sides, we obtain

$4A=4 \Rightarrow A=1$

$A + B = 1 \Rightarrow B = 0$

Let $2x^2 + x - 3 = t$

$\therefore (4x + 1)\, dx = dt$

$$\Rightarrow \int \frac{4x+1}{\sqrt{2x^2+x-3}}dx = \int \frac{1}{\sqrt{t}}dt$$
$$= 2\sqrt{t} + C$$
$$= 2\sqrt{2x^2+x-3} + C$$

Answer needs Correction? Click Here

Q17 : $\frac{x+2}{\sqrt{x^2-1}}$

Answer :

$$\text{Let } x+2 = A\frac{d}{dx}\left(x^2-1\right)+B \qquad ...(1)$$
$$\Rightarrow x+2 = A(2x)+B$$

Equating the coefficients of x and constant term on both sides, we obtain

$$2A = 1 \Rightarrow A = \frac{1}{2}$$
$$B = 2$$

From (1), we obtain

$$(x+2) = \frac{1}{2}(2x)+2$$
$$\text{Then, } \int \frac{x+2}{\sqrt{x^2-1}}dx = \int \frac{\frac{1}{2}(2x)+2}{\sqrt{x^2-1}}dx$$
$$= \frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx + \int \frac{2}{\sqrt{x^2-1}}dx \qquad ...(2)$$
$$\text{In } \frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx, \text{ let } x^2-1 = t \Rightarrow 2xdx = dt$$
$$\frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$
$$= \frac{1}{2}\left[2\sqrt{t}\right]$$
$$= \sqrt{t}$$
$$= \sqrt{x^2-1}$$
$$\text{Then, } \int \frac{2}{\sqrt{x^2-1}}dx = 2\int \frac{1}{\sqrt{x^2-1}}dx = 2\log\left|x+\sqrt{x^2-1}\right|$$

From equation (2), we obtain

$$\int \frac{x+2}{\sqrt{x^2-1}}dx = \sqrt{x^2-1} + 2\log\left|x+\sqrt{x^2-1}\right| + C$$

Answer needs Correction? Click Here

Q18 : $\frac{6x+7}{\sqrt{(x-5)(x-4)}}$

Answer :

$$\frac{6x+7}{\sqrt{(x-5)(x-4)}} = \frac{6x+7}{\sqrt{x^2-9x+20}}$$

$$\text{Let } 6x+7 = A\frac{d}{dx}\left(x^2-9x+20\right)+B$$
$$\Rightarrow 6x+7 = A(2x-9)+B$$

Equating the coefficients of x and constant term, we obtain

$2A = 6 \Rightarrow A = 3$

$-9A + B = 7 \Rightarrow B = 34$

$\therefore 6x + 7 = 3(2x - 9) + 34$

$$\int \frac{6x+7}{\sqrt{x^2-9x+20}} = \int \frac{3(2x-9)+34}{\sqrt{x^2-9x+20}}dx$$
$$= 3\int \frac{2x-9}{\sqrt{x^2-9x+20}}dx + 34\int \frac{1}{\sqrt{x^2-9x+20}}dx$$
$$\text{Let } I_1 = \int \frac{2x-9}{\sqrt{x^2-9x+20}}dx \text{ and } I_2 = \int \frac{1}{\sqrt{x^2-9x+20}}dx$$
$$\therefore \int \frac{6x+7}{\sqrt{x^2-9x+20}} = 3I_1 + 34I_2 \qquad ...(1)$$

Then,

$$I_1 = \int \frac{2x-9}{\sqrt{x^2-9x+20}}dx$$
$$\text{Let } x^2-9x+20 = t$$
$$\Rightarrow (2x-9)dx = dt$$
$$\Rightarrow I_1 = \frac{dt}{\sqrt{t}}$$
$$I_1 = 2\sqrt{t}$$
$$I_1 = 2\sqrt{x^2-9x+20} \qquad ...(2)$$
$$\text{and } I_2 = \int \frac{1}{\sqrt{x^2-9x+20}}dx$$

$\sqrt{x^2-9x+20}$

$x^2-9x+20$ can be written as $x^2-9x+20+\frac{81}{4}-\frac{81}{4}$.

Therefore,

$$x^2-9x+20+\frac{81}{4}-\frac{81}{4}$$
$$=\left(x-\frac{9}{2}\right)^2-\frac{1}{4}$$
$$=\left(x-\frac{9}{2}\right)^2-\left(\frac{1}{2}\right)^2$$
$$\Rightarrow I_2=\int\frac{1}{\sqrt{\left(x-\frac{9}{2}\right)^2-\left(\frac{1}{2}\right)^2}}dx$$
$$I_2=\log\left|\left(x-\frac{9}{2}\right)+\sqrt{x^2-9x+20}\right| \quad ...(3)$$

Substituting equations (2) and (3) in (1), we obtain

$$\int\frac{6x+7}{\sqrt{x^2-9x+20}}dx=3\left[2\sqrt{x^2-9x+20}\right]+34\log\left[\left(x-\frac{9}{2}\right)+\sqrt{x^2-9x+20}\right]+C$$
$$=6\sqrt{x^2-9x+20}+34\log\left[\left(x-\frac{9}{2}\right)+\sqrt{x^2-9x+20}\right]+C$$

Answer needs Correction? Click Here

Q19 : $\frac{x+2}{\sqrt{4x-x^2}}$

Answer :

$$\text{Let } x+2=A\frac{d}{dx}\left(4x-x^2\right)+B$$
$$\Rightarrow x+2=A(4-2x)+B$$

Equating the coefficients of *x* and constant term on both sides, we obtain

$$-2A=1\Rightarrow A=-\frac{1}{2}$$
$$4A+B=2\Rightarrow B=4$$
$$\Rightarrow (x+2)=-\frac{1}{2}(4-2x)+4$$
$$\therefore \int\frac{x+2}{\sqrt{4x-x^2}}dx=\int\frac{-\frac{1}{2}(4-2x)+4}{\sqrt{4x-x^2}}dx$$
$$=-\frac{1}{2}\int\frac{4-2x}{\sqrt{4x-x^2}}dx+4\int\frac{1}{\sqrt{4x-x^2}}dx$$
$$\text{Let } I_1=\int\frac{4-2x}{\sqrt{4x-x^2}}dx \text{ and } I_2\int\frac{1}{\sqrt{4x-x^2}}dx$$
$$\therefore \int\frac{x+2}{\sqrt{4x-x^2}}dx=-\frac{1}{2}I_1+4I_2 \quad ...(1)$$

$$\text{Then, } I_1=\int\frac{4-2x}{\sqrt{4x-x^2}}dx$$
$$\text{Let } 4x-x^2=t$$
$$\Rightarrow (4-2x)dx=dt$$
$$\Rightarrow I_1=\int\frac{dt}{\sqrt{t}}=2\sqrt{t}=2\sqrt{4x-x^2} \quad ...(2)$$

$$I_2=\int\frac{1}{\sqrt{4x-x^2}}dx$$
$$\Rightarrow 4x-x^2=-\left(-4x+x^2\right)$$
$$=\left(-4x+x^2+4-4\right)$$
$$=4-(x-2)^2$$
$$=(2)^2-(x-2)^2$$
$$\therefore I_2=\int\frac{1}{\sqrt{(2)^2-(x-2)^2}}dx=\sin^{-1}\left(\frac{x-2}{2}\right) \quad ...(3)$$

Using equations (2) and (3) in (1), we obtain

$$\int\frac{x+2}{\sqrt{4x-x^2}}dx=-\frac{1}{2}\left(2\sqrt{4x-x^2}\right)+4\sin^{-1}\left(\frac{x-2}{2}\right)+C$$
$$=-\sqrt{4x-x^2}+4\sin^{-1}\left(\frac{x-2}{2}\right)+C$$

Answer needs Correction? Click Here

Q20 : $\frac{x+2}{\sqrt{x^2+2x+3}}$

Answer :

$$\int\frac{(x+2)}{\sqrt{x^2+2x+3}}dx=\frac{1}{2}\int\frac{2(x+2)}{\sqrt{x^2+2x+3}}dx$$
$$=\frac{1}{2}\int\frac{2x+4}{\sqrt{x^2+2x+3}}dx$$
$$=\frac{1}{2}\int\frac{2x+2}{\sqrt{x^2+2x+3}}dx+\frac{1}{2}\int\frac{2}{\sqrt{x^2+2x+3}}dx$$

$$= \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}}dx + \int \frac{1}{\sqrt{x^2+2x+3}}dx$$

Let $I_1 = \int \frac{2x+2}{\sqrt{x^2+2x+3}}dx$ and $I_2 = \int \frac{1}{\sqrt{x^2+2x+3}}dx$

$$\therefore \int \frac{x+2}{\sqrt{x^2+2x+3}}dx = \frac{1}{2}I_1 + I_2 \quad ...(1)$$

Then, $I_1 = \int \frac{2x+2}{\sqrt{x^2+2x+3}}dx$

Let $x^2 + 2x + 3 = t$

$\Rightarrow (2x+2)\,dx = dt$

$$I_1 = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} = 2\sqrt{x^2+2x+3} \quad ...(2)$$

$$I_2 = \int \frac{1}{\sqrt{x^2+2x+3}}dx$$

$$\Rightarrow x^2+2x+3 = x^2+2x+1+2 = (x+1)^2 + \left(\sqrt{2}\right)^2$$

$$\therefore I_2 = \int \frac{1}{\sqrt{(x+1)^2+\left(\sqrt{2}\right)^2}}dx = \log\left|(x+1)+\sqrt{x^2+2x+3}\right| \quad ...(3)$$

Using equations (2) and (3) in (1), we obtain

$$\int \frac{x+2}{\sqrt{x^2+2x+3}}dx = \frac{1}{2}\left[2\sqrt{x^2+2x+3}\right] + \log\left|(x+1)+\sqrt{x^2+2x+3}\right| + C$$
$$= \sqrt{x^2+2x+3} + \log\left|(x+1)+\sqrt{x^2+2x+3}\right| + C$$

Answer needs Correction? **Click Here**

Q21 : $\frac{x+3}{x^2-2x-5}$

Answer :

Let $(x+3) = A\frac{d}{dx}\left(x^2-2x-5\right) + B$

$(x+3) = A(2x-2) + B$

Equating the coefficients of x and constant term on both sides, we obtain

$$2A = 1 \Rightarrow A = \frac{1}{2}$$
$$-2A + B = 3 \Rightarrow B = 4$$
$$\therefore (x+3) = \frac{1}{2}(2x-2) + 4$$
$$\Rightarrow \int \frac{x+3}{x^2-2x-5}dx = \int \frac{\frac{1}{2}(2x-2)+4}{x^2-2x-5}dx$$
$$= \frac{1}{2}\int \frac{2x-2}{x^2-2x-5}dx + 4\int \frac{1}{x^2-2x-5}dx$$

Let $I_1 = \int \frac{2x-2}{x^2-2x-5}dx$ and $I_2 = \int \frac{1}{x^2-2x-5}dx$

$$\therefore \int \frac{x+3}{\left(x^2-2x-5\right)}dx = \frac{1}{2}I_1 + 4I_2 \quad ...(1)$$

Then, $I_1 = \int \frac{2x-2}{x^2-2x-5}dx$

Let $x^2 - 2x - 5 = t$

$\Rightarrow (2x-2)dx = dt$

$$\Rightarrow I_1 = \int \frac{dt}{t} = \log|t| = \log\left|x^2-2x-5\right| \quad ...(2)$$

$$I_2 = \int \frac{1}{x^2-2x-5}dx$$
$$= \int \frac{1}{\left(x^2-2x+1\right)-6}dx$$
$$= \int \frac{1}{(x-1)^2+\left(\sqrt{6}\right)^2}dx$$
$$= \frac{1}{2\sqrt{6}}\log\left(\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right) \quad ...(3)$$

Substituting (2) and (3) in (1), we obtain

$$\int \frac{x+3}{x^2-2x-5}dx = \frac{1}{2}\log\left|x^2-2x-5\right| + \frac{4}{2\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + C$$
$$= \frac{1}{2}\log\left|x^2-2x-5\right| + \frac{2}{\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + C$$

Answer needs Correction? **Click Here**

Q22 : $\frac{5x+3}{\sqrt{x^2+4x+10}}$

Answer :

Let $5x+3 = A\frac{d}{dx}\left(x^2+4x+10\right) + B$

$\Rightarrow 5x+3 = A(2x+4) + B$

Equating the coefficients of x and constant term, we obtain

$$2A=5\Rightarrow A=\frac{5}{2}$$
$$4A+B=3\Rightarrow B=-7$$
$$\therefore 5x+3=\frac{5}{2}(2x+4)-7$$
$$\Rightarrow \int\frac{5x+3}{\sqrt{x^2+4x+10}}dx=\int\frac{\frac{5}{2}(2x+4)-7}{\sqrt{x^2+4x+10}}dx$$
$$=\frac{5}{2}\int\frac{2x+4}{\sqrt{x^2+4x+10}}dx-7\int\frac{1}{\sqrt{x^2+4x+10}}dx$$
$$\text{Let } I_1=\int\frac{2x+4}{\sqrt{x^2+4x+10}}dx \text{ and } I_2=\int\frac{1}{\sqrt{x^2+4x+10}}dx$$
$$\therefore \int\frac{5x+3}{\sqrt{x^2+4x+10}}dx=\frac{5}{2}I_1-7I_2 \quad ...(1)$$
$$\text{Then, } I_1=\int\frac{2x+4}{\sqrt{x^2+4x+10}}dx$$

$$\text{Let } x^2+4x+10=t$$
$$\therefore (2x+4)dx=dt$$
$$\Rightarrow I_1=\int\frac{dt}{t}=2\sqrt{t}=2\sqrt{x^2+4x+10} \quad ...(2)$$
$$I_2=\int\frac{1}{\sqrt{x^2+4x+10}}dx$$
$$=\int\frac{1}{\sqrt{(x^2+4x+4)+6}}dx$$
$$=\int\frac{1}{(x+2)^2+(\sqrt{6})^2}dx$$
$$=\log\left|(x+2)\sqrt{x^2+4x+10}\right| \quad ...(3)$$

Using equations (2) and (3) in (1), we obtain

$$\int\frac{5x+3}{\sqrt{x^2+4x+10}}dx=\frac{5}{2}\left[2\sqrt{x^2+4x+10}\right]-7\log\left|(x+2)+\sqrt{x^2+4x+10}\right|+C$$
$$=5\sqrt{x^2+4x+10}-7\log\left|(x+2)+\sqrt{x^2+4x+10}\right|+C$$

Answer needs Correction? Click Here

Q23 : $\int\frac{dx}{x^2+2x+2}$ equals

A. $x\tan^{-1}(x+1)+C$

B. $\tan^{-1}(x+1)+C$

C. $(x+1)\tan^{-1}x+C$

D. $\tan^{-1}x+C$

Answer :

$$\int\frac{dx}{x^2+2x+2}=\int\frac{dx}{(x^2+2x+1)+1}$$
$$=\int\frac{1}{(x+1)^2+(1)^2}dx$$
$$=\left[\tan^{-1}(x+1)\right]+C$$

Hence, the correct answer is B.

Answer needs Correction? Click Here

Q24 : $\int\frac{dx}{\sqrt{9x-4x^2}}$ equals

A. $\frac{1}{9}\sin^{-1}\left(\frac{9x-8}{8}\right)+C$

B. $\frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right)+C$

C. $\frac{1}{3}\sin^{-1}\left(\frac{9x-8}{8}\right)+C$

D. $\frac{1}{2}\sin^{-1}\left(\frac{9x-8}{9}\right)+C$

Answer :

$$\int\frac{dx}{\sqrt{9x-4x^2}}$$
$$=\int\frac{1}{\sqrt{-4\left(x^2-\frac{9}{4}x\right)}}dx$$
$$=\int\frac{1}{-4\left(x^2-\frac{9}{4}x+\frac{81}{64}-\frac{81}{64}\right)}dx$$
$$=\int\frac{1}{\sqrt{-4\left[\left(x-\frac{9}{8}\right)^2-\left(\frac{9}{8}\right)^2\right]}}dx$$
$$=\frac{1}{}\int\frac{1}{}dx$$

$$=\frac{1}{2}\int\frac{1}{\sqrt{\left(\frac{9}{8}\right)^2-\left(x-\frac{9}{8}\right)^2}}dx$$

$$=\frac{1}{2}\left[\sin^{-1}\left(\frac{x-\frac{9}{8}}{\frac{9}{8}}\right)\right]+C \qquad \left(\int\frac{dy}{\sqrt{a^2-y^2}}=\sin^{-1}\frac{y}{a}+C\right)$$

$$=\frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right)+C$$

Hence, the correct answer is B.

*********** END **********