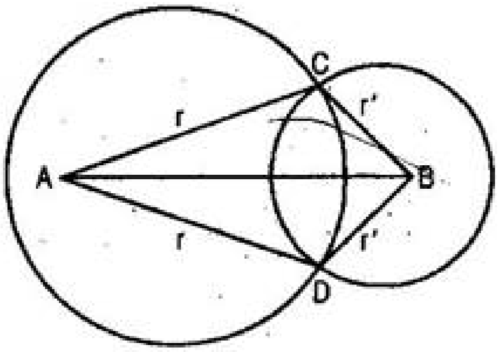
Q1. Prove that the line of centres of two intersecting circles subtends equal angles at the two points of intersection.

Ans. Let two circles with respective centers A and B intersect each other at points C and D.



We have to prove $\angle$ ACB = $\angle$ ADB

Proof: In triangles ABC and ABD,

AC = AD = r

BC = BD = r'

AB = AB [Common]

$\therefore \Delta$ ABC $\cong \Delta$ ABD

[SSS rule of congruency]

$\Rightarrow \angle$ ACB = $\angle$ ADB [By CPCT]

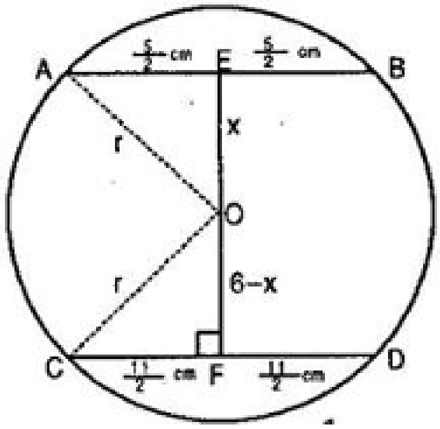
Q2. Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find he radius of the circle.

Ans. Let O be the centre of the circle. Join OA and OC.

Since perpendicular from the centre of the circle to the chord bisects the chord.

$\therefore AE = EB = \frac{1}{2} AB = \frac{1}{2} \times 5 = \frac{5}{2}$ cm

And $CF = FD = \frac{1}{2} CD = \frac{1}{2} \times 11 = \frac{11}{2}$ cm



Let OE = x

$\therefore OF = 6 - x$

Let radius of the circle be r

In right angled triangle AEO,

$AO^2 = AE^2 + OE^2$

[Using Pythagoras theorem]

$\Rightarrow r^2 = \left(\frac{5}{2}\right)^2 + x^2$ ....(i)

Again In right angled triangle CFO,

$OC^2 = CF^2 + OF^2$

[Using Pythagoras theorem]

$\Rightarrow r^2 = \left(\frac{11}{2}\right)^2 + (6 - x)^2$ ....(ii)

Equating eq. (i) and (ii),

$$\left(\frac{5}{2}\right)^2 + x^2 = \left(\frac{11}{2}\right)^2 + (6-x)^2$$

$$\Rightarrow \frac{25}{4} + x^2 = \frac{121}{4} + 36 + x^2 - 12x$$

$$\Rightarrow 12x = \frac{121}{4} - \frac{25}{4} + 36$$

$$\Rightarrow 12x = \frac{96}{4} + 36$$

$$\Rightarrow 12x = 24 + 36$$

$$\Rightarrow 12x = 60$$

$$\Rightarrow x = 5$$

Now from eq. (i),

$$r^2 = \frac{25}{4} + x^2$$

$$\Rightarrow r^2 = \frac{25}{4} + 5^2$$

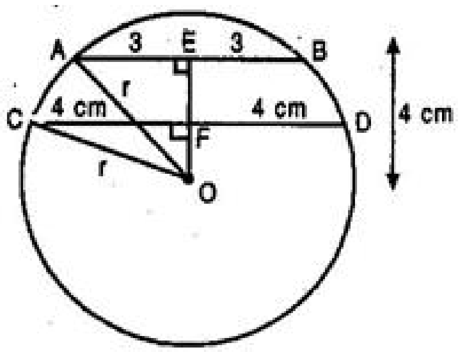
$$\Rightarrow r^2 = \frac{125}{4}$$

$$\Rightarrow r = \frac{5\sqrt{5}}{2} \text{ cm}$$

Hence radius of the circle is $\frac{5\sqrt{5}}{2}$ cm.

Q3. The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at a distance of 4 cm from the centre, what is the distance of the other chord form the centre?

Ans. Let AB = 6 cm and CD = 8 cm are the chords of circle with centre O. Join OA and OC.



Since perpendicular from the centre of the circle to the chord bisects the chord.

$\therefore$ AE = EB = $\frac{1}{2}$ AB = $\frac{1}{2}$ x 6 = 3 cm

And CF = FD = $\frac{1}{2}$ CD = $\frac{1}{2}$ x 8 = 4 cm

Perpendicular distance of chord AB from the centre O is OE.

$\therefore$ OE = 4 cm

Now in right angled triangle AOE,

$OA^2 = AE^2 + OE^2$ [Using Pythagoras theorem]

$\Rightarrow r^2 = 3^2 + 4^2$

$\Rightarrow r^2 = 9 + 16 = 25$

$\Rightarrow r = 5$ cm

Perpendicular distance of chord CD from the center O is OF.

Now in right angled triangle OFC,

$OC^2 = CF^2 + OF^2$ [Using Pythagoras theorem]

$\Rightarrow r^2 = 4^2 + OF^2$

$\Rightarrow 5^2 = 16 + OF^2$

$\Rightarrow OF^2 = 16$

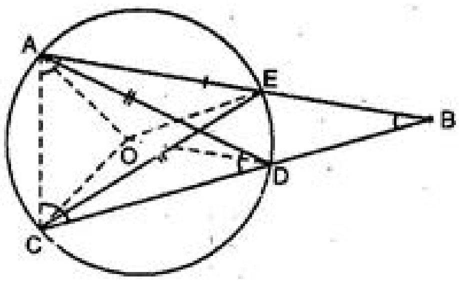
$\Rightarrow OF = 3cm$

Hence distance of other chord from the centre is 3 cm.

Q4. Let vertex of an angle ABC be located outside a circle and let the sides of the angle intersect chords AD and CE with the circle. Prove that $\angle$ ABC is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

Ans. Vertex B of $\angle$ ABC is located outside the circle with centre O.

Side AB intersects chord CE at point E and side BC intersects chord AD at point D with the circle.



We have to prove that

$\angle ABC = \frac{1}{2}[\Delta AOC - \Delta DOE]$

Join OA, OC, OE and OD.

Now $\angle AOC = 2\angle AEC$

[Angle subtended by an arc at the centre of the circle is twice the angle subtended by the same arc at any point in the alternate segment of the circle]

$\Rightarrow \frac{1}{2}\angle AOC = \angle AEC$ ...(i)

Similarly $\frac{1}{2}\angle DOE = \angle DCE$ ....(ii)

Subtracting eq. (ii) from eq. (i),

$\frac{1}{2}[\Delta \text{ AOC} - \Delta \text{ DOE}] = \angle \text{AEC} - \angle \text{DCE}$ ....(iii)

Now $\angle$ AEC = $\angle$ ADC

[Angles in same segment in circle] ....(iv)

Also $\angle$ DCE = $\angle$ DAE

[Angles in same segment in circle] ....(v)

Using eq. (iv) and (v) in eq. (iii),

$\frac{1}{2}[\Delta\text{AOC} - \Delta\text{DOE}]$

$= \angle \text{DAE} + \angle \text{ABD} - \angle \text{DAE}$

$\Rightarrow \frac{1}{2}[\Delta\text{AOC} - \Delta\text{DOE}] = \angle \text{ABD}$

Or $\frac{1}{2}[\Delta\text{AOC} - \Delta\text{DOE}] = \angle \text{ABC}$

Hence proved.

Q5. Prove that the circle drawn with any drawn with any side of a rhombus as a diameter, passes through the point of intersection of its diagonals.

Ans. Let ABCD be a rhombus in which diagonals AC and BD intersect each other at point O.

As we know that diagonals of a rhombus bisect and perpendicular to each other.

$\therefore \angle \text{AOB} = 90^\circ$

And if we draw a circle with side AB as diameter, it will definitely **pass through point O** (the point intersection of diagonals) because then $\angle \text{AOB} = 90^\circ$ will be the angle in a semi-circle.

*********** END *********