Areas of Parallelograms and Triangles Ex 15.3 Q17

Answer :

Given:

(1) ABCD is a parallelogram.

(2) E is a point on BA such that $BE = 2EA$ $BE = 2EA$

(3) F is a point on DC such that $DF = 2FC$.

To find:

Area of parallelogram $AECF = \frac{1}{3}$ Area of parallelogram ABCD

Proof: We have,

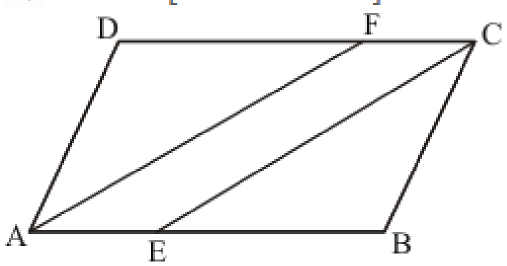
$BE = 2EA$ and $DF = 2FC$

$\Rightarrow AB - AE = 2AE$ and $DC - FC = 2FC$

$\Rightarrow AB = 3AE$ and $DC = 3FC$

$\Rightarrow AE = \frac{1}{3}AB$ and $FC = \frac{1}{3}DC$

$\Rightarrow AE = FC$ [since $AB = DC$]



Thus, $AE \parallel FC$ such that $AE = FC$

Therefore AECF is a parallelogram.

Clearly, parallelograms ABCD and AECF have the same altitude and

$AE = \frac{1}{3}AB$.

Therefore

Area of a parallelogram $AECF = \frac{1}{3}$ Area of parallelogram ABCD

Hence proved that $\boxed{\text{Area of parallelogram AECF} = \frac{1}{3}\text{ Area of parallelogram ABCD}}$

Areas of Parallelograms and Triangles Ex 15.3 Q18

Answer :

Given:

(1) In a triangle ABC, P is the mid-point of AB.

(2) Q is mid-point of BC.

(3) R is mid-point of AP.

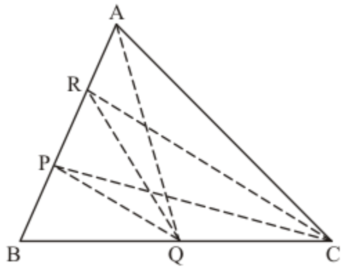
To prove:

(a) Area of ΔPBQ = Area of ΔARC

(b) Area of $\Delta PRQ = \frac{1}{2}$ Area of ΔARC

(c) Area of $\Delta RQC = \frac{3}{8}$ Area of ΔABC

Proof: We know that each median of a triangle divides it into two triangles of equal area.



(a) Since CR is a median of ΔCAP

Therefore $\operatorname{ar}(\Delta CRA) = \frac{1}{2}\operatorname{ar}(\Delta CAP)$ ...... (1)

Also, CP is a median of ΔCAB.

Therefore $\operatorname{ar}(\Delta CRA) = \frac{1}{2}\operatorname{ar}(\Delta CAP)$ ...... (2)

From equation (1) and (2), we get

Therefore $\operatorname{ar}(\Delta ARC) = \frac{1}{4}\operatorname{ar}(\Delta CAB)$ ...... (3)

PQ is a median of ΔABQ

Therefore $\operatorname{ar}(\Delta CPB) = 2\operatorname{ar}(\Delta PBQ)$

Since $\operatorname{ar}(\Delta CPB) = \frac{1}{2}\operatorname{ar}(\Delta ABC)$

Put this value in the above equation we get

$$\frac{1}{2}\operatorname{ar}(\Delta ABC) = 2\operatorname{ar}(\Delta PBQ)$$

$$\Rightarrow \operatorname{ar}(\Delta PBQ) = \frac{1}{4}\operatorname{ar}(\Delta ABC) \quad \text{...... (4)}$$

From equation (3) and (4), we get

Therefore $\operatorname{ar}(\Delta ARC) = \operatorname{ar}(\Delta PBQ)$ ...... (5)

(b) Since QP is a median of ΔQAB and QR medians of ΔQAP

Therefore $\operatorname{ar}(\Delta QAP) = \operatorname{ar}(\Delta QBP)$ ...... (6)

and, $ar(\Delta QAP) = 2\ ar(\Delta QRP)$ ...... (7)

From equation (6) and (7)

$ar(\Delta PRQ) = \frac{1}{2} ar(\Delta PBQ)$ ...... (8)

From equation (7) and (8)

$$\boxed{ar(\Delta PRQ) = \frac{1}{2} ar(\Delta ARC)}$$

(c) Since CR is a median of ΔCAP.

$$ar(\Delta ARC) = \frac{1}{2} ar(\Delta CAP)$$

$$\Rightarrow \quad = \frac{1}{2}\left\{\frac{1}{2} ar(\Delta ABC)\right\} \quad [\text{Since CP is a median of DABC}]$$

$= \frac{1}{4} ar(\Delta ABC)$ ...... (9)

Since RQ is a median of ΔRBC.

$$\Rightarrow \quad ar(\Delta RQC) = \frac{1}{2} ar(\Delta RBC)$$

$$= \frac{1}{2}\{ar(\Delta ABC) - ar(\Delta ARC)\}$$

$$= \frac{1}{2}\left\{ar(\Delta ABC) - \frac{1}{4} ar(\Delta ABC)\right\} \quad [\text{from equation (9)}]$$

$$\boxed{= \frac{3}{8} ar(\Delta ABC)}$$

*********** END **********