



Co-Ordinate Geometry Ex 14.2 Q13

Answer :

The distance d between two points (x_1, y_1) and (x_2, y_2) is given by the formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

In an equilateral triangle all the sides are of equal length.

Here we are given that $A(3, 4)$ and $B(-2, 3)$ are two vertices of an equilateral triangle. Let $C(x, y)$ be the third vertex of the equilateral triangle.

First let us find out the length of the side of the equilateral triangle.

$$\begin{aligned} AB &= \sqrt{(3+2)^2 + (4-3)^2} \\ &= \sqrt{(5)^2 + (1)^2} \\ &= \sqrt{25+1} \end{aligned}$$

$$AB = \sqrt{26}$$

Hence the side of the equilateral triangle measures $\sqrt{26}$ units.

Now, since it is an equilateral triangle, all the sides need to measure the same length.

Hence we have $BC = AC$

$$BC = \sqrt{(-2-x)^2 + (3-y)^2}$$

$$AC = \sqrt{(3-x)^2 + (4-y)^2}$$

Equating both these equations we have,

$$\sqrt{(-2-x)^2 + (3-y)^2} = \sqrt{(3-x)^2 + (4-y)^2}$$

Squaring on both sides we have,

$$(-2-x)^2 + (3-y)^2 = (3-x)^2 + (4-y)^2$$

$$4 + x^2 + 4x + 9 + y^2 - 6y = 9 + x^2 - 6x + 16 + y^2 - 8y$$

$$10x + 2y = 12$$

$$5x + y = 6$$

From the above equation we have, $y = 6 - 5x$

Substituting this and the value of the side of the triangle in the equation for one of the sides we have,

$$BC = \sqrt{(-2-x)^2 + (3-y)^2}$$

$$\sqrt{26} = \sqrt{(-2-x)^2 + (3-6+5x)^2}$$

Squaring on both sides,

$$26 = (-2-x)^2 + (-3+5x)^2$$

$$26 = 4 + x^2 + 4x + 9 + 25x^2 - 30x$$

$$13 = 26x^2 - 26x$$

$$1 = 2x^2 - 2x$$

Now we have a quadratic equation for 'x'. Solving for the roots of this equation,

$$2x^2 - 2x - 1 = 0$$

$$x = \frac{2 \pm \sqrt{4 - 4(2)(-1)}}{4}$$

$$= \frac{2 \pm \sqrt{12}}{4}$$

$$x = \frac{1 \pm \sqrt{3}}{2}$$

We know that $y = 6 - 5x$. Substituting the value of 'x' we have,

$$y = 6 - 5\left(\frac{1 \pm \sqrt{3}}{2}\right)$$

$$= \frac{12 - 5 \mp 5\sqrt{3}}{2}$$

$$y = \frac{7 \mp 5\sqrt{3}}{2}$$

Hence the two possible values of the third vertex are $\frac{1+\sqrt{3}}{2}, \frac{7-5\sqrt{3}}{2}$ and $\frac{1-\sqrt{3}}{2}, \frac{7+5\sqrt{3}}{2}$.

Co-Ordinate Geometry Ex 14.2 Q14

Answer :

The distance d between two points (x_1, y_1) and (x_2, y_2) is given by the formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

In a rhombus all the sides are equal in length.

Here the four points are $A(2, -1)$, $B(3, 4)$, $C(-2, 3)$ and $D(-3, -2)$.

First let us check if all the four sides are equal.

$$\begin{aligned} AB &= \sqrt{(2-3)^2 + (-1-4)^2} \\ &= \sqrt{(-1)^2 + (-5)^2} \\ &= \sqrt{1+25} \end{aligned}$$

$$AB = \sqrt{26}$$

$$\begin{aligned} BC &= \sqrt{(3+2)^2 + (4-3)^2} \\ &= \sqrt{(5)^2 + (1)^2} \\ &= \sqrt{25+1} \end{aligned}$$

$$BC = \sqrt{26}$$

$$\begin{aligned} CD &= \sqrt{(-2-3)^2 + (-3+2)^2} \\ &= \sqrt{(-5)^2 + (-1)^2} \\ &= \sqrt{25+1} \end{aligned}$$

$$CD = \sqrt{26}$$

$$\begin{aligned} AD &= \sqrt{(2+3)^2 + (-1+2)^2} \\ &= \sqrt{(5)^2 + (1)^2} \\ &= \sqrt{25+1} \end{aligned}$$

$$AD = \sqrt{26}$$

Here, we see that all the sides are equal, so it has to be a rhombus.

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