Definite Integrals Ex 20.1 Q47

We have,

$$\int_1^2 \left(\frac{x-1}{x^2}\right) e^x \, dx = \int_1^2 \frac{xe^x}{x^2} - \int_1^2 \frac{e^x}{x^2} \, dx = \int_1^2 \frac{e^x \, dx}{x} - \int_1^2 \frac{e^x}{x^2} \, dx$$

Expanding 1st integral by by parts we get

$$= \frac{1}{x} \int_1^2 e^x \, dx - \int_1^2 \left(\int e^x \cdot \frac{d\left(1/x\right)}{dx} \, dx \right) - \int_1^2 \frac{e^x}{x^2}$$

$$= \left[\frac{e^x}{x}\right]_1^2 + \int_1^2 \frac{e^x}{x^2} \, dx - \int_1^2 \frac{e^x}{x^2} \, dx$$

$$= \left[\frac{e^x}{x}\right]_1^2$$

$$= \frac{e^2}{2} - e$$

$$\therefore \int_1^2 \left(\frac{x-1}{x^2}\right) e^x \, dx = \frac{e^2}{2} - e$$

Definite Integrals Ex 20.1 Q48

We have,

$$\int_0^1 \left(xe^{2x} + \sin\frac{\pi x}{2}\right) dx = \int_0^1 \underset{I}{x} \, \underset{II}{e^{2x}} \, dx + \int_0^1 \sin\frac{\pi x}{2} \, dx$$

Applying by parts in first integral

$$= x \int_0^1 e^{2x} \, dx - \int_0^1 \left(\int e^{2x} \, dx \right) \frac{dx}{dx} \, dx + \left[\frac{-\cos\frac{\pi x}{2}}{\frac{\pi}{2}}\right]_0^1$$

$$= \frac{xe^{2x}}{2} - \frac{1}{2} \int_0^1 e^{2x} \, dx + \frac{2}{\pi}[1-0]$$

$$= \frac{xe^{2x}}{2} - \frac{1}{2} \int_0^1 e^{2x} \, dx + \frac{2}{\pi}[1-0]$$

$$= \left[\frac{xe^{2x}}{2} - \frac{1}{4}e^{2x}\right]_0^1 + \frac{2}{\pi}[1-0]$$

$$= \frac{e^2}{2} - \frac{1}{4}e^2 + \frac{1}{4} + \frac{2}{\pi}[1-0]$$

$$= \frac{e^2}{4} + \frac{2}{\pi} + \frac{1}{4}$$

$$= \frac{e^2}{4} + \frac{1}{4} + \frac{2}{\pi}$$

$$\therefore \int_0^1 \left(xe^{2x} + \sin\frac{\pi x}{2}\right) dx = \frac{e^2}{4} + \frac{1}{4} + \frac{2}{\pi}$$

Definite Integrals Ex 20.1 Q49

We have,

$$\int_0^1 \left(xe^x + \cos\frac{\pi x}{4} \right) dx$$

$$= \int_0^1 \underset{I\ \ II}{x\ e^x}\ dx + \int_0^1 \cos\frac{\pi x}{4}\ dx$$

Applying by by parts in 1st integral we get,

$$= x\int_0^1 e^x dx - \int_0^1 \left(\int e^x dx\right)\frac{dx}{dx}.dx + \int_0^1 \cos\frac{\pi x}{4}\ dx$$

$$= \left[xe^x \right]_0^1 - \int_0^1 e^x dx + \left[\frac{\sin\frac{\pi x}{4}}{\frac{\pi}{4}} \right]_0^1$$

$$= \left[xe^x - e^x \right]_0^1 + \frac{4}{\pi}\left[\frac{1}{\sqrt{2}} - 0 \right]$$

$$= \left[e^x(x-1) \right]_0^1 + \frac{4}{\pi}\left[\frac{1}{\sqrt{2}} \right]$$

$$= 0 + 1 + \frac{4}{\pi\sqrt{2}}$$

$$= 1 + \frac{2\sqrt{2}}{\pi}$$

$$\therefore \int_0^1 \left(xe^x + \cos\frac{\pi x}{4} \right) dx = 1 + \frac{2\sqrt{2}}{\pi}$$

Definite Integrals Ex 20.1 Q50

$$\int_{\frac{\pi}{2}}^{\pi} e^x \frac{1-\sin x}{1-\cos x} dx = \int_{\frac{\pi}{2}}^{\pi} e^x \frac{1-2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}} dx \qquad \left[1-\cos x = 2\sin^2\frac{x}{2} \right]$$

$$= -\int_{\frac{\pi}{2}}^{\pi} e^x \left(-\frac{1}{2}\csc^2\frac{x}{2} + \cot\frac{x}{2} \right) dx$$

$$= -e^x \cot\frac{x}{2}\Big|_{\frac{\pi}{2}}^{\pi} \qquad \left[\int e^x \left(F(x) + F'(x) \right) dx = e^x F(x) \right]$$

$$= e^{\frac{\pi}{2}}$$

********** END *********