



### EXERCISE 9.3

Question 1:

Find the 20<sup>th</sup> and  $n^{\text{th}}$  terms of the G.P.  $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \dots$

Ans:

The given G.P. is  $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \dots$

Here,  $a$  = First term =  $\frac{5}{2}$

$r$  = Common ratio =  $\frac{\frac{5}{4}}{\frac{5}{2}} = \frac{1}{2}$

$$a_{20} = ar^{20-1} = \frac{5}{2} \left( \frac{1}{2} \right)^{19} = \frac{5}{(2)(2)^{19}} = \frac{5}{(2)^{20}}$$

$$a_n = ar^{n-1} = \frac{5}{2} \left( \frac{1}{2} \right)^{n-1} = \frac{5}{(2)(2)^{n-1}} = \frac{5}{(2)^n}$$

Question 2:

Find the 12<sup>th</sup> term of a G.P. whose 8<sup>th</sup> term is 192 and the common ratio is 2.

Ans:

Common ratio,  $r = 2$

Let  $a$  be the first term of the G.P.

$$a_8 = ar^{8-1} = ar^7$$

$$ar^7 = 192$$

$$a(2)^7 = 192$$

$$a(2)^7 = (2)^6 (3)$$

$$\Rightarrow a = \frac{(2)^6 \times 3}{(2)^7} = \frac{3}{2}$$

$$\therefore a_{12} = ar^{12-1} = \left(\frac{3}{2}\right)(2)^{11} = (3)(2)^{10} = 3072$$

Question 3:

The 5<sup>th</sup>, 8<sup>th</sup> and 11<sup>th</sup> terms of a G.P. are  $p$ ,  $q$  and  $s$ , respectively. Show that  $q^2 = ps$ .

Ans:

Let  $a$  be the first term and  $r$  be the common ratio of the G.P.

According to the given condition,

$$a_5 = ar^{5-1} = ar^4 = p \dots (1)$$

$$a_8 = ar^{8-1} = ar^7 = q \dots (2)$$

$$a_{11} = ar^{11-1} = ar^{10} = s \dots (3)$$

Dividing equation (2) by (1), we obtain

$$\frac{ar^7}{ar^4} = \frac{q}{p}$$
$$r^3 = \frac{q}{p} \dots (4)$$

Dividing equation (3) by (2), we obtain

$$\frac{ar^{10}}{ar^7} = \frac{s}{q}$$
$$\Rightarrow r^3 = \frac{s}{q} \dots (5)$$

Equating the values of  $r^3$  obtained in (4) and (5), we obtain

$$\frac{q}{p} = \frac{s}{q}$$
$$\Rightarrow q^2 = ps$$

Thus, the given result is proved.

Question 4:

The 4<sup>th</sup> term of a G.P. is square of its second term, and the first term is  $-3$ . Determine its 7<sup>th</sup> term.

Ans:

Let  $a$  be the first term and  $r$  be the common ratio of the G.P.

$$a = -3$$

It is known that,  $a_n = ar^{n-1}$

$$a_4 = ar^3 = (-3) r^3$$

$$a_2 = ar^1 = (-3) r$$

According to the given condition,

$$(-3) r^3 = [(-3) r]^2$$

$$-3r^3 = 9r^2$$

$$r = -3$$

$$a_7 = ar^{7-1} = ar^6 = (-3) (-3)^6 = -(3)^7 = -2187$$

Thus, the seventh term of the G.P. is -2187.

Question 5:

Which term of the following sequences:

$$(a) \ 2, 2\sqrt{2}, 4, \dots \text{ is } 128? (b) \ \sqrt{3}, 3, 3\sqrt{3}, \dots \text{ is } 729? (c) \ \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots \text{ is } \frac{1}{19683}?$$

Ans:

**(a)** The given sequence is  $2, 2\sqrt{2}, 4, \dots$

$$\text{Here, } a = 2 \text{ and } r = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

Let the  $n^{\text{th}}$  term of the given sequence be 128.

$$a_n = ar^{n-1}$$

$$\Rightarrow (2)(\sqrt{2})^{n-1} = 128$$

$$\Rightarrow (2)(2)^{\frac{n-1}{2}} = (2)^7$$

$$\Rightarrow (2)^{\frac{n-1}{2}+1} = (2)^7$$

$$\therefore \frac{n-1}{2} + 1 = 7$$

$$\Rightarrow \frac{n-1}{2} = 6$$

$$\Rightarrow n-1 = 12$$

$$\Rightarrow n = 13$$

Thus, the 13<sup>th</sup> term of the given sequence is 128.

**(b)** The given sequence is  $\sqrt{3}, 3, 3\sqrt{3}, \dots$

Here,  $a = \sqrt{3}$  and  $r = \frac{3}{\sqrt{3}} = \sqrt{3}$

Let the  $n^{\text{th}}$  term of the given sequence be 729.

$$a_n = ar^{n-1}$$

$$\therefore ar^{n-1} = 729$$

$$\Rightarrow (\sqrt{3})(\sqrt{3})^{n-1} = 729$$

$$\Rightarrow (3)^{\frac{1}{2}}(3)^{\frac{n-1}{2}} = (3)^6$$

$$\Rightarrow (3)^{\frac{1+n-1}{2}} = (3)^6$$

$$\therefore \frac{1}{2} + \frac{n-1}{2} = 6$$

$$\Rightarrow \frac{1+n-1}{2} = 6$$

$$\Rightarrow n = 12$$

Thus, the 12<sup>th</sup> term of the given sequence is 729.

**(c)** The given sequence is  $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots$

Here,  $a = \frac{1}{3}$  and  $r = \frac{1}{9} \div \frac{1}{3} = \frac{1}{3}$

Let the  $n^{\text{th}}$  term of the given sequence be  $\frac{1}{19683}$ .

$$a_n = ar^{n-1}$$

$$\therefore ar^{n-1} = \frac{1}{19683}$$

$$\Rightarrow \left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^{n-1} = \frac{1}{19683}$$

$$\Rightarrow \left(\frac{1}{3}\right)^n = \left(\frac{1}{3}\right)^9$$

$$\Rightarrow n = 9$$

Thus, the 9<sup>th</sup> term of the given sequence is  $\frac{1}{19683}$ .

Question 6:

For what values of  $x$ , the numbers  $\frac{2}{7}, x, -\frac{7}{2}$  are in G.P?

Ans:

The given numbers are  $\frac{-2}{7}, x, \frac{-7}{2}$ .

$$\text{Common ratio} = \frac{\frac{x}{-2}}{\frac{-2}{7}} = \frac{-7x}{2}$$

$$\text{Also, common ratio} = \frac{\frac{-7}{2}}{\frac{x}{2}} = \frac{-7}{2x}$$

$$\therefore \frac{-7x}{2} = \frac{-7}{2x}$$

$$\Rightarrow x^2 = \frac{-2 \times 7}{-2 \times 7} = 1$$

$$\Rightarrow x = \sqrt{1}$$

$$\Rightarrow x = \pm 1$$

Thus, for  $x = \pm 1$ , the given numbers will be in G.P.

Question 7:

Find the sum to 20 terms in the geometric progression 0.15, 0.015, 0.0015 ...

Ans:

The given G.P. is 0.15, 0.015, 0.00015, ...

$$\text{Here, } a = 0.15 \text{ and } r = \frac{0.015}{0.15} = 0.1$$

$$\begin{aligned} S_n &= \frac{a(1-r^n)}{1-r} \\ \therefore S_{20} &= \frac{0.15[1-(0.1)^{20}]}{1-0.1} \\ &= \frac{0.15}{0.9}[1-(0.1)^{20}] \\ &= \frac{15}{90}[1-(0.1)^{20}] \\ &= \frac{1}{6}[1-(0.1)^{20}] \end{aligned}$$

Question 8:

Find the sum to  $n$  terms in the geometric progression  $\sqrt{7}, \sqrt{21}, 3\sqrt{7}, \dots$

Ans:

The given G.P. is  $\sqrt{7}, \sqrt{21}, 3\sqrt{7}, \dots$

Here,  $a = \sqrt{7}$

$$\begin{aligned} r &= \frac{\sqrt{21}}{\sqrt{7}} = \sqrt{3} \\ S_n &= \frac{a(1-r^n)}{1-r} \\ \therefore S_n &= \frac{\sqrt{7}[1-(\sqrt{3})^n]}{1-\sqrt{3}} \\ &= \frac{\sqrt{7}[1-(\sqrt{3})^n]}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}} \quad \text{(By rationalizing)} \\ &= \frac{\sqrt{7}(1+\sqrt{3})[1-(\sqrt{3})^n]}{1-3} \\ &= \frac{-\sqrt{7}(1+\sqrt{3})}{2} \left[ 1 - (3)^{\frac{n}{2}} \right] \\ &= \frac{\sqrt{7}(1+\sqrt{3})}{2} \left[ (3)^{\frac{n}{2}} - 1 \right] \end{aligned}$$

Question 9:

Find the sum to  $n$  terms in the geometric progression  $1, -a, a^2, -a^3 \dots$  (if  $a \neq -1$ )

Ans:

The given G.P. is  $1, -a, a^2, -a^3, \dots$

Here, first term  $= a_1 = 1$

Common ratio  $= r = -a$

$$\begin{aligned} S_n &= \frac{a_1(1-r^n)}{1-r} \\ \therefore S_n &= \frac{1[1-(-a)^n]}{1-(-a)} = \frac{[1-(-a)^n]}{1+a} \end{aligned}$$

Question 10:

Find the sum to  $n$  terms in the geometric progression  $x^3, x^5, x^7 \dots$  (if  $x \neq \pm 1$ )

Ans:

The given G.P. is  $x^3, x^5, x^7, \dots$

Here,  $a = x^3$  and  $r = x^2$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{x^3[1-(x^2)^n]}{1-x^2} = \frac{x^3(1-x^{2n})}{1-x^2}$$

Question 11:

$$\text{Evaluate } \sum_{k=1}^{11} (2 + 3^k)$$

Ans:

$$\begin{aligned} \sum_{k=1}^{11} (2 + 3^k) &= \sum_{k=1}^{11} (2) + \sum_{k=1}^{11} 3^k = 2(11) + \sum_{k=1}^{11} 3^k = 22 + \sum_{k=1}^{11} 3^k \quad \dots(1) \\ \sum_{k=1}^{11} 3^k &= 3^1 + 3^2 + 3^3 + \dots + 3^{11} \end{aligned}$$

The terms of this sequence  $3, 3^2, 3^3, \dots$  forms a G.P.

$$\begin{aligned} S_n &= \frac{a(r^n - 1)}{r - 1} \\ \Rightarrow S_{11} &= \frac{3[(3)^{11} - 1]}{3 - 1} \\ \Rightarrow S_{11} &= \frac{3}{2}(3^{11} - 1) \\ \therefore \sum_{k=1}^{11} 3^k &= \frac{3}{2}(3^{11} - 1) \end{aligned}$$

Substituting this value in equation (1), we obtain

$$\sum_{k=1}^{11} (2 + 3^k) = 22 + \frac{3}{2}(3^{11} - 1)$$

Question 12:

The sum of first three terms of a G.P. is  $\frac{39}{10}$  and their product is 1. Find the common ratio and the terms.

Ans:

Let  $\frac{a}{r}, a, ar$  be the first three terms of the G.P.

$$\frac{a}{r} + a + ar = \frac{39}{10} \quad \dots(1)$$

$$\left(\frac{a}{r}\right)(a)(ar) = 1 \quad \dots(2)$$

From (2), we obtain

$$a^3 = 1$$

$$a = 1 \text{ (Considering real roots only)}$$

Substituting  $a = 1$  in equation (1), we obtain

$$\frac{1}{r} + 1 + r = \frac{39}{10}$$

$$\Rightarrow 1 + r + r^2 = \frac{39}{10}r$$

$$\Rightarrow 10 + 10r + 10r^2 - 39r = 0$$

$$\Rightarrow 10r^2 - 29r + 10 = 0$$

$$\Rightarrow 10r^2 - 25r - 4r + 10 = 0$$

$$\Rightarrow 5r(2r - 5) - 2(2r - 5) = 0$$

$$\Rightarrow (5r - 2)(2r - 5) = 0$$

$$\Rightarrow r = \frac{2}{5} \text{ or } \frac{5}{2}$$

Thus, the three terms of G.P. are  $\frac{5}{2}, 1, \text{ and } \frac{2}{5}$ .

Question 13:

How many terms of G.P.  $3, 3^2, 3^3, \dots$  are needed to give the sum 120?

Ans:

The given G.P. is  $3, 3^2, 3^3, \dots$

Let  $n$  terms of this G.P. be required to obtain the sum as 120.

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

Here,  $a = 3$  and  $r = 3$

$$\therefore S_n = 120 = \frac{3(3^n - 1)}{3 - 1}$$

$$\Rightarrow 120 = \frac{3(3^n - 1)}{2}$$

$$\Rightarrow \frac{120 \times 2}{3} = 3^n - 1$$

$$\Rightarrow 3^n - 1 = 80$$

$$\Rightarrow 3^n = 81$$

$$\Rightarrow 3^n = 3^4$$

$$n = 4$$

Thus, four terms of the given G.P. are required to obtain the sum as 120.



Question 14:

The sum of first three terms of a G.P. is 16 and the sum of the next three terms is 128. Determine the first term, the common ratio and the sum to  $n$  terms of the G.P.

Ans:

According to the given condition,

$$a + ar + ar^2 = 16 \text{ and } ar^3 + ar^4 + ar^5 = 128$$

$$a(1 + r + r^2) = 16 \dots (1)$$

$$ar^3(1 + r + r^2) = 128 \dots (2)$$

Dividing equation (2) by (1), we obtain

$$\frac{ar^3(1+r+r^2)}{a(1+r+r^2)} = \frac{128}{16}$$

$$\Rightarrow r^3 = 8$$

$$\therefore r = 2$$

Substituting  $r = 2$  in (1), we obtain

$$a(1 + 2 + 4) = 16$$

$$a(7) = 16$$

$$\Rightarrow a = \frac{16}{7}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$\Rightarrow S_n = \frac{16}{7} \frac{(2^n - 1)}{2 - 1} = \frac{16}{7} (2^n - 1)$$

Question 15:

Given a G.P. with  $a = 729$  and 7<sup>th</sup> term 64, determine  $S_7$ .

Ans:

Let  $r$  be the common ratio of the G.P.

It is known that,  $a_n = a r^{n-1}$

$$a_7 = ar^{7-1} = (729)r^6$$

$$64 = 729 r^6$$

$$\Rightarrow r^6 = \frac{64}{729}$$

$$\Rightarrow r^6 = \left(\frac{2}{3}\right)^6$$

$$\Rightarrow r = \frac{2}{3}$$

$$\text{Also, it is known that, } S_n = \frac{a(1-r^n)}{1-r}$$

$$\begin{aligned}
 \therefore S_7 &= \frac{729 \left[ 1 - \left( \frac{2}{3} \right)^7 \right]}{1 - \frac{2}{3}} \\
 &= 3 \times 729 \left[ 1 - \left( \frac{2}{3} \right)^7 \right] \\
 &= (3)^7 \left[ \frac{(3)^7 - (2)^7}{(3)^7} \right] \\
 &= (3)^7 - (2)^7 \\
 &= 2187 - 128 \\
 &= 2059
 \end{aligned}$$

Question 16:

Find a G.P. for which sum of the first two terms is  $-4$  and the fifth term is 4 times the third term.

Ans:

Let  $a$  be the first term and  $r$  be the common ratio of the G.P.

According to the given conditions,

$$S_2 = -4 = \frac{a(1-r^2)}{1-r} \quad \dots(1)$$

$$a_5 = 4 \times a_3$$

$$ar^4 = 4ar^2$$

$$r^2 = 4$$

$$r = \pm 2$$

From (1), we obtain

$$-4 = \frac{a[1-(2)^2]}{1-2} \text{ for } r = 2$$

$$\Rightarrow -4 = \frac{a(1-4)}{-1}$$

$$\Rightarrow -4 = a(3)$$

$$\Rightarrow a = \frac{-4}{3}$$

$$\text{Also, } -4 = \frac{a[1-(-2)^2]}{1-(-2)} \text{ for } r = -2$$

$$\Rightarrow -4 = \frac{a(1-4)}{1+2}$$

$$\Rightarrow -4 = \frac{a(-3)}{3}$$

$$\Rightarrow a = 4$$

Thus, the required G.P. is

$$\frac{-4}{3}, \frac{-8}{3}, \frac{-16}{3}, \dots \text{ or } 4, -8, 16, -32, \dots$$

Question 17:

If the 4<sup>th</sup>, 10<sup>th</sup> and 16<sup>th</sup> terms of a G.P. are  $x, y$  and  $z$ , respectively. Prove that  $x, y, z$  are in G.P.

Ans:

Let  $a$  be the first term and  $r$  be the common ratio of the G.P.

According to the given condition,

$$a_4 = a r^3 = x \dots (1)$$

$$a_{10} = a r^9 = y \dots (2)$$

$$a_{16} = a r^{15} = z \dots (3)$$

Dividing (2) by (1), we obtain

$$\frac{y}{x} = \frac{ar^9}{ar^3} \Rightarrow \frac{y}{x} = r^6$$

Dividing (3) by (2), we obtain

$$\frac{z}{y} = \frac{ar^{15}}{ar^9} \Rightarrow \frac{z}{y} = r^6$$

$$\frac{y}{x} = \frac{z}{y}$$

Thus,  $x, y, z$  are in G. P.

Question 18:

Find the sum to  $n$  terms of the sequence, 8, 88, 888, 8888...

Ans:

The given sequence is 8, 88, 888, 8888...

This sequence is not a G.P. However, it can be changed to G.P. by writing the terms as

$$S_n = 8 + 88 + 888 + 8888 + \dots \text{to } n \text{ terms}$$

$$= \frac{8}{9} [9 + 99 + 999 + 9999 + \dots \text{to } n \text{ terms}]$$

$$= \frac{8}{9} [(10 - 1) + (10^2 - 1) + (10^3 - 1) + (10^4 - 1) + \dots \text{to } n \text{ terms}]$$

$$= \frac{8}{9} [(10 + 10^2 + \dots n \text{ terms}) - (1 + 1 + 1 + \dots n \text{ terms})]$$

$$= \frac{8}{9} \left[ \frac{10(10^n - 1)}{10 - 1} - n \right]$$

$$= \frac{8}{9} \left[ \frac{10(10^n - 1)}{9} - n \right]$$

$$= \frac{80}{81} (10^n - 1) - \frac{8}{9} n$$

Question 19:

Find the sum of the products of the corresponding terms of the sequences 2, 4, 8, 16, 32 and 128, 32, 8, 2,  $\frac{1}{2}$ .

Ans:

$$\begin{aligned}\text{Required sum} &= 2 \times 128 + 4 \times 32 + 8 \times 8 + 16 \times 2 + 32 \times \frac{1}{2} \\ &= 64 \left[ 4 + 2 + 1 + \frac{1}{2} + \frac{1}{2^2} \right]\end{aligned}$$

Here,  $4, 2, 1, \frac{1}{2}, \frac{1}{2^2}$  is a G.P.

First term,  $a = 4$

Common ratio,  $r = \frac{1}{2}$

It is known that,  $S_n = \frac{a(1-r^n)}{1-r}$

$$\therefore S_5 = \frac{4 \left[ 1 - \left( \frac{1}{2} \right)^5 \right]}{1 - \frac{1}{2}} = \frac{4 \left[ 1 - \frac{1}{32} \right]}{\frac{1}{2}} = 8 \left( \frac{32-1}{32} \right) = \frac{31}{4}$$

$$\text{Required sum} = 64 \left( \frac{31}{4} \right) = (16)(31) = 496$$

Question 20:

Show that the products of the corresponding terms of the sequences  $a, ar, ar^2, \dots, ar^{n-1}$  and  $A, AR, AR^2, \dots, AR^{n-1}$  form a G.P. and find the common ratio.

Ans:

It has to be proved that the sequence,  $aA, arAR, ar^2AR^2, \dots, ar^{n-1}AR^{n-1}$ , forms a G.P.

$$\begin{aligned}\frac{\text{Second term}}{\text{First term}} &= \frac{arAR}{aA} = rR \\ \frac{\text{Third term}}{\text{Second term}} &= \frac{ar^2AR^2}{arAR} = rR\end{aligned}$$

Thus, the above sequence forms a G.P. and the common ratio is  $rR$ .

Question 21:

Find four numbers forming a geometric progression in which third term is greater than the first term by 9, and the second term is greater than the 4<sup>th</sup> by 18.

Ans:

Let  $a$  be the first term and  $r$  be the common ratio of the G.P.

$$a_1 = a, a_2 = ar, a_3 = ar^2, a_4 = ar^3$$

By the given condition,

$$a_3 = a_1 + 9$$

$$ar^2 = a + 9 \dots (1)$$

$$a_2 = a_4 + 18$$

$$ar = ar^3 + 18 \dots (2)$$

From (1) and (2), we obtain

$$a(r^2 - 1) = 9 \dots (3)$$

$$ar(1 - r^2) = 18 \dots (4)$$

Dividing (4) by (3), we obtain

$$\frac{ar(1-r^2)}{a(r^2-1)} = \frac{18}{9}$$

$$\Rightarrow -r = 2$$

$$\Rightarrow r = -2$$

Substituting the value of  $r$  in (1), we obtain

$$4a = a + 9$$

$$3a = 9$$

$$a = 3$$

Thus, the first four numbers of the G.P. are 3,  $3(-2)$ ,  $3(-2)^2$ , and  $3(-2)^3$  i.e., 3, -6, 12, and -24.

Question 22:

If the  $p^{\text{th}}$ ,  $q^{\text{th}}$  and  $r^{\text{th}}$  terms of a G.P. are  $a$ ,  $b$  and  $c$ , respectively. Prove that  $a^{q-r} b^{r-p} c^{p-q} = 1$

Ans:

Let  $A$  be the first term and  $R$  be the common ratio of the G.P.

According to the given information,

$$AR^{p-1} = a$$

$$AR^{q-1} = b$$

$$AR^{r-1} = c$$

$$a^{q-r} b^{r-p} c^{p-q}$$

$$= A^{q-r} \times R^{(p-1)(q-r)} \times A^{r-p} \times R^{(q-1)(r-p)} \times A^{p-q} \times R^{(r-1)(p-q)}$$

$$= A^{q-r+r-p+p-q} \times R^{(pr-pr-q+r)+(rq-r+p-pq)+(pr-p-qr+q)}$$

$$= A^0 \times R^0$$

$$= 1$$

Thus, the given result is proved.

Question 23:

If the first and the  $n^{\text{th}}$  term of a G.P. are  $a$  and  $b$ , respectively, and if  $P$  is the product of  $n$  terms, prove that  $P^2 = (ab)^n$ .

Ans:

The first term of the G.P is  $a$  and the last term is  $b$ .

Therefore, the G.P. is  $a, ar, ar^2, ar^3, \dots, ar^{m-1}$ , where  $r$  is the common ratio.

$$b = ar^{m-1} \dots (1)$$

$P$  = Product of  $n$  terms

$$= (a) (ar) (ar^2) \dots (ar^{m-1})$$

$$= (a \times a \times \dots a) (r \times r^2 \times \dots r^{m-1})$$

$$= a^n r^{1+2+\dots(n-1)} \dots (2)$$

Here,  $1, 2, \dots, (n-1)$  is an A.P.

$$1 + 2 + \dots + (n-1) = \frac{n-1}{2} [2 + (n-1) \times 1] = \frac{n-1}{2} [2 + n - 2] = \frac{n(n-1)}{2}$$

$$P = a^n r^{\frac{n(n-1)}{2}}$$

$$\therefore P^2 = a^{2n} r^{n(n-1)}$$

$$= [a^2 r^{(n-1)}]^n$$

$$= [a \times ar^{n-1}]^n$$

$$= (ab)^n \quad [\text{Using (1)}]$$

Thus, the given result is proved.

Question 24:

Show that the ratio of the sum of first  $n$  terms of a G.P. to the sum of terms from  $(n+1)^{\text{th}}$  to  $(2n)^{\text{th}}$  term is  $\frac{1}{r^n}$ .

Ans:

Let  $a$  be the first term and  $r$  be the common ratio of the G.P.

$$\text{Sum of first } n \text{ terms} = \frac{a(1-r^n)}{(1-r)}$$

Since there are  $n$  terms from  $(n+1)^{\text{th}}$  to  $(2n)^{\text{th}}$  term,

$$\text{Sum of terms from } (n+1)^{\text{th}} \text{ to } (2n)^{\text{th}} \text{ term} = \frac{a_{n+1}(1-r^n)}{(1-r)}$$

$$a_{n+1} = ar^{n+1-1} = ar^n$$

$$\text{Thus, required ratio} = \frac{a(1-r^n)}{(1-r)} \times \frac{(1-r)}{ar^n(1-r^n)} = \frac{1}{r^n}$$

Thus, the ratio of the sum of first  $n$  terms of a G.P. to the sum of terms from  $(n+1)^{\text{th}}$  to  $(2n)^{\text{th}}$  term is  $\frac{1}{r^n}$ .

Question 25:

If  $a, b, c$  and  $d$  are in G.P. show that  $(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc + cd)^2$ .

Ans:



$a, b, c, d$  are in G.P.

Therefore,

$$bc = ad \dots (1)$$

$$b^2 = ac \dots (2)$$

$$c^2 = bd \dots (3)$$

It has to be proved that,

$$(a^2 + b^2 + c^2) (b^2 + c^2 + d^2) = (ab + bc + cd)^2$$

R.H.S.

$$= (ab + bc + cd)^2$$

$$= (ab + ad + cd)^2 \text{ [Using (1)]}$$

$$= [ab + d(a + c)]^2$$

$$= a^2b^2 + 2abd(a + c) + d^2(a + c)^2$$

$$= a^2b^2 + 2a^2bd + 2acbd + d^2(a^2 + 2ac + c^2)$$

$$= a^2b^2 + 2a^2c^2 + 2b^2c^2 + d^2a^2 + 2d^2b^2 + d^2c^2 \text{ [Using (1) and (2)]}$$

$$= a^2b^2 + a^2c^2 + a^2d^2 + b^2c^2 + b^2c^2 + d^2a^2 + d^2b^2 + d^2b^2 + d^2c^2$$

$$= a^2b^2 + a^2c^2 + a^2d^2 + b^2 \times b^2 + b^2c^2 + b^2d^2 + c^2b^2 + c^2 \times c^2 + c^2d^2$$

[Using (2) and (3) and rearranging terms]

$$= a^2(b^2 + c^2 + d^2) + b^2(b^2 + c^2 + d^2) + c^2(b^2 + c^2 + d^2)$$

$$= (a^2 + b^2 + c^2) (b^2 + c^2 + d^2)$$

= L.H.S.

L.H.S. = R.H.S.

$$(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc + cd)^2$$

Question 26:

Insert two numbers between 3 and 81 so that the resulting sequence is G.P.

Ans:

Let  $G_1$  and  $G_2$  be two numbers between 3 and 81 such that the series, 3,  $G_1$ ,  $G_2$ , 81, forms a G.P.

Let  $a$  be the first term and  $r$  be the common ratio of the G.P.

$$81 = (3) (r)^3$$

$$r^3 = 27$$

$$r = 3 \text{ (Taking real roots only)}$$

$$\text{For } r = 3,$$

$$G_1 = ar = (3) (3) = 9$$

$$G_2 = ar^2 = (3) (3)^2 = 27$$

Thus, the required two numbers are 9 and 27.

Question 27:

Find the value of  $n$  so that  $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$  may be the geometric mean between  $a$  and  $b$ .

Ans:

G. M. of  $a$  and  $b$  is  $\sqrt{ab}$ .

$$\text{By the given condition, } \frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \sqrt{ab}$$

Squaring both sides, we obtain

$$\begin{aligned} \frac{(a^{n+1} + b^{n+1})^2}{(a^n + b^n)^2} &= ab \\ \Rightarrow a^{2n+2} + 2a^{n+1}b^{n+1} + b^{2n+2} &= (ab)(a^{2n} + 2a^n b^n + b^{2n}) \\ \Rightarrow a^{2n+2} + 2a^{n+1}b^{n+1} + b^{2n+2} &= a^{2n+1}b + 2a^{n+1}b^{n+1} + ab^{2n+1} \\ \Rightarrow a^{2n+2} + b^{2n+2} &= a^{2n+1}b + ab^{2n+1} \\ \Rightarrow a^{2n+2} - a^{2n+1}b &= ab^{2n+1} - b^{2n+2} \\ \Rightarrow a^{2n+1}(a - b) &= b^{2n+1}(a - b) \\ \Rightarrow \left(\frac{a}{b}\right)^{2n+1} &= 1 = \left(\frac{a}{b}\right)^0 \\ \Rightarrow 2n+1 &= 0 \\ \Rightarrow n &= \frac{-1}{2} \end{aligned}$$

Question 28:

The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio  $(3+2\sqrt{2}) : (3-2\sqrt{2})$ .

Ans:

Let the two numbers be  $a$  and  $b$ .

$$\text{G.M.} = \sqrt{ab}$$

According to the given condition,

$$a + b = 6\sqrt{ab} \quad \dots(1)$$

$$\Rightarrow (a + b)^2 = 36(ab)$$

Also,

$$(a - b)^2 = (a + b)^2 - 4ab = 36ab - 4ab = 32ab$$

$$\Rightarrow a - b = \sqrt{32}\sqrt{ab}$$

$$= 4\sqrt{2}\sqrt{ab} \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2a = (6 + 4\sqrt{2})\sqrt{ab}$$

$$\Rightarrow a = (3 + 2\sqrt{2})\sqrt{ab}$$

Substituting the value of  $a$  in (1), we obtain

$$b = 6\sqrt{ab} - (3 + 2\sqrt{2})\sqrt{ab}$$

$$\Rightarrow b = (3 - 2\sqrt{2})\sqrt{ab}$$

$$\frac{a}{b} = \frac{(3 + 2\sqrt{2})\sqrt{ab}}{(3 - 2\sqrt{2})\sqrt{ab}} = \frac{3 + 2\sqrt{2}}{3 - 2\sqrt{2}}$$

Thus, the required ratio is  $(3 + 2\sqrt{2}) : (3 - 2\sqrt{2})$ .

Question 29:

If  $A$  and  $G$  be A.M. and G.M., respectively between two positive numbers, prove that the numbers are  $A \pm \sqrt{(A+G)(A-G)}$ .

Ans:

It is given that  $A$  and  $G$  are A.M. and G.M. between two positive numbers. Let these two positive numbers be  $a$  and  $b$ .

$$\therefore AM = A = \frac{a+b}{2} \quad \dots(1)$$

$$GM = G = \sqrt{ab} \quad \dots(2)$$

From (1) and (2), we obtain

$$a + b = 2A \quad \dots (3)$$

$$ab = G^2 \quad \dots (4)$$

Substituting the value of  $a$  and  $b$  from (3) and (4) in the identity  $(a - b)^2 = (a + b)^2 - 4ab$ , we obtain

$$(a - b)^2 = 4A^2 - 4G^2 = 4(A^2 - G^2)$$

$$(a - b)^2 = 4(A + G)(A - G)$$

$$(a - b) = 2\sqrt{(A + G)(A - G)} \quad \dots(5)$$

From (3) and (5), we obtain

$$2a = 2A + 2\sqrt{(A + G)(A - G)}$$

$$\Rightarrow a = A + \sqrt{(A + G)(A - G)}$$

Substituting the value of  $a$  in (3), we obtain

$$b = 2A - a - \sqrt{(A + G)(A - G)} = A - \sqrt{(A + G)(A - G)}$$

Thus, the two numbers are  $A \pm \sqrt{(A + G)(A - G)}$ .

**Question 30:**

The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2<sup>nd</sup> hour, 4<sup>th</sup> hour and 2<sup>th</sup> hour?

**Ans:**

It is given that the number of bacteria doubles every hour. Therefore, the number of bacteria after every hour will form a G.P.

Here,  $a = 30$  and  $r = 2$

$$a_3 = ar^2 = (30)(2)^2 = 120$$

Therefore, the number of bacteria at the end of 2<sup>nd</sup> hour will be 120.

$$a_5 = ar^4 = (30)(2)^4 = 480$$

The number of bacteria at the end of 4<sup>th</sup> hour will be 480.

$$a_{n+1} = ar^n = (30)2^n$$

Thus, number of bacteria at the end of  $n^{\text{th}}$  hour will be  $30(2)^n$ .

**Question 31:**

What will Rs 500 amounts to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually?

**Ans:**

The amount deposited in the bank is Rs 500.

At the end of first year, amount =  $\text{Rs } 500 \left(1 + \frac{1}{10}\right) = \text{Rs } 500 (1.1)$

At the end of 2<sup>nd</sup> year, amount = Rs 500 (1.1) (1.1)

At the end of 3<sup>rd</sup> year, amount = Rs 500 (1.1) (1.1) (1.1) and so on

□ Amount at the end of 10 years = Rs 500 (1.1) (1.1) ... (10 times)

$$= \text{Rs } 500(1.1)^{10}$$

Question 32:

If A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively, then obtain the quadratic equation.

Ans:

Let the root of the quadratic equation be  $a$  and  $b$ .

According to the given condition,

$$\text{A.M.} = \frac{a+b}{2} = 8 \Rightarrow a+b = 16 \quad \dots(1)$$

$$\text{G.M.} = \sqrt{ab} = 5 \Rightarrow ab = 25 \quad \dots(2)$$

The quadratic equation is given by,

$$x^2 - x (\text{Sum of roots}) + (\text{Product of roots}) = 0$$

$$x^2 - x (a + b) + (ab) = 0$$

$$x^2 - 16x + 25 = 0 \text{ [Using (1) and (2)]}$$

Thus, the required quadratic equation is  $x^2 - 16x + 25 = 0$

\*\*\*\*\* END \*\*\*\*\*