Question 6. 11. A body constrained to move along the z-axis of a coordinate system is subject to a constant force F given by

$$F = -\hat{i} + 2\hat{j} + 3\hat{k} N$$

where i, j, k, are unit vectors along the x- y- and z-axis of the system respectively. What is the work done by this force in moving the body a distance of 4 m along the z-axis?
Answer:

Since the body is displaced 4 m along z-axis only,

$\therefore \quad \vec{S} = 0\hat{i} + 0\hat{j} + 4\hat{k}$

Also $\quad \vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$

Work done, $\quad W = \vec{F}.\vec{S}$

$= (-\hat{i} + 2\hat{j} + 3\hat{k}).(0\hat{i} + 0\hat{j} + 4\hat{k})$

$= 12\ (\hat{k}.\hat{k})$ Joule = 12 Joule.

Question 6. 12. An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV, and the second with 100 keV. Which is faster, the electron or the proton? Obtain the ratio of their speeds, (electron mass = 9.11×10^{-31} kg, proton mass = 1.67×10^{-27} kg, 1 eV= 1.60×10^{19} J).
Answer:

Here $\quad K_e = 10\ keV$ and $K_p = 100\ keV$

$m_e = 9.11 \times 10^{-31}$ kg and $m_p = 1.67 \times 10^{-27}$ kg

As $\quad K = \frac{1}{2}mv^2$ or $v = \sqrt{\frac{2K}{m}}$,

Hence, $\quad \frac{v_e}{v_p} = \sqrt{\frac{K_e}{K_p} \times \frac{m_p}{m_e}} = \sqrt{\frac{10\ keV}{100\ keV} \times \frac{1.67 \times 10^{-27}\ kg}{9.11 \times 10^{-31}\ kg}}$

$= 13.54$

$\Rightarrow \quad v_e = 13.54\ v_p$.

Thus, electron is travelling faster.

Question 6. 13. A rain drop of radius 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration (due to viscous resistance of the air) until at half its original height, it attains its maximum (terminal) speed, and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first and second half of its journey? What is the work done by the resistive force in the entire journey if its speed on reaching the ground is 10 ms^{-1} ?
Answer:
Here, r = 2 mm = 2×10^{-3} m.
Distance moved in each half of the journey, S=500/2= 250 m.
Density of water, p = 10^3 kg/ m^3

Mass of rain drop = volume of drop x density

$m = 4/3\ \pi r^2 \times \rho$

$= 4/3 \times 22/7\ (2 \times 10^{-3})^3 \times 10^3$

$= 3.35 \times 10^{-5}$ kg

$\therefore W = mg \times s = 3.35 \times 10^{-5} \times 9.8 \times 250 = 0.082$ J

Note: Whether the drop moves with decreasing acceleration or with uniform speed, work done by the gravitational force on the drop remains the same.

If there was no resistive forces, energy of drop on reaching the ground.

$E_1 = mgh = 3.35 \times 10^{-5} \times 9.8 \times 500 = 0.164$ J

Actual energy, $E_2 = 1/2mv^2 = 1/2 \times 3.35 \times 10^{-5}\ (10)^2 = 1.675 \times 10^{-3}$J

Work done by the resistive forces, $W = E_1 - E_2 = 0.164 - 1.675 \times 10^{-3}$ W

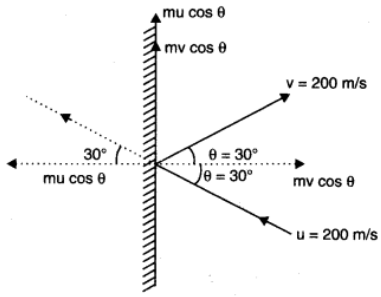
= 0.1623 joule.

Question 6. 14. A molecule in a gas container hits a horizontal wall with speed 200 msA and angle 30° with the normal, and rebounds with the same speed. Is momentum conserved in the collision? Is the collision elastic or inelastic?

Answer: Let us consider the mass of the molecule be m and that of wall be M. The wall remains at rest due to its large mass. Resolving momentum of the molecule along x-axis and y-axis, we get

The x-component of momentum of molecule

$$= mu \cos\theta = -m\ 200 \cos 30° = -100\sqrt{3}\ m$$



y-component of the molecule

$$= mu \sin\theta = m \times 200 \times \sin 30° = 100\ m$$

Before collision: x-component of total momentum (wall + molecule)

$$= 0 + (-100\sqrt{3}\ m) = -100\sqrt{3}\ m$$

y- component of momentum (wall + molecule) = 0 + 100 m = 100 m

After collision: x-component of the momentum (wall + molecule)

$$= 0 + m\ 200 \cos 30° = 100\sqrt{3}\ m$$

and y-component $= 0 + m\ 100 \sin 30° = 100\ m$

We find that momentum of the (molecule + wall) system is conserved. The wall has a recoil momentum such that momentum of the wall + momentum of outgoing molecule equals the momentum of the incoming molecule.

Initial kinetic energy $\left(\frac{1}{2}mu^2\right)$ is the same as final K.E. $\left(\frac{1}{2}mv^2\right)$ of the molecule as

$u = v = 200$ m/s *i.e.*, thus, the collision is elastic collision.

Question 6. 15. A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m^3 in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 30%, how much electric power is consumed by the pump?

Answer:

Here, volume of water = 30 m^3 ; t = 15 min = 15 x 60 = 900s

h = 40 m ; n = 30%

As the density of water = p = 10^3 kg m^{-3}

Mass of water pumped, m = volume x density = 30 x 10^3 kg

Actual power consumed or output power p_0 = W/t = mgh/t

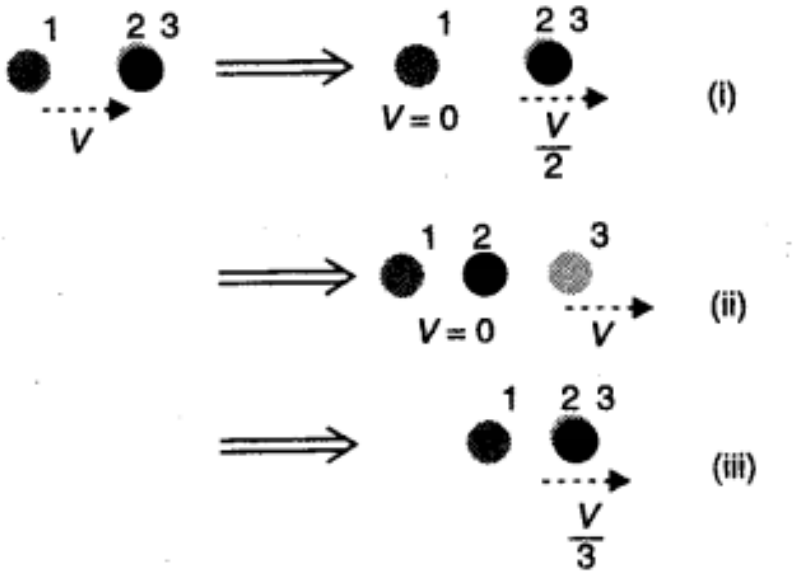
$\Rightarrow p_0 = (30 \times 10^3 \times 9.8 \times 40)/900 = 13070$ watt

If pi is input power (required), then as

$\eta = p_0/p_i \Rightarrow p_i = p_0/\eta = 13070/(30/100) = 43567$ W =43.56 KW

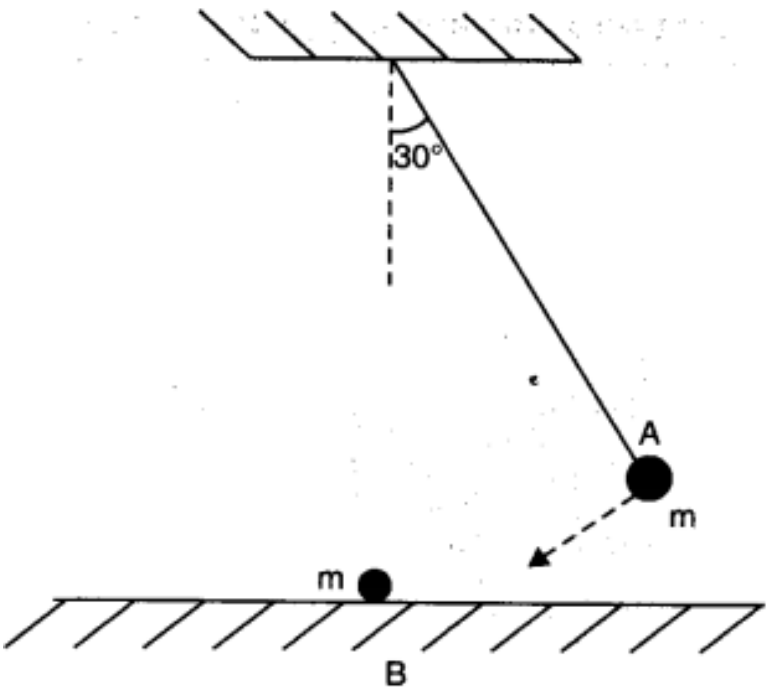
Question 6. 16. Two identical ball bearings in contact with each other and resting on a friction less table are hit head-on by another ball

bearing of the same mass moving initially with a speed V. If the collision is elastic, which of the following (Fig.) is a possible result after collision?



Answer:
Let m be the mass of each ball bearing. Before collision, total K.E. of the system
$= 1/2mv^2 + 0 = 1/2\ mv^2$
After collision, K.E. of the system is
Case I, $E_1 = 1/2\ (2m)\ (v/2)^2 = 1/4\ mv^2$
Case II, $E_2 = 1/2\ mv^2$
Case III, $E_3 = 1/2(3m)\ (v/3)^2 = 1/6mv^2$
Thus, case II is the only possibility since K.E. is conserved in this case.

Question 6.17. The bob A of a pendulum released from 30° to the vertical hits another bob B of the same mass at rest on a table as shown in Fig. How high does the bob A rise after the collision? Neglect the size of the bobs and assume the collision to be elastic.
Answer: Since collision is elastic therefore A would come to rest and B would begin to move with the velocity of A.



The bob transfers its entire momentum to the ball on the table. The bob does not rise at all.

Question 6. 18. The bob of a pendulum is released from a horizontal position. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowermost point, given that it dissipated 5% of its initial energy against air resistance?
Answer: On releasing the bob of pendulum from horizontal position, it falls vertically downward by a distance equal to length of pendulum i.e., h = l = 1.5 m .
As 5% of loss in P.E. is dissipated against air resistance, the balance 95% energy is transformed into K.E. Hence,

$$\frac{1}{2}mv^2 = \frac{95}{100} \times mgh$$

$$\Rightarrow \quad v = \sqrt{2 \times \frac{95}{100} \times gh} = \sqrt{\frac{2 \times 95 \times 9.8 \times 1.5}{100}} = 5.3 \text{ ms}^{-1}$$

Question 6. 19. A trolley of mass 300 kg carrying a sandbag of 25 kg is moving uniformly with a speed of 27 km/h on a friction less track. After a while, sand starts leaking out of a hole on the trolley's floor at the rate of 0.05 kg s^{-1}. What is the speed of the trolley after the entire sand bag is empty?
Answer: The system of trolley and sandbag is moving with a uniform speed. Clearly, the system is not being acted upon by an external force. If the sand leaks out, even then no external force acts. So there shall be no change in the speed of the trolley.

Question 6. 20. A particle of mass 0.5 kg travels in a straight line with velocity $u = a\,x^{3/2}$, where $a = 5\ m^{-1/2}\ s^{-1}$. What is the work done by the net force during its displacement from x = 0 to x = 2 m?
Answer:
Here m = 0.5 kg
$u = a\,x^{3/2}$, $a = 5\ m^{-1/2}\ s^{-1}$.
Initial velocity at x = 0, $v_1 = a \times 0 = 0$
Final velocity at x = 2, $v_2 = a\,(2)^{3/2} = 5 \times (2)^{3/2}$
Work done = increase in K.E
$= 1/2\ m(v_2^2 - v_1^2) = 1/2 \times 0.5[(5 \times (2)^{3/2})^2 - 0] = 50$ J.

********** END **********