Indefinite Integrals Ex 19.25 Q51

Let $\quad I = \int \left(\tan^{-1} x^2\right) x dx$

Let $\quad x^2 = t$

$$2x\,dx = dt$$

$$I = \frac{1}{2}\int \tan^{-1} t\,dt$$

$$= \frac{1}{2}\int 1 \tan^{-1} t\,dt$$

$$= \frac{1}{2}\left[\tan^{-1} t \int dt - \left(\int \frac{1}{1+t^2}\int dt\right)dt\right]$$

$$= \frac{1}{2}\left[t\tan^{-1} t - \int \frac{t}{1+t^2}dt\right]$$

$$= \frac{1}{2}t\tan^{-1} t - \frac{1}{4}\int \frac{2t}{1+t^2}dt$$

$$= \frac{1}{2}t\tan^{-1} t - \frac{1}{4}\log\left|1+t^2\right| + c$$

$$I = \frac{1}{2}x^2\tan^{-1} x^2 - \frac{1}{4}\log\left|1+x^4\right| + c$$

Indefinite Integrals Ex 19.25 Q52

Let first function be $\sin^{-1} x$ and second dunction be $\frac{x}{\sqrt{1-x^2}}$.

First we find the intergral of the second function, i.e., $\int \frac{xdx}{\sqrt{1-x^2}}$.

Put $t = 1 - x^2$. Then $dt = -2x\,dx$

Therefore, $\quad \int \frac{xdx}{\sqrt{1-x^2}} = -\frac{1}{2}\int \frac{dt}{\sqrt{t}} = -\sqrt{t} = -\sqrt{1-x^2}$

Hence, $\quad \int \frac{x\sin^{-1} x}{\sqrt{1-x^2}}dx = \left(\sin^{-1} x\right)\left(-\sqrt{1-x^2}\right) - \int \frac{1}{\sqrt{1-x^2}}\left(-\sqrt{1-x^2}\right)dx$

$$= -\sqrt{1-x^2}\sin^{-1} x + x + C = x - \sqrt{1-x^2}\sin^{-1} x + C$$

Indefinite Integrals Ex 19.25 Q53

Let $I = \int \sin^3 \sqrt{x}\, dx$

$\sqrt{x} = t$

$x = t^2$

$dx = 2t\, dt$

$I = 2\int t \sin^3 t\, dt$

$= 2\int t \left(\dfrac{3\sin t - \sin 3t}{4}\right) dt$

$= \dfrac{1}{2}\int t\left(3\sin t - \sin 3t\right) dt$

Using integration by parts,

$I = \dfrac{1}{2}\left[t\left(-3\cos t + \dfrac{1}{3}\cos 3t\right) - \int\left(-3\cos t + \dfrac{\cos 3t}{3}\right) dt\right]$

$= \dfrac{1}{2}\left[\dfrac{-9t\cos t + t\cos 3t}{3} - \left\{-3\sin t + \dfrac{\sin 3t}{9}\right\}\right] + c$

$= \dfrac{1}{2}\left[\dfrac{-9t\cos t + t\cos 3t}{3} + \dfrac{27\sin t - 3\sin 3t}{9}\right] + c$

$= \dfrac{1}{18}\left[-27t\cos t + 3t\cos 3t + 27\sin t - 3\sin 3t\right] + c$

$I = \dfrac{1}{18}\left[3\sqrt{x}\cos 3\sqrt{x} + 27\sin\sqrt{x} - 27\sqrt{x}\cos\sqrt{x} - 3\sin 3\sqrt{x}\right] + c$

Indefinite Integrals Ex 19.25 Q54

Let $I = \int x \sin^3 x\, dx$

$= \int x\left(\dfrac{3\sin x - \sin 3x}{4}\right) dx$

$= \dfrac{1}{4}\int x\left(3\sin x - \sin 3x\right) dx$

Using integration by parts,

$I = \dfrac{1}{4}\left[x\int\left(3\sin x - \sin 3x\right) dx - \int\left(1\int\left(3\sin x - \sin 3x\right) dx\right) dx\right]$

$= \dfrac{1}{4}\left[x\left(-3\cos x + \dfrac{\cos 3x}{3}\right) - \int\left(-3\cos x + \dfrac{\cos 3x}{3}\right) dx\right]$

$= \dfrac{1}{4}\left[-3x\cos x + \dfrac{x\cos 3x}{3} + 3\sin x - \dfrac{\sin 3x}{9}\right] + c$

$I = \dfrac{1}{36}\left[3x\cos 3x - 27x\cos x + 27\sin x - \sin 3x\right] + c$

Indefinite Integrals Ex 19.25 Q55

Let $I = \int \cos^3 \sqrt{x}\, dx$

Let $x = t^2$

$dx = 2t\,dt$

$= 2\int t \cos^3 t\, dt$

$= 2\int t \left(\dfrac{3\cos t + \cos 3t}{4}\right) dt$

$= \dfrac{1}{2}\int t\left(3\cos t + \cos 3t\right) dt$

Using integration by parts,

$$I = \frac{1}{2}\left[t\left(3\sin t + \frac{1}{3}\sin 3t\right) + \int \left(1 \times 3\sin t + \frac{\sin 3t}{3}\right) dt\right]$$

$$= \frac{1}{2}\left[t\left(\frac{9\sin t + \sin 3t}{3}\right) + 3\cos t + \frac{\cos 3t}{9}\right] + c$$

$$= \frac{1}{18}\left[27t\sin t + 3t\sin 3t + 9\cos t + \cos 3t\right] + c$$

$$I = \frac{1}{18}\left[27\sqrt{x}\,\sin\sqrt{x} + 3\sqrt{x}\,\sin 3\sqrt{x} + 9\cos\sqrt{x} + \cos 3\sqrt{x}\right] + c$$

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