Sine and Cosine Formulae and their Applications Ex-10.2 Q15

$$\text{Let } \frac{b+c}{12}=\frac{c+a}{13}=\frac{a+b}{15}=\lambda(\text{say})$$

$$b+c=12\lambda, c+a=13\lambda, a+b=15\lambda$$

$$(b+c+c+a+a+b)=12\lambda+13\lambda+15\lambda$$

$$2(a+b+c)=40\lambda$$

$$a+b+c=20\lambda$$

$$b+c=12\lambda \text{ and } a+b+c=20\lambda \Rightarrow a=8\lambda$$

$$c+a=13\lambda \text{ and } a+b+c=20\lambda \Rightarrow b=7\lambda$$

$$a+b=15\lambda \text{ and } a+b+c=20\lambda \Rightarrow c=5\lambda$$

$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{49\lambda^2+25\lambda^2-64\lambda^2}{70\lambda^2}=\frac{1}{7}$$

$$\cos B=\frac{a^2+c^2-b^2}{2ac}=\frac{64\lambda^2+25\lambda^2-49\lambda^2}{80\lambda^2}=\frac{1}{2}$$

$$\cos C=\frac{a^2+b^2-c^2}{2ab}=\frac{64\lambda^2+49\lambda^2-25\lambda^2}{112\lambda^2}=\frac{11}{14}$$

$$\cos A:\cos B:\cos C=\frac{1}{7}:\frac{1}{2}:\frac{11}{14}=2:7:11$$

Sine and Cosine Formulae and their Applications Ex-10.2 Q16

We have, $\angle B=60^\circ$

$$\cos B=\frac{1}{2}\Rightarrow\frac{a^2+c^2-b^2}{2ac}=\frac{1}{2}$$

$$\Rightarrow a^2+c^2-b^2=ac$$

$$\Rightarrow a^2+c^2-ac=b^2 \quad \ldots\ldots(i)$$

$$(a+b+c)(a-b+c)=3ca$$

$$a^2-ab+ac+ab-b^2+bc+ac-bc+c^2=3ac$$

$$a^2+c^2-b^2+2ac-3ac=0$$

$$a^2+c^2-ac=b^2$$

which is given.

Sine and Cosine Formulae and their Applications Ex-10.2 Q17

Consider the given equation:

$$\cos^2 A+\cos^2 B+\cos^2 C=1$$

$$\Rightarrow 1-\sin^2 A+1-\sin^2 B+1-\sin^2 C=1$$

$$\Rightarrow 3-\sin^2 A+1-\sin^2 B+1-\sin^2 C=1$$

Sine and Cosine Formulae and their Applications Ex-10.2 Q18

$\text{Let}\dfrac{\sin A}{a}=\dfrac{\sin B}{b}=\dfrac{\sin C}{c}=k.\text{Then},\sin A=ka,\sin B=kb,\sin C=kc$

$\text{Now},\cos C=\dfrac{\sin A}{2\sin B}$

$2\sin B\cos C=\sin A$

$2\left(\dfrac{a^2+b^2-c^2}{2ab}\right)kb=ka$

$a^2+b^2-c^2=a^2$

$b^2=c^2$

$b=c$

$\triangle ABC\text{is isosceles.}$

Sine and Cosine Formulae and their Applications Ex-10.2 Q19

Let P and Q be the position of two ships at the end of 3 hours.

Then,

$OP=3\times24=72km\text{ and }OQ=3\times32=96km$

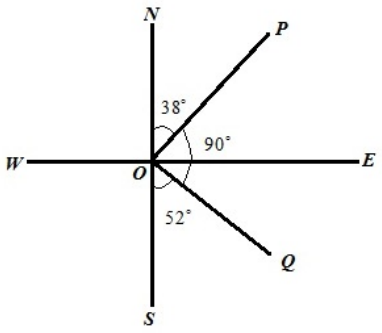
Using cosine formula in ΔOPQ, we get

$PQ^2=OP^2+OQ^2-2OP\times OQ\cos90^\circ$

$PQ^2=72^2+96^2-2\times72\times96\cos90^\circ$

$PQ^2=14400$

$PQ=120km$



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