Indefinite Integrals Ex 19.30 Q1

Let $\int \frac{2x+1}{(x+1)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x-2)}$

$\Rightarrow \quad 2x+1 = A(x-2) + B(x+1)$

Put $x = 2$

$\Rightarrow \quad 5 = 3B \Rightarrow \quad B = \frac{5}{3}$

Put $x = -1$

$\Rightarrow \quad -1 = -3A \quad \Rightarrow \quad A = \frac{1}{3}$

So,

$$\int \frac{2x+1}{(x+1)(x-2)}dx = \frac{1}{3}\int \frac{dx}{x+1} + \frac{5}{3}\int \frac{dx}{x-2}$$

$$= \frac{1}{3}\log|x+1| + \frac{5}{3}\log|x-2| + c$$

Thus,

$$I = \frac{1}{3}\log|x+1| + \frac{5}{3}\log|x-2| + c$$

Indefinite Integrals Ex 19.30 Q2

Let $\int \frac{1}{x(x-2)(x-4)}dx = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x-4}$

$\Rightarrow \quad 1 = A(x-2)(x-4) + B(x)(x-4) + Cx(x-2)$

Put $x = 0$

$\Rightarrow \quad 1 = 8A \Rightarrow \quad A = \frac{1}{8}$

Put $x = 2$

$\Rightarrow \quad 1 = -4B \quad \Rightarrow \quad B = -\frac{1}{4}$

Put $x = 4$

$\Rightarrow \quad 1 = 8C \Rightarrow \quad C = \frac{1}{8}$

So,

$$\int \frac{1}{x(x-2)(x-4)}dx = \frac{1}{8}\int \frac{dx}{x} + \left(-\frac{1}{4}\right)\int \frac{dx}{x-2} + \frac{1}{8}\int \frac{dx}{x-4}$$
$$= \frac{1}{8}\log|x| - \frac{1}{4}\log|x-2| + \frac{1}{8}\log|x-4| + c$$
$$= \frac{1}{8}\log\left|\frac{x(x-4)}{(x-2)^2}\right| + c$$

Thus,

$$I = \frac{1}{8}\log\left|\frac{x(x-4)}{(x-2)^2}\right| + c$$

Indefinite Integrals Ex 19.30 Q3

Let $I = \int \frac{x^2 + x - 1}{x^2 + x - 6} dx$

$= \int 1 + \frac{5}{x^2 + x - 6} dx$

$\Rightarrow \quad I = \int dx + \int \frac{5dx}{(x+3)(x-2)} \quad ---(1)$

Let $\frac{5}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2}$

$\Rightarrow \quad 5 = A(x-2) + B(x+3)$

Put $x = 2$

$\Rightarrow \quad 5 = 5B \Rightarrow \quad B = 1$

Put $x = -3$

$\Rightarrow \quad 5 = -5A \Rightarrow \quad A = -1$

$\therefore \quad I = \int dx + \int \frac{-dx}{x+3} + \int \frac{dx}{x-2}$

$= x - \log|x+3| + \log|x-2| + c$

Hence,

$I = x - \log|x+3| + \log|x-2| + c$

Indefinite Integrals Ex 19.30 Q4

Let $I = \int \frac{3+4x-x^2}{(x+2)(x-1)}dx$

$= \int -1 + \frac{5x+1}{(x+2)(x-1)}dx$

$\Rightarrow \quad I = -\int dx + \int \frac{5x+1}{(x+2)(x-1)}dx \qquad ---(1)$

Let $\frac{5x+1}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$

$\Rightarrow \quad 5x+1 = A(x-1) + B(x+2)$

Put $x = 1$

$\Rightarrow \quad 6 = 3B \Rightarrow \quad B = 2$

Put $x = -2$

$\Rightarrow \quad -9 = -3A \Rightarrow \quad A = 3$

So,

$I = -\int dx + 3\int \frac{dx}{x+2} + 2\int \frac{dx}{x-1}$

$I = -x + 3\log|x+2| + 2\log|x-1| + c$

Indefinite Integrals Ex 19.30 Q5

$$\text{Let } I = \int \frac{x^2+1}{x^2-1}dx$$

$$= \int 1 + \frac{2}{x^2-1}dx$$

$$= \int dx + \int \frac{2dx}{(x+1)(x-1)}$$

$$= \int dx + \int \frac{-1}{x+1} + \frac{1}{x-1}dx$$

$$= x - \log|x+1| + \log|x-1| + c$$

$$I = x + \log\left|\frac{x-1}{x+1}\right| + c$$

********** END **********