Indefinite Integrals Ex 19.2 Q43

$$\frac{1-\cos x}{1+\cos x}=\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}} \qquad \left[2\sin^2\frac{x}{2}=1-\cos x \text{ and } 2\cos^2\frac{x}{2}=1+\cos x\right]$$

$$=\tan^2\frac{x}{2}$$

$$=\left(\sec^2\frac{x}{2}-1\right)$$

$$\therefore \int\frac{1-\cos x}{1+\cos x}dx=\int\left(\sec^2\frac{x}{2}-1\right)dx$$

$$=\left[\frac{\tan\frac{x}{2}}{\frac{1}{2}}-x\right]+C$$

$$=2\tan\frac{x}{2}-x+C$$

Indefinite Integrals Ex 19.2 Q44

$$\int\left\{3\sin x-4\cos x+\frac{5}{\cos^2 x}-\frac{6}{\sin^2 x}+\tan^2 x-\cot^2 x\right\}dx$$

$$=3\int\sin x dx-4\int\cos x dx+5\int\sec^2 dx-6\int cosec^2 x+\int\tan^2 x dx-\int\cot^2 x dx$$

$$=3\int\sin x dx-4\int\cos x dx+5\int\sec^2 x dx-6\int cosec^2 x+\int\left(\sec^2 x-1\right)dx-\int\left(cosec^2 x-1\right)dx$$

$$=3\int\sin x dx-4\int\cos x dx+6\int\sec^2 x dx-7\int cosec^2 x dx$$

$$=-3\cos x-4\sin x+6\tan x+7\cot x+c$$

Indefinite Integrals Ex 19.2 Q45

It is given that $f'(x) = x - \frac{1}{x^2}$

$$\therefore \quad \int f'(x) = \int \left(x - \frac{1}{x^2}\right) dx$$

$$\Rightarrow \quad f(x) = \int x dx - \int \frac{1}{x^2} dx$$

$$= \frac{x^2}{2} + x^{-1} + c$$

$$= \frac{x^2}{2} + \frac{1}{x} + c$$

$$\Rightarrow \quad f(x) = \frac{x^2}{2} + \frac{1}{x} + c \qquad \text{---(i)}$$

Now,

$$f(1) = \frac{1}{2} \qquad [\text{given}]$$

$$\Rightarrow \quad \frac{1^2}{2} + \frac{1}{1} + c = \frac{1}{2}$$

$$\Rightarrow \quad c = -1$$

Putting $c = -1$ in (i), we get

$$f(x) = \frac{x^2}{2} + \frac{1}{x} - 1.$$

Indefinite Integrals Ex 19.2 Q46

It is given that $f'(x) = x + b$

$$\therefore \quad \int f'(x) = \int (x + b) dx$$

$$\Rightarrow \quad f(x) = \frac{x^2}{2} + bx + c \qquad \text{---(i)}$$

Since,

$$f(1) = 5$$

$$\therefore \quad \frac{1^2}{2} + b \times 1 + c = 5$$

$$\Rightarrow \quad \frac{1}{2} + b + c = 5$$

$$\Rightarrow \quad b + c = \frac{9}{2} \qquad \text{---(ii)}$$

and, $f(2) = 13$

$$\Rightarrow \quad \frac{(2)^2}{2} + b \times 2 + c = 13$$

$$\Rightarrow \quad 2 + 2b + c = 13$$

$$\Rightarrow \quad 2b + c = 11 \qquad \text{---(iii)}$$

Subtracting equation (ii) from equatio n(iii), we get

$$b = 11 - \frac{9}{2}$$

$$\Rightarrow \quad b = \frac{13}{2}$$

Putting $b = \frac{13}{2}$ in equation (ii), we get

$$\frac{13}{2} + c = \frac{9}{2}$$

$$\Rightarrow \quad c = \frac{9}{2} - \frac{13}{2}$$

$$\Rightarrow \quad c = \frac{9 - 13}{2} = \frac{-4}{2} = -2$$

Putting $b = \frac{13}{2}$ and $c = -2$ in equation (i), we get

$$f(x) = \frac{x^2}{x} + \frac{13}{2}x - 2$$

$$\therefore \quad f(x) = \frac{x^2}{2} + \frac{13}{2}x - 2$$

*********** END **********