Question 11:

Show that the relation R in the set A of points in a plane given by R = {(P, Q): distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further, show that the set of all point related to a point P ≠ (0, 0) is the circle passing through P with origin as centre.

Answer

R = {(P, Q): distance of point P from the origin is the same as the distance of point Q from the origin}

Clearly, (P, P) ∈ R since the distance of point P from the origin is always the same as the distance of the same point P from the origin.

∴R is reflexive.

Now,

Let (P, Q) ∈ R.

⇒ The distance of point P from the origin is the same as the distance of point Q from the origin.

⇒ The distance of point Q from the origin is the same as the distance of point P from the origin.

⇒ (Q, P) ∈ R

∴R is symmetric.

Now,

Let (P, Q), (Q, S) ∈ R.

⇒ The distance of points P and Q from the origin is the same and also, the distance of points Q and S from the origin is the same.

⇒ The distance of points P and S from the origin is the same.

⇒ (P, S) ∈ R

∴R is transitive.

Therefore, R is an equivalence relation.

The set of all points related to P ≠ (0, 0) will be those points whose distance from the origin is the same as the distance of point P from the origin.

In other words, if O (0, 0) is the origin and OP = k, then the set of all points related to P is at a distance of k from the origin.

Hence, this set of points forms a circle with the centre as the origin and this circle passes through point P.

Question 12:

Show that the relation R defined in the set A of all triangles as R = {(T_1, T_2): T_1 is similar to T_2}, is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

Answer

R = {(T_1, T_2): T_1 is similar to T_2}

R is reflexive since every triangle is similar to itself.

Further, if (T_1, T_2) ∈ R, then T_1 is similar to T_2.

⇒ T_2 is similar to T_1.

⇒ (T_2, T_1) ∈R

∴R is symmetric.

Now,

Let (T_1, T_2), (T_2, T_3) ∈ R.

⇒ T_1 is similar to T_2 and T_2 is similar to T_3.

⇒ T_1 is similar to T_3.

⇒ (T_1, T_3) ∈ R

∴ R is transitive.

Thus, R is an equivalence relation.

Now, we can observe that:

$$\frac{3}{6}=\frac{4}{8}=\frac{5}{10}\left(=\frac{1}{2}\right)$$

∴The corresponding sides of triangles T_1 and T_3 are in the same ratio.

Then, triangle T_1 is similar to triangle T_3.

Hence, T_1 is related to T_3.

Question 13:

Show that the relation R defined in the set A of all polygons as R = $\{(P_1, P_2): P_1$ and P_2 have same number of sides$\}$, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5?

Answer

R = $\{(P_1, P_2): P_1$ and P_2 have same the number of sides$\}$

R is reflexive since $(P_1, P_1) \in$ R as the same polygon has the same number of sides with itself.

Let $(P_1, P_2) \in$ R.

$\Rightarrow P_1$ and P_2 have the same number of sides.

$\Rightarrow P_2$ and P_1 have the same number of sides.

$\Rightarrow (P_2, P_1) \in$ R

$\therefore$R is symmetric.

Now,

Let $(P_1, P_2), (P_2, P_3) \in$ R.

$\Rightarrow P_1$ and P_2 have the same number of sides. Also, P_2 and P_3 have the same number of sides.

$\Rightarrow P_1$ and P_3 have the same number of sides.

$\Rightarrow (P_1, P_3) \in$ R

$\therefore$R is transitive.

Hence, R is an equivalence relation.

The elements in A related to the right-angled triangle (T) with sides 3, 4, and 5 are those polygons which have 3 sides (since T is a polygon with 3 sides).

Hence, the set of all elements in A related to triangle T is the set of all triangles.

Question 14:

Let L be the set of all lines in XY plane and R be the relation in L defined as R = $\{(L_1, L_2): L_1$ is parallel to $L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$.

Answer

R = $\{(L_1, L_2)$: L_1 is parallel to $L_2\}$

R is reflexive as any line L_1 is parallel to itself i.e., $(L_1, L_1) \in$ R.

Now,

Let $(L_1, L_2) \in$ R.

$\Rightarrow L_1$ is parallel to L_2.

$\Rightarrow L_2$ is parallel to L_1.

$\Rightarrow (L_2, L_1) \in$ R

$\therefore$ R is symmetric.

Now,

Let $(L_1, L_2), (L_2, L_3) \in$R.

$\Rightarrow L_1$ is parallel to L_2. Also, L_2 is parallel to L_3.

$\Rightarrow L_1$ is parallel to L_3.

$\therefore$R is transitive.

Hence, R is an equivalence relation.

The set of all lines related to the line $y = 2x + 4$ is the set of all lines that are parallel to the line $y = 2x + 4$.

Slope of line $y = 2x + 4$ is $m = 2$

It is known that parallel lines have the same slopes.

The line parallel to the given line is of the form $y = 2x + c$, where $c \in \mathbf{R}$.

Hence, the set of all lines related to the given line is given by $y = 2x + c$, where $c \in \mathbf{R}$.

Question 15:

Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}. Choose the correct answer.

(A) R is reflexive and symmetric but not transitive.

(B) R is reflexive and transitive but not symmetric.

(C) R is symmetric and transitive but not reflexive.

(D) R is an equivalence relation.

Answer

R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}

It is seen that $(a, a) \in$ **R,** for every $a \in \{1, 2, 3, 4\}$.

$\therefore$ R is reflexive.

It is seen that $(1, 2) \in$ R, but $(2, 1) \notin$ R.

$\therefore$R is not symmetric.

Also, it is observed that $(a, b), (b, c) \in$ R $\Rightarrow (a, c) \in$ R for all $a, b, c \in \{1, 2, 3, 4\}$.

$\therefore$ R is transitive.

Hence, R is reflexive and transitive but not symmetric.

The correct answer is B.

Question 16:

Let R be the relation in the set **N** given by R = $\{(a, b): a = b - 2, b > 6\}$. Choose the correct answer.

(A) (2, 4) ∈ R (B) (3, 8) ∈R (C) (6, 8) ∈R (D) (8, 7) ∈ R

Answer

R = {(*a*, *b*): *a* = *b* − 2, *b* > 6}

Now, since *b* > 6, (2, 4) ∉ R

Also, as 3 ≠ 8 − 2, (3, 8) ∉ R

And, as 8 ≠ 7 − 2

∴ (8, 7) ∉ R

Now, consider (6, 8).

We have 8 > 6 and also, 6 = 8 − 2.

∴(6, 8) ∈ R

The correct answer is C.

Exercise 1.2

Question 1:

Show that the function *f*: **R**∗ → **R**∗ defined by $f(x)=\frac{1}{x}$ is one-one and onto, where **R**∗ is the set of all non-zero real numbers. Is the result true, if the domain **R**∗ is replaced by **N** with co-domain being same as **R**∗?

Answer

It is given that *f*: **R**∗ → **R**∗ is defined by $f(x)=\frac{1}{x}$.

One-one:

$$f(x)=f(y)$$
$$\Rightarrow \frac{1}{x}=\frac{1}{y}$$
$$\Rightarrow x=y$$

∴*f* is one-one.

Onto:

It is clear that for *y*∈ **R**∗, there exists $x=\frac{1}{y}\in R_*\ (\text{Exists as } y\neq 0)$ such that

$$f(x)=\frac{1}{\left(\frac{1}{y}\right)}=y.$$

∴*f* is onto.

Thus, the given function (*f*) is one-one and onto.

Now, consider function *g*: **N** → **R**∗defined by

$$g(x)=\frac{1}{x}.$$

We have,

$$g(x_1)=g(x_2)\Rightarrow \frac{1}{x_1}=\frac{1}{x_2}\Rightarrow x_1=x_2$$

∴*g* is one-one.

Further, it is clear that *g* is not onto as for 1.2 ∈**R**∗ there does not exit any *x* in **N** such that *g*(*x*) = $\frac{1}{1.2}$.

Hence, function *g* is one-one but not onto.

*********** END **********