Indefinite Integrals Ex 19.12 Q6

Let $I = \int \cos^7 x dx$. Then

$$I = \int \cos^6 x \cos x dx$$

$$= \int \left(\cos^2 x\right)^3 \cos x dx$$

$$= \int \left(1 - \sin^2 x\right)^3 \cos x dx$$

$$= \int \left[1 - \sin^6 x - 3\sin^2 x + 3\sin^4 x\right] \cos x dx$$

$$= \int \left[\cos x - \sin^6 x \cos x - 3\sin^2 x \cos x + 3\sin^4 x \cos x\right] dx + c$$

$$\Rightarrow \quad I = \int \cos x dx - \int \sin^6 x \cos x dx - 3\int \sin^2 x \cos x dx + 3\int \sin^4 x \cos x dx$$

Putting $\sin x = t$ and $\cos x dx = dt$ in 2nd and 3rd and 4th integral, we get

$$I = \int \cos x dx - \int t^6 dt - 3\int t^2 dt + 3\int t^4 dt$$

$$= \sin x - \frac{t^7}{7} - \frac{3}{3}t^3 + \frac{3}{5}t^5 + c$$

$$= \sin x - \frac{1}{7}\sin^7 x - \sin^3 x + \frac{3}{5}\sin^5 x + c$$

$$\therefore \quad I = \sin x - \sin^3 x + \frac{3}{5}\sin^5 x - \frac{1}{7}\sin^7 x + c$$

Indefinite Integrals Ex 19.12 Q7

Let $I = \int x \cos^3 x^2 \sin x^2 dx$

Let $\cos x^2 = t$. Then

$$d\left(\cos x^2\right) = dt$$

$$\Rightarrow \quad -2x \sin x^2 x = dt$$

$$\Rightarrow \quad x \sin x^2 dx = -\frac{dt}{2}$$

$$\therefore \quad I = \int t^3 \times \frac{-dt}{2}$$

$$= -\frac{t^4}{8} + c$$

$$= -\frac{1}{8}\cos^4 x^2 + c$$

$$\therefore \quad I = -\frac{1}{8}\cos^4 x^2 + c$$

Indefinite Integrals Ex 19.12 Q8

Let $I = \int \sin^7 x dx$. Then

$$I = \int \sin^6 x \sin x dx$$
$$= \int \left(\sin^2 x\right)^3 \sin x dx$$
$$= \int \left(1 - \cos^2 x\right)^3 \sin x dx$$
$$= \int \left(1 - \cos^6 x + 3\cos^4 x - 3\cos^2 x\right) \sin x dx$$

$\Rightarrow \quad I = \int \sin x dx - \int \cos^6 x \sin x dx + 3\int \cos^4 x \sin x dx - 3\int \cos^2 x \sin x dx$

Putting $\cos x = t$ and $-\sin x dx = dt$ in 2ne, 3rd and 4th integral, we get

$$I = \int \sin x dx - \int t^6 (-dt) + 3\int t^4 (-dt) - 3\int t^2 (-dt)$$
$$= -\cos x + \frac{t^7}{7} - \frac{3}{5}t^5 + \frac{3}{3}t^3 + c$$
$$= -\cos x + \frac{\cos^7 x}{7} - \frac{3}{5}\cos^5 x + \cos^3 x + c$$

$\therefore \quad I = -\cos x + \cos^3 x - \frac{3}{5}\cos^5 x + \frac{1}{7}\cos^7 x + c$

Indefinite Integrals Ex 19.12 Q9

Let $I = \int \sin^3 x \cos^5 x dx$. Then

Let $\cos x = t$. Then

$$d(\cos x) = dt$$

$\Rightarrow \quad -\sin x dx = dt$

$\Rightarrow \quad dx = \frac{-dt}{\sin x}$

$\therefore \quad I = \int \sin^3 x t^5 \frac{-dt}{\sin x}$

$$= -\int \sin^2 x t^5 dt$$
$$= -\int \left(1 - \cos^2 x\right) t^5 dt$$
$$= -\int \left(1 - t^2\right) t^5 dt$$
$$= -\int \left(t^5 - t^7\right) dt$$
$$= -\frac{t^6}{6} + \frac{t^8}{8} + c$$
$$= -\frac{1}{6}\cos^6 x + \frac{1}{8}\cos^8 x + c$$

$\therefore \quad I = \frac{-1}{6}\cos^6 x + \frac{1}{8}\cos^8 x + c$

*********** END **********