Exercise 7B

Question 12:

$(\cos ec\theta - \sin\theta) = a^3$ and $(\sec\theta - \cos\theta) = b^3$

$a^3 = \left(\dfrac{1}{\sin\theta} - \sin\theta\right) = \left(\dfrac{1 - \sin^2\theta}{\sin\theta}\right) = \dfrac{\cos^2\theta}{\sin\theta}$

$\Rightarrow a = \dfrac{\cos^{2/3}\theta}{\sin^{1/3}\theta}$

$b^3 = \left(\dfrac{1}{\cos\theta} - \cos\theta\right) = \dfrac{\left(1 - \cos^2\theta\right)}{\cos\theta} = \dfrac{\sin^2\theta}{\cos\theta}$

$\Rightarrow b = \dfrac{\sin^{2/3}\theta}{\cos^{1/3}\theta}$

$\therefore a^2b^2\left(a^2 + b^2\right) = a^4b^2 + a^2b^4$

$\qquad\qquad = a^3\left(ab^2\right) + \left(a^2b\right)b^3$

$\qquad = \dfrac{\cos^2\theta}{\sin\theta}\left[\dfrac{\cos^{2/3}\theta}{\sin^{1/3}\theta} \times \dfrac{\sin^{4/3}\theta}{\cos^{2/3}}\right] + \left[\dfrac{\cos^{4/3}\theta}{\sin^{2/3}\theta} \times \dfrac{\sin^{2/3}\theta}{\cos^{2/3}\theta}\right] \times \dfrac{\sin^2\theta}{\cos\theta}$

$\qquad = \dfrac{\cos^2\theta}{\sin\theta} \times \sin\theta + \cos\theta \times \dfrac{\sin^2\theta}{\cos\theta} = \left(\cos^2\theta + \sin^2\theta\right) = 1$

Question 13:

$a\cos^3\theta + 3a\sin^2\theta\cos\theta = m$ and $a\sin^3\theta + 3a\sin\theta\cos^2\theta = n$

$\text{LHS} = (m+n)^{2/3} + (m-n)^{2/3}$

$= \left(a\cos^3\theta + 3a\sin^2\theta\cos\theta + a\sin^3\theta + 3a\sin\theta\cos^2\theta\right)^{2/3}$

$+ \left(a\cos^3\theta + 3a\sin^2\theta\cos\theta - a\sin^3\theta - 3a\sin\theta\cos^2\theta\right)^{2/3}$

$$\left[\because (a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2, \atop (a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2 \right]$$

$= \left[\left(a^{\frac{1}{3}}\cos\theta + a^{\frac{1}{3}}\sin\theta\right)^3\right]^{\frac{2}{3}} + \left[\left(a^{\frac{1}{3}}\cos\theta - a^{\frac{1}{3}}\sin\theta\right)^3\right]^{\frac{2}{3}}$

$= \left(a^{\frac{1}{3}}\cos\theta + a^{\frac{1}{3}}\sin\theta\right)^2 + \left(a^{\frac{1}{3}}\cos\theta - a^{\frac{1}{3}}\sin\theta\right)^2$

$= a^{2/3}\cos^2\theta + a^{2/3}\sin^2\theta + 2a^{2/3}\cos\theta\sin\theta + a^{2/3}\cos^2 a$
$\qquad + a^{2/3}\sin^2\theta - 2a^{2/3}\sin\theta\cos\theta$

$= a^{2/3}\left(\cos^2\theta + \sin^2\theta\right) + a^{2/3}\left(\cos^2\theta + \sin^2\theta\right)$

$= a^{2/3} + a^{2/3} = 2a^{2/3} = \text{RHS}$

$\therefore$ LHS = RHS

********** END **********