



Indefinite Integrals Ex 19.9 Q1

$$\text{Let } I = \int \frac{\log x}{x} dx$$

$$\begin{aligned} \text{Let } \log x &= t & \text{then,} \\ d(\log x) &= dt \end{aligned}$$

$$\Rightarrow \frac{1}{x} dx = dt$$

$$\Rightarrow dx = x dt$$

Putting $\log x = t$ and $dx = x dt$, we get

$$\begin{aligned} I &= \int \frac{t}{x} \times x dt \\ &= \int t dt \\ &= \frac{t^2}{2} + c \\ &= \frac{(\log x)^2}{2} + c \end{aligned}$$

$$\therefore I = \frac{(\log x)^2}{2} + c$$

Indefinite Integrals Ex 19.9 Q2

$$\text{Let } I = \int \frac{\log\left(1 + \frac{1}{x}\right)}{x(1+x)} dx \text{ ----- (i)}$$

$$\text{Let } \log\left(1 + \frac{1}{x}\right) = t \quad \text{then,}$$

$$d\left[\log\left(1 + \frac{1}{x}\right)\right] = dt$$

$$\Rightarrow \frac{1}{1 + \frac{1}{x}} \times \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \frac{1}{\frac{x+1}{x}} \times \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \frac{-x}{x^2(x+1)} dx = -dt$$

$$\Rightarrow \frac{dx}{x(x+1)} = -dt$$

Putting $\log\left(1 + \frac{1}{x}\right) = t$ and $\frac{dx}{x(x+1)} = -dt$ in equation (i), we get

$$\begin{aligned} I &= \int t \times -dt \\ &= -\frac{t^2}{2} + c \\ &= -\frac{1}{2} \left[\log\left(1 + \frac{1}{x}\right) \right]^2 + c \end{aligned}$$

$$\therefore I = -\frac{1}{2} \left[\log\left(1 + \frac{1}{x}\right) \right]^2 + c$$

$$\text{Let } I = \int \frac{\log\left(1 + \frac{1}{x}\right)}{x(1+x)} dx \text{ ----- (i)}$$

$$\text{Let } \log\left(1 + \frac{1}{x}\right) = t \quad \text{then,}$$

$$d\left[\log\left(1 + \frac{1}{x}\right)\right] = dt$$

$$\begin{aligned}\Rightarrow \quad & \frac{1}{1+\frac{1}{x}} \times \frac{-1}{x^2} dx = dt \\ \Rightarrow \quad & \frac{1}{\frac{x+1}{x}} \times \frac{-1}{x^2} dx = dt \\ \Rightarrow \quad & \frac{-x}{x^2(x+1)} dx = dt \\ \Rightarrow \quad & \frac{dx}{x(x+1)} = -dt\end{aligned}$$

Putting $\log\left(1+\frac{1}{x}\right) = t$ and $\frac{dx}{x(x+1)} = -dt$ in equation (i), we get

$$\begin{aligned}I &= \int t \times -dt \\ &= -\frac{t^2}{2} + c \\ &= -\frac{1}{2} \left[\log\left(1+\frac{1}{x}\right) \right]^2 + c\end{aligned}$$

$$\therefore \quad I = -\frac{1}{2} \left[\log\left(1+\frac{1}{x}\right) \right]^2 + c$$

Indefinite Integrals Ex 19.9 Q3

$$\text{Let} \quad I = \int \frac{(1+\sqrt{x})^2}{\sqrt{x}} dx$$

$$\begin{aligned}\text{Let } (1+\sqrt{x}) &= t \quad \text{then,} \\ d(1+\sqrt{x}) &= dt\end{aligned}$$

$$\begin{aligned}\Rightarrow \quad & \frac{1}{2\sqrt{x}} dx = dt \\ \Rightarrow \quad & dx = dt \times 2\sqrt{x}\end{aligned}$$

Putting $(1+\sqrt{x}) = t$ and $dx = dt \times 2\sqrt{x}$, we get

$$\begin{aligned}I &= \int \frac{t^2}{\sqrt{x}} \times dt \times 2\sqrt{x} \\ &= 2 \int t^2 dt \\ &= 2 \times \frac{t^3}{3} + c \\ &= \frac{2}{3} [1+\sqrt{x}]^3 + c\end{aligned}$$

$$\therefore \quad I = \frac{2}{3} (1+\sqrt{x})^3 + c$$

Indefinite Integrals Ex 19.9 Q4

Let $I = \int \sqrt{1+e^x} e^x dx$ ----- (i)

Let $1+e^x = t$ then,
 $d(1+e^x) = dt$

$\Rightarrow e^x dx = dt$

$\Rightarrow dx = \frac{dt}{e^x}$

Putting $1+e^x = t$ and $dx = \frac{dt}{e^x}$ in equation (i), we get

$$I = \int \sqrt{t} e^x \frac{dt}{e^x}$$

$$= \int t^{\frac{1}{2}} dt$$

$$= \frac{2}{3} \times \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{2}{3} (1+e^x)^{\frac{3}{2}} + c$$

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