



### Binomial Theorem Ex 18.2 Q27

We are given,

$$T_3 = a, T_4 = b, T_5 = c, T_6 = d$$

We have to prove that

$$\begin{aligned} \frac{b^2 - ac}{c^2 - bd} &= \frac{5a}{3c} \\ \Rightarrow \frac{b^2 - ac}{a} &= \frac{5}{3} \left[ \frac{c^2 - bd}{c} \right] \\ \Rightarrow \frac{1}{b} \left[ \frac{b^2 - ac}{a} \right] &= \frac{5}{3} \left[ \frac{c^2 - bd}{bc} \right] \\ \Rightarrow \frac{b}{a} - \frac{c}{b} &= \frac{5}{3} \left[ \frac{c}{b} - \frac{d}{c} \right] \quad \text{---(i)} \end{aligned}$$

Now we know,

$$a = {}^nC_2 x^{n-2} a^2$$

$$b = {}^nC_3 x^{n-3} a^3$$

$$c = {}^nC_4 x^{n-4} a^4$$

$$d = {}^nC_5 x^{n-5} a^5$$

Putting these values in equation (i), we get

$$\begin{aligned} \frac{{}^nC_3 x^{n-3} a^3}{{}^nC_2 x^{n-2} a^2} - \frac{{}^nC_4 x^{n-4} a^4}{{}^nC_3 x^{n-3} a^3} &= \frac{5}{3} \left[ \frac{{}^nC_4 x^{n-4} a^4}{{}^nC_3 x^{n-3} a^3} - \frac{{}^nC_5 x^{n-5} a^5}{{}^nC_4 x^{n-4} a^4} \right] \\ \Rightarrow \left[ \frac{{}^nC_3}{{}^nC_2} - \frac{{}^nC_4}{{}^nC_3} \right] \frac{a}{x} &= \frac{5a}{3x} \left[ \frac{{}^nC_4}{{}^nC_3} - \frac{{}^nC_5}{{}^nC_4} \right] \end{aligned}$$

We know that,

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$$

∴ The given equation above becomes,

$$\begin{aligned} \left[ \frac{n-2}{3} - \frac{n-3}{4} \right] &= \frac{5}{3} \left[ \frac{n-3}{4} - \frac{n-4}{5} \right] \\ \Rightarrow \frac{4n-8-3n+9}{3 \times 4} &= \frac{5n-15-4n+16}{3 \times 4} \\ \Rightarrow \frac{n+1}{12} &= \frac{n+1}{12} \end{aligned}$$

Which is true.

Hence proved.

### Binomial Theorem Ex 18.2 Q28

Suppose the binomial is  $(x+\alpha)^n$

We are given,

$$T_6 = a, T_7 = b, T_8 = c, T_9 = d$$

We have to prove that

$$\begin{aligned} \frac{b^2 - ac}{c^2 - bd} &= \frac{4a}{3c} \\ \Rightarrow \frac{b^2 - ac}{a} &= \frac{4}{3} \left[ \frac{c^2 - bd}{c} \right] \\ \Rightarrow \frac{1}{b} \left[ \frac{b^2 - ac}{a} \right] &= \frac{4}{3} \left[ \frac{c^2 - bd}{bc} \right] \\ \Rightarrow \frac{b}{a} - \frac{c}{b} &= \frac{4}{3} \left[ \frac{c}{b} - \frac{d}{c} \right] \quad \text{---(i)} \end{aligned}$$

Now we know,

$$a = {}^nC_5 x^{n-5} \alpha^5$$

$$b = {}^nC_6 x^{n-6} \alpha^6$$

$$c = {}^nC_7 x^{n-7} \alpha^7$$

$$d = {}^nC_8 x^{n-8} \alpha^8$$

Putting these values in equation (i), we get

$$\begin{aligned} \frac{{}^nC_6 x^{n-6} \alpha^6}{{}^nC_5 x^{n-5} \alpha^5} - \frac{{}^nC_7 x^{n-7} \alpha^7}{{}^nC_6 x^{n-6} \alpha^6} &= \frac{4}{3} \left[ \frac{{}^nC_7 x^{n-7} \alpha^7}{{}^nC_6 x^{n-6} \alpha^6} - \frac{{}^nC_8 x^{n-8} \alpha^8}{{}^nC_7 x^{n-7} \alpha^7} \right] \\ \Rightarrow \left[ \frac{{}^nC_6}{{}^nC_5} - \frac{{}^nC_7}{{}^nC_6} \right] \frac{\alpha}{x} &= \frac{4\alpha}{3x} \left[ \frac{{}^nC_7}{{}^nC_6} - \frac{{}^nC_8}{{}^nC_7} \right] \end{aligned}$$

We know that,

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$$

$\therefore$  The given equation above becomes,

$$\begin{aligned} \left[ \frac{n-5}{6} - \frac{n-6}{7} \right] &= \frac{4}{3} \left[ \frac{n-6}{7} - \frac{n-7}{8} \right] \\ \Rightarrow \frac{7n-35-6n+36}{6 \times 7} &= \frac{8n-48-7n+49}{3 \times 7 \times 2} \\ \Rightarrow \frac{n+1}{42} &= \frac{n+1}{42} \end{aligned}$$

Which is true.

Hence proved.

## Binomial Theorem Ex 18.2 Q29

We have,

$$(1+x)^n$$

Let the three consecutive terms are  $T_r, T_{r+1}$  and  $T_{r+2}$

$\therefore$  Coefficients of  $r$ th term =  ${}^nC_{r-1} = 76$

Coefficients of  $(r+1)$ th term =  ${}^nC_{r+1-1} = {}^nC_r = 95$

and, Coefficients of  $(r+2)$ th term =  ${}^nC_{r+2-1} = {}^nC_{r+1} = 76$

Now,

$$\begin{aligned} \frac{{}^nC_{r+1}}{{}^nC_r} &= \frac{76}{95} \\ \Rightarrow \frac{n-(r+1)+1}{r+1} &= \frac{76}{95} \quad \left[ \because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r} \right] \\ \Rightarrow \frac{n-r-1}{r+1} &= \frac{4}{5} \\ \Rightarrow \frac{n-r}{r+1} &= \frac{4}{5} \\ \Rightarrow 5n-5r &= 4r+4 \\ \Rightarrow 5n-5r-4r &= 4 \\ \Rightarrow 5n-9r &= 4 \quad \text{---(i)} \end{aligned}$$

and,

$$\begin{aligned} \frac{{}^nC_r}{{}^nC_{r-1}} &= \frac{95}{76} \\ \Rightarrow \frac{n-r+1}{r} &= \frac{5}{4} \\ \Rightarrow 4n-4r+4 &= 5r \\ \Rightarrow 4n-9r &= -4 \quad \text{---(ii)} \end{aligned}$$

Subtracting equation (ii) from (i), we get

$$\begin{aligned} n &= 4+4 \\ \Rightarrow n &= 8 \end{aligned}$$

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