Binomial Theorem Ex 18.2 Q32

It is given that

$T_1 = 729$

$T_2 = 7290$

and, $T_3 = 30375$

$\therefore$ $T_1 = {}^nC_0 \times a^n = 729$

$T_2 = {}^nC_{n-1} \times a^{n-1} \times b = 7290$

and, $T_3 = {}^nC_{n-2} \times a^{n-2} \times b^2 = 30375$

Now,

$$\frac{T_2}{T_1} = \frac{{}^nC_{n-1} \times a^{n-1} \times b}{{}^nC_0 \times a^n} = \frac{7290}{729}$$

$$\Rightarrow \frac{{}^nC_{n-1} \times a^{n-1} \times b}{{}^nC_0 \times a^n} = 10$$

$$\Rightarrow \frac{{}^nC_{n-1}}{1} \times \frac{b}{a} = 10$$

$$\Rightarrow \frac{n!}{(n-n+1)!\,(n-1)!} \times \frac{b}{a} = 10$$

$$\Rightarrow \frac{n!}{(n-1)!} \times \frac{b}{a} = 10$$

$$\Rightarrow \frac{n(n-1)!}{(n-1)!} \times \frac{b}{a} = 10$$

$$\Rightarrow \frac{b}{a} = \frac{10}{n} \qquad \text{---(i)}$$

and,

$$\frac{T_3}{T_2} = \frac{{}^nC_{n-2} \times a^{n-2} \times b^2}{{}^nC_{n-1} \times a^{n-1} \times b} = \frac{30375}{7290}$$

$$\Rightarrow \frac{{}^nC_{n-2}}{{}^nC_{n-1}} \times \frac{b}{a} = \frac{25}{6}$$

$$\Rightarrow \frac{n-2+1}{n-(n-1)+1} \times \frac{b}{a} = \frac{26}{6} \qquad \left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}\right]$$

$$\Rightarrow \frac{n-1}{2} \times \frac{b}{a} = \frac{25}{6}$$

$$\Rightarrow \frac{b}{a} = \frac{25}{6} \times \frac{2}{(n-1)}$$

$$\Rightarrow \frac{b}{a} = \frac{25}{6} \times \frac{2}{(n-1)}$$

$$\Rightarrow \frac{b}{a} = \frac{25}{3(n-1)} \qquad \text{---(ii)}$$

Comparing equation (i) and equation (ii), we get

$$\frac{10}{n} = \frac{25}{3(n-1)}$$

$\Rightarrow$ $30(n-1) = 25n$

$\Rightarrow$ $30n - 30 = 25n$

$\Rightarrow$ $5n = 30$

$\Rightarrow$ $n = 6$

Now,

$T_1 = {}^nC_0 \times a^n = 729$

$\Rightarrow$ $a^n = 729$

$\Rightarrow$ $a^6 = 729$ $\qquad [\because n = 6]$

$\Rightarrow$ $a^6 = 3^6$

$\Rightarrow$ $a = 3$

Putting $a = 3$ in $n = 6$ in equation (i), we get

$$\frac{b}{3} = \frac{10}{6}$$

$$\Rightarrow b = \frac{10}{2} = 5$$

Hence, $a = 3$, $b = 5$ and $n = 6$.

Binomial Theorem Ex 18.2 Q33

We have,

$$\left(3+ax\right)^9 = {}^9C_0 \times 3^9 + {}^9C_1 \times 3^8 \times \left(ax\right)^1 + {}^9C_2 \times 3^7 \times \left(ax\right)^2 + {}^9C_3 \times 3^6 \times \left(ax\right)^3 + \ldots$$

$\therefore$ Coefficient of $x^2 = {}^9C_2 \times 3^7 \times a^2$

and, Coefficient of $x^3 = {}^9C_3 \times 3^6 \times a^3$

Now, Coefficient of x^2 = Coefficient of x^3

$\Rightarrow \quad {}^9C_2 \times 3^7 \times a^2 = {}^9C_3 \times 3^6 \times a^3$

$\Rightarrow \quad 36 \times 3^7 \times a^2 = 84 \times 3^6 \times a^3$

$\Rightarrow \quad a = \dfrac{36 \times 3^7}{84 \times 3^6} = \dfrac{9}{7}$

*********** END **********