Indefinite Integrals Ex 19.32 Q1

$$\text{Let} \quad I = \int \frac{1}{(x-1)\sqrt{x+2}}\,dx$$

$$\text{Let } x + 2 = t^2$$

$$\therefore \qquad I = \int \frac{2t\,dt}{\left(t^2-3\right)t}$$

$$= 2\int \frac{dt}{t^2-3}$$

$$= \frac{2}{2\sqrt{3}}\log\left|\frac{t-\sqrt{3}}{t+\sqrt{3}}\right| + c$$

Thus,

$$I = \frac{1}{\sqrt{3}}\log\left|\frac{\sqrt{x-2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| + c$$

Indefinite Integrals Ex 19.32 Q2

$$\text{Let} \quad I = \int \frac{1}{(x-1)\sqrt{2x+3}}\,dx$$

$$\text{Let} \quad I = \int \frac{1}{(x-1)\sqrt{2x+3}}\,dx$$

$$\text{Let } 2x + 3 = t^2$$

$$\Rightarrow \qquad 2dx = 2t\,dt$$

$$\therefore \qquad I = \int \frac{t\,dt}{\left(\frac{t^2-3}{2}-1\right)t}$$

$$= 2\int \frac{dt}{t^2-5}$$

$$= \frac{2}{2\sqrt{5}}\log\left|\frac{t-\sqrt{5}}{t+\sqrt{5}}\right| + c$$

Thus,

$$I = \frac{1}{\sqrt{5}}\log\left|\frac{\sqrt{2x+3}-\sqrt{5}}{\sqrt{2x+3}+\sqrt{5}}\right| + c$$

Indefinite Integrals Ex 19.32 Q3

Let $I = \int \frac{1}{(x-1)\sqrt{x+2}} dx$

$$I = \int \frac{(x-1)+2}{(x-1)\sqrt{x+2}} dx$$

$$I = \int \frac{dx}{\sqrt{x+2}} + 2\int \frac{dx}{(x-1)\sqrt{x+2}} \qquad \text{---(A)}$$

Now,

$$\int \frac{dx}{\sqrt{x+2}} = 2\sqrt{x+2} + c_1$$

and,

$$\int \frac{dx}{(x-1)\sqrt{x+2}}$$

Let $x + 2 = t^2$

$\Rightarrow \quad dx = 2tdt$

$$\therefore \quad \int \frac{dx}{(x-1)\sqrt{x+2}} = 2\int \frac{tdt}{(t^2-3)t} = 2\int \frac{dt}{t^2-3}$$

$$= \frac{2 \times 1}{2\sqrt{3}} log \left|\frac{t-\sqrt{3}}{t+\sqrt{3}}\right| + c_2$$

$$= \frac{1}{\sqrt{3}} log \left|\frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| +_2$$

Thus, form (A),

$$I = 2\sqrt{x+2} + c_1 + \frac{2}{\sqrt{3}} log \left|\frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| + c_2$$

Hence,

$$I = 2\sqrt{x+2} + \frac{2}{\sqrt{3}} log \left|\frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| + c$$

## Indefinite Integrals Ex 19.32 Q4

Let $I = \int \frac{x^2}{(x-1)\sqrt{x+2}} dx$

$$= \int \frac{(x^2-1+1)}{(x-1)\sqrt{x+2}} dx$$

$$= \int \frac{(x+1)(x-1)}{(x-1)\sqrt{x+2}} dx + \int \frac{dx}{(x-1)\sqrt{x+2}}$$

$$= \int \frac{(x+1)}{\sqrt{x+2}} dx + \int \frac{dx}{(x-1)\sqrt{x+2}}$$

$$= \int \frac{(x+2)-1}{\sqrt{x+2}} dx + \int \frac{dx}{(x-1)\sqrt{x+2}}$$

$$I = \int \sqrt{x+2}dx - \int \frac{dx}{\sqrt{x+2}} + \int \frac{dx}{(x-1)\sqrt{x+2}} \qquad \text{---(A)}$$

Now,

$$\int \sqrt{x+2}dx = \frac{2}{3}(x+2)^{\frac{3}{2}} + c_1$$

and,

$$\int \frac{dx}{\sqrt{x+2}} = 2\sqrt{x+2} + c_2$$

$$\int \frac{dx}{(x-1)\sqrt{x+2}}$$

Let $x + 2 = t^2$

$\Rightarrow \quad dx = 2tdt$

$$\therefore \quad 2\int \frac{tdt}{(t^2-3)t} = 2\int \frac{dt}{t^2-3}$$

$$= \frac{2}{2\sqrt{3}} log \left|\frac{t-\sqrt{3}}{t+\sqrt{3}}\right| + c_3$$

$$\therefore \quad \int \frac{dx}{(x-1)\sqrt{x+2}} = \frac{1}{\sqrt{3}} log \left|\frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| + c_3$$

Thus, form (A)

$$I = \frac{2}{3}(x+2)^{\frac{3}{2}} - 2\sqrt{x+2} + \frac{1}{\sqrt{3}} log \left|\frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\right| + c \qquad [\text{when } c = c_1 + c_2 + c_3]$$

## Indefinite Integrals Ex 19.32 Q5

Let $I = \int \frac{x}{(x-3)\sqrt{x+1}}dx$

$$= \int \frac{(x-3)+3}{(x-3)\sqrt{x+1}}dx$$

$$I = \int \frac{dx}{\sqrt{x+1}} + 3\int \frac{dx}{(x-3)\sqrt{x+1}} \qquad \text{---(A)}$$

Now,

$$\int \frac{dx}{\sqrt{x+1}} = 2\sqrt{x+1} + c_1$$

and,

$$\int \frac{dx}{\sqrt{x+2}} = 2\sqrt{x+2} + c_2$$

$$\int \frac{dx}{(x-3)\sqrt{x+1}}$$

Let $x+1 = t^2$

$\Rightarrow \quad dx = 2tdt$

$$\therefore \quad \int \frac{dx}{(x-3)\sqrt{x+1}} = 2\int \frac{tdt}{\left(t^2-4\right)t}$$

$$= 2\left|\frac{dt}{t^2-4}\right|$$

$$= \frac{2}{2\times 2}log\left|\frac{t-2}{t+2}\right| + c_2$$

$$\therefore \quad \int \frac{dx}{(x-3)\sqrt{x+1}} = \frac{1}{2}log\left|\frac{\sqrt{x+1}-2}{\sqrt{x+1}+2}\right| + c_2$$

Thus, form (A)

$$I = 2\sqrt{x+1} + \frac{3}{2}log\left|\frac{\sqrt{x+1}-2}{\sqrt{x+1}+2}\right| + c \qquad [\text{when } c = c_1 + c_2]$$

*********** END **********