



Indefinite Integrals Ex 19.31 Q9

$$\begin{aligned}\text{Let } I &= \int \frac{(x-1)^2}{x^4 + x^2 + 1} dx \\ &= \int \frac{x^2 - 2x + 1}{x^4 + x^2 + 1} dx\end{aligned}$$

Dividing numerator and denominator by x^2

$$\begin{aligned}\therefore I &= \int \frac{1 - \frac{2}{x} + \frac{1}{x^2}}{x^2 + 1 + \frac{1}{x^2}} dx \\ &= \int \frac{\left(1 + \frac{1}{x^2}\right)}{\left(x - \frac{1}{x}\right)^2 + 3} dx - \int \frac{2x}{x^4 + x^2 + 1} dx\end{aligned}$$

$$\text{Let } \left(x - \frac{1}{x}\right) = t$$

$$\Rightarrow \left(1 + \frac{1}{x^2}\right) dx = dt \quad \text{[For Ist part]}$$

$$\text{Let } x^2 = z$$

$$\Rightarrow 2x dx = dz \quad \text{[For IInd part]}$$

$$\begin{aligned}\therefore I &= \int \frac{dt}{t^2 + 3} - \int \frac{dz}{z^2 + z + 1} \\ &= \int \frac{dt}{t^2 + 3} - \int \frac{dz}{\left(z + \frac{1}{2}\right)^2 + \frac{3}{4}}\end{aligned}$$

$$\Rightarrow I = \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{t}{\sqrt{3}} \right) - \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{z + \frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) + c$$

$$\Rightarrow I = \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{3}x} \right) - \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{2x^2 + 1}{\sqrt{3}} \right) + c$$

Hence,

$$I = \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{3}x} \right) - \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{2x^2 + 1}{\sqrt{3}} \right) + c$$

Indefinite Integrals Ex 19.31 Q10

$$\text{Let } I = \int \frac{1}{x^4 + 3x^2 + 1} dx$$

Dividing numerator and denominator by x^2

$$\begin{aligned} \therefore I &= \int \frac{\frac{1}{x^2}}{x^2 + 3 + \frac{1}{x^2}} dx \\ &= \frac{1}{2} \int \frac{\left(1 + \frac{1}{x^2}\right) - \left(1 - \frac{1}{x^2}\right)}{x^2 + 3 + \frac{1}{x^2}} dx \\ &= \frac{1}{2} \int \frac{\left(1 + \frac{1}{x^2}\right)}{\left(x - \frac{1}{x}\right)^2 + 5} dx - \frac{1}{2} \int \frac{\left(1 - \frac{1}{x^2}\right)}{\left(x + \frac{1}{x}\right)^2 + 1} dx \end{aligned}$$

$$\text{Let } \left(x - \frac{1}{x}\right) = t$$

$$\Rightarrow \left(1 + \frac{1}{x^2}\right) dx = dt$$

$$\text{and } x + \frac{1}{x} = z$$

$$\Rightarrow \left(1 - \frac{1}{x^2}\right) dx = dz$$

$$\begin{aligned} \therefore I &= \frac{1}{2} \int \frac{dt}{t^2 + 5} - \frac{1}{2} \int \frac{dz}{z^2 + 1} \\ &= \frac{1}{2\sqrt{5}} \tan^{-1}\left(\frac{t}{\sqrt{5}}\right) - \frac{1}{2} \tan^{-1}(z) + c \end{aligned}$$

Hence,

$$I = \frac{1}{2\sqrt{5}} \tan^{-1}\left(\frac{x^2 - 1}{\sqrt{5}x}\right) - \frac{1}{2} \tan^{-1}\left(\frac{x^2 + 1}{x}\right) + c$$

Indefinite Integrals Ex 19.31 Q11

Consider the integral

$$I = \int \frac{1}{\sin^4 x + \sin^2 x \cos^2 x + \cos^4 x} dx$$

Divide both the numerator and the denominator by $\cos^4 x$, we have,

$$I = \int \frac{\frac{1}{\cos^4 x}}{\frac{\sin^4 x + \sin^2 x \cos^2 x + \cos^4 x}{\cos^4 x}} dx$$

$$= \int \frac{\sec^4 x}{\tan^4 x + \tan^2 x + 1} dx$$

$$= \int \frac{\sec^2 x \times \sec^2 x}{\tan^4 x + \tan^2 x + 1} dx$$

$$= \int \frac{(\tan^2 x + 1) \times \sec^2 x}{\tan^4 x + \tan^2 x + 1} dx$$

Substituting $\tan x = t$; $\sec^2 x dx = dt$

Thus,

$$I = \int \frac{(1+t^2) dt}{t^4 + t^2 + 1}$$

$$= \int \frac{\left(1 + \frac{1}{t^2}\right) dt}{\left(t^2 + \frac{1}{t^2} + 1\right)}$$

$$= \int \frac{\left(1 + \frac{1}{t^2}\right) dt}{\left(t^2 + \frac{1}{t^2} - 2 + 2 + 1\right)}$$

$$= \int \frac{\left(1 + \frac{1}{t^2}\right) dt}{\left(t - \frac{1}{t}\right)^2 + 3}$$

$$= \int \frac{\left(1 + \frac{1}{t^2}\right) dt}{\left(t - \frac{1}{t}\right)^2 + 3}$$

Substituting $z = t - \frac{1}{t}$; $dz = \left(1 + \frac{1}{t^2}\right) dt$,

$$I = \int \frac{dz}{z^2 + 3}$$

$$\Rightarrow I = \int \frac{dz}{z^2 + (\sqrt{3})^2}$$

$$I = \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{z}{\sqrt{3}} \right) + C$$

$$= \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{t - \frac{1}{t}}{\sqrt{3}} \right) + C$$

$$= \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{\tan x - \frac{1}{\tan x}}{\sqrt{3}} \right) + C$$

$$I = \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{\tan x - \cot x}{\sqrt{3}} \right) + C$$

***** END *****

