Mean Value Theorems Ex 15.2 Q1(v)

Here,

$$f(x) = 2x^2 - 3x + 1 \text{ on } [1,3]$$

We know that, polynomial function is continuous and differentiable. So, $f(x)$ is continuous in $[1,3]$ and $f(x)$ is differentiable in $(1,3)$. So, Lagrange's mean value theorem is applicable, so there exist a point $c \in (1,3)$ such that

$$f'(c) = \frac{f(3) - f(1)}{3 - 1}$$

$$\Rightarrow \quad 4c - 3 = \frac{\left(2(3)^2 - 3(3) + 1\right) - (2 - 3 + 1)}{3 - 1}$$

$$\Rightarrow \quad 4c - 3 = \frac{10}{2}$$

$$\Rightarrow \quad 4c = 5 + 3$$

$$\Rightarrow \quad 4c = 8$$

$$\Rightarrow \quad c = 2 \in (1,3)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(vi)

Here,

$$f(x) = x^2 - 2x + 4 \text{ on } [1,5]$$

We know that, polynomial is always continuous and differentiable. So, $f(x)$ is continuous in $[1,5]$ and it is differentiable in $(1,5)$. So, Lagrange's mean value theorem is applicable. Thus, there exists a point $c \in (1,5)$ such that

$$f'(c) = \frac{f(5) - f(1)}{5 - 1}$$

$$\Rightarrow \quad 2c - 2 = \frac{\left((5)^2 - 2(5) + 4\right) - (1 - 2 + 4)}{4}$$

$$\Rightarrow \quad 2c - 2 = \frac{19 - 3}{4}$$

$$\Rightarrow \quad 2c - 2 = 4$$

$$\Rightarrow \quad 2c = 6$$

$$\Rightarrow \quad c = 3 \in (1,5)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(vii)

Here,

$$f(x) = 2x - x^2 \text{ on } [0,1]$$

We know that, polynomial is continuous and differentiable. So, $f(x)$ is continuous in $[0,1]$ and differentiable in $(0,1)$. So, Lagrange's mean value theorem is applicable. Thus, there exists a point $c \in (0,1)$ such that

$$f'(c) = \frac{f(1) - f(0)}{1 - 0}$$

$$\Rightarrow \quad 2 - 2c = \frac{\left(2(1) - (1)^2\right) - (0)}{1}$$

$$\Rightarrow \quad 2 - 2c = 1$$

$$\Rightarrow \quad 1 = 2c$$

$$\Rightarrow \quad c = \frac{1}{2} \in (0,1)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(viii)

$$f(x) = (x-1)(x-2)(x-3) \text{ on } [0, 4]$$

We know that, polynomial is continuous and differentiable every where. So, $f(x)$ is continuous in $[0, 4]$ and differentiable in $(0, 4)$. So, Lagrange's mean value theorem is applicable. Thus, there exists a point $c \in (0, 4)$ such that

$$f'(c) = \frac{f(4) - f(0)}{4 - 0}$$

$$\Rightarrow \quad (c-1)(c-2) + (c-2)(c-3) + (c-1)(c-3) = \frac{(3)(2)(1) - (-1)(-2)(-3)}{4 - 0}$$

$$\Rightarrow \quad c^2 - 3c + 2 + c^2 + 5c + 6 + c^2 - 4c + 3 = \frac{6 + 6}{4}$$

$$\Rightarrow \quad 3c^2 - 12c + 11 = 3$$

$$\Rightarrow \quad 3c^2 = 12c + 8 = 0$$

$$\Rightarrow \quad c = \frac{-(-12) \pm \sqrt{144 - 4 \times 3 \times 8}}{6}$$

$$\Rightarrow \quad c = \frac{12 \pm \sqrt{48}}{6}$$

$$\Rightarrow \quad c = 2 \pm \cdot \frac{2\sqrt{3}}{3} \in (0, 4)$$

$$\Rightarrow \quad c = 2 \pm \cdot \frac{2}{\sqrt{3}} \in (0, 4)$$

Hence, Lagrange's mean value theorem is verified.

*********** END **********