Increasing and Decreasing Functions Ex 17.2 Q38

As $f(0) = f(1)$ and f is differentiable, hence by Rolles theorem:

$f'(c) = 0$ for some $c \in [0, 1]$

Let us now apply LMVT (as function is twice differentiable) for point c and $x \in [0, 1]$, hence

$$\frac{|f'(x) - f(c)|}{x - c} = f''(d)$$

$$\Rightarrow \frac{|f'(x) - 0|}{x - c} = f''(d)$$

$$\Rightarrow \frac{|f'(x)|}{x - c} = f''(d)$$

As given that $|f''(d)| \le 1$ for $x \in [0, 1]$

$$\Rightarrow \frac{|f'(x)|}{x - c} \le 1$$

$$\Rightarrow |f'(x)| \le x - c$$

Now as both x and c lie in $[0, 1]$, hence $x - c \in (0, 1)$

$$\Rightarrow |f'(x)| < 1 \text{ for all } x \in [0, 1]$$

Increasing and Decreasing Functions Ex 17.2 Q39(i)

Consider the given function,

$f(x) = x|x|,\ x \in R$

$$\Rightarrow f(x) = \begin{cases} -x^2, & x<0 \\ x^2, & x>0 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} -2x, & x<0 \\ 2x, & x>0 \end{cases}$$

$\Rightarrow f'(x) > 0,$ for values of x

Therefore, $f(x)$ is an increasing function for all real values.

Increasing and Decreasing Functions Ex 17.2 Q39(ii)

Consider the function

$f(x) = \sin x + |\sin x|,\ 0 < x \le 2\pi$

$\Rightarrow f(x) = \begin{cases} 2\sin x, & 0 < x < \pi \\ 0, & \pi < x < 2\pi \end{cases}$

$\Rightarrow f'(x) = \begin{cases} 2\cos x, & 0 < x < \pi \\ 0, & \pi < x < 2\pi \end{cases}$

The function 2cosx will be positive between $\left(0, \frac{\pi}{2}\right)$.

Hence the function $f(x)$ is increasing in the interval $\left(0, \frac{\pi}{2}\right)$.

The function 2cosx will be negative between $\left(\frac{\pi}{2}, \pi\right)$.

Hence the function $f(x)$ is decreasing in the interval $\left(\frac{\pi}{2}, \pi\right)$.

The value of $f'(x) = 0$, when $\pi \le x < 2\pi$.

Therefore, the function $f(x)$ is neither increasing nor decreasing in the interval $(\pi, 2\pi)$

Increasing and Decreasing Functions Ex 17.2 Q39(iii)

Consider the function,

$f(x) = \sin x(1 + \cos x),\ 0 < x < \frac{\pi}{2}$

$\Rightarrow f'(x) = \cos x + \sin x(-\sin x) + \cos x(\cos x)$

$\Rightarrow f'(x) = \cos x - \sin^2 x + \cos^2 x$

$\Rightarrow f'(x) = \cos x + (\cos^2 x - 1) + \cos^2 x$

$\Rightarrow f'(x) = \cos x + 2\cos^2 x - 1$

$\Rightarrow f'(x) = 2\cos^2 x + \cos x - 1$

$\Rightarrow f'(x) = 2\cos^2 x + 2\cos x - \cos x - 1$

$\Rightarrow f'(x) = 2\cos x(\cos x + 1) - 1(\cos x + 1)$

$\Rightarrow f'(x) = (2\cos x - 1)(\cos x + 1)$

For $f(x)$ to be increasing, we must have,

$f'(x) > 0$

$\Rightarrow f'(x) = (2\cos x - 1)(\cos x + 1) > 0$

$\Rightarrow 0 < x < \frac{\pi}{3}$

$\Rightarrow x \in \left(0, \frac{\pi}{3}\right)$

So, $f(x)$ is *increasing* in $\left(0, \frac{\pi}{3}\right)$

For $f(x)$ to be decreasing, we must have,

$f'(x) < 0$

$\Rightarrow f'(x) = (2\cos x - 1)(\cos x + 1) < 0$

$\Rightarrow \frac{\pi}{3} < x < \frac{\pi}{2}$

$\Rightarrow x \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

So, $f(x)$ is decreasing in $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

*********** END **********