Sine and Cosine Formulae and their Applications Ex-10.2 Q5

$b(c\cos A-a\cos C)=c^2-a^2$

RHS

$=c^2-a^2$

$=k^2\sin^2 C-k^2\sin^2 A$

$=k^2(\sin^2 C-\sin^2 A)$

$=k^2\sin(C+A).\sin(C-A)$

$=k^2\sin(\pi-B).\sin(C-A)$

$=k^2\sin B.\sin(C-A)$

$=k\sin B.k\sin(C-A)$

$=bk\sin(C-A)$

$=bk(\sin C.\cos A-\sin A.\cos C)$

$=b(k\sin C.\cos A-k\sin A.\cos C)$

$=b(c\cos A-a\cos C)=LHS$

Sine and Cosine Formulae and their Applications Ex-10.2 Q6

$c(a\cos B-b\cos A)$

$=ac.\cos B-bc\cos A$

$=ac.\dfrac{a^2+c^2-b^2}{2ac}-bc.\dfrac{b^2+c^2-a^2}{2bc}$

$=\dfrac{a^2+c^2-b^2}{2}-\dfrac{b^2+c^2-a^2}{2}$

$=\dfrac{a^2+c^2-b^2-b^2-c^2+a^2}{2}$

$=\dfrac{2a^2-2b^2}{2}=\left(a^2-b^2\right)=RHS$

Sine and Cosine Formulae and their Applications Ex-10.2 Q7

$2(bc\cos A+ca\cos B+ab\cos C)=a^2+b^2+c^2$

LHS

$=2bc\cos A+2ca\cos B+2ab\cos C$

$=2bc\dfrac{b^2+c^2-a^2}{2bc}+2ca\dfrac{a^2+c^2-b^2}{2ca}+2ab\dfrac{a^2+b^2-c^2}{2ab}$

$=b^2+c^2-a^2+a^2+c^2-b^2+a^2+b^2-c^2$

$=a^2+b^2+c^2=RHS$

Sine and Cosine Formulae and their Applications Ex-10.2 Q8

For any $\triangle ABC$, we have

$\cos A = \dfrac{b^2+c^2-a^2}{2bc}$

$\cos B = \dfrac{a^2+c^2-b^2}{2ac}$

$\cos C = \dfrac{a^2+b^2-c^2}{2ab}$

therefore,

$$\left(c^2+b^2-a^2\right)\tan A = \left(c^2+b^2-a^2\right)\frac{\sin A}{\cos A}$$

$$= \left(c^2+b^2-a^2\right)\frac{ka}{\frac{b^2+c^2-a^2}{2bc}}$$

$$= 2kabc$$

Also,

$$\left(a^2+c^2-b^2\right)\tan B = \left(a^2+c^2-b^2\right)\frac{\sin B}{\cos B}$$

$$= \left(a^2+c^2-b^2\right)\frac{kb}{\frac{a^2+c^2-b^2}{2ac}}$$

$$= 2kabc$$

Now,

$$\left(a^2+b^2-c^2\right)\tan C = \left(a^2+b^2-c^2\right)\frac{\sin C}{\cos C}$$

$$= \left(a^2+b^2-c^2\right)\frac{kc}{\frac{a^2+b^2-c^2}{2ab}}$$

$$= 2kabc$$

Hence proved.

Sine and Cosine Formulae and their Applications Ex-10.2 Q9

$$\frac{c-b\cos A}{b-c\cos A} = \frac{\cos B}{\cos C}$$

LHS

$$= \frac{c-b\cos A}{b-c\cos A}$$

$$= \frac{k\sin C - k\sin B\cos A}{k\sin B - k\sin C\cos A}$$

$$= \frac{\sin(\pi-(A+B)) - \sin B\cos A}{\sin(\pi-(A+C)) - \sin C\cos A}$$

$$= \frac{\sin(A+B) - \sin B\cos A}{\sin(A+C) - \sin C\cos A}$$

$$= \frac{\sin A\cos B + \cos A\sin B - \sin B\cos A}{\sin A\cos C + \cos A\sin C - \sin C\cos A}$$

$$= \frac{\sin A\cos B}{\sin A\cos C}$$

$$= \frac{\cos B}{\cos C} = RHS$$

********** END **********