Transformation Formulae Ex 8.1 Q1

(i) $2\sin 3\theta \cos\theta$

$= \sin(3\theta+\theta)+\sin(3\theta-\theta)$ $\quad [\because 2\sin A\cos B = \sin(A+B)+\sin(A-B)]$

$= \sin 4\theta + \sin 2\theta$

(ii) $2\cos 3\theta \sin 2\theta$

$\because \quad 2\cos A\sin B = \sin(A+B)-\sin(A-B)$

$\Rightarrow \quad 2\cos 3\theta\sin 2\theta = \sin(3\theta+2\theta)-\sin(3\theta-2\theta)$

$= \quad \sin 5\theta - \sin\theta$

(iii) $2\sin 4\theta\sin 3\theta$

$\because \quad 2\sin A\sin B = \cos(A-B)-\cos(A+B)$

$\Rightarrow \quad 2\sin 4\theta\sin 3\theta = \cos(4\theta-3\theta)-\cos(4\theta+3\theta)$

$= \cos\theta - \cos 7\theta$

(iv) $2\cos 7\theta\cos 3\theta$

$\because \quad 2\cos A\cos B = \cos(A+B)+\cos(A-B)$

$\Rightarrow \quad 2\cos 7\theta\cos 3\theta = \cos(7\theta+3\theta)+\cos(7\theta-3\theta)$

$= \cos 10\theta + \cos 4\theta$

Transformation Formulae Ex 8.1 Q2

(i) $2\sin\frac{5\pi}{12}\sin\frac{\pi}{12}$

$\because \quad 2\sin A\sin B = \cos(A-B)-\cos(A+B)$

$\Rightarrow \quad 2\sin\frac{5\pi}{12}\sin\frac{\pi}{12} = \cos\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)-\cos\left(\frac{5\pi}{12}+\frac{\pi}{12}\right)$

$= \cos\left(\frac{4\pi}{12}\right)-\cos\left(\frac{6\pi}{12}\right)$

$= \cos\left(\frac{\pi}{3}\right)-\cos\left(\frac{\pi}{2}\right)$

$= \frac{1}{2}-0 = \frac{1}{2} = \text{RHS}$

(ii) $2\cos\frac{5\pi}{12}\cos\frac{\pi}{12} = \frac{1}{2}$

$\because \quad 2\cos A\cos B = \cos(A+B)+\cos(A-B)$

$= \cos\left(\frac{5\pi}{12}+\frac{\pi}{12}\right)+\cos\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)$

$= \cos(\left(\frac{\pi}{2}\right)+\cos\left(\frac{\pi}{3}\right)$

$= 0+\frac{1}{2} = \frac{1}{2} = \text{RHS}$

(iii) $2\sin\frac{5\pi}{12}\cos\frac{\pi}{12}$

$\because \quad 2\sin A\cos B = \sin(A+B)+\sin(A-B)$

$= \left(\frac{5\pi}{12}+\frac{\pi}{12}\right)+\sin\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)$

$= \sin\frac{\pi}{2}+\sin\frac{\pi}{3}$

$= 1+\frac{\sqrt{3}}{2} = \frac{2+\sqrt{3}}{2} = \text{RHS}$ (Taking LCM)

Transformation Formulae Ex 8.1 Q3(i)

$$\sin 50^\circ \cos 85^\circ = \frac{1-\sqrt{2}\sin 35^\circ}{2\sqrt{2}}$$

$$\text{LHS} = \sin 50^\circ \cos 85^\circ = \frac{2\sin 50^\circ \cos 85^\circ}{2}$$

$$\because \quad 2\sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$\Rightarrow \quad \frac{2\sin 50^\circ \cos 85^\circ}{2} = \frac{1}{2}\left[\sin(50^\circ + 85^\circ) + \sin(50^\circ - 85^\circ)\right]$$

$$= \frac{1}{2}\left[\sin 135^\circ + \sin(-35^\circ)\right]$$

$$= \frac{1}{2}\left[\sin(90^\circ + 45^\circ) - \sin 35^\circ\right] \qquad [\because \sin(-\theta) = -\sin\theta]$$

$$= \frac{1}{2}\left[\cos 45^\circ - \sin 35^\circ\right] \qquad [\because \sin(90^\circ + \theta) = \cos\theta]$$

Now,

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$= \frac{1}{2}\left[\frac{1}{\sqrt{2}} - \sin 35^\circ\right]$$

$$= \frac{1-\sqrt{2}\sin 35^\circ}{2\sqrt{2}}$$

Transformation Formulae Ex 8.1 Q3(ii)

$$\text{LHS} = \sin 25^\circ \cos 115^\circ$$

$$= \frac{2\sin 25^\circ \cos 115^\circ}{2}$$

We Know that

$$2\sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$= \frac{1}{2}\left[\sin(25^\circ + 115^\circ) + \sin(25^\circ - 115^\circ)\right]$$

$$= \frac{1}{2}\left[\sin 140^\circ + \sin(-90^\circ)\right]$$

$$\sin(-\theta) = -\sin\theta$$

And, $\sin(90^\circ + \theta) = \cos\theta$

$$\Rightarrow \quad \frac{1}{2}\left[\sin(90^\circ + 50^\circ) - \sin 90^\circ\right]$$

$$= \frac{1}{2}\left[\cos 50^\circ - 1\right]$$

Also,

$$\cos\theta = \sin(90^\circ - \theta)$$

$$\cos 50^\circ = \sin(90^\circ - 50^\circ) = \sin 40^\circ$$

$$\frac{1}{2}\left[\sin 40^\circ - 1\right]$$

Transformation Formulae Ex 8.1 Q4.

We have,

$$
\begin{aligned}
\text{LHS} &= 4\cos\theta\cos\left(\frac{\pi}{3}+\theta\right)\cos\left(\frac{\pi}{3}-\theta\right)\\
&= 2\cos\theta\left[2\cos\left(\frac{\pi}{3}+\theta\right)\cos\left(\frac{\pi}{3}-\theta\right)\right]\\
&= 2\cos\theta\left[2\cos\left(\frac{\pi}{3}+\theta+\frac{\pi}{3}-\theta\right)+\cos\left(\frac{\pi}{3}+\theta-\frac{\pi}{3}+\theta\right)\right]\\
&= 2\cos\theta\left[\cos\frac{2\pi}{3}+\cos 2\theta\right]\\
&= 2\cos\theta\left[\cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right)+\cos 2\theta\right]\\
&= 2\cos\theta\left[-\sin\frac{\pi}{6}+\cos 2\theta\right]\\
&= 2\cos\theta\left[-\frac{1}{2}+\cos 2\theta\right]\\
&= -2\cos\theta\times\frac{1}{2}+2\cos\theta\cos 2\theta\\
&= -\cos\theta+\left[\cos(\theta+2\theta)+\cos(2\theta-\theta)\right]\\
&= -\cos\theta+\cos 3\theta+\cos\theta\\
&= \cos 3\theta\\
&= \text{RHS}
\end{aligned}
$$

$\therefore$ LHS = RHS Hence proved.

*********** END **********