Indefinite Integrals Ex 19.10 Q1

Let $I = \int x^2 \sqrt{x+2}\,dx$

Substituting $x+2 = t$ and $dx = dt$ we get,

$$I = \int (t-2)^2 \sqrt{t}\,dt$$

$$= \int \left(t^2 + 4 - 4t\right)\sqrt{t}\,dt$$

$$= \int \left(t^{\frac{5}{2}} - 4t^{\frac{3}{2}} + 4t^{\frac{1}{2}}\right)dt$$

$$= \frac{2}{7}t^{\frac{7}{2}} - \frac{8}{5}t^{\frac{5}{2}} + \frac{8}{3}t^{\frac{3}{2}} + c$$

$$= \frac{2}{7}(x+2)^{\frac{7}{2}} - \frac{8}{5}(x+2)^{\frac{5}{2}} + \frac{8}{3}(x+2)^{\frac{3}{2}} + c$$

$$\therefore \quad I = \frac{2}{7}(x+2)^{\frac{7}{2}} - \frac{8}{5}(x+2)^{\frac{5}{2}} + \frac{8}{3}(x+2)^{\frac{3}{2}} + c$$

Indefinite Integrals Ex 19.10 Q2

Let $I = \int \frac{x^2}{\sqrt{x-1}}dx$

Substituting $x - 1 = t$ and $dx = dt$ we get,

$$I = \int \frac{(t+1)^2}{\sqrt{t}}dx$$

$$= \int \frac{t^2+1+2t}{\sqrt{t}}dt$$

$$= \int \left(t^{\frac{3}{2}} + t^{\frac{-1}{2}} + 2t^{\frac{-1}{2}}\right)dt$$

$$= \frac{2}{5}t^{\frac{5}{2}} + 2t^{\frac{1}{2}} + \frac{4}{3}t^{\frac{3}{2}} + c$$

$$= \frac{6t^{\frac{5}{2}} + 30t^{\frac{1}{2}} + 20t^{\frac{3}{2}}}{15} + c$$

$$= \frac{2}{15}t^{\frac{1}{2}}\left(3t^2+15+10t\right) + c$$

$$= \frac{2}{15}\sqrt{x-1}\left(3(x-1)^2+15+10(x-1)\right) + x$$

$$= \frac{2}{15}\sqrt{x-1}\left(3\left(x^2+1-2x\right)+15+10x-10\right) + c$$

$$= \frac{2}{15}\sqrt{x-1}\left(3x^2+3-6x+15+10x-10\right) + c$$

$$= \frac{2}{15}\sqrt{x-1}\left(3x^2+4x+8\right) + c$$

$$\therefore \quad I = \frac{2}{15}\left(3x^2+4x+8\right)\sqrt{x-1} + c$$

Indefinite Integrals Ex 19.10 Q3

Let $I = \int \frac{x^2}{\sqrt{3x+4}}dx$

Substituting $3x + 4 = t$ and $dx = \frac{dt}{3}$ we get,

$$I = \int \frac{\left(\frac{t-4}{3}\right)^2}{\sqrt{t}} \times \frac{dt}{3} \qquad \left[\because x = \frac{t-4}{3}\right]$$

$$= \int \frac{(t-4)^2}{9\sqrt{t}x^3}dt$$

$$= \frac{1}{27}\int \frac{t^2+16-8t}{\sqrt{t}}dt$$

$$= \frac{1}{27}\int \left(t^{\frac{3}{2}} - 8t^{\frac{1}{2}} + 16t^{\frac{1}{2}}16t^{\frac{-1}{2}}\right)dt$$

$$= \frac{1}{27}\left[\frac{2}{5}t^{\frac{5}{2}} - \frac{16}{3}t^{\frac{3}{2}} + 32t^{\frac{1}{1}}\right] + c$$

$$= \frac{2}{135}(3x+4)^{\frac{5}{2}} - \frac{16}{81}(3x+4)^{\frac{3}{2}} + \frac{32}{27}(3x+4)^{\frac{1}{2}} + c$$

$$\therefore \quad I = \frac{2}{135}(3x+4)^{\frac{5}{2}} - \frac{16}{81}(3x+4)^{\frac{3}{2}} + \frac{32}{27}(3x+4)^{\frac{1}{2}} + c$$

Indefinite Integrals Ex 19.10 Q4

Let $I = \int \frac{2x-1}{(x-1)^2}dx$

Substituting $x-1 = t$ and $dx = dt$, we get

$$I = \int \frac{2(t+1)}{t^2}dt$$

$$= \int \frac{2t+2-1}{t^2}dt$$

$$= \int \frac{2t+1}{t^2}dt$$

$$= \int \left(\frac{2t}{t^2} + \frac{1}{t^2}\right)dt$$

$$= 2\int \frac{1}{t}dt + \int t^{-2}dt$$

$$= 2\log|t| - t^{-1} + c$$

$$= 2\log|x-1| - \frac{1}{x-1} + c$$

$$\int \frac{2x-1}{(x-1)^2}dx = 2\log|x-1| - \frac{1}{x-1} + c$$

*********** END **********