Definite Integrals Ex 20.5 Q9

We have,

$$\int_a^b f(x)dx = \lim_{h \to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 1$, $b = 2$ and $f(x) = x^2$

$$\therefore h = \frac{1}{n} \Rightarrow nh = 1$$

Thus, we have,

$$I = \int_1^2 x^2 dx$$

$$= \lim_{h \to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(1+(n-1)h)\right]$$

$$= \lim_{h \to 0} h\left[1+(1+h)^2+(1+2h)^2+----+(1+(n-1)h\ )^2\right]$$

$$= \lim_{h \to 0} h\left[1+(1+2h+h^2)+(1+2\times 2h+2\times 2h^2)+----(1+2\times(n-1)h+(1-n)^2h^2)\right]$$

$$= \lim_{h \to 0} h\left[n+2h\{1+2+3---(n-1)\}+h^2\{1^2+2^2+3^2+---(n-1)^2\}\right]$$

$$\because h = \frac{1}{n} \ \&\text{ if } h \to 0 \Rightarrow n \to \infty$$

$$= \lim_{n \to \infty} \frac{1}{n}\left[n+\frac{2}{n}\frac{n(n-1)}{2}+\frac{1}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n \to \infty} 1+\frac{n^2}{n^2}\left(1-\frac{1}{n}\right)+\frac{1}{6n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= 1+1+\frac{2}{6} = \frac{7}{3}$$

$$\therefore \int_1^2 x^2 dx = \frac{7}{3}$$

Definite Integrals Ex 20.5 Q10

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 2,\ b = 3$ and $f(x) = 2x^2+1$

$\therefore\ h = \frac{1}{n} \Rightarrow nh = 1$

Thus, we have,

$$I = \int_2^3 \left(2x^2+1\right)dx$$

$$= \lim_{h\to 0} h\left[f(2)+f(2+h)+f(2+2h)+----f(2+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[\left(2\times 2^2+1\right)\left\{2(2+h)^2+1\right\}+\left\{2(2+2h)^2+1\right\}+-----+\left\{2(2+(n-1)h)^2+1\right\}\right]$$

$$= \lim_{h\to 0} h\left[9n+8h(1+2+3---)+2h^2\left(1^2+2^2+3^2+--\right)\right]$$

$\because h = \frac{1}{n}$ & if $h\to 0 \Rightarrow n\to\infty$

$$= \lim_{n\to\infty} \frac{1}{n}\left[9n+\frac{8}{n}\frac{n(n-1)}{2}+\frac{2}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty}\ 9+\frac{4}{n^2}n^2\left(1-\frac{1}{n}\right)+\frac{1}{3n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= 9+4+\frac{2}{3} = \frac{41}{3}$$

$$\therefore \int_2^3\left(2x^2+1\right)dx = \frac{41}{3}$$

Definite Integrals Ex 20.5 Q11

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 1,\ b = 2$ and $f(x) = x^2-1$

$\therefore\ h = \frac{1}{n} \Rightarrow nh = 1$

Thus, we have,

$$I = \int_1^2\left(x^2-1\right)dx$$

$$= \lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(1+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[\left(1^2-1\right)\left\{(1+h)^2-1\right\}+\left\{(1+2h)^2-1\right\}+-----+\left\{(1+(n-1)h)^2-1\right\}\right]$$

$$= \lim_{h\to 0} h\left[0+2h(1+2+3+---)+h^2\left(1+2^2+3^2+--\right)\right]$$

$\because h = \frac{1}{n}$ & if $h\to 0 \Rightarrow n\to\infty$

$$= \lim_{n\to\infty}\frac{1}{n}\left[\frac{2}{n}\frac{n(n-1)}{2}+\frac{1}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n^2}n^2\left(1-\frac{1}{n}\right)+\frac{1}{6n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= 1+\frac{2}{6} = \frac{4}{3}$$

$$\therefore \int_1^2\left(x^2-1\right)dx = \frac{4}{3}$$

Definite Integrals Ex 20.5 Q12

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 0$, $b = 2$ and $f(x) = x^2 + 4$

$\therefore h = \frac{2}{n} \Rightarrow nh = 2$

Thus, we have,

$$I = \int_0^2 (x^2+4)dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f(0+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[4(h^2+4)+\{(2h)^2+4\}+----\{(n-1)h^2+4\}\right]$$

$$\because h = \frac{2}{n} \ \& \text{ if } h \to 0 \Rightarrow n \to \infty$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[4n+\frac{4}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} 8+\frac{4}{3n^2}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= 8+\frac{4\times 2}{3} = \frac{32}{3}$$

$$\therefore \int_0^2 (x^2+4)dx = \frac{32}{3}$$

*********** END **********