Trigonometric Ratios of Compound Angles Ex 7.1 Q17

We have,

$$8\theta = 6\theta + 2\theta$$

$$\Rightarrow \quad \tan 8\theta = \tan(6\theta + 2\theta)$$

$$\Rightarrow \quad \tan 8\theta = \frac{\tan 6\theta + \tan 2\theta}{1 - \tan 6\theta \tan 2\theta} \qquad \left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right]$$

$$\Rightarrow \quad \tan 8\theta(1 - \tan 6\theta \tan 2\theta) = \tan 6\theta + \tan 2\theta$$

$$\Rightarrow \quad \tan 8\theta - \tan 8\theta \tan 6\theta \tan 2\theta = \tan 6\theta + \tan 2\theta$$

$$\Rightarrow \quad \tan 8\theta - \tan 6\theta - \tan 2\theta = \tan 8\theta \tan 6\theta \tan 2\theta$$

Hence proved.

We have,

$$45° = 30° + 15°$$

$$\Rightarrow \quad \tan 45° = \tan(30° + 15°)$$

$$\Rightarrow \quad 1 = \frac{\tan 30° + \tan 15°}{1 - \tan 30° \tan 15°} \qquad \left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right]$$

$$\Rightarrow \quad 1 - \tan 30° \tan 15° = \tan 15° + \tan 30°$$

$$\Rightarrow \quad 1 = \tan 15° + \tan 30° + \tan 30° \tan 15°$$

$$\Rightarrow \quad \tan 15° + \tan 30° + \tan 15° \tan 30° = 1$$

Hence proved.

We have,

$$45° = 9° + 36°$$

$$\Rightarrow \quad \tan 45° = \tan(9° + 36°)$$

$$\Rightarrow \quad 1 = \frac{\tan 9° + \tan 36°}{1 - \tan 9° \tan 36°} \qquad \left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right]$$

$$\Rightarrow \quad 1 - \tan 9° \tan 36° = \tan 9° + \tan 36°$$

$$\Rightarrow \quad 1 = \tan 9° + \tan 36° + \tan 9° \tan 36°$$

$$\Rightarrow \quad \tan 9° + \tan 36° + \tan 9° \tan 36° = 1$$

Hence proved.

We have,

$$13\theta = 9\theta + 4\theta$$

$$\Rightarrow \quad \tan 13\theta = \tan(9\theta + 4\theta)$$

$$\Rightarrow \quad \tan 13\theta = \frac{\tan 9\theta + \tan 4\theta}{1 - \tan 9\theta \tan 4\theta} \qquad \left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right]$$

$$\Rightarrow \quad \tan 13\theta(1 - \tan 9\theta \tan 4\theta) = \tan 9\theta + \tan 4\theta$$

$$\Rightarrow \quad \tan 13\theta - \tan 13\theta \tan 9\theta \tan 4\theta = \tan 9\theta + \tan 4\theta$$

$$\Rightarrow \quad \tan 13\theta - \tan 9\theta - \tan 4\theta = \tan 13\theta \tan 9\theta \tan 4\theta$$

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q18

We have,

$$\begin{aligned}
\text{RHS} &= \tan 3\theta \tan\theta \\
&= \tan(2\theta+\theta)\times\tan(2\theta-\theta) \\
&= \left[\frac{\tan 2\theta+\tan\theta}{1-\tan 2\theta\tan\theta}\right]\times\left[\frac{\tan 2\theta-\tan\theta}{1+\tan 2\theta\tan\theta}\right] \\
&= \frac{(\tan 2\theta+\tan\theta)(\tan 2\theta-\tan\theta)}{(1-\tan 2\theta\tan\theta)(1+\tan 2\theta\tan\theta)} \\
&= \frac{\tan^2 2\theta-\tan^2\theta}{1-\tan^2 2\theta\tan^2\theta} \qquad \left[\because (a-b)(a+b)=a^2-b^2\right] \\
&= \text{LHS}
\end{aligned}$$

$\therefore$ LHS = RHS

Hence proved

Trigonometric Ratios of Compound Angles Ex 7.1 Q19

$$\frac{\sin x.\cos y+\sin y.\cos x}{\sin x.\cos y-\sin y.\cos x}=\frac{a+b}{a-b}$$

$$\Rightarrow \frac{\sin x.\cos y+\sin y.\cos x+\sin x.\cos y-\sin y.\cos x}{\sin x.\cos y+\sin y.\cos x-\sin x.\cos y+\sin y.\cos x}=\frac{a+b+a-b}{a+b-a+b}$$ [*Using Componendo and Dividendo*]

$$\Rightarrow \frac{2\sin x.\cos y}{2\sin y.\cos x}=\frac{2a}{2b}$$

$$\Rightarrow \frac{\tan x}{\tan y}=\frac{a}{b}$$

Hence Proved

********** END **********