



## EXERCISE.14.5

### Question-1

Show that the statement

$p$ : "If  $x$  is a real number such that  $x^3 + 4x = 0$ , then  $x$  is 0" is true by

- (i) direct method
- (ii) method of contradiction
- (iii) method of contrapositive

Ans.

$p$ : "If  $x$  is a real number such that  $x^3 + 4x = 0$ , then  $x$  is 0".

Let  $q$ :  $x$  is a real number such that  $x^3 + 4x = 0$

$r$ :  $x$  is 0.

(i) To show that statement  $p$  is true, we assume that  $q$  is true and then show that  $r$  is true.

Therefore, let statement  $q$  be true.

$$\therefore x^3 + 4x = 0$$

$$x(x^2 + 4) = 0$$

$$\Rightarrow x = 0 \text{ or } x^2 + 4 = 0$$

However, since  $x$  is real, it is 0.

Thus, statement  $r$  is true.

Therefore, the given statement is true.

(ii) To show statement  $p$  to be true by contradiction, we assume that  $p$  is not true.

Let  $x$  be a real number such that  $x^3 + 4x = 0$  and let  $x$  is not 0.

Therefore,  $x^3 + 4x = 0$

$$x(x^2 + 4) = 0$$

$$x = 0 \text{ or } x^2 + 4 = 0$$

$$x = 0 \text{ or } x^2 = -4$$

However,  $x$  is real. Therefore,  $x = 0$ , which is a contradiction since we have assumed that  $x$  is not 0.

Thus, the given statement  $p$  is true.

(iii) To prove statement  $p$  to be true by contrapositive method, we assume that  $r$  is false and prove that  $q$  must be false.

Here,  $r$  is false implies that it is required to consider the negation of statement  $r$ . This obtains the following statement.

$\sim r$ :  $x$  is not 0.

It can be seen that  $(x^2 + 4)$  will always be positive.

$x \neq 0$  implies that the product of any positive real number with  $x$  is not zero.

Let us consider the product of  $x$  with  $(x^2 + 4)$ .

$$\therefore x(x^2 + 4) \neq 0$$

$$\Rightarrow x^3 + 4x \neq 0$$

This shows that statement  $q$  is not true.

Thus, it has been proved that

$$\sim r \Rightarrow \sim q$$

Therefore, the given statement  $p$  is true.

### Question-2

Show that the statement "For any real numbers  $a$  and  $b$ ,  $a^2 = b^2$  implies that  $a = b$ " is not true by giving a counter-example

Ans.

The given statement can be written in the form of "if-then" as follows.

If  $a$  and  $b$  are real numbers such that  $a^2 = b^2$ , then  $a = b$ .

Let  $p$ :  $a$  and  $b$  are real numbers such that  $a^2 = b^2$ .

$q$ :  $a = b$

The given statement has to be proved false. For this purpose, it has to be proved that if  $p$ , then  $\sim q$ . To show this, two real numbers,  $a$  and  $b$ , with  $a^2 = b^2$  are required such that  $a \neq b$ .

Let  $a = 1$  and  $b = -1$

$a^2 = (1)^2 = 1$  and  $b^2 = (-1)^2 = 1$

$\therefore a^2 = b^2$

However,  $a \neq b$

Thus, it can be concluded that the given statement is false.

### Question-3

Show that the following statement is true by the method of contrapositive.

$p$ : If  $x$  is an integer and  $x^2$  is even, then  $x$  is also even.

Ans.

$p$ : If  $x$  is an integer and  $x^2$  is even, then  $x$  is also even.

Let  $q$ :  $x$  is an integer and  $x^2$  is even.

$r$ :  $x$  is even.

To prove that  $p$  is true by contrapositive method, we assume that  $r$  is false, and prove that  $q$  is also false.

Let  $x$  is not even.

To prove that  $q$  is false, it has to be proved that  $x$  is not an integer or  $x^2$  is not even.

$x$  is not even implies that  $x^2$  is also not even.

Therefore, statement  $q$  is false.

Thus, the given statement  $p$  is true.

### Question-4

By giving a counter example, show that the following statements are not true.

(i)  $p$ : If all the angles of a triangle are equal, then the triangle is an obtuse angled triangle.

(ii)  $q$ : The equation  $x^2 - 1 = 0$  does not have a root lying between 0 and 2.

Ans.

(i) The given statement is of the form "if  $q$  then  $r$ ".

$q$ : All the angles of a triangle are equal.

$r$ : The triangle is an obtuse-angled triangle.

The given statement  $p$  has to be proved false. For this purpose, it has to be proved that if  $q$ , then  $\sim r$ .

To show this, angles of a triangle are required such that none of them is an obtuse angle.

It is known that the sum of all angles of a triangle is  $180^\circ$ . Therefore, if all the three angles are equal, then each of them is of measure  $60^\circ$ , which is not an obtuse angle.

In an equilateral triangle, the measure of all angles is equal. However, the triangle is not an obtuse-angled triangle.

Thus, it can be concluded that the given statement  $p$  is false.

(ii) The given statement is as follows.

$q$ : The equation  $x^2 - 1 = 0$  does not have a root lying between 0 and 2.

This statement has to be proved false. To show this, a counter example is required.

Consider  $x^2 - 1 = 0$

$x^2 = 1$

$x = \pm 1$

One root of the equation  $x^2 - 1 = 0$ , i.e. the root  $x = 1$ , lies between 0 and 2.

Thus, the given statement is false.

### Question-5

Which of the following statements are true and which are false? In each case give a valid reason for saying so.

(i)  $p$ : Each radius of a circle is a chord of the circle.

(ii)  $q$ : The centre of a circle bisects each chord of the circle.

(iii)  $r$ : Circle is a particular case of an ellipse.

(iv)  $s$ : If  $x$  and  $y$  are integers such that  $x > y$ , then  $-x < -y$ .

(v)  $t$ :  $\sqrt{11}$  is a rational number.

Ans.

(i) The given statement  $p$  is false.

According to the definition of chord, it should intersect the circle at two distinct points.

(ii) The given statement  $q$  is false.

If the chord is not the diameter of the circle, then the centre will not bisect that chord.

In other words, the centre of a circle only bisects the diameter, which is the chord of the circle.

(iii) The equation of an ellipse is,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

If we put  $a = b = 1$ , then we obtain

$x^2 + y^2 = 1$ , which is an equation of a circle

Therefore, circle is a particular case of an ellipse.

Thus, statement  $r$  is true.

(iv)  $x > y$

$\Rightarrow -x < -y$  (By a rule of inequality)

Thus, the given statement  $s$  is true.

(v) 11 is a prime number and we know that the square root of any prime number is an irrational number. Therefore,  $\sqrt{11}$  is an irrational number.

Thus, the given statement  $t$  is false.

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