Quadratic Equations Ex 8.6 Q7

Answer :

The given quadric equation is $(b-c)x^2+(c-a)x+(a-b)=0$, and roots are real

Then prove that $2b=a+c$.

Here,

$a=(b-c), b=(c-a) \text{ and}, c=(a-b)$

As we know that $D=b^2-4ac$

Putting the value of $a=(b-c), b=(c-a) \text{ and}, c=(a-b)$

$$
\begin{aligned}
D &= b^2-4ac \\
&= (c-a)^2-4\times(b-c)\times(a-b) \\
&= c^2-2ca+a^2-4\left(ab-b^2-ca+bc\right) \\
&= c^2-2ca+a^2-4ab+4b^2+4ca-4bc \\
&= c^2+2ca+a^2-4ab+4b^2-4bc \\
&= a^2+4b^2+c^2+2ca-4ab-4bc
\end{aligned}
$$

As we know that $\left(a^2+4b^2+c^2+2ca-4ab-4bc\right)=(a+c-2b)^2$

$D=(a+c-2b)^2$

The given equation will have real roots, if $D=0$

$(a+c-2b)^2=0$

Square root both side we get

$\sqrt{(a+c-2b)^2}=0$

$$a+c-2b=0$$

$$a+c=2b$$

Hence $\boxed{2b=a+c}$

Quadratic Equations Ex 8.6 Q8

Answer :

The given quadric equation is $\left(a^2+b^2\right)x^2-2\left(ac+bd\right)x+\left(c^2+d^2\right)=0$, and roots are real

Then prove that $\frac{a}{b}=\frac{c}{d}$.

Here,

$a=\left(a^2+b^2\right), b=-2\left(ac+bd\right) \text{and}, c=\left(c^2+d^2\right)$

As we know that $D=b^2-4ac$

Putting the value of $a=\left(a^2+b^2\right), b=-2\left(ac+bd\right) \text{and}, c=\left(c^2+d^2\right)$

$$
\begin{aligned}
D &= b^2-4ac \\
&= \left\{-2\left(ac+bd\right)\right\}^2-4\times\left(a^2+b^2\right)\times\left(c^2+d^2\right) \\
&= 4\left(a^2c^2+2abcd+b^2d^2\right)-4\left(a^2c^2+a^2d^2+b^2c^2+b^2d^2\right) \\
&= \cancel{4a^2c^2}+8abcd\cancel{+4b^2d^2}\cancel{-4a^2c^2}-4a^2d^2-4b^2c^2\cancel{-4b^2d^2} \\
&= -4a^2d^2-4b^2c^2+8abcd \\
&= -4\left(a^2d^2+b^2c^2-2abcd\right)
\end{aligned}
$$

The given equation will have real roots, if $D=0$

$$-4\left(a^2d^2+b^2c^2-2abcd\right)=0$$

$$\left(a^2d^2+b^2c^2-2abcd\right)=0$$

$$(ad)^2+(bc)^2-2(ad)(bc)=0$$

$$(ad-bc)^2=0$$

Square root both sides we get,

$$ad-bc=0$$

$$ad=bc$$

$$\frac{a}{b}=\frac{c}{d}$$

Hence $\boxed{\frac{a}{b}=\frac{c}{d}}$

Quadratic Equations Ex 8.6 Q9

Answer :

The given equations are

$ax^2+2bx+c=0$ ...... (1)

$bx^2-2\sqrt{ac}x+b=0$ ...... (2)

Roots are simultaneously real

Then prove that $b^2=ac$.

Let D_1 and D_2 be the discriminants of equation (1) and (2) respectively,

Then,

$$D_1=(2b)^2-4ac$$
$$=4b^2-4ac$$

And

$$D_2=\left(-2\sqrt{ac}\right)^2-4\times b\times b$$
$$=4ac-4b^2$$

Both the given equation will have real roots, if $D_1\geq 0$ and $D_2\geq 0$

$$4b^2-4ac\geq 0$$
$$4b^2\geq 4ac$$
$$b^2\geq ac \text{ ...... (3)}$$
$$4ac-4b^2\geq 0$$
$$4ac\geq 4b^2$$
$$ac\geq b^2 \text{ ...... (4)}$$

From equations (3) and (4) we get

$$b^2=ac$$

Hence, $\boxed{b^2=ac}$

*********** END **********