



Exercise 7.5 : Solutions of Questions on Page Number : 322

Q1 : $\frac{x}{(x+1)(x+2)}$

Answer :

Let $\frac{x}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$

$\Rightarrow x = A(x+2) + B(x+1)$

Equating the coefficients of x and constant term, we obtain

$A + B = 1$

$2A + B = 0$

On solving, we obtain

$A = -1$ and $B = 2$

$$\begin{aligned} \therefore \frac{x}{(x+1)(x+2)} &= \frac{-1}{(x+1)} + \frac{2}{(x+2)} \\ \Rightarrow \int \frac{x}{(x+1)(x+2)} dx &= \int \frac{-1}{(x+1)} + \frac{2}{(x+2)} dx \\ &= -\log|x+1| + 2\log|x+2| + C \\ &= \log(x+2)^2 - \log|x+1| + C \\ &= \log \frac{(x+2)^2}{(x+1)} + C \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q2 : $\frac{1}{x^2-9}$

Answer :

Let $\frac{1}{(x+3)(x-3)} = \frac{A}{(x+3)} + \frac{B}{(x-3)}$

$1 = A(x-3) + B(x+3)$

Equating the coefficients of x and constant term, we obtain

$A + B = 0$

$-3A + 3B = 1$

On solving, we obtain

$A = -\frac{1}{6}$ and $B = \frac{1}{6}$

$$\begin{aligned} \therefore \frac{1}{(x+3)(x-3)} &= \frac{-1}{6(x+3)} + \frac{1}{6(x-3)} \\ \Rightarrow \int \frac{1}{(x^2-9)} dx &= \int \left(\frac{-1}{6(x+3)} + \frac{1}{6(x-3)} \right) dx \\ &= -\frac{1}{6} \log|x+3| + \frac{1}{6} \log|x-3| + C \\ &= \frac{1}{6} \log \left| \frac{(x-3)}{(x+3)} \right| + C \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q3 : $\frac{3x-1}{(x-1)(x-2)(x-3)}$

Answer :

Let $\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2) \quad \dots(1)$

Substituting $x = 1, 2,$ and 3 respectively in equation (1), we obtain

$A = 1, B = -5,$ and $C = 4$

$$\therefore \frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{1}{(x-1)} - \frac{5}{(x-2)} + \frac{4}{(x-3)}$$

$$\Rightarrow \int \frac{x}{(x-1)(x-2)(x-3)} dx = \int \left\{ \frac{1}{(x-1)} - \frac{2}{(x-2)} + \frac{1}{(x-3)} \right\} dx$$

$$= \log|x-1| - 2\log|x-2| + \log|x-3| + C$$

Answer needs Correction? [Click Here](#)

Q4: $\frac{x}{(x-1)(x-2)(x-3)}$

Answer :

$$\text{Let } \frac{x}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$$

$$x = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2) \quad \dots(1)$$

Substituting $x = 1, 2$, and 3 respectively in equation (1), we obtain $A = \frac{1}{2}$, $B = -2$, and $C = \frac{3}{2}$

$$\therefore \frac{x}{(x-1)(x-2)(x-3)} = \frac{1}{2(x-1)} - \frac{2}{(x-2)} + \frac{3}{2(x-3)}$$

$$\Rightarrow \int \frac{x}{(x-1)(x-2)(x-3)} dx = \int \left\{ \frac{1}{2(x-1)} - \frac{2}{(x-2)} + \frac{3}{2(x-3)} \right\} dx$$

$$= \frac{1}{2} \log|x-1| - 2\log|x-2| + \frac{3}{2} \log|x-3| + C$$

Answer needs Correction? [Click Here](#)

Q5: $\frac{2x}{x^2+3x+2}$

Answer :

$$\text{Let } \frac{2x}{x^2+3x+2} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$$

$$2x = A(x+2) + B(x+1) \quad \dots(1)$$

Substituting $x = -1$ and -2 in equation (1), we obtain

$$A = -2 \text{ and } B = 4$$

$$\therefore \frac{2x}{(x+1)(x+2)} = \frac{-2}{(x+1)} + \frac{4}{(x+2)}$$

$$\Rightarrow \int \frac{2x}{(x+1)(x+2)} dx = \int \left\{ \frac{4}{(x+2)} - \frac{2}{(x+1)} \right\} dx$$

$$= 4\log|x+2| - 2\log|x+1| + C$$

Answer needs Correction? [Click Here](#)

Q6: $\frac{1-x^2}{x(1-2x)}$

Answer :

It can be seen that the given integrand is not a proper fraction.

Therefore, on dividing $(1-x^2)$ by $x(1-2x)$, we obtain

$$\frac{1-x^2}{x(1-2x)} = \frac{1}{2} + \frac{1}{2} \left(\frac{2-x}{x(1-2x)} \right)$$

$$\text{Let } \frac{2-x}{x(1-2x)} = \frac{A}{x} + \frac{B}{(1-2x)}$$

$$\Rightarrow (2-x) = A(1-2x) + Bx \quad \dots(1)$$

Substituting $x = 0$ and $\frac{1}{2}$ in equation (1), we obtain

$$A = 2 \text{ and } B = 3$$

$$\therefore \frac{2-x}{x(1-2x)} = \frac{2}{x} + \frac{3}{1-2x}$$

Substituting in equation (1), we obtain

$$\frac{1-x^2}{x(1-2x)} = \frac{1}{2} + \frac{1}{2} \left\{ \frac{2}{x} + \frac{3}{(1-2x)} \right\}$$

$$\Rightarrow \int \frac{1-x^2}{x(1-2x)} dx = \int \left\{ \frac{1}{2} + \frac{1}{2} \left(\frac{2}{x} + \frac{3}{1-2x} \right) \right\} dx$$

$$= \frac{x}{2} + \log|x| + \frac{3}{2(-2)} \log|1-2x| + C$$

$$= \frac{x}{2} + \log|x| - \frac{3}{4} \log|1-2x| + C$$

Answer needs Correction? [Click Here](#)

Q7: $\frac{x}{(x^2+1)(x-1)}$

Answer :

$$\text{Let } \frac{x}{(x^2+1)(x-1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1}$$

$$\text{Let } \frac{x}{(x^2+1)(x-1)} = \frac{A}{(x^2+1)} + \frac{B}{(x-1)}$$

$$x = (Ax+B)(x-1) + C(x^2+1)$$

$$x = Ax^2 - Ax + Bx - B + Cx^2 + C$$

Equating the coefficients of x^2 , x , and constant term, we obtain

$$A + C = 0$$

$$-A + B = 1$$

$$-B + C = 0$$

On solving these equations, we obtain

$$A = -\frac{1}{2}, B = \frac{1}{2}, \text{ and } C = \frac{1}{2}$$

From equation (1), we obtain

$$\begin{aligned} \therefore \frac{x}{(x^2+1)(x-1)} &= \frac{\left(-\frac{1}{2}x + \frac{1}{2}\right)}{x^2+1} + \frac{\frac{1}{2}}{(x-1)} \\ \Rightarrow \int \frac{x}{(x^2+1)(x-1)} dx &= -\frac{1}{2} \int \frac{x}{x^2+1} dx + \frac{1}{2} \int \frac{1}{x^2+1} dx + \frac{1}{2} \int \frac{1}{x-1} dx \\ &= -\frac{1}{4} \int \frac{2x}{x^2+1} dx + \frac{1}{2} \tan^{-1} x + \frac{1}{2} \log|x-1| + C \end{aligned}$$

$$\text{Consider } \int \frac{2x}{x^2+1} dx, \text{ let } (x^2+1) = t \Rightarrow 2x dx = dt$$

$$\Rightarrow \int \frac{2x}{x^2+1} dx = \int \frac{dt}{t} = \log|t| = \log|x^2+1|$$

$$\begin{aligned} \therefore \int \frac{x}{(x^2+1)(x-1)} dx &= -\frac{1}{4} \log|x^2+1| + \frac{1}{2} \tan^{-1} x + \frac{1}{2} \log|x-1| + C \\ &= \frac{1}{2} \log|x-1| - \frac{1}{4} \log|x^2+1| + \frac{1}{2} \tan^{-1} x + C \end{aligned}$$

Answer needs Correction? [Click Here](#)

$$\text{Q8 : } \frac{x}{(x-1)^2(x+2)}$$

Answer :

$$\text{Let } \frac{x}{(x-1)^2(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$$

$$x = A(x-1)(x+2) + B(x+2) + C(x-1)^2$$

Substituting $x = 1$, we obtain

$$B = \frac{1}{3}$$

Equating the coefficients of x^2 and constant term, we obtain

$$A + C = 0$$

$$-2A + 2B + C = 0$$

On solving, we obtain

$$A = \frac{2}{9} \text{ and } C = \frac{-2}{9}$$

$$\begin{aligned} \therefore \frac{x}{(x-1)^2(x+2)} &= \frac{2}{9(x-1)} + \frac{1}{3(x-1)^2} - \frac{2}{9(x+2)} \\ \Rightarrow \int \frac{x}{(x-1)^2(x+2)} dx &= \frac{2}{9} \int \frac{1}{(x-1)} dx + \frac{1}{3} \int \frac{1}{(x-1)^2} dx - \frac{2}{9} \int \frac{1}{(x+2)} dx \\ &= \frac{2}{9} \log|x-1| + \frac{1}{3} \left(\frac{-1}{x-1} \right) - \frac{2}{9} \log|x+2| + C \\ &= \frac{2}{9} \log \left| \frac{x-1}{x+2} \right| - \frac{1}{3(x-1)} + C \end{aligned}$$

Answer needs Correction? [Click Here](#)

$$\text{Q9 : } \frac{3x+5}{x^3-x^2-x+1}$$

Answer :

$$\frac{3x+5}{x^3-x^2-x+1} = \frac{3x+5}{(x-1)^2(x+1)}$$

$$\text{Let } \frac{3x+5}{(x-1)^2(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)}$$

$$3x+5 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$$

$$3x+5 = A(x^2-1) + B(x+1) + C(x^2-2x+1) \quad \dots(1)$$

Substituting $x = 1$ in equation (1), we obtain

$$B = 4$$

Equating the coefficients of x^2 and x , we obtain

$$A + C = 0$$

$$B - 2C = 3$$

On solving, we obtain

On solving, we obtain

$$A = -\frac{1}{2} \text{ and } C = \frac{1}{2}$$

$$\begin{aligned} \therefore \frac{3x+5}{(x-1)^2(x+1)} &= \frac{-1}{2(x-1)} + \frac{4}{(x-1)^2} + \frac{1}{2(x+1)} \\ \Rightarrow \int \frac{3x+5}{(x-1)^2(x+1)} dx &= -\frac{1}{2} \int \frac{1}{x-1} dx + 4 \int \frac{1}{(x-1)^2} dx + \frac{1}{2} \int \frac{1}{x+1} dx \\ &= -\frac{1}{2} \log|x-1| + 4 \left(\frac{-1}{x-1} \right) + \frac{1}{2} \log|x+1| + C \\ &= \frac{1}{2} \log \left| \frac{x+1}{x-1} \right| - \frac{4}{(x-1)} + C \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q10: $\frac{2x-3}{(x^2-1)(2x+3)}$

Answer :

$$\frac{2x-3}{(x^2-1)(2x+3)} = \frac{2x-3}{(x+1)(x-1)(2x+3)}$$

$$\text{Let } \frac{2x-3}{(x+1)(x-1)(2x+3)} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{2x+3}$$

$$\Rightarrow (2x-3) = A(x-1)(2x+3) + B(x+1)(2x+3) + C(x+1)(x-1)$$

$$\Rightarrow (2x-3) = A(2x^2+x-3) + B(2x^2+5x+3) + C(x^2-1)$$

$$\Rightarrow (2x-3) = (2A+2B+C)x^2 + (A+5B)x + (-3A+3B-C)$$

Equating the coefficients of x^2 and x , we obtain

$$B = -\frac{1}{10}, A = \frac{5}{2}, \text{ and } C = -\frac{24}{5}$$

$$\begin{aligned} \therefore \frac{2x-3}{(x+1)(x-1)(2x+3)} &= \frac{5}{2(x+1)} - \frac{1}{10(x-1)} - \frac{24}{5(2x+3)} \\ \Rightarrow \int \frac{2x-3}{(x^2-1)(2x+3)} dx &= \frac{5}{2} \int \frac{1}{x+1} dx - \frac{1}{10} \int \frac{1}{x-1} dx - \frac{24}{5} \int \frac{1}{2x+3} dx \\ &= \frac{5}{2} \log|x+1| - \frac{1}{10} \log|x-1| - \frac{24}{5 \times 2} \log|2x+3| \\ &= \frac{5}{2} \log|x+1| - \frac{1}{10} \log|x-1| - \frac{12}{5} \log|2x+3| + C \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q11: $\frac{5x}{(x+1)(x^2-4)}$

Answer :

$$\frac{5x}{(x+1)(x^2-4)} = \frac{5x}{(x+1)(x+2)(x-2)}$$

$$\text{Let } \frac{5x}{(x+1)(x+2)(x-2)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x-2}$$

$$5x = A(x+2)(x-2) + B(x+1)(x-2) + C(x+1)(x+2) \quad \dots(1)$$

Substituting $x = -1, -2$, and 2 respectively in equation (1), we obtain

$$A = \frac{5}{3}, B = -\frac{5}{2}, \text{ and } C = \frac{5}{6}$$

$$\begin{aligned} \therefore \frac{5x}{(x+1)(x+2)(x-2)} &= \frac{5}{3(x+1)} - \frac{5}{2(x+2)} + \frac{5}{6(x-2)} \\ \Rightarrow \int \frac{5x}{(x+1)(x^2-4)} dx &= \frac{5}{3} \int \frac{1}{x+1} dx - \frac{5}{2} \int \frac{1}{x+2} dx + \frac{5}{6} \int \frac{1}{x-2} dx \\ &= \frac{5}{3} \log|x+1| - \frac{5}{2} \log|x+2| + \frac{5}{6} \log|x-2| + C \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q12: $\frac{x^3+x+1}{x^2-1}$

Answer :

It can be seen that the given integrand is not a proper fraction.

Therefore, on dividing $(x^3 + x + 1)$ by $x^2 - 1$, we obtain

$$\frac{x^3+x+1}{x^2-1} = x + \frac{2x+1}{x^2-1}$$

$$\text{Let } \frac{2x+1}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}$$

$$2x+1 = A(x-1) + B(x+1) \quad \dots(1)$$

Substituting $x = 1$ and -1 in equation (1), we obtain

$$A = \frac{1}{2} \text{ and } B = \frac{3}{2}$$

$$\therefore \frac{x^3+x+1}{x^2-1} = x + \frac{1}{2} \log \left| \frac{x+1}{x-1} \right| + \frac{3}{2} \log|x-1| + C$$

$$\begin{aligned}\therefore \frac{x+x+1}{x^2-1} &= x + \frac{1}{2(x+1)} + \frac{3}{2(x-1)} \\ \Rightarrow \int \frac{x^2+x+1}{x^2-1} dx &= \int x dx + \frac{1}{2} \int \frac{1}{(x+1)} dx + \frac{3}{2} \int \frac{1}{(x-1)} dx \\ &= \frac{x^2}{2} + \frac{1}{2} \log|x+1| + \frac{3}{2} \log|x-1| + C\end{aligned}$$

Answer needs Correction? [Click Here](#)

Q13: $\frac{2}{(1-x)(1+x^2)}$

Answer :

$$\begin{aligned}\text{Let } \frac{2}{(1-x)(1+x^2)} &= \frac{A}{(1-x)} + \frac{Bx+C}{(1+x^2)} \\ 2 &= A(1+x^2) + (Bx+C)(1-x) \\ 2 &= A + Ax^2 + Bx - Bx^2 + C - Cx\end{aligned}$$

Equating the coefficient of x^2 , x , and constant term, we obtain

$$A - B = 0$$

$$B - C = 0$$

$$A + C = 2$$

On solving these equations, we obtain

$$A = 1, B = 1, \text{ and } C = 1$$

$$\begin{aligned}\therefore \frac{2}{(1-x)(1+x^2)} &= \frac{1}{1-x} + \frac{x+1}{1+x^2} \\ \Rightarrow \int \frac{2}{(1-x)(1+x^2)} dx &= \int \frac{1}{1-x} dx + \int \frac{x}{1+x^2} dx + \int \frac{1}{1+x^2} dx \\ &= -\int \frac{1}{x-1} dx + \frac{1}{2} \int \frac{2x}{1+x^2} dx + \int \frac{1}{1+x^2} dx \\ &= -\log|x-1| + \frac{1}{2} \log|1+x^2| + \tan^{-1} x + C\end{aligned}$$

Answer needs Correction? [Click Here](#)

Q14: $\frac{3x-1}{(x+2)^2}$

Answer :

$$\begin{aligned}\text{Let } \frac{3x-1}{(x+2)^2} &= \frac{A}{(x+2)} + \frac{B}{(x+2)^2} \\ \Rightarrow 3x-1 &= A(x+2) + B\end{aligned}$$

Equating the coefficient of x and constant term, we obtain

$$A = 3$$

$$2A + B = -1 \Rightarrow B = -7$$

$$\begin{aligned}\therefore \frac{3x-1}{(x+2)^2} &= \frac{3}{(x+2)} - \frac{7}{(x+2)^2} \\ \Rightarrow \int \frac{3x-1}{(x+2)^2} dx &= 3 \int \frac{1}{(x+2)} dx - 7 \int \frac{x}{(x+2)^2} dx \\ &= 3 \log|x+2| - 7 \left(\frac{-1}{(x+2)} \right) + C \\ &= 3 \log|x+2| + \frac{7}{(x+2)} + C\end{aligned}$$

Answer needs Correction? [Click Here](#)

Q15: $\frac{1}{x^4-1}$

Answer :

$$\begin{aligned}\frac{1}{(x^4-1)} &= \frac{1}{(x^2-1)(x^2+1)} = \frac{1}{(x+1)(x-1)(1+x^2)} \\ \text{Let } \frac{1}{(x+1)(x-1)(1+x^2)} &= \frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{Cx+D}{(x^2+1)} \\ 1 &= A(x-1)(x^2+1) + B(x+1)(x^2+1) + (Cx+D)(x^2-1) \\ 1 &= A(x^3+x-x^2-1) + B(x^3+x+x^2+1) + Cx^3 + Dx^2 - Cx - D \\ 1 &= (A+B+C)x^3 + (-A+B+D)x^2 + (A+B-C)x + (-A+B-D)\end{aligned}$$

Equating the coefficient of x^3 , x^2 , x , and constant term, we obtain

$$A + B + C = 0$$

$$-A + B + D = 0$$

$$A + B - C = 0$$

$$-A + B - D = 1$$

On solving these equations, we obtain

$$A = -\frac{1}{4}, B = \frac{1}{4}, C = 0, \text{ and } D = -\frac{1}{2}$$

$$\begin{aligned}\therefore \frac{1}{x^4-1} &= \frac{-1}{4(x+1)} + \frac{1}{4(x-1)} - \frac{1}{2(x^2+1)} \\ \Rightarrow \int \frac{1}{x^4-1} dx &= -\frac{1}{4} \log|x-1| + \frac{1}{4} \log|x+1| - \frac{1}{2} \tan^{-1} x + C \\ &= \frac{1}{4} \log \left| \frac{x-1}{x+1} \right| - \frac{1}{2} \tan^{-1} x + C\end{aligned}$$

Answer needs Correction? [Click Here](#)

Q16 : $\frac{1}{x(x^n+1)}$ [Hint: multiply numerator and denominator by x^{n-1} and put $x^n = t$]

Answer :

$$\frac{1}{x(x^n+1)}$$

Multiplying numerator and denominator by x^{n-1} , we obtain

$$\begin{aligned}\frac{1}{x(x^n+1)} &= \frac{x^{n-1}}{x^{n-1}x(x^n+1)} = \frac{x^{n-1}}{x^n(x^n+1)} \\ \text{Let } x^n &= t \Rightarrow x^{n-1} dx = dt \\ \therefore \int \frac{1}{x(x^n+1)} dx &= \int \frac{x^{n-1}}{x^n(x^n+1)} dx = \frac{1}{n} \int \frac{1}{t(t+1)} dt \\ \text{Let } \frac{1}{t(t+1)} &= \frac{A}{t} + \frac{B}{(t+1)} \\ 1 &= A(1+t) + Bt \quad \dots(1) \\ \text{Substituting } t &= 0, -1 \text{ in equation (1), we obtain} \\ A &= 1 \text{ and } B = -1 \\ \therefore \frac{1}{t(t+1)} &= \frac{1}{t} - \frac{1}{(t+1)} \\ \Rightarrow \int \frac{1}{x(x^n+1)} dx &= \frac{1}{n} \int \left(\frac{1}{t} - \frac{1}{(t+1)} \right) dx \\ &= \frac{1}{n} [\log|t| - \log|t+1|] + C \\ &= -\frac{1}{n} [\log|x^n| - \log|x^n+1|] + C \\ &= \frac{1}{n} \log \left| \frac{x^n}{x^n+1} \right| + C\end{aligned}$$

Answer needs Correction? [Click Here](#)

Q17 : $\frac{\cos x}{(1-\sin x)(2-\sin x)}$ [Hint: Put $\sin x = t$]

Answer :

$$\begin{aligned}\frac{\cos x}{(1-\sin x)(2-\sin x)} \\ \text{Let } \sin x &= t \Rightarrow \cos x dx = dt \\ \therefore \int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx &= \int \frac{dt}{(1-t)(2-t)} \\ \text{Let } \frac{1}{(1-t)(2-t)} &= \frac{A}{(1-t)} + \frac{B}{(2-t)} \\ 1 &= A(2-t) + B(1-t) \quad \dots(1) \\ \text{Substituting } t &= 2 \text{ and then } t = 1 \text{ in equation (1), we obtain} \\ A &= 1 \text{ and } B = -1 \\ \therefore \frac{1}{(1-t)(2-t)} &= \frac{1}{(1-t)} - \frac{1}{(2-t)} \\ \Rightarrow \int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx &= \int \left(\frac{1}{1-t} - \frac{1}{(2-t)} \right) dt \\ &= -\log|1-t| + \log|2-t| + C \\ &= \log \left| \frac{2-t}{1-t} \right| + C \\ &= \log \left| \frac{2-\sin x}{1-\sin x} \right| + C\end{aligned}$$

Answer needs Correction? [Click Here](#)

Q18 : $\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$

Answer :

$$\begin{aligned}\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} &= 1 - \frac{(4x^2+10)}{(x^2+3)(x^2+4)} \\ \text{Let } \frac{4x^2+10}{(x^2+3)(x^2+4)} &= \frac{Ax+B}{(x^2+3)} + \frac{Cx+D}{(x^2+4)} \\ 4x^2+10 &= (Ax+B)(x^2+4) + (Cx+D)(x^2+3)\end{aligned}$$

$$4x^2 + 10 = Ax^3 + 4Ax + Bx^2 + 4B + Cx^3 + 3Cx + Dx^2 + 3D$$

$$4x^2 + 10 = (A+C)x^3 + (B+D)x^2 + (4A+3C)x + (4B+3D)$$

Equating the coefficients of x^3 , x^2 , x , and constant term, we obtain

$$A + C = 0$$

$$B + D = 4$$

$$4A + 3C = 0$$

$$4B + 3D = 10$$

On solving these equations, we obtain

$$A = 0, B = -2, C = 0, \text{ and } D = 6$$

$$\begin{aligned} \therefore \frac{4x^2 + 10}{(x^2 + 3)(x^2 + 4)} &= \frac{-2}{(x^2 + 3)} + \frac{6}{(x^2 + 4)} \\ \frac{(x^2 + 1)(x^2 + 2)}{(x^2 + 3)(x^2 + 4)} &= 1 - \left(\frac{-2}{(x^2 + 3)} + \frac{6}{(x^2 + 4)} \right) \\ \Rightarrow \int \frac{(x^2 + 1)(x^2 + 2)}{(x^2 + 3)(x^2 + 4)} dx &= \int \left\{ 1 + \frac{2}{(x^2 + 3)} - \frac{6}{(x^2 + 4)} \right\} dx \\ &= \int \left\{ 1 + \frac{2}{x^2 + (\sqrt{3})^2} - \frac{6}{x^2 + 2^2} \right\} dx \\ &= x + 2 \left(\frac{1}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} \right) - 6 \left(\frac{1}{2} \tan^{-1} \frac{x}{2} \right) + C \\ &= x + \frac{2}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} - 3 \tan^{-1} \frac{x}{2} + C \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q19: $\frac{2x}{(x^2 + 1)(x^2 + 3)}$

Answer :

$$\frac{2x}{(x^2 + 1)(x^2 + 3)}$$

$$\text{Let } x^2 = t \Rightarrow 2x dx = dt$$

$$\therefore \int \frac{2x}{(x^2 + 1)(x^2 + 3)} dx = \int \frac{dt}{(t + 1)(t + 3)} \quad \dots(1)$$

$$\text{Let } \frac{1}{(t + 1)(t + 3)} = \frac{A}{(t + 1)} + \frac{B}{(t + 3)}$$

$$1 = A(t + 3) + B(t + 1) \quad \dots(1)$$

Substituting $t = -3$ and $t = -1$ in equation (1), we obtain

$$A = \frac{1}{2} \text{ and } B = -\frac{1}{2}$$

$$\therefore \frac{1}{(t + 1)(t + 3)} = \frac{1}{2(t + 1)} - \frac{1}{2(t + 3)}$$

$$\begin{aligned} \Rightarrow \int \frac{2x}{(x^2 + 1)(x^2 + 3)} dx &= \int \left\{ \frac{1}{2(t + 1)} - \frac{1}{2(t + 3)} \right\} dt \\ &= \frac{1}{2} \log |t + 1| - \frac{1}{2} \log |t + 3| + C \\ &= \frac{1}{2} \log \left| \frac{t + 1}{t + 3} \right| + C \\ &= \frac{1}{2} \log \left| \frac{x^2 + 1}{x^2 + 3} \right| + C \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q20: $\frac{1}{x(x^4 - 1)}$

Answer :

$$\frac{1}{x(x^4 - 1)}$$

Multiplying numerator and denominator by x^3 , we obtain

$$\frac{1}{x(x^4 - 1)} = \frac{x^3}{x^4(x^4 - 1)}$$

$$\therefore \int \frac{1}{x(x^4 - 1)} dx = \int \frac{x^3}{x^4(x^4 - 1)} dx$$

$$\text{Let } x^4 = t \Rightarrow 4x^3 dx = dt$$

$$\therefore \int \frac{1}{x(x^4 - 1)} dx = \frac{1}{4} \int \frac{dt}{t(t - 1)}$$

$$\text{Let } \frac{1}{t(t - 1)} = \frac{A}{t} + \frac{B}{(t - 1)}$$

$$1 = A(t - 1) + Bt \quad \dots(1)$$

Substituting $t = 0$ and 1 in (1), we obtain

Substituting $t = 1$ and $t = 0$ in the system

$$A = -1 \text{ and } B = 1$$

$$\Rightarrow \frac{1}{t(t+1)} = \frac{-1}{t} + \frac{1}{t+1}$$

$$\begin{aligned}\Rightarrow \int \frac{1}{x(x^4-1)} dx &= \frac{1}{4} \int \left(\frac{-1}{t} + \frac{1}{t+1} \right) dt \\ &= \frac{1}{4} [-\log|t| + \log|t+1|] + C \\ &= \frac{1}{4} \log \left| \frac{t+1}{t} \right| + C \\ &= \frac{1}{4} \log \left| \frac{x^4-1}{x^4} \right| + C\end{aligned}$$

Answer needs Correction? [Click Here](#)

Q21 : $\frac{1}{(e^x-1)}$ [Hint: Put $e^x = t$]

Answer :

$$\frac{1}{(e^x-1)}$$

$$\text{Let } e^x = t \Rightarrow e^x dx = dt$$

$$\Rightarrow \int \frac{1}{e^x-1} dx = \int \frac{1}{t-1} \times \frac{dt}{t} = \int \frac{1}{t(t-1)} dt$$

$$\text{Let } \frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1}$$

$$1 = A(t-1) + Bt \quad \dots(1)$$

Substituting $t = 1$ and $t = 0$ in equation (1), we obtain

$$A = -1 \text{ and } B = 1$$

$$\therefore \frac{1}{t(t-1)} = \frac{-1}{t} + \frac{1}{t-1}$$

$$\begin{aligned}\Rightarrow \int \frac{1}{t(t-1)} dt &= \log \left| \frac{t-1}{t} \right| + C \\ &= \log \left| \frac{e^x-1}{e^x} \right| + C\end{aligned}$$

Answer needs Correction? [Click Here](#)

Q22 : $\int \frac{xdx}{(x-1)(x-2)}$ equals

A. $\log \left| \frac{(x-1)^2}{x-2} \right| + C$

B. $\log \left| \frac{(x-2)^2}{x-1} \right| + C$

C. $\log \left| \left(\frac{x-1}{x-2} \right)^2 \right| + C$

D. $\log |(x-1)(x-2)| + C$

Answer :

$$\text{Let } \frac{x}{(x-1)(x-2)} = \frac{A}{(x-1)} + \frac{B}{(x-2)}$$

$$x = A(x-2) + B(x-1) \quad \dots(1)$$

Substituting $x = 1$ and $x = 2$ in (1), we obtain

$$A = -1 \text{ and } B = 2$$

$$\begin{aligned}\therefore \frac{x}{(x-1)(x-2)} &= -\frac{1}{(x-1)} + \frac{2}{(x-2)} \\ \Rightarrow \int \frac{x}{(x-1)(x-2)} dx &= \int \left\{ \frac{-1}{(x-1)} + \frac{2}{(x-2)} \right\} dx \\ &= -\log|x-1| + 2\log|x-2| + C \\ &= \log \left| \frac{(x-2)^2}{x-1} \right| + C\end{aligned}$$

Hence, the correct answer is B.

Answer needs Correction? [Click Here](#)

Q23 : $\int \frac{dx}{x(x^2+1)}$ equals

A. $\log|x| - \frac{1}{2} \log(x^2+1) + C$

B. $\log|x| + \frac{1}{2} \log(x^2+1) + C$

C. $-\log|x| + \frac{1}{2} \log(x^2+1) + C$

D. $\frac{1}{2} \log|x| + \log(x^2 + 1) + C$

Answer :

Let $\frac{1}{x(x^2 + 1)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 1}$

$1 = A(x^2 + 1) + (Bx + C)x$

Equating the coefficients of x^2 , x , and constant term, we obtain

$A + B = 0$

$C = 0$

$A = 1$

On solving these equations, we obtain

$A = 1$, $B = -1$, and $C = 0$

$\therefore \frac{1}{x(x^2 + 1)} = \frac{1}{x} - \frac{x}{x^2 + 1}$

$\Rightarrow \int \frac{1}{x(x^2 + 1)} dx = \int \left(\frac{1}{x} - \frac{x}{x^2 + 1} \right) dx$
 $= \log|x| - \frac{1}{2} \log|x^2 + 1| + C$

Hence, the correct answer is A.

***** END *****