Binomial Theorem Ex 18.2 Q16(ix)

We have,

$$\left(\sqrt[3]{x}+\frac{1}{2\sqrt[3]{x}}\right)^{18}, x>0$$

Let $(r+1)^{th}$ term be independent of x.

$$\therefore \quad T_{r+1} = {}^{18}C_r \left(\sqrt[3]{x}\right)^{18-r} \times \left(\frac{1}{2\sqrt[3]{x}}\right)^r$$

$$= {}^{18}C_r \left(\left(x\right)^{\frac{1}{3}}\right)^{18-r} \times \left(\frac{1}{2}\right)^r \times \left(\frac{1}{x^{\frac{1}{3}}}\right)^r$$

$$= {}^{18}C_r \left(x\right)^{\frac{8-r}{3}} \times \left(\frac{1}{x^{\frac{r}{3}}}\right) \times \left(\frac{1}{2}\right)^r$$

$$= {}^{18}C_r \left(x\right)^{\frac{18-r}{3}} \times \left(\frac{1}{2}\right)^r$$

$$= {}^{18}C_r \left(x\right)^{\frac{18-r}{3}} \times \left(\frac{1}{2}\right)^r$$

If it is independent of x, we must have

$$\frac{18-2r}{3} = 0$$

$$\Rightarrow \quad 18 = 2r$$

$$\Rightarrow \quad r = 9$$

$\therefore$ Term independet of x = $T_{9+1} = T_{10}$

Now,

$$T_{10} = {}^{18}C_9 \left(\sqrt[3]{x}\right)^{18-9} \left(\frac{1}{2\sqrt[3]{x}}\right)^9$$

$$= {}^{18}C_9 \left(\sqrt[3]{x}\right)^9 \times \frac{1}{2^9} \times \left(\frac{1}{\sqrt[3]{x}}\right)^9$$

$$= \frac{{}^{18}C_9}{2^9}$$

Hence, required term = $\frac{{}^{18}C_9}{2^9}$.

Binomial Theorem Ex 18.2 Q16(x)

$$\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^6$$

In expansion

$$T_{r+1}={}^6C_r\left(\frac{3x^2}{2}\right)^{6-r}\left(-\frac{1}{3x}\right)^r$$

$$={}^6C_r\left(\frac{3}{2}\right)^{6-r}\left(x^{12-3r}\right)\left(-\frac{1}{3}\right)^r$$

Let T_{r+1} be independent of x,

$$12-3r=0 \text{ or } r=4$$

$\therefore$ Required term

$$\Rightarrow T_{r+1}=T_{4+1}=T_5={}^6C_4\left(\frac{3}{2}\right)^{6-4}\left(\frac{-1}{3}\right)^4 x^{12-3(4)}$$

$$=15\left(\frac{9}{4}\right)\left(\frac{1}{81}\right)x^0=\frac{5}{12}$$

Binomial Theorem Ex 18.2 Q17

We know that the coefficient of rth term in the expansion of $(1+x)^n$ is ${}^nC_{r-1}$

$\therefore$ Coefficient of $(2r+4)$ th term of the expansion $(1+x)^{18}$ = ${}^{18}C_{2r+4-1}$ = ${}^{18}C_{2r+3}$

and, coefficient of $(r-2)$ th term of the expansion $(1+x)^{18}$ = ${}^{18}C_{r-2-1}$ = ${}^{18}C_{r-3}$

It is given that these coefficients are equal.

$\therefore$ ${}^{18}C_{2r+3}={}^{18}C_{r-3}$

$\Rightarrow$ $2r+3=r-3$ or, $2r+3+r-3=18$ $\left[\begin{array}{l}\because {}^nC_r={}^nC_s\\ \Rightarrow r=s \text{ or, } r+s=n\end{array}\right]$

$\Rightarrow$ $r=-6$ or, $3r=18$

$\Rightarrow$ $r=-6$ or, $r=6$

$\Rightarrow$ $r=6$ $[\because r=-6 \text{ is not possible}]$

Binomial Theorem Ex 18.2 Q18

$$(1+x)^{43}$$

$$\binom{43}{2r}=\binom{43}{r+1}$$

$$2r+r+1=43$$

$$3r=42$$

$$r=14$$

*********** END **********