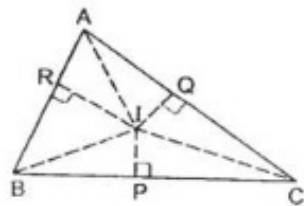




Exercise 5A

Question 30:

Given: A triangle ABC in which bisectors of $\angle B$ and $\angle C$ meet at I.



Also, we have $IP \perp BC$, $IQ \perp CA$ and $IR \perp AB$

To Prove: (i) $IP = IQ = IR$

(ii) $\angle IAR = \angle IAQ$

Proof: (i) In $\triangle BIP$ and $\triangle BIR$ we have,

$$\angle PBI = \angle RBI \quad [\text{Given}]$$

$$\angle IRB = \angle IPB = 90^\circ \quad [\text{Given}]$$

$$\text{and, } IB = IB \quad [\text{Common}]$$

Thus by Angle-Angle-Side criterion of congruence, we have

$$\therefore \triangle BIP \cong \triangle BIR \quad [\text{By AAS}]$$

The corresponding parts of the congruent triangles are equal.

$$\text{So, } IP = IR$$

$$\text{Similarly } IP = IQ$$

$$\therefore IP = IQ = IR$$

(ii) Now in $\triangle AIR$ and $\triangle AIQ$ we have

$$IR = IQ \quad [\text{Proved above}]$$

$$IA = IA \quad [\text{Common}]$$

$$\text{and, } \angle IRA = \angle IQA = 90^\circ$$

Thus by Side-Angle-Side criterion of congruence, we have

$$\therefore \triangle AIR \cong \triangle AIQ \quad [\text{By SAS}]$$

The corresponding parts of the congruent triangles are equal.

$$\text{So, } \angle IAR = \angle IAQ \quad [\text{by c.p.c.t.}]$$

\Rightarrow IA bisects $\angle A$

***** END *****

