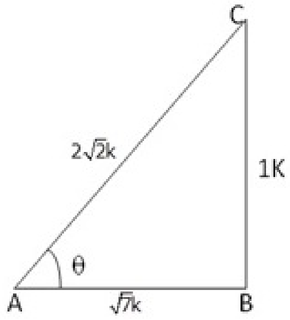
Question 8

Given: $tan\theta = \frac{BC}{AB} = \frac{1}{\sqrt{7}}$

Let BC = 1k and AB = $\sqrt{7}$k,

Where k is positive

Let us draw a ΔABC in which $\angle B = 90^0$ and $\angle BAC = \theta$



By pythagoras theorem, we have

$$AC^2 = \left(AB^2 + BC^2\right)$$

$$\Rightarrow AC^2 = \left[\left(\sqrt{7}k\right)^2 + \left(1k\right)^2\right]$$

$$= 7k^2 + 1k^2 = 8k^2$$

$$\Rightarrow AC = 2\sqrt{2}k$$

$$\operatorname{cosec}\theta = \frac{AC}{BC} = \frac{2\sqrt{2}k}{1k} = 2\sqrt{2}$$

$$\sec\theta = \frac{AC}{AB} = \frac{2\sqrt{2}k}{\sqrt{7}k} = \frac{2\sqrt{2}}{\sqrt{7}}$$

$$\Rightarrow \frac{\left(\operatorname{cosec}^2\theta - \sec^2\theta\right)}{\left(\operatorname{cosec}^2\theta + \sec^2\theta\right)} = \left[\frac{\left(2\sqrt{2}\right)^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{\left(2\sqrt{2}\right)^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}\right]$$

$$= \frac{\left(8 - \frac{8}{7}\right)}{\left(8 + \frac{8}{7}\right)} = \frac{\left(\frac{48}{7}\right)}{\left(\frac{64}{7}\right)} = \frac{48}{64} = \frac{3}{4}$$

Hence, $\left(\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}\right) = \frac{3}{4}$

*********** END **********