Functions Ex 2.5 Q 1.

i) $f : \{1, 2, 3, 4\} \to \{10\}$ given by

$f\{(1, 10), (2, 10), (3, 10), (4, 10)\}$

clearly f is many-one function

$\Rightarrow$ f is not bijective

$\Rightarrow$ f is not invertible

ii) $g : \{5, 6, 7, 8\} \to \{1, 2, 3, 4\}$ given by

$g\{(5, 4), (6, 3), (7, 4), (8, 2)\}$

Since, 5 and 7 have same image 4

∴ g is not bijectible

$\Rightarrow$ g is not bijective

$\Rightarrow$ g is not invertible

iii) $h : \{2, 3, 4, 5\} \to \{7, 9, 11, 13\}$ given by

$h\{(2, 7), (3, 9), (4, 11), (5, 13)\}$

We can observe that different element of domain have defferent image is co-domain.

Functions Ex 2.5 Q2

$A = \{0, -1, -3, 2\}, \quad B = \{-9, -3, 0, 6\}$

$f : A \to B$ is defined by $f(x) = 3x$

Since different elements of A have different images in B.

∴ f is one-one

Again, each element in B has a preimage in A.

∴ f is onto

∵ f in one-one bijective

$\Rightarrow$ $f^{-1} : B \to A$ exists and is given by

$f^{-1}(x) = \dfrac{x}{3}$

$A = \{1, 3, 5, 7, 9\}, \quad B = \{0, 1, 9, 25, 49, 81\}$

$f : A \to B$ be a function defined by $f(x) = x^2$

Since different elements of A have different images in B.

∴ f is one-one

Again, $0 \in B$ does not have a preimage in A.

∴ f is not onto

Hence, f^{-1} does not exist.

Functions Ex 2.5 Q3

Given that $f:\{1,2,3\}\to\{a,b,c\}$ and $g:\{a,b,c\}\to\{apple, ball, cat\}$ such that

$f(1)=a, f(2)=b, f(3)=c, g(a)=apple, g(b)=ball$ and $g(c)=cat$

We need to prove that f, g and $g\circ f$ are invertible.

In order to prove that f is invertiblem is is sufficient to show that

$f:\{1,2,3\}\to\{a,b,c\}$ is a bijection.

f is one $-$ one:

Each and every element of the set $\{1,2,3\}$ is having an image in the set $\{a,b,c\}$

Thus, f is one $-$ one.

Obviously, the number of element of the sets $\{1,2,3\}$ and $\{a,b,c\}$ are equal and hence

f is onto.

Thus, the function f is invertible.

Similarly, let us observe for the function g:

g is one $-$ one:

Each and every element of the set $\{a,b,c\}$ is having an image in the set $\{apple, ball, cat\}$

Thus, g is one $-$ one.

Obviously, the number of element of the sets $\{a,b,c\}$ and $\{apple, ball, cat\}$ are equal and hence

g is onto.

Thus, the function g is invertible.

Now let us consider the function $g\circ f=g[f(x)]$

Each and every element of of the set $\{1,2,3\}$ is having an image in the set

$\{apple, ball, cat\}$.

Therefore, $g\circ f=\{(1,apple),(2,ball),(3,cat)\}$

Thus, $g\circ f$ is one $-$ one.

Since the number of elemenets in the sets $\{1,2,3\}$ and $\{apple, ball, cat\}$ are equal.

Hence $g\circ f$ is onto.

Therefore, function $g\circ f$ is invertible.

Let us now find f^{-1}:

We have $f:\{1,2,3\}\to\{a,b,c\}$

Thus, $f^{-1}:\{a,b,c\}\to\{1,2,3\}$

$\Rightarrow f^{-1}=\{(a,1),(b,2),(c,3)\}$

Let us now find g^{-1}:

We have $g:\{a,b,c\}\to\{apple,ball,cat\}$

Thus, $g^{-1}:\{apple,ball,cat\}\to\{a,b,c\}$

$\Rightarrow g^{-1}=\{(apple,a),(ball,b),(cat,c)\}$

Let us now find $f^{-1}\circ g^{-1}$:

$\Rightarrow f^{-1}\circ g^{-1}=\{(apple,1),(ball,2),(cat,3)\}.....(1)$

Also, let us find, $(g\circ f)^{-1}$:

$\Rightarrow (g\circ f)^{-1}=\{(apple,1),(ball,2),(cat,3)\}....(2)$

From (1) and (2), we have,

$(g\circ f)^{-1}=f^{-1}\circ g^{-1}$

********** END *********