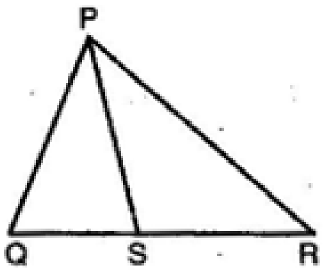
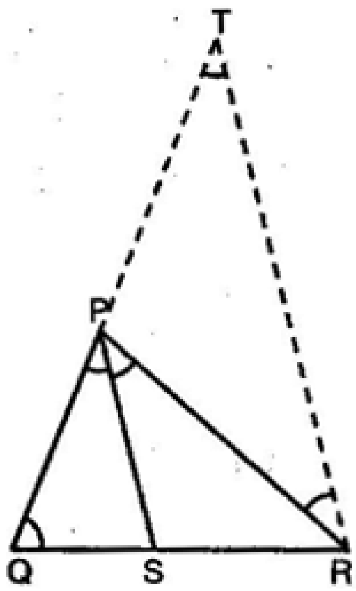
1. In figure, PS is the bisector of $\angle$ QPR of Δ PQR. Prove that $\frac{QS}{SR} = \frac{PQ}{PR}$.



Ans. Given: PQR is a triangle and PS is the internal bisector of $\angle$ QPR

meeting QR at S.



$\therefore \angle QPS = \angle SPR$

To prove: $\frac{QS}{SR} = \frac{PQ}{PR}$

Construction: Draw RT $\parallel$ SP to cut QP produced at T.

Proof: Since PS $\parallel$ TR and PR cuts them, hence,

$\angle$ SPR = $\angle$ PRT ....(i) [Alternate $\angle$ s]

And $\angle$ QPS = $\angle$ PTR ....(ii)[Corresponding

$\angle$ s]

But $\angle$ QPS = $\angle$ SPR [Given]

$\therefore$ $\angle$ PRT = $\angle$ PTR[From eq. (i) & (ii)]

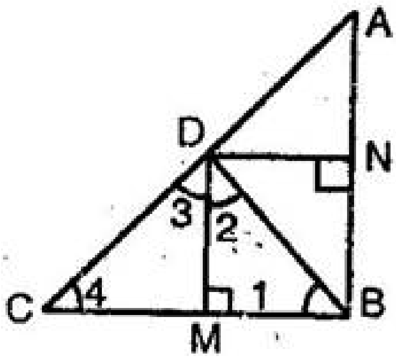
$\Rightarrow$ PT = PR..........(iii)

[Sides opposite to equal angles are equal]

Now, in ΔQRT,

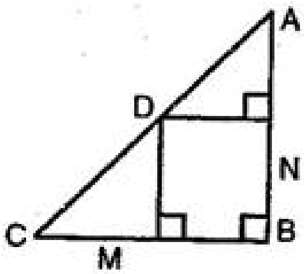
RT $\parallel$ SP[By construction]

$\therefore \frac{QS}{SR} = \frac{PQ}{PT}$ [Thales theorem]



$\Rightarrow \frac{QS}{SR} = \frac{PQ}{PR}$ [From eq. (iii)]

2. In figure, D is a point on hypotenuse AC of Δ ABC, BD $\perp$ AC, DM $\perp$ BC and DN $\perp$ AB. Prove that:



(i) $DM^2 = DN.MC$

(ii) $DN^2 = DM.AN$

Ans. Since, AB $\perp$ BC and DM $\perp$ BC

$\Rightarrow$ AB $\parallel$ DM

Similarly, BC $\perp$ AB and DN $\perp$ AB

$\Rightarrow$ CB $\parallel$ DN

$\therefore$ quadrilateral BMDN is a rectangle.

$\therefore$ BM = ND

(i) In ΔBMD, $\angle 1 + \angle BMD + \angle 2 = 180^\circ$

$\Rightarrow \angle 1 + 90^\circ + \angle 2 = 180^\circ$

$\Rightarrow \angle 1 + \angle 2 = 90^\circ$

Similarly, in ΔDMC, $\angle 3 + \angle 4 = 90^\circ$

Since BD $\perp$ AC,

$\therefore \angle 2 + \angle 3 = 90^\circ$

Now, $\angle 1 + \angle 2 = 90^\circ$ and $\angle 2 + \angle 3 = 90^\circ$

$\Rightarrow \angle 1 + \angle 2 = \angle 2 + \angle 3$

$\Rightarrow \angle 1 = \angle 3$

Also, $\angle 3 + \angle 4 = 90^\circ$ and $\angle 2 + \angle 3 = 90^\circ$

$\Rightarrow \angle 3 + \angle 4 = \angle 2 + \angle 3$

$\Rightarrow \angle 4 = \angle 2$

Thus, in ΔBMD and ΔDMC,

$\angle 1 = \angle 3$ and $\angle 4 = \angle 2$

$\therefore \Delta BMD \sim \Delta DMC$

$\Rightarrow \frac{BM}{DM} = \frac{MD}{MC}$

$\Rightarrow \frac{DN}{DM} = \frac{DM}{MC}$ [BM = ND]

$\Rightarrow DM^2 = DN.MC$

(ii) Processing as in (i), we can prove that

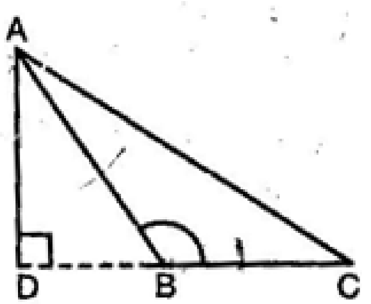
$\Delta BND \sim \Delta DNA$

$\Rightarrow \frac{BN}{DN} = \frac{ND}{NA}$

$\Rightarrow \frac{DM}{DN} = \frac{DN}{AN}$ [BN = DM]

$\Rightarrow DN^2 = DM.AN$

3. In figure, ABC is a triangle in which $\angle ABC > 90^\circ$ and AD$\perp$ CB produced. Prove that:



$AC^2 = AB^2 + BC^2 + 2BC.BD$

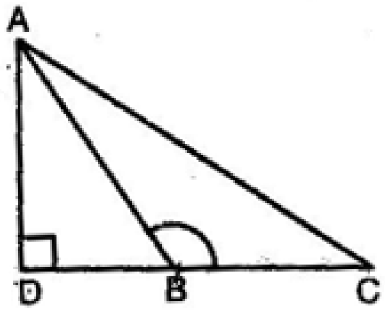
Ans. Given: ABC is a triangle in which $\angle ABC > 90^\circ$ and AD$\perp$ CB produced.

To prove: $AC^2 = AB^2 + BC^2 + 2BC$ .BD

Proof: Since Δ ADB is a right triangle, right angled at D, therefore, by Pythagoras theorem,

$AB^2 = AD^2 + DB^2$ ....(i)

Again, Δ ADB is a right triangle, right angled at D, therefore, by Pythagoras theorem,



$AC^2 = AD^2 + DC^2$

$\Rightarrow AC^2 = AD^2 + (DB + BC)^2$

$\Rightarrow AC^2 = AD^2 + DB^2 + BC^2 + 2DB$ .BC

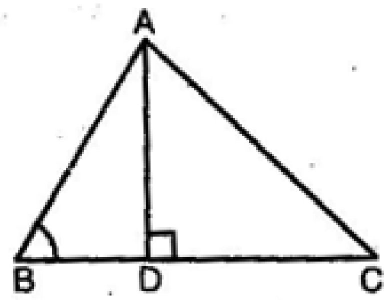
$\Rightarrow AC^2 = (AD^2 + DB^2) + BC^2 + 2DB$ .BC

$\Rightarrow AC^2 = AB^2 + BC^2 + 2DB$ .BC

[Using eq. (i)]

4. In figure, ABC is a triangle in which $\angle$ABC < 90° and AD$\perp$ BC produced. Prove that:

$AC^2 = AB^2 + BC^2 - 2BC$ .BD



Ans. Given: ABC is a triangle in which $\angle$ABC < 90° and AD$\perp$ BC produced.

To prove: $AC^2 = AB^2 + BC^2 - 2BC$ .BD

Proof: Since Δ ADB is a right triangle, right angled at D, therefore, by Pythagoras theorem,

$AB^2 = AD^2 + BD^2$ ....(i)

..........(i)

Again, Δ ADB is a right triangle, right angled at D, therefore, by Pythagoras theorem,

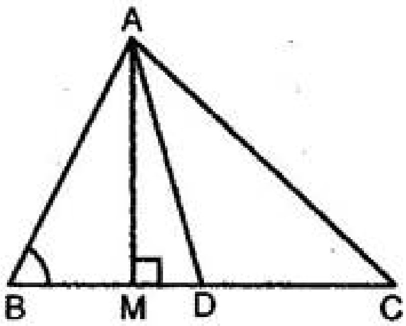
$AC^2 = AD^2 + DC^2$

$\Rightarrow AC^2 = AD^2 + (BC - BD)^2$

$\Rightarrow AC^2 = AD^2 + BC^2 + BD^2 - 2BC.BD$

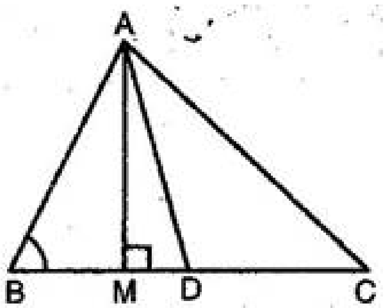
$\Rightarrow AC^2 = (AD^2 + DB^2) + BC^2 - 2DB.BC$

$\Rightarrow AC^2 = AB^2 + BC^2 - 2DB.BC$



[Using eq. (i)]

5. In figure, AD is a median of a triangle ABC and AM$\perp$ BC. Prove that:



(i) $\mathbf{AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2}$

(ii) $\mathbf{AB^2 = AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2}$

(iii) $\mathbf{AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC}$

Ans. Since $\angle$AMD = 90°, therefore $\angle$ADM < 90° and $\angle$ADC > 90°

Thus, $\angle$ ADC is acute angle and $\angle$ ADC is obtuse angle.

*********** END **********