Complex Numbers Ex 13.4 Q1(iii)

$$\text{Modulus, } |1-i| = \sqrt{1^2+1^2} = \sqrt{2}$$

$$\text{Argument, } \arg(1-i) = \tan^{-1}\left(\frac{-1}{1}\right) = \tan^{-1}(-1) = -\frac{\pi}{4}$$

$$\text{Polar form, } \sqrt{2}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)$$

Complex Numbers Ex 13.4 Q1(iv)

$$\frac{1-i}{1+i} = \frac{(1-i)(1-i)}{(1+i)(1-i)} = \frac{(1-i)^2}{1^2-i^2} = \frac{1-2i-1}{1+1} = \frac{-2i}{2} = -i$$

$$\text{Modulus, } \left|\frac{1-i}{1+i}\right| = |-i| = 1$$

$$\text{Argument, } \tan^{-1}\left(\frac{-1}{0}\right) = -\frac{\pi}{2}$$

$$\text{Polar Form, } z = r(\cos\theta + i\sin\theta)$$

$$z = \left(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}\right)$$

Complex Numbers Ex 13.4 Q1(v)

$$\text{Modulus, } \left|\frac{1}{1+i}\right|$$

$$= \left|\frac{1(1-i)}{(1+i)(1-i)}\right| [\text{Rationalizing the denominator}]$$

$$= \left|\frac{1-i}{1^2-i^2}\right| = \left|\frac{1-i}{2}\right| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

$$\text{Argument, } \tan^{-1}(-1) = -\frac{\pi}{4}$$

$$\text{Polar Form} = \cos\left(\frac{\pi}{4}\right) - i\sin\left(\frac{\pi}{4}\right)$$

Complex Numbers Ex 13.4 Q1(vi)

The polar form of a complex number $z = x + iy$, is given by $z = |z|(\cos\theta + i\sin\theta)$

where,

$|z| = \sqrt{x^2 + y^2}$ and

$\arg(z) = \theta = \tan^{-1}\left(\frac{b}{a}\right)$

$$\text{let } z = \frac{1+2i}{1-3i}$$
$$= \frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i}$$
$$= \frac{1(1+3i) + 2i(1+3i)}{1^2 + 3^2}$$
$$= \frac{1+3i+2i-6}{1+9}$$
$$= \frac{-5+5i}{10}$$
$$= \frac{-5}{10} + \frac{5}{10}i$$
$$= \frac{-1}{2} + \frac{1}{2}i$$

$$\therefore |z| = \sqrt{\left(\frac{-1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$
$$= \sqrt{\frac{1}{4} + \frac{1}{4}}$$
$$= \sqrt{\frac{2}{4}}$$
$$= \frac{1}{\sqrt{2}}$$

Here $x = \frac{-1}{2} < 0$ & $y = \frac{1}{2} > 0, \therefore \theta$ lies in quadrant II

$$\theta = \arg(z) = \tan^{-1}\frac{1/2}{-1/2}$$
$$= \tan^{-1}(-1)$$
$$= \tan^{-1}\left(-\tan\frac{\pi}{4}\right)$$
$$= \tan^{-1}\left(\tan\left(\pi - \frac{\pi}{4}\right)\right) \qquad (\because \tan(\pi - \theta) = -\tan\theta)$$
$$= \pi - \frac{\pi}{4}$$
$$= \frac{3\pi}{4}$$

The polar form is given by $z = \frac{1}{\sqrt{2}}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

*********** END **********