



Some Applications of Trigonometry Ex 12.1 Q52

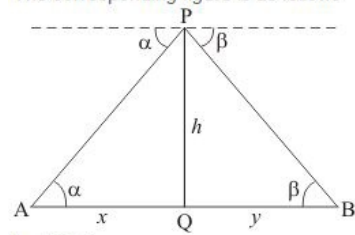
Answer :

Let h be the height of aero plane P above the road. And A and B be the two consecutive milestone, then $AB = 1$ mile. We have $\angle PAQ = \alpha$ and $\angle PBQ = \beta$.

We have to prove that

$$h = \frac{\tan \alpha \tan \beta}{\tan \alpha + \tan \beta}$$

The corresponding figure is as follows



In $\triangle PAQ$

$$\Rightarrow \tan \alpha = \frac{PQ}{AQ}$$

$$\Rightarrow \tan \alpha = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\tan \alpha}$$

$$\Rightarrow x = h \cot \alpha$$

Again in ΔPBQ

$$\Rightarrow \tan \beta = \frac{PQ}{BQ}$$

$$\Rightarrow \tan \beta = \frac{h}{y}$$

$$\Rightarrow y = \frac{h}{\tan \beta}$$

$$\Rightarrow y = h \cot \beta$$

Now,

$$\Rightarrow AB = x + y$$

$$\Rightarrow AB = h(\cot \alpha + \cot \beta)$$

$$\Rightarrow AB = h \left(\frac{1}{\tan \alpha} + \frac{1}{\tan \beta} \right)$$

$$\Rightarrow AB = h \left(\frac{\tan \alpha + \tan \beta}{\tan \alpha \tan \beta} \right)$$

$$\text{Therefore } h = \frac{\tan \alpha \tan \beta}{\tan \alpha + \tan \beta} \text{ (since } AB = 1)$$

Hence height of aero plane is

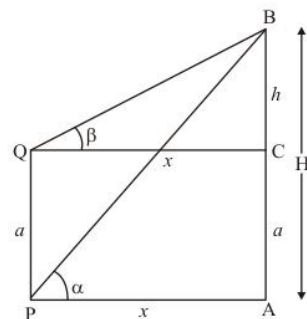
$$\boxed{\frac{\tan \alpha \tan \beta}{\tan \alpha + \tan \beta}}$$

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Answer :

Let AB be the tower of height H and PQ is a given post of height a , α and β are angles of elevation of top of tower AB from P and Q . Let $PA = x$, $PQ = a$ and $BC = h$.

The corresponding figure is as follows



In ΔQCB ,

$$\Rightarrow \tan \beta = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\tan \beta}$$

Again in $\triangle PAB$,

$$\begin{aligned}\Rightarrow \tan \alpha &= \frac{h+a}{x} \\ \Rightarrow \tan \alpha &= \frac{(h+a) \tan \beta}{h} \\ \Rightarrow h \tan \alpha &= (h+a) \tan \beta \\ \Rightarrow h(\tan \alpha - \tan \beta) &= a \tan \beta \\ \Rightarrow h &= \frac{a \tan \beta}{\tan \alpha - \tan \beta}\end{aligned}$$

Now

$$\begin{aligned}\Rightarrow x &= \frac{a \tan \beta}{(\tan \alpha - \tan \beta) \times \tan \beta} \\ \Rightarrow x &= \frac{a}{\tan \alpha - \tan \beta} \\ \Rightarrow H &= a + \frac{a \tan \beta}{\tan \alpha - \tan \beta} \\ \Rightarrow H &= \frac{a \tan \alpha}{\tan \alpha - \tan \beta}\end{aligned}$$

Hence required height is $\boxed{\frac{a \tan \alpha}{\tan \alpha - \tan \beta}}$. And distance is $\boxed{\frac{a}{\tan \alpha - \tan \beta}}$

Some Applications of Trigonometry Ex 12.1 Q54

Answer :

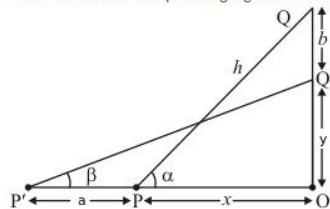
Let PQ be the ladder such that its top Q is on the wall OQ and bottom P is on the ground. The ladder is pulled away from the wall through a distance a , so that its top Q slides and takes position Q' . So $PQ = P'Q'$

$\angle OPQ = \alpha$ And $\angle OP'Q' = \beta$. Let $PQ = h$

We have to prove that

$$\frac{a}{b} = \frac{\cos \alpha - \cos \beta}{\sin \beta - \sin \alpha}$$

We have the corresponding figure as follows



We use trigonometric ratios.

In $\triangle POQ$

$$\Rightarrow \sin \alpha = \frac{OQ}{PQ}$$

$$\Rightarrow \sin \alpha = \frac{b+y}{h}$$

And

$$\Rightarrow \cos \alpha = \frac{OP}{PQ}$$

$$\Rightarrow \cos \alpha = \frac{x}{h}$$

Again in $\Delta P'OQ'$

$$\Rightarrow \sin \beta = \frac{OQ'}{P'Q'}$$

$$\Rightarrow \sin \beta = \frac{y}{h}$$

And

$$\Rightarrow \cos \beta = \frac{OP'}{P'Q'}$$

$$\Rightarrow \cos \beta = \frac{a+x}{h}$$

Now,

$$\Rightarrow \sin \alpha - \sin \beta = \frac{b+y}{h} - \frac{y}{h}$$

$$\Rightarrow \sin \alpha - \sin \beta = \frac{b}{h}$$

And

$$\Rightarrow \cos \beta - \cos \alpha = \frac{a+x}{h} - \frac{x}{h}$$

$$\Rightarrow \cos \beta - \cos \alpha = \frac{a}{h}$$

So

$$\Rightarrow \frac{\sin \alpha - \sin \beta}{\cos \beta - \cos \alpha} = \frac{b}{a}$$

$$\Rightarrow \frac{a}{b} = \frac{\cos \beta - \cos \alpha}{\sin \alpha - \sin \beta}$$

$$\Rightarrow \frac{a}{b} = \frac{\cos \alpha - \cos \beta}{\sin \beta - \sin \alpha}$$

Hence $\boxed{\frac{a}{b} = \frac{\cos \alpha - \cos \beta}{\sin \beta - \sin \alpha}}$.

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