



Differentiation Ex 11.8 Q1

Let $u = x^2$, $v = x^3$

Differentiating u with respect to x ,

$$\frac{du}{dx} = 2x \quad \text{---(i)}$$

Differentiating v with respect to x ,

$$\frac{dv}{dx} = 3x^2 \quad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2x}{3x^2}$$

$$\frac{du}{dv} = \frac{2}{3x}$$

Differentiation Ex 11.8 Q2

Let $u = \log(1+x^2)$

Differentiating it with respect to x using chain rule,

$$\begin{aligned}\frac{du}{dx} &= \frac{1}{(1+x^2)} \frac{d}{dx}(1+x^2) \\ &= \frac{1}{(1+x^2)} (2x) \\ \frac{du}{dx} &= \frac{2x}{(1+x^2)} \quad \text{--- (i)}\end{aligned}$$

Let $v = \tan^{-1} x$

Differentiating it with respect to x ,

$$\frac{dv}{dx} = \frac{1}{1+x^2} \quad \text{--- (ii)}$$

Dividing equation (i) by (ii),

$$\begin{aligned}\frac{\frac{du}{dx}}{\frac{dv}{dx}} &= \frac{2x}{(1+x^2)} \times \frac{(1+x^2)}{1} \\ \frac{du}{dv} &= 2x\end{aligned}$$

Let $u = (\log x)^x$

Taking log on both the sides,

$$\begin{aligned}\log u &= \log(\log x)^x \\ \log u &= x \log(\log x) \quad \quad \quad \left[\text{Since, } \log a^b = b \log a \right]\end{aligned}$$

Differentiating it with respect to x using chain rule, product rule,

$$\begin{aligned}\frac{1}{u} \frac{du}{dx} &= x \frac{d}{dx} \log(\log x) + \log(\log x) \frac{d}{dx}(x) \\ \frac{1}{u} \frac{du}{dx} &= x \left(\frac{1}{\log x} \right) \frac{d}{dx}(\log x) + \log \log x (1) \\ \frac{du}{dx} &= u \left[\frac{x}{\log x} \left(\frac{1}{x} \right) + \log \log x \right] \\ \frac{du}{dx} &= (\log x)^x \left[\frac{1}{\log x} + \log \log x \right] \quad \quad \quad \text{---(i)}\end{aligned}$$

Again, let $v = \log x$

Differentiating it with respect to x ,

$$\frac{dv}{dx} = \frac{1}{x} \quad \quad \quad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\begin{aligned}\frac{\frac{du}{dx}}{\frac{dv}{dx}} &= \frac{(\log x)^x \left[\frac{1}{\log x} + \log \log x \right]}{\frac{1}{x}} \\ \frac{du}{dv} &= \frac{(\log x)^x \left[\frac{1 + \log x \times \log \log x}{\log x} \right]}{\frac{1}{x}} \\ \frac{du}{dv} &= (\log x)^{x-1} (1 + \log x \times \log \log x) \times x\end{aligned}$$

Differentiation Ex 11.8 Q4(i)

Let $u = \sin^{-1} \sqrt{1-x^2}$

Put $x = \cos \theta$, so,

$$u = \sin^{-1} \sqrt{1 - \cos^2 \theta}$$

$$u = \sin^{-1} (\sin \theta) \quad \text{---(i)}$$

And, $v = \cos^{-1} x \quad \text{---(ii)}$

Now, $x \in (0, 1)$

$$\Rightarrow \cos \theta \in (0, 1)$$

$$\Rightarrow \theta \in \left(0, \frac{\pi}{2}\right)$$

So, from equation (i),

$$\begin{aligned} u &= \theta && \left[\text{Since, } \sin^{-1}(\sin \theta) = \theta \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \right] \\ u &= \cos^{-1} x && [\text{Since, } \cos \theta = x] \end{aligned}$$

Differentiating it with respect to x ,

$$\frac{du}{dx} = \frac{-1}{\sqrt{1-x^2}} \quad \text{---(ii)}$$

From equation (ii),

$$v = \cos^{-1} x$$

Differentiating it with respect to x ,

$$\frac{dv}{dx} = \frac{-1}{\sqrt{1-x^2}} \quad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\begin{aligned} \frac{\frac{du}{dx}}{\frac{dv}{dx}} &= \frac{-1}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{-1} \\ \frac{du}{dv} &= 1 \end{aligned}$$

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