Exercise 9.3 : Solutions of Questions on Page Number : **391**

Q1 : $\frac{x}{a}+\frac{y}{b}=1$

Answer :

$\frac{x}{a}+\frac{y}{b}=1$

Differentiating both sides of the given equation with respect to *x*, we get:

$$\frac{1}{a}+\frac{1}{b}\frac{dy}{dx}=0$$
$$\Rightarrow \frac{1}{a}+\frac{1}{b}y'=0$$

Again, differentiating both sides with respect to *x*, we get:

$$0+\frac{1}{b}y''=0$$
$$\Rightarrow \frac{1}{b}y''=0$$
$$\Rightarrow y''=0$$

Hence, the required differential equation of the given curve is $y''=0$.

Answer needs Correction? **Click Here**

Q2 : $y^2=a\left(b^2-x^2\right)$

Answer :

$y^2=a\left(b^2-x^2\right)$

Differentiating both sides with respect to *x*, we get:

$$2y\frac{dy}{dx}=a(-2x)$$
$$\Rightarrow 2yy'=-2ax$$
$$\Rightarrow yy'=-ax \qquad ...(1)$$

Again, differentiating both sides with respect to *x*, we get:

$$y'\cdot y'+yy''=-a$$
$$\Rightarrow (y')^2+yy''=-a \qquad ...(2)$$

Dividing equation (2) by equation (1), we get:

$$\frac{(y')^2+yy''}{yy'}=\frac{-a}{-ax}$$
$$\Rightarrow xyy''+x(y')^2-yy''=0$$

This is the required differential equation of the given curve.

Answer needs Correction? **Click Here**

Q3 : $y=ae^{3x}+be^{-2x}$

Answer :

$$y=ae^{3x}+be^{-2x} \qquad ...(1)$$

Differentiating both sides with respect to *x*, we get:

$$y'=3ae^{3x}-2be^{-2x} \qquad ...(2)$$

Again, differentiating both sides with respect to *x*, we get:

$$y''=9ae^{3x}+4be^{-2x} \qquad ...(3)$$

Multiplying equation (1) with (2) and then adding it to equation (2), we get:

$$\left(2ae^{3x}+2be^{-2x}\right)+\left(3ae^{3x}-2bc^{-2x}\right)=2y+y'$$
$$\Rightarrow 5ae^{3x}=2y+y'$$
$$\Rightarrow ae^{3x}=\frac{2y+y'}{5}$$

Now, multiplying equation (1) with 3 and subtracting equation (2) from it, we get:

$$\left(3ae^{3x}+3be^{-2x}\right)-\left(3ae^{3x}-2be^{-2x}\right)=3y-y'$$
$$\Rightarrow 5be^{-2x}=3y-y'$$
$$\Rightarrow be^{-2x}=\frac{3y-y'}{5}$$

Substituting the values of ae^{3x} and be^{-2x} in equation (3), we get:

$$y'' = 9 \cdot \frac{(2y+y')}{5} + 4\frac{(3y-y')}{5}$$
$$\Rightarrow y'' = \frac{18y+9y'}{5} + \frac{12y-4y'}{5}$$
$$\Rightarrow y'' = \frac{30y+5y'}{5}$$
$$\Rightarrow y'' = 6y+y'$$
$$\Rightarrow y''-y'-6y=0$$

This is the required differential equation of the given curve.

Answer needs Correction? **Click Here**

Q4 : $y = e^{2x}(a+bx)$

Answer :

$$y = e^{2x}(a+bx) \qquad ...(1)$$

Differentiating both sides with respect to *x*, we get:

$$y' = 2e^{2x}(a+bx) + e^{2x} \cdot b$$
$$\Rightarrow y' = e^{2x}(2a+2bx+b) \qquad ...(2)$$

Multiplying equation (1) with equation (2) and then subtracting it from equation (2), we get:

$$y'-2y = e^{2x}(2a+2bx+b) - e^{2x}(2a+2bx)$$
$$\Rightarrow y'-2 = be^{2x} \qquad ...(3)$$

Differentiating both sides with respect to *x*, we get:

$$y''k - 2y' = 2be^{2x} \qquad ...(4)$$

Dividing equation (4) by equation (3), we get:

$$\frac{y''-2y'}{y'-2y} = 2$$
$$\Rightarrow y''-2y' = 2y'-4y$$
$$\Rightarrow y''-4y'+4y=0$$

This is the required differential equation of the given curve.

Answer needs Correction? **Click Here**

Q5 : $y = e^{x}(a\cos x + b\sin x)$

Answer :

$$y = e^{x}(a\cos x + b\sin x) \qquad ...(1)$$

Differentiating both sides with respect to *x*, we get:

$$y' = e^{x}(a\cos x + b\sin x) + e^{x}(-a\sin x + b\cos x)$$
$$\Rightarrow y' = e^{x}\left[(a+b)\cos x - (a-b)\sin x\right] \qquad ...(2)$$

Again, differentiating with respect to *x*, we get:

$$y'' = e^{x}\left[(a+b)\cos x - (a-b)\sin x\right] + e^{x}\left[-(a+b)\sin x - (a-b)\cos x\right]$$
$$y'' = e^{x}\left[2b\cos x - 2a\sin x\right]$$
$$y'' = 2e^{x}(b\cos x - a\sin x)$$
$$\Rightarrow \frac{y''}{2} = e^{x}(b\cos x - a\sin x) \qquad ...(3)$$

Adding equations (1) and (3), we get:

$$y + \frac{y''}{2} = e^{x}\left[(a+b)\cos x - (a-b)\sin x\right]$$
$$\Rightarrow y + \frac{y''}{2} = y'$$
$$\Rightarrow 2y + y'' = 2y'$$
$$\Rightarrow y'' - 2y' + 2y = 0$$

This is the required differential equation of the given curve.

Answer needs Correction? **Click Here**

Q6 : Form the differential equation of the family of circles touching the *y*-axis at the origin.

Answer :

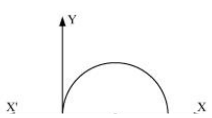
The centre of the circle touching the *y*-axis at origin lies on the *x*-axis.

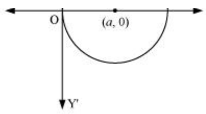
Let (*a*, 0) be the centre of the circle.

Since it touches the *y*-axis at origin, its radius is *a*.

Now, the equation of the circle with centre (*a*, 0) and radius (*a*) is

$$(x-a)^2 + y^2 = a^2.$$
$$\Rightarrow x^2 + y^2 = 2ax \qquad ...(1)$$





Differentiating equation (1) with respect to *x*, we get:

$$2x+2yy'=2a$$
$$\Rightarrow x+yy'=a$$

Now, on substituting the value of *a* in equation (1), we get:

$$x^2+y^2=2(x+yy')x$$
$$\Rightarrow x^2+y^2=2x^2+2xyy'$$
$$\Rightarrow 2xyy'+x^2=y^2$$

This is the required differential equation.

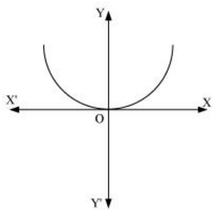
Answer needs Correction? **Click Here**

Q7 : Form the differential equation of the family of parabolas having vertex at origin and axis along positive *y*-axis.

Answer :

The equation of the parabola having the vertex at origin and the axis along the positive *y*-axis is:

$$x^2=4ay \qquad ...(1)$$



Differentiating equation (1) with respect to *x*, we get:

$$2x=4ay' \qquad ...(2)$$

Dividing equation (2) by equation (1), we get:

$$\frac{2x}{x^2}=\frac{4ay'}{4ay}$$
$$\Rightarrow \frac{2}{x}=\frac{y'}{y}$$
$$\Rightarrow xy'=2y$$
$$\Rightarrow xy'-2y=0$$

This is the required differential equation.

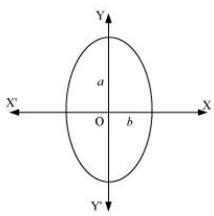
Answer needs Correction? **Click Here**

Q8 : Form the differential equation of the family of ellipses having foci on *y*-axis and centre at origin.

Answer :

The equation of the family of ellipses having foci on the *y*-axis and the centre at origin is as follows:

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1 \qquad ..(1)$$



Differentiating equation (1) with respect to *x*, we get:

$$\frac{2x}{b^2}+\frac{2yy'}{b^2}=0$$
$$\Rightarrow \frac{x}{b^2}+\frac{yy'}{a^2}=0 \qquad ...(2)$$

Again, differentiating with respect to *x*, we get:

$$\frac{1}{b^2}+\frac{y'.y'+y.y''}{a^2}=0$$
$$\Rightarrow \frac{1}{b^2}+\frac{1}{a^2}\left(y'^2+yy''\right)=0$$
$$\Rightarrow \frac{1}{b^2}=-\frac{1}{a^2}\left(y'^2+yy''\right)$$

Substituting this value in equation (2), we get:

$$x\left[-\frac{1}{a^2}\left((y')^2+yy''\right)\right]+\frac{yy'}{a^2}=0$$
$$\Rightarrow -x(y')^2-xyy''+yy'=0$$
$$\Rightarrow xyy''+x(y')^2-yy'=0$$

This is the required differential equation.

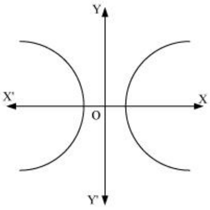
Answer needs Correction? **Click Here**

Q9 : Form the differential equation of the family of hyperbolas having foci on *x*-axis and centre at origin.

Answer :

The equation of the family of hyperbolas with the centre at origin and foci along the *x*-axis is:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad \text{...(1)}$$



Differentiating both sides of equation (1) with respect to *x*, we get:

$$\frac{2x}{a^2}-\frac{2yy'}{b^2}=0$$
$$\Rightarrow \frac{x}{a^2}-\frac{yy'}{b^2}=0 \qquad \text{...(2)}$$

Again, differentiating both sides with respect to *x*, we get:

$$\frac{1}{a^2}-\frac{y'\cdot y'+yy''}{b^2}=0$$
$$\Rightarrow \frac{1}{a^2}=\frac{1}{b^2}\left((y')^2+yy''\right)$$

Substituting the value of $\frac{1}{a^2}$ in equation (2), we get:

$$\frac{x}{b^2}\left((y')^2+yy''\right)-\frac{yy'}{b^2}=0$$
$$\Rightarrow x(y')^2+xyy''-yy'=0$$
$$\Rightarrow xyy''+x(y')^2-yy'=0$$

This is the required differential equation.

Answer needs Correction? **Click Here**

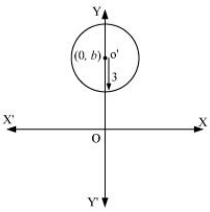
Q10 : Form the differential equation of the family of circles having centre on *y*-axis and radius 3 units.

Answer :

Let the centre of the circle on *y*-axis be (0, *b*).

The differential equation of the family of circles with centre at (0, *b*) and radius 3 is as follows:

$$x^2+(y-b)^2=3^2$$
$$\Rightarrow x^2+(y-b)^2=9 \qquad \text{...(1)}$$



Differentiating equation (1) with respect to *x*, we get:

$$2x+2(y-b)\cdot y'=0$$
$$\Rightarrow (y-b)\cdot y'=-x$$
$$\Rightarrow y-b=\frac{-x}{y'}$$

Substituting the value of (*y* â€“ *b*) in equation (1), we get:

$$x^2+\left(\frac{-x}{y'}\right)^2=9$$
$$\Rightarrow x^2\left[1+\frac{1}{(y')^2}\right]=9$$
$$\Rightarrow x^2\left((y')^2+1\right)=9(y')^2$$
$$\Rightarrow (x^2-9)(y')^2+x^2=0$$

This is the required differential equation.

Answer needs Correction? **Click Here**

Q11 : Which of the following differential equations has $y=c_1e^x+c_2e^{-x}$ as the general solution?

A. $\frac{d^2y}{dx^2}+y=0$

B. $\frac{d^2y}{dx^2}-y=0$

C. $\frac{d^2y}{dx^2}+1=0$

D. $\frac{d^2y}{dx^2}-1=0$

Answer :

The given equation is:

$y=c_1e^x+c_2e^{-x}$ ...(1)

Differentiating with respect to *x*, we get:

$\frac{dy}{dx}=c_1e^x-c_2e^{-x}$

Again, differentiating with respect to *x*, we get:

$$\frac{d^2y}{dx^2}=c_1e^x+c_2e^{-x}$$
$$\Rightarrow \frac{d^2y}{dx^2}=y$$
$$\Rightarrow \frac{d^2y}{dx^2}-y=0$$

This is the required differential equation of the given equation of curve.

Hence, the correct answer is B.

Answer needs Correction? **Click Here**

Q12 : Which of the following differential equation has $y=x$ as one of its particular solution?

A. $\frac{d^2y}{dx^2}-x^2\frac{dy}{dx}+xy=x$

B. $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+xy=x$

C. $\frac{d^2y}{dx^2}-x^2\frac{dy}{dx}+xy=0$

D. $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+xy=0$

Answer :

The given equation of curve is *y* = *x*.

Differentiating with respect to *x*, we get:

$\frac{dy}{dx}=1$ ...(1)

Again, differentiating with respect to *x*, we get:

$\frac{d^2y}{dx^2}=0$ ...(2)

Now, on substituting the values of *y*, $\frac{d^2y}{dx^2}$, and $\frac{dy}{dx}$ from equation (1) and (2) in each of the given alternatives, we find that only the differential equation given in alternative **C** is correct.

$$\begin{aligned}\frac{d^2y}{dx^2}-x^2\frac{dy}{dx}+xy&=0-x^2\cdot 1+x\cdot x\\&=-x^2+x^2\\&=0\end{aligned}$$

Hence, the correct answer is C.

Answer needs Correction? **Click Here**

*********** END **********