Tangents and Normals Ex 16.2 Q3(xi)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad (A) \qquad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad (B) \qquad \text{Normal}$$

Where m is the slope

We have,

$$xy = c^2 \qquad P = \left(ct, \frac{c}{t}\right)$$

Differentiating with respect to x, we get

$$y + x\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{y}{x}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{\frac{-c}{t}}{ct} = \frac{-1}{t^2}$$

From (A)

Equation of tangent is

$$\left(y - \frac{c}{t}\right) = \frac{-1}{t^2}(x - ct)$$

$$\Rightarrow \quad x + t^2y = tc + ct$$

$$\Rightarrow \quad x + t^2y = 2ct$$

From (B)

Equation of normal is

$$\left(y - \frac{c}{t}\right) = t^2(x - ct)$$

$$\Rightarrow \quad xt^3 - ty = ct^3 \times t - c$$

$$\Rightarrow \quad xt^3 - ty = ct^4 - c$$

Tangents and Normals Ex 16.2 Q3(xii)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad \text{(A)} \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{(B)} \quad \text{Normal}$$

Where m is the slope

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{---(i)} \qquad P = (x_1, y_1)$$

Differentiating with resect to x, we get

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{xb^2}{ya^2}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = -\frac{x_1 b^2}{y_1 a^2}$$

From (A)

Equation of tangent is

$$(y - y_1) = -\frac{x_1 b^2}{y_1 a^2}(x - x_1)$$

$$\Rightarrow \quad xx_1 b^2 + yy_1 a^2 = x_1^2 b^2 + y_1^2 a^2$$

Divide by $a^2 b^2$ both side

$$\Rightarrow \quad \frac{xx_1}{a^2} + \frac{yy_1}{b^2} = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2}$$

$$= 1 \qquad [\because (x_1, y_1) \text{ lies on (i)}]$$

$$\therefore \quad \frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

From (B)

Equation of normal is

$$(y - y_1) = \frac{y_1 a^2}{x_1 b^2}(x - x_1)$$

$$xy_1 a^2 - yx_1 b^2 = x_1 y_1 a^2 - y_1 x_1 b^2$$

Dividing by $x_1 y_1$ both side

$$\frac{xa^2}{x_1} - \frac{yb^2}{y_1} = a^2 - b^2$$

Tangents and Normals Ex 16.2 Q3(xiii)

Differentiating $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ with respect to x, we have:

$$\frac{2x}{a^2}-\frac{2y}{b^2}\frac{dy}{dx}=0$$
$$\Rightarrow \frac{2y}{b^2}\frac{dy}{dx}=\frac{2x}{a^2}$$
$$\Rightarrow \frac{dy}{dx}=\frac{b^2x}{a^2y}$$

Therefore, the slope of the tangent at (x_0, y_0) is $\left.\frac{dy}{dx}\right]_{(x_0,\, y_0)}=\frac{b^2x_0}{a^2y_0}$.

Then, the equation of the tangent at (x_0, y_0) is given by,

$$y-y_0=\frac{b^2x_0}{a^2y_0}(x-x_0)$$
$$\Rightarrow a^2yy_0-a^2y_0^2=b^2xx_0-b^2x_0^2$$
$$\Rightarrow b^2xx_0-a^2yy_0-b^2x_0^2+a^2y_0^2=0$$

*********** END **********