Exercise 9.4 : Solutions of Questions on Page Number : **395**

Q1 : $\frac{dy}{dx}=\frac{1-\cos x}{1+\cos x}$

Answer :

The given differential equation is:

$$\frac{dy}{dx}=\frac{1-\cos x}{1+\cos x}$$

$$\Rightarrow \frac{dy}{dx}=\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}=\tan^2\frac{x}{2}$$

$$\Rightarrow \frac{dy}{dx}=\left(\sec^2\frac{x}{2}-1\right)$$

Separating the variables,we get:

$$dy=\left(\sec^2\frac{x}{2}-1\right)dx$$

Now, integrating both sides of this equation, we get:

$$\int dy=\int\left(\sec^2\frac{x}{2}-1\right)dx=\int\sec^2\frac{x}{2}dx-\int dx$$

$$\Rightarrow y=2\tan\frac{x}{2}-x+C$$

This is the required general solution of the given differential equation.

Answer needs Correction? **Click Here**

Q2 : $\frac{dy}{dx}=\sqrt{4-y^2}\ (-2<y<2)$

Answer :

The given differential equation is:

$$\frac{dy}{dx}=\sqrt{4-y^2}$$

Separating the variables, we get:

$$\Rightarrow \frac{dy}{\sqrt{4-y^2}}=dx$$

Now, integrating both sides of this equation, we get:

$$\int\frac{dy}{\sqrt{4-y^2}}=\int dx$$

$$\Rightarrow \sin^{-1}\frac{y}{2}=x+C$$

$$\Rightarrow \frac{y}{2}=\sin(x+C)$$

$$\Rightarrow y=2\sin(x+C)$$

This is the required general solution of the given differential equation.

Answer needs Correction? **Click Here**

Q3 : $\frac{dy}{dx}+y=1\ (y\neq 1)$

Answer :

The given differential equation is:

$$\frac{dy}{dx}+y=1$$

$$\Rightarrow dy+y\ dx=dx$$

$$\Rightarrow dy=(1-y)dx$$

Separating the variables, we get:

$$\Rightarrow \frac{dy}{1-y}=dx$$

Now, integrating both sides, we get:

$$\int\frac{dy}{1-y}=\int dx$$

$$\Rightarrow \log(1-y)=x+\log C$$

$$\Rightarrow -\log C-\log(1-y)=x$$

$$\Rightarrow \log C(1-y)=-x$$

$\Rightarrow C(1-y)=e^{-x}$

$\Rightarrow 1-y=\frac{1}{C}e^{-x}$

$\Rightarrow y=1-\frac{1}{C}e^{-x}$

$\Rightarrow y=1+Ae^{-x}$ (where $A=-\frac{1}{C}$)

This is the required general solution of the given differential equation.

Answer needs Correction? **Click Here**

Q4 : $\sec^2 x\tan y\,dx+\sec^2 y\tan x\,dy=0$

Answer :

The given differential equation is:

$$\sec^2 x\tan y\,dx+\sec^2 y\tan x\,dy=0$$
$$\Rightarrow \frac{\sec^2 x\tan y\,dx+\sec^2 y\tan x\,dy}{\tan x\tan y}=0$$
$$\Rightarrow \frac{\sec^2 x}{\tan x}dx+\frac{\sec^2 y}{\tan y}dy=0$$
$$\Rightarrow \frac{\sec^2 x}{\tan x}dx=-\frac{\sec^2 y}{\tan y}dy$$

Integrating both sides of this equation, we get:

$$\int\frac{\sec^2 x}{\tan x}dx=-\int\frac{\sec^2 y}{\tan y}dy \qquad ...(1)$$

Let $\tan x=t$.

$$\therefore \frac{d}{dx}(\tan x)=\frac{dt}{dx}$$
$$\Rightarrow \sec^2 x=\frac{dt}{dx}$$
$$\Rightarrow \sec^2 x\,dx=dt$$

Now, $\int\frac{\sec^2 x}{\tan x}dx=\int\frac{1}{t}dt.$

$$=\log t$$
$$=\log(\tan x)$$

Similarly, $\int\frac{\sec^2 x}{\tan x}dy=\log(\tan y).$

Substituting these values in equation (1), we get:

$$\log(\tan x)=-\log(\tan y)+\log C$$
$$\Rightarrow \log(\tan x)=\log\left(\frac{C}{\tan y}\right)$$
$$\Rightarrow \tan x=\frac{C}{\tan y}$$
$$\Rightarrow \tan x\tan y=C$$

This is the required general solution of the given differential equation.

Answer needs Correction? **Click Here**

Q5 : $(e^x+e^{-x})dy-(e^x-e^{-x})dx=0$

Answer :

The given differential equation is:

$$(e^x+e^{-x})dy-(e^x-e^{-x})dx=0$$
$$\Rightarrow (e^x+e^{-x})dy=(e^x-e^{-x})dx$$
$$\Rightarrow dy=\left[\frac{e^x-e^{-x}}{e^x+e^{-x}}\right]dx$$

Integrating both sides of this equation, we get:

$$\int dy=\int\left[\frac{e^x-e^{-x}}{e^x+e^{-x}}\right]dx+C$$
$$\Rightarrow y=\int\left[\frac{e^x-e^{-x}}{e^x+e^{-x}}\right]dx+C \qquad ...(1)$$

Let (e^x + eâ€"x) = *t.*

Differentiating both sides with respect to *x*, we get:

$$\frac{d}{dx}(e^x+e^{-x})=\frac{dt}{dx}$$
$$\Rightarrow e^x-e^{-x}=\frac{dt}{dt}$$
$$\Rightarrow (e^x-e^{-x})dx=dt$$

Substituting this value in equation (1), we get:

$$y=\int\frac{1}{t}dt+C$$
$$\Rightarrow y=\log(t)+C$$
$$\Rightarrow y=\log(e^x+e^{-x})+C$$

This is the required general solution of the given differential equation.

Answer needs Correction? **Click Here**

Answer needs Correction? Click Here

Q6 : $\frac{dy}{dx}=\left(1+x^2\right)\left(1+y^2\right)$

Answer :

The given differential equation is:

$$\frac{dy}{dx}=\left(1+x^2\right)\left(1+y^2\right)$$
$$\Rightarrow \frac{dy}{1+y^2}=\left(1+x^2\right)dx$$

Integrating both sides of this equation, we get:

$$\int\frac{dy}{1+y^2}=\int\left(1+x^2\right)dx$$
$$\Rightarrow \tan^{-1}y=\int dx+\int x^2dx$$
$$\Rightarrow \tan^{-1}y=x+\frac{x^3}{3}+C$$

This is the required general solution of the given differential equation.

Answer needs Correction? Click Here

Q7 : $y\log y\,dx-x\,dy=0$

Answer :

The given differential equation is:

$$y\log y\,dx-x\,dy=0$$
$$\Rightarrow y\log y\,dx=x\,dy$$
$$\Rightarrow \frac{dy}{y\log y}=\frac{dx}{x}$$

Integrating both sides, we get:

$$\int\frac{dy}{y\log y}=\int\frac{dx}{x} \qquad ...(1)$$

Let $\log y=t$.

$$\therefore \frac{d}{dy}(\log y)=\frac{dt}{dy}$$
$$\Rightarrow \frac{1}{y}=\frac{dt}{dy}$$
$$\Rightarrow \frac{1}{y}dy=dt$$

Substituting this value in equation (1), we get:

$$\int\frac{dt}{t}=\int\frac{dx}{x}$$
$$\Rightarrow \log t=\log x+\log C$$
$$\Rightarrow \log(\log y)=\log Cx$$
$$\Rightarrow \log y=Cx$$
$$\Rightarrow y=e^{Cx}$$

This is the required general solution of the given differential equation.

Answer needs Correction? Click Here

Q8 : $x^5\frac{dy}{dx}=-y^5$

Answer :

The given differential equation is:

$$x^5\frac{dy}{dx}=-y^5$$
$$\Rightarrow \frac{dy}{y^5}=-\frac{dx}{x^5}$$
$$\Rightarrow \frac{dx}{x^5}+\frac{dy}{y^5}=0$$

Integrating both sides, we get:

$$\int\frac{dx}{x^5}+\int\frac{dy}{y^5}=k \quad (\text{where } k \text{ is any constant})$$
$$\Rightarrow \int x^{-5}dx+\int y^{-5}dy=k$$
$$\Rightarrow \frac{x^{-4}}{-4}+\frac{y^{-4}}{-4}=k$$
$$\Rightarrow x^{-4}+y^{-4}=-4k$$
$$\Rightarrow x^{-4}+y^{-4}=C \qquad (C=-4k)$$

This is the required general solution of the given differential equation.

Answer needs Correction? Click Here

Q9 : $\frac{dy}{dx}=\sin^{-1}x$

Answer :

The given differential equation is:

$$\frac{dy}{dx} = \sin^{-1} x$$
$$\Rightarrow dy = \sin^{-1} x \, dx$$

Integrating both sides, we get:

$$\int dy = \int \sin^{-1} x \, dx$$
$$\Rightarrow y = \int \left(\sin^{-1} x \cdot 1\right) dx$$
$$\Rightarrow y = \sin^{-1} x \cdot \int (1)\, dx - \int \left[\left(\frac{d}{dx}\left(\sin^{-1} x\right) \cdot \int (1)\, dx\right)\right] dx$$
$$\Rightarrow y = \sin^{-1} x \cdot x - \int \left(\frac{1}{\sqrt{1-x^2}} \cdot x\right) dx$$
$$\Rightarrow y = x \sin^{-1} x + \int \frac{-x}{\sqrt{1-x^2}}\, dx \qquad ...(1)$$

Let $1 - x^2 = t$.

$$\Rightarrow \frac{d}{dx}\left(1-x^2\right) = \frac{dt}{dx}$$
$$\Rightarrow -2x = \frac{dt}{dx}$$
$$\Rightarrow x\,dx = -\frac{1}{2} dt$$

Substituting this value in equation (1), we get:

$$y = x \sin^{-1} x + \int \frac{1}{2\sqrt{t}}\, dt$$
$$\Rightarrow y = x \sin^{-1} x + \frac{1}{2} \cdot \int (t)^{-\frac{1}{2}}\, dt$$
$$\Rightarrow y = x \sin^{-1} x + \frac{1}{2} \cdot \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$$
$$\Rightarrow y = x \sin^{-1} x + \sqrt{t} + C$$
$$\Rightarrow y = x \sin^{-1} x + \sqrt{1-x^2} + C$$

This is the required general solution of the given differential equation.

Answer needs Correction? **Click Here**

Q10 : $e^x \tan y\, dx + \left(1-e^x\right)\sec^2 y\, dy = 0$

Answer :

The given differential equation is:

$$e^x \tan y\, dx + \left(1-e^x\right)\sec^2 y\, dy = 0$$
$$\left(1-e^x\right)\sec^2 y\, dy = -e^x \tan y\, dx$$

Separating the variables, we get:

$$\frac{\sec^2 y}{\tan y} dy = \frac{-e^x}{1-e^x} dx$$

Integrating both sides, we get:

$$\int \frac{\sec^2 y}{\tan y} dy = \int \frac{-e^x}{1-e^x} dx \qquad ...(1)$$

Let $\tan y = u$.

$$\Rightarrow \frac{d}{dy}(\tan y) = \frac{du}{dy}$$
$$\Rightarrow \sec^2 y = \frac{du}{dy}$$
$$\Rightarrow \sec^2 y dy = du$$
$$\therefore \int \frac{\sec^2 y}{\tan y} dy = \int \frac{du}{u} = \log u = \log(\tan y)$$

Now, let $1 - e^x = t$.

$$\therefore \frac{d}{dx}\left(1-e^x\right) = \frac{dt}{dx}$$
$$\Rightarrow -e^x = \frac{dt}{dx}$$
$$\Rightarrow -e^x dx = dt$$
$$\Rightarrow \int \frac{-e^x}{1-e^x} dx = \int \frac{dt}{t} = \log t = \log\left(1-e^x\right)$$

Substituting the values of $\int \frac{\sec^2 y}{\tan y} dy$ and $\int \frac{-e^x}{1-e^x} dx$ in equation (1), we get:

$$\Rightarrow \log(\tan y) = \log\left(1-e^x\right) + \log C$$
$$\Rightarrow \log(\tan y) = \log\left[C\left(1-e^x\right)\right]$$
$$\Rightarrow \tan y = C\left(1-e^x\right)$$

This is the required general solution of the given differential equation.

Answer needs Correction? **Click Here**

Q11 : $\left(x^3 + x^2 + x + 1\right)\frac{dy}{dx} = 2x^2 + x;\ y = 1$ when $x = 0$

.

Answer :

The given differential equation is:

$$\left(x^3+x^2+x+1\right)\frac{dy}{dx}=2x^2+x$$
$$\Rightarrow \frac{dy}{dx}=\frac{2x^2+x}{\left(x^3+x^2+x+1\right)}$$
$$\Rightarrow dy=\frac{2x^2+x}{(x+1)\left(x^2+1\right)}dx$$

Integrating both sides, we get:

$$\int dy=\int\frac{2x^2+x}{(x+1)\left(x^2+1\right)}dx \qquad ...(1)$$
$$\text{Let } \frac{2x^2+x}{(x+1)\left(x^2+1\right)}=\frac{A}{x+1}+\frac{Bx+C}{x^2+1}. \qquad ...(2)$$
$$\Rightarrow \frac{2x^2+x}{(x+1)\left(x^2+1\right)}=\frac{Ax^2+A+(Bx+C)(x+1)}{(x+1)\left(x^2+1\right)}$$
$$\Rightarrow 2x^2+x=Ax^2+A+Bx^2+Bx+Cx+C$$
$$\Rightarrow 2x^2+x=(A+B)x^2+(B+C)x+(A+C)$$

Comparing the coefficients of x^2 and x, we get:

$A+B=2$

$B+C=1$

$A+C=0$

Solving these equations, we get:

$$A=\frac{1}{2},\ B=\frac{3}{2} \text{ and } C=\frac{-1}{2}$$

Substituting the values of A, B, and C in equation (2), we get:

$$\frac{2x^2+x}{(x+1)\left(x^2+1\right)}=\frac{1}{2}\cdot\frac{1}{(x+1)}+\frac{1}{2}\frac{(3x-1)}{\left(x^2+1\right)}$$

Therefore, equation (1) becomes:

$$\int dy=\frac{1}{2}\int\frac{1}{x+1}dx+\frac{1}{2}\int\frac{3x-1}{x^2+1}dx$$
$$\Rightarrow y=\frac{1}{2}\log(x+1)+\frac{3}{2}\int\frac{x}{x^2+1}dx-\frac{1}{2}\int\frac{1}{x^2+1}dx$$
$$\Rightarrow y=\frac{1}{2}\log(x+1)+\frac{3}{4}\cdot\int\frac{2x}{x^2+1}dx-\frac{1}{2}\tan^{-1}x+\text{C}$$
$$\Rightarrow y=\frac{1}{2}\log(x+1)+\frac{3}{4}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}x+\text{C}$$
$$\Rightarrow y=\frac{1}{4}\left[2\log(x+1)+3\log\left(x^2+1\right)\right]-\frac{1}{2}\tan^{-1}x+\text{C}$$
$$\Rightarrow y=\frac{1}{4}\left[(x+1)^2\left(x^2+1\right)^3\right]-\frac{1}{2}\tan^{-1}x+\text{C} \qquad ...(3)$$

Now, $y=1$ when $x=0$.

$$\Rightarrow 1=\frac{1}{4}\log(1)-\frac{1}{2}\tan^{-1}0+\text{C}$$
$$\Rightarrow 1=\frac{1}{4}\times 0-\frac{1}{2}\times 0+\text{C}$$
$$\Rightarrow \text{C}=1$$

Substituting C = 1 in equation (3), we get:

$$y=\frac{1}{4}\left[\log(x+1)^2\left(x^2+1\right)^3\right]-\frac{1}{2}\tan^{-1}x+1$$

Answer needs Correction? **Click Here**

Q12 : $x\left(x^2-1\right)\frac{dy}{dx}=1;\ y=0$ when $x=2$

Answer :

$$x\left(x^2-1\right)\frac{dy}{dx}=1$$
$$\Rightarrow dy=\frac{dx}{x\left(x^2-1\right)}$$
$$\Rightarrow dy=\frac{1}{x(x-1)(x+1)}dx$$

Integrating both sides, we get:

$$\int dy=\int\frac{1}{x(x-1)(x+1)}dx \qquad ...(1)$$
$$\text{Let } \frac{1}{x(x-1)(x+1)}=\frac{A}{x}+\frac{B}{x-1}+\frac{C}{x+1}. \qquad ...(2)$$
$$\Rightarrow \frac{1}{x(x-1)(x+1)}=\frac{A(x-1)(x+1)+Bx(x+1)+Cx(x-1)}{x(x-1)(x+1)}$$
$$=\frac{(A+B+C)x^2+(B-C)x-A}{x(x-1)(x+1)}$$

Comparing the coefficients of x^2, x, and constant, we get:

$A=-1$

$B-C=0$

$A+B+C=0$

Solving these equations, we get $B=\frac{1}{2}$ and $C=\frac{1}{2}$.

Solving these equations, we get $B = \frac{1}{2}$ and $C = \frac{1}{2}$.

Substituting the values of *A*, *B*, and *C* in equation (2), we get:

$$\frac{1}{x(x-1)(x+1)} = \frac{-1}{x} + \frac{1}{2(x-1)} + \frac{1}{2(x+1)}$$

Therefore, equation (1) becomes:

$$\int dy = -\int \frac{1}{x}dx + \frac{1}{2}\int \frac{1}{x-1}dx + \frac{1}{2}\int \frac{1}{x+1}dx$$

$$\Rightarrow y = -\log x + \frac{1}{2}\log(x-1) + \frac{1}{2}\log(x+1) + \log k$$

$$\Rightarrow y = \frac{1}{2}\log\left[\frac{k^2(x-1)(x+1)}{x^2}\right] \qquad ...(3)$$

Now, $y = 0$ when $x = 2$.

$$\Rightarrow 0 = \frac{1}{2}\log\left[\frac{k^2(2-1)(2+1)}{4}\right]$$

$$\Rightarrow \log\left(\frac{3k^2}{4}\right) = 0$$

$$\Rightarrow \frac{3k^2}{4} = 1$$

$$\Rightarrow 3k^2 = 4$$

$$\Rightarrow k^2 = \frac{4}{3}$$

Substituting the value of k^2 in equation (3), we get:

$$y = \frac{1}{2}\log\left[\frac{4(x-1)(x+1)}{3x^2}\right]$$

$$y = \frac{1}{2}\log\left[\frac{4(x^2-1)}{3x^2}\right]$$

Answer needs Correction? **Click Here**

Q13 : $\cos\left(\frac{dy}{dx}\right) = a\,(a \in R);\; y = 1 \text{ when } x = 0$

Answer :

$$\cos\left(\frac{dy}{dx}\right) = a$$

$$\Rightarrow \frac{dy}{dx} = \cos^{-1} a$$

$$\Rightarrow dy = \cos^{-1} a\, dx$$

Integrating both sides, we get:

$$\int dy = \cos^{-1} a \int dx$$

$$\Rightarrow y = \cos^{-1} a \cdot x + C$$

$$\Rightarrow y = x\cos^{-1} a + C \qquad ...(1)$$

Now, $y = 1$ when $x = 0$.

$$\Rightarrow 1 = 0 \cdot \cos^{-1} a + C$$

$$\Rightarrow C = 1$$

Substituting C = 1 in equation (1), we get:

$$y = x\cos^{-1} a + 1$$

$$\Rightarrow \frac{y-1}{x} = \cos^{-1} a$$

$$\Rightarrow \cos\left(\frac{y-1}{x}\right) = a$$

*********** END **********