Indefinite Integrals Ex 19.9 Q1

Let $I = \int \frac{\log x}{x} dx$

Let $\log x = t$ then,

$d(\log x) = dt$

$\Rightarrow \frac{1}{x} dx = dt$

$\Rightarrow dx = x\,dt$

Putting $\log x = t$ and $dx = x\,dt$, we get

$$I = \int \frac{t}{x} \times x\,dt$$

$$= \int t\,dt$$

$$= \frac{t^2}{2} + c$$

$$= \frac{(\log x)^2}{2} + c$$

$$\therefore \quad I = \frac{(\log x)^2}{2} + c$$

Indefinite Integrals Ex 19.9 Q2

Let $\quad I = \int \frac{\log\left(1+\frac{1}{x}\right)}{x\left(1+x\right)} dx \text{ - - - - - - } (i)$

Let $\log\left(1+\frac{1}{x}\right) = t$ then,

$$d\left[\log\left(1+\frac{1}{x}\right)\right] = dt$$

$$\Rightarrow \quad \frac{1}{1+\frac{1}{x}} \times \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \quad \frac{1}{\frac{x+1}{x}} \times \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \quad \frac{-x}{x^2\left(x+1\right)} dx = -dt$$

$$\Rightarrow \quad \frac{dx}{x\left(x+1\right)} = -dt$$

Putting $\log\left(1+\frac{1}{x}\right) = t$ and $\frac{dx}{x\left(x+1\right)} = -dt$ in equation (i), we get

$$I = \int t \times -dt$$

$$= -\frac{t^2}{2} + c$$

$$= -\frac{1}{2}\left[\log\left(1+\frac{1}{x}\right)\right]^2 + c$$

$$\therefore \quad I = -\frac{1}{2}\left[\log\left(1+\frac{1}{x}\right)\right]^2 + c$$

Let $\quad I = \int \frac{\log\left(1+\frac{1}{x}\right)}{x\left(1+x\right)} dx \text{ - - - - - - } (i)$

Let $\log\left(1+\frac{1}{x}\right) = t$ then,

$$d\left[\log\left(1+\frac{1}{x}\right)\right] = dt$$

$$\Rightarrow \quad \frac{1}{1+\frac{1}{x}} \times \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \quad \frac{1}{\frac{x+1}{x}} \times \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \quad \frac{-x}{x^2(x+1)} dx = dt$$

$$\Rightarrow \quad \frac{dx}{x(x+1)} = -dt$$

Putting $\log\left(1+\frac{1}{x}\right) = t$ and $\frac{dx}{x(x+1)} = -dt$ in equation (i), we get

$$I = \int t \times -dt$$
$$= -\frac{t^2}{2} + c$$
$$= -\frac{1}{2}\left[\log\left(1+\frac{1}{x}\right)\right]^2 + c$$

$$\therefore \quad I = -\frac{1}{2}\left[\log\left(1+\frac{1}{x}\right)\right]^2 + c$$

Indefinite Integrals Ex 19.9 Q3

Let $\quad I = \int \frac{\left(1+\sqrt{x}\right)^2}{\sqrt{x}} dx$

Let $\left(1+\sqrt{x}\right) = t$ then,

$d\left(1+\sqrt{x}\right) = dt$

$$\Rightarrow \quad \frac{1}{2\sqrt{x}} dx = dt$$

$$\Rightarrow \quad dx = dt \times 2\sqrt{x}$$

Putting $\left(1+\sqrt{x}\right) = t$ and $dx = dt \times 2\sqrt{x}$, we get

$$I = \int \frac{t^2}{\sqrt{x}} \times dt \times 2\sqrt{x}$$
$$= 2\int t^2 dt$$
$$= 2 \times \frac{t^3}{3} + c$$
$$= \frac{2}{3}\left[1+\sqrt{x}\right]^3 + c$$

$$\therefore \quad I = \frac{2}{3}\left(1+\sqrt{x}\right)^3 + c$$

Indefinite Integrals Ex 19.9 Q4

Let $\quad I = \int \sqrt{1+e^x}\, e^x dx \text{ ------ (i)}$

$\qquad$ Let $1+e^x = t \qquad$ then,

$\qquad d\left(1+e^x\right) = dt$

$\Rightarrow \qquad e^x dx = dt$

$\Rightarrow \qquad dx = \dfrac{dt}{e^x}$

Putting $1+e^x = t$ and $dx = \dfrac{dt}{e^x}$ in equation (i), we get

$$I = \int \sqrt{t}\, e^x \frac{dt}{e^x}$$

$$= \int t^{\frac{1}{2}} dt$$

$$= \frac{2}{3} \times \frac{t^{\frac{3}{2}}}{} + c$$

$$= \frac{2}{3}\left(1+e^x\right)^{\frac{3}{2}} + c$$

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