Maxima and Minima 18.4 Q1(i)

The given function is $f(x) = 4x - \frac{1}{2}x^2$.

$\therefore f'(x) = 4 - \frac{1}{2}(2x) = 4 - x$

Now,

$f'(x) = 0 \Rightarrow x = 4$

Then, we evaluate the value of f at critical point $x = 4$ and at the end points of the interval $\left[-2, \frac{9}{2}\right]$.

$f(4) = 16 - \frac{1}{2}(16) = 16 - 8 = 8$

$f(-2) = -8 - \frac{1}{2}(4) = -8 - 2 = -10$

$f\left(\frac{9}{2}\right) = 4\left(\frac{9}{2}\right) - \frac{1}{2}\left(\frac{9}{2}\right)^2 = 18 - \frac{81}{8} = 18 - 10.125 = 7.875$

Hence, we can conclude that the absolute maximum value of f on $\left[-2, \frac{9}{2}\right]$ is 8 occurring at $x = 4$ and the absolute minimum value of f on $\left[-2, \frac{9}{2}\right]$ is -10 occurring at $x = -2$.

Maxima and Minima 18.4 Q1(ii)

The given function is $f(x) = (x-1)^2 + 3$.

$\therefore f'(x) = 2(x-1)$

Now,

$f'(x) = 0 \Rightarrow 2(x - 1) = 0 \Rightarrow x = 1$

Then, we evaluate the value of f at critical point $x = 1$ and at the end points of the interval $[-3, 1]$.

$f(1) = (1-1)^2 + 3 = 0 + 3 = 3$

$f(-3) = (-3-1)^2 + 3 = 16 + 3 = 19$

Hence, we can conclude that the absolute maximum value of f on $[-3, 1]$ is 19 occurring at $x = -3$ and the minimum value of f on $[-3, 1]$ is 3 occurring at $x = 1$.

Maxima and Minima 18.4 Q1(iii)

Let $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$.

$$\therefore f'(x) = 12x^3 - 24x^2 + 24x - 48$$
$$= 12(x^3 - 2x^2 + 2x - 4)$$
$$= 12[x^2(x-2) + 2(x-2)]$$
$$= 12(x-2)(x^2+2)$$

Now, $f'(x) = 0$ gives $x = 2$ or $x^2 + 2 = 0$ for which there are no real roots.

Therefore, we consider only $x = 2 \in [0, 3]$.

Now, we evaluate the value of f at critical point $x = 2$ and at the end points of the interval [0, 3].

$$f(2) = 3(16) - 8(8) + 12(4) - 48(2) + 25$$
$$= 48 - 64 + 48 - 96 + 25$$
$$= -39$$
$$f(0) = 3(0) - 8(0) + 12(0) - 48(0) + 25$$
$$= 25$$
$$f(3) = 3(81) - 8(27) + 12(9) - 48(3) + 25$$
$$= 243 - 216 + 108 - 144 + 25 = 16$$

Hence, we can conclude that the absolute maximum value of f on [0, 3] is 25 occurring at $x = 0$ and the absolute minimum value of f at [0, 3] is -39 occurring at $x = 2$.

Maxima and Minima 18.4 Q1(iv)

$f(x) = (x-2)\sqrt{x-1}$

$\Rightarrow \quad f'(x) = \sqrt{x-1} + (x-2)\dfrac{1}{2\sqrt{x-1}}$

Put $f'(x) = 0$

$\Rightarrow \quad \sqrt{x-1} + \dfrac{x-2}{2\sqrt{x-1}} = 0$

$\Rightarrow \quad \dfrac{2(x-1) + (x-2)}{2\sqrt{x-1}} = 0$

$\Rightarrow \quad \dfrac{3x-4}{2\sqrt{x-1}} = 0$

$\Rightarrow \quad x = \dfrac{4}{3}$

Now,

$f(1) = 0$

$f\left(\dfrac{4}{3}\right) = \left(\dfrac{4}{3} - 2\right)\sqrt{\dfrac{4}{3} - 1} = \dfrac{4-6}{3\sqrt{3}} = \dfrac{-2}{3\sqrt{3}} = \dfrac{-2\sqrt{3}}{9}$

$f(9) = (9-2)\sqrt{9-1} = 7\sqrt{8} = 14\sqrt{2}$

$\therefore$ The absolute maximum value of $f(x)$ is $14\sqrt{2}$ at $x = 9$ and the absolute minimum value is $\dfrac{-2\sqrt{3}}{9}$ at $x = \dfrac{4}{3}$.

*********** END **********