



### Trigonometric Functions Ex 5.1 Q1

$$\begin{aligned}
 \text{LHS} &= \sec^4 \theta - \sec^2 \theta \\
 &= \sec^2 \theta (\sec^2 \theta - 1) \\
 &= (1 + \tan^2 \theta) \tan^2 \theta & [\because \sec^2 \theta = 1 + \tan^2 \theta] \\
 &= \tan^2 \theta + \tan^4 \theta \\
 &= \tan^4 \theta + \tan^2 \theta \\
 &= \text{RHS} \\
 \text{LHS} &= \text{RHS} \\
 &\text{Proved}
 \end{aligned}$$

### Trigonometric Functions Ex 5.1 Q2

$$\begin{aligned}
 \text{LHS} &= \sin^6 \theta + \cos^6 \theta \\
 &= (\sin^2 \theta)^3 + (\cos^2 \theta)^3 \\
 &= (\sin^2 \theta + \cos^2 \theta) \left[ (\sin^2 \theta)^2 - \sin^2 \theta \cos^2 \theta + (\cos^2 \theta)^2 \right] & (\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)) \\
 &= (\sin^2 \theta)^2 + (\cos^2 \theta)^2 + 2 \sin^2 \theta \cos^2 \theta - 2 \sin^2 \theta \cos^2 \theta - \sin^2 \theta \cos^2 \theta \\
 & & \left[ \text{adding and subtracting } 2 \sin^2 \theta \cos^2 \theta \text{ and} \right. \\
 & & \left. \text{using identity } \sin^2 \theta + \cos^2 \theta = 1 \right] \\
 &= (\sin^2 \theta + \cos^2 \theta)^2 - 3 \sin^2 \theta \cos^2 \theta \\
 &= (\sin^2 \theta + \cos^2 \theta)^2 - 3 \sin^2 \theta \cos^2 \theta \\
 &= 1^2 - 3 \sin^2 \theta \cos^2 \theta & (\because \sin^2 \theta + \cos^2 \theta = 1) \\
 &= 1 - 3 \sin^2 \theta \cos^2 \theta \\
 &= \text{RHS} \\
 \therefore \text{LHS} &= \text{RHS} \\
 &\text{Proved}
 \end{aligned}$$

### Trigonometric Functions Ex 5.1 Q3

$$\begin{aligned}
 \text{LHS} &= (\cos \sec \theta - \sin \theta) (\sec \theta - \cos \theta) (\tan \theta + \cot \theta) \\
 &= \left( \frac{1}{\sin \theta} - \sin \theta \right) \left( \frac{1}{\cos \theta} - \cos \theta \right) \left( \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right) & \left[ \because \csc \theta = \frac{1}{\sin \theta}, \sec \theta = \frac{1}{\cos \theta}, \right. \\
 & & \left. \tan \theta = \frac{\sin \theta}{\cos \theta} \text{ and } \cot \theta = \frac{\cos \theta}{\sin \theta} \right] \\
 &= \left( \frac{1 - \sin^2 \theta}{\sin \theta} \right) \left( \frac{1 - \cos^2 \theta}{\cos \theta} \right) \left( \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} \right) \\
 &= \frac{\cos^2 \theta \cdot \sin^2 \theta \cdot 1}{\sin^2 \theta \cdot \cos^2 \theta} \left( \begin{array}{l} \because \sin^2 \theta + \cos^2 \theta = 1 \\ \Rightarrow 1 - \sin^2 \theta = \cos^2 \theta, \text{ and} \\ 1 - \cos^2 \theta = \sin^2 \theta \end{array} \right) \\
 &= 1 \\
 &= \text{RHS} \\
 &\text{Proved}
 \end{aligned}$$

### Trigonometric Functions Ex 5.1 Q4

$$\begin{aligned}
\text{LHS} &= \cos \sec \theta (\sec \theta - 1) - \cot \theta (1 - \cos \theta) \\
&= \frac{1}{\sin \theta} \left( \frac{1}{\cos \theta} - 1 \right) - \frac{\cos \theta}{\sin \theta} (1 - \cos \theta) \quad \left[ \because \cos \sec \theta = \frac{1}{\sin \theta}, \sec \theta = \frac{1}{\cos \theta}, \cot \theta = \frac{\cos \theta}{\sin \theta} \right] \\
&= \frac{(1 - \cos \theta)}{\sin \theta \cos \theta} - \frac{\cos \theta (1 - \cos \theta)}{\sin \theta} \\
&= \frac{(1 - \cos \theta) - \cos^2 \theta (1 - \cos \theta)}{\sin \theta \cos \theta} \\
&= \frac{(1 - \cos \theta)(1 - \cos^2 \theta)}{\sin \theta \cos \theta} \\
&= \frac{(1 - \cos \theta) \sin^2 \theta}{\sin \theta \cos \theta} \quad (\because 1 - \cos^2 \theta = \sin^2 \theta) \\
&= (1 - \cos \theta) \frac{\sin \theta}{\cos \theta} \\
&= \frac{\sin \theta}{\cos \theta} - \sin \theta \\
&= \tan \theta - \sin \theta \quad (\because \tan \theta = \sin \theta - \cos \theta) \\
&= \text{RHS} \\
&\text{Proved}
\end{aligned}$$

### Trigonometric Functions Ex 5.1 Q5

$$\begin{aligned}
\text{LHS} &= \frac{1 - \sin A \cos A}{\cos A (\sec A - \cos \sec A)} \cdot \frac{\sin^2 A - \cos^2 A}{\sin^3 A + \cos^3 A} \\
&= \frac{1 - \sin A \cos A}{\cos A \left( \frac{1}{\cos A} - \frac{1}{\sin A} \right)} \cdot \frac{(\sin A + \cos A)(\sin A - \cos A)}{(\sin A + \cos A)(\sin^2 A + \cos^2 A - \sin A \cos A)} \\
&\quad \left[ \begin{array}{l} \text{Using } a^2 - b^2 = (a - b)(a + b) \\ \text{and } a^3 + b^3 = (a + b)(a^2 + b^2 - ab) \end{array} \right] \\
&= \frac{(1 - \sin A \cos A)}{\cos A \left( \frac{\sin A - \cos A}{\cos A \sin A} \right)} \cdot \frac{(\sin A - \cos A)}{(1 - \sin A \cos A)} \quad (\because \sin^2 A + \cos^2 A = 1) \\
&= \frac{\cos A \sin A}{\cos A} \\
&= \sin A \\
&= \text{RHS} \\
&\text{Proved}
\end{aligned}$$

\*\*\*\*\* END \*\*\*\*\*