Chapter 10 Differentiability Ex 10.2 Q5

$f(x) = x^3 + 7x^2 + 8x - 9$ is a polynomial function. So, it is differentiable every where.

$$f'(4) = \lim_{h \to 0} \frac{f(4+h) - h(4)}{h}$$

$$= \lim_{h \to 0} \frac{\left[(4+h)^3 + 7(4+h)^2 + 8(4+h) - 9\right] - [64 + 112 + 32 - 9]}{h}$$

$$= \lim_{h \to 0} \frac{\left[64 + h^3 + 48h + 12h^2 + 112 + 7h^2 + 56h + 32 + 8h - 9\right] - [210 - 9]}{h}$$

$$= \lim_{h \to 0} \frac{h^3 + 19h^2 + 112h + 210 - 9 - 210 + 9}{h}$$

$$= \lim_{h \to 0} \frac{h^3 + 19h^2 + 112h}{h}$$

$$= \lim_{h \to 0} \frac{h\left(h^2 + 19h + 112\right)}{h}$$

$$f'(4) = 112$$

Chapter 10 Differentiability Ex 10.2 Q6

$$f(x) = mx + c$$

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{f(h) - h(0)}{h}$$

$$= \lim_{h \to 0} \frac{(mh + c) - (m \times 0 + c)}{h}$$

$$= \lim_{h \to 0} \frac{mh + c - c}{h}$$

$$= \lim_{h \to 0} \frac{mh}{h}$$

$$= m$$

$$f'(0) = m$$

Chapter 10 Differentiability Ex 10.2 Q7

$$f(x)=\begin{cases}2x+3, \text{ if } -3\le x<-2\\ x+1, \text{ if } -2\le x<0\\ x+2, \text{ if } 0\le x\le 1\end{cases}$$

We know that polynomial funtiona are continuous and differentiable everywhere.
So f(x) is differentiable on $x\in[-3,2)$, $x\in(-2,0)$ and $x\in(0,1]$.
We need to check the differentiability at x = -2 and x = 0

Differentiability at x = -2

$$(\text{LHD at } x=-2)=\lim_{x\to-2^-}\frac{f(x)-f(-2)}{x-(-2)}=\lim_{x\to-2^-}\frac{2x+3+1}{x+2}=\lim_{x\to-2^-}\frac{2(x+2)}{x+2}=2$$

$$(\text{RHD at } x=-2)=\lim_{x\to-2^+}\frac{f(x)-f(-2)}{x-(-2)}=\lim_{x\to-2^+}\frac{x+1+1}{x+2}=\lim_{x\to-2^+}\frac{x+2}{x+2}=1$$

$\therefore$ (LHD at x = -2) $\neq$ (RHD at x = -2)
So, $f(x)$ is not differentiable at x = -2.

Differentiability at x = 0

$$(\text{LHD at } x=0)=\lim_{x\to0^-}\frac{f(x)-f(0)}{x-0}=\lim_{x\to0^-}\frac{x+1-2}{x}=\lim_{x\to0^-}\frac{x-1}{x}\to\infty$$

$$(\text{RHD at } x=0)=\lim_{x\to0^+}\frac{f(x)-f(0)}{x-0}=\lim_{x\to0^+}\frac{x+2-2}{x}=\lim_{x\to0^+}\frac{x}{x}=1$$

$\therefore$ (LHD at x = 0) $\neq$ (RHD at x = 0)
So, $f(x)$ is not differentiable at x = 0.

Chapter 10 Differentiability Ex 10.2 Q8

We know that, modulus function

$f(x)=|x|$ is continuous but bot differentiable at $x=0$,

So,

$f(x)=|x|+|x-1|+|x-2|+|x-3|+|x-4|$ is continuous but not differentiable $x=0,1,2,3,4$.

********** END **********