Definite Integrals Ex 20.4B Q33

Let $I=\int_0^2 x\sqrt{2-x}\,dx$

$I=\int_0^2 (2-x)\sqrt{x}\,dx \qquad \left(\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx\right)$

$=\int_0^2 \left\{2x^{\frac{1}{2}}-x^{\frac{3}{2}}\right\}dx$

$=\left[2\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)-\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^2$

$=\left[\frac{4}{3}x^{\frac{3}{2}}-\frac{2}{5}x^{\frac{5}{2}}\right]_0^2$

$=\frac{4}{3}(2)^{\frac{3}{2}}-\frac{2}{5}(2)^{\frac{5}{2}}$

$=\frac{4\times2\sqrt{2}}{3}-\frac{2}{5}\times4\sqrt{2}$

$=\frac{8\sqrt{2}}{3}-\frac{8\sqrt{2}}{5}$

$=\frac{40\sqrt{2}-24\sqrt{2}}{15}$

$=\frac{16\sqrt{2}}{15}$

Definite Integrals Ex 20.4B Q34

Let $I=\int_0^1 \log\left(\frac{1}{x}-1\right)dx$

$=\int_0^1 \log\left(\frac{1-x}{x}\right)dx$

$=\int_0^1 \log(1-x)dx-\int_0^1 \log(x)dx$

Applying the property, $\int_0^a f(x)dx=\int_0^a f(a-x)dx$

Thus, $I=\int_0^1 \log(1-(1-x))dx-\int_0^1 \log(x)dx$

$=\int_0^1 \log(1-1+x)dx-\int_0^1 \log(x)dx$

$=\int_0^1 \log(x)dx-\int_0^1 \log(x)dx$

$=0$

Definite Integrals Ex 20.4B Q35

$$I = \int_{-1}^{1} |x \cos \pi x| \, dx$$

Let $f(x) = |x \cos \pi x|$

$$f(-x) = |-x \cos(-\pi x)| = |-x \cos(\pi x)| = |x \cos \pi x| = f(x)$$

$$\therefore I = \int_{-1}^{1} |x \cos \pi x| \, dx = 2\int_{0}^{1} |x \cos \pi x| \, dx$$

Now,

$$f(x) = |x \cos \pi x| = \begin{cases} x \cos \pi x \text{ , if } 0 \le x \le \frac{1}{2} \\ -x \cos \pi x \text{ , if } \frac{1}{2} < x < 1 \end{cases}$$

$$\therefore I = 2\int_{0}^{1} |x \cos \pi x| \, dx$$

$$\Rightarrow I = 2\left[\int_{0}^{\frac{1}{2}} x \cos \pi x \, dx + \int_{\frac{1}{2}}^{1} -x \cos \pi x \, dx\right]$$

$$\Rightarrow I = 2\left\{\left[\frac{x}{\pi}\sin \pi x + \frac{1}{\pi^2}\cos \pi x\right]_0^{\frac{1}{2}} - \left[\frac{x}{\pi}\sin \pi x + \frac{1}{\pi^2}\cos \pi x\right]_{\frac{1}{2}}^{1}\right\}$$

$$\Rightarrow I = 2\left\{\left[\frac{1}{2\pi} - \frac{1}{\pi^2}\right] - \left[-\frac{1}{\pi^2} - \frac{1}{2\pi}\right]\right\}$$

$$\Rightarrow I = \frac{2}{\pi}$$

Definite Integrals Ex 20.4B Q36

$$I = \int_0^{\pi}\left(\frac{x}{1+\sin^2 x} + \cos^7 x\right)dx$$

$$I = \int_0^{\pi}\left(\frac{\Pi - x}{1+\sin^2(\Pi - x)} + \cos^7(\Pi - x)\right)dx$$

$$I = \int_0^{\pi}\left(\frac{\Pi - x}{1+\sin^2 x} - \cos^7 x\right)dx$$

$$2I = \int_0^{\pi}\left(\frac{\Pi}{1+\sin^2 x}\right)dx$$

$$2I = \Pi\int_0^{\pi}\frac{1}{1+\sin^2 x}dx$$

$$2I = \Pi\int_0^{\pi}\frac{1}{1+2\tan^2 x}\sec^2 x dx$$

$$I = \pi\int_0^{\frac{\pi}{2}}\frac{1}{1+2\tan^2 x}\sec^2 x dx \ldots\ldots\ldots\left[\because \int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx, \text{ if } f(2a - x) = f(x)\right]$$

Let $\tan x = v$

$dv = \sec^2 x \, dx$

$$\Rightarrow I = \pi\int_0^{\infty}\frac{1}{1+2v^2}dv$$

$$\Rightarrow I = \pi\left[\frac{\tan^{-1}(\sqrt{2}v)}{\sqrt{2}}\right]_0^{\infty}$$

$$\Rightarrow I = \pi\left[\frac{\pi}{2\sqrt{2}}\right]$$

$$\Rightarrow I = \frac{\pi^2}{2\sqrt{2}}$$

*********** END **********