Indefinite Integrals Ex 19.17 Q1

$$\text{Let } I = \int \frac{1}{\sqrt{2x - x^2}}dx$$

$$= \int \frac{1}{\sqrt{-\left[x^2 - 2x\right]}}dx$$

$$= \int \frac{1}{\sqrt{-\left[x^2 - 2x\left(1\right) + 1^2 - 1^2\right]}}dx$$

$$= \int \frac{1}{\sqrt{-\left[\left(x - 1\right)^2 - 1\right]}}dx$$

$$= \int \frac{1}{\sqrt{1 - \left(x - 1\right)^2}}dx$$

Let $\left(x - 1\right) = t$

$\Rightarrow \quad dx = dt$

so, $\quad I = \int \frac{1}{\sqrt{1 - t^2}}dt$

$$= \sin^{-1} t + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{1 - x^2}}dx = \sin^{-1} x + c\right]$$

$$I = \sin^{-1}\left(x - 1\right) + c$$

Indefinite Integrals Ex 19.17 Q2

$8+3x-x^2$ can be written as $8-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)$.

Therefore,

$$8-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)$$

$$=\frac{41}{4}-\left(x-\frac{3}{2}\right)^2$$

$$\Rightarrow \int \frac{1}{\sqrt{8+3x-x^2}}\,dx=\int \frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}}dx$$

Let $x-\frac{3}{2}=t$

$\therefore\ dx=dt$

$$\Rightarrow \int \frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}}dx=\int \frac{1}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2-t^2}}dt$$

$$=\sin^{-1}\left(\frac{t}{\frac{\sqrt{41}}{2}}\right)+C$$

$$=\sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{41}}{2}}\right)+C$$

$$=\sin^{-1}\left(\frac{2x-3}{\sqrt{41}}\right)+C$$

Indefinite Integrals Ex 19.17 Q3

$$\text{Let } I = \int \frac{1}{\sqrt{5-4x-2x^2}}dx$$

$$= \int \frac{1}{\sqrt{-2\left[x^2+2x-\frac{5}{2}\right]}}dx$$

$$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{-\left[x^2+2x+(1)^2-(1)^2-\frac{5}{2}\right]}}dx$$

$$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{-\left[(x+1)^2-\frac{7}{2}\right]}}dx$$

$$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{\frac{7}{2}-(x+1)^2}}dx$$

Let $(x+1) = t$

$\Rightarrow \quad dx = dt$

so, $\quad I = \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{\left(\sqrt{\frac{7}{2}}\right)^2 - t^2}}\,dt$

$$= \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{t}{\sqrt{\frac{7}{2}}}\right)+c \quad \left[\text{Since } \int\frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)+c\right]$$

$$I = \frac{1}{\sqrt{2}}\sin^{-1}\left(\sqrt{\frac{2}{7}}\times(x+1)\right)+c$$

Indefinite Integrals Ex 19.17 Q4

$$\text{Let } I = \int \frac{1}{\sqrt{3x^2+5x+7}}dx$$

$$= \frac{1}{\sqrt{3}}\int \frac{1}{\sqrt{x^2+\frac{5}{3}x+\frac{7}{3}}}dx$$

$$= \frac{1}{\sqrt{3}}\int \frac{1}{\sqrt{x^2+2x\left(\frac{5}{6}\right)+\left(\frac{5}{6}\right)^2-\left(\frac{5}{6}\right)^2+\frac{7}{3}}}dx$$

$$= \frac{1}{\sqrt{3}}\int \frac{1}{\sqrt{\left(x+\frac{5}{6}\right)^2-\frac{59}{36}}}dx$$

Let $\left(x+\frac{5}{6}\right) = t$

$\Rightarrow \quad dx = dt$

$$I = \frac{1}{\sqrt{3}}\int \frac{1}{\sqrt{t^2-\left(\frac{\sqrt{59}}{6}\right)^2}}dt$$

$$= \frac{1}{\sqrt{3}}\log\left|t+\sqrt{t^2-\left(\frac{\sqrt{59}}{6}\right)}\right|+c \quad \left[\text{Since } \int\frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right|+c\right]$$

$$I = \frac{1}{\sqrt{3}}\log\left|x+\frac{5}{6}+\sqrt{\left(x+\frac{5}{6}\right)^2-\left(\frac{\sqrt{59}}{6}\right)^2}\right|+c$$

$$I = \frac{1}{\sqrt{3}}\log\left|x+\frac{5}{6}+\sqrt{x^2+\frac{5x}{3}+\frac{7}{3}}\right|+c$$

*********** END **********