$$\begin{array}{l} 10x = 4.7777..... \\ -\ x = 0.4777.... \\ \hline 9x = 4.3 \end{array}$$

We can also write $9x = 4.3$ as $x = \frac{4.3}{9}$ or $x = \frac{43}{90}$.

Therefore, on converting $0.4\overline{7}$ in the $\frac{p}{q}$ form, we get the answer as $\frac{43}{90}$.

(iii) Let $x = 0.\overline{001}$

$\Rightarrow x = 0.001001........(a)$

We need to multiply both sides by 1000 to get

$1000x = 1.001001.....$ $\qquad ....(b)$

We need to subtract (a) from (b), to get

$$\begin{array}{l} 1000x = 1.001001..... \\ \quad -\ x = 0.001001.... \\ \hline 999x = 1 \end{array}$$

We can also write $999x = 1$ as $x = \frac{1}{999}$.

Therefore, on converting $0.\overline{001}$ in the $\frac{p}{q}$ form, we get the answer as $\frac{1}{999}$.

Q4. Express $0.99999....$ in the form $\frac{p}{q}$. Are you surprised by your answer? Discuss why the answer makes sense with your teacher and classmates.

Ans: Let $x = 0.99999......(a)$

We need to multiply both sides by 10 to get

$10x = 9.9999......(b)$

We need to subtract $(a)$from (b), to get

$$\begin{array}{r} 10x = 9.99999..... \\ -\ x = 0.99999.... \\ \hline 9x = 9 \end{array}$$

We can also write $9x = 9$ as $x = \frac{9}{9}$ or $x = 1$.

Therefore, on converting $0.99999....$ in the $\frac{p}{q}$ form, we get the answer as 1.

Yes, at a glance we are surprised at our answer.

But the answer makes sense when we observe that 0.9999......... goes on forever. SO there is not gap between 1 and 0.9999....... and hence they are equal.

Q5. What can the maximum number of digits be in the recurring block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Ans: We need to find the number of digits in the recurring block of $\frac{1}{17}$.

Let us perform the long division to get the recurring block of $\frac{1}{17}$.

We need to divide 1 by 17, to get

We can observe that while dividing 1 by 17 we got the remainder as 1, which will continue to be 1 after carrying out 16 continuous divisions.

Therefore, we conclude that

$\frac{1}{17} = 0.0588235294117647.....$ or $\frac{1}{17} = 0.\overline{0588235294117647}$

, which is a non-terminating decimal and recurring decimal.

Q6. Look at several examples of rational numbers in the form $\frac{p}{q}$ $(q \neq 0)$, where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Ans: Let us consider the examples of the form $\frac{p}{q}$ that are terminating decimals.

$$\frac{5}{2} = 2.5$$

$$\frac{5}{4} = 1.25$$

$$\frac{2}{5} = 0.4$$

$$\frac{2}{10} = 0.2$$

$$\frac{5}{16} = 0.3125$$

We can observe that the denominators of the above rational numbers have powers of 2, 5 or both.

Therefore, we can conclude that the property, which q must satisfy in $\frac{p}{q}$, so that the rational number $\frac{p}{q}$ is a terminating decimal is that q must have powers of 2, 5 or both.

Q7. Write three numbers whose decimal expansions are non-terminating non-recurring.

Ans: The three numbers that have their expansions as non-terminating on recurring decimal are given below.

0.04004000400004....

0.07007000700007....

0.013001300013000013....

Q8. Find three different irrational numbers between the rational numbers $\frac{5}{11}$ and $\frac{9}{11}$.

Ans: Let us convert $\frac{5}{7}$ and $\frac{9}{11}$ into decimal form, to get

$$\frac{5}{7} = 0.714285\ldots. \text{ and } \frac{9}{11} = 0.818181\ldots.$$

Three irrational numbers that lie between $0.714285\ldots.$ and $0.818181\ldots.$ are:

*********** END **********