Indefinite Integrals Ex 19.8 Q31

Let $\quad I = \int \frac{\sec x}{\log(\sec x + \tan x)} dx \text{ - - - - - - - } (i)$

Let $\log(\sec x + \tan x) = t$ then,

$d[\log(\sec x + \tan x)] = dt$

$\Rightarrow \quad \sec x \, dx = dt \qquad \left[\because \quad \frac{d}{dx}(\log(\sec x + \tan x)) = \sec x\right]$

$\Rightarrow \quad dx = \frac{dt}{\sec x}$

Putting $\log(\sec x + \tan x) = t$ and $dx = \frac{dt}{\sec x}$ in equation (i), we get,

$$I = \int \frac{\sec x}{t} \times \frac{dt}{\sec x}$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + c$$

$$= \log|\log(\sec x + \tan x)| + c$$

$$\therefore \quad I = \log|\log(\sec x + \tan x)| + c$$

Indefinite Integrals Ex 19.8 Q32

Let $\quad I = \int \frac{\operatorname{cosec} x}{\log \tan \frac{x}{2}} dx \text{ - - - - - - - (i)}$

Let $\log \tan \frac{x}{2} = t$ then,

$d\left[\log \tan \frac{x}{2}\right] = dt$

$\Rightarrow \quad \operatorname{cosec} x \, dx = dt$

$\Rightarrow \quad dx = \frac{dt}{\operatorname{cosec} x}$

Putting $\log \tan \frac{x}{2} = t$ and $dx = \frac{dt}{\operatorname{cosec} x}$ in equation (i), we get,

$I = \int \frac{\operatorname{cosec} x}{t} \times \frac{dt}{\operatorname{cosec} x}$

$= \int \frac{dt}{t}$

$= \log|t| + c$

$= \log\left|\log \tan \frac{x}{2}\right| + c$

$\therefore \quad I = \log\left|\log \tan \frac{x}{2}\right| + c$

Indefinite Integrals Ex 19.8 Q33

Let $\quad I = \int \frac{1}{x \log x \log(\log x)} dx \text{ - - - - - - - (i)}$

Let $\log(\log x) = t$ then,

$d[\log(\log x)] = dt$

$\Rightarrow \quad \frac{1}{x} \times \frac{1}{\log x} dx = dt$

$\Rightarrow \quad dx = x \log x \, dt$

Putting $\log(\log x) = t$ and $dx = x \log x \, dt$ in equation (i), we get,

$I = \int \frac{1}{x \log x \, t} \times x \log x \, dt$

$= \int \frac{1}{t} dt$

$= \log|t| + c$

$= \log|\log(\log x)| + c$

$\therefore \quad I = \log|\log(\log x)| + c$

Indefinite Integrals Ex 19.8 Q34

Let $\quad I = \int \frac{cosec^2 x}{1+\cot x} dx$ - - - - - - - (i)

Let $1+\cot x = t$ then,

$d\left[1+\cot x\right] = dt$

$\Rightarrow \quad -cosec^2 x\, dx = dt$

$\Rightarrow \quad dx = -\frac{dt}{cosec^2 x}$

Putting $1+\cot x = t$ and $dx = \frac{-dt}{cosec^2 x}$ in equation (i), we get,

$$I = \int \frac{cosec^2 x}{t} \times -\frac{dt}{cosec^2 x}$$

$$= -\int \frac{1}{t} dt$$

$$= -\log|t| + c$$

$$= -\log|1+\cot x| + c$$

$\therefore \quad I = -\log|1+\cot x| + c$

Indefinite Integrals Ex 19.8 Q35

Let $\quad I = \int \frac{10x^9 + 10^x \log_e 10}{10^x + x^{10}} dx$ - - - - - - - (i)

Let $10^x + x^{10} = t$ then,

$d\left(10^x + x^{10}\right) = dt$

$\Rightarrow \quad \left(10^x \log_e 10 + 10x^9\right) dx = dt$

$\Rightarrow \quad dx = \frac{dt}{10x^9 + 10^x \log_e 10}$

Putting $10^x + x^{10} = t$ and $dx = \frac{dt}{10x^9 + 10^x \log_e 10}$ in equation (i), we get,

$$I = \int \frac{10x^9 + 10^x \log_e 10}{t} \times \frac{dt}{10x^9 + 10^x \log_e 10}$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + c$$

$$= \log|10^x + x^{10}| + c$$

$\therefore \quad I = \log|10^x + x^{10}| + c$

*********** END **********