Question 9. 11. A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1 m, is whirled in a vertical circle with an angular velocity of 2 rero./s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm^2. Calculate the elongation of the zvire when the mass is at the lowest point of its path. Ysteel = 2 x 10^{11} Nm^{-2}.

Ans.Here, m = 14.5 kg; l = r = 1 m; v = 2 rps; A = 0.065 x 10^4 m^2 Total pulling force on mass, when it is at the lowest position of the vertical circle is F = mg + mr w^2 = mg + mr $4\pi^2v^2$

$$= 14.5 \times 9.8 + 14.5 \times 1 \times 4 \times (22/7)^2 \times 2^2$$
$$= 142.1 + 2291.6 = 2433.9 \text{ N}$$
$$Y = \frac{F}{A} \times \frac{l}{\Delta l}$$

or
$$\Delta l = \frac{Fl}{AY} = \frac{2433.7 \times 1}{(0.065 \times 10^{-4}) \times (2 \times 10^{11})} = 1.87 \times 10^{-3} \text{ m} = 1.87 \text{ m.}$$

9.12. Compute the bulk modulus of water from the following data: Initial volume = 100.0 litre, Pressure increase = 100.0 atm (1 atm = 1.013 x 10^5 Pa), Final volume = 100.5 litre. Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large.

Answer:

Here P = 100 atmosphere

$= 100 \times 1.013 \times 10^5$ Pa $\quad (\because 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa})$

Initial volume, V_1 = 100 litre = 100×10^{-3} m^3

Final volume, V_2 = 100.5 litre = 100.5×10^{-3} m^3

$\therefore$ Change in volume = $\Delta V = V_2 - V_1$

$= (100.5 - 100) \times 10^{-3}$ m^3

$= 0.5 \times 10^{-3}$ m^3

Using formula of bulk modulus,

$$B = \frac{P}{\frac{\Delta V}{V}} = \frac{PV}{\Delta V}$$
$$= \frac{100 \times 1.013 \times 10^5 \times 100 \times 10^{-3}}{0.5 \times 10^{-3}}$$
$$B = 2.026 \times 10^9 \text{ Pa}$$

Also we know that the bulk modulus of air = 1.0×10^5 Pa

$$\text{Now, } \frac{\text{Bulk modulus of water}}{\text{Bulk modulus of air}} = \frac{2.026 \times 10^9}{1.0 \times 10^5}$$
$$= 2.026 \times 10^4$$
$$= 20260$$

The ratio is too large. This is due to the fact that the strain for air is much larger than for water at the same temperature. In other words, the intermolecular distances in case of liquids are very small as compared to the corresponding distances in the case of gases. Hence there are larger interatomic forces in liquids than in gases.

Question 9. 13. What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is 1.03 x 10^3 kg m^{-3}?

Answer:

Compressibility of water,

$$k = \frac{1}{B} = 45.8 \times 10^{-11} \text{ Pa}^{-1}$$

Change in pressure,

$$\Delta p = 80 \text{ atm} - 1 \text{ atm}$$
$$= 79 \text{ atm} = 79 \times 1.013 \times 10^5 \text{ Pa}$$

Density of water at the surface,

$$\rho = 1.03 \times 10^3 \text{ kg m}^{-3}$$

As $$B = \frac{\Delta p . V}{\Delta V} \quad \text{or} \quad \frac{\Delta V}{V} = \frac{\Delta p}{B} = \Delta p \times \frac{1}{B} = \Delta p \times k$$

or $$\frac{\Delta V}{V} = 79 \times 1.013 \times 10^5 \times 45.8 \times 10^{-11} = 3.665 \times 10^{-5}$$

Now $$\frac{\Delta V}{V} = \frac{(M/\rho)-(M/\rho')}{(M/\rho)} = 1 - \frac{\rho}{\rho'}$$

or $$\frac{\rho}{\rho'} = 1 - \frac{\Delta V}{V}$$

or $$\rho' = \frac{\rho}{1-(\Delta V/V)}$$

or $$\rho' = \frac{1.03 \times 10^3}{1-3.665 \times 10^{-3}} = \frac{1.03 \times 10^3}{0.996}$$
$$= 1.034 \times 10^3 \text{ kg/m}^3.$$

Question 9. 14. Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm.

Answer:

Here, $P = 10 \text{ atm} = 10 \times 1.013 \times 10^5 \text{ Pa}$; $k = 37 \times 10^9 \text{ Nm}^{-2}$

$$\text{Volumetric strain} = \frac{\Delta V}{V} = \frac{P}{K} = \frac{10 \times 1.013 \times 10^5}{37 \times 10^9} = 2.74 \times 10^{-5}$$

$$\therefore \text{ Fractional change in volume} = \frac{\Delta V}{V} = 2.74 \times 10^{-5}.$$

Question 9. 15. Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of 7.0×10^6 Pa.

Answer: Here a side of copper cube a = 10 cm, hence volume V = $a^3 = 10^{-3} \text{ m}^3$, hydraulic pressure applied p = 7.0×10^6 Pa and from table we find that bulk modulus of copper B = 140 G Pa = 140×10^9 Pa.

Using the relation $B = -\frac{P}{\frac{\Delta V}{V}}$, we have decrease in

$$\text{volume } \Delta V = \frac{PV}{B}$$

$$\therefore \quad \Delta V = \frac{7.0 \times 10^6 \times 10^{-3}}{140 \times 10^9} = 5 \times 10^{-8} \text{ m}^3 = 5 \times 10^{-2} \text{ cm}^3.$$

Question 9. 16. How much should be pressure the a litre of water be changed to compress it by 0.10 %? Bulk modulus of elasticity of water = $2.2 \times 10^9 \text{ Nm}^{-2}$.

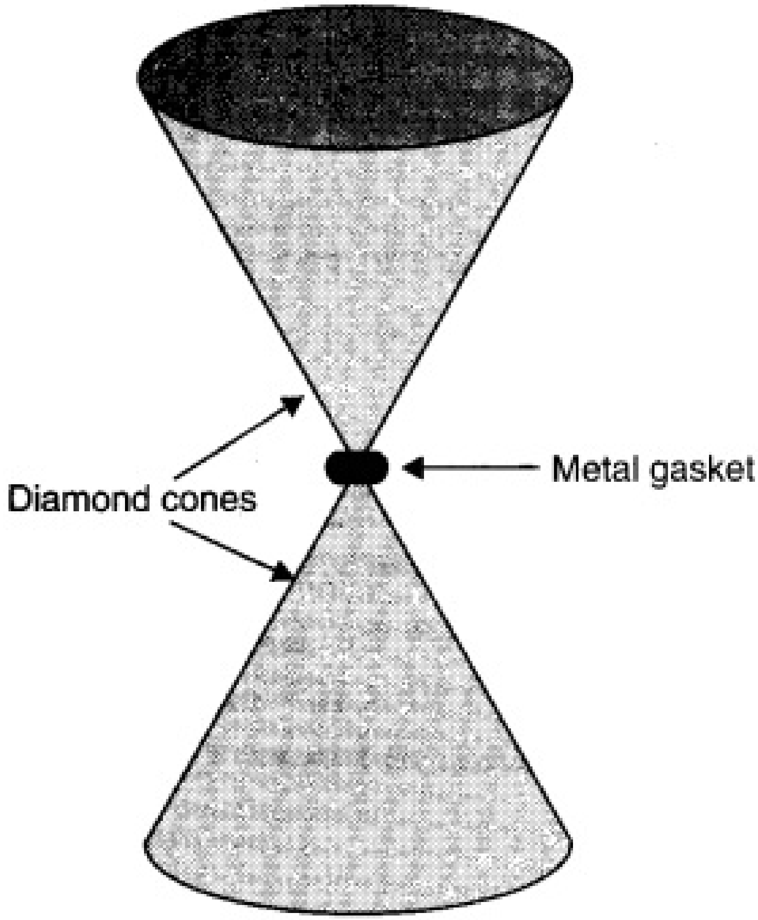
Answer:

Here, $V = 1 \text{ litre} = 10^{-3} \text{ m}^3$; $\Delta V/V = 0.10/100 = 10^{-3}$

$$K = \frac{pV}{\Delta V}$$

or $$p = K\frac{\Delta V}{V} = (2.2 \times 10^9) \times 10^{-3} = 2.2 \times 10^6 \text{ Pa}.$$

Question 9. 17. Anvils made of single crystals of diamond, with the shape as shown in figure are used to investigate behaviour of materials under very high pressures. Flat faces at the narrow end of the anvil have a diameter of 0.50 mm, and the wide ends are subjected to a compressional force of 50,000 N. What is the pressure at the tip of the anvil?



Answer:

Diameter of the corner end of the anvil,

$$d = 0.50 \text{ mm} = 0.50 \times 10^{-3} \text{ m}$$

Area of cross-section of tip,

$$A = \frac{\pi d^2}{4}$$

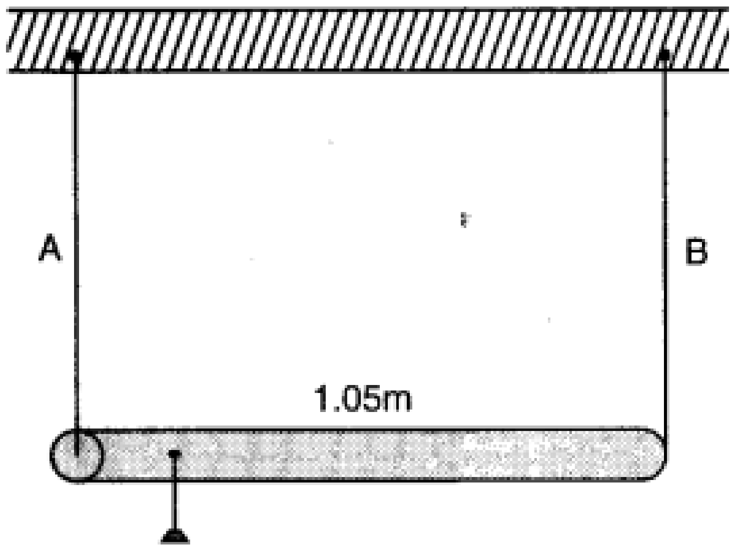
$$= \frac{22 \times (0.50 \times 10^{-3})^2}{7 \times 4} \text{m}^2$$

Stress (= pressure at the tip of the anvil)

$$= \frac{F}{A} = \frac{50{,}000 \times 4 \times 7}{22 \times (0.50)^2 \times 10^{-6}} \text{Nm}^{-2}$$

$$= 2.54 \times 10^{11} \text{ Nm}^{-2} \text{ (or Pa).}$$

Question 9. 18. A rod of length 1.05 m having negligible mass is supported at its ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths as shown in figure.



The cross-sectional areas of wires A and B are 1.0 mm^2 and 2.0 mm^2, respectively. At what point along the rod should a mass m be suspended in order to produce (a) equal stresses and (b) equal strains in both steel and aluminium wires.

Answer:

For steel wire A, $l_1=l$; A_z = 1 mm^2; Y_1= 2 x 1011 Nm^{-2}

For aluminium wire B, l_2 = l; A_2 = 2mm^2; Y_2 = 7 x 1010 Nm^{-2}

(a) Let mass m be suspended from the rod at distance x from the end where wire A is connected. Let F_1 and F_2 be the tensions in two wires and there is equal stress in two wires, then

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow \frac{F_1}{F_2} = \frac{A_1}{A_2} = \frac{1}{2} \quad \text{...}(i)$$

Taking moment of forces about the point of suspension of mass from the rod, we have

$$F_1 x = F_2 (1.05 - x) \quad \text{or} \quad \frac{1.05 - x}{x} = \frac{F_1}{F_2} = \frac{1}{2}$$

or $$2.10 - 2x = x \Rightarrow x = 0.70 \text{ m} = 70 \text{ cm}$$

(b) Let mass m be suspended from the rod at distance x from the end where wire A is connected. Let F_1 and F_2 be the tension in the wires and there is equal strain in the two wires i.e.,

$$\frac{F_1}{A_1 Y_1} = \frac{F_2}{A_2 Y_2} \Rightarrow \frac{F_1}{F_2} = \frac{A_1 Y_1}{A_2 Y_2} = \frac{1}{2} \times \frac{2 \times 10^{11}}{7 \times 10^{10}} = \frac{10}{7}$$

As the rod is stationary, so $F_1 x = F_2 (1.05 - x)$ or $\frac{1.05 - x}{x} = \frac{F_1}{F_2} = \frac{10}{7}$

$$\Rightarrow \quad 10x = 7.35 - 7x \quad \text{or} \quad x = 0.4324 \text{ m} = 43.2 \text{ cm.}$$

Question 9. 19. A mild steel wire of length 1.0 m and cross-sectional area 0.50 x 10^{-2} cm^2 is stretched, well within its elastic limit, horizontally between two pillars. A mass of 100g is suspended from the mid-point of the wire. Calculate the depression at the mid-point.

Answer: Let AB be a mild steel wire of length 2L = lm and its cross-section area A = 0.50 x 10^{-2} cm^2. A mass m = 100 g = 0.1 kg is suspended at mid-point C of wire as shown in figure. Let x be the depression at mid-point i.e., CD = x

$$\therefore \quad AD = DB = \sqrt{AC^2 + CD^2} = \sqrt{L^2 + x^2}$$

$$\therefore \text{Increase in length } \Delta L = (AD + DB) - AB = 2\sqrt{L^2 + x^2} - 2L$$

$$= 2L\left[\left(1 + \frac{x^2}{L^2}\right)^{\frac{1}{2}} - 1\right] = 2L.\frac{x^2}{2L^2} = \frac{x^2}{L}$$

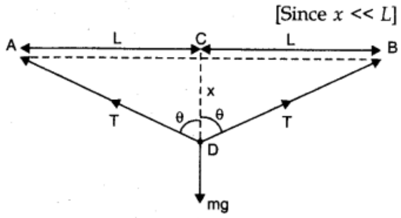
$$\therefore \text{ Longitudinal strain} = \frac{\Delta L}{2L} = \frac{x^2}{2L^2}.$$

If T be the tension in the wire as shown in Fig., then in equilibrium $2T \cos \theta = mg$

$$\text{or} \quad T = \frac{mg}{2\cos\theta}$$

$$= \frac{mg}{2\frac{x}{\sqrt{x^2 + L^2}}} = \frac{mg\sqrt{x^2 + L^2}}{2x} = \frac{mgL}{2x}$$

[Since $x << L$]



$$\therefore \quad \text{Stress} = \frac{T}{A} = \frac{mgL}{2\,x\,A}$$

As Young's modulus $Y = \dfrac{\text{stress}}{\text{strain}}$

$$= \frac{\left(\frac{mgL}{2\,x\,A}\right)}{\left(\frac{x^2}{2L^2}\right)} = \frac{mgL}{2\,x\,A} \times \frac{2L^2}{x^2} = \frac{mgL^3}{A\,x^3}$$

$$\Rightarrow \quad x = \left[\frac{mgL^3}{YA}\right]^{\frac{1}{3}} = L\left[\frac{mg}{YA}\right]^{\frac{1}{3}}$$

$$= \frac{1}{2}\left[\frac{0.1 \times 9.8}{2 \times 10^{11} \times 0.50 \times 10^{-2} \times 10^{-4}}\right]^{\frac{1}{3}} = 1.074 \times 10^{-2} \text{ m}$$

$$= 1.074 \text{ cm} \simeq 1.07 \text{ cm} \quad \text{or} \quad 0.01 \text{ m}.$$

Question 9. 20. Two strips of metal are riveted together at their ends by four rivets, each of diameter 6.0 mm. What is the maximum tension that can be exerted by the riveted strip if the shearing stress on the rivet is not to exceed 6.9×10^7 Pa? Assume that each rivet is to carry one quarter of the load.

Answer:

Diameter = 6mm; Radius, r = 3×10^{-3} m;

Maximum stress = 6.9×10^7 Pa

Maximum load on a rivet

= Maximum stress x cross-sectional area

= $6.9 \times 10^7 \times 22/7\ (3 \times 10^{-3})^2$ N = 1952 N

Maximum tension = 4 x 1951.7 N = 7.8×10^3 N.

*********** END **********