Mean Value Theorems Ex 15.2 Q1(ix)

Here,

$$f(x) = \sqrt{25 - x^2} \text{ on } [-3, 4]$$

Given function is continuous as it has unique value for each $x \in [-3, 4]$ and

$$f'(x) = \frac{-2x}{2\sqrt{25 - x^2}}$$

$$f'(x) = \frac{-x}{\sqrt{25 - x^2}}$$

So, $f'(x)$ exists for all values for $x \in (-3, 4)$ so, $f(x)$ is differentiable in $(-3, 4)$. So, Lagrange's mean value theorem is applicable. Thus, there exists a point $c \in (-3, 4)$ such that

$$f'(c) = \frac{f(4) - f(-3)}{4 + 3}$$

$$\Rightarrow \quad \frac{-2c}{2\sqrt{25 - c^2}} = \frac{\sqrt{9} - \sqrt{16}}{7}$$

$$\Rightarrow \quad -7c = -\sqrt{25 - c^2}$$

Squaring both the sides,

$$49c^2 = 25 - c^2$$

$$c^2 = \frac{1}{2}$$

$$c = \pm\frac{1}{\sqrt{2}} \in (-3, 4)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(x)

Here,

$$f(x) = \tan^{-1} x \text{ on } [0, 1]$$

We know that, $\tan^{-1} x$ has unique value in $[0, 1]$ so, it is continuous in $[0, 1]$

$$f'(x) = \frac{1}{1 + x^2}$$

So, $f'(x)$ exists for each $x \in (0, 1)$

So, $f'(x)$ is differentiable in $(0, 1)$, thus Lagrange's mean value theorem is applicable, so there exist a point $c \in (0, 1)$ such that

$$f'(c) = \frac{f(1) - f(0)}{1 - 0}$$

$$\Rightarrow \quad \frac{1}{1 + c^2} = \frac{\tan^{-1}(1) - \tan^{-1}(0)}{1}$$

$$\Rightarrow \quad \frac{1}{1 + c^2} = \frac{\frac{\pi}{4} - 0}{1}$$

$$\Rightarrow \quad \frac{4}{\pi} = 1 + c^2$$

$$\Rightarrow \quad c = \sqrt{\frac{4}{\pi} - 1}$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(xi)

Here,

$$f(x) = x + \frac{1}{x} \text{ on } [1,3]$$

$f(x)$ attiams a unique value for each $x \in [1,3]$, so it is continuous

$f'(x) = 1 - \frac{1}{x^2}$ is definded for each $x \in (1,3)$

$\Rightarrow$ $f(x)$ is differentiable in $(1,3)$, so Lagrange's mean value theorem is a applicable, so there exist a point $c \in (1,3)$ such that

$$f'(c) = \frac{f(3) - f(1)}{3 - 1}$$

$$\Rightarrow \quad 1 - \frac{1}{c^2} = \frac{\left(3 + \frac{1}{3} - (1+1)\right)}{2}$$

$$\Rightarrow \quad 1 - \frac{1}{c^2} = \frac{\frac{10}{3} - 2}{2}$$

$$\Rightarrow \quad 1 - \frac{1}{c^2} = \frac{4}{3 \times 2}$$

$$\Rightarrow \quad 1 - \frac{2}{3} = \frac{1}{c^2}$$

$$\Rightarrow \quad c^2 = 3$$

$$\Rightarrow \quad c = \sqrt{3} \in (1,3)$$

So, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(xii)

Here,

$$f(x) = x(x+4)^2 \text{ on } [0,4]$$

We know that every polynomial function is continuous and differentiable every wher, so, $f(x)$ is continuous in $[0,4]$ and differentiable in $(0,4)$, so, Lagrange's mean value theorem is applicable, thus there exist a point $c \in (0,4)$ such that

$$f'(c) = \frac{f(4) - f(0)}{4 - 0}$$

$$\Rightarrow \quad 3c^2 + 16c + 16 = \frac{4 \times (8)^2 - 0}{4}$$

$$\Rightarrow \quad 3c^2 + 16c + 16 = 64$$

$$\Rightarrow \quad 3c^2 + 16c - 48 = 0$$

$$\Rightarrow \quad c = \frac{-16 \pm \sqrt{256 + 576}}{6}$$

$$\Rightarrow \quad = \frac{-16 \pm \sqrt{832}}{6}$$

$$\Rightarrow \quad = \frac{-16 \pm 8\sqrt{13}}{6}$$

$$\Rightarrow \quad c = \frac{-8 \pm 4\sqrt{13}}{3}$$

$$c = \frac{-8 + 4\sqrt{13}}{3} \in (0,4)$$

Hence, Lagrange's mean value theorem is verified.

*********** END **********