Exercise 7A

Question 7

$$\text{LHS} = \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{\sin\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)} + \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{\sin\theta(1-\cos\theta)}{1-\cos^2\theta} + \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{\sin\theta(1-\cos\theta)}{\sin^2\theta} + \frac{1+\cos\theta}{\sin\theta} = \frac{(1-\cos\theta)}{\sin\theta} + \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{1-\cos\theta+1+\cos\theta}{\sin\theta} = \frac{2}{\sin\theta} = 2\operatorname{cosec}\theta = \text{RHS}$$

$\therefore$ LHS = RHS

Question 8

$$\text{LHS} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$$

$$= \frac{(\sin\theta-\cos\theta)\left(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta\right)}{(\sin\theta-\cos\theta)\sin\theta\cos\theta}$$

$$\left[\because a^3-b^3 = (a-b)\left(a^2+ab+b^2\right)\right]$$

$$= \frac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} + 1 = 1 + \sec\theta\operatorname{cosec}\theta = \text{RHS}$$

$\therefore$ LHS = RHS

********** END **********