



Question 11:

By using properties of determinants, show that:

$$(i) \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

$$(ii) \begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$$

Answer

$$(i) \Delta = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$, we have:

$$\Delta = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$, $C_3 \rightarrow C_3 - C_1$, we have:

$$\Delta = (a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+b+c) \end{vmatrix}$$

$$= (a+b+c)^3 \begin{vmatrix} 1 & 0 & 0 \\ 2b & -1 & 0 \\ 2c & 0 & -1 \end{vmatrix}$$

Expanding along C_3 , we have:

$$\Delta = (a+b+c)^3 (-1)(-1) = (a+b+c)^3$$

Hence, the given result is proved.

$$(ii) \Delta = \begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$, we have:

$$\Delta = \begin{vmatrix} 2(x+y+z) & x & y \\ 2(x+y+z) & y+z+2x & y \\ 2(x+y+z) & x & z+x+2y \end{vmatrix}$$

$$= 2(x+y+z) \begin{vmatrix} 1 & x & y \\ 1 & y+z+2x & y \\ 1 & x & z+x+2y \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we have:

$$\Delta = 2(x+y+z) \begin{vmatrix} 1 & x & y \\ 0 & x+y+z & 0 \\ 0 & 0 & x+y+z \end{vmatrix}$$

$$= 2(x+y+z)^3 \begin{vmatrix} 1 & x & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding along R_3 , we have:

$$\Delta = 2(x+y+z)^3 (1)(1-0) = 2(x+y+z)^3$$

Hence, the given result is proved.

Question 12:

By using properties of determinants, show that:

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$$

Answer

$$\Delta = \begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} x & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$, we have:

$$\begin{aligned} \Delta &= \begin{vmatrix} 1+x+x^2 & 1+x+x^2 & 1+x+x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} \\ &= (1+x+x^2) \begin{vmatrix} 1 & 1 & 1 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} \end{aligned}$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, we have:

$$\begin{aligned} \Delta &= (1+x+x^2) \begin{vmatrix} 1 & 0 & 0 \\ x^2 & 1-x^2 & x-x^2 \\ x & x^2-x & 1-x \end{vmatrix} \\ &= (1+x+x^2)(1-x)(1-x) \begin{vmatrix} 1 & 0 & 0 \\ x^2 & 1+x & x \\ x & -x & 1 \end{vmatrix} \\ &= (1-x^3)(1-x) \begin{vmatrix} 1 & 0 & 0 \\ x^2 & 1+x & x \\ x & -x & 1 \end{vmatrix} \end{aligned}$$

Expanding along R_1 , we have:

$$\begin{aligned} \Delta &= (1-x^3)(1-x)(1) \begin{vmatrix} 1+x & x \\ -x & 1 \end{vmatrix} \\ &= (1-x^3)(1-x)(1+x+x^2) \\ &= (1-x^3)(1-x^3) \\ &= (1-x^3)^2 \end{aligned}$$

Hence, the given result is proved.

Question 13:

By using properties of determinants, show that:

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

Answer

$$\Delta = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + bR_3$ and $R_2 \rightarrow R_2 - aR_3$, we have:

$$\begin{aligned} \Delta &= \begin{vmatrix} 1+a^2+b^2 & 0 & -b(1+a^2+b^2) \\ 0 & 1+a^2+b^2 & a(1+a^2+b^2) \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} \\ &= (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} \end{aligned}$$

Expanding along R_1 , we have:

$$\begin{aligned} \Delta &= (1+a^2+b^2)^2 \left[(1) \begin{vmatrix} 1 & a \\ -2a & 1-a^2-b^2 \end{vmatrix} - b \begin{vmatrix} 0 & 1 \\ 2b & -2a \end{vmatrix} \right] \\ &= (1+a^2+b^2)^2 [1-a^2-b^2+2a^2-b(-2b)] \\ &= (1+a^2+b^2)^2 (1+a^2+b^2) \\ &= (1+a^2+b^2)^3 \end{aligned}$$

Question 14:

By using properties of determinants, show that:

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

Answer

$$\Delta = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$$

Taking out common factors a , b , and c from R_1 , R_2 , and R_3 respectively, we have:

$$\Delta = abc \begin{vmatrix} a+\frac{1}{a} & b & c \\ a & b+\frac{1}{b} & c \\ a & b & c+\frac{1}{c} \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we have:

$$\Delta = abc \begin{vmatrix} a + \frac{1}{a} & b & c \\ -\frac{1}{a} & \frac{1}{b} & 0 \\ -\frac{1}{a} & 0 & \frac{1}{c} \end{vmatrix}$$

Applying $C_1 \rightarrow aC_1$, $C_2 \rightarrow bC_2$, and $C_3 \rightarrow cC_3$, we have:

$$\begin{aligned} \Delta &= abc \times \frac{1}{abc} \begin{vmatrix} a^2 + 1 & b^2 & c^2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} \\ &= \begin{vmatrix} a^2 + 1 & b^2 & c^2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} \end{aligned}$$

Expanding along R_3 , we have:

$$\begin{aligned} \Delta &= -1 \begin{vmatrix} b^2 & c^2 \\ 1 & 0 \end{vmatrix} + 1 \begin{vmatrix} a^2 + 1 & b^2 \\ -1 & 1 \end{vmatrix} \\ &= -1(-c^2) + (a^2 + 1 + b^2) = 1 + a^2 + b^2 + c^2 \end{aligned}$$

Hence, the given result is proved.

Question 15:

Choose the correct answer.

Let A be a square matrix of order 3×3 , then $|kA|$ is equal to

A. $k|A|$ **B.** $k^2|A|$ **C.** $k^3|A|$ **D.** $3k|A|$

Answer

Answer: C

A is a square matrix of order 3×3 .

***** END *****