Answer

Frequency of the electromagnetic wave, $\nu = 2.0 \times 10^{10}$ Hz

Electric field amplitude, $E_0 = 48 \text{ V m}^{-1}$

Speed of light, $c = 3 \times 10^8$ m/s

(a) Wavelength of a wave is given as:

$$\lambda = \frac{c}{\nu}$$

$$= \frac{3\times10^8}{2\times10^{10}} = 0.015 \text{ m}$$

(b) Magnetic field strength is given as:

$$B_0 = \frac{E_0}{c}$$

$$= \frac{48}{3\times10^8} = 1.6\times10^{-7} \text{ T}$$

(c) Energy density of the electric field is given as:

$$U_E = \frac{1}{2}\epsilon_0 E^2$$

And, energy density of the magnetic field is given as:

$$U_B = \frac{1}{2\mu_0}B^2$$

Where,

ϵ_0 = Permittivity of free space

μ_0 = Permeability of free space

We have the relation connecting E and B as:

$E = cB$ ... (1)

Where,

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \quad \ldots (2)$$

Putting equation (2) in equation (1), we get

$$E = \frac{1}{\sqrt{\epsilon_0 \mu_0}} B$$

Squaring both sides, we get

$$E^2 = \frac{1}{\epsilon_0 \mu_0} B^2$$

$$\epsilon_0 E^2 = \frac{B^2}{\mu_0}$$

$$\frac{1}{2} \epsilon_0 E^2 = \frac{1}{2} \frac{B^2}{\mu_0}$$

$$\Rightarrow U_E = U_B$$

Question 8.11:

Suppose that the electric field part of an electromagnetic wave in vacuum is **E** = {(3.1 N/C) cos [(1.8 rad/m) y + (5.4 × 10^6 rad/s)t]}$\hat{i}$.

(a) What is the direction of propagation?

(b) What is the wavelength λ?

(c) What is the frequency v?

(d) What is the amplitude of the magnetic field part of the wave?

(e) Write an expression for the magnetic field part of the wave.

Answer

(a) From the given electric field vector, it can be inferred that the electric field is directed along the negative x direction. Hence, the direction of motion is along the negative y direction i.e., $-\hat{j}$.

(b) It is given that,

$$\vec{E} = 3.1 \text{ N/C} \cos\left[\left(1.8 \text{ rad/m}\right) y + \left(5.4 \times 10^8 \text{ rad/s}\right) t\right] \hat{i} \quad \ldots (1)$$

The general equation for the electric field vector in the positive x direction can be written as:

$$\vec{E} = E_0 \sin\left(kx - \omega t\right) \hat{i} \quad \ldots (2)$$

On comparing equations (1) and (2), we get

Electric field amplitude, E_0 = 3.1 N/C

Angular frequency, ω = 5.4 × 10^8 rad/s

Wave number, k = 1.8 rad/m

Wavelength, $\lambda = \frac{2\pi}{1.8}$ = 3.490 m

(c) Frequency of wave is given as:

$$v = \frac{\omega}{2\pi}$$

$$= \frac{5.4 \times 10^8}{2\pi} = 8.6 \times 10^7 \text{ Hz}$$

(d) Magnetic field strength is given as:

$$B_0 = \frac{E_0}{c}$$

Where,

c = Speed of light = 3 × 10^8 m/s

$$\therefore B_0 = \frac{3.1}{3 \times 10^8} = 1.03 \times 10^{-7} \text{ T}$$

(e) On observing the given vector field, it can be observed that the magnetic field vector is directed along the negative z direction. Hence, the general equation for the magnetic field vector is written as:

$$\vec{B} = B_0 \cos\left(ky + \omega t\right) \hat{k}$$

$$= \left\{\left(1.03 \times 10^{-7} \text{ T}\right) \cos\left[\left(1.8 \text{ rad/m}\right) y + \left(5.4 \times 10^6 \text{ rad/s}\right) t\right]\right\} \hat{k}$$

Question 8.12:

About 5% of the power of a 100 W light bulb is converted to visible radiation. What is the average intensity of visible radiation

(a) at a distance of 1 m from the bulb?

(b) at a distance of 10 m?

Assume that the radiation is emitted isotropically and neglect reflection.

Answer

Power rating of bulb, P = 100 W

It is given that about 5% of its power is converted into visible radiation.

∴ Power of visible radiation,

$$P' = \frac{5}{100} \times 100 = 5 \text{ W}$$

Hence, the power of visible radiation is 5W.

(a) Distance of a point from the bulb, d = 1 m

Hence, intensity of radiation at that point is given as:

$$I = \frac{P'}{4\pi d^2}$$
$$= \frac{5}{4\pi (1)^2} = 0.398 \text{ W/m}^2$$

(b) Distance of a point from the bulb, d_1 = 10 m

Hence, intensity of radiation at that point is given as:

$$I = \frac{P'}{4\pi (d_1)^2}$$
$$= \frac{5}{4\pi (10)^2} = 0.00398 \text{ W/m}^2$$

*********** END **********