Exercise 11C

Question 31:
First term 'a' of an AP = 2
The last term l = 29

Sum of n terms $= S_n = \frac{n}{2}(a+l) = 155$

$\therefore \frac{n}{2}(2+29) = 155$

$n = \frac{155 \times 2}{31} = 10$

Also, $l = a + (n-1)d$

or $29 = 2 + (10-2)d = 2 + 9d$

$\Rightarrow 9d = 29 - 2 = 27 \quad \therefore d = \frac{27}{9} = 3$

$\therefore$ common difference = 3

Question 32:

The given AP is $\left(1-\frac{1}{n}\right)+\left(1-\frac{2}{n}\right)+\left(1-\frac{3}{n}\right)+\ldots$

First term $a = 1 - \frac{1}{n}$

Common difference $d = \left(1-\frac{2}{n}\right)-\left(1-\frac{1}{n}\right) = -\frac{1}{n}$

Sum of n terms $= S_n = \frac{n}{2}\left[2a+(n-1)d\right]$

$$\therefore \quad S_n = \frac{n}{2}\left[2\left(1-\frac{1}{n}\right)+(n-1)\left(-\frac{1}{n}\right)\right]$$

$$= \frac{n}{2}\left(2-\frac{2}{n}-\frac{n}{n}+\frac{1}{n}\right)$$

$$= \frac{n}{2}\left(1-\frac{1}{n}\right) = \frac{n}{2} \times \frac{n-1}{n} = \frac{n-1}{2}$$

Question 33:
TV production every year forms an AP
Let a be the TV production in first year and d be the common difference
Production in nth year = a + (n - 1)d
production in 6^{th} year = a + 5d = 8000
a + 5d = 8000 ----(1)
Production in 9^{th} year = a + (9 - 1)d = 11300
a + 8d = 11300 ----(2)
Subtracting (1) from (2), we get

$$3d = 11300 - 8000 = 3300$$

$$\therefore\ d = \frac{3300}{3} = 1100$$

From (1), $a + 5 \times 1100 = 8000$

$$\Rightarrow\ a = 8000 - 5500 = 2500$$

$$\text{Production in } n^{th} \text{ year} = a + (n-1)d$$

$$\therefore\ \text{Production in } 8^{th} \text{ year} = 2500 + (8-1) \times 1100$$
$$= 2500 + 7700 = 10200$$

$$\text{Total Production in } n \text{ years} = \frac{n}{2}(a + l)$$

$$\therefore \text{Production in 6 years} = \frac{6}{2}(2500 + 8000)$$
$$= 3 \times 10500 = 31500$$

Thus,
(i) TV production in first year = 2500
(ii) Production in 8^{th} year = 10200
(iii) Total production in 6 years = 31500.

Question 34:
Let the value of first prize be Rs. a
Subsequent prizes are Rs (a - 200), Rs (a - 400) and Rs (a - 600)
Total value of these prizes

$$= \text{Rs}\left[a + (a - 200) + (a - 400) + (a - 600)\right]$$
$$= \text{Rs}\left[4a - 1200\right]$$

Value of these prizes = Rs. 2800 (given)

$$\therefore\ 4a - 1200 = 2800$$
$$4a = 2800 + 1200 = 4000$$
$$\therefore\ a = \frac{4000}{4} = 1000$$

Hence the first prize is Rs. 1000 and subsequent prizes are worth Rs. 800, Rs. 600 and Rs. 400.

Question 35:
Number of logs in 1^{st} row (from bottom) = 20
Number of logs in 2^{nd} row = 19
Number of logs in 3^{rd} row = 18
Let there are n number of rows
20 + 19 + 18 + ... to n terms = 200

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = 200$$

$$\Rightarrow \frac{n}{2}\left[2 \times 20 + (n-1) \times (-1)\right] = 200$$

$$\Rightarrow \frac{n}{2}\left[40 - n + 1\right] = 200$$

$$\Rightarrow \frac{n}{2}(41 - n) = 200 \quad \text{or} \quad 41n - n^2 = 400$$

$$\Rightarrow n^2 - 41n + 400 = 0$$

$$\Rightarrow n^2 - 16n - 25n + 400 = 0$$

$$\Rightarrow n(n-16) - 25(n-16) = 0 \quad \text{or} \quad (n-16)(n-25) = 0$$

$$\therefore\ n = 16 \text{ or } 25$$

For n = 16, number of logs in nth row
= a + (n - 1)d
= 20 + (16- 1)(-1)
= 20 -15
= 5
n = 25, since number of logs in 25^{th} row is negative which is not admissible.
Thus, there are 16 rows and number of logs at the top is 5.

*********** END **********