



Differentials Errors and Approximation Ex 14.1 Q9(xxvi)

Let $y = f(x) = \sqrt{x}$, $x = 49$ and $x + \Delta x = 49.5$

Then $\Delta x = 0.5$

For $x = 49$ we have

$$y = \sqrt{49} = 7$$

$$dx = \Delta x = 0.5$$

$$y = \sqrt{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\Rightarrow \left(\frac{dy}{dx} \right)_{x=49} = \frac{1}{2 \times 7} = \frac{1}{14}$$

$$\therefore dy = \frac{dy}{dx} dx$$

$$\Rightarrow dy = \frac{1}{14} (0.5) = \frac{5}{140}$$

$$\Rightarrow \Delta y = \frac{1}{28}$$

Hence,

$$\sqrt{49.5} = y + \Delta y = 7 + \frac{1}{28} = 7 + 0.0357 = 7.0357$$

Differentials Errors and Approximation Ex 14.1 Q9(xxvii)

Define a function $y = x^{3/2}$

For $x = 4$, $y = 8$

$$x + \Delta x = 3.968 \Rightarrow \Delta x = 3.968 - 4 = -0.032$$

$$\frac{dy}{dx} = \frac{3}{2} x^{1/2}$$

$$\Rightarrow dy = \left(\frac{3}{2} x^{1/2} \right) dx$$

$$\Rightarrow \Delta y|_{x=4} \approx (3) \Delta x$$

$$\Rightarrow \Delta y|_{x=4} \approx 3 \times (-0.032) = -0.096$$

$$\begin{aligned} (3.968)^{3/2} &= y + \Delta y = 8 - 0.096 \\ &= 7.904 \end{aligned}$$

Differentials Errors and Approximation Ex 14.1 Q9(xxviii)

Let $y = f(x) = x^5$, $x = 2$ and $x + \Delta x = 1.999$

Then $\Delta x = -0.001$

For $x = 2$ we have

$$y = (2)^5 = 32$$

$$dx = \Delta x = -0.001$$

$$y = x^5$$

$$\Rightarrow \frac{dy}{dx} = 5x^4$$

$$\Rightarrow \left(\frac{dy}{dx} \right)_{x=2} = 5(2)^4 = 80$$

$$\therefore dy = \frac{dy}{dx} dx$$

$$\Rightarrow dy = 80(-0.001) = -0.080$$

$$\Rightarrow \Delta y = -0.080$$

Hence,

$$(1.999)^5 = y + \Delta y = 32 - 0.080 = 31.920$$

Differentials Errors and Approximation Ex 14.1 Q9(xxix)

Let $y = f(x) = \sqrt{x}$, $x = 0.09$ and $x + \Delta x = 0.082$

Then $\Delta x = -0.008$

For $x = 0.09$ we have

$$y = \sqrt{0.09} = 0.3$$

$$dx = \Delta x = -0.008$$

$$y = \sqrt{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\Rightarrow \left(\frac{dy}{dx} \right)_{x=1} = \frac{1}{2 \times \sqrt{0.09}} = \frac{1}{2 \times 0.3} = \frac{1}{0.6}$$

$$\therefore dy = \frac{dy}{dx} dx$$

$$\Rightarrow dy = \frac{1}{0.6} (-0.008)$$

$$\Rightarrow \Delta y = -\frac{8}{600}$$

Hence,

$$\sqrt{0.082} = y + \Delta y = 0.3 - \frac{8}{600} = 0.3 - 0.0133 = 0.2867$$

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