Differentiation Ex 11.4 Q21

Here,

$$y = x\sin(a+y) \qquad \text{---(i)}$$

Differentiating with respect to y,

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left[x\sin(a+y)\right]$$

$$\Rightarrow \frac{dy}{dx} = x\frac{d}{dx}\sin(a+y) + \sin(a+y)\frac{d}{dx}(x) \qquad \text{[Using product rule, chain ruel]}$$

$$\Rightarrow \frac{dy}{dx} = x\cos(a+y)\frac{d}{dx}(a+y) + \sin(a+y)(1)$$

$$= x\cos(a+y)\left(0+\frac{dy}{dx}\right) + \sin(a+y)$$

$$\Rightarrow \frac{dy}{dx}\left(1 - x\cos(a+y)\right) = \sin(a+y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin(a+y)}{1 - x\cos(a+y)}$$

Put x from equation (i), $x = \dfrac{y}{\sin(a+y)}$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin(a+y)}{1 - \dfrac{y}{\sin(a+y)}\cos(a+y)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin(a+y) - y\cos(a+y)}$$

Differentiation Ex 11.4 Q22

Here,

$$x\sin(a+y) + \sin a\cos(a+y) = 0 \qquad \text{---(i)}$$

Differentiating with respect to x,

$$\Rightarrow \frac{d}{dx}\left[x\sin(a+y)\right] + \frac{d}{dx}\left[\sin a\cos(a+y)\right] = 0$$

$$\Rightarrow \left[x\frac{d}{dx}\sin(a+y) + \sin(a+y)\frac{d}{dx}(x)\right] + \sin a\frac{d}{dx}\cos(a+y) = 0$$

[Using product rule and chain rule]

$$\Rightarrow \left[x\cos(a+y)\frac{d}{dx}(a+y) + \sin(a+y)(1)\right] + \sin a\left[-\sin(a+y)\frac{d}{dx}(a+y)\right] = 0$$

$$\Rightarrow \left[x\cos(a+y)\left(0+\frac{dy}{dx}\right) + \sin(a+y)\right] - \sin a\sin(a+y)\left(0+\frac{dy}{dx}\right) = 0$$

$$\Rightarrow x\cos(a+y)\frac{dy}{dx} + \sin(a+y) - \sin a\sin(a+y)\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx}\left[x\cos(a+y) - \sin a\sin(a+y)\right] = -\sin(a+y)$$

Put $x = -\sin a\dfrac{\cos(a+y)}{\sin(a+y)}$ from equation (i),

$$\Rightarrow \frac{dy}{dx}\left[-\sin a\frac{\cos^2(a+y)}{\sin(a+y)} - \sin a\sin(a+y)\right] = -\sin(a+y)$$

$$\Rightarrow -\frac{dy}{dx}\left[\frac{\sin a\cos^2(a+y) + \sin a\sin^2(a+y)}{\sin(a+y)}\right] = -\sin(a+y)$$

$$\Rightarrow \frac{dy}{dx} = \sin(a+y)^2\left[\frac{\sin(a+y)}{\sin a\left(\cos^2(a+y) + \sin^2(a+y)\right)}\right]$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a} \qquad \left[\text{Since } \sin^2\theta + \cos^2\theta = 1\right]$$

Differentiation Ex 11.4 Q23

Here,

$$y = x\sin y$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{dy}{dx} = \frac{d}{dx}(x\sin y)$$

$$\Rightarrow \quad \frac{dy}{dx} = x\frac{d}{dx}(\sin y) + \sin y\frac{d}{dx}(x) \qquad \text{[Using product rule]}$$

$$\Rightarrow \quad \frac{dy}{dx} = x\cos\frac{dy}{dx} + \sin y\,(1)$$

$$\Rightarrow \quad \frac{dy}{dx}(1 - x\cos y) = \sin y$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\sin y}{1 - x\cos y}$$

Differentiation Ex 11.4 Q24

Here,

$$y\sqrt{x^2+1} = \log\left(\sqrt{x^2+1} - x\right)$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{d}{dx}\left(y\sqrt{x^2+1}\right) = \frac{d}{dx}\log\left(\sqrt{x^2+1} - x\right) \qquad \text{[Using product rule and chain rule]}$$

$$\Rightarrow \quad y\frac{d}{dx}\left(\sqrt{x^2+1}\right) + \sqrt{x^2+1}\frac{dy}{dx} = \frac{1}{\left(\sqrt{x^2+1} - x\right)} \times \frac{d}{dx}\left(\sqrt{x^2+1} - x\right)$$

$$\Rightarrow \quad y\frac{1}{2\sqrt{x^2+1}} \times \frac{d}{dx}\left(x^2+1\right) + \sqrt{x^2+1}\frac{dy}{dx} = \frac{1}{\left(\sqrt{x^2+1} - x\right)} \times \left[\frac{1}{2\sqrt{x^2+1}}\frac{d}{dx}\left(x^2+1\right) - 1\right]$$

$$\Rightarrow \quad \frac{2xy}{2\sqrt{x^2+1}} + \sqrt{x^2+1}\frac{dy}{dx} = \frac{1}{\left(\sqrt{x^2+1} - x\right)}\left[\frac{2x}{2\sqrt{x^2+1}} - 1\right]$$

$$\Rightarrow \quad \sqrt{x^2+1}\frac{dy}{dx} = \left[\frac{1}{\sqrt{x^2+1} - x}\right]\left[\frac{x - \sqrt{x^2+1}}{\sqrt{x^2+1}}\right] - \frac{xy}{\sqrt{x^2+1}}$$

$$\Rightarrow \quad \sqrt{x^2+1}\frac{dy}{dx} = \frac{-1}{\sqrt{x^2+1}} - \frac{xy}{\sqrt{x^2+1}}$$

$$\Rightarrow \quad \sqrt{x^2+1}\frac{dy}{dx} = \frac{-(1+xy)}{\sqrt{x^2+1}}$$

$$\Rightarrow \quad \left(x^2+1\right)\frac{dy}{dx} = -(1+xy)$$

$$\Rightarrow \quad \left(x^2+1\right)\frac{dy}{dx} + 1 + xy = 0$$

Differentiation Ex 11.4 Q25

Here,

$$y = [\log_{\cos x} \sin x][\log_{\sin x} \cos x]^{-1} + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$y = [\log_{\cos x} \sin x][\log_{\cos x} \sin x] + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$\left[\text{Since, } \log_a a = (\log_b a)^{-1}\right]$$

$$y = \left[\frac{\log \sin x}{\log \cos x}\right]^2 + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$\left[\text{Sicne, } \log_a b = \frac{\log b}{\log a}\right]$$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{\log \sin x}{\log \cos x}\right]^2 + \frac{d}{dx}\left(\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right)$$

$$= 2\left[\frac{\log \sin x}{\log \cos x}\right]\frac{d}{dx}\left(\frac{\log \sin x}{\log \cos x}\right) + \frac{1}{\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}} \times \frac{d}{dx}\left[\frac{2x}{1+x^2}\right]$$

$$\frac{dy}{dx} = 2\left[\frac{\log \sin x}{\log \cos x}\right]\left(\frac{(\log \cos x)\frac{d}{dx}(\log \sin x) - \log \sin x \frac{d}{dx}(\log \cos x)}{(\log \cos x)^2}\right) +$$

$$[\text{Using chain rule, quotient rule}] \quad \left(\frac{(1+x^2)}{\sqrt{1+x^4-2x^2}}\right)\left(\frac{(1+x^2)(2)-(2x)(2x)}{(1+x^2)^2}\right)$$

$$= 2\left(\frac{\log \sin x}{\log \cos x}\right)\left(\frac{\log \cos x \times \frac{1}{\sin x}\frac{d}{dx}(\sin x) - \log \sin x \times \frac{1}{\cos x}\frac{d}{dx}(\cos x)}{(\log \cos x)^2}\right) +$$

$$\left(\frac{(1+x^2)}{\sqrt{1+x^4-2x^2}}\right)\left(\frac{(1+x^2)(2)-(2x)(2x)}{(1+x^2)^2}\right)$$

$$= 2\left(\frac{\log\sin x}{\log\cos x}\right)\left(\frac{\log\cos x\left(\frac{\cos x}{\sin x}\right)+\log\sin x\left(\frac{\sin x}{\cos x}\right)}{(\log\cos x)^2}\right)+$$

$$\left(\frac{1+x^2}{\sqrt{\left(1-x^2\right)^2}}\right)\left(\frac{2+2x^2-4x^2}{\left(1+x^2\right)^2}\right)$$

$$\frac{dy}{dx} = 2\,\frac{\log\sin x}{(\log\cos x)^3}\left(\cot x\log\cos x+\tan x\log\sin x\right)+\frac{2}{1+x^2}$$

Put $x = \frac{\pi}{4}$

$$\frac{dy}{dx} = 2\left(\frac{\log\sin\frac{\pi}{4}}{\left(\log\cos\frac{\pi}{4}\right)^3}\right)\left(\cot\frac{\pi}{4}\log\cos\frac{\pi}{4}+\tan\frac{\pi}{4}\log\sin\frac{\pi}{4}\right)+2\left(\frac{1}{1+\left(\frac{\pi}{4}\right)^2}\right)$$

$$= 2\left(\frac{1}{\left(\log\frac{1}{\sqrt{2}}\right)^2}\right)\left(1\times\log\frac{1}{\sqrt{2}}+1\times\log\frac{1}{\sqrt{2}}\right)+2\left(\frac{16}{16+\pi}\right)$$

$$= 2\times\frac{2\log\left(\frac{1}{\sqrt{2}}\right)}{\left(\log\left(\frac{1}{\sqrt{2}}\right)\right)}+\frac{32}{16+\pi^2}$$

$$= 4\,\frac{1}{\log\left(\frac{1}{\sqrt{2}}\right)}+\frac{32}{16+\pi^2}$$

$$= 4\,\frac{1}{-\frac{1}{2}\log^2}+\frac{32}{16+\pi^2}$$

$$= -\frac{8}{\log 2}+\frac{32}{16+\pi^2}$$

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = 8\left[\frac{4}{16+\pi^2}-\frac{1}{\log 2}\right]$$

*********** END **********