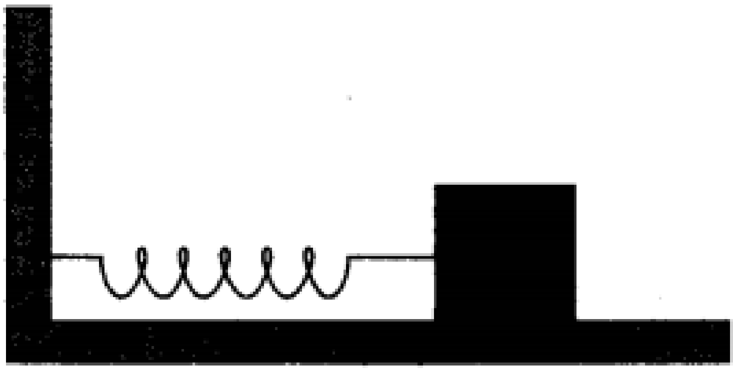
Question 14. 9. A spring having with a spring constant 1200 Nm^{-1} is mounted on a horizontal table as shown in Fig. A mass of 3 kg is attached to the free end of the spring. The mass is then pulled sideways to a distance of 2.0 cm and released. Determine (i) the frequency of oscillations, (ii) maximum acceleration of the mass, and (iii) the maximum speed of the mass.



Answer:

Here, $K = 1200\ Nm^{-1};\ m = 3.0\ kg,\ a = 2.0\ cm = 0.02\ m$

(i) Frequency, $\nu = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\times 3.14}\sqrt{\frac{1200}{3}} = 3.2\ s^{-1}$

(ii) Acceleration, $A = \omega^2\ y = \frac{k}{m}y$

Acceleration will be maximum when y is maximum *i.e.*, $y = a$

$\therefore$ max. acceleration, $A_{max} = \frac{ka}{m} = \frac{1200\times 0.02}{3} = 8\ ms^{-2}$

(iii) Max. speed of the mass will be when it is passing through mean position

$$V_{max} = a\omega = a\sqrt{\frac{k}{m}} = 0.02\times\sqrt{\frac{1200}{3}} = 0.4\ ms^{-1}$$

Question 14. 10. In Exercise 9, let us take the position of mass when the spring is unstreched as x = 0, and the direction from left to right as the positive direction of x - axis. Give x as a function of time t for the oscillating mass if at the moment we start the stopwatch (t = 0), the mass is (a) at the mean position, (b) at the maximum stretched position, and (c)at the maximum compressed position.In what way do these functions for SHM differ from each other, in frequency, in amplitude or the initial phase?

Answer:

$$a = 2\ cm,\quad \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1200}{3}}s^{-1} = 20s^{-1}$$

(a) Since time is measured from mean position,

$$x = a\sin\omega t = 2\sin 20t$$

(b) At the maximum stretched position, the body is at the extreme right position. The initial phase is $\frac{\pi}{2}$.

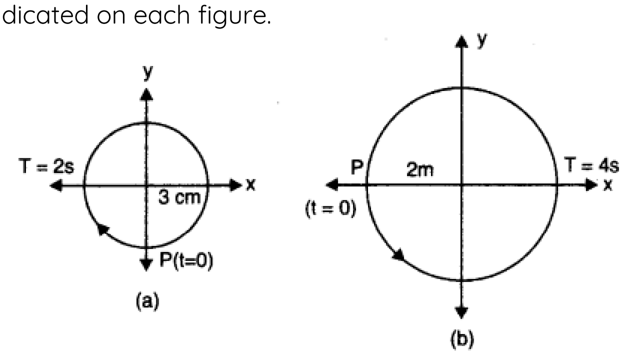
$$\therefore\quad x = a\sin(\omega t + \frac{\pi}{2}) = a\cos\omega t = 2\cos 20\,t$$

(c) At the maximum compressed position, the body is at the extreme left position. The initial phase is $\frac{3\pi}{2}$.

$$\therefore\quad x = a\sin(\omega t+\frac{3\pi}{2}) = -a\cos\omega t = -2\cos 20t$$

Note: The functions neither differ in amplitude nor in frequency. They differ only in initial phase.

Question 14. 11. Following figures correspond to two circular motions. The radius of the circle, the period of revolution, the initial position, and the sense of revolution (i.e., clockwise or anticlockwise) are indicated on each figure.



Obtain the corresponding simple harmonic motions of the x-projection of the radius vector of the revolving particle P in each case.

Answer:

(1) Let A be any point on the circle of reference of the fig. (a) From A, draw BN perpen-dicular on x-axis.

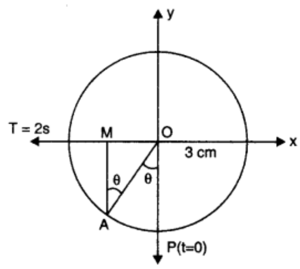
If $\angle POA = \theta$, then

$\angle OAM = \theta = \omega t$

$\therefore$ In triangle OAM,

$$\frac{OM}{OA} = \sin\theta$$

$$\therefore \quad \frac{-x}{3} = \sin\omega t = \sin\frac{2\pi}{T}t$$



[x is –ve in fourth quadrant]

$$\therefore \quad x = -3\sin\frac{2\pi}{2}t \quad \text{or} \quad x = -3$$

$\sin\pi t$

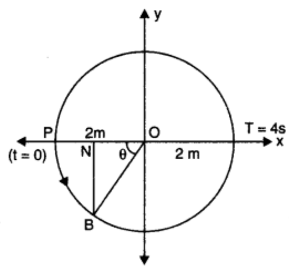
which is the equation of SHM.

(2) Let B be any point on the circle of reference of fig. (b). From B, draw BN perpendicular on x-axis.

Then $\angle BON = \theta = \omega t$

$$\therefore \text{ In } \Delta ONB, \quad \cos\theta = \frac{ON}{OB}$$

or $ON = OB\cos\theta$



$$\therefore \quad -x = 2\cos\omega t$$

$$\Rightarrow \quad x = -2\cos\frac{2\pi}{T}t = -2\cos\frac{2\pi}{4}t$$

$$\therefore \quad x = -2\cos\frac{\pi}{4}t \quad \text{which is equation of SHM}$$

Question 14. 12. Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial (t = 0) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anti-clockwise in every case:

(x is in cm and t is in s)

(a) $x = -2\sin(3t + \pi/3)$

(b) $x = \cos(\pi/6 - t)$

(c) $x = 3\sin(2\pi t + \pi/4)$

(d) $x = 2\cos\pi t$.

Answer:

(*a*) $$x = 2\cos\left(3t + \frac{\pi}{3} + \frac{\pi}{2}\right)$$

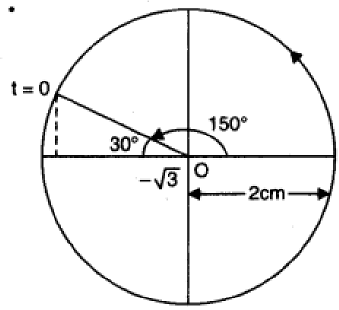
Radius of the reference circle, r = amplitude of SHM = 2 cm,

At $\quad t = 0, \quad x = -2\sin\frac{\pi}{3} = \frac{-2\sqrt{3}}{2} = -\sqrt{3}$ cm

Also $\quad \omega t = 3t \quad \therefore \omega = 3$ rad/s

$$\cos\phi_0 = -\frac{\sqrt{3}}{2}, \quad \phi_0 = 150°$$

The reference circle is, thus, as plotted below.



(*b*) $$x = \cos\left(t - \frac{\pi}{6}\right)$$

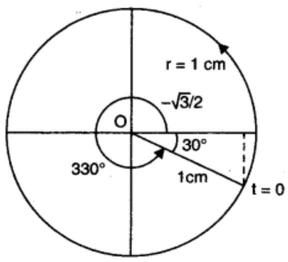
Radius of circle, r = amplitude of SHM = 1 cm.

At $\quad t = 0, \quad x = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ cm

Also $\quad \omega t = 1t \Rightarrow \omega = 1$ rad/s

$$\cos\phi_0 = \frac{\sqrt{3}}{2}, \quad \phi_0 = -\frac{\pi}{6}$$

The reference circle is, thus as plotted below



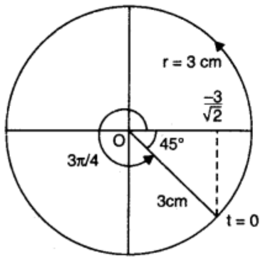
(*c*) $$x = 3\cos\left(2\pi t + \frac{\pi}{4} + \frac{\pi}{2}\right)$$

Here, radius of reference circle, r = 3 cm and at $t = 0$, $x = 3\sin\frac{\pi}{4} = \frac{\sqrt{3}}{2}$ cm

$$\omega t = 2\pi t \Rightarrow \omega = 2\pi \text{ rad/s}$$

$$\cos \quad \phi_0 = \frac{\sqrt{\frac{3}{2}}}{3} = \frac{1}{\sqrt{2}}$$

Therefore, the reference circle is being shown below.



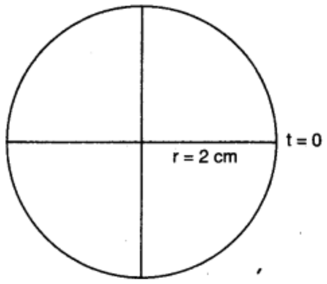
(d) $x = 2 \cos \pi t$

Radius of reference circle, $r = 2$ cm and at $t = 0, x = 2$ cm

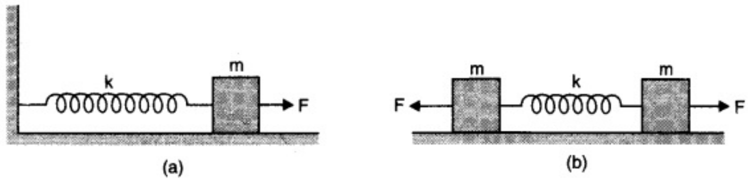
$\therefore \quad \omega t = \pi t$, or $\omega = \pi$ rad/s

$\cos \phi_0 = 1, \quad \phi_0 = 0$

The reference circle is plotted below.



Question 14. 13. Figure (a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force F applied at the free end stretches the spring. Figure (b) shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in Figure - (b) is stretched by the same force F.



(a) What is the maximum extension of the spring in the two cases?
(b) If the mass in Fig. (a) and the two masses in Fig. (b) are released free, what is the period of oscillation in each case?
Answer:

(a) Let y be the maximum extension produced in the spring in Fig. (a)

Then $F = ky$ (in magnitude) $\therefore \quad y = \frac{F}{k}$

If fig. (b), the force on one mass acts as the force of reaction due to the force on the other mass. Therefore, each mass behaves as if it is fixed with respect to the other.

Therefore, $F = ky \Rightarrow y = \frac{F}{k}$

(b) In fig. (a), $F = -ky$

$\Rightarrow \quad ma = -ky \Rightarrow a = -\frac{k}{m}y \quad \therefore \quad \omega^2 = \frac{k}{m}$ i.e., $\omega = \sqrt{\frac{k}{m}}$

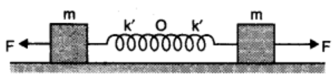
Therefore, period $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

In fig. (b), we may consider that the centre of the system is O and there are two springs each of length $\frac{l}{2}$ attached to the two masses, each m, so that k' is the spring factor of each of the springs.

Then, $K' = 2k$

$\therefore \quad T = 2\pi\sqrt{\frac{m}{k'}}$

$= 2\pi\sqrt{\frac{m}{2k}}$



*********** END **********