## EXERCISE - 2.3

### Question-1

Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}

(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

(iii) {(1, 3), (1, 5), (2, 5)}

### Ans.

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}

Since 2, 5, 8, 11, 14, and 17 are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain = {2, 5, 8, 11, 14, 17} and range = {1}

(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

Since 2, 4, 6, 8, 10, 12, and 14 are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain = {2, 4, 6, 8, 10, 12, 14} and range = {1, 2, 3, 4, 5, 6, 7}

(iii) {(1, 3), (1, 5), (2, 5)}

Since the same first element i.e., 1 corresponds to two different images i.e., 3 and 5, this relation is not a function.

### Question-2

Find the domain and range of the following real function:

(i) $f(x) = -|x|$ (ii) $f(x) = \sqrt{9-x^2}$

### Ans.

(i) $f(x) = -|x|$, $x \in \mathbf{R}$

We know that $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

$$\therefore f(x) = -|x| = \begin{cases} -x, & x \geq 0 \\ x, & x < 0 \end{cases}$$

Since $f(x)$ is defined for $x \in \mathbf{R}$, the domain of $f$ is **R**.

It can be observed that the range of $f(x) = -|x|$ is all real numbers except positive real numbers.

$\therefore$ The range of $f$ is $(-\infty, 0]$.

(ii) $f(x) = \sqrt{9-x^2}$

Since $\sqrt{9-x^2}$ is defined for all real numbers that are greater than or equal to $-3$ and less than or equal to 3, the domain of $f(x)$ is $\{x : -3 \leq x \leq 3\}$ or $[-3, 3]$.

For any value of $x$ such that $-3 \leq x \leq 3$, the value of $f(x)$ will lie between 0 and 3.

$\therefore$ The range of $f(x)$ is $\{x: 0 \leq x \leq 3\}$ or $[0, 3]$.

Question-3

A function $f$ is defined by $f(x) = 2x - 5$. Write down the values of

(i) $f(0)$, (ii) $f(7)$, (iii) $f(-3)$

Ans.

The given function is $f(x) = 2x - 5$.

Therefore,

(i) $f(0) = 2 \times 0 - 5 = 0 - 5 = -5$

(ii) $f(7) = 2 \times 7 - 5 = 14 - 5 = 9$

(iii) $f(-3) = 2 \times (-3) - 5 = -6 - 5 = -11$

Question-4

The function '$t$' which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32$.

Find (i) $t$ (0) (ii) $t$ (28) (iii) $t$ (−10) (iv) The value of C, when $t(C) = 212$

Ans.

The given function is $t(C) = \frac{9C}{5} + 32$.

Therefore,

(i) $t(0) = \frac{9 \times 0}{5} + 32 = 0 + 32 = 32$

(ii) $t(28) = \frac{9 \times 28}{5} + 32 = \frac{252 + 160}{5} = \frac{412}{5}$

(iii) $t(-10) = \frac{9 \times (-10)}{5} + 32 = 9 \times (-2) + 32 = -18 + 32 = 14$

(iv) It is given that $t(C) = 212$

$$\therefore 212 = \frac{9C}{5} + 32$$
$$\Rightarrow \frac{9C}{5} = 212 - 32$$
$$\Rightarrow \frac{9C}{5} = 180$$
$$\Rightarrow 9C = 180 \times 5$$
$$\Rightarrow C = \frac{180 \times 5}{9} = 100$$

Thus, the value of $t$, when $t(C) = 212$, is 100.

Question-5

Find the range of each of the following functions.

(i) $f(x) = 2 - 3x$, $x \in \mathbf{R}$, $x > 0$.

(ii) $f(x) = x^2 + 2$, $x$, is a real number.

(iii) $f(x) = x$, $x$ is a real number

Ans.

(i) $f(x) = 2 - 3x$, $x \in \mathbf{R}$, $x > 0$

The values of $f(x)$ for various values of real numbers $x > 0$ can be written in the tabular form as

| $x$ | 0.01 | 0.1 | 0.9 | 1 | 2 | 2.5 | 4 | 5 | … |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 1.97 | 1.7 | –0.7 | –1 | –4 | –5.5 | –10 | –13 | … |

Thus, it can be clearly observed that the range of $f$ is the set of all real numbers less than 2.

i.e., range of $f = (-\infty, 2)$

**Alter:**

Let $x > 0$

$\Rightarrow 3x > 0$

$\Rightarrow 2 - 3x < 2$

$\Rightarrow f(x) < 2$

$\therefore$ Range of $f = (-\infty, 2)$

(ii) $f(x) = x^2 + 2$, $x$, is a real number

The values of $f(x)$ for various values of real numbers $x$ can be written in the tabular form as

| $x$ | 0 | ±0.3 | ±0.8 | ±1 | ±2 | ±3 | | … |
|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 2 | 2.09 | 2.64 | 3 | 6 | 11 | | ….. |

Thus, it can be clearly observed that the range of $f$ is the set of all real numbers greater than 2.

i.e., range of $f = [2, \infty)$

**Alter:**

Let $x$ be any real number.

Accordingly,

$x^2 \geq 0$

$\Rightarrow x^2 + 2 \geq 0 + 2$

$\Rightarrow x^2 + 2 \geq 2$

$\Rightarrow f(x) \geq 2$

$\therefore$ Range of $f = [2, \infty)$

(iii) $f(x) = x$, $x$ is a real number

It is clear that the range of $f$ is the set of all real numbers.

$\therefore$ Range of $f = \mathbf{R}$

*********** END **********