Derivatives as a Rate Measurer Ex 13.2 Q21

Here, $\frac{dA}{dt} = 2 \text{ cm}^2/\text{sec}$

To find $\frac{dV}{dt}$ at $r = 6$ cm

$$A = 4\pi r^2$$

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

$$2 = 8\pi r \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{4\pi r} \text{ cm/sec}$$

Now, $V = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$= 4\pi r^2 \left(\frac{1}{4\pi r}\right)$$

$$= r$$

$$\frac{dV}{dt} = 6 \text{ cm}^3/\text{sec}$$

So, volume of bubble is increasing at the rate of 6 cm^3/sec.

Derivatives as a Rate Measurer Ex 13.2 Q22

Here, $\frac{dr}{dt} = 2$ cm/sec, $\frac{dh}{dt} = -3$ cm/sec

To find $\frac{dV}{dt}$ when $r = 3$ cm, $h = 5$ cm

Now, V = volume of cylinder

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = \pi \left[2r \frac{dr}{dt} \times h + r^2 \frac{dh}{dt}\right]$$

$$= \pi\left[2(3)(2)(5) + (3)^2(-3)^2\right]$$

$$= \pi[60 - 27]$$

$$\frac{dV}{dt} = 33\pi \text{ cm}^3/\text{sec}$$

So, volume of cylinder is increasing at the rate of 33π cm^3/sec.

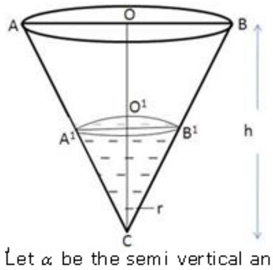
Derivatives as a Rate Measurer Ex 13.2 Q23

Let V be volume of sphere with miner radius r and onter radius R, then

$$V = \frac{4}{3}\pi\left(R^3 - r^3\right)$$
$$\frac{dV}{dt} = \frac{4}{3}\pi\left(3R^2\frac{dR}{dt} - 3r^2\frac{dr}{dt}\right)$$
$$0 = \frac{4\pi}{3}3\left(R^2\frac{dR}{dt} - r^2\frac{dr}{dt}\right) \qquad \left[\text{Since volume } V \text{ is constant}\right]$$
$$R^2\frac{dR}{dt} = r^2\frac{dr}{dt}$$
$$(8)^2\frac{dR}{dt} = (4)^2(1)$$
$$\frac{dR}{dt} = \frac{16}{64}$$
$$\frac{dR}{dt} = \frac{1}{4} \text{ cm/sec}$$

Rate of increasing of onter radius = $\frac{1}{4}$ cm/sec.

Derivatives as a Rate Measurer Ex 13.2 Q24



Let α be the semi vertical angle of the cone CAB whose height CO is half of radius OB.

Now,

$$\tan\alpha = \frac{OB}{CO}$$
$$= \frac{OB}{2OB} \qquad \left[\because CO = 2OB\right]$$
$$\tan\alpha = \frac{1}{2}$$

Let V be the volume of the sand in the cone

$$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$$
$$= \frac{\pi}{12}h^3$$
$$\frac{dV}{dt} = \frac{3\pi}{12}h^2\frac{dh}{dt}$$
$$50 = \frac{3\pi}{12}h^2\frac{dh}{dt} \qquad \left[\because \frac{dV}{dt} = 50 \text{ cm}^3/\text{min}\right]$$
$$\frac{dh}{dt} = \frac{200}{\pi h^2}$$
$$= \frac{200}{\pi(5)^2}$$
$$\frac{dh}{dt} = \frac{8}{3.14} \text{cm/min}$$

Rate of increasing of height = $\frac{8}{\pi}$ cm/min

*********** END **********