Functions Ex 3.4 Q4

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \geq -1$

So, $\text{domain}(f) = [-1, \infty]$

Clearly, $g(x) = \sqrt{9-x^2}$ is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$

$\Rightarrow \quad x^2 - 3^2 \leq 0$

$\Rightarrow \quad (x-3)(x+3) \leq 0$

$\Rightarrow \quad x \in [-3, 3]$

$\therefore \quad \text{domain}(g) = [-3, 3]$

Now,

$$\text{domain}(f) \cap \text{domain}(g) = [-1, \infty] \cap [-3, 3]$$
$$= [-1, 3]$$

$f + g : [-1, 3] \rightarrow R$ is given by $(f+g)(x) = f(x) + g(x) = \sqrt{x+1} + \sqrt{9-x^2}$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \geq -1$

So, $\text{domain}(f) = [-1, \infty]$

Clearly, $g(x) = \sqrt{9-x^2}$ is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$

$\Rightarrow \quad x^2 - 3^2 \leq 0$

$\Rightarrow \quad (x-3)(x+3) \leq 0$

$\Rightarrow \quad x \in [-3, 3]$

$\therefore \quad \text{domain}(g) = [-3, 3]$

Now,

$$\text{domain}(f) \cap \text{domain}(g) = [-1, \infty] \cap [-3, 3]$$
$$= [-1, 3]$$

$g - f : [-, 3] \rightarrow R$ is given by $(g-f)(x) = g(x) - f(x) = \sqrt{9-x^2} - \sqrt{x+1}$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \geq -1$

So, $\text{domain}(f) = [-1, \infty)$

Clearly, $g(x) = \sqrt{9-x^2}$ is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$

$\Rightarrow \quad x^2 - 3^2 \leq 0$

$\Rightarrow \quad (x-3)(x+3) \leq 0$

$\Rightarrow \quad x \in [-3, 3]$

$\therefore \quad \text{domain}(g) = [-3, 3]$

Now,

$$\text{domain}(f) \cap \text{domain}(g) = [-1, \infty) \cap [-3, 3]$$
$$= [-1, 3]$$

$fg : [-, 3] \to R$ is given by $(fg)(x) = f(x) \times g(x) = \sqrt{x+1} \times \sqrt{9-x^2}$

$$= \sqrt{9 + 9x - x^2 - x^3}$$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \geq -1$

So, $\text{domain}(f) = [-1, \infty]$

Clearly, $g(x) = \sqrt{9-x^2}$ is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$

$\Rightarrow \quad x^2 - 3^2 \leq 0$

$\Rightarrow \quad (x-3)(x+3) \leq 0$

$\Rightarrow \quad x \in [-3, 3]$

$\therefore \quad \text{domain}(g) = [-3, 3]$

Now,

$$\text{domain}(f) \cap \text{domain}(g) = [-1, \infty] \cap [-3, 3]$$
$$= [-1, 3]$$

We have, $g(x) = \sqrt{9-x^2}$

$\therefore \quad 9 - x^2 = 0 \Rightarrow x^2 - 9 = 0$

$\Rightarrow \quad (x-3)(x+3) = 0$

$\Rightarrow \quad x = \pm 3$

So, $\text{domain}\left(\frac{f}{g}\right) = [-1, 3] - [-3, 3] = [-1, 3]$

$\therefore \quad \frac{f}{g} : [-1, 3] \to R$ is given by $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x+1}}{\sqrt{9-x^2}}$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \geq -1$

So, $\text{domain}(f) = [-1, \infty]$

Clearly, $g(x) = \sqrt{9-x^2}$ is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$

$\Rightarrow \quad x^2 - 3^2 \leq 0$

$\Rightarrow \quad (x-3)(x+3) \leq 0$

$\Rightarrow \quad x \in [-3, 3]$

$\therefore \quad \text{domain}(g) = [-3, 3]$

Now,

$$\text{domain}(f) \cap \text{domain}(g) = [-1, \infty] \cap [-3, 3]$$
$$= [-1, 3]$$

We have,

$$f(x) = \sqrt{x+1}$$

$\therefore \quad \sqrt{x+1} = 0$

$\Rightarrow \quad x + 1 = 0$

$\Rightarrow \quad x = -1$

So, $\text{domain}\left(\frac{g}{f}\right) = [-1, 3] - \{-1\}$

$$= [-1, 3]$$

$\therefore \quad \frac{g}{f} : [-1, 3] \to R$ is given by $\frac{g}{f}(x) = \frac{g(x)}{f(x)} = \frac{\sqrt{9-x^2}}{\sqrt{x+1}}$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \geq -1$

So, $\text{domain}(f) = [-1, \infty]$

Clearly, $g(x) = \sqrt{9-x^2}$ is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$

$\Rightarrow \quad x^2 - 3^2 \leq 0$

$\Rightarrow \quad (x-3)(x+3) \leq 0$

$\Rightarrow \quad x \in [-3, 3]$

$\therefore \quad \text{domain}(g) = [-3, 3]$

Now,

$$\text{domain}(f) \cap \text{domain}(g) = [-1, \infty] \cap [-3, 3]$$
$$= [-1, 3]$$

$2f - \sqrt{5}g : [-, 3] \to R$ defined by $(2f - \sqrt{5}g)(x) = 2\sqrt{x+1} - \sqrt{5}\sqrt{9-x^2}$

$$= 2\sqrt{x+1} - \sqrt{45 - 5x^2}.$$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \geq -1$

So, domain$(f) = [-1, \infty]$

Clearly, $g(x) = \sqrt{9-x^2}$ is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$

$\Rightarrow \quad x^2 - 3^2 \leq 0$

$\Rightarrow \quad (x-3)(x+3) \leq 0$

$\Rightarrow \quad x \in [-3, 3]$

$\therefore$ domain$(g) = [-3, 3]$

Now,

$$\text{domain}(f) \cap \text{domain}(g) = [-1, \infty] \cap [-3, 3] = [-1, 3]$$

$f^2 + 7f : [-1, \infty] \to R$ defined by $(f^2 + 7f)(x) = f^2(x) + 7f(x)$ $\quad [\because D(f) = [-1, \infty]]$

$$= \left(\sqrt{x+1}\right)^2 + 7\sqrt{x+1}$$

$$= x + 1 + 7\sqrt{x+1}$$

We have,

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We observe that $f(x) = \sqrt{x+1}$ is defined for all $x \geq -1$

So, domain$(f) = [-1, \infty]$

Clearly, $g(x) = \sqrt{9-x^2}$ is defined for

$$9 - x^2 \geq 0 \Rightarrow x^2 - 9 \leq 0$$

$\Rightarrow \quad x^2 - 3^2 \leq 0$

$\Rightarrow \quad (x-3)(x+3) \leq 0$

$\Rightarrow \quad x \in [-3, 3]$

$\therefore$ domain$(g) = [-3, 3]$

Now,

$$\text{domain}(f) \cap \text{domain}(g) = [-1, \infty] \cap [-3, 3] = [-1, 3]$$

We have,

$$g(x) = \sqrt{9-x^2}$$

$\therefore \quad 9 - x^2 = 0 \Rightarrow x^2 - 9 = 0$

$\Rightarrow \quad (x-3)(x+3) = 0$

$\Rightarrow \quad x = \pm 3$

So, domain$\left(\frac{1}{g}\right) = [-3, 3] - \{-3, 3\}$

$$= (-3, 3)$$

$\therefore \quad \frac{5}{g} = (-3, 3) \to R$ defined by $\left(\frac{5}{g}\right)(x) = \frac{5}{\sqrt{9-x^2}}$

*********** END **********