Exercise 6.4 : Solutions of Questions on Page Number : **216**

Q1 : 1. Using differentials, find the approximate value of each of the following up to 3 places of decimal

(i) $\sqrt{25.3}$ **(ii)** $\sqrt{49.5}$ **(iii)** $\sqrt{0.6}$

(iv) $(0.009)^{\frac{1}{3}}$ **(v)** $(0.999)^{\frac{1}{10}}$ **(vi)** $(15)^{\frac{1}{4}}$

(vii) $(26)^{\frac{1}{3}}$ **(viii)** $(255)^{\frac{1}{4}}$ **(ix)** $(82)^{\frac{1}{4}}$

(x) $(401)^{\frac{1}{2}}$

Answer :

(i) $\sqrt{25.3}$

Consider $y=\sqrt{x}$. Let x = 25 and Δx = 0.3.

Then,

$$\Delta y=\sqrt{x+\Delta x}-\sqrt{x}=\sqrt{25.3}-\sqrt{25}=\sqrt{25.3}-5$$
$$\Rightarrow \sqrt{25.3}=\Delta y+5$$

Now, dy is approximately equal to Δy and is given by,

$$dy=\left(\frac{dy}{dx}\right)\Delta x=\frac{1}{2\sqrt{x}}(0.3) \qquad \left[\text{as } y=\sqrt{x}\right]$$

$$=\frac{1}{2\sqrt{25}}(0.3)=0.03$$

Hence, the approximate value of $\sqrt{25.3}$ is 0.03 + 5 = 5.03.

(ii) $\sqrt{49.5}$

Consider $y=\sqrt{x}$. Let x = 49 and Δx = 0.5.

Then,

$$\Delta y=\sqrt{x+\Delta x}-\sqrt{x}=\sqrt{49.5}-\sqrt{49}=\sqrt{49.5}-7$$
$$\Rightarrow \sqrt{49.5}=7+\Delta y$$

Now, dy is approximately equal to Δy and is given by,

$$dy=\left(\frac{dy}{dx}\right)\Delta x=\frac{1}{2\sqrt{x}}(0.5) \qquad \left[\text{as } y=\sqrt{x}\right]$$
$$=\frac{1}{2\sqrt{49}}(0.5)=\frac{1}{14}(0.5)=0.035$$

Hence, the approximate value of $\sqrt{49.5}$ is 7 + 0.035 = 7.035.

(iii) $\sqrt{0.6}$

Consider $y=\sqrt{x}$. Let x = 1 and Δx = - 0.4.

Then,

$$\Delta y=\sqrt{x+\Delta x}-\sqrt{x}=\sqrt{0.6}-1$$
$$\Rightarrow \sqrt{0.6}=1+\Delta y$$

Now, dy is approximately equal to Δy and is given by,

$$dy=\left(\frac{dy}{dx}\right)\Delta x=\frac{1}{2\sqrt{x}}(\Delta x) \qquad \left[\text{as } y=\sqrt{x}\right]$$
$$=\frac{1}{2}(-0.4)=-0.2$$

Hence, the approximate value of $\sqrt{0.6}$ is 1 + (- 0.2) = 1 - 0.2 = 0.8.

(iv) $(0.009)^{\frac{1}{3}}$

Consider $y=x^{\frac{1}{3}}$. Let x = 0.008 and Δx = 0.001.

Then,

$$\Delta y=(x+\Delta x)^{\frac{1}{3}}-(x)^{\frac{1}{3}}=(0.009)^{\frac{1}{3}}-(0.008)^{\frac{1}{3}}=(0.009)^{\frac{1}{3}}-0.2$$
$$\Rightarrow (0.009)^{\frac{1}{3}}=0.2+\Delta y$$

Now, dy is approximately equal to Δy and is given by,

$$dy=\left(\frac{dy}{dx}\right)\Delta x=\frac{1}{3(x)^{\frac{2}{3}}}(\Delta x) \qquad \left[\text{as } y=x^{\frac{1}{3}}\right]$$
$$=\frac{1}{3\times 0.04}(0.001)=\frac{0.001}{0.12}=0.008$$

Hence, the approximate value of $(0.009)^{\frac{1}{3}}$ is 0.2 + 0.008 = 0.208.

(v) $(0.999)^{\frac{1}{10}}$

Answer needs Correction? **Click Here**

Q2 : Find the approximate value of $f(2.01)$, where $f(x) = 4x^2 + 5x + 2$

Answer :

Let $x = 2$ and $\Delta x = 0.01$. Then, we have:

$f(2.01) = f(x + \Delta x) = 4(x + \Delta x)^2 + 5(x + \Delta x) + 2$

Now, $\Delta y = f(x + \Delta x) - f(x)$

$\therefore f(x + \Delta x) = f(x) + \Delta y$

Ã¢â€°Ë† $f(x)+f'(x)\cdot\Delta x \qquad (\text{as } dx = \Delta x)$

$$\begin{aligned}\Rightarrow f(2.01) &\approx (4x^2+5x+2)+(8x+5)\Delta x\\ &=\left[4(2)^2+5(2)+2\right]+\left[8(2)+5\right](0.01) \qquad [\text{as } x=2,\ \Delta x=0.01]\\ &=(16+10+2)+(16+5)(0.01)\\ &=28+(21)(0.01)\\ &=28+0.21\\ &=28.21\end{aligned}$$

Hence, the approximate value of $f(2.01)$ is 28.21.

Answer needs Correction? **Click Here**

Q3 : Find the approximate value of $f(5.001)$, where $f(x) = x^3 - 7x^2 + 15$.

Answer :

Let $x = 5$ and $\Delta x = 0.001$. Then, we have:

$$\begin{aligned}&f(5.001)=f(x+\Delta x)=(x+\Delta x)^3-7(x+\Delta x)^2+15\\ &\text{Now, } \Delta y=f(x+\Delta x)-f(x)\\ &\therefore f(x+\Delta x)=f(x)+\Delta y\\ &\qquad\qquad \approx f(x)+f'(x)\cdot\Delta x \qquad (\text{as } dx=\Delta x)\\ &\Rightarrow f(5.001)\approx(x^3-7x^2+15)+(3x^2-14x)\Delta x\\ &\qquad=\left[(5)^3-7(5)^2+15\right]+\left[3(5)^2-14(5)\right](0.001) \qquad [x=5, \Delta x=0.001]\\ &\qquad=(125-175+15)+(75-70)(0.001)\\ &\qquad=-35+(5)(0.001)\\ &\qquad=-35+0.005\\ &\qquad=-34.995\end{aligned}$$

Hence, the approximate value of $f(5.001)$ is - 34.995.

Answer needs Correction? **Click Here**

Q4 : Find the approximate change in the volume V of a cube of side x metres caused by increasing side by 1%.

Answer :

The volume of a cube (V) of side x is given by $V = x^3$.

$$\begin{aligned}\therefore dV &= \left(\frac{dV}{dx}\right)\Delta x\\ &=(3x^2)\Delta x\\ &=(3x^2)(0.01x) \qquad [\text{as } 1\% \text{ of } x \text{ is } 0.01x]\\ &=0.03x^3\end{aligned}$$

Hence, the approximate change in the volume of the cube is $0.03x^3$ m^3.

Answer needs Correction? **Click Here**

Q5 : Find the approximate change in the surface area of a cube of side x metres caused by decreasing the side by 1%

Answer :

The surface area of a cube (S) of side x is given by $S = 6x^2$.

$$\begin{aligned}\therefore \frac{dS}{dx} &= \left(\frac{dS}{dx}\right)\Delta x\\ &=(12x)\Delta x\\ &=(12x)(0.01x) \qquad [\text{as } 1\% \text{ of } x \text{ is } 0.01x]\\ &=0.12x^2\end{aligned}$$

Hence, the approximate change in the surface area of the cube is $0.12x^2$ m^2.

Answer needs Correction? **Click Here**

Q6 : If the radius of a sphere is measured as 7 m with an error of 0.02m, then find the approximate error in calculating its volume.

Answer :

Let r be the radius of the sphere and Δr be the error in measuring the radius.

Then,

r = 7 m and Δr = 0.02 m

Now, the volume V of the sphere is given by,

$$V = \frac{4}{3}\pi r^3$$
$$\therefore \frac{dV}{dr} = 4\pi r^2$$
$$\therefore dV = \left(\frac{dV}{dr}\right)\Delta r$$
$$= \left(4\pi r^2\right)\Delta r$$
$$= 4\pi(7)^2(0.02)\ \text{m}^3 = 3.92\pi\ \text{m}^3$$

Hence, the approximate error in calculating the volume is 3.92 π m^3.

Answer needs Correction? **Click Here**

Q7 : If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating in surface area.

Answer :

Let r be the radius of the sphere and Δr be the error in measuring the radius.

Then,

r = 9 m and Δr = 0.03 m

Now, the surface area of the sphere (S) is given by,

$S = 4\pi r^2$

$$\therefore \frac{dS}{dr} = 8\pi r$$
$$\therefore dS = \left(\frac{dS}{dr}\right)\Delta r$$
$$= (8\pi r)\Delta r$$
$$= 8\pi(9)(0.03)\ \text{m}^2$$
$$= 2.16\pi\ \text{m}^2$$

Hence, the approximate error in calculating the surface area is 2.16π m^2.

Answer needs Correction? **Click Here**

Q8 : If $f(x) = 3x^2 + 15x + 5$, then the approximate value of $f(3.02)$ is

A. 47.66 B. 57.66 C. 67.66 D. 77.66

Answer :

Let x = 3 and Δx = 0.02. Then, we have:

$$f(3.02) = f(x+\Delta x) = 3(x+\Delta x)^2 + 15(x+\Delta x) + 5$$
$$\text{Now, } \Delta y = f(x+\Delta x) - f(x)$$
$$\Rightarrow f(x+\Delta x) = f(x) + \Delta y$$
$$\approx f(x) + f'(x)\Delta x \qquad (\text{As } dx = \Delta x)$$
$$\Rightarrow f(3.02) \approx (3x^2+15x+5) + (6x+15)\Delta x$$
$$= \left[3(3)^2+15(3)+5\right] + \left[6(3)+15\right](0.02) \qquad [\text{As } x = 3,\ \Delta x = 0.02]$$
$$= (27+45+5) + (18+15)(0.02)$$
$$= 77 + (33)(0.02)$$
$$= 77 + 0.66$$
$$= 77.66$$

Hence, the approximate value of $f(3.02)$ is 77.66.

The correct answer is D.

Answer needs Correction? **Click Here**

Q9 : The approximate change in the volume of a cube of side x metres caused by increasing the side by 3% is

A. 0.06 x^3 m^3 B. 0.6 x^3 m^3 C. 0.09 x^3 m^3 D. 0.9 x^3 m^3

Answer :

The volume of a cube (V) of side x is given by $V = x^3$.

$$\therefore dV = \left(\frac{dV}{dx}\right)\Delta x$$
$$= \left(3x^2\right)\Delta x$$
$$= \left(3x^2\right)(0.03x) \qquad [\text{As } 3\% \text{ of } x \text{ is } 0.03x]$$
$$= 0.09x^3\ \text{m}^3$$

Hence, the approximate change in the volume of the cube is $0.09x^3$ m^3.

The correct answer is C.

Answer needs Correction? **Click Here**

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