Indefinite Integrals Ex 19.29 Q5

Let $I = \int (4x+1)\sqrt{x^2 - x - 2}\,dx$

Let $4x + 1 = \lambda \frac{d}{dx}\left(x^2 - x - 2\right) + \mu$

$= \lambda(2x - 1) + \mu$

Equating similar terms, we get,

$2\lambda = 4 \quad \Rightarrow \quad \lambda = 2$

$-\lambda + \mu = 1 \quad \Rightarrow \quad \mu = 3$

So,

$I = \int \left(2(2x-1) + 3\right)\sqrt{x^2 - x - 2}\,dx$

$= 2\int (2x-1)\sqrt{x^2 - x - 2}\,dx + 3\int \sqrt{x^2 - x - 2}\,dx$

Let $x^2 - x - 2 = t$

$(2x - 1)\,dx = dt$

$$\therefore \quad I = 2\int \sqrt{t}\,dt + 3\int \sqrt{\left(x - \frac{1}{2}\right)^2 - \left(\frac{3}{2}\right)^2}\,dx$$

$$\Rightarrow \quad I = 2\,\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + 3\left\{\frac{\left(x - \frac{1}{2}\right)}{2}\sqrt{x^2 - x - 2} - \frac{9}{8}\log\left|\left(x - \frac{1}{2}\right) + \sqrt{x^2 - x - 2}\right| + c\right.$$

Hence,

$$I = \frac{4}{3}\left(x^2 - x - 2\right)^{\frac{3}{2}} + \frac{3}{4}(2x - 1)\sqrt{x^2 - x - 2} - \frac{27}{8}\log\left|\left(x - \frac{1}{2}\right) + \sqrt{x^2 - x - 2}\right| + c$$

Indefinite Integrals Ex 19.29 Q6

Let $I = \int (x-2)\sqrt{2x^2-6x+5}dx$

Let $x - 2 = \lambda \frac{d}{dx}\left(2x^2 - 6x + 5\right) + \mu$

$= \lambda\left(4x-6\right) + \mu$

Equating similar terms, we get,

$4\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{4}$

$-6\lambda + \mu = -2 \quad \Rightarrow \quad \mu = -2 + 6\lambda = -\frac{2}{4} = -\frac{1}{2}$

$\therefore \quad \mu = -\frac{1}{2}$

So,

$I = \int\left(\frac{1}{4}\left(4x-6\right) + \left(-\frac{1}{2}\right)\right)\sqrt{2x^2-6x+5}dx$

$= \frac{1}{4}\int\left(4x-6\right)\sqrt{2x^2-6x+5} - \frac{1}{2}\int\sqrt{2x^2-6x+5}dx$

Let $2x^2 - 6x + 5 = t$

$\left(4x-6\right)dx = dt$

$\therefore \quad I = \frac{1}{4}\int\sqrt{t}dt - \frac{\sqrt{2}}{2}\int\sqrt{x^2-3x+\frac{5}{2}}dx$

$\Rightarrow \quad I = \frac{1}{4}\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{\sqrt{2}}\int\sqrt{\left(x-\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2}dx$

$= \frac{1}{6}\left(2x^2-6x+5\right)^{\frac{3}{2}} - \frac{1}{\sqrt{2}}\left\{\frac{\left(x-\frac{3}{2}\right)}{2}\sqrt{x^2-3x+\frac{5}{2}} + \frac{1}{8}\log\left|\left(x-\frac{3}{2}\right) + \sqrt{x^2-3x+\frac{5}{2}}\right| + c\right.$

Hence,

$I = \frac{1}{6}\left(2x^2-6x+5\right)^{\frac{3}{2}} - \frac{1}{8}\left(2x-3\right)\sqrt{2x^2-6x+5} - \frac{1}{8\sqrt{2}}\log\left|\left(x-\frac{3}{2}\right) + \sqrt{x^2-3x+\frac{5}{2}}\right| + c$

Indefinite Integrals Ex 19.29 Q7

Let $I = \int (x+1)\sqrt{x^2+x+1}dx$

Let $x+1 = \lambda \frac{d}{dx}(x^2+x+1) + \mu$

$= \lambda(2x+1) + \mu$

Equating similar terms, we get,

$2\lambda = 1 \Rightarrow \quad \lambda = \frac{1}{2}$

$\lambda + \mu = 1 \Rightarrow \quad \mu = \frac{1}{2}$

So,

$I = \int \left(\frac{1}{2}(2x+1) + \frac{1}{2}\right)\sqrt{x^2+x+1}dx$

$= \frac{1}{2}\int (2x+1)\sqrt{x^2+x+1}dx + \frac{1}{2}\int \sqrt{x^2+x+1}dx$

Let $x^2+x+1 = t$

$\Rightarrow \quad (2x+1)dx = dt$

$= \frac{1}{2}\int \sqrt{t}dt + \frac{1}{2}\int \sqrt{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}dx$

$= \frac{1}{2}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{1}{2}\left\{\frac{\left(x+\frac{1}{2}\right)}{2}\sqrt{x^2+x+1} + \frac{3}{8}\log\left|\left(x+\frac{1}{2}\right) + \sqrt{x^2+x+1}\right| + c\right.$

$\Rightarrow \quad I = \frac{1}{3}(x^2+x+1)^{\frac{3}{2}} + \frac{1}{8}(2x+1)\sqrt{x^2+x+1} + \frac{3}{16}\log\left|\left(x+\frac{1}{2}\right) + \sqrt{x^2+x+1}\right| + c$

Hence,

$I = \frac{1}{3}(x^2+x+1)^{\frac{3}{2}} + \frac{1}{8}(2x+1)\sqrt{x^2+x+1} + \frac{3}{16}\log\left|\left(x+\frac{1}{2}\right) + \sqrt{x^2+x+1}\right| + c$

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