



Polynomials Ex 2.1 Q19

Answer :

(i) Since α and β are the zeros of the quadratic polynomial $f(x) = x^2 - 2x + 3$

$$\begin{aligned}\alpha + \beta &= \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} \\ &= \frac{-(-2)}{1} \\ &= 2\end{aligned}$$

$$\begin{aligned}\text{Product of the zeros} &= \frac{\text{Constant term}}{\text{Coefficient of } x^2} \\ &= \frac{3}{1} \\ &= 3\end{aligned}$$

Let S and P denote respectively the sums and product of the polynomial whose zeros $\alpha + 2, \beta + 2$

$$S = (\alpha + 2) + (\beta + 2)$$

$$S = \alpha + \beta + 2 + 2$$

$$S = 2 + 2 + 2$$

$$S = 6$$

$$P = (\alpha + 2)(\beta + 2)$$

$$P = \alpha\beta + 2\beta + 2\alpha + 4$$

$$P = \alpha\beta + 2(\alpha + \beta) + 4$$

$$P = 3 + 2(2) + 4$$

$$P = 3 + 4 + 4$$

$$P = 11$$

Therefore the required polynomial $f(x)$ is given by

$$\begin{aligned}f(x) &= k(x^2 - Sx + P) \\ &= k(x^2 - 6x + 11)\end{aligned}$$

Hence, the required equation is $f(x) = k(x^2 - 6x + 11)$.

(ii) Since α and β are the zeros of the quadratic polynomial $f(x) = x^2 - 2x + 3$

$$\begin{aligned}\alpha + \beta &= \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} \\ &= \frac{-(-2)}{1} \\ &= 2\end{aligned}$$

$$\begin{aligned}\text{Product of the zeros} &= \frac{\text{Constant term}}{\text{Coefficient of } x^2} \\ &= \frac{3}{1} \\ &= 3\end{aligned}$$

Let S and P denote respectively the sums and product of the polynomial whose zeros $\frac{\alpha-1}{\alpha+1}, \frac{\beta-1}{\beta+1}$

$$S = \frac{\alpha-1}{\alpha+1} + \frac{\beta-1}{\beta+1}$$

$$S = \frac{(\alpha-1)(\beta+1) + (\beta-1)(\alpha+1)}{(\alpha+1)(\beta+1)}$$

$$S = \frac{\alpha\beta - \beta + \alpha - 1 + \alpha\beta - \alpha + \beta - 1}{\alpha\beta + \beta + \alpha + 1}$$

$$S = \frac{\alpha\beta - \cancel{\beta} + \cancel{\alpha} + 1 - \alpha\beta - \cancel{\alpha} + \cancel{\beta} - 1}{\alpha\beta + (\alpha + \beta) + 1}$$

$$S = \frac{\alpha\beta + \alpha\beta - 1 - 1}{\alpha\beta + (\alpha + \beta) + 1}$$

By substituting $\alpha + \beta = 2$ and $\alpha\beta = 3$ we get ,

$$S = \frac{3 + 3 - 1 - 1}{3 + 2 + 1}$$

$$S = \frac{6 - 2}{6}$$

$$S = \frac{4^2}{6^3}$$

$$P = \left(\frac{\alpha - 1}{\alpha + 1} \right) \left(\frac{\beta - 1}{\beta + 1} \right)$$

$$P = \frac{\alpha\beta - \beta - \alpha + 1}{\alpha\beta + \beta + \alpha + 1}$$

$$P = \frac{\alpha\beta - (\beta + \alpha) + 1}{\alpha\beta + (\alpha + \beta) + 1}$$

$$P = \frac{3 - 2 + 1}{3 + 2 + 1}$$

$$P = \frac{2}{6}$$

$$P = \frac{2^1}{6^1}$$

$$P = \frac{1}{3}$$

The required polynomial $f(x)$ is given by,

$$f(x) = k(x^2 - Sx + P)$$

$$f(x) = k\left(x^2 - \frac{2}{3}x + \frac{1}{3}\right)$$

Hence, the required equation is $f(x) = k\left(x^2 - \frac{2}{3}x + \frac{1}{3}\right)$, where k is any non zero real number

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