



Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k}$.

It is observed that $\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k} = -2(2\hat{i} - 3\hat{j} + 4\hat{k}) = -2\vec{a}$

$$\therefore \vec{b} = \lambda \vec{a}$$

where,

$$\lambda = -2$$

Hence, the given vectors are collinear.

Question 12:

Find the direction cosines of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$

Answer

Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$.

$$\therefore |\vec{a}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$$

Hence, the direction cosines of \vec{a} are $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$.

Question 13:

Find the direction cosines of the vector joining the points A (1, 2, -3) and B (-1, -2, 1) directed from A to B.

Answer

The given points are A (1, 2, -3) and B (-1, -2, 1).

$$\therefore \vec{AB} = (-1-1)\hat{i} + (-2-2)\hat{j} + \{1-(-3)\}\hat{k}$$

$$\Rightarrow \vec{AB} = -2\hat{i} - 4\hat{j} + 4\hat{k}$$

$$\therefore |\vec{AB}| = \sqrt{(-2)^2 + (-4)^2 + 4^2} = \sqrt{4 + 16 + 16} = \sqrt{36} = 6$$

Hence, the direction cosines of \vec{AB} are $\left(-\frac{2}{6}, -\frac{4}{6}, \frac{4}{6}\right) = \left(-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right)$.

Question 14:

Show that the vector $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the axes OX, OY, and OZ.

Answer

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$.

Then,

$$|\vec{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

Therefore, the direction cosines of \vec{a} are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.

Now, let α , β , and γ be the angles formed by \vec{a} with the positive directions of x, y, and z axes.

$$\cos \alpha = \frac{1}{\sqrt{3}}, \cos \beta = \frac{1}{\sqrt{3}}, \cos \gamma = \frac{1}{\sqrt{3}}.$$

Then, we have

Hence, the given vector is equally inclined to axes OX, OY, and OZ.

Question 15:

Find the position vector of a point R which divides the line joining two points P and Q

whose position vectors are $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively, in the ratio 2:1

(i) internally

(ii) externally

Answer

The position vector of point R dividing the line segment joining two points

P and Q in the ratio $m:n$ is given by:

i. Internally:

$$\frac{m\vec{b} + n\vec{a}}{m+n}$$

ii. Externally:

$$\frac{m\vec{b} - n\vec{a}}{m-n}$$

$$\overline{m-n}$$

Position vectors of P and Q are given as:

$$\overline{OP} = \hat{i} + 2\hat{j} - \hat{k} \text{ and } \overline{OQ} = -\hat{i} + \hat{j} + \hat{k}$$

(i) The position vector of point R which divides the line joining two points P and Q internally in the ratio 2:1 is given by,

$$\begin{aligned}\overline{OR} &= \frac{2(-\hat{i} + \hat{j} + \hat{k}) + 1(\hat{i} + 2\hat{j} - \hat{k})}{2+1} = \frac{(-2\hat{i} + 2\hat{j} + 2\hat{k}) + (\hat{i} + 2\hat{j} - \hat{k})}{3} \\ &= \frac{-\hat{i} + 4\hat{j} + \hat{k}}{3} = -\frac{1}{3}\hat{i} + \frac{4}{3}\hat{j} + \frac{1}{3}\hat{k}\end{aligned}$$

(ii) The position vector of point R which divides the line joining two points P and Q externally in the ratio 2:1 is given by,

$$\begin{aligned}\overline{OR} &= \frac{2(-\hat{i} + \hat{j} + \hat{k}) - 1(\hat{i} + 2\hat{j} - \hat{k})}{2-1} = \frac{(-2\hat{i} + 2\hat{j} + 2\hat{k}) - (\hat{i} + 2\hat{j} - \hat{k})}{1} \\ &= -3\hat{i} + 3\hat{k}\end{aligned}$$

Question 16:

Find the position vector of the mid point of the vector joining the points P (2, 3, 4) and Q (4, 1, -2).

Answer

The position vector of mid-point R of the vector joining points P (2, 3, 4) and Q (4, 1, -2) is given by,

$$\begin{aligned}\overline{OR} &= \frac{(2\hat{i} + 3\hat{j} + 4\hat{k}) + (4\hat{i} + \hat{j} - 2\hat{k})}{2} = \frac{(2+4)\hat{i} + (3+1)\hat{j} + (4-2)\hat{k}}{2} \\ &= \frac{6\hat{i} + 4\hat{j} + 2\hat{k}}{2} = 3\hat{i} + 2\hat{j} + \hat{k}\end{aligned}$$

Question 17:

Show that the points A, B and C with position vectors, $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$,

$\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$, respectively form the vertices of a right angled triangle.

Answer

Position vectors of points A, B, and C are respectively given as:

$$\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k} \text{ and } \vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$$

$$\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}, \vec{b} = 2\hat{i} - \hat{j} + \hat{k} \text{ and } \vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$$

$$\therefore \overline{AB} = \vec{b} - \vec{a} = (2-3)\hat{i} + (-1+4)\hat{j} + (1+4)\hat{k} = -\hat{i} + 3\hat{j} + 5\hat{k}$$

$$\overline{BC} = \vec{c} - \vec{b} = (1-2)\hat{i} + (-3+1)\hat{j} + (-5-1)\hat{k} = -\hat{i} - 2\hat{j} - 6\hat{k}$$

$$\overline{CA} = \vec{a} - \vec{c} = (3-1)\hat{i} + (-4+3)\hat{j} + (-4+5)\hat{k} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\therefore |\overline{AB}|^2 = (-1)^2 + 3^2 + 5^2 = 1 + 9 + 25 = 35$$

$$|\overline{BC}|^2 = (-1)^2 + (-2)^2 + (-6)^2 = 1 + 4 + 36 = 41$$

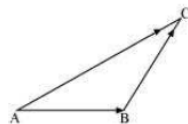
$$|\overline{CA}|^2 = 2^2 + (-1)^2 + 1^2 = 4 + 1 + 1 = 6$$

$$\therefore |\overline{AB}|^2 + |\overline{CA}|^2 = 35 + 6 = 41 = |\overline{BC}|^2$$

Hence, ABC is a right-angled triangle.

Question 18:

In triangle ABC which of the following is **not** true:



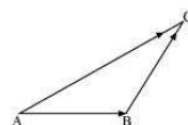
A. $\overline{AB} + \overline{BC} + \overline{CA} = \vec{0}$

B. $\overline{AB} + \overline{BC} - \overline{AC} = \vec{0}$

C. $\overline{AB} + \overline{BC} - \overline{CA} = \vec{0}$

D. $\overline{AB} - \overline{CB} + \overline{CA} = \vec{0}$

Answer



On applying the triangle law of addition in the given triangle, we have:

$$\overline{AB} + \overline{BC} = \overline{AC} \quad \dots(1)$$

$$\Rightarrow \overline{AB} + \overline{BC} = \overline{CA}$$

$$\Rightarrow \overline{AB} + \overline{BC} + \overline{CA} = \vec{0} \quad \dots(2)$$

\therefore The equation given in alternative A is true.

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

$$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$$

\therefore The equation given in alternative B is true.

From equation (2), we have:

$$\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$$

\therefore The equation given in alternative D is true.

Now, consider the equation given in alternative C:

$$\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \vec{0}$$

$$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{CA} \quad \dots(3)$$

From equations (1) and (3), we have:

$$\overrightarrow{AC} = \overrightarrow{CA}$$

$$\Rightarrow \overrightarrow{AC} = -\overrightarrow{AC}$$

$$\Rightarrow \overrightarrow{AC} + \overrightarrow{AC} = \vec{0}$$

$$\Rightarrow 2\overrightarrow{AC} = \vec{0}$$

$$\Rightarrow \overrightarrow{AC} = \vec{0}, \text{ which is not true.}$$

Hence, the equation given in alternative C is **incorrect**.

The correct answer is **C**.

Question 19:

If \vec{a} and \vec{b} are two collinear vectors, then which of the following are **incorrect**:

A. $\vec{b} = \lambda \vec{a}$, for some scalar λ

B. $\vec{a} = \pm \vec{b}$

C. the respective components of \vec{a} and \vec{b} are proportional

D. both the vectors \vec{a} and \vec{b} have same direction, but different magnitudes

Answer

If \vec{a} and \vec{b} are two collinear vectors, then they are parallel.

Therefore, we have:

$$\vec{b} = \lambda \vec{a} \text{ (For some scalar } \lambda)$$

If $\lambda = \pm 1$, then $\vec{a} = \pm \vec{b}$.

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then

$$\vec{b} = \lambda \vec{a}.$$

$$\Rightarrow b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = \lambda(a_1\hat{i} + a_2\hat{j} + a_3\hat{k})$$

$$\Rightarrow b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = (\lambda a_1)\hat{i} + (\lambda a_2)\hat{j} + (\lambda a_3)\hat{k}$$

$$\Rightarrow b_1 = \lambda a_1, b_2 = \lambda a_2, b_3 = \lambda a_3$$

$$\Rightarrow \frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3} = \lambda$$

Thus, the respective components of \vec{a} and \vec{b} are proportional.

However, vectors \vec{a} and \vec{b} can have different directions.

Hence, the statement given in **D** is **incorrect**.

The correct answer is **D**.

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