Algebraic Identities Ex 4.5 Q4

Answer :

In the given problem, we have to find value of $a^3+b^3+c^3-3abc$

Given $a+b+c=9, a^2+b^2+c^2=35$

We shall use the identity

$$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$$

$$(a+b+c)^2=35+2(ab+bc+ca)$$

$$(9)^2=35+2(ab+bc+ca)$$

$$81-35=2(ab+bc+ca)$$

$$\frac{46}{2}=ab+bc+ca$$

$$23=ab+bc+ca$$

We know that

$$a^3+b^3+c^3-3abc=(a+b+c)\left(a^2+b^2+c^2-ab-bc-ca\right)$$

$$a^3+b^3+c^3-3abc=(a+b+c)\left[\left(a^2+b^2+c^2\right)-(ab+bc+ca)\right]$$

Here substituting $a+b+c=9, a^2+b^2+c^2=35, ab+bc+ca=23$ we get

$$a^3+b^3+c^3-3abc=9[(35-23)]$$

$$=9\times 12$$

$$=108$$

Hence the value of $a^3+b^3+c^3-3abc$ is $\boxed{108}$.

Algebraic Identities Ex 4.5 Q5

Answer :

In the given problem we have to evaluate the following

(i) Given $25^3-75^3+50^3$

We shall use the identity $a^3+b^3+c^3-3abc=(a+b+c)\left(a^2+b^2+c^2-ab-bc-ca\right)$

Let Take $a=25, b=75, c=50$

$$a^3+b^3+c^3-3abc=(a+b+c)\left(a^2+b^2+c^2-ab-bc-ca\right)$$

$$a^3+b^3+c^3=(a+b+c)\left(a^2+b^2+c^2-ab-bc-ca\right)+3abc$$

$$a^3+b^3+c^3=(-75+25+50)\left(a^2+b^2+c^2-ab-bc-ca\right)+3abc$$

$$a^3+b^3+c^3=0\times\left(a^2+b^2+c^2-ab-bc-ca\right)+3abc$$

$$a^3+b^3+c^3=+3abc$$

$$25^3-75^3+50^3=3\times 25\times 50\times -75$$

$$=-281250$$

Hence the value of $25^3-75^3+50^3$ is $\boxed{-281250}$

(ii) Given $48^3-30^3-18^3$

We shall use the identity $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

Let Take $a=48, b=-30, c=-18$

$$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$
$$a^3+b^3+c^3=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)+3abc$$
$$a^3+b^3+c^3=(48-30-18)(a^2+b^2+c^2-ab-bc-ca)+3abc$$
$$a^3+b^3+c^3=0\times(a^2+b^2+c^2-ab-bc-ca)+3abc$$
$$a^3-b^3-c^3=+3abc$$
$$48^3-30^3-18^3=3\times 48\times -30\times -18$$
$$=77760$$

Hence the value of $25^3-75^3+50^3$ is $\boxed{77760}$

Applying least common multiple we get,

$$a^3+b^3+c^3=\left(\frac{1}{2}+\frac{1}{3}-\frac{5}{6}\right)(a^2+b^2+c^2-ab-bc-ca)+3abc$$
$$a^3+b^3+c^3=\left(\frac{1\times 6}{2\times 6}+\frac{1\times 4}{3\times 4}-\frac{5}{6}\right)(a^2+b^2+c^2-ab-bc-ca)+3abc$$
$$a^3+b^3+c^3=\frac{6}{12}+\frac{4}{12}-\frac{10}{12}(a^2+b^2+c^2-ab-bc-ca)+3abc$$
$$a^3+b^3+c^3=0(a^2+b^2+c^2-ab-bc-ca)+3abc$$
$$a^3+b^3+c^3=+3abc$$
$$\left(\frac{1}{2}\right)^3+\left(\frac{1}{3}\right)^3-\left(\frac{5}{6}\right)^3=3\times\frac{1}{2}\times\frac{1}{3}\times-\frac{5}{6}$$
$$=\cancel{3}\times\frac{1}{2}\times\frac{1}{\cancel{3}}\times-\frac{5}{6}$$
$$=-\frac{5}{12}$$

Hence the value of $\left(\frac{1}{2}\right)^3+\left(\frac{1}{3}\right)^3-\left(\frac{5}{6}\right)^3$ is $\boxed{-\frac{5}{12}}$

(iv) Given $(0.2)^3-(0.3)^3+(0.1)^3$

We shall use the identity $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

Let Take $a=0.2, b=0.3, c=0.1$

$$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$
$$a^3+b^3+c^3=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)+3abc$$
$$a^3+b^3+c^3=(0.2-0.3+0.1)(a^2+b^2+c^2-ab-bc-ca)+3abc$$
$$a^3+b^3+c^3=0\times(a^2+b^2+c^2-ab-bc-ca)+3abc$$
$$a^3+b^3+c^3=+3abc$$
$$(0.2)^3-(0.3)^3+(0.1)^3=3\times 0.2\times 0.3\times 0.1$$
$$=-0.018$$

Hence the value of $(0.2)^3-(0.3)^3+(0.1)^3$ is $\boxed{-0.018}$

*********** END **********