Algebra of Matrices Ex 5.3 Q46

Given,

$$A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$A^2 - 7A + 10I_3$

$$= \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} - 7 \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} + 10 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9+2+0 & 6+8+0 & 0+0+0 \\ 3+4+0 & 2+16+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+25 \end{bmatrix} - \begin{bmatrix} 21 & 14 & 0 \\ 7 & 28 & 0 \\ 0 & 0 & 35 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{bmatrix} - \begin{bmatrix} 21 & 14 & 0 \\ 7 & 28 & 0 \\ 0 & 0 & 35 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 11-21+10 & 14-14+0 & 0-0+0 \\ 7-7+0 & 18-28+10 & 0-0+0 \\ 0-0+0 & 0-0+0 & 25-35+10 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$= 0$

Hence,

$$A^2 - 7A + 10I_3 = 0$$

Algebra of Matrices Ex 5.3 Q47

Given,

$$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} x & y \\ z & u \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5x - 7z & 5y - 7u \\ -2x + 3z & -2y + 3u \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$$5x - 7z = -16 \qquad\qquad ---(i)$$
$$-2x + 3z = 7 \qquad\qquad ---(ii)$$
$$5y - 7u = -6 \qquad\qquad ---(iii)$$
$$-2y + 3u = 2 \qquad\qquad ---(iv)$$

Solving equation (i) and (ii)

$$10x - 14z = -32$$
$$\underline{-10x + 15z = 35}$$
$$z = 3$$

Put the value of z in equation (i)

$$5x - 7(3) = -16$$
$$\Rightarrow \quad 5x = 16 + 21$$
$$\Rightarrow \quad 5x = 5$$
$$\Rightarrow \quad x = 1$$

Solving equation (iii) and (iv)

$$10y - 14u = -12$$
$$\underline{-10y + 15u = 10}$$
$$u = -2$$

Put the value of u in equation (iii)

$$5y - 7u = -6$$
$$\Rightarrow \quad 5y - 7(-2) = -6$$
$$\Rightarrow \quad 5y + 14 = -6$$
$$\Rightarrow \quad 5y = -20$$
$$\Rightarrow \quad y = -4$$

Algebra of Matrices Ex 5.3 Q48

Given,

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

Since, $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}_{2 \times 2}$ $A = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}_{2 \times 3}$

$\Rightarrow$ A is a matrix of order 2×3

So,

Let $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

$\Rightarrow$ $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$

$\Rightarrow$ $\begin{bmatrix} a+d & b+e & c+c \\ 0+d & 0+e & 0+f \end{bmatrix} = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$

Since, corresponding entries of equal matrices are equal, so
$d = 1, e = 0, f = 1$

And $a + d = 3$

$a + 1 = 3$

$a = 3 - 1$

$a = 2$

$b + e = 3$

$b + 0 = 3$

$b = 3$

And $c + f = 5$

$c + 1 = 5$

$c = 4$

Hence,

$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 1 \end{bmatrix}$

Algebra of Matrices Ex 5.3 Q48(ii)

It is given that:

$$A\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

The matrix given on the R.H.S. of the equation is a 2×3 matrix and the one given on the L.H.S. of the equation is a 2×3 matrix. Therefore, X has to be a 2×2 matrix.

Now, let $X = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

Therefore, we have:

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+4c & 2a+5c & 3a+6c \\ b+4d & 2b+5d & 3b+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

Equating the corresponding elements of the two matrices, we have:

$a+4c = -7$, $2a+5c = -8$, $3a+6c = -9$

$b+4d = 2$, $2b+5d = 4$, $3b+6d = 6$

Now, $a+4c = -7 \Rightarrow a = -7-4c$

$\therefore 2a+5c = -8 \Rightarrow -14-8c+5c = -8$

$\Rightarrow -3c = 6$

$\Rightarrow c = -2$

$\therefore a = -7-4(-2) = -7+8 = 1$

Now, $b+4d = 2 \Rightarrow b = 2-4d$

$\therefore 2b+5d = 4 \Rightarrow 4-8d+5d = 4$

$\Rightarrow -3d = 0$

$\Rightarrow d = 0$

$\therefore b = 2-4(0) = 2$

Thus, $a = 1$, $b = 2$, $c = -2$, $d = 0$

Hence, the required matrix X is $\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$.

Algebra of Matrices Ex 5.3 Q48(iii)

We know that two matrices B and C are eligible
for the product BC only when number of columns of B
is equal to number of rows in C. So, from the given definition
we can conclude that the order of matrix A is 1 x 3 i.e. we can
assume $A = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$.

Therefore,

$$\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}_{3\times 1} \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}_{1\times 3} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}_{3\times 3},$$

$$\Rightarrow \begin{bmatrix} 4\times(x_1) & 4\times(x_2) & 4\times(x_3) \\ 1\times(x_1) & 1\times(x_2) & 1\times(x_3) \\ 3\times(x_1) & 3\times(x_2) & 3\times(x_3) \end{bmatrix}_{3\times 3} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}_{3\times 3}$$

$$\Rightarrow \begin{bmatrix} 4x_1 & 4x_2 & 4x_3 \\ x_1 & x_2 & x_3 \\ 3x_1 & 3x_2 & 3x_3 \end{bmatrix}_{3\times 3} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}_{3\times 3}$$

$\Rightarrow \quad 4x_1 = -4$, $4x_2 = 8$, $4x_3 = 4$

Solving $x_1 = -1$, $x_2 = 2$, $x_3 = 1$

So, matrix $A = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$.

Algebra of Matrices Ex 5.3 Q48(iv)

Using matrix multiplication,

Let, $A_1 = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}$, $A_2 = \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and $A_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

Now, $A_1 \cdot A_2 = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} =$

$= \begin{bmatrix} (2 \times -1) + (1 \times -1) + (3 \times 0) & (2 \times 0) + (1 \times 1) + (3 \times 1) & (2 \times -1) + (1 \times 0) + (3 \times 1) \end{bmatrix}$

$= \begin{bmatrix} -3 & 4 & 1 \end{bmatrix}$

and $(A_1 \cdot A_2) A_3 = \begin{bmatrix} -3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$= \begin{bmatrix} (-3 \times 1) + (4 \times 0) + (1 \times -1) \end{bmatrix}$

$(A_1 \cdot A_2) A_3 = \begin{bmatrix} -4 \end{bmatrix} = A$

Therefore matrix $A = \begin{bmatrix} -4 \end{bmatrix}$

Note : The problem can also be solved by calculating $(A_2 A_3)$ first
then pre multiplying it with A_1 as matrix multiplication is
associative but one must not change the order of multiplication.

*********** END *********

-1 0 -1

Let, $A_1 = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}$, $A_2 = \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and $A_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

Now, $A_1 \cdot A_2 = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} =$

$= \begin{bmatrix} (2 \times -1) + (1 \times -1) + (3 \times 0) & (2 \times 0) + (1 \times 1) + (3 \times 1) & (2 \times -1) + (1 \times 0) + (3 \times 1) \end{bmatrix}$

$= \begin{bmatrix} -3 & 4 & 1 \end{bmatrix}$

and $(A_1 \cdot A_2) A_3 = \begin{bmatrix} -3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$= \begin{bmatrix} (-3 \times 1) + (4 \times 0) + (1 \times -1) \end{bmatrix}$