Definite Integrals Ex 20.3 Q3

We have,

$$\int_{-3}^{3} |x + 1|dx$$

$$= \int_{-3}^{-1} -(x+1)dx + \int_{-1}^{3} (x+1)dx$$

$$= -\left[\frac{x^2}{2}+x\right]_{-3}^{-1} + \left[\frac{x^2}{2}+x\right]_{-1}^{3}$$

$$= -\left[\left(\frac{1}{2}-1\right)-\left(\frac{9}{2}-3\right)\right]+\left[\left(\frac{9}{2}+3\right)-\left(\frac{1}{2}-1\right)\right]$$

$$= -\left[\left(-\frac{1}{2}\right)-\left(1\frac{1}{2}\right)\right]+\left[\left(7\frac{1}{2}\right)-\left(-\frac{1}{2}\right)\right]$$

$$= -\left[-\frac{1}{2}-1\frac{1}{2}\right]+\left[7\frac{1}{2}+\frac{1}{2}\right]$$

$$= [-2]+[8]$$

$$= 2+8$$

$$= 10$$

$$\therefore \int_{-3}^{3} |x+1|dx = 10$$

Definite Integrals Ex 20.3 Q4

We have,

$$\int_{-1}^{1} |2x+1| \, dx$$

$$= \int_{-1}^{-\frac{1}{2}} -(2x+1) \, dx + \int_{-\frac{1}{2}}^{1} (2x+1) \, dx$$

$$= -\left[\frac{2x^2}{2} + x\right]_{-1}^{-\frac{1}{2}} + \left[\frac{2x^2}{2} + x\right]_{-\frac{1}{2}}^{1}$$

$$= -\left[\left(\frac{2}{8} - \frac{1}{2}\right) - \left(\frac{2}{2} - 1\right)\right] + \left[\left(\frac{2}{2} + 1\right) - \left(\frac{2}{8} - \frac{1}{2}\right)\right]$$

$$= -\left[\left(\frac{1}{4} - \frac{1}{2}\right) - (1-1)\right] + \left[(1+1) - \left(\frac{1}{4} - \frac{1}{2}\right)\right]$$

$$= -\left[-\frac{1}{4}\right] + \left[2 + \frac{1}{4}\right]$$

$$= \frac{1}{4} + 2 + \frac{1}{4}$$

$$= 2\frac{1}{2}$$

$$\therefore \int_{-1}^{1} |2x+1| \, dx = \frac{5}{2}$$

Definite Integrals Ex 20.3 Q5

(i)

$$\int_{-2}^{2} |2x + 3| dx$$

$$= \int_{-2}^{-\frac{3}{2}} -(2x + 3) dx + \int_{-\frac{3}{2}}^{2} (2x + 3) dx$$

$$= -\left[\frac{2x^2}{2} + 3x\right]_{-2}^{-\frac{3}{2}} + \left[\frac{2x^2}{2} + 3x\right]_{-\frac{3}{2}}^{2}$$

$$= -\left[\left(\frac{2 \times 9}{2 \times 4} - \frac{9}{2}\right) - \left(\frac{2 \times 4}{2} - 6\right)\right] + \left[\left(\frac{2 \times 4}{2} + 6\right) - \left(\frac{2 \times 9}{2 \times 4} - \frac{9}{2}\right)\right]$$

$$= -\left[\left(\frac{18}{8} - \frac{9}{2}\right) - \left(\frac{8}{2} - 6\right)\right] + \left[\left(\frac{8}{2} + 6\right) - \left(\frac{18}{8} - \frac{9}{2}\right)\right]$$

$$= -\left[\left(\frac{9}{4} - \frac{9}{2}\right) - (-2)\right] + \left[(10) - \left(\frac{9}{4} - \frac{9}{2}\right)\right]$$

$$= \left[-\frac{9}{4} + 2\right] + \left[10 + \frac{9}{4}\right]$$

$$= \frac{9}{4} - 2 + 10 + \frac{9}{4}$$

$$\Rightarrow 8\frac{9}{2}$$

$$= 12\frac{1}{2}$$

$$\therefore \int_{-2}^{2} |2x + 3| dx = \frac{25}{2}$$

Definite Integrals Ex 20.3 Q6

(ii)

We have,

$$f(x) = |x^2 - 3x + 2|$$

$$= |(x-1)(x-2)|$$

$$= \begin{cases} x^2 - 3x + 2 & 0 \le x \le 1 \\ -(x^2 - 3x + 2) & 1 \le x \le 2 \end{cases}$$

Hence,

$$\int_0^2 |x^2 - 3x + 2| \, dx$$

$$= \int_0^1 (x^2 - 3x + 2) \, dx + \int_1^2 -(x^2 - 3x + 2) \, dx$$

$$= \left[ \frac{x^3}{3} - \frac{3x^2}{2} + 2x \right]_0^1 - \left[ \frac{x^3}{3} - \frac{3x^2}{2} + 2x \right]_1^2$$

$$= \left[ \frac{1}{3} - \frac{3}{2} + 2 - 0 \right] - \left[ \frac{8}{3} - \frac{12}{2} + 4 - \frac{1}{3} + \frac{3}{2} + 2 \right]$$

$$= \left[ \frac{1}{6} \right] - \left[ -\frac{5}{6} \right]$$

$$= \frac{1}{6} + \frac{5}{6}$$

$$= 1$$

$$\therefore \int_0^2 |x^2 - 3x + 2| \, dx = 1$$

********** END *********