Q4. Simplify:

(i) $\left(a^2-b^2\right)^2$

(ii) $(2x+5)^2-(2x-5)^2$

(iii) $(7m-8n)^2+(7m+8n)^2$

(iv) $(4m+5n)^2+(5m+4n)^2$

(v) $(2.5p-1.5q)^2-(1.5p-2.5q)^2$

(vi) $(ab+bc)^2-2ab^2c$

(vii) $\left(m^2-n^2m\right)^2+2m^3n^2$

Ans: (i) $\left(a^2-b^2\right)^2$

$=\left(a^2\right)^2-2\times a^2\times b^2+\left(b^2\right)^2$

[Using identity $(a-b)^2=a^2-2ab+b^2$]

$=a^4-2a^2b^2+b^4$

(ii) $(2x+5)^2-(2x-5)^2$

$=(2x)^2+2\times 2x\times 5+(5)^2-\left[(2x)^2-2\times 2x\times 5+(5)^2\right]$

[Using identities $(a+b)^2=a^2+2ab+b^2$ and $(a-b)^2=a^2-2ab+b^2$]

$= 4x^2 + 20x + 25 - \left[4x^2 - 20x + 25\right]$

$= 4x^2 + 20x + 25 - 4x^2 + 20x - 25$

$= 40x$

(iii) $(7m - 8n)^2 + (7m + 8n)^2$

$= (7m)^2 - 2 \times 7m \times 8n + (8n)^2$

$+ \left[(7m)^2 + 2 \times 7m \times 8n + (8n)^2\right]$

[Using identities $(a+b)^2 = a^2 + 2ab + b^2$ and

$(a-b)^2 = a^2 - 2ab + b^2$]

$= 49m^2 - 112mn + 64n^2 + \left[49m^2 + 112mn + 64n^2\right]$

$= 49m^2 - 112mn + 64n^2 + 49m^2 + 112mn + 64n^2$

$= 98m^2 + 128n^2$

(iv) $(4m + 5n)^2 + (5m + 4n)^2$

$= (4m)^2 + 2 \times 4m \times 5n + (5n)^2 + (5m)^2 + 2 \times 5m \times 4n + (4n)^2$

$= (4m)^2 + 2 \times 4m \times 5n + (5n)^2 + (5m)^2 + 2 \times 5m \times 4n + (4n)^2$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

$= 16m^2 + 40mn + 25n^2 + 25m^2 + 40mn + 16n^2$

$= 16m^2 + 25m^2 + 40mn + 40mn + 25n^2 + 16n^2$

$= 41m^2 + 80mn + 41n^2$

(v) $(2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$

$= (2.5p)^2 - 2 \times 2.5p \times 1.5q + (1.5q)^2 - \left[(1.5p)^2 - 2 \times 1.5p \times 2.5q + (2.5q)^2\right]$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= 6.25p^2 - 7.50pq + 2.25q^2 - \left[2.25p^2 - 7.50pq + 6.25q^2\right]$

$= 6.25p^2 - 7.50pq + 2.25q^2 - 2.25p^2 + 7.50pq - 6.25q^2$

$= 4p^2 - 4q^2$

(vi)

$(ab + bc)^2 - 2ab^2c = (ab)^2 + 2 \times ab \times bc + (bc)^2 - 2ab^2c$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

$= a^2b^2 + 2ab^2c + b^2c^2 - 2ab^2c$

$= a^2b^2 + b^2c^2$

(vii) $(m^2 - n^2m)^2 + 2m^3n^2$

$= \left(m^2\right)^2 - 2 \times m^2 \times n^2 m + \left(n^2 m\right)^2 + 2m^3 n^2$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= m^4 - 2m^3 n^2 + n^4 m^2 + 2m^3 n^2$

$= m^4 + n^4 m^2$

Q5. Show that:

(i) $(3x+7)^2 - 84x = (3x-7)^2$

(ii) $(9p-5q)^2 + 180\,pq = (9p+5q)^2$

(iii) $\left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$

(iv) $(4pq+3q)^2 - (4pq-3q)^2 = 48pq^2$

(v) $(a-b)(a+b) + (b-c)(b+c) + (c-a)(c+a) = 0$

Ans: (i) L.H.S. $= (3x+7)^2 - 84x$

$= (3x)^2 + 2 \times 3x \times 7 + (7)^2 - 84x$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

$= 9x^2 + 42x + 49 - 84x$

$= 9x^2 - 42x + 49$

$= (3x-7)^2$ [$\because (a-b)^2 = a^2 - 2ab + b^2$]

= R.H.S.

(ii) L.H.S. $= (9p-5q)^2+180pq$

$=(9p)^2-2\times 9p\times 5q+(5q)^2+180pq$

[Using identity $(a-b)^2=a^2-2ab+b^2$]

$= 81p^2-90pq+25q^2+180pq$

$= 81p^2+90pq+25q^2$

$= (9p+5q)^2$ [$\because (a+b)^2=a^2+2ab+b^2$]

(iii) L.H.S. $= \left(\frac{4}{3}m-\frac{3}{4}n\right)^2+2mn$

$=\left(\frac{4}{3}m\right)^2-2\times\frac{4}{3}m\times\frac{3}{4}n+\left(\frac{3}{4}n\right)^2+2mn$

[Using identity $(a-b)^2=a^2-2ab+b^2$]

$=\frac{16}{9}m^2-2mn+\frac{9}{16}n^2+2mn$

$=\frac{16}{9}m^2+\frac{9}{16}n^2$

= R.H.S.

(iv) L.H.S. $= (4pq+3q)^2-(4pq-3q)^2$

$=(4pq)^2+2\times 4pq\times 3q+(3q)^2-\left[(4pq)^2-2\times 4pq\times 3q+(3q)^2\right]$

[Using identities $(a+b)^2=a^2+2ab+b^2$ and
$(a-b)^2=a^2-2ab+b^2$]

$= 16p^2q^2+24pq^2+9q^2-\left[16p^2q^2-24pq^2+9q^2\right]$

$= 16p^2q^2+24pq^2+9q^2-16p^2q^2+24pq^2-9q^2$

$= 48pq^2$

= R.H.S.

(v) L.H.S.=

$(a-b)(a+b)+(b-c)(b+c)+(c-a)(c+a)$

$= a^2-b^2+b^2-c^2+c^2-a^2$

[Using identity $(a-b)(a+b)=a^2-b^2$]

= 0

= R.H.S.

Q6. Using identities, evaluate:

(i) 71^2

(ii) 99^2

(iii) 102^2

(iv) 998^2

(v) 5.2^2

(vi) 297×303

(vii) 78×82

(viii) 8.9^2

(ix) 1.05×9.5

Ans: (i) $71^2 = (70+1)^2$

$= (70)^2 + 2 \times 70 \times 1 + (1)^2$

[Using identity $(a+b)^2 = a^2 - 2ab + b^2$]

$= 4900 + 140 + 1 = 5041$

(ii) $99^2 = (100-1)^2$

$= (100)^2 - 2 \times 100 \times 1 + (1)^2$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

= 10000 − 200 + 1 = 9801

(iii) $102^2 = (100+2)^2$

$= (100)^2 + 2\times100\times2 + (2)^2$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

= 10000 + 400 + 4 = 10404

(iv) $998^2 = (1000-2)^2$

$= (1000)^2 - 2\times1000\times2 + (2)^2$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

= 1000000 − 4000 + 4 = 996004

(v) $5.2^2 = (5+0.2)^2$

$= (5)^2 + 2\times5\times0.2 + (0.2)^2$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

= 25 + 2.0 + 0.04 = 27.04

(vi) 297×303

$= (300 - 3)\times(300 + 3)$

$= (300)^2 - (3)^2$

[Using identity $(a-b)(a+b)=a^2-b^2$]

= 90000 – 9 = 89991

(vii) $78\times 82=(80\ -\ 2)\times(80\ +\ 2)$

$=(80)^2-(2)^2$

[Using identity $(a-b)(a+b)=a^2-b^2$]

= 6400 – 4 = 6396

(viii) $8.9^2=(8+0.9)^2$

$=(8)^2+2\times 8\times 0.9+(0.9)^2$

[Using identity $(a+b)^2=a^2+2ab+b^2$]

= 64 + 14.4 + 0.81 = 79.21

(ix) $1.05\times 9.5=(10\ +\ 0.5)\times(10\ -\ 0.5)$

$=(10)^2-(0.5)^2$

[Using identity $(a-b)(a+b)=a^2-b^2$]

= 100 – 0.25 = 99.75

Q7. Using $a^2 - b^2 = (a+b)(a-b)$, find

(i) $51^2 - 49^2$

(ii) $(1.02)^2 - (0.98)^2$

(iii) $153^2 - 147^2$

(iv) $12.1^2 - 7.9^2$

Ans: (i) $51^2 - 49^2 = (51+49)(51-49)$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

$= 100 \times 2 = 200$

(ii) $(1.02)^2 - (0.98)^2$

$= (1.02+0.98)(1.02-0.98)$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

$= 2.00 \times 0.04 = 0.08$

(iii) $153^2 - 147^2 = (153+147)(153-147)$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

$= 300 \times 6 = 1800$

(iv) $12.1^2 - 7.9^2 = (12.1+7.9)(12.1-7.9)$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

$= 20.0 \times 4.2 = 84.0 = 84$

Q8. Using $(x+a)(x+b)=x^2+(a+b)x+ab$, find

(i) 103×104

(ii) 5.1×5.2

(iii) 103×98

(iv) 9.7×9.8

Ans: (i) $103\times104=(100+3)\times(100+4)$

$=(100)^2+(3+4)\times100+3\times4$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$=10000+7\times100+12$

$=10000+700+12=10712$

(ii) $5.1\times5.2=(5+0.1)\times(5+0.2)$

$=(5)^2+(0.1+0.2)\times5+0.1\times0.2$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$=25+0.3\times5+0.02$

$=25+1.5+0.02=26.52$

(iii) $103\times98=(100+3)\times(100-2)$

$=(100)^2+(3-2)\times100+3\times(-2)$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$=10000+(3-2)\times100-6$

$=10000+100-6=10094$

(iv) $9.7\times9.8=(10-0.3)\times(10-0.2)$

$=(10)^2+\{(-0.3)+(-0.2)\}\times10+(-0.3)\times(-0.2)$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$=100+\{-0.3-0.2\}\times10+0.06$

$=100-0.5\times10+0.06$

$=100-5+0.06=95.06$

********** END **********