Trigonometric Ratios Ex 5.1 Q15

Answer :

Given: $\cot \theta = \dfrac{1}{\sqrt{3}}$ ......(1)

To show that $\dfrac{1 - \cos^2 \theta}{2 - \sin^2 \theta} = \dfrac{3}{5}$

Now, we know that $\cot \theta = \dfrac{1}{\tan \theta}$

Since $\tan \theta = \dfrac{\text{Perpendicular side opposite to} \angle \theta}{\text{Base side adjacent to} \angle \theta}$

Therefore,

$\cot \theta = \dfrac{1}{\dfrac{\text{Perpendicular side opposite to} \angle \theta}{\text{Base side adjacent to} \angle \theta}}$

Therefore,

$\cot \theta = \dfrac{\text{Base side adjacent to} \angle \theta}{\text{Perpendicular side opposite to} \angle \theta}$ ...... (2)

Comparing Equation (1) and (2)

We get,
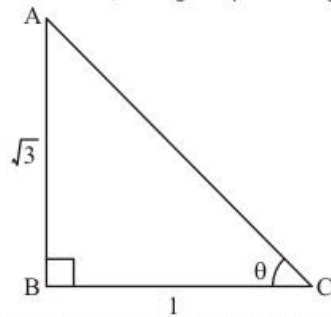
Base side adjacent to $\angle \theta = 1$

Perpendicular side opposite to $\angle \theta = \sqrt{3}$

Therefore, Triangle representing angle θ is as shown below

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Hypotenuse AC is unknown and it can be found by using Pythagoras theorem
Therefore by applying Pythagoras theorem
We get,

$$AC^2 = AB^2 + BC^2$$

Therefore by substituting the values of known sides
We get,

$$AC^2 = \left(\sqrt{3}\right)^2 + 1^2$$

Therefore,

$$AC^2 = 3 + 1$$
$$AC^2 = 4$$
$$AC = \sqrt{4}$$

Therefore,

$$AC = 2 \quad \text{...... (3)}$$

Now, we know that

$$\sin\theta = \frac{\text{Perpendicular side opposite to} \angle\theta}{\text{Hypotenuse}}$$

Now from figure (a)
We get,

$$\sin\theta = \frac{AB}{AC}$$

Therefore from figure (a) and equation (3),

$$\sin\theta = \frac{\sqrt{3}}{2} \quad \text{...... (4)}$$

Now, we know that

$$\cos\theta = \frac{\text{Base side adjacent to} \angle\theta}{\text{Hypotenuse}}$$

Now from figure (a)
We get,

$$\cos\theta = \frac{BC}{AC}$$

Therefore from figure (a) and equation (3),

$$\cos\theta = \frac{1}{2} \quad \text{...... (5)}$$

Now, L.H.S of the equation to be proved is as follows

$$L.H.S = \frac{1 - \cos^2\theta}{2 - \sin^2\theta}$$

Substituting the value of $\sin\theta$ and $\cos\theta$ from equation (4) and (5)

We get,

$$L.H.S = \frac{1-\left(\frac{1}{2}\right)^2}{2-\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$L.H.S = \frac{1-\frac{(1)^2}{(2)^2}}{2-\frac{(\sqrt{3})^2}{(2)^2}}$$

$$L.H.S = \frac{1-\frac{1}{4}}{2-\frac{3}{4}}$$

Now by taking L.C.M. in numerator as well as denominator
We get,

$$L.H.S = \frac{\frac{(4\times 1)-1}{4}}{\frac{(4\times 2)-3}{4}}$$

Therefore,

$$L.H.S = \dfrac{\dfrac{4-1}{4}}{\dfrac{8-3}{4}}$$

$$L.H.S = \dfrac{\dfrac{3}{4}}{\dfrac{5}{4}}$$

Therefore,

$$L.H.S = \dfrac{3}{4} \times \dfrac{4}{5}$$

$$L.H.S = \dfrac{3}{5} = R.H.S$$

Therefore,

$$\dfrac{1-\cos^2\theta}{2-\sin^2\theta} = \dfrac{3}{5}$$

Hence proved that $\boxed{\dfrac{1-\cos^2\theta}{2-\sin^2\theta} = \dfrac{3}{5}}$

********** END *********