Q12 : A point on the hypotenuse of a triangle is at distance *a* and *b* from the sides of the triangle.

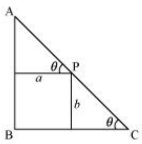
Show that the minimum length of the hypotenuse is $\left(a^{\frac{2}{3}}+b^{\frac{2}{3}}\right)^{\frac{3}{2}}$

Answer :

Let ΔABC be right-angled at B. Let AB = *x* and BC = *y*.

Let P be a point on the hypotenuse of the triangle such that P is at a distance of *a* and *b* from the sides AB and BC respectively.

Let ∠C = *θ*.



We have,

$\text{AC}=\sqrt{x^2+y^2}$

Now,

PC = *b* cosec *θ*

And, AP = *a* sec *θ*

∴AC = AP + PC

⇒ AC = *b* cosec *θ* + *a* sec *θ* ... (1)

$$\therefore \frac{d(\text{AC})}{d\theta}=-b\operatorname{cosec}\theta\cot\theta+a\sec\theta\tan\theta$$

$$\therefore \frac{d(\text{AC})}{d\theta}=0$$

$$\Rightarrow a\sec\theta\tan\theta=b\operatorname{cosec}\theta\cot\theta$$

$$\Rightarrow \frac{a}{\cos\theta}\cdot\frac{\sin\theta}{\cos\theta}=\frac{b}{\sin\theta}\frac{\cos\theta}{\sin\theta}$$

$$\Rightarrow a\sin^3\theta=b\cos^3\theta$$

$$\Rightarrow (a)^{\frac{1}{3}}\sin\theta=(b)^{\frac{1}{3}}\cos\theta$$

$$\Rightarrow \tan\theta=\left(\frac{b}{a}\right)^{\frac{1}{3}}$$

$$\therefore \sin\theta=\frac{(b)^{\frac{1}{3}}}{\sqrt{a^{\frac{2}{3}}+b^{\frac{2}{3}}}} \text{ and } \cos\theta=\frac{(a)^{\frac{1}{3}}}{\sqrt{a^{\frac{2}{3}}+b^{\frac{2}{3}}}} \qquad ...(2)$$

It can be clearly shown that $\frac{d^2(\text{AC})}{d\theta^2}<0$ when $\tan\theta=\left(\frac{b}{a}\right)^{\frac{1}{3}}$.

Therefore, by second derivative test, the length of the hypotenuse is the maximum when $\tan\theta=\left(\frac{b}{a}\right)^{\frac{1}{3}}$.

Now, when $\tan\theta=\left(\frac{b}{a}\right)^{\frac{1}{3}}$, we have:

$$\text{AC}=\frac{b\sqrt{a^{\frac{2}{3}}+b^{\frac{2}{3}}}}{b^{\frac{1}{3}}}+\frac{a\sqrt{a^{\frac{2}{3}}+b^{\frac{2}{3}}}}{a^{\frac{1}{3}}} \qquad [\text{Using (1) and (2)}]$$

$$=\sqrt{a^{\frac{2}{3}}+b^{\frac{2}{3}}}\left(b^{\frac{2}{3}}+a^{\frac{2}{3}}\right)$$

$$=\left(a^{\frac{2}{3}}+b^{\frac{2}{3}}\right)^{\frac{3}{2}}$$

Hence, the maximum length of the hypotenuses is $\left(a^{\frac{2}{3}}+b^{\frac{2}{3}}\right)^{\frac{3}{2}}$.

Q13 : Find the points at which the function *f* given by $f(x)=(x-2)^4(x+1)^3$ has

(i) local maxima (ii) local minima

(ii) point of inflexion

Answer :

Answer :

The given function is $f(x)=(x-2)^4(x+1)^3$.

$$\therefore f'(x)=4(x-2)^3(x+1)^3+3(x+1)^2(x-2)^4$$
$$=(x-2)^3(x+1)^2\left[4(x+1)+3(x-2)\right]$$
$$=(x-2)^3(x+1)^2(7x-2)$$
$$\text{Now, } f'(x)=0 \Rightarrow x=-1 \text{ and } x=\frac{2}{7} \text{ or } x=2$$

Now, for values of x close to $\frac{2}{7}$ and to the left of $\frac{2}{7}$, $f'(x)>0$. Also, for values of x close to $\frac{2}{7}$ and to the right of $\frac{2}{7}$, $f'(x)<0$.

Thus, $x=\frac{2}{7}$ is the point of local maxima.

Now, for values of x close to 2 and to the left of 2, $f'(x)<0$. Also, for values of x close to 2 and to the right of 2, $f'(x)>0$.

Thus, x = 2 is the point of local minima.

Now, as the value of x varies through - 1, $f'(x)$ does not changes its sign.

Thus, x = - 1 is the point of inflexion.

Answer needs Correction? Click Here

Q14 : Find the absolute maximum and minimum values of the function f given by

$f(x)=\cos^2 x+\sin x, x\in[0,\pi]$

Answer :

$$f(x)=\cos^2 x+\sin x$$
$$f'(x)=2\cos x(-\sin x)+\cos x$$
$$=-2\sin x\cos x+\cos x$$
$$\text{Now, } f'(x)=0$$
$$\Rightarrow 2\sin x\cos x=\cos x \Rightarrow \cos x(2\sin x-1)=0$$
$$\Rightarrow \sin x=\frac{1}{2} \text{ or } \cos x=0$$
$$\Rightarrow x=\frac{\pi}{6}, \text{ or } \frac{\pi}{2} \text{ as } x\in[0,\pi]$$

Now, evaluating the value of f at critical points $x=\frac{\pi}{2}$ and $x=\frac{\pi}{6}$ and at the end points of the interval $[0,\pi]$ (i.e., at x = 0 and x = π), we have:

$$f\left(\frac{\pi}{6}\right)=\cos^2\frac{\pi}{6}+\sin\frac{\pi}{6}=\left(\frac{\sqrt{3}}{2}\right)^2+\frac{1}{2}=\frac{5}{4}$$
$$f(0)=\cos^2 0+\sin 0=1+0=1$$
$$f(\pi)=\cos^2\pi+\sin\pi=(-1)^2+0=1$$
$$f\left(\frac{\pi}{2}\right)=\cos^2\frac{\pi}{2}+\sin\frac{\pi}{2}=0+1=1$$

Hence, the absolute maximum value of f is $\frac{5}{4}$ occurring at $x=\frac{\pi}{6}$ and the absolute minimum value of f is 1 occurring at $x=0,\frac{\pi}{2},\text{and}\pi.$

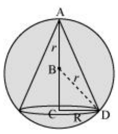
Answer needs Correction? Click Here

Q15 : Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.

Answer :

A sphere of fixed radius (r) is given.

Let R and h be the radius and the height of the cone respectively.



The volume (V) of the cone is given by,

$$V=\frac{1}{3}\pi R^2 h$$

Now, from the right triangle BCD, we have:

$$BC=\sqrt{r^2-R^2}$$
$$\therefore h=r+\sqrt{r^2-R^2}$$

$$\therefore V=\frac{1}{3}\pi R^2\left(r+\sqrt{r^2-R^2}\right)=\frac{1}{3}\pi R^2 r+\frac{1}{3}\pi R^2\sqrt{r^2-R^2}$$
$$\therefore \frac{dV}{dR}=\frac{2}{3}\pi Rr+\frac{2}{3}\pi R\sqrt{r^2-R^2}+\frac{\pi R^2}{3}\cdot\frac{(-2R)}{2\sqrt{r^2-R^2}}$$
$$=\frac{2}{3}\pi Rr+\frac{2}{3}\pi R\sqrt{r^2-R^2}-\frac{\pi R^3}{3\sqrt{r^2-R^2}}$$

$$= \frac{2}{3}\pi R r + \frac{2\pi R\left(r^2 - R^2\right) - \pi R^3}{3\sqrt{r^2 - R^2}}$$

$$= \frac{2}{3}\pi R r + \frac{2\pi R r^2 - 3\pi R^3}{3\sqrt{r^2 - R^2}}$$

Now, $\frac{dV}{dR^2} = 0$

$$\Rightarrow \frac{2\pi r R}{3} = \frac{3\pi R^3 - 2\pi R r^2}{3\sqrt{r^2 - R^2}}$$

$$\Rightarrow 2r\sqrt{r^2 - R^2} = 3R^2 - 2r^2$$

$$\Rightarrow 4r^2\left(r^2 - R^2\right) = \left(3R^2 - 2r^2\right)^2$$

$$\Rightarrow 4r^4 - 4r^2R^2 = 9R^4 + 4r^4 - 12R^2r^2$$

$$\Rightarrow 9R^4 - 8r^2R^2 = 0$$

$$\Rightarrow 9R^2 = 8r^2$$

$$\Rightarrow R^2 = \frac{8r^2}{9}$$

$$\text{Now, } \frac{d^2V}{dR^2} = \frac{2\pi r}{3} + \frac{3\sqrt{r^2 - R^2}\left(2\pi r^2 - 9\pi R^2\right) - \left(2\pi R r^2 - 3\pi R^3\right)\left(-6R\right)\frac{1}{2\sqrt{r^2 - R^2}}}{9\left(r^2 - R^2\right)}$$

$$= \frac{2\pi r}{3} + \frac{3\sqrt{r^2 - R^2}\left(2\pi r^2 - 9\pi R^2\right) + \left(2\pi R r^2 - 3\pi R^3\right)\left(3R\right)\frac{1}{2\sqrt{r^2 - R^2}}}{9\left(r^2 - R^2\right)}$$

Now, when $R^2 = \frac{8r^2}{9}$, it can be shown that $\frac{d^2V}{dR^2} < 0$.

$\therefore$ The volume is the maximum when $R^2 = \frac{8r^2}{9}$.

When $R^2 = \frac{8r^2}{9}$, height of the cone $= r + \sqrt{r^2 - \frac{8r^2}{9}} = r + \sqrt{\frac{r^2}{9}} = r + \frac{r}{3} = \frac{4r}{3}$.

Hence, it can be seen that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.

Answer needs Correction? Click Here

Q16 : Let f be a function defined on [a, b] such that $f'(x) > 0$, for all $x \in (a, b)$. Then prove that f is an increasing function on (a, b).

Answer :

Let such that .
Consider the sub-interval [

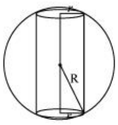
Answer needs Correction? Click Here

Q17 : Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

Answer :

A sphere of fixed radius (R) is given.

Let r and h be the radius and the height of the cylinder respectively.



From the given figure, we have $h = 2\sqrt{R^2 - r^2}$.

The volume (V) of the cylinder is given by,

$$V = \pi r^2 h = 2\pi r^2\sqrt{R^2 - r^2}$$

$$\therefore \frac{dV}{dr} = 4\pi r\sqrt{R^2 - r^2} + \frac{2\pi r^2(-2r)}{2\sqrt{R^2 - r^2}}$$

$$= 4\pi r\sqrt{R^2 - r^2} - \frac{2\pi r^3}{\sqrt{R^2 - r^2}}$$

$$= \frac{4\pi r\left(R^2 - r^2\right) - 2\pi r^3}{\sqrt{R^2 - r^2}}$$

$$= \frac{4\pi r R^2 - 6\pi r^3}{\sqrt{R^2 - r^2}}$$

Now, $\frac{dV}{dr} = 0 \Rightarrow 4\pi r R^2 - 6\pi r^3 = 0$

$$\Rightarrow r^2 = \frac{2R^2}{3}$$

$$\text{Now, } \frac{d^2V}{dr^2} = \frac{\sqrt{R^2 - r^2}\left(4\pi R^2 - 18\pi r^2\right) - \left(4\pi r R^2 - 6\pi r^3\right)\frac{(-2r)}{2\sqrt{R^2 - r^2}}}{\left(R^2 - r^2\right)}$$

$$= \frac{\left(R^2 - r^2\right)\left(4\pi R^2 - 18\pi r^2\right) + r\left(4\pi r R^2 - 6\pi r^3\right)}{\left(R^2 - r^2\right)^{\frac{3}{2}}}$$

$$= \frac{4\pi R^4 - 22\pi r^2R^2 + 12\pi r^4 + 4\pi r^2R^2}{\left(R^2 - r^2\right)^{\frac{3}{2}}}$$

Now, it can be observed that at $r^2 = \frac{2R^2}{3}, \frac{d^2V}{dr^2} < 0$.

$\therefore$The volume is the maximum when $r^2 = \frac{2R^2}{3}$.

When $r^2 = \frac{2R^2}{3}$, the height of the cylinder is $2\sqrt{R^2 - \frac{2R^2}{3}} = 2\sqrt{\frac{R^2}{3}} = \frac{2R}{\sqrt{3}}$.

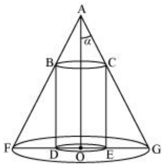
Hence, the volume of the cylinder is the maximum when the height of the cylinder is $\frac{2R}{\sqrt{3}}$.

Answer needs Correction? **Click Here**

Q18 : Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height *h* and semi vertical angle α is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 \tan^2\alpha$.

Answer :

The given right circular cone of fixed height (*h*) and semi-vertical angle (α) can be drawn as:



Here, a cylinder of radius *R* and height *H* is inscribed in the cone.

Then, $\angle$GAO = α, OG = *r*, OA = *h*, OE = *R*, and CE = *H*.

We have,

$r = h \tan\alpha$

Now, since ΔAOG is similar to ΔCEG, we have:

$$\frac{AO}{OG} = \frac{CE}{EG}$$
$$\Rightarrow \frac{h}{r} = \frac{H}{r-R} \qquad [EG = OG - OE]$$
$$\Rightarrow H = \frac{h}{r}(r-R) = \frac{h}{h\tan\alpha}(h\tan\alpha - R) = \frac{1}{\tan\alpha}(h\tan\alpha - R)$$

Now, the volume (*V*) of the cylinder is given by,

$$V = \pi R^2 H = \frac{\pi R^2}{\tan\alpha}(h\tan\alpha - R) = \pi R^2 h - \frac{\pi R^3}{\tan\alpha}$$
$$\therefore \frac{dV}{dR} = 2\pi R h - \frac{3\pi R^2}{\tan\alpha}$$
$$\text{Now}, \frac{dV}{dR} = 0$$
$$\Rightarrow 2\pi R h = \frac{3\pi R^2}{\tan\alpha}$$
$$\Rightarrow 2h\tan\alpha = 3R$$
$$\Rightarrow R = \frac{2h}{3}\tan\alpha$$

$$\text{Now}, \frac{d^2V}{dR^2} = 2\pi h - \frac{6\pi R}{\tan\alpha}$$

And, for $R = \frac{2h}{3}\tan\alpha$, we have:

$$\frac{d^2V}{dR^2} = 2\pi h - \frac{6\pi}{\tan\alpha}\left(\frac{2h}{3}\tan\alpha\right) = 2\pi h - 4\pi h = -2\pi h < 0$$

$\therefore$By second derivative test, the volume of the cylinder is the greatest when

$R = \frac{2h}{3}\tan\alpha$.

When $R = \frac{2h}{3}\tan\alpha$, $H = \frac{1}{\tan\alpha}\left(h\tan\alpha - \frac{2h}{3}\tan\alpha\right) = \frac{1}{\tan\alpha}\left(\frac{h\tan\alpha}{3}\right) = \frac{h}{3}$.

Thus, the height of the cylinder is one-third the height of the cone when the volume of the cylinder is the greatest.

Now, the maximum volume of the cylinder can be obtained as:

$$\pi\left(\frac{2h}{3}\tan\alpha\right)^2\left(\frac{h}{3}\right) = \pi\left(\frac{4h^2}{9}\tan^2\alpha\right)\left(\frac{h}{3}\right) = \frac{4}{27}\pi h^3\tan^2\alpha$$

Hence, the given result is proved.

Answer needs Correction? **Click Here**

Q19 : A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic mere per hour. Then the depth of the wheat is increasing at the rate of

(A) 1 m/h (B) 0.1 m/h

(C) 1.1 m/h (D) 0.5 m/h

Answer :

Let *r* be the radius of the cylinder.

Then, volume (V) of the cylinder is given by,

$$V = \pi(\text{radius})^2 \times \text{height}$$
$$= \pi(10)^2 h \qquad (\text{radius} = 10 \text{ m})$$
$$= 100\pi h$$

Differentiating with respect to time t, we have:

$$\frac{dV}{dt} = 100\pi \frac{dh}{dt}$$

The tank is being filled with wheat at the rate of 314 cubic metres per hour.

$$\therefore \frac{dV}{dt} = 314 \text{ m}^3/\text{h}$$

Thus, we have:

$$314 = 100\pi \frac{dh}{dt}$$
$$\Rightarrow \frac{dh}{dt} = \frac{314}{100(3.14)} = \frac{314}{314} = 1$$

Hence, the depth of wheat is increasing at the rate of 1 m/h.

The correct answer is A.

Answer needs Correction? **Click Here**

Q20 : The slope of the tangent to the curve $x = t^2 + 3t - 8,\ y = 2t^2 - 2t - 5$ at the point (2, - 1) is

(A) $\frac{22}{7}$ (B) $\frac{6}{7}$ (C) $\frac{7}{6}$ (D) $\frac{-6}{7}$

Answer :

The given curve is $x = t^2 + 3t - 8$ and $y = 2t^2 - 2t - 5$.

$$\therefore \frac{dx}{dt} = 2t + 3 \text{ and } \frac{dy}{dt} = 4t - 2$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{4t-2}{2t+3}$$

The given point is (2, - 1).

At x = 2, we have:

$$t^2 + 3t - 8 = 2$$
$$\Rightarrow t^2 + 3t - 10 = 0$$
$$\Rightarrow (t-2)(t+5) = 0$$
$$\Rightarrow t = 2 \text{ or } t = -5$$
$$\text{At } y = -1, \text{ we have:}$$
$$2t^2 - 2t - 5 = -1$$
$$\Rightarrow 2t^2 - 2t - 4 = 0$$
$$\Rightarrow 2(t^2 - t - 2) = 0$$
$$\Rightarrow (t-2)(t+1) = 0$$
$$\Rightarrow t = 2 \text{ or } t = -1$$

The common value of t is 2.

Hence, the slope of the tangent to the given curve at point (2, - 1) is

$$\left.\frac{dy}{dx}\right]_{t=2} = \frac{4(2)-2}{2(2)+3} = \frac{8-2}{4+3} = \frac{6}{7}.$$

The correct answer is B.

Answer needs Correction? **Click Here**

Q21 : The line $y = mx + 1$ is a tangent to the curve $y^2 = 4x$ if the value of m is

(A) 1 (B) 2 (C) 3 (D) $\frac{1}{2}$

Answer :

The equation of the tangent to the given curve is $y = mx + 1$.

Now, substituting $y = mx + 1$ in $y^2 = 4x$, we get:

$$\Rightarrow (mx+1)^2 = 4x$$
$$\Rightarrow m^2x^2 + 1 + 2mx - 4x = 0$$
$$\Rightarrow m^2x^2 + x(2m-4) + 1 = 0 \qquad \text{...(i)}$$

Since a tangent touches the curve at one point, the roots of equation (i) must be equal.

Therefore, we have:

$$\text{Discriminant} = 0$$
$$(2m-4)^2 - 4(m^2)(1) = 0$$
$$\Rightarrow 4m^2 + 16 - 16m - 4m^2 = 0$$
$$\Rightarrow 16 - 16m = 0$$
$$\Rightarrow m = 1$$

Hence, the required value of m is 1.

The correct answer is A.

Answer needs Correction? **Click Here**

Q22 : The normal at the point (1, 1) on the curve $2y + x^2 = 3$ is

(A) $x + y = 0$ (B) $x - y = 0$

(C) $x + y + 1 = 0$ (D) $x - y = 1$

Answer :

The equation of the given curve is $2y + x^2 = 3$.

Differentiating with respect to x, we have:

$$\frac{2dy}{dx} + 2x = 0$$
$$\Rightarrow \frac{dy}{dx} = -x$$
$$\therefore \left.\frac{dy}{dx}\right]_{(1,1)} = -1$$

The slope of the normal to the given curve at point (1, 1) is

$$\frac{-1}{\left.\frac{dy}{dx}\right]_{(1,1)}} = 1.$$

Hence, the equation of the normal to the given curve at (1, 1) is given as:

$$\Rightarrow y - 1 = 1(x - 1)$$
$$\Rightarrow y - 1 = x - 1$$
$$\Rightarrow x - y = 0$$

The correct answer is B.

Answer needs Correction? Click Here

Q23 : The normal to the curve $x^2 = 4y$ passing (1, 2) is

(A) $x + y = 3$ (B) $x - y = 3$

(C) $x + y = 1$ (D) $x - y = 1$

Answer :

The equation of the given curve is $x^2 = 4y$.

Differentiating with respect to x, we have:

$$2x = 4 \cdot \frac{dy}{dx}$$
$$\Rightarrow \frac{dy}{dx} = \frac{x}{2}$$

The slope of the normal to the given curve at point (h, k) is given by,

$$\frac{-1}{\left.\frac{dy}{dx}\right]_{(h,k)}} = -\frac{2}{h}$$

$\therefore$Equation of the normal at point (h, k) is given as:

$$y - k = \frac{-2}{h}(x - h)$$

Now, it is given that the normal passes through the point (1, 2).

Therefore, we have:

$$2 - k = \frac{-2}{h}(1 - h) \text{ or } k = 2 + \frac{2}{h}(1 - h) \qquad \text{... (i)}$$

Since (h, k) lies on the curve $x^2 = 4y$, we have $h^2 = 4k$.

$$\Rightarrow k = \frac{h^2}{4}$$

From equation (i), we have:

$$\frac{h^2}{4} = 2 + \frac{2}{h}(1 - h)$$
$$\Rightarrow \frac{h^3}{4} = 2h + 2 - 2h = 2$$
$$\Rightarrow h^3 = 8$$
$$\Rightarrow h = 2$$
$$\therefore k = \frac{h^2}{4} \Rightarrow k = 1$$

Hence, the equation of the normal is given as:

$$\Rightarrow y - 1 = \frac{-2}{2}(x - 2)$$
$$\Rightarrow y - 1 = -(x - 2)$$
$$\Rightarrow x + y = 3$$

The correct answer is A.

Answer needs Correction? Click Here

Q24 : The points on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts with the axes are

(A) $\left(4, \pm\frac{8}{3}\right)$ (B) $\left(4, \frac{-8}{3}\right)$

(C) $\left(4, \pm\frac{3}{8}\right)$ (D) $\left(\pm 4, \frac{3}{8}\right)$

$(\quad 8) \quad (\quad 8)$

Answer :

The equation of the given curve is $9y^2 = x^3$.

Differentiating with respect to x, we have:

$$9(2y)\frac{dy}{dx} = 3x^2$$
$$\Rightarrow \frac{dy}{dx} = \frac{x^2}{6y}$$

The slope of the normal to the given curve at point (x_1, y_1) is

$$\frac{-1}{\left.\frac{dy}{dx}\right]_{(x_1,y_1)}} = -\frac{6y_1}{x_1^2}.$$

$\therefore$ The equation of the normal to the curve at (x_1, y_1) is

$$y - y_1 = \frac{-6y_1}{x_1^2}(x - x_1).$$
$$\Rightarrow x_1^2 y - x_1^2 y_1 = -6xy_1 + 6x_1y_1$$
$$\Rightarrow 6xy_1 + x_1^2 y = 6x_1y_1 + x_1^2 y_1$$
$$\Rightarrow \frac{6xy_1}{6x_1y_1 + x_1^2y_1} + \frac{x_1^2 y}{6x_1y_1 + x_1^2y_1} = 1$$
$$\Rightarrow \frac{x}{\frac{x_1(6+x_1)}{6}} + \frac{y}{\frac{y_1(6+x_1)}{x_1}} = 1$$

It is given that the normal makes equal intercepts with the axes.

Therefore, We have:

$$\therefore \frac{x_1(6+x_1)}{6} = \frac{y_1(6+x_1)}{x_1}$$
$$\Rightarrow \frac{x_1}{6} = \frac{y_1}{x_1}$$
$$\Rightarrow x_1^2 = 6y_1 \qquad \text{...(i)}$$

Also, the point (x_1, y_1) lies on the curve, so we have

$$9y_1^2 = x_1^3 \qquad \text{...(ii)}$$

From (i) and (ii), we have:

$$9\left(\frac{x_1^2}{6}\right)^2 = x_1^3 \Rightarrow \frac{x_1^4}{4} = x_1^3 \Rightarrow x_1 = 4$$

From (ii), we have:

$$9y_1^2 = (4)^3 = 64$$
$$\Rightarrow y_1^2 = \frac{64}{9}$$
$$\Rightarrow y_1 = \pm\frac{8}{3}$$

Hence, the required points are $\left(4, \pm\frac{8}{3}\right)$.

The correct answer is A.

*********** END **********