Chapter 10 Differentiability Ex 10.1 Q10

Her,e $f(x) = \begin{cases} ax^2 - b & , \text{ if } |x| < 1 \\ \dfrac{1}{|x|} & , \text{ if } |x| \geq 1 \end{cases}$

$$= \begin{cases} -\dfrac{1}{x} & , \text{ if } x \leq -1 \\ ax^2 - b & , \text{ if } -1 < x < 1 \\ \dfrac{1}{x} & , \text{ if } x \geq 1 \end{cases}$$

LHL $= \lim_{x \to 1^-} f(x)$

$= \lim_{h \to 0} f(1-h)$

$= \lim_{h \to 0} a(1-h)^2 - b$

$= a - b$

RHL $= \lim_{x \to 1^+} f(x)$

$= \lim_{h \to 0} f(1+h)$

$= \lim_{h \to 0} \dfrac{1}{1+h}$

$= 1$

Since, $f(x)$ is continuous, so

LHS = RHS

$a - b = 1$ ---(i)

$$(\text{LHD at } x = 1) = \lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1}$$

$$= \lim_{h \to 0} \frac{f(1-h) - 1}{1 - h - 1}$$

$$= \lim_{h \to 0} \frac{a(1-h)^2 - b - 1}{-h}$$

$$= \lim_{h \to 0} \frac{a(1-h)^2 - (a-1) - 1}{-h}$$

Using equation (i),

$$= \lim_{h \to 0} \frac{a + ah^2 - 2ah - a + 1 - 1}{-h}$$

$$= \lim_{h \to 0} \frac{ah^2 - 2ah}{-h}$$

$$= \lim_{h \to 0} (2a - ah)$$

$$= 2a$$

$$(\text{RHD at } x = 1) = \lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1}$$

$$= \lim_{h \to 0} \frac{f(1+h) - f(1)}{1 + h - 1}$$

$$= \lim_{h \to 0} \frac{\frac{1}{1+h} - 1}{h}$$

$$= \lim_{h \to 0} \frac{1 - 1 - h}{(1+h)h}$$

$$= \lim_{h \to 0} \frac{-1}{1+h}$$

$$= -1$$

Since $f(x)$ is differentiable at $x = 1$,

$$(\text{LHD at } x = 1) = (\text{RHD at } x = 1)$$

$$2a = -1$$

$$a = \frac{-1}{2}$$

Put $a = \frac{-1}{2}$ in equation (i),

$$a - b = 1$$

$$\left(\frac{-1}{2}\right) - b = 1$$

$$b = \frac{-1}{2} - 1$$

$$b = \frac{-3}{2}$$

$$a = \frac{-1}{2}$$

*********** END **********