

Differentiation Ex 11.2 Q39

Let
$$y = \frac{2^x \cos x}{\left(x^2 + 3\right)^2}$$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx} \left[\frac{2^x \cos x}{(x^2 + 3)^2} \right]$$

$$= \left[\frac{\left(x^2 + 3\right)^2 \frac{d}{dx} \left(2^x \cos x\right) - \left(2^x \cos x\right) \frac{d}{dx} \left(x^2 + 3\right)^2}{\left[\left(x^2 + 3\right)^2\right]^2} \right]$$
[Using quotient rule, product rule and chain rule]
$$= \left[\frac{\left(x^2 + 3\right)^2 \left[2^x \frac{d}{dx} \cos x + \cos x \frac{d}{dx} 2^x\right] - \left(2^x \cos x\right) 2 \left(x^2 + 3\right) \frac{d}{dx} \left(x^2 + 3\right)}{\left(x^2 + 3\right)^4} \right]$$

$$= \left[\frac{\left(x^2 + 3\right)^2 \left[-2^x \sin x + \cos x 2^x \log 2\right] - 2 \left(2^x \cos x\right) \left(x^2 + 3\right) \left(2x\right)}{\left(x^2 + 3\right)^4} \right]$$

$$= \left[\frac{2^x \left(x^2 + 3\right) \left[\left(x^2 + 3\right) \left(\cos x \log 2 - \sin x\right)\right] - 4x \cos x}{\left(x^2 + 3\right)^4} \right]$$

$$= \frac{2^x}{\left(x^2 + 3\right)^2} \left[\cos x \log 2 - \sin x - \frac{4x \cos x}{\left(x^2 + 3\right)} \right]$$

So,

$$\frac{d}{dx}\left(\frac{2^{x}\cos x}{\left(x^{2}+3\right)^{2}}\right) = \frac{2^{x}}{\left(x^{2}+3\right)^{2}}\left[\cos x \log 2 - \sin x - \frac{4x\cos x}{\left(x^{2}+3\right)}\right].$$

Differentiation Ex 11.2 Q40

Let
$$y = x \sin 2x + 5^x + k^k + (\tan^2 x)^3$$

Diffferentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx} \left[x \sin 2x + 5^x + k^k + \left(\tan^6 x \right) \right]$$

$$= \frac{d}{dx} \left(x \sin 2x \right) + \frac{d}{dx} \left(5^x \right) + \frac{d}{dx} \left(k^k \right) + \frac{d}{dx} \left(\tan^6 x \right)$$

$$= \left[x \frac{d}{dx} \left(\sin 2x \right) + \sin 2x \frac{d}{dx} \left(x \right) \right] + 5^x \log 5 + 0 + 6 \tan^5 x \frac{d}{dx} \left(\tan x \right)$$
[Using product rule and chain rule]
$$= \left[x \cos 2x \frac{d}{dx} \left(2x \right) + \sin 2x \right] + 5^x \log 5 + 6 \tan^5 x \sec^2 x$$

$$= 2x \cos 2x + \sin 2x + 5^x \log 5 + 6 \tan^5 x \sec^2 x$$

so, $\frac{d}{dx} \left(x \sin 2x + 5^x + k^k + \left(\tan^2 x \right)^3 \right) = 2x \cos 2x + \sin 2x + 5^x \log 5 + 6 \tan^5 x \sec^2 x.$

Differentiation Ex 11.2 Q41

Let
$$y = \log(3x + 2) - x^2 \log(2x - 1)$$

Differentiate with respect to x,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \Big[\log(3x+2) - x^2 \log(2x-1) \Big] \\ &= \frac{d}{dx} \log(3x+2) - \frac{d}{dx} \Big(x^2 \log(2x-1) \Big) \\ &= \frac{1}{(3x+2)} \frac{d}{dx} (3x+2) - \Big[x^2 \frac{d}{dx} \log(2x-1) + \log(2x-1) \frac{d}{dx} \Big(x^2 \Big) \Big] \\ &= [\text{Using product rule and chain rule}] \\ &= \frac{3}{3x+2} - \Big[x^2 \times \frac{1}{(2x-1)} \frac{d}{dx} (2x-1) + \log(2x-1) \times 2x \Big] \\ &= \frac{3}{3x+2} - \frac{2x^2}{(2x-1)} - 2x \log(2x-1) \end{aligned}$$

So,

$$\frac{d}{dx} \Big(\log \big(3x + 2 \big) - x^2 \log \big(2x - 1 \big) \Big) = \frac{3}{3x + 2} - \frac{2x^2}{(2x - 1)} - 2x \log \big(2x - 1 \big)$$

Differentiation Ex 11.2 Q42

Let
$$y = \frac{3x^2 \sin x}{\sqrt{7 - x^2}}$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{3x^2 \sin x}{(7 - x^2)^{\frac{1}{2}}} \right)$$

$$= \frac{\left(7 - x^2 \right)^{\frac{1}{2}} \times \frac{d}{dx} \left(3x^2 \sin x \right) - 3x^2 \sin x \frac{d}{dx} \left(7 - x^2 \right)^{\frac{1}{2}}}{\left[\left(7 - x^2 \right)^{\frac{1}{2}} \right]^2}$$

[Using quotient rule, chain and product rule]

$$= \frac{\left[(7 - x^2)^{\frac{1}{2}} \times 3 \times \left[x^2 \frac{d}{dx} \sin x + \sin x \frac{d}{dx} x^2 \right] - 3x^2 \sin x \times \frac{1}{2} \left(7 - x^2 \right)^{\frac{1}{2}} \frac{d}{dx} \left(7 - x^2 \right) \right]}{\left(7 - x^2 \right)}$$

$$= \frac{\left[(7 - x^2)^{\frac{1}{2}} 3 \left(x^2 \cos x + 2x \sin x \right) - 3x^2 \sin x \times \frac{1}{2} \left(7 - x^2 \right)^{\frac{1}{2}} \left(-2x \right) \right]}{\left(7 - x^2 \right)}$$

$$= \frac{\left[(7 - x^2)^{\frac{1}{2}} \times 3 \left(x^2 \cos x + 2x \sin x \right) + 3x^3 \sin x \left(7 - x^2 \right)^{\frac{1}{2}} \right]}{\left(7 - x^2 \right)}$$

$$= \frac{\left[6x \sin x + 3x^2 \cos x + 3x^2 \cos x + 3x^3 \sin x \left(7 - x^2 \right)^{\frac{3}{2}} \right]}{\left(7 - x^2 \right)}$$

So.

$$\frac{d}{dx} \left(\frac{3x^2 \sin x}{\sqrt{7 - x^2}} \right) = \left[\frac{6x \sin x + 3x^2 \cos x}{\sqrt{7 - x^2}} + \frac{3x^3 \sin x}{\left(7 - x^2\right)^{\frac{3}{2}}} \right].$$

Differentiation Ex 11.2 Q43

Let
$$y = \sin^2 \left[\log \left(2x + 3 \right) \right]$$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx} \Big[\sin^2 \big(\log(2x+3) \big) \Big]$$

$$= 2 \sin \big(\log(2x+3) \big) \frac{d}{dx} \sin \big(\log(2x+3) \big) \quad \text{Using chain rule}$$

$$= 2 \sin \big(\log(2x+3) \big) \cos \big(\log(2x+3) \big) \frac{d}{dx} \log(2x+3)$$

$$= \sin \big(2 \log(2x+3) \big) \times \frac{1}{(2x+3)} \frac{d}{dx} (2x+3)$$

$$\Big[\text{Since, } 2 \sin A \cos A = \sin^2 A \Big]$$

$$= \sin \big(2 \log(2x+3) \big) \times \frac{2}{(2x+3)}$$

So,
$$\frac{d}{dx} \left(\sin^2 \log (2x+3) \right) = \sin \left(2 \log (2x+3) \right) \times \frac{2}{(2x+3)}.$$
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