Exercise 7.3 : Solutions of Questions on Page Number : **307**

Q1 : $\sin^2(2x+5)$

Answer :

$$\sin^2(2x+5) = \frac{1-\cos 2(2x+5)}{2} = \frac{1-\cos(4x+10)}{2}$$

$$\Rightarrow \int \sin^2(2x+5)\,dx = \int \frac{1-\cos(4x+10)}{2}dx$$

$$= \frac{1}{2}\int 1\,dx - \frac{1}{2}\int \cos(4x+10)\,dx$$

$$= \frac{1}{2}x - \frac{1}{2}\left(\frac{\sin(4x+10)}{4}\right) + C$$

$$= \frac{1}{2}x - \frac{1}{8}\sin(4x+10) + C$$

Answer needs Correction? **Click Here**

Q2 : $\sin 3x \cos 4x$

Answer :

It is known that, $\sin A \cos B = \frac{1}{2}\{\sin(A+B) + \sin(A-B)\}$

$$\therefore \int \sin 3x \cos 4x\,dx = \frac{1}{2}\int \{\sin(3x+4x) + \sin(3x-4x)\}\,dx$$

$$= \frac{1}{2}\int \{\sin 7x + \sin(-x)\}\,dx$$

$$= \frac{1}{2}\int \{\sin 7x - \sin x\}\,dx$$

$$= \frac{1}{2}\int \sin 7x\,dx - \frac{1}{2}\int \sin x\,dx$$

$$= \frac{1}{2}\left(\frac{-\cos 7x}{7}\right) - \frac{1}{2}(-\cos x) + C$$

$$= \frac{-\cos 7x}{14} + \frac{\cos x}{2} + C$$

Answer needs Correction? **Click Here**

Q3 : $\cos 2x \cos 4x \cos 6x$

Answer :

It is known that, $\cos A \cos B = \frac{1}{2}\{\cos(A+B) + \cos(A-B)\}$

$$\therefore \int \cos 2x(\cos 4x \cos 6x)\,dx = \int \cos 2x\left[\frac{1}{2}\{\cos(4x+6x) + \cos(4x-6x)\}\right]dx$$

$$= \frac{1}{2}\int \{\cos 2x \cos 10x + \cos 2x \cos(-2x)\}\,dx$$

$$= \frac{1}{2}\int \{\cos 2x \cos 10x + \cos^2 2x\}\,dx$$

$$= \frac{1}{2}\int \left[\left\{\frac{1}{2}\cos(2x+10x) + \cos(2x-10x)\right\} + \left(\frac{1+\cos 4x}{2}\right)\right]dx$$

$$= \frac{1}{4}\int (\cos 12x + \cos 8x + 1 + \cos 4x)\,dx$$

$$= \frac{1}{4}\left[\frac{\sin 12x}{12} + \frac{\sin 8x}{8} + x + \frac{\sin 4x}{4}\right] + C$$

Answer needs Correction? **Click Here**

Q4 : $\sin^3(2x+1)$

Answer :

Let $I = \int \sin^3(2x+1)$

$$\Rightarrow \int \sin^3(2x+1)\,dx = \int \sin^2(2x+1)\cdot\sin(2x+1)\,dx$$

$$= \int (1-\cos^2(2x+1))\sin(2x+1)\,dx$$

Let $\cos(2x+1) = t$

$\Rightarrow -2\sin(2x+1)\,dx = dt$

$\Rightarrow \sin(2x+1)\,dx = \frac{-dt}{2}$

$$\Rightarrow I = \frac{-1}{2}\int\left(1-t^2\right)dt$$
$$= \frac{-1}{2}\left\{t-\frac{t^3}{3}\right\}$$
$$= \frac{-1}{2}\left\{\cos(2x+1)-\frac{\cos^3(2x+1)}{3}\right\}$$
$$= \frac{-\cos(2x+1)}{2}+\frac{\cos^3(2x+1)}{6}+C$$

Answer needs Correction? **Click Here**

Q5 : $\sin^3 x\cos^3 x$

Answer :

$$\text{Let } I = \int \sin^3 x\cos^3 x\cdot dx$$
$$= \int \cos^3 x\cdot\sin^2 x\cdot\sin x\cdot dx$$
$$= \int \cos^3 x\left(1-\cos^2 x\right)\sin x\cdot dx$$

Let $\cos x = t$

$$\Rightarrow -\sin x\cdot dx = dt$$
$$\Rightarrow I = -\int t^3\left(1-t^2\right)dt$$
$$= -\int\left(t^3-t^5\right)dt$$
$$= -\left\{\frac{t^4}{4}-\frac{t^6}{6}\right\}+C$$
$$= -\left\{\frac{\cos^4 x}{4}-\frac{\cos^6 x}{6}\right\}+C$$
$$= \frac{\cos^6 x}{6}-\frac{\cos^4 x}{4}+C$$

Answer needs Correction? **Click Here**

Q6 : $\sin x\sin 2x\sin 3x$

Answer :

It is known that, $\sin A\sin B = \frac{1}{2}\left\{\cos(A-B)-\cos(A+B)\right\}$

$$\therefore \int \sin x\sin 2x\sin 3x\ dx = \int\left[\sin x\cdot\frac{1}{2}\left\{\cos(2x-3x)-\cos(2x+3x)\right\}\right]dx$$
$$= \frac{1}{2}\int\left(\sin x\cos(-x)-\sin x\cos 5x\right)dx$$
$$= \frac{1}{2}\int\left(\sin x\cos x-\sin x\cos 5x\right)dx$$
$$= \frac{1}{2}\int\frac{\sin 2x}{2}\ dx-\frac{1}{2}\int\sin x\cos 5x\ dx$$
$$= \frac{1}{4}\left[\frac{-\cos 2x}{2}\right]-\frac{1}{2}\int\left\{\frac{1}{2}\sin(x+5x)+\sin(x-5x)\right\}dx$$
$$= \frac{-\cos 2x}{8}-\frac{1}{4}\int\left(\sin 6x+\sin(-4x)\right)dx$$
$$= \frac{-\cos 2x}{8}-\frac{1}{4}\left[\frac{-\cos 6x}{3}+\frac{\cos 4x}{4}\right]+C$$
$$= \frac{-\cos 2x}{8}-\frac{1}{8}\left[\frac{-\cos 6x}{3}+\frac{\cos 4x}{2}\right]+C$$
$$= \frac{1}{8}\left[\frac{\cos 6x}{3}-\frac{\cos 4x}{2}-\cos 2x\right]+C$$

Answer needs Correction? **Click Here**

Q7 : $\sin 4x\sin 8x$

Answer :

It is known that,

$\sin A\sin B = \frac{1}{2}\cos(A-B)-\cos(A+B)$

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Answer needs Correction? **Click Here**

Q8 : $\frac{1-\cos x}{1+\cos x}$

Answer :

$$\frac{1-\cos x}{1+\cos x} = \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}} \qquad \left[2\sin^2\frac{x}{2} = 1-\cos x \text{ and } 2\cos^2\frac{x}{2} = 1+\cos x\right]$$
$$= \tan^2\frac{x}{2}$$
$$= \left(\sec^2\frac{x}{2}-1\right)$$
$$\therefore \int\frac{1-\cos x}{1+\cos x}dx = \int\left(\sec^2\frac{x}{2}-1\right)dx$$

$$= \left[\frac{\tan\frac{x}{2}}{\frac{1}{2}} - x\right] + C$$

$$= 2\tan\frac{x}{2} - x + C$$

Answer needs Correction? Click Here

Q9 : $\frac{\cos x}{1+\cos x}$

Answer :

$$\frac{\cos x}{1+\cos x} = \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}} \qquad \left[\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} \text{ and } \cos x = 2\cos^2\frac{x}{2} - 1\right]$$

$$= \frac{1}{2}\left[1 - \tan^2\frac{x}{2}\right]$$

$$\therefore \int \frac{\cos x}{1+\cos x}dx = \frac{1}{2}\int\left(1 - \tan^2\frac{x}{2}\right)dx$$

$$= \frac{1}{2}\int\left(1 - \sec^2\frac{x}{2} + 1\right)dx$$

$$= \frac{1}{2}\int\left(2 - \sec^2\frac{x}{2}\right)dx$$

$$= \frac{1}{2}\left[2x - \frac{\tan\frac{x}{2}}{\frac{1}{2}}\right] + C$$

$$= x - \tan\frac{x}{2} + C$$

Answer needs Correction? Click Here

Q10 : $\sin^4 x$

Answer :

$$\sin^4 x = \sin^2 x \sin^2 x$$

$$= \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right)$$

$$= \frac{1}{4}(1-\cos 2x)^2$$

$$= \frac{1}{4}\left[1 + \cos^2 2x - 2\cos 2x\right]$$

$$= \frac{1}{4}\left[1 + \left(\frac{1+\cos 4x}{2}\right) - 2\cos 2x\right]$$

$$= \frac{1}{4}\left[1 + \frac{1}{2} + \frac{1}{2}\cos 4x - 2\cos 2x\right]$$

$$= \frac{1}{4}\left[\frac{3}{2} + \frac{1}{2}\cos 4x - 2\cos 2x\right]$$

$$\therefore \int \sin^4 x\, dx = \frac{1}{4}\int\left[\frac{3}{2} + \frac{1}{2}\cos 4x - 2\cos 2x\right]dx$$

$$= \frac{1}{4}\left[\frac{3}{2}x + \frac{1}{2}\left(\frac{\sin 4x}{4}\right) - \frac{2\sin 2x}{2}\right] + C$$

$$= \frac{1}{8}\left[3x + \frac{\sin 4x}{4} - 2\sin 2x\right] + C$$

$$= \frac{3x}{8} - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$$

Answer needs Correction? Click Here

Q11 : $\cos^4 2x$

Answer :

$$\cos^4 2x = \left(\cos^2 2x\right)^2$$

$$= \left(\frac{1+\cos 4x}{2}\right)^2$$

$$= \frac{1}{4}\left[1 + \cos^2 4x + 2\cos 4x\right]$$

$$= \frac{1}{4}\left[1 + \left(\frac{1+\cos 8x}{2}\right) + 2\cos 4x\right]$$

$$= \frac{1}{4}\left[1 + \frac{1}{2} + \frac{\cos 8x}{2} + 2\cos 4x\right]$$

$$= \frac{1}{4}\left[\frac{3}{2} + \frac{\cos 8x}{2} + 2\cos 4x\right]$$

$$\therefore \int \cos^4 2x\, dx = \int\left(\frac{3}{8} + \frac{\cos 8x}{8} + \frac{\cos 4x}{2}\right)dx$$

$$= \frac{3}{8}x + \frac{\sin 8x}{64} + \frac{\sin 4x}{8} + C$$

Answer needs Correction? Click Here

Q12 : $\sin^2 x$

$\overline{1+\cos x}$

Answer :

$$\frac{\sin^2 x}{1+\cos x}=\frac{\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}}\quad\left[\sin x=2\sin\frac{x}{2}\cos\frac{x}{2};\ \cos x=2\cos^2\frac{x}{2}-1\right]$$

$$=\frac{4\sin^2\frac{x}{2}\cos^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$$

$$=2\sin^2\frac{x}{2}$$

$$=1-\cos x$$

$$\therefore\int\frac{\sin^2 x}{1+\cos x}dx=\int(1-\cos x)dx$$

$$=x-\sin x+\mathrm{C}$$

Answer needs Correction? **Click Here**

Q13 : $\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}$

Answer :

$$\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}=\frac{-2\sin\frac{2x+2\alpha}{2}\sin\frac{2x-2\alpha}{2}}{-2\sin\frac{x+\alpha}{2}\sin\frac{x-\alpha}{2}}\qquad\left[\cos C-\cos D=-2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right]$$

$$=\frac{\sin(x+\alpha)\sin(x-\alpha)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$=\frac{\left[2\sin\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x+\alpha}{2}\right)\right]\left[2\sin\left(\frac{x-\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)\right]}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$=4\cos\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)$$

$$=2\left[\cos\left(\frac{x+\alpha}{2}+\frac{x-\alpha}{2}\right)+\cos\frac{x+\alpha}{2}-\frac{x-\alpha}{2}\right]$$

$$=2\left[\cos(x)+\cos\alpha\right]$$

$$=2\cos x+2\cos\alpha$$

$$\therefore\int\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx=\int 2\cos x+2\cos\alpha$$

$$=2\left[\sin x+x\cos\alpha\right]+\mathrm{C}$$

Answer needs Correction? **Click Here**

Q14 : $\frac{\cos x-\sin x}{1+\sin 2x}$

Answer :

$$\frac{\cos x-\sin x}{1+\sin 2x}=\frac{\cos x-\sin x}{\left(\sin^2 x+\cos^2 x\right)+2\sin x\cos x}$$

$$\left[\sin^2 x+\cos^2 x=1;\ \ \sin 2x=2\sin x\cos x\right]$$

$$=\frac{\cos x-\sin x}{(\sin x+\cos x)^2}$$

Let $\sin x+\cos x=t$

$\therefore(\cos x-\sin x)dx=dt$

$$\Rightarrow\int\frac{\cos x-\sin x}{1+\sin 2x}dx=\int\frac{\cos x-\sin x}{(\sin x+\cos x)^2}dx$$

$$=\int\frac{dt}{t^2}$$

$$=\int t^{-2}dt$$

$$=-t^{-1}+\mathrm{C}$$

$$=-\frac{1}{t}+\mathrm{C}$$

$$=\frac{-1}{\sin x+\cos x}+\mathrm{C}$$

Answer needs Correction? **Click Here**

Q15 : $\tan^3 2x\sec 2x$

Answer :

$$\tan^3 2x\sec 2x=\tan^2 2x\tan 2x\sec 2x$$

$$=\left(\sec^2 2x-1\right)\tan 2x\sec 2x$$

$$=\sec^2 2x\cdot\tan 2x\sec 2x-\tan 2x\sec 2x$$

$$\therefore\int\tan^3 2x\sec 2x\,dx=\int\sec^2 2x\tan 2x\sec 2x\,dx-\int\tan 2x\sec 2x\,dx$$

$$=\int\sec^2 2x\tan 2x\sec 2x\,dx-\frac{\sec 2x}{2}+\mathrm{C}$$

Let $\sec 2x=t$

$\therefore 2\sec 2x\tan 2x\,dx=dt$

$$\therefore \int \tan^3 2x \sec 2x \, dx = \frac{1}{2}\int t^2 dt - \frac{\sec 2x}{2} + C$$
$$= \frac{t^3}{6} - \frac{\sec 2x}{2} + C$$
$$= \frac{(\sec 2x)^3}{6} - \frac{\sec 2x}{2} + C$$

Answer needs Correction? **Click Here**

Q16 : $\tan^4 x$

Answer :

$\tan^4 x$
$= \tan^2 x \cdot \tan^2 x$
$= (\sec^2 x - 1)\tan^2 x$
$= \sec^2 x \tan^2 x - \tan^2 x$
$= \sec^2 x \tan^2 x - (\sec^2 x - 1)$
$= \sec^2 x \tan^2 x - \sec^2 x + 1$

$$\therefore \int \tan^4 x \, dx = \int \sec^2 x \tan^2 x \, dx - \int \sec^2 x \, dx + \int 1 \cdot dx$$
$$= \int \sec^2 x \tan^2 x \, dx - \tan x + x + C \qquad ...(1)$$

Consider $\int \sec^2 x \tan^2 x \, dx$
Let $\tan x = t \Rightarrow \sec^2 x \, dx = dt$
$$\Rightarrow \int \sec^2 x \tan^2 x dx = \int t^2 dt = \frac{t^3}{3} = \frac{\tan^3 x}{3}$$

From equation (1), we obtain

$$\int \tan^4 x \, dx = \frac{1}{3}\tan^3 x - \tan x + x + C$$

Answer needs Correction? **Click Here**

Q17 : $\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$

Answer :

$$\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} = \frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x}$$
$$= \frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x}$$
$$= \tan x \sec x + \cot x \operatorname{cosec} x$$
$$\therefore \int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int (\tan x \sec x + \cot x \operatorname{cosec} x) dx$$
$$= \sec x - \operatorname{cosec} x + C$$

Answer needs Correction? **Click Here**

Q18 : $\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$

Answer :

$$\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$$
$$= \frac{\cos 2x + (1 - \cos 2x)}{\cos^2 x} \qquad [\cos 2x = 1 - 2\sin^2 x]$$
$$= \frac{1}{\cos^2 x}$$
$$= \sec^2 x$$
$$\therefore \int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx = \int \sec^2 x \, dx = \tan x + C$$

Answer needs Correction? **Click Here**

Q19 : $\frac{1}{\sin x \cos^3 x}$

Answer :

Answer needs Correction? **Click Here**

Q20 : $\frac{\cos 2x}{(\cos x + \sin x)^2}$

Answer :

$$\frac{\cos 2x}{(\cos x + \sin x)^2} = \frac{\cos 2x}{\cos^2 x + \sin^2 x + 2\sin x \cos x} = \frac{\cos 2x}{1 + \sin 2x}$$

$$\therefore \int \frac{\cos 2x}{(\cos x+\sin x)^2}\,dx = \int \frac{\cos 2x}{(1+\sin 2x)}\,dx$$

Let $1+\sin 2x = t$

$\Rightarrow 2\cos 2x\,dx = dt$

$$\therefore \int \frac{\cos 2x}{(\cos x+\sin x)^2}\,dx = \frac{1}{2}\int \frac{1}{t}dt$$
$$= \frac{1}{2}\log|t| + C$$
$$= \frac{1}{2}\log|1+\sin 2x| + C$$
$$= \frac{1}{2}\log\left|(\sin x+\cos x)^2\right| + C$$
$$= \log|\sin x+\cos x| + C$$

Answer needs Correction? **Click Here**

Q21 : $\sin^{-1}(\cos x)$

Answer :

$\sin^{-1}(\cos x)$

Let $\cos x = t$

Then, $\sin x = \sqrt{1-t^2}$

$\Rightarrow (-\sin x)dx = dt$

$$dx = \frac{-dt}{\sin x}$$

$$dx = \frac{-dt}{\sqrt{1-t^2}}$$

$$\therefore \int \sin^{-1}(\cos x)dx = \int \sin^{-1} t\left(\frac{-dt}{\sqrt{1-t^2}}\right)$$
$$= -\int \frac{\sin^{-1} t}{\sqrt{1-t^2}}dt$$

Let $\sin^{-1} t = u$

$$\Rightarrow \frac{1}{\sqrt{1-t^2}}dt = du$$

$$\therefore \int \sin^{-1}(\cos x)dx = \int 4du$$
$$= -\frac{u^2}{2} + C$$
$$= \frac{-(\sin^{1} t)^2}{2} + C$$
$$= \frac{-\left[\sin^{-1}(\cos x)\right]^2}{2} + C \quad \ldots(1)$$

It is known that,

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$\therefore \sin^{-1}(\cos x) = \frac{\pi}{2} - \cos^{-1}(\cos x) = \left(\frac{\pi}{2} - x\right)$$

Substituting in equation (1), we obtain

$$\int \sin^{-1}(\cos x)\,dx = \frac{-\left[\frac{\pi}{2}-x\right]^2}{2} + C$$
$$= -\frac{1}{2}\left(\frac{\pi^2}{2} + x^2 - \pi x\right) + C$$
$$= -\frac{\pi^2}{8} - \frac{x^2}{2} + \frac{1}{2}\pi x + C$$
$$= \frac{\pi x}{2} - \frac{x^2}{2} + \left(C - \frac{\pi^2}{8}\right)$$
$$= \frac{\pi x}{2} - \frac{x^2}{2} + C_1$$

Answer needs Correction? **Click Here**

Q22 : $\dfrac{1}{\cos(x-a)\cos(x-b)}$

Answer :

$$\frac{1}{\cos(x-a)\cos(x-b)} = \frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)}\right]$$
$$= \frac{1}{\sin(a-b)}\left[\frac{\sin[(x-b)-(x-a)]}{\cos(x-a)\cos(x-b)}\right]$$
$$= \frac{1}{\sin(a-b)}\frac{[\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)]}{\cos(x-a)\cos(x-b)}$$
$$= \frac{1}{\sin(a-b)}[\tan(x-b)-\tan(x-a)]$$

$$\Rightarrow \int \frac{1}{\cos(x-a)\cos(x-b)}dx = \frac{1}{\sin(a-b)}\int[\tan(x-b)-\tan(x-a)]dx$$
$$= \frac{1}{\sin(a-b)}\left[-\log|\cos(x-b)| + \log|\cos(x-a)|\right]$$

$$= \frac{1}{\sin(a-b)}\left[\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right|\right] + C$$

Answer needs Correction? **Click Here**

Q23 : $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x}\, dx$ is equal to

A. tan x + cot x + C

B. tan x + cosec x + C

C. - tan x + cot x + C

D. tan x + sec x + C

Answer :

$$\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x}\, dx = \int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} - \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) dx$$
$$= \int \left(\sec^2 x - \operatorname{cosec}^2 x \right) dx$$
$$= \tan x + \cot x + C$$

Hence, the correct answer is A.

Answer needs Correction? **Click Here**

Q24 : $\int \frac{e^x (1+x)}{\cos^2 (e^x x)} dx$ equals

A. - cot ($e^x x$) + C

B. tan (xe^x) + C

C. tan (e^x) + C

D. cot (e^x) + C

Answer :

$$\int \frac{e^x (1+x)}{\cos^2 (e^x x)} dx$$

Let $e^x x = t$

$$\Rightarrow \left(e^x \cdot x + e^x \cdot 1\right) dx = dt$$
$$e^x (x+1)\, dx = dt$$

$$\therefore \int \frac{e^x (1+x)}{\cos^2 (e^x x)}\, dx = \int \frac{dt}{\cos^2 t}$$
$$= \int \sec^2 t \; dt$$
$$= \tan t + C$$
$$= \tan\left(e^x \cdot x\right) + C$$

Hence, the correct answer is B.

Answer needs Correction? **Click Here**

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