Adjoint and Inverse of Matrix Ex 7.1 Q18

$$A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$$

Now $A^2 + 4A - 42I = 0$

For this $A^2 = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix}$

Hence,

$$A^2 + 4A - 42I = \begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix} + \begin{bmatrix} -32 & 20 \\ 8 & 16 \end{bmatrix} - \begin{bmatrix} 42 & 0 \\ 0 & 42 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence proved.

Now, $A^2 + 4A - 42I = 0$

$\Rightarrow \quad A^{-1}.A.A + 4A^{-1}.A - 42A^{-1}.I = 0$

$\Rightarrow \quad IA + 4I - 42A^{-1} = 0$

$\Rightarrow \quad 42A^{-1} = A + 4I$

$\Rightarrow \quad A^{-1} = \frac{1}{42}[A + 4I] = \frac{1}{42}\left\{\begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}\right\} = \frac{1}{42}\begin{bmatrix} -4 & 5 \\ 2 & 8 \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q19

Here

$$A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$$

$$A^2=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}$$

Now,

$$A^2-5A+7I=0$$

$$=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}-5\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}+7\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

So,

$$A^2-5A+7I=0$$

Px-multipling with A^{-1}

$$A^{-1}A^2-5A^{-1}A+7IA^{-1}=0$$

$$A-5I+7A^{-1}=0$$

$$A^{-1}=\frac{1}{7}[5I-A]$$

$$=\frac{1}{7}\left\{\begin{bmatrix}5 & 0\\ 0 & 5\end{bmatrix}-\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\right\}$$

$$=\frac{1}{7}\begin{bmatrix}2 & -1\\ 1 & 3\end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q20

$$A=\begin{bmatrix}4 & 3\\ 2 & 5\end{bmatrix}$$

$$\therefore\ A^2=\begin{bmatrix}4 & 3\\ 2 & 5\end{bmatrix}\begin{bmatrix}4 & 3\\ 2 & 5\end{bmatrix}=\begin{bmatrix}22 & 27\\ 18 & 31\end{bmatrix}$$

Now $A^2-xA+yI=0$

$$\Rightarrow\quad \begin{bmatrix}22 & 27\\ 18 & 31\end{bmatrix}-\begin{bmatrix}4x & 3x\\ 2x & 5x\end{bmatrix}+\begin{bmatrix}y & 0\\ 0 & y\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$\Rightarrow\quad 22-4x+y=0 \qquad$ or $\qquad 4x-y=22$

$\Rightarrow\quad 18-2x=0 \qquad$ or $\qquad x=9$

$\therefore\quad y=14$

Again,

$$A^2-9A+14I=0$$

$$\Rightarrow\quad 9A=A^2+14I=0$$

$$\Rightarrow\quad 9A^{-1}A=A^{-1}.A.A+14A^{-1}$$

$$\Rightarrow\quad 9I=IA+14A^{-1}$$

$$\Rightarrow\quad A^{-1}=\frac{1}{14}\{9I-A\}=\frac{1}{14}\left\{\begin{bmatrix}9 & 0\\ 0 & 9\end{bmatrix}-\begin{bmatrix}4 & 3\\ 2 & 5\end{bmatrix}\right\}=\frac{1}{14}\left\{\begin{bmatrix}5 & -3\\ -2 & 4\end{bmatrix}\right\}$$

Adjoint and Inverse of Matrix Ex 7.1 Q21

$$A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

If $A^2 = \lambda A - 2I$

$$\lambda A = A^2 + 2I$$

$$= \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\lambda \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 3\lambda & -2\lambda \\ 4\lambda & -2\lambda \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$3\lambda = 3$$

$$\lambda = 1 \qquad \text{Ans } \lambda = 1$$

$$A^2 = A - 2I$$

Px multiplying by A^{-1}

$$A^{-1}.AA = A^{-1}.A - 2A^{-1}.I$$

$$A = I - 2A^{-1}$$

$$2A^{-1} = I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ -4 & 3 \end{bmatrix}$$

$$\therefore \qquad A^{-1} = \frac{1}{2}\begin{bmatrix} -2 & 2 \\ -4 & 3 \end{bmatrix}$$

********** END **********