Q17 : Find the relationship between *a* and *b* so that the function *f* defined by

$$f(x)=\begin{cases}ax+1, & \text{if } x\le 3\\ bx+3, & \text{if } x>3\end{cases}$$

is continuous at $x = 3$.

Answer :

The given function *f* is $f(x)=\begin{cases}ax+1, & \text{if } x\le 3\\ bx+3, & \text{if } x>3\end{cases}$

If *f* is continuous at $x = 3$, then

$$\lim_{x\to 3^-} f(x)=\lim_{x\to 3^+} f(x)=f(3) \qquad ...(1)$$

Also,

$$\lim_{x\to 3^-} f(x)=\lim_{x\to 3^-}(ax+1)=3a+1$$
$$\lim_{x\to 3^+} f(x)=\lim_{x\to 3^+}(bx+3)=3b+3$$
$$f(3)=3a+1$$

Therefore, from (1), we obtain

$$3a+1=3b+3=3a+1$$
$$\Rightarrow 3a+1=3b+3$$
$$\Rightarrow 3a=3b+2$$
$$\Rightarrow a=b+\frac{2}{3}$$

Therefore, the required relationship is given by, $a=b+\frac{2}{3}$

Answer needs Correction? **Click Here**

Q18 : For what value of λ is the function defined by

$$f(x)=\begin{cases}\lambda(x^2-2x), & \text{if } x\le 0\\ 4x+1, & \text{if } x>0\end{cases}$$

continuous at $x = 0$? What about continuity at $x = 1$?

Answer :

The given function *f* is $f(x)=\begin{cases}\lambda(x^2-2x), & \text{if } x\le 0\\ 4x+1, & \text{if } x>0\end{cases}$

If *f* is continuous at $x = 0$, then

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=f(0)$$
$$\Rightarrow \lim_{x\to 0^-}\lambda(x^2-2x)=\lim_{x\to 0^+}(4x+1)=\lambda(0^2-2\times 0)$$
$$\Rightarrow \lambda(0^2-2\times 0)=4\times 0+1=0$$
$$\Rightarrow 0=1=0, \text{ which is not possible}$$

Therefore, there is no value of ÃŽÂ» for which *f* is continuous at $x = 0$

At $x = 1$,

$f(1) = 4x + 1 = 4 \times 1 + 1 = 5$

$$\lim_{x\to 1}(4x+1)=4\times 1+1=5$$
$$\therefore \lim_{x\to 1} f(x)=f(1)$$

Therefore, for any values of ÃŽÂ», *f* is continuous at $x = 1$

Answer needs Correction? **Click Here**

Q19 : Show that the function defined by $g(x)=x-[x]$ is discontinuous at all integral point. Here $[x]$ denotes the greatest integer less than or equal to *x*.

Answer :

The given function is $g(x)=x-[x]$

It is evident that *g* is defined at all integral points.

Let *n* be an integer.

Then,

$$g(n)=n-[n]=n-n=0$$

The left hand limit of *f* at $x = n$ is,

$$\lim_{x\to n^-} g(x) = \lim_{x\to n^-}(x-[x]) = \lim_{x\to n^-}(x) - \lim_{x\to n^-}[x] = n-(n-1) = 1$$

The right hand limit of *f* at *x* = *n* is,

$$\lim_{x\to n^+} g(x) = \lim_{x\to n^+}(x-[x]) = \lim_{x\to n^+}(x) - \lim_{x\to n^+}[x] = n-n = 0$$

It is observed that the left and right hand limits of *f* at *x* = *n* do not coincide.

Therefore, *f* is not continuous at *x* = *n*

Hence, *g* is discontinuous at all integral points.

Answer needs Correction? **Click Here**

Q20 : Is the function defined by $f(x) = x^2 - \sin x + 5$ continuous at *x* = ?

Answer :

The given function is $f(x) = x^2 - \sin x + 5$

It is evident that *f* is defined at *x* = .

$$\text{At } x=\pi,\ f(x) = f(\pi) = \pi^2 - \sin\pi + 5 = \pi^2 - 0 + 5 = \pi^2 + 5$$

$$\text{Consider } \lim_{x\to\pi} f(x) = \lim_{x\to\pi}(x^2 - \sin x + 5)$$

$$\text{Put } x = \pi + h$$

$$\text{If } x\to\pi, \textit{ then it is evident that } h\to 0$$

$$\begin{aligned}
\therefore \lim_{x\to\pi} f(x) &= \lim_{x\to\pi}(x^2 - \sin x + 5)\\
&= \lim_{h\to0}\left[(\pi+h)^2 - \sin(\pi+h) + 5\right]\\
&= \lim_{h\to0}(\pi+h)^2 - \lim_{h\to0}\sin(\pi+h) + \lim_{h\to0}5\\
&= (\pi+0)^2 - \lim_{h\to0}[\sin\pi\cosh + \cos\pi\sinh] + 5\\
&= \pi^2 - \lim_{h\to0}\sin\pi\cosh - \lim_{h\to0}\cos\pi\sinh + 5\\
&= \pi^2 - \sin\pi\cos0 - \cos\pi\sin0 + 5\\
&= \pi^2 - 0\times1 - (-1)\times0 + 5\\
&= \pi^2 + 5
\end{aligned}$$

$$\therefore \lim_{x\to\pi} f(x) = f(\pi)$$

Therefore, the given function *f* is continuous at $x = \pi$

Answer needs Correction? **Click Here**

Q21 : Discuss the continuity of the following functions.

(a) *f*(*x*) = sin *x* + cos *x*

(b) *f*(*x*) = sin *x* - cos *x*

(c) *f*(*x*) = sin *x* x cos x

Answer :

It is known that if *g* and *h* are two continuous functions, then

$g+h,\ g-h,$ and $g.h$ are also continuous.

It has to proved first that *g*(*x*) = sin *x* and *h*(*x*) = cos *x* are continuous functions.

Let *g* (*x*) = sin *x*

It is evident that *g*(*x*) = sin *x* is defined for every real number.

Let *c* be a real number. Put *x* = *c* + *h*

If *x* Ã¢â€ ' *c*, then *h* Ã¢â€ '0

$$g(c) = \sin c$$

$$\begin{aligned}
\lim_{x\to c} g(x) &= \lim_{x\to c}\sin x\\
&= \lim_{h\to0}\sin(c+h)\\
&= \lim_{h\to0}[\sin c\cos h + \cos c\sin h]\\
&= \lim_{h\to0}(\sin c\cos h) + \lim_{h\to0}(\cos c\sin h)\\
&= \sin c\cos0 + \cos c\sin0\\
&= \sin c + 0\\
&= \sin c
\end{aligned}$$

$$\therefore \lim_{x\to c} g(x) = g(c)$$

Therefore, *g* is a continuous function.

Let *h* (*x*) = cos *x*

It is evident that *h*(*x*) = cos *x* is defined for every real number.

Let *c* be a real number. Put *x* = *c* + *h*

If *x* Ã¢â€ ' *c*, then *h* Ã¢â€ '0

h (*c*) = cos *c*

$$\begin{aligned}
\lim_{x\to c} h(x) &= \lim_{x\to c}\cos x\\
&= \lim_{h\to0}\cos(c+h)\\
&= \lim_{h\to0}[\cos c\cos h - \sin c\sin h]\\
&= \lim_{h\to0}\cos c\cos h - \lim_{h\to0}\sin c\sin h\\
&= \cos c\cos0 - \sin c\sin0\\
&= \cos c\times1 - \sin c\times0\\
&= \cos c
\end{aligned}$$

$\therefore \lim_{x\to c} h(x) = h(c)$

Therefore, *h* is a continuous function.

Therefore, it can be concluded that

(a) *f*(*x*) = *g*(*x*) + *h*(*x*) = sin *x* + cos *x* is a continuous function

(b) *f*(*x*) = *g*(*x*) - *h* (*x*) = sin *x* - cos *x* is a continuous function

(c) *f*(*x*) = *g*(*x*) × *h* (*x*) = sin *x* ×cos *x* is a continuous function

Answer needs Correction? Click Here

Q22 : Discuss the continuity of the cosine, cosecant, secant and cotangent functions,

Answer :

It is known that if *g* and *h* are two continuous functions, then

$$(i)\ \frac{h(x)}{g(x)},\ g(x) \neq 0 \text{ is continuous}$$

$$(ii)\ \frac{1}{g(x)},\ g(x) \neq 0 \text{ is continuous}$$

$$(iii)\ \frac{1}{h(x)},\ h(x) \neq 0 \text{ is continuous}$$

It has to be proved first that *g*(*x*) = sin *x* and *h*(*x*) = cos *x* are continuous functions.

Let *g* (*x*) = sin *x*

It is evident that *g*(*x*) = sin *x* is defined for every real number.

Let *c* be a real number. Put *x* = *c* + *h*

If *x* → *c*, then *h* →0

$$\begin{aligned}
&g(c) = \sin c \\
&\lim_{x\to c} g(x) = \lim_{x\to c} \sin x \\
&\qquad = \lim_{h\to 0} \sin(c+h) \\
&\qquad = \lim_{h\to 0} [\sin c \cos h + \cos c \sin h] \\
&\qquad = \lim_{h\to 0} (\sin c \cos h) + \lim_{h\to 0} (\cos c \sin h) \\
&\qquad = \sin c \cos 0 + \cos c \sin 0 \\
&\qquad = \sin c + 0 \\
&\qquad = \sin c \\
&\therefore\ \lim_{x\to c} g(x) = g(c)
\end{aligned}$$

Therefore, *g* is a continuous function.

Let *h*(*x*) = cos *x*

It is evident that *h*(*x*) = cos *x* is defined for every real number.

Let *c* be a real number. Put *x* = *c* + *h*

If *x* Â® *c*, then *h* Â®0

h (*c*) = cos *c*

$$\begin{aligned}
&\lim_{x\to c} h(x) = \lim_{x\to c} \cos x \\
&\qquad = \lim_{h\to 0} \cos(c+h) \\
&\qquad = \lim_{h\to 0} [\cos c \cos h - \sin c \sin h] \\
&\qquad = \lim_{h\to 0} \cos c \cos h - \lim_{h\to 0} \sin c \sin h \\
&\qquad = \cos c \cos 0 - \sin c \sin 0 \\
&\qquad = \cos c \times 1 - \sin c \times 0 \\
&\qquad = \cos c \\
&\therefore \lim_{x\to c} h(x) = h(c)
\end{aligned}$$

Therefore, *h*(*x*) = cos *x* is continuous function.

It can be concluded that,

$$\operatorname{cosec} x = \frac{1}{\sin x},\ \sin x \neq 0 \text{ is continuous}$$

$$\Rightarrow \operatorname{cosec} x,\ x \neq n\pi\ (n \in Z) \text{ is continuous}$$

Therefore, cosecant is continuous except at *x* = *n*p, *n* ÃƒÂ½ Z

$$\sec x = \frac{1}{\cos x},\ \cos x \neq 0 \text{ is continuous}$$

$$\Rightarrow \sec x,\ x \neq (2n+1)\frac{\pi}{2}\ (n \in \mathbf{Z}) \text{ is continuous}$$

Therefore, secant is continuous except at $x = (2n+1)\frac{\pi}{2}\ (n \in \mathbf{Z})$

$$\cot x = \frac{\cos x}{\sin x},\ \sin x \neq 0 \text{ is continuous}$$

$$\Rightarrow \cot x,\ x \neq n\pi\ (n \in Z) \text{ is continuous}$$

Therefore, cotangent is continuous except at *x* = *n*p, *n* ÃƒÂ½ Z

Answer needs Correction? Click Here

Q23 : Find the points of discontinuity of *f*, where

$$f(x) = \begin{cases} \frac{\sin x}{x}, & \text{if } x < 0 \\ x+1, & \text{if } x \geq 0 \end{cases}$$

Answer :

The given function *f* is $f(x)=\begin{cases}\dfrac{\sin x}{x}, & \text{if } x<0\\ x+1, & \text{if } x\geq 0\end{cases}$

It is evident that *f* is defined at all points of the real line.

Let *c* be a real number.

Case I:

$$\text{If } c<0, \text{ then } f(c)=\frac{\sin c}{c} \text{ and } \lim_{x\to c} f(x)=\lim_{x\to c}\left(\frac{\sin x}{x}\right)=\frac{\sin c}{c}$$
$$\therefore \lim_{x\to c} f(x)=f(c)$$

Therefore, *f* is continuous at all points *x*, such that $x<0$

Case II:

$$\text{If } c>0, \text{ then } f(c)=c+1 \text{ and } \lim_{x\to c} f(x)=\lim_{x\to c}(x+1)=c+1$$
$$\therefore \lim_{x\to c} f(x)=f(c)$$

Therefore, *f* is continuous at all points *x*, such that $x>0$

Case III:

$$\text{If } c=0, \text{ then } f(c)=f(0)=0+1=1$$

The left hand limit of *f* at $x=0$ is,

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0}\frac{\sin x}{x}=1$$

The right hand limit of *f* at $x=0$ is,

$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}(x+1)=1$$
$$\therefore \lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=f(0)$$

Therefore, *f* is continuous at $x=0$

From the above observations, it can be concluded that *f* is continuous at all points of the real line.

Thus, *f* has no point of discontinuity.

Answer needs Correction? **Click Here**

Q24 : Determine if *f* defined by

$$f(x)=\begin{cases}x^2\sin\dfrac{1}{x}, & \text{if } x\neq 0\\ 0, & \text{if } x=0\end{cases}$$

is a continuous function?

Answer :

The given function *f* is $f(x)=\begin{cases}x^2\sin\dfrac{1}{x}, & \text{if } x\neq 0\\ 0, & \text{if } x=0\end{cases}$

It is evident that *f* is defined at all points of the real line.

Let *c* be a real number.

Case I:

$$\text{If } c\neq 0, \text{ then } f(c)=c^2\sin\frac{1}{c}$$
$$\lim_{x\to c} f(x)=\lim_{x\to c}\left(x^2\sin\frac{1}{x}\right)=\left(\lim_{x\to c}x^2\right)\left(\lim_{x\to c}\sin\frac{1}{x}\right)=c^2\sin\frac{1}{c}$$
$$\therefore \lim_{x\to c} f(x)=f(c)$$

Therefore, *f* is continuous at all points $x\neq 0$

Case II:

$$\text{If } c=0, \text{ then } f(0)=0$$

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}\left(x^2\sin\frac{1}{x}\right)=\lim_{x\to 0}\left(x^2\sin\frac{1}{x}\right)$$

$$\text{It is known that, } -1\leq \sin\frac{1}{x}\leq 1,\ x\neq 0$$

$$\Rightarrow -x^2\leq \sin\frac{1}{x}\leq x^2$$

$$\Rightarrow \lim_{x\to 0}\left(-x^2\right)\leq \lim_{x\to 0}\left(x^2\sin\frac{1}{x}\right)\leq \lim_{x\to 0}x^2$$

$$\Rightarrow 0\leq \lim_{x\to 0}\left(x^2\sin\frac{1}{x}\right)\leq 0$$

$$\Rightarrow \lim_{x\to 0}\left(x^2\sin\frac{1}{x}\right)=0$$

$$\therefore \lim_{x\to 0^-} f(x)=0$$

$$\text{Similarly, } \lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}\left(x^2\sin\frac{1}{x}\right)=\lim_{x\to 0}\left(x^2\sin\frac{1}{x}\right)=0$$

$$\therefore \lim_{x\to 0^-} f(x)=f(0)=\lim_{x\to 0^+} f(x)$$

Therefore, *f* is continuous at $x=0$

From the above observations, it can be concluded that *f* is continuous at every point of the real line.

Thus, *f* is a continuous function.

Answer needs Correction? Click Here

Q25 : Examine the continuity of f, where f is defined by

$$f(x)=\begin{cases}\sin x-\cos x, & \text{if } x\neq 0\\ -1 & \text{if } x=0\end{cases}$$

Answer :

The given function f is $f(x)=\begin{cases}\sin x-\cos x, & \text{if } x\neq 0\\ -1 & \text{if } x=0\end{cases}$

It is evident that f is defined at all points of the real line.

Let c be a real number.

Case I:

$$\text{If } c\neq 0, \text{ then } f(c)=\sin c-\cos c$$
$$\lim_{x\to c} f(x)=\lim_{x\to c}(\sin x-\cos x)=\sin c-\cos c$$
$$\therefore \lim_{x\to c} f(x)=f(c)$$

Therefore, f is continuous at all points x, such that $x\neq 0$

Case II:

$$\text{If } c=0, \text{ then } f(0)=-1$$

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0}(\sin x-\cos x)=\sin 0-\cos 0=0-1=-1$$
$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0}(\sin x-\cos x)=\sin 0-\cos 0=0-1=-1$$
$$\therefore \lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=f(0)$$

Therefore, f is continuous at $x=0$

From the above observations, it can be concluded that f is continuous at every point of the real line.

Thus, f is a continuous function.

Answer needs Correction? Click Here

Q26 : Find the values of k so that the function f is continuous at the indicated point.

$$f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x}, & \text{if } x\neq\dfrac{\pi}{2}\\ 3, & \text{if } x=\dfrac{\pi}{2}\end{cases} \quad \text{at } x=\frac{\pi}{2}$$

Answer :

The given function f is $f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x}, & \text{if } x\neq\dfrac{\pi}{2}\\ 3, & \text{if } x=\dfrac{\pi}{2}\end{cases}$

The given function f is continuous at $x=\frac{\pi}{2}$, if f is defined at $x=\frac{\pi}{2}$ and if the value of the f at $x=\frac{\pi}{2}$ equals the limit of f at $x=\frac{\pi}{2}$.

It is evident that f is defined at $x=\frac{\pi}{2}$ and $f\left(\frac{\pi}{2}\right)=3$

$$\lim_{x\to\frac{\pi}{2}} f(x)=\lim_{x\to\frac{\pi}{2}}\frac{k\cos x}{\pi-2x}$$

$$\text{Put } x=\frac{\pi}{2}+h$$

$$\text{Then, } x\to\frac{\pi}{2}\Rightarrow h\to 0$$

$$\therefore \lim_{x\to\frac{\pi}{2}} f(x)=\lim_{x\to\frac{\pi}{2}}\frac{k\cos x}{\pi-2x}=\lim_{h\to 0}\frac{k\cos\left(\frac{\pi}{2}+h\right)}{\pi-2\left(\frac{\pi}{2}+h\right)}$$
$$=k\lim_{h\to 0}\frac{-\sin h}{-2h}=\frac{k}{2}\lim_{h\to 0}\frac{\sin h}{h}=\frac{k}{2}.1=\frac{k}{2}$$
$$\therefore \lim_{x\to\frac{\pi}{2}} f(x)=f\left(\frac{\pi}{2}\right)$$
$$\Rightarrow\frac{k}{2}=3$$
$$\Rightarrow k=6$$

Therefore, the required value of k is 6.

Answer needs Correction? Click Here

Q27 : Find the values of k so that the function f is continuous at the indicated point.

$$f(x)=\begin{cases}kx^2, & \text{if } x\leq 2\\ 3, & \text{if } x>2\end{cases} \quad \text{at } x=2$$

Answer :

The given function is $f(x)=\begin{cases}kx^2, & \text{if } x\leq 2\\ 3, & \text{if } x>2\end{cases}$

The given function f is continuous at $x=2$, if f is defined at $x=2$ and if the value of f at $x=2$ equals the limit of f at $x=2$

limit of f at $x = 2$

It is evident that f is defined at x= 2 and $f(2) = k(2)^2 = 4k$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$$
$$\Rightarrow \lim_{x \to 2^-} (kx^2) = \lim_{x \to 2^+} (3) = 4k$$
$$\Rightarrow k \times 2^2 = 3 = 4k$$
$$\Rightarrow 4k = 3 = 4k$$
$$\Rightarrow 4k = 3$$
$$\Rightarrow k = \frac{3}{4}$$

Therefore, the required value of k is $\frac{3}{4}$.

Answer needs Correction? Click Here

Q28 : Find the values of k so that the function f is continuous at the indicated point.

$$f(x) = \begin{cases} kx+1, & \text{if } x \le \pi \\ \cos x, & \text{if } x > \pi \end{cases} \quad \text{at } x = \pi$$

Answer :

The given function is $f(x) = \begin{cases} kx+1, & \text{if } x \le \pi \\ \cos x, & \text{if } x > \pi \end{cases}$

The given function f is continuous at x= p, if f is defined at x= pand if the value of f at x= pequals the limit of $f$at$x$= p

It is evident that f is defined at$x$= pand $f(\pi) = k\pi + 1$

$$\lim_{x \to \pi^-} f(x) = \lim_{x \to \pi^+} f(x) = f(\pi)$$
$$\Rightarrow \lim_{x \to \pi^-} (kx+1) = \lim_{x \to \pi^+} \cos x = k\pi + 1$$
$$\Rightarrow k\pi + 1 = \cos \pi = k\pi + 1$$
$$\Rightarrow k\pi + 1 = -1 = k\pi + 1$$
$$\Rightarrow k = -\frac{2}{\pi}$$

Therefore, the required value of k is $-\frac{2}{\pi}$.

Answer needs Correction? Click Here

Q29 : Find the values of k so that the function f is continuous at the indicated point.

$$f(x) = \begin{cases} kx+1, & \text{if } x \le 5 \\ 3x-5, & \text{if } x > 5 \end{cases} \quad \text{at } x = 5$$

Answer :

The given function f is $f(x) = \begin{cases} kx+1, & \text{if } x \le 5 \\ 3x-5, & \text{if } x > 5 \end{cases}$

The given function f is continuous at x= 5, if f is defined at x= 5 and if the value of f at x = 5 equals the limit of f at x = 5

It is evident that f is defined at x= 5 and $f(5) = kx + 1 = 5k + 1$

$$\lim_{x \to 5^-} f(x) = \lim_{x \to 5^+} f(x) = f(5)$$
$$\Rightarrow \lim_{x \to 5^-} (kx+1) = \lim_{x \to 5^+} (3x-5) = 5k+1$$
$$\Rightarrow 5k + 1 = 15 - 5 = 5k + 1$$
$$\Rightarrow 5k + 1 = 10$$
$$\Rightarrow 5k = 9$$
$$\Rightarrow k = \frac{9}{5}$$

Therefore, the required value of k is $\frac{9}{5}$.

Answer needs Correction? Click Here

Q30 : Find the values of $a$and b such that the function defined by

$$f(x) = \begin{cases} 5, & \text{if } x \le 2 \\ ax+b, & \text{if } 2 < x < 10 \\ 21, & \text{if } x \ge 10 \end{cases}$$

is a continuous function.

Answer :

The given function f is $f(x) = \begin{cases} 5, & \text{if } x \le 2 \\ ax+b, & \text{if } 2 < x < 10 \\ 21, & \text{if } x \ge 10 \end{cases}$

It is evident that the given function $f$is defined at all points of the real line.

If $f$is a continuous function, then $f$is continuous at all real numbers.

In particular, $f$is continuous at x = 2 and x = 10

Since f is continuous at x = 2. we obtain

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$$
$$\Rightarrow \lim_{x \to 2^-} (5) = \lim_{x \to 2^+} (ax+b) = 5$$
$$\Rightarrow 5 = 2a + b = 5$$
$$\Rightarrow 2a + b = 5 \qquad \ldots(1)$$

Since f is continuous at $x = 10$, we obtain

$$\lim_{x \to 10^-} f(x) = \lim_{x \to 10^+} f(x) = f(10)$$
$$\Rightarrow \lim_{x \to 10^-} (ax+b) = \lim_{x \to 10^+} (21) = 21$$
$$\Rightarrow 10a + b = 21 = 21$$
$$\Rightarrow 10a + b = 21 \qquad \ldots(2)$$

On subtracting equation (1) from equation (2), we obtain

$8a = 16$

$\Rightarrow a = 2$

Byputting $a = 2$ in equation (1), we obtain

$2 \times 2 + b = 5$

$\Rightarrow 4 + b = 5$

$\Rightarrow b = 1$

Therefore, the values of a and b for which f is a continuous function are 2 and 1 respectively.

Answer needs Correction? **Click Here**

Q31 : Show that the function defined by $f(x) = \cos(x^2)$ is a continuous function.

Answer :

The given function is $f(x) = \cos(x^2)$

This function f is defined for every real number and f can be written as the composition of two functions as,

$f = g \circ h$, where $g(x) = \cos x$ and $h(x) = x^2$

$$\left[\because (goh)(x) = g(h(x)) = g(x^2) = \cos(x^2) = f(x)\right]$$

It has to be first proved that $g(x) = \cos x$ and $h(x) = x^2$ are continuous functions.

It is evident that g is defined for every real number.

Let c be a real number.

Then, $g(c) = \cos c$

$$\text{Put } x = c + h$$
$$\text{If } x \to c, \text{ then } h \to 0$$
$$\lim_{x \to c} g(x) = \lim_{x \to c} \cos x$$
$$= \lim_{h \to 0} \cos(c+h)$$
$$= \lim_{h \to 0} [\cos c \cos h - \sin c \sin h]$$
$$= \lim_{h \to 0} \cos c \cos h - \lim_{h \to 0} \sin c \sin h$$
$$= \cos c \cos 0 - \sin c \sin 0$$
$$= \cos c \times 1 - \sin c \times 0$$
$$= \cos c$$
$$\therefore \lim_{x \to c} g(x) = g(c)$$

Therefore, $g(x) = \cos x$ is continuous function.

$h(x) = x^2$

Clearly, h is defined for every real number.

Let k be a real number, then $h(k) = k^2$

$$\lim_{x \to k} h(x) = \lim_{x \to k} x^2 = k^2$$
$$\therefore \lim_{x \to k} h(x) = h(k)$$

Therefore, h is a continuous function.

It is known that for real valued functions g and h, such that $(g \circ h)$ is defined at c, if g is continuous at c and if f is continuous at $g(c)$, then $(f \circ g)$ is continuous at c.

Therefore, $f(x) = (goh)(x) = \cos(x^2)$ is a continuous function.

Answer needs Correction? **Click Here**

Q32 : Show that the function defined by $f(x) = |\cos x|$ is a continuous function.

Answer :

The given function is $f(x) = |\cos x|$

This function f is defined for every real number and f can be written as the composition of two functions as,

$f = g \circ h$, where $g(x) = |x|$ and $h(x) = \cos x$

$$\left[\because (goh)(x) = g(h(x)) = g(\cos x) = |\cos x| = f(x)\right]$$

It has to be first proved that $g(x) = |x|$ and $h(x) = \cos x$ are continuous functions.

$g(x)=|x|$ can be written as

$$g(x)=\begin{cases}-x, & \text{if } x<0\\ x, & \text{if } x\geq 0\end{cases}$$

Clearly, *g* is defined for all real numbers.

Let *c* be a real number.

Case I:

If $c<0$, then $g(c)=-c$ and $\lim\limits_{x\to c} g(x)=\lim\limits_{x\to c}(-x)=-c$

$\therefore \lim\limits_{x\to c} g(x)=g(c)$

Therefore, *g* is continuous at all points *x*, such that $x<0$

Case II:

If $c>0$, then $g(c)=c$ and $\lim\limits_{x\to c} g(x)=\lim\limits_{x\to c} x=c$

$\therefore \lim\limits_{x\to c} g(x)=g(c)$

Therefore, *g* is continuous at all points *x*, such that $x>0$

Case III:

If $c=0$, then $g(c)=g(0)=0$

$$\lim_{x\to 0^-} g(x)=\lim_{x\to 0^-}(-x)=0$$
$$\lim_{x\to 0^+} g(x)=\lim_{x\to 0^+}(x)=0$$
$$\therefore \lim_{x\to 0^-} g(x)=\lim_{x\to 0^+}(x)=g(0)$$

Therefore, *g* is continuous at $x=0$

From the above three observations, it can be concluded that *g* is continuous at all points.

$h(x) = \cos x$

It is evident that $h(x) = \cos x$ is defined for every real number.

Let *c* be a real number. Put $x = c + h$

If *x* Ã¢â€ ' *c*, then *h* Ã¢â€ '0

$h(c) = \cos c$

$$\begin{aligned}\lim_{x\to c} h(x) &= \lim_{x\to c}\cos x\\ &= \lim_{h\to 0}\cos(c+h)\\ &= \lim_{h\to 0}[\cos c\cos h-\sin c\sin h]\\ &= \lim_{h\to 0}\cos c\cos h-\lim_{h\to 0}\sin c\sin h\\ &= \cos c\cos 0-\sin c\sin 0\\ &= \cos c\times 1-\sin c\times 0\\ &= \cos c\end{aligned}$$
$$\therefore \lim_{x\to c} h(x)=h(c)$$

Therefore, $h(x) = \cos x$ is a continuous function.

It is known that for real valued functions *g* and *h*, such that (*g* o *h*) is defined at *c*, if *g* is continuous at *c* and if *f* is continuous at *g*(*c*), then (*f* o *g*) is continuous at *c*.

Therefore, $f(x)=(goh)(x)=g(h(x))=g(\cos x)=|\cos x|$ is a continuous function.

Answer needs Correction? **Click Here**

Q33 : Examine that $\sin|x|$ is a continuous function.

Answer :

Let $f(x)=\sin|x|$

This function *f* is defined for every real number and *f* can be written as the composition of two functions as,

f = *g* o *h*, where $g(x)=|x|$ and $h(x)=\sin x$

$[\because (goh)(x)=g(h(x))=g(\sin x)=|\sin x|=f(x)]$

It has to be proved first that $g(x)=|x|$ and $h(x)=\sin x$ are continuous functions.

$g(x)=|x|$ can be written as

$$g(x)=\begin{cases}-x, & \text{if } x<0\\ x, & \text{if } x\geq 0\end{cases}$$

Clearly, *g* is defined for all real numbers.

Let *c* be a real number.

Case I:

If $c<0$, then $g(c)=-c$ and $\lim\limits_{x\to c} g(x)=\lim\limits_{x\to c}(-x)=-c$

$\therefore \lim\limits_{x\to c} g(x)=g(c)$

Therefore, *g* is continuous at all points *x*, such that $x<0$

Case II:

If $c>0$, then $g(c)=c$ and $\lim\limits_{x\to c} g(x)=\lim\limits_{x\to c} x=c$

$\therefore \lim\limits_{x\to c} g(x)=g(c)$

Therefore, *g* is continuous at all points *x*, such that $x>0$

Case III:

If $c=0$, then $g(c)=g(0)=0$

$$\lim_{x\to0^-} g(x)=\lim_{x\to0^-}(-x)=0$$
$$\lim_{x\to0^+} g(x)=\lim_{x\to0^+}(x)=0$$
$$\therefore \lim_{x\to0^-} g(x)=\lim_{x\to0^+}(x)=g(0)$$

Therefore, *g* is continuous at *x*= 0

From the above three observations, it can be concluded that *g*is continuous at all points.

h (*x*) = sin *x*

It is evident that *h*(*x*) = sin*x* is defined for every real number.

Let *c* be a real number. Put *x*= *c* + *k*

If *x* Ã¢â€ ' *c*, then *k* Ã¢â€ '0

h (*c*) = sin *c*

$$h(c)=\sin c$$
$$\begin{aligned}\lim_{x\to c} h(x) &= \lim_{x\to c}\sin x\\ &= \lim_{k\to0}\sin(c+k)\\ &= \lim_{k\to0}[\sin c\cos k+\cos c\sin k]\\ &= \lim_{k\to0}(\sin c\cos k)+\lim_{h\to0}(\cos c\sin k)\\ &= \sin c\cos0+\cos c\sin0\\ &= \sin c+0\\ &= \sin c\end{aligned}$$
$$\therefore \lim_{x\to c} h(x)=g(c)$$

Therefore, *h* is a continuous function.

It is known that for real valued functions *g* and *h*,such that (*g* o *h*) is defined at *c*, if *g* is continuous at *c* and if *f* is continuous at *g* (*c*), then (*f* o *g*) is continuous at *c*.

Therefore, $f(x)=(goh)(x)=g(h(x))=g(\sin x)=|\sin x|$ is a continuous function.

Answer needs Correction? Click Here

Q34 : Find all the points of discontinuity of *f* defined by $f(x)=|x|-|x+1|$.

Answer :

The given function is $f(x)=|x|-|x+1|$

Thetwo functions, *g*and *h*, are defined as

$g(x)=|x|$ and $h(x)=|x+1|$

Then, *f* = *g* - *h*

Thecontinuity of *g*and *h* is examined first.

$g(x)=|x|$ can be written as

$$g(x)=\begin{cases}-x, & \text{if } x<0\\ x, & \text{if } x\ge0\end{cases}$$

Clearly, *g* is defined for all real numbers.

Let *c* be a real number.

Case I:

If $c<0$, then $g(c)=-c$ and $\lim_{x\to c} g(x)=\lim_{x\to c}(-x)=-c$

$\therefore \lim_{x\to c} g(x)=g(c)$

Therefore, *g* is continuous at all points *x*, such that *x* < 0

Case II:

If $c>0$, then $g(c)=c$ and $\lim_{x\to c} g(x)=\lim_{x\to c} x=c$

$\therefore \lim_{x\to c} g(x)=g(c)$

Therefore, *g* is continuous at all points *x*, such that *x*> 0

Case III:

If $c=0$, then $g(c)=g(0)=0$

$$\lim_{x\to0^-} g(x)=\lim_{x\to0^-}(-x)=0$$
$$\lim_{x\to0^+} g(x)=\lim_{x\to0^+}(x)=0$$
$$\therefore \lim_{x\to0^-} g(x)=\lim_{x\to0^+}(x)=g(0)$$

Therefore, *g* is continuous at *x*= 0

From the above three observations, it can be concluded that *g*is continuous at all points.

$h(x)=|x+1|$ can be written as

$$h(x)=\begin{cases}-(x+1), & \text{if, } x<-1\\ x+1, & \text{if } x\ge-1\end{cases}$$

Clearly, *h* is defined for every real number.

Let *c* be a real number.

Case I:

If $c<-1$, then $h(c)=-(c+1)$ and $\lim_{x\to c} h(x)=\lim_{x\to c}[-(x+1)]=-(c+1)$

$\therefore \lim_{x\to c} h(x)=h(c)$

$\therefore \lim\limits_{x \to c} h(x) = h(c)$

Therefore, *h* is continuous at all points *x*, such that $x < -1$

Case II:

If $c > -1$, then $h(c) = c + 1$ and $\lim\limits_{x \to c} h(x) = \lim\limits_{x \to c}(x+1) = c + 1$

$\therefore \lim\limits_{x \to c} h(x) = h(c)$

Therefore, *h* is continuous at all points *x*, such that $x > -1$

Case III:

If $c = -1$, then $h(c) = h(-1) = -1 + 1 = 0$

$\lim\limits_{x \to -1^-} h(x) = \lim\limits_{x \to -1^-}\left[-(x+1)\right] = -(-1+1) = 0$

$\lim\limits_{x \to -1^+} h(x) = \lim\limits_{x \to -1^+}(x+1) = (-1+1) = 0$

$\therefore \lim\limits_{x \to -1^-} h(x) = \lim\limits_{h \to -1^+} h(x) = h(-1)$

Therefore, *h* is continuous at $x = -1$

From the above three observations, it can be concluded that *h* is continuous at all points of the real line.

Answer needs Correction? **Click Here**

*********** END **********