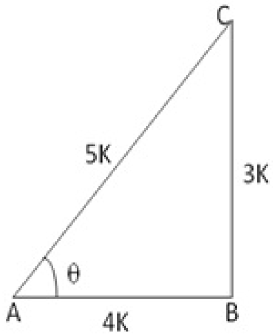
Question 10

Given: $sec\theta = \frac{AC}{AB} = \frac{5}{4}$

Let AC = 5k and AB = 4k,

Where k is positive

Let us draw a ΔABC in which $\angle$B = 90^0 and $\angle$BAC = θ



By pythagoras theorem, we have

$$\Rightarrow AC^2 = (AB)^2 + (BC)^2 \Rightarrow BC^2 = \left(AC^2 - AB^2\right)$$

$$BC^2 = \left[(5k)^2 - (4k)^2\right]$$

$$= \left(25k^2 - 16k^2\right)$$

$$\Rightarrow BC^2 = 9k^2$$

$$\Rightarrow BC = 3k$$

$$\sin\theta = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}, \sec\theta = \frac{5}{4}$$

$$\text{and} \quad \cos\theta = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \left(\frac{3}{5} \times \frac{5}{4}\right) = \frac{3}{4}$$

$$\text{L.H.S.} = \frac{\tan\theta}{1 + \tan^2\theta} = \frac{\left(\frac{3}{4}\right)}{\left(1 + \frac{9}{16}\right)} = \frac{\left(\frac{3}{4}\right)}{\left(\frac{25}{16}\right)} = \frac{3}{4} \times \frac{16}{25} = \frac{12}{25}$$

$$\text{R.H.S.} = \frac{\sin\theta}{\sec\theta} = \frac{\left(\frac{3}{5}\right)}{\left(\frac{5}{4}\right)} = \left(\frac{3}{5} \times \frac{4}{5}\right) = \frac{12}{25}$$

Thus, L.H.S. = R.H.S.

$$\text{Hence,} \quad \frac{\tan\theta}{1 + \tan^2\theta} = \frac{\sin\theta}{\cos\theta}$$

*********** END **********