Trigonometric Functions Ex 5.1 Q11

$$\text{LHS} = 1 - \frac{\sin^2\theta}{1+\cot\theta} - \frac{\cos^2\theta}{1+\tan\theta}$$

$$= 1 - \frac{\sin^2\theta}{1+\frac{\cos\theta}{\sin\theta}} - \frac{\cos^2\theta}{1+\frac{\sin\theta}{\cos\theta}} \quad \left(\because\ \cot\theta = \frac{\cos\theta}{\sin\theta}, \tan\theta = \frac{\sin\theta}{\cos\theta}\right)$$

$$= 1 - \frac{\sin^2\theta}{\frac{\sin\theta+\cos\theta}{\sin\theta}} - \frac{\cos^2\theta}{\frac{\cos\theta+\sin\theta}{\cos\theta}}$$

$$= 1 - \frac{\sin^3\theta}{\sin\theta+\cos\theta} - \frac{\cos^3\theta}{\cos\theta+\sin\theta}$$

$$= \frac{\sin\theta+\cos\theta-\left(\sin^3+\cos^3\theta\right)}{\sin\theta+\cos\theta}$$

$$= \frac{\sin\theta+\cos\theta-\left(\sin\theta+\cos\theta\right)\left(\sin^2\theta+\cos^2\theta-\sin\theta\cos\theta\right)}{\sin\theta+\cos\theta}$$

$$\left(\text{Using } a^3+b^3 = (a+b)\left(a^2+b^2-ab\right)\right)$$

$$\frac{\left(\sin\theta+\cos\theta\right)\left(1-\left(1-\sin\theta\cos\theta\right)\right)}{\sin\theta+\cos\theta} \qquad \left(\text{Using}\sin^2\theta+\cos^2\theta = 1\right)$$

$= \sin\theta\cos\theta$

$=$ RHS

Proved

Trigonometric Functions Ex 5.1 Q12

$$\text{LHS} = \left(\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{\operatorname{cosec}^2\theta - \sin^2\theta}\right)\sin^2\theta\cos^2\theta$$

$$= \left(\frac{1}{\frac{1}{\cos^2\theta} - \cos^2\theta} + \frac{1}{\left(\frac{1}{\sin^2\theta} - \sin^2\theta\right)}\right)\sin^2\theta\cos^2\theta$$

$$= \left(\frac{1}{\frac{1-\cos^4\theta}{\cos^2\theta}} + \frac{1}{\frac{1-\sin^4\theta}{\sin^2\theta}}\right)\sin^2\theta\cos^2\theta$$

$$= \left(\frac{\cos^2\theta}{(1-\cos^2\theta)(1+\cos^2\theta)} + \frac{\sin^2\theta}{(1-\sin^2\theta)(1+\sin^2\theta)}\right)\sin^2\theta\cos^2\theta$$

$$\left(\begin{aligned}\text{Using } 1-a^4 &= 1-(a^2)^2 \\ &= (1-a^2)(1+a^2)\end{aligned}\right)$$

$$= \left(\frac{\cos^2\theta}{\sin^2\theta(1+\cos^2\theta)} + \frac{\sin^2\theta}{\cos^2\theta(1+\sin^2)}\right)\sin^2\theta\cos^2\theta \qquad \left(\begin{aligned}&\text{Using } 1-\cos^2\theta = \sin^2\theta \\ &\ \& \quad 1-\sin^2\theta = \cos^2\theta\end{aligned}\right)$$

$$= \left(\frac{\cos^4\theta(1+\sin^2\theta) + \sin^4\theta(1+\cos^2\theta)}{\sin^2\theta\cos^2\theta(1+\cos^2\theta)(1+\sin^2\theta)}\right).\sin^2\theta\cos^2\theta$$

$$= \frac{\cos^4\theta + \sin^2\theta\cos^4\theta + \sin^4\theta + \cos^2\theta\sin^4\theta}{(1+\cos^2\theta)(1+\sin^2\theta)}$$

$$= \frac{(\cos^2\theta)^2 + (\sin^2\theta)^2 + 2\cos^2\theta\sin^2\theta - 2\cos^2\theta\sin^2\theta + \sin^2\theta\cos^4\theta + \cos^2\theta\sin^4\theta}{(1+\cos^2\theta)(1+\sin^2\theta)}$$

$$\left(\text{adding and subtracting } 2\cos^2\theta\sin^2\theta\right)$$

$$= \frac{(\cos^2\theta + \sin^2\theta)^2 - 2\cos^2\theta\sin^2\theta + \sin^2\theta\cos^2\theta(\cos^2\theta + \sin^2\theta)}{1 + \sin^2\theta + \cos^2\theta + \sin^2\theta\cos^2\theta}$$

$$= \frac{1^2 - 2\cos^2\theta\sin^2\theta + \sin^2\theta\cos^2\theta.1}{1 + 1 + \sin^2\theta\cos^2\theta}$$

$$= \frac{1 - \sin^2\theta\cos^2\theta}{2 + \sin^2\theta\cos^2\theta}$$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q13

$$\text{LHS} = (1 + \tan\alpha\tan\beta)^2 + (\tan\alpha - \tan\beta)^2$$

$$= 1 + (\tan\alpha + \tan\beta)^2 + 2.1\tan\alpha\tan\beta + (\tan\alpha)^2 + (\tan\beta)^2 - 2\tan\alpha.\tan\beta$$

$$\left(\text{Using } (a+b)^2 = a^2 + b^2 + 2ab \text{ and } (a-b)^2 = a^2 + b^2 - 2ab\right)$$

$$= 1 + \tan^2\alpha + \tan^2\beta + 2\tan\alpha\tan\beta + \tan^2\alpha + \tan^2\beta - 2\tan\alpha\tan\beta$$

$$= 1 + \tan^2\alpha + \tan^2\alpha + \tan^2\beta + \tan^2\beta$$

$$= \sec^2\alpha + \tan^2\beta(1 + \tan^2\alpha) \qquad (\because 1 + \tan^2\alpha = \sec^2\alpha)$$

$$= \sec^2\alpha + \tan^2\beta.\sec^2\alpha$$

$$= \sec^2\alpha(1 + \tan^2\beta)$$

$$= \sec^2\alpha.\sec^2\beta \qquad (\because 1 + \tan^2\beta = \sec^2\beta)$$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q14

$$\text{LHS} = \frac{(1+\cot\theta+\tan\theta)(\sin\theta-\cos\theta)}{\sec^3\theta-\operatorname{cosec}^3\theta}$$

$$= \frac{\left(1+\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}\right)}{\left(\frac{1}{\cos^3\theta}-\frac{1}{\sin^3\theta}\right)} \qquad \left(\because \cot\theta=\frac{\cos\theta}{\sin\theta}, \tan\theta=\frac{\sin\theta}{\cos\theta} \atop \sec\theta=\frac{1}{\cos\theta}, \operatorname{cosec}\theta=\frac{1}{\sin\theta}\right)$$

$$= \left(\frac{1+\frac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta}}{\frac{\sin^3\theta-\cos^3\theta}{\cos^3\theta\sin^3\theta}}\right)(\sin\theta-\cos\theta)$$

$$= \frac{(\sin\theta\cos\theta+1)\sin^3\theta\cos^3\theta}{\sin\theta\cos\theta.(\sin^3\theta-\cos^3\theta)}.(\sin\theta-\cos\theta) \qquad (\because \sin^2\theta+\cos^2\theta=1)$$

$$= \frac{(1+\sin\theta\cos\theta)\sin^2\theta\cos^2\theta.(\sin\theta-\cos\theta)}{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)} \qquad (\because a^3-b^3=(a-b)(a^2+b^2+ab))$$

$$= \frac{(1+\sin\theta\cos\theta).\sin^2\theta\cos^2\theta}{(1+\sin\theta\cos\theta)}$$

$= \sin^2\theta\cos^2\theta$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q15

$$\text{LHS} = \frac{2\sin\theta\cos\theta-\cos\theta}{1-\sin\theta+\sin^2\theta-\cos^2\theta}$$

$$= \frac{\cos\theta(2\sin\theta-1)}{1-\cos^2\theta+\sin^2\theta-\sin\theta}$$

$$= \frac{\cos\theta(2\sin\theta-1)}{\sin^2\theta+\sin^2\theta-\sin\theta} \qquad (\because 1-\cos^2\theta=\sin^2\theta)$$

$$= \frac{\cos\theta(2\sin\theta-1)}{2\sin^2\theta-\sin\theta}$$

$$= \frac{\cos\theta(2\sin\theta-1)}{\sin\theta(2\sin\theta-1)}$$

$$= \frac{\cos\theta}{\sin\theta}$$

$= \cot\theta$

= RHS

Proved

*********** END **********