Transformation Formulae Ex 8.2 Q3(i)

$\cos 55 + \cos 65 + \cos 175 = 0$

$\cos 175 = -\cos 5$

substitute above value in the equation we get

$\cos 55 + \cos 65 = \cos 5$

applying rule $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\cos 55 + \cos 65 = 2\cos\left(\frac{65+55}{2}\right)\cos\left(\frac{65-55}{2}\right) = 2\cos 60 \cos 5 = 2\times\frac{1}{2}\times\cos 5 = \cos 5$

Hence Proved

Transformation Formulae Ex 8.2 Q3(ii)

$$\sin 50^\circ - \sin 70^\circ + \sin 10^\circ = 0$$

$$(\sin 50^\circ - \sin 70^\circ) + \sin 10^\circ$$

$$\Rightarrow \left(2\sin\left(\frac{50^\circ - 70^\circ}{2}\right)\cos\left(\frac{50^\circ + 70^\circ}{2}\right)\right) + \sin 10^\circ \qquad \left[\because \sin C - \sin D = 2\sin\left(\frac{C-D}{2}\right)\cos\left(\frac{C+D}{2}\right)\right]$$

$$= 2\sin(-10^\circ)\cos 60^\circ + \sin 10^\circ$$

$$= -2\sin 10^\circ \times \frac{1}{2} + \sin 10^\circ \qquad \left[\because \cos 60^\circ = \frac{1}{2}\right]$$

$$= 0$$

$$= \text{RHS}$$

Transformation Formulae Ex 8.2 Q3(iii)

$$\cos 80^\circ + \cos 40^\circ - \cos 20^\circ = 0$$

$$(\cos 80^\circ + \cos 40^\circ) - \cos 20^\circ$$

$$= 2\cos\left(\frac{80^\circ + 40^\circ}{2}\right)\cos\left(\frac{80^\circ - 40^\circ}{2}\right) - \cos 20^\circ \qquad \left[\because \cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\right]$$

$$= 2\cos 60^\circ \cos 20^\circ - \cos 20^\circ$$

$$= 2\times\frac{1}{2}\cos 20^\circ - \cos 20^\circ$$

$$= \cos 20^\circ - \cos 20^\circ$$

$$= 0$$

$$= \text{RHS}$$

Transformation Formulae Ex 8.2 Q3(iv)

$$\cos 20^\circ + \cos 100^\circ + \cos 140^\circ = 0$$

$$\Rightarrow (\cos 20^\circ + \cos 100^\circ) + \cos 140^\circ$$

$$= 2\cos\left(\frac{20^\circ + 100^\circ}{2}\right)\cos\left(\frac{20^\circ - 100^\circ}{2}\right) + \cos 140^\circ \qquad \left[\because \cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\right]$$

$$= 2\cos 60^\circ \cos(-40^\circ) + \cos 140^\circ$$

$$= 2\times\frac{1}{2}\cos 40^\circ + \cos 140^\circ \qquad \left[\because \cos 60^\circ = \frac{1}{2}\right]$$

$$= \cos 40^\circ + \cos(180^\circ - 40^\circ)$$

$$= \cos 40^\circ - \cos 40^\circ$$

$$= 0$$

$$= \text{RHS}$$

Transformation Formulae Ex 8.2 Q3(v)

$$\sin\frac{5\pi}{18} - \cos\frac{4\pi}{9} = \sqrt{3}\sin\frac{\pi}{9}$$

$$\begin{aligned}
\text{LHS} &= \sin\frac{5\pi}{18} - \cos\frac{4\pi}{9} \\
&= \sin 50° - \cos 80° \\
&= \sin 50° - \sin 10° \\
&= 2\sin\left(\frac{50° - 10°}{2}\right)\cos\left(\frac{50° + 10°}{2}\right) \\
&= 2\sin 20° \cos 30° \\
&= 2\sin 20° \times \frac{\sqrt{3}}{2} \\
&= \sqrt{3}\sin\frac{\pi}{9}
\end{aligned}$$

Transformation Formulae Ex 8.2 Q3(vi)

$\cos\frac{\pi}{12} - \sin\frac{\pi}{12} = \frac{1}{\sqrt{2}}$

Multiplying and dividing by $\sqrt{2}$ on LHS

$$\begin{aligned}
&= \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos\frac{\pi}{12} - \frac{1}{\sqrt{2}}\sin\frac{\pi}{12}\right) \\
&= \sqrt{2}\left(\sin\frac{\pi}{4}\cos\frac{\pi}{12} - \cos\frac{\pi}{4}\sin\frac{\pi}{12}\right) && \left[\because \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4} = \sin\frac{\pi}{4}\right] \\
&= \sqrt{2}\left(\sin\left(\frac{\pi}{4} - \frac{\pi}{12}\right)\right) && \left[\because \sin(A-B) = \sin A\cos B - \cos A\sin B\right] \\
&= \sqrt{2}\left(\sin\frac{\pi}{6}\right) \\
&= \sqrt{2}\times\frac{1}{2} \\
&= \frac{1}{\sqrt{2}} \\
&= \text{RHS}
\end{aligned}$$

Transformation Formulae Ex 8.2 Q3(vii)

$\sin 80° - \cos 70° = \cos 50°$

LHS $= \sin 80° = \cos 50° + \cos 70°$

Now,

$$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\begin{aligned}
\text{RHS} &= \cos 50° + \cos 70° \\
&= 2\cos\left(\frac{50° + 70°}{2}\right)\cos\left(\frac{50° - 70°}{2}\right) \\
&= 2\cos 60° \cos(-10°) \\
&= 2\times\frac{1}{2}\cos 10° && \left[\cos(-\theta) = \cos\theta\right] \\
&= \cos 10° \\
&= \sin 80° \\
&= \text{LHS} && \left[\because \cos\theta = \sin(90 - \theta)\right]
\end{aligned}$$

Transformation Formulae Ex 8.2 Q3(viii)

$\sin 51° + \cos 81° = \cos 21°$

$\sin 51° = \cos 21° - \cos 81°$

$$\begin{aligned}
\text{RHS} &= \cos 21° - \cos 81° \\
&= -2\sin(51°)\sin(-30°) && \left[\because \cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right] \\
&= +2\sin 51° \sin 30° \\
&= 2\sin 51° \times \frac{1}{2} \\
&= \sin 51° \\
&= \text{LHS}
\end{aligned}$$

*********** END **********