EXERCISE.7.4

Question-1

If $^{n}C_{8} = {}^{n}C_{2}$, find $^{n}C_{2}$.

Ans.

It is known that, $^{n}C_{a} = {}^{n}C_{b} \Rightarrow a = b \text{ or } n = a + b$

Therefore,

$$^{n}C_{8} = {}^{n}C_{2} \Rightarrow n = 8 + 2 = 10$$

$$\therefore {}^{n}C_{2} = {}^{10}C_{2} = \frac{10!}{2!(10-2)!} = \frac{10!}{2!8!} = \frac{10 \times 9 \times 8!}{2 \times 1 \times 8!} = 45$$

Question-2

Determine n if

(i) $^{2n}C_{3} : {}^{n}C_{3} = 12:1$ (ii) $^{2n}C_{3} : {}^{n}C_{3} = 11:1$

Ans.

$$\frac{{}^{2n}C_3}{{}^{n}C_3}=\frac{12}{1}$$

$$\Rightarrow \frac{(2n)!}{3!(2n-3)!}\times\frac{3!(n-3)!}{n!}=\frac{12}{1}$$

$$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!}\times\frac{(n-3)!}{n(n-1)(n-2)(n-3)!}=12$$

$$\Rightarrow \frac{2(2n-1)(2n-2)}{(n-1)(n-2)}=12$$

$$\Rightarrow \frac{4(2n-1)(n-1)}{(n-1)(n-2)}=12$$

$$\Rightarrow \frac{(2n-1)}{(n-2)}=3$$

$$\Rightarrow 2n-1=3(n-2)$$

$$\Rightarrow 2n-1=3n-6$$

$$\Rightarrow 3n-2n=-1+6$$

$$\Rightarrow n=5$$

(ii)

$$\frac{{}^{2n}C_3}{{}^{n}C_3}=\frac{11}{1}$$

$$\Rightarrow \frac{(2n)!}{3!(2n-3)!}\times\frac{3!(n-3)!}{n!}=11$$

$$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!}\times\frac{(n-3)!}{n(n-1)(n-2)(n-3)!}=11$$

$$\Rightarrow \frac{2(2n-1)(2n-2)}{(n-1)(n-2)}=11$$

$$\Rightarrow \frac{4(2n-1)(n-1)}{(n-1)(n-2)}=11$$

$$\Rightarrow \frac{4(2n-1)}{n-2}=11$$

$$\Rightarrow 4(2n-1)=11(n-2)$$

$$\Rightarrow 8n-4=11n-22$$

$$\Rightarrow 11n-8n=-4+22$$

$$\Rightarrow 3n=18$$

$$\Rightarrow n=6$$

Question-3

How many chords can be drawn through 21 points on a circle?

Ans.

For drawing one chord on a circle, only 2 points are required.

To know the number of chords that can be drawn through the given 21 points on a circle, the number of combinations have to be counted.

Therefore, there will be as many chords as there are combinations of 21 points taken 2 at a time.

Thus, required number of chords = ${}^{21}C_2=\frac{21!}{2!(21-2)!}=\frac{21!}{2!19!}=\frac{21\times 20}{2}=210$

Question-4

In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4 girls?

Ans.

A team of 3 boys and 3 girls is to be selected from 5 boys and 4 girls.

3 boys can be selected from 5 boys in ${}^5C_3$ ways.

3 girls can be selected from 4 girls in ${}^4C_3$ ways.

Therefore, by multiplication principle, number of ways in which a team of 3 boys and 3 girls can be selected = ${}^5C_3 \times {}^4C_3 = \frac{5!}{3!2!} \times \frac{4!}{3!1!}$

$$= \frac{5\times4\times3!}{3!\times2} \times \frac{4\times3!}{3!}$$
$$= 10\times4 = 40$$

Question-5

Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour.

Ans.

There are a total of 6 red balls, 5 white balls, and 5 blue balls.

9 balls have to be selected in such a way that each selection consists of 3 balls of each colour.

Here,

3 balls can be selected from 6 red balls in ${}^6C_3$ ways.

3 balls can be selected from 5 white balls in ${}^5C_3$ ways.

3 balls can be selected from 5 blue balls in ${}^5C_3$ ways.

Thus, by multiplication principle, required number of ways of selecting 9 balls

$$= {}^6C_3 \times {}^5C_3 \times {}^5C_3 = \frac{6!}{3!3!} \times \frac{5!}{3!2!} \times \frac{5!}{3!2!}$$
$$= \frac{6\times5\times4\times3!}{3!\times3\times2} \times \frac{5\times4\times3!}{3!\times2\times1} \times \frac{5\times4\times3!}{3!\times2\times1}$$
$$= 20\times10\times10 = 2000$$

Question-6

Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination.

Ans.

In a deck of 52 cards, there are 4 aces. A combination of 5 cards have to be made in which there is exactly one ace.

Then, one ace can be selected in ${}^4C_1$ ways and the remaining 4 cards can be selected out of the 48 cards in ${}^{48}C_4$ ways.

Thus, by multiplication principle, required number of 5 card combinations

$$= {}^{48}C_4 \times {}^4C_1 = \frac{48!}{4!44!} \times \frac{4!}{1!3!}$$
$$= \frac{48\times47\times46\times45}{4\times3\times2\times1} \times 4$$
$$= 778320$$

Question-7

In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

Ans.

Out of 17 players, 5 players are bowlers.

A cricket team of 11 players is to be selected in such a way that there are exactly 4 bowlers.

4 bowlers can be selected in ${}^{5}C_{4}$ ways and the remaining 7 players can be selected out of the 12 players in ${}^{12}C_{7}$ ways.

Thus, by multiplication principle, required number of ways of selecting cricket team

$$= {}^{5}C_{4} \times {}^{12}C_{7} = \frac{5!}{4!1!} \times \frac{12!}{7!5!} = 5 \times \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1} = 3960$$

Question-8

A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.

Ans.

There are 5 black and 6 red balls in the bag.

2 black balls can be selected out of 5 black balls in ${}^{5}C_{2}$ ways and 3 red balls can be selected out of 6 red balls in ${}^{6}C_{3}$ ways.

Thus, by multiplication principle, required number of ways of selecting 2 black and 3 red balls $= {}^{5}C_{2} \times {}^{6}C_{3} = \frac{5!}{2!3!} \times \frac{6!}{3!3!} = \frac{5 \times 4}{2} \times \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 10 \times 20 = 200$

Question-9

In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 specific courses are compulsory for every student?

Ans.

There are 9 courses available out of which, 2 specific courses are compulsory for every student.

Therefore, every student has to choose 3 courses out of the remaining 7 courses. This can be chosen in ${}^{7}C_{3}$ ways.

Thus, required number of ways of choosing the programme

$$= {}^{7}C_{3} = \frac{7!}{3!4!} = \frac{7 \times 6 \times 5 \times 4!}{3 \times 2 \times 1 \times 4!} = 35$$

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