Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 29

$\because x$ lies in II^{nd} quad.

$$\Rightarrow \quad \frac{\pi}{2} < x < \pi$$

$$\Rightarrow \quad \frac{\pi}{4} < x\frac{x}{2} < \frac{\pi}{2}$$

Which means $\frac{x}{2}$ lies in first quad.

Now, $\sin x = \frac{\sqrt{5}}{3} = \frac{p}{h} \Rightarrow \quad p = \sqrt{5} \quad \Rightarrow b = 2$

$h = 3$

so, $\cos x = \frac{b}{h} = \frac{-2}{3}$ $\left(-\text{ve due to II}^{\text{nd}}\text{ quad}\right)$

Thus,

$$\cos\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1-\frac{2}{3}}{2}} = \frac{1}{\sqrt{6}}$$

$$\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1+\frac{2}{3}}{2}} = \sqrt{\frac{5}{6}}$$

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\sqrt{\frac{5}{6}}}{\frac{1}{\sqrt{6}}} = \sqrt{5}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q30(i)

Since x lies in II^{nd} quadrant

$$\Rightarrow \frac{\pi}{2} < x < \pi$$

$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}, \text{ which means } \frac{x}{2} \text{ lies in I}^{st} \text{ quad.}$$

Now,

$$\sin x = \frac{1}{4} = \frac{p}{h} \Rightarrow p = 1 \qquad \Rightarrow b = \sqrt{15}$$
$$h = 4$$

so, $\cos x = \frac{b}{h} = \frac{-\sqrt{15}}{4}$ $\qquad$ $\left(-\text{ve due to II}^{nd}\text{ quad}\right)$

Thus,

$$\cos x/2 = \sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1-\frac{\sqrt{15}}{14}}{2}} = \frac{\sqrt{4-\sqrt{15}}}{8}$$

$$\sin x/2 = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1+\frac{\sqrt{15}}{4}}{2}} = \sqrt{\frac{4+\sqrt{15}}{8}}$$

$$\tan x/2 = \frac{\sin x/2}{\cos x/2} = \frac{\sqrt{\frac{4+\sqrt{15}}{8}}}{\sqrt{\frac{4-\sqrt{15}}{8}}} = \sqrt{\frac{4+\sqrt{15}}{4-\sqrt{15}}}$$

$$= \sqrt{\frac{\left(4+\sqrt{15}\right)\left(4+\sqrt{15}\right)}{\left(4-\sqrt{15}\right)\left(4+\sqrt{15}\right)}}$$

$$= 4+\sqrt{15}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 30(ii)

Since θ in acute, so $0 \le 2\theta < \pi$

$$\text{Now, } \cos\theta = \frac{4}{5} = \frac{b}{h} \quad \Rightarrow b = 4 \qquad \Rightarrow p = 3$$
$$h = 5$$

$$\therefore \sin\theta = \frac{p}{h} = \frac{3}{5}$$

$$\tan\theta = \frac{p}{b} = \frac{3}{4}$$

$$\text{so, } \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2.3/4}{1-\left(\frac{3}{4}\right)^2}$$

$$= \frac{6/4}{7/16} = \frac{24}{7}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 30(iii)

$$\sin\theta = \frac{4}{5} = \frac{p}{h} \Rightarrow p = 4 \qquad \Rightarrow b = 3$$

$$h = 5$$

$$\therefore \cos\theta = \frac{b}{h} = \frac{3}{5}$$

Now, $\sin\theta = 2\sin\theta.\cos\theta = 2.\frac{4}{5}.\ \frac{3}{5} = \frac{24}{25}$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2 = \frac{-7}{25}$$

so, $\sin 4\theta = \sin 2.2\theta = 2\sin 2\theta.\ \cos 2\theta$

$$= 2.\frac{24}{25}.\left(\frac{-7}{25}\right)$$

$$= \frac{-336}{625}$$

*********** END **********