



Geometric Progressions Ex 20.5 Q 15

a, b, c, d are in G.P.

$$a, b = ar, c = ar^2, d = ar^3$$

Now,

$$(b^2 + c^2)^2 = (a^2 + b^2)(c^2 + d^2)$$

$$(a^2r^2 + a^2r^4)^2 = (a^2 + a^2r^2)(a^2r^4 + a^2r^6)$$

$$a^4(r^2 + r^4)^2 = a^2(1 + r^2)a^2r^4(1 + r^2)$$

$$a^4r^4(1 + r^2)^2 = a^4r^4(1 + r^2)^2$$

LHS = RHS

$$\Rightarrow (b^2 + c^2)^2 = (a^2 + b^2)(c^2 + d^2)$$

$$\Rightarrow (a^2 + b^2), (b^2 + c^2), (c^2 + d^2) \text{ are in G.P.}$$

a, b, c, d are in G.P.

$$a, b = ar, c = ar^2, d = ar^3$$

Now,

$$(b^2 - c^2)^2 = (a^2 - b^2)(c^2 - d^2)$$

$$(a^2r^2 - a^2r^4)^2 = (a^2 - a^2r^2)(a^2r^4 - a^2r^6)$$

$$a^4(r^2 - r^4)^2 = a^2(1 - r^2)a^2r^4(1 - r^2)$$

$$a^4r^4(1 - r^2)^2 = a^4r^4(1 - r^2)^2$$

LHS = RHS

$$\Rightarrow (b^2 - c^2)^2 = (a^2 - b^2)(c^2 - d^2)$$

$$\Rightarrow (a^2 - b^2), (b^2 - c^2), (c^2 - d^2) \text{ are in G.P.}$$

a, b, c, d are in G.P.

$$a, b = ar, c = ar^2, d = ar^3$$

Now,

$$\left(\frac{1}{b^2 + c^2}\right)^2 = \left(\frac{1}{a^2 + b^2}\right)\left(\frac{1}{c^2 + d^2}\right)$$

$$\left(\frac{1}{a^2r^2 + a^2r^4}\right)^2 = \left(\frac{1}{a^2 + a^2r^2}\right)\left(\frac{1}{a^2r^4 + a^2r^6}\right)$$

$$\frac{1}{a^4(r^2 + r^4)^2} = \frac{1}{a^2(1 + r^2)} \times \frac{1}{a^2(r^4 + r^6)}$$

$$\frac{1}{a^4r^4(1 + r^2)^2} = \frac{1}{a^2r^4(1 + r^2)(1 + r^2)}$$

$$\frac{1}{a^4r^4(1 + r^2)^2} = \frac{1}{a^2r^4(1 + r^2)^2}$$

LHS = RHS

$$\Rightarrow \left(\frac{1}{b^2 + c^2}\right)^2 = \left(\frac{1}{a^2 + b^2}\right)\left(\frac{1}{c^2 + d^2}\right)$$

$$\Rightarrow \left(\frac{1}{a^2 + b^2}\right), \left(\frac{1}{b^2 + c^2}\right), \left(\frac{1}{c^2 + d^2}\right) \text{ are in G.P.}$$

a, b, c, d are in G.P.

$$a, b = ar, c = ar^2, d = ar^3$$

Now,

$$(ab + bc + cd)^2 = (a^2 + b^2 + c^2)(b^2 + c^2 + d^2)$$

$$(a^2r + a^2r^3 + a^2r^5)^2 = (a^2 + a^2r^2 + a^2r^4)(a^2r^2 + a^2r^4 + a^2r^6)$$

$$a^4(r + r^3 + r^5)^2 = a^2(1 + r^2 + r^4)a^2r^2(1 + r^2 + r^4)$$

$$a^4r^2(1 + r^2 + r^4)^2 = a^4r^2(1 + r^2 + r^4)^2$$

$$\text{LHS} = \text{RHS}$$

$$\Rightarrow (ab + bc + cd)^2 = (a^2 + b^2 + c^2)(b^2 + c^2 + d^2)$$

$$\Rightarrow (a^2 + b^2 + c^2), (ab + bc + cd), (b^2 + c^2 + d^2) \text{ are in G.P.}$$

Geometric Progressions Ex 20.5 Q 16

$(a - b), (b - c), (c - a)$ are in G.P.

$$(b - c)^2 = (a - b)(c - a)$$

$$b^2 + c^2 - 2bc = ac - a^2 - bc + ab$$

$$b^2 + c^2 + a^2 = ac + bc + ab \quad \text{---(i)}$$

Now,

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$= ac + bc + ab + 2ab + 2bc + 2ca$$

Using equation (i)

$$= 3ab + 3bc + 3ca$$

$$(a + b + c)^2 = 3(ab + bc + ca)$$

***** END *****