Differentiation Ex 11.3 Q45

Here, $y = \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

Put $x = \cos 2\theta$, so

$$y = \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2}\,(\cos\theta-\sin\theta)}{\sqrt{2}\,(\cos\theta+\sin\theta)}\right)$$

$$= \tan^{\{1\}}\left(\frac{\frac{\cos\theta-\sin\theta}{\cos\theta}}{\frac{\cos\theta+\sin\theta}{\cos\theta}}\right)$$

[Dividing numerator and denominator by $\cos\theta$]

$$= \tan^{-}1\left(\frac{\frac{\cos\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}\right)$$

$$= \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}+\tan\theta}\right)$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}-\theta\right)\right]$$

$$= \frac{\pi}{4}-\theta$$

$$y = \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x \qquad \left[\text{Using } x = \cos 2\theta\right]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 0-\frac{1}{2}\left(\frac{-1}{\sqrt{1-x^2}}\right)$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q46

Here, $y = \cos^{-1}\left\{\frac{2x - 3\sqrt{1-x^2}}{\sqrt{13}}\right\}$

Let $x = \cos\theta$, so,

$$y = \cos^{-1}\left\{\frac{2\cos\theta - 3\sqrt{1-\cos^2\theta}}{\sqrt{13}}\right\}$$

$$= \cos^{-1}\left\{\frac{2}{\sqrt{13}}\cos\theta - \frac{3}{\sqrt{13}}\sin\theta\right\}$$

Let $\cos\phi = \frac{2}{\sqrt{13}}$

$\Rightarrow$ $\sin\phi = \sqrt{1-\cos^2\phi}$

$$= \sqrt{1-\left(\frac{2}{\sqrt{13}}\right)^2}$$

$$= \sqrt{\frac{13-4}{13}}$$

$$= \sqrt{\frac{9}{13}}$$

$$\sin\phi = \frac{3}{\sqrt{13}}$$

So,

$$y = \cos^{-1}\{\cos\phi\cos\theta - \sin\phi\sin\theta\}$$

$$= \cos^{-1}[\cos(\theta+\phi)]$$

$$y = \phi + \theta$$

$$y = \cos^{-1}\left(\frac{2}{\sqrt{13}}\right) + \cos^{-1}x \qquad \left[\text{Since, } x = \cos\theta, \cos\phi = \frac{2}{\sqrt{13}}\right]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 0 + \left(-\frac{1}{\sqrt{1-x^2}}\right)$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q47

Consider the given expression:

$$y = \sin^{-1}\left\{\frac{2^{x+1} \times 3^x}{1+(36)^x}\right\}$$

$$= \sin^{-1}\left\{\frac{2 \times 2^x \times 3^x}{1+(6^2)^x}\right\}$$

$$y = \sin^{-1}\left\{\frac{2 \times 6^x}{1+6^{2x}}\right\}....(1)$$

Substituting $6^x = \tan\theta$ in the above equation, we get,

$$y=\sin^{-1}\left\{\frac{2 \times 6^x}{1+6^{2x}}\right\}$$

$$= \sin^{-1}\left\{\frac{2 \times \tan\theta}{1+\tan^2\theta}\right\}$$

$$= \sin^{-1}(\sin 2\theta)$$

$$= 2\theta$$

$$= 2\tan^{-1}(6^x)$$

Differentiating the above function with respect to x, we have,

$$\frac{d}{dx}\left[\sin^{-1}\left\{\frac{2^{x+1} \times 3^x}{1+(36)^x}\right\}\right] = \frac{d}{dx}\left[2\tan^{-1}(6^x)\right]$$

$$= 2 \times \frac{1}{1+(6^x)^2} \times 6^x \log 6$$

$$= \frac{2 \times 6^x \log 6}{1+6^{2x}}$$

*********** END **********