



### Algebra of Matrices Ex 5.3 Q55

We have,

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

Then ,

$$A^2 = A \cdot A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 2 \times 2 + 0 \times 2 + 1 \times 1 & 2 \times 0 + 0 \times 1 + 1 \times -1 & 2 \times 1 + 0 \times 3 + 1 \times 0 \\ 2 \times 2 + 1 \times 2 + 3 \times 1 & 2 \times 0 + 0 \times 1 + 1 \times -1 & 2 \times 1 + 1 \times 3 + 3 \times 0 \\ 1 \times 2 + -1 \times 2 + 0 \times 1 & 1 \times 0 + -1 \times 1 + 0 \times -1 & 1 \times 1 + -1 \times 3 + 0 \times 0 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix}, \quad 4I = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\text{Hence, } A^2 - 5A + 4I = \begin{bmatrix} 5 & -1 & 5 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} + \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$A^2 - 5A + 4I = \begin{bmatrix} 5-10+4 & -1+0+0 & 5-5+0 \\ 9-10+0 & -2-5+4 & 5-15+0 \\ 0-5+0 & -1+5+0 & -2+0+4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$$

Now, given is  $A^2 - 5A + 4I + X = 0$

$$\Rightarrow X = -(A^2 - 5A + 4I)$$

$$X = - \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & 4 & 2 \end{bmatrix}$$

### Algebra of Matrices Ex 5.3 Q56

Given,

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

To prove  $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$  we will use the principle of mathematical induction.

Step 1: Put  $n = 1$

$$A^1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

So,

$A^n$  is true for  $n = 1$

Step 2: Let,  $A^n$  be true for  $n = k$ , then

$$A^k = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \quad \text{---(i)}$$

Step 3: We have to show that  $A^{k+1} = \begin{bmatrix} 1 & k+1 \\ 0 & 1 \end{bmatrix}$

So,

$$\begin{aligned} A^{k+1} &= A^k \times A \\ &= \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} && \text{(using equation (i) and given)} \\ &= \begin{bmatrix} 1+0 & 1+k \\ 0+0 & 0+1 \end{bmatrix} \end{aligned}$$

$$A^{k+1} = \begin{bmatrix} 1 & 1+k \\ 0 & 1 \end{bmatrix}$$

This shows that  $A^n$  is true for  $n = k + 1$  whenever it is true for  $n = k$

Hence, by the principle of mathematical induction  $A^n$  is true for all positive integer.

\*\*\*\*\* END \*\*\*\*\*

