Pair of Linear Equations in Two varibles Ex 3.10 Q14

Answer:

Let the speed of the train be x km/hour that of the taxi be y km/hr, we have the following cases

Case I: When Abdul travels 300 Km by train and the 200 Km by taxi

Time taken by Abdul to travel 300 Km by train = $\dfrac{300}{x}$ hrs

Time taken by Abdul to travel 200 Km by taxi = $\dfrac{200}{y}$ hrs

Total time taken by Abdul to cover 500 Km = $\dfrac{300}{x} + \dfrac{200}{y}$

It is given that total time taken in 5 hours 30 minutes

$$\dfrac{300}{x} + \dfrac{200}{y} = 5\,hours\,30\,min\,utes$$

$$100\left(\dfrac{3}{x} + \dfrac{2}{y}\right) = 5\dfrac{30}{60}$$

$$100\left(\dfrac{3}{x} + \dfrac{2}{y}\right) = 5\dfrac{3\cancel{0}}{6\cancel{0}}$$

$$100\left(\dfrac{3}{x} + \dfrac{2}{y}\right) = 5\dfrac{1}{2}$$

$$100\left(\dfrac{3}{x} + \dfrac{2}{y}\right) = \dfrac{11}{2}$$

$$\left(\dfrac{3}{x} + \dfrac{2}{y}\right) = \dfrac{11}{2} \times \dfrac{1}{100}$$

$$\dfrac{3}{x} + \dfrac{2}{y} = \dfrac{11}{200} \cdots(i)$$

Case II: When Abdul travels 260 Km by train and the 240 km by taxi

Time taken by Abdul to travel 260 Km by train = $\dfrac{260}{x}$ hrs

Time taken by Abdul to travel 240 Km by taxi = $\dfrac{240}{y}$ hrs

In this case total time of the journey is 5 hours 36 minutes

$$\dfrac{260}{x} + \dfrac{240}{y} = 5hrs\,36\,min\,utes$$

$$\dfrac{260}{x} + \dfrac{240}{y} = 5\dfrac{36}{60}$$

$$\dfrac{260}{x} + \dfrac{240}{y} = 5\dfrac{6}{10}$$

$$\dfrac{260}{x} + \dfrac{240}{y} = 5\dfrac{3}{5}$$

$$20\left(\dfrac{13}{x} + \dfrac{12}{y}\right) = \dfrac{28}{5}$$

$$\left(\dfrac{13}{x} + \dfrac{12}{y}\right) = \dfrac{28}{5} \times \dfrac{1}{20}$$

$$\dfrac{13}{x} + \dfrac{12}{y} = \dfrac{7}{25}$$

Putting $\frac{1}{x} = u$ and, $\frac{1}{y} = v$, the equations (i) and (ii) reduces to

$$3u + 2v = \frac{11}{200} \quad \cdots(iii)$$

$$13u + 12v = \frac{7}{25} \quad \cdots(iv)$$

Multiplying equation (iii) by 6 the above system of equation becomes

$$18u + 12v = \frac{33}{100} \quad \cdots(v)$$

Subtracting equation (iv) from (v) we get

$$18u \cancel{+ 12v} = \frac{33}{100}$$

$$-13u \cancel{- 12v} = -\frac{7}{25}$$

$$\overline{5u = \frac{33}{100} - \frac{7}{25}}$$

$$5u = \frac{33}{100} - \frac{7 \times 4}{25 \times 4}$$

$$5u = \frac{33}{100} - \frac{28}{100}$$

$$5u = \frac{33 - 28}{100}$$

$$5u = \frac{5}{100}$$

$$u = \frac{5}{100} \times \frac{1}{5}$$

$$u = \frac{\cancel{5}}{100} \times \frac{1}{\cancel{5}}$$

$$u = \frac{1}{100}$$

Putting $u = \frac{1}{100}$ in equation $(\overline{\rule{1em}{0.4pt}})$, we get

$$3u + 2v = \frac{11}{200}$$

$$3 \times \frac{1}{100} + 2v = \frac{11}{200}$$

$$\frac{3}{100} + 2v = \frac{11}{200}$$

$$2v = \frac{11}{200} - \frac{3}{100}$$

$$2v = \frac{11}{200} - \frac{3 \times 2}{100 \times 2}$$

$$2v = \frac{11 - 6}{200}$$

$$2v = \frac{5}{200}$$

$$v = \frac{\cancel{5}}{\cancel{200}_{40}} \times \frac{1}{2}$$

$$v = \frac{1}{80}$$

Now

$$u = \frac{1}{100}$$

$$\frac{1}{x} = \frac{1}{100}$$

$$x = 100$$

and

$$v = \frac{1}{80}$$

$$\frac{1}{y} = \frac{1}{80}$$

$$y = 80$$

Hence, the speed of the train is $\boxed{100 \ km/hr}$.

The speed of the taxi is $\boxed{80 \ km/hr}$.

********** END *********