



Exercise 5.1 : Solutions of Questions on Page Number : 159

Q1 : Prove that the function $f(x) = 5x - 3$ is continuous at $x = 0$, at $x = -3$ and at $x = 5$.

Answer :

The given function is $f(x) = 5x - 3$

At $x = 0$, $f(0) = 5 \times 0 - 3 = -3$

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} (5x - 3) = 5 \times 0 - 3 = -3$$

$$\therefore \lim_{x \rightarrow 0} f(x) = f(0)$$

Therefore, f is continuous at $x = 0$

At $x = -3$, $f(-3) = 5 \times (-3) - 3 = -18$

$$\lim_{x \rightarrow -3} f(x) = \lim_{x \rightarrow -3} (5x - 3) = 5 \times (-3) - 3 = -18$$

$$\therefore \lim_{x \rightarrow -3} f(x) = f(-3)$$

Therefore, f is continuous at $x = -3$

At $x = 5$, $f(5) = 5 \times 5 - 3 = 25 - 3 = 22$

$$\lim_{x \rightarrow 5} f(x) = \lim_{x \rightarrow 5} (5x - 3) = 5 \times 5 - 3 = 22$$

$$\therefore \lim_{x \rightarrow 5} f(x) = f(5)$$

Therefore, f is continuous at $x = 5$

Answer needs Correction? [Click Here](#)

Q2 : Examine the continuity of the function $f(x) = 2x^2 - 1$ at $x = 3$.

Answer :

The given function is $f(x) = 2x^2 - 1$

At $x = 3$, $f(3) = 2 \times 3^2 - 1 = 17$

$$\lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3} (2x^2 - 1) = 2 \times 3^2 - 1 = 17$$

$$\therefore \lim_{x \rightarrow 3} f(x) = f(3)$$

Thus, f is continuous at $x = 3$

Answer needs Correction? [Click Here](#)

Q3 : Examine the following functions for continuity.

(a) $f(x) = x - 5$ (b) $f(x) = \frac{1}{x - 5}, x \neq 5$

(c) $f(x) = \frac{x^2 - 25}{x + 5}, x \neq -5$ (d) $f(x) = |x - 5|$

Answer :

(a) The given function is $f(x) = x - 5$

It is evident that f is defined at every real number k and its value at k is $k - 5$.

$$\text{It is also observed that, } \lim_{x \rightarrow k} f(x) = \lim_{x \rightarrow k} (x - 5) = k - 5 = f(k)$$

$$\therefore \lim_{x \rightarrow k} f(x) = f(k)$$

Hence, f is continuous at every real number and therefore, it is a continuous function.

(b) The given function is $f(x) = \frac{1}{x - 5}, x \neq 5$

For any real number $k \neq 5$, we obtain

$$\lim_{x \rightarrow k} f(x) = \lim_{x \rightarrow k} \frac{1}{x - 5} = \frac{1}{k - 5}$$

$$\text{Also, } f(k) = \frac{1}{k - 5} \quad (\text{As } k \neq 5)$$

$$\therefore \lim_{x \rightarrow k} f(x) = f(k)$$

Hence, f is continuous at every point in the domain of f and therefore, it is a continuous function.

(c) The given function is $f(x) = \frac{x^2 - 25}{x + 5}, x \neq -5$

For any real number $c \neq -5$, we obtain

$$\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} \frac{x^2 - 25}{x + 5} = \lim_{x \rightarrow c} \frac{(x+5)(x-5)}{x+5} = \lim_{x \rightarrow c} (x-5) = (c-5)$$

$$\text{Also, } f(c) = \frac{(c+5)(c-5)}{c+5} = (c-5) \quad (\text{as } c \neq -5)$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Hence, f is continuous at every point in the domain of f and therefore, it is a continuous function.

$$(d) \text{ The given function is } f(x) = |x-5| = \begin{cases} 5-x, & \text{if } x < 5 \\ x-5, & \text{if } x \geq 5 \end{cases}$$

This function f is defined at all points of the real line.

Let c be a point on a real line. Then, $c < 5$ or $c = 5$ or $c > 5$

Case I: $c < 5$

$$\text{Then, } f(c) = 5 - c$$

$$\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (5 - x) = 5 - c$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all real numbers less than 5.

Case II: $c = 5$

$$\text{Then, } f(c) = f(5) = (5-5) = 0$$

$$\lim_{x \rightarrow 5} f(x) = \lim_{x \rightarrow 5} (5 - x) = (5-5) = 0$$

$$\lim_{x \rightarrow 5^+} f(x) = \lim_{x \rightarrow 5^+} (x-5) = 0$$

$$\therefore \lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at $x = 5$

Case III: $c > 5$

$$\text{Then, } f(c) = f(5) = c - 5$$

$$\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (x-5) = c - 5$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Answer needs Correction? [Click Here](#)

Q4: Prove that the function $f(x) = x^n$ is continuous at $x = n$, where n is a positive integer.

Answer :

The given function is $f(x) = x^n$

It is evident that f is defined at all positive integers, n , and its value at n is n^n .

$$\text{Then, } \lim_{x \rightarrow n} f(x) = \lim_{x \rightarrow n} (x^n) = n^n$$

$$\therefore \lim_{x \rightarrow n} f(x) = f(n)$$

Therefore, f is continuous at n , where n is a positive integer.

Answer needs Correction? [Click Here](#)

Q5: Is the function f defined by

$$f(x) = \begin{cases} x, & \text{if } x \leq 1 \\ 5, & \text{if } x > 1 \end{cases}$$

continuous at $x = 0$? At $x = 1$? At $x = 2$?

Answer :

$$\text{The given function } f \text{ is } f(x) = \begin{cases} x, & \text{if } x \leq 1 \\ 5, & \text{if } x > 1 \end{cases}$$

At $x = 0$,

It is evident that f is defined at 0 and its value at 0 is 0.

$$\text{Then, } \lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} x = 0$$

$$\therefore \lim_{x \rightarrow 0} f(x) = f(0)$$

Therefore, f is continuous at $x = 0$

At $x = 1$,

f is defined at 1 and its value at 1 is 1.

The left hand limit of f at $x = 1$ is,

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} x = 1$$

The right hand limit of f at $x = 1$ is,

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (5) = 5$$

$$\therefore \lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$$

Therefore, f is not continuous at $x = 1$

At $x = 2$,

f is defined at 2 and its value at 2 is 5.

$$\text{Then, } \lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} (5) = 5$$

$$\therefore \lim_{x \rightarrow 2} f(x) = f(2)$$

Therefore, f is continuous at $x = 2$

Answer needs Correction? [Click Here](#)

Q6 : Find all points of discontinuity of f , where f is defined by

$$f(x) = \begin{cases} 2x+3, & \text{if } x \leq 2 \\ 2x-3, & \text{if } x > 2 \end{cases}$$

Answer :

$$\text{The given function is } f(x) = \begin{cases} 2x+3, & \text{if } x \leq 2 \\ 2x-3, & \text{if } x > 2 \end{cases}$$

It is evident that the given function f is defined at all the points of the real line.

Let c be a point on the real line. Then, three cases arise.

(i) $c < 2$

(ii) $c > 2$

(iii) $c = 2$

Case (i) $c < 2$

$$\text{Then, } f(c) = 2c+3$$

$$\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (2x+3) = 2c+3$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points x , such that $x < 2$

Case (ii) $c > 2$

$$\text{Then, } f(c) = 2c-3$$

$$\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (2x-3) = 2c-3$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points x , such that $x > 2$

Case (iii) $c = 2$

Then, the left hand limit of f at $x = 2$ is,

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (2x+3) = 2 \times 2 + 3 = 7$$

The right hand limit of f at $x = 2$ is,

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (2x-3) = 2 \times 2 - 3 = 1$$

It is observed that the left and right hand limit of f at $x = 2$ do not coincide.

Therefore, f is not continuous at $x = 2$

Hence, $x = 2$ is the only point of discontinuity of f .

Answer needs Correction? [Click Here](#)

Q7 : Find all points of discontinuity of f , where f is defined by

$$f(x) = \begin{cases} |x|+3, & \text{if } x \leq -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x+2, & \text{if } x \geq 3 \end{cases}$$

Answer :

$$\text{The given function is } f(x) = \begin{cases} |x|+3 = -x+3, & \text{if } x \leq -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x+2, & \text{if } x \geq 3 \end{cases}$$

The given function f is defined at all the points of the real line.

Let c be a point on the real line.

Case I:

$$\text{If } c < -3, \text{ then } f(c) = -c+3$$

$$\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (-x+3) = -c+3$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points x , such that $x < -3$

Case II:

$$\text{If } c = -3, \text{ then } f(-3) = -(-3)+3 = 6$$

$$\lim_{x \rightarrow -3^-} f(x) = \lim_{x \rightarrow -3^-} (-x+3) = -(-3)+3 = 6$$

$$\lim_{x \rightarrow -3^+} f(x) = \lim_{x \rightarrow -3^+} (-2x) = -2 \times (-3) = 6$$

$$\therefore \lim_{x \rightarrow -3} f(x) = f(-3)$$

Therefore, f is continuous at $x = -3$

Case III:

$$\text{If } -3 < c < 3, \text{ then } f(c) = -2c \text{ and } \lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (-2x) = -2c$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous in $(-3, 3)$.

Case IV:

If $c = 3$, then the left hand limit of f at $x = 3$ is,

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} (-2x) = -2 \times 3 = -6$$

The right hand limit of f at $x = 3$ is,

$$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} (6x + 2) = 6 \times 3 + 2 = 20$$

It is observed that the left and right hand limit of f at $x = 3$ do not coincide.

Therefore, f is not continuous at $x = 3$

Case V:

If $c > 3$, then $f(c) = 6c + 2$ and $\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^-} (6x + 2) = 6c + 2$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points x , such that $x > 3$

Hence, $x = 3$ is the only point of discontinuity of f .

Answer needs Correction? [Click Here](#)

Q8 : Find all points of discontinuity of f , where f is defined by

$$f(x) = \begin{cases} \frac{|x|}{x} & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

Answer :

$$\text{The given function is } f(x) = \begin{cases} \frac{|x|}{x} & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

It is known that, $x < 0 \Rightarrow |x| = -x$ and $x > 0 \Rightarrow |x| = x$

Therefore, the given function can be rewritten as

$$f(x) = \begin{cases} \frac{|x|}{x} = \frac{-x}{x} = -1 & \text{if } x < 0 \\ 0, & \text{if } x = 0 \\ \frac{|x|}{x} = \frac{x}{x} = 1, & \text{if } x > 0 \end{cases}$$

The given function f is defined at all the points of the real line.

Let c be a point on the real line.

Case I:

If $c < 0$, then $f(c) = -1$

$$\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (-1) = -1$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points $x < 0$

Case II:

If $c = 0$, then the left hand limit of f at $x = 0$ is,

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (-1) = -1$$

The right hand limit of f at $x = 0$ is,

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (1) = 1$$

It is observed that the left and right hand limit of f at $x = 0$ do not coincide.

Therefore, f is not continuous at $x = 0$

Case III:

If $c > 0$, then $f(c) = 1$

$$\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (1) = 1$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points x , such that $x > 0$

Hence, $x = 0$ is the only point of discontinuity of f .

Answer needs Correction? [Click Here](#)

Q9 : Find all points of discontinuity of f , where f is defined by

$$f(x) = \begin{cases} \frac{x}{|x|}, & \text{if } x < 0 \\ -1, & \text{if } x \geq 0 \end{cases}$$

Answer :

$$\text{The given function is } f(x) = \begin{cases} \frac{x}{|x|}, & \text{if } x < 0 \\ -1, & \text{if } x \geq 0 \end{cases}$$

It is known that, $x < 0 \Rightarrow |x| = -x$

Therefore, the given function can be rewritten as

$$f(x) = \begin{cases} \frac{x}{|x|} = \frac{x}{-x} = -1, & \text{if } x < 0 \\ -1, & \text{if } x \geq 0 \end{cases}$$

$$\Rightarrow f(x) = -1 \text{ for all } x \in \mathbf{R}$$

Let c be any real number. Then, $\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (-1) = -1$

$$\text{Also, } f(c) = -1 = \lim_{x \rightarrow c} f(x)$$

Therefore, the given function is a continuous function.

Hence, the given function has no point of discontinuity.

Answer needs Correction? [Click Here](#)

Q10 : Find all points of discontinuity of f , where f is defined by

$$f(x) = \begin{cases} x+1, & \text{if } x \geq 1 \\ x^2+1, & \text{if } x < 1 \end{cases}$$

Answer :

$$\text{The given function is } f(x) = \begin{cases} x+1, & \text{if } x \geq 1 \\ x^2+1, & \text{if } x < 1 \end{cases}$$

The given function f is defined at all the points of the real line.

Let c be a point on the real line.

Case I:

$$\text{If } c < 1, \text{ then } f(c) = c^2 + 1 \text{ and } \lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (x^2 + 1) = c^2 + 1$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points x , such that $x < 1$

Case II:

$$\text{If } c = 1, \text{ then } f(c) = f(1) = 1 + 1 = 2$$

The left hand limit of f at $x = 1$ is,

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (x^2 + 1) = 1^2 + 1 = 2$$

The right hand limit of f at $x = 1$ is,

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (x + 1) = 1 + 1 = 2$$

$$\therefore \lim_{x \rightarrow 1} f(x) = f(1)$$

Therefore, f is continuous at $x = 1$

Case III:

$$\text{If } c > 1, \text{ then } f(c) = c + 1$$

$$\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (x + 1) = c + 1$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points x , such that $x > 1$

Hence, the given function f has no point of discontinuity.

Answer needs Correction? [Click Here](#)

Q11 : Find all points of discontinuity of f , where f is defined by

$$f(x) = \begin{cases} x^3 - 3, & \text{if } x \leq 2 \\ x^2 + 1, & \text{if } x > 2 \end{cases}$$

Answer :

$$\text{The given function is } f(x) = \begin{cases} x^3 - 3, & \text{if } x \leq 2 \\ x^2 + 1, & \text{if } x > 2 \end{cases}$$

The given function f is defined at all the points of the real line.

Let c be a point on the real line.

Case I:

$$\text{If } c < 2, \text{ then } f(c) = c^3 - 3 \text{ and } \lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (x^3 - 3) = c^3 - 3$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points x , such that $x < 2$

Case II:

$$\text{If } c = 2, \text{ then } f(c) = f(2) = 2^3 - 3 = 5$$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (x^3 - 3) = 2^3 - 3 = 5$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (x^2 + 1) = 2^2 + 1 = 5$$

$$\therefore \lim_{x \rightarrow 2} f(x) = f(2)$$

Therefore, f is continuous at $x = 2$

Case III:

$$\text{If } c > 2, \text{ then } f(c) = c^2 + 1$$

$$\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (x^2 + 1) = c^2 + 1$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points x , such that $x > 2$

Thus, the given function f is continuous at every point on the real line.

Hence, f has no point of discontinuity.

Answer needs Correction? [Click Here](#)

Q12 : Find all points of discontinuity of f , where f is defined by

$$f(x) = \begin{cases} x^{10} - 1, & \text{if } x \leq 1 \\ x^2, & \text{if } x > 1 \end{cases}$$

Answer :

$$\text{The given function } f \text{ is } f(x) = \begin{cases} x^{10} - 1, & \text{if } x \leq 1 \\ x^2, & \text{if } x > 1 \end{cases}$$

The given function f is defined at all the points of the real line.

Let c be a point on the real line.

Case I:

$$\text{If } c < 1, \text{ then } f(c) = c^{10} - 1 \text{ and } \lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (x^{10} - 1) = c^{10} - 1$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points x , such that $x < 1$

Case II:

If $c = 1$, then the left hand limit of f at $x = 1$ is,

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (x^{10} - 1) = 1^{10} - 1 = 1 - 1 = 0$$

The right hand limit of f at $x = 1$ is,

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (x^2) = 1^2 = 1$$

It is observed that the left and right hand limit of f at $x = 1$ do not coincide.

Therefore, f is not continuous at $x = 1$

Case III:

$$\text{If } c > 1, \text{ then } f(c) = c^2$$

$$\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (x^2) = c^2$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points x , such that $x > 1$

Thus, from the above observation, it can be concluded that $x = 1$ is the only point of discontinuity of f .

Answer needs Correction? [Click Here](#)

Q13 : Is the function defined by

$$f(x) = \begin{cases} x + 5, & \text{if } x \leq 1 \\ x - 5, & \text{if } x > 1 \end{cases}$$

a continuous function?

Answer :

$$\text{The given function is } f(x) = \begin{cases} x + 5, & \text{if } x \leq 1 \\ x - 5, & \text{if } x > 1 \end{cases}$$

The given function f is defined at all the points of the real line.

Let c be a point on the real line.

Case I:

$$\text{If } c < 1, \text{ then } f(c) = c + 5 \text{ and } \lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (x + 5) = c + 5$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points x , such that $x < 1$

Case II:

$$\text{If } c = 1, \text{ then } f(1) = 1 + 5 = 6$$

The left hand limit of f at $x = 1$ is,

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (x + 5) = 1 + 5 = 6$$

The right hand limit of f at $x = 1$ is,

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (x - 5) = 1 - 5 = -4$$

It is observed that the left and right hand limit of f at $x = 1$ do not coincide.

Therefore, f is not continuous at $x = 1$

Case III:

$$\text{If } c > 1, \text{ then } f(c) = c - 5 \text{ and } \lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (x - 5) = c - 5$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points x , such that $x > 1$

Thus, from the above observation, it can be concluded that $x = 1$ is the only point of discontinuity of f .

Answer needs Correction? [Click Here](#)

Q14 : Discuss the continuity of the function f , where f is defined by

$$f(x) = \begin{cases} 3, & \text{if } 0 \leq x \leq 1 \\ 4, & \text{if } 1 < x < 3 \\ 5, & \text{if } 3 \leq x \leq 10 \end{cases}$$

Answer :

$$\text{The given function is } f(x) = \begin{cases} 3, & \text{if } 0 \leq x \leq 1 \\ 4, & \text{if } 1 < x < 3 \\ 5, & \text{if } 3 \leq x \leq 10 \end{cases}$$

The given function is defined at all points of the interval $[0, 10]$.

Let c be a point in the interval $[0, 10]$.

Case I:

$$\text{If } 0 \leq c < 1, \text{ then } f(c) = 3 \text{ and } \lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (3) = 3$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous in the interval $[0, 1)$.

Case II:

$$\text{If } c = 1, \text{ then } f(3) = 3$$

The left hand limit of f at $x = 1$ is,

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (3) = 3$$

The right hand limit of f at $x = 1$ is,

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (4) = 4$$

It is observed that the left and right hand limits of f at $x = 1$ do not coincide.

Therefore, f is not continuous at $x = 1$

Case III:

$$\text{If } 1 < c < 3, \text{ then } f(c) = 4 \text{ and } \lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (4) = 4$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points of the interval $(1, 3)$.

Case IV:

$$\text{If } c = 3, \text{ then } f(c) = 5$$

The left hand limit of f at $x = 3$ is,

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} (4) = 4$$

The right hand limit of f at $x = 3$ is,

$$\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} (5) = 5$$

It is observed that the left and right hand limits of f at $x = 3$ do not coincide.

Therefore, f is not continuous at $x = 3$

Case V:

$$\text{If } 3 < c \leq 10, \text{ then } f(c) = 5 \text{ and } \lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (5) = 5$$

$$\lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points of the interval $(3, 10]$.

Hence, f is not continuous at $x = 1$ and $x = 3$

Answer needs Correction? [Click Here](#)

Q15 : Discuss the continuity of the function f , where f is defined by

$$f(x) = \begin{cases} 2x, & \text{if } x < 0 \\ 0, & \text{if } 0 \leq x \leq 1 \\ 4x, & \text{if } x > 1 \end{cases}$$

Answer :

$$\text{The given function is } f(x) = \begin{cases} 2x, & \text{if } x < 0 \\ 0, & \text{if } 0 \leq x \leq 1 \\ 4x, & \text{if } x > 1 \end{cases}$$

The given function is defined at all points of the real line.

Let c be a point on the real line.

Case I:

$$\text{If } c < 0, \text{ then } f(c) = 2c$$

$$\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (2x) = 2c$$

∴ $\lim_{x \rightarrow c} f(x) = f(c)$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points x , such that $x < 0$

Case II:

$$\text{If } c = 0, \text{ then } f'(c) = f(0) = 0$$

The left hand limit of f at $x = 0$ is,

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (2x) = 2 \times 0 = 0$$

The right hand limit of f at $x = 0$ is,

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (0) = 0$$

$$\therefore \lim_{x \rightarrow 0} f(x) = f(0)$$

Therefore, f is continuous at $x = 0$

Case III:

$$\text{If } 0 < c < 1, \text{ then } f(x) = 0 \text{ and } \lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (0) = 0$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points of the interval $(0, 1)$.

Case IV:

$$\text{If } c = 1, \text{ then } f(c) = f(1) = 0$$

The left hand limit of f at $x = 1$ is,

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (0) = 0$$

The right hand limit of f at $x = 1$ is,

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (4x) = 4 \times 1 = 4$$

It is observed that the left and right hand limits of f at $x = 1$ do not coincide.

Therefore, f is not continuous at $x = 1$

Case V:

$$\text{If } c < 1, \text{ then } f(c) = 4c \text{ and } \lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (4x) = 4c$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points x , such that $x > 1$

Hence, f is not continuous only at $x = 1$

Answer needs Correction? [Click Here](#)

Q16 : Discuss the continuity of the function f , where f is defined by

$$f(x) = \begin{cases} -2, & \text{if } x \leq -1 \\ 2x, & \text{if } -1 < x \leq 1 \\ 2, & \text{if } x > 1 \end{cases}$$

Answer :

$$\text{The given function is } f(x) = \begin{cases} -2, & \text{if } x \leq -1 \\ 2x, & \text{if } -1 < x \leq 1 \\ 2, & \text{if } x > 1 \end{cases}$$

The given function is defined at all points of the real line.

Let c be a point on the real line.

Case I:

$$\text{If } c < -1, \text{ then } f(c) = -2 \text{ and } \lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (-2) = -2$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points x , such that $x < -1$

Case II:

$$\text{If } c = -1, \text{ then } f(c) = f(-1) = -2$$

The left hand limit of f at $x = -1$ is,

$$\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^-} (-2) = -2$$

The right hand limit of f at $x = -1$ is,

$$\lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^+} (2x) = 2 \times (-1) = -2$$

$$\therefore \lim_{x \rightarrow -1} f(x) = f(-1)$$

Therefore, f is continuous at $x = -1$

Case III:

$$\text{If } -1 < c < 1, \text{ then } f(c) = 2c$$

$$\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} (2x) = 2c$$

$$\therefore \lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points of the interval $(-1, 1)$.

Case IV:

$$\text{If } c = 1, \text{ then } f(c) = f(1) = 2 \times 1 = 2$$

The left hand limit of f at $x = 1$ is,

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (2x) = 2 \times 1 = 2$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (2x) = 2 \times 1 = 2$$

The right hand limit of f at $x = 1$ is,

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} 2 = 2$$

$$\therefore \lim_{x \rightarrow 1} f(x) = f(1)$$

Therefore, f is continuous at $x = 2$

Case V:

$$\text{If } c > 1, \text{ then } f(c) = 2 \text{ and } \lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^-} (2) = 2$$

$$\lim_{x \rightarrow c} f(x) = f(c)$$

Therefore, f is continuous at all points x , such that $x > 1$

Thus, from the above observations, it can be concluded that f is continuous at all points of the real line.

Answer needs Correction? [Click Here](#)

*****END*****