Functions Ex 2.5 Q4

Given that

$$A = \{1, 2, 3, 4\}, \quad B = \{3, 5, 7, 9\}, \quad C = \{7, 23, 47, 79\}$$

$f : A \to B$ and $g : B \to C$ are two functions defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2$

Now,

$$g \circ f(x) = g(f(x)) = g(2x + 1) = (2x + 1)^2 - 2$$

$$\Rightarrow \quad g \circ f(x) = 4x^2 + 4x - 1$$

Now,

$f : A \to B$ given by $f(x) = 2x + 1$

Clearly f in one-one and onto, $\therefore$ f in bijective

$\Rightarrow \quad f^{-1}$ exist

$\therefore \quad f^{-1} = \{(3, 1), (5, 2), (7, 3), (9, 7_1)\}$

Again, $g : B \to C$ given by $g(x) = x^2 - 2$

Clearly g in one-one and onto $\Rightarrow g^{-1}$ exists

$g^{-1} = \{(7, 3), (23, 5), (47, 7), (79, 9)\}$

$f \circ^{-1} g^{-1} = \{(7, 1), (23, 2), (47, 3), (79, 4)\}$ ....(A)

Now, $g \circ f(x) = 4x^2 + 4x - 1$

Clearly gof in one-one and onto $\Rightarrow (g \circ f)^{-1}$ exists.

Hence,

$$(g \circ f)^{-1} = \{(7, 1), (23, 2), (47, 3), (79, 4)\} \text{ ....(B)}$$

From (A) & (B) we have $\quad g \circ f^{-1} = f \circ^{-1} g^{-1}$

Functions Ex 2.5 Q5

Given that $f : Q \to Q$ *defined by* $f(x) = 3x + 5$.

To prove that f *is invertible, we need to prove that* f *is one – one and onto.*

Let $(x, y) \in Q$ *be such that,* $f(x) = f(y)$

$\Rightarrow 3x + 5 = 3y + 5$

$\Rightarrow x = y$

So, f *is an injection.*

Let y *be an arbitrary element of* Q *such that* $f(x) = y$.

$\Rightarrow 3x + 5 = y$

$\Rightarrow 3x = y - 5$

$$\Rightarrow x = \frac{y - 5}{3}$$

Thus, for any $y \in Q$ *there exists* $x = \dfrac{y - 5}{3} \in Q$ *such that*

$$f(x) = f\left(\frac{y - 5}{3}\right) = 3\frac{y - 5}{3} + 5 = y$$

Thus, $f : Q \to Q$ *is a bijection and hence invertible.*

Let f^{-1} *denotes the inverse of* f.

Thus, $f \circ f^{-1}(x) = x$ *for all* $x \in Q$

$\Rightarrow f[f^{-1}(x)] = x$ *for all* $x \in Q$.

$\Rightarrow 3f^{-1}(x) + 5 = x$ *for all* $x \in Q$.

$$\Rightarrow f^{-1}(x) = \frac{x - 5}{3} \text{ for all } x \in Q$$

Functions Ex 2.5 Q6

f: **R** → **R** is given by, $f(x) = 4x + 3$

One-one:

Let $f(x) = f(y)$.

$$\Rightarrow 4x+3=4y+3$$

$$\Rightarrow 4x=4y$$

$$\Rightarrow x=y$$

Therefore f is a one-one function.

Onto:

For $y \in \mathbf{R}$, let $y = 4x + 3$.

$$\Rightarrow x=\frac{y-3}{4}\in \mathbf{R}$$

Therefore, for any $y \in \mathbf{R}$, there exists $x=\frac{y-3}{4}\in \mathbf{R}$ such that

$$f(x)=f\left(\frac{y-3}{4}\right)=4\left(\frac{y-3}{4}\right)+3=y.$$

Therefore, f is onto.

Thus, f is one-one and onto and therefore, f^{-1} exists.

Let us define g: **R**→ **R** by $g(x) = \frac{x-3}{4}$

$$\text{Now,}(gof)(x)=g(f(x))=g(4x+3)=\frac{(4x+3)-3}{4}=x$$

$$(fo\,g)(y)=f(g(y))=f\left(\frac{y-3}{4}\right)=4\left(\frac{y-3}{4}\right)+3=y-3+3=y$$

Therefore, gof = fog = I_R

Hence, f is invertible and the inverse of f is given by

$$f^{-1}(y)=g(y)=\frac{y-3}{4}.$$

Functions Ex 2.5 Q7

f: $\mathbf{R}_+ \to [4, \infty)$ is given as $f(x) = x^2 + 4$.
One-one:
Let $f(x) = f(y)$.

$\Rightarrow x^2 + 4 = y^2 + 4$

$\Rightarrow x^2 = y^2$

$\Rightarrow x = y \qquad [\text{as } x = y \in \mathbf{R}_+]$

Therefore, f is a one-one function.
Onto:
For $y \in [4, \infty)$, let $y = x^2 + 4$.

$\Rightarrow x^2 = y - 4 \geq 0 \qquad [\text{as } y \geq 4]$

$\Rightarrow x = \sqrt{y-4} \geq 0$

Therefore, for any $y \in \mathbf{R}$, there exists $x = \sqrt{y-4} \in \mathbf{R}$ such that

$f(x) = f\left(\sqrt{y-4}\right) = \left(\sqrt{y-4}\right)^2 + 4 = y - 4 + 4 = y$.

Therefore, f is onto.
Thus, f is one-one and onto and therefore, f^{-1} exists.
Let us define g: $[4, \infty) \to \mathbf{R}_+$ by,

$g(y) = \sqrt{y-4}$

$\text{Now, } gof(x) = g(f(x)) = g(x^2+4) = \sqrt{(x^2+4)-4} = \sqrt{x^2} = x$

$\text{And, } fog(y) = f(g(y)) = f\left(\sqrt{y-4}\right) = \left(\sqrt{y-4}\right)^2 + 4 = (y-4) + 4 = y$

Therefore, gof = fog = I_R
Hence, f is invertible and the inverse of f is given by

$f^{-1}(y) = g(y) = \sqrt{y-4}.$

*********** END **********