Definite Integrals Ex 20.5 Q13

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 1$, $b = 4$ and $f(x) = x^2 - x$

$$h = \frac{3}{n} \Rightarrow nh = 3$$

Thus, we have,

$$I = \int_1^4 \left(x^2 - x\right)dx$$

$$= \lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(1+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[\left(1^2-1\right)+\left\{(1+h)^2-(1+h)\right\}+\left\{(1+2h)^2-(1+h)\right\}+----\right]$$

$$= \lim_{h\to 0} h\left[0+\left(h+h^2\right)+\left\{2h+(2h)^2\right\}+----\right]$$

$$= \lim_{h\to 0} h\left[h+(1+2+3+---(n-1))+h^2\left\{\left(1+2^2+3^2+---(n-1)^2\right\}\right]$$

$$\because h = \frac{3}{n} \ \&\ \text{if } h\to 0 \Rightarrow n\to\infty$$

$$= \lim_{n\to\infty} \frac{3}{n}\left[\frac{3}{n}\frac{n(n-1)}{2}+\frac{9}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} \frac{9}{n^2}n^2\left(1-\frac{1}{n}\right)+\frac{3}{2n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= \frac{9}{2}+3 = \frac{27}{2}$$

$$\therefore \int_1^4 \left(x^2-x\right)dx = \frac{27}{2}$$

Definite Integrals Ex 20.5 Q14

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here, $a = 0,\ b = 1$ and $f(x) = 3x^2+5x$

$$h = \frac{1}{n} \Rightarrow nh = 1$$

Thus, we have,

$$I = \int_0^1 \left(3x^2+5x\right)dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(0+h)+f(0+2h)+----f(0+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[\left\{0+\left(3h^2+5h\right)+\left\{3(2h)^2+5(2h)\right\}+----\right]$$

$$= \lim_{h\to 0} h\left[\left\{3h^2\left(1+2^2+3^2+---(n-1)^2\right)\right\}+5h\left\{1+2+3+---(n-1)\right\}\right]$$

$$\because h = \frac{1}{n} \text{ if } h\to 0 \Rightarrow n\to\infty$$

$$= \lim_{n\to\infty} \frac{1}{n}\left[\frac{3}{n^2}\frac{n(n-1)(2n-1)}{6}+\frac{5}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} \frac{3}{n^3}\frac{n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}{6}+\frac{5}{2n^2}n^2\left(1-\frac{1}{n}\right)$$

$$= \frac{3\times 2}{6}+\frac{5}{2} = \frac{7}{2}$$

$$\therefore \int_0^1 \left(3x^2+5x\right)dx = \frac{7}{2}$$

Definite Integrals Ex 20.5 Q15

We have

$$\int_a^b f(x) = dx \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+\ldots+f(a+(n-1)h)\right]$$

Where $h = \dfrac{b-a}{n}$

Here

$a = 0, b = 2$ and $f(x) = e^x$

Now

$$h = \frac{2}{n}$$

$$nh = 2$$

Thus, we have

$$I = \int_0^2 e^x dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+\ldots+f((n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[1+e^h+e^{2h}+\ldots+e^{(n-1)h}\right]$$

$$= \lim_{h\to 0} h\left\{\frac{\left(e^h\right)^n-1}{e^h-1}\right\}$$

$$= \lim_{h\to 0} h\left\{\frac{e^{nh}-1}{e^h-1}\right\}$$

$$= \lim_{h\to 0} h\left\{\frac{e^2-1}{e^h-1}\right\} \quad [nh = 2]$$

$$= \lim_{h\to 0}\left\{\frac{e^2-1}{\frac{e^h-1}{h}}\right\}$$

$$= e^2-1$$

Definite Integrals Ex 20.5 Q16

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here, $a = a,\ b = b$ and $f(x) = e^x$

$\therefore\ h = \dfrac{b-a}{n} \Rightarrow nh = b-a$

Thus, we have,

$$I = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[e^a + e^{a+h} + e^{a+2h} + ----e^{a+(n-1)h}\right]$$

$$= \lim_{h\to 0} he^a\left[1+e^h+e^{2h}+e^{3h}+----e^{(n-1)h}\right]$$

$$= \lim_{h\to 0} he^a\left[1+e^h+\left(e^h\right)^2+\left(e^h\right)^3+----\left(e^h\right)^{n-1}\right]$$

$$= \lim_{h\to 0} he^a\left\{\frac{\left(e^h\right)^n-1}{e^h-1}\right\} \qquad \left[\because a+ar+ar^2+----ar^{n-1} = a\left\{\frac{r^n-1}{r-1}\right\} \text{ if } r>1\right]$$

$$= \lim_{h\to 0} he^a n\left\{\frac{e^{nh}-1}{nh}\right\}\times\left(\frac{h}{e^{h-1}}\right) \qquad \left[\because \lim_{\theta\to 0}\frac{e^\theta-1}{\theta} = 1 \quad \&\quad nh = b-a\right]$$

$$\therefore \lim_{h\to 0}\left(e^{b-a}-1\right) = e^b - e^a$$

$$\therefore \int_a^b e^x\,dx = e^b - e^a$$

*********** END **********