Exercise 5.3

$\Rightarrow 35 = 7 + 12d$

$\Rightarrow 28 = 12d \Rightarrow d = \dfrac{7}{3}$

Applying formula, $S_n = \frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP,

$$S_{13} = \frac{13}{2}\left[14+(13-1)\frac{7}{3}\right] = \frac{13}{2}(14+28) = \frac{13}{2}\times 42 = 273$$

Therefore, $d = \dfrac{7}{3}$ and $S_{13} = 273$

(iii) Given $a_{12} = 37$, $d = 3$, find a and S_{12} .

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$a_{12} = a + (12 - 1)\,3$

$\Rightarrow 37 = a + 33 \Rightarrow a = 4$

Applying formula, $S_n = \frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP,

$$S_{12} = \frac{12}{2}[8+(12-1)3] = 6(8+33) = 6\times 41 = 246$$

Therefore, $a = 4$ and $S_{12} = 246$

(iv) Given $a_3 = 15, S_{10} = 125$, find d and a_{10}.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$a_3 = a + (3 - 1)(d)$

$\Rightarrow 15 = a + 2d$

$\Rightarrow a = 15 - 2d \ldots (1)$

Applying formula, $S_n = \frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP,

$$S_{10} = \frac{10}{2}[2a+(10-1)d]$$

$\Rightarrow 125 = 5\,(2a + 9d) = 10a + 45d$

Putting (1) in the above equation,

$125 = 5\,[2\,(15 - 2d) + 9d] = 5\,(30 - 4d + 9d)$

$\Rightarrow 125 = 150 + 25d$

$\Rightarrow 125 - 150 = 25d$

$\Rightarrow -25 = 25d \Rightarrow d = -1$

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$a_{10} = a + (10 - 1)\,d$

Putting value of d and equation (1) in the above equation,

$a_{10} = 15 - 2d + 9d = 15 + 7d$

$= 15 + 7\,(-1) = 15 - 7 = 8$

Therefore, $d = -1$ and $a_{10} = 8$

(v) Given $d = 5$, $S_9 = 75$, *find a and* a_9.

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,

$S_9 = \frac{9}{2}[2a + (9-1)5]$

$\Rightarrow 75 = \frac{9}{2}[2a + 40]$

$\Rightarrow 150 = 18a + 360$

$\Rightarrow -210 = 18a$

$\Rightarrow a = \frac{-35}{3}$

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$a_9 = \frac{-35}{3} + (9 - 1)\,(5)$

$= \frac{-35}{3} + 40 = \frac{-35 + 120}{3} = \frac{85}{3}$

Therefore, $a = \frac{-35}{3}$ and $a_9 = \frac{85}{3}$

Therefore, $a = \frac{-35}{3}$ and $a_9 = \frac{85}{3}$

(vi) Given $a = 2, d = 8, S_n = 90$, *find n and* a_n.

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,

$90 = \frac{n}{2}[4 + (n-1)8]$

$\Rightarrow 90 = \frac{n}{2}[4 + 8n - 8]$

$\Rightarrow 90 = \frac{n}{2}[8n - 4]$

$\Rightarrow 8n^2 - 4n - 180 = 0$

$\Rightarrow 2n^2 - n - 45 = 0$

$\Rightarrow 2n^2 - 10n + 9n - 45 = 0$

$\Rightarrow 2n(n-5) + 9(n-5) = 0$

$\Rightarrow (n-5)(2n+9) = 0$

$\Rightarrow n = 5, -9/2$

We discard negative value of n because here n cannot be in negative or fraction.

The value of n must be a positive integer.

Therefore, $n = 5$

Using formula $a_n = a + (n-1)d$,to find nth term of arithmetic progression,

$a_5 = 2 + (5-1)(8) = 2 + 32 = 34$

Therefore, $n = 5$ and $a_n = 34$

(vii) Given $a = 8$, $a_n = 62, S_n = 210$, find n and d.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$62 = 8 + (n-1)(d) = 8 + nd - d$

$\Rightarrow 62 = 8 + nd - d$

$\Rightarrow nd - d = 54$

$\Rightarrow nd = 54 + d$... (1)

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,

$$210 = \frac{n}{2}[16 + (n-1)d] = \frac{n}{2}(16 + nd - d)$$

Putting equation (1) in the above equation,

$$210 = \frac{n}{2}[16 + 54 + d - d] = \frac{n}{2} \times 70$$

$$\Rightarrow n = \frac{210 \times 2}{70} = 6 \Rightarrow n = 6$$

Putting value of n in equation (1),

$$6d = 54 + d \Rightarrow d = \frac{54}{5}$$

Therefore, $n = 6$ and $d = \frac{54}{5}$

(viii) Given $a_n = 4, d = 2, S_n = -14$, find n and a.

Using formula $a_n = a + (n-1)d$, to find n[th] term of arithmetic progression,

$4 = a + (n-1)(2) = a + 2n - 2$

$\Rightarrow 4 = a + 2n - 2$

$\Rightarrow 6 = a + 2n$

$\Rightarrow a = 6 - 2n \ldots (1)$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,

$$-14 = \frac{n}{2}[2a + (n-1)2] = \frac{n}{2}(2a + 2n - 2)$$

$$\Rightarrow -14 = \frac{n}{2}(2a + 2n - 2)$$

Putting equation (1) in the above equation, we get

$-28 = n\,[2(6 - 2n) + 2n - 2]$

$\Rightarrow -28 = n\,(12 - 4n + 2n - 2)$

$\Rightarrow -28 = n\,(10 - 2n)$

$\Rightarrow 2n^2 - 10n - 28 = 0$

$\Rightarrow n^2 - 5n - 14 = 0$

$\Rightarrow n^2 - 7n + 2n - 14 = 0$

$\Rightarrow n(n-7) + 2(n-7) = 0$

$\Rightarrow (n+2)(n-7) = 0$

$\Rightarrow n = -2, 7$

Here, we cannot have negative value of n.

Therefore, we discard negative value of n which means $n = 7$.

Putting value of n in equation (1), we get

$a = 6 - 2n = 6 - 2(7) = 6 - 14 = -8$

Therefore, $n = 7$ and $a = -8$

(ix) Given $a = 3$, $n = 8$, $S = 192$, find d.

Using formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$192 = \frac{8}{2}[6 + (8-1)d] = 4(6 + 7d)$

$\Rightarrow 192 = 24 + 28d$

$\Rightarrow 168 = 28d \Rightarrow d = 6$

(x) Given $l = 28$, $S = 144$, and there are total of 9 terms. Find a.

Applying formula, $S_n = \frac{n}{2}[a + l]$, to find sum of n terms, we get

$144 = \frac{9}{2}[a + 28]$

$\Rightarrow 288 = 9[a + 28]$

$\Rightarrow 32 = a + 28 \Rightarrow a = 4$

4. How many terms of the AP: 9, 17, 25, ... must be taken to give a sum of 636?

Ans. First term = a = 9, Common difference = d = 17 − 9 = 8, $S_n = 636$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$636 = \frac{n}{2}[18 + (n-1)(8)]$

$$\Rightarrow 1272 = n\ (18 + 8n - 8)$$

*********** END **********