

Definite Integrals Ex 20.4A Q4

We know

$$\int_{a}^{b} f(x)dx = \int_{a}^{b} f(a+b-x)dx$$

Hence

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} \, dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin(\frac{\Pi}{2} - x)}}{\sqrt{\sin(\frac{\Pi}{2} - x)} + \sqrt{\cos(\frac{\Pi}{2} - x)}} \, dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} \, dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} \, dx$$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

Then

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

Hence

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{6}} 1dx$$

$$2I = \frac{\Pi}{6}$$

$$I = \frac{\Pi}{12}$$

Definite Integrals Ex 20.4A Q5

We know
$$\int_{a}^{b} f(x)dx = \int_{a}^{b} f(a+b-x)dx$$
Hence
$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^{2}x}{1+e^{x}} dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^{2}(-x)}{1+e^{-x}} dx$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^{2}x}{1+e^{x}} dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^{2}x}{1+e^{-x}} dx$$
If
$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^{2}x}{1+e^{x}} dx$$
Then
$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^{2}x}{1+e^{-x}} dx$$
So
$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^{2}x}{1+e^{x}} + \frac{\tan^{2}x}{1+e^{-x}} dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^{2}x}{1+e^{x}} + \frac{\tan^{2}x}{1+e^{-x}} dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^{2} x}{1 + e^{x}} + \frac{\tan^{2} x}{1 + e^{-x}} dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^{2} x}{1 + e^{x}} + \frac{\tan^{2} x}{1 + e^{-x}} dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^{2} x}{1 + e^{x}} + \frac{e^{x} \tan^{2} x}{1 + e^{x}} dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^{2} x + e^{x} \tan^{2} x}{1 + e^{x}} dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{(1 + e^{x}) \tan^{2} x}{1 + e^{x}} dx$$

$$2I = \int_{\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x + e^x \tan^2 x}{1 + e^x} dx$$

$$2I = \int_{\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{(1 + e^x) \tan^2 x}{1 + e^x} dx$$

$$2I = \int_{\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x dx$$

$$I = \frac{1}{2} \int_{\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x dx$$
We know

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$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$$

If
$$f(x)$$
 is odd
$$\int_{-a}^{a} f(x) dx = 0$$

$$f(x) = \tan^2 x$$

f(x) is even, hence
$$I = \int_{0}^{\frac{\pi}{4}} \tan^{2} x dx$$

$$I = \int_{0}^{\frac{\pi}{4}} \sec^{2} x - 1 dx$$

$$I = \left\{ \tan x - x \right\}_0^{\frac{\pi}{4}}$$

$$I = 1 - \frac{\Pi}{4}$$

Note: Answer given in the book is incorrect.

Definite Integrals Ex 20.4A Q6

We know
$$\int_{a}^{b} f(x)dx = \int_{a}^{b} f(a+b-x)dx$$
Hence
$$\int_{a}^{a} \frac{1}{1+a^{x}} dx = \int_{a}^{a} \frac{1}{1+a^{-x}} dx$$
If
$$I = \int_{-a}^{a} \frac{1}{1+a^{x}} dx$$
Then
$$I = \int_{-a}^{a} \frac{1}{1+a^{-x}} dx$$
So
$$2I = \int_{-a}^{a} \frac{1}{1+a^{x}} + \frac{1}{1+a^{-x}} dx$$

$$2I = \int_{-a}^{a} \frac{1}{1+a^{x}} + \frac{a^{x}}{1+a^{x}} dx$$

$$2I = \int_{-a}^{a} 1 dx$$

$$2I = 2a$$

$$I = a$$

********* END *******