$\therefore$Mass defect of this nucleus, $\Delta m' = 29 \times m_H + 34 \times m_n - m$

Where,

Mass of a proton, $m_H = 1.007825$ u

Mass of a neutron, $m_n = 1.008665$ u

$\therefore \Delta m' = 29 \times 1.007825 + 34 \times 1.008665 - 62.9296$

$= 0.591935$ u

Mass defect of all the atoms present in the coin, $\Delta m = 0.591935 \times 2.868 \times 10^{22}$

$= 1.69766958 \times 10^{22}$ u

But 1 u = 931.5 MeV/c^2

$\therefore \Delta m = 1.69766958 \times 10^{22} \times 931.5$ MeV/c^2

Hence, the binding energy of the nuclei of the coin is given as:

$E_b = \Delta mc^2$

$= 1.69766958 \times 10^{22} \times 931.5 \left(\frac{\text{MeV}}{c^2}\right) \times c^2$

$= 1.581 \times 10^{25}$ MeV

But 1 MeV = 1.6×10^{-13} J

$E_b = 1.581 \times 10^{25} \times 1.6 \times 10^{-13}$

$= 2.5296 \times 10^{12}$ J

This much energy is required to separate all the neutrons and protons from the given coin.

Question 13.6:

Write nuclear reaction equations for

(i) α-decay of ${}^{226}_{88}\text{Ra}$ **(ii)** α-decay of ${}^{242}_{94}\text{Pu}$

(iii) β^--decay of ${}^{32}_{15}\text{P}$ **(iv)** β^--decay of ${}^{210}_{83}\text{Bi}$

(v) β^+-decay of ${}^{11}_{6}\text{C}$ **(vi)** β^+-decay of ${}^{97}_{43}\text{Tc}$

(vii) Electron capture of ${}^{120}_{54}\text{Xe}$

Answer

α is a nucleus of helium $\left({}_2\text{He}^4\right)$ and β is an electron (e^- for β^- and e^+ for β^+). In every α-decay, there is a loss of 2 protons and 4 neutrons. In every β^+-decay, there is a loss of 1 proton and a neutrino is emitted from the nucleus. In every β^--decay, there is a gain of 1 proton and an antineutrino is emitted from the nucleus.

For the given cases, the various nuclear reactions can be written as:

(i) $\quad {}_{88}\text{Ra}^{226} \longrightarrow {}_{86}\text{Rn}^{222} + {}_2\text{He}^4$

(ii) $\quad {}^{242}_{94}\text{Pu} \longrightarrow {}^{238}_{92}\text{U} + {}^{4}_{2}\text{He}$

(iii) $\quad {}^{32}_{15}\text{P} \longrightarrow {}^{32}_{16}\text{S} + e^- + \overline{\nu}$

(iv) $\quad {}^{210}_{83}\text{B} \longrightarrow {}^{210}_{84}\text{PO} + e^- + \overline{\nu}$

(v) $\quad {}^{11}_{6}\text{C} \longrightarrow {}^{11}_{5}\text{B} + e^+ + \nu$

(vi) $\quad {}^{97}_{43}\text{Tc} \longrightarrow {}^{97}_{42}\text{MO} + e^+ + \nu$

(vii) $\quad {}^{120}_{54}\text{Xe} + e^+ \longrightarrow {}^{120}_{53}\text{I} + \nu$

Question 13.7:

A radioactive isotope has a half-life of *T* years. How long will it take the activity to reduce to a) 3.125%, b) 1% of its original value?

Answer

Half-life of the radioactive isotope = *T* years

Original amount of the radioactive isotope = N_0

(a) After decay, the amount of the radioactive isotope = *N*

It is given that only 3.125% of N_0 remains after decay. Hence, we can write:

$$\frac{N}{N_0} = 3.125\% = \frac{3.125}{100} = \frac{1}{32}$$

$$\text{But } \frac{N}{N_0} = e^{-\lambda t}$$

Where,

λ = Decay constant

t = Time

$$\therefore -\lambda t = \frac{1}{32}$$

$$-\lambda t = \ln 1 - \ln 32$$
$$-\lambda t = 0 - 3.4657$$

$$t = \frac{3.4657}{\lambda}$$

$$\text{Since } \lambda = \frac{0.693}{T}$$

$$\therefore t = \frac{3.466}{\frac{0.693}{T}} \approx 5T \text{ years}$$

Hence, the isotope will take about 5*T* years to reduce to 3.125% of its original value.

(b) After decay, the amount of the radioactive isotope = *N*

It is given that only 1% of N_0 remains after decay. Hence, we can write:

$$\frac{N}{N_0} = 1\% = \frac{1}{100}$$

$$\text{But } \frac{N}{N_0} = e^{-\lambda t}$$

$$\therefore e^{-\lambda t} = \frac{1}{100}$$

$$-\lambda t = \ln 1 - \ln 100$$
$$-\lambda t = 0 - 4.6052$$
$$t = \frac{4.6052}{\lambda}$$

Since, $\lambda = 0.693/T$

$$\therefore t = \frac{4.6052}{\frac{0.693}{T}} = 6.645T \text{ years}$$

Hence, the isotope will take about 6.645*T* years to reduce to 1% of its original value.

Question 13.8:

The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive $^{14}_{6}C$ present with the stable carbon isotope $^{12}_{6}C$. When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life (5730 years) of $^{14}_{6}C$, and the measured activity, the age of the specimen can be approximately estimated. This is the principle of $^{14}_{6}C$ dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilisation.

Answer

Decay rate of living carbon-containing matter, *R* = 15 decay/min

Let N be the number of radioactive atoms present in a normal carbon- containing matter.

Half life of $^{14}_{6}C$, $T_{1/2}$ = 5730 years

The decay rate of the specimen obtained from the Mohenjodaro site:

R' = 9 decays/min

Let N' be the number of radioactive atoms present in the specimen during the Mohenjodaro period.

Therefore, we can relate the decay constant, $\lambda$and time, t as:

$$\frac{N}{N'} = \frac{R}{R'} = e^{-\lambda t}$$

$$e^{-\lambda t} = \frac{9}{15} = \frac{3}{5}$$

$$-\lambda t = \log_e \frac{3}{5} = -0.5108$$

$$\therefore t = \frac{0.5108}{\lambda}$$

$$\text{But } \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{5730}$$

$$\therefore t = \frac{0.5108}{\frac{0.693}{5730}} = 4223.5 \text{ years}$$

Hence, the approximate age of the Indus-Valley civilisation is 4223.5 years.

Question 13.9:

Obtain the amount of ${}^{60}_{27}\text{Co}$ necessary to provide a radioactive source of 8.0 mCi strength. The half-life of ${}^{60}_{27}\text{Co}$ is 5.3 years.

Answer

The strength of the radioactive source is given as:

$$\frac{dN}{dt} = 8.0 \text{ mCi}$$
$$= 8 \times 10^{-3} \times 3.7 \times 10^{10}$$
$$= 29.6 \times 10^{7} \text{ decay/s}$$

Where,

N = Required number of atoms

Half-life of ${}^{60}_{27}\text{Co}$, $T_{1/2}$ = 5.3 years

= 5.3 × 365 × 24 × 60 × 60

= 1.67 × 10^8 s

For decay constant λ, we have the rate of decay as:

$$\frac{dN}{dt} = \lambda N$$

Where, $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1.67 \times 10^{8}} \text{ s}^{-1}$

$$\therefore N = \frac{1}{\lambda}\frac{dN}{dt}$$

$$= \frac{29.6 \times 10^{7}}{\frac{0.693}{1.67 \times 10^{8}}} = 7.133 \times 10^{16} \text{ atoms}$$

For ${}_{27}\text{Co}^{60}$:

Mass of 6.023 × 10^{23} (Avogadro's number) atoms = 60 g

∴Mass of 7.133×10^{16} atoms $= \frac{60 \times 7.133 \times 10^{16}}{6.023 \times 10^{23}} = 7.106 \times 10^{-6} \text{ g}$

Hence, the amount of ${}_{27}\text{Co}^{60}$ necessary for the purpose is 7.106 × 10^{-6} g.

Question 13.10:

The half-life of ${}^{90}_{38}\text{Sr}$ is 28 years. What is the disintegration rate of 15 mg of this isotope?

Answer

*********** END **********