

Q21:
$$\int_{-\pi}^{\frac{\pi}{2}} \sin^2 x \, dx$$

Answer:

Let
$$I = \int_{-\pi}^{\frac{\pi}{2}} \sin^2 x \, dx$$

As $\sin^2(-x) = (\sin(-x))^2 = (-\sin x)^2 = \sin^2 x$, therefore, $\sin^2 x$ is an even function.

It is known that if f(x) is an even function, then $\int_{a}^{a} f(x) dx = 2 \int_{0}^{a} f(x) dx$

$$I = 2 \int_0^{\frac{\pi}{2}} \sin^2 x \, dx$$

$$= 2 \int_0^{\frac{\pi}{2}} \frac{1 - \cos 2x}{2} \, dx$$

$$= \int_0^{\frac{\pi}{2}} (1 - \cos 2x) \, dx$$

$$= \left[x - \frac{\sin 2x}{2} \right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2}$$

Answer needs Correction? Click Here

Q22:
$$\int_0^{\pi} \frac{x \, dx}{1 + \sin x}$$

Answer:

Let
$$I = \int_0^{\pi} \frac{x \, dx}{1 + \sin x}$$
 ...(1)

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1 + \sin(\pi - x)} dx \qquad \left(\int_0^{\infty} f(x) dx = \int_0^{\infty} f(a - x) dx \right)$$

$$\Rightarrow I = \int_{0}^{\pi} \frac{(\pi - x)}{1 + \sin x} dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\pi} \frac{\pi}{1 + \sin x} dx$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \frac{(1 - \sin x)}{(1 + \sin x)(1 - \sin x)} dx$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \frac{1 - \sin x}{\cos^2 x} dx$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \{\sec^2 x - \tan x \sec x\} dx$$

$$\Rightarrow 2I = \pi \left[\tan x - \sec x\right]_0^{\pi}$$

$$\Rightarrow 2I = \pi \left[2\right]$$

Answer needs Correction? Click Here

Q23:
$$\int_0^{\pi} \frac{x \, dx}{1 + \sin x}$$

Answer:

Let
$$I = \int_0^\pi \frac{x \, dx}{1 + \sin x}$$
 ...(1)

$$\Rightarrow I = \int_0^\pi \frac{(\pi - x)}{1 + \sin(\pi - x)} dx \qquad \left(\int_0^\pi f(x) dx = \int_0^\pi f(a - x) dx \right)$$

$$\Rightarrow I = \int_0^\pi \frac{(\pi - x)}{1 + \sin x} dx \qquad \dots (2)$$

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1 + \sin x} dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\pi} \frac{\pi}{1 + \sin x} dx$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \frac{(1 - \sin x)}{(1 + \sin x)(1 - \sin x)} dx$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \frac{1 - \sin x}{\cos^2 x} dx$$

$$\Rightarrow 2I = \pi \left\{ \sec^2 x - \tan x \sec x \right\} dx$$

$$\Rightarrow 2I = \pi \left[\tan x - \sec x \right]_0^{\pi}$$

$$\Rightarrow 2I = \pi \left[2 \right]$$

$$\Rightarrow I = \pi$$

Answer needs Correction? Click Here

Q24:
$$\int_{\frac{\pi}{2}}^{x} \sin^{7} x \, dx$$

Answer:

Let
$$I = \int_{-\pi}^{\pi} \sin^7 x dx$$
 ...(1)

As $\sin^7 (-x) = (\sin (-x))^7 = (-\sin x)^7 = -\sin^7 x$, therefore, $\sin^2 x$ is an odd function.

It is known that, if f(x) is an odd function, then $\int_{-x}^{a} f(x) dx = 0$

$$\therefore I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x \ dx = 0$$

Answer needs Correction? Click Here

Q25:
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x \, dx$$

Answer:

Let
$$I = \int_{-\pi}^{\pi} \sin^7 x dx$$
 ...(1

As $\sin^7(-x) = (\sin(-x))^7 = (-\sin x)^7 = -\sin^7 x$, therefore, $\sin^2 x$ is an odd function.

It is known that, if f(x) is an odd function, then $\int_{-a}^{a} f(x) dx = 0$

$$\therefore I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x \ dx = 0$$

Answer needs Correction? Click Here

Q26:
$$\int_{0}^{2\pi} \cos^{5} x dx$$

Answer:

Let
$$I = \int_0^{2\pi} \cos^5 x dx$$
 ...(
 $\cos^5 (2\pi - x) = \cos^5 x$

It is known that,

$$\int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx, \text{ if } f(2a - x) = f(x)$$

$$= 0 \text{ if } f(2a - x) = -f(x)$$

$$\therefore I = 2 \int_0^{\pi} \cos^5 x dx$$

$$\Rightarrow I = 2(0) = 0 \qquad \left[\cos^5(\pi - x) = -\cos^5 x\right]$$

Answer needs Correction? Click Here

Q27:
$$\int_0^{2\pi} \cos^5 x dx$$

Answer:

Let
$$I = \int_0^{2\pi} \cos^5 x dx$$
 ...(1)
 $\cos^5 (2\pi - x) = \cos^5 x$

It is known that,

$$\int_{0}^{2a} f(x) dx = 2 \int_{0}^{a} f(x) dx, \text{ if } f(2a - x) = f(x)$$

$$= 0 \text{ if } f(2a - x) = -f(x)$$

$$\therefore I = 2 \int_0^{\pi} \cos^5 x dx$$

$$\Rightarrow I = 2(0) = 0 \qquad \left[\cos^5(\pi - x) = -\cos^5 x\right]$$

Answer needs Correction? Click Here

Q28:
$$\int_0^{\pi} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$$

Answer:

$$\int_{2}^{\pi} \sin x - \cos x \qquad (1)$$

Let
$$I = \int_0^x \frac{\sin\left(\frac{\pi}{2} - x\right) - \cos\left(\frac{\pi}{2} - x\right)}{1 + \sin\left(\frac{\pi}{2} - x\right) \cos\left(\frac{\pi}{2} - x\right)} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right) - \cos\left(\frac{\pi}{2} - x\right)}{1 + \sin\left(\frac{\pi}{2} - x\right) \cos\left(\frac{\pi}{2} - x\right)} dx$$
 ...(2)

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1 + \sin x \cos x} dx$$
 ...(2)

Answer needs Correction? Click Here

$$Q29: \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$$
 ...(1)

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$$
 ...(1)

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right) - \cos\left(\frac{\pi}{2} - x\right)}{1 + \sin\left(\frac{\pi}{2} - x\right) \cos\left(\frac{\pi}{2} - x\right)} dx$$

$$(\int_0^x f(x) dx = \int_0^x f(a - x) dx$$
)

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1 + \sin x \cos x} dx$$
 ...(2)

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1 + \sin x \cos x} dx$$
 ...(2)

Answer needs Correction? Click Here

Answer:

Let $I = \int_0^{\pi} \log(1 + \cos x) dx$...(1)

$$\Rightarrow I = \int_{0}^{\pi} \log(1 + \cos(\pi - x)) dx \qquad \qquad \left(\int_{0}^{\pi} f(x) dx = \int_{0}^{\pi} f(a - x) dx\right)$$

$$\Rightarrow I = \int_{0}^{\pi} \log(1 - \cos x) dx \qquad \qquad \dots(2)$$
Adding (1) and (2), we obtain
$$2I = \int_{0}^{\pi} \left\{\log(1 + \cos x) + \log(1 - \cos x)\right\} dx$$

$$\Rightarrow 2I = \int_{0}^{\pi} \log(1 - \cos^{2} x) dx$$

$$\Rightarrow 2I = \int_{0}^{\pi} \log\sin^{2} x dx$$

$$\Rightarrow 2I = 2 \int_{0}^{\pi} \log\sin x dx \qquad \qquad \dots(3)$$

$$\sin(\pi - x) = \sin x$$

$$\therefore I = 2 \int_{0}^{\pi} \log\sin x dx \qquad \qquad \dots(4)$$

$$\Rightarrow I = 2 \int_{0}^{\pi} \log\sin \left(\frac{\pi}{2} - x\right) dx = 2 \int_{0}^{\pi} \log\cos x dx \qquad \qquad \dots(5)$$
Adding (4) and (5), we obtain
$$2I = 2 \int_{0}^{\pi} (\log\sin x + \log\cos x) dx$$

$$\Rightarrow I = \int_{0}^{\pi} (\log\sin x + \log\cos x + \log 2 - \log 2) dx$$

$$\Rightarrow I = \int_{0}^{\pi} (\log\sin x + \cos x - \log 2) dx$$

$$\Rightarrow I = \int_{0}^{\pi} (\log\sin x + \cos x - \log 2) dx$$

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$$\Rightarrow I = \int_{0}^{\pi} (\sin\sin x + \cos x) dx$$

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$$\Rightarrow I = \int_{0}^{\pi} (\sin\sin x + \cos$$

Q31: $\int_{0}^{\pi} \log(1+\cos x) dx$

Let $I = \int_0^{\pi} \log(1 + \cos x) dx$...(1)

Answer:

$$\Rightarrow I = \int_{0}^{\pi} \log(1 + \cos(\pi - x)) dx \qquad \left(\int_{0}^{\pi} f(x) dx = \int_{0}^{\pi} f(a - x) dx \right)$$
$$\Rightarrow I = \int_{0}^{\pi} \log(1 - \cos x) dx \qquad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\pi} \{ \log(1 + \cos x) + \log(1 - \cos x) \} dx$$

$$\Rightarrow 2I = \int_{0}^{\pi} \log(1 - \cos^{2} x) dx$$

$$\Rightarrow 2I = \int_{0}^{\pi} \log \sin^2 x \, dx$$

$$\Rightarrow 2I = 2 \int_{0}^{\pi} \log \sin x \, dx$$

$$\Rightarrow I = \int_{-\infty}^{\infty} \log \sin x \, dx \qquad \dots (3)$$

 $\sin(\pi - x) = \sin x$

$$\therefore I = 2 \int_0^{\frac{\pi}{2}} \log \sin x \, dx$$

$$\Rightarrow I = 2\int_{0}^{\frac{\pi}{2}}\log\sin\left(\frac{\pi}{2} - x\right)dx = 2\int_{0}^{\frac{\pi}{2}}\log\cos x \, dx \qquad ...(5)$$

Adding (4) and (5), we obtain

$$2I = 2\int_0^{\pi} (\log \sin x + \log \cos x) dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x + \log 2 - \log 2) dx$$

$$\Rightarrow I = \int_{0}^{\frac{\pi}{2}} (\log 2 \sin x \cos x - \log 2) dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log \sin 2x \, dx - \int_0^{\frac{\pi}{2}} \log 2 \, dx$$

Let
$$2x = t \Rightarrow 2dx = dt$$

When x = 0, t = 0

and when

2.

Answer needs Correction? Click Here

Q32:
$$\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a - x}} dx$$

Answer:

Let
$$I = \int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a - x}} dx$$
 ...(1)

It is known that,
$$\left(\int_{0}^{a} f(x)dx = \int_{0}^{a} f(a-x)dx\right)$$

$$I = \int_0^a \frac{\sqrt{a - x}}{\sqrt{a - x} + \sqrt{x}} dx \qquad \dots (2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^a \frac{\sqrt{x} + \sqrt{a - x}}{\sqrt{x} + \sqrt{a - x}} dx$$

$$\Rightarrow 2I = \int_{a}^{a} 1 dx$$

$$\Rightarrow 2I = [x]_0^a$$

$$\Rightarrow 2I = a$$

$$\Rightarrow I = \frac{a}{2}$$

Answer needs Correction? Click Here

Q33:
$$\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a - x}} dx$$

Answer:

Let
$$I = \int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a - x}} dx$$
 ...(1)

It is known that,
$$\left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$I = \int_0^a \frac{\sqrt{a - x}}{\sqrt{a - x} + \sqrt{x}} dx \qquad \dots (2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^a \frac{\sqrt{x} + \sqrt{a - x}}{\sqrt{x} + \sqrt{a - x}} dx$$

$$\Rightarrow 2I = \int_{0}^{a} 1 \, dx$$

$$\Rightarrow 2I = [x]_0^a$$

$$\Rightarrow 2I = a$$

$$\Rightarrow I = \frac{a}{2}$$

Answer needs Correction? Click Here

Q34:
$$\int_{0}^{4} |x-1| dx$$

Answer:

$$I = \int_0^4 |x - 1| dx$$

It can be seen that, $(x-1) \le 0$ when $0 \le x \le 1$ and $(x-1) \ge 0$ when $1 \le x \le 4$

$$I = \int_{0}^{1} |x - 1| dx + \int_{0}^{1} |x - 1| dx \qquad \left(\int_{x}^{6} f(x) = \int_{x}^{6} f(x) + \int_{0}^{6} f(x) \right)$$

$$= \int_{0}^{1} - (x - 1) dx + \int_{0}^{1} (x - 1) dx$$

$$= \left[x - \frac{x^{2}}{2} \right]_{0}^{1} + \left[\frac{x^{2}}{2} - x \right]_{1}^{1}$$

$$= 1 - \frac{1}{2} + \frac{(4)^{2}}{2} - 4 - \frac{1}{2} + 1$$

$$= 1 - \frac{1}{2} + 8 - 4 - \frac{1}{2} + 1$$

$$= 5$$

Answer needs Correction? Click Here

Q35:
$$\int_{0}^{4} |x-1| dx$$

Answer:

$$I = \int_0^4 |x-1| \, dx$$

It can be seen that, $(x-1) \le 0$ when $0 \le x \le 1$ and $(x-1) \ge 0$ when $1 \le x \le 4$

$$I = \int_{0}^{1} |x - 1| dx + \int_{0}^{1} |x - 1| dx \qquad \left(\int_{x}^{6} f(x) = \int_{x}^{6} f(x) + \int_{0}^{6} f(x) \right)$$

$$= \int_{0}^{1} - (x - 1) dx + \int_{0}^{1} (x - 1) dx$$

$$= \left[x - \frac{x^{2}}{2} \right]_{0}^{1} + \left[\frac{x^{2}}{2} - x \right]_{1}^{1}$$

$$= 1 - \frac{1}{2} + \frac{(4)^{2}}{2} - 4 - \frac{1}{2} + 1$$

$$= 1 - \frac{1}{2} + 8 - 4 - \frac{1}{2} + 1$$

$$= 5$$

Answer needs Correction? Click Here

Q36: Show that $\int_0^x f(x)g(x)dx = 2\int_0^x f(x)dx$, if f and g are defined as f(x) = f(a-x) and g(x) + g(a-x) = 4

Answer:

Let
$$I = \int_0^a f(x)g(x)dx$$
 ...(1)

$$\Rightarrow I = \int_0^x f(a-x)g(a-x)dx \qquad \left(\int_0^x f(x)dx = \int_0^x f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^x f(x)g(a-x)dx \qquad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^x \{f(x)g(x) + f(x)g(a-x)\} dx$$

$$\Rightarrow 2I = \int_0^x f(x)\{g(x) + g(a-x)\} dx$$

$$\Rightarrow 2I = \int_0^x f(x) \times 4 dx \qquad \left[g(x) + g(a-x) = 4\right]$$

$$\Rightarrow I = 2\int_0^x f(x) dx$$

Answer needs Correction? Click Here

Q37 : Show that $\int_0^x f(x)g(x)dx = 2\int_0^x f(x)dx$, if f and g are defined as f(x) = f(a-x) and g(x) + g(a-x) = 4

Answer:

Let
$$I = \int_0^a f(x)g(x)dx$$
 ...(1)

$$\Rightarrow I = \int_0^a f(a-x)g(a-x)dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^a f(x)g(a-x)dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^x \{f(x)g(x) + f(x)g(a-x)\} dx$$

$$\Rightarrow 2I = \int_0^x f(x)\{g(x) + g(a-x)\} dx$$

$$\Rightarrow 2I = \int_0^x f(x) \times 4 dx \qquad \left[g(x) + g(a-x) + 4\right]$$

$$\Rightarrow I = \int_0^x f(x) dx$$

Answer needs Correction? Click Here

Q38: The value of
$$\int_{-\pi}^{\pi} (x^3 + x \cos x + \tan^5 x + 1) dx$$
 is

A. 0

B. 2

C. π

D. 1

Answer:

Let
$$I = \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3 + x \cos x + \tan^5 x + 1) dx$$

$$\Rightarrow I = \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} x^3 dx + \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x + \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \tan^5 x dx + \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} 1 \cdot dx$$

It is known that if f(x) is an even function, then $\int_{a}^{b} f(x) dx = 2 \int_{0}^{b} f(x) dx$ and

if f(x) is an odd function, then $\int_{a}^{x} f(x) dx = 0$

$$I = 0 + 0 + 0 + 2 \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$=2\left[x\right]_0^{\frac{\pi}{2}}$$
$$=\frac{2\pi}{2}$$

π=

Hence, the correct answer is C.

Answer needs Correction? Click Here

Q39: The value of
$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(x^3 + x\cos x + \tan^5 x + 1\right) dx$$
 is

A. 0

B. 2

C. π

D. 1

Answer:

Let
$$I = \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3 + x \cos x + \tan^5 x + 1) dx$$

$$\Rightarrow I = \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} x^3 dx + \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x + \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \tan^5 x dx + \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} 1 \cdot dx$$

It is known that if f(x) is an even function, then $\int_a^a f(x) dx = 2 \int_a^a f(x) dx$ and

if f(x) is an odd function, then $\int_{a}^{x} f(x) dx = 0$

$$I = 0 + 0 + 0 + 2 \int_{0}^{\frac{\pi}{2}} 1 \cdot dx$$

$$= 2\left[x\right]_0^{\frac{\pi}{2}}$$
$$= \frac{2\pi}{2}$$

Hence, the correct answer is C.

Answer needs Correction? Click Here

Q40: The value of
$$\int_0^{\frac{x}{2}} \log \left(\frac{4+3\sin x}{4+3\cos x} \right) dx$$
 is

A. 2

B. $\frac{3}{4}$

C. 0

D. -2

Answer

Let
$$I = \int_0^{\pi} \log \left(\frac{4 + 3\sin x}{4 + 3\cos x} \right) dx$$
 ...(1)

$$\Rightarrow I = \int_0^{\pi} \log \left[\frac{4 + 3\sin\left(\frac{\pi}{2} - x\right)}{4 + 3\cos\left(\frac{\pi}{2} - x\right)} \right] dx \qquad \left(\int_0^{\infty} f(x) dx = \int_0^{\infty} f(a - x) dx \right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log \left(\frac{4 + 3\cos x}{4 + 3\sin x} \right) dx \qquad \dots (2)$$

Adding (1) and (2), we obtain

$$2I = \int_{0}^{x} \left\{ \log\left(\frac{4 + 3\sin x}{4 + 3\cos x}\right) + \log\left(\frac{4 + 3\cos x}{4 + 3\sin x}\right) \right\} dx$$

$$\Rightarrow 2I = \int_{0}^{x} \log\left(\frac{4 + 3\sin x}{4 + 3\cos x} \times \frac{4 + 3\cos x}{4 + 3\sin x}\right) dx$$

$$\Rightarrow 2I = \int_{0}^{x} \log 1 dx$$

$$\Rightarrow I = 0$$

Hence, the correct answer is C.

Answer needs Correction? Click Here

Q41: The value of
$$\int_0^{\frac{\pi}{2}} \log \left(\frac{4+3\sin x}{4+3\cos x} \right) dx$$
 is

A. 2

B. $\frac{3}{4}$

C. 0

D. -2

Answer:

Let
$$I = \int_0^{\frac{\pi}{2}} \log \left(\frac{4 + 3\sin x}{4 + 3\cos x} \right) dx$$
 ...(1)

$$\Rightarrow I = \int_0^{\pi} \log \left[\frac{4 + 3\sin\left(\frac{\pi}{2} - x\right)}{4 + 3\cos\left(\frac{\pi}{2} - x\right)} \right] dx \qquad \left(\int_0^{\infty} f(x) dx = \int_0^{\infty} f(a - x) dx \right)$$

$$\Rightarrow I = \int_0^{\pi} \log \left(\frac{4 + 3\cos x}{4 + 3\sin x} \right) dx \qquad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\pi} \left\{ \log\left(\frac{4+3\sin x}{4+3\cos x}\right) + \log\left(\frac{4+3\cos x}{4+3\sin x}\right) \right\} dx$$

$$\Rightarrow 2I = \int_0^{\pi} \log\left(\frac{4+3\sin x}{4+3\cos x}\right) + \frac{4+3\cos x}{4+3\sin x} dx$$

$$\Rightarrow 2I = \int_0^{\pi} \log 1 dx$$

$$\Rightarrow 2I = \int_0^{\pi} 0 dx$$

$$\Rightarrow I = 0$$

Hence, the correct answer is C.

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