Increasing and Decreasing Functions Ex 17.2 Q1(xxi)

$f(x) = x^4 - 4x^3 + 4x^2 + 15$

$\therefore \quad f'(x) = 4x^3 - 12x^2 + 8x$

Critical points

$$f'(x) = 0$$

$$\Rightarrow \quad 4x\left(x^2 - 3x + 2\right) = 0$$

$$\Rightarrow \quad 4x(x-2)(x-1) = 0$$

$$\Rightarrow \quad x = 0,\ 2,\ 1$$

Clealry, $f'(x) > 0$ if $0 < x < 1$ and $x > 2$

$f'(x) < 0$ if $x < 0$ and $1 < x < 2$

Thus, $f(x)$ increases in $(0,1) \cup (2,\infty)$, decreases in $(-\infty,0) \cup (1,2)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xxii)

We have,

$$f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}};\ x > 0$$

$$\therefore \quad f'(x) = \frac{15}{2}x^{\frac{1}{2}} - \frac{15}{2}x^{\frac{3}{2}}$$

Critical points

$$f'(x) = 0$$

$$\Rightarrow \quad \frac{15}{2}x^{\frac{1}{2}} - \frac{15}{2}x^{\frac{3}{2}} = 0$$

$$\Rightarrow \quad \frac{15}{2}x^{\frac{1}{2}}(1-x) = 0$$

$$\Rightarrow \quad x = 0,\ 1$$

Clearly, $f'(x) > 0$ if $0 < x < 1$

and $f'(x) < 0$ if $x > 1$

Thus, $f(x)$ increases in $(0,1)$, decreases in $(1,\infty)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xxiii)

We have,

$$f(x) = x^8 + 6x^2$$

$$\therefore \quad f'(x) = 8x^7 + 12x$$

Critical points

$$f'(x) = 0$$

$$\Rightarrow \quad 8x^7 + 12x = 0$$

$$\Rightarrow \quad 4x\left(2x^6 + 3\right) = 0$$

$$\Rightarrow \quad x = 0$$

Clearly, $f'(x) > 0$ if $x > 0$

$$f'(x) < 0 \text{ if } x < 0$$

Thus, $f(x)$ increases in $(0, \infty)$, decreases in $(-\infty, 0)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xxiv)

We have,

$$f(x) = x^3 - 6x^2 + 9x + 15$$

$$\therefore \quad f'(x) = 3x^2 - 12x + 9$$

Critical points

$$f'(x) = 0$$

$$\Rightarrow \quad 3\left(x^2 - 4x + 3\right) = 0$$

$$\Rightarrow \quad (x-3)(x-1) = 0$$

$$\Rightarrow \quad x = 3,\ 1$$

Clearly, $f'(x) > 0$ if $x < 1$ and $x > 3$

$$f'(x) < 0 \text{ if } 1 < x < 3$$

Thus, $f(x)$ increases in $(-\infty, 1) \cup (3, \infty)$, decreases in $(1, 3)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xxv)

We have,

$$y = \left[x(x-2)\right]^2 = \left[x^2 - 2x\right]^2$$

$$\therefore \frac{dy}{dx} = y' = 2\left(x^2 - 2x\right)(2x-2) = 4x(x-2)(x-1)$$

$$\therefore \frac{dy}{dx} = 0 \;\Rightarrow\; x = 0, x = 2, x = 1.$$

The points $x = 0$, $x = 1$, and $x = 2$ divide the real line into four disjoint intervals i.e., $(-\infty, 0)$, $(0,1)$ $(1,2)$, and $(2,\infty)$.

In intervals $(-\infty, 0)$ and $(1,2)$, $\frac{dy}{dx} < 0$.

$\therefore y$ is strictly decreasing in intervals $(-\infty, 0)$ and $(1,2)$.

However, in intervals $(0, 1)$ and $(2, \infty)$, $\frac{dy}{dx} > 0$.

$\therefore y$ is strictly increasing in intervals $(0, 1)$ and $(2, \infty)$.

$\therefore$ y is strictly increasing for $0 < x < 1$ and $x > 2$.

*********** END **********