Chapter 10 Differentiability Ex 10.1 Q7(i)

$$f(x)=\begin{cases} x^m \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0 & , x = 0 \end{cases}$$

$$(\text{LHD at } x=0) = \lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0}$$

$$= \lim_{h\to 0} \frac{f(0-h)-f(0)}{(0-h)-0}$$

$$= \lim_{h\to 0} \frac{(0-h)^m \sin\left(\frac{1}{-h}\right)-0}{-h}$$

$$= \lim_{h\to 0} \frac{(-h)^m \sin\left(-\frac{1}{h}\right)}{-h}$$

$$= \lim_{h\to 0} (-h)^{m-1} \sin\left(-\frac{1}{h}\right)$$

$$= \lim_{h\to 0} -(-h)^{m-1} \sin\left(\frac{1}{h}\right)$$

$$= 0 \times k \qquad [\text{When } -1 \le k \le 1]$$

$$= 0$$

$$(\text{RHD at } x=0) = \lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0}$$

$$= \lim_{h\to 0} \frac{f(0+h)-f(0)}{(0+h)-0}$$

$$= \lim_{h\to 0} \frac{h^m \sin\left(\frac{1}{h}\right)-0}{h}$$

$$= \lim_{h\to 0} \left(h^{m-1}\right) \sin\left(\frac{1}{h}\right)$$

$$= 0 \times k' \qquad [\text{Since } -1 \le k' \le 1]$$

$$= 0$$

$$(\text{LHD at } x=0) = (\text{RHD at } x=0)$$

$\therefore$ $f(x)$ is differentiable at $x = 0$

Chapter 10 Differentiability Ex 10.1 Q7(ii)

LHL $= \lim_{x \to 0^-} f(x)$

$= \lim_{h \to 0} f(0-h)$

$= \lim_{h \to 0} (-h)^m \sin\left(-\frac{1}{h}\right)$

$= -\lim_{h \to 0} (-h)^m \sin\left(\frac{1}{h}\right)$

$= 0 \times k$ $\quad$ [When $-1 \le k \le 1$]

$= 0$

RHL $= \lim_{x \to 0^+} f(x)$

$= \lim_{h \to 0} f(0+h)$

$= \lim_{h \to 0} (0+h)^m \sin \frac{1}{(0+h)}$

$= \lim_{h \to 0} h^m \sin\left(\frac{1}{h}\right)$

$= 0 \times k'$ $\quad$ [Where $-1 \le k' \le 1$]

$= 0$

LHL $= f(0) =$ RHL

$\therefore$ $f(x)$ is continuons at $x = 0$

For differentiablility at $x = 0$

(LHD at $x = 0$) $= \lim_{x \to 0^{-1}} \frac{f(x) - f(0)}{x - 0}$

$= \lim_{h \to 0} \frac{(0-h) - f(0)}{(0-h) - 0}$

$= \lim_{h \to 0} \frac{(-h)^m \sin\left(-\frac{1}{h}\right)}{-h}$

$= \lim_{h \to 0} - (-h)^{m-1} \sin\left(\frac{1}{h}\right)$

$=$ Not definded $\quad$ [Since $0 < m < 1$]

(RHD at $x = 0$) $= \lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0}$

$= \lim_{h \to 0} \frac{f(0+h) - f(0)}{0 + h - 0}$

$= \lim_{h \to 0} \frac{h^m \sin\left(\frac{1}{h}\right)}{h}$

Chapter 10 Differentiability Ex 10.1 Q7(iii)

$$\text{LHL} = \lim_{x\to 0^-} f(x)$$

$$= \lim_{h\to 0} f(0-h)$$

$$= \lim_{h\to 0} (-h)^m \sin\left(-\frac{1}{h}\right)$$

= Not defined as $m \le 0$

$$\text{RHL} = \lim_{x\to 0^+} f(x)$$

$$= \lim_{h\to 0} f(0+h)$$

$$= \lim_{h\to 0} h^m \sin\left(\frac{1}{h}\right)$$

= Not defined, as $m \le 0$

Since RHL and LHL are not difined, so $f(x)$ is not continuous

Let $x = 0$ for $m \le 0$.

Now,

$$(\text{LHD at } x = 0) = \lim_{x\to 0^{-1}} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{h\to 0} \frac{f(0-h) - 0}{0 - h - 0}$$

$$= \lim_{h\to 0} \frac{(-h)^m \sin\left(-\frac{1}{h}\right)}{-h}$$

$$= \lim_{h\to 0} -(-h)^{m-1} \sin\left(\frac{1}{h}\right)$$

= Not defined, as $m \le 0$

$$\text{RHD} = \lim_{x\to 0^+} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{h\to 0} \frac{f(0+h) - f(0)}{0 + h - 0}$$

$$= \lim_{h\to 0} \frac{h^m \sin\left(\frac{1}{h}\right)}{h}$$

$$= \lim_{h\to 0} \left(h^{m-1}\right) \sin\left(\frac{1}{h}\right)$$

= Not defined, as $m \le 0$

Thus,

$f(x)$ is neither continuous not differentiable at $x = 0$ for $m \le 0$.

*********** END **********