

Indefinite Integrals Ex 19.28 Q12

Let
$$x^2 = t$$

 $\Rightarrow 2xdx = dt$

$$I = \frac{1}{2} \int \sqrt{t^2 + 1^2} dt$$

$$= \frac{1}{2} \left\{ \frac{t}{2} \sqrt{t^2 + 1} + \frac{1}{2} \log \left| t + \sqrt{t^2 + 1} \right| \right\} + c$$

$$I = \frac{1}{2} \left\{ \frac{x^2}{2} \sqrt{x^4 + 1} + \frac{1}{2} \log |x^2 + \sqrt{x^4 + 1}| \right\} + c$$

Indefinite Integrals Ex 19.28 Q13

Let
$$I = \int x^2 \sqrt{a^6 - x^6} dx$$

Let
$$x^3 = t$$

$$\Rightarrow 3x^2 dx = dt$$

$$I = \frac{1}{3} \int \sqrt{a^6 - t^2} dt$$
$$= \frac{1}{3} \left\{ \frac{t}{2} \sqrt{a^6 - t^2} + \frac{a^6}{2} \sin^{-1} \left(\frac{t}{a^3} \right) \right\} + C$$

$$I = \frac{x^3}{6} \sqrt{a^6 - x^6} + \frac{a^6}{6} \sin^{-1} \left(\frac{x^3}{a^3} \right) + C$$

Indefinite Integrals Ex 19.28 Q14

Let
$$I = \int \frac{\sqrt{16 + (\log x)^2}}{x} dx$$

$$\Rightarrow \frac{1}{x}dx = dt$$

$$I = \int \sqrt{16 + t^2} dt$$

$$= \int \sqrt{4^2 + t^2} dt$$

$$= \frac{t}{2} \sqrt{16 + t^2} + \frac{16}{2} \log \left| t + \sqrt{16 + t^2} \right| + c$$

$$I = \frac{\log x}{2} \sqrt{16 + (\log x)^2} + 8\log \left| \log x + \sqrt{16 + (\log x)^2} \right| + c$$

Indefinite Integrals Ex 19.28 Q15

Let
$$I = \int \sqrt{2ax - x^2} dx$$

$$= \int \sqrt{a^2 - (a^2 - 2ax + x^2)} dx \qquad \left[\text{Adding and subtracting } a^2 \right]$$

$$= \int \sqrt{a^2 - (a - x)^2} dx$$

$$= \int \sqrt{a^2 - (x - a)^2} dx$$

$$= \frac{(x - a)}{2} \sqrt{2ax - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x - a}{a} \right) + c$$

$$\therefore I = \frac{1}{2} (x - a) \sqrt{2ax - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x - a}{a} \right) + c$$

Indefinite Integrals Ex 19.28 Q16

Let
$$I = \int \sqrt{3 - x^2} dx$$

$$= \int \sqrt{\left(\sqrt{3}\right)^2 - x^2} dx$$

$$I = \frac{x}{2} \sqrt{3 - x^2} + \frac{3}{2} \sin^{-1} \left(\frac{x}{\sqrt{3}}\right) + C$$

********* END *******