



Geometric Progressions Ex 20.3 Q16.

Let the G.P. be  $2n, 2, 2n+4, \dots$

$$\text{Then, } S_n = \frac{a(r^n - 1)}{r - 1}, \quad a = 2n, \quad r = 2$$

$$\therefore S_n = \frac{2n(2^n - 1)}{2 - 1} = 2n^{n+1} - 2n$$

Then the G.P. of odd term

$$a_1 + a_3 + a_5 + \dots + a_{2n-1}$$

According to the question

Sum of all terms = 5 (sum of terms occupying the odd places)

$$a_1 + a_2 + a_3 + \dots + a_{2n} = 5(a_1 + a_3 + a_5 + \dots + a_{2n-1})$$

$$a + ar + ar^2 + \dots + ar^{2n-1} = 5(a + ar^2 + ar^4 + \dots + ar^{2n-2})$$

$$\frac{a(1 - r^{2n})}{1 - r} = 5 \left( \frac{a(1 - (r^2)^n)}{1 - r^2} \right)$$

$$\frac{a}{1 - r} \text{ is cancelled on both side}$$

$$1 - r^{2n} = \frac{5(1 - r^{2n})}{1 + r}$$

$$1 + r - r^{2n} - r^{2n+1} = 5 - 5r^{2n}$$

$$r^{2n+1} - 4r^{2n} - r + 4 = 0$$

$$r^{2n}(r - 4) - 1(r - 4) = 0$$

$$r^{2n} = 1, \quad r = 4$$

$$\Rightarrow r = 4$$

Geometric Progressions Ex 20.3 Q17

$$\text{Given } \sum_{n=1}^{100} a_{2n} = \alpha$$

$$\Rightarrow a_2 + a_4 + a_6 + \dots + a_{200} = \alpha \quad \text{---(i)}$$

$$\text{also, } \sum_{n=1}^{100} a_{2n-1} = \beta$$

$$\Rightarrow a_1 + a_3 + a_5 + \dots + a_{199} = \beta \quad \text{---(ii)}$$

Sum of G.P.,

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$= a = a_2, r = r^2, n = 100$$

$$ar + ar^3 + ar^5 + \dots + ar^{199} = \alpha$$

$$ar \frac{(1 - (r^2)^{100})}{1 - r^2} = \alpha \quad \text{---(iii)}$$

$$a + ar^2 + ar^4 + \dots + ar^{198} = \beta$$

$$\frac{a(1 - (r^2)^{100})}{1 - r^2} = \beta \quad \text{---(iv)}$$

$$r(\beta) = \alpha$$

$$r = \frac{\alpha}{\beta} \quad \text{[From (iv) and (v)]}$$

Geometric Progressions Ex 20.3 18

Let the series be  $a_1 + a_2 + a_3 + \dots + a_{2n}$

It is given that  $a_1 = 1, a_2 = a, a_3 = ac, a_4 = a^2c, a_5 = a^2c^2, \dots$

$\therefore$  Sum of  $2n$  term

$$a_1 + a_2 + a_3 + \dots + a_{2n}$$

$$= 1 + a + ac + a^2c + a^2c^2 + \dots + 2n \text{ term}$$

$$= (1+a) + ac(1+a) + a^2c^2(1+a) + \dots + n \text{ term}$$

$$= (1+a) \frac{(1-(ac)^n)}{1-ac}$$

$$= (a+1) \frac{((ac)^n - 1)}{ac - 1}$$

Geometric Progressions Ex 20.3 Q19.

Sum of first  $n$  term of G.P.

$$= a + a_2 + a_3 + \dots + a_n$$

$$= a + ar + ar^2 + \dots + ar^{n-1}$$

$$[\because t_n = ar^{n-1}] \text{ --- (i)}$$

Also sum of term from

$$(n+1)^{\text{th}} \text{ to } (2n)^{\text{th}} \text{ term is}$$

$$= a_{n+1} + a_{n+2} + \dots + a_{2n}$$

$$= ar^n + ar^{n+1} + \dots + ar^{2n-1}$$

$$\text{--- (ii)}$$

Ratio of (i) and (ii) is

$$= \frac{a + ar + ar^2 + \dots + ar^{n-1}}{ar^n + ar^{n+1} + \dots + ar^{2n-1}}$$

$$\left[ \because S_n = \frac{a(1-r^n)}{1-r} \right]$$

$$= \frac{a(1-r^n)}{1-r}$$

$$= \frac{ar^n(1-r^n)}{1-r}$$

$$= \frac{1}{r^n}$$

Geometric Progressions Ex 20.3 Q20

Given,

$$a, b \text{ are roots of the equation } x^2 - 3x + p = 0$$

$$\Rightarrow a + b = 3, ab = p$$

$$\text{and } c, d \text{ are roots of the equation } x^2 - 12x + q = 0$$

$$\Rightarrow c + d = 12, cd = q$$

$$\text{Let } b = ar, c = ar^2 \text{ and } d = ar^3, \text{ then } a + b = 3 \text{ and } c + d = 12$$

$$a(1+r) = 3 \text{ and } ar^2(1+r) = 12$$

$$\Rightarrow \frac{ar^2(1+r)}{a(1+r)} = \frac{12}{3}$$

$$\Rightarrow r = 2$$

$$\text{and } a(r+1) = 3$$

$$\Rightarrow a = 1$$

$$p = ab$$

$$= a \times ar$$

$$p = 2$$

$$q = cd$$

$$= ar^2 \times ar^3$$

$$= 2^5$$

$$a = 32$$

$$\frac{q+p}{q-p} = \frac{32+2}{32-2}$$

$$= \frac{34}{30}$$

$$(q+p) : (q-p) = 17 : 15$$

Geometric Progressions Ex 20.3 Q21.

$$\begin{aligned} \text{Sum} &= \frac{3069}{512} = \frac{3(1 - \frac{1}{2^8})}{\frac{1}{2}} \\ 1 - \frac{1}{2^8} &= \frac{3069}{512 \times 6} = \frac{1023}{512 \times 2} \\ 1 - \frac{1023}{1024} &= \frac{1}{2^8} \\ \frac{1}{2^8} &= \frac{1}{1024} \\ n &= 10 \end{aligned}$$

Geometric Progressions Ex 20.3 Q22.

To find number of ancestors, we will find the sum of  $2, 2^2, 2^3, \dots$

$$\begin{aligned} \text{Number of ancestors} &= \frac{2(2^{10} - 1)}{2 - 1} \\ &= 2(1024 - 1) \\ &= 2 \times 1023 \\ &= 2046 \end{aligned}$$

\*\*\*\*\* END \*\*\*\*\*