Increasing and Decreasing Functions Ex 17.2 Q2

We have,

$$f(x) = x^2 - 6x + 9$$

$$\therefore \quad f'(x) = 2x - 6$$

Critical points

$$f'(x) = 0$$

$$\Rightarrow \quad 2(x-3) = 0$$

$$\Rightarrow \quad x = 3$$

Clearly, $f'(x) > 0$ if $x > 3$

$$f'(x) < 0 \text{ if } x < 3$$

Thus, $f(x)$ increases in $(3, \infty)$, decreases in $(-\infty, 3)$

IInd part

The given equation of curves

$$y = x^2 - 6x + 9 \qquad \text{---(i)}$$

$$y = x + 5 \qquad \text{---(ii)}$$

Slope of (i)

$$m_1 = \frac{dy}{dx} = 2x - 6$$

Slope of (ii)

$$m_2 = 1$$

Given that slope of normal to (i) is parallelt to (ii)

$$\therefore \quad \frac{-1}{2x-6} = 1$$

$$\Rightarrow \quad 2x - 6 = -1$$

$$\Rightarrow \quad x = \frac{5}{2}$$

From (i)

$$y = \frac{25}{4} - 15 + 9$$

$$= \frac{25}{4} - 6$$

$$= \frac{1}{4}$$

Thus, the required point is $\left(\frac{5}{2}, \frac{1}{4}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q3

We have,

$$f(x) = \sin x - \cos x, \quad 0 < x < 2\pi$$

$\therefore \quad f'(x) = \cos x + \sin x$

Critical points

$$f'(x) = 0$$

$\Rightarrow \quad \cos x + \sin x = 0$

$\Rightarrow \quad \tan x = -1$

$\Rightarrow \quad x = \frac{3\pi}{4}, \frac{7\pi}{4}$

Clearly, $f'(x) > 0$ if $0 < x < \frac{3\pi}{4}$ and $\frac{7\pi}{4} < x < 2\pi$

$$f'(x) < 0 \text{ if } \frac{3\pi}{4} < x < \frac{7\pi}{4}$$

Thus, $f(x)$ increases in $\left(0, \frac{3\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right)$, decreases in $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q4

We have,

$$f(x) = e^{2x}$$

$\therefore \quad f'(x) = 2e^{2x}$

We know that

$$f(x) \text{ is increasing if } f'(x) > 0$$

$\Rightarrow \quad 2e^{2x} > 0$

$\Rightarrow \quad e^{2x} > 0$

Since, the value of $e$ lies between 2 and 3

So, any power of $e$ will be greater than zero.

Thus, $f(x)$ is increasing on $R$.

Increasing and Decreasing Functions Ex 17.2 Q5

We have,

$$f(x) = e^{\frac{1}{x}}, \qquad x \neq 0$$

$$f'(x) = e^{\frac{1}{x}} \times \left(\frac{-1}{x^2}\right)$$

$$\therefore \qquad f'(x) = -\frac{e^{\frac{1}{x}}}{x^2}$$

Now,

$$x \in R,\ x \neq 0$$

$$\Rightarrow \qquad \frac{1}{x^2} > 0 \text{ and } e^{\frac{1}{x}} > 0$$

$$\Rightarrow \qquad \frac{e^{\frac{1}{x}}}{x^2} > 0$$

$$\Rightarrow \qquad -\frac{e^{\frac{1}{x}}}{x^2} < 0$$

$$\Rightarrow \qquad f'(x) < 0$$

Hence, $f(x)$ is a decreasing function for all $x \neq 0$.

*********** END **********