Maxima and Minima 18.5 Q1

Let x and y be the two numbers.

Given that $x + y = 16$ ---(i)

Let $s = x^2 + y^2$ ---(ii)

From (i) and (ii)

$$S = x^2 + (15 - x)^2$$

$$\therefore \quad \frac{ds}{dx} = 2x + 2(15 - x)(-1)$$

$$= 2x - 30 + 2x$$

$$= 4x - 30$$

Now, $\frac{ds}{dx} = 0$

$$\Rightarrow \quad 4x - 30 = 0$$

$$\Rightarrow \quad x = \frac{15}{2}$$

Since,

$$\frac{d^2s}{dx^2} = 4 > 0$$

$\therefore \quad x = \frac{15}{2}$ is the point of local minima.

So, from (i)

$$y = 15 - \frac{15}{2} = \frac{15}{2}$$

Hence, the required numbers are $\frac{15}{2}, \frac{15}{2}$.

Maxima and Minima 18.5 Q2

Let x and y be the two parts of 64.

$\therefore \quad x + y = 64 \quad$ ---(i)

Let $\quad S = x^3 + y^3 \quad$ ---(ii)

From (i) and (ii), we get

$$S = x^3 + (64 - x)^3$$

$$\therefore \quad \frac{dS}{dx} = 3x^2 + 3(64 - x)^2 \times (-1)$$

$$= 3x^2 - 3\left(4096 - 128x + x^2\right)$$

$$= -3(4096 - 128x)$$

For maxima and minima,

$$\frac{dS}{dx} = 0$$

$$\Rightarrow \quad -3(4096 - 128x) = 0$$

$$\Rightarrow \quad x = 32$$

Now,

$$\frac{d^2s}{dx^2} = 384 > 0$$

$\therefore \quad x = 32$ is the point of local minima.

Thus, the two parts of 64 are $(32, 32)$.

Maxima and Minima 18.5 Q3

Let x and y be the two numbers, such that, $x, y \geq -2$ and

$$x + y = \frac{1}{2} \qquad \text{---(i)}$$

Let $S = x + y^3$ ---(ii)

From (i) and (ii), weget

$$S = x + \left(\frac{1}{2} - x\right)^3$$

$$\therefore \quad \frac{dS}{dx} = 1 + 3\left(\frac{1}{2} - x\right)^2 \times (-1)$$

$$= 1 - 3\left(\frac{1}{4} - x + x^2\right)$$

$$= \frac{1}{4} + 3x - 3x^2$$

For maximum and minimum,

$$\frac{dS}{dx} = 0$$

$$\Rightarrow \quad \frac{1}{4} + 3x - 3x^2 = 0$$

$$\Rightarrow \quad 1 + 12x - 12x^2 = 0$$

$$\Rightarrow \quad 12x^2 - 12x - 1 = 0$$

$$\Rightarrow \quad x = \frac{12 \pm \sqrt{144 + 48}}{24}$$

$$\Rightarrow \quad x = \frac{1}{2} \pm \frac{8\sqrt{3}}{24}$$

$$\Rightarrow \quad x = \frac{1}{2} \pm \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad x = \frac{1}{2} - \frac{1}{\sqrt{3}}, \frac{1}{2} + \frac{1}{\sqrt{3}}$$

Now,

$$\frac{d^2S}{dx^2} = 3 - 6x$$

At $x = \frac{1}{2} - \frac{1}{\sqrt{3}}$, $\frac{d^2S}{dx^2} = 3\left(1 - 2\left(\frac{1}{2} - \frac{1}{\sqrt{3}}\right)\right)$

$$= 3\left(+\frac{2}{\sqrt{3}}\right) = 2\sqrt{3} > 0$$

$\therefore \quad x = \frac{1}{2} - \frac{1}{\sqrt{3}}$ is point of local minima

$\therefore$ from (i)

$$y = \frac{1}{2} - \left(\frac{1}{2} - \frac{1}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}}$$

Hence, the required numbers are $\frac{1}{2} - \frac{1}{\sqrt{3}}$, $\frac{1}{\sqrt{3}}$.

Maxima and Minima 18.5 Q4

Let x and y be the two parts of 15, such that

$\therefore$ $x + y = 15$ ---(i)

Also, $S = x^2 y^3$ ---(ii)

From (i) and (ii), weget

$$S = x^2 (15 - x)^3$$

$$\therefore \quad \frac{dS}{dx} = 2x(15 - x)^3 - 3x^2(15 - x)^2$$

$$= (15 - x)^2 \left[30x - 2x^2 - 3x^2\right]$$

$$= 5x(15 - x)^2 (6 - x)$$

For maxima and minima,

$$\frac{dS}{dx} = 0$$

$$\Rightarrow \quad 5x(15 - x)^2(6 - x) = 0$$

$$\Rightarrow \quad x = 0,\ 15,\ 6$$

Now,

$$\frac{d^2S}{dx^2} = 5(15 - x)^2(6 - x) - 5x \times 2(15 - x)(6 - x) - 5x(15 - x)^2$$

$\therefore$ At $x = 0$, $\frac{dS^2}{dx^2} = 1125 > 0$

$\therefore$ $x = 0$ is point of local minima

At $x = 15$, $\frac{d^2s}{dx^2} = 0$

$\therefore$ $x = 15$ is an inflection point.

At $x = 6$, $\frac{ds^2}{dx^2} = -2430 < 0$

$\therefore$ $x = 6$ is the point of local maxima

Thus the numbers are 6 and 9.

*********** END **********