Exercise 6.2 : Solutions of Questions on Page Number : **205**

Q1 : Show that the function given by $f(x) = 3x + 17$ is strictly increasing on **R**.

Answer :

Let x_1 and x_2 be any two numbers in **R**.

Then, we have:

$x_1 < x_2 \Rightarrow 3x_1 < 3x_2 \Rightarrow 3x_1 + 17 < 3x_2 + 17 \Rightarrow f(x_1) < f(x_2)$

Hence, f is strictly increasing on **R**.

Answer needs Correction? **Click Here**

Q2 : Show that the function given by $f(x) = e^{2x}$ is strictly increasing on **R**.

Answer :

Let x_1 and x_2 be any two numbers in **R**.

Then, we have:

$x_1 < x_2 \Rightarrow 2x_1 < 2x_2 \Rightarrow e^{2x_1} < e^{2x_2} \Rightarrow f(x_1) < f(x_2)$

Hence, f is strictly increasing on **R**.

Answer needs Correction? **Click Here**

Q3 : Show that the function given by $f(x) = \sin x$ is

(a) strictly increasing in $\left(0, \frac{\pi}{2}\right)$ (b) strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$

(c) neither increasing nor decreasing in $(0, \pi)$

Answer :

The given function is $f(x) = \sin x$.

$\therefore f'(x) = \cos x$

(a) Since for each $x \in \left(0, \frac{\pi}{2}\right), \cos x > 0$, we have $f'(x) > 0$.

Hence, f is strictly increasing in $\left(0, \frac{\pi}{2}\right)$.

(b) Since for each $x \in \left(\frac{\pi}{2}, \pi\right), \cos x < 0$, we have $f'(x) < 0$.

Hence, f is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

(c) From the results obtained in (a) and (b), it is clear that f is neither increasing nor decreasing in $(0, \pi)$.

Answer needs Correction? **Click Here**

Q4 : Find the intervals in which the function f given by $f(x) = 2x^2 - 3x$ is

(a) strictly increasing (b) strictly decreasing

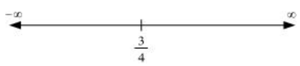
Answer :

The given function is $f(x) = 2x^2 - 3x$.

$f'(x) = 4x - 3$

$\therefore f'(x) = 0 \Rightarrow x = \frac{3}{4}$

Now, the point $\frac{3}{4}$ divides the real line into two disjoint intervals i.e., $\left(-\infty, \frac{3}{4}\right)$ and $\left(\frac{3}{4}, \infty\right)$.



In interval $\left(-\infty, \frac{3}{4}\right), f'(x) = 4x - 3 < 0.$

Hence, the given function (f) is strictly decreasing in interval $\left(-\infty, \frac{3}{4}\right)$.

In interval $\left(\frac{3}{4},\infty\right), f'(x)=4x-3>0.$

Hence, the given function (f) is strictly increasing in interval $\left(\frac{3}{4},\infty\right)$.

Answer needs Correction? **Click Here**

Q5 : Find the intervals in which the function f given by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is

(a) strictly increasing (b) strictly decreasing

Answer :

The given function is $f(x) = 2x^3 - 3x^2 - 36x + 7$.

$f'(x)=6x^2-6x-36=6(x^2-x-6)=6(x+2)(x-3)$

$\therefore f'(x)=0 \Rightarrow x = -2, 3$

The points $x = -2$ and $x = 3$ divide the real line into three disjoint intervals i.e.,

$(-\infty,-2),(-2,3)$, and $(3,\infty)$.

In intervals $(-\infty,-2)$ and $(3,\infty)$, $f'(x)$ is positive while in interval

$(-2, 3)$, $f'(x)$ is negative.

Hence, the given function (f) is strictly increasing in intervals

$(-\infty,-2)$ and $(3,\infty)$, while function (f) is strictly decreasing in interval

$(-2, 3)$.

Answer needs Correction? **Click Here**

Q6 : Find the intervals in which the following functions are strictly increasing or decreasing:

(a) $x^2 + 2x - 5$ (b) $10 - 6x - 2x^2$

(c) $-2x^3 - 9x^2 - 12x + 1$ (d) $6 - 9x - x^2$

(e) $(x + 1)^3 (x - 3)^3$

Answer :

(a) We have,

$f(x)=x^2+2x-5$

$\therefore f'(x)=2x+2$

Now,

$f'(x)=0 \Rightarrow x = -1$

Point $x = -1$ divides the real line into two disjoint intervals i.e., $(-\infty,-1)$ and $(-1,\infty)$.

In interval $(-\infty,-1), f'(x)=2x+2<0$.

$\therefore f$ is strictly decreasing in interval $(-\infty,-1)$.

Thus, f is strictly decreasing for $x < -1$.

In interval $(-1,\infty), f'(x)=2x+2>0$.

$\therefore f$ is strictly increasing in interval $(-1,\infty)$.

Thus, f is strictly increasing for $x > -1$.

(b) We have,

$f(x) = 10 - 6x - 2x^2$

$\therefore f'(x)=-6-4x$

Now,

$f'(x)=0 \Rightarrow x=-\frac{3}{2}$

The point $x=-\frac{3}{2}$ divides the real line into two disjoint intervals i.e., $\left(-\infty,-\frac{3}{2}\right)$ and $\left(-\frac{3}{2},\infty\right)$.

In interval $\left(-\infty,-\frac{3}{2}\right)$ i.e., when $x<-\frac{3}{2}$,

Answer needs Correction? **Click Here**

Q7 : Show that $y=\log(1+x)-\frac{2x}{2+x}, x>-1$, is an increasing function of x throughout its domain.

Answer :

We have,

$y=\log(1+x)-\frac{2x}{2+x}$

$\therefore$

Answer needs Correction? **Click Here**

Q8 : Find the values of x for which $y=[x(x-2)]^2$ is an increasing function.

Answer :

We have,

$y=[x(x-2)]^2=[x^2-2x]^2$

$\therefore \frac{dy}{dx}=y'=2(x^2-2x)(2x-2)=4x(x-2)(x-1)$

$\therefore \frac{dy}{dx}=0 \Rightarrow x=0, x=2, x=1.$

The points $x = 0$, $x = 1$, and $x = 2$ divide the real line into four disjoint intervals i.e., $(-\infty,0)$, $(0,1)$ $(1,2)$, and $(2,\infty)$.

In intervals $(-\infty,0)$ and $(1,2)$, $\frac{dy}{dx}<0$.

$\therefore$ y is strictly decreasing in intervals $(-\infty,0)$ and $(1,2)$.

However, in intervals (0, 1) and (2, ∞), $\frac{dy}{dx}>0$.

$\therefore$ y is strictly increasing in intervals (0, 1) and (2, ∞).

$\therefore$ y is strictly increasing for $0 < x < 1$ and $x > 2$.

Answer needs Correction? Click Here

Q9 : Prove that $y=\frac{4\sin\theta}{(2+\cos\theta)}-\theta$ is an increasing function of θ in $\left[0,\frac{\pi}{2}\right]$.

Answer :

We have,

$$y=\frac{4\sin\theta}{(2+\cos\theta)}-\theta$$

$$\therefore \frac{dy}{dx}=\frac{(2+\cos\theta)(4\cos\theta)-4\sin\theta(-\sin\theta)}{(2+\cos\theta)^2}-1$$

$$=\frac{8\cos\theta+4\cos^2\theta+4\sin^2\theta}{(2+\cos\theta)^2}-1$$

$$=\frac{8\cos\theta+4}{(2+\cos\theta)^2}-1$$

Now, $\frac{dy}{dx}=0.$

$$\Rightarrow \frac{8\cos\theta+4}{(2+\cos\theta)^2}=1$$

$$\Rightarrow 8\cos\theta+4=4+\cos^2\theta+4\cos\theta$$

$$\Rightarrow \cos^2\theta-4\cos\theta=0$$

$$\Rightarrow \cos\theta(\cos\theta-4)=0$$

$$\Rightarrow \cos\theta=0 \text{ or } \cos\theta=4$$

Since cos θ ≠ 4, cos θ = 0.

$$\cos\theta=0 \Rightarrow \theta=\frac{\pi}{2}$$

Now,

$$\frac{dy}{dx}=\frac{8\cos\theta+4-(4+\cos^2\theta+4\cos\theta)}{(2+\cos\theta)^2}=\frac{4\cos\theta-\cos^2\theta}{(2+\cos\theta)^2}=\frac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2}$$

In interval $\left(0, \frac{\pi}{2}\right)$, we have cos θ > 0. Also, 4 > cos θ ⇒ 4 - cos θ > 0.

$$\therefore \cos\theta(4-\cos\theta)>0 \text{ and also } (2+\cos\theta)^2>0$$

$$\Rightarrow \frac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2}>0$$

$$\Rightarrow \frac{dy}{dx}>0$$

Therefore, y is strictly increasing in interval $\left(0, \frac{\pi}{2}\right)$.

Also, the given function is continuous at $x=0$ and $x=\frac{\pi}{2}$.

Hence, y is increasing in interval $\left[0,\frac{\pi}{2}\right]$.

Answer needs Correction? Click Here

Q10 : Prove that the logarithmic function is strictly increasing on (0, ∠Â¾).

Answer :

The given function is $f(x)=\log x$.

$$\therefore f'(x)=\frac{1}{x}$$

It is clear that for x > 0, $f'(x)=\frac{1}{x}>0$.

Hence, $f(x)$ = log x is strictly increasing in interval (0, ∞).

Answer needs Correction? Click Here

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