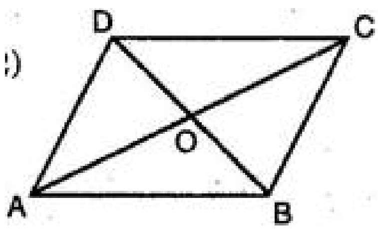
Exercise 6.6

(i) In Δ ADC, $\angle$ ADC is an obtuse angle.

$$\therefore AC^2 = AD^2 + DC^2 + 2DC.DM$$

$$\Rightarrow AC^2 = AD^2 + \left(\frac{BC}{2}\right)^2 + 2.\frac{BC}{2}.DM$$

$$\Rightarrow AC^2 = AD^2 + \left(\frac{BC}{2}\right)^2 + BC.DM$$



$$\Rightarrow AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2 \quad ....(i)$$

(ii) In Δ ABD, $\angle$ ADM is an acute angle.

$$AB^2 = AD^2 + BD^2 - 2BD.DM$$

$$\Rightarrow AB^2 = AD^2 + \left(\frac{BC}{2}\right)^2 - 2.\frac{BC}{2}.DM$$

$$\Rightarrow AB^2 = AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2 \quad ....(ii)$$

(iii) From eq. (i) and eq. (ii),

$$AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2$$

6. Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

Ans. If AD is a median of ΔABC, then

$$AB^2 + AC^2 = 2AD^2 + \frac{1}{2}BC^2 \text{ [See Q.5 (iii)]}$$

Since the diagonals of a parallelogram bisect each other, therefore, BO and DO are medians of triangles ABC and ADC respectively.

$\therefore AB^2 + BC^2 = 2BO^2 + \frac{1}{2}AC^2$ ....(i)

And $AD^2 + CD^2 = 2DO^2 + \frac{1}{2}AC^2$ ....(ii)

Adding eq. (i) and (ii),

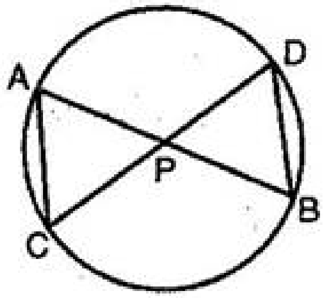
$AB^2 + BC^2 + AD^2 + CD^2 = 2(BO^2 + DO^2) + AC^2$

$\Rightarrow AB^2 + BC^2 + AD^2 + CD^2 = 2\left(\frac{1}{4}BD^2 + \frac{1}{4}BD^2\right)$

$+ AC^2 \left[DO = \frac{1}{2}BD\right]$

$\Rightarrow AB^2 + BC^2 + AD^2 + CD^2 = AC^2 + BD^2$

7. In figure, two chords AB and CD intersect each other at the point P. Prove that:



(i) $\Delta APC \sim \Delta DPB$

(ii) AP.PB = CP.DP

Ans. (i) In the triangles APC and DPB,

$\angle APC = \angle DPB$ [Vertically opposite angles]

$\angle CAP = \angle BDP$ [Angles in same segment of a circle are equal]

$\therefore$ By AA-criterion of similarity,

$\Delta APC \sim \Delta DPB$

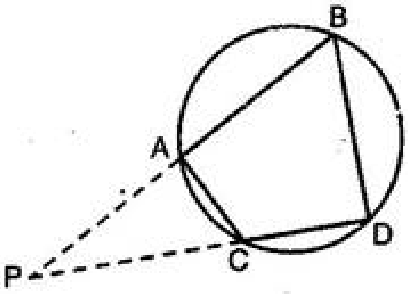
(ii) Since $\Delta APC \sim \Delta DPB$

$\therefore \frac{AP}{DP} = \frac{CP}{PB} \Rightarrow AP \times PB = CP \times DP$

8. In figure, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that:

(i) ΔPAC ~ Δ PDB

(ii) PA.PB = PC.PD



Ans. (i) In the triangles PAC and PDB,

$\angle$ APC = $\angle$ DPB [Common]

$\angle$ CAP = $\angle$ BDP [$\because$ $\angle$ BAC = $180° - \angle$ PAC and $\angle$ PDB = $\angle$ CDB]

$= 180° - \angle$ BAC $= 180° - (180° - \angle PAC)$ $= \angle$ PAC]

$\therefore$ By AA-criterion of similarity,

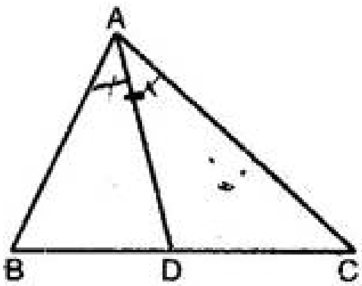
Δ APC ~ Δ DPB

(ii) Since Δ APC ~ Δ DPB

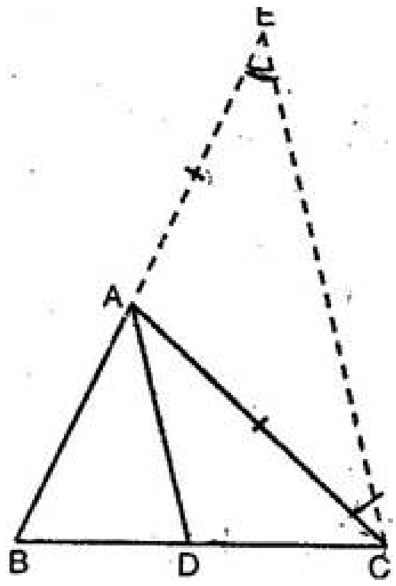
$\therefore \dfrac{AP}{DP} = \dfrac{CP}{PB}$

$\Rightarrow$ PA.PB = PC.PD

9. In figure, D is appointing on side BC of ΔABC such that $\dfrac{BD}{CD} = \dfrac{AB}{AC}$. Prove that AD is the bisector of $\angle$ BAC.



Ans. Given: ABC is a triangle and D is a point on BC such that $\dfrac{BD}{CD} = \dfrac{AB}{AC}$



To prove: AD is the internal bisector of $\angle$ BAC.

Construction: Produce BA to E such that AE = AC. Join CE.

Proof: In ΔAEC, since AE = AC

$\therefore \angle$ AEC = $\angle$ ACE ....(i)

[Angles opposite to equal side of a triangle are equal]

Now, $\frac{BD}{CD} = \frac{AB}{AC}$ [Given]

$\Rightarrow \frac{BD}{CD} = \frac{AB}{AE}$ [$\because$ AE = AC, by construction]

$\therefore$ By converse of Basic Proportionality Theorem,

DA $\parallel$ CE

Now, since CA is a transversal,

$\therefore \angle$ BAD = $\angle$ AEC ....(ii) [Corresponding $\angle$ s]

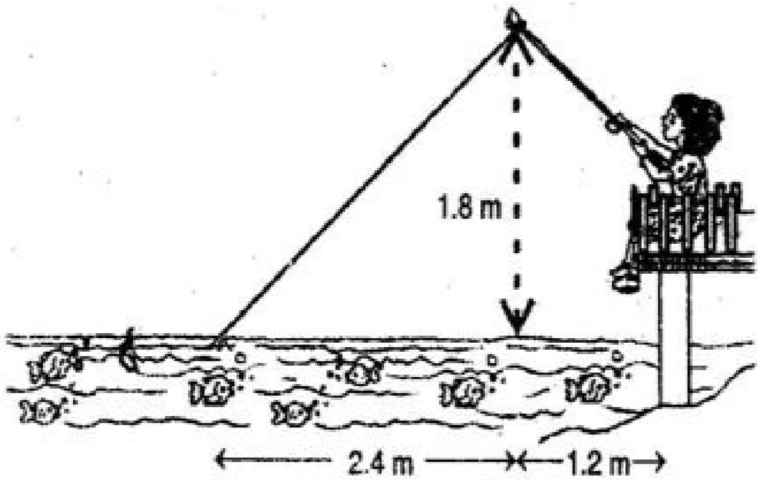
And $\angle$ DAC = $\angle$ ACE ....(iii) [Alternate $\angle$ s]

Also $\angle$ AEC = $\angle$ ACE [From eq. (i)]

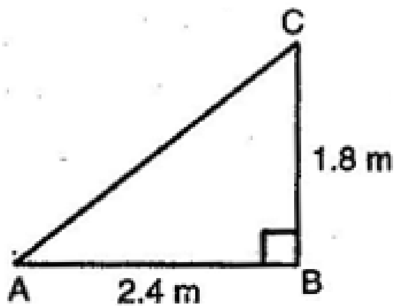
Hence, $\angle$ BAD = $\angle$ DAC [From eq. (ii) and (iii)]

Thus, AD bisects $\angle$ BAC internally.

10. Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taur, how much string does she have out (see figure)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?



Ans. I. To find: The length of AC.



By Pythagoras theorem,

$AC^2 = (2.4)^2 + (1.8)^2$

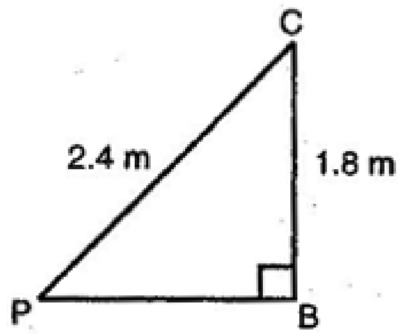
$\Rightarrow AC^2 = 5.76 + 3.24 = 9.00$

$\Rightarrow AC = 3$ m

$\therefore$ Length of string she has out= 3 m

Length of the string pulled at the rate of 5 cm/sec in 12 seconds

Length of the string pulled at the rate of 5 cm/sec in 12 seconds



$= (5 \times 12)$ cm = 60 cm = 0.60 m

$\therefore$ Remaining string left out = 3 – 0.6 = 2.4 m

II. To find: The length of PB

$PB^2 = PC^2 - BC^2$

$= (2.4)^2 - (1.8)^2$

$= 5.76 - 3.24 = 2.52$

$\Rightarrow PB = \sqrt{2.52} = 1.59$ (approx.)

Hence, the horizontal distance of the fly from Nazima after 12 seconds

= 1.59 + 1.2 = 2.79 m (approx.)

*********** END **********