Trigonometric Identities Ex 6.1 Q48

Answer :

In the given question, we need to prove $\frac{1}{\sec A+\tan A}-\frac{1}{\cos A}=\frac{1}{\cos A}-\frac{1}{\sec A-\tan A}$.

Here, we will first solve the L.H.S.

Now, using $\sec\theta=\frac{1}{\cos\theta}$ and $\tan\theta=\frac{\sin\theta}{\cos\theta}$, we get

$$\begin{aligned}\frac{1}{\sec A+\tan A}-\frac{1}{\cos A}&=\frac{1}{\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)}-\left(\frac{1}{\cos A}\right)\\&=\frac{1}{\left(\frac{1+\sin A}{\cos A}\right)}-\left(\frac{1}{\cos A}\right)\\&=\left(\frac{\cos A}{1+\sin A}\right)-\left(\frac{1}{\cos A}\right)\\&=\frac{\cos^2 A-(1+\sin A)}{(1+\sin A)(\cos A)}\end{aligned}$$

On further solving, we get

$$\begin{aligned}\frac{\cos^2 A-(1+\sin A)}{(1+\sin A)(\cos A)}&=\frac{\cos^2 A-1-\sin A}{(1+\sin A)(\cos A)}\\&=\frac{-\sin^2 A-\sin A}{(1+\sin A)(\cos A)}\qquad\left(\text{Using }\sin^2\theta=1-\cos^2\theta\right)\\&=\frac{-\sin A(\sin A+1)}{(1+\sin A)(\cos A)}\\&=\frac{-\sin A}{\cos A}\\&=-\tan A\end{aligned}$$

Similarly we solve the R.H.S.

Now, using $\sec\theta=\frac{1}{\cos\theta}$ and $\tan\theta=\frac{\sin\theta}{\cos\theta}$, we get

$$\begin{aligned}\frac{1}{\cos A}-\frac{1}{\sec A-\tan A}&=\left(\frac{1}{\cos A}\right)-\frac{1}{\left(\frac{1}{\cos A}-\frac{\sin A}{\cos A}\right)}\\&=\left(\frac{1}{\cos A}\right)-\frac{1}{\left(\frac{1-\sin A}{\cos A}\right)}\\&=\left(\frac{1}{\cos A}\right)-\left(\frac{\cos A}{1-\sin A}\right)\\&=\frac{(1-\sin A)-\cos^2 A}{(\cos A)(1-\sin A)}\end{aligned}$$

On further solving, we get

$$\frac{(1-\sin A)-\cos^2 A}{(\cos A)(1-\sin A)} = \frac{1-\sin A-\cos^2 A}{(\cos A)(1-\sin A)}$$

$$= \frac{\sin^2 A-\sin A}{(\cos A)(1-\sin A)} \qquad \left(\text{Using } \sin^2\theta = 1-\cos^2\theta\right)$$

$$= \frac{-\sin A(1-\sin A)}{(\cos A)(1-\sin A)}$$

$$= \frac{-\sin A}{\cos A}$$

$$= -\tan A$$

So, L.H.S = R.H.S

Hence proved.

*********** END **********