Question-34

Solve the following pairs of equations by reducing them to a pair of linear equation:

(i) $\frac{10}{x+y} + \frac{2}{x-y} = 4$

$\frac{15}{x+y} - \frac{5}{x-y} = -2$

(ii) $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$

$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = -\frac{1}{8}$

Solution:

(i) $\frac{10}{x+y} + \frac{2}{x-y} = 4$

$\frac{15}{x+y} - \frac{5}{x-y} = -2$

Let $\frac{1}{x+y} = a$, $\frac{1}{x-y} = b$

10a + 2b = 4 ..... (1)

15a - 5b = -2 .....(2)

Multiply (1) with 3 and (2) with 2, we get

30a + 6b = 12

30a - 10b = -4

16b = 16

b = 1

Substituting b = 1 in (1) we get,

10a + 2(1) = 4

10a = 4 - 2

10a = 2

$a = \frac{2}{10} = \frac{1}{5}$

as $\frac{1}{x+y} = a = \frac{1}{5}$

x + y = 5 ..... (3)

and $\frac{1}{x-y} = b$

$\frac{1}{x-y} = 1$

x – y = 1 ..... (4)

Solving (3) and (4)

x + y = 5

x – y = 1

2x = 6

x = 3

when x = 3

3 + y = 5

y = 5 – 3 = 2

Thus x = 3, y = 2.

(ii) $\frac{1}{3x + y} + \frac{1}{3x - y} = \frac{3}{4}$

$\frac{1}{2(3x + y)} - \frac{1}{2(3x - y)} = -\frac{1}{8}$

Let $\frac{1}{3x + y} = a$ and $\frac{1}{3x - y} = b$

$a + b = \frac{3}{4}$

$\Rightarrow 4a + 4b = 3$ ..... (1)

$\frac{a}{2} - \frac{b}{2} = \frac{-1}{8}$

$\frac{a - b}{2} = \frac{-1}{8}$

$a - b = \frac{-2}{8} = \frac{-1}{4}$

4a + 4b = 3

4a - 4b = -1

8a = 2

$a = \frac{2}{8}$

$a = \frac{1}{4}$

substituting in (1)

$4(\frac{1}{4}) + 4b = 3$

4b = 2

$b = \frac{2}{4} = \frac{1}{2}$

$\frac{1}{3x + y} = \frac{1}{4}$

3x + y = 4 ..... (3)

$\frac{1}{3x - y} = \frac{1}{2}$

3x – y = 2 ..... (4)

(3) + (4)⇒

3x + y = 4

3x - y = 2

6x = 6

x = 1

substituting x = 1 in (3) we get

3(1) + y = 4

y = 1

Thus x = 1, y = 1.

Question-35

Formulate the following problems as a pair of equations, and hence find their solutions:

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.
(ii) 2 woman and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

Solution:

(i) Let the speed of rowing in still water be 'x' and the speed of the current be y km/hr.

∴ Speed upstream = (x – y) km/hr

∴ Speed downstream = (x + y) km/hr

case 1

Where Ritu rows 20 km downstream in 2 hours, the equation is

$\frac{20}{x+y} = 2$

$2(x + y) = 20$

$x + y = 10$ ……..(1)

case 2

When Ritu rows 4 km downstream in 2 hours, the equation is

$\frac{4}{x-y} = 2$

$2(x – y) = 4$

$x – y = \frac{4}{2} = 2$

$x – y = 2$ …… (2)

solving (1) and (2)

$2x = 12$

$x = 6$

Substitute x = 6 in (1)

$x + y = 10$

$6 + y = 10$

$y = 10 – 6$

$= 4$

Therefore speed of the rowing in still water = 6 km/hr

And speed of the current = 4 km/hr.

(ii) 2 woman and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

Let a woman take x days to complete the work.

One women's one day work = $\left(\frac{1}{x}\right)$

Similarly let one man take 'y' days to complete the work.

Therefore one man's one day work = $\frac{1}{y}$

$\therefore \frac{2}{x} + \frac{5}{y} = \frac{1}{4}$

Let $\frac{1}{x} = a$, $\frac{1}{y} = b$

$2a + 5b = \frac{1}{4}$

$8a + 20b = 1$ ..... (1)

$\frac{3}{x} + \frac{6}{y} = \frac{1}{3}$

Let $\frac{1}{x} = a$, $\frac{1}{y} = b$

$3a + 6b = \frac{1}{3}$

$9a + 18b = 1$ ..... (2)

$((1) \times 9) \Rightarrow 72a + 180\ b = 9$

$((2) \times 8) \Rightarrow \underline{72a + 144\ b = 8}$

$36b = 1$

$b = \frac{1}{36}$, Substitute $b = \frac{1}{36}$ in (1)

$3a + 6\left(\frac{1}{36}\right) = \frac{1}{3}$

$3a = \frac{1}{3} - \frac{1}{6}$

$3a = \frac{2-1}{6}$

$3a = \frac{1}{6}$

$a = \frac{1}{18}$

when $a = \frac{1}{18}$

$\frac{1}{x} = \frac{1}{18}$ or $x = 18$

when $b = \frac{1}{36}$

$\frac{1}{y} = \frac{1}{36}$

$y = 36$

$\therefore$ Time taken by 1 woman to do the work = 18 days.

$\therefore$ Time taken by 1 man to do the work = 36 days.

Question-36

Formulate the following problem as a pair of equations, and hence find it's solution

Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Solution:

Let the spread the train be 'x' km/hr

Let the spread the bus be 'y' km/hr

Case (i)

Distance traveled by train = 60 km

Distance traveled by bus = 300 - 60 = 240 km

Time taken by Roohi in train = $\frac{60}{x}$ (Time taken = $\frac{\text{Dis tance}}{\text{Speed}}$)

Time taken by Roohi in bus = $\frac{240}{y}$

Total time = 4 hrs

$= \frac{60}{x} + \frac{240}{y} = 4$ ..... (1)

case (ii)

Distance traveled by train = 100 km

Distance traveled by bus = 300 – 100 = 200 km

Time taken by Roohi in Train = $\frac{100}{x}$ (Time taken = $\frac{\text{Dis tance}}{\text{Speed}}$)

Time taken by Roohi in Bus = $\frac{200}{y}$

Total time is 10 more minutes

Longer than time taken in case (i) $\therefore 4 + \frac{10}{60} = 4 + \frac{1}{6} = \frac{25}{6}$

$\therefore \frac{100}{x} + \frac{200}{y} = \frac{25}{6}$ ..... (2)

Solving (1) and (2)

$\frac{60}{x} + \frac{240}{y} = 4$ ---------- (1)

$\frac{100}{x} + \frac{200}{y} = \frac{25}{6}$ ---------- (2)

Let $\frac{1}{x} = a$ and $\frac{1}{y} = b$

Rewrite (1) and (2) as follows

60 a + 240 b = 4

$100\ a + 200\ b = \frac{25}{6}$

Multiply (1) with 5 , and (2) with 6

300a + 1200b = 20 ..... (3)

600a + 1200b = 25 .....(4)

(3) –(4) =>-300a = -5

$a = \frac{-5}{-300} = \frac{1}{60}$ in (1)

Substituting $a = \frac{1}{60}$ in (1)

$60 \times \frac{1}{60} + 240b = 4$

240 b = 4 – 1

240 b = 3

$\Rightarrow b = \frac{1}{80}$

when $a = \frac{1}{60}$

(or) $\frac{1}{x} = \frac{1}{60} \Rightarrow x = 60$ km/hr

when $b = \frac{1}{80}$

(or) $\frac{1}{y} = \frac{1}{80} \Rightarrow y = 80$ km/hr

$\therefore$ speed of train = 60 km/hr

and speed of bus = 80 km/hr

********** END **********