Algebraic Identities Ex 4.3 Q2

Answer :

In the given problem, we have to simplify equation

(i) Given $(x+3)^3+(x-3)^3$

We shall use the identity $a^3+b^3=(a+b)(a^2+b^2-ab)$

Here $a=(x+3), b=(x-3)$

By applying identity we get

$$=(x+\cancel{3}+x-\cancel{3})\left[(x+3)^2+(x-3)^2-(x+3)(x-3)\right]$$

$$=2x\left[(x^2+3^2+2\times x\times 3)+(x^2+3^2-2\times x\times 3)-(x^2-3^2)\right]$$

$$=2x\left[(x^2+9+6x)+(x^2+9-6x)-(x^2-3^2)\right]$$

$$=2x\left[x^2+9+6x+x^2+9-6x-x^2+9\right]$$

$$=2x\left[x^2+\cancel{x^2}-\cancel{x^2}-\cancel{6x}+\cancel{6x}+9+9+9\right]$$

$$=2x\left[x^2+27\right]$$

$$=2x^3+54x$$

Hence simplified form of expression $(x+3)^3+(x-3)^3$ is $\boxed{2x^3+54x}$.

(ii) Given $\left(\frac{x}{2}+\frac{y}{3}\right)^3-\left(\frac{x}{2}-\frac{y}{3}\right)^3$

We shall use the identity $a^3-b^3=(a-b)(a^2+b^2+ab)$

Here $a=\left(\frac{x}{2}+\frac{y}{3}\right), b=\left(\frac{x}{2}-\frac{y}{3}\right)$

By applying identity we get

$$=\left(\left(\frac{x}{2}+\frac{y}{3}\right)-\left(\frac{x}{2}-\frac{y}{3}\right)\right)\left[\left(\frac{x}{2}+\frac{y}{3}\right)^2+\left(\frac{x}{2}-\frac{y}{3}\right)^2-\left(\frac{x}{2}+\frac{y}{3}\right)\left(\frac{x}{2}-\frac{y}{3}\right)\right]$$

$$=\left(\cancel{\frac{x}{2}}+\frac{y}{3}-\cancel{\frac{x}{2}}+\frac{y}{3}\right)\left[\left(\left(\frac{x}{2}\right)^2+\left(\frac{y}{3}\right)^2+\frac{2xy}{6}\right)^2+\left(\left(\frac{x}{2}\right)^2+\left(\frac{y}{3}\right)^2-\frac{2xy}{6}\right)^2+\left(\left(\frac{x}{2}\right)^2-\left(\frac{y}{3}\right)^2\right)\right]$$

$$=\frac{2y}{3}\left[\left(\frac{x^2}{4}+\frac{y^2}{9}+\frac{2xy}{6}\right)+\left(\frac{x^2}{4}+\frac{y^2}{9}-\frac{2xy}{6}\right)+\frac{x^2}{4}-\frac{y^2}{9}\right]$$

$$=\frac{2y}{3}\left[\frac{x^2}{4}+\frac{y^2}{9}+\cancel{\frac{2xy}{6}}+\frac{x^2}{4}+\cancel{\frac{y^2}{9}}-\cancel{\frac{2xy}{6}}+\frac{x^2}{4}-\cancel{\frac{y^2}{9}}\right]$$

By rearranging the variable we get

$$=\frac{2y}{3}\left[\frac{x^2}{4}+\frac{y^2}{9}+\frac{x^2}{4}+\frac{x^2}{4}\right]$$
$$=\frac{2y}{3}\left[\frac{3x^2}{4}+\frac{y^2}{9}\right]$$
$$=\frac{x^2y}{2}+\frac{2y^3}{27}$$

Hence the simplified value of $\left(\frac{x}{2}+\frac{y}{3}\right)^3-\left(\frac{x}{2}-\frac{y}{3}\right)^3$ is $\boxed{\frac{x^2y}{2}+\frac{2y^3}{27}}$

(iii) Given $\left(x+\frac{2}{x}\right)^3+\left(x-\frac{2}{x}\right)^3$

We shall use the identity $a^3+b^3=(a+b)\left(a^2+b^2-ab\right)$

Here $a=\left(x+\frac{2}{x}\right), b=\left(x-\frac{2}{x}\right)$

By applying identity we get

$$=\left(x+\frac{2}{x}+x-\frac{2}{x}\right)\left[\left(x+\frac{2}{x}\right)^2+\left(x-\frac{2}{x}\right)^2-\left(\left(x+\frac{2}{x}\right)\times\left(x-\frac{2}{x}\right)\right)\right]$$
$$=\left(x+\cancel{\frac{2}{x}}+x-\cancel{\frac{2}{x}}\right)\left[\left(x\times x+\frac{2}{x}\times\frac{2}{x}+2\times x\times\frac{2}{x}\right)+\left(x\times x+\frac{2}{x}\times\frac{2}{x}-2\times x\times\frac{2}{x}\right)-\left(x^2+\frac{4}{x^2}\right)\right]$$
$$=(2x)\left[\left(x^2+\frac{4}{x^2}+\frac{4x}{x}\right)+\left(x^2+\frac{4}{x^2}-\frac{4x}{x}\right)-\left(x^2-\frac{4}{x^2}\right)\right]$$
$$=(2x)\left[x^2+\frac{4}{x^2}+\cancel{\frac{4x}{x}}+\cancel{x^2}+\frac{4}{x^2}-\cancel{\frac{4x}{x}}-\cancel{x^2}+\frac{4}{x^2}\right]$$

By rearranging the variable we get,

$$=(2x)\left[x^2+\frac{4}{x^2}+\frac{4}{x^2}+\frac{4}{x^2}\right]$$
$$=2x\times\left[x^2+\frac{12}{x^2}\right]$$
$$=2x^3+\frac{24}{x}$$

Hence the simplified value of $\left(x+\frac{2}{x}\right)^3+\left(x-\frac{2}{x}\right)^3$ is $\boxed{2x^3+\frac{24}{x}}$

(iv) Given $(2x-5y)^3-(2x+5y)^3$

We shall use the identity $a^3-b^3=(a-b)\left(a^2+b^2+ab\right)$

Here $a=(2x-5y), b=(2x+5y)$

By applying the identity we get

$$=(2x-5y-2x+5y)\left[(2x-5y)^2+(2x+5y)^2+((2x-5y)\times(2x+5y))\right]$$
$$=(\cancel{2x}-5y-\cancel{2x}-5y)\left[(2x\times 2x+5y\times 5y-2\times 2x\times 5y)+(2x\times 2x+5y\times 5y+2\times 2x\times 5y)+\left(4x^2-25y^2\right)\right]$$
$$=(-10y)\left[\left(4x^2+25y^2-20xy\right)+\left(4x^2+25y^2+20xy\right)+4x^2-25y^2\right]$$
$$=(-10y)\left[4x^2+25y^2-\cancel{20xy}+4x^2+\cancel{25y^2}+\cancel{20xy}+4x^2-\cancel{25y^2}\right]$$

By rearranging the variable we get,

$$=(-10y)\left[4x^2+4x^2+4x^2+25y^2\right]$$
$$=-10y\times\left[12x^2+25y^2\right]$$
$$=-120x^2y-250y^3$$

Hence the simplified value of $(2x-5y)^3-(2x+5y)^3$ is $\boxed{-120x^2y-250y^3}$.

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