Indefinite Integrals Ex 19.21 Q6

Let $I = \int \frac{x}{\sqrt{8+x-x^2}}dx$

Let $x = \lambda \frac{d}{dx}\left(8+x-x^2\right)+\mu$

$= \lambda\left(1-2x\right)+\mu$

$x = \left(-2\lambda\right)x+\lambda+\mu$

Comparing the coefficients of like powers of x,

$-2\lambda = 1 \quad \Rightarrow \quad \lambda = -\frac{1}{2}$

$\lambda+\mu = 0 \quad \Rightarrow \quad \left(-\frac{1}{2}\right)+\mu = 0$

$\mu = \frac{1}{2}$

so, $I = \int \frac{-\frac{1}{2}\left(1-2x\right)+\frac{1}{2}}{\sqrt{8+x-x^2}}dx$

$= -\frac{1}{2}\int \frac{\left(1-2x\right)}{\sqrt{8+x-x^2}}dx + \frac{1}{2}\int \frac{1}{\sqrt{-\left[x^2-x-8\right]}}dx$

$I = -\frac{1}{2}\int \frac{\left(1-2x\right)}{\sqrt{8+x-x^2}}dx + \frac{1}{2}\int \frac{1}{\sqrt{-\left[x^2-2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2-8\right]}}dx$

$I = -\frac{1}{2}\int \frac{\left(1-2x\right)}{\sqrt{8+x-x^2}}dx + \frac{1}{2}\int \frac{1}{\sqrt{-\left[\left(x-\frac{1}{2}\right)^2-\left(\frac{33}{4}\right)^2\right]}}dx$

$I = -\frac{1}{2}\int \frac{\left(1-2x\right)}{\sqrt{8+x-x^2}}dx + \frac{1}{2}\int \frac{1}{\sqrt{\left[\left(\frac{\sqrt{33}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2\right]}}dx$

$I = -\frac{1}{2}\times 2\sqrt{8+x-x^2}+\frac{1}{2}\sin^{-1}\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{33}}{2}}\right)+c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)+c\right]$

$I = -\sqrt{8+x-x^2}+\frac{1}{2}\sin^{-1}\left(\frac{2x-1}{\sqrt{33}}\right)+c$

Indefinite Integrals Ex 19.21 Q7

Let $I = \int \frac{x+2}{\sqrt{x^2+2x-1}}dx$

Let $x+2 = \lambda \frac{d}{dx}\left(x^2+2x-1\right)+\mu$

$x+2 = \lambda\left(2x+2\right)+\mu$

$x+2 = \left(2\lambda\right)x+2\lambda+\mu$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$2\lambda+\mu = 2 \quad \Rightarrow \quad 2\left(\frac{1}{2}\right)+\mu = 2$

$\Rightarrow \quad \mu = 1$

so, $I_1 = \int \frac{\frac{1}{2}\left(2x+2\right)+1}{\sqrt{x^2+2x-1}}dx$

$= \frac{1}{2}\int \frac{\left(2x+2\right)}{\sqrt{x^2+2x-1}}dx + \int \frac{1}{\sqrt{x^2+2x+\left(1\right)^2-\left(1\right)^2-1}}dx$

$I = \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x-1}}dx + \int \frac{1}{\sqrt{\left(x+1\right)^2-\left(\sqrt{2}\right)^2}}dx$

$I = \frac{1}{2}2\sqrt{x^2+2x-1}+\log\left|x+1+\sqrt{\left(x+1\right)^2-\left(\sqrt{2}\right)^2}\right|+c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x},\ \int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right|+c\right]$

$I = \sqrt{x^2+2x-1}+\log\left|x+1+\sqrt{x^2+2x-1}\right|+c$

Indefinite Integrals Ex 19.21 Q8

$$\text{Let } x+2 = A\frac{d}{dx}\left(x^2-1\right)+B \qquad ...(1)$$
$$\Rightarrow x+2 = A(2x)+B$$

Equating the coefficients of x and constant term on both sides, we obtain

$$2A = 1 \Rightarrow A = \frac{1}{2}$$
$$B = 2$$

From (1), we obtain

$$(x+2) = \frac{1}{2}(2x)+2$$

$$\text{Then, } \int \frac{x+2}{\sqrt{x^2-1}}dx = \int \frac{\frac{1}{2}(2x)+2}{\sqrt{x^2-1}}dx$$
$$= \frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx + \int \frac{2}{\sqrt{x^2-1}}dx \qquad ...(2)$$

$$\text{In } \frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx, \text{ let } x^2-1 = t \Rightarrow 2xdx = dt$$

$$\text{In } \frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx, \text{ let } x^2-1 = t \Rightarrow 2xdx = dt$$

$$\frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$
$$= \frac{1}{2}\left[2\sqrt{t}\right]$$
$$= \sqrt{t}$$
$$= \sqrt{x^2-1}$$

$$\text{Then, } \int \frac{2}{\sqrt{x^2-1}}dx = 2\int \frac{1}{\sqrt{x^2-1}}dx = 2\log\left|x+\sqrt{x^2-1}\right|$$

From equation (2), we obtain

$$\int \frac{x+2}{\sqrt{x^2-1}}dx = \sqrt{x^2-1} + 2\log\left|x+\sqrt{x^2-1}\right| + C$$

Indefinite Integrals Ex 19.21 Q9

$$\int \frac{x-1}{\sqrt{x^2+1}}dx =$$

$$\int \frac{x-1}{\sqrt{x^2+1}}dx = \int \frac{x}{\sqrt{x^2+1}} - \frac{1}{\sqrt{x^2+1}}dx = \frac{1}{2}\int \frac{2x}{\sqrt{x^2+1}}dx - \int \frac{1}{\sqrt{x^2+1}}dx$$

$$= \frac{1}{2}\int \frac{dt}{\sqrt{t}} - \int \frac{1}{\sqrt{x^2+1}}dx = \frac{1}{2}\left(2\sqrt{t}\right) - \int \frac{1}{\sqrt{x^2+1}}dx = \sqrt{t} - \ln\left|x+\sqrt{x^2+1}\right| + C$$

$$= \sqrt{x^2+1} - \ln\left|x+\sqrt{x^2+1}\right| + C$$

Indefinite Integrals Ex 19.21 Q10

Let $I = \int \frac{x}{\sqrt{x^2+x+1}}dx$

Let $x = \lambda \frac{d}{dx}\left(x^2+x+1\right) + \mu$

$= \lambda\left(2x+1\right) + \mu$

$x = \left(2\lambda\right)x + \lambda + \mu$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$\lambda + \mu = 0 \quad \Rightarrow \quad \left(\frac{1}{2}\right) + \mu = 0$

$\Rightarrow \quad \mu = -\frac{1}{2}$

so, $I = \int \frac{\frac{1}{2}\left(2x+1\right) + \frac{1}{2}}{\sqrt{x^2+x+1}}dx$

$= \frac{1}{2}\int \frac{\left(2x+1\right)}{\sqrt{x^2+x+1}}dx - \frac{1}{2}\int \frac{1}{\sqrt{x^2+2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}}dx$

$I = \frac{1}{2}\int \frac{2x+1}{\sqrt{x^2+x+1}}dx - \frac{1}{2}\int \frac{1}{\sqrt{\left(x+\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2}}dx$

$I = \frac{1}{2}\times 2\sqrt{x^2+x+1} - \frac{1}{2}\log\left|x+\frac{1}{2}+\sqrt{\left(x+\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2}\right| + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right| + c\right]$

$I = \sqrt{x^2+x+1} - \frac{1}{2}\log\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right| + c$

*********** END **********