Exercise 7A

Question 5

(i) LHS = $1+\dfrac{\cot^2\theta}{(1+\operatorname{cosec}\theta)}$

$=1+\dfrac{\operatorname{cosec}^2\theta-1}{(1+\operatorname{cosec}\theta)}$

$=\dfrac{1+\operatorname{cosec}\theta+\operatorname{cosec}^2\theta-1^2}{(1+\operatorname{cosec}\theta)}$

$=\dfrac{(1+\operatorname{cosec}\theta)+(\operatorname{cosec}\theta+1)(\operatorname{cosec}\theta-1)}{(1+\operatorname{cosec}\theta)}$

$=\dfrac{(1+\operatorname{cosec}\theta)[1+(\operatorname{cosec}\theta-1)]}{(1+\operatorname{cosec}\theta)}$

$=1+\operatorname{cosec}\theta-1$

$=\operatorname{cosec}\theta=\text{RHS}$

$\therefore$ LHS = RHS

(ii) LHS $=1+\dfrac{\tan^2\theta}{(1+\sec\theta)}=\dfrac{1+\sec\theta+\tan^2\theta}{(1+\sec\theta)}$

$=\dfrac{\sec^2\theta+\sec\theta}{(1+\sec\theta)}\quad\left[\because(1+\tan^2\theta)=\sec^2\theta\right]$

$=\dfrac{\sec\theta(1+\sec\theta)}{(1+\sec\theta)}=\sec\theta=\text{RHS}$

$\therefore$ LHS = RHS

Question 6

$$\text{(i) } LHS = \sec\theta(1-\sin\theta)(\sec\theta+\tan\theta)$$

$$= \left[\sec\theta - \frac{\sin\theta}{\cos\theta}\right] \times (\sec\theta+\tan\theta)$$

$$= (\sec\theta-\tan\theta)(\sec\theta+\tan\theta)$$

$$= \left(\sec^2\theta-\tan^2\theta\right) = 1 = RHS$$

$\therefore$ LHS = RHS

$$\text{(ii) } LHS = \sin\theta(1+\tan\theta)+\cos\theta(1+\cot\theta)$$

$$= \left(\sin\theta+\frac{\sin^2\theta}{\cos\theta}\right)+\cos\theta\left(1+\frac{\cos\theta}{\sin\theta}\right)$$

$$= \left(\frac{\sin\theta\cos\theta+\sin^2\theta}{\cos\theta}\right)+\left(\cos\theta+\frac{\cos^2\theta}{\sin\theta}\right)$$

$$= \left(\frac{\sin\theta\cos\theta+\sin^2\theta}{\cos\theta}\right)+\left(\frac{\cos\theta\sin\theta+\cos^2\theta}{\sin\theta}\right)$$

$$= \frac{\sin\theta\cos\theta}{\cos\theta}+\frac{\sin^2\theta}{\cos\theta}+\frac{\cos\theta\sin\theta}{\sin\theta}+\frac{\cos^2\theta}{\sin\theta}$$

$$= \frac{\sin\theta\cos\theta}{\cos\theta}+\frac{\cos\theta\sin\theta}{\sin\theta}+\frac{\sin^2\theta}{\cos\theta}+\frac{\cos^2\theta}{\sin\theta}$$

$$= \sin\theta\cos\theta\left(\frac{\sin\theta+\cos\theta}{\cos\theta\sin\theta}\right)+\frac{\sin^3\theta+\cos^3\theta}{\cos\theta\sin\theta}$$

$$= \sin\theta\cos\theta\left(\frac{\sin\theta+\cos\theta}{\cos\theta\sin\theta}\right) + \frac{(\sin\theta+\cos\theta)\left(\sin^2\theta-\sin\theta\cos\theta+\cos^2\theta\right)}{\cos\theta\sin\theta}$$

$$\left[\because a^3+b^3=(a+b)\left(a^2-ab+b^2\right)\right]$$

$$= (\sin\theta+\cos\theta)\left[\frac{\sin\theta\cos\theta}{\cos\theta\sin\theta}+\frac{(1-\sin\theta\cos\theta)}{\cos\theta\sin\theta}\right]$$

$$= (\sin\theta+\cos\theta)\left[\frac{\sin\cos\theta+1-\sin\theta\cos\theta}{\cos\theta\sin\theta}\right]$$

$$= (\sin\theta+\cos\theta)\left(\frac{1}{\cos\theta\sin\theta}\right)$$

$$= \frac{\sin}{\cos\theta\sin\theta}+\frac{\cos\theta}{\cos\theta\sin\theta} = \sec\theta+\operatorname{cosec}\theta = RHS$$

$\therefore$ LHS = RHS

*********** END **********