

Maxima and Minima 18.5 Q10

ABC is a right angled triangle. Hypotenuse h = AC = 5 cm.

Let x and y one the other two side of the triangle.

$$x^2 + y^2 = 25$$
 ---(i)

$$\therefore \qquad \text{Area of } \triangle ABC = \frac{1}{2}BC \times AB$$

$$\Rightarrow S = \frac{1}{2}xy \qquad ---(ii)$$

$$S = \frac{1}{2} x \sqrt{25 - x}$$

$$\frac{ds}{dx} = \frac{1}{2} \left[\sqrt{25 - x^2} - \frac{2x^2}{2\sqrt{25 - x^2}} \right]$$

$$= \frac{1}{2} \frac{\left[25 - x^2 - x^2 \right]}{\sqrt{25 - x^2}}$$

$$= \frac{1}{2} \left[\frac{25 - 2x^2}{\sqrt{25 - x^2}} \right]$$

For maxima and minima,

$$\frac{ds}{dx} = 0$$

$$\Rightarrow \frac{1}{2} \left[\frac{25 - 2x^2}{\sqrt{25 - x^2}} \right] = 0$$

$$\Rightarrow x = 5\sqrt{2}$$

Now,

$$\frac{d^2s}{dx^2} = \frac{1}{2} \frac{\sqrt{25 - x^2} \times \left(-4x\right) + \frac{\left(25 - 2x^2\right)2x}{2\sqrt{25 - x^2}}}{\left(25 - x^2\right)}$$

At
$$x = \frac{5}{\sqrt{2}}$$
, $\frac{d^2s}{dx^2} = \frac{1}{2} \frac{\left[-\frac{25}{\sqrt{2}} \times \frac{5}{\sqrt{2}} + 0 \right]}{\frac{25}{2}}$
= $-\frac{5}{2} < 0$

$$x = \frac{5}{\sqrt{2}}$$
 is a point local maxima,

Maxima and Minima 18.5 Q11

ABC is a given triangle with AB = a, BC = b and $\angle ABC = \theta$.

AD in perpendicular to BC.

$$BD = a \sin \theta$$

Now,

Area of
$$\triangle ABC = \frac{1}{2} \times BC \times AD$$

$$\Rightarrow A = \frac{1}{2}b \times a \sin \theta \qquad ---(i)$$

$$\therefore \frac{dA}{d\theta} = \frac{1}{2}ab \cos \theta$$

For maxima and minima,

$$\frac{dA}{d\theta} = 0$$

$$\Rightarrow \frac{1}{2}ab\cos\theta = 0$$

$$\Rightarrow \cos\theta = 0$$

$$\Rightarrow \theta = \frac{\pi}{2}$$

Now,

$$\frac{d^2A}{d\theta^2} = -\frac{1}{2}ab\sin\theta$$
At $\theta = \frac{\pi}{2}$, $\frac{d^2A}{d\theta^2} = -\frac{1}{2}ab < 0$
 $\therefore \theta = \frac{\pi}{2}$ is point of local maxima

:. Maximum area of
$$\Delta = \frac{1}{2}ab \sin \frac{\pi}{2} = \frac{1}{2}ab$$
.

Maxima and Minima 18.5 Q12

Let the side of the square to be cut off be x cm. Then, the length and the breadth of the box will be (18-2x) cm each and the height of the box is x cm.

Therefore, the volume V(x) of the box is given by,

$$V(x) = x(18 - 2x)^2$$

$$\therefore V'(x) = (18-2x)^2 - 4x(18-2x)$$

$$= (18-2x)[18-2x-4x]$$

$$= (18-2x)(18-6x)$$

$$= 6 \times 2(9-x)(3-x)$$

$$= 12(9-x)(3-x)$$
And, $V''(x) = 12[-(9-x)-(3-x)]$

$$= -12(9-x+3-x)$$

$$= -12(12-2x)$$

$$= -24(6-x)$$

Maximum volume is $V_{x=3} = 3 \times (18 - 2 \times 3)^2$

$$\Rightarrow V = 3 \times 12^2$$

$$\Rightarrow V = 3 \times 144$$

$$\Rightarrow V = 432 \text{ cm}^3$$

******* END ******