Mathematical Induction Ex 12.2 Q44

$S_n = 1^2 + 2\times2^2 + 3^2 + 2\times4^2 + ...$

Using induction we first show this is true for n=2,

we get $S_2 = 1^2 + 2\times2^2 = 1+8 = 9$

From RHS, we have if n is even $S_n = \frac{n(n+1)^2}{2}$

$S_2 = \frac{2\times9}{2} = 9$

Now using induction we first show this is true also

for n=3, we get $S_3 = 1+8+9 = 18$

From RHS, we have if n is odd $S_n = \frac{n^2(n+1)}{2}$

$S_3 = \frac{9\times4}{2} = 18$

Lets assume above is true for n=k, we get

k is even, $S_k = 1^2 + 2\times2^2 + 3^2 + 2\times4^2 + ...+2\times k^2$ ---1

k is odd, $S_k = 1^2 + 2\times2^2 + 3^2 + 2\times4^2 + ...+k^2$ -----2

Now lets prove for n=k+1

If k is even, k+1 is odd we get

$S_{k+1} = 1^2 + 2\times2^2 + 3^2 + ...+2\times k^2 + (k+1)^2$ ---3

From above relation, we get

$S_k = 1^2 + 2\times2^2 + 3^2 + 2\times4^2 + ...+2\times k^2 = \frac{k(k+1)^2}{2}$

Substitute this in 3, we get

$S_{k+1} = \frac{k(k+1)^2}{2} + (k+1)^2 = \frac{(k+1)^2(k+2)}{2}$

$= RHS$ (when '$k+1$' is odd)

Hence Proved

Mathematical Induction Ex 12.2 Q45

Let $P(n)$ be the statement given by

$P(n)$: The number of subsets of a set containing n distinct elements is 2^n for all $n \in N$.

Step I:

$P(1) : 2^1 = 2$

For any set A contaiining 1 element, empty set and set A are two sets always subsets of A.

$\therefore P(1)$ is true.

Step II:

Let $P(m)$ is true. Then,

A set containing m distinct elements has 2^m subsets..........(i)

We have to prove that $P(m+1)$ is true.

Let the set A has $(m+1)$ elements.

$A = \{1,2,.......,m,m+1\}$

$A = \{1,2,.......,m\} \cup \{m+1\}$

Now using (i) we can say that $\{1,2,.......,m\}$ being m elemets has 2^m subsets.

For $\{m+1\}$, empty set and set itself $\{m+1\}$ are subsets.

So, $\{m+1\}$ has 2 subsets.

$\Rightarrow$ Set A has $2^m + 2$ subsets

$\Rightarrow$ Set A has 2^{m+1} subsets

$\Rightarrow P(m+1)$ is true.

Hence by the principle of mathematical induction, the given result is true for all $n \in N$.

Mathematical Induction Ex 12.2 Q46

Let $P(n)$ be the statement given by

$P(n) : a_n = 3 \times 7^{n-1}$ for all $n \in N$.

Step I:

$P(2) : a_2 = 3 \times 7^{2-1} = 21$

Given that $a_k = 7\ a_{k-1}$ for all natural numbers $k \geq 2$

$a_2 = 7a_1 = 7 \times 3 = 21$

$\therefore P(2)$ is true.

Step II:

Let $P(m)$ is true. Then,

$a_m = 3 \times 7^{m-1}$..........(i)

We have to prove that $P(m+1)$ is true.

$a_{m+1} = 7a_m$

$a_{m+1} = 7 \times a_m$

$a_{m+1} = 7^1 \times 3 \times 7^{m-1}$............$\left[\text{from(i)}\right]$

$a_{m+1} = 3 \times 7^{m-1+1}$

$a_{m+1} = 3 \times 7^m$

$\Rightarrow P(m+1)$ is true.

Hence by the principle of mathematical induction, the given result is true for all $n \in N$.

*********** END **********