



Chapter 9 Continuity Ex 9.2 Q14

The given function is $f(x) = \cos(x^2)$

This function f is defined for every real number and f can be written as the composition of two functions as,

$f = g \circ h$, where $g(x) = \cos x$ and $h(x) = x^2$

$$[\because (goh)(x) = g(h(x)) = g(x^2) = \cos(x^2) = f(x)]$$

It has to be first proved that $g(x) = \cos x$ and $h(x) = x^2$ are continuous functions.

It is evident that g is defined for every real number.

Let c be a real number.

Then, $g(c) = \cos c$

Put $x = c + h$

If $x \rightarrow c$, then $h \rightarrow 0$

$$\begin{aligned} \lim_{x \rightarrow c} g(x) &= \lim_{x \rightarrow c} \cos x \\ &= \lim_{h \rightarrow 0} \cos(c + h) \\ &= \lim_{h \rightarrow 0} [\cos c \cos h - \sin c \sin h] \\ &= \lim_{h \rightarrow 0} \cos c \cos h - \lim_{h \rightarrow 0} \sin c \sin h \\ &= \cos c \cos 0 - \sin c \sin 0 \\ &= \cos c \times 1 - \sin c \times 0 \\ &= \cos c \end{aligned}$$

$$\therefore \lim_{x \rightarrow c} g(x) = g(c)$$

Therefore, $g(x) = \cos x$ is continuous function.

$$h(x) = x^2$$

Clearly, h is defined for every real number.

Let k be a real number, then $h(k) = k^2$

$$\lim_{x \rightarrow k} h(x) = \lim_{x \rightarrow k} x^2 = k^2$$

$$\therefore \lim_{x \rightarrow k} h(x) = h(k)$$

Therefore, h is a continuous function.

It is known that for real valued functions g and h , such that $(g \circ h)$ is defined at c , if g is continuous at c and if f is continuous at $g(c)$, then $(f \circ g)$ is continuous at c .

Therefore, $f(x) = (goh)(x) = \cos(x^2)$ is a continuous function.

Chapter 9 Continuity Ex 9.2 Q15

The given function is $f(x) = |\cos x|$

This function f is defined for every real number and f can be written as the composition of two functions as,

$f = g \circ h$, where $g(x) = |x|$ and $h(x) = \cos x$

$$[\because (goh)(x) = g(h(x)) = g(\cos x) = |\cos x| = f(x)]$$

It has to be first proved that $g(x) = |x|$ and $h(x) = \cos x$ are continuous functions.

$g(x) = |x|$ can be written as

$$g(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

Clearly, g is defined for all real numbers.

Let c be a real number.

Case I:

$$\text{If } c < 0, \text{ then } g(c) = -c \text{ and } \lim_{x \rightarrow c} g(x) = \lim_{x \rightarrow c} (-x) = -c$$

$$\therefore \lim_{x \rightarrow c} g(x) = g(c)$$

Therefore, g is continuous at all points x , such that $x < 0$

Case II:

$$\text{If } c > 0, \text{ then } g(c) = c \text{ and } \lim_{x \rightarrow c} g(x) = \lim_{x \rightarrow c} x = c$$

$$\therefore \lim_{x \rightarrow c} g(x) = g(c)$$

Therefore, g is continuous at all points x , such that $x > 0$

Case III:

$$\text{If } c = 0, \text{ then } g(c) = g(0) = 0$$

$$\lim_{x \rightarrow 0^-} g(x) = \lim_{x \rightarrow 0^-} (-x) = 0$$

$$\lim_{x \rightarrow 0^+} g(x) = \lim_{x \rightarrow 0^+} (x) = 0$$

$$\therefore \lim_{x \rightarrow 0} g(x) = \lim_{x \rightarrow 0} (x) = g(0)$$

Therefore, g is continuous at $x = 0$

From the above three observations, it can be concluded that g is continuous at all points.

$$h(x) = \cos x$$

It is evident that $h(x) = \cos x$ is defined for every real number.

Let c be a real number. Put $x = c + h$

$$\text{If } x \rightarrow c, \text{ then } h \rightarrow 0$$

$$h(c) = \cos c$$

$$\begin{aligned} \lim_{x \rightarrow c} h(x) &= \lim_{x \rightarrow c} \cos x \\ &= \lim_{h \rightarrow 0} \cos(c + h) \\ &= \lim_{h \rightarrow 0} [\cos c \cos h - \sin c \sin h] \\ &= \lim_{h \rightarrow 0} \cos c \cos h - \lim_{h \rightarrow 0} \sin c \sin h \\ &= \cos c \cos 0 - \sin c \sin 0 \\ &= \cos c \times 1 - \sin c \times 0 \\ &= \cos c \\ \therefore \lim_{x \rightarrow c} h(x) &= h(c) \end{aligned}$$

Therefore, $h(x) = \cos x$ is a continuous function.

It is known that for real valued functions g and h , such that $(g \circ h)$ is defined at c , if g is continuous at c and if h is continuous at $g(c)$, then $(f \circ g)$ is continuous at c .

Therefore, $f(x) = (g \circ h)(x) = g(h(x)) = g(\cos x) = |\cos x|$ is a continuous function

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