Differentiation Ex 11.5 Q16

Let $\quad y = (\tan x)^{\frac{1}{x}}$ ---(i)

Taking log on both the sides,

$$\log y = \log(\tan x)^{\frac{1}{x}}$$

$$\log y = \frac{1}{x}\log(\tan x) \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\frac{d}{dx}\log(\tan x) + \log(\tan x)\frac{d}{dx}\left(\frac{1}{x}\right)$$

$$= \frac{1}{x} \times \frac{1}{\tan x}\frac{d}{dx}(\tan x) + \log(\tan x)\left(-\frac{1}{x^2}\right)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x\tan x}\left(\sec^2 x\right) - \frac{\log(\tan x)}{x^2}$$

$$\frac{dy}{dx} = y\left[\frac{\sec^2 x}{x\tan x} - \frac{\log(\tan x)}{x^2}\right]$$

$$\frac{dy}{dx} = (\tan x)^{\frac{1}{x}}\left[\frac{\sec^2 x}{x\tan x} - \frac{\log \tan x}{x^2}\right] \qquad \left[\text{Using equation (i)}\right]$$

Differentiation Ex 11.5 Q17

Let $\quad y = x^{\tan^{-1} x}$ ---(i)

Taking log on both the sides,

$$\log y = \log x^{\tan^{-1} x}$$

$$\log y = \tan^{-1} x \log x \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating it with respect to x using product rule,

$$\frac{1}{y}\frac{dy}{dx} = \tan^{-1} x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}\left(\tan^{-1} x\right)$$

$$\frac{1}{y}\frac{dy}{dx} = \tan^{-1} x\left(\frac{1}{x}\right) + \log x\left(\frac{1}{1+x^2}\right)$$

$$\frac{dy}{dx} = y\left[\frac{\tan^{-1} x}{x} + \frac{\log x}{1+x^2}\right]$$

$$\frac{dy}{dx} = x^{\tan^{-1} x}\left[\frac{\tan^{-1} x}{x} + \frac{\log x}{1+x^2}\right] \qquad \left[\text{Using equation (i)}\right]$$

Differentiation Ex 11.5 Q18(i)

Let $\quad y = x^x\sqrt{x} \qquad$ ---(i)

Taking log on both the sides,

$$\log y = \log\left(x^x\sqrt{x}\right)$$
$$= \log x^x + \log x^{\frac{1}{2}} \qquad \left[\text{Since, } \log^{(ab)} = \log a + \log b\right]$$
$$\log y = x\log x + \frac{1}{2}\log x \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating it with respect to x using product rule,

$$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x) + \frac{1}{2}\frac{d}{dx}(\log x)$$
$$= x\left(\frac{1}{x}\right) + \log x\,(1) + \frac{1}{2}\left(\frac{1}{x}\right)$$
$$\frac{1}{y}\frac{dy}{dx} = 1 + \log x + \frac{1}{2x}$$
$$\frac{dy}{dx} = y\left(1 + \log x + \frac{1}{2x}\right)$$
$$\frac{dy}{dx} = x^x\sqrt{x}\left(1 + \log x + \frac{1}{2x}\right) \qquad \left[\text{Using equation (i)}\right]$$

Differentiation Ex 11.5 Q18(ii)

Let $\quad y = x^{(\sin x - \cos x)} + \left(\frac{x^2-1}{x^2+1}\right)$

$$y = e^{\log x^{(\sin x - \cos x)}} + \left(\frac{x^2-1}{x^2+1}\right)$$
$$y = e^{(\sin x - \cos x)\log x} + \left(\frac{x^2-1}{x^2+1}\right) \qquad \left[\text{Since, } e^{\log a} = a,\ \log a^b = b\log a\right]$$

Differentiating it with respect to x using chain rule and quotient rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left[e^{(\sin x - \cos x)\log x}\right] + \frac{d}{dx}\left[\frac{x^2-1}{x^2+1}\right]$$
$$= e^{(\sin x - \cos x)\log x}\frac{d}{dx}\{(\sin x - \cos x)\log x\} + \left[\frac{\left(x^2+1\right)\frac{d}{dx}\left(x^2-1\right) - \left(x^2-1\right)\frac{d}{dx}\left(x^2+1\right)}{\left(x^2+1\right)^2}\right]$$
$$= e^{\log x^{(\sin x - \cos x)}}\left[(\sin x - \cos x)\frac{d}{dx}(\log x) + (\log x)\frac{d}{dx}(\sin x - \cos x)\right] + \left[\frac{\left(x^2+1\right)(2x) - \left(x^2-1\right)(2x)}{\left(x^2+1\right)^2}\right]$$
$$= x^{(\sin x - \cos x)}\left[(\sin x - \cos x)\left(\frac{1}{x}\right) + \log x\,(\sin x + \cos x)\right] + \left[\frac{2x^3 + 2x - 2x^3 + 2x}{\left(x^2+1\right)^2}\right]$$
$$\frac{dy}{dx} = x^{(\sin x - \cos x)}\left[\frac{(\sin x - \cos x)}{x} + \log x\,(\sin x + \cos x)\right] + \frac{4x}{\left(x^2+1\right)^2}$$

Differentiation Ex 11.5 Q18(iii)

Let $y = x^{x\cos x} + \dfrac{x^2+1}{x^2-1}$

Also, let $u = x^{x\cos x}$ and $v = \dfrac{x^2+1}{x^2-1}$

$\therefore y = u + v$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad ...(1)$$

$u = x^{x\cos x}$

$\Rightarrow \log u = \log\left(x^{x\cos x}\right)$

$\Rightarrow \log u = x\cos x \log x$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(x)\cdot\cos x\cdot\log x + x\cdot\frac{d}{dx}(\cos x)\cdot\log x + x\cos x\cdot\frac{d}{dx}(\log x)$$

$$\Rightarrow \frac{du}{dx} = u\left[1\cdot\cos x\cdot\log x + x\cdot(-\sin x)\log x + x\cos x\cdot\frac{1}{x}\right]$$

$$\Rightarrow \frac{du}{dx} = x^{x\cos x}\left(\cos x\log x - x\sin x\log x + \cos x\right)$$

$$\Rightarrow \frac{du}{dx} = x^{x\cos x}\left[\cos x(1+\log x) - x\sin x\log x\right] \qquad ...(2)$$

$v = \dfrac{x^2+1}{x^2-1}$

$\Rightarrow \log v = \log\left(x^2+1\right) - \log\left(x^2-1\right)$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v}\frac{dv}{dx} = \frac{2x}{x^2+1} - \frac{2x}{x^2-1}$$

$$\Rightarrow \frac{dv}{dx} = v\left[\frac{2x\left(x^2-1\right) - 2x\left(x^2+1\right)}{\left(x^2+1\right)\left(x^2-1\right)}\right]$$

$$\Rightarrow \frac{dv}{dx} = \frac{x^2+1}{x^2-1}\times\left[\frac{-4x}{\left(x^2+1\right)\left(x^2-1\right)}\right]$$

$$\Rightarrow \frac{dv}{dx} = \frac{-4x}{\left(x^2-1\right)^2} \qquad ...(3)$$

From (1), (2), and (3), we obtain

$$\frac{dy}{dx} = x^{x\cos x}\left[\cos x(1+\log x) - x\sin x\log x\right] - \frac{4x}{\left(x^2-1\right)^2}$$

*********** END **********