EXERCISE.10.3

Question-1

Reduce the following equations into slope-intercept form and find their slopes and the y-intercepts.

(i) $x + 7y = 0$ (ii) $6x + 3y - 5 = 0$ (iii) $y = 0$

Ans.

(i) The given equation is $x + 7y = 0$.

It can be written as

$$y = -\frac{1}{7}x + 0 \qquad ...(1)$$

This equation is of the form $y = mx + c$, where $m = -\frac{1}{7}$ and $c = 0$.

Therefore, equation (1) is in the slope-intercept form, where the slope and the y-intercept are $-\frac{1}{7}$ and 0 respectively.

(ii) The given equation is $6x + 3y - 5 = 0$.

It can be written as

$$y = \frac{1}{3}(-6x + 5)$$

$$y = -2x + \frac{5}{3} \qquad ...(2)$$

This equation is of the form $y = mx + c$, where $m = -2$ and $c = \frac{5}{3}$.

Therefore, equation (2) is in the slope-intercept form, where the slope and the y-intercept are–2 and $\frac{5}{3}$ respectively.

(iii) The given equation is $y = 0$.

It can be written as

$y = 0.x + 0$ ... (3)

This equation is of the form $y = mx + c$, where $m = 0$ and $c = 0$.

Therefore, equation (3) is in the slope-intercept form, where the slope and the y-intercept are 0 and 0 respectively.

Question-2

Reduce the following equations into intercept form and find their intercepts on the axes.

(i) $3x + 2y - 12 = 0$ (ii) $4x - 3y = 6$ (iii) $3y + 2 = 0$.

Ans.

(i) The given equation is $3x + 2y - 12 = 0$.

It can be written as

$$3x+2y=12$$
$$\frac{3x}{12}+\frac{2y}{12}=1$$

$$\text{i.e., } \frac{x}{4}+\frac{y}{6}=1 \qquad ...(1)$$

This equation is of the form $\frac{x}{a}+\frac{y}{b}=1$, where $a = 4$ and $b = 6$.

Therefore, equation (1) is in the intercept form, where the intercepts on the x and y axes are 4 and 6 respectively.

(ii) The given equation is $4x - 3y = 6$.

It can be written as

$$\frac{4x}{6}-\frac{3y}{6}=1$$
$$\frac{2x}{3}-\frac{y}{2}=1$$

$$\text{i.e., } \frac{x}{\left(\frac{3}{2}\right)}+\frac{y}{(-2)}=1 \qquad ...(2)$$

This equation is of the form $\frac{x}{a}+\frac{y}{b}=1$, where $a = \frac{3}{2}$ and $b = -2$.

Therefore, equation (2) is in the intercept form, where the intercepts on the x and y axes are $\frac{3}{2}$ and -2 respectively.

(iii) The given equation is $3y + 2 = 0$.

It can be written as

$$3y=-2$$

$$\text{i.e., } \frac{y}{\left(-\frac{2}{3}\right)}=1 \qquad ...(3)$$

This equation is of the form $\frac{x}{a}+\frac{y}{b}=1$, where $a = 0$ and $b = -\frac{2}{3}$.

Therefore, equation (3) is in the intercept form, where the intercept on the y-axis is $-\frac{2}{3}$ and it has no intercept on the x-axis.

Question-3

Reduce the following equations into normal form. Find their perpendicular distances from the origin and angle between perpendicular and the positive x-axis.

(i) $x-\sqrt{3}y+8=0$ (ii) $y - 2 = 0$ (iii) $x - y = 4$

Ans.

(i) The given equation is $x-\sqrt{3}y+8=0$.

It can be reduced as:

$$x-\sqrt{3}y=-8$$
$$\Rightarrow -x+\sqrt{3}y=8$$

On dividing both sides by $\sqrt{(-1)^2+(\sqrt{3})^2}=\sqrt{4}=2$, we obtain

$$-\frac{x}{2}+\frac{\sqrt{3}}{2}y=\frac{8}{2}$$
$$\Rightarrow\left(-\frac{1}{2}\right)x+\left(\frac{\sqrt{3}}{2}\right)y=4$$
$$\Rightarrow x\cos 120^\circ+y\sin 120^\circ=4 \qquad ...(1)$$

Equation (1) is in the normal form.

On comparing equation (1) with the normal form of equation of line

$x\cos\omega+y\sin\omega=p$, we obtain $\omega=120°$ and $p=4$.

Thus, the perpendicular distance of the line from the origin is 4, while the angle between the perpendicular and the positive x-axis is 120°.

(ii) The given equation is $y-2=0$.

It can be reduced as $0.x+1.y=2$

On dividing both sides by $\sqrt{0^2+1^2}=1$, we obtain $0.x+1.y=2$

$\Rightarrow x\cos 90°+y\sin 90°=2$ ... (1)

Equation (1) is in the normal form.

On comparing equation (1) with the normal form of equation of line

$x\cos\omega+y\sin\omega=p$, we obtain $\omega=90°$ and $p=2$.

Thus, the perpendicular distance of the line from the origin is 2, while the angle between the perpendicular and the positive x-axis is 90°.

(iii) The given equation is $x-y=4$.

It can be reduced as $1.x+(-1)y=4$

On dividing both sides by $\sqrt{1^2+(-1)^2}=\sqrt{2}$, we obtain

$$\frac{1}{\sqrt{2}}x+\left(-\frac{1}{\sqrt{2}}\right)y=\frac{4}{\sqrt{2}}$$
$$\Rightarrow x\cos\left(2\pi-\frac{\pi}{4}\right)+y\sin\left(2\pi-\frac{\pi}{4}\right)=2\sqrt{2}$$
$$\Rightarrow x\cos 315^\circ+y\sin 315^\circ=2\sqrt{2} \qquad ...(1)$$

Equation (1) is in the normal form.

On comparing equation (1) with the normal form of equation of line

$x\cos\omega+y\sin\omega=p$, we obtain $\omega=315°$ and $p=2\sqrt{2}$.

Thus, the perpendicular distance of the line from the origin is $2\sqrt{2}$, while the angle between the perpendicular and the positive x-axis is 315°.

Question-4

Find the distance of the point $(-1, 1)$ from the line $12(x+6)=5(y-2)$.

Ans.

The given equation of the line is $12(x + 6) = 5(y - 2)$.

$\Rightarrow 12x + 72 = 5y - 10$

$\Rightarrow 12x - 5y + 82 = 0$ ... (1)

On comparing equation (1) with general equation of line $Ax + By + C = 0$, we obtain $A = 12$, $B = -5$, and $C = 82$.

It is known that the perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by $d = \dfrac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$.

The given point is $(x_1, y_1) = (-1, 1)$.

Therefore, the distance of point $(-1, 1)$ from the given line

$$= \frac{|12(-1) + (-5)(1) + 82|}{\sqrt{(12)^2 + (-5)^2}} \text{ units} = \frac{|-12 - 5 + 82|}{\sqrt{169}} \text{ units} = \frac{|65|}{13} \text{ units} = 5 \text{ units}$$

Question-5

Find the points on the x-axis, whose distances from the line $\dfrac{x}{3} + \dfrac{y}{4} = 1$ are 4 units.

Ans.

The given equation of line is

$$\frac{x}{3} + \frac{y}{4} = 1$$

$$\text{or, } 4x + 3y - 12 = 0 \qquad ...(1)$$

On comparing equation (1) with general equation of line $Ax + By + C = 0$, we obtain $A = 4$, $B = 3$, and $C = -12$.

Let $(a, 0)$ be the point on the x-axis whose distance from the given line is 4 units.

It is known that the perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by $d = \dfrac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$.

Therefore,

$$4 = \frac{|4a + 3 \times 0 - 12|}{\sqrt{4^2 + 3^2}}$$

$$\Rightarrow 4 = \frac{|4a - 12|}{5}$$

$$\Rightarrow |4a - 12| = 20$$

$$\Rightarrow \pm(4a - 12) = 20$$

$$\Rightarrow (4a - 12) = 20 \text{ or } -(4a - 12) = 20$$

$$\Rightarrow 4a = 20 + 12 \text{ or } 4a = -20 + 12$$

$$\Rightarrow a = 8 \text{ or } -2$$

Thus, the required points on the x-axis are $(-2, 0)$ and $(8, 0)$.

Question-6

Find the distance between parallel lines

(i) $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$

(ii) $l(x + y) + p = 0$ and $l(x + y) - r = 0$

Ans.

It is known that the distance (d) between parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by $d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$.

(i) The given parallel lines are $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$.

Here, $A = 15$, $B = 8$, $C_1 = -34$, and $C_2 = 31$.

Therefore, the distance between the parallel lines is

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}} = \frac{|-34-31|}{\sqrt{(15)^2 + (8)^2}} \text{ units} = \frac{|-65|}{17} \text{ units} = \frac{65}{17} \text{ units}$$

(ii) The given parallel lines are $l(x + y) + p = 0$ and $l(x + y) - r = 0$.

$lx + ly + p = 0$ and $lx + ly - r = 0$

Here, $A = l$, $B = l$, $C_1 = p$, and $C_2 = -r$.

Therefore, the distance between the parallel lines is

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}} = \frac{|p+r|}{\sqrt{l^2 + l^2}} \text{ units} = \frac{|p+r|}{\sqrt{2l^2}} \text{ units} = \frac{|p+r|}{l\sqrt{2}} \text{ units} = \frac{1}{\sqrt{2}}\left|\frac{p+r}{l}\right| \text{ units}$$

Question-7

Find equation of the line parallel to the line $3x - 4y + 2 = 0$ and passing through the point $(-2, 3)$.

Ans.

The equation of the given line is

$$3x - 4y + 2 = 0$$
$$\text{or } y = \frac{3x}{4} + \frac{2}{4}$$

$\text{or } y = \frac{3}{4}x + \frac{1}{2}$, which is of the form $y = mx + c$

$\therefore$ Slope of the given line $= \frac{3}{4}$

It is known that parallel lines have the same slope.

$\therefore$ Slope of the other line $= m = \frac{3}{4}$

Now, the equation of the line that has a slope of $\frac{3}{4}$ and passes through the point $(-2, 3)$ is

$$(y - 3) = \frac{3}{4}\{x - (-2)\}$$
$$4y - 12 = 3x + 6$$

$$\text{i.e., } 3x - 4y + 18 = 0$$

Question-8

Find equation of the line perpendicular to the line $x - 7y + 5 = 0$ and having x intercept 3.

Ans.

The given equation of line is $x - 7y + 5 = 0$.

Or, $y = \frac{1}{7}x + \frac{5}{7}$, which is of the form $y = mx + c$

$\therefore$ Slope of the given line $= \frac{1}{7}$

The slope of the line perpendicular to the line having a slope of $\frac{1}{7}$ is

$m = -\frac{1}{\left(\frac{1}{7}\right)} = -7$

The equation of the line with slope –7 and x-intercept 3 is given by

$y = m\ (x - d)$

$\Rightarrow y = -7\ (x - 3)$

$\Rightarrow y = -7x + 21$

$\Rightarrow 7x + y = 21$

Question-9

Find angles between the lines $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$

Ans.

The given lines are $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$.

$y = -\sqrt{3}x + 1 \quad ...(1) \qquad$ and $y = -\frac{1}{\sqrt{3}}x + \frac{1}{\sqrt{3}} \quad ...(2)$

The slope of line (1) is $m_1 = -\sqrt{3}$, while the slope of line (2) is $m_2 = -\frac{1}{\sqrt{3}}$.

The acute angle i.e., θ between the two lines is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan\theta = \left|\frac{-\sqrt{3} + \frac{1}{\sqrt{3}}}{1 + \left(-\sqrt{3}\right)\left(-\frac{1}{\sqrt{3}}\right)}\right|$$

$$\tan\theta = \left|\frac{\frac{-3+1}{\sqrt{3}}}{1+1}\right| = \left|\frac{-2}{2\times\sqrt{3}}\right|$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ$$

Thus, the angle between the given lines is either 30° or 180° – 30° = 150°.

Question-10

The line through the points (h, 3) and (4, 1) intersects the line $7x - 9y - 19 = 0$. at right angle. Find the value of h.

Ans.

The slope of the line passing through points (h, 3) and (4, 1) is

$$m_1 = \frac{1-3}{4-h} = \frac{-2}{4-h}$$

The slope of line $7x - 9y - 19 = 0$ or $y = \frac{7}{9}x - \frac{19}{9}$ is $m_2 = \frac{7}{9}$.

It is given that the two lines are perpendicular.

$$\therefore m_1 \times m_2 = -1$$
$$\Rightarrow \left(\frac{-2}{4-h}\right) \times \left(\frac{7}{9}\right) = -1$$
$$\Rightarrow \frac{-14}{36-9h} = -1$$
$$\Rightarrow 14 = 36 - 9h$$
$$\Rightarrow 9h = 36 - 14$$
$$\Rightarrow h = \frac{22}{9}$$

Thus, the value of h is $\frac{22}{9}$.

Question-11

Prove that the line through the point (x_1, y_1) and parallel to the line $Ax + By + C = 0$ is $A(x - x_1) + B(y - y_1) = 0$.

Ans.

The slope of line $Ax + By + C = 0$ or $y = \left(\frac{-A}{B}\right)x + \left(\frac{-C}{B}\right)$ is $m = -\frac{A}{B}$

It is known that parallel lines have the same slope.

$\therefore$ Slope of the other line = $m = -\frac{A}{B}$

The equation of the line passing through point (x_1, y_1) and having a slope $m = -\frac{A}{B}$ is

$$y - y_1 = m(x - x_1)$$
$$y - y_1 = -\frac{A}{B}(x - x_1)$$
$$B(y - y_1) = -A(x - x_1)$$
$$A(x - x_1) + B(y - y_1) = 0$$

Hence, the line through point (x_1, y_1) and parallel to line $Ax + By + C = 0$ is

$A(x - x_1) + B(y - y_1) = 0$

Question-12

Two lines passing through the point (2, 3) intersects each other at an angle of 60°. If slope of one line is 2, find equation of the other line.

Ans.

It is given that the slope of the first line, $m_1 = 2$.

Let the slope of the other line be m_2.

The angle between the two lines is 60°.

$$\therefore \tan 60° = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$
$$\Rightarrow \sqrt{3} = \left|\frac{2 - m_2}{1 + 2m_2}\right|$$
$$\Rightarrow \sqrt{3} = \pm\left(\frac{2 - m_2}{1 + 2m_2}\right)$$
$$\Rightarrow \sqrt{3} = \frac{2 - m_2}{1 + 2m_2} \text{ or } \sqrt{3} = -\left(\frac{2 - m_2}{1 + 2m_2}\right)$$
$$\Rightarrow \sqrt{3}(1 + 2m_2) = 2 - m_2 \text{ or } \sqrt{3}(1 + 2m_2) = -(2 - m_2)$$
$$\Rightarrow \sqrt{3} + 2\sqrt{3}m_2 + m_2 = 2 \text{ or } \sqrt{3} + 2\sqrt{3}m_2 - m_2 = -2$$
$$\Rightarrow \sqrt{3} + (2\sqrt{3} + 1)m_2 = 2 \text{ or } \sqrt{3} + (2\sqrt{3} - 1)m_2 = -2$$
$$\Rightarrow m_2 = \frac{2 - \sqrt{3}}{(2\sqrt{3} + 1)} \text{ or } m_2 = \frac{-(2 + \sqrt{3})}{(2\sqrt{3} - 1)}$$

Case I : $m_2 = \left(\frac{2 - \sqrt{3}}{2\sqrt{3} + 1}\right)$

The equation of the line passing through point (2, 3) and having a slope of $\frac{(2 - \sqrt{3})}{(2\sqrt{3} + 1)}$ is

$$(y - 3) = \frac{2 - \sqrt{3}}{2\sqrt{3} + 1}(x - 2)$$
$$(2\sqrt{3} + 1)y - 3(2\sqrt{3} + 1) = (2 - \sqrt{3})x - 2(2 - \sqrt{3})$$
$$(\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = -4 + 2\sqrt{3} + 6\sqrt{3} + 3$$
$$(\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = -1 + 8\sqrt{3}$$

In this case, the equation of the other line is $(\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = -1 + 8\sqrt{3}$.

Case II : $m_2 = \frac{-(2 + \sqrt{3})}{(2\sqrt{3} - 1)}$

The equation of the line passing through point (2, 3) and having a slope of $\frac{-(2 + \sqrt{3})}{(2\sqrt{3} - 1)}$ is

$$(y - 3) = \frac{-(2 + \sqrt{3})}{(2\sqrt{3} - 1)}(x - 2)$$
$$(2\sqrt{3} - 1)y - 3(2\sqrt{3} - 1) = -(2 + \sqrt{3})x + 2(2 + \sqrt{3})$$
$$(2\sqrt{3} - 1)y + (2 + \sqrt{3})x = 4 + 2\sqrt{3} + 6\sqrt{3} - 3$$
$$(2 + \sqrt{3})x + (2\sqrt{3} - 1)y = 1 + 8\sqrt{3}$$

In this case, the equation of the other line is $(2 + \sqrt{3})x + (2\sqrt{3} - 1)y = 1 + 8\sqrt{3}$.

Thus, the required equation of the other line is $(\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = -1 + 8\sqrt{3}$ or $(2 + \sqrt{3})x + (2\sqrt{3} - 1)y = 1 + 8\sqrt{3}$.

Question-13

Find the equation of the right bisector of the line segment joining the points (3, 4) and (–1, 2).

Ans.

The right bisector of a line segment bisects the line segment at 90°.

The end-points of the line segment are given as A (3, 4) and B (–1, 2).

Accordingly, mid-point of AB $=\left(\frac{3-1}{2},\frac{4+2}{2}\right)=(1,3)$

Slope of AB $=\frac{2-4}{-1-3}=\frac{-2}{-4}=\frac{1}{2}$

$\therefore$ Slope of the line perpendicular to AB $=-\frac{1}{\left(\frac{1}{2}\right)}=-2$

The equation of the line passing through (1, 3) and having a slope of –2 is

$(y - 3) = -2\,(x - 1)$

$y - 3 = -2x + 2$

$2x + y = 5$

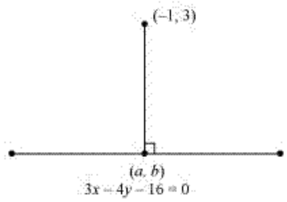
Thus, the required equation of the line is $2x + y = 5$.

Question-14

Find the coordinates of the foot of perpendicular from the point (–1, 3) to the line $3x - 4y - 16 = 0$.

Ans.

Let (a, b) be the coordinates of the foot of the perpendicular from the point (–1, 3) to the line $3x - 4y - 16 = 0$.



Slope of the line joining (–1, 3) and (a, b), $m_1=\frac{b-3}{a+1}$

Slope of the line $3x - 4y - 16 = 0$ or $y=\frac{3}{4}x-4,\ m_2=\frac{3}{4}$

Since these two lines are perpendicular, $m_1m_2 = -1$

$$\therefore\left(\frac{b-3}{a+1}\right)\times\left(\frac{3}{4}\right)=-1$$
$$\Rightarrow\frac{3b-9}{4a+4}=-1$$
$$\Rightarrow 3b-9=-4a-4$$
$$\Rightarrow 4a+3b=5 \qquad ...(1)$$

Point (a, b) lies on line $3x - 4y = 16$.

$\therefore 3a - 4b = 16$ ... (2)

On solving equations (1) and (2), we obtain

$$a=\frac{68}{25}\text{ and }b=-\frac{49}{25}$$

Thus, the required coordinates of the foot of the perpendicular are $\left(\frac{68}{25},-\frac{49}{25}\right)$.

Question-15

The perpendicular from the origin to the line $y = mx + c$ meets it at the point

$(-1, 2)$. Find the values of m and c.

Ans.

The given equation of line is $y = mx + c$.

It is given that the perpendicular from the origin meets the given line at $(-1, 2)$.

Therefore, the line joining the points $(0, 0)$ and $(-1, 2)$ is perpendicular to the given line.

$\therefore$ Slope of the line joining $(0, 0)$ and $(-1, 2)$ $= \frac{2}{-1} = -2$

The slope of the given line is m.

$$\therefore m \times -2 = -1 \qquad \left[\text{The two lines are perpendicular}\right]$$
$$\Rightarrow m = \frac{1}{2}$$

Since point $(-1, 2)$ lies on the given line, it satisfies the equation $y = mx + c$.

$$\therefore 2 = m(-1) + c$$
$$\Rightarrow 2 = \frac{1}{2}(-1) + c$$
$$\Rightarrow c = 2 + \frac{1}{2} = \frac{5}{2}$$

Thus, the respective values of m and c are $\frac{1}{2}$ and $\frac{5}{2}$.

Question-16

If p and q are the lengths of perpendiculars from the origin to the lines $x \cos\theta - y \sin\theta = k \cos 2\theta$ and $x \sec\theta + y \operatorname{cosec}\theta = k$, respectively, prove that $p^2 + 4q^2 = k^2$

Ans.

The equations of given lines are

$x \cos\theta - y \sin\theta = k \cos 2\theta$ ... (1)

$x \sec\theta + y \operatorname{cosec}\theta = k$ ... (2)

The perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$.

On comparing equation (1) to the general equation of line i.e., $Ax + By + C = 0$, we obtain $A = \cos\theta$, $B = -\sin\theta$, and $C = -k \cos 2\theta$.

It is given that p is the length of the perpendicular from $(0, 0)$ to line (1).

$$\therefore p = \frac{|A(0) + B(0) + C|}{\sqrt{A^2 + B^2}} = \frac{|C|}{\sqrt{A^2 + B^2}} = \frac{|-k \cos 2\theta|}{\sqrt{\cos^2\theta + \sin^2\theta}} = |-k \cos 2\theta| \qquad ...(3)$$

On comparing equation (2) to the general equation of line i.e., $Ax + By + C = 0$, we obtain $A = \sec\theta$, $B = \operatorname{cosec}\theta$, and $C = -k$.

It is given that q is the length of the perpendicular from $(0, 0)$ to line (2).

$$\therefore q=\frac{|A(0)+B(0)+C|}{\sqrt{A^2+B^2}}=\frac{|C|}{\sqrt{A^2+B^2}}=\frac{|-k|}{\sqrt{\sec^2\theta+\operatorname{cosec}^2\theta}} \quad ...(4)$$

From (3) and (4), we have

$$p^2+4q^2=(|-k\cos 2\theta|)^2+4\left(\frac{|-k|}{\sqrt{\sec^2\theta+\operatorname{cosec}^2\theta}}\right)^2$$

$$=k^2\cos^2 2\theta+\frac{4k^2}{(\sec^2\theta+\operatorname{cosec}^2\theta)}$$

$$=k^2\cos^2 2\theta+\frac{4k^2}{\left(\frac{1}{\cos^2\theta}+\frac{1}{\sin^2\theta}\right)}$$

$$=k^2\cos^2 2\theta+\frac{4k^2}{\left(\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta\cos^2\theta}\right)}$$

$$=k^2\cos^2 2\theta+\frac{4k^2}{\left(\frac{1}{\sin^2\theta\cos^2\theta}\right)}$$

$$=k^2\cos^2 2\theta+4k^2\sin^2\theta\cos^2\theta$$

$$=k^2\cos^2 2\theta+k^2(2\sin\theta\cos\theta)^2$$

$$=k^2\cos^2 2\theta+k^2\sin^2 2\theta$$

$$=k^2(\cos^2 2\theta+\sin^2 2\theta)$$

$$=k^2$$

Hence, we proved that $p^2+4q^2=k^2$.

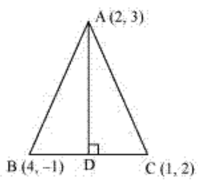
Question-17

In the triangle ABC with vertices A (2, 3), B (4, −1) and C (1, 2), find the equation and length of altitude from the vertex A.

Ans.

Let AD be the altitude of triangle ABC from vertex A.

Accordingly, AD⊥BC



The equation of the line passing through point (2, 3) and having a slope of 1 is

$(y-3)=1(x-2)$

$\Rightarrow x-y+1=0$

$\Rightarrow y-x=1$

Therefore, equation of the altitude from vertex A = $y-x=1$.

Length of AD = Length of the perpendicular from A (2, 3) to BC

The equation of BC is

$$(y+1)=\frac{2+1}{1-4}(x-4)$$
$$\Rightarrow (y+1)=-1(x-4)$$
$$\Rightarrow y+1=-x+4$$
$$\Rightarrow x+y-3=0 \qquad ...(1)$$

The perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by $d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$.

On comparing equation (1) to the general equation of line $Ax + By + C = 0$, we obtain $A = 1$, $B = 1$, and $C = -3$.

$\therefore$ Length of AD $=\frac{|1\times 2+1\times 3-3|}{\sqrt{1^2+1^2}}$ units $=\frac{|2|}{\sqrt{2}}$ units $=\frac{2}{\sqrt{2}}$ units $=\sqrt{2}$ units

Thus, the equation and the length of the altitude from vertex A are $y - x = 1$ and $\sqrt{2}$ units respectively.

Question-18

If p is the length of perpendicular from the origin to the line whose intercepts on the axes are a and b, then show that $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$.

Ans.

It is known that the equation of a line whose intercepts on the axes are a and b is

$$\frac{x}{a}+\frac{y}{b}=1$$
$$\text{or } bx+ay=ab$$
$$\text{or } bx+ay-ab=0 \qquad ...(1)$$

The perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by $d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$.

On comparing equation (1) to the general equation of line $Ax + By + C = 0$, we obtain $A = b$, $B = a$, and $C = -ab$.

Therefore, if p is the length of the perpendicular from point $(x_1, y_1) = (0, 0)$ to line (1), we obtain

$$p=\frac{|A(0)+B(0)-ab|}{\sqrt{b^2+a^2}}$$
$$\Rightarrow p=\frac{|-ab|}{\sqrt{a^2+b^2}}$$

On squaring both sides, we obtain

$$p^2=\frac{(-ab)^2}{a^2+b^2}$$
$$\Rightarrow p^2\left(a^2+b^2\right)=a^2b^2$$
$$\Rightarrow \frac{a^2+b^2}{a^2b^2}=\frac{1}{p^2}$$
$$\Rightarrow \frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$$

Hence, we showed that $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$.

*********** END **********