Exponents of Real Numbers Ex 2.1 Q1

Answer :

We have to simplify the following, assuming that x, y, z are positive real numbers

(i) Given $\left(\sqrt{x^{-3}}\right)^5$

As x is positive real number then we have

$$\left(\sqrt{x^{-3}}\right)^5 = \left(\sqrt{\frac{1}{x^3}}\right)^5$$

$$= \left(\frac{\sqrt{1}}{\sqrt{x^3}}\right)^5$$

$$= \left(\frac{1}{x^{3\times\frac{1}{2}}}\right)^5$$

$$= \left(\frac{1}{x^{3\times\frac{1}{2}}}\right)^5$$

$$\left(\sqrt{x^{-3}}\right)^5 = \left(\frac{1^5}{x^{\frac{3}{2}\times 5}}\right)$$

$$= \frac{1}{x^{\frac{15}{2}}}$$

Hence the simplified value of $\left(\sqrt{x^{-3}}\right)^5$ is $\boxed{\dfrac{1}{x^{\frac{15}{2}}}}$

(ii) Given $\sqrt{x^3y^{-2}}$

As x and y are positive real numbers then we can write

$$\begin{aligned}\sqrt{x^3y^{-2}} &= \left(x^3y^{-2}\right)^{\frac{1}{2}}\\ &= \left(x^{3\times\frac{1}{2}}\times y^{-2\times\frac{1}{2}}\right)\\ &= \left(x^{3\times\frac{1}{2}}\times y^{-\cancel{2}\times\frac{1}{\cancel{2}}}\right)\\ &= \left(x^{\frac{3}{2}}y^{-1}\right)\end{aligned}$$

By using law of rational exponents $a^{-n}=\dfrac{1}{a^n}$ we have

$$\begin{aligned}\sqrt{x^3y^{-2}} &= x^{\frac{3}{2}}\times\frac{1}{y}\\ &= \frac{x^{\frac{3}{2}}}{y}\end{aligned}$$

Hence the simplified value of $\sqrt{x^3y^{-2}}$ is $\boxed{\dfrac{x^{\frac{3}{2}}}{y}}$

(iii) Given $\left(x^{\frac{-2}{3}}y^{\frac{-1}{2}}\right)^2$

As x and y are positive real numbers then we have

$$\left(x^{\frac{-2}{3}}y^{\frac{-1}{2}}\right)^2=\left(x^{\frac{-2}{3}}\times x^{\frac{-2}{3}}\times y^{\frac{-1}{2}}\times y^{\frac{-1}{2}}\right)$$

By using law of rational exponents $a^{-n}=\dfrac{1}{a^n}$ we have

$$\left(x^{\frac{-2}{3}}y^{\frac{-1}{2}}\right)^2=\frac{1}{x^{\frac{2}{3}}}\times\frac{1}{x^{\frac{2}{3}}}\times\frac{1}{y^{\frac{1}{2}}}\times\frac{1}{y^{\frac{1}{2}}}$$

$$\Rightarrow\left(x^{\frac{-2}{3}}y^{\frac{-1}{2}}\right)^2=\frac{1}{x^{\frac{2}{3}}\times x^{\frac{2}{3}}}\times\frac{1}{y^{\frac{1}{2}}\times y^{\frac{1}{2}}}$$

By using law of rational exponents $a^m\times a^n=a^{m+n}$ we have

$$\left(x^{\frac{-2}{3}} y^{\frac{-1}{2}}\right)^2 = \frac{1}{x^{\frac{2}{3}+\frac{2}{3}}} \times \frac{1}{y^{\frac{1}{2}+\frac{1}{2}}}$$

$$= \frac{1}{x^{\frac{4}{3}}} \times \frac{1}{y^{\frac{2}{2}}} = \frac{1}{x^{\frac{4}{3}}} \times \frac{1}{y}$$

$$= \frac{1}{x^{\frac{4}{3}} y}$$

Hence the simplified value of $\left(x^{\frac{-2}{3}} y^{\frac{-1}{2}}\right)^2$ is $\boxed{\frac{1}{x^{\frac{4}{3}} y}}$.

(iv) $\left(\sqrt{x}\right)^{-\frac{2}{3}} \sqrt{y^4} \div \sqrt{xy^{-\frac{1}{2}}}$

$$= \left(x^{\frac{1}{2}}\right)^{-\frac{2}{3}} \left(y^4\right)^{\frac{1}{2}} \div \left(x \times y^{-\frac{1}{2}}\right)^{\frac{1}{2}}$$

$$= \frac{x^{\frac{1}{2} \times -\frac{2}{3}} \times y^{4 \times \frac{1}{2}}}{x^{\frac{1}{2}} \times y^{-\frac{1}{2} \times \frac{1}{2}}}$$

$$= \frac{x^{-\frac{1}{3}} \times y^2}{x^{\frac{1}{2}} \times y^{-\frac{1}{4}}}$$

by using the law of rational exponents, $a^m \div a^n = a^{m-n}$, we have

$$x^{-\frac{1}{3}-\frac{1}{2}} \times y^{2+\frac{1}{4}}$$

$$= x^{-\frac{5}{6}} \times y^{\frac{9}{4}}$$

$$= \frac{y^{\frac{9}{4}}}{x^{\frac{5}{6}}}$$

(v). $\sqrt[5]{243 \; x^{10} \, y^5 \, z^{10}}$

$$= \left(243 \times x^{10} \times y^5 \times z^{10}\right)^{\frac{1}{5}}$$

$$= (243)^{\frac{1}{5}} \times \left(x^{10}\right)^{\frac{1}{5}} \times \left(y^5\right)^{\frac{1}{5}} \times \left(z^{10}\right)^{\frac{1}{5}}$$

$$= \left(3^5\right)^{\frac{1}{5}} \times x^{10 \times \frac{1}{5}} \times y^{5 \times \frac{1}{5}} \times z^{10 \times \frac{1}{5}}$$

$$= 3 \times x^2 \times y \times z^2$$

$$= 3x^2yz^2$$

$$(\text{vi})\ \left(\frac{x^{-4}}{y^{-10}}\right)^{\frac{5}{4}}$$

$$=\frac{\left(x^{-4}\right)^{\frac{5}{4}}}{\left(y^{-10}\right)^{\frac{5}{4}}}$$

$$=\frac{x^{-4\times\frac{5}{4}}}{y^{-10\times\frac{5}{4}}}$$

$$=\frac{x^{-5}}{y^{-\frac{25}{2}}}$$

$$=\frac{y^{\frac{25}{2}}}{x^{5}}$$

*********** END **********