

coordinates, (2, -1, 2). The point is (-1, -5, -10).

The distance d between the points, (2, -1, 2) and (-1, -5, -10), is

$$d = \sqrt{(-1-2)^2 + (-5+1)^2 + (-10-2)^2} = \sqrt{9+16+144} = \sqrt{169} = 13$$

Question 19:

Find the vector equation of the line passing through (1, 2, 3) and parallel to the planes

$$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) = 5$$
 and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$

Let the required line be parallel to vector \vec{b} given by,

$$\vec{b} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$$

The position vector of the point (1, 2, 3) is $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

The equation of line passing through (1, 2, 3) and parallel to \vec{b} is given by,

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

$$\Rightarrow \vec{r} \left(\hat{i} + 2\hat{j} + 3\hat{k} \right) + \lambda \left(b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k} \right) \qquad \dots (1)$$

The equations of the given planes are

$$\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5 \qquad \dots (2)$$

$$\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$$
 ...(3)

The line in equation (1) and plane in equation (2) are parallel. Therefore, the normal to the plane of equation (2) and the given line are perpendicular.

$$\Rightarrow (\hat{i} - \hat{j} + 2\hat{k}) \cdot \lambda (b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}) = 0$$

$$\Rightarrow \lambda (b_1 - b_2 + 2b_3) = 0$$

$$\Rightarrow b_1 - b_2 + 2b_3 = 0 \qquad \dots (4)$$

Similarly,
$$(3\hat{i} + \hat{j} + \hat{k}) \cdot \lambda (b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) = 0$$

$$\Rightarrow \lambda (3b_1 + b_2 + b_3) = 0$$

$$\Rightarrow 3b_1 + b_2 + b_3 = 0 \qquad \dots (5)$$

From equations (4) and (5), we obtain

$$\frac{b_1}{(-1)\times 1 - 1\times 2} = \frac{b_2}{2\times 3 - 1\times 1} = \frac{b_3}{1\times 1 - 3(-1)}$$

$$\Rightarrow \frac{b_1}{-3} = \frac{b_2}{5} = \frac{b_3}{4}$$

Therefore, the direction ratios of \vec{b} are -3, 5, and 4.

$$\vec{b} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k} = -3\hat{i} + 5\hat{j} + 4\hat{k}$$

Substituting the value of \vec{b} in equation (1), we obtain

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda \left(-3\hat{i} + 5\hat{j} + 4\hat{k}\right)$$

This is the equation of the required line.

Ouestion 20:

Find the vector equation of the line passing through the point (1, 2, -4) and

perpendicular to the two lines:
$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$$
 and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

Let the required line be parallel to the vector \vec{b} given by, $\vec{b}=b_{\rm l}\hat{i}+b_{\rm 2}\hat{j}+b_{\rm 3}\hat{k}$

The position vector of the point (1, 2, - 4) is $\vec{a}=\hat{l}+2\hat{j}-4\hat{k}$

The equation of the line passing through (1, 2, -4) and parallel to vector \vec{b} is

$$\vec{r} = \vec{a} + 4\vec{b}$$

$$\Rightarrow \vec{r} (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda (b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$$
 ...(1)

The equations of the lines are

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \qquad \dots (2)$$

$$\frac{x-15}{2} = \frac{y-29}{2} = \frac{z-5}{5} \qquad \dots (3)$$

$$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$
 ...(3)

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Line (1) and line (2) are perpendicular to each other.

$$\therefore 3b_1 - 16b_2 + 7b_3 = 0$$

Also, line (1) and line (3) are perpendicular to each other.

$$3b_1 + 8b_2 - 5b_3 = 0$$

From equations (4) and (5), we obtain

$$\frac{b_1}{(-16)(-5) - 8 \times 7} = \frac{b_2}{7 \times 3 - 3(-5)} = \frac{b_3}{3 \times 8 - 3(-16)}$$

$$\Rightarrow \frac{b_1}{24} = \frac{b_2}{36} = \frac{b_3}{72}$$

$$\Rightarrow \frac{b_1}{2} = \frac{b_2}{3} = \frac{b_3}{6}$$

 ${\scriptstyle ...} {\rm Direction}$ ratios of \vec{b} are 2, 3, and 6.

$$\therefore \vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

Substituting $\vec{b}=2\hat{i}+3\hat{j}+6\hat{k}$ in equation (1), we obtain

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda (2\hat{i} + 3\hat{j} + 6\hat{k})$$

This is the equation of the required line.

Question 21:

Prove that if a plane has the intercepts a, b, c and is at a distance of P units from the

origin, then
$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$$

Answer

The equation of a plane having intercepts a, b, c with x, y, and z axes respectively is given by.

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$
 ...(1)

The distance (p) of the plane from the origin is given by,

$$p = \frac{\frac{0}{a} + \frac{0}{b} + \frac{0}{c} - 1}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2}}$$

$$\Rightarrow p = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}$$

$$\Rightarrow p^2 = \frac{1}{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}$$

$$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$$

Question 22:

Distance between the two planes: 2x+3y+4z=4 and 4x+6y+8z=12 is

(A)2 units (B)4 units (C)8 units

(D)
$$\frac{2}{\sqrt{29}}$$
 units

Answer

The equations of the planes are

$$2x + 3y + 4z = 4$$

$$4x + 6y + 8z = 12$$

$$\Rightarrow 2x + 3y + 4z = 6 \qquad \dots (2)$$

It can be seen that the given planes are parallel.

It is known that the distance between two parallel planes, $ax + by + cz = d_1$ and $ax + by + cz = d_2$, is given by,

$$D = \left| \frac{d_2 - d_1}{\sqrt{a^2 + b^2 + c^2}} \right|$$

$$\Rightarrow D = \left| \frac{6 - 4}{\sqrt{(2)^2 + (3)^2 + (4)^2}} \right|$$

$$D = \frac{2}{\sqrt{(2)^2 + (3)^2 + (4)^2}}$$

Thus, the distance between the lines is $\frac{2}{\sqrt{29}}$ units.

Hence, the correct answer is D.

Question 23:

The planes: 2x - y + 4z = 5 and 5x - 2.5y + 10z = 6 are

(A) Perpendicular (B) Parallel (C) intersect y-axis

(C) passes through
$$\left(0,0,\frac{5}{4}\right)$$

The equations of the planes are

$$2x - y + 4z = 5 \dots (1)$$

$$5x - 2.5y + 10z = 6 \dots (2)$$

It can be seen that,

$$\begin{aligned} \frac{a_1}{a_2} &= \frac{2}{5} \\ \frac{b_1}{b_2} &= \frac{-1}{-2.5} = \frac{2}{5} \\ \frac{c_1}{c_2} &= \frac{4}{10} = \frac{2}{5} \\ \frac{a_1}{a_2} &= \frac{b_1}{b_2} = \frac{c_1}{c_2} \end{aligned}$$

Therefore, the given planes are parallel. Hence, the correct answer is B.

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