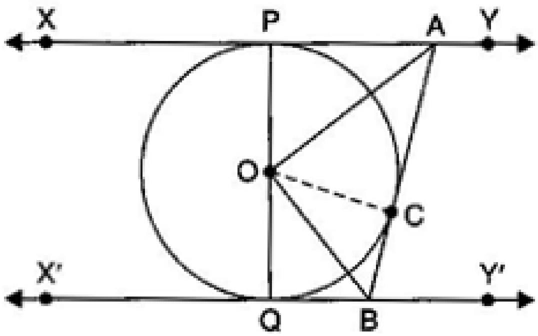
To Prove: $\angle AOB = 90°$

Construction: Join OC

Proof: $\angle OPA = 90°$..........(i)



$\angle OCA = 90°$..........(ii)

[Tangent at any point of a circle is $\perp$ to the radius through the point of contact]

In right angled triangles OPA and OCA,

OA = OA [Common]

AP = AC [Tangents from an external point to a circle are equal]

$\therefore \Delta OPA \cong \Delta OCA$

[RHS congruence criterion]

$\therefore \angle OAP = \angle OAC$ [By C.P.C.T.]

$\Rightarrow \angle OAC = \frac{1}{2} \angle PAB$ ....(iii)

Similarly, $\angle OBQ = \angle OBC$

$\Rightarrow \angle OBC = \frac{1}{2}\angle QBA$ ....(iv)

$\because$ XY || X'Y' and a transversal AB intersects them.

$\therefore \angle PAB + \angle QBA = 180°$

[Sum of the consecutive interior angles on the same side of the transversal is 180°]

$\Rightarrow \frac{1}{2}\angle PAB + \frac{1}{2}\angle QBA$

$= \frac{1}{2}\times 180°$ ....(v)

$\Rightarrow \angle OAC + \angle OBC = 90°$

[From eq. (iii) & (iv)]

In ΔAOB,

$\angle OAC + \angle OBC + \angle AOB = 180°$

[Angel sum property of a triangle]

$\Rightarrow 90° + \angle AOB = 180°$ [From eq. (v)]

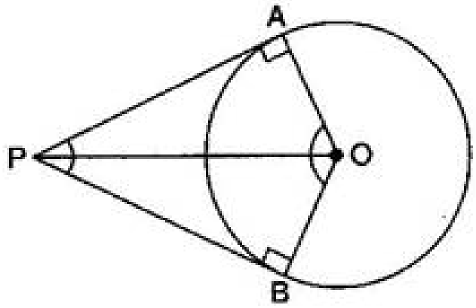
$\Rightarrow \angle AOB = 90^\circ$

Hence proved.

10. Prove that the angel between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Ans: $\angle OPA = 90^\circ$..........(i)

$\angle OCA = 90^\circ$..........(ii)



[Tangent at any point of a circle is $\perp$ to the radius through the point of contact]

$\because$ OAPB is quadrilateral.

$\therefore \angle APB + \angle AOB + \angle OAP + \angle OBP = 360^\circ$

[Angle sum property of a quadrilateral]

$\Rightarrow \angle APB + \angle AOB + 90^\circ + 90^\circ = 360^\circ$

[From eq. (i) & (ii)]

$\Rightarrow \angle APB + \angle AOB = 180^\circ$

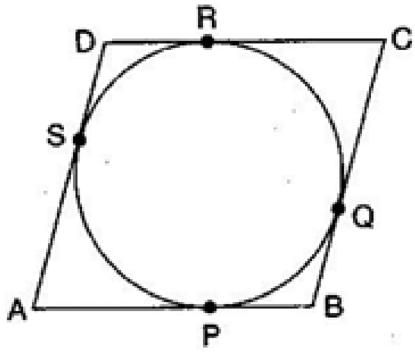
$\therefore \angle APB$ and $\angle AOB$ are supplementary.

Q11. Prove that the parallelogram circumscribing a circle is a rhombus.
Ans: Given: ABCD is a parallelogram circumscribing a circle.

To Prove: ABCD is a rhombus.

Proof: Since, the tangents from an external point to a circle are equal.

$\therefore AP = AS$ ....(i)



$BP = BQ$ ....(ii)

$CR = CQ$ ....(iii)

$DR = DS$..........(iv)

On adding eq. (i), (ii), (iii) and (iv), we get

$(AP + BP) + (CR + DR)$

$= (AS + BQ) + (CQ + DS)$

$\Rightarrow AB + CD = (AS + DS) + (BQ + CQ)$

$\Rightarrow AB + CD = AD + BC$

$\Rightarrow AB + AB = AD + AD$

[Opposite sides of $\parallel$gm are equal]

$\Rightarrow 2AB = 2AD$

$\Rightarrow AB = AD$

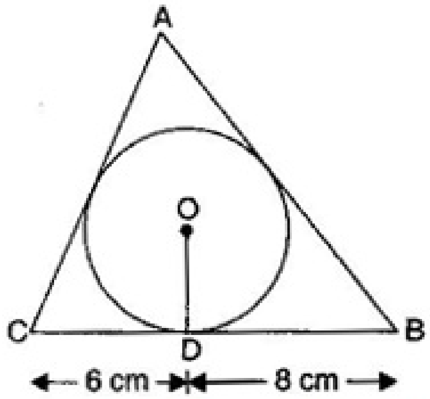
But $AB = CD$ and $AD = BC$

[Opposite sides of $\parallel$gm]

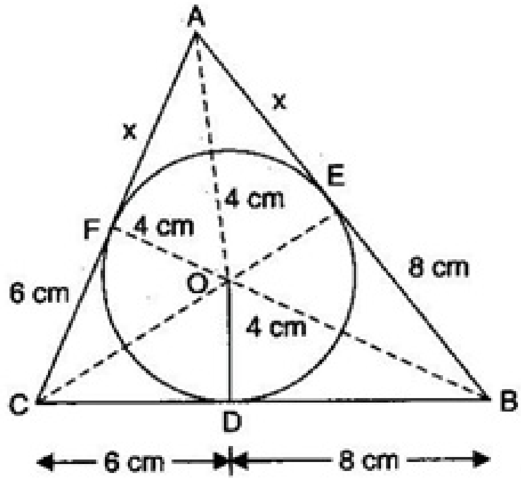
$\therefore AB = BC = CD = AD$

$\therefore$ Parallelogram ABCD is a rhombus.

Q12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see figure). Find the sides AB and AC.



Ans: Join OE and OF. Also join OA, OB and OC.



Since BD = 8 cm

∴ BE = 8 cm

[Tangents from an external point to a circle are equal]

Since CD = 6 cm

$\therefore$ CF = 6 cm

[Tangents from an external point to a circle are equal]

Let AE = AF = x

Since OD = OE = OF = 4 cm

[Radii of a circle are equal]

$\therefore$ Semi-perimeter of ΔABC = $\frac{(x+6)+(x+8)+(6+8)}{2}$ = $(x+14)$ cm

$\therefore$ Area of Δ ABC = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{(x+14)(x+14-14)\left(x+14-\overline{x+8}\right)\left(x+14-\overline{x+6}\right)}$

$= \sqrt{(x+14)(x)(6)(8)}$ cm^2

Now, Area of ΔABC = Area of ΔOBC + Area of ΔOCA + Area of ΔOAB

$\Rightarrow \sqrt{(x+14)(x)(6)(8)}$

$= \frac{(6+8)4}{2} + \frac{(x+6)4}{2} + \frac{(x+8)4}{2}$

$\Rightarrow \sqrt{(x+14)(x)(6)(8)}$

$= 28 + 2x + 12 + 2x + 16$

$\Rightarrow \sqrt{(x+14)(x)(6)(8)} = 4x+56$

$\Rightarrow \sqrt{(x+14)(x)(6)(8)} = 4(x+14)$

Squaring both sides,

$(x+14)(x)(6)(8) = 16(x+14)^2$

$\Rightarrow 3x = x+14$

$\Rightarrow 2x = 14$

$\Rightarrow x = 7$

$\therefore$ AB = $x+8$ = 7 + 8 = 15 cm

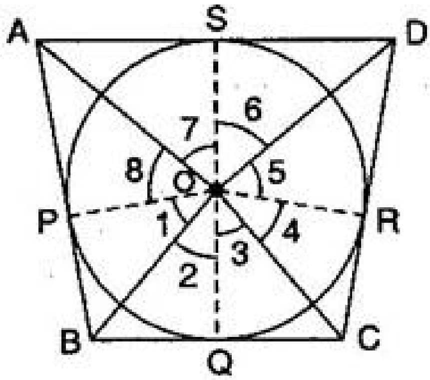
And AC = $x+6$ = 7 + 6 = 13 cm

13. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Ans: Given: ABCD is a quadrilateral circumscribing a circle whose centre is O.

To prove: (i) ∠AOB + ∠COD = (ii) ∠BOC + ∠AOD = 180°

Construction: Join OP, OQ, OR and OS.



Proof: Since tangents from an external point to a circle are equal.

∴ AP = AS,

BP = BQ ....(i)

CQ = CR

DR = DS

In ΔOBP and ΔOBQ,

OP = OQ [Radii of the same circle]

OB = OB [Common]

BP = BQ [From eq. (i)]

∴ ΔOPB ≅ ΔOBQ [By SSS congruence criterion]

∴ $\angle 1 = \angle 2$ [By C.P.C.T.]

Similarly, $\angle 3 = \angle 4$, $\angle 5 = \angle 6$, $\angle 7 = \angle 8$

Since, the sum of all the angles round a point is equal to $360°$.

$\therefore \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

$\Rightarrow \angle 1 + \angle 1 + \angle 4 + \angle 4 + \angle 5 + \angle 5 + \angle 8 + \angle 8 = 360°$

$\Rightarrow 2(\angle 1 + \angle 4 + \angle 5 + \angle 8) = 360°$

$\Rightarrow \angle 1 + \angle 4 + \angle 5 + \angle 8 = 180°$

$\Rightarrow (\angle 1 + \angle 5) + (\angle 4 + \angle 8) = 180°$

$\Rightarrow$ ∠AOB + ∠COD = $180°$

Similarly, we can prove that

∠BOC + ∠AOD = 180°

*********** END **********