Complex Numbers Ex 13.4 Q1(vii)

The polar form of a complex number $z = x + iy$, is given by $z = |z|(\cos\theta + i\sin\theta)$ where,

$|z| = \sqrt{x^2 + y^2}$ and

$\arg(z) = \theta = \tan^{-1}\left(\dfrac{b}{a}\right)$

let $z = \sin 120° - i\cos 120°$

$\qquad = \sin\left(\dfrac{\pi}{2} + \dfrac{\pi}{6}\right) - i\cos\left(\dfrac{\pi}{2} + \dfrac{\pi}{6}\right) \qquad \left(\because 120° = \dfrac{\pi}{2} + \dfrac{\pi}{6}\right)$

$\Rightarrow z = \cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6} \qquad\qquad \left(\because \sin\left(\dfrac{\pi}{2} + \theta\right) = \cos\theta \ \& \ \cos\left(\dfrac{\pi}{2} + \theta\right) = -\sin\theta\right)$

Here z is already in polar form

with $|z| = 1$ & $\theta = \arg(z) = \dfrac{\pi}{6}$

Complex Numbers Ex 13.4 Q1(viii)

The polar form of a complex number $z = x + iy$, is given by $z = |z|(\cos\theta + i\sin\theta)$ where,

$|z| = \sqrt{x^2 + y^2}$ and

$\arg(z) = \theta = \tan^{-1}\left(\dfrac{b}{a}\right)$

let $z = \dfrac{-16}{1 + i\sqrt{3}}$

$= \dfrac{-16}{1 + i\sqrt{3}} \times \dfrac{1 - i\sqrt{3}}{1 - i\sqrt{3}}$

$= \dfrac{-16\left(1 - i\sqrt{3}\right)}{(1)^2 + \left(\sqrt{3}\right)^2}$

$= \dfrac{-16\left(1 - i\sqrt{3}\right)}{1 + 3}$

$= \dfrac{-16}{4}\left(1 - i\sqrt{3}\right)$

$= -4\left(1 - i\sqrt{3}\right)$

$= -4 + 4\sqrt{3}i$

$\therefore |z| = \sqrt{(-4)^2 + \left(4\sqrt{3}\right)^2}$

$= \sqrt{16 + 48}$

$= \sqrt{64}$

$= 8$

Here $x = -4 < 0$ & $y = 4\sqrt{3} > 0$, $\therefore \theta$ lies in quadrant II

$\theta = \arg(z) = \tan^{-1}\left(\dfrac{4\sqrt{3}}{-4}\right)$

$= \tan^{-1}\left(-\sqrt{3}\right)$

$= \tan^{-1}\left(-\tan\dfrac{\pi}{3}\right)$

$= \tan^{-1}\left(\tan\left(\pi - \dfrac{\pi}{3}\right)\right)$ $\left(\because \tan(\pi - \theta) = -\tan\theta\right)$

$= \pi - \dfrac{\pi}{3}$

$= \dfrac{2\pi}{3}$

The polar form is given by $z = 8\left(\cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3}\right)$

Complex Numbers Ex 13.4 Q2

$z = \left(i^{25}\right)^3 = (i)^3 = -i$

$|z| = 1,$

$\arg(z) = \tan^{-1}\left(\dfrac{-1}{0}\right) = -\dfrac{\pi}{2}$

Polar Form: $\cos\left(\dfrac{\pi}{2}\right) - i\sin\left(\dfrac{\pi}{2}\right) = \cos\left(-\dfrac{\pi}{2}\right) + i\sin\left(-\dfrac{\pi}{2}\right)$

Complex Numbers Ex 13.4 Q3(i)

Let $z = 1 + i\tan\alpha$

$\tan\alpha$ is periodic function with period π.

So, let us take α lying in the interval $\left[0, \dfrac{\pi}{2}\right) \cup \left(\dfrac{\pi}{2}, \pi\right]$.

Case – I : When $\alpha \in \left[0, \dfrac{\pi}{2}\right)$

$|z| = \sqrt{1 + \tan^2\alpha} = \sqrt{\sec^2\alpha} = |\sec\alpha| = \sec\alpha$

Let β be acute angle given by $\tan\beta = \dfrac{|Im(z)|}{|Re(z)|}$.

$\tan\beta = |\tan\alpha| = \tan\alpha$

$\Rightarrow \beta = \alpha$

As z is represented by a point in first quadrant.

$\therefore \arg(z) = \beta = \alpha$.

So polar form of z is $\sec\alpha(\cos\alpha + i\sin\alpha)$

Case – II : When $\alpha \in \left(\dfrac{\pi}{2}, \pi\right]$

$|z| = \sqrt{1 + \tan^2\alpha} = \sqrt{\sec^2\alpha} = |\sec\alpha| = -\sec\alpha$

Let β be acute angle given by $\tan\beta = \dfrac{|Im(z)|}{|Re(z)|}$.

$\tan\beta = |\tan\alpha| = -\tan\alpha = \tan(\pi - \alpha)$

$\Rightarrow \beta = \pi - \alpha$

As z is represented by a point in fourth quadrant.

$\therefore \arg(z) = -\beta = \alpha - \pi$.

So polar form of z is $-\sec\alpha\big(\cos(\alpha - \pi) + i\sin(\alpha - \pi)\big)$.

********** END *********