Indefinite Integrals Ex 19.30 Q17

Let $I = \int \frac{1}{(x-1)(x+1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+2}$

$$\Rightarrow \quad 1 = A(x+1)(x+2) + B(x-1)(x+2) + C(x^2-1)$$

Put $x = 1$

$$\Rightarrow \quad 1 = 6A \Rightarrow \quad A = \frac{1}{6}$$

Put $x = -1$

$$\Rightarrow \quad 1 = -2B \quad \Rightarrow \quad B = -\frac{1}{2}$$

Put $x = -2$

$$\Rightarrow \quad 1 = 3C \Rightarrow \quad C = \frac{1}{3}$$

So,

$$I = \frac{1}{6}\int \frac{dx}{x-1} - \frac{1}{2}\int \frac{dx}{x+1} + \frac{1}{3}\int \frac{dx}{x+2}$$

$$I = \frac{1}{6}\log|x-1| - \frac{1}{2}\log|x+1| + \frac{1}{3}\log|x+2| + c$$

Indefinite Integrals Ex 19.30 Q18

Consider the integral

$$I=\int \frac{x^2}{(x^2+4)(x^2+9)}dx$$

Now let us separate the fraction $\frac{x^2}{(x^2+4)(x^2+9)}$ through partial fractions.

Substitute $x^2 = t$

$$\frac{x^2}{(x^2+4)(x^2+9)} = \frac{t}{(t+4)(t+9)}$$

$$\Rightarrow \frac{t}{(t+4)(t+9)} = \frac{A}{t+4} + \frac{B}{t+9}$$

$$\Rightarrow \frac{t}{(t+4)(t+9)} = \frac{A(t+9)+B(t+4)}{(t+4)(t+9)}$$

$$\Rightarrow t = A(t+9)+B(t+4)$$

$$\Rightarrow t = At + 9A + Bt + 4B$$

Comparing the coefficients, we have,

A+B=1 and 9A+4B=0

$$\Rightarrow A=-\frac{4}{5} \text{ and } B=\frac{9}{5}$$

$$\Rightarrow \frac{x^2}{(x^2+4)(x^2+9)} = -\frac{4}{5(t+4)} + \frac{9}{5(t+9)}$$

$$\Rightarrow \frac{x^2}{(x^2+4)(x^2+9)} = -\frac{4}{5(x^2+4)} + \frac{9}{5(x^2+9)}$$

Thus, we have,

$$I=\int \frac{x^2}{(x^2+4)(x^2+9)}dx$$

$$= \int \left[-\frac{4}{5(x^2+4)} + \frac{9}{5(x^2+9)}\right]dx$$

$$= -\int \frac{4dx}{5(x^2+4)} + \int \frac{9dx}{5(x^2+9)}$$

$$= -\frac{4}{5}\int \frac{dx}{(x^2+4)} + \frac{9}{5}\int \frac{dx}{(x^2+9)}$$

$$= -\frac{4}{5} \times \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + \frac{9}{5} \times \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$$

$$= -\frac{2}{5}\tan^{-1}\left(\frac{x}{2}\right) + \frac{3}{5}\tan^{-1}\left(\frac{x}{3}\right) + C$$

Indefinite Integrals Ex 19.30 Q19

Let $\int \frac{5x^2-1}{x(x-1)(x+1)}dx = \frac{A}{x}+\frac{B}{x-1}+\frac{C}{x+1}$

$\Rightarrow \quad 5x^2-1 = A(x^2-1)+B(x+1)x+C(x-1)x$

Put $x = 0$

$\Rightarrow \quad -1 = -A \Rightarrow \quad A = 1$

Put $x = +1$

$\Rightarrow \quad 4 = 2B \Rightarrow \quad B = 2$

Put $x = -1$

$\Rightarrow \quad 4 = 2C \quad \Rightarrow \quad C = 2$

So,

$I = \int \frac{dx}{x} + \int \frac{2dx}{x-1} \cdot + \int \frac{2dx}{x+1}$

$= \log|x| + 2\log|x-1| + 2\log|x+1| + c$

$I = \log\left|x\left(x^2-1\right)^2\right|$

Indefinite Integrals Ex 19.30 Q20

Let $I = \int \frac{x^2 + 6x - 8}{x^3 - 4x} dx$

$\Rightarrow \quad I = \int \frac{x^2 + 6x - 8}{x(x+2)(x-2)} dx$

Now,

Let $\frac{x^2 + 6x - 8}{x(x+2)(x-2)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-2}$

$\Rightarrow \quad x^2 + 6x - 8 = A(x^2 - 4) + B(x-2)x + C(x+2)x$

Put $x = 0$

$\Rightarrow \quad -8 = -4A \Rightarrow \quad A = 2$

Put $x = -2$

$\Rightarrow \quad -16 = 8B \quad \Rightarrow \quad B = -2$

Put $x = 2$

$\Rightarrow \quad 8 = 8C \Rightarrow \quad C = 1$

Thus,

$I = \int \frac{2dx}{x} - \int \frac{2dx}{x+2} + \int \frac{dx}{x-2}$

$= 2\log|x| - 2\log|x+2| + \log|x-2| + c$

$\therefore \quad I = \log\left|\frac{x^2(x-2)}{(x+2)^2}\right| + c$

Indefinite Integrals Ex 19.30 Q21

Let $\int \frac{x^2+1}{(2x+1)(x^2-1)} = \frac{A}{2x+1} + \frac{Bx+C}{x^2-1}$

$\Rightarrow \quad x^2+1 = A(x^2-1) + (Bx+C)(2x+1)$

$= (A+2B)x^2 + (B+2C)x + (-A+C)$

Equating similar terms, we get,

$A+2B = 1$, $B+2C = 0$ and $-A+C = 1$

Solving we get,

$A = -\frac{5}{3} \qquad B = \frac{4}{3} \qquad C = -\frac{2}{3}$

Thus,

$$I = -\frac{5}{3}\int \frac{dx}{2x+1} + \int \frac{\frac{4}{3}x - \frac{2}{3}}{x^2-1}dx$$

$$= -\frac{5}{3}\int \frac{dx}{2x+1} + \frac{2}{3}\int \frac{2xdx}{x^2-1} - \frac{2}{3}\int \frac{dx}{x^2-1}$$

$$= -\frac{5}{3}\int \frac{dx}{2x+1} + \frac{2}{3}\int \frac{2x-1}{(x+1)(x-1)}dx$$

$$= -\frac{5}{3}\int \frac{dx}{2x+1} + \frac{2}{3}\left(\int \left(\frac{\frac{3}{2}}{(x+1)} + \frac{\frac{1}{2}}{(x-1)}\right)dx\right)$$

$$I = -\frac{5}{6}\log|2x+1| + \log|x+1| + \frac{1}{3}\log|x-1| + c$$

*********** END **********