Transformation Formulae Ex 8.1 Q6(i)

We have,

$$\begin{aligned}
\text{LHS} &= \sin A \sin(B-C) + \sin B \sin(C-A) + \sin C \sin(A-B) \\
&= \frac{1}{2}\left[2\sin A \sin(B-C) + 2\sin B \sin(C-A) + 2\sin C \sin(A-B)\right] \\
&= \frac{1}{2}\begin{bmatrix} \cos(A-B+C) - \cos(A+B-C) + \cos(B-C+A) - \cos(B+C-A) \\ + \cos(C-A+B) - \cos(C+A-B) \end{bmatrix} \\
&= \frac{1}{2}\begin{bmatrix} \cos(A-B+C) - \cos(A-B+C) - \cos(A+B-C) + \cos(A+B-C) \\ - \cos(B+C-A) + \cos(B+C-A) \end{bmatrix} \\
&= \frac{1}{2}\times 0 \\
&= 0 \\
&= \text{RHS}
\end{aligned}$$

$\therefore \sin A \sin(B-C) + \sin B \sin(C-A) + \sin C \sin(A-B) = 0$ Hence proved.

Transformation Formulae Ex 8.1 Q6(ii)

We have,

$$\begin{aligned}
\text{LHS} &= \sin(B-C)\cos(A-D) + \sin(C-A)\cos(B-D) + \sin(A-B)\cos(C-D) \\
&= \frac{1}{2}\left[2\sin(B-C)\cos(A-D) + 2\sin(C-A)\cos(B-D) + 2\sin(A-B)\cos(C-D)\right] \\
&= \frac{1}{2}\begin{bmatrix} \sin(B-C+A-D) + \sin(B-C-A+D) + \sin(C-A+B-D) + \sin(C-A-B+D) \\ + \sin(A-B+C-D) + \sin(A-B-C+D) \end{bmatrix} \\
&= \frac{1}{2}\begin{bmatrix} \sin(A+B-C-D) + \sin(B+D-C-A) + \sin(B+C-A-D) + \sin(C+D-A-B) \\ + \sin(A+C-B-D) + \sin(A+D-B-C) \end{bmatrix} \\
&= \frac{1}{2}\begin{bmatrix} \sin(A+B-C-D) + \sin(B+D-C-A) + \sin\{-(A+D-B-C)\} + \sin\{-(A+B-C-D)\} \\ + \sin\{-(B+D-A-C)\} + \sin(A+D-B-C) \end{bmatrix} \\
&= \frac{1}{2}\begin{bmatrix} \sin(A+B-C-D) + \sin(B+D-C-A) - \sin(A+D-B-C) - \sin(A+B-C-D) \\ - \sin(B+D-A-C) + \sin(A+D-B-C) \end{bmatrix} \\
&= \frac{1}{2}\times 0 \qquad [\because \sin(-\theta) = -\sin\theta] \\
&= 0 \\
&= \text{RHS}
\end{aligned}$$

$\therefore \sin(B-C)\cos(A-D) + \sin(C-A)\cos(B-D) + \sin(A-B)\cos(C-D) = 0$ Hence proved.

Transformation Formulae Ex 8.1 Q7

We have,

$$\begin{aligned}
\text{LHS} &= \tan\theta \tan(60^\circ-\theta)\tan(60^\circ+\theta) \\
&= \frac{\sin\theta \sin(60^\circ-\theta)\sin(60^\circ+\theta)}{\cos\theta\cos(60^\circ-\theta)\cos(60^\circ+\theta)} \\
&= \frac{2\sin\theta \sin(60^\circ-\theta)\sin(60^\circ+\theta)}{2\cos\theta\cos(60^\circ-\theta)\cos(60^\circ+\theta)} \\
&= \frac{\sin\theta\left[2\sin(60^\circ-\theta)\sin(60^\circ+\theta)\right]}{\cos\theta\left[2\cos(60^\circ-\theta)\cos(60^\circ+\theta)\right]} \\
&= \frac{\sin\theta\left[\cos\{(60^\circ-\theta)-(60^\circ+\theta)\} - \cos\{(60^\circ-\theta)+(60^\circ+\theta)\}\right]}{\cos\theta\left[\cos\{(60^\circ-\theta)+(60^\circ+\theta)\} + \cos\{(60^\circ-\theta)-(60^\circ+\theta)\}\right]} \\
&= \frac{\sin\theta\left[\cos(-2\theta) - \cos 120^\circ\right]}{\cos\theta\left[\cos 120^\circ + \cos(-2\theta)\right]} \\
&= \frac{\sin\theta\left[\cos 2\theta - \cos 120^\circ\right]}{\cos\theta\left[\cos 120^\circ + \cos 2\theta\right]} \qquad [\because \cos(-\theta) = \cos\theta] \\
&= \frac{\sin\theta\left[\cos 2\theta - \cos(90^\circ+30^\circ)\right]}{\cos\theta\left[\cos(90^\circ+30^\circ) + \cos 2\theta\right]} \\
&= \frac{\sin\theta\left[\cos 2\theta + \sin 30^\circ\right]}{\cos\theta\left[-\sin 30^\circ + \cos 2\theta\right]} \qquad [\because \cos \text{ is negative in IInd quadrant}] \\
&= \frac{\sin\theta\left[\cos 2\theta + \frac{1}{2}\right]}{\cos\theta\left[\frac{-1}{2} + \cos 2\theta\right]} \\
&= \frac{\sin\theta\cos 2\theta + \frac{1}{2}\sin\theta}{\frac{-1}{2}\cos\theta + \cos\theta\cos 2\theta}
\end{aligned}$$

Transformation Formulae Ex 8.1 Q8

Let $y = \cos\alpha \cos\beta$ then,

$$y = \frac{1}{2}(2\cos\alpha\cos\beta)$$

$$= \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$$

$$= \frac{1}{2}\left[\cos 90^\circ + \cos(\alpha-\beta)\right] \qquad \left[\because \alpha+\beta = 90^\circ\right]$$

$$= \frac{1}{2}\left[0 + \cos(\alpha-\beta)\right]$$

$$= \frac{1}{2}\cos(\alpha-\beta)$$

$$\Rightarrow \quad y = \frac{1}{2}\cos(\alpha-\beta)$$

Now,

$$-1 \le \cos(\alpha-\beta) \le 1$$

$$\Rightarrow \quad \frac{-1}{2} \le \frac{1}{2}\cos(\alpha-\beta) \le \frac{1}{2}$$

$$\Rightarrow \quad \frac{-1}{2} \le y \le \frac{1}{2}$$

$$\Rightarrow \quad \frac{-1}{2} \le \cos\alpha\cos\beta \le \frac{1}{2}$$

Hence, the maximum values of $\cos\alpha \cos\beta$ is $\frac{1}{2}$.

*********** END **********