Indefinite Integrals Ex 19.25 Q40

$$\text{Let} \quad I = \int \frac{x^2 \tan^{-1} x}{1+x^2} dx$$

$$\text{Let} \quad \tan^{-1} x = t$$

$$x = \tan t$$

$$\frac{1}{1+x^2} dx = dt$$

$$I = \int t \tan^2 t \, dt$$

$$= \int t \left(\sec^2 t - 1\right) dt$$

$$= \int \left(t \sec^2 t - t\right) dt$$

$$= \int t \sec^2 t \, dt - \int t \, dt$$

$$= \left[t \int \sec^2 t \, dt - \int \left(1 \int \sec^2 t \, dt\right) dt\right] - \frac{t^2}{2}$$

$$= \left[t \tan t - \int \tan t \, dt\right] - \frac{t^2}{2}$$

$$= t \tan t - \log \sec t - \frac{t^2}{2} + c$$

$$= x \tan^{-1} x - \log \sqrt{1+x^2} - \frac{\tan^2 x}{2} + c$$

$$I = x \tan^{-1} x - \frac{1}{2} \log \left|1 + x^2\right| - \frac{\tan^2 x}{2} + c$$

Indefinite Integrals Ex 19.25 Q41

$$\text{Let} \quad I = \int \cos^{-1} \left(4x^3 - 3x\right) dx$$

$$\text{Let} \quad x = \cos \theta$$

$$dx = -\sin \theta \, d\theta$$

$$I = -\int \cos^{-1} \left(4 \cos^3 \theta - 3 \cos \theta\right) \sin \theta \, d\theta$$

$$= -\int \cos^{-1} \left(\cos 3\theta\right) \sin \theta \, d\theta$$

$$= -\int 3\theta \sin \theta \, d\theta$$

$$= -3 \left[\theta \int \sin \theta \, d\theta - \int \left(1 \int \sin \theta \, d\theta\right) d\theta\right]$$

$$= -3 \left[-\theta \cos \theta + \int \cos \theta \, d\theta\right]$$

$$= 3\theta \cos \theta - 3 \sin \theta + c$$

$$I = 3x \cos^{-1} x - 3\sqrt{1 - x^2} + c$$

Indefinite Integrals Ex 19.25 Q42

Let $I = \int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$

Let $x = \tan t$

$dx = \sec^2 t\,dt$

$I = \int \cos^{-1}\left(\frac{1-\tan^2 t}{1+\tan^2 t}\right)\sec^2 t\,dt$

$= \int \cos^{-1}(\cos 2t)\sec^2 t\,dt$

$= \int 2t \sec^2 t\,dt$

$= 2\left[t\int sce^2 t\,dt - \int\left(1\int \sec^2 t\,dt\right)dt\right]$

$= 2\left[t\tan^2 t - \int \tan t\,dt\right]$

$= 2\left[t\tan^2 t - \log\sec t\right] + c$

$= 2\left[x\tan^{-1}x - \log\sqrt{1+x^2}\right] + c$

$I = 2x\tan^{-1}x - \log\left|1+x^2\right| + c$

Indefinite Integrals Ex 19.25 Q43

Let $I = \int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$

Let $x = \tan\theta$

$dx = \sec^2\theta\,d\theta$

$I = \int \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)\sec^2\theta\,d\theta$

$= \int \tan^{-1}(\tan 2\theta)\sec^2\theta\,d\theta$

$= \int 2\theta\sec^2\theta\,d\theta$

$= 2\left[\theta\int \sec^2\theta\,d\theta - \int\left(1\int \sec^2\theta\,d\theta\right)d\theta\right]$

$= 2\left[\theta\tan\theta - \int \tan\theta\,d\theta\right]$

$= 2\left[\theta\tan\theta - \log\sec\theta\right] + c$

$= 2\left[x\tan^{-1}x - \log\sqrt{1+x^2}\right] + c$

$I = 2x\tan^{-1}x - \log\left|1+x^2\right| + c$

Indefinite Integrals Ex 19.25 Q44

Let $I = \int (x+1)\log x\,dx$

$= \log x \int (x+1)dx - \int \left(\frac{1}{x}\int (x+1)\,dx\right)dx$

$= \left(\frac{x^2}{2}+x\right)\log x - \int \frac{1}{x}\left(\frac{x^2}{2}+x\right)dx$

$= \left(\frac{x^2}{2}+x\right)\log x - \frac{1}{2}\int x\,dx - \int dx$

$= \left(x+\frac{x^2}{2}\right)\log x - \frac{1}{2}\times\frac{x^2}{2} - x + c$

$I = \left(x+\frac{x^2}{2}\right)\log x - \left(\frac{x^2}{4}+x\right)+c$

Indefinite Integrals Ex 19.25 Q45

Let $I = \int x^2 \tan^{-1} x\,dx$

$= \tan^{-1} x \int x^2\,dx - \int \left(\frac{1}{1+x^2}\int x^2 dx\right)$

$= \tan^{-1} x\left(\frac{x^3}{3}\right) - \frac{1}{3}\int \frac{x^3}{1+x^2}dx$

$= \frac{1}{3}x^3\tan^{-1}x - \frac{1}{3}\int\left(x - \frac{x}{1+x^2}\right)dx$

$= \frac{1}{3}x^3\tan^{-1}x - \frac{1}{3}\times\frac{x^2}{2} + \frac{1}{3}\int\frac{x}{1+x^2}dx$

$I = \frac{1}{3}x^3\tan^{-1}x - \frac{1}{6}x^2 + \frac{1}{6}\log\left|1+x^2\right| + c$

********** END **********