



Adjoint and Inverse of Matrix Ex 7.1 Q1(i)

$$\text{Here, } A = \begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix}$$

Cofactors of A are:

$$C_{11} = 4$$

$$C_{12} = -2$$

$$C_{21} = -5$$

$$C_{22} = -3$$

$$\therefore \text{adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$$

$$\begin{aligned} \therefore (\text{adj } A) &= \begin{bmatrix} 4 & -2 \\ -5 & -3 \end{bmatrix}^T \\ &= \begin{bmatrix} 4 & -5 \\ -2 & -3 \end{bmatrix} \end{aligned}$$

$$\text{Now, } (\text{adj } A)A = \begin{bmatrix} 4 & -5 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -22 & 0 \\ 0 & -22 \end{bmatrix}$$

$$\text{And, } |A|.I = \begin{vmatrix} -3 & 5 \\ 2 & 4 \end{vmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (-22) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -22 & 0 \\ 0 & -22 \end{bmatrix}$$

Also,

$$A.(\text{adj } A) = \begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 4 & -5 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} -22 & 0 \\ 0 & -22 \end{bmatrix}$$

$$\text{Therefore, } (\text{adj } A)A = |A|.I = A.(\text{adj } A)$$

Adjoint and Inverse of Matrix Ex 7.1 Q1(ii)

Here, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Cofactors of A are:

$$\begin{aligned} C_{11} &= d \\ C_{12} &= -c \\ C_{21} &= -b \\ C_{22} &= a \end{aligned}$$

$$\therefore \text{adj} A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$$

$$\begin{aligned} \therefore \text{adj} A &= \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}^T \\ &= \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \end{aligned}$$

$$\text{Now, } (\text{adj} A)(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ad-bc & bd-bd \\ -ac+ac & ad-bc \end{bmatrix} = \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$$

$$\text{And, } |A|.I = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (ad-bc) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$$

Also,

$$A(\text{adj} A) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$$

$$\therefore (\text{adj} A)(A) = |A|.I = A(\text{adj} A)$$

Adjoint and Inverse of Matrix Ex 7.1 Q1(iii)

Here, $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$

Cofactors of A are:

$$C_{11} = \cos \alpha$$

$$C_{12} = -\sin \alpha$$

$$C_{21} = -\sin \alpha$$

$$C_{22} = \cos \alpha$$

$$\therefore \text{Adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$$

$$\begin{aligned} \text{Adj } A &= \begin{bmatrix} \cos \alpha & -\sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}^T \\ &= \begin{bmatrix} \cos \alpha & -\sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{Now, } (\text{adj } A) \cdot (A) &= \begin{bmatrix} \cos \alpha & -\sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \alpha - \sin^2 \alpha & \cos \alpha \sin \alpha - \sin \alpha \cos \alpha \\ -\cos \alpha \sin \alpha + \sin \alpha \cos \alpha & -\sin^2 \alpha + \cos^2 \alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos 2\alpha & 0 \\ 0 & \cos 2\alpha \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{And, } A(\text{adj } A) &= \begin{bmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \alpha - \sin^2 \alpha & -\cos \alpha \sin \alpha + \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha - \cos \alpha \sin \alpha & -\sin^2 \alpha + \cos^2 \alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos 2\alpha & 0 \\ 0 & \cos 2\alpha \end{bmatrix} \end{aligned}$$

Also,

$$\begin{aligned} |A|.I &= \begin{vmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & \cos \alpha \end{vmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (\cos^2 \alpha - \sin^2 \alpha) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \alpha - \sin^2 \alpha & 0 \\ 0 & \cos^2 \alpha - \sin^2 \alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos 2\alpha & 0 \\ 0 & \cos 2\alpha \end{bmatrix} \end{aligned}$$

Adjoint and Inverse of Matrix Ex 7.1 Q1(iv)

We have,

$$A = \begin{bmatrix} 1 & \tan \frac{\alpha}{2} \\ -\tan \frac{\alpha}{2} & 1 \end{bmatrix}$$

Cofactors of A are:

$$c_{11} = 1, \quad c_{12} = -\left(-\tan \frac{\alpha}{2}\right) = \tan \frac{\alpha}{2}$$

$$c_{21} = -\tan \frac{\alpha}{2}, \quad c_{22} = 1$$

$$\therefore \text{adj } A = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & \tan \frac{\alpha}{2} \\ -\tan \frac{\alpha}{2} & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 1 \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} 1 & \tan \frac{\alpha}{2} \\ -\tan \frac{\alpha}{2} & 1 \end{vmatrix}$$

$$= 1 + \tan^2 \frac{\alpha}{2}$$

$$= \sec^2 \frac{\alpha}{2}$$

We have,

$$A = \begin{bmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{bmatrix}$$

Cofactors of A are:

$$c_{11} = 1, \quad c_{12} = -(-\tan \alpha/2) = \tan \alpha/2$$

$$c_{21} = -\tan \alpha/2, \quad c_{22} = 1$$

$$\therefore \text{adj } A = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -\tan \alpha/2 \\ \tan \alpha/2 & 1 \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{vmatrix}$$

$$= 1 + \tan^2 \alpha/2$$

$$= \sec^2 \alpha/2$$

***** END *****