Number System Ex 1.4 Q1

Answer :

An irrational number is a real number that cannot be reduced to any ratio between an integer p and a natural number q.

If the decimal representation of an irrational number is non-terminating and non-repeating, then it is called irrational number. For example $\sqrt{3} = 1.732$.......

Number System Ex 1.4 Q2

Answer :

Every rational number must have either terminating or non-terminating but irrational number must have non- terminating and non-repeating decimal representation.

A rational number is a number that can be written as simple fraction (ratio) and denominator is not equal to zero while an irrational is a number that cannot be written as a ratio.

Number System Ex 1.4 Q3

Answer :

(i) Let $x = \sqrt{7}$

Therefore,

$x = 2.645751311064...$

It is non-terminating and non-repeating

Hence $\sqrt{7}$ is an irrational number

(ii) Let $x = \sqrt{4}$

Therefore,

$x = 2$

It is terminating.

Hence $\sqrt{4}$ is a rational number.

(iii) Let $x = 2 + \sqrt{3}$ be the rational

Squaring on both sides

$$\Rightarrow x^2 = \left(2 + \sqrt{3}\right)^2$$

$$\Rightarrow x^2 = 4 + 3 + 4\sqrt{3}$$

$$\Rightarrow x^2 = 7 + 4\sqrt{3}$$

$$\Rightarrow x^2 - 7 = 4\sqrt{3}$$

$$\Rightarrow \frac{x^2 - 7}{4} = \sqrt{3}$$

Since, x is rational

$\Rightarrow x^2$ is rational

$\Rightarrow x^2 - 7$ is rational

$\Rightarrow \dfrac{x^2 - 7}{4}$ is rational

$\Rightarrow \sqrt{3}$ is rational

But, $\sqrt{3}$ is irrational

So, we arrive at a contradiction.

Hence $2 + \sqrt{3}$ is an irrational number

(iv) Let $x = \sqrt{3} + \sqrt{2}$ be the rational number

Squaring on both sides, we get

$\Rightarrow x^2 = \left(\sqrt{3} + \sqrt{2}\right)^2$

$\Rightarrow x^2 = 3 + 2 + 2\sqrt{6}$

$\Rightarrow x^2 = 5 + 2\sqrt{6}$

$\Rightarrow x^2 - 5 = 2\sqrt{6}$

$\Rightarrow \dfrac{x^2 - 5}{2} = \sqrt{6}$

Since, x is a rational number

$\Rightarrow x^2$ is rational number

$\Rightarrow x^2 - 5$ is rational number

$\Rightarrow \dfrac{x^2 - 5}{2}$ is rational number

$\Rightarrow \sqrt{6}$ is rational number

But $\sqrt{6}$ is an irrational number

So, we arrive at contradiction

Hence $\sqrt{3} + \sqrt{2}$ is an irrational number

(v) Let $x = \sqrt{3} + \sqrt{5}$ be the rational number

Squaring on both sides, we get

$\Rightarrow x^2 = \left(\sqrt{3} + \sqrt{5}\right)^2$

$\Rightarrow x^2 = 3 + 5 + 2\sqrt{15}$

$\Rightarrow x^2 = 8 + 2\sqrt{15}$

$\Rightarrow x^2 - 8 = 2\sqrt{15}$

$\Rightarrow \dfrac{x^2 - 8}{2} = \sqrt{15}$

Now, x is rational number

$\Rightarrow x^2$ is rational number

$\Rightarrow x^2 - 8$ is rational number

$\Rightarrow \dfrac{x^2 - 8}{2}$ is rational number

$\Rightarrow \sqrt{15}$ is rational number

But $\sqrt{15}$ is an irrational number

So, we arrive at a contradiction

Hence $\sqrt{3} + \sqrt{5}$ is an irrational number

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