Exercise 5.7 : Solutions of Questions on Page Number : **183**

Q1 : Find the second order derivatives of the function.

x^2+3x+2

Answer :

Let $y=x^2+3x+2$

Then,

$$\frac{dy}{dx}=\frac{d}{dx}\left(x^2\right)+\frac{d}{dx}(3x)+\frac{d}{dx}(2)=2x+3+0=2x+3$$
$$\therefore \frac{d^2y}{dx^2}=\frac{d}{dx}(2x+3)=\frac{d}{dx}(2x)+\frac{d}{dx}(3)=2+0=2$$

Answer needs Correction? **Click Here**

Q2 : Find the second order derivatives of the function.

x^{20}

Answer :

Let $y=x^{20}$

Then,

$$\frac{dy}{dx}=\frac{d}{dx}\left(x^{20}\right)=20x^{19}$$
$$\therefore \frac{d^2y}{dx^2}=\frac{d}{dx}\left(20x^{19}\right)=20\frac{d}{dx}\left(x^{19}\right)=20\cdot19\cdot x^{18}=380x^{18}$$

Answer needs Correction? **Click Here**

Q3 : Find the second order derivatives of the function.

$x\cdot\cos x$

Answer :

Let $y=x\cdot\cos x$

Then,

$$\frac{dy}{dx}=\frac{d}{dx}(x\cdot\cos x)=\cos x\cdot\frac{d}{dx}(x)+x\frac{d}{dx}(\cos x)=\cos x\cdot1+x(-\sin x)=\cos x-x\sin x$$
$$\begin{aligned}\therefore \frac{d^2y}{dx^2}&=\frac{d}{dx}[\cos x-x\sin x]=\frac{d}{dx}(\cos x)-\frac{d}{dx}(x\sin x)\\&=-\sin x-\left[\sin x\cdot\frac{d}{dx}(x)+x\cdot\frac{d}{dx}(\sin x)\right]\\&=-\sin x-(\sin x+x\cos x)\\&=-(x\cos x+2\sin x)\end{aligned}$$

Answer needs Correction? **Click Here**

Q4 : Find the second order derivatives of the function.

$\log x$

Answer :

Let $y=\log x$

Then,

$$\frac{dy}{dx}=\frac{d}{dx}(\log x)=\frac{1}{x}$$
$$\therefore \frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{1}{x}\right)=\frac{-1}{x^2}$$

Answer needs Correction? **Click Here**

Q5 : Find the second order derivatives of the function.

$x^3 \log x$

Answer :

Let $y = x^3 \log x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}\left[x^3 \log x\right] = \log x \cdot \frac{d}{dx}\left(x^3\right) + x^3 \cdot \frac{d}{dx}(\log x)$$
$$= \log x \cdot 3x^2 + x^3 \cdot \frac{1}{x} = \log x \cdot 3x^2 + x^2$$
$$= x^2(1 + 3\log x)$$
$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left[x^2(1 + 3\log x)\right]$$
$$= (1 + 3\log x) \cdot \frac{d}{dx}\left(x^2\right) + x^2 \frac{d}{dx}(1 + 3\log x)$$
$$= (1 + 3\log x) \cdot 2x + x^2 \cdot \frac{3}{x}$$
$$= 2x + 6x\log x + 3x$$
$$= 5x + 6x\log x$$
$$= x(5 + 6\log x)$$

Answer needs Correction? **Click Here**

Q6 : Find the second order derivatives of the function.

$e^x \sin 5x$

Answer :

Let $y = e^x \sin 5x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^x \sin 5x\right) = \sin 5x \cdot \frac{d}{dx}\left(e^x\right) + e^x \frac{d}{dx}(\sin 5x)$$
$$= \sin 5x \cdot e^x + e^x \cdot \cos 5x \cdot \frac{d}{dx}(5x) = e^x \sin 5x + e^x \cos 5x \cdot 5$$
$$= e^x(\sin 5x + 5\cos 5x)$$
$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left[e^x(\sin 5x + 5\cos 5x)\right]$$
$$= (\sin 5x + 5\cos 5x) \cdot \frac{d}{dx}\left(e^x\right) + e^x \cdot \frac{d}{dx}(\sin 5x + 5\cos 5x)$$
$$= (\sin 5x + 5\cos 5x)e^x + e^x\left[\cos 5x \cdot \frac{d}{dx}(5x) + 5(-\sin 5x) \cdot \frac{d}{dx}(5x)\right]$$
$$= e^x(\sin 5x + 5\cos 5x) + e^x(5\cos 5x - 25\sin 5x)$$
$$= e^x(10\cos 5x - 24\sin 5x) = 2e^x(5\cos 5x - 12\sin 5x)$$

Answer needs Correction? **Click Here**

Q7 : Find the second order derivatives of the function.

$e^{6x} \cos 3x$

Answer :

Let $y = e^{6x} \cos 3x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{6x} \cdot \cos 3x\right) = \cos 3x \cdot \frac{d}{dx}\left(e^{6x}\right) + e^{6x} \cdot \frac{d}{dx}(\cos 3x)$$
$$= \cos 3x \cdot e^{6x} \cdot \frac{d}{dx}(6x) + e^{6x} \cdot (-\sin 3x) \cdot \frac{d}{dx}(3x)$$
$$= 6e^{6x}\cos 3x - 3e^{6x}\sin 3x \qquad \ldots(1)$$
$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left(6e^{6x}\cos 3x - 3e^{6x}\sin 3x\right) = 6 \cdot \frac{d}{dx}\left(e^{6x}\cos 3x\right) - 3 \cdot \frac{d}{dx}\left(e^{6x}\sin 3x\right)$$
$$= 6 \cdot \left[6e^{6x}\cos 3x - 3e^{6x}\sin 3x\right] - 3 \cdot \left[\sin 3x \cdot \frac{d}{dx}\left(e^{6x}\right) + e^{6x} \cdot \frac{d}{dx}(\sin 3x)\right] \qquad [\text{Using } (1)]$$
$$= 36e^{6x}\cos 3x - 18e^{6x}\sin 3x - 3\left[\sin 3x \cdot e^{6x} \cdot 6 + e^{6x} \cdot \cos 3x \cdot 3\right]$$
$$= 36e^{6x}\cos 3x - 18e^{6x}\sin 3x - 18e^{6x}\sin 3x - 9e^{6x}\cos 3x$$
$$= 27e^{6x}\cos 3x - 36e^{6x}\sin 3x$$
$$= 9e^{6x}(3\cos 3x - 4\sin 3x)$$

Answer needs Correction? **Click Here**

Q8 : Find the second order derivatives of the function.

$\tan^{-1} x$

Answer :

Let $y = \tan^{-1} x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$$
$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{1+x^2}\right) = \frac{d}{dx}\left(1+x^2\right)^{-1} = (-1) \cdot \left(1+x^2\right)^{-2} \cdot \frac{d}{dx}\left(1+x^2\right)$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{1+x^2}\right) = \frac{d}{dx}\left(1+x^2\right)^{-1} = (-1)\cdot\left(1+x^2\right)^{-2}\cdot\frac{d}{dx}\left(1+x^2\right)$$

$$= \frac{-1}{\left(1+x^2\right)^2}\times 2x = \frac{-2x}{\left(1+x^2\right)^2}$$

Answer needs Correction? **Click Here**

Q9 : Find the second order derivatives of the function.

$\log(\log x)$

Answer :

Let $y = \log(\log x)$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\log(\log x)\right] = \frac{1}{\log x}\cdot\frac{d}{dx}(\log x) = \frac{1}{x\log x} = (x\log x)^{-1}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left[(x\log x)^{-1}\right] = (-1)\cdot(x\log x)^{-2}\cdot\frac{d}{dx}(x\log x)$$

$$= \frac{-1}{(x\log x)^2}\cdot\left[\log x\cdot\frac{d}{dx}(x) + x\cdot\frac{d}{dx}(\log x)\right]$$

$$= \frac{-1}{(x\log x)^2}\cdot\left[\log x\cdot 1 + x\cdot\frac{1}{x}\right] = \frac{-(1+\log x)}{(x\log x)^2}$$

Answer needs Correction? **Click Here**

Q10 : Find the second order derivatives of the function.

$\sin(\log x)$

Answer :

Let $y = \sin(\log x)$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\sin(\log x)\right] = \cos(\log x)\cdot\frac{d}{dx}(\log x) = \frac{\cos(\log x)}{x}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{\cos(\log x)}{x}\right]$$

$$= \frac{x\cdot\frac{d}{dx}\left[\cos(\log x)\right] - \cos(\log x)\cdot\frac{d}{dx}(x)}{x^2}$$

$$= \frac{x\cdot\left[-\sin(\log x)\cdot\frac{d}{dx}(\log x)\right] - \cos(\log x).1}{x^2}$$

$$= \frac{-x\sin(\log x).\frac{1}{x} - \cos(\log x)}{x^2}$$

$$= \frac{-\left[\sin(\log x) + \cos(\log x)\right]}{x^2}$$

Answer needs Correction? **Click Here**

Q11 : If $y = 5\cos x - 3\sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$

Answer :

It is given that, $y = 5\cos x - 3\sin x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}(5\cos x) - \frac{d}{dx}(3\sin x) = 5\frac{d}{dx}(\cos x) - 3\frac{d}{dx}(\sin x)$$

$$= 5(-\sin x) - 3\cos x = -(5\sin x + 3\cos x)$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left[-(5\sin x + 3\cos x)\right]$$

$$= -\left[5\cdot\frac{d}{dx}(\sin x) + 3\cdot\frac{d}{dx}(\cos x)\right]$$

$$= -\left[5\cos x + 3(-\sin x)\right]$$

$$= -\left[5\cos x - 3\sin x\right]$$

$$= -y$$

$$\therefore \frac{d^2y}{dx^2} + y = 0$$

Hence, proved.

Answer needs Correction? **Click Here**

Q12 : If $y = \cos^{-1} x$, find $\frac{d^2y}{dx^2}$ in terms of y alone.

Answer :

It is given that

It is given that, $y=\cos^{-1}x$

Then,

$$\frac{dy}{dx}=\frac{d}{dx}\left(\cos^{-1}x\right)=\frac{-1}{\sqrt{1-x^2}}=-\left(1-x^2\right)^{\frac{-1}{2}}$$

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\left[-\left(1-x^2\right)^{\frac{-1}{2}}\right]$$

$$=-\left(-\frac{1}{2}\right)\cdot\left(1-x^2\right)^{\frac{-3}{2}}\cdot\frac{d}{dx}\left(1-x^2\right)$$

$$=\frac{1}{2\sqrt{\left(1-x^2\right)^3}}\times(-2x)$$

$$\Rightarrow\frac{d^2y}{dx^2}=\frac{-x}{\sqrt{\left(1-x^2\right)^3}}\qquad\ldots\text{(i)}$$

$y=\cos^{-1}x\Rightarrow x=\cos y$

Putting $x=\cos y$ in equation (i), we obtain

$$\frac{d^2y}{dx^2}=\frac{-\cos y}{\sqrt{\left(1-\cos^2 y\right)^3}}$$

$$\Rightarrow\frac{d^2y}{dx^2}=\frac{-\cos y}{\sqrt{\left(\sin^2 y\right)^3}}$$

$$=\frac{-\cos y}{\sin^3 y}$$

$$=\frac{-\cos y}{\sin y}\times\frac{1}{\sin^2 y}$$

$$\Rightarrow\frac{d^2y}{dx^2}=-\cot y\cdot\operatorname{cosec}^2 y$$

Answer needs Correction? **Click Here**

Q13 : If $y=3\cos(\log x)+4\sin(\log x)$, show that $x^2y_2+xy_1+y=0$

Answer :

It is given that, $y=3\cos(\log x)+4\sin(\log x)$

Then,

$$y_1=3\cdot\frac{d}{dx}\left[\cos(\log x)\right]+4\cdot\frac{d}{dx}\left[\sin(\log x)\right]$$

$$=3\cdot\left[-\sin(\log x)\cdot\frac{d}{dx}(\log x)\right]+4\cdot\left[\cos(\log x)\cdot\frac{d}{dx}(\log x)\right]$$

$$\therefore y_1=\frac{-3\sin(\log x)}{x}+\frac{4\cos(\log x)}{x}=\frac{4\cos(\log x)-3\sin(\log x)}{x}$$

$$\therefore y_2=\frac{d}{dx}\left(\frac{4\cos(\log x)-3\sin(\log x)}{x}\right)$$

$$=\frac{x\{4\cos(\log x)-3\sin(\log x)\}'-\{4\cos(\log x)-3\sin(\log x)\}(x)'}{x^2}$$

$$=\frac{x\left[4\{\cos(\log x)\}'-3\{\sin(\log x)\}'\right]-\{4\cos(\log x)-3\sin(\log x)\}.1}{x^2}$$

$$=\frac{x\left[-4\sin(\log x).(\log x)'-3\cos(\log x).(\log x)'\right]-4\cos(\log x)+3\sin(\log x)}{x^2}$$

$$=\frac{x\left[-4\sin(\log x).\frac{1}{x}-3\cos(\log x).\frac{1}{x}\right]-4\cos(\log x)+3\sin(\log x)}{x^2}$$

$$=\frac{-4\sin(\log x)-3\cos(\log x)-4\cos(\log x)+3\sin(\log x)}{x^2}$$

$$=\frac{-\sin(\log x)-7\cos(\log x)}{x^2}$$

$\therefore x^2y_2+xy_1+y$

$$=x^2\left(\frac{-\sin(\log x)-7\cos(\log x)}{x^2}\right)+x\left(\frac{4\cos(\log x)-3\sin(\log x)}{x}\right)+3\cos(\log x)+4\sin(\log x)$$

$$=-\sin(\log x)-7\cos(\log x)+4\cos(\log x)-3\sin(\log x)+3\cos(\log x)+4\sin(\log x)$$

$$=0$$

Hence, proved.

Answer needs Correction? **Click Here**

Q14 : If $y=Ae^{mx}+Be^{nx}$, show that $\frac{d^2y}{dx^2}-(m+n)\frac{dy}{dx}+mny=0$

Answer :

It is given that, $y=Ae^{mx}+Be^{nx}$

Then,

$$\frac{dy}{dx}=A\cdot\frac{d}{dx}\left(e^{mx}\right)+B\cdot\frac{d}{dx}\left(e^{nx}\right)=A\cdot e^{mx}\cdot\frac{d}{dx}(mx)+B\cdot e^{nx}\cdot\frac{d}{dx}(nx)=Ame^{mx}+Bne^{nx}$$

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(Ame^{mx}+Bne^{nx}\right)=Am\cdot\frac{d}{dx}\left(e^{mx}\right)+Bn\cdot\frac{d}{dx}\left(e^{nx}\right)$$

$$=Am\cdot e^{mx}\cdot\frac{d}{dx}(mx)+Bn\cdot e^{nx}\cdot\frac{d}{dx}(nx)=Am^2e^{mx}+Bn^2e^{nx}$$

$$\begin{aligned}
&\therefore \frac{d^2y}{dx^2}-(m+n)\frac{dy}{dx}+mny \\
&= Am^2e^{mx}+Bn^2e^{nx}-(m+n)\cdot\left(Ame^{mx}+Bne^{nx}\right)+mn\left(Ae^{mx}+Be^{nx}\right) \\
&= Am^2e^{mx}+Bn^2e^{nx}-Am^2e^{mx}-Bmne^{nx}-Amne^{mx}-Bn^2e^{nx}+Amne^{mx}+Bmne^{nx} \\
&= 0
\end{aligned}$$

Hence, proved.

Q15 : If $y=500e^{7x}+600e^{-7x}$, show that $\frac{d^2y}{dx^2}=49y$

Answer :

It is given that, $y=500e^{7x}+600e^{-7x}$

Then,

$$\begin{aligned}
\frac{dy}{dx} &= 500.\frac{d}{dx}\left(e^{7x}\right)+600.\frac{d}{dx}\left(e^{-7x}\right) \\
&= 500\cdot e^{7x}\cdot\frac{d}{dx}(7x)+600\cdot e^{-7x}\cdot\frac{d}{dx}(-7x) \\
&= 3500e^{7x}-4200e^{-7x} \\
\therefore \frac{d^2y}{dx^2} &= 3500\cdot\frac{d}{dx}\left(e^{7x}\right)-4200.\frac{d}{dx}\left(e^{-7x}\right) \\
&= 3500\cdot e^{7x}.\frac{d}{dx}(7x)-4200\cdot e^{-7x}\cdot\frac{d}{dx}(-7x) \\
&= 7\times3500\cdot e^{7x}+7\times4200\cdot e^{-7x} \\
&= 49\times500e^{7x}+49\times600e^{-7x} \\
&= 49\left(500e^{7x}+600e^{-7x}\right) \\
&= 49y
\end{aligned}$$

Hence, proved.

Q16 : If $e^y(x+1)=1$, show that $\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$

Answer :

The given relationship is $e^y(x+1)=1$

$$\begin{aligned}
&e^y(x+1)=1 \\
&\Rightarrow e^y=\frac{1}{x+1}
\end{aligned}$$

Taking logarithm on both the sides, we obtain

$$y=\log\frac{1}{(x+1)}$$

Differentiating this relationship with respect to x, we obtain

$$\begin{aligned}
&\frac{dy}{dx}=(x+1)\frac{d}{dx}\left(\frac{1}{x+1}\right)=(x+1).\frac{-1}{(x+1)^2}=\frac{-1}{x+1} \\
&\therefore \frac{d^2y}{dx^2}=-\frac{d}{dx}\left(\frac{1}{x+1}\right)=-\left(\frac{-1}{(x+1)^2}\right)=\frac{1}{(x+1)^2} \\
&\Rightarrow \frac{d^2y}{dx^2}=\left(\frac{-1}{x+1}\right)^2 \\
&\Rightarrow \frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2
\end{aligned}$$

Hence, proved.

Q17 : If $y=\left(\tan^{-1}x\right)^2$, show that $\left(x^2+1\right)^2y_2+2x\left(x^2+1\right)y_1=2$

Answer :

The given relationship is $y=\left(\tan^{-1}x\right)^2$

Then,

$$\begin{aligned}
&y_1=2\tan^{-1}x\frac{d}{dx}\left(\tan^{-1}x\right) \\
&\Rightarrow y_1=2\tan^{-1}x.\frac{1}{1+x^2} \\
&\Rightarrow \left(1+x^2\right)y_1=2\tan^{-1}x
\end{aligned}$$

Again differentiating with respect to x on both the sides, we obtain

$$\begin{aligned}
&\left(1+x^2\right)y_2+2xy_1=2\left(\frac{1}{1+x^2}\right) \\
&\Rightarrow \left(1+x^2\right)^2y_2+2x\left(1+x^2\right)y_1=2
\end{aligned}$$

Hence, proved.

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