Trigonometric Identities Ex 6.1 Q24

Answer :

We have to prove $\frac{\cos^2\theta}{\sin\theta} - \operatorname{cosec}\theta + \sin\theta = 0$

We know that, $\sin^2\theta + \cos^2\theta = 1$

So,

$$\begin{aligned}
\frac{\cos^2\theta}{\sin\theta} - \operatorname{cosec}\theta + \sin\theta &= \left(\frac{\cos^2\theta}{\sin\theta} - \operatorname{cosec}\theta\right) + \sin\theta \\
&= \left(\frac{\cos^2\theta}{\sin\theta} - \frac{1}{\sin\theta}\right) + \sin\theta \\
&= \left(\frac{\cos^2\theta - 1}{\sin\theta}\right) + \sin\theta \\
&= \left(\frac{-\sin^2\theta}{\sin\theta}\right) + \sin\theta \\
&= -\sin\theta + \sin\theta \\
&= 0
\end{aligned}$$

Trigonometric Identities Ex 6.1 Q25

Answer :

We have to prove $\frac{1}{1+\sin A} + \frac{1}{1-\sin A} = 2\sec^2 A$

We know that, $\sin^2 A + \cos^2 A = 1$

So,

$$\begin{aligned}
\frac{1}{1+\sin A} + \frac{1}{1-\sin A} &= \frac{(1-\sin A)+(1+\sin A)}{(1+\sin A)(1-\sin A)} \\
&= \frac{1-\sin A+1+\sin A}{1-\sin^2 A} \\
&= \frac{2}{\cos^2 A} \\
&= 2\sec^2 A
\end{aligned}$$

Trigonometric Identities Ex 6.1 Q26

Answer :

We have to prove $\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}-2\sec\theta$

We know that, $\sin^2\theta+\cos^2\theta=1$

Multiplying the denominator and numerator of the second term by $(1-\sin\theta)$, we have

$$\begin{aligned}\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}&=\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}\\&=\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}\\&=\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta(1-\sin\theta)}{\cos^2\theta}\\&=\frac{1+\sin\theta}{\cos\theta}+\frac{1-\sin\theta}{\cos\theta}\\&=\frac{1+\sin\theta+1-\sin\theta}{\cos\theta}\\&=\frac{2}{\cos\theta}\\&=2\sec\theta\end{aligned}$$

Trigonometric Identities Ex 6.1 Q27

Answer :

We have to prove that $\frac{(1+\sin\theta)^2+(1-\sin\theta)^2}{2\cos^2\theta}=\frac{1+\sin^2\theta}{1-\sin^2\theta}$.

We know that, $\sin^2\theta+\cos^2\theta=1$

So,

$$\begin{aligned}\frac{(1+\sin\theta)^2+(1-\sin\theta)^2}{2\cos^2\theta}&=\frac{(1+2\sin\theta+\sin^2\theta)+(1-2\sin\theta+\sin^2\theta)}{2\cos^2\theta}\\&=\frac{1+2\sin\theta+\sin^2\theta+1-2\sin\theta+\sin^2\theta}{2\cos^2\theta}\\&=\frac{2+2\sin^2\theta}{2\cos^2\theta}\\&=\frac{2(1+\sin^2\theta)}{2(1-\sin^2\theta)}\\&=\frac{1+\sin^2\theta}{1-\sin^2\theta}\end{aligned}$$

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