Higher Order Derivatives Ex 12.1 Q34

It is given that, $y = e^{a\cos^{-1}x}$

Taking logarithm on both the sides, we obtain

$\log y = a\cos^{-1}x \log e$

$\log y = a\cos^{-1}x$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{y}\frac{dy}{dx} = a \times \frac{-1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-ay}{\sqrt{1-x^2}}$$

By squaring both the sides, we obtain

$$\left(\frac{dy}{dx}\right)^2 = \frac{a^2y^2}{1-x^2}$$

$$\Rightarrow \left(1-x^2\right)\left(\frac{dy}{dx}\right)^2 = a^2y^2$$

$$\left(1-x^2\right)\left(\frac{dy}{dx}\right)^2 = a^2y^2$$

Again differentiating both sides with respect to x, we obtain

$$\left(\frac{dy}{dx}\right)^2 \frac{d}{dx}\left(1-x^2\right) + \left(1-x^2\right) \times \frac{d}{dx}\left[\left(\frac{dy}{dx}\right)^2\right] = a^2\frac{d}{dx}\left(y^2\right)$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2(-2x) + \left(1-x^2\right) \times 2\frac{dy}{dx}.\frac{d^2y}{dx^2} = a^2.2y.\frac{dy}{dx}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2(-2x) + \left(1-x^2\right) \times 2\frac{dy}{dx}.\frac{d^2y}{dx^2} = a^2.2y.\frac{dy}{dx}$$

$$\Rightarrow -x\frac{dy}{dx} + \left(1-x^2\right)\frac{d^2y}{dx^2} = a^2.y \qquad \left[\frac{dy}{dx} \neq 0\right]$$

$$\Rightarrow \left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$$

Hence, proved.

Higher Order Derivatives Ex 12.1 Q35

It is given that, $y = 500e^{7x} + 600e^{-7x}$

Then,

$$\frac{dy}{dx} = 500.\frac{d}{dx}\left(e^{7x}\right) + 600.\frac{d}{dx}\left(e^{-7x}\right)$$
$$= 500 \cdot e^{7x} \cdot \frac{d}{dx}(7x) + 600 \cdot e^{-7x} \cdot \frac{d}{dx}(-7x)$$
$$= 3500e^{7x} - 4200e^{-7x}$$
$$\therefore \frac{d^2y}{dx^2} = 3500 \cdot \frac{d}{dx}\left(e^{7x}\right) - 4200.\frac{d}{dx}\left(e^{-7x}\right)$$
$$= 3500 \cdot e^{7x}.\frac{d}{dx}(7x) - 4200 \cdot e^{-7x} \cdot \frac{d}{dx}(-7x)$$
$$= 7 \times 3500 \cdot e^{7x} + 7 \times 4200 \cdot e^{-7x}$$
$$= 49 \times 500e^{7x} + 49 \times 600e^{-7x}$$
$$= 49\left(500e^{7x} + 600e^{-7x}\right)$$
$$= 49y$$

Hence, proved

Higher Order Derivatives Ex 12.1 Q36

$y = 2\cos t - \cos 2t; \quad y = 2\sin t - \sin 2t$

differentiating w.r.t. t

$\Rightarrow \quad \frac{dy}{dt} = 2(-\sin t) - 2(-\sin 2t); \; \frac{dy}{dt} = 2\cos t - 2\cos 2t$

dividing (2) by (1)

$\Rightarrow \quad \frac{dy}{dx} = \frac{2(\cos t - \cos 2t)}{2(\sin 2t - \sin t)}$

differentiating w.r.t. t

$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{dt} = \frac{(\sin 2t - \sin t)(-\sin t + 2\sin 2t) - (\cos t - \cos 2t)(2\cos 2t - \cos t)}{(\sin 2t - \sin t)^2}$ ....(3)

dividing (3) by (1)

$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{(\sin 2t - \sin t)(2\sin 2t - \sin t) - (\cos t - \cos 2t)(2\cos 2t - \cos t)}{2(\sin 2t - \sin t)^3}$

Putting $t = \pi/2$

$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{(0-1)(0-1) - (0-(-1))(2(-1)-0)}{2(0-1)^3} = \frac{1+2}{-2} = \frac{-3}{2}$

Higher Order Derivatives Ex 12.1 Q37

$x = 4z^2 + 5$ $\qquad\qquad$ $y = 6z^2 + 72 + 3$

differentiating both w.r.t. z

$$\Rightarrow \qquad \frac{dx}{dz} = 8z + 0 \qquad\qquad \frac{dy}{dz} = 12z + 7$$

$$\Rightarrow \qquad \frac{dx}{dz} = \frac{12z + 7}{8z} = \frac{12z}{8z} + \frac{7}{8z}$$

differentiating w.r.t. z

$$\Rightarrow \qquad \frac{d\left(\frac{dy}{dx}\right)}{dz} = 0 + \frac{7}{8}\left(\frac{-1}{z^2}\right) \qquad\qquad .....(3)$$

dividing (3) by (1)

$$\Rightarrow \qquad \frac{d^2y}{dx^2} = \frac{-7}{8z^2 \times 8z} = \frac{-7}{64z^3}$$

*********** END **********