Functions Ex 2.3 Q2

We have, $f(x) = x^2 + x + 1$ and $g(x) = \sin x$

Now,

$$f \circ g(x) = f(g(x)) = f(\sin x)$$

$$\Rightarrow \quad f \circ g(x) = \sin^2 x + \sin x + 1$$

Again, $g \circ f(x) = g(f(x)) = g(x^2 + x + 1)$

$$\Rightarrow \quad g \circ f(x) = \sin(x^2 + x + 1)$$

Clearly

$$f \circ g \neq g \circ f$$

Functions Ex 2.3 Q3

We have $\quad f(x) = |x|$

We assume the domain of $f = R$

Range of $f = (0, \infty)$

$\therefore$ Range of f $\subset$ domain of f

$\therefore$ $f \circ f$ exists.

Now,

$$f \circ f(x) = f(f(x)) = f(|x|) = ||x|| = f(x)$$

$\therefore \quad f \circ f = f$

Functions Ex 2.3 Q4

$f(x) = 2x + 5$ and $g(x) = x^2 + 1$

$\therefore$ Range of $f = R$ and range of $g = [1, \infty]$

$\therefore$ Range of $f \subseteq$ Domain of $g\,(R)$ and range of $g \subseteq$ domain of $f\,(R)$

$\therefore$ both fog and gof exist.

i) $f \circ g(x) = f(g(x)) = f(x^2 + 1)$

$= 2(x^2 + 1) + 5$

$\Rightarrow \quad f \circ g(x) = 2x^2 + 7$

ii) $g \circ f(x) = g(f(x)) = g(2x + 5)$

$= (2x + 5)^2 + 1$

$\Rightarrow \quad g \circ f(x) = 4x^2 + 20x + 26$

iii) $f \circ f(x) = f(f(x)) = f(2x + 5)$

$= 2(2x + 5) + 5$

$f \circ f(x) = 4x + 15$

iv) $f^2(x) = [f(x)]^2 = (2x + 5)^2$

$= 4x^2 + 20x + 25$

$\therefore$ from (iii) & (iv)

$f \circ f \neq f^2$

Functions Ex 2.3 Q5

We have, $f(x) = \sin x$ and $g(x) = 2x$.

Domain of f and g is R

Range of $f = [-1, 1]$

Range of $g = R$

$\therefore$ Range of $f \subset$ Domain g and

Range of $g \subseteq$ Domain f

$\therefore$ fog and gof both exist.

i) $g \circ f(x) = g(f(x)) = g(\sin x) = g \circ f(x) = 2\sin x$

ii) $f \circ g(x) = f(g(x)) = f(2x) = \sin 2x$

$\therefore \quad g \circ f \neq f \circ g$

Functions Ex 2.3 Q6

$f, g,$ and h are real functions given by $f(x) = \sin x$, $g(x) = 2x$ and $h(x) = \cos x$

To prove: $f \circ g = g \circ (fh)$

L.H.S

$$f \circ g(x) = f(g(x))$$
$$= f(2x) = \sin 2x$$
$$\Rightarrow f \circ g(x) = 2\sin x \cos x \text{ ....}(A)$$

R.H.S

$$g \circ (fh)(x) = go(f(x).h(x))$$
$$= g(\sin x \cos x)$$
$$g \circ (fh)(x) = 2\sin x \cos x \text{ ....}(B)$$

from A & B

$$f \circ g(x) = g \circ (fh)(x)$$

*********** END **********