



$$\frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b} = \frac{a\left(\frac{b^2}{a^2} - 2 \times \frac{c}{a}\right) + \left(\frac{-b^2}{a}\right)}{a \times c - b^2 + b^2}$$

$$\frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b} = \frac{a\left(\frac{b^2}{a^2} - \frac{2c \times a}{a \times a}\right) + \left(\frac{-b^2}{a}\right)}{a \times c - \cancel{b^2} + \cancel{b^2}}$$

$$\frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b} = \frac{a\left(\frac{b^2}{a^2} - \frac{2ca}{a^2}\right) + \left(\frac{-b^2}{a}\right)}{ac}$$

$$\frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b} = \frac{a\left(\frac{b^2 - 2ca}{a^2}\right) + \left(\frac{-b^2}{a}\right)}{ac}$$

$$\frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b} = \frac{\cancel{a}\left(\frac{b^2 - 2ca}{a^{\cancel{2}1}}\right) + \left(\frac{-b^2}{a}\right)}{ac}$$

$$\frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b} = \frac{\left(\frac{b^2 - 2ca}{a}\right) + \left(\frac{-b^2}{a}\right)}{ac}$$

$$\frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b} = \frac{\left(\frac{b^2 - 2ca - b^2}{a}\right)}{ac}$$

$$\frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b} = \frac{\left(\frac{b^2-2ca-b^2}{a}\right)}{ac}$$

$$\frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b} = \left(\frac{-2\cancel{a}}{a}\right) \times \frac{1}{\cancel{ac}}$$

$$\frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b} = \frac{-2}{a}$$

Hence, the value of $\frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b}$ is $\boxed{\frac{-2}{a}}$.

(viii) Since α and β are the zeros of the quadratic polynomial $f(x) = ax^2 + bx + c$

$$\alpha + \beta = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$= \frac{-b}{a}$$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$= \frac{c}{a}$$

$$\text{We have, } a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right) + b\left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right)$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{\alpha\beta}\right) + \left(\frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}\right)$$

By substituting $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$ we get ,

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{\left(\frac{-b}{a}\right)^3 - 3 \times \frac{c}{a} \left(\frac{-b}{a}\right)}{\frac{c}{a}}\right) + b\left(\frac{\left(\frac{-b}{a}\right)^2 - 2 \times \frac{c}{a}}{\frac{c}{a}}\right)$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{\frac{-b^3}{a^3} - \frac{3bc}{a^2}}{\frac{c}{a}}\right) + b\left(\frac{\frac{-b^2}{a} - \frac{2c}{a}}{\frac{c}{a}}\right)$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{-b^3 + 3bca}{a^2} \times \frac{\cancel{a}}{c}\right) + b\left(\frac{b^2 - 2ca}{a^2} \times \frac{\cancel{a}}{c}\right)$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = \cancel{a}\left(\frac{-b^3 + 3abc}{a^2 c}\right) + b\left(\frac{b^2 - 2ca}{ac}\right)$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = \frac{-b^3 + 3abc}{ac} + \frac{b^3 - 2abc}{ac}$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = \frac{\cancel{a^2} + 3abc + \cancel{\beta^2} - 2abc}{ac}$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = \frac{3abc - 2abc}{ac}$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = \frac{\cancel{a}b\cancel{\alpha}}{\cancel{a}\cancel{\alpha}}$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = b$$

Hence, the value of $a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$ is \boxed{b} .

***** END *****