Pair of Linear Equations in Two varibles Ex 3.10 Q8

Answer :

Let the speed of the train be x km/hr that of the car be y km/hr, we have the following cases:

Case I: When a man travels 600Km by train and the rest by car

Time taken by a man to travel 400 Km by train = $\frac{400}{x} hrs$

Time taken by a man to travel (600-400) =200Km by car = $\frac{200}{y}$ hrs

Total time taken by a man to cover 600Km = $\frac{400}{x}+\frac{200}{y}$

It is given that total time taken in 8 hours

$$\frac{400}{x}+\frac{200}{y}=6hrs30\min$$

$$\frac{400}{x}+\frac{200}{y}=6\times\frac{30}{60}$$

$$\frac{400}{x}+\frac{200}{y}=6\times\frac{1}{2}$$

$$\frac{400}{x}+\frac{200}{y}=\frac{13}{2}$$

$$200\left(\frac{2}{x}+\frac{1}{y}\right)=\frac{13}{2}$$

$$\left(\frac{2}{x}+\frac{1}{y}\right)=\frac{13}{2}\times\frac{1}{200}$$

$$\frac{2}{x}+\frac{1}{y}=\frac{13}{400}\cdots(i)$$

Case II: When a man travels 200Km by train and the rest by car

Time taken by a man to travel 200 Km by train = $\frac{200}{x} hrs$

Time taken by a man to travel (600-200) = 400 Km by car = $\frac{400}{y} hrs$

In this case, total time of the journey in 6 hours 30 minutes + 30 minutes that is 7 hours,

$$\frac{200}{x}+\frac{400}{y}=7$$

$$200\left(\frac{1}{x}+\frac{2}{y}\right)=7$$

$$\frac{1}{x}+\frac{2}{y}=\frac{7}{200} \quad \ldots(ii)$$

Putting $\frac{1}{x}=u$ and, $\frac{1}{y}=u$, the equations (i) and (ii) reduces to

$$2u+1v=\frac{13}{400}\cdots(iii)$$

$$1u+2v=\frac{7}{200}\cdots(vi)$$

Multiplying equation (iii) by 6 the above system of equation becomes

$$4u+2v=\frac{13}{200}\cdots(v)$$

$$1u + 2v = \frac{7}{200} \cdots (vi)$$

Substituting equation (vi) and (v), we get

$$4u + 2v = \frac{13}{200}$$

$$\underline{1u + 2v = \frac{7}{200}}$$

$$3u = \frac{6}{200}$$

$$u = \frac{6}{200} \times \frac{1}{3}$$

$$u = \frac{2}{200}$$

$$u = \frac{1}{100}$$

Putting $u = \frac{1}{100}$ in equation (vi), we get

$$1u + 2v = \frac{7}{200}$$

$$\frac{1}{100} + 2v = \frac{7}{200}$$

$$2v = \frac{7}{200} - \frac{1}{100}$$

$$2v = \frac{7}{200} - \frac{2}{200}$$

$$2v = \frac{7-2}{200}$$

$$2v = \frac{5}{200}$$

$$v = \frac{5}{200} \times \frac{1}{2}$$

$$v = \frac{1}{80}$$

Now

$$u = \frac{1}{100}$$

$$\frac{1}{x} = \frac{1}{100}$$

$$x = 100$$

and

$$v = \frac{1}{80}$$

$$\frac{1}{y} = \frac{1}{80}$$

$$y = 80$$

Hence, the speed of the train is $\boxed{100\ km/hr}$,

The speed of the car is $\boxed{80\ km/hr}$.

********** END **********