## Real Numbers Ex 1.1 Q1

**Answer :**

To Prove: that the product of two consecutive integers is divisible by 2.
Proof: Let $n - 1$ and $n$ be two consecutive positive integers.
Then their product is $n(n - 1) = n^2 - n$
We know that every positive integer is of the form $2q$ or $2q + 1$ for some integer $q$.
So let $n = 2q$
So, $n^2 - n = (2q)^2 - (2q)$

$$\Rightarrow n^2 - n = (2q)^2 - 2q$$
$$\Rightarrow n^2 - n = 4q^2 - 2q$$
$$\Rightarrow n^2 - n = 2q(2q - 1)$$
$$\Rightarrow n^2 - n = 2r \text{ (where } r = q(2q-1))$$
$$\Rightarrow \boxed{n^2 - n \text{ is even and divisible by } 2}$$

Let $n = 2q + 1$
So, $n^2 - n = (2q + 1)^2 - (2q + 1)$

$$\Rightarrow n^2 - n = (2q+1)((2q+1)-1)$$
$$\Rightarrow n^2 - n = (2q+1)(2q)$$
$$\Rightarrow n^2 - n = 2r(r = q(2q+1))$$
$$\Rightarrow \boxed{n^2 - n \text{ is even and divisible by } 2}$$

Hence it is proved that that the product of two consecutive integers is divisible by 2

## Real Numbers Ex 1.1 Q2

**Answer :**

Given: If $a$ and $b$ are two odd positive integers such that $a > b$.

To Prove: That one of the two numbers $\frac{a+b}{2}$ and $\frac{a-b}{2}$ is odd and the other is even.

Proof: Let $a$ and $b$ be any odd odd positive integer such that $a > b$.
Since any positive integer is of the form $q$, $2q + 1$
Let $a = 2q + 1$ and $b = 2m + 1$, where, $q$ and $m$ are some whole numbers

$$\Rightarrow \frac{a+b}{2} = \frac{(2q+1)+(2m+1)}{2}$$
$$\Rightarrow \frac{a+b}{2} = \frac{2(q+m)+1)}{2}$$
$$\Rightarrow \frac{a+b}{2} = \left(q+m+1\right)$$

which is a positive integer.
Also,

$$\Rightarrow \frac{a-b}{2} = \frac{(2q+1)-(2m+1)}{2}$$
$$\Rightarrow \frac{a-b}{2} = \frac{2(q-m)}{2}$$
$$\Rightarrow \frac{a-b}{2} = \left(q-m\right)$$

Given, $a > b$
$\therefore 2q + 1 > 2m + 1$
$\Rightarrow 2q > 2m$
$\Rightarrow q > m$

$\therefore \frac{a-b}{2} = \left(q-m\right) > 0$

Thus, $\frac{(a-b)}{2}$ is a positive integer.
Now, we need to prove that one of the two numbers $\frac{(a+b)}{2}$ and $\frac{(a-b)}{2}$ is odd and other is even.

Consider, $\frac{(a+b)}{2} - \frac{(a-b)}{2} = \frac{(a+b)-(a-b)}{2} = \frac{2b}{2} = b$, which is odd positive integer.

Also, we know from the proof above that $\frac{(a+b)}{2}$ and $\frac{(a-b)}{2}$ are positive integers.

We know that the difference of two positive integers is an odd number if one of them is odd and another is even. (Also, difference between two odd and two even integers is even)
Hence it is proved that If $a$ and $b$ are two odd positive integers such that $a > b$ then one of the two numbers $\frac{a+b}{2}$ and $\frac{a-b}{2}$ is odd and the other is even.

## Real Numbers Ex 1.1 Q3

**Answer :**

To Prove: that the square of an odd positive integer is of the form $8q + 1$, for some integer $q$.
Proof: Since any positive integer $n$ is of the form $4m + 1$ and $4m + 3$
If $n = 4m + 1$

$$\Rightarrow n^2 = (4m+1)^2$$
$$\Rightarrow n^2 = (4m)^2 + 8m + 1$$
$$\Rightarrow n^2 = 16m^2 + 8m + 1$$
$$\Rightarrow n^2 = 8m(2m+1) + 1$$
$$\Rightarrow n^2 + 8q + 1 \left(q = m(2m+1)\right)$$

If $n = 4m + 3$

$$\Rightarrow n^2 = (4m+3)^2$$
$$\Rightarrow n^2 = (4m)^2 + 24m + 9$$
$$\Rightarrow n^2 = 16m^2 + 24m + 9$$
$$\Rightarrow n^2 = 8(2m^2 + 3m + 1) + 1$$
$$\boxed{\Rightarrow n^2 = 8q + 1} \left(q = (2m^2 + 3m + 1)\right)$$

Hence $n^2$ integer is of the form $8q + 1$, for some integer $q$.

*********** END **********