



Exercise 2.1

Question 1:

$$\sin^{-1}\left(-\frac{1}{2}\right)$$

Find the principal value of

Answer

$$\text{Let } \sin^{-1}\left(-\frac{1}{2}\right) = y. \text{ Then } \sin y = -\frac{1}{2} = -\sin\left(\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right).$$

We know that the range of the principal value branch of \sin^{-1} is

$$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \text{ and } \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}.$$

$$\sin^{-1}\left(-\frac{1}{2}\right) \text{ is } -\frac{\pi}{6}.$$

Therefore, the principal value of

Question 2:

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

Find the principal value of

Answer

$$\text{Let } \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = y. \text{ Then, } \cos y = \frac{\sqrt{3}}{2} = \cos\left(\frac{\pi}{6}\right).$$

We know that the range of the principal value branch of \cos^{-1} is

$$[0, \pi] \text{ and } \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}.$$

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \text{ is } \frac{\pi}{6}.$$

Therefore, the principal value of

Question 3:

Find the principal value of $\operatorname{cosec}^{-1}(2)$

Answer

$$\operatorname{cosec} y = 2 = \operatorname{cosec}\left(\frac{\pi}{6}\right).$$

Let $\operatorname{cosec}^{-1}(2) = y$. Then,

We know that the range of the principal value branch of $\operatorname{cosec}^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$.

$$\operatorname{cosec}^{-1}(2) \text{ is } \frac{\pi}{6}.$$

Therefore, the principal value of

Question 4:

$$\tan^{-1}(-\sqrt{3})$$

Find the principal value of

Answer

$$\text{Let } \tan^{-1}(-\sqrt{3}) = y. \text{ Then, } \tan y = -\sqrt{3} = -\tan\frac{\pi}{3} = \tan\left(-\frac{\pi}{3}\right).$$

We know that the range of the principal value branch of \tan^{-1} is

$$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \text{ and } \tan\left(-\frac{\pi}{3}\right) = -\sqrt{3}.$$

$$\tan^{-1}(\sqrt{3}) \text{ is } \frac{\pi}{3}.$$

Therefore, the principal value of

Question 5:

$$\cos^{-1}\left(-\frac{1}{2}\right)$$

Find the principal value of

Answer

$$\text{Let } \cos^{-1}\left(-\frac{1}{2}\right) = y. \text{ Then, } \cos y = -\frac{1}{2} = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right).$$

We know that the range of the principal value branch of \cos^{-1} is

$$[0, \pi] \text{ and } \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}.$$

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is $\frac{2\pi}{3}$.

Question 6:

Find the principal value of $\tan^{-1}(-1)$

Answer

$$\tan y = -1 = -\tan\left(\frac{\pi}{4}\right) = \tan\left(-\frac{\pi}{4}\right).$$

Let $\tan^{-1}(-1) = y$. Then,

We know that the range of the principal value branch of \tan^{-1} is

$$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \text{ and } \tan\left(-\frac{\pi}{4}\right) = -1.$$

Therefore, the principal value of $\tan^{-1}(-1)$ is $-\frac{\pi}{4}$.

Question 7:

$$\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

Find the principal value of

Answer

$$\text{Let } \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = y. \text{ Then, } \sec y = \frac{2}{\sqrt{3}} = \sec\left(\frac{\pi}{6}\right).$$

We know that the range of the principal value branch of \sec^{-1} is

$$[0, \pi] - \left\{\frac{\pi}{2}\right\} \text{ and } \sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}.$$

Therefore, the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{6}$.

Question 8:

$$\cot^{-1}(\sqrt{3})$$

Find the principal value of

Answer

$$\text{Let } \cot^{-1}(\sqrt{3}) = y. \text{ Then, } \cot y = \sqrt{3} = \cot\left(\frac{\pi}{6}\right).$$

We know that the range of the principal value branch of \cot^{-1} is $(0, \pi)$ and

$$\cot\left(\frac{\pi}{6}\right) = \sqrt{3}.$$

Therefore, the principal value of $\cot^{-1}(\sqrt{3})$ is $\frac{\pi}{6}$.

Question 9:

$$\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$$

Find the principal value of

Answer

$$\text{Let } \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = y. \text{ Then, } \cos y = -\frac{1}{\sqrt{2}} = -\cos\left(\frac{\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right).$$

We know that the range of the principal value branch of \cos^{-1} is $[0, \pi]$ and

$$\cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}.$$

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$.

Question 10:

$$\operatorname{cosec}^{-1}(-\sqrt{2})$$

Find the principal value of

Answer

$$\text{Let } \operatorname{cosec}^{-1}(-\sqrt{2}) = y. \text{ Then, } \operatorname{cosec} y = -\sqrt{2} = -\operatorname{cosec}\left(\frac{\pi}{4}\right) = \operatorname{cosec}\left(-\frac{\pi}{4}\right).$$

We know that the range of the principal value branch of $\operatorname{cosec}^{-1}$ is

$$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\} \text{ and } \operatorname{cosec}\left(-\frac{\pi}{4}\right) = -\sqrt{2}.$$

Therefore, the principal value of $\operatorname{cosec}^{-1}(-\sqrt{2})$ is $-\frac{\pi}{4}$.

Question 11:

$$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$$

Find the value of

Answer

Let $\tan^{-1}(1) = x$. Then, $\tan x = 1 = \tan \frac{\pi}{4}$.

$$\therefore \tan^{-1}(1) = \frac{\pi}{4}$$

Let $\cos^{-1}\left(-\frac{1}{2}\right) = y$. Then, $\cos y = -\frac{1}{2} = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$.

$$\therefore \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$$

Let $\sin^{-1}\left(-\frac{1}{2}\right) = z$. Then, $\sin z = -\frac{1}{2} = -\sin\left(\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right)$.

$$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

$$\begin{aligned}\therefore \tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right) \\&= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6} \\&= \frac{3\pi + 8\pi - 2\pi}{12} = \frac{9\pi}{12} = \frac{3\pi}{4}\end{aligned}$$

Question 12:

$$\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$$

Find the value of

Answer

Let $\cos^{-1}\left(\frac{1}{2}\right) = x$. Then, $\cos x = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$.

$$\therefore \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

Let $\sin^{-1}\left(\frac{1}{2}\right) = y$. Then, $\sin y = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$.

$$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$\therefore \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + \frac{2\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$

Question 13:

Find the value of if $\sin^{-1} x = y$, then

$$\text{(A) } 0 \leq y \leq \pi \quad \text{(B) } -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

$$\text{(C) } 0 < y < \pi \quad \text{(D) } -\frac{\pi}{2} < y < \frac{\pi}{2}$$

Answer

It is given that $\sin^{-1} x = y$.

We know that the range of the principal value branch of \sin^{-1} is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Therefore, $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$.

Question 14:

Find the value of $\tan^{-1} \sqrt{3} - \sec^{-1}(-2)$ is equal to

$$\text{(A) } \frac{\pi}{3} \quad \text{(B) } -\frac{\pi}{3} \quad \text{(C) } \frac{\pi}{3} \quad \text{(D) } \frac{2\pi}{3}$$

Answer

***** END *****