Trigonometric Ratios of Compound Angles Ex 7.2 Q3

We have,

$$\sin 100^\circ - \sin 10^\circ$$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}} \times \sin 100^\circ - \frac{1}{\sqrt{2}} \times \cos 100^\circ\right) \quad \left[\begin{array}{l}\text{Multiplying and dividing} \\ \text{by } \sqrt{1^2+1^2} \text{ ie., by } \sqrt{2}\end{array}\right]$$

$$= \sqrt{2}\left(\cos 45^\circ \times \sin 100^\circ - \sin 45^\circ \times \cos 100^\circ\right)$$

$$= \sqrt{2}\left(\sin 100^\circ \times \cos 45^\circ - \cos 100^\circ \times \sin 45^\circ\right)$$

$$= \sqrt{2}\left(\sin\left(100^\circ - 45^\circ\right)\right)$$

$= \sqrt{2} \sin 55^\circ$, which is positive real number.

$[\because \sin\theta$ is positive in first quadrant$]$

Trigonometric Ratios of Compound Angles Ex 7.2 Q4

$(2\sqrt{3}+3)\sin\theta + 2\sqrt{3}\cos\theta$

assume $a = 2\sqrt{3}+3$, $b = 2\sqrt{3}$

$\sqrt{a^2+b^2} = \sqrt{12+9+12\sqrt{3}+12} = \sqrt{33+12\sqrt{3}}$

Dividing and multiplying the above equation with above value

we get, $\sqrt{33+12\sqrt{3}}\left(\dfrac{2\sqrt{3}+3}{\sqrt{33+12\sqrt{3}}}\sin\theta + \dfrac{2\sqrt{3}}{\sqrt{33+12\sqrt{3}}}\cos\theta\right)$

Assume $\tan\phi = \dfrac{a}{b}$, we have $\sin\phi = \dfrac{a}{\sqrt{a^2+b^2}}$, $\cos\phi = \dfrac{b}{\sqrt{a^2+b^2}}$

so above expressions changes to $\sqrt{33+12\sqrt{3}}\,(\sin\phi\sin\theta + \cos\phi\cos\theta)$

which is equal to $\sqrt{33+12\sqrt{3}}\cos(\theta - \phi)$

We know that maximum and minimum value of any cosine term is +1 and -1

$\sqrt{33+12\sqrt{3}} = \sqrt{15+12+6+12\sqrt{3}}$

we know that $12\sqrt{3}+6 < 12\sqrt{5}$ becasue value of $\sqrt{5}-\sqrt{3}$ is more than 0.5

so if we replace $12\sqrt{3}+6$ with $12\sqrt{5}$ the above inequality still holds

So range of above expression can be $\sqrt{15+12+12\sqrt{5}} = 2\sqrt{3}+\sqrt{15}$

$-(2\sqrt{3}+\sqrt{15}) < \sqrt{33+12\sqrt{3}}\cos(\theta-\phi) < 2\sqrt{3}+\sqrt{15}$

*********** END **********