



Trigonometric Functions Ex 5.2 Q 4

We have,

$$\sin \theta + \cos \theta = 0$$

$$\Rightarrow \sin \theta = -\cos \theta \text{ --- (i)}$$

$$\Rightarrow \frac{\sin \theta}{\cos \theta} = -1$$

$$\Rightarrow \tan \theta = -1$$

We know that,

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$\Rightarrow \sec^2 \theta = 1 + \tan^2 \theta$$

$$\Rightarrow \sec \theta = \pm \sqrt{1 + \tan^2 \theta}$$

In the 4th quadrant $\sec \theta$ is positive.

$$\begin{aligned} \therefore \sec \theta &= \sqrt{1 + \tan^2 \theta} \\ &= \sqrt{1 + (-1)^2} \\ &= \sqrt{1 + 1} \\ &= \sqrt{2} \end{aligned}$$

$$\therefore \cos \theta = \frac{1}{\sec \theta} = \frac{1}{\sqrt{2}}$$

putting $\cos \theta = \frac{1}{\sqrt{2}}$ in equation (i), we get,

$$\sin \theta = -\left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{\sqrt{2}}$$

Hence, $\sin \theta = -\frac{1}{\sqrt{2}}$ and $\cos \theta = \frac{1}{\sqrt{2}}$.

We have,

$$\cos \theta = -\frac{3}{5}, \quad \text{and } \pi < \theta < \frac{3\pi}{2}$$

\Rightarrow θ lies in the 3rd quadrant

We know that,

$$\Rightarrow \sin \theta = \pm \sqrt{1 - \cos^2 \theta}$$

In the 3rd quadrant $\sin \theta$ is negative and $\tan \theta$ is positive.

$$\begin{aligned} \therefore \sin \theta &= -\sqrt{1 - \cos^2 \theta} \\ &= -\sqrt{1 - \left(-\frac{3}{5}\right)^2} \quad \left[\because \cos \theta = -\frac{3}{5} \right] \\ &= -\sqrt{1 - \frac{9}{25}} \\ &= -\sqrt{\frac{16}{25}} \\ &= -\frac{4}{5} \end{aligned}$$

$$\Rightarrow \sin \theta = -\frac{4}{5}$$

$$\text{and, } \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{-\frac{4}{5}}{-\frac{3}{5}} = \frac{4}{3}$$

$$\text{Now, } \operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{1}{-\frac{4}{5}} = -\frac{5}{4}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{-\frac{3}{5}} = -\frac{5}{3}$$

$$\text{and, } \cot \theta = \frac{1}{\tan \theta} = \frac{1}{\frac{4}{3}} = \frac{3}{4}$$

$$\begin{aligned}
 \therefore \frac{\operatorname{cosec} \theta + \cot \theta}{\sec \theta - \tan \theta} &= \frac{\frac{-5}{4} + \frac{3}{4}}{\frac{-5}{3} - \frac{4}{3}} \\
 &= \frac{\frac{-5+3}{4}}{\frac{-5-4}{3}} \\
 &= \frac{-\frac{2}{4}}{-\frac{9}{3}} \\
 &= \frac{2}{4} \times \frac{3}{9} \\
 &= \frac{1}{6}
 \end{aligned}$$

$$\therefore \frac{\operatorname{cosec} \theta + \cot \theta}{\sec \theta - \tan \theta} = \frac{1}{6}$$

***** END *****