Maxima and Minima 18.5 Q42

Let l be a line through the point $P(1, 4)$ that cuts *the* x-axis and y-axis.

Now, equation of l is

$$y - 4 = m(x - 1)$$

$\therefore$ x – Intercept is $\dfrac{m-4}{m}$ and y – Intercept is $4 - m$

Let $S = \dfrac{m-4}{m} + 4 - m$

$\therefore$ $\dfrac{dS}{dm} = +\dfrac{4}{m^2} - 1$

For maxima and minima,

$$\frac{dS}{dm} = 0$$

$\Rightarrow$ $\dfrac{4}{m^2} - 1 = 0$

$\Rightarrow$ $m = \pm 2$

Now,

$$\frac{d^2S}{dm^2} = -\frac{8}{m^3}$$

At $m = 2$, $\dfrac{d^2S}{dm^2} = -1 < 0$

$m = -2$ $\dfrac{d^2S}{dm^2} = 1 > 0$

$\therefore$ $m = -2$ is point of local minima.

$\therefore$ least value of sum of intercept is

$$\frac{m-4}{m} + 4 - m$$

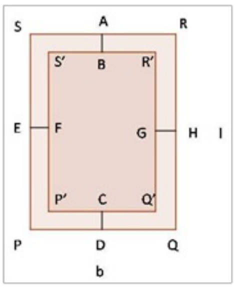
$$= 3 + 6 = 9$$

Maxima and Minima 18.5 Q43

The area of the page $PQRS$ in 150 cm^2

Also, $AB + CD = 3$ cm

$EF + GH = 2$ Cm



Let x and y be the combined width of margin at the top and bottom and the sides respectively.

$\therefore \quad x = 3$ cm and $y = 2$ cm.

Now, area of printed matter = area of $P'Q'R'S'$

$\Rightarrow \quad A = P'Q' \times Q'R'$

$\Rightarrow \quad A = (b - y)(l - x)$

$\Rightarrow \quad A = (b - 2)(l - 3) \qquad$ ---(i)

Also,

Area of $PQRS = 150 \text{ cm}^2$

$\Rightarrow \quad lb = 150 \qquad$ ---(ii)

From (i) and (ii)

$$A = (b - 2)\left(\frac{150}{b} - 3\right)$$

$\therefore$ For maximum and minimum,

$$\frac{dA}{db} = \left(\frac{150}{b} - 3\right) + (b - 2)\left(-\frac{150}{b^2}\right) = 0$$

$$\Rightarrow \quad \frac{(150 - 3b)}{b} + (-150)\frac{(b - 2)}{b^2} = 0$$

$\Rightarrow \quad 150b - 3b^2 - 150b + 300 = 0$

$\Rightarrow \quad -3b^2 + 300 = 0$

$\Rightarrow \quad b = 10$

From (ii)

$l = 15$

Now,

$$\frac{d^2A}{db^2} = \frac{-150}{b^2} - 150\left[-\frac{1}{b^2} + \frac{4}{b^3}\right]$$

At $b = 10$

$$\frac{d^2A}{db^2} = -\frac{15}{10} - 150\left[-\frac{1}{100} + \frac{4}{1000}\right]$$

$= -1.5 - .15[-10 + 4]$

$= -1.5 + .9$

$= -0.6 < 0$

$\therefore \quad b = 10$ is point of local maxima.

Hence,

The required dimension will be $l = 15$ cm, $b = 10$ cm.

Maxima and Minima 18.5 Q44

The space s described in time t by a moving particle is given by

$$s = t^5 - 40t^3 + 30t^2 + 80t - 250$$

$\therefore$ velocity $= \frac{ds}{dt} = 5t^4 - 120t^2 + 60t + 80$

Acceleration $= a = \frac{d^2s}{dt^2} = 20t^3 - 240t + 60t$ ---(i)

Now,

$$\frac{da}{dt} = 60t^2 - 240$$

For maxima and minima,

$$\frac{da}{dt} = 0$$

$\Rightarrow$ $60t^2 - 240 = 0$

$\Rightarrow$ $60\left(t^2 - 4\right) = 0$

$\Rightarrow$ $t = 2$

Now,

$$\frac{d^2a}{dt^2} = 120t$$

At $t = 2$, $\frac{d^2a}{dt^2} = 240 > 0$

$\therefore$ $t = 2$ is point of local minima

Hence, minimum acceleration is $160 - 480 + 60 = -260$.

Maxima and Minima 18.5 Q45

We have,

$$\text{Distance, } s = \frac{t^4}{4} - 2t^3 + 4t^2 - 7$$

$$\text{Velocity,} v = \frac{ds}{dt} = t^3 - 6t^2 + 8t$$

$$\text{Acceleration,} a = \frac{d^2s}{dt^2} = 3t^2 - 12t + 8$$

For velocity to be maximum and minimum,

$$\frac{dv}{dt} = 0$$

$$\Rightarrow \quad 3t^2 - 12t + 8 = 0$$

$$\Rightarrow \quad t = \frac{12 \pm \sqrt{144 - 96}}{6}$$

$$= 2 \pm \frac{4\sqrt{3}}{6}$$

$$\therefore \quad t = 2 + \frac{2}{\sqrt{3}},\ 2 - \frac{2}{\sqrt{3}}$$

Now,

$$\frac{d^2v}{dt^2} = 6t - 12$$

$$\text{At} \quad t = 2 - \frac{2}{\sqrt{3}},\ \frac{d^2v}{dt^2} = 6\left(2 - \frac{2}{\sqrt{3}}\right) - 12 = \frac{-12}{\sqrt{3}} < 0$$

$$t = 2 + \frac{2}{\sqrt{3}},\ \frac{d^2r}{dt^2} = 6\left(2 + \frac{2}{\sqrt{3}}\right) - 12 = \frac{12}{\sqrt{3}} > 0$$

$$\therefore \quad \text{At } t = 2 - \frac{2}{\sqrt{3}}, \text{ velocity is maximum}$$

For acceleration to be maximum and minimum

$$\frac{da}{dt} = 0$$

$$\Rightarrow \quad 6t - 12 = 0$$

$$\Rightarrow \quad t = 2$$

Now,

$$\frac{d^2a}{dt^2} = 6 > 0$$

$\therefore$ At, $t = 2$ Acceleration is minimum.

*********** END **********