



Exercise 5.1

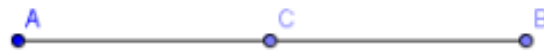
Q4. If a point C lies between two points A and B such that $AC = BC$, then prove that $AC = \frac{1}{2} AB$.

Explain by drawing the figure.

Ans: We are given that a point C lies between two points A and B , such that $AC = BC$.

We need to prove that $AC = \frac{1}{2} AB$.

Let us consider the given below figure.



We are given that $AC = BC$(i)

An axiom of the Euclid says that "If equals are added to equals, the wholes are equal."

Let us add AC to both sides of equation (i).

$$AC + AC = BC + AC.$$

An axiom of the Euclid says that "Things which coincide with one another are equal to one another."

We can conclude that $BC + AC$ coincide with AB ,
or

$$AB = BC + AC$$
....(ii)

An axiom of the Euclid says that "Things which are equal to the same thing are equal to one another."

From equations (i) and (ii), we can conclude that $AC + AC = AB$, or $2AC = AB$.

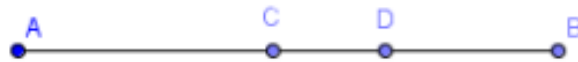
An axiom of the Euclid says that “Things which are halves of the same things are equal to one another.”

Therefore, we can conclude that $AC = \frac{1}{2} AB$.

Q5. In the above question, point C is called a mid-point of line segment AB, prove that every line segment has one and only one mid-point.

Ans: We need to prove that every line segment has one and only one mid-point.

Let us consider the given below line segment AB and assume that C and D are the mid-points of the line segment AB.



If C is the mid-point of line segment AB, then $AC = CB$.

An axiom of the Euclid says that “If equals are added to equals, the wholes are equal.”

$$AC + AC = CB + AC \dots (i)$$

From the figure, we can conclude that $CB + AC$ will coincide with AB.

An axiom of the Euclid says that “Things which coincide with one another are equal to one another.”

$$AC + AC = AB \dots (ii)$$

An axiom of the Euclid says that “Things which are equal to the same thing are equal to one another.”

Let us compare equations (i) and (ii), to get
 $AC + AC = AB$, or $2AC = AB$ (iii)

If D is the mid-point of line segment AB , then
 $AD = DB$.

An axiom of the Euclid says that “If equals are added to equals, the wholes are equal.”

$AD + AD = DB + AD$ (iv)

From the figure, we can conclude that $DB + AD$ will coincide with AB .

An axiom of the Euclid says that “Things which coincide with one another are equal to one another.”

$AD + AD = AB$ (v)

An axiom of the Euclid says that “Things which are equal to the same thing are equal to one another.”

Let us compare equations (iv) and (v), to get

$$AD + AD = AB, \text{ or}$$

$$2AD = AB. \text{(vi)}$$

An axiom of the Euclid says that “Things which are equal to the same thing are equal to one another.”

Let us compare equations (iii) and (vi), to get

$$2AC = 2AD.$$

An axiom of the Euclid says that “Things which are halves of the same things are equal to one another.”

$$AC = AD.$$

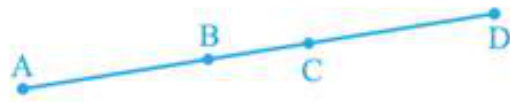
Therefore, we can conclude that the assumption that we made previously is false and a line segment has one and only one mid-point.

Q6. In the following figure, if $AC = BD$, then prove that $AB = CD$.



Ans: We are given that $AC = BD$.

We need to prove that $AB = CD$ in the figure given below.



From the figure, we can conclude that

$AC = AB + BC$, and

$BD = CD + BC$.

An axiom of the Euclid says that “Things which are equal to the same thing are equal to one another.”

$AB + BC = CD + BC$. (i)

An axiom of the Euclid says that “when equals are subtracted from equals, the remainders are also equal.”

We need to subtract BC from equation (i), to get

$AB + BC - BC = CD + BC - BC$

$AB = CD$.

Therefore, we can conclude that the desired result is proved.

Q7. Why is axiom 5, in the list of Euclid’s axioms, considered as a ‘universal truth’? (Note that the question is not about fifth postulate)

Ans: We need to prove that Euclid's fifth axiom is considered as a universal truth.

Euclid's fifth axiom states that "the whole is greater than the part."

The above given axiom is a universal truth. We can apply the fifth axiom not only mathematically but also universally in daily life.

Mathematical proof:

Let us consider a quantity z , which has different parts as a , b , x and y .

$$z = a + b + x + y .$$

Therefore, we can conclude that z will always be greater than its corresponding parts a , b , x and y .

Universal proof:

We know that Mumbai is located in Maharashtra and Maharashtra is located in India.

In other words, we can conclude that Mumbai is a part of Maharashtra and Maharashtra is a part of India.

Therefore, we can conclude that whole India will be greater than Mumbai or Maharashtra or both.

Therefore, we can conclude that Euclid's fifth axiom is considered as a 'Universal truth'.

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