



#### Differentials Errors and Approximation Ex14.1 Q10

Let  $x = 2$  and  $\Delta x = 0.01$ . Then, we have:

$$f(2.01) = f(x + \Delta x) = 4(x + \Delta x)^2 + 5(x + \Delta x) + 2$$

$$\text{Now, } \Delta y = f(x + \Delta x) - f(x)$$

$$\therefore f(x + \Delta x) = f(x) + \Delta y$$

$$\approx f(x) + f'(x) \cdot \Delta x \quad (\text{as } dx = \Delta x)$$

$$\begin{aligned} \Rightarrow f(2.01) &\approx (4x^2 + 5x + 2) + (8x + 5)\Delta x \\ &= [4(2)^2 + 5(2) + 2] + [8(2) + 5](0.01) \quad [x = 2, \Delta x = 0.01] \\ &= (16 + 10 + 2) + (16 + 5)(0.01) \\ &= 28 + (21)(0.01) \\ &= 28 + 0.21 \\ &= 28.21 \end{aligned}$$

Hence, the approximate value of  $f(2.01)$  is 28.21.

#### Differentials Errors and Approximation Ex14.1 Q11

Let  $x = 5$  and  $\Delta x = 0.001$ . Then, we have:

$$f(5.001) = f(x + \Delta x) = (x + \Delta x)^3 - 7(x + \Delta x)^2 + 15$$

$$\text{Now, } \Delta y = f(x + \Delta x) - f(x)$$

$$\therefore f(x + \Delta x) = f(x) + \Delta y$$

$$\approx f(x) + f'(x) \cdot \Delta x \quad (\text{as } dx = \Delta x)$$

$$\begin{aligned} \Rightarrow f(5.001) &\approx (x^3 - 7x^2 + 15) + (3x^2 - 14x)\Delta x \\ &= [(5)^3 - 7(5)^2 + 15] + [3(5)^2 - 14(5)](0.001) \quad [x = 5, \Delta x = 0.001] \\ &= (125 - 175 + 15) + (75 - 70)(0.001) \\ &= -35 + (5)(0.001) \\ &= -35 + 0.005 \\ &= -34.995 \end{aligned}$$

Hence, the approximate value of  $f(5.001)$  is -34.995.

#### Differentials Errors and Approximation Ex14.1 Q12

$$\begin{aligned}\text{Let } x &= 1000, x + \Delta x = 1005 \\ \Delta x &= 1005 - 1000 \\ &= 5\end{aligned}$$

$$\begin{aligned}\text{Let } y &= \log_{10} x \\ \frac{dy}{dx} &= \frac{\log_e x}{\log_e 10} \\ \frac{dy}{dx} &= \frac{1}{x \log_e 10}\end{aligned}$$

$$\left[ \because \log_a b = \frac{\log_e b}{\log_e a} \right]$$

$$\begin{aligned}\left( \frac{dy}{dx} \right)_{x=1000} &= \frac{\log_{10} e}{1000} \\ &= \frac{0.4343}{1000} \\ &= (0.0004343)\end{aligned}$$

$$\left[ \because \log_a b = \frac{1}{\log_b a} \right]$$

$$\begin{aligned}\therefore \Delta y &= \left( \frac{dy}{dx} \right)_{x=1000} \times (\Delta x) \\ &= (0.0004343)(5) \\ &= 0.0021715\end{aligned}$$

$$\begin{aligned}\log_{10} 1005 &= y + \Delta y \\ &= \log_{10} x + 0.0021715 \\ &= \log_{10} 1000 + 0.0021715 \\ &= \log_{10} 10^3 + 0.0021715 \\ &= 3 \log_{10} 10 + 0.0021715\end{aligned}$$

$$\log_{10} 1005 = 3.0021715$$

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