Definite Integrals Ex 20.5 Q5

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 0,\ b = 5$

and $f(x) = (x+1)$

$\therefore\ h = \frac{5}{n} \Rightarrow nh = 5$

Thus, we have,

$$I = \int_0^5 (x+1)dx$$

$$I = \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f\{(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[1+(h+1)+(2h+1)+-----+((n-1)h+1)\right]$$

$$= \lim_{h\to 0} h\left[n+h(1+2+3+-----(n-1))\right]$$

$\because h = \frac{5}{n}$ and if $h \to 0,\ n \to \infty$

$$= \lim_{n\to\infty} \frac{5}{n}\left[n+\frac{5}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} 5+\frac{25}{2n^2}n^2\left(1-\frac{1}{n}\right)$$

$$= 5+\frac{25}{2}$$

$$\therefore \int_0^5 (x+1)dx = \frac{35}{2}$$

Definite Integrals Ex 20.5 Q6

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 1$, $b = 3$

and $f(x) = (2x+3)$

$\therefore\ h = \frac{2}{n} \Rightarrow nh = 2$

Thus, we have,

$$I = \int_1^3 (2x+3)\,dx$$

$$= \lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(1+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[2+3+\{2(1+h)+3\}+\{2(1+2h)+3\}----+2\{1+(n-1)+3\}\right]$$

$$= \lim_{h\to 0} h\left[5+(5+2h)+(5+4h)+----5+2(n-1)h\right]$$

$$= \lim_{h\to 0} h\left[5n+2h(1+2+3+----(n-1))\right]$$

$\because h = \frac{2}{n}$ and if $h \to 0 \Rightarrow n \to \infty$

$$\therefore \lim_{n\to\infty} \frac{2}{n}\left[5n+\frac{4}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty}\left[10+\frac{4}{n^2}\frac{n(n-1)}{2}\right] = 14$$

$$\therefore \int_1^3 (2x+3)\,dx = 14$$

Definite Integrals Ex 20.5 Q7

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 3$, $b = 5$

and $f(x) = (2-x)$

$\therefore\ h = \frac{2}{n} \Rightarrow nh = 2$

Thus, we have,

$$I = \int_3^5 (2-x)\,dx$$

$$= \lim_{h\to 0} h\left[f(3)+f(3+h)+f(3+2h)+----f(3+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[(2-3)+\{2-(3+h)\}+\{2-(3+2h)\}+----\{2-(3+(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[-1+(-1-h)+(-1-2h)+----\{-1-(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[-n-h(1+2+----(n-1)h\right]$$

$\because h = \frac{2}{n}$ & if $h \to 0 \Rightarrow n \to \infty$

$$\therefore \lim_{n\to\infty} \frac{2}{n}\left[-n-\frac{2}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} -2-\frac{2}{n^2}n^2\left(1-\frac{1}{n}\right) = -2-2 = -4$$

$$\therefore \int_3^5 (2-x)\,dx = -4$$

Definite Integrals Ex 20.5 Q8

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a) + f(a+h) + f(a+2h) + ---- f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 0$, $b = 2$ and $f(x) = (x^2+1)$

$$\therefore\ h = \frac{2}{n} \Rightarrow nh = 2$$

Thus, we have,

$$I = \int_0^2 (x^2+1)\,dx$$

$$= \lim_{h\to 0} h\left[f(0) + f(h) + f(2h) + ---- f\{(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[1 + (h^2+1) + \{(2h)^2+1\} + ---- \{((n-1)h)^2+1\}\right]$$

$$= \lim_{h\to 0} h\left[n + h^2\left(1 + 2^2 + 3^2 + ----- + (n-1)^2\right)\right]$$

$$\because h = \frac{2}{n} \ \&\ \text{if } h \Rightarrow 0 \Rightarrow n \to \infty$$

$$\therefore \lim_{n\to\infty} \frac{2}{n}\left[n + \frac{4}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} 2 + \frac{4}{3n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= 2 + \frac{4}{3}\times 2 = \frac{14}{3}$$

$$\therefore \int_0^2 (x^2+1)\,dx = \frac{14}{3}$$

*********** END **********