Derivatives as a Rate Measurer Ex 13.2 Q5

Let r be the radius of the spherical soap bubble.

Here, $\frac{dr}{dt} = 0.2$ cm/sec, $r = 7$ cm

Surface Area $(A) = 4\pi r^2$

$$\frac{dA}{dt} = 4\pi(2r)\frac{dr}{dt}$$

$$\left(\frac{dA}{dt}\right)_{r=7} = 4\pi(2 \times 7) \times 0.$$

$$= 11.2\pi \text{ cm}^2/\text{sec.}$$

So, area of bubble increases at the rate of 11.2π cm^2/sec.

Derivatives as a Rate Measurer Ex 13.2 Q6

The volume of a sphere (V) with radius (r) is given by,

$$V = \frac{4}{3}\pi r^3$$

$\therefore$Rate of change of volume (V) with respect to time (t) is given by,

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt} \text{ [By chain rule]}$$

$$= \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right) \cdot \frac{dr}{dt}$$

$$= 4\pi r^2 \cdot \frac{dr}{dt}$$

It is given that $\frac{dV}{dt} = 900 \text{ cm}^3/\text{s}$.

$$\therefore 900 = 4\pi r^2 \cdot \frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = \frac{900}{4\pi r^2} = \frac{225}{\pi r^2}$$

Therefore, when radius = 15 cm,

$$\frac{dr}{dt} = \frac{225}{\pi(15)^2} = \frac{1}{\pi}$$

Hence, the rate at which the radius of the balloon increases when the radius is 15 cm is $\frac{1}{\pi}$ cm/s.

Derivatives as a Rate Measurer Ex 13.2 Q7

Let r be the radius of the air bubble.

Here, $\frac{dr}{dt} = 0.5$ cm/sec, $r = 1$ cm

Volume $(V) = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = \frac{4}{3}\pi\left(3r^2\right)\frac{dr}{dt}$$
$$= 4\pi r^2 \frac{dr}{dt}$$
$$= 4\pi\left(1\right)^2 \times \left(0.5\right)$$
$$\frac{dV}{dt} = 2\pi \text{ cm}^3/\text{sec.}$$

So,volume of air bubble increases at the rate of 2π cm^3/sec.

*********** END **********