

Trigonometric Ratios of Compound Angles Ex 7.1 Q5

We have,

$$\sin A = \frac{1}{2}$$
 and $\cos B = \frac{12}{13}$

$$\cos A = -\sqrt{1 - \sin^2 A} \text{ and } \sin B = -\sqrt{1 - \cos^2 B}$$

$$\begin{bmatrix} \because \text{ cosine is negative in second quadrant and} \\ \text{sine is negative in fourth quadrant} \end{bmatrix}$$

$$\Rightarrow \cos A = -\sqrt{1 - \left(\frac{1}{2}\right)^2} \text{ and } \sin B = -\sqrt{1 - \left(\frac{12}{13}\right)^2}$$

$$\Rightarrow \cos A = -\sqrt{1 - \frac{1}{4}} \text{ and } \sin B = -\sqrt{1 - \frac{144}{169}}$$

$$\Rightarrow \cos A = -\sqrt{\frac{3}{4}} \text{ and } \sin B = -\sqrt{\frac{25}{169}}$$

$$\Rightarrow \cos A = -\frac{\sqrt{3}}{2} \text{ and } \sin B = -\frac{5}{13}$$

$$\Rightarrow \cos A = -\sqrt{\frac{3}{4}} \text{ and } \sin B = -\sqrt{\frac{25}{169}}$$

$$\Rightarrow \cos A = -\frac{\sqrt{3}}{2} \text{ and } \sin B = -\frac{5}{13}$$

and,
$$\tan B = \frac{\sin B}{\cos B} = \frac{-\frac{5}{13}}{\frac{12}{13}} = \frac{-5}{12}$$

Now,
$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{-\frac{1}{\sqrt{3}} - \left(\frac{-5}{12}\right)}{1 + \left(\frac{-1}{\sqrt{3}}\right) \times \left(\frac{-5}{12}\right)}$$

$$= \frac{-\frac{1}{\sqrt{3}} + \frac{5}{12}}{1 + \frac{5}{12\sqrt{3}}}$$

$$= \frac{-12 + 5\sqrt{3}}{12\sqrt{3}}$$

$$= \frac{-12 + 5\sqrt{3}}{12\sqrt{3}}$$

$$= \frac{5\sqrt{3} - 12}{5 + 12\sqrt{3}}$$

$$\Rightarrow \tan (A - B) = \frac{5\sqrt{3} - 12}{5 + 12\sqrt{3}}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q6

$$\sin A = \frac{1}{2}$$
 and $\cos B = \frac{\sqrt{3}}{2}$

$$\cos A = -\sqrt{1 - \sin^2 A} \text{ and } \sin B = \sqrt{1 - \cos^2 B}$$

$$\left[\because \text{ cosine is negative in second quadrant}\right]$$

$$\Rightarrow \cos A = -\sqrt{1 - \left(\frac{1}{2}\right)^2} \text{ and } \sin B = \sqrt{1 - \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow$$
 $\cos A = -\sqrt{1 - \frac{1}{4}}$ and $\sin B = \sqrt{1 - \frac{3}{4}}$

$$\Rightarrow \cos A = -\sqrt{\frac{3}{4}} \text{ and } \sin B = \sqrt{\frac{1}{4}}$$

$$\Rightarrow \cos A = -\frac{\sqrt{3}}{2} \text{ and } \sin B = \frac{1}{2}$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{2}}{\frac{-\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}}$$

and,
$$\tan B = \frac{\sin B}{\cos B} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

Now,

$$\tan \left(A + B\right) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$
$$= \frac{\frac{-1}{\sqrt{3}} + \frac{1}{\sqrt{3}}}{1 - \left(\frac{-1}{\sqrt{3}}\right) \times \left(\frac{1}{\sqrt{3}}\right)}$$
$$= 0$$

$$\therefore \tan(A+B)=0$$

We have,

$$\sin A = \frac{1}{2}$$
 and $\cos B = \frac{\sqrt{3}}{2}$

 $\sin A = \frac{1}{2}$ and $\cos B = \frac{\sqrt{3}}{2}$ $\cos A = -\sqrt{1 - \sin^2 A}$ and $\sin B = \sqrt{1 - \cos^2 B}$ [$\cdot \cdot \cdot$ cosine is negative in second quadrant]

$$\Rightarrow \cos A = -\sqrt{1 - \left(\frac{1}{2}\right)^2} \text{ and } \sin B = \sqrt{1 - \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow \cos A = -\sqrt{1 - \frac{1}{4}} \text{ and } \sin B = \sqrt{1 - \frac{3}{4}}$$

$$\Rightarrow \cos A = -\sqrt{\frac{3}{4}} \text{ and } \sin B = \sqrt{\frac{1}{4}}$$

$$\Rightarrow \cos A = -\frac{\sqrt{3}}{2} \text{ and } \sin B = \frac{1}{2}$$

$$\therefore \tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{2}}{\frac{-\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}}$$

and,
$$\tan B = \frac{\sin B}{\cos B} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

Now,
$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\frac{\sqrt{3} - \left(\sqrt{3}\right)}{1 + \left(\frac{-1}{\sqrt{3}}\right) \times \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{-2}{\sqrt{3}}}{1 - \frac{1}{3}}$$

$$= \frac{\frac{-2}{\sqrt{3}}}{\frac{3 - 1}{3}}$$

$$= \frac{\frac{-2}{\sqrt{3}}}{\frac{2}{3}}$$

$$= \frac{-\sqrt{3}}{\sqrt{3}} \times \sqrt{3} = -\sqrt{3}$$

$$\therefore \tan(A-B) = -\sqrt{3}$$

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(i)
          sin 78° cos 18° - cos 78° sin 18°
                                                          \left[\sin(A-B) = \sin A \cos B - \cos A \sin B\right]
          = sin (78° - 18°)
          = sin 60°
          =\frac{\sqrt{3}}{2}
(ii)
          cos 47° cos 13° - sin 47° sin 13°
                                                          \left[\cos\left(A+B\right)=\cos A\cos B-\sin A\sin B\right]
          = \cos(47^{\circ} + 13^{\circ})
          = cos60°
          =\frac{1}{2}
(iii)
                                                          \left[\sin(A+B) = \sin A \cos B + \cos A \sin B\right]
          sin 36° cos 9° + cos 36° sin 9°
          = sin(36° + 9°)
          = sin 45°
          =\frac{1}{\sqrt{2}}
(iv)
          cos 80° cos 20° + sin 80° sin 20°
                                                          \left[\cos\left(A-B\right)=\cos A\cos B+\sin A\sin B\right]
          = cos(80° - 20°)
          = cos60°
          =\frac{1}{2}
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