



Indefinite Integrals Ex 19.8 Q6

$$\frac{\cos 2x}{(\cos x + \sin x)^2} = \frac{\cos 2x}{\cos^2 x + \sin^2 x + 2 \sin x \cos x} = \frac{\cos 2x}{1 + \sin 2x}$$

$$\therefore \int \frac{\cos 2x}{(\cos x + \sin x)^2} dx = \int \frac{\cos 2x}{1 + \sin 2x} dx$$

$$\text{Let } 1 + \sin 2x = t$$

$$\Rightarrow 2 \cos 2x dx = dt$$

$$\begin{aligned} \therefore \int \frac{\cos 2x}{(\cos x + \sin x)^2} dx &= \frac{1}{2} \int \frac{1}{t} dt \\ &= \frac{1}{2} \log |t| + C \\ &= \frac{1}{2} \log |1 + \sin 2x| + C \\ &= \frac{1}{2} \log |(\sin x + \cos x)^2| + C \\ &= \log |\sin x + \cos x| + C \end{aligned}$$

Indefinite Integrals Ex 19.8 Q7

$$\text{Let } I = \int \frac{\sin(x-a)}{\sin(x-b)} dx \text{ then}$$

$$\begin{aligned} I &= \int \frac{\sin(x-a+b-b)}{\sin(x-b)} dx \\ &= \int \frac{\sin(x-b+b-a)}{\sin(x-b)} dx \\ &= \int \frac{\sin(x-b) \cos(b-a) + \cos(x-b) \sin(b-a)}{\sin(x-b)} dx \\ &= \int (\cos(b-a) + \cot(x-b) \sin(b-a)) dx \\ &= \cos(b-a) \int dx + \sin(b-a) \int \cot(x-b) dx \\ &= x \cos(b-a) + \sin(b-a) \log |\sin(x-b)| + c \end{aligned}$$

$$\therefore I = x \cos(b-a) + \sin(b-a) \log |\sin(x-b)| + c$$

Indefinite Integrals Ex 19.8 Q8

Let $I = \int \frac{\sin(x - \alpha)}{\sin(x + \alpha)} dx$ then,

$$\begin{aligned}
 I &= \int \frac{\sin(x - \alpha + \alpha - \alpha)}{\sin(x + \alpha)} dx \\
 &= \int \frac{\sin(x + \alpha - 2\alpha)}{\sin(x + \alpha)} dx \\
 &= \int \frac{\sin(x + \alpha) \cos 2\alpha - \cos(x + \alpha) \sin 2\alpha}{\sin(x + \alpha)} dx \\
 &= \int \left[\frac{\sin(x + \alpha) \cos 2\alpha}{\sin(x + \alpha)} - \frac{\cos(x + \alpha) \sin 2\alpha}{\sin(x + \alpha)} \right] dx \\
 &= \int (\cos 2\alpha - \cot(x + \alpha) \sin 2\alpha) dx \\
 &= \cos 2\alpha \int dx - \sin 2\alpha \int \cot(x + \alpha) dx \\
 &= x \cos 2\alpha - \sin 2\alpha \log |\sin(x + \alpha)| + c
 \end{aligned}$$

$\therefore I = x \cos 2\alpha - \sin 2\alpha \log |\sin(x + \alpha)| + c$

Indefinite Integrals Ex 19.8 Q9

Let $I = \int \frac{1 + \tan x}{1 - \tan x} dx$

$$\begin{aligned}
 I &= \int \frac{1 + \frac{\sin x}{\cos x}}{1 - \frac{\sin x}{\cos x}} dx \\
 &= \int \frac{\frac{\cos x + \sin x}{\cos x}}{\frac{\cos x - \sin x}{\cos x}} dx
 \end{aligned}$$

$\Rightarrow I = \int \frac{\cos x + \sin x}{\cos x - \sin x} dx \dots \dots (i)$

Let $\cos x - \sin x = t$ then,
 $d(\cos x - \sin x) = dt$

$\Rightarrow (-\sin x - \cos x) dx = dt$

$\Rightarrow -(\sin x + \cos x) dx = dt$

$\Rightarrow dx = -\frac{dt}{\sin x + \cos x}$

Putting $\cos x - \sin x = t$ and $dx = \frac{-dt}{\sin x + \cos x}$ in equation (i), we get

$$\begin{aligned}
 I &= \int \frac{\cos x + \sin x}{t} \times \frac{-dt}{\sin x + \cos x} \\
 &= -\int \frac{dt}{t} \\
 &= -\log |t| + c \\
 &= -\log |\cos x - \sin x| + c
 \end{aligned}$$

$\therefore I = -\log |\cos x - \sin x| + c$

Indefinite Integrals Ex 19.8 Q10

Let $I = \int \frac{\cos x}{\cos(x-a)} dx$ then,

$$\begin{aligned} I &= \int \frac{\cos(x+a-a)}{\cos(x-a)} dx \\ &= \int \frac{\cos(x-a+a)}{\cos(x-a)} dx \\ &= \int \frac{\cos(x-a)\cos a - \sin(x-a)\sin a}{\cos(x-a)} dx \\ &= \int \frac{\cos(x-a)\cos a}{\cos(x-a)} dx - \int \frac{\sin(x-a)\sin a}{\cos(x-a)} dx \\ &= \cos a \int dx - \sin a \int \tan(x-a) dx \end{aligned}$$

$$= x \cos a - \sin a \log |\sec(x-a)| + c$$

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