Functions Ex 3.4 Q5

We have,

$$f(x) = \log_e (1-x)$$

and $g(x) = [x]$

$f(x) = \log_e (1-x)$ is defined, if $1-x > 0$

$\Rightarrow \quad 1 > x$

$\Rightarrow \quad x < 1$

$\Rightarrow \quad x \in (-\infty, 1)$

$\therefore$ Domain$(f) = (-\infty, 1)$

$g(x) = [x]$ is defined for all $x \in R$

$\therefore$ Domain$(g) = R$

$\therefore$ Domain$(f) \cap R$ Domain$(g) = (-\infty, 1) \cap R$

$$= (-\infty, 1)$$

(i) $f + g : (-\infty, 1) \to R$ defined by $(f+g)(x) = f(x) + g(x)$

$$= \log_e (1-x) + [x]$$

(ii) $fg : (-\infty, 1) \to R$ defined by $(fg)(x) = f(x) \times g(x)$

$$= \log_e (1-x) \times [x]$$

$$= [x] \log_e (1-x)$$

(iii) $g(x) = [x]$

$\therefore \quad [x] = 0$

$\Rightarrow \quad x \in (0, 1)$

So, domain$\left(\dfrac{f}{g}\right) = $ domain$(f) \cap$ domain$(g) - \{x : g(x) = 0\}$

$$= (-\infty, 0)$$

$\therefore \quad \dfrac{f}{g} : (-\infty, 0) \to R$ defined by $\left(\dfrac{f}{g}\right)(x) = \dfrac{\log_e (1-x)}{[x]}$

(iv) We have,

$$f(x) = \log_e (1 - x)$$

$$\Rightarrow \quad \frac{1}{f(x)} = \frac{1}{\log_e (1-x)}$$

$\therefore \quad \dfrac{1}{f(x)}$ is defined if $\log_e (1-x)$ is defined and $\log_e (1-x) \neq 0$

$$\Rightarrow \quad 1 - x > 0 \qquad \text{and} \qquad 1 - x \neq 0$$

$$\Rightarrow \quad x < 1 \qquad \text{and} \qquad x \neq 0$$

$$\Rightarrow \quad x \in (-\infty, 0) \cup (0, 1)$$

$$\therefore \quad \text{domain}\left(\frac{g}{f}\right) = (-\infty, 0) \cup (0, 1)$$

$$\frac{g}{f} : (-\infty, 0) \cup (0, 1) \to R \text{ defined by } \left(\frac{g}{f}\right)(x) = \frac{[x]}{\log_e (1-x)}$$

Now,

$$(f + g)(-1) = f(-1) + g(-1)$$

$$= \log_e (1 - (-1)) + [-1]$$

$$= \log_e 2 - 1$$

$$\Rightarrow \quad (f + g)(-1) = \log_e 2 - 1$$

(v) $\quad fg(0) = \log_e (1 - 0) \times [0]$

$$= 0$$

(vi) $\quad \left(\dfrac{f}{g}\right)\left(\dfrac{1}{2}\right) = \text{does not exist}$

(vii) $\quad \left(\dfrac{g}{f}\right)\left(\dfrac{1}{2}\right) = \dfrac{\left[\dfrac{1}{2}\right]}{\log_e \left(1 - \dfrac{1}{2}\right)} = 0$

Functions Ex 3.4 Q6

We have,

$$f(x) = \sqrt{x + 1}, \quad g(x) = \frac{1}{x}$$

and $\quad h(x) = 2x^2 - 3$

Clearly, $f(x)$ is defined for $x + 1 \geq 0$

$$\Rightarrow \quad x \geq -1$$

$$\Rightarrow \quad x \in [-1, \infty]$$

$\therefore \quad \text{Domain}(f) = [-1, \infty]$

$g(x)$ is defined for $x \neq 0$

$$\Rightarrow \quad x \in R - \{0\}$$

and, $\quad h(x)$ is defined for all $x \in R$

$\therefore \quad \text{Domain}(f) \cap \text{Domain}(g) \cap \text{Domain}(h) = [-1, \infty] - \{0\}$

Clearly,

$$2f + g - h : [-1, \infty] - \{0\} \to R \text{ is given by}$$

$$(2f + g - h)(x) = 2f(x) + g(x) - h(x)$$

$$= 2\sqrt{x + 1} + \frac{1}{x} - 2x^2 + 3$$

$$\therefore \quad (2f + g - h)(1) = 2\sqrt{1 + 1} + \frac{1}{1} - 2 \times (1)^2 + 3$$

$$= 2\sqrt{2} + 1 - 2 + 3$$

$$= 2\sqrt{2} + 4 - 2$$

$$= 2\sqrt{2} + 2$$

and, $\quad (2f + g - h)(0)$ does not exist, it is not lies in the domain $x \in [-1, \infty] - \{0\}$.

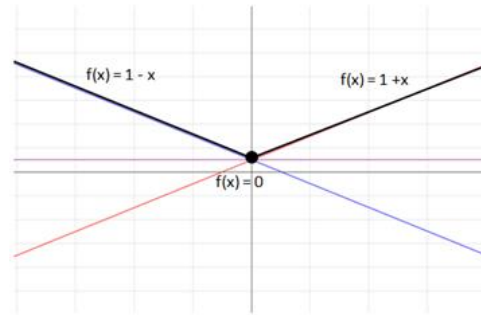## Functions Ex 3.4 Q7

Let,

$$y = f(x) = \begin{cases} 1 - x, & x < 0 \\ 1, & x = 0 \\ x + 1, & x > 0 \end{cases}$$

The graph of f(x) for x < 0 is the part of the line y = 1-x that lies to the left of origin.
The graph of f(x) for x > 0 is the part of the line y = 1+x that lies to the right of origin.
For x = 0, the graph of f(x) represents the point (0,1)

The graph of f(x) is shown below.

********** END *********