$$\frac{2}{\sqrt{14}}x+\frac{3}{\sqrt{14}}y-\frac{1}{\sqrt{14}}z=\frac{5}{\sqrt{14}}$$

This equation is of the form *lx* + *my* + *nz* = *d*, where *l*, *m*, *n* are the direction cosines of normal to the plane and *d* is the distance of normal from the origin.

Therefore, the direction cosines of the normal to the plane are $\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$, and $\frac{-1}{\sqrt{14}}$ and the distance of normal from the origin is $\frac{5}{\sqrt{14}}$ units.

(d) 5*y* + 8 = 0

⇒ 0*x* − 5*y* + 0*z* = 8 ... (1)

The direction ratios of normal are 0, −5, and 0.

$$\therefore \sqrt{0+(-5)^2+0}=5$$

Dividing both sides of equation (1) by 5, we obtain

$$-y=\frac{8}{5}$$

This equation is of the form *lx* + *my* + *nz* = *d*, where *l*, *m*, *n* are the direction cosines of normal to the plane and *d* is the distance of normal from the origin.

Therefore, the direction cosines of the normal to the plane are 0, −1, and 0 and the distance of normal from the origin is $\frac{8}{5}$ units.

Question 2:

Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector $3\hat{i}+5\hat{j}-6\hat{k}$.

Answer

The normal vector is, $\vec{n}=3\hat{i}+5\hat{j}-6\hat{k}$

$$\therefore \hat{n}=\frac{\vec{n}}{|\vec{n}|}=\frac{3\hat{i}+5\hat{j}-6\hat{k}}{\sqrt{(3)^2+(5)^2+(6)^2}}=\frac{3\hat{i}+5\hat{j}-6\hat{k}}{\sqrt{70}}$$

It is known that the equation of the plane with position vector $\vec{r}$ is given by, $\vec{r}\cdot\hat{n}=d$

$$\Rightarrow \vec{r}\cdot\left(\frac{3\hat{i}+5\hat{j}-6\hat{k}}{\sqrt{70}}\right)=7$$

This is the vector equation of the required plane.

Question 3:

Find the Cartesian equation of the following planes:

(a) $\vec{r}\cdot(\hat{i}+\hat{j}-\hat{k})=2$ (b) $\vec{r}\cdot(2\hat{i}+3\hat{j}-4\hat{k})=1$

(c) $\vec{r}\cdot\left[(s-2t)\hat{i}+(3-t)\hat{j}+(2s+t)\hat{k}\right]=15$

Answer

(a) It is given that equation of the plane is

$$\vec{r}\cdot(\hat{i}+\hat{j}-\hat{k})=2 \qquad ...(1)$$

For any arbitrary point P (*x*, *y*, *z*) on the plane, position vector $\vec{r}$ is given by,

$$\vec{r}=x\hat{i}+y\hat{j}-z\hat{k}$$

Substituting the value of $\vec{r}$ in equation (1), we obtain

$$(x\hat{i}+y\hat{j}-z\hat{k})\cdot(\hat{i}+\hat{j}-\hat{k})=2$$

$$\Rightarrow x+y-z=2$$

This is the Cartesian equation of the plane.

(b) $\vec{r}\cdot(2\hat{i}+3\hat{j}-4\hat{k})=1 \qquad ...(1)$

For any arbitrary point P (*x*, *y*, *z*) on the plane, position vector $\vec{r}$ is given by,

$\vec{r} = x\hat{i} + y\hat{j} - z\hat{k}$

Substituting the value of $\vec{r}$ in equation (1), we obtain

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\cdot\left(2\hat{i} + 3\hat{j} - 4\hat{k}\right) = 1$

$\Rightarrow 2x + 3y - 4z = 1$

This is the Cartesian equation of the plane.

(c) $\vec{r}\cdot\left[(s-2t)\hat{i} + (3-t)\hat{j} + (2s+t)\hat{k}\right] = 15 \quad \ldots(1)$

For any arbitrary point P (*x*, *y*, *z*) on the plane, position vector $\vec{r}$ is given by,

$\vec{r} = x\hat{i} + y\hat{j} - z\hat{k}$

Substituting the value of $\vec{r}$ in equation (1), we obtain

$\left(x\hat{i} + y\hat{j} - z\hat{k}\right)\cdot\left[(s-2t)\hat{i} + (3-t)\hat{j} + (2s+t)\hat{k}\right] = 15$

$\Rightarrow (s-2t)x + (3-t)y + (2s+t)z = 15$

This is the Cartesian equation of the given plane.

Question 4:

In the following cases, find the coordinates of the foot of the perpendicular drawn from the origin.

(a) $2x + 3y + 4z - 12 = 0$ **(b)** $3y + 4z - 6 = 0$

(c) $x + y + z = 1$ **(d)** $5y + 8 = 0$

Answer

(a) Let the coordinates of the foot of perpendicular P from the origin to the plane be (x_1, y_1, z_1).

2*x* + 3*y* + 4*z* − 12 = 0

⇒ 2*x* + 3*y* + 4*z* = 12 … (1)

The direction ratios of normal are 2, 3, and 4.

$\therefore \sqrt{(2)^2 + (3)^2 + (4)^2} = \sqrt{29}$

Dividing both sides of equation (1) by $\sqrt{29}$, we obtain

$\frac{2}{\sqrt{29}}x + \frac{3}{\sqrt{29}}y + \frac{4}{\sqrt{29}}z = \frac{12}{\sqrt{29}}$

This equation is of the form *lx* + *my* + *nz* = *d*, where *l*, *m*, *n* are the direction cosines of normal to the plane and *d* is the distance of normal from the origin.

The coordinates of the foot of the perpendicular are given by

(*ld*, *md*, *nd*).

Therefore, the coordinates of the foot of the perpendicular are

$\left(\frac{2}{\sqrt{29}}\cdot\frac{12}{\sqrt{29}}, \frac{3}{\sqrt{29}}\cdot\frac{12}{\sqrt{29}}, \frac{4}{\sqrt{29}}\cdot\frac{12}{\sqrt{29}}\right)$ i.e., $\left(\frac{24}{29}, \frac{36}{49}, \frac{48}{29}\right)$.

(b) Let the coordinates of the foot of perpendicular P from the origin to the plane be (x_1, y_1, z_1).

$3y + 4z - 6 = 0$

⇒ $0x + 3y + 4z = 6$ … (1)

The direction ratios of the normal are 0, 3, and 4.

$\therefore \sqrt{0 + 3^2 + 4^2} = 5$

Dividing both sides of equation (1) by 5, we obtain

$0x + \frac{3}{5}y + \frac{4}{5}z = \frac{6}{5}$

This equation is of the form *lx* + *my* + *nz* = *d*, where *l*, *m*, *n* are the direction cosines of normal to the plane and *d* is the distance of normal from the origin.

The coordinates of the foot of the perpendicular are given by

(*ld*, *md*, *nd*).

Therefore, the coordinates of the foot of the perpendicular are

$\left(0, \frac{3}{5}\cdot\frac{6}{5}, \frac{4}{5}\cdot\frac{6}{5}\right)$ i.e., $\left(0, \frac{18}{25}, \frac{24}{25}\right)$.

(c) Let the coordinates of the foot of perpendicular P from the origin to the plane be (x_1, y_1, z_1).

$x + y + z = 1$ … (1)

The direction ratios of the normal are 1, 1, and 1.

$\therefore \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

Dividing both sides of equation (1) by $\sqrt{3}$, we obtain

$\frac{1}{\sqrt{3}}x + \frac{1}{\sqrt{3}}y + \frac{1}{\sqrt{3}}z = \frac{1}{\sqrt{3}}$

This equation is of the form *lx* + *my* + *nz* = *d*, where *l*, *m*, *n* are the direction cosines of

normal to the plane and d is the distance of normal from the origin.
The coordinates of the foot of the perpendicular are given by
(ld, md, nd).
Therefore, the coordinates of the foot of the perpendicular are

$$\left(\frac{1}{\sqrt{3}}\cdot\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\cdot\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\cdot\frac{1}{\sqrt{3}}\right) \text{ i.e., } \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right).$$

(d) Let the coordinates of the foot of perpendicular P from the origin to the plane be (x_1, y_1, z_1).

$$5y + 8 = 0$$

$\Rightarrow 0x - 5y + 0z = 8$ … (1)

The direction ratios of the normal are 0, −5, and 0.

$$\therefore \sqrt{0 + (-5)^2 + 0} = 5$$

Dividing both sides of equation (1) by 5, we obtain

$$-y = \frac{8}{5}$$

This equation is of the form $lx + my + nz = d$, where l, m, n are the direction cosines of normal to the plane and d is the distance of normal from the origin.
The coordinates of the foot of the perpendicular are given by
(ld, md, nd).
Therefore, the coordinates of the foot of the perpendicular are

$$\left(0, -1\left(\frac{8}{5}\right), 0\right) \text{ i.e., } \left(0, -\frac{8}{5}, 0\right).$$

*********** END **********