Geometric Progressions Ex 20.5 Q 17

a,b,c are in A.P. $\Rightarrow 2b = a+c$ ---(i)

b,c,d are in G.P. $\Rightarrow c^2 = bd$ ---(ii)

$\frac{1}{c},\frac{1}{d},\frac{1}{e}$ are in A.P. $\Rightarrow \frac{2}{d} = \frac{1}{c}+\frac{1}{e}$ ---(iv)

We need to prove that

a,b,c are in G.P.

$\Rightarrow \quad c^2 = ae$

Now,

$$c^2 = bd = 2b \times \frac{d}{2}$$

$$\Rightarrow \quad c^2 = (a+c) \times \frac{ce}{c+e} \qquad \left[\because \frac{2}{d} = \frac{e+c}{ce}\right]$$

$$\Rightarrow \quad c^2 = \frac{(a+c)ce}{c+e}$$

$$\Rightarrow \quad c^2(c+e) = ace + c^2e$$

$$\Rightarrow \quad c^3 + c^2e = ace + c^2e$$

$$\Rightarrow \quad c^3 = ace$$

$$\Rightarrow \quad c^2 = ae$$

Hence proved.

Geometric Progressions Ex 20.5 Q 18

a,b,c are in A.P.

$\Rightarrow \quad 2b = a+c$ ------ (1)

a,x,b are in GP

$\Rightarrow \quad x^2 = ab$ ------ (2)

b,y,c are in G.P.

$\Rightarrow \quad y^2 = bc$ ------- (3)

Now

$$2b^2 = x^2 + y^2$$

$$= (ab) + (bc) \qquad [\text{Using (2) and (3)}]$$

$$2b^2 = b(a+c)$$

$$2b^2 = b(2b) \qquad [\text{Using (1)}]$$

$$2b^2 = 2b^2$$

$$LHS = RHS$$

$$\Rightarrow \quad 2b^2 = x^2 + y^2$$

$\Rightarrow \quad x^2, b^2, y^2$ are in A.P.

Geometric Progressions Ex 20.5 Q 19

a, b, c are in A.P.

$\Rightarrow \quad 2b = a + c - - - - - (1)$

a, b, d are in GP

$\Rightarrow \quad b^2 = ad - - - - - - (2)$

Now

$$(a-b)^2 = a(d-c)$$

$$\text{[Using (2)]}$$

$$a^2 - 2ab = -ac$$

$$a^2 - 2ab = ab - ac$$

$$a(a-b) = a(b-c)$$

$$a - b = a - c$$

$$2b = a + c$$

$$a + c = a + c, \qquad \text{[Using equation (1)]}$$

$$LHS = RHS$$

$\Rightarrow \quad a, (a-b), (d-c)$ are in G.P.

Geometric Progressions Ex 20.5 Q 20

a, b, c are in G.P.

$$a, b = ar, \qquad c = ar^2$$

$$\frac{a^2 + ab + b^2}{bc + ca + ab} = \frac{b + a}{c + b}$$

$$\frac{a^2 + a(ar) + a^2r^2}{(ar)(ar^2) + (ar^2)a + a(ar)} = \frac{ar + a}{ar^2 + ar}$$

$$\frac{a^2(1 + r + r^2)}{a^2(r^3 + r^2 + r)} = \frac{a(1+r)}{a(r^2 + r)}$$

$$\frac{1 + r + r^2}{r(1 + r + r^2)} = \frac{1 + r}{r(1 + r)}$$

$$\frac{1}{r} = \frac{1}{r}$$

$$LHS = RHS$$

so,

$$\frac{a^2 + ab + b^2}{bc + ca + ab} = \frac{b + a}{c + b}$$

Geometric Progressions Ex 20.5 Q 21

Let r be the common ratio of G.P.

$$a, \qquad b = ar, c = ar^2$$

$$a + b + c = xb$$

$$a + ar + ar^2 = x(ar)$$

$$a(1 + r + r^2) = xar$$

$$r^2 + (1 - x)r + 1 = 0$$

Here, r is real, so

$$D \geq 0$$

$$(1-x)^2 - 4(1)(1) \geq 0$$

$$1 + x^2 - 2x - 4 \geq 0$$

$$x^2 - 2x - 3 \geq 0$$

$$(x-3)(x+1) \geq 0$$

$\Rightarrow \quad x < -1$ or $x > 3$

Geometric Progressions Ex 20.5 Q 22

Let the 4th term be ar^3

10th term be ar^9

16th term be ar^{15}

$$ar^9 = \sqrt{\left(ar^3\right)\left(ar^{15}\right)} = ar^9$$

$\therefore$ *4th, 10th, 16th terms are also in GP*

Hence Proved

Geometric Progressions Ex 20.5 Q 23

Let the A.P. be A, A +D, A +2 D, ... and G.P. be x, xR, xR^2, ... then

$a = A + (p - 1)D, b = A + (q - 1)D, c = A + (r - 1)D$

$\Rightarrow a - b = (p - q)D, b - c = (q - r)D, c - a = (r - p)D$

Also $a = xR^{p-1}, b = xR^{q-1}, c = xR^{r-1}$

Hence $a^{b-c}.b^{c-a}.c^{a-b} = (xR^{p-1})^{(q-r)D}.(xR^{q-1})^{(r-p)D}.(xR^{r-1})^{(p-q)D}$

$= x^{(q-r+r-p+p-q)D}.R^{[(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)]D}$

$= x^0.R^0 = 1.1 = 1$

*********** END **********