Definite Integrals Ex 20.1 Q31

We have,

$$\int x^2 \cos^2 x dx = \int x^2\left(\frac{1+\cos 2x}{2}\right)dx = \frac{1}{2}\int\left(x^2 + x^2\cos 2x\right)dx = \frac{1}{2}\left[\int x^2 dx + \int x^2 \cos 2x dx\right] \quad ..(A)$$

Now,

$$\int_0^{\frac{\pi}{2}} x^2 dx = \left[\frac{x^3}{3}\right]_0^{\frac{\pi}{2}} = \frac{\pi^3}{24} \quad ...(B)$$

$$\int x^2 \cos 2x\, dx = x^2\int \cos 2x dx - \int 2x\left(\int \cos 2x dx\right)dx = \frac{x^2 \sin 2x}{2} - \int \frac{\sin 2x}{2}.2x dx$$

$$= \frac{x^2 \sin 2x}{2} - \left[x\int \sin 2x - \int\left(\int \sin 2x dx\right)dx\right]$$

$$= \frac{x^2 \sin 2x}{2} + \frac{x \cos 2x}{2} - \int \frac{\cos 2x}{2}dx$$

$$\therefore \quad \int_0^{\frac{\pi}{2}} x^2 \cos 2x\, dx = \left[\frac{x^2 \sin 2x}{2} + \frac{x \cos 2x}{2} - \frac{\sin 2x}{4}\right]_0^{\frac{\pi}{2}} = \frac{-\pi}{4} \quad -(C)$$

Now, Put (B) & (C) in (A), we get,

$$\int_0^{\frac{\pi}{2}} x^2 \cos^2 x dx = \int_0^{\frac{\pi}{2}} x^2 dx + \int_0^{\frac{\pi}{2}} x^2 \cos 2x\, dx = \frac{1}{2}\left[\frac{\pi^3}{24} - \frac{\pi}{4}\right] = \frac{\pi^3}{48} - \frac{\pi}{8}$$

Definite Integrals Ex 20.1 Q32

We have,

$$\int \log x dx = \int 1.\log x dx = \log x\int dx - \int\left(\int dx\right).\frac{1}{x}dx = x\log x - \int x.\frac{1}{x}dx = x\log x - \int dx$$

$$\therefore \quad \int_1^2 \log x dx = \left[x\log x - x\right]_1^2 = 2\log 2 - 2 - 0 + 1 = 2\log 2 - 1$$

Definite Integrals Ex 20.1 Q33

We have,

$$\int \frac{\log x}{(x+1)^2}dx = \int \frac{1}{(x+1)^2}\log x dx = \log x\int \frac{1}{(x+1)^2}dx - \int\left(\int \frac{1}{(x+1)^2}dx\right)\frac{1}{x}dx$$

$$= \frac{-\log x}{(x+1)} + \int \frac{1}{x(x+1)}dx$$

$$= \frac{-\log x}{(x+1)} + \int\left(\frac{1}{x} - \frac{1}{x+1}\right)dx$$

$$\therefore \quad \int_1^3 \frac{\log x}{(x+1)^2}dx = \left[\frac{-\log x}{x+1} + \log x - \log(x+1)\right]_1^3 = \frac{3}{4}\log 3 - \log 2$$

Definite Integrals Ex 20.1 Q34

Let $I = \int_1^e \frac{e^x}{x}\left(1 + x\log x\right)dx$

$$I = \int_1^e \frac{e^x}{x}dx + \int_1^e e^x \log x dx$$

$$I = \left[e^x \log x\right]_1^e - \int_1^e e^x.\log x + \int_1^e e^x \log x$$

$$I = \left[e^x \log x\right]_1^e$$

$$I = \left[e^x \log e - e^1.\log 1\right]$$

$$I = \left[e^e.1 - 0\right]$$

$$I = e^e$$

$$\therefore \int_1^e \frac{e^x}{x}\left(1 + x\log x\right)dx = e^e$$

*********** END **********