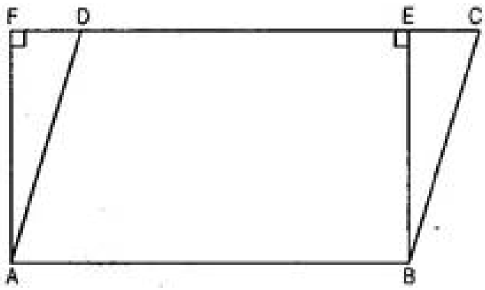
Q1. Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.

Ans. Given: Parallelogram ABCD and rectangle ABEF are on same base AB and between the same parallels AB and CF.



$\therefore$ ar ($\parallel$gm ABCD) = ar (rect. ABEF)

To prove: $AB + BC + CD + AD > AB + BE + EF + AF$

Proof: $AB = CD$ [$\because$ opposites sides of a parallelogram are always equal]

$AB = EF$ [$\because$ opposites sides of a rectangle are always equal]

$\therefore CD = EF$

Adding AB both sides,

$AB + CD = AB + EF$ ....(i)

$\therefore$ Off all the segments that can be drawn to a given line from a point not lying on it, the perpendicular segment is the shortest.

$\therefore BE < BC$ and $AF < AD$

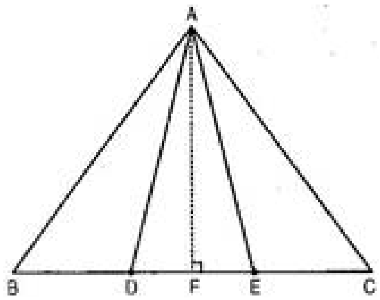
$\Rightarrow BC > BE$ and $AD > AF$

$\therefore BC + AD > BE + AF$ ....(ii)

From eq. (i) and (ii),

AB + CD + BC + AD = AB + EF + BE + AF

Q2. In figure, D and E are two points on BC such that BD = DE = EC. Show that ar (ABD) = ar (ADE) = ar (AEC). Can you know answer the question that you have left in the 'introduction' of this chapter, whether the field of Budhia has been actually divided into three parts of equal area?



Ans. In ΔABC, points D and E divides BC in three equal parts such that BD = DE = EC.

$\therefore$ BD = DE = EC = $\frac{1}{3}$ BC

Draw AF$\perp$BC

ar (ΔABC) = $\frac{1}{2}\times BC\times AF$ ....(i)

and ar (ΔABD) = $\frac{1}{2}\times BC\times AF$ ....(ii)

$= \frac{1}{2}\times\frac{BC}{3}\times AF = \frac{1}{3}\times\left[\frac{1}{2}\times BC\times AF\right]$

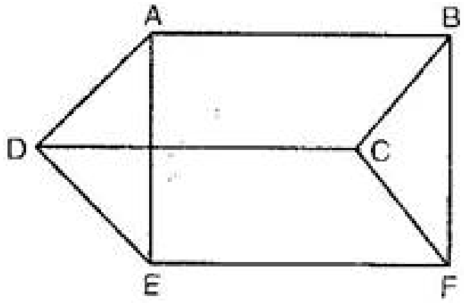
$= \frac{1}{3}$ ar (ΔABC) ....(iii)

And ar (Δ AEC) = $\frac{1}{3}$ ar (Δ ABC) ....(iv)

From (ii), (iii) and (iv),

ar (Δ ABD) = ar (Δ ADE) = ar (Δ AEC)

Q3. In figure, ABCD, DCFE and ABFE are parallelograms. Show that ar (ADE) = ar (BCF).



Ans. As we know that opposite sides of a parallelogram are always equal.

$\therefore$ In parallelogram ABFE, AE = BF and AB = EF

In parallelogram DCFE, DE = CF and DC = EF

In parallelogram ABCD, AD = BC and AB = DC

Now in Δ ADE and Δ BCF,

AE = BF [Opposite sides of parallelogram ABFE]

DE = CF [Opposite sides of parallelogram DCFE]

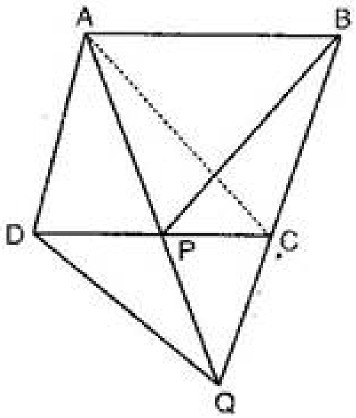
And AD = BC [Opposite sides of parallelogram ABCD]

$\therefore$ Δ ADE $\cong$ Δ BCF [By SSS congruency]

$\therefore$ ar (Δ ADE) = ar (Δ BCF)

[$\because$ Area of two congruent figures is always equal]

Q4. In figure, ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersects DC at P, show that ar (BPC) = ar (DPQ).



Ans. Join A and C.

Δ APC and Δ BPC are on the same base PC and between the same parallels PC and AB.

$\therefore$ ar (Δ APC) = ar (Δ BPC) ....(i)

Now ACBD is a parallelogram.

AD = BC [opposite sides of a parallelogram are always equal]

Also BC = CQ [given]

$\therefore$ AD = CQ

Now AD $\parallel$ CQ [Since CQ is the extension of BC]

And AD = CQ

$\therefore$ ADQC is a parallelogram.

[$\because$ If one pair of opposite sides of a quadrilateral is equal and parallel then it is a parallelogram]

Since diagonals of a parallelogram bisect each other.

$\therefore$ AP = PQ and CP = DP

Now in ΔAPC and ΔDPQ,

AP = PQ [Proved above]

$\angle$ APC = $\angle$ DPQ [Vertically opposite angles]

PC = PD [Prove above]

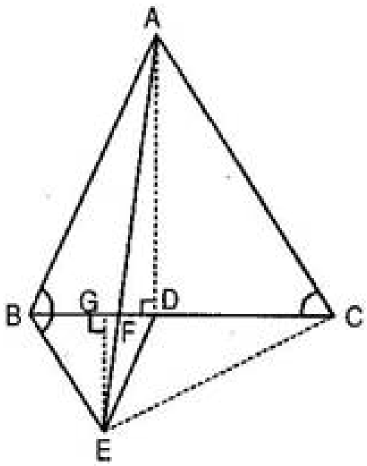
$\therefore$ ΔAPC $\cong$ ΔDPQ ....(ii)

$\Rightarrow$ ar (ΔAPC) = ar (ΔDPQ) [area of congruent figures is always equal]

From eq. (i) and (ii),

ar (ΔBPC) = ar (ΔDPQ)

Q5. In figure, ABC and BDF are two equilateral triangles such that D is the mid-point of BC. If AE intersects BC at F, show that:



(i) ar (BDE) = $\frac{1}{4}$ ar (ABC)

(ii) ar (BDE) = $\frac{1}{2}$ ar (BAE)

(iii) ar (ABC) = 2 ar (BEC)

(iv) ar (BFE) = ar (AFD)

(v) ar (BFE) = 2 ar (FED)

(vi) ar (FED) = $\frac{1}{8}$ ar (AFC)

Ans. Join EC and AD.

Since ΔABC is an equilateral triangle.

$\therefore \angle A = \angle B = \angle C = 60^\circ$

Also ΔBDE is an equilateral triangle.

$\therefore \angle B = \angle D = \angle E = 60^\circ$

If we take two lines, AC and BE and BC as a transversal.

Then $\angle B = \angle C = 60^\circ$ [Alternate angles]

$\Rightarrow$ BE || AC

Similarly, for lines AB and DE and BF as transversal.

Then $\angle B = \angle C = 60^\circ$ [Alternate angles]

$\Rightarrow$ BE || AC

(i) Area of equilateral triangle BDE = $\frac{\sqrt{3}}{4}$ (BD)^2

..........(i)

Area of equilateral triangle ABC = $\frac{\sqrt{3}}{4}$ (BC)^2

..........(ii)

Dividing eq. (i) by (ii),

$$\frac{ar(\Delta BDE)}{ar(\Delta ABC)} = \frac{\frac{\sqrt{3}}{4}(BD)^2}{\frac{\sqrt{3}}{4}(BC)^2} \Rightarrow \frac{ar(\Delta BDE)}{ar(\Delta ABC)} = \frac{\frac{\sqrt{3}}{4}(BD)^2}{\frac{\sqrt{3}}{4}(2BD)^2}$$

[$\because$ BD = DC]

$$\Rightarrow \frac{ar(\Delta BDE)}{ar(\Delta ABC)} = \frac{(BD)^2}{(2BD)^2} \Rightarrow \frac{ar(\Delta BDE)}{ar(\Delta ABC)} = \frac{1}{4}$$

$$\Rightarrow ar(\Delta BDE) = \frac{1}{4} ar(\Delta ABC)$$

(ii) In ΔBEC, ED is the median.

$\therefore$ ar (ΔBEC) = ar (ΔBAE) ....(i)

[Median divides the triangle in two triangles having equal area]

Now BE || AC

And ΔBEC and ΔBAE are on the same base BE and between the same parallels BE and AC.

$\therefore$ ar (ΔBEC) = ar (ΔBAE) ....(ii)

Using eq. (i) and (ii), we get

Ar (ΔBDE) = $\frac{1}{2}$ ar (ΔBAE)

(iii) We have ar (ΔBDE) = $\frac{1}{4}$ ar (ΔABC)

[Proved in part (i)] ....(iii)

ar (ΔBDE) = $\frac{1}{4}$ ar (ΔBAE) [Proved in part (ii)] ar (ΔBDE) = $\frac{1}{4}$ ar (ΔBEC) [Using eq. (iii)]

..........(iv)

From eq. (iii) and (iv), we het

$\frac{1}{4}$ ar (ΔABC) = $\frac{1}{4}$ ar (ΔBEC)

$\Rightarrow$ ar (ΔABC) = 2 ar (ΔBEC)

(iv) ΔBDE and ΔAED are on the same base DE and between same parallels AB and DE.

$\therefore$ ar (ΔBDE) = ar (ΔAED)

Subtracting ΔFED from both the sides,

ar (ΔBDE) – ar (ΔFED) = ar (ΔAED) – ar (Δ FED)

$\Rightarrow$ ar (ΔBFE) = ar (ΔAFD) ....(v)

(v) An in equilateral triangle, median drawn is also perpendicular to the side,

$\therefore$ AD $\perp$ BC

Now ar (ΔAFD) = $\frac{1}{2} \times FD \times AD$ ....(vi)

Draw EG $\perp$ BC

$\therefore$ ar (ΔFED) = $\frac{1}{2} \times FD \times EG$ ....(vii)

Dividing eq. (vi) by (vii), we get

$$\frac{\text{ar}(\Delta \text{AFD})}{\text{ar}(\Delta \text{FED})} \frac{\frac{1}{2} \times \text{FD} \times \text{AD}}{\frac{1}{2} \times \text{FD} \times \text{EG}} \Rightarrow \frac{\text{ar}(\Delta \text{AFD})}{\text{ar}(\Delta \text{FED})} = \frac{\text{AD}}{\text{EG}}$$

$$\Rightarrow \frac{ar(\Delta AFD)}{ar(\Delta FED)} = \frac{\frac{\sqrt{3}}{4}BC}{\frac{\sqrt{3}}{4}BD}$$ [Altitude of equilateral triangle = $\frac{\sqrt{3}}{4}$ side]

$$\Rightarrow \frac{ar(\Delta AFD)}{ar(\Delta FED)} = \frac{2BD}{BD}$$ [D is the mid-point of BC]

$$\Rightarrow \frac{ar(\Delta AFD)}{ar(\Delta FED)} = 2 \Rightarrow ar(\Delta AFD) = 2\, ar(\Delta FED)$$ ......(viii)

Using the value of eq. (viii) in eq. (v),

Ar (ΔBFE) = 2 ar (ΔFED)

(vi) ar (ΔAFC) = ar (ΔAFD) + ar (ΔADC) = 2 ar (ΔFED) + $\frac{1}{2}$ ar (ΔABC) [using (v)

= 2 ar (ΔFED) + $\frac{1}{2}$ [4 $\times$ ar (ΔBDE)] [Using result of part (i)]

= 2 ar (ΔFED) + 2 ar (ΔBDE) = 2 ar (ΔFED) + 2 ar (ΔAED)

[ΔBDE and ΔAED are on the same base and between same parallels]

= 2 ar (ΔFED) + 2 [ar (ΔAFD) + ar (ΔFED)]

= 2 ar (ΔFED) + 2 ar (ΔAFD) + 2 ar (ΔFED) [Using (viii)]

= 4 ar (ΔFED) + 4 ar (ΔFED)

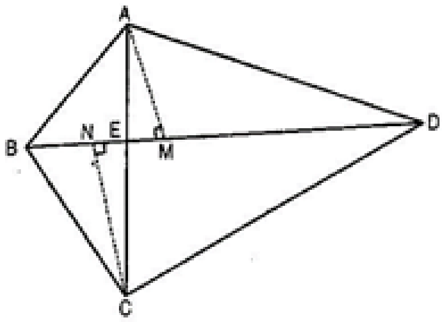
$\Rightarrow$ ar (ΔAFC) = 8 ar (ΔFED)

$\Rightarrow$ ar (ΔFED) = $\frac{1}{8}$ ar (ΔAFC)

Q6. Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that:

ar (APB) $\times$ ar (CPD) = ar (APD) $\times$ ar (BPC)

Ans. Given: A quadrilateral ABCD, in which diagonals AC and BD intersect each other at point E.



To Prove: ar (ΔAED) $\times$ ar (ΔBEC)

= ar (ΔABE) $\times$ ar (ΔCDE)

Construction: From A, draw AM $\perp$ BD and from C, draw CN $\perp$ BD.

Proof: ar (ΔABE) = $\frac{1}{2} \times BE \times AM$ ....(i)

And ar (ΔAED) = $\frac{1}{2} \times DE \times AM$ ....(ii)

Dividing eq. (ii) by (i), we get,

$$\frac{ar(\Delta AED)}{ar(\Delta ABE)} = \frac{\frac{1}{2} \times DE \times AM}{\frac{1}{2} \times BE \times AM} \Rightarrow \frac{ar(\Delta AED)}{ar(\Delta ABE)} = \frac{DE}{BE}$$

..........(iii)

Similarly $\frac{ar(\Delta CDE)}{ar(\Delta BEC)} = \frac{DE}{BE}$ ....(iv)

From eq. (iii) and (iv), we get

$$\frac{ar(\Delta AED)}{ar(\Delta ABE)} = \frac{ar(\Delta CDE)}{ar(\Delta BEC)}$$

$\Rightarrow$ ar (ΔAED) $\times$ ar (ΔBEC) = ar (ΔABE) $\times$ ar (ΔCDE)

Hence proved.

Q7. P and Q are respectively the mid-points of sides AB and BC or a triangle ABC and R is the mid-point of AP, show that:

(i) ar (PRQ) = $\frac{1}{2}$ ar (ARC)

(ii) ar (RQC) = $\frac{3}{8}$ ar (ABC)

(iii) ar (PBQ) = ar (ARC)

Ans. (i) PC is the median of ΔABC.

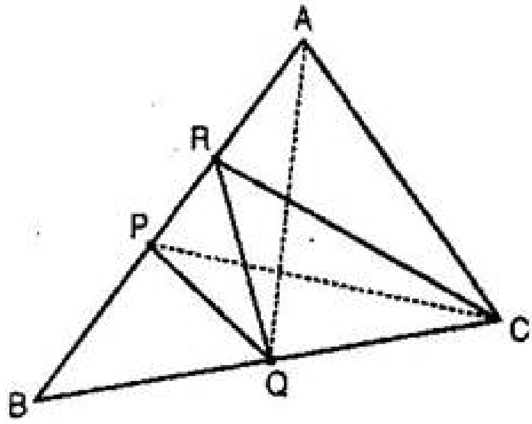
$\therefore$ ar (ΔBPC) = ar (ΔAPC) ....(i)

RC is the median of ΔAPC.

$\therefore$ ar (ΔARC) = $\frac{1}{2}$ ar (ΔAPC) ....(ii)

[Median divides the triangle into two triangles of equal area]

PQ is the median of ΔBPC.



$\therefore$ ar (ΔPQC) = $\frac{1}{2}$ ar (ΔBPC) ....(iii)

From eq. (i) and (iii), we get,

ar (ΔPQC) = $\frac{1}{2}$ ar (ΔAPC) ....(iv)

From eq. (ii) and (iv), we get,

ar (ΔPQC) = ar (ΔARC) ....(v)

We are given that P and Q are the mid-points of AB and BC respectively.

$\therefore$ PQ $\parallel$ AC and PA = $\frac{1}{2}$ AC

$\Rightarrow$ ar (ΔAPQ) = ar (ΔPQC) ....(vi) [triangles between same parallel are equal in area]

From eq. (v) and (vi), we get

ar (ΔAPQ) = ar (ΔARC) ....(vii)

R is the mid-point of AP. Therefore RQ is the median of ΔAPQ.

$\therefore$ ar (ΔPRQ) = $\frac{1}{2}$ ar (ΔAPQ) ....(viii)

From (vii) and (viii), we get,

ar (ΔPRQ) = $\frac{1}{2}$ ar (ΔARC)

(ii) PQ is the median of ΔBPC

$\therefore$ ar (ΔPQC) = $\frac{1}{2}$ ar (ΔBPC) = $\frac{1}{2} \times \frac{1}{2}$ ar (ΔABC)

= $\frac{1}{4}$ ar (ΔABC) ....(ix)

Also ar (ΔPRC) = $\frac{1}{2}$ ar (ΔAPC) [Using (iv)]

$\Rightarrow$ ar (ΔPRC) = $\frac{1}{2} \times \frac{1}{2}$ ar (ΔABC) = $\frac{1}{4}$ ar (ΔABC) ....(x)

Adding eq. (ix) and (x), we get,

ar (ΔPQC) + ar (ΔPRC) = $\left(\frac{1}{4} + \frac{1}{4}\right)$ ar (ΔABC)

$\Rightarrow$ ar (quad. PQCR) = $\frac{1}{2}$ ar (ΔABC) ....(xi)

Subtracting ar (ΔPRQ) from the both sides,

ar (quad. PQCR) – ar (ΔPRQ) = $\frac{1}{2}$ ar (ΔABC) –

ar (ΔPRQ)

$\Rightarrow$ ar (ΔRQC) = $\frac{1}{2}$ ar (ΔABC) – $\frac{1}{2}$ ar (ΔARC) [Using result (i)]

$\Rightarrow$ ar (ΔARC) = $\frac{1}{2}$ ar (ΔABC) – $\frac{1}{2}\times\frac{1}{2}$ ar (Δ APC)

$\Rightarrow$ ar (ΔRQC) = $\frac{1}{2}$ ar (ΔABC) – $\frac{1}{4}$ ar (ΔAPC)

$\Rightarrow$ ar (ΔRQC) = $\frac{1}{2}$ ar (ΔABC) – $\frac{1}{4}\times\frac{1}{2}$ ar (Δ ABC) [PC is median of ΔABC]

$\Rightarrow$ ar (ΔRQC) = $\frac{1}{2}$ ar (ΔABC) – $\frac{1}{8}$ ar (ΔABC)

$\Rightarrow$ ar (ΔRQC) = $\left(\frac{1}{2}-\frac{1}{8}\right)$ $\times$ ar (ΔABC)

$\Rightarrow$ ar (ΔRQC) = $\frac{3}{8}$ ar (ΔABC)

(iii) ar (ΔPRQ) = $\frac{1}{2}$ ar (ΔARC) [Using result (i)]

$\Rightarrow$ 2 ar (ΔPRQ) = ar (ΔARC) ..(xii)

ar (ΔPRQ) = $\frac{1}{2}$ ar (ΔAPQ) [RQ is the median of ΔAPQ] ....(xiii)

But ar (ΔAPQ) = ar (ΔPQC) [Using reason of eq. (vi)] ....(xiv)

From eq. (xiii) and (xiv), we get,

$ar(\Delta PRQ) = \frac{1}{2} ar(\Delta PQC)$ ....(xv)

But $ar(\Delta BPQ) = ar(\Delta PQC)$ [PQ is the median of ΔBPC] ....(xvi)

From eq. (xv) and (xvi), we get,

$ar(\Delta PRQ) = \frac{1}{2} ar(\Delta BPQ)$ ....(xvii)

Now from (xii) and (xvii), we get,

$2\left(\frac{1}{2} ar(\Delta BPQ)\right) = ar(\Delta ARC) \Rightarrow ar(\Delta BPQ) = ar(\Delta ARC)$

Q8. In figure, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment $AX \perp DE$ meets BC at Y. Show that:

(i) $\Delta MBC \cong \Delta ABD$

(ii) ar (BYXD) = 2 ar (MBC)

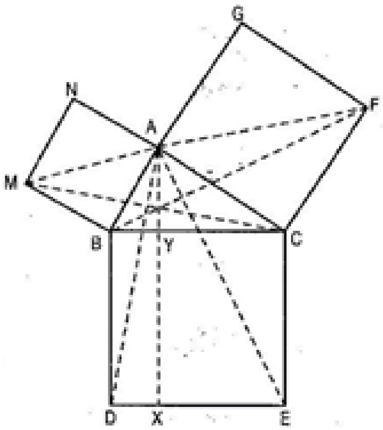
(iii) ar (BYXD) = ar (ABMN)

(iv) $\Delta FCB \cong \Delta ACE$

(v) ar (CYXE) = 2 ar (FCB)

(vi) ar (CYXE) = ar (ACFG)

(vii) ar (BCED) = ar (ABMN) + ar (ACFG)



Ans. (i) $\angle ABM = \angle CBD = 90^\circ$

Adding $\angle ABC$ both sides, we get,

$\angle ABM + \angle ABC = \angle CBD + \angle ABC \Rightarrow \angle MBC = ABD$ ....(i)

Now in ΔMBC and ΔABD,

MB = AB [equal sides of square ABMN]

BC = BD [sides of square BCED]

$\angle MBC = \angle ABD$ [proved above]

$\therefore \Delta MBC \cong \Delta ABD$ [By SAS congruency]

(ii) From above, $\Delta MBC \cong \Delta ABD$

$\Rightarrow$ ar (ΔMBC) = ar (ΔABD) $\Rightarrow$ ar (Δ MBC) = ar (trap. ABDX) – ar (ΔADX)

$\Rightarrow$ ar (ΔMBC) = $\frac{1}{2}$ (BD + AX) BY – $\frac{1}{2}$ DX.AX

$\Rightarrow$ ar (ΔMBC) = $\frac{1}{2}$ BD.BY + $\frac{1}{2}$ AX.BY – $\frac{1}{2}$ DX.AX

$\Rightarrow$ ar (ΔMBC) = $\frac{1}{2}$ BD.BY + $\frac{1}{2}$ AX (BY – DX)

$\Rightarrow$ ar (ΔMBC) = $\frac{1}{2}$ BD.BY + $\frac{1}{2}$ AX. 0 [BY = DX]

$\Rightarrow$ ar (ΔMBC) = $\frac{1}{2}$ BD.BY

$\Rightarrow$ 2 ar (ΔMBC) = BD.BY $\Rightarrow$ 2 ar (ΔMBC) = ar (rect. BYXD)

Hence ar (BYXD) = 2 ar (ΔMBC)

(iii) Join AM. ABMN is a square.

Therefore, NA || MB $\Rightarrow$ AC || MB

Now ΔAMB and ΔMBC are on the same base and between the same parallels MB and AC.

$\therefore$ ar (ΔAMB) = ar (ΔMBC) ....(ii)

From result (ii), we have ar (BYXD) = 2 ar (ΔMBC) ....(iii)

Using eq. (ii) and (iii), we get, ar (BYXD) = 2 ar (ΔAMB)

$\Rightarrow$ ar (BYXD) = ar (square ABMN)

[Diagonal AM of square ABMN divides it in two triangles of equal area]

(iv) In ΔFCB and ΔACE,

FC = AC [sides of square ACFG]

BC = CE [sides of square BCED]

$\angle$BCF = $\angle$ACE [$\because$ $\angle$ACF = $\angle$BCE = 90°]

Adding $\angle$ACB both sides,

$\angle BCF + \angle ACB = \angle ACE + \angle ACB \Rightarrow \angle BCF = \angle ACE$

$\therefore \Delta FCB \cong \Delta ACE$ [By SAS congruency]

(v) From (iv), we have, $\Delta FCB \cong \Delta ACE$

$\Rightarrow$ ar (ΔFCB) = ar (ΔACE) $\Rightarrow$ ar (ΔFCB) = ar (trap. ACEX) – ar (ΔAEX)

$\Rightarrow$ ar (ΔFCB) = $\frac{1}{2}$ (CE + AX) CY – $\frac{1}{2}$ XE.AX

$\Rightarrow$ ar (ΔFCB) = $\frac{1}{2}$ CE.CY + $\frac{1}{2}$ AX.CY – $\frac{1}{2}$ XE.AX

$\Rightarrow$ ar (ΔFCB) = $\frac{1}{2}$ CE.CY + $\frac{1}{2}$ AX (CY – XE)

$\Rightarrow$ ar (ΔFCB) = $\frac{1}{2}$ CE.CY + $\frac{1}{2}$ AX. 0 [CY = XE]

$\Rightarrow$ ar (ΔFCB) = $\frac{1}{2}$ CE.CY

$\Rightarrow$ 2 ar (ΔFCB) = CE.CY $\Rightarrow$ 2 ar (ΔFCB) = ar (rect. CYXE)

Hence ar (BYXD) = 2 ar (ΔFCB)

(vi) Join AF. ACFG is a square.

$\therefore$ FC $\parallel$ AG $\Rightarrow$ FC $\parallel$ AB

Now ΔACF and ΔFCB are on the same base FC and between the same parallels FC and AB.

$\therefore$ ar (ΔACF) = ar (ΔFCB) ....(v)

From result (v), we get, ar (CYXE) = 2 ar (ΔFCB) ....(vi)

Using eq. (v) in (vi), we get, ar (CYXE) = 2 ar (ΔACF)

Diagonal AF of square ACFG divides it in two triangles of equal area.

$\therefore$ ar (CYXE) = ar (sq. ACFG) ....(vii)

(vii) Adding eq. (iv) and (vii), we get,

ar (BYXD) + ar (CYXE) = ar (ABMN) + ar (ACFG)

$\Rightarrow$ ar (BCED) = ar (ABMN) + ar (ACFG)

*********** END **********