Question 15. 12. (i) For the wave on a string described in Question 11, do all the points on the string oscillate with the same (a) frequency, (b) phase, (c) amplitude? Explain your answers, (ii) What is the amplitude of a point 0.375 m away from one end?

Answer:

(i) For the wave on the string described in questions we have seen that l = 1.5 m and λ = 3 m. So, it is clear that λ = λ /2 and for a string clamped at both ends, it is possible only when both ends behave as nodes and there is only one antinode in between i.e., whole string is vibrating in one segment only.

(a) Yes, all the sring particles, except nodes, vibrate with the same frequency v = 60 Hz.

(b) As all string particles lie in one segment, all of them are in same phase.

(c) Amplitude varies from particle to particle. At antinode, amplitude = 2A = 0.06 m. It gradually falls on going towards nodes and at nodes, and at nodes, it is zero.

(ii) Amplitude at a point x = 0.375 m will be obtained by putting cos (120 πt) as + 1 in the wave equation.

$$\therefore \quad A\ (x)\ =\ 0.06\ \sin\left(\frac{2\pi}{3}\times 0.375\right)\times 1\ =\ 0.06\ \sin\frac{\pi}{4}\ =\ 0.042\ \text{m.}$$

Question 15. 13. Given below are some functions of x and t to represent the displacement (transverse or longitudinal) of an elastic wave. State which of these represent (i) a travelling wave, (ii) a stationary wave or (iii) none at all

(a) $y = 2\ \cos\ (3x)\ \sin\ (10t)$

(b) $y = 2\sqrt{x-vt}$

(c) $y = 3\ \sin\ (5x - 0.5t) + 4\ \cos\ (5x - 0.5t)$

(d) $y = \cos\ x\ \sin\ t + \cos\ 2x\ \sin\ 2t.$

Answer:

(a) It represents a stationary wave.

(b) It does not represent either a travelling wave or a stationary wave.

(c) It is a representation for the travelling wave.

(d) It is a superposition of two stationary wave.

Question 15. 14. A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5 x 10^{-2} kg and its linear mass density is 4.0 x 10^2 kg m^{-3}. What is (a) the speed of a transverse wave on the string, and (b) the tension in the string?

Answer:

Here, $n = 45$ Hz, $M = 3.5 \times 10^{-2}$ kg

Mass per unit length $= m = 4.0 \times 10^{-2}$ kg m^{-1}

$\therefore$ $l = \dfrac{M}{m} = \dfrac{3.5 \times 10^{-2}}{4.0 \times 10^{-2}} = \dfrac{7}{8}$

As $\dfrac{l}{2} = \lambda = \dfrac{7}{8}$ $\therefore$ $\lambda = \dfrac{7}{4}$ m $= 1.75$ m.

(a) The speed of the transverse wave, $v = v\lambda = 45 \times 1.75 = 78.75$ m/s

(b) As $v = \sqrt{\dfrac{T}{m}}$

$\therefore$ $T = v^2 \times m = (78.75)^2 \times 4.0 \times 10^{-2} = 248.06$ N.

Question 15. 15. A metre-long tube open at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a turning fork of frequency 340 Hz) when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. The edge effect may be neglected.
Answer:

Frequency of nth mode of vibration of the closed organ pipe of legnth

$$l_1 = (2n - 1)\,\dfrac{v}{4l_1}$$

Frequency of $(n + 1)^{th}$ mode of vibration of closed pipe of length

$$'l_2' = [2(n + 1) - 1]\,\dfrac{v}{4l_2} = (2n + 1)\,\dfrac{v}{4l_2}$$

Both the modes are given to resonate with a frequency of 340 Hz.

$\therefore$ $(2n - 1)\dfrac{v}{4l_1} = (2n + 1)\dfrac{v}{4l_2}$

or $\dfrac{2n - 1}{2n + 2} = \dfrac{l_1}{l_2} = \dfrac{25.5}{79.3} = \dfrac{1}{3}$

[Approximation has been used because edge effect is being ignored. Moreover, we know that in the case of a closed organ pipe, the second resonance length is three times the first resonance length.]
On simplification, n = 1
Now, $(2n-1)v/4l_1 = 340$. Substituting values

$(2 \times 1 - 1)$ v $\times 100/4 \times 25.5 = 340$ or v $= 346.8$ ms^{-1}

Question 15. 16. A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod is given to be 2.53 k Hz. What is the speed of sound in steel?
Answer: Here, L = 100 cm = 1m, v = 2.53 k Hz = 2.53 x 10^3 Hz
When the rod is clamped at the middle, then in the fundamental mode of vibration of the rod, a node is formed at the middle and ant mode is formed at each end.
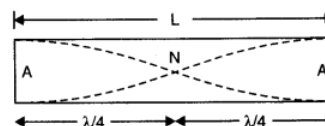
Therefore, as is clear from Fig.

$L = \dfrac{\lambda}{4} + \dfrac{\lambda}{4} = \dfrac{\lambda}{2}$

$\lambda = 2L = 2$ m

As $v = v\lambda$

$\therefore$ $v = 2.53 \times 10^3 \times 2$

$= 5.06 \times 10^3$ ms^{-1}

Question 15. 17. A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source? Will the same source be in resonance with the pipe if both ends are open? (speed of sound in air is 340 ms^{-1}).
Answer: Here length of pipe, 1 = 20 cm = 0.20 m, frequency v = 430 Hz and speed of sound in air v = 340 ms^{-1}

For closed end pipe, $v = \dfrac{(2n-1)\,v}{4l}$, where $n = 1, 2, 3........$

$\therefore \qquad (2n-1) = \dfrac{4vl}{v} = \dfrac{4 \times 430 \times 0.20}{340} = 1.02$

$\Rightarrow \qquad 2n = 1.02 + 1 = 2.02 \Rightarrow n = \dfrac{0.20}{2} = 1.01$

Hence, resonance can occur only for first (or fundamental) mode of vibration.

As for an open pipe $v = \dfrac{nv}{2l}$, where $n = 1, 2, 3...........$

$\therefore \qquad n = \dfrac{2lv}{v} = \dfrac{2 \times 430 \times 0.20}{340} = 0.51.$

As $n < 1$, hence, in this case resonance position cannot be obtained.

Question 15. 18. Two sitar strings A and B playing the note 'Ga' are slightly out of tune and produce beats of frequency 6Hz. The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3Hz. If the original frequency of A is 324 Hz, what is the frequency of B?
Answer:
Let v_1 and v_2 be the frequencies of strings A and B respectively.
Then, $v_1 = 324$ Hz, $v_2 = ?$
Number of beats, b = 6
$v_2 = v_1 \pm b = 324 \pm 6$ i.e., $v_2 = 330$ Hz or 318 Hz
Since the frequency is directly proportional to square root of tension, on decreasing the tension in the string A, its frequency v_1 will be reduced i.e., number of beats will increase if $v_2 = 330$ Hz.
This is not so because number of beats become 3.
Therefore, it is concluded that the frequency $v_2 = 318$ Hz. because on reducing the tension in the string A, its frequency may be reduced to 321 Hz, thereby giving 3 beats with $v_2 = 318$ Hz.

Question 15. 19. Explain why (or how):
(a) in a sound wave, a displacement node is a pressure antinode and vice versa.
(b) bats can ascertain distances, directions, nature and sizes of the obstacles without any "eyes".
(c) a violin note and sitar note may have the same frequency, yet we can distinguish between the two notes.
(d) solids can support both longitudinal and transverse waves, but only longitudinal waves can propagate in gases, and
(e) the shape of a pulse gets distorted during propagation in a dispersive medium.
Answer:
(a) In a sound wave, a decrease in displacement i.e., displacement node causes an increase in the pressure there i.e., a pressure antinode is formed. Also, an increase in displacement is due to the decrease in pressure.
(b) Bats emit ultrasonic waves of high frequency from their mouths. These waves after being reflected back from the obstacles on their path are observed by the bats. These waves give them an idea of distance, direction, nature and size of the obstacles.
(c) The quality of a violin note is different from the quality of sitar. Therefore, they emit different harmonics which can be observed by human ear and used to differentiate between the two notes.
(d) This is due to the fact that gases have only the bulk modulus of elasticity whereas solids have both, the shear modulus as well as the bulk modulus of elasticity.
(e) A pulse of sound consists of a combination of waves of different wavelength. In a dispersive medium, these waves travel with different velocities giving rise to the distortion in the wave.

Question 15. 20. A train, standing at the outer signal of a railway station blows a whistle of frequency 400 Hz in still air. (i) What is the frequency of the whistle for a platform observer when the train (a)

approaches the platform with a speed of 10 ms~1. (b) recedes from the platform with a speed of 10 ms^{-1}(ii) What is the speed of sound in each case? The speed of sound in still air can be taken as 340 ms^{-1} .

Answer:

Frequency of whistle, v = 400 Hz; speed of sound, υ= 340 ms^{-1} speed of train, υ_s= 10 ms^{-1}

(i) (a) When the train approaches the platform (i.e., the observer at rest),

$$v' = \frac{\upsilon}{\upsilon - \upsilon_s} \times v = \frac{340}{340 - 10} \times 400 = 412 \text{ Hz.}$$

(b) When the train recedes from the platform (i.e., from the observer at rest),

$$v = \frac{\upsilon}{\upsilon + \upsilon_s} \times v = \frac{340}{340 + 10} \times 400 = 389 \text{ Hz.}$$

(ii) The speed of sound in each case does not change.It is 340 ms^{-1} in each case.

Question 15. 21. A train, standing in a station-yard, blows a whistle of frequency 400 Hz in still air. The wind starts blowing in the direction from the yard to the station with a speed of 10 ms^{-1}. What are the frequency, wavelength, and speed of sound for an observer standing on the station's platform? Is the situation exactly identical to the case when the air is still and the observer runs towards the yard at a speed of 10 ms^{-1}? The speed of sound in still air can be taken as 340 ms^{-1}?

Answer:

Here actual frequency of whistle of train v = 400 Hz, speed of sound in still air v = 340 ms^{-1}.

As wind is blowing in the direction from the yard to the station with a speed of v_m = 10 ms^{-1}

∴ For an observer standing on the platform, the effective speed of sound

$v' = v + v_m$ = 340 + 10 = 350 ms^{-1}

As there is no relative motion between the sound source (rail engine) and the observer, the frequency of sound for the observer, v = 400 Hz

∴ Wavelength of sound heard by the observer $\lambda' = \frac{v'}{v} = \frac{350}{400}$

= 0.875 m.

The situation is not identical to the case when the air is still and observer runs towards the yard at a speed of v_0 = 10 ms^{-1}. In this situation as medium is at rest. Hence $v' = v$ = 340 ms^{-1}.

$$v' = \frac{v + v_0}{v} v = \frac{340 + 10}{340} \times 400 = 412 \text{ Hz}$$

and $\lambda' = \lambda = \frac{v}{v} = \frac{340}{400} = 0.85 \text{ m}$

Question 15. 22. A travelling harmonic wave on a string is described by y (x, t) = 7.5 sin (0.0050x + 12t + π /4)
(a) what are the displacement and velocity of oscillation of a point at x = 1 cm, and t = 1s? Is this velocity equal to the velocity of wave propagation?
(b) Locate the points of the string which have the same transverse displacement and velocity as the x = 1 cm point at t = 2s, 5s and 11s.

Answer:

The travelling harmonic wave is y (x, t) = 7.5 sin (0.0050x + 12t + π/4)

At   x = 1 cm   and   t = 1 sec,

y (1, 1) = 7.5 sin (0.005 × 1 + 12 × 1 π/4) = 7.5 sin (12.005 + π/4)   ...(i)

Now,   θ = (12.005 + π/4) radian

$$= \frac{180}{\pi} (12.005 + \pi/4) \text{ degree} = \frac{12.005 \times 180}{\frac{22}{7}} + 45 = 732.55°.$$

∴ From (i), y (1, 1) = 7.5 sin (732.55°) = 7.5 sin (720 + 12.55°)

= 7.5 sin 12.55° = 7.5 × 0.2173 = 1.63 cm

Velocity of oscillation, $v = \dfrac{dy}{dt}(1,1) = \dfrac{d}{dt}\left[7.5\sin\left(0.005x+12+\dfrac{\pi}{4}\right)\right]$

$= 7.5 \times 12\cos\left[0.005x+12t+\dfrac{\pi}{4}\right]$

At $\qquad x = 1cm, \quad t = 1sec.$

$v = 7.5 \times 12\cos(0.005 + 12 + \pi/4) = 90\cos(732.35°)$

$= 90\cos(720 + 12.55)$

$v = 90\cos(12.55°) = 90 \times 0.9765 = 87.89$ cm/s.

Comparing the given eqn. with the standard form $y(x,t) = t\sin\left[\dfrac{\pi}{4}(vt+x)+\phi_0\right]$

We get $\qquad r = 7.5$ cm, $\quad \dfrac{2\pi v}{\lambda} = 12 \quad$ or $\quad 2\pi v = 12$

$v = \dfrac{6}{\pi}$

$\dfrac{2\pi}{\lambda} = 0.005.$

$\therefore \qquad \lambda = \dfrac{2\pi}{0.005} = \dfrac{2\times 3.14}{0.005} = 1256$ cm $= 12.56$ m

Velocity of wave propagation, $v = v\lambda = \dfrac{6}{\pi}\times 12.56 = $ **24 m/s.**

We find that velocity at $x = 1$ cm, $t = 1$ sec is not equal to velocity of wave propagation.

(b) Now, all points which are at a distance of $\pm\lambda, \pm 2\lambda, \pm 3\lambda$ from $x = 1$ cm will have same transverse displacement and velocity. As $\lambda = 12.56$ m, therefore, all points at distances ± 12.6 m, ± 25.2 m, ± 37.8 m.... from $x = 1$ cm will have same displacement and velocity, as at $x = 1$ point $t = 2$ s, 5 s and 11 s.

Question 15. 23. A narrow sound pulse (for example, a short pip by a whistle) is sent across a medium, (a) Does the pulse have a definite (i) frequency, (ii) wavelength, (iii) speed of propagation? (b) If the pulse rate is 1 after every 20 s, (that is the whistle is blown for a split of second after every 20 s), is the frequency of the note produced by the whistle equal to 1/20 or 0.05 Hz?
Answer:
(a) In a non dispersive medium, the wave propagates with definite speed but its wavelength of frequency is not definite.
(b) No, the frequency of the note is not 1/20 or 0.50 Hz. 0.005 Hz is only the frequency ' of repetition of the pip of the whistle.

Question 15. 24. One end of a long string of linear mass density 8.0 x 10^{-3} kg m^{-1} is connected to an electrically driven tuning fork of frequency 256 Hz. The other end passes over a pulley and is tied to a pan containing a mass of 90 kg. The pulley end absorbs all the incoming energy so that reflected waves at this end have negligible amplitude. At t = 0, the left end (fork end) of the string x = 0 has zero transverse displacement (y = 0) and is moving along positive y-direction. The amplitude of the wave is 5.0 cm. Write down the transverse displacement y as function of x and t that describes the wave on the string.
Answer:
Here, mass per unit length, g = linear mass density = 8 x 10^{-3} kg m^{-1};
Tension in the string, T = 90 kg = 90 x 9.8 N= 882 N;

Frequency, $\qquad v = 256$ Hz
and amplitude, $A = 5.0$ cm $= 0.05$ m
As the wave propagating along the string is a transverse travelling wave, the velocity of the wave,

$$v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{882}{8\times 10^{-3}}} \text{ ms}^{-1} = 3.32 \times 10^2 \text{ ms}^{-1}$$

Now, $\qquad \omega = 2\pi v = 2 \times 3.142 \times 256 = 1.61 \times 10^3$ rad s^{-1}

Also, $\qquad v = v\lambda \quad$ or $\quad \lambda = \dfrac{v}{v} = \dfrac{3.32\times 10^2}{256}$ m

Propagation constant, $k = \dfrac{2\pi}{\lambda} = \dfrac{2\times 3.142\times 256}{3.32\times 10^2}$

$= 4.84$ m^{-1}

$\therefore$ The equation of the wave is ,
$\qquad v(x, t) = A\sin(\omega t - kx)$
$\qquad\qquad = 0.05\sin(1.61 \times 10^3 t - 4.84 x)$
Here, x, y are in metre and t is in second.

Question 15. 25. A SONAR system fixed in a submarine operates at a frequency 40.0 kHz. An enemy submarine moves towards the

SONAR with a speed of 360 km h^{-1}. What is the frequency of sound reflected by the submarine? Take the speed of sound in water to be 1450 ms^{-1}.

Answer:

Here, frequency of SONAR (source) = 40.0 kHz = 40 × 10^3 Hz

Speed of sound waves, v = 1450 ms^{-1}

Speed of observers, v_0 = 360 km/h = 360 × $\dfrac{5}{18}$ = 100 ms^{-1}.

Since the source is at rest and observer moves towards the source (SONAR),

$$\therefore \qquad v' = \dfrac{v+v_0}{v} \cdot v = \dfrac{1450+100}{1450} \times 40 \times 10^3 = 4.276 \times 10^4 \text{ Hz.}$$

This frequency (v') is reflected by the enemy ship and is observed by the SONAR (which now acts as observer). Therefore, in this case, v_s = 360 km/h = 100 ms^{-1}.

$$\therefore \text{ Apparent frequency, } v'' = \dfrac{v}{v-v_s} v' = \dfrac{1450}{1450-100} \times 4.276 \times 10^4$$

$$= 4.59 \times 10^4 \text{ Hz} = 45.9 \text{ kHz.}$$

Question 15. 26. Earthquakes generate sound waves inside the earth. Unlike a gas, the earth can experience both transverse (S) and longitudinal (P) sound waves. Typically the speed of S wave is about 4.0 km s^{-1}. A seismograph records P and S waves from an earthquake. The first P wave arrives 4 min before the first S wave. Assuming the waves travel in straight line, at what distance does the earthquake occur?

Answer:

Here speed of S wave, v_S = 4.0 km s^{-1} and speed of P wave, v_p = 8.0 km s^{-1}. Time gap between P and S waves reaching the resimograph, t = 40 min = 240 s.

Let distance of earthquake centre = sKm

$$\therefore \qquad t = t_s - t_p = \dfrac{S}{v_s} - \dfrac{S}{v_p} = \dfrac{S}{4.0} - \dfrac{S}{8.0} = \dfrac{S}{8.0} = 240 \text{ s}$$

or $\qquad$ s = 240 × 8.0 = 1920 km.

Question 15. 27. A bat is flitting about in a cave, navigating via ultrasonic beeps. Assume that the sound emission frequency of the bat is 40 kHz. During one fast swoop directly toward a flat wall surface, the bat is moving at 0.03 times the speed of sound in air. What frequency does the bat hear reflected off the wall ?

Answer: Here, the frequency of sound emitted by the bat, v = 40 kHz. Velocity of bat, v_s = 0.03 v, where v is velocity of sound. Apparent frequency of sound striking the wall

$$v' = \dfrac{v}{v-v_s} \times v = \dfrac{v}{v-0.03v} \times 40 \text{ kHz}$$

$$= \dfrac{40}{0.97} \text{ kHz}$$

This frequency is reflected by the wall and is received by the bat moving towards the wall. So v_s = 0,

$$v_L = 0.03 \ v.$$

$$v'' = \dfrac{(v+v_L)}{v} \times v' = \dfrac{(v+0.03v)}{v} \left(\dfrac{40}{0.97} \right)$$

$$= \dfrac{1.03}{0.97} \times 40 \text{ kHz} = 42.47 \text{ kHz.}$$

********** END *********