Transformation Formulae Ex 8.2 Q 7(i)

We have,

$$\text{LHS} = \frac{\sin A + \sin 3A}{\cos A - \cos 3A}$$

$$= \frac{2\sin\left(\frac{A+3A}{2}\right)\cos\left(\frac{A-3A}{2}\right)}{-2\sin\left(\frac{A+3A}{2}\right)\sin\left(\frac{A-3A}{2}\right)}$$

$$= \frac{-\sin 2A \times \cos(-A)}{\sin 2A \sin(-A)}$$

$$= \frac{-\cos(-A)}{\sin(-A)}$$

$$= \frac{-\cos A}{-\sin A} \qquad [\because \cos(-\theta) = \cos\theta \text{ and } \sin(-\theta) = -\sin\theta]$$

$$= \frac{\cos A}{\sin A}$$

$$= \cot A$$

$= \text{RHS}$

$\therefore \frac{\sin A + \sin 3A}{\cos A - \cos 3A} = \cot A.$ Hence proved.

Transformation Formulae Ex 8.2 Q 7(ii)

We have,

$$\text{LHS} = \frac{\sin 9A - \sin 7A}{\cos 7A - \cos 9A}$$

$$= \frac{2\sin\left(\frac{9A-7A}{2}\right)\cos\left(\frac{9A+7A}{2}\right)}{-2\sin\left(\frac{7A+9A}{2}\right)\sin\left(\frac{7A-9A}{2}\right)}$$

$$= \frac{-\sin A \cos 8A}{\sin 8A \sin(-A)}$$

$$= \frac{-\sin A \cos 8A}{-\sin A \times \sin 8A} \qquad [\because \sin(-\theta) = -\sin\theta]$$

$$= \frac{\cos 8A}{\sin 8A}$$

$$= \cot 8A$$

$= \text{RHS}$

$\therefore \frac{\sin 9A - \sin 7A}{\cos 7A - \cos 9A} = \cot 8A.$ Hence proved.

Transformation Formulae Ex 8.2 Q 7(iii)

We have,

$$\begin{aligned}
\text{LHS} &= \frac{\sin A - \sin B}{\cos A + \cos B} \\
&= \frac{2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)} \\
&= \frac{\sin\left(\frac{A-B}{2}\right)}{\cos\left(\frac{A-B}{2}\right)} \\
&= \tan\left(\frac{A-B}{2}\right) \\
&= \text{RHS}
\end{aligned}$$

$\therefore \dfrac{\sin A - \sin B}{\cos A + \cos B} = \tan\left(\dfrac{A-B}{2}\right).$ Hence proved.

Transformation Formulae Ex 8.2 Q7(iv)

We have,

$$\begin{aligned}
\text{LHS} &= \frac{\sin A + \sin B}{\sin A - \sin B} \\
&= \frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)} \\
&= \frac{\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)} \\
&= \tan\left(\frac{A+B}{2}\right)\cot\left(\frac{A-B}{2}\right) \\
&= \text{RHS}
\end{aligned}$$

$\therefore \dfrac{\sin A + \sin B}{\sin A - \sin B} = \tan\left(\dfrac{A+B}{2}\right)\cot\left(\dfrac{A-B}{2}\right).$ Hence proved.

Transformation Formulae Ex 8.2 Q 7(v)

We have,

$$\begin{aligned}
\text{LHS} &= \frac{\cos A + \cos B}{\cos B - \cos A} \\
&= \frac{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{-2\sin\left(\frac{B+A}{2}\right)\sin\left(\frac{B-A}{2}\right)} \\
&= \frac{-\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)} && [\because \sin(-\theta) = -\sin\theta] \\
&= \frac{-\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{-\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)} \\
&= \cot\left(\frac{A+B}{2}\right)\cot\left(\frac{A-B}{2}\right) \\
&= \text{RHS}
\end{aligned}$$

$\therefore \dfrac{\cos A + \cos B}{\cos B - \cos A} = \cot\left(\dfrac{A+B}{2}\right)\cot\left(\dfrac{A-B}{2}\right).$ Hence proved.

*********** END **********