Indefinite Integrals Ex 19.24 Q8

Let $\quad I = \int \frac{2\tan x + 3}{3\tan x + 4} dx$

$= \int \frac{2\sin x + 3\cos x}{3\sin x + 4\cos x} dx$

Let $\quad 2\sin x + 3\cos x = \lambda \frac{d}{dx}(3\sin x + 4\cos x) + \mu(3\sin x + 4\cos x) + \nu$

$2\sin x + 3\cos x = \lambda(3\cos x - 4\sin x) + \mu(3\sin x + 4\cos x) + \nu$

$2\sin x + 3\cos x = (3\lambda + 4\mu)\cos x + (-4\lambda + 3\mu)\sin x + \nu$

Camparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$3\lambda + 4\mu = 3 ------(1)$

$-4\lambda + 3\mu = 2 -----(2)$

$\nu = 0$

Solving the equation $(1),(2)$ and (3),

$\mu = \frac{18}{25}$

$\lambda = \frac{1}{25}$

$\nu = 0$

$I = \frac{1}{25}\int \frac{(3\cos x - 4\sin x)}{(3\sin x + 4\cos x)} dx + \frac{18}{25}\int dx$

$I = \frac{18}{25}x + \frac{1}{25}\log|3\sin x + 4\cos x| + c$

Indefinite Integrals Ex 19.24 Q9

Let $\quad I = \int \frac{1}{4 + 3\tan x} dx$

$I = \int \frac{\cos x}{4\cos x + 3\sin x} dx$

Let $\quad \cos x = \lambda \frac{d}{dx}(4\cos x + 3\sin x) + \mu(4\cos x + 3\sin x) + \nu$

$\cos x = \lambda(-4\sin x + 3\cos x) + \mu(4\cos x + 3\sin x) + \nu$

$\cos x = (-4\lambda + 3\mu)\sin x + (3\lambda + 4\mu)\cos x + \nu$

Camparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$-4\lambda + 3\mu = 0 -----(1)$

$3\lambda + 4\mu = 1 ------(2)$

$\nu = 0 -------(3)$

Solving the equation $(1),(2)$ and (3),

$\lambda = \frac{3}{25}$

$\mu = \frac{4}{25}$

$\nu = 0$

$I = \int \frac{3}{25} \frac{(-4\sin x + 3\cos x)}{(4\cos x + 3\sin x)} dx + \frac{4}{25}\int dx$

$I = \frac{3}{25}\log|4\cos x + 3\sin x| + \frac{4}{25}x + c$

Indefinite Integrals Ex 19.24 Q10

Let $\quad I = \int \frac{8\cot x + 1}{3\cot x + 2} dx$

$I = \int \frac{8\cos x + \sin x}{3\cos x + 2\sin x} dx$

Let $\quad 8\cos x + \sin x = \lambda \frac{d}{dx}(3\cos x + 2\sin x) + \mu(3\cos x + 2\sin x) + \nu$

$8\cos x + \sin x = \lambda(-3\sin x + 2\cos x) + \mu(3\cos x + 2\sin x) + \nu$

$8\cos x + \sin x = (-3\lambda + 2\mu)\sin x + (2\lambda + 3\mu)\cos x + \nu$

Camparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$2\lambda + 3\mu = 8 - - - - - (1)$

$-3\lambda + 2\mu = 1 - - - - - - (2)$

$\nu = 0 - - - - - - - (3)$

Solving equation $(1), (2)$ and (3),

$\lambda = 1$

$\mu = 2$

$\nu = 0$

$I = \int \frac{(-3\sin x + 2\cos x)}{(3\cos x + 2\sin x)} dx + 2\int dx$

$I = \log|3\cos x + 2\sin x| + 2x + c$

Indefinite Integrals Ex 19.24 Q11

Let $\quad I = \int \frac{4\sin x + 5\cos x}{5\sin x + 4\cos x} dx$

Let $\quad 4\sin x + 5\cos x = \lambda \frac{d}{dx}(5\sin x + 4\cos x) + \mu(5\sin x + 4\cos x) + \nu$

$4\sin x + 5\cos x = \lambda(5\cos x - 4\sin x) + \mu(5\sin x + 4\cos x) + \nu$

$4\sin x + 5\cos x = (5\lambda + 4\mu)\cos x + (-4\lambda + 5\mu)\sin x + \nu$

Camparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$-4\lambda + 5\mu = 4 - - - - - (1)$

$5\lambda + 4\mu = 5 - - - - - - (2)$

$\nu = 0 - - - - - - - (3)$

Solving equation $(1), (2)$ and (3),

$\lambda = \frac{9}{41}$

$\mu = \frac{40}{41}$

$\nu = 0$

Now,

$I = \frac{40}{41}\int dx + \frac{9}{41}\int \frac{(5\cos x - 4\sin x)}{(5\sin x + 4\cos x)} dx$

$I = \frac{40}{41}x + \frac{9}{41}\log|5\sin x + 4\cos x| + c$

*********** END **********