



Definite Integrals Ex 20.4B Q25

We have,

$$I = \int_{-1}^1 \log\left(\frac{2-x}{2+x}\right) dx$$

Since, $\log\left\{\frac{2-(-x)}{2+(-x)}\right\} = -\log\left(\frac{2-x}{2+x}\right) \therefore$ This is an odd function.

Hence,

$$I = 0$$

Definite Integrals Ex 20.4B Q26

We have,

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x \, dx$$

$\sin^2 x$ is even function.

Hence,

$$\begin{aligned} I &= 2 \int_0^{\frac{\pi}{4}} \sin^2 x \, dx = 2 \int_0^{\frac{\pi}{4}} \left(\frac{1 - \cos 2x}{2} \right) dx = \frac{2}{2} \left[x - \frac{\sin 2x}{2} \right]_0^{\frac{\pi}{4}} = \frac{1}{2} \left[\frac{2\pi}{4} - \sin \frac{\pi}{2} - 0 + \sin 0 \right] \\ &= \frac{1}{2} \left[\frac{2\pi}{4} - 1 \right] \\ &= \frac{\pi}{4} - \frac{1}{2} \end{aligned}$$

$$\therefore \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x \, dx = \frac{\pi}{4} - \frac{1}{2}$$

Definite Integrals Ex 20.4B Q27

$$\begin{aligned}
 I &= \int_0^{\pi} \log(1 - \cos x) dx \\
 &= \int_0^{\pi} \log\left(2 \sin^2 \frac{x}{2}\right) dx \\
 &= \int_0^{\pi} \log 2 dx + \int_0^{\pi} \log \sin^2 \frac{x}{2} dx \\
 &= \int_0^{\pi} \log 2 dx + 2 \int_0^{\pi} \log \sin \frac{x}{2} dx
 \end{aligned}$$

$$I = \log 2 [x]_0^{\pi} + 4 \int_0^{\frac{\pi}{2}} \log \sin t dt \quad \left[\text{Put } t = \frac{x}{2} \Rightarrow dt = \frac{1}{2} dx \right]$$

$$I = \pi \log 2 + 4I_1 \quad \dots(i)$$

$$I_1 = \int_0^{\frac{\pi}{2}} \log \sin t dt \quad \dots(ii)$$

$$= \int_0^{\frac{\pi}{2}} \log \cos t dt \quad \dots(iii)$$

Adding (ii) & (iii) we get

$$2I_1 = \int_0^{\frac{\pi}{2}} \log \sin t \cdot \cos t dt = \int_0^{\frac{\pi}{2}} \log \left(\frac{\sin 2t}{2} \right) dt = \int_0^{\frac{\pi}{2}} \log \sin 2t dt - \frac{\pi}{2} \log 2$$

We know the property $\int_a^b f(x) = \int_a^b f(t)$

$$2I_1 = I_1 - \frac{\pi}{2} \log 2$$

$$\Rightarrow I_1 = -\frac{\pi}{2} \log 2 \quad \dots(iv)$$

Putting the value from (iv) to (i)

$$I = \pi \log 2 + 4 \left(-\frac{\pi}{2} \log 2 \right) = \pi \log 2 - 2\pi \log 2 = -\pi \log 2$$

$$I = -\pi \log 2$$

Definite Integrals Ex 20.4B Q28

We have,

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \log \left(\frac{2 - \sin x}{2 + \sin x} \right) dx$$

$$\text{Let } f(x) = \log \left(\frac{2 - \sin x}{2 + \sin x} \right)$$

Then,

$$f(-x) = \log \left(\frac{2 - \sin(-x)}{2 + \sin(-x)} \right) = -\log \left(\frac{2 - \sin x}{2 + \sin x} \right) = -f(x)$$

Thus, $f(x)$ is an odd function.

$$\therefore I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \log \left(\frac{2 - \sin x}{2 + \sin x} \right) dx = 0$$

***** END *****