Geometric Progressions Ex 20.3 Q 5

Here,

$3,\ \frac{3}{2},\ \frac{3}{4},\ \ldots$ is a G.P.

and $S_n = \frac{3069}{512},\ a = 3, r = \frac{1}{2}$

$$S_n = \frac{a\left(1 - r^n\right)}{1 - r}$$

$$\frac{3069}{512} = \frac{3\left(1 - \left(\frac{1}{2}\right)^n\right)}{1 - \frac{1}{2}}$$

$$\frac{3069}{512} = \frac{3\left(2^n - 1\right)}{2^n \times \frac{1}{2}}$$

$$\frac{1023}{512} = \frac{2\left(2^n - 1\right)}{2^n}$$

$$10232^n = 1024.2^n - 1024$$

$$1024 = 2^n$$

$\Rightarrow \quad 2^{10} = 2^n$

$\Rightarrow \quad n = 10$

Geometric Progressions Ex 20.3 Q 6

$2 + 6 + 18 + \ldots$

$$S_n = 728$$

Now,

$$S_n = \frac{a\left(r^n - 1\right)}{r - 1}$$

$$a = 2,\ r = \frac{6}{2} = 3$$

$$728 = \frac{2\left(3^n - 1\right)}{3 - 1}$$

$$728 = \frac{2\left(3^n - 1\right)}{2} = \left(3^n - 1\right)$$

$$728 + 1 = 3^n$$

$$729 = 3^n$$

$$(3)^6 = 3^n$$

$\Rightarrow \quad n = 6$

Geometric Progressions Ex 20.3 Q 7

$\sqrt{3}, 3, 3\sqrt{3}, \ldots$

$$S_n = \frac{a\left(r^n - 1\right)}{r - 1}$$

$$a = \sqrt{3},\ r = \frac{3}{\sqrt{3}} = \sqrt{3},\ S_n = 39 + 13\sqrt{3}$$

Putting into formula

$$39 + 13\sqrt{3} = \frac{\sqrt{3}\left(\left(\sqrt{3}\right)^n - 1\right)}{\sqrt{3} - 1}$$

$$39 + 13\sqrt{3} = \frac{\left(\sqrt{3}\right)^{n+1} - \sqrt{3}}{\sqrt{3} - 1}$$

$$\left(39 + 13\sqrt{3}\right)\left(\sqrt{3} - 1\right) = \left(\sqrt{3}\right)^{n+1} - \sqrt{3}$$

$$39\sqrt{3} - 39 + 39 - 13\sqrt{3} = \left(\sqrt{3}\right)^{n+1} - \sqrt{3}$$

$$26\sqrt{3} + \sqrt{3} = \left(\sqrt{3}\right)^{n+1}$$

$$\left(27\sqrt{3}\right)^1 = \left(\sqrt{3}\right)^{n+1}$$

$$\left(\sqrt{3}\right)^6\left(\sqrt{3}\right)^1 = \left(\sqrt{3}\right)^{n+1}$$

$$7 = n + 1$$

$$\Rightarrow \quad n = 6$$

Geometric Progressions Ex 20.3 Q 8

$3, 6, 12, \ldots\ n$ 381

$$a = 3,\ r = \frac{6}{3} = 2,\ n = ?\ S_n = 381$$

We know that

$$S_n = \frac{a\left(r^n - 1\right)}{r - 1}$$

$$381 = \frac{3(2)^n - 1}{2 - 1}$$

$$\frac{381}{3} = 2^n - 1$$

$$127 = 2^n - 1$$

$$128 = 2^n$$

$$2^7 = 2^n$$

$$n = 7$$

Geometric Progressions Ex 20.3 Q 9

$r = 3$, last term is 486

Sum of terms $= S_n = 728,\ a = ?$

We know that

$$S_n = \frac{a\left(r^n - 1\right)}{r - 1}$$

$$728 = \frac{a\left(3^n - 1\right)}{3 - 1}$$

Also, $t_n = ar^{n-1}$

$t_n = 486$

$\therefore \quad 486 = a(3)^{n-1}$

$$a\left(3^{n-1}\right) = 3^5 \times 2$$

$$3^{n-1} = 3^5$$

$$n = 6$$

and $a = 2$

Geometric Progressions Ex 20.3 Q 10

Let Sum of first three terms $=a+ar+ar^2$

The ratio $=\dfrac{a+ar+ar^2}{a+ar+ar^2+ar^3+ar^4+ar^5}$

$=\dfrac{1+r+r^2}{1+r+r^2+r^3+r^4+r^5}$

$=\dfrac{1+r+r^2}{1+r+r^2+r^3(1+r+r^2)}$ .....(1)

Let A=$1+r+r^2$ .....(2)

$Ratio=\dfrac{A}{A+r^3A}=\dfrac{125}{152}$

$\dfrac{1}{1+r^3}=\dfrac{125}{152}$

$152=125+125\,r^3$

$r^3=\dfrac{27}{125}$

$r=\dfrac{3}{5}$

********** END **********