



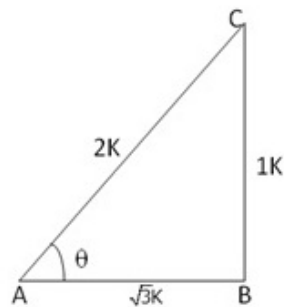
Question 9

Given: $\operatorname{cosec} \theta = \frac{AC}{BC} = \frac{2}{1}$

Let $AC = 2k$ and $BC = 1k$,

Where k is positive

Let us draw a ΔABC in which $\angle B = 90^\circ$ and $\angle BAC = \theta$



By Pythagoras theorem, we have

$$AC^2 = (AB)^2 + (BC)^2$$

$$\Rightarrow (AB)^2 = (AC)^2 - (BC)^2$$

$$= \left[(2k)^2 - (1k)^2 \right] = (4k^2 - 1k^2) = 3k^2$$

$$\Rightarrow (AB) = \sqrt{3}k$$

$$\sin \theta = \frac{BC}{AC} = \frac{1k}{2k} = \frac{1}{2}$$

$$\cos \theta = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta} = \left(\frac{\frac{\sqrt{3}}{2} \times 2}{1} \right) = \sqrt{3}$$

$$\begin{aligned} \Rightarrow \left[\cot \theta + \frac{\sin \theta}{1 + \cos \theta} \right] &= \left[\sqrt{3} + \frac{\frac{1}{2}}{1 + \frac{\sqrt{3}}{2}} \right] \\ &= \left(\sqrt{3} + \frac{1}{2 + \sqrt{3}} \right) = \left(\frac{2\sqrt{3} + 3 + 1}{2 + \sqrt{3}} \right) \\ &= \left(\frac{2\sqrt{3} + 4}{2 + \sqrt{3}} \right) = 2 \left(\frac{\sqrt{3} + 2}{2 + \sqrt{3}} \right) = 2 \end{aligned}$$

Hence, $\left[\cot \theta + \frac{\sin \theta}{1 + \cos \theta} \right] = 2$

***** END *****