Geometric Progressions Ex 20.5 Q 13

a,b,c,d are in G.P.

$a,\ b = ar,\ c = ar^2,\ d = ar^3$

$$\frac{ab - cd}{b^2 - c^2} = \frac{a + c}{b}$$

$$\frac{a(ar) - (ar^2)(ar^3)}{a^2r^2 - a^2r^4} = \frac{a + ar^2}{ar}$$

$$\frac{a^2r - a^2r^5}{a^2r^2(1 - r^2)} = \frac{a(1 + r^2)}{ar}$$

$$\frac{a^2r(1 - r^4)}{a^2r^2(1 - r^2)} = \frac{a(1 + r^2)}{ar}$$

$$\frac{1 + r^2}{r} = \frac{1 + r^2}{r}$$

LHS = RHS

a,b,c,d are in G.P.

$a,\ b = ar,\ c = ar^2,\ d = ar^3$

$$(a + b + c + d)^2 = (a + b)^2 + 2(b + c)^2 + (c + d)^2$$

$$\Rightarrow \quad (a + ar + ar^2 + ar^3)^2 = (a + ar)^2 + 2(ar + ar^2)^2 + (ar^2 + ar^3)^2$$

$$\Rightarrow \quad a^2(1 + r + r^2 + r^3)^2 = a^2\left[(1 + r)^2 + 2(r + r^2)^2 + (r^2 + r^3)^2\right]$$

$$\Rightarrow \quad (1 + r + r^2 + r^3)^2 = 1 + r^2 + 2r + 2(r^2 + r^4 + 2r^3) + r^4 + r^6 + 2r^5$$

$$\Rightarrow \quad (1 + r + r^2 + r^3 + r + r^2 + r^3 + r^4 + r^2 + r^3 + r^4 + r^5 + r^3 + r^4 + r^5 + r^6)$$
$$= (1 + r^2 + 2r + 2r^2 + 2r^4 + 4r^3 + r^4 + r^6 + 2r^5)$$

$$\Rightarrow \quad (r^6 + 2r^5 + 3r^4 + 4r^3 + 3r^2 + 2r + 1) = (r^6 + 2r^5 + 3r^4 + 4r^3 + 3r^2 + 2r + 1)$$

LHS = RHS

a,b,c,d are in G.P.

$a,\ b = ar,\ c = ar^2,\ d = ar^3$

$$(b + c)(b + d) = (c + a)(c + d)$$

$$\Rightarrow \quad (ar + ar^2)(ar + ar^3) = (ar^2 + a)(ar^2 + ar^3)$$

$$\Rightarrow \quad a^2(r + r^2)(r + r^3) = a^2(r^2 + 1)(r^2 + r^3)$$

$$\Rightarrow \quad r^2(1 + r)(1 + r^2) = r^2(1 + r^2)(1 + r)$$

$\therefore$ LHS = RHS

a,b,c are in G.P.

$$\Rightarrow \quad b^2 = ac \qquad \text{---(i)}$$

$$(b^2)^2 = (ac)^2$$

$$(b^2)^2 = a^2c^2$$

$$\Rightarrow \quad a^2, b^2, c^2 \text{ are in G.P.}$$

a, b, c are in G.P.

$a,\ b = ar,\ c = ar^2$

$\left(b^3\right)^2 = a^3c^3$

$\left((ar)^3\right)^2 = a^3\left(ar^2\right)^3$

$a^6r^6 = a^3\left(a^3r^6\right)$

$a^6r^6 = a^6r^6$

LHS = RHS

$\Rightarrow \quad \left(b^3\right)^2 = a^3c^3$

So,

a^3, b^3, c^3 are in G.P.

a, b, c are in G.P.

$a,\ b = ar,\ c = ar^2$

$(ab+bc)^2 = \left(a^2+b^2\right)\left(b^2+c^2\right)$

$\left(a\times ar + ar\times ar^2\right)^2 \left(a^2+(ar)^2\right)\left((ar)^2+\left(ar^2\right)^2\right)$

$\left(a^2r + a^2r^3\right)^2 = \left(a^2+a^2r^2\right)\left(a^2r^2+a^2r^4\right)$

$a^4\left(r+r^3\right)^2 = a^4\left(1+r^2\right)\left(r^2+r^4\right)$

$a^4r^2\left(1+r^2\right)^2 = a^4\left(1+r^2\right)r^2\left(1+r^2\right)$

$a^4r^2\left(1+r^2\right)^2 = a^4r^2\left(1+r^2\right)^2$

LHS = RHS

$\Rightarrow \quad (ab+bc)^2 = \left(a^2+b^2\right)\left(b^2+c^2\right)$

$\Rightarrow \quad \left(a^2+b^2\right),\ (ab+bc),\ \left(b^2+c^2\right)$ are in G.P.

********** END **********