Relations Ex 1.1 Q2

We have, $A = \{a, b, c\}$

$R_1 = \{(a,a)(a,b)(a,c)(b,b)(b,c)(c,a)(c,b)(c,c)\}$

R_1 is reflexive *as* $(a,a) \in R_1, (b,b) \in R_1$ & $(c,c) \in R_1$

R_1 is not symmetric *as* $(a,b) \in R_1$ *but* $(b,a) \in R_1$

R_1 is not transitive as $(b,c) \in R_1$ and $(c,a) \in R_1$ but $(b,a) \notin R_1$

$R_2 = \{(a,a)\}$

R_2 is not reflexive as $(b,b) \notin R_2$

R_2 is symmetric and transitive.

$R_3 = \{(b,c)\}$

R_3 is not reflexive as $(b,b) \notin R_3$

R_3 is not symmetric

R_3 is not transitive.

$R_4 = \{(a,b)(b,c)(c,a)\}$

R_4 is not reflexive on set A as $(a,a) \notin R_4$

R_4 is not symmetric as $(a,b) \in R_4$ but $(b,a) \notin R_4$

R_4 is not transitive as $(a,b) \in R_4$ and $(b,c) \in R_4$ but $(a,c) \notin R_4$

Relations Ex 1.1 Q3

$$R_1 = \left\{ (x, y), x, y \in Q_0, x = \frac{1}{y} \right\}$$

Reflexivity: Let, $x \in Q_0$

$\Rightarrow \quad x \neq \frac{1}{x}$

$\Rightarrow \quad (x, x) \in R_1$

$\therefore \quad R_1$ is not reflexive

Symmetric: Let, $(x, y) \in R_1$

$\Rightarrow \quad x = \frac{1}{y}$

$\Rightarrow \quad y = \frac{1}{x}$

$\Rightarrow \quad (y, x) \in R_1$

$\therefore \quad R_1$ is symmetric symmetric

Transitive: Let, $(x, y) \in R_1$ and $(y, z) \in R_1$

$\Rightarrow \quad x = \frac{1}{y}$ and $y = \frac{1}{z}$

$\Rightarrow \quad x = z$

$\Rightarrow \quad (x, z) \notin R_1$

$\therefore \quad R_1$ is not trasitive

Relations Ex 1.1 Q3(ii)

Reflexivity: Let, $a \in z$

$\Rightarrow \quad |a-a| = 0 \le 5$

$\therefore \quad (a,a) \in R_2 \Rightarrow \quad R_2$ is reflexive

Symmetricity:Let, $(a,b) \in R_2$

$\Rightarrow \quad |a-a| \le 5$

$\Rightarrow \quad |b-a| \le 5$

$\Rightarrow \quad |b,a| \in R_2 \quad \Rightarrow \quad R_2$ is symmetric

Transitivity: Let, $(a,b) \in R_2$ and $(b,c) \in R_2$

$\Rightarrow \quad |a-b| \le 5$ and $|b-c| \le 5$

$\not\Rightarrow \quad |a-c| \le 5$

$\Rightarrow \quad R_2$ is not transitive

$$\left[\begin{array}{l} \therefore \ \text{if } a = 15, b = 11, c = 7 \\ \qquad \Rightarrow \quad |15-11| \le 5 \text{ and } |11-7| \le 5 \\ \qquad\qquad \text{but } |15-7| \ge 5 \end{array}\right]$$

Relations Ex 1.1 Q4

(i) We have, $A = \{1, 2, 3\}$ and

$R_1 = \{(1,1)(1,3)(3,1)(2,2)(2,1)(3,3)\}$

$\therefore \quad (1,1), (2,2)$ and $(3,3) \in R_1$

$\therefore \quad R_1$ is not Reflexive

Now,

$\therefore \quad (2,1) \in R_1$ but $(1,2) \notin R_1$

$\therefore \quad R_1$ is not Symmetric

Again,

$\therefore \quad (2,1) \in R_1$ and $(1,3) \in R_1$ but $(2,3) \notin R_1$

$\therefore \quad R_1$ is not Transitive

$\therefore \quad R_1$ is not transitive

(ii) $R_2 = \{(2,2), (3,1), (1,3)\}$

$\therefore \quad (1,1) \notin R_2$

$\Rightarrow \quad R_2$ is not reflexive

Now, $(1,3) \in R_2$
$\Rightarrow \quad (3,1) \in R_2$

$\Rightarrow \quad R_2$ is symmetric

Again, $(3,1) \in R_2$ and $(1,3) \in R_2$ but $(3,3) \notin R_1$

$\therefore \quad R_2$ is not transitive

(iii) $R_3 = \{(1,3)\,(3,3)\}$

$\therefore \quad (1,1) \notin R_3$

$\Rightarrow \quad R_3$ is not reflexive

Now, $(1,3) \in R_3$ but $(3,1) \in R_3$
$\Rightarrow \quad R_3$ is not symmetric

Again, It is clear that R_3 is transitive

*********** END **********