Differentiation Ex 11.3 Q31

Let $\quad y = \tan^{-1}\left(\frac{5x}{1-6x^2}\right)$

$$= \tan^{-1}\left(\frac{3x+2x}{1-(3x)(2x)}\right)$$

$$y = \tan^{-1}(3x) + \tan^{-1}(2x) \qquad \left[\text{Since, } \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{1+(3x)^2}\frac{d}{dx}(3x) + \frac{1}{1+(2x)^2}\frac{d}{dx}(2x)$$

$$= \frac{1}{1+9x^2}(3) + \frac{1}{1+4x^2}(2)$$

$$\frac{dy}{dx} = \frac{3}{1+9x^2} + \frac{2}{1+4x^2}.$$

Differentiation Ex 11.3 Q32

Let $y = \tan^{-1}\left[\dfrac{\cos x + \sin x}{\cos x - \sin x}\right]$

$$= \tan^{-1}\left[\dfrac{\dfrac{\cos x + \sin x}{\cos x}}{\dfrac{\cos x - \sin x}{\cos x}}\right]$$

$$= \tan^{-1}\left[\dfrac{\dfrac{\cos x}{\cos x} + \dfrac{\sin x}{\cos x}}{\dfrac{\cos x}{\cos x} - \dfrac{\sin x}{\cos x}}\right]$$

$$= \tan^{-1}\left[\dfrac{1 + \tan x}{1 - \tan x}\right]$$

$$= \tan^{-1}\left[\dfrac{\dfrac{\tan \pi}{4} + \tan x}{1 - \dfrac{\tan \pi}{4}\tan x}\right]$$

$$= \tan^{-1}\left[\tan\left(\dfrac{\pi}{4} + x\right)\right]$$

$$y = \dfrac{\pi}{4} + x$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 0 + 1$$

$$\frac{dy}{dx} = 1.$$

Differentiation Ex 11.3 Q33

Let $y = \tan^{-1}\left[\frac{x^{\frac{1}{3}} + a^{\frac{1}{3}}}{1 - (ax)^{\frac{1}{3}}}\right]$

$y = \tan^{-1}\left(x^{\frac{1}{3}}\right) + \tan^{-1}\left(a^{\frac{1}{3}}\right)$ $\left[\text{Since, } \tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x + y}{1 - xy}\right)\right]$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{1 + \left(x^{\frac{1}{3}}\right)^2} \times \frac{d}{dx}\left(x^{\frac{1}{3}}\right) + 0$$

$$= \frac{\left(\frac{1}{3} \times x^{\frac{1}{3}-1}\right)}{1 + x^{\frac{2}{3}}}$$

$$\frac{dy}{dx} = \frac{1}{3x^{\frac{2}{3}}\left(1 + x^{\frac{2}{3}}\right)}.$$

Differentiation Ex 11.3 Q34

Let $f(x) = \sin^{-1}\left(\frac{2^{x+1}}{1 + 4^x}\right)$

To find the domain, we need to find all x such that

$$-1 \le \frac{2^{x+1}}{1 + 4^x} \le 1$$

Since the quantity in the middle is always positive, we need to find all x such that $\frac{2^{x+1}}{1 + 4^x} \le 1$

i.e all x such that $2^{x+1} \le 1 + 4^x$

We may rewrite as $2 \le \frac{1}{2^x} + 2^x$, which is true for all x

Hence the function is defined at all real numbers.

Putting $2^x = \tan\theta$

$$f(x) = \sin^{-1}\left(\frac{2^{x+1}}{1 + 4^x}\right) = \sin^{-1}\left(\frac{2^x.2}{1 + (2^x)^2}\right)$$

$$= \sin^{-1}\left[\frac{2\tan\theta}{1 + \tan^2\theta}\right] = \sin^{-1}(\sin 2\theta) = 2\theta = 2\tan^{-1}\left(2^x\right)$$

Thus, $f'(x) = 2.\frac{1}{1 + \left(2^x\right)^2}\frac{d}{dx}\left(2^x\right)$

$$= \frac{2}{1 + 4^x}.\left(2^x\right)\log 2 = \frac{2^{x+1}\log 2}{1 + 4^x}$$

*********** END **********