

Definite Integrals Ex 20.3 Q19

$$\begin{aligned} |x| &= \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases} \\ |x - 2| &= \begin{cases} x - 2, & x \ge 2 \\ 2 - x, & x < 2 \end{cases} \\ |x - 4| &= \begin{cases} x - 4, & x \ge 4 \\ 4 - x, & x < 4 \end{cases} \end{aligned}$$

Splitting the limits of the integral, we get

$$\int_{0}^{4} (|x| + |x - 2| + |x - 4|) dx$$

$$= \int_{0}^{2} (|x| + |x - 2| + |x - 4|) dx + \int_{2}^{4} (|x| + |x - 2| + |x - 4|) dx$$

$$= \int_{0}^{2} (x + 2 - x + 4 - x) dx + \int_{2}^{4} (x + x - 2 + 4 - x) dx$$

$$= \int_{0}^{2} (6 - x) dx + \int_{2}^{4} (2 + x) dx$$

$$= \left[6x - \frac{x^{2}}{2} \right]_{0}^{2} + \left[2x + \frac{x^{2}}{2} \right]_{2}^{4}$$

$$= \left[12 - 2 \right] + \left[16 - 6 \right]$$

$$= 10 + 10$$

$$= 20$$

Definite Integrals Ex 20.3 Q20

$$\begin{split} & \int_{-1}^{2} |x+1| \, dx + \int_{-1}^{2} |x| \, dx + \int_{-1}^{2} |x-1| \, dx \\ & \int_{-1}^{2} (x+1) \, dx - \int_{-1}^{0} x \, dx + \int_{0}^{2} x \, dx - \int_{-1}^{1} (x-1) \, dx + \int_{1}^{2} (x-1) \, dx \\ & \left\{ \frac{x^{2}}{2} + x \right\}_{-1}^{2} - \left\{ \frac{x^{2}}{2} \right\}_{-1}^{0} + \left\{ \frac{x^{2}}{2} \right\}_{0}^{2} - \left\{ \frac{x^{2}}{2} - x \right\}_{-1}^{1} + \left\{ \frac{x^{2}}{2} - x \right\}_{1}^{2} \\ & \left\{ (4) - (-\frac{1}{2}) \right\} - \left\{ -\frac{1}{2} \right\} + \left\{ 2 \right\} - \left\{ (-\frac{1}{2}) - (\frac{3}{2}) \right\} + \left\{ (0) - (-\frac{1}{2}) \right\} \\ & \left\{ 4 + \frac{1}{2} \right\} + \left\{ \frac{1}{2} \right\} + \left\{ 2 \right\} + \left\{ 2 \right\} + \left\{ \frac{1}{2} \right\} \end{split}$$

Definite Integrals Ex 20.3 Q21

For
$$\int_{-2}^{0} xe^{-x} dx + \int_{0}^{2} xe^{x} dx$$
For
$$\int_{-2}^{0} xe^{-x} dx$$
Using Integration By parts
$$\int f'g = fg - \int fg'$$

$$f' = e^{-x}, g = x$$

$$f = -e^{-x}, g' = 1$$

$$\int_{-2}^{0} xe^{-x} dx = \left\{-xe^{-x}\right\}_{-2}^{0} + \int_{-2}^{0} e^{-x} dx$$

$$\int_{-2}^{0} xe^{-x} dx = \left\{(-1) - (2e^{2} - e^{2})\right\}$$

$$\int_{-2}^{0} xe^{-x} dx = \left\{(-1) - (2e^{2} - e^{2})\right\}$$
For
$$\int_{-2}^{2} xe^{x} dx$$
Using Integration By parts
$$\int f'g = fg - \int fg'$$

$$f' = e^{x}, g = x$$

$$f = e^{x}, g' = 1$$

$$\int_{0}^{2} xe^{x} dx = \left\{xe^{x}\right\}_{0}^{2} - \int_{0}^{2} e^{x} dx$$

$$\int_{0}^{2} xe^{x} dx = \left\{xe^{x} - e^{x}\right\}_{0}^{2}$$

$$\int_{0}^{2} xe^{x} dx = 2e^{2} - e^{2} + 1$$

$$\int_{0}^{2} xe^{x} dx = e^{2} + 1$$

Hence answer is,

$$\int_{-2}^{2} x e^{|x|} dx = -1 - e^{2} + e^{2} + 1 = 0$$

Definite Integrals Ex 20.3 Q22

$$-\int_{-\frac{\pi}{4}}^{0} \sin^{2}x dx + \int_{0}^{\frac{\pi}{2}} \sin^{2}x dx$$

$$\sin^{2}x = \frac{1 - \cos 2x}{2}$$

$$-\int_{-\frac{\pi}{4}}^{0} \frac{1 - \cos 2x}{2} dx + \int_{0}^{\frac{\pi}{2}} \frac{1 - \cos 2x}{2} dx$$

$$-\frac{1}{2} \left\{ x - \frac{\sin 2x}{2} \right\}_{-\frac{\pi}{4}}^{0} + \frac{1}{2} \left\{ x - \frac{\sin 2x}{2} \right\}_{0}^{\frac{\pi}{2}}$$

$$-\frac{1}{2} \left\{ -(-\frac{\Pi}{4} + \frac{1}{2}) \right\} + \frac{1}{2} \left\{ \frac{\Pi}{2} \right\}$$

$$\left\{ -\frac{\Pi}{8} + \frac{1}{4} \right\} + \left\{ \frac{\Pi}{4} \right\}$$

$$\frac{\Pi}{8} + \frac{1}{4}$$

$$\frac{\Pi + 2}{2}$$

******* END *******