Exercise 12.2

Question 1:

Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs Rs 60/kg and Food Q costs Rs 80/kg. Food P contains 3 units /kg of vitamin A and 5 units /kg of vitamin B while food Q contains 4 units /kg of vitamin A and 2 units /kg of vitamin B. Determine the minimum cost of the mixture?

Answer

Let the mixture contain x kg of food P and y kg of food Q. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be compiled in a table as follows.

	Vitamin A (units/kg)	Vitamin B (units/kg)	Cost (Rs/kg)
Food P	3	5	60
Food Q	4	2	80
Requirement (units/kg)	8	11	

The mixture must contain at least 8 units of vitamin A and 11 units of vitamin B.

Therefore, the constraints are

$3x + 4y \geq 8$

$5x + 2y \geq 11$

Total cost, Z, of purchasing food is, $Z = 60x + 80y$

The mathematical formulation of the given problem is

Minimise $Z = 60x + 80y$ … (1)

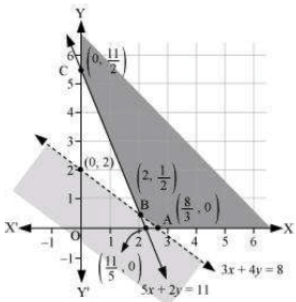
subject to the constraints,

$3x + 4y \geq 8$ … (2)

$5x + 2y \geq 11$ … (3)

$x, y \geq 0$ … (4)

The feasible region determined by the system of constraints is as follows.



It can be seen that the feasible region is unbounded.

The corner points of the feasible region are $A\left(\frac{8}{3}, 0\right)$, $B\left(2, \frac{1}{2}\right)$, and $C\left(0, \frac{11}{2}\right)$.

The values of Z at these corner points are as follows.

Corner point	Z = 60x + 80y	
$A\left(\frac{8}{3}, 0\right)$	160	$\}\rightarrow$ Minimum
$B\left(2, \frac{1}{2}\right)$	160	
$C\left(0, \frac{11}{2}\right)$	440	

As the feasible region is unbounded, therefore, 160 may or may not be the minimum value of Z.

For this, we graph the inequality, $60x + 80y < 160$ or $3x + 4y < 8$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $3x + 4y < 8$

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Therefore, the minimum cost of the mixture will be Rs 160 at the line segment joining the points $\left(\frac{8}{3}, 0\right)$ and $\left(2, \frac{1}{2}\right)$.

Question 2:

One kind of cake requires 200g flour and 25g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes?

Answer

Let there be x cakes of first kind and y cakes of second kind. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be complied in a table as follows.

	Flour (g)	**Fat (g)**
Cakes of first kind, *x*	200	25
Cakes of second kind, *y*	100	50
Availability	5000	1000

$\therefore 200x + 100y \leq 5000$

$\Rightarrow 2x + y \leq 50$

$25x + 50y \leq 1000$

$\Rightarrow x + 2y \leq 40$

Total numbers of cakes, Z, that can be made are, Z = $x + y$

The mathematical formulation of the given problem is

Maximize Z = $x + y$ ... (1)

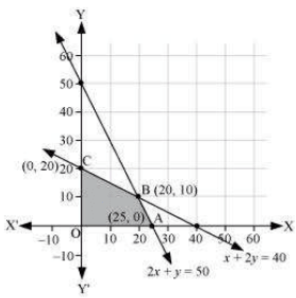
subject to the constraints,

$2x + y \leq 50$ ...(2)

$x + 2y \leq 40$ ...(3)

$x, y \geq 0$ ...(4)

The feasible region determined by the system of constraints is as follows.



The corner points are A (25, 0), B (20, 10), O (0, 0), and C (0, 20).

The values of Z at these corner points are as follows.

Corner point	**Z = *x* + *y***	
A(25, 0)	25	
B(20, 10)	30	→ Maximum
C(0, 20)	20	
O(0, 0)	0	

Thus, the maximum numbers of cakes that can be made are 30 (20 of one kind and 10 of the other kind).

Question 3:

A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftsman's time in its making while a cricket bat takes 3 hour of machine time and 1 hour of craftsman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time.

(ii) What number of rackets and bats must be made if the factory is to work at full capacity?

(ii) If the profit on a racket and on a bat is Rs 20 and Rs 10 respectively, find the maximum profit of the factory when it works at full capacity.

Answer

(i) Let the number of rackets and the number of bats to be made be x and y respectively.

The machine time is not available for more than 42 hours.

$\therefore 1.5x + 3y \leq 42$ ...(1)

The craftsman's time is not available for more than 24 hours.

$\therefore 3x + y \leq 24$ ...(2)

The factory is to work at full capacity. Therefore,

$1.5x + 3y = 42$

$3x + y = 24$

On solving these equations, we obtain

$x = 4$ and $y = 12$

Thus, 4 rackets and 12 bats must be made.

(i) The given information can be complied in a table as follows.

	Tennis Racket	**Cricket Bat**	**Availability**
Machine Time (h)	1.5	3	42
Craftsman's Time (h)	3	1	24

$\therefore 1.5x + 3y \leq 42$

$3x + y \leq 24$

$x, y \geq 0$

The profit on a racket is Rs 20 and on a bat is Rs 10.

$\therefore Z = 20x + 10y$

The mathematical formulation of the given problem is

Maximize $Z = 20x + 10y$ … (1)

subject to the constraints,

$1.5x + 3y \leq 42$ … (2)

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