Indefinite Integrals Ex 19.27 Q1

Let $I = \int e^{ax} \cos bx dx$

Intergrating by parts,

$$I = e^{ax} \frac{\sin bx}{b} - a \int e^{ax} \frac{\sin bx}{b} fx$$

$$= \frac{1}{b} e^{ax} \sin bx - \frac{a}{b} \int e^{ax} \sin bx dx$$

$$= \frac{1}{b} e^{ax} \sin bx - \frac{a}{b}\left[-e^{ax} \frac{\cos bx}{b} + \int a e^{ax} \frac{\cos bx}{b} dx\right]$$

$$= \frac{1}{b} e^{ax} \sin bx + \frac{a}{b^2} e^{ax} \cos bx - \frac{a^2}{b^2} \int e^{ax} \cos bx dx$$

$$\Rightarrow \qquad I = \frac{e^{ax}}{b^2}\left[b \sin bx + a \cos bx\right] - \frac{a^2}{b^2} I + c$$

$$\Rightarrow \qquad I.\left\{\frac{a^2 + b^2}{b^2}\right\} = \frac{e^{ax}}{b^2}\left[b \cos bx + a \cos bx\right] + c$$

Thus,

$$I = \frac{e^{ax}}{a^2 + b^2}\left[a \cos bx + a \cos bx\right] + c$$

Indefinite Integrals Ex 19.27 Q2

Let $I = \int e^{ax} \sin(bx + c) dx$

$$\Rightarrow \qquad -e^{ax} \frac{\cos(bx + c)}{b} + \int a e^{ax} \frac{\cos(bx + c)}{b} dx$$

$$= -\frac{1}{b} e^{ax} \cos(bx + c) + \frac{a}{b} \int e^{ax} \cos(bx + c) dx$$

$$= -\frac{1}{b} e^{ax} \cos(bx + c) + \frac{a}{b}\left[\int e^{ax} \frac{\sin(bx + c)}{b} - \int a e^{ax} \frac{\sin(bx + c)}{b} dx\right] + c_1$$

$$= \frac{e^{ax}}{b^2}\left\{a \sin(bx + c) - b \cos(bx + c)\right\} - \frac{a^2}{b^2} \int e^{ax} \sin(bx + c) dx + c_1$$

$$\Rightarrow \qquad I = \frac{e^{ax}}{b^2}\left\{a \sin(bx + c) - b \cos(bx + c)\right\} - \frac{a^2}{b^2} I + c_1$$

$$\Rightarrow \qquad I = \left\{\frac{a^2 + b^2}{b^2}\right\} - \frac{e^{ax}}{b^2}\left\{a \sin(bx + c) - b \cos(bx + c)\right\} + c_1$$

$$\Rightarrow \qquad I = \frac{e^{ax}}{a^2 + b^2}\left\{a \sin(bx + c) - b \cos(bx + c)\right\} + c_1$$

Indefinite Integrals Ex 19.27 Q3

Let $\log x = t$

$\Rightarrow \quad \frac{1}{x}dx = dt$

$\Rightarrow \quad dx = xdt$

$\Rightarrow \quad dx = e^t dt$

$\therefore \quad I = \int \cos(\log x)dx = \int e^t \cos t dt$

We know that

$$\int e^{ax}\cos bx dx = \frac{e^{ax}}{a^2+b^2}\{a\cos bx + b\sin bx\} + c$$

Here $a = 1$, $b = 1$

So, $I = \frac{e^t}{2}\{\cos t + \sin t\} + c$

Hence,

$$I = \int \cos(\log x)dx = \frac{e^{\log x}}{2}\{\cos(\log x) + \sin(\log x)\} + c$$

$$\Rightarrow \quad I = \frac{x}{2}\{\cos(\log x) + \sin(\log x)\} + c$$

Indefinite Integrals Ex 19.27 Q4

Let $I = \int e^{2x}\cos(3x+4)dx$

Integrating by parts

$$I = e^{2x}\frac{\sin(3x+4)}{3} - \int 2e^{2x}\frac{\sin(3x+4)}{3}dx$$

$$= \frac{1}{3}e^{2x}\sin(3x+4) - \frac{2}{3}\int e^{2x}\sin(3x+4)dx$$

$$= \frac{1}{3}e^{2x}\sin(3x+4) - \frac{2}{3}\left[-e^{2x}\frac{\cos(3x+4)}{3} + \int 2e^{2x}\frac{\cos(3x+4)}{3}dx\right] + c$$

$$I = \frac{e^{2x}}{9}\{2\cos(3x+4) + 3\sin(3x+4)\} + c$$

Hence,

$$I = \frac{e^{2x}}{13}\{2\cos(3x+4) + 3\sin(3x+4)\} + c$$

*********** END **********