Differentiation Ex 11.5 Q1

Let $\quad y = x^{\frac{1}{x}}$ $\qquad\qquad$ ---(i)

Taking log on both the sides,

$\Rightarrow \quad \log y = \log x^{\frac{1}{x}}$

$\Rightarrow \quad \log y = \dfrac{1}{x}\log x$ $\qquad\qquad\qquad$ $\left[\text{Since, } \log a^b = b\log a\right]$

Differentiating with respect to x,

$\Rightarrow \quad \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{x}\dfrac{d}{dx}(\log x) + \log x\dfrac{d}{dx}(x^{-1})$ $\qquad$ [Using product rule]

$\Rightarrow \quad \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{x}\times\dfrac{1}{x} + (\log x)\times\left(-\dfrac{1}{x^2}\right)$

$\Rightarrow \quad \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{x^2} - \dfrac{\log x}{x^2}$

$\Rightarrow \quad \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{(1-\log x)}{x^2}$

$\Rightarrow \quad \dfrac{dy}{dx} = y\left[\dfrac{1-\log x}{x^2}\right]$

Put the value of y from equation (i),

$\dfrac{dy}{dx} = x^{\frac{1}{x}}\left[\dfrac{1-\log x}{x}\right]$

Differentiation Ex 11.5 Q2

Let $\quad y = x^{\sin x}$ $\qquad\qquad$ ---(i)

Taking log on both the sides,

$\log y = \log x^{\sin x}$

$\log y = \sin x\log x$ $\qquad\qquad\qquad$ $\left[\text{Since, } \log a^b = b\log a\right]$

Differentiating with respect to x,

$\dfrac{1}{y}\dfrac{dy}{dx} = \sin x\dfrac{d}{dx}\log x + \log x\dfrac{d}{dx}\sin x$ $\qquad$ [Using product rule]

$\dfrac{1}{y}\dfrac{dy}{dx} = \sin x\left(\dfrac{1}{x}\right) + \log x(\cos x)$

$\dfrac{dy}{dx} = y\left[\dfrac{\sin x}{x} + (\log x)(\cos x)\right]$

Put the value of y,

$\dfrac{dy}{dx} = x^{\sin x}\left[\dfrac{\sin x}{x} + (\log x)(\cos x)\right]$

Differentiation Ex 11.5 Q3

Let $\quad y = (1 + \cos x)^x \qquad\qquad$ ---(i)

Taking log on both the sides,

$\log y = \log(1 + \cos x)^x$

$\log y = x \log(1 + \cos x)$

Differentiating with respect to x,

$\dfrac{1}{y}\dfrac{dy}{dx} = x\dfrac{d}{dx}\log(1+\cos x) + \log(1+\cos x)\dfrac{d}{dx}(x) \qquad$ [Using product rule and chain rule]

$\dfrac{1}{y}\dfrac{dy}{dx} = x\dfrac{1}{(1+\cos x)}\dfrac{d}{dx}(1+\cos x) + \log(1+\cos x)(1)$

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{x}{(1+\cos x)}(0-\sin x) + \log(1+\cos x)$

$\dfrac{1}{y}\dfrac{dy}{dx} = \log(1+\cos x) - \dfrac{x \text{som} x}{(1+\cos x)}$

$\dfrac{dy}{dx} = y\left[\log(1+\cos x) - \dfrac{x\sin x}{1+\cos x}\right]$

$\dfrac{dy}{dx} = (1+\cos x)^x\left[\log(1+\cos x) - \dfrac{x\sin x}{(1+\cos x)}\right] \qquad$ [Using equation (i)]

Differentiation Ex 11.5 Q4

Let $\quad y = x^{\cos^{-1}x} \qquad\qquad$ ---(i)

Taking log on both the sides,

$\log y = \log x^{\cos^{-1}x}$

$\log y = \cos^{-1}x \log x \qquad\qquad$ $\left[\text{Since, } \log a^b = b\log a\right]$

Differentiating it with respect to x using product rule,

$\dfrac{1}{y}\dfrac{dy}{dx} = \cos^{-1}x\dfrac{d}{dx}(\log x) + \log x\dfrac{d}{dx}(\cos^{-1}x)$

$\qquad = \cos^{-1}x\left(\dfrac{1}{x}\right) + \log x\left(\dfrac{-1}{\sqrt{1-x^2}}\right)$

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{\cos^{-1}x}{x} - \dfrac{\log x}{\sqrt{1-x^2}}$

$\dfrac{dy}{dx} = y\left[\dfrac{\cos^{-1}x}{x} - \dfrac{\log x}{\sqrt{1-x^2}}\right]$

$\dfrac{dy}{dx} = x^{\cos^{-1}x}\left[\dfrac{\cos^{-1}x}{x} - \dfrac{\log x}{\sqrt{1-x^2}}\right] \qquad$ [Using equation (i)]

Differentiation Ex 11.5 Q5

Let $\quad y = (\log x)^x \qquad\qquad$ ---(i)

Taking log on both the sides,

$\log y = \log(\log x)^x$

$\log y = x\log(\log x) \qquad\qquad$ $\left[\text{Since, } \log a^b = b\log a\right]$

Differentiating with respect to x, using product rule, chain rule,

$\dfrac{1}{y}\dfrac{dy}{dx} = x\dfrac{d}{dx}\log(\log x) + \log\log x\dfrac{d}{dx}(x)$

$\qquad = x\dfrac{1}{\log x}\dfrac{d}{dx}(\log x) + \log\log x\,(1)$

$\qquad = \dfrac{x}{\log x}\left(\dfrac{1}{x}\right) + \log\log x$

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{\log x} + \log\log x$

$\dfrac{dy}{dx} = y\left[\dfrac{1}{\log x} + \log\log x\right]$

$\dfrac{dy}{dx} = (\log x)^x\left[\dfrac{1}{\log x} + \log\log x\right] \qquad$ [Using equation (i)]

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