Q11 : $\int_0^2 \frac{dx}{x+4-x^2}$

Answer :

$$\int_0^2 \frac{dx}{x+4-x^2} = \int_0^2 \frac{dx}{-\left(x^2-x-4\right)}$$
$$= \int_0^2 \frac{dx}{-\left(x^2-x+\frac{1}{4}-\frac{1}{4}-4\right)}$$
$$= \int_0^2 \frac{dx}{-\left[\left(x-\frac{1}{2}\right)^2-\frac{17}{4}\right]}$$
$$= \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}$$

Let $x-\frac{1}{2}=t \Rightarrow dx = dt$

When $x=0,\ t=-\frac{1}{2}$ and when $x=2,\ t=\frac{3}{2}$

$$\therefore \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2} = \int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{dt}{\left(\frac{\sqrt{17}}{2}\right)^2-t^2}$$
$$= \left[\frac{1}{2\left(\frac{\sqrt{17}}{2}\right)} \log \frac{\frac{\sqrt{17}}{2}+t}{\frac{\sqrt{17}}{2}-t}\right]_{-\frac{1}{2}}^{\frac{3}{2}}$$
$$= \frac{1}{\sqrt{17}}\left[\log \frac{\frac{\sqrt{17}}{2}+\frac{3}{2}}{\frac{\sqrt{17}}{2}-\frac{3}{2}} - \log \frac{\frac{\sqrt{17}}{2}-\frac{1}{2}}{\frac{\sqrt{17}}{2}+\frac{1}{2}}\right]$$
$$= \frac{1}{\sqrt{17}}\left[\log \frac{\sqrt{17}+3}{\sqrt{17}-3} - \log \frac{\sqrt{17}-1}{\sqrt{17}+1}\right]$$
$$= \frac{1}{\sqrt{17}} \log \frac{\sqrt{17}+3}{\sqrt{17}-3} \times \frac{\sqrt{17}+1}{\sqrt{17}-1}$$
$$= \frac{1}{\sqrt{17}} \log \left[\frac{17+3+4\sqrt{17}}{17+3-4\sqrt{17}}\right]$$
$$= \frac{1}{\sqrt{17}} \log \left[\frac{20+4\sqrt{17}}{20-4\sqrt{17}}\right]$$
$$= \frac{1}{\sqrt{17}} \log \left(\frac{5+\sqrt{17}}{5-\sqrt{17}}\right)$$
$$= \frac{1}{\sqrt{17}} \log \left[\frac{\left(5+\sqrt{17}\right)\left(5+\sqrt{17}\right)}{25-17}\right]$$
$$= \frac{1}{\sqrt{17}} \log \left[\frac{25+17+10\sqrt{17}}{8}\right]$$
$$= \frac{1}{\sqrt{17}} \log \left(\frac{42+10\sqrt{17}}{8}\right)$$
$$= \frac{1}{\sqrt{17}} \log \left(\frac{21+5\sqrt{17}}{4}\right)$$

Q12 : $\int_0^2 \frac{dx}{x+4-x^2}$

Answer :

$$\int_0^2 \frac{dx}{x+4-x^2} = \int_0^2 \frac{dx}{-\left(x^2-x-4\right)}$$
$$= \int_0^2 \frac{dx}{-\left(x^2-x+\frac{1}{4}-\frac{1}{4}-4\right)}$$
$$= \int_0^2 \frac{dx}{-\left[\left(x-\frac{1}{2}\right)^2-\frac{17}{4}\right]}$$

$$= \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2}$$

Let $x - \frac{1}{2} = t \Rightarrow dx = dt$

When $x = 0,\ t = -\frac{1}{2}$ and when $x = 2,\ t = \frac{3}{2}$

$$\therefore \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2} = \int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{dt}{\left(\frac{\sqrt{17}}{2}\right)^2 - t^2}$$

$$= \left[\frac{1}{2\left(\frac{\sqrt{17}}{2}\right)} \log \frac{\frac{\sqrt{17}}{2} + t}{\frac{\sqrt{17}}{2} - t}\right]_{-\frac{1}{2}}^{\frac{3}{2}}$$

$$= \frac{1}{\sqrt{17}}\left[\log \frac{\frac{\sqrt{17}}{2} + \frac{3}{2}}{\frac{\sqrt{17}}{2} - \frac{3}{2}} - \log \frac{\frac{\sqrt{17}}{2} - \frac{1}{2}}{\frac{\sqrt{17}}{2} + \frac{1}{2}}\right]$$

$$= \frac{1}{\sqrt{17}}\left[\log \frac{\sqrt{17}+3}{\sqrt{17}-3} - \log \frac{\sqrt{17}-1}{\sqrt{17}+1}\right]$$

$$= \frac{1}{\sqrt{17}} \log \frac{\sqrt{17}+3}{\sqrt{17}-3} \times \frac{\sqrt{17}+1}{\sqrt{17}-1}$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{17+3+4\sqrt{17}}{17+3-4\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{20+4\sqrt{17}}{20-4\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{5+\sqrt{17}}{5-\sqrt{17}}\right)$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{\left(5+\sqrt{17}\right)\left(5+\sqrt{17}\right)}{25-17}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{25+17+10\sqrt{17}}{8}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{42+10\sqrt{17}}{8}\right)$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{21+5\sqrt{17}}{4}\right)$$

Q13 : $\int_{-1}^{1} \frac{dx}{x^2+2x+5}$

Answer :

$$\int_{-1}^{1} \frac{dx}{x^2+2x+5} = \int_{-1}^{1} \frac{dx}{\left(x^2+2x+1\right)+4} = \int_{-1}^{1} \frac{dx}{\left(x+1\right)^2+\left(2\right)^2}$$

Let $x + 1 = t \Rightarrow dx = dt$

When $x = -1$, $t = 0$ and when $x = 1$, $t = 2$

$$\therefore \int_{-1}^{1} \frac{dx}{\left(x+1\right)^2+\left(2\right)^2} = \int_0^2 \frac{dt}{t^2+2^2}$$

$$= \left[\frac{1}{2}\tan^{-1}\frac{t}{2}\right]_0^2$$

$$= \frac{1}{2}\tan^{-1}1 - \frac{1}{2}\tan^{-1}0$$

$$= \frac{1}{2}\left(\frac{\pi}{4}\right) = \frac{\pi}{8}$$

Q14 : $\int_{-1}^{1} \frac{dx}{x^2+2x+5}$

Answer :

$$\int_{-1}^{1} \frac{dx}{x^2+2x+5} = \int_{-1}^{1} \frac{dx}{\left(x^2+2x+1\right)+4} = \int_{-1}^{1} \frac{dx}{\left(x+1\right)^2+\left(2\right)^2}$$

Let $x + 1 = t \Rightarrow dx = dt$

When $x = -1$, $t = 0$ and when $x = 1$, $t = 2$

$$\therefore \int_{-1}^{1} \frac{dx}{\left(x+1\right)^2+\left(2\right)^2} = \int_0^2 \frac{dt}{t^2+2^2}$$

$$= \left[\frac{1}{2}\tan^{-1}\frac{t}{2}\right]_0^2$$

$$= \frac{1}{2}\tan^{-1}1 - \frac{1}{2}\tan^{-1}0$$

$$= \frac{1}{2}\left(\frac{\pi}{4}\right) = \frac{\pi}{8}$$

Answer needs Correction? **Click Here**

Q15 : $\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx$

Answer :

$\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx$

Let $2x = t \Rightarrow 2dx = dt$

When $x = 1$, $t = 2$ and when $x = 2$, $t = 4$

$$\therefore \int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx = \frac{1}{2}\int_2^4 \left(\frac{2}{t} - \frac{2}{t^2}\right) e^t dt$$
$$= \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2}\right) e^t dt$$

Let $\frac{1}{t} = f(t)$

Then, $f'(t) = -\frac{1}{t^2}$

$$\Rightarrow \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2}\right) e^t dt = \int_2^4 e^t \left[f(t) + f'(t)\right] dt$$
$$= \left[e^t f(t)\right]_2^4$$
$$= \left[e^t \cdot \frac{2}{t}\right]_2^4$$
$$= \left[\frac{e^t}{t}\right]_2^4$$
$$= \frac{e^4}{4} - \frac{e^2}{2}$$
$$= \frac{e^2\left(e^2 - 2\right)}{4}$$

Answer needs Correction? **Click Here**

Q16 : $\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx$

Answer :

$\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx$

Let $2x = t \Rightarrow 2dx = dt$

When $x = 1$, $t = 2$ and when $x = 2$, $t = 4$

$$\therefore \int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx = \frac{1}{2}\int_2^4 \left(\frac{2}{t} - \frac{2}{t^2}\right) e^t dt$$
$$= \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2}\right) e^t dt$$

Let $\frac{1}{t} = f(t)$

Then, $f'(t) = -\frac{1}{t^2}$

$$\Rightarrow \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2}\right) e^t dt = \int_2^4 e^t \left[f(t) + f'(t)\right] dt$$
$$= \left[e^t f(t)\right]_2^4$$
$$= \left[e^t \cdot \frac{2}{t}\right]_2^4$$
$$= \left[\frac{e^t}{t}\right]_2^4$$
$$= \frac{e^4}{4} - \frac{e^2}{2}$$
$$= \frac{e^2\left(e^2 - 2\right)}{4}$$

Answer needs Correction? **Click Here**

Q17 : The value of the integral $\int_{\frac{1}{3}}^{1} \frac{\left(x - x^3\right)^{\frac{1}{3}}}{x^4} dx$ is

A. 6

B. 0

C. 3

D. 4

Answer :

Let $I = \int_{\frac{1}{3}}^{1} \frac{\left(x - x^3\right)^{\frac{1}{3}}}{x^4} dx$

Also, let $x=\sin\theta \Rightarrow dx=\cos\theta\, d\theta$

When $x=\frac{1}{3}$, $\theta=\sin^{-1}\left(\frac{1}{3}\right)$ and when $x=1$, $\theta=\frac{\pi}{2}$

$$\Rightarrow I=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{\left(\sin\theta-\sin^3\theta\right)^{\frac{1}{3}}}{\sin^4\theta}\cos\theta\, d\theta$$

$$=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{(\sin\theta)^{\frac{1}{3}}\left(1-\sin^2\theta\right)^{\frac{1}{3}}}{\sin^4\theta}\cos\theta\, d\theta$$

$$=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{(\sin\theta)^{\frac{1}{3}}(\cos\theta)^{\frac{2}{3}}}{\sin^4\theta}\cos\theta\, d\theta$$

$$=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{(\sin\theta)^{\frac{1}{3}}(\cos\theta)^{\frac{2}{3}}}{\sin^2\theta\sin^2\theta}\cos\theta\, d\theta$$

$$=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{(\cos\theta)^{\frac{5}{3}}}{(\sin\theta)^{\frac{5}{3}}}\operatorname{cosec}^2\theta\, d\theta$$

$$=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}(\cot\theta)^{\frac{5}{3}}\operatorname{cosec}^2\theta\, d\theta$$

Let cotθ = t ⇒ - cosec2θ $d\theta$= dt

When $\theta=\sin^{-1}\left(\frac{1}{3}\right)$, $t=2\sqrt{2}$ and when $\theta=\frac{\pi}{2}$, $t=0$

$$\therefore I=-\int_{2\sqrt{2}}^{0}(t)^{\frac{5}{3}}dt$$

$$=-\left[\frac{3}{8}(t)^{\frac{8}{3}}\right]_{2\sqrt{2}}^{0}$$

$$=-\frac{3}{8}\left[(t)^{\frac{8}{3}}\right]_{2\sqrt{2}}^{0}$$

$$=-\frac{3}{8}\left[-\left(2\sqrt{2}\right)^{\frac{8}{3}}\right]$$

$$=\frac{3}{8}\left[\left(\sqrt{8}\right)^{\frac{8}{3}}\right]$$

$$=\frac{3}{8}\left[(8)^{\frac{4}{3}}\right]$$

$$=\frac{3}{8}[16]$$

$$=3\times 2$$

$$=6$$

Hence, the correct answer is A.

Answer needs Correction? **Click Here**

Q18 : The value of the integral $\int_{\frac{1}{3}}^{1}\frac{\left(x-x^3\right)^{\frac{1}{3}}}{x^4}dx$ is

A. 6

B. 0

C. 3

D. 4

Answer :

Let $I=\int_{\frac{1}{3}}^{1}\frac{\left(x-x^3\right)^{\frac{1}{3}}}{x^4}dx$

Also, let $x=\sin\theta \Rightarrow dx=\cos\theta\, d\theta$

When $x=\frac{1}{3}$, $\theta=\sin^{-1}\left(\frac{1}{3}\right)$ and when $x=1$, $\theta=\frac{\pi}{2}$

$$\Rightarrow I=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{\left(\sin\theta-\sin^3\theta\right)^{\frac{1}{3}}}{\sin^4\theta}\cos\theta\, d\theta$$

$$=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{(\sin\theta)^{\frac{1}{3}}\left(1-\sin^2\theta\right)^{\frac{1}{3}}}{\sin^4\theta}\cos\theta\, d\theta$$

$$=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{(\sin\theta)^{\frac{1}{3}}(\cos\theta)^{\frac{2}{3}}}{\sin^4\theta}\cos\theta\, d\theta$$

$$=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{(\sin\theta)^{\frac{1}{3}}(\cos\theta)^{\frac{2}{3}}}{\sin^2\theta\sin^2\theta}\cos\theta\, d\theta$$

$$=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\frac{(\cos\theta)^{\frac{5}{3}}}{(\sin\theta)^{\frac{5}{3}}}\operatorname{cosec}^2\theta\, d\theta$$

$$=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}(\cot\theta)^{\frac{5}{3}}\operatorname{cosec}^2\theta\, d\theta$$

Let cotθ = t ⇒ - cosec2θ $d\theta$= dt

When $\theta=\sin^{-1}\left(\frac{1}{3}\right)$, $t=2\sqrt{2}$ and when $\theta=\frac{\pi}{2}$, $t=0$

$$\therefore I=-\int_{2\sqrt{2}}^{0}(t)^{\frac{5}{3}}dt$$

$$=-\left[\frac{3}{8}(t)^{\frac{8}{3}}\right]^{0}$$

$$= -\frac{3}{8}\left[(t)^{\frac{8}{3}}\right]_{2\sqrt{2}}^{0}$$
$$= -\frac{3}{8}\left[-\left(2\sqrt{2}\right)^{\frac{8}{3}}\right]$$
$$= \frac{3}{8}\left[\left(\sqrt{8}\right)^{\frac{8}{3}}\right]$$
$$= \frac{3}{8}\left[(8)^{\frac{4}{3}}\right]$$
$$= \frac{3}{8}[16]$$
$$= 3 \times 2$$
$$= 6$$

Hence, the correct answer is A.

Answer needs Correction? Click Here

Q19 : If $f(x) = \int_0^x t \sin t \, dt$, then $f'(x)$ is

A. cos x + x sin x

B. x sin$x$

C. x cos x

D. sin x + x cos x

Answer :

$f(x) = \int_0^x t \sin t dt$

Integrating by parts, we obtain

$$f(x) = t\int_0^x \sin t \, dt - \int_0^x \left\{\left(\frac{d}{dt}t\right)\int \sin t \, dt\right\} dt$$
$$= \left[t(-\cos t)\right]_0^x - \int_0^x (-\cos t) \, dt$$
$$= \left[-t\cos t + \sin t\right]_0^x$$
$$= -x\cos x + \sin x$$

$$\Rightarrow f'(x) = -\left[\{x(-\sin x)\} + \cos x\right] + \cos x$$
$$= x\sin x - \cos x + \cos x$$
$$= x\sin x$$

Hence, the correct answer is B.

Answer needs Correction? Click Here

Q20 : If $f(x) = \int_0^x t \sin t \, dt$, then $f'(x)$ is

A. cos x + x sin x

B. x sin$x$

C. x cos x

D. sin x + x cos x

Answer :

$f(x) = \int_0^x t \sin t dt$

Integrating by parts, we obtain

$$f(x) = t\int_0^x \sin t \, dt - \int_0^x \left\{\left(\frac{d}{dt}t\right)\int \sin t \, dt\right\} dt$$
$$= \left[t(-\cos t)\right]_0^x - \int_0^x (-\cos t) \, dt$$
$$= \left[-t\cos t + \sin t\right]_0^x$$
$$= -x\cos x + \sin x$$

$$\Rightarrow f'(x) = -\left[\{x(-\sin x)\} + \cos x\right] + \cos x$$
$$= x\sin x - \cos x + \cos x$$
$$= x\sin x$$

Hence, the correct answer is B.

Answer needs Correction? Click Here

*********** END **********