Trigonometric Identities Ex 6.1 Q36

Answer :

We need to prove $\frac{1+\cos A}{\sin A}=\frac{\sin A}{1-\cos A}$

Now, multiplying the numerator and denominator of LHS by $1-\cos A$, we get

$$\frac{1+\cos A}{\sin A}=\frac{1+\cos A}{\sin A}\times\frac{1-\cos A}{1-\cos A}$$

Further using the identity, $a^2-b^2=(a+b)(a-b)$, we get

$$\begin{aligned}\frac{1+\cos A}{\sin A}\times\frac{1-\cos A}{1-\cos A}&=\frac{1-\cos^2 A}{\sin A(1-\cos A)}\\&=\frac{\sin^2 A}{\sin A(1-\cos A)} \qquad (\text{using } \sin^2\theta+\cos^2\theta=1)\\&=\frac{\sin A}{1-\cos A}\end{aligned}$$

Hence proved.

Trigonometric Identities Ex 6.1 Q37

Answer :

We need to prove $\sqrt{\frac{1+\sin A}{1-\sin A}}=\sec A+\tan A$

Here, rationalising the L.H.S, we get

$$\begin{aligned}\sqrt{\frac{1+\sin A}{1-\sin A}}&=\sqrt{\frac{1+\sin A}{1-\sin A}}\times\sqrt{\frac{1+\sin A}{1+\sin A}}\\&=\sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}\end{aligned}$$

Further using the property, $\sin^2\theta+\cos^2\theta=1$, we get

So,

$$\begin{aligned}\sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}&=\sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}\\&=\frac{1+\sin A}{\cos A}\\&=\frac{1}{\cos A}+\frac{\sin A}{\cos A}\\&=\sec A+\tan A\end{aligned}$$

Hence proved.

Trigonometric Identities Ex 6.1 Q38

Answer :

We need to prove $\sqrt{\frac{1-\cos A}{1+\cos A}} = \text{cosec}A - \cot A$

Here, rationaliaing the L.H.S, we get

$$\sqrt{\frac{1-\cos A}{1+\cos A}} = \sqrt{\frac{1-\cos A}{1+\cos A}} \times \sqrt{\frac{1-\cos A}{1-\cos A}}$$
$$= \sqrt{\frac{(1-\cos A)^2}{1-\cos^2 A}}$$

Further using the property, $\sin^2\theta + \cos^2\theta = 1$, we get

So,

$$\sqrt{\frac{(1-\cos A)^2}{1-\cos^2 A}} = \sqrt{\frac{(1-\cos A)^2}{\sin^2 A}}$$
$$= \frac{1-\cos A}{\sin A}$$
$$= \frac{1}{\sin A} - \frac{\cos A}{\sin A}$$
$$= \text{cosec}A - \cot A$$

Hence proved.

Trigonometric Identities Ex 6.1 Q39

Answer :

We need to prove $(\sec A - \tan A)^2 = \frac{1-\sin A}{1+\sin A}$

Here, we will first solve the L.H.S.

Now, using $\sec\theta = \frac{1}{\cos\theta}$ and $\tan\theta = \frac{\sin\theta}{\cos\theta}$, we get

$$(\sec A - \tan A)^2 = \left(\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right)^2$$
$$= \left(\frac{1-\sin A}{\cos A}\right)^2$$
$$= \frac{(1-\sin A)^2}{(\cos A)^2}$$

Further using the property $\sin^2\theta + \cos^2\theta = 1$, we get

So,

$$\frac{(1-\sin A)^2}{(\cos A)^2} = \frac{(1-\sin A)^2}{1-\sin^2 A}$$
$$= \frac{(1-\sin A)^2}{(1-\sin A)(1+\sin A)} \qquad \left(\text{Using } a^2 - b^2 = (a+b)(a-b)\right)$$
$$= \frac{1-\sin A}{1+\sin A}$$

Hence proved.

********** END **********