EXERCISE 11.3

Question 1:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$

Ans:

The given equation is $\frac{x^2}{36}+\frac{y^2}{16}=1$.

Here, the denominator of $\frac{x^2}{36}$ is greater than the denominator of $\frac{y^2}{16}$.

Therefore, the major axis is along the *x*-axis, while the minor axis is along the *y*-axis.

On comparing the given equation with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we obtain *a* = 6 and *b* = 4.

$$\therefore c=\sqrt{a^2-b^2}=\sqrt{36-16}=\sqrt{20}=2\sqrt{5}$$

Therefore,

The coordinates of the foci are $\left(2\sqrt{5},0\right)$ and $\left(-2\sqrt{5},0\right)$.

The coordinates of the vertices are (6, 0) and (–6, 0).

Length of major axis = 2*a* = 12

Length of minor axis = 2*b* = 8

$$\text{Eccentricity, } e=\frac{c}{a}=\frac{2\sqrt{5}}{6}=\frac{\sqrt{5}}{3}$$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times16}{6}=\frac{16}{3}$

Question 2:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{4}+\frac{y^2}{25}=1$

Ans:

The given equation is $\frac{x^2}{4}+\frac{y^2}{25}=1$ or $\frac{x^2}{2^2}+\frac{y^2}{5^2}=1$.

Here, the denominator of $\frac{y^2}{25}$ is greater than the denominator of $\frac{x^2}{4}$.

Therefore, the major axis is along the *y*-axis, while the minor axis is along the *x*-axis.

On comparing the given equation with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, we obtain b = 2 and a = 5.

$\therefore c=\sqrt{a^2-b^2}=\sqrt{25-4}=\sqrt{21}$

Therefore,

The coordinates of the foci are $\left(0,\sqrt{21}\right)$ and $\left(0,-\sqrt{21}\right)$.

The coordinates of the vertices are (0, 5) and (0, –5)

Length of major axis = 2a = 10

Length of minor axis = 2b = 4

Eccentricity, $e=\frac{c}{a}=\frac{\sqrt{21}}{5}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 4}{5}=\frac{8}{5}$

Question 3:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$

Ans:

The given equation is $\frac{x^2}{16}+\frac{y^2}{9}=1$ or $\frac{x^2}{4^2}+\frac{y^2}{3^2}=1$.

Here, the denominator of $\frac{x^2}{16}$ is greater than the denominator of $\frac{y^2}{9}$.

Therefore, the major axis is along the *x*-axis, while the minor axis is along the *y*-axis.

On comparing the given equation with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we obtain a = 4 and b = 3.

$\therefore c=\sqrt{a^2-b^2}=\sqrt{16-9}=\sqrt{7}$

Therefore,

The coordinates of the foci are $\left(\pm\sqrt{7},0\right)$.

The coordinates of the vertices are $(\pm 4,0)$.

Length of major axis = 2a = 8

Length of minor axis = 2b = 6

Eccentricity, $e=\frac{c}{a}=\frac{\sqrt{7}}{4}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 9}{4}=\frac{9}{2}$

Question 4:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{25}+\frac{y^2}{100}=1$

Ans:

The given equation is $\frac{x^2}{25}+\frac{y^2}{100}=1$ or $\frac{x^2}{5^2}+\frac{y^2}{10^2}=1$.

Here, the denominator of $\frac{y^2}{100}$ is greater than the denominator of $\frac{x^2}{25}$.

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing the given equation with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, we obtain $b = 5$ and $a = 10$.

$\therefore\ c=\sqrt{a^2-b^2}=\sqrt{100-25}=\sqrt{75}=5\sqrt{3}$

Therefore,

The coordinates of the foci are $\left(0,\pm 5\sqrt{3}\right)$.

The coordinates of the vertices are (0, ±10).

Length of major axis = $2a$ = 20

Length of minor axis = $2b$ = 10

Eccentricity, $e=\frac{c}{a}=\frac{5\sqrt{3}}{10}=\frac{\sqrt{3}}{2}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 25}{10}=5$

Question 5:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{49}+\frac{y^2}{36}=1$

Ans:

The given equation is $\frac{x^2}{49}+\frac{y^2}{36}=1$ or $\frac{x^2}{7^2}+\frac{y^2}{6^2}=1$.

Here, the denominator of $\frac{x^2}{49}$ is greater than the denominator of $\frac{y^2}{36}$.

Therefore, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we obtain $a = 7$ and $b = 6$.

$\therefore c=\sqrt{a^2-b^2}=\sqrt{49-36}=\sqrt{13}$

Therefore,

The coordinates of the foci are $\left(\pm\sqrt{13},0\right)$.

The coordinates of the vertices are (± 7, 0).

Length of major axis = $2a$ = 14

Length of minor axis = $2b$ = 12

Eccentricity, $e=\frac{c}{a}=\frac{\sqrt{13}}{7}$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 36}{7}=\frac{72}{7}$

Question 6:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{100}+\frac{y^2}{400}=1$

Ans:

The given equation is $\frac{x^2}{100}+\frac{y^2}{400}=1$ or $\frac{x^2}{10^2}+\frac{y^2}{20^2}=1$.

Here, the denominator of $\frac{y^2}{400}$ is greater than the denominator of $\frac{x^2}{100}$.

Therefore, the major axis is along the *y*-axis, while the minor axis is along the *x*-axis.

On comparing the given equation with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, we obtain b = 10 and a = 20.

$$\therefore\ c=\sqrt{a^2-b^2}=\sqrt{400-100}=\sqrt{300}=10\sqrt{3}$$

Therefore,

The coordinates of the foci are $\left(0,\pm10\sqrt{3}\right)$.

The coordinates of the vertices are (0, ±20)

Length of major axis = 2a = 40

Length of minor axis = 2b = 20

$$\text{Eccentricity, } e=\frac{c}{a}=\frac{10\sqrt{3}}{20}=\frac{\sqrt{3}}{2}$$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times100}{20}=10$

Question 7:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $36x^2 + 4y^2 = 144$

Ans:

The given equation is $36x^2 + 4y^2 = 144$.

It can be written as

$$36x^2+4y^2=144$$

$$\text{Or, } \frac{x^2}{4}+\frac{y^2}{36}=1$$

$$\text{Or, } \frac{x^2}{2^2}+\frac{y^2}{6^2}=1 \qquad ...(1)$$

Here, the denominator of $\frac{y^2}{6^2}$ is greater than the denominator of $\frac{x^2}{2^2}$.

Therefore, the major axis is along the *y*-axis, while the minor axis is along the *x*-axis.

On comparing equation (1) with $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, we obtain b = 2 and a = 6.

$$\therefore c=\sqrt{a^2-b^2}=\sqrt{36-4}=\sqrt{32}=4\sqrt{2}$$

Therefore,

The coordinates of the foci are $\left(0,\ \pm4\sqrt{2}\right)$.

The coordinates of the vertices are (0, ±6).

Length of major axis = 2a = 12

Length of minor axis = 2b = 4

$$\text{Eccentricity, } e=\frac{c}{a}=\frac{4\sqrt{2}}{6}=\frac{2\sqrt{2}}{3}$$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times4}{6}=\frac{4}{3}$

Question 8:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $16x^2 + y^2 = 16$

Ans:

The given equation is $16x^2 + y^2 = 16$.

It can be written as

$$16x^2 + y^2 = 16$$

$$\text{Or, } \frac{x^2}{1} + \frac{y^2}{16} = 1$$

$$\text{Or, } \frac{x^2}{1^2} + \frac{y^2}{4^2} = 1 \qquad \ldots(1)$$

Here, the denominator of $\frac{y^2}{4^2}$ is greater than the denominator of $\frac{x^2}{1^2}$.

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing equation (1) with $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, we obtain $b = 1$ and $a = 4$.

$$\therefore c = \sqrt{a^2 - b^2} = \sqrt{16-1} = \sqrt{15}$$

Therefore,

The coordinates of the foci are $\left(0, \pm\sqrt{15}\right)$.

The coordinates of the vertices are $(0, \pm 4)$.

Length of major axis = $2a = 8$

Length of minor axis = $2b = 2$

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{\sqrt{15}}{4}$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2\times 1}{4} = \frac{1}{2}$$

Question 9:

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $4x^2 + 9y^2 = 36$

Ans:

The given equation is $4x^2 + 9y^2 = 36$.

It can be written as

$$4x^2 + 9y^2 = 36$$

$$\text{Or, } \frac{x^2}{9} + \frac{y^2}{4} = 1$$

$$\text{Or, } \frac{x^2}{3^2} + \frac{y^2}{2^2} = 1 \qquad \ldots(1)$$

Here, the denominator of $\frac{x^2}{3^2}$ is greater than the denominator of $\frac{y^2}{2^2}$.

Therefore, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we obtain $a = 3$ and $b = 2$.

$$\therefore c = \sqrt{a^2 - b^2} = \sqrt{9-4} = \sqrt{5}$$

Therefore,

The coordinates of the foci are $\left(\pm\sqrt{5}, 0\right)$.

The coordinates of the vertices are $(\pm 3, 0)$.

Length of major axis = $2a = 6$

Length of minor axis = $2b = 4$

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{\sqrt{5}}{3}$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2\times 4}{3} = \frac{8}{3}$$

Question 10:

Find the equation for the ellipse that satisfies the given conditions: Vertices $(\pm 5, 0)$, foci $(\pm 4, 0)$

Ans:

Vertices (±5, 0), foci (±4, 0)

Here, the vertices are on the *x*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is the semi-major axis.

Accordingly, a = 5 and c = 4.

It is known that $a^2=b^2+c^2$.

$$\therefore 5^2=b^2+4^2$$
$$\Rightarrow 25=b^2+16$$
$$\Rightarrow b^2=25-16$$
$$\Rightarrow b=\sqrt{9}=3$$

Thus, the equation of the ellipse is $\frac{x^2}{5^2}+\frac{y^2}{3^2}=1$ or $\frac{x^2}{25}+\frac{y^2}{9}=1$.

Question 11:

Find the equation for the ellipse that satisfies the given conditions: Vertices (0, ±13), foci (0, ±5)

Ans:

Vertices (0, ±13), foci (0, ±5)

Here, the vertices are on the *y*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, where a is the semi-major axis.

Accordingly, a = 13 and c = 5.

It is known that $a^2=b^2+c^2$.

$$\therefore 13^2=b^2+5^2$$
$$\Rightarrow 169=b^2+25$$
$$\Rightarrow b^2=169-25$$
$$\Rightarrow b=\sqrt{144}=12$$

Thus, the equation of the ellipse is $\frac{x^2}{12^2}+\frac{y^2}{13^2}=1$ or $\frac{x^2}{144}+\frac{y^2}{169}=1$.

Question 12:

Find the equation for the ellipse that satisfies the given conditions: Vertices (±6, 0), foci (±4, 0)

Ans:

Vertices (±6, 0), foci (±4, 0)

Here, the vertices are on the *x*-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is the semi-major axis.

Accordingly, a = 6, c = 4.

It is known that $a^2=b^2+c^2$.

$$\therefore 6^2=b^2+4^2$$
$$\Rightarrow 36=b^2+16$$
$$\Rightarrow b^2=36-16$$
$$\Rightarrow b=\sqrt{20}$$

Thus, the equation of the ellipse is $\frac{x^2}{6^2}+\frac{y^2}{\left(\sqrt{20}\right)^2}=1$ or $\frac{x^2}{36}+\frac{y^2}{20}=1$.

Question 13:

Find the equation for the ellipse that satisfies the given conditions: Ends of major axis (±3, 0), ends of minor axis (0, ±2)

Ans:

Ends of major axis (±3, 0), ends of minor axis (0, ±2)

Here, the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is the semi-major axis.

Accordingly, a = 3 and b = 2.

Thus, the equation of the ellipse is $\frac{x^2}{3^2}+\frac{y^2}{2^2}=1$ i.e., $\frac{x^2}{9}+\frac{y^2}{4}=1$.

Question 14:

Find the equation for the ellipse that satisfies the given conditions: Ends of major axis $\left(0,\ \pm\sqrt{5}\right)$, ends of minor axis (±1, 0)

Ans:

Ends of major axis $\left(0,\ \pm\sqrt{5}\right)$, ends of minor axis (±1, 0)

Here, the major axis is along the y-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, where a is the semi-major axis.

Accordingly, a = $\sqrt{5}$ and b = 1.

Thus, the equation of the ellipse is $\frac{x^2}{1^2}+\frac{y^2}{\left(\sqrt{5}\right)^2}=1$ or $\frac{x^2}{1}+\frac{y^2}{5}=1$.

Question 15:

Find the equation for the ellipse that satisfies the given conditions: Length of major axis 26, foci (±5, 0)

Ans:

Length of major axis = 26; foci = (±5, 0).

Since the foci are on the x-axis, the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is the semi-major axis.

Accordingly, 2a = 26 □ a = 13 and c = 5.

It is known that $a^2=b^2+c^2$.

$$\therefore 13^2=b^2+5^2$$
$$\Rightarrow 169=b^2+25$$
$$\Rightarrow b^2=169-25$$
$$\Rightarrow b=\sqrt{144}=12$$

Thus, the equation of the ellipse is $\frac{x^2}{13^2}+\frac{y^2}{12^2}=1$ or $\frac{x^2}{169}+\frac{y^2}{144}=1$.

Question 16:

Find the equation for the ellipse that satisfies the given conditions: Length of minor axis 16, foci (0, ±6)

Ans:

Length of minor axis = 16; foci = (0, ±6).

Since the foci are on the y-axis, the major axis is along the y-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$, where a is the semi-major axis.

Accordingly, $2b = 16$ □ $b = 8$ and $c = 6$.

It is known that $a^2 = b^2 + c^2$.

$$\therefore a^2 = 8^2 + 6^2 = 64 + 36 = 100$$
$$\Rightarrow a = \sqrt{100} = 10$$

Thus, the equation of the ellipse is $\frac{x^2}{8^2}+\frac{y^2}{10^2}=1$ or $\frac{x^2}{64}+\frac{y^2}{100}=1$.

Question 17:

Find the equation for the ellipse that satisfies the given conditions: Foci (±3, 0), $a = 4$

Ans:

Foci (±3, 0), $a = 4$

Since the foci are on the x-axis, the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is the semi-major axis.

Accordingly, $c = 3$ and $a = 4$.

It is known that $a^2 = b^2 + c^2$.

$$\therefore 4^2 = b^2 + 3^2$$
$$\Rightarrow 16 = b^2 + 9$$
$$\Rightarrow b^2 = 16 - 9 = 7$$

Thus, the equation of the ellipse is $\frac{x^2}{16}+\frac{y^2}{7}=1$.

Question 18:

Find the equation for the ellipse that satisfies the given conditions: $b = 3$, $c = 4$, centre at the origin; foci on the x axis.

Ans:

It is given that $b = 3$, $c = 4$, centre at the origin; foci on the x axis.

Since the foci are on the x-axis, the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a is the semi-major axis.

Accordingly, $b = 3$, $c = 4$.

It is known that $a^2 = b^2 + c^2$.

$$\therefore a^2 = 3^2 + 4^2 = 9 + 16 = 25$$
$$\Rightarrow a = 5$$

Thus, the equation of the ellipse is $\frac{x^2}{5^2}+\frac{y^2}{3^2}=1$ or $\frac{x^2}{25}+\frac{y^2}{9}=1$.

Question 19:

Find the equation for the ellipse that satisfies the given conditions: Centre at (0, 0), major axis on the y-axis and passes through the points (3, 2) and (1, 6).

Ans:

Since the centre is at (0, 0) and the major axis is on the y-axis, the equation of the ellipse will be of the form

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1 \qquad ...(1)$$

Where, a is the semi-major axis

The ellipse passes through points (3, 2) and (1, 6). Hence,

$$\frac{9}{b^2}+\frac{4}{a^2}=1 \qquad ...(2)$$

$$\frac{1}{b^2}+\frac{36}{a^2}=1 \qquad ...(3)$$

On solving equations (2) and (3), we obtain $b^2 = 10$ and $a^2 = 40$.

Thus, the equation of the ellipse is $\frac{x^2}{10}+\frac{y^2}{40}=1$ or $4x^2+y^2=40$.

Question 20:

Find the equation for the ellipse that satisfies the given conditions: Major axis on the x-axis and passes through the points (4, 3) and (6, 2).

Ans:

Since the major axis is on the x-axis, the equation of the ellipse will be of the form

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad ...(1)$$

Where, a is the semi-major axis

The ellipse passes through points (4, 3) and (6, 2). Hence,

$$\frac{16}{a^2}+\frac{9}{b^2}=1 \qquad ...(2)$$

$$\frac{36}{a^2}+\frac{4}{b^2}=1 \qquad ...(3)$$

On solving equations (2) and (3), we obtain $a^2 = 52$ and $b^2 = 13$.

Thus, the equation of the ellipse is $\frac{x^2}{52}+\frac{y^2}{13}=1$ or $x^2+4y^2=52$.

*********** END **********