Trigonometric Functions Ex 5.1 Q21

Given, $\cos ec\theta - \sin\theta = a^3, \sec\theta - \cos\theta = b^3$

To show: $a^2b^2\left(a^2+b^2\right) = 1$

Since, $\cos ec\theta - \sin\theta = a^3$

$$\Rightarrow \quad \frac{1}{\sin\theta} - \sin\theta = a^3 \qquad \left(\because \cos ec\theta = \frac{1}{\sin\theta}\right)$$

$$\Rightarrow \quad \frac{1-\sin^2\theta}{\sin\theta} = a^3$$

$$\Rightarrow \quad \frac{\cos^2\theta}{\sin\theta} = a^3 \qquad \left(\because 1-\sin^2\theta = \cos^2\theta\right)$$

$$\Rightarrow \quad a = \frac{\cos^{2/3}\theta}{\sin^{1/3}\theta}$$

Since, $\frac{1}{\cos\theta} - \cos\theta = b^3 \qquad \left(\because \sec\theta = \frac{1}{\cos\theta}\right)$

$$\Rightarrow \quad \frac{1-\cos^2\theta}{\cos\theta} = b^3$$

$$\Rightarrow \quad \frac{\sin^2\theta}{\cos\theta} = b^3 \qquad \left(\because 1-\cos^2\theta = \sin^2\theta\right)$$

$$\Rightarrow \quad b = \frac{\sin^{2/3}\theta}{\cos^{1/3}\theta}$$

Now, $a^2b^2\left(a^2+b^2\right) = \frac{\cos^{4/3}\theta}{\sin^{2/3}\theta} \times \frac{\sin^{4/3}\theta}{\cos^{2/3}\theta}\left(\frac{\cos^{4/3}\theta}{\sin^{2/3}\theta} + \frac{\sin^{4/3}\theta}{\cos^{2/3}\theta}\right)$

$$= \cos^{2/3}\theta \times \sin^{2/3}\theta \, \frac{\left(\cos^{6/3}\theta + \sin^{6/3}\theta\right)}{\sin^{2/3}\theta . \cos^{2/3}\theta}$$

$$= \cos^2\theta + \sin^2\theta$$

$$= 1$$

Proved

Trigonometric Functions Ex 5.1 Q22

Let,

$$\cot\theta(1+\sin\theta) = 4m \qquad \text{---(i)}$$

and, $$\cot\theta(1-\sin\theta) = 4n \qquad \text{---(ii)}$$

To show: $(m^2-n^2)^2 = mn$

From (i) and(ii), we get

$$m = \frac{\cot\theta(1+\sin\theta)}{4} \quad \& \quad n = \frac{\cot\theta(1-\sin\theta)}{4}$$

$$\begin{aligned}
\text{LHS} &= (m^2-n^2)^2 \\
&= ((m+n)(m-n))^2 \\
&= (m+n)^2(m-n)^2 \\
&= \left(\frac{\cot\theta(1+\sin\theta)+\cot\theta(1-\sin\theta)}{4}\right)^2\left(\frac{\cot\theta(1+\sin\theta)-\cot\theta(1-\sin\theta)}{4}\right)^2 \\
&= \left(\frac{\cot\theta(1+\sin\theta+1-\sin\theta)}{4}\right)^2 \times \left(\frac{\cot\theta(1+\sin\theta-1+\sin\theta)}{4}\right)^2 \\
&= \frac{\cot^2\theta}{16}\times 4\times\frac{\cot^2\theta}{16}\times 4\sin^2\theta \\
&= \frac{\cot^2\theta}{16}\times\frac{\cos^2\theta}{\sin^2\theta}\sin^2\theta && \left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right] \\
&= \frac{\cot\theta}{4}\times\frac{\cot\theta}{4}\times(1-\sin^2\theta) && \left[\because \cos^2\theta = 1-\sin^2\theta\right] \\
&= \frac{\cot\theta(1+\sin\theta)}{4}\times\frac{\cot\theta(1-\sin\theta)}{4} \\
&= mn
\end{aligned}$$

Trigonometric Functions Ex 5.1 Q23

To show: $\sin^6\theta+\cos^6\theta = \dfrac{4-3(m^2-1)^2}{4}$, where $m^2 \le 2$

$Since,\ \sin\theta+\cos\theta = m \qquad \ldots(i)$

$$\begin{aligned}
&\Rightarrow && (\sin\theta+\cos\theta)^2 = m^2 \\
&\Rightarrow && \sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta = m^2 \\
&\Rightarrow && 1+2\sin\theta\cos\theta = m^2 \quad (\because \sin^2\theta+\cos^2\theta = 1) \\
&\Rightarrow && 2\sin\theta\cos\theta = m^2-1 \\
&\Rightarrow && \sin\theta\cos\theta = \frac{m^2-1}{2} \qquad \ldots(ii)
\end{aligned}$$

$$\begin{aligned}
\therefore \text{LHS} &= \sin^6\theta+\cos^2\theta \\
&= (\sin^2\theta)^3+(\cos^2\theta)^3 \\
&= (\sin^2\theta+\cos^2\theta)\left((\sin^2\theta)^2+(\cos^2\theta)^2-\sin^2\theta\cos^2\theta\right) \\
&= 1.\left((\sin^2\theta)^2+(\cos^2\theta)^2+2\sin^2\theta\cos^2\theta-2\sin^2\theta\cos^2\theta-\sin^2\theta\cos^2\theta\right) \\
&\qquad (\text{adding and subtracting } 2\sin^2\theta\cos^2\theta) \\
&= (\sin^2\theta+\cos^2\theta)^2-3\sin^2\theta\cos^2\theta \\
&= 1-3\sin^2\theta\cos^2\theta \\
&= 1-3(\sin\theta\cos\theta)^2 \\
&= 1-3\frac{(m^2-1)^2}{4}\ (\text{from (ii)}) \\
&= \frac{4-3(m^2-1)^2}{4}, \text{ where } m^2 \le 2 \\
&= \text{RHS}
\end{aligned}$$

Proved

Trigonometric Functions Ex 5.1 Q24

$LHS = ab + a - b + 1$

$= (\sec\theta - \tan\theta)(\operatorname{cosec} + \cot) + \sec\theta - \tan\theta - \operatorname{cosec}\theta - \cot\theta + 1$

$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)\left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right) + \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} - \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} + 1$

$= \frac{1}{\sin\theta\cos\theta} + \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} \times \frac{1}{\sin\theta} - \tan\theta \times \cot\theta + \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} - \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} + 1$

$= \frac{1}{\sin\theta\cos\theta} + \frac{1}{\sin\theta} - \frac{1}{\cos\theta} - 1 + \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} - \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} + 1$

$= \frac{1}{\sin\theta\cos\theta} - \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}$

$= \frac{1}{\sin\theta\cos\theta} - \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}$

$= \frac{1 - \sin^2\theta - \cos^2\theta}{\sin\theta.\cos\theta}$

$= \frac{1 - (\cos^2\theta + \sin^2\theta)}{\sin\theta.\cos\theta}$

$= \frac{1-1}{\sin\theta.\cos\theta} = 0 = RHS.$ *Hence Proved*

Trigonometric Functions Ex 5.1 Q25

$$\text{LHS} = \left|\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} + \sqrt{\frac{1+\sin\theta}{1-\sin\theta}}\right|$$

$$= \left|\frac{\left(\sqrt{1-\sin\theta}\right)^2 + \left(\sqrt{1+\sin\theta}\right)^2}{\sqrt{(1+\sin\theta)(1-\sin\theta)}}\right|$$

$$= \left|\frac{1-\sin\theta+1+\sin\theta}{\sqrt{1-\sin^2\theta}}\right|$$

$$= \left|\frac{2}{\cos\theta}\right| \qquad \left(\because\ 1-\sin^2\theta = \cos^2\theta \Rightarrow \sqrt{1-\sin^2\theta} = \cos\theta\right)$$

$$= \frac{-2}{\cos\theta} \qquad \left(\because \frac{\pi}{2} < \theta < \pi \Rightarrow \cos\theta < 0\right)$$

$= \text{RHS}$

*********** END **********