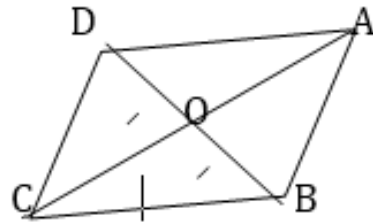




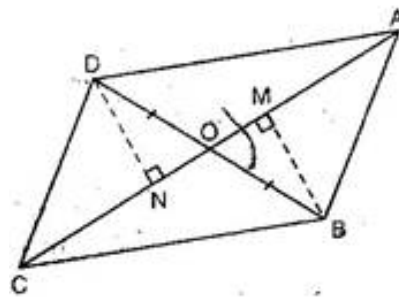
Exercise 9.3

**Q6.** In figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that  $OB = OD$ . If  $AB = CD$ , then show that:



- (i)  $\text{ar}(\text{DOC}) = \text{ar}(\text{AOB})$
- (ii)  $\text{ar}(\text{DCB}) = \text{ar}(\text{ACB})$
- (iii)  $DA \parallel CB$  or ABCD is a parallelogram.

**Ans. (i)** Draw  $BM \perp AC$  and  $DN \perp AC$ .



In  $\triangle DON$  and  $\triangle BOM$ ,

$OD = OB$  [Given]

$\angle DNO = \angle BMO = 90^\circ$  [By construction]

$\angle DON = \angle BOM$  [Vertically opposite]

$\therefore \triangle DON \cong \triangle BOM$  [By RHS congruency]

$\Rightarrow DN = BM$  [By CPCT]

Also  $\text{ar}(\triangle DON) = \text{ar}(\triangle BOM)$  .....(i)

Again, In  $\triangle DCN$  and  $\triangle ABM$ ,

$CD = AB$  [Given]

$$\angle DNC = \angle BMA = 90^\circ \text{ [By construction]}$$

$$DN = BM \text{ [Prove above]}$$

$$\therefore \triangle DCN \cong \triangle BAM \text{ [By RHS congruency]}$$

$$\therefore \text{ar}(\triangle DCN) = \text{ar}(\triangle BAM) \dots\dots\dots(ii)$$

Adding eq. (i) and (ii),

$$\text{ar}(\triangle DON) + \text{ar}(\triangle DCN) = \text{ar}(\triangle BOM) + \text{ar}(\triangle BAM)$$

$$\Rightarrow \text{ar}(\triangle DOC) = \text{ar}(\triangle AOB)$$

$$(ii) \text{ Since } \text{ar}(\triangle DOC) = \text{ar}(\triangle AOB)$$

Adding ar  $\triangle BOC$  both sides,

$$\text{ar}(\triangle DOC) + \text{ar} \triangle BOC = \text{ar}(\triangle AOB) + \text{ar} \triangle BOC$$

$$\Rightarrow \text{ar}(\triangle DCB) = \text{ar}(\triangle ACB)$$

$$(iii) \text{ Since } \text{ar}(\triangle DCB) = \text{ar}(\triangle ACB)$$

Therefore, these two triangles in addition to be on the same base CB lie between two same parallels CB and DA.

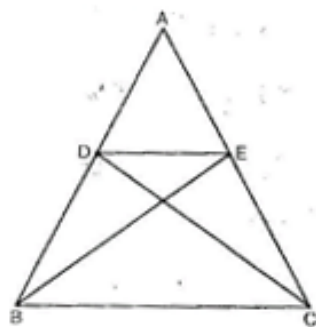
$$\therefore DA \parallel CB$$

$$\text{Now } AB = CD \text{ and } DA \parallel CB$$

Therefore, ABCD is a parallelogram.

**Q7.** D and E are points on sides AB and AC respectively of  $\triangle ABC$  such that  $\text{ar}(\triangle DBC) = \text{ar}(\triangle EBC)$ . Prove that  $DE \parallel BC$ .

$$\text{Ans. Given: } \text{ar}(\triangle DBC) = \text{ar}(\triangle EBC)$$

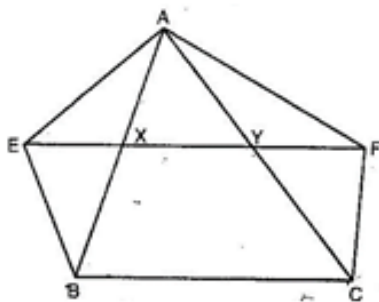


Since two triangles of equal area have common base BC.

Therefore,  $DE \parallel BC$  [ $\because$  Two triangles having same base (or equal bases) and equal areas lie between the same parallel]

**Q8.** XY is a line parallel to side BC of triangle ABC. If  $BE \parallel AC$  and  $CF \parallel AB$  meet XY at E and F respectively, show that  $\text{ar}(\triangle ABE) = \text{ar}(\triangle ACF)$ .

**Ans.**  $\triangle ABE$  and parallelogram BCYE lie on the same base BE and between the same parallels BE and AC.



$$\therefore \text{ar}(\triangle ABE) = \frac{1}{2} \text{ar}(\parallel \text{gm BCYE}) \dots\dots\dots(i)$$

Also  $\triangle ACF$  and  $\parallel \text{gm BCFX}$  lie on the same base CF and between same parallel BX and CF.

$$\therefore \text{ar}(\triangle ACF) = \frac{1}{2} \text{ar}(\parallel \text{gm BCFX}) \dots\dots\dots(\text{ii})$$

But  $\parallel \text{gm BCYE}$  and  $\parallel \text{gm BCFX}$  lie on the same base BC and between the same parallels BC and EF.

$$\therefore \text{ar}(\parallel \text{gm BCYE}) = \text{ar}(\parallel \text{gm BCFX}) \dots\dots\dots(\text{iii})$$

From eq. (i), (ii) and (iii), we get,

$$\text{ar}(\triangle ABE) = \text{ar}(\triangle ACF)$$

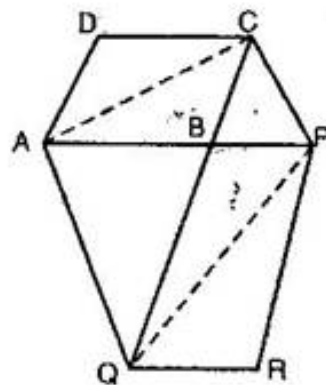
**Q9.** The side AB of parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed. Show that  $\text{ar}(\text{ABCD}) = \text{ar}(\text{PBQR})$ .

**Ans. Given:** ABCD is a parallelogram,  $CP \parallel AQ$  and PBQR is a parallelogram.

**To prove:**  $\text{ar}(\text{ABCD}) = \text{ar}(\text{PBQR})$

**Construction:** Join AC and QP.

**Proof:** Since  $AQ \parallel CP$



$$\therefore \text{ar}(\triangle AQC) = \text{ar}(\triangle AQP)$$

[Triangles on the same base and between the same parallels are equal in area]

Subtracting  $\text{ar}(\triangle ABQ)$  from both sides, we get

$$\text{ar}(\triangle AQC) - \text{ar}(\triangle ABQ) = \text{ar}(\triangle AQP) - \text{ar}(\triangle ABQ)$$

$$\Rightarrow \text{ar}(\triangle ABC) = \text{ar}(\triangle QBP) \dots\dots\dots(i)$$

$$\text{Now } \text{ar}(\triangle ABC) = \frac{1}{2} \text{ar}(\parallel \text{gm } ABCD)$$

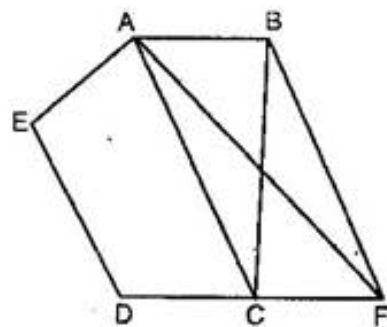
[Diagonal divides a parallelogram in two parts of equal area]

$$\text{And } \text{ar}(\triangle PQB) = \frac{1}{2} \text{ar}(\parallel \text{gm } PBQR)$$

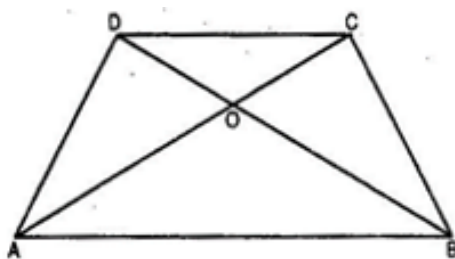
From eq. (i), (ii) and (iii), we get

$$\text{ar}(\parallel \text{gm } ABCD) = \text{ar}(\parallel \text{gm } PBQR)$$

**Q10.** Diagonals AC and BD of a trapezium ABCD with  $AB \parallel DC$  intersect each other at O. Prove that  $\text{ar}(\triangle AOD) = \text{ar}(\triangle BOC)$ .



**Ans.**  $\triangle ABD$  and  $\triangle ABC$  lie on the same base AB and between the same parallels AB and DC.



$$\therefore \text{ar}(\triangle ABD) = \text{ar}(\triangle ABC)$$

Subtracting  $\text{ar}(\triangle AOB)$  from both sides,

$$\text{ar}(\triangle ABD) - \text{ar}(\triangle AOB)$$

$$= \text{ar}(\triangle ABC) - \text{ar}(\triangle AOB)$$

$$\Rightarrow \text{ar}(\triangle AOD) = \text{ar}(\triangle BOC)$$

**Q11.** In figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that:

$$(i) \text{ar}(\triangle ACB) = \text{ar}(\triangle ACF)$$

$$(ii) \text{ar}(\text{AEDF}) = \text{ar}(\text{ABCDE})$$

**Ans. (i)** Given that  $BF \parallel AC$

$\triangle ACB$  and  $\triangle ACF$  lie on the same base AC and between the same parallels AC and BF.

$$\therefore \text{ar}(\triangle ACB) = \text{ar}(\triangle ACF) \dots\dots\dots(i)$$

$$(ii) \text{ Now } \text{ar}(\text{ABCDE}) = \text{ar}(\text{trap. AEDC}) + \text{ar}(\triangle ABC) \dots\dots\dots(ii)$$

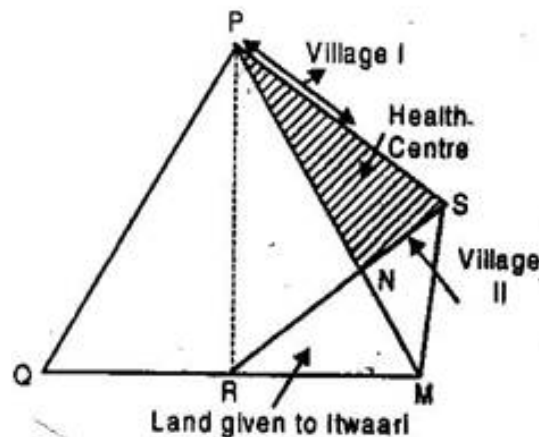
$$\Rightarrow \text{ar}(\text{ABCDE}) = \text{ar}(\text{trap. AEDC}) + \text{ar}(\triangle ACF) = \text{ar}(\text{quad. AEDF}) \text{ [Using (i)]}$$

$$\Rightarrow \text{ar}(\text{AEDF}) = \text{ar}(\text{ABCDE})$$

**Q12.** A villager Itwaari has a plot of land of the shape of quadrilateral. The Gram Panchayat of two villages decided to take over some portion of his plot from one of the corners to construct a health centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

**Ans.** Let Itwari has land in shape of quadrilateral PQRS.

Draw a line through S parallel to PR, which meets QR produced at M.



Let diagonals PM and RS of new formed quadrilateral intersect each other at point N.

We have  $PR \parallel SM$  [By construction]

$$\therefore \text{ar}(\triangle PRS) = \text{ar}(\triangle PMR)$$

[Triangles on the same base and same parallel are equal in area]

Subtracting  $\text{ar}(\triangle PNR)$  from both sides,

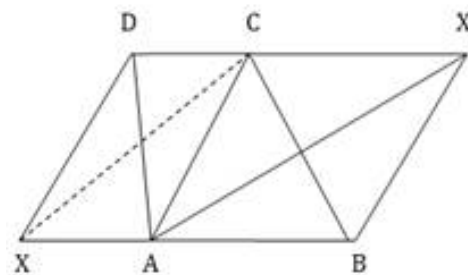
$$\text{ar}(\triangle PRS) - \text{ar}(\triangle PNR) = \text{ar}(\triangle PMR) - \text{ar}(\triangle PNR)$$

$$\Rightarrow \text{ar} (\triangle PSN) = \text{ar} (\triangle MNR)$$

It implies that Itwari will give corner triangular shaped plot PSN to the Grampanchayat for health centre and will take equal amount of land (denoted by  $\triangle MNR$ ) adjoining his plot so as to form a triangular plot PQM.

**Q13.** ABCD is a trapezium with  $AB \parallel DC$ . A line parallel to AC intersects AB at X and BC at Y. Prove that  $\text{ar} (\triangle ADX) = \text{ar} (\triangle ACY)$ .

**Ans.** Join CX,  $\triangle ADX$  and  $\triangle ACX$  lie on the same base XA and between the same parallels XA and DC.



$$\therefore \text{ar} (\triangle ADX) = \text{ar} (\triangle ACX) \dots\dots\dots(i)$$

Also  $\triangle ACX$  and  $\triangle ACY$  lie on the same base AC and between same parallels CY and XA.

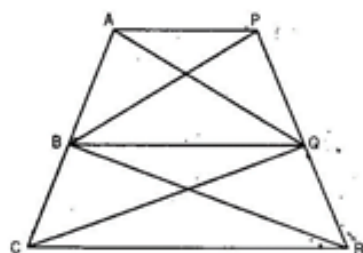
$$\therefore \text{ar} (\triangle ACX) = \text{ar} (\triangle ACY) \dots\dots\dots(ii)$$

From (i) and (ii),

$$\text{ar} (\triangle ADX) = \text{ar} (\triangle ACY)$$



**Q14.** In figure,  $AP \parallel BQ \parallel CR$ . Prove that  $\text{ar}(\triangle AQC) = \text{ar}(\triangle PBR)$ .



**Ans.**  $\triangle ABQ$  and  $\triangle BPQ$  lie on the same base BQ and between same parallels AP and BQ.

$$\therefore \text{ar}(\triangle ABQ) = \text{ar}(\triangle BPQ) \dots\dots\dots(i)$$

$\triangle BQC$  and  $\triangle BQR$  lie on the same base BQ and between same parallels BQ and CR.

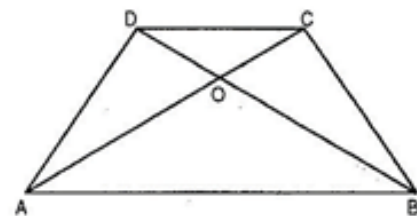
$$\therefore \text{ar}(\triangle BQC) = \text{ar}(\triangle BQR) \dots\dots\dots(ii)$$

$$\text{Adding eq (i) and (ii), } \text{ar}(\triangle ABQ) + \text{ar}(\triangle BQC) = \text{ar}(\triangle BPQ) + \text{ar}(\triangle BQR)$$

$$\Rightarrow \text{ar}(\triangle AQC) = \text{ar}(\triangle PBR)$$

**15.** Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that  $\text{ar}(\triangle AOD) = \text{ar}(\triangle BOC)$ . Prove that ABCD is a trapezium.

**Ans.** Given that  $\text{ar}(\triangle AOD) = \text{ar}(\triangle BOC)$



Adding  $\triangle AOB$  both sides,

$$\text{ar}(\triangle AOD) + \text{ar}(\triangle AOB) = \text{ar}(\triangle BOC) + \text{ar}(\triangle AOB)$$

$$\Rightarrow \text{ar}(\triangle ABD) = \text{ar}(\triangle ABC)$$

Since if two triangles equal in area, lie on the same base then, they lie between same parallels. We have  $\triangle ABD$  and  $\triangle ABC$  lie on common base AB and are equal in area.

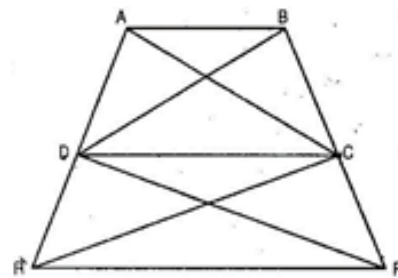
$\therefore$  They lie in same parallels AB and DC.

$$\Rightarrow AB \parallel DC$$

Now in quadrilateral ABCD, we have  $AB \parallel DC$

Therefore, ABCD is trapezium. [ $\because$  In trapezium one pair of opposite sides is parallel]

**Q16.** In figure,  $\text{ar}(\triangle DRC) = \text{ar}(\triangle DPC)$  and  $\text{ar}(\triangle BDP) = \text{ar}(\triangle ARC)$ . Show that both the quadrilaterals ABCD and DCPR are trapeziums.



**Ans.** Given that  $\triangle DRC$  and  $\triangle DPC$  lie on the same base DC and  $\text{ar}(\triangle DPC) = \text{ar}(\triangle DRC)$  .....(i)

$$\therefore DC \parallel RP$$

[If two triangles equal in area, lie on the same base then, they lie between same parallels]

Therefore, DCPR is trapezium. [ $\because$  In trapezium one pair of opposite sides is parallel]

$$\text{Also } \text{ar}(\triangle BDP) = \text{ar}(\triangle ARC) \text{ .....(ii)}$$

Subtracting eq. (i) from (ii),

$$\text{ar}(\triangle BDP) - \text{ar}(\triangle DPC) = \text{ar}(\triangle ARC) - \text{ar}(\triangle DRC)$$

$$\Rightarrow \text{ar}(\triangle BDC) = \text{ar}(\triangle ADC)$$

Therefore,  $AB \parallel DC$  [If two triangles equal in area, lie on the same base then, they lie between same parallels]

Therefore, ABCD is trapezium.

\*\*\*\*\* END \*\*\*\*\*

