Q14 : $\frac{dy}{dx} = y\tan x; y = 1 \text{ when } x = 0$

Answer :

$$\frac{dy}{dx} = y\tan x$$
$$\Rightarrow \frac{dy}{y} = \tan x\, dx$$

Integrating both sides, we get:

$$\int\frac{dy}{y} = -\int\tan x\, dx$$
$$\Rightarrow \log y = \log(\sec x) + \log C$$
$$\Rightarrow \log y = \log(C\sec x)$$
$$\Rightarrow y = C\sec x \qquad ...(1)$$

$$\text{Now, } y = 1 \text{ when } x = 0.$$
$$\Rightarrow 1 = C\times\sec 0$$
$$\Rightarrow 1 = C\times 1$$
$$\Rightarrow C = 1$$

Substituting C = 1 in equation (1), we get:

$y = \sec x$

Answer needs Correction? **Click Here**

Q15 : Find the equation of a curve passing through the point (0, 0) and whose differential equation is $y' = e^x \sin x$.

Answer :

The differential equation of the curve is:

$$y' = e^x \sin x$$
$$\Rightarrow \frac{dy}{dx} = e^x \sin x$$
$$\Rightarrow dy = e^x \sin x$$

Integrating both sides, we get:

$$\int dy = \int e^x \sin x\, dx \qquad ...(1)$$

$$\text{Let } I = \int e^x \sin x\, dx.$$
$$\Rightarrow I = \sin x\int e^x dx - \int\left(\frac{d}{dx}(\sin x)\cdot\int e^x dx\right)dx$$
$$\Rightarrow I = \sin x\cdot e^x - \int\cos x\cdot e^x dx$$
$$\Rightarrow I = \sin x\cdot e^x - \left[\cos x\cdot\int e^x dx - \int\left(\frac{d}{dx}(\cos x)\cdot\int e^x dx\right)dx\right]$$
$$\Rightarrow I = \sin x\cdot e^x - \left[\cos x\cdot e^x - \int(-\sin x)\cdot e^x dx\right]$$
$$\Rightarrow I = e^x\sin x - e^x\cos x - I$$
$$\Rightarrow 2I = e^x(\sin x - \cos x)$$
$$\Rightarrow I = \frac{e^x(\sin x - \cos x)}{2}$$

Substituting this value in equation (1), we get:

$$y = \frac{e^x(\sin x - \cos x)}{2} + C \qquad ...(2)$$

Now, the curve passes through point (0, 0).

$$\therefore 0 = \frac{e^0(\sin 0 - \cos 0)}{2} + C$$
$$\Rightarrow 0 = \frac{1(0-1)}{2} + C$$
$$\Rightarrow C = \frac{1}{2}$$

Substituting $C = \frac{1}{2}$ in equation (2), we get:

$$y = \frac{e^x(\sin x - \cos x)}{2} + \frac{1}{2}$$
$$\Rightarrow 2y = e^x(\sin x - \cos x) + 1$$
$$\Rightarrow 2y - 1 = e^x(\sin x - \cos x)$$

$\Rightarrow 2y-1=e^x(\sin x-\cos x)$

Hence, the required equation of the curve is $2y-1=e^x(\sin x-\cos x)$.

Answer needs Correction? **Click Here**

Q16 : **For the differential equation $xy\frac{dy}{dx}=(x+2)(y+2)$, find the solution curve passing through the point (1, â€“1).**

Answer :

The differential equation of the given curve is:

$$xy\frac{dy}{dx}=(x+2)(y+2)$$
$$\Rightarrow \left(\frac{y}{y+2}\right)dy=\left(\frac{x+2}{x}\right)dx$$
$$\Rightarrow \left(1-\frac{2}{y+2}\right)dy=\left(1+\frac{2}{x}\right)dx$$

Integrating both sides, we get:

$$\int\left(1-\frac{2}{y+2}\right)dy=\int\left(1+\frac{2}{x}\right)dx$$
$$\Rightarrow \int dy-2\int\frac{1}{y+2}dy=\int dx+2\int\frac{1}{x}dx$$
$$\Rightarrow y-2\log(y+2)=x+2\log x+C$$
$$\Rightarrow y-x-C=\log x^2+\log(y+2)^2$$
$$\Rightarrow y-x-C=\log\left[x^2(y+2)^2\right] \qquad ...(1)$$

Now, the curve passes through point (1, â€“1).

$$\Rightarrow -1-1-C=\log\left[(1)^2(-1+2)^2\right]$$
$$\Rightarrow -2-C=\log 1=0$$
$$\Rightarrow C=-2$$

Substituting C = â€“2 in equation (1), we get:

$$y-x+2=\log\left[x^2(y+2)^2\right]$$

This is the required solution of the given curve.

Answer needs Correction? **Click Here**

Q17 : **Find the equation of a curve passing through the point (0, â€“2) given that at any point (x,y) on the curve, the product of the slope of its tangent and *y*-coordinate of the point is equal to the *x*-coordinate of the point.**

Answer :

Let *x* and *y* be the *x*-coordinate and *y*-coordinate of the curve respectively.

We know that the slope of a tangent to the curve in the coordinate axis is given by the relation,

$$\frac{dy}{dx}$$

According to the given information, we get:

$$y\cdot\frac{dy}{dx}=x$$
$$\Rightarrow y\,dy=x\,dx$$

Integrating both sides, we get:

$$\int y\,dy=\int x\,dx$$
$$\Rightarrow \frac{y^2}{2}=\frac{x^2}{2}+C$$
$$\Rightarrow y^2-x^2=2C \qquad ...(1)$$

Now, the curve passes through point (0, â€“2).

∴ (â€“2)^2 â€“ 0^2 = 2C

⇒ 2C = 4

Substituting 2C = 4 in equation (1), we get:

y^2 â€“ *x*^{2} = 4

This is the required equation of the curve.

Answer needs Correction? **Click Here**

Q18 : **At any point (*x, y*) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point (-4, -3). Find the equation of the curve given that it passes through (-2, 1).**

Answer :

It is given that (*x, y*) is the point of contact of the curve and its tangent.

The slope (m_1) of the line segment joining (*x, y*) and (â€“4, â€“3) is $\frac{y+3}{x+4}$.

We know that the slope of the tangent to the curve is given by the relation,

$\frac{dy}{dx}$

$\therefore$ Slope (m_2) of the tangent $= \frac{dy}{dx}$

According to the given information:

$$m_2 = 2m_1$$
$$\Rightarrow \frac{dy}{dx} = \frac{2(y+3)}{x+4}$$
$$\Rightarrow \frac{dy}{y+3} = \frac{2dx}{x+4}$$

Integrating both sides, we get:

$$\int \frac{dy}{y+3} = 2\int \frac{dx}{x+4}$$
$$\Rightarrow \log(y+3) = 2\log(x+4) + \log C$$
$$\Rightarrow \log(y+3) \log C(x+4)^2$$
$$\Rightarrow y+3 = C(x+4)^2 \qquad ...(1)$$

This is the general equation of the curve.

It is given that it passes through point (â€“2, 1).

$$\Rightarrow 1+3 = C(-2+4)^2$$
$$\Rightarrow 4 = 4C$$
$$\Rightarrow C = 1$$

Substituting C = 1 in equation (1), we get:

$y + 3 = (x + 4)^2$

This is the required equation of the curve.

Answer needs Correction? **Click Here**

Q19 : The volume of spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of balloon after *t* seconds.

Answer :

Let the rate of change of the volume of the balloon be *k* (where *k* is a constant).

$$\Rightarrow \frac{dv}{dt} = k$$
$$\Rightarrow \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = k \qquad \left[\text{Volume of sphere} = \frac{4}{3}\pi r^3\right]$$
$$\Rightarrow \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt} = k$$
$$\Rightarrow 4\pi r^2\, dr = k\, dt$$

Integrating both sides, we get:

$$4\pi \int r^2 dr = k \int dt$$
$$\Rightarrow 4\pi \cdot \frac{r^3}{3} = kt + C$$
$$\Rightarrow 4\pi r^3 = 3(kt + C) \qquad ...(1)$$

Now, at $t = 0$, $r = 3$:

$\Rightarrow 4\pi \times 3^3 = 3 (k \times 0 + C)$

$\Rightarrow 108\pi = 3C$

$\Rightarrow C = 36\pi$

At $t = 3$, $r = 6$:

$\Rightarrow 4\pi \times 6^3 = 3 (k \times 3 + C)$

$\Rightarrow 864\pi = 3 (3k + 36\pi)$

⇒ $3k$ = â€“288π â€“ 36π = 252π

$\Rightarrow k = 84\pi$

Substituting the values of *k* and C in equation (1), we get:

$$4\pi r^3 = 3[84\pi t + 36\pi]$$
$$\Rightarrow 4\pi r^3 = 4\pi(63t + 27)$$
$$\Rightarrow r^3 = 63t + 27$$
$$\Rightarrow r = (63t + 27)^{\frac{1}{3}}$$

Thus, the radius of the balloon after *t* seconds is $(63t+27)^{\frac{1}{3}}$.

Answer needs Correction? **Click Here**

Q20 : In a bank, principal increases continuously at the rate of *r*% per year. Find the value of *r* if Rs 100 doubles itself in 10 years ($\log_e 2 = 0.6931$).

Answer :

Let *p*, *t*, and *r* represent the principal, time, and rate of interest respectively.

It is given that the principal increases continuously at the rate of *r*% per year.

$$\Rightarrow \frac{dp}{dt} = \left(\frac{r}{100}\right)p$$
$$\Rightarrow \frac{dp}{p} = \left(\frac{r}{100}\right)dt$$

Integrating both sides, we get:

$$\int \frac{dp}{p} = \frac{r}{100}\int dt$$
$$\Rightarrow \log p = \frac{rt}{100} + k$$
$$\Rightarrow p = e^{\frac{rt}{100}+k} \qquad ...(1)$$

It is given that when $t = 0$, $p = 100$.

$\Rightarrow 100 = e^k$ ... (2)

Now, if $t = 10$, then $p = 2 \times 100 = 200$.

Therefore, equation (1) becomes:

$$200 = e^{\frac{r}{10}+k}$$
$$\Rightarrow 200 = e^{\frac{r}{10}} \cdot e^k$$
$$\Rightarrow 200 = e^{\frac{r}{10}} \cdot 100 \qquad (\text{From } (2))$$
$$\Rightarrow e^{\frac{r}{10}} = 2$$
$$\Rightarrow \frac{r}{10} = \log_e 2$$
$$\Rightarrow \frac{r}{10} = 0.6931$$
$$\Rightarrow r = 6.931$$

Hence, the value of r is 6.93%.

Answer needs Correction? **Click Here**

Q21 : In a bank, principal increases continuously at the rate of 5% per year. An amount of Rs 1000 is deposited with this bank, how much will it worth after 10 years $\left(e^{0.5} = 1.648\right)$.

Answer :

Let p and t be the principal and time respectively.

It is given that the principal increases continuously at the rate of 5% per year.

$$\Rightarrow \frac{dp}{dt} = \left(\frac{5}{100}\right)p$$
$$\Rightarrow \frac{dp}{dt} = \frac{p}{20}$$
$$\Rightarrow \frac{dp}{p} = \frac{dt}{20}$$

Integrating both sides, we get:

$$\int \frac{dp}{p} = \frac{1}{20}\int dt$$
$$\Rightarrow \log p = \frac{t}{20} + C$$
$$\Rightarrow p = e^{\frac{t}{20}+C} \qquad ...(1)$$

Now, when $t = 0$, $p = 1000$.

$\Rightarrow 1000 = e^C$ ... (2)

At $t = 10$, equation (1) becomes:

$$p = e^{\frac{1}{2}+C}$$
$$\Rightarrow p = e^{0.5} \times e^C$$
$$\Rightarrow p = 1.648 \times 1000$$
$$\Rightarrow p = 1648$$

Hence, after 10 years the amount will worth Rs 1648.

Answer needs Correction? **Click Here**

Q22 : In a culture, the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number present?

Answer :

Let y be the number of bacteria at any instant t.

It is given that the rate of growth of the bacteria is proportional to the number present.

$$\therefore \frac{dy}{dt} \propto y$$
$$\Rightarrow \frac{dy}{dt} = ky \text{ (where } k \text{ is a constant)}$$
$$\Rightarrow \frac{dy}{y} = kdt$$

Integrating both sides, we get:

$$\int \frac{dy}{y} = k\int dt$$
$$\Rightarrow \log y = kt + C \qquad ...(1)$$

Let y_0 be the number of bacteria at $t = 0$.

$\Rightarrow \log y_0 = C$

Substituting the value of C in equation (1), we get:

$$\log y = kt + \log y_0$$
$$\Rightarrow \log y - \log y_0 = kt$$
$$\Rightarrow \log\left(\frac{y}{y_0}\right) = kt$$
$$\Rightarrow kt = \log\left(\frac{y}{y_0}\right) \qquad ...(2)$$

Also, it is given that the number of bacteria increases by 10% in 2 hours.

$$\Rightarrow y = \frac{110}{100}y_0$$
$$\Rightarrow \frac{y}{y_0} = \frac{11}{10} \qquad ...(3)$$

Substituting this value in equation (2), we get:

$$k \cdot 2 = \log\left(\frac{11}{10}\right)$$
$$\Rightarrow k = \frac{1}{2}\log\left(\frac{11}{10}\right)$$

Therefore, equation (2) becomes:

$$\frac{1}{2}\log\left(\frac{11}{10}\right)\cdot t = \log\left(\frac{y}{y_0}\right)$$
$$\Rightarrow t = \frac{2\log\left(\frac{y}{y_0}\right)}{\log\left(\frac{11}{10}\right)} \qquad ...(4)$$

Now, let the time when the number of bacteria increases from 100000 to 200000 be t_1.

$\Rightarrow y = 2y_0$ at $t = t_1$

From equation (4), we get:

$$t_1 = \frac{2\log\left(\frac{y}{y_0}\right)}{\log\left(\frac{11}{10}\right)} = \frac{2\log 2}{\log\left(\frac{11}{10}\right)}$$

Hence, in $\frac{2\log 2}{\log\left(\frac{11}{10}\right)}$ hours the number of bacteria increases from 100000 to 200000.

Answer needs Correction? **Click Here**

Q23 : The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is

A. $e^x + e^{-y} = C$

B. $e^x + e^y = C$

C. $e^{-x} + e^y = C$

D. $e^{-x} + e^{-y} = C$

Answer :

$$\frac{dy}{dx} = e^{x+y} = e^x \cdot e^y$$
$$\Rightarrow \frac{dy}{e^y} = e^x dx$$
$$\Rightarrow e^{-y}dy = e^x dx$$

Integrating both sides, we get:

$$\int e^{-y}dy = \int e^x dx$$
$$\Rightarrow -e^{-y} = e^x + k$$
$$\Rightarrow e^x + e^{-y} = -k$$
$$\Rightarrow e^x + e^{-y} = c \qquad (c = -k)$$

Hence, the correct answer is A.

*********** END **********