Areas of Parallelograms and Triangles Ex 15.3 Q22

Answer :

Given: ABCD IS A trapezium in which

(a) $AB \| DC$

(b) $DC = 40$ cm

(c) $AB = 60$ cm

(d) X is the midpoint of AD

(e) Y is the midpoint of BC

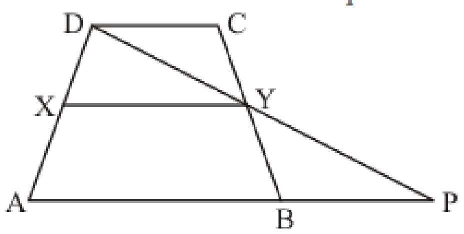
To prove:

(i) $XY = 50$ cm

(ii) DCYX is a trapezium

(iii) $\operatorname{ar}(\text{trap.DCYX}) = \frac{9}{11} \operatorname{ar}(\text{trap.XYAB})$

Construction: Join DY and produce it to meet AB produced at P.



Proof:

(i) In ΔBYP and ΔCYD

$$\angle BYP = \angle CYD \text{ (vertically opposite angles)}$$

$$BY = CY$$

$$\angle DCY = \angle PBY \text{ (alternate angles)}$$

$$\Delta BYP \cong \Delta CYD \text{ (A.S.A congruence condition)}$$

$DY = YP$ and

$DC = BP$

Y is the midpoint of BC also X is the midpoint of AD

Therefore $XY \| AP$ and $XY = \frac{1}{2} AP$

$$\Rightarrow XY = \frac{1}{2}(AB + BP)$$

$$\Rightarrow XY = \frac{1}{2}(AB + DC)$$

$$\Rightarrow XY = \frac{1}{2}(60 + 40)$$

$$\Rightarrow XY = \boxed{50 \text{ cm}}$$

(ii) We have proved above that $XY \| AP$

$\Rightarrow XY \| AP$ and $AB \| DC$ (Given in question)

$\Rightarrow XY \| DC$

$\Rightarrow$ $\boxed{\text{DCYX is a trapezium}}$

(iii) Since X and Y are the midpoints of AD and BC respectively.

Therefore DCYX and ABYX are of the same height say h cm.

$$ar(trap.DCYX) = \frac{1}{2}(DC + XY) \times h$$

$$ar(trap.DCYX) = \frac{1}{2}(40 + 50) \times h$$

$$ar(trap.DCYX) = 45 \text{ h cm}^2$$

$$ar(trap.ABYX) = \frac{1}{2}(AB + XY) \times h$$

$$ar(trap.ABYX) = \frac{1}{2}(60 + 50) \times h$$

$$ar(trap.ABYX) = 55 \text{ h cm}^2$$

$$\frac{ar(trap.DCYX)}{ar(trap.ABYX)} = \frac{45 h}{55 h}$$

$$\frac{ar(trap.DCYX)}{ar(trap.ABYX)} = \frac{9}{11}$$

$$\boxed{ar(trap.DCYX) = \frac{9}{11} ar(trap.ABYX)}$$

*********** END **********