Functions Ex 2.1 Q1(i)

Example of a function which is one-one but not only.

let $f : N \to N$ given by $f(x) = x^2$

Check for injectivity:

let $x, y \in N$ such that

$f(x) = f(y)$

$\Rightarrow \quad x^2 = y^2$

$\Rightarrow \quad (x - y)(x + y) = 0 \qquad [\because x, y \in N \Rightarrow x + y > 0]$

$\Rightarrow \quad x - y = 0$

$\Rightarrow \quad x = y$

$\therefore \quad f$ is one-one

Surjectivity: let $y \in N$ be arbitrary, then

$f(x) = y$

$\Rightarrow \quad x^2 = y$

$\Rightarrow \quad x = \sqrt{y} \notin N$ for non-perfect square value of y.

$\therefore$ No non-perfect square value of y has a pre image in domain N.

$\therefore$ $f : N \to N$ given by $f(x) = x^2$ is one-one but not onto.

Functions Ex 2.1 Q1(ii)

Example of a function which is onto but not one-one.

let $f : R \to R$ defined by $f(x) = x^3 - x$

Check for injectivity:

let $x, y \in R$ such that

$f(x) = f(y)$

$\Rightarrow \quad x^3 - x = y^3 - y$

$\Rightarrow \quad x^3 - y^3 - (x - y) = 0$

$\Rightarrow \quad (x - y)(x^2 + xy + y^2 - 1) = 0$

$\because \quad x^2 + xy + y^2 \geq 0 \quad \Rightarrow \quad x^2 + xy + y^2 - 1 \geq -1$

$\therefore \quad x \neq y$ for some $x, y \in R$

$\therefore \quad f$ is not one-one.

Surjectivity: let $y \in R$ be arbitrary

then, $f(x) = y$

$\Rightarrow \quad x^3 - x = y$

$\Rightarrow \quad x^3 - x - y = 0$

we know that a degree 3 equation has a real root.

let $x = \alpha$ be that root

$\therefore \quad \alpha^3 - \alpha = y$

$\Rightarrow \quad f(\alpha) = y$

Thus for clearly $y \in R$. there exist $\alpha \in R$ such that $f(x) = y$

$\therefore \quad f$ is onto

$\therefore$ Hence $f : R \to R$ defined by $f(x) = x^3 - x$ is not one-one but onto.

Functions Ex 2.1 Q1(iii)

Example of a function which is neither one-one nor onto.

let $f : R \to R$ defined by $f(x) = 2$

We know that a constant function in neither one-one nor onto

Here $f(x) = 2$ is a constant function

$\therefore \quad f : R \to R$ defined by $f(x) = 2$ is neither one-one nor onto.

Functions Ex 2.1 Q2

i) $f_1 = \{(1,3),(2,5),(3,7)\}$

$A = \{1,2,3\}$, $B = \{3,5,7\}$

We can earily observe that in f_1 every element of A has different image from B.

$\therefore$ f_1 in one-one

also, each element of B is the image of some element of A.

$\therefore$ f_1 in onto.

ii)

$f_2 = \{(2,a),(3,b),(4,c)\}$

$A = \{2,3,4\}$ $B = \{a,b,c\}$

It in clear that different elements of A have different images in B

$\therefore$ f_2 in one-one

Again, each element of B is the image of some element of A.

$\therefore$ f_2 in onto

iii) $f_3 = \{(a,x),(b,x),(c,z)(d,z)\}$

$A = \{a,b,c,d\}$ $B = \{x,y,z\}$

Since, $f_3(a) = x = f_3(b)$ and $f_3(c) = z = f_3(d)$

$\therefore$ f_3 in not one-one

Again, $y \in B$ in not the image of any of the element of A

$\therefore$ f_3 in not onto

Functions Ex 2.1 Q3

We have, $f: N \rightarrow N$ defined by $f(x) = x^2 + x + 1$

Check for injectivity:

Let $x, y \in N$ such that

$$f(x) = f(y)$$

$$\Rightarrow \quad x^2 + x + 1 = y^2 + y + 1$$

$$\Rightarrow \quad x^2 - y^2 + x - y = 0$$

$$\Rightarrow \quad (x - y)(x + y + 1) = 0$$

$$\Rightarrow \quad x - y = 0 \qquad [\because x, y \in N \Rightarrow x + y + 1 > 0]$$

$$\Rightarrow \quad x = y$$

$\therefore \quad f$ is one-one.

Surjectivity:

Let $y \in N$, then

$$f(x) = y$$

$$\Rightarrow \quad x^2 + x + 1 - y = 0$$

$$\Rightarrow \quad x = \frac{-1 \pm \sqrt{1 - 4(1 - y)}}{2} \notin N \quad \text{for} \quad y > 1$$

$\therefore$ for $y > 1$, we do not have any pre-image in domain N.

$\therefore$ f is not onto.

Functions Ex 2.1 Q4.

We have, $A = \{-1, 0, 1\}$ and $f: A \rightarrow A$

defined by $f = \{(x, x^2) : x \in A\}$

clearly $f(1) = 1$ and $f(-1) = 1$

$\therefore \quad f(1) = f(-1)$

$\therefore \quad f$ is not one-one

Again $y = -1 \in A$ in the co-domain does not have any pre image in domain A.

$\therefore$ f is not onto.

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