Sine and Cosine Formulae and their Applications Ex-10.1 Q13

$$\frac{\sqrt{\sin A}-\sqrt{\sin B}}{\sqrt{\sin A}+\sqrt{\sin B}}=\frac{a+b-2\sqrt{ab}}{a-b}$$

RHS

$$\frac{a+b-2\sqrt{ab}}{a-b}$$

$$=\frac{\left(\sqrt{a}\right)^2+\left(\sqrt{b}\right)^2-2\sqrt{ab}}{\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2}$$

$$=\frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2}$$

$$=\frac{\left(\sqrt{a}-\sqrt{b}\right)}{\left(\sqrt{a}+\sqrt{b}\right)}$$

$$=\frac{\left(\sqrt{k\sin A}-\sqrt{k\sin B}\right)}{\left(\sqrt{k\sin A}+\sqrt{k\sin B}\right)}$$

$$=\frac{\left(\sqrt{\sin A}-\sqrt{\sin B}\right)}{\left(\sqrt{\sin A}+\sqrt{\sin B}\right)}[\text{taking } k \text{ common and cancelling them}]$$

$$=LHS$$

Hence Proved

Sine and Cosine Formulae and their Applications Ex-10.1 Q14

$LHS,$

$a(\sin B-\sin C)+b(\sin C-\sin A)+c(\sin A-\sin B)$

$=a\sin B-a\sin C+b\sin C-b\sin A+c\sin A-c\sin B$

$=b\sin A-c\sin A+c\sin B-b\sin A+c\sin A-c\sin B[\because b\sin A=a\sin B, b\sin C=c\sin B, c\sin A=a\sin C]$

$=0=RHS$

Hence Proved

Sine and Cosine Formulae and their Applications Ex-10.1 Q15

$$\frac{a^2\sin(B-C)}{\sin A}+\frac{b^2\sin(C-A)}{\sin B}+\frac{c^2\sin(A-B)}{\sin C}=0$$

$$\frac{a}{\sin A}=\frac{b}{b\sin C}=\frac{c}{\sin C}=k$$

LHS

$$\frac{a^2\sin(B-C)}{\sin A}+\frac{b^2\sin(C-A)}{\sin B}+\frac{c^2\sin(A-B)}{\sin C}$$

$$=ak\sin(B-C)+bk\sin(C-A)+ck\sin(A-B)$$

$$=\sin A\sin(B-C)+\sin B\sin(C-A)+\sin C\sin(A-B)$$

$$=\sin(\pi-(B+C))\sin(B-C)+\sin(\pi-(C+A))\sin(C-A)$$

$$+\sin(\pi-(A+B))\sin(A-B)$$

$$=\sin(B+C)\sin(B-C)+\sin(C+A)\sin(C-A)$$

$$+\sin(A+B)\sin(A-B)$$

$$=\sin^2 B-\sin^2 C+\sin^2 C-\sin^2 A+\sin^2 A-\sin^2 B=0=RHS$$

Sine and Cosine Formulae and their Applications Ex-10.1 Q16

$a^2(\cos^2 B - \cos^2 C) + b^2(\cos^2 C - \cos^2 A) + c^2(\cos^2 A - \cos^2 B) = 0$

LHS

$$=a^2(1-\sin^2 B-1+\sin^2 C)+b^2(1-\sin^2 C-1+\sin^2 A)$$

$$+c^2(1-\sin^2 A-1+\sin^2 B)$$

$$=a^2(\sin^2 C-\sin^2 B)+b^2(\sin^2 A-\sin^2 C)+c^2(\sin^2 B-\sin^2 A)$$

$$=a^2(k^2c^2-k^2b^2)+b^2(k^2a^2-k^2c^2)+c^2(k^2b^2-k^2a^2)$$

$$=k^2(a^2c^2-a^2b^2+b^2a^2-b^2c^2+b^2c^2-a^2c^2)$$

$$=k^2\times 0=0=RHS$$

********** END **********