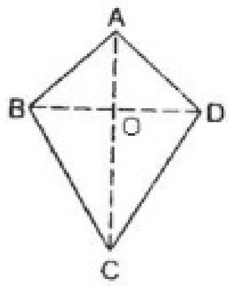
Exercise 5A

Question 29:

Given: a quarilateral ABCD in which AB=AD and BC=DC



To Prove: (i) AC bisects $\angle A$ and $\angle C$

(ii) $AC \perp BD$ and AC bisects BD

Proof: In ΔABC and ΔADC,we have

$AB=AD$ [Given]

$BC=DC$ [Given]

$AC=AC$ [Common]

Thus by Side-Side-Side criterion of congruence,

$\Delta ABC \cong \Delta ADC$ [By SSS]

The corresponding parts of the congruent triangles are equal.

So, $\angle BAC = \angle DAC$ [C.P.C.T]

$\Rightarrow$ $\angle BAO = \angle DAO$ .....(1)

It means that AC bisects $\angle BAD$, that is $\angle A$

Also, $\angle BCA = \angle DCA$ [C.P.C.T]

It means that AC bisects $\angle BCD$, that is $\angle C$

(ii)

Now in ΔABO and ΔADO

$AB = AD$ [Given]

$\angle BAO = \angle DAO$ [from (1)]

$AO = AO$ [Common]

Thus, by Side-Angle-Side criterion of congruence, we have

$\Delta ABO \cong \Delta ADO$ [By SAS]

The corresponding parts of the congruent triangles are equal.

$\therefore$ $\angle BOA = \angle DOA$

But $\angle BOA + \angle DOA = 180^\circ$

Or $2\angle BOA = 180^\circ$

$$\Rightarrow \quad \angle BOA = \frac{180^\circ}{2} = 90^\circ$$

Also, as $\Delta ABO \cong \Delta ADO$

So, $BO = OD$

which means that AC bisects BD.

*********** END **********