Relations Ex 1.2 Q13

$S = \{(a,b) : \ a^2 + b^2 = 1\}$

Now,

Reflexivity: Let $a = \frac{1}{2} \in R$

Then, $a^2 + a^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \neq 1$

$\Rightarrow \quad (a,a) \notin S$

$\Rightarrow \quad S$ is not reflexive

Hence, S in not an equivalenve relation on R

Relations Ex 1.2 Q14

We have, Z be set of integers and Z_0 be the set of non-zero integers.

$R = \{(a,b)(c,d) : ad = bc\}$ be a relation on $z \times z_0$

Now,

Reflexivity: $(a,b) \in Z \times Z_0$

$\Rightarrow \quad ab = ba$

$\Rightarrow \quad ((a,b),(a,b)) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $((a,b),(c,d)) \in R$

$\Rightarrow \quad ad = bc$

$\Rightarrow \quad cd = da$

$\Rightarrow \quad ((c,d),(a,b)) \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let $(a,b),(c,d) \in R$ and $(c,d),(e,f) \in R$

$\Rightarrow \quad ad = bc$ and $cf = de$

$\Rightarrow \quad \frac{a}{b} = \frac{c}{d}$ and $\frac{c}{d} = \frac{e}{f}$

$\Rightarrow \quad \frac{a}{b} = \frac{e}{f}$

$\Rightarrow \quad af = be$

We have, Z be set of integers and Z_0 be the set of non-zero integers.

$R = \{(a,b)(c,d) : ad = bc\}$ be a relation on Z and Z_0.

Now,

Reflexivity: $(a,b) \in Z \times Z_0$

$\Rightarrow \quad ab = ba$

$\Rightarrow \quad ((a,b),(a,b)) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $((a,b),(c,d)) \in R$

$\Rightarrow \quad ad = bc$

$\Rightarrow \quad cd = da$

$\Rightarrow \quad ((c,d),(a,b)) \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let $(a,b),(c,d) \in R$ and $(c,d),(e,f) \in R$

$\Rightarrow \quad ad = bc$ and $cf = de$

$\Rightarrow \quad \frac{a}{b} = \frac{c}{d}$ and $\frac{c}{d} = \frac{e}{f}$

$\Rightarrow \quad \frac{a}{b} = \frac{e}{f}$

$\Rightarrow \quad af = be$

$\Rightarrow \quad (a,b)(e,f) \in R$

$\Rightarrow \quad R$ is transitive

Hence, R is an equivalence relation on $Z \times Z_0$

Relations Ex 1.2 Q15

R and S are two symmetric relations on set A

(i) To prove: $R \cap S$ is symmetric

Let $(a,b) \in R \cap S$

$\Rightarrow \quad (a,b) \in R$ and $(a,b) \in S$

$\Rightarrow \quad (b,a) \in R$ and $(b,a) \in S \qquad [\because R$ and S are symmetric$]$

$\Rightarrow \quad (b,a) \in R \cap S$

$\Rightarrow \quad R \cap S$ is symmetric

To prove: $R \cup S$ is symmetric.

Let $(a,b) \in R \cup S$

$\Rightarrow \quad (a,b) \in R$ or $(a,b) \in S$

$\Rightarrow \quad (b,a) \in R$ or $(b,a) \in S \qquad [\because R$ and S are symmetric$]$

$\Rightarrow \quad (b,a) \in R \cup S$

$\Rightarrow \quad R \cup S$ is symmetric

(ii) R and S are two relations on A such that R is reflexive.

To prove: $R \cup S$ is reflexive

Suppose $R \cup S$ is not reflexive.

This means that there is an $a \in R \cup S$ such that $(a,a) \notin R \cup S$

Since $a \in R \cup S$,

$\therefore \quad a \in R$ or $a \in S$

If $a \in R$, then $(a,a) \in R \qquad [\because R$ is reflexive$]$

$\Rightarrow \quad (a,a) \in R \cup S$

Hence, $R \cup S$ is reflexive

Relations Ex 1.2 Q16

We will prove this by means of an example.

Let $A = \{a,b,c\}$ be a set and

$R = \{(a,a)(b,b)(c,c)(a,b)(b,a)\}$ and

$S = \{(a,a)(b,b)(c,c)(b,c)(c,b)\}$ are two relations on A

Clearly R and S are transitive relation on A

Now, $R \cup S = \{(a,a)(b,b)(c,c)(a,b)(b,a)(b,c)(c,b)\}$

Here, $(a,b) \in R \cup S$ and $(b,c) \in R \cup S$

but $(a,c) \notin R \cup S$

$\therefore \quad R \cup S$ is not transitive

*********** END **********