Differentials Errors and Approximation Ex14.1 Q8

Let error in radius $(r) = x\%$ of r

$\Delta r = 0.0xr$

Let $v =$ volume of sphere

$$v = \frac{4}{3}\pi r^3$$

Differentiating it with respect to r,

$$\frac{dv}{dr} = 4\pi r^2$$

So,

$$\Delta v = \left(\frac{dv}{dr}\right) \times \Delta r$$

$$= \left(4\pi r^2\right)(0.0x)r$$

$$\Delta v = 0.0x \times 4\pi r^3$$

$$\text{Percentage of error in volume} = \frac{\Delta v \times 100}{v}$$

$$= \frac{(0.0x)\,4\pi r^3 \times 100}{\frac{4}{3}\pi r^3}$$

$$= 3x\%$$

Percentage of error in volume = 3 (percentage of error in radius).

Differentials Errors and Approximation Ex14.1 Q9(i)

Let $x = 25,\ x + \Delta x = 25.02$

$\Delta x = 25.02 - 25$

$\Delta x = 0.02$

Let $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\left(\frac{dy}{dx}\right)_{x=25} = \frac{1}{2\sqrt{25}}$$

$$\left(\frac{dy}{dx}\right)_{x=25} = \frac{1}{10}$$

Now,

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=25} \times x$$

$$= \frac{1}{10}(0.02)$$

$\Delta y = 0.002$

$\sqrt{25.02} = y + \Delta y$

$= \sqrt{25} + 0.002$

$= 5 + 0.002$

$\sqrt{25.02} = 5.002$

Differentials Errors and Approximation Ex14.1 Q9(ii)

Let $x = 0.008,\ x + \Delta x = 0.009$

$\Delta x = 0.009 - 0.008$

$\Delta x = 0.001$

Let $y = x^{\frac{1}{3}}$

$$\frac{dy}{dx} = \frac{1}{3x^{\frac{2}{3}}}$$

$$\left(\frac{dy}{dx}\right)_{x=0.008} = \frac{1}{3(0.008)^{\frac{2}{3}}}$$

$$= \frac{1}{3(0.04)}$$

$$= \frac{100}{12}$$

$$= 0.8333$$

So,

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=0.008} \times \Delta x$$

$$= (0.8333)(0.001)$$

$$\Delta y = 0.008333$$

$$(0.009)^{\frac{1}{3}} = y + \Delta y$$

$$= (x)^{\frac{1}{3}} + 0.008333$$

$$= (0.008)^{\frac{1}{3}} + 0.008333$$

$$= 0.52 + 0.008333$$

$$(0.009)^{\frac{1}{3}} = 0.208333$$

Let $x = 0.008,\ x + \Delta x = 0.007$

$$\Delta x = 0.007 - 0.008$$

$$\Delta x = -0.001$$

Let $y = x^{\frac{1}{3}}$

$$\frac{dy}{dx} = \frac{1}{3(x)^{\frac{2}{3}}}$$

$$\left(\frac{dy}{dx}\right)_{x=0.008} = \frac{1}{3(0.008)^{\frac{2}{3}}}$$

$$= \frac{100}{12}$$

$$\left(\frac{dy}{dx}\right)_{x=0.008} = 8.333$$

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=0.008} \times \Delta x$$

$$= (8.333)(-0.001)$$

$$\Delta y = -0.008333$$

$$(0.007)^{\frac{1}{3}} = y + \Delta y$$

$$= x^{\frac{1}{3}} - 0.008333$$

$$= (0.008)^{\frac{1}{3}} - 0.008333$$

$$= 0.2 - 0.008333$$

$$(0.007)^{\frac{1}{3}} = 0.191667$$

********** END **********