Indefinite Integrals Ex 19.13 Q1

Let $I = \int \frac{x}{\sqrt{x^4 + a^4}} dx$

$= \int \frac{x}{\sqrt{(x^2)^2 + (a^2)^2}} dx$

Let $x^2 = t$

$\Rightarrow \quad 2x\, dx = dt$

$\Rightarrow \quad x\, dx = \frac{dt}{2}$

$I = \frac{1}{2} \int \frac{dt}{\sqrt{t^2 + (a^2)^2}}$

$= \frac{1}{2} \log\left|t + \sqrt{t^2 + (a^2)^2}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2 + a^2}} dx = \log\left|x + \sqrt{x^2 + a^2}\right| + c\right]$

$I = \frac{1}{2} \log\left|x^2 + \sqrt{(x^2)^2 + (a^2)^2}\right| + c$

$I = \frac{1}{2} \log\left|x^2 + \sqrt{x^4 + a^4}\right| + c$

Indefinite Integrals Ex 19.13 Q2

Let $\tan x = t$

$\Rightarrow \sec^2 x\, dx = dt$

$$\Rightarrow \int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx = \int \frac{dt}{\sqrt{t^2 + 2^2}}$$

$$= \log\left|t + \sqrt{t^2 + 4}\right| + C$$

$$= \log\left|\tan x + \sqrt{\tan^2 x + 4}\right| + C$$

Indefinite Integrals Ex 19.13 Q3

Let $I = \int \frac{e^x}{\sqrt{16 - e^{2x}}} dx$

Let $e^x = t$

$\Rightarrow \quad e^x\, dx = dt$

$I = \int \frac{dt}{\sqrt{(4)^2 - t^2}}$

$= \sin^{-1}\left(\frac{t}{4}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = \sin^{-1}\left(\frac{e^x}{4}\right) + c$

Indefinite Integrals Ex 19.13 Q4

Let $I = \int \frac{\cos x}{\sqrt{4+\sin^2 x}} dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x \, dx = dt$

$$I = \int \frac{dt}{\sqrt{(2)^2 + t^2}}$$

$$= \log\left|t + \sqrt{(2)^2 + t^2}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2+x^2}} dx = \log\left|x + \sqrt{a^2+x^2}\right| + c\right]$$

$$I = \log\left|\sin x + \sqrt{4+\sin^2 x}\right| + c$$

Indefinite Integrals Ex 19.13 Q5

Let $I = \int \frac{\sin x}{\sqrt{4\cos^2 x - 1}} dx$

Let $2\cos x = t$

$\Rightarrow \quad -2\sin x \, dx = dt$

$\Rightarrow \quad \sin x \, dx = -\frac{dt}{2}$

$$I = -\frac{1}{2}\int \frac{dt}{\sqrt{t^2 - 1}}$$

$$= -\frac{1}{2}\log\left|t + \sqrt{t^2 - 1}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2-a^2}} dx = \log\left|x + \sqrt{x^2+a^2}\right| + c\right]$$

$$I = -\frac{1}{2}\log\left|2\cos x + \sqrt{4\cos^2 x - 1}\right| + c$$

*********** END **********