Exercise 5.2

4. Which term of the AP: 3, 8, 13, 18 ... is 78?

Ans. First term = a = 3, Common difference = d = 8 − 3 = 13 − 8 = 5 and a_n = 78

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

a_n = 3 + (n − 1) 5,

⇒ 78 = 3 + (n − 1) 5

⇒ 75 = 5n − 5

⇒ 80 = 5n⇒ n = 16

It means 16th term of the given AP is equal to 78.

5. Find the number of terms in each of the following APs:

(i) 7, 13, 19...., 205

(ii) 18, $15\frac{1}{2}$, 13..., −47

Ans. (i) 7, 13, 19 ..., 205

First term = a = 7, Common difference = d = 13 − 7 = 19 − 13 = 6

And a_n = 205

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

205 = 7 + (n − 1) 6 = 7 + 6n − 6

⇒ 205 = 6n + 1

⇒ 204 = 6n⇒ n = 34

Therefore, there are 34 terms in the given arithmetic progression.

(ii) 18, $15\frac{1}{2}$, 13 ..., −47

First term = a =18, Common difference = d = $15\frac{1}{2} - 18 = \frac{31}{2} - 18 = \frac{31-36}{2} = \frac{-5}{2}$

And a_n = −47

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$-47 = 18 + (n-1)\left(-\frac{5}{2}\right)$

$= 36 - \frac{5}{2}n + \frac{5}{2}$

$\Rightarrow -94 = 36 - 5n + 5$

$\Rightarrow 5n = 135 \Rightarrow n = 27$

Therefore, there are 27 terms in the given arithmetic progression.

6. Check whether −150 is a term of the AP: 11, 8, 5, 2...

Ans. Let −150 is the n^{th} of AP 11, 8, 5, 2... which means that $a_n = -150$

Here, First term = a = 11, Common difference = d = 8 − 11 = −3

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$-150 = 11 + (n-1)(-3)$

$\Rightarrow -150 = 11 - 3n + 3$

$\Rightarrow 3n = 164 \Rightarrow n = \frac{164}{3}$

But, n cannot be in fraction.

Therefore, our supposition is wrong. −150 cannot be term in AP.

7. Find the 31^{st} term of an AP whose 11^{th} term is 38 and 16^{th} term is 73.

Ans. Here $a_{11} = 38 and\ a_{16} = 73$

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$38 = a + (11-1)(d) And 73 = a + (16-1)(d)$

$\Rightarrow 38 = a + 10d$ And $73 = a + 15d$

These are equations consisting of two variables.

We have, $38 = a + 10d$

$\Rightarrow a = 38 - 10d$

Let us put value of a in equation $(73 = a + 15d)$,

$73 = 38 - 10d + 15d$

$\Rightarrow 35 = 5d$

Therefore, Common difference = $d = 7$

Putting value of d in equation $38 = a + 10d$,

$38 = a + 70$

$\Rightarrow a = -32$

Therefore, common difference = d = 7 and First term = a = −32

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$a_{31} = -32 + (31 - 1)(7)$

$= -32 + 210 = 178$

Therefore, 31^{st} term of AP is 178.

8. An AP consists of 50 terms of which 3^{rd} term is 12 and the last term is 106. Find the 29^{th} term.

Ans. An AP consists of 50 terms and the 50^{th} term is equal to 106 and $a_3 = 12$

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$a_{50} = a + (50 - 1)d$ And $a_3 = a + (3 - 1)d$

$\Rightarrow 106 = a + 49d$ And $12 = a + 2d$

These are equations consisting of two variables.

Using equation $106 = a + 49d$, we get $a = 106 - 49d$

Putting value of a in the equation $12 = a + 2d$,

$12 = 106 - 49d + 2d$

$\Rightarrow 47d = 94 \Rightarrow d = 2$

Putting value of d in the equation, $a = 106 - 49d$,

$a = 106 - 49(2) = 106 - 98 = 8$

Therefore, First term = $a = 8$ and Common difference = $d = 2$

To find 29^{th} term, we use formula $a_n = a + (n-1)d$ which is used to find n^{th} term of arithmetic progression,

$a_{29} = 8 + (29 - 1)2 = 8 + 56 = 64$

Therefore, 29th term of AP is equal to 64.

9. If the third and the ninth terms of an AP are 4 and −8 respectively, which term of this AP is zero?

Ans. It is given that 3rd and 9th term of AP are 4 and −8 respectively.

It means $a_3 = 4 and\ a_9 = -8$

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$4 = a + (3-1)d$ And, $-8 = a + (9-1)d$

$\Rightarrow 4 = a + 2d$ And, $-8 = a + 8d$

These are equations in two variables.

Using equation $4 = a + 2d$, we can say that $a = 4 - 2d$

Putting value of a in other equation $-8 = a + 8d$,

$-8 = 4 - 2d + 8d$

$\Rightarrow -12 = 6d \Rightarrow d = -2$

Putting value of d in equation $-8 = a + 8d$,

$-8 = a + 8(-2)$

$\Rightarrow -8 = a - 16 \Rightarrow a = 8$

Therefore, first term = $a = 8$ and Common Difference = $d = -2$

We want to know which term is equal to zero.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$0 = 8 + (n-1)(-2)$

$\Rightarrow 0 = 8 - 2n + 2$

$\Rightarrow 0 = 10 - 2n$

$\Rightarrow 2n = 10 \Rightarrow n = 5$

Therefore, 5th term is equal to 0.

10. The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

Ans. $a_{17} = a_{10} + 7 \ldots (1)$

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$a_{17} = a + 16d$... (2)

$a_{10} = a + 9d$... (3)

Putting (2) and (3) in equation (1),

$a + 16d = a + 9d + 7$

$\Rightarrow 7d = 7 \Rightarrow d = 1$

11. Which term of the AP: 3, 15, 27, 39... will be 132 more than its 54^{th} term?

Ans. Lets first calculate 54^{th} of the given AP.

First term = a = 3, Common difference = d = 15 − 3 = 12

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$a_{54} = a + (54 - 1)\,d = 3 + 53\,(12) = 3 + 636 = 639$

We want to find which term is 132 more than its 54^{th} term.

Let us suppose it is n^{th} term which is 132 more than 54^{th} term.

$a_n = a_{54} + 132$

$\Rightarrow 3 + (n - 1)\,12 = 639 + 132$

$\Rightarrow 3 + 12n - 12 = 771$

$\Rightarrow 12n - 9 = 771$

$\Rightarrow 12n = 780 \Rightarrow n = 65$

Therefore, 65^{th} term is 132 more than its 54^{th} term.

12. Two AP's have the same common difference. The difference between their 100^{th} terms is 100, what is the difference between their 1000^{th} terms.

Ans. Let first term of 1^{st} AP = a

Let first term of 2^{nd} AP = a'

It is given that their common difference is same.

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