



Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 26

$$\begin{aligned}\tan 82 \frac{1^\circ}{2} &= \tan \left( 90 - 7 \frac{1^\circ}{2} \right) \\ &= \cot 7 \frac{1^\circ}{2} \\ &= \cot A \quad \text{If } A = 7 \frac{1^\circ}{2}\end{aligned}$$

Now

$$\begin{aligned}\cot A &= \frac{\cos A}{\sin A} \\ &= \frac{2 \cos^2 A}{2 \sin A \cos A} \\ &= \frac{1 + \cos^2 A}{\sin^2 A} \\ \cot A &= \frac{1 + \cos 15}{\sin 15} \\ &= \frac{1 + \cos (45 - 30)}{\sin 15} \\ &= \frac{1 + \left( \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} \right)}{\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}} \\ &= \frac{2\sqrt{2} + (\sqrt{3} + 1)}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} \\ &= \frac{2\sqrt{2}(\sqrt{3} + 1) + (\sqrt{3} + 1)^2}{3 - 1} \\ &= \frac{2\sqrt{6} + 2\sqrt{2} + 4 + 2\sqrt{3}}{2}\end{aligned}$$

$$\begin{aligned}\cot A &= \sqrt{6} + \sqrt{2} + 2 + \sqrt{3} \text{ ----- (1)} \\ &= \sqrt{2} + 2 + \sqrt{6} + \sqrt{3} \\ &= \sqrt{2} (1 + \sqrt{2}) + \sqrt{3} (\sqrt{2} + 1)\end{aligned}$$

$$\cot A = (\sqrt{2} + 1)(\sqrt{2} + \sqrt{3}) \text{ ----- (2)}$$

From equation (1) and (2)

$$\begin{aligned}\tan 82 \frac{1^\circ}{2} &= \cot 7 \frac{1^\circ}{2} = \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6} \\ &= (\sqrt{2} + 1)(\sqrt{2} + \sqrt{3})\end{aligned}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 27

We know that,

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

Put  $A = 45^\circ$ ,

$$\sin 22 \frac{1^\circ}{2} = \sqrt{\frac{1 - \cos 45^\circ}{2}} \quad \left\{ \text{since } \sin 22 \frac{1^\circ}{2}, \text{ is positive} \right\}$$

$$= \sqrt{\frac{1 - \frac{1}{\sqrt{2}}}{2}}$$

$$\sin 22 \frac{1^\circ}{2} = \sqrt{\frac{\sqrt{2} - 1}{2\sqrt{2}}}$$

And

$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$$

Put  $A = 45^\circ$

$$\cos 22 \frac{1^\circ}{2} = \sqrt{\frac{1 + \cos 45^\circ}{2}}$$

$$= \sqrt{\frac{1 + \frac{1}{\sqrt{2}}}{2}}$$

$$\cos 22 \frac{1^\circ}{2} = \sqrt{\frac{\sqrt{2} + 1}{2\sqrt{2}}}$$

Now,

$$\cot 22 \frac{1^\circ}{2} = \frac{\cos 22 \frac{1^\circ}{2}}{\sin 22 \frac{1^\circ}{2}}$$

$$= \sqrt{\frac{\sqrt{2} + 1}{2\sqrt{2}} \times \frac{2\sqrt{2}}{\sqrt{2} - 1}}$$

$$= \sqrt{\frac{\sqrt{2} + 1}{\sqrt{2} - 1}}$$

Rationalizing denominator,

$$= \sqrt{\frac{\sqrt{2} + 1 \times \sqrt{2} + 1}{\sqrt{2} - 1 \times \sqrt{2} + 1}}$$

$$= \sqrt{\frac{(\sqrt{2} + 1)^2}{2 - 1}}$$

$$\cot 22 \frac{1^\circ}{2} = \sqrt{2} + 1$$

\*\*\*\*\* END \*\*\*\*\*