Indefinite Integrals Ex 19.23 Q9

Let $\quad I = \int \dfrac{1}{\sin x + \cos x}\,dx$

Put $\quad \sin x = \dfrac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}},\ \cos x = \dfrac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$

$$I = \int \dfrac{1}{\dfrac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}} + \dfrac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}}\,dx$$

$$= \int \dfrac{1 + \tan^2\frac{x}{2}}{2\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2}}\,dx$$

$$= \int \dfrac{\sec^2\frac{x}{2}}{2\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2}}\,dx$$

Let $\quad \tan\dfrac{x}{2} = t$

$$\dfrac{1}{2}\sec^2\dfrac{x}{2}\,dx = dt$$

$$I = \int \dfrac{2dt}{2t + 1 - t^2}$$

$$= -2\int \dfrac{dt}{t^2 - 2t - 1}$$

$$I = -2\int \dfrac{dt}{t^2 - 2t + 1 - 1 - 1}$$

$$I = -2\int \dfrac{dt}{(t - 1)^2 - \left(\sqrt{2}\right)^2}$$

$$= 2\int \dfrac{2dt}{\left(\sqrt{2}\right)^2 - (t - 1)^2}$$

$$= \dfrac{2}{2\sqrt{2}}\log\left|\dfrac{\sqrt{2} + t - 1}{\sqrt{2} - t + 1}\right| + c$$

$$I = \dfrac{1}{\sqrt{2}}\log\left|\dfrac{\sqrt{2} + \tan\frac{x}{2} - 1}{\sqrt{2} - \tan\frac{x}{2} + 1}\right| + c$$

Indefinite Integrals Ex 19.23 Q10

Let $\quad I = \int \dfrac{1}{5 - 4\cos x}\,dx$

Put $\quad \cos x = \dfrac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$I = \int \dfrac{1}{5 - 4\left(\dfrac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right)}\,dx$$

$$= \int \dfrac{\left(1 + \tan^2 \frac{x}{2}\right)}{5 + 5\tan^2 \frac{x}{2} - 4 + 4\tan^2 \frac{x}{2}}\,dx$$

$$= \int \dfrac{\sec^2 \frac{x}{2}}{9\tan^2 \frac{x}{2} + 1}\,dx$$

Let $\quad 3\tan\dfrac{x}{2} = t$

$$\dfrac{3}{2}\sec^2 \dfrac{x}{2}\,dx = dt$$

$$I = \dfrac{2}{3}\int \dfrac{dt}{t^2 + 1}$$

$$= \dfrac{2}{3}\tan^{-1}(t) + c$$

$$I = \dfrac{2}{3}\tan^{-1}\left(3\tan\dfrac{x}{2}\right) + c$$

Indefinite Integrals Ex 19.23 Q11

Let $\quad I = \int \dfrac{1}{2 + \sin x + \cos x}\,dx$

Put $\quad \sin x = \dfrac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}, \cos x = \dfrac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$

$$I = \int \dfrac{1}{2 + \left(\dfrac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) + \left(\dfrac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right)}\,dx$$

$$= \int \dfrac{\left(1 + \tan^2\frac{x}{2}\right)}{2 + 2\tan^2\frac{x}{2} + 2\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2}}\,dx$$

$$= \int \dfrac{\sec^2\frac{x}{2}}{\tan^2\frac{x}{2} + 2\tan\frac{x}{2} + 3}\,dx$$

Let $\quad \tan\dfrac{x}{2} = t$

$$\dfrac{1}{2}\sec^2\dfrac{x}{2}\,dx = dt$$

$$I = \int \dfrac{2dt}{t^2 + 2t + 3}$$

$$= 2\int \dfrac{dt}{t^2 + 2t + 1 - 1 + 3}$$

$$I = 2\int \dfrac{dt}{(t + 1)^2 + \left(\sqrt{2}\right)^2}$$

$$= \dfrac{2}{\sqrt{2}}\tan^{-1}\left(\dfrac{t + 1}{\sqrt{2}}\right) + c$$

$$I = \sqrt{2}\tan^{-1}\left(\dfrac{\tan\frac{x}{2} + 1}{\sqrt{2}}\right) + c$$

Indefinite Integrals Ex 19.23 Q12

Let $\quad I = \int \dfrac{1}{\sin x + \sqrt{3}\cos x}\,dx$

Put $\quad \sin x = \dfrac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}, \cos x = \dfrac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$

$$I = \int \dfrac{1}{\left(\dfrac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) + \sqrt{3}\left(\dfrac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right)}\,dx$$

$$= \int \dfrac{\left(1 + \tan^2\frac{x}{2}\right)}{2\tan\frac{x}{2} + \sqrt{3} - \sqrt{3}\tan^2\frac{x}{2}}\,dx$$

$$= \int \dfrac{\sec^2\frac{x}{2}}{2\tan\frac{x}{2} + \sqrt{3} - \sqrt{3}\tan^2\frac{x}{2}}\,dx$$

Let $\quad \tan\dfrac{x}{2} = t$

$$\dfrac{1}{2}\sec^2\dfrac{x}{2}\,dx = dt$$

$$I = \int \dfrac{2\,dt}{2t + \sqrt{3} - \sqrt{3}t^2}$$

$$= -\dfrac{2}{\sqrt{3}}\int \dfrac{dt}{t^2 - \dfrac{2}{\sqrt{3}}t + \left(\dfrac{1}{\sqrt{3}}\right)^2 - \left(\dfrac{1}{\sqrt{3}}\right)^2 - 1}$$

$$I = -\dfrac{2}{\sqrt{3}}\int \dfrac{dt}{\left(t - \dfrac{1}{\sqrt{3}}\right)^2 - \left(\dfrac{2}{\sqrt{3}}\right)^2}$$

$$= \dfrac{2}{\sqrt{3}}\int \dfrac{dt}{\left(\dfrac{2}{\sqrt{3}}\right)^2 - \left(t - \dfrac{1}{\sqrt{3}}\right)^2}$$

$$= \dfrac{2}{\sqrt{3}} \times \dfrac{1}{2\left(\dfrac{2}{\sqrt{3}}\right)}\log\left|\dfrac{\dfrac{2}{\sqrt{3}} + t + \dfrac{1}{\sqrt{3}}}{\dfrac{2}{\sqrt{3}} - t + \dfrac{1}{\sqrt{3}}}\right| + c$$

$$I = \dfrac{1}{2}\log\left|\dfrac{\sqrt{3}t + 1}{3 - \sqrt{3}t}\right| + c$$

$$I = \dfrac{1}{2}\log\left|\dfrac{1 + \sqrt{3}\tan\frac{x}{2}}{3 - \sqrt{3}\tan\frac{x}{2}}\right| + c$$

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