Indefinite Integrals Ex 19.11 Q1

Let $I = \int \tan^3 x \sec^2 x dx$ ---(i)

Let $\tan x = t$. Then

$$d(\tan x) = dt$$

$$\Rightarrow \quad \sec^2 x dx = dt$$

$$\Rightarrow \quad dx = \frac{dt}{\sec^2 x}$$

Puting $\tan x = t$ and $dx = \dfrac{dt}{\sec^2 x}$ in equation (i), we get

$$I = \int t^3 \sec^2 x \times \frac{dt}{\sec^2 x}$$

$$= \int t^3 dt$$

$$= \frac{t^{3+1}}{3+1} + c$$

$$= \frac{t^4}{4} + c$$

$$= \frac{(\tan x)^4}{4} + c$$

$$\therefore \quad I = \frac{(\tan x)^4}{4} + c$$

$$= \frac{1}{4} \times \tan^4 x + c.$$

Indefinite Integrals Ex 19.11 Q2

Let $I = \int \tan x \sec^4 x dx$. Then

$$I = \int \tan x \sec^2 x \sec^2 x dx$$

$$= \int \tan x \left(1 + \tan^2 x\right) \sec^2 x dx$$

$$\Rightarrow \quad I = \int \left(\tan x + \tan^3 x\right) \sec^2 x dx$$

Substituting $\tan x = t$ and $\sec^2 x dx = dt$, we get

$$I = \int \left(t + t^3\right) dt$$

$$= \frac{t^2}{2} + \frac{t^4}{4} + c$$

$$= \frac{\tan^2 x}{2} + \frac{\tan^4}{4} + c$$

$$\therefore \quad I = \frac{1}{2} \times \tan^2 x + \frac{1}{4} \times \tan^4 x + c.$$

Indefinite Integrals Ex 19.11 Q3

Let $I = \int \tan^5 x \sec^4 x dx$. Then

$$I = \int \tan^4 x \sec^2 x \sec^2 x dx$$

$$= \int \tan^5 x \left(1 + \tan^2 x\right) \sec^2 x dx$$

$$= \int \left(\tan^5 x + \tan^7 x\right) \sec^2 x dx$$

Substituting $\tan x = t$ and $\sec^2 x dx = dt$, we get

$$I = \int \left(t^5 + t^7\right) dt$$

$$= \frac{t^6}{6} + \frac{t^8}{8} + c$$

$$= \frac{\left(\tan x\right)^6}{6} + \frac{\left(\tan x\right)^8}{8} + c$$

$$\therefore \quad I = \frac{1}{6} \times \tan^6 x + \frac{1}{8} \times \tan^8 x + c.$$

Indefinite Integrals Ex 19.11 Q4

Let $I = \int \sec^6 x \tan x dx$. Then

$$I = \int \sec^5 x \left(\sec x \tan x\right) dx$$

Substituting $\sec x = t$ and $\sec x \tan x = dt$, we get

$$I = \int t^5 dt$$

$$= \frac{t^6}{6} + c$$

$$= \frac{\left(\sec x\right)^6}{6} + c$$

$$\therefore \quad I = \frac{1}{6} \sec^6 x + c$$

*********** END **********