Definite Integrals Ex 20.4A Q1

We know

$$\int_0^{2\Pi} f(x)dx = \int_0^{2\Pi} f(2\Pi - x)dx$$

Hence

$$\int_0^{2\Pi} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}} dx = \int_0^{2\Pi} \frac{e^{\sin(2\Pi - x)}}{e^{\sin(2\Pi - x)} + e^{-\sin(2\Pi - x)}} dx$$

We know

$$\sin(2\Pi - x) = -\sin x$$

$$\int_0^{2\Pi} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}} dx = \int_0^{2\Pi} \frac{e^{-\sin x}}{e^{-\sin x} + e^{\sin x}} dx$$

If

$$I = \int_0^{2\Pi} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}} dx$$

Then also

$$I = \int_0^{2\Pi} \frac{e^{-\sin x}}{e^{-\sin x} + e^{\sin x}} dx$$

Hence

$$2I = \int_0^{2\Pi} \frac{e^{-\sin x}}{e^{-\sin x} + e^{\sin x}} dx + \int_0^{2\Pi} \frac{e^{\sin x}}{e^{-\sin x} + e^{\sin x}} dx$$

$$2I = \int_0^{2\Pi} \frac{e^{-\sin x}}{e^{-\sin x} + e^{\sin x}} + \frac{e^{\sin x}}{e^{-\sin x} + e^{\sin x}} dx$$

$$2I = \int_0^{2\Pi} dx$$

$$2I = 2\Pi$$

$$I = \Pi$$

Definite Integrals Ex 20.4A Q2

We know

$$\int_0^{2\pi} f(x)dx = \int_0^{2\pi} f(2\Pi - x)dx$$

Hence

$$\int_0^{2\pi} \log(\sec x + \tan x)dx = \int_0^{2\pi} \log(\sec(2\Pi - x) + \tan(2\Pi - x))dx$$

$$\int_0^{2\pi} \log(\sec x + \tan x)dx = \int_0^{2\pi} \log(\sec x - \tan x)dx$$

If

$$I = \int_0^{2\pi} \log(\sec x + \tan x)dx$$

Then

$$I = \int_0^{2\pi} \log(\sec x - \tan x)dx$$

$$2I = \int_0^{2\pi} \log(\sec x + \tan x)dx + \int_0^{2\pi} \log(\sec x - \tan x)dx$$

$$2I = \int_0^{2\pi} \log(\sec x + \tan x) + \log(\sec x - \tan x)dx$$

$$2I = \int_0^{2\pi} \log(\sec^2 x - \tan^2 x)dx$$

$$2I = \int_0^{2\pi} \log(1)dx$$

$$2I = 0$$

$$I = 0$$

Definite Integrals Ex 20.4A Q3

We know

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

Hence

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan(\frac{\Pi}{2}-x)}}{\sqrt{\tan(\frac{\Pi}{2}-x)}+\sqrt{\cot(\frac{\Pi}{2}-x)}}dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$$

If

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$$

Then

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$$

So

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}} + \frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1dx$$

$$2I = \frac{\Pi}{6}$$

$$I = \frac{\Pi}{12}$$

*********** END **********