Algebra of Matrices Ex 5.3 Q52

Solving the LHS i.e.

$$A^2 + A = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}^2 + \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & -6 \\ 4 & 4 & 0 \\ 2 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & -9 \\ 6 & 5 & 3 \\ 2 & 3 & 5 \end{bmatrix}$$

Solving the RHS i.e.

$$A(A+I) = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & -3 \\ 2 & 2 & 3 \\ 2 & 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & -9 \\ 6 & 5 & 3 \\ 2 & 3 & 5 \end{bmatrix}$$

So, LHS = RHS verified.

Algebra of Matrices Ex 5.3 Q53

We have,

$$A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

Now, $A^2 = AA = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} (3 \times 3) + (-5 \times -4) & (3 \times -5) + (-5 \times 2) \\ (-4 \times 3) + (2 \times -4) & (-4 \times -5) + (2 \times 2) \end{bmatrix}$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix},$$

$$-5A = \begin{bmatrix} -15 & 25 \\ 20 & -10 \end{bmatrix} \quad \text{and} \quad -14I = \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix}$$

$$\therefore \quad A^2 - 5A - 14I = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + \begin{bmatrix} -15 & 25 \\ 20 & -10 \end{bmatrix} + \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix}$$

$$= \begin{bmatrix} 29 - 15 - 14 & -25 + 25 + 0 \\ -20 + 20 + 0 & 24 - 10 + -14 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Now,

$$A^2 - 5A - 14I = 0$$

$\Rightarrow \quad A^2 = 5A + 14I$

$\Rightarrow \quad A^3 = A^2.A = (5A + 14I)A$

$\Rightarrow \quad A^3 = A^2.A = 5A^2 + 14A \qquad \begin{bmatrix} \text{By using dist. of matrices over} \\ \text{matrix addition} \end{bmatrix}$

$\Rightarrow \quad A^3 = 5\begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + 14\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$\Rightarrow \quad A^3 = \begin{bmatrix} 145 & -125 \\ -100 & 120 \end{bmatrix} + \begin{bmatrix} 42 & -70 \\ -56 & 28 \end{bmatrix}$

$\Rightarrow \quad A^3 = \begin{bmatrix} 187 & -195 \\ -156 & 148 \end{bmatrix}$

Algebra of Matrices Ex 5.3 Q54

We have,

$$P(x).P(y) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}\begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$$

$\Rightarrow P(x).P(y) = \begin{bmatrix} \cos x \cos y - \sin x \sin y & \sin y \cos x + \sin x \cos y \\ -\sin x \cos y - \cos x \sin y & -\sin x \sin y + \cos x \cos y \end{bmatrix}$

$\Rightarrow P(x).P(y) = \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} = P(x+y)$

Now,

$$P(y).P(x) = \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

$\Rightarrow P(y).P(x) = \begin{bmatrix} \cos y \cos x - \sin y \sin x & \sin x \cos y + \sin y \cos x \\ -\sin y \cos x - \cos y \sin x & -\sin y \sin x + \cos y \cos x \end{bmatrix}$

$\Rightarrow P(y).P(x) = \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} = P(x+y)$

$\therefore \quad P(x).P(y) = P(x+y) = P(y).P(x)$

We have,

$$P = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}, \ Q = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

So, $\quad PQ = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

$$= \begin{bmatrix} x \times a & 0 & 0 \\ 0 & y \times b & 0 \\ 0 & 0 & z \times c \end{bmatrix}$$

$$= \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix}$$

and $\ QP = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

$$= \begin{bmatrix} a \times x & 0 & 0 \\ 0 & b \times y & 0 \\ 0 & 0 & c \times z \end{bmatrix}$$

$$= \begin{bmatrix} ax & 0 & 0 \\ 0 & by & 0 \\ 0 & 0 & zc \end{bmatrix}$$

as, $xa = ax$, $yb = by$, $zc = cz$

$\therefore \ PQ = \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix} = QP.$

********** END *********