Binomial Theorem Ex 18.1 Q1(vi)

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $\left(\sqrt{\frac{x}{a}}-\sqrt{\frac{a}{x}}\right)^6$ has 7 terms.

Using binomial theorem to expand, we get

$$\left(\sqrt{\frac{x}{a}}-\sqrt{\frac{a}{x}}\right)^6 = {}^6C_0\left(\sqrt{\frac{x}{a}}\right)^6\left(\sqrt{\frac{a}{x}}\right)^0 - {}^6C_1\left(\sqrt{\frac{x}{a}}\right)^5\left(\sqrt{\frac{a}{x}}\right)^1 + {}^6C_2\left(\sqrt{\frac{x}{a}}\right)^4\left(\sqrt{\frac{a}{x}}\right)^2 - {}^6C_3\left(\sqrt{\frac{x}{a}}\right)^3\left(\sqrt{\frac{a}{x}}\right)^3$$

$$+{}^6C_4\left(\sqrt{\frac{x}{a}}\right)^2\left(\sqrt{\frac{a}{x}}\right)^4 - {}^6C_5\left(\sqrt{\frac{x}{a}}\right)\left(\sqrt{\frac{a}{x}}\right)^5 + {}^6C_6\left(\sqrt{\frac{x}{a}}\right)^0\left(\sqrt{\frac{a}{x}}\right)^6$$

$$=\left(\frac{x}{a}\right)^{\frac{1}{2}\times 6} - 6\left(\frac{x}{a}\right)^{\frac{1}{2}\times 5}\left(\frac{a}{x}\right)^{\frac{1}{2}} + 15\left(\frac{x}{a}\right)^{\frac{1}{2}\times 4}\left(\frac{a}{x}\right)^{2\times\frac{1}{2}} - 20\left(\frac{x}{a}\right)^{3\times\frac{1}{2}}\left(\frac{a}{x}\right)^{3\times\frac{1}{2}} + 15\left(\frac{x}{a}\right)^{2\times\frac{1}{2}}\left(\frac{a}{x}\right)^{4\times\frac{1}{2}}$$

$$-6\left(\frac{x}{a}\right)^{\frac{1}{2}}\left(\frac{a}{x}\right)^{5\times\frac{1}{2}} + \left(\frac{a}{x}\right)^{6\times\frac{1}{2}}$$

$$=\frac{x^3}{a^3} - 6\frac{x^{\frac{5}{2}-\frac{1}{2}}}{a^{\frac{5}{2}-\frac{1}{2}}} + 15\times\frac{x^2\times a}{a^2\times x} - 20\times\frac{x^{\frac{3}{2}-\frac{3}{2}}}{a^{\frac{3}{2}-\frac{3}{2}}} + 15\times\frac{x}{a}\times\frac{a^2}{x^2} - 6\times\frac{x^{\frac{1}{2}-\frac{5}{2}}}{a^{\frac{1}{2}-\frac{5}{2}}} + \frac{a^3}{x^3}$$

$$=\frac{x^3}{a^3} - \frac{6x^2}{a^2} + \frac{15x}{a} - 20 + \frac{15a}{x} - \frac{6a^2}{x^2} + \frac{a^3}{x^3}$$

Binomial Theorem Ex 18.1 Q1(vii)

$$\left(\sqrt[3]{x}-\sqrt[3]{a}\right)^6$$

$$=\binom{6}{0}\left(\sqrt[3]{x}\right)^6\left(-\sqrt[3]{a}\right)^0 + \binom{6}{1}\left(\sqrt[3]{x}\right)^5\left(-\sqrt[3]{a}\right)^1 + \binom{6}{2}\left(\sqrt[3]{x}\right)^4\left(-\sqrt[3]{a}\right)^2$$

$$\binom{6}{3}\left(\sqrt[3]{x}\right)^3\left(-\sqrt[3]{a}\right)^3 + \binom{6}{4}\left(\sqrt[3]{x}\right)^2\left(-\sqrt[3]{a}\right)^4 + \binom{6}{5}\left(\sqrt[3]{x}\right)^1\left(-\sqrt[3]{a}\right)^5$$

$$\binom{6}{6}\left(\sqrt[3]{x}\right)^0\left(-\sqrt[3]{a}\right)^6$$

$$= x^2 - 6x^{\frac{5}{3}}a^{\frac{1}{3}} + 15x^{\frac{4}{3}}a^{\frac{2}{3}} - 20ax + 15x^{\frac{2}{3}}a^{\frac{4}{3}} - 6x^{\frac{1}{3}}a^{\frac{5}{3}} + a^2$$

Binomial Theorem Ex 18.1 Q1(viii)

Let y = $1+2x$, then

$$\left(1+2x-3x^2\right)^5 = \left(y-3x^2\right)^5$$

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $\left(y-3x^2\right)^5$ has 6 terms.

Using binomial theorem to expand, we get

$$\begin{aligned}\left(y-3x^2\right)^5 &= {}^5C_0 y^5\left(3x^2\right)^0 - {}^5C_1 y^4\left(3x^2\right)^1 + {}^5C_2 y^3\left(3x^2\right)^2 - {}^5C_3 y^2\left(3x^2\right)^3 + {}^5C_4 y\left(3x^2\right)^4 - {}^5C_5 y^0\left(3x^2\right)^5 \\ &= y^5 - 5y^4\ 3x^2\ + 10y^3\ 9x\ ^4 - 10y^2\left(27x^6\right) + 5y81x^8 - 243x^{10}\end{aligned}$$

Now,

$$\begin{aligned}y^5 &= (1+2x)^5 = {}^5C_0 + {}^5C_1(2x)^1 + {}^5C_2(2x)^2 + {}^5C_3(2x)^3 + {}^5C_4(2x)^4 + {}^5C_5(2x)^5 \\ y^4 &= (1+2x)^4 = {}^4C_0 + {}^4C_1(2x)^1 + {}^4C_2(2x)^2 + {}^4C_3(2x)^3 + {}^4C_4(2x)^4 \\ y^3 &= (1+2x)^3 = {}^3C_0 + {}^3C_1(2x) + {}^3C_2(2x)^2 + {}^3C_3(2x)^3 \\ y^2 &= (1+2x)^2 = {}^2C_0 + {}^2C_1(2x) + {}^2C_2(2x)^2 \\ y &= (1+2x)\end{aligned}$$

Substituting the valus of powers of y in the equation above, we get,

$$\begin{aligned}\left(1+2x-3x^2\right)^5 = &\left[{}^5C_0 + {}^5C_1(2x)^1 + {}^5C_2(2x)^2 + {}^5C_3(2x)^3 + {}^5C_4(2x)^4 + {}^5C_5(2x)^5\right] \\ &-15x^2\left[{}^4C_0 + {}^4C_1(2x)^1 + {}^4C_2(2x)^2 + {}^4C_3(2x)^3 + {}^4C_4(2x)^4\right] \\ &+90x^4\left[{}^3C_0 + {}^3C_1(2x) + {}^3C_2(2x)^2 + {}^3C_3(2x)^3\right] - 270x^6 \\ &\left[{}^2C_0 + {}^2C_1(2x) + {}^2C_2(2x)^2 + 5 \times 81x^8(1+2x) - 243x^{10}\right]\end{aligned}$$

$$\begin{aligned}= \quad & 10 + 10x + 10 \times 4x^2 + 10 \times 8x^3 + 5 \times 16x^4 + 32x^5 - 15x^2 - 120x^3 \\ & -180x^4 + 480x^5 - 240x^6 + 90x^4 + 540x^5 + 1080x^6 + 720x^7 - 270x^6 \\ & -1080x^7 - 1080x^8 + 405x^8 + 810x^9 - 243x^{10}\end{aligned}$$

$$= 1 + 10x + 25x^2 - 40x^3 - 190x^4 + 92x^5 + 570x^6 - 360x^7 - 675x^8 + 810x^9 - 243x^{10}$$

*********** END **********