



Differentiation Ex 11.3 Q16

Let $y = \tan^{-1} \left\{ \frac{4x}{1-4x^2} \right\}$

Put $2x = \tan \theta$, so

$$y = \tan^{-1} \left\{ \frac{2 \tan \theta}{1 - \tan^2 \theta} \right\}$$

$$y = \tan^{-1} \{ \tan 2\theta \} \quad \text{---(i)}$$

Here, $-\frac{1}{2} < x < \frac{1}{2}$

$$\Rightarrow -1 < 2x < 1$$

$$\Rightarrow -1 < \tan \theta < 1$$

$$\Rightarrow -\frac{\pi}{4} < \theta < \frac{\pi}{4}$$

$$\Rightarrow -\frac{\pi}{2} < (2\theta) < \frac{\pi}{2}$$

So, from equation (i),

$$y = 2\theta$$

$$\left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right]$$

$$y = 2 \tan^{-1}(2x)$$

$$[\text{Since, } 2x = \tan \theta]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = 2 \left(\frac{1}{1+(2x)^2} \right) \frac{d}{dx}(2x)$$

$$\frac{dy}{dx} = \frac{4}{1+4x^2}.$$

Differentiation Ex 11.3 Q17

$$\text{Let } y = \tan^{-1} \left\{ \frac{2^{x+1}}{1-4^x} \right\}$$

$$\text{Put } 2^x = \tan \theta, \text{ so,}$$

$$= \tan^{-1} \left\{ \frac{2^x \times 2}{1 - (2^x)^2} \right\}$$

$$= \tan^{-1} \left\{ \frac{2 \tan \theta}{1 - \tan^2 \theta} \right\}$$

$$y = \tan^{-1} \{ \tan(2\theta) \} \quad \text{--- (i)}$$

$$\text{Here, } -\infty < x < \infty$$

$$\Rightarrow 2^{-\infty} < 2^x < 2^{\infty}$$

$$\Rightarrow 0 < 2^x < 1$$

$$\Rightarrow 0 < \theta < \frac{\pi}{4}$$

$$\Rightarrow 0 < (2\theta) < \frac{\pi}{2}$$

$$\text{From equatoin (i),}$$

$$y = 2\theta \quad \left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) \right]$$

$$y = 2 \tan^{-1} (2^x)$$

Differentiate it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{2}{1 + (2^x)^2} \frac{d}{dx} (2^x)$$

$$= \frac{2 \times 2^x \log 2}{1 + 4^x}$$

$$\frac{dy}{dx} = \frac{2^{x+1} \log 2}{1 + 4^x}.$$

Differentiation Ex 11.3 Q18

$$\text{Let } y = \tan^{-1} \left\{ \frac{2a^x}{1-a^{2x}} \right\}$$

$$\text{Put } a^x = \tan \theta,$$

$$y = \tan^{-1} \left\{ \frac{2 \tan \theta}{1 - \tan^2 \theta} \right\}$$

$$y = \tan^{-1} \{ \tan(2\theta) \} \quad \text{--- (i)}$$

$$\text{Here, } -\infty < x < \infty$$

$$\Rightarrow a^{-\infty} < a^x < a^{\infty}$$

$$\Rightarrow 0 < \tan \theta < 1$$

$$\Rightarrow 0 < \theta < \frac{\pi}{4}$$

$$\Rightarrow 0 < (2\theta) < \frac{\pi}{2}$$

$$\text{From equatoin (i),}$$

$$y = 2\theta \quad \left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) \right]$$

$$y = 2 \tan^{-1} (a^x)$$

Differentiate it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{2}{1 + (a^x)^2} \frac{d}{dx} (a^x)$$

$$\frac{dy}{dx} = \frac{2a^x \log a}{1 + a^{2x}}.$$

Differentiation Ex 11.3 Q19

$$\text{Let } y = \sin^{-1} \left\{ \frac{\sqrt{1+x} + \sqrt{1-x}}{2} \right\}$$

$$\text{Put } x = \cos 2\theta, \text{ so,}$$

$$= \sin^{-1} \left\{ \frac{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}}{2} \right\}$$

$$= \sin^{-1} \left\{ \frac{\sqrt{2\cos^2 \theta} + \sqrt{2\sin^2 \theta}}{2} \right\}$$

$$= \sin^{-1} \left\{ \frac{\sqrt{2}\cos \theta + \sqrt{2}\sin \theta}{2} \right\}$$

$$= \sin^{-1} \left\{ \cos \theta \left(\frac{1}{\sqrt{2}} \right) + \left(\frac{1}{\sqrt{2}} \right) \sin \theta \right\}$$

$$= \sin^{-1} \left\{ \cos \theta \sin \left(\frac{\pi}{4} \right) + \cos \frac{\pi}{4} \sin \theta \right\}$$

$$y = \sin^{-1} \left\{ \sin \left(\theta + \frac{\pi}{4} \right) \right\} \quad \text{---(i)}$$

$$\text{Here, } 0 < x < 1$$

$$\Rightarrow 0 < \cos 2\theta < 1$$

$$\Rightarrow 0 < 2\theta < \frac{\pi}{2}$$

$$\Rightarrow 0 < \theta < \frac{\pi}{4}$$

$$\Rightarrow \frac{\pi}{4} < \left(\theta + \frac{\pi}{4} \right) < \frac{\pi}{2}$$

So, from equatoin (i),

$$y = \theta + \frac{\pi}{4}$$

$$y = \frac{1}{2} \cos^{-1} x + \frac{\pi}{4}$$

$$\left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) \right]$$

Differentiate it with respect to x ,

$$\frac{dy}{dx} = \frac{1}{2} \left(\frac{-1}{\sqrt{1-x^2}} \right) + 0$$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q20

$$\text{Let } y = \tan^{-1} \left(\frac{\sqrt{1+a^2x^2} - 1}{ax} \right)$$

$$\text{Put } ax = \tan \theta$$

$$y = \tan^{-1} \left(\frac{\sqrt{1+a^2x^2} - 1}{ax} \right)$$

$$= \tan^{-1} \left(\frac{\sec \theta - 1}{\tan \theta} \right)$$

$$= \tan^{-1} \left(\frac{1 - \cos \theta}{\sin \theta} \right)$$

$$= \tan^{-1} \left(\frac{\frac{2 \sin^2 \theta}{2}}{\frac{2 \sin \theta}{2} \frac{\cos \theta}{2}} \right)$$

$$y = \tan^{-1} \left(\frac{\tan \theta}{2} \right)$$

$$= \frac{\theta}{2}$$

$$y = \frac{1}{2} \tan^{-1}(ax)$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{2} \times \left(\frac{1}{1+(ax)^2} \right) \frac{d}{dx}(ax)$$

$$\frac{dy}{dx} = \frac{1}{2(1+a^2x^2)} (a)$$

$$\frac{dy}{dx} = \frac{a}{2(1+a^2x^2)}.$$

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