Mathematical Induction Ex 12.2 Q47

Let $P(n)$ be the statement given by

$P(n): x_n = \frac{2}{n!}$ for all $n \in N$.

Step I:

$P(2): x_2 = \frac{2}{2!} = 1$

Given that $x_k = \frac{x_{k-1}}{n}$ for all natural numbers $k \geq 2$

$x_2 = \frac{x_1}{2} = \frac{2}{2} = 1$

$\therefore$ $P(2)$ is true.

Step II:

Let $P(m)$ is true. Then,

$x_m = \frac{2}{m!}$..........(i)

We have to prove that $P(m+1)$ is true.

$$x_{m+1} = \frac{x_{m+1-1}}{m+1}$$

$$x_{m+1} = \frac{x_m}{m+1}$$

$$x_{m+1} = \frac{\frac{2}{m!}}{m+1}\text{............[from(i)]}$$

$$x_{m+1} = \frac{2}{m!(m+1)}$$

$$x_{m+1} = \frac{2}{(m+1)!}$$

$\Rightarrow P(m+1)$ is true.

Hence by the principle of mathematical induction, the given result is true for all $n \in N$.

Mathematical Induction Ex 12.2 Q48

Let $P(n)$ be the statement given by

$P(n): x_n = 5 + 4n$ for all $n \in N$.

Step I:

$P(1): x_1 = 5 + 4(1) = 5 + 4 = 9$

Given that $x_k = 4 + x_{k-1}$ for all natural numbers k

$x_1 = 4 + x_0 = 4 + 5 = 9$

$\therefore P(1)$ is true.

Step II:

Let $P(m)$ is true. Then,

$x_m = 5 + 4m$.........(i)

We have to prove that $P(m+1)$ is true.

$x_{m+1} = 4 + x_{m+1-1}$

$x_{m+1} = 4 + x_m$

$x_{m+1} = 4 + 5 + 4m$...........[from(i)]

$x_{m+1} = 5 + 4(m+1)$

$\Rightarrow P(m+1)$ is true.

Hence by the principle of mathematical induction, the given result is true for all $n \in N$.

Mathematical Induction Ex 12.2 Q49

Let $P(n)$ be the statement given by

$P(n): \sqrt{n} < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots\ldots + \frac{1}{\sqrt{n}}$ for all natural numbers $n \geq 2$.

Step I:

$P(2): \sqrt{2} = 1.4142$

$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} = 1 + \frac{1}{1.4142} = 1 + 0.7071 = 1.7071$

$\therefore \sqrt{2} < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}}$

$\therefore P(2)$ is true.

Step II:

Let $P(m)$ is true. Then,

$\sqrt{m} < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots\ldots + \frac{1}{\sqrt{m}}$..........(i)

We have to prove that $P(m+1)$ is true.

$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots\ldots + \frac{1}{\sqrt{m}} > \sqrt{m}$...........[from(i)]

$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots\ldots + \frac{1}{\sqrt{m}} + \frac{1}{\sqrt{m+1}} > \sqrt{m} + \frac{1}{\sqrt{m+1}}$

$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots\ldots + \frac{1}{\sqrt{m}} + \frac{1}{\sqrt{m+1}} > \frac{\sqrt{m^2 + m} + 1}{\sqrt{m+1}}$

$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots\ldots + \frac{1}{\sqrt{m}} + \frac{1}{\sqrt{m+1}} > \frac{\sqrt{m^2} + 1}{\sqrt{m+1}}$

$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots\ldots + \frac{1}{\sqrt{m}} + \frac{1}{\sqrt{m+1}} > \frac{m+1}{\sqrt{m+1}}$

$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots\ldots + \frac{1}{\sqrt{m}} + \frac{1}{\sqrt{m+1}} > \sqrt{m+1}$

$\Rightarrow P(m+1)$ is true.

Hence by the principle of mathematical induction, the given result is true for all $n \in N$.

Mathematical Induction Ex 12.2 Q50

The distributive law from algebra states that for all real numbers c, a1 and a2, we have c (a1 + a2) = ca1 + ca2
Use this law and mathematical induction to prove that, for all natural numbers, n   2, if c (a1 + a2 + &. + an)

= ca1 + ca2 + ... + can

Let P(n) be the statement given by
$P(n): c(a_1 + a_2 + ... + a_n) = ca_1 + ca_2 + ca_3 + ... + ca_n$ for all natural numbers $n \geq 2$.

Step I:
$P(2): c(a_1 + a_2) = ca_1 + ca_2$

$\therefore$ P(2) is true.

Step II:
Let P(m) is true. Then,
$c(a_1 + a_2 + ... + a_m) = ca_1 + ca_2 + ca_3 + ... + ca_m$..........(i)

We have to prove that P(m + 1) is true.
$c(a_1 + a_2 + ... + a_m + a_{m+1}) = c[(a_1 + a_2 + ... + a_m) + a_{m+1}]$
$c(a_1 + a_2 + ... + a_m + a_{m+1}) = c(a_1 + a_2 + ... + a_m) + ca_{m+1}$
$c(a_1 + a_2 + ... + a_m + a_{m+1}) = ca_1 + ca_2 + ca_3 + ... + ca_m + ca_{m+1}$
...........[from(i)]

$\Rightarrow$ P(m+1) is true.

Hence by the principle of mathematical induction, the given result is true for all $n \in N$.

*********** END **********