



Arithmetic Progressions Ex 9.5 Q5

Answer :

In the given problem, we have the sum of the certain number of terms of an A.P. and we need to find the number of terms.

(i) A.P. is 18, 16, 14, ...

So here, let us find the number of terms whose sum is 0. For that, we will use the formula,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

Where; a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

The first term (a) = 18

The sum of n terms (S_n) = 0

Common difference of the A.P. (d) = $a_2 - a_1$

$$= 16 - 18$$

$$= -2$$

So, on substituting the values in the formula for the sum of n terms of an A.P., we get,

$$0 = \frac{n}{2} [2(18) + (n-1)(-2)]$$

$$0 = \left(\frac{n}{2}\right) [36 + (-2n+2)]$$

$$0 = \left(\frac{n}{2}\right) [38 - 2n]$$

Further,

$$\frac{n}{2} = 0$$

$$n = 0$$

Or,

$$38 - 2n = 0$$

$$n = \frac{-38}{-2}$$

$$n = 19$$

Since, the number of terms cannot be zero, the number of terms (n) is $\boxed{n = 19}$.

(ii) Here, let us take the common difference as d .

So, we are given,

First term (a_1) = -14

Fifth term (a_5) = 2

Sum of terms (s_n) = 40

Now,

$$a_5 = a_1 + 4d$$

$$2 = -14 + 4d$$

$$2 + 14 = 4d$$

$$d = \frac{16}{4}$$

$$d = 4$$

Further, let us find the number of terms whose sum is 40. For that, we will use the formula,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

Where; a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

The first term (a_1) = -14

The sum of n terms (S_n) = 40

Common difference of the A.P. (d) = 4

So, on substituting the values in the formula for the sum of n terms of an A.P., we get,

$$40 = \frac{n}{2} [2(-14) + (n-1)(4)]$$

$$40 = \left(\frac{n}{2}\right) [-28 + (4n-4)]$$

$$40 = \left(\frac{n}{2}\right) [-32 + 4n]$$

$$40(2) = -32n + 4n^2$$

So, we get the following quadratic equation,

$$4n^2 - 32n - 80 = 0$$

$$n^2 - 8n + 20 = 0$$

On solving by splitting the middle term, we get,

$$n^2 - 10n + 2n + 20 = 0$$

$$n(n-10) + 2(n-10) = 0$$

$$(n+2)(n-10) = 0$$

Further,

$$n+2 = 0$$

$$n = -2$$

Or,

$$n-10 = 0$$

$$n = 10$$

Since the number of terms cannot be negative. Therefore, the number of terms (n) is $\boxed{n = 10}$

(iii) A.P. is 9, 17, 25, ...

So here, let us find the number of terms whose sum is 636. For that, we will use the formula,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

Where; a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

The first term (a) = 9

The sum of n terms (S_n) = 636

Common difference of the A.P. (d) = $a_2 - a_1$

$$= 17 - 9$$

$$= 8$$

So, on substituting the values in the formula for the sum of n terms of an A.P., we get,

$$636 = \frac{n}{2} [2(9) + (n-1)(8)]$$

$$636 = \left(\frac{n}{2}\right)[18 + (8n - 8)]$$

$$636 = \left(\frac{n}{2}\right)[10 + 8n]$$

$$636(2) = 10n + 8n^2$$

So, we get the following quadratic equation,

$$8n^2 + 10n - 1272 = 0$$

$$4n^2 + 5n - 636 = 0$$

On solving by splitting the middle term, we get,

$$4n^2 - 48n + 53n - 636 = 0$$

$$4n(n - 12) - 53(n - 12) = 0$$

$$(4n - 53)(n - 12) = 0$$

Further,

$$4n - 53 = 0$$

$$n = \frac{53}{4}$$

Or,

$$n - 12 = 0$$

$$n = 12$$

Since, the number of terms cannot be a fraction, the number of terms (n) is 12.

(iv) A.P. is 63, 60, 57, ...

So here, let us find the number of terms whose sum is 693. For that, we will use the formula,

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Where; a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

The first term (a) = 63

The sum of n terms (S_n) = 693

Common difference of the A.P. (d) = $a_2 - a_1$

$$= 60 - 63$$

$$= -3$$

So, on substituting the values in the formula for the sum of n terms of an A.P., we get,

$$693 = \frac{n}{2}[2(63) + (n - 1)(-3)]$$

$$693 = \left(\frac{n}{2}\right)[126 + (-3n + 3)]$$

$$693 = \left(\frac{n}{2}\right)[129 - 3n]$$

$$693(2) = 129n - 3n^2$$

So, we get the following quadratic equation,

$$3n^2 - 129n + 1386 = 0$$

$$n^2 - 43n + 462 = 0$$

On solving by splitting the middle term, we get,

$$n^2 - 22n - 21n + 462 = 0$$

$$n(n - 22) - 21(n - 22) = 0$$

$$(n - 22)(n - 21) = 0$$

Further,

$$n - 22 = 0$$

$$n = 22$$

Or,

$$n - 21 = 0$$

$$n = 21$$

Here, 22nd term will be

$$a_{22} = a_1 + 21d$$

$$= 63 + 21(-3)$$

$$= 63 - 63$$

$$= 0$$

So, the sum of 22 as well as 21 terms is 693. Therefore, the number of terms (n) is 21 or 22.

***** END *****

