Indefinite Integrals Ex 19.24 Q4

Let $\quad I = \int \frac{1}{p + q\tan x} dx$

$$= \int \frac{1}{p + q\left(\frac{\sin x}{\cos x}\right)} dx$$

$$= \int \frac{\cos x}{p\cos x + q\sin x} dx$$

Let $\quad \cos x = \lambda \frac{d}{dx}\left(p\cos x + q\sin x\right) + \mu\left(p\cos x + q\sin x\right) + \nu$

$$\cos x = \lambda\left(-p\sin x + q\cos x\right) + \mu\left(p\cos x + q\sin x\right) + \nu$$

$$\cos x = \left(-p\lambda + q\mu\right)\sin x + \left(q\lambda + p\mu\right)\cos x + \nu$$

Camparing the coefficients of $\sin x, \cos x$ on the both the sides,

$$-p\lambda + q\mu = 0 ------ (1)$$

$$q\lambda + p\mu = 1 ------- (2)$$

$$\nu = 0 ---------- (3)$$

Solving equation $(1), (2)$ and (3),

$$\lambda = \frac{q}{\left(p^2 + q^2\right)}$$

$$\mu = \frac{p}{\left(p^2 + q^2\right)}$$

$$\nu = 0$$

Now,

$$I = \int \frac{q}{\left(p^2 + q^2\right)} \frac{\left(-p\sin x + q\cos x\right)}{\left(p\cos x + q\sin x\right)} dx + \int \frac{p}{\left(p^2 + q^2\right)} \frac{\left(p\cos x + q\sin x\right)}{\left(p\cos x + q\sin x\right)} dx$$

$$I = \frac{q}{\left(p^2 + q^2\right)}\left(\log\left|p\cos x + q\sin x\right|\right) + \frac{p}{\left(p^2 + q^2\right)} x + c$$

Indefinite Integrals Ex 19.24 Q5

Let $I = \int \frac{5\cos x + 6}{2\cos x + \sin x + 3} dx$

Let $(5\cos x + 6) = \lambda \frac{d}{dx}(2\cos x + \sin x + 3) + \mu(2\cos x + \sin x + 3) + \nu$

$(5\cos x + 6) = \lambda(-2\sin x + \cos x) + \mu(2\cos x + \sin x + 3) + \nu$

$(5\cos x + 6) = (-2\lambda + \mu)\sin x + (\lambda + 2\mu)\cos x + (3\mu + \nu)$

Camparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$-2\lambda + \mu = 0 ------(1)$

$\lambda + 2\mu = 5 -------(2)$

$3\mu + \nu = 6 -------(3)$

Solving equation $(1),(2)$ and (3),

$\lambda = 1$

$\mu = 2$

$\nu = 0$

Now,

$I = \int \frac{(-2\sin x + \cos x)}{(2\cos x + \sin x + 3)} dx + 2\int dx$

$I = \log|2\cos x + \sin x + 3| + 2x + c$

Indefinite Integrals Ex 19.24 Q6

Let $I = \int \frac{2\sin x + 3\cos x}{3\sin x + 4\cos x} dx$

Let $(2\sin x + 3\cos x) = \lambda \frac{d}{dx}(3\sin x + 4\cos x) + \mu(3\sin x + 4\cos x) + \nu$

$(2\sin x + 3\cos x) = \lambda(3\cos x - 4\sin x) + \mu(3\sin x + 4\cos x) + \nu$

$(2\sin x + 3\cos x) = (3\lambda + 4\mu)\cos x + (-4\lambda + 3\mu)\sin x + \nu$

Camparing the coefficients of $\sin x, \cos x$ on the both the sides,

$3\lambda + 4\mu = 3 ------(1)$

$-4\lambda + 3\mu = 2 -----(2)$

$\nu = 0 -------(3)$

Solving the equation $(1),(2)$ and (3),

$\lambda = \frac{1}{25}$

$\mu = \frac{18}{25}$

$\nu = 0$

$I = \frac{1}{25}\int \frac{(3\cos x - 4\sin x)}{(3\sin x + 4\cos x)} dx + \frac{18}{25}\int dx$

$I = \frac{1}{25}\log|3\sin x + 4\cos x| + \frac{18}{25}x + c$

Indefinite Integrals Ex 19.24 Q7

Let $\quad I = \int \frac{1}{3 + 4\cot x} dx$

$$= \int \frac{\sin x}{3\sin x + 4\cos x} dx$$

Let $\quad \sin x = \lambda \frac{d}{dx}\left(3\sin x + 4\cos x\right) + \mu\left(3\sin x + 4\cos x\right) + \nu$

$$\sin x = \lambda\left(3\cos x - 4\sin x\right) + \mu\left(3\sin x + 4\cos x\right) + \nu$$

$$\sin x = \left(3\lambda + 4\mu\right)\cos x + \left(-4\lambda + 3\mu\right)\sin x + \nu$$

Camparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$$3\lambda + 4\mu = 0 ------ (1)$$

$$-4\lambda + 3\mu = 1 ----- (2)$$

$$\nu = 0 ------- (3)$$

Solving the equation $(1),(2)$ and (3), we get

$$\lambda = -\frac{4}{25}$$

$$\mu = \frac{3}{25}$$

$$\nu = 0$$

$$I = -\frac{4}{25}\int \frac{\left(3\cos x - 4\sin x\right)}{\left(3\sin x + 4\cos x\right)} dx + \frac{3}{25}\int dx$$

$$I = -\frac{4}{25}\log\left|3\sin x + 4\cos x\right| + \frac{3}{25}x + c$$

*********** END **********