



Indefinite Integrals Ex 19.25 Q46

$$\begin{aligned}
 \text{Let } I &= \int (e^{\log x} + \sin x) \cos x \, dx \\
 &= \int (x + \sin x) \cos x \, dx \\
 &= \int x \cos x \, dx + \int \sin x \cos x \, dx \\
 &= \left[x \int \cos x \, dx - \int (1 \int \cos x \, dx) dx \right] + \frac{1}{2} \int \sin 2x \, dx \\
 &= [x \sin x - \int \sin x \, dx] + \frac{1}{2} \left(-\frac{\cos 2x}{2} \right) + c
 \end{aligned}$$

$$I = x \sin x + \cos x - \frac{1}{4} \cos 2x + c$$

$$I = x \sin x + \cos x - \frac{1}{4} [1 - 2 \sin^2 x] + c$$

$$I = x \sin x + \cos x - \frac{1}{4} + \frac{1}{2} \sin^2 x + c$$

$$I = x \sin x + \cos x + \frac{1}{2} \sin^2 x + c - \frac{1}{4}$$

$$I = x \sin x + \cos x + \frac{1}{2} \sin^2 x + k, \text{ where } k = c - \frac{1}{4}$$

Indefinite Integrals Ex 19.25 Q47

$$\text{Let } I = \int \frac{(x \tan^{-1} x)}{(1+x^2)^{\frac{3}{2}}} dx$$

$$\text{Let } \tan^{-1} x = t$$

$$\frac{1}{1+x^2} dx = dt$$

$$I = \int \frac{t \tan t}{\sqrt{1+\tan^2 t}} dt$$

$$= \int \frac{t \tan t}{\sec t} dt$$

$$= \int t \frac{\sin t}{\cos t} \cos t \, dt$$

$$= \int t \sin t \, dt$$

$$= [t \int \sin t \, dt - \int (1 \int \sin t \, dt) dt]$$

$$= [-t \cos t + \int \cos t \, dt]$$

$$= [-t \cos t + \sin t] + c$$

$$I = -\frac{\tan^{-1} x}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} + c$$

Indefinite Integrals Ex 19.25 Q48

$$\text{Let } I = \int \tan^{-1}(\sqrt{x}) dx$$

$$\text{Let } x = t^2$$

$$dx = 2t dt$$

$$I = \int 2t \tan^{-1} t dt$$

$$= 2 \left[\tan^{-1} t \int t dt - \int \left(\frac{1}{1+t^2} \int t dt \right) dt \right]$$

$$= 2 \left[\frac{t^2}{2} \tan^{-1} t - \int \frac{t^2}{2(1+t^2)} dt \right]$$

$$= t^2 \tan^{-1} t - \int \frac{t^2 + 1 - 1}{1+t^2} dt$$

$$= t^2 \tan^{-1} t - \int \left(1 - \frac{1}{1+t^2} \right) dt$$

$$= t^2 \tan^{-1} t - t + \tan^{-1} t + C$$

$$= (t^2 + 1) \tan^{-1} t - t + C$$

$$I = (x + 1) \tan^{-1} \sqrt{x} - \sqrt{x} + C$$

Indefinite Integrals Ex 19.25 Q49

$$\int x^3 \tan^{-1} x dx =$$

$$\int x^3 \tan^{-1} x dx = \tan^{-1} x \int x^3 dx - \left(\int \frac{d \tan^{-1} x}{dx} \left(\int x^3 dx \right) dx \right)$$

$$= \tan^{-1} x \frac{x^4}{4} - \left(\int \frac{1}{1+x^2} \left(\frac{x^4}{4} \right) dx \right)$$

$$= \tan^{-1} x \frac{x^4}{4} - \left(\int \frac{1}{1+x^2} \left(\frac{x^4}{4} \right) dx \right)$$

$$\int \frac{1}{1+x^2} \left(\frac{x^4}{4} \right) dx = \frac{1}{4} \left[\int \frac{1}{1+x^2} dx + (x^2 - 1) dx \right]$$

$$\int \frac{1}{1+x^2} \left(\frac{x^4}{4} \right) dx = \frac{1}{4} \left[\tan^{-1} x + \frac{x^3}{3} - x \right]$$

$$\int x^3 \tan^{-1} x dx = \frac{x^4}{4} \tan^{-1} x - \frac{1}{4} \left[\tan^{-1} x + \frac{x^3}{3} - x \right] + C$$

Indefinite Integrals Ex 19.25 Q50

$$\begin{aligned}
\text{Let } I &= \int x \sin x \cos 2x \, dx \\
&= \frac{1}{2} \int x (2 \sin x \cos 2x) \, dx \\
&= \frac{1}{2} \int x (\sin(x + 2x) - \sin(2x - x)) \, dx \\
&= \frac{1}{2} \int x (\sin 3x - \sin x) \, dx \\
&= \frac{1}{2} \left[x \int (\sin 3x - \sin x) \, dx - \int (1 \int (\sin 3x - \sin x) \, dx) \, dx \right] \\
&= \frac{1}{2} \left[x \left(-\frac{\cos 3x}{3} + \cos x \right) - \int \left(-\frac{\cos 3x}{3} + \cos x \right) \, dx \right] \\
I &= \frac{1}{2} \left[-x \frac{\cos 3x}{3} + x \cos x + \frac{1}{9} \sin 3x - \sin x \right] + c
\end{aligned}$$

***** END *****