Indefinite Integrals Ex 19.23 Q13

Let $I = \int \frac{1}{\sqrt{3}\sin x + \cos x} dx$

Let $\sqrt{3} = r\cos\theta$, and $1 = r\sin\theta$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6}$

$r = \sqrt{3+1} = 2$

$I = \int \frac{1}{r\cos\theta \sin x + r\sin\theta\cos x} dx$

$= \frac{1}{r}\int \frac{1}{\sin(x+\theta)} dx$

$= \frac{1}{2}\int \operatorname{cosec}(x+\theta) dx$

$= \frac{1}{2}\log\left|\tan\left(\frac{x}{2}+\frac{\theta}{2}\right)\right| + c$

$I = \frac{1}{2}\log\left|\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)\right| + c$

Indefinite Integrals Ex 19.23 Q14

Let $I = \int \frac{1}{\sin x - \sqrt{3}\cos x} dx$

Let $1 = r\cos\theta$, and $\sqrt{3} = r\sin\theta$

$r = \sqrt{3+1} = 2$

$\tan\theta = \sqrt{3}$

$\theta = \frac{\pi}{3}$

$I = \int \frac{1}{r\cos\theta \sin x - r\sin\theta\cos x} dx$

$= \frac{1}{2}\int \frac{1}{\sin(x-\theta)} dx$

$= \frac{1}{2}\int \operatorname{cosec}(x-\theta) dx$

$= \frac{1}{2}\log\left|\tan\left(\frac{x}{2}-\frac{\theta}{2}\right)\right| + c$

$I = \frac{1}{2}\log\left|\tan\left(\frac{x}{2}-\frac{\pi}{6}\right)\right| + c$

Indefinite Integrals Ex 19.23 Q15

Let $I = \int \frac{1}{5 + 7\cos x + \sin x} dx$

Put $\sin x = \frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}, \cos x = \frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$

Now,

$$I = \int \frac{1}{5 + \frac{7\left(1 - \tan^2\frac{x}{2}\right)}{\left(1 + \tan^2\frac{x}{2}\right)} + \left(\frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right)} dx$$

$$= \int \frac{\left(1 + \tan^2\frac{x}{2}\right)}{5\left(1 + \tan^2\frac{x}{2}\right) + 7 - 7\tan^2\frac{x}{2} + 2\tan\frac{x}{2}} dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{-2\tan^2\frac{x}{2} + 12 + 2\tan\frac{x}{2}} dx$$

Let $\tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2} dx = dt$$

$$I = \int \frac{2dt}{-2t^2 + 12 + 2t}$$

$$= -\int \frac{dt}{t^2 - t - 6}$$

$$= -\int \frac{dt}{t^2 - 2t\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 6}$$

$$= -\int \frac{dt}{\left(t - \frac{1}{2}\right)^2 - \left(\frac{5}{2}\right)^2}$$

$$= -\frac{1}{2\left(\frac{5}{2}\right)} \log\left|\frac{t - \frac{1}{2} - \frac{5}{2}}{t - \frac{1}{2} + \frac{5}{2}}\right| + c$$

$$= -\frac{1}{5}\log\left|\frac{t - 3}{t + 2}\right| + c$$

$$I = \frac{1}{5}\log\left|\frac{\tan\frac{x}{2} + 2}{\tan\frac{x}{2} - 3}\right| + c$$

********** END **********