Polynomials Ex 2.3 Q1

Answer :

(i) We have

$f(x) = x^3 - 6x^2 + 11x - 6$

$g(x) = x^2 + x + 1$

Here, degree $[f(x)] = 3$ and

Degree $(g(x)) = 2$

Therefore, quotient $q(x)$ is of degree $3 - 2 = 1$ and the remainder $r(x)$ is of degree less than 2

Let $q(x) = ax + b$ and

$r(x) = cx + d$

Using division algorithm, we have

$$f(x) = g(x) \times q(x) + r(x)$$

$$x^3 - 6x^2 + 11x - 6 = (x^2 + x + 1)(ax + b) + cx + d$$

$$x^3 - 6x^2 + 11x - 6 = ax^3 + ax^2 + ax + bx^2 + bx + b + cx + d$$

$$x^3 - 6x^2 + 11x - 6 = ax^3 + ax^2 + bx^2 + ax + bx + cx + b + d$$

$$x^3 - 6x^2 + 11x - 6 = ax^3 + (a + b)x^2 + (a + b + c)x + b + d$$

Equating the co-efficients of various powers of x on both sides, we get

On equating the co-efficient of x^3

$x^3 = ax^3$

$\cancel{x^3} = a\cancel{x^3}$

$$1 = a$$

On equating the co-efficient of x^2

$$-6x^2 = (a+b)x^2$$

$$-6\cancel{x^2} = (a+b)\cancel{x^2}$$

$$-6 = a+b$$

Substituting $a=1$

$$-6 = 1+b$$

$$-6-1 = b$$

$$-7 = b$$

On equating the co-efficient of x

$$11x = (a+b+c)x$$

$$11\cancel{x} = (a+b+c)\cancel{x}$$

$$11 = a+b+c$$

Substituting $a=1$; and $b=-7$ we get,

$$11 = 1+(-7)+c$$

$$11 = -6+c$$

$$11+6 = c$$

$$17 = c$$

On equating the constant terms

$$-6 = b+d$$

Substituting $b=-7$ we get,

$$-6 = -7+d$$

$-6+7=d$

$1=d$

Therefore,

Quotient $q(x)=ax+b$

$=(1x-7)$

And remainder $r(x)=cx+d$

$=(17x+1)$

Hence, the quotient and remainder is given by,

$$\boxed{\begin{aligned} q(x) &= (x-7) \\ r(x) &= 17x+1 \end{aligned}}$$

(ii) We have

$f(x)=10x^4+17x^3-62x^2+30x-3$

$g(x)=2x^2+7x+1$

Here, Degree $(f(x))=4$ and

Degree $(g(x))=2$

Therefore, quotient $q(x)$ is of degree $4-2=2$ and remainder $r(x)$ is of degree less than 2 $(=\deg ree(g(x)))$

Let $g(x)=ax^2+bx+c$ and

$r(x)=px+q$

Using division algorithm, we have

$$f(x)=g(x)\times q(x)+r(x)$$

$10x^4+17x^3-62x^2+30x-3=(2x^2+7x+1)(ax^2+bx+c)+px+q$

$10x^4+17x^3-62x^2+30x-3=2ax^4+7ax^3+ax^2+2bx^3+7bx^2+bx+2cx^2+7xc+c+px+q$

$10x^4+17x^3-62x^2+30x-3=2ax^4+7ax^3+2bx^3+ax^2+7bx^2+2cx^2+bx+7xc+px+c+q$

$10x^4+17x^3-62x^2+30x-3=2ax^4+x^3(7a+2b)+x^2(a+7b+2c)+x(b+7c+p)+c+q$

Equating the co-efficients of various powers x on both sides, we get

On equating the co-efficient of x^4

$2a=10$

$a=\frac{10}{2}$

$a=5$

On equating the co-efficient of x^3

$7a+2b=17$

Substituting $a=5$ we get

$7\times5+2b=17$

$35+2b=17$

$2b=17-35$

$2b=-18$

$b=\frac{-18}{2}$

$b=-9$

*********** END **********