Adjoint and Inverse of Matrix Ex 7.1 Q11

$$A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} \qquad \therefore |A| = 1 \neq 0 \text{ and adj } A = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{\text{adj} A}{|A|} = \frac{1}{1}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & 7 \\ 7 & 9 \end{bmatrix} \qquad \therefore |B| = -2 \neq 0 \text{ and adj} B = \begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}$$

$$\therefore \quad B^{-1} = \frac{\text{adj} B}{|B|} = \frac{1}{(-2)}\begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}$$

Now, $(AB)^{-1} = B^{-1}.A^{-1}$

$$(AB)^{-1} = \frac{1}{(-2)}\begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$(AB)^{-1} = -\frac{1}{2}\begin{bmatrix} 94 & -39 \\ -82 & 34 \end{bmatrix}$$

$$(AB)^{-1} = \begin{bmatrix} -47 & \frac{39}{2} \\ 41 & -17 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q12

$$A=\begin{bmatrix}2 & -3\\ -4 & 7\end{bmatrix} \quad \therefore \ |A| = 2 \neq 0$$

$$\text{adj}\,A=\begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix}$$

$$\therefore \ A^{-1}=\frac{1}{2}\begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix}$$

To show: $\quad 2A^{-1} = 9I - A$

LHS: $\quad 2A^{-1} = 2.\frac{1}{2}\begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix} = \begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix}$

RHS: $\quad 9I - A = \begin{bmatrix}9 & 0\\ 0 & 9\end{bmatrix} - \begin{bmatrix}2 & -3\\ -4 & 7\end{bmatrix} = \begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix}$

Therefore, $\quad 2A^{-1} = 9I - A$

Adjoint and Inverse of Matrix Ex 7.1 Q13

$A=\begin{bmatrix}4 & 5\\ 2 & 1\end{bmatrix} \quad \therefore \ |A| = -6$ and $\text{adj}\,A = \begin{bmatrix}1 & -5\\ -2 & 4\end{bmatrix} \quad \therefore \ A^{-1} = \frac{1}{-6}\begin{bmatrix}1 & -5\\ -2 & 4\end{bmatrix}$

To show: $\quad A - 3I = 2\left(I + 3A^{-1}\right)$

$\therefore \quad$ LHS$=A - 3I = \begin{bmatrix}4 & 5\\ 2 & 1\end{bmatrix} - \begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix} = \begin{bmatrix}1 & 5\\ 2 & -2\end{bmatrix}$

RHS: $\quad 2\left(I + 3A^{-1}\right) = 2I + 2.3.A^{-1} = \begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix} + 2.3.\frac{1}{6}\begin{bmatrix}-1 & 5\\ 2 & -4\end{bmatrix}$

$$= \begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix} + \begin{bmatrix}-1 & 5\\ 2 & -4\end{bmatrix}$$

$$= \begin{bmatrix}1 & 5\\ 2 & -2\end{bmatrix}$$

$\therefore \quad A - 3I = 2\left(I + 3A^{-1}\right)$

Adjoint and Inverse of Matrix Ex 7.1 Q14

$$A=\begin{bmatrix} a & b \\ c & \frac{1+bc}{a} \end{bmatrix}$$

$$\Rightarrow |A| = (1+bc) - bc = 1 \neq 0$$

$$A^{-1} = \frac{1}{|A|}.adjA = \frac{1}{1}\begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$$

Now $aA^{-1} = (a^2+bc+1)I - aA$

$$LHS : aA^{-1} = a\begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} 1+bc & -ab \\ -ac & a^2 \end{bmatrix}$$

$$RHS : (a^2+bc+1)I - aA = \begin{bmatrix} a^2+bc+1 & 0 \\ 0 & a^2+bc+1 \end{bmatrix} - \begin{bmatrix} a^2 & ab \\ ac & 1+bc \end{bmatrix} = \begin{bmatrix} 1+bc & -ab \\ -ac & a^2 \end{bmatrix}$$

Since, $L.H.S = RHS$

Hence, proved

*********** END **********