Transformation Formulae Ex 8.2 Q 6(i)

We have,

$$\begin{aligned}
\text{LHS} &= \cos 3A + \cos 5A + \cos 7A + \cos 15A \\
&= [\cos 5A + \cos 3A] + [\cos 15A + \cos 7A] \\
&= \left[2\cos\frac{(5A+3A)}{2}\cos\frac{(5A-3A)}{2}\right] + \left[2\cos\frac{(15A+7A)}{2}\cos\frac{(15A-7A)}{2}\right] \\
&= 2\cos 4A \cos A + 2\cos 11A \cos 4A \\
&= 2\cos 4A[\cos A + \cos 11A] \\
&= 2\cos 4A[\cos 11A + \cos A] \\
&= 2\cos 4A\left[2\cos\frac{(11A+A)}{2}\cos\frac{(11A-A)}{2}\right] \\
&= 4\cos A[\cos 6A \cos 5A] \\
&= 4\cos 4A \cos 5A \cos 6A \\
&= \text{RHS}
\end{aligned}$$

$\therefore \quad \cos 3A + \cos 5A + \cos 7A + \cos 15A = 4\cos 4A \cos 5A \cos 6A$ Hence proved.

Transformation Formulae Ex 8.2 Q 6(ii)

We have,

$$\begin{aligned}
\text{LHS} &= \cos A + \cos 3A + \cos 5A + \cos 7A \\
&= (\cos 3A + \cos A) + (\cos 7A + \cos 5A) \\
&= \left[2\cos\left(\frac{3A+A}{2}\right)\cos\left(\frac{3A-A}{2}\right)\right] + \left[2\cos\left(\frac{7A+5A}{2}\right)\cos\left(\frac{7A-5A}{2}\right)\right] \\
&= 2\cos 2A \cos A + 2\cos 6A \cos A \\
&= 2\cos A[\cos 2A + \cos 6A] \\
&= 2\cos A[\cos 6A + \cos 2A] \\
&= 2\cos A\left[2\cos\left(\frac{6A+2A}{2}\right)\cos\left(\frac{6A-2A}{2}\right)\right] \\
&= 4\cos A[\cos 4A \cos 2A] \\
&= \text{RHS}
\end{aligned}$$

$\therefore \quad \cos A + \cos 3A + \cos 5A + \cos 7A = 4\cos A \cos 2A \cos 4A$. Hence proved.

Transformation Formulae Ex 8.2 Q 6(iii)

We have,

$$\begin{aligned}
\text{LHS} &= \sin A + \sin 2A + \sin 4A + \sin 5A \\
&= (\sin 2A + \sin A) + (\sin 5A + \sin 4a) \\
&= \left[2\sin\left(\frac{2A+A}{2}\right)\cos\left(\frac{2A-A}{2}\right)\right] + \left[2\sin\left(\frac{5A+4A}{2}\right)\cos\left(\frac{5A-4A}{2}\right)\right] \\
&= 2\sin\frac{3A}{2}\cos\frac{A}{2} + 2\sin\frac{9A}{2}\cos\frac{A}{2} \\
&= 2\cos\frac{A}{2}\left[\sin\frac{3A}{2} + \sin\frac{9A}{2}\right] \\
&= 2\cos\frac{A}{2}\left[\sin\frac{9A}{2} + \sin\frac{3A}{2}\right] \\
&= 2\cos\frac{A}{2}\left[2\sin\left\{\frac{1}{2}\left(\frac{9A}{2}+\frac{3A}{2}\right)\right\}\cos\left\{\frac{1}{2}\left(\frac{9A}{2}-\frac{3A}{2}\right)\right\}\right] \\
&= 4\cos\frac{A}{2}\left[\sin\frac{12A}{4}\cos\frac{6A}{4}\right] \\
&= 4\cos\frac{A}{2}\sin 3A\cos\frac{3A}{2} \\
&= 4\cos\frac{A}{2}\cos\frac{3A}{2}\sin 3A \\
&= \text{RHS}
\end{aligned}$$

$\therefore \quad \sin A + \sin 2A + \sin 4A + \sin 5A = 4\cos\frac{A}{2}\cos\frac{3A}{2}\sin 3A$. Hence proved.

Transformation Formulae Ex 8.2 Q 6(iv)

We have,

$$\begin{aligned}
\text{LHS} &= \sin 3A + \sin 2A - \sin A \\
&= \sin 3A - \sin A + \sin 2A \\
&= 2\sin\left(\frac{3A-A}{2}\right)\cos\left(\frac{3A+A}{2}\right) + \sin 2A \\
&= 2\sin A\cos 2A + \sin 2A \\
&= 2\sin A\cos 2A + 2\sin A\cos A \\
&= 2\sin A\left[\cos 2A + \cos A\right] \\
&= 2\sin A\left[2\cos\left(\frac{2A+A}{2}\right)\cos\left(\frac{2A-A}{2}\right)\right] \\
&= 4\sin A\cos\frac{3A}{2}\cos\frac{A}{2} \\
&= 4\sin A\cos\frac{A}{2}\cos\frac{3A}{2} \\
&= \text{RHS}
\end{aligned}$$

$\therefore \quad \sin 3A + \sin 2A - \sin A = 4\sin A\cos\frac{A}{2}\cos\frac{3A}{2}$. Hence proved.

Transformation Formulae Ex 8.2 Q 6(v)

We have,

$$\begin{aligned}
\text{LHS} &= \cos 20^\circ\cos 100^\circ + \cos 100^\circ\cos 140^\circ - \cos 140^\circ\cos 200^\circ \\
&= \frac{1}{2}\left[2\cos 100^\circ\cos 20^\circ + 2\cos 140^\circ\cos 100^\circ - 2\cos 200^\circ\cos 140^\circ\right] \\
&= \frac{1}{2}\left[\begin{array}{l}\cos(100^\circ+20^\circ) + \cos(100^\circ-20^\circ) + \cos(140^\circ+100^\circ) + \cos(140^\circ-100^\circ) \\ \qquad\qquad - \left\{\cos(200^\circ+140^\circ) + \cos(200^\circ-140^\circ)\right\}\end{array}\right] \\
&= \frac{1}{2}\left[\cos 120^\circ + \cos 80^\circ + \cos 240^\circ + \cos 40^\circ - \cos 340^\circ - \cos 60^\circ\right] \\
&= \frac{1}{2}\left[\cos(90^\circ+30^\circ) + \cos 80^\circ + \cos 40^\circ - \cos(180^\circ+60^\circ) - \cos(360^\circ-20^\circ) - \frac{1}{2}\right] \\
&= \frac{1}{2}\left[-\sin 30^\circ + 2\cos\left(\frac{80^\circ+40^\circ}{2}\right)\cos\left(\frac{80^\circ-40^\circ}{2}\right) - \cos 60^\circ - \cos 20^\circ - \frac{1}{2}\right] \\
&= \frac{1}{2}\left[-\frac{1}{2} + 2\cos 60^\circ\cos 20^\circ - \frac{1}{2} - \cos 20^\circ - \frac{1}{2}\right] \\
&= \frac{1}{2}\left[-\frac{3}{2} + 2\times\frac{1}{2}\times\cos 20^\circ - \cos 20^\circ\right] \\
&= \frac{1}{2}\left[-\frac{3}{2} + \cos 20^\circ - \cos 20^\circ\right] \\
&= \frac{1}{2}\left[-\frac{3}{2} + 0\right] \\
&= -\frac{3}{4} \\
&= \text{RHS}
\end{aligned}$$

$\therefore \quad \cos 20^\circ\cos 100^\circ + \cos 100^\circ\cos 140^\circ - \cos 140^\circ\cos 200^\circ = -\frac{3}{4}$. Hence proved.

Transformation Formulae Ex 8.2 Q 6(vi)

We have,

$$\text{LHS} = \sin\frac{\theta}{2}\sin\frac{7\theta}{2} + \sin\frac{3\theta}{2}\sin\frac{11\theta}{2}$$

$$= \frac{1}{2}\left[2\sin\frac{7\theta}{2}\sin\frac{\theta}{2} + 2\sin\frac{11\theta}{2}\sin\frac{3\theta}{2}\right]$$

$$= \frac{1}{2}\left[\cos\left(\frac{7\theta}{2} - \frac{\theta}{2}\right) - \cos\left(\frac{7\theta}{2} + \frac{\theta}{2}\right) + \cos\left(\frac{11\theta}{2} - \frac{3\theta}{2}\right) - \cos\left(\frac{11\theta}{2} + \frac{3\theta}{2}\right)\right]$$

$$= \frac{1}{2}\left[\cos\frac{6\theta}{2} - \cos\frac{8\theta}{2} + \cos\frac{8\theta}{2} - \cos\frac{14\theta}{2}\right]$$

$$= \frac{1}{2}\left[\cos 3\theta - \cos 4\theta + \cos 4\theta - \cos 7\theta\right]$$

$$= \frac{1}{2}\left[\cos 3\theta - \cos 7\theta\right]$$

$$= \frac{-1}{2}\left[\cos 7\theta - \cos 3\theta\right]$$

$$= \frac{-1}{2}\left[-2\sin\left(\frac{7\theta + 3\theta}{2}\right)\sin\left(\frac{7\theta - 3\theta}{2}\right)\right]$$

$$= \sin\frac{10\theta}{2}\sin\frac{4\theta}{2}$$

$$= \sin 5\theta \sin 2\theta$$

$$= \sin 2\theta \sin 5\theta$$

$$= \text{RHS}$$

$$\therefore \quad \sin\frac{\theta}{2}\sin\frac{7\theta}{2} + \sin\frac{3\theta}{2}\sin\frac{11\theta}{2} = \sin 2\theta \sin 5\theta.$$ Hence proved.

*********** END **********