Arithmetic Progressions Ex 9.5 Q18

Answer :

In this problem, we need to find the sum of all the multiples of 9 lying between 100 and 550.
So, we know that the first multiple of 9 after 100 is 108 and the last multiple of 9 before 550 is 549.
Also, all these terms will form an A.P. with the common difference of 9.
So here,
First term (a) = 108
Last term (l) = 549
Common difference (d) = 9
So, here the first step is to find the total number of terms. Let us take the number of terms as n.
Now, as we know,

$$a_n = a+(n-1)d$$

So, for the last term,

$$549 = 108+(n-1)9$$
$$549 = 108+9n-9$$
$$549 = 99+9n$$
$$549-99 = 9n$$

Further simplifying,

$$450 = 9n$$
$$n = \frac{450}{9}$$
$$n = 50$$

Now, using the formula for the sum of n terms,

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

We get,

$$S_n = \frac{50}{2}\left[2(108)+(50-1)9\right]$$
$$= 25\left[216+(49)9\right]$$
$$= 25(216+441)$$
$$= 25(657)$$
$$= 16425$$

Therefore, the sum of all the multiples of 9 lying between 100 and 550 is $\boxed{S_n = 16425}$

Arithmetic Progressions Ex 9.5 Q19

Answer :

In the given problem, we need to find the number of terms of an A.P. Let us take the number of terms as n.
Here, we are given that,

$$a = 22$$
$$d = -4$$
$$S_n = 64$$

So, as we know the formula for the sum of n terms of an A.P. is given by,

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

Where; a = first term for the given A.P.
d = common difference of the given A.P.
n = number of terms
So, using the formula we get,

$$S_n = \frac{n}{2}\left[2(22)+(n-1)(-4)\right]$$
$$64 = \frac{n}{2}\left[44-4n+4\right]$$
$$64(2) = n(48-4n)$$
$$128 = 48n-4n^2$$

Further rearranging the terms, we get a quadratic equation,

$$4n^2-48n+128=0$$

On taking 4 common, we get,

$$n^2-12n+32=0$$

Further, on solving the equation for *n* by splitting the middle term, we get,

$$n^2-12n+32=0$$
$$n^2-8n-4n+32=0$$
$$n(n-8)-4(n-8)=0$$
$$(n-8)(n-4)=0$$

So, we get,

$$(n-8)=0$$
$$n=8$$

Also,

$$(n-4)=0$$
$$n=4$$

Therefore, $\boxed{n=4 \text{ or } 8}$

Arithmetic Progressions Ex 9.5 Q20

Answer :

In the given problem, let us take the first term as *a* and the common difference *d*

Here, we are given that,

$$a_5=30 \qquad ......(1)$$
$$a_{12}=65 \qquad ......(2)$$

Also, we know,

$$a_n=a+(n-1)d$$

For the 5th term (n = 5),

$$a_5=a+(5-1)d$$
$$30=a+4d \qquad (\text{Using } 1)$$
$$a=30-4d \qquad ......(3)$$

Similarly, for the 12th term (n = 12),

$$a_{12}=a+(12-1)d$$
$$65=a+11d \qquad (\text{Using } 2)$$
$$a=65-11d \qquad ......(4)$$

Subtracting (3) from (4), we get,

$$a-a=(65-11d)-(30-4d)$$
$$0=65-11d-30+4d$$
$$0=35-7d$$
$$7d=35$$
$$d=5$$

Now, to find *a*, we substitute the value of *d* in (4),

$$a=30-4(5)$$
$$a=30-20$$
$$a=10$$

So, for the given A.P $d=5$ and $a=10$

So, to find the sum of first 20 terms of this A.P., we use the following formula for the sum of *n* terms of an A.P.,

$$S_n=\frac{n}{2}[2a+(n-1)d]$$

Where; *a* = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

So, using the formula for n = 20, we get,

$$S_{20}=\frac{20}{2}[2(10)+(20-1)(5)]$$
$$=(10)[20+(19)(5)]$$
$$=(10)[20+95]$$
$$=(10)[115]$$
$$=1150$$

Therefore, the sum of first 20 terms for the given A.P. is $\boxed{S_{20}=1150}$.

*********** END **********