

Exercise 7A

Question 19

:: LHS = RHS

(i) LHS =
$$\frac{\tan \theta}{(\sec \theta - 1)} + \frac{\tan \theta}{(\sec \theta + 1)}$$

= $\frac{\sin \theta}{\cos \theta}$ + $\frac{\sin \theta}{\cos \theta}$
= $\frac{1}{(\cos \theta)} + \frac{\sin \theta}{(\cos \theta)}$ + $\frac{\sin \theta}{(\cos \theta)}$
= $\frac{\sin \theta}{(\cos \theta)} + \frac{\sin \theta}{(\cos \theta)}$ + $\frac{\sin \theta}{(\cos \theta)}$
= $\frac{\sin \theta}{1 - \cos \theta} + \frac{\sin \theta}{1 + \cos \theta}$
= $\frac{\sin \theta (1 + \cos \theta) + \sin \theta (1 - \cos \theta)}{1 - \cos^2 \theta}$
= $\frac{\sin \theta + \sin \theta \cos \theta + \sin \theta - \sin \theta \cos \theta \cos \theta}{\sin^2 \theta}$
= $\frac{2\sin \theta}{\sin^2 \theta} = \frac{2}{\sin \theta} = 2\cos \theta \cos \theta$ RHS

$$\therefore LHS = RHS$$
(ii) LHS = $\frac{\cot \theta}{(\cos \theta + 1)} + \frac{(\cos \theta + 1)}{\cot \theta}$
= $\frac{\cos \theta}{\sin \theta} + \frac{(\frac{1}{\sin \theta} + 1)}{(\frac{1}{\sin \theta})}$
= $\frac{(\cos \theta)}{\sin \theta} + \frac{(\frac{1}{\sin \theta} + 1)}{(\frac{\cos \theta}{\sin \theta})}$
= $\frac{\cos \theta}{1 + \sin \theta} + \frac{(1 + \sin \theta)}{\cos \theta} = \frac{\cos^2 \theta + (1 + \sin \theta)^2}{\cos \theta (1 + \sin \theta)}$
= $\frac{\cos^2 \theta + 1 + \sin^2 \theta + 2\sin \theta}{\cos \theta (1 + \sin \theta)} = \frac{2}{\cos \theta} = 2\sec \theta = RHS$

******* END ********