



Tangents and Normals Ex 16.3 Q8(i)

We have,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{--- (i)}$$

$$xy = c^2 \quad \text{--- (ii)}$$

Slope of (i)

$$\frac{2x}{a^2} - \frac{2y}{b^2} \frac{dy}{dx} = 0$$

$$\therefore m_1 = \frac{dy}{dx} = \frac{x}{y} \times \frac{b^2}{a^2}$$

Slope of (ii)

$$y + x \frac{dy}{dx} = 0$$

$$\therefore m_2 = \frac{dy}{dx} = \frac{-y}{x}$$

\therefore (i) and (ii) cuts orthogonally

$$\therefore m_1 \times m_2 = -1$$

$$\Rightarrow \frac{x}{y} \times \frac{-y}{x} \times \frac{a^2}{b^2} = -1$$

$$\Rightarrow a^2 = b^2$$

Tangents and Normals Ex 16.3 Q8(ii)

We have,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{--- (i)}$$

$$\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1 \quad \text{--- (ii)}$$

Slope of (i)

$$\frac{2x}{a^2} + \frac{2y}{b^2} \times \frac{dy}{dx} = 0$$

$$a^2 - b^2 = ax$$

$$\therefore m_1 = \frac{dy}{dx} = -\frac{x}{y} \frac{b^2}{a^2}$$

Slope of (ii)

$$\frac{2x}{A^2} - \frac{2y}{B^2} \times \frac{dy}{dx} = 0$$

$$\therefore m_2 = \frac{dy}{dx} = \frac{x}{y} \times \frac{B^2}{A^2}$$

(i) and (ii) cuts orthogonally

$$\therefore m_1 \times m_2 = -1$$

$$\therefore \frac{-x}{y} \frac{b^2}{a^2} \times \frac{x}{y} \times \frac{B^2}{A^2} = -1$$

$$\Rightarrow \frac{x^2}{y^2} \times \frac{b^2 B^2}{a^2 A^2} = 1$$

$$\Rightarrow \frac{x^2}{y^2} = \frac{a^2 A^2}{b^2 B^2} \quad \text{--- (iii)}$$

Now,

(i) - (ii) gives

$$x^2 \left[\frac{1}{a^2} - \frac{1}{A^2} \right] + y^2 \left[\frac{1}{b^2} + \frac{1}{B^2} \right] = 0$$

$$\Rightarrow \frac{x^2}{y^2} = \frac{B^2 + b^2}{b^2 B^2} \times \frac{a^2 A^2}{a^2 - A^2}$$

Put in (iii), we get

$$\frac{(B^2 + b^2)}{b^2 B^2} \times \frac{a^2 A^2}{(a^2 - A^2)} = \frac{a^2 A^2}{b^2 B^2}$$

$$\Rightarrow B^2 + b^2 = a^2 - A^2$$

$$\Rightarrow a^2 - b^2 = A^2 + B^2$$

We have,

$$\frac{x^2}{a^2 + \lambda_1} + \frac{y^2}{b^2 + \lambda_1} = 1 \quad \text{---(i)}$$

$$\frac{x^2}{a^2 + \lambda_2} + \frac{y^2}{b^2 + \lambda_2} = 1 \quad \text{---(ii)}$$

slope of (i)

$$\frac{2x}{a^2 + \lambda_1} + \frac{2y}{b^2 + \lambda_1} \times \frac{dy}{dx} = 0$$

$$\Rightarrow m_1 = \frac{dy}{dx} = \frac{-x}{y} \times \frac{b^2 + \lambda_1}{a^2 + \lambda_1}$$

Slope of (ii)

$$\frac{2x}{a^2 + \lambda_2} + \frac{2y}{b^2 + \lambda_2} \times \frac{dy}{dx} = 0$$

$$\Rightarrow m_2 = \frac{dy}{dx} = \frac{-x}{y} \times \frac{b^2 + \lambda_2}{a^2 + \lambda_2}$$

Now,

Subtracting (ii) from (i), we get

$$x^2 \left[\frac{1}{a^2 + \lambda_1} - \frac{1}{a^2 + \lambda_2} \right] + y^2 \left[\frac{1}{b^2 + \lambda_1} - \frac{1}{b^2 + \lambda_2} \right] = 0$$

$$\Rightarrow \frac{x^2}{y^2} = \frac{\lambda_2 - \lambda_1}{(b^2 + \lambda_1)(b^2 + \lambda_2)} \times \frac{1}{\frac{\lambda_1 - \lambda_2}{(a^2 + \lambda_1)(a^2 + \lambda_2)}}$$

Now,

$$\begin{aligned} m_1 \times m_2 &= \frac{x^2}{y^2} \times \frac{(b^2 + \lambda_1)(b^2 + \lambda_2)}{(a^2 + \lambda_1)(a^2 + \lambda_2)} \\ &= \frac{(\lambda_2 - \lambda_1)}{(b^2 + \lambda_1)(b^2 + \lambda_2)} \times - \frac{(a^2 + \lambda_1)(a^2 + \lambda_2)}{\lambda_2 - \lambda_1} \times \frac{(b^2 + \lambda_1)(b^2 + \lambda_2)}{(a^2 + \lambda_1)(a^2 + \lambda_2)} \\ &= -1 \end{aligned}$$

\therefore (i) and (ii) cuts orthogonally

Tangents and Normals Ex 16.3 Q10

Suppose the straight line $x \cos \alpha + y \sin \alpha = p$ touches the curve at $Q(x_1, y_1)$.

But equation of tangent to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $Q(x_1, y_1)$ is

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

Thus equation $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$ and $x \cos \alpha + y \sin \alpha = p$ represent the same line.

$$\therefore \frac{x_1/a^2}{\cos \alpha} + \frac{y_1/b^2}{\sin \alpha} = \frac{1}{p}$$

$$\Rightarrow x_1 = \frac{a^2 \cos \alpha}{p}, \quad y_1 = \frac{b^2 \sin \alpha}{p} \quad \text{.....(i)}$$

The point $Q(x_1, y_1)$ lies on the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\therefore \frac{a^4 \cos^2 \alpha}{p^2 a^2} + \frac{b^4 \sin^2 \alpha}{p^2 b^2} = 1$$

$$\Rightarrow a^2 \cos^2 \alpha - b^2 \sin^2 \alpha = p^2$$

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