Question 13. 1. Estimate the fraction of molecular volume to the actual volume occupied by oxygen gas at STP.' Take the diameter of an oxygen molecule to be 3 A.

Answer: Diameter of an oxygen molecule, $d = 3A = 3 \times 10^{-10}$ m. Consider one mole of oxygen gas at STP, which contain total NA = 6.023×10^{23} molecules.

Actual molecular volume of 6.023×10^{23} oxygen molecules

$$V_{actual} = \frac{4}{3}\pi r^3 \cdot N_A$$

$$= \frac{4}{3} \times 3.14 \times (1.5)^3 \times 10^{-3} \times 6.02 \times 10^{23} \text{ m}^3$$

$$= 8.51 \times 10^{-6} \text{ m}^3$$

$$= 8.51 \times 10^{-3} \text{ litre} \qquad [\because 1 \text{ m}^3 = 10^3 \text{ litre}]$$

$\therefore$ Molecular volume of one mole of oxygen

$$V_{actual} = 8.51 \times 10^{-3} \text{ litre}$$

At STP, the volume of one mole of oxygen

$$V_{molar} = 22.4 \text{ litre}$$

$$\frac{V_{actual}}{V_{molar}} = \frac{8.51 \times 10^{-3}}{22.4} = 3.8 \times 10^{-4} \simeq 4 \times 10^{-4}.$$

Question 13. 2. Molar volume is the volume occupied by 1 mol of any (ideal) gas at standard temperature and pressure (STP : 1 atmospheric pressure, 0 °C). Show that it is 22.4 litres.
Answer:

For one mole of an ideal gas, we have

$$PV = RT \Rightarrow V = \frac{RT}{P}$$

Putting $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$, $T = 273K$ and $P = 1$ atmosphere $= 1.013 \times 10^5 \text{ Nm}^{-2}$

$$\therefore \qquad V = \frac{8.31 \times 273}{1.013 \times 10^5} = 0.0224 \text{ m}^3$$

$$= 0.0224 \times 10^6 \text{ cm}^3 = 22400 \text{ ml} \qquad [1 \text{ cm}^3 = 1\text{ml}]$$

Question 13. 3. Following figure shows plot of PV/T versus P for 1.00 x 10^{-3} kg of oxygen gas at two different temperatures.
(a) What does the dotted plot signify?
(b) Which is true : $T_1 > T_2$ or $T_1 < T_2$?
(c) What is the value of PV/T where the curves meet on the y-axis?
(d) If we obtained similar plots for 1.00 x 10^{-3} kg of hydrogen, would we get the same value of PV/T at the point where the curves meet on the y-axis? If not, what mass of hydrogen yields the same value of PV/T (for low pressure high temperature region of the plot) ? (Molecular mass of H_2 = 2.02 u, of O_2 = 32.0 u, R = 8.31 J mol^{-1} K^{-1}.)
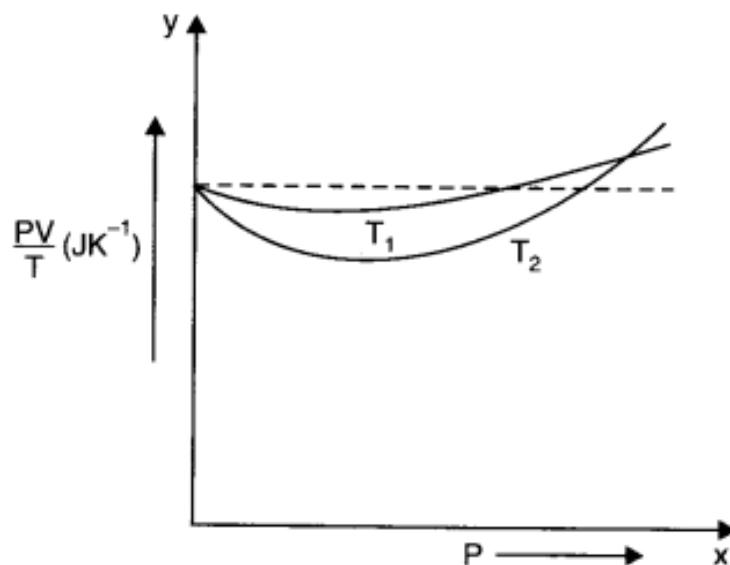
Answer:
(a) The dotted plot corresponds to 'ideal' gas behaviour as it is parallel to P-axis and it tells that value of PV/T remains same even when P is changed.
(b) The upper position of PV/T shows that its value is lesser for T_1 thus $T_1 > T_2$. This is because the curve at T_1 is more close to dotted plot than the curve at T_2 Since the behaviour of a real gas approaches the perfect gas behaviour, as the temperature is increased.
(c) Where the two curves meet, the value of PV/T on y-axis is equal to μR. Since ideal gas equation for μ moles is PV = μRT

where, $$\mu = \frac{1.00 \times 10^{-3} kg}{32 \times 10^{-3} kg} = \frac{1}{32}$$

$\therefore$ Value of $\frac{PV}{T} = \mu R = \frac{1}{32} \times 8.31 \text{ JK}^{-1} = 0.26 \text{ JK}^{-1}$

(d) If we obtained similar plots for 1.00 × 10⁻³ kg of hydrogen, we will not get the same value of $\frac{PV}{T}$ at the point, where the curves meet on the y-axis. This is because molecular mass of hydrogen is different from that of oxygen.

For the same value of $\frac{PV}{T}$, mass of hydrogen required is obtained from

$$\frac{PV}{T} = nR = \frac{m}{2.02} \times 8.31 = 0.26$$

$$m = \frac{2.02 \times 0.26}{8.31} \text{ gram} = 6.32 \times 10^{-2} \text{ gram.}$$

Question 13. 4. An oxygen cylinder of volume 30 Hire has an initial gauge pressure of 15 atmosphere and a temperature of 27 °C. After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atmosphere and its temperature drops to 17 °C. Estimate the mass of oxygen taken out of the cylinder. (R = 8.31 J mol⁻¹ K⁻¹, molecular mass of O_2 = 32 u.)
Answer:

Initial volume, V_1 = 30 litre = 30×10^3 cm^3

$= 30 \times 10^3 \times 10^{-6}$ m^3 = 30×10^{-3} m^3

Initial pressure, P_1 = 15 atm

$= 15 \times 1.013 \times 10^5$ N m^{-2}

Initial temperature, T_1 = (27 + 273) K = 300 K

Initial number of moles,

$$\mu_1 = \frac{P_1 V_1}{RT_1} = \frac{15 \times 1.013 \times 10^5 \times 30 \times 10^{-3}}{8.31 \times 300} = 18.3$$

Final pressure, P_2 = 11 atm

$= 11 \times 1.013 \times 10^5$ N m^{-2}

Final volume, V_2 = 30 litre = 30×10^{-3} cm^3

Final temperature, T_2 = 17 + 273 = 290 K

Final number of moles,

$$\mu_2 = \frac{P_2 V_2}{RT_2} = \frac{11 \times 1.013 \times 10^5 \times 30 \times 10^{-3}}{8.31 \times 290} = 13.9$$

Number of moles taken out of cylinder

$= 18.3 - 13.9 = 4.4$

Mass of gas taken out of cylinder

$= 4.4 \times 32$ g = 140.8 g = 0.141 kg.

Question 13. 5. An air bubble of volume 1.0 cm^3 rises from the bottom of a lake 40 m deep at a temperature of 12°C. To what volume does it grow when it reaches the surface, which is at a temperature of 35 °C.

Answer:

Volume of the bubble inside, V_1 = 1.0 cm^3 = 1×10^{-6} m^3

Pressure on the bubble, P_1 = Pressure of water + Atmospheric pressure

$= \rho g h + 1.01 \times 10^5 = 1000 \times 9.8 \times 40 + 1.01 \times 10^5$

$= 3.92 \times 10^5 + 1.01 \times 10^5 = 4.93 \times 10^5$ Pa

Temperature, T_1 = 12 °C = 273 + 12 = 285 K

Also, pressure outside the lake, P_2, = 1.01×10^5 N m^{-2}

Temperature, T_2 = 35 °C = 273 + 35 = 308 K, volume V_2 =?

Now $\dfrac{P_1 V_1}{T_1} = \dfrac{P_2 V_2}{T_2}$

∴ $V_2 = \dfrac{P_1 V_1}{T_1} \cdot \dfrac{T_2}{P_2} = \dfrac{4.93 \times 10^5 \times 1 \times 10^{-6} \times 308}{285 \times 1.01 \times 10^5}$ = 5.3×10^{-6} m^{-3}

Question 13. 6. Estimate the total number of air molecules (inclusive of oxygen, nitrogen, water vapour and other constituents) in a room of capacity 25.0 m^3 at a temperature of 27 °C and 1 atm pressure.

Answer:

Here, volume of room, V = 25.0 m^3, temperature, T = 27 °C = 300 K and

Pressure, P = 1 atm = 1.01×10^5 Pa

According to gas equation,

$PV = \mu RT = \mu \, N_A \cdot k_B T$

Hence, total number of air molecules in the volume of given gas,

$$N = \mu \cdot N_A = \frac{PV}{k_B T}$$

∴ $$N = \frac{1.01 \times 10^5 \times 25.0}{\left(1.38 \times 10^{-23}\right) \times 300} = 6.1 \times 10^{26}.$$

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