

Exercise 5A

$$\begin{array}{c} \text{(V)} \left\{ \left(\frac{-3}{4} \right)^3 - \left(\frac{-5}{2} \right)^3 \right\} \times 4^2 = \left\{ \left(\frac{-3^3}{4^3} \right) - \left(\frac{-5^2}{2^3} \right) \right\} \times 4^2 \\ = \left\{ \left(\frac{-27}{64} \right) - \left(\frac{-125}{8} \right) \right\} \times 16 \\ = \left\{ \frac{-27}{64} + \frac{125}{8} \right\} \times 16 \\ = \left(\frac{-27 + 1000}{64} \right) \times 16 \\ = \left(\frac{973}{64} \times 16 \right) = \frac{973}{4} \end{array}$$

Q8

Answer

$$\begin{array}{l} \text{(i) } \left(\frac{4}{9}\right)^{6} \times \left(\frac{4}{9}\right)^{-4} = \left(\frac{4}{9}\right)^{6+\left(-4\right)} \\ = \left(\frac{4}{9}\right)^{2} = \frac{\left(4\right)^{2}}{\left(9\right)^{2}} = \frac{4\times4}{9\times9} = \frac{16}{81} \end{array} \hspace{2cm} \left[since \ \mathbf{a^{n}} \times \mathbf{a^{m}} = \mathbf{a^{n+m}}\right] \end{array}$$

$$\begin{aligned} & \text{(ii)} \left(\frac{-7}{8}\right)^{-3} \times \left(\frac{-7}{8}\right)^2 = \left(\frac{-7}{8}\right)^{\left(-3\right)+2} & \left[since\ \mathbf{a}^{\mathbf{n}} \times \mathbf{a}^{\mathbf{m}} = \mathbf{a}^{\mathbf{n}+\mathbf{m}}\right] \\ & = \left(\frac{-7}{8}\right)^{-1} \\ & = \left(\frac{8}{-7}\right)^1 & \left[since\ \left(\frac{a}{b}\right)^{-1} = \left(\frac{b}{a}\right)^1\right] \\ & = \left(\frac{8\times -1}{-7\times -1}\right) = \frac{-8}{7} \end{aligned}$$

$$\begin{array}{l} \text{(iii)} \left(\frac{4}{3}\right)^{-3} \times \left(\frac{4}{3}\right)^{-2} = \left(\frac{4}{3}\right)^{\left(-3\right) + \left(-2\right)} & \left[s \, \text{ince} \, \, \mathbf{a}^{\mathbf{n}} \times \mathbf{a}^{\mathbf{m}} = \, \mathbf{a}^{\mathbf{n} + \mathbf{m}}\right] \\ & = \left(\frac{4}{3}\right)^{-5} \\ & = \left(\frac{3}{4}\right)^{5} \\ & = \frac{(3)^{5}}{(4)^{5}} = \frac{3 \times 3 \times 3 \times 3 \times 3}{4 \times 4 \times 4 \times 4 \times 4} = \frac{243}{1024} \end{array} \right]$$

$$\left[s \, \text{ince} \, \left(\frac{a}{b}\right)^{-1} = \left(\frac{b}{a}\right)^{1}\right]$$

09

(i)
$$5^{-3} = \left(\frac{5}{1}\right)^{-3} = \left(\frac{1}{5}\right)^3 = \frac{\left(1\right)^3}{\left(5\right)^3} = \frac{1}{125}$$

(ii)
$$(-2)^{-5} = \left(\frac{-2}{1}\right)^{-5} = \left(\frac{1}{-2}\right)^5 = \frac{\left(1\right)^5}{\left(-2\right)^5} = \frac{1 \times -1}{-32 \times -1} = \frac{-1}{32}$$

(iii)
$$\left(\frac{1}{4}\right)^{-4} = \left(\frac{4}{1}\right)^4 = \frac{\left(4\right)^4}{\left(1\right)^4} = \frac{256}{1} = 256$$

(iv)
$$\left(\frac{-3}{4}\right)^{-3} = \left(\frac{4}{-3}\right)^3 = \frac{\left(4\right)^3}{\left(-3\right)^3} = \frac{64}{-27} = \frac{64 \times -1}{-27 \times -1} = \frac{-64}{27}$$

$$\text{(v) } \left(-3\right)^{-1} \times \left(\tfrac{1}{3}\right)^{-1} = \left(\tfrac{1}{-3}\right)^1 \times \left(\tfrac{3}{1}\right)^1 = \left(\tfrac{1\times3}{-3\times1}\right)^1 = \left(\tfrac{3}{-3}\right)^1 = \tfrac{1}{-1} = \tfrac{1\times-1}{-1\times-1} = \tfrac{-1}{1} = -1$$

$$\text{(Vi)} \left(\frac{5}{7}\right)^{-1} \times \left(\frac{7}{4}\right)^{-1} = \left(\frac{7}{5}\right)^1 \times \left(\frac{4}{7}\right)^1 = \left(\frac{7\times 4}{5\times 7}\right)^1 = \frac{4}{5}$$

$$\text{(vii) } \left(5^{-1}-7^{-1}\right)^{-1} = \left(\frac{1}{5}-\frac{1}{7}\right)^{-1} = \left(\frac{7-5}{35}\right)^{-1} \\ = \left(\frac{2}{35}\right)^{-1} = \left(\frac{35}{2}\right)^1 = \frac{35}{2}$$

$$\begin{aligned} \text{(Viii)} & \left\{ \left(\frac{4}{3} \right)^{-1} - \left(\frac{1}{4} \right)^{-1} \right\}^{-1} = \left\{ \left(\frac{3}{4} \right)^1 - \left(\frac{4}{1} \right)^1 \right\}^{-1} = \left(\frac{3}{4} - \frac{4}{1} \right)^{-1} \\ & = \left(\frac{3 - 16}{4} \right)^{-1} = \left(\frac{-13}{4} \right)^{-1} \\ & = \left(\frac{4}{-13} \right)^1 = \left(\frac{4 \times -1}{-13 \times -1} \right) \\ & = \frac{-4}{13} \end{aligned}$$

$$\begin{aligned} \text{(iX)} &\left\{ \left(\frac{3}{2} \right)^{-1} \div \left(\frac{-2}{5} \right)^{-1} \right\} = \left\{ \left(\frac{2}{3} \right)^1 \div \left(\frac{5}{-2} \right)^1 \right\} \\ &= \left(\frac{2}{3} \times \frac{-2}{5} \right) \\ &= \frac{-4}{15} \end{aligned}$$

(x)
$$\left(\frac{23}{25}\right)^0=1$$
 [since a^0 = 1 for every integer a]

Q10

Answer:

(i)

$$\left[\left\{\left(-\frac{1}{4}\right)^{2}\right\}^{-2}\right]^{-1} = \left[\left(-\frac{1}{4}\right)^{2\times-2}\right]^{-1} \qquad \left[since \left\{\left(\frac{\mathbf{a}}{\mathbf{b}}\right)^{\mathbf{m}}\right\}^{\mathbf{n}} = \left(\frac{\mathbf{a}}{\mathbf{b}}\right)^{\mathbf{m}\mathbf{n}}\right]$$

$$= \left[\left(-\frac{1}{4}\right)^{-4}\right]^{-1}$$

$$= \left(-\frac{1}{4}\right)^{(-4)\times(-1)}$$

$$= \left(-\frac{1}{4}\right)^{4} = \frac{(-1)^{4}}{(4)^{4}}$$

(ii)

$$\left\{ \left(\frac{-2}{3} \right)^2 \right\}^3 = \left(\frac{-2}{3} \right)^{2 \times 3} \qquad \left[s \text{ ince } \left\{ \left(\frac{\mathbf{a}}{\mathbf{b}} \right)^{\mathbf{m}} \right\}^{\mathbf{n}} = \left(\frac{\mathbf{a}}{\mathbf{b}} \right)^{\mathbf{m} \mathbf{n}} \right]$$

$$= \left(\frac{-2}{3} \right)^6$$

$$= \frac{\left(-2 \right)^6}{\left(3 \right)^6} = \frac{64}{729} \qquad [\text{since } (-2)^6 = 64 \text{ and } (3)^6 = 729]$$

(iii)

(iv)

$$\left(\frac{-2}{3}\right)^7 \div \left(\frac{-2}{3}\right)^4 = \left(\frac{-2}{3}\right)^{7-4} \qquad [since a^m \div a^n = a^{m-n}]$$
$$= \left(\frac{-2}{3}\right)^3$$
$$= \frac{\left(-2\right)^3}{\left(3\right)^3} = \frac{-8}{27}$$

Q11

Answer:

Let the required number be x.

$$(-5)^{-1} \times x = (8)^{-1}$$

 $\Rightarrow \frac{1}{-5} \times x = \frac{1}{8}$
 $\therefore x = \frac{1}{8} \times (-5) = \frac{-5}{8}$
Hence, the required number is $\frac{-5}{8}$.

Q12

Answer:

Let the required number be x.

(3)⁻³ x x = 4
⇒
$$\frac{1}{3^3}$$
 × $x = 4$
⇒ $\frac{1}{27}$ × $x = 4$
∴ $x = 4$ x 27 = 108
Hence, the required number is 108.

Q13

Answer:

Let the required number be x.

$$(-30)^{-1} \div \chi = 6^{-1}$$

$$\Rightarrow \frac{1}{(-30)} \times \frac{1}{z} = \frac{1}{6}$$

$$\Rightarrow \frac{1}{(-30z)} = \frac{1}{6}$$

$$\therefore \chi = \frac{6}{(-30)} = \frac{1}{-5}$$

$$= \frac{-1}{5}$$

Hence, the required number is $\frac{-1}{5}$.

Answer:

$$\begin{split} & \left(\frac{3}{5}\right)^3 \times \left(\frac{3}{5}\right)^{-6} = \left(\frac{3}{5}\right)^{2\boldsymbol{x}-1} \\ & \Rightarrow \left(\frac{3}{5}\right)^{3+\left(-6\right)} = \left(\frac{3}{5}\right)^{2\boldsymbol{x}-1} & \left[s \text{ ince } \mathbf{a}^{\mathbf{m}} \times \mathbf{a}^{\mathbf{n}} = \mathbf{a}^{\mathbf{m}+\mathbf{n}}\right] \\ & \Rightarrow \left(\frac{3}{5}\right)^{-3} = \left(\frac{3}{5}\right)^{2\boldsymbol{x}-1} \end{split}$$

On equating the exponents:

$$-3 = 2x - 1$$

$$\Rightarrow 2x = -3 + 1$$

$$\Rightarrow 2x = -2$$

$$\therefore x = \left(\frac{-2}{2}\right) = -1$$

Q15

Answer:

$$\begin{split} \frac{3^{5}\times10^{5}\times25}{5^{7}\times6^{5}} &= \frac{3^{5}\times\left(2\times5\right)^{5}\times5^{2}}{5^{7}\times\left(2\times3\right)^{5}} \\ &= \frac{3^{5}\times2^{5}\times5^{5}\times5^{2}}{5^{7}\times2^{5}\times3^{5}} \\ &= \frac{3^{5}\times2^{5}\times5^{7}}{3^{5}\times2^{5}\times5^{7}} \\ &= 3^{5-5}\times2^{5-5}\times5^{7-7} \\ &= 3^{0}\times2^{0}\times5^{0} \\ &= 1\times1\times1=1 \end{split}$$

Q16

Answer:

$$\begin{array}{l} \frac{16\times2^{n+1}-4\times2^{n}}{16\times2^{n+2}-2\times2^{n+2}} \\ \Rightarrow \frac{2^{4}\times2^{n+1}-2^{2}\times2^{n}}{2^{4}\times2^{n+2}-2^{n+1}\times2^{2}} \\ \Rightarrow \frac{2^{2}\times\left(2^{n+2}-2^{n}\right)}{2^{2}\times\left(2^{n+4}-2^{n+1}\right)} \\ \Rightarrow \frac{2^{n}\times2^{3}-2^{n}}{2^{n}\times2^{4}-2^{n}\times2} \\ \Rightarrow \frac{2^{n}\left(2^{3}-1\right)}{2^{n}\left(2^{4}-2\right)} = \frac{8-1}{16-2} = \frac{7}{14} = \frac{1}{2} \end{array}$$

Q17

Answer:

(i)
$$5^{2n} \times 5^3 = 5^9$$

 $5^{2n+3} = 5^9$ [since $a^n \times a^m = a^{m+n}$]

On equating the coefficients:

$$2n + 3 = 9$$

$$\Rightarrow 2n = 9 - 3$$

$$\Rightarrow 2n = 6$$

$$\therefore n = \frac{6}{2} = 3$$

(ii)
$$8 \times 2^{n+2} = 32$$

 $\Rightarrow (2)^3 \times 2^{n+2} = (2)^5$ [since $2^3 = 8$ and $2^5 = 32$]
 $\Rightarrow (2)^{3+(n+2)} = (2)^5$

On equating the coefficients:

$$3 + n + 2 = 5$$

$$\Rightarrow n + 5 = 5$$

$$\Rightarrow n = 5 - 5$$

$$\therefore n = 0$$

(iii)
$$6^{2n+1} \div 36 = 6^3$$

 $\Rightarrow 6^{2n+1} \div 6^2 = 6^3$ [since $36 = 6^2$]
 $\Rightarrow \frac{6^{2n+1}}{6^2} = 6^3$ [since $\frac{a^m}{a^n} = a^{m-n}$]
 $\Rightarrow 6^{2n+1-2} = 6^3$ [since $\frac{a^m}{a^n} = a^{m-n}$]
 $\Rightarrow 6^{2n-1} = 6^3$ On equating the coefficients:
 $2n - 1 = 3$
 $\Rightarrow 2n = 3 + 1$
 $\Rightarrow 2n = 4$
 $\therefore n = \frac{4}{2} = 2$

Q18

Answer:

$$2^{n-7} \times 5^{n-4} = 1250$$

$$\Rightarrow \frac{2^n}{2^7} \times \frac{5^n}{5^4} = 2 \times 5^4 \qquad \text{[since } 1250 = 2 \times 5^4\text{]}$$

$$\Rightarrow \frac{2^n \times 5^n}{2^7 \times 5^4} = 2 \times 5^4$$

$$\Rightarrow 2^n \times 5^n = 2 \times 5^4 \times 2^7 \times 5^4 \qquad \text{[using cross multiplication]}$$

$$\Rightarrow 2^n \times 5^n = 2^{1+7} \times 5^{4+4} \qquad \text{[since } a^m \times a^n = a^{m+n}\text{]}$$

$$\Rightarrow 2^n \times 5^n = 2^8 \times 5^8$$

$$\Rightarrow (2 \times 5)^n = (2 \times 5)^8 \qquad \text{[since } a^n \times b^n = (a \times b)^n\text{]}$$

$$\Rightarrow 10^n = 10^8$$

$$\Rightarrow n = 8$$

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