Quadratic Equations Ex 8.5 Q3

Answer :

(i) We have been given,

$\frac{x-1}{x-2}+\frac{x-3}{x-4}=3\frac{1}{3}, x\neq 2,4$

Now we solve the above equation as follows,

$$\frac{(x-1)(x-4)+(x-3)(x-2)}{(x-2)(x-4)}=\frac{10}{3}$$

$$\frac{x^2-5x+4+x^2-5x+6}{x^2-6x+8}=\frac{10}{3}$$

$$6x^2-30x+30=10x^2-60x+80$$

$$4x^2-30x+50=0$$

$$2x^2-15x+25=0$$

Now we also know that for an equation $ax^2+bx+c=0$, the discriminant is given by the following equation:

$D=b^2-4ac$

Now, according to the equation given to us, we have, $a=2$, $b=-15$ and $c=25$.

Therefore, the discriminant is given as,

$$D=(-15)^2-4(2)(25)$$

$$=225-200$$

$$=25$$

Now, the roots of an equation is given by the following equation,

$$x=\frac{-b\pm\sqrt{D}}{2a}$$

Therefore, the roots of the equation are given as follows,

$$x=\frac{-(-15)\pm\sqrt{25}}{2(2)}$$

$$=\frac{15\pm5}{4}$$

Now we solve both cases for the two values of x. So, we have,

$$x=\frac{15+5}{4}$$

$$=5$$

Also,

$$x=\frac{15-5}{4}$$

$$=\frac{5}{2}$$

Therefore, the value of $\boxed{x=5,\frac{5}{2}}$

(ii) We have been given,

$\frac{1}{x}-\frac{1}{x-2}=3,\ x\neq 0,2$

Now we solve the above equation as follows,

$$\frac{(x-2)-x}{(x-2)(x)}=3$$

$$\frac{-2}{x^2-2x}=3$$

$$-2=3x^2-6x$$

$$3x^2-6x+2=0$$

Now we also know that for an equation $ax^2+bx+c=0$, the discriminant is given by the following equation:

$D=b^2-4ac$

Now, according to the equation given to us, we have, $a=3$, $b=-6$ and $c=2$.

Therefore, the discriminant is given as,

$$D=(-6)^2-4(3)(2)$$

$$=36-24$$

$$=12$$

Now, the roots of an equation is given by the following equation,

$x=\frac{-b\pm\sqrt{D}}{2a}$

Therefore, the roots of the equation are given as follows,

$$x=\frac{-(-6)\pm\sqrt{12}}{2(3)}$$

$$=\frac{6\pm 2\sqrt{3}}{6}$$

$$=\frac{3\pm\sqrt{3}}{3}$$

Now we solve both cases for the two values of x. So, we have,

$x=\frac{3+\sqrt{3}}{3}$

Also,

$x=\frac{3-\sqrt{3}}{3}$

Therefore, the value of $\boxed{x=\frac{3+\sqrt{3}}{3},\frac{3-\sqrt{3}}{3}}$

(iii) We have been given,

$x+\frac{1}{x}=3,\ x\neq 0$

Now, we solve the equation as follows:

$$\frac{x^2+1}{x}=3$$

$$x^2+1=3x$$

$$x^2-3x+1=0$$

Now we also know that for an equation $ax^2+bx+c=0$, the discriminant is given by the following equation:

$D=b^2-4ac$

Now, according to the equation given to us, we have, $a=1$, $b=-3$ and $c=1$.

Therefore, the discriminant is given as,

$$D=(-3)^2-4(1)(1)$$

$$=9-4$$

$$=5$$

Now, the roots of an equation is given by the following equation,

$x=\frac{-b\pm\sqrt{D}}{2a}$

Therefore, the roots of the equation are given as follows,

$x=\frac{-(-3)\pm\sqrt{5}}{2(1)}$

$$= \frac{3 \pm \sqrt{5}}{2}$$

Now we solve both cases for the two values of x. So, we have,

$$x = \frac{3+\sqrt{5}}{2}$$

Also,

$$x = \frac{3-\sqrt{5}}{2}$$

Therefore, the value of $\boxed{x = \frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}}$

(iv) We have been given,

$\frac{16}{x} - 1 = \frac{15}{x+1}$, $x \neq 0, \ -1$

Now we solve the above equation as follows,

$\frac{16-x}{x} = \frac{15}{x+1}$

$\Rightarrow (16 - x)(x + 1) = 15x$

$\Rightarrow 16x + 16 - x^2 - x = 15x$

$\Rightarrow 15x + 16 - x^2 - 15x = 0$

$\Rightarrow 16 - x^2 = 0$

$\Rightarrow x^2 - 16 = 0$

Now we also know that for an equation $ax^2 + bx + c = 0$, the discriminant is given by the following equation:

$D = b^2 - 4ac$

Now, according to the equation given to us, we have, $a = 1$, $b = 0$ and $c = -16$.

Therefore, the discriminant is given as,

$D = (0)^2 - 4(1)(-16)$

$= 64$

Now, the roots of an equation is given by the following equation,

$x = \frac{-b \pm \sqrt{D}}{2a}$

Therefore, the roots of the equation are given as follows,

$x = \frac{-0 \pm \sqrt{64}}{2(1)}$

$= \frac{\pm 8}{2}$

$= \pm 4$

Therefore, the value of $x = \pm 4$.

********** END **********