Differentiation Ex 11.5 Q6

Let $\quad y = (\log x)^{\cos x} \qquad$ ---(i)

Taking log on both the sides,

$$\log y = \log(\log x)^{\cos x}$$

$$\log y = \cos x \log(\log x) \qquad \left[\text{Since, } \log a^b = b \log a\right]$$

Differentiating with respect to x, using product rule, chain rule,

$$\frac{1}{y}\frac{dy}{dx} = \cos x \frac{d}{dx}\log(\log x) + \log\log x \frac{d}{dx}(\cos x)$$

$$= \frac{\cos x}{\log x}\frac{d}{dx}(\log x) + \log\log x \times (-\sin x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{\cos x}{\log x} \times \left(\frac{1}{x}\right) - \sin x \log\log x$$

$$\frac{dy}{dx} = y\left[\frac{\cos x}{x \log x} - \sin x \log\log x\right]$$

$$\frac{dy}{dx} = (\log x)^{\cos x}\left[\frac{\cos x}{x \log x} - \sin x \log\log x\right] \qquad \left[\text{Using equation (i)}\right]$$

Differentiation Ex 11.5 Q7

Let $\quad y = (\sin x)^{\cos x} \qquad$ --- (i)

Taking log on both the sides,

$$\log y = \log(\sin x)^{\cos x}$$

$$\log y = \cos x \log \sin x \qquad \left[\text{Since, } \log a^b = b \log a\right]$$

Differentiating with respect to x, using product rule, chain rule,

$$\frac{1}{y}\frac{dy}{dx} = \cos x \frac{d}{dx}\log \sin x + \log \sin x \frac{d}{dx}\cos x$$

$$= \cos x \frac{1}{\sin x}\frac{d}{dx}(\sin x) + \log \sin x(-\sin x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{\cos x}{\sin x}(\cos x) - \sin x \log \sin x$$

$$\frac{dy}{dx} = y\left[\cos x \cot x - \sin x \log \sin x\right]$$

$$\frac{dy}{dx} = (\sin x)^{\cos x}\left[\cos x \cot x - \sin x \log \sin x\right]$$

Differentiation Ex 11.5 Q8

Let $y = e^{x\log x}$

$\Rightarrow \quad y = e^{\log x^x} \qquad \left[\text{Since, } \log a^b = b\log a\right]$

$\Rightarrow \quad y = x^x \qquad \text{---(i)} \qquad \left[\text{Since, } e^{\log a} = a\right]$

Taking log both the sides,

$$\log y = \log x^x$$
$$\log y = x\log x$$

Differentiating with respect to x, using product rule,

$$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x)$$
$$= x\left(\frac{1}{x}\right) + \log x(1)$$
$$\frac{1}{y}\frac{dy}{dx} = 1 + \log x$$
$$\frac{dy}{dx} = y\left[1 + \log x\right]$$
$$\frac{dy}{dx} = x^x(1 + \log x) \qquad \left[\text{Using equation (i)}\right]$$

Differentiation Ex 11.5 Q9

Let $y = (\sin x)^{\log x}$ ---(i)

Taking log on both the sides,

$$\log y = \log(\sin x)^{\log x}$$
$$\log y = \log x\log(\sin x) \qquad \left[\text{Using } \log a^b = b\log a\right]$$

Differentiating with respect to x, using product rule and chain rule,

$$\frac{1}{y}\frac{dy}{dx} = \log x\frac{d}{dx}(\log\sin x) + \log\sin x\frac{d}{dx}(\log x)$$
$$= \log x\left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x) + \log\sin x\left(\frac{1}{x}\right)$$
$$= \frac{\log x}{\sin x}\times\cos x + \frac{\log\sin x}{x}$$
$$\frac{1}{y}\frac{dy}{dx} = \log x\cot x + \frac{\log\sin x}{x}$$
$$\frac{dy}{dx} = y\left[\log x\cot x + \frac{\log\sin x}{x}\right]$$
$$\frac{dy}{dx} = (\sin x)^{\log x}\left[\log x\cot x + \frac{\log\sin x}{x}\right] \qquad \left[\text{Using equation (i)}\right]$$

Differentiation Ex 11.5 Q10

Let $y = 10^{\log\sin x}$ ---(i)

Taking log on both the sides,

$$\log y = \log 10^{\log\sin x}$$
$$\log y = \log\sin x\log 10 \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating with respect to x, using chain rule,

$$\frac{1}{y}\frac{dy}{dx} = \log 10\frac{d}{dx}(\log\sin x)$$
$$= \log 10\left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x)$$
$$\frac{1}{y}\frac{dy}{dx} = \log 10\left(\frac{1}{\sin x}\right)(\cos x)$$
$$\frac{dy}{dx} = y\left[\log 10\cot x\right]$$
$$\frac{dy}{dx} = 10^{\log\sin x}\left[\log 10\times\cot x\right] \qquad \left[\text{Using equation (i)}\right]$$

*********** END **********