



Differentiation Ex 11.5 Q11

$$\text{Let } y = (\log x)^{\log x}$$

Taking logarithm on both the sides, we obtain

$$\log y = \log x \cdot \log(\log x)$$

Differentiating both sides with respect to x , we obtain

$$\begin{aligned} \frac{1}{y} \frac{dy}{dx} &= \frac{d}{dx} [\log x \cdot \log(\log x)] \\ \Rightarrow \frac{1}{y} \frac{dy}{dx} &= \log(\log x) \cdot \frac{d}{dx} (\log x) + \log x \cdot \frac{d}{dx} [\log(\log x)] \\ \Rightarrow \frac{dy}{dx} &= y \left[\log(\log x) \cdot \frac{1}{x} + \log x \cdot \frac{1}{\log x} \cdot \frac{d}{dx} (\log x) \right] \\ \Rightarrow \frac{dy}{dx} &= y \left[\frac{1}{x} \log(\log x) + \frac{1}{x} \right] \\ \therefore \frac{dy}{dx} &= (\log x)^{\log x} \left[\frac{1}{x} + \frac{\log(\log x)}{x} \right] \end{aligned}$$

Differentiation Ex 11.5 Q12

$$\text{Let } y = 10^{(10^x)} \quad \text{---(i)}$$

Taking \log on both the sides,

$$\begin{aligned} \log y &= \log 10^{(10^x)} \\ \log y &= 10^x \log 10 \end{aligned}$$

Differentiating it with respect to x ,

$$\begin{aligned} \frac{1}{y} \frac{dy}{dx} &= \log 10 \times 10^x \log 10 \\ \frac{1}{y} \frac{dy}{dx} &= 10^x \times (\log 10)^2 \\ \frac{dy}{dx} &= 10^{(10^x)} \times 10^x (\log 10)^2 \quad \quad \quad [\text{Using equation (i)}] \end{aligned}$$

Differentiation Ex 11.5 Q13

$$\begin{aligned}\text{Let } y &= \sin x^x \\ \Rightarrow \sin^{-1} y &= x^x\end{aligned}$$

Taking log on both the sides,

$$\begin{aligned}\log(\sin^{-1} y) &= \log x^x \\ \log(\sin^{-1} y) &= x \log x \quad \quad \quad [\text{Since, } \log a^b = b \log a]\end{aligned}$$

Differentiating it with respect to x using chain rule and product rule,

$$\begin{aligned}\frac{1}{\sin^{-1} y} \frac{dy}{dx} &= (\sin^{-1} y)' = x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x) \\ \frac{1}{\sin^{-1} y} \times \left(\frac{1}{\sqrt{1-y^2}} \right) \frac{dy}{dx} &= x \left(\frac{1}{x} \right) + \log x (1) \\ \frac{dy}{dx} &= \sin^{-1} y \sqrt{1-y^2} (1 + \log x) \\ &= \sin^{-1} (\sin x^x) \sqrt{1 - (\sin x^x)^2} (1 + \log x) \\ &= x^x \sqrt{\cos^2 x^x} (1 + \log x) \quad \quad \quad [\text{Using equation (i)}] \\ \frac{dy}{dx} &= x^x \cos x^x (1 + \log x)\end{aligned}$$

Differentiation Ex 11.5 Q14

$$\text{Let } y = (\sin^{-1} x)^x$$

Taking log on both the sides,

$$\begin{aligned}\log y &= \log (\sin^{-1} x)^x \\ \log y &= x \log (\sin^{-1} x) \quad \quad \quad [\text{Since, } \log a^b = b \log a]\end{aligned}$$

Differentiating it with respect to x using product rule and chain rule,

$$\begin{aligned}\frac{1}{y} \frac{dy}{dx} &= x \frac{d}{dx} (\log \sin^{-1} x) + \log \sin^{-1} x \frac{d}{dx}(x) \\ &= x \times \frac{1}{\sin^{-1} x} \frac{d}{dx} (\sin^{-1} x) + \log \sin^{-1} x (1) \\ \frac{1}{y} \frac{dy}{dx} &= \frac{x}{\sin^{-1} x} \left(\frac{1}{\sqrt{1-x^2}} \right) + \log \sin^{-1} x \\ \frac{dy}{dx} &= y \left[\log \sin^{-1} x + \frac{x}{\sin^{-1} x \sqrt{1-x^2}} \right] \\ \frac{dy}{dx} &= (\sin^{-1} x)^2 \left[\log \sin^{-1} x + \frac{x}{\sin^{-1} x \sqrt{1-x^2}} \right] \quad \quad \quad [\text{Using equation (i)}]\end{aligned}$$

Differentiation Ex 11.5 Q15

$$\text{Let } y = x^{\sin^{-1} x} \quad \quad \quad \text{---(i)}$$

Taking log on both the sides,

$$\begin{aligned}\log y &= \log x^{\sin^{-1} x} \\ \log y &= \sin^{-1} x \log x \quad \quad \quad [\text{Since, } \log a^b = b \log a]\end{aligned}$$

Differentiating it with respect to x using product rule,

$$\begin{aligned}\frac{1}{y} \frac{dy}{dx} &= \sin^{-1} x \frac{d}{dx} (\log x) + (\log x) \frac{d}{dx} (\sin^{-1} x) \\ \frac{1}{y} \frac{dy}{dx} &= \sin^{-1} x \left(\frac{1}{x} \right) + (\log x) \left(\frac{1}{\sqrt{1-x^2}} \right) \\ \frac{dy}{dx} &= y \left[\frac{\sin^{-1} x}{x} + \frac{\log x}{\sqrt{1-x^2}} \right] \\ \frac{dy}{dx} &= x^{\sin^{-1} x} \left[\frac{\sin^{-1} x}{x} + \frac{\log x}{\sqrt{1-x^2}} \right] \quad \quad \quad [\text{Using equation (i)}]\end{aligned}$$

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