Differentiation Ex 11.3 Q38

$$\text{Let } y=\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right] \qquad \ldots(1)$$

$$\text{Then, } \frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}$$

$$=\frac{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)^2}{\left(\sqrt{1+\sin x}-\sqrt{1-\sin x}\right)\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}$$

$$=\frac{(1+\sin x)+(1-\sin x)+2\sqrt{(1-\sin x)(1+\sin x)}}{(1+\sin x)-(1-\sin x)}$$

$$=\frac{2+2\sqrt{1-\sin^2 x}}{2\sin x}$$

$$=\frac{1+\cos x}{\sin x}$$

$$=\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$=\cot\frac{x}{2}$$

Therefore, equation (1) becomes

$$y=\cot^{-1}\left(\cot\frac{x}{2}\right)$$

$$\Rightarrow y=\frac{x}{2}$$

$$\therefore \frac{dy}{dx}=\frac{1}{2}\frac{d}{dx}(x)$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{2}$$

Differentiation Ex 11.3 Q39

Here, $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right) + \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

$$y = \tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

Put $x = \tan\theta$,

$$y = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) + \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$y = \tan^{-1}(\tan 2\theta) + \cos^{-1}(\cos 2\theta) \qquad \text{---(i)}$$

Here, $< x < \infty$

$\Rightarrow \quad 0 < \tan\theta < \infty$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{2}$

$\Rightarrow \quad 0 < 2\theta < \pi$

So, from equation (i),

$$y = 2\theta + 2\theta \qquad \left[\begin{array}{l}\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ \text{and } \cos^{-1}(\cos\theta) = \theta, \text{ if } \theta \in [0, \pi]\end{array}\right]$$

$$y = 4\theta$$

$$y = 4\tan^{-1}x \qquad [\text{Using } x = \tan\theta]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{4}{1+x^2}$$

Differentiation Ex 11.3 Q40

Here, $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$

$$y = \cos^{-1}\left(\frac{x-1}{x+1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right) \qquad \left[\text{Since, } \sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)\right]$$

$$y = \frac{\pi}{2} \qquad \left[\text{Since, } \cos^{-1}x + \sin^{-1}x = \frac{\pi}{2}\right]$$

Differentiating with respect to x,

$$\frac{dy}{dx} = 0$$

Differentiation Ex 11.3 Q41

Here, $y = \sin\left[2\tan^{-1}\left[\sqrt{\frac{1-x}{1+x}}\right]\right]$

Put $x = \cos 2\theta$, so,

$$y = \sin\left[2\tan^{-1}\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}\right]$$

$$= \sin\left[2\tan^{-1}\sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}}\right]$$

$$= \sin\left[2\tan^{-1}\sqrt{\tan^2\theta}\right]$$

$$= \sin\left[2\tan^{-1}(\tan\theta)\right]$$

$$= \sin(2\theta)$$

$$= \sin\left[2 \times \frac{1}{2}\cos^{-1}x\right] \qquad [\text{Since, } x = \cos 2\theta]$$

$$= \sin\left(\sin^{-1}\sqrt{1-x^2}\right)$$

$$y = \sqrt{1-x^2}$$

Differentiating with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}\frac{d}{dx}\left(1-x^2\right).$$

*********** END **********