



Co-Ordinate Geometry Ex 14.2 Q37

Answer :

The distance d between two points (x_1, y_1) and (x_2, y_2) is given by the formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

In a square all the sides are of equal length. The diagonals are also equal to each other. Also in a square the diagonal is equal to $\sqrt{2}$ times the side of the square.

Here let the two points which are said to be the opposite vertices of a diagonal of a square be $A(-1, 2)$ and $C(3, 2)$.

Let us find the distance between them which is the length of the diagonal of the square.

$$\begin{aligned} AC &= \sqrt{(-1 - 3)^2 + (2 - 2)^2} \\ &= \sqrt{(-4)^2 + (0)^2} \\ &= \sqrt{16} \end{aligned}$$

$$AC = 4$$

Now we know that in a square,

$$\text{Side of the square} = \frac{\text{Diagonal of the square}}{\sqrt{2}}$$

Substituting the value of the diagonal we found out earlier in this equation we have,

$$\text{Side of the square} = \frac{4}{\sqrt{2}}$$

$$\text{Side of the square} = 2\sqrt{2}$$

Now, a vertex of a square has to be at equal distances from each of its adjacent vertices.

Let $P(x, y)$ represent another vertex of the same square adjacent to both 'A' and 'C'.

$$AP = \sqrt{(-1 - x)^2 + (2 - y)^2}$$

$$CP = \sqrt{(3 - x)^2 + (2 - y)^2}$$

But these two are nothing but the sides of the square and need to be equal to each other.

$$AP = CP$$

$$\sqrt{(-1 - x)^2 + (2 - y)^2} = \sqrt{(3 - x)^2 + (2 - y)^2}$$

Squaring on both sides we have,

$$(-1 - x)^2 + (2 - y)^2 = (3 - x)^2 + (2 - y)^2$$

$$1 + x^2 + 2x + 4 + y^2 - 4y = 9 + x^2 - 6x + 4 + y^2 - 4y$$

$$8x = 8$$

$$x = 1$$

Substituting this value of 'x' and the length of the side in the equation for 'AP' we have,

$$AP = \sqrt{(-1 - x)^2 + (2 - y)^2}$$

$$2\sqrt{2} = \sqrt{(-1 - 1)^2 + (2 - y)^2}$$

$$2\sqrt{2} = \sqrt{(-2)^2 + (2 - y)^2}$$

Squaring on both sides,

$$8 = (-2)^2 + (2 - y)^2$$

$$8 = 4 + 4 + y^2 - 4y$$

$$0 = y^2 - 4y$$

We have a quadratic equation. Solving for the roots of the equation we have,

$$y^2 - 4y = 0$$

$$y(y - 4) = 0$$

The roots of this equation are 0 and 4.

Therefore the other two vertices of the square are $(1, 0)$ and $(1, 4)$.

***** END *****