Question 11. The force experienced by a mass moving with a uniform speed v in a circular path of radius r experiences a force which depends on its mass, speed and radius. Prove that the relation is $f = mv^2/r$.

Answer:

$$f \propto m^a v^b r^c$$

$$\therefore \quad [f] = k\,[m]^a\,[v]^b\,[r]^c$$

where k is a constant

or $$[MLT^{-2}] = [M]^a\,[LT^{-1}]^b\,[L]^c$$

Compare the powers of M, L and T, we have

$$a = 1, \quad b + c = 1 \quad \text{and} \quad -b = -2$$

Solving above equations, we get

$$a = 1, \quad b = 2 \quad \text{and} \quad c = -1$$

$$\therefore \quad f = k\,M^1\,v^2\,r^{-1}$$

or $$f = k\,\frac{mv^2}{r}$$

Here $$k = 1$$

$$\therefore \quad \boxed{f = \frac{mv^2}{r}}$$

Question 12. The distance of the Sun from the Earth is 1.496×10^{11} m (i.e., 1 A.U.). If the angular diameter of the Sun is 2000″, find the diameter of the Sun.

Answer:

Here, θ = 2000

$= 2000/3600 \times \pi/180$ rad

$= 9.7 \times 10^{-3}$ rad d

$= 1.496 \times 10^{11}$ m

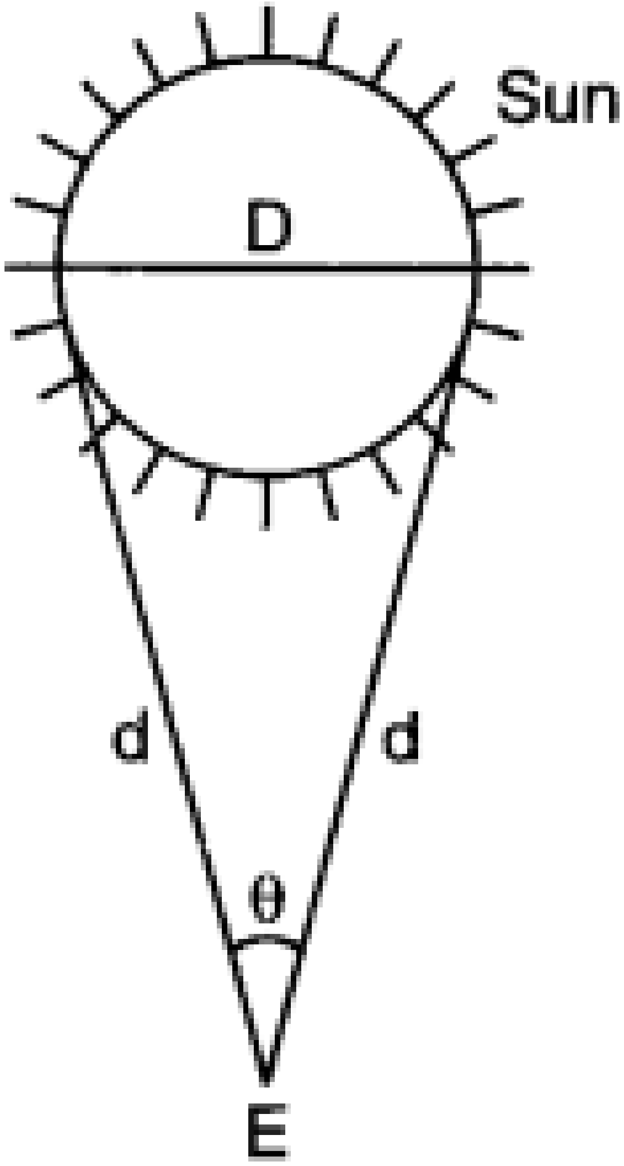
From the figure,

θ = D/d

D = θ d

$= 9.7 \times 10^{-3} \times 1.496 \times 10^{11}$

$= 1.45 \times 10^{9}$m



Question 13. Experiments show that the frequency (n) of a tuning fork depends upon the length (l) of the prong, the density (d) and the Young's modulus (Y) of its material. From dimensional considerations, find a possible relation for the frequency of a tuning fork.
Answer:

We are given that

$$n = f(l, d, Y)$$

Assuming that

$$n = k\, l^a d^b Y^c$$

and substituting dimensions of all the quantities involved, we have

$$[T^{-1}] = [L]^a [ML^{-3}]^b [ML^{-1} T^{-2}]^c$$

Equating powers of M, L and T on both sides, we have

$$b + c = 0$$

$$a - 3b - c = 0$$

$$-2c = -1$$

These give $c = \frac{1}{2}$, $b = -\frac{1}{2}$ and $a = -1$

$$\therefore \quad n = kl^{-1} d^{-1/2} Y^{1/2}$$

or

$$n = \frac{k}{l}\sqrt{\frac{Y}{d}}$$

This is the required relation for the frequency of a tuning fork.

Question 14. Calculate focal length of a spherical mirror from the following observations: object distance u = (50.1 ± 0.5) cm and image distance v = (20.1 ± 0.2) cm.
Answer:

As
$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = \frac{v+u}{uv}$$

$$\therefore \quad f = \frac{uv}{u+v} = \frac{(50.1)(20.1)}{(50.1+20.1)} = 14.3 \text{ cm}$$

Also,
$$\frac{\Delta f}{f} = \pm\left[\frac{\Delta u}{u} + \frac{\Delta v}{v} + \frac{\Delta u + \Delta v}{u+v}\right] = \pm\left[\frac{0.5}{50.1} + \frac{0.2}{20.1} + \frac{0.5+0.2}{50.1+20.1}\right]$$

$$\frac{\Delta f}{f} = \pm\left[\frac{1}{100.2} + \frac{1}{100.5} + \frac{0.7}{70.2}\right] = \pm\,[0.00998 + 0.00995 + 0.00997]$$

$$\Delta f = 0.02990 \times f = 0.0299 \times 14.3 = 0.428 \text{ cm} = 0.4 \text{ cm}$$

$$\therefore \quad f = \mathbf{(14.3 \pm 0.4)\ cm}$$

Question 15. The radius of the Earth is 6.37×10^6 m and its mass is 5.975×10^{24} kg. Find the Earth's average density to appropriate significant figures.
Answer:
Radius of the Earth (R) = 6.37×10^6 m
Volume of the Earth (V) = $4/3\ \pi R^3 m^3$
= $4/3 \times (3.142) \times (6.37 \times 10^6)^3\ m^3$
Average density (D)=Mass/Volume=M/V= 0.005517×10^6 kg m^{-3}
The density is accurate only up to three significant figures which is the accuracy of the least accurate factor, namely, the radius of the earth.

Question 16. The orbital velocity v of a satellite may depend on its mass m, distance r from the centre of Earth and acceleration due to gravity g. Obtain an expression for orbital velocity.
Answer: Let orbital velocity of satellite be given by the relation
v = kma rb gc where k is a dimensionless constant and a, b and c are the unknown powers. Writing dimensions on two sides of equation, we have
$[M^\circ L^1 T^{-1}] = [M]^a [L]^b [L T^{-2}]^c = [M^a L^{b+c} T^{-2c}]$
Applying principle of homogeneity of dimensional equation, we find that
a = 0 ...(i)
b + c = 1 ...(ii)
- 2c = - 1 ...(iii)
On solving these equations, we find that
a = 0, b = +(1/2) and c = +(1/2)

$$\therefore \qquad v = kr^{\frac{1}{2}}g^{\frac{1}{2}}$$

$$\text{or} \qquad v = k\sqrt{rg}\,.$$

Question 17. Check by the method of dimensional analysis whether the following relations are correct.

(i) $v = \sqrt{\frac{P}{D}}$ *where* v = *velocity of sound and*

P = *pressure*, D = *density of medium.*

(ii) $n = \frac{1}{2l}\sqrt{\frac{F}{m}}$, where n = frequency of vibration

l = length of the string

F = stretching force

m = mass per unit length of the string.

Answer:

(i)

$$[\text{R.H.S.}] = \sqrt{\frac{[P]}{[D]}}$$

$$= \sqrt{\frac{ML^{-1}T^{-2}}{ML^{-3}}} = LT^{-1}$$

$$[\text{L.H.S.}] = [v] = LT^{-1}$$

$$[\text{R.H.S.}] = [\text{L.H.S.}]$$

Hence, the relation is correct.

(ii)

$$[\text{R.H.S.}] = \frac{1}{[l]}\sqrt{\frac{[F]}{[m]}}$$

$$= \frac{1}{L}\sqrt{\frac{MLT^{-2}}{ML^{-1}}} = \frac{1}{L}LT^{-1} = T^{-1}$$

$$[\text{L.H.S.}] = \left[\frac{1}{\text{Time}}\right] = \frac{1}{T} = T^{-1}$$

Hence, the relation is correct.

Question 18. Given that the time period T of oscillation of a gas bubble from an explosion under water depends upon P, d and E where P is the static pressure, d the density of water and E is the total energy of explosion, find dimensionally a relation for T.

Answer: We are given that

T = f (P, d, E)

Assuming that T = k Pa db Ec and substituting dimensions of all the quantities involved, we have

$[T] = [M L^{-1} T^{-2}]^a [M L^{-3}]^b [M L^2 T^{-2}]^c$ Equating powers of M, L and T on both sides,

we have a + b + c = 0

-a - 3b + 2c = 0

-2a - 2 c = 1

Solving these equations, we get

a = -5/6 b = 1/2 c = 1/3

$$\therefore \qquad T = k\, P^{-5/6}\, d^{1/2}\, E^{1/3}$$

$$\text{or} \qquad T = k\left(\frac{d^{1/2}\, E^{1/3}}{P^{5/6}}\right)$$

This is the required relation for T.

Question 19. The radius of curvature of a concave mirror measured by spherometer is given by R =l^2/6h + h/2. The values of l and h are 4 cm and 0.065 cm respectively. Compute the error in measurement of radius of curvature.

Answer:

We are given

$$l = 4 \text{ cm}, \quad \Delta l = 0.1 \text{ cm} \qquad \text{(least count of the metre scale)}$$

here l is the distance between the legs of the spherometer.

$$\text{As} \qquad R = \frac{l^2}{6h} + \frac{h}{2}$$

$$\therefore \qquad \frac{\Delta R}{R} = \frac{2\Delta l}{l} + \left(-\frac{\Delta h}{h}\right) + \frac{\Delta h}{h}$$

Considering the magnitudes only, we get

$$\frac{\Delta R}{R} = 2\frac{\Delta l}{l} + \frac{\Delta h}{h} + \frac{\Delta h}{h}$$

$$= 2\left(\frac{\Delta l}{l} + \frac{\Delta h}{h}\right)$$

$$= 2 \times \frac{0.1}{4} + \frac{2 \times 0.001}{0.065}$$

$$= 0.05 + 0.03 = 0.08.$$

Question 20. The radius of the Earth is 6.37 x10^6 m and its average density is 5.517 x 10^3 kg m^{-3}. Calculate the mass of earth to correct significant figures.

Answer:

Mass = Volume x density

Volume of earth = 4/3π R^3

= 4/3 x 3.142 x $(6.37 \times 10^6)^3 m^3$

Mass of earth = 4/3 x 3.142 x $(6.37 \times 10^6)^3$ x 5.517 x 10^3 kg

= 5974.01 x 10^{21} kg = 5.97401 x 10^{24} kg

The radius has three significant figures and the density has four. Therefore, the final result should be rounded up to three significant figures. Hence, mass of the earth = 5.97 x 10^{24} kg.

*********** END **********