Indefinite Integrals Ex 19.32 Q6

$$\text{Let} \quad I = \int \frac{x}{\left(x^2+1\right)\sqrt{x}}\,dx$$

Let $x = t^2$

$$\Rightarrow \qquad dx = 2t\,dt$$

$$\therefore \qquad 2\int \frac{t\,dt}{\left(t^2+1\right)t}$$

$$= 2\left|\frac{dt}{t^4+1}\right|$$

Dividing numerator and denominator by t^2

$$I = 2\int \frac{\frac{t}{t^2}}{\left(t^2+\frac{1}{t^2}\right)}dt$$

$$= \int \frac{\left(1+\frac{1}{t^2}\right)-\left(1-\frac{1}{t^2}\right)}{\left(t^2+\frac{1}{t^2}\right)}dt$$

$$= \int \frac{\left(1+\frac{1}{t^2}\right)}{\left(t-\frac{1}{t}\right)^2+2}dt - \int \frac{\left(1-\frac{1}{t^2}\right)}{\left(1+\frac{1}{t}\right)^2-2}dt$$

Let $t - \frac{1}{t} = z$

$$\Rightarrow \qquad \left(1+\frac{1}{t^2}\right)dt = dz \qquad \qquad [\text{For Ist part}]$$

and,

$$t + \frac{1}{t} = y$$

$$\Rightarrow \qquad \left(1-\frac{1}{t^2}\right)dt = dy \qquad \qquad [\text{For IInd part}]$$

$$\therefore \qquad I = \int \frac{dz}{z^2+2} - \int \frac{dy}{y^2-2}$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{z}{\sqrt{2}}\right) - \frac{1}{2\sqrt{2}}\log\left|\frac{y-\sqrt{2}}{y+\sqrt{2}}\right| + c$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t^2-1}{\sqrt{2}t}\right) - \frac{1}{2\sqrt{2}}\log\left|\frac{x+1-\sqrt{2x}}{x+1+\sqrt{2x}}\right| + c$$

Indefinite Integrals Ex 19.32 Q7

Let $\quad I = \int \frac{x}{\left(x^2 + 2X + 2\right)\sqrt{x+1}} dx$

Let $x + 1 = t^2$

$\Rightarrow \qquad dx = 2tdt$

$$= 2\int \frac{\left(t^2 - 1\right)tdt}{\left(t^4 + 1\right)t}$$

$$= 2\int \frac{\left(t^2 - 1\right)dt}{\left(t^4 + 1\right)}$$

$$= 2\int \frac{\left(1 - \frac{1}{t^2}\right)dt}{t^2 + \frac{1}{t^2}}$$

$$= 2\int \frac{\left(1 - \frac{1}{t^2}\right)dt}{\left(t + \frac{1}{t}\right)^2 - 2}$$

Let $t + \frac{1}{t} = y$

$\Rightarrow \qquad \left(1 - \frac{1}{t^2}\right)dt = dy$

$\therefore \qquad I = 2\int \frac{dy}{y^2 - 2}$

$$= \frac{2}{2\sqrt{2}} log \left|\frac{y - \sqrt{2}}{y + \sqrt{2}}\right| + c$$

Thus,

$$I = \frac{1}{\sqrt{2}} log \left|\frac{t^2 + 1 - \sqrt{2}t}{t^2 + 1 + \sqrt{2}t}\right| + c$$

Hence,

$$I = \frac{1}{\sqrt{2}} log \left|\frac{x + 2 - \sqrt{2\left(x + 1\right)}}{x + 2 + \sqrt{2\left(x + 1\right)}}\right| + c$$

Indefinite Integrals Ex 19.32 Q8

Let $\quad I = \int \frac{1}{(x-1)\sqrt{x^2+1}}dx$

Let $x - 1 = \frac{1}{t}$

$\Rightarrow \quad dx = -\frac{1}{t^2}dt$

$\therefore \quad I = -\int \frac{\frac{1}{t^2}dt}{\frac{1}{t}\sqrt{\left(\frac{1}{t}+1\right)^2+1}}$

$= -\int \frac{dt}{\sqrt{2t^2+2t+1}}$

$= -\frac{1}{\sqrt{2}}\int \frac{dt}{\sqrt{t^2+t+\frac{1}{2}}}$

$= -\frac{1}{\sqrt{2}}\int \frac{dt}{\sqrt{\left(t+\frac{1}{2}\right)^2+\frac{1}{4}}}$

$\therefore \quad I = -\frac{1}{\sqrt{2}}\log\left|\left(t+\frac{1}{2}\right)+\sqrt{\left(t+\frac{1}{2}\right)^2+\frac{1}{4}}\right|+c \quad \left[\text{When } t = \frac{1}{x-1}\right]$

Indefinite Integrals Ex 19.32 Q9

Let $\quad I = \int \frac{1}{(x+1)\sqrt{x^2+x+1}}dx$

Let $x + 1 = \frac{1}{t}$

$\Rightarrow \quad dx = -\frac{1}{t^2}dt$

$\therefore \quad I = -\int \frac{\frac{1}{t^2}dt}{\frac{1}{t}\sqrt{\left(\frac{1}{t^2}+\frac{1}{t}-1\right)}}$

$= -\int \frac{dt}{\sqrt{1+t-t^2}}$

$= -\int \frac{dt}{\sqrt{\frac{5}{4}-\left(\frac{1}{4}-t+t^2\right)}}$

$= -\int \frac{dt}{\sqrt{\frac{5}{4}-\left(t-\frac{1}{2}\right)^2}}$

$= -\sin^{-1}\left(\frac{t-\frac{1}{2}}{\frac{\sqrt{5}}{2}}\right)+c$

$\therefore \quad I = -\sin^{-1}\left(\frac{2t-1}{\sqrt{5}}\right)+c \qquad \left[\text{When } t = \frac{1}{x+1}\right]$

Indefinite Integrals Ex 19.32 Q10

Let $I = \int \frac{1}{\left(x^2-1\right)\sqrt{x^2+1}}dx$

Let $x = \frac{1}{t}$

$\Rightarrow \quad dx = -\frac{1}{t^2}dt$

$$\therefore \quad I = -\int \frac{\frac{1}{t^2}dt}{\left(\frac{1}{t^2}-1\right)\sqrt{\left(\frac{1}{t^2}+1\right)}}$$

$$= -\int \frac{tdt}{\left(1-t^2\right)\sqrt{1+t^2}}$$

Let $1+t^2 = u^2$

$\Rightarrow \quad 2tdt = 2udu$

$$I = \int \frac{u\,du}{\left(u^2-2\right)u}$$

$$= \int \frac{du}{u^2-2}$$

$$\therefore \quad I = \frac{1}{2\sqrt{2}} log\left|\frac{u-\sqrt{2}}{u+\sqrt{2}}\right| + c$$

$$= \frac{1}{2\sqrt{2}} log\left|\frac{\sqrt{1+t^2}-\sqrt{2}}{\sqrt{1+t^2}+\sqrt{2}}\right| + c$$

Hence,

$$I = -\frac{1}{2\sqrt{2}} log\left|\frac{\sqrt{2}x+\sqrt{x^2+1}}{\sqrt{2}x-\sqrt{x^2+1}}\right| + c$$

*********** END **********