Question 7. 31. A cylinder of mass 10 kg and radius 15 cm is rolling perfectly on a plane of inclination 30°. The coefficient of static friction us = 0.25.
(a) How much is the force of friction acting on the cylinder?
(b) What is the work done against friction during rolling?
(c) If the inclination O of the plane is increased, at what value of 8 does the cylinder begin to skid, and not roll perfectly?
Answer:

(a)
$$f = \frac{1}{2} mg \sin\theta$$
$$= \frac{1}{3} \times 10 \times 9.8 \times \sin 30° \text{ N} = \mathbf{16.3\ N.}$$

(b) **No work is done** against friction during rolling.

(c)
$$\mu = \frac{1}{3}\tan\theta \quad \text{or} \quad \tan\theta = 3\mu$$
$$\text{tam}\ \theta = 3 \times 0.25 = 0.75$$
$$\theta = \tan^{-1}(0.75)$$
$$= 36.87° = \mathbf{37°.}$$

Question 7. 32. Read each statement below carefully, and state, with reasons, if it is true or false:
(a) During rolling, the force of friction acts in the same direction as the direction of motion of the CM of the body.
(b) The instantaneous speed of the point of contact during rolling is zero.
(c) The instantaneous acceleration of the point of contact during rolloing is zero.
(d) For perfect rolling motion, work done against friction is zero.
(e) A wheel moving down a perfectly frictionless inclined plane will undergo slipping (not rolling) motion.
Answer:
(a) True. When a body rolls without slipping, the force of friction acts in the same direction as the direction of motion of the centre of mass of rolling body.
(b) True. This is because rolling body can be imagined to be rotating about an axis passing through the point of contact of the body with the ground. Hence its instantaneous speed is zero.
(c) False. This is because when the body is rotating, its instantaneous acceleration is not zero.
(d) True. For perfect rolling motion as there is no relative motion at the point of contact, hence work done against friction is zero.
(e) True. This is because rolling occurs only on account of friction which is a tangential force capable of providing torque. When the inclined plane is perfectly smooth, it will simply slip under the effect of its own weight.

Question 7. 33. Separation of Motion of a system of particles into motion of the centre of mass and motion about the centre of mass:

(a) *Show* $\vec{p_i} = \vec{p_i}' + m_i \vec{V}$

where p_i *is the momentum of the* i^{th} *particle (of mass* m_i*) and* $\vec{p_i}' + m_i \overrightarrow{v'_i}$. *Note* $\overrightarrow{v'_i}$ *is the veloity of* i^{th} *particle relative to the centre of mass.*

Also, prove using the definition of the centre of mass $\sum \vec{p_i} = 0$

(b) *Show* $K = K' + \frac{1}{2}MV^2$

where K is the total kinteic energy of the system of particles, K′ is the total kinetic energy of the system when the particle velocities are taken with respect to the centre of mass and $MV^2/2$ *is the kinetic energy of the translation of the system as a whole (i.e., of the centre of mass motion of the system). The result has been used in Sec. 7.14.*

(c) *Show* $\vec{L} = m_i \overrightarrow{L'} + M\vec{R} \times \vec{V}$

where $\vec{L} = \sum \vec{r'_i} \times \vec{p'_i}$ *is the angular momentum of the system about the centre of mass with velocities taken relative to the centre of mass. Remember* $\vec{r'_i} = \vec{r_i} - \vec{R}$; *rest of the notation is the standard notation used in the chapter. Note* $\vec{L'}$ **and** $\overrightarrow{MR} \times \vec{V}$ *can be said to be angular momenta respectively, about and of the centre of mass of the system of particles.*

(d) *Show* $$\frac{d\vec{L'}}{dt} = \sum \vec{r'_i} \times \frac{d\vec{p'}}{dt}$$

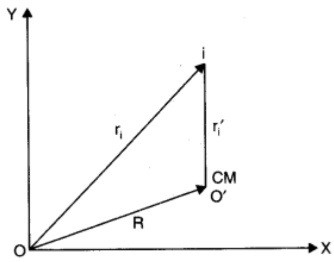
Further, show that:

$$\frac{d\vec{L'}}{dt} = \overrightarrow{\tau'_{ext}}$$

where $\overrightarrow{\tau'_{ext}}$ *is the sum of all external torques acting on the system about the centre of mass.*

[Hint: use the definition of centre of mass and Newton's Third Law. Assume the internal forces between any two particles act along the line joining the particles.]

Answer:



Here $\vec{r_i} = \vec{r'_i} + \vec{R}$ + R and $\vec{V_i} = \overrightarrow{V'_i} + \vec{V}$

where $\vec{r'_i}$ and $\vec{v'_i}$ denote the radius vector and velocity of the i^{th} particle referred to centre of mass O' as the new origin and $\vec{V}$ is the velocity of centre of mass relative to O.

(a) Momentum of i^{th} particle

$$\vec{P} = m_i \vec{V_i'}$$

$$= m_i (\vec{V_i'} + \vec{V}) \text{ (since } \vec{V_i} = \vec{V_i'} + \vec{V})$$

or $$\vec{P} = m_i \vec{V_i'} + m_i \vec{V}$$

$$\vec{P} = \vec{P_i} + m_i \vec{V}$$

(b) Kinetic energy of the system of particles.

$$K = \frac{1}{2}\sum m_i V_i^2$$

$$= \frac{1}{2}\sum m_i \vec{V_i} \cdot \vec{V_i}$$

$$= \frac{1}{2}\sum m_i \left(\overrightarrow{V_i'} + \vec{V}\right) \cdot \left(\overrightarrow{V_i'} + \vec{V}\right)$$

$$= \frac{1}{2}\sum m_i \left(V_i'^2 + V^2 + 2\overrightarrow{V_i'} \cdot \vec{V}\right)$$

$$= \frac{1}{2}\sum m_i V_i'^2 + \frac{1}{2}\sum m_i V_i'^2 + \sum m_i \vec{V_i} \cdot \vec{V}$$

$$= \frac{1}{2}MV^2 + K'$$

where $$M = \sum m_i$$

= total mass of the system

$$K' = \frac{1}{2}\sum_i m_i V_i'^2$$

= kinetic energy of motion about the centre of mass

or $\frac{1}{2}Mv^2$ = kinetic energy of motion of centre of mass. **(Proved)**

since $$\sum_i m_i \overrightarrow{V'_i} \cdot \vec{V} = \sum m_i \frac{d\vec{r_i}}{dt} \cdot \vec{V}$$

$$= \frac{d}{dt}\left(\sum m_i \overrightarrow{r'_i}\right) \cdot \vec{V} = \frac{d}{dt}\left(M\vec{R} \cdot \vec{V}\right)$$

$$= 0$$

(c) Total angular momentum of the system of particles.

$$\vec{L} = \vec{r}_i \times \vec{p}$$
$$= (\vec{r}_i + \vec{R}) \times \sum_i m_i \vec{V}_i$$
$$= (\vec{r}'_i + \vec{R}) \times \sum_i m_i (\vec{V}_i' + \vec{V})$$
$$= \sum_i \left(\vec{R} \times m_i \vec{V}\right) + \sum_i \vec{r}'_i \times m_i \vec{V}'_i + \left(\sum_i m_i \vec{r}'_i\right) \times \vec{V} + \vec{R} \times \sum_i m_i \vec{V}_i$$
$$= \sum_i \left(\vec{R} \times m_i \vec{V}\right) + \sum_i \vec{r}_i' \times m_i \vec{V}_i + \left(\sum_i m_i \vec{r}_i'\right) \times \vec{V} + \vec{R} \times \frac{d}{dt}\left(\sum_i m_i \vec{r}_i'\right)$$

The last two terms vanish for both contain the factor $\sum m_i \vec{r}'_i$ which is equal to

$$\sum_i m_i \vec{r}_i = \sum_i m_i \left(\vec{r}'_i - \vec{R}\right) = M\vec{R} - M\vec{R} = \vec{0}$$

from the definition of centre of mass. Also

$$\sum_i \left(\vec{R} \times m_i \vec{V}\right) = \vec{R} \times M\vec{V}$$

so that $$\vec{L} = \vec{R} \times M\vec{V} + \sum_i \vec{r}'_i \times \vec{P}_i$$

or $$\vec{L} = \vec{R} \times M\vec{V} + \vec{L}'$$

where $$\vec{L}' = \sum \vec{r}'_i \times \vec{P}_i$$

(*d*) From previous solution

$$\vec{L}' = \sum \vec{r}'_i \times \vec{P}_i$$
$$\frac{d\vec{L}'}{dt} = \sum \vec{r}_i' \times \frac{d\vec{P}_i}{dt} + \sum \frac{d\vec{r}'_i}{dt} \times \vec{P}_i$$
$$= \sum \vec{r}'_i \times \frac{d\vec{P}_i}{dt}$$
$$= \sum \vec{r}_i \times \vec{F}_i^{ext} = \vec{\tau}'_{ext}$$

Since $$\sum \frac{d\vec{r}'_i}{dt} \times \vec{P}_i = \sum \frac{d\vec{r}'_i}{dt} \times m\vec{v}_i = 0$$

Total torque $$\vec{\tau} = \sum \vec{r}_i \times \vec{F}_i^{ext}$$
$$= \sum \left(\vec{r}_i + \vec{R}\right) \times \vec{F}_i^{ext}$$
$$= \sum \vec{r}'_i \times \vec{F}_i^{ext} + \vec{R} \times \sum_i \vec{F}_i^{ext}$$
$$= \vec{\tau}'_{ext} + \vec{\tau}_0^{(ext)}$$

where $\vec{\tau}_{ext}$ is the total torque about the centre of mass as origin and $\vec{\tau}_0^{ext}$, that about the origin O.

$$\vec{\tau}'_{ext} = \sum \vec{r}'_i \times \vec{F}_i^{ext}$$
$$= \sum \vec{r}_i \times \frac{d\vec{P}'_i}{dt} = \frac{d}{dt} \sum_i \left(\vec{r}_i \times \vec{P}_i\right) = \frac{d\vec{L}'}{dt}$$

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