Exercise 7.4

7. Let A (4, 2), B (6, 5) and C (1, 4) be the vertices of ΔABC.

(i) The median from A meets BC at D. Find the coordinates of the point D.

(ii) Find the coordinates of the point P on AD such that AP: PD = 2: 1.

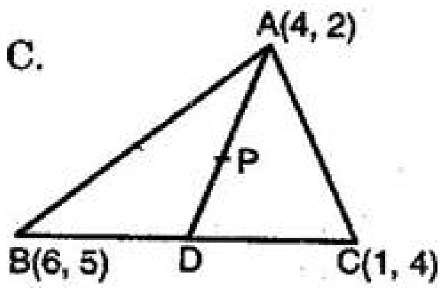
(iii) Find the coordinates of points Q and R on medians BE and CF respectively such that BQ: QE = 2: 1 and CR : RF = 2 : 1.

(iv) What do you observe?

(Note: The point which is common to all the three medians is called *centroid* and this point divides each median in the ratio 2: 1)

(v) If A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are the vertices of ΔABC, find the coordinates of the centroid of the triangle.

Ans. Let A (4, 2), B (6, 5) and C (1, 4) be the vertices of ΔABC.



(i) Since AD is the median of ΔABC.

$\therefore$ D is the mid-point of BC.

$\therefore$ Its coordinates are $\left(\frac{6+1}{2}, \frac{5+4}{2}\right) = \left(\frac{7}{2}, \frac{9}{2}\right)$

(ii) Since P divides AD in the ratio 2: 1

$\therefore$ Its coordinates are $\left(\frac{2\times\frac{7}{2}+1\times4}{2+1}, \frac{2\times\frac{9}{2}+1\times2}{2+1}\right) =$

$\left(\frac{11}{3}, \frac{11}{3}\right)$

(iii) Since BE is the median of ΔABC.

$\therefore$ E is the mid-point of AD.

$\therefore$ Its coordinates are $\left(\frac{4+1}{2}, \frac{2+4}{2}\right) = \left(\frac{5}{2}, 3\right)$

Since Q divides BE in the ratio 2: 1.

$\therefore$ Its coordinates are $\left(\frac{2\times\frac{5}{2}+1\times 6}{2+1}, \frac{2\times 3+1\times 5}{2+1}\right) =$

$\left(\frac{11}{3}, \frac{11}{3}\right)$

Since CF is the median of ΔABC.

$\therefore$ F is the mid-point of AB.

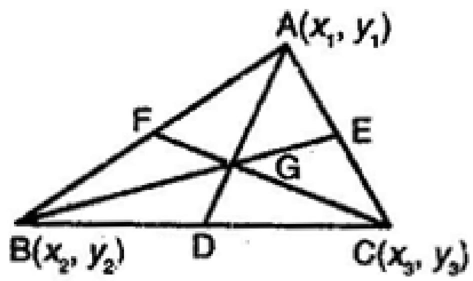
$\therefore$ Its coordinates are $\left(\frac{4+6}{2}, \frac{2+5}{2}\right) = \left(5, \frac{7}{2}\right)$

Since R divides CF in the ratio 2: 1.

$\therefore$ Its coordinates are $\left(\frac{2\times 5+1\times 1}{2+1}, \frac{2\times\frac{7}{2}+1\times 4}{2+1}\right) =$

$\left(\frac{11}{3}, \frac{11}{3}\right)$

(iv) We observe that the points P, Q and R coincide, i.e., the medians AD, BE and CF are concurrent at the point $\left(\frac{11}{3}, \frac{11}{3}\right)$. This point is known as the centroid of the triangle.



(v) According to the question, D, E, and F are the mid-points of BC, CA and AB respectively.

$\therefore$ Coordinates of D are $\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$

Coordinates of a point dividing AD in the ratio 2: 1 are

$$\left(\frac{1.x_1 + 2\left(\frac{x_2 + x_2}{2}\right)}{1+2}, \frac{1.y_1 + 2\left(\frac{y_2 + y_3}{2}\right)}{1+2}\right) =$$

$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

The coordinates of E are $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$.

$\therefore$ The coordinates of a point dividing BE in the ratio 2: 1 are

$$\left(\frac{1.x_2+2\left(\frac{x_1+x_3}{2}\right)}{1+2}, \frac{1.y_2+2\left(\frac{y_1+y_3}{2}\right)}{1+2}\right) =$$

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Similarly, the coordinates of a point dividing CF in the ratio 2: 1 are

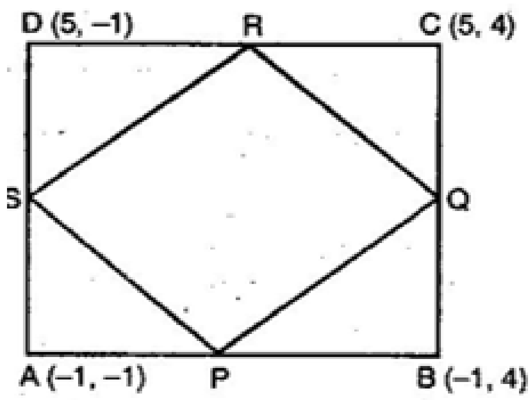
$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Thus, the point $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$ is common to AD, BE and CF and divides them in the ratio 2: 1.

$\therefore$ The median of a triangle are concurrent and the coordinates of the centroid are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

8. ABCD is a rectangle formed by joining points A$(-1,-1)$, B$(-1,4)$, C$(5,4)$ and D$(5,-1)$. P, Q, R and S are the mid-points of AB, BC, CD and DA respectively. Is the quadrilateral PQRS a square? Or a rhombus? Justify your answer.

Ans. Using distance formula, PQ = $\sqrt{(2+1)^2+\left(4-\frac{3}{2}\right)^2}$



$= \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$

$QR = \sqrt{(5-2)^2+\left(\frac{3}{2}-4\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$

$RS = \sqrt{(2-5)^2+\left(-1-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$

$SP = \sqrt{(-1-2)^2+\left(\frac{3}{2}+1\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$

$\Rightarrow PQ = QR = RS = SP$

Now, $PR = \sqrt{(5+1)^2+\left(\frac{3}{2}-\frac{3}{2}\right)^2} = \sqrt{36} = 6$

And $SQ = \sqrt{(2-2)^2+(4+1)^2} = \sqrt{25} = 5$

$\Rightarrow PR \neq SQ$

Since all the sides are equal but the diagonals are not equal.

$\therefore$ PQRS is a rhombus.

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