Continuity Ex 9.1 Q37
It is given that the function is continuous at $x=3$ and at $x=5$

$\therefore$ LHL = RHL = $f(3)\ldots\ldots(1)$ and
LHL = RHL = $f(5)\ldots\ldots(2)$

Now,
$f(3)=1$
RHL = $\lim\limits_{x\to 3^+} f(x) = \lim\limits_{h\to 0} f(3+h) = \lim\limits_{h\to 0} a(3+h)+b = 3a+b$

Thus, using (1), we get,
$3a+b=1\ldots.(3)$

$f(5)=7$
LHL = $\lim\limits_{x\to 5^-} f(x) = \lim\limits_{h\to 0} f(5-h) = \lim\limits_{h\to 0} a(5-h)+b = 5a+b$

Thus, using (2), we get
$5a+b=7\ \ldots\ldots(4)$

Now, solving (3) and (4) we get,
$a=3$ and $b=-8$
Continuity Ex 9.1 Q38
We want to discuss the continuity of the function at $x=1$

We need to prove that

LHL = RHL = $f(1)$

$f(1)=\frac{1^2}{2}=\frac{1}{2}$

LHL = $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{h\to 0} f(1-h) = \lim\limits_{h\to 0}\frac{(1-h)^2}{2}=\frac{1}{2}$

RHL = $\lim\limits_{x\to 1^+} f(x) = \lim\limits_{h\to 0} f(1+h) = \lim\limits_{h\to 0} 2(1+h)^2-3(1+h)+\frac{3}{2} = 2-3+\frac{3}{2}=\frac{1}{2}$

Thus, LHL = RHL = $f(1)=\frac{1}{2}$
Hence, function is continuous at $x=1$
Continuity Ex 9.1 Q39

We want to discuss the continuity at $x = 0$ and $x = 1$

Now,

$f(0) = 1$

LHL = $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} |-h| + |-h - 1| = 1.$

RHL = $\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} |h| + |h - 1| = 1$

$\because$ LHL = RHL = $f(0) = 1$, function is continuous at $x = 0$.

For $x = 1$.

$f(1) = 1$

LHL = $\lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1 - h) = \lim_{h \to 0} |1 - h| + |1 - h - 1| = 1$

RHL = $\lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1 + h) = \lim_{h \to 0} |1 + h| + |1 + h - 1| = 1$

$\because$ LHL = RHL = $f(1) = 1$ function is continuous at $x = 1$.

For $x = -1$

$f(-1) = |-1 - 1| + |-1 + 1| = 2$

LHL = $\lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(-1 - h) = \lim_{h \to 0} |-1 - h - 1| + |-1 - h + 1| = 2$

RHL = $\lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(-1 + h) = \lim_{h \to 0} |-1 + h - 1| + |-1 + h + 1| = 2$

Thus, LHL = RHL = $f(-1) = 2$

Hence, function is continuous at $x = -1$

For $x = 1$

$f(1) = |1 - 1| + |1 + 1| = 2$

LHL = $\lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1 - h) = \lim_{h \to 0} |1 - h - 1| + |1 - h + 1| = 2$

RHL = $\lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1 + h) = \lim_{h \to 0} |1 + h - 1| + |1 + h + 1| = 2$

Thus, LHL = RHL = $f(1) = 2$

Hence, function is continuous at $x = 1$

Continuity Ex 9.1 Q40

Since $f(x)$ *is continuous at* $x=0$, *L.H.Limit = R.H.Limit.*
Thus, we have

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)$$

$$\Rightarrow \lim_{x\to 0^-} a\sin\frac{\pi}{2}(x+1)=\lim_{x\to 0^+}\frac{\tan x-\sin x}{x^3}$$

$$\Rightarrow a\times 1=\lim_{x\to 0}\frac{\tan x-\sin x}{x^3}$$

$$\Rightarrow a=\lim_{x\to 0}\frac{\frac{\sin x}{\cos x}-\sin x}{x^3}$$

$$\Rightarrow a=\lim_{x\to 0}\frac{\frac{\sin x}{x}\left(\frac{1}{\cos x}-1\right)}{x^2}$$

$$\Rightarrow a=\lim_{x\to 0}\frac{\frac{\sin x}{x}\left(\frac{1-\cos x}{\cos x}\right)}{x^2}$$

$$\Rightarrow a=\lim_{x\to 0}\frac{\sin x}{x}\times\lim_{x\to 0}\frac{1}{\cos x}\times\lim_{x\to 0}\frac{1-\cos x}{x^2}$$

$$\Rightarrow a=1\times 1\times\lim_{x\to 0}\frac{1-\cos x}{x^2}$$

$$\Rightarrow a=\lim_{x\to 0}\frac{1-\cos x}{x^2}\times\frac{1+\cos x}{1+\cos x}$$

$$\Rightarrow a=\lim_{x\to 0}\frac{1-\cos^2 x}{x^2(1+\cos x)}$$

$$\Rightarrow a=\lim_{x\to 0}\frac{\sin^2 x}{x^2(1+\cos x)}$$

$$\Rightarrow a=\lim_{x\to 0}\frac{\sin^2 x}{x^2}\times\lim_{x\to 0}\frac{1}{1+\cos x}$$

$$\Rightarrow a=1\times\lim_{x\to 0}\frac{1}{1+\cos x}$$

$$\Rightarrow a=1\times\frac{1}{1+1}$$

$$\Rightarrow a=\frac{1}{2}$$

*********** END **********