Trigonometric Identities Ex 6.1 Q49

Answer :

In the given question, we need to prove $\tan^2 A + \cot^2 A = \sec^2 A \operatorname{cosec}^2 A - 2$

Now, using $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ in L.H.S, we get

$$\begin{aligned}\tan^2 A + \cot^2 A &= \frac{\sin^2 A}{\cos^2 A} + \frac{\cos^2 A}{\sin^2 A}\\ &= \frac{\sin^4 A + \cos^4 A}{\cos^2 A \sin^2 A}\\ &= \frac{(\sin^2 A)^2 + (\cos^2 A)^2}{\cos^2 A \sin^2 A}\end{aligned}$$

Further, using the identity $a^2 + b^2 = (a+b)^2 - 2ab$, we get

$$\begin{aligned}\frac{(\sin^2 A)^2 + (\cos^2 A)^2}{\cos^2 A \sin^2 A} &= \frac{(\sin^2 A + \cos^2 A)^2 - 2\sin^2 A\cos^2 A}{\sin^2 A\cos^2 A}\\ &= \frac{(1)^2 - 2\sin^2 A\cos^2 A}{\sin^2 A\cos^2 A}\\ &= \frac{1}{\sin^2 A\cos^2 A} - \frac{2\sin^2 A\cos^2 A}{\sin^2 A\cos^2 A}\\ &= \operatorname{cosec}^2 A\sec^2 A - 2\end{aligned}$$

Since L.H.S = R.H.S

Hence proved.

Trigonometric Identities Ex 6.1 Q50

Answer :

In the given question, we need to prove $\dfrac{1-\tan^2 A}{\cot^2 A - 1} = \tan^2 A$

Now, using $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ in the L.H.S, we get

$$\begin{aligned}\frac{1-\tan^2 A}{\cot^2 A - 1} &= \frac{1 - \frac{\sin^2 A}{\cos^2 A}}{\frac{\cos^2 A}{\sin^2 A} - 1}\\ &= \frac{\frac{\cos^2 A - \sin^2 A}{\cos^2 A}}{\frac{\cos^2 A - \sin^2 A}{\sin^2 A}}\\ &= \frac{\cos^2 A - \sin^2 A}{\cos^2 A} \times \frac{\sin^2 A}{\cos^2 A - \sin^2 A}\end{aligned}$$

Solving further, we get

$$\begin{aligned}\frac{\cos^2 A - \sin^2 A}{\cos^2 A} \times \frac{\sin^2 A}{\cos^2 A - \sin^2 A} &= \frac{\sin^2 A}{\cos^2 A}\\ &= \tan^2 A\end{aligned}$$

Hence proved.

Trigonometric Identities Ex 6.1 Q51

Answer :

In the given question, we need to prove $1+\frac{\cot^2\theta}{1+\operatorname{cosec}\theta}=\operatorname{cosec}\theta$

Using $\cot\theta=\frac{\cos\theta}{\sin\theta}$ and $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$, we get

$$1+\frac{\cot^2\theta}{1+\operatorname{cosec}\theta}=\frac{1+\operatorname{cosec}\theta+\cot^2\theta}{1+\operatorname{cosec}\theta}$$

$$=\frac{\left(1+\frac{1}{\sin\theta}+\frac{\cos^2\theta}{\sin^2\theta}\right)}{\left(1+\frac{1}{\sin\theta}\right)}$$

$$=\frac{\left(\frac{\sin^2\theta+\sin\theta+\cos^2\theta}{\sin^2\theta}\right)}{\left(\frac{\sin\theta+1}{\sin\theta}\right)}$$

Further, using the property $\sin^2\theta+\cos^2\theta=1$, we get

$$\frac{\left(\frac{\sin^2\theta+\sin\theta+\cos^2\theta}{\sin^2\theta}\right)}{\left(\frac{\sin\theta+1}{\sin\theta}\right)}=\frac{\left(\frac{1+\sin\theta}{\sin^2\theta}\right)}{\left(\frac{\sin\theta+1}{\sin\theta}\right)}$$

$$=\left(\frac{1+\sin\theta}{\sin^2\theta}\right)\left(\frac{\sin\theta}{1+\sin\theta}\right)$$

$$=\frac{1}{\sin\theta}$$

$$=\operatorname{cosec}\theta$$

Hence proved.

*********** END **********