Adjoint and Inverse of Matrix Ex 7.1 Q8(i)

Here, $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

Expanding using 1^{st} row, we get

$$|A| = 1\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} + 3\begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix}$$

$$= 1(6-1) - 2(4-3) + 3(2-9)$$

$$= 5 - 2\times 1 + 3\times(-7)$$

$$= 5 - 2 - 21 = -18 \neq 0$$

Therefore, A^{-1} exists.

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 5 & C_{21} = -1 & C_{31} = -7 \\ C_{12} = -1 & C_{22} = -7 & C_{32} = 5 \\ C_{13} = -7 & C_{23} = 5 & C_{33} = -1 \end{array}$$

$$\therefore \quad \operatorname{adj} A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{bmatrix}$$

Now,

$$A^{-1} = \frac{1}{|A|}.\operatorname{adj} A$$

Hence, $A^{-1} = \frac{1}{(-18)}\begin{bmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{bmatrix} = \begin{bmatrix} \frac{-5}{18} & \frac{1}{18} & \frac{7}{18} \\ \frac{1}{18} & \frac{7}{18} & \frac{-5}{18} \\ \frac{7}{18} & \frac{-5}{18} & \frac{1}{18} \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q8(ii)

Here, $A=\begin{bmatrix}1 & 2 & 5\\ 1 & -1 & -1\\ 2 & 3 & -1\end{bmatrix}$

Expanding using first column, we get

$$|A| = 1\begin{vmatrix}-1 & -1\\ 3 & -1\end{vmatrix} - 2\begin{vmatrix}1 & -1\\ 2 & -1\end{vmatrix} + 5\begin{vmatrix}1 & -1\\ 2 & 3\end{vmatrix}$$
$$= 1\times(1+3) - 2(-1+2) + 5(3+2)$$
$$= 4 - 2(1) + 5(5) = 27 \neq 0$$

Therefore, A^{-1} exists.

Cofactors of A are:

$C_{11} = 4$ $\quad C_{21} = +17$ $\quad C_{31} = 3$

$C_{12} = -1$ $\quad C_{22} = -11$ $\quad C_{32} = +6$

$C_{13} = 5$ $\quad C_{23} = +1$ $\quad C_{33} = -3$

$$\therefore \quad \operatorname{adj} A = \begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^T$$
$$= \begin{bmatrix}4 & -1 & 5\\ 17 & -11 & 1\\ 3 & 6 & -3\end{bmatrix}^T$$
$$= \begin{bmatrix}4 & 17 & 3\\ -1 & -11 & 6\\ 5 & 1 & -3\end{bmatrix}$$

Now, $A^{-1} = \dfrac{1}{|A|}.\operatorname{adj} A$

$$\therefore \quad A^{-1} = \frac{1}{27}\begin{bmatrix}4 & 17 & 3\\ -1 & -11 & 6\\ 5 & 1 & -3\end{bmatrix} = \begin{bmatrix}\frac{4}{27} & \frac{17}{27} & \frac{3}{27}\\ \frac{-1}{27} & \frac{-11}{27} & \frac{6}{27}\\ \frac{5}{27} & \frac{1}{27} & \frac{-3}{27}\end{bmatrix} = \begin{bmatrix}\frac{4}{27} & \frac{17}{27} & \frac{1}{9}\\ \frac{-1}{27} & \frac{-11}{27} & \frac{2}{9}\\ \frac{5}{27} & \frac{1}{27} & \frac{-1}{9}\end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q8(iii)

Here, $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$

Expanding using first column, we get

$$|A|=2\begin{vmatrix}2 & -1\\ -1 & 2\end{vmatrix}+1\begin{vmatrix}-1 & -1\\ 1 & 2\end{vmatrix}+1\begin{vmatrix}-1 & 2\\ 1 & -1\end{vmatrix}$$
$$=2(4-1)+1(-2+1)+1(1-2)$$
$$=2(3)+1(-1)+1(-1)=6-2=4\neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$C_{11}=3$ $\quad C_{21}=+1$ $\quad C_{31}=-1$

$C_{12}=+1$ $\quad C_{22}=3$ $\quad C_{32}=+1$

$C_{13}=-1$ $\quad C_{23}=+1$ $\quad C_{33}=3$

$$\therefore \quad \operatorname{adj}A=\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^T$$
$$=\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}^T$$
$$=\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}$$

Now, $A^{-1}=\frac{1}{|A|}.\operatorname{adj}A$

$$\therefore \quad A^{-1}=\frac{1}{4}\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}=\begin{bmatrix}\frac{3}{4} & \frac{1}{4} & \frac{-1}{4}\\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4}\\ \frac{-1}{4} & \frac{1}{4} & \frac{3}{4}\end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q8(iv)

Here, $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

Expanding using first column, we get

$$|A| = 2\begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix} - 0\begin{vmatrix} 5 & 0 \\ 0 & 3 \end{vmatrix} - 1\begin{vmatrix} 5 & 1 \\ 0 & 1 \end{vmatrix}$$
$$= 2(3-0) - 0 - 1(5)$$
$$= 2(3) - 1(5) = 1 \neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$C_{11} = 3$ $\quad C_{21} = -1$ $\quad C_{31} = 1$

$C_{12} = -15$ $\quad C_{22} = 6$ $\quad C_{32} = -5$

$C_{13} = 5$ $\quad C_{23} = -2$ $\quad C_{33} = 2$

$$\therefore \quad \text{adj}A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$
$$= \begin{bmatrix} 3 & -15 & 5 \\ -1 & 6 & -2 \\ 1 & -5 & 2 \end{bmatrix}^T$$
$$= \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}.\text{adj}A$$
$$= \frac{1}{1}\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

Hence, $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

Here, $A=\begin{bmatrix}0 & 1 & -1\\ 4 & -3 & 4\\ 3 & -3 & 4\end{bmatrix}$

Expanding using first column, we get

$$|A| = 0\begin{vmatrix}-3 & 4\\ -3 & 4\end{vmatrix} - 1\begin{vmatrix}4 & 4\\ 3 & 4\end{vmatrix} - 1\begin{vmatrix}4 & -3\\ 3 & -3\end{vmatrix}$$

$$= 0 - 1(6-12) - 1(-12+9)$$

$$= -1(4) - 1(-3) = -1 \neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$C_{11} = 0$ $\quad C_{21} = -1$ $\quad C_{31} = 1$

$C_{12} = -4$ $\quad C_{22} = 3$ $\quad C_{32} = -4$

$C_{13} = -3$ $\quad C_{23} = +3$ $\quad C_{33} = -4$

$$\therefore \quad \text{adj}A = \begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^T$$

$$= \begin{bmatrix}0 & -4 & -3\\ -1 & 3 & 3\\ 1 & -4 & -4\end{bmatrix}^T$$

$$= \begin{bmatrix}0 & -1 & 1\\ -4 & 3 & -4\\ -3 & 3 & -4\end{bmatrix}$$

Now, $A^{-1} = \dfrac{1}{|A|}.\text{adj}A$

$$A^{-1} = \frac{1}{(-1)}\begin{bmatrix}0 & -1 & 1\\ -4 & 3 & -4\\ -3 & 3 & -4\end{bmatrix}$$

$$A^{-1} = \begin{bmatrix}0 & 1 & -1\\ 4 & -3 & 4\\ 3 & -3 & 4\end{bmatrix}$$

Here, $A = \begin{bmatrix} 0 & 0 & -1 \\ 3 & 4 & 5 \\ -2 & -4 & -7 \end{bmatrix}$

Expanding using first column, we get

$$|A| = 0\begin{vmatrix} 4 & 5 \\ -4 & -7 \end{vmatrix} - 0\begin{vmatrix} 3 & 5 \\ -2 & -7 \end{vmatrix} - 1\begin{vmatrix} 3 & 4 \\ -2 & -4 \end{vmatrix}$$
$$= 0 - 0 - 1(-12 + 8)$$
$$= -1(-4) = 4 \neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$C_{11} = -8$ $\qquad C_{21} = +4$ $\qquad C_{31} = 4$

$C_{12} = +11$ $\qquad C_{22} = -2$ $\qquad C_{32} = -3$

$C_{13} = -4$ $\qquad C_{23} = +0$ $\qquad C_{33} = 0$

$$\therefore \qquad \text{adj}\, A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$
$$= \begin{bmatrix} -8 & 11 & -4 \\ 4 & -2 & 0 \\ 4 & -3 & 0 \end{bmatrix}^T$$
$$= \begin{bmatrix} -8 & 4 & 4 \\ 11 & -2 & -3 \\ -4 & 0 & 0 \end{bmatrix}$$

$$\therefore \qquad A^{-1} = \frac{1}{|A|}.\text{adj}\, A$$
$$A^{-1} = \frac{1}{4}\begin{bmatrix} -8 & 4 & 4 \\ 11 & -2 & -3 \\ -4 & 0 & 0 \end{bmatrix}$$

Hence, $A^{-1} = \frac{1}{4}\begin{bmatrix} -8 & 4 & 4 \\ 11 & -2 & -3 \\ -4 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 1 \\ \frac{11}{4} & \frac{-1}{2} & \frac{-3}{4} \\ -1 & 0 & 0 \end{bmatrix}$

Here, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

Expanding using first column, we get

$$|A| = 1\begin{vmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{vmatrix} - 0 + 0$$

$$= -\cos^2\alpha + \sin^2\alpha$$

$$= -\left(\cos^2\alpha + \sin^2\alpha\right)$$

$$|A| = -1 \neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$C_{11} = -1$ $\quad C_{21} = 0$ $\quad C_{31} = 0$

$C_{12} = 0$ $\quad C_{22} = -\cos\alpha$ $\quad C_{32} = -\sin\alpha$

$C_{13} = 0$ $\quad C_{23} = -\sin\alpha$ $\quad C_{33} = \cos\alpha$

$$\therefore \quad \text{adj} A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}^T$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

Now, $A^{-1} = \dfrac{1}{|A|}.\text{adj} A$

$$\therefore \quad A^{-1} = \frac{1}{(-1)}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

Hence, $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

*********** END **********