Question 20:

$$\left(\sin^2 25^\circ + \sin^2 65^\circ\right) + \sqrt{3}\left(\tan 5^\circ \tan 15^\circ \tan 30^\circ \tan 75^\circ \tan 85^\circ\right)$$

$$= \left(\sin^2 25^\circ + \sin^2 (90^\circ - 25^\circ)\right) + \sqrt{3}\left(\tan 5^\circ \tan 15^\circ \tan 30^\circ \tan(90^\circ - 15^\circ) \tan(90^\circ - 5^\circ)\right)$$

$$= \left(\sin^2 25^\circ + \cos^2 25\right) + \sqrt{3}\left[\tan 5^\circ \tan 15^\circ \times \frac{1}{\sqrt{3}} \cot 15^\circ \cot 5^\circ\right]$$

$$= 1 + \left(\tan 5^\circ \cot 5^\circ\right)\left(\tan 15^\circ \cot 15^\circ\right) \quad [\tan\theta \times \cot\theta = 1]$$

$$= 1 + 1 \times 1 = 2$$

Question 21:

$$\frac{\sin^2 40^\circ + \sin^2 50^\circ}{\cos^2 20 + \cos^2 70^\circ} + \tan 10^\circ \tan 20^\circ \tan 60^\circ \tan 70^\circ \tan 80^\circ$$

$$= \frac{\sin^2 40^\circ + \sin^2 (90^\circ - 40^\circ)}{\cos^2 20^\circ + \cos^2 (90^\circ - 20^\circ)} + \tan 10^\circ \tan 20^\circ \tan 60^\circ \times \tan(90^\circ - 20^\circ) \tan(90^\circ - 10^\circ)$$

$$= \frac{\sin^2 40^\circ + \cos^2 40^\circ}{\cos^2 20^\circ + \sin^2 20^\circ} + \tan 10^\circ \tan 20^\circ \tan 60^\circ \cot 20^\circ \cot 10^\circ$$

$$= \frac{1}{1} + \left(\tan 10^\circ \times \frac{1}{\tan 10^\circ}\right)\left(\tan 20^\circ \times \frac{1}{\tan 20^\circ}\right) \tan 60^\circ$$

$$= 1 + 1 \times 1 \times \sqrt{3} = \sqrt{3} + 1$$

********** END **********