Indefinite Integrals Ex 19.31 Q1

$$I = \int \frac{x^2+1}{x^4+x^2+1}dx$$

$$= \int \frac{1+\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}}dx$$

Dividing numerator and denominator by x^2

$$= \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x-\frac{1}{x}\right)^2+3}dx$$

Let $x-\frac{1}{x} = t \Rightarrow \qquad \left(1+\frac{1}{x^2}\right)dx = dt$

$$\therefore \qquad I = \int \frac{dt}{t^2+3}$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{t}{\sqrt{3}}\right)+c$$

$$\therefore \qquad I = \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right)+c$$

Indefinite Integrals Ex 19.31 Q2

$$\int \sqrt{\cot\theta}\,d\theta$$

Let $\cot\theta = x^2$

$\Rightarrow \quad -\operatorname{cosec}^2\theta\, d\theta = 2x\,dx$

$\Rightarrow \quad d\theta = \dfrac{-2x}{\operatorname{cosec}^2\theta}dx$

$= \dfrac{-2x}{1+\cot^2\theta}dx$

$= \dfrac{-2x}{1+x^4}dx$

$\therefore \quad I = -\int \dfrac{2x^2}{1+x^4}dx$

$= -\int \dfrac{2}{\dfrac{1}{x^2}+x^2}dx$

Dividing numerator and denominator by x^2

$= -\int \dfrac{1+\dfrac{1}{x^2}+1-\dfrac{1}{x^2}}{x^2+\dfrac{1}{x^2}}dx$

$= -\int \dfrac{\left(1+\dfrac{1}{x^2}\right)dx}{\left(x-\dfrac{1}{x}\right)^2+2} - \int \dfrac{\left(1-\dfrac{1}{x^2}\right)dx}{\left(x+\dfrac{1}{x}\right)^2-2}$

Let $x - \dfrac{1}{x} = t \Rightarrow \quad \left(1+\dfrac{1}{x^2}\right)dx = dt$

and $\quad x + \dfrac{1}{x} = z \Rightarrow \quad \left(1-\dfrac{1}{x^2}\right)dx = dz$

$\Rightarrow \quad I = -\int \dfrac{dt}{t^2+2} - \int \dfrac{dz}{z^2-2}$

$= -\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{t}{\sqrt{2}}\right) - \dfrac{1}{2\sqrt{2}}\log\left|\dfrac{z-\sqrt{2}}{z+\sqrt{2}}\right| + c$

$= -\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{x^2-1}{\sqrt{2}x}\right) - \dfrac{1}{2\sqrt{2}}\log\left|\dfrac{x^2+1-\sqrt{2}x}{x^2+1+\sqrt{2}x}\right| + c$

$I = -\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{\cot\theta-1}{\sqrt{2\cot\theta}}\right) - \dfrac{1}{2\sqrt{2}}\log\left|\dfrac{\cot\theta+1-\sqrt{2\cot\theta}}{\cot\theta+1-\sqrt{2\cot\theta}}\right| + c$

Indefinite Integrals Ex 19.31 Q3

Let $I = \int \dfrac{x^2+9}{x^4+81}dx$

Dividing numerator and denominator by x^2

$I = \int \dfrac{1+\dfrac{9}{x^2}}{x^2+\dfrac{81}{x^2}}dx$

$= \int \dfrac{1+\dfrac{9}{x^2}}{\left(x-\dfrac{9}{x}\right)^2+18}dx$

Let $\left(x-\dfrac{9}{x}\right) = t \Rightarrow \quad \left(1+\dfrac{9}{x^2}\right)dx = dt$

$\therefore \quad I = \int \dfrac{dt}{t^2+18}$

$\Rightarrow \quad I = \dfrac{1}{3\sqrt{2}}\tan^{-1}\left(\dfrac{t}{3\sqrt{2}}\right) + c$

Thus,

$I = \dfrac{1}{3\sqrt{2}}\tan^{-1}\left(\dfrac{x^2-9}{3\sqrt{2}x}\right) + c$

Indefinite Integrals Ex 19.31 Q4

Let $I = \int \frac{1}{x^4 + x^2 + 1} dx$

Dividing numerator and denominator by x^2

$$I = \int \frac{\frac{1}{x^2}}{x^2 + 1 + \frac{1}{x^2}} dx$$

$$= \frac{1}{2} \int \frac{1 + \frac{1}{x^2} - 1 + \frac{1}{x^2}}{x^2 + 1 + \frac{1}{x^2}} dx$$

$$= \frac{1}{2} \left\{ \int \frac{\left(1 + \frac{1}{x^2}\right) dx}{\left(x - \frac{1}{x}\right)^2 + 3} - \int \frac{\left(1 - \frac{1}{x^2}\right)}{\left(x + \frac{1}{x}\right)^2 - 1} dx \right\}$$

Let $\left(x - \frac{1}{x}\right) = t$

$\Rightarrow \quad \left(1 + \frac{1}{x^2}\right) dx = dt$

and $\quad \left(x + \frac{1}{x}\right) = z$

$\Rightarrow \quad \left(1 - \frac{1}{x^2}\right) dx = dz$

$\therefore \quad I = \frac{1}{2} \int \frac{dt}{t^2 + 3} - \frac{1}{2} \int \frac{dz}{z^2 - 1}$

$\Rightarrow \quad I = \frac{1}{2\sqrt{3}} \tan^{-1} \left(\frac{t}{\sqrt{3}}\right) - \frac{1}{4} \log \left|\frac{z - 1}{z + 1}\right| + c$

$\Rightarrow \quad I = \frac{1}{2\sqrt{3}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{3}x}\right) - \frac{1}{4} \log \left|\frac{x^2 + 1 - x}{x^2 + 1 + x}\right| + c$

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