Real Numbers Ex 1.1 Q7

Answer :

To Prove: that the square of any positive integer is of the form $5q$ or $5q+1$, $5q+4$ for some integer q.

Proof: Since positive integer n is of the form of $5q$ or $5q+1$, $5q+4$

If $n = 5q$

Then, $n^2 = (5q)^2$

$\Rightarrow \quad n^2 = 25q^2$

$\Rightarrow \quad n^2 = 5(5q)$

$\Rightarrow \quad n^2 = 5m$ (where $m = 5q$)

If $n = 5q+1$

Then, $n^2 = (5q+1)^2$

$\Rightarrow \quad n^2 = (5q)^2 + 10q + 1$

$\Rightarrow \quad n^2 = 25q^2 + 10q + 1$

$\Rightarrow \quad n^2 = 5q(5q+2)+1$

$\boxed{\Rightarrow n^2 = 5m+1}$ (where $m = q(5q+2)$)

If $n = 5q+2$

Then, $n^2 = (5q+4)^2$

$\Rightarrow \quad n^2 = (5q)^2 + 40q + 16$

$\Rightarrow \quad n^2 = 25q^2 + 40q + 16$

$\Rightarrow \quad n^2 = 5(5q^2+8q+3)+1$

$\boxed{\Rightarrow n^2 = 5m+1}$ (where $m = (5q^2+8q+3)$)

Hence it is proved that the square of a positive integer is of the form $5q$ or $5q+1$, $5q+4$ for some integer q.

Real Numbers Ex 1.1 Q8

Answer :

To Prove: that if a positive integer is of the form $6q+5$ then it is of the form $3q+2$ for some integer q, but not conversely.

Proof: Let $n = 6q+5$

Since any positive integer n is of the form of $3k$ or $3k+1$, $3k+2$

If $q = 3k$

Then, $n = 6q+5$

$\Rightarrow \quad n = 18k+5 \; (q = 3k)$

$\Rightarrow \quad n = 3(6k+1)+2$

$\Rightarrow \quad n = 3m+2$ (where $m = (6k+1)$)

If $q = 3k+1$

Then, $n = (6q+5)$

$\Rightarrow \quad n = (6(3k+1)+5) \; (q = 3k+1)$

$\Rightarrow \quad n = 18k+6+5$

$\Rightarrow \quad n = 18k+11$

$\Rightarrow \quad n = 3(6k+3)+2$

$\Rightarrow \quad n = 3m+2$ (where $m = (6k+3)$)

If $q = 3k+2$

Then, $n = (6q+5)$

$\Rightarrow \quad n = (6(3k+2)+5) \; (q = 3k+2)$

$\Rightarrow \quad n = 18k+12+5$

$\Rightarrow \quad n = 18k+17$

$\Rightarrow \quad n = 3(6k+5)+2$

$\Rightarrow \quad n = 3m+2$ (where $m = (6k+5)$)

Consider here 8 which is the form $3q+2$ *i.e.* $3 \times 2 + 2$ but it can't be written in the form $6q+5$.

Hence the converse is not true

*********** END **********