Chapter 6 Determinants Ex 6.4 Q24

Here $D = \begin{vmatrix} 3 & -1 & 2 \\ 2 & 1 & 3 \\ 1 & -2 & -1 \end{vmatrix}$

Expanding along R_1

$$= 3(5) + 1(-5) + 2(-5)$$
$$= 15 - 5 - 10 = 15 - 15 = 0$$

Also $D_1 = \begin{vmatrix} 3 & -1 & 2 \\ 5 & 1 & 3 \\ 1 & -2 & -1 \end{vmatrix}$

Expanding along R_1

$$= 3(5) + 1(-8) + 2(-11)$$
$$= 15 - 8 - 22$$
$$= -15 \neq 0$$

Since $D = 0$ and $D_1 \neq 0$

Hence the given system of equations is inconsistent.

Chapter 6 Determinants Ex 6.4 Q25

Here $D = \begin{vmatrix} 3 & -1 & 2 \\ 2 & -1 & 1 \\ 3 & 6 & 5 \end{vmatrix} = 3(-11) + 1(7) + 2(15) = -33 + 7 + 30 = 4$

$$D_1 = \begin{vmatrix} 6 & -1 & 2 \\ 2 & -1 & 1 \\ 20 & 6 & 5 \end{vmatrix} = 12$$

$$D_2 = \begin{vmatrix} 3 & 6 & 2 \\ 2 & 2 & 1 \\ 3 & 20 & 5 \end{vmatrix} = -4$$

$$D_3 = \begin{vmatrix} 3 & -1 & 6 \\ 2 & -1 & 2 \\ 3 & 6 & 20 \end{vmatrix} = 28$$

Now $x = \frac{D_1}{D} = \frac{12}{4} = -3$

$$y = \frac{D_2}{D} = \frac{-4}{4} = -1$$

$$z = \frac{D_3}{D} = \frac{28}{4} = 7$$

Chapter 6 Determinants Ex 6.4 Q26

We have,

$$D = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ -1 & -2 & 2 \end{vmatrix} = \begin{vmatrix} 0 & -1 & 0 \\ 3 & 1 & 0 \\ -3 & -2 & 0 \end{vmatrix} = 0$$

$$D_1 = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & -2 & 2 \end{vmatrix} = \begin{vmatrix} 3 & -1 & 0 \\ 2 & 1 & 0 \\ 1 & -2 & 0 \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} 1 & 3 & 1 \\ 2 & 2 & -1 \\ -1 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & -4 & -3 \\ -1 & 4 & 3 \end{vmatrix} = 1(-12 + 12) = 0$$

$$D_3 = \begin{vmatrix} 1 & -1 & 3 \\ 2 & 1 & 2 \\ -1 & -2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 3 & -4 \\ -1 & -3 & 4 \end{vmatrix} = 1(12 - 12) = 0$$

$\therefore \quad D = D_1 = D_2 = D_3 = 0$

So, either the system is consistent with infinitely many solutions or it is inconsistent.
Consider the first two equations, written as

$$x - y = 3 - z$$
$$2x + y = 2 + z$$

To solve these equations we use Cramer's rule.
Here,

$$D = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 1 + 2 = 3$$

$$D_1 = \begin{vmatrix} 3 - z & -1 \\ 2 + z & 1 \end{vmatrix} = (3 - z) + (2 + z) = 5$$

$$D_2 = \begin{vmatrix} 1 & 3 - z \\ 2 & 2 + z \end{vmatrix} = (2 + z) - (6 - 2z) = -4 + 3z$$

$\therefore \quad x = \frac{D_1}{D} = \frac{5}{3}$

$$y = \frac{D_2}{D} = \frac{-4 + 3z}{3}$$

Let $z = k$, then the equations have the solution.

$$x = \frac{5}{3}, \ y = \frac{-4 + 3k}{3}, \ z = k$$

Chapter 6 Determinants Ex 6.4 Q27

Here,

$$D = \begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix} = 6 - 6 = 0$$

$$D_1 = \begin{vmatrix} 5 & 2 \\ 15 & 6 \end{vmatrix} = 30 - 30 = 0$$

$$D_2 = \begin{vmatrix} 1 & 5 \\ 3 & 15 \end{vmatrix} = 15 - 15 = 0$$

So, $D = D_1 = D_2 = 0$

Let $y = k$, then we have,

$$x + 2y = 5$$

$$\Rightarrow \quad x = 5 - 2y = 5 - 2k$$

$\therefore \quad x = 5 - 2k,\ y = k$ are the infinite solutions of the given system.

Chapter 6 Determinants Ex 6.4 Q28

Here,

$$D = \begin{vmatrix} 1 & 1 & -1 \\ 1 & -2 & 1 \\ 3 & 6 & -5 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & -3 & 2 \\ 3 & 3 & -2 \end{vmatrix} = 1(6 - 6) = 0$$

$$D_1 = \begin{vmatrix} 0 & 1 & -1 \\ 0 & -2 & 1 \\ 0 & 6 & -5 \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} 1 & 0 & -1 \\ 1 & 0 & 1 \\ 3 & 0 & -5 \end{vmatrix} = 0$$

$$D_3 = \begin{vmatrix} 1 & 1 & 0 \\ 1 & -2 & 0 \\ 3 & 6 & 0 \end{vmatrix} = 0$$

So, $D = D_1 = D_2 = D_3 = 0$

The given system either has infinite solutions or it is inconsistent.

Consider the first two equations, written as

$$x + y = z$$

$$x - 2y = -z$$

To solve this we will use Cramer's rule

Here,

$$D = \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = -2 - 1 = -3$$

$$D_1 = \begin{vmatrix} z & 1 \\ -z & -2 \end{vmatrix} = -2z - z = -z$$

$$D_2 = \begin{vmatrix} 1 & z \\ 1 & -z \end{vmatrix} = -z - z = -2z$$

$$\therefore \quad x = \frac{D_1}{D} = \frac{-z}{-3} = \frac{z}{3}$$

$$y = \frac{D_2}{D} = \frac{-2z}{-3} = \frac{2z}{3}$$

Let $z = k$, then the solutions of the given system are

$$x = \frac{k}{3},\ y = \frac{2k}{3},\ z = k$$

*********** END **********