Definite Integrals Ex 20.2 Q5

Let $a^2 + x^2 = t^2$

Differentiating w.r.t. x, we get

$$2x\,dx = 2t\,dt$$
$$x\,dx = t\,dt$$

Now, $x = 0 \Rightarrow t = 0$

$x = a \Rightarrow t = \sqrt{2}a$

$$\therefore \int_0^a \frac{x\,dx}{\sqrt{a^2 + x^2}}dx = \int_a^{\sqrt{2}a} \frac{t\,dt}{t}$$
$$= \int_a^{\sqrt{2}a} dt$$
$$= \left[t\right]_a^{\sqrt{2}a}$$
$$= \left[\sqrt{2}a - a\right]$$
$$= a\left(\sqrt{2} - 1\right)$$

$$\therefore \int_0^a \frac{x}{\sqrt{a^2 + x^2}}dx = a\left(\sqrt{2} - 1\right)$$

Definite Integrals Ex 20.2 Q6

Let $e^x = t$

Differentiating w.r.t. x, we get

$$e^x\,dx = dt$$

Now, $x = 0 \Rightarrow t = 1$

$x = 1 \Rightarrow t = e$

$$\therefore \int_0^1 \frac{e^x}{1 + e^{2x}}dx = \int_1^e \frac{dt}{1 + t^2}$$
$$= \left[\tan^{-1} t\right]_1^e \qquad \left[\because \int \frac{dt}{1 + t^2} = \tan^{-1} t\right]$$
$$= \left[\tan^{-1} e - \tan^{-1} 1\right] \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$$
$$= \tan^{-1} e - \frac{\pi}{4}$$

$$\therefore \int_0^1 \frac{e^x}{1 + e^{2x}}dx = \tan^{-1} e - \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q7

Let $x^2 = t$

Differentiating w.r.t. x, we get

$$2x\,dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = 1 \Rightarrow t = 1$

$$\therefore \int_0^1 x e^{x^2} dx = \int_0^1 \frac{e^t dt}{2}$$

$$= \frac{1}{2}\int_0^1 e^t dt$$

$= \frac{1}{2}\left[e^t\right]_0^1$

$= \frac{1}{2}\left[e^1 - e^0\right] \qquad \left[\because e^0 = 1\right]$

$= \frac{1}{2}(e - 1)$

$\therefore \int_0^1 x e^{x^2} dx = \frac{1}{2}(e - 1)$

Definite Integrals Ex 20.2 Q8

Let $\log x = t$

Differentiating w.r.t. x, we get

$$\frac{1}{x}dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = 3 \Rightarrow t = \log 3$

$$\int_1^3 \frac{\cos(\log x)}{x}dx$$

$$= \int_0^{\log 3} \cos t \, dt \qquad \left[\because \int \cos t = \sin t\right]$$

$$= \left[\sin t\right]_0^{\log 3}$$

$$= \sin(\log 3) - \sin 0$$

$$= \sin(\log 3)$$

$$\int_1^3 \frac{\cos(\log x)}{x}dx = \sin(\log 3)$$

*********** END **********