Polynomials Ex 2.1 Q10

Answer :

Since α and β are the zeros of the quadratic polynomial $p(s)=3s^2-6s+4$

$$\alpha+\beta=\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\alpha+\beta=\frac{-(-6)}{3}$$

$$\alpha+\beta=\frac{\cancel{6}^{2}}{\cancel{3}}$$

$$\alpha+\beta=2$$

$$\alpha\beta=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\alpha\beta=\frac{4}{3}$$

We have, $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta$

$$=\frac{\alpha^2+\beta^2}{\alpha\beta}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta$$

$$=\frac{\left(\alpha+\beta^2\right)-2\alpha\beta}{\alpha\beta}+2\left(\frac{\alpha+\beta}{\alpha\beta}\right)+3\alpha\beta$$

By substituting $\alpha+\beta=2$ and $\alpha\beta=\frac{4}{3}$ we get ,

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta=\frac{(2)^2-2\left(\frac{4}{3}\right)}{\frac{4}{3}}+2\frac{(2)}{\frac{4}{3}}+3\left(\frac{4}{3}\right)$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta=\frac{4-\frac{8}{3}}{\frac{4}{3}}+\frac{4}{\frac{4}{3}}+\frac{12}{3}$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta=\frac{\frac{4\times3}{1\times3}-\frac{8}{3}}{\frac{4}{3}}+\frac{4}{\frac{4}{3}}+\frac{12}{3}$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta=\frac{\frac{12-8}{3}}{\frac{4}{3}}+\frac{4}{\frac{4}{3}}+\frac{12}{3}$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta=\frac{\frac{4}{3}}{\frac{4}{3}}+\frac{4}{\frac{4}{3}}+\frac{12}{3}$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta=\frac{\cancel{4}}{\cancel{3}}\times\frac{\cancel{3}}{\cancel{4}}+\frac{4\times3}{4}+\frac{12}{3}$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta=1+\frac{12}{4}+\frac{12}{3}$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta=\frac{1\times12}{1\times12}+\frac{12\times3}{4\times3}+\frac{12\times4}{3\times4}$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta=\frac{12+36+48}{12}$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta=\frac{48+48}{12}$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta=\frac{\cancel{96}^{8}}{\cancel{12}}$$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta=8$$

Hence, the value of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+3\alpha\beta$ is $\boxed{8}$

*********** END **********