



Exponents of Real Numbers Ex 2.1 Q2

Answer :

$$(1) \text{ Given } \left(16^{-\frac{1}{5}}\right)^{\frac{5}{2}}$$

$$\begin{aligned}\left(16^{-\frac{1}{5}}\right)^{\frac{5}{2}} &= 16^{-\frac{1}{5} \times \frac{5}{2}} \\ &= 16^{-\frac{1}{\cancel{5}} \times \frac{\cancel{5}}{2}} \\ &= 16^{-\frac{1}{2}}\end{aligned}$$

By using law of rational exponents $a^{-n} = \frac{1}{a^n}$ we have

$$\begin{aligned}\left(16^{-\frac{1}{5}}\right)^{\frac{5}{2}} &= \frac{1}{16^{\frac{1}{2}}} \\ &= \frac{1}{4^{2 \times \frac{1}{2}}} \\ &= \frac{1}{4^{\cancel{2} \times \frac{1}{\cancel{2}}}} \\ &= \frac{1}{4}\end{aligned}$$

Hence the value of $\left(16^{-\frac{1}{5}}\right)^{\frac{5}{2}}$ is $\boxed{\frac{1}{4}}$

(ii) Given $\sqrt[3]{343^{-2}}$

$$\begin{aligned}\sqrt[3]{343^{-2}} &= \sqrt[3]{\frac{1}{343^2}} \\&= \sqrt[3]{\frac{1}{(7^3)^2}} \\&= \frac{1^{\frac{1}{3}}}{7^{3 \times \frac{1}{3} \times 2}} \\&= \frac{1}{7^{\cancel{3} \times \frac{1}{\cancel{3}} \times 2}} \\\sqrt[3]{343^{-2}} &= \frac{1}{7^2} \\&= \frac{1}{49}\end{aligned}$$

Hence the value of $\sqrt[3]{343^{-2}}$ is $\boxed{\frac{1}{49}}$

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(iii) Given $(0.001)^{\frac{1}{3}}$

$$(0.001)^{\frac{1}{3}} = \left(\frac{0.001 \times 1000}{1 \times 1000} \right)^{\frac{1}{3}}$$

$$= \left(\frac{1}{1000} \right)^{\frac{1}{3}}$$

$$= \left(\frac{1 \times 1 \times 1}{10 \times 10 \times 10} \right)^{\frac{1}{3}}$$

$$= \left(\frac{1^3}{10^3} \right)^{\frac{1}{3}}$$

$$(0.001)^{\frac{1}{3}} = \left(\frac{1^{3 \times \frac{1}{3}}}{10^{3 \times \frac{1}{3}}} \right)$$

$$= \left(\frac{1^{\cancel{3} \times \frac{1}{\cancel{3}}}}{10^{\cancel{3} \times \frac{1}{\cancel{3}}}} \right)$$

$$= \frac{1}{10}$$

The value of $(0.001)^{\frac{1}{3}}$ is $\boxed{\frac{1}{10}}$

(iv) Given $\frac{25^{\frac{3}{2}} \times 243^{\frac{3}{5}}}{16^{\frac{5}{4}} \times 8^{\frac{4}{3}}}$

$$\begin{aligned} \frac{25^{\frac{3}{2}} \times 243^{\frac{3}{5}}}{16^{\frac{5}{4}} \times 8^{\frac{4}{3}}} &= \frac{5^{2 \times \frac{3}{2}} \times 3^{5 \times \frac{3}{5}}}{2^{4 \times \frac{5}{4}} \times 2^{3 \times \frac{4}{3}}} \\ &= \frac{5^{\cancel{2} \times \frac{3}{\cancel{2}}} \times 3^{\cancel{5} \times \frac{3}{\cancel{5}}}}{2^{\cancel{4} \times \frac{5}{\cancel{4}}} \times 2^{\cancel{3} \times \frac{4}{\cancel{3}}}} \\ &= \frac{5^3 \times 3^3}{2^5 \times 2^4} \\ &= \frac{125 \times 27}{32 \times 16} = \frac{3375}{512} \end{aligned}$$

Hence the value of $\frac{25^{\frac{3}{2}} \times 243^{\frac{3}{5}}}{16^{\frac{5}{4}} \times 8^{\frac{4}{3}}}$ is $\frac{3375}{512}$

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(v) Given $\left(\frac{\sqrt{2}}{5}\right)^8 \div \left(\frac{\sqrt{2}}{5}\right)^{13}$. So,

$$\begin{aligned}\left(\frac{\sqrt{2}}{5}\right)^8 \div \left(\frac{\sqrt{2}}{5}\right)^{13} &= \frac{2^{\frac{1}{2} \times 8}}{5^8} \div \frac{2^{\frac{1}{2} \times 13}}{5^{13}} \\ &= \frac{2^{\frac{1}{2} \times 8}}{5^8} \div \frac{2^{\frac{1}{2} \times 13}}{5^{13}} \\ &= \frac{2^4}{5^8} \div \frac{2^{\frac{13}{2}}}{5^{13}} \\ &= \frac{2^4}{5^8} \times \frac{5^{13}}{2^{\frac{13}{2}}}\end{aligned}$$

$$\begin{aligned}\left(\frac{\sqrt{2}}{5}\right)^8 \div \left(\frac{\sqrt{2}}{5}\right)^{13} &= \frac{2^4}{5^8} \times \frac{5^{13}}{2^{\frac{13}{2}}} \\ &= \frac{5^{13}}{5^8} \times \frac{2^4}{2^{\frac{13}{2}}}\end{aligned}$$

By using the law of rational exponents $\frac{a^m}{a^n} = a^{m-n}$

$$\Rightarrow \left(\frac{\sqrt{2}}{5}\right)^8 \div \left(\frac{\sqrt{2}}{5}\right)^{13} = 5^{13-8} \times 2^{4-\frac{13}{2}}$$

$$\left(\frac{\sqrt{2}}{5}\right)^8 \div \left(\frac{\sqrt{2}}{5}\right)^{13} = 5^5 \times 2^{\frac{4 \times 2}{1 \times 2} - \frac{13}{2}}$$

$$= 5^5 \times 2^{-\frac{5}{2}}$$

$$= \frac{5^5}{2^{\frac{5}{2}}}$$

$$= \frac{5^5}{\sqrt[2]{2 \times 2 \times 2 \times 2 \times 2}}$$

$$= \frac{5^5}{4\sqrt{2}}$$

Hence the value of $\left(\frac{\sqrt{2}}{5}\right)^8 \div \left(\frac{\sqrt{2}}{5}\right)^{13}$ is $\boxed{\frac{5^5}{4\sqrt{2}}}$

(vi) Given $\left(\frac{5^{-1} \times 7^2}{5^2 \times 7^{-4}}\right)^{\frac{7}{2}} \times \left(\frac{5^{-2} \times 7^3}{5^3 \times 7^{-5}}\right)^{-\frac{5}{2}}$. So,

$$\left(\frac{5^{-1} \times 7^2}{5^2 \times 7^{-4}}\right)^{\frac{7}{2}} \times \left(\frac{5^{-2} \times 7^3}{5^3 \times 7^{-5}}\right)^{-\frac{5}{2}} = \left(\frac{5^{-1 \times \frac{7}{2}} \times 7^{2 \times \frac{7}{2}}}{5^{2 \times \frac{7}{2}} \times 7^{-4 \times \frac{7}{2}}}\right) \times \left(\frac{5^{-2 \times -\frac{5}{2}} \times 7^{3 \times -\frac{5}{2}}}{5^{3 \times -\frac{5}{2}} \times 7^{-5 \times -\frac{5}{2}}}\right)$$

$$\Rightarrow \left(\frac{5^{-1} \times 7^2}{5^2 \times 7^{-4}}\right)^{\frac{7}{2}} \times \left(\frac{5^{-2} \times 7^3}{5^3 \times 7^{-5}}\right)^{-\frac{5}{2}} = \left(\frac{5^{-1 \times \frac{7}{2}} \times 7^{2 \times \frac{7}{2}}}{5^{2 \times \frac{7}{2}} \times 7^{-4 \times \frac{7}{2}}}\right) \times \left(\frac{5^{2 \times \frac{5}{2}} \times 7^{3 \times -\frac{5}{2}}}{5^{3 \times -\frac{5}{2}} \times 7^{-5 \times -\frac{5}{2}}}\right)$$

$$\Rightarrow \left(\frac{5^{-1} \times 7^2}{5^2 \times 7^{-4}}\right)^{\frac{7}{2}} \times \left(\frac{5^{-2} \times 7^3}{5^3 \times 7^{-5}}\right)^{-\frac{5}{2}} = \frac{5^{-\frac{7}{2}} \times 7^7}{5^7 \times 7^{-14}} \times \frac{5^{-5} \times 7^{-\frac{15}{2}}}{5^{-\frac{15}{2}} \times 7^{\frac{25}{2}}}$$

$$\begin{aligned} \left(\frac{5^{-1} \times 7^2}{5^2 \times 7^{-4}}\right)^{\frac{7}{2}} \times \left(\frac{5^{-2} \times 7^3}{5^3 \times 7^{-5}}\right)^{-\frac{5}{2}} &= \frac{5^{-\frac{7}{2}} \times 7^7}{5^7 \times 7^{-14}} \times \frac{5^{-5} \times 7^{-\frac{15}{2}}}{5^{-\frac{15}{2}} \times 7^{\frac{25}{2}}} \\ &= 5^{\frac{-7}{2} - 7} \times 7^{7+14} \times 5^{+5 + \frac{15}{2}} \times 7^{\frac{15}{2} - \frac{25}{2}} \\ &= 5^{\frac{7}{2} - \frac{14}{2} + \frac{10}{2} + \frac{15}{2}} \times 7^{21 - \frac{40}{2}} \\ &= 5^{\frac{-7-14+10+15}{2}} \times 7^{\frac{42-40}{2}} \end{aligned}$$

$$\begin{aligned}
 \left(\frac{5^{-1} \times 7^2}{5^2 \times 7^{-4}}\right)^{\frac{7}{2}} \times \left(\frac{5^{-2} \times 7^3}{5^3 \times 7^{-5}}\right)^{-\frac{5}{2}} &= 5^{\frac{4}{2}} \times 7^{\frac{2}{2}} \\
 &= 5^2 \times 7^1 \\
 &= 25 \times 7 \\
 &= 175
 \end{aligned}$$

Hence the value of $\left(\frac{5^{-1} \times 7^2}{5^2 \times 7^{-4}}\right)^{\frac{7}{2}} \times \left(\frac{5^{-2} \times 7^3}{5^3 \times 7^{-5}}\right)^{-\frac{5}{2}}$ is 175

***** END *****