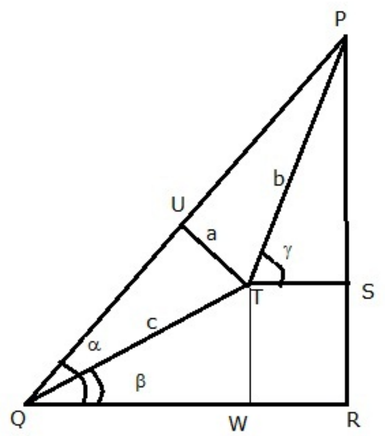
Sine and Cosine Formulae and their Applications Ex-10.1 Q30

Consider the following figure.



The person is observing the peak P from the point Q.

The distance he travelled is $QT = c$ *metres and the angle of inclination of* QT *is* β.

He is observing the peak from the point and the angle of inclination is γ.

Now consider the triangle ΔQUT.

$\angle TQU = \beta - \alpha$

Thus, $\sin(\alpha - \beta) = \frac{a}{c}$

$\Rightarrow a = c \times \sin(\alpha - \beta)....(1)$

Now consider the triangle ΔPQR.

We know that $\angle QPR = 90° - \alpha$

In triangle ΔPTS, $\angle TPS = 90° - \gamma$

Thus, $\angle TPU = \angle QPR - \angle TPS$

$\Rightarrow \angle TPU = (90° - \alpha) - (90° - \gamma)$

$\Rightarrow \angle TPU = \gamma - \alpha$

Now consider the ΔTPU,

Thus, $\sin(\gamma - \alpha) = \dfrac{a}{b}$

$\Rightarrow b = \dfrac{a}{\sin(\gamma - \alpha)}$

Substituting the value of a from equation (1), *we have*,

$b = \dfrac{c \times \sin(\alpha - \beta)}{\sin(\gamma - \alpha)} \ldots(2)$

We need to find the total height of the peak PR.

Here, $PR = PS + SR \ldots.(3)$

From the triangle PST,

$\sin\gamma = \dfrac{PS}{PT} = \dfrac{PS}{b}$

$\Rightarrow PS = b\sin\gamma \ldots.(4)$

From the triangle QTW,

$\sin\beta = \dfrac{TW}{QT} = \dfrac{TW}{c}$

$\Rightarrow TW = SR = c\sin\beta \ldots.(5)$

Substituting the values of PS *and* SR *from equations* (4) *and* (5)

in equation (3), *we have*

$PR = PS + SR$

$\Rightarrow PR = b\sin\gamma + c\sin\beta$

$\Rightarrow PR = \dfrac{c \times \sin(\alpha - \beta)}{\sin(\gamma - \alpha)}\sin\gamma + c\sin\beta \quad$ [*from equation* (2)]

$\Rightarrow PR = \dfrac{c \times \sin(\alpha - \beta) \times \sin\gamma + c\sin\beta \times \sin(\gamma - \alpha)}{\sin(\gamma - \alpha)}$

$\Rightarrow PR = c\left[\dfrac{\sin\alpha \times \cos\beta \times \sin\gamma - \cos\alpha \times \sin\beta \times \sin\gamma + \sin\beta \times \sin\gamma \times \cos\alpha - \sin\beta \times \sin\alpha \times \cos\gamma}{\sin(\gamma - \alpha)}\right]$

$\Rightarrow PR = c\left[\dfrac{\sin\alpha \times \cos\beta \times \sin\gamma - \sin\beta \times \sin\alpha \times \cos\gamma}{\sin(\gamma - \alpha)}\right]$

$\Rightarrow PR = \dfrac{c\sin\alpha \times (\cos\beta \times \sin\gamma - \sin\beta \times \cos\gamma)}{\sin(\gamma - \alpha)}$

$\Rightarrow PR = \dfrac{c\sin\alpha \times \sin(\gamma - \beta)}{\sin(\gamma - \alpha)}$

Sine and Cosine Formulae and their Applications Ex-10.1 Q31

If the sides a, b, c of a Δ ABC are in H.P.

$\therefore \frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are in AP

$\therefore \frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$

$\Rightarrow \frac{a-b}{ba} = \frac{b-c}{ca}$

$\Rightarrow \frac{\sin A - \sin B}{\sin B \sin A} = \frac{\sin B - \sin C}{\sin C \sin B}$........[Using sine rule]

$\Rightarrow \frac{2\sin\frac{A-B}{2}\cos\frac{A+B}{2}}{\sin A} = \frac{2\sin\frac{B-C}{2}\cos\frac{B+C}{2}}{\sin C}$

But A + B + C = π

A + B = π − C

$\cos\frac{A+B}{2} = \cos\left(\frac{\pi}{2} - \frac{C}{2}\right) = \sin\frac{C}{2}$

$\sin^2\frac{C}{2}\cos\frac{C}{2}\sin\frac{A-B}{2} = \sin\frac{B-C}{2}\cos\frac{A}{2}\sin^2\frac{A}{2}$

$\sin^2\frac{C}{2}\sin\frac{A+B}{2}\sin\frac{A-B}{2} = \sin\frac{B-C}{2}\cos\frac{B+C}{2}\sin^2\frac{A}{2}$

$\sin^2\frac{C}{2}\left[\sin^2\frac{A}{2} - \sin^2\frac{B}{2}\right] = \sin^2\frac{A}{2}\left[\sin^2\frac{B}{2} - \sin^2\frac{C}{2}\right]$

$\sin^2\frac{C}{2}\sin^2\frac{A}{2} - \sin^2\frac{C}{2}\sin^2\frac{B}{2} = \sin^2\frac{A}{2}\sin^2\frac{B}{2} - \sin^2\frac{A}{2}\sin^2\frac{C}{2}$

$\frac{1}{\sin^2\frac{B}{2}} - \frac{1}{\sin^2\frac{A}{2}} = \frac{1}{\sin^2\frac{C}{2}} - \frac{1}{\sin^2\frac{B}{2}}$

Hence $\frac{1}{\sin^2\frac{A}{2}}, \frac{1}{\sin^2\frac{B}{2}}, \frac{1}{\sin^2\frac{C}{2}}$ are in AP.

$\therefore \sin^2\frac{A}{2}, \sin^2\frac{B}{2}, \sin^2\frac{C}{2}$ are in HP.

*********** END **********