Indefinite Integrals Ex 19.30 Q6

$$\text{Let } I = \int \frac{x^2}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$$\Rightarrow \quad x^2 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

Put $x = 1$

$$\Rightarrow \quad 1 = 2A \Rightarrow \quad A = \frac{1}{2}$$

Put $x = 2$

$$\Rightarrow \quad 4 = -B \quad \Rightarrow \quad B = -4$$

Put $x = 3$

$$\Rightarrow \quad 9 = 2C \Rightarrow \quad C = \frac{9}{2}$$

Thus,

$$I = \int \frac{x^2}{(x-1)(x-2)(x-3)} dx = \frac{1}{2}\int \frac{dx}{x-1} - 4\int \frac{dx}{x-2} + \frac{9}{2}\int \frac{dx}{x-3}$$

$$= \frac{1}{2}\log|x-1| - 4\log|x-2| + \frac{9}{2}\log|x-3| + c$$

Hence,

$$I = \frac{1}{2}\log|x-1| - 4\log|x-2| + \frac{9}{2}\log|x-3| + c$$

Indefinite Integrals Ex 19.30 Q7

$$\frac{5x}{(x+1)(x^2-4)} = \frac{5x}{(x+1)(x+2)(x-2)}$$

Let $\frac{5x}{(x+1)(x+2)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)} + \frac{C}{(x-2)}$

$$5x = A(x+2)(x-2) + B(x+1)(x-2) + C(x+1)(x+2) \qquad ...(1)$$

Substituting $x = -1, -2$, and 2 respectively in equation (1), we obtain

$$A = \frac{5}{3},\ B = -\frac{5}{2}, \text{ and } C = \frac{5}{6}$$

$$\therefore \frac{5x}{(x+1)(x+2)(x-2)} = \frac{5}{3(x+1)} - \frac{5}{2(x+2)} + \frac{5}{6(x-2)}$$

$$\Rightarrow \int \frac{5x}{(x+1)(x^2-4)}dx = \frac{5}{3}\int \frac{1}{(x+1)}dx - \frac{5}{2}\int \frac{1}{(x+2)}dx + \frac{5}{6}\int \frac{1}{(x-2)}dx$$

$$= \frac{5}{3}\log|x+1| - \frac{5}{2}\log|x+2| + \frac{5}{6}\log|x-2| + C$$

Indefinite Integrals Ex 19.30 Q8

Let $I = \int \frac{x^2+1}{x(x^2-1)}dx = \int \frac{x^2+1}{x(x+1)(x-1)}dx$

Let $\frac{x^2+1}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}$

$\Rightarrow \quad x^2+1 = A(x+1)(x-1) + B.x(x-1) + Cx(x+1)$

Put $x = 0$

$\Rightarrow \quad 1 = -A \Rightarrow \quad A = -1$

Put $x = -1$

$\Rightarrow \quad 2 = 2B \Rightarrow \quad B = 1$

Put $x = 1$

$\Rightarrow \quad 2 = 2C \Rightarrow \quad C = 1$

Thus,

$$I = -\int \frac{dx}{x} + \int \frac{dx}{x+1} + \int \frac{dx}{x-1}$$

$$= -\log|x| + \log|x+1| + \log|x-1| + c$$

$$I = \log\left|\frac{x^2-1}{x}\right| + c$$

Indefinite Integrals Ex 19.30 Q9

Let $I = \int \frac{2x-3}{\left(x^2-1\right)\left(2x+3\right)}dx = \int \frac{2x-3}{\left(x+1\right)\left(x-1\right)\left(2x+3\right)}dx$

Let $\frac{2x-3}{\left(x+1\right)\left(x-1\right)\left(2x+3\right)} = \frac{A}{\left(x+1\right)} + \frac{B}{\left(x-1\right)} + \frac{C}{\left(2x+3\right)}$

$\Rightarrow \quad 2x-3 = A\left(x-1\right)\left(2x+3\right) + B\left(x+1\right)\left(2x+3\right) + C\left(x^2-1\right)$

Put $x = -1$

$\Rightarrow \quad -5 = -2A \quad \Rightarrow \quad A = \frac{5}{2}$

Put $x = 1$

$\Rightarrow \quad -1 = 10B \quad \Rightarrow \quad B = -\frac{1}{10}$

Put $x = -\frac{3}{2}$

$\Rightarrow \quad -6 = \frac{5}{4}C \quad \Rightarrow \quad C = -\frac{24}{5}$

Thus,

$I = \frac{5}{2}\int \frac{dx}{x+1} - \frac{1}{10}\int \frac{dx}{x-1} - \frac{24}{5}\int \frac{dx}{2x+3}$

$= \frac{5}{2}\log\left|x+1\right| - \frac{1}{10}\log\left|x-1\right| - \frac{24}{5}.\frac{1}{2}\log\left|2x+3\right| + c$

Hence,

$I = \frac{5}{2}\log\left|x+1\right| - \frac{1}{10}\log\left|x-1\right| - \frac{12}{5}\log\left|2x+3\right| + c$

Indefinite Integrals Ex 19.30 Q10

Let $I = \int \frac{x^3}{(x-1)(x-2)(x-3)} dx$

$$= \int 1 + \frac{6x^2 - 9x + 6}{(x-1)(x-2)(x-3)} dx$$

Let $\frac{6x^2 - 11x + 6}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$

$\Rightarrow \quad 6x^2 - 11x + 6 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

Put $x = 1$

$\Rightarrow \quad 1 = 2A \Rightarrow \quad A = \frac{1}{2}$

Put $x = 2$

$\Rightarrow \quad 8 = -B \quad \Rightarrow \quad B = -8$

Put $x = 3$

$\Rightarrow \quad 27 = 2C \quad \Rightarrow \quad C = \frac{27}{2}$

Thus,

$I = \int dx + \frac{1}{2}\int \frac{dx}{x-1} \; -8\int \frac{dx}{x-2} + \frac{27}{2} \cdot \int \frac{dx}{x-3}$

$$= x + \frac{1}{2}\log|x-1| \; -8 \log|x-2| + \frac{27}{2} \cdot \log|x-3| + c$$

Hence,

$I = x + \frac{1}{2}\log|x-1| \; -8 \log|x-2| + \frac{27}{2} \cdot \log|x-3| + c$

*********** END **********