Higher Order Derivatives Ex 12.1 Q15

$x = a(1 - \cos\theta)\,; \quad y = a(\theta + \sin\theta)$

Differentiating both w.r.t.θ

$$\Rightarrow \quad \frac{dx}{d\theta} = a(0 + \sin\theta)\,; \quad \frac{dy}{d\theta} = a(1 + \cos\theta)$$

Dividing (2) by (1)

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{a(1+\cos\theta)}{a\sin\theta}$$

Differentiating w.r.t.θ

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \frac{\sin\theta(0 - \sin\theta) - (1 + \cos\theta)\cos\theta}{\sin^2\theta} = -\frac{\sin^2\theta - \cos\theta - \cos^2\theta}{\sin^2\theta}$$

$$= -\frac{(1+\cos\theta)}{\sin^2\theta} \qquad \ldots\ldots(3)$$

dividing (3) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\frac{(1+\cos\theta)}{\sin^2\theta \times a\sin\theta}$$

Putting $\theta = \pi/2$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\frac{1}{a}$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q17

$x = \cos\theta;\ y = \sin^3\theta$

Differentiating both w.r.t.θ

$$\Rightarrow \quad \frac{dx}{d\theta} = -\sin\theta; \qquad (1)$$

$$\frac{dy}{d\theta} = 3\sin^2\theta\cos\theta \qquad (2)$$

Dividing (2) by (1)

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = -\frac{3\sin^2\theta\cos\theta}{\sin\theta} = -3\sin\theta\cos\theta$$

Differentiating w.r.t.θ

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = -3\left\{\sin\theta\left(-\sin\theta\right) + \cos\theta\left(\cos\theta\right)\right\} = -3\left(\cos^2\theta - \sin^2\theta\right) \ldots\ldots\ldots (3)$$

Dividing (3) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{+3\left(\cos^2\theta - \sin^2\theta\right)}{\sin\theta} \times \frac{\sin^2\theta}{\sin^2\theta}$$

$$\Rightarrow \quad \sin^3\theta\frac{d^2y}{dx^2} = 3\sin^2\theta\left(\cos^2\theta - \sin^2\theta\right)$$

$$\Rightarrow \quad y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 3\sin^2\theta\left(\cos^2\theta - \sin^2\theta\right) + \left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow \quad y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 3\sin^2\theta\cos^2\theta - 3\sin^4\theta + 9\sin^2\theta\cos^2\theta$$

adding and subtracting $3\sin^2\theta\cos^2\theta$ on RHS

$$\Rightarrow \quad y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 12\sin^2\theta\cos^2\theta - 3\sin^4\theta + 3\sin^2\theta\cos^2\theta - 3\sin^2\theta\cos^2\theta$$

$$\Rightarrow \quad y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 15\sin^2\theta\cos^2\theta - 3\sin^2\theta\left(\sin^2\theta + \cos^2\theta\right)$$

$$= 15\sin^2\theta\cos^2\theta - 3\sin^2\theta$$

$$\Rightarrow \quad y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 3\sin^2\theta\left\{5\cos^2\theta - 1\right\}$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q18

$y = sin\left(sin\,x\right)$

differentiating w.r.t. x

$$\Rightarrow \qquad \frac{dy}{dx} = \frac{d\left(sin\left(sin\,x\right)\right)}{d\left(sin\,x\right)} \times \frac{d\left(sin\,x\right)}{dx} = cos\left(sin\,x\right) \times cos\,x$$

differentiating w.r.t. x

$$\Rightarrow \qquad \frac{d^2y}{dx^2} = \left(cos\left(sin\,x\right)\right)\left(-sin\,x\right) + \left(cos\,x\right)\left(-sin\left(sin\,x\right)\right)\left(cos\,x\right)$$

$$\Rightarrow \qquad \frac{d^2y}{dx^2} = -sin\,x\,cos\left(sin\,x\right) \times \frac{cos\,x}{cos\,x} - y\,cos^2 x$$

$$\Rightarrow \qquad \frac{d^2y}{dx^2} = -tan\,x\,\frac{dy}{dx} - y\,cos^2 x$$

$$\Rightarrow \qquad \frac{d^2y}{dx^2} + tan\,x\,\frac{dy}{dx} + y\,cos^2 x = 0$$

Hence proved!

*********** END **********