Definite Integrals Ex 20.2 Q1

$$\text{Let } I=\int_2^4 \frac{x}{x^2+1}dx$$

$$\int \frac{x}{x^2+1}dx=\frac{1}{2}\int \frac{2x}{x^2+1}dx=\frac{1}{2}\log\left(1+x^2\right)=F(x)$$

By the second funamental theorem of calculus, we obtain

$$I=F(4)-F(2)$$

$$=\frac{1}{2}\left[\log\left(1+4^2\right)-\log\left(1+2^2\right)\right]$$

$$=\frac{1}{2}[\log 17-\log 5]$$

$$=\frac{1}{2}\log\left(\frac{17}{5}\right)$$

Definite Integrals Ex 20.2 Q2

Let $1+\log x = t$

Differentiating w.r.t. x, we get

$$\frac{1}{x}dx = dt$$

Now, $x = 1 \Rightarrow t = 1$

$x = 2 \Rightarrow t = 1+\log 2$

$$\therefore \int_1^2 \frac{1}{x\left(1+\log x\right)^2}dx = \int_1^{1+\log 2}\frac{dt}{t^2}$$

$$=\left[\frac{-1}{t}\right]_1^{1+\log 2}$$

$$=\left[\frac{-1}{1+\log 2}+1\right]$$

$$=\left[\frac{-1+1+\log 2}{1+\log 2}\right]$$

$$=\left[\frac{\log 2}{1+\log 2}\right] \qquad [\because \log e = 1]$$

$$=\frac{\log 2}{\log e+\log 2} \qquad [\log a+\log b = \log ab]$$

$$=\frac{\log 2}{\log 2e}$$

$$\therefore \int_1^2 \frac{1}{x\left(1+\log x\right)^2}dx = \frac{\log 2}{\log 2e}$$

Definite Integrals Ex 20.2 Q3

Let $9x^2 - 1 = t$

Differentiating w.r.t. x, we get

$$18x\,dx = dt$$

$$3x\,dx = \frac{dt}{6}$$

Now, $x = 1 \Rightarrow t = 8$

$x = 2 \Rightarrow t = 35$

$$\therefore \int_1^2 \frac{3x}{9x^2 - 1}dx = \int_8^{35} \frac{dt}{6t}$$

$$= \frac{1}{6}\left[\log t\right]_8^{35}$$

$$= \frac{1}{6}\left(\log 35 - \log 8\right)$$

$$\therefore \int_1^2 \frac{3x}{9x^2 - 1}dx = \frac{1}{6}\left(\log 35 - \log 8\right)$$

Definite Integrals Ex 20.2 Q4

Put $\sin x = \dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$\cos x = \dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$\int_0^{\frac{\pi}{2}} \frac{dx}{5\cos x + 3\sin x} = \int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}dx}{5\left(1-\tan^2\frac{x}{2}\right)+6\tan\frac{x}{2}}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}dx}{5-5\tan^2\frac{x}{2}+6\tan\frac{x}{2}}$$

Let $\tan\frac{x}{2} = t$

Differentiating w.r.t. x, we get

$$\frac{1}{2}\sec^2\frac{x}{2}dx = dt$$

Now, $x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\int_0^{\frac{\pi}{2}} \frac{\sec^2\frac{x}{2}dx}{5-5\tan^2\frac{x}{2}+6\tan\frac{x}{2}} = \int_0^1 \frac{2dt}{5-5t^2+6t} = \frac{2}{5}\int \frac{dt}{1-t^2+\frac{6}{5}t}$$

Forming perfect square by adding and subtracting $\frac{9}{25}$

$$\frac{2}{5}\int_0^1 \frac{dt}{1-t^2+\frac{6}{5}t}$$

$$= \frac{2}{5}\int_0^1 \frac{dt}{\frac{34}{25}-\left(t-\frac{3}{5}\right)^2}$$

$$= \frac{2}{5}\cdot\frac{1}{2}\sqrt{\frac{25}{34}}\log\left(\frac{\sqrt{\frac{34}{25}}+t-\frac{3}{5}}{\sqrt{\frac{34}{25}}-t+\frac{3}{5}}\right)_0^1 \qquad \left[\because \int\frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left(\frac{x+a}{x-a}\right)\right]$$

$$= \frac{1}{\sqrt{34}}\left\{\log\left(\frac{\sqrt{34}+2}{\sqrt{34}-2}\right) - \log\left(\frac{\sqrt{34}-3}{\sqrt{34}+3}\right)\right\}$$

$$= \frac{1}{\sqrt{34}}\log\left(\frac{\left(\sqrt{34}+2\right)\left(\sqrt{34}-3\right)}{\left(\sqrt{34}-2\right)\left(\sqrt{34}-3\right)}\right)$$

$$= \frac{1}{\sqrt{34}}\log\left(\frac{40+5\sqrt{34}}{40-5\sqrt{34}}\right)$$

$$= \frac{1}{\sqrt{34}}\log\left(\frac{8+\sqrt{34}}{8-\sqrt{34}}\right)$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{dx}{5\cos x + 3\sin x} = \frac{1}{\sqrt{34}}\log\left(\frac{8+\sqrt{34}}{8-\sqrt{34}}\right)$$

*********** END **********