



## Chapter 9 Continuity Ex 9.2 Q10

Let  $f(x) = \sin|x|$

This function  $f$  is defined for every real number and  $f$  can be written as the composition of two functions as,

$f = g \circ h$ , where  $g(x) = |x|$  and  $h(x) = \sin x$

$$[\because (g \circ h)(x) = g(h(x)) = g(\sin x) = |\sin x| = f(x)]$$

It has to be proved first that  $g(x) = |x|$  and  $h(x) = \sin x$  are continuous functions.

$g(x) = |x|$  can be written as

$$g(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

Clearly,  $g$  is defined for all real numbers.

Let  $c$  be a real number.

Case I:

If  $c < 0$ , then  $g(c) = -c$  and  $\lim_{x \rightarrow c} g(x) = \lim_{x \rightarrow c} (-x) = -c$

$$\therefore \lim_{x \rightarrow c} g(x) = g(c)$$

Therefore,  $g$  is continuous at all points  $x$ , such that  $x < 0$

Case II:

If  $c > 0$ , then  $g(c) = c$  and  $\lim_{x \rightarrow c} g(x) = \lim_{x \rightarrow c} x = c$

$$\therefore \lim_{x \rightarrow c} g(x) = g(c)$$

Therefore,  $g$  is continuous at all points  $x$ , such that  $x > 0$

Case III:

If  $c = 0$ , then  $g(c) = g(0) = 0$

$$\lim_{x \rightarrow 0^-} g(x) = \lim_{x \rightarrow 0^-} (-x) = 0$$

$$\lim_{x \rightarrow 0^+} g(x) = \lim_{x \rightarrow 0^+} (x) = 0$$

$$\therefore \lim_{x \rightarrow 0} g(x) = \lim_{x \rightarrow 0} (x) = g(0)$$

Therefore,  $g$  is continuous at  $x = 0$

From the above three observations, it can be concluded that  $g$  is continuous at all points.

$$h(x) = \sin x$$

It is evident that  $h(x) = \sin x$  is defined for every real number.

Let  $c$  be a real number. Put  $x = c + k$

If  $x \rightarrow c$ , then  $k \rightarrow 0$

$$h(c) = \sin c$$

$$h(c) = \sin c$$

$$\lim_{x \rightarrow c} h(x) = \lim_{x \rightarrow c} \sin x$$

$$= \lim_{k \rightarrow 0} \sin(c + k)$$

$$= \lim_{k \rightarrow 0} [\sin c \cos k + \cos c \sin k]$$

$$= \lim_{k \rightarrow 0} (\sin c \cos k) + \lim_{k \rightarrow 0} (\cos c \sin k)$$

$$= \sin c \cos 0 + \cos c \sin 0$$

$$= \sin c + 0$$

$$= \sin c$$

$$\therefore \lim_{x \rightarrow c} h(x) = g(c)$$

Therefore,  $h$  is a continuous function.

It is known that for real valued functions  $g$  and  $h$ , such that  $(g \circ h)$  is defined at  $c$ , if  $g$  is continuous at  $c$  and if  $f$  is continuous at  $g(c)$ , then  $(f \circ g)$  is continuous at  $c$ .

Therefore,  $f(x) = (g \circ h)(x) = g(h(x)) = g(\sin x) = |\sin x|$  is a continuous function.

## Chapter 9 Continuity Ex 9.2 Q11

When  $x < 0$ , we have,

$$f(x) = \frac{\sin x}{x}$$

We know that the  $\sin x$  and the identity function  $x$  are continuous for  $x < 0$ .

So, the quotient function  $f(x) = \frac{\sin x}{x}$  is continuous for  $x < 0$ .

When  $x > 0$ , we have,

$f(x) = x + 1$ , which is a polynomial of degree 1. So,  $f(x)$  is continuous for  $x > 0$

Now, consider the point  $x = 0$ .

$$f(0) = 0 + 1 = 1.$$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} \frac{\sin(-h)}{-h} = \lim_{h \rightarrow 0} \frac{-\sin h}{-h} = 1$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0 + h) = \lim_{h \rightarrow 0} h + 1 = 1$$

$$\text{Thus, LHL} = \text{RHL} = f(0) = 1$$

So,  $f(x)$  is continuous at  $x = 0$ .

Hence,  $f(x)$  is continuous everywhere

## Chapter 9 Continuity Ex 9.2 Q12

The given function is  $g(x) = x - [x]$

It is evident that  $g$  is defined at all integral points.

Let  $n$  be an integer.

Then,

$$g(n) = n - [n] = n - n = 0$$

The left hand limit of  $f$  at  $x = n$  is,

$$\lim_{x \rightarrow n^-} g(x) = \lim_{x \rightarrow n^-} (x - [x]) = \lim_{x \rightarrow n^-} (x) - \lim_{x \rightarrow n^-} [x] = n - (n-1) = 1$$

The right hand limit of  $f$  at  $x = n$  is,

$$\lim_{x \rightarrow n^+} g(x) = \lim_{x \rightarrow n^+} (x - [x]) = \lim_{x \rightarrow n^+} (x) - \lim_{x \rightarrow n^+} [x] = n - n = 0$$

It is observed that the left and right hand limits of  $f$  at  $x = n$  do not coincide.

Therefore,  $f$  is not continuous at  $x = n$

Hence,  $g$  is discontinuous at all integral points

#### Chapter 9 Continuity Ex 9.2 Q13

It is known that if  $g$  and  $h$  are two continuous functions, then

$g + h$ ,  $g - h$ , and  $g.h$  are also continuous.

It has to be proved first that  $g(x) = \sin x$  and  $h(x) = \cos x$  are continuous functions.

Let  $g(x) = \sin x$

It is evident that  $g(x) = \sin x$  is defined for every real number.

Let  $c$  be a real number. Put  $x = c + h$

If  $x \rightarrow c$ , then  $h \rightarrow 0$

$$g(c) = \sin c$$

$$\begin{aligned}\lim_{x \rightarrow c} g(x) &= \lim_{x \rightarrow c} \sin x \\ &= \lim_{h \rightarrow 0} \sin(c + h) \\ &= \lim_{h \rightarrow 0} [\sin c \cos h + \cos c \sin h] \\ &= \lim_{h \rightarrow 0} (\sin c \cos h) + \lim_{h \rightarrow 0} (\cos c \sin h) \\ &= \sin c \cos 0 + \cos c \sin 0 \\ &= \sin c + 0 \\ &= \sin c\end{aligned}$$

$$\therefore \lim_{x \rightarrow c} g(x) = g(c)$$

Therefore,  $g$  is a continuous function.

Let  $h(x) = \cos x$

It is evident that  $h(x) = \cos x$  is defined for every real number.

Let  $c$  be a real number. Put  $x = c + h$

If  $x \rightarrow c$ , then  $h \rightarrow 0$

$h(c) = \cos c$

$$\begin{aligned}\lim_{x \rightarrow c} h(x) &= \lim_{x \rightarrow c} \cos x \\ &= \lim_{h \rightarrow 0} \cos(c + h) \\ &= \lim_{h \rightarrow 0} [\cos c \cos h - \sin c \sin h] \\ &= \lim_{h \rightarrow 0} \cos c \cos h - \lim_{h \rightarrow 0} \sin c \sin h \\ &= \cos c \cos 0 - \sin c \sin 0 \\ &= \cos c \times 1 - \sin c \times 0 \\ &= \cos c\end{aligned}$$

$$\therefore \lim_{x \rightarrow c} h(x) = h(c)$$

Therefore,  $h$  is a continuous function.

Therefore, it can be concluded that

- (a)  $f(x) = g(x) + h(x) = \sin x + \cos x$  is a continuous function
- (b)  $f(x) = g(x) - h(x) = \sin x - \cos x$  is a continuous function
- (c)  $f(x) = g(x) \times h(x) = \sin x \times \cos x$  is a continuous function

\*\*\*\*\* END \*\*\*\*\*