Binomial Theorem Ex 18.2 Q16(vi)

$$\left(x-\frac{1}{x^2}\right)^{3n}$$

$$T_{r+1}=(-1)^r\ {}^{3n}C_r x^{3n-r}\left(\frac{1}{x^2}\right)^r$$

$$=(-1)^r\ {}^{3n}C_r x^{3n-r-2r}$$

Independent of $x \Rightarrow x^0$

$$x^{3n-3r}=x^0 \Rightarrow r=n$$

$$=(-1)^n\ {}^{3n}C_r$$

Binomial Theorem Ex 18.2 Q16(vii)

We have,

$$\left(\frac{1}{2}x^{\frac{1}{3}} + x^{\frac{-1}{5}}\right)^8$$

Let $(r+1)^{th}$ term be independent of x.

$$\therefore \quad T_{r+1} = {}^8C_r \left(\frac{1}{2}x^{\frac{1}{3}}\right)^{8-r} \left(x^{\frac{-1}{5}}\right)^r$$

$$= {}^8C_r \left(\frac{1}{2}\right)^{8-r} \times \left(x^{\frac{1}{3}}\right)^{8-r} \times \left(\frac{1}{x^{\frac{1}{5}}}\right)^r$$

$$= {}^8C_r \left(\frac{1}{2}\right)^{8-r} \times (x)^{\frac{8-r}{3}} \times \left(\frac{1}{x^{\frac{1}{5}}}\right)^r$$

$$= {}^8C_r \left(\frac{1}{2}\right)^{8-r} \times (x)^{\frac{8-r}{3}-\frac{r}{5}}$$

$$= {}^8C_r \left(\frac{1}{2}\right)^{8-r} \times (x)^{\frac{40-5r-3r}{15}}$$

$$= {}^8C_r \left(\frac{1}{2}\right)^{8-r} \times (x)^{\frac{40-8r}{15}}$$

If it is independent of x, we must have

$$\frac{40-8r}{15} = 0$$

$\Rightarrow \quad 8r = 40$

$\Rightarrow \quad r = 5$

$\therefore \quad$ The term independet of $x = T_6$

Now,

$$T_6 = {}^8C_r \left(\frac{1}{2}x^{\frac{1}{3}}\right)^{8-5} \left(x^{\frac{-1}{5}}\right)^5$$

$$= 56 \times \left(\frac{1}{2}\right)^3$$

$$= 56 \times \frac{1}{8}$$

$$= 7$$

Hence, required term = 7

Binomial Theorem Ex 18.2 Q16(viii)

$$\left(1+x+2x^3\right)\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$$

$$=\left(1+x+2x^3\right)\left[\left(\frac{3}{2}x^2\right)^9-{}^9C_1\left(\frac{3}{2}x^2\right)^8\frac{1}{3x}\ldots\ldots+{}^9C_6\left(\frac{3}{2}x^2\right)^3\left(\frac{1}{3x}\right)^6-{}^9C_7\left(\frac{3}{2}x^2\right)^2\left(\frac{1}{3x}\right)^7\right]$$

In the second bracket, we have to search the term so x° and $\frac{1}{x^3}$ which when multiplying by 1 and $2x^3$ is first bracket will give the term in dependent of x. The term containing $\frac{1}{x}$ will not occur is second bracket.

The term independent of x

$$=1\left[{}^9C_6\frac{3^3}{2^3}\times\frac{1}{3^6}\right]-2x^3\left[{}^9C_7\frac{3^3}{2^3}\times\frac{1}{3^7}\times\frac{1}{x^3}\right]$$

$$=\left[\frac{9\times8\times7}{1\times2\times3}\times\frac{1}{8\times27}\right]-2\left[\frac{9\times8}{1\times2}-\frac{1}{4\times243}\right]$$

$$=\frac{7}{18}-\frac{2}{27}$$

$$=\frac{17}{54}$$

Required term $=\frac{17}{54}$

*********** END **********