Trigonometric Ratios of Compound Angles Ex 7.1 Q14

We have,

$$\tan A = \frac{5}{6} \text{ and } \tan B = \frac{1}{11}$$

Now,

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\dfrac{5}{6} + \dfrac{1}{11}}{1 - \dfrac{5}{6} \times \dfrac{1}{11}}$$

$$= \frac{\dfrac{55+6}{66}}{1 - \dfrac{5}{66}}$$

$$= \frac{\dfrac{61}{66}}{\dfrac{66-5}{66}}$$

$$= \frac{\dfrac{61}{66}}{\dfrac{61}{66}}$$

$$= \frac{61}{66} \times \frac{66}{61}$$

$$= 1$$

$$= \tan\frac{\pi}{4} \qquad\qquad \left[\because \tan\frac{\pi}{4} = 1\right]$$

$\Rightarrow \qquad \tan(A+B) = \tan\frac{\pi}{4}$

$\Rightarrow \qquad A + B = \frac{\pi}{4}$

Hence proved.

We have,

$$\tan A = \frac{m}{m-1} \quad \text{and} \quad \tan B = \frac{1}{2m-1}$$

Now,
$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{\dfrac{m}{m-1} - \dfrac{1}{2m-1}}{1 + \dfrac{m}{m-1} \times \dfrac{1}{2m-1}}$$

$$= \frac{\dfrac{m(2m-1) - (m-1)}{(m-1)(2m-1)}}{1 + \dfrac{m}{(m-1)(2m-1)}}$$

$$= \frac{\dfrac{m(2m-1) - (m-1)}{(m-1)(2m-1)}}{\dfrac{(m-1)(2m-1) + (m)}{(m-1)(2m-1)}}$$

$$= \frac{m(2m-1) - (m-1)}{(m-1)(2m-1) + (m)}$$

$$= \frac{2m^2 - m - m + 1}{2m^2 - m - 2m + 1 + m}$$

$$= \frac{2m^2 - m - m + 1}{2m^2 - 2m + 1}$$

$$= \frac{2m^2 - 2m + 1}{2m^2 - 2m + 1}$$

$$= 1$$

$\therefore \quad \tan(A-B) = 1 = \tan\left(\dfrac{\pi}{4}\right)$ $\qquad\qquad \left[\because \tan\dfrac{\pi}{4} = 1\right]$

$\Rightarrow \quad \tan(A-B) = \tan\left(\dfrac{\pi}{4}\right)$

$\Rightarrow \quad A - B = \left(\dfrac{\pi}{4}\right)$

Trigonometric Ratios of Compound Angles Ex 7.1 Q15

LHS: $\cos^2 45° - \sin^2 15°$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 - \sin^2 15° \qquad\qquad \left[\because \cos 45 = \frac{1}{\sqrt{2}}\right]$$

$$= \frac{1}{2} - \left(\frac{1 - \cos 2 \times 15°}{2}\right) \qquad\qquad \left[\because \cos 2\theta = 1 - \sin^2\theta\right]$$

$$= \frac{1}{2} - \left(\frac{1 - \cos 30°}{2}\right)$$

$$= \frac{1 - 1 + \cos 30°}{2}$$

$$= \frac{\cos 30°}{2}$$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{4}$$

$$= \text{RHS}$$

$\therefore \quad$ LHS = RHS

Hence proved.

We have,

LHS $\sin^2(n+1)A - \sin^2 nA$

$= \sin\big[(n+1)A + nA\big]\sin\big[(n+1)A - nA\big]$

$\Big[\because \sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)\Big]$

$= \sin\big[nA + A + nA\big]\sin\big[nA + A - nA\big]$

$= \sin(2nA + A)\sin(A)$

$= \sin(2n+1)A\sin A$

$=$ RHS

$\therefore$ LHS $=$ RHS

Hence proved.

*********** END **********