



Differentiation Ex 11.3 Q21

$$\text{Let } f(x) = \tan^{-1} \left(\frac{\sin x}{1 + \cos x} \right)$$

This function is defined for all real numbers where $\cos x \neq -1$
i.e. at all odd multiples of π

$$\begin{aligned} f(x) &= \tan^{-1} \left(\frac{\sin x}{1 + \cos x} \right) \\ &= \tan^{-1} \left[\frac{2 \sin \left(\frac{x}{2} \right) \cos \left(\frac{x}{2} \right)}{2 \cos^2 \left(\frac{x}{2} \right)} \right] \\ &= \tan^{-1} \left[\tan \left(\frac{x}{2} \right) \right] = \frac{x}{2} \end{aligned}$$

$$\text{Thus, } f'(x) = \frac{d}{dx} \left(\frac{x}{2} \right) = \frac{1}{2}$$

Differentiation Ex 11.3 Q22

$$\text{Let } y = \sin^{-1} \left(\frac{1}{\sqrt{1+x^2}} \right)$$

$$\text{Put } x = \cot \theta$$

$$\begin{aligned} y &= \sin^{-1} \left(\frac{1}{\sqrt{1 + \cot^2 \theta}} \right) \\ &= \sin^{-1} \left(\frac{1}{\sqrt{\operatorname{cosec}^2 \theta}} \right) \\ &= \sin^{-1} (\sin \theta) \\ &= \theta \\ y &= \cot^{-1} x \end{aligned}$$

[Since, $\cot \theta = x$]

Differentiating it with respect to x ,

$$\frac{dy}{dx} = - \frac{1}{(1+x^2)}.$$

Differentiation Ex 11.3 Q23

$$\text{Let } y = \cos^{-1} \left(\frac{1 - x^{2n}}{1 + x^{2n}} \right)$$

$$\text{Put } x^n = \tan \theta, \text{ so,}$$

$$\begin{aligned} y &= \cos^{-1} \left(\frac{1 - (x^n)^2}{1 + (x^n)^2} \right) \\ &= \cos^{-1} \left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right) \end{aligned}$$

$$y = \cos^{-1} (\cos 2\theta) \quad \text{---(i)}$$

$$\text{Here, } 0 < x < \infty$$

$$\Rightarrow 0 < x^n < \infty$$

$$\Rightarrow 0 < \theta < \frac{\pi}{2}$$

$$\Rightarrow 0 < (2\theta) < \pi$$

So, from equation (i),

$$y = 2\theta \quad \left[\text{Since, } \cos^{-1}(\cos \theta) = \theta, \text{ if } \theta \in [0, \pi] \right]$$

$$y = 2 \tan^{-1} (x^n)$$

Differentiating it with respect to x using chain rule,

$$\begin{aligned} \frac{dy}{dx} &= 2 \left(\frac{1}{1 + (x^n)^2} \right) \frac{d}{dx} (x^n) \\ &= \frac{2}{1 + x^{2n}} \times (nx^{n-1}) \end{aligned}$$

$$\frac{dy}{dx} = \frac{2nx^{n-1}}{1 + x^{2n}}.$$

Differentiation Ex 11.3 Q24

$$\begin{aligned} \text{Let } y &= \sin^{-1} \left(\frac{1 - x^2}{1 + x^2} \right) + \sec^{-1} \left(\frac{1 + x^2}{1 - x^2} \right) \\ &= \sin^{-1} \left(\frac{1 - x^2}{1 + x^2} \right) + \cos^{-1} \left(\frac{1 - x^2}{1 + x^2} \right) \end{aligned}$$

$$y = \frac{\pi}{2}$$

$$\left[\text{Since, } \sec^{-1} x = \cos^{-1} \left(\frac{1}{x} \right) \right]$$

$$\left[\text{Since, } \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \right]$$

Differentiating it with respect to x ,

$$\frac{dy}{dx} = 0.$$

Differentiation Ex 11.3 Q25

$$\text{Let } y = \tan^{-1} \left(\frac{a+x}{1-ax} \right)$$

$$y = \tan^{-1} a + \tan^{-1} x$$

$$\left[\text{Since, } \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right) \right]$$

Differentiating it with respect to x ,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} (\tan^{-1} a) + \frac{d}{dx} (\tan^{-1} x) \\ &= 0 + \frac{1}{1+x^2} \end{aligned}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}.$$

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