NCERT Solutions for Class 10th Maths Chapter 3 Pair of Linear Equations in Two Variables Ex 3.5

Question-23

Which of the following pairs of linear equations has unique solution, no solution, or infinitely many solutions. In case there is a unique solution, find it by using cross multiplication method.

(i) x – 3y – 3 = 0
3x – 9y – 2 = 0

(ii) 2x + y = 5
3x +2y = 8

(iii) 3x – 5y = 20
6x – 10y = 40

(iv) x - 3y – 7 = 0
3x - 3y – 15 = 0

Solution:

(i) $\frac{a_1}{a_2} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{3}{2}$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, there is no solution.

(ii) 2x + y = 5
3x +2y = 8

$\frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-5}{-8}$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, we have a unique solution.

By cross multiplication method

2x + y = 5

3x +2y = 8

x	y	1
1	-5	2	1
2	-8	3	2

$\frac{x}{-8+10} = \frac{y}{-15+16} = \frac{1}{4-3}$

$\Rightarrow \frac{x}{2} = \frac{y}{1} = \frac{1}{1}$

$\Rightarrow x = 2, y = 1.$

(iii) $3x - 5y = 20$

$6x - 10y = 40$

$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{5}{10} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{20}{40} = \frac{1}{2}$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, there are infinitely many solutions

(iv) $x - 3y - 7 = 0$

$3x - 3y - 15 = 0$

$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-3}{-3} = 1,$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

∴ It has a unique solution

By cross multiplication method

$x - 3y - 7 = 0$

$3x - 3y - 15 = 0$

x y 1

-3 -7 1 -3

-3 -15 3 -3

$\frac{x}{45 - 21} = \frac{y}{-21 + 15} = \frac{1}{-3 + 9}$

$\Rightarrow \frac{x}{24} = \frac{y}{-6} = \frac{1}{6}$

$\Rightarrow x = 4, y = -1.$

Question-24

(i) For which values of a and b does the following pair of linear equations have an infinite number of solutions?

$2x + 3y = 7$

$(a - b)x + (a + b)y = 3a + b - 2$

(ii) For which value of k will the following pair of the linear equations have no solution?

$3x + y = 1$

$(2k - 1)x + (k - 1)y = 2k + 1$

Solution:

(i) If the above equation should have an infinite number of solution.

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(i.e.,) $\frac{2}{a - b} = \frac{3}{a + b} = \frac{7}{3a + b - 2}$

$\frac{2}{a - b} = \frac{3}{a + b}$

(i.e.,) $2(a+b) = 3(a-b)$

$2a - 3a = -3b - 2b$

$-a = -5b$

$a = 5b$

$\frac{3}{a + b} = \frac{7}{3a + b - 2}$ .....(1)

Substitute $a = 5b$ in (1)

$\frac{3}{6b} = \frac{7}{3(5b) + b - 2}$

$16b - 2 = 14b$

$2b = 2$

$b = 1$

Substitute $b = 1$ in $a = 5b$

$a = 5(1)$

$= 5$

$a = 5, b = 1.$

(ii) For which value of k will the following pair of the linear equations have no solution?

$3x + y = 1$

$(2k - 1)x + (k - 1)y = 2k + 1$

If the two equation should have no solution then $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

(i.e.,) $\frac{3}{2k - 1} = \frac{1}{k - 1}$

(i.e.,) $3(k - 1) = (2k - 1)$

$3k - 3 = 2k - 1$

$3k - 2k = 3 - 1$

$k = 2$

Hence k = 2 satifies the above condition

Question-25

Solve the following pair of linear equations by the substitutions and cross multiplication methods:

8x + 5y = 9

3x + 2y = 4

Solution:

By substitution method

$8x + 5y = 9$ ..... (1)

$3x + 2y = 4$ ..... (2)

from(2) $y = \frac{4 - 3x}{2}$ .....(3)

Substitute in (1)

$8x + 5(\frac{4 - 3x}{2}) = 9$

$16x + 20 - 15x = 18$

$x = 18 - 20$

$= -2$

Substitute x = -2 in (3)

$y = \frac{4 - 3(-2)}{2}$

$= \frac{4 + 6}{2} = 5$

$\therefore x = -2, y = 5$

By cross multiplication method:

$8x + 5y = 9$

$3x + 2y = 4$

$\frac{x}{-20 + 18} = \frac{y}{-27 + 32} = \frac{1}{16 - 15}$

$\frac{x}{-2} = \frac{y}{5} = 1$

$\Rightarrow x = -2, y = 5$

Question-26

Form the pair of linear equations in the following problem and find it's solution (if they exist) by any algebraic method:

A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay `1000 as hostel charges whereas a student B, who takes food for 26 days, pays `1180 as hostel charges. Find the fixed charges and the cost of food per day.

Solution:

Let the fixed charges be x and cost of food per day be y

In the case of student A

$x + 20y = 1000$ ..... (1)

In the case of student B

$x + 26y = 1180$ ..... (2)

Subtracting

(1) from (2)

$26y - 20y = 1180 - 1000$

$6y = 180$

$y = 30$

substituting y = 30 in (1) we get,

$x + 20(30) = 1000$

$x = 1000 - 600 = 400$

∴ Fixed charges = `400 and cost of food per day = `30

Question-27

Form the pair of linear equations in the following problem and find it's solution (if they exist) by any algebraic method:

A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

Solution:

Let $\frac{x}{y}$ be the fraction

$\frac{x-1}{y} = \frac{1}{3}$(given) .....(1)

$\frac{x}{y+8} = \frac{1}{4}$(given) .....(2)

from (1) $3(x - 1) = y$

$3x - y = 3$ .....(3)

from (2) $4x = y + 8$

$4x - y = 8$ ..... (4)

subtracting (4) from (3)

$(3x - y) - (4x - y) = 3 - 8$

$-x = -5$

$x = 5$

sub x = 5 in (3)

$3x - y = 3$

$3 \times 5 - y = 3$

$y = -3 + 15 = 12$

∴ The fraction is $\frac{5}{12}$

Question-28

Form the pair of linear equations in the following problem and find it's solution (if they exist) by any algebraic method:
Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

Solution:

Let x be the no of right answers and y be the no of wrong answers written by Yash respectively.

$3x - y = 40$ ..... (1)

$4x - 2y = 50$ .....(2)

$(1) \times 2 \Rightarrow 6x - 2y = 80$.....(3)

$(2) \Rightarrow \ 4x - 2y = 50$

$(3) - (2) \Rightarrow 2x = 30$

$\therefore x = 15$

From (1), $3(15) - y = 40$

$y = 45 - 40 = 5$

$\therefore$ Number of right answers be 15 and number of wrong answers be 5.

Question-29

Form the pair of linear equations in the following problem and find it's solution (if they exist) by any algebraic method:
Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

Solution:

Let x be the speed of 1^{st} car and y be the speed of the 2^{nd} car

If car (1) meets (2) at C

Distance traveled by car(1) is 5 x

Distance traveled by car(2) is 5 y

We know that

$5x - 5y = 100$

when they travel in the opposite direction (towards each other) they meet at C in 1 hour.

Since distance between A and B = 100

AB = AC - BC

Distance traveled by car (1) = x

Distance traveled by car (2) = y

We know that $x + y = 100$ (AC + BC = 100)

$5x - 5y = 100$ .....(1)

$x + y = 100$ .....(2)

$(1) \Rightarrow 5x - 5y = 100$

$5 \times (2) \Rightarrow \underline{5x + 5y = 500}$

$10x = 600$

$x = 60$

from (1) $5y = 5x - 100$

$= 5(60) - 100 = 300 - 100$

$5y = 200$

$\therefore y = 40$

$\therefore$ Speed of car (1) = 60 km/ hr

Speed of car (2) = 40 km/ hr

Question-30

Form the pair of linear equations in the following problem and find it's solution (if they exist) by any algebraic method:
The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Solution:

Area of rectangle = lb

Let l be the length and b be the breadth of the rectangle.

$(l - 5)(b + 3) = lb - 9$ ……(1)

$(l + 3)(b + 2) = lb + 67$ ……(2)

becomes

$(l - 5)(b + 3) = lb - 9$

$lb + 3l - 5b - 15 = lb - 9$

$3l - 5b = 6$ ……(3)

becomes

$(l + 3)(b + 2) = lb + 67$

$lb + 2l + 3b + 6 = lb + 67$

$2l + 3b = 61$ ……(4)

Multiply (3) by 2 and (4) by 3,

$6l - 10b = 12$

$6l + 9b = 183$

$-19b = -171$

$b = 9$

from (3) $\Rightarrow 3l - 5(9) = 6$

$3l = 45 + 6$

$l = \frac{51}{3} = 17$

$\therefore l = 17, b = 9.$

Thus the length and breadth of the rectangle are 17units and 9 units respectively.

********** END **********