Derivatives as a Rate Measurer Ex 13.2 Q11

Let AB be the height of source of light. Suppose at time t, the man CD is at a distance of x meters from the lamp-post and y meters be the length of his shadow CE, then

$$\frac{dx}{dt} = 2 \text{ m/sec}$$

ΔABE is similar to ΔCDE,

$$\frac{AB}{CD} = \frac{AE}{CE}$$

$$\frac{900}{180} = \frac{x+y}{y}$$

$$5y = x + y$$

$$4y = x$$

$$4\frac{dy}{dt} = \frac{dx}{dt}$$

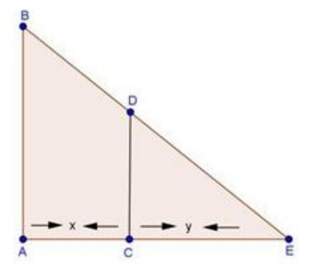
$$\frac{dy}{dt} = \frac{2}{4}$$

$$= \frac{1}{2}$$

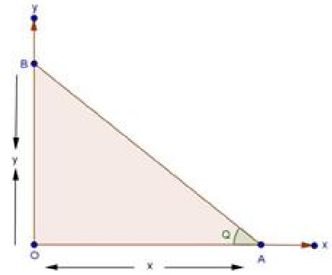
$$\frac{dy}{dt} = 0.5 \text{ m/sec}$$

So, rate of increase of shadow is 0.5 m/sec.

The diagram of the problem is shown below



Derivatives as a Rate Measurer Ex 13.2 Q12



Let AB be the position of the ladder, at time t, such that $OA = x$ and $OB = y$

Here,

$$OA^2 + OB^2 = AB^2$$
$$x^2 + y^2 = (13)^2$$
$$x^2 + y^2 = 169 \qquad \text{---(i)}$$

And $\dfrac{dx}{dt} = 1.5$ m/sec

From figure, $\tan\theta = \dfrac{y}{x}$

Differentiating equation (i) with respect to t,

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$
$$2(1.5)x + 2y\frac{dy}{dt} = 0$$
$$3x + 2y\frac{dy}{dt} = 0$$
$$\frac{dy}{dt} = -\frac{3x}{2y}$$

Differentiating equation (ii) with respect to t,

$$\sec^2\theta\frac{d\theta}{dt} = \frac{d\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}$$
$$= \frac{x \times \left(-\frac{3x}{2y}\right) - y\,(1.5)}{x^2}$$
$$= \frac{-1.5x^2 - 1.5y^2}{yx^2}$$
$$\frac{d\theta}{dt} = \frac{-1.5\left(x^2 + y^2\right)}{x^2 y \sec^2\theta}$$
$$= \frac{-1.5\left(x^2 + y^2\right)}{x^2 y\left(1 + \tan^2\theta\right)}$$
$$\frac{d\theta}{dt} = \frac{-1.5\left(x^2 + y^2\right)}{x^2 y\left(1 + \frac{y^2}{x^2}\right)}$$
$$= \frac{-1.5\left(x^2 + y^2\right) \times x^2}{x^2 y\left(x^2 + y^2\right)}$$
$$= \frac{-1.5}{y}$$
$$= \frac{-1.5}{\sqrt{169 - x^2}}$$
$$= \frac{-1.5}{\sqrt{169 - 144}}$$
$$= \frac{-1.5}{5}$$
$$= -0.3 \text{ radian/sec}$$

So, angle between ladder and ground is decreasing at the rate of 0.3 radian/sec.

*********** END **********