Mathematical Induction Ex 12.2 Q36

$$P(n): \frac{(2n)!}{2^{2n}(n!)^2} \leq \frac{1}{\sqrt{3n+1}}$$

For $n = 1$

$$\frac{2!}{2^2.1} \leq \frac{1}{\sqrt{4}}$$

$$= \frac{1}{2} \leq \frac{1}{2}$$

$\Rightarrow \quad P(n)$ is true for $n = 1$

Let $P(n)$ is true for $n = k$, so

$$\frac{(2k)!}{2^{2k}(k!)^2} \leq \frac{1}{\sqrt{3k+1}} \qquad ---(1)$$

We have to show that,

$$\frac{2(k+1)!}{2^{2(k+1)}[(k+1)!]^2} \leq \frac{1}{\sqrt{3k+4}}$$

Now,

$$\frac{2(k+1)!}{2^{2(k+1)}[(k+1)!]^2}$$

$$= \frac{(2k+2)!}{2^{2k}.2^2(k+1)!(k+1)!}$$

$$= \frac{(2k+2)(2k+1)(2k)!}{4.2^2(k+1)(k!)(k+1)(k!)}$$

$$= \frac{2(k+1)(2k+1)(2k)!}{4.(k+1)^2.2^{2k}.(k!)^2}$$

$$\leq \frac{2(2k+1)}{4(k+1)} \cdot \frac{1}{\sqrt{3k+1}} \qquad \text{[Using equation (1)]}$$

$$\leq \frac{(2k+1)}{2(k+1)} \cdot \frac{1}{\sqrt{3k+1}}$$

$$\leq \frac{(2k+2)}{2(k+1)} \cdot \frac{1}{\sqrt{3k+3+1}}$$

$$\leq \frac{1}{\sqrt{3k+4}} \qquad \begin{bmatrix}\text{Since, } 2k+1 < 2k+2 \\ 3k+1 \leq 3k+4\end{bmatrix}$$

$\Rightarrow \quad P(n)$ is true for $n = k+1$

$\Rightarrow \quad P(n)$ is true for all $n \in N$ by PMI

Mathematical Induction Ex 12.2 Q37

Let $P(n): 1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\ldots+\frac{1}{n^2}<2-\frac{1}{n}$ for all $n \geq 2$

For $n=2$

$1+\frac{1}{4}<2-\frac{1}{4}$

$=\frac{5}{4}<\frac{7}{4}$

$\Rightarrow \quad P(n)$ is true for $n=2$

Let $P(n)$ is true for $n=k$,

$$1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\ldots+\frac{1}{k^2}<2-\frac{1}{k} \qquad ---(1)$$

Now, we have to show that,

$$1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\ldots+\frac{1}{k^2}+\frac{1}{(k+1)^2}<2-\frac{1}{(k+1)}$$

Now,

$$1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\ldots+\frac{1}{k^2}+\frac{1}{(k+1)^2}$$

$$<2-\frac{1}{k}+\frac{1}{(k+1)^2} \qquad [\text{Using }(1)]$$

$$<2-\frac{k^2+2k+1-k}{k(k+1)^2}$$

$$<2-\frac{k^2+k+1}{k(k+1)^2}$$

$$<2-\frac{k^2+k}{k(k+1)^2}$$

$$<2-\frac{k(k+1)}{k(k+1)^2}$$

$$<2-\frac{1}{k+1}$$

$\Rightarrow \quad P(n)$ is true for $n=k+1$

$\Rightarrow \quad P(n)$ is true for all $n \in N$ by PMI

Mathematical Induction Ex 12.2 Q38

*********** END **********