



The adjacent sides \overline{AB} and \overline{BC} of the given rectangle are given as:

$$\overline{AB} = (1+1)\hat{i} + \left(\frac{1}{2} - \frac{1}{2}\right)\hat{j} + (4-4)\hat{k} = 2\hat{i}$$

$$\overline{BC} = (1-1)\hat{i} + \left(-\frac{1}{2} - \frac{1}{2}\right)\hat{j} + (4-4)\hat{k} = -\hat{j}$$

$$\therefore \overline{AB} \times \overline{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = \hat{k}(-2) = -2\hat{k}$$

$$|\overline{AB} \times \overline{BC}| = \sqrt{(-2)^2} = 2$$

Now, it is known that the area of a parallelogram whose adjacent sides are

$$\vec{a} \text{ and } \vec{b} \text{ is } |\vec{a} \times \vec{b}|.$$

Hence, the area of the given rectangle is $|\overline{AB} \times \overline{BC}| = 2$ square units.

The correct answer is C.

Miscellaneous Solutions

Question 1:

Write down a unit vector in XY-plane, making an angle of 30° with the positive direction of x-axis.

Answer

If \vec{r} is a unit vector in the XY-plane, then $\vec{r} = \cos \theta \hat{i} + \sin \theta \hat{j}$.

Here, θ is the angle made by the unit vector with the positive direction of the x-axis.

Therefore, for $\theta = 30^\circ$:

$$\vec{r} = \cos 30^\circ \hat{i} + \sin 30^\circ \hat{j} = \frac{\sqrt{3}}{2} \hat{i} + \frac{1}{2} \hat{j}$$

$$\text{Hence, the required unit vector is } \frac{\sqrt{3}}{2} \hat{i} + \frac{1}{2} \hat{j}$$

Question 2:

Find the scalar components and magnitude of the vector joining the points

$$P(x_1, y_1, z_1) \text{ and } Q(x_2, y_2, z_2).$$

Answer

The vector joining the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ can be obtained by,

$$\overline{PQ} = \text{Position vector of } Q - \text{Position vector of } P$$

$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$|\overline{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Hence, the scalar components and the magnitude of the vector joining the given points

are respectively $\{(x_2 - x_1), (y_2 - y_1), (z_2 - z_1)\}$ and $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

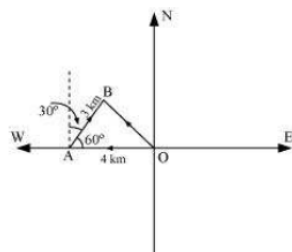
Question 3:

A girl walks 4 km towards west, then she walks 3 km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure.

Answer

Let O and B be the initial and final positions of the girl respectively.

Then, the girl's position can be shown as:





Now, we have:

$$\begin{aligned}\overrightarrow{OA} &= -4\hat{i} \\ \overrightarrow{AB} &= \hat{i}|\overrightarrow{AB}|\cos 60^\circ + \hat{j}|\overrightarrow{AB}|\sin 60^\circ \\ &= \hat{i}3 \times \frac{1}{2} + \hat{j}3 \times \frac{\sqrt{3}}{2} \\ &= \frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}\end{aligned}$$

By the triangle law of vector addition, we have:

$$\begin{aligned}\overrightarrow{OB} &= \overrightarrow{OA} + \overrightarrow{AB} \\ &= (-4\hat{i}) + \left(\frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}\right) \\ &= \left(-4 + \frac{3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j} \\ &= \left(\frac{-8+3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j} \\ &= \frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}\end{aligned}$$

Hence, the girl's displacement from her initial point of departure is

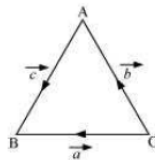
$$\frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}.$$

Question 4:

If $\vec{a} = \vec{b} + \vec{c}$, then is it true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$? Justify your answer.

Answer

In $\triangle ABC$, let $\overrightarrow{CB} = \vec{a}$, $\overrightarrow{CA} = \vec{b}$, and $\overrightarrow{AB} = \vec{c}$ (as shown in the following figure).



Now, by the triangle law of vector addition, we have $\vec{a} = \vec{b} + \vec{c}$.

It is clearly known that $|\vec{a}|$, $|\vec{b}|$, and $|\vec{c}|$ represent the sides of $\triangle ABC$.

Also, it is known that the sum of the lengths of any two sides of a triangle is greater than the third side.

$$\therefore |\vec{a}| < |\vec{b}| + |\vec{c}|$$

Hence, it is not true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$.

Question 5:

Find the value of x for which $x(\hat{i} + \hat{j} + \hat{k})$ is a unit vector.

Answer

$$x(\hat{i} + \hat{j} + \hat{k}) \text{ is a unit vector if } |x(\hat{i} + \hat{j} + \hat{k})| = 1.$$

Now,

$$\begin{aligned}|x(\hat{i} + \hat{j} + \hat{k})| &= 1 \\ \Rightarrow \sqrt{x^2 + x^2 + x^2} &= 1 \\ \Rightarrow \sqrt{3x^2} &= 1 \\ \Rightarrow \sqrt{3}x &= 1 \\ \Rightarrow x &= \pm \frac{1}{\sqrt{3}}\end{aligned}$$

Hence, the required value of x is $\pm \frac{1}{\sqrt{3}}$.

Question 6:

Find a vector of magnitude 5 units, and parallel to the resultant of the vectors

$$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k} \text{ and } \vec{b} = \hat{i} - 2\hat{j} + \hat{k}.$$

Answer

We have,

$$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k} \text{ and } \vec{b} = \hat{i} - 2\hat{j} + \hat{k}$$

Let \vec{c} be the resultant of \vec{a} and \vec{b} .

Then,

$$\vec{c} = \vec{a} + \vec{b} = (2+1)\hat{i} + (3-2)\hat{j} + (-1+1)\hat{k} = 3\hat{i} + \hat{j}$$

$$\therefore |\vec{c}| = \sqrt{3^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$$

$$\therefore \hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{(3\hat{i} + \hat{j})}{\sqrt{10}}$$

Hence, the vector of magnitude 5 units and parallel to the resultant of vectors \vec{a} and \vec{b} is

$$\pm 5 \cdot \hat{c} = \pm 5 \cdot \frac{1}{\sqrt{10}}(3\hat{i} + \hat{j}) = \pm \frac{3\sqrt{10}\hat{i}}{2} \pm \frac{\sqrt{10}\hat{j}}{2}.$$

Question 7:

If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

Answer

We have,

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} - \hat{j} + 3\hat{k} \text{ and } \vec{c} = \hat{i} - 2\hat{j} + \hat{k}$$

$$\begin{aligned} 2\vec{a} - \vec{b} + 3\vec{c} &= 2(\hat{i} + \hat{j} + \hat{k}) - (2\hat{i} - \hat{j} + 3\hat{k}) + 3(\hat{i} - 2\hat{j} + \hat{k}) \\ &= 2\hat{i} + 2\hat{j} + 2\hat{k} - 2\hat{i} + \hat{j} - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k} \\ &= 3\hat{i} - 3\hat{j} + 2\hat{k} \end{aligned}$$

$$|2\vec{a} - \vec{b} + 3\vec{c}| = \sqrt{3^2 + (-3)^2 + 2^2} = \sqrt{9+9+4} = \sqrt{22}$$

Hence, the unit vector along $2\vec{a} - \vec{b} + 3\vec{c}$ is

$$\frac{2\vec{a} - \vec{b} + 3\vec{c}}{|2\vec{a} - \vec{b} + 3\vec{c}|} = \frac{3\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{22}} = \frac{3}{\sqrt{22}}\hat{i} - \frac{3}{\sqrt{22}}\hat{j} + \frac{2}{\sqrt{22}}\hat{k}.$$

Question 8:

Show that the points A (1, -2, -8), B (5, 0, -2) and C (11, 3, 7) are collinear, and find the ratio in which B divides AC.

Answer

The given points are A (1, -2, -8), B (5, 0, -2), and C (11, 3, 7).

$$\therefore \vec{AB} = (5-1)\hat{i} + (0+2)\hat{j} + (-2+8)\hat{k} = 4\hat{i} + 2\hat{j} + 6\hat{k}$$

$$\vec{BC} = (11-5)\hat{i} + (3-0)\hat{j} + (7+2)\hat{k} = 6\hat{i} + 3\hat{j} + 9\hat{k}$$

$$\vec{AC} = (11-1)\hat{i} + (3+2)\hat{j} + (7+8)\hat{k} = 10\hat{i} + 5\hat{j} + 15\hat{k}$$

$$|\vec{AB}| = \sqrt{4^2 + 2^2 + 6^2} = \sqrt{16+4+36} = \sqrt{56} = 2\sqrt{14}$$

$$|\vec{BC}| = \sqrt{6^2 + 3^2 + 9^2} = \sqrt{36+9+81} = \sqrt{126} = 3\sqrt{14}$$

$$|\vec{AC}| = \sqrt{10^2 + 5^2 + 15^2} = \sqrt{100+25+225} = \sqrt{350} = 5\sqrt{14}$$

$$\therefore |\vec{AC}| = |\vec{AB}| + |\vec{BC}|$$

Thus, the given points A, B, and C are collinear.

Now, let point B divide AC in the ratio $\lambda:1$. Then, we have:

$$\vec{OB} = \frac{\lambda\vec{OC} + \vec{OA}}{(\lambda+1)}$$

$$\Rightarrow 5\hat{i} - 2\hat{k} = \frac{\lambda(11\hat{i} + 3\hat{j} + 7\hat{k}) + (\hat{i} - 2\hat{j} - 8\hat{k})}{\lambda+1}$$

$$\Rightarrow (\lambda+1)(5\hat{i} - 2\hat{k}) = 11\lambda\hat{i} + 3\lambda\hat{j} + 7\lambda\hat{k} + \hat{i} - 2\hat{j} - 8\hat{k}$$

$$\Rightarrow 5(\lambda+1)\hat{i} - 2(\lambda+1)\hat{k} = (11\lambda+1)\hat{i} + (3\lambda-2)\hat{j} + (7\lambda-8)\hat{k}$$

On equating the corresponding components, we get:

$$5(\lambda+1) = 11\lambda+1$$

$$\Rightarrow 5\lambda+5 = 11\lambda+1$$

$$\Rightarrow 6\lambda = 4$$

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