



Differentiation Ex 11.8 Q7(i)

Let $u = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Put $x = \sin\theta$, so

$$\begin{aligned} u &= \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right) \\ &= \sin^{-1}(2\sin\theta\cos\theta) \\ u &= \sin^{-1}(\sin 2\theta) \end{aligned}$$

---(i)

And,

Let $v = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

$$\begin{aligned} &= \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right) \\ &= \sec^{-1}\left(\frac{1}{\cos\theta}\right) \\ &= \sec^{-1}(\sec\theta) \\ &= \cos^{-1}\left(\frac{1}{\cos\theta}\right) \\ v &= \cos^{-1}(\cos\theta) \end{aligned}$$

$$\left[\text{Since, } \sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right) \right]$$

---(ii)

Here,

$$\begin{aligned} x &\in \left(0, \frac{1}{\sqrt{2}}\right) \\ \Rightarrow \sin\theta &\in \left(0, \frac{1}{\sqrt{2}}\right) \\ \Rightarrow \theta &\in \left(0, \frac{\pi}{4}\right) \end{aligned}$$

So, from equation (i),

$$u = 2\theta$$

$$\left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \right]$$

Let $u = 2\sin^{-1}x$

$$[\text{Since, } x = \sin\theta]$$

Differentiating it with respect to x ,

$$\begin{aligned} \frac{du}{dx} &= 2\left(\frac{1}{\sqrt{1-x^2}}\right) \\ \frac{du}{dx} &= \frac{2}{\sqrt{1-x^2}} \end{aligned}$$

---(iii)

And, from equation (ii),

$$v = \theta$$

$$[\text{Since, } \cos^{-1}(\cos\theta) = \theta, \text{ if } \theta \in [0, \pi]]$$

$$v = \sin^{-1}x$$

$$[\text{Since, } x = \sin\theta]$$

Differentiating it with respect to x ,

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}}$$

---(iv)

Dividing equation (iii) by (iv),

$$\begin{aligned} \frac{\frac{du}{dx}}{\frac{dv}{dx}} &= \frac{2}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{1} \\ \frac{du}{dv} &= 2 \end{aligned}$$

Differentiation Ex 11.8 Q7(ii)

$$\text{Let } u = \sin^{-1} \left(2x\sqrt{1-x^2} \right)$$

$$\text{Put } x = \sin \theta, \text{ so}$$

$$\begin{aligned} u &= \sin^{-1} \left(2 \sin \theta \sqrt{1 - \sin^2 \theta} \right) \\ &= \sin^{-1} (2 \sin \theta \cos \theta) \\ u &= \sin^{-1} (\sin 2\theta) \end{aligned} \quad \text{---(i)}$$

And,

$$\begin{aligned} \text{Let } v &= \sec^{-1} \left(\frac{1}{\sqrt{1-x^2}} \right) \\ &= \sec^{-1} \left(\frac{1}{\sqrt{1-\sin^2 \theta}} \right) \\ &= \sec^{-1} \left(\frac{1}{\cos \theta} \right) \\ &= \sec^{-1} (\sec \theta) \\ &= \cos^{-1} \left(\frac{1}{\sec \theta} \right) \quad \left[\text{Since, } \sec^{-1} x = \cos^{-1} \left(\frac{1}{x} \right) \right] \\ v &= \cos^{-1} (\cos \theta) \end{aligned} \quad \text{---(ii)}$$

Here,

$$\begin{aligned} x &\in \left(\frac{1}{\sqrt{2}}, 1 \right) \\ \Rightarrow \sin \theta &\in \left(\frac{1}{\sqrt{2}}, 1 \right) \\ \Rightarrow \theta &\in \left(\frac{\pi}{4}, \frac{\pi}{2} \right) \end{aligned}$$

So, from equation (i),

$$\begin{aligned} u &= 2\theta \quad \left[\text{Since, } \sin^{-1} (\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right] \\ u &= 2 \sin^{-1} x \quad \left[\text{Since, } x = \sin \theta \right] \end{aligned}$$

Differentiating it with respect to x ,

$$\frac{du}{dx} = 2 \left(\frac{1}{\sqrt{1-x^2}} \right) \quad \text{---(iv)}$$

From equation (ii)

$$\begin{aligned} v &= \theta \quad \left[\text{Since, } \cos^{-1} (\cos \theta) = \theta, \text{ if } \theta \in [0, \pi] \right] \\ v &= \sin^{-1} x \end{aligned}$$

Differentiating it with respect to x ,

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}} \quad \text{---(v)}$$

Dividing equation (iv) by (v),

$$\begin{aligned} \frac{\frac{du}{dx}}{\frac{dv}{dx}} &= \frac{2}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{1} \\ \frac{du}{dv} &= 2 \end{aligned}$$

Differentiation Ex 11.8 Q8

Let $u = (\cos x)^{\sin x}$

Taking log on both the sides,

$$\begin{aligned}\log u &= \log (\cos x)^{\sin x} \\ \log u &= \sin x \log (\cos x)\end{aligned}$$

Differentiating it with respect to x using product and chain rule,

$$\begin{aligned}\frac{1}{u} \frac{du}{dx} &= \sin x \frac{d}{dx} (\log \cos x) + \log \cos x \frac{d}{dx} (\sin x) \\ \frac{1}{u} \frac{du}{dx} &= \sin x \left(\frac{1}{\cos x} \right) \frac{d}{dx} (\cos x) + \log \cos x (\cos x) \\ \frac{du}{dx} &= 4 [(\tan x) \times (-\sin x) + \log \log x \times (\cos x)] \\ \frac{du}{dx} &= (\cos x)^{\sin x} [\cos x \log \cos x - \sin x \tan x] \quad \text{---(i)}\end{aligned}$$

Let $v = (\sin x)^{\cos x}$

Taking log on both the sides,

$$\begin{aligned}\log v &= \log (\sin x)^{\cos x} \\ \log v &= \cos x \log (\sin x)\end{aligned}$$

Differentiating it with respect to x using product rule and chain rule,

$$\begin{aligned}\frac{1}{v} \frac{dv}{dx} &= \cos x \frac{d}{dx} (\log \sin x) + \log \sin x \frac{d}{dx} (\cos x) \\ \frac{1}{v} \frac{dv}{dx} &= \cos x \left(\frac{1}{\sin x} \right) \frac{d}{dx} (\sin x) + \log \sin x (-\sin x) \\ \frac{dv}{dx} &= v [\cot x (\cos x) - \sin x \log \sin x] \\ \frac{dv}{dx} &= (\sin x)^{\cos x} [\cot x (\cos x) - \sin x \log \sin x] \quad \text{---(ii)}\end{aligned}$$

Dividing equation (i) by (ii),

$$\frac{du}{dv} = \frac{(\cos x)^{\sin x} [\cos x \log \cos x - \sin x \tan x]}{(\sin x)^{\cos x} [\cot x (\cos x) - \sin x \log \sin x]}$$

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