



$$\Rightarrow \lambda = \frac{10}{9}$$

Substituting $\lambda = \frac{10}{9}$ in equation (3), we obtain

$$\vec{r} \cdot \left(\frac{38}{9} \hat{i} + \frac{68}{9} \hat{j} + \frac{3}{9} \hat{k} \right) = 17$$

$$\Rightarrow \vec{r} \cdot (38\hat{i} + 68\hat{j} + 3\hat{k}) = 153$$

This is the vector equation of the required plane.

Question 11:

Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$

Answer

The equation of the plane through the intersection of the planes, $x + y + z = 1$ and $2x + 3y + 4z = 5$, is

$$(x + y + z - 1) + \lambda(2x + 3y + 4z - 5) = 0$$

$$\Rightarrow (2\lambda + 1)x + (3\lambda + 1)y + (4\lambda + 1)z - (5\lambda + 1) = 0 \quad \dots(1)$$

The direction ratios, a_1, b_1, c_1 , of this plane are $(2\lambda + 1)$, $(3\lambda + 1)$, and $(4\lambda + 1)$.

The plane in equation (1) is perpendicular to $x - y + z = 0$

Its direction ratios, a_2, b_2, c_2 , are 1, -1, and 1.

Since the planes are perpendicular,

$$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

$$\Rightarrow (2\lambda + 1) - (3\lambda + 1) + (4\lambda + 1) = 0$$

$$\Rightarrow 3\lambda + 1 = 0$$

$$\Rightarrow \lambda = -\frac{1}{3}$$

Substituting $\lambda = -\frac{1}{3}$ in equation (1), we obtain

$$\frac{1}{3}x - \frac{1}{3}z + \frac{2}{3} = 0$$

$$\Rightarrow x - z + 2 = 0$$

This is the required equation of the plane.

Question 12:

Find the angle between the planes whose vector equations are

$$\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 5 \quad \text{and} \quad \vec{r} \cdot (3\hat{i} - 3\hat{j} + 5\hat{k}) = 3$$

Answer

The equations of the given planes are $\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} - 3\hat{j} + 5\hat{k}) = 3$

It is known that if \vec{n}_1 and \vec{n}_2 are normal to the planes, $\vec{r} \cdot \vec{n}_1 = d_1$ and $\vec{r} \cdot \vec{n}_2 = d_2$, then the angle between them, Q , is given by,

$$\cos Q = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1| |\vec{n}_2|} \quad \dots(1)$$

Here, $\vec{n}_1 = 2\hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{n}_2 = 3\hat{i} - 3\hat{j} + 5\hat{k}$

$$\therefore \vec{n}_1 \cdot \vec{n}_2 = (2\hat{i} + 2\hat{j} - 3\hat{k}) \cdot (3\hat{i} - 3\hat{j} + 5\hat{k}) = 2.3 + 2.(-3) + (-3).5 = -15$$

$$|\vec{n}_1| = \sqrt{(2)^2 + (2)^2 + (-3)^2} = \sqrt{17}$$

$$|\vec{n}_2| = \sqrt{(3)^2 + (-3)^2 + (5)^2} = \sqrt{43}$$

Substituting the value of $\vec{n}_1 \cdot \vec{n}_2$, $|\vec{n}_1|$ and $|\vec{n}_2|$ in equation (1), we obtain

$$\cos Q = \frac{|-15|}{\sqrt{17} \cdot \sqrt{43}}$$

$$\Rightarrow \cos Q = \frac{15}{\sqrt{731}}$$

$$\Rightarrow \cos Q^{-1} = \left(\frac{15}{\sqrt{731}} \right)$$

Question 13:

In the following cases, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between them.

- (a) $7x + 5y + 6z + 30 = 0$ and $3x - y - 10z + 4 = 0$
 (b) $2x + y + 3z - 2 = 0$ and $x - 2y + 5 = 0$
 (c) $2x - 2y + 4z + 5 = 0$ and $3x - 3y + 6z - 1 = 0$
 (d) $2x - y + 3z - 1 = 0$ and $2x - y + 3z + 3 = 0$
 (e) $4x + 8y + z - 8 = 0$ and $y + z - 4 = 0$

Answer

The direction ratios of normal to the plane, $L_1 : a_1x + b_1y + c_1z = 0$, are a_1, b_1, c_1 and $L_2 : a_2x + b_2y + c_2z = 0$ are a_2, b_2, c_2 .

$$L_1 \parallel L_2, \text{ if } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$L_1 \perp L_2, \text{ if } a_1a_2 + b_1b_2 + c_1c_2 = 0$$

The angle between L_1 and L_2 is given by,

$$Q = \cos^{-1} \left| \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \cdot \sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$$

(a) The equations of the planes are $7x + 5y + 6z + 30 = 0$ and $3x - y - 10z + 4 = 0$

Here, $a_1 = 7, b_1 = 5, c_1 = 6$

$$a_2 = 3, b_2 = -1, c_2 = -10$$

$$a_1a_2 + b_1b_2 + c_1c_2 = 7 \times 3 + 5 \times (-1) + 6 \times (-10) = -44 \neq 0$$

Therefore, the given planes are not perpendicular.

$$\frac{a_1}{a_2} = \frac{7}{3}, \frac{b_1}{b_2} = \frac{5}{-1} = -5, \frac{c_1}{c_2} = \frac{6}{-10} = \frac{-3}{5}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

It can be seen that,

Therefore, the given planes are not parallel.

The angle between them is given by,

$$\begin{aligned} Q &= \cos^{-1} \left| \frac{7 \times 3 + 5 \times (-1) + 6 \times (-10)}{\sqrt{(7)^2 + (5)^2 + (6)^2} \times \sqrt{(3)^2 + (-1)^2 + (-10)^2}} \right| \\ &= \cos^{-1} \left| \frac{21 - 5 - 60}{\sqrt{110} \times \sqrt{110}} \right| \\ &= \cos^{-1} \frac{44}{110} \\ &= \cos^{-1} \frac{2}{5} \end{aligned}$$

(b) The equations of the planes are $2x + y + 3z - 2 = 0$ and $x - 2y + 5 = 0$

Here, $a_1 = 2, b_1 = 1, c_1 = 3$ and $a_2 = 1, b_2 = -2, c_2 = 0$

$$\therefore a_1a_2 + b_1b_2 + c_1c_2 = 2 \times 1 + 1 \times (-2) + 3 \times 0 = 0$$

Thus, the given planes are perpendicular to each other.

(c) The equations of the given planes are $2x - 2y + 4z + 5 = 0$ and $3x - 3y + 6z - 1 = 0$

Here, $a_1 = 2, b_1 = -2, c_1 = 4$ and

$$a_2 = 3, b_2 = -3, c_2 = 6 \quad a_1a_2 + b_1b_2 + c_1c_2 = 2 \times 3 + (-2)(-3) + 4 \times 6 = 6 + 6 + 24 = 36 \neq 0$$

Thus, the given planes are not perpendicular to each other.

$$\frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{-2}{-3} = \frac{2}{3} \text{ and } \frac{c_1}{c_2} = \frac{4}{6} = \frac{2}{3}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Thus, the given planes are parallel to each other.

(d) The equations of the planes are $2x - y + 3z - 1 = 0$ and $2x - y + 3z + 3 = 0$

Here, $a_1 = 2, b_1 = -1, c_1 = 3$ and $a_2 = 2, b_2 = -1, c_2 = 3$

$$\frac{a_1}{a_2} = \frac{2}{2} = 1, \frac{b_1}{b_2} = \frac{-1}{-1} = 1 \text{ and } \frac{c_1}{c_2} = \frac{3}{3} = 1$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Thus, the given lines are parallel to each other.

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(e) The equations of the given planes are $4x + 8y + z - 8 = 0$ and $y + z - 4 = 0$

Here, $a_1 = 4, b_1 = 8, c_1 = 1$ and $a_2 = 0, b_2 = 1, c_2 = 1$

$$a_1a_2 + b_1b_2 + c_1c_2 = 4 \times 0 + 8 \times 1 + 1 = 9 \neq 0$$

Therefore, the given lines are not perpendicular to each other.

$$\frac{a_1}{a_2} = \frac{4}{0}, \frac{b_1}{b_2} = \frac{8}{1} = 8, \frac{c_1}{c_2} = \frac{1}{1} = 1$$

$$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, the given lines are not parallel to each other.

The angle between the planes is given by,

$$Q = \cos^{-1} \left| \frac{4 \times 0 + 8 \times 1 + 1 \times 1}{\sqrt{4^2 + 8^2 + 1^2} \times \sqrt{0^2 + 1^2 + 1^2}} \right| = \cos^{-1} \left| \frac{9}{9\sqrt{2}} \right| = \cos^{-1} \left(\frac{1}{\sqrt{2}} \right) = 45^\circ$$

Question 14:

In the following cases, find the distance of each of the given points from the corresponding given plane.

Point Plane

(a) $(0, 0, 0)$ $3x - 4y + 12z = 3$

(b) $(3, -2, 1)$ $2x - y + 2z + 3 = 0$

(c) $(2, 3, -5)$ $x + 2y - 2z = 9$

(d) $(-6, 0, 0)$ $2x - 3y + 6z - 2 = 0$

Answer

It is known that the distance between a point, $p(x_1, y_1, z_1)$, and a plane, $Ax + By + Cz = D$, is given by,

$$d = \left| \frac{Ax_1 + By_1 + Cz_1 - D}{\sqrt{A^2 + B^2 + C^2}} \right| \quad \dots(1)$$

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