Functions Ex 3.2 Q6

We have,

$$f(x) = \begin{cases} x^2, & \text{when } x < 0 \\ x, & \text{when } 0 \le x < 1 \\ \dfrac{1}{x}, & \text{when } x \ge 1 \end{cases}$$

(a) $f(1/2) = \dfrac{1}{2}$

(b) $f(-2) = (-2)^2 = 4$

(c) $f(1) = \dfrac{1}{1} = 1$

(d) $f(\sqrt{3}) = \dfrac{1}{\sqrt{3}}$

(e) $f(\sqrt{-3}) =$ does not exist because $\sqrt{-3} \notin \text{domain}(f)$.

Functions Ex 3.2 Q7

We have,

$$f(x) = x^3 - \frac{1}{x^3} \qquad \text{---(i)}$$

Now,

$$f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^3 - \frac{1}{\left(\frac{1}{x}\right)^3}$$

$$= \frac{1}{x^3} - \frac{1}{\frac{1}{x^3}}$$

$$\Rightarrow \quad f\left(\frac{1}{x}\right) = \frac{1}{x^3} - x^3 \qquad \text{---(ii)}$$

Adding equation (i) and equation (ii), we get

$$f(x) + f\left(\frac{1}{x}\right) = \left(x^3 - \frac{1}{x^3}\right) + \left(\frac{1}{x^3} - x^3\right)$$

$$= x^3 - \frac{1}{x^3} + \frac{1}{x^3} - x^3$$

$$= 0$$

$$\therefore \quad f(x) + f\left(\frac{1}{x}\right) = 0 \qquad \text{Hence, proved.}$$

Functions Ex 3.2 Q8

We have,

$$f(x) = \frac{2x}{1+x^2}$$

Now,

$$f(\tan\theta) = \frac{2(\tan\theta)}{1+\tan^2\theta}$$

$$= \sin 2\theta \qquad \left[\because \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}\right]$$

$\therefore \quad f(\tan\theta) = \sin 2\theta \qquad$ Hence, proved.

Functions Ex 3.2 Q9

i. $= \dfrac{x-1}{x+1}$

$$f\left(\frac{1}{x}\right) = \frac{\frac{1}{x}-1}{\frac{1}{x}+1} = \frac{\frac{1-x}{x}}{\frac{1+x}{x}} = \frac{1-x}{1+x} = -f(x)$$

ii. $f(x) = \dfrac{x-1}{x+1}$

$$f\left(-\frac{1}{x}\right) = \frac{-\frac{1}{x}-1}{-\frac{1}{x}+1} = \frac{\frac{-1-x}{x}}{\frac{-1+x}{x}} = \frac{-1-x}{-1+x} = -\frac{1}{\frac{1+x}{x-1}} = -\frac{1}{f(x)}$$

Functions Ex 3.2 Q10

We have,

$$f(x) = (a - x^n)^{1/n}, \quad a > 0$$

Now,

$$f(f(x)) = f(a - x^n)^{1/n}$$

$$= \left[a - \left\{(a - x^n)^{1/n}\right\}^n\right]^{1/n}$$

$$= \left[a - (a - x^n)\right]^{1/n}$$

$$= \left[a - a + x^n\right]^{1/n}$$

$$= (x^n)^{1/n}$$

$$= (x)^{n \times \frac{1}{n}}$$

$$= x$$

$\therefore \quad f(f(x)) = x \qquad$ Hence, proved.

Functions Ex 3.2 Q11

We have,

$$af\left(x\right)+bf\left(\frac{1}{x}\right)=\frac{1}{x}-5 \qquad \text{---(i)}$$

$$\Rightarrow \quad af\left(\frac{1}{x}\right)+bf\left(x\right)=\frac{1}{\frac{1}{x}}-5$$

$$=x-5$$

$$\Rightarrow \quad af\left(\frac{1}{x}\right)+bf\left(x\right)=x-5 \qquad \text{---(ii)}$$

Adding equations (i) and (ii), we get

$$af\left(x\right)+bf\left(x\right)+bf\left(\frac{1}{x}\right)+af\left(\frac{1}{x}\right)=\frac{1}{x}-5+x-5$$

$$\Rightarrow \quad \left(a+b\right)f\left(x\right)+f\left(\frac{1}{x}\right)\left(a+b\right)=\frac{1}{x}+x-10$$

$$\Rightarrow \quad f\left(x\right)+f\left(\frac{1}{x}\right)=\frac{1}{a+b}\left[\frac{1}{x}+x-10\right] \qquad \text{---(iii)}$$

Subtracting equation (ii) from equation (i), we get

$$af\left(x\right)-bf\left(x\right)+bf\left(\frac{1}{x}\right)-af\left(\frac{1}{x}\right)=\frac{1}{x}-5-x+5$$

$$\Rightarrow \quad \left(a-b\right)f\left(x\right)-f\left(\frac{1}{x}\right)\left(a-b\right)=\frac{1}{x}-x$$

$$\Rightarrow \quad f\left(x\right)-f\left(\frac{1}{x}\right)=\frac{1}{a-b}\left[\frac{1}{x}-x\right]$$

Adding equations (iii) and (iv), we get

$$2f\left(x\right)=\frac{1}{a+b}\left[\frac{1}{x}+x-10\right]+\frac{1}{a-b}\left[\frac{1}{x}-x\right]$$

$$\Rightarrow \quad 2f\left(x\right)=\frac{\left(a-b\right)\left[\frac{1}{x}+x-10\right]+\left(a+b\right)\left[\frac{1}{x}-x\right]}{\left(a+b\right)\left(a-b\right)}$$

$$\Rightarrow \quad 2f\left(x\right)=\frac{\frac{a}{x}+ax-10a-\frac{b}{x}-bx+10b+\frac{a}{x}-ax+\frac{b}{x}-bx}{a^2-b^2}$$

$$\Rightarrow \quad 2f\left(x\right)=\frac{\frac{2a}{x}-10a+10b-2bx}{a^2-b^2}$$

$$\Rightarrow \quad f\left(x\right)=\frac{1}{a^2-b^2}\times\frac{1}{2}\left[\frac{2a}{x}-10a+10b-2bx\right]$$

$$=\frac{1}{a^2-b^2}\left[\frac{a}{x}-5a+5b-bx\right]$$

$$\therefore \quad f\left(x\right)=\frac{1}{a^2-b^2}\left[\frac{a}{x}-bx-5a+5b\right]$$

$$=\frac{1}{a^2-b^2}\left[\frac{a}{x}-bx\right]-\frac{5\left(a-b\right)}{a^2-b^2}$$

$$=\frac{1}{a^2-b^2}\left[\frac{a}{x}-bx\right]-\frac{5\left(a-b\right)}{\left(a-b\right)\left(a+b\right)}$$

$$=\frac{1}{a^2-b^2}\left[\frac{a}{x}-bx\right]-\frac{5}{a+b}$$

*********** END **********