Differentiation Ex 11.2 Q29

Let $\quad y = \frac{e^x \log x}{x^2}$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{x^2 \frac{d}{dx}\left(e^x \log x\right) - \left(e^x \log x\right)\frac{d}{dx}\left(x^2\right)}{\left(x^2\right)^2} \qquad \text{[Using quotient rule]}$$

$$= \frac{x^2\left[e^x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}\left(e^x\right) - e^x \log x \times 2x\right]}{x^4} \qquad \text{[Using product rule]}$$

$$= \frac{x^2\left[\frac{e^x}{x} + e^x \log x\right] - 2xe^x \log x}{x^4}$$

$$= \frac{\frac{x^2 e^x\left(1 + x\log x\right)}{x} - 2xe^x \log x}{x^4}$$

$$= \frac{xe^x\left[1 + x\log x - 2\log x\right]}{x^4}$$

$$= \frac{xe^x}{x^3}\left[\frac{1}{x} + \frac{x\log x}{x} - \frac{2\log x}{x}\right]$$

$$= e^x x^{-2}\left[\frac{1}{x} + \log x - \frac{2}{x}\log x\right]$$

So,

$$\frac{d}{dx}\left[\frac{e^x \log x}{x^2}\right] = e^x x^{-2}\left[\frac{1}{x} + \log x - \frac{2}{x}\log x\right].$$

Differentiation Ex 11.2 Q30

Let $\quad y = \log\left(\operatorname{cosec} x - \cot x\right)$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\log\left(\operatorname{cosec} x - \cot x\right)$$

$$= \frac{1}{\left(\operatorname{cosec} x - \cot x\right)}\frac{d}{dx}\left(\operatorname{cosec} x - \cot x\right) \qquad \text{[Using chain rule]}$$

$$= \frac{1}{\left(\operatorname{cosec} x - \cot x\right)} \times \left(-\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x\right)$$

$$= \frac{\operatorname{cosec} x\left(\operatorname{cosec} x - \cot x\right)}{\left(\operatorname{cosec} x - \cot x\right)}$$

$$= \operatorname{cosec} x$$

So,

$$\frac{d}{dx}\left(\log\left(\operatorname{cosec} x - \cot x\right)\right) = \operatorname{cosec} x.$$

Differentiation Ex 11.2 Q31

Let $\quad y = \frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}\right]$$

$$= \left[\frac{\left(e^{2x} - e^{-2x}\right)\frac{d}{dx}\left(e^{2x} + e^{-2x}\right) - \left(e^{2x} + e^{-2x}\right)\frac{d}{dx}\left(e^{2x} - e^{-2x}\right)}{\left(e^{2x} - e^{-2x}\right)^2}\right] \qquad \left[\begin{array}{l}\text{Using quotient rule}\\ \text{and chain rule}\end{array}\right]$$

$$= \frac{\left(e^{2x} - e^{-2x}\right)\left[e^{2x}\frac{d}{dx}(2x) + e^{-2x}\frac{d}{dx}(-2x)\right] - \left(e^{2x} + e^{-2x}\right)\left[\begin{array}{l}e^{2x}\frac{d}{dx}(2x) - \\ e^{-2x}\frac{d}{dx}(-2x)\end{array}\right]}{\left(e^{2x} - e^{-2x}\right)^2}$$

$$= \frac{\left(e^{2x} - e^{-2x}\right)\left(2e^{2x} - 2e^{-2x}\right) - \left(e^{2x} + e^{-2x}\right)\left(2e^{2x} + 2e^{-2x}\right)}{\left(e^{2x} - e^{-2x}\right)^2}.$$

$$= \frac{2\left(e^{2x} - e^{-2x}\right)^2 - 2\left(e^{2x} + e^{-2x}\right)^2}{\left(e^{2x} - e^{-2x}\right)^2}$$

$$= \frac{2\left[e^{4x} + e^{-4x} - 2e^{2x}e^{-2x} - e^{4x} - e^{-4x} - 2e^{2x}e^{-2x}\right]}{\left(e^{2x} - e^{-2x}\right)^2}$$

$$= \frac{-8}{\left(e^{2x} - e^{-2x}\right)^2}$$

So,

$$\frac{d}{dx}\left(\frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}\right) = \frac{-8}{\left(e^{2x} - e^{-2x}\right)^2}.$$

Differentiation Ex 11.2 Q32

Let $\quad y = \log\left(\frac{x^2 + x + 1}{x^2 - x + 1}\right)$

Differentiating with respect to x, .

$$\frac{dy}{dx} = \frac{d}{dx}\left[\log\left(\frac{x^2 + x + 1}{x^2 - x + 1}\right)\right]$$

$$= \frac{1}{\left(\frac{x^2 + x + 1}{x^2 - x + 1}\right)}\frac{d}{dx}\left(\frac{x^2 + x + 1}{x^2 - x + 1}\right) \qquad [\text{Using chain rule and quotient rule}]$$

$$= \left(\frac{x^2 - x + 1}{x^2 + x + 1}\right)\left[\frac{\left(x^2 - x + 1\right)\frac{d}{dx}\left(x^2 + x + 1\right) - \left(x^2 + x + 1\right)\frac{d}{dx}\left(x^2 - x + 1\right)}{\left(x^2 - x + 1\right)^2}\right]$$

$$= \left(\frac{x^2 - x + 1}{x^2 + x + 1}\right)\left[\frac{\left(x^2 - x + 1\right)(2x + 1) - \left(x^2 + x + 1\right)(2x - 1)}{\left(x^2 - x + 1\right)^2}\right]$$

$$= \frac{\left(x^2 - x + 1\right)}{\left(x^2 + x + 1\right)}\left[\frac{2x^3 - 2x^2 + 2x + x^2 - x + 1 - 2x^3 - 2x^2 - 2x + x^2 + x + 1}{\left(x^2 - x + 1\right)^2}\right]$$

$$= \frac{-4x^2 + 2x^2 + 2}{\left(x^2 + x + 1\right)\left(x^2 - x + 1\right)}$$

$$= \frac{-2\left(x^2 - 1\right)}{x^4 + 1 + 2x^2 - x^2}$$

$$= \frac{-2\left(x^2 - 1\right)}{x^4 + x^2 + 1}$$

So,

$$\frac{d}{dx}\left(\log\frac{x^2 + x + 1}{x^2 - x + 1}\right) = \frac{-2\left(x^2 - 1\right)}{x^4 + x^2 + 1}$$

Differentiation Ex 11.2 Q33

Let $\quad y = \tan^{-1}\left(e^{x}\right)$

Differentiate it with respect to x,.

$$\begin{aligned}
\frac{dy}{dx} &= \frac{d}{dx}\left(\tan^{-1} e^{x}\right) \\
&= \frac{1}{1+\left(e^{x}\right)^{2}} \frac{d}{dx}\left(e^{x}\right) && \text{[Using chain rule]} \\
&= \frac{1}{1+e^{2x}} \times e^{x} \\
&= \frac{e^{x}}{1+e^{2x}}
\end{aligned}$$

So,

$$\frac{d}{dx}\left(\tan^{-1} e^{x}\right) = \frac{e^{x}}{1+e^{2x}}.$$

*********** END **********