Question 15:

Find $A^2-5A+6I$ if $A=\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$

Answer

We have $A^2 = A \times A$

$$A^2 = AA=\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}2(2)+0(2)+1(1) & 2(0)+0(1)+1(-1) & 2(1)+0(3)+1(0)\\ 2(2)+1(2)+3(1) & 2(0)+1(1)+3(-1) & 2(1)+1(3)+3(0)\\ 1(2)+(-1)(2)+0(1) & 1(0)+(-1)(1)+0(-1) & 1(1)+(-1)(3)+0(0)\end{bmatrix}$$

$$=\begin{bmatrix}4+0+1 & 0+0-1 & 2+0+0\\ 4+2+3 & 0+1-3 & 2+3+0\\ 2-2+0 & 0-1+0 & 1-3+0\end{bmatrix}$$

$$=\begin{bmatrix}5 & -1 & 2\\ 9 & -2 & 5\\ 0 & -1 & -2\end{bmatrix}$$

$$\therefore A^2-5A+6I$$

$$=\begin{bmatrix}5 & -1 & 2\\ 9 & -2 & 5\\ 0 & -1 & -2\end{bmatrix}-5\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}+6\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}5 & -1 & 2\\ 9 & -2 & 5\\ 0 & -1 & -2\end{bmatrix}-\begin{bmatrix}10 & 0 & 5\\ 10 & 5 & 15\\ 5 & -5 & 0\end{bmatrix}+\begin{bmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{bmatrix}$$

$$=\begin{bmatrix}5-10 & -1-0 & 2-5\\ 9-10 & -2-5 & 5-15\\ 0-5 & -1+5 & -2-0\end{bmatrix}+\begin{bmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{bmatrix}$$

$$=\begin{bmatrix}-5 & -1 & -3\\ -1 & -7 & -10\\ -5 & 4 & -2\end{bmatrix}+\begin{bmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{bmatrix}$$

$$=\begin{bmatrix}-5+6 & -1+0 & -3+0\\ -1+0 & -7+6 & -10+0\\ -5+0 & 4+0 & -2+6\end{bmatrix}$$

$$=\begin{bmatrix}1 & -1 & -3\\ -1 & -1 & -10\\ -5 & 4 & 4\end{bmatrix}$$

Question 16:

If $A=\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$, prove that $A^3-6A^2+7A+2I=O$

Answer

$$A^2 = AA=\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}1+0+4 & 0+0+0 & 2+0+6\\ 0+0+2 & 0+4+0 & 0+2+3\\ 2+0+6 & 0+0+0 & 4+0+9\end{bmatrix}=\begin{bmatrix}5 & 0 & 8\\ 2 & 4 & 5\\ 8 & 0 & 13\end{bmatrix}$$

Now $A^3 = A^2 \cdot A$

$$=\begin{bmatrix}5 & 0 & 8\\ 2 & 4 & 5\\ 8 & 0 & 13\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}5+0+16 & 0+0+0 & 10+0+24\\ 2+0+10 & 0+8+0 & 4+4+15\\ 8+0+26 & 0+0+0 & 16+0+39\end{bmatrix}$$

$$\begin{bmatrix}21 & 0 & 34\end{bmatrix}$$

$$=\begin{bmatrix} 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

$\therefore A^3-6A^2+7A+2I$

$$=\begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}-6\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}+7\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}+2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}-\begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix}+\begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix}+\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$=\begin{bmatrix} 21+7+2 & 0+0+0 & 34+14+0 \\ 12+0+0 & 8+14+2 & 23+7+0 \\ 34+14+0 & 0+0+0 & 55+21+2 \end{bmatrix}-\begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix}$$

$$=\begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix}-\begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}=O$$

$\therefore A^3-6A^2+7A+2I=O$

Question 17:

If $A=\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find *k* so that $A^2=kA-2I$

Answer

$$A^2=A\cdot A=\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$=\begin{bmatrix} 3(3)+(-2)(4) & 3(-2)+(-2)(-2) \\ 4(3)+(-2)(4) & 4(-2)+(-2)(-2) \end{bmatrix}=\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

Now $A^2=kA-2I$

$$\Rightarrow\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}=k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}-2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}=\begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix}-\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}=\begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

Comparing the corresponding elements, we have:

$$\mathbf{3k-2=1}$$
$$\mathbf{\Rightarrow 3k=3}$$
$$\mathbf{\Rightarrow k=1}$$

Thus, the value of *k* is 1.

Question 18:

If $A=\begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and *I* is the identity matrix of order 2, show that $I+A=(I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Answer

<u>On the L.H.S.</u>

$I+A$

$$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}+\begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$$

$$=\begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix} \quad ...(1)$$

<u>On the R.H.S.</u>

$$(I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix} \right) \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha + \sin\alpha \tan\frac{\alpha}{2} & -\sin\alpha + \cos\alpha \tan\frac{\alpha}{2} \\ -\cos\alpha \tan\frac{\alpha}{2} + \sin\alpha & \sin\alpha \tan\frac{\alpha}{2} + \cos\alpha \end{bmatrix} \quad \ldots(2)$$

$$= \begin{bmatrix} 1-2\sin^2\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\tan\frac{\alpha}{2} & -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + \left(2\cos^2\frac{\alpha}{2} - 1\right)\tan\frac{\alpha}{2} \\ -\left(2\cos^2\frac{\alpha}{2} - 1\right)\tan\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\tan\frac{\alpha}{2} + 1 - 2\sin^2\frac{\alpha}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1-2\sin^2\frac{\alpha}{2} + 2\sin^2\frac{\alpha}{2} & -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} - \tan\frac{\alpha}{2} \\ -2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + \tan\frac{\alpha}{2} + 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} & 2\sin^2\frac{\alpha}{2} + 1 - 2\sin^2\frac{\alpha}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

Thus, from (1) and (2), we get L.H.S. = R.H.S.

Question 19:

A trust fund has Rs 30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs 30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of:

(a) Rs 1,800 (b) Rs 2,000

Answer

(a) Let Rs *x* be invested in the first bond. Then, the sum of money invested in the second bond will be Rs (30000 − *x*).

It is given that the first bond pays 5% interest per year and the second bond pays 7% interest per year.

Therefore, in order to obtain an annual total interest of Rs 1800, we have:

$$\begin{bmatrix} x & (30000-x) \end{bmatrix} \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 1800 \qquad \left[\text{S.I. for 1 year} = \frac{\text{Principal} \times \text{Rate}}{100} \right]$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000-x)}{100} = 1800$$
$$\Rightarrow 5x + 210000 - 7x = 180000$$
$$\Rightarrow 210000 - 2x = 180000$$
$$\Rightarrow 2x = 210000 - 180000$$
$$\Rightarrow 2x = 30000$$
$$\Rightarrow x = 15000$$

*********** END **********