Binary Operations Ex 3.1 Q2

(i) On $\mathbf{Z}^+$, $*$ is defined by $a * b = a - b$.
It is not a binary operation as the image of (1, 2) under $*$ is $1 * 2 = 1 - 2$
$= -1 \notin \mathbf{Z}^+$.

(ii) On $\mathbf{Z}^+$, $*$ is defined by $a * b = ab$.
It is seen that for each $a, b \in \mathbf{Z}^+$, there is a unique element ab in $\mathbf{Z}^+$.
This means that $*$ carries each pair (a, b) to a unique element $a * b = ab$ in $\mathbf{Z}^+$.
Therefore, $*$ is a binary operation.

(iii) On $\mathbf{R}$, $*$ is defined by $a * b = ab^2$.
It is seen that for each $a, b \in \mathbf{R}$, there is a unique element ab^2 in $\mathbf{R}$.
This means that $*$ carries each pair (a, b) to a unique element $a * b = ab^2$ in $\mathbf{R}$.
Therefore, $*$ is a binary operation.

(iv) On $\mathbf{Z}^+$, $*$ is defined by $a * b = |a - b|$.
It is seen that for each $a, b \in \mathbf{Z}^+$, there is a unique element $|a - b|$ in $\mathbf{Z}^+$.
This means that $*$ carries each pair (a, b) to a unique element $a * b = |a - b|$ in $\mathbf{Z}^+$.
Therefore, $*$ is a binary operation.

(v) On $\mathbf{Z}^+$, $*$ is defined by $a * b = a$.
$*$ carries each pair (a, b) to a unique element $a * b = a$ in $\mathbf{Z}^+$.
Therefore, $*$ is a binary operation.

(vi) on R, $*$ is defined by $a * b = a + 4b^2$
it is seen that for each element $a, b \in R$, there is unique element $a + 4b^2$ in R
This means that $*$ carries each pair (a, b) to a unique element $a * b = a + 4b^2$ in R.
Therefore, $*$ is a binary operation.

Binary Operations Ex 3.1 Q3

It is given that, $a * b = 2a + b - 3$
Now

$$\begin{aligned} 3 * 4 &= 2 \times 3 + 4 - 3 \\ &= 10 - 3 \\ &= 7 \end{aligned}$$

Binary Operations Ex 3.1 Q4

The operation $*$ on the set $A = \{1, 2, 3, 4, 5\}$ is defined as
$a * b =$ L.C.M. of a and b.

2*3 = L.C.M of 2 and 3 = 6. But 6 does not belong to the given set.
Hence, the given operation $*$ is not a binary operation.

Binary Operations Ex 3.1 Q5

We have,

$$S = \{a, b, c\}$$

We know that the total number of binary operation on a set S with n element is n^{n^2}

$\Rightarrow$ Total number of binary operation on $S = \{a, b, c\} = 3^{3^2} = 3^9$

Binary Operations Ex 3.1 Q6

We have,

$$S = \{a, b\}$$

The total number of binary operation on $S = \{a, b\}$ in $2^{2^2} = 2^4 = 16$

*********** END **********