



Trigonometric Ratios Ex 5.1 Q8

Answer :

Given: $3 \cot A = 4$

To check whether $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$ or not

$3 \cot A = 4$

Dividing by 3 on both sides,

We get,

$$\cot A = \frac{4}{3} \dots\dots (1)$$

By definition,

$$\cot A = \frac{1}{\tan A}$$

Therefore,

$$\cot A = \frac{1}{\frac{\text{Perpendicular side opposite to } \angle A}{\text{Base side adjacent to } \angle A}}$$

$$\cot A = \frac{\text{Base side adjacent to } \angle A}{\text{Perpendicular side opposite to } \angle A} \dots\dots (2)$$

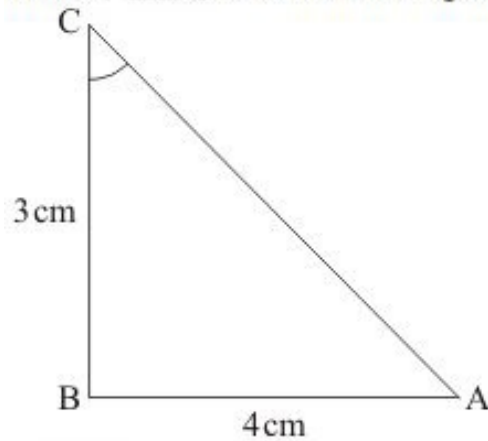
Comparing Equation (1) and (2)

We get,

Base side adjacent to $\angle A = 4$

Perpendicular side opposite to $\angle A = 3$

Hence, $\triangle ABC$ is as shown in figure below



In $\triangle ABC$, Hypotenuse is unknown

Hence, It can be found by using Pythagoras theorem

Therefore by applying Pythagoras theorem in $\triangle ABC$

We get

$$AC^2 = AB^2 + BC^2$$

Substituting values of sides from the above figure

$$AC^2 = 4^2 + 3^2$$

$$AC^2 = 16 + 9$$

$$AC^2 = 25$$

$$AC = \sqrt{25}$$

$$AC = 5$$

Hence, Hypotenuse = 5

To check whether $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$ or not

We get the values of $\tan A, \cos A, \sin A$

By definition,

$$\tan A = \frac{1}{\cot A}$$

Substituting the value $\cot A$ from Equation (1)

We get,

$$\tan A = \frac{1}{\frac{4}{3}}$$

$$\tan A = \frac{3}{4} \dots\dots (3)$$

Now by definition,

$$\sin A = \frac{\text{Perpendicular side opposite to } \angle A}{\text{Hypotenuse}}$$

$$\sin A = \frac{BC}{AC}$$

Substituting values of sides from the above figure

$$\sin A = \frac{3}{5} \dots\dots (4)$$

Now by definition,

$$\cos A = \frac{\text{Base side adjacent to } \angle A}{\text{Hypotenuse}}$$

$$\cos A = \frac{AB}{AC}$$

Substituting values of sides from the above figure

$$\cos A = \frac{4}{5} \dots\dots (5)$$

Now we first take L.H.S of Equation $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$

$$L.H.S = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

Substituting value of $\tan A$ from equation (3)

We get,

$$L.H.S = \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2}$$

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2}$$

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \left(\frac{3^2}{4^2}\right)}{1 + \left(\frac{3^2}{4^2}\right)}$$

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}}$$

Taking L.C.M on both numerator and denominator

We get,

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{16 - 9}{16 + 9}$$

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{7}{25} \dots\dots (6)$$

Now we take R.H.S of Equation $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$

$$R.H.S = \cos^2 A - \sin^2 A$$

Substituting value of $\sin A$ and $\cos A$ from equation (4) and (5)

We get,

$$R.H.S = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$\cos^2 A - \sin^2 A = \frac{4^2}{5^2} - \frac{3^2}{5^2}$$

$$\cos^2 A - \sin^2 A = \frac{16}{25} - \frac{9}{25}$$

$$\cos^2 A - \sin^2 A = \frac{16 - 9}{25}$$

$$\cos^2 A - \sin^2 A = \frac{7}{25} \dots\dots (7)$$

Comparing Equation (6) and (7)

We get,

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$$

$$\text{Answer: Yes } \frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$$

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