Differentials Errors and Approximation Ex14.1 Q9(xii)

Let $x = 25,\ x + \Delta x = 25.1$

$\Delta x = 25.1 - 25$

$\Delta x = 0.1$

Let $y = \dfrac{1}{\sqrt{x}}$

$\dfrac{dy}{dx} = \dfrac{2}{2x^{\frac{3}{2}}}$

$\left(\dfrac{dy}{dx}\right)_{x=25} = -\dfrac{1}{2(25)^{\frac{3}{2}}}$

$= -\dfrac{1}{250}$

$= -0.004$

Now,

$\Delta y = \left(\dfrac{dy}{dx}\right)_{x=25} \times (\Delta x)$

$= (-0.004)(0.1)$

$= -0.0004$

$\dfrac{1}{\quad} = v + \Delta v$

$\sqrt{25.1}$

$$= \frac{1}{\sqrt{x}} + (-0.0004)$$

$$= \frac{1}{\sqrt{25}} - 0.0004$$

$$= \frac{1}{5} - 0.0004$$

$$= 0.2 - 0.0004$$

$$\frac{1}{\sqrt{25.1}} = 0.1996$$

Differentials Errors and Approximation Ex14.1 Q9(xiii)

Let $x = \frac{\pi}{2}$, $x + \Delta x = \frac{22}{14}$

$$\Delta x = \left(\frac{22}{14} - \frac{\pi}{2}\right)$$

$$\Delta x = \sin x$$

Let $y = \sin x$

$$\frac{dy}{dx} = \cos x$$

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} = \cos\frac{\pi}{2}$$

$$\left(\frac{dy}{dx}\right)_{\frac{\pi}{2}} = 0$$

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} \times (\Delta x)$$

$$= 0 \times \left(\frac{22}{14} - \frac{\pi}{2}\right)$$

$$= 0$$

So,

$$\sin\left(\frac{22}{14}\right) = y + \Delta y$$

$$= \sin x + 0$$

$$= \sin\left(\frac{\pi}{2}\right)$$

$$\sin\left(\frac{22}{14}\right) = 1$$

Differentials Errors and Approximation Ex14.1 Q9(xiv)

Let $x = \frac{\pi}{3}$, $x + \Delta x = \frac{11\pi}{36}$

$$\Delta x = \frac{11\pi}{36} - \frac{\pi}{3}$$

$$= -\frac{\pi}{36}$$

$$= -\frac{22}{7 \times 36}$$

$$= -0.0873$$

Let $y = \cos x$

$$\frac{dy}{dx} = -\sin x$$

$$dx$$

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{3}} = -\sin\frac{\pi}{3}$$

$$= -\frac{\sqrt{3}}{2}$$

$$= -0.866$$

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{3}} \times (\Delta x)$$

$$= (-0.866)(-0.0873)$$

$$= 0.0756$$

$$\cos\left(\frac{11\pi}{36}\right) = y + \Delta y$$

$$= \cos x + (0.0756)$$

$$= \cos\frac{\pi}{3} + 0.0756$$

$$= \frac{1}{2} + 0.0756$$

$$= 0.5 + 0.0756$$

$$\cos\frac{11\pi}{36} = 0.7546$$

Differentials Errors and Approximation Ex14.1 Q9(xix)

Let $x = 36,\ x + \Delta x = 37$

$$\Delta x = 37 - 36$$
$$= 1$$

Let $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\left(\frac{dy}{dx}\right)_{x=36} = \frac{1}{2\sqrt{36}}$$
$$= \frac{1}{12}$$
$$= 0.0833$$

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=36} \times (\Delta x)$$
$$= (0.0833)(1)$$
$$= 0.0833$$

$$\sqrt{37} = y + \Delta y$$
$$= \sqrt{x} + 0.0833$$
$$= \sqrt{36} + 0.0833$$

$$\sqrt{37} = 6.0833$$

********** END **********