

Indefinite Integrals Ex 19.16 Q11

Let 
$$I = \int \frac{1}{x(x^6 + 1)} dx$$
  

$$= \int \frac{x^5}{x^6(x^6 + 1)} dx$$
Let  $x^6 = t$   

$$\Rightarrow 6x^5 dx = dt$$

$$\Rightarrow x^5 dx = \frac{dt}{6}$$

$$I = \frac{1}{6} \int \frac{dt}{t^2 + t}$$

$$= \frac{1}{6} \int \frac{dt}{t^2 + 2t} \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$= \frac{1}{6} \int \frac{dt}{\left(t + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$$
Let  $t + \frac{1}{2} = u$   

$$\Rightarrow dt = du$$

$$I = \frac{1}{6} \int \frac{du}{u^2 - \left(\frac{1}{2}\right)^2}$$

$$= \frac{1}{6} \times \frac{1}{2\left(\frac{1}{2}\right)} \log \left| \frac{u - \frac{1}{2}}{u + \frac{1}{2}} \right| + c$$

$$\int \sin c \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x - a}{x + a} \right| + c$$

$$I = \frac{1}{6} \log \left| \frac{t + \frac{1}{2} - \frac{1}{2}}{t + \frac{1}{2} + \frac{1}{2}} \right| + c$$

$$I = \frac{1}{6} \log \left| \frac{x^6}{x^6 + 1} \right| + C$$

Indefinite Integrals Ex 19.16 Q12

Let 
$$I = \int \frac{x}{x^4 - x^2 + 1} dx$$
  
Let  $X^2 = t$   
 $\Rightarrow 2x dx = dt$   
 $\Rightarrow x dx = \frac{dt}{2}$   
so,  $I = \frac{1}{2} \int \frac{dt}{t^2 - t + 1}$   
 $= \frac{1}{2} \int \frac{dt}{t^2 - 2t \times (\frac{1}{2}) + (\frac{1}{2})^2 - (\frac{1}{2})^2 + 1}$   
 $= \frac{1}{2} \int \frac{dt}{(t - \frac{1}{2})^2 + (\frac{3}{4})}$   
Let  $t - \frac{1}{2} = u$   
 $\Rightarrow dt = du$   
 $I = \frac{1}{2} \int \frac{du}{u^2 + (\frac{\sqrt{3}}{2})^2}$   
 $= \frac{1}{2} \times \frac{1}{(\frac{\sqrt{3}}{2})} \tan^{-1} \left( \frac{u}{\sqrt{\frac{3}{2}}} \right) + c$  [Since  $\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + c$ ]  
 $I = \frac{1}{\sqrt{3}} \tan^{-1} \left( \frac{t - \frac{1}{2}}{\sqrt{\frac{3}{2}}} \right) + c$ 

Indefinite Integrals Ex 19.16 Q13

Let 
$$I = \int \frac{x}{3x^4 - 18x^2 + 11} dx$$
  
 $= \frac{1}{3} \int \frac{x}{x^4 - 6x^2 + \frac{11}{3}} dx$   
Let  $x^2 = t$   
 $\Rightarrow 2x dx = dt$   
 $\Rightarrow x dx = \frac{dt}{2}$   
 $I = \frac{1}{3} \times \frac{1}{2} \int \frac{dt}{t^2 - 6t + \frac{11}{3}}$   
 $= \frac{1}{6} \int \frac{dt}{t^2 - 2t(3) + (3)^2 - (3)^2 + \frac{11}{3}}$   
 $= \frac{1}{6} \int \frac{dt}{(t - 3)^2 - (\frac{16}{3})}$   
Let  $t - 3 = u$   
 $\Rightarrow dt = du$   
 $I = \frac{1}{6} \int \frac{du}{u^2 - (\frac{4}{\sqrt{3}})^2}$   
 $= \frac{1}{6} \times \frac{1}{2(\frac{4}{\sqrt{3}})} \log \left| \frac{u - \frac{4}{\sqrt{3}}}{u + \frac{4}{\sqrt{3}}} \right| + c \left[ \text{Since } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x - a}{x + a} \right| + c \right]$   
 $I = \frac{\sqrt{3}}{48} \log \left| \frac{t - 3 - \frac{4}{\sqrt{3}}}{t - 3 + \frac{4}{\sqrt{3}}} \right| + c$ 

$$I = \frac{\sqrt{3}}{48} \log \left| \frac{x^2 - 3 - \frac{4}{\sqrt{3}}}{x^2 - 3 + \frac{4}{\sqrt{3}}} \right| + c$$

Indefinite Integrals Ex 19.16 Q14

To evaluate the following integral follow the steps:

Let  $e^x = t$  therefore  $e^x dx = dt$ 

Now

$$\int \frac{e^x}{(1+e^x)(2+e^x)} dx = \int \frac{dt}{(1+t)(2+t)}$$

$$= \int \frac{dt}{(1+t)} - \int \frac{dt}{(2+t)}$$

$$= \ln|1+t| - \ln|2+t| + c$$

$$= \ln\left|\frac{1+t}{2+t}\right| + c$$

$$= \ln\left|\frac{1+e^x}{2+e^x}\right| + c$$

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