Differentiation Ex 11.5 Q18(vii)

Let $y = (\cos x)^{x} + (\sin x)^{\frac{1}{x}}$

$y = e^{\log(\cos x)^{x}} + e^{\log(\sin)^{\frac{1}{x}}}$ $\qquad\qquad \left[\text{Since, } \log a^{b} = b\log a, e^{\log a} = a\right]$

$y = e^{x\log(\cos x)} + e^{\frac{1}{x}\log\sin x}$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}e^{x\log\cos x} + \frac{d}{dx}e^{\frac{1}{x}\log\sin x}$$

$$= e^{x\log\cos x} \times \frac{d}{dx}(x\log x) + e^{\frac{1}{x}\log\sin x}\frac{d}{dx}\left(\frac{1}{x}\log\sin x\right)$$

$$= e^{\log(\cos x)} \times \left[x\frac{d}{dx}\log\cos x + \log\cos x \times \frac{d}{dx}(x)\right] + e^{\log(\sin x)^{\frac{1}{x}}} \times \left[\frac{1}{x}\frac{d}{dx}\log\sin x + \log\sin x\frac{d}{dx}\left(\frac{1}{x}\right)\right]$$

$$= (\cos x)^{x}\left[x \times \left(\frac{1}{\cos x}\right)\frac{d}{dx}\cos x + \log\cos x\,(1)\right] + (\sin)^{\frac{1}{x}}\left[\frac{1}{x} \times \frac{1}{\sin x} \times \frac{d}{dx}(\sin x) + \log\sin x\left(-\frac{1}{x^{2}}\right)\right]$$

$$= (\cos x)^{x}\left[x\left(\frac{1}{\cos x}\right)(-\sin x) + \log\cos x\right] + (\sin x)^{\frac{1}{x}}\left[\frac{1}{x} \times \frac{1}{\sin x}(\cos x) - \frac{1}{x^{2}}\log\sin x\right]$$

$$\frac{dy}{dx} = (\cos x)^{x}[\log\cos x - x\tan x] + (\sin x)^{\frac{1}{x}}\left[\frac{\cot x}{x} - \frac{1}{x^{2}}\log\sin x\right]$$

Differentiation Ex 11.5 Q18(vii)

Let $y = x^{x^2-3} + (x-3)^{x^2}$

Also, let $u = x^{x^2-3}$ and $v = (x-3)^{x^2}$

$\therefore y = u + v$

Differentiating both sides with respect to x, we obtain

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad ...(1)$$

$u = x^{x^2-3}$

$\therefore \log u = \log\left(x^{x^2-3}\right)$

$\log u = \left(x^2-3\right)\log x$

Differentiating with respect to x, we obtain

$$\frac{1}{u}\cdot\frac{du}{dx} = \log x\cdot\frac{d}{dx}\left(x^2-3\right) + \left(x^2-3\right)\cdot\frac{d}{dx}(\log x)$$

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = \log x\cdot 2x + \left(x^2-3\right)\cdot\frac{1}{x}$$

$$\Rightarrow \frac{du}{dx} = x^{x^2-3}\cdot\left[\frac{x^2-3}{x} + 2x\log x\right]$$

Also,

$v = (x-3)^{x^2}$

$\therefore \log v = \log(x-3)^{x^2}$

$\Rightarrow \log v = x^2\log(x-3)$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v}\cdot\frac{dv}{dx}=\log(x-3)\cdot\frac{d}{dx}(x^2)+x^2\cdot\frac{d}{dx}[\log(x-3)]$$
$$\Rightarrow\frac{1}{v}\frac{dv}{dx}=\log(x-3)\cdot 2x+x^2\cdot\frac{1}{x-3}\cdot\frac{d}{dx}(x-3)$$
$$\Rightarrow\frac{dv}{dx}=v\left[2x\log(x-3)+\frac{x^2}{x-3}\cdot 1\right]$$
$$\Rightarrow\frac{dv}{dx}=(x-3)^{x^2}\left[\frac{x^2}{x-3}+2x\log(x-3)\right]$$

Substituting the expressions of $\frac{du}{dx}$ and $\frac{dv}{dx}$ in equation (1), we obtain

$$\frac{dy}{dx}=x^{x^2-3}\left[\frac{x^2-3}{x}+2x\log x\right]+(x-3)^{x^2}\left[\frac{x^2}{x-3}+2x\log(x-3)\right]$$

Here,

$$y=e^x+10^x+x^x$$
$$=e^x+10^x+e^{\log x^x}\qquad\left[\text{Since, } e^{\log_e a}=a, \log a^b=b\log a\right]$$
$$y=e^x+10^x+e^{x\log x}$$

Differentiating it with respect to x using product rule, chain rule,

$$\frac{dy}{dx}=\frac{d}{dx}(e^x)+\frac{d}{dx}(10^x)+\frac{d}{dx}(e^{x\log x})$$
$$=e^x+10^x\log 10+e^{x\log x}\frac{d}{dx}(x\log x)$$
$$=e^x+10^x\log 10+e^{x\log x}\left[x\times\frac{d}{dx}(\log x)+\log x\frac{d}{dx}(x)\right]$$
$$=e^x+10^x\log 10+e^{\log x^x}\left[x\left(\frac{1}{x}\right)+\log x\,(1)\right]$$
$$=e^x+10^x\log 10+x^x[1+\log x]$$
$$=e^x+10^x\log 10+x^x[\log e+\log x]\qquad\left[\text{Since, }\log_e e=1\right]$$

$$\frac{dy}{dx}=e^x+10^x\log 10+x^x(\log ex)\qquad\left[\text{Sicne }\log A+\log B=\log AB\right]$$

Differentiation Ex 11.5 Q20

Here,

$$y=x^n+n^x+x^x+n^n$$
$$y=x^n+n^n+e^{\log x^x}+n^n\qquad\left[\text{Since, } e^{\log_e a}=a\text{ and }\log a^b=b\log a\right]$$
$$y=x^n+n^x+e^{x\log x}+n^n$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx}=\frac{d}{dx}(x^n)+\frac{d}{dx}(n^x)+\frac{d}{dx}(e^{x\log x})+\frac{d}{dx}(n^n)$$
$$=nx^{n-1}+n^x\log n+e^{\log x^x}\left[d\frac{d}{dx}\log x+\log x\frac{d}{dx}(1)\right]$$
$$=nx^{n-1}+n^x\log n+x^x\left[x\left(\frac{1}{x}\right)+\log x\right]$$
$$=nx^{n-1}+n^x\log n+x^x[1+\log x]$$
$$=nx^{n-1}+n^x\log n+x^x[\log e+\log x]\qquad\left[\text{Since, }\log_e e=1\text{ and }\log A+\log B=\log(AB)\right]$$

$$\frac{dy}{dx}=nx^{n-1}+n^x\log n+x^x\log(ex)$$

*********** END **********