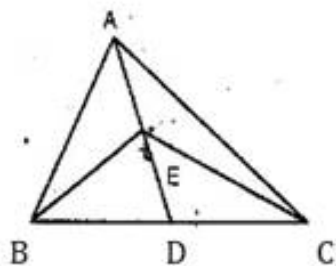




NCERT solutions for class-9 maths Areas of Parallelograms and Triangles Ex-9.3

Q1. In figure, E is any point on median AD of a $\triangle ABC$. Show that $\text{ar}(\triangle ABE) = \text{ar}(\triangle ACE)$.



Ans. In $\triangle ABC$, AD is a median.

$$\text{ar}(\triangle ABD) = \text{ar}(\triangle ACD) \dots\dots\dots(i)$$

[\because Median divides a \triangle into two \triangle s of equal area]

Again in $\triangle EBC$, ED is a median

$$\text{ar}(\triangle EBD) = \text{ar}(\triangle ECD) \dots\dots\dots(ii)$$

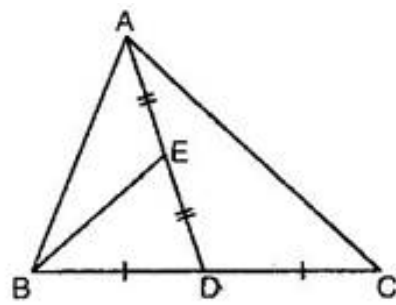
Subtracting eq. (ii) from (i),

$$\text{ar}(\triangle ABD) - \text{ar}(\triangle EBD) = \text{ar}(\triangle ACD) - \text{ar}(\triangle ECD)$$

$$\Rightarrow \text{ar}(\triangle ABE) = \text{ar}(\triangle ACE)$$

Q2. In a triangle ABC, E is the mid-point of median AD. Show that $\text{ar}(\triangle BED) = \frac{1}{4} \text{ar}(\triangle ABC)$.

Ans. Given: A $\triangle ABC$, AD is the median and E is the mid-point of median AD.



To prove: $\text{ar}(\triangle BED) = \frac{1}{4} \text{ar}(\triangle ABC)$

Proof: In $\triangle ABC$, AD is the median.

$$\therefore \text{ar}(\triangle ABD) = \text{ar}(\triangle ADC)$$

[\because Median divides a \triangle into two \triangle s of equal area]

$$\Rightarrow \text{ar}(\triangle ABD) = \frac{1}{2} \text{ar}(\triangle ABC) \dots\dots\dots(i)$$

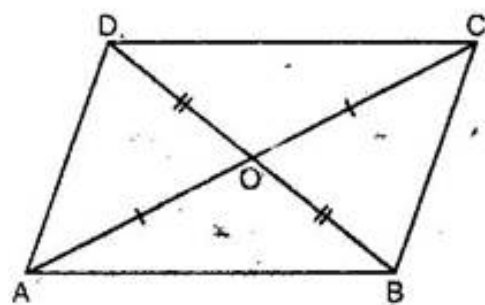
In $\triangle ABD$, BE is the median.

$$\therefore \text{ar}(\triangle BED) = \text{ar}(\triangle BAE)$$

$$\Rightarrow \text{ar}(\triangle BED) = \frac{1}{2} \text{ar}(\triangle ABD)$$

$$\Rightarrow \text{ar}(\triangle BED) = \frac{1}{2} \times \frac{1}{2} \text{ar}(\triangle ABC) = \frac{1}{4} \text{ar}(\triangle ABC)$$

Q3. Show that the diagonals of a parallelogram divide it into four triangles of equal area.



Ans. Let parallelogram be ABCD and its diagonals AC and BD intersect each other at O.

In $\triangle ABC$ and $\triangle ADC$,

$AB = DC$ [Opposite sides of a parallelogram]

$BC = AD$ [Opposite sides of a parallelogram]

And $AC = AC$ [Common]

$\therefore \triangle ABC \cong \triangle CDA$ [By SSS congruency]

Since, diagonals of a parallelogram bisect each other.

$\therefore O$ is the mid-point of bisection.

Now in $\triangle ADC$, DO is the median.

$\therefore \text{ar}(\triangle AOD) = \text{ar}(\triangle COD)$ (i)

[Median divides a triangle into two equal areas]

Similarly, in $\triangle ABC$, OB is the median.

$\therefore \text{ar}(\triangle AOB) = \text{ar}(\triangle BOC)$ (ii)

And in $\triangle AOB$ and $\triangle AOD$, AO is the median.

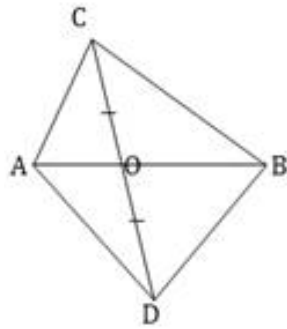
$\therefore \text{ar}(\triangle AOB) = \text{ar}(\triangle AOD)$ (iii)

From eq. (i), (ii) and (iii),

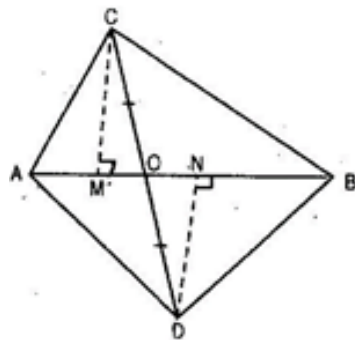
$$\text{ar}(\triangle AOB) = \text{ar}(\triangle AOD) = \text{ar}(\triangle BOC) = \text{ar}(\triangle COD)$$

Thus diagonals of parallelogram divide it into four triangles of equal area.

Q4. In figure, ABC and ABD are two triangles on the same base AB. If line-segment CD is bisected by AB at O, show that $\text{ar}(\triangle ABC) = \text{ar}(\triangle ABD)$.



Ans. Draw $CM \perp AB$ and $DN \perp AB$.



In $\triangle CMO$ and $\triangle DNO$,

$$\angle CMO = \angle DNO = 90^\circ \text{ [By construction]}$$

$$\angle COM = \angle DON \text{ [Vertically opposite]}$$

$$OC = OD \text{ [Given]}$$

$$\therefore \triangle CMO \cong \triangle DNO \text{ [By ASA congruency]}$$

$$\therefore AM = DN \text{ [By CPCT](i)}$$

$$\text{Now ar} (\triangle ABC) = \frac{1}{2} \times AB \times CM \text{(ii)}$$

$$\text{ar} (\triangle ADB) = \frac{1}{2} \times AB \times DN \text{(iii)}$$

Using eq. (i) and (iii),

$$\text{ar} (\triangle ADB) = \frac{1}{2} \times AB \times CM \text{(iv)}$$

From eq. (ii) and (iv),

$$\text{ar} (\triangle ABC) = \text{ar} (\triangle ADB)$$

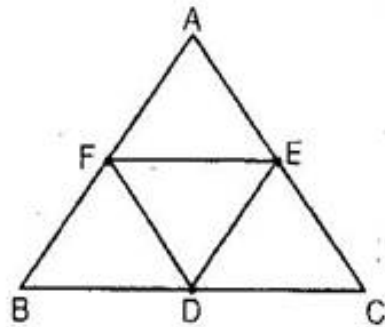
Q5. D, E and F are respectively the mid-points of the sides BC, CA and AB of a $\triangle ABC$. Show that:

(i) BDEF is a parallelogram.

$$\text{(ii) ar} (\triangle DEF) = \frac{1}{4} \text{ ar} (\triangle ABC)$$

$$\text{(iii) ar} (\triangle BDEF) = \frac{1}{2} \text{ ar} (\triangle ABC)$$

Ans. (i) F is the mid-point of AB and E is the mid-point of AC.



$$\therefore FE \parallel BC \text{ and } FE = \frac{1}{2} BC$$

[\because Line joining the mid-points of two sides of a triangle is parallel to the third and half of it]

$$\Rightarrow FE \parallel BD \text{ [BD is the part of BC]}$$

$$\text{And } FE = BD$$

Also, D is the mid-point of BC.

$$\therefore BD = \frac{1}{2} BC$$

$$\text{And } FE \parallel BC \text{ and } FE = BD$$

Again E is the mid-point of AC and D is the mid-point of BC.

$$\therefore DE \parallel AB \text{ and } DE = \frac{1}{2} AB$$

$$\Rightarrow DE \parallel AB \text{ [BF is the part of AB]}$$

$$\text{And } DE = BF$$

Again F is the mid-point of AB.

$$\therefore BF = \frac{1}{2} AB$$

$$\text{But } DE = \frac{1}{2} AB$$

$$\therefore DE = BF$$

Now we have $FE \parallel BD$ and $DE \parallel BF$

$$\text{And } FE = BD \text{ and } DE = BF$$

Therefore, BDEF is a parallelogram.

(ii) BDEF is a parallelogram.

[diagonals of parallelogram divides it in two triangles of equal area]

DCEF is also parallelogram.

$$\therefore \text{ar} (\triangle DEF) = \text{ar} (\triangle DEC) \dots\dots\dots(\text{ii})$$

Also, AEDF is also parallelogram.

$$\therefore \text{ar} (\triangle AFE) = \text{ar} (\triangle DEF) \dots\dots\dots(\text{iii})$$

From eq. (i), (ii) and (iii),

$$\text{ar} (\triangle DEF) = \text{ar} (\triangle BDF) = \text{ar} (\triangle DEC) = \text{ar} (\triangle AFE) \dots\dots\dots(\text{iv})$$

$$\text{Now, ar} (\triangle ABC) = \text{ar} (\triangle DEF) + \text{ar} (\triangle BDF) + \text{ar} (\triangle DEC) + \text{ar} (\triangle AFE) \dots\dots\dots(\text{v})$$

$$\Rightarrow \text{ar} (\triangle ABC) = \text{ar} (\triangle DEF) + \text{ar} (\triangle DEF) + \text{ar} (\triangle DEF) + \text{ar} (\triangle DEF)$$

[Using (iv) & (v)]

$$\Rightarrow \text{ar} (\triangle ABC) = 4 \times \text{ar} (\triangle DEF)$$

$$\Rightarrow \text{ar} (\triangle DEF) = \frac{1}{4} \text{ar} (\triangle ABC)$$

$$\text{(iii) ar} (\parallel \text{gm BDEF}) = \text{ar} (\triangle BDF) + \text{ar} (\triangle DEF) = \text{ar} (\triangle DEF) + \text{ar} (\triangle DEF) \text{ [Using (iv)]}$$

$$\Rightarrow \text{ar} (\parallel \text{gm BDEF}) = 2 \text{ar} (\triangle DEF)$$

$$\Rightarrow \text{ar} (\parallel \text{gm BDEF}) = 2 \times \frac{1}{4} \text{ar} (\triangle ABC)$$

$$\Rightarrow \text{ar} (\parallel \text{gm BDEF}) = \frac{1}{2} \text{ar} (\triangle ABC)$$

***** END *****