Indefinite Integrals Ex 19.9 Q45

Let $\quad I = \int \frac{e^{\sqrt{x}} \cos\left(e^{\sqrt{x}}\right)}{\sqrt{x}} dx \text{ - - - - - - } (i)$

Let $\quad e^{\sqrt{x}} = t \qquad$ then,

$d\left(e^{\sqrt{x}}\right) = dt$

$\Rightarrow \quad e^{\sqrt{x}} \frac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow \quad \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2dt$

Putting $e^{\sqrt{x}} = t$ and $\frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2dt$ in equation (i),

we get

$$I = \int \cos t \times 2dt$$
$$= 2\int \cos t\, dt$$
$$= 2\sin t + c$$
$$= 2\sin\left(e^{\sqrt{x}}\right) + c$$

$\therefore \quad I = 2\sin\left(e^{\sqrt{x}}\right) + c$

Indefinite Integrals Ex 19.9 Q46

Let $\quad I = \int \frac{\cos^5 x}{\sin x} dx \text{ ----(i)}$

Let $\quad \sin x = t \qquad$ then,

$d(\sin x) = dt$

$\Rightarrow \quad \cos x\, dx = dt$

$\Rightarrow \quad dx = \frac{dt}{\cos x}$

Putting $\sin x = t$ and $dx = \frac{dt}{\cos x}$ in equation (i),

we get

$$I = \int \frac{\cos^5 x}{t} \times \frac{dt}{\cos x}$$
$$= \int \frac{\cos^4 x}{t} dt$$
$$= \int \frac{\left(1 - \sin^2 x\right)^2}{t} dt$$
$$= \int \frac{\left(1 - t^2\right)^2}{t} dt$$
$$= \int \frac{1 + t^4 - 2t^2}{t} dt$$
$$= \int \frac{1}{t} dt + \int \frac{t^4}{t} dt - 2\int \frac{t^2}{t} dt$$
$$= \log|t| + \frac{t^4}{4} - \frac{2t^2}{2} + c$$
$$= \log|\sin x| + \frac{\sin^4 x}{4} - \sin^2 x + c$$

$$\therefore \quad I = \frac{1}{4}\sin^4 x - \sin^2 x + \log|\sin x| + c$$

Indefinite Integrals Ex 19.9 Q47

Let $\sqrt{x} = t$

$\Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow \frac{1}{\sqrt{x}} dx = 2dt$

Therefore,

$\int \frac{\sin \sqrt{x}}{\sqrt{x}}\, dx$

$= 2\int \sin t\, dt$

$= -2 \cos t + C$

$= -2 \cos \sqrt{x} + C$

Indefinite Integrals Ex 19.9 Q48

Let $\quad I = \int \frac{(x+1)e^x}{\sin^2(xe^x)} dx$ - - - - - (i)

Let $\quad xe^x = t$ then,

$d(xe^x) = dt$

$\Rightarrow \quad (xe^x + e^x)dx = dt$

$\Rightarrow \quad (x+1)e^x dx = dt$

Putting $xe^x = t$ and $(x+1)e^x dx = dt$ in equation (i), we get

$$I = \int \frac{dt}{\sin^2 t}$$
$$= \int \operatorname{cosec}^2 t \, dt$$
$$= -\cot t + c$$
$$= -\cot(xe^x) + c$$

$\therefore \quad I = -\cot(xe^x) + c$

Indefinite Integrals Ex 19.9 Q49

Let $\quad I = \int 5^{x+\tan^{-1}x}\left(\frac{x^2+2}{x^2+1}\right)dx$ - - - - - (i)

Let $\quad x + \tan^{-1}x = t \qquad$ then,

$$d\left(x + \tan^{-1}x\right) = dt$$

$$\Rightarrow \quad \left(1+\frac{1}{1+x^2}\right)dx = dt$$

$$\Rightarrow \quad \left(\frac{1+x^2+1}{1+x^2}\right)dx = dt$$

$$\Rightarrow \quad \frac{\left(x^2+2\right)}{\left(x^2+1\right)}dx = dt$$

Putting $x + \tan^{-1}x = t$ and $\left(\frac{x^2+2}{x^2+1}\right)dx = dt$ in equation (i),

we get

$$I = \int 5^t\,dt$$

$$= \frac{5^t}{\log 5} + c$$

$$= \frac{5^{x+\tan^{-1}x}}{\log 5} + c$$

$$\therefore \quad I = \frac{5^{x+\tan^{-1}x}}{\log 5} + c$$

*********** END **********