Maxima and Minima 18.4 Q2

Let $f(x) = 2x^3 - 24x + 107$.

$$\therefore f'(x) = 6x^2 - 24 = 6(x^2 - 4)$$

Now,

$$f'(x) = 0 \Rightarrow 6(x^2 - 4) = 0 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$$

We first consider the interval $[1, 3]$.

Then, we evaluate the value of f at the critical point $x = 2 \in [1, 3]$ and at the end points of the interval $[1, 3]$.

$f(2) = 2(8) - 24(2) + 107 = 16 - 48 + 107 = 75$

$f(1) = 2(1) - 24(1) + 107 = 2 - 24 + 107 = 85$

$f(3) = 2(27) - 24(3) + 107 = 54 - 72 + 107 = 89$

Hence, the absolute maximum value of $f(x)$ in the interval $[1, 3]$ is 89 occurring at $x = 3$.

Next, we consider the interval $[-3, -1]$.

Evaluate the value of f at the critical point $x = -2 \in [-3, -1]$ and at the end points of the interval $[1, 3]$.

$f(-3) = 2(-27) - 24(-3) + 107 = -54 + 72 + 107 = 125$

Maxima and Minima 18.4 Q3

$$f(x) = \cos^2 x + \sin x$$
$$f'(x) = 2\cos x(-\sin x) + \cos x$$
$$= -2\sin x\cos x + \cos x$$

Now, $f'(x) = 0$

$$\Rightarrow 2\sin x\cos x = \cos x \Rightarrow \cos x(2\sin x - 1) = 0$$
$$\Rightarrow \sin x = \frac{1}{2} \text{ or } \cos x = 0$$
$$\Rightarrow x = \frac{\pi}{6}, \text{ or } \frac{\pi}{2} \text{ as } x \in [0, \pi]$$

Now, evaluating the value of f at critical points $x = \frac{\pi}{2}$ and $x = \frac{\pi}{6}$ and at the end points of the interval $[0, \pi]$ (i.e., at $x = 0$ and $x = \pi$), we have:

$$f\left(\frac{\pi}{6}\right) = \cos^2\frac{\pi}{6} + \sin\frac{\pi}{6} = \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{1}{2} = \frac{5}{4}$$
$$f(0) = \cos^2 0 + \sin 0 = 1 + 0 = 1$$
$$f(\pi) = \cos^2\pi + \sin\pi = (-1)^2 + 0 = 1$$
$$f\left(\frac{\pi}{2}\right) = \cos^2\frac{\pi}{2} + \sin\frac{\pi}{2} = 0 + 1 = 1$$

Hence, the absolute maximum value of f is $\frac{5}{4}$ occurring at $x = \frac{\pi}{6}$ and the absolute minimum value of f is 1 occurring at $x = 0, \frac{\pi}{2}$, and π.

Maxima and Minima 18.4 Q4

We have

$$f(x) = 12x^{\frac{4}{3}} - 6x^{\frac{1}{3}}$$

$$\therefore \quad f'(x) = 16x^{\frac{1}{3}} - \frac{2}{x^{\frac{2}{3}}} = \frac{2(8x-1)}{x^{\frac{2}{3}}}$$

Thus, $f'(x) = 0$

$$\Rightarrow \quad x = \frac{1}{8}$$

Further note that $f'(x)$ is not defined at $x = 0$.

So, the critical points are $x = 0$ and $x = \frac{1}{8}$.

Evaluating the value of f at critical points $x = 0, \frac{1}{8}$ and at end points of the interval $x = -1$ and $x = 1$

$f(-1) = 12(-1)^{4/3} - 6(-1)^{1/3} = 18$

$f(0) = 12(0) - 6(0) = 0$

$f\left(\frac{1}{8}\right) = 12\left(\frac{1}{8}\right)^{4/3} - 6\left(\frac{1}{8}\right)^{1/3} = \frac{-9}{4}$

$f(1) = 12(1)^{4/3} - 6(1)^{1/3} = 6$

Hence we conclude that absolute maximum value of f is 18 at $x=-1$

and absolute minimum value of f is $\frac{-9}{4}$ at $x = \frac{1}{8}$.

Maxima and Minima 18.4 Q5

Given,

$$f(x) = 2x^3 - 15x^2 + 36x + 1$$

$$\therefore \quad f'(x) = 6x^2 - 30x + 36 = 6\left(x^2 - 5x + 6\right) = 6(x-2)(x-3)$$

Note that $f'(x) = 0$ gives $x = 2$ and $x = 3$

We shall now evaluate the value of f at these points and at the end points of the interval [1,5], i.e at $x=$ 1, 2, 3 and 5

At $x = 1$, $f(1) = 2\left(1^3\right) - 15\left(1^2\right) + 36(1) + 1 = 24$

At $x = 2$, $f(2) = 2\left(2^3\right) - 15\left(2^2\right) + 36(2) + 1 = 29$

At $x = 3$, $f(3) = 2\left(3^3\right) - 15\left(3^2\right) + 36(3) + 1 = 28$

At $x = 5$, $f(5) = 2\left(5^3\right) - 15\left(5^2\right) + 36(5) + 1 = 56$

Thus we conclude that the absolute maximum value of f on [1,5] is 56, occurring at $x=5$, and absolute minimum value of f on [1,5] is 24 which occurs at $x=1$.

*********** END **********