Indefinite Integrals Ex 19.30 Q47

Consider the integral

$$I=\int \frac{1}{x\left(x^3+8\right)}dx$$

Rewriting the above integrand, we have,

$$I=\int \frac{x^2}{x^3\left(x^3+8\right)}dx$$

$$=\frac{1}{3}\int \frac{3x^2}{x^3\left(x^3+8\right)}dx$$

Now substituting $x^3 = t$, we have,

$3x^2 dx = dt$

$$\Rightarrow I = \frac{1}{3}\int \frac{dt}{t\left(t+8\right)}$$

Let us separata the integrand by partial fractions.

Thus,

$$\frac{1}{t\left(t+8\right)} = \frac{A}{t} + \frac{B}{t+8}$$

$$\Rightarrow \frac{1}{t\left(t+8\right)} = \frac{A\left(t+8\right)+Bt}{t\left(t+8\right)}$$

$$\Rightarrow 1 = A\left(t+8\right)+Bt$$

$$\Rightarrow 1 = At + 8A + Bt$$

Comparing the coefficients, we have,

$A+B=0$ and $8A=1$

$$\Rightarrow A = \frac{1}{8} \text{ and } B=-\frac{1}{8}$$

Therefore,

$$I = \frac{1}{3}\int \frac{dt}{t(t+8)}$$

$$= \frac{1}{3}\int \left[\frac{\frac{1}{8}}{t} - \frac{\frac{1}{8}}{t+8}\right] dt$$

$$= \frac{1}{3} \times \frac{1}{8}\int \frac{dt}{t} dt - \frac{1}{3} \times \frac{1}{8}\int \frac{dt}{t+8}$$

$$= \frac{1}{24}\log t - \frac{1}{24} \times \log(t+8) + C$$

$$= \frac{1}{24}\log x^3 - \frac{1}{24} \times \log(x^3+8) + C$$

$$= \frac{3}{24}\log x - \frac{1}{24} \times \log(x^3+8) + C$$

$$= \frac{1}{8}\log x - \frac{1}{24} \times \log(x^3+8) + C$$

Indefinite Integrals Ex 19.30 Q48

Let $\dfrac{3}{(1-x)(1+x^2)} = \dfrac{A}{1-x} + \dfrac{Bx+C}{1+x^2}$

$$\Rightarrow \quad 3 = A(1+x^2) + (Bx+C)(1-x)$$

$$= (A-B)x^2 + (B-C)x + (A+C)$$

Equating similar terms, we get,

$A - B = 0,\ B - C = 0,\ A + C = 3$

Solving we get,

$A = C = \dfrac{3}{2}$ and $B = \dfrac{3}{2}$

Thus,

$$I = \frac{3}{2}\int \frac{dx}{1-x} + \frac{3}{2}\int \frac{xdx}{1+x^2} + \frac{3}{2}\int \frac{dx}{1+x^2}$$

$$= -\frac{3}{2}\log|1-x| + \frac{3}{2}\log\left|1+x^2\right| + \frac{3}{2}\tan^{-1}x + c$$

$$I = \frac{3}{4}\left[\log\left|\frac{1+x^2}{(1-x)^2}\right| + 2\tan^{-1}x + c\right]$$

Indefinite Integrals Ex 19.30 Q49

Let

$$\sin x = t$$

$$\Rightarrow \quad \cos x = dt$$

$$\therefore \quad \int \frac{\cos x}{(1-\sin x)^3 (2+\sin x)} = \int \frac{1}{(1-t)^3 (2+t)} dt$$

Let $f(t) = \dfrac{1}{(1-t)^3 (2+t)}$

Then, suppose

$$\frac{1}{(1-t)^3 (2+t)} = \frac{A}{1-t} + \frac{B}{(1-t)^2} + \frac{C}{(1-t)^3} + \frac{D}{(2+t)}$$

$$\Rightarrow \quad 1 = A(1-t)^2 (2+t) + B(1-t)(2+t) + C(2+t) + D(1-t)^3$$

Put $t = 1$

$$1 = 3C$$

$$\Rightarrow \quad C = \frac{1}{3}$$

Put $t = -2$

$$1 = 27D$$

$$\Rightarrow \quad D = \frac{1}{27}$$

Similarly, we can find that $A = \dfrac{-1}{27}$ and $B = \dfrac{+1}{9}$

$$\therefore \quad \int \frac{1}{(1-t)^3 (2+t)} dt = \frac{-1}{27} \int \frac{1}{1-t} dt + \frac{1}{9} \int \frac{dt}{(1-t)^2} + \frac{1}{3} \int \frac{dt}{(1-t)^3} + \frac{1}{27} \int \frac{dt}{2+t}$$

$$= \frac{-1}{27} \log|1-t| + \frac{1}{9(1-t)} + \frac{1}{6(1-t)^2} + \frac{1}{27} \log|2+t| + c$$

Putting $t = \sin x$, we get

$$\int \frac{\cos x}{(1-\sin x)^3 (2+\sin x)} dx$$

$$= \frac{-1}{27} \log|1-\sin x| + \frac{1}{9(1-\sin x)} + \frac{1}{6(1-\sin x)^2} + \frac{1}{27} \log|2+\sin x| + C$$

Indefinite Integrals Ex 19.30 Q50

Consider the integral

$$I=\int \frac{2x^2+1}{x^2\left(x^2+4\right)}dx$$

Now let us separate the fraction $\frac{2x^2+1}{x^2\left(x^2+4\right)}$ through partial fractions.

Substitute $x^2 = t$, then

$$\frac{2x^2+1}{x^2\left(x^2+4\right)} = \frac{2t+1}{t\left(t+4\right)}$$

$$\Rightarrow \frac{2t+1}{t\left(t+4\right)} = \frac{A}{t} + \frac{B}{t+4}$$

$$\Rightarrow \frac{2t+1}{t\left(t+4\right)} = \frac{A\left(t+4\right)+Bt}{t\left(t+4\right)}$$

$$\Rightarrow 2t+1 = A\left(t+4\right)+Bt$$

$$\Rightarrow 2t+1 = At+4A+Bt$$

Comparing the coefficients, we have,

A+B=2 and 4A=1

$$\Rightarrow A=\frac{1}{4} \text{ and } B=\frac{7}{4}$$

$$\Rightarrow \frac{2x^2+1}{x^2\left(x^2+4\right)} = \frac{1}{4x^2} + \frac{7}{4\left(x^2+4\right)}$$

Thus, we have,

$$I= \int \frac{2x^2+1}{x^2\left(x^2+4\right)}dx$$

$$= \frac{1}{4}\int \frac{dx}{x^2} + \frac{7}{4}\int \frac{\mathrm{dx}}{\left(x^2+4\right)}$$

$$= -\frac{1}{4x} + \frac{7}{4} \times \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

$$= -\frac{1}{4x} + \frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right) + C$$

*********** END **********