



Indefinite Integrals Ex 19.26 Q14

$$\text{Let } I = \int \frac{\sqrt{1 - \sin x}}{1 + \cos x} e^{-\frac{x}{2}} dx$$

$$\text{Put } \frac{x}{2} = t$$

$$\Rightarrow x = 2t$$

$$dx = 2dt$$

$$\therefore \int \frac{\sqrt{1 - \sin x}}{1 + \cos x} e^{-\frac{x}{2}} dx$$

$$= 2 \int \frac{\sqrt{1 - \sin 2t}}{1 + \cos 2t} e^{-t} dt \quad \left[\because \sin^2 t + \cos^2 t = 1 \right]$$

$$= 2 \int \frac{\sqrt{\sin^2 t + \cos^2 t - 2 \sin t \cos t}}{1 + \cos 2t} e^{-t} dt$$

$$= 2 \int \frac{\sqrt{(\cos t - \sin t)^2}}{2 \cos^2 t} e^{-t} dt$$

$$= 2 \int \frac{(\cos t - \sin t)}{2 \cos^2 t} e^{-t} dt$$

$$= \int (\sec t - \tan t \sec t) e^{-t} dt$$

$$= \int \sec t e^{-t} dt - \int \tan t \sec t e^{-t} dt$$

Integrating by parts

$$= e^{-t} \sec t + \int e^{-t} \frac{d}{dt} (\sec t) dt - \int \tan t \sec t e^{-t} dt$$

$$= -e^{-t} \sec t + \int e^{-t} \sec t \tan t dt - \int \sec t \tan t e^{-t} dt$$

$$= -e^{-t} \sec t + c$$

Putting the value of t

$$= -e^{-\frac{x}{2}} \sec \frac{x}{2} + c$$

Indefinite Integrals Ex 19.26 Q15

We have,

$$I = \int e^x \left(\log x + \frac{1}{x} \right) dx$$

We know that

$$\int e^{ax} \left(af(x) + f'(x) \right) dx = e^{ax} f(x) + c$$

$$\text{Here } f(x) = \log x \text{ and } f'(x) = \frac{1}{x}$$

$$\therefore \int e^x \left(\log x + \frac{1}{x} \right) dx = e^x \log x + c$$

Indefinite Integrals Ex 19.26 Q16

We have,

$$\begin{aligned} I &= \int e^x \left(\log x + \frac{1}{x^2} \right) dx \\ &= \int e^x \left(\log x + \frac{1}{x} - \frac{1}{x} + \frac{1}{x^2} \right) dx \\ &= \int e^x \left(\log x - \frac{1}{x} \right) dx + \int e^x \left(\frac{1}{x} + \frac{1}{x^2} \right) dx \end{aligned}$$

Integrating by parts

$$\begin{aligned} &= e^x \left(\log x - \frac{1}{x} \right) - \int e^x \frac{d}{dx} \left(\log x - \frac{1}{x} \right) dx + \int e^x \left(\frac{1}{x} + \frac{1}{x^2} \right) dx \\ &= e^x \left(\log x - \frac{1}{x} \right) - \int e^x \left(\frac{1}{x} + \frac{1}{x^2} \right) dx + \int e^x \left(\frac{1}{x} + \frac{1}{x^2} \right) dx \\ &= e^x \left(\log x - \frac{1}{x} \right) + c \end{aligned}$$

Indefinite Integrals Ex 19.26 Q17

We have,

$$\begin{aligned} I &= \int \frac{e^x}{x} \left\{ x (\log x)^2 + 2 \log x \right\} dx \\ &= \int e^x \left\{ (\log x)^2 + \frac{2}{x} \log x \right\} dx \\ &= \int e^x (\log x)^2 + 2 \int \frac{e^x}{x} \log x dx \end{aligned}$$

Integrating by parts

$$\begin{aligned} &= e^x (\log x)^2 - \int e^x \frac{d}{dx} (\log x)^2 dx + 2 \int e^x \frac{1}{x} \log x dx \\ &= e^x (\log x)^2 - \int e^x \frac{2 \log x}{x} dx + 2 \int e^x \frac{\log x}{x} dx \\ &= e^x (\log x)^2 + c \end{aligned}$$

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