Solution of Simultaneous Linear Equations Ex 8.1 Q17

Let x, y and z be the prize amount per student for sincerity, truthfulness and helpfulness respectively. As per the data in the question, we get

$3x + 2y + z = 1600$

$4x + y + 3z = 2300$

$x + y + z = 900$

The above three simulataneous equations can be written in matrix form as

$$\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix} \quad \ldots(1)$$

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$|A| = 3(-2) - 2(1) + 1(3) = -5$

$$cofA = \begin{bmatrix} -2 & -1 & 3 \\ -1 & 2 & -1 \\ 5 & -5 & -5 \end{bmatrix}$$

$$adjA = (cofA)^T = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{adjA}{|A|} = \frac{\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}}{-5}$$

From (1)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}}{-5} \begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix} \begin{bmatrix} -320 \\ -460 \\ -180 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 200 \\ 300 \\ 400 \end{bmatrix}$$

Excellence in extra-curricular activities should be another value considered for an award.

Solution of Simultaneous Linear Equations Ex 8.1 Q18

x, y and z be prize amount per student for Discipline, Politeness and Punctuality respectively.
As per the data in the question, we get

3x+2y+z=1000

4x+y+3z=1500

x+y+z=600

The above three simulataneous equations can be written in matrix form as

$$\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1000 \\ 1500 \\ 600 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1000 \\ 1500 \\ 600 \end{bmatrix} \quad ...(1)$$

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$|A| = 3(-2) - 2(1) + 1(3) = -5$$

$$cofA = \begin{bmatrix} -2 & -1 & 3 \\ -1 & 2 & -1 \\ 5 & -5 & -5 \end{bmatrix}$$

$$adjA = (cofA)^T = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{adjA}{|A|} = \frac{\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}}{-5}$$

From (1)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}}{-5} \begin{bmatrix} 1000 \\ 1500 \\ 600 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix} \begin{bmatrix} -200 \\ -300 \\ -120 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 100 \\ 200 \\ 300 \end{bmatrix}$$

********** END *********