Differentiation Ex 11.7 Q13

Here,

$$x = \frac{1-t^2}{1+t^2}$$

Differentiating it with respect to t using quotient rule,

$$\frac{dx}{dt} = \left[\frac{\left(1+t^2\right)\frac{d}{dt}\left(1-t^2\right) - \left(1-t^2\right)\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{\left(1+t^2\right)(-2t) - \left(1-t^2\right)(2t)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{-2t - 2t^3 - 2t + 2t^3}{\left(1+t^2\right)^2}\right]$$

$$\frac{dx}{dt} = \left(\frac{-4t}{\left(1+t^2\right)^2}\right) \qquad \text{---(i)}$$

And, $y = \dfrac{2t}{1+t^2}$

Differentiating it with respect to t using quotient rule,

$$\frac{dy}{dt} = \left[\frac{\left(1+t^2\right)\frac{d}{dt}(2t) - (2t)\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right]$$

$$\frac{dy}{dt} = \left[\frac{\left(1+t^2\right)(2) - (2t)(2t)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{2 + 2t^2 - 4t^2}{\left(1+t^2\right)^2}\right]$$

$$\frac{dy}{dt} = \frac{2\left(1-t^2\right)}{\left(1+t^2\right)^2} \qquad \text{---(ii)}$$

Differentiation Ex 11.7 Q14 Here, x= 2cosθ - cos2θ

Differentiating it with respect to θ using chain rule,

$$\frac{dx}{d\theta} = 2(-\sin\theta) - (-\sin 2\theta)\frac{d}{d\theta}(2\theta)$$
$$= -2\sin\theta + 2\sin 2\theta$$
$$\frac{dx}{d\theta} = 2(\sin 2\theta - \sin\theta) \qquad \text{---(i)}$$

And, $y = 2\sin\theta - \sin 2\theta$

Differentiating it with respect to θ using chain rule,

$$\frac{dy}{d\theta} = 2\cos\theta - \cos 2\theta\frac{d}{d\theta}(2\theta)$$
$$= 2\cos\theta - \cos 2\theta(2)$$
$$= 2\cos\theta - 2\cos 2\theta$$
$$\frac{dy}{d\theta} = 2(\cos\theta - \cos 2\theta) \qquad \text{---(ii)}$$

Dividing equation (ii) by equation (i),

$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2(\cos\theta - \cos 2\theta)}{2(\sin 2\theta - \sin\theta)}$$
$$= \frac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta}$$
$$\frac{dy}{dx} = \frac{-2\sin\left(\frac{\theta+2\theta}{2}\right)\sin\left(\frac{\theta-2\theta}{2}\right)}{2\cos\left(\frac{2\theta+\theta}{2}\right)\sin\left(\frac{2\theta-\theta}{2}\right)} \qquad \left[\begin{aligned}\text{Since, } \sin A - \sin B &= 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\\ \cos A - \cos B &= -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\end{aligned}\right]$$
$$= \frac{-\sin\left(\frac{3\theta}{2}\right)\left(\sin\left(\frac{-\theta}{2}\right)\right)}{\cos\left(\frac{3\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)}$$

$$= \frac{-\sin\left(\frac{3\theta}{2}\right)\left(-\sin\frac{\theta}{2}\right)}{\cos\left(\frac{3\theta}{2}\right)\left(\sin\frac{\theta}{2}\right)}$$
$$= \frac{\sin\left(\frac{3\theta}{2}\right)}{\cos\left(\frac{3\theta}{2}\right)}$$

$$\frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right)$$

Differentiation Ex 11.7 Q15

Here,

$$x = e^{\cos 2t}$$

Differentiating it with respect to t using chain rule,

$$\frac{dx}{dt} = \frac{d}{dt}\left(e^{\cos 2t}\right)$$
$$= e^{\cos 2t}\frac{d}{dt}(\cos 2t)$$
$$= e^{\cos 2t}(-\sin 2t)\frac{d}{dt}(2t)$$
$$= -\sin 2te^{\cos 2t}(2)$$
$$\frac{dx}{dt} = -2\sin 2te^{\cos 2t} \qquad \text{---(i)}$$

And, $y = e^{\sin 2t}$

Differentiating it with respect to t using chain rule,

$$\frac{dy}{dt} = \frac{d}{dt}\left(e^{\sin 2t}\right)$$
$$= e^{\sin 2t}\frac{d}{dt}(\sin 2t)$$
$$= e^{\sin 2t}(\cos 2t)\frac{d}{dt}(2t)$$
$$= e^{\sin 2t}(\cos 2t)(2)$$
$$\frac{dy}{dt} = 2\cos 2te^{\sin 2t} \qquad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2\cos 2te^{\sin 2t}}{-2\sin 2te^{\cos 2t}}$$

$$\frac{dy}{dx} = -\frac{y\log x}{x\log y} \qquad \left[\begin{aligned}\text{Since, } x &= e^{\cos 2t} \Rightarrow \log x = \cos 2t\\ y &= e^{\sin 2t} \Rightarrow \log y = \sin 2t\end{aligned}\right]$$

Differentiation Ex 11.7 Q16

Here,

$$x = \cos t$$

Differentiating it with respect to t,

$$\frac{dx}{dt} = \frac{d}{dt}(\cos t)$$

$$\frac{dx}{dt} = -\sin t \qquad \text{---(i)}$$

and, $y = \sin t$

Differentiating it with respect to t,

$$\frac{dy}{dt} = \frac{d}{dt}(\sin t)$$

$$\frac{dy}{dt} = \cos t \qquad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t}{-\sin t}$$

$$\frac{dy}{dx} = -\cot t$$

$$\left(\frac{dy}{dx}\right) = -\cot\left(\frac{2\pi}{3}\right)$$

$$= -\cot\left(\pi - \frac{\pi}{3}\right)$$

$$= -\left[-\cot\left(\frac{\pi}{3}\right)\right]$$

$$= \cot\left(\frac{\pi}{3}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{3}}$$

*********** END **********