Q21 : $\frac{2+\sin 2x}{1+\cos 2x}e^x$

Answer :

$$I=\int\left(\frac{2+\sin 2x}{1+\cos 2x}\right)e^x$$
$$=\int\left(\frac{2+2\sin x\cos x}{2\cos^2 x}\right)e^x$$
$$=\int\left(\frac{1+\sin x\cos x}{\cos^2 x}\right)e^x$$
$$=\int\left(\sec^2 x+\tan x\right)e^x$$

$$\text{Let } f(x)=\tan x\Rightarrow f'(x)=\sec^2 x$$
$$\therefore I=\int\left[f(x)+f'(x)\right]e^x dx$$
$$=e^x f(x)+\text{C}$$
$$=e^x\tan x+\text{C}$$

Answer needs Correction? **Click Here**

Q22 : $\frac{x^2+x+1}{(x+1)^2(x+2)}$

Answer :

$$\text{Let } \frac{x^2+x+1}{(x+1)^2(x+2)}=\frac{A}{(x+1)}+\frac{B}{(x+1)^2}+\frac{C}{(x+2)} \qquad ...(1)$$
$$\Rightarrow x^2+x+1=A(x+1)(x+2)+B(x+2)+C\left(x^2+2x+1\right)$$
$$\Rightarrow x^2+x+1=A\left(x^2+3x+2\right)+B(x+2)+C\left(x^2+2x+1\right)$$
$$\Rightarrow x^2+x+1=(A+C)x^2+(3A+B+2C)x+(2A+2B+C)$$

Equating the coefficients of x^2, x,and constant term, we obtain

$A+C=1$

$3A+B+2C=1$

$2A+2B+C=1$

On solving these equations, we obtain

$A=-2$, $B=1$, and $C=3$

From equation (1), we obtain

$$\frac{x^2+x+1}{(x+1)^2(x+2)}=\frac{-2}{(x+1)}+\frac{3}{(x+2)}+\frac{1}{(x+1)^2}$$
$$\int\frac{x^2+x+1}{(x+1)^2(x+2)}dx=-2\int\frac{1}{x+1}dx+3\int\frac{1}{(x+2)}dx+\int\frac{1}{(x+1)^2}dx$$
$$=-2\log|x+1|+3\log|x+2|-\frac{1}{(x+1)}+\text{C}$$

Answer needs Correction? **Click Here**

Q23 : $\tan^{-1}\sqrt{\frac{1-x}{1+x}}$

Answer :

$$I=\tan^{-1}\sqrt{\frac{1-x}{1+x}}dx$$
$$\text{Let } x=\cos\theta \;\Rightarrow\; dx=-\sin\theta d\theta$$
$$I=\int\tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}(-\sin\theta d\theta)$$
$$=-\int\tan^{-1}\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}}\sin\theta d\theta$$
$$=-\int\tan^{-1}\tan\frac{\theta}{2}\cdot\sin\theta d\theta$$
$$=-\frac{1}{2}\int\theta\cdot\sin\theta d\theta$$

$$=-\frac{1}{2}\left[\theta\cdot(-\cos\theta)-\int 1\cdot(-\cos\theta)d\theta\right]$$
$$=-\frac{1}{2}[-\theta\cos\theta+\sin\theta]$$
$$=+\frac{1}{2}\theta\cos\theta-\frac{1}{2}\sin\theta$$
$$=\frac{1}{2}\cos^{-1}x\cdot x-\frac{1}{2}\sqrt{1-x^2}+C$$
$$=\frac{x}{2}\cos^{-1}x-\frac{1}{2}\sqrt{1-x^2}+C$$
$$=\frac{1}{2}\left(x\cos^{-1}x-\sqrt{1-x^2}\right)+C$$

Answer needs Correction? **Click Here**

Q24 : $\dfrac{\sqrt{x^2+1}\left[\log(x^2+1)-2\log x\right]}{x^4}$

Answer :

$$\frac{\sqrt{x^2+1}\left[\log(x^2+1)-2\log x\right]}{x^4}=\frac{\sqrt{x^2+1}}{x^4}\left[\log(x^2+1)-\log x^2\right]$$
$$=\frac{\sqrt{x^2+1}}{x^4}\left[\log\left(\frac{x^2+1}{x^2}\right)\right]$$
$$=\frac{\sqrt{x^2+1}}{x^4}\log\left(1+\frac{1}{x^2}\right)$$
$$=\frac{1}{x^3}\sqrt{\frac{x^2+1}{x^2}}\log\left(1+\frac{1}{x^2}\right)$$
$$=\frac{1}{x^3}\sqrt{1+\frac{1}{x^2}}\log\left(1+\frac{1}{x^2}\right)$$

Let $1+\frac{1}{x^2}=t\Rightarrow\ \frac{-2}{x^3}dx=dt$

$$\therefore I=\int\frac{1}{x^3}\sqrt{1+\frac{1}{x^2}}\log\left(1+\frac{1}{x^2}\right)dx$$
$$=-\frac{1}{2}\int\sqrt{t}\log t\,dt$$
$$=-\frac{1}{2}\int t^{\frac{1}{2}}\cdot\log t\,dt$$

Integrating by parts, we obtain

$$I=-\frac{1}{2}\left[\log t\cdot\int t^{\frac{1}{2}}dt-\left\{\left(\frac{d}{dt}\log t\right)\int t^{\frac{1}{2}}dt\right\}dt\right]$$
$$=-\frac{1}{2}\left[\log t\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}}-\int\frac{1}{t}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}}dt\right]$$
$$=-\frac{1}{2}\left[\frac{2}{3}t^{\frac{3}{2}}\log t-\frac{2}{3}\int t^{\frac{1}{2}}dt\right]$$
$$=-\frac{1}{2}\left[\frac{2}{3}t^{\frac{3}{2}}\log t-\frac{4}{9}t^{\frac{3}{2}}\right]$$
$$=-\frac{1}{3}t^{\frac{3}{2}}\log t+\frac{2}{9}t^{\frac{3}{2}}$$
$$=-\frac{1}{3}t^{\frac{3}{2}}\left[\log t-\frac{2}{3}\right]$$
$$=-\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{\frac{3}{2}}\left[\log\left(1+\frac{1}{x^2}\right)-\frac{2}{3}\right]+C$$

Answer needs Correction? **Click Here**

Q25 : $\int_{\frac{\pi}{2}}^{\pi}e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$

Answer :

$$I=\int_{\frac{\pi}{2}}^{\pi}e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$$
$$=\int_{\frac{\pi}{2}}^{\pi}e^x\left(\frac{1-2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}\right)dx$$
$$=\int_{\frac{\pi}{2}}^{\pi}e^x\left(\frac{\operatorname{cosec}^2\frac{x}{2}}{2}-\cot\frac{x}{2}\right)dx$$

Let $f(x)=-\cot\frac{x}{2}$

$$\Rightarrow f'(x)=-\left(-\frac{1}{2}\operatorname{cosec}^2\frac{x}{2}\right)=\frac{1}{2}\operatorname{cosec}^2\frac{x}{2}$$
$$\therefore I=\int_{\frac{\pi}{2}}^{\pi}e^x\left(f(x)+f'(x)\right]dx$$
$$=\left[e^x\cdot f(x)dx\right]_{\frac{\pi}{2}}^{\pi}$$

$$= -\left[e^x \cdot \cot \frac{x}{2}\right]_{\frac{\pi}{2}}$$

$$= -\left[e^{\pi} \times \cot \frac{\pi}{2} - e^{\frac{\pi}{2}} \times \cot \frac{\pi}{4}\right]$$

$$= -\left[e^{\pi} \times 0 - e^{\frac{\pi}{2}} \times 1\right]$$

$$= e^{\frac{\pi}{2}}$$

Answer needs Correction? **Click Here**

Q26 : $\int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x} dx$

Answer :

$$\text{Let } I = \int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \frac{\frac{(\sin x \cos x)}{\cos^4 x}}{\frac{(\cos^4 x + \sin^4 x)}{\cos^4 x}} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \frac{\tan x \sec^2 x}{1 + \tan^4 x} dx$$

$$\text{Let } \tan^2 x = t \Rightarrow 2 \tan x \sec^2 x\, dx = dt$$

When $x = 0$, $t = 0$ and when $x = \frac{\pi}{4}, t = 1$

$$\therefore I = \frac{1}{2} \int_0^1 \frac{dt}{1+t^2}$$

$$= \frac{1}{2}\left[\tan^{-1} t\right]_0^1$$

$$= \frac{1}{2}\left[\tan^{-1} 1 - \tan^{-1} 0\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{4}\right]$$

$$= \frac{\pi}{8}$$

Answer needs Correction? **Click Here**

Q27 : $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x\, dx}{\cos^2 x + 4 \sin^2 x}$

Answer :

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 x + 4 \sin^2 x} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 x + 4(1 - \cos^2 x)} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 x + 4 - 4\cos^2 x} dx$$

$$\Rightarrow I = \frac{-1}{3} \int_0^{\frac{\pi}{2}} \frac{4 - 3\cos^2 x - 4}{4 - 3\cos^2 x} dx$$

$$\Rightarrow I = \frac{-1}{3} \int_0^{\frac{\pi}{2}} \frac{4 - 3\cos^2 x}{4 - 3\cos^2 x} dx + \frac{1}{3} \int_0^{\frac{\pi}{2}} \frac{4}{4 - 3\cos^2 x} dx$$

$$\Rightarrow I = \frac{-1}{3} \int_0^{\frac{\pi}{2}} 1\, dx + \frac{1}{3} \int_0^{\frac{\pi}{2}} \frac{4 \sec^2 x}{4 \sec^2 x - 3} dx$$

$$\Rightarrow I = \frac{-1}{3}[x]_0^{\frac{\pi}{2}} + \frac{1}{3} \int_0^{\frac{\pi}{2}} \frac{4 \sec^2 x}{4(1 + \tan^2 x) - 3} dx$$

$$\Rightarrow I = -\frac{\pi}{6} + \frac{2}{3} \int_0^{\frac{\pi}{2}} \frac{2 \sec^2 x}{1 + 4 \tan^2 x} dx \qquad \ldots(1)$$

$$\text{Consider, } \int_0^{\frac{\pi}{2}} \frac{2 \sec^2 x}{1 + 4 \tan^2 x} dx$$

$$\text{Let } 2 \tan x = t \Rightarrow 2 \sec^2 x\, dx = dt$$

When $x = 0, t = 0$ and when $x = \frac{\pi}{2}, t = \infty$

$$\Rightarrow \int_0^{\frac{\pi}{2}} \frac{2 \sec^2 x}{1 + 4 \tan^2 x} dx = \int_0^{\infty} \frac{dt}{1+t^2}$$

$$= \left[\tan^{-1} t\right]_0^{\infty}$$

$$= \left[\tan^{-1}(\infty) - \tan^{-1}(0)\right]$$

$$= \frac{\pi}{2}$$

Therefore, from (1), we obtain

$$I = -\frac{\pi}{6} + \frac{2}{3}\left[\frac{\pi}{2}\right] = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$$

Answer needs Correction? **Click Here**

Q28 : $\int^{\frac{\pi}{3}} \sin x + \cos x$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x+\cos x}{\sqrt{\sin 2x}}dx$$

Answer :

Let $I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x+\cos x}{\sqrt{\sin 2x}}dx$

$\Rightarrow I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(\sin x+\cos x)}{\sqrt{-(-\sin 2x)}}dx$

$\Rightarrow I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x+\cos x}{\sqrt{-(-1+1-2\sin x\cos x)}}dx$

$\Rightarrow I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(\sin x+\cos x)}{\sqrt{1-(\sin^2 x+\cos^2 x-2\sin x\cos x)}}dx$

$\Rightarrow I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{(\sin x+\cos x)dx}{\sqrt{1-(\sin x-\cos x)^2}}$

Let $(\sin x-\cos x)=t \Rightarrow (\sin x+\cos x)dx=dt$

When $x=\frac{\pi}{6}, t=\left(\frac{1-\sqrt{3}}{2}\right)$ and when $x=\frac{\pi}{3}, t=\left(\frac{\sqrt{3}-1}{2}\right)$

$I=\int_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}} \frac{dt}{\sqrt{1-t^2}}$

$\Rightarrow I=\int_{-\left(\frac{\sqrt{3}-1}{2}\right)}^{\frac{\sqrt{3}-1}{2}} \frac{dt}{\sqrt{1-t^2}}$

As $\frac{1}{\sqrt{1-(-t)^2}}=\frac{1}{\sqrt{1-t^2}}$, therefore, $\frac{1}{\sqrt{1-t^2}}$ is an even function.

It is known that if $f(x)$ is an even function, then $\int_{-a}^{a} f(x)dx=2\int_0^a f(x)dx$

$\Rightarrow I=2\int_0^{\frac{\sqrt{3}-1}{2}} \frac{dt}{\sqrt{1-t^2}}$

$=\left[2\sin^{-1}t\right]_0^{\frac{\sqrt{3}-1}{2}}$

$=2\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$

Answer needs Correction? **Click Here**

Q29 : $\int_0^1 \frac{dx}{\sqrt{1+x}-\sqrt{x}}$

Answer :

Let $I=\int_0^1 \frac{dx}{\sqrt{1+x}-\sqrt{x}}$

$I=\int_0^1 \frac{1}{(\sqrt{1+x}-\sqrt{x})}\times\frac{(\sqrt{1+x}+\sqrt{x})}{(\sqrt{1+x}+\sqrt{x})}dx$

$=\int_0^1 \frac{\sqrt{1+x}+\sqrt{x}}{1+x-x}dx$

$=\int_0^1 \sqrt{1+x}\,dx+\int_0^1 \sqrt{x}\,dx$

$=\left[\frac{2}{3}(1+x)^{\frac{3}{2}}\right]_0^1+\left[\frac{2}{3}(x)^{\frac{3}{2}}\right]_0^1$

$=\frac{2}{3}\left[(2)^{\frac{3}{2}}-1\right]+\frac{2}{3}[1]$

$=\frac{2}{3}(2)^{\frac{3}{2}}$

$=\frac{2\cdot 2\sqrt{2}}{3}$

$=\frac{4\sqrt{2}}{3}$

Answer needs Correction? **Click Here**

Q30 : $\int_0^{\frac{\pi}{4}} \frac{\sin x+\cos x}{9+16\sin 2x}dx$

Answer :

Let $I=\int_0^{\frac{\pi}{4}} \frac{\sin x+\cos x}{9+16\sin 2x}dx$

Also, let $\sin x-\cos x=t \Rightarrow (\cos x+\sin x)dx=dt$

When $x=0$, $t=-1$ and when $x=\frac{\pi}{4}$, $t=0$

$\Rightarrow (\sin x-\cos x)^2=t^2$

$\Rightarrow \sin^2 x+\cos^2 x-2\sin x\cos x=t^2$

$\Rightarrow 1-\sin 2x=t^2$

$\Rightarrow \sin 2x=1-t^2$

$\therefore I=\int^0 \frac{dt}{\ldots}$

$$\int_{-1}^{0} \frac{dt}{9+16(1-t^2)}$$

$$= \int_{-1}^{0} \frac{dt}{9+16-16t^2}$$

$$= \int_{-1}^{0} \frac{dt}{25-16t^2} = \int_{-1}^{0} \frac{dt}{(5)^2-(4t)^2}$$

$$= \frac{1}{4}\left[\frac{1}{2(5)}\log\left|\frac{5+4t}{5-4t}\right|\right]_{-1}^{0}$$

$$= \frac{1}{40}\left[\log(1)-\log\left|\frac{1}{9}\right|\right]$$

$$= \frac{1}{40}\log 9$$

Answer needs Correction? **Click Here**

Q31 : $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x)\,dx$

Answer :

Let $I = \int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x)\,dx = \int_0^{\frac{\pi}{2}} 2\sin x\cos x \tan^{-1}(\sin x)\,dx$

Also, let $\sin x = t \Rightarrow \cos x\,dx = dt$

When $x = 0, t = 0$ and when $x = \frac{\pi}{2}, t = 1$

$$\Rightarrow I = 2\int_0^1 t\tan^{-1}(t)\,dt \qquad \ldots(1)$$

Consider $\int t\cdot\tan^{-1} t\,dt = \tan^{-1} t\cdot\int t\,dt - \int\left\{\frac{d}{dt}(\tan^{-1} t)\int t\,dt\right\}dt$

$$= \tan^{-1} t\cdot\frac{t^2}{2} - \int\frac{1}{1+t^2}\cdot\frac{t^2}{2}dt$$

$$= \frac{t^2\tan^{-1} t}{2} - \frac{1}{2}\int\frac{t^2+1-1}{1+t^2}dt$$

$$= \frac{t^2\tan^{-1} t}{2} - \frac{1}{2}\int 1\,dt + \frac{1}{2}\int\frac{1}{1+t^2}dt$$

$$= \frac{t^2\tan^{-1} t}{2} - \frac{1}{2}\cdot t + \frac{1}{2}\tan^{-1} t$$

$$\Rightarrow \int_0^1 t\cdot\tan^{-1} t\,dt = \left[\frac{t^2\cdot\tan^{-1} t}{2} - \frac{t}{2} + \frac{1}{2}\tan^{-1} t\right]_0^1$$

$$= \frac{1}{2}\left[\frac{\pi}{4} - 1 + \frac{\pi}{4}\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{2} - 1\right] = \frac{\pi}{4} - \frac{1}{2}$$

From equation (1), we obtain

$$I = 2\left[\frac{\pi}{4} - \frac{1}{2}\right] = \frac{\pi}{2} - 1$$

Answer needs Correction? **Click Here**

Q32 : $\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}dx$

Answer :

$$Let\ I = \int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}dx \qquad \ldots(1)$$

$$I = \int_0^{\pi}\left\{\frac{(\pi-x)\tan(\pi-x)}{\sec(\pi-x)+\tan(\pi-x)}\right\}dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I = \int_0^{\pi}\left\{\frac{-(\pi-x)\tan x}{-(\sec x+\tan x)}\right\}dx$$

$$\Rightarrow I = \int_0^{\pi}\frac{(\pi-x)\tan x}{\sec x+\tan x}dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\pi}\frac{\pi\tan x}{\sec x+\tan x}dx$$

$$\Rightarrow 2I = \pi\int_0^{\pi}\frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}+\frac{\sin x}{\cos x}}dx$$

$$\Rightarrow 2I = \pi\int_0^{\pi}\frac{\sin x+1-1}{1+\sin x}dx$$

$$\Rightarrow 2I = \pi\int_0^{\pi}1.dx - \pi\int_0^{\pi}\frac{1}{1+\sin x}dx$$

$$\Rightarrow 2I = \pi[x]_0^{\pi} - \pi\int_0^{\pi}\frac{1-\sin x}{\cos^2 x}dx$$

$$\Rightarrow 2I = \pi^2 - \pi\int_0^{\pi}(\sec^2 x - \tan x\sec x)\,dx$$

$$\Rightarrow 2I = \pi^2 - \pi[\tan x - \sec x]_0^{\pi}$$

$$\Rightarrow 2I = \pi^2 - \pi[\tan\pi - \sec\pi - \tan 0 + \sec 0]$$

$$\Rightarrow 2I = \pi^2 - \pi[0-(-1)-0+1]$$

$$\Rightarrow 2I = \pi^2 - 2\pi$$

$$\Rightarrow 2I = \pi(\pi-2)$$

$$\Rightarrow I = \frac{\pi}{2}(\pi - 2)$$

Answer needs Correction? **Click Here**

Q33 : $\int_1^4 \left[|x-1| + |x-2| + |x-3|\right] dx$

Answer :

Let $I = \int_1^4 \left[|x-1| + |x-2| + |x-3|\right] dx$

$$\Rightarrow I = \int_1^4 |x-1|\,dx + \int_1^4 |x-2|\,dx + \int_1^4 |x-3|\,dx$$

$$I = I_1 + I_2 + I_3 \qquad \ldots(1)$$

where, $I_1 = \int_1^4 |x-1|\,dx$, $I_2 = \int_1^4 |x-2|\,dx$, and $I_3 = \int_1^4 |x-3|\,dx$

$$I_1 = \int_1^4 |x-1|\,dx$$

$(x-1) \geq 0$ for $1 \leq x \leq 4$

$$\therefore I_1 = \int_1^4 (x-1)\,dx$$

$$\Rightarrow I_1 = \left[\frac{x^2}{x} - x\right]_1^4$$

$$\Rightarrow I_1 = \left[8 - 4 - \frac{1}{2} + 1\right] = \frac{9}{2} \qquad \ldots(2)$$

$$I_2 = \int_1^4 |x-2|\,dx$$

$x - 2 \geq 0$ for $2 \leq x \leq 4$ and $x - 2 \leq 0$ for $1 \leq x \leq 2$

$$\therefore I_2 = \int_1^2 (2-x)\,dx + \int_2^4 (x-2)\,dx$$

$$\Rightarrow I_2 = \left[2x - \frac{x^2}{2}\right]_1^2 + \left[\frac{x^2}{2} - 2x\right]_2^4$$

$$\Rightarrow I_2 = \left[4 - 2 - 2 + \frac{1}{2}\right] + \left[8 - 8 - 2 + 4\right]$$

$$\Rightarrow I_2 = \frac{1}{2} + 2 = \frac{5}{2} \qquad \ldots(3)$$

$$I_3 = \int_1^4 |x-3|\,dx$$

$x - 3 \geq 0$ for $3 \leq x \leq 4$ and $x - 3 \leq 0$ for $1 \leq x \leq 3$

$$\therefore I_3 = \int_1^3 (3-x)\,dx + \int_3^4 (x-3)\,dx$$

$$\Rightarrow I_3 = \left[3x - \frac{x^2}{2}\right]_1^3 + \left[\frac{x^2}{2} - 3x\right]_3^4$$

$$\Rightarrow I_3 = \left[9 - \frac{9}{2} - 3 + \frac{1}{2}\right] + \left[8 - 12 - \frac{9}{2} + 9\right]$$

$$\Rightarrow I_3 = \left[6 - 4\right] + \left[\frac{1}{2}\right] = \frac{5}{2} \qquad \ldots(4)$$

From equations (1), (2), (3), and (4), we obtain

$$I = \frac{9}{2} + \frac{5}{2} + \frac{5}{2} = \frac{19}{2}$$

Answer needs Correction? **Click Here**

Q34 : $\int_1^3 \frac{dx}{x^2(x+1)} = \frac{2}{3} + \log\frac{2}{3}$

Answer :

Let $I = \int_1^3 \frac{dx}{x^2(x+1)}$

Also, let $\frac{1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$

$$\Rightarrow 1 = Ax(x+1) + B(x+1) + C(x^2)$$

$$\Rightarrow 1 = Ax^2 + Ax + Bx + B + Cx^2$$

Equating the coefficients of x^2, x, and constant term, we obtain

$A + C = 0$

$A + B = 0$

$B = 1$

On solving these equations, we obtain

$A = -1$, $C = 1$, and $B = 1$

$$\therefore \frac{1}{x^2(x+1)} = \frac{-1}{x} + \frac{1}{x^2} + \frac{1}{(x+1)}$$

$$\Rightarrow I = \int_1^3 \left\{-\frac{1}{x} + \frac{1}{x^2} + \frac{1}{(x+1)}\right\} dx$$

$$= \left[-\log x - \frac{1}{x} + \log(x+1)\right]_1^3$$

$$= \left[\log\left(\frac{x+1}{x}\right) - \frac{1}{x}\right]_1^3$$

$$= \log\left(\frac{4}{3}\right) - \frac{1}{3} - \log\left(\frac{2}{1}\right) + 1$$

$$= \log 4 - \log 3 - \log 2 + \frac{2}{3}$$
$$= \log 2 - \log 3 + \frac{2}{3}$$
$$= \log\left(\frac{2}{3}\right) + \frac{2}{3}$$

Hence, the given result is proved.

Answer needs Correction? **Click Here**

Q35 : $\int_0^1 xe^x dx = 1$

Answer :

Let $I = \int_0^1 xe^x \, dx$

Integrating by parts, we obtain

$$I = x\int_0^1 e^x dx - \int_0^1 \left\{\left(\frac{d}{dx}(x)\right)\int e^x dx\right\}dx$$
$$= \left[xe^x\right]_0^1 - \int_0^1 e^x dx$$
$$= \left[xe^x\right]_0^1 - \left[e^x\right]_0^1$$
$$= e - e + 1$$
$$= 1$$

Hence, the given result is proved.

Answer needs Correction? **Click Here**

Q36 : $\int_{-1}^1 x^{17} \cos^4 x dx = 0$

Answer :

Let $I = \int_{-1}^1 x^{17} \cos^4 x dx$

Also, let $f(x) = x^{17} \cos^4 x$

$\Rightarrow f(-x) = (-x)^{17} \cos^4(-x) = -x^{17} \cos^4 x = -f(x)$

Therefore, $f(x)$ is an odd function.

It is known that if $f(x)$ is an odd function, then $\int_{-a}^a f(x) dx = 0$

$\therefore I = \int_{-1}^1 x^{17} \cos^4 x \, dx = 0$

Hence, the given result is proved.

Answer needs Correction? **Click Here**

Q37 : $\int_0^{\frac{\pi}{2}} \sin^3 x \, dx = \frac{2}{3}$

Answer :

Let $I = \int_0^{\frac{\pi}{2}} \sin^3 x \, dx$

$$I = \int_0^{\frac{\pi}{2}} \sin^2 x \cdot \sin x \, dx$$
$$= \int_0^{\frac{\pi}{2}} (1 - \cos^2 x) \sin x \, dx$$
$$= \int_0^{\frac{\pi}{2}} \sin x \, dx - \int_0^{\frac{\pi}{2}} \cos^2 x \cdot \sin x \, dx$$
$$= [-\cos x]_0^{\frac{\pi}{2}} + \left[\frac{\cos^3 x}{3}\right]_0^{\frac{\pi}{2}}$$
$$= 1 + \frac{1}{3}[-1] = 1 - \frac{1}{3} = \frac{2}{3}$$

Hence, the given result is proved.

Answer needs Correction? **Click Here**

Q38 : $\int_0^{\frac{\pi}{4}} 2\tan^3 x dx = 1 - \log 2$

Answer :

Let $I = \int_0^{\frac{\pi}{4}} 2\tan^3 x \, dx$

$$I = 2\int_0^{\frac{\pi}{4}} \tan^2 x \tan x \, dx = 2\int_0^{\frac{\pi}{4}} (\sec^2 x - 1) \tan x \, dx$$
$$= 2\int_0^{\frac{\pi}{4}} \sec^2 x \tan x \, dx - 2\int_0^{\frac{\pi}{4}} \tan x \, dx$$

$$= 2\left[\frac{\tan x}{2}\right]_0 + 2\left[\log\cos x\right]_0^{\frac{\pi}{4}}$$

$$= 1 + 2\left[\log\cos\frac{\pi}{4} - \log\cos 0\right]$$

$$= 1 + 2\left[\log\frac{1}{\sqrt{2}} - \log 1\right]$$

$$= 1 - \log 2 - \log 1 = 1 - \log 2$$

Hence, the given result is proved.

Answer needs Correction? **Click Here**

Q39 : $\int_0^1 \sin^{-1} x\, dx = \frac{\pi}{2} - 1$

Answer :

$$\text{Let } I = \int_0^1 \sin^{-1} x\, dx$$

$$\Rightarrow I = \int_0^1 \sin^{-1} x \cdot 1 \cdot dx$$

Integrating by parts, we obtain

$$I = \left[\sin^{-1} x \cdot x\right]_0^1 - \int_0^1 \frac{1}{\sqrt{1-x^2}} \cdot x\, dx$$

$$= \left[x\sin^{-1} x\right]_0^1 + \frac{1}{2}\int_0^1 \frac{(-2x)}{\sqrt{1-x^2}}\, dx$$

Let $1 - x^2 = t \Rightarrow -2x\, dx = dt$

When $x = 0$, $t = 1$ and when $x = 1$, $t = 0$

$$I = \left[x\sin^{-1} x\right]_0^1 + \frac{1}{2}\int_1^0 \frac{dt}{\sqrt{t}}$$

$$= \left[x\sin^{-1} x\right]_0^1 + \frac{1}{2}\left[2\sqrt{t}\right]_1^0$$

$$= \sin^{-1}(1) + \left[-\sqrt{1}\right]$$

$$= \frac{\pi}{2} - 1$$

Hence, the given result is proved.

Answer needs Correction? **Click Here**

Q40 : Evaluate $\int_0^1 e^{2-3x} dx$ as a limit of a sum.

Answer :

$$\text{Let } I = \int_0^1 e^{2-3x} dx$$

It is known that,

$$\int_a^b f(x)\, dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a) + f(a+h) + \ldots + f\left(a + (n-1)h\right)\right]$$

$$\text{Where, } h = \frac{b-a}{n}$$

$$\text{Here, } a = 0, b = 1, \text{ and } f(x) = e^{2-3x}$$

$$\Rightarrow h = \frac{1-0}{n} = \frac{1}{n}$$

$$\therefore \int_0^1 e^{2-3x} dx = (1-0)\lim_{n\to\infty}\frac{1}{n}\left[f(0) + f(0+h) + \ldots + f\left(0 + (n-1)h\right)\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[e^2 + e^{2-3h} + \ldots e^{2-3(n-1)h}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[e^2\left\{1 + e^{-3h} + e^{-6h} + e^{-9h} + \ldots e^{-3(n-1)h}\right\}\right]$$

$$= \lim_{h\to\infty}\frac{1}{n}\left[e^2\left\{\frac{1-\left(e^{-3h}\right)^n}{1-\left(e^{-3h}\right)}\right\}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[e^2\left\{\frac{1-e^{-\frac{3}{n}\times n}}{1-e^{-\frac{3}{n}}}\right\}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[\frac{e^2\left(1-e^{-3}\right)}{1-e^{-\frac{3}{n}}}\right]$$

$$= e^2\left(e^{-3}-1\right)\lim_{n\to\infty}\frac{1}{n}\left[\frac{1}{e^{-\frac{3}{n}}-1}\right]$$

$$= e^2\left(e^{-3}-1\right)\lim_{n\to\infty}\left(-\frac{1}{3}\right)\left[\frac{-\frac{3}{n}}{e^{-\frac{3}{n}}-1}\right]$$

$$= \frac{-e^2\left(e^{-3}-1\right)}{3}\lim_{n\to\infty}\left[\frac{-\frac{3}{n}}{e^{-\frac{3}{n}}-1}\right]$$

$$= \frac{-e^2\left(e^{-3}-1\right)}{3}(1) \qquad \left[\lim_{n\to\infty}\frac{x}{e^x-1}\right]$$

$$-e^{-1} + e^2$$

$= \frac{}{3}$

$= \frac{1}{3}\left(e^2 - \frac{1}{e}\right)$

Answer needs Correction? Click Here

Q41 : $\int \frac{dx}{e^x + e^{-x}}$ is equal to

A. $\tan^{-1}\left(e^x\right) + C$

B. $\tan^{-1}\left(e^{-x}\right) + C$

C. $\log\left(e^x - e^{-x}\right) + C$

D. $\log\left(e^x + e^{-x}\right) + C$

Answer :

Let $I = \int \frac{dx}{e^x + e^{-x}} dx = \int \frac{e^x}{e^{2x} + 1} dx$

Also, let $e^x = t \Rightarrow e^x\,dx = dt$

$\therefore I = \int \frac{dt}{1+t^2}$

$= \tan^{-1} t + C$

$= \tan^{-1}\left(e^x\right) + C$

Hence, the correct answer is A.

Answer needs Correction? Click Here

Q42 : $\int \frac{\cos 2x}{\left(\sin x + \cos x\right)^2} dx$ is equal to

A. $\frac{-1}{\sin x + \cos x} + C$

B. $\log\left|\sin x + \cos x\right| + C$

C. $\log\left|\sin x - \cos x\right| + C$

D. $\frac{1}{\left(\sin x + \cos x\right)^2}$

Answer :

Let $I = \frac{\cos 2x}{\left(\cos x + \sin x\right)^2}$

$I = \int \frac{\cos^2 x - \sin^2 x}{\left(\cos x + \sin x\right)^2} dx$

$= \int \frac{\left(\cos x + \sin x\right)\left(\cos x - \sin x\right)}{\left(\cos x + \sin x\right)^2} dx$

$= \int \frac{\cos x - \sin x}{\cos + \sin x} dx$

Let $\cos x + \sin x = t \Rightarrow \left(\cos x - \sin x\right) dx = dt$

$\therefore I = \int \frac{dt}{t}$

$= \log|t| + C$

$= \log\left|\cos x + \sin x\right| + C$

Hence, the correct answer is B.

Answer needs Correction? Click Here

Q43 : If $f(a+b-x) = f(x)$, then $\int_a^b x f(x)\,dx$ is equal to

A. $\frac{a+b}{2}\int_a^b f(b-x)\,dx$

B. $\frac{a+b}{2}\int_a^b f(b+x)\,dx$

C. $\frac{b-a}{2}\int_a^b f(x)\,dx$

D. $\frac{a+b}{2}\int_a^b f(x)\,dx$

Answer :

Let $I = \int_a^b x f(x)\,dx$ ...(1)

$I = \int_a^b (a+b-x) f(a+b-x)\,dx$ $\left(\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right)$

$\Rightarrow I = \int_a^b (a+b-x) f(x)\,dx$

$\Rightarrow I = (a+b)\int_a^b f(x)\,dx \quad - I$ $\left[\text{Using }(1)\right]$

$\Rightarrow I + I = (a+b)\int_a^b f(x)\,dx$

$$\Rightarrow 2I=(a+b)\int_a^b f(x)\,dx$$
$$\Rightarrow I=\left(\frac{a+b}{2}\right)\int_a^b f(x)\,dx$$

Hence, the correct answer is D.

Answer needs Correction? **Click Here**

Q44 : The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$ is

A. 1

B. 0

C. - 1

D. $\frac{\pi}{4}$

Answer :

$$\text{Let } I=\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$$
$$\Rightarrow I=\int_0^1 \tan^{-1}\left(\frac{x-(1-x)}{1+x(1-x)}\right)dx$$
$$\Rightarrow I=\int_0^1\left[\tan^{-1}x-\tan^{-1}(1-x)\right]dx \qquad ...(1)$$
$$\Rightarrow I=\int_0^1\left[\tan^{-1}(1-x)-\tan^{-1}(1-1+x)\right]dx$$
$$\Rightarrow I=\int_0^1\left[\tan^{-1}(1-x)-\tan^{-1}(x)\right]dx$$
$$\Rightarrow I=\int_0^1\left[\tan^{-1}(1-x)-\tan^{-1}(x)\right]dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^1\left(\tan^{-1}x+\tan^{-1}(1-x)-\tan^{-1}(1-x)-\tan^{-1}x\right)dx$$
$$\Rightarrow 2I=0$$
$$\Rightarrow I=0$$

Hence, the correct answer is B.

*********** END **********