Q 1 . If the zeroes of polynomial x^3-3x^2+x+1 are $a-b, a, a+b$, find a and b.

Answer :

$p(x)=x^3-3x^2+x+1$

Zeroes are a - b, a + a + b

Comparing the given polynomial with px^3+qx^2+rx+t, we obtain

p = 1, q = - 3, r = 1, t = 1

$\text{Sum of zeroes} = a-b+a+a+b$

$\frac{-q}{p}=3a$

$\frac{-(-3)}{1}=3a$

$3=3a$

$a=1$

The zeroes are $1-b, 1, 1+b$.

$\text{Multiplication of zeroes} = 1(1-b)(1+b)$

$\frac{-t}{p}=1-b^2$

$\frac{-1}{1}=1-b^2$

$1-b^2=-1$

$1+1=b^2$

$b=\pm\sqrt{2}$

Hence, a = 1 and b = $\sqrt{2}$ or $-\sqrt{2}$.

Q 2 . It two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

Answer :

Given that 2 + $\sqrt{3}$ and 2 − $\sqrt{3}$ are zeroes of the given polynomial.

Therefore, $\left(x-2-\sqrt{3}\right)\left(x-2+\sqrt{3}\right)$ = *x*2 + 4 - 4*x* - 3

= *x*2 - 4*x* + 1 is a factor of the given polynomial

For finding the remaining zeroes of the given polynomial, we will find the quotient by dividing $x^4 - 6x^3 - 26x^2 + 138x - 35$ by $x^2 - 4x + 1$.

$$
\begin{array}{r|l}
 & x^2 - 2x - 35 \\
\hline
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 - 4x^3 + x^2 \\
 & - \quad + \quad - \\
\hline
 & -2x^3 - 27x^2 + 138x - 35 \\
 & -2x^3 + 8x^2 - 2x \\
 & + \quad - \quad + \\
\hline
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & + \quad - \quad + \\
\hline
 & 0
\end{array}
$$

Clearly, $x^4 - 6x^3 - 26x^2 + 138x - 35 = (x^2 - 4x + 1)(x^2 - 2x - 35)$

It can be observed that $(x^2 - 2x - 35)$ is also a factor of the given polynomial.

And $(x^2 - 2x - 35) = (x - 7)(x + 5)$

Therefore, the value of the polynomial is also zero when $x - 7 = 0$ or $x + 5 = 0$

Or x = 7 or - 5

Hence, 7 and - 5 are also zeroes of this polynomial.

Q 3. If the polynomial $x^4-6x^3+16x^2-25x+10$ is divided by another polynomial x^2-2x+k, the remainder comes out to be $x + a$, find k and a.

Answer :

By division algorithm,

Dividend = Divisor × Quotient + Remainder

Dividend - Remainder = Divisor × Quotient

$x^4-6x^3+16x^2-25x+10-x-a=x^4-6x^3+16x^2-26x+10-a$ will be perfectly divisible by x^2-2x+k.

Let us divide $x^4-6x^3+16x^2-26x+10-a$ by x^2-2x+k

$$
\begin{array}{r|l}
 & x^2-4x+(8-k) \\
\hline
x^2-2x+k & x^4-6x^3+16x^2-26x+10-a \\
 & x^4-2x^3+kx^2 \\
 & -\quad +\quad - \\
\hline
 & -4x^3+(16-k)x^2-26x \\
 & -4x^3+8x^2-4kx \\
 & +\quad -\quad + \\
\hline
 & (8-k)x^2-(26-4k)x+10-a \\
 & (8-k)x^2-(16-2k)x+(8k-k^2) \\
 & -\quad +\quad - \\
\hline
 & (-10+2k)x+(10-a-8k+k^2)
\end{array}
$$

It can be observed that $(-10+2k)x+(10-a-8k+k^2)$ will be 0.

Therefore, $(-10+2k) = 0$ and $(10-a-8k+k^2) = 0$

For $(-10+2k) = 0$,

$2k = 10$

And thus, $k = 5$

For $(10-a-8k+k^2) = 0$

$10 - a - 8 \times 5 + 25 = 0$

$10 - a - 40 + 25 = 0$

$-5 - a = 0$

Therefore, $a = -5$

Hence, $k = 5$ and $a = -5$

********** END *********