Definite Integrals Ex 20.2 Q40

$$I = \int_0^{\pi/2} \frac{\cos^2 x}{1+3\sin^2 x}\,dx$$

$$I = \int_0^{\pi/2} \frac{\sec^2 x}{\sec^2 x\left(\sec^2 x + 3\tan^2 x\right)}\,dx$$

Put $\tan x = t$

$\sec^2 x\, dx = dt$

$x = 0 \Rightarrow t = 0$ and $x = \frac{\pi}{2} \Rightarrow t = \infty$

$$I = \int_0^{\infty} \frac{1}{\left(1+t^2\right)\left(1+4t^2\right)}\,dt$$

$$I = -\frac{1}{3}\int_0^{\infty}\left[\frac{1}{\left(1+t^2\right)} - \frac{1}{\left(1+4t^2\right)}\right]dt$$

$$I = -\frac{1}{3}\left[\tan^{-1} t - 2\tan^{-1} 2t\right]_0^{\infty}$$

$$I = \frac{\pi}{6}$$

Definite Integrals Ex 20.2 Q41

Let $I = \int_0^{\frac{\pi}{4}} \sin^3 2t \cos 2t\, dt$. consider $\int \sin^3 2t \cos 2t\, dt$

Put $\sin 2t = u$ so that $2\cos 2t\, dt = du$ or $\cos 2t\, dt = \frac{1}{2}\, du$

So $\int \sin^3 2t \cos 2t\, dt = \frac{1}{2}\int u^3\, du$

$= \frac{1}{8}\left[u^4\right] = \frac{1}{8}\sin^4 2t = F(t)$ say

Therefore, by the second fundamental theorem of integrals calculus

$I = F\left(\frac{\pi}{4}\right) - F(0) = \frac{1}{8}\left[\sin^4 \frac{\pi}{2} - \sin^4 0\right] = \frac{1}{8}$

Definite Integrals Ex 20.2 Q42

Let $5 - 4\cos\theta = t$

Differentiating w.r.t. x, we get

$$4\sin\theta d\theta = dt$$

Now, $\theta = 0 \Rightarrow t = 1$

$\theta = \pi \Rightarrow t = 9$

$$\therefore \int_0^{\pi} 5\left(5 - 4\cos\theta\right)^{\frac{1}{4}} \sin\theta d\theta$$

$$= \frac{5}{4}\int_1^9 t^{\frac{1}{4}} dt$$

$$= \frac{5}{4}\left[\frac{4}{5}.t^{\frac{5}{4}}\right]_1^9$$

$$= 3^{\frac{5}{2}} - 1$$

$$= 9\sqrt{3} - 1$$

$$\therefore \int_0^{\pi} 5\left(5 - 4\cos\theta\right)^{\frac{1}{4}} \sin\theta d\theta = 9\sqrt{3} - 1$$

Definite Integrals Ex 20.2 Q43

We have,

$$\int_0^{\frac{\pi}{6}} \cos^{-3} 2\theta \sin 2\theta d\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{\sin 2\theta}{\cos^3 2\theta} d\theta$$

$$= \int_0^{\frac{\pi}{6}} \tan 2\theta . \sec^2 2\theta d\theta$$

Let $\tan 2\theta = t$

Differentiating w.r.t. x, we get

$$2\sec^2 2\theta d\theta = dt$$

Now, $\theta = 0 \Rightarrow t = 0$

$$\theta = \frac{\pi}{6} \Rightarrow t = \sqrt{3}$$

$$\therefore \int_0^{\frac{\pi}{6}} \tan 2\theta . \sec^2 2\theta d\theta = \frac{1}{2}\int_0^{\sqrt{3}} t dt = \frac{1}{2}\left[\frac{t^2}{2}\right]_0^{\sqrt{3}}$$

$$= \frac{3}{4}$$

$$\therefore \int_0^{\frac{\pi}{6}} \cos^{-3} 2\theta \sin 2\theta d\theta = \frac{3}{4}$$

*********** END **********