



Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q31

$$\begin{aligned}
 \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} &= \frac{(a+b) + (a-b)}{\sqrt{(a-b)(a+b)}} \\
 &= \frac{2a}{\sqrt{a^2 - b^2}} \\
 &= \frac{2}{\sqrt{1 - \left(\frac{b}{a}\right)^2}} \\
 &= \frac{2}{\sqrt{1 - \tan^2 x}} \dots\dots\dots \left[\because \tan x = \frac{b}{a} \right] \\
 &= \frac{2 \cos x}{\sqrt{\cos^2 x - \sin^2 x}} \\
 &= \frac{2 \cos x}{\sqrt{\cos 2x}}
 \end{aligned}$$

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We have,

$$\tan A = \frac{1}{7} \quad \& \quad \tan B = \frac{1}{3}$$

$$\begin{aligned} \therefore \cos 2A &= \frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \left(\frac{1}{7}\right)^2}{1 + \left(\frac{1}{7}\right)^2} = \frac{48}{\frac{50}{49}} \\ &= \frac{48}{50} = \frac{24}{25} \dots\dots\dots (A) \end{aligned}$$

Also,

$$\begin{aligned} \sin 4B &= \sin 2 \cdot 2B \\ &= 2 \sin 2B \cdot \cos 2B \\ &= 2 \cdot \left(\frac{2 \tan B}{1 + \tan^2 B} \right) \cdot \left(\frac{1 - \tan^2 B}{1 + \tan^2 B} \right) \\ &= 4 \cdot \left(\frac{\frac{1}{3}}{1 + \frac{1}{9}} \right) \cdot \left(\frac{1 - \frac{1}{9}}{1 + \frac{1}{9}} \right) \\ &= \frac{4 \cdot \frac{1}{3} \cdot \frac{8}{9}}{\frac{10}{9} \times \frac{10}{9}} \\ &= \frac{32 \times 3}{100} \\ &= \frac{8 \times 3}{25} = \frac{24}{25} \dots\dots\dots (B) \end{aligned}$$

from (A) & (B)

$$\cos 2A = \sin 4B$$

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LHS,

$$\cos 7^\circ \cos 14^\circ \cos 28^\circ \cos 56^\circ$$

Divide and multiply by $2 \sin 7^\circ$, we get

$$\begin{aligned} &\frac{1}{2 \sin 7^\circ} \cdot 2 \sin 7^\circ \cdot \cos 7^\circ \cdot \cos 14^\circ \cdot \cos 28^\circ \cdot \cos 56^\circ \\ &= \frac{2 \sin 14^\circ}{2 \cdot 2 \sin 7^\circ} \cdot \cos 14^\circ \cdot \cos 28^\circ \cdot \cos 56^\circ \quad [\because 2 \sin A \cos A = \sin 2A] \\ &= \frac{2 \sin 28^\circ}{2 \cdot 4 \sin 7^\circ} \cdot \cos 28^\circ \cdot \cos 56^\circ \\ &= \frac{2 \sin 56^\circ}{2 \cdot 8 \sin 7^\circ} \cdot \cos 56^\circ \\ &= \frac{\sin 112^\circ}{16 \sin 7^\circ} \\ &= \frac{\sin (180^\circ - 68^\circ)}{16 \sin (90^\circ - 83^\circ)} \\ &= \frac{\sin 68^\circ}{16 \cos 83^\circ} \\ &= \text{RHS} \end{aligned}$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q34

$$\sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a) \quad (1)$$

$$\text{for } a=b, \sin(2a) = 2 \sin(a) \cos(a) \quad (2)$$

$$\text{let } a = 16 \pi/15 \quad (3)$$

$$(\text{so } 2a = 32 \pi/15)$$

then using (3) in (2), we have

$$\begin{aligned} \sin(2a) &= 2 \sin(a) \cos(a) \\ &= 2 (2 \sin(a/2) \cos(a/2)) \cos(a) \\ &= 2 (2 (2 \sin(a/4) \cos(a/4)) \cos(a/2)) \cos(a) \\ &= 2 (2 (2 (2 \sin(a/8) \cos(a/8)) \cos(a/4)) \cos(a/2)) \cos(a) \\ &= 16 \sin(a/8) (\cos(a/8) \cos(a/4) \cos(a/2) \cos(a)) \end{aligned}$$

$$\text{now note } \sin(2a) = \sin(2 \pi/15) \text{ and } \sin(a/8) = \sin(2 \pi/15)$$

so,

$$\cos(a/8) \cos(a/4) \cos(a/2) \cos(a) = 1/16$$

or, replacing a with 16 pi/15,

$$\cos(2\pi/15) * \cos(4\pi/15) * \cos(8\pi/15) * \cos(16\pi/15) = 1/16$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q35

$$\begin{aligned} \cos \frac{\pi}{5} \cos \frac{2\pi}{5} \cos \frac{4\pi}{5} \cos \frac{8\pi}{5} &= \frac{\sin \frac{2^4 \pi}{5}}{2^4 \sin \frac{\pi}{5}} \\ &= \frac{\sin \frac{16\pi}{5}}{16 \sin \frac{\pi}{5}} \\ &= \frac{\sin \left(3\pi + \frac{\pi}{5} \right)}{16 \sin \frac{\pi}{5}} \\ &= \frac{1 \left\{ -\sin \left(\frac{\pi}{5} \right) \right\}}{16 \sin \frac{\pi}{5}} \\ &= \frac{-1}{16} \end{aligned}$$

***** END *****