



## NCERT MISCELLANEOUS SOLUTIONS

Question-1

Find the derivative of the following functions from first principle:

(i)  $-x$  (ii)  $(-x)^{-1}$  (iii)  $\sin(x + 1)$

(iv)  $\cos\left(x - \frac{\pi}{8}\right)$

Ans.

(i) Let  $f(x) = -x$ . Accordingly,  $f(x + h) = -(x + h)$

By first principle,

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{-(x+h) - (-x)}{h} \\&= \lim_{h \rightarrow 0} \frac{-x - h + x}{h} \\&= \lim_{h \rightarrow 0} \frac{-h}{h} \\&= \lim_{h \rightarrow 0} (-1) = -1\end{aligned}$$

(ii) Let  $f(x) = (-x)^{-1} = \frac{1}{-x} = -\frac{1}{x}$ . Accordingly,  $f(x+h) = \frac{-1}{(x+h)}$

By first principle,

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{-1}{x+h} - \left( \frac{-1}{x} \right) \right] \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{-1}{x+h} + \frac{1}{x} \right] \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{-x + (x+h)}{x(x+h)} \right] \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{-x + x + h}{x(x+h)} \right] \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{h}{x(x+h)} \right] \\
 &= \lim_{h \rightarrow 0} \frac{1}{x(x+h)} \\
 &= \frac{1}{x \cdot x} = \frac{1}{x^2}
 \end{aligned}$$

(iii) Let  $f(x) = \sin(x+1)$ . Accordingly,  $f(x+h) = \sin(x+h+1)$

By first principle,

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} [\sin(x+h+1) - \sin(x+1)] \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \left[ 2 \cos\left(\frac{x+h+1+x+1}{2}\right) \sin\left(\frac{x+h+1-x-1}{2}\right) \right] \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \left[ 2 \cos\left(\frac{2x+h+2}{2}\right) \sin\left(\frac{h}{2}\right) \right] \\
 &= \lim_{h \rightarrow 0} \left[ \cos\left(\frac{2x+h+2}{2}\right) \cdot \frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)} \right] \\
 &= \lim_{h \rightarrow 0} \cos\left(\frac{2x+h+2}{2}\right) \cdot \lim_{\frac{h}{2} \rightarrow 0} \frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)} \quad \left[ \text{As } h \rightarrow 0 \Rightarrow \frac{h}{2} \rightarrow 0 \right] \\
 &= \cos\left(\frac{2x+0+2}{2}\right) \cdot 1 \quad \left[ \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right] \\
 &= \cos(x+1)
 \end{aligned}$$

(iv) Let  $f(x) = \cos\left(x - \frac{\pi}{8}\right)$ . Accordingly,  $f(x+h) = \cos\left(x+h - \frac{\pi}{8}\right)$

By first principle,

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \cos\left(x+h - \frac{\pi}{8}\right) - \cos\left(x - \frac{\pi}{8}\right) \right] \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \left[ -2 \sin\left(\frac{x+h - \frac{\pi}{8} + x - \frac{\pi}{8}}{2}\right) \sin\left(\frac{x+h - \frac{\pi}{8} - x + \frac{\pi}{8}}{2}\right) \right] \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \left[ -2 \sin\left(\frac{2x+h - \frac{\pi}{4}}{2}\right) \sin\left(\frac{h}{2}\right) \right] \\
 &= \lim_{h \rightarrow 0} \left[ -\sin\left(\frac{2x+h - \frac{\pi}{4}}{2}\right) \sin\left(\frac{h}{2}\right) \right] \\
 &= \lim_{h \rightarrow 0} \left[ -\sin\left(\frac{2x+h - \frac{\pi}{4}}{2}\right) \right] \lim_{\frac{h}{2} \rightarrow 0} \frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)} \quad \left[ \text{As } h \rightarrow 0 \Rightarrow \frac{h}{2} \rightarrow 0 \right] \\
 &= -\sin\left(\frac{2x+0 - \frac{\pi}{4}}{2}\right) \cdot 1 \\
 &= -\sin\left(x - \frac{\pi}{8}\right)
 \end{aligned}$$

### Question-2

Find the derivative of the following functions (it is to be understood that  $a, b, c, d, p, q, r$  and  $s$  are fixed non-zero constants and  $m$  and  $n$  are integers):  $(x+a)$

Ans.

Let  $f(x) = x + a$ . Accordingly,  $f(x+h) = x+h+a$

By first principle,

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{x+h+a - x-a}{h} \\
 &= \lim_{h \rightarrow 0} \left( \frac{h}{h} \right) \\
 &= \lim_{h \rightarrow 0} (1) \\
 &= 1
 \end{aligned}$$

### Question-3

Find the derivative of the following functions (it is to be understood that  $a, b, c, d, p, q, r$  and  $s$  are fixed non-zero constants and  $m$  and  $n$  are integers):  $(px+q)\left(\frac{r}{x}+s\right)$

Ans.

$$\text{Let } f(x) = (px + q) \left( \frac{r}{x} + s \right)$$

By Leibnitz product rule,

$$\begin{aligned} f'(x) &= (px + q) \left( \frac{r}{x} + s \right)' + \left( \frac{r}{x} + s \right) (px + q)' \\ &= (px + q) (rx^{-1} + s)' + \left( \frac{r}{x} + s \right) (p) \\ &= (px + q) (-rx^{-2}) + \left( \frac{r}{x} + s \right) p \\ &= (px + q) \left( \frac{-r}{x^2} \right) + \left( \frac{r}{x} + s \right) p \\ &= \frac{-pr}{x} - \frac{qr}{x^2} + \frac{pr}{x} + ps \\ &= ps - \frac{qr}{x^2} \end{aligned}$$

#### Question-4

Find the derivative of the following functions (it is to be understood that  $a, b, c, d, p, q, r$  and  $s$  are fixed non-zero constants and  $m$  and  $n$  are integers):  $(ax + b)(cx + d)^2$

Ans.

$$\text{Let } f(x) = (ax + b)(cx + d)^2$$

By Leibnitz product rule,

$$\begin{aligned} f'(x) &= (ax + b) \frac{d}{dx} (cx + d)^2 + (cx + d)^2 \frac{d}{dx} (ax + b) \\ &= (ax + b) \frac{d}{dx} (c^2x^2 + 2cdx + d^2) + (cx + d)^2 \frac{d}{dx} (ax + b) \\ &= (ax + b) \left[ \frac{d}{dx} (c^2x^2) + \frac{d}{dx} (2cdx) + \frac{d}{dx} d^2 \right] + (cx + d)^2 \left[ \frac{d}{dx} ax + \frac{d}{dx} b \right] \\ &= (ax + b) (2c^2x + 2cd) + (cx + d)^2 a \\ &= 2c(ax + b)(cx + d) + a(cx + d)^2 \end{aligned}$$

#### Question-5

Find the derivative of the following functions (it is to be understood that  $a, b, c, d, p, q, r$  and  $s$  are fixed non-zero constants and  $m$  and  $n$  are integers):  $\frac{ax+b}{cx+d}$

Ans.

$$\text{Let } f(x) = \frac{ax+b}{cx+d}$$

By quotient rule,

$$\begin{aligned} f'(x) &= \frac{(cx+d) \frac{d}{dx}(ax+b) - (ax+b) \frac{d}{dx}(cx+d)}{(cx+d)^2} \\ &= \frac{(cx+d)(a) - (ax+b)(c)}{(cx+d)^2} \\ &= \frac{acx+ad-acx-bc}{(cx+d)^2} \\ &= \frac{ad-bc}{(cx+d)^2} \end{aligned}$$

#### Question-6

Find the derivative of the following functions (it is to be understood that  $a, b, c, d, p, q, r$  and  $s$  are fixed non-zero constants and  $m$  and  $n$  are integers):

$$\frac{1+\frac{1}{x}}{1-\frac{1}{x}}$$

Ans.

$$\text{Let } f(x) = \frac{1+\frac{1}{x}}{1-\frac{1}{x}} = \frac{\frac{x+1}{x}}{\frac{x-1}{x}} = \frac{x+1}{x-1}, \text{ where } x \neq 0$$

By quotient rule,

$$\begin{aligned} f'(x) &= \frac{(x-1) \frac{d}{dx}(x+1) - (x+1) \frac{d}{dx}(x-1)}{(x-1)^2}, x \neq 0, 1 \\ &= \frac{(x-1)(1) - (x+1)(1)}{(x-1)^2}, x \neq 0, 1 \\ &= \frac{x-1-x-1}{(x-1)^2}, x \neq 0, 1 \\ &= \frac{-2}{(x-1)^2}, x \neq 0, 1 \end{aligned}$$

#### Question-7

Find the derivative of the following functions (it is to be understood that  $a, b, c, d, p, q, r$  and  $s$  are fixed non-zero constants and  $m$  and  $n$  are integers):  $\frac{1}{ax^2+bx+c}$

Ans.

$$\text{Let } f(x) = \frac{1}{ax^2 + bx + c}$$

By quotient rule,

$$\begin{aligned} f'(x) &= \frac{(ax^2 + bx + c) \frac{d}{dx}(1) - \frac{d}{dx}(ax^2 + bx + c)}{(ax^2 + bx + c)^2} \\ &= \frac{(ax^2 + bx + c)(0) - (2ax + b)}{(ax^2 + bx + c)^2} \\ &= \frac{-(2ax + b)}{(ax^2 + bx + c)^2} \end{aligned}$$

Question-8

Find the derivative of the following functions (it is to be understood that  $a, b, c, d, p, q, r$  and  $s$  are fixed non-zero constants and  $m$  and  $n$  are integers):  $\frac{ax+b}{px^2+qx+r}$

Ans.

$$\text{Let } f(x) = \frac{ax+b}{px^2+qx+r}$$

By quotient rule,

$$\begin{aligned} f'(x) &= \frac{(px^2 + qx + r) \frac{d}{dx}(ax + b) - (ax + b) \frac{d}{dx}(px^2 + qx + r)}{(px^2 + qx + r)^2} \\ &= \frac{(px^2 + qx + r)(a) - (ax + b)(2px + q)}{(px^2 + qx + r)^2} \\ &= \frac{apx^2 + aqx + ar - 2apx^2 - aqx - 2bpx - bq}{(px^2 + qx + r)^2} \\ &= \frac{-apx^2 - 2bpx + ar - bq}{(px^2 + qx + r)^2} \end{aligned}$$

Question-9

Find the derivative of the following functions (it is to be understood that  $a, b, c, d, p, q, r$  and  $s$  are fixed non-zero constants and  $m$  and  $n$  are integers):  $\frac{px^2+qx+r}{ax+b}$

Ans.

$$\text{Let } f(x) = \frac{px^2 + qx + r}{ax + b}$$

By quotient rule,

$$\begin{aligned} f'(x) &= \frac{(ax+b) \frac{d}{dx}(px^2 + qx + r) - (px^2 + qx + r) \frac{d}{dx}(ax+b)}{(ax+b)^2} \\ &= \frac{(ax+b)(2px+q) - (px^2 + qx + r)(a)}{(ax+b)^2} \\ &= \frac{2apx^2 + aqx + 2bpx + bq - apx^2 - aqx - ar}{(ax+b)^2} \\ &= \frac{apx^2 + 2bpx + bq - ar}{(ax+b)^2} \end{aligned}$$

#### Question-10

Find the derivative of the following functions (it is to be understood that  $a, b, c, d, p, q, r$  and  $s$  are fixed non-zero constants and  $m$  and  $n$  are integers):  $\frac{a}{x^4} - \frac{b}{x^2} + \cos x$

Ans.

$$\begin{aligned} \text{Let } f(x) &= \frac{a}{x^4} - \frac{b}{x^2} + \cos x \\ f'(x) &= \frac{d}{dx}\left(\frac{a}{x^4}\right) - \frac{d}{dx}\left(\frac{b}{x^2}\right) + \frac{d}{dx}(\cos x) \\ &= a \frac{d}{dx}(x^{-4}) - b \frac{d}{dx}(x^{-2}) + \frac{d}{dx}(\cos x) \\ &= a(-4x^{-5}) - b(-2x^{-3}) + (-\sin x) \quad \left[ \frac{d}{dx}(x^n) = nx^{n-1} \text{ and } \frac{d}{dx}(\cos x) = -\sin x \right] \\ &= -\frac{4a}{x^5} + \frac{2b}{x^3} - \sin x \end{aligned}$$

#### Question-11

Find the derivative of the following functions (it is to be understood that  $a, b, c, d, p, q, r$  and  $s$  are fixed non-zero constants and  $m$  and  $n$  are integers):  $4\sqrt{x} - 2$

Ans.

$$\begin{aligned} \text{Let } f(x) &= 4\sqrt{x} - 2 \\ f'(x) &= \frac{d}{dx}(4\sqrt{x} - 2) = \frac{d}{dx}(4\sqrt{x}) - \frac{d}{dx}(2) \\ &= 4 \frac{d}{dx}\left(x^{\frac{1}{2}}\right) - 0 = 4\left(\frac{1}{2}x^{\frac{1}{2}-1}\right) \\ &= \left(2x^{-\frac{1}{2}}\right) = \frac{2}{\sqrt{x}} \end{aligned}$$

#### Question-12

Find the derivative of the following functions (it is to be understood that  $a, b, c, d, p, q, r$  and  $s$  are fixed non-zero constants and  $m$  and  $n$  are integers):  $(ax + b)^n$

Ans.

Let  $f(x) = (ax + b)^n$ . Accordingly,  $f(x+h) = \{a(x+h) + b\}^n = (ax + ah + b)^n$

By first principle,

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(ax + ah + b)^n - (ax + b)^n}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(ax + b)^n \left(1 + \frac{ah}{ax + b}\right)^n - (ax + b)^n}{h} \\
 &= (ax + b)^n \lim_{h \rightarrow 0} \frac{\left(1 + \frac{ah}{ax + b}\right)^n - 1}{h} \\
 &= (ax + b)^n \lim_{h \rightarrow 0} \frac{1}{h} \left[ 1 + n \left(\frac{ah}{ax + b}\right) + \frac{n(n-1)}{2} \left(\frac{ah}{ax + b}\right)^2 + \dots \right] - 1 \\
 &\quad \text{(Using binomial theorem)} \\
 &= (ax + b)^n \lim_{h \rightarrow 0} \frac{1}{h} \left[ n \left(\frac{ah}{ax + b}\right) + \frac{n(n-1)a^2h^2}{2(ax + b)^2} + \dots \text{(Terms containing higher degrees of } h) \right] \\
 &= (ax + b)^n \lim_{h \rightarrow 0} \left[ \frac{na}{(ax + b)} + \frac{n(n-1)a^2h}{2(ax + b)^2} + \dots \right] \\
 &= (ax + b)^n \left[ \frac{na}{(ax + b)} + 0 \right] \\
 &= na \frac{(ax + b)^n}{(ax + b)} \\
 &= na(ax + b)^{n-1}
 \end{aligned}$$

### Question-13

Find the derivative of the following functions (it is to be understood that  $a, b, c, d, p, q, r$  and  $s$  are fixed non-zero constants and  $m$  and  $n$  are integers):  $(ax + b)^n (cx + d)^m$

Ans.

$$\text{Let } f(x) = (ax + b)^n (cx + d)^m$$

By Leibnitz product rule,

$$f'(x) = (ax + b)^n \frac{d}{dx}(cx + d)^m + (cx + d)^m \frac{d}{dx}(ax + b)^n \quad \dots(1)$$

$$\text{Now, let } f_1(x) = (cx + d)^m$$

$$f_1(x+h) = (cx + ch + d)^m$$

$$\begin{aligned}
 f_1'(x) &= \lim_{h \rightarrow 0} \frac{f_1(x+h) - f_1(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(cx + ch + d)^m - (cx + d)^m}{h} \\
 &= (cx + d)^m \lim_{h \rightarrow 0} \frac{1}{h} \left[ \left(1 + \frac{ch}{cx + d}\right)^m - 1 \right] \\
 &= (cx + d)^m \lim_{h \rightarrow 0} \frac{1}{h} \left[ 1 + \frac{mch}{(cx + d)} + \frac{m(m-1)}{2} \frac{(c^2h^2)}{(cx + d)^2} + \dots \right] - 1 \\
 &= (cx + d)^m \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{mch}{(cx + d)} + \frac{m(m-1)c^2h^2}{2(cx + d)^2} + \dots \text{(Terms containing higher degrees of } h) \right] \\
 &= (cx + d)^m \lim_{h \rightarrow 0} \left[ \frac{mc}{(cx + d)} + \frac{m(m-1)c^2h}{2(cx + d)^2} + \dots \right] \\
 &= (cx + d)^m \left[ \frac{mc}{cx + d} + 0 \right]
 \end{aligned}$$



$$\begin{aligned}
&= \frac{mc(cx+d)^m}{(cx+d)} \\
&= mc(cx+d)^{m-1} \\
\frac{d}{dx}(cx+d)^m &= mc(cx+d)^{m-1} \quad \dots(2) \\
\text{Similarly, } \frac{d}{dx}(ax+b)^n &= na(ax+b)^{n-1} \quad \dots(3)
\end{aligned}$$

Therefore, from (1), (2), and (3), we obtain

$$\begin{aligned}
f'(x) &= (ax+b)^n \{mc(cx+d)^{m-1}\} + (cx+d)^m \{na(ax+b)^{n-1}\} \\
&= (ax+b)^{n-1} (cx+d)^{m-1} [mc(ax+b) + na(cx+d)]
\end{aligned}$$

#### Question-14

Find the derivative of the following functions (it is to be understood that  $a, b, c, d, p, q, r$  and  $s$  are fixed non-zero constants and  $m$  and  $n$  are integers):  $\sin(x+a)$

Ans.

$$\text{Let } f(x) = \sin(x+a)$$

$$f(x+h) = \sin(x+h+a)$$

By first principle,

$$\begin{aligned}
f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{\sin(x+h+a) - \sin(x+a)}{h} \\
&= \lim_{h \rightarrow 0} \frac{1}{h} \left[ 2 \cos\left(\frac{x+h+a+x+a}{2}\right) \sin\left(\frac{x+h+a-x-a}{2}\right) \right] \\
&= \lim_{h \rightarrow 0} \frac{1}{h} \left[ 2 \cos\left(\frac{2x+2a+h}{2}\right) \sin\left(\frac{h}{2}\right) \right] \\
&= \lim_{h \rightarrow 0} \left[ \cos\left(\frac{2x+2a+h}{2}\right) \left\{ \frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)} \right\} \right] \\
&= \lim_{h \rightarrow 0} \cos\left(\frac{2x+2a+h}{2}\right) \lim_{\frac{h}{2} \rightarrow 0} \left\{ \frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)} \right\} \quad \left[ \text{As } h \rightarrow 0 \Rightarrow \frac{h}{2} \rightarrow 0 \right] \\
&= \cos\left(\frac{2x+2a}{2}\right) \times 1 \quad \left[ \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right] \\
&= \cos(x+a)
\end{aligned}$$

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