Polynomials Ex 2.1 Q15

Answer :

Since α and β are the zeros of the quadratic polynomial $f(x) = x^2 - 1$

The roots are $\alpha\, and\, \beta$

$$\alpha + \beta = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\alpha + \beta = \frac{0}{1}$$

$$\alpha + \beta = 0$$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\alpha\beta = \frac{-1}{1}$$

$$\alpha\beta = -1$$

Let S and P denote respectively the sum and product of zeros of the required polynomial. Then,

$$S = \frac{2\alpha}{\beta} + \frac{2\beta}{\alpha}$$

Taking least common factor we get,

$$S = \frac{2\alpha^2 + 2\beta^2}{\alpha\beta}$$

$$S = \frac{2(\alpha^2 + \beta^2)}{\alpha\beta}$$

$$S = \frac{2[(\alpha + \beta) - 2\alpha\beta]}{\alpha\beta}$$

$$S = \frac{2[0 - 2 \times -1]}{-1}$$

$$S = \frac{2[-2 \times -1]}{-1}$$

$$S = \frac{2 \times 2}{-1}$$

$$S = \frac{4}{-1}$$

$$S = -4$$

$$P = \frac{2\alpha}{\beta} \times \frac{2\beta}{\alpha}$$

$$P = \frac{2\alpha}{\beta} \times \frac{2\beta}{\alpha}$$

$$P = 4$$

Hence, the required polynomial $f(x)$ is given by,

$$f(x) = k(x^2 - Sx + p)$$

$$f(x) = k(x^2 - (-4)x + 4)$$

$$f(x) = k(x^2 + 4x + 4)$$

Hence, required equation is $\boxed{f(x) = k(x^2 + 4x + 4)}$ Where k is any non zero real number.

Polynomials Ex 2.1 Q16

Answer :

Since α and β are the zeros of the quadratic polynomial $f(x) = x^2 - 3x - 2$

The roots are α and β

$$\alpha + \beta = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\alpha + \beta = -\left(\frac{-3}{1}\right)$$

$$\alpha + \beta = -(-3)$$

$$\alpha + \beta = 3$$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\alpha\beta = \frac{-2}{1}$$

$$\alpha\beta = -2$$

Let S and P denote respectively the sum and the product of zero of the required polynomial . Then,

$$S = \frac{1}{2\alpha + \beta} + \frac{1}{2\beta + \alpha}$$

Taking least common factor then we have ,

$$S = \frac{1}{2\alpha + \beta} \times \frac{2\beta + \alpha}{2\beta + \alpha} + \frac{1}{2\beta + \alpha} \times \frac{2\alpha + \beta}{2\alpha + \beta}$$

$$S = \frac{2\beta + \alpha}{(2\alpha + \beta)(2\beta + \alpha)} + \frac{2\alpha + \beta}{(2\beta + \alpha)(2\alpha + \beta)}$$

$$S = \frac{2\beta + \alpha + 2\alpha + \beta}{(2\alpha + \beta)(2\beta + \alpha)}$$

$$S = \frac{3\beta + 3\alpha}{4\alpha\beta + 2\beta^2 + 2\alpha^2 + \beta\alpha}$$

$$S = \frac{3(\beta + \alpha)}{5\alpha\beta + 2(\alpha^2 + \beta^2)}$$

$$S = \frac{3(\beta + \alpha)}{5\alpha\beta + 2\left[(\alpha + \beta)^2 - 2\alpha\beta\right]}$$

By substituting $\alpha + \beta = 3$ and $\alpha\beta = -2$ we get ,

$$S = \frac{3(3)}{5(-2) + 2\left[(3)^2 - 2 \times -2\right]}$$

$$S = \frac{9}{-10 + 2(13)}$$

$$S = \frac{9}{-10 + 26}$$

$$S = \frac{9}{16}$$

$$P = \frac{1}{2\alpha + \beta} \times \frac{1}{2\beta + \alpha}$$

$$P = \frac{1}{(2\alpha + \beta)(2\beta + \alpha)}$$

$$P = \frac{1}{4\alpha\beta + 2\beta^2 + 2\alpha^2 + \beta\alpha}$$

$$P = \frac{1}{5\alpha\beta + 2(\alpha^2 + \beta^2)}$$

$$P = \frac{1}{5\alpha\beta + 2\left[(\alpha + \beta)^2 - 2\alpha\beta\right]}$$

By substituting $\alpha + \beta = 3$ and $\alpha\beta = -2$ we get,

$$P = \frac{1}{5(-2) + 2\left[(3)^2 - 2 \times -2\right]}$$

$$P = \frac{1}{10 + 2[9 + 4]}$$

$$P = \frac{1}{10 + 2(13)}$$

$$P = \frac{1}{-10 + 26}$$

$$P = \frac{1}{16}$$

Hence ,the required polynomial $f(x)$ is given by

$$f(x) = k(x^2 - Sx + P)$$

$$f(x) = k\left(x^2 - \frac{9}{16}x + \frac{1}{16}\right)$$

Hence, the required equation is $\boxed{f(x) = k\left(x^2 - \frac{9}{16}x + \frac{1}{16}\right)}$ Where k is any non zero real number.

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