Mathematical Induction Ex 12.2 Q15

Let $P(n): \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^n}=1-\frac{1}{2^n}$

For $n=1$

$$\frac{1}{2}=1-\frac{1}{2^1}$$

$$\frac{1}{2}=\frac{1}{2}$$

$\Rightarrow \quad P(n)$ is true for $n=1$

Let $P(n)$ is true for $n=k$, so

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^k}=1-\frac{1}{2^k} \qquad ---(1)$$

We have to show that,

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^k}+\frac{1}{2^{k+1}}=1-\frac{1}{2^{k+1}}$$

Now,

$$\left\{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^k}\right\}+\frac{1}{2^{k+1}}$$

$$=1-\frac{1}{2^k}+\frac{1}{2^{k+1}} \qquad \left[\text{Using equation } (1)\right]$$

$$=1-\left(\frac{2-1}{2^{k+1}}\right)$$

$$=1-\frac{1}{2^{k+1}}$$

$\Rightarrow \quad P(n)$ is true for $n=k+1$

$\Rightarrow \quad P(n)$ is true for all $n \in N$ by PMI

Mathematical Induction Ex 12.2 Q16

Let $P(n): 1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 = \frac{1}{3}n\left(4n^2 - 1\right)$

For $n = 1$

$$1 = \frac{1}{3}.1.(4-1)$$

$$1 = 1$$

$\Rightarrow$ $P(n)$ is true for $n = 1$

Let $P(n)$ is true for $n = k$, so

$$1^2 + 3^2 + 5^2 + \ldots + (2k-1)^2 = \frac{1}{3}k\left(4k^2 - 1\right) \qquad ---(1)$$

We have to show that,

$$1^2 + 3^2 + 5^2 + \ldots + (2k-1)^2 + (2k+1)^2 = \frac{1}{3}(k+1)\left[4(k+1)^2 - 1\right]$$

Now,

$$\left\{1^2 + 3^2 + 5^2 + \ldots + (2k-1)^2\right\} + (2k+1)^2$$

$$= \frac{1}{3}k\left(4k^2 - 1\right) + (2k+1)^2 \qquad \left[\text{Using equation } (1)\right]$$

$$= \frac{1}{3}k(2k+1)(2k-1) + (2k+1)^2$$

$$= (2k+1)\left[\frac{k(2k-1)}{3} + (2k+1)\right]$$

$$= (2k+1)\left[\frac{2k^2 - k + 3(2k+1)}{3}\right]$$

$$= (2k+1)\left[\frac{2k^2 - k + 6k + 3}{3}\right]$$

$$= \frac{(2k+1)\left(2k^2 + 5k + 3\right)}{3}$$

$$= \frac{(2k+1)\left(2k^2 + 5k + 3\right)}{3}$$

$$= \frac{(2k+1)(2k(k+1) + 3(k+1))}{3}$$

$$= \frac{(2k+1)(2k+3)(k+1)}{3}$$

$$= \frac{(k+1)}{2}\left[4k^2 + 6k + 2k + 3\right]$$

$$= \frac{(k+1)}{2}\left[4k^2 + 8k + 4 - 1\right]$$

$$= \frac{(k+1)}{2}\left[4(k+1)^2 - 1\right]$$

$\Rightarrow$ $P(n)$ is true for $n = k + 1$

$\Rightarrow$ $P(n)$ is true for all $n \in N$ by PMI

Mathematical Induction Ex 12.2 Q17

Let $P(n): a + ar + ar^2 + \ldots + ar^{n-1} = a\left(\frac{r^n - 1}{r - 1}\right), r \neq 1$

For $n = 1$

$$a = a\left(\frac{r^1 - 1}{r - 1}\right)$$

$$a = a$$

$\Rightarrow$ $P(n)$ is true for $n = 1$

Let $P(n)$ is true for $n = k$, so

$$a + ar + ar^2 + \ldots + ar^{k-1} = a\left(\frac{r^k - 1}{r - 1}\right), r \neq 1 \qquad ---(1)$$

We have to show that,

$$a + ar + ar^2 + \ldots + ar^{k-1} + ar^k = a\left(\frac{r^{k+1} - 1}{r - 1}\right)$$

Now,

$$\left\{a + ar + ar^2 + \ldots + ar^{k-1}\right\} + ar^k$$

$$= a\left(\frac{r^k - 1}{r - 1}\right) + ar^k \qquad \left[\text{Using equation } (1)\right]$$

$$= \frac{a\left[r^k - 1 + r^k(r - 1)\right]}{r - 1}$$

$$= \frac{a\left[r^k - 1 + r^{k+1} - r^k\right]}{r - 1}$$

$$= \frac{a\left(r^{k+1} - 1\right)}{r + 1}$$

$\Rightarrow$ $P(n)$ is true for $n = k + 1$

$\Rightarrow$ $P(n)$ is true for all $n \in N$ by PMI

*********** END **********