Trigonometric Identities Ex 6.1 Q83

Answer :

(i) We have,

$$
\begin{aligned}
\sqrt{\frac{\sec\theta-1}{\sec\theta+1}}+\sqrt{\frac{\sec\theta+1}{\sec\theta-1}} &= \frac{\sqrt{\sec\theta-1}}{\sqrt{\sec\theta+1}}+\frac{\sqrt{\sec\theta+1}}{\sqrt{\sec\theta-1}}\\
&= \frac{\sqrt{\sec\theta-1}\sqrt{\sec\theta-1}+\sqrt{\sec\theta+1}\sqrt{\sec\theta+1}}{\sqrt{\sec\theta+1}\sqrt{\sec\theta-1}}\\
&= \frac{\left(\sqrt{\sec\theta-1}\right)^2+\left(\sqrt{\sec\theta+1}\right)^2}{\sqrt{(\sec\theta-1)(\sec\theta+1)}}\\
&= \frac{\sec\theta-1+\sec\theta+1}{\sqrt{\sec^2\theta-1}}\\
&= \frac{2\sec\theta}{\sqrt{\tan^2\theta}}\\
&= \frac{2\sec\theta}{\tan\theta}\\
&= \frac{2\dfrac{1}{\cos\theta}}{\dfrac{\sin\theta}{\cos\theta}}\\
&= 2\frac{1}{\sin\theta}\\
&= 2\cos ec\theta
\end{aligned}
$$

(ii) We have,

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=\frac{\sqrt{1+\sin\theta}}{\sqrt{1-\sin\theta}}+\frac{\sqrt{1-\sin\theta}}{\sqrt{1+\sin\theta}}$$
$$=\frac{\sqrt{1+\sin\theta}\sqrt{1+\sin\theta}+\sqrt{1-\sin\theta}\sqrt{1-\sin\theta}}{\sqrt{1-\sin\theta}\sqrt{1+\sin\theta}}$$
$$=\frac{\left(\sqrt{1+\sin\theta}\right)^2+\left(\sqrt{1-\sin\theta}\right)^2}{\sqrt{(1-\sin\theta)(1+\sin\theta)}}$$
$$=\frac{1+\sin\theta+1-\sin\theta}{\sqrt{1-\sin^2\theta}}$$
$$=\frac{2}{\sqrt{\cos^2\theta}}$$
$$=\frac{2}{\cos\theta}$$
$$=2\sec\theta$$

(iii) We have,

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}+\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=\frac{\sqrt{1+\cos\theta}}{\sqrt{1-\cos\theta}}+\frac{\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}}$$
$$=\frac{\sqrt{1+\cos\theta}\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}\sqrt{1-\cos\theta}}{\sqrt{1-\cos\theta}\sqrt{1+\cos\theta}}$$
$$=\frac{\left(\sqrt{1+\cos\theta}\right)^2+\left(\sqrt{1-\cos\theta}\right)^2}{\sqrt{(1-\cos\theta)(1+\cos\theta)}}$$
$$=\frac{1+\cos\theta+1-\cos\theta}{\sqrt{1-\cos^2\theta}}$$
$$=\frac{2}{\sqrt{\sin^2\theta}}$$
$$=\frac{2}{\sin\theta}$$
$$=2\operatorname{cosec}\theta$$

(iv) We have,

$$\frac{\sec\theta-1}{\sec\theta+1}=\frac{\frac{1}{\cos\theta}-1}{\frac{1}{\cos\theta}+1}$$
$$=\frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{1+\cos\theta}{\cos\theta}}$$
$$=\frac{1-\cos\theta}{1+\cos\theta}$$

Multiplying both the numerator and the denominator by $(1+\cos\theta)$, we have

$$\frac{\sec\theta-1}{\sec\theta+1}=\frac{(1-\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(1+\cos\theta)}$$
$$=\frac{(1-\cos^2\theta)}{(1+\cos\theta)^2}$$

(v) We have,

$$\frac{\sin\theta+1-\cos\theta}{\cos\theta-1+\sin\theta}=\frac{\sin\theta+(1-\cos\theta)}{\sin\theta-(1-\cos\theta)}$$

Multiplying both the numerator and the denominator by $\sin\theta+(1-\cos\theta)$, we have

$$\frac{\sin\theta+1-\cos\theta}{\cos\theta-1+\sin\theta}=\frac{\{\sin\theta+(1-\cos\theta)\}\{\sin\theta+(1-\cos\theta)\}}{\{\sin\theta-(1-\cos\theta)\}\{\sin\theta+(1-\cos\theta)\}}$$

$$= \frac{\{\sin\theta + (1-\cos\theta)\}^2}{\{\sin^2\theta - (1-\cos\theta)^2\}}$$

$$= \frac{\sin^2\theta + 2\sin\theta(1-\cos\theta) + (1-\cos\theta)^2}{\sin^2\theta - (1 - 2\cos\theta + \cos^2\theta)}$$

$$= \frac{\sin^2\theta + 2\sin\theta - 2\sin\theta\cos\theta + (1 - 2\cos\theta + \cos^2\theta)}{\sin^2\theta - (\sin^2\theta + \cos^2\theta - 2\cos\theta + \cos^2\theta)}$$

$$= \frac{(\sin^2\theta + \cos^2\theta) + 2\sin\theta - 2\sin\theta\cos\theta + 1 - 2\cos\theta}{-2\cos^2\theta + 2\cos\theta}$$

$$= \frac{1 + 2\sin\theta - 2\sin\theta\cos\theta + 1 - 2\cos\theta}{-2\cos^2\theta + 2\cos\theta}$$

$$= \frac{2 + 2\sin\theta - 2\sin\theta\cos\theta - 2\cos\theta}{2\cos\theta(1-\cos\theta)}$$

$$= \frac{2(1+\sin\theta) - 2\cos\theta(\sin\theta + 1)}{2\cos\theta(\cos\theta - 1)}$$

$$= \frac{(1+\sin\theta)(2-2\cos\theta)}{2\cos\theta(1-\cos\theta)}$$

$$= \frac{2(1+\sin\theta)(1-\cos\theta)}{2\cos\theta(1-\cos\theta)}$$

$$= \frac{1+\sin\theta}{\cos\theta}$$

*********** END **********