Co-Ordinate Geometry Ex 14.3 Q27

Answer :

The co-ordinates of the midpoint (x_m, y_m) between two points (x_1, y_1) and (x_2, y_2) is given by,

$$(x_m, y_m) = \left(\left(\frac{x_1 + x_2}{2}\right), \left(\frac{y_1 + y_2}{2}\right)\right)$$

Let the three vertices of the triangle be $A(x_A, y_A)$, $B(x_B, y_B)$ and $C(x_C, y_C)$.

The three midpoints are given. Let these points be $M_{AB}(3, -2)$, $M_{BC}(-3, 1)$ and $M_{CA}(4, -3)$.

Let us now equate these points using the earlier mentioned formula,

$$(3, -2) = \left(\left(\frac{x_A + x_B}{2}\right), \left(\frac{y_A + y_B}{2}\right)\right)$$

Equating the individual components we get,

$$x_A + x_B = 6$$
$$y_A + y_B = -4$$

Using the midpoint of another side we have,

$$(-3, 1) = \left(\left(\frac{x_B + x_C}{2}\right), \left(\frac{y_B + y_C}{2}\right)\right)$$

Equating the individual components we get,

$$x_B + x_C = -6$$
$$y_B + y_C = 2$$

Using the midpoint of the last side we have,

$$(4, -3) = \left(\left(\frac{x_A + x_C}{2}\right), \left(\frac{y_A + y_C}{2}\right)\right)$$

Equating the individual components we get,

$$x_A + x_C = 8$$
$$y_A + y_C = -6$$

Adding up all the three equations which have variable 'x' alone we have,

$$x_A + x_B + x_B + x_C + x_A + x_C = 6 - 6 + 8$$
$$2(x_A + x_B + x_C) = 8$$
$$x_A + x_B + x_C = 4$$

Substituting $x_B + x_C = -6$ in the above equation we have,

$$x_A + x_B + x_C = 4$$
$$x_A - 6 = 4$$
$$x_A = 10$$

Therefore,

$$x_A + x_C = 8$$
$$x_C = 8 - 10$$
$$x_C = -2$$

And

$$x_A + x_B = 6$$
$$x_B = 6 - 10$$
$$x_B = -4$$

Adding up all the three equations which have variable 'y' alone we have,

$$y_A + y_B + y_B + y_C + y_A + y_C = -4 + 2 - 6$$
$$2(y_A + y_B + y_C) = -8$$
$$y_A + y_B + y_C = -4$$

Substituting $y_B + y_C = 2$ in the above equation we have,

$$y_A + y_B + y_C = -4$$
$$y_A + 2 = -4$$
$$y_A = -6$$

Therefore,

$$y_A + y_C = -6$$
$$y_C = -6 + 6$$
$$y_C = 0$$

And

$$y_A + y_B = -4$$
$$y_B = -4 + 6$$
$$y_B = 2$$

Therefore the co-ordinates of the three vertices of the triangle are $\boxed{\begin{array}{l} A(10,-6) \\ B(-4,2) \\ C(-2,0) \end{array}}$.

Co-Ordinate Geometry Ex 14.3 Q28

Answer :

We have to find the lengths of the medians of a triangle whose co-ordinates of the vertices are A (0,−1); B (2, 1) and C (0, 3).

So we should find the mid-points of the sides of the triangle.

In general to find the mid-point $P(x, y)$ of two points $A(x_1, y_1)$ and $B(x_2, y_2)$ we use section formula as,

$$P(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Therefore mid-point P of side AB can be written as,

$$P(x, y) = \left(\frac{2+0}{2}, \frac{1-1}{2}\right)$$

Now equate the individual terms to get,

$x = 1$

$y = 0$

So co-ordinates of P is (1, 0)

Similarly mid-point Q of side BC can be written as,

$$Q(x, y) = \left(\frac{2+0}{2}, \frac{3+1}{2}\right)$$

Now equate the individual terms to get,

$x = 1$

$y = 2$

So co-ordinates of Q is (1, 2)

Similarly mid-point R of side AC can be written as,

$$R(x, y) = \left(\frac{0+0}{2}, \frac{3-1}{2}\right)$$

Now equate the individual terms to get,

$$x = 0$$

$$y = 1$$

So co-ordinates of R is (0, 1)

Therefore length of median from A to the side BC is,

$$\begin{aligned} \mathrm{AQ} &= \sqrt{(0-1)^2 + (-1-2)^2} \\ &= \sqrt{1+9} \\ &= \boxed{\sqrt{10}} \end{aligned}$$

Similarly length of median from B to the side AC is,

$$\begin{aligned} \mathrm{BR} &= \sqrt{(2-0)^2 + (1-1)^2} \\ &= \sqrt{4} \\ &= \boxed{2} \end{aligned}$$

Similarly length of median from C to the side AB is

$$\begin{aligned} \mathrm{CP} &= \sqrt{(0-1)^2 + (3-0)^2} \\ &= \sqrt{1+9} \\ &= \boxed{\sqrt{10}} \end{aligned}$$

*********** END **********