

Indefinite Integrals Ex 19.30 Q22

Let
$$I = \int \frac{dx}{x \left[6 \left(\log x\right)^2 + 7 \log x + 2\right]}$$
$$= \int \frac{1}{x \left(2 \log x + 1\right) \left(3 \log x + 2\right)} dx$$

Now,

Let
$$\frac{1}{x(2\log x + 1)(3\log x + 2)} = \frac{A}{x(2\log x + 1)} + \frac{B}{x(3\log x + 2)}$$

$$\Rightarrow 1 = A (3 \log x + 2) + B (2 \log x + 1)$$

Put
$$x = 10^{-\frac{1}{2}}$$

$$\Rightarrow 1 = \frac{1}{2}A \Rightarrow A = 2$$

Put
$$x = 10^{-\frac{2}{3}}$$

$$\Rightarrow 1 = -\frac{1}{3}B \Rightarrow B = -3$$

$$I = \int \frac{2dx}{x \left(2\log x + 1\right)} - \int \frac{3dx}{x \left(3\log x + 2\right)}$$
$$= \log \left|2\log x + 1\right| - \log \left|3\log x + 2\right| + c$$

$$\therefore I = \log \left| \frac{2 \log x + 1}{3 \log x + 2} \right| + c$$

Indefinite Integrals Ex 19.30 Q23

$$\frac{1}{x(x^n+1)}$$

Multiplying numerator and denominator by x^{n-1} , we obtain

$$\frac{1}{x(x^{n}+1)} = \frac{x^{n-1}}{x^{n-1}x(x^{n}+1)} = \frac{x^{n-1}}{x^{n}(x^{n}+1)}$$

Let $x^n = t \implies x^{n-1}dx = dt$

$$\therefore \int \frac{1}{x(x^n+1)} dx = \int \frac{x^{n-1}}{x^n(x^n+1)} dx = \frac{1}{n} \int \frac{1}{t(t+1)} dt$$

Let
$$\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{(t+1)}$$

 $1 = A(1+t) + Bt$...(1)

Substituting t = 0, -1 in equation (1), we obtain

$$A = 1$$
 and $B = -1$

$$\therefore \frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{(1+t)}$$

$$\Rightarrow \int \frac{1}{x(x^n+1)} dx = \frac{1}{n} \int \left\{ \frac{1}{t} - \frac{1}{(t+1)} \right\} dx$$

$$= \frac{1}{n} \left[\log|t| - \log|t+1| \right] + C$$

$$= -\frac{1}{n} \left[\log|x^n| - \log|x^n+1| \right] + C$$

$$= \frac{1}{n} \log\left| \frac{x^n}{x^n+1} \right| + C$$

Indefinite Integrals Ex 19.30 Q24

Let
$$\int \frac{x}{(x^2 - a^2)(x^2 - b^2)} = \frac{Ax + B}{(x^2 - a^2)} + \frac{Cx + D}{(x^2 - b^2)}$$

$$\Rightarrow \qquad x = (Ax + B)(x^2 - b^2) + (Cx + D)(x^2 - a^2)$$

$$x = (A + C)x^3 + (B + D)x^2 + (-Ab^2 - Ca^2)x + (-Bb^2 - Da^2)$$

$$\Rightarrow \qquad A + C = 0, \ B + D = 0, \ -Ab^2 - Ca^2 = 1, \ -Bb^2 - Da^2 = 0$$
We get $B = 0, \ D = 0, \ C = \frac{1}{b^2 - a^2}, \ A = -\frac{1}{b^2 - a^2}$

Thus,

$$I = -\frac{1}{b^2 - a^2} \int \frac{x dx}{x^2 - a^2} + \frac{1}{b^2 - a^2} \int \frac{x dx}{x^2 - b^2}$$

$$I = -\frac{1}{2\left(b^2 - a^2\right)} \log \left|x^2 - a^2\right| + \frac{1}{2\left(b^2 - a^2\right)} \log \left|x^2 - b^2\right| + c$$

Indefinite Integrals Ex 19.30 Q25

Consider the integral

$$I = \int \frac{x^2 + 1}{(x^2 + 4)(x^2 + 25)} dx$$

Let $y=x^2$

Thus,

$$\frac{x^2 + 1}{(x^2 + 4)(x^2 + 25)} = \frac{y + 1}{(y + 4)(y + 25)}$$

$$\Rightarrow \frac{y + 1}{(y + 4)(y + 25)} = \frac{A}{y + 4} + \frac{B}{y + 25}$$

$$\Rightarrow \frac{y + 1}{(y + 4)(y + 25)} = \frac{A(y + 25) + B(y + 4)}{(y + 4)(y + 25)}$$

 \Rightarrow y + 1=Ay+25A+By+4B

Comparing the coefficients, we have,

A+B=1 and 25A+4B=1

Solving the above equations, we have,

$$A = \frac{-1}{7}$$
 and $B = \frac{8}{7}$

Thus,
$$\int \frac{x^2 + 1}{(x^2 + 4)(x^2 + 25)} dx$$

$$= \int \frac{\frac{-1}{7}}{x^2 + 4} dx + \int \frac{\frac{\circ}{7}}{x^2 + 25} dx$$
$$= \frac{-1}{7} \int \frac{1}{x^2 + 4} dx + \frac{8}{7} \int \frac{1}{x^2 + 25} dx$$

$$= \frac{-1}{7} \times \frac{1}{2} \tan^{-1} \frac{x}{2} + \frac{8}{7} \times \frac{1}{5} \tan^{-1} \frac{x}{5} + C$$

$$= \frac{-1}{14} \tan^{-1} \frac{x}{2} + \frac{8}{35} \tan^{-1} \frac{x}{5} + C$$

Indefinite Integrals Ex 19.30 Q26

Let
$$I = \int \frac{x^3 + x + 1}{x^2 - 1} dx$$
$$= \int \left(x + \frac{2x + 1}{x^2 - 1}\right) dx$$

Let
$$\frac{2x+1}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}$$

$$\Rightarrow 2x+1=A\left(x-1\right)+B\left(x+1\right)$$

Put *x* = 1

$$\Rightarrow 3 = 2B \Rightarrow B = \frac{3}{2}$$

Put
$$x = -1$$

$$\Rightarrow -1 = -2A \Rightarrow A = \frac{1}{2}$$

$$I = \int x dx + \frac{1}{2} \int \frac{dx}{x+1} + \frac{3}{2} \int \frac{dx}{x-1}$$

$$I = \frac{x^2}{2} + \frac{1}{2} \log |x + 1| + \frac{3}{2} \log |x - 1| + c$$

********** END ********