Exercise 2.5

Q6. Write the following cubes in expanded form:

(i) $(2x+1)^3$

(ii) $(2a-3b)^3$

(iii) $\left(\frac{3}{2}x+1\right)^3$

(iv) $\left(x-\frac{2}{3}y\right)^3$

Ans: (i) $(2x+1)^3$

We know that $(x+y)^3 = x^3+y^3+3xy(x+y)$.

$$\begin{aligned}\therefore (2x+1)^3 &= (2x)^3+(1)^3+3\times 2x\times 1(2x+1)\\ &= 8x^3+1+6x(2x+1)\\ &= 8x^3+12x^2+6x+1.\end{aligned}$$

Therefore, the expansion of the expression $(2x+1)^3$ is $8x^3+12x^2+6x+1$.

(ii) $(2a-3b)^3$

We know that $(x-y)^3 = x^3-y^3-3xy(x-y)$.

$$\begin{aligned}\therefore (2a-3b)^3 &= (2a)^3-(3b)^3-3\times 2a\times 3b(2a-3b)\\ &= 8a^3-27b^3-18ab(2a-3b)\\ &= 8a^3-36a^2b+54ab^2-27b^3.\end{aligned}$$

Therefore, the expansion of the expression $(2a-3b)^3$ is $8a^3-36a^2b+54ab^2-27b^3$.

(iii) $\left(\frac{3}{2}x+1\right)^3$

We know that $(x+y)^3 = x^3+y^3+3xy(x+y)$

We know that $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$.

$$\left(\frac{3}{2}x+1\right)^3 = \left(\frac{3}{2}x\right)^3 + (1)^3 + 3\times\frac{3}{2}x\times1\left(\frac{3}{2}x+1\right) \therefore$$

$$= \frac{27}{8}x^3 + 1 + \frac{9}{2}x\left(\frac{3}{2}x+1\right)$$

$$= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1.$$

Therefore, the expansion of the expression

$\left(\frac{3}{2}x+1\right)^3$ is $\frac{27}{8}x^3+\frac{27}{4}x^2+\frac{9}{2}x+1$.

(iv) $\left(x-\frac{2}{3}y\right)^3$

We know that $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$.

$$\therefore \left(x-\frac{2}{3}y\right)^3 = (x)^3 - \left(\frac{2}{3}y\right)^3 - 3\times x\times\frac{2}{3}y\left(x-\frac{2}{3}y\right)$$

$$= x^3 - \frac{8}{27}y^3 - 2xy\left(x-\frac{2}{3}y\right)$$

$$= x^3 - 2x^2y + \frac{4}{3}xy^2 - \frac{8}{27}y^3.$$

Therefore, the expansion of the expression

$\left(x-\frac{2}{3}y\right)^3$ is $x^3 - 2x^2y + \frac{4}{3}xy^2 - \frac{8}{27}y^3$.

Q7. Evaluate the following using suitable identities:

(i) $(99)^3$

(ii) $(102)^3$

(iii) $(998)^3$

Ans: (i) $(99)^3$

$(99)^3$ can also be written as $(100-1)^3$.

Using identity, $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$(100-1)^3 = (100)^3 - (1)^3 - 3\times100\times1(100-1)$

=1000000-1-300(99)

=999999-29700

=970299

(ii) $(102)^3$

$(102)^3$ can also be written as $(100+2)^3$.

Using identity $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

$(100+2)^3 = (100)^3 + (2)^3 + 3\times 100\times 2(100+2)$

=1000000+8+600(102)

=1000008+61200

=1061208

(iii) $(998)^3$

$(998)^3$ can also be written as $(1000-2)^3$.

Using identity $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$(1000-2)^3 = (1000)^3 - (2)^3 - 3\times 1000\times 2(1000-2)$

=1000000000-8-6000(998)

=999999992-5988000

=994011992

Q8. Factorize each of the following:

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$ (ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$ (iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Ans: (i) $8a^3 + b^3 + 12a^2b + 6ab^2$

The expression $8a^3 + b^3 + 12a^2b + 6ab^2$ can also be written as

$= (2a)^3 + (b)^3 + 3\times 2a\times 2a\times b + 3\times 2a\times b\times b$

$= (2a)^3 + (b)^3 + 3\times 2a\times b(2a+b)$.

Using identity $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$ with respect to the expression $(2a)^3 + (b)^3 + 3\times 2a\times b(2a+b)$, we get $(2a+b)^3$.

Therefore, after factorizing the expression $8a^3 + b^3 + 12a^2b + 6ab^2$, we get $(2a+b)^3$.

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

The expression $8a^3 - b^3 - 12a^2b + 6ab^2$ can also be written as

$= (2a)^3 - (b)^3 - 3\times 2a\times 2a\times b + 3\times 2a\times b\times b$

$= (2a)^3 - (b)^3 - 3\times 2a\times b(2a-b).$

Using identity $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$ with respect to the expression $(2a)^3 - (b)^3 - 3\times 2a\times b(2a-b)$, we get $(2a-b)^3$.

Therefore, after factorizing the expression $8a^3 - b^3 - 12a^2b + 6ab^2$, we get $(2a-b)^3$.

(iii) $27 - 125a^3 - 135a + 225a^2$

The expression $27 - 125a^3 - 135a + 225a^2$ can also be written as

$= (3)^3 - (5a)^3 - 3\times 3\times 3\times 5a + 3\times 3\times 5a\times 5a$

$= (3)^3 - (5a)^3 + 3\times 3\times 5a(3-5a).$

Using identity $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$ with respect to the expression $(3)^3 - (5a)^3 + 3\times 3\times 5a(3-5a)$, we get $(3-5a)^3$.

Therefore, after factorizing the expression $27 - 125a^3 - 135a + 225a^2$, we get $(3-5a)^3$.

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

The expression $64a^3 - 27b^3 - 144a^2b + 108ab^2$ can also be written as

$= (4a)^3 - (3b)^3 - 3\times 4a\times 4a\times 3b + 3\times 4a\times 3b\times 3b$

$= (4a)^3 - (3b)^3 - 3\times 4a\times 3b(4a-3b).$

Using identity $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$ with respect to the expression $(4a)^3 - (3b)^3 - 3\times 4a\times 3b(4a-3b)$, we get $(4a-3b)^3$.

Therefore, after factorizing the expression $64a^3 - 27b^3 - 144a^2b + 108ab^2$, we get $(4a-3b)^3$.

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

The expression $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$ can also be written as

$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3\times 3p \times 3p \times \frac{1}{6} + 3\times 3p \times \frac{1}{6} \times \frac{1}{6}$

$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3\times 3p \times \frac{1}{6}\left(3p - \frac{1}{6}\right).$

Using identity $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$ with respect to the expression

$(3p)^3 - \left(\frac{1}{6}\right)^3 - 3\times 3p \times \frac{1}{6}\left(3p - \frac{1}{6}\right)$, to get $\left(3p - \frac{1}{6}\right)^3$.

Therefore, after factorizing the expression

$27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$, we get $\left(3p - \frac{1}{6}\right)^3$.

Q9. Verify:

(i) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

(ii) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

Ans: (i) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

We know that $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$.

$\Rightarrow x^3 + y^3 = (x+y)^3 - 3xy(x+y)$

$= (x+y)\left[(x+y)^2 - 3xy\right]$

$\because$ We know that $(x+y)^2 = x^2 + 2xy + y^2$

$\therefore\ x^3 + y^3 = (x+y)(x^2 + 2xy + y^2 - 3xy)$

$= (x+y)(x^2 - xy + y^2)$

Therefore, the desired result has been verified.

(ii) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

We know that $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$.

$\Rightarrow x^3 - y^3 = (x-y)^3 + 3xy(x-y)$

$= (x-y)\left[(x-y)^2+3xy\right]$

$\because$ We know that $(x-y)^2 = x^2-2xy+y^2$

$\therefore x^3-y^3 = (x-y)(x^2-2xy+y^2+3xy)$

$= (x-y)(x^2+xy+y^2)$

Therefore, the desired result has been verified.

Q10. Factorize:

(i) $27y^3+125z^3$

(ii) $64m^3-343n^3$

Ans: (i) $27y^3+125z^3$

The expression $27y^3+125z^3$ can also be written as $(3y)^3+(5z)^3$.

We know that $x^3+y^3=(x+y)(x^2-xy+y^2)$.

$(3y)^3+(5z)^3=(3y+5z)\left[(3y)^2-3y\times 5z+(5z)^2\right]$

$=(3y+5z)(9y^2-15yz+25z^2)$.

(ii) $64m^3-343n^3$

The expression $64m^3-343n^3$ can also be written as $(4m)^3-(7n)^3$.

We know that $x^3-y^3=(x-y)(x^2+xy+y^2)$.

$(4m)^3-(7n)^3=(4m-7n)\left[(4m)^2+4m\times 7n+(7n)^2\right]$

$=(4m-7n)(16m^2+28mn+49n^2)$

.

Therefore, we conclude that after factorizing the expression $64m^3-343n^3$, we get $(4m-7n)(16m^2+28mn+49n^2)$.

Q11. Factorize: $27x^3+y^3+z^3-9xyz$

Ans:

The expression $27x^3+y^3+z^3-9xyz$ can also be written as

$(3x)^3+(y)^3+(z)^3-3\times 3x\times y\times z.$

We know that
$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$.

$$\therefore (3x)^3+(y)^3+(z)^3-3\times 3x\times y\times z=(3x+y+z)\left[(3x)^2+(y)^2+(z)^2-3x\times y-y\times z-z\times 3x\right]$$
$$=(3x+y+z)(9x^2+y^2+z^2-3xy-yz-3xz).$$

Therefore, we conclude that after factorizing the expression $27x^3+y^3+z^3-9xyz$, we get
$(3x+y+z)(9x^2+y^2+z^2-3xy-yz-3xz)$.

Q12. Verify that
$x^3+y^3+z^3-3xyz=\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$

Ans:

LHS is $x^3+y^3+z^3-3xyz$ and RHS is $\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$.

We know that
$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$
.

And also, we know that $(x-y)^2=x^2-2xy+y^2$.

$$\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$$
$$\frac{1}{2}(x+y+z)\left[(x^2-2xy+y^2)+(y^2-2yz+z^2)+(z^2-2xz+x^2)\right]$$
$$\frac{1}{2}(x+y+z)(2x^2+2y^2+2z^2-2xy-2yz-2zx)$$
$$(x+y+z)(x^2+y^2+z^2-xy-yz-zx).$$

Therefore, we can conclude that the desired result is verified.

Q13. If $x+y+z=0$, show that $x^3+y^3+z^3=0$.

Ans: We know that
$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$.

We need to substitute $x^3+y^3+z^3=0$ in
$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$,

to get

$x^3+y^3+z^3-3xy$ =(0)=$\left(x^2+y^2+z^2-xy-yz-zx\right)$,

$x^3+y^3+z^3-3xyz=0$

$\Rightarrow x^3+y^3+z^3=3xyz.$

Therefore, the desired result is verified.

Q14. Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3+(7)^3+(5)^3$

(ii) $(28)^3+(-15)^3+(-13)^3$

Ans: (i) $(-12)^3+(7)^3+(5)^3$

Let $a=-12, b=7$ and $c=5$

We know that, if $a+b+c=0,$ then $a^3+b^3+c^3=3abc$

Here, $a+b+c=-12+7+5=0$

$\therefore (-12)^3+(7)^3+(5)^3=3(-12)(7)(5)$

$=-1260$

(ii) $(28)^3+(-15)^3+(-13)^3$

Let $a=28, b=-15$ and $c=-13$

We know that, if $a+b+c=0,$ then $a^3+b^3+c^3=3abc$

Here, $a+b+c=28-15-13=0$

$\therefore (28)^3+(-15)^3+(-13)^3=3(28)(-15)(-13)$

$=16380$

Q15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) $Are: 25a^2-35a+12$

(ii) $Are: 35y^2+13y-12$

Ans: (i) Area : $25a^2-35a+12$

The expression $25a^2-35a+12$ can also be written as $25a^2-15a-20a+12$.

$25a^2-15a-20a+12=5a(5a-3)-4(5a-3)$

$=(5a-4)(5a-3)$.

Therefore, we can conclude that a possible expression for the length and breadth of a rectangle of area $25a^2-35a+12$ is
Length $=(5a-4)$ and Breadth $=(5a-3)$.

(ii) Area : $35y^2+13y-12$

The expression $35y^2+13y-12$ can also be written as $35y^2+28y-15y-12$.

$35y^2+28y-15y-12=7y(5y+4)-3(5y+4)$

$=(7y-3)(5y+4)$.

Therefore, we can conclude that a possible expression for the length and breadth of a rectangle of area $35y^2+13y-12$ is
Length $=(7y-3)$ and Breadth $=(5y+4)$.

Q16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) $Volume: 3x^2-12x$

(ii) $Volume: 12ky^2+8ky-20k$

Ans: (i) Volume : $3x^2-12x$

The expression $3x^2-12x$ can also be written as $3\times x\times(x-4)$.

Therefore, we can conclude that a possible expression for the dimension of a cuboid of volume $3x^2-12x$ is $3, x$ and $(x-4)$.

(ii) Volume : $12ky^2+8ky-20k$

The expression $12ky^2+8ky-20k$ can also be written as $k(12y^2+8y-20)$.

$= k\left[12y(y-1) + 20(y-1)\right]$

$= k(12y+20)(y-1)$

$= 4k \times (3y+5) \times (y-1)$.

Therefore, we can conclude that a possible expression for the dimension of a cuboid of volume $12ky^2 + 8ky - 20k$ is $4k, (3y+5)$ and $(y-1)$.

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