Indefinite Integrals Ex 19.14 Q1

Let $I = \int \dfrac{1}{a^2 - b^2 x^2} \, dx$

$= \dfrac{1}{b^2} \int \dfrac{1}{\dfrac{a^2}{b^2} - x^2} \, dx$

$= \dfrac{1}{b^2} \int \dfrac{1}{\left(\dfrac{a}{b}\right)^2 - x^2} \, dx$

$I = \dfrac{1}{b^2} \times \dfrac{1}{2 \times \left(\dfrac{a}{b}\right)} \log \left| \dfrac{\dfrac{a}{b} + x}{\dfrac{a}{b} - x} \right| + c \qquad \left[\text{Since } \int \dfrac{1}{a^2 - x^2} \, dx = \dfrac{1}{2a} \log \left| \dfrac{x+a}{x-a} \right| + c \right]$

$I = \dfrac{1}{2ab} \log \left| \dfrac{a + bx}{a - bx} \right| + c$

Indefinite Integrals Ex 19.14 Q2

Let $I = \int \dfrac{1}{a^2 x^2 - b^2} \, dx$

$= \dfrac{1}{a^2} \int \dfrac{1}{x^2 - \dfrac{b^2}{a^2}} \, dx$

$= \dfrac{1}{a^2} \int \dfrac{1}{x^2 - \left(\dfrac{b}{a}\right)^2} \, dx$

$I = \dfrac{1}{a^2} \times \dfrac{1}{2 \times \left(\dfrac{b}{a}\right)} \times \log \left| \dfrac{x - \dfrac{b}{a}}{x + \dfrac{b}{a}} \right| + c \qquad \left[\text{Since } \int \dfrac{1}{x^2 - a^2} \, dx = \dfrac{1}{2a} \log \left| \dfrac{x-a}{x+a} \right| + c \right]$

$I = \dfrac{1}{2ab} \log \left| \dfrac{ax - b}{ax + b} \right| + c$

Indefinite Integrals Ex 19.14 Q3

Let $I = \int \dfrac{1}{a^2 x^2 + b^2} \, dx$

$= \dfrac{1}{a^2} \int \dfrac{1}{x^2 + \dfrac{b^2}{a^2}} \, dx$

$= \dfrac{1}{a^2} \int \dfrac{1}{x^2 + \left(\dfrac{b}{a}\right)^2} \, dx$

$I = \dfrac{1}{a^2} \times \dfrac{1}{\left(\dfrac{b}{a}\right)} \tan^{-1} \left(\dfrac{x}{\dfrac{b}{a}} \right) + c \qquad \left[\text{Since } \int \dfrac{1}{x^2 + a^2} \, dx = \dfrac{1}{a} \tan^{-1} \left(\dfrac{x}{a} \right) + c \right]$

$I = \dfrac{1}{ab} \tan^{-1} \left(\dfrac{ax}{b} \right) + c$

Indefinite Integrals Ex 19.14 Q4

Let $I = \int \dfrac{x^2 - 1}{x^2 - 4} dx$

Let $I = \int \dfrac{x^2 - 1}{x^2 + 4} dx$

$= \int \dfrac{(x^2 + 4) - 4 - 1}{x^2 + 4} dx$

$= \int \dfrac{x^2 + 4}{x^2 + 4} dx - \int \dfrac{5}{x^2 + 4} dx$

$= \int dx - 5\int \dfrac{1}{x^2 + (2)^2} dx$

$I = x - 5 \times \dfrac{1}{2} \tan^{-1}\left(\dfrac{x}{2}\right) + c$ $\qquad \left[\text{Since } \int \dfrac{1}{x^2 + a^2} dx = \dfrac{1}{a} \tan^{-1}\left(\dfrac{x}{a}\right)\right]$

$I = x - \dfrac{5}{2} \tan^{-1}\left(\dfrac{x}{2}\right) + c$

Indefinite Integrals Ex 19.14 Q5

Let $2x = t$

$\Rightarrow 2dx = dt$

$\Rightarrow \int \dfrac{1}{\sqrt{1 + 4x^2}} dx = \dfrac{1}{2} \int \dfrac{dt}{\sqrt{1 + t^2}}$

$= \dfrac{1}{2}\left[\log\left|t + \sqrt{t^2 + 1}\right|\right] + C$ $\qquad \left[\int \dfrac{1}{\sqrt{x^2 + a^2}} dt = \log\left|x + \sqrt{x^2 + a^2}\right|\right]$

$= \dfrac{1}{2} \log\left|2x + \sqrt{4x^2 + 1}\right| + C$

********** END **********