



Indefinite Integrals Ex 19.30 Q57

$$\begin{aligned}\text{Let } I &= \int \frac{dx}{\cos x (5 - 4 \sin x)} \\ &= \int \frac{\cos x dx}{\cos^2 x (5 - 4 \sin x)} \\ &= \int \frac{\cos x dx}{(1 - \sin^2 x)(5 - 4 \sin x)}\end{aligned}$$

$$\text{Let } \sin x = t \Rightarrow \cos x dx = dt$$

$$\therefore I = \int \frac{dt}{(1 - t^2)(5 - 4t)}$$

Now,

$$\text{Let } \frac{1}{(1 - t^2)(5 - 4t)} = \frac{A}{1 - t} + \frac{B}{1 + t} + \frac{C}{5 - 4t}$$

$$\Rightarrow 1 = A(1 + t)(5 - 4t) + B(1 - t)(5 - 4t) + C(1 - t^2)$$

$$\text{Put } t = 1$$

$$\Rightarrow 1 = 2A \Rightarrow A = \frac{1}{2}$$

$$\text{Put } t = -1$$

$$\Rightarrow 1 = 18B \Rightarrow B = \frac{1}{18}$$

$$\text{Put } t = \frac{5}{4}$$

$$\Rightarrow 1 = -\frac{9C}{16} \Rightarrow C = -\frac{16}{9}$$

Thus,

$$\begin{aligned}I &= \frac{1}{2} \int \frac{dt}{1 - t} + \frac{1}{18} \int \frac{dt}{1 + t} - \frac{16}{9} \int \frac{dt}{5 - 4t} \\ &= -\frac{1}{2} \log|1 - t| + \frac{1}{18} \log|1 + t| + \frac{4}{9} \log|5 - 4t| + c\end{aligned}$$

Hence,

$$I = -\frac{1}{2} \log|1 - \sin x| + \frac{1}{18} \log|1 + \sin x| + \frac{4}{9} \log|5 - 4 \sin x| + c$$

Indefinite Integrals Ex 19.30 Q58

$$\begin{aligned}
 \text{Let } I &= \int \frac{1}{\sin x (3 + 2 \cos x)} dx \\
 &= \int \frac{\sin x dx}{\sin^2 x (3 + 2 \cos x)} \\
 &= \int \frac{\sin x dx}{(1 - \cos^2 x) (3 + 2 \cos x)}
 \end{aligned}$$

$$\begin{aligned}
 \text{Let } \cos x &= t \\
 \Rightarrow -\sin x dx &= dt
 \end{aligned}$$

$$\therefore I = \int \frac{dt}{(t^2 - 1)(3 + 2t)}$$

Now,

$$\text{Let } \frac{1}{(t^2 - 1)(3 + 2t)} = \frac{A}{t - 1} + \frac{B}{t + 1} + \frac{C}{3 + 2t}$$

$$\Rightarrow 1 = A(t + 1)(3 + 2t) + B(t - 1)(3 + 2t) + C(t^2 - 1)$$

$$\text{Put } t = 1$$

$$\Rightarrow 1 = 10A \Rightarrow A = \frac{1}{10}$$

$$\text{Put } t = -1$$

$$\Rightarrow 1 = -2B \Rightarrow B = -\frac{1}{2}$$

$$\text{Put } t = -\frac{3}{2}$$

$$\Rightarrow 1 = \frac{5}{4}C \Rightarrow C = \frac{4}{5}$$

Thus,

$$\begin{aligned}
 I &= \frac{1}{10} \int \frac{dt}{t - 1} - \frac{1}{2} \int \frac{dt}{t + 1} + \frac{5}{4} \int \frac{dt}{3 + 2t} \\
 &= \frac{1}{10} \log|t - 1| - \frac{1}{2} \log|t + 1| + \frac{2}{5} \log|3 + 2t| + c
 \end{aligned}$$

Hence,

$$I = \frac{1}{10} \log|\cos x - 1| - \frac{1}{2} \log|\cos x + 1| + \frac{2}{5} \log|3 + 2 \cos x| + c$$

Indefinite Integrals Ex 19.30 Q59

$$\begin{aligned}
 \text{Let } I &= \int \frac{1}{\sin x + \sin 2x} dx \\
 &= \int \frac{dx}{\sin x + 2 \sin x \cos x} \\
 &= \int \frac{\sin x dx}{(1 - \cos^2 x) + 2(1 - \cos^2 x) \cos x}
 \end{aligned}$$

$$\text{Let } \cos x = t \Rightarrow -\sin x dx = dt$$

$$\begin{aligned}
 \therefore I &= \int \frac{dt}{(t^2 - 1) + 2(t^2 - 1)t} \\
 &= \int \frac{dt}{(t^2 - 1)(1 + 2t)}
 \end{aligned}$$

$$\text{Let } \int \frac{1}{(t^2 - 1)(1 + 2t)} = \frac{A}{t - 1} + \frac{B}{t + 1} + \frac{C}{1 + 2t}$$

$$\Rightarrow 1 = A(t + 1)(1 + 2t) + B(t - 1)(1 + 2t) + C(t^2 - 1)$$

$$\text{Put } t = 1$$

$$\Rightarrow 1 = 6A \Rightarrow A = \frac{1}{6}$$

$$\text{Put } t = -1$$

$$\Rightarrow 1 = 2B \Rightarrow B = \frac{1}{2}$$

$$\text{Put } t = -\frac{1}{2}$$

$$\Rightarrow 1 = -\frac{3}{4}C \Rightarrow C = -\frac{4}{3}$$

Thus,

$$\begin{aligned}
 I &= \frac{1}{6} \int \frac{dt}{t - 1} + \frac{1}{2} \int \frac{dt}{t + 1} - \frac{4}{3} \int \frac{dt}{1 + 2t} \\
 &= \frac{1}{6} \log|t - 1| + \frac{1}{2} \log|t + 1| - \frac{2}{3} \log|1 + 2t| + c
 \end{aligned}$$

Hence,

$$I = \frac{1}{6} \log|\cos x - 1| + \frac{1}{2} \log|\cos x + 1| - \frac{2}{3} \log|1 + 2 \cos x| + c$$

Indefinite Integrals Ex 19.30 Q60

$$\text{Let } I = \int \frac{x+1}{x(1+xe^x)} dx$$

$$= \int \frac{(x+1)(1+xe^x - xe^x)}{x(1+xe^x)} dx$$

$$= \int \frac{(x+1)(1+xe^x)}{x(1+xe^x)} dx - \int \frac{(x+1)(xe^x)}{x(1+xe^x)} dx$$

$$= \int \frac{(x+1)}{x} dx - \int \frac{e^x(x+1)}{1+xe^x} dx$$

$$= \int \frac{(x+1)e^x}{xe^x} dx - \int \frac{e^x(x+1)}{1+xe^x} dx$$

$$= \log|xe^x| - \log|1+xe^x| + c$$

$$\therefore I = \log \left| \frac{xe^x}{1+xe^x} \right| + c$$

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