



Adjoint and Inverse of Matrix Ex 7.1 Q9(i)

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

Expanding using 1st row, we get

$$\begin{aligned} |A| &= 1 \begin{vmatrix} 4 & 3 \\ 3 & 4 \end{vmatrix} - 3 \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} + 3 \begin{vmatrix} 1 & 4 \\ 1 & 3 \end{vmatrix} \\ &= 1(16 - 9) - 3(4 - 3) + 3(3 - 4) \\ &= 7 - 3(1) + 3(-1) \\ &= 7 - 3 - 3 = +1 = 1 \neq 0 \end{aligned}$$

Therefore, A^{-1} exists

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 7 & C_{21} = -3 & C_{31} = -3 \\ C_{12} = -1 & C_{22} = 1 & C_{32} = -0 \\ C_{13} = -1 & C_{23} = -0 & C_{33} = 1 \end{array}$$

$$\begin{aligned} \therefore \text{adj } A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} 7 & -1 & -1 \\ -3 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}^T \\ &= \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$\text{Now, } A^{-1} = \frac{1}{|A|} \cdot \text{adj } A$$

$$\therefore A^{-1} = \frac{1}{1} \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\text{Also, } A^{-1}.A = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q9(ii)

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$$

Expanding 1st row, we get

$$\begin{aligned} |A| &= 2 \begin{vmatrix} 4 & 1 \\ 7 & 2 \end{vmatrix} - 3 \begin{vmatrix} 3 & 1 \\ 3 & 2 \end{vmatrix} + 1 \begin{vmatrix} 3 & 4 \\ 3 & 7 \end{vmatrix} \\ &= 2(8-7) - 3(6-3) + 1(21-12) \\ &= 2 - 3(3) + 1(9) = 2 \neq 0 \end{aligned}$$

Therefore, A^{-1} exists

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 1 & C_{21} = +1 & C_{31} = -1 \\ C_{12} = -3 & C_{22} = 1 & C_{32} = +1 \\ C_{13} = 9 & C_{23} = -5 & C_{33} = -1 \end{array}$$

$$\begin{aligned} \therefore \text{adj } A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} 1 & -3 & 9 \\ 1 & 1 & -5 \\ -1 & 1 & -1 \end{bmatrix}^T = \begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix} \end{aligned}$$

$$\text{Now, } A^{-1} = \frac{1}{|A|} \cdot \text{adj } A \Rightarrow A^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$$

$$\begin{aligned} \text{Also, } A^{-1} \cdot A &= \frac{1}{2} \begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 2+3-3 & 3+4-7 & 1+1-2 \\ -6+3+3 & -9+4+7 & -3+1+2 \\ 18-15-3 & 27-20-7 & 9-5-2 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$\therefore A^{-1} \cdot A = I_3$$

Adjoint and Inverse of Matrix Ex 7.1 Q10(i)

$$A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} \quad \therefore |A| = 1 \neq 0$$

$$\text{adj } A = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} \Rightarrow A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{1} \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 & 6 \\ 3 & 2 \end{bmatrix} \quad \therefore |B| = -10 \neq 0$$

$$\text{adj } B = \begin{bmatrix} 2 & -6 \\ -3 & 4 \end{bmatrix} \Rightarrow B^{-1} = \frac{\text{adj } B}{|B|} = \frac{1}{-10} \begin{bmatrix} 2 & -6 \\ -3 & 4 \end{bmatrix}$$

$$\text{Also, } AB = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} \begin{bmatrix} 4 & 6 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 18 & 22 \\ 43 & 52 \end{bmatrix}$$

$$|AB| = 936 - 946 = -10 \neq 0$$

$$\text{adj}(AB) = \begin{bmatrix} 52 & -22 \\ -43 & 18 \end{bmatrix}$$

$$(AB)^{-1} = \frac{\text{adj}(AB)}{|AB|} = \frac{1}{|AB|} \begin{bmatrix} 52 & -22 \\ -43 & 18 \end{bmatrix} = \frac{1}{-10} \begin{bmatrix} +52 & -22 \\ -43 & +18 \end{bmatrix} = \frac{1}{10} \begin{bmatrix} -52 & 22 \\ 43 & -18 \end{bmatrix}$$

$$B^{-1}.A^{-1} = \frac{-1}{10} \begin{bmatrix} 2 & -6 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} = \frac{-1}{10} \begin{bmatrix} 52 & -22 \\ -43 & 18 \end{bmatrix} = \frac{1}{10} \begin{bmatrix} -52 & 22 \\ 43 & -18 \end{bmatrix}$$

$$\text{Hence, } (AB)^{-1} = B^{-1}.A^{-1}$$

Adjoint and Inverse of Matrix Ex 7.1 Q10(ii)

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \quad \therefore |A| = 1 \neq 0 \text{ and } \text{adj } A = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{1} \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} \quad \therefore |B| = -1 \neq 0 \text{ and } \text{adj } B = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$$

$$\therefore B^{-1} = \frac{\text{adj } B}{|B|} = \frac{1}{-1} \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$$

$$\text{Also, } AB = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 11 & 14 \\ 29 & 27 \end{bmatrix}$$

$$|AB| = 407 - 406 = 1 \neq 0$$

$$\text{and, } \text{adj}(AB) = \begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix}$$

$$\begin{aligned} \therefore (AB)^{-1} &= \frac{1}{|AB|} \cdot \text{adj}(AB) \\ &= \frac{1}{1} \begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix} = \begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{Again, } B^{-1} \cdot A^{-1} &= \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix} \end{aligned}$$

$$\text{Hence, } (AB)^{-1} = B^{-1} \cdot A^{-1}$$

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