



Binary Operations Ex 3.2 Q9

The binary operator $*$ defined as

$$a * b = \frac{a - b}{2} \text{ for all } a, b \in \mathbb{Q}.$$

Now,

Associativity: Let $a, b, c \in \mathbb{Q}$, then

$$\begin{aligned} (a * b) * c &= \frac{a - b}{2} * c = \frac{\frac{a - b}{2} - c}{2} \\ &= \frac{a - b - 2c}{4} \end{aligned} \quad \text{--- (i)}$$

$$\begin{aligned} \text{and, } a * (b * c) &= a * \frac{b - c}{2} = \frac{a - \frac{b - c}{2}}{2} \\ &= \frac{2a - b + c}{4} \end{aligned} \quad \text{--- (ii)}$$

From (i) & (ii)

$$(a * b) * c \neq a * (b * c)$$

Hence, ' $*$ ' is not associative on \mathbb{Q} .

Binary Operations Ex 3.2 Q10

The binary operator $*$ defined as

$$a * b = a + 3b - 4 \text{ for all } a, b \in \mathbb{Z}$$

Now,

Commutativity: Let $a, b \in \mathbb{Z}$, then

$$a * b = a + 3b - 4 \neq b + 3a - 4 = b * a$$

$$\Rightarrow a * b \neq b * a$$

$$\Rightarrow \text{'*'} \text{ is not commutative on } \mathbb{Z}.$$

Associativity: Let $a, b, c \in \mathbb{Z}$, then

$$\begin{aligned} (a * b) * c &= (a + 3b - 4) * c = a + 3b - 4 + 3c - 4 \\ &= a + 3b + 3c - 8 \end{aligned} \quad \text{--- (i)}$$

$$\begin{aligned} \text{and, } a * (b * c) &= a * (b + 3c - 4) = a + 3(b + 3c - 4) - 4 \\ &= a + 3b + 9c - 16 \end{aligned} \quad \text{--- (ii)}$$

From (i) & (ii)

$$(a * b) * c \neq a * (b * c)$$

Hence, ' $*$ ' is not associative on \mathbb{Z} .

Binary Operations Ex 3.2 Q11

Q be the set of rational numbers and $*$ be a binary operation defined as

$$a * b = \frac{ab}{5} \text{ for all } a, b \in Q$$

Now,

Associativity: Let $a, b, c \in Q$, then

$$(a * b) * c = \frac{ab}{5} * c = \frac{abc}{25} \quad \text{--- (i)}$$

$$\text{and, } a * (b * c) = a * \frac{bc}{5} = \frac{abc}{25} \quad \text{--- (ii)}$$

From (i) & (ii)

$$\therefore (a * b) * c = a * (b * c)$$

\Rightarrow $*$ is associative on Q .

Binary Operations Ex 3.2 Q12

The binary operator $*$ is defined as

$$a * b = \frac{ab}{7} \text{ for all } a, b \in Q$$

Now,

Associativity: Let $a, b, c \in Q$, then

$$(a * b) * c = \frac{ab}{7} * c = \frac{abc}{49} \quad \text{--- (i)}$$

$$\text{and, } a * (b * c) = a * \frac{bc}{7} = \frac{abc}{49} \quad \text{--- (ii)}$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

\Rightarrow $'*$ is associative on Q .

Binary Operations Ex 3.2 Q13

The binary operator $*$ defined as

$$a * b = \frac{a+b}{2} \text{ for all } a, b \in Q.$$

Now,

Associativity: Let $a, b, c \in Q$, then

$$\begin{aligned}(a * b) * c &= \frac{a+b}{2} * c = \frac{\frac{a+b}{2} + c}{2} \\ &= \frac{a+b+2c}{4} \quad \text{--- (i)}\end{aligned}$$

$$\begin{aligned}\text{and, } a * (b * c) &= a * \frac{b+c}{2} \\ &= \frac{a + \frac{b+c}{2}}{2} \\ &= \frac{2a+b+c}{4} = \quad \text{--- (ii)}\end{aligned}$$

From (i) & (ii)

$$(a * b) * c \neq a * (b * c)$$

Hence, ' $*$ ' is not associative on Q .

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