Differentiation Ex 11.2 Q25

Let $\quad y = \log\left(\frac{\sin x}{1+\cos x}\right)$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\log\left(\frac{\sin x}{1+\cos x}\right)$$

$$= \frac{1}{\left(\frac{\sin x}{1+\cos x}\right)} \times \frac{d}{dx}\left(\frac{\sin x}{1+\cos x}\right) \qquad \text{[Using chain rule]}$$

$$= \left(\frac{1+\cos x}{\sin x}\right)\left[\frac{(1+\cos x)\frac{d}{dx}(\sin x) - \sin x\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}\right] \qquad \text{[Using quotient rule]}$$

$$= \frac{(1+\cos x)}{\sin x}\left[\frac{(1+cox)(\cos x) - \sin x(-\sin x)}{(1+\cos x)^2}\right]$$

$$= \frac{(1+\cos x)}{\sin x}\left[\frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}\right]$$

$$= \frac{(1+\cos x)}{\sin x}\left[\frac{(1+\cos x)}{(1+\cos x)^2}\right]$$

$$= \frac{1}{\sin x}$$

$$= \operatorname{cosec} x$$

So,

$$\frac{d}{dx}\left(\log\left(\frac{\sin x}{1+\cos x}\right)\right) = \operatorname{cosec} x.$$

Differentiation Ex 11.2 Q26

Let $\quad y = \log\sqrt{\frac{1-\cos x}{1+\cos x}}$

$\Rightarrow \quad y = \log\left(\frac{1-\cos x}{1+\cos x}\right)^{\frac{1}{2}}$

$\Rightarrow \quad y = \frac{1}{2}\log\left(\frac{1-\cos x}{1+\cos x}\right) \qquad \left[\text{Using } \log a^b = b\log a\right]$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left\{\frac{1}{2}\log\left(\frac{1-\cos x}{1+\cos x}\right)\right\}$$

$$= \frac{1}{2} \times \frac{1}{\left(\frac{1-\cos x}{1+\cos x}\right)} \times \frac{d}{dx}\left(\frac{1-\cos x}{1+\cos x}\right) \qquad \text{[Using chain rule]}$$

$$= \frac{1}{2}\left(\frac{1+\cos x}{1-\cos x}\right)\left[\frac{(1+\cos x)\frac{d}{dx}(1-\cos x) - (1-\cos x)\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}\right] \qquad \text{[Using quotient rule]}$$

$$= \frac{1}{2}\left(\frac{1+\cos x}{1-\cos x}\right)\left[\frac{(1+\cos x)(\sin x) - (1-\cos x)(-\sin x)}{(1+\cos x)^2}\right]$$

$$= \frac{1}{2}\frac{(1+\cos x)}{(1-\cos x)}\left[\frac{\sin x + \sin x\cos x + \sin x - \sin x\cos x}{(1+\cos x)^2}\right]$$

$$= \frac{1}{2}\frac{(1+\cos x)}{(1-\cos x)}\left[\frac{2\sin x}{(1+\cos x)^2}\right]$$

$$= \frac{\sin x}{(1-\cos x)(1+\cos x)}$$

$$= \frac{\sin x}{1-\cos^2 x}$$

$$= \frac{\sin x}{\sin^2 x} \qquad \left[\text{Sicne } 1-\cos^2 x = \sin^2 x\right]$$

$$= \frac{1}{\sin x}$$

$$= \operatorname{cosec} x$$

So,

$$\frac{d}{dx}\left(\log\sqrt{\frac{1-\cos x}{1+\cos x}}\right) = \operatorname{cosec} x.$$

Differentiation Ex 11.2 Q27

Let $y = \tan\left(e^{\sin x}\right)$

Differentiate it with respect to x,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left[\tan e^{\sin x}\right]\\ &= \sec^2\left(e^{\sin x}\right)\frac{d}{dx}\left(e^{\sin x}\right) && \text{[Using chain rule]}\\ &= \sec^2\left(e^{\sin x}\right) \times e^{\sin x} \times \frac{d}{dx}(\sin x)\\ &= \cos x \sec^2\left(e^{\sin x}\right) \times e^{\sin x}\end{aligned}$$

So,

$$\frac{d}{dx}\left(\tan e^{\sin x}\right) = \sec^2\left(e^{\sin x}\right) \times e^{\sin x} \times \cos x.$$

Differentiation Ex 11.2 Q28

Let $y = \log\left(x + \sqrt{x^2+1}\right)$

Differentiate with respect to x,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\log\left(x + \sqrt{x^2+1}\right)\\ &= \frac{1}{x + \sqrt{x^2+1}}\frac{d}{dx}\left(x + \left(x^2+1\right)^{\frac{1}{2}}\right) && \text{[Using chain rule]}\\ &= \frac{1}{x + \sqrt{x^2+1}}\left[1 + \frac{1}{2}\left(x^2+1\right)^{\frac{1}{2}-1}\frac{d}{dx}\left(x^2+1\right)\right]\\ &= \frac{1}{x + \sqrt{x^2+1}}\left[1 + \frac{1}{2\sqrt{x^2+1}} \times 2x\right]\\ &= \frac{1}{x + \sqrt{x^2+1}}\left[\frac{\sqrt{x^2+1} + x}{\sqrt{x^2+1}}\right]\\ &= \frac{1}{\sqrt{x^2+1}}\end{aligned}$$

So,

$$\frac{d}{dx}\left(\log\left(x + \sqrt{x^2+1}\right)\right) = \frac{1}{\sqrt{x^2+1}}.$$

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