Sets Ex 1.6 Q4(i)

i. Let $x \in B$. Then

$\Rightarrow x \in B \cup A$

$\Rightarrow x \in A \cup B$

$\therefore B \subset (A \cup B)$

Sets Ex 1.6 Q4(ii)

ii. Let $x \in A \cap B$. Then

$\Rightarrow x \in A$ and $x \in B$

$\Rightarrow x \in B$

$\therefore (A \cap B) \subset B$

Sets Ex 1.6 Q4(iii)

iii. Let $x \in A \subset B$. Then

$\Rightarrow x \in B$

Let and $x \in A \cap B$

$\Leftrightarrow x \in A$ and $x \in B$

$\Leftrightarrow x \in A$ and $x \in A \quad (\because A \subset B)$

$\therefore (A \cap B) = A$

Sets Ex 1.6 Q5

(i)

In order to show that the following four statements are equivalent, we need to show that $(1) \Rightarrow (2)$, $(2) \Rightarrow (3)$, $(3) \Rightarrow (4)$ and $(4) \Rightarrow (1)$

We first show that $(1) \Rightarrow (2)$

We assume that $A \subset B$, and use this to show that $A - B = \phi$

Now $A - B = \{x \in A : x \notin B\}$. As $A \subset B$,

$\therefore$ Each element of A is an element of B,

$\therefore$ $A - B = \phi$

Hence, we have proved that $(1) \Rightarrow (2)$.

(ii)

We new show that $(2) \Rightarrow (3)$

So assume that $A - B = \phi$

To show: $A \cup B = B$

$\because$ $A - B = \phi$

$\therefore$ Every element of A is an element of B

$[\because A - B = \phi$ only when ther is some element in A which is not in $B]$

So $A \subset B$ and therefore $A \cup B = B$

So $(2) \Rightarrow (3)$ is true.

(iii)

We new show that $(3) \Rightarrow (4)$

Assume that $A \cup B = B$

To show: $A \cap B = A$

$\because$ $A \cup B = B$

$\therefore$ $A \subset B$ and so $A \cap B = A$

So $(3) \Rightarrow (4)$ is true.

(iv)

Finally we show that $(4) \Rightarrow (1)$, which will prove the equivalence of the four statements.

So, assume that $A \cap B = A$

To show: $A \subset B$

$\because$ $A \cap B = A$, therefore $A \subset B$, and so $(4) \Rightarrow (1)$ is true.

Hence, $(1) \Leftrightarrow (2) \Leftrightarrow (3) \Leftrightarrow (4)$.

*********** END **********