Chapter 6 Determinants Ex 6.5 Q1

$$\text{Here } D = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$$

$$= 1(3) - 1(-3) - 2(3)$$

$$= 3 + 3 - 6$$

$$= 0$$

Since $D = 0$, so the system has infinite solutions:

Now let $z = k$,

$$x + y = 2k$$

$$2x + y = 3k$$

Solving there equations by cramer's Rule

$$x = \frac{D_1}{D} = \frac{\begin{vmatrix} 2k & 1 \\ 3k & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}} = \frac{-k}{-1} = k$$

$$y = \frac{D_2}{D} = \frac{\begin{vmatrix} 1 & 2k \\ 2 & 3k \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}} = \frac{-k}{-1} = k$$

thus, we have $x = k, y = k, z = k$

and there values satisfy eq.(3)

Hence $x = k, y = k, z = k$

Chapter 6 Determinants Ex 6.5 Q2

$$\text{Here } D = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{vmatrix}$$

$$= 2(4) - 3(1) + 4(-3)$$

$$= 8 - 3 - 7$$

$$= -2$$

$$\neq 0$$

So, the given system of equations has only the trivial solutions i.e $x = 0 = y = z$:

Hence $x = 0$

$y = 0$

$z = 0$

Chapter 6 Determinants Ex 6.5 Q3

Here $D = \begin{vmatrix} 3 & 1 & 1 \\ 1 & -4 & 3 \\ 2 & 5 & -2 \end{vmatrix}$

$$= 3(8-15) - 1(-2-6) + 1(13)$$
$$= -21 + 8 + 13$$
$$= 0$$

So, the system has infinite solutions:

Let $z = k$,

so, $\quad 3x + y = -k$

$\quad\quad x - 4y = -3k$

Now,

$$x = \frac{D_1}{D} = \frac{\begin{vmatrix} -k & 1 \\ -3k & -4 \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 1 & -4 \end{vmatrix}} = \frac{7k}{-13}$$

$$y = \frac{D_2}{D} = \frac{\begin{vmatrix} 3 & -k \\ 1 & -3k \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 1 & -4 \end{vmatrix}} = \frac{-8k}{-13}$$

$$x = \frac{-7k}{13}, y = \frac{8k}{13}, z = k$$

and there values satisfy eq.(3)

Hence $x = -7k, y = 8k, z = 13k$

Chapter 6 Determinants Ex 6.5 Q4

$$D=\begin{vmatrix}2\lambda & -2 & 3\\ 1 & \lambda & 2\\ 2 & 0 & \lambda\end{vmatrix}$$
$$=3\lambda^3+2\lambda-8-6\lambda$$
$$=2\lambda^3-4\lambda-8$$

which is satisfied by $\lambda = 2$ $\left[\because \quad \text{for non-trivial solutions } \lambda = 2\right]$

Now Let $z = k$,

$$4x-2y=-3k$$
$$x+2y=-3k$$

$$x=\frac{D_1}{D}=\frac{\begin{vmatrix}-3k & -2\\ -2k & 2\end{vmatrix}}{\begin{vmatrix}4 & -2\\ 1 & 2\end{vmatrix}}=\frac{-10k}{10}=-k$$

$$y=\frac{D_2}{D}=\frac{\begin{vmatrix}4 & -3k\\ 1 & -2k\end{vmatrix}}{\begin{vmatrix}4 & -2\\ 1 & 2\end{vmatrix}}=\frac{-5k}{10}=\frac{-k}{2}$$

Hence solution is $x=-k, y=\frac{-k}{2}, z=k$

Chapter 6 Determinants Ex 6.5 Q5

$$D=\begin{vmatrix}(a-1) & -1 & -1\\ -1 & (b-1) & -1\\ -1 & -1 & (c-1)\end{vmatrix}$$

Now for non-trivial solution, $D = 0$

$$0=(a-1)\left[(b-1)(c-1)-1\right]+1\left[-c+\cancel{1}-\cancel{1}\right]-\left[\cancel{1}+b-\cancel{1}\right]$$
$$0=(a-1)\left[bc-b-c+\cancel{1}-\cancel{1}\right]-c-b$$
$$0=abc-ab-ac+\cancel{b}+\cancel{c}-\cancel{c}-\cancel{b}$$
$$ab+bc+ac=abc$$

Hence proved

*********** END **********