



Definite Integrals Ex 20.1 Q19

We have,

$$\int_0^{\frac{\pi}{6}} \cos x \cos 2x dx \quad [\because 2 \cos C \cos D = \cos(C+D) + \cos(C-D)]$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{6}} 2 \cos x \cos 2x dx$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{6}} (\cos 3x + \cos x) dx$$

$$= \frac{1}{2} \left[ \frac{\sin 3x}{3} + \sin x \right]_0^{\frac{\pi}{6}}$$

$$= \frac{1}{2} \left[ \left( \frac{\sin 3 \frac{\pi}{6}}{3} + \sin \frac{\pi}{6} \right) - (\sin 0 + \sin 0) \right]$$

$$= \frac{1}{2} \left[ \frac{\sin \frac{\pi}{2}}{3} + \sin \frac{\pi}{6} \right]$$

$$= \frac{1}{2} \left( \frac{1}{3} + \frac{1}{2} \right)$$

$$= \frac{1}{2} \left( \frac{5}{6} \right)$$

$$= \frac{5}{12}$$

$$\therefore \int_0^{\frac{\pi}{6}} \cos x \cos 2x dx = \frac{5}{12}$$

Definite Integrals Ex 20.1 Q20

We have,

$$\int_0^{\frac{\pi}{2}} \sin x \sin 2x dx$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} 2 \sin x \sin 2x dx \quad [\because 2 \sin C \times \sin D = \cos(D - C) - \cos(D + C)]$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} (\cos x - \cos 3x) dx$$

$$= \frac{1}{2} \left[ \sin x - \frac{\sin 3x}{3} \right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2} \left[ \left( \sin \frac{\pi}{2} - \sin 0 \right) - \left( \frac{\sin 3 \frac{\pi}{2}}{3} - \frac{\sin 0}{3} \right) \right]$$

$$= \frac{1}{2} \left[ (1 - 0) - \left( \frac{-1}{3} - 0 \right) \right] \quad \left[ \because \sin 3 \frac{\pi}{2} = -1 \right]$$

$$= \frac{1}{2} \times \frac{4}{3}$$

$$= \frac{2}{3}$$

$$\therefore \int_0^{\frac{\pi}{2}} \sin x \sin 2x dx = \frac{2}{3}$$

Definite Integrals Ex 20.1 Q21

We have,

$$\begin{aligned} & \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} (\tan x + \cot x)^2 dx \\ &= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left( \frac{\sin^2 x + \cot^2 x}{\sin x \cos x} \right)^2 dx \\ &= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left( \frac{1}{\sin x \cos x} \right)^2 dx \end{aligned}$$

Multiplying numerator and denominator by 2

$$\begin{aligned} &= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left( \frac{2}{2 \sin x \cos x} \right)^2 dx \\ &= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left( \frac{2}{\sin 2x} \right)^2 dx \quad [\because 2 \sin x \cos x = \sin 2x] \\ &= 4 \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \operatorname{cosec}^2 x dx \\ &= 4 \left[ -\frac{\cot 2x}{2} \right]_{\frac{\pi}{3}}^{\frac{\pi}{4}} \\ &= 2 \left[ -\cot \frac{\pi}{2} + \cot 2 \frac{\pi}{3} \right] \\ &= 2 \left[ \frac{-1}{\sqrt{3}} - 0 \right] \\ &= \frac{-2}{\sqrt{3}} \end{aligned}$$

$$\therefore \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} (\tan x + \cot x)^2 dx = \frac{-2}{\sqrt{3}}$$

Definite Integrals Ex 20.1 Q22

We have,

$$\int_0^{\frac{\pi}{2}} \cos^4 x dx$$

$$= \frac{1}{4} \int_0^{\frac{\pi}{2}} (1 + \cos 2x)^2 dx \quad \left[ \because 2 \cos^2 x = 1 + \cos 2x \right]$$

$$= \frac{1}{4} \int_0^{\frac{\pi}{2}} (1 + \cos^2 2x + 2 \cos 2x) dx$$

$$= \frac{1}{4} \int_0^{\frac{\pi}{2}} \left( 1 + \frac{1 + \cos 4x}{2} + 2 \cos 2x \right) dx$$

$$= \frac{1}{4} \left[ x + \frac{1}{2}x + \frac{\sin 4x}{8} + \sin 2x \right]_0^{\frac{\pi}{2}} \quad \left[ \because \int \cos 4x dx = \frac{\sin 4x}{4} \right]$$

$$= \frac{1}{4} \left[ \frac{\pi}{2} + \frac{\pi}{4} + 0 + 0 - 0 - 0 - 0 - 0 \right]$$

$$= \frac{1}{4} \times \frac{3\pi}{4}$$

$$= \frac{3\pi}{16}$$

$$\therefore \int_0^{\frac{\pi}{2}} \cos^4 x dx = \frac{3\pi}{16}$$

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