

Maxima and Minima 18.3 Q1(i)

$$f(x) = x^4 - 62x^2 + 120x + 9$$

$$f'(x) = 4x^3 - 124x + 120 = 4(x^3 - 31x + 30)$$
$$f''(x) = 12x^2 - 124 = 4(3x^2 - 31)$$

For maxima and minima,

$$f^+(x) = 0$$

$$\Rightarrow 4\left(x^3 - 31x + 30\right) = 0$$

$$\Rightarrow 4\left(x^3 - 31x + 30\right) = 0$$

$$\Rightarrow x = 5, 1, -6$$

Now,

$$f''(5) = 176 > 0$$

$$\Rightarrow$$
 $x = 5$ is point of local minima
 $f''(1) = -112 < 0$

$$\Rightarrow x = 1 \text{ is point of local maxima}$$

$$f''(-6) = 308 > 0$$

$$\Rightarrow$$
 $x = -6$ is point of local minima

$$| local max value = f(1) = 68$$

$$| local min value = f(5) = -316$$

$$| and = f(-6) = -1647.$$

Maxima and Minima 18.3 Q1(ii)

We have,

$$f(x) = x^{3} - 6x^{2} + 9x + 15$$

$$f'(x) = 3x^{2} - 12x + 9$$

$$= 3(x^{2} - 4x + 3)$$

$$f''(x) = 6x - 12$$

$$= 6(x - 2)$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow 3(x^2 - 4x + 3) = 0$$

$$\Rightarrow 3(x - 3)(x - 1) = 0$$

$$\Rightarrow x = 3, 1$$

Now,

$$f^{\prime\prime}(3) = 6 > 0$$

- x = 3 is point of local minima f''(1) = -6 < 0
- $\therefore \qquad x = 1 \text{ is point of local maxima}$
- local max value = f(1) = 19 local min value = f(3) = 15.

Maxima and Minima 18.3 Q1(iii)

We have,

$$f(x) = (x - 1)(x + 2)^{2}$$

$$f'(x) = (x + 2)^{2} + 2(x - 1)(x + 2)$$

$$= (x + 2)(x + 2 + 2x - 2)$$

$$= (x + 2)(3x)$$
and, $f''(x) = 3(x + 2) + 3x$

$$= 6x + 6$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow 3x(x+2) = 0$$

$$\Rightarrow x = 0, -2$$

Now,

$$f''(0) = 6 > 0$$

$$x = 0 \text{ is point of local minima}$$

$$f''(-2) = -6 < 0$$

x = -2 is point of local maxima

Maxima and Minima 18.3 Q1(iv)

We have,

$$f(x) = \frac{2}{x} - \frac{2}{x^2}, \ x > 0$$

$$f'(x) = \frac{-2}{x^2} + \frac{4}{x^3}$$
and,
$$f''(x) = \frac{+4}{x^3} - \frac{12}{x^4}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \frac{-2}{x^2} + \frac{4}{x^3} = 0$$

$$\Rightarrow \frac{-2(x-2)}{x^3} = 0$$

$$\Rightarrow x = 2$$

Now,

$$f''(2) = \frac{4}{8} - \frac{12}{6} = \frac{1}{2} - \frac{3}{4} = \frac{-1}{4} < 0$$

 $\therefore x = 2 \text{ is point of local maxima}$

local max value =
$$f(2) = \frac{1}{2}$$
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****** END ******