Indefinite Integrals Ex 19.8 Q46

Let $I = \int \tan 2x \tan 3x \tan 5x dx$ ---------(i)

Now,

$$\tan(5x) = \tan(2x + 3x)$$

$$= \frac{\tan 2x + \tan 3x}{1 - \tan 2x \tan 3x}$$

$$\Rightarrow \quad \tan 5x = \frac{\tan 2x + \tan 3x}{1 - \tan 2x \tan 3x}$$

$$\Rightarrow \quad \tan 5x - \tan 2x \tan 3x \tan 5x = \tan 2x + \tan 3x$$

$$\Rightarrow \quad \tan 5x - \tan 2x - \tan 3x = \tan 2x \tan 3x \tan 5x \text{ -----(ii)}$$

Using equation (i) and equation (ii), we get

$$I = \int [\tan 5x - \tan 2x - \tan 3x] dx$$

$$= \frac{1}{5}\log|\sec 5x| - \frac{1}{2}\log|\sec 2x| - \frac{1}{3}\log|\sec 3x| + c$$

$$\therefore \quad I = \frac{1}{5}\log|\sec 5x| - \frac{1}{2}\log|\sec 2x| - \frac{1}{3}\log|\sec 3x| + c$$

Indefinite Integrals Ex 19.8 Q47

Since,

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\therefore \quad \tan(x + \theta - x) = \frac{\tan(x + \theta) - \tan x}{1 + \tan(x + \theta)\tan x}$$

$$\Rightarrow \quad 1 + \tan(x + \theta)\tan x = \frac{\tan(x + \theta) - \tan x}{\tan\theta}$$

$$\Rightarrow \quad \int 1 + \tan(x + \theta)\tan x dx$$

$$= \frac{1}{\tan\theta}\left[\int \tan(x + \theta) dx - \int \tan x dx\right]$$

$$= \frac{1}{\tan\theta}\left[-\log|\cos(x + \theta)| + \log|\cos x|\right] + C$$

$$= \frac{1}{\tan\theta}\left[\log|\cos x| - \log|\cos(x + \theta)|\right] + C$$

$$= \frac{1}{\tan\theta}\log\left|\frac{\cos x}{\cos(x + \theta)}\right| + C$$

Indefinite Integrals Ex 19.8 Q48

$$\text{Consider } I = \int\left(\frac{\sin 2x}{\sin\left(x-\frac{\pi}{6}\right)\sin\left(x+\frac{\pi}{6}\right)}\right)dx$$

$$= \int\left(\frac{\sin 2x}{\left(\frac{3}{4}\sin^2 x \ -\frac{1}{4}\cos^2 x\right)}\right)dx$$

$$= \int\left(\frac{\sin 2x}{\left(\frac{3}{4}\left(1-\cos^2 x\right) \ -\frac{1}{4}\cos^2 x\right)}\right)dx$$

$$= \int\left(\frac{\sin 2x}{\left(\frac{3}{4}-\cos^2 x\right)}\right)dx$$

$$let \ \cos^2 x = t \rightarrow \sin 2x dx = -dt$$

$$I = \int\left(\frac{-dt}{\left(\frac{3}{4}-t\right)}\right)$$

$$I = \log\left|\sin^2 x - \frac{1}{4}\right| + C$$

*********** END **********