NCERT Solutions For Class 10 Chapter 6 Triangles Exercise 6.5

1. Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm

(ii) 3 cm, 8 cm, 6 cm

(iii) 50 cm, 80 cm, 100 cm

(iv) 13 cm, 12 cm, 5 cm

Ans. (i) Let a = 7 cm, b = 24 cm and c = 25 cm

Here the larger side is c = 25 cm.

We have, $a^2 + b^2 = 7^2 + 24^2 = 49 + 576 = 625 = c^2$

So, the triangle with the given sides is a right triangle. Its hypotenuse = 25 cm

(ii) Let a = 3 cm, b = 8 cm and c = 6 cm

Here the larger side is b = 8 cm.

We have, $a^2 + c^2 = 3^2 + 6^2 = 9 + 36 = 45 \neq b^2$

So, the triangle with the given sides is not a right triangle.

(iii) Let a = 50 cm, b = 80 cm and c = 100 cm

Here the larger side is c = 100 cm.

We have, $a^2 + b^2 = 50^2 + 80^2 = 2500 + 6400 = 8900 \neq c^2$

So, the triangle with the given sides is not a right triangle.

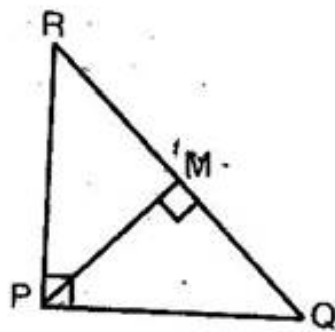**(iv)** Let a = 13 cm, b = 12 cm and c = 5 cm

Here the larger side is a = 13 cm.

We have, $b^2 + c^2 = 12^2 + 5^2 = 144 + 25 = 169 = a^2$

So, the triangle with the given sides is a right triangle. Its hypotenuse = 13 cm

2. PQR is a triangle right angled at P and M is a point on QR such that PM$\perp$ QR. Show that PM^2 = QM.MR.

Ans. Given: PQR is a triangle right angles at P and PM$\perp$ QR

To Prove: $PM^2 = QM.MR$

Proof: Since $PM \perp QR$

$\therefore \triangle PQM \sim \triangle PRM$

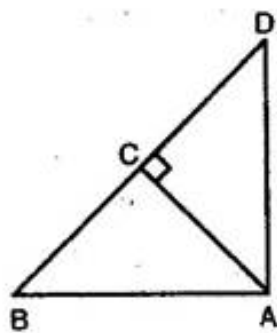$\Rightarrow \dfrac{PM}{QM} = \dfrac{MR}{PM}$

$\Rightarrow PM^2 = QM.MR$

3. In figure, ABD is a triangle right angled at A and $AC \perp BD$. Show that:

(i) $AB^2 = BC.BD$

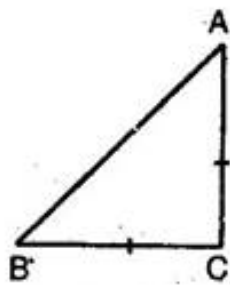(ii) $AC^2 = BC.DC$

(iii) $AD^2 = BD.CD$

Ans. Given: ABD is a triangle right angled at A and $AC \perp BD$.

To Prove: (i) $AB^2 = BC.BD$, **(ii)** $AC^2 = BC.DC$, **(iii)** $AD^2 = BD.CD$

Proof:(i) Since $AC \perp BD$

$\therefore \triangle ABC \sim \triangle ADC$ and each triangle is similar to $\triangle ABD$

$\therefore \triangle ABC \sim \triangle ABD$

$$\Rightarrow \frac{AB}{BD} = \frac{BC}{AB}$$

$$\Rightarrow AB^2 = BC.BD$$

(ii) Since $\triangle ABC \sim \triangle ADC$

$$\Rightarrow \frac{AC}{BC} = \frac{DC}{AC}$$

$$\Rightarrow AC^2 = BC.DC$$

(iii) Since $\triangle ACD \sim \triangle ABD$

$$\Rightarrow \frac{AD}{CD} = \frac{BD}{AD}$$

$$\Rightarrow AD^2 = BD.CD$$

4. ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.

Ans. Since ABC is an isosceles right triangle, right angled at C.

$$\Rightarrow AB^2 = AC^2 + BC^2$$

$$\Rightarrow AB^2 = AC^2 + AC^2 \text{ [BC = AC, given]}$$

$$\Rightarrow AB^2 = 2AC^2$$

5. ABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, prove that ABC is a right triangle.

Ans. Since ABC is an isosceles right triangle with AC = BC and $AB^2 = 2AC^2$
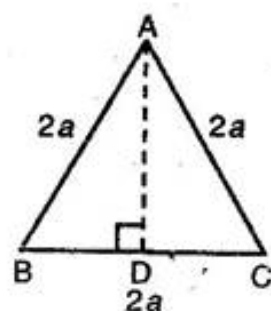
$$\Rightarrow AB^2 = AC^2 + AC^2$$

$$\Rightarrow AB^2 = AC^2 + BC^2 \text{ [BC = AC, given]}$$

$\therefore$ $\triangle ABC$ is right angled at C.

6. ABC is an equilateral triangle of side $2a$. Find each of its altitudes.

Ans. Let ABC be an equilateral triangle of side $2a$ units.

Draw $AD \perp BC$. Then, D is the mid-point of BC.

$$\Rightarrow BD = \frac{1}{2} BC = \frac{1}{2} \times 2a = a$$

Since, ABD is a right triangle, right triangle at D.

$$\therefore AB^2 = AD^2 + BD^2$$
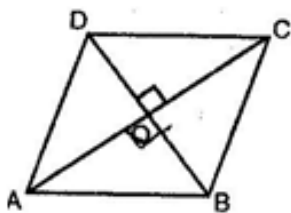
$$\Rightarrow (2a)^2 = AD^2 + (a)^2$$

$$\Rightarrow AD^2 = 4a^2 - a^2 = 3a^2$$

$\therefore$ Each of its altitude = $\sqrt{3}a$

7. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of squares of its diagonals.

Ans. Let the diagonals AC and BD of rhombus ABCD intersect each other at O. Since the diagonals of a rhombus bisect each other at right angles.

$\therefore \angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$ and
AO = CO, BO = OD

Since AOB is a right triangle, right angled at O.

$\therefore$ $AB^2 = OA^2 + OB^2$

$\Rightarrow$ $AB^2 = \left(\dfrac{1}{2}AC\right)^2 + \left(\dfrac{1}{2}BD\right)^2$

[$\because$ OA = OC and OB = OD]

$\Rightarrow$ $4AB^2 = AC^2 + BD^2$ ....(1)

Similarly, we have $4BC^2 = AC^2 + BD^2$ ....(2)

$4CD^2 = AC^2 + BD^2$ ....(3)

$4AD^2 = AC^2 + BD^2$ ....(4)

Adding all these results, we get

$4\left(AB^2 + BC^2 + CD^2 + DA^2\right) = 4\left(AC^2 + BD^2\right)$

$\Rightarrow$ $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

8. In figure, O is a point in the interior of a triangle ABC, OD$\perp$ BC,

OE$\perp$ AC and OF$\perp$ AB. Show that:
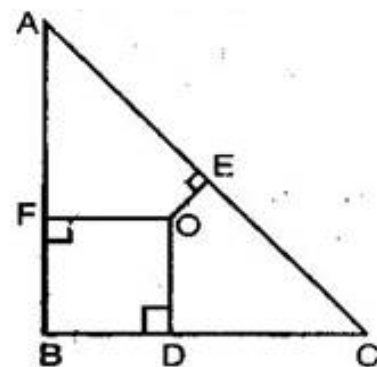
(i) $\quad OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 =$
$AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Ans. Join AO, BO and CO.

(i) In right $\triangle$ s OFA, ODB and OEC, we have

$OA^2 = AF^2 + OF^2$, $OB^2 = BD^2 + OD^2$ and
$OC^2 = CE^2 + OE^2$

********** END *********