EXERCISE 11.4

Question 1:

Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$

Ans:

The given equation is $\frac{x^2}{16} - \frac{y^2}{9} = 1$ or $\frac{x^2}{4^2} - \frac{y^2}{3^2} = 1$.

On comparing this equation with the standard equation of hyperbola i.e., $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we obtain $a = 4$ and $b = 3$.

We know that $a^2 + b^2 = c^2$.

$$\therefore c^2 = 4^2 + 3^2 = 25$$
$$\Rightarrow c = 5$$

Therefore,

The coordinates of the foci are (±5, 0).

The coordinates of the vertices are (±4, 0).

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{5}{4}$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2 \times 9}{4} = \frac{9}{2}$$

Question 2:

Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $\frac{y^2}{9} - \frac{x^2}{27} = 1$

Ans:

The given equation is $\frac{y^2}{9}-\frac{x^2}{27}=1$ or $\frac{y^2}{3^2}-\frac{x^2}{\left(\sqrt{27}\right)^2}=1$.

On comparing this equation with the standard equation of hyperbola i.e., $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$, we obtain a = 3 and $b=\sqrt{27}$.

We know that $a^2 + b^2 = c^2$.

$$\therefore c^2=3^2+\left(\sqrt{27}\right)^2=9+27=36$$
$$\Rightarrow c=6$$

Therefore,

The coordinates of the foci are (0, ±6).

The coordinates of the vertices are (0, ±3).

$$\text{Eccentricity, } e=\frac{c}{a}=\frac{6}{3}=2$$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 27}{3}=18$

Question 3:

Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $9y^2 - 4x^2 = 36$

Ans:

The given equation is $9y^2 - 4x^2 = 36$.

It can be written as

$9y^2 - 4x^2 = 36$

$$\text{Or, } \frac{y^2}{4}-\frac{x^2}{9}=1$$
$$\text{Or, } \frac{y^2}{2^2}-\frac{x^2}{3^2}=1 \qquad \ldots(1)$$

On comparing equation (1) with the standard equation of hyperbola i.e., $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$, we obtain a = 2 and b = 3.

We know that $a^2 + b^2 = c^2$.

$$\therefore c^2=4+9=13$$
$$\Rightarrow c=\sqrt{13}$$

Therefore,

The coordinates of the foci are $\left(0,\ \pm\sqrt{13}\right)$.

The coordinates of the vertices are $\left(0,\ \pm 2\right)$.

$$\text{Eccentricity, } e=\frac{c}{a}=\frac{\sqrt{13}}{2}$$

Length of latus rectum $=\frac{2b^2}{a}=\frac{2\times 9}{2}=9$

Question 4:

Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $16x^2 - 9y^2 = 576$

Ans:

The given equation is $16x^2 - 9y^2 = 576$.

It can be written as

$16x^2 - 9y^2 = 576$

$$\Rightarrow \frac{x^2}{36} - \frac{y^2}{64} = 1$$
$$\Rightarrow \frac{x^2}{6^2} - \frac{y^2}{8^2} = 1 \qquad ...(1)$$

On comparing equation (1) with the standard equation of hyperbola i.e., $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we obtain $a = 6$ and $b = 8$.

We know that $a^2 + b^2 = c^2$.

$$\therefore c^2 = 36 + 64 = 100$$
$$\Rightarrow c = 10$$

Therefore,

The coordinates of the foci are $(\pm 10, 0)$.

The coordinates of the vertices are $(\pm 6, 0)$.

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{10}{6} = \frac{5}{3}$$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 64}{6} = \frac{64}{3}$

Question 5:

Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $5y^2 - 9x^2 = 36$

Ans:

The given equation is $5y^2 - 9x^2 = 36$.

$$\Rightarrow \frac{y^2}{\left(\frac{36}{5}\right)} - \frac{x^2}{4} = 1$$
$$\Rightarrow \frac{y^2}{\left(\frac{6}{\sqrt{5}}\right)^2} - \frac{x^2}{2^2} = 1 \qquad ...(1)$$

On comparing equation (1) with the standard equation of hyperbola i.e., $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, we obtain $a = \frac{6}{\sqrt{5}}$ and $b = 2$.

We know that $a^2 + b^2 = c^2$.

$$\therefore c^2 = \frac{36}{5} + 4 = \frac{56}{5}$$
$$\Rightarrow c = \sqrt{\frac{56}{5}} = \frac{2\sqrt{14}}{\sqrt{5}}$$

Therefore, the coordinates of the foci are $\left(0, \pm\frac{2\sqrt{14}}{\sqrt{5}}\right)$.

The coordinates of the vertices are $\left(0, \pm\frac{6}{\sqrt{5}}\right)$.

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{\left(\frac{2\sqrt{14}}{\sqrt{5}}\right)}{\left(\frac{6}{\sqrt{5}}\right)} = \frac{\sqrt{14}}{3}$$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 4}{\left(\frac{6}{\sqrt{5}}\right)} = \frac{4\sqrt{5}}{3}$

Question 6:

Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $49y^2 - 16x^2 = 784$

Ans:

The given equation is $49y^2 - 16x^2 = 784$.

It can be written as
$49y^2 - 16x^2 = 784$

$$\text{Or, } \frac{y^2}{16} - \frac{x^2}{49} = 1$$
$$\text{Or, } \frac{y^2}{4^2} - \frac{x^2}{7^2} = 1 \qquad \ldots(1)$$

On comparing equation (1) with the standard equation of hyperbola i.e., $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, we obtain $a = 4$ and $b = 7$.

We know that $a^2 + b^2 = c^2$.

$$\therefore c^2 = 16 + 49 = 65$$
$$\Rightarrow c = \sqrt{65}$$

Therefore,

The coordinates of the foci are $\left(0, \pm\sqrt{65}\right)$.

The coordinates of the vertices are $(0, \pm 4)$.

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{\sqrt{65}}{4}$$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 49}{4} = \frac{49}{2}$

Question 7:

Find the equation of the hyperbola satisfying the give conditions: Vertices $(\pm 2, 0)$, foci $(\pm 3, 0)$

Ans:

Vertices $(\pm 2, 0)$, foci $(\pm 3, 0)$

Here, the vertices are on the x-axis.

Therefore, the equation of the hyperbola is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Since the vertices are $(\pm 2, 0)$, $a = 2$.

Since the foci are $(\pm 3, 0)$, $c = 3$.

We know that $a^2 + b^2 = c^2$.

$$\therefore 2^2 + b^2 = 3^2$$
$$b^2 = 9 - 4 = 5$$

Thus, the equation of the hyperbola is $\frac{x^2}{4} - \frac{y^2}{5} = 1$.

Question 8:

Find the equation of the hyperbola satisfying the give conditions: Vertices $(0, \pm 5)$, foci $(0, \pm 8)$

Ans:

Vertices (0, ±5), foci (0, ±8)

Here, the vertices are on the y-axis.

Therefore, the equation of the hyperbola is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$.

Since the vertices are (0, ±5), $a = 5$.

Since the foci are (0, ±8), $c = 8$.

We know that $a^2 + b^2 = c^2$.

$$\therefore 5^2 + b^2 = 8^2$$
$$b^2 = 64 - 25 = 39$$

Thus, the equation of the hyperbola is $\frac{y^2}{25} - \frac{x^2}{39} = 1$.

Question 9:

Find the equation of the hyperbola satisfying the give conditions: Vertices (0, ±5), foci (0, ±8)

Ans:

Vertices (0, ±5), foci (0, ±8)

Here, the vertices are on the y-axis.

Therefore, the equation of the hyperbola is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$.

Since the vertices are (0, ±5), $a = 5$.

Since the foci are (0, ±8), $c = 8$.

We know that $a^2 + b^2 = c^2$.

$$\therefore 5^2 + b^2 = 8^2$$
$$b^2 = 64 - 25 = 39$$

Thus, the equation of the hyperbola is $\frac{y^2}{25} - \frac{x^2}{39} = 1$.

Question 10:

Find the equation of the hyperbola satisfying the give conditions: Foci (±5, 0), the transverse axis is of length 8.

Ans:

Foci (±5, 0), the transverse axis is of length 8.

Here, the foci are on the x-axis.

Therefore, the equation of the hyperbola is of the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

Since the foci are (±5, 0), $c = 5$.

Since the length of the transverse axis is 8, $2a = 8$ □ $a = 4$.

We know that $a^2 + b^2 = c^2$.

$4^2 + b^2 = 5^2$

$b^2 = 25 - 16 = 9$

Thus, the equation of the hyperbola is $\frac{x^2}{16}-\frac{y^2}{9}=1$.

Question 11:

Find the equation of the hyperbola satisfying the give conditions: Foci (0, ±13), the conjugate axis is of length 24.

Ans:

Foci (0, ±13), the conjugate axis is of length 24.

Here, the foci are on the y-axis.

Therefore, the equation of the hyperbola is of the form $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$.

Since the foci are (0, ±13), $c = 13$.

Since the length of the conjugate axis is 24, $2b = 24$ □ $b = 12$.

We know that $a^2 + b^2 = c^2$.

$a^2 + 12^2 = 13^2$

$a^2 = 169 - 144 = 25$

Thus, the equation of the hyperbola is $\frac{y^2}{25}-\frac{x^2}{144}=1$.

Question 12:

Find the equation of the hyperbola satisfying the give conditions: Foci $\left(\pm 3\sqrt{5},\ 0\right)$, the latus rectum is of length 8.

Ans:

Foci $\left(\pm 3\sqrt{5},\ 0\right)$, the latus rectum is of length 8.

Here, the foci are on the x-axis.

Therefore, the equation of the hyperbola is of the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

Since the foci are $\left(\pm 3\sqrt{5},\ 0\right)$, $c = \pm 3\sqrt{5}$.

Length of latus rectum = 8

$\Rightarrow \frac{2b^2}{a}=8$

$\Rightarrow b^2 = 4a$

We know that $a^2 + b^2 = c^2$.

$a^2 + 4a = 45$

$a^2 + 4a - 45 = 0$

$a^2 + 9a - 5a - 45 = 0$

$(a + 9)(a - 5) = 0$

$a = -9, 5$

Since a is non-negative, $a = 5$.

$b^2 = 4a = 4 \times 5 = 20$

Thus, the equation of the hyperbola is $\frac{x^2}{25}-\frac{y^2}{20}=1$.

Question 13:

Find the equation of the hyperbola satisfying the give conditions: Foci (±4, 0), the latus rectum is of length 12

Ans:

Foci (±4, 0), the latus rectum is of length 12.

Here, the foci are on the x-axis.

Therefore, the equation of the hyperbola is of the form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

Since the foci are (±4, 0), c = 4.

Length of latus rectum = 12

$$\Rightarrow \frac{2b^2}{a}=12$$
$$\Rightarrow b^2=6a$$

We know that $a^2 + b^2 = c^2$.

$a^2 + 6a = 16$

$a^2 + 6a - 16 = 0$

$a^2 + 8a - 2a - 16 = 0$

$(a + 8) (a - 2) = 0$

$a = -8, 2$

Since a is non-negative, a = 2.

$b^2 = 6a = 6 \times 2 = 12$

Thus, the equation of the hyperbola is $\frac{x^2}{4}-\frac{y^2}{12}=1$.

Question 14:

Find the equation of the hyperbola satisfying the give conditions: Vertices (±7, 0), $e=\frac{4}{3}$

Ans:

Vertices (±7, 0), $e = \frac{4}{3}$

Here, the vertices are on the x-axis.

Therefore, the equation of the hyperbola is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Since the vertices are (±7, 0), $a = 7$.

It is given that $e = \frac{4}{3}$

$$\therefore \frac{c}{a} = \frac{4}{3} \qquad \left[e = \frac{c}{a}\right]$$

$$\Rightarrow \frac{c}{7} = \frac{4}{3}$$

$$\Rightarrow c = \frac{28}{3}$$

We know that $a^2 + b^2 = c^2$.

$$\therefore 7^2 + b^2 = \left(\frac{28}{3}\right)^2$$

$$\Rightarrow b^2 = \frac{784}{9} - 49$$

$$\Rightarrow b^2 = \frac{784 - 441}{9} = \frac{343}{9}$$

Thus, the equation of the hyperbola is $\frac{x^2}{49} - \frac{9y^2}{343} = 1$.

Question 15:

Find the equation of the hyperbola satisfying the give conditions: Foci $\left(0, \pm\sqrt{10}\right)$, passing through (2, 3)

Ans:

Foci $\left(0,\ \pm\sqrt{10}\right)$, passing through (2, 3)

Here, the foci are on the y-axis.

Therefore, the equation of the hyperbola is of the form $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$.

Since the foci are $\left(0,\ \pm\sqrt{10}\right)$, $c=\sqrt{10}$.

We know that $a^2 + b^2 = c^2$.

$a^2 + b^2 = 10$

$b^2 = 10 - a^2$ ... (1)

Since the hyperbola passes through point (2, 3),

$$\frac{9}{a^2}-\frac{4}{b^2}=1 \qquad \ldots(2)$$

From equations (1) and (2), we obtain

$$\frac{9}{a^2}-\frac{4}{\left(10-a^2\right)}=1$$
$$\Rightarrow 9\left(10-a^2\right)-4a^2=a^2\left(10-a^2\right)$$
$$\Rightarrow 90-9a^2-4a^2=10a^2-a^4$$
$$\Rightarrow a^4-23a^2+90=0$$
$$\Rightarrow a^4-18a^2-5a^2+90=0$$
$$\Rightarrow a^2\left(a^2-18\right)-5\left(a^2-18\right)=0$$
$$\Rightarrow \left(a^2-18\right)\left(a^2-5\right)=0$$
$$\Rightarrow a^2=18 \text{ or } 5$$

In hyperbola, $c > a$, i.e., $c^2 > a^2$

$a^2 = 5$

$b^2 = 10 - a^2 = 10 - 5 = 5$

Thus, the equation of the hyperbola is $\frac{y^2}{5}-\frac{x^2}{5}=1$.

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