$$= [2 \times 12.0 - 19.992439 - 4.002603]c^2$$
$$= (0.004958\ c^2)\ \text{u}$$
$$= 0.004958 \times 931.5 = 4.618377\ \text{MeV}$$

The positive *Q*-value of the reaction shows that the reaction is exothermic.

Question 13.16:

Suppose, we think of fission of a $^{56}_{26}\text{Fe}$ nucleus into two equal fragments, $^{28}_{13}\text{Al}$. Is the fission energetically possible? Argue by working out *Q* of the process. Given $m(^{56}_{26}\text{Fe}) = 55.93494\ \text{u}$ and $m(^{28}_{13}\text{Al}) = 27.98191\ \text{u}$.

Answer

The fission of $^{56}_{26}\text{Fe}$ can be given as:

$$^{56}_{13}\text{Fe} \longrightarrow 2\,^{28}_{13}\text{Al}$$

It is given that:

Atomic mass of $m(^{56}_{26}\text{Fe})$ = 55.93494 u

Atomic mass of $m(^{28}_{13}\text{Al}) = 27.98191\ \text{u}$

The *Q*-value of this nuclear reaction is given as:

$$Q = \left[m(^{56}_{26}\text{Fe}) - 2m(^{28}_{13}\text{Al})\right]c^2$$
$$= [55.93494 - 2 \times 27.98191]c^2$$
$$= (-0.02888\ c^2)\ \text{u}$$

But $1\,\text{u} = 931.5\ \text{MeV}/c^2$

$\therefore Q = -0.02888 \times 931.5 = -26.902\ \text{MeV}$

The *Q*-value of the fission is negative. Therefore, the fission is not possible energetically. For an energetically-possible fission reaction, the *Q*-value must be positive.

Question 13.17:

The fission properties of $^{239}_{94}\text{Pu}$ are very similar to those of $^{235}_{92}\text{U}$.
The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure $^{239}_{94}\text{Pu}$ undergo fission?

Answer

Average energy released per fission of $^{239}_{94}\text{Pu}$, $E_{av} = 180\ \text{MeV}$

Amount of pure $_{94}\text{Pu}^{239}$, *m* = 1 kg = 1000 g

N_A= Avogadro number = 6.023 × 10^{23}

Mass number of $^{239}_{94}\text{Pu}$ = 239 g

1 mole of $_{94}\text{Pu}^{239}$ contains N_A atoms.

$\therefore$*m* g of $_{94}\text{Pu}^{239}$ contains $\left(\dfrac{N_A}{\text{Mass number}} \times m\right)$ atoms

$$= \frac{6.023 \times 10^{23}}{239} \times 1000 = 2.52 \times 10^{24}\ \text{atoms}$$

$\therefore$Total energy released during the fission of 1 kg of $^{239}_{94}\text{Pu}$ is calculated as:

$$E = E_{av} \times 2.52 \times 10^{24}$$
$$= 180 \times 2.52 \times 10^{24} = 4.536 \times 10^{26}\ \text{MeV}$$

Hence, 4.536×10^{26} MeV is released if all the atoms in 1 kg of pure $_{94}\text{Pu}^{239}$ undergo fission.

Question 13.18:

A 1000 MW fission reactor consumes half of its fuel in 5.00 y. How much $^{235}_{92}\mathrm{U}$ did it contain initially? Assume that the reactor operates 80% of the time, that all the energy generated arises from the fission of $^{235}_{92}\mathrm{U}$ and that this nuclide is consumed only by the fission process.

Answer

Half life of the fuel of the fission reactor, $t_{\frac{1}{2}} = 5$ years

= 5 × 365 × 24 × 60 × 60 s

We know that in the fission of 1 g of $^{235}_{92}\mathrm{U}$ nucleus, the energy released is equal to 200 MeV.

1 mole, i.e., 235 g of $^{235}_{92}\mathrm{U}$ contains 6.023 × 10^{23} atoms.

∴1 g $^{235}_{92}\mathrm{U}$ contains $\frac{6.023 \times 10^{23}}{235}$ atoms

The total energy generated per gram of $^{235}_{92}\mathrm{U}$ is calculated as:

$$E = \frac{6.023 \times 10^{23}}{235} \times 200 \text{ MeV/g}$$

$$= \frac{200 \times 6.023 \times 10^{23} \times 1.6 \times 10^{-19} \times 10^{6}}{235} = 8.20 \times 10^{10} \text{ J/g}$$

The reactor operates only 80% of the time.

Hence, the amount of $^{235}_{92}\mathrm{U}$ consumed in 5 years by the 1000 MW fission reactor is calculated as:

$$= \frac{5 \times 80 \times 60 \times 60 \times 365 \times 24 \times 1000 \times 10^{6}}{100 \times 8.20 \times 10^{10}} \text{ g}$$

$$\approx 1538 \text{ kg}$$

∴Initial amount of $^{235}_{92}\mathrm{U}$ = 2 × 1538 = 3076 kg

Question 13.19:

How long can an electric lamp of 100W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as

$$^{2}_{1}\mathrm{H} + ^{2}_{1}\mathrm{H} \longrightarrow ^{3}_{1}\mathrm{He} + \mathrm{n} + 3.27 \text{ MeV}$$

Answer

The given fusion reaction is:

$$^{2}_{1}\mathrm{H} + ^{2}_{1}\mathrm{H} \longrightarrow ^{3}_{2}\mathrm{He} + \mathrm{n} + 3.27 \text{ MeV}$$

Amount of deuterium, m = 2 kg

1 mole, i.e., 2 g of deuterium contains 6.023 × 10^{23} atoms.

∴2.0 kg of deuterium contains $= \frac{6.023 \times 10^{23}}{2} \times 2000 = 6.023 \times 10^{26}$ atoms

It can be inferred from the given reaction that when two atoms of deuterium fuse, 3.27 MeV energy is released.

∴Total energy per nucleus released in the fusion reaction:

$$E = \frac{3.27}{2} \times 6.023 \times 10^{26} \text{ MeV}$$

$$= \frac{3.27}{2} \times 6.023 \times 10^{26} \times 1.6 \times 10^{-19} \times 10^{6}$$

$$= 1.576 \times 10^{14} \text{ J}$$

Power of the electric lamp, P = 100 W = 100 J/s

Hence, the energy consumed by the lamp per second = 100 J

The total time for which the electric lamp will glow is calculated as:

$$\frac{1.576 \times 10^{14}}{100} \text{ s}$$

$$\frac{1.576 \times 10^{[illegible]}}{100 \times 60 \times 60 \times 24 \times 365} \approx 4.9 \times 10^4 \text{ years}$$

Question 13.20:

Calculate the height of the potential barrier for a head on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0 fm.)

Answer

When two deuterons collide head-on, the distance between their centres, d is given as:

Radius of 1st deuteron + Radius of 2nd deuteron

Radius of a deuteron nucleus = 2 fm = 2×10^{-15} m

$\therefore d = 2 \times 10^{-15} + 2 \times 10^{-15} = 4 \times 10^{-15}$ m

Charge on a deuteron nucleus = Charge on an electron = $e = 1.6 \times 10^{-19}$ C

Potential energy of the two-deuteron system:

$$V = \frac{e^2}{4\pi \epsilon_0 d}$$

Where,

ϵ_0 = Permittivity of free space

$$\frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{C}^{-2}$$

$$\therefore V = \frac{9 \times 10^9 \times \left(1.6 \times 10^{-19}\right)^2}{4 \times 10^{-15}} \text{ J}$$

$$= \frac{9 \times 10^9 \times \left(1.6 \times 10^{-19}\right)^2}{4 \times 10^{-15} \times \left(1.6 \times 10^{-19}\right)} \text{ eV}$$

$$= 360 \text{ keV}$$

Hence, the height of the potential barrier of the two-deuteron system is 360 keV.

Question 13.21:

From the relation $R = R_0 A^{1/3}$, where R_0 is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e. independent of A).

Answer

We have the expression for nuclear radius as:

$R = R_0 A^{1/3}$

Where,

R_0 = Constant.

A = Mass number of the nucleus

Nuclear matter density, $\rho = \frac{\text{Mass of the nucleus}}{\text{Volume of the nucleus}}$

Let m be the average mass of the nucleus.

Hence, mass of the nucleus = mA

$$\therefore \rho = \frac{mA}{\frac{4}{3}\pi R^3} = \frac{3mA}{4\pi\left(R_0 A^{\frac{1}{3}}\right)^3} = \frac{3mA}{4\pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$$

Hence, the nuclear matter density is independent of A. It is nearly constant.

*********** END **********