Co-Ordinate Geometry Ex 14.5 Q6

Answer :

The formula for the area '*A*' encompassed by three points (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by the formula,

$$A = \frac{1}{2}\begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_2 - x_3 & y_2 - y_3 \end{vmatrix}$$

$$A = \frac{1}{2}\{(x_1 - x_2)(y_2 - y_3) - (x_2 - x_3)(y_1 - y_2)\}$$

If three points are collinear the area encompassed by them is equal to 0.

The three given points are *A*(*a*,0), *B*(0,*b*) and *C*(1,1).

$$A = \frac{1}{2}\begin{vmatrix} a - 0 & 0 - b \\ 0 - 1 & b - 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} a & -b \\ -1 & b - 1 \end{vmatrix}$$

$$= \frac{1}{2}\{(a)(b-1) - (-1)(-b)\}$$

$$= \frac{1}{2}\{ab - a - b\}$$

$$A = ab - (a + b)$$

It is given that $\frac{1}{a} + \frac{1}{b} = 1$

So we have,

$$\frac{1}{a} + \frac{1}{b} = 1$$

$$\frac{a+b}{ab} = 1$$

$$a + b = ab$$

Using this in the previously arrived equation for area we have,

$$A = ab - (a + b)$$

$$A = ab - ab$$

$$A = 0$$

Since the area enclosed by the three points is equal to 0, the three points need to be $\boxed{\text{collinear}}$.

Co-Ordinate Geometry Ex 14.5 Q7

Answer :

GIVEN: point A divides the line segment joining P (−5, 1) and Q (3, −5) in the ratio *k*: 1

Coordinates of point B (1, 5) and C (7, −2)

TO FIND: The value of *k*

PROOF: point A divides the line segment joining P (−5, 1) and Q (3, −5) in the ratio *k*: 1

So the coordinates of A are $\left[\frac{3k+1}{k+1}, \frac{5k+1}{k+1}\right]$

We know area of triangle formed by three points $(x_1, y_1), (x_2, y_2) \, and \, (x_3, y_3)$ is given by

$$\Delta = \frac{1}{2}\left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

Now Area of ΔABC= 2 sq units.

Taking three points A $\left[\frac{3k+1}{k+1}, \frac{5k+1}{k+1}\right]$, B (1, 5) and C (7, −2)

$$2 = \frac{1}{2}\left|\left\{\left(\frac{3k-5}{k+1}\right)5 - 2 + 7\left(\frac{5k+1}{k+1}\right)\right\} - \left(\frac{5k+1}{k+1}\right) + 35 - 2\left(\frac{3k-5}{k+1}\right)\right|$$

$$2 = \frac{1}{2}\left|\left\{\left(\frac{15k-25}{k+1}\right) - 2 + \left(\frac{35k+7}{k+1}\right)\right\} - \left(\frac{5k+1}{k+1}\right) + 35 - \left(\frac{6k-10}{k+1}\right)\right|$$

$$2 = \frac{1}{2}\left|\left(\frac{15k - 25 - 2k + 35k + 7}{k+1}\right) - \left(\frac{5k + 1 + 35k + 35 - 6k + 10}{k+1}\right)\right|$$

$$2 = \frac{1}{2}\left|\left(\frac{(48k - 20) - (34k + 46)}{k+1}\right)\right|$$

$$2 = \frac{1}{2}\left|\left(\frac{14k-66}{k+1}\right)\right|$$

$$2 = \left|\left(\frac{7k-33}{k+1}\right)\right|$$

$$\pm 2 = \left(\frac{7k-33}{k+1}\right)$$

$$\Rightarrow 7k-33 = \pm 2(k+1)$$

$$\Rightarrow 7k-33 = 2(k+1), 7k-33 = -2(k+1)$$

$$\Rightarrow 7k-33 = 2k+2, 7k-33 = -2k-2$$

$$\Rightarrow 7k-2k = 33+2, 7k+2k = +33-2$$

$$\Rightarrow 5k = 35, 9k = 31$$

$$\Rightarrow k = \frac{35}{5}, k = \frac{31}{9}$$

$$\Rightarrow k = 7, k = \frac{31}{9}$$

Hence $\boxed{k = 7 \text{ or } \frac{31}{9}}$

*********** END **********