Indefinite Integrals Ex 19.30 Q27

$$\text{Let } \frac{3x-2}{(x+1)^2(x+3)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+3)}$$

$$\Rightarrow \quad 3x - 2 = A(x+1)(x+3) + B(x+3) + C(x+1)^2$$
$$= (A+C)x^2 + (4A+B+2C)x + (3A+3B+C)$$

Equating similar terms, we get,

$$A + C = 0 \quad \Rightarrow \quad A = -C$$
$$4A + B + 2C = 3 \quad \Rightarrow \quad B = -2C = 3$$
$$3A + 3B + C = -2 \Rightarrow \quad 3B - 2C = -2$$

Solving, we get, $B = -\frac{5}{2}$, $C = -\frac{11}{4}$ & $A = \frac{11}{4}$

Thus,

$$I = \frac{11}{4}\int \frac{dx}{x+1} - \frac{5}{2}\int \frac{dx}{(x+1)^2} - \frac{11}{4}\int \frac{dx}{x+3}$$

$$I = \frac{11}{4}\log|x+1| + \frac{5}{2(x+1)} - \frac{11}{4}\log|x+3| + c$$

Indefinite Integrals Ex 19.30 Q28

Let $\frac{2x+1}{(x+2)(x-3)^2} = \frac{A}{x+2} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$

$$\Rightarrow \quad 2x+1 = A(x-3)^2 + B(x+2)(x-3) + C(x+2)$$
$$= (A+B)x^2 + (-6A-B+C)x + (9A-6B+2C)$$

Equating similar terms, we get,

$A+B=0 \quad \Rightarrow \quad A=-B$

$-6A-B+C=2 \quad \Rightarrow \quad 5B+C=2$

$9A-6B+2C=1 \Rightarrow \quad -15B+2C=1$

Solving, we get, $B=\frac{3}{25}$, $C=\frac{7}{5}$, $A=-\frac{3}{25}$

Thus,

$$I = -\frac{3}{25}\int\frac{dx}{x+2} + \frac{3}{25}\int\frac{dx}{x-3} + \frac{7}{5}\int\frac{dx}{(x-3)^2}$$

$$I = -\frac{3}{25}\log|x+2| + \frac{3}{25}\log|x-3| - \frac{7}{5(x-3)} + c$$

Indefinite Integrals Ex 19.30 Q29

Let $\frac{x^2+1}{(x-2)^2(x+3)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+3}$

$$\Rightarrow \quad x^2+1 = A(x-2)(x+3) + B(x+3) + C(x-2)^2$$
$$= (A+C)x^2 + (A+B-4C)x + (-6A+3B+4C)$$

Equating similar terms, we get,

$A+C=1$, $A+B-4C=0$, $-6A+3B+4C=1$

Solving, we get, $A=\frac{3}{5}$, $B=1$, $C=\frac{2}{5}$

Thus,

$$I = \frac{3}{5}\int\frac{dx}{x-2} + \int\frac{dx}{(x-2)^2} + \frac{2}{5}\int\frac{dx}{x+3}$$

$$I = \frac{3}{5}\log|x-2| - \frac{1}{(x-2)} + \frac{2}{5}\log|x+3| + c$$

Indefinite Integrals Ex 19.30 Q30

Let $\frac{x}{(x-1)^2(x+2)}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(x+2)}$

$$x=A(x-1)(x+2)+B(x+2)+C(x-1)^2$$

Substituting $x = 1$, we obtain

$$B=\frac{1}{3}$$

Equating the coefficients of x^2 and constant term, we obtain

$A + C = 0$

$-2A + 2B + C = 0$

On solving, we obtain

$$A=\frac{2}{9} \text{ and } C=\frac{-2}{9}$$

$$\therefore \frac{x}{(x-1)^2(x+2)}=\frac{2}{9(x-1)}+\frac{1}{3(x-1)^2}-\frac{2}{9(x+2)}$$

$$\Rightarrow \int\frac{x}{(x-1)^2(x+2)}dx=\frac{2}{9}\int\frac{1}{(x-1)}dx+\frac{1}{3}\int\frac{1}{(x-1)^2}dx-\frac{2}{9}\int\frac{1}{(x+2)}dx$$

$$=\frac{2}{9}\log|x-1|+\frac{1}{3}\left(\frac{-1}{x-1}\right)-\frac{2}{9}\log|x+2|+C$$

$$=\frac{2}{9}\log\left|\frac{x-1}{x+2}\right|-\frac{1}{3(x-1)}+C$$

Indefinite Integrals Ex 19.30 Q31

Let $\dfrac{x^2}{(x-1)(x+1)^2} = \dfrac{A}{x-1} + \dfrac{B}{(x+1)} + \dfrac{C}{(x+1)^2}$

$$\Rightarrow \qquad x^2 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$$
$$= (A+B)x^2 + (2A+C)x + (A-B-C)$$

Equating similar terms,

$A + B = 1,\ 2A + C = 0,\ A - B - C = 0$

Solving, we get, $A = \dfrac{1}{4},\ B = \dfrac{3}{4},\ C = -\dfrac{1}{2}$

Thus,

$$I = \frac{1}{4}\int \frac{dx}{x-1} + \frac{3}{4}\int \frac{dx}{x+1} - \frac{1}{2}\int \frac{dx}{(x+1)^2}$$
$$= \frac{1}{4}\log|x-1| + \frac{3}{4}\log|x+1| + \frac{1}{2(x+1)} + c$$

$$I = \frac{1}{4}\log|x-1| + \frac{3}{4}\log|x+1| + \frac{1}{2(x+1)} + c$$

*********** END **********