Indefinite Integrals Ex 19.9 Q35

Let $\quad I = \int \frac{x \sin^{-1} x^2}{\sqrt{1 - x^4}} dx \text{ - - - - (i)}$

Let $\quad \sin^{-1} x^2 = t \qquad$ then,

$d\left(\sin^{-1} x^2\right) = dt$

$\Rightarrow \quad 2x \times \frac{1}{\sqrt{1 - x^4}} dx = dt$

$\Rightarrow \quad \frac{x}{\sqrt{1 - x^4}} dx = \frac{dt}{2}$

Putting $\sin^{-1} x^2 = t$ and $\frac{x}{\sqrt{1 - x^4}} dx = \frac{dt}{2}$ in equation (i),

we get

$$I = \int t \frac{dt}{2}$$

$$= \frac{1}{2} \times \frac{t^2}{2} + c$$

$$= \frac{1}{4}\left(\sin^{-1} x^2\right)^2 + c$$

$$\therefore \quad I = \frac{1}{4}\left(\sin^{-1} x^2\right)^2 + c$$

Indefinite Integrals Ex 19.9 Q36

Let $\quad I = \int x^3 \sin\left(x^4+1\right)dx$ - - - - - (i)

Let $\quad x^4+1 = t$ then,

$d\left(x^4+1\right) = dt$

$\Rightarrow \quad x^4\,dx = dt$

$\Rightarrow \quad x^3dx = \frac{dt}{4}$

Putting $x^4+1 = t$ and $x^3dx = \frac{dt}{4}$ in equation (i), we get

$$I = \int \sin t \frac{dt}{4}$$
$$= -\frac{1}{4}\cos t + c$$
$$= -\frac{1}{4}\cos\left(x^4+1\right) + c$$

$\therefore \quad I = -\frac{1}{4}\cos\left(x^4+1\right) + c$

Indefinite Integrals Ex 19.9 Q37

Let $\quad I = \int \frac{(x+1)e^x}{\cos^2\left(xe^x\right)}dx$ - - - - - (i)

Let $\quad xe^x = t$ then,

$d\left(xe^x\right) = dt$

$\Rightarrow \quad \left(e^x + xe^x\right)dx = dt$

$\Rightarrow \quad (x+1)e^x dx = dt$

Putting $xe^x = t$ and $(x+1)e^x\,dx = dt$ in equation (i), we get

$$I = \int \frac{dt}{\cos^2 t}$$
$$= \int \sec^2 t\,dt$$
$$= \tan t + c$$
$$= \tan\left(xe^x\right) + c$$

$\therefore \quad I = \tan\left(xe^x\right) + c$

Indefinite Integrals Ex 19.9 Q38

Let $\quad I = \int x^2 e^{x^3} \cos\left(e^{x^3}\right) dx - - - - - (i)$

Let $\quad e^{x^3} = t \qquad$ then,

$d\left(e^{x^3}\right) = dt$

$\Rightarrow \quad 3x^2 e^{x^3}\, dx = dt$

$\Rightarrow \quad x^2 e^{x^3} dx = \dfrac{dt}{3}$

Putting $e^{x^3} = t$ and $x^2 e^{x^3}\, dx = \dfrac{dt}{3}$ in equation (i), we get

$$I = \int \cos t \frac{dt}{3}$$
$$= \frac{\sin t}{3} + c$$
$$= \frac{\sin\left(e^{x^3}\right)}{3} + c$$

$\therefore \quad I = \dfrac{1}{3}\sin\left(e^{x^3}\right) + c$

Indefinite Integrals Ex 19.9 Q39

Let $\quad I = \int 2x \sec^3\left(x^2+3\right)\tan\left(x^2+3\right) dx - - - - - (i)$

Let $\quad \sec\left(x^2+3\right) = t \qquad$ then,

$d\left[\sec\left(x^2+3\right)\right] = dt$

$\Rightarrow \quad 2x \sec\left(x^2+3\right)\tan\left(x^2+3\right) dx = dt$

Putting $\sec\left(x^2+3\right) = t$ and $2x \sec\left(x^2+3\right)\tan\left(x^2+3\right) dx = dt$ in equation (i), we get

$$I = \int t^2 dt$$
$$= \frac{t^3}{3} + c$$
$$= \frac{1}{3}\left[\sec\left(x^2+3\right)\right]^3 + c$$

$\therefore \quad I = \dfrac{1}{3}\left[\sec\left(x^2+3\right)\right]^3 + c$

*********** END **********