Adjoint and Inverse of Matrix Ex 7.1 Q3

Here

$$A=\begin{bmatrix}1 & -1 & 1\\ 2 & 3 & 0\\ 18 & 2 & 10\end{bmatrix}$$

Cofactors of A are

$$\begin{array}{lll} C_{11}=30 & C_{21}=12 & C_{31}=-3\\ C_{12}=-20 & C_{22}=-8 & C_{32}=2\\ C_{13}=-50 & C_{23}=-20 & C_{33}=5\end{array}$$

Therefore,

$$adj\ A=\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^T$$

$$=\begin{bmatrix}30 & -20 & -50\\ 12 & -8 & -20\\ -3 & 2 & 5\end{bmatrix}^T$$

So,

$$adjA=\begin{bmatrix}30 & 12 & -3\\ -20 & -8 & 2\\ -50 & -20 & 5\end{bmatrix}$$

Now,

$$A(adjA)=\begin{bmatrix}1 & -1 & 1\\ 2 & 3 & 0\\ 18 & 2 & 10\end{bmatrix}\begin{bmatrix}30 & 12 & -3\\ -20 & -8 & 2\\ -50 & -20 & 5\end{bmatrix}$$

$$=\begin{bmatrix}1 & -1 & 1\\ 2 & 3 & 0\\ 18 & 2 & 10\end{bmatrix}(0)$$

$$=0$$

Hence proved.

Adjoint and Inverse of Matrix Ex 7.1 Q4

Here, $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

Cofactors of A are:

$C_{11} = -4 \qquad C_{21} = -3 \qquad C_{31} = -3$

$C_{12} = 1 \qquad C_{22} = 0 \qquad C_{32} = 1$

$C_{13} = 4 \qquad C_{23} = 4 \qquad C_{33} = 3$

$$\therefore \quad \text{adj}A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} -4 & 1 & 4 \\ -3 & 0 & 4 \\ -3 & 1 & 3 \end{bmatrix}^T$$

Therefore, $\text{adj}A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

So, $\text{adj}A = A$

Adjoint and Inverse of Matrix Ex 7.1 Q5

Here $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

Cofactors of A are:

$C_{11} = -3 \qquad C_{21} = 6 \qquad C_{31} = 6$

$C_{12} = -6 \qquad C_{22} = 3 \qquad C_{32} = -6$

$C_{13} = -6 \qquad C_{23} = -6 \qquad C_{33} = 3$

$$\therefore \quad \text{adj}A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix}^T$$

Therefore, $\text{adj}A = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix} \qquad ---(i)$

Now, $3.A^T = 3\begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix} \qquad ---(ii)$

$\therefore \quad \text{adj}A = 3.A^T$

Adjoint and Inverse of Matrix Ex 7.1 Q6

Here, $A=\begin{bmatrix}1 & -2 & 3\\ 0 & 2 & -1\\ -4 & 5 & 2\end{bmatrix}$

Cofactors of A are:

$$\begin{array}{lll} C_{11}=9 & C_{21}=19 & C_{31}=-4\\ C_{12}=4 & C_{22}=14 & C_{32}=1\\ C_{13}=8 & C_{23}=3 & C_{33}=2\end{array}$$

$$\therefore \quad \operatorname{adj}A=\begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^T$$

$$=\begin{bmatrix}9 & 4 & 8\\ 19 & 14 & 3\\ -4 & 1 & 2\end{bmatrix}^T$$

Therefore,

$$\operatorname{adj}A=\begin{bmatrix}9 & 19 & -4\\ 4 & 14 & 1\\ 8 & 3 & 2\end{bmatrix}$$

Now, $A\operatorname{adj}A=\begin{bmatrix}1 & -2 & 3\\ 0 & 2 & -1\\ -4 & 5 & 2\end{bmatrix}\begin{bmatrix}9 & 19 & -4\\ 4 & 14 & 1\\ 8 & 3 & 2\end{bmatrix}$

$$=\begin{bmatrix}25 & 0 & 0\\ 0 & 25 & 0\\ 0 & 0 & 25\end{bmatrix}$$

$$=25\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=25I_3$$

*********** END **********