Differentiation Ex 11.5 Q44

Here,

$$e^y = y^x$$

Taking log on both the sides,

$$\log e^y = \log y^x$$
$$y \log e = x \log y \qquad\qquad \left[\text{Since, } \log a^b = b \log a, \log_e e = 1\right]$$
$$y = x \log y \qquad\qquad\qquad ---(i)$$

Differentiating it with respect to x using product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(x \log y\right)$$
$$= x \frac{dy}{dx}\left(\log y\right) + \log y \frac{d}{dx}\left(x\right)$$
$$\frac{dy}{dx} = \frac{x}{y}\frac{dy}{dx} + \log y\,(1)$$
$$\frac{dy}{dx}\left(1 - \frac{x}{y}\right) = \log y$$
$$\frac{dy}{dx}\left(\frac{y - x}{y}\right) = \log y$$
$$\frac{dy}{dx} = \frac{y \log y}{y - x}$$
$$\frac{dy}{dx} = \frac{y \log y}{\left(y - \dfrac{y}{\log y}\right)} \qquad\qquad \left[\text{Since, using equation (i)}\right]$$
$$= \frac{y \log y \times \log y}{y \log y - y}$$
$$= \frac{y\,(\log y)^2}{y\,(\log y - 1)}$$

$$\frac{dy}{dx} = \frac{(\log y)^2}{(\log y - 1)}$$

Differentiation Ex 11.5 Q45

Here,

$$e^{x+y} - x = 0$$
$$e^{x+y} = x \qquad\qquad\qquad ---(i)$$

Differentiating it with respect to x using chain rule,

$$\frac{d}{dx}\left(e^{x+y}\right) = \frac{d}{dx}\left(x\right)$$
$$e^{x+y}\frac{d}{dx}\left(x + y\right) = 1$$
$$x\left[1 + \frac{dy}{dx}\right] = 1 \qquad\qquad \left[\text{Using equatoin (i)}\right]$$
$$1 + \frac{dy}{dx} = \frac{1}{x}$$
$$\frac{dy}{dx} = \frac{1}{x} - 1$$

$$\frac{dy}{dx} = \frac{1 - x}{x}$$

Differentiation Ex 11.5 Q46

Here $y = x\sin(a+y)$

Differentiating it with respect to x using the chain rule and product rule,

$$\frac{dy}{dx} = x\frac{d}{dx}\sin(a+y) + \sin(a+y)\frac{dx}{dx}$$

$$\frac{dy}{dx} = x\cos(a+y)\frac{dy}{dx} + \sin(a+y)$$

$$(1 - x\cos(a+y))\frac{dy}{dx} = \sin(a+y)$$

$$\frac{dy}{dx} = \frac{\sin(a+y)}{(1 - x\cos(a+y))}$$

$$\frac{dy}{dx} = \frac{\sin(a+y)}{\left(1 - \dfrac{y}{\sin(a+y)}\cos(a+y)\right)} \qquad \left[\text{Since } \frac{y}{\sin(a+y)} = x\right]$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin(a+y) - y\cos(a+y)}$$

Differentiation Ex 11.5 Q47

Here $x\sin(a+y) + \sin a\cos(a+y) = 0$

Differentiating it with respect to x using the chain rule and product rule,

$$\frac{d}{dx}\left[x\sin(a+y) + \sin a\cos(a+y)\right] = 0$$

$$x\frac{d}{dx}\sin(a+y) + \sin(a+y)\frac{dx}{dx} + \sin a\frac{d}{dx}\cos(a+y) + \cos(a+y)\frac{d}{dx}\sin a = 0$$

$$x\cos(a+y)\left(0 + \frac{dy}{dx}\right) + \sin(a+y) + \sin a\left(-\sin(a+y)\frac{dy}{dx}\right) + 0 = 0$$

$$\left[x\cos(a+y) - \sin a\sin(a+y)\right]\frac{dy}{dx} + \sin(a+y) = 0$$

$$\frac{dy}{dx} = -\frac{\sin(a+y)}{x\cos(a+y) - \sin a\sin(a+y)}$$

$$\frac{dy}{dx} = \frac{-\sin(a+y)}{\left(-\dfrac{\sin a\cos(a+y)}{\sin(a+y)}\right)\cos(a+y) - \sin a\sin(a+y)} \qquad \left[\text{Since } x = -\frac{\sin a\cos(a+y)}{\sin(a+y)}\right]$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{(\sin a)\cos^2(a+y) + \sin a\sin^2(a+y)}$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{(\sin a)\left[\cos^2(a+y) + \sin^2(a+y)\right]}$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{(\sin a)} \qquad \left[\text{Since } \cos^2(a+y) + \sin^2(a+y) = 1\right]$$

Differentiation Ex 11.5 Q48

Here,

$$(\sin x)^y = x + y$$

Taking log on both the sides,

$$\log(\sin x)^y = \log(x + y)$$

$$y\log(\sin x) = \log(x + y) \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating it with respect to x using chain rule, product rule,

$$\frac{d}{dx}\left(y\log(\sin x)\right) = \frac{d}{dx}\log(x + y)$$

$$y\frac{d}{dx}\log\sin x + \log\sin x\frac{dy}{dx} = \frac{1}{x+y}\frac{d}{dx}(x + y)$$

$$\frac{y}{\sin x}\frac{d}{dx}(\sin x) + \log\sin x\frac{dy}{dx} = \frac{1}{(x+y)}\left[1 + \frac{dy}{dx}\right]$$

$$\frac{y(\cos x)}{(\sin x)} + \log\sin x\frac{dy}{dx} = \frac{1}{(x+y)} + \frac{1}{(x+y)}\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(\log\sin x - \frac{1}{x+y}\right) = \frac{1}{(x+y)} - y\cot x$$

$$\frac{dy}{dx}\left(\frac{(x+y)\log\sin x - 1}{(x+y)}\right) = \left(\frac{1 - y(x+y)\cot x}{x+y}\right)$$

$$\frac{dy}{dx} = \left(\frac{1 - y(x+y)\cot x}{(x+y)\log\sin x - 1}\right)$$

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