Exercise 5.2

Let their common difference be d.

It is given that difference between their 100^{th} terms is 100.

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$a + (100 - 1)\,d - [a' + (100 - 1)\,d]$

$= a + 99d - a' - 99d = 100$

$\Rightarrow a - a' = 100 \ldots (1)$

We want to find difference between their 1000^{th} terms which means we want to calculate:

$a + (1000 - 1)\,d - [a' + (1000 - 1)\,d]$

$= a + 999d - a' - 999d = a - a'$

Putting equation (1) in the above equation,

$a + (1000 - 1)\,d - [a' + (1000 - 1)\,d]$

$= a + 999d - a' + 999d = a - a' = 100$

Therefore, difference between their 1000^{th} terms would be equal to 100.

13. How many three digit numbers are divisible by 7?

Ans. We have AP starting from 105 because it is the first three digit number divisible by 7.

AP will end at 994 because it is the last three digit number divisible by 7.

Therefore, we have AP of the form 105, 112, 119..., 994

Let 994 is the n^{th} term of AP.

We need to find n here.

First term = a = 105, Common difference = d = 112 − 105 = 7

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$994 = 105 + (n - 1)\,(7)$

$\Rightarrow 994 = 105 + 7n - 7$

$\Rightarrow 896 = 7n \Rightarrow n = 128$

It means 994 is the 128^{th} term of AP.

Therefore, there are 128 terms in AP.

14. How many multiples of 4 lie between 10 and 250?

Ans. First multiple of 4 which lie between 10 and 250 is 12.

The last multiple of 4 which lie between 10 and 250 is 248.

Therefore, AP is of the form 12, 16, 20... ,248

First term = a = 12, Common difference = d = 4

Using formula $a_n = a + (n - 1)\,d$, to find n^{th} term of arithmetic progression,

$248 = 12 + (n - 1)\,(4)$

$\Rightarrow 248 = 12 + 4n - 4$

$\Rightarrow 240 = 4n \Rightarrow n = 60$

It means that 248 is the 60^{th} term of AP.

So, we can say that there are 60 multiples of 4 which lie between 10 and 250.

15. For what value of n, are the nth terms of two AP's: 63, 65, 67... and 3, 10, 17... equal?

Ans. Lets first consider AP 63, 65, 67...

First term = a = 63, Common difference = d = 65 − 63 = 2

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$a_n = 63 + (n - 1)\,(2)$... (1)

Now, consider second AP 3, 10, 17...

First term = a = 3, Common difference = d = 10 − 3 = 7

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$a_n = 3 + (n - 1)\,(7)$... (2)

According to the given condition:

(1) = (2)

$\Rightarrow 63 + (n - 1)\,(2) = 3 + (n - 1)\,(7)$

$\Rightarrow 63 + 2n - 2 = 3 + 7n - 7$

$\Rightarrow 65 = 5n \Rightarrow n = 13$

Therefore, 13^{th} term of both the AP's are equal.

Therefore, 13th terms of both the AP's are equal.

16. Determine the AP whose third term is 16 and the 7^{th} term exceeds the 5^{th} term by 12.

Ans. Let first term of AP = a

Let common difference of AP = d

It is given that its 3^{rd} term is equal to 16.

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$16 = a + (3 - 1)(d)$

$\Rightarrow 16 = a + 2d \ldots (1)$

It is also given that 7^{th} term exceeds 5^{th} term by12.

According to the given condition:

$a_7 = a_5 + 12$

$\Rightarrow a + (7 - 1)d = a + (5 - 1)d + 12$

$\Rightarrow 2d = 12 \Rightarrow d = 6$

Putting value of d in equation $16 = a + 2d$,

$16 = a + 2(6) \Rightarrow a = 4$

Therefore, first term = a = 4

And, common difference = d = 6

Therefore, AP is 4, 10, 16, 22...

17. Find the 20^{th} term from the last term of the AP: 3, 8, 13..., 253.

Ans. We want to find 20^{th} term from the last term of given AP.

So, let us write given AP in this way:253 ... 13, 8, 3

Here First term = a = 253, Common Difference = d = 8 − 13 = −5

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$a_{20} = 253 + (20 - 1)(-5)$

$\Rightarrow a_{20} = 253 + 19(-5) = 253 - 95 = 158$

Therefore, the 20^{th} term from the last term of

given AP is 158.

18. The sum of the 4^{th} and 8^{th} terms of an AP is 24 and the sum of 6^{th} and 10^{th} terms is 44. Find the three terms of the AP.

Ans. The sum of 4^{th} and 8^{th} terms of an AP is 24 and sum of 6^{th} and 10^{th} terms is 44.

$a_4 + a_8 = 24$ and $a_6 + a_{10} = 44$

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$a + (4 - 1)\,d + [a + (8 - 1)\,d] = 24$

And, $a + (6 - 1)\,d + [a + (10 - 1)\,d] = 44$

$\Rightarrow a + 3d + a + 7d$

$= 24$ And $a + 5d + a + 9d = 44$

$\Rightarrow 2a + 10d = 24$ And $2a + 14d = 44$

$\Rightarrow a + 5d = 12$ And $a + 7d = 22$

These are equations in two variables.

Using equation, $a + 5d = 12$, we can say that $a = 12 - 5d$... (1)

Putting (1) in equation $a + 7d = 22$,

$12 - 5d + 7d = 22$

$\Rightarrow 12 + 2d = 22$

$\Rightarrow 2d = 10 \Rightarrow d = 5$

Putting value of d in equation $a = 12 - 5d$,

$a = 12 - 5\,(5) = 12 - 25 = -13$

Therefore, first term = $a = -13$ and, Common difference = $d = 5$

Therefore, AP is $-13, -8, -3, 2$...

Its first three terms are -13, -8 and -3.

19. Subba Rao started work in 1995 at an annual salary of Rs 5000 and received an increment of Rs 200 each year. In which year did his income reach Rs 7000?

Ans. Subba Rao's starting salary = Rs 5000

It means, first term = a = 5000

He gets an increment of Rs 200 after every year.

Therefore, common difference = d = 200

His salary after 1 year = 5000 + 200 = Rs 5200

His salary after two years = 5200 + 200 = Rs 5400

Therefore, it is an AP of the form 5000, 5200, 5400, 5600... , 7000

We want to know in which year his income reaches Rs 7000.

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$7000 = 5000 + (n - 1)(200)$

$\Rightarrow 7000 = 5000 + 200n - 200$

$\Rightarrow 7000 - 5000 + 200 = 200n$

$\Rightarrow 2200 = 200n$

$\Rightarrow n = 11$

It means after 11 years, Subba Rao's income would be Rs 7000.

20. Ramkali saved Rs. 5 in the first week of a year and then increased her weekly savings by Rs. 1.75. If in the nth week, her weekly savings become Rs 20.75, find n.

Ans. Ramkali saved Rs. 5 in the first week of year. It means first term = a = 5

Ramkali increased her weekly savings by Rs 1.75.

Therefore, common difference = d = Rs 1.75

Money saved by Ramkali in the second week = a + d = 5 + 1.75 = Rs 6.75

Money saved by Ramkali in the third week = 6.75 + 1.75 = Rs 8.5

Therefore, it is an AP of the form:5, 6.75, 8.5 ... , 20.75

We want to know in which year her weekly savings become 20.75.

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$20.75 = 5 + (n - 1)(1.75)$

$\Rightarrow 20.75 = 5 + 1.75n - 1.75$

$\Rightarrow 17.5 = 1.75n \Rightarrow n = 10$

It means in the 10^{th} week her savings become Rs 20.75.

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