Geometric Progressions Ex 20.4 Q 1

$$S_\infty = 1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \ldots$$

$$\Rightarrow \quad a = 1,\ r = -\frac{1}{3}$$

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{1}{1+\frac{1}{3}}$$

$$S_\infty = \frac{3}{4}$$

$$S_\infty = 8 + 4\sqrt{2} + 4 + \ldots$$

$$\Rightarrow \quad a = 8,\ r = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{8}{1-\frac{1}{\sqrt{2}}}$$

$$= \frac{8\sqrt{2}}{\sqrt{2}-1} \times \frac{\left(\sqrt{2}+1\right)}{\left(\sqrt{2}+1\right)}$$

$$= \frac{8\left(2+\sqrt{2}\right)}{2-1}$$

$$S_\infty = 8\left(2+\sqrt{2}\right)$$

$$S_\infty = \frac{2}{5}+\frac{3}{5^2}+\frac{2}{5^3}+\frac{3}{5^4}+\ldots$$
$$= \left(\frac{2}{5}+\frac{2}{5^3}+\ldots\right)+\left(\frac{3}{5^2}+\frac{3}{5^4}+\ldots\right)$$
$$S_\infty = S'_\infty + S''_\infty$$

For

$$S'_\infty = \frac{a}{1-r}$$
$$= \frac{\frac{2}{5}}{1-\frac{1}{25}}$$
$$= \frac{2}{5}\times\frac{25}{24}$$
$$S'_\infty = \frac{5}{12}$$
$$S''_\infty = \frac{\frac{3}{25}}{1-\frac{1}{25}}$$
$$= \frac{3}{25}\times\frac{25}{24}$$
$$= \frac{3}{24}$$
$$S_\infty = S'_\infty + S''_\infty$$
$$= \frac{5}{12}+\frac{3}{24}$$
$$= \frac{13}{24}$$
$$S_\infty = \frac{13}{24}$$

This infinite G.P has first term $a=10$ and common ratio $r=-\frac{9}{10}=-0.9$

Thus the sum of the infinite G.P will be:

$$10-9+8.9-7.29+\cdots\infty = \frac{a}{1-r} \quad \left[\text{Since } |r|<1\right]$$
$$= \frac{10}{1-(-0.9)}$$
$$= \frac{10}{1.9}$$
$$= \frac{100}{19}$$

The G.P can be written as follows:

$$\frac{1}{3}+\frac{1}{5^2}+\frac{1}{3^3}+\frac{1}{5^4}+\frac{1}{3^5}+\frac{1}{5^6}+\cdots\infty = \left(\frac{1}{3}+\frac{1}{3^3}+\frac{1}{3^5}+\cdots\infty\right)+\left(\frac{1}{5^2}+\frac{1}{5^4}+\frac{1}{5^6}+\cdots\infty\right)$$
$$= \frac{\frac{1}{3}}{1-\frac{1}{3^2}}+\frac{\frac{1}{5^2}}{1-\frac{1}{5^2}}$$
$$= \frac{3}{8}+\frac{1}{24}$$
$$= \frac{10}{24}$$
$$= \frac{5}{12}$$

*********** END **********