Geometric Progressions Ex 20.3 Q 2

$$0.15+0.015+0.0015+\ldots\ldots\text{upto 8 terms}$$

$$=15(0.1+0.01+0.001+\ldots\ldots\text{upto 8 terms})$$

$$=15\left(\frac{1}{10}+\frac{1}{100}+\ldots\ldots\right)$$

$$r=\frac{1}{10}, a=\frac{1}{10}$$

$$Sum=15\left(\frac{\frac{1}{10}\left(1-\frac{1}{10^8}\right)}{1-\frac{1}{10}}\right)$$

$$=\frac{5}{3}\left(1-\frac{1}{10^8}\right)$$

Here the first term of the series is $a=\sqrt{2}$ and the common ration is $r=\frac{\frac{1}{\sqrt{2}}}{\sqrt{2}}=\frac{1}{2}$

Thus the sum of the G.P up to 8^{th} terms is:

$$S_8=\frac{a\left(1-r^8\right)}{1-r}=\frac{\sqrt{2}\left(1-\left(\frac{1}{2}\right)^8\right)}{1-\frac{1}{2}}=2\sqrt{2}\left(1-\frac{1}{256}\right)=\frac{255\sqrt{2}}{128}$$

$$\frac{2}{9}-\frac{1}{3}+\frac{1}{2}-\frac{3}{4}+\ldots\text{ to 5 terms.}$$

$$a=\frac{2}{9},\ r=\frac{\frac{-1}{3}}{\frac{2}{9}}=\frac{-1}{3}\times\frac{9}{2}=\frac{-3}{2},\ n=5$$

$$S_5=a\frac{\left(1-r^5\right)}{1-r}$$

$$=\frac{2}{9}\frac{\left(1-\left(\frac{-3}{2}\right)^5\right)}{1-\left(\frac{-3}{2}\right)}$$

$$=\frac{2}{9}\frac{\left(1+\frac{243}{32}\right)}{1+\frac{3}{2}}$$

$$=\frac{2}{9}\frac{(275)}{32}\times\frac{2}{5}$$

$$=\frac{55}{72}$$

$$(x+y)+(x^2+xy+y^2)+(x^3+x^2y+xy^2+y^3)+\ldots\ldots$$

$$=\frac{1}{x-y}\{(x^2-y^2)+(x^3-y^3)+\ldots\ldots..to\ \infty\}\ldots\ldots[\because\frac{x^n-y^n}{x-y}=x^{n-1}+x^{n-2}y+\ldots.+y^{n-1}]$$

$$=\frac{1}{x-y}\{(x^2+x^3+\ldots..to\ \infty)-(y^2+y^3+\ldots..to\ \infty)\ \}$$

$$=\frac{1}{x-y}\left\{\frac{x^2}{1-x}-\frac{y^2}{1-y}\right\}$$

$$=\frac{1}{x-y}\left\{\frac{x^2-x^2y-y^2+xy^2}{(1-x)(1-y)}-\right\}$$

$$=\frac{x+y-xy}{(1-x)(1-y)}$$

The series can be written as:

$$3\left(\frac{1}{5}+\frac{1}{5^3}+\frac{1}{5^5}+\cdots\ n\text{ terms}\right)+4\left(\frac{1}{5^2}+\frac{1}{5^4}+\frac{1}{5^6}+\cdots\ n\text{ terms}\right)$$

For the first part $a=\frac{1}{5}$ and the common ratio $r=\frac{1}{5^2}=\frac{1}{25}$

Thus the sum is:

$$3\left(\frac{1}{5}+\frac{1}{5^3}+\frac{1}{5^5}+\cdots\ n\text{ terms}\right)=3\cdot\frac{\frac{1}{5}\left(1-\left(\frac{1}{25}\right)^n\right)}{1-\frac{1}{25}}$$

$$=\frac{5}{8}\left(1-\frac{1}{5^{2n}}\right)$$

For the second part $a=\frac{1}{25}$ and common ratio $r=\frac{1}{25}$ then

$$4\left(\frac{1}{5^2}+\frac{1}{5^4}+\frac{1}{5^6}+\cdots\ n\text{ terms}\right)=4\cdot\frac{\frac{1}{25}\left(1-\left(\frac{1}{25}\right)^n\right)}{1-\frac{1}{25}}$$

$$=\frac{1}{6}\left(1-\frac{1}{5^{2n}}\right)$$

Thus the sum is:

$$\frac{3}{5}+\frac{4}{5^2}+\frac{3}{5^3}+\cdots\ 2n\text{ terms}=\frac{5}{8}\left(1-\frac{1}{5^{2n}}\right)+\frac{1}{6}\left(1-\frac{1}{5^{2n}}\right)$$

$$\frac{a}{1+i}+\frac{a}{(1+i)^2}+\frac{a}{(1+i)^3}+\ldots+\frac{a}{(1+i)^n}$$

$$a=\frac{a}{1+i},\quad r=\frac{\frac{a}{(1+i)^2}}{\frac{a}{1+i}}=\frac{1}{1+i}$$

$$S_n=a\frac{(1-r^n)}{1-r}$$

$$=\frac{a}{1+i}\frac{\left(1-\left(\frac{1}{1+i}\right)^n\right)}{1-\frac{1}{1+i}}$$

$$=\frac{a}{1+i}\times\frac{1+i}{(-i)}\left(1-(1+i)^n\right)$$

$$=-ai\left(1-(1+i)^{-n}\right)$$

Re writing the sequence and sum we get,

Sum= $1-a+a^2-a^3+a^4-a^5+\ldots\ldots\ldots$

Here, r = $-a$ and first term =1

$$\text{Sum} = \frac{[1-(-a)^n]}{1+a}$$

Here the first term of the G.P is $a = x^3$ and the common ratio is $r = \frac{x^5}{x^3} = x^2$

Thus the sum of the G.P is:

$$x^3 + x^5 + x^7 + \cdots \text{ to } n \text{ terms} = \frac{x^3\left(\left(x^2\right)^n - 1\right)}{x^2-1} = \frac{x^3\left(x^{2n}-1\right)}{x^2-1}$$

Here the first term of the G.P is $a = \sqrt{7}$ and the common ratio is $r = \frac{\sqrt{21}}{\sqrt{7}} = \sqrt{3}$

Thus the sum of the G.P is:

$$\sqrt{7} + \sqrt{21} + 3\sqrt{7} + \cdots \text{ to } n \text{ terms} = \frac{\sqrt{7}\left(\left(\sqrt{3}\right)^n - 1\right)}{\sqrt{3}-1} = \frac{\sqrt{7}\left(3^{\frac{n}{2}} - 1\right)}{\sqrt{3}-1}$$

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