Definite Integrals Ex 20.1 Q23

We have,

$$\int_0^{\frac{\pi}{2}} \left\{a^2 \cos^2 x + b^2\left(1 - \cos^2 x\right)\right\} dx$$

$$= \int_0^{\frac{\pi}{2}} \left\{\left(a^2 - b^2\right)\cos^2 x + b^2\right\} dx$$

$$= \frac{a^2 - b^2}{2} \int_0^{\frac{\pi}{2}} \left(1 + \cos 2x\right) dx + b^2 \int_0^{\frac{\pi}{2}} dx$$

$$= \frac{a^2 - b^2}{2}\left[x + \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}} + b^2[x]_0^{\frac{\pi}{2}}$$

$$= \frac{a^2 - b^2}{2}\left[\frac{\pi}{2} + 0 - 0 - 0\right] + b^2\left[\frac{\pi}{2} - 0\right]$$

$$= \frac{a^2 - b^2}{2}\left[\frac{\pi}{2}\right] + b^2\left[\frac{\pi}{2}\right]$$

$$= a^2 \frac{\pi}{4} + b^2\left[\frac{\pi}{2} - \frac{\pi}{4}\right]$$

$$= \frac{\pi a^2}{4} + \frac{\pi b^2}{4}$$

$$= \frac{\pi}{4}\left(a^2 + b^2\right)$$

$$\therefore \int_0^{\frac{\pi}{2}} (a^2 \cos^2 x + b^2 \sin^2 x) dx = \frac{\pi}{4}\left(a^2 + b^2\right)$$

Definite Integrals Ex 20.1 Q24

We have,

$$\int_0^{\frac{\pi}{2}} \sqrt{1+\sin x}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{1+\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}\, dx \qquad \text{We use } \sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

$$\int_0^{\frac{\pi}{2}} \sqrt{1+\sin x}\, dx = \int_0^{\frac{\pi}{2}} \sqrt{\frac{\left(1+\tan\frac{x}{2}\right)^2}{1+\tan^2\frac{x}{2}}}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{\frac{\left(1+\tan\frac{x}{2}\right)^2}{\sec^2\frac{x}{2}}}\, dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{1+\tan\frac{x}{2}}{\sec\frac{x}{2}}\right) dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\cos\frac{x}{2}+\sin\frac{x}{2}\right) dx$$

$$= \left[2\sin\frac{x}{2} - 2\cos\frac{x}{2}\right]_0^{\frac{\pi}{2}}$$

$$= 2\left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - 0 + 1\right]$$

$$\therefore \int_0^{\frac{\pi}{2}} \sqrt{1+\sin x}\, dx = 2$$

Definite Integrals Ex 20.1 Q25

We have,

$$\int_0^{\frac{\pi}{2}} \sqrt{1+\cos x}$$

We use $1+\cos x = 2\cos^2\frac{x}{2}$

$$= \int_0^{\frac{\pi}{2}} \sqrt{2\cos^2\frac{x}{2}}\,dx$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{2}\cos\frac{x}{2}dx$$

$$= \sqrt{2}\left[2\sin\frac{x}{2}\right]_0^{\frac{\pi}{2}}$$

$$= 2\sqrt{2}\left[\frac{1}{\sqrt{2}}\right]$$

$$= 2$$

$$\therefore \int_0^{\frac{\pi}{2}} \sqrt{1+\cos x} = 2$$

Definite Integrals Ex 20.1 Q26

We have,

$$\int x\sin x\,dx = x\int \sin x dx - \int\left(\int \sin x dx\right)\left(\frac{dx}{dx}\right)dx$$

$$= -x\cos x + \int \cos x dx$$

$$\therefore \int_0^{\frac{\pi}{2}} x\sin x\,dx = \left[-x\cos x + \sin x\right]_0^{\frac{\pi}{2}} = \left(-\frac{\pi}{2}\times 0\right) + 1 + 0 - 0 = 1$$

$$\therefore \int_0^{\frac{\pi}{2}} x\sin x dx = 1$$

*********** END **********