Exercise 1.2

Relations Ex 1.2 Q1

We have,

$R = \{(a,b): \ a-b \text{ is divisible by } 3; \ a,b, \in Z\}$

To prove: R is an equivalence relation

Proff:

Reflexivity: Let $a \in Z$

$\Rightarrow \quad a - a = 0$

$\Rightarrow \quad a - a$ is divisible by 3

$\Rightarrow \quad (a, a) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $a,b \in Z$ and $(a,b) \in R$

$\Rightarrow \quad a - b$ is divisible by 3

$\Rightarrow \quad a - b = 3p \qquad$ For some $p \in Z$

$\Rightarrow \quad b - a = 3 \times (-p)$

$\Rightarrow \quad b - a \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let $a,b,c \in Z$ and such that $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow \quad a - b = 3p$ and $b - c = 3q$ For some $p, q \in Z$

$\Rightarrow \quad a - c = 3(p + q)$

$\Rightarrow \quad a - c$ is divisible by 3

$\Rightarrow \quad (a, c) \in R$

$\Rightarrow \quad R$ is transitive

Since, R is reflexive, symmetric and transitive, so R is equivalence relation.

Relations Ex 1.2 Q2

We have,

$R = \{(a,b):\ a-b \text{ is divisible by } 2;\ a,b, \in Z\}$

To prove: R is an equivalence relation

Proff:

Reflexivity: Let $a \in Z$

$\Rightarrow \quad a - a = 0$

$\Rightarrow \quad a - a$ is divisible by 2

$\Rightarrow \quad (a,a) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $a,b \in Z$ and $(a,b) \in R$

$\Rightarrow \quad a - b$ is divisible by $\mathbf{2}$

$\Rightarrow \quad a - b = 2p \qquad$ For some $p \in Z$

$\Rightarrow \quad b - a = \mathbf{2} \times (-p)$

$\Rightarrow \quad b - a \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let $a,b,c \in Z$ and such that $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow \quad a - b = \mathbf{2}p$ and $b - c = q$ For some $p,q \in Z$

$\Rightarrow \quad a - c = 2(p+q)$

$\Rightarrow \quad a - c$ is divisible by $\mathbf{2}$

$\Rightarrow \quad (a,c) \in R$

$\Rightarrow \quad R$ is transitive

Relations Ex 1.2 Q3

We have,

$R = \{(a,b) : (\text{a-b}) \text{ is divisible by } 5\}$ on Z.

We want to prove that R is an equivalence relation on Z.

Now,

Reflexivity: Let $a \in Z$

$\Rightarrow \quad a - a = 0$

$\Rightarrow \quad a - a$ is divisible by 5.

$\therefore \quad (a,a) \in R$, so R is reflexive

Symmetric: Let $(a,b) \in R$

$\Rightarrow \quad a - b = 5P \quad$ For some $P \in Z$

$\Rightarrow \quad b - a = 5 \times (-P)$

$\Rightarrow \quad b - a$ is divisible by 5

$\Rightarrow \quad (b,a) \in R$, so R is symmetric

Transitive: Let $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow \quad a - b = 5p$ and $b - c = 5q$ For some $p,q \in Z$

$\Rightarrow \quad a - c = 5(p + q)$

$\Rightarrow \quad a - c$ is divisible by 5.

$\Rightarrow \quad R$ is transitive.

Thus, R being reflexive, symmetric and transitive on Z.

Hence, R is equivalence relation on Z

Relations Ex 1.2 Q4

$R = \{(a,b): \text{ a–b is divisible by n}\}$ on Z.

Now,

Reflexivity: Let $a \in Z$

$\Rightarrow \quad a - a = 0 \times n$

$\Rightarrow \quad a - a$ is divisible by n

$\Rightarrow \quad (a,a) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $(a,b) \in R$

$\Rightarrow \quad a - b = np$ For some p $\in$ Z

$\Rightarrow \quad b - a = n(-p)$

$\Rightarrow \quad b - a$ is divisible by n

$\Rightarrow \quad (b,a) \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow \quad a - b = xp$ and $b - c = xq$ For some p,q $\in$ Z

$\Rightarrow \quad a - c = n(p + q)$

$\Rightarrow \quad a - c$ is divisible by n

$\Rightarrow \quad (a,c) \in R$

$\Rightarrow \quad R$ is transitive

Thus, R being reflexive, symmetric and transitive on Z.

Hence, R is an equivalence relation on Z

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