



Differentials Errors and Approximation Ex14.1 Q9(viii)

Let $x = 4$, $x + \Delta x = 4.04$

$$\Delta x = 4.04 - 4$$

$$\Delta x = 0.04$$

Let $y = \log x$

$$\frac{dy}{dx} = \frac{1}{x}$$

$$\left(\frac{dy}{dx}\right)_{x=4} = \frac{1}{4}$$
$$= 0.25$$

Now,

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=4} \times \Delta x$$
$$= (0.25)(0.04)$$
$$\Delta y = 0.01$$

$$\log_e 4.04 = y + \Delta y$$

$$= \log x + (0.01)$$

$$= \log_e 4 + 0.01$$

$$= \frac{\log_e 4}{\log_{10} e} + 0.01$$

$$= \frac{0.6021}{0.4343} + 0.01$$

$$= 1.38637 + 0.01$$

$$\left[\text{Since, } \log_a b = \frac{\log_c b}{\log_c a} \right]$$

$$\log_e 4.04 = 1.39637$$

Differentials Errors and Approximation Ex14.1 Q9(ix)

$$\text{Let } x = 10, x + \Delta x = 10.02$$

$$\Delta x = 10.02 - 10$$

$$\Delta x = 0.02$$

$$\text{Let } y = \log_e x$$

$$\frac{dy}{dx} = \frac{1}{x}$$

$$\left(\frac{dy}{dx}\right)_{x=10} = \frac{1}{10}$$

$$\left(\frac{dy}{dx}\right)_{x=10} = 0.1$$

Now,

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=10} \times \Delta x$$

$$= (0.1)(0.02)$$

$$\Delta y = 0.002$$

$$\log_e (10.02) = y + \Delta y$$

$$= \log_e x + 0.002$$

$$= \log_e 10 + 0.002$$

$$= 2.3026 + 0.002$$

$$\log_e (10.02) = 2.3046$$

Differentials Errors and Approximation Ex14.1 Q9(x)

$$\text{Let } x = 10, x + \Delta x = 10.1$$

$$\Delta x = 10.1 - 10$$

$$\Delta x = 0.1$$

$$\text{Let } y = \log_{10} x$$

$$= \frac{\log_e x}{\log_e 10}$$

$$\left[\text{Since, } \log_a b = \frac{\log_c a}{\log_c b} \right]$$

$$\left(\frac{dy}{dx}\right) = \frac{1}{x \log_e 10}$$

$$\left(\frac{dy}{dx}\right)_{x=10} = \frac{1}{10 \log_e 10}$$

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=10} \times \Delta x$$

$$= \frac{1}{10(\log_e 10)} \times 0.1$$

$$\Delta y = \frac{0.01}{(\log_e 10)}$$

$$\log_{10} (10.1) = y + \Delta y$$

$$= \log_{10} x + \frac{0.01}{\log_e 10}$$

$$= \log_{10} 10 + 0.01 \log_{10} e$$

$$= 1 + (0.01)(0.4343)$$

$$\left[\text{Since, } \log_a b = \frac{1}{\log_b a} \right]$$

$$\log_{10} (10.1) = 1.004343$$

Differentials Errors and Approximation Ex14.1 Q9(xi)

$$\begin{aligned}\text{Let } x &= 60^\circ, x + \Delta x = 61^\circ \\ \Delta x &= 61^\circ - 60^\circ \\ \Delta x &= 1^\circ = \frac{\pi}{18^\circ} = 0.01745\end{aligned}$$

$$\begin{aligned}\text{Let } y &= \cos x \\ \frac{dy}{dx} &= -\sin x \\ \left(\frac{dy}{dx}\right)_{x=60^\circ} &= -\sin(60^\circ) \\ &= -\frac{\sqrt{3}}{2} \\ &= -0.866\end{aligned}$$

$$\begin{aligned}\Delta y &= \left(\frac{dy}{dx}\right)_{x=60^\circ} \times (\Delta x) \\ &= (-0.866)(0.01745) \\ &= -0.01511\end{aligned}$$

So,

$$\begin{aligned}\cos 61^\circ &= y + \Delta y \\ &= \cos 60^\circ - 0.01511 \\ &= \frac{1}{2} - 0.01511 \\ &= 0.5 - 0.01511\end{aligned}$$

$$\cos 61^\circ = 0.48489$$

***** END *****