Maxima and Minima 18.5 Q33

Let $P(x, y)$ be a point on the curve $y^2 = 2x$ which is minimum distance from the point $A(1, 4)$.

Let

$$S = \text{square of the length of } AP$$

$$S = (x-1)^2 + (y-4)^2$$

Using this equation, we have

$$S = x^2 + 1 - 2x + y^2 + 16 - 8y$$

$$S = x^2 - \cancel{2x} + \cancel{2x} + 17 - 8y$$

$$S = \frac{y^4}{4} - 8y + 17 \qquad \left[\text{Since } x = \frac{y^2}{2}\right]$$

$$\frac{dS}{dy} = y^3 - 8$$

For maxima and minima, we have

$$\frac{dS}{dy} = 0$$

$$y^3 - 8 = 0$$

$$y^3 = 2^3$$

$$y = 2$$

Now,

$$\frac{d^2S}{dy^2} = 3y^2$$

$$\frac{d^2S}{dy^2} = 12 > 0$$

$\therefore y = 2$ is minimum point

We have

$$x = \frac{y^2}{2}$$

$$= \frac{4}{2}$$

$$= 2$$

Hence, $\boxed{(2,2)}$ is at a minimum distance from the point $(1, 4)$.

Maxima and Minima 18.5 Q34

The given equation of curve is

$$y = x^3 + 3x^2 + 2x - 27 \qquad \text{---(i)}$$

Slope of (i)

$$m = \frac{dy}{dx} = -3x^2 + 6x + 2 \qquad \text{---(ii)}$$

Now,

$$\frac{dm}{dx} = -6x + 6$$

and $\dfrac{d^2m}{dx^2} = -6 < 0$

For maxima and minima,

$$\frac{dm}{dx} = 0$$

$\Rightarrow \quad -6x + 6 = 0$

$\Rightarrow \quad x = 1$

$\therefore \quad \dfrac{d^2m}{dx^2} = -6 < 0$

$\therefore \quad x = 1$ is point of local maxima

Hence, maximum slope $= -3 + 6 + 2 = 5$

Maxima and Minima 18.5 Q35

We have,

Cost of producing x radio sets is Rs. $\dfrac{x^2}{4} + 35x + 25$

Selling price of x radio is Rs. $x\left(50 - \dfrac{x}{2}\right)$

So,

Profit on x radio sets is

$$P = \text{Rs}\left(50x - \frac{x^2}{2} - \frac{x^2}{4} - 35x - 25\right)$$

$\therefore \quad \dfrac{dP}{dx} = 50 - x - \dfrac{x}{2} - 35$

$$= 15 - \frac{3}{2}x$$

For maxima and minima,

$$\frac{dP}{dx} = 0$$

$\Rightarrow \quad 15 - \dfrac{3}{2}x = 0$

$\Rightarrow \quad x = 10$

Also,

$$\frac{d^2P}{dx^2} = \frac{-3}{2} < 0$$

$\therefore \quad x = 10$ is the point of local maxima

Hence, the daily output should be 10 radio sets.

Maxima and Minima 18.5 Q35

We have,

Cost of producing x radio sets is Rs. $\frac{x^2}{4} + 35x + 25$

Selling price of x radio is Rs. $x\left(50 - \frac{x}{2}\right)$

So,

Profit on x radio sets is

$$P = \text{Rs}\left(50x - \frac{x^2}{2} - \frac{x^2}{4} - 35x - 25\right)$$

$$\therefore \quad \frac{dP}{dx} = 50 - x - \frac{x}{2} - 35$$

$$= 15 - \frac{3}{2}x$$

For maxima and minima,

$$\frac{dP}{dx} = 0$$

$$\Rightarrow \quad 15 - \frac{3}{2}x = 0$$

$$\Rightarrow \quad x = 10$$

Also,

$$\frac{d^2P}{dx^2} = \frac{-3}{2} < 0$$

$\therefore$ $x = 10$ is the point of local maxima

Hence, the daily output should be 10 radio sets.

*********** END **********