Question 13. 7. Estimate the average thermal energy of a helium atom at (i) room temperature (27 °C), (ii) the temperature on the surface of the Sun (6000 K), (iii) the temperature of 10 million kelvin (the typical core temperature in the case of a star).

Answer:

(*i*) Here, $T = 27\ ^\circ\text{C} = 27 + 273 = 300\ \text{K}$

$$\text{Average thermal energy} = \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300 = 6.2 \times 10^{-21}\ \text{J}.$$

(*ii*) At $T = 6000\ \text{K}$,

$$\text{Average thermal energy} = \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 6000 = 1.24 \times 10^{-19}\ \text{J}.$$

(*iii*) At $T = 10$ million $K = 10^7$ K

$$\text{Average thermal energy} = \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 10^7 = 2.1 \times 10^{-16}\ \text{J}$$

Question 13. 8. Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (mono atomic), the second contains chlorine (diatomic), and the third contains uranium hexafluoride (polyatomic). Do the vessels contain equal number of respective molecules? Is the root mean square speed of molecules the same in the three cases? If not, in which case is v_{rms} the largest?

Answer: Equal volumes of all the gases under similar conditions of pressure and temperature contains equal number of molecules (according to Avogadro's hypothesis). Therefore, the number of molecules in each case is same.

The rms velocity of molecules is given by

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

Clearly

$$v_{\text{rms}} \propto \frac{1}{\sqrt{m}}$$

Since neon has minimum atomic mass m, its rms velocity is maximum.

Question 13. 9. At what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the rms speed of a helium gas atom at -20 °C? (atomic mass of Ar = 39.9 u, of He = 4.0 u).

Answer:

Let C and C′ be the rms velocity of argon and a helium gas atoms at temperature T K and T K respectively.

Here, M = 39.9; M′ = 4.0; T =?; T = -20 + 273 = 253 K

Now, $$C = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3RT}{39.9}} \quad \text{and} \quad C' = \sqrt{\frac{3RT'}{M'}} = \sqrt{\frac{3R \times 253}{4}}$$

Since $$C = C'$$

Therefore, $$\sqrt{\frac{3RT}{39.9}} = \sqrt{\frac{3R \times 253}{4}}$$

or $$T = \frac{39.9 \times 253}{4} = 2523.7\ \text{K}.$$

Question 13. 10. Estimate the mean free path and collision frequency of a nitrogen molecule in a cylinder containing nitrogen at 2.0 atm and temperature 17 °C. Take the radius of a nitrogen molecule to be roughly 1.0 A. Compare the collision time with the time the molecule moves freely between two successive collisions (Molecular mass of N_2 = 28.0 u).

Answer:

Here, $P = 2.0 \text{ atm} = 2 \times 1.013 \times 10^5 \text{ Pa} = 2.026 \times 10^5 \text{ Pa}$

$T = 17\ °C = 17 + 273 = 290$

Radius, $R = 1.0\ Å = 1 \times 10^{-10}$ m, Molecular mass = 28 u

$\therefore$ $m = 28 \times 1.66 \times 10^{-27} = 4.65 \times 10^{-26}$ kg

Also, $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$, $k = 1.38 \times 10^{-23} \text{ JK}^{-1}$

Now for one mole of a gas,

$$PV = RT \Rightarrow V = \frac{RT}{P} = \frac{8.31 \times 290}{2.026 \times 10^5}$$

$$\Rightarrow \quad V = 1.189 \times 10^{-2} \text{ m}^3$$

$\therefore$ Number of molecules per unit volume, $n = \frac{N}{V}$

$$\therefore \quad n = \frac{6.023 \times 10^{23}}{1.189 \times 10^{-2}} = 5.06 \times 10^{25} \text{ m}^{-3}$$

Now, mean free path,

$$\lambda = \frac{1}{\sqrt{2}\pi n d^2} = \frac{1}{\sqrt{2}\pi n (2r)^2}$$

$$= \frac{1}{1.414 \times 3.14 \times 5.06 \times 10^{25} \times \left(2 \times 1 \times 10^{-10}\right)^2}$$

$$= 1.1 \times 10^{-7} \text{ m.}$$

Also, $$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times 290}{28 \times 10^{-3}}} = 5.08 \times 10^2 \text{ ms}^{-1}$$

$\therefore$ Collision frequency,

$$v = \frac{v_{rms}}{\lambda} = \frac{5.08 \times 10^2}{1.1 \times 10^{-7}} = 4.62 \times 10^9 \text{ s}^{-1}$$

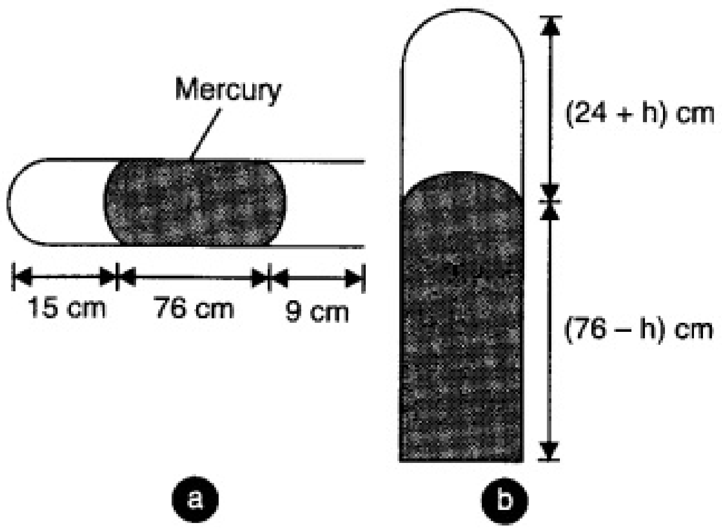
$$\text{Time between successive collisions} = \frac{1}{v} = \frac{1}{4.62 \times 10^9} = 2.17 \times 10^{-10} \text{ s}$$

$$\text{Also the collision time} = \frac{d}{v_{rms}} = \frac{2 \times 1 \times 10^{-10}}{5.08 \times 10^2} \text{ s} = 3.92 \times 10^{-13} \text{ s.}$$

Question 13. 11. A meter long narrow bore held horizontally (and closed at one end) contains a 76 cm long mercury thread which traps a 15 cm column of air. What happens if the tube is held vertically with the open end at the bottom?

Answer: When the tube is held horizontally, the mercury thread of length 76 cm traps a length of air = 15 cm. A length of 9 cm of the tube will be left at the open end. The pressure of air enclosed in tube will be atmospheric pressure. Let area of cross-section of the tube be 1 sq. cm.

$\therefore P_1 = 76$ cm and $V_1 = 15$ cm^3



When the tube is held vertically, 15 cm air gets another 9 cm of air (filled in the right handside in the horizontal position) and let h cm of mercury flows out to balance the atmospheric pressure. Then the heights of air column and mercury column are (24 + h) cm and (76 - h) cm respectively.

The pressure of air $= 76 - (76 - h) = h$ cm of mercury.

$\therefore \quad V_2 = (24 + h)\ \text{cm}^3$ and $P_2 = h$ cm

If we assume that temperature remains constant, then

$$P_1V_1 = P_2V_2 \quad \text{or} \quad 76 \times 15 = h \times (24 + h) \quad \text{or} \quad h^2 + 24h - 1140 = 0$$

or
$$h = \frac{-24 \pm \sqrt{(24)^2 + 4 \times 1140}}{2} = 23.8 \text{ cm} \quad \text{or} \quad -47.8 \text{ cm}$$

Since h cannot be negative (because more mercury cannot flow into the tube), therefore $h = 23.8$ cm.

Thus in the vertical position of the tube, 23.8 cm of mercury flows out.

Question 13. 12. From a certain apparatus, the diffusion rate of hydrogen has an average value of 28.7 $cm^3\ s^{-1}$. The diffusion of another gas under the same conditions is measured to have an average rate of 7.2 $cm^3\ s^{-1}$. Identify the gas.

Answer: According to Graham's law of diffusion of gases, the rate of diffusion of a gas is inversely proportional to the square root of its molecular mass.

If R_1 and R_2 be the rates of diffusion of two gases having molecular masses M_1 and M_2 respectively, then

$$\frac{R_1}{R_2} = \sqrt{\frac{M_2}{M_1}}$$

Now,
$$R_1 = 28.7\ \text{cm}^3\ \text{s}^{-1}, \quad R_2 = 7.2\ \text{cm}^3\ \text{s}^{-1}, \quad M_1 = 2,\ M_2 = ?$$

$\therefore$
$$\frac{28.7}{7.2} = \sqrt{\frac{M_2}{2}}$$

or
$$\frac{M_2}{2} = \frac{28.7 \times 28.7}{7.2 \times 7.2}$$

or
$$M_2 = \frac{2 \times 28.7 \times 28.7}{7.2 \times 7.2} = 31.78 \approx 32$$

This is molecular mass of oxygen. Therefore, the seoond gas is oxygen.

Question 13. 13. A gas in equilibrium has uniform density and pressure throughout its volume. This is strictly true only if there are no external influences. A gas column under gravity, for example, does not have uniform density (and pressure). As you might expect, its density decreases with height. The precise dependence is given by the so-called law of atmospheres $n_2 = n_1 \exp [- mg (h_2 - h_1)/k_BT]$ where n_2, n_1 refer to number density at heights h_2 and h_1 respectively. Use this relation to derive the equation for sedimentation equilibrium of a suspension in a liquid column :
$n_2 = n_1 \exp [- mg N_A(\rho - P) (h_2 - h_1)/(\rho RT)]$ where ρ is the density of the suspended particle, and ρ that of surrounding medium. [N_A is Avogadro's number, and R the universal gas constant.]

[Hint: Use Archimedes principle to find the apparent weight of the suspended particle.]

Answer: Considering the particles and molecules to be spherical, the weight of the particle is

$$W = mg = \frac{4}{3}\pi r^3 \rho g \qquad ...(i)$$

where r = radius of the particle and ρ = density of the particle. Its motion under gravity causes buoyant force to act upward which is equal to

$$B = \text{Volume of particle} \times \text{density of the surrounding medium} \times g$$
$$= \frac{4}{3}\pi r^3 \rho' g \qquad ...(ii)$$

If F be the downward force acting on the particle, then

$$F = W - B = \frac{4}{3}\pi r^3 (\rho - \rho')g \qquad ...(iii)$$

Also
$$n_2 = n_1 \exp\left[\frac{-mg}{k_B T}(h_2 - h_1)\right] \qquad ...(iv)$$

where k_B = Boltzman constant

n_1 and n_2 are number densities at heights h_1 and h_2 respectively. Here mg can be replaced by effective force F given by equation (*iii*).

∴ From (*iii*) and (*iv*), we get

$$n_2 = n_1 \exp\left[-\frac{4\pi}{3} r^3 \frac{(\rho - \rho')}{k_B T} g(h_2 - h_1)\right]$$

$$= n_1 \exp\left[-\frac{4\pi}{3} r^3 \frac{\rho g\left(1 - \frac{\rho'}{\rho}\right)(h_2 - h_1)}{\left(\frac{RT}{N_A}\right)}\right] \qquad \left[\because k_B = \frac{R}{N_A}\right]$$

$$n_2 = n_1 \exp\left[-\frac{mg\, N_A\left(1 - \frac{\rho'}{\rho}\right)(h_2 - h_1)}{RT}\right]$$

which is required relation

where, $\frac{4}{3}\pi r^3 \rho g$ = mass of the particle × g = mg.

Question 13. 14. Given below are densities of some solids and liquids. Give rough estimates of the size of their atoms

Substance	*Atomic Mass (u)*	*Density (10^3 Kg m^3)*
Carbon (diamond)	*12.01*	*2.22*
Gold	*197.00*	*19.32*
Nitrogen (liquid)	*14.01*	*1.00*
Lithium	*6.94*	*0.53*
Fluorine (liquid)	*19.00*	*1.14*

[Hint: Assume the atoms to be 'tightly packed' in a solid or liquid phase, and use the known value of Avogadro's number. You should, however, not take the actual numbers you obtain for various atomic sizes too literally. Because of the crudeness of the tight packing approximation, the results only indicate that atomic sizes are in the range of a few A].

Answer: In one mole of a substance, there are 6.023 x 10^{23} atoms

$$\therefore \left(\frac{4}{3}\pi R^3\right) \times 6.023 \times 10^{23} = \frac{M}{\rho}$$

or $$R = \left[\frac{3M}{4\pi\rho \times 6.023 \times 10^{23}}\right]^{1/3}$$

For carbon, $M = 12.01 \times 10^{-3}$ kg and $\rho = 2.22 \times 10^3$ kg m^{-3}

$$\therefore \quad R = \left[\frac{3 \times 12.01 \times 10^{-3}}{4 \times 3.14 \times 2.22 \times 10^3 \times 6.023 \times 10^{23}}\right]^{1/3}$$

$$= 1.29 \times 10^{-10} \text{ m} = 1.29 \text{ Å}$$

For gold, $M = 197 \times 10^{-3}$ kg and $\rho = 19.32 \times 10^3$ kg m^{-3}

$$\therefore \quad R = \left[\frac{3 \times 197 \times 10^{-3}}{4 \times 3.14 \times 19.32 \times 10^3 \times 6.023 \times 10^{23}}\right]^{1/3}$$

$$= 1.59 \times 10^{-10} \text{ m} = 1.59 \text{ Å}$$

For nitrogen (liquid)

$M = 14.01 \times 10^{-3}$ kg and $\rho = 1.00 \times 10^3$ kg m^{-3}

$$\therefore \quad R = \left[\frac{3 \times 14.01 \times 10^{-3}}{4 \times 3.14 \times 1.00 \times 10^3 \times 6.023 \times 10^{23}}\right]^{1/3}$$

$$= 1.77 \times 10^{-10} \text{ m} = 1.77 \text{ Å}$$

For lithium, $M = 6.94 \times 10^{-3}$ kg, $\rho = 0.53 \times 10^3$ kg m^{-3}

$$\therefore \quad R = \left[\frac{3 \times 6.94 \times 10^{-3}}{4 \times 3.14 \times 0.53 \times 10^3 \times 6.023 \times 10^{23}}\right]^{1/3}$$

$$= 1.73 \times 10^{-10} \text{ m} = 1.73 \text{ Å}$$

For fluorine (liquid)

$M = 19.00 \times 10^{-3}$ kg, $\rho = 1.14 \times 10^3$ kg m^{-3}

$$\therefore \quad R = \left[\frac{3 \times 19.00 \times 10^{-3}}{4 \times 3.14 \times 1.14 \times 10^3 \times 6.023 \times 10^{23}}\right]^{1/3}$$

$$= 1.88 \times 10^{-10} \text{ m} = 1.88 \text{ Å}$$

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