



Trigonometric Identities Ex 6.1 Q58

Answer :

In the given question, we need to prove $\left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

Taking $\sin\theta$ common from the numerator and the denominator of the L.H.S, we get

$$\begin{aligned}\left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2 &= \left(\frac{(\sin\theta)(\operatorname{cosec}\theta+1-\cot\theta)}{(\sin\theta)(\operatorname{cosec}\theta+1+\cot\theta)}\right)^2 \\ &= \left(\frac{1+\operatorname{cosec}\theta-\cot\theta}{1+\operatorname{cosec}\theta+\cot\theta}\right)^2\end{aligned}$$

Now, using the property $1+\cot^2\theta = \operatorname{cosec}^2\theta$, we get

$$\left(\frac{1+\operatorname{cosec}\theta-\cot\theta}{1+\operatorname{cosec}\theta+\cot\theta}\right)^2 = \left(\frac{(\operatorname{cosec}^2\theta-\cot^2\theta)+\operatorname{cosec}\theta-\cot\theta}{1+\operatorname{cosec}\theta+\cot\theta}\right)^2$$

Using $a^2-b^2 = (a+b)(a-b)$, we get

$$\left(\frac{(\operatorname{cosec}^2\theta-\cot^2\theta)+\operatorname{cosec}\theta-\cot\theta}{1+\operatorname{cosec}\theta+\cot\theta}\right)^2 = \left(\frac{(\operatorname{cosec}\theta+\cot\theta)(\operatorname{cosec}\theta-\cot\theta)+(\operatorname{cosec}\theta-\cot\theta)}{1+\operatorname{cosec}\theta+\cot\theta}\right)^2$$

Taking $\operatorname{cosec}\theta-\cot\theta$ common from the numerator, we get

$$\begin{aligned}\left(\frac{(\operatorname{cosec}\theta+\cot\theta)(\operatorname{cosec}\theta-\cot\theta)+\operatorname{cosec}\theta-\cot\theta}{1+\operatorname{cosec}\theta+\cot\theta}\right)^2 &= \left(\frac{(\operatorname{cosec}\theta-\cot\theta)(\operatorname{cosec}\theta+\cot\theta+1)}{1+\operatorname{cosec}\theta+\cot\theta}\right)^2 \\ &= (\operatorname{cosec}\theta-\cot\theta)^2\end{aligned}$$

Using $\cot\theta = \frac{\cos\theta}{\sin\theta}$ and $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$, we get

$$\begin{aligned}(\operatorname{cosec}\theta-\cot\theta)^2 &= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 \\ &= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2 \\ &= \frac{(1-\cos\theta)^2}{\sin^2\theta}\end{aligned}$$

Now, using the property $\sin^2\theta + \cos^2\theta = 1$, we get

$$\begin{aligned}\frac{(1-\cos\theta)^2}{\sin^2\theta} &= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} \\ &= \frac{(1-\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)} \\ &= \frac{1-\cos\theta}{1+\cos\theta}\end{aligned}$$

Hence proved.

Trigonometric Identities Ex 6.1 Q59

Answer :

We have to prove $(\sec A + \tan A - 1)(\sec A - \tan A + 1) = 2 \tan A$

We know that, $\sec^2 A - \tan^2 A = 1$

So, we have

$$\begin{aligned}(\sec A + \tan A - 1)(\sec A - \tan A + 1) &= \{\sec A + (\tan A - 1)\} \{\sec A - (\tan A - 1)\} \\&= \sec^2 A - (\tan A - 1)^2 \\&= \sec^2 A - (\tan^2 A - 2 \tan A + 1) \\&= (\sec^2 A - \tan^2 A) + 2 \tan A - 1\end{aligned}$$

So, we have

$$\begin{aligned}(\sec A + \tan A - 1)(\sec A - \tan A + 1) &= 1 + 2 \tan A - 1 \\&= 2 \tan A\end{aligned}$$

Hence proved.

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