Differentials Errors and Approximation Ex14.1 Q9(iv)

Let $\quad x = 400,\ x + \Delta x = 401$

$\qquad \Delta x = 401 - 400$

$\qquad \Delta x = 1$

Let $\quad y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\left(\frac{dy}{dx}\right)_{x=400} = \frac{1}{2\sqrt{400}}$$

$$= \frac{1}{40}$$

$$\left(\frac{dy}{dx}\right)_{x=400} = 0.025$$

So,

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=400} \times \Delta x$$

$$= (0.025)(1)$$

$$= 0.025$$

$$\sqrt{401} = y + \Delta y$$

$$= \sqrt{x} + 0.025$$

$$= \sqrt{400} + 0.025$$

$$= 20 + 0.025$$

$$\sqrt{401} = 20.025$$

Let $\quad x = 16, \; x + \Delta x = 15$

$\Delta x = 15 - 16$

$\Delta x = -1$

Let $\quad y = x^{\frac{1}{4}}$

$\dfrac{dy}{dx} = \dfrac{1}{4x^{\frac{3}{4}}}$

$\left(\dfrac{dy}{dx}\right)_{x=16} = \dfrac{1}{4\,(16)^{\frac{3}{4}}}$

$= \dfrac{1}{32}$

$= 0.03125$

Now,

$\Delta y = \left(\dfrac{dy}{dx}\right)_{x=16} \times \Delta x$

$= (0.03125)(-1)$

$\Delta y = -0.03125$

$(15)^{\frac{1}{4}} = y + \Delta y$

$= (x)^{\frac{1}{4}} - 0.03125$

$= (16)^{\frac{1}{4}} - 0.03125$

$= 2 - 0.03125$

$$(15)^{\frac{1}{4}} = 1.96875$$

Let $\quad x = 256, \quad x + \Delta x = 255$

$$\Delta x = 255 - 256$$

$$\Delta x = -1$$

Let $\quad y = x^{\frac{1}{4}}$

$$\frac{dy}{dx} = \frac{1}{4x^{\frac{3}{4}}}$$

$$\left(\frac{dy}{dx}\right)_{x=256} = \frac{1}{4(256)^{\frac{3}{4}}}$$

$$= \frac{1}{256}$$

$$= 0.00391$$

Now,

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=256} \times \Delta x$$

$$= (0.00391)(-1)$$

$$\Delta y = -0.00391$$

$$(255)^{\frac{1}{4}} = y + \Delta y$$

$$= (x)^{\frac{1}{4}} + (-0.00391)$$

$$= (256)^{\frac{1}{4}} - 0.00391$$

$$= 4 - 0.00391$$

$$(255)^{\frac{1}{4}} = 3.99609$$

Differentials Errors and Approximation Ex14.1 Q9(vii)

Let $\quad x = 2, \ x + \Delta x = 2.002$

$$\Delta x = 2.002 - 2$$

$$\Delta x = 0.002$$

Let $\quad y = \dfrac{1}{x^2}$

$$\dfrac{dy}{dx} = -\dfrac{2}{x^3}$$

$$\left(\dfrac{dy}{dx}\right)_{x=2} = -\dfrac{2}{8}$$

$$= -0.25$$

Now,

$$\Delta y = \left(\dfrac{dy}{dx}\right)_{x=2} \times \Delta x$$

$$= (-0.25)(0.002)$$

$$\Delta y = -0.0005$$

Now,

$$\dfrac{1}{(2.002)^3} = y + \Delta y$$

$$= \dfrac{1}{.2} + (-0.0005)$$

$$x^{-}$$

$$= \frac{1}{4} - 0.0005$$

$$= 0.25 - 0.0005$$

$$\frac{1}{(2.002)^3} = 0.2495$$

********** END *********