



EXERCISE 11.1

Question 1:

Find the equation of the circle with centre $(0, 2)$ and radius 2

Ans:

The equation of a circle with centre (h, k) and radius r is given as

$$(x - h)^2 + (y - k)^2 = r^2$$

It is given that centre $(h, k) = (0, 2)$ and radius $(r) = 2$.

Therefore, the equation of the circle is

$$(x - 0)^2 + (y - 2)^2 = 2^2$$

$$x^2 + y^2 + 4 - 4y = 4$$

$$x^2 + y^2 - 4y = 0$$

Question 2:

Find the equation of the circle with centre $(-2, 3)$ and radius 4

Ans:

The equation of a circle with centre (h, k) and radius r is given as

$$(x - h)^2 + (y - k)^2 = r^2$$

It is given that centre $(h, k) = (-2, 3)$ and radius $(r) = 4$.

Therefore, the equation of the circle is

$$(x + 2)^2 + (y - 3)^2 = (4)^2$$

$$x^2 + 4x + 4 + y^2 - 6y + 9 = 16$$

$$x^2 + y^2 + 4x - 6y - 3 = 0$$

Question 3:

Find the equation of the circle with centre $\left(\frac{1}{2}, \frac{1}{4}\right)$ and radius $\frac{1}{12}$

Ans:

The equation of a circle with centre (h, k) and radius r is given as

$$(x - h)^2 + (y - k)^2 = r^2$$

It is given that centre $(h, k) = \left(\frac{1}{2}, \frac{1}{4}\right)$ and radius $(r) = \frac{1}{12}$.

Therefore, the equation of the circle is

$$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{4}\right)^2 = \left(\frac{1}{12}\right)^2$$

$$x^2 - x + \frac{1}{4} + y^2 - \frac{y}{2} + \frac{1}{16} = \frac{1}{144}$$

$$x^2 - x + \frac{1}{4} + y^2 - \frac{y}{2} + \frac{1}{16} - \frac{1}{144} = 0$$

$$144x^2 - 144x + 36 + 144y^2 - 72y + 9 - 1 = 0$$

$$144x^2 - 144x + 144y^2 - 72y + 44 = 0$$

$$36x^2 - 36x + 36y^2 - 18y + 11 = 0$$

$$36x^2 + 36y^2 - 36x - 18y + 11 = 0$$

Question 4:

Find the equation of the circle with centre $(1, 1)$ and radius $\sqrt{2}$

Ans:

The equation of a circle with centre (h, k) and radius r is given as

$$(x - h)^2 + (y - k)^2 = r^2$$

It is given that centre $(h, k) = (1, 1)$ and radius $(r) = \sqrt{2}$.

Therefore, the equation of the circle is

$$(x - 1)^2 + (y - 1)^2 = (\sqrt{2})^2$$

$$x^2 - 2x + 1 + y^2 - 2y + 1 = 2$$

$$x^2 + y^2 - 2x - 2y = 0$$

Question 5:

Find the equation of the circle with centre $(-a, -b)$ and radius $\sqrt{a^2 - b^2}$

Ans:

The equation of a circle with centre (h, k) and radius r is given as

$$(x - h)^2 + (y - k)^2 = r^2$$

It is given that centre $(h, k) = (-a, -b)$ and radius $(r) = \sqrt{a^2 - b^2}$

Therefore, the equation of the circle is

$$(x + a)^2 + (y + b)^2 = (\sqrt{a^2 - b^2})^2$$

$$x^2 + 2ax + a^2 + y^2 + 2by + b^2 = a^2 - b^2$$

$$x^2 + y^2 + 2ax + 2by + 2b^2 = 0$$

Question 6:

Find the centre and radius of the circle $(x + 5)^2 + (y - 3)^2 = 36$

Ans:

The equation of the given circle is $(x + 5)^2 + (y - 3)^2 = 36$.

$$(x + 5)^2 + (y - 3)^2 = 36$$

$\{x - (-5)\}^2 + \{y - 3\}^2 = 6^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = -5$, $k = 3$, and $r = 6$.

Thus, the centre of the given circle is $(-5, 3)$, while its radius is 6.

Question 7:

Find the centre and radius of the circle $x^2 + y^2 - 4x - 8y - 45 = 0$

Ans:

The equation of the given circle is $x^2 + y^2 - 4x - 8y - 45 = 0$.

$$x^2 + y^2 - 4x - 8y - 45 = 0$$

$$(x^2 - 4x) + (y^2 - 8y) = 45$$

$$\{x^2 - 2(x)(2) + 2^2\} + \{y^2 - 2(y)(4) + 4^2\} - 4 - 16 = 45$$

$$(x - 2)^2 + (y - 4)^2 = 65$$

$(x - 2)^2 + (y - 4)^2 = (\sqrt{65})^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = 2$, $k = 4$, and $r = \sqrt{65}$.

Thus, the centre of the given circle is $(2, 4)$, while its radius is $\sqrt{65}$.

Question 8:

Find the centre and radius of the circle $x^2 + y^2 - 8x + 10y - 12 = 0$

Ans:

The equation of the given circle is $x^2 + y^2 - 8x + 10y - 12 = 0$.

$$x^2 + y^2 - 8x + 10y - 12 = 0$$

$$(x^2 - 8x) + (y^2 + 10y) = 12$$

$$\{x^2 - 2(x)(4) + 4^2\} + \{y^2 + 2(y)(5) + 5^2\} - 16 - 25 = 12$$

$$(x - 4)^2 + (y + 5)^2 = 53$$

$$\Rightarrow (x - 4)^2 + \{y - (-5)\}^2 = (\sqrt{53})^2,$$

which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = 4$, $k = -5$, and $r = \sqrt{53}$.

Thus, the centre of the given circle is $(4, -5)$, while its radius is $\sqrt{53}$.

Question 9:

Find the centre and radius of the circle $2x^2 + 2y^2 - x = 0$

Ans:

The equation of the given circle is $2x^2 + 2y^2 - x = 0$.

$$2x^2 + 2y^2 - x = 0$$

$$\Rightarrow (2x^2 - x) + 2y^2 = 0$$

$$\Rightarrow 2\left[\left(x^2 - \frac{x}{2}\right) + y^2\right] = 0$$

$$\Rightarrow \left\{x^2 - 2 \cdot x\left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2\right\} + y^2 - \left(\frac{1}{4}\right)^2 = 0$$

$$\Rightarrow \left(x - \frac{1}{4}\right)^2 + (y - 0)^2 = \left(\frac{1}{4}\right)^2, \text{ which is of the form } (x - h)^2 + (y - k)^2 = r^2, \text{ where } h = \frac{1}{4}, k$$

$$= 0, \text{ and } r = \frac{1}{4}.$$

Thus, the centre of the given circle is $\left(\frac{1}{4}, 0\right)$, while its radius is $\frac{1}{4}$.

Question 10:

Find the equation of the circle passing through the points $(4, 1)$ and $(6, 5)$ and whose centre is on the line $4x + y = 16$.

Ans:

Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the circle passes through points (4, 1) and (6, 5),

$$(4 - h)^2 + (1 - k)^2 = r^2 \dots (1)$$

$$(6 - h)^2 + (5 - k)^2 = r^2 \dots (2)$$

Since the centre (h, k) of the circle lies on line $4x + y = 16$,

$$4h + k = 16 \dots (3)$$

From equations (1) and (2), we obtain

$$(4 - h)^2 + (1 - k)^2 = (6 - h)^2 + (5 - k)^2$$

$$16 - 8h + h^2 + 1 - 2k + k^2 = 36 - 12h + h^2 + 25 - 10k + k^2$$

$$16 - 8h + 1 - 2k = 36 - 12h + 25 - 10k$$

$$4h + 8k = 44$$

$$h + 2k = 11 \dots (4)$$

On solving equations (3) and (4), we obtain $h = 3$ and $k = 4$.

On substituting the values of h and k in equation (1), we obtain

$$(4 - 3)^2 + (1 - 4)^2 = r^2$$

$$(1)^2 + (-3)^2 = r^2$$

$$1 + 9 = r^2$$

$$r^2 = 10$$

$$r = \sqrt{10}$$

Thus, the equation of the required circle is

$$(x - 3)^2 + (y - 4)^2 = (\sqrt{10})^2$$

$$x^2 - 6x + 9 + y^2 - 8y + 16 = 10$$

$$x^2 + y^2 - 6x - 8y + 15 = 0$$

Question 11:

Find the equation of the circle passing through the points (2, 3) and (-1, 1) and whose centre is on the line $x - 3y - 11 = 0$.

Ans:

Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the circle passes through points (2, 3) and (-1, 1),

$$(2 - h)^2 + (3 - k)^2 = r^2 \dots (1)$$

$$(-1 - h)^2 + (1 - k)^2 = r^2 \dots (2)$$

Since the centre (h, k) of the circle lies on line $x - 3y - 11 = 0$,

$$h - 3k = 11 \dots (3)$$

From equations (1) and (2), we obtain

$$(2 - h)^2 + (3 - k)^2 = (-1 - h)^2 + (1 - k)^2$$

$$4 - 4h + h^2 + 9 - 6k + k^2 = 1 + 2h + h^2 + 1 - 2k + k^2$$

$$4 - 4h + 9 - 6k = 1 + 2h + 1 - 2k$$

$$6h + 4k = 11 \dots (4)$$

On solving equations (3) and (4), we obtain $h = \frac{7}{2}$ and $k = \frac{-5}{2}$.

On substituting the values of h and k in equation (1), we obtain

$$\begin{aligned} \left(2 - \frac{7}{2}\right)^2 + \left(3 + \frac{5}{2}\right)^2 &= r^2 \\ \Rightarrow \left(\frac{4-7}{2}\right)^2 + \left(\frac{6+5}{2}\right)^2 &= r^2 \\ \Rightarrow \left(\frac{-3}{2}\right)^2 + \left(\frac{11}{2}\right)^2 &= r^2 \\ \Rightarrow \frac{9}{4} + \frac{121}{4} &= r^2 \\ \Rightarrow \frac{130}{4} &= r^2 \end{aligned}$$

Question 12:

Find the equation of the circle with radius 5 whose centre lies on x-axis and passes through the point (2, 3).

Ans:

Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the radius of the circle is 5 and its centre lies on the x -axis, $k = 0$ and $r = 5$.

Now, the equation of the circle becomes $(x - h)^2 + y^2 = 25$.

It is given that the circle passes through point (2, 3).

$$\therefore (2 - h)^2 + 3^2 = 25$$

$$\Rightarrow (2 - h)^2 = 25 - 9$$

$$\Rightarrow (2 - h)^2 = 16$$

$$\Rightarrow 2 - h = \pm\sqrt{16} = \pm 4$$

If $2 - h = 4$, then $h = -2$.

If $2 - h = -4$, then $h = 6$.

When $h = -2$, the equation of the circle becomes

$$(x + 2)^2 + y^2 = 25$$

$$x^2 + 4x + 4 + y^2 = 25$$

$$x^2 + y^2 + 4x - 21 = 0$$

When $h = 6$, the equation of the circle becomes

$$(x - 6)^2 + y^2 = 25$$

$$x^2 - 12x + 36 + y^2 = 25$$

$$x^2 + y^2 - 12x + 11 = 0$$

Question 13:

Find the equation of the circle passing through (0, 0) and making intercepts a and b on the coordinate axes.

Ans:

Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the centre of the circle passes through (0, 0),

$$(0 - h)^2 + (0 - k)^2 = r^2$$

$$h^2 + k^2 = r^2$$

The equation of the circle now becomes $(x - h)^2 + (y - k)^2 = h^2 + k^2$.

It is given that the circle makes intercepts a and b on the coordinate axes. This means that the circle passes through points $(a, 0)$ and $(0, b)$. Therefore,

$$(a - h)^2 + (0 - k)^2 = h^2 + k^2 \dots (1)$$

$$(0 - h)^2 + (b - k)^2 = h^2 + k^2 \dots (2)$$

From equation (1), we obtain

$$a^2 - 2ah + h^2 + k^2 = h^2 + k^2$$

$$a^2 - 2ah = 0$$

$$a(a - 2h) = 0$$

$$a = 0 \text{ or } (a - 2h) = 0$$

However, $a \neq 0$; hence, $(a - 2h) = 0 \Rightarrow h = \frac{a}{2}$.

From equation (2), we obtain

$$h^2 + b^2 - 2bk + k^2 = h^2 + k^2$$

$$b^2 - 2bk = 0$$

$$b(b - 2k) = 0$$

$$b = 0 \text{ or } (b - 2k) = 0$$

However, $b \neq 0$; hence, $(b - 2k) = 0 \Rightarrow k = \frac{b}{2}$.

Thus, the equation of the required circle is

$$\begin{aligned}\left(x - \frac{a}{2}\right)^2 + \left(y - \frac{b}{2}\right)^2 &= \left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2 \\ \Rightarrow \left(\frac{2x-a}{2}\right)^2 + \left(\frac{2y-b}{2}\right)^2 &= \frac{a^2+b^2}{4} \\ \Rightarrow 4x^2 - 4ax + a^2 + 4y^2 - 4by + b^2 &= a^2 + b^2 \\ \Rightarrow 4x^2 + 4y^2 - 4ax - 4by &= 0 \\ \Rightarrow x^2 + y^2 - ax - by &= 0\end{aligned}$$

Question 14:

Find the equation of a circle with centre (2, 2) and passes through the point (4, 5).

Ans:

The centre of the circle is given as $(h, k) = (2, 2)$.

Since the circle passes through point (4, 5), the radius (r) of the circle is the distance between the points (2, 2) and (4, 5).

$$\therefore r = \sqrt{(2-4)^2 + (2-5)^2} = \sqrt{(-2)^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13}$$

Thus, the equation of the circle is

$$\begin{aligned}(x-h)^2 + (y-k)^2 &= r^2 \\ (x-2)^2 + (y-2)^2 &= (\sqrt{13})^2 \\ x^2 - 4x + 4 + y^2 - 4y + 4 &= 13 \\ x^2 + y^2 - 4x - 4y - 5 &= 0\end{aligned}$$

Question 15:

Does the point (-2.5, 3.5) lie inside, outside or on the circle $x^2 + y^2 = 25$?

Ans:

The equation of the given circle is $x^2 + y^2 = 25$.

$$x^2 + y^2 = 25$$

$(x-0)^2 + (y-0)^2 = 5^2$, which is of the form $(x-h)^2 + (y-k)^2 = r^2$, where $h = 0$, $k = 0$, and $r = 5$.

Centre = (0, 0) and radius = 5

Distance between point (-2.5, 3.5) and centre (0, 0)

$$\begin{aligned}&= \sqrt{(-2.5-0)^2 + (3.5-0)^2} \\ &= \sqrt{6.25 + 12.25} \\ &= \sqrt{18.5} \\ &= 4.3 \text{ (approx.)} < 5\end{aligned}$$

Since the distance between point (-2.5, 3.5) and centre (0, 0) of the circle is less than the radius of the circle, point (-2.5, 3.5) lies inside the circle.

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