Exercise 4D

Question 22:

Given : In ΔABC, bisectors of ∠B and ∠C meet at O and ∠A = 70°

In ΔBOC, we have,

$$\Rightarrow \angle BOC + \frac{1}{2}\angle B + \frac{1}{2}\angle C = 180^\circ$$

$$\Rightarrow \angle BOC = 180^\circ - \frac{1}{2}\angle B - \frac{1}{2}\angle C$$

$$= 180^\circ - \frac{1}{2}(\angle B + \angle C)$$

$$= 180^\circ - \frac{1}{2}\left[180^\circ - \angle A\right]$$

$$\left[\because \angle A + \angle B + \angle C = 180^\circ\right]$$

$$= 180^\circ - \frac{1}{2}\left[180^\circ - 70^\circ\right]$$

$$= 180^\circ - \frac{1}{2} \times 110^\circ$$

$$\angle BOC + \angle OBC + \angle OCB = 180^\circ$$

$$= 180^\circ - 55^\circ = 125^\circ$$

$$\therefore \angle BOC = 125^\circ.$$

Question 23:

We have a ΔABC whose sides AB and AC have been procued to D and E. A = 40° and bisectors of ∠CBD and ∠BCE meet at O.

In ΔABC, we have,

Exterior $\angle CBD = C + 40^\circ$

$$\Rightarrow \quad \angle CBO = \frac{1}{2}\text{ Ext. } \angle CBD$$
$$= \frac{1}{2}\left(\angle C + 40^\circ\right)$$
$$= \frac{1}{2}\angle C + 20^\circ$$

And exterior $\angle BCE = B + 40^\circ$

$$\Rightarrow \quad \angle BCO = \frac{1}{2}\text{ Ext. } \angle BCE$$
$$= \frac{1}{2}\left(\angle B + 40^\circ\right)$$
$$= \frac{1}{2}\angle B + 20^\circ.$$

Now, in ΔBCO, we have,

$$\angle BOC = 180^\circ - \angle CBO - \angle BCO$$
$$= 180^\circ - \frac{1}{2}\angle C - 20^\circ - \frac{1}{2}\angle B - 20^\circ$$
$$= 180^\circ - \frac{1}{2}\angle C - \frac{1}{2}\angle B - 20^\circ - 20^\circ$$
$$= 180^\circ - \frac{1}{2}(\angle B + \angle C) - 40^\circ$$
$$= 140^\circ - \frac{1}{2}(\angle B + \angle C)$$
$$= 140^\circ - \frac{1}{2}\left[180^\circ - \angle A\right]$$

$$= 140^\circ - 90^\circ + \frac{1}{2}\angle A$$

$$= 50^\circ + \frac{1}{2}\angle A$$

$$= 50^\circ + \frac{1}{2} \times 40^\circ$$

$$= 50^\circ + 20^\circ$$

$$= 70^\circ$$

Thus, $\angle BOC = 70^\circ$

********** END **********