Trigonometric Ratios of multiple and Submultiple Angles Ex 9.3 Q 6

$$\cos\frac{5\pi}{15} = \cos\frac{\pi}{3} = \frac{1}{2}$$

$$\cos\frac{7\pi}{15} = \cos\left(\pi - \frac{8\pi}{15}\right)$$

$$\cos\frac{7\pi}{15} = -\cos\frac{8\pi}{15}$$

Now $\text{LHS} = \cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{3\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}$

$$= \left[\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\left(-\cos\frac{8\pi}{15}\right)\right]\left(\cos\frac{3\pi}{15}\cos\frac{6\pi}{15}\right)\frac{1}{2}$$

$$= -\frac{2^3}{2^4\sin\frac{\pi}{15}}\left[2\sin\frac{\pi}{15}\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\right]$$

$$\times\frac{2}{8\sin\frac{3\pi}{15}}\left(2\sin\frac{3\pi}{15}\cos\frac{3\pi}{15}\cos\frac{6\pi}{15}\right)$$

$$= -\frac{2^3}{16\sin\frac{\pi}{15}}\left[\sin\frac{2\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\right]\times\frac{2}{8\sin\frac{3\pi}{15}}\left(\sin\frac{6\pi}{15}\cos\frac{6\pi}{15}\right)$$

$$= -\frac{2^2}{16\sin\frac{\pi}{15}}\left[2\sin\frac{2\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\right]\times\frac{1}{8\sin\frac{3\pi}{15}}\left(2\sin\frac{6\pi}{15}\cos\frac{6\pi}{15}\right)$$

$$= -\frac{2}{16\sin\frac{\pi}{15}}\left[\sin\frac{8\pi}{15}\cos\frac{8\pi}{15}\right]\frac{\sin\frac{12\pi}{15}}{8\sin\frac{3\pi}{15}}$$

$$= -\frac{1}{16\sin\frac{\pi}{15}}\left(\sin\frac{16\pi}{15}\right)\frac{\sin\frac{12\pi}{15}}{8\sin\frac{3\pi}{15}}$$

$$= -\frac{\sin\left(\pi + \frac{\pi}{15}\right)}{128\sin\frac{\pi}{15}}\times\frac{\sin\left(\pi - \frac{3\pi}{15}\right)}{\sin\frac{3\pi}{15}}$$

$$= -\frac{-\sin\frac{\pi}{15}}{128\sin\frac{\pi}{15}}\times\frac{\sin\frac{3\pi}{15}}{\sin\frac{3\pi}{15}}$$

$$= \frac{1}{128}$$

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.3 Q 7

$$\text{L.H.S} = \cos 6^\circ . \cos 42^\circ . \cos 66^\circ . \cos 78^\circ$$

$$= \frac{1}{4}\left(2\cos 6^\circ . \cos 66^\circ\right)\left(2\cos 42^\circ . \cos 78^\circ\right)$$

$$= \frac{1}{4}\left(\cos 72^\circ + \cos 60^\circ\right)\left(\cos 120^\circ + \cos 36^\circ\right)$$

$$= \frac{1}{4}\left(\sin 18^\circ + \frac{1}{2}\right)\left(-\frac{2}{2} + \frac{\sqrt{5}+1}{4}\right)$$

$$= \frac{1}{4}\left(\frac{\sqrt{5}-1}{4} + \frac{1}{2}\right)\left(\frac{\sqrt{5}+1}{4} - \frac{1}{2}\right)$$

$$= \frac{1}{4}\left(\frac{\sqrt{5}-1+2}{4}\right)\left(\frac{\sqrt{5}+1-2}{4}\right)$$

$$= \frac{1}{64}\left(\sqrt{5}+1\right)\left(\sqrt{5}-1\right)$$

$$= \frac{1}{64}\left(\left(\sqrt{5}\right)^2 - 1^2\right)$$

$$= \frac{1}{64}(5-1)$$

$$= \frac{1}{16}$$

$$= \text{RHS}$$

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.3 Q 8

$$\text{L.H.S} = \sin 6^\circ . \ \sin 42^\circ . \sin 66^\circ . \sin 78^\circ$$

$$= \frac{1}{4}\left(2\sin 6^\circ . \sin 66^\circ\right)\left(2\sin 42^\circ . \sin 78^\circ\right)$$

$$= \frac{1}{4}\left(\cos 60 - \cos 72^\circ\right)\left(\cos 36^\circ - \cos 120^\circ\right)$$

$$= \frac{1}{4}\left(\frac{1}{2} - \sin 18^\circ\right)\left(\frac{\sqrt{5}+1}{4} + \frac{1}{2}\right)$$

$$= \frac{1}{4}\left(\frac{1}{2} - \frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4} + \frac{1}{2}\right)$$

$$= \frac{1}{4}\left(\frac{2-\sqrt{5}+1}{4}\right)\left(\frac{\sqrt{5}+1+2}{4}\right)$$

$$= \frac{1}{64}\left(3^2 - \sqrt{5}^2\right)$$

$$= \frac{1}{64}(9-5)$$

$$= \frac{1}{16}$$

$$= \text{RHS}$$

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.3 Q 9

$$\text{L.H.S} = \cos 36^\circ . \cos 42^\circ . \cos 60^\circ . \cos 78^\circ$$

$$= \frac{1}{2}\cos 36^\circ . \ \cos 60^\circ . \ \left(2\cos 42^\circ . \cos 78^\circ\right)$$
$$= \frac{1}{2}\left(\frac{\sqrt{5}+1}{4}\right) . \frac{1}{2}\left(\cos 120^\circ + \cos 36^\circ\right)$$
$$= \frac{\left(\sqrt{5}+1\right)}{16}\left(\frac{-1}{2} + \frac{\sqrt{5}+1}{4}\right)$$
$$= \frac{\left(\sqrt{5}+1\right)}{16}\left(\frac{-2+\sqrt{5}+1}{4}\right)$$
$$= \frac{\left(\sqrt{5}+1\right)\left(\sqrt{5}-1\right)}{64}$$
$$= \frac{5-1}{64}$$
$$= \frac{1}{16}$$
$$= \text{RHS}$$

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.3 Q 10

L.H.S,

$$\sin 36^\circ . \sin 72^\circ . \sin 108^\circ . \sin 144^\circ \qquad \left[\begin{array}{l} \because \ \sin 144^\circ = \sin\left(180^\circ - 36^\circ\right) = \sin 36^\circ \\ \quad \text{and } \sin 108^\circ = \sin\left(180^\circ - 78^\circ\right) = \sin 72^\circ \end{array}\right]$$

$$= \sin 36^\circ . \sin 72^\circ . \sin 72^\circ . \sin 36^\circ$$
$$= \frac{1}{4}\left(2\sin 36^\circ . \sin 72^\circ\right)^2$$
$$= \frac{1}{4}\left(2\sin 36^\circ \cos 18^\circ\right)^2 \qquad \left[\because \ \sin 72^\circ = \cos 18^\circ\right]$$
$$= \frac{4}{4}\left(\frac{\sqrt{10-2\sqrt{5}}}{4} . \frac{\sqrt{10+2\sqrt{5}}}{4}\right)^2$$
$$= \frac{1}{64}\left(10-2\sqrt{5}\right)\left(10+2\sqrt{5}\right)$$
$$= \frac{100-20}{64\times 4}$$
$$= \frac{80}{256}$$
$$= \frac{5}{16}$$
$$= \text{RHS}$$

*********** END **********