Indefinite Integrals Ex 19.8 Q21

Let $I = \int \frac{1-\sin x}{x+\cos x}dx$ ----- (i)

Let $x + \cos x = t$ then,

$d(x + \cos x) = dt$

$\Rightarrow \quad (1 - \sin x)dx = dt$

$\Rightarrow \quad dx = \frac{dt}{1-\sin x}$

Putting $x + \cos x = t$ and $dx = \frac{dt}{1-\sin x}$ in equation (i), we get,

$$I = \int \frac{1-\sin x}{t} \times \frac{dt}{1-\sin x}$$
$$= \int \frac{dt}{t}$$
$$= \log|t| + c$$
$$= \log|x + \cos x| + c$$

$\therefore \quad I = \log|x + \cos x| + c$

Indefinite Integrals Ex 19.8 Q22

Let $I = \int \frac{a}{b + ce^x} dx$ then,

$$I = \int \frac{a}{e^x \left[\frac{b}{e^x} + c\right]} dx$$

$$\Rightarrow \quad I = \int \frac{a}{e^x \left[be^{-x} + c\right]} dx \text{ - - - - - - - - (i)}$$

Let $be^{-x} + c = t$ then,

$d\left(be^{-x} + c\right) = dt$

$$\Rightarrow \quad -be^{-x} dx = dt$$

$$\Rightarrow \quad dx = \frac{-dt}{be^{-x}}$$

$$= -\frac{e^x dt}{b}$$

Putting $be^{-x} + c = t$ and $dx = \frac{-e^x\, dt}{b}$ in equation (i), we get,

$$I = \int \frac{a}{e^x \times t} \times \frac{-e^x\, dt}{b}$$

$$= -\frac{a}{b} \int \frac{dt}{t}$$

$$= -\frac{a}{b} \log |t| + c$$

$$= -\frac{a}{b} \log \left|be^{-x} + c\right| + c$$

Indefinite Integrals Ex 19.8 Q23

Let $I = \int \frac{1}{e^x + 1} dx$ then,

$$I = \int \frac{1}{e^x \left[1 + \frac{1}{e^x}\right]} dx$$

$$\Rightarrow \quad I = \int \frac{1}{e^x \left[1 + e^{-x}\right]} dx \text{ -------- (i)}$$

Let $1 + e^{-x} = t$ then,

$$d\left(1 + e^{-x}\right) = dt$$

$$\Rightarrow \quad -e^{-x} dx = dt$$

$$\Rightarrow \quad dx = \frac{-dt}{e^{-x}}$$

$$dx = -dt \times e^x$$

Putting $1 + e^{-x} = t$ and $dx = -e^x dt$ in equation (i), we get,

$$I = \int \frac{1}{e^x \times t} \times -e^x \, dt$$

$$= -\int \frac{dt}{t}$$

$$= -\log|t| + c$$

$$= -\log\left|1 + e^{-x}\right| + c$$

$$\therefore \quad = -\log\left|1 + e^{-x}\right| + c$$

Indefinite Integrals Ex 19.8 Q24

Let $I = \int \frac{\cot x}{\log \sin x} dx$ - - - - - - - (i)

Let $\log \sin x = t$ then,

$d(\log \sin x) = dt$

$\Rightarrow \quad \frac{\cos x}{\sin x} dx = dt$

$\Rightarrow \quad \cot x \, dx = dt$

$\Rightarrow \quad dx = \frac{dt}{\cot x}$

Putting $\log \sin x = t$ and $dx = \frac{dt}{\cot x}$ in equation (i), we get,

$$I = \int \frac{\cot x}{t} \times \frac{dt}{\cot x}$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + c$$

$$= \log|\log \sin x| + c$$

********** END **********