Factorisation of Algebraic Expressions Ex 5.4 Q9

Answer :

The given expression to be factorized is

$$\left(\frac{x}{2}+y+\frac{z}{3}\right)^3+\left(\frac{x}{3}-\frac{2y}{3}+z\right)^3+\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right)^3$$

Let $a=\left(\frac{x}{2}+y+\frac{z}{3}\right)$, $b=\left(\frac{x}{3}-\frac{2y}{3}+z\right)$ and $c=\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right)$. Then the given expression becomes

$$\left(\frac{x}{2}+y+\frac{z}{3}\right)^3+\left(\frac{x}{3}-\frac{2y}{3}+z\right)^3+\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right)^3=a^3+b^3+c^3$$

Note that

$$\begin{aligned} a+b+c &= \left(\frac{x}{2}+y+\frac{z}{3}\right)+\left(\frac{x}{3}-\frac{2y}{3}+z\right)+\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right) \\ &= \frac{x}{2}+y+\frac{z}{3}+\frac{x}{3}-\frac{2y}{3}+z-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3} \\ &= 0 \end{aligned}$$

Recall the formula

$$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

When $a+b+c=0$, this becomes

$$\begin{aligned} a^3+b^3+c^3-3abc &= 0.(a^2+b^2+c^2-ab-bc-ca) \\ &= 0 \end{aligned}$$

$$\Rightarrow a^3+b^3+c^3=3abc$$

So, we have the new formula

$a^3+b^3+c^3=3abc$, when $a+b+c=0$.

Using the above formula, the given expression can be written as $a^3+b^3+c^3=3abc$

Put $a=\left(\frac{x}{2}+y+\frac{z}{3}\right)$, $b=\left(\frac{x}{3}-\frac{2y}{3}+z\right)$ and $c=\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right)$.

Then we have

$$\left(\frac{x}{2}+y+\frac{z}{3}\right)^3+\left(\frac{x}{3}-\frac{2y}{3}+z\right)^3+\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right)^3=3\left(\frac{x}{2}+y+\frac{z}{3}\right)\left(\frac{x}{3}-\frac{2y}{3}+z\right)\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right)$$

We cannot further factorize the expression.

So, the required factorization is of $\left(\frac{x}{2}+y+\frac{z}{3}\right)^3+\left(\frac{x}{3}-\frac{2y}{3}+z\right)^3+\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right)^3$ is

$$\boxed{3\left(\frac{x}{2}+y+\frac{z}{3}\right)\left(\frac{x}{3}-\frac{2y}{3}+z\right)\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right)}$$

Factorisation of Algebraic Expressions Ex 5.4 Q10

Answer :

The given expression to be factorized is

$$(a-3b)^3+(3b-c)^3+(c-a)^3$$

Let $x=(a-3b)$, $y=(3b-c)$ and $z=(c-a)$. Then the given expression becomes

$$(a-3b)^3+(3b-c)^3+(c-a)^3=x^3+y^3+z^3$$

Note that

$$\begin{aligned} x+y+z &= (a-3b)+(3b-c)+(c-a) \\ &= a-3b+3b-c+c-a \\ &= 0 \end{aligned}$$

Recall the formula $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

When $a+b+c=0$, this becomes

$$\begin{aligned} a^3+b^3+c^3-3abc &= 0.(a^2+b^2+c^2-ab-bc-ca) \\ &= 0 \end{aligned}$$

$$\Rightarrow a^3+b^3+c^3=3abc$$

So, we have the new formula

$a^3+b^3+c^3=3abc$, when $a+b+c=0$.

Using the above formula, the given expression can be written as

$x^3+y^3+z^3=3xyz$

Put $x=(a-3b)$, $y=(3b-c)$ and $z=(c-a)$. Then we have

$(a-3b)^3+(3b-c)^3+(c-a)^3=3(a-3b)(3b-c)(c-a)$

We cannot further factorize the expression.

So, the required factorization is of $(a-3b)^3+(3b-c)^3+(c-a)^3$ is $\boxed{3(a-3b)(3b-c)(c-a)}$.

Factorisation of Algebraic Expressions Ex 5.4 Q11

Answer :

The given expression to be factorized is

$2\sqrt{2}a^3+3\sqrt{3}b^3+c^3-3\sqrt{6}abc$

This can be written in the form

$2\sqrt{2}a^3+3\sqrt{3}b^3+c^3-3\sqrt{6}abc=(\sqrt{2}a)^3+(\sqrt{3}b)^3+(c)^3-3.(\sqrt{2}a).(\sqrt{3}b).(c)$

Recall the formula

$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

Using the above formula, we have

$$2\sqrt{2}a^3+3\sqrt{3}b^3+c^3-3\sqrt{6}abc$$
$$=\{(\sqrt{2}a)+(\sqrt{3}b)+c)\}\{(\sqrt{2}a)^2+(\sqrt{3}b)^2+(c)^2-(\sqrt{2}a).(\sqrt{3}b)-(\sqrt{3}b).(c)-(c).(\sqrt{2}a)\}$$
$$=(\sqrt{2}a+\sqrt{3}b+c)(2a^2+3b^2+c^2-\sqrt{6}ab-\sqrt{3}bc-\sqrt{2}ca)$$

We cannot further factorize the expression.

So, the required factorization is of $2\sqrt{2}a^3+3\sqrt{3}b^3+c^3-3\sqrt{6}abc$ is

$\boxed{(\sqrt{2}a+\sqrt{3}b+c)(2a^2+3b^2+c^2-\sqrt{6}ab-\sqrt{3}bc-\sqrt{2}ca)}$.

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