Combinations Ex 17.1 Q10

${}^{n+2}C_8 : {}^{n-2}P_4 = 57 : 16$

$$\frac{\dfrac{(n+2)!}{8!\,(n-6)!}}{\dfrac{(n-2)!}{(n-6)!}} = \frac{57}{16}$$

$$\Rightarrow \quad \frac{(n+2)(n+1)(n)(n-1)(n-2)!}{8!\,(n-2)!} = \frac{57}{16}$$

Cancelling $(n-2)!$ from numerator and denominator

$$\Rightarrow \quad (n+2)(n+1)(n)(n-1) = \frac{57\times7\times6\times5\times4\times3\times1\times16}{16}$$

$$\Rightarrow \quad (n+2)(n+1)(n)(n-1) = 21\times20\times19\times18$$

comparing both sides $n = 19$

Combinations Ex 17.1 Q11

$$\frac{\dfrac{28!}{(2r)!(28-2r)!}}{\dfrac{24!}{(2r-4)!(24-(2r-4))!}} = \frac{225}{11}$$

$$\Rightarrow \quad \frac{28\times27\times26\times25\times24!\,(2r-4)!(28-2r)!}{(2r)!\,(28-2r)!\;\;24!} = \frac{225}{11}$$

$$\Rightarrow \quad \frac{28\times27\times26\times25}{2r\times(2r-1)\times(2r-2)(2r-3)} = \frac{225}{11}$$

$$\Rightarrow \quad \frac{28\times27\times26\times25\times11}{15\times15} = 2r(2r-1)(2r-2)(2r-3)$$

$$\Rightarrow \quad 11\times12\times13\times14 = 2r(2r-1)(2r-2)(2r-3)$$

Composing both sides $r = 7$

Combinations Ex 17.1 Q12

$$\frac{\frac{4n!}{(2n)!(2n)!}}{\frac{2n!}{n!\ n!}} \qquad \left(\because\ {}^nC_r = \frac{n!}{r!(n-r)!}\right)$$

$$= \frac{(4n)!}{(2n)!(2n)!}\frac{\times(n!)^2}{\times(2n)!^2}$$

$$= \frac{[1.2.3.4\ldots(4n-1)(4n)](n!^2)}{(2n)!\ [1.2.3.4\ldots\ (2n-2)(2n-1)(2n)]^2}$$

$$= \frac{[1.3.5\ldots(4n-1)]\times[2.4.6\ldots4n]\times(n!)^2}{(2n)!\,[1.3.5\ldots\ (2n-1)]^2\times[2.4.6\ldots\ (2n-2)(2n)]^2}$$

$$= \frac{[1.3.5\ldots(4n-1)]\times 2^{2n}\times[1.2.3\ldots2n]\times n!^2}{(2n)!\times\left[1.3.5\ldots(2n-1)^2\times 2^{2n}\times n!^2\right]}$$

$$= \frac{[1.3.5\ldots.(4n-1)]}{[1.3.5\ldots.(2n-1)]^2}$$

Hence Proved

Combinations Ex 17.1 Q13

$$\frac{\frac{2n!}{3!\ (2n-3)!}}{\frac{n!}{2!\ (n-2)!}} = \frac{44}{3}$$

$$\Rightarrow \qquad \frac{2n!\ 2!(n-2)!}{3!\ (2n-3)!\,n!} = \frac{44}{3}$$

$$\Rightarrow \qquad \frac{2n!}{3n!\ (n-1)(2n-3)!} = \frac{44}{3}$$

$$\Rightarrow \qquad 2n(2n-1)(2n-2) = 44n(n-1)$$

$$\Rightarrow \qquad (2n-1)(n-1) = 11(n-1)$$

$$Q \qquad n = 6$$

$$\therefore \qquad n = 6$$

*********** END **********