



Differentiation Ex 11.3 Q11

$$\begin{aligned}
 \text{Let } y &= \cos^{-1} \left\{ \frac{\cos x + \sin x}{\sqrt{2}} \right\} \\
 y &= \cos^{-1} \left\{ \cos x \left(\frac{1}{\sqrt{2}} \right) + \sin x \left(\frac{1}{\sqrt{2}} \right) \right\} \\
 &= \cos^{-1} \left\{ \cos x \cos \left(\frac{\pi}{4} \right) + \sin x \sin \left(\frac{\pi}{4} \right) \right\} \\
 y &= \cos^{-1} \left[\cos \left(x - \frac{\pi}{4} \right) \right] \quad \text{---(i)}
 \end{aligned}$$

$$\begin{aligned}
 \text{Here, } -\frac{\pi}{4} < x < \frac{\pi}{4} \\
 \Rightarrow \left(-\frac{\pi}{4} - \frac{\pi}{4} \right) < \left(x - \frac{\pi}{4} \right) < \left(\frac{\pi}{4} - \frac{\pi}{4} \right) \\
 \Rightarrow -\frac{\pi}{2} < \left(x - \frac{\pi}{4} \right) < 0
 \end{aligned}$$

So, from equation (i),

$$\begin{aligned}
 y &= - \left(x - \frac{\pi}{4} \right) & \left[\text{Since, } \cos^{-1}(\cos \theta) = -\theta, \text{ if } \theta \in [-\pi, 0] \right] \\
 y &= -x + \frac{\pi}{4}
 \end{aligned}$$

Differentiating it with respect to x ,

$$\frac{dy}{dx} = -1.$$

Differentiation Ex 11.3 Q12

$$\begin{aligned}
 \text{Let } y &= \tan^{-1} \left\{ \frac{x}{1 + \sqrt{1 - x^2}} \right\} \\
 \text{Put } x &= \sin \theta, \text{ so} \\
 y &= \tan^{-1} \left\{ \frac{\sin \theta}{1 + \sqrt{1 - \sin^2 \theta}} \right\} \\
 &= \tan^{-1} \left\{ \frac{\sin \theta}{1 + \cos \theta} \right\} \\
 &= \tan^{-1} \left\{ \frac{\frac{2 \sin \theta \cos \theta}{2}}{\frac{2 \cos^2 \theta}{2}} \right\} \\
 y &= \tan^{-1} \left\{ \frac{\tan \theta}{2} \right\} \quad \text{---(i)}
 \end{aligned}$$

$$\begin{aligned}
 \text{Here, } -1 < x < 1 \\
 \Rightarrow -1 < \sin \theta < 1 \\
 \Rightarrow -\frac{\pi}{2} < \theta < \frac{\pi}{2} \\
 \Rightarrow -\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{4}
 \end{aligned}$$

So, from equation (i),

$$\begin{aligned}
 y &= \frac{\theta}{2} & \left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right] \\
 y &= \frac{1}{2} \sin^{-1} x & \left[\text{Since, } x = \sin \theta \right]
 \end{aligned}$$

Differentiating it with respect to x ,

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q13

$$\text{Let } y = \tan^{-1} \left\{ \frac{x}{a + \sqrt{a^2 - x^2}} \right\}$$

$$\text{Put } x = a \sin \theta, \text{ so}$$

$$y = \tan^{-1} \left\{ \frac{a \sin \theta}{a + \sqrt{a^2 - a^2 \sin^2 \theta}} \right\}$$

$$= \tan^{-1} \left\{ \frac{a \sin \theta}{a + \sqrt{a^2 (1 - \sin^2 \theta)}} \right\}$$

$$= \tan^{-1} \left\{ \frac{a \sin \theta}{a + a \cos \theta} \right\}$$

$$= \tan^{-1} \left\{ \frac{a \sin \theta}{a (1 + \cos \theta)} \right\}$$

$$= \tan^{-1} \left(\frac{\sin \theta}{1 + \cos \theta} \right)$$

$$= \tan^{-1} \left(\frac{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2}} \right)$$

$$y = \tan^{-1} \left(\tan \frac{\theta}{2} \right) \quad \text{--- (i)}$$

$$\text{Here, } -a < x < a$$

$$\Rightarrow -1 < \frac{x}{a} < 1$$

$$\Rightarrow -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$\Rightarrow -\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{4}$$

So, from equation (i),

$$y = \frac{\theta}{2}$$

$$\left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right]$$

$$y = \frac{1}{2} + \sin^{-1} \left(\frac{x}{a} \right)$$

$$[\text{Since, } x = a \sin \theta]$$

Differentiating it with respect to x using chain rule,

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2} \times \frac{1}{\sqrt{1 - \left(\frac{x}{a} \right)^2}} \frac{d}{dx} \left(\frac{x}{a} \right) \\ &= \frac{a}{2\sqrt{a^2 - x^2}} \times \left(\frac{1}{a} \right) \end{aligned}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{a^2 - x^2}}.$$

Differentiation Ex 11.3 Q14

$$\text{Let } y = \sin^{-1} \left\{ \frac{x + \sqrt{1-x^2}}{\sqrt{2}} \right\}$$

$$\begin{aligned} \text{Put } x &= \sin \theta, \text{ so} \\ &= \sin^{-1} \left\{ \frac{\sin \theta + \sqrt{1-\sin^2 \theta}}{\sqrt{2}} \right\} \\ &= \sin^{-1} \left\{ \frac{\sin \theta + \cos \theta}{\sqrt{2}} \right\} \\ &= \sin^{-1} \left\{ \sin \theta \left(\frac{1}{\sqrt{2}} \right) + \cos \theta \left(\frac{1}{\sqrt{2}} \right) \right\} \\ &= \sin^{-1} \left\{ \sin \theta \cos \frac{\pi}{4} + \cos \theta \sin \frac{\pi}{4} \right\} \\ y &= \sin^{-1} \left\{ \sin \left(\theta + \frac{\pi}{4} \right) \right\} \end{aligned}$$

---(i)

$$\begin{aligned} \text{Here, } -1 < x < 1 \\ \Rightarrow -1 < \sin \theta < 1 \\ \Rightarrow -\frac{\pi}{2} < \theta < \frac{\pi}{2} \\ \Rightarrow \left(-\frac{\pi}{2} + \frac{\pi}{4} \right) < \left(\frac{\pi}{4} + \theta \right) < \frac{3\pi}{4} \end{aligned}$$

So, from equation (i),

$$y = \theta + \frac{\pi}{4}$$

$$\left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ as } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right]$$

$$y = \sin^{-1} x + \frac{\pi}{4}$$

$$[\text{Since, } \sin \theta = x]$$

Differentiating it with respect to x ,

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + 0$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q15

$$\text{Let } y = \cos^{-1} \left\{ \frac{x + \sqrt{1-x^2}}{\sqrt{2}} \right\}$$

$$\begin{aligned} \text{Put } x &= \sin \theta, \text{ so} \\ y &= \cos^{-1} \left\{ \frac{\sin \theta + \sqrt{1-\sin^2 \theta}}{\sqrt{2}} \right\} \\ &= \cos^{-1} \left\{ \frac{\sin \theta + \cos \theta}{\sqrt{2}} \right\} \\ &= \cos^{-1} \left\{ \sin \theta \left(\frac{1}{\sqrt{2}} \right) + \cos \theta \left(\frac{1}{\sqrt{2}} \right) \right\} \\ &= \cos^{-1} \left\{ \sin \theta \times \sin \frac{\pi}{4} + \cos \theta \times \cos \frac{\pi}{4} \right\} \\ y &= \cos^{-1} \left\{ \cos \left(\theta - \frac{\pi}{4} \right) \right\} \end{aligned}$$

---(i)

$$\begin{aligned} \text{Here, } -1 < x < 1 \\ \Rightarrow -1 < \sin \theta < 1 \\ \Rightarrow -\frac{\pi}{2} < \theta < \frac{\pi}{2} \\ \Rightarrow -\frac{\pi}{2} + \frac{\pi}{4} < \left(\theta - \frac{\pi}{4} \right) < \frac{\pi}{2} - \frac{\pi}{4} \\ \Rightarrow \left(-\frac{3\pi}{4} \right) < \left(\theta - \frac{\pi}{4} \right) < \left(\frac{\pi}{4} \right) \end{aligned}$$

So, from equation (i),

$$y = - \left(\theta - \frac{\pi}{4} \right)$$

$$[\text{Since, } \cos^{-1}(\cos \theta) = -\theta, \text{ if } \theta \in [-\pi, 0]]$$

$$y = -\theta + \frac{\pi}{4}$$

$$y = -\sin^{-1} x + \frac{\pi}{4}$$

$$[\text{Since, } x = \sin \theta]$$

Differentiating it with respect to x ,

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}} + 0$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}.$$

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