Trigonometric Functions Ex 5.2 Q 1

We have,

$$\operatorname{cosec}^2\theta - \cot^2\theta = 1$$

$$\Rightarrow \quad \operatorname{cosec}^2\theta = 1 + \cot^2\theta$$

$$\Rightarrow \quad \operatorname{cosec}\theta = \pm\sqrt{1 + \cot^2\theta}$$

In the third quadrant $\operatorname{cosec}\theta$ is negative

$$\therefore \quad \operatorname{cosec}\theta = -\sqrt{1 + \cot^2\theta}$$

$$= -\sqrt{1 + \left(\frac{12}{5}\right)^2} \qquad \left[\because \quad \cot\theta = \frac{12}{5}\right]$$

$$= -\sqrt{1 + \frac{144}{25}}$$

$$= -\sqrt{\frac{169}{25}}$$

$$= -\frac{13}{5}$$

$$\therefore \quad \operatorname{cosec}\theta = -\frac{13}{5}$$

Now, $\tan\theta = \dfrac{1}{\cot\theta}$

$$= \frac{1}{\frac{12}{5}}$$

$$= \frac{5}{12}$$

We have,

$$\sin^2\theta + \cos^2\theta = 1$$

$$\Rightarrow \quad \sin^2\theta = 1 - \cos^2\theta$$

$$\Rightarrow \quad \sin\theta = \pm\sqrt{1-\cos^2\theta}$$

In the 2nd quadrant $\sin\theta$ is positive and $\tan\theta$ is negative

$$\therefore \quad \sin\theta = \sqrt{1-\cos^2\theta}$$

$$= \sqrt{1-\left(-\frac{1}{2}\right)^2} \qquad \left[\because \quad \cos\theta = -\frac{1}{2}\right]$$

$$= \sqrt{1-\frac{1}{4}}$$

$$= \sqrt{\frac{3}{4}}$$

$$= \frac{\sqrt{3}}{2}$$

and, $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$

Now, $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\frac{\sqrt{3}}{2}} = \dfrac{2}{\sqrt{3}}$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{-\frac{1}{2}} = -2$

and $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{-\sqrt{3}} = \dfrac{-1}{\sqrt{3}}$

Hence, $\sin\theta = \dfrac{\sqrt{3}}{2}$, $\quad \tan\theta = -\sqrt{3}$,

$\operatorname{cosec}\theta = \dfrac{2}{\sqrt{3}}$, $\sec\theta = -2$ and $\cot\theta = \dfrac{-1}{\sqrt{3}}$

In the third quadrant $cosec\theta$ is negative

$$\therefore \quad cosec\theta = -\sqrt{1+\cot^2\theta}$$
$$= -\sqrt{1+\left(\frac{4}{3}\right)^2}$$
$$= -\sqrt{1+\frac{16}{9}}$$
$$= -\sqrt{\frac{25}{9}}$$
$$= -\frac{5}{3}$$

Now, $\sin\theta = \dfrac{1}{cosec\theta} = \dfrac{1}{\frac{-5}{3}} = \dfrac{-3}{5}$

and, $\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{1}{\frac{-5}{4}} = \dfrac{-4}{5}$

Hence, $\sin\theta = \dfrac{-3}{5}$, $\cos\theta = \dfrac{-4}{5}$,

$cosec\theta = -\dfrac{5}{3}$, $\sec\theta = \dfrac{-5}{4}$ and $\cot\theta = \dfrac{4}{3}$

We have,

$$\sin^2\theta + \cos^2\theta = 1$$

$$\Rightarrow \quad \cos^2\theta = 1 - \sin^2\theta$$

$$\Rightarrow \quad \cos\theta = \pm\sqrt{1 - \sin^2\theta}$$

In the 1^{st} quadrant $\cos\theta$ is positive and $\tan\theta$ is also positive

$$\therefore \quad \cos\theta = \sqrt{1 - \sin^2\theta}$$

$$= \sqrt{1 - \left(\frac{3}{5}\right)^2} \qquad \left[\because \quad \sin\theta = \frac{3}{5}\right]$$

$$= \sqrt{1 - \frac{9}{25}}$$

$$= \sqrt{\frac{16}{25}}$$

$$= \frac{4}{5}$$

and, $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{3}{5}}{\frac{4}{5}} = \dfrac{3}{4}$

Now, $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\frac{3}{5}} = \dfrac{5}{3}$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{\frac{4}{5}} = \dfrac{5}{4}$

and, $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{\frac{3}{4}} = \dfrac{4}{3}$

Hence, $\cos\theta = \dfrac{4}{5}$, $\operatorname{cosec}\theta = \dfrac{5}{3}$, $\tan\theta = \dfrac{3}{4}$,

$\sec\theta = \dfrac{5}{4}$, and $\cot\theta = \dfrac{4}{3}$

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