Q12 : **Find the equations of all lines having slope 0 which are tangent to the curve** $y=\frac{1}{x^2-2x+3}$.

Answer :

The equation of the given curve is $y=\frac{1}{x^2-2x+3}$.

The slope of the tangent to the given curve at any point (*x*, *y*) is given by,

$$\frac{dy}{dx}=\frac{-(2x-2)}{\left(x^2-2x+3\right)^2}=\frac{-2(x-1)}{\left(x^2-2x+3\right)^2}$$

If the slope of the tangent is 0, then we have:

$$\frac{-2(x-1)}{\left(x^2-2x+3\right)^2}=0$$

$$\Rightarrow -2(x-1)=0$$
$$\Rightarrow x=1$$

When *x* = 1, $y=\frac{1}{1-2+3}=\frac{1}{2}$.

$\therefore$The equation of the tangent through $\left(1,\ \frac{1}{2}\right)$ is given by,

$$y-\frac{1}{2}=0(x-1)$$
$$\Rightarrow y-\frac{1}{2}=0$$
$$\Rightarrow y=\frac{1}{2}$$

Hence, the equation of the required line is $y=\frac{1}{2}$.

Answer needs Correction? **Click Here**

Q13 : **Find points on the curve** $\frac{x^2}{9}+\frac{y^2}{16}=1$ **at which the tangents are**

(i) parallel to *x*-axis (ii) parallel to *y*-axis

Answer :

The equation of the given curve is $\frac{x^2}{9}+\frac{y^2}{16}=1$.

On differentiating both sides with respect to *x*, we have:

$$\frac{2x}{9}+\frac{2y}{16}\cdot\frac{dy}{dx}=0$$
$$\Rightarrow \frac{dy}{dx}=\frac{-16x}{9y}$$

(i) The tangent is parallel to the *x*-axis if the slope of the tangent is i.e., $0\frac{-16x}{9y}=0,$ which is possible if *x* = 0.

Then, $\frac{x^2}{9}+\frac{y^2}{16}=1$ for *x* = 0

$$\Rightarrow y^2=16 \Rightarrow y=\pm 4$$

Hence, the points at which the tangents are parallel to the *x*-axis are (0, 4) and (0, - 4).

(ii) The tangent is parallel to the *y*-axis if the slope of the normal is 0, which gives $\frac{-1}{\left(\frac{-16x}{9y}\right)}=\frac{9y}{16x}=0$ ⇒ *y* = 0.

Then, $\frac{x^2}{9}+\frac{y^2}{16}=1$ for *y* = 0.

$$\Rightarrow x=\pm 3$$

Hence, the points at which the tangents are parallel to the *y*-axis are (3, 0) and (- 3, 0).

Answer needs Correction? **Click Here**

Q14 : **Find the equations of the tangent and normal to the given curves at the indicated points:**

(i) $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ **at (0, 5)**

(ii) $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at (1, 3)

(iii) $y = x^3$ at (1, 1)

(iv) $y = x^2$ at (0, 0)

(v) $x = \cos t$, $y = \sin t$ at $t = \frac{\pi}{4}$

Answer :

(i) The equation of the curve is $y = x^4 - 6x^3 + 13x^2 - 10x + 5$.

On differentiating with respect to x, we get:

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$
$$\left.\frac{dy}{dx}\right]_{(0,\,5)} = -10$$

Thus, the slope of the tangent at (0, 5) is - 10. The equation of the tangent is given as:

$y - 5 = -10(x - 0)$

$\Rightarrow y - 5 = -10x$

$\Rightarrow 10x + y = 5$

The slope of the normal at (0, 5) is $\frac{-1}{\text{Slope of the tangent at }(0,\,5)} = \frac{1}{10}$.

Therefore, the equation of the normal at (0, 5) is given as:

$$y - 5 = \frac{1}{10}(x - 0)$$
$$\Rightarrow 10y - 50 = x$$
$$\Rightarrow x - 10y + 50 = 0$$

(ii) The equation of the curve is $y = x^4 - 6x^3 + 13x^2 - 10x + 5$.

On differentiating with respect to x, we get:

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$
$$\left.\frac{dy}{dx}\right]_{(1,\,3)} = 4 - 18 + 26 - 10 = 2$$

Thus, the slope of the tangent at (1, 3) is 2. The equation of the tangent is given as:

$$y - 3 = 2(x - 1)$$
$$\Rightarrow y - 3 = 2x - 2$$
$$\Rightarrow y = 2x + 1$$

The slope of the normal at (1, 3) is $\frac{-1}{\text{Slope of the tangent at }(1,\,3)} = \frac{-1}{2}$.

Therefore, the equation of the normal at (1, 3) is given as:

$$y - 3 = -\frac{1}{2}(x - 1)$$
$$\Rightarrow 2y - 6 = -x + 1$$
$$\Rightarrow x + 2y - 7 = 0$$

(iii) The equation of the curve is $y = x^3$.

On differentiating with respect to x, we get:

$$\frac{dy}{dx} = 3x^2$$
$$\left.\frac{dy}{dx}\right]_{(1,\,1)} = 3(1)^2 = 3$$

Thus, the slope of the tangent at (1, 1) is 3 and the equation of the tangent is given as:

$$y - 1 = 3(x - 1)$$
$$\Rightarrow y = 3x - 2$$

The slope of the normal at (1, 1) is $\frac{-1}{\text{Slope of the tangent at }(1,\,1)} = \frac{-1}{3}$.

Therefore, the equation of the normal at (1, 1) is given as:

$$y - 1 = \frac{-1}{3}(x - 1)$$
$$\Rightarrow 3y - 3 = -x + 1$$
$$\Rightarrow x + 3y - 4 = 0$$

(iv) The equation of the curve is $y = x^2$.

On differentiating with respect to x, we get:

$$\frac{dy}{dx} = 2x$$
$$\left.\frac{dy}{dx}\right]_{(0,\,0)} = 0$$

Thus, the slope of the tangent at (0, 0) is 0 and the equation of the tangent is given as:

$y - 0 = 0\,(x - 0)$

$\Rightarrow y = 0$

The slope of the normal at (0, 0) is $\frac{-1}{\text{Slope of the tangent at }(0,\,0)} = -\frac{1}{0}$, which is not defined.

Therefore, the equation of the normal at (x_0, y_0) = (0, 0) is given by

$x = x_0 = 0$.

(v) The equation of the curve is $x = \cos t$, $y = \sin t$.

$x = \cos t$ and $y = \sin t$

$\therefore \frac{dx}{dt} = -\sin t, \ \frac{dy}{dt} = \cos t$

$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{\cos t}{-\sin t} = -\cot t$

$\left.\frac{dy}{dx}\right]_{t=\frac{\pi}{4}} = -\cot t = -1$

$\therefore$The slope of the tangent at $t = \frac{\pi}{4}$

Answer needs Correction? **Click Here**

Q15 : Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is

(a) parallel to the line $2x - y + 9 = 0$

(b) perpendicular to the line $5y - 15x = 13$.

Answer :

The equation of the given curve is $y = x^2 - 2x + 7$.

On differentiating with respect to x, we get:

$\frac{dy}{dx} = 2x - 2$

(a) The equation of the line is $2x - y + 9 = 0$.

$2x - y + 9 = 0 \Rightarrow y = 2x + 9$

This is of the form $y = mx + c$.

$\therefore$Slope of the line = 2

If a tangent is parallel to the line $2x - y + 9 = 0$, then the slope of the tangent is equal to the slope of the line.

Therefore, we have:

$2 = 2x - 2$

$\Rightarrow 2x = 4$

$\Rightarrow x = 2$

Now, $x = 2$

$\Rightarrow y = 4 - 4 + 7 = 7$

Thus, the equation of the tangent passing through (2, 7) is given by,

$y - 7 = 2(x - 2)$

$\Rightarrow y - 2x - 3 = 0$

Hence, the equation of the tangent line to the given curve (which is parallel to line $2x - y + 9 = 0$) is $y - 2x - 3 = 0$.

(b) The equation of the line is $5y - 15x = 13$.

$5y - 15x = 13 \Rightarrow y = 3x + \frac{13}{5}$

This is of the form $y = mx + c$.

$\therefore$Slope of the line = 3

If a tangent is perpendicular to the line $5y - 15x = 13$, then the slope of the tangent is $\frac{-1}{\text{slope of the line}} = \frac{-1}{3}$.

$\Rightarrow 2x - 2 = \frac{-1}{3}$

$\Rightarrow 2x = \frac{-1}{3} + 2$

$\Rightarrow 2x = \frac{5}{3}$

$\Rightarrow x = \frac{5}{6}$

$\text{Now, } x = \frac{5}{6}$

$\Rightarrow y = \frac{25}{36} - \frac{10}{6} + 7 = \frac{25 - 60 + 252}{36} = \frac{217}{36}$

Thus, the equation of the tangent passing through $\left(\frac{5}{6}, \frac{217}{36}\right)$ is given by,

$y - \frac{217}{36} = -\frac{1}{3}\left(x - \frac{5}{6}\right)$

$\Rightarrow \frac{36y - 217}{36} = \frac{-1}{18}(6x - 5)$

$\Rightarrow 36y - 217 = -2(6x - 5)$

$\Rightarrow 36y - 217 = -12x + 10$

$\Rightarrow 36y + 12x - 227 = 0$

Hence, the equation of the tangent line to the given curve (which is perpendicular to line $5y - 15x = 13$) is $36y + 12x - 227 = 0$.

Answer needs Correction? **Click Here**

Q16 : Show that the tangents to the curve $y = 7x^3 + 11$ at the points where $x = 2$ and $x = -2$ are parallel.

Answer :

The equation of the given curve is $y = 7x^3 + 11$.

$$\therefore \frac{dy}{dx} = 21x^2$$

The slope of the tangent to a curve at (x_0, y_0) is $\left.\frac{dy}{dx}\right]_{(x_0, y_0)}$.

Therefore, the slope of the tangent at the point where $x = 2$ is given by,

$$\left.\frac{dy}{dx}\right]_{x=-2} = 21(2)^2 = 84$$

It is observed that the slopes of the tangents at the points where $x = 2$ and $x = -2$ are equal.

Hence, the two tangents are parallel.

Answer needs Correction? **Click Here**

Q17 : Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to the y-coordinate of the point.

Answer :

The equation of the given curve is $y = x^3$.

$$\therefore \frac{dy}{dx} = 3x^2$$

The slope of the tangent at the point (x, y) is given by,

$$\left.\frac{dy}{dx}\right]_{(x,y)} = 3x^2$$

When the slope of the tangent is equal to the y-coordinate of the point, then $y = 3x^2$.

Also, we have $y = x^3$.

$\therefore 3x^2 = x^3$

$\Rightarrow x^2 (x - 3) = 0$

$\Rightarrow x = 0, x = 3$

When $x = 0$, then $y = 0$ and when $x = 3$, then $y = 3(3)^2 = 27$.

Hence, the required points are (0, 0) and (3, 27).

Answer needs Correction? **Click Here**

Q18 : For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangents passes through the origin.

Answer :

The equation of the given curve is $y = 4x^3 - 2x^5$.

$$\therefore \frac{dy}{dx} = 12x^2 - 10x^4$$

Therefore, the slope of the tangent at a point (x, y) is $12x^2 - 10x^4$.

The equation of the tangent at (x, y) is given by,

$$Y - y = \left(12x^2 - 10x^4\right)(X - x) \qquad ...(1)$$

When the tangent passes through the origin (0, 0), then $X = Y = 0$.

Therefore, equation (1) reduces to:

$$-y = \left(12x^2 - 10x^4\right)(-x)$$
$$y = 12x^3 - 10x^5$$

Also, we have $y = 4x^3 - 2x^5$.

$$\therefore 12x^3 - 10x^5 = 4x^3 - 2x^5$$
$$\Rightarrow 8x^5 - 8x^3 = 0$$
$$\Rightarrow x^5 - x^3 = 0$$
$$\Rightarrow x^3\left(x^2 - 1\right) = 0$$
$$\Rightarrow x = 0, \ \pm 1$$

When $x = 0$, $y = 4(0)^3 - 2(0)^5 = 0$.

When $x = 1$, $y = 4(1)^3 - 2(1)^5 = 2$.

When $x = -1$, $y = 4(-1)^3 - 2(-1)^5 = -2$.

Hence, the required points are (0, 0), (1, 2), and (- 1, - 2).

Answer needs Correction? **Click Here**

Q19 : Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangents are parallel to the x-axis.

Answer :

The equation of the given curve is $x^2 + y^2 - 2x - 3 = 0$.

On differentiating with respect to x, we have:

$$2x + 2y\frac{dy}{dx} - 2 = 0$$
$$\Rightarrow y\frac{dy}{dx} = 1 - x$$
$$\frac{dy}{dx} = \frac{1-x}{}$$

$\Rightarrow \frac{\cdot}{dx} = \frac{}{y}$

Now, the tangents are parallel to the *x*-axis if the slope of the tangent is 0.

$\therefore \frac{1-x}{y} = 0 \Rightarrow 1-x = 0 \Rightarrow x = 1$

But, $x^2 + y^2 - 2x - 3 = 0$ for $x = 1$.

$\Rightarrow y^2 = 4 \Rightarrow y = \pm 2$

Hence, the points at which the tangents are parallel to the *x*-axis are (1, 2) and (1, - 2).

Answer needs Correction? Click Here

Q20 : Find the equation of the normal at the point (am^2, am^3) for the curve $ay^2 = x^3$.

Answer :

The equation of the given curve is $ay^2 = x^3$.

On differentiating with respect to *x*, we have:

$$2ay\frac{dy}{dx} = 3x^2$$
$$\Rightarrow \frac{dy}{dx} = \frac{3x^2}{2ay}$$

The slope of a tangent to the curve at (x_0, y_0) is $\left.\frac{dy}{dx}\right]_{(x_0, y_0)}$.

$\Rightarrow$ The slope of the tangent to the given curve at (am^2, am^3) is

$$\left.\frac{dy}{dx}\right]_{(am^2, am^3)} = \frac{3(am^2)^2}{2a(am^3)} = \frac{3a^2m^4}{2a^2m^3} = \frac{3m}{2}.$$

$\therefore$ Slope of normal at (am^2, am^3) = $\frac{-1}{\text{slope of the tangent at } (am^2, am^3)} = \frac{-2}{3m}$

Hence, the equation of the normal at (am^2, am^3) is given by,

$y - am^3 = \frac{-2}{3m}(x - am^2)$

$$\Rightarrow 3my - 3am^4 = -2x + 2am^2$$
$$\Rightarrow 2x + 3my - am^2(2 + 3m^2) = 0$$

Answer needs Correction? Click Here

Q21 : Find the equation of the normals to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$.

Answer :

The equation of the given curve is $y = x^3 + 2x + 6$.

The slope of the tangent to the given curve at any point (x, y) is given by,

$$\frac{dy}{dx} = 3x^2 + 2$$

$\therefore$ Slope of the normal to the given curve at any point (x, y) =

$$\frac{-1}{\text{Slope of the tangent at the point } (x, y)}$$
$$= \frac{-1}{3x^2 + 2}$$

The equation of the given line is $x + 14y + 4 = 0$.

$x + 14y + 4 = 0 \Rightarrow y = -\frac{1}{14}x - \frac{4}{14}$ (which is of the form $y = mx + c$)

$\therefore$Slope of the given line = $\frac{-1}{14}$

If the normal is parallel to the line, then we must have the slope of the normal being equal to the slope of the line.

$$\therefore \frac{-1}{3x^2+2} = \frac{-1}{14}$$
$$\Rightarrow 3x^2 + 2 = 14$$
$$\Rightarrow 3x^2 = 12$$
$$\Rightarrow x^2 = 4$$
$$\Rightarrow x = \pm 2$$

When $x = 2$, $y = 8 + 4 + 6 = 18$.

When $x = -2$, $y = -8 - 4 + 6 = -6$.

Therefore, there are two normals to the given curve with slope $\frac{-1}{14}$ and passing through the points (2, 18) and (- 2, - 6).

Thus, the equation of the normal through (2, 18) is given by,

$$y - 18 = \frac{-1}{14}(x - 2)$$
$$\Rightarrow 14y - 252 = -x + 2$$
$$\Rightarrow x + 14y - 254 = 0$$

And, the equation of the normal through (- 2, - 6) is given by,

$$y - (-6) = \frac{-1}{14}[x - (-2)]$$

$$\Rightarrow y+6=\frac{-1}{14}(x+2)$$
$$\Rightarrow 14y+84=-x-2$$
$$\Rightarrow x+14y+86=0$$

Hence, the equations of the normals to the given curve (which are parallel to the given line) are $x+14y-254=0$ and $x+14y+86=0$.

Answer needs Correction? **Click Here**

Q22 : Find the equations of the tangent and normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$.

Answer :

The equation of the given parabola is $y^2 = 4ax$.

On differentiating $y^2 = 4ax$ with respect to x, we have:

$$2y\frac{dy}{dx}=4a$$
$$\Rightarrow \frac{dy}{dx}=\frac{2a}{y}$$

$\therefore$The slope of the tangent at $(at^2,\ 2at)$ is $\left.\frac{dy}{dx}\right]_{(at^2,\ 2at)}=\frac{2a}{2at}=\frac{1}{t}$.

Then, the equation of the tangent at $(at^2,\ 2at)$ is given by,

$$y-2at=\frac{1}{t}\left(x-at^2\right)$$
$$\Rightarrow ty-2at^2=x-at^2$$
$$\Rightarrow ty=x+at^2$$

Now, the slope of the normal at $(at^2,\ 2at)$ is given by,

$$\frac{-1}{\text{Slope of the tangent at }\left(at^2,2at\right)}=-t$$

Thus, the equation of the normal at $(at^2, 2at)$ is given as:

$$y-2at=-t\left(x-at^2\right)$$
$$\Rightarrow y-2at=-tx+at^3$$
$$\Rightarrow y=-tx+2at+at^3$$

Answer needs Correction? **Click Here**

Q23 : Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$. [Hint: Two curves intersect at right angle if the tangents to the curves at the point of intersection are perpendicular to each other.]

Answer :

The equations of the given curves are given as $x=y^2$ and $xy=k$.

Putting $x = y^2$ in $xy = k$, we get:

$$y^3=k\Rightarrow y=k^{\frac{1}{3}}$$
$$\therefore x=k^{\frac{2}{3}}$$

Thus, the point of intersection of the given curves is $\left(k^{\frac{2}{3}},\ k^{\frac{1}{3}}\right)$.

Differentiating $x = y^2$ with respect to x, we have:

$$1=2y\frac{dy}{dx}\Rightarrow\frac{dy}{dx}=\frac{1}{2y}$$

Therefore, the slope of the tangent to the curve $x = y^2$ at $\left(k^{\frac{2}{3}},\ k^{\frac{1}{3}}\right)$ is $\left.\frac{dy}{dx}\right]_{\left(k^{\frac{2}{3}},k^{\frac{1}{3}}\right)}=\frac{1}{2k^{\frac{1}{3}}}$.

On differentiating $xy = k$ with respect to x, we have:

$$x\frac{dy}{dx}+y=0\Rightarrow\frac{dy}{dx}=\frac{-y}{x}$$

$\therefore$ Slope of the tangent to the curve $xy = k$ at $\left(k^{\frac{2}{3}},\ k^{\frac{1}{3}}\right)$ is given by,

$$\left.\frac{dy}{dx}\right]_{\left(k^{\frac{2}{3}},k^{\frac{1}{3}}\right)}=\left.\frac{-y}{x}\right]_{\left(k^{\frac{2}{3}},k^{\frac{1}{3}}\right)}=-\frac{k^{\frac{1}{3}}}{k^{\frac{2}{3}}}=\frac{-1}{k^{\frac{1}{3}}}$$

We know that two curves intersect at right angles if the tangents to the curves at the point of intersection i.e., at $\left(k^{\frac{2}{3}},\ k^{\frac{1}{3}}\right)$ are perpendicular to each other.

This implies that we should have the product of the tangents as - 1.

Thus, the given two curves cut at right angles if the product of the slopes of their respective tangents at $\left(k^{\frac{2}{3}},\ k^{\frac{1}{3}}\right)$ is - 1.

$$\text{i.e.,}\left(\frac{1}{2k^{\frac{1}{3}}}\right)\left(\frac{-1}{k^{\frac{1}{3}}}\right)=-1$$
$$\Rightarrow 2k^{\frac{2}{3}}=1$$

$$\Rightarrow \left(2k^{\frac{2}{3}}\right)^3 = (1)^3$$
$$\Rightarrow 8k^2 = 1$$

Hence, the given two curves cut at right angels if $8k^2 = 1$.

Answer needs Correction? **Click Here**

Q24 : Find the equations of the tangent and normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_0, y_0).

Answer :

Differentiating $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with respect to x, we have:

$$\frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0$$
$$\Rightarrow \frac{2y}{b^2}\frac{dy}{dx} = \frac{2x}{a^2}$$
$$\Rightarrow \frac{dy}{dx} = \frac{b^2 x}{a^2 y}$$

Therefore, the slope of the tangent at (x_0, y_0) is $\left.\frac{dy}{dx}\right]_{(x_0, y_0)} = \frac{b^2 x_0}{a^2 y_0}$.

Then, the equation of the tangent at (x_0, y_0) is given by,

$$y - y_0 = \frac{b^2 x_0}{a^2 y_0}(x - x_0)$$
$$\Rightarrow a^2 y y_0 - a^2 y_0^2 = b^2 x x_0 - b^2 x_0^2$$
$$\Rightarrow b^2 x x_0 - a^2 y y_0 - b^2 x_0^2 + a^2 y_0^2 = 0$$
$$\Rightarrow \frac{xx_0}{a^2} - \frac{yy_0}{b^2} - \left(\frac{x_0^2}{a^2} - \frac{y_0^2}{b^2}\right) = 0 \qquad \left[\text{On dividing both sides by } a^2 b^2\right]$$
$$\Rightarrow \frac{xx_0}{a^2} - \frac{yy_0}{b^2} - 1 = 0 \qquad \left[(x_0, y_0) \text{ lies on the hyperbola } \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1\right]$$
$$\Rightarrow \frac{xx_0}{a^2} - \frac{yy_0}{b^2} = 1$$

Now, the slope of the normal at (x_0, y_0) is given by,

$$\frac{-1}{\text{Slope of the tangent at } (x_0, y_0)} = \frac{-a^2 y_0}{b^2 x_0}$$

Hence, the equation of the normal at (x_0, y_0) is given by,

$$y - y_0 = \frac{-a^2 y_0}{b^2 x_0}(x - x_0)$$
$$\Rightarrow \frac{y - y_0}{a^2 y_0} = \frac{-(x - x_0)}{b^2 x_0}$$
$$\Rightarrow \frac{y - y_0}{a^2 y_0} + \frac{(x - x_0)}{b^2 x_0} = 0$$

Answer needs Correction? **Click Here**

Q25 : Find the equation of the tangent to the curve $y = \sqrt{3x - 2}$ which is parallel to the line $4x - 2y + 5 = 0$.

Answer :

The equation of the given curve is $y = \sqrt{3x - 2}$.

The slope of the tangent to the given curve at any point (x, y) is given by,

$$\frac{dy}{dx} = \frac{3}{2\sqrt{3x - 2}}$$

The equation of the given line is $4x - 2y + 5 = 0$.

$4x - 2y + 5 = 0 \Rightarrow y = 2x + \frac{5}{2}$ (which is of the form $y = mx + c$)

$\therefore$Slope of the line = 2

Now, the tangent to the given curve is parallel to the line $4x - 2y - 5 = 0$ if the slope of the tangent is equal to the slope of the line.

$$\frac{3}{2\sqrt{3x - 2}} = 2$$
$$\Rightarrow \sqrt{3x - 2} = \frac{3}{4}$$
$$\Rightarrow 3x - 2 = \frac{9}{16}$$
$$\Rightarrow 3x = \frac{9}{16} + 2 = \frac{41}{16}$$
$$\Rightarrow x = \frac{41}{48}$$

$$\text{When } x = \frac{41}{48},\ y = \sqrt{3\left(\frac{41}{48}\right) - 2} = \sqrt{\frac{41}{16} - 2} = \sqrt{\frac{41 - 32}{16}} = \sqrt{\frac{9}{16}} = \frac{3}{4}.$$

$\therefore$Equation of the tangent passing through the point $\left(\frac{41}{48}, \frac{3}{4}\right)$ is given by,

$$y - \frac{3}{4} = 2\left(x - \frac{41}{48}\right)$$
$$\Rightarrow \frac{4y - 3}{4} = 2\left(\frac{48x - 41}{\ldots}\right)$$

$$\Rightarrow 4y-3=\frac{48x-41}{6}$$
$$\Rightarrow 24y-18=48x-41$$
$$\Rightarrow 48x-24y=23$$

Hence, the equation of the required tangent is $48x-24y=23$.

Answer needs Correction? **Click Here**

Q26 : The slope of the normal to the curve $y = 2x^2 + 3 \sin x$ at $x = 0$ is

(A) 3 (B) $\frac{1}{3}$ (C) - 3 (D) $-\frac{1}{3}$

Answer :

The equation of the given curve is $y=2x^2+3\sin x$.

Slope of the tangent to the given curve at $x = 0$ is given by,

$$\left.\frac{dy}{dx}\right]_{x=0}=4x+3\cos x\Big]_{x=0}=0+3\cos 0=3$$

Hence, the slope of the normal to the given curve at $x = 0$ is

$$\frac{-1}{\text{Slope of the tangent at } x=0}=\frac{-1}{3}.$$

The correct answer is D.

Answer needs Correction? **Click Here**

Q27 : The line $y = x + 1$ is a tangent to the curve $y^2 = 4x$ at the point

(A) (1, 2) (B) (2, 1) (C) (1, -2) (D) (-1, 2)

Answer :

The equation of the given curve is $y^2=4x$.

Differentiating with respect to *x*, we have:

$$2y\frac{dy}{dx}=4\Rightarrow\frac{dy}{dx}=\frac{2}{y}$$

Therefore, the slope of the tangent to the given curve at any point (x, y) is given by,

$$\frac{dy}{dx}=\frac{2}{y}$$

The given line is $y = x + 1$ (which is of the form $y = mx + c$)

∴ Slope of the line = 1

The line $y = x + 1$ is a tangent to the given curve if the slope of the line is equal to the slope of the tangent. Also, the line must intersect the curve.

Thus, we must have:

$$\frac{2}{y}=1$$
$$\Rightarrow y=2$$
$$\text{Now, } y=x+1\Rightarrow x=y-1\Rightarrow x=2-1=1$$

Hence, the line $y = x + 1$ is a tangent to the given curve at the point (1, 2).

The correct answer is A.

Answer needs Correction? **Click Here**

*********** END **********