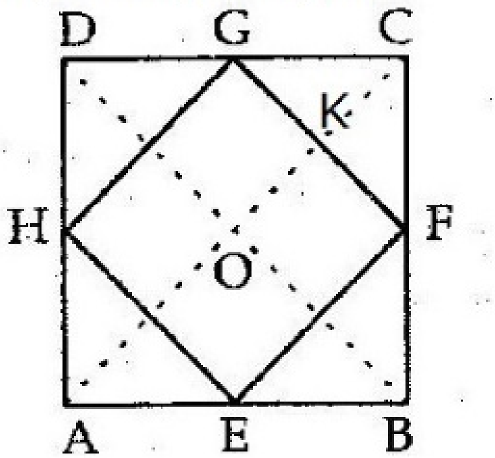
Exercise 9C

Question 11:

Given : ABCD is a square in which E, F, G and H are the mid points of AB, BC , CD and AD, respectively.

The mid points are joined together.



To prove :EFGH is a square.

Construction :Join AC and BD

Proof :

Midpoint Theorem: The line segment joining the midpoints of any two sides of a triangle is parallel to the third side and equal to half of it.

In $\triangle ABC$

E and F are the mid – points and by the

Mid points Theorem , we have

$EF \parallel AC$ and $EF = \frac{1}{2}AC$

Similarly, in $\triangle ADC$,

H and G are the midpoints and by the

Mid points Theorem , we have

$HG \parallel AC$ and $HG = \frac{1}{2}AC$

Thus, we have,

$EF \parallel HG$ and $EF = HG = \frac{1}{2}AC$.....(1)

In ΔBAD,
H and E are the midpoints and by the Mid points Theorem , we have,

$$HE \parallel BD \text{ and } HE = \frac{1}{2}BD$$

In ΔBCD,
G and F are the midpoints and by the Mid points Theorem , we have,

$$GF \parallel BD \text{ and } GF = \frac{1}{2} BD$$

Thus, we have,

$$HE \parallel GF \text{ and } HE = GF = \frac{1}{2}BD \ldots..(2)$$

The diagonals of a square are equal.
$\Rightarrow AC=BD$ ......(3)

From (1), (2) and (3), we have
$GF \parallel BD$ and $HE \parallel GF$.
Also, we have $EF = GF = GH = HE$
So, EFGH is a rhombus
Now, as diagonals of a square are equal and intersect at right angles.
So, $\angle DOC = 90^0$
In a parallelogram the sum of adjacent angles is 180^0.
So, $\angle DOC + \angle GKO = 180^0$
$\Rightarrow \angle GKO = 180^0 - 90^0 = 90^0$
But $\angle GKO = \angle EFG$ [Corresponding angles]
$\therefore \angle EFG = 90^0$
$\therefore$ EFGH is a square.

*********** END **********