Trigonometric Ratios Ex 5.1 Q28

Answer :

Given:

$\sin\theta = \frac{a}{b}$ ...... (1)

To find: $\sec\theta + \tan\theta$

Now we know, $\sin\theta$ is defined as follows

$\sin\theta = \frac{\text{Perpendicular side opposite to}\angle\theta}{\text{Hypotenuse}}$ ...... (2)

Now by comparing (1) and (2)

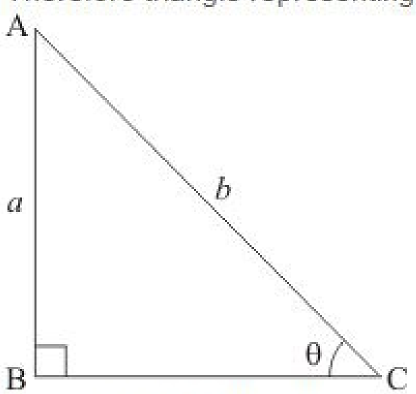
We get,

$\text{Perpendicular side opposite to}\angle\theta$ = a

and

Hypotenuse = b

Therefore triangle representing angle θ is as shown below



Here side *BC* is unknown

Now we find side *BC* by applying Pythagoras theorem to right angled ΔABC

Therefore,

$$AC^2 = AB^2 + BC^2$$

Now by substituting the value of sides *AB* and *AC* from figure (a)

We get,

$$b^2 = a^2 + BC^2$$

Therefore,

$$BC^2 = b^2 - a^2$$

Now by taking square root on both sides

We get,

$$BC = \sqrt{b^2 - a^2}$$

Therefore,

Base side $BC = \sqrt{b^2 - a^2}$ …… (3)

Now we know, $\cos\theta$ is defined as follows

$$\cos\theta = \frac{\text{Base side adjacent to} \angle\theta}{\text{Hypotenuse}}$$

Therefore from figure (*a*) and equation (3)

We get,

$$\cos\theta = \frac{BC}{AC}$$
$$= \frac{\sqrt{b^2 - a^2}}{b}$$

$$\cos\theta = \frac{\sqrt{b^2 - a^2}}{b} \quad \text{...... (4)}$$

Now we know, $\sec\theta = \frac{1}{\cos\theta}$

Therefore,

$$\sec\theta = \frac{1}{\frac{\sqrt{b^2 - a^2}}{b}}$$

Therefore,

$$\sec\theta = \frac{b}{\sqrt{b^2 - a^2}} \quad \text{...... (5)}$$

Now we know, $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Now by substituting the values from equation (1) and (3)

We get,

$$\tan\theta = \frac{\frac{a}{b}}{\frac{\sqrt{b^2 - a^2}}{b}}$$
$$= \frac{a}{b} \times \frac{b}{\sqrt{b^2 - a^2}}$$
$$= \frac{a}{\sqrt{b^2 - a^2}}$$

Therefore,

$$\tan\theta = \frac{a}{\sqrt{b^2 - a^2}} \quad \ldots\ldots (6)$$

Now we need to find $\sec\theta + \tan\theta$

Now by substituting the value of $\sec\theta$ and $\tan\theta$ from equation (5) and (6) respectively

We get,

$$\sec\theta + \tan\theta = \frac{b}{\sqrt{b^2 - a^2}} + \frac{a}{\sqrt{b^2 - a^2}}$$

$$\sec\theta + \tan\theta = \frac{b + a}{\sqrt{b^2 - a^2}} \quad \ldots\ldots (7)$$

Now we have the following formula which says

$$x^2 - y^2 = (x + y)\times(x - y)$$

Therefore by applying above formula in equation (7)

We get,

$$\sec\theta + \tan\theta = \frac{b + a}{\sqrt{(b+a)\times(b-a)}}$$

$$= \frac{b + a}{\sqrt{(b+a)}\times\sqrt{(b-a)}}$$

Now by substituting $(b + a) = \sqrt{(b+a)}\times\sqrt{(b+a)}$ in above expression

We get,

$$\sec\theta + \tan\theta = \frac{\sqrt{(b+a)}\times\sqrt{(b+a)}}{\sqrt{(b+a)}\times\sqrt{(b-a)}}$$

Now $\sqrt{(b+a)}$ present in the numerator as well as denominator of above expression gets cancels and we get,

$$\sec\theta + \tan\theta = \frac{\sqrt{(b+a)}}{\sqrt{(b-a)}}$$

Square root is present in the numerator as well as denominator of above expression

Therefore we can place both numerator as well as denominator under a common square root sign

Therefore, $\sec\theta + \tan\theta = \sqrt{\frac{b+a}{b-a}}$

*********** END **********