Algebraic Identities Ex 4.3 Q1

Answer :

In the given problem, we have to find cube of the binomial expressions

(i) Given $\left(\frac{1}{x}+\frac{y}{3}\right)^3$

We shall use the identity $(a+b)^3=a^3+b^3+3ab(a+b)$

Here $a=\frac{1}{x}, b=\frac{y}{3}$

By applying the identity we get

$$\left(\frac{1}{x}+\frac{y}{3}\right)^3=\left(\frac{1}{x}\right)^3+\left(\frac{y}{3}\right)^3+3\left(\frac{1}{x}\right)\left(\frac{y}{3}\right)\left(\frac{1}{x}+\frac{y}{3}\right)$$

$$=\frac{1}{x^3}+\frac{y^3}{27}+\cancel{3}\times\frac{1}{x}\times\frac{y}{\cancel{3}}\left(\frac{1}{x}+\frac{y}{3}\right)$$

$$=\frac{1}{x^3}+\frac{y^3}{27}+\frac{y}{x}\left(\frac{1}{x}+\frac{y}{3}\right)$$

$$=\frac{1}{x^3}+\frac{y^3}{27}+\frac{y}{x}\times\frac{1}{x}+\frac{y}{x}\times\frac{y}{3}$$

$$=\frac{1}{x^3}+\frac{y^3}{27}+\frac{y}{x^2}+\frac{y^2}{3x}$$

Hence cube of the binomial expression $\frac{1}{x}+\frac{y}{3}$ is $\boxed{\frac{1}{x^3}+\frac{y^3}{27}+\frac{y}{x^2}+\frac{y^2}{3x}}$

(ii) Given $\left(\frac{3}{x}-\frac{2}{x^2}\right)^3$

We shall use the identity $(a-b)^3=a^3-b^3-3ab(a-b)$

Here $a=\frac{3}{x}, b=\frac{2}{x^2}$

By applying the identity we get

$$\left(\frac{3}{x}-\frac{2}{x^2}\right)^3=\left(\frac{3}{x}\right)^3-\left(\frac{2}{x^2}\right)^3-3\left(\frac{3}{x}\right)\left(\frac{2}{x^2}\right)\left(\frac{3}{x}-\frac{2}{x^2}\right)$$

$$=\frac{27}{x^3}-\frac{8}{x^6}-3\times\frac{3}{x}\times\frac{2}{x^2}\left(\frac{3}{x}-\frac{2}{x^2}\right)$$

$$=\frac{27}{x^3}-\frac{8}{x^6}-\frac{18}{x^3}\left(\frac{3}{x}-\frac{2}{x^2}\right)$$

$$=\frac{27}{x^3}-\frac{8}{x^6}-\left(\frac{18}{x^3}\times\frac{3}{x}\right)-\left(\frac{18}{x^3}\times\frac{2}{x^2}\right)$$

$$=\frac{27}{x^3}-\frac{8}{x^6}-\left(\frac{54}{x^4}-\frac{36}{x^5}\right)$$

$$=\frac{27}{x^3}-\frac{8}{x^6}-\frac{54}{x^4}+\frac{36}{x^5}$$

Hence cube of the binomial expression of $\left(\frac{3}{x}-\frac{2}{x^2}\right)$ is $\boxed{\frac{27}{x^3}-\frac{8}{x^6}-\frac{54}{x^4}+\frac{36}{x^5}}$

(iii) Given $\left(2x+\dfrac{3}{x}\right)^3$

We shall use the identity $(a+b)^3 = a^3+b^3+3ab(a+b)$.

Here $a = 2x, b = \dfrac{3}{x}$.

By applying identity we get

$$\begin{aligned}\left(2x+\frac{3}{x}\right)^3 &= (2x)^3+\left(\frac{3}{x}\right)^3+3(2x)\left(\frac{3}{x}\right)\left(2x+\frac{3}{x}\right)\\ &= 2x\times 2x\times 2x+\frac{3}{x}\times\frac{3}{x}\times\frac{3}{x}+\frac{18x}{x}\left(2x+\frac{3}{x}\right)\\ &= 8x^3+\frac{27}{x^3}+\frac{18\cancel{x}}{\cancel{x}}\left(2x+\frac{3}{x}\right)\\ &= 8x^3+\frac{27}{x^3}+(18\times 2x)+\left(18\times\frac{3}{x}\right)\\ &= 8x^3+\frac{27}{x^3}+36x+\frac{54}{x}\end{aligned}$$

Hence cube of the binomial expression of $\left(2x+\dfrac{3}{x}\right)$ is $\boxed{8x^3+\dfrac{27}{x^3}+36x+\dfrac{54}{x}}$

(iv) Given $\left(4-\dfrac{1}{3x}\right)^3$

We shall use the identity $(a-b)^3 = a^3-b^3-3ab(a-b)$

Here $a = 4, b = \dfrac{1}{3x}$

By applying in identity we get

$$\begin{aligned}\left(4-\frac{1}{3x}\right)^3 &= (4)^3-\left(\frac{1}{3x}\right)^3-3(4)\left(\frac{1}{3x}\right)\left(4-\frac{1}{3x}\right)\\ &= 4\times 4\times 4-\frac{1\times 1\times 1}{3x\times 3x\times 3x}-\frac{12}{3x}\left(4-\frac{1}{3x}\right)\\ &= 64-\frac{1}{27x^3}-\frac{4}{x}\left(4-\frac{1}{3x}\right)\\ &= 64-\frac{1}{27x^3}-\left(\frac{4}{x}\times 4\right)-\left(\frac{4}{x}\times\frac{1}{3x}\right)\\ &= 64-\frac{1}{27x^3}-\left(\frac{16}{x}-\frac{4}{3x^2}\right)\\ &= 64-\frac{1}{27x^3}-\frac{16}{x}+\frac{4}{3x^2}\end{aligned}$$

Hence cube of the binomial expression of $\left(4-\dfrac{1}{3x}\right)^3$ is $\boxed{64-\dfrac{1}{27x^3}-\dfrac{16}{x}+\dfrac{4}{3x^2}}$.

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