



Mean Value Theorems Ex 15.1 Q2(viii)

Here,  $f(x) = x^2 + 5x + 6$  on  $[-3, -2]$

$f(x)$  is continuous in  $[-3, -2]$  and  $f(x)$  is differentiable in  $(-3, -2)$  since it is a polynomial function.

Now,

$$f(x) = x^2 + 5x + 6$$

$$\begin{aligned} f(-3) &= (-3)^2 + 5(-3) + 6 \\ &= 9 - 15 + 6 \end{aligned}$$

$$f(-3) = 0 \quad \text{--- (i)}$$

$$\begin{aligned} f(-2) &= (-2)^2 + 5(-2) + 6 \\ &= 4 - 10 + 6 \end{aligned}$$

$$f(-2) = 0 \quad \text{--- (ii)}$$

From equation (i) and (ii),

$$f(-3) = f(-2)$$

So, Rolle's theorem is applicable in  $[-3, -2]$ , we have to show that

$f'(c) = 0$  as  $c \in (-3, -2)$ .

Now,

$$f(x) = x^2 + 5x + 6$$

$$f'(x) = 2x + 5$$

$$\Rightarrow f'(c) = 0$$

$$2c + 5 = 0$$

$$c = \frac{-5}{2} \in (-3, -2)$$

So, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(i)

Here,

$$f(x) = \cos 2\left(x - \frac{\pi}{4}\right) \text{ on } \left[0, \frac{\pi}{2}\right]$$

We know that cosine function is continuous and differentiable

every where, so  $f(x)$  is continuous is  $\left[0, \frac{\pi}{2}\right]$  and differentiable is  $\left[0, \frac{\pi}{2}\right]$ .

Now,

$$f(0) = \cos 2\left(0 - \frac{\pi}{4}\right) = 0$$

$$f\left(\frac{\pi}{2}\right) = \cos 2\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = 0$$

$$\Rightarrow f(0) = f\left(\frac{\pi}{2}\right)$$

So, Rolle's theorem is applicable.

Hence, there must exists a  $c \in \left(0, \frac{\pi}{2}\right)$  such that  $f'(c) = 0$ .

Now,

$$f'(x) = -\sin 2\left(x - \frac{\pi}{4}\right) \times 2$$

$$f'(x) = -2 \sin\left(2x - \frac{\pi}{2}\right)$$

$$\Rightarrow -2 \sin\left(2c - \frac{\pi}{2}\right) = 0$$

$$\Rightarrow \sin\left(2c - \frac{\pi}{2}\right) = \sin 0$$

$$\Rightarrow 2c - \frac{\pi}{2} = 0$$

$$c = \frac{\pi}{4} \in \left(0, \frac{\pi}{2}\right)$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(ii)

Here,

$$f(x) = \sin 2x \text{ on } \left[0, \frac{\pi}{2}\right]$$

We know that  $\sin x$  is a continuous and differentiable every where. So,

$f(x)$  is continuous in  $\left[0, \frac{\pi}{2}\right]$  and differentiable is  $\left(0, \frac{\pi}{2}\right)$ .

Now,

$$f(0) = \sin 0 = 0$$

$$f\left(\frac{\pi}{2}\right) = \sin \pi = 0$$

$$\Rightarrow f(0) = f\left(\frac{\pi}{2}\right)$$

So, Rolle's theorem is applicable, so, there must exist a  $c \in \left(0, \frac{\pi}{2}\right)$

such that  $f'(c) = 0$

Now,

$$f'(x) = 2 \cos 2x$$

$$f'(c) = 2 \cos 2c = 0$$

$$\Rightarrow \cos 2c = 0$$

$$\Rightarrow 2c = \frac{\pi}{2}$$

$$\Rightarrow c = \frac{\pi}{4} \in \left(0, \frac{\pi}{2}\right)$$

Thus, Rolle's theorem verified.

Mean Value Theorems Ex 15.1 Q3(iii)

Here,

$$f(x) = \cos 2x \text{ on } \left[ -\frac{\pi}{4}, \frac{\pi}{4} \right]$$

We know that  $\cos x$  is a continuous and differentiable every where. So,

$f(x)$  is continuous in  $\left[ -\frac{\pi}{4}, \frac{\pi}{4} \right]$  and differentiable is  $\left( -\frac{\pi}{4}, \frac{\pi}{4} \right)$ .

$$\text{Now, } f\left(-\frac{\pi}{4}\right) = \cos 2\left(-\frac{\pi}{4}\right) = \cos\left(-\frac{\pi}{2}\right) = 0$$

$$f\left(\frac{\pi}{4}\right) = \cos 2\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{2}\right) = 0$$

$$\Rightarrow f\left(-\frac{\pi}{4}\right) = f\left(\frac{\pi}{4}\right)$$

So, Rolle's theorem is applicable, so, there must exist a  $c \in \left(0, \frac{\pi}{2}\right)$

such that  $f'(c) = 0$

Now,

$$f'(x) = 2 \sin 2x$$

$$f'(c) = 2 \sin 2c = 0$$

$$\Rightarrow \sin 2c = 0$$

$$\Rightarrow 2c = 0$$

$$\Rightarrow c = 0 \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$$

Thus, Rolle's theorem verified.

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