Arithmetic Progressions Ex 9.5 Q1

Answer :

In the given problem, we need to find the sum of terms for different arithmetic progressions. So, here we use the following formula for the sum of n terms of an A.P.,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Where; a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

(i) $50, 46, 42, \ldots$ To 10 terms

Common difference of the A.P. (d)

$= a_2 - a_1$

$= 46 - 50$

$= -4$

Number of terms (n) = 10

First term for the given A.P. (a) = 50

So, using the formula we get,

$$\begin{aligned} S_{10} &= \frac{10}{2}\left[2(50) + (10-1)(-4)\right] \\ &= (5)\left[100 + (9)(-4)\right] \\ &= (5)[100 - 36] \\ &= (5)[64] \\ &= 320 \end{aligned}$$

Therefore, the sum of first 10 terms for the given A.P. is $\boxed{320}$.

(ii) $1, 3, 5, 7, \ldots - 26$ To 12 terms.

Common difference of the A.P. (d)

$= a_2 - a_1$

$= 3 - 1$

$= 2$

Number of terms (n) = 12

First term for the given A.P. (a) = 1

So, using the formula we get,

$$\begin{aligned} S_n &= \frac{12}{2}\left[2(1) + (12-1)(2)\right] \\ &= (6)\left[2 + (11)(2)\right] \\ &= (6)[2 + 22] \\ &= (6)[24] \\ &= 144 \end{aligned}$$

Therefore, the sum of first 12 terms for the given A.P. is $\boxed{144}$.

(iii) $3, 9/2, 6, 15/2, \ldots$ To 25 terms.

Common difference of the A.P. (d) = $a_2 - a_1$

$$= \frac{9}{2} - 3$$

$$= \frac{9-6}{2}$$

$= \frac{3}{2}$

Number of terms (n) = 25

First term for the given A.P. (a) = 3

So, using the formula we get,

$$S_{25} = \frac{25}{2}\left[2(3) + (25-1)\left(\frac{3}{2}\right)\right]$$
$$= \left(\frac{25}{2}\right)\left[6 + (24)\left(\frac{3}{2}\right)\right]$$
$$= \left(\frac{25}{2}\right)\left[6 + \left(\frac{72}{2}\right)\right]$$
$$= \left(\frac{25}{2}\right)[6 + 36]$$
$$= \left(\frac{25}{2}\right)[42]$$
$$= (25)(21)$$
$$= 525$$

On further simplifying, we get,

$S_{25} = 525$

Therefore, the sum of first 25 terms for the given A.P. is $\boxed{525}$.

(iv) $41, 36, 31, \ldots$ To 12 terms.

Common difference of the A.P. (d) = $a_2 - a_1$

$= 36 - 41$

$= -5$

Number of terms (n) = 12

First term for the given A.P. (a) = 41

So, using the formula we get,

$$S_{12} = \frac{12}{2}\left[2(41) + (12-1)(-5)\right]$$
$$= (6)\left[82 + (11)(-5)\right]$$
$$= (6)[82 - 55]$$
$$= (6)[27]$$
$$= 162$$

Therefore, the sum of first 12 terms for the given A.P. is $\boxed{162}$.

(v) $a+b, a-b, a-3b, \ldots$ To 22 terms.

Common difference of the A.P. (d) = $a_2 - a_1$

$= (a-b) - (a+b)$

$= a - b - a - b$

$= -2b$

Number of terms (n) = 22

First term for the given A.P. (a) = $a+b$

So, using the formula we get,

$$S_{22} = \frac{22}{2}\left[2(a+b)+(22-1)(-2b)\right]$$
$$=(11)\left[2a+2b+(21)(-2b)\right]$$
$$=(11)\left[2a+2b-42b\right]$$
$$=(11)\left[2a-40b\right]$$
$$=22a-440b$$

Therefore, the sum of first 22 terms for the given A.P. is $\boxed{22a-440b}$.

(vi) $(x-y)^2, (x^2+y^2), (x+y)^2, \ldots$ To n terms.

Common difference of the A.P. (d) = $a_2 - a_1$

$$=(x^2+y^2)-(x-y)^2$$
$$=x^2+y^2-(x^2+y^2-2xy)$$
$$=x^2+y^2-x^2-y^2+2xy$$
$$=2xy$$

Number of terms (n) = n

First term for the given A.P. (a) = $(x-y)^2$

So, using the formula we get,

$$S_n = \frac{n}{2}\left[2(x-y)^2+(n-1)2xy\right]$$

Now, taking 2 common from both the terms inside the bracket we get,

$$=\left(\frac{n}{2}\right)\left[(2)(x-y)^2+(2)(n-1)xy\right]$$
$$=\left(\frac{n}{2}\right)(2)\left[(x-y)^2+(n-1)xy\right]$$
$$=(n)\left[(x-y)^2+(n-1)xy\right]$$

Therefore, the sum of first n terms for the given A.P. is $\boxed{n\left[(x-y)^2+(n-1)xy\right]}$

(vii) $\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}, \ldots$ To n terms.

Number of terms (n) = n

First term for the given A.P. (a) = $\left(\frac{x-y}{x+y}\right)$

Common difference of the A.P. (d) = $a_2 - a_1$

$$=\left(\frac{3x-2y}{x+y}\right)-\left(\frac{x-y}{x+y}\right)$$
$$=\frac{(3x-2y)-(x-y)}{x+y}$$
$$=\frac{3x-2y-x+y}{x+y}$$
$$=\frac{2x-y}{x+y}$$

So, using the formula we get,

$$S_n = \frac{n}{2}\left[2\left(\frac{x-y}{x+y}\right)+(n-1)\left(\frac{2x-y}{x+y}\right)\right]$$

$$=\left(\frac{n}{2}\right)\left[\left(\frac{2x-2y}{x+y}\right)+\left(\frac{(n-1)(2x-y)}{x+y}\right)\right]$$

$$=\left(\frac{n}{2}\right)\left[\left(\frac{2x-2y}{x+y}\right)+\left(\frac{n(2x-y)-1(2x-y)}{x+y}\right)\right]$$

$$=\left(\frac{n}{2}\right)\left[\left(\frac{2x-2y}{x+y}\right)+\left(\frac{n(2x-y)-2x+y}{x+y}\right)\right]$$

Now, on further solving the above equation we get,

$$=\left(\frac{n}{2}\right)\left(\frac{2x-2y+n(2x-y)-2x+y}{x+y}\right)$$

$$=\left(\frac{n}{2}\right)\left(\frac{n(2x-y)-y}{x+y}\right)$$

Therefore, the sum of first n terms for the given A.P. is $\boxed{\left(\frac{n}{2}\right)\left(\frac{n(2x-y)-y}{x+y}\right)}$.

(viii) $-26,-24,-22,\ldots$ To 36 terms.

Common difference of the A.P. (d) = a_2-a_1

$=(-24)-(-26)$

$=-24+26$

$=2$

Number of terms (n) = 36

First term for the given A.P. (a) = −26

So, using the formula we get,

$$S_{36}=\frac{36}{2}\left[2(-26)+(36-1)(2)\right]$$

$$=(18)\left[-52+(35)(2)\right]$$

$$=(18)\left[-52+70\right]$$

$$=(18)[18]$$

$$=324$$

Therefore, the sum of first 36 terms for the given A.P. is $\boxed{324}$.

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