Solution of Simultaneous Linear Equations Ex 8.2 Q5

$x + y + z = 0$

$x - y - 5z = 0$

$x + 2y + 4z = 0$

$$|A| = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -5 \\ 1 & 2 & 4 \end{bmatrix}$$

$$= 1(6) - 1(9) + 1(3) = 9 - 9 = 0$$

Hence, the system has infinite solutions.

Let $z = k$

$x + y = -k$

$x - y = 5k$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -k \\ 5k \end{bmatrix}$$

or $A \quad X = B$

$|A| = -2 \neq 0$, hence $A^{-1}$ exists.

$$adj\ A = \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\text{so, } X = A^{-1}B = \frac{1}{|A|}(adj\ A)B = \frac{1}{-2}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -k \\ 5k \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \left(\frac{1}{-2}\right)\begin{bmatrix} k - 5k \\ k + 5k \end{bmatrix} = \begin{bmatrix} 2k \\ -3k \end{bmatrix}$$

$x = 2k,\ y = -3k, z = k$

Solution of Simultaneous Linear Equations Ex 8.2 Q6

$x + y - z = 0$

$x - 2y + z = 0$

$3x + 6y - 5z = 0$

Hence, $|A| = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -2 & 1 \\ 3 & 6 & -5 \end{bmatrix}$

$= 1(4) - 1(-8) - 1(12)$

$= 4 + 8 - 12 = 0$

Hence, the system will have infinite solutions.

Let $z = k$

$x + y = -k$

$x - 2y = -k$

or $\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} k \\ -k \end{bmatrix}$

or $A \quad x = B$

$|A| = -3 \neq 0$, hence $A^{-1}$ exists.

Now, $adj\ A = \begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix}' = \begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix}$

Next $x = A^{-1}B$

$= \frac{1}{|A|}(adj\ A)(B) = \frac{1}{-3}\begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} k \\ -k \end{bmatrix}$

$= \frac{-1}{3}\begin{bmatrix} -2k + k \\ -2k \end{bmatrix}$

$= \frac{-1}{3}\begin{bmatrix} -k \\ -2k \end{bmatrix} = \begin{bmatrix} \frac{k}{3} \\ \frac{2k}{3} \end{bmatrix}$

Hence, $x = \frac{k}{3}$, $y = \frac{2k}{3}$, $z = k$

or $x = k$, $y = 2k$, $z = 3k$

$3x + y - 2z = 0$
$x + y + z = 0$
$x - 2y + z = 0$

Hence, $|A| = \begin{bmatrix} 3 & 1 & -2 \\ 1 & 1 & 1 \\ 1 & -2 & 1 \end{bmatrix}$

$$|A| = B(1+2) - 1(1-1) - 2(-3)$$
$$= 9 - 0 + 6$$
$$= 15 \neq 0$$

Hence, the given system has only trivial solutions given by $x = y = z = 0$

Solution of Simultaneous Linear Equations Ex 8.2 Q8

$2x + 3y - z = 0$
$x - y - 2z = 0$
$3x + y + 3z = 0$

Hence, $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & -2 \\ 3 & 1 & 3 \end{bmatrix}$

$$|A| = \begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & -2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= 2(-3+2) - 3(3+6) - 1(4)$$
$$= -2 - 27 - 4$$
$$\neq 0$$

Hence, the system has only trivial solutions given by $x = y = z = 0$

********** END **********