

Differentiation Ex 11.2 Q47

Let
$$y = \left(\sin^{-1} x^4\right)^4$$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx} \left(\sin^{-1} x^{4} \right)^{4}$$

$$= 4 \left(\sin^{-1} x^{4} \right) \frac{d}{dx} \left(\sin^{-1} x^{4} \right)$$

$$= 4 \left(\sin^{-1} x^{4} \right)^{3} \frac{1}{\sqrt{1 - \left(x^{4} \right)^{2}}} \frac{d}{dx} \left(x^{4} \right)$$

$$= 4 \left(\sin^{-1} x^{4} \right)^{3} \frac{4x^{3}}{\sqrt{1 - x^{8}}}$$

$$= \frac{16x^{3} \left(\sin^{-1} x^{4} \right)^{3}}{\sqrt{1 - x^{8}}}$$

So,

$$\frac{d}{dx} \left(\sin^{-1} x^4 \right) = \frac{16x^3 \left(\sin^{-1} x^4 \right)^3}{\sqrt{1 - x^8}}.$$

Differentiation Ex 11.2 Q48

Let
$$y = \sin^{-1}\left(\frac{x}{\sqrt{x^2 + a^2}}\right)$$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx} \sin^{-1} \left(\frac{x}{\sqrt{x^2 + a^2}} \right)$$

$$= \frac{1}{\sqrt{1 - \left(\frac{x}{\sqrt{x^2 + a^2}} \right)^2}} \left[\text{Using chain rule and quotient rule} \right]$$

$$= \frac{1}{\sqrt{1 - \left(\frac{x}{\sqrt{x^2 + a^2}} \right)^2}} \left[\frac{\left(x^2 + a^2 \right)^{\frac{1}{2}} \frac{d}{dx} (x) - \frac{d}{dx} \left(x^2 + a^2 \right)^{\frac{1}{2}}}{\left(\left(x^2 + a^2 \right)^{\frac{1}{2}} \right)^2} \right]$$

$$= \frac{\sqrt{x^2 + a^2}}{\sqrt{x^2 + a^2 - x^2}} \left[\frac{\sqrt{x^2 + a^2} - x \times \frac{1}{2\sqrt{x^2 + a^2}} \frac{d}{dx} \left(x^2 + a^2 \right)}{\left(x^2 + a^2 \right)} \right]$$

$$= \frac{\sqrt{x^2 + a^2}}{a \left(x^2 + a^2 \right)} \left[\sqrt{x^2 + a^2} - \frac{x}{2\sqrt{x^2 + a^2}} \times 2x \right]$$

$$= \frac{\sqrt{x^2 + a^2}}{a \left(x^2 + a^2 \right)} \left[\frac{x^2 + a^2 - x^2}{\sqrt{x^2 + a^2}} \right]$$

$$= \frac{a^2}{a \left(x^2 + a^2 \right)}$$

$$= \frac{a}{a^2 + x^2}$$

So,

$$\frac{d}{dx}\left(\sin^{-1}\frac{x}{\sqrt{x^2+a^2}}\right) = \frac{a}{a^2+x^2}$$

Differentiation Ex 11.2 Q49

Consider

$$y = \frac{e^x \sin x}{\left(x^2 + 2\right)^3}$$

Differentiating it with respect toxand applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{\left(x^{2}+2\right)^{3} \frac{d}{dx} \left(e^{x} \sin x\right) - e^{x} \sin x \frac{d}{dx} \left(x^{2}+2\right)^{3}}{\left[\left(x^{2}+2\right)^{3}\right]^{2}}$$

$$= \frac{\left(x^{2}+2\right)^{3} \left[e^{x} \cos x + \sin x e^{x}\right] - e^{x} \sin x 3 \left(x^{2}+2\right)^{2} (2x)}{\left(x^{2}+2\right)^{6}}$$

$$= \frac{\left(x^{2}+2\right)^{3} \left[e^{x} \cos x + \sin x e^{x}\right] - 6x e^{x} \sin x \left(x^{2}+2\right)^{2}}{\left(x^{2}+2\right)^{6}}$$

$$= \frac{\left(x^{2}+2\right)^{2} \left[\left(x^{2}+2\right) \left(e^{x} \cos x + \sin x e^{x}\right) - 6x e^{x} \sin x\right]}{\left(x^{2}+2\right)^{6}}$$

$$= \frac{x^{2} e^{x} \cos x + x^{2} \sin x e^{x} + 2e^{x} \cos x + 2 \sin x e^{x} - 6x e^{x} \sin x}{\left(x^{2}+2\right)^{4}}$$

$$= \frac{e^{x} \sin x}{\left(x^{2}+2\right)^{3}} + \frac{e^{x} \cos x}{\left(x^{2}+2\right)^{3}} - \frac{6x e^{x} \sin x}{\left(x^{2}+2\right)^{4}}$$
Therefore,
$$\frac{dy}{dx} = \frac{e^{x} \sin x}{\left(x^{2}+2\right)^{3}} + \frac{e^{x} \cos x}{\left(x^{2}+2\right)^{3}} - \frac{6x e^{x} \sin x}{\left(x^{2}+2\right)^{4}}$$

Differentiation Ex 11.2 Q50

Consider

$$y = 3e^{-3x}\log(1+x)$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\begin{aligned}
\frac{dy}{dx} &= 3\frac{d}{dx} \left[e^{-3x} \log (1+x) \right] \\
\frac{dy}{dx} &= 3 \left(e^{-3x} \frac{1}{1+x} + \log (1+x) \left(-3e^{-3x} \right) \right) \\
&= 3 \left(\frac{e^{-3x}}{1+x} - 3e^{-3x} \log (1+x) \right) \\
&= 3e^{-3x} \left(\frac{1}{1+x} - 3\log (1+x) \right)
\end{aligned}$$

Differentiation Ex 11.2 Q51 Consider

$$y = \frac{x^2 + 2}{\sqrt{\cos x}}$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{\sqrt{\cos x} \frac{d}{dx} (x^2 + 2) - (x^2 + 2) \frac{d}{dx} \sqrt{\cos x}}{(\sqrt{\cos x})^2}$$

$$= \frac{2x\sqrt{\cos x} - (x^2 + 2) \left(-\frac{1}{2} \frac{\sin x}{\sqrt{\cos x}}\right)}{\cos x}$$

$$= \frac{2x\sqrt{\cos x} + \frac{(x^2 + 2)\sin x}{2\sqrt{\cos x}}}{\cos x}$$

$$= \frac{4x\cos x + (x^2 + 2)\sin x}{2(\cos x)^{\frac{3}{2}}}$$

$$= \frac{2x}{\sqrt{\cos x}} + \frac{1}{2} \frac{(x^2 + 2)\sin x}{(\cos x)^{\frac{3}{2}}}$$
Therefore,
$$\frac{dy}{dx} = \frac{2x}{\sqrt{\cos x}} + \frac{1}{2} \frac{(x^2 + 2)\sin x}{(\cos x)^{\frac{3}{2}}}$$

$$\frac{dy}{dx} = \frac{2x}{\sqrt{\cos x}} + \frac{1}{2} \frac{(x^2 + 2)\sin x}{(\cos x)^{\frac{3}{2}}}$$

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