Geometric Progressions Ex 20.3 Q 3

$$\sum_{n=1}^{11}\left(2+3^n\right)$$
$$=\left(2+3^1\right)+\left(2+3^2\right)+\left(2+3^3\right)+\ldots+\left(2+3^{11}\right)$$
$$=2\times 11+3^1+3^2+3^3+\ldots+3^{11}$$
$$=22+\frac{3\left(3^{11}-1\right)}{(3-1)}$$
$$=22+\frac{3\left(3^{11}-1\right)}{2}$$
$$=\frac{44+3(177147-1)}{2}$$
$$=\frac{44+3(177146)}{2}$$
$$=265741$$

So,

$$\sum_{n=1}^{11}\left(2+3^n\right)=265741$$

$$\sum_{k=1}^{n}\left(2^k+3^{k-1}\right)$$

$$=\left(2+3^0\right)+\left(2^2+3\right)+\left(2^3+3^2\right)+\ldots+\left(2^n+3^{n-1}\right)$$
$$=\left(2+2^2+2^3+\ldots+2^n\right)+\left(3^0+3^1+3^2+\ldots+3^{n-1}\right)$$
$$=S_n+S_m$$

$S_n \Rightarrow a=2,\ n=n,\ r=\frac{2^2}{2}=2$

$$s_n=\frac{a\left(r^n-1\right)}{r-1}=\frac{2\left(2^n-1\right)}{2-1}=2\left(2^n-1\right)$$

Also, $S_m=S_{n-1}$

$a=1,\ r=3,\ n=n-1$

$$S_{n-1}=\frac{1\left(3^{n-1}-1\right)}{3-1}=\frac{1}{2}\left(3^n-1\right)$$

$\therefore$ $$\sum_{k=1}^{n}\left(2^k+3^{k-1}\right)=2\left(2^n-1\right)+\frac{1}{2}\left(3^n-1\right)$$
$$=\frac{1}{2}\left[2^{n+2}+3^n-4-1\right]$$
$$=\frac{1}{2}\left[2^{n+2}+3^n-5\right]$$

$$\sum_{n=2}^{10} 4^n$$

$$= 4^2 + 4^3 + 4^4 + \ldots + 4^{10}$$

$$a = 4^2,\ r = \frac{4^3}{4} = 4,\ n = 9$$

$$S_{10} = \frac{a\left(r^9 - 1\right)}{1 - r}$$

$$= \frac{4^2\left(4^9 - 1\right)}{4 - 1}$$

$$= \frac{1}{3}\left[4^{11} - 16\right]$$

$$= \frac{16}{3}\left[4^9 - 1\right]$$

********** END **********