Exercise 8.1

$= \sqrt{25k^2} = 5k$

$\therefore \sin A = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$

$\cos A = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$

And $\tan A = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$

Now, L.H.S. $\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}}$

$= \frac{16-9}{16+9} = \frac{7}{25}$

R.H.S. $\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

$\because$ L.H.S. = R.H.S.

$\therefore \frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

Q9. In ΔABC right angles at B, if $\tan A = \frac{1}{\sqrt{3}}$,

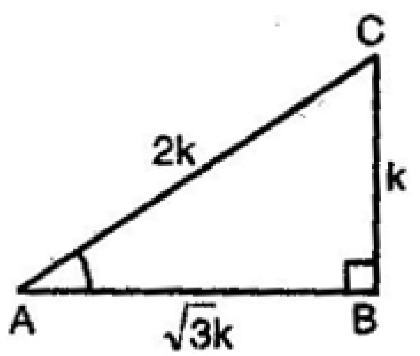
find value of:

(i) $\sin A \cos C + \cos A \sin C$

(ii) $\cos A \cos C - \sin A \sin C$

Ans: Consider a triangle ABC in which $\angle$B = 90°.

Let BC = k and AB = $\sqrt{3}k$



Then, using Pythagoras theorem,

$$AC = \sqrt{(BC)^2 + (AB)^2}$$

$$= \sqrt{(k)^2 + \left(\sqrt{3}k\right)^2}$$

$$= \sqrt{k^2 + 3k^2} = \sqrt{4k^2} = 2k$$

$$\therefore \sin A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$\cos A = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

For $\angle$C, Base = BC, Perpendicular = AB and Hypotenuse = AC

$$\therefore \sin C = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

(i) $\sin A \cos C + \cos A \sin C = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

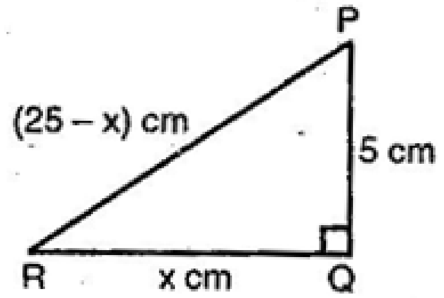
$$= \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$

(ii) $\cos A \cos C - \sin A \sin C = \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

Q10. In Δ PQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of $\sin P, \cos P$ and $\tan P$.

Ans: In Δ PQR, right angled at Q.



PR + QR = 25 cm and PQ = 5 cm

Let QR = x cm and PR = $(25 - x)$ cm

Using Pythagoras theorem,

$RP^2 = RQ^2 + QP^2$

$\Rightarrow (25-x)^2 = (x)^2 + (5)^2$

$\Rightarrow\ 625 - 50x + x^2 = x^2 + 25$

$\Rightarrow\ -50x = -600$

$\Rightarrow\ x = 12$

$\therefore$ RQ = 12 cm and RP = 25 − 12 = 13 cm

$\therefore\ \sin P = \frac{\text{RQ}}{\text{RP}} = \frac{12}{13}$

$\cos P = \frac{\text{PQ}}{\text{RP}} = \frac{5}{13}$

And $\tan P = \frac{\text{RQ}}{\text{PQ}} = \frac{12}{5}$

Q11. State whether the following are true or false. Justify your answer.

(i) The value of $\tan A$ is always less than 1.

(ii) $\sec A = \frac{12}{5}$ for some value of angle A.

(iii) $\cos A$ is the abbreviation used for the cosecant of angle A.

(iv) $\cot A$ is the product of $\cot$ and A.

(v) $\sin\theta = \frac{4}{3}$ for some angle θ.

Ans: (i) False because sides of a right triangle may have any length, so $\tan A$ may have any value.

(ii) True as $\sec A$ is always greater than 1.

(iii) False as $\cos A$ is the abbreviation of cosine A.

(iv) False as $\cot A$ is not the product of 'cot' and A. 'cot' is separated from A has no meaning.

(v) False as $\sin\theta$ cannot be > 1.

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