Definite Integrals Ex 20.4A Q10

$$I = \int_a^b \frac{x^{1/n}}{x^{1/n} + (a+b-x)^{1/n}}\,dx$$

$$I = \int_a^b \frac{(a+b-x)^{1/n}}{(a+b-x)^{1/n} + x^{1/n}}\,dx$$

$$2I = \int_a^b \frac{x^{1/n}}{x^{1/n} + (a+b-x)^{1/n}}\,dx + \int_a^b \frac{(a+b-x)^{1/n}}{(a+b-x)^{1/n} + x^{1/n}}\,dx$$

$$2I = \int_a^b \frac{x^{1/n} + (a+b-x)^{1/n}}{x^{1/n} + (a+b-x)^{1/n}}\,dx$$

$$I = \frac{1}{2}\int_a^b dx$$

$$I = \frac{b-a}{2}$$

Definite Integrals Ex 20.4A Q11

We have,

$$I = \int_0^{\frac{\pi}{2}} (2\log\cos x - \log\sin 2x)\,dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\log\cos^2 x - \log\sin 2x\right)dx$$

$$= \int_0^{\frac{\pi}{2}} \log\frac{\cos^2 x}{\sin x}\,dx$$

$$= \int_0^{\frac{\pi}{2}} \log\frac{\cos^2 x}{2\sin x.\cos x}\,dx$$

$$= \int_0^{\frac{\pi}{2}} \log\frac{\cos x}{2\sin x}\,dx$$

$$= \int_0^{\frac{\pi}{2}} (\log\cos x - \log\sin x - \log 2)\,dx$$

$$= \int_0^{\frac{\pi}{2}} \log\cos x\,dx - \int_0^{\frac{\pi}{2}} \log\sin x\,dx - \int_0^{\frac{\pi}{2}} \log 2$$

We know that $\int_0^{\frac{\pi}{2}} \log\cos x\,dx = \int_0^{\frac{\pi}{2}} \log\sin x\,dx$ - (i)

Hence from equation (i)

$$I = -\int_0^{\frac{\pi}{2}} \log 2 = -\frac{\pi}{2}\log 2$$

Definite Integrals Ex 20.4A Q12

Let $I=\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$ ...(1)

It is known that, $\left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$

$I=\int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}dx$ ...(2)

Adding (1) and (2), we obtain

$$2I=\int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}dx$$
$$\Rightarrow 2I=\int_0^a 1\cdot dx$$
$$\Rightarrow 2I=[x]_0^a$$
$$\Rightarrow 2I=a$$
$$\Rightarrow I=\frac{a}{2}$$

Definite Integrals Ex 20.4A Q13

Let $I=\int_0^5 \frac{\sqrt[4]{x+4}}{\sqrt[4]{x+4}+\sqrt[4]{9-x}}dx$ --(i)

We know that $\int_0^a f(x)=\int_0^a f(a-x)$

So,

$$I=\int_0^5 \frac{\sqrt[4]{(5-x)+4}}{\sqrt[4]{(5-x)+4}+\sqrt[4]{9-(5-x)}}dx$$

$I=\int_0^5 \frac{\sqrt[4]{9-x}}{\sqrt[4]{9-x}+\sqrt[4]{4+x}}dx$ --(ii)

Adding (i) & (ii)

$$2I=\int_0^5 \frac{\sqrt[4]{x+4}}{\sqrt[4]{x+4}+\sqrt[4]{9-x}}dx+\int_0^5 \frac{\sqrt[4]{9-x}}{\sqrt[4]{9-x}+\sqrt[4]{4+x}}dx$$
$$2I=\int_0^5 \frac{\sqrt[4]{x+4}+\sqrt[4]{9-x}}{\sqrt[4]{x+4}+\sqrt[4]{9-x}}dx$$
$$2I=\int_0^5 dx$$
$$2I=[x]_0^5$$
$$I=\frac{1}{2}[5-0] = \frac{5}{2}$$

$$\therefore \int_0^5 \frac{\sqrt[4]{x+4}}{\sqrt[4]{x+4}+\sqrt[4]{9-x}}dx = \frac{5}{2}$$

*********** END **********