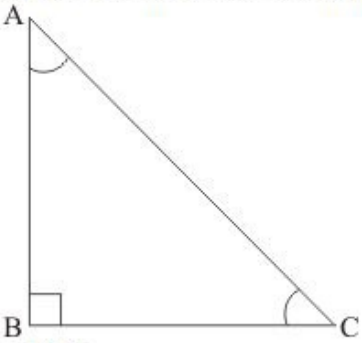
Trigonometric Ratios Ex 5.2 Q36

Answer :

(i) We have drawn the following figure related to given information



To find:

$\sin A\cos C+\cos A\sin C$ ..... (1)

Now we have,

$\sin A=\frac{BC}{AC}$, $\sin A=\frac{AB}{AC}$

$\cos A=\frac{AB}{AC}$, $\cos C=\frac{BC}{AC}$

Now by substituting the above values in equation (1)

We get,

$$\sin A\cos C+\cos A\sin C=\frac{BC}{AC}\times\frac{BC}{AC}+\frac{AB}{AC}\times\frac{AB}{AC}$$

$$=\frac{BC^2}{AC^2}+\frac{AB^2}{AC^2}$$

$$=\frac{BC^2+AB^2}{AC^2}$$

Therefore,

$$\sin A\cos C+\cos A\sin C=\frac{BC^2+AB^2}{AC^2} \quad \text{..... (2)}$$

Now in right angled ΔABC

By applying Pythagoras theorem

We get,

$AC^2=AB^2+BC^2$

Now, by substituting above value of AC^2 in equation (2)

We get,

$$\sin A\cos C+\cos A\sin C=\frac{BC^2+AB^2}{AB^2+BC^2}$$

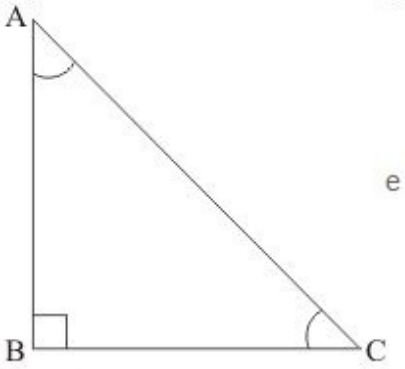
$$\Rightarrow \sin A\cos C+\cos A\sin C=\frac{AB^2+BC^2}{AB^2+BC^2}$$

Now both numerator and denominator contains AB^2+BC^2

Therefore it gets cancelled and 1 remains

Hence $\boxed{\sin A\cos C+\cos A\sin C=1}$

(ii) We have drawn the following figure



To find:

$\sin A \sin B + \cos A \cos B$ ..... (1)

Now we know that sum of all the angles of any triangle is 180°

Therefore,

$\angle A + \angle B + \angle C = 180°$

Since $\angle A = \angle C$ and $\angle B = 90°$

Therefore,

$\angle A + 90° + \angle A = 180°$

$\Rightarrow \quad 2\angle A + 90° = 180°$

$\Rightarrow \quad 2\angle A = 180° - 90°$

$\Rightarrow \quad 2\angle A = 90°$

$\Rightarrow \quad \angle A = \frac{90°}{2}$

$\Rightarrow \quad \angle A = 45°$

It is given that $\angle A = \angle C$

Therefore,

$\angle A = \angle C = 45°$ ..... (2)

Now we have,

$\sin A = \frac{BC}{AC}$, $\sin B = \sin 90° = 1$

$\cos A = \frac{AB}{AC}$, $\cos B = \cos 90° = 0$

Now by substituting the above values in equation (1)

We get,

$\sin A \sin B + \cos A \cos B = \sin 45° \times 1 + \cos 45° \times 0$

$= \sin 45°$

Since $\sin 45° = \frac{1}{\sqrt{2}}$

Therefore $\boxed{\sin A \sin B + \cos A \cos B = \frac{1}{\sqrt{2}}}$

********** END **********