Definite Integrals Ex 20.2 Q9

Let $x^2 = t$

Differentiating w.r.t. x, we get

$2x\,dx = dt$

Now,

$x = 0 \Rightarrow t = 0$

$x = 1 \Rightarrow t = 1$

$\therefore \displaystyle\int_0^1 \frac{2x}{1+x^4}\,dx$

$= \displaystyle\int_0^1 \frac{dt}{1+t^2}$

$= \left[\tan^{-1} t\right]_0^1$

$= \left[\tan^{-1} 1 - \tan^{-1} 0\right] \qquad\qquad \left[\because \tan\frac{\pi}{4} = 1\right]$

$= \dfrac{\pi}{4}$

$\therefore \displaystyle\int_0^1 \frac{2x}{1+x^4}\,dx = \frac{\pi}{4}$

Definite Integrals Ex 20.2 Q10

Let $x = a\sin\theta$

Differentiating w.r.t. x, we get

$dx = a\cos\theta d\theta$

Now,

$x = 0 \Rightarrow \theta = 0$

$x = a \Rightarrow \theta = \dfrac{\pi}{2}$

$\therefore \displaystyle\int_0^a \sqrt{a^2 - x^2}\, dx$

$= \displaystyle\int_0^{\frac{\pi}{2}} \sqrt{a^2\left(1 - \sin^2\theta\right)}\, a\cos\theta d\theta$

$= a^2 \displaystyle\int_0^{\frac{\pi}{2}} \cos^2\theta d\theta \qquad \left[\because \left(1 - \sin^2\theta\right) = \cos^2\theta \text{ and } \dfrac{1 + \cos 2\theta}{2} = \cos 2\theta\right]$

$= \dfrac{a^2}{2} \displaystyle\int_0^{\frac{\pi}{2}} (1 + \cos 2\theta)\, d\theta$

$= \dfrac{a^2}{2}\left[\theta + \dfrac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$

$= \dfrac{a^2}{2}\left[\dfrac{\pi}{2} + 0 - 0 - 0\right]$

$= \dfrac{\pi a^2}{4}$

$\therefore \displaystyle\int_0^a \sqrt{a^2 - x^2}\, dx = \dfrac{\pi a^2}{4}$

Definite Integrals Ex 20.2 Q11

Let $I = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\cos^5\phi\, d\phi = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\cos^4\phi\cos\phi\, d\phi$

Also, let $\sin\phi = t \Rightarrow \cos\phi\, d\phi = dt$

When $\phi = 0$, $t = 0$ and when $\phi = \dfrac{\pi}{2}$, $t = 1$

$\therefore I = \int_0^1 \sqrt{t}\left(1-t^2\right)^2 dt$

$= \int_0^1 t^{\frac{1}{2}}\left(1+t^4 - 2t^2\right) dt$

$= \int_0^1 \left[t^{\frac{1}{2}} + t^{\frac{9}{2}} - 2t^{\frac{5}{2}}\right] dt$

$= \left[\dfrac{t^{\frac{3}{2}}}{\dfrac{3}{2}} + \dfrac{t^{\frac{11}{2}}}{\dfrac{11}{2}} - \dfrac{2t^{\frac{7}{2}}}{\dfrac{7}{2}}\right]_0^1$

$= \dfrac{2}{3} + \dfrac{2}{11} - \dfrac{4}{7}$

$= \dfrac{154 + 42 - 132}{231}$

$= \dfrac{64}{231}$

Definite Integrals Ex 20.2 Q12

Let $\sin x = t$

Differentiating w.r.t. x, we get

$$\cos x \, dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = \dfrac{\pi}{2} \Rightarrow t = 1$

$$\int_0^{\frac{\pi}{2}} \frac{\cos x}{1 + \sin^2 x} dx$$

$$= \int_0^1 \frac{dt}{1 + t^2}$$

$$= \left[\tan^{-1} t \right]_0^1$$

$$= \left[\tan^{-1} 1 - \tan^{-1} 0 \right] \qquad \left[\because \tan \frac{\pi}{4} = 1 \right]$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\cos x}{1 + \sin^2 x} dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q13

Let $1 + \cos\theta = t^2$

Differentiating w.r.t. x, we get

$$-\sin\theta\, d\theta = 2t\, dt$$

$$\sin\theta\, d\theta = -2t\, dt$$

Now,

$x = 0 \Rightarrow t = \sqrt{2}$

$x = \dfrac{\pi}{2} \Rightarrow t = 1$

$\therefore \displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin\theta\, d\theta}{\sqrt{1 + \cos\theta}}$

$= \displaystyle\int_{\sqrt{2}}^{1} \frac{-2t\, dt}{t}$

$= -2 \displaystyle\int_{\sqrt{2}}^{1} dt$

$= -2\left[t\right]_{\sqrt{2}}^{1}$

$= -2\left[1 - \sqrt{2}\right]$

$= 2\left[\sqrt{2} - 1\right]$

$\therefore \displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin\theta\, d\theta}{\sqrt{1 + \cos\theta}} = 2\left[\sqrt{2} - 1\right]$

********** END *********