Question 7. 11. Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. The cylinder is free to rotate about its standard axis of symmetry, and the sphere is free to rotate about an axis passing through its centre. Which of the two will acquire a greater angular speed after a given time?

Answer: Let M be the mass and R the radius of the hollow cylinder, and also of the solid sphere. Their moments of inertia about the respective axes are $I_1 = MR^2$ and $I_2 = 2/5\ MR^2$

Let τ be the magnitude of the torque applied to the cylinder and the sphere, producing angular accelerations $\alpha_1$and α_2 respectively.

Then $\tau = I_1\ \alpha_1 = I_2\ \alpha_2$

The angular acceleration 04 produced in the sphere is larger. Hence, the sphere will acquire larger angular speed after a given time.

Question 7. 12. A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad s^{-1}. The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis?

Answer: M = 20 kg

Angular speed, w = 100 rad s^{-1}; R = 0.25 m

Moment of inertia of the cylinder about its axis

$=1/2\ MR^2 = 1/2 \times 20\ (0.25)^2\ kg\ m^2 = 0.625\ kg\ m^2$

Rotational kinetic energy,

$E_r = 1/2\ Iw2 = 1/2 \times 0.625 \times (100)^2\ J = 3125\ J$

Angular momentum,

$L = I_w = 0.625 \times 100\ J_s = 62.5\ J_s$.

Question 7. 13. (a) A child stands at the centre of a turntable with his arms outstretched. The turntable is set rotating with an angular speed of 40 rev/min. How much is the angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to 2/5 times the initial value? Assume that the turntable rotates without friction, (b) Show that the child's new kinetic energy of rotation is more than the initial kinetic energy of rotation. How do you account of this increase in kinetic energy?

Answer: (a) Suppose, initial moment of inertia of the child is I Then final moment of inertia,

$$I_2 = \frac{2}{5} I_l$$

Also, $\nu_1 = 40$ rev min^{-1}

By using the principle of conservation of angular momentum, we get

$$I_1\omega_1 = I_2\omega_2 \quad \text{or} \quad I_1\ (2\pi\nu_1) = I_2(2\pi\nu_2)$$

or

$$\nu_2 = \frac{I_1\nu_1}{I_2} = \frac{I_1 \times 40}{\frac{2}{5} \times I_1} = 100 \text{ rev min}^{-1}$$

(b)

$$\frac{\text{Final K.E. of rotation}}{\text{Initial K.E. of rotation}} = \frac{\frac{1}{2}I_2\omega_2^2}{\frac{1}{2}I_1\omega_1^2} = \frac{\frac{1}{2}I_2(2\pi\nu_2)^2}{\frac{1}{2}I_1(2\pi\nu_1)^2} = \frac{I_2\nu_2^2}{I_1\nu_1^2} = \frac{\frac{2}{5}I_1 \times (100)^2}{\frac{2}{5}I_1 \times (40)^2} = 2.5$$

Clearly, final $(K.E.)_{rot}$ becomes more because the child uses his internal energy when he folds his hands to increase the kinetic energy.

Question 7. 14. A rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N? What is the linear acceleration of the rope? Assume that there is no slipping.
Answer:
Here, M = 3 kg, R = 40 cm = 0.4 m
Moment of inertia of the hollow cylinder about its axis.
$I = MR^2 = 3(0.4)^2 = 0.48$ kg m^2

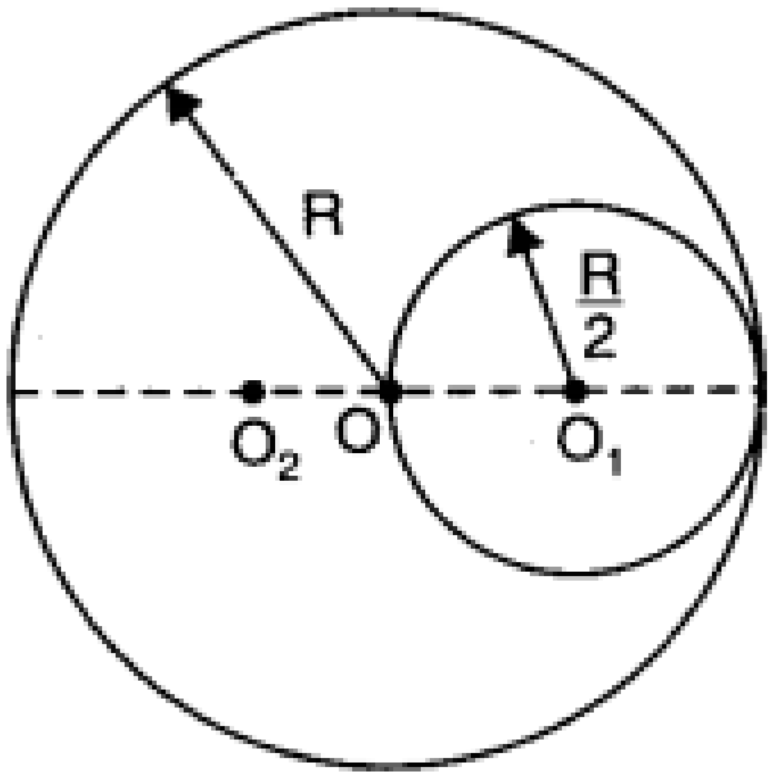
Force applied $F = 30$ N
$\therefore$ Torque, $\tau = F \times R = 30 \times 0.4 = 12$ N–m.
If α is angular acceleration produced, then from $\tau = I\alpha$

$$\alpha = \frac{\tau}{I} = \frac{12}{0.48} = 25 \text{ rad s}^{-2}$$

Linear acceleration, $a = R\alpha = 0.4 \times 25 = 10$ ms^{-2}.

Question 7. 15. To maintain a rotor at a uniform angular speed of 200 rad s^{-1}, an engine needs to transmit a torque of 180 Nm. What is the power required by the engine?
Note: Uniform angular velocity in the absence of friction implies zero torque. In practice, applied torque is needed to counter fricitional torque). Assume that the engine is 100 efficient.
Answer:
Here, a = 200 rad s^{-1}; Torque, τ = 180 N-m
Since,Power, P = Torque (τ) x angular speed (w)
= 180 x 200 = 36000 watt = 36 KW.

Question 7. 16. From a uniform disk of radius R, a circular hole of radius R/2 is cut out. The centre of the hole is at R/2 from the centre of the original disc.Locate the centre of gravity of the resulting flat body.



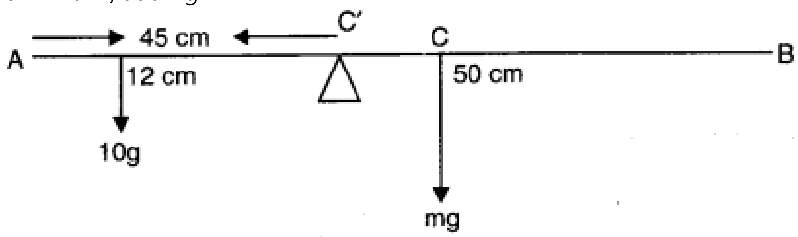
Answer: Let from a bigger uniform disc of radius R with centre O a smaller circular hole of radius R/2 with its centre at O_1 (where R OO_1 = R/2) is cut out.
Let centre of gravity or the centre of mass of remaining flat body be at O_2, where OO_2 = x. If σ be mass per unit area, then mass of whole disc $M_1 = \pi R^2 \sigma$ and mass of cut out part

$$M_2 = \pi\left(\frac{R}{2}\right)^2 \sigma = \frac{1}{4}\pi R^2 \sigma = \frac{M_1}{4}$$

$$\therefore \quad x = \frac{M_1 \times (0) - M_2(OO_1)}{M_1 - M_2} = \frac{0 - \frac{M_1}{4} \times \frac{R}{2}}{M_1 - \frac{M_1}{4}} = -\frac{R}{6}$$

i.e., O_2 is at a distance R/6 from centre of disc on diametrically opposite side to centre of hole.

Question 7. 17. A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm. What is the mass of the metre stick?
Answer: Let m be the mass of the stick concentrated at C, the 50 cm mark, see fig.



For equilibrium about C, the 45 cm mark,
10 g (45 - 12) = mg (50 - 45)
10 g x 33 = mg x 5
⇒ m = 10 x 33/5
or m = 66 grams.

Question 7. 18. A solid sphere rolls down two different inclined planes of the same heights but different angles of inclination, (a) Will it reach the bottom with the same speed in each case? (b) Will it take longer to roll down one plane than the other? (c) If so, which one and why?
Answer: (a) Using law of conservation of energy,

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

$$\text{or} \quad \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\frac{v^2}{R^2} = mgh$$

$$\text{or} \quad \frac{7}{10}v^2 = gh \quad \text{or} \quad v = \sqrt{\frac{10gh}{7}}$$

Since h is same for both the inclined planes therefore v is the same.

$$(b) \quad l = \frac{1}{2}\left(\frac{g\sin\theta}{1+\frac{K^2}{R^2}}\right)t^2 = \frac{g\sin\theta}{2\left(1+\frac{2}{5}\right)}t^2 = \frac{5g\sin\theta}{14}t^2$$

$$\text{or} \quad t = \sqrt{\frac{14\,l}{5g\sin\theta}}$$

$$\text{Now,} \quad \sin\theta = \frac{h}{l} \quad \text{or} \quad l = \frac{h}{\sin\theta}$$

$$\therefore \quad t = \frac{1}{\sin\theta}\sqrt{\frac{14\,h}{5g}}$$

Lesser the value of θ, more will be t.

(c) Clearly, the solid sphere will take longer to roll down the plane with smaller inclination.

Question 7. 19. A hoop of radius 2 m weighs 100 kg. It rolls along a horizontal floor so that its centre of mass has a speed of 20 cm/s. How much work has to be done to stop it?
Answer:
Here, R = 2 m, M = 100 kg

v = 20 cm/s = 0.2 m/s
Total energy of the hoop =$1/2Mv^2 + 1/2Iw^2$
= $1/2Mv^2 + 1/2(MR^2)w^2$
= $1/2Mv^2 + 1/2Mv^2 = Mv^2$
Work required to stop the hoop = total energy of the hoop W = $Mv^2 = 100\ (0.2)^2$= 4 Joule.

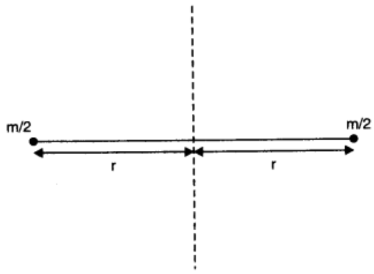
Question 7. 20. The oxygen molecule has a mass of 5.30 x 10^{26} kg and a moment of inertia of 1.94 x 10^{-45} kg m^2 about an axis through its centre perpendicular to the lines joining the two atoms. Suppose the mean speed of such a molecule in a gas is 500 m/s and that its kinetic energy of rotation is two thirds of its kinetic energy of translation. Find the average angular velocity of the molecule.
Answer:

Here,
$$m = 5.30 \times 10^{-26} \text{ kg}$$
$$I = 1.94 \times 10^{-46} \text{ kg m}^2$$
$$v = 500 \text{ m/s}$$

If $\frac{m}{2}$ is mass of each atom of oxygen and $2r$ is distance between the two atoms as shown in Fig, then

$$I = \frac{m}{2}r^2 + \frac{m}{2}r^2 = mr^2$$

$$r = \sqrt{\frac{I}{m}} = \sqrt{\frac{1.94 \times 10^{-46}}{5.30 \times 10^{-26}}}$$
$$= 0.61 \times 10^{-10} \text{ m}$$



As K.E. of rotation= $\frac{2}{3}$ K.E. of translation

$$\therefore \quad \frac{1}{2}I\omega^2 = \frac{2}{3} \times \frac{1}{2}m\omega^2$$

$$\frac{1}{2}(mr^2)\,\omega^2 = \frac{1}{2}mv^2$$

$$\omega = \sqrt{\frac{2}{3}}\frac{v}{r} = \sqrt{\frac{2}{3}} \times \frac{500}{0.61 \times 10^{-10}} = 6.7 \times 10^{12} \text{ rad/s.}$$

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