Trigonometric Ratios Ex 5.1 Q23

Answer :

Given:

$$\sec A = \frac{5}{4} \quad \ldots\ldots (1)$$

To verify:

$$\frac{3\sin A - 4\sin^3 A}{4\cos^3 A - 3\cos A} = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A} \quad \ldots\ldots (2)$$

Now we know that $\sec A = \dfrac{1}{\cos A}$

Therefore $\cos A = \dfrac{1}{\sec A}$

Now, by substituting the value of $\sec A$ from equation (1)

We get,

$$\cos A = \frac{1}{\frac{5}{4}}$$

$$= \frac{4}{5}$$

Therefore,

$$\cos A = \frac{4}{5} \quad \ldots\ldots (3)$$

Now, we know the following trigonometric identity

$$\cos^2 A + \sin^2 A = 1$$

Therefore,

$$\sin^2 A = 1 - \cos^2 A$$

Now by substituting the value of $\cos A$ from equation (3)

We get,

$$\sin^2 A = 1 - \left(\frac{4}{5}\right)^2$$

$$= 1 - \frac{(4)^2}{(5)^2}$$

$$= 1 - \frac{16}{25}$$

Now by taking L.C.M

We get,

$$\sin^2 A = \frac{25-16}{25}$$

$$= \frac{9}{25}$$

Now, by taking square root on both sides

We get,

$$\sin A = \sqrt{\frac{9}{25}}$$

$$= \frac{\sqrt{9}}{\sqrt{25}}$$

$$= \frac{3}{5}$$

Therefore,

$\sin A = \frac{3}{5}$ ...... (4)

Now, we know that $\tan A = \frac{\sin A}{\cos A}$

Now by substituting the value of $\sin A$ and $\cos A$ from equation (3) and (4) respectively

We get,

$$\tan A = \frac{\frac{3}{5}}{\frac{4}{5}}$$

$$= \frac{3}{5} \times \frac{5}{4}$$

$$= \frac{3}{4}$$

Therefore

$\tan A = \frac{3}{4}$ ...... (5)

Now from the expression of equation (2)

$$\text{L.H.S} = \frac{3\sin A - 4\sin^3 A}{4\cos^3 A - 3\cos A}$$

Now by substituting the value of $\cos A$ and $\sin A$ from equation (3) and (4)

We get,

$$\text{L.H.S} = \frac{3\left(\frac{3}{5}\right) - 4\left(\frac{3}{5}\right)^3}{4\left(\frac{4}{5}\right)^3 - 3\left(\frac{4}{5}\right)}$$

Therefore,

$$\text{L.H.S} = \frac{\frac{9}{5} - 4\left(\frac{27}{125}\right)}{4\left(\frac{64}{125}\right) - \frac{12}{5}}$$

$$= \frac{\frac{9}{5} - \frac{108}{125}}{\frac{256}{125} - \frac{12}{5}}$$

Now by taking L.C.M of both numerator and denominator

We get,

$$\text{L.H.S} = \frac{\frac{9\times 25}{5\times 25} - \frac{108}{125}}{\frac{256}{125} - \frac{12\times 25}{5\times 25}}$$

$$= \frac{\frac{225}{125} - \frac{108}{125}}{\frac{256}{125} - \frac{300}{125}}$$

$$= \frac{\frac{225-108}{125}}{\frac{256-300}{125}}$$

$$= \frac{\frac{117}{125}}{\frac{-44}{125}}$$

$$= \frac{-117}{44}$$

$$\frac{3\sin A - 4\sin^3 A}{4\cos^3 A - 3\cos A} = \frac{-117}{44} \quad \ldots\ldots (6)$$

Now from the expression of equation (2)

$$\text{R.H.S} = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$$

Now by substituting the value of $\tan A$ from equation (5)

We get,

$$\text{R.H.S} = \frac{3\left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^3}{}$$

R.H.S. =

$$1-3\left(\frac{3}{4}\right)^2$$

$$=\frac{\frac{9}{4}-\frac{27}{64}}{1-\frac{3\times 9}{16}}$$

Now by taking L.C.M

We get,

$$\text{R.H.S}=\frac{\frac{9\times 16}{4\times 16}-\frac{27}{64}}{\frac{16-27}{16}}$$

$$=\frac{\frac{144}{64}-\frac{27}{64}}{\frac{-11}{16}}$$

$$=\frac{\frac{144-27}{64}}{\frac{-11}{16}}$$

$$= \frac{\frac{117}{64}}{\frac{-11}{16}}$$

$$= \frac{117}{64} \times \frac{16}{-11}$$

Now, $16 \times 4 = 64$

Therefore,

$$\text{R.H.S} = \frac{117}{4} \times \frac{1}{-11}$$

$$= \frac{117 \times 1}{4 \times -11}$$

$$= \frac{117}{-44}$$

$$= \frac{-117}{44}$$

Therefore,

$$\frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A} = \frac{-117}{44} \quad \ldots\ldots (7)$$

Now by comparing equation (6) and (7)

We get, $\frac{3\sin A - 4\sin^3 A}{4\cos^3 A - 3\cos A} = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

*********** END **********