Trigonometric Ratios Ex 5.1 Q24

Answer :

Given:

$$\sin\theta = \frac{3}{4} \text{ ...... (1)}$$

To prove:

$$\sqrt{\frac{\operatorname{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} = \frac{\sqrt{7}}{3} \text{ ...... (2)}$$

By definition,

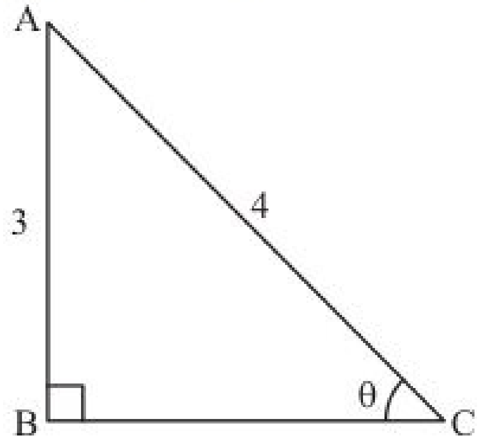
$$\sin A = \frac{\text{Perpendicular side opposite to } \angle A}{\text{Hypotenuse}} \text{ ...... (3)}$$

By Comparing (1) and (3)

We get,

Perpendicular side = 3 and

Hypotenuse = 4



Side BC is unknown.
So we find BC by applying Pythagoras theorem to right angled ΔABC ,
Hence,
$AC^2 = AB^2 + BC^2$
Now we substitute the value of perpendicular side (AB) and hypotenuse (AC) and get the base side (BC)
Therefore,
$$4^2 = 3^2 + BC^2$$
$$BC^2 = 4^2 - 3^2$$
$$BC^2 = 16 - 9$$
$$BC^2 = 7$$
$$BC = \sqrt{7}$$
$$BC^2 = 4^2 - 3^2$$
$$BC^2 = 16 - 9$$
$$BC^2 = 7$$
$$BC = \sqrt{7}$$
Hence, Base side BC = $\sqrt{7}$ ...... (3)

Now, $\cos A = \frac{\text{Base}}{\text{Hypotenuse}}$

Therefore from fig. a and equation (3)
$$\cos A = \frac{BC}{AC}$$
$$= \frac{\sqrt{7}}{4}$$
Therefore,
$$\cos A = \frac{\sqrt{7}}{4} \quad \text{......} \ (4)$$
Now, $\operatorname{cosec} A = \frac{1}{\sin A}$

Therefore from fig. a and equation (1) ,
$$\operatorname{cosec} A = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$$
$$\operatorname{cosec} A = \frac{4}{3} \quad \text{......} \ (5)$$
Now, $\sec A = \frac{1}{\text{cosA}}$

Therefore from fig. a and equation (4) ,
$$\sec A = \frac{4}{\sqrt{7}} \quad \text{......} \ (6)$$
Now, $\cot A = \frac{\text{cosA}}{\sin A}$

Therefore by substituting the values from equation (1) and (4) , We get,

$$\cot A = \frac{\frac{\sqrt{7}}{4}}{\frac{3}{4}}$$
$$= \frac{\sqrt{7}}{4} \times \frac{4}{3}$$
$$= \frac{\sqrt{7}}{3}$$

Therefore,

$$\cot A = \frac{\sqrt{7}}{3} \quad \ldots\ldots (7)$$

Now by substituting the value of $\operatorname{cosec} A$, $\sec A$ and $\cot A$ from equation (5) ,(6) and (7) respectively in the L.H.S of expression (2) ,

We get,

$$\sqrt{\frac{\operatorname{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} = \sqrt{\frac{\left(\frac{4}{3}\right)^2 - \left(\frac{\sqrt{7}}{3}\right)^2}{\left(\frac{4}{\sqrt{7}}\right)^2 - 1}}$$

$$= \sqrt{\frac{\frac{(4)^2}{(3)^2} - \frac{\left(\sqrt{7}\right)^2}{(3)^2}}{\frac{(4)^2}{\left(\sqrt{7}\right)^2} - 1}}$$

$$= \sqrt{\frac{\frac{16}{9} - \frac{7}{9}}{\frac{16}{7} - 1}}$$

$$= \sqrt{\frac{\frac{16-7}{9}}{\frac{16-7}{7}}}$$

$$= \sqrt{\frac{\frac{9}{9}}{\frac{9}{7}}}$$

$$= \sqrt{\frac{9}{9} \times \frac{7}{9}}$$

Therefore,

$$\sqrt{\frac{\operatorname{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} = \sqrt{\frac{7}{9}}$$

$$= \frac{\sqrt{7}}{\sqrt{9}}$$

$$= \frac{\sqrt{7}}{3}$$

Hence it is proved that

$$\sqrt{\frac{\operatorname{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} = \frac{\sqrt{7}}{3}$$

*********** END **********