Maxima and Minima 18.5 Q29

Let $P(x, y)$ be a point on

$$y^2 = 4x \qquad \text{---(i)}$$

Let S be the square of the distance between $A(2, -8)$ and P.

$$\therefore \quad S = (x-2)^2 + (y+8)^2 \qquad \text{---(ii)}$$

Using (i),

$$S = \left(\frac{y^2}{4} - 2\right)^2 + (y+8)^2$$

$$\therefore \quad \frac{dS}{dy} = 2\left(\frac{y^2}{4} - 2\right) \times \frac{y}{2} + 2(y+8)$$

$$= \frac{y^3 - 8y}{4} + 2y + 16$$

$$= \frac{y^3}{4} + 16$$

For maxima and minima,

$$\frac{dS}{dy} = 0$$

$$\Rightarrow \quad \frac{y^3}{4} + 16 = 0$$

$$\Rightarrow \quad y = -4$$

Now,

$$\frac{d^2S}{dy^2} = \frac{3y^2}{4}$$

At $y = -4$, $\dfrac{d^2S}{dy^2} = 12 > 0$

$\therefore$ $y = -4$ is the point of local minima

From (i)

$$x = \frac{y^2}{4} = 4$$

Thus, the required point is $(4, -4)$ nearest to $(2, -8)$.

Maxima and Minima 18.5 Q30

Let $P(x, y)$ be a point on the curve,

$$x^2 = 8y \qquad \text{---(i)}$$

Let $A = (2, 4)$ be a point and

let $S =$ square of the distance between P and A

$$\therefore \quad S = (x-2)^2 + (y-4)^2 \qquad \text{---(ii)}$$

Using (i), we get

$$S = (x-2)^2 + \left(\frac{x^2}{8} - 4\right)^2$$

$$\therefore \quad \frac{dS}{dy} = 2(x-2) + 2\left(\frac{x^2}{8} - 4\right) \times \frac{2x}{8}$$

$$= 2(x-2) + \frac{\left(x^2 - 32\right)x}{16}$$

Also, $$\frac{d^2S}{dx^2} = 2 + \frac{1}{16}\left[x^2 - 32 + 2x^2\right]$$

$$= 2 + \frac{1}{16}\left[3x^2 - 32\right]$$

For maxima and minima,

$$\frac{dS}{dx} = 0$$

$$\Rightarrow \quad 2(x-2) + \frac{x\left(x^2 - 32\right)}{16} = 0$$

$$\Rightarrow \quad 32x - 64 + x^3 - 32x = 0$$

$$\Rightarrow \quad x^3 - 64 = 0$$

$$\Rightarrow \quad x = 4$$

Now,

At $x = 4$, $\dfrac{d^2S}{dx^2} = 2 + \dfrac{1}{16}\left[16 \times 3 - 32\right] = 2 + 1 = 3 > 0$

$\therefore$ $x = 4$ is point of local minima

From (i)

$$y = \frac{x^2}{8} = 2$$

Thus, $P(4, 2)$ is the nearest point.

Maxima and Minima 18.5 Q31

Let $P(x, y)$ be a point on the curve $x^2 = 2y$ which is closest to $A(0, 5)$

Let $\quad S$ = square of the length of AP

$\Rightarrow \quad S = x^2 + (y-5)^2 \qquad$ ---(ii)

Using (i),

$$S = 2y + (y-5)^2$$

$$\therefore \quad \frac{dS}{dy} = 2 + 2(y-5)$$

For maxima and minima,

$$\frac{dS}{dy} = 0$$

$\Rightarrow \quad 2 + 2y - 10 = 0$

$\Rightarrow \quad y = 4$

Now,

$$\frac{d^2S}{dy^2} = 2 > 0$$

$\therefore \quad y = 4$ is the point of local minima

From (i)

$$r = \pm 2\sqrt{2}$$

Hence, $\left(\pm 2\sqrt{2}, 4\right)$ is the closest point on the curve to $A(0, 5)$.

Maxima and Minima 18.5 Q32

The given equations are

$y = x^2 + 7x + 2 \qquad$ ---(i)

and $\quad y = 3x - 3 \qquad$ ---(ii)

Let $P(x, y)$ be the point on parabola (i) which is closest to the line (ii)

Let S be the perpendicular distance from P to the line (ii).

$$\therefore \quad S = \frac{|y - 3x + 3|}{\sqrt{1^2 + (-3)^2}}$$

$\Rightarrow \quad S = \dfrac{\left|x^2 + 7x + 2 - 3x + 3\right|}{\sqrt{10}} \qquad$ ---(iii)

$$\Rightarrow \quad \frac{dS}{dx} = \frac{2x + 4}{\sqrt{10}}$$

For maxima or minima, we have

$$\frac{dS}{dx} = 0$$

$$\Rightarrow \quad \frac{2x + y}{\sqrt{10}} = 0$$

$\Rightarrow \quad x = -2$

From (i)

$$y = 4 - 14 + 2 = -8$$

Now,

$$\frac{d^2S}{dx^2} = \frac{2}{\sqrt{10}} > 0$$

$\therefore .(x = -2, y = -8)$ is the point of local minima,

Hence,

The closest point on the parabola to the line $y = 3x - 3$ is $(-2, -8)$.

*********** END **********