Trigonometric Ratios Ex 5.1 Q17

Answer :

Given: $\sin\theta = \dfrac{12}{13}$ ......(1)

To Find: The value of expression $\dfrac{\sin^2\theta - \cos^2\theta}{2\sin\theta\cos\theta} \times \dfrac{1}{\tan^2\theta}$

Now, we know that

$$\sin\theta = \frac{\text{Perpendicular side opposite to} \angle\theta}{\text{Hypotenuse}} \quad \text{...... (2)}$$

Now when we compare equation (1) and (2)

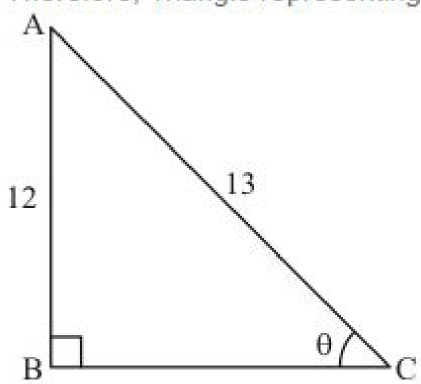
We get,

Perpendicular side opposite to $\angle\theta$ = 12

And

Hypotenuse = 13

Therefore, Triangle representing angle θ is as shown below



Base side BC is unknown and it can be found by using Pythagoras theorem

Therefore by applying Pythagoras theorem

We get,

$$AC^2 = AB^2 + BC^2$$

Therefore by substituting the values of known sides

We get,

$$13^2 = 12^2 + BC^2$$

Therefore,

$$BC^2 = 13^2 - 12^2$$

$$BC^2 = 169 - 144$$

$$BC^2 = 25$$

$$BC = \sqrt{25}$$

Therefore,

$BC = 5$ ...... (3)

Now, we know that

$$\cos\theta = \frac{\text{Base side adjacent to} \angle\theta}{\text{Hypotenuse}}$$

Now from figure (a)

We get,

$$\cos\theta = \frac{BC}{AC}$$

Therefore from figure (*a*) and equation (3) ,

$\cos\theta = \frac{5}{13}$ ...... (4)

Now we know that,

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Therefore, substituting the value of $\sin\theta$ and $\cos\theta$ from equation (1) and (4)

We get,

$$\tan\theta = \frac{\frac{12}{13}}{\frac{5}{13}}$$

$$\tan\theta = \frac{12}{13} \times \frac{13}{5}$$

Therefore 13 gets cancelled and we get

$\tan\theta = \frac{12}{5}$ ...... (5)

Now we substitute the value of $\sin\theta$, $\cos\theta$ and $\tan\theta$ from equation (1) , (4) and (5) respectively in the expression below

$$\frac{\sin^2\theta - \cos^2\theta}{2\sin\theta\cos\theta} \times \frac{1}{\tan^2\theta}$$

Therefore,

We get,

$$\frac{\sin^2\theta - \cos^2\theta}{2\sin\theta\cos\theta} \times \frac{1}{\tan^2\theta} = \frac{\left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2}{2 \times \left(\frac{12}{13}\right) \times \left(\frac{5}{13}\right)} \times \frac{1}{\left(\frac{12}{5}\right)^2}$$

Therefore by further simplifying we get,

$$\frac{\sin^2\theta-\cos^2\theta}{2\sin\theta\cos\theta}\times\frac{1}{\tan^2\theta}=\frac{\frac{(12)^2}{(13)^2}-\frac{(5)^2}{(13)^2}}{2\times\left(\frac{12}{13}\right)\times\left(\frac{5}{13}\right)}\times\frac{1}{\frac{(12)^2}{(5)^2}}$$

$$=\frac{\frac{144}{169}-\frac{25}{169}}{\frac{2\times12\times5}{13\times13}}\times\frac{25}{144}$$

$$=\frac{\frac{144-25}{169}}{\frac{120}{169}}\times\frac{25}{144}$$

$$=\frac{119}{169}\times\frac{169}{120}\times\frac{25}{144}$$

Now 169 gets cancelled and $\frac{25}{120}$ gets reduced to $\frac{5}{24}$

Therefore

$$\frac{\sin^2\theta-\cos^2\theta}{2\sin\theta\cos\theta}\times\frac{1}{\tan^2\theta}=\frac{119}{1}\times\frac{1}{24}\times\frac{5}{144}$$

$$=\frac{119\times5}{24\times144}$$

$$=\frac{595}{3456}$$

Therefore the value of $\frac{\sin^2\theta-\cos^2\theta}{2\sin\theta\cos\theta}\times\frac{1}{\tan^2\theta}$ is $\frac{595}{3456}$

That is $\frac{\sin^2\theta-\cos^2\theta}{2\sin\theta\cos\theta}\times\frac{1}{\tan^2\theta}=\frac{595}{3456}$

*********** END **********