Tangents and Normals Ex 16.2 Q3(vii)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad \text{(A)} \qquad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{(B)} \qquad \text{Normal}$$

Where m is the slope

We have,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{and} \quad P = (a\cos\theta, b\sin\theta)$$

Differentiating with respect to x, we get

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{xb^2}{ya^2}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{-a\cos\theta b^2}{b\sin\theta a^2}$$

$$= \frac{-b}{a}\cot\theta$$

From (A)

Equation of tangent is,

$$(y - b\sin\theta) = \frac{-b}{a}\cot\theta(x - a\cos\theta)$$

$$\Rightarrow \quad \frac{b}{a}x\cot\theta + y = b\sin\theta + b\cot\theta \times \cos\theta$$

$$\Rightarrow \quad \frac{x}{a}\cot\theta + \frac{y}{b} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta}$$

$$\Rightarrow \quad \frac{x}{a}\cot\theta + \frac{y}{b} = \frac{1}{\sin\theta}$$

$$\Rightarrow \quad \frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$$

From (B)

Equation of normal is

$$(y - b\sin\theta) = \frac{a}{b}\frac{\sin\theta}{\cos\theta}(x - a\cos\theta)$$

$$\Rightarrow \quad \frac{a}{b}x\tan\theta - y = \frac{a^2}{b}\sin\theta - b\sin\theta$$

$$\Rightarrow \quad \frac{a}{b}x\tan\theta - y = \frac{a^2 - b^2}{b}\sin\theta$$

$$\Rightarrow \quad \frac{a}{b}x\sec\theta - y\,\mathrm{cosec}\theta = \frac{a^2 - b^2}{b}$$

$$\Rightarrow \quad ax\sec\theta - by\,\mathrm{cosec}\theta = a^2 - b^2$$

Tangents and Normals Ex 16.2 Q3(viii)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad \text{(A)} \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{(B)} \quad \text{Normal}$$

Where m is the slope

We have,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad P = (a\sec\theta, b\tan\theta)$$

Differentiating with respect to x, we get

$$\frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{xb^2}{ya^2}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{a\sec\theta b^2}{b\tan\theta a^2}$$

$$= \frac{b}{a\sin\theta}$$

From (A)

Equation of tangent is,

$$(y - b\tan\theta) = \frac{b}{a\sin\theta}(x - a\sec\theta)$$

$$\Rightarrow \quad \frac{b}{a}\frac{x}{\sin\theta} - y = \frac{b\sec\theta}{\sin\theta} - b\tan\theta$$

$$\Rightarrow \quad \frac{bx}{a\sin\theta} - y = \frac{b\sec\theta}{\sin\theta}\left(1 - \sin^2\theta\right)$$

$$\Rightarrow \quad \frac{x}{a} - \frac{y}{b}\sin\theta = \cos\theta$$

$$\Rightarrow \quad \frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$$

From (B)

Equation of normal is

$$y - b\tan\theta = \frac{-a\sin\theta}{b}(x - a\sec\theta)$$

$$\Rightarrow \quad ax\sin\theta + by = b^2\tan\theta + a^2\tan\theta$$

$$\Rightarrow \quad ax\cos\theta + by\cot\theta = a^2 + b^2$$

Tangents and Normals Ex 16.2 Q3(ix)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad \text{(A)} \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{(B)} \quad \text{Normal}$$

Where m is the slope

We have,

$$y^2 = 4ax \qquad p\left(\frac{a}{m^2}, \frac{2a}{m}\right)$$

Differentiating with respect to x, we get

$$2y\frac{dy}{dx} = 4a$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2a}{y}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_p = m$$

From (A)

Equation of tangent is

$$\left(y - \frac{2a}{m}\right) = m\left(x - \frac{a}{m^2}\right)$$

$$\Rightarrow \quad m^2x - my = 2a - a$$

$$\Rightarrow \quad m^2x - my = a$$

From (B)

Equation of normal is

$$\left(y - \frac{2a}{m}\right) = \frac{-1}{m}\left(x - \frac{a}{m^2}\right)$$

$$\Rightarrow \quad (my - 2a) = \frac{-m^2x + a}{m^2}$$

$$\Rightarrow \quad m^2x + m^3y = 2am^2 + a$$

$$\Rightarrow \quad m^2x + m^3y - 2am^2 - a = 0$$

Tangents and Normals Ex 16.2 Q3(x)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad \text{(A)} \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{(B)} \quad \text{Normal}$$

Where m is the slope

We have,

$$c^2\left(x^2 + y^2\right) = x^2y^2 \qquad P = \left(\frac{c}{\cos\theta}, \frac{c}{\sin\theta}\right)$$

Differentiating with respect to x, we get

$$c^2\left(2x + 2y\frac{dy}{dx}\right) = 2xy^2 + 2x^2y\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx}\left(2yc^2 - 2x^2y\right) = 2xy^2 - 2xc^2$$

$$\therefore \quad \frac{dy}{dx} = \frac{x\left(y^2 - c^2\right)}{y\left(c^2 - x^2\right)}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{\frac{c}{\cos\theta}\left(\frac{c^2}{\sin^2\theta} - c^2\right)}{\frac{c}{\sin\theta}\left(c^2 - \frac{c^2}{\cos^2\theta}\right)}$$

$$= \frac{c^2\tan\theta\left(1 - \sin^2\theta\right)}{c^2\tan^2\theta\left(\cos^2\theta - 1\right)}$$

$$= \frac{1}{-\tan\theta} \times \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \frac{-\cos^3\theta}{\sin^3\theta}$$

From (A)

Equation of tangent is

$$\left(y - \frac{c}{\sin\theta}\right) = \frac{-\cos^3\theta}{\sin^3\theta}\left(x - \frac{c}{\cos\theta}\right)$$

$$\Rightarrow \quad x\cos^3\theta + y\sin^3\theta = c\sin^2\theta + c\cos^2\theta$$

$$\Rightarrow \quad x\cos^3\theta + y\sin^3\theta = c$$

From (B)

Equation of normal is

$$\left(y-\frac{c}{\sin\theta}\right)=\frac{\sin^3\theta}{\cos^3\theta}\left(x-\frac{c}{\cos\theta}\right)$$

$$\Rightarrow \quad x\sin^3\theta-y\cos^3\theta=\frac{c\sin^3\theta}{\cos\theta}-\frac{c\cos^3\theta}{\sin\theta}$$

$$\Rightarrow \quad x\sin^3\theta-y\cos^3\theta=\frac{c\left(\sin^4\theta-\cos^4\theta\right)}{\cos\theta\times\sin\theta}$$

$$=\frac{c\left(\sin^2\theta-\cos^2\theta\right)\left(\sin^2\theta+\cos^2\theta\right)}{\frac{1}{2}\sin2\theta}$$

$$=\frac{-2c\cos2\theta}{\sin2\theta}=-2c\cot2\theta$$

$$\therefore \quad x\sin^3\theta-y\cos^3\theta+2c\cot2\theta=0$$

*********** END **********