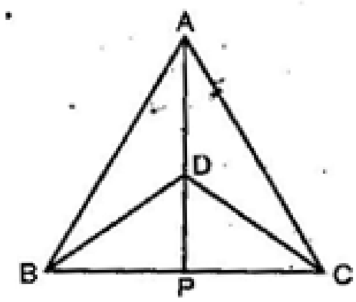
Q1. ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (See figure). If AD is extended to intersect BC at P, show that:



(i) $\Delta ABD \cong \Delta ACD$

(ii) $\Delta ABP \cong \Delta ACP$

(iii) AP bisects $\angle$A as well as $\angle$D.

(iv) AP is the perpendicular bisector of BC.

Ans. (i) ΔABC is an isosceles triangle.

$\therefore$ AB = AC

ΔDBC is an isosceles triangle.

$\therefore$ BD = CD

Now in ΔABD and ΔACD,

AB = AC [Given]

BD = CD [Given]

AD = AD [Common]

$\therefore$ $\Delta ABD \cong \Delta ACD$ [By SSS congruency]

$\Rightarrow$ $\angle BAD = \angle CAD$ [By C.P.C.T.] ....(i)

(ii) Now in ΔABP and ΔACP,

AB = AC [Given]

$\angle BAD = \angle CAD$ [From eq. (i)]

AP = AP

$\therefore \Delta ABP \cong \Delta ACP$ [By SAS congruency]

(iii) Since $\Delta ABP \cong \Delta ACP$ [From part (ii)]

$\Rightarrow \angle BAP = \angle CAP$ [By C.P.C.T.]

$\Rightarrow$ AP bisects $\angle A$.

Since $\Delta ABD \cong \Delta ACD$ [From part (i)]

$\Rightarrow \angle ADB = \angle ADC$ [By C.P.C.T.] ....(ii)

Now $\angle ADB + \angle BDP = 180^\circ$ [Linear pair] ....(iii)

And $\angle ADC + \angle CDP = 180^\circ$ [Linear pair] ....(iv)

From eq. (iii) and (iv),

$\angle ADB + \angle BDP = \angle ADC + \angle CDP$

$\Rightarrow \angle ADB + \angle BDP = \angle ADB + \angle CDP$ [Using (ii)]

$\Rightarrow \angle BDP = \angle CDP$

$\Rightarrow$ DP bisects $\angle D$ or AP bisects $\angle D$.

(iv) Since $\Delta ABP \cong \Delta ACP$ [From part (ii)]

$\therefore$ BP = PC [By C.P.C.T.] ....(v)

And $\angle APB = \angle APC$ [By C.P.C.T.] ....(vi)

Now $\angle APB + \angle APC = 180^\circ$ [Linear pair]

$\Rightarrow \angle APB + \angle APC = 180^\circ$ [Using eq. (vi)]

$\Rightarrow 2\angle APB = 180^\circ$

$\Rightarrow \angle APB = 90^\circ$

$\Rightarrow AP \perp BC$ ....(vii)

From eq. (v), we have BP PC and from (vii), we have proved AP $\perp$ B. So, collectively AP is perpendicular bisector of BC.

Q2. AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that:

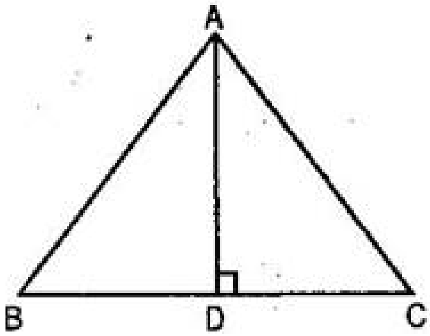
(i) AD bisects BC.

(ii) AD bisects $\angle$A.

Ans. In $\triangle$ABD and $\triangle$ACD,

AB = AC [Given]

$\angle$ ADB = $\angle$ ADC = 90° [AD $\perp$ BC]



AD = AD [Common]

$\therefore$ $\triangle$ABD $\cong$ $\triangle$ ACD [RHS rule of congruency]

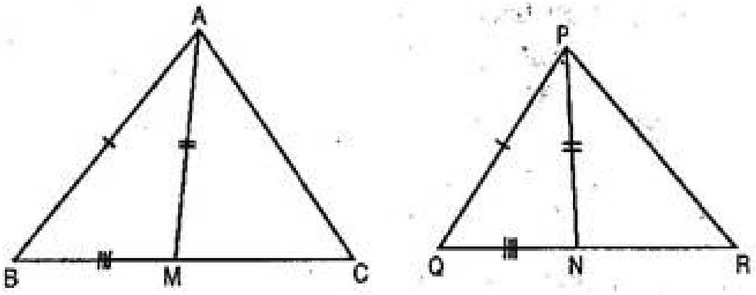
$\Rightarrow$ BD = DC [By C.P.C.T.]

$\Rightarrow$ AD bisects BC

Also $\angle$ BAD = $\angle$ CAD [By C.P.C.T.]

$\Rightarrow$ AD bisects $\angle$A.

Q3. Two sides AB and BC and median AM of the triangle ABC are respectively equal to side PQ and QR and median PN of Δ PQR (See figure). Show that:



(i) $\Delta ABM \cong \Delta PQN$

(ii) $\Delta ABC \cong \Delta PQR$

Ans. AM is the median of Δ ABC.

$\therefore$ $BM = MC = \frac{1}{2} BC$ ....(i)

PN is the median of Δ PQR.

$\therefore$ $QN = NR = \frac{1}{2} QR$ ....(ii)

Now BC = QR [Given] $\Rightarrow \frac{1}{2} BC = \frac{1}{2} QR$

$\therefore$ BM = QN ....(iii)

(i) Now in Δ ABM and Δ PQN,

AB = PQ [Given]

AM = PN [Given]

BM = QN [From eq. (iii)]

$\therefore$ $\Delta ABM \cong \Delta PQN$ [By SSS congruency]

$\Rightarrow \angle B = \angle Q$ [By C.P.C.T.] ....(iv)

(ii) In ΔABC and ΔPQR,

AB = PQ [Given]

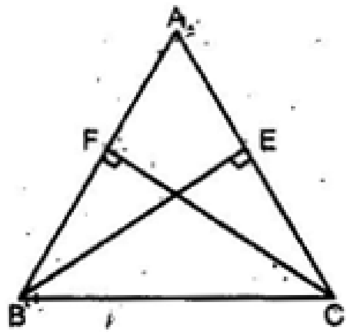
$\angle$B = $\angle$Q [Prove above]

BC = QR [Given]

$\therefore$ ΔABC $\cong$ ΔPQR [By SAS congruency]

Q4. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Ans. In ΔBEC and ΔCFB,



$\angle$BEC = $\angle$CFB [Each 90°]

BC = BC [Common]

BE = CF [Given]

$\therefore$ ΔBEC $\cong$ ΔCFB [RHS congruency]

$\Rightarrow$ EC = FB [By C.P.C.T.] .....(i)

Now In ΔAEB and ΔAFC

$\angle$AEB = $\angle$AFC [Each 90°]

$\angle$A = $\angle$A [Common]

BE = CF [Given]

$\therefore$ ΔAEB $\cong$ ΔAFC [ASA congruency]

$\Rightarrow$ AE = AF [By C.P.C.T.] ....(ii)

Adding eq. (i) and (ii), we get,

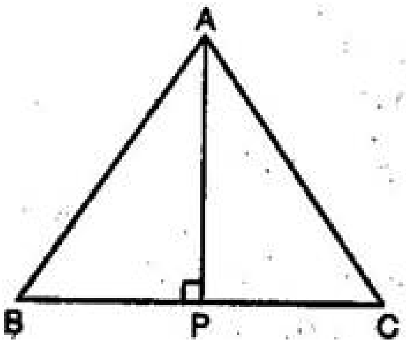
EC + AE = FB + AF

$\Rightarrow$ AB = AC

$\Rightarrow$ ABC is an isosceles triangle.

Q5. ABC is an isosceles triangles with AB = AC. Draw AP $\perp$ BC and show that $\angle$B = $\angle$C.

Ans. Given: ABC is an isosceles triangle in which AB = AC



To prove: $\angle$B = $\angle$C

Construction: Draw AP $\perp$ BC

Proof: In ΔABP and ΔACP

$\angle$APB = $\angle$APC = 90° [By construction]

AB = AC [Given]

AP = AP [Common]

$\therefore$ ΔABP $\cong$ ΔACP [RHS congruency]

$\Rightarrow$ $\angle$B = $\angle$C [By C.P.C.T.]

*********** END **********