Differentiation Ex 11.3 Q35

Let $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Put, $x = \tan\theta$

$y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) + \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$y = \sin^{-1}(\sin 2\theta) + \cos^{-1}(\cos 2\theta)$ ---(i)

Here, $0 < x < 1$

$\Rightarrow$ $0 < \tan\theta < 1$

$\Rightarrow$ $0 < \theta < \frac{\pi}{4}$

$\Rightarrow$ $0 < (2\theta) < \frac{\pi}{2}$

So, from eqauation (i),

$y = 2\theta + 2\theta$

$\left[\begin{array}{l} \text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ \cos^{-1}(\cos\theta) = \theta, \text{ if } \theta \in [0, \pi] \end{array}\right]$

$y = 4\theta$

$y = 4\tan^{-1} x$ $[\text{Since, } x = \tan\theta]$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{4}{1+x^2}.$$

Differentiation Ex 11.3 Q36

Here, $y = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) + \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

Put $x = \tan\theta$

$$y = \sin^{-1}\left(\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}\right) + \cos^{-1}\left(\frac{1}{\sqrt{1+\tan^2\theta}}\right)$$

$$= \sin^{-1}\left(\frac{\frac{\sin\theta}{\cos\theta}}{\sec\theta}\right) + \cos^{-1}\left(\frac{1}{\sec\theta}\right)$$

$$= \sin^{-1}\left(\frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}\right) + \cos^{-1}(\cos\theta)$$

$$y = \sin^{-1}(\sin\theta) + \cos^{-1}(\cos\theta) \qquad \text{---(i)}$$

Here, $0 < x < \infty$

$\Rightarrow \quad 0 < \tan\theta < \infty$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{2}$

So, from eqation (i),

$$y = \theta + \theta \qquad \left[\begin{array}{l}\text{Since, } \sin^{-1}(\sin\theta)\theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ \text{and } \cos^{-1}(\cos\theta) = \theta, \text{ if } \theta \in [0, \pi]\end{array}\right]$$

$$= 2\theta$$

$$y = 2\tan^{-1} x \qquad [\text{Since, } x = \tan\theta]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{2}{1+x^2}.$$

Differentiation Ex 11.3 Q37

Let $f(x) = \cos^{-1}(\sin x)$

We observe that this function is defined for all real numbers.

$$f(x) = \cos^{-1}(\sin x)$$

$$= \cos^{-1}\left[\cos\left(\frac{\pi}{2} - x\right)\right] = \frac{\pi}{2} - x$$

Thus, $f'(x) = \frac{d}{dx}\left(\frac{\pi}{2} - x\right) = -1$

Let $\quad y = \cot^{-1}\left(\frac{1-x}{1+x}\right)$

Put $\quad x = \tan\theta$, so,

$$y = \cot^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$

$$= \cot^{-1}\left(\frac{\tan\frac{\pi}{4} - \tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}\right)$$

$$= \cot^{-1}\left[\tan\left(\frac{\pi}{4} - \theta\right)\right]$$

$$= \cot^{-1}\left[\cot\left(\frac{\pi}{2} - \frac{\pi}{4} + \theta\right)\right]$$

$$= \frac{\pi}{4} + \theta$$

$$y = \frac{\pi}{4} + \tan^{-1} x \qquad \left[\text{Since } x = \tan\theta\right]$$

Differentiating it with respect do x,

$$\frac{dy}{dx} = 0 + \frac{1}{1+x^2}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}.$$

*********** END **********