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Question 9. 1. A steel wire of length 4.7 m and cross-sectional area 3.0×10^{-5} m^2 stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area of 4.0×10^{-5} m^2 under a given load. What is the ratio of the Young's modulus of steel to that of copper?

Answer:

For steel

$$l_1 = 4.7\text{m}, \quad A_1 = 3.0 \times 10^{-5} \text{ m}^2$$

If F newton is the stretching force and Δl metre the extension in each case, then

$$Y_1 = \frac{Fl_1}{A_1 \Delta l}$$

$$\Rightarrow \quad Y_1 = \frac{F \times 4.7}{3.0 \times 10^{-5} \times \Delta l} \quad ...(i)$$

For copper

$$l_2 = 3.5\text{m}, \quad A_2 = 4.0 \times 10^{-5} \text{ m}^2$$

Now,

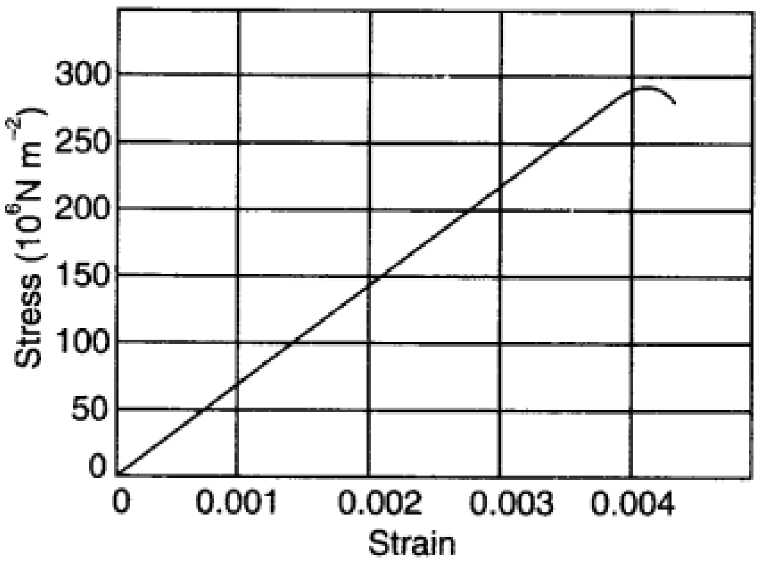
$$Y_2 = \frac{F \times 3.5}{4.0 \times 10^{-5} \times \Delta l} \quad ...(ii)$$

Dividing (i) by (ii), we get

$$\frac{Y_1}{Y_2} = \frac{4.7}{3.0 \times 10^{-5}} \times \frac{4.0 \times 10^{-5}}{3.5} = \frac{4.7 \times 4.0}{3.0 \times 3.5} = 1.79.$$

Question 9. 2. Figure shows the strain-stress curve for a given material. What are (a) Young's modulus and (b) approximate yield strength for this material?

Answer:



(a) Young's modulus of the material (Y) is given by

Y = Stress/Strain

= $150 \times 10^6/0.002$

$150 \times 10^6/2 \times 10$-3

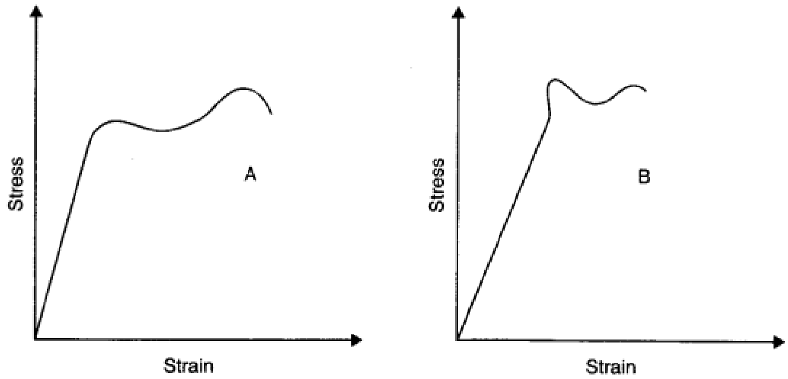
= 75×10^9 Nm^{-2}

= 75×10^{10} Nm^{-2}

(a) Yield strength of a material is defined as the maximum stress it

can sustain. From graph, the approximate yield strength of the given material
$= 300 \times 10^6\ Nm^{-2}$
$= 3 \times 10^8\ Nm^{-2}$.

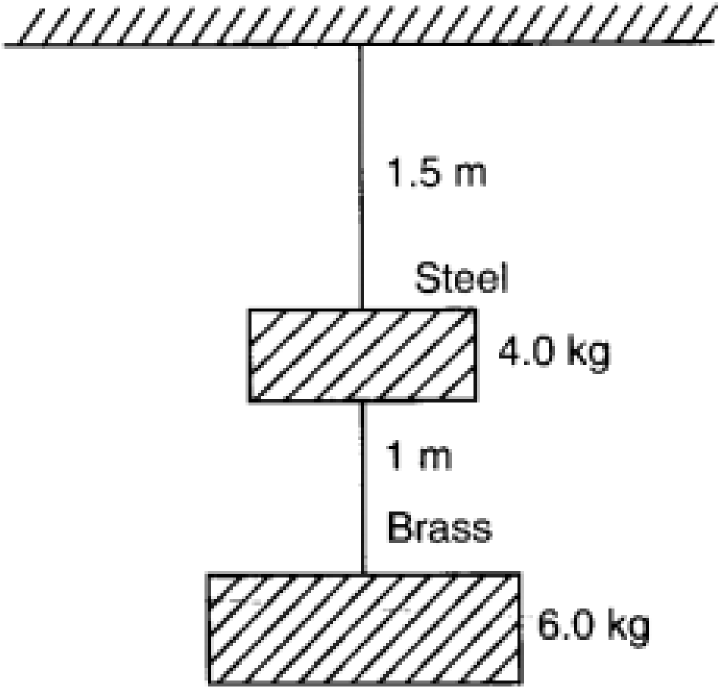
Question 9. 3. The stress-strain graphs for materials A and B are shown in figure.



The graphs are drawn to the same scale.
(a) Which of the materials has the greater Young's modulus?
(b) Which of the two is the stronger material?
Answer:
(a) From the two graphs we note that for a given strain, stress for A is more than that of B. Hence Young's modulus = (Stress /Strain) is greater for A than that of B.
(b) Strength of a material is determined by the amount of stress required to cause fracture. This stress corresponds to the point of fracture. The stress corresponding to the point of fracture in A is more than for B. So, material A is stronger than material B.

Question 9. 4. Read the 'allowing two statements below carefully and state, with reasons, if it is true or false.
(a) The Young's modulus of rubber is greater than that of steel;
(b) The stretching of a coil is determined by its shear modulus.
Answer:
(a) False. The-Young's modulus is defined as the ratio of stress to the strain within elastic limit. For a given stretching force elongation is more in rubber and quite less in steel. Hence, rubber is less elastic than steel.
(b) True. Stretching of a coil is determined by its shear modulus. When equal and opposite forces are applied at opposite ends of a coil, the distance as well as shape of helicals of the coil change and it involves shear modulus.

Question 9. 5. Two wires of diameter 0.25 cm, one made of steel and other made of brass are loaded as shown in figure. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m.Young's modulus of steel is 2.0×10^{11} Pa. Compute the elongations of steel and brass wires. ($1\ Pa = 1\ N\ m^2$).



Answer:

For steel wire; total force on steel wire;

$$F_1 = 4 + 6 = 10 \text{ kg} f = 10 \times 9.8 \, N;$$

$$l_1 = 1.5 \text{ m}, \quad \Delta l_1 = ?; \quad 2r_1 = 0.25$$

cm

or $$r_1 = \left(\frac{0.25}{2}\right) \text{cm} = 0.125 \times 10^{-2} \text{ m}$$

$$Y_1 = 2.0 \times 10^{11} \text{ Pa}$$

For brass wire, $$F_2 = 6.0 \text{ kg} f = 6 \times 9.8 \text{ N};$$

$$2r_2 = 0.25 \text{ cm}$$

or $$r_2 = \left(\frac{0.25}{2}\right) = 0.125 \times 10^{-2} \text{ m};$$

$$Y_2 = 0.91 \times 10^{11} \text{ Pa}, \quad l_2 = 1.0 \text{ m}, \quad \Delta l_2 = ?$$

Since, $$Y_1 = \frac{F_1 \times l_1}{A_1 \times \Delta l_1} = \frac{F_1 \times l_1}{\pi r_1^2 \times \Delta l_1} \Rightarrow \Delta l_1 = \frac{F_1 \times l_1}{\pi r_1^2 \times \Delta l_1}$$

or $$\Delta l_1 = \frac{(10 \times 9.8) \times 1.5 \times 7}{22 \times (0.125 \times 10^{-2})^2 \times 2 \times 10^{11}} = 1.49 \times 10^{-4} \text{ m.}$$

And $$\Delta l_2 = \frac{F_2 \times l_2}{\pi r_2^2 \times Y_2} = \frac{(6 \times 9.8) \times 1 \times 7}{22 \times (0.125 \times 10^{-2})^2 \times (0.91 \times 10^{11})} = 1.3 \times 10^{-4} \text{ m.}$$

Question 9. 6. The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa. What is the vertical deflection of this face?

Answer: Here, side of cube, L = 10 cm =10/100= 0.1 m

∴ Area of each face, A = $(0.1)^2$ = 0.01 m^2

Tangential force acting on the face,

$F = 100 \text{ kg} = 100 \times 9.8 = 980 \text{ N}$

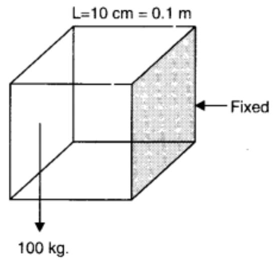
Shear modulus, $\eta = 25 \text{ GPa} = 25 \times 10^9 \text{ Nm}^{-2}$

Since shear modulus is given as:

$$\eta = \frac{\text{Tangential stress}}{\text{Shearing strain}}$$

$$\therefore \text{Shearing strain} = \frac{\text{Tangential stress}}{\text{Shear modulus}}$$

$$= \frac{F}{A\eta} = \frac{980}{0.01 \times 25 \times 10^9} = 3.92 \times 10^{-6}$$



Now, $\frac{\text{Lateral Strain}}{\text{Side of cube}} = \text{Shearing strain}$

$\therefore$ Lateral Strain = Shearing strain × Side of the cube

$= 3.92 \times 10^{-6} \times 0.1$

$= 3.92 \times 10^{-7} \text{ m} \approx 4 \times 10^{-7} \text{ m}.$

Question 9. 7. Four identical hollow cylindrical columns of mild steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 cm and 60 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column. Young's modulus, Y = 2.0 x 10^{11} Pa.
Answer:

Here total mass to be supported, M = 50,000 kg

$\therefore$ Total weight of the structure to be supported = Mg

$= 50,000 \times 9.8 \text{ N}$

Since this weight is to be supported by 4 columns,

$\therefore$ Compressional force on each column (F) is given by

$$F = \frac{Mg}{4} = \frac{50,000 \times 9.8}{4} \text{ N}$$

Inner radius of a column, $r_1 = 30 \text{ cm} = 0.3 \text{ m}$

Outer radius of a column, $r_2 = 60 \text{ cm} = 0.6 \text{ m}.$

$\therefore$ Area of cross-section of each column is given by

$$A = \pi(r_2^2 - r_1^2)$$

$$= \pi[(0.6)^2 - (0.3)^2] = 0.27\pi \text{ m}^2$$

Young's modulus, $Y = 2 \times 10^{11}$ Pa

Compressional strain of each column =?

$$\therefore \quad Y = \frac{\text{Compressional force / area}}{\text{Compressional Strain}}$$

$$= \frac{F/A}{\text{Compressional Strain}}$$

or Compressional strain of each column

$$= \frac{F}{AY} = \frac{50,000 \times 9.8 \times 7}{4 \times 0.27 \times 22 \times 2 \times 10^{11}}$$

$$= 0.722 \times 10^{-6}$$

$\therefore$ Compressional strain of all columns is given by

$$= 0.722 \times 10^{-6} \times 4 = 2.88 \times 10^{-6}$$

$$= 2.88 \times 10^{-6}$$

Question 9. 8. A piece of copper having a rectangular cross-section of 15.2 mm x 19.1 mm is pulled in tension with 44,500 N force, producing only elastic deformation. Calculate the resulting strain? Shear modulus of elasticity of copper is 42 x 10^9 N/m^2.
Answer:

Here, $A = 15.2 \times 19.2 \times 10^{-6}\ m^2$; $F = 44500$ N; $\eta = 42 \times 10^9\ Nm^{-2}$

$$\text{Strain} = \frac{\text{Stress}}{\text{modulus of elasticity}} = \frac{F/A}{\eta}$$

$$= \frac{F}{A\eta} = \frac{44500}{(15.2 \times 19.2 \times 10^{-6}) \times 42 \times 10^9} = 3.65 \times 10^{-3}.$$

Question 9. 9. A steel cable with a radius of 1.5 cm supports a chairlift at a ski area. If the maximum stress is not to exceed 10^8 Nm^{-2} what is the maximum load the cable can support ?

Answer:

$$\text{Maximum load} = \text{Maximum stress} \times \text{Cross-sectional area}$$

$$= 10^8\ Nm^{-2} \times \frac{22}{7} \times (1.5 \times 10^{-2}\ m)^2$$

$$= 7.07 \times 10^4\ N.$$

Question 9. 10. A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension.

Answer: Since each wire is to have same tension therefore, each wire has same extension. Moreover, each wire has the same initial length.So, strain is same for each wire.

Now, $$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/(\pi D^2/4)}{\text{Strain}}$$

or $$Y \propto \frac{1}{D^2} \Rightarrow D \propto \frac{1}{\sqrt{Y}}$$

$$\frac{D_{copper}}{D_{iron}} = \sqrt{\frac{Y_{iron}}{Y_{copper}}} = \sqrt{\frac{190 \times 10^9}{110 \times 10^9}} = \sqrt{\frac{19}{11}} = 1.314$$

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