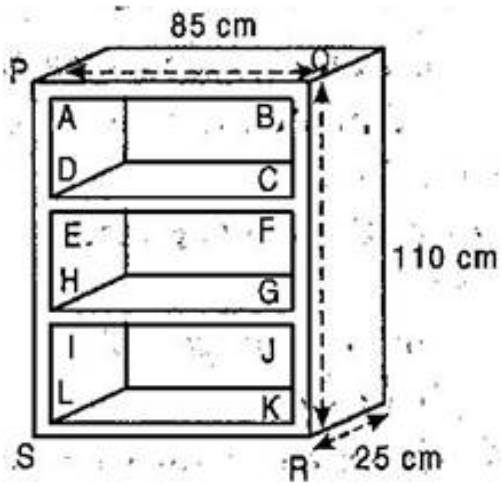
Surface Areas and Volumes Ex 13.9

Q1. A wooden bookshelf has external dimensions as follows: Height = 110 cm, Depth = 25 cm, Breadth = 85 cm [See fig.]. The thickness of the planks is 5 cm everywhere. The external faces are to be polished and the inner faces are to be painted. If the rate of polishing is 20 paise per cm^2 and the rate of painting is 10 paise per cm2, find the total expenses required for polishing and painting the surface of the bookshelf.



Ans: External faces to be polished

= Area of six faces of cuboidal bookshelf – 3 (Area of open portion ABCD)

= 2 (110 x 25 + 25 x 85 + 85 x 110) – 3 (75 x 30)

[∵ AB = 85 – 5 – 5 = 75 cm and AD

$= \frac{1}{3}$ x 110 – 5 – 5 – 5 – 5 = 30 cm]

= 2 (2750 + 2125 + 9350) – 3 x 2250

= 2 x 14225 – 6750

= 28450 – 6750

= 21700 cm^2

Now, cost of painting outer faces of wodden bookshelf at the rate of 20 paise.

= Rs. 0.20 per cm^2 = Rs. 0.20 x 21700 = Rs. 4340

Here, three equal five sides inner faces.

Therefore total surface area

= 3 [2 (30 + 75) 20 + 30 x 75] [∵ Depth = 25 – 5 = 20 cm]

= 3 [2 x 105 x 20 + 2250] = 3

[4200 + 2250]

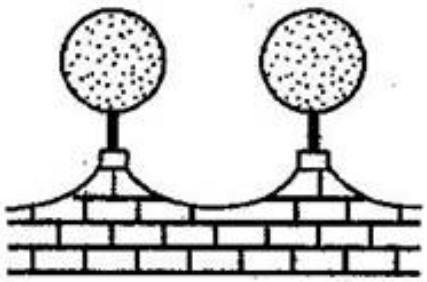
= 3 x 6450 = 19350 cm^2

Now, cost of painting inner faces at the rate of 10 paise i.e. Rs. 0.10 per cm^2.

= Rs. 0.10 x 19350 = Rs. 1935

Total expenses required = Rs. 4340 + Rs. 1935 = Rs. 6275

Q2. The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed on small supports as shown in figure. Eight such spheres are used for this purpose and are to be painted silver. Each support is a cylinder of radius 1.5 cm and height 7 cm and is to be painted black. Find he cost of paint required if silver paint costs 25 paise per cm^2 and black paint costs 5 paise per cm^2.



Ans: Diameter of a wooden sphere

= 21 cm.

$\therefore$ Radius of wooden sphere (R)

$= \frac{21}{2}$ cm

And Radius of the cylinder (r)

= 1.5 cm

Surface area of silver painted part

= Surface area of sphere – Upper part of cylinder for support

$= 4\pi R^2 - \pi r^2$

$= \pi\left(4R^2 - r^2\right)$

$= \frac{22}{7} \times \left[4 \times \left(\frac{21}{2}\right)^2 - \left(\frac{15}{10}\right)^2\right]$

$$= \frac{22}{7} \times \left[\frac{4 \times 441}{4} - \frac{9}{4}\right]$$

$$= \frac{22}{7}\left[\frac{1764 - 9}{4}\right]$$

$$= \frac{22}{7} \times \frac{1755}{4} = 1378.928 \text{ cm}^2$$

Surface area of such type of 8 spherical part = 8 x 1378.928

$= 11031.424 \text{ cm}^2$

$\because$ Cost of silver paint over 1 cm^2

= Rs. 0.25

$\therefore$ Cost of silver paint over 11031.928 cm^2 = 0.25 x 11031.928

= Rs. 2757.85

Now, curved surface area of a cylindrical support

$= 2\pi rh$

$$= 2 \times \frac{22}{7} \times \frac{15}{10} \times 7 = 66 \text{ cm}^2$$

Curved surface area of 8 such cylindrical supports = 66 x 8 = 528 cm^2

$\because$ Cost of black paint over 1 cm^2 of cylindrical support = Rs. 0.50

$\therefore$ Cost of black paint over 528 cm^2 of cylindrical support = 0.50 x 528

= Rs. 26.40

$\therefore$ Total cost of paint required

= Rs. 2757.85 + Rs. 26.4

= Rs. 2784.25

Q3. If diameter of a sphere is decreased by 25% then what percent does its curved surface area decrease?

Ans: Diameter of original sphere

= D = 2R

$$\Rightarrow R = \frac{D}{2}$$

Curved surface area of original sphere = $4\pi R^2 = 4\pi\left(\frac{D}{2}\right)^2 = \pi D^2$

According to the question,

Decreased diameter = 25% of D

$= \frac{25}{100}D = \frac{D}{4}$

$\therefore$ Diameter of new sphere = $D - \frac{D}{4} = \frac{3D}{4}$

$\therefore$ Radius of new sphere = $\frac{3D}{8}$

Now, curved surface area of new sphere = $4\pi r^2 = 4\pi\left(\frac{3D}{8}\right)^2 = \frac{9\pi}{16}D^2$

Change in curved surface area

$= \pi D^2 - \frac{9\pi}{16}D^2$

$= \frac{7}{16}\pi D^2$

Percent change in the curved surface area

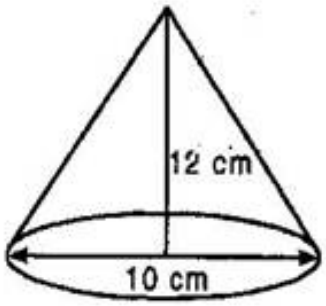
$$\frac{\text{Change in curved surface area}}{\text{Curved surface area of origianal sphere}} \text{ x } 100$$

$= \frac{\frac{7}{16}\pi D^2}{\pi D^2}$ x 100 = $\frac{7}{16} \times 100 = 43.75\%$

Q4. Sameera wants to celebrate the fifth birthday of her daughter with a party. She bought thick paper to make the conical party caps. Each cap is to have a base diameter of 10 cm and height 12 cm. A sheet of the paper is 25 cm by 40 cm and approximately 82% of the sheet can be effectively used for making the caps after cutting. What is the minimum number of sheets of paper that Sameera would need to buy, if there are to be 15 children at the party?
(Use $\pi = 3.14$)

Ans: Diameter of base of conical cap

= 10 cm



$\therefore$ Radius of conical cap (r) = 5 cm

Slant height of cone $(l) = \sqrt{r^2 + h^2}$

$= \sqrt{(5)^2 + (12)^2}$

$= \sqrt{25+144} = \sqrt{169}$ = 13 cm

Curved surface area of a cap

$= \pi rl$ = 3.14 x 5 x 13 = 204.1 cm^2

Curved surface area of 15 caps

= 15 x 204.1 = 3061.5 cm^2

Area of a sheet of paper used for making caps = 25 x 40 = 1000 cm^2

82% of sheet is used after cutting

= 82% of 1000 cm^2

$= \frac{82}{100} \times 1000$ = 820 cm^2

Number of sheet = $\frac{3061.5}{820}$ = 3.73

Hence 4 sheets area needed .

********** END **********