Q-value of this nuclear reaction is given as:

$Q = (m_1 - m_2 - m_3)\, c^2$

$= (223.01850 - 219.00948 - 4.00260)\, C^2$

$= (0.00642\, c^2)$ u

$= 0.00642 \times 931.5 = 5.98$ MeV

Hence, the Q value of the second nuclear reaction is 5.98 MeV. Since the value is positive, the reaction is energetically allowed.

Question 13.27:

Consider the fission of ${}^{238}_{92}\mathrm{U}$ by fast neutrons. In one fission event, no neutrons are emitted and the final end products, after the beta decay of the primary fragments, are ${}^{140}_{58}\mathrm{Ce}$ and ${}^{99}_{44}\mathrm{Ru}$. Calculate Q for this fission process. The relevant atomic and particle masses are

$m\left({}^{238}_{92}\mathrm{U}\right) = 238.05079$ u

$m\left({}^{140}_{58}\mathrm{Ce}\right) = 139.90543$ u

$m\left({}^{99}_{44}\mathrm{Ru}\right) = 98.90594$ u

Answer

In the fission of ${}^{238}_{92}\mathrm{U}$, 10 β^- particles decay from the parent nucleus. The nuclear reaction can be written as:

$${}^{238}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \longrightarrow {}^{140}_{58}\mathrm{Ce} + {}^{99}_{44}\mathrm{Ru} + 10\, {}^{0}_{-1}e$$

It is given that:

Mass of a nucleus ${}^{238}_{92}\mathrm{U}$, $m_1 = 238.05079$ u

Mass of a nucleus ${}^{140}_{58}\mathrm{Ce}$, $m_2 = 139.90543$ u

Mass of a nucleus ${}^{99}_{44}\mathrm{Ru}$, $m_3 = 98.90594$ u

Mass of a neutron ${}^{1}_{0}\mathrm{n}$, $m_4 = 1.008665$ u

Q-value of the above equation,

$$Q = \left[m'\left({}^{238}_{92}\mathrm{U}\right) + m\left({}^{1}_{0}\mathrm{n}\right) - m'\left({}^{140}_{58}\mathrm{Ce}\right) - m'\left({}^{99}_{44}\mathrm{Ru}\right) - 10m_e\right]c^2$$

Where,

m' = Represents the corresponding atomic masses of the nuclei

$m'\left({}^{238}_{92}\mathrm{U}\right) = m_1 - 92m_e$

$m'\left({}^{140}_{58}\mathrm{Ce}\right) = m_2 - 58m_e$

$m'\left({}^{99}_{44}\mathrm{Ru}\right) = m_3 - 44m_e$

$m\left({}^{1}_{0}\mathrm{n}\right) = m_4$

$$\begin{aligned} Q &= \left[m_1 - 92m_e + m_4 - m_2 + 58m_e - m_3 + 44m_e - 10m_e\right]c^2 \\ &= \left[m_1 + m_4 - m_2 - m_3\right]c^2 \\ &= \left[238.0507 + 1.008665 - 139.90543 - 98.90594\right]c^2 \\ &= \left[0.247995\, c^2\right] \mathrm{u} \end{aligned}$$

$$\text{But } 1\ \mathrm{u} = 931.5\ \mathrm{MeV}/c^2$$

$$\therefore Q = 0.247995 \times 931.5 = 231.007\ \mathrm{MeV}$$

Hence, the Q-value of the fission process is 231.007 MeV.

Question 13.28:

Consider the D–T reaction (deuterium–tritium fusion)

$${}^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \longrightarrow {}^{4}_{2}\mathrm{He} + \mathrm{n}$$

(a) Calculate the energy released in MeV in this reaction from the data:

$m\left({}_1^2\text{H}\right) = 2.014102$ u

$m\left({}_1^3\text{H}\right) = 3.016049$ u

(b) Consider the radius of both deuterium and tritium to be approximately 2.0 fm. What is the kinetic energy needed to overcome the coulomb repulsion between the two nuclei? To what temperature must the gas be heated to initiate the reaction? (Hint: Kinetic energy required for one fusion event =average thermal kinetic energy available with the interacting particles = $2(3kT/2)$; k = Boltzman's constant, T = absolute temperature.)

Answer

(a) Take the D-T nuclear reaction: ${}_1^2\text{H} + {}_1^3\text{H} \longrightarrow {}_2^4\text{He} + \text{n}$

It is given that:

Mass of ${}_1^2\text{H}$, $m_1 = 2.014102$ u

Mass of ${}_1^3\text{H}$, $m_2 = 3.016049$ u

Mass of ${}_2^4\text{He}$ $m_3 = 4.002603$ u

Mass of ${}_0^1\text{n}$. $m_4 = 1.008665$ u

Q-value of the given D-T reaction is:

$Q = [m_1 + m_2 - m_3 - m_4]\, c^2$

$= [2.014102 + 3.016049 - 4.002603 - 1.008665]\, c^2$

$= [0.018883\, c^2]$ u

But 1 u = 931.5 MeV/c^2

$\therefore Q = 0.018883 \times 931.5 = 17.59$ MeV

(b) Radius of deuterium and tritium, $r \approx 2.0$ fm $= 2 \times 10^{-15}$ m

Distance between the two nuclei at the moment when they touch each other, $d = r + r = 4 \times 10^{-15}$ m

Charge on the deuterium nucleus = e

Charge on the tritium nucleus = e

Hence, the repulsive potential energy between the two nuclei is given as:

$$V = \frac{e^2}{4\pi \epsilon_0 (d)}$$

Where,

ϵ_0 = Permittivity of free space

$$\frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\therefore V = \frac{9 \times 10^9 \times \left(1.6 \times 10^{-19}\right)^2}{4 \times 10^{-15}} = 5.76 \times 10^{-14} \text{ J}$$

$$= \frac{5.76 \times 10^{-14}}{1.6 \times 10^{-19}} = 3.6 \times 10^5 \text{ eV} = 360 \text{ keV}$$

Hence, 5.76×10^{-14} J or 360 keV of kinetic energy (KE) is needed to overcome the Coulomb repulsion between the two nuclei.

However, it is given that:

$$\text{KE} = 2 \times \frac{3}{2} kT$$

Where,

k = Boltzmann constant = 1.38×10^{-23} m^2 kg s^{-2} K^{-1}

T = Temperature required for triggering the reaction

$$\therefore T = \frac{\text{KE}}{3K}$$

$$= \frac{5.76 \times 10^{-14}}{3 \times 1.38 \times 10^{-23}} = 1.39 \times 10^9 \text{ K}$$

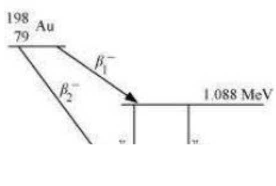
Hence, the gas must be heated to a temperature of 1.39×10^9 K to initiate the reaction.

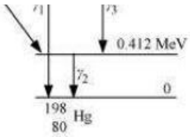
Question 13.29:

Obtain the maximum kinetic energy of β-particles, and the radiation frequencies of γ decays in the decay scheme shown in Fig. 13.6. You are given that

$m\,({}^{198}\text{Au}) = 197.968233$ u

$m\,({}^{198}\text{Hg}) = 197.966760$ u





Answer

It can be observed from the given γ-decay diagram that γ_1 decays from the 1.088 MeV energy level to the 0 MeV energy level.

Hence, the energy corresponding to γ_1-decay is given as:

$E_1 = 1.088 - 0 = 1.088$ MeV

$h\nu_1 = 1.088 \times 1.6 \times 10^{-19} \times 10^{6}$ J

Where,

h = Planck's constant = 6.6×10^{-34} Js

ν_1 = Frequency of radiation radiated by γ_1-decay

$$\therefore \nu_1 = \frac{E_1}{h}$$

$$= \frac{1.088 \times 1.6 \times 10^{-19} \times 10^{6}}{6.6 \times 10^{-34}} = 2.637 \times 10^{20} \text{ Hz}$$

It can be observed from the given γ-decay diagram that γ_2 decays from the 0.412 MeV energy level to the 0 MeV energy level.

Hence, the energy corresponding to γ_2-decay is given as:

$E_2 = 0.412 - 0 = 0.412$ MeV

$h\nu_2 = 0.412 \times 1.6 \times 10^{-19} \times 10^{6}$ J

Where,

ν_2 = Frequency of radiation radiated by γ_2-decay

$$\therefore \nu_2 = \frac{E_2}{h}$$

$$= \frac{0.412 \times 1.6 \times 10^{-19} \times 10^{6}}{6.6 \times 10^{-34}} = 9.988 \times 10^{19} \text{ Hz}$$

It can be observed from the given γ-decay diagram that γ_3 decays from the 1.088 MeV energy level to the 0.412 MeV energy level.

Hence, the energy corresponding to γ_3-decay is given as:

$E_3 = 1.088 - 0.412 = 0.676$ MeV

$h\nu_3 = 0.676 \times 10^{-19} \times 10^{6}$ J

Where,

ν_3 = Frequency of radiation radiated by γ_3-decay

$$\therefore \nu_3 = \frac{E_3}{h}$$

$$= \frac{0.676 \times 1.6 \times 10^{-19} \times 10^{6}}{6.6 \times 10^{-34}} = 1.639 \times 10^{20} \text{ Hz}$$

Mass of $m\left({}^{198}_{78}\text{Au}\right)$ = 197.968233 u

Mass of $m\left({}^{198}_{80}\text{Hg}\right)$ = 197.966760 u

*********** END **********