Differentials Errors and Approximation Ex14.1 Q9(xxii)

Let $x = \frac{16}{81}$, $x + \Delta x = \frac{17}{81}$

$$\Delta x = \frac{17}{81} - \frac{16}{81}$$

$$= \frac{1}{81}$$

Let $y = x^{\frac{1}{4}}$

$$\frac{dy}{dx} = \frac{1}{4x^{\frac{3}{4}}}$$

$$\left(\frac{dy}{dx}\right)_{x=\frac{16}{81}} = \frac{1}{4\left(\frac{16}{81}\right)^{\frac{3}{4}}}$$

$$= \frac{27}{32}$$

$$= 0.84375$$

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=\frac{16}{81}} \times (\Delta x)$$

$$= (0.84375)\left(\frac{1}{81}\right)$$

$$= 0.01041$$

$$\left(\frac{17}{81}\right)^{\frac{1}{4}} = y + \Delta y$$

$$= \left(\frac{16}{81}\right)^{\frac{1}{4}} + 0.01041$$

$$= 0.6666 + 0.01041$$

$$\left(\frac{17}{81}\right)^{\frac{1}{4}} = 0.67707$$

Differentials Errors and Approximation Ex14.1 Q9(xxiii)

Let $x = 32,\ x + \Delta x = 33$

$$\Delta x = 33 - 32$$

$$= 1$$

Let $y = x^{\frac{1}{5}}$

$$\frac{dy}{dx} = \frac{1}{5x^{\frac{4}{5}}}$$

$$\left(\frac{dy}{dx}\right)_{x=32} = \frac{1}{5(32)^{\frac{4}{5}}}$$

$$= \frac{1}{80}$$

$$= 0.0125$$

$\therefore$ $\Delta y = \left(\frac{dy}{dx}\right) \times (\Delta x)$

$$\left(\frac{dx}{dy}\right)_{x=32} = (0.0125)(1)$$

$$\Delta y = 0.0125$$

$$(33)^{\frac{1}{5}} = y + \Delta y$$

$$= x^{\frac{1}{5}} + 0.0125$$

$$= (32)^{\frac{1}{5}} + 0.0125$$

$$(33)^{\frac{1}{5}} = 2.0125$$

Differentials Errors and Approximation Ex14.1 Q9(xxiv)

Let $x = 36,\ x + \Delta x = 36.6$

$\Delta x = 36.6 - 36$

$= 0.6$

Let $y = \sqrt{x}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$\left(\frac{dy}{dx}\right)_{x=36} = \frac{1}{2\sqrt{36}}$

$= \frac{1}{12}$

$= 0.0833$

$\therefore \quad \Delta y = \left(\frac{dy}{dx}\right)_{x=36} \times (\Delta x)$

$= (0.0833)(0.6)$

$= 0.04998$

$\sqrt{36.6} = y + \Delta y$

$= \sqrt{x} + 0.04998$

$= \sqrt{36} + 0.04998$

$\sqrt{36.6} = 6.04998$

Differentials Errors and Approximation Ex14.1 Q9(xxv)

Let $x = 27,\ x + \Delta x = 25$

$\Delta x = 25 - 27$

$= -2$

Let $y = x^{\frac{1}{3}}$

$$\frac{dy}{dx} = \frac{1}{3x^{\frac{2}{3}}}$$

$$\left(\frac{dy}{dx}\right)_{x=27} = \frac{1}{3(27)^{\frac{2}{3}}}$$

$$= \frac{1}{27}$$

$$= 0.037$$

$\therefore$ $\Delta y = \left(\frac{dy}{dx}\right)_{x=27} \times (\Delta x)$

$$= (0.037)(-2)$$

$$= -0.074$$

$$(25)^{\frac{1}{3}} = y + \Delta y$$

$$= x^{\frac{1}{3}} + (-0.074)$$

$$= (27)^{\frac{1}{3}} - 0.074$$

$$= 3 - 0.074$$

$$(25)^{\frac{1}{3}} = 2.926$$

********** END **********