Definite Integrals Ex 20.5 Q1

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----+f(a+(n-1)h)\right], \text{ where } h=\frac{b-a}{n}$$

Here, $a=0,\ b=3$ and $f(x)=(x+4)$

$h=\frac{3}{n} \Rightarrow nh=3$

Thus, we have,

$$\Rightarrow I=\int_0^3 (x+4)\,dx$$

$$\Rightarrow I=\lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f((n-1)h)\right]$$

$$\Rightarrow I=\lim_{h\to 0} h\left[4+(h+4)+(2h+4)+-----+((n-1)h+4)\right]$$

$$\Rightarrow I=\lim_{h\to 0} h\left[4n+h(1+2+3+-----+(n-1))\right]$$

$$\Rightarrow I=\lim_{h\to 0} h\left[4n+h\left(\frac{n(n-1)}{2}\right)\right] \qquad \left[\because h\to 0\ \&\ h=\frac{3}{n}\Rightarrow n\to\infty\right]$$

$$\Rightarrow I=\lim_{n\to\infty}\frac{3}{n}\left[4n+\frac{3}{n}\frac{(n^2-1)}{2}\right]$$

$$\Rightarrow I=\lim_{n\to\infty} 12+\frac{9}{2}\left(1-\frac{1}{n}\right)$$

$$=12+\frac{9}{2}=\frac{33}{2}$$

$$\therefore \int_0^3 (x+4)\,dx=\frac{33}{2}$$

Definite Integrals Ex 20.5 Q2

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a) + f(a+h) + f(a+2h) + ----+f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 0,\ b = 2$

$$\Rightarrow h = \frac{2}{n}\ \&\ f(x) = x + 3$$

Thus, we have,

$$I = \int_0^2 (x+3)\,dx$$

$$\Rightarrow I = \lim_{h\to 0} h\left[f(0) + f(h) + f(2h) + ----f((n-1)h)\right]$$

$$\Rightarrow I = \lim_{h\to 0} h\left[3 + (h+3) + (2h+3) + (3h+3) ----+(n-1)h + 3\right]$$

$$= \lim_{h\to 0} h\left[3n + h(1+2+3+----(n-1))\right]$$

$$= \lim_{h\to 0} h\left[3n + h\frac{n(n-1)}{2}\right]$$

$$\because h = \frac{2}{n}\ \&\text{if } h \to 0 \Rightarrow n \to \infty$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[3n + \frac{2}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} \left[6 + \frac{2}{n}n^2\left(1 - \frac{1}{n}\right)\right]$$

$$= 6 + 2 = 8$$

$$\therefore \int_0^2 (x+3)\,dx = 8$$

Definite Integrals Ex 20.5 Q3

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a) + f(a+h) + f(a+2h) + ----+f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 1,\ b = 3$ and $f(x) = 3x - 2$

$$h = \frac{2}{n} \Rightarrow nh = 2$$

Thus, we have,

$$I = \int_1^3 (3x-2)\,dx$$

$$\Rightarrow I = \lim_{h\to 0} h\left[f(1) + f(1+h) + f(1+2h) + ----f(1+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[1 + \{3(1+h) - 2\} + \{3(1+2h) - 2\} + -----+\ \{3(1+(n-1)h) - 2\}\right]$$

$$= \lim_{h\to 0} h\left[n + 3h(1+2+3+----(n-1))\right]$$

$$= \lim_{h\to 0} h\left[n + 3h\frac{n(n-1)}{2}\right]$$

$$\because h = \frac{2}{n} \qquad \therefore \text{if } h \to 0 \Rightarrow n \to \infty$$

$$\therefore \lim_{n\to\infty} \frac{2}{n}\left[n + \frac{6}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty}\ 2 + \frac{6}{n^2}n^2\left(1 - \frac{1}{n}\right)$$

$$= \lim_{n\to\infty}\ 2 + 6 = 8$$

$$\therefore \int_1^3 (3x-2)\,dx = 8$$

Definite Integrals Ex 20.5 Q4

We have,

$$\int_a^b f(x)dx = \lim_{h \to 0} h\left[f(a) + f(a+h) + f(a+2h) + ----+f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = -1,\ b = 1$ and $f(x) = x + 3$

$$\therefore h = \frac{2}{n} \Rightarrow nh = 2$$

Thus, we have,

$$I = \int_{-1}^{1} (x+3)dx$$

$$I = \lim_{h \to 0} h\left[f(-1) + f(-1+h) + f(-1+2h) + ----+f(-1+(n-1)h)\right]$$

$$= \lim_{h \to 0} h\left[2 + (2+h) + (2+2h) + ----+\{(n-1)h+2\}\right]$$

$$= \lim_{h \to 0} h\left[2n + h(1+2+3+----)\right]$$

$$= \lim_{h \to 0} h\left[2n + h\frac{n(n-1)}{2}\right] \qquad \left[\because h = \frac{2}{n} \ \&\text{ if } h \to 0 \Rightarrow n \to \infty\right]$$

$$= \lim_{n \to \infty} \frac{2}{n}\left[2n + \frac{2}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n \to \infty} 4 + \frac{2n^2}{n^2}\left(1 - \frac{1}{n}\right)$$

$$= 4 + 2 = 6$$

$$\therefore \int_{-1}^{1} (x+3)dx = 6$$

*********** END **********