Question 2. 31. The farthest objects in our Universe discovered by modem astronomers are so distant that light emitted by them takes billions of years to reach the Earth. These objects (known as quasars) have many puzzling features, which have not yet been satisfactorily explained. What is the distance in km of a quasar from which light takes 3.0 billion years to reach us?
Answer:
The time taken by light from the quasar to the observer t = 3.0 billion years
$= 3.0 \times 10^9$ years As 1 ly
$= 9.46 \times 10^{15}$ m
$\therefore$ Distance of quasar from the observer $d = 3.0 \times 10^9 \times 9.46 \times 10^{15}$ m
$= 28.38 \times 10^{24}$ m
$= 2.8 \times 10^{25}$ m or 2.8×10^{22} km.

Question 2. 32. It is a well known fact that during a total solar eclipse the disk of the Moon almost completely covers the disk of the Sun. From this fact and from the information you can gather from examples 2.3 and 2.4, determine the approximate diameter of the Moon.
Answer:
From examples 2.3 and 2.4, we get $\theta = 1920''$ and $S = 3.8452 \times 10^8$ m.
During the total solar eclipse, the disc of the moon completely covers the disc of the sun, so the angular diameter of both the sun and the moon must be equal.
Angular diameter of the moon, θ = Angular diameter of the sun
$= 1920'' = 1920 \times 4.85 \times 10^{-6}$ rad $[1'' = 4.85 \times 10^{-6}$ rad$]$
The earth-moon distance, $S = 3.8452 \times 10^8$ m
$\therefore$ The diameter of the moon, $D = \theta \times S$
$= 1920 \times 4.85 \times 10^{-6} \times 3.8452 \times 10^8$ m
$= 35806.5024 \times 10^2$ m
$= 3581 \times 10^3$ m 3581 km.

Question 2. 33. A great physicist of this century (P.A.M. Dirac) loved playing with numerical values of fundamental constants of nature. This led him to an interesting observation. Dirac found that from the basic constants of atomic physics (c, e, mass of electron, mass of proton) and the gravitational constant G, he could arrive at a number with the dimension of time. Further, it was a very large number, its magnitude being close to the present estimate on the age of the universe (-15 billion years). From the table of fundamental constants in this book, try to see if you too can construct this number (or any other interesting number you can think of). If its coincidence with the age of the universe were significant, what would this imply for the constancy of fundamental constants?
Answer: The values of different fundamental constants are given below:

Charge on an electron, $e = 1.6 \times 10^{-19}$ C
Mass of an electron, $m_e = 9.1 \times 10^{-31}$ kg
Mass of a proton, $m_p = 1.67 \times 10^{-27}$ kg
Speed of light, $c = 3 \times 10^8$ m/s
Gravitational constant, $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$$

We have to try to make permutations and combinations of the universal constants and see if there can be any such combination whose dimensions come out to be the dimensions of time. One such combination is:

$$\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \cdot \frac{1}{m_p\, m_e^2\, c^3\, G}$$

According to Coulomb's law of electrostatics,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{(e)(e)}{r^2}$$

or,

$$\frac{1}{4\pi\varepsilon_0} = \frac{Fr^2}{e^2} \quad \text{or} \quad \left(\frac{1}{4\pi\varepsilon_0}\right)^2 = \frac{F^2r^4}{e^4}$$

According to Newton's law of gravitation,

$$F = G\frac{m_1\, m_2}{r^2} \quad \text{or} \quad G = \frac{Fr^2}{m_1\, m_2}$$

Now,

$$\left[\frac{e^4}{(4\pi\varepsilon_0)^2\, m_p\, m_e^2\, c^3 G}\right] = \left[e^4\left(\frac{F^2r^4}{e^4}\right)\frac{1}{m_p\, m_e^2\, c^3}\,\frac{m_1\, m_2}{Fr^2}\right]$$

$$= \left[\frac{Fr^2}{mc^3}\right] = \left[\frac{MLT^{-2}L^2}{ML^3T^{-3}}\right] = [T]$$

Clearly, the quantity under discussion has the dimensions of time.
Substituting values in the quantity under discussion, we get

$$\frac{\left(1.6\times10^{-19}\right)^4\left(9\times10^9\right)^2}{\left(1.69\times10^{-27}\right)\left(9.1\times10^{-31}\right)^2\left(3\times10^8\right)^3\left(6.67\times10^{-11}\right)}$$

$$= 2.1 \times 10^{16} \text{ second}$$

$$= \frac{2.1\times10^{16}}{60\times60\times24\times365.25}\text{ years}$$

$$= 6.65 \times 10^8 \text{ years}$$

$$= 10^9 \text{ years}$$

The estimated time is nearly one billion years.

QUESTIONS BASED ON SUPPLEMENTARY CONTENTS

Question 1. The radius of a sphere is measured as (2.1 ± 0.5) cm calculate its surface area with error limits.
Answer:
Radius of the sphere = (2.1 ± 0.5) cm
.-. r = 2.1 and Ar = ± 0.5
S.A. = 4 πr^2
= 4 x 3.14 x 2.1 x 2.1
= 55.4 cm^2

As per the principle of error

$$\frac{\Delta s}{s} = \pm 2 \cdot \frac{\Delta r}{r}$$

$$\frac{\Delta s}{55.4} = \pm \frac{2 \times 0.5}{2.1}$$

$$\therefore \qquad \Delta s = \pm\ 26.4\ \text{cm}$$

$\therefore$ Error limits are ± 26.4 cm

$\therefore$ Surface area of the sphere $= (55.4 \pm 26.4)\ \text{cm}^2$

Question 2. The voltage across a lamp is (6.0 ± 0.1) volt and the current passing through it is (4.0 ± 0.2) ampere. Find the power consumed by the lamp.
Answer: Power P = V x I

$$P = 6 \times 4 = 24\ \text{watt}$$

Here $\Delta V = \pm\ 0.1$ volts

and $\Delta I = \pm 1.6$ A

As per the principle of error;

$$\therefore \qquad \frac{\Delta P}{P} = \pm\left(\frac{\Delta V}{V} + \frac{\Delta I}{I}\right)$$

$$\frac{\Delta P}{24} = \pm\left(\frac{0.1}{6} + \frac{0.2}{4}\right)$$

$$\frac{\Delta P}{24} = \pm\frac{0.8}{12}$$

$$\therefore \qquad \Delta P = \pm 1.6\ \text{watt}$$

$\therefore$ Power with error limit is (24 ± 1.6) watt .

Question 3. The length and breadth of a rectangular block are 25.2 cm and 16.8 cm, which have both been measured to an accuracy of 0.1 cm. Find the area of the rectangular block.
Answer:
Here l = (25.2 ± 0.1) cm
b = (16.8 ± 0.1) cm
Area= l x b
= 25.2 x 16.8
= 423.4 cm^2

As per the principle of error,

$$\frac{\Delta A}{A} = \pm\left(\frac{\Delta P}{P}+\frac{\Delta b}{b}\right)$$

$$\Rightarrow \quad \frac{\Delta A}{423.4} = \pm\left(\frac{0.1}{25.2}+\frac{0.1}{16.8}\right)$$

$$\Rightarrow \quad \Delta A = \pm\frac{423.4\times 0.1\times 42}{25.2\times 16.8}$$

$$\Rightarrow \quad \Delta A = \pm 4.2 \text{ cm}^2$$

Hence the area with error limit = (423.4 ± 4.2) cm^2 .

Question 4. A force of (2500 ±5) N is applied over an area of (0.32 ± 0.02) m^2. Calculate the pressure exerted over the area.
Answer:

Here force $F = (2500 \pm 5)$ N

and area $A = (0.32 \pm 0.02)$ m^2

Pressure $P = \frac{F}{A}$

$$P = \frac{2500}{0.32}$$

$$\therefore \quad P = 7812.5 \text{ Nm}^{-2}$$

As per the principle of error,

$$\frac{\Delta P}{P} = \pm\left(\frac{\Delta F}{F}+\frac{\Delta A}{A}\right)$$

$$\frac{\Delta P}{7812.5} = \pm\left(\frac{5}{2500}+\frac{0.02}{0.32}\right)$$

$$\Delta P = \pm 7812.5\left(\frac{1}{500}+\frac{1}{16}\right)$$

$$= \pm 7812.5\times\frac{516}{8000}$$

$$= \pm 503.9 \text{ Nm}^{-2}$$

Hence, the pressure with error limit = (7812.5 ± 503.9) Nm^{-2}

Question 5. To find the value of 'g by using a simple pendulum, the following observations were made :
Length of thread l = (100 ± 0.1) cm
Time period of oscillation T = (2 ± 0.1) sec
Calculate the maximum permissible error in measurement of 'g'.
Which quantity should be measured more accurately and why?
Answer:

Here $\quad l = (100 \pm 0.1)$ cm

and $\quad T = (2 \pm 0.1)$ sec

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T^2 = 4\pi^2 \times \frac{l}{g}$$

$$\therefore \quad g = \frac{4\pi^2 l}{T^2}$$

As per the principle of error,

$$\frac{\Delta g}{g} = \pm\left(\frac{\Delta l}{l} + 2\frac{\Delta T}{T}\right)$$

$$\frac{\Delta g}{9.8} = \pm\left(\frac{0.1}{100} + 2\times\frac{0.1}{2}\right)$$

$$\Delta g = \pm 9.8 \times 0.101$$

$$= \pm 0.99$$

$\therefore$ Maximum permissible error in the measurement of $g = \pm 0.99$. Time period of the pendulum should be measured more accurately as $g \propto \frac{1}{T^2}$.

Question 6. For a glass prism of refracting angle 60°, the minimum angle of deviation Dm is found to be 36° with a maximum error of 1.05°. When a beam of parallel light is incident on the prism, find the range of experimental value of refractive index 'μ'. It is known that the refractive index 'μ' of the material of the prism is given by

$$\mu = \frac{\sin\left(\frac{A + D_m}{2}\right)}{\sin(A/2)}$$

Answer:

Error in calculated D_m is $\pm 1.05°$

Given : $$\mu = \frac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\frac{A}{2}}$$

$$\mu = \frac{\sin\left(\frac{36° \pm 1.05°}{2}\right)}{\sin\left(\frac{60}{2}\right)°} = \frac{\sin\left(\frac{37.05}{2}\right)}{\sin 30°}$$

$$= \frac{\sin(18.525°)}{\frac{1}{2}} \text{ or } \frac{\sin(17.475°)}{\frac{1}{2}}$$

$\Rightarrow \quad 2 \times 0.755 \quad$ or $\quad 2 \times 0.73$

$\Rightarrow \quad 1.51 \quad$ or $\quad 1.46$

Here range of μ is

$$1.46 \le \mu \le 1.51$$

Question 7. The radius of curvature of a concave mirror, measured by a spherometer is given by $R = l^2/6h + h/2$. The value of l and h are 4.0 cm and 0.065 cm respectively where l is measured by a metre scale and h by the spherometer. Find the relative error in the measurement of R.

Answer: Given that l = 4 cm and Al = 0.1 cm (least count of the metre scale) here l is the distance between the legs of the spherometer.

As $$R = \frac{l^2}{6h}+\frac{h}{2}$$

$$\therefore \quad \frac{\Delta R}{R} = \frac{2\Delta l}{l}+\left(-\frac{\Delta h}{h}\right)+\frac{\Delta h}{h}$$

$$\Rightarrow \quad \frac{\Delta R}{R} = 2\frac{\Delta l}{l}+\frac{\Delta h}{h}+\frac{\Delta h}{h} \quad \text{(Considering the magnitude only)}$$

$$= 2\left(\frac{\Delta l}{l}+\frac{\Delta h}{h}\right)$$

$$= 2\left(\frac{0.1}{4}\right)+2\times\left(\frac{0.001}{0.065}\right)$$

$$= 0.05 + 0.03$$

$$= 0.08$$

Question 8. In Searle's experiment, the diameter of the wire as measured by a screw guage of least count 0.001 cm, is 0.500 cm. The length, measured by a scale of least count 0.1 cm is 110.0 cm. When a weight of 40 N is suspended from the wire, its extension is measured to be 0.125 cm by a micrometer of least count 0.001 cm. Find the Young's modulus of the material of the wire from this data.
Answer: Young's modulus of the material of the wire is given as

$$Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}}$$

$$= \frac{F/A}{l/L}$$

$$= \frac{FL}{A\cdot l}$$

$$= \frac{F\cdot L}{\frac{\pi d^2}{4}\times l}$$

$$= \frac{4FL}{\pi d^2 l}$$

Here $F = 40$ N, $L = 110$ cm $= 1.1$ m

$l = 0.125$ cm $= 0.00125$ m and $d = 0.500$ cm $= 0.005$ m

$$\therefore \quad Y = \frac{4\times 40\times 1.1}{0.00125\times 3.14\times(0.005)^2}$$

$$= 2.2 \times 10^{11}\ \text{N/m}^2$$

Now $$\frac{\Delta Y}{Y} = \frac{\Delta L}{L}+\frac{\Delta l}{l}+2\frac{\Delta d}{d}$$

$$\frac{\Delta Y}{2.2\times 10^{11}} = \frac{0.1}{110}\times\frac{0.001}{0.125}+2\frac{0.001}{0.5}$$

$$= \frac{1}{1100}+\frac{1}{125}+\frac{1}{250}$$

$$\therefore \quad \Delta Y = 2.2\times 10^{11}\times\left[\frac{1}{1100}+\frac{1}{125}+\frac{1}{250}\right]$$

$$= 0.10758 \times 10^{11}$$

$$= 10.758 \times 10^{9}\ \text{N/m}^2$$

Hence the Young's modulus of the wire is

$$= (2.2 \times 10^{11} \pm 10.758 \times 10^{9})\ \text{N/m}^2.$$

Question 9. A small error in the measurement of the quantity having the highest power (in a given formula) will contribute maximum percentage error in the value of the physical quantity to whom it is related. Explain why?

Answer:

Let $Z = A^m \times B^n \times C^l$

where $m > n > l$

$\therefore$ Maximum fractional error in Z is given by

$$\frac{\Delta Z}{Z} = m \cdot \frac{\Delta A}{A} + n \cdot \frac{\Delta B}{B} + l \cdot \frac{\Delta C}{C}$$

as $m > n > l$

$\therefore m \times \frac{\Delta A}{A}$ will contribute the maximum percentage error in the value of A.

Question 10. The two specific heat capacities of a gas are measured as Cp = (12.28 ± 0.2) units and Cv = (3.97 ± 0.3) units. Find the value of the gas constant R.

Answer:

Here $C_p = (12.28 \pm 0.2)$ units

and $C_v = (3.97 \pm 0.3)$ units

We know that

$$C_p - C_v = R$$

$$(12.28 \pm 0.2) \pm (3.97 \pm 0.3) = R$$

$$\Rightarrow (12.28 - 3.97) \pm (0.2 + 0.3) = R$$

$$\Rightarrow (8.31 \pm 0.5) = R$$

Hence $R = (8.31 \pm 0.5)$ units

*********** END **********