Exercise 16A

Q5

Answer :

Given :
$AB = AC$, $BD = DC$
To prove : $\triangle ADB \cong \triangle ADC$
Proof :
(i) In $\triangle ADB$ and $\triangle ADC$:
$AB = AC$ (given)
$\mathrm{BD} = \mathrm{DC}$ (given)
$\mathrm{DA} = \mathrm{DA}$ (common)
By SSS congruence property :
$\triangle ADB \cong \triangle ADC$
$\angle ADB = \angle ADC$ (corresponding parts of the congruent triangles) $\ldots(1)$
$\angle ADB$ and $\angle ADC$ are on the straight line.
$\therefore \angle ADB + \angle ADC = 180^\circ$
$\angle ADB + \angle ADB = 180^\circ$
$=> 2\angle ADB = 180^\circ$
$=> \angle ADB = 90^\circ$
$From$ (1) :
$\angle ADB = \angle ADC = 90^\circ$
(ii) $\angle BAD = \angle CAD$ (corresponding parts of the congruent triangles)

Q6

Answer :

Given :
AD is a bisector of $\angle A$.
$=> \angle DAB = \angle DAC$ $\ldots(1)$
$AD \perp BC$
$=> \angle BDA = \angle CDA$ (90° each)
To prove :
$\triangle ABC$ is isosceles.
Proof :
In $\triangle DAB$ and $\triangle DAC$:
$\angle BDA = \angle CDA$ (90° each)
$DA = DA$ (common)
$\angle \mathrm{DAB} = \angle \mathrm{DAC}$ (from 1)
By ASA congruence property :
$\triangle DAB \cong \triangle DAC$
$=> AB = AC$ (corresponding parts of the congruent triangles)
Therefore, $\triangle \mathrm{ABC}$ is isosceles.

Q7

Answer :

Given :

$AB = AD$

$CB = CD$

To prove :

$\triangle ABC \cong \triangle ADC$

Proof :

In $\triangle ABC$ and $\triangle ADC$:

$AB = AD$ (given)

$BC = DC$ (given)

$AC = AC$ (common)

$\therefore \triangle ABC \cong \triangle ADC$ (*by* SSS congruence property)

Q8

Answer :

Given :

$PA \perp AB$

$QB \perp AB$

$PA = QB$

To prove : $\triangle OAP \cong \triangle OBQ$

Find whether $OA = OB$.

Proof :

In $\triangle OAP$ and $\triangle OBQ$:

$\angle POA = \angle QOB$ ($v$ertically opposite angles)

$\angle OAP = \angle OBQ$ (90° each)

$PA = QB$ ($g$iven)

By AAS congruence property :

$\triangle OAP \cong \triangle OBQ$

$=> OA = OB$ (corresponding parts of the congruent triangles)

Q9

Answer :

Given :

Triangles ABC *and* DCB are right angled at A and D, respectively.

$AC = DB$

To prove : $\triangle ABC \cong \triangle DCB$

In $\triangle ABC$ and $\triangle DCB$:

$\angle CAB = \angle BDC$ (90° each)

$BC = BC$ (common)

$AC = DB$ (given)

By R.H.S. congruence property :

$\triangle ABC \cong \triangle DCB$

Q10

Answer :

Given :

$\triangle ABC$ is an isosceles triangle in which $AB = AC$.

E and F are midpoints of AC and AB, respectively.

To prove :

$BE = CF$

Proof :

E and F are midpoints of AC and AB, respectively.

$=> AF = FB,\ AE = EC$
$AB = AC$
$=> \frac{1}{2} AB = \frac{1}{2} AC$
$=> FB = EC$
$\angle ABC = \angle ACB \quad$ (angle opposite to equal sides are equal)
$=> \angle FBC = \angle ECB$
Consider $\triangle BCF$ and $\triangle CBE$:
$BC = BC \quad$ (common)

$=> \angle FBC = \angle ECB$
Consider $\triangle BCF$ and $\triangle CBE$:
$BC = BC \quad$ (common)
$\angle FBC = \angle ECB \quad$ (proved above)
$FB = EC \quad$ (proved above)
By SAS congruence property:
$\triangle BCF \cong \triangle CBE$
$BE = CF \quad$ (corresponding parts of the congruent triangles)

Q11
Answer :

Given:
$AB = AC$
$\triangle ABC$ is an isosceles triangle.

*********** END **********