Trigonometric Identities Ex 6.1 Q55

Answer :

In the given question, we are given $T_n = \sin^n \theta + \cos^n \theta$

We need to prove $\frac{T_3 - T_5}{T_1} = \frac{T_5 - T_7}{T_3}$

Here L.H.S is

$$\frac{T_3 - T_5}{T_1} = \frac{\left(\sin^3 \theta + \cos^3 \theta\right) - \left(\sin^5 \theta + \cos^5 \theta\right)}{\left(\sin \theta + \cos \theta\right)}$$

Now, solving the L.H.S, we get

$$\frac{\left(\sin^3 \theta + \cos^3 \theta\right) - \left(\sin^5 \theta + \cos^5 \theta\right)}{\left(\sin \theta + \cos \theta\right)} = \frac{\sin^3 \theta - \sin^5 \theta + \cos^3 \theta - \cos^5 \theta}{\sin \theta + \cos \theta}$$

$$= \frac{\sin^3 \theta(1 - \sin^2 \theta) + \cos^3 \theta(1 - \cos^2 \theta)}{\sin \theta + \cos \theta}$$

Further using the property $\sin^2 \theta + \cos^2 \theta = 1$, we get

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

So,

$$\frac{\sin^3 \theta(1 - \sin^2 \theta) + \cos^3 \theta(1 - \cos^2 \theta)}{\sin \theta + \cos \theta} = \frac{\sin^3 \theta \cos^2 \theta + \cos^3 \theta \sin^2 \theta}{\sin \theta + \cos \theta}$$

$$= \frac{\sin^2 \theta \cos^2 \theta(\sin \theta + \cos \theta)}{\sin \theta + \cos \theta}$$

$$= \sin^2 \theta \cos^2 \theta$$

Now, solving the R.H.S, we get

$$\frac{T_5 - T_7}{T_3} = \frac{\left(\sin^5 \theta + \cos^5 \theta\right) - \left(\sin^7 \theta + \cos^7 \theta\right)}{\left(\sin^3 \theta + \cos^3 \theta\right)}$$

So,

$$\frac{\left(\sin^5 \theta + \cos^5 \theta\right) - \left(\sin^7 \theta + \cos^7 \theta\right)}{\left(\sin^3 \theta + \cos^3 \theta\right)} = \frac{\sin^5 \theta - \sin^7 \theta + \cos^5 \theta - \cos^7 \theta}{\sin^3 \theta + \cos^3 \theta}$$

$$= \frac{\sin^5 \theta(1 - \sin^2 \theta) + \cos^5 \theta(1 - \cos^2 \theta)}{\sin^3 \theta + \cos^3 \theta}$$

Further using the property $\sin^2 \theta + \cos^2 \theta = 1$, we get,

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

So,

$$\frac{\sin^5 \theta(1 - \sin^2 \theta) + \cos^5 \theta(1 - \cos^2 \theta)}{\sin^3 \theta + \cos^3 \theta} = \frac{\sin^5 \theta \cos^2 \theta + \cos^5 \theta \sin^2 \theta}{\sin^3 \theta + \cos^3 \theta}$$

$$= \frac{\sin^2 \theta \cos^2 \theta(\sin^3 \theta + \cos^3 \theta)}{\sin^3 \theta + \cos^3 \theta}$$

$$= \sin^2 \theta \cos^2 \theta$$

Hence proved.

Trigonometric Identities Ex 6.1 Q56

Answer :

In the given question, we need to prove

$$\left(\tan\theta+\frac{1}{\cos\theta}\right)^2+\left(\tan\theta-\frac{1}{\cos\theta}\right)^2=2\left(\frac{1+\sin^2\theta}{1-\sin^2\theta}\right)$$

Now, using the identity $(a+b)^2=a^2+b^2+2ab$ in L.H.S, we get

$$\begin{aligned}\left(\tan\theta+\frac{1}{\cos\theta}\right)^2+\left(\tan\theta-\frac{1}{\cos\theta}\right)^2&=\left(\tan^2\theta+\frac{1}{\cos^2\theta}+2\frac{\tan\theta}{\cos\theta}\right)+\left(\tan^2\theta+\frac{1}{\cos^2\theta}-2\frac{\tan\theta}{\cos\theta}\right)\\&=\tan^2\theta+\frac{1}{\cos^2\theta}+2\frac{\tan\theta}{\cos\theta}+\tan^2\theta+\frac{1}{\cos^2\theta}-2\frac{\tan\theta}{\cos\theta}\\&=2\tan^2\theta+\frac{2}{\cos^2\theta}\end{aligned}$$

Further using $\tan\theta=\frac{\sin\theta}{\cos\theta}$, we get

$$\begin{aligned}2\tan^2\theta+\frac{2}{\cos^2\theta}&=\frac{2\sin^2\theta}{\cos^2\theta}+\frac{2}{\cos^2\theta}\\&=\frac{2\sin^2\theta+2}{\cos^2\theta}\\&=\frac{2(\sin^2\theta+1)}{\cos^2\theta}\end{aligned}$$

Also, from the identity $\sin^2\theta+\cos^2\theta=1$, we get

$$\begin{aligned}\frac{2(\sin^2\theta+1)}{\cos^2\theta}&=\frac{2(\sin^2\theta+1)}{1-\sin^2\theta}\\&=2\left(\frac{1+\sin^2\theta}{1-\sin^2\theta}\right)\end{aligned}$$

Hence proved.

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