Exercise 16C

Question 3:

(i) Let A(0, 1), B(1, 2) and C(-2, -1) be the given points. Then,

$(x_1 = 0, y_1 = 1)(x_2 = 1, y_2 = 2), (x_3 = -2, y_3 = -1)$

$$\therefore \text{Area of } \Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$
$$= \frac{1}{2}[0 \times (2+1) + 1(-1-1) + (-2) \times (1-2)]$$
$$= \frac{1}{2}(0 - 2 + 2) = 0$$

Hence the given points are collinear

(ii) Let A(-5, 1), B(5,5) and C(10, 7) be the given points.

$\therefore$ Then, $(x_1 = -5, y_1 = 1)(x_2 = 5, y_2 = 5)(x_3 = 10, y_3 = 7)$

$$\therefore \text{Area of } \Delta ABC = \frac{1}{2}[x_1(y_2 - y_1) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$
$$= \frac{1}{2}[(-5)(5-7) + 5(7-1) + 10(1-5)]$$
$$= \frac{1}{2}[10 + 30 - 40] = 0$$

Hence the given points are collinear

(iii) Let P(a, b + c), Q(b, c + a) and R(c, a + B) be the given points.

Then, $(x_1 = 1, y_1 = b + c), (x_2 = b, y_2 = c + a), (x_3 = c, y_3 = a + b)$

$$\therefore \text{Area of } \Delta PQR = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_3)]$$
$$= \frac{1}{2} a(c + a - a - b) + b(a + b - b - c) + c(b + c - c - a)$$
$$= \frac{1}{2}[a(c - b) + b(a - c) + c(b - a)] = 0$$

Hence the given points are collinear

Question 4:

(i) The given points are A(-1, 3), B(2, p) and C(5, -1)

$\therefore (x_1 = -1, y_1 = 3)(x_2 = 2, y_2 = p)(x_3 = 5, y_3 = -1)$

The given point A, B, C are collinear if

$$x_1(y_2 - y_1) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$
$$\Rightarrow (-1)(p + 1) + 2(-1 - 3) + 5(3 - p) = 0$$
$$\Rightarrow -p - 1 - 8 + 15 - 5p = 0$$
$$\Rightarrow -6p + 6 = 0 \Rightarrow -6p = -6$$
$$p = \frac{6}{6} = 1$$

Hence, $p = 1$

(ii) The given points are A(3, 2), B(4, p) and C(5, 3)

$(x_1 = 3, y_1 = 2), (x_2 = 4, y_2 = p)$ and $(x_3 = 5, y_3 = 3)$

The given point s A, B, C are collinear if

$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

$\Rightarrow 3(p-3) + 4(3-2) + 5(2-p) = 0$

$3p - 9 + 4 + 10 - 5p = 0$

$-2p = -5 \Rightarrow p = \frac{5}{2}$

Hence, $p = \frac{5}{2}$

(iii) The three points are A(-3, 9), B(2, p), C(4, -5)

Let $x_1 = -3,\ y_1 = 9;\ x_2 = 2, y_2 = p;\ x_3 = 4, y_3 = -5$

Three points $(x_1 y_1), (x_2 y_2)$ and $(x_3 y_3)$ are collinear if

$x_1(y_2 - y_3) + x_2(y_3 - y_2) + (y_1 - y_2) = 0$

$\Rightarrow -3(p+5) + 2(-5-9) + 4(9-p) = 0$

$\Rightarrow -3p - 15 - 28 + 36 - 4p = 0$ or $-7p - 7 = 0$

$\therefore \quad 7p = -7$

$p = \frac{-7}{7} = -1$

*********** END **********