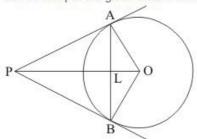


Circles Ex 10.2 Q12

Answer:

Let us first put the given data in the form of a diagram.



Consider ΔPOA and ΔPOB . We have,

PO is the common side for both the triangles.

PA = PB(Tangents drawn from an external point will be equal in length)

OB = OA(Radii of the same circle)

Therefore, by SSS postulate of congruency, we have

ΔΡΟΑ ≅ ΔΡΟΒ

Hence,

 $\angle OPA = \angle OPB \dots (1)$

Now let us consider ΔPLA and ΔPLB . We have,

PL is the common side for both the triangles.

 $\angle OPA = \angle OPB$ (From equation (1))

PA = PB (Tangents drawn from an external point will be equal in length)

From SAS postulate of congruent triangles,

 $\Delta PLA \cong \Delta PLB$

Therefore,

PL = LB (2)

 $\angle PLA = \angle PLB$

Since AB is a straight line,

 $\angle ALB = 180^{\circ}$

 $\angle PLA + \angle PLB = 180^{\circ}$

 $2\angle PLA = 180^{\circ}$

 $\angle PLA = 90^{\circ}$

 $\angle PLB = 90^{\circ}$

Let us now take up Δ OPB. We know that the radius of a circle will always be perpendicular to the tangent at the point of contact. Therefore,

 $\angle OBP = 90$

By Pythagoras theorem we have,

 $PB^2 = OP^2 - OB^2$

It is given that

OP = diameter of the circle

Therefore,

OP = 20B

Hence,

 $PB^2 = (2OB)^2 - OB^2$

 $PB^2 = 4OB^2 - OB^2$

$$PB^2 = 3OB^2$$

 $PB = \sqrt{3}OB$

Consider ΔPLB . We have,

$$LB^2 = PB^2 - PL^2$$

But we have found that,

$PB = \sqrt{3}OB$

Also from the figure, we can say

$$PL = PO - OL$$

Therefore,

$$LB^2 = 3OB^2 - [PO - OL]^2$$
 (3)

Also, from ΔOLB , we have

$$LB^2 = OB^2 - OL^2 \quad \dots \quad (4)$$

Since Left Hand Sides of equation (3) and equation (4) are same, we can equate the Right Hand Sides of the two equations. Thus we have,

$$OB^2 - OL^2 = 3OB^2 - [PO - OL]^2$$

$$OB^2 - OL^2 = 3OB^2 - [PO^2 + OL^2 - 2.PO.OL]$$

$$OB^2 - OL^2 = 3OB^2 - PO^2 - OL^2 + 2.PO.OL$$

We know from the given data, that OP = 2.0B. Let us substitute 20B in place of PO in the above equation. We get,

$$OB^2 - OL^2 = 3OB^2 - (2OB)^2 - OL^2 + 2.2.OB.OL$$

$$OB^2 - OL^2 = 3OB^2 - 4OB^2 - OL^2 + 4.OB.OL$$

 $2OB^2 = 4.OB.OL$

$$OL = \frac{OB}{2}$$

Substituting the value of OL and also PO in equation (3), we get,

$$LB^2 = 3OB^2 - \left[2OB - \left(\frac{OB}{2}\right)\right]^2$$

$$LB^2 = 3OB^2 - [4OB^2 + \left(\frac{OB^2}{4}\right) - 2.OB^2]$$

$$LB^2 = 3OB^2 - 4OB^2 - \left(\frac{OB^2}{4}\right) + 2.OB^2$$

$$LB^2 = \frac{3OB^2}{4}$$

$$LB = \frac{\sqrt{3}OB}{2}$$

Also from the figure we get,

$$AB = PL + LB$$

From equation (2), we know that PL = LB. Therefore,

$$AB = 2.LB$$

$$AB = 2 \times \frac{\sqrt{3}OB}{2}$$

$$AB = \sqrt{3}OB$$

We have also found that $PB = \sqrt{3}OB$. We know that tangents drawn from an external point will be equal in length. Therefore, we have

PA = PB

Hence,

$$PA = \sqrt{3}OB$$

Now, consider ΔPAB . We have,

 $PA = \sqrt{3}OB$

 $PB = \sqrt{3}OB$

$$AB = \sqrt{30}$$

Since all the sides of the triangle are of equal length, ΔPAB is an equilateral triangle. Thus we have proved.