

Definite Integrals Ex 20.1 Q63

$$\begin{split} I &= \int\limits_0^{\pi/4} \left( tan x + \cot x \right)^{-2} dx \\ I &= \int\limits_0^{\pi/4} \frac{1}{\left( tan x + \cot x \right)^2} dx \\ I &= \int\limits_0^{\pi/4} \frac{1}{\left( \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \right)^2} dx \\ I &= \int\limits_0^{\pi/4} \left( \sin x \cos x \right)^2 dx \\ I &= \int\limits_0^{\pi/4} \sin^2 x \left( 1 - \sin^2 x \right) dx \\ I &= \int\limits_0^{\pi/4} \sin^2 x dx - \int\limits_0^{\pi/4} \sin^4 x dx \end{aligned}$$

We know that by reduction formula,

$$\int \sin^n x \, dx = \frac{n-1}{n} \int \sin^{n-2} x \, dx - \frac{\cos x \sin^{n-1} x}{n}$$

For 
$$n = 2$$

$$\int \sin^2 x \, dx = \frac{2-1}{2} \int 1 \, dx - \frac{\cos x \sin x}{2}$$
$$\int \sin^2 x \, dx = \frac{1}{2} x - \frac{\cos x \sin x}{2}$$

For n = 4

$$\int \sin^4 x \ dx = \frac{4-1}{4} \int \sin^2 x \ dx - \frac{\cos x \sin^3 x}{4}$$
$$\int \sin^4 x \ dx = \frac{3}{4} \left\{ \frac{1}{2} x - \frac{\cos x \sin x}{2} \right\} - \frac{\cos x \sin^3 x}{4}$$

Hence,

$$\begin{split} I &= \left\{ \frac{1}{2} \times -\frac{\cos \times \sin \times}{2} \right\}_{0}^{\frac{\pi}{4}} - \left\{ \frac{3}{4} \left\{ \frac{1}{2} \times -\frac{\cos \times \sin \times}{2} \right\} - \frac{\cos \times \sin^{3} \times}{4} \right\}_{0}^{\frac{\pi}{4}} \\ &= \left\{ \frac{\pi}{8} - \frac{1}{4} \right\} - \left\{ \frac{3}{4} \left( \frac{\pi}{8} - \frac{1}{4} \right) - \frac{1}{16} \right\} \\ &= \frac{\pi}{32} \end{split}$$

$$\frac{\pi}{2} \int_{0}^{\frac{\pi}{2}} (\sin x \cos x)^{2} dx$$

$$\frac{\pi}{2} \int_{0}^{\frac{\pi}{2}} \sin^{2} x (1 - \sin^{2} x) dx$$

$$\frac{\pi}{2} \int_{0}^{\frac{\pi}{2}} \sin^{2} x dx - \sin^{4} x dx$$
We know, By reduction formula
$$\int \sin^{8} x dx = \frac{n-1}{n} \int \sin^{8-2} x dx - \frac{\cos x \sin^{8-1} x}{n}$$
For n=2
$$\int \sin^{2} x dx = \frac{2-1}{2} \int 1 dx - \frac{\cos x \sin x}{2}$$

$$\int \sin^{2} x dx = \frac{1}{2} x - \frac{\cos x \sin x}{2}$$
For n=4
$$\int \sin^{4} x dx = \frac{4-1}{4} \int \sin^{2} x dx - \frac{\cos x \sin^{3} x}{4}$$
Hence
$$\left\{ \frac{1}{2} x - \frac{\cos x \sin x}{2} \right\}_{0}^{\frac{\pi}{2}} - \left\{ \frac{3}{4} \left\{ \frac{1}{2} x - \frac{\cos x \sin x}{2} \right\} - \frac{\cos x \sin^{3} x}{4} \right\}_{0}^{\frac{\pi}{2}}$$

$$\frac{\pi}{4} - \frac{3}{4} \left\{ \frac{\pi}{4} \right\}$$

$$\frac{\pi}{4} = \frac{3}{4} \left\{ \frac{\pi}{4} \right\}$$

$$\frac{\pi}{4} = \frac{3}{4} \left\{ \frac{\pi}{4} \right\}$$

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Using Integration By parts

$$\int f'g = fg - \int fg'$$

$$f' = x, g = \log(2x+1)$$

$$f = \frac{x^2}{2}, g' = \frac{2}{2x+1}$$

$$\begin{split} &\int_{0}^{1} x \log(1+2x) dx \\ &= \left[ \frac{x^{2} \log(1+2x)}{2} \right]_{0}^{1} - \int_{0}^{1} \frac{2x^{2}}{2(2x+1)} dx \\ &= \frac{\log(3)}{2} - \int_{0}^{1} \frac{x}{2} - \frac{1}{4} + \frac{1}{4(2x+1)} dx \\ &= \frac{\log(3)}{2} - \left[ \frac{x^{2}}{4} - \frac{x}{4} + \frac{1}{8} \log|2x+1| \right]_{0}^{1} \\ &= \frac{\log(3)}{2} - \frac{1}{8} \log(3) \\ &= \frac{3}{8} \log_{e}(3) \end{split}$$

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