Arithmetic Progressions Ex 9.3 Q11

Answer :

Here, we are given that $(m+1)^{th}$ term is twice the $(n+1)^{th}$ term, for a certain A.P. Here, let us take the first term of the A.P. as a and the common difference as d

We need to prove that $a_{3m+1} = 2a_{m+n+1}$

So, let us first find the two terms.

As we know,

$a_{n'} = a + (n'-1)d$

For $(m+1)^{th}$ term ($n' = m+1$)

$$a_{m+1} = a + (m+1-1)d$$
$$= a + md$$

For $(n+1)^{th}$ term ($n' = n+1$),

$$a_{n+1} = a + (n+1-1)d$$
$$= a + nd$$

Now, we are given that $a_{m+1} = 2a_{n+1}$

So, we get,

$$a + md = 2(a + nd)$$
$$a + md = 2a + 2nd$$
$$md - 2nd = 2a - a$$
$$(m-2n)d = a \qquad ......(1)$$

Further, we need to prove that the $(3m+1)^{th}$ term is twice of $(m+n+1)^{th}$ term. So let us now find these two terms,

For $(m+n+1)^{th}$ term ($n' = m+n+1$),

$$a_{m+n+1} = a + (m+n+1-1)d$$
$$= (m-2n)d + (m+n)d \quad \text{(Using 1)}$$
$$= md - 2nd + md + nd$$
$$= 2md - nd$$

For $(3m+1)^{th}$ term ($n' = 3m+1$),

$$a_{3m+1} = a + (3m+1-1)d$$
$$= (m-2n)d + 3md \qquad \text{(Using 1)}$$
$$= md - 2nd + 3md$$
$$= 4md - 2nd$$
$$= 2(2md - nd)$$

Therefore, $a_{3m+1} = 2a_{m+n+1}$

Hence proved

Arithmetic Progressions Ex 9.3 Q12

Answer :

Here, we are given two A.P. sequences whose n^{th} terms are equal. We need to find n.

So let us first find the n^{th} term for both the A.P.

First A.P. is 9, 7, 5 ...

Here,

First term (a) = 9

Common difference of the A.P. (d) $= 7 - 9$

$= -2$

Now, as we know,

$a_n = a + (n-1)d$

So, for n^{th} term,

$$a_n = 9 + (n-1)(-2)$$
$$= 9 - 2n + 2$$
$$= 11 - 2n \qquad ......(1)$$

Second A.P. is 15, 12, 9 ...

Here,

First term (a) = 15

Common difference of the A.P. (d) $= 12 - 15$

$= -3$

Now, as we know,

$a_n = a + (n-1)d$

So, for n^{th} term,

$$a_n = 15 + (n-1)(-3)$$
$$= 15 - 3n + 3$$
$$= 18 - 3n \qquad \text{......}(2)$$

Now, we are given that the n^{th} terms for both the A.P. sequences are equal, we equate (1) and (2),

$$11 - 2n = 18 - 3n$$
$$3n - 2n = 18 - 11$$
$$n = 7$$

Therefore, $\boxed{n = 7}$

*********** END **********