



Definite Integrals Ex 20.4B Q9

Let $x = \tan \theta$

$$\Rightarrow dx = \sec^2 \theta d\theta$$

$$\text{If } x = 0, \theta = 0$$

$$\text{If } x = 1, \theta = \frac{\pi}{4}$$

$$\therefore \int_0^1 \frac{\log(1+x)}{1+x^2} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log(1 + \tan \theta) d\theta$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log \left\{ 1 + \tan \left(\frac{\pi}{4} - \theta \right) \right\} d\theta$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log \left\{ 1 + \frac{1 - \tan \theta}{1 + \tan \theta} \right\} d\theta$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log \left(\frac{2}{1 + \tan \theta} \right) d\theta$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} (\log 2 - \log(1 + \tan \theta)) d\theta$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{4}} \log 2 \times d\theta = \frac{\pi}{4} \log 2$$

$$\Rightarrow I = \frac{\pi}{8} \log 2$$

Definite Integrals Ex 20.4B Q10

$$I = \int_0^{\infty} \frac{x}{(1+x)(1+x^2)} dx$$

Let,

$$\frac{x}{(1+x)(1+x^2)} = \frac{A}{1+x} + \frac{Bx+C}{1+x^2}$$

$$\Rightarrow x = A(1+x^2) + (Bx+C)(1+x)$$

Equating coefficients, we get

$$A+B=0 \Rightarrow A=-B$$

$$B+C=1 \Rightarrow -2A=1$$

$$A+C=0 \Rightarrow A=-C$$

$$\therefore A = -\frac{1}{2}, B = \frac{1}{2}, C = \frac{1}{2}$$

So,

$$\begin{aligned} I &= \int_0^{\infty} \left(\frac{-\frac{1}{2}}{1+x} + \frac{1}{2} \frac{x+1}{x^2+1} \right) dx \\ &= \int_0^{\infty} -\frac{1}{2} \frac{dx}{1+x} + \frac{1}{2} \int_0^{\infty} \frac{x}{x^2+1} dx + \frac{1}{2} \int_0^{\infty} \frac{dx}{1+x^2} \\ &= \left[-\frac{1}{2} \log|1+x| + \frac{1}{4} \log|x^2+1| + \frac{1}{2} \tan^{-1} x \right]_0^{\infty} \\ &= 0 + 0 + \frac{\pi}{4} + 0 - 0 - 0 \\ &= \frac{\pi}{4} \end{aligned}$$

$$\therefore \int_0^{\infty} \frac{x}{(1+x)(1+x^2)} dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q11

We have,

$$I = \int_0^{\pi} \frac{x \tan x}{\sec x \operatorname{cosec} x} dx$$

$$I = \int_0^{\pi} \frac{x \left(\frac{\sin x}{\cos x} \right)}{\left(\frac{1}{\cos x} \right) \left(\frac{1}{\sin x} \right)} dx$$

$$I = \int_0^{\pi} x \sin^2 x dx \quad \dots (i)$$

$$I = \int_0^{\pi} (\pi - x) \sin^2 (\pi - x) dx \quad \left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right]$$

$$I = \int_0^{\pi} (\pi - x) \sin^2 x dx \quad \dots (ii)$$

Add (i) and (ii), we get

$$2I = \int_0^{\pi} (\pi) \sin^2 x dx = \pi \int_0^{\pi} \frac{1 - \cos 2x}{2} dx = \frac{\pi}{2} \left[x - \frac{\sin 2x}{2} \right]_0^{\pi} = \frac{\pi}{2} [\pi - 0 - 0 + 0] = \frac{\pi^2}{2}$$

$$\therefore \int_0^{\pi} \frac{x \tan x}{\sec x \operatorname{cosec} x} dx = \frac{\pi^2}{4}$$

Definite Integrals Ex 20.4B Q12

$$\text{Let } I = \int_0^{\pi} x \sin x \cdot \cos^4 x \, dx \quad \text{--- (i)}$$

So,

$$\begin{aligned} I &= \int_0^{\pi} (\pi - x) \sin(\pi - x) \cdot \cos^4(\pi - x) \, dx & \left[\because \int_0^a f(x) \, dx = \int_0^a f(a-x) \, dx \right] \\ &= \int_0^{\pi} (\pi - x) \sin x \cdot \cos^4 x \, dx \\ &= \int_0^{\pi} \pi \sin x \cdot \cos^4 x \, dx - \int_0^{\pi} x \sin x \cdot \cos^4 x \, dx \end{aligned}$$

So from equation (i)

$$I = \int_0^{\pi} \pi \sin x \cdot \cos^4 x \, dx - I$$

$$2I = \pi \int_0^{\pi} \sin x \cdot \cos^4 x \, dx$$

$$\text{Let } t = \cos x \, dx$$

$$dt = -\sin x \, dx$$

As,

$$x = 0 \quad t = 1$$

$$x = \pi \quad t = -1$$

Hence

$$2I = \pi \int_{-1}^{+1} t^4 \, dt = \pi \left[\frac{t^5}{5} \right]_{-1}^{+1} = \pi \left[\frac{1}{5} + \frac{1}{5} \right]$$

$$I = \frac{\pi}{5}$$

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