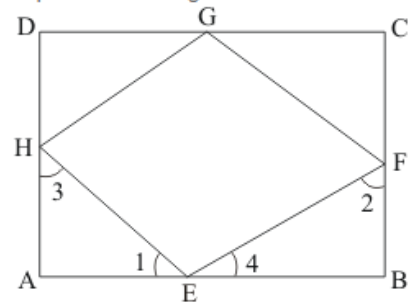




Quadrilaterals Ex 14.3 Q7

Answer :

Square $ABCD$ is given:



E , F , G and H are the points on AB , BC , CD and DA respectively, such that :

$$AE = BF = CG = DH$$

We need to prove that $EFGH$ is a square.

Say, $AE = BF = CG = DH = x$

As sides of a square are equal. Then, we can also say that:

$$BE = CF = DG = AH = y$$

In $\triangle AEH$ and $\triangle BFE$, we have:

$$AE = BF \text{ (Given)}$$

$$\angle A = \angle B \text{ (Each equal to } 90^\circ \text{)}$$

$$BE = AH \text{ (Each equal to } y \text{)}$$

By SAS Congruence criteria, we have:

$$\triangle AEH \cong \triangle BFE$$

Therefore, $EH = EF$

Similarly, $EF = FG$, $FG = HG$ and $HG = HE$

Thus, $HE = EF = FG = HG$

Also,

$$\angle 1 = \angle 2 \text{ and } \angle 3 = \angle 4$$

But,

$$\angle 1 + \angle 3 = 90^\circ \text{ and } \angle 2 + \angle 4 = 90^\circ$$

Therefore,

$$\angle 1 + \angle 3 + \angle 2 + \angle 4 = 90^\circ + 90^\circ$$

$$\angle 1 + \angle 4 + \angle 1 + \angle 4 = 180^\circ$$

$$2(\angle 1 + \angle 4) = 180^\circ$$

$$\angle 1 + \angle 4 = 90^\circ$$

i.e., $\angle HEF = 90^\circ$

Similarly,

$$\angle F = 90^\circ$$

$$\angle G = 90^\circ$$

$$\angle H = 90^\circ$$

Thus, $EFGH$ is a square.

Hence proved.

***** END *****