



Chapter 6 Determinants Ex 6.2 Q52-i

$$\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0$$

Apply $C_1 \rightarrow C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & c \\ x+a+b+c & b & x+c \end{vmatrix} = 0$$

$$\Rightarrow (x+a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix} = 0$$

$$\Rightarrow (x+a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix} = 0$$

$$\Rightarrow (x+a+b+c) x^2 = 0$$

$$\Rightarrow x = -(a+b+c) \quad \text{or} \quad x = 0$$

Chapter 6 Determinants Ex 6.2 Q52-ii

$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$, we get:

$$\begin{vmatrix} 3x+a & 3x+a & 3x+a \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

$$\Rightarrow (3x+a) \begin{vmatrix} 1 & 1 & 1 \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, we have:

$$(3x+a) \begin{vmatrix} 1 & 0 & 0 \\ x & a & 0 \\ x & 0 & a \end{vmatrix} = 0$$

Expanding along R_1 , we have:

$$(3x+a) [1 \times a^2] = 0$$

$$\Rightarrow a^2 (3x+a) = 0$$

But $a \neq 0$.

Therefore, we have:

$$3x+a=0$$

$$\Rightarrow x = -\frac{a}{3}$$

Chapter 6 Determinants Ex 6.2 Q52-iii

$$\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$$

Apply $C_1 \rightarrow C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix} 3x-2 & 3 & 3 \\ 3x-2 & 3x-8 & 3 \\ 3x-2 & 3 & 3x-8 \end{vmatrix} = 0$$

$$\Rightarrow (3x-2) \begin{vmatrix} 1 & 3 & 3 \\ 1 & 3x-8 & 3 \\ 1 & 3 & 3x-8 \end{vmatrix} = 0$$

$$\Rightarrow (3x-2) \begin{vmatrix} 1 & 3 & 3 \\ 0 & 3x-11 & 0 \\ 0 & 0 & 3x-11 \end{vmatrix} = 0$$

$$\Rightarrow (3x-2)(3x-11)^2 = 0$$

$$\Rightarrow (3x-2) = 0 \quad \text{or} \quad (3x-11)^2 = 0$$

$$\Rightarrow x = \frac{2}{3} \quad \text{or} \quad x = \pm \frac{11}{3}$$

Chapter 6 Determinants Ex 6.2 Q52-iv

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & a & a^2 \\ 1 & b & b^2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & x & x^2 \\ 0 & a-x & a^2-x^2 \\ 0 & b-x & b^2-x^2 \end{vmatrix} = 0$$

$$\Rightarrow (a-x)(b-x) \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & a+x \\ 0 & 1 & b+x \end{vmatrix} = 0$$

$$\Rightarrow (a-x)(b-x) \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & a+x \\ 0 & 0 & b-a \end{vmatrix} = 0$$

$$\Rightarrow (a-x)(b-x)(b-a) = 0$$

$$\Rightarrow (a-x) = 0 \quad \text{or} \quad (b-x) = 0$$

$$\Rightarrow a = x \quad \text{or} \quad b = x$$

Chapter 6 Determinants Ex 6.2 Q52-v

$$\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$$

Apply $C_1 \rightarrow C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix} x+9 & 3 & 5 \\ x+9 & x+2 & 5 \\ x+9 & 3 & x+4 \end{vmatrix} = 0$$

$$\Rightarrow (x+9) \begin{vmatrix} 1 & 3 & 5 \\ 1 & x+2 & 5 \\ 1 & 3 & x+4 \end{vmatrix} = 0$$

$$\Rightarrow (x+9) \begin{vmatrix} 1 & 3 & 5 \\ 0 & x-1 & 0 \\ 0 & 0 & x-1 \end{vmatrix} = 0$$

$$\Rightarrow (x+9)(x-1)^2 = 0$$

$$\Rightarrow (x+9) = 0 \quad \text{or} \quad (x-1)^2 = 0$$

$$\Rightarrow x = -9 \quad \text{or} \quad x = 1$$

Chapter 6 Determinants Ex 6.2 Q52-vi

$$\begin{aligned}
&\Rightarrow \begin{vmatrix} 1 & x & x^3 \\ 0 & b-x & b^3-x^3 \\ 0 & c-x & c^3-x^3 \end{vmatrix} = 0 \\
&\Rightarrow (b-x)(c-x) \begin{vmatrix} 1 & x & x^3 \\ 0 & 1 & b^2+x^2+bx \\ 0 & 1 & c^2+x^2+cx \end{vmatrix} = 0 \\
&\Rightarrow (b-x)(c-x) \begin{vmatrix} 1 & x & x^3 \\ 0 & 1 & b^2+x^2+bx \\ 0 & 0 & c^2+x^2+cx - (b^2+x^2+bx) \end{vmatrix} = 0 \\
&\Rightarrow (b-x)(c-x) \begin{vmatrix} 1 & x & x^3 \\ 0 & 1 & b^2+x^2+bx \\ 0 & 0 & c^2-b^2+cx-bx \end{vmatrix} = 0 \\
&\Rightarrow (b-x)(c-x)(c-b) \begin{vmatrix} 1 & x & x^3 \\ 0 & 1 & b^2+x^2+bx \\ 0 & 0 & b+c+x \end{vmatrix} = 0 \\
&\Rightarrow (b-x)(c-x)(c-b)(b+c+x) = 0 \\
&\Rightarrow (b-x) = 0 \quad (c-x) = 0 \quad (b+c+x) = 0 \\
&\Rightarrow x = b \quad x = c \quad x = -(b+c)
\end{aligned}$$

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$$\begin{aligned}
&\begin{vmatrix} 15-2x & 11-3x & 7-x \\ 11 & 17 & 14 \\ 10 & 16 & 13 \end{vmatrix} = 0 \\
&\Rightarrow \begin{vmatrix} 15-2x & 11-3x & 7-x \\ 1 & 1 & 1 \\ 10 & 16 & 13 \end{vmatrix} = 0 \\
&\Rightarrow \begin{vmatrix} 15-2x & -x-4 & 7-x \\ 1 & 0 & 1 \\ 10 & 6 & 13 \end{vmatrix} = 0 \\
&\Rightarrow \begin{vmatrix} 8-x & -x-4 & 7-x \\ 0 & 0 & 1 \\ -3 & 6 & 13 \end{vmatrix} = 0 \\
&\Rightarrow -[(8-x)(6) - (-x-4)(-3)] = 0 \\
&\Rightarrow -[36 - 9x] = 0 \\
&\Rightarrow x = 4
\end{aligned}$$

Chapter 6 Determinants Ex 6.2 Q52-viii

$$\begin{aligned}
&\begin{vmatrix} 1 & 1 & x \\ p+1 & p+1 & p+x \\ 3 & x+1 & x+2 \end{vmatrix} = 0 \\
&\Rightarrow \begin{vmatrix} 1 & 1 & x \\ p & p & p \\ 2 & x & 2 \end{vmatrix} = 0 \\
&\Rightarrow p \begin{vmatrix} 1 & 1 & x \\ 1 & 1 & 1 \\ 2 & x & 2 \end{vmatrix} = 0 \\
&\Rightarrow p \begin{vmatrix} 1 & 1 & x \\ 0 & 0 & 1-x \\ 2 & x & 2 \end{vmatrix} = 0 \\
&\Rightarrow p(x-1)(x-2) = 0 \\
&\Rightarrow (x-1) = 0 \quad (x-2) = 0 \\
&\Rightarrow x = 1 \quad x = 2
\end{aligned}$$

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$$\begin{vmatrix} 3 & -2 & \sin 3\theta \\ -7 & 8 & \cos 2\theta \\ -11 & 14 & 2 \end{vmatrix} = 0$$

$$\Rightarrow 3(16 - 14\cos 2\theta) + 2(-14 + 11\cos 2\theta) + \sin 3\theta(-98 + 88) = 0$$

$$\Rightarrow 20(1 - \cos 2\theta) + 10\sin 3\theta = 0$$

$$\Rightarrow 20(2\sin^2 \theta) + 10(3\sin \theta - 4\sin^3 \theta) = 0$$

$$\Rightarrow 4\sin^2 \theta + 3\sin \theta - 4\sin^3 \theta = 0$$

$$\Rightarrow 4\sin \theta + 3 - 4\sin^2 \theta = 0$$

$$\Rightarrow 4\sin^2 \theta - 4\sin \theta - 3 = 0$$

$$\Rightarrow (2\sin \theta + 1)(2\sin \theta - 3) = 0$$

$$\Rightarrow \sin \theta = -\frac{1}{2} \text{ or } \sin \theta = \frac{3}{2} = 1.5$$

$$\text{As } \sin \theta \in [-1, 1]$$

$$\therefore \sin \theta = -\frac{1}{2}$$

$$\Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{6}, n \in \mathbb{Z}$$

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