Chapter 6 Determinants Ex 6.2 Q29

We need to prove the following identity:

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

Let us consider the L.H.S of the above equation.

$$\Delta = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$$

Applying $R_1 \to R_1(a)$, $R_2 \to R_2(b)$ and $R_3 \to R_3(c)$, we get

$$\Delta = \frac{1}{abc}\begin{vmatrix} a(a^2+1) & a^2b & a^2c \\ ab^2 & b(b^2+1) & b^2c \\ c^2a & c^2b & c(c^2+1) \end{vmatrix}$$

Taking a,b, and c common from C_1,C_2 and C_3, respectively, we get,

$$\Delta = \frac{abc}{abc}\begin{vmatrix} (a^2+1) & a^2 & a^2 \\ b^2 & (b^2+1) & b^2 \\ c^2 & c^2 & (c^2+1) \end{vmatrix}$$

Applying $R_1 \to R_1+R_2+R_3$, we get,

$$\Delta = \frac{abc}{abc}\begin{vmatrix} (a^2+b^2+c^2+1) & (a^2+b^2+c^2+1) & (a^2+b^2+c^2+1) \\ b^2 & (b^2+1) & b^2 \\ c^2 & c^2 & (c^2+1) \end{vmatrix}$$

Taking the term, $(a^2+b^2+c^2+1)$ common from the above equation, we have,

$$\Delta = (a^2+b^2+c^2+1)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & (b^2+1) & b^2 \\ c^2 & c^2 & (c^2+1) \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$, we get,

$$\Delta = (a^2+b^2+c^2+1)\begin{vmatrix} 1 & 0 & 1 \\ b^2 & 1 & 0 \\ c^2 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = (a^2+b^2+c^2+1)$$

Chapter 6 Determinants Ex 6.2 Q30

Let us consider the L.H.S of the given equation.

$$\text{Let } \Delta = \begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, *we have,*

$$\Delta = \begin{vmatrix} 1+a+a^2 & a & a^2 \\ 1+a+a^2 & 1 & a \\ 1+a+a^2 & a^2 & 1 \end{vmatrix}$$

Taking the term $(1+a+a^2)$ *common, we have,*

$$\Delta = (1+a+a^2)\begin{vmatrix} 1 & a & a^2 \\ 1 & 1 & a \\ 1 & a^2 & 1 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ *and* $R_3 \to R_3 - R_1$, *we have*

$$\Delta = (1+a+a^2)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1-a & a(1-a) \\ 0 & -a(1-a) & (1-a)(1+a) \end{vmatrix}$$

Taking the term $(1-a)$ *common from* R_2 *and* R_3, *we have*

$$\Rightarrow \Delta = (1+a+a^2)(1-a)^2\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & a \\ 0 & -a & (1+a) \end{vmatrix}$$

$$\Rightarrow \Delta = (1+a+a^2)(1-a)^2(1+a+a^2)$$

$$\Rightarrow \Delta = (1+a+a^2)^2(1-a)^2$$

$$\Rightarrow \Delta = [(1+a+a^2)(1-a)]^2$$

$$\Rightarrow \Delta = [(a^3-1)]^2$$

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$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a)$$

$$\text{LHS} = \begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$$

Apply: $C_1 \to C_1 + C_3$ and $C_2 \to C_2 + C_3$

$$= \begin{vmatrix} a+c & -(c+b) & -b \\ -(c+a) & b+c & -a \\ a+c & b+c & a+b+c \end{vmatrix}$$

$$=(c+a)(c+b)\begin{vmatrix} 1 & -1 & -b \\ -1 & 1 & -a \\ 1 & 1 & a+b+c \end{vmatrix}$$

$$=(c+a)(c+b)\begin{vmatrix} 1 & -1 & -b \\ 0 & 0 & -a-b \\ 0 & 2 & a+c \end{vmatrix}$$

$$= 2(a+b)(b+c)(c+a)$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q32

We need to prove the following identity:

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$$

Let us consider the L.H.S of the above equation.

$$\Delta = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, *we have,*

$$\Delta = \begin{vmatrix} 2(b+c) & 2(a+c) & 2(a+b) \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Taking 2 common from the above equation, we have,

$$\Delta = 2\begin{vmatrix} (b+c) & (a+c) & (a+b) \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ *and* $R_3 \to R_3 - R_1$, *we have,*

$$\Delta = 2\begin{vmatrix} (b+c) & (a+c) & (a+b) \\ -c & 0 & -a \\ -b & -a & 0 \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, *we have,*

$$\Delta = 2\begin{vmatrix} 0 & c & b \\ -c & 0 & -a \\ -b & -a & 0 \end{vmatrix}$$

$$\Rightarrow \Delta = 2(0 + 2abc + abc)$$

$$\Rightarrow \Delta = 4abc$$

*********** END **********