

Exercise 7A

Question 34

(i)
$$LHS = \sqrt{\frac{(1-\cos\theta)}{(1+\cos\theta)}} \times \frac{(1+\cos\theta)}{(1+\cos\theta)} = \sqrt{\frac{1-\cos^2\theta}{(1+\cos\theta)^2}}$$

$$= \sqrt{\frac{\sin^2\theta}{(1+\cos\theta)^2}} = \frac{\sin\theta}{1+\cos\theta} = RHS$$

$$\therefore LHS = RHS$$
(iii)
$$LHS = \sqrt{\frac{(1+\sin\theta)}{(1-\sin\theta)}} \times \frac{(1-\sin\theta)}{(1-\sin\theta)} = \sqrt{\frac{1-\sin^2\theta}{(1-\sin\theta)^2}}$$

$$= \sqrt{\frac{\cos^2\theta}{(1-\sin\theta)^2}} = \frac{\cos\theta}{1-\sin\theta} = RHS$$

$$\therefore LHS = RHS$$
(iii)
$$LHS = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$\therefore LHS = RHS$$
(iiii)
$$LHS = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$= \sqrt{\frac{(1+\cos\theta)\times(1-\cos\theta)}{(1-\cos\theta)\times(1-\cos\theta)}} + \sqrt{\frac{(1-\cos\theta)\times(1+\cos\theta)}{(1+\cos\theta)\times(1+\cos\theta)}}$$

$$= \sqrt{\frac{(1-\cos^2\theta)}{(1-\cos^2\theta)}} = \sqrt{\frac{(1-\cos^2\theta)}{(1-\cos^2\theta)}}$$

$$= \sqrt{\frac{(1 + \cos \theta) \times (1 - \cos \theta)}{(1 - \cos \theta) \times (1 - \cos \theta)}} + \sqrt{\frac{(1 - \cos \theta) \times (1 + \cos \theta)}{(1 + \cos \theta)}}$$

$$= \sqrt{\frac{(1 - \cos^2 \theta)}{(1 - \cos \theta)^2}} + \sqrt{\frac{(1 - \cos^2 \theta)}{(1 + \cos \theta)^2}}$$

$$= \sqrt{\frac{\sin^2 \theta}{(1 - \cos \theta)^2}} + \sqrt{\frac{\sin^2 \theta}{(1 + \cos \theta)^2}} = \frac{\sin \theta}{1 - \cos \theta} + \frac{\sin \theta}{1 + \cos \theta}$$

$$= \frac{\sin \theta (1 + \cos \theta) + \sin \theta (1 - \cos \theta)}{1 - \cos^2 \theta}$$

$$= \frac{\sin \theta + \cos \theta \sin \theta + \sin \theta - \sin \theta \cos \theta}{\sin^2 \theta}$$

$$= \frac{2\sin \theta}{\sin^2 \theta} = \frac{2}{\sin \theta} = 2\cos \theta \text{ RHS}$$

LHS =
$$\frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B}$$
=
$$\frac{(\sin A + \sin B)(\sin A - \sin B) + (\cos A + \cos B)(\cos A - \cos B)}{(\cos A + \cos B)(\sin A + \sin B)}$$
=
$$\frac{\sin^2 A - \sin^2 B + \cos^2 A - \cos^2 B}{(\cos A + \cos B)(\sin A + \sin B)}$$
=
$$\frac{(\sin^2 A + \cos^2 A) - (\sin^2 A + \cos^2 B)}{(\cos A + \cos B)(\sin A + \sin B)}$$
=
$$\frac{1 - 1}{(\cos A + \cos B)(\sin A + \sin B)} = 0 = RHS$$
∴ LHS = RHS

******** END *******