Chapter 10 Differentiability Ex 10.1 Q5

$f(x) = |x|+|x-1|$ in theinterval $(-1, 2)$.

$$f(x) = \begin{cases} x + x + 1 & -1 < x < 0 \\ 1 & 0 \le x \le 1 \\ -x - x + 1 & 1 < x < 2 \end{cases}$$

$$f(x) = \begin{cases} 2x + 1 & -1 < x < 0 \\ 1 & 0 \le x \le 1 \\ -2x + 1 & 1 < x < 2 \end{cases}$$

We know that a polynomial and a constant function is continuous and differentiable everywhere.
So, $f(x)$ is continuous and differentiable for $x \in (-1, 0)$, $x \in (0, 1)$ and $(1, 2)$.

We need to check continuity and differentiability at $x = 0$ and $x = 1$.

Continuity at $x = 0$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 2x + 1 = 1$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 1 = 1$$

$$f(0) = 1$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$$
$\therefore$ $f(x)$ is continuous at $x = 0$.

Continuity at $x = 1$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 1 = 1$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 1 = 1$$

$$f(1) = 1$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(1)$$
$\therefore$ $f(x)$ is continuous at $x = 1$.

Differentiability at $x = 0$

$$(\text{LHD at } x = 0) = \lim_{x\to 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x\to 0^-} \frac{2x + 1 - 1}{x - 0} = \lim_{x\to 0^-} \frac{2x}{x} = 2$$

$$(\text{RHD at } x = 0) = \lim_{x\to 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x\to 0^+} \frac{1 - 1}{x} = \lim_{x\to 0^+} \frac{0}{x} = 0$$

$\therefore$ $(\text{LHD at } x = 0) \ne (\text{RHD at } x = 0)$
So, $f(x)$ is differentiable at $x = 0$.

Differentiability at $x = 1$

$$(\text{LHD at } x = 1) = \lim_{x\to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^-} \frac{1 - 1}{x - 1} = 0$$

$$(\text{RHD at } x = 1) = \lim_{x\to 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^+} \frac{-2x + 1 - 1}{x - 1} \to \infty$$

$\therefore$ (LHD at $x = 1$) $\neq$ (RHD at $x = 1$)

So, $f(x)$ is not differentiable at $x = 1$.

So, $f(x)$ is continuous on $(-1, 2)$ but not differentiable at $x = 0, 1$.

Chapter 10 Differentiability Ex 10.1 Q6

$$f(x) = \begin{cases} x, & x \le 1 \\ 2 - x, & 1 \le x \le 2 \\ -2 + 3x - x^2, & x > 2 \end{cases}$$

Differentiability at $x = 1$

$$(\text{LHD at } x = 1) = \lim_{x\to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^-} \frac{x - 1}{x - 1} = 1$$

$$(\text{RHD at } x = 1) = \lim_{x\to 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^+} \frac{2 - x - 1}{x - 1} = \lim_{x\to 1^+} \frac{1 - x}{x - 1} = -1$$

$\therefore$ (LHD at $x = 1$) $\neq$ (RHD at $x = 1$)

So, $f(x)$ is not differentiable at $x = 1$.

Differentiability at $x = 2$

$$(\text{LHD at } x = 2) = \lim_{x\to 2^-} \frac{f(x) - f(2)}{x - 2} = \lim_{x\to 2^-} \frac{2 - x - 0}{x - 2} = \lim_{x\to 2^-} \frac{2 - x}{x - 2} = -1$$

$$(\text{RHD at } x = 2) = \lim_{x\to 2^+} \frac{f(x) - f(2)}{x - 2} = \lim_{x\to 2^+} \frac{-2 + 3x - x^2 - 0}{x - 2} = \lim_{x\to 2^+} \frac{(1 - x)(x - 2)}{x - 2} = -1$$

$\therefore$ (LHD at $x = 2$) = (RHD at $x = 2$)

So, $f(x)$ is differentiable at $x = 2$.

*********** END **********