Differentiation Ex 11.2 Q44

Let $\quad y = e^x \log \sin 2x$

Differentiate with respect to $x$,

$$\frac{dy}{dx} = \frac{d}{dx}\left[e^x \log \sin 2x\right]$$

$$= e^x \frac{d}{dx} \log \sin 2x + \log \sin 2x \frac{d}{dx}\left(e^x\right)$$

[Using product rule and chain rule]

$$= e^x \frac{1}{\sin 2x} \frac{d}{dx}\left(\sin 2x\right) + \log \sin 2x \left(e^x\right)$$

$$= \frac{e^x}{\sin 2x} \cos 2x \frac{d}{dx}\left(2x\right) + e^x \log \sin 2x$$

$$= \frac{2\cos 2x e^x}{\sin 2x} + e^x \log \sin 2x$$

$$= e^x \left(2\cot 2x + \log \sin 2x\right)$$

so,

$$\frac{d}{dx}\left(e^x \log \sin 2x\right) = e^x \left(2\cot 2x + \log \sin 2x\right).$$

Differentiation Ex 11.2 Q45

Let $\quad y = \dfrac{\sqrt{x^2+1}+\sqrt{x^2-1}}{\sqrt{x^2+1}-\sqrt{x^2-1}}$

$\Rightarrow \quad y = \dfrac{\left(x^2+1\right)^{\frac{1}{2}}+\left(x^2-1\right)^{\frac{1}{2}}}{\left(x^2+1\right)^{\frac{1}{2}}-\left(x^2-1\right)^{\frac{1}{2}}}$

Differentiate it with respect to $x$,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{\left(x^2+1\right)^{\frac{1}{2}}+\left(x^2-1\right)^{\frac{1}{2}}}{\left(x^2+1\right)^{\frac{1}{2}}-\left(x^2-1\right)^{\frac{1}{2}}}\right]$$

$$= \left[\frac{\left\{\left(x^2+1\right)^{\frac{1}{2}}-\left(x^2-1\right)^{\frac{1}{2}}\right\}\frac{d}{dx}\left\{\left(x^2+1\right)^{\frac{1}{2}}+\left(x^2-1\right)^{\frac{1}{2}}\right\}-\left\{\left(x^2+1\right)^{\frac{1}{2}}+\left(x^2-1\right)^{\frac{1}{2}}\right\}\frac{d}{dx}\left\{\left(x^2+1\right)^{\frac{1}{2}}-\left(x^2-1\right)^{\frac{1}{2}}\right\}}{\left\{\left(x^2+1\right)^{\frac{1}{2}}-\left(x^2-1\right)^{\frac{1}{2}}\right\}^2}\right]$$

[Using quotient rule and chian rule]

$$= \left[\frac{\left\{\left(x^2+1\right)^{\frac{1}{2}}-\left(x^2-1\right)^{\frac{1}{2}}\right\}\left[\frac{1}{2}\left(x^2+1\right)^{\frac{-1}{2}}\frac{d}{dx}\left(x^2+1\right)+\frac{1}{2}\left(x^2-1\right)^{\frac{-1}{2}}\frac{d}{dx}\left(x^2-1\right)\right]}{\left[\left(x^2+1\right)+\left(x^2-1\right)-2\sqrt{x^4-1}\right]}\right]$$

$$-\left[\frac{\left\{\left(x^2+1\right)^{\frac{1}{2}}+\left(x^2-1\right)^{\frac{1}{2}}\right\}\frac{1}{2}\left[\left(x^2+1\right)^{\frac{-1}{2}}\frac{d}{dx}\left(x^2+1\right)-\frac{1}{2}\left(x^2-1\right)^{\frac{-1}{2}}\frac{d}{dx}\left(x^2-1\right)\right]}{\left[\left(x^2+1\right)\left(x^2-1\right)-2\sqrt{x^4-1}\right]}\right]$$

$$= \left[\frac{\left(\sqrt{x^2+1}-\sqrt{x^2-1}\right)\left(\frac{2x}{2\sqrt{x^2+1}}+\frac{2x}{2\sqrt{x^2-1}}\right)}{\left[2x^2-2\sqrt{x^4-1}\right]}\right] - \left[\frac{\left(\sqrt{x^2+1}+\sqrt{x^2-1}\right)\left(\frac{2x}{2\sqrt{x^2+1}}-\frac{2x}{2\sqrt{x^2-1}}\right)}{\left[2x^2-2\sqrt{x^4-1}\right]}\right]$$

$$= \left[\frac{x\left(\sqrt{x^2+1}-\sqrt{x^2-1}\right)\left(\sqrt{x^2-1}+\sqrt{x^2+1}\right)}{2\left[x^2-\sqrt{x^4-1}\right]\left(\sqrt{x^2+1}\sqrt{x^2-1}\right)}\right] - \left[\frac{x\left(\sqrt{x^2+1}+\sqrt{x^2-1}\right)\left(\sqrt{x^2-1}-\sqrt{x^2+1}\right)}{2\left[x^2-\sqrt{x^4-1}\right]\left(\sqrt{x^2+1}\sqrt{x^2-1}\right)}\right]$$

$$= \left[\frac{x\left(x^2+1-x^2+1\right)-x\left(x^2-1-x^2-1\right)}{2\left[x^2-\sqrt{x^4-1}\right]\sqrt{x^4-1}}\right]$$

$$= \left[\frac{4x}{2\left(x^2-\sqrt{x^4-1}\right)\sqrt{x^4-1}}\right]$$

$$= 2x\left[\frac{1\times\left(x^2+\sqrt{x^4-1}\right)}{\left(\left(x^2-\sqrt{x^4-1}\sqrt{x^4-1}\times\left(x^2+\sqrt{x^4-1}\right)\right)\right)}\right]$$

Multiplying and divide by $\left\{x^2+\sqrt{x^4-1}\right\}$,

$$= 2x\left[\frac{x^2+\sqrt{x^4-1}}{\left(x^4-x^4+1\right)\sqrt{x^4-1}}\right]$$

$$= 2x\left[\frac{x^2+\sqrt{x^4-1}}{\sqrt{x^4-1}}\right]$$

$$= \frac{2x^3}{\sqrt{x^4-1}}+2x$$

So,

$$\frac{d}{dx}\left[\frac{\sqrt{x^2+1}+\sqrt{x^2-1}}{\sqrt{x^2+1}-\sqrt{x^2-1}}\right] = \frac{2x^3}{\sqrt{x^4-1}}+2x.$$

Differentiation Ex 11.2 Q46

Let $\quad y = \log\left[x + 2 + \sqrt{x^2 + 4x + 1}\right]$

Differentiate with respect to $x$,

$$\frac{dy}{dx} = \frac{d}{dx}\log\left[x + 2 + \sqrt{x^2 + 4x + 1}\right]$$

$$= \frac{1}{\left[x + 2 + \sqrt{x^4 + 4x + 1}\right]}\frac{d}{dx}\left[x + 2 + \left(x^2 + 4x + 1\right)^{\frac{1}{2}}\right]$$

[using chain rule]

$$= \frac{1}{\left[x + 2 + \sqrt{x^4 + 4x + 1}\right]} \times \left[1 + 0 + \frac{1}{2}\left(x^2 + 4x + 1\right)^{\frac{-1}{2}}\frac{d}{dx}\left(x^2 + 4x + 1\right)\right]$$

$$= \frac{1 + \frac{(2x + 4)}{2\left(\sqrt{x^2 + 4x + 1}\right)}}{\left[x + 2 + \sqrt{x^4 + 4x + 1}\right]}$$

$$= \frac{\sqrt{x^2 + 4x + 1} + x + 2}{\left[x + 2 + \sqrt{x^2 + 4x + 1}\right] \times \sqrt{x^2 + 4x + 1}}$$

$$= \frac{1}{\sqrt{x^2 + 4x + 1}}$$

So,

$$\frac{d}{dx}\left[\log\left\{x + 2 + \sqrt{x^2 + 4x + 1}\right\}\right] = \frac{1}{\sqrt{x^2 + 4x + 1}}.$$

*********** END **********