Trigonometric Identities Ex 6.1 Q43

Answer :

We need to prove $\frac{\operatorname{cosec} A}{\operatorname{cosec} A-1}+\frac{\operatorname{cosec} A}{\operatorname{cosec} A+1}=2\sec^2 A$

Using the identity $a^2-b^2=(a+b)(a-b)$, we get

$$\begin{aligned}\frac{\operatorname{cosec} A}{\operatorname{cosec} A-1}+\frac{\operatorname{cosec} A}{\operatorname{cosec} A+1}&=\frac{\operatorname{cosec} A(\operatorname{cosec} A+1)+\operatorname{cosec} A(\operatorname{cosec} A-1)}{\operatorname{cosec}^2 A-1}\\&=\frac{\operatorname{cosec} A(\operatorname{cosec} A+1+\operatorname{cosec} A-1)}{\operatorname{cosec}^2 A-1}\end{aligned}$$

Further, using the property $1+\cot^2\theta=\operatorname{cosec}^2\theta$, we get

So,

$$\begin{aligned}\frac{\operatorname{cosec} A(\operatorname{cosec} A+1+\operatorname{cosec} A-1)}{\operatorname{cosec}^2 A-1}&=\frac{\operatorname{cosec} A(2\operatorname{cosec} A)}{\cot^2 A}\\&=\frac{2\operatorname{cosec}^2 A}{\cot^2 A}\\&=\frac{(2)\left(\frac{1}{\sin^2 A}\right)}{\left(\frac{\cos^2 A}{\sin^2 A}\right)}\\&=2\left(\frac{1}{\sin^2 A}\right)\left(\frac{\sin^2 A}{\cos^2 A}\right)\\&=2\left(\frac{1}{\cos^2 A}\right)\\&=2\sec^2 A\end{aligned}$$

Hence proved.

Trigonometric Identities Ex 6.1 Q44

Answer :

We need to prove $(1+\tan^2 A)+\left(1+\dfrac{1}{\tan^2 A}\right)=\dfrac{1}{\sin^2 A-\sin^4 A}$.

Using the property $1+\tan^2\theta=\sec^2\theta$, we get

$$\begin{aligned}(1+\tan^2 A)+\left(1+\frac{1}{\tan^2 A}\right)&=\sec^2 A+\left(\frac{\tan^2 A+1}{\tan^2 A}\right)\\&=\sec^2 A+\left(\frac{\sec^2 A}{\tan^2 A}\right)\end{aligned}$$

Now, using $\sec\theta=\dfrac{1}{\cos\theta}$ and $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$, we get

$$\begin{aligned}\sec^2 A+\left(\frac{\sec^2 A}{\tan^2 A}\right)&=\frac{1}{\cos^2 A}+\left(\frac{\frac{1}{\cos^2 A}}{\frac{\sin^2 A}{\cos^2 A}}\right)\\&=\frac{1}{\cos^2 A}+\left(\frac{1}{\cos^2 A}\times\frac{\cos^2 A}{\sin^2 A}\right)\\&=\frac{1}{\cos^2 A}+\frac{1}{\sin^2 A}\\&=\frac{\sin^2 A+\cos^2 A}{\cos^2 A(\sin^2 A)}\end{aligned}$$

Further, using the property, $\sin^2\theta+\cos^2\theta=1$, we get

$$\begin{aligned}\frac{\sin^2 A+\cos^2 A}{\cos^2 A(\sin^2 A)}&=\frac{1}{\cos^2 A(\sin^2 A)}\\&=\frac{1}{(1-\sin^2 A)(\sin^2 A)}\qquad(\text{using }\cos^2\theta=1-\sin^2\theta)\\&=\frac{1}{\sin^2 A-\sin^4 A)}\end{aligned}$$

Hence proved.

*********** END **********