Definite Integrals Ex 20.3 Q15

We have,

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \{\sin|x| + \cos|x|\} dx$$

Let $f(x) = \sin|x| + \cos|x|$

Then, $f(x) = f(-x)$

$\therefore$ f(x) is an even function.

So, $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \{\sin|x| + \cos|x|\} dx = 2\int_{0}^{\frac{\pi}{2}} (\sin x + \cos x) dx = 2[-\cos x + \sin x]_0^{\frac{\pi}{2}} = 4$

$$\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \{\sin|x| + \cos|x|\} dx = 4$$

Definite Integrals Ex 20.3 Q16

$$I = \int_0^4 |x-1| dx$$

It can be seen that, $(x-1) \le 0$ when $0 \le x \le 1$ and $(x-1) \ge 0$ when $1 \le x \le 4$

$$I = \int_0^1 |x-1| dx + \int_1^4 |x-1| dx \qquad \left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x)\right)$$

$$= \int_0^1 -(x-1) dx + \int_1^4 (x-1) dx$$

$$= \left[x - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^4$$

$$= 1 - \frac{1}{2} + \frac{(4)^2}{2} - 4 - \frac{1}{2} + 1$$

$$= 1 - \frac{1}{2} + 8 - 4 - \frac{1}{2} + 1$$

$$= 5$$

Definite Integrals Ex 20.3 Q17

$$\text{Let } I = \int_1^4 \{|x-1| + |x-2| + |x-4|\} dx$$

$$= \int_1^2 \{(x-1) - (x-2) - (x-4)\} dx + \int_2^4 \{(x-1) + (x-2) - (x-4)\} dx$$

$$= \int_1^2 \{(x-1-x+2-x+4)\} dx + \int_2^4 \{(x-1+x-2-x+4)\} dx$$

$$= \int_1^2 (5-x) dx + \int_2^4 (x+1) dx$$

$$= \left[5x - \frac{x^2}{2}\right]_1^2 + \left[\frac{x^2}{2} + x\right]_2^4$$

$$= \left[10 - 2 - 5 + \frac{1}{2}\right] + [8 + 4 - 2 - 2]$$

$$= \frac{7}{2} + 8$$

$$I = \frac{23}{2}$$

Definite Integrals Ex 20.3 Q18

We have,

$$I = \int_{-5}^{0}\left(|x| + |x+2| + |x+5|\right)dx = \int_{-5}^{0}|x|dx + \int_{-5}^{0}|x+2|dx + \int_{-5}^{0}|x+5|dx$$

$$\Rightarrow I = \int_{-5}^{0} -x\,dx + \int_{-5}^{-2} -(x+2)\,dx + \int_{-2}^{0}(x+2)\,dx + \int_{-5}^{0}(x+5)\,dx$$

$$= \left[\frac{-x^2}{2}\right]_{-5}^{0} + \left[\frac{-x^2}{2} - 2x\right]_{-5}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^{0} + \left[\frac{x^2}{2} + 5x\right]_{-5}^{0}$$

$$= \left[+\frac{25}{2}\right] - \left[\frac{4}{2} - 4 - \frac{25}{2} + 10\right] + \left[0 + 0 - \frac{4}{2} + 4\right] + \left[0 + 0 - \frac{25}{2} + 25\right]$$

$$= \frac{25}{2} - \left[8 - \frac{25}{2}\right] + [2] + \left[25 - \frac{25}{2}\right]$$

$$= \frac{25}{2} - 8 + \frac{25}{2} + 2 + 25 - \frac{25}{2}$$

$$= 19 + \frac{25}{2} = 31\frac{1}{2}$$

$$I = \frac{63}{2}$$

*********** END **********