Definite Integrals Ex 20.5 Q21

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 0,\ b = 2$ and $f(x) = 3x^2 - 2$

$$\therefore \quad h = \frac{2}{n} \quad \Rightarrow \quad nh = 2$$

Thus, we have,

$$I = \int_0^2 (3x^2 - 2)\,dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(0+h)+f(0+2h)+----f\{0+(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[-2+(3h^2-2)+\{3(2h)^2-2\}+----\right]$$

$$= \lim_{h\to 0} h\left[-2h+3h^2(1+2^2+3^2+----)\right]$$

$$\because h = \frac{2}{n} \quad \text{if } h \to 0 \Rightarrow n \to \infty$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[-2n+\frac{12}{n^2}\,\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} -4+\frac{4}{n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right) = -4+8 = 4$$

$$\therefore \int_0^2 (3x^2-2)\,dx = 4$$

Definite Integrals Ex 20.5 Q22

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 0$, $b = 2$ and $f(x) = x^2 + 2$

$$\therefore \quad h = \frac{2}{n} \Rightarrow nh = 2$$

Thus, we have,

$$I = \int_0^2 \left(x^2+2\right)dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(0+h)+f(2h)+----f(0+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[2+\left(h^2+2\right)+\left\{(2h)^2+2\right\}+----\right]$$

$$= \lim_{h\to 0} h\left[2h+h^2\left(1+2^2+3^2+----(n-1)^2\right]\right.$$

$$\because h = \frac{2}{n} \ \&\ \text{if } h\to 0 \Rightarrow n\to\infty$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[2n+\frac{4}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} 4+\frac{4}{3n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= 4+\frac{8}{3} = \frac{20}{3}$$

$$\therefore \int_0^2 \left(x^2+2\right)dx = \frac{20}{3}$$

Definite Integrals Ex 20.5 Q23

It is known that,

$$\int_a^b f(x)\,dx=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+\ldots+f(a+(n-1)h)\right],\text{ where } h=\frac{b-a}{n}$$

Here, $a=0$, $b=4$, and $f(x)=x+e^{2x}$

$$\therefore h=\frac{4-0}{n}=\frac{4}{n}$$

$$\Rightarrow \int_0^4\left(x+e^{2x}\right)dx=(4-0)\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f(h)+f(2h)+\ldots+f((n-1)h)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\left(0+e^{0}\right)+\left(h+e^{2h}\right)+\left(2h+e^{2\cdot 2h}\right)+\ldots+\left\{(n-1)h+e^{2(n-1)h}\right\}\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[1+\left(h+e^{2h}\right)+\left(2h+e^{4h}\right)+\ldots+\left\{(n-1)h+e^{2(n-1)h}\right\}\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\left\{h+2h+3h+\ldots+(n-1)h\right\}+\left(1+e^{2h}+e^{4h}+\ldots+e^{2(n-1)h}\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[h\left\{1+2+\ldots(n-1)\right\}+\left(\frac{e^{2hn}-1}{e^{2h}-1}\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\frac{\left(h(n-1)n\right)}{2}+\left(\frac{e^{2hn}-1}{e^{2h}-1}\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\frac{4}{n}\cdot\frac{(n-1)n}{2}+\left(\frac{e^{8}-1}{e^{\frac{8}{n}}-1}\right)\right]$$

$$=4(2)+4\lim_{n\to\infty}\frac{\left(e^{8}-1\right)}{\left(\frac{e^{\frac{8}{n}}-1}{\frac{8}{n}}\right)8}$$

$$=8+\frac{4\cdot\left(e^{8}-1\right)}{8}\qquad\qquad\left(\lim_{x\to 0}\frac{e^{x}-1}{x}=1\right)$$

$$=8+\frac{e^{8}-1}{2}$$

$$=\frac{15+e^{8}}{2}$$

Definite Integrals Ex 20.5 Q24

We have,

$$\int_a^b f(x)dx=\lim_{h\to 0}h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h=\dfrac{b-a}{n}$

Thus, we have,

$$I=\int_0^2\left(x^2+x\right)dx$$

$$=\lim_{h\to 0}h\left[f(0)+f(0+h)+f(0+2h)+----f\{(n-1)h\}\right]$$

$$=\lim_{h\to 0}h\left[0+\left(h^2+h\right)+\left\{(2h)^2+2h\right\}+----\right]$$

$$=\lim_{h\to 0}h\left[\left\{h^2\left(1+2^2+3^2+----(n-1)^2\right\}+h\left\{1+2+3----(n-1)\right\}\right]$$

$$\because h=\frac{2}{n}\ \ \&\text{ if } h\to 0\Rightarrow n\to\infty$$

$$=\lim_{n\to\infty}\frac{2}{n}\left[\frac{4}{n^2}\frac{n(n-1)(2n-1)}{6}+\frac{2}{n}\frac{n(n-1)}{2}\right]$$

$$=\lim_{n\to\infty}\frac{4}{3n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+\frac{2}{n^2}n^2\left(1-\frac{1}{n}\right)$$

$$=\frac{8}{3}+2=\frac{14}{3}$$

$$\therefore\int_0^2\left(x^2+x\right)dx=\frac{14}{3}$$

*********** END **********