Definite Integrals Ex 20.2 Q32

We have,

$$\int_0^1 x\tan^{-1}x\,dx = \tan^{-1}x\int_0^1 x\,dx - \int_0^1\left(\int x\,dx\right)\frac{d}{dx}\left(\tan^{-1}x\right)dx$$

$$= \left[\frac{x^2}{2}\tan^{-1}x\right]_0^1 - \frac{1}{2}\int_0^1\frac{x^2}{1+x^2}dx$$

$$= \left[\frac{x^2}{2}\tan^{-1}x\right]_0^1 - \frac{1}{2}\int_0^1\frac{1+x^2-1}{1+x^2}dx$$

$$= \frac{1}{2}\left(\frac{\pi}{4}\right) - \frac{1}{2}\left[\int_0^1 dx - \int_0^1\frac{dx}{1+x^2}\right]$$

$$= \frac{\pi}{8} - \frac{1}{2}\left[x - \tan^{-1}x\right]_0^1$$

$$= \frac{\pi}{8} - \frac{1}{2}\left[1 - \frac{\pi}{4}\right]$$

$$= \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8}$$

$$= \frac{\pi}{4} - \frac{1}{2}$$

$$\therefore \int_0^1 x\tan^{-1}x\,dx = \frac{\pi}{4} - \frac{1}{2}$$

Definite Integrals Ex 20.2 Q33

Let $I = \int\frac{1-x^2}{x^4+x^2+1}dx = -\int\frac{x^2-1}{x^4+x^2+1}dx.$

Then,

$$I = -\int\frac{1-\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}}dx \qquad \left[\text{Dividing the numerator and denominator by } x^2\right]$$

$$\Rightarrow \quad I = -\int\frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)^2 - 1^2}dx$$

Let, $x+\frac{1}{x} = u$. Then, $d\left(x+\frac{1}{x}\right) = du \Rightarrow \left(1-\frac{1}{x^2}\right)dx = du$

$$\therefore \quad I = -\int\frac{du}{u^2-1^2}$$

$$\Rightarrow \quad I = -\frac{1}{2(1)}\log\left|\frac{u-1}{u+1}\right| + C$$

$$\Rightarrow \quad I = -\frac{1}{2}\log\left|\frac{x+\frac{1}{x}-1}{x+\frac{1}{x}+1}\right| + C = -\frac{1}{2}\log\left|\frac{x^2-x+1}{x^2+x+1}\right| + C$$

$$\therefore \int_0^1\frac{1-x^2}{x^4+x^2+1}dx = \left[-\frac{1}{2}\log\left|\frac{x^2-x+1}{x^2+x+1}\right|\right]_0^1 = \left(-\frac{1}{2}\log\left|\frac{1}{3}\right|\right) - \left(-\frac{1}{2}\log|1|\right) = \log\sqrt{3}$$

$$= \log 3^{\frac{1}{2}}$$

$$= \frac{1}{2}\log 3$$

Definite Integrals Ex 20.2 Q34

Let $1 + x^2 = t$

Differentiating w.r.t. x, we get

$$2x\,dx = dt$$

Now, $x = 0 \Rightarrow t = 1$

$x = 1 \Rightarrow t = 2$

$$\int_0^1 \frac{24x^3}{\left(1 + x^2\right)^4} dx = \int_1^2 \frac{12\left(t - 1\right)}{t^4} dt$$

$$= 12\int_1^2 \left(\frac{1}{t^3} - \frac{1}{t^4}\right) dt$$

$$= 12\left[-\frac{1}{2t^2} - \frac{1}{3t^3}\right]_1^2$$

$$= 12\left[-\frac{1}{8} + \frac{1}{24} + \frac{1}{2} - \frac{1}{3}\right]$$

$$= 12\left[\frac{-3 + 1 + 12 - 8}{24}\right]$$

$$= \frac{12 \times 2}{24} = 1$$

$$\therefore \int_0^1 \frac{24x^3}{\left(1 + x^2\right)^4} dx = 1$$

Definite Integrals Ex 20.2 Q35

Let $x - 4 = t^3$

Differentiating w.r.t. x, we get

$$dx = 3t^2 dt$$

Now, $x = 4 \Rightarrow t = 0$

$x = 12 \Rightarrow t = 2$

$$\therefore \int_4^{12} x\left(x-4\right)^{\frac{1}{3}} dx = \int_0^2 \left(t^3+1\right) t.3t^2 dt$$

$$= 3\int_0^2 \left(t^6 + 4t^3\right) dt$$

$$= 3\left[\frac{t^7}{7} + t^4\right]_0^2$$

$$= 3\left[\frac{128}{7} + 16\right]$$

$$= \frac{720}{7}$$

$$\therefore \int_4^{12} x\left(x-4\right)^{\frac{1}{3}} dx = \frac{720}{7}$$

*********** END **********