



### Geometric Progressions Ex 20.6 Q 7

Given,

$$A.M = 25$$

$$G.M = 20$$

$$\text{Now, } A.M = \frac{a+b}{2} = 25$$

$$\text{and, } G.M = \sqrt{ab} = 20$$

$$a+b = 50, ab = 400$$

$$(a-b) = \sqrt{(a+b)^2 - 4ab}$$

$$= \sqrt{(50)^2 - 1600}$$

$$= \sqrt{2500 - 1600}$$

$$= \pm 30$$

$$a-b = \pm 30$$

$$\underline{a+b = 50}$$

$$2a = 80$$

$$a = 40$$

$$\text{Also, } -2b = -20$$

$$b = 10$$

$\therefore$  The numbers are 40, 10.

### Geometric Progressions Ex 20.6 Q 8

A.M. between two numbers  $a$  and  $b$  ( $a > b$ ) is  $\frac{a+b}{2}$

Also, geometric mean between 2 numbers is  $\sqrt{ab}$

Given,

$$A.M = 2G.M$$

$$\Rightarrow \frac{a+b}{2} = 2\sqrt{ab}$$

$$a+b = 4\sqrt{ab}$$

$$\frac{a+b}{2\sqrt{ab}} = \frac{2}{1}$$

$$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{2+1}{2-1} = \frac{3}{1}$$

[By componendo and dividendo]

$$\frac{(\sqrt{a} + \sqrt{b})^2}{(\sqrt{a} - \sqrt{b})^2} = \frac{(\sqrt{3})^2}{(1)^2}$$

$$\frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}} = \frac{\sqrt{3}}{1}$$

By componendo and dividendo, we get

$$\frac{(\sqrt{a} + \sqrt{b}) + (\sqrt{a} - \sqrt{b})}{(\sqrt{a} + \sqrt{b}) - (\sqrt{a} - \sqrt{b})} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

$$\frac{a}{b} = \frac{(\sqrt{3} + 1)^2}{(\sqrt{3} - 1)^2} = \frac{3 + 1 + 2\sqrt{3}}{3 + 1 - 2\sqrt{3}}$$

$$= \frac{4 + 2\sqrt{3}}{4 - 2\sqrt{3}}$$

$$\frac{a}{b} = \frac{2 + \sqrt{3}}{2 - \sqrt{3}}$$

Thus,  $a : b = (2 + \sqrt{3}) : (2 - \sqrt{3})$ .

### Geometric Progressions Ex 20.6 Q 9

Let A.M =  $A$  between  $a$  and  $b$   
 G.M =  $G_1$  and  $G_2$  between  $a$  and  $b$

$$\Rightarrow A = \frac{a+b}{2}$$

$a, G_1, G_2, b$  is G.P. with common ratio  $r = \left(\frac{b}{a}\right)^{\frac{1}{3}}$

$$G_1 = ar = a \left(\frac{b}{a}\right)^{\frac{1}{3}}$$

$$G_2 = ar^2 = a \left(\frac{b}{a}\right)^{\frac{2}{3}} = a^{\frac{1}{3}} b^{\frac{2}{3}}$$

Now,

$$G_1^2 = a^2 \left(\frac{b}{a}\right)^{\frac{2}{3}}$$

$$G_2^2 = a^{\frac{2}{3}} b^{\frac{4}{3}}$$

Then,

$$\begin{aligned} \frac{G_1^2}{G_2} + \frac{G_2^2}{G_1} &= \frac{a^2 \left(\frac{b}{a}\right)^{\frac{2}{3}}}{a^{\frac{1}{3}} b^{\frac{2}{3}}} + \frac{a^{\frac{2}{3}} b^{\frac{4}{3}}}{a^2 \left(\frac{b}{a}\right)^{\frac{2}{3}}} \\ &= a^{2-\frac{2}{3}-\frac{1}{3}} b^{\frac{2}{3}-\frac{2}{3}} + a^{\frac{2}{3}-2+\frac{2}{3}} b^{\frac{4}{3}-\frac{2}{3}} \\ &= a^{\frac{3}{3}} b^0 + a^0 b \\ &= a + b \\ &= 2a \\ &= \text{RHS} \end{aligned}$$

Geometric Progressions Ex 20.6 Q 10

A.M. of root of quadratic equation is  $A$ .

G.M. of root of quadratic equation is  $G$ .

Then,

$$\frac{a+b}{2} = A, F = \sqrt{ab}$$

The equation having  $a$  and  $b$  as roots of quadratic equation is

$$x^2 - Sx + P = 0$$

$$x^2 - (a+b)x + ab = 0$$

$$x^2 - 2Ax + G^2 = 0$$

Geometric Progressions Ex 20.6 Q 11

Let  $a, b$  be the numbers.

$$a + b = 6 \text{ (G.M of } a, b)$$

$$a + b = 6\sqrt{ab}$$

$$\frac{a+b}{2\sqrt{ab}} = \frac{3}{1}$$

Applying componendo and dividendo,

$$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{3+1}{3-1}$$

$$\left( \frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}} \right)^2 = \frac{4}{2}$$

$$\frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}} = \frac{\sqrt{2}}{1}$$

Again applying componendo and dividendo,

$$\frac{\sqrt{a} + \sqrt{b} + \sqrt{a} - \sqrt{b}}{\sqrt{a} + \sqrt{b} - \sqrt{a} + \sqrt{b}} = \frac{\sqrt{2} + 1}{\sqrt{2} - 1}$$

$$\frac{2\sqrt{a}}{2\sqrt{b}} = \frac{\sqrt{2} + 1}{\sqrt{2} - 1}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{2} + 1}{\sqrt{2} - 1}$$

$$\left( \frac{a}{b} \right) = \left( \frac{\sqrt{2} + 1}{\sqrt{2} - 1} \right)^2$$

$$= \frac{2+1+2\sqrt{2}}{2+1-2\sqrt{2}}$$

$$= \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

$$a : b = (3+2\sqrt{2}) : (3-2\sqrt{2})$$

Geometric Progressions Ex 20.6 Q 12

Let quadratic equation be  $(x - \alpha)(x - \beta) = 0$

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

Roots are  $\alpha, \beta$

Here,

$$\frac{\alpha + \beta}{2} = 8, \sqrt{\alpha\beta} = 5$$

$$\alpha + \beta = 16, \alpha\beta = 25$$

$\therefore$  Required quadratic equation is,

$$x^2 - 16x + 25 = 0$$

Geometric Progressions Ex 20.6 Q 13

The AM and GM of  $a$  and  $b$  will be:

$$\frac{a+b}{2} = 10 \Rightarrow a+b = 20 \quad \text{.....(1)}$$

$$\sqrt{ab} = 8 \Rightarrow ab = 64$$

Now

$$\begin{aligned} a-b &= \sqrt{(a+b)^2 - 4ab} \\ &= \sqrt{20^2 - 4 \cdot 64} \\ &= \sqrt{400 - 256} \\ &= \sqrt{144} \end{aligned}$$

$$a-b = 12 \quad \text{.....(2)}$$

Adding (1) and (2)

$$2a = 32$$

$$a = 16$$

From (1)

$$b = 20 - 16 = 4$$

Thus the numbers are  $a = 16$  and  $b = 4$ .

[RD Sharma Class 11 Solutions](#)

\*\*\*\*\* END \*\*\*\*\*