**Exercise 7.7 :** Solutions of Questions on Page Number : **330**

**Q1 :** $\sqrt{4-x^2}$

**Answer :**

$$\text{Let } I = \int \sqrt{4-x^2}\,dx = \int \sqrt{(2)^2-(x)^2}\,dx$$

$$\text{It is known that, } \int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$$

$$\therefore I = \frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}+C$$

$$= \frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}+C$$

Answer needs Correction? **Click Here**

**Q2 :** $\sqrt{1-4x^2}$

**Answer :**

$$\text{Let } I = \int \sqrt{1-4x^2}\,dx = \int \sqrt{(1)^2-(2x)^2}\,dx$$

$$\text{Let } 2x = t \Rightarrow 2\,dx = dt$$

$$\therefore I = \frac{1}{2}\int \sqrt{(1)^2-(t)^2}\,dt$$

$$\text{It is known that, } \int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$$

$$\Rightarrow I = \frac{1}{2}\left[\frac{t}{2}\sqrt{1-t^2}+\frac{1}{2}\sin^{-1}t\right]+C$$

$$= \frac{t}{4}\sqrt{1-t^2}+\frac{1}{4}\sin^{-1}t+C$$

$$= \frac{2x}{4}\sqrt{1-4x^2}+\frac{1}{4}\sin^{-1}2x+C$$

$$= \frac{x}{2}\sqrt{1-4x^2}+\frac{1}{4}\sin^{-1}2x+C$$

Answer needs Correction? **Click Here**

**Q3 :** $\sqrt{x^2+4x+6}$

**Answer :**

$$\text{Let } I = \int \sqrt{x^2+4x+6}\;dx$$

$$= \int \sqrt{x^2+4x+4+2}\;dx$$

$$= \int \sqrt{\left(x^2+4x+4\right)+2}\;dx$$

$$= \int \sqrt{(x+2)^2+\left(\sqrt{2}\right)^2}\;dx$$

$$\text{It is known that, } \int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$$

$$\therefore I = \frac{(x+2)}{2}\sqrt{x^2+4x+6}+\frac{2}{2}\log\left|(x+2)+\sqrt{x^2+4x+6}\right|+C$$

$$= \frac{(x+2)}{2}\sqrt{x^2+4x+6}+\log\left|(x+2)+\sqrt{x^2+4x+6}\right|+C$$

Answer needs Correction? **Click Here**

**Q4 :** $\sqrt{x^2+4x+1}$

**Answer :**

$$\text{Let } I = \int \sqrt{x^2+4x+1}\,dx$$

$$= \int \sqrt{\left(x^2+4x+4\right)-3}\,dx$$

$$= \int \sqrt{(x+2)^2-\left(\sqrt{3}\right)^2}\,dx$$

$$\text{It is known that, } \int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$$

$$\therefore I=\frac{(x+2)}{2}\sqrt{x^2+4x+1}-\frac{3}{2}\log\left|(x+2)+\sqrt{x^2+4x+1}\right|+C$$

Answer needs Correction? Click Here

Q5 : $\sqrt{1-4x-x^2}$

Answer :

$$\text{Let } I=\int\sqrt{1-4x-x^2}\,dx$$
$$=\int\sqrt{1-\left(x^2+4x+4-4\right)}\,dx$$
$$=\int\sqrt{1+4-(x+2)^2}\,dx$$
$$=\int\sqrt{\left(\sqrt{5}\right)^2-(x+2)^2}\,dx$$

It is known that, $\int\sqrt{a^2-x^2}\,dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

$$\therefore I=\frac{(x+2)}{2}\sqrt{1-4x-x^2}+\frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+C$$

Answer needs Correction? Click Here

Q6 : $\sqrt{x^2+4x-5}$

Answer :

$$\text{Let } I=\int\sqrt{x^2+4x-5}\,dx$$
$$=\int\sqrt{\left(x^2+4x+4\right)-9}\,dx$$
$$=\int\sqrt{(x+2)^2-(3)^2}\,dx$$

It is known that, $\int\sqrt{x^2-a^2}\,dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$

$$\therefore I=\frac{(x+2)}{2}\sqrt{x^2+4x-5}-\frac{9}{2}\log\left|(x+2)+\sqrt{x^2+4x-5}\right|+C$$

Answer needs Correction? Click Here

Q7 : $\sqrt{1+3x-x^2}$

Answer :

$$\text{Let } I=\int\sqrt{1+3x-x^2}\,dx$$
$$=\int\sqrt{1-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)}\,dx$$
$$=\int\sqrt{\left(1+\frac{9}{4}\right)-\left(x-\frac{3}{2}\right)^2}\,dx$$
$$=\int\sqrt{\left(\frac{\sqrt{13}}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}\,dx$$

It is known that, $\int\sqrt{a^2-x^2}\,dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

$$\therefore I=\frac{x-\frac{3}{2}}{2}\sqrt{1+3x-x^2}+\frac{13}{4\times 2}\sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{13}}{2}}\right)+C$$
$$=\frac{2x-3}{4}\sqrt{1+3x-x^2}+\frac{13}{8}\sin^{-1}\left(\frac{2x-3}{\sqrt{13}}\right)+C$$

Answer needs Correction? Click Here

Q8 : $\sqrt{x^2+3x}$

Answer :

$$\text{Let } I=\int\sqrt{x^2+3x}\,dx$$
$$=\int\sqrt{x^2+3x+\frac{9}{4}-\frac{9}{4}}\,dx$$
$$=\int\sqrt{\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2}\,dx$$

It is known that, $\int\sqrt{x^2-a^2}\,dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$

$$\therefore I=\frac{\left(x+\frac{3}{2}\right)}{2}\sqrt{x^2+3x}-\frac{\frac{9}{4}}{2}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{x^2+3x}\right|+C$$
$$=\frac{(2x+3)}{4}\sqrt{x^2+3x}-\frac{9}{8}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{x^2+3x}\right|+C$$

Answer needs Correction? Click Here

Q9 : $\sqrt{1+\frac{x^2}{9}}$

Answer :

Let $I=\int\sqrt{1+\frac{x^2}{9}}dx=\frac{1}{3}\int\sqrt{9+x^2}\,dx=\frac{1}{3}\int\sqrt{(3)^2+x^2}\,dx$

It is known that, $\int\sqrt{x^2+a^2}dx=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$

$$\therefore I=\frac{1}{3}\left[\frac{x}{2}\sqrt{x^2+9}+\frac{9}{2}\log\left|x+\sqrt{x^2+9}\right|\right]+C$$
$$=\frac{x}{6}\sqrt{x^2+9}+\frac{3}{2}\log\left|x+\sqrt{x^2+9}\right|+C$$

Answer needs Correction? Click Here

Q10 : $\int\sqrt{1+x^2}\,dx$ is equal to

A. $\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+C$

B. $\frac{2}{3}\left(1+x^2\right)^{\frac{2}{3}}+C$

C. $\frac{2}{3}x\left(1+x^2\right)^{\frac{3}{2}}+C$

D. $\frac{x^2}{2}\sqrt{1+x^2}+\frac{1}{2}x^2\log\left|x+\sqrt{1+x^2}\right|+C$

Answer :

It is known that, $\int\sqrt{a^2+x^2}dx=\frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$

$\therefore \int\sqrt{1+x^2}dx=\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+C$

Hence, the correct answer is A.

Answer needs Correction? Click Here

Q11 : $\int\sqrt{x^2-8x+7}dx$ is equal to

A. $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}+9\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

B. $\frac{1}{2}(x+4)\sqrt{x^2-8x+7}+9\log\left|x+4+\sqrt{x^2-8x+7}\right|+C$

C. $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-3\sqrt{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

D. $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$

Answer :

$$\text{Let } I=\int\sqrt{x^2-8x+7}\,dx$$
$$=\int\sqrt{\left(x^2-8x+16\right)-9}\,dx$$
$$=\int\sqrt{(x-4)^2-(3)^2}\,dx$$

It is known that, $\int\sqrt{x^2-a^2}dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$

$\therefore I=\frac{(x-4)}{2}\sqrt{x^2-8x+7}-\frac{9}{2}\log\left|(x-4)+\sqrt{x^2-8x+7}\right|+C$

Hence, the correct answer is D.

Answer needs Correction? Click Here

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