



Mean Value Theorems Ex 15.2 Q2

Here,

$$f(x) = |x| \text{ on } [-1, 1]$$
$$f(x) = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

For differentiability at  $x = 0$

$$\begin{aligned} \text{LHD} &= \lim_{x \rightarrow 0^-} \frac{f(0-h) - f(0)}{-h} \\ &= \lim_{h \rightarrow 0} \frac{-(0-h) - 0}{-h} \\ &= \lim_{h \rightarrow 0} \frac{h}{-h} \end{aligned}$$

$$\text{LHD} = -1$$

$$\begin{aligned} \text{RHD} &= \lim_{x \rightarrow 0^+} \frac{f(0+h) - f(0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(0+h) - 0}{h} \\ &= \lim_{h \rightarrow 0} \frac{h}{h} \\ &= 1 \end{aligned}$$

$$\therefore \text{LHD} \neq \text{RHD}$$

$$\Rightarrow f(x) \text{ is not differentiable at } x = 0 \in (-1, 1)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q3

Here,

$$f(x) = \frac{1}{x} \text{ on } [-1, 1]$$
$$f'(x) = -\frac{1}{x^2}$$

$$\Rightarrow f'(x) \text{ doesnot exist at } x = 0 \in (-1, 1)$$

$$\Rightarrow f(x) \text{ is not differentiable in } (-1, 1)$$

Hence, **LMVT** is verified

Mean Value Theorems Ex 15.2 Q4

Here,

$$f(x) = \frac{1}{4x-1}, x \in [1, 4]$$

$f(x)$  attain unique value for each  $x \in [1, 4]$ , so  $f(x)$  is continuous in  $[1, 4]$ .

$$f'(x) = -\frac{4}{(4x-1)^2}$$

$\Rightarrow f'(x)$  exists for each  $x \in (1, 4)$

$\Rightarrow f'(x)$  is differentiable in  $(1, 4)$

So, Lagranges mean value theroem is applicable.

So, there exist a point  $c \in (1, 4)$  such that,

$$f'(c) = \frac{f(4) - f(1)}{4 - 1}$$

$$\Rightarrow -\frac{4}{(4x-1)^2} = \frac{\frac{1}{15} - \frac{1}{3}}{3}$$

$$\Rightarrow -\frac{4}{(4x-1)^2} = -\frac{4}{45}$$

$$\Rightarrow (4x-1)^2 = 45$$

$$\Rightarrow 4x-1 = \pm 3\sqrt{5}$$

$$\Rightarrow x = \frac{3\sqrt{5}+1}{4} \in [1, 4]$$

\*\*\*\*\* END \*\*\*\*\*