Differentiation Ex 11.7 Q20

Here,

$$x = \left(t + \frac{1}{t}\right)^a$$

Differentiating it with respect to t using chain rule,

$$\frac{dx}{dt} = \frac{d}{dt}\left(\left(t + \frac{1}{t}\right)^a\right)$$

$$= a\left(t + \frac{1}{t}\right)^{a-1} \frac{d}{dt}\left(t + \frac{1}{t}\right)$$

$$\frac{dx}{dt} = a\left(t + \frac{1}{t}\right)^{1-1}\left(1 - \frac{1}{t^2}\right) \qquad \text{---(i)}$$

And, $y = a^{\left(t+\frac{1}{t}\right)}$

Differentiating it with respect to t using chain rule,

$$\frac{dy}{dt} = \frac{d}{dt}\left(a^{\left(t+\frac{1}{t}\right)}\right)$$

$$= a^{\left(t+\frac{1}{t}\right)} \times \log a \frac{d}{dt}\left(t + \frac{1}{t}\right)$$

$$\frac{dy}{dt} = a^{\left(t+\frac{1}{t}\right)} \times \log a\left(1 - \frac{1}{t^2}\right) \qquad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a^{\left(t+\frac{1}{t}\right)} \times \log a\left(1 - \frac{1}{t^2}\right)}{a\left(t + \frac{1}{t}\right)^{a-1}\left(1 - \frac{1}{t^2}\right)}$$

$$\frac{dy}{dx} = \frac{a^{\left(t+\frac{1}{t}\right)} \times \log a}{a\left(t + \frac{1}{t}\right)^{a-1}}$$

Differentiation Ex 11.7 Q21

Here,

$$x = a\left(\frac{1+t^2}{1-t^2}\right)$$

Differentiating it with respect to t using chain rule,

$$\frac{dx}{dt} = a\left[\frac{\left(1+t^2\right)\frac{d}{dt}\left(1+t^2\right) - \left(1+t^2\right)\frac{d}{dt}\left(1-t^2\right)}{\left(1-t^2\right)^2}\right]$$

$$= a\left[\frac{\left(1-t^2\right)(2t) - \left(1+t^2\right)(-2t)}{\left(1-t^2\right)^2}\right]$$

$$= a\left[\frac{2t - 2t^2 + 2t + 2t^3}{\left(1-t^2\right)^2}\right]$$

$$\frac{dy}{dt} = \frac{4at}{\left(1-t^2\right)^2} \qquad \text{---(i)}$$

And, $\quad y = \dfrac{2t}{1-t^2}$

Differentiating it with respect to t using quotient rule,

$$\frac{dy}{dt} = 2\left[\frac{\left(1-t^2\right)\frac{d}{dt}(t) - t\frac{d}{dt}\left(1-t^2\right)}{\left(1-t^2\right)^2}\right]$$

$$= 2\left[\frac{\left(1-t^2\right)(1) - t(-2t)}{\left(1-t^2\right)^2}\right]$$

$$= 2\left[\frac{1-t^2+2t^2}{\left(1-t^2\right)^2}\right]$$

$$\frac{dy}{dt} = \frac{2\left(1+t^2\right)}{\left(1-t^2\right)} \qquad \text{---(ii)}$$

Differentiation Ex 11.7 Q22

It is given that, $y = 12(1-\cos t), x = 10(t-\sin t)$

$$\therefore \frac{dx}{dt} = \frac{d}{dt}\left[10(t-\sin t)\right] = 10 \cdot \frac{d}{dt}(t-\sin t) = 10(1-\cos t)$$

$$\frac{dy}{dt} = \frac{d}{dt}\left[12(1-\cos t)\right] = 12 \cdot \frac{d}{dt}(1-\cos t) = 12 \cdot \left[0-(-\sin t)\right] = 12\sin t$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{12\sin t}{10(1-\cos t)} = \frac{12 \cdot 2\sin\frac{t}{2} \cdot \cos\frac{t}{2}}{10 \cdot 2\sin^2\frac{t}{2}} = \frac{6}{5}\cot\frac{t}{2}$$

Differentiation Ex 11.7 Q23

Here $x = a(\theta - \sin\theta)$ and $y = a(1+\cos\theta)$

Then,

$$\frac{dx}{d\theta} = \frac{d}{d\theta}\left[a(\theta - \sin\theta)\right] = a(1-\cos\theta)$$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}\left[a(1+\cos\theta)\right] = a(-\sin\theta)$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \left.\frac{-a\sin\theta}{a(1-\cos\theta)}\right|_{\theta=\frac{\pi}{3}} = -\frac{\sin\frac{\pi}{3}}{1-\cos\frac{\pi}{3}} = \frac{\frac{\sqrt{3}}{2}}{1-\frac{1}{2}} = -\sqrt{3}$$

Differentiation Ex 11.7 Q24

Consider the given functions,

$x = a \sin 2t \ (1 + \cos 2t)$ and $y = b \cos 2t \ (1-\cos 2t)$

Rewriting the above function, we have,

$$x = a \sin 2t + \frac{a}{2}\sin 4t$$

Differentiating the above function w.r.t. 't', we have,

$$\frac{dx}{dt} = 2a\cos 2t + 2a\cos 4t \ldots (1)$$

$y = b \cos 2t \ (1-\cos 2t)$

$y = b \cos 2t - b\cos^2 2t$

$$\frac{dy}{dt} = -2b\sin 2t + 2b\cos 2t \sin 2t = -2b\sin 2t + b\sin 4t \ldots (2)$$

From (1) and (2),

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2b\sin 2t + b\sin 4t}{2a\cos 2t + 2a\cos 4t}$$

$$\therefore \left.\frac{dy}{dx}\right|_{\pi/4} = \left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_{t=\pi/4} = \frac{-2b}{-2a} = \frac{b}{a}$$

*********** END **********