Exercise 13.5

Ans. Slant height of the frustum of the cone

$$(l)=\sqrt{h^2+(r_1-r_2)^2}$$

$$=\sqrt{(22-10)^2+\left(\frac{18}{2}-\frac{8}{2}\right)^2} = 13 \text{ cm}$$

Area of the tin sheet required

= CSA of cylinder + CSA of the frustum

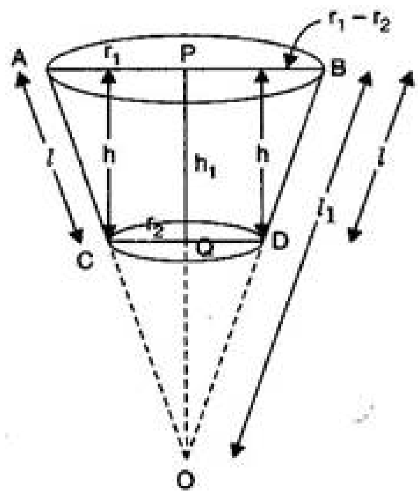
$$=2\pi(4)(10)+\pi(4+9)13$$

$$=80\pi+169\pi$$

$$=249\pi = 249\times\frac{22}{7} = 782\frac{4}{7}\,cm^2$$

6. Derive the formula for the curved surface area and total surface area of the frustum of a cone, given to you in Section 13.5, using the symbols as explained.

Ans. According to the question, the frustum if difference of the two cones OAB and OCD (in figure).



For frustum, height = h, slant height = l and radii of the bases = r_1 and r_2 $(r_1 > r_2)$

OP = h_1, OA = OB = l

$\therefore$ Height of the cone = $h_1 - h$

$\because$ Δ OQD ~ Δ OPB [AA similarity]

$\therefore$ $\dfrac{h_1 - h}{h_1} = \dfrac{r_2}{r_1}$

$\Rightarrow 1 - \dfrac{h}{h_1} = \dfrac{r_2}{r_1}$

$\Rightarrow 1 - \dfrac{r_2}{r_1} = \dfrac{h}{h_1}$

$\Rightarrow h_1 = \dfrac{hr_1}{r_1 - r_2}$ ....(i)

$\therefore$ height of the cone OCD = $h_1 - h$

$= \dfrac{hr_1}{r_1 - r_2} - h = \dfrac{hr_2}{r_1 - r_2}$ ....(ii)

$\therefore$ height of the cone OCD = $h_1 - h$

$$= \frac{hr_1}{r_1 - r_2} - h = \frac{hr_2}{r_1 - r_2} \quad \text{..........(ii)}$$

$\therefore$ V of the frustum

= V of cone OAB – V of cone OCD

$$= \frac{1}{3}\pi r_1^2 h_1 - \frac{1}{3}\pi r_2^2 (h_1 - h)$$

$$= \frac{\pi}{3}\left[r_1^2 . \frac{hr_1}{r_1 - r_2} - r_2^2 . \frac{hr_2}{r_1 - r_2}\right]$$

[From eq. (i) & (ii)]

$$= \frac{\pi h}{3}\left(\frac{r_1^3 - r_2^3}{r_1 - r_2}\right)$$

$$= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

7. Derive the formula for the volume for the volume of the frustum of a cone, given to you in Section 13.5, using the symbols as explained.

Ans. If $A_1 and\ A_2$ are the surface areas of two circular bases, then

$A_1 = \pi r_1^2$ and $A_2 = \pi r_2^2$

$\therefore$ V of the frustum

$$= \frac{h}{3}\left(\pi r_1^2 + \pi r_2^2 + \sqrt{\pi r_1^2}.\sqrt{\pi r_2^2}\right)$$

$$= \frac{h}{3}\left(A_1 + A_2 + \sqrt{A_1 A_2}\right)$$

Again, from ΔDEB, $l = \sqrt{h^2 + (r_1 - r_2)^2}$

$\because$ ΔOQD ~ ΔOPB [AA similarity]

$$\therefore \quad \frac{l_1 - l}{l_1} = \frac{r_2}{r_1}$$

$$\Rightarrow l_1 = \frac{lr_1}{r_1 - r_2} \quad ....(iii)$$

$$\therefore \; l_1 - l = \frac{lr_1}{r_1 - r_2} - l = \frac{lr_2}{r_1 - r_2} \quad ....(iv)$$

Hence, CSA of the frustum of the cone = $\pi r_1 l_1 - \pi r_2 (l_1 - l)$

$$= \pi r_1 . \frac{lr_1}{r_1 - r_2} - \pi r_2 \frac{lr_2}{r_1 - r_2}$$

[From eq. (i) and (ii)]

$$= \pi l \left(\frac{r_1^2 - r_2^2}{r_1 - r_2}\right) = \pi l (r_1 + r_2),$$

where $l = \sqrt{h^2 + (r_1 - r_2)^2}$

$\therefore$ TSA of the frustum of the cone

$$= \pi l (r_1 + r_2) + \pi r_1^2 + \pi r_2^2$$

*********** END **********