ω_2 = Angular frequency for capacitor C_2

$$= 2\pi\nu_2 = 2\pi \times 1200 \times 10^3 \text{ rad s}^{-1}$$

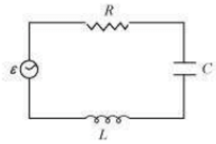
$$\therefore C_2 = \frac{1}{\left(2\pi \times 1200 \times 10^3\right)^2 \times 200 \times 10^{-6}}$$

$$= 88.04 \text{ pF}$$

Hence, the range of the variable capacitor is from 88.04 pF to 198.1 pF.

Question 7.11:

Figure 7.21 shows a series *LCR* circuit connected to a variable frequency 230 V source. L = 5.0 H, C = 80μF, R = 40 Ω



(a) Determine the source frequency which drives the circuit in resonance.

(b) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.

(c) Determine the rms potential drops across the three elements of the circuit. Show that the potential drop across the *LC* combination is zero at the resonating frequency.

Answer

Inductance of the inductor, L = 5.0 H

Capacitance of the capacitor, C = 80 μH = 80 × 10^{-6} F

Resistance of the resistor, R = 40 Ω

Potential of the variable voltage source, V = 230 V

(a) Resonance angular frequency is given as:

$$\omega_R = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = \frac{10^3}{20} = 50 \text{ rad/s}$$

Hence, the circuit will come in resonance for a source frequency of 50 rad/s.

(b) Impedance of the circuit is given by the relation,

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

At resonance,

$$\omega L = \frac{1}{\omega C}$$

$$\therefore Z = R = 40\ \Omega$$

Amplitude of the current at the resonating frequency is given as: $I_0 = \frac{V_0}{Z}$

Where,

V_0 = Peak voltage

$$= \sqrt{2}V$$

$$\therefore I_0 = \frac{\sqrt{2}V}{Z}$$

$$= \frac{\sqrt{2} \times 230}{40} = 8.13 \text{ A}$$

Hence, at resonance, the impedance of the circuit is 40 Ω and the amplitude of the current is 8.13 A.

(c) Rms potential drop across the inductor,

$(V_L)_{rms}$ = I × $\omega_R L$

Where,

I = rms current

$$= \frac{I_0}{\sqrt{2}} = \frac{\sqrt{2}V}{\sqrt{2}Z} = \frac{230}{40} \text{ A}$$

$$\therefore (V_L)_{\text{rms}} = \frac{230}{40} \times 50 \times 5 = 1437.5 \text{ V}$$

Potential drop across the capacitor,

$$(V_c)_{\text{rms}} = I \times \frac{1}{\omega_R C}$$

$$= \frac{230}{40} \times \frac{1}{50 \times 80 \times 10^{-6}} = 1437.5 \text{ V}$$

Potential drop across the resistor,

$(V_R)_{rms} = IR$

$= \frac{230}{40} \times 40 = 230$ V

Potential drop across the LC combination,

$$V_{LC} = I\left(\omega_R L - \frac{1}{\omega_R C}\right)$$

At resonance, $\omega_R L = \frac{1}{\omega_R C}$

$\therefore V_{LC} = 0$

Hence, it is proved that the potential drop across the LC combination is zero at resonating frequency.

Question 7.12:

An LC circuit contains a 20 mH inductor and a 50 μF capacitor with an initial charge of 10 mC. The resistance of the circuit is negligible. Let the instant the circuit is closed be t = 0.

(a) What is the total energy stored initially? Is it conserved during LC oscillations?

(b) What is the natural frequency of the circuit?

(c) At what time is the energy stored

(i) completely electrical (i.e., stored in the capacitor)? (ii) completely magnetic (i.e., stored in the inductor)?

(d) At what times is the total energy shared equally between the inductor and the capacitor?

(e) If a resistor is inserted in the circuit, how much energy is eventually dissipated as heat?

Answer

Inductance of the inductor, $L = 20 \text{ mH} = 20 \times 10^{-3}$ H

Capacitance of the capacitor, $C = 50 \text{ μF} = 50 \times 10^{-6}$ F

Initial charge on the capacitor, $Q = 10 \text{ mC} = 10 \times 10^{-3}$ C

(a) Total energy stored initially in the circuit is given as:

$$E = \frac{1}{2}\frac{Q^2}{C}$$

$$= \frac{(10 \times 10^{-3})^2}{2 \times 50 \times 10^{-6}} = 1 \text{ J}$$

Hence, the total energy stored in the LC circuit will be conserved because there is no resistor connected in the circuit.

(b) Natural frequency of the circuit is given by the relation,

$$\nu = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{20 \times 10^{-3} \times 50 \times 10^{-6}}}$$

$$= \frac{10^3}{2\pi} = 159.24 \text{ Hz}$$

Natural angular frequency,

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{20 \times 10^{-3} \times 50 \times 10^{-6}}} = \frac{1}{\sqrt{10^{-6}}} = 10^3 \text{ rad/s}$$

Hence, the natural frequency of the circuit is 10^3 rad/s.

(c) (i) For time period ($T = \frac{1}{\nu} = \frac{1}{159.24} = 6.28 \text{ ms}$), total charge on the capacitor at time t, $Q' = Q\cos\frac{2\pi}{T}t$

For energy stored is electrical, we can write $Q' = Q$.

Hence, it can be inferred that the energy stored in the capacitor is completely electrical at time, $t = 0, \frac{T}{2}, T, \frac{3T}{2}, \ldots\ldots$

(ii) Magnetic energy is the maximum when electrical energy, Q' is equal to 0.

Hence, it can be inferred that the energy stored in the capacitor is completely magnetic

Hence, it can be inferred that the energy stored in the capacitor is completely magnetic

at time, $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4} ......$

(d) Q^1 = Charge on the capacitor when total energy is equally shared between the capacitor and the inductor at time *t*.

When total energy is equally shared between the inductor and capacitor, the energy stored in the capacitor = $\frac{1}{2}$ (maximum energy).

$$\Rightarrow \frac{1}{2}\frac{(Q^1)^2}{C} = \frac{1}{2}\left(\frac{1}{2}\frac{Q^2}{C}\right) = \frac{1}{4}\frac{Q^2}{C}$$

$$Q^1 = \frac{Q}{\sqrt{2}}$$

$$\text{But } Q^1 = Q\cos\frac{2\pi}{T}t.$$

$$\frac{Q}{\sqrt{2}} = Q\cos\frac{2\pi}{T}t$$

$$\cos\frac{2\pi}{T}t = \frac{1}{\sqrt{2}} = \cos(2n+1)\frac{\pi}{4}; \qquad \text{where } n = 0, 1, 2, ...$$

$$t = (2n+1)\frac{T}{8}$$

Hence, total energy is equally shared between the inductor and the capacity at time,

$$t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8} ......$$

(e) If a resistor is inserted in the circuit, then total initial energy is dissipated as heat energy in the circuit. The resistance damps out the *LC* oscillation.

Question 7.13:

A coil of inductance 0.50 H and resistance 100 Ω is connected to a 240 V, 50 Hz ac supply.

(a) What is the maximum current in the coil?

(b) What is the time lag between the voltage maximum and the current maximum?

Answer

Inductance of the inductor, L = 0.50 H

Resistance of the resistor, R = 100 Ω

Potential of the supply voltage, V = 240 V

Frequency of the supply, ν = 50 Hz

(a) Peak voltage is given as:

$$V_0 = \sqrt{2}V$$

$$= \sqrt{2} \times 240 = 339.41 \text{ V}$$

Angular frequency of the supply,

$\omega = 2\pi\nu$

$= 2\pi \times 50 = 100\pi$ rad/s

Maximum current in the circuit is given as:

$$I_0 = \frac{V_0}{\sqrt{R^2 + \omega^2 L^2}}$$

$$= \frac{339.41}{\sqrt{(100)^2 + (100\pi)^2 (0.50)^2}} = 1.82 \text{ A}$$

(b) Equation for voltage is given as:

$V = V_0 \cos \omega t$

Equation for current is given as:

$I = I_0 \cos(\omega t - \Phi)$

Where,

Φ = Phase difference between voltage and current

At time, t = 0.

$V = V_0$(voltage is maximum)

For $\omega t - \Phi = 0$ i.e., at time $t = \frac{\phi}{\omega}$,

$I = I_0$ (current is maximum)

Hence, the time lag between maximum voltage and maximum current is $\frac{\phi}{\omega}$.

Now, phase angle Φ is given by the relation,

*********** END **********