Algebraic Identities Ex 4.3 Q16

Answer :

In the given problem, we have to find the value of $x^3+\frac{1}{x^3}, x^2+\frac{1}{x^2}, x+\frac{1}{x}$

Given $x^4+\frac{1}{x^4}=194$

By adding and subtracting $2\times x^2\times\frac{1}{x^2}$ in left hand side of $x^4+\frac{1}{x^4}=194$ we get,

$$x^4+\frac{1}{x^4}+2\times x^2\times\frac{1}{x^2}-2\times x^2\times\frac{1}{x^2}=194$$

$$x^4+\frac{1}{x^4}+2\times x^2\times\frac{1}{x^2}-2\times\left(\cancel{x^2}\times\frac{1}{\cancel{x^2}}\right)=194$$

$$\left(x^2+\frac{1}{x^2}\right)^2-2=194$$

$$\left(x^2+\frac{1}{x^2}\right)^2=194+2$$

$$\left(x^2+\frac{1}{x^2}\right)^2=196$$

$$\left(x^2+\frac{1}{x^2}\right)^2=(14)^2$$

$$\left(x^2+\frac{1}{x^2}\right)=14$$

Again by adding and subtracting $2\times x\times\frac{1}{x}$ in left hand side of $\left(x^2+\frac{1}{x^2}\right)=14$ we get,

$$x^2+\frac{1}{x^2}+2\times x\times\frac{1}{x}-2\times x\times\frac{1}{x}=14$$

$$\left(x+\frac{1}{x}\right)^2-2\times\cancel{x}\times\frac{1}{\cancel{x}}=14$$

$$\left(x+\frac{1}{x}\right)^2-2=14$$

$$\left(x+\frac{1}{x}\right)^2=14+2$$

$$\left(x+\frac{1}{x}\right)^2=16$$

$$\left(x+\frac{1}{x}\right)^2=4\times 4$$

$$\left(x+\frac{1}{x}\right)=4$$

Now cubing on both sides of $\left(x+\frac{1}{x}\right)=4$ we get

$$\left(x+\frac{1}{x}\right)^3=4^3$$

we shall use identity $(a+b)^3 = a^3+b^3+3ab(a+b)$

$$x^3+\frac{1}{x^3}+3\times x\times\frac{1}{x}\left(x+\frac{1}{x}\right)=4\times4\times4$$

$$x^3+\frac{1}{x^3}+3\times\cancel{x}\times\frac{1}{\cancel{x}}\times4=64$$

$$x^3+\frac{1}{x^3}+12=64$$

$$x^3+\frac{1}{x^3}=64-12$$

$$x^3+\frac{1}{x^3}=52$$

Hence the value of $x^3+\frac{1}{x^3}, x^2+\frac{1}{x^2}, x+\frac{1}{x}$ is $\boxed{52,14,4}$ respectively.

Algebraic Identities Ex 4.3 Q17

Answer :

In the given problem, we have to find the value of $27x^3+8y^3$

(i) Given $3x+2y=14, xy=8$

On cubing both sides we get,

$$(3x+2y)^3=(14)^3$$

We shall use identity $(a+b)^3=a^3+b^3+3ab(a+b)$

$$27x^3+8y^3+3(3x)(2y)(3x+2y)=14\times14\times14$$

$$27x^3+8y^3+18(xy)(3x+2y)=14\times14\times14$$

$$27x^3+8y^3+18(8)(14)=2744$$

$$27x^3+8y^3+2016=2744$$

$$27x^3+8y^3=2744-2016$$

$$27x^3+8y^3=728$$

Hence the value of $27x^3+8y^3$ is $\boxed{728}$

(ii) Given $3x+2y=20, xy=\frac{14}{9}$

On cubing both sides we get,

$$(3x+2y)^3=(20)^3$$

We shall use identity $(a+b)^3=a^3+b^3+3ab(a+b)$

$$27x^3+8y^3+3(3x)(2y)(3x+2y)=20\times20\times20$$

$$27x^3+8y^3+18(xy)(3x+2y)=8000$$

$$27x^3+8y^3+18\left(\frac{14}{9}\right)(20)=8000$$

$$27x^3+8y^3+560=8000$$

$$27x^3+8y^3=8000-560$$

$$27x^3+8y^3=7440$$

Hence the value of $27x^3+8y^3$ is $\boxed{7440}$.

*********** END **********