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Question 15. 1. A string of mass 2.50 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end?

Answer:

Tension, $T = 200\text{ N};$

Length, $l = 20.0\text{ m};\quad$ Mass, $M = 2.50\text{ kg}$

Mass per unit length, $\mu = \dfrac{2.50}{20.0}\text{ kg m}^{-1} = 0.125\text{ kgm}^{-1}$

Wave velocity, $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{200\ N}{0.125\text{ kg m}^{-1}}}$

or $v = \sqrt{1600}\text{ ms}^{-1} = 40\text{ ms}^{-1}$

Time, $t = \dfrac{l}{v} = \dfrac{20.0}{40}s = \dfrac{1}{2}s = 0.5\ s.$

Question 15. 2. A stone dropped from the top of a tower of height 300 m high splashes into the water of a pond near the base of the tower. When is the splash heard at the top given that the speed of sound in air is 340 ms^{-1}? ($g = 9.8\ ms^{-2}$)

Answer:

Here, h = 300 m, g = 9.8 ms^{-2} and velocity of sound, v = 340 ms^{-1}

Let t_1 be the time taken by the stone to reach at the surface of pond.

Then, using $s = ut + \frac{1}{2}at^2 \frac{1}{2}at^2 \Rightarrow h = 0 \times t + \frac{1}{2}gt_1^2$

$\therefore\quad t_1 = \sqrt{\dfrac{2 \times 300}{9.8}} = 7.82\text{ s}$

Also, if t_2 is the time taken by the sound to reach at a height h, then

$$t_2 = \frac{h}{v} = \frac{300}{340} = 0.88\text{ s}$$

$\therefore$ Total time after which sound of splash is heard $= t_1 + t_2$

$= 7.82 + 0.88 = 8.7\text{s}.$

Question 15. 3. A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that speed of a transverse wave on the wire equals the speed of sound in dry air at 2 0°C = 340 ms^{-1}.

Answer:

Here, $l = 12.0$ m, $M = 2.10$ kg

$v = 343 \text{ ms}^{-1}$

$$\text{Mass per unit length} = \frac{M}{l} = \frac{2.10}{12.0} = 0.175 \text{ kg m}^{-1}$$

As $v = \sqrt{\frac{T}{m}}$

$\therefore$ $T = v^2 . \text{m} = (343)^2 \times 0.175 = 2.06 \times 10^4$ N.

Question 15. 4.

Use the formula $v = \sqrt{\frac{\gamma P}{\rho}}$ to explain why the speed of sound in air

(a) is independent of pressure. *(b) increases with temperature.*

(c) increases with humidity.

Answer:

We are given that $v = \sqrt{\frac{\gamma P}{\rho}}$

We know $PV = nRT$ (for n moles of ideal gas)

$\Rightarrow$ $PV = \frac{m}{M} RT$

where m is the total mass and M is the molecular mass of the gas.

$$\therefore \quad P = \frac{m}{M} \cdot \frac{RT}{M} = \frac{\rho RT}{M} \Rightarrow \frac{P}{\rho} = \frac{RT}{M}$$

(*a*) For a gas at constant temperature, $\frac{P}{\rho}$ = constant

$\therefore$ As P increase, ρ also increases and vice versa. This implies that $v = \sqrt{\frac{\gamma P}{\rho}}$ = constant, *i.e.,* velocity is independent of pressure of the gas.

(*b*) Since $\frac{P}{\rho} = \frac{RT}{M}$, therfore, $v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$

Clearly $v \propto \sqrt{T}$ *i.e.,* speed of sound in air increases with increase in temperature.

(*c*) Increase in humidity decreases the effective density of air. Therefore the velocity $\left(v \propto \frac{1}{\sqrt{\rho}}\right)$ increases.

Question 15. 5. You have learnt, that a travelling wave in one dimension is represented by a function y = f(x, t), where x and t must appear in the combination x-vt or x + vt i.e., y =f (x ± vt). Is the converse true? That is, does every function of (x - vt) or (x + vt) represent a travelling wave? Examine, if the following functions for y can possibly represent a travelling wave?

$$(a)\ (x - vt)^2 \quad (b)\ \log\left[\frac{(x+vt)}{x_0}\right] \quad (c)\ \frac{1}{x+vt}$$

Answer: No, the converse is not true. The basic requirement for a wave function to represent a travelling wave is that for all values of x and t, wave function must have a finite value. Out of the given functions for y none satisfies this condition. Therefore, none can represent a travelling wave.

Question 15. 6. A bat emits ultrasonic sound of frequency 1000 kHz in air. If this sound meets a water surface, what is the wavelength of (a) the reflected sound, (b) the transmitted sound? Speed of sound in air = 340 ms^{-1} and in water = 1486 ms^{-1}.

Answer:

Here, $\nu = 1000 \times 10^3$ Hz $= 10^6$ Hz, $v_a = 340\ ms^{-1}$,

$v_w = 1486\ ms^{-1}$

Wavelength of reflected sound, λ_a

$$= \frac{v_a}{\nu} = \frac{340}{10^6} m = 3.4 \times 10^{-4} m$$

Wavelength of transmitted sound, λ_ω

$$= \frac{v_w}{\nu} = \frac{1486}{10^6} m = 1.486 \times 10^{-3} m$$

Question 15. 7. A hospital uses an ultrasonic scanner to locate tumours in a tissue. What is the wavelength of sound in a tissue in which the speed of sound is 1.7 km s^{-1}? The operating frequency of the scanner is 4.2 MHz.

Answer:

Here speed of sound ⇒ $\upsilon = 1.7$ km $s^{-1} = 1700\ ms^{-1}$ and

frequency $\upsilon = 4.2$ MHz $= 4.2 \times 10^6$ Hz

∴ Wavelength, A = υ/V = 1700/(4.2 x 106) =4.1×10^{-4} m.

Question 15. 8. A transverse harmonic wave on a string is described by

y(x, t) = 3.0 sin (36t + 0.018x + π/4)

where x and y are in cm and t in s. The positive direction of x is from left to right.

(a) Is this a travelling wave or a stationary wave? If it is travelling, what are the speed and direction of its propagation ?

(b) What are its amplitude and frequency?

(c) What is the initial phase at the origin?

(d) What is the least distance between two successive crests in the wave?

Answer:

The given equation is $y(x, t) = 3.0 \sin (36t + 0.018x + \frac{\pi}{4})$, where x and y are in cm and t in s.

(*a*) The equation is the equation of a travelling wave, travelling from right to left (*i.e.,* along $-ve$ direction of x because it is an equation of the type

$$y(x, t) = A \sin (\omega t + kx + \phi)$$

Here, $A = 3.0$ cm, $\omega = 36$ rad s^{-1}, $k = 0.018$ cm and $\phi = \frac{\pi}{4}$.

∴ Speed of wave propagation,

$$v = \frac{\omega}{k} = \frac{36 \text{ rad s}^{-1}}{0.018 \text{ cm}^{-1}} = \frac{36 \text{ rad s}^{-1}}{0.018 \times 10^2 \text{ ms}^{-1}} = 20 \text{ ms}^{-1}$$

(*b*) Amplitude of wave, $A = 3.0$ cm $= 0.03$ m

Frequency of wave $\nu = \frac{\omega}{2\pi} = \frac{36}{2\pi} = 5.7$ Hz

(*c*) Initial phase at the origin, $\phi = \frac{\pi}{4}$

(*d*) Least distance between two successive crests in the wave

$$= \lambda = \frac{2\pi}{k} = \frac{2\pi}{0.018}$$

$$= 349 \text{ cm} = 3.5 \text{ m}$$

Question 15. 9. For the wave described in Exercise 8, plot the displacement (y) versus (t) graphs for x = 0, 2 and 4 cm. What are the shape of these graphs? In which aspects does the oscillatory motion in travelling wave differ from one point to another : amplitude, frequency or phase?

Answer: The transverse harmonic wave is

$$y(x, t) = 3.0 \sin\left(36t + 0.018x + \frac{\pi}{4}\right)$$

For $x = 0$,

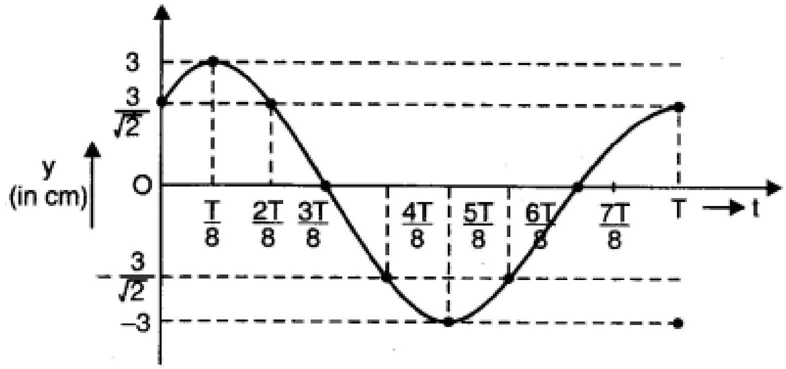
$$y(0, t) = 3 \sin\left(36t + 0 + \frac{\pi}{4}\right) = 3 \sin\left(36t + \frac{\pi}{4}\right) \quad ...(1)$$

Here $\omega = \frac{2\pi}{T} = 36 \Rightarrow T = \frac{2\pi}{36}$

To plot a (y) versus (t) graph, different values of y corresponding to the values of t may be tabulated as under (by making use of eqn. (1)).

t	0	$\frac{T}{8}$	$\frac{2T}{8}$	$\frac{3T}{8}$	$\frac{4T}{8}$	$\frac{5T}{8}$	$\frac{6T}{8}$	$\frac{7T}{8}$	T
y	$\frac{3}{\sqrt{2}}$	3	$\frac{3}{\sqrt{2}}$	0	$\frac{-3}{\sqrt{2}}$	-3	$\frac{-3}{\sqrt{2}}$	0	$\frac{3}{\sqrt{2}}$

Using the values of t and y (as in the table), a graph is plotted as under The graph obtained is sinusoidal.
Similar graphs are obtained for y x = 2 cm and x = 4 cm. The (in cm) oscillatory motion in the travelling wave only differs in respect of phase. Amplitude and frequency of oscillatory motion remains the same in all the cases.



Question 15. 10. For the travelling harmonic wave
y(x, t) = 2.0 cos 2π (10t - 0.0080x + 0.35)
where x and y are in cm and t in s. Calculate the phase difference between oscillatory motion of two points separated by a distance of
(a) 4 m (b) 0.5 m
(c) λ/2 (d) 3λ/4.
Answer:

The given equation can be rewritten as under:

$$y(x, t) = 2.0 \cos [2\pi (10t - 0.0080\, x) + 2\pi \times 0.35]$$

or

$$y(x, t) = 2.0 \cos \left[2\pi \times 0.0080 \left(\frac{10t}{0.0080} - x\right) + 0.7\pi\right]$$

Comparing this equation with the standard equation of a travelling harmonic wave,

$$\frac{2\pi}{\lambda} = 2\pi \times 0.0080 \quad \text{or} \quad \lambda = \frac{1}{0.0080}\text{ cm} = 125\text{ cm}$$

The phase difference between oscillatory motion of two points separated by a distance Δx is given by

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$$

(*a*) When $\Delta x = 4\text{ m} = 400\text{ cm}$, then

$$\Delta\phi = \frac{2\pi}{125} \times 400 = 6.4\,\pi\text{ rad}$$

(*b*) When $\Delta x = 0.5\text{ m} = 50\text{ cm}$, then

$$\Delta\phi = \frac{2\pi}{125} \times 50 = 0.8\,\pi\text{ rad}$$

(*c*) When $\Delta x = \frac{\lambda}{2} = \frac{125}{2}$ cm, then

$$\Delta\phi = \frac{2\pi}{125} \times \frac{125}{2} = \pi\text{ rad}$$

(*d*) When $\Delta x = \frac{3\lambda}{4} = \frac{3 \times 125}{4}$ cm, then

$$\Delta\phi = \frac{2\pi}{125} \times \frac{3 \times 125}{4} = \frac{3\pi}{2}\text{ rad.}$$

Question 15. 11. The transverse displacement of a string (clamped at its two ends) is given by y(x, t) = 0.06 sin 2π /3 x cos (120π t)
where x, y are in m and t in s. The length of the string is 1.5 m and its

mass is 3 x 10^{-2} kg. Answer the following:
(i) Does the function represent a travelling or a stationary wave?
(ii) Interpret the wave as a superimposition of two waves travelling in opposite directions. What are the wavelength, frequency and speed of propagation of each wave?
(iii) Determine the tension in the string.
Answer: The given equation is
y(x, t) = 0.06 sin 2π/3 x cos 120 πt ...(1)
(i) As the equation involves harmonic functions of x and t separately, it represents a stationary wave.

(*ii*) We know that when a wave pulse

$$y_1 = r \sin \frac{2\pi}{\lambda}(vt - x)$$

travelling along + direction of x-axis is superimposed by the reflected wave

$$y_2 = -r \sin \frac{2\pi}{\lambda}(vt + x)$$

travelling in opposite direction, a stationary wave

$$y = y_1 + y_2 = -2r \sin \frac{2\pi}{\lambda} x \cos \frac{2\pi}{\lambda} vt \text{ is formed.} \quad ...(2)$$

Comparing eqns. (1) and (2), we find that

$$\frac{2\pi}{\lambda} = \frac{2\pi}{3} \Rightarrow \lambda = 3\text{m}$$

Also, $\frac{2\pi}{\lambda} v = 120\pi$ or $v = 60\lambda = 60 \times 3 = 180 \text{ ms}^{-1}$

Frequency, $\nu = \frac{v}{\lambda} = \frac{180}{3} = 60$ Hz

Note that both the waves have same wavelength, same frequency and same speed.

(*iii*) Velocity of transverse waves is

$$v = \sqrt{\frac{T}{m}} \text{ or } v^2 = \frac{T}{m}$$

$$T = mv^2, \text{ where } m = \frac{3 \times 10^{-2}}{1.5} = 2 \times 10^{-2} \text{ kg/m}$$

$$\therefore \quad T = (180)^2 \times 2 \times 10^{-2} = 648 \text{ N.}$$

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