Trigonometric Identities Ex 6.1 Q84

Answer :

Given: $\cos\theta + \cos^2\theta = 1$

We have to prove $\sin^{12}\theta + 3\sin^{10}\theta + 3\sin^8\theta + \sin^6\theta + 2\sin^4\theta + 2\sin^2\theta - 2 = 1$

From the given equation, we have

$$\cos\theta + \cos^2\theta = 1$$
$$\Rightarrow \quad \cos\theta = 1 - \cos^2\theta$$
$$\Rightarrow \quad \cos\theta = \sin^2\theta$$
$$\Rightarrow \quad \sin^2\theta = \cos\theta$$

Therefore, we have

$$\begin{aligned}
&\sin^{12}\theta + 3\sin^{10}\theta + 3\sin^8\theta + \sin^6\theta + 2\sin^4\theta + 2\sin^2\theta - 2 \\
&= (\sin^{12}\theta + 3\sin^{10}\theta + 3\sin^8\theta + \sin^6\theta) + (2\sin^4\theta + 2\sin^2\theta) - 2 \\
&= \{(\sin^4\theta)^3 + 3(\sin^4\theta)^2\sin^2\theta + 3\sin^4\theta(\sin^2\theta)^2 + (\sin^2\theta)^3\} + 2(\sin^4\theta + \sin^2\theta) - 2 \\
&= (\sin^4\theta + \sin^2\theta)^3 + 2(\sin^4\theta + \sin^2\theta) - 2 \\
&= (\cos^2\theta + \cos\theta)^3 + 2(\cos^2\theta + \cos\theta) - 2 \\
&= (1)^3 + 2(1) - 2 \\
&= 1
\end{aligned}$$

Hence proved.

Trigonometric Identities Ex 6.1 Q85

Answer :

Given: $(1+\cos\alpha)(1+\cos\beta)(1+\cos\gamma) = (1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma)$

Let us assume that

$$(1+\cos\alpha)(1+\cos\beta)(1+\cos\gamma) = (1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma) = L$$

We know that, $\sin^2\theta + \cos^2\theta = 1$

Then, we have

$$L \times L = (1+\cos\alpha)(1+\cos\beta)(1+\cos\gamma) \times (1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma)$$
$$\Rightarrow L^2 = \{(1+\cos\alpha)(1-\cos\alpha)\}\{(1+\cos\beta)(1-\cos\beta)\}\{(1+\cos\gamma)(1-\cos\gamma)\}$$
$$\Rightarrow L^2 = (1-\cos^2\alpha)(1-\cos^2\beta)(1-\cos^2\gamma)$$
$$\Rightarrow L^2 = \sin^2\alpha\sin^2\beta\sin^2\gamma$$
$$\Rightarrow L = \pm\sin\alpha\sin\beta\sin\gamma$$

Therefore, we have

$$(1+\cos\alpha)(1+\cos\beta)(1+\cos\gamma) = (1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma) = \pm\sin\alpha\sin\beta\sin\gamma$$

Taking the expression with the positive sign, we have

$$(1+\cos\alpha)(1+\cos\beta)(1+\cos\gamma) = (1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma) = \sin\alpha\sin\beta\sin\gamma$$

*********** END **********