Differentiation Ex 11.3 Q26

Let $y = \tan^{-1}\left(\frac{\sqrt{x}+\sqrt{a}}{1-\sqrt{xa}}\right)$

$y = \tan^{-1}\sqrt{x} + \tan^{-1}\sqrt{a}$ $\left[\text{Since, } \tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1}\sqrt{x}\right) + \frac{d}{dx}\left(\tan^{-1}\sqrt{a}\right)$$

$$= \frac{1}{1+\left(\sqrt{x}\right)^2}\frac{d}{dx}\left(\sqrt{x}\right) + 0$$

$$= \left(\frac{1}{1+x}\right)\left(\frac{1}{2\sqrt{x}}\right)$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}\,(1+x)}.$$

Differentiation Ex 11.3 Q27

Let $y = \tan^{-1}\left[\frac{a+b\tan x}{b-a\tan x}\right]$

$$= \tan^{-1}\left[\frac{\frac{a+b\tan x}{b}}{\frac{b-a\tan x}{b}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{a}{b}+\tan x}{1+\frac{a}{b}\tan x}\right]$$

$$= \tan^{-1}\left[\frac{\tan\left(\tan^{-1}\frac{a}{b}\right)+\tan x}{1-\tan\left(\tan^{-1}\frac{a}{b}\right)+\tan x}\right]$$

$$= \tan^{-1}\left[\tan\left(\tan^{-1}\frac{a}{b} + x.\right)\right]$$

$$y = \tan^{-1}\left(\frac{a}{b}\right) + x$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = 0 + 1$$

$$\frac{dy}{dx} = 1.$$

Differentiation Ex 11.3 Q28

Let $y = \tan^{-1}\left(\frac{a+bx}{b-ax}\right)$

$= \tan^{-1}\left(\frac{\frac{a+bx}{b}}{\frac{b-ax}{b}}\right)$

$= \tan^{-}\left(\frac{\frac{a}{b}+\frac{bx}{b}}{\frac{b}{a}-\frac{ax}{b}}\right)$

$= \tan^{-1}\left(\frac{\frac{a}{b}+x}{1-\left(\frac{a}{b}\right)x}\right)$

$y = \tan^{-1}\left(\frac{a}{b}\right) + \tan^{-1}x$ $\left[\text{Since, } \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$

Differentiating it with respect to x,

$\frac{dy}{dx} = 0 + \frac{1}{1+x^2}$

$\frac{dy}{dx} = \frac{1}{1+x^2}.$

Differentiation Ex 11.3 Q29

Let $y = \tan^{-1}\left(\frac{x-a}{x+a}\right)$

$= \tan^{-1}\left(\frac{\frac{x-a}{x}}{\frac{x+a}{x}}\right)$

$= \tan^{-1}\left(\frac{\frac{x}{x}-\frac{a}{x}}{\frac{x}{x}+\frac{a}{x}}\right)$

$= \tan^{-1}\left(\frac{1-\frac{a}{x}}{1+1\times\frac{a}{x}}\right)$

$y = \tan^{-1}(1) - \tan^{-1}\left(\frac{a}{x}\right)$

Differentiating it with respect to x using chain rule,

$\frac{dy}{dx} = 0 - \frac{1}{1+\left(\frac{a}{x}\right)^2}\frac{d}{dx}\left(\frac{a}{x}\right)$

$= -\frac{x^2}{x^2+a^2}\left(\frac{-a}{x^2}\right)$

$\frac{dy}{dx} = \frac{a}{a^2+x^2}.$

Differentiation Ex 11.3 Q30

Let $y = \tan^{-1}\left(\frac{x}{1+6x^2}\right)$

$= \tan^{-1}\left(\frac{3x-2x}{1+(3x)(2x)}\right)$

$y = \tan^{-1} 3x - \tan^{-1} 2x$ $\left[\text{Since, } \tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{1+(3x)^2}\frac{d}{dx}(3x) - \frac{1}{1+(2x)^2}\frac{d}{dx}(2x)$$

$$= \frac{1}{1+9x^2}(3) - \frac{1}{1+4x^2}(2)$$

$$\frac{dy}{dx} = \frac{3}{1+9x^2} - \frac{2}{1+4x^2}.$$

********** END **********