Definite Integrals Ex 20.1 Q51

We have,

$$\int_0^{2\pi} e^{x/2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)dx = \int_0^{2\pi} e^{x/2}\left(\sin\frac{x}{2}\cos\frac{\pi}{4}+\cos\frac{x}{2}\sin\frac{\pi}{4}\right)dx$$

$$= \int_0^{2\pi} e^{x/2}\sin\frac{x}{2}\cdot\frac{1}{\sqrt{2}}dx + \int_0^{2\pi} e^{x/2}\cos\frac{x}{2}\cdot\frac{1}{\sqrt{2}}dx$$

Expanding 1st part by parts, we get,

$$\int_0^{2\pi} e^{x/2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)dx = \frac{1}{\sqrt{2}}\left\{\sin\frac{x}{2}\int_0^{2\pi} e^{x/2}dx - \int_0^{2\pi}\left(\int_0^{2\pi} e^{x/2}dx\right)\cdot\frac{d\left(\sin\frac{x}{2}\right)}{dx}dx\right\}+\frac{1}{\sqrt{2}}\int_0^{2\pi} e^{x/2}.\cos\frac{x}{2}dx$$

$$= \frac{1}{\sqrt{2}}\left\{\sin\frac{x}{2}.2e^{x/2}\right\}_0^{2\pi} - \frac{1}{\sqrt{2}}\int_0^{2\pi} e^{x/2}2.\frac{1}{2}\cos\frac{x}{2}dx + \frac{1}{\sqrt{2}}\int_0^{2\pi} e^{x/2}\cos\frac{x}{2}dx$$

$$= \frac{1}{\sqrt{2}}\left\{\sin\frac{x}{2}.2e^{x/2}\right\}_0^{2\pi} = \frac{1}{\sqrt{2}}\{0-0\} = 0$$

$$\therefore \int_0^{2\pi} e^{x/2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)dx = 0$$

Definite Integrals Ex 20.1 Q52

Let $I = \int_0^{2\pi} e^x\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)dx = \left[\cos\left(\frac{\pi}{4}+\frac{x}{2}\right).e^x\right]_0^{2\pi} + \frac{1}{2}\int_0^{2\pi} e^x\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)dx$

$$\Rightarrow I = \left[\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)e^x\right]_0^{2\pi} + \frac{1}{2}\left[\left\{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right).e^x\right\}_0^{2\pi} - \frac{1}{2}\int_0^{2\pi} e^x\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]dx$$

$$I = \left[\cos\left(\pi+\frac{\pi}{4}\right)e^{2\pi} - \cos\frac{\pi}{4}\right] + \frac{1}{2}\left[\sin\left(\pi+\frac{\pi}{4}\right).e^{2\pi} - \sin\frac{\pi}{4} - \frac{1}{2}I\right]$$

$$I = \left[-\cos\frac{\pi}{4}.e^{2\pi} - \cos\frac{\pi}{4}\right] + \frac{1}{2}\left[-\sin\frac{\pi}{4}.e^{2\pi} - \sin\frac{\pi}{4}\right] - \frac{I}{4}$$

$$\frac{5I}{4} = -\frac{1}{\sqrt{2}}\left(e^{2\pi}+1\right) - \frac{1}{2}\cdot\frac{1}{\sqrt{2}}\left(e^{2\pi}+1\right) = \frac{-3}{2\sqrt{2}}\left(e^{2\pi}+1\right)$$

$$I = \frac{-3\sqrt{2}}{5}\left(e^{2\pi}+1\right)$$

$$\therefore \int_0^{2\pi} e^x\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)dx = \frac{-3\sqrt{2}}{5}\left(e^{2\pi}+1\right)$$

Definite Integrals Ex 20.1 Q53

$$\text{Let } I = \int_0^1 \frac{dx}{\sqrt{1+x}-\sqrt{x}}$$

$$I = \int_0^1 \frac{1}{\left(\sqrt{1+x}-\sqrt{x}\right)} \times \frac{\left(\sqrt{1+x}+\sqrt{x}\right)}{\left(\sqrt{1+x}+\sqrt{x}\right)}\, dx$$

$$= \int_0^1 \frac{\sqrt{1+x}+\sqrt{x}}{1+x-x}\, dx$$

$$= \int_0^1 \sqrt{1+x}\, dx + \int_0^1 \sqrt{x}\, dx$$

$$= \left[\frac{2}{3}(1+x)^{\frac{3}{2}}\right]_0^1 + \left[\frac{2}{3}(x)^{\frac{3}{2}}\right]_0^1$$

$$= \frac{2}{3}\left[(2)^{\frac{3}{2}} - 1\right] + \frac{2}{3}[1]$$

$$= \frac{2}{3}(2)^{\frac{3}{2}}$$

$$= \frac{2 \cdot 2\sqrt{2}}{3}$$

$$= \frac{4\sqrt{2}}{3}$$

Definite Integrals Ex 20.1 Q54

$$\int_1^2 \frac{x}{(x+1)(x+2)}\, dx = -\int_1^2 \frac{1}{x+1}\, dx + \int_1^2 \frac{2}{x+2}\, dx \qquad \text{[Using Partial Fraction]}$$

$$= -\log(x+1)\Big|_1^2 + 2\log(x+2)\Big|_1^2$$

$$= -(\log 3 - \log 2) + 2(\log 4 - \log 3)$$

$$= -3\log 3 + 5\log 2$$

$$= \log \frac{32}{27}$$

*********** END **********