



Exercise 1E

Question 14:

For rationalising the denominator of a number, we multiply its numerator and denominator by its rationalising factor.

If a and b are integers, then

$(a+\sqrt{b})$ and $(a-\sqrt{b})$ are rationalising factor of each other,

as $(a+\sqrt{b})(a-\sqrt{b}) = (a^2 - b)$, which is rational.

Let us rationalise the denominator of the first term on the Left hand side.

We have,

$$\begin{aligned}\frac{\sqrt{5}-1}{\sqrt{5}+1} &= \frac{\sqrt{5}-1}{\sqrt{5}+1} \times \frac{\sqrt{5}-1}{\sqrt{5}-1} \\ &= \frac{(\sqrt{5}-1)^2}{(\sqrt{5})^2 - (1)^2} \\ &= \frac{(\sqrt{5})^2 - 2(\sqrt{5})(1) + 1}{5 - 1} \\ &= \frac{5 - 2\sqrt{5} + 1}{4} = \frac{6 - 2\sqrt{5}}{4} \dots\dots(1)\end{aligned}$$

Now consider the denominator of the second term on the left hand side:

$$\begin{aligned}\frac{\sqrt{5}+1}{\sqrt{5}-1} &= \frac{\sqrt{5}+1}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} \\ &= \frac{(\sqrt{5}+1)^2}{(\sqrt{5})^2 - (1)^2} \\ &= \frac{(\sqrt{5})^2 + 2(\sqrt{5})(1) + (1)^2}{5 - 1} \\ &= \frac{5 + 2\sqrt{5} + 1}{4} = \frac{6 + 2\sqrt{5}}{4} \dots\dots\dots(2)\end{aligned}$$

Adding equations (1) and (2), we have

$$\begin{aligned}\therefore \frac{\sqrt{5}-1}{\sqrt{5}+1} + \frac{\sqrt{5}+1}{\sqrt{5}-1} &= \frac{6-2\sqrt{5}}{4} + \frac{6+2\sqrt{5}}{4} \\ &= \frac{6-2\sqrt{5}+6+2\sqrt{5}}{4} = \frac{12}{4} = 3.\end{aligned}$$

***** END *****