Length of a co-axial cylinder, l = 15 cm = 0.15 m
Radius of outer cylinder, r_1 = 1.5 cm = 0.015 m
Radius of inner cylinder, r_2 = 1.4 cm = 0.014 m
Charge on the inner cylinder, q = 3.5 μC = 3.5 × 10^{-6} C

Capacitance of a co-axial cylinder of radii r_1 and r_2 is given by the relation,

$$C = \frac{2\pi \epsilon_0 l}{\log_e \frac{r_1}{r_2}}$$

Where,

ϵ_0 = Permittivity of free space = $8.85 \times 10^{-12}\ \text{N}^{-1}\ \text{m}^{-2}\ \text{C}^2$

$$\therefore C = \frac{2\pi \times 8.85 \times 10^{-12} \times 0.15}{2.3026\ \log_{10}\left(\frac{0.15}{0.14}\right)}$$

$$= \frac{2\pi \times 8.85 \times 10^{-12} \times 0.15}{2.3026 \times 0.0299} = 1.2 \times 10^{-10}\ \text{F}$$

Potential difference of the inner cylinder is given by,

$$V = \frac{q}{C}$$

$$= \frac{3.5 \times 10^{-6}}{1.2 \times 10^{-10}} = 2.92 \times 10^4\ \text{V}$$

Question 2.33:

A parallel plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength about 10^7 Vm^{-1}. (Dielectric strength is the maximum electric field a material can tolerate without breakdown, i.e., without starting to conduct electricity through partial ionisation.) For safety, we should like the field never to exceed, say 10% of the dielectric strength. What minimum area of the plates is required to have a capacitance of 50 pF?

Answer

Potential rating of a parallel plate capacitor, V = 1 kV = 1000 V

Dielectric constant of a material, $\epsilon_r = 3$
Dielectric strength = 10^7 V/m
For safety, the field intensity never exceeds 10% of the dielectric strength.
Hence, electric field intensity, E = 10% of 10^7 = 10^6 V/m
Capacitance of the parallel plate capacitor, C = 50 pF = 50 × 10^{-12} F
Distance between the plates is given by,

$$d = \frac{V}{E}$$

$$= \frac{1000}{10^6} = 10^{-3}\ \text{m}$$

Capacitance is given by the relation,

$$C = \frac{\epsilon_0 \epsilon_r A}{d}$$

Where,

A = Area of each plate

ϵ_0 = Permittivity of free space = $8.85 \times 10^{-12}\ \text{N}^{-1}\ \text{C}^2\text{m}^{-2}$

$$\therefore A = \frac{Cd}{\epsilon_0 \epsilon_r}$$

$$= \frac{50 \times 10^{-12} \times 10^{-3}}{8.85 \times 10^{-12} \times 3} \approx 19\ \text{cm}^2$$

Hence, the area of each plate is about 19 cm^2.

Question 2.34:

Describe schematically the equipotential surfaces corresponding to

(a) a constant electric field in the z-direction,

(b) a field that uniformly increases in magnitude but remains in a constant (say, z) direction,

(c) a single positive charge at the origin, and

(d) a uniform grid consisting of long equally spaced parallel charged wires in a plane.

Answer

(a) Equidistant planes parallel to the x-y plane are the equipotential surfaces.

(b) Planes parallel to the x-y plane are the equipotential surfaces with the exception that when the planes get closer, the field increases.

(c) Concentric spheres centered at the origin are equipotential surfaces.

(d) A periodically varying shape near the given grid is the equipotential surface. This shape gradually reaches the shape of planes parallel to the grid at a larger distance.

Question 2.35:

In a Van de Graaff type generator a spherical metal shell is to be a 15 × 10^6 V electrode. The dielectric strength of the gas surrounding the electrode is 5 × 10^7 Vm^{-1}. What is the minimum radius of the spherical shell required? (You will learn from this exercise why one cannot build an electrostatic generator using a very small shell which requires a small charge to acquire a high potential.)

Answer

Potential difference, V = 15 × 10^6 V

Dielectric strength of the surrounding gas = 5 × 10^7 V/m

Electric field intensity, E = Dielectric strength = 5 × 10^7 V/m

Minimum radius of the spherical shell required for the purpose is given by,

$$r = \frac{V}{E}$$

$$= \frac{15\times10^6}{5\times10^7} = 0.3\text{ m} = 30\text{ cm}$$

Hence, the minimum radius of the spherical shell required is 30 cm.

Question 2.36:

A small sphere of radius r_1 and charge q_1 is enclosed by a spherical shell of radius r_2 and charge q_2. Show that if q_1 is positive, charge will necessarily flow from the sphere to the shell (when the two are connected by a wire) no matter what the charge q_2 on the shell is.

Answer

According to Gauss's law, the electric field between a sphere and a shell is determined by the charge q_1 on a small sphere. Hence, the potential difference, V, between the sphere and the shell is independent of charge q_2. For positive charge q_1, potential difference V is always positive.

Question 2.37:

Answer the following:

(a) The top of the atmosphere is at about 400 kV with respect to the surface of the earth, corresponding to an electric field that decreases with altitude. Near the surface of the earth, the field is about 100 Vm^{-1}. Why then do we not get an electric shock as we step out of our house into the open? (Assume the house to be a steel cage so there is no field inside!)

(b) A man fixes outside his house one evening a two metre high insulating slab carrying on its top a large aluminium sheet of area 1m^2. Will he get an electric shock if he touches the metal sheet next morning?

(c) The discharging current in the atmosphere due to the small conductivity of air is known to be 1800 A on an average over the globe. Why then does the atmosphere not discharge itself completely in due course and become electrically neutral? In other words, what keeps the atmosphere charged?

(d) What are the forms of energy into which the electrical energy of the atmosphere is dissipated during a lightning? (Hint: The earth has an electric field of about 100 Vm^{-1} at its surface in the downward direction, corresponding to a surface charge density = -10^{-9} C m^{-2}. Due to the slight conductivity of the atmosphere up to about 50 km (beyond which it is good conductor), about + 1800 C is pumped every second into the earth as a whole. The earth, however, does not get discharged since thunderstorms and lightning occurring continually all over the globe pump an equal amount of negative charge on the earth.)

Answer

(a) We do not get an electric shock as we step out of our house because the original equipotential surfaces of open air changes, keeping our body and the ground at the same potential.

(b) Yes, the man will get an electric shock if he touches the metal slab next morning. The steady discharging current in the atmosphere charges up the aluminium sheet. As a result, its voltage rises gradually. The raise in the voltage depends on the capacitance of the capacitor formed by the aluminium slab and the ground.

(c) The occurrence of thunderstorms and lightning charges the atmosphere continuously. Hence, even with the presence of discharging current of 1800 A, the atmosphere is not discharged completely. The two opposing currents are in equilibrium and the atmosphere remains electrically neutral.

(d) During lightning and thunderstorm, light energy, heat energy, and sound energy are dissipated in the atmosphere.

*********** END **********