NCERT solutions for class 9 Maths Polynomials Ex 2.4

Q1. Determine which of the following polynomials has $(x+1)$ a factor:

(i) x^3+x^2+x+1

(ii) $x^4+x^3+x^2+x+1$

(iii) $x^4+3x^3+3x^2+x+1$

(iv) $x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$

Ans: (i) x^3+x^2+x+1

While applying the factor theorem, we get

$p(x)=x^3+x^2+x+1$

$p(-1)=(-1)^3+(-1)^2+(-1)+1$

$=-1+1-1+1$

$=0$

We conclude that on dividing the polynomial x^3+x^2+x+1 by $(x+1)$, we get the remainder as 0.

Therefore, we conclude that $(x+1)$ is a factor of x^3+x^2+x+1.

(ii) $x^4+x^3+x^2+x+1$

While applying the factor theorem, we get

$p(x)=x^4+x^3+x^2+x+1$

$p(-1)=(-1)^4+(-1)^3+(-1)^2+(-1)+1$

$=1-1+1-1+1$

$=1$

We conclude that on dividing the polynomial $x^4+x^3+x^2+x+1$ by $(x+1)$, we will get the remainder as 1, which is not 0.

Therefore, we conclude that $(x+1)$ is not a factor of $x^4+x^3+x^2+x+1$.

(iii) $x^4+3x^3+3x^2+x+1$

While applying the factor theorem, we get

$p(x)=x^4+3x^3+3x^2+x+1$

$p(-1)=(-1)^4+3(-1)^3+3(-1)^2+(-1)+1$

$=1-3+3-1+1$

$=1$

We conclude that on dividing the polynomial $x^4+3x^3+3x^2+x+1$ by $(x+1)$, we will get the remainder as 1, which is not 0.

Therefore, we conclude that $(x+1)$ is not a factor of $x^4+3x^3+3x^2+x+1$.

(iv) $x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$

While applying the factor theorem, we get

$p(x)=x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$

$p(-1)=(-1)^3-(-1)^2-\left(2+\sqrt{2}\right)(-1)+\sqrt{2}$

$=-1-1+2+\sqrt{2}+\sqrt{2}$

$=2\sqrt{2}$.

We conclude that on dividing the polynomial $x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$ by $(x+1)$, we will get the remainder as $2\sqrt{2}$, which is not 0.

Therefore, we conclude that $(x+1)$ is not a factor of $x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$.

Q2. Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x)=2x^3+x^2-2x-1$, $g(x)=x+1$

(ii) $p(x)=x^3+3x^2+3x+1$, $g(x)=x+2$

(iii) $p(x)=x^3-4x^2+x+6$, $g(x)=x-3$

Ans: (i) $p(x)=2x^3+x^2-2x-1$, $g(x)=x+1$

We know that according to the factor theorem, $(x-a)$ is a factor of $p(x)$, if $p(a)=0$.

We can conclude that $g(x)$ is a factor of $p(x)$, if $p(-1)=0$.

$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$

=2+1-1-2

=0

Therefore, we conclude that the *g(x)* is a factor of *p(x)*.

(ii) $p(x) = x^3 + 3x^2 + 3x + 1, g(x) = x + 2$

We know that according to the factor theorem, $(x-a)$ is a factor of $p(x)$, if $p(a) = 0$.

We can conclude that *g(x)* is a factor of *p(x)*, if *p(-2)*=0.

$p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$

=-8+12-6+1

=-1

Therefore, we conclude that the *g(x)* is not a factor of *p(x)*.

(iii) $p(x) = x^3 - 4x^2 + x + 6, g(x) = x - 3$

We know that according to the factor theorem, $(x-a)$ is a factor of $p(x)$, if $p(a) = 0$.

We can conclude that *g(x)* is a factor of *p(x)*, if *p(3)*=0.

$p(3) = (3)^3 - 4(3)^2 + (3) + 6$

=27-36+3+6

=0

Therefore, we conclude that the *g(x)* is a factor of *p(x)*.

Q3. Find the value of k, if $x - 1$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = x^2 + x + k$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

(iv) $p(x) = kx^2 - 3x + k$

Ans: (i) $p(x)=x^2+x+k$

We know that according to the factor theorem

$p(a)=0$, if $x-a$ is a factor of $p(x)$.

We conclude that if $(x-1)$ is a factor of

$p(x)=x^2+x+k$, then $p(1)=0$.

$p(1)=(1)^2+(1)+k=0$, or

$k+2=0$

$k=-2$

Therefore, we can conclude that the value of k is -2.

(ii) $p(x)=2x^2+kx+\sqrt{2}$

We know that according to the factor theorem

$p(a)=0$, if $x-a$ is a factor of $p(x)$.

We conclude that if $(x-1)$ is a factor of

$p(x)=2x^2+kx+\sqrt{2}$, then $p(1)=0$.

$p(1)=2(1)^2+k(1)+\sqrt{2}=0$, or

$2+k+\sqrt{2}=0$

$k=-\left(2+\sqrt{2}\right)$.

Therefore, we can conclude that the value of k is $-\left(2+\sqrt{2}\right)$.

(iii) $p(x)=kx^2-\sqrt{2}x+1$

We know that according to the factor theorem

$p(a)=0$, if $x-a$ is a factor of $p(x)$.

We conclude that if $(x-1)$ is a factor of

$p(x)=kx^2-\sqrt{2}x+1$, then $p(1)=0$.

$p(1)=k(1)^2-\sqrt{2}(1)+1=0$, or

$k-\sqrt{2}+1=0$

$k=\sqrt{2}-1$.

Therefore, we can conclude that the value of k is $\sqrt{2}-1$.

(iv) $p(x)=kx^2-3x+k$

We know that according to the factor theorem

$p(a)=0$, if $x-a$ is a factor of p(x)

We conclude that if $(x-1)$ is a factor of $p(x)=kx^2-3x+k$, then $p(1)=0$.

$p(1)=k(1)^2-3(1)+k$, or $2k-3=0 \quad \Rightarrow k=\frac{3}{2}$

Therefore, we can conclude that the value of k is $\frac{3}{2}$.

Q4. Factorize:

(i) $12x^2-7x+1$

(ii) $2x^2+7x+3$

(iii) $6x^2+5x-6$

(iv) $3x^2-x-4$

Ans: (i) $12x^2-7x+1$

$12x^2-7x+1=12x^2-3x-4x+1$

$=3x(4x-1)-1(4x-1)$

$=(3x-1)(4x-1)$.

Therefore, we conclude that on factorizing the polynomial $12x^2-7x+1$, we get $(3x-1)(4x-1)$.

(ii) $2x^2+7x+3$

$2x^2+7x+3=2x^2+6x+x+3$

$=2x(x+3)+1(x+3)$

$=(2x+1)(x+3)$.

Therefore, we conclude that on factorizing the polynomial $2x^2+7x+3$, we get $(2x+1)(x+3)$.

(iii) $6x^2+5x-6$

$6x^2+5x-6=6x^2+9x-4x-6$

$=3x(2x+3)-2(2x+3)$

$=(3x-2)(2x+3)$.

Therefore, we conclude that on factorizing the polynomial $6x^2+5x-6$, we get $(3x-2)(2x+3)$.

(iv) $3x^2-x-4$

$3x^2-x-4=3x^2+3x-4x-4$

$=3x(x+1)-4(x+1)$

$=(3x-4)(x+1)$.

Therefore, we conclude that on factorizing the polynomial $3x^2-x-4$, we get $(3x-4)(x+1)$.

Q5. Factorize:

(i) x^3-2x^2-x+2

(ii) x^3-3x^2-9x-5

(iii) $x^3+13x^2+32x+20$

(iv) $2y^3+y^2-2y-1$

Ans: (i) x^3-2x^2-x+2

We need to consider the factors of 2, which are $\pm1, \pm2$.

Let us substitute 1 in the polynomial x^3-2x^2-x+2, to get

$(1)^3-2(1)^2-(1)+2$ =1-1-2+2=0

Thus, according to factor theorem, we can conclude that $(x-1)$ is a factor of the polynomial x^3-2x^2-x+2.

Let us divide the polynomial x^3-2x^2-x+2 by $(x-1)$, to get

$$
\begin{array}{r|l}
 & x^2-x-2 \\
\hline
x-1 & x^3-2x^2-x+2 \\
 & \underline{\overset{-}{x^3}\overset{+}{-x^2}} \\
 & -x^2-x \\
 & \underline{\overset{+}{-x^2}\overset{-}{+x}} \\
 & -2x+2 \\
 & \underline{\overset{+}{-2x}\overset{-}{+2}} \\
 & \underline{\quad 0 \quad}
\end{array}
$$

$x^3-2x^2-x+2=(x-1)(x^2-x-2)$.

$x^3-2x^2-x+2=(x-1)(x^2-x-2)$.

$=(x-1)(x^2+x-2x-2)$

$=(x-1)[x(x+1)-2(x+1)]$

$=(x-1)(x-2)(x+1)$.

Therefore, we can conclude that on factorizing the polynomial x^3-2x^2-x+2, we get $(x-1)(x-2)(x+1)$.

(ii) x^3-3x^2-9x-5

We need to consider the factors of -5, which are $\pm1, \pm5$.

Let us substitute 1 in the polynomial x^3-3x^2-9x-5, to get

$(-1)^3-3(-1)^2-9(-1)-5=-1-3+9-5=0$

Thus, according to factor theorem, we can conclude that $(x+1)$ is a factor of the polynomial x^3-3x^2-9x-5.

Let us divide the polynomial x^3-3x^2-9x-5 by $(x+1)$, to get

$$
\begin{array}{r|l}
 & x^2-4x-5 \\
\hline
x+1 & x^3-3x^2-9x-5 \\
 & \underline{x^3+x^2} \\
 & \quad -\;\;\; - \\
 & -4x^2-9x \\
 & \underline{-4x^2-4x} \\
 & \quad +\;\;\; + \\
 & -5x-5 \\
 & \underline{-5x-5} \\
 & \quad +\;\;\; + \\
 & \underline{\quad 0 \quad}
\end{array}
$$

$x^3-3x^2-9x-5=(x+1)(x^2-4x-5)$

$=(x+1)(x^2+x-5x-5)$

$=(x+1)[x(x+1)-5(x+1)]$

$=(x+1)(x-5)(x+1).$

Therefore, we can conclude that on factorizing the polynomial x^3-3x^2-9x-5, we get $(x+1)(x-5)(x+1)$.

(iii) $x^3+13x^2+32x+20$

We need to consider the factors of 20, which are $\pm5, \pm4, \pm2, \pm1$.

Let us substitute -1 in the polynomial $x^3+13x^2+32x+20$, to get

$(-1)^3+13(-1)^2+32(-1)+20$ =-1+13-32+20=-20+20=0

Thus, according to factor theorem, we can conclude that $(x+1)$ is a factor of the polynomial $x^3+13x^2+32x+20$.

Let us divide the polynomial $x^3+13x^2+32x+20$ by $(x+1)$, to get

$$\begin{array}{r|l} & x^2+12x+20 \\ \hline x+1 & x^3+13x^2+32x+20 \\ & \underline{\overset{-}{x^3}+\overset{-}{x^2}} \\ & 12x^2+32x \\ & \underline{\overset{-}{12x^2}+\overset{-}{12x}} \\ & 20x+20 \\ & \underline{\overset{-}{20x}+\overset{-}{20}} \\ & \underline{\quad 0} \end{array}$$

$x^3+13x^2+32x+20=(x+1)(x^2+12x+20)$

$=(x+1)(x^2+2x+10x+20)$

$=(x+1)[x(x+2)+10(x+2)]$

$=(x+1)(x+10)(x+2).$

Therefore, we can conclude that on factorizing the polynomial $x^3+13x^2+32x+20$, we get $(x+1)(x-10)(x+2)$.

(iv) $2y^3+y^2-2y-1$

We need to consider the factors of -1, which are ± 1.

Let us substitute 1 in the polynomial $2y^3+y^2-2y-1$, to get

$2(1)^3+(1)^2-2(1)-1$ =2+1-2-1=3-3=0

Thus, according to factor theorem, we can conclude that $(y-1)$ is a factor of the polynomial $2y^3+y^2-2y-1$.

Let us divide the polynomial $2y^3+y^2-2y-1$ by $(y-1)$, to get

$$
\begin{array}{r|l}
 & \quad 2y^2+3y+1 \\
\hline
y-1 & 2y^3+y^2-2y-1 \\
 & 2y^3-2y^2 \\
 & \underline{-\quad\quad +} \\
 & 3y^2-2y \\
 & 3y^2-3y \\
 & \underline{-\quad\quad +} \\
 & y-1 \\
 & y-1 \\
 & \underline{-\quad +} \\
 & \quad 0 \\
\end{array}
$$

$$2y^3+y^2-2y-1=(y-1)(2y^2+3y+1)$$

$$=(y-1)(2y^2+2y+y+1)$$

$$=(y-1)[2y(y+1)+1(y+1)]$$

$$=(y-1)(2y+1)(y+1).$$

Therefore, we can conclude that on factorizing the polynomial $2y^3+y^2-2y-1$, we get $(y-1)(2y+1)(y+1)$

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