



Definite Integrals Ex 20.1 Q27

We have,

$$\int x \cos x \, dx = x \int \cos x \, dx - \int \left( \int \cos x \, dx \right) \frac{dx}{dx} \, dx = x \sin x - \int \sin x \, dx$$

$$\therefore \int_0^{\frac{\pi}{2}} x \cos x \, dx = \left[ x \sin x + \cos x \right]_0^{\frac{\pi}{2}} = \left[ \frac{\pi}{2} + 0 - 0 - 1 \right] = \frac{\pi}{2} - 1$$

$$\therefore \int_0^{\frac{\pi}{2}} x \cos x \, dx = \frac{\pi}{2} - 1$$

Definite Integrals Ex 20.1 Q28

We have,

$$\begin{aligned} \int x^2 \cos x \, dx &= x^2 \int \cos x \, dx - \int (2x) \left( \int \cos x \, dx \right) \, dx = x^2 \sin x - \int \sin x \cdot 2x \, dx \\ &= x^2 \sin x - 2 \left[ x \int \sin x \, dx - \int \left( \int \sin x \, dx \right) \, dx \right] \\ &= x^2 \sin x - 2 \left[ -x \cos x + \int \cos x \, dx \right] \end{aligned}$$

$$\begin{aligned} \therefore \int_0^{\frac{\pi}{2}} x^2 \cos x \, dx &= \left[ x^2 \sin x + 2x \cos x - 2 \sin x \right]_0^{\frac{\pi}{2}} \\ &= \left[ \frac{\pi^2}{4} + 0 - 2 - 0 - 0 + 0 \right] \\ &= \frac{\pi^2}{4} - 2 \end{aligned}$$

$$\therefore \int_0^{\frac{\pi}{2}} x^2 \cos x \, dx = \frac{\pi^2}{4} - 2$$

Definite Integrals Ex 20.1 Q29

We have,

$$\begin{aligned} \int x^2 \sin x \, dx &= x^2 \int \sin x \, dx - \int 2x \left( \int \sin x \, dx \right) \, dx = x^2 \cos x + \int 2x \cos x \, dx \\ &= x^2 \cos x + 2 \left[ x \int \cos x \, dx - \int \left( \int \cos x \, dx \right) \, dx \right] \\ &= -x^2 \cos x + 2 \left[ x \sin x - \int \sin x \, dx \right] \end{aligned}$$

$$\begin{aligned} \therefore \int_0^{\frac{\pi}{4}} x^2 \sin x \, dx &= \left[ -x^2 \cos x + 2x \sin x + 2 \cos x \right]_0^{\frac{\pi}{4}} \\ &= \frac{-\pi^2}{16} \cdot \frac{1}{\sqrt{2}} + \frac{\pi}{2} \cdot \frac{1}{\sqrt{2}} + 2 \cdot \frac{1}{\sqrt{2}} + 0 - 0 - 2 \\ &= \frac{1}{\sqrt{2}} \left[ \frac{-\pi^2}{16} + \frac{\pi}{2} + 2 \right] - 2 \\ &= \sqrt{2} + \frac{\pi}{2\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} - 2 \end{aligned}$$

$$\therefore \int_0^{\frac{\pi}{4}} x^2 \sin x \, dx = \sqrt{2} + \frac{\pi}{2\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} - 2$$

Definite Integrals Ex 20.1 Q30

We have,

$$\begin{aligned}\int x^2 \cos 2x \, dx &= x^2 \int \cos 2x \, dx - \int 2x \left( \int \cos 2x \, dx \right) dx \\&= \frac{x^2 \sin 2x}{2} - \int 2x \times \frac{\sin 2x}{2} \, dx \\&= \frac{x^2 \sin 2x}{2} - \left[ x \int \sin 2x \, dx - \int \left( \int \sin 2x \, dx \right) dx \right] \\&= \frac{x^2 \sin 2x}{2} + \left[ \frac{x \cos 2x}{2} - \int \frac{x \cos 2x}{2} \right] \\\therefore \int_0^{\frac{\pi}{2}} x^2 \cos 2x \, dx &= \left[ \frac{x^2 \sin 2x}{2} + \frac{x \cos 2x}{2} - \frac{\sin 2x}{4} \right]_0^{\frac{\pi}{2}} \\&= \left[ \frac{\pi^2}{8} \times 0 + \frac{\pi}{4}(-1) - 0 - 0 - 0 + 0 \right] \\&= \frac{-\pi}{4}\end{aligned}$$

$$\therefore \int_0^{\frac{\pi}{2}} x^2 \cos 2x \, dx = \frac{-\pi}{4}$$

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