



NCERT Solutions For Class 10 Chapter 8 Introduction to Trigonometry Exercise 8.4

Q1. Express the trigonometric ratios $\sin A$, $\sec A$ and $\tan A$ in terms of $\cot A$

Ans: For $\sin A$,

By using identity $\sec^2 A - \cot^2 A = 1$

$$\Rightarrow \sec^2 A = 1 + \cot^2 A$$

$$\Rightarrow \frac{1}{\sin^2 A} = 1 + \cot^2 A$$

$$\Rightarrow \sin^2 A = \frac{1}{1 + \cot^2 A}$$

$$\Rightarrow \sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$$

For $\sec A$,

By using identity $\sec^2 A - \tan^2 A = 1$

$$\Rightarrow \sec^2 A = 1 + \tan^2 A$$

$$\Rightarrow \sec^2 A = 1 + \frac{1}{\cot^2 A} = \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\Rightarrow \sec^2 A = \frac{1 + \cot^2 A}{\cot^2 A}$$

$$\Rightarrow \sec A = \frac{\sqrt{1 + \cot^2 A}}{\cot A}$$

For $\tan A$,

$$\tan A = \frac{1}{\cot A}$$

Q2. Write the other trigonometric ratios of A in terms of $\sec A$

Ans: For $\sin A$,

By using identity, $\sin^2 A + \cos^2 A = 1$

$$\Rightarrow \sin^2 A = 1 - \cos^2 A$$

$$\Rightarrow \sin^2 A = 1 - \frac{1}{\sec^2 A} = \frac{\sec^2 A - 1}{\sec^2 A}$$

$$\Rightarrow \sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

For $\cos A$,

$$= \sin 25^\circ \cdot \sin 25^\circ + \cos 25^\circ \cdot \cos 25^\circ$$

$$\left[\because \sin(90^\circ - \theta) = \cos \theta, \cos(90^\circ - \theta) = \sin \theta \right]$$

$$= \sin^2 25^\circ + \cos^2 25^\circ = 1$$

$$\left[\because \sin^2 \theta + \cos^2 \theta = 1 \right]$$

Q4. Choose the correct option. Justify your choice:

(i) $9\sec^2 A - 9\tan^2 A =$

(A) 1

(B) 9

(C) 8

(D) 0

(ii) $(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta) =$

(A) 0

(B) 1

(C) 2

(D) none of these

(iii) $(\sec A + \tan A)(1 - \sin A) =$

(A) $\sec A$

(B) $\sin A$

(C) $\operatorname{cosec} A$

(D) $\cos A$

(iv) $\frac{1 + \tan^2 A}{1 + \cot^2 A} =$

(A) $\sec^2 A$

✓ ✓

(B) -1

(C) $\cot^2 A$

(D) none of these

Ans: (i) (B) $9\sec^2 A - 9\tan^2 A$

$$= 9(\sec^2 A - \tan^2 A)$$

$$= 9 \times 1 = 9$$

(ii) (C) $(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta)$

$$\begin{aligned}
&= \left(1 + \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}\right) \left(1 + \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}\right) \\
&= \left(\frac{\cos \theta + \sin \theta + 1}{\cos \theta}\right) \left(\frac{\sin \theta + \cos \theta - 1}{\sin \theta}\right) \\
&= \frac{(\cos \theta + \sin \theta)^2 - (1)^2}{\cos \theta \cdot \sin \theta} \\
&= \frac{\cos^2 \theta + \sin^2 \theta + 2 \cos \theta \sin \theta - 1}{\cos \theta \cdot \sin \theta} \\
&= \frac{1 + 2 \cos \theta \sin \theta - 1}{\cos \theta \cdot \sin \theta} \\
&\left[\because \sin^2 \theta + \cos^2 \theta = 1 \right] \\
&= \frac{2 \cos \theta \sin \theta}{\cos \theta \cdot \sin \theta} = 2
\end{aligned}$$

(iii)(D) $(\sec A + \tan A)(1 - \sin A)$

$$\begin{aligned}
&= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A) \\
&= \left(\frac{1 + \sin A}{\cos A}\right)(1 - \sin A) \\
&= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} \\
&= \cos A \left[\because 1 - \sin^2 A = \cos^2 A \right]
\end{aligned}$$

(iv)(D) $\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{\sec^2 A - \tan^2 A + \tan^2 A}{\sec^2 A - \cot^2 A + \cot^2 A}$

$$\begin{aligned}
&= \frac{1}{\cos^2 A} = \frac{\frac{\cos^2 A}{1}}{\sin^2 A} \\
&= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A
\end{aligned}$$

Q5. Prove the following identities, where the angles involved are acute angles for which the expressions are defined:

(i) $(\sec \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$

$$(ii) \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$$

$$(iii) \frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$$

$$(iv) \frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$$

$$(v) \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A, \text{ using the identity } \operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$(vi) \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$$

$$(vii) \frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta$$

$$(viii) (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$$

$$(ix) (\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$$

$$(x) \left(\frac{1 + \tan^2 A}{1 + \cot^2 A} \right) = \left(\frac{1 - \tan A}{1 - \cot A} \right)^2 = \tan^2 A$$

$$\text{Ans: (i) L.H.S. } (\operatorname{cosec} \theta - \cot \theta)^2$$

$$= \operatorname{cosec}^2 \theta + \cot^2 \theta - 2 \operatorname{cosec} \theta \cot \theta$$

$$= \frac{1}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} - 2 \times \frac{1}{\sin \theta} \cdot \frac{\cos \theta}{\sin \theta}$$

$$= \frac{1 + \cos^2 \theta}{\sin^2 \theta} - \frac{2 \cos \theta}{\sin^2 \theta}$$

$$= \frac{1 + \cos^2 \theta - 2 \cos \theta}{\sin^2 \theta}$$

$$= \frac{(1 - \cos \theta)^2}{\sin^2 \theta} \left[\because a^2 + b^2 - 2ab = (a - b)^2 \right]$$

***** END *****