Question 2. 21. Precise measurements of physical quantities are a need of science. For example, to ascertain the speed of an aircraft, one must have an accurate method to find its positions at closely separated instants of time. This was the actual motivation behind the discovery of radar in World War II. Think of different examples in modem science where precise measurements of length, time, mass etc., are needed. Also, wherever you can, give a quantitative idea of the precision needed.

Answer: Extremely precise measurements are needed in modem science. As an example, while launching a satellite using a space launch rocket system we must measure time to a precision of 1 micro second. Again working with lasers we require length measurements to an angstrom unit (1 A° = 10^{-10}m) or even a fraction of it. For estimating nuclear sizes we require a precision of 10^{-15} m. To measure atomic masses using mass spectrograph we require a precision of 10^{-30}kg and so on.

Question 2.22. Just as precise measurements are necessary in science, it is equally important to be able to make rough estimates of quantities using rudimentary ideas and common observations. Think of ways by which you can estimate the following (where an estimate is difficult to obtain, try to get an upper bound on the quantity):

(a) the total mass of rain-bearing clouds over India during the Monsoon

(b) the mass of an elephant

(c) the wind speed during a storm

(d) the number of strands of hair on your head

(e) the number of air molecules in your classroom.

Answer:

(a) The average rainfall of nearly 100 cm or 1 m is recorded by meteorologists, during Monsoon, in India.

If A is the area of the country, then A = 3.3 million sq. km

= 3.3×10^6 (km)^2

= $3.3 \times 10^6 \times 10^6$m^2

= 3.3×10^{12} m^2

Mass of rain-bearing clouds

= area x height x density

= $3.3 \times 10^{12} \times 1 \times 1000$ kg

= 3.3×10^{15} kg.

(b) Measure the depth of an empty boat in water. Let it be d1. If A be the base area of the boat, then volume of water displaced by boat, $V_1 = Ad_2$

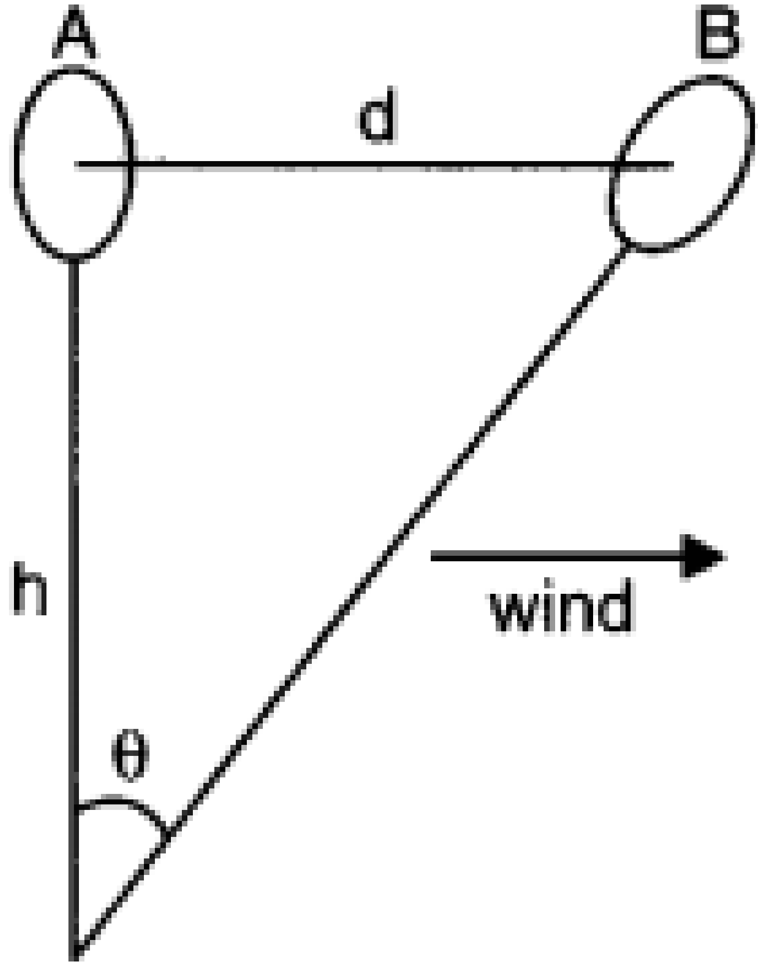
Let d_2 be the depth of boat in water when the elephant is moved into the boat. Volume of water displaced by (boat + elephant), $V_2 = Ad_2$ Volume of water displaced by elephant,

$V = V_2 - V_1 = A(d_2 - d_1)$

If p be the density of water, then mass of elephant = mass of water displaced by it = $A(d_2 - d_1)$ p.

(c) Wind speed can be estimated by floating a gas-filled balloon in air at a known height h. When there is no wind, the balloon is at A. Suppose the wind starts blowing to the right such that the balloon drifts to position B in 1 second.

Now, AB = d = hθ.



The value of d directly gives the wind speed.
(d) Let us assume that the man is not partially bald. Let us further assume that the hair on the head are uniformly distributed. We can estimate the area of the head. The thickness of a hair can be measured by using a screw gauge. The number of hair on the head is clearly the ratio of the area of head to the cross-sectional area of a hair.
Assume that the human head is a circle of radius 0.08 m i.e., 8 cm.
Let us further assume that the thickness of a human air is 5×10^{-5}m.
Number of hair on the head
= Area of the head/Area of cross - section of a hair
= π (0.08)2/π(5×10^{-5})
= 64 x 10-4/25 x 10^{-10}
= 2.56×10^{6}
The number of hair on the human head is of the order of one million.
(e) We can determine the volume of the class-room by measuring its length, breadth and height.
Consider a class room of size 10 m x 8 m x 4 m.
Volume of this room is 320 m^3.
We know that 22.4l or 22.4×10^{-3} m^3 of air has 6.02×10^{23} molecules (equal to Avogadro's number).
Number of molecules of air in the class room
= (6.02×10^{23} /22.4×10^{-3}) x 320
= 8.6×10^{27}

Question 2. 23. The Sun is a hot plasma (ionized matter) with its inner core at a temperature exceeding 107 K, and its outer surface at a temperature of about 6000 K. At these high temperatures, no substance remains in a solid or liquid phase. In what range do you

expect the mass density of the Sun to be, in the range of densities of solids and liquids or gases? Check if your guess is correct from the following data: mass of the Sun = 2.0×10^{30} kg, radius of the Sun = 7.0×10^8 m.
Answer:
Given M = 2×10^{30} kg, r = 7×10^8 m
$\therefore$ Volume of Sun = $4/3\pi r^3 \times 3.14 \times (7 \times 10^8)^3$
= 1.437×10^{27} m^3
As p = M/V,
$\therefore$ p = $2 \times 10^{30}/1.437 \times 10^{27}$
= 1391.8 kg m^{-3}
= 1.4×10^3 kg m^{-3}
Mass density of Sun is in the range of mass densities of solids/liquids and not gases.

Question 2. 24. When the planet Jupiter is at a distance of 824.7 million kilometres from the Earth, its angular diameter is measured to be 35.72″ of arc. Calculate the diameter of Jupiter.
Answer:
Given angular diameter θ = 35.72
= $35.72 \times 4.85 \times 10^{-6}$ rad
= 173.242×10^{-6}
= 1.73×10^{-4} rad
Diameter of Jupiter D = θ x d
= $1.73 \times 10^{-4} \times 824.7 \times 10^9$ m
= 1426.731×10^3
= 1.43×10^8 m

Question 2. 25. A man walking briskly in rain with speed v must slant his umbrella forward making an angle θ with the vertical. A student derives the following relation between θ and v: tanθ = v and checks that the relation has a correct limit: as v—>θ, θ —>0, as expected. (We are assuming there is no strong wind and that the rain falls vertically for a stationary man). Do you think this relation can be correct? If not, guess the correct relation.
Answer: According to principle of homogenity of dimensional equations,
Dimensions of L.H.S. = Dimensions of R.H.S.
Here, v = tan θ
i. e., [$L^1 T^{-1}$] = dimensionless, which is incorrect.
Correcting the L.H.S., we. get
v/u= tan θ, where u is velocity of rain.

Question 2. 26. It is claimed that two cesium clocks, if allowed to run for 180 years, free from any disturbance, may differ by only about 0.02 s. What does this imply for the accuracy of the standard cesium clock in measuring a time-interval of 1 s?
Answer:
Total time = 100 years = 100 x 365 x 24 x 60 x 60 s
Error in 1 second=0.02/100 x 365 x 24 x 60 x 60
= 6.34×10^{-12} s
$\therefore$ Accuracy of 1 part in 10^{11} to 10^{12}.

Question 2. 27. Estimate the average mass density of a sodium atom assuming its size to be about 2.5 A. (Use the known values of Avogadro's number and the atomic mass of sodium). Compare it with the density of sodium in its crystalline phase: 970 kg m^{3-}. Are the two densities of the same order of magnitude? If so, why?
Answer: It is given that radius of sodium atom, R = 2.5 A = 2.5×10^{-10} m
Volume of one mole atom of sodium, V = NA .4/3 π R^3
V = $6.023 \times 10^{23} \times 4/3 \times 3.14 \times (2.5 \times 10^{-10})^3$ m^3 and

mass of one mole atom of sodium, M = 23 g = 23 x 10^{-3} kg
∴ Average mass density of sodium atom, p = M/V
=(23 x 10^{-3}/6.023 x 10^{23} x 4/3 x 3.14 x (2.5 x 10^{-10}))
= 6.96 x 10^{2} kg m^{-3}
= 0.7 x 10^{-3} kg m^{-3}
The density of sodium in its crystalline phase = 970 kg m^{-3}
= 0.97 x 10^{3} kg m^{-3}
Obviously the two densities are of the same order of magnitude (= 10^{3} kg m^{-3}). It is on account of the fact that in solid phase atoms are tightly packed and so the atomic mass density is close to the mass density of solid.

Question 2. 28. The unit of length convenient on the nuclear scale is afermi: I f=10^{-15} m. Nuclear sizes obey roughly the following empirical relation:
r = r_0 $A^{1/3}$
where r is the radius of the nucleus, A its mass number, and r0 is a constant equal to about,1.2 f. Show that the rule implies that nuclear mass density is nearly constant for different nuclei. Estimate the mass density of sodium nucleus. Compare it with the average mass density of a sodium atom obtained in Exercise 2.27.
Answer:
Assume that the nucleus is spherical. Volume of nucleus
= 4/3 πr^3 = 4/3 π [r_0 $A^{1/3}$]^3
= 4/3 $\pi r_0^3 A$
Mass of nucleus = A
∴ Nuclear mass density = Mass of nucleus/Volume of nucleus
= A/(4/3πr_0^3A) = 3/4πr_0^3
Since r0 is a constant therefore the right hand side is a constant. So, the nuclear mass density is independent of mass number. Thus, nuclear mass density is constant for different nuclei.
For sodium, A = 23
∴ radius of sodium nucleus,
r = 1.2 x 10^{-15} $(23)^{1/3}$ m
= 1.2 x 2.844 x 10^{-15} m
=3.4128 x 10^{-15}

Volume of nucleus = $\frac{4}{3}\pi r^3$

$$= \frac{4}{3}\times\frac{22}{7}\ (3.4128 \times 10^{-15})^3\ m^3 = 1.66 \times 10^{-43}\ m^3$$

If we neglect the mass of electrons of a sodium atom, then the mass of its nucleus can be taken to be the mass of its atom.
∴ Mass of sodium nucleus = 3.82×10^{-26} kg
(Refer to Q. 2.27)
Mass density of sodium nucleus

$$= \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}}$$

$$= \frac{3.82\times10^{-26}}{1.66\times10^{-43}}\ kg\ m^{-3} = 2.3 \times 10^{17}\ kg\ m^{-3}$$

Mass density of sodium atom = 4.67×10^3 kg m^{-3}
(Refer to Q. 2.27)
The ratio of the mass density of sodium nucleus to the average mass density of a sodium atom is

$$\frac{2.3\times10^{17}}{4.67\times10^{3}}\ \textit{i.e.,}\ 4.92 \times 10^{13}.$$

So, the nuclear mass density is nearly 50 million times more than the atomic mass density for a sodium atom.

Question 2. 29. A LASER is source of very intense, monochromatic, and unidirectional beam of light. These properties of a laser light can be exploited to measure long distances. The distance of the Moon from the Earth has been already determined very precisely using a laser as a source of light. A laser light beamed at the Moon takes 2.56 s to return after reflection at the Moon's surface. How

much is the radius of the lunar orbit around the Earth?
Answer:
We known that speed of laser light = c = 3 x 10^8 m/s.
If d be the distance of Moon from the earth, the time taken by laser signal to return after reflection at the Moon's surface

$$t = 2.56 \text{ s} = \frac{2d}{c} = \frac{2d}{3 \times 10^8 \text{ ms}^{-1}}$$

$$\Rightarrow \quad d = \frac{1}{2} \times 2.56 \times 3 \times 10^8 \text{ m} = 3.84 \times 10^8 \text{ m.}$$

Question 2. 30. A SONAR (sound navigation and ranging) uses ultrasonic waves to detect and locate objects under water. In a submarine equipped with a SONAR the time delay between generation of a probe wave and the reception of its echo after reflection from an enemy submarine is found to be 77.0 s. What is the distance of the enemy submarine? (Speed of sound in water = 1450 m s^{-1}).
Answer:
Here speed of sound in water v = 1450 m s^{-1} and time of echo t = 77.0 s.
If distance of enemy submarine be d, then t = 2d/v
∴ d = vt/2 = 1450 x 77.0/2
= 55825 m = 55.8 x 10^3 m or 55.8 km.

*********** END **********