Quadratic Equations Ex 8.6 Q13

Answer :

The quadric equation is $(x-a)(x-b)+(x+b)(x-c)+(x-c)(x-a)=0$

Here,

After simplifying the equation

$$x^2-(a+b)x+ab+x^2-(b+c)x+bc+x^2-(c+a)x+ca=0$$
$$3x^2-2(a+b+c)x+(ab+bc+ca)=0$$

$a=3, b=2(a+b+c)\text{and}, c=(ab+bc+ca)$

As we know that $D=b^2-4ac$

Putting the value of $a=3, b=2(a+b+c)\text{and}, c=(ab+bc+ca)$

$$\begin{aligned}D&=\{2(a+b+c)\}^2-4\times3\times(ab+bc+ca)\\&=4(a^2+b^2+c^2+2ab+2bc+2ca)-12(ab+bc+ca)\\&=4(a^2+b^2+c^2+2ab+2bc+2ca-3ab-3bc-3ca)\\&=4(a^2+b^2+c^2-ab-bc-ca)\end{aligned}$$

$$\begin{aligned}D&=4(a^2+b^2+c^2-ab-bc-ca)\\&=2\left[2a^2+2b^2+2c^2-2ab-2ac-2bc\right]\\&=2\left[(a-b)^2+(b-c)^2+(c-a)^2\right]\end{aligned}$$

Since, $D>0$. So the solutions are real

Let $a=b=c$

Then

$$\begin{aligned}D&=4(a^2+b^2+c^2-ab-bc-ca)\\&=4(a^2+b^2+c^2-aa-bb-cc)\\&=4(\cancel{a^2}+\cancel{b^2}+\cancel{c^2}-\cancel{a^2}-\cancel{b^2}-\cancel{c^2})\\&=4\times0\end{aligned}$$

Thus, the value of $D=0$

Therefore, the roots of the given equation are real and but they are equal only when, $a=b=c$

Hence proved

Quadratic Equations Ex 8.6 Q14

Answer :

The given equations are

$ax^2+bx+c=0$ ...... (1)

$-ax^2+bx+c=0$ ...... (2)

Roots are simultaneously real

Let D_1 and D_2 be the discriminants of equation (1) and (2) respectively,

Then,

$$D_1=(b)^2-4ac$$
$$=b^2-4ac$$

And

$$D_2=(b)^2-4\times(-a)\times c$$
$$=b^2+4ac$$

Both the given equation will have real roots, if $D_1\geq 0$ and $D_2\geq 0$.

Thus,

b2-4ac≥0

b2≥4ac ...... (3)

And,

$b^2+4ac\geq 0$ ...... (4)

Now given that a,b,c are real number and $ac\neq 0$ as well as from equations (3) and (4) we get

At least one of the given equation has real roots

Hence, proved

Quadratic Equations Ex 8.6 Q15

Answer :

The given equation $(1+m^2)x^2+2mcx+(c^2-a^2)=0$, has equal roots

Then prove that $c^2=(1+m^2)$.

Here,

$a=(1+m^2), b=2mc$ and, $c=(c^2-a^2)$

As we know that $D=b^2-4ac$

Putting the value of $a=(1+m^2), b=2mc$ and, $c=(c^2-a^2)$

$$D=b^2-4ac$$
$$=\{2mc\}^2-4\times(1+m^2)\times(c^2-a^2)$$
$$=4(m^2c^2)-4(c^2-a^2+m^2c^2-m^2a^2)$$
$$=\cancel{4m^2c^2}-4c^2+4a^2-\cancel{4m^2c^2}+4m^2a^2$$
$$=4a^2+4m^2a^2-4c^2$$

The given equation will have real roots, if $D=0$

$$4a^2+4m^2a^2-4c^2=0$$
$$4a^2+4m^2a^2=4c^2$$
$$4a^2(1+m^2)=4c^2$$
$$a^2(1+m^2)=c^2$$

Hence, $\boxed{c^2=a^2(1+m^2)}$

*********** END **********