Differentiation Ex 11.7 Q1

Given that $x = at^2$, $y = 2at$

$$\text{So, } \frac{dx}{dt} = \frac{d}{dt}\left(at^2\right) = 2at$$

$$\frac{dy}{dt} = \frac{d}{dt}(2at) = 2a$$

$$\text{Therefore, } \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$$

Differentiation Ex 11.7 Q2

Here,

$$x = a(\theta + \sin\theta)$$

Differentiating it with respect to θ,

$$\frac{dx}{d\theta} = a(1 + \cos\theta) \qquad \text{---(i)}$$

And,

$$y = a(1 - \cos\theta)$$

Differentiating it with respect to θ,

$$\frac{dy}{d\theta} = a(\theta + \sin\theta)$$

$$\frac{dy}{d\theta} = a\sin\theta \qquad \text{---(ii)}$$

Using equation (i) and (ii),

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$= \frac{a\sin\theta}{a(1-\cos\theta)}$$

$$= \frac{2\frac{\sin\theta}{2}\frac{\cos\theta}{2}}{\frac{2\sin^2\theta}{2}}, \qquad \left\{\begin{array}{l}\text{Since, } 1-\cos\theta = \frac{2\sin^{2\theta}}{2}, \\ \frac{2\sin\theta}{2}\frac{\cos\theta}{2} = \sin\theta\end{array}\right\}$$

$$= \frac{dy}{dx} = \frac{\tan\theta}{2}$$

Differentiation Ex 11.7 Q3

Here $x = a\cos\theta$ and $y = b\sin\theta$

Then,

$$\frac{dx}{d\theta} = \frac{d}{d\theta}(a\cos\theta) = -a\sin\theta$$
$$\frac{dy}{d\theta} = \frac{d}{d\theta}(b\sin\theta) = b\cos\theta$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{b\cos\theta}{-a\sin\theta} = -\frac{b}{a}\cot\theta$$

Differentiation Ex 11.7 Q4

Here,

$$x = ae^{\theta}(\sin\theta - \cos\theta)$$

Differentiating it with respcet to θ,

$$\frac{dx}{d\theta} = a\left[e^{\theta}\frac{d}{d\theta}(\sin\theta - \cos\theta) + (\sin\theta - \cos\theta)\frac{d}{d\theta}\left(e^{\theta}\right)\right]$$
$$= a\left[e^{\theta}(\cos\theta + \sin\theta) + (\sin\theta - \cos\theta)e^{\theta}\right]$$
$$\frac{dx}{d\theta} = a\left[2e^{\theta}\sin\theta\right] \qquad \text{---(i)}$$

And, $y = ae^{\theta}(\sin\theta + \cos\theta)$

Differentiating it with respect to θ,

$$\frac{dy}{d\theta} = a\left[e^{\theta}\frac{d}{d\theta}(\sin\theta + \cos\theta) + (\sin\theta + \cos\theta)\frac{d}{d\theta}\left(e^{\theta}\right)\right]$$
$$= a\left[e^{\theta}(\cos\theta - \sin\theta) + (\sin\theta + \cos\theta)e^{\theta}\right]$$
$$\frac{dy}{d\theta} = a\left[2e^{\theta}\cos\theta\right] \qquad \text{---(ii)}$$

Dividing equation (ii) by equation (i),

$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\left(2e^{\theta}\cos\theta\right)}{a\left(2e^{\theta}\sin\theta\right)}$$
$$\frac{dy}{dx} = \cot\theta$$

Differentiation Ex 11.7 Q5

Here $x = b\sin^2\theta$ and $y = a\cos^2\theta$

Then,

$$\frac{dx}{d\theta} = \frac{d}{d\theta}\left(b\sin^2\theta\right) = 2b\sin\theta\cos\theta$$
$$\frac{dy}{d\theta} = \frac{d}{d\theta}\left(a\cos^2\theta\right) = -2a\cos\theta\sin\theta$$
$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-2a\cos\theta\sin\theta}{2b\sin\theta\cos\theta} = -\frac{a}{b}$$

*********** END **********