



Differentiation Ex 11.5 Q39

Here,

$$x^m y^n = 1$$

Taking \log on both the side,

$$\log(x^m y^n) = \log(1)$$

$$m \log x + n \log y = \log(1)$$

Differentiating it with respect to x ,

$$\frac{dy}{dx} (m \log x) + \frac{d}{dx} (n \log y) = \frac{d}{dx} (\log(1))$$

$$\frac{m}{x} + \frac{n}{y} \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{m}{x} \times \frac{y}{n}$$

$$\frac{dy}{dx} = -\frac{my}{nx}$$

Differentiation Ex 11.5 Q40

Here,

$$y^x = e^{y-x}$$

Taking \log on both the sides,

$$\log y^x = \log e^{(y-x)}$$

$$x \log y = (y-x) \log e$$

$$x \log y = y - x \quad \text{---(i)}$$

[Since, $\log a^b = b \log a$ and $\log_e e = 1$]

Differentiating it with respect to x using product rule,

$$\frac{d}{dx} (x \log y) = \frac{d}{dx} (y - x)$$

$$\left[x \frac{d}{dx} (\log y) + \log y \frac{d}{dx} (x) \right] = \frac{dy}{dx} - 1$$

$$x \left(\frac{1}{y} \right) \frac{dy}{dx} + \log y (1) = \frac{dy}{dx} - 1$$

$$\frac{dy}{dx} \left(\frac{x}{y} - 1 \right) = -1 - \log y$$

$$\frac{dy}{dx} \left(\frac{y - x}{y} \right) = -(1 + \log y)$$

[Since, from equation (i), $x = \frac{y}{(1 + \log y)}$]

$$\frac{dy}{dx} \left[\frac{1 - 1 - \log y}{(1 + \log y)} \right] = -(1 + \log y)$$

$$\frac{dy}{dx} = -\frac{(1 + \log y)^2}{-\log y}$$

$$\frac{dy}{dx} = \frac{(1 + \log y)^2}{\log y}$$

Differentiation Ex 11.5 Q41

Here,

$$(\sin x)^y = (\cos y)^x$$

Taking log on both the sides,

$$\begin{aligned}\log(\sin x)^y &= \log(\cos y)^x & [\text{Using } \log a^b &= b \log a] \\ y \log(\sin x) &= x \log(\cos y)\end{aligned}$$

Differentiating it with respect to x using product rule and chain rule,

$$\begin{aligned}\frac{d}{dx}[y \log \sin x] &= \frac{d}{dx}[x \log \cos y] \\ y \frac{d}{dx}(\log \sin x) + \log \sin x \frac{dy}{dx} &= x \frac{dy}{dx} \log \cos y + \log \cos y \frac{d}{dx}(x) \\ y \left(\frac{1}{\sin x} \right) \frac{d}{dx}(\sin x) + \log \sin x \frac{dy}{dx} &= \frac{x}{\cos y} \frac{d}{dx}(\cos y) + \log \cos y (1) \\ \frac{y}{\sin x}(\cos x) + \log \sin x \frac{dy}{dx} &= \frac{x}{\cos y}(-\sin y) \frac{dy}{dx} + \log \cos y \\ y \cot x + \log \sin x \frac{dy}{dx} &= -x \tan y \frac{dy}{dx} + \log \cos y \\ \frac{dy}{dx}(\log \sin x + x \tan y) &= \log \cos y - y \cot x \\ \frac{dy}{dx} &= \frac{\log \cos y - y \cot x}{\log \sin x + x \tan y}\end{aligned}$$

Differentiation Ex 11.5 Q42

Here,

$$(\cos x)^y = (\tan y)^x$$

Taking log on both the sides,

$$\begin{aligned}\log(\cos x)^y &= \log(\tan y)^x \\ y \log \cos x &= x \log \tan y & [\text{Since, } \log a^b &= b \log a]\end{aligned}$$

Differentiating it with respect to x using chain rule and product rule,

$$\begin{aligned}\frac{d}{dx}(y \log \cos x) &= \frac{d}{dx}(x \log \tan y) \\ \left(y \frac{d}{dx} \log \cos x + \log \cos x \frac{dy}{dx} \right) &= \left(x \frac{d}{dx} \log \tan y + \log \tan y \frac{d}{dx}(x) \right) \\ \left(y \left(\frac{1}{\cos x} \right) \frac{d}{dx}(\cos x) + \log \cos x \frac{dy}{dx} \right) &= \left(x \frac{1}{\tan y} \frac{d}{dx}(\tan y) + \log \tan y (1) \right) \\ \left(\frac{y}{\cos x}(-\sin x) + \log \cos x \frac{dy}{dx} \right) &= \left(\frac{x}{\tan y}(\sec^2 y) \right) \frac{dy}{dx} + \log \tan y - y \tan x + \log \cos x \frac{dy}{dx} \\ &= \left(\sec y \operatorname{cosec} y \times x \frac{dy}{dx} + \log \tan y \right) \\ \frac{dy}{dx}[\log \cos x - x \sec y \operatorname{cosec} y] &= \log \tan y + y \tan x \\ \frac{dy}{dx} &= \left[\frac{\log \tan y + y \tan x}{\log \cos x - x \sec y \operatorname{cosec} y} \right]\end{aligned}$$

Differentiation Ex 11.5 Q43

Here,

$$e^x + e^y = e^{x+y} \quad \text{---(i)}$$

Differentiating both the sides using chain rule,

$$\frac{d}{dx}(e^x) + \frac{d}{dx}(e^y) = \frac{d}{dx}(e^{x+y})$$

$$e^x + e^y \frac{dy}{dx} = e^{x+y} \frac{d}{dx}(x+y)$$

$$e^x + e^y \frac{dy}{dx} = e^{x+y} \left[1 + \frac{dy}{dx} \right]$$

$$e^y \frac{dy}{dx} - e^{x+y} \frac{dy}{dx} = e^{x+y} - e^x$$

$$\frac{dy}{dx} = \frac{e^{x+y} - e^x}{e^y - e^{x+y}}$$

$$= \left(\frac{e^x + e^y - e^x}{e^y - e^x - e^y} \right)$$

[Using equation (i)]

$$\frac{dy}{dx} = -e^{y-x}$$

$$\frac{dy}{dx} + e^{y-x} = 0$$

***** END *****