Polynomials Ex 2.3 Q10

Answer :

We know that if $x=\alpha$ is a zero of a polynomial, and then $x-\alpha$ is a factor of $f(x)$.

Since $\sqrt{2}$ and $-\sqrt{2}$ are zeros of $f(x)$.

Therefore

$$\left(x+\sqrt{2}\right)\left(x-\sqrt{2}\right)=x^2-\left(\sqrt{2}\right)^2$$
$$=x^2-2$$

x^2-2 is a factor of $f(x)$. Now, we divide $2x^4+7x^3-19x^2-14x+30$ by $g(x)=x^2-2$ to find the zero of $f(x)$.

$$\begin{array}{r|l} & 2x^2+7x-15 \\ \hline x^2-2 & +\cancel{2x^4}+7x^3-19x^2-14x+30 \\ & +\cancel{2x^4}+0-4x^2 \\ & \underline{-\quad\; -\quad +} \\ & +\cancel{7x^3}-15x^2-\cancel{14x} \\ & +\cancel{7x^3}-0\quad-\cancel{14x} \\ & \underline{-\qquad +\qquad +} \\ & -\cancel{15x^2}+\cancel{30} \\ & -\cancel{15x^2}+\cancel{30} \\ & \underline{+\qquad -} \\ & 0 \end{array}$$

By using division algorithm we have $f(x)=g(x)\times q(x)-r(x)$

$$2x^4+7x^3-19x^2-14x+30=\left(x^2-2\right)\left(2x^2+7x-15\right)+0$$
$$2x^4+7x^3-19x^2-14x+30=\left(x+\sqrt{2}\right)\left(x-\sqrt{2}\right)\left(2x^2+10x-3x-15\right)$$
$$2x^4+7x^3-19x^2-14x+30=\left(x+\sqrt{2}\right)\left(x-\sqrt{2}\right)\left[2x(x+5)-3(x+5)\right]$$
$$2x^4+7x^3-19x^2-14x+30=\left(x+\sqrt{2}\right)\left(x-\sqrt{2}\right)(2x-3)(x+5)$$

Hence, the zeros of the given polynomial are $\boxed{-\sqrt{2},+\sqrt{2},\frac{+3}{2},-5}$.

Polynomials Ex 2.3 Q11

Answer :

We know that if $x=\alpha$ is a zero of a polynomial, and then $x-\alpha$ is a factor of $f(x)$.

Since $\sqrt{3}$ and $-\sqrt{3}$ are zeros of $f(x)$.

Therefore

$$\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)=x^2-3$$
$$=x^2-3$$

x^2-3 is a factor of $f(x)$. Now, we divide $2x^3+x^2-6x-3$ by $g(x)=x^2-3$ to find the other zeros of $f(x)$.

$$\begin{array}{r|l} & 2x+1 \\ \hline x^2-3 & +\cancel{2x^3}+x^2-6x-3 \\ & +\cancel{2x^3}-0-6x \\ & \underline{-\qquad\quad +} \\ & +\cancel{x^2}+0\;-\cancel{3} \\ & +\cancel{x^2}+0\;\;\cancel{3} \\ & \underline{-} \\ & 0 \end{array}$$

By using division algorithm we have $f(x)=g(x)\times q(x)-r(x)$

$$2x^3+x^2-6x-3=\left(x^2-3\right)\times(2x+1)+0$$
$$2x^3+x^2-6x-3=\left(x^2+\sqrt{3}\right)\left(x-\sqrt{3}\right)(2x+1)$$

Hence, the zeros of the given polynomial are $\boxed{-\sqrt{3},+\sqrt{3},\frac{-1}{2}}$.

Polynomials Ex 2.3 Q12

Answer :

We know that if $x=\alpha$ is a zero of a polynomial, and then $x-\alpha$ is a factor of $f(x)$.

Since $\sqrt{2}$ and $-\sqrt{2}$ are zeros of $f(x)$.

Therefore

$$\begin{aligned}(x+\sqrt{2})(x-\sqrt{2}) &= x^2-(\sqrt{2})^2 \\ &= x^2-2\end{aligned}$$

x^2-2 is a factor of $f(x)$. Now, we divide x^3+3x^2-2x-6 by $g(x)=x^2-2$ to find the other zeros of $f(x)$.

$$\begin{array}{r|l} & x+3 \\ \hline x^2-2 & +\cancel{x^3}+3x^2-\cancel{2x}-6 \\ & \underset{-}{+}\cancel{x^3}-0\ \underset{+}{-}\cancel{2x} \\ \hline & +\cancel{3x^2}-\cancel{6} \\ & \underset{-}{+}\cancel{3x^2}\ \underset{+}{-}\cancel{6} \\ \hline & 0 \end{array}$$

By using division algorithm we have $f(x)=g(x)\times q(x)-r(x)$

$$\begin{aligned}x^3+3x^2-2x-6 &= (x^2-2)(x+3)-0 \\ &= (x+\sqrt{2})(x-\sqrt{2})(x+3)\end{aligned}$$

Hence, the zeros of the given polynomials are $\boxed{-\sqrt{2},+\sqrt{2},\text{ and }-3}$.

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