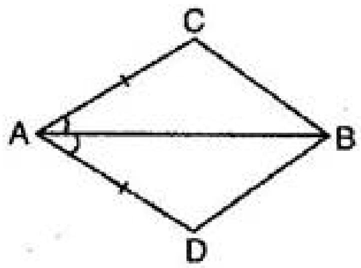
Q1. In quadrilateral ABCD (See figure). AC = AD and AB bisects $\angle$A. Show that ΔABC $\cong$ ΔABD. What can you say about BC and BD?



Ans. Given: In quadrilateral ABCD, AC = AD and AB bisects $\angle$A.

To prove: ΔABC $\cong$ ΔABD

Proof: In ΔABC and ΔABD,

AC = AD [Given]

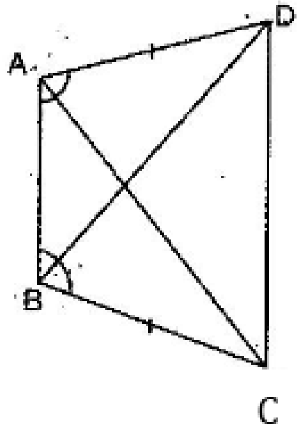
$\angle$BAC = $\angle$BAD [$\because$ AB bisects $\angle$A]

AB = AB [Common]

$\therefore$ ΔABC $\cong$ ΔABD [By SAS congruency]

Thus BC = BD [By C.P.C.T.]

Q2. ABCD is a quadrilateral in which AD = BC and $\angle$DAB = $\angle$CBA. (See figure). Prove that:



(i) Δ **ABD** $\cong \Delta$ **BAC**

(ii) BD = AC

(iii) $\angle$ **ABD** = $\angle$ **BAC**

Ans. (i) In ΔABC and ΔABD,

BC = AD [Given]

$\angle$ DAB = $\angle$ CBA [Given]

AB = AB [Common]

$\therefore$ ΔABC $\cong \Delta$ABD [By SAS congruency]

Thus AC = BD [By C.P.C.T.]

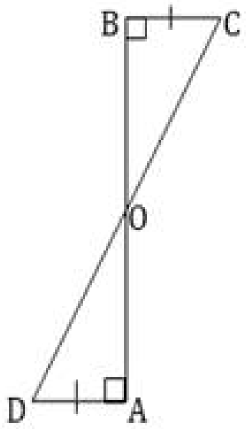
(ii) Since ΔABC $\cong \Delta$ABD

$\therefore$ AC = BD [By C.P.C.T.]

(iii) Since ΔABC $\cong \Delta$ABD

$\therefore$ $\angle$ABD = $\angle$BAC [By C.P.C.T.]

Q3. AD and BC are equal perpendiculars to a line segment AB. Show that CD bisects AB (See figure)



Ans. In ΔBOC and ΔAOD,

$\angle$ OBC = $\angle$ OAD = 90° [Given]

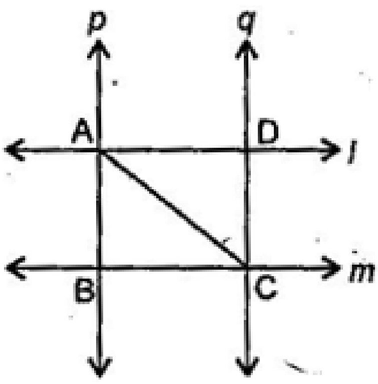
$\angle$ BOC = $\angle$ AOD [Vertically Opposite angles]

BC = AD [Given]

$\therefore$ Δ BOC $\cong$ Δ AOD [By ASA congruency]

$\Rightarrow$ OB = OA and OC = OD [By C.P.C.T.]

Q4. l **and** m **are two parallel lines intersected by another pair of parallel lines** p **and** q **(See figure). Show that** Δ **ABC** $\cong$ Δ **CDA.**



Ans. AC being a transversal. [Given]

Therefore $\angle$ DAC = $\angle$ ACB [Alternate angles]

Now $p \parallel q$ [Given]

And AC being a transversal. [Given]

Therefore $\angle$ BAC = $\angle$ ACD [Alternate angles]

Now In Δ ABC and Δ ADC,

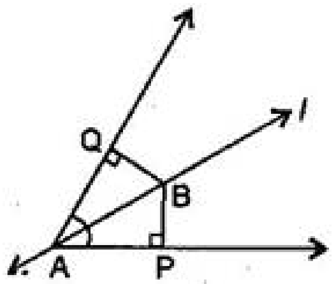
$\angle$ ACB = $\angle$ DAC [Proved above]

$\angle$ BAC = $\angle$ ACD [Proved above]

AC = AC [Common]

$\therefore$ Δ ABC $\cong$ Δ CDA [By ASA congruency]

Q5. Line l is the bisector of the angle A and B is any point on l. BP and BQ are perpendiculars from B to the arms of $\angle$ A. Show that:



(i) Δ APB $\cong$ Δ AQB

(ii) BP = BQ or P is equidistant from the arms of $\angle$ A (See figure).

Ans. Given: Line l bisects $\angle$ A.

$\therefore$ $\angle$ BAP = $\angle$ BAQ

(i) In ΔABP and ΔABQ,

$\angle BAP = \angle BAQ$ [Given]

$\angle BPA = \angle BQA = 90^\circ$ [Given]

AB = AB [Common]

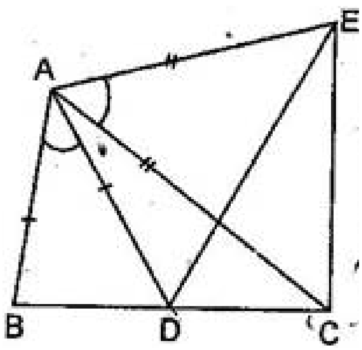
$\therefore$ $\Delta APB \cong \Delta AQB$ [By ASA congruency]

(ii) Since $\Delta APB \cong \Delta AQB$

$\therefore$ BP = BQ [By C.P.C.T.]

$\Rightarrow$ B is equidistant from the arms of $\angle A$.

Q6. In figure, AC = AB, AB = AD and $\angle BAD = \angle EAC$. Show that BC = DE.



Ans. Given that $\angle BAD = \angle EAC$

Adding $\angle DAC$ on both sides, we get

$\angle BAD + \angle DAC = \angle EAC + \angle DAC$

$\Rightarrow$ $\angle BAC = \angle EAD$ ....(i)

Now in ΔABC and ΔAED,

AB = AD [Given]

AC = AE [Given]

$\angle BAC = \angle DAE$ [From eq. (i)]

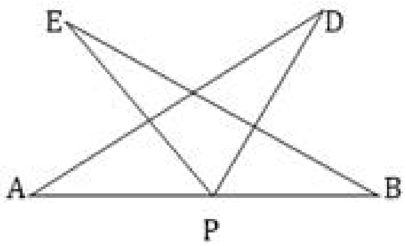
$\therefore \Delta ABC \cong \Delta ADE$ [By SAS congruency]

$\Rightarrow BC = DE$ [By C.P.C.T.]

Q7. AB is a line segment and P is the mid-point. D and E are points on the same side of AB such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$. Show that:

(i) $\Delta DAF \cong \Delta FBP$

(ii) AD = BE (See figure)



Ans. Given that $\angle EPA = \angle DPB$

Adding $\angle EPD$ on both sides, we get

$\angle EPA + \angle EPD = \angle DPB + \angle EPD$

$\Rightarrow \angle APD = \angle BPE$ ....(i)

Now in ΔAPD and ΔBPE,

$\angle PAD = \angle PBE$ [$\because$ $\angle BAD = \angle ABE$ (given),

$\therefore$ $\angle PAD = \angle PBE$]

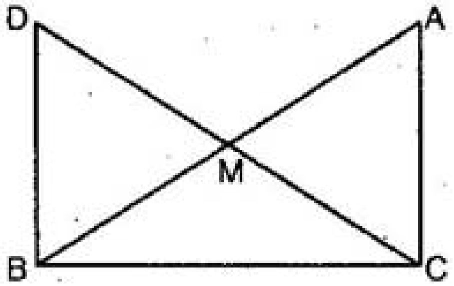
AP = PB [P is the mid-point of AB]

$\angle APD = \angle BPE$ [From eq. (i)]

$\therefore$ $\Delta DPA \cong \Delta EBP$ [By ASA congruency]

$\Rightarrow AD = BE$ [By C.P.C.T.]

Q8. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B. (See figure)



Show that:

(i) $\Delta AMC \cong \Delta BMD$

(ii) $\angle DBC$ is a right angle.

(iii) $\Delta DBC \cong \Delta ACB$

(iv) $CM = \frac{1}{2} AB$

Ans. (i) In ΔAMC and ΔBMD,

AM = BM [AB is the mid-point of AB]

$\angle AMC = \angle BMD$ [Vertically opposite angles]

CM = DM [Given]

$\therefore$ $\Delta AMC \cong \Delta BMD$ [By SAS congruency]

$\therefore$ $\angle ACM = \angle BDM$ ....(i)

$\angle CAM = \angle DBM$ and AC = BD [By C.P.C.T.]

(ii) For two lines AC and DB and transversal DC, we have,

$\angle ACD = \angle BDC$ [Alternate angles]

$\therefore$ AC || DB

Now for parallel lines AC and DB and for transversal BC.

$\angle$ DBC = $\angle$ ACB [Alternate angles] ....(ii)

But Δ ABC is a right angled triangle, right angled at C.

$\therefore$ $\angle$ ACB = 90° ....(iii)

Therefore $\angle$ DBC = 90° [Using eq. (ii) and (iii)]

$\Rightarrow$ $\angle$ DBC is a right angle.

(iii) Now in Δ DBC and Δ ABC,

DB = AC [Proved in part (i)]

$\angle$ DBC = $\angle$ ACB = 90° [Proved in part (ii)]

BC = BC [Common]

$\therefore$ Δ DBC $\cong$ Δ ACB [By SAS congruency]

(iv) Since Δ DBC $\cong$ Δ ACB [Proved above]

$\therefore$ DC = AB

$\Rightarrow$ AM + CM = AB

$\Rightarrow$ CM + CM = AB [$\because$ DM = CM]

$\Rightarrow$ 2CM = AB

$\Rightarrow$ CM = $\frac{1}{2}$ AB

*********** END **********