Mathematical Induction Ex 12.2 Q4

Let $P(n): \frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots+\frac{1}{n(n+1)}=\frac{n}{n+1}$

For $n=1$

$$P(1): \frac{1}{1.2}=\frac{1}{1+1}$$
$$\frac{1}{2}=\frac{1}{2}$$

$\Rightarrow \quad P(n)$ is true for $n=1$

Let $P(n)$ is true for $n=k$, so

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots+\frac{1}{k(k+1)}=\frac{k}{k+1} \qquad ---(1)$$

We have to show that

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots+\frac{1}{k(k+1)}+\frac{k}{(k+1)(k+2)}=\frac{k+1}{(k+2)}$$

Now,

$$\left\{\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots+\frac{1}{k(k+1)}\right\}+\frac{1}{(k+1)(k+2)}$$

$$=\frac{k}{k+1}+\frac{1}{(k+1)(k+2)} \qquad \text{[Using equation (1)]}$$

$$=\frac{1}{k+1}\left[\frac{k(k+2)+1}{(k+2)}\right]$$
$$=\frac{1}{k+1}\left[\frac{k^2+2k+1}{(k+2)}\right]$$
$$=\frac{1}{k+1}\left[\frac{(k+1)(k+1)}{(k+2)}\right]$$

$$=\frac{(k+1)}{(k+2)}$$

$\Rightarrow \quad P(n)$ is true for $n=k+1$

$\Rightarrow \quad P(n)$ is true for all $n \in N$ by PMI

Mathematical Induction Ex 12.2 Q5

Let $P(n): 1+3+5+\ldots+(2n-1)=n^2$

For $n=1$

$P(1): 1=1^2$
$1=1$

$\Rightarrow \quad P(n)$ is true for $n=1$
Let $P(n)$ is true for $n=k$, so

$P(k): 1+3+5+\ldots+(2k-1)=k^2 \qquad ---(1)$

We have to show that
$1+3+5+\ldots+(2k-1)+2(k+1)-1=(k+1)^2$

Now,
$\{1+3+5+\ldots+(2k-1)\}+(2k+1)$

$=k^2+(2k+1) \qquad$ [Using equation (1)]

$=k^2+2k+1$

$=(k+1)^2$

$\Rightarrow \quad P(n)$ is true for $n=k+1$
$\Rightarrow \quad P(n)$ is true for all $n \in N$ by PMI

Mathematical Induction Ex 12.2 Q6

Let $P(n): \frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \ldots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{(6n+4)}$

Put $n = 1$

$P(1): \frac{1}{2.5} = \frac{1}{6+4}$

$\frac{1}{10} = \frac{1}{10}$

$\Rightarrow \quad P(n)$ is true for $n = 1$

Let $P(n)$ is true for $n = k$, so

$$\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \ldots + \frac{1}{(3k-1)(3k+2)} = \frac{k}{(6k+4)} \qquad ---(1)$$

We have to show that,

$$\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \ldots + \frac{1}{(3k-1)(3k+2)} + \frac{1}{(3k+2)(3k+5)} = \frac{(k+1)}{(6k+10)}$$

Now,

$$\left\{\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \frac{1}{(3k-1)(3k+2)}\right\} + \frac{1}{(3k+2)(3k+5)}$$

$$= \frac{k}{6k+4} + \frac{1}{(3k+2)(3k+5)}$$

$$= \frac{k}{2(3k+2)} + \frac{1}{(3k+2)(3k+5)}$$

$$= \frac{k(3k+5)+2}{2(3k+2)(3k+5)}$$

$$= \frac{3k^2+5k+2}{2(3k+2)(3k+5)}$$

$$= \frac{3k^2+3k+2k+2}{2(3k+2)(3k+5)}$$

$$= \frac{3k(k+1)+2(k+1)}{2(3k+2)(3k+5)}$$

$$= \frac{(k+1)\cancel{(3k+2)}}{2\cancel{(3k+2)}(3k+5)}$$

$$= \frac{(k+1)}{2(3k+5)}$$

$P(n)$ is true for $n = k+1$

$P(n)$ is true for all $n \in N$ by PMI

*********** END **********