Chapter 6 Determinants Ex 6.2 Q1-i

$$\begin{vmatrix} 1 & 3 & 5 \\ 2 & 6 & 10 \\ 31 & 11 & 38 \end{vmatrix} = 2\begin{vmatrix} 1 & 3 & 5 \\ 1 & 3 & 5 \\ 31 & 11 & 38 \end{vmatrix} = 0$$

Chapter 6 Determinants Ex 6.2 Q1-ii

Consider the determinant

$$\Delta = \begin{vmatrix} 67 & 19 & 21 \\ 39 & 13 & 14 \\ 81 & 24 & 26 \end{vmatrix}$$

Applying $C_1 \to C_1 - 4C_3$, *we get*,

$$\Delta = \begin{vmatrix} 4 & 19 & 21 \\ -3 & 13 & 14 \\ -3 & 24 & 26 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 4 & 19 & 21 \\ -3 & 13 & 14 \\ -3 & 24 & 26 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 32 & 35 \\ -3 & 13 & 14 \\ 0 & 11 & 12 \end{vmatrix} \quad [\text{Applying } R_3 \to R_3 - R_2 \text{ and } R_1 \to R_1 + R_2]$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 32 & 35 \\ 0 & 109 & 119 \\ 0 & 11 & 12 \end{vmatrix} \quad [\text{Applying } R_2 \to 3R_1 + R_2]$$

$$\Rightarrow \Delta = 1(109 \times 12 - 119 \times 11)$$

$$\Rightarrow \Delta = -1$$

Chapter 6 Determinants Ex 6.2 Q1-iii

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

$$= a(bc - f^2) - h(hc - fg) + g(hf - bg)$$

$$= abc - af^2 - h^2c + hfg + ghf - bg^2$$

Chapter 6 Determinants Ex 6.2 Q1-iv

$$\begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix} = 2\begin{vmatrix} 1 & -3 & 1 \\ 4 & -1 & 1 \\ 3 & 5 & 1 \end{vmatrix} = 2\begin{vmatrix} 1 & -3 & 1 \\ 3 & 2 & 0 \\ 2 & 8 & 0 \end{vmatrix} = 2(24 - 4) = 40$$

Chapter 6 Determinants Ex 6.2 Q1-v

Let Δ be the determinant.

$$\Delta = \begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$$

Applying $R_3 \to R_3 - R_2$, we get,

$$\Delta = \begin{vmatrix} 1 & 4 & 9-4 \\ 4 & 9 & 16-9 \\ 9 & 16 & 25-16 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 4 & 5 \\ 4 & 9 & 7 \\ 9 & 16 & 9 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 5 & 5 \\ 4 & 13 & 7 \\ 9 & 25 & 9 \end{vmatrix} \qquad [\text{Applying } C_2 \to C_1 + C_2]$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 0 & 0 \\ 4 & -7 & -13 \\ 9 & -20 & -36 \end{vmatrix} \qquad [\text{Applying } C_2 \to 5C_1 - C_2 \text{ and } C_3 \to 5C_1 - C_3]$$

$$\Rightarrow \Delta = 1(7 \times 36 - 13 \times 20) = 252 - 260 = -8$$

Chapter 6 Determinants Ex 6.2 Q1-vi

$$\begin{vmatrix} 6 & -3 & 2 \\ 2 & -1 & 2 \\ -10 & 5 & 2 \end{vmatrix}$$

Apply: $R_1 \to R_1 + (-3)R_2$ and $R_3 \to R_3 + 5R_2$

$$= \begin{vmatrix} 0 & 0 & -4 \\ 2 & -1 & 2 \\ 0 & 0 & 12 \end{vmatrix} = 0$$

Chapter 6 Determinants Ex 6.2 Q1-vii

$$\begin{vmatrix} 1 & 3 & 9 & 27 \\ 3 & 9 & 27 & 1 \\ 9 & 27 & 1 & 3 \\ 27 & 1 & 3 & 9 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 3^2 & 3^3 \\ 3 & 3^2 & 3^3 & 1 \\ 3^2 & 3^3 & 1 & 3 \\ 3^3 & 1 & 3 & 3^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1+3+3^2+3^3 & 3 & 3^2 & 3^3 \\ 1+3+3^2+3^3 & 3^2 & 3^3 & 1 \\ 1+3+3^2+3^3 & 3^3 & 1 & 3 \\ 1+3+3^2+3^3 & 1 & 3 & 3^2 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right) \begin{vmatrix} 1 & 3 & 3^2 & 3^3 \\ 1 & 3^2 & 3^3 & 1 \\ 1 & 3^3 & 1 & 3 \\ 1 & 1 & 3 & 3^2 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right) \begin{vmatrix} 1 & 3 & 3^2 & 3^3 \\ 0 & 3^2-3 & 3^3-3^2 & 1-3^3 \\ 0 & 3^3-3 & 1-3^2 & 3-3^3 \\ 0 & 1-3 & 3-3^2 & 3^2-3^3 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right) \begin{vmatrix} 6 & 18 & -26 \\ 24 & -8 & -24 \\ -2 & -6 & -18 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right) 2^3 \begin{vmatrix} 3 & -9 & 13 \\ 12 & 4 & 12 \\ -1 & 3 & 9 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right) 2^3 \begin{vmatrix} 0 & 0 & 40 \\ 12 & 4 & 12 \\ -1 & 3 & 9 \end{vmatrix}$$

$$= \left(1+3+3^2+3^3\right) 2^3 \times 40(36+4) = 512000$$

Chapter 6 Determinants Ex 6.2 Q1-viii

$$Let\ \Delta = \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

$Applying\ R_3 \to 17R_2 - R_3,\ we\ get,$

$$\Delta = \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 0 & 48 & 62 \end{vmatrix}$$

$Applying\ R_2 \to 102R_2 - R_1,\ we\ get,$

$$\Delta = \begin{vmatrix} 102 & 18 & 36 \\ 0 & 288 & 372 \\ 0 & 48 & 62 \end{vmatrix}$$

$Thus,$

$$\Delta = 102(288 \times 62 - 372 \times 48)$$

$$\Rightarrow \Delta = 0$$

********** END **********