Geometric Progressions Ex 20.5 Q 12

a, b, c are in G.P.

$a,\ b = ar,\ c = ar^2$

$a\left(b^2 + c^2\right) = c\left(a^2 + b^2\right)$

$a\left(a^2r^2 + a^2r^4\right) = ar^2\left(a^2 + a^2r^2\right)$

$a^3r^2\left(1 + r^2\right) = a^3r^2\left(1 + r^2\right)$

LHS = RHS

a, b, c are in G.P.

$a,\ b = ar,\ c = ar^2$

LHS $= a^2b^2c^2\left(\frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3}\right)$

$= a^2 \times a^2r^2 \times a^2r^4\left(\frac{1}{a^3} + \frac{1}{a^3r^3} + \frac{1}{a^3r^6}\right)$

$= a^6r^6\left(\frac{r^6 + r^3 + 1}{a^3r^6}\right)$

$= a^3\left(r^6 + r^3 + 1\right)$

$= a^3 + a^3r^3 + a^3r^6$

$= a^3 + (ar)^3 + \left(ar^2\right)^3$

$= a^3 + b^3 + c^3$

= RHS

$\therefore$ LHS = RHS

a, b, c are in G.P.

$a,\ b = ar,\ c = ar^2$

$$\text{LHS} = \frac{(a+b+c)^2}{a^2+b^2+c^2}$$

$$= \frac{\left(a+ar+ar^2\right)^2}{a^2+a^2r^2+a^2r^4}$$

$$= \frac{a^2\left(1+r+r^2\right)^2}{a^2\left(1+r^2+r^4\right)}$$

$$= \frac{a^2\left(1+r+r^2\right)^2}{a^2\left[\left(1+2r^2+r^4\right)-r^2\right]}$$

$$= \frac{a^2\left(1+r+r^2\right)^2}{a^2\left[\left(1+r^2-r\right)\left(1+r^2+r\right)\right]}$$

$$= \frac{a\left(1+r+r^2\right)}{a\left(1+r^2-r\right)}$$

$$= \frac{a+ar+ar^2}{a+ar^2-ar}$$

$$= \frac{a+b+c}{a-b+c}$$

= RHS

$\therefore$ LHS = RHS

a, b, c are in G.P.

$a,\ b = ar,\ c = ar^2$

$$\text{LHS} = \frac{1}{a^2-b^2}+\frac{1}{b^2}$$

$$= \frac{1}{a^2-a^2r^2}+\frac{1}{a^2r^2}$$

$$= \frac{1}{a^2}\left[\frac{1}{1-r^2}+\frac{1}{r^2}\right]$$

$$= \frac{1}{a^2}\left[\frac{r^2+1-r^2}{\left(1-r^2\right)r^2}\right]$$

$$= \frac{1}{a^2}\left[\frac{1}{r^2-r^4}\right]$$

$$= \frac{1}{(ar)^2-\left(ar^2\right)^2}$$

$$= \frac{1}{b^2-c^2}$$

= RHS

$\therefore$ LHS = RHS

a, b, c are in G.P.

$a,\ b = ar,\ c = ar^2$

LHS $= (a + 2b + 2c)(a - 2b + 2c)$

$= \left(a + 2ar + 2ar^2\right)\left(a - 2ar + 2ar^2\right)$

$= a^2\left(1 + 2r + 2r^2\right)\left(1 - 2r + 2r^2\right)$

$= a^2\left[\left(1 + 2r^2\right)^2 - (2r)^2\right]$

$= a^2\left[1 + 4r^4 + 4r^2 - 4r^2\right]$

$= a^2\left[1 + 4r^4\right]$

$= a^2 + 4\left(ar^2\right)^2$

$= a^2 + 4c^2$

= RHS

$\therefore$ LHS = RHS

********** END **********