



Trigonometric Ratios Ex 5.2 Q26

Answer :

(i) Given:

$$\theta = 30^\circ \dots\dots (1)$$

To verify:

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} \dots\dots (2)$$

Now consider LHS of the expression to be verified in equation (2)

Therefore,

$$\text{LHS} = \tan 2\theta$$

Now by substituting the value of θ from equation (1) in the above expression

We get,

$$\begin{aligned} \text{LHS} &= \tan 2 \times (30^\circ) \\ &= \tan 60^\circ \\ &= \sqrt{3} \end{aligned}$$

Now by substituting the value of θ from equation (1) in the expression $\frac{2 \tan \theta}{1 - \tan^2 \theta}$

We get,

$$\text{RHS} = \frac{2 \tan (30^\circ)}{1 - \tan^2 (30^\circ)} \dots\dots (4)$$

$$\begin{aligned}
 \text{RHS} &= \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} \\
 &= \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1^2}{(\sqrt{3})^2}} \\
 &= \frac{\frac{2}{\sqrt{3}}}{\frac{3-1}{3}} \\
 &= \sqrt{3}
 \end{aligned}$$

Now by comparing equation (3) and (4)

We get,

$$\text{LHS} = \text{RHS} = \sqrt{3}$$

$$\text{Hence } \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

(ii) Given:

$$\theta = 30^\circ \dots\dots (1)$$

To verify:

$$\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta} \dots\dots (2)$$

$$\sin 2\theta = \sin 2 \times 30$$

$$= \sin 60$$

$$= \frac{\sqrt{3}}{2}$$

Now consider right hand side

$$\frac{2 \tan \theta}{1 + \tan^2 \theta} = \frac{2 \tan 30}{1 + \tan^2 30}$$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{\sqrt{3}}{2}$$

Hence it is verified that,

$$\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$$

(iii) Given:

$$\theta = 30^\circ \dots\dots (1)$$

To verify:

$$\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \dots\dots (2)$$

Now consider left hand side of the equation (2)

Therefore,

$$\begin{aligned}\cos 2\theta &= \cos 2 \times 30 \\ &= \cos 60 \\ &= \frac{1}{2}\end{aligned}$$

Now consider right hand side of equation (2)

Therefore,

$$\begin{aligned}\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} &= \frac{1 - (\tan 30)^2}{1 + (\tan 30)^2} \\ &= \frac{1 - \left(\frac{1}{\sqrt{3}}\right)^2}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} \\ &= \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}} \\ &= \frac{1}{2}\end{aligned}$$

Hence it is verified that,

$$\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

(iv) Given:

$$\theta = 30^\circ \dots\dots (1)$$

To verify:

$$\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta \dots\dots (2)$$

Now consider left hand side of the expression in equation (2)

Therefore

$$\begin{aligned}\cos 3\theta &= \cos 3 \times 30 \\ &= \cos 90 \\ &= 0\end{aligned}$$

Now consider right hand side of the expression to be verified in equation (2)

Therefore,

$$\begin{aligned}4 \cos^3 \theta - 3 \cos \theta &= 4 \cos^3 30 - 3 \cos 30 \\ &= 4 \times \left(\frac{\sqrt{3}}{2} \right)^3 - 3 \times \frac{\sqrt{3}}{2} \\ &= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} \\ &= 0\end{aligned}$$

Hence it is verified that,

$$\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$$

***** END *****