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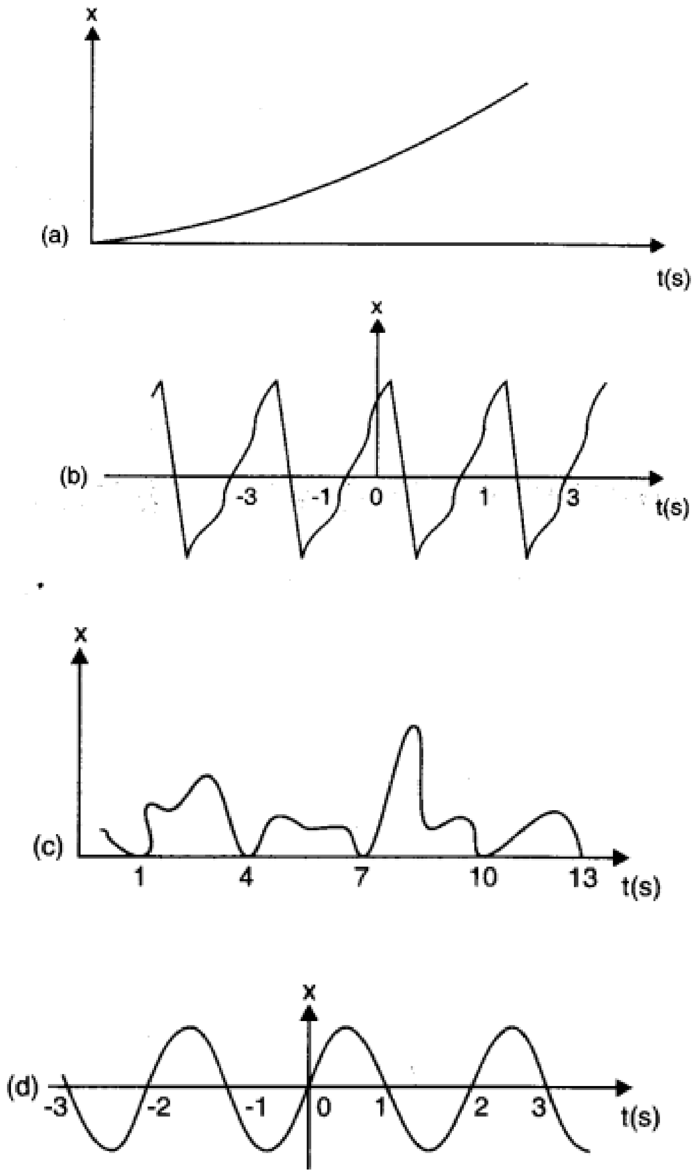
Question 14. 1. Which of the following examples represent periodic motion ?
(a) A swimmer completing one (return) trip from one bank of a river to the other and back.
(b) A freely suspended bar magnet displaced from its'N-S direction and released.
(c) A hydrogen molecule rotating about its centre of mass.
(d) An arrow released from a bow.
Answer:
(a) It is not a periodic motion. Though the motion of a swimmer is to and fro but will not have a definite period.
(b) Since a freely suspended magnet if once displaced from N-S direction and released, it oscillates about this position, it is a periodic motion.
(c) The rotating motion of a hydrogen molecule about its centre of mass is periodic.
(d) Motion of an arrow released from a bow is non-periodic.

Question 14. 2. Which of the following examples represent (nearly) simple harmonic motion and which represent periodic but not simple harmonic motion ?
(a) the rotations of earth about its axis.
(b) motion of an oscillating mercury column in a U-tube.
(c) motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower most point.
(d) general vibrations of a polyatomic molecule about its equilibrium position.
Answer:
(a) Since the rotation of earth is not to and fro motion about a fixed point, thus it is periodic but not S.H.M.
(b) It is S.H.M.
(c) It is S.H.M.
(d) General vibrations of a polyatomic molecule about its equilibrium position is periodic but non SHM. In fact, it is a result of superposition of SHMs executed by individual vibrations of atoms of the molecule.

Question 14. 3. Fig. depicts four x-t plots for linear motion of a particle. Which of ike plots represent periodic motion? What is the period of motion (in case of periodic motion)?




Answer: Figure (b) and (d) represent periodic motions and the time period of each of these is 2 seconds, (a) and (c) are non-periodic motions.

Question 14. 4. Which of the following function of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of periodic motion (to is any positive constant).

(a) sin wt - cos wt

(b) $\sin^2$ wt

(c) 3 cos -2 cos ($\pi$/4-2 wt)

(d) cos wt + cos 3 wt + cos 5 wt

(e) exp (- $w^2t^2$) (f) 1 + wt + $w^2t^2$.

Answer:  The function will represent a periodic motion, if it is identically repeated after a fixed interval of time and will represent S.H.M if it can be written uniquely in the form of a cos

$\left(\frac{2\pi t}{T}+\phi\right)$ or a sin $\left(\frac{2\pi t}{T}+\phi\right)$, where $T$ is the time period.

(a) $\sin \omega t - \cos \omega t = \sqrt{2}\left[\frac{1}{\sqrt{2}}\sin\omega t - \frac{1}{\sqrt{2}}\cos\omega t\right]$

$$= \sqrt{2}\left[\sin\omega t \cos\frac{\pi}{4} - \cos\omega t \sin\frac{\pi}{4}\right]$$

$$= \sqrt{2}\,\sin\left(\omega t - \frac{\pi}{4}\right)$$

It is a S.H.M. and its period is $2\pi/\omega$

(b) $\sin^3 \omega t = \frac{1}{3}[3\sin\omega t - \sin 3\omega t]$

Here each term $\sin \omega t$ and $\sin 3\omega t$ individually represents S.H.M. But (ii) which is the outcome of the superposition of two SHMs will only be periodic but not **SHMs.** Its time period is $2\pi/\omega$.

(c) $3\cos\left(\frac{\pi}{4}-2\omega t\right) = 3\cos\left(2\omega t - \frac{\pi}{4}\right)$. $[\because \cos(-\theta) = \cos\theta]$

Clearly it represents SHM and its time period is $2\pi/2\omega$.

(d) $\cos \omega t + \cos 3\omega t + \cos 5\omega t$. It represents the periodic but not S.H.M. Its time period is $2\pi/\omega$

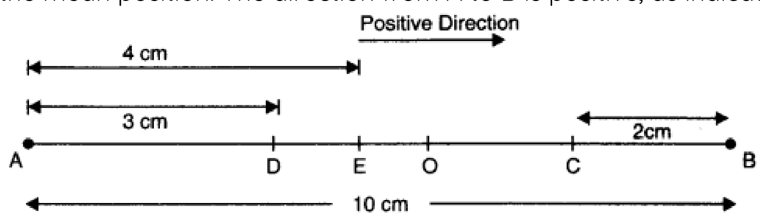
(e) $e^{-w2t2}$ . It is an exponential function which never repeats itself. Therefore it represents non-periodic motion.

(f) $1 + wt + w^2t^2$ also represents non periodic motion.

Question 14. 5. A particle is in linear simple harmonic motion between two points, A and B, 10 cm apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is

(a) at the end A,
(b) at the end B,
(c) at the mid-point of AB going towards A,
(d) at 2 cm away from B going towards A,
(e) at 3 cm away from A going towards B, and
(f) at 4 cm away from B going towards A.

Answer: In the fig. (given below), the points A and B, 10 cm apart, are the extreme positions of the particle in SHM, and the point O is the mean position. The direction from A to B is positive, as indicated.




(a) At the end A, i.e., extreme position, velocity is zero, acceleration and force are directed towards O and are positive.

(b) At the end B, i.e., second extreme position, velocity is zero whereas the acceleration and force are directed towards the point O and are negative.

(c) At the mid point O, while going towards A, velocity is negative and maximum. The acceleration and force both are zero.

(d) At 2 cm away from B, that is, at C and going towards A: v is negative; acceleration and F, being directed towards O, are also negative.

(e) At 3 cm away from A, that is, at D and going towards B: v is positive; acceleration and F, being directed towards O, are also positive.

(f) At a distance of 4 cm away from A and going towards A, velocity is directed along BA, therefore, it is positive. Since acceleration and force are directed towards OB, both of them are positive.

Question 14. 6. Which of the following relationships between the acceleration a and the displacement x of a particle involve simple harmonic motion?

(a) a = 0.7x
(b) a = - 200 $x^2$
(c) a = - 10x

(d) a = 100 $x^3$
Answer: Only (c) i.e., a = - 10x represents SHM. This is because acceleration is proportional and opposite to displacement (x).

Question 14. 7. The motion of a particle executing simple harmonic motion is described by the displacement function. x(t) = A cos (wt +Φ ). If the initial (t = 0) position of the particle is 1 cm and its initial velocity is w cm/s, what are its amplitude and initial phase angle? The angular frequency of the particle is $\pi s^{-1}$. If instead of the cosine function, we choose the sine function to describe the SHM: x = B sin (wt + α), what are the amplitude and initial phase of the particle with the above initial conditions?
Answer:

The given displacement function is

$$x(t) = A\cos(\omega t + \phi) \quad \text{...}(i)$$

At $t = 0$, $x(0) = 1$ cm. Also, $\omega = \pi\ s^{-1}$

$\therefore$ $1 = A\cos(\pi \times 0 + \phi)$

$\Rightarrow$ $A\cos\phi = 1 \quad \text{...}(ii)$

Also, differentiating eqn. (i) w.r.t. '$t$'.

$$v = \frac{d}{dt}x(t) = -A\omega\sin(\omega t + \phi) \quad \text{...}(iii)$$

Now at $t = 0$, $v = \omega$

$\therefore$ from eqn. (iii), $\omega = -A\omega\sin(\pi \times 0 + \phi)$

or $A\sin\phi = -1 \quad \text{...}(iv)$

Squaring and adding eqns. (ii) and (iv).

$$A^2\cos^2\phi + A^2\sin^2\phi = 1^2 + 1^2 \quad \text{or} \quad A = \sqrt{2}\ \text{cm}$$

Dividing eqns. (ii) and (iv),

$$\frac{A\sin\phi}{A\cos\phi} = \frac{-1}{1} \quad \therefore \quad \tan\phi = -1 \quad \Rightarrow \quad \phi = \frac{3\pi}{4}$$

If instead we use the sine function, i.e.,

$$x = B\sin(\omega t + \alpha), \quad \text{then } v = \frac{d}{dt}B\omega\cos(\omega t + \alpha)$$

$\therefore$ At $t = 0$, using $x = 1$ and $v = \omega$, we get $1 = B\sin(\omega \times 0 + \alpha)$

or $B\sin\alpha = 1 \quad \text{...}(v)$

and $\omega = B\omega\cos(\omega \times 0 + \alpha)$ or $B\cos\alpha = 1 \quad \text{...}(vi)$

Dividing (v) by (vi),

$$\tan\alpha = 1 \quad \text{or} \quad \alpha = \frac{\pi}{4} \quad \text{or} \quad \frac{5\pi}{4}$$

Squaring (v) and (vi), we get

$$B^2\sin^2\alpha + B^2\cos^2\alpha = 1^2 + 1^2$$

$\Rightarrow$ $B = \sqrt{2}$ cm.

Question 14. 8. A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s. What is the weight of the body?
Answer:

$$M = 50\text{kg}, \quad y = 20\text{cm} = 0.2\ \text{m}, \ T = 0.60\ \text{s}$$

$$F = ky \quad \text{or} \quad Mg = ky \quad \text{or} \quad k = \frac{Mg}{0.2} = \frac{50 \times 9.8}{0.2}\text{Nm}^{-1}$$

or $K = 2450\ \text{Nm}^{-1}$

Now, $T = 2\pi\sqrt{\frac{m}{k}}$

or $T^2 = 4\pi^2\frac{m}{k}$ or $m = \frac{T^2k}{4\pi^2}$

or $m = \frac{0.6 \times 0.6 \times 2460 \times 49}{4 \times 484}\text{kg} = 22.3\ \text{kg}$

$\Rightarrow$ $mg = 22.3 \times 9.8\ \text{N} = 218.5\ \text{N} = 22.3\ \text{kgf}.$

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