Exercise 10D

Question 1:
Let two numbers be x and 8 - x

$$\text{Sum of their reciprocals} = \frac{1}{x} + \frac{1}{8-x} = \frac{8}{15}$$

$$\Rightarrow \frac{8-x+x}{x(8-x)} = \frac{8}{15} \text{ or } \frac{8}{x(8-x)} = \frac{8}{15}$$

$$\Rightarrow x(8-x) = 15 \text{ or } 8x - x^2 = 15$$

$$x^2 - 8x + 15 = 0 \text{ or } x^2 - 5x - 3x + 15 = 0$$

$$\Rightarrow x(x-5) - 3(x-5) = 0 \text{ or } (x-5)(x-3) = 0$$

$$\therefore x = 5, 3$$

Thus, two numbers are 3 and 5.

Question 2:
Let the two numbers are x and x + 4

$$\text{Difference of their reciprocals} = \frac{1}{x} - \frac{1}{x+4} = \frac{4}{21}$$

$$\therefore \frac{x+4-x}{x(x+4)} = \frac{4}{21} \text{ or } \frac{4}{x(x+4)} = \frac{4}{21}$$

$$\Rightarrow x(x+4) = 21 \text{ or } x^2 + 4x - 21 = 0$$

$$\Rightarrow x^2 + 7x - 3x - 21 = 0 \text{ or } x(x+7) - 3(x+7) = 0$$

$$\Rightarrow (x+7)(x-3) = 0$$

$$\therefore x = 3 \text{ or } -7$$

But, $x \neq -7$

Hence, the two required numbers are 3 and 7.

Question 3:
Let the required number be x and (18- x)

$$\text{Then, } \frac{1}{x}+\frac{1}{18-x}=\frac{1}{4}$$

$$\Rightarrow \frac{18-x+x}{18x-x^2}=\frac{1}{4}$$

$$\Rightarrow 18-x^2=72$$

$$\Rightarrow x^2-18x+72=0$$

$$\Rightarrow x^2-12x-6x+72=0$$

$$\Rightarrow x(x-12)-6(x-12)=0$$

$$\Rightarrow (x-12)(x-6)=0$$

x = 12, x = 6
hence, the required numbers are 12 and 6.

Question 4:
Let the number be x and y
x - y = 5 -----(1)
Difference of reciprocal

$$\frac{1}{y}-\frac{1}{x}=\frac{1}{10}$$

$$\text{or } \frac{x-y}{xy}=\frac{1}{10} \text{ ------(2)}$$

$$\text{Put } x-y=5 \text{ in } (2)$$

$$\frac{5}{xy}=\frac{1}{10} \quad \therefore xy=50 \ (3)$$

$$\text{from}(1), x=y+5, \text{ put value of } x \text{ in } (3)$$

$$(y+5)y=50 \quad \text{or} \quad y^2+5y-50=0$$

$$\Rightarrow y^2+10y-5y-50=0 \quad \Rightarrow y(y+10)-5(y+10)=0$$

$$\text{or } (y+10)(y-5)=0$$

$$\therefore y+10=0 \text{ or } y=-10$$

$$y-5=0 \text{ or } y=5$$

$$\text{When } y=-10, \text{from } (1)$$

$$x=y+5=-10+5=-5$$

$$\therefore x=-5, y=-10$$

$$\text{when } y=5, \quad x+5=10$$

$$\therefore x=10, \ y=5$$

Hence, the two numbers are 10 and 5.

Question 5:
Let the number be x

Then, $x + \frac{1}{x} = 3\frac{41}{80} \Rightarrow \frac{x^2+1}{x} = \frac{281}{80}$

$\Rightarrow 80x^2 + 80 = 281x$

$\Rightarrow 80x^2 - 281x + 80 = 0$

$\Rightarrow x = \frac{281 \pm \sqrt{78961 - 25600}}{160} = \frac{281 \pm \sqrt{53361}}{160}$

$\Rightarrow \frac{(281 \pm 231)}{160}$

$\Rightarrow x = \frac{(281+231)}{160} = \frac{512}{160} = \frac{16}{5}$ or $x = \frac{(281-231)}{160} = \frac{50}{160} = \frac{5}{16}$

Hence, the required number is 5/16 or 16/5.

Question 6:

Let the required number be x and (57 - x), then

$x(57 - x) = 782 \Rightarrow x^2 - 57x + 782 = 0$

$\Rightarrow \frac{57 \pm \sqrt{(57)^2 - 4 \times 1 \times 782}}{2} = \frac{57 \pm \sqrt{(3249 - 3128)}}{2}$

$\Rightarrow \quad x = \frac{57 \pm \sqrt{121}}{2}$

$x = \frac{(57+11)}{2} = \frac{68}{2} = 34$ or $x = \frac{(57-11)}{2} = \frac{46}{2} = 23$

The required numbers are 34 and 23.

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