



### Binary Operations Ex 3.5 Q6

$a \times_7 b$  = the remainder when the product of  $ab$  is divided by 7.

The composition table for  $\times_7$  on  $S = \{1, 2, 3, 4, 5, 6\}$

$\times_7$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

We know that 1 is the identity element with respect to multiplication

Also,  $b$  will be the inverse of  $a$   
if,  $a \times_7 b = e = 1$

$$\Rightarrow 3 \times_7 b = 1$$

From the above table  $3 \times_7 5 = 1$

$$\therefore b = 3^{-1} = 5$$

$$\text{Now, } 3^{-1} \times_7 4 = 5 \times_7 4 = 6$$

### Binary Operations Ex 3.5 Q7

$a \times_{11} b$  = the remainder when the product of  $ab$  is divided by 11.

The composition table for  $\times_{11}$  on  $Z_{11}$

$\times_{11}$	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	1	3	5	7	9
3	3	6	9	1	4	7	10	3	5	8
4	4	8	1	5	9	2	6	10	3	7
5	5	10	4	9	3	8	2	7	1	6
6	6	1	7	2	8	3	9	4	10	5
7	7	3	10	6	2	9	5	1	8	4
8	8	5	3	10	7	4	1	9	6	3
9	9	7	5	3	1	10	8	6	4	2
10	10	9	8	7	6	5	4	3	2	1

From the above table

$$5 \times_{11} 9 = 1$$

[ $\therefore 1$  is the identity element]

$\therefore$  Inverse of 5 is 9.

### Binary Operations Ex 3.5 Q8

$$Z_5 = \{0, 1, 2, 3, 4\}$$

$a \times_5 b$  = the remainder when the product of  $ab$  is divided by 5.

The composition table for  $\times_5$  on  $Z_5 = \{0, 1, 2, 3, 4\}$

$\times_5$	0	1	2	3	4
<b>0</b>	0	0	0	0	0
<b>1</b>	0	1	2	3	4
<b>2</b>	0	2	4	1	3
<b>3</b>	0	3	1	4	2
<b>4</b>	0	4	3	2	1

Binary Operations Ex 3.5 Q9

(i)

From the above table we can say that

$$a * b = b * a = b$$

$$a * c = c * a = c$$

$$a * d = d * a = d$$

$$b * c = c * b = d$$

$$b * d = d * b = c$$

$$c * d = d * c = b$$

$\therefore$   $*$  is commutative

Again,  $a, b, c \in S$

$$\Rightarrow (a * b) * c = b * c = d \text{ and } a * (b * c) = a * d = d$$

$$\therefore (a * b) * c = a * (b * c)$$

$\therefore$   $*$  is associative

We know that  $e$  will be identity element with respect to  $*$  if

$$a * e = e * a = a \text{ for all } a \in S$$

$$\Rightarrow a * a = a, a * b = b, a * c = c, a * d = d$$

$\therefore$   $a$  will be the identity element

Again,

$b$  will be the inverse of  $a$  if

$$b * a = a * b = e$$

From the above table

$$a * a = a, \quad b * b = b, c * c = c \text{ and } d * d = d$$

$\therefore$  Inverse of  $a = a$

$$b = b$$

$$c = c$$

$$d = d$$

(ii)

From the above table, we can observe

$$\begin{aligned} aob &= boa, & boc &= cob \\ aoc &= caa, & bod &= dob \\ aod &= daa, & cod &= doc \end{aligned}$$

$\therefore$  'o' is commutative on S

Again, for  $a, b, c \in S$

$$(aob)oc = aoc = a \quad \text{--- (i)}$$

$$ao(boc) = aoc = a \quad \text{--- (ii)}$$

From (i) & (ii)

$$(aob)oc = ao(boc)$$

So, 'o' is associative on S

Now, we have,

$$aob = a$$

$$bob = b$$

$$cob = c$$

$$dob = d$$

$\Rightarrow$  b is the identity element with respect to 'o'

We know that x will be inverse of y

If  $xy = yx = e$

$$\Rightarrow \quad xoy = yox = b \quad \quad \quad [\because e = b]$$

Now, from the above table we find that

$$bob = b$$

$$cod = b$$

$$doc = b$$

$\therefore$   $b^{-1} = b$ ,  $c^{-1} = d$ , and  $d^{-1} = c$

**Note:**  $a^{-1}$  does not exist.

Let  $X = \{0, 1, 2, 3, 4, 5\}$ .

The operation  $*$  on  $X$  is defined as:

$$a * b = \begin{cases} a + b & \text{if } a + b < 6 \\ a + b - 6 & \text{if } a + b \geq 6 \end{cases}$$

An element  $e \in X$  is the identity element for the operation  $*$ , if

$$a * e = a = e * a \quad \forall a \in X.$$

For  $a \in X$ , we observed that:

$$a * 0 = a + 0 = a \quad [a \in X \Rightarrow a + 0 < 6]$$

$$0 * a = 0 + a = a \quad [a \in X \Rightarrow 0 + a < 6]$$

$$\therefore a * 0 = a = 0 * a \quad \forall a \in X$$

Thus, 0 is the identity element for the given operation  $*$ .

An element  $a \in X$  is invertible if there exists  $b \in X$  such that  $a * b = 0 = b * a$ .

$$\text{i.e., } \begin{cases} a + b = 0 = b + a, & \text{if } a + b < 6 \\ a + b - 6 = 0 = b + a - 6, & \text{if } a + b \geq 6 \end{cases}$$

i.e.,

$$a = -b \text{ or } b = 6 - a$$

But,  $X = \{0, 1, 2, 3, 4, 5\}$  and  $a, b \in X$ . Then,  $a \neq -b$ .

Therefore,  $b = 6 - a$  is the inverse of  $a \in X$ .

Hence, the inverse of an element  $a \in X$ ,  $a \neq 0$  is  $6 - a$  i.e.,  $a^{-1} = 6 - a$ .

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