Differentiation Ex 11.1 Q7

Let $\quad f(x) = e^{\sqrt{\cot x}}$

$\Rightarrow \quad f(x+h) = e^{\sqrt{\cot(x+h)}}$

$$\frac{d}{dx}\left(f(x)\right) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{e^{\sqrt{\cot(x+h)}} - e^{\sqrt{\cot x}}}{h}$$

$$= \lim_{h\to 0} \frac{e^{\sqrt{\cot x}}\left(e^{\sqrt{\cot(x+h)}-\sqrt{\cot x}} - 1\right)}{h}$$

$$= e^{\sqrt{\cot x}} \lim_{h\to 0} \left(\frac{e^{\sqrt{\cot(x+h)}-\sqrt{\cot x}} - 1}{\sqrt{\cot(x+h)} - \sqrt{\cot x}}\right) \times \left(\frac{\sqrt{\cot(x+h)} - \sqrt{\cot x}}{h}\right)$$

$$= e^{\sqrt{\cot x}} \lim_{h\to 0} \frac{\left(\sqrt{\cot(x+h)} - \sqrt{\cot x}\right)}{h} \times \frac{\sqrt{\cot(x+h)} + \sqrt{\cot x}}{\sqrt{\cot(x+h)} + \sqrt{\cot x}}$$

$\left[\text{Since, } \lim_{x\to 0} \frac{e^x - 1}{x} = 1 \text{ and rationalizing numerator}\right]$

$$= e^{\sqrt{\cot x}} \lim_{h\to 0} \frac{\cot(x+h) - \cot x}{h\left(\sqrt{\cot(x+h)} + \sqrt{\cot x}\right)}$$

$$= e^{\sqrt{\cot x}} \lim_{h\to 0} \frac{\frac{\cot(x+h)\cot x + 1}{\cot(x+h-x)}}{h\left(\sqrt{\cot(x+h)} + \sqrt{\cot x}\right)} \qquad \left[\text{Since, } \cot(A-B) = \frac{\cot A \cot B + 1}{\cot A - \cot B}\right]$$

$$= e^{\sqrt{\cot x}} \lim_{h\to 0} \frac{\cot(x+h)\cot x + 1}{\text{cothx}\; h\left(\sqrt{\cot(x+h)} + \sqrt{\cot x}\right)}$$

$$= e^{\sqrt{\cot x}} \lim_{h\to 0} \frac{\left(\cot(x+h)\cot x + 1\right)}{\left(\frac{h}{\tanh}\right)\left(\sqrt{\cot(x+h)} + \sqrt{\cot x}\right)}$$

$$= \frac{e^{\sqrt{\cot x}} \times \left(\cot^2 x + 1\right)}{2\sqrt{\cot x}} \qquad \left[\text{Since, } \lim_{x\to 0} \frac{\tan x}{x} = 1\right]$$

$$= \frac{e^{\sqrt{\cot x}} \times \operatorname{cosec}^2 x}{2\sqrt{\cot x}} \qquad \left[\text{Since, } \left(1 + \cot^2 x\right) = \operatorname{cosec}^2 x\right]$$

So,

$$\frac{d}{dx}\left(e^{\sqrt{\cot x}}\right) = \frac{e^{\sqrt{\cot x}} \times \operatorname{cosec}^2 x}{2\sqrt{\cot x}}$$

Differentiation Ex 11.1 Q8

Let $\quad f(x) = x^2 e^x$

$\Rightarrow \quad f(x+h) = (x+h)^2 e^{(x+h)}$

$$\therefore \quad \frac{d}{dx}\left(f(x)\right) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^2 e^{(x+h)} - x^2 e^x}{h}$$

$$= \lim_{h\to 0} \left(\frac{x^2 e^{(x+h)} - x^2 e^x}{h} + \frac{2xhe^{(x+h)}}{h} + \frac{h^2 e^{(x+h)}}{h}\right)$$

$$= \lim_{h\to 0} \left(\frac{x^2 e^x \left(e^{(x+h)-x} - 1\right)}{h} + 2xe^{(x+h)} + he^{(x+h)}\right)$$

$$= \lim_{h\to 0} \left[x^2 e^x \frac{\left(e^h - 1\right)}{h} + 2xe^{(x+h)} + he(x+h)\right]$$

$$= x^2 e^x + 2xe^x + 0 \times e^x \qquad \left[\text{Since, } \lim_{x\to 0} \frac{e^x - 1}{x} = 1\right]$$

So,

$$\frac{d}{dx}\left(x^2 e^x\right) = e^x\left(x^2 + 2x\right)$$

Differentiation Ex 11.1 Q9

Let $\quad f(x) = \log \operatorname{cosec} x$

$\Rightarrow \quad f(x+h) = \log \operatorname{cosec}(x+h)$

$$\frac{d}{dx}\left(f(x)\right) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\log \operatorname{cosec}(x+h) - \log \operatorname{cosec} x}{h}$$

$$= \lim_{h\to 0} \frac{\log\left(\frac{\operatorname{cosec}(x+h)}{\operatorname{cosec} x}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\log\left(1 + \left(\frac{\sin x}{\sin(x+h)} - 1\right)\right)}{h}$$

$$= \lim_{h\to 0} \left\{\frac{\log\left(1 + \left(\frac{\sin x - \sin(x+h)}{\sin(x+h)}\right)\right)}{\left(\frac{\sin x - \sin(x+h)}{\sin(x+h)}\right)}\right\} \frac{\left(\frac{\sin x - \sin(x+h)}{\sin(x+h)}\right)}{h}$$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{x+x+h}{2}\right)\sin\left(\frac{x-x-h}{2}\right)}{\sin(x+h)h}$$

$$\left[\text{Since, } \lim_{x\to 0} \frac{\log(1+x)}{x} = 1 \text{ and } \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{2x+h}{2}\right)}{\sin(x+h)(-2)} \left\{\frac{\sin\left(-\frac{h}{2}\right)}{-\frac{h}{2}}\right\} \qquad \left[\text{Since, } \lim_{x\to 0} \frac{\sin x}{x} = 1\right]$$

$$= -\cot x$$

So,

$$\frac{d}{dx}(\log \operatorname{cosec} x) = -\cot x.$$

Differentiation Ex 11.1 Q10

Let $\quad f(x) = \sin^{-1}(2x+3)$

$\Rightarrow \quad f(x+h) = \sin^{-1}\left(2(x+h)+3\right)$

$\Rightarrow \quad f(x+h) = \sin^{-1}(2x+2h+3)$

$$\therefore \quad \frac{d}{dx}\left(f(x)\right) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin^{-1}(2x+2h+3) - \sin^{-1}(2x+3)}{h}$$

$$= \lim_{h\to 0} \frac{\sin^{-1}\left[(2x+2h+3)\sqrt{1-(2x+3)^2} - (2x+3)\sqrt{1-(2x+2h+3)^2}\right]}{h}$$

$$\left[\text{Since, } \sin^{-1}x - \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} - y\sqrt{1-x^2}\right]\right]$$

$$= \lim_{h\to 0} \frac{\sin^{-1}z}{z} \times \frac{z}{h}$$

Where, $z = (2x+2h+3)\sqrt{1-(2x+3)^2} - (2x+3)\sqrt{1-(2x+2h+3)^2}$ and $\lim_{h\to 0} \frac{\sin^{-1}h}{h} = 1$

$$= \lim_{h\to 0} \frac{z}{h}$$

$$= \lim_{h\to 0} \frac{(2x+2h+3)\sqrt{1-(2x+3)^2} - (2x+3)\sqrt{1-(2x+2h+3)^2}}{h}$$

$$= \lim_{h\to 0} \frac{(2x+2h+3)^2 - (2x+3)^2 - (2x+3)^2\left(1-(2x+2h+3)^2\right)}{h\left\{(2x+2h+3)\sqrt{1-(2x+3)^2} + (2x+3)\sqrt{1-(2x+2h+3)^2}\right\}}$$

[Since, rationalizing numerator]

$$= \lim_{h\to 0} \frac{\left[(2x+3)^2 + 4h^2 + 4h(2x+3)\right]\left(1-(2x+3)^2\right) - (2x+3)^2\left[1-(2x+3)^2 - 4h^2 - 4h(2x+3)\right]}{h\left\{(2x+2h+3)\sqrt{1-(2x+3)^2} + (2x+3)\sqrt{1-(2x+2h+3)^2}\right\}}$$

$$= \lim_{h\to 0} \frac{\left[(2x+3)^2 + 4h^2 + 4h(2x+3) - (2x+3)^4 - 4h^2(2x+3)^2 - 4h(2x+3)^3 - (2x+3)^2 + (2x+3)^4 + 4h^2(2x+3)^2 + 4h(2x+3)^3\right]}{h\left\{(2x+2h+3)\sqrt{1-(2x+3)^2} + (2x+3)\sqrt{1-(2x+2h+3)^2}\right\}}$$

$$= \lim_{h \to 0} \frac{4h\left[h + (2x+3)\right]}{h\left\{(2x+2h+3)\sqrt{1-(2x+3)^2} + (2x+3)\sqrt{1-(2x+2h+3)^2}\right\}}$$

$$= \frac{4(2x+3)}{(2x+3)\sqrt{1-(2x+3)^2} + (2x+3)\sqrt{1-(2x+3)^2}}$$

$$= \frac{4(2x+3)}{2(2x+3)\sqrt{1-(2x+3)^2}}$$

$$= \frac{2}{\sqrt{1-(2x+3)^2}}$$

So,

$$\frac{d}{dx}\left(\sin^{-1}(2x+3)\right) = \frac{2}{\sqrt{1-(2x+3)^2}}$$

*********** END **********