Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 7

$$\text{LHS} = \cot A + \cot\left(60^\circ + A\right) - \cot\left(60^\circ - A\right)$$

$$= \frac{1}{\tan A} + \frac{1}{\tan\left(60^\circ + A\right)} - \frac{1}{\tan\left(60^\circ - A\right)}$$

$$= \frac{1}{\tan A} + \frac{1 - \sqrt{3}\tan A}{\sqrt{3} + \tan A} - \frac{1 + \sqrt{3}\tan A}{\sqrt{3} - \tan A}$$

$$= \frac{1}{\tan A} - \frac{8\tan A}{3 - \tan^2 A}$$

$$= \frac{3 - \tan^2 A - 8\tan^2 A}{3\tan A - \tan^3 A}$$

$$= \frac{3 - 9\tan^2 A}{3\tan A - \tan^3 A}$$

$$= 3\left(\frac{1 - 3\tan^2 A}{3\tan A - \tan^3 A}\right)$$

$$= \frac{3}{\tan 3A}$$

$$= 3\cot 3A$$

$$= \text{RHS}$$

LHS = RHS

Hence proved.

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 8

$$\text{LHS} = \cot A + \cot\left(60^\circ + A\right) + \cot\left(120^\circ + A\right)$$

$$= \cot A + \cot\left(60^\circ + A\right) - \cot\left[180^\circ - \left(120^\circ + A\right)\right]$$

$$\left\{\text{since } -\cot\theta = \cot\left(180^\circ - \theta\right)\right\}$$

$$= \cot A + \cot\left(60^\circ + A\right) - \cot\left(60^\circ - A\right)$$

$$= \frac{1}{\tan A} + \frac{1}{\tan\left(60^\circ + A\right)} - \frac{1}{\tan\left(60^\circ - A\right)}$$

$$= \frac{1}{\tan A} + \frac{1 - \sqrt{3}\tan A}{\sqrt{3} + \tan A} - \frac{1 + \sqrt{3}\tan A}{\sqrt{3} - \tan A}$$

$$= \frac{1}{\tan A} - \frac{8\tan A}{3 - \tan^2 A}$$

$$= \frac{3 - \tan^2 A - 8\tan^2 A}{3\tan A - \tan^3 A}$$

$$= \frac{3 - 9\tan^2 A}{3\tan A - \tan^3 A}$$

$$= \frac{3\left(1 - 3\tan^2 A\right)}{3\tan A - \tan^3 A}$$

$$= \frac{3}{\tan 3A}$$

$$= 3\cot 3A$$

LHS = RHS

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 9

$$\text{LHS} = \sin^3 A + \sin^3\left(\frac{2\pi}{3} + A\right) + \sin^3\left(\frac{4\pi}{3} + A\right)$$

$$\left\{\text{we know that } \sin^3 A = \frac{3\sin A - \sin 3A}{4}\right\}$$

$$= \left(\frac{3\sin A - \sin 3A}{4}\right) + \left\{\frac{3\sin\left(\frac{2\pi}{3} + A\right) - \sin 3\left(\frac{2\pi}{3} + A\right)}{4}\right\} + \left\{\frac{3\sin\left(\frac{4\pi}{3} + A\right) - \sin 3\left(\frac{4\pi}{3} + A\right)}{4}\right\}$$

$$= \left[\frac{3\sin A - \sin 3A}{4}\right] + \left[\frac{3\sin\left[\pi\left(\frac{2\pi}{3} + A\right)\right] - \sin\left(2\pi + 3A\right)}{4}\right] + \left[\frac{3\sin\left(\pi + \left(\frac{\pi}{3} + A\right)\right) - \sin\left(4\pi + 3A\right)}{4}\right]$$

$$= \frac{1}{4}\left\{\left[3\sin A - \sin 3A\right] + \left[3\sin\left(\frac{\pi}{3} - A\right) - \sin 3A\right] - \left[3\sin\left(\frac{\pi}{3} + A\right) + \sin 3A\right]\right\}$$

$$= \frac{1}{4}\left[3\sin A - \sin 3A + 3\sin\left(\frac{\pi}{3} - A\right) - 3\sin\left(\frac{\pi}{3} + A\right) - \sin 3A - \sin 3A\right]$$

$$= \frac{1}{4}\left[3\sin A - 3\sin 3A + 3\left(\sin\left(\frac{\pi}{3} - A\right) - \sin\left(\frac{\pi}{3} + A\right)\right)\right]$$

$$= \frac{1}{4}\left[3\sin A - 3\sin 3A + 3\left\{2\cos\frac{\frac{\pi}{3} - A + \frac{\pi}{3} + A}{2}\sin\frac{\frac{\pi}{3} - A - \frac{\pi}{3} - A}{2}\right\}\right]$$

$$= \frac{1}{4}\left[3\sin A - 3\sin 3A + 6\cos\frac{\pi}{3}\sin(-A)\right]$$

$$= \frac{1}{4}\left[3\sin A - 3\sin 3A - 3\sin A\right]$$

$$= -\frac{3}{4}\sin 3A$$

$= \text{RHS}$

$\text{LHS} = \text{RHS}$

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 10

$$\left|\sin\theta \sin\left(60 - \theta\right)\sin\left(60 + \theta\right)\right|$$

$$= \left|\sin\theta\left(\sin^2 60 - \sin^2\theta\right)\right|$$

$$\left\{\text{since } \sin\left(A + B\right)\sin\left(A - B\right) = \sin^2 A - \sin^2 B\right\}$$

$$= \left|\sin\theta\left(\frac{3}{4} - \sin^2\theta\right)\right|$$

$$= \left|\frac{1}{4}\sin\theta\left(3 - 4\sin^2\theta\right)\right|$$

$$= \left|\frac{1}{4}\sin 3\theta\right|$$

$$= \frac{1}{4}\left|\sin 3\theta\right|$$

$$\le \frac{1}{4} \qquad \left\{\sin\theta\left|\sin 3\theta\right|\right|\le\left|\right\}$$

So,

$$\left|\sin\theta\sin\left(60^\circ - \theta\right)\sin\left(60^\circ + \theta\right)\right| \le \frac{1}{4}$$

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 11

$$\left|\cos\theta\cos\left(60^\circ-\theta\right)\cos\left(60^\circ+\theta\right)\right|$$

$$=\left|\cos\theta\left(\cos^2 60^\circ-\sin^2\theta\right)\right|$$

$$\left\{\text{since } \cos\left(A-B\right)\cos\left(A+B\right)=\cos^2 A-\sin^2 B\right\}$$

$$=\left|\cos\theta\left(\frac{1}{4}-\sin^2\theta\right)\right|$$

$$=\left|\cos\theta\,\frac{1}{4}\left(1-4\sin^2\theta\right)\right|$$

$$=\left|\frac{1}{4}\cos\theta\left(1-4\left(1-\cos^2\theta\right)\right)\right|$$

$$=\left|\frac{1}{4}\cos\theta\left(-3+4\cos^2\theta\right)\right|$$

$$=\left|\frac{1}{4}\left(4\cos 3\theta-3\cos\theta\right)\right|$$

$$=\left|\frac{1}{4}\cos 3\theta\right|$$

$$\le\frac{1}{4} \qquad \left\{\text{since } \left|\cos 3\theta\right|\le 1\right\}$$

So,

$$\left|\cos\theta\cos\left(60^\circ-\theta\right)\cos\left(60^\circ+\theta\right)\right|\le\frac{1}{4}$$

*********** END **********