Tangents and Normals Ex 16.3 Q4

We have,

$$4x = y^2 \qquad \text{---(i)}$$

$$4xy = k \qquad \text{---(ii)}$$

Slope of (i)

$$4 = 2y\frac{dy}{dx}$$

$$\Rightarrow \quad m_1 = \frac{dy}{dx} = \frac{2}{y}$$

Slope of (ii)

$$y + x\frac{dy}{dx} = 0$$

$$\Rightarrow \quad m_2 = \frac{dy}{dx} = \frac{-y}{x}$$

Solving (i) and (ii)

$$\frac{k}{y} = y^2$$

$$\Rightarrow \quad y^3 = k$$

$$k = \frac{k^{\frac{2}{3}}}{4}$$

$\therefore$ (i) and (ii) cuts orthogonolly

$\therefore \quad m_1 \times m_2 = -1$

$$\Rightarrow \quad \frac{2}{y} \times \frac{-y}{x} = -1$$

$$\Rightarrow \quad \frac{2}{x} = 1$$

$$\Rightarrow \quad x = 2$$

$$\Rightarrow \quad \frac{k^{\frac{2}{3}}}{4} = 2$$

$$\Rightarrow \quad k^{\frac{2}{3}} = 8$$

$\therefore \quad k^2 = 512$

We have,

$$2x = y^2 \qquad \text{---(i)}$$

$$2xy = k \qquad \text{---(ii)}$$

Slope of (i)

$$2 = 2y\frac{dy}{dx}$$

$$\Rightarrow \quad m_1 = \frac{dy}{dx} = \frac{1}{y}$$

Slope of (ii)

$$y + x\left(\frac{dy}{dx}\right) = 0$$

$$\therefore \quad m_2 = \frac{dy}{dx} = \frac{-y}{x}$$

Now,

Solving (i) and (ii)

$$\frac{k}{y} = y^2$$

$$\Rightarrow \quad y^3 = k$$

$$\therefore \quad x = \frac{y^2}{2} = \frac{k^{\frac{2}{3}}}{2}$$

$\therefore$ (i) and (ii) cuts orthogonolly

$$\therefore \quad m_1 \times m_2 = -1$$

$$\frac{1}{y} \times \frac{-y}{x} = -1$$

$$\Rightarrow \quad \frac{1}{x} = 1$$

$$\Rightarrow \quad x = 1$$

$$\Rightarrow \quad \frac{k^{\frac{2}{3}}}{2} = 1$$

$$\Rightarrow \quad k^{\frac{2}{3}} = 2$$

Closing both side, we get

$$k^2 = 8$$

********** END **********