Indefinite Integrals Ex 19.16 Q6

$$\text{Let } I = \int \frac{dx}{e^x + e^{-x}}$$

$$= \int \frac{dx}{e^x + \frac{1}{e^x}}$$

$$= \int \frac{e^x dx}{\left(e^x\right)^2 + 1}$$

Let $e^x = t$

$$\Rightarrow \quad e^x dx = dt$$

$$I = \int \frac{dt}{t^2 + 1}$$

$$I = \tan^{-1} t + c \qquad \left[\text{Since } \int \frac{1}{1+x^2} dx = \tan^{-1} x + c\right]$$

$$I = \tan^{-1}\left(e^x\right) + c$$

Indefinite Integrals Ex 19.16 Q7

$$\text{Let } I = \int \frac{x}{x^4 + 2x^2 + 3} dx$$

Let $x^2 = t$

$$\Rightarrow \quad 2x\, dx = dt$$

$$\Rightarrow \quad x\, dx = \frac{dt}{2}$$

$$I = \frac{1}{2} \int \frac{dt}{t^2 + 2t + 3}$$

$$= \frac{1}{2} \int \frac{dt}{t^2 + 2t + 1 - 1 + 3}$$

$$= \frac{1}{2} \int \frac{dt}{(t+1)^2 + 2}$$

put $t + 1 = u$

$$\Rightarrow \quad dt = du$$

$$I = \frac{1}{2} \int \frac{du}{u^2 + \left(\sqrt{2}\right)^2}$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + c \qquad \left[\text{Since } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{t+1}{\sqrt{2}}\right) + c$$

$$I = \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right) + c$$

Indefinite Integrals Ex 19.16 Q8

Let $I = \int \frac{3x^5}{1+x^{12}}dx$

$= \int \frac{3x^5}{1+\left(x^6\right)^2}dx$

Let $x^6 = t$

$\Rightarrow \quad 6x^5\,dx = dt$

$\Rightarrow \quad x^5\,dx = \frac{dt}{6}$

$I = \frac{3}{6}\int \frac{dt}{1+t^2}$

$= \frac{1}{2}\tan^{-1}\left(t\right)+c \qquad \left[\text{Since } \int \frac{1}{x^2+1}dx = \tan^{-1}x + c\right]$

$I = \frac{1}{2}\tan^{-1}\left(x^6\right)+c$

Indefinite Integrals Ex 19.16 Q9

Let $I = \int \frac{x^2}{x^6-a^6}dx$

$= \int \frac{x^2}{\left(x^3\right)^2-\left(a^3\right)^2}dx$

Let $x^3 = t$

$\Rightarrow \quad 3x^2dx = dt$

$\Rightarrow \quad x^2\,dx = \frac{dt}{3}$

so, $\quad I = \frac{1}{3}\int \frac{dt}{t^2-\left(a^3\right)^2}$

$= \frac{1}{3}\times\frac{1}{2a^3}\log\left|\frac{t-a^3}{t+a^3}\right| \qquad \left[\text{Since } \int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c\right]$

$I = \frac{1}{6a^3}\log\left|\frac{x^3-a^3}{x^3+a^3}\right|+c$

Indefinite Integrals Ex 19.16 Q10

Let $I = \int \frac{x^2}{x^6+\left(a^3\right)^2}dx$

$= \int \frac{x^2}{\left(x^3\right)^2+\left(a^3\right)^2}dx$

Let $x^3 = t$

$\Rightarrow \quad 3x^2dx = dt$

$\Rightarrow \quad x^2\,dx = \frac{dt}{3}$

so, $\quad I = \frac{1}{3}\int \frac{dt}{t^2+\left(a^3\right)^2}$

$= \frac{1}{3}\times\frac{1}{\left(a^3\right)}\tan^{-1}\left(\frac{t}{a^3}\right)+c \qquad \left[\text{Since } \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$

$I = \frac{1}{3a^3}\tan^{-1}\left(\frac{x^3}{a^3}\right)+c$

*********** END **********