$$\text{Now, } A-A'=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}-\begin{bmatrix}0 & -a & -b\\ a & 0 & -c\\ b & c & 0\end{bmatrix}=\begin{bmatrix}0 & 2a & 2b\\ -2a & 0 & 2c\\ -2b & -2c & 0\end{bmatrix}$$

$$\therefore \frac{1}{2}(A-A')=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}$$

**Question 10:**

Express the following matrices as the sum of a symmetric and a skew symmetric matrix:

(i) $\begin{bmatrix}3 & 5\\ 1 & -1\end{bmatrix}$

(ii) $\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}$

(iii) $\begin{bmatrix}3 & 3 & -1\\ -2 & -2 & 1\\ -4 & -5 & 2\end{bmatrix}$

(iv) $\begin{bmatrix}1 & 5\\ -1 & 2\end{bmatrix}$

Answer

**(i)**

$$\text{Let } A=\begin{bmatrix}3 & 5\\ 1 & -1\end{bmatrix}, \text{then } A'=\begin{bmatrix}3 & 1\\ 5 & -1\end{bmatrix}$$

$$\text{Now, } A+A'=\begin{bmatrix}3 & 5\\ 1 & -1\end{bmatrix}+\begin{bmatrix}3 & 1\\ 5 & -1\end{bmatrix}=\begin{bmatrix}6 & 6\\ 6 & -2\end{bmatrix}$$

$$\text{Let } P=\frac{1}{2}(A+A')=\frac{1}{2}\begin{bmatrix}6 & 6\\ 6 & -2\end{bmatrix}=\begin{bmatrix}3 & 3\\ 3 & -1\end{bmatrix}$$

$$\text{Now, } P'=\begin{bmatrix}3 & 3\\ 3 & -1\end{bmatrix}=P$$

Thus, $P=\frac{1}{2}(A+A')$ is a symmetric matrix.

$$\text{Now, } A-A'=\begin{bmatrix}3 & 5\\ 1 & -1\end{bmatrix}-\begin{bmatrix}3 & 1\\ 5 & -1\end{bmatrix}=\begin{bmatrix}0 & 4\\ -4 & 0\end{bmatrix}$$

$$\text{Let } Q=\frac{1}{2}(A-A')=\frac{1}{2}\begin{bmatrix}0 & 4\\ -4 & 0\end{bmatrix}=\begin{bmatrix}0 & 2\\ -2 & 0\end{bmatrix}$$

$$\text{Now, } Q'=\begin{bmatrix}0 & 2\\ -2 & 0\end{bmatrix}=-Q$$

Thus, $Q=\frac{1}{2}(A-A')$ is a skew-symmetric matrix.

Representing $A$ as the sum of $P$ and $Q$:

$$P+Q=\begin{bmatrix}3 & 3\\ 3 & -1\end{bmatrix}+\begin{bmatrix}0 & 2\\ -2 & 0\end{bmatrix}=\begin{bmatrix}3 & 5\\ 1 & -1\end{bmatrix}=A$$

**(ii)**

$$\text{Let } A=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}, \text{ then } A'=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}$$

$$\text{Now, } A+A'=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}+\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}=\begin{bmatrix}12 & -4 & 4\\ -4 & 6 & -2\\ 4 & -2 & 6\end{bmatrix}$$

$$\text{Let } P=\frac{1}{2}(A+A')=\frac{1}{2}\begin{bmatrix}12 & -4 & 4\\ -4 & 6 & -2\\ 4 & -2 & 6\end{bmatrix}=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}$$

$$\text{Now, } P'=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}=P$$

Thus, $P=\frac{1}{2}(A+A')$ is a symmetric matrix.

$$\text{Now, } A-A'=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}+\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$\text{Let } Q=\frac{1}{2}(A-A')=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$\text{Now, } Q'=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}=-Q$$

Thus, $Q=\frac{1}{2}(A-A')$ is a skew-symmetric matrix.

Representing *A* as the sum of *P* and *Q*:

$$P+Q=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}+\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}=A$$

**(iii)**

$$\text{Let } A=\begin{bmatrix}3 & 3 & -1\\ -2 & -2 & 1\\ -4 & -5 & 2\end{bmatrix},\ \text{then } A'=\begin{bmatrix}3 & -2 & -4\\ 3 & -2 & -5\\ -1 & 1 & 2\end{bmatrix}$$

$$\text{Now, } A+A'=\begin{bmatrix}3 & 3 & -1\\ -2 & -2 & 1\\ -4 & -5 & 2\end{bmatrix}+\begin{bmatrix}3 & -2 & -4\\ 3 & -2 & -5\\ -1 & 1 & 2\end{bmatrix}=\begin{bmatrix}6 & 1 & -5\\ 1 & -4 & -4\\ -5 & -4 & 4\end{bmatrix}$$

$$\text{Let } P=\frac{1}{2}(A+A')=\frac{1}{2}\begin{bmatrix}6 & 1 & -5\\ 1 & -4 & -4\\ -5 & -4 & 4\end{bmatrix}=\begin{bmatrix}3 & \frac{1}{2} & -\frac{5}{2}\\ \frac{1}{2} & -2 & -2\\ -\frac{5}{2} & -2 & 2\end{bmatrix}$$

$$\text{Now, } P'=\begin{bmatrix}3 & \frac{1}{2} & -\frac{5}{2}\\ \frac{1}{2} & -2 & -2\\ -\frac{5}{2} & -2 & 2\end{bmatrix}=P$$

Thus, $P=\frac{1}{2}(A+A')$ is a symmetric matrix.

$$\text{Now, } A-A'=\begin{bmatrix}3 & 3 & -1\\ -2 & -2 & 1\\ -4 & -5 & 2\end{bmatrix}-\begin{bmatrix}3 & -2 & -4\\ 3 & -2 & -5\\ -1 & 1 & 2\end{bmatrix}=\begin{bmatrix}0 & 5 & 3\\ -5 & 0 & 6\\ -3 & -6 & 0\end{bmatrix}$$

$$\text{Let } Q=\frac{1}{2}(A-A')=\frac{1}{2}\begin{bmatrix}0 & 5 & 3\\ -5 & 0 & 6\\ -3 & -6 & 0\end{bmatrix}=\begin{bmatrix}0 & \frac{5}{2} & \frac{3}{2}\\ -\frac{5}{2} & 0 & 3\\ -\frac{3}{2} & -3 & 0\end{bmatrix}$$

$$\text{Now, } Q'=\begin{bmatrix}0 & -\frac{5}{2} & -\frac{3}{2}\\ \frac{5}{2} & 0 & -3\\ \frac{3}{2} & 3 & 0\end{bmatrix}=-Q$$

Thus, $Q=\frac{1}{2}(A-A')$ is a skew-symmetric matrix.

Representing $A$ as the sum of $P$ and $Q$:

$$P+Q=\begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix}+\begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}=\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}=A$$

**(iv)**

Let $A=\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$, then $A'=\begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$

Now $A+A'=\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}+\begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}=\begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix}$

Let $P=\frac{1}{2}(A+A')=\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$

Now, $P'=\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}=P$

Thus, $P=\frac{1}{2}(A+A')$ is a symmetric matrix.

Now, $A-A'=\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}-\begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}=\begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix}$

Let $Q=\frac{1}{2}(A-A')=\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

Now, $Q'=\begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}=-Q$

Thus, $Q=\frac{1}{2}(A-A')$ is a skew-symmetric matrix.

Representing $A$ as the sum of $P$ and $Q$:

$$P+Q=\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}+\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}=\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}=A$$

**Question 11:**

If $A$, $B$ are symmetric matrices of same order, then $AB - BA$ is a

**A.** Skew symmetric matrix **B.** Symmetric matrix

**C.** Zero matrix **D.** Identity matrix

Answer

The correct answer is A.

$A$ and $B$ are symmetric matrices, therefore, we have:

$A' = A$ and $B' = B$ ...(1)

$$\begin{aligned} \text{Consider } (AB-BA)' &= (AB)'-(BA)' && \left[(A-B)' = A'-B'\right] \\ &= B'A'-A'B' && \left[(AB)' = B'A'\right] \\ &= BA-AB && \left[\text{by } (1)\right] \end{aligned}$$

*********** END **********