Maxima and Minima 18.5 Q21

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$\Rightarrow V = \frac{1}{3}\pi r^2 h$

Squaring both the sides, we have,

$V^2 = \left(\frac{1}{3}\pi r^2 h\right)^2$

$= \frac{1}{9}\pi^2 r^4 h^2 \ldots(1)$

$\Rightarrow \pi^2 r^2 h^2 = \frac{9V^2}{r^2} \ldots(2)$

Consider the curved surface area of the cone.

Thus,

$C = \pi r l$

Squaring both the sides, we have,

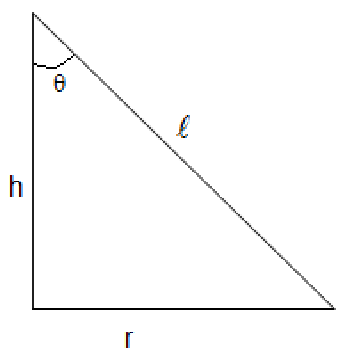
$C^2 = \pi^2 r^2 l^2$

We know that $l^2 = r^2 + h^2$

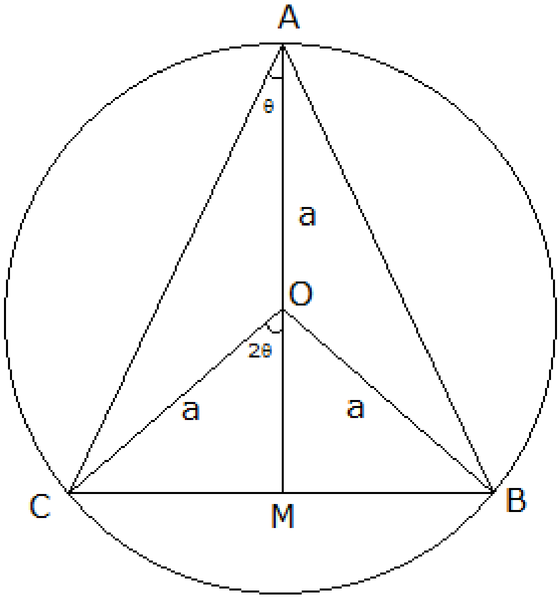
$\Rightarrow C^2 = \pi^2 r^2 \left(r^2 + h^2\right)$

$\Rightarrow C^2 = \pi^2 r^4 + \pi^2 r^2 h^2$

$\Rightarrow C^2 = \pi^2 r^4 + \frac{9V^2}{r^2} \ldots$(From equation (2))



Maxima and Minima 18.5 Q22



ABC is an isosceles triangle such that AB = AC.
The vertical angle $\angle BAC = 2\theta$.
Triangle is inscribed in the circle with center O and radius a.

Draw AM perpendicular to BC.
$\because$ ΔABC is an isoscales triangle the circumcentre of the circle will lie or the perpendicular from A to BC.

Let O be the circumcentre.
$\angle BOC = 2 \times 2\theta = 4\theta$ ....[Using central angle theorem]
$\angle COM = 2\theta$ ....[$\because$ ΔOMB and ΔOMC are congruent triangles]
$OA = OB = OC = a$.......[Radius of the circle]

In ΔOMC,
$CM = a\sin 2\theta$ and $OM = a\cos 2\theta$
$BC = 2CM$...[Perpendicular from the center bisects the chord]
$BC = 2a\sin 2\theta$..........(1)
Height of $\Delta ABC = AM = AO + OM$
$AM = a + a\cos 2\theta$........(2)

Area of ΔABC is,
$A = \frac{1}{2} \times BC \times AM$

Differentiating equation (3) with respect to θ

$$\frac{dA}{d\theta} = a^2\left(2\cos 2\theta + \frac{1}{2} \times 4\cos 4\theta\right)$$

$$\frac{dA}{d\theta} = 2a^2\left(\cos 2\theta + \cos 4\theta\right)$$

Differentiating agin with respect to θ

$$\frac{d^2A}{d\theta^2} = 2a^2\left(-2\sin 2\theta - 4\sin 4\theta\right)$$

For maximum value of area equating $\frac{dA}{d\theta} = 0$

$$2a^2\left(\cos 2\theta + \cos 4\theta\right) = 0$$

$$\cos 2\theta + \cos 4\theta = 0$$

$$\cos 2\theta + 2\cos^2 2\theta - 1 = 0$$

$$\left(2\cos 2\theta - 1\right)\left(2\cos 2\theta + 1\right) = 0$$

$$\cos 2\theta = \frac{1}{2} \text{ or } \cos 2\theta = -1$$

$$2\theta = \frac{\pi}{3} \text{ or } 2\theta = \pi$$

$$\theta = \frac{\pi}{6} \text{ or } \theta = \frac{\pi}{2}$$

If $2\theta = \pi$ it will not form a triangle.

$$\therefore \theta = \frac{\pi}{6}$$

Also $\frac{d^2A}{d\theta^2}$ is negative for $\theta = \frac{\pi}{6}$.

Thus the area of the triangle is maximum when $\theta = \frac{\pi}{6}$.

*********** END **********