Binomial Theorem Ex 18.2 Q19

Now, Coefficient of $(r+1)$ th term in the expansion of $(4+x)^{n+1} = {}^{n+1}C_{r+1-1} = {}^{n+1}C_r$

and, Coefficient of rth term in $(1+x)^n$ + Coefficient of $(r+1)$ th term in $(1+x)^n$

$$= {}^nC_{r-1} + {}^nC_{r+1-1}$$

$$= {}^nC_{r-1} + {}^nC_r$$

$$= \frac{n!}{\{n-(r-1)\}!(r-1)!} + \frac{n!}{(n-r)!r!}$$

$$= \frac{n!}{\{n-r+1\}!(r-1)!} + \frac{n!}{(n-r)!r!}$$

$$= \frac{n!}{(n-r+1)(n-r)!(r-1)!} + \frac{n!}{(n-r)!r(r-1)!}$$

$$= \frac{n!}{(n-r+1)(n-r)!(r-1)!} + \frac{n!}{(n-r)!(r-1)!r}$$

$$= \frac{n!}{(n-r)!(r-1)!}\left[\frac{1}{n-r+1} + \frac{1}{r}\right]$$

$$= \frac{n!}{(n-r)!(r-1)!}\left[\frac{r+n-r+1}{(n-r+1)r}\right]$$

$$= \frac{n!}{(n-r)!(r-1)!}\left[\frac{n+1}{(n-r+1)r}\right]$$

$$= \frac{n!(n+1)}{(n-r)!(n-r+1)(r-1)!r}$$

$$= \frac{(n+1)!}{(n-r+1)!r!}$$

$$= \frac{(n+1)!}{(n+1-r)!r!}$$

$$= {}^{n+1}C_r$$

$$\therefore \quad {}^{n+1}C_r = {}^nC_{r-1} + {}^nC_r$$

The coefficient of $(r+1)$ th term in the expansion of $(1+x)^{n+1}$ is equal to the sum of the coefficients of rth and $(r+1)$th terms in the expansion of $(1+x)^n$.

Binomial Theorem Ex 18.2 Q20

We have,

$$\left(x+\frac{1}{x}\right)^{2n}$$

Let $(r+1)^{\text{th}}$ term be independent of x.

$$\therefore \quad T_{r+1} = {}^{2n}C_r\,(x)^{2n-r}\left(\frac{1}{x}\right)^r$$

$$= {}^{2n}C_r\,(x)^{2n-r-r}$$

$$= {}^{2n}C_r x^{2n-2r}$$

If it is independent of x, we must have,

$$2n-2r=0$$

$$\Rightarrow \quad 2n=2r$$

$$\Rightarrow \quad r=n$$

$\therefore$ Term independent of $x = T_{n+1}$

Now,

$$T_{n+1} = {}^{2n}C_n\,(x-1)^{2n-n}\left(\frac{1}{x}\right)^n$$

$$= {}^{2n}C_n$$

$$= \frac{(2n)!}{(2n-n)!\,n!}$$

$$= \frac{(2n)!}{n!\,n!}$$

$$= \frac{(2n)(2n-1)(2n-2)\ldots5\times4\times3\times2\times1}{n!\,n!}$$

$$= \frac{\{1\times3\times5\times\ldots(2n-1)\}\{2\times4\times6\times\ldots2n\}}{n!\,n!}$$

$$= \frac{\{1\times3\times5\times\ldots(2n-1)\}\times2^n\{1\times2\times3\times\ldots n\}}{n!\,n!}$$

$$= \frac{\{1\times3\times5\times\ldots(2n-1)\}\times2^n\times n!}{n!\,n!}$$

$$= 2^n\times\frac{\{1\times3\times5\times\ldots(2n-1)\}}{n!}$$

$\therefore$ The term independent to $x = \dfrac{\{1\times3\times5\times\ldots(2n-1)\}}{n!}\times2^n$ Hence proved.

Binomial Theorem Ex 18.2 Q21

We have,

$$(1+x)^n$$

Now,

Coefficient of 5th term $= {}^nC_{5-1} = {}^nC_4$

Coefficient of 5th term $= {}^nC_{6-1} = {}^nC_5$

and, Coefficient of 5th term $= {}^nC_{7-1} = {}^nC_6$

It is given that these coefficients are in A.P.

$$\therefore \quad 2\,{}^nC_5 = {}^nC_4 + {}^nC_6$$

$$\Rightarrow \quad 2\left[\frac{n!}{(n-5)!\,5!}\right] = \frac{n!}{(n-4)!\,4!}+\frac{n!}{(n-6)!\,6!}$$

$$\Rightarrow \quad \frac{2}{(n-5)!\,5!} = \frac{1}{(n-4)!\,4!}+\frac{1}{(n-6)!\,6!}$$

$$\Rightarrow \quad \frac{2}{(n-5)(n-6)!\,5\times4!} = \frac{1}{(n-4)(n-5)(n-6)!\,4!}+\frac{1}{(n-6)!\,6\times5\times4!}$$

$$\Rightarrow \quad \frac{2}{(n-5)\times5} = \frac{1}{(n-4)(n-5)}+\frac{1}{6\times5}$$

$$\Rightarrow \quad \frac{2}{5(x-5)}-\frac{1}{30} = \frac{1}{(n-4)(n-5)}$$

$$\Rightarrow \quad \frac{12-(n-5)}{30(n-5)} = \frac{1}{(n-4)(n-5)}$$

$$\Rightarrow \quad \frac{12-n+5}{30} = \frac{1}{(n-4)(n-5)}$$

$$\Rightarrow \quad \frac{17-n}{30} = \frac{1}{n-4}$$

$$\Rightarrow \quad 17n-68-n^2+4n=30$$

$$\Rightarrow \quad 21n-68-m^2-30=0$$

$$\Rightarrow \quad 21n-n^2-98=0$$

$$\Rightarrow \quad n^2-21n+98=0$$

$$\Rightarrow \quad n^2-7n-14n+98=0$$

$$\Rightarrow \quad n(n-7)-17(n-7)=0$$

$$\Rightarrow \quad (n-7)(n-14)=0$$

$$\Rightarrow \quad n=7 \text{ or, } n=14$$

Binomial Theorem Ex 18.2 Q22

We have,

$$(1+x)^{2n}$$

Now,

Coefficient 2nd term = ${}^{2n}C_{2-1} = {}^{2n}C_1$

Coefficient 3rd term = ${}^{2n}C_{3-1} = {}^{2n}C_2$

and, Coefficient 4th term = ${}^{2n}C_{4-1} = {}^{2n}C_3$

It is given that these coefficients are in A.P.

$\therefore \quad 2\,{}^{2n}C_2 = {}^{2n}C_1 + {}^{2n}C_3$

$\Rightarrow \quad 2 = \dfrac{{}^{2n}C_1}{{}^{2n}C_2} + \dfrac{{}^{2n}C_3}{{}^{2n}C_2}$

$\Rightarrow \quad 2 = \dfrac{2}{2n-2+1} + \dfrac{2n-3+1}{3} \qquad \left[\because \dfrac{{}^{n}C_r}{{}^{n}C_{r-1}} = \dfrac{n-r+1}{r}\right]$

$\Rightarrow \quad 2 = \dfrac{2}{2n-1} + \dfrac{2n-2}{3}$

$\Rightarrow \quad 2 = \dfrac{6+(2n-1)(2n-2)}{3(2n-1)}$

$\Rightarrow \quad 6(2n-1) = 6 + 4n^2 - 4n - 2n + 2$

$\Rightarrow \quad 12n - 6 = 8 + 4n^2 - 6n$

$\Rightarrow \quad 4n^2 - 6n - 12n + 8 + 6 = 0$

$\Rightarrow \quad 4n^2 - 18n + 14 = 0$

$\Rightarrow \quad 2(2n^2 - 9n + 7) = 0$

$\Rightarrow \quad 2n^2 - 9n + 7 = 0 \qquad$ Hence proved.

*********** END **********