Exercise 5.3

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$S_{15} = \frac{15}{2}[14 + (15-1)4] = \frac{15}{2}(14+56) = \frac{15}{2} \times 70 = 15 \times 35 = 525$$

Therefore, sum of first 15 terms of AP is equal to 525.

(ii) We need to show that $a_1, a_2 \ldots a_n$ form an AP where $a_n = 9 - 5n$

Let us calculate values of $a_1, a_2, a_3 \ldots$ using $a_n = 9 - 5n$

$a_1 = 9 - 5(1) = 9 - 5 = 4 a_2 = 9 - 5(2) = 9 - 10 = -1$

$a_3 = 9 - 5(3) = 9 - 15 = -6 a_4 = 9 - 5(4) = 9 - 20 = -11$

So, the sequence is of the form 4, −1, −6, −11 ...

Let us check difference between consecutive terms of this sequence.

−1 − (4) = −5,−6 − (−1)

= −6 + 1 = −5,−11 − (−6)

= −11 + 6 = −5

Therefore, the difference between consecutive terms is constant which means terms $a_1, a_2 \ldots a_n$ form an AP.

We have sequence 4, −1, −6, −11 ...

First term = a = 4 and Common difference = d = −5

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP , we get

$$S_{15} = \frac{15}{2}[8 + (15-1)(-5)] = \frac{15}{2}(8-70) = \frac{15}{2} \times (-62) = 15 \times (-31) = -465$$

Therefore, sum of first 15 terms of AP is equal to −465

11. If the sum of the first n terms of an AP is $(4n - n^2)$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.

Ans. It is given that the sum of n terms of an AP is equal to $(4n - n^2)$

It means $S_n = 4n - n^2$

Let us calculate $S_1 and\ S_2$ using $S_n = 4n - n^2$

$S_1 = 4(1) - (1)^2 = 4 - 1 = 3$

$S_2 = 4(2) - (2)^2 = 8 - 4 = 4$

First term = a = $S_1 = 3$ ... (1)

Let us find common difference now.

We can write any AP in the form of general terms like a , a + d, a + 2d ...

We have calculated that sum of first two terms is equal to 4 i.e. $S_2 = 4$

Therefore, we can say that a + (a + d) = 4

Putting value of a from equation (1), we get

2a + d = 4

⇒ 2 (3) + d = 4

⇒ 6 + d = 4

⇒ d = −2

Using formula a_n = a + (n − 1) d, to find n^{th} term of arithmetic progression,

Second term of AP = a_2 = a + (2 − 1) d = 3 + (2 − 1) (−2) = 3 − 2 = 1

Third term of AP = a_3 = a + (3 − 1) d = 3 + (3 − 1) (−2) = 3 − 4 = −1

Tenth term of AP = a_{10} = a + (10 − 1) d = 3 + (10 − 1) (−2) = 3 − 18 = −15

n^{th} term of AP = a_n = a + (n − 1) d = 3 + (n − 1) (−2) = 3 − 2n + 2 = 5 − 2n

12. Find the sum of the first 40 positive integers divisible by 6.

Ans. The first 40 positive integers divisible by 6 are 6, 12, 18, 24 ... 40 terms.

Therefore, we want to find sum of 40 terms of sequence of the form:

6, 12, 18, 24 ... 40 terms

Here, first term = a = 6 and Common difference = d = 12 – 6 = 6, n = 40

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$S_{40} = \frac{40}{2}[12 + (40-1)6]$$

$$= 20\ (12 + 39 \times 6)$$

$$= 20\ (12 + 234)$$

$$= 20 \times 246 = 4920$$

13. Find the sum of the first 15 multiples of 8.

Ans. The first 15 multiples of 8 are 8,16, 24, 32 ... 15 times

First term = a = 8 and Common difference = d = 16 – 8 = 8, n = 15

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$S_{15} = \frac{15}{2}[16 + (15-1)8] = \frac{15}{2}(16 + 14 \times 8) = \frac{15}{2}(16 + 112) = \frac{15}{2} \times 128 = 15 \times 64 = 960$$

14. Find the sum of the odd numbers between 0 and 50.

Ans. The odd numbers between 0 and 50 are 1, 3, 5, 7 ... 49

It is an arithmetic progression because the difference between consecutive terms is constant.

First term = a = 1, Common difference = 3 – 1 = 2, Last term = l = 49

We do not know how many odd numbers are present between 0 and 50.

Therefore, we need to find n first.

Using formula $a_n = a + (n - 1) d$, to find nth term of arithmetic progression, we get

$49 = 1 + (n - 1) 2$

$\Rightarrow 49 = 1 + 2n - 2$

$\Rightarrow 50 = 2n \Rightarrow n = 25$

Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP, we get

$$S_{25} = \frac{25}{2}(1 + 49) = \frac{25}{2} \times 50 = 25 \times 25 = 625$$

15. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs 250 for the second day, Rs 300 for the third day, etc., the penalty for each succeeding day being Rs 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Ans. Penalty for first day = Rs 200, Penalty for second day = Rs 250

Penalty for third day = Rs 300

It is given that penalty for each succeeding day is Rs 50 more than the preceding day.

It makes it an arithmetic progression because the difference between consecutive terms is constant.

We want to know how much money the contractor has to pay as penalty, if he has delayed the work by 30 days.

So, we have an AP of the form200, 250, 300, 350 ... 30 terms

First term = a = 200, Common difference = d = 50, n = 30

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$S_n = \frac{30}{2}[400 + (30 - 1)50]$$

$\Rightarrow S_n = 15\ (400 + 29 \times 50)$

$\Rightarrow S_n = 15\ (400 + 1450) = 27750$

Therefore, penalty for 30 days is Rs. 27750.

16. A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If, each prize is Rs 20 less than its preceding term, find the value of each of the prizes.

Ans. It is given that sum of seven cash prizes is equal to Rs 700.

And, each prize is Rs 20 less than its preceding term.

Let value of first prize = Rs. a

Let value of second prize = Rs (a – 20)

Let value of third prize = Rs (a – 40)

So, we have sequence of the form:

a, a – 20, a – 40, a – 60 ...

It is an arithmetic progression because the difference between consecutive terms is constant.

First term = a, Common difference = d = (a – 20) – a = –20

n = 7 (Because there are total of seven prizes)

$S_7 = Rs\ 700$ {given}

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$S_7 = \frac{7}{2}[2a + (7-1)(-20)]$$

$$\Rightarrow 700 = \frac{7}{2}[2a - 120]$$

$\Rightarrow$ 200 = 2a – 120

********** END **********