## Maxima and Minima 18.5 Q13

Let the side of the square to be cut off be $x$ cm. Then, the height of the box is $x$, the length is $45 - 2x$, and the breadth is $24 - 2x$.

Therefore, the volume $V(x)$ of the box is given by,

$$
\begin{aligned}
V(x) &= x(45-2x)(24-2x) \\
&= x(1080 - 90x - 48x + 4x^2) \\
&= 4x^3 - 138x^2 + 1080x \\
\therefore V'(x) &= 12x^2 - 276x + 1080 \\
&= 12(x^2 - 23x + 90) \\
&= 12(x-18)(x-5) \\
V''(x) &= 24x - 276 = 12(2x - 23)
\end{aligned}
$$

Now, $V'(x) = 0 \Rightarrow x = 18$ and $x = 5$

It is not possible to cut off a square of side 18 cm from each corner of the rectangular sheet.
Thus, $x$ cannot be equal to 18.

$\therefore x = 5$

Now, $V''(5) = 12(10-23) = 12(-13) = -156 < 0$

$\therefore$ By second derivative test, $x = 5$ is the point of maxima.

Hence, the side of the square to be cut off to make the volume of the box maximum possible is 5 cm.

## Maxima and Minima 18.5 Q14

Let $l$, $b$, and $h$ represent the length, breadth, and height of the tank respectively.

Then, we have height $(h) = 2$ m

Volume of the tank $= 8\text{m}^3$

Volume of the tank $= l \times b \times h$

$\therefore 8 = l \times b \times 2$

$\Rightarrow lb = 4 \Rightarrow b = \dfrac{4}{l}$

Now, area of the base $= lb = 4$

Area of the 4 walls $(A) = 2h\,(l + b)$

$$
\begin{aligned}
&\therefore A = 4\left(l + \frac{4}{l}\right) \\
&\Rightarrow \frac{dA}{dl} = 4\left(1 - \frac{4}{l^2}\right)
\end{aligned}
$$

Now, $\dfrac{dA}{dl} = 0$

$$
\begin{aligned}
&\Rightarrow 1 - \frac{4}{l^2} = 0 \\
&\Rightarrow l^2 = 4
\end{aligned}
$$

$\Rightarrow l = \pm 2$

However, the length cannot be negative.

Therefore, we have $l = 4$.

$$\therefore b = \frac{4}{l} = \frac{4}{2} = 2$$

Now, $\frac{d^2A}{dl^2} = \frac{32}{l^3}$

When $l = 2$, $\frac{d^2A}{dl^2} = \frac{32}{8} = 4 > 0.$

Thus, by second derivative test, the area is the minimum when $l = 2$.

We have $l = b = h = 2$.

$\therefore$Cost of building the base = Rs 70 × ($lb$) = Rs 70 (4) = Rs 280

Cost of building the walls = Rs $2h$ ($l$ + $b$) × 45 = Rs 90 (2) (2 + 2)

= Rs 8 (90) = Rs 720

Required total cost = Rs (280 + 720) = Rs 1000

Hence, the total cost of the tank will be Rs 1000.

*********** END **********