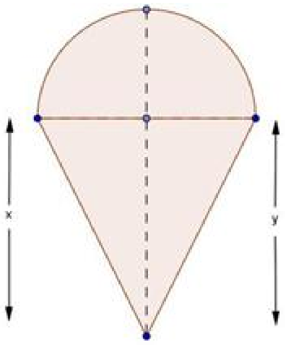
Derivatives as a Rate Measurer Ex 13.2 Q18



Let height of the cone is x cm. and radius of sphere is r cm.

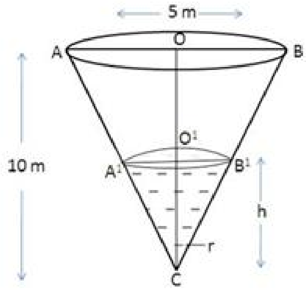
Here given,

$$x = 2r \qquad ---(i)$$
$$h = x + r$$
$$h = 2r + r$$
$$h = 3r \qquad ---(ii)$$

$$v = \text{volume of cone} + \text{volume of hemisphere}$$
$$= \frac{1}{3}\pi r^2 x + \frac{2}{3}\pi r^3$$
$$= \frac{1}{3}\pi r^2 (2r) + \frac{2}{3}\pi r^3 \qquad [\text{Using equation (ii)}]$$
$$v = \frac{2}{3}\pi r^3 + \frac{2}{3}\pi r^3$$
$$= \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi \left(\frac{h}{3}\right)^3$$
$$v = \frac{4}{81}\pi h^3$$
$$\frac{dv}{dh} = \frac{4}{81}\pi \times 3h^2$$
$$\left(\frac{dv}{dh}\right)_{h=9} = \frac{12}{81}\pi (9)^2$$
$$\left(\frac{dv}{dh}\right)_{h=9} = 12\pi \text{ cm}^2$$

Volume is changing at the rate 12π cm^2 with respect to total height.

Derivatives as a Rate Measurer Ex 13.2 Q19



Let α be the semi vertical angle of the cone CAB whose height CO is 10 m and radius OB = 5 m.

Now,

$$\tan\alpha = \frac{OB}{CO}$$
$$= \frac{5}{10}$$
$$\tan\alpha = \frac{1}{2}$$

Let V be the volume of the water in the cone, then

$$v = \frac{1}{3}\pi\left(O'B'\right)^2\left(CO'\right)$$
$$= \frac{1}{3}\pi\left(h\tan\alpha\right)^2(h)$$
$$v = \frac{1}{3}\pi h^3\tan^2\alpha$$
$$v = \frac{\pi}{12}h^2 \qquad \left[\because \tan\alpha = \frac{1}{2}\right]$$
$$\frac{dv}{dt} = \frac{\pi}{12}3h^2\frac{dh}{dt}$$
$$\pi = \frac{\pi}{4}h^2\frac{dh}{dt} \qquad \left[\because \frac{dV}{dt} = \text{m}^3/\text{min}\right]$$
$$\frac{dh}{dt} = \frac{4}{h^2}$$
$$\left(\frac{dh}{dt}\right)_{2.5} = \frac{4}{(2.5)^2} \qquad \left[\because h = 10 - 7.5 = 2.5 \text{ m}\right]$$
$$= \frac{4}{6.25}$$
$$= 0.64 \text{ m/min}$$

So, water level is rising at the rate of 0.64 m/min.

Derivatives as a Rate Measurer Ex 13.2 Q20

Let AB be the lamp-post. Suppose at time t, the man CD is at a distance x m. from the lamp-post and y m be the length of the shadow CE.

Here, $\frac{dx}{dt} = 6$ km/hr

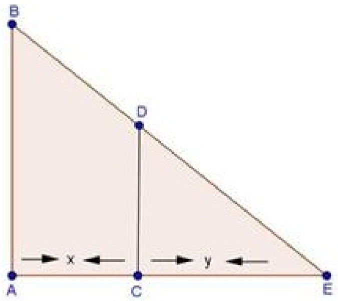
$CD = 2$ m, $AB = 6$ m

Here, ΔABE and ΔCDE are similar

So,
$$\frac{AB}{CD} = \frac{AE}{CE}$$
$$\frac{6}{2} = \frac{x+y}{y}$$
$$3y = x + y$$
$$2y = x$$
$$2\frac{dy}{dt} = \frac{dx}{dt}$$
$$2\frac{dy}{dt} = 6$$
$$\frac{dy}{dt} = 3 \text{ km/hr}$$

So, length of his shadow increases at the rate of 3 km/hr.

The diagram of the problem is shown below



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