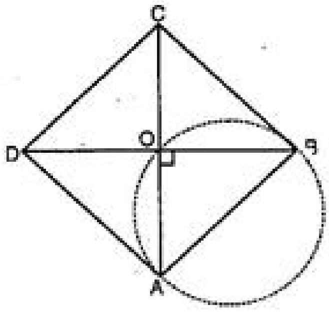
Exercise 10.6

Q6. ABCD is a parallelogram. The circle through A, B and C intersect CD (produced it necessary) at E. Prove that AE = AD.



Ans. In figure (a),

ABCD is a parallelogram.

$\Rightarrow \angle 1 = \angle 3$ ....(i)

ABCE is a cyclic quadrilateral.

$\therefore \angle 1 + \angle 6 = 180^\circ$ ....(ii)

And $\angle 5 + \angle 6 = 180^\circ$ ....(iii)

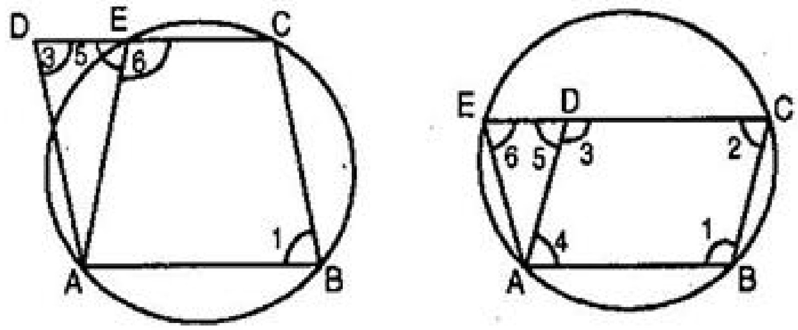
[Linear pair]

From eq. (ii) and (iii), $\angle 1 = \angle 5$ ....(iv)

Now, from eq. (i) and (iv),

$\angle 3 = \angle 5$

$\Rightarrow$ AE = AD [Sides opposite to equal angles are equal]



(a) (b)

In figure (b),

ABCD is a parallelogram.

$\therefore \angle 1 = \angle 3$ and $\angle 2 = \angle 4$

Also AB $\parallel$ CD and BC meets them.

$\therefore \angle 1 + \angle 2 = 180^\circ$ ....(i)

And AD $\parallel$ BC and EC meets them.

$\therefore \angle 5 = \angle 2$ ....(ii) [Corresponding angles]

Since ABCE is a cyclic quadrilateral.

$\therefore \angle 1 + \angle 6 = 180^\circ$ ....(iii)

From eq. (i) and (iii),

$\angle 1 + \angle 2 = \angle 1 + \angle 6$

$\Rightarrow \angle 2 = \angle 6$

But from eq. (ii), $\angle 2 = \angle 5$

$\therefore \angle 5 = \angle 6$

Now in triangle AED,

$\angle 5 = \angle 6$

$\Rightarrow$ AE = AD [Sides opposite to equal angles]

Hence in both the cases, AE = AD

Q7. AC and BD are chords of a circle which bisect each other. Prove that:

(i) AC and BD are diameters.

(ii) ABCD is a rectangle.

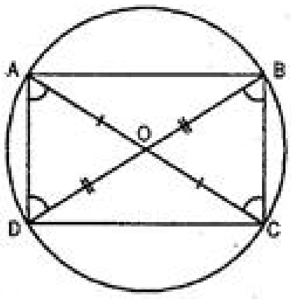
Ans. Given: AC and BD of a circle bisect each other at O.

Then OA = OC and OB = OD

To prove: (i) AC and BD are the diameters. In other words, O is the centre of the circle.

(ii) ABCD is a rectangle.

Proof: (i) In triangles AOD and BOC,



$AO = OC$ [given]

$\angle AOD = \angle BOC$ [Vertically opp.]

$OD = OB$ [given]

$\therefore \Delta AOD \cong \Delta COB$ [SAS congruency]

$\Rightarrow AD = CB$ [By CPCT]

Similarly $\Delta AOB \cong \Delta COD$

$\Rightarrow AB = CD$

$\Rightarrow \overset{\frown}{AB} \cong \overset{\frown}{CD}$ [Arcs opposite to equal chords]

$\Rightarrow \overset{\frown}{AB} + \overset{\frown}{BC} \cong \overset{\frown}{CD} + \overset{\frown}{BC}$

$\Rightarrow \overset{\frown}{ABC} \cong \overset{\frown}{BCD}$

$\Rightarrow AC = BD$ [Chords opposites to equal arcs]

$\therefore$ AC and BD are the diameters as only diameters can bisects each other as the chords of the circle.

(ii) Ac is the diameter. [Proved in (i)]

$\therefore \angle B = \angle D = 90^\circ$ ....(i) [Angle in semi-circle]

Similarly BD is the diameter.

$\therefore \angle A = \angle C = 90^\circ$ ...(ii) [Angle in semi-circle]

Now diameters $AC = BD$

Now diameters AC = BD

$\Rightarrow \overset{\frown}{AC} \cong \overset{\frown}{BD}$ [Arcs opposite to equal chords]

$\Rightarrow \overset{\frown}{AC} - \overset{\frown}{DC} \cong \overset{\frown}{BD} - \overset{\frown}{DC}$

$\Rightarrow \overset{\frown}{AD} \cong \overset{\frown}{BC}$

$\Rightarrow$ AD = BC [Chords corresponding to the equal arcs] ….(iii)

Similarly AB = DC ….(iv)

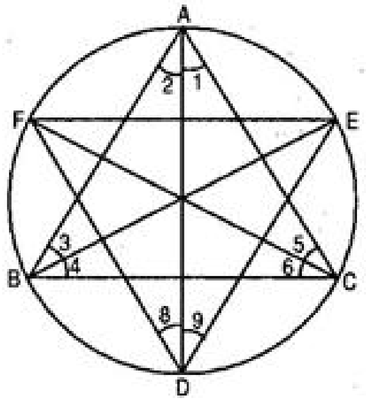
From eq. (i), (ii), (iii) and (iv), we observe that each angle of the quadrilateral is 90° and opposite sides are equal.

Hence ABCD is a rectangle.

Q8. Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that angles of the triangle are $\left(90^\circ - \frac{A}{2}\right), \left(90^\circ - \frac{B}{2}\right)$ and $\left(90^\circ - \frac{C}{2}\right)$ respectively.

Ans. According to question, AD is bisector of $\angle$A.

$\therefore \angle 1 = \angle 2 = \frac{A}{2}$



And BE is the bisector of $\angle$B.

$\therefore \angle 3 = \angle 4 = \frac{B}{2}$

Also CF is the bisector of $\angle$C.

$\therefore \angle 5 = \angle 6 = \frac{C}{2}$

Since the angles in the same segment of a circle are equal.

$\therefore \angle 9 = \angle 3$ [angles subtended by $\widehat{AE}$] ….(i)

And $\angle 8 = \angle 5$ [angles subtended by $\widehat{FA}$] ….(ii)

Adding both equations,

$\angle 9 + \angle 8 = \angle 3 + \angle 5$

$\Rightarrow \angle D = \frac{B}{2} + \frac{C}{2}$

Similarly $\angle E = \frac{A}{2} + \frac{C}{2}$

And $\angle F = \frac{A}{2} + \frac{B}{2}$

In triangle DEF,

$\angle D + \angle E + \angle F = 180^\circ$

$\Rightarrow \angle D = 180^\circ - (\angle E + \angle F)$

$\Rightarrow \angle D = 180^\circ - \left(\frac{A}{2} + \frac{C}{2} + \frac{A}{2} + \frac{B}{2}\right)$

$\Rightarrow \angle D = 180^\circ - \left(\frac{A}{2} + \frac{B}{2} + \frac{C}{2}\right) - \frac{A}{2}$

$\Rightarrow \angle D = 180^\circ - 90^\circ - \frac{A}{2}$ [$\because \angle A + \angle B + \angle C = 180^\circ$]

$\Rightarrow \angle D = 90^\circ - \frac{A}{2}$

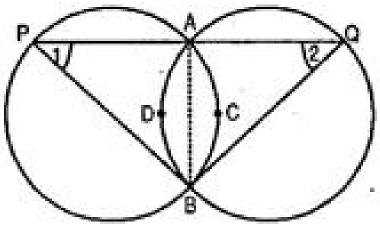
Similarly, we can prove that

$\angle E = 90^\circ - \frac{B}{2}$ and $\angle F = 90^\circ - \frac{C}{2}$

Q9. Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.

Ans. Given: Two equal circles intersect in A and B.

A straight line through A meets the circles in P and Q.



To prove: BP = BQ

Construction: Join A and B.

Proof: AB is a common chord and the circles are equal.

∴ Arc about the common chord are equal, i.e.,

$\overset{\frown}{ACB} = \overset{\frown}{ADB}$

Since equal arcs of two equal circles subtend equal angles at any point on the remaining part of the circle, then we have,

$\angle 1 = \angle 2$

In triangle PBQ,

$\angle 1 = \angle 2$ [proved]

∴ Sides opposite to equal angles of a triangle are equal.

Then we have, BP = BQ

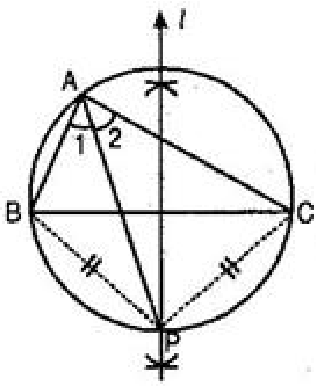
Q10. In any triangle ABC, if the angle bisector of $\angle A$ and perpendicular bisector of BC intersect, prove that they intersect on the circum circle of the triangle ABC.

Ans. Given: ABC is a triangle and a circle passes through its vertices.

Angle bisector of $\angle$ A and the perpendicular bisector (say l) of its opposite side BC intersect each other at a point P.

To prove: Circumcircle of triangle ABC also passes through point P.

Proof: Since any point on the perpendicular bisector is equidistant from the end points of the corresponding side,



$\therefore$ BP = PC ....(i)

Also we have $\angle$ 1 = $\angle$ 2 [$\because$ AP is the bisector of $\angle$ A (given)] ....(ii)

From eq. (i) and (ii) we observe that equal line segments are subtending equal angles in the same segment i.e., at point A of circumcircle of ΔABC. Therefore BP and PC acts as chords of circumcircle of ΔABC and the corresponding congruent arcs $\widehat{BP}$ and $\widehat{PC}$ acts as parts of circumcircle. Hence point P lies on the circumcircle. In other words, points A, B, P and C are concyclic (proved).

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