Geometric Progressions Ex 20.2 Q5

Let the three numbers in G.P. be $\frac{a}{r}, a, ar$

then product of them is $\left(\frac{a}{r}\right)(a)(ar) = 21$ ---(i)

$$= \frac{a}{r}\left(1 + r + r^2\right) = 21$$

and sum of their squares

$$\frac{a^2}{r^2} + a^2 + a^2r^2 = a^2\frac{\left(1 + r^2 + r^4\right)}{r^2} = 189 \quad \text{---(ii)}$$

Now,

$$a\left(1 + r + r^2\right) = 21r \quad \text{---(iii)}$$

Then, $a^2\left(1 + r + r^2\right)^2 = 441r^2$ [suqaring]

$$a^2\left(1 + r^2 + r^4\right) + 2a^2r\left(1 + r + r^2\right) = 441r$$

$$189r^2 + 2ar \times 21r = 441r^2$$

Dividing both sides by $21r^2$

$$9 + 2a = 21$$

$$2a = 21 - 9 = 12$$

$$a = 6 \Rightarrow a = 6$$

Putting in (iii)

$$6\left(1 + r + r^2\right) = 21r$$

$$6 + 6r + 6r^2 - 21r = 0$$

$$6r^2 - 15r + 6 = 0$$

$$6r^2 - 12r - 3r + 6 = 0$$

$$\Rightarrow \quad 6r(r - 2) - 3(r - 2) = 0$$

$$r = 2, \ \frac{1}{2}$$

$\therefore$ G.P. is $3, 6, 12$ or $12, 6, 3$.

Geometric Progressions Ex 20.2 Q6

Let the numbers are: $\frac{a}{r}, a$ and ar.

Then

$$\frac{a}{r}+a+ar=14$$

Again the numbers $a+1, ar+1$ and ar^2-1 are in A.P, therefore

$$2(a+1)=(ar-1)+\left(\frac{a}{r}+1\right)$$

$$2(a+1)=ar+\frac{a}{r}$$

$$2(a+1)=14-a$$

$$3a=12$$

$$a=4$$

Now we have

$$\frac{4}{r}+4+4r=14$$

$$2-5r+2r^2=0$$

$$2r^2-4r-r+2=0$$

$$2r(r-2)-1(r-2)=0$$

$$(r-2)(2r-1)=0$$

$$r=2,\frac{1}{2}$$

Thus the numbers are: 2, 4, 8 or 8, 4, 2.

Geometric Progressions Ex 20.2 Q7

Let the number in G.P. are $\frac{a}{r}$, a, ar

So,

$$\frac{a}{r}\times a\times ar = 216$$

$$\Rightarrow \quad a^3 = 216$$

$$\Rightarrow \quad a = 6$$

And also given,

$$\frac{a}{r}+2,\ a+8,\ ar+6 \text{ are in A.P.}$$

$$2(a+8) = \left(\frac{a}{r}+2\right)+(ar+6)$$

$$\Rightarrow \quad 2(6+8) = \left(\frac{6+2r}{r}\right)+6r+6$$

$$\Rightarrow \quad 28r = 6+2r+6r^2+6r$$

$$\Rightarrow \quad 6r^2-20r+6=0$$

$$\Rightarrow \quad 6r^2-18r-2r+6=0$$

$$\Rightarrow \quad 6r(r-3)-2(r-3)=0$$

$$\Rightarrow \quad (r-3)(6r-2)=0$$

$$r = 3,\ r = \frac{1}{3}$$

So,

Required G.P. is 18, 6, 2, ...

or, 2, 6, 18, ...

Geometric Progressions Ex 20.2 Q8

Let three numbers in G.P. are $\frac{a}{r}$, a, ar

Here,

$$\frac{a}{r} \times a \times ar = 729$$

$\Rightarrow$ $a^3 = 729$

$\Rightarrow$ $a = 9$

And,

$$\left(\frac{a}{r} \times a\right) + (a \times ar) + \left(\frac{a}{r} \times ar\right) = 819$$

$\Rightarrow$ $\frac{81}{r} + 81r + 81 = 819$

$\Rightarrow$ $\frac{9}{r} + 9r + 9 = 91$

$\Rightarrow$ $9 + 9r^2 + 9r = 91r$

$\Rightarrow$ $9r^2 - 82r + 9 = 0$

$\Rightarrow$ $9r^2 - 81r - r + 9 = 0$

$\Rightarrow$ $9r(r-9) - 1(r-9) = 0$

$r = 9, \ \frac{1}{9}$

So, required G.P. are

$81, \ 9, \ 1, \ \ldots$

or, $1, \ 9, \ 81, \ \ldots$

Geometric Progressions Ex 20.2 Q9

Let the numbers are $\frac{a}{r}, a$ and ar. Then we have

$$\frac{a}{r} + a + ar = \frac{39}{10}$$

And

$$\frac{a}{r} \cdot a \cdot ar = 1$$

$$a^3 = 1$$

$$a = 1$$

Now we have

$$\frac{1}{r} + 1 + r = \frac{39}{10}$$

$$1 + r + r^2 = \frac{39}{10}r$$

$$r^2 - \frac{29}{10}r + 1 = 0$$

$$10r^2 - 29r + 10 = 0$$

$$(2r-5)(5r-2) = 0$$

$$r = \frac{5}{2}, \frac{2}{5}$$

Thus the numbers are: either $\frac{2}{5}, 1, \frac{5}{2}$ or $\frac{5}{2}, 1, \frac{2}{5}$.

********** END **********