Definite Integrals Ex 20.4B Q5

$$\int_{0}^{\frac{\pi}{2}} \frac{\sin^{n} x}{\sin^{n} x + \cos^{n} x}\, dx$$

Let $I = \int_{0}^{\frac{\pi}{2}} \dfrac{\sin^{n} x}{\sin^{n} x + \cos^{n} x}\, dx \qquad --(i)$

So,

$$I = \int_{0}^{\frac{\pi}{2}} \frac{\sin^{n}\left(\frac{\pi}{2} - x\right)}{\sin^{n}\left(\frac{\pi}{2} - x\right) + \cos^{n}\left(\frac{\pi}{2} - x\right)}\, dx \qquad \left[\because \int_{0}^{a} f(x)dx = \int_{0}^{a} f(a - x)dx\right]$$

$$= \int_{0}^{\frac{\pi}{2}} \frac{\cos^{n} x}{\sin^{n} x + \cos^{n} x}\, dx \qquad --(II)$$

Adding (I) & (II)

$$2I = \int_{0}^{\frac{\pi}{2}} \frac{\sin^{n} x}{\sin^{n} x + \cos^{n} x}\, dx + \int_{0}^{\frac{\pi}{2}} \frac{\cos^{n} x}{\sin^{n} x + \cos^{n} x}\, dx$$

$$2I = \int_{0}^{\frac{\pi}{2}} \frac{\sin^{n} + \cos^{n} x}{\sin^{n} + \cos^{n} x}\, dx$$

$$2I = \int_{0}^{\frac{\pi}{2}} dx$$

$$2I = \left[x\right]_{0}^{\frac{\pi}{2}}$$

$$2I = \left[\frac{\pi}{2} - 0\right]$$

$$I = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q6

We have,

$$\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\tan x}}\,dx = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$$

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \qquad --(i)$$

So

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}\,dx \qquad \left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} \qquad --(ii)$$

Adding (i) & (ii)

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx + \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}+\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$$

$$2I = \int_0^{\frac{\pi}{2}} dx$$

$$2I = \left[x\right]_0^{\frac{\pi}{2}}$$

$$I = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q7

Let $I = \int_0^a \dfrac{dx}{x + \sqrt{a^2 - x^2}}$

Let $x = a\sin\theta$

$\qquad dx = a\cos\theta\,d\theta$

Now, $\quad x = 0 \Rightarrow \theta = 0$

$\qquad\qquad x = a \Rightarrow \theta = \dfrac{\pi}{2}$

$I = \int_0^{\frac{\pi}{2}} \dfrac{a\cos\theta\,d\theta}{a\sin\theta + a\cos\theta}$

$= \int_0^{\frac{\pi}{2}} \dfrac{\cos\theta}{\sin\theta + \cos\theta} \qquad - -(i)$

So,

$I = \int_0^{\frac{\pi}{2}} \dfrac{\cos\left(\dfrac{\pi}{2} - \theta\right)}{\sin\left(\dfrac{\pi}{2} - \theta\right) + \cos\left(\dfrac{\pi}{2} - \theta\right)}\,d\theta \qquad \left[\because \int_0^a f(x)\,dx = \int_0^a f(a - x)\,dx\right]$

$= \int_0^{\frac{\pi}{2}} \dfrac{\sin\theta}{\cos\theta + \sin\theta} \qquad - -(ii)$

Adding (i) & (ii) we get

$2I = \int_0^{\frac{\pi}{2}} \dfrac{\cos\theta + \sin\theta}{\cos\theta + \sin\theta}\,d\theta$

$2I = \int_0^{\frac{\pi}{2}} d\theta$

$2I = \dfrac{1}{2}\left[\theta\right]_0^{\frac{\pi}{2}}$

$\quad I = \dfrac{\pi}{4}$

Definite Integrals Ex 20.4B Q8

Put $x = \tan\theta$

$\Rightarrow$ $dx = \sec^2\theta \, d\theta$

If $x = 0, \theta = 0$

If $x = \infty, \theta = \dfrac{\pi}{2}$

$\therefore$ $I = \displaystyle\int_0^\infty \dfrac{\log x}{1+x^2} dx$

$= \displaystyle\int_0^{\frac{\pi}{2}} \dfrac{\log(\tan\theta)\sec^2\theta \, d\theta}{1+\tan^2\theta}$

$\Rightarrow$ $I = \displaystyle\int_0^{\frac{\pi}{2}} \log(\tan\theta) \, d\theta$ $---$ (i)

$\Rightarrow$ $I = \displaystyle\int_0^{\frac{\pi}{2}} \log\tan\left(\dfrac{\pi}{2}-\theta\right) d\theta$

$\Rightarrow$ $I = \displaystyle\int_0^{\frac{\pi}{2}} \log\cot(\theta) \, d\theta$ $---$ (ii)

Adding (i) and (ii), we get

$2I = \displaystyle\int_0^{\frac{\pi}{2}} \big(\log\tan\theta + \log\cot\theta\big) d\theta$

$\Rightarrow$ $2I = \displaystyle\int_0^{\frac{\pi}{2}} \log 1 \times dx = \displaystyle\int_0^{\frac{\pi}{2}} 0 \times dx = 0$

$\Rightarrow$ $I = 0$

********** END *********