Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 1

L.H.S,

$$\sin 5\theta = \sin(3\theta + 2\theta)$$
$$= \sin 3\theta \cos 2\theta + \cos 3\theta . \sin 2\theta$$
$$= (3\sin\theta - 4\sin^3\theta)(1 - 2\sin^2\theta) + (4\cos^3\theta - 3\cos\theta)\, 2\sin\theta\cos\theta.$$
$$= 3\sin\theta - 4\sin^3\theta - 6\sin^3\theta + 8\sin^5\theta + (8\cos^4\theta - 6\cos^2\theta)\sin\theta$$
$$= 3\sin\theta - 10\sin^3\theta + 8\sin^5\theta + 8\sin\theta\left((1-\sin^2\theta)^2 - 6\sin\theta(1-\sin^2\theta)\right)$$
$$= 3\sin\theta - 10\sin^3\theta + 8\sin^5\theta + 8\sin\theta - 16\sin^3\theta + 8\sin^5\theta - 6\sin\theta + 6\sin^3\theta$$
$$= 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta = \text{RHS}$$

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 2

Consider the L.H.S of the given equation

$$4(\cos^3 10^\circ + \sin^3 20^\circ) = 3(\cos 10^\circ + \sin 20^\circ)$$

Since $\sin 30^\circ = \cos 60^\circ = \frac{1}{2}$

and $\sin 60^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \sin 3.20^\circ = \cos 3.10^\circ$

$\Rightarrow \quad 3\sin 20^\circ - 4\sin^3 20^\circ = 4\cos^3 10^\circ - 3\cos 10^\circ$

$\Rightarrow \quad 4(\cos^3 10^\circ + \sin^3 20^\circ) = 3(\cos 10^\circ + \sin 20^\circ)$

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 3

$$\cos^3\theta \sin 3\theta + \sin^3\theta\cos 3\theta = \frac{3}{4}\sin 4\theta$$

LHS $= \cos^3\theta\sin 3\theta + \sin^3\theta\cos 3\theta$

$$= \left(\frac{\cos 3\theta + 3\cos\theta}{4}\right)\sin 3\theta + \left(\frac{3\sin\theta - \sin 3\theta}{4}\right)\cos 3\theta \quad \left\{\begin{array}{l} \because \sin 3\theta = 3\sin\theta - 4\sin^3\theta \\ \cos 3\theta = 4\cos^3\theta - 3\cos\theta \end{array}\right\}$$

$$= \frac{1}{4}\left[3(\sin 3\theta\cos\theta + \sin\theta\cos 3\theta) + \cos 3\theta\sin 3\theta - \sin 3\theta\cos 3\theta\right]$$

$$= \frac{1}{4}\left[3\sin(3\theta + \theta) + 0\right]$$

$$= \frac{3}{4}\sin 4\theta$$

So,

$$\cos^3\theta\sin 3\theta + \sin^3\theta\cos 3\theta = \frac{3}{4}\sin 4\theta$$

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 4

We have to prove that

$\sin 5A = 5\cos^4 A \sin A - 10\cos^2 A \sin^3 A + \sin^5 A$

L.H.S= $\sin 5A = \sin(3A + 2A)$

$$
\begin{aligned}
&= \sin 3A \cos 2A + \cos 3A . \sin 2A \\
&= (3\sin A - 4\sin^3 A)(2\cos^2 A - 1) + (4\cos^3 A - 3\cos A)2\sin A \cos A. \\
&= -3\sin A + 4\sin^3 A + 6\sin A\cos^2 A - 8\sin^3 A\cos^2 A + 8\cos^4 A\sin A - 6\cos^2 A\sin A \\
&= 8\cos^4 A\sin A - 8\sin^3 A\cos^2 A - 3\sin A + 4\sin^3 A \\
&= 5\cos^4 A\sin A - 10\sin^3 A\cos^2 A - 3\sin A + 3\cos^4 A\sin A + 4\sin^3 A + 2\sin^3 A\cos^2 A \\
&= 5\cos^4 A\sin A - 10\sin^3 A\cos^2 A - 3\sin A(1 - \cos^4 A) + 2\sin^3 A(2 + \cos^2 A) \\
&= 5\cos^4 A\sin A - 10\sin^3 A\cos^2 A - 3\sin A(1 - \cos^2 A)(1 + \cos^2 A) + 2\sin^3 A(2 + \cos^2 A) \\
&= 5\cos^4 A\sin A - 10\sin^3 A\cos^2 A - 3\sin^3 A(1 + \cos^2 A) + 2\sin^3 A(2 + \cos^2 A) \\
&= 5\cos^4 A\sin A - 10\sin^3 A\cos^2 A - \sin^3 A\left[3(1 + \cos^2 A) - 2(2 + \cos^2 A)\right] \\
&= 5\cos^4 A\sin A - 10\sin^3 A\cos^2 A - \sin^3 A\left[3 + 3\cos^2 A - 4 - 2\cos^2 A\right] \\
&= 5\cos^4 A\sin A - 10\sin^3 A\cos^2 A - \sin^3 A\left[\cos^2 A - 1\right] \\
&= 5\cos^4 A\sin A - 10\sin^3 A\cos^2 A + \sin^5 A \\
&= \text{RHS}
\end{aligned}
$$

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 5

$$\tan A \times \tan(A+60^\circ) + \tan A \times \tan(A - 60^\circ) + \tan(A+60^\circ)\tan(A - 60^\circ)$$

$$= \tan(A)\frac{[\tan(A) - \tan(60^\circ)]}{[1 + \tan(A)\tan(60^\circ)]}$$

$$+ \tan(A)\frac{[\tan(A) + \tan(60^\circ)]}{[1 - \tan(A)\tan(60^\circ)]}$$

$$+ \left\{\frac{[\tan(A) - \tan(60^\circ)]}{[1 + \tan(A)\tan(60^\circ)]}\right\}\left\{\frac{[\tan(A) + \tan(60^\circ)]}{[1 - \tan(A)\tan(60^\circ)]}\right\}$$

$$= \tan(A)\frac{[\tan(A) - \tan(60^\circ)][1 - \tan(A)\tan(60^\circ)]}{[1 - \tan^2(A)\tan^2(60^\circ)]}$$

$$+ \tan(A)\frac{[\tan(A) + \tan(60^\circ)][1 + \tan(A)\tan(60^\circ)]}{[1 - \tan^2(A)\tan^2(60^\circ)]}$$

$$+ \frac{[\tan(A) - \tan(60^\circ)][\tan(A) + \tan(60^\circ)]}{[1 - \tan^2(A)\tan^2(60^\circ)]}$$

$$= \tan(A)\frac{[\tan(A) - \sqrt{3}][1 - \sqrt{3}\tan(A)]}{[1 - 3\tan^2(A)]}$$

$$+ \tan(A)\frac{[\tan(A) + \sqrt{3}][1 + \sqrt{3}\tan(A)]}{[1 - 3\tan^2(A)]}$$

$$+ \frac{[\tan(A) - \sqrt{3}][\tan(A) + \sqrt{3}]}{[1 - 3\tan^2(A)]}$$

$$= \tan(A)\frac{[4\tan(A) - \sqrt{3} - \sqrt{3}\tan^2(A)]}{[1 - 3\tan^2(A)]}$$

$$+ \tan(A)\frac{[4\tan(A) + \sqrt{3} + \sqrt{3}\tan^2(A)]}{[1 - 3\tan^2(A)]}$$

$$+ \frac{[\tan^2(A) - 3]}{[1 - 3\tan^2(A)]}$$

$$= \frac{[9\tan^2(A) - 3]}{[1 - 3\tan^2(A)]}$$

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 6

$$\tan A+\tan\left(60^\circ+A\right)-\tan\left(60^\circ-A\right)=3\tan 3A$$

$$\text{LHS}=\tan A+\tan\left(60^\circ+A\right)-\tan\left(60^\circ-A\right)$$

$$=\tan A+\frac{\tan 60^\circ+\tan A}{1-\tan 60^\circ\tan A}-\frac{\tan 60^\circ-\tan A}{1+\tan 60^\circ\tan A}$$

$$=\tan A+\frac{\sqrt{3}+\tan A}{1-\sqrt{3}\tan A}-\frac{\sqrt{3}-\tan A}{1+\sqrt{3}\tan A}$$

$$=\tan A+\left[\frac{\sqrt{3}+3\tan A+\tan A+\sqrt{3}\tan^2 A-\sqrt{3}+3\tan A+\tan A-\sqrt{3}\tan^2 A}{\left(1-\sqrt{3}\tan A\right)\left(1+\sqrt{3}\tan A\right)}\right]$$

$$=\tan A+\frac{8\tan A}{1-3\tan^2 A}$$

$$=\frac{\tan A-3\tan^3 A+8\tan A}{1-3\tan^2 A}$$

$$=\frac{9\tan A-3\tan^3 A}{1-3\tan^2 A}$$

$$=3\left(\frac{3\tan A-\tan^3 A}{1-3\tan^2 A}\right)$$

$$=3\tan 3A$$

so,

$$\tan A+\tan\left(60^\circ+A\right)-\tan\left(60^\circ-A\right)=3\tan 3A$$

*********** END **********