



Differentiation Ex 11.8 Q14

$$\begin{aligned}
\text{Let } u &= \tan^{-1} \left(\frac{\cos x}{1 + \sin x} \right) \\
&= \tan^{-1} \left(\frac{\frac{\cos^2 x}{2} - \frac{\sin^2 x}{2}}{\frac{\cos^2 x}{2} + \frac{\sin^2 x}{2} + \frac{2 \sin x \cos x}{2}} \right) \\
&= \tan^{-1} \left(\frac{\left(\frac{\cos x}{2} + \frac{\sin x}{2} \right) \left(\frac{\cos x}{2} - \frac{\sin x}{2} \right)}{\left(\frac{\cos x}{2} + \frac{\sin x}{2} \right)^2} \right) \\
&= \tan^{-1} \left(\frac{\frac{\cos x}{2} - \frac{\sin x}{2}}{\frac{\cos x}{2} + \frac{\sin x}{2}} \right) \\
&= \tan^{-1} \left[\frac{\frac{\cos x}{2} - \frac{\sin x}{2}}{\frac{\cos x}{2} + \frac{\sin x}{2}} \right] \\
&= \tan^{-1} \left[\frac{1 - \frac{\tan x}{2}}{1 + \frac{\tan x}{2}} \right] \\
&= \tan^{-1} \left[\frac{\frac{\tan \frac{\pi}{4}}{4} - \frac{\tan x}{2}}{1 + \frac{\tan \frac{\pi}{4}}{4} \times \frac{\tan x}{2}} \right] \\
&= \tan^{-1} \left[\tan \left(\frac{\pi}{4} - \frac{x}{2} \right) \right] \\
u &= \frac{\pi}{4} - \frac{x}{2}
\end{aligned}$$

Differentiating it with respect to x ,

$$\begin{aligned}
\frac{du}{dx} &= 0 - \left(\frac{1}{2} \right) \\
\frac{du}{dx} &= -\frac{1}{2} \quad \text{---(i)}
\end{aligned}$$

$$\text{Let } v = \sec^{-1} x$$

Differentiating it with respect to x ,

$$\frac{dv}{dx} = \frac{1}{x\sqrt{x^2-1}} \quad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = -\frac{1}{2} \times \frac{x\sqrt{x^2-1}}{1}$$

$$\frac{du}{dv} = \frac{-x\sqrt{x^2-1}}{2}$$

Differentiation Ex 11.8 Q15

$$\text{Let } u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$\text{Put } x = \tan \theta \Rightarrow \theta = \tan^{-1} x, \text{ so}$$

$$u = \sin^{-1}\left(\frac{2 \tan \theta}{1 + \tan^2 \theta}\right)$$

$$u = \sin^{-1}(\sin 2\theta) \quad \text{---(i)}$$

$$\text{Let } v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$= \tan^{-1}\left(\frac{2 \tan \theta}{1 - \tan^2 \theta}\right)$$

$$v = \tan^{-1}(\tan 2\theta) \quad \text{---(ii)}$$

$$\text{Here, } -1 < x < 1$$

$$\Rightarrow -1 < \tan \theta < 1$$

$$\Rightarrow -\frac{\pi}{4} < \theta < \frac{\pi}{4}$$

So, from equation (i),

$$u = 2\theta \quad \left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \right]$$

$$u = 2 \tan^{-1} x$$

Differentiating it with respect to x ,

$$\frac{du}{dx} = \frac{2}{(1+x^2)} \quad \text{---(iii)}$$

From equation (ii),

$$v = 2\theta \quad \left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \right]$$

$$v = 2 \tan^{-1} x$$

Differentiating it with respect to x ,

$$\frac{dv}{dx} = \frac{2}{1+x^2} \quad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{1+x^2} \times \frac{1+x^2}{2}$$

$$\frac{du}{dv} = 1$$

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