



#### Differentiation Ex 11.4 Q6

Given,

$$x^5 + y^5 = 5xy$$

Differentiating with respect to  $x$ ,

$$\begin{aligned} \frac{d}{dx}(x^5) + \frac{d}{dx}(y^5) &= \frac{d}{dx}(5xy) \\ \Rightarrow 5x^4 + 5y^4 \frac{dy}{dx} &= 5 \left[ x \frac{dy}{dx} + y \frac{dy}{dx}(x) \right] && \text{[Using product rule]} \\ \Rightarrow 5x^4 + 5y^4 \frac{dy}{dx} &= 5 \left[ x \frac{dy}{dx} + y(1) \right] \\ \Rightarrow 5x^4 + 5y^4 \frac{dy}{dx} &= 5x \frac{dy}{dx} + 5y \\ \Rightarrow 5y^4 \frac{dy}{dx} - 5x \frac{dy}{dx} &= 5y - 5x^4 \\ \Rightarrow 5 \frac{dy}{dx} (y^4 - x) &= 5(y - x^4) \\ \Rightarrow \frac{dy}{dx} &= \frac{5(y - x^4)}{5(y^4 - x)} \\ \Rightarrow \frac{dy}{dx} &= \frac{y - x^4}{y^4 - x} \end{aligned}$$

#### Differentiation Ex 11.4 Q7

Given,

$$(x + y)^2 = 2axy$$

Differentiating with respect to  $x$ ,

$$\begin{aligned} \Rightarrow \frac{d}{dx}(x + y)^2 &= \frac{d}{dx}(2axy) \\ \Rightarrow 2(x + y) \frac{d}{dx}(x + y) &= 2a \left[ x \frac{dy}{dx} + y \frac{d}{dx}(x) \right] && \text{[Using chain rule and product rule]} \\ \Rightarrow 2(x + y) \left[ 1 + \frac{dy}{dx} \right] &= 2a \left[ x \frac{dy}{dx} + y(1) \right] \\ \Rightarrow 2(x + y) + 2(x + y) \frac{dy}{dx} &= 2ax \frac{dy}{dx} + 2ay \\ \Rightarrow \frac{dy}{dx} [2(x + y) - 2ax] &= 2ay - 2(x + y) \\ \Rightarrow \frac{dy}{dx} &= \frac{2[ay - x - y]}{2[x + y - ax]} \\ \Rightarrow \frac{dy}{dx} &= \frac{ay - x - y}{x + y - ax} \end{aligned}$$

#### Differentiation Ex 11.4 Q8

Given,

$$(x^2 + y^2)^2 = xy$$

Differentiating with respect to  $x$ ,

$$\begin{aligned} \Rightarrow \frac{d}{dx} \left( (x^2 + y^2)^2 \right) &= \frac{d}{dx}(xy) \\ \Rightarrow 2(x^2 + y^2) \frac{d}{dx}(x^2 + y^2) &= x \frac{dy}{dx} + y \frac{d}{dx}(x) && \text{[Using chain rule and product rule]} \\ \Rightarrow 2(x^2 + y^2) \left( 2x + 2y \frac{dy}{dx} \right) &= x \frac{dy}{dx} + y(1) \\ \Rightarrow 4x(x^2 + y^2) + 4y(x^2 + y^2) \frac{dy}{dx} &= x \frac{dy}{dx} + y \\ \Rightarrow 4y(x^2 + y^2) \frac{dy}{dx} - x \frac{dy}{dx} &= y - 4x(x^2 + y^2) \\ \Rightarrow \frac{dy}{dx} [4yx^2 + 4y^3 - x] &= y - 4x^3 - 4xy^2 \\ \Rightarrow \frac{dy}{dx} &= \frac{y - 4x^3 - 4xy^2}{4yx^2 + 4y^3 - x} \end{aligned}$$

#### Differentiation Ex 11.4 Q9

Here,

$$\tan^{-1}(x^2 + y^2) = a$$

Differentiating with respect to  $x$ ,

$$\begin{aligned} \frac{d}{dx}(\tan^{-1}(x^2 + y^2)) &= \frac{d}{dx}(a) \\ \Rightarrow \frac{1}{1 + (x^2 + y^2)^2} \times \frac{d}{dx}(x^2 + y^2) &= 0 && \text{[Using chain rule]} \\ \Rightarrow \left[ \frac{1}{1 + (x^2 + y^2)^2} \right] \left( 2x + 2y \frac{dy}{dx} \right) &= 0 \\ \Rightarrow \left\{ \frac{2x}{1 + (x^2 + y^2)^2} \right\} + \left\{ \frac{2y}{1 + (x^2 + y^2)^2} \right\} \frac{dy}{dx} &= 0 \\ \Rightarrow \frac{2y}{1 + (x^2 + y^2)^2} \frac{dy}{dx} &= - \frac{2x}{1 + (x^2 + y^2)^2} \\ \Rightarrow \frac{dy}{dx} &= - \left( \frac{2x}{1 + (x^2 + y^2)^2} \right) \left( \frac{1 + (x^2 + y^2)^2}{2y} \right) \\ \Rightarrow \frac{dy}{dx} &= - \left( \frac{x}{y} \right) \end{aligned}$$

Differentiation Ex 11.4 Q10

Given,

$$e^{x-y} = \log\left(\frac{x}{y}\right)$$

Differentiating with respect to  $x$ ,

$$\begin{aligned} \frac{d}{dx}(e^{x-y}) &= \frac{d}{dx} \log\left(\frac{x}{y}\right) \\ \Rightarrow e^{(x-y)} \frac{d}{dx}(x-y) &= \frac{1}{\left(\frac{x}{y}\right)} \times \frac{d}{dx}\left(\frac{x}{y}\right) && \text{[Using chain rule and quotient rule]} \\ \Rightarrow e^{(x-y)} \left( 1 - \frac{dy}{dx} \right) &= \frac{y}{x} \left[ \frac{y \frac{d}{dx}(x) - x \frac{dy}{dx}}{y^2} \right] \\ \Rightarrow e^{(x-y)} - e^{(x-y)} \frac{dy}{dx} &= \frac{1}{xy} \left[ y(1) - x \frac{dy}{dx} \right] \\ \Rightarrow e^{(x-y)} - e^{(x-y)} \frac{dy}{dx} &= \frac{y}{xy} - \frac{x}{xy} \frac{dy}{dx} \\ \Rightarrow e^{(x-y)} - e^{(x-y)} \frac{dy}{dx} &= \frac{1}{x} - \frac{1}{y} \frac{dy}{dx} \\ \Rightarrow \frac{1}{y} \frac{dy}{dx} - e^{(x-y)} \frac{dy}{dx} &= \frac{1}{x} - e^{(x-y)} \\ \Rightarrow \frac{dy}{dx} \left[ \frac{1}{y} - \frac{e^{(x-y)}}{1} \right] &= \frac{1}{x} - \frac{e^{(x-y)}}{1} \\ \Rightarrow \frac{dy}{dx} \left[ \frac{1 - ye^{(x-y)}}{y} \right] &= \frac{(1 - xe^{(x-y)})}{x} \\ \Rightarrow \frac{dy}{dx} &= \frac{y}{x} \left[ \frac{1 - xe^{(x-y)}}{1 - ye^{(x-y)}} \right] \\ &= \frac{-y}{-x} \left[ \frac{xe^{(x-y)} - 1}{ye^{(x-y)} - 1} \right] \\ \frac{dy}{dx} &= \frac{y}{x} \left[ \frac{xe^{(x-y)} - 1}{ye^{(x-y)} - 1} \right] \end{aligned}$$

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