Higher Order Derivatives Ex 12.1 Q1(vi)

Let $y = x^3 \log x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}\left[x^3 \log x\right] = \log x \cdot \frac{d}{dx}\left(x^3\right) + x^3 \cdot \frac{d}{dx}\left(\log x\right)$$

$$= \log x \cdot 3x^2 + x^3 \cdot \frac{1}{x} = \log x \cdot 3x^2 + x^2$$

$$= x^2\left(1 + 3\log x\right)$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left[x^2\left(1+3\log x\right)\right]$$

$$= \left(1+3\log x\right) \cdot \frac{d}{dx}\left(x^2\right) + x^2 \frac{d}{dx}\left(1+3\log x\right)$$

$$= \left(1+3\log x\right) \cdot 2x + x^2 \cdot \frac{3}{x}$$

$$= 2x + 6x\log x + 3x$$

$$= 5x + 6x\log x$$

$$= x\left(5 + 6\log x\right)$$

Higher Order Derivatives Ex 12.1 Q1(vii)

Let $y = \tan^{-1} x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{1+x^2}\right) = \frac{d}{dx}\left(1+x^2\right)^{-1} = (-1)\cdot\left(1+x^2\right)^{-2} \cdot \frac{d}{dx}\left(1+x^2\right)$$

$$= \frac{-1}{\left(1+x^2\right)^2} \times 2x = \frac{-2x}{\left(1+x^2\right)^2}$$

Higher Order Derivatives Ex 12.1 Q1(viii)

Let $y = x \cdot \cos x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}(x \cdot \cos x) = \cos x \cdot \frac{d}{dx}(x) + x\frac{d}{dx}(\cos x) = \cos x \cdot 1 + x(-\sin x) = \cos x - x\sin x$$

$$\begin{aligned} \therefore \frac{d^2y}{dx^2} &= \frac{d}{dx}[\cos x - x\sin x] = \frac{d}{dx}(\cos x) - \frac{d}{dx}(x\sin x) \\ &= -\sin x - \left[\sin x \cdot \frac{d}{dx}(x) + x \cdot \frac{d}{dx}(\sin x)\right] \\ &= -\sin x - (\sin x + x\cos x) \\ &= -(x\cos x + 2\sin x) \end{aligned}$$

Higher Order Derivatives Ex 12.1 Q1(ix)

Let $y = \log(\log x)$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\log(\log x)\right] = \frac{1}{\log x} \cdot \frac{d}{dx}(\log x) = \frac{1}{x\log x} = (x\log x)^{-1}$$

$$\begin{aligned} \therefore \frac{d^2y}{dx^2} &= \frac{d}{dx}\left[(x\log x)^{-1}\right] = (-1)\cdot(x\log x)^{-2} \cdot \frac{d}{dx}(x\log x) \\ &= \frac{-1}{(x\log x)^2} \cdot \left[\log x \cdot \frac{d}{dx}(x) + x \cdot \frac{d}{dx}(\log x)\right] \\ &= \frac{-1}{(x\log x)^2} \cdot \left[\log x \cdot 1 + x \cdot \frac{1}{x}\right] = \frac{-(1+\log x)}{(x\log x)^2} \end{aligned}$$

Higher Order Derivatives Ex 12.1 Q2

$$y = e^{-x}\cos x$$

differentiating both sides w.r.t x

$$\Rightarrow \quad \frac{dy}{dx} = e^{-x}(-\sin x) + (\cos x)(-e^{-x})$$

$$\Rightarrow \quad \frac{dy}{dx} = -e^{-x}\sin x - e^{-x}\cos x = -e^{-x}(\sin x + \cos x)$$

again differentiating both sides w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -e^{-x}(\cos x - \sin x) + e^{-x}(\sin x + \cos x)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 2e^{-x}\sin x$$

*********** END **********