Tangents and Normals Ex 16.2 Q17

The given equations are,

$$x^2 + 3y - 3 = 0 \qquad \text{---(i)}$$

$$y = 4x - 5 \qquad \text{---(ii)}$$

Slope m_1 of (i)

$$m_1 = \frac{dy}{dx} = -\frac{2x}{3}$$

Slope m_2 of (ii)

$$m_2 = 4$$

According to the question

$$m_1 = m_2$$

$$\Rightarrow \quad -\frac{2x}{3} = 4$$

$$\Rightarrow \quad x = -6$$

From (i)

$$36 + 3y - 3 = 0$$

$$\Rightarrow \quad 3y = -33$$

$$\therefore \quad y = -11$$

So, $P = (-6, -11)$

Thus, the equation of tangent is

$$(y + 11) = 4(x + 6)$$

$$\Rightarrow \quad 4x - y + 13 = 0$$

Tangents and Normals Ex 16.2 Q18

The equations are

$$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2 \qquad ---(i)$$

$$\frac{x}{a} + \frac{y}{b} = 2 \qquad ---(ii)$$

$$P = (a, b)$$

We need to prove (ii) is the tangent to (i)

Differentiating (i) with respect to x, we get

$$n\left(\frac{x}{a}\right)^n \times \frac{1}{a} + n\left(\frac{y}{b}\right)^{n-1} \times \frac{1}{b} \times \frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{x^{n-1}}{a^n} + \frac{y^{n-1}}{b^n} \times \frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\left(\frac{x}{y}\right)^{n-1} \times \left(\frac{b}{a}\right)^n$$

$$\therefore \text{ Slope } m = \left(\frac{dy}{dx}\right)_P = -\left(\frac{a}{b}\right)^{n-1} \times \left(\frac{b}{a}\right)^n$$

$$= -\frac{b}{a}$$

Thus, the equation of tangent is

$$(y - b) = -\frac{b}{a}(x - a)$$

$$\Rightarrow \quad bx + ay = ab + ab$$

$$\Rightarrow \quad bx + ay = 2ab$$

$$\Rightarrow \quad \frac{x}{a} + \frac{y}{b} = 2$$

Tangents and Normals Ex 16.2 Q19

We have,

$$x = \sin 3t, \qquad y = \cos 2t, \qquad t = \frac{\pi}{4}$$

$$\therefore \qquad P = \left(x = \frac{1}{\sqrt{2}}, y = 0\right)$$

Now,

$$\frac{dx}{dt} = 3\cos 3t, \quad \frac{dy}{dt} = -2\sin 2t$$

$$\therefore \quad \text{Slope } m = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2\sin 2t}{3\cos 3t}$$

$$= \frac{-2}{-3 \times \frac{1}{\sqrt{2}}}$$

$$= \frac{+2\sqrt{2}}{3}$$

Thus, equation of tangent is

$$(y - 0) = \frac{+2\sqrt{2}}{3}\left(x - \frac{1}{\sqrt{2}}\right)$$

$$2\sqrt{2}x - 3y = 2$$

*********** END **********