Co-Ordinate Geometry Ex 14.5 Q10

Answer :

GIVEN: four points A (6, 3), B (−3, 5) C (4, −2) and D(x, 3x) such that $\frac{\Delta DBC}{\Delta ABC}=\frac{1}{2}$

TO FIND: the value of x

PROOF:

We know area of the triangles formed by three points $(x_1,y_1),(x_2,y_2)\,and\,(x_3,y_3)$ is given by

$$\text{Area of triangle}=\frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$

Now

Area of triangle DBC taking D(x, 3x), B (−3, 5), C (4, −2)

$$\Delta DBC\Rightarrow\frac{1}{2}\left|x(5-(-2))+(-3)((-2)-3x)+(4)(3x-5)\right|$$

$$\Delta DBC\Rightarrow\frac{1}{2}\left|7x+6+9x+12x-20\right|$$

$$\Delta DBC\Rightarrow\frac{1}{2}\left|28x-14\right|$$

$$\Delta DBC\Rightarrow\frac{1}{2}\left|14(2x-1)\right|$$

$$\Delta DBC\Rightarrow\left|7(2x-1)\right|\qquad\ldots\ldots(1)$$

Area of triangle ABC taking, A (6, 3), B (−3, 5), C (4, −2)

$$\Rightarrow\frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$

$$\Rightarrow \frac{1}{2}\left|6(5-(-2))+(-3)((-2)-3)+(4)(3-5)\right|$$

$$\Rightarrow \frac{1}{2}\left|6(7)+(-3)(-5)+(4)(-2)\right|$$

$$\Rightarrow \frac{1}{2}|42+15-8|$$

$$\Rightarrow \frac{49}{2} \qquad \ldots\ldots(2)$$

Also it is given that

$$\frac{\Delta DBC}{\Delta ABC}=\frac{1}{2}$$

Substituting the values from (1) and (2) we get

$$\frac{\Delta DBC}{\Delta ABC}=\frac{1}{2}$$

$$\frac{\pm 7(2x-1)}{\frac{49}{2}}=\frac{1}{2}$$

$$\frac{2\times 7(2x-1)}{49}=\frac{1}{2} \text{ or } \frac{-2\times 7(2x-1)}{49}=\frac{1}{2}$$

$$(2x-1)=\frac{1}{2}\times\frac{7}{2} \text{ or } (-2x+1)=\frac{1}{2}\times\frac{7}{2}$$

$$2x=\frac{7}{4}+1 \qquad \text{or } 2x=\frac{7}{4}-1$$

$$2x=\frac{11}{4} \qquad \text{or} 2x=\frac{-3}{4}$$

$$\boxed{x=\frac{11}{8}} \text{ or } \boxed{x=\frac{-3}{8}}$$

Co-Ordinate Geometry Ex 14.5 Q11

Answer :

The formula for the area 'A' encompassed by three points (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by the formula,

$$A = \frac{1}{2}\begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_2 - x_3 & y_2 - y_3 \end{vmatrix}$$

$$A = \frac{1}{2}\left\{(x_1 - x_2)(y_2 - y_3) - (x_2 - x_3)(y_1 - y_2)\right\}$$

If three points are collinear the area encompassed by them is equal to 0.

The three given points are *A*(*a*,1), *B*(1,-1) and *C*(11,4). It is also said that they are collinear and hence the area enclosed by them should be 0.

$$A = \frac{1}{2}\begin{vmatrix} a-1 & 1+1 \\ 1-11 & -1-4 \end{vmatrix}$$

$$A = \frac{1}{2}\begin{vmatrix} a-1 & 2 \\ -10 & -5 \end{vmatrix}$$

$$0 = \frac{1}{2}\left|(a-1)(-5) - (-10)(2)\right|$$

$$0 = \frac{1}{2}\left|-5a + 5 + 20\right|$$

$$0 = -5a + 5 + 20$$

$$5a = 25$$

$$a = 5$$

Hence the value of '*a*' for which the given points are collinear is $\boxed{a = 5}$.

Co-Ordinate Geometry Ex 14.5 Q12

Answer :

The formula for the area 'A' encompassed by three points (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by the formula,

$$A = \frac{1}{2}\begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_2 - x_3 & y_2 - y_3 \end{vmatrix}$$

$$A = \frac{1}{2}\left\{(x_1 - x_2)(y_2 - y_3) - (x_2 - x_3)(y_1 - y_2)\right\}$$

If three points are collinear the area encompassed by them is equal to 0.

The three given points are (a, b), (a_1, b_1) and $(a - a_1, b - b_1)$. If they are collinear then the area enclosed by them should be 0.

$$A = \frac{1}{2}\begin{vmatrix} a - a_1 & b - b_1 \\ a_1 - a + a_1 & b_1 - b + b_1 \end{vmatrix}$$

$$0 = \frac{1}{2}\left|(a - a_1)(2b_1 - b) - (2a_1 - a)(b - b_1)\right|$$

$$0 = \frac{1}{2}\left|2ab_1 - ab - 2a_1b_1 + a_1b - 2a_1b + 2a_1b_1 + ab - ab_1\right|$$

$$0 = 2ab_1 + a_1b - 2a_1b - ab_1$$

$$a_1b = ab_1$$

Hence we have proved that for the given conditions to be satisfied we need to have $\boxed{a_1b = ab_1}$.

*********** END **********