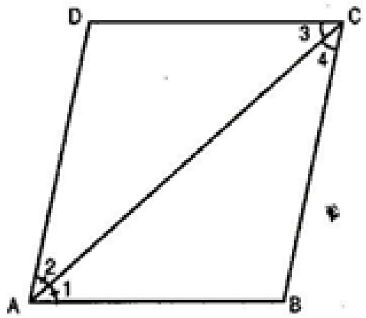
Exercise 8.1

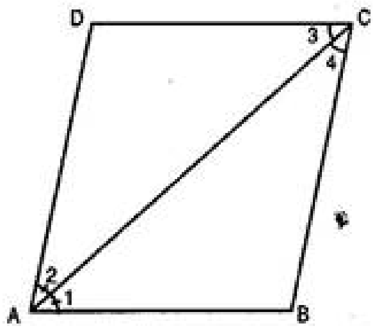
Q6. Diagonal AC of a parallelogram ABCD bisects ∠A (See figure). Show that:



(i) It bisects ∠C also.

(ii) ABCD is a rhombus.

Ans. Diagonal AC bisects ∠A of the parallelogram ABCD.



(i) Since AB $\parallel$ DC and AC intersects them.

∴ ∠1 = ∠3 [Alternate angles] ....(i)

Similarly ∠2 = ∠4 ....(ii)

But ∠1 = ∠2 [Given]..........(iii)

∴ ∠3 = ∠4[Using eq. (i), (ii) and (iii)]

Thus AC bisects ∠C.

(ii) ∠2 = ∠3 = ∠4 = ∠1

⇒ AD = CD[Sides opposite to equal angles]

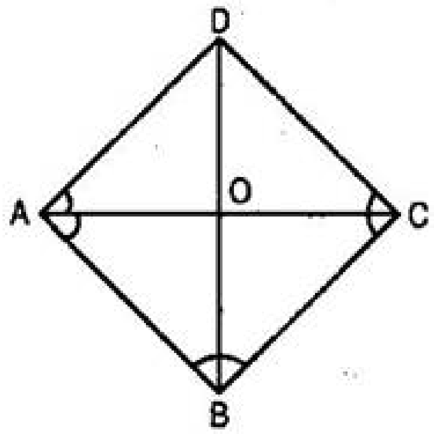
∴ AB = CD = AD = BC

Hence ABCD is a rhombus.

Q7. ABCD is a rhombus. Show that the diagonal AC bisects $\angle A$ as well as $\angle C$ and diagonal BD bisects $\angle B$ as well as $\angle D$.

Ans. ABCD is a rhombus. Therefore, AB = BC = CD = AD

Let O be the point of bisection of diagonals.



$\therefore$ OA = OC and OB = OD

In ΔAOB and ΔAOD,

OA = OA [Common]

AB = AD [Equal sides of rhombus]

OB = OD (diagonals of rhombus bisect each other]

$\therefore$ ΔAOB $\cong$ ΔAOD [By SSS congruency]

$\Rightarrow$ $\angle$OAD = $\angle$OAB [By C.P.C.T.]

$\Rightarrow$ OA bisects $\angle$A ....(i)

Similarly, ΔBOC $\cong$ ΔDOC [By SSS congruency]

$\Rightarrow \angle OCB = \angle OCD$[By C.P.C.T.]

$\Rightarrow$ OC bisects $\angle C$ ....(ii)

From eq. (i) and (ii), we can say that diagonal AC bisects $\angle A$ and $\angle C$.

Now in ΔAOB and ΔBOC,

OB = OB [Common]

AB = BC [Equal sides of rhombus]

OA = OC (diagonals of rhombus bisect each other]

$\therefore \Delta AOB \cong \Delta COB$ [By SSS congruency]

$\Rightarrow \angle OBA = \angle OBC$ [By C.P.C.T.]

$\Rightarrow$ OB bisects $\angle B$ ....(iii)

Similarly, $\Delta AOD \cong \Delta COD$ [By SSS congruency]

$\Rightarrow \angle ODA = \angle ODC$[By C.P.C.T.]

$\Rightarrow$ BD bisects $\angle D$ ....(iv)

From eq. (iii) and (iv), we can say that diagonal BD bisects $\angle B$ and $\angle D$.

Q8. ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$. Show that:

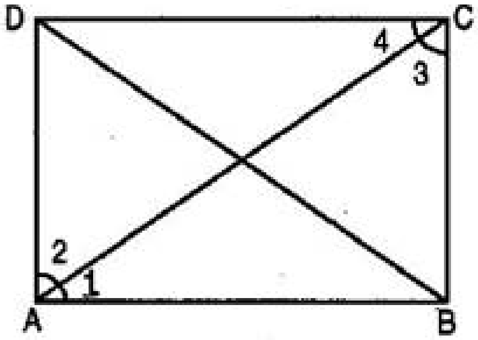
(i) ABCD is a square.

(ii) Diagonal BD bisects both $\angle B$ as well as $\angle D$.

Ans. ABCD is a rectangle. Therefore AB = DC..........(i)

And BC = AD

Also $\angle A = \angle B = \angle C = \angle D = 90^\circ$



(i) In ΔABC and ΔADC

$\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

[AC bisects $\angle A$ and $\angle C$ (given)]

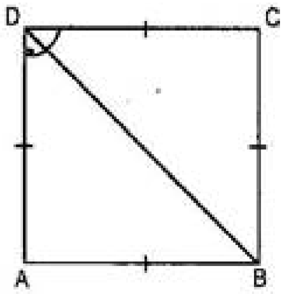
AC = AC [Common]

$\therefore$ $\Delta ABC \cong \Delta ADC$[By ASA congruency]

$\Rightarrow$ AB = AD ....(ii)

From eq. (i) and (ii), AB = BC = CD = AD

Hence ABCD is a square.



(ii) In ΔABC and ΔADC

AB = BA [Since ABCD is a square]

AD = DC [Since ABCD is a square]

BD = BD [Common]

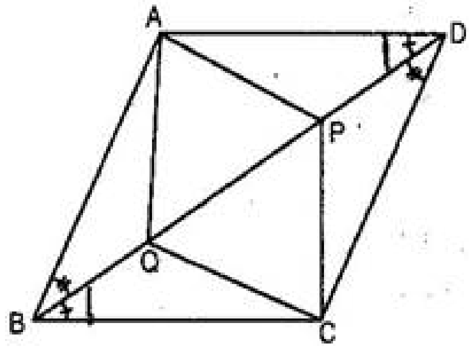
$\therefore$ $\Delta ABD \cong \Delta CBD$ [By SSS congruency]

$\Rightarrow$ $\angle ABD = \angle CBD$ [By C.P.C.T.]..........(iii)

And $\angle ADB = \angle CDB$[By C.P.C.T.]..........(iv)

From eq. (iii) and (iv), it is clear that diagonal BD bisects both $\angle$ B and $\angle$ D.

Q9. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (See figure). Show that:



(i) ΔAPD $\cong$ Δ CQB

(ii) AP = CQ

(iii) ΔAQB $\cong$ Δ CPD

(iv) AQ = CP

(v) APCQ is a parallelogram.

Ans. (i) In ΔAPD and ΔCQB,

DP = BQ[Given]

$\angle$ ADP = $\angle$ QBC [Alternate angles (AD$\parallel$ BC and BD is transversal)]

AD = CB [Opposite sides of parallelogram]

$\therefore$ ΔAPD $\cong$ Δ CQB [By SAS congruency]

(ii) Since ΔAPD $\cong$ ΔCQB

$\Rightarrow$ AP = CQ[By C.P.C.T.]

(iii) In ΔAQB and ΔCPD, BQ = DP[Given]

$\angle ABQ = \angle PDC$ [Alternate angles (AB || CD and BD is transversal)]

AB = CD[Opposite sides of parallelogram]

$\therefore \Delta AQB \cong \Delta CPD$ [By SAS congruency]

(iv) Since $\Delta AQB \cong \Delta CPD$

$\Rightarrow$ AQ = CP[By C.P.C.T.]

(v) In quadrilateral APCQ,

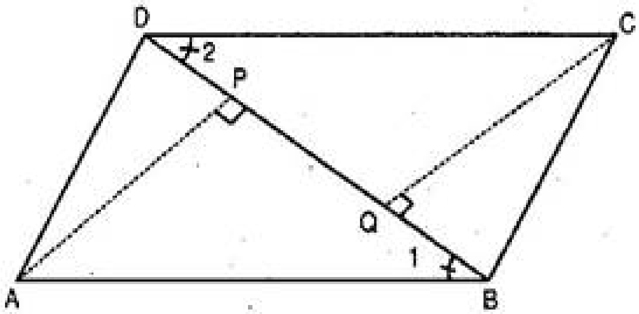
AP = CQ[proved in part (i)]

AQ = CP[proved in part (iv)]

Since opposite sides of quadrilateral APCQ are equal.

Hence APCQ is a parallelogram.

Q10. ABCD is a parallelogram and AP and CQ are the perpendiculars from vertices A and C on its diagonal BD (See figure). Show that:



(i) $\Delta APB \cong \Delta CQD$

(ii) AP = CQ

Ans. Given: ABCD is a parallelogram. AP $\perp$ BD and CQ $\perp$ BD

Proof: (i) In ΔAPB and ΔCQD,

$\angle 1 = \angle 2$[Alternate interior angles]

AB = CD[Opposite sides of a parallelogram are equal]

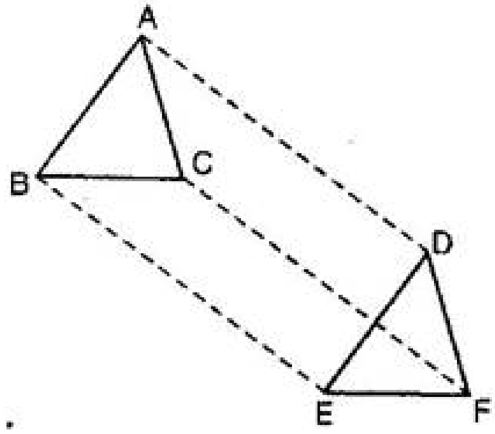
$\angle APB = \angle CQD = 90^\circ$

$\therefore \Delta APB \cong \Delta CQD$ [By ASA Congruency]

(ii) Since $\Delta APB \cong \Delta CQD$

$\therefore$ AP = CQ [By C.P.C.T.]

Q11. An ΔABC and ΔDEF, AB = DE, AB || DE, BC = EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F respectively (See figure). Show that:



(i) Quadrilateral ABED is a parallelogram.

(ii) Quadrilateral BEFC is a parallelogram.

(iii) AD || CF and AD = CF

(iv) Quadrilateral ACFD is a parallelogram.

(v) AC = DF

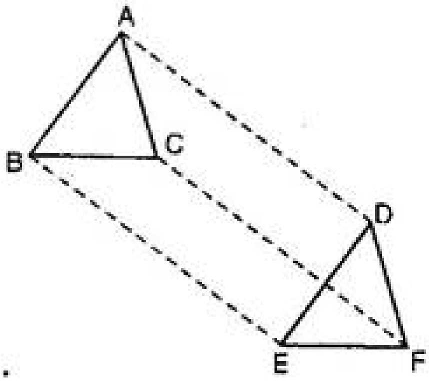
(vi) $\Delta ABC \cong \Delta DEF$

Ans. (i) In ΔABC and ΔDEF

AB = DE[Given]

And AB $\parallel$ DE[Given]

$\therefore$ ABED is a parallelogram.



(ii) In ΔABC and ΔDEF

BC = EF[Given]

And BC $\parallel$ EF[Given]

$\therefore$ BEFC is a parallelogram.

(iii) As ABED is a parallelogram.

$\therefore$ AD $\parallel$ BE and AD = BE ....(i)

Also BEFC is a parallelogram.

$\therefore$ CF $\parallel$ BE and CF = BE..........(ii)

From (i) and (ii), we get

$\therefore$ AD $\parallel$ CF and AD = CF

(iv) As AD $\parallel$ CF and AD = CF

$\Rightarrow$ ACFD is a parallelogram.

(v) As ACFD is a parallelogram.

$\therefore$ AC = DF

(vi) In ΔABC and ΔDEF,

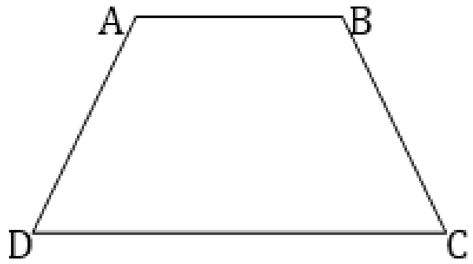
AB = DE [Given]

BC = EF [Given]

AC = DF [Proved]

$\therefore \Delta ABC \cong \Delta DEF$[By SSS congruency]

Q12. ABCD is a trapezium in which AB $\parallel$ CD and AD = BC (See figure). Show that:



(i) $\angle A = \angle B$

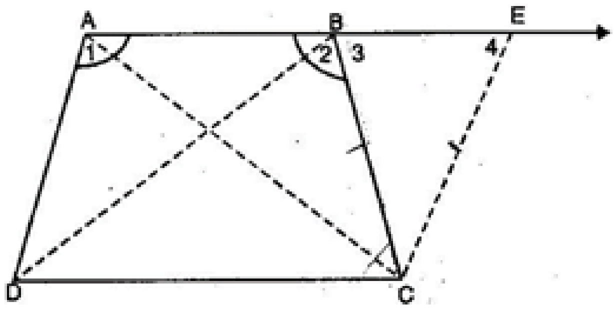
(ii) $\angle C = \angle D$

(iii) $\Delta ABC \cong \Delta BAD$

(iv) Diagonal AC = Diagonal BD

Ans. Given: ABCD is a trapezium.

AB $\parallel$ CD and AD = BC



To prove: (i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\Delta ABC \cong \Delta BAD$

(iv) Diag. AC = Diag. BD

Construction: Draw CE || AD and extend AB to intersect CE at E.

Proof: (i) As AECD is a parallelogram. [By construction]

$\therefore$ AD = EC

But AD = BC [Given]

$\therefore$ BC = EC

$\Rightarrow \angle 3 = \angle 4$ [Angles opposite to equal sides are equal]

Now $\angle 1 + \angle 4 = 180^\circ$ [Interior angles]

And $\angle 2 + \angle 3 = 180^\circ$ [Linear pair]

$\Rightarrow \angle 1 + \angle 4 = \angle 2 + \angle 3$

$\Rightarrow \angle 1 = \angle 2$ [$\because \angle 3 = \angle 4$]

$\Rightarrow \angle A = \angle B$

(ii) $\angle 3 = \angle C$ [Alternate interior angles]

And $\angle D = \angle 4$ [Opposite angles of a parallelogram]

But $\angle 3 = \angle 4$ [ΔBCE is an isosceles triangle]

$\therefore \angle C = \angle D$

(iii) In ΔABC and ΔBAD,

AB = AB [Common]

$\angle 1 = \angle 2$ [Proved]

AD = BC [Given]

$\therefore \Delta ABC \cong \Delta BAD$ [By SAS congruency]

(iv) We had observed that,

$\therefore \Delta ABC \cong \Delta BAD$

$\Rightarrow$ AC = BD [By C.P.C.T.]

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