



Binomial Theorem Ex 18.2 Q30

It is given that,

$$T_6 = 112, T_7 = 7, T_8 = \frac{1}{4}$$

$$\therefore T_6 = {}^nC_{n-5} x^{n-5} \times a^5 = 112$$

$$T_7 = {}^nC_{n-6} x^{n-6} \times a^6 = 7$$

$$\text{and, } T_8 = {}^nC_{n-7} x^{n-7} \times a^7 = \frac{1}{4}$$

Now,

$$\frac{T_7}{T_6} = \frac{{}^nC_{n-6} x^{n-6} \times a^6}{{}^nC_{n-5} x^{n-5} \times a^5} = \frac{7}{112}$$

$$\Rightarrow \frac{{}^nC_{n-6} \times a}{{}^nC_{n-5} \times x} = \frac{1}{16}$$

$$\Rightarrow \frac{n-6+1}{n-(n-5)+1} \times \frac{a}{x} = \frac{1}{16}$$

$$\Rightarrow \frac{n-5}{6} \times \frac{a}{x} = \frac{1}{16}$$

$$\Rightarrow \frac{a}{x} = \frac{6}{16} \times \frac{1}{n-5}$$

$$\Rightarrow \frac{a}{x} = \frac{3}{8} \times \frac{1}{(n-5)} \quad \text{---(i)}$$

and,

$$\frac{T_8}{T_7} = \frac{{}^nC_{n-7} x^{n-7} \times a^7}{{}^nC_{n-6} x^{n-6} \times a^6} = \frac{1}{4}$$

$$\Rightarrow \frac{T_8}{T_7} = \frac{{}^nC_{n-7} \times a}{{}^nC_{n-6} \times x} = \frac{1}{28}$$

$$\Rightarrow \frac{{}^nC_{n-7} \times a}{{}^nC_{n-6} \times x} = \frac{1}{28}$$

$$\Rightarrow \frac{n-7+1}{n-(n-6)+1} \times \frac{a}{x} = \frac{1}{28}$$

$$\Rightarrow \frac{n-6}{7} \times \frac{a}{x} = \frac{1}{28}$$

$$\Rightarrow \frac{a}{x} = \frac{1}{4(n-6)} \quad \text{---(ii)}$$

$$\left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r} \right]$$

Comparing equation (i) and (ii), we get

$$\begin{aligned}\frac{3}{8} \times \frac{1}{(n-5)} &= \frac{1}{4(n-6)} \\ \Rightarrow \frac{3}{2} \times \frac{1}{(n-5)} &= \frac{1}{(n-6)} \\ \Rightarrow 3(n-6) &= 2(n-5) \\ \Rightarrow 3n-18 &= 2n-10 \\ \Rightarrow 3n-2n &= 18-10 \\ \Rightarrow n &= 8\end{aligned}$$

Putting $n = 8$ in equation (ii), we get

$$\begin{aligned}\frac{a}{x} &= \frac{1}{4(8-6)} \\ \Rightarrow \frac{a}{x} &= \frac{1}{8} \\ \Rightarrow x &= 8a\end{aligned}$$

Now,

$$\begin{aligned}76 &= 112 \\ \Rightarrow {}^nC_{n-5} \times x^{n-5} \times a^5 &= 112 \\ \Rightarrow {}^8C_3 \times x^3 \times a^5 &= 112 & [\because n = 8] \\ \Rightarrow {}^8C_3 \times (8a)^3 \times a^5 &= 112 & [\because x = 8a] \\ \Rightarrow \frac{8!}{(8-3)!3!} \times 8^3 \times a^8 &= 112 \\ \Rightarrow \frac{8!}{5!3!} \times 512 \times a^8 &= 112 \\ \Rightarrow \frac{8 \times 7 \times 6 \times 5!}{5!3!} \times 512 \times a^8 &= 112 \\ \Rightarrow a^8 &= \frac{112}{56 \times 512} \\ \Rightarrow a^8 &= \frac{2}{512} \\ \Rightarrow a^8 &= \frac{1}{256} \\ \Rightarrow a^8 &= \left(\frac{1}{2}\right)^8 \\ \Rightarrow a &= \frac{1}{2}\end{aligned}$$

Putting $a = \frac{1}{2}$ in $x = 8a$, we get

$$x = 8 \times \frac{1}{2} = 4$$

Hence, $x = 4$, $a = \frac{1}{2}$ and $n = 8$.

Binomial Theorem Ex 18.2 Q31

It is given that

$$T_2 = 240$$

$$T_3 = 720$$

$$T_4 = 1080$$

$$\therefore T_2 = {}^nC_1 \times x^{n-1} \times a = 240$$

$$T_3 = {}^nC_2 \times x^{n-2} \times a^2 = 720$$

$$\text{and, } T_4 = {}^nC_3 \times x^{n-3} \times a^3 = 1080$$

Now,

$$\frac{T_4}{T_3} = \frac{{}^nC_3 \times x^{n-3} \times a^3}{{}^nC_2 \times x^{n-2} \times a^2} = \frac{1080}{720}$$

$$\Rightarrow \frac{{}^nC_3 a}{{}^nC_2 x} = \frac{3}{2}$$

$$\Rightarrow \frac{n-3+1}{2+1} \times \frac{a}{x} = \frac{3}{2}$$

$$\Rightarrow \frac{n-2}{3} \times \frac{a}{x} = \frac{3}{2}$$

$$\Rightarrow \frac{a}{x} = \frac{9}{2(n-2)}$$

---(i)

and,

$$\frac{T_3}{T_2} = \frac{{}^nC_2 \times x^{n-2} \times a^2}{{}^nC_1 \times x^{n-1} \times a} = \frac{720}{240}$$

$$\Rightarrow \frac{{}^nC_2 \times a}{{}^nC_1 \times x} = 3$$

$$\Rightarrow \frac{n-2+1}{2} \times \frac{a}{x} = 3$$

$$\Rightarrow \frac{n-1}{2} \times \frac{a}{x} = 3$$

$$\Rightarrow \frac{a}{x} = \frac{6}{n-1}$$

---(ii)

Comparing equation (i) and equation (ii), we get

$$\frac{6}{n-1} = \frac{9}{2(n-2)}$$

$$\Rightarrow 12(n-2) = 9(n-1)$$

$$\Rightarrow 12n - 24 = 9n - 9$$

$$\Rightarrow 3n = 24 - 9$$

$$\Rightarrow 3n = 15$$

$$\Rightarrow n = 5$$

Putting $n = 5$ in equation (ii), we get

$$\frac{a}{x} = \frac{6}{5-1}$$

$$\Rightarrow \frac{a}{x} = \frac{6}{4}$$

$$\Rightarrow \frac{a}{x} = \frac{3}{2}$$

$$\Rightarrow a = \frac{3}{2}x$$

Now,

$$T_2 = {}^nC_1 \times x^{n-1} \times a = 240$$

$$\Rightarrow {}^5C_1 \times x^4 \times \left(\frac{3}{2}x\right) = 240$$

$$\left[\because n = 5 \text{ and } a = \frac{3}{2}x \right]$$

$$\Rightarrow x^5 = \frac{240 \times 2}{5 \times 3}$$

$$\Rightarrow x^5 = 32$$

$$\Rightarrow x^5 = 2^5$$

$$\Rightarrow x = 2$$

Putting $x = 2$ in $a = \frac{3}{2}x$, we get

$$a = \frac{3}{2} \times 2 = 3$$

Hence, $x = 2$, $a = 3$ and $n = 5$.

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