Algebraic Identities Ex 4.3 Q12

Answer :

In the given problem, we have to find the value of $x^3 - \frac{1}{x^3}$

Given $x^4 + \frac{1}{x^4} = 119$

We shall use the identity $(x + y)^2 = x^2 + y^2 + 2xy$

Here putting $x^4 + \frac{1}{x^4} = 119$,

$$\left(x^2 + \frac{1}{x^2}\right)^2 = x^4 + \frac{1}{x^4} + 2 \times x^2 \times \frac{1}{x^2}$$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = x^4 + \frac{1}{x^4} + 2 \times \cancel{x^2} \times \frac{1}{\cancel{x^2}}$$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = x^4 + \frac{1}{x^4} + 2$$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 119 + 2$$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 121$$

$$x^2 + \frac{1}{x^2} = \sqrt{11 \times 11}$$

$$x^2+\frac{1}{x^2}=\pm 11$$

In order to find $\left(x-\frac{1}{x}\right)$ we are using identity $(x-y)^2=x^2+y^2-2xy$

$$\left(x-\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}-2\times x\times\frac{1}{x}$$

$$\left(x-\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}-2$$

$$\left(x-\frac{1}{x}\right)^2=11-2$$

$$\left(x-\frac{1}{x}\right)^2=9$$

$$\left(x-\frac{1}{x}\right)=\sqrt{9}$$

$$\left(x-\frac{1}{x}\right)=\sqrt{3\times 3}$$

$$\left(x-\frac{1}{x}\right)=\pm 3$$

In order to find $x^3-\frac{1}{x^3}$ we are using identity $a^3-b^3=(a-b)\left(a^2+b^2+ab\right)$

$$x^3-\frac{1}{x^3}=\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+x\times\frac{1}{x}\right)$$ Here $x^2+\frac{1}{x^2}=11$ and $\left(x-\frac{1}{x}\right)=3$

$$\begin{aligned}x^3-\frac{1}{x^3}&=\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+\cancel{x}\times\frac{1}{\cancel{x}}\right)\\&=3(11+1)\\&=3\times 12\\&=36\end{aligned}$$

Hence the value of $x^3-\frac{1}{x^3}$ is $\boxed{36}$.

*********** END **********