Chapter 6 Determinants Ex 6.2 Q8

$$\begin{vmatrix} 0 & xy^2 & xz^2 \\ x^2y & 0 & yz^2 \\ x^2z & zy^2 & 0 \end{vmatrix}$$

$$= 0\left(0-y^3z^3\right)-xy^2\left(0-x^2yz^3\right)+xz^2\left(x^2y^3z-0\right)$$

$$= 0+x^3y^3z^3+x^3y^3z^3$$

$$= 2x^3y^3z^3$$

Chapter 6 Determinants Ex 6.2 Q9

$$\text{Let } \Delta = \begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$ and $R_3 \to R_3 - R_2$

$$\Delta = \begin{vmatrix} a & -a & 0 \\ x & a+y & z \\ 0 & -a & a \end{vmatrix}$$

Applying $C_2 \to C_2 + C_1$

$$\Delta = \begin{vmatrix} a & 0 & 0 \\ x & a+x+y & z \\ 0 & -a & a \end{vmatrix}$$

$$\Delta = a\left[a\left(a+x+y\right)+az\right]+0+0$$

$$\Delta = a^2\left(a+x+y+z\right)$$

. Chapter 6 Determinants Ex 6.2 Q10

$$\Delta+\Delta_1 = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ yz & zx & xy \\ x & y & z \end{vmatrix}$$

$$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + \begin{vmatrix} 1 & yz & x \\ 1 & zx & y \\ 1 & xy & z \end{vmatrix} \ldots\ldots\ldots\left[\because |A| = \left|A^T\right|\right]$$

$$= \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} - \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$$

$$\ldots\ldots\begin{bmatrix}\text{If any two rows (columns) of the determinant are interchanged} \\ \text{then value of the determinant changes in sign.}\end{bmatrix}$$

$$= \begin{vmatrix} 0 & 0 & x^2-yz \\ 0 & 0 & y^2-zx \\ 0 & 0 & z^2-xy \end{vmatrix}$$

$$= 0\ldots\ldots\ldots\left[\because C_1 \text{ and } C_2 \text{ are identical}\right]$$

Chapter 6 Determinants Ex 6.2 Q11

$$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3+b^3+c^3-3abc$$

$$\text{LHS} = \begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$$

Apply: $C_1 \to C_1 + C_2 + C_3$.

$$= \begin{vmatrix} a+b+c & b & c \\ 0 & b-c & c-a \\ 2(a+b+c) & c+a & a+b \end{vmatrix}$$

Take $(a+b+c)$ common from C_1

$$= (a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 2 & c+a & a+b \end{vmatrix}$$

Apply: $R_3 \to R_3 - 2R_1$

$$= (a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 0 & c+a-2b & a+b-2c \end{vmatrix}$$

$$= (a+b+c)\left[(b-c)(a+b-2c)-(c-a)(c+a-2b)\right]$$

$$= a^3+b^3+c^3-3abc$$

$$= RHS$$

*********** END **********