

Complex Numbers Ex 13.2 Q23

Let
$$z = x + iy$$

$$|z| = z + 1 + 2i$$

$$\Rightarrow |x+iy| = x+iy+1+2i$$

$$\Rightarrow \sqrt{x^2 + y^2} = (x+1) + i(y+2)$$

$$\Rightarrow x^2 + y^2 = (x+1)^2 + 2i(x+1)(y+2) - (y+2)^2$$
 [Squaring both sides]

$$\Rightarrow x^2 + y^2 = x^2 + 2x + 1 + 2i(xy + 2x + y + 2) - (y^2 + 4y + 4)$$

$$\Rightarrow 2y^2 - 2x + 4y + 4 = 2i(xy + 2x + y + 2)$$

$$\Rightarrow y^2 - x + 2y + 2 = i(xy + 2x + y + 2)$$

$$\Rightarrow (y^2 - x + 2y + 2) - i(xy + 2x + y + 2) = 0$$

Comparing we get,

$$(xy+2x+y+2)=0$$

$$\Rightarrow (x+1)(y+2)=0$$

$$\Rightarrow x=-1 \& y=-2$$

Also,
$$(y^2-x+2y+2)=0$$

Taking
$$x=-1$$
, $(y^2-(-1)+2y+2)=0$

$$\Rightarrow (y^2+2y+3)=0$$

Doesnot have a solution since roots will be imaginary

Taking
$$y=-2$$
, $(4-x-4+2)=0$

$$\Rightarrow x=2$$

$$z = x + iy = 2 - 2i$$

Complex Numbers Ex 13.2 Q24

$$(1+i)^{2n} = (1-i)^{2n}$$

$$\Rightarrow \left(\frac{1+i}{1-i}\right)^{2n} = 1$$

$$\Rightarrow \left(\frac{(1+i)(1+i)}{(1-i)(1+i)}\right)^{2n} = 1 \quad [Rationalizing the denominator]$$

$$\Rightarrow \left(\frac{1+2i-1}{1+1}\right)^{2n} = 1$$

$$\Rightarrow \left(\frac{2i}{2}\right)^{2n} = 1$$

$$\Rightarrow i^{2n} = 1$$

$$\Rightarrow i^{2n} = 1$$

$$\therefore n = 2$$

Complex Numbers Ex 13.2 Q25

$$\begin{aligned} & \left| Z_{1} + Z_{2} + Z_{3} \right| = \left| \frac{Z_{1}\overline{Z_{1}}}{\overline{Z_{1}}} + \frac{Z_{2}\overline{Z_{2}}}{\overline{Z_{2}}} + \frac{Z_{3}\overline{Z_{3}}}{\overline{Z_{3}}} \right| \\ & = \left| \frac{\left| Z_{1} \right|^{2}}{\overline{Z_{1}}} + \frac{\left| Z_{2} \right|^{2}}{\overline{Z_{2}}} + \frac{\left| Z_{3} \right|^{2}}{\overline{Z_{3}}} \right| \\ & = \frac{\left| \frac{1}{\overline{Z_{1}}} + \frac{1}{\overline{Z_{2}}} + \frac{1}{\overline{Z_{3}}} \right|}{\left| \frac{1}{\overline{Z_{1}}} + \frac{1}{\overline{Z_{2}}} + \frac{1}{\overline{Z_{3}}} \right|} \\ & = \left| \frac{1}{\overline{Z_{1}}} + \frac{1}{\overline{Z_{2}}} + \frac{1}{\overline{Z_{3}}} \right| \\ & = \left| \frac{1}{\overline{Z_{1}}} + \frac{1}{\overline{Z_{2}}} + \frac{1}{\overline{Z_{3}}} \right| \\ & = 1 \end{aligned}$$

Complex Numbers Ex 13.2 Q26

Let
$$z = x + iy$$

 $z^2 = (x + iy)^2 = x^2 - y^2 + 2xyi$
 $|z|^2 = \overline{zz} = (x + iy)(x - iy) = x^2 + y^2$

$$z^{2} + |z|^{2} = 0$$

$$x^{2} - y^{2} + 2xyi + x^{2} + y^{2} = 0$$

$$2x^{2} + 2xyi = 0$$

$$\Rightarrow 2x^{2} = 0 \text{ and } 2xy = 0$$

$$\Rightarrow x = 0 \text{ and } y \in \mathbb{R}$$

$$\therefore z = 0 + iy \text{ where } y \in \mathbb{R}$$

********* END *******