



Mathematical Induction Ex 12.2 Q9

$$\text{Let } P(n) : \frac{1}{3 \cdot 7} + \frac{1}{7 \cdot 11} + \frac{1}{11 \cdot 15} + \dots + \frac{1}{(4n-1)(4n+3)} = \frac{n}{3(4n+3)}$$

For $n = 1$

$$\frac{1}{3 \cdot 7} = \frac{1}{3(7)}$$

$$\frac{1}{21} = \frac{1}{21}$$

$\Rightarrow P(n)$ is true for $n = 1$

Let $P(n)$ is true for $n = k$, so

$$\frac{1}{3 \cdot 7} + \frac{1}{7 \cdot 11} + \frac{1}{11 \cdot 15} + \dots + \frac{1}{(4n-1)(4n+3)} = \frac{k}{3(4k+3)} \quad \dots (1)$$

We have to show that,

$$\frac{1}{3 \cdot 7} + \frac{1}{7 \cdot 11} + \frac{1}{11 \cdot 15} + \dots + \frac{1}{(4k-1)(4k+3)} + \frac{1}{(4k+3)(4k+7)} = \frac{(k+1)}{3(4k+7)}$$

Now,

$$\left\{ \frac{1}{3 \cdot 7} + \frac{1}{7 \cdot 11} + \frac{1}{11 \cdot 15} + \dots + \frac{1}{(4k-1)(4k+3)} \right\} + \frac{1}{(4k+3)(4k+7)}$$

Now,

$$\left\{ \frac{1}{3 \cdot 7} + \frac{1}{7 \cdot 11} + \frac{1}{11 \cdot 15} + \dots + \frac{1}{(4k-1)(4k+3)} \right\} + \frac{1}{(4k+3)(4k+7)}$$

$$= \frac{k}{3(4k+3)} + \frac{1}{(4k+3)(4k+7)}$$

$$= \frac{1}{(4k+3)} \left[\frac{k}{3} + \frac{1}{4k+7} \right]$$

$$= \frac{1}{(4k+3)} \left[\frac{k(4k+7)+3}{3(4k+7)} \right]$$

$$= \frac{1}{(4k+3)} \left[\frac{4k^2+7k+3}{3(4k+7)} \right]$$

$$= \frac{1}{(4k+3)} \left[\frac{4k^2+4k+3k+3}{3(4k+7)} \right]$$

$$= \frac{1}{(4k+3)} \left[\frac{4k(k+1)+3(k+1)}{3(4k+7)} \right]$$

$$= \frac{1}{(4k+3)} \left[\frac{(4k+3)(k+1)}{3(4k+7)} \right]$$

$$= \frac{(k+1)}{3(4k+7)}$$

$\Rightarrow P(n)$ is true for $n = k+1$

$\Rightarrow P(n)$ is true for all $n \in N$ by *PMI*

Mathematical Induction Ex 12.2 Q10

$$\text{Let } P(n) : 1.2 + 2.2^2 + 3.2^3 + \dots + n.2^n = (n-1)2^{n+1} + 2$$

For $n = 1$

$$1.2 = 0.2^0 + 2$$

$$2 = 2$$

$\Rightarrow P(n)$ is true for $n = 1$

Let $P(n)$ is true for $n = k$, so

$$1.2 + 2.2^2 + 3.2^3 + \dots + k.2^k = (k-1)2^{k+1} + 2 \quad \text{--- (1)}$$

We have to show that,

$$\{1.2 + 2.2^2 + 3.2^3 + \dots + k.2^k\} + (k+1)2^{k+1} = k2^{k+2} + 2$$

Now,

$$\{1.2 + 2.2^2 + 3.2^3 + \dots + k.2^k\} + (k+1)2^{k+1}$$

$$= [(k-1)2^{k+1} + 2] + (k+1)2^{k+1} \quad \text{[Using equation (1)]}$$

$$= (k-1)2^{k+1} + 2 + (k+1)2^{k+1}$$

$$= 2^{k+1}(k-1+k+1) + 2$$

$$= 2^{k+1}.2k + 2$$

$$= k2^{k+2} + 2$$

$\Rightarrow P(n)$ is true for $n = k+1$

$\Rightarrow P(n)$ is true for all $n \in N$ by *PMI*

Mathematical Induction Ex 12.2 Q11

$$\text{Let } P(n) : 2 + 5 + 8 + 11 + \dots + (3n - 1) = \frac{1}{2}n(3n + 1)$$

For $n = 1$

$$P(1) \quad 2 = \frac{1}{2} \cdot 1 \cdot (4) \\ 2 = 2$$

$\Rightarrow P(n)$ is true for $n = 1$

Let $P(n)$ is true for $n = k$, so

$$2 + 5 + 8 + 11 + \dots + (3k - 1) = \frac{1}{2}k(3k + 1) \quad \text{--- (1)}$$

We have to show that,

$$2 + 5 + 8 + 11 + \dots + (3k - 1) + (3k + 2) = \frac{1}{2}(k + 1)(3k + 4)$$

Now,

$$\{2 + 5 + 8 + 11 + \dots + (3k - 1)\} + (3k + 2)$$

$$= \frac{1}{2}k(3k + 1) + (3k + 2)$$

$$= \frac{3k^2 + k + 2(3k + 2)}{2}$$

$$= \frac{3k^2 + k + 6k + 4}{2}$$

$$= \frac{3k^2 + 7k + 4}{2}$$

$$= \frac{3k^2 + 3k + 4k + k}{2}$$

$$= \frac{3k(k + 1) + 4(k + 1)}{2}$$

$$= \frac{(k + 1)(3k + 4)}{2}$$

$\Rightarrow P(n)$ is true for $n = k + 1$

$\Rightarrow P(n)$ is true for all $n \in N$ by *PMI*

***** END *****