



Differentiation Ex 11.8 Q18

Let $u = \sin^{-1}(\sqrt{1-x^2})$

Put $x = \cos \theta \Rightarrow \theta = \cos^{-1} x$, so

$$u = \sin^{-1}(\sin \theta) \quad \text{---(i)}$$

And,

$$\begin{aligned} \text{Let } v &= \cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) \\ &= \cot^{-1}\left(\frac{\cos \theta}{\sqrt{1-\cos^2 \theta}}\right) \\ &= \cot^{-1}\left(\frac{\cos \theta}{\sin \theta}\right) \end{aligned}$$

$$v = \cot^{-1}(\cot \theta) \quad \text{---(ii)}$$

Here, $0 < x < 1$

$$\Rightarrow 0 < \cos \theta < 1$$

$$\Rightarrow 0 < \theta < \frac{\pi}{2}$$

So, from equation (i),

$$u = \theta$$

$$\left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \right]$$

$$u = \cos^{-1} x$$

Differentiation Ex 11.8 Q19

$$\text{Let } u = \sin^{-1} \left\{ 2ax\sqrt{1-a^2x^2} \right\}$$

$$\text{Put } ax = \sin \theta \Rightarrow \theta = \sin^{-1}(ax)$$

$$u = \sin^{-1} \left\{ 2 \sin \theta \sqrt{1 - \sin^2 \theta} \right\}$$

$$= \sin^{-1} \{ 2 \sin \theta \cos \theta \}$$

$$u = \sin^{-1}(\sin 2\theta) \quad \text{---(i)}$$

And,

$$\text{Let } v = \sqrt{1-a^2x^2}$$

Differentiating it with respect to x using chain rule,

$$\frac{dv}{dx} = \frac{1}{2\sqrt{1-a^2x^2}} \times \frac{d}{dx}(1-a^2x^2)$$

$$= \left(\frac{0 - 2a^2x}{2\sqrt{1-a^2x^2}} \right)$$

$$\frac{dv}{dx} = \frac{-a^2x}{\sqrt{1-a^2x^2}} \quad \text{---(ii)}$$

$$\text{Here, } -\frac{1}{\sqrt{2}} < ax < \frac{1}{\sqrt{2}}$$

$$\Rightarrow -\frac{1}{\sqrt{2}} < \sin \theta < \frac{1}{\sqrt{2}}$$

$$\Rightarrow -\frac{\pi}{4} < \theta < \frac{\pi}{4}$$

So, from equation (i),

$$u = 2\theta$$

$$\left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \right]$$

$$u = 2 \sin^{-1} ax$$

Differentiation Ex 11.8 Q20

Let $u = \tan^{-1}\left(\frac{1-x}{1+x}\right)$

Put $x = \tan \theta \Rightarrow \theta = \tan^{-1} x$, so

$$\begin{aligned} u &= \tan^{-1}\left(\frac{1-\tan \theta}{1+\tan \theta}\right) \\ &= \tan^{-1}\left(\frac{\frac{\tan \pi}{4} - \tan \theta}{1 + \frac{\tan \pi}{4} \tan \theta}\right) \\ u &= \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \theta\right)\right) \end{aligned} \quad \text{---(i)}$$

Here, $-1 < x < 1$

$\Rightarrow -1 < \tan \theta < 1$

$\Rightarrow -\frac{\pi}{4} < \theta < \frac{\pi}{4}$

So, from equation (i),

$$\begin{aligned} u &= \left(\frac{\pi}{4} - \theta\right) && \left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right] \\ u &= \frac{\pi}{4} - \tan^{-1} x \end{aligned}$$

Differentiating it with respect to x ,

$$\begin{aligned} \frac{du}{dx} &= 0 - \left(\frac{1}{1+x^2}\right) \\ \frac{du}{dx} &= -\frac{1}{1+x^2} \end{aligned} \quad \text{---(ii)}$$

And,

Let $v = \sqrt{1-x^2}$

Differentiating it with respect to x using chain rule,

$$\begin{aligned} \frac{dv}{dx} &= \frac{1}{2\sqrt{1-x^2}} \times \frac{d}{dx}(1-x^2) \\ &= \frac{1}{2\sqrt{1-x^2}} (-2x) \\ \frac{dv}{dx} &= \frac{-x}{\sqrt{1-x^2}} \end{aligned} \quad \text{---(iii)}$$

Dividing equation (ii) by (iii),

$$\begin{aligned} \frac{\frac{du}{dx}}{\frac{dv}{dx}} &= -\frac{1}{(1+x^2)} \times \frac{\sqrt{1-x^2}}{-x} \\ \frac{du}{dv} &= \frac{\sqrt{1-x^2}}{x(1+x^2)} \end{aligned}$$

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