Differentials Errors and Approximation Ex14.1 Q9(xviii)

Let $x = 25,\ x + \Delta x = 26$

$$\Delta x = 26 - 25$$
$$= 1$$

Let $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\left(\frac{dy}{dx}\right)_{x=25} = \frac{1}{2\sqrt{25}}$$
$$= \frac{1}{10}$$
$$= 0.1$$

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=25} \times (\Delta x)$$
$$= (0.1)(1)$$
$$= 0.1$$

$$\sqrt{26} = y + \Delta y$$
$$= \sqrt{x} + 0.01$$
$$= \sqrt{25} + 0.1$$

$$\sqrt{26} = 5.1$$

Let $x = 0.49,\ x + \Delta x = 0.487$

$$\Delta x = 0.48 - 0.49$$
$$= -0.01$$

Let $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\left(\frac{dy}{dx}\right)_{x=0.49} = \frac{1}{2\sqrt{0.49}}$$
$$= \frac{1}{1.4}$$
$$= 0.71428$$

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=0.49} \times (\Delta x)$$
$$= (0.71428)(-0.01)$$
$$\Delta y = -0.0071428$$

$$\sqrt{37} = y + \Delta y$$
$$= \sqrt{0.49} - 0.0071428$$
$$= 0.7 - 0071428$$

$$\sqrt{0.48} = 0.6928572$$

Let $x = 81,\ x + \Delta x = 82$

$$\Delta x = 82 - 81$$
$$= 1$$

Let $y = x^{\frac{1}{4}}$

$$\frac{dy}{dx} = \frac{1}{4x^{\frac{3}{4}}}$$

$$\left(\frac{dy}{dx}\right)_{x=81} = \frac{1}{4(81)^{\frac{3}{4}}}$$
$$= \frac{1}{108}$$
$$= 0.009259$$

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=81} \times (\Delta x)$$
$$= (0.008259)(1)$$
$$= 0.009259$$

$$(82)^{\frac{1}{4}} = y + \Delta y$$
$$= x^{\frac{1}{4}} + 0.009259$$
$$= (81)^{\frac{1}{4}} + 0.009259$$

$$(82)^{\frac{1}{4}} = 3.009259$$

********** END **********