Differentials Errors and Approximation Ex14.1 Q5

Let $x$ be the radius of sphere,

$$\Delta x = 0.1\% \text{ of } x$$
$$\Delta x = 0.001x$$

Now,

Let $y$ = volume of sphere

$$y = \frac{4}{3}\pi x^3$$

$$\frac{dy}{dx} = 4\pi x^2$$

$$\Delta y = \left(\frac{dy}{dx}\right) \times \Delta x$$

$$= 4\pi x^2 \times 0.001x$$

$$= \frac{4}{3}\pi x^3 (0.003)$$

$$= \frac{0.3}{100} \times y$$

$$\Delta y = 0.3\% \text{ of } y$$

So, percentage error in volume of error = 0.3%.

Differentials Errors and Approximation Ex14.1 Q6

Given, $\Delta v = -\dfrac{1}{2}\%$

$\qquad\qquad = -0.5\%$

$\Delta v = -0.005$

Here,

$$pv^{1.4} = k$$

Taking log on both the sides,

$$\log\left(pv^{1.4}\right) = \log k$$

$$\log p + 1.4\log v = \log k$$

Differentiate it with respect to $v$,

$$\frac{1}{p}\frac{dp}{dv} + \frac{1.4}{v} = 0$$

$$\frac{dp}{dv} = -\frac{1.4}{v}p$$

$$\Delta p = \left(\frac{dp}{dv}\right)\Delta v$$

$$\qquad = -\frac{1.4p}{v} \times (-0.005)$$

$$\Delta p = \frac{1.4p\,(0.005)}{v}$$

$$\Delta p \text{ in } \% = \frac{\Delta p}{p} \times 100$$

$$\qquad = \frac{1.4p\,(0.005)}{p} \times 100$$

$$\qquad = 0.7\%$$

So, percentage error in $p$ = 0.7% .

Differentials Errors and Approximation Ex14.1 Q7

Let $h$ be the height of the cone, and $\alpha$ be the semivertide angle.

Here vertgide angle $\alpha$ is fixed.

$$\Delta h = k\% \text{ of } h$$
$$= \frac{k}{100} \times h$$
$$\Delta h = (0.0k)h$$

(i)
$$A = \pi r\,(r+l)$$
$$= \pi\left(r^2 + rl\right)$$
$$= \pi\left(r^2\right) + r\sqrt{h^2 + r^2} \qquad \left[\text{Since, in a cone } l^2 = h^2 + r^2\right]$$

$$r = h\tan\alpha \qquad \left[\text{from figure}\right]$$
$$A = \pi\left[h^2\tan^2\alpha + h\tan\alpha\sqrt{h^2 + h^2\tan^2\alpha}\,\right]$$
$$= \pi\left[h^2\tan^2\alpha + h\tan\alpha\sqrt{h^2\left(1 + \tan^2\alpha\right)}\,\right]$$
$$= \pi\left[h^2\tan^2\alpha + h\tan\alpha \times h\sec\alpha\right]$$
$$= \pi h^2\left[\frac{\sin^2\alpha}{\cos^2\alpha} + \frac{\sin\alpha}{\cos^2\alpha}\right]$$
$$A = \pi h^2\frac{\sin\alpha\,(\sin\alpha + 1)}{\cos^2\alpha}$$

Differentiating with respect to $h$ as $\alpha$ is fixed.

$$\frac{dA}{dh} = 2\pi h\,\frac{\sin\alpha\,(\sin\alpha + 1)}{\cos^2\alpha}$$

So,
$$\Delta A = \frac{dA}{dh} \times \Delta h$$
$$\Delta a = \frac{2\pi h\,(0.0kh)\sin\alpha\,(\sin\alpha + 1)}{\cos^2\alpha}$$
$$\Delta A \text{ in } \% \text{ of } A = \frac{2\pi h\,(0.0kh)\sin\alpha\,(\sin\alpha + 1)}{\cos^2\alpha} \times \frac{100}{A}$$
$$= \frac{2\pi k h^2 \times \sin\alpha\,(\sin\alpha + 1)}{\cos^2\alpha} \times \frac{\cos^2\alpha}{\pi h^2\sin\alpha\,(\sin\alpha + 1)}$$
$$= 2k\,\%$$

So, percentage increase in area $= 2k\%$.

(ii)

Let $v$ = volume of cone

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi (h\tan\alpha)^2 h$$

$$v = \frac{\pi}{3}\tan^2\alpha h^2$$

Differentiating it with respect to $h$ treating $\alpha$ as constant,

$$\frac{dv}{dh} = \pi\tan^2\alpha \times h^2$$

$$\Delta v = \left(\frac{dv}{dh}\right)\Delta h$$

$$= \pi\tan^2\alpha h^2 \times (0.0kh)$$

$$\Delta v = 0.0k\pi h^3 \tan^2\alpha$$

Percentage increase in $v = \dfrac{\Delta v \times 100}{v}$

$$= \frac{0.0k\pi h^3 \tan^2\alpha \times 100}{\dfrac{\pi}{3}\tan^2\alpha \times h^3}$$

$$= 3k\%$$

So, percentage increase in volume = $3k\%$.

********** END *********