Geometric Progressions Ex 20.2 Q 1

Let the three number in G.P be $\frac{a}{r}, a, ar$

Sum of these numbers $= \frac{a}{r} + a + ar = 65$

3375 = Product of these numbers

$3375 = \left(\frac{a}{r}\right)(a)(ar) = a^3$

$a^3 = (5)^3 \times (3)^3 = (15)^3$

$\Rightarrow \quad a = 15$

$a\left(\frac{1}{r} + 1 + r\right) = 65$

$15\left(\frac{1}{r} + 1 + r\right) = \frac{65}{15} = \frac{13}{3}$

$\frac{1 + r + r^2}{r} = \frac{13}{3}$

$3 + 3r + 3r^2 = 13r$

$3r^2 - 10r + 3 = 0$

$3r^2 - r - 9r + 3 = 0$

$r(3r - 1) - 3(3r - 1) = 0$

$r = 3, \ \frac{1}{3} \qquad r = \frac{1}{3}$ or $r = 3$

$\therefore$ G.P. is a, ar, ar^2

$\therefore$ G.P. is 45, 15, 5 or 5, 15, 45

Geometric Progressions Ex 20.2 Q 2

Let the three numbers be a, ar, ar^2 in G.P., where a is first teror and r is the common ratio.

Then,

$$a + ar + ar^2 = 38$$

$$a\left(1 + r + r^2\right) = 38 \qquad \text{---(i)}$$

and

$$(a)(ar)(ar)^2 = 1728$$

$$a^3r^3 = 1728 = 4^3 3^3 = (12)^3$$

$$a^3 = \frac{12^3}{r^3} \Rightarrow \frac{12}{r} = a$$

Putting $a = \frac{12}{r}$ in (i)

$$\frac{12}{r}\left(1 + r + r^2\right) = 38$$

$$12 + 12r + 12r^2 = 38r$$

$$12r^2 - 26r + 12 = 0$$

$$6r^2 - 13r + 6 = 0$$

$$6r^2 - 9r - 4r + 6 = 0$$

$$3r(3r - 3) - 2(3r - 3) = 0$$

$$r = \frac{3}{2},\ \frac{2}{3}$$

$$a = \frac{12}{\frac{3}{2}} = 8 \text{ or } \frac{12}{\frac{2}{3}} = 18$$

$\therefore$ G.P. is $8, 12, 18$.

Geometric Progressions Ex 20.2 Q 3

Let the first three terms of G.P. are $\frac{a}{r},\ a,\ ar$

Here,

$$\frac{a}{r} + a + ar = \frac{13}{12} \qquad \text{---(i)}$$

and $\frac{a}{r} \times a \times ar = -1$

$\Rightarrow$ $a^3 = -1$

$\Rightarrow$ $a = -1$

Put $a = -1$ in equation (i),

$$\frac{-1}{r} + (-1) - r = \frac{13}{12}$$

$\Rightarrow$ $-1 - r - r^2 = \frac{13}{12}r$

$\Rightarrow$ $-12 - 12r - 12r^2 = 13r$

$\Rightarrow$ $12r^2 + 12r + 13r + 12 = 0$

$\Rightarrow$ $12r^2 + 25r + 12 = 0$

$\Rightarrow$ $12r^2 + 16r + 9r + 12 = 0$

$\Rightarrow$ $4r(3r + 4) + 3(3r + 4) = 0$

$\Rightarrow$ $(4r + 3)(3r + 4) = 0$

$$r = \frac{-3}{4},\ \frac{-4}{3}$$

So,

Required G.P. is, $\frac{4}{3},\ -1,\ \frac{3}{4},\ \ldots$

or $\frac{3}{4},\ -1,\ \frac{4}{3},\ \ldots$

Geometric Progressions Ex 20.2 Q4

Let the three numbers in G.P. be $\frac{a}{r}, a, ar$ then product of these numbers $\left(\frac{a}{r}\right)(a)(ar)$

$\Rightarrow \quad a^3 = 125 = 5^3$

$a = 5$

Also, sum of these products in pair

$\left(\frac{a}{r}\right)(a) + (a)(ar) + \left(\frac{a}{r}\right)(ar) = 87\frac{1}{2} = \frac{195}{2}$

$\frac{a^2}{r} + a^2 r + a^2 = a^2\left(\frac{1}{r} + r + 1\right)$

$= (5)^2\left(\frac{1 + r^2 + r}{r}\right) = \frac{195}{2}$

$1 + r^2 + r = \left(\frac{195}{2 \times 25}\right)r$

$2\left(1 + r^2 + r\right) = \frac{39}{5}r$

$10 + 10r^2 + 10r = 39r$

$10r^2 - 29r + 10 = 0$

$10r^2 - 25r - 4r + 10 = 0$

$5r(2r - 5) - 2(2r - 5) = 0$

$r = \frac{5}{2}, \quad \frac{2}{5}$

$\therefore$ G.P. is $\frac{a}{r}, a, ar$

$10, 5, \frac{5}{2}, \ldots$ or $\frac{5}{2}, 5, 10, \ldots$

********** END **********