Determinants Ex 6.1 Q1(i)

Let M_{ij} and C_{ij} represents the minor and co-factor respectively of an element which is placed at the i^{th} row and j^{th} column.

Now,

$M_{11} = -1$ $\left[\text{In a } 2 \times 2 \text{ matrix, the minor is obtained for a particular element, by deleting that row and column where the element is present.}\right]$

$M_{21} = 20$

$$C_{11} = (-1)^{1+1} \times M_{11} \qquad \left[\because C_{ij} = (-1)^{i+j} \times M_{ij}\right]$$
$$= (+1)(-1)$$
$$= -1$$

$$C_{21} = (-1)^{2+1} M_{21}$$
$$= (-1)^3 \times 20$$
$$= -20$$

Also,

$$|A| = 5(-1) - (0) \times (20) \qquad \left[\text{If } A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{ then } |A| = a_{11}a_{22} - a_{21}a_{12}\right]$$
$$= -5$$

Determinants Ex 6.1 Q1(ii)

Let M_{ij} and C_{ij} represents the minor and co-factor respectively of an element which is present at the i^{th} row and j^{th} column.

Now,

$M_{11} = 3$ $\left[\text{In a } 2 \times 2 \text{ matrix, the minor of an element is obtained by deleting that row and that column in which it is present.}\right]$

$M_{21} = 4$

$$C_{11} = (-1)^{1+1} \times M_{11} \qquad \left[C_{ij} = (-1)^{i+j} \times M_{ij}\right]$$

$$C_{21} = (-1)^{2+1} \times M_{21}$$
$$= (-1)^3 \times 4$$
$$= -4$$

Also,

$$|A| = (-1) \times (3) - (2) \times (4)$$
$$= -3 - 8$$
$$= -11$$

Determinants Ex 6.1 Q1(iii)

Let M_{ij} and C_{ij} represents the minor and co-factor respectively of an element which is placed at the i^{th} row and j^{th} column.

Now,

$$M_{11} = \begin{bmatrix} -1 & 2 \\ 5 & 2 \end{bmatrix} \qquad \left[\begin{matrix}\text{In a } 3\times 3 \text{ matrix, } M_{ij} \text{ equals to the determinant of the } 2\times 2 \\ \text{sub-matrix obtained by leaving the } i^{th} \text{ row and } j^{th} \text{ column of } A.\end{matrix}\right]$$

$$= (-1)\times(2) - (5)\times(2)$$
$$= -2 - 10$$
$$= -12$$

$$M_{21} = \begin{bmatrix} -3 & 2 \\ 5 & 2 \end{bmatrix} = (-3)\times(2) - (5)\times(2) = -6 - 10 = -16$$

$$M_{31} = \begin{bmatrix} -3 & 2 \\ -1 & 2 \end{bmatrix} = (-3)(2) - (-1)(2) = -6 + 2 = -4$$

$$C_{11} = (-1)^{1+1} M_{11} \qquad \left(C_{ij} = (-1)^{i+j} \times M_{ij}\right)$$
$$= (+)(-12) = -12$$
$$C_{21} = (-1)^{2+1} M_{21} = (-1)^3 (-16) = 16$$
$$C_{31} = (-1)^{3+1} M_{31} = (-1)^4 (-4) = -4$$

Also, expanding the determinant along the first column.

$$|A| = a_{11} \times \left((-1)^{1+1} \times M_{11}\right) + a_{21} \times \left((-1)^{2+1} \times M_{21}\right) + a_{31} \times \left((-1)^{3+1} \times M_{31}\right)$$
$$= a_{11} \times C_{11} + a_{21} \times C_{21} + a_{31} \times C_{31}$$
$$= 1 \times (-12) + 4 \times 16 + 3 \times (-4)$$
$$= -12 + 48 - 12 = 24$$

Determinants Ex 6.1 Q1(iv)

Let M_{ij} and C_{ij} are respectively the minor and co-factor of the element a_{ij}.

Now,

$$M_{11} = \begin{bmatrix} b & ca \\ c & ab \end{bmatrix}$$
$$= ab^2 - ac^2$$

$$M_{21} = \begin{bmatrix} a & bc \\ c & ab \end{bmatrix}$$
$$= a^2b - c^2b$$

$$M_{31} = \begin{bmatrix} a & bc \\ b & ca \end{bmatrix}$$
$$= a^2c - b^2c$$

$$C_{11} = (-1)^{1+1} \times M_{11} = +\left(ab^2 - ac^2\right)$$
$$C_{21} = (-1)^{2+1} \times M_{21} = -\left(a^2b - c^2b\right)$$
$$C_{31} = (-1)^{3+1} \times M_{31} = +\left(a^2c - b^2c\right)$$

Also, expanding the determinant, along the first column.

$$|A| = a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31}$$
$$= 1\left(ab^2 - ac^2\right) + 1\left(c^2b - a^2b\right) + 1 \times \left(a^2c - b^2c\right)$$
$$= ab^2 - ac^2 + c^2b - a^2b + a^2c - b^2c$$

Determinants Ex 6.1 Q1(v)

Let M_{ij} and C_{ij} are respectively the minor and co-factor of the element a_{ij}.

Now,

$$M_{11} = \begin{bmatrix} 5 & 0 \\ 7 & 1 \end{bmatrix} = 5 - 0 = 5$$

$$M_{21} = \begin{bmatrix} 2 & 6 \\ 7 & 1 \end{bmatrix} = 2 - 42 = -40$$

$$M_{31} = \begin{bmatrix} 2 & 6 \\ 5 & 0 \end{bmatrix} = 0 - 30 = -30$$

$$C_{11} = (-1)^{1+1} \times M_{11} = +5$$

$$C_{21} = (-1)^{2+1} \times M_{21} = (-)(-40) = 40$$

$$C_{31} = (-1)^{3+1} \times M_{31} = +(-30) = -30$$

Now, expanding the determinant along the first column.

$$\begin{aligned} |A| &= a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31} \\ &= 0 \times 5 + 1 \times (40) + 3 \times (-30) \\ &= 40 - 90 \\ &= -50 \end{aligned}$$

Determinants Ex 6.1 Q1(vi)

Let M_{ij} and C_{ij} are respectively the minor and co-factor of the element a_{ij}.

Now,

$$M_{11} = \begin{bmatrix} b & f \\ f & c \end{bmatrix} = bc - f^2$$

$$M_{21} = \begin{bmatrix} h & g \\ f & c \end{bmatrix} = hc - gf$$

$$M_{31} = \begin{bmatrix} h & g \\ b & f \end{bmatrix} = hf - bg$$

$$Also\, C_{11} = (-1)^{1+1} M_{11} = bc - f^2$$

$$C_{21} = (-1)^{2+1} M_{21} = -(hc - gf)$$

$$C_{31} = (-1)^{3+1} M_{31} = hf - bg$$

Also, expanding along the first column.

$$\begin{aligned} |A| &= a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31} \\ &= a\left(bc - f^2\right) + h(-)(hc - gf) + g(hf - bg) \\ &= abc - af^2 + hgf - h^2c + ghf - bg^2 \end{aligned}$$

Determinants Ex 6.1 Q1(vii)

We have,

$$\begin{bmatrix} 2 & -1 & 0 & 1 \\ -3 & 0 & 1 & -2 \\ 1 & 1 & -1 & 1 \\ 2 & -1 & 5 & 0 \end{bmatrix}$$

Here, $M_{11} = \begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & 1 \\ -1 & 5 & 0 \end{bmatrix} = -1(0+10) - 1(1-2) = -9$

$$M_{21} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 1 \\ -1 & 5 & 0 \end{bmatrix} = 9$$

$$M_{31} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & -2 \\ -1 & 5 & 0 \end{bmatrix} = -9$$

$$M_{41} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & - & 1 \end{bmatrix} = 0$$

$\therefore$ $C_{11} = (-1)^{1+1} M_{11} = -9$

$C_{21} = (-1)^{3} M_{21} = -9$

$C_{31} = (-4)^{4} M_{31} = -9$

$C_{41} = (-1)^{5} M_{41} = 0$

Hence,

$$\begin{bmatrix} 2 & -1 & 0 & 1 \\ -3 & 0 & 1 & -2 \\ 1 & 1 & -1 & 1 \\ 2 & -1 & 5 & 0 \end{bmatrix} = 2 \times C_{11} + (-3) C_{21} + 1 \times C_{31} + 2 \times C_{41} = -9[2 - 3 + 1] = 0$$

*********** END **********