

Rationalisation Ex 3.2 Q3

Answer:

(i) We know that rationalization factor for $3+\sqrt{2}$ is $3-\sqrt{2}$. We will multiply numerator and denominator of the given expression $\frac{1}{3+\sqrt{2}}$ by $3-\sqrt{2}$, to get

$$\frac{1}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}} = \frac{3-\sqrt{2}}{3^2 - (\sqrt{2})^2}$$
$$= \frac{3-\sqrt{2}}{9-2}$$
$$= \frac{3-\sqrt{2}}{7}$$

Hence the given expression is simplified with rational denominator to $\frac{3-\sqrt{2}}{7}$

(ii) We know that rationalization factor for $\sqrt{6}-\sqrt{5}$ is $\sqrt{6}+\sqrt{5}$. We will multiply numerator and denominator of the given expression $\frac{1}{\sqrt{6}-\sqrt{5}}$ by $\sqrt{6}+\sqrt{5}$, to get

$$\frac{1}{\sqrt{6} - \sqrt{5}} \times \frac{\sqrt{6} + \sqrt{5}}{\sqrt{6} + \sqrt{5}} = \frac{\sqrt{6} + \sqrt{5}}{\left(\sqrt{6}\right)^2 - \left(\sqrt{5}\right)^2}$$
$$= \frac{\sqrt{6} + \sqrt{5}}{6 - 5}$$
$$= \frac{\sqrt{6} + \sqrt{5}}{1}$$
$$= \sqrt{6} + \sqrt{5}$$

Hence the given expression is simplified with rational denominator to $\sqrt{6+\sqrt{5}}$

(iii) We know that rationalization factor for $\sqrt{41}-5$ is $\sqrt{41}+5$. We will multiply numerator and denominator of the given expression $\frac{16}{\sqrt{41}-5}$ by $\sqrt{41}+5$, to get

$$\frac{16}{\sqrt{41} - 5} \times \frac{\sqrt{41} + 5}{\sqrt{41} + 5} = \frac{16(\sqrt{41} + 5)}{(\sqrt{41})^2 - (5)^2}$$
$$= \frac{16(\sqrt{41} + 5)}{41 - 25}$$
$$= \frac{16(\sqrt{41} + 5)}{16}$$
$$= \sqrt{41} + 5$$

Hence the given expression is simplified with rational denominator to $\sqrt{41+5}$

(iv) We know that rationalization factor for $5\sqrt{3}-3\sqrt{5}$ is $5\sqrt{3}+3\sqrt{5}$. We will multiply numerator and denominator of the given expression $\frac{30}{5\sqrt{3}-3\sqrt{5}}$ by $5\sqrt{3}+3\sqrt{5}$, to get

$$\frac{30}{5\sqrt{3} - 3\sqrt{5}} \times \frac{5\sqrt{3} + 3\sqrt{5}}{5\sqrt{3} + 3\sqrt{5}} = \frac{30 \times 5 \times \sqrt{3} + 30 \times 3 \times \sqrt{5}}{\left(5\sqrt{3}\right)^2 - \left(3\sqrt{5}\right)^2}$$

$$= \frac{30 \times 5 \times \sqrt{3} + 30 \times 3 \times \sqrt{5}}{25 \times 3 - 9 \times 5}$$

$$= \frac{30 \times 5 \times \sqrt{3} + 30 \times 3 \times \sqrt{5}}{75 - 45}$$

$$= \frac{30 \times 5 \times \sqrt{3} + 30 \times 3 \times \sqrt{5}}{30}$$

$$= 5\sqrt{3} + 3\sqrt{5}$$

Hence the given expression is simplified with rational denominator to $\sqrt{5\sqrt{3}+3\sqrt{5}}$

(v) We know that rationalization factor for $2\sqrt{5}-\sqrt{3}$ is $2\sqrt{5}+\sqrt{3}$. We will multiply numerator and denominator of the given expression $\frac{1}{2\sqrt{5}-\sqrt{3}}$ by $2\sqrt{5}+\sqrt{3}$, to get

$$\begin{split} \frac{1}{2\sqrt{5} - \sqrt{3}} \times \frac{2\sqrt{5} + \sqrt{3}}{2\sqrt{5} + \sqrt{3}} &= \frac{2\sqrt{5} + \sqrt{3}}{\left(2\sqrt{5}\right)^2 - \left(\sqrt{3}\right)^2} \\ &= \frac{2\sqrt{5} + \sqrt{3}}{4 \times 5 - 3} \\ &= \frac{2\sqrt{5} + \sqrt{3}}{20 - 3} \\ &= \frac{2\sqrt{5} + \sqrt{3}}{17} \\ &= 5\sqrt{3} + 3\sqrt{5} \end{split}$$

Hence the given expression is simplified with rational denominator to $\frac{2\sqrt{5} + \sqrt{3}}{17}$

(vi) We know that rationalization factor for $2\sqrt{2}-\sqrt{3}$ is $2\sqrt{2}+\sqrt{3}$. We will multiply numerator and denominator of the given expression $\frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}}$ by $2\sqrt{2}+\sqrt{3}$, to get

$$\frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}} \times \frac{2\sqrt{2}+\sqrt{3}}{2\sqrt{2}+\sqrt{3}} = \frac{2\times\sqrt{3}\times\sqrt{2}+\sqrt{3}\times\sqrt{3}+2\sqrt{2}+\sqrt{3}}{\left(2\sqrt{2}\right)^2 - \left(\sqrt{3}\right)^2}$$

$$= \frac{2\sqrt{3\times2}+3+2\sqrt{2}+\sqrt{3}}{4\times2-3}$$

$$= \frac{2\sqrt{6}+3+2\sqrt{2}+\sqrt{3}}{8-3}$$

$$= \frac{2\sqrt{6}+3+2\sqrt{2}+\sqrt{3}}{5}$$

$$= \frac{2\sqrt{6}+3+2\sqrt{2}+\sqrt{3}}{5}$$

Hence the given expression is simplified with rational denominator to $\boxed{\frac{2\sqrt{6}+3+2\sqrt{2}+\sqrt{3}}{5}}$

(vii) We know that rationalization factor for $6+4\sqrt{2}$ is $6-4\sqrt{2}$. We will multiply numerator and denominator of the given expression $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$ by $6-4\sqrt{2}$, to get

$$\begin{split} \frac{6-4\sqrt{2}}{6+4\sqrt{2}} \times \frac{6-4\sqrt{2}}{6-4\sqrt{2}} &= \frac{6^2 + \left(4\sqrt{2}\right)^2 - 2 \times 6 \times 4\sqrt{2}}{\left(6\right)^2 - \left(4\sqrt{2}\right)^2} \\ &= \frac{36+16 \times 2 - 48\sqrt{2}}{36-16 \times 2} \\ &= \frac{36+32-48\sqrt{2}}{36-32} \\ &= \frac{68-48\sqrt{2}}{4} \\ &= 17-12\sqrt{2} \end{split}$$

Hence the given expression is simplified with rational denominator to $17-12\sqrt{2}$

(viii) We know that rationalization factor for $2\sqrt{5}-3$ is $2\sqrt{5}+3$. We will multiply numerator and denominator of the given expression $\frac{3\sqrt{2}+1}{2\sqrt{5}-3}$ by $2\sqrt{5}+3$, to get

$$\frac{3\sqrt{2}+1}{2\sqrt{5}-3} \times \frac{2\sqrt{5}+3}{2\sqrt{5}+3} = \frac{3\sqrt{2} \times 2\sqrt{5} + 3 \times 3\sqrt{2} + 2\sqrt{5} + 3}{\left(2\sqrt{5}\right)^2 - \left(3\right)^2}$$

$$= \frac{3 \times 2 \times \sqrt{2} \times \sqrt{5} + 3 \times 3\sqrt{2} + 2\sqrt{5} + 3}{4 \times 5 - 9}$$

$$= \frac{6\sqrt{2 \times 5} + 9\sqrt{2} + 2\sqrt{5} + 3}{4 \times 5 - 9}$$

$$= \frac{6\sqrt{10} + 9\sqrt{2} + 2\sqrt{5} + 3}{11}$$

Hence the given expression is simplified with rational denominator to $6\sqrt{10+9\sqrt{2}+2\sqrt{5}+3}$

(ix) We know that rationalization factor for $\sqrt{a^2+b^2}+a^{-is}\sqrt{a^2+b^2}-a$. We will multiply numerator and denominator of the given expression $\frac{b^2}{\sqrt{a^2+b^2}+a}$ by $\sqrt{a^2+b^2}-a$, to get

$$\frac{b^2}{\sqrt{a^2 + b^2} + a} \times \frac{\sqrt{a^2 + b^2} - a}{\sqrt{a^2 + b^2} - a} = \frac{b^2 \left(\sqrt{a^2 + b^2} - a\right)}{\left(\sqrt{a^2 + b^2}\right)^2 - a^2}$$

$$= \frac{b^2 \left(\sqrt{a^2 + b^2} - a\right)}{a^2 + b^2 - a^2}$$

$$= \frac{b^2 \left(\sqrt{a^2 + b^2} - a\right)}{b^2}$$

$$= \sqrt{a^2 + b^2} - a$$

Hence the given expression is simplified with rational denominator to $\sqrt{a^2+b^2}-a$

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