Trigonometric Ratios of Compound Angles Ex 7.1 Q29

LHS $\dfrac{1}{\sin(x-a)\sin(x-b)}$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\sin(x-a)\sin(x-b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin\{(x-b)-(x-a)\}}{\sin(x-a)\sin(x-b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin(x-b)\cos(x-a)}{\sin(x-a)\sin(x-b)}-\frac{\cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\cot(x-a)-\cot(x-b)\right]$$

$$= \frac{\cot(x-a)-\cot(x-b)}{\sin(a-b)}$$

= RHS

$\therefore$ LHS= RHS

Hence proved

LHS $\dfrac{1}{\sin(x-a)\cos(x-b)}$

$$= \frac{1}{\cos(a-b)}\left[\frac{\cos(a-b)}{\sin(x-a)\cos(x-b)}\right]$$

$$= \frac{1}{\cos(a-b)}\left[\frac{\cos\{(x-b)-(x-a)\}}{\sin(x-a)\cos(x-b)}\right]$$

$$= \frac{1}{\cos(a-b)}\left[\frac{\cos(x-b)\cos(x-a)+\sin(x-b)\sin(x-a)}{\sin(x-a)\cos(x-b)}\right]$$

$$= \frac{1}{\cos(a-b)}\left[\frac{\cos(x-b)\cos(x-a)}{\sin(x-a)\cos(x-b)}+\frac{\sin(x-b)\sin(x-a)}{\sin(x-a)\cos(x-b)}\right]$$

$$= \frac{1}{\cos(a-b)}\left[\frac{\cos(x-a)}{\sin(x-a)}+\frac{\sin(x-b)}{\cos(x-b)}\right]$$

$$= \frac{1}{\cos(a-b)}\left[\cot(x-a)+\tan(x-b)\right]$$

$$= \frac{\cot(x-a)+\tan(x-b)}{\cos(a-b)}$$

= RHS

$\therefore$ LHS= RHS

Hence proved

$$\text{LHS} \quad \frac{1}{\cos(x-a)\cos(x-b)}$$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin\{(x-b)-(x-a)\}}{\cos(x-a)\cos(x-b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin(x-b)\cos(x-a)}{\cos(x-a)\cos(x-b)} - \frac{\cos(x-b)\ \sin(x-a)}{\cos(x-a)\cos(x-b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin(x-b)}{\cos(x-b)} - \frac{\sin(x-a)}{\cos(x-a)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\tan(x-b) - \tan(x-s)\right]$$

$$= \frac{\tan(x-b)-\tan(x-a)}{\sin(a-b)}$$

$= \text{RHS}$

$\therefore \quad \text{LHS} = \text{RHS}$

Hence proved

Trigonometric Ratios of Compound Angles Ex 7.1 Q30

We have,

$$\sin\alpha\sin\beta - \cos\alpha\cos\beta + 1 = 0$$

$$\Rightarrow \quad -(\cos\alpha\cos\beta - \sin\alpha\sin\beta) = -1$$

$$\Rightarrow \quad \cos(\alpha+\beta) = 1 \qquad ---(i)$$

$$\therefore \quad \sin(\alpha+\beta) = \sqrt{1-\cos^2(\alpha+\beta)}$$

$$= \sqrt{1-1^2}$$

$$= 0$$

$$\Rightarrow \quad \sin(\alpha+\beta) = 0 \qquad ---(ii)$$

Now,

$$1+\cot\alpha\tan\beta = 1 + \frac{\cos\alpha}{\sin\alpha}\times\frac{\sin\beta}{\cos\beta}$$

$$= \frac{\sin\alpha\times\cos\beta + \cos\alpha\times\sin\beta}{\sin\alpha\times\cos\beta}$$

$$= \frac{\sin(\alpha+\beta)}{\sin\alpha\times\cos\beta}$$

$$= \frac{0}{\sin\alpha\times\cos\beta} \qquad [\text{Using equation (ii)}]$$

$$= 0$$

$$\therefore \quad 1+\cot\alpha\tan\beta = 0$$

Hence proved

*********** END **********