Trigonometric Ratios Ex 5.1 Q25

Answer :

Given:

$$\sec A = \frac{17}{8} \quad \ldots\ldots (1)$$

To verify:

$$\frac{3-4\sin^2 A}{4\cos^2 A-3} = \frac{3-\tan^2 A}{1-3\tan^2 A} \quad \ldots\ldots (2)$$

Now we know that $\sec A = \frac{1}{\cos A}$

Therefore $\cos A = \frac{1}{\sec A}$

Now, by substituting the value of $\sec A$ from equation (1)

We get,

$$\cos A = \frac{1}{\frac{17}{8}}$$

$$= \frac{8}{17}$$

Therefore,

$$\cos A = \frac{8}{17} \quad \ldots\ldots (3)$$

Now, we know the following trigonometric identity

$$\cos^2 A + \sin^2 A = 1$$

Therefore,

$$\sin^2 A = 1 - \cos^2 A$$

Now by substituting the value of $\cos A$ from equation (3)

We get,

$$\sin^2 A = 1 - \left(\frac{8}{17}\right)^2$$

$$= 1 - \frac{(8)^2}{(17)^2}$$

$$= 1 - \frac{64}{289}$$

Now by taking L.C.M

We get,

$$\sin^2 A = \frac{289 - 64}{289}$$

$$= \frac{225}{289}$$

Now, by taking square root on both sides

We get,

$$\sin A = \sqrt{\frac{225}{289}}$$

$$= \frac{\sqrt{225}}{\sqrt{289}}$$

$$= \frac{15}{17}$$

Therefore,

$$\sin A = \frac{15}{17} \quad \ldots\ldots (4)$$

Now, we know that $\tan A = \frac{\sin A}{\cos A}$

Now by substituting the value of $\cos A$ and $\sin A$ from equation (3) and (4) respectively

We get,

$$\tan A = \frac{\frac{15}{17}}{\frac{8}{17}}$$

$$= \frac{15}{17} \times \frac{17}{8}$$

$$= \frac{15}{8}$$

Therefore

$$\tan A = \frac{15}{8} \quad \ldots\ldots (5)$$

Now from the expression of equation (2)

$$\text{L.H.S} = \frac{3-4\sin^2 A}{4\cos^2 A-3}$$

Now by substituting the value of $\cos A$ and $\sin A$ from equation (3) and (4)

We get,

$$\text{L.H.S} = \frac{3-4\left(\frac{15}{17}\right)^2}{4\left(\frac{8}{17}\right)^2-3}$$

Therefore,

$$\text{L.H.S} = \frac{3-4\left(\frac{225}{289}\right)}{4\left(\frac{64}{289}\right)-3}$$

$$= \frac{3-\frac{900}{289}}{\frac{256}{289}-3}$$

Now by taking L.C.M of both numerator and denominator

We get,

$$\text{L.H.S} = \frac{\frac{3\times 289}{1\times 289}-\frac{900}{289}}{\frac{256}{289}-\frac{3\times 289}{1\times 289}}$$

$$= \frac{\frac{867}{289}-\frac{900}{289}}{\frac{256}{289}-\frac{867}{289}}$$

$$= \frac{\frac{867-900}{289}}{\frac{256-867}{289}}$$

$$= \frac{\frac{-33}{289}}{\frac{-611}{289}}$$

$$= \frac{33}{611}$$

Therefore,

$$\frac{3-4\sin^2 A}{4\cos^2 A-3} = \frac{33}{611} \quad \ldots\ldots (6)$$

Now from the expression of equation (2)

$$R.\text{H.S} = \frac{3-\tan^2 A}{1-3\tan^2 A}$$

Now by substituting the value of $\tan A$ from equation (5)

We get,

$$3-\left(\frac{15}{8}\right)^2$$

$$R.H.S. = \frac{\left(\frac{15}{8}\right)}{1-3\left(\frac{15}{8}\right)^2}$$

$$= \frac{3-\frac{225}{64}}{1-\frac{3\times 225}{64}}$$

Now by taking L.C.M

We get,

$$R.H.S = \frac{\frac{3\times 64}{1\times 64} - \frac{225}{64}}{\frac{64-675}{64}}$$

$$= \frac{\frac{192}{64} - \frac{225}{64}}{\frac{-611}{64}}$$

$$= \frac{\frac{192-225}{64}}{\frac{-611}{64}}$$

Therefore

$$R.H.S = \frac{\frac{-33}{64}}{\frac{-611}{64}}$$

$$= \frac{-33}{64} \times \frac{64}{-611}$$

$$= \frac{33}{611}$$

Therefore,

$$\frac{3-\tan^2 A}{1-3\tan^2 A} = \frac{33}{611} \quad \ldots\ldots (7)$$

Now by comparing equation (6) and (7)

We get,

$$\frac{3-4\sin^2 A}{4\cos^2 A-3} = \frac{3-\tan^2 A}{1-3\tan^2 A}$$

********** END **********