Inverse Trigonometric Functions Ex 4.1 Q3.

$\text{Let } \sin^{-1}\left(\frac{1}{2}\right) = x. \text{ Then, } \sin x = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$

$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\text{Let } \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = y. \text{ Then, } \sin y = \frac{1}{\sqrt{2}} = \sin\left(\frac{\pi}{4}\right)$

$\therefore \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

$\therefore \sin^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{6} - \frac{2\pi}{4} = \frac{\pi}{6} - \frac{\pi}{2} = \frac{\pi - 3\pi}{6} = -\frac{\pi}{3}$

$\text{Let } \sin^{-1}\left(-\frac{1}{2}\right) = x. \text{ Then, } \sin x = -\frac{1}{2} = \sin\left(-\frac{\pi}{6}\right)$

$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

$\text{Let } \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = y. \text{ Then, } \cos y = \frac{-\sqrt{3}}{2} = \cos\left(\pi - \frac{\pi}{6}\right)$

$\therefore \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

$\therefore \sin^{-1}\left(-\frac{1}{2}\right) + 2\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = -\frac{\pi}{6} + \frac{10\pi}{6} = \frac{-\pi + 10\pi}{6} = \frac{9\pi}{6} = \frac{3\pi}{2}$

$\text{Let } \tan^{-1}(-1) = x. \text{ Then, } \tan x = -1 = -\tan\left(\frac{\pi}{4}\right) = \tan\left(\pi - \frac{\pi}{4}\right)$

$\therefore \tan^{-1}(-1) = \frac{3\pi}{4}$

$\text{Let } \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = y. \text{ Then, } \cos y = \frac{-1}{\sqrt{2}} = -\cos\left(\frac{\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{4}\right)$

$\therefore \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \frac{3\pi}{4}$

$\therefore \tan^{-1}(-1) + \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \frac{3\pi}{4} + \frac{3\pi}{4} = \frac{6\pi}{4} = \frac{3\pi}{2}$

Let $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)=x$. Then, $\sin x=-\frac{\sqrt{3}}{2}=-\sin\left(\frac{\pi}{3}\right)=\sin\left(\pi-\frac{\pi}{3}\right)$

$\therefore \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)=\frac{2\pi}{3}$

Let $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=y$. Then, $\cos y=\frac{\sqrt{3}}{2}=\cos\left(\frac{\pi}{6}\right)$

$\therefore \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{6}$

$\therefore \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)+\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=\frac{2\pi}{3}+\frac{\pi}{6}=\frac{4\pi+\pi}{6}=\frac{5\pi}{6}$

Let $\tan^{-1}\left(\sqrt{3}\right)=x$. Then, $\tan x=\sqrt{3}=\tan\left(\frac{\pi}{3}\right)$

$\therefore \tan^{-1}\left(\sqrt{3}\right)=\frac{\pi}{3}$

Let $\sec^{-1}(-2)=y$. Then, $\sec y=-2=\sec\left(\pi-\frac{\pi}{3}\right)$

$\therefore \sec^{-1}(-2)=\frac{2\pi}{3}$

$\therefore \tan^{-1}\left(\sqrt{3}\right)-\sec^{-1}(-2)=\frac{\pi}{3}-\frac{2\pi}{3}=\frac{\pi-2\pi}{3}=-\frac{\pi}{3}$

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