Indefinite Integrals Ex 19.28 Q1

$$\int \sqrt{3+2x-x^2}\ dx = \int \sqrt{4-(x-1)^2}\ dx$$

Let $x-1=t$, so that $dx = dt$

Thus, $\int \sqrt{3+2x-x^2}\ dx = \int \sqrt{4-t^2}\ dt$

$$= \frac{1}{2}t\sqrt{4-t^2} + \frac{4}{2}\sin^{-1}\left(\frac{t}{2}\right) + C$$

$$= \frac{1}{2}(x-1)\sqrt{3+2x-x^2} + 2\sin^{-1}\left(\frac{x-1}{2}\right) + C$$

Indefinite Integrals Ex 19.28 Q2

Let $I = \int \sqrt{x^2+x+1}dx$

$$= \int \sqrt{x^2+x+\frac{1}{4}+\frac{3}{4}}dx$$

$$= \int \sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dx$$

$$= \frac{\left(x+\frac{1}{2}\right)}{2}\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} + \frac{\left(\frac{\sqrt{3}}{2}\right)^2}{2}.\log\left|\left(x+\frac{1}{2}\right)+\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right| + c$$

$$= \left(\frac{2x+1}{4}\right)\sqrt{x^2+x+1} + \frac{3}{8}\log\left|\left(\frac{2x+1}{2}\right)+\frac{1}{2}\sqrt{x^2+x+1}\right| + c$$

$$\therefore \quad I = \left(\frac{2x+1}{4}\right)\sqrt{x^2+x+1} + \frac{3}{8}\log\left|2x+1+\sqrt{x^2+x+1}\right| + c$$

Indefinite Integrals Ex 19.28 Q3

Let $I = \int \sqrt{x-x^2}dx$

$$= \int \sqrt{\frac{1}{4}-\frac{1}{4}+x-x^2}dx \qquad \left[\text{Add and subtract } \frac{1}{4}\right]$$

$$= \int \sqrt{\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}-x\right)^2}dx$$

$$= -\left(\frac{1-2x}{4}\right)\sqrt{\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}-x\right)^2} - \frac{\left(\frac{1}{2}\right)^2}{2}\sin^{-1}\left(\frac{\frac{1-2x}{2}}{\frac{1}{2}}\right) + c$$

$$\therefore \quad I = \left(\frac{2x-1}{4}\right)\sqrt{x-x^2} + \frac{1}{8}\sin^{-1}(2x-1) + c$$

Indefinite Integrals Ex 19.28 Q4

Let $I = \int \sqrt{1 + x - 2x^2}dx$

$$= \sqrt{2}\int \sqrt{\frac{1}{2} + \frac{x}{2} - x^2}dx$$

$$= \sqrt{2}\int \sqrt{\frac{9}{16} - \left(\frac{1}{16} - \frac{x}{2} + x^2\right)}dx$$

$$= \sqrt{2}\int \sqrt{\left(\frac{3}{4}\right)^2 - \left(x - \frac{1}{4}\right)^2}dx$$

$$= \sqrt{2}\left\{\frac{\left(x - \frac{1}{4}\right)}{2}\sqrt{\frac{1}{2} + \frac{x}{2} - x^2} + \frac{9}{32}\sin^{-1}\left(\frac{x - \frac{1}{4}}{\frac{3}{4}}\right)\right\} + c$$

$$I = \frac{1}{8}(4x - 1)\sqrt{1 + x - 2x^2} + \frac{9\sqrt{2}}{32}\sin^{-1}\left(\frac{4x - 1}{3}\right) + c$$

Indefinite Integrals Ex 19.28 Q5

Let $I = \int \cos x\sqrt{4 - \sin^2 x}dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x dx = dt$

$\Rightarrow \quad I = \int \sqrt{4 - t^2}dt$

$= \int \sqrt{2^2 - t^2}dt$

$= \frac{t}{2}\sqrt{2^2 - t^2} + \frac{4}{2}\sin^{-1}\frac{t}{2} + c$

$\therefore \quad I = \frac{1}{2}\sin x\sqrt{4 - \sin^2 x} + 2\sin^{-1}\left(\frac{\sin x}{2}\right) + c$

********** END **********