



Question 13:

$$A = 30^\circ \Rightarrow 2A = 60^\circ$$

$$(i) \sin 2A = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\begin{aligned} \text{Also } \frac{2 \tan A}{1 + \tan^2 A} &= \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} \\ &= \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2} \end{aligned}$$

$$\text{Hence, } \sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$$

$$(ii) \cos 2A = \cos 60^\circ = \frac{1}{2}$$

$$\begin{aligned} \text{Also, } \frac{1 - \tan^2 A}{1 + \tan^2 A} &= \frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ} = \frac{1 - \left(\frac{1}{\sqrt{3}}\right)^2}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} \\ &= \frac{\left(1 - \frac{1}{3}\right)}{\left(1 + \frac{1}{3}\right)} = \frac{\left(\frac{2}{3}\right)}{\left(\frac{4}{3}\right)} = \left(\frac{2}{3} \times \frac{3}{4}\right) = \frac{1}{2} \end{aligned}$$

$$\text{Hence, } \cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$(iii) \tan 2A = \tan 60^\circ = \sqrt{3}$$

$$\begin{aligned} \text{Also, } \frac{2 \tan A}{1 - \tan^2 A} &= \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \frac{1}{3}} \\ &= \frac{\left(\frac{2}{\sqrt{3}}\right)}{\left(\frac{2}{3}\right)} = \left(\frac{2}{\sqrt{3}} \times \frac{3}{2}\right) = \sqrt{3} \end{aligned}$$

$$\text{Hence, } \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

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