Algebraic Identities Ex 4.1 Q12

Answer :

(i) In the given problem, we have to find product of $\left(\frac{1}{2}a-3b\right)\left(3b+\frac{1}{2}a\right)\left(\frac{1}{4}a^2+9b^2\right)$

We have been given $\left(\frac{1}{2}a-3b\right)\left(3b+\frac{1}{2}a\right)\left(\frac{1}{4}a^2+9b^2\right)$

On rearranging we get, $\left(\frac{1}{2}a-3b\right)\left(\frac{1}{2}a+3b\right)\left(\frac{1}{4}a^2+9b^2\right)$

We shall use the identity $(x-y)(x+y)=x^2-y^2$

By substituting $x=\frac{1}{2}a$, $y=3b$ we get,

$$\left(\frac{1}{2}a-3b\right)\left(3b+\frac{1}{2}a\right)\left(\frac{1}{4}a^2+9b^2\right)=\left(\frac{1}{2}a\right)^2-(3b)^2\left(\frac{1}{4}a^2+9b^2\right)$$

$$=\left(\frac{1}{4}a^2-9b^2\right)\left(\frac{1}{4}a^2+9b^2\right)$$

We shall use the identity $(x-y)(x+y)=x^2-y^2$

$$\left(\frac{1}{2}a-3b\right)\left(3b+\frac{1}{2}a\right)\left(\frac{1}{4}a^2+9b^2\right)=\left(\frac{1}{4}a^2\right)^2-\left(9b^2\right)^2$$

$$=\frac{1}{16}a^4-81b^4$$

Hence the value of $\left(\frac{1}{2}a-3b\right)\left(3b+\frac{1}{2}a\right)\left(\frac{1}{4}a^2+9b^2\right)$ is $\boxed{\frac{1}{16}a^4-81b^4}$

(ii) In the given problem, we have to find product of $\left(m+\frac{n}{7}\right)^3\left(m-\frac{n}{7}\right)$

We have been given $\left(m+\frac{n}{7}\right)^3\left(m-\frac{n}{7}\right)$

On rearranging we get

$$\left(m+\frac{n}{7}\right)^3\left(m-\frac{n}{7}\right)=\left(m+\frac{n}{7}\right)^2\left(m+\frac{n}{7}\right)\left(m-\frac{n}{7}\right)$$

We shall use the identity $(x-y)(x+y)=x^2-y^2$

By substituting $x=m$, $y=\frac{n}{7}$, we get ,

$$\left(m+\frac{n}{7}\right)^3\left(m-\frac{n}{7}\right)=\left(m+\frac{n}{7}\right)^2\left(m^2-\left(\frac{n}{7}\right)^2\right)$$

$$=\left(m+\frac{n}{7}\right)^2\left(m^2-\frac{n^2}{49}\right)$$

Hence the value of $\left(m+\frac{n}{7}\right)^3\left(m-\frac{n}{7}\right)$ is $\boxed{\left(m+\frac{n}{7}\right)^2\left(m^2-\frac{n^2}{49}\right)}$

(iii) In the given problem, we have to find product of $\left(\frac{x}{2}-\frac{2}{5}\right)\left(\frac{2}{5}-\frac{x}{2}\right)-x^2+2x$

On rearranging we get

$$\left(\frac{x}{2}-\frac{2}{5}\right)\left(\frac{2}{5}-\frac{x}{2}\right)-x^2+2x=\left(\frac{x}{2}-\frac{2}{5}\right)\left[-\left(\frac{x}{2}-\frac{2}{5}\right)\right]-x^2+2x$$

$$\Rightarrow\left(\frac{x}{2}-\frac{2}{5}\right)\left(\frac{2}{5}-\frac{x}{2}\right)-x^2+2x=-\left(\frac{x}{2}-\frac{2}{5}\right)^2-x^2+2x$$

We shall use the identity $(x-y)^2=x^2-2xy+y^2$

By substituting $x=\frac{x}{2},\ y=\frac{2}{5}$

$$\left(\frac{x}{2}-\frac{2}{5}\right)\left(\frac{2}{5}-\frac{x}{2}\right)-x^2+2x=\left(\frac{x^2}{4}+\frac{4}{25}-\frac{2\times x}{\cancel{2}}\times\frac{\cancel{2}}{5}\right)-x^2+2x$$

$$=-\left(\frac{x^2}{4}+\frac{4}{25}-\frac{2x}{5}\right)-x^2+2x$$

$$=-\frac{x^2}{4}-\frac{4}{25}+\frac{2x}{5}-x^2+2x$$

$$=\left[-\frac{x^2}{4}-x^2\right]-\frac{4}{25}+\left[\frac{2x}{5}+2x\right]$$

$$=\left[-\frac{x^2}{4}-\frac{x^2}{1}\frac{\times 4}{\times 4}\right]-\frac{4}{25}+\left[\frac{2x}{5}+\frac{2x}{1}\frac{\times 5}{\times 5}\right]$$

$$=\left[-\frac{x^2}{4}-\frac{4x^2}{4}\right]-\frac{4}{25}+\left[\frac{2x}{5}+\frac{10x}{5}\right]$$

$$=\left[\frac{-x^2-4x^2}{4}\right]-\frac{4}{25}+\left[\frac{2x+10x}{5}\right]$$

$$=\frac{-5x^2}{4}-\frac{4}{25}+\frac{12x}{5}$$

Hence the value of $\left(\frac{x}{2}-\frac{2}{5}\right)\left(\frac{x}{2}+\frac{2}{5}\right)-x^2+2x$ is $\boxed{\frac{-5x^2}{4}-\frac{4}{25}+\frac{12x}{5}}$

(iv) In the given problem, we have to find product of $(x^2+x-2)(x^2-x+2)$

On rearranging we get

$$(x^2+x-2)(x^2-x+2)=\left[x^2+(x-2)\right]\left[x^2-(x-2)\right]$$

We shall use the identity $(x-y)(x+y)=x^2-y^2$

$$(x^2+x-2)(x^2-x+2)=\left[(x^2)^2-(x-2)^2\right]$$

$$=x^4-(x^2-2\times 2\times x+2^2)$$

$$=x^4-(x^2-4x+4)$$

$$=x^4-x^2+4x-4$$

Hence the value of $(x^2+x-2)(x^2-x+2)$ is $\boxed{x^4-x^2+4x-4}$

(v) In the given problem, we have to find product of $(x^3-3x^2-x)(x^2-3x+1)$

Taking x as common factor $=x(x^2-3x-1)(x^2-3x+1)$

$$(x^3-3x^2-x)(x^2-3x+1)=\left[x(x^2-3x-1)(x^2-3x+1)\right]$$

$$=x\left[\{(x^2-3x)-1\}\{(x^2-3x)+1\}\right]$$

We shall use the identity $(x-y)(x+y)=x^2-y^2$

$$(x^3-3x^2-x)(x^2-3x+1)=x\left[(x^2-3x)^2-1^2\right]$$

$$=x(x^4-6x^3+9x^2-1)$$

$$=x^5-6x^4+9x^3-x$$

Hence the value of $(x^3-3x^2-x)(x^3-3x+1)$ is $\boxed{x^5-6x^4+9x^3-x}$

(vi) In the given problem, we have to find product of $\left(2x^4-4x^2+1\right)\left(2x^4-4x^2-1\right)$

On rearranging we get $\left(\left[2x^4-4x^2\right]+1\right)\left(\left[2x^4-4x^2\right]-1\right)$

We shall use the identity $(x-y)(x+y)=x^2-y^2$

$$
\begin{aligned}
\left(2x^4-4x^2+1\right)\left(2x^4-4x^2-1\right) &= \left[2x^4-4x^2\right]^2-1^2 \\
&= \left[4x^8+16x^4-2\times 2x^4\times 4x^2-1\right] \\
&= 4x^8+16x^4-16x^6-1
\end{aligned}
$$

Hence the value of $\left(2x^4-4x^2+1\right)\left(2x^4-4x^2-1\right)$ is $\boxed{4x^8+16x^4-16x^6-1}$.

Answer :

In the given problem, we have to prove $a^2+b^2+c^2-ab-bc-ca$ is always non negative for all a,b,c that is we have to prove that $a^2+b^2+c^2-ab-bc-ca\geq 0$

Consider,

$a^2+b^2+c^2-ab-bc-ca$

$$
\begin{aligned}
a^2+b^2+c^2-ab-bc-ca &= \frac{1}{2}\left(2a^2+2b^2+2c^2-2ab-2bc-2ca\right) \\
&= \frac{1}{2}\left[\left(a^2-2ab+b^2\right)+\left(b^2-2bc+c^2\right)+\left(c^2-2ac+a^2\right)\right] \\
&= \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]
\end{aligned}
$$

$\Rightarrow a^2+b^2+c^2-ab-bc-ca\geq 0$

Hence $\boxed{(a-b)^2+(b-c)^2+(c-a)^2}$ is always non negative for all $a,b,c.$

Note: Square of all negative numbers is always positive or non negative.

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