Mathematical Induction Ex 12.2 Q21

Let $P(n): 5^{2n+2} - 24n - 25$ is divisible by 576

For $n = 1$

$5^4 - 24 - 25$

$= 625 - 49$

$= 576$

Which is divisible by 576

Let $P(n)$ is true for $n = k$, so

$5^{2k+2} - 24k - 25$ is divisible by 576

$$5^{2k+2} - 24k - 25 = 576\lambda \qquad ---(1)$$

We have to show that,

$5^{2k+4} - 24(k+1) - 25$ is divisible by 576

$5^{(2k+2)+2} - 24(k+1) - 25 = 576\mu$

Now,

$5^{(2k+2)+2} - 24(k+1) - 25$

$= 5^{(2k+2)}.5^2 - 24k - 24 - 25$

$= (576\lambda + 24k + 25)25 - 24k - 49$ [Using equation (1)]

$= 25.576\lambda + 600k + 625 - 24k - 49$

$= 25.576\lambda + 576k + 576$

$= 576(25\lambda + k + 1)$

$= 576\mu$

$\Rightarrow$ $P(n)$ is true for $n = k + 1$

$\Rightarrow$ $P(n)$ is true for all $n \in N$ by PMI

Mathematical Induction Ex 12.2 Q22

Let $P(n): 3^{2n+2} - 8n - 9$ is divisible by 8

For $n = 1$
$3^{2+2} - 8 - 9$
$= 81 - 17$

$= 64$

It is divisible by 8
$\Rightarrow \quad P(n)$ is true for $n = 1$
Let $P(n)$ is true for $n = k$, so

$\left(3^{2k+2} - 8k - 9\right)$ is divisible by 8
$\Rightarrow \quad 3^{2k+2} - 8k - 9 = 8\lambda \qquad ---(1)$

We have to show that,

$3^{2(k+1)+2} - 8(k+1) - 9$ is divisible by 8
$3^{2(k+1)}.3^2 - 8(k+1) - 9 = 8\mu$

Now,
$3^{2(k+1)}.9 - 8k - 8 - 9$

$= (8\lambda + 8k + 9)9 - 8k - 8 - 9$
$= 72\lambda + 72k + 81 - 8k - 17$
$= 72\lambda + 64k + 64$
$= 8(9\lambda + 8k + 8)$

$= 8\mu$

$\Rightarrow \quad P(n)$ is true for $n = 1$
$\Rightarrow \quad P(n)$ is true for all $n \in N$ by PMI

Mathematical Induction Ex 12.2 Q23

Let $P(n): (ab)^n = a^n b^n$

For $n = 1$

$(ab)^1 = a^1 b^1$

$ab = ab$

$\Rightarrow \quad P(n)$ is true for $n = 1$

Let $P(n)$ is true for $n = k$,

$$(ab)^k = a^k b^k \qquad ---(1)$$

We have to show that,

$$(ab)^{k+1} = a^{k+1} b^{k+1}$$

Now,

$(ab)^{k+1}$

$= (ab)^k (ab)$

$= (a^k b^k)(ab) \qquad$ [Using equation (1)]

$= (a^{k+1})(b^{k+1})$

$\Rightarrow \quad P(n)$ is true for $n = k + 1$

$\Rightarrow \quad P(n)$ is true for all $n \in N$ by PMI

*********** END **********