Algebraic Identities Ex 4.2 Q6

Answer :

In the given problem, we have to find value of $4x^2+y^2+25z^2+4xy-10yz-20zx$

Given $x=4, y=3, z=2$

We have $4x^2+y^2+25z^2+4xy-10yz-20zx$

This equation can also be written as $(2x)^2+y^2+(5z)^2+2\times x\times y-2\times y\times 5z-2\times 5z\times x$

Using the identity

x+y-z2=x2+y2+z2+2xy-2yz-2xz

$$\begin{aligned}(2x)^2+y^2+(5z)^2+2\times x\times y-2\times y\times 5z-2\times 5z\times x &= (2x+y-5z)^2\\ &=(4\times 2+3-5\times 2)\\ &=(8+3-10)\\ &=1\end{aligned}$$

Hence the value of $4x^2+y^2+25z^2+4xy-10yz-20zx$ is $\boxed{1}$.

Algebraic Identities Ex 4.2 Q7

Answer :

In the given problem, we have to simplify the value of each expression

(i) Given $(x+y+z)^2+\left(x+\frac{y}{2}+\frac{z}{3}\right)^2-\left(\frac{x}{2}+\frac{y}{3}+\frac{z}{4}\right)^2$

We shall use the identity $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$ for each bracket

$$=\left\{x^2+y^2+z^2+2xy+2yz+2zx\right\}+\left\{x^2+\left(\frac{y}{2}\right)^2+\left(\frac{z}{3}\right)^2+2\times x\times\frac{y}{2}+2\times\frac{y}{2}\times\frac{z}{3}+2\times\frac{z}{3}\times x\right\}$$

-x22+y32+z42+2x2y3+2y3z4+2x2z4

$$=x^2+y^2+z^2+2xy+2yz+2zx+x^2+\frac{y^2}{4}+\frac{z^2}{9}+\frac{2xy}{2}+\frac{2yz}{6}+\frac{2zx}{3}$$

$$-\frac{x^2}{4}-\frac{y^2}{9}-\frac{z^2}{16}-\frac{2xy}{6}+\frac{2yz}{12}+\frac{2zx}{8}$$

By arranging the like terms we get

$$=x^2+x^2-\frac{x^2}{4}+y^2+\left(\frac{y}{2}\right)^2-\left(\frac{y}{3}\right)^2+z^2+\left(\frac{z}{3}\right)^2-\left(\frac{z}{4}\right)^2+2xy+\frac{2xy}{2}-\frac{2xy}{6}$$

$$+2yz+\frac{2yz}{6}+\frac{2yz}{12}+2zx+\frac{2zx}{3}+\frac{2zx}{8}$$

Now adding or subtracting like terms,

$$=\frac{8x^2-x^2}{4}+\frac{36y^2+9y^2-4y^2}{36}+\frac{144z^2+16z^2-9z^2}{144}+\frac{8xy}{3}+\frac{12yz+2yz-yz}{6}+\frac{24zx+8zx-3zx}{12}$$

$$=\frac{7x^2}{4}+\frac{41y^2}{36}+\frac{151z^2}{144}+\frac{8xy}{3}+\frac{13yz}{6}+\frac{29zx}{12}$$

Hence the value of $(x+y+z)^2+\left(x+\frac{y}{2}+\frac{z}{3}\right)^2-\left(\frac{x}{2}+\frac{y}{3}+\frac{z}{4}\right)^2$ is

$$\boxed{\frac{7x^2}{4}+\frac{41y^2}{36}+\frac{151z^2}{144}+\frac{8xy}{3}+\frac{13yz}{6}+\frac{29zx}{12}}$$

(ii) Given $(x+y-2z)^2-x^2-y^2-3z^2+4xy$

We shall use the identity $(x+y-z)^2=x^2+y^2+z^2+2xy-2yz-2zx$ for expanding the brackets

$$=x^2+y^2+(2z)^2+2xy-2y(2z)-2(2z)x-x^2-y^2-3z^2+4xy$$

$$=\cancel{x^2}+\cancel{y^2}+(2z)^2+2xy-2y(2z)-2(2z)x-\cancel{x^2}-\cancel{y^2}-3z^2+4xy$$

$$=4z^2+2xy-4yz-4zx-3z^2+4xy$$

Now arranging liked terms we get,

$$=4z^2-3z^2+2xy+4xy-4yz-4zx$$

$$=z^2+6xy-4yz-4zx$$

Hence the value of $(x+y-2z)^2-x^2-y^2-3z^2+4xy$ is $\boxed{z^2+6xy-4yz-4zx}$

(iii) Given $\left(x^2-x+1\right)^2-\left(x^2+x+1\right)^2$

We shall use the identity $\left(x+y+z\right)^2=x^2+y^2+z^2+2xy+2yz+2zx$ for each brackets

$$\left(x^2-x+1\right)^2-\left(x^2+x+1\right)^2=\left[\left(x^2\right)^2+(-x)^2+1^2-2x^3-2x+2x^2\right]$$
$$-\left[\left(x^2\right)^2+(x)^2+1^2+2x^3+2x+2x^2\right]$$

$$=x^4+x^2+1-2x^3-2x^2+2x^2-x^4-x^2-1-2x^3-2x-2x^2$$

Canceling the opposite term and simplifies

$$=\cancel{x^4}+\cancel{x^2}+\cancel{1}-2x^3-\cancel{2x^2}+\cancel{2x^2}-\cancel{x^4}-\cancel{x^2}-\cancel{1}-2x^3-2x-2x^2$$
$$=-4x^3-4x$$
$$=-4x\left(x^2+1\right)$$

Hence the value of $\left(x^2-x+1\right)^2-\left(x^2+x+1\right)^2$ is $\boxed{-4x\left(x^2+1\right)}$.

*********** END **********