Maxima and Minima 18.3 Q2(i)

$$f(x) = (x-1)(x-2)^2$$

$$\therefore \quad f'(x) = (x-2)^2 + 2(x-1)(x-2)$$
$$= (x-2)(x-2+2x-2)$$
$$= (x-2)(3x-4)$$
$$f''(x) = (3x-4) + 3(x-2)$$

For maxima and minima,

$$f'(x) = 0$$
$$\Rightarrow \quad (x-2)(3x-4) = 0$$
$$\Rightarrow \quad x = 2, \frac{4}{3}$$

Now,

$$f''(2) > 0$$

$\therefore \quad x = 2$ is local minima

$$f''\left(\frac{4}{3}\right) = -2 < 0$$

$\therefore \quad x = \frac{4}{3}$ is point of local maxima

$\therefore \quad$ local max value $= f\left(\frac{4}{3}\right) = \frac{4}{27}$

local min value $= f(2) = 0.$

Maxima and Minima 18.3 Q2(ii)

$$f(x) = x\sqrt{1-x}$$

$$\therefore \quad f'(x) = \sqrt{1-x} + \frac{x}{2\sqrt{1-x}}(-1)$$

$$= \frac{2(1-x) - x}{2\sqrt{1-x}}$$

$$= \frac{2-3x}{2\sqrt{1-x}}$$

$$f''(x) = \frac{2\sqrt{1-x}\,(-3) + \frac{(2-3x)}{\sqrt{1-x}}}{4(1-x)}$$

For maximum and minimum,

$$f'(x) = 0$$

$$\Rightarrow \quad \frac{2-3x}{2\sqrt{1-x}} = 0$$

$$\Rightarrow \quad x = \frac{2}{3}$$

Now,

$$f''\left(\frac{2}{3}\right) < 0$$

$$\therefore \quad x = \frac{2}{3} \text{ is point ofmaxima}$$

$$\therefore \quad \text{local max value} = f\left(\frac{2}{3}\right) = \frac{2}{3\sqrt{3}}.$$

Maxima and Minima 18.3 Q2(iii)

$$f(x) = -(x-1)^3(x+1)^2$$

$$\therefore \quad f'(x) = -3(x-1)^2(x+1)^2 - 2(x-1)^3(x+1)$$

$$= -(x-1)^2(x+1)(3x+3+2x-2)$$

$$= -(x-1)^2(x+1)(5x+1)$$

$$\therefore \quad f''(x) = -2(x-1)(x+1)(5x+1) - (x-1)^2(5x+1) - 5(x-1)^2(x+1)$$

For maximum and minimum value,

$$f'(x) = 0$$

$$\Rightarrow \quad -(x-1)^2(x+1)(5x+1) = 0$$

$$\Rightarrow \quad x = 1, -1, -\frac{1}{5}$$

Now,

$$f''(1) = 0$$

$$\therefore \quad x = 1 \text{ is inflection point}$$

$$f''(-1) = -4 \times -4 = 16 > 0$$

$$\therefore \quad x = -1 \text{ is point of minima}$$

$$f''\left(\frac{-1}{5}\right) = -5\left(\frac{36}{25}\right) \times \frac{4}{5} = \frac{-144}{25} < 0$$

$$\therefore \quad x = \frac{-1}{5} \text{ is point of maxima}$$

Hence,

$$\text{local max value} = f\left(-\frac{1}{5}\right) = \frac{3456}{3125}$$

$$\text{local min value} = f(-1) = 0.$$

Maxima and Minima 18.3 Q3

We have,

$$y = a\log x + bx^2 + x$$

$$\therefore \quad \frac{dy}{dx} = \frac{a}{x} + 2bx + 1$$

$$\text{and} \quad \frac{d^2y}{dx^2} = \frac{-a}{x^2} + 2b$$

For maximum and minimum value,

$$\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{a}{x} + 2bx + 1 = 0$$

Given that extreme value exist at $x = 1, 2$

$$\Rightarrow \quad a + 2b = -1 \qquad \text{---(i)}$$

$$\frac{a}{2} + 4b = -1$$

$$\Rightarrow \quad a + 8b = -2 \qquad \text{---(ii)}$$

Solving (i) and (ii), we get

$$a = \frac{-2}{3}, \; b = \frac{-1}{6}.$$

*********** END **********