Binary Operations Ex 3.1 Q7

We have,

$$M = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : \ a, b \in R - \{0\} \right\} \text{ and }$$

$A * B = AB$ for all $A, B \in M$

Let $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \in M$ and $B = \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} \in M$

Now, $AB = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} = \begin{bmatrix} ac & 0 \\ 0 & bd \end{bmatrix}$

$\therefore$ $\quad a \in R, \ b \in R, \ c \in R, \ \& \ d \in R$

$\Rightarrow$ $\quad ac \in R$ and $bd \in R$

$\Rightarrow$ $\begin{bmatrix} ac & 0 \\ 0 & bd \end{bmatrix} \in M$

$\Rightarrow$ $\quad A * B \in M$

Thus, the operater $*$ difines a binary operation on M

Binary Operations Ex 3.1 Q8

S = set of rational numbers of the form $\dfrac{m}{n}$ where $m \in Z$ and $n = 1, 2, 3$

Also, $\quad a * b = ab$

Let $a \in S$ and $b \in S$
$\Rightarrow$ $\quad ab \notin S$

For example $\quad a = \dfrac{7}{3}$ and $b = \dfrac{5}{2}$

$\Rightarrow$ $\quad ab = \dfrac{35}{6} \notin S$

$\therefore$ $\quad a * b \notin S$

Hence, the operater $*$ does not define a binary operation on S

Binary Operations Ex 3.1 Q9

It is given that, a*b = 2a + b

Now

$(2*3) = 2 \times 2 + 3$

$= 4 + 3$

$= 7$

$(2*3)*4 = 7*4 = 2 \times 7 + 4$

$= 14 + 4$

$= 18$

Binary Operations Ex 3.1 Q10

It is given that, a*b = LCM (a, b)

Now

$5*7 = LCM (5, 7)$

$= 35$

********** END *********