

Indefinite Integrals Ex 19.8 Q41

Let
$$I = \int \frac{\sec^2 x}{\tan x + 2} dx - - - - - - (i)$$

Let
$$\tan x + 2 = t$$
 then,
 $d(\tan x + 2) = dt$

$$\Rightarrow$$
 $\sec^2 x dx = dt$

$$\Rightarrow dx = \frac{1}{\sec^2 x} dt$$

Putting $\tan x + 2 = t$ and $dx = \frac{dt}{\sec^2 x}$ in equation (i), we get,

$$I = \int \frac{\sec^2 x}{t} \times \frac{1}{\sec^2 x} dt$$
$$= \int \frac{dt}{t}$$
$$= \log|t| + c$$
$$= \log|\tan x + 2| + c$$

$$\Rightarrow$$
 $I = \log |\tan x + 2| + c$

Indefinite Integrals Ex 19.8 Q42
Let
$$I = \int \frac{2\cos 2x + \sec^2 x}{\sin 2x + \tan x - 5} dx - - - - - - (i)$$

Let $\sin 2x + \tan x - 5 = t$ then, $d\left(\sin 2x + \tan x - 5\right) = dt$

$$\Rightarrow \qquad \left(2\cos 2x + \sec^2 x\right) dx = dt$$

$$\Rightarrow dx = \frac{1}{2\cos 2x + \sec^2 x} dt$$

Putting $\sin 2x + \tan x - 5 = t$ and $dx = \frac{dt}{2\cos 2x + \sec^2 x}$ in equation (i), we get,

$$\begin{split} I &= \int \frac{2\cos 2x + \sec^2 x}{t} \times \frac{1}{2\cos 2x + \sec^2 x} \, dt \\ &= \int \frac{dt}{t} \\ &= \log|t| + c \\ &= \log|\sin 2x + \tan x - 5| + c \end{split}$$

$$I = \log |\sin 2x + \tan x - 5| + c$$

Indefinite Integrals Ex 19.8 Q43

Let
$$I = \int \frac{\sin 2x}{\sin 5x \sin 3x} dx \quad \text{then,}$$

$$I = \int \frac{\sin (5x - 3x)}{\sin 5x \sin 3x} dx$$

$$= \int \frac{\sin 5x \cos 3x - \cos 5x \sin 3x}{\sin 5x \sin 3x} dx$$

$$= \int \frac{\sin 5x \cos 3x}{\sin 5x \sin 3x} dx - \int \frac{\cos 5x \sin 3x}{\sin 5x \sin 3x} dx$$

$$= \int \frac{\cos 3x}{\sin 3x} dx - \int \frac{\cos 5x}{\sin 5x} dx$$

 $= \frac{1}{3} \log |\sin 3x| - \frac{1}{5} \log |\sin 5x| + c$

$$I = \frac{1}{3} \log \left| \sin 3x \right| - \frac{1}{5} \log \left| \sin 5x \right| + c$$

Indefinite Integrals Ex 19.8 Q44

Let
$$I = \int \frac{1 + \cot x}{x + \log \sin x} dx - - - - (i)$$

Let
$$x + \log \sin x = t$$
 then,
d($x + \log \sin x$) = dt

$$\Rightarrow \qquad (1 + \cot x)dx = dt \qquad \qquad \left[\because \qquad \frac{d}{dx} \left(\log \sin x \right) = \cot x \right]$$

$$\Rightarrow \qquad dx = \frac{dt}{1 + \cot x}$$

Putting $x + \log \sin x = t$ and $dx = \frac{dt}{1 + \cot x}$ in equation (i), we get

$$I = \int \frac{1 + \cot x}{t} \times \frac{dt}{1 + \cot x}$$
$$= \int \frac{dt}{t}$$
$$= \log |t| + c$$
$$= \log |x + \log \sin x| + c$$

$$\therefore I = \log |x| + \log \sin x + c$$

Indefinite Integrals Ex 19.8 Q45

Let
$$I = \int \frac{1}{\sqrt{x} \left(\sqrt{x} + 1\right)} dx - \cdots - i$$

Let
$$\sqrt{x} + 1 = t$$
 then,

$$d(\sqrt{x} + 1) = dt$$

$$\Rightarrow \frac{1}{2\sqrt{x}}dx = dt$$

$$\Rightarrow dx = 2\sqrt{x}dt$$

$$\Rightarrow dx = 2\sqrt{x} dt$$

Putting $\sqrt{x} + 1 = t$ and $dx = 2\sqrt{x} dt$ in equation (i), we get

$$I = \int \frac{1}{\sqrt{x} t} \times 2\sqrt{x} dt$$

$$= 2\int \frac{dt}{t}$$

$$= 2\log|t| + c$$

$$= 2\log|\sqrt{x} + 1| + c$$

$$\therefore I = 2\log|\sqrt{x} + 1| + c$$

********* END ********