Differentials Errors and Approximation Ex14.1 Q1

Let $x = \dfrac{\pi}{2}$, $x + \Delta x = \dfrac{22}{14}$

$$\Delta x = \dfrac{22}{14} - x$$

$$\Delta x = \left(\dfrac{22}{14} - \dfrac{\pi}{2}\right)$$

$$y = \sin x$$

$$\dfrac{dy}{dx} = \cos x$$

$$\left(\dfrac{dy}{dx}\right)_{x=\frac{\pi}{2}} = \dfrac{\cos \pi}{2}$$

$$\left(\dfrac{dy}{dx}\right)_{x=\frac{\pi}{2}} = 0$$

$\therefore$ $$\Delta y = \left(\dfrac{dy}{dx}\right)_{x=\frac{\pi}{2}} \times \Delta ax$$

$$= 0 \times \left(\dfrac{22}{14} - \dfrac{\pi}{2}\right)$$

$$\Delta y = 0$$

So, there is no change in y.

Differentials Errors and Approximation Ex14.1 Q2

Let $x = 10$, $x + \Delta x = 9.8$

$$\Delta x = 9.8 - x$$
$$= 9.8 - 10$$
$$\Delta x = -0.2$$

$$y = \frac{4}{3}\pi x^3 \qquad \text{[volume of sphere]}$$

$$\frac{dy}{dx} = 4\pi r^2$$

$$\left(\frac{dy}{dx}\right)_{x=10} = 4\pi (10)^2$$

$$\left(\frac{dy}{dx}\right)_{x=10} = 400\pi \ cm^2$$

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=10} \times \Delta x$$
$$= 400\pi \times (-0.2)$$
$$\Delta y = -80\pi \ cm^3$$

So, approximate diecocase in volume is 80π cm^3.

Differentials Errors and Approximation Ex14.1 Q3

Let $x = 10$, $x + \Delta x = 10 + \dfrac{k}{100} \times 10$

$$x + \Delta x = 10 + 0.k$$
$$\Rightarrow \qquad \Delta x = 10 + 0.k - 10$$
$$\Delta x = 0.k$$

$$y = \pi r^2$$

$$\frac{dy}{dx} = 2\pi r$$

$$\left(\frac{dy}{dx}\right)_{x=10} = 2\pi (10)$$

$$\left(\frac{dy}{dx}\right)_{x=10} = 20\pi \ cm$$

So,

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=10} \times \Delta x$$
$$= (20\pi) \times (0.k)$$
$$\Delta y = 2k\pi cm^2$$

Area of the plate increases by $2k\pi$ cm^2.

Differentials Errors and Approximation Ex14.1 Q4

Let   length(L) = x

$$x + \Delta x = x + \frac{x}{100}$$

$$\Delta x = 0.01x$$

Now,

$$y = 6x^2$$

$$\frac{dy}{dx} = 12x \text{ cm}$$

So,

$$\Delta y = \left(\frac{dy}{dx}\right) \times \Delta x$$

$$= (12x)(0.01x)$$

$$\Delta y = 0.12x^2 \text{ cm}^2$$

$$= 6(0.02)x^2$$

$$= 2\% \text{ of } 6x^2$$

Percentage error in area is 2%.

********** END *********