Functions Ex 3.3 Q3

We have,

$$f(x) = \frac{ax+b}{bx-a}$$

We observe that $f(x)$ is a rational function of x as $\frac{ax+b}{bx-a}$ is a rational expression.

Clearly, $f(x)$ assumes real values for all x except for the values of x for which $bx - a = 0$ i.e., $bx = a$

$\Rightarrow \quad x = \frac{a}{b}$

$\therefore$ Domain$(f) = R - \left\{\frac{a}{b}\right\}$

Range of f: Let $f(x) = y$

$\Rightarrow \quad \frac{ax+b}{bx-a} = y$

$\Rightarrow \quad ax + b = y(bx - a)$

$\Rightarrow \quad ax + b = bxy - ax$

$\Rightarrow \quad b + ay = bxy - ax$

$\Rightarrow \quad b + ay = x(by - a)$

$\Rightarrow \quad \frac{b+ay}{b-ay} = x$

$\Rightarrow \quad x = \frac{b+ay}{by-a}$

Clearly, x will take real value for all $x \in R$ except for

$by - a = 0$

$\Rightarrow \quad by = a$

$\Rightarrow \quad y = \frac{a}{b}$

$\therefore$ Range$(f) = R - \left\{\frac{a}{b}\right\}$.

We have,

$$f(x) = \frac{ax-b}{cx-d}$$

We observe that $f(x)$ is a rational function of x as $\frac{ax-b}{cx-d}$ is a rational expression.

Clearly, $f(x)$ assumes real values for all x except for all those values of x for which $cx - d = 0$ i.e., $cx = d$

$\Rightarrow \quad x = \frac{d}{c}$

$\therefore$ Domain$(f) = R - \left\{\frac{d}{c}\right\}$

Range: Let $f(x) = y$

$\Rightarrow \quad \frac{ax-b}{cx-d} = y$

$\Rightarrow \quad ax - b = y(cx - d)$

$\Rightarrow \quad ax - b = cxy - dy$

$\Rightarrow \quad dy - b = cxy - 9x$

$\Rightarrow \quad dy - b = x(cy - a)$

$\Rightarrow \quad \frac{dy-b}{cy-a} = x$

Clearly, x assumes real values for all y except

$cy - a = 0$ i.e., $y = \frac{a}{c}$

Hence, range$(f) = R - \left\{\frac{a}{c}\right\}$

We have,

$$f(x) = \sqrt{x-1}$$

Clearly, $f(x)$ assumes real values, if

$$x - 1 \geq 0$$

$\Rightarrow \quad x \geq 1$

$\Rightarrow \quad x \in [1, \infty)$

Hence, domain$(f) = [1, \infty)$

Range: For $x \geq 1$, we have,

$$x - 1 \geq 0$$

$\Rightarrow \quad \sqrt{x-1} \geq 0$

$\Rightarrow \quad f(x) \geq 0$

Thus, $f(x)$ takes all real values greater than zero.

Hence, range$(f) = [0, \infty)$

We have,

$$f(x) = \sqrt{x-3}$$

Clearly, $f(x)$ assumes real values, if

$$x - 3 \geq 0$$

$\Rightarrow \quad x \geq 3$

$\Rightarrow \quad x \in [3, \infty)$

Hence, domain$(f) = [3, \infty)$

Range: For $x \geq 3$, we have,

$$x - 3 \geq 0$$

$\Rightarrow \quad \sqrt{x-3} \geq 0$

$\Rightarrow \quad f(x) \geq 3$

Thus, $f(x)$ takes all real values greater than zero.

Hence, range$(f) = [0, \infty)$

We have,

$$f(x) = \frac{x-2}{2-x}$$

Domain of f: Clearly, $f(x)$ is defined for all $x \in R$ except for which

$2 - x \neq 0$ i.e., $x \neq 2$

Hence, domain$(f) = R - \{2\}$

Range of f: Let $f(x) = y$

$\Rightarrow \quad \frac{x-2}{2-x} = y$

$\Rightarrow \quad \frac{-1(2-x)}{2-x} = y$

$\Rightarrow \quad -1 = y$

$\Rightarrow \quad y = -1$

$\therefore$ Range$(f) = \{-1\}$

We have,

$$f(x) = |x - 1|$$

Clearly, $f(x)$ is defined for all $x \in R$

$\Rightarrow \quad \text{Domain}(f) = R$

Range: Let $f(x) = y$

$\Rightarrow \quad |x - 1| = y$

$\Rightarrow \quad f(x) \geq 0 \ \forall \ x \in R$

It follows from the above relation that y takes all real values greater or equal to zero.

$\therefore \quad \text{Range}(f) = [0, \infty)$

As |x|is defined for all real numbers, its domain is R and range is only negative numbers because, |x| is always positive real number for all real numbers and -|x| is always negative real numbers.

In order to have F(x) has defined value, term inside square root should always be greater than or equal to zero which gives domain as $-3 \leq x \leq 3$

Where as Range of above function is limited to [0 , 3]

*********** END **********