

Exercise 1E

Question 12:

Consider the given equation

$$\frac{5 - \sqrt{6}}{5 + \sqrt{6}} = a - b\sqrt{6}$$

For rationalising the denominator of a number, we multiply its numerator and denominator by its rationalising factor.

If a and b are integers, then $(a+\sqrt{b})$ and $(a-\sqrt{b})$ are rationalising factor of each other, as $(a+\sqrt{b})(a-\sqrt{b})=(a^2-b)$, which is rational.

Let us rationalise the denominator of the Left hand side.
$$\Rightarrow \frac{5 - \sqrt{6}}{5 + \sqrt{6}} \times \frac{5 - \sqrt{6}}{5 - \sqrt{6}} = a - b\sqrt{6}$$

$$\Rightarrow \frac{\left(5 - \sqrt{6}\right)^2}{\left(5\right)^2 - \left(\sqrt{6}\right)^2} = a - b\sqrt{6}$$

$$\Rightarrow \frac{\left(5\right)^2 - 2\left(5\right)\left(\sqrt{6}\right) + \left(\sqrt{6}\right)^2}{25 - 6} = a - b\sqrt{6}$$

$$\Rightarrow \frac{25 - 10\sqrt{6} + 6}{19} = a - b\sqrt{6}$$

$$\Rightarrow \frac{31 - 10\sqrt{6}}{19} = a - b\sqrt{6}$$

$$\Rightarrow \frac{31}{19} - \frac{10\sqrt{6}}{19} = a - b\sqrt{6}$$

$$\therefore a = \frac{31}{19} \text{ and } b = \frac{10}{19}.$$

Question 13:

Consider the given equation
$$\frac{5 + 2\sqrt{3}}{7 + 4\sqrt{3}} = a - b\sqrt{3}$$

For rationalising the denominator of a number, we multiply its numerator and denominator by its rationalising factor.

If a and b are integers and x is a natural number, then $(a+b\sqrt{x})$ and $(a-b\sqrt{x})$ are rationalising factor of each other, as $(a+b\sqrt{x})(a-b\sqrt{x}) = (a^2-b^2x)$, which is rational.

Let us rationalise the denominator of the Left hand side.
$$\Rightarrow \frac{5+2\sqrt{3}}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = a-b\sqrt{3}$$

$$\Rightarrow \frac{\left(5+2\sqrt{3}\right)\left(7-4\sqrt{3}\right)}{\left(7\right)^2-\left(4\sqrt{3}\right)^2} = a-b\sqrt{3}$$

$$\Rightarrow \frac{5\left(7-4\sqrt{3}\right)+2\sqrt{3}\left(7-4\sqrt{3}\right)}{49-48} = a-b\sqrt{3}$$

$$\Rightarrow \frac{35-20\sqrt{3}+14\sqrt{3}-24=a-b\sqrt{3}}{11-6\sqrt{3}=a-b\sqrt{3}}$$

$$\Rightarrow a=11 \text{ and } b=6.$$

******* END *******