A(250, 0)	1125000	
B(200, 50)	1150000	→ Maximum
C(0, 175)	875000	

The maximum value of Z is 1150000 at (200, 50).

Thus, the merchant should stock 200 desktop models and 50 portable models to get the maximum profit of Rs 1150000.

Question 9:

A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods $F_1$and F_2 are available. Food F_1 costs Rs 4 per unit food and F_2 costs Rs 6 per unit. One unit of food F_1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements?

Answer

Let the diet contain x units of food F_1 and y units of food F_2. Therefore,

$x \geq 0$ and $y \geq 0$

The given information can be complied in a table as follows.

	Vitamin A (units)	**Mineral (units)**	**Cost per unit (Rs)**
Food F_1 (x)	3	4	4
Food F_2 (y)	6	3	6
Requirement	80	100	

The cost of food F_1 is Rs 4 per unit and of Food F_2 is Rs 6 per unit. Therefore, the constraints are

$3x + 6y \geq 80$

$4x + 3y \geq 100$

$x, y \geq 0$

Total cost of the diet, $Z = 4x + 6y$

The mathematical formulation of the given problem is

Minimise $Z = 4x + 6y$ … (1)

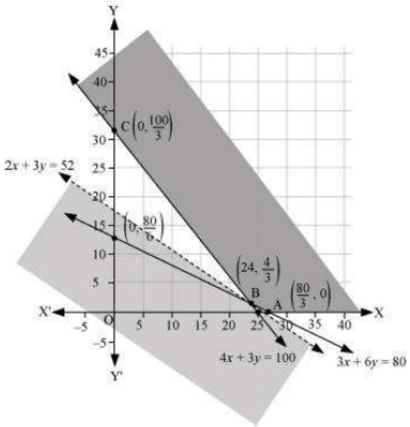
subject to the constraints,

$3x + 6y \geq 80$ … (2)

$4x + 3y \geq 100$ … (3)

$x, y \geq 0$ … (4)

The feasible region determined by the constraints is as follows.



It can be seen that the feasible region is unbounded.

The corner points of the feasible region are $A\left(\frac{8}{3},0\right)$, $B\left(2,\frac{1}{2}\right)$, and $C\left(0,\frac{11}{2}\right)$.

The corner points are $A\left(\frac{80}{3},0\right)$, $B\left(24,\frac{4}{3}\right)$, and $C\left(0,\frac{100}{3}\right)$.

The values of Z at these corner points are as follows.

Corner point	$Z = 4x + 6y$	
$A\left(\frac{80}{3}, 0\right)$	$\frac{320}{3} = 106.67$	
$B\left(24, \frac{4}{3}\right)$	104	→ Minimum
$C\left(0, \frac{100}{3}\right)$	200	

As the feasible region is unbounded, therefore, 104 may or may not be the minimum value of Z.

For this, we draw a graph of the inequality, $4x + 6y < 104$ or $2x + 3y < 52$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $2x + 3y < 52$

Therefore, the minimum cost of the mixture will be Rs 104.

Question 10:

There are two types of fertilizers F_1 and F_2. F_1 consists of 10% nitrogen and 6% phosphoric acid and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If F_1 cost Rs 6/kg and F_2 costs Rs 5/kg, determine how much of each type of fertilizer should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?

Answer

Let the farmer buy x kg of fertilizer F_1 and y kg of fertilizer F_2. Therefore,

$x \ge 0$ and $y \ge 0$

The given information can be complied in a table as follows.

	Nitrogen (%)	Phosphoric Acid (%)	Cost (Rs/kg)
F_1 (x)	10	6	6
F_2 (y)	5	10	5
Requirement (kg)	14	14	

F_1 consists of 10% nitrogen and F_2 consists of 5% nitrogen. However, the farmer requires at least 14 kg of nitrogen.

∴ 10% of x + 5% of $y \ge 14$

$$\frac{x}{10} + \frac{y}{20} \ge 14$$
$$2x + y \ge 280$$

F_1 consists of 6% phosphoric acid and F_2 consists of 10% phosphoric acid. However, the farmer requires at least 14 kg of phosphoric acid.

∴ 6% of x + 10% of $y \ge 14$

$$\frac{6x}{100} + \frac{10y}{100} \ge 14$$
$$3x + 56y \ge 700$$

Total cost of fertilizers, $Z = 6x + 5y$

The mathematical formulation of the given problem is

Minimize $Z = 6x + 5y$ … (1)

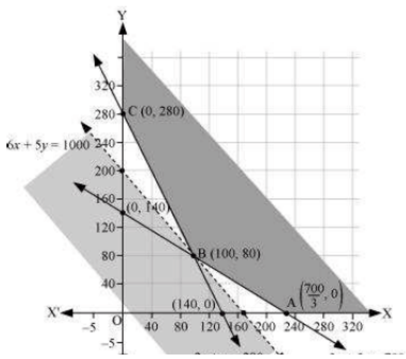
subject to the constraints,

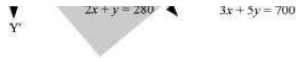
$2x + y \ge 280$ … (2)

$3x + 5y \ge 700$ … (3)

$x, y \ge 0$ … (4)

The feasible region determined by the system of constraints is as follows.





It can be seen that the feasible region is unbounded.

The corner points are $A\left(\frac{700}{3},0\right), B(100,80), \text{ and } C(0,280)$.

The values of Z at these points are as follows.

Corner point	Z = 6*x* + 5*y*	
$A\left(\frac{700}{3},0\right)$	1400	
B(100, 80)	1000	→ Minimum
C(0, 280)	1400	

As the feasible region is unbounded, therefore, 1000 may or may not be the minimum value of Z.

For this, we draw a graph of the inequality, $6x + 5y < 1000$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with

$6x + 5y < 1000$

*********** END **********