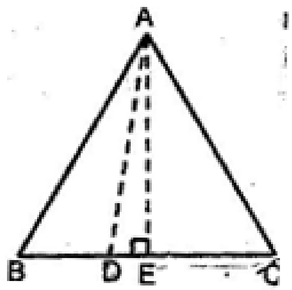
Exercise 6.5

15. In an equilateral triangle ABC, D is a point on side BC such that BD = $\frac{1}{3}$ BC. Prove that $9AD^2 = 7AB^2$.

Ans. Let ABC be an equilateral triangle and let D be a point on BC such that BD = $\frac{1}{3}$ BC



Draw AE$\perp$ BC, Join AD.

In Δ s AEB and AEC, we have,

AB = AC[$\because$ Δ ABC is equilateral]

$\angle$ AEB = $\angle$ AEC [$\because$ each 90°]

And AE = AE

$\therefore$ By SAS-criterion of similarity, we have

Δ AEB ~ Δ AEC

$\Rightarrow$ BE = EC

Thus, we have, BD = $\frac{1}{3}$ BC, DC = $\frac{2}{3}$ BC and BE = EC = $\frac{1}{3}$ BC ....(1)

Since, $\angle$ C = 60°

$\therefore$ Δ ADC is an acute angle triangle.

$\therefore$ $AD^2 = AC^2 + DC^2 - 2DC \times EC$

$= AC^2 + \left(\frac{2}{3}BC\right)^2 - 2\times\frac{2}{3}BC \times\frac{1}{3}BC$ [using eq.(1)]

$$\Rightarrow \quad AD^2 = AC^2 + \frac{4}{9}BC^2 - \frac{2}{3}BC^2$$

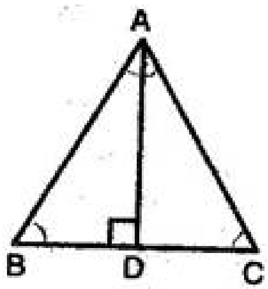
$$= AB^2 + \frac{4}{9}AB^2 - \frac{2}{3}AB^2 \quad [\because \ AB = BC = AC]$$

$$\Rightarrow \quad AD^2 = \frac{(9+4-6)AB^2}{9} = \frac{7}{9}AB^2$$

$$\Rightarrow \quad 9AD^2 = 7AB^2$$

16. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Ans. Let ABC be an equilateral triangle and let AD$\perp$ BC. In Δ s ADB and ADC, we have,



AB = AC [Given]

$\angle$ B = $\angle$ C = 60° [Given]

And $\angle$ ADB = $\angle$ ADC[Each = 90°]

$\therefore$ Δ ADB $\cong$ Δ ADC[By RHS criterion of congruence]

$\Rightarrow$ BD = DC

$\Rightarrow$ BD = DC = $\frac{1}{2}$ BC

Since Δ ADB is a right triangle, right angled at D, by Pythagoras theorem, we have,

$$AB^2 = AD^2 + BD^2$$

$$\Rightarrow \quad AB^2 = AD^2 + \left(\frac{1}{2}BC\right)^2$$

$$\Rightarrow \quad AB^2 = AD^2 + \frac{1}{4}BC^2$$

$$\Rightarrow \quad AB^2 = AD^2 + \frac{AB^2}{4} \quad [\because \ BC = AB]$$

$$\Rightarrow \quad \frac{3}{4}AB^2 = AD^2$$

$$\Rightarrow \quad 3AB^2 = 4AD^2$$

17. Tick the correct answer and justify: In Δ ABC, AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm. the angles A and B are respectively:

(A) 90° and 30°

(B) 90° and 60°

(C) 30° and 90°

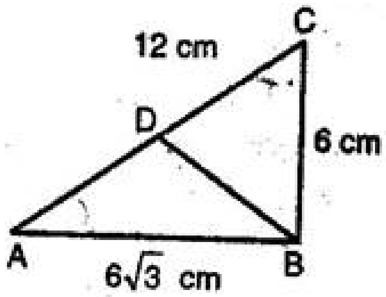
(D) 60° and 90°

Ans. (C) In Δ ABC, we have, AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm.

Now, $AB^2 + BC^2$ = $\left(6\sqrt{3}\right)^2 + (6)^2 = 36 \times 3 + 36 = 108 + 36$ = 144 = $(AC)^2$

Thus, Δ ABC is a right triangle, right angled at B.

$\therefore$ $\angle$ B = 90°



Let D be the mid-point of AC. We know that the mid-point of the hypotenuse of a right triangle is equidistant from the vertices.

AD = BD = CD

$\Rightarrow$ CD = BD = 6 cm [$\because$ CD = $\frac{1}{2}$ AC]

Also, BC = 6 cm

$\therefore$ In Δ BDC, we have, BD = CD = BC

$\Rightarrow$ Δ BDC is equilateral

$\Rightarrow$ $\angle$ ACB = 60°

$\therefore$ $\angle$A= $180^\circ - (\angle B + \angle C) = 180^\circ - \left(90^\circ + 60^\circ\right) = 30^\circ$

Thus, $\angle$ A = 30° and $\angle$ B = 90°

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