Polynomials Ex 2.2 Q4

Answer :

Let $a-d, a$ and $a+d$ be the zeros of the polynomials $f(x)$. Then,

Sum of the zeros = $\dfrac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

$$a-d+a+a+d=\frac{-3p}{1}$$
$$a\cancel{-d}+a+a\cancel{+d}=-3p$$
$$3a=-3p$$
$$a=\frac{-3\times p}{3}$$
$$a=\frac{-\cancel{3}\times p}{\cancel{3}}$$
$$a=-p$$

Since a is a zero of the polynomial $f(x)$. Therefore,

$$f(x)=x^3+3px^2+3qx+r$$
$$f(a)=0$$
$$f(a)=a^3+3pa^2+3qa+r$$
$$a^3+3pa^2+3qa+r=0$$

Substituting $a=-p$ we get,

$$(-p)^3+3p(-p)^2+3q(-p)+r=0$$
$$-p^3+3p^3-3pq+r=0$$
$$2p^3-3pq+r=0$$

Hence, the condition for the given polynomial is $\boxed{2p^3-3pq+r=0}$.

Polynomials Ex 2.2 Q5

Answer :

Let $a-d, a$ and $a+d$ be the zeros of the polynomial $f(x)$. Then,

Sum of the zeros = $\dfrac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

$$(a+d)+a+(a-d)=-\frac{3b}{a}$$

$$a+\cancel{d}+a+a-\cancel{d}=\frac{-3b}{a}$$

$$3a=\frac{-3b}{a}$$

$$a=\frac{-\cancel{3}b}{a}\times\frac{1}{\cancel{3}}$$

$$a=\frac{-b}{a}$$

Since a is a zero of the polynomial $f(x)$.

Therefore,

$$f(x)=ax^3+3bx^2+3cx+d$$

$$f(a)=0$$

$$f(a)=aa^3+3ba^2+3ca+d$$

$$aa^3+3ba^2+3ca+d=0$$

$$a\left(\frac{-b}{a}\right)^3+3b\times\left(\frac{-b}{a}\right)^2 3\times c\left(\frac{-b}{a}\right)+d=0$$

$$a\times\frac{-b}{a}\times\frac{-b}{a}\times\frac{-b}{a}+3\times b\times\frac{-b}{a}\times\frac{-b}{a}+3\times c\times\frac{-b}{a}+d=0$$

$$\cancel{a}\times\frac{-b}{\cancel{a}}\times\frac{-b}{a}\times\frac{-b}{a}+3\times b\times\frac{-b}{a}\times\frac{-b}{a}+3\times c\times\frac{-b}{a}+d=0$$

$$\frac{-b^3}{a^2}+\frac{3b^3}{a^2}-3\frac{cb}{a}+d=0$$

$$\frac{-b^3+3b^3-3abc+a^2d}{a^2}=0$$

$$2b^3-3abc+a^2d=0\times a^2$$

$$2b^3-3abc+a^2d=0$$

Hence, it is proved that $\boxed{2b^3-3abc+a^2d=0}$.

Polynomials Ex 2.2 Q6

Answer :

Let $a-d, a$ and $a+d$ be the zeros of the polynomial $f(x)$.

Then,

Sum of the zeros = $\dfrac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

$$a-d+a+a+d=\frac{-(-12)}{1}$$

$$a-\cancel{d}+a+a+\cancel{d}=12$$

$$3a=12$$

$$a=\frac{12}{3}$$

$a=4$

Since a is a zero of the polynomial $f(x)$

$$f(x)=x-12x+39x+k$$

$$f(a)=0$$

$$f(a)=4^3-12\times4^2+39\times4+k$$

$$0=64-192+156+k$$

$$0=220-192+k$$

$$0=28+k$$

$$-28=k$$

Hence, the value of k is $\boxed{-28}$.

*********** END **********