Sine and Cosine Formulae and their Applications Ex-10.2 Q10

In any $\triangle ABC$, we have

$a = b\cos C + c\cos B$

$b = c\cos A + a\cos C$

$c = a\cos B + b\cos A$

Therefore,

$$\begin{aligned} L.H.S &= a(\cos B + \cos C - 1) + b(\cos C + \cos A - 1) + c(\cos A + \cos B - 1) \\ &= a\cos B + a\cos C - a + b\cos C + b\cos A - b + c\cos A + c\cos B - c \\ &= c - b\cos A + a\cos C - a + a - c\cos B + b\cos A - b + b - a\cos C + c\cos B - c \\ &= 0 \\ &= R.H.S \end{aligned}$$

Hence proved.

Sine and Cosine Formulae and their Applications Ex-10.2 Q11

By sine rule we have

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$k\sin A = a, k\sin B = b, k\sin C = c$

$$a\cos A + b\cos B + c\cos C = k\sin A\cos A + k\sin B\cos B + k\sin C\cos C$$

$$= \left(\frac{1}{2}\right)k[2\sin A\cos A + 2\sin B\cos B + 2\sin C\cos C]$$

$$= \left(\frac{1}{2}\right)k[\sin 2A + \sin 2B + \sin 2C]$$

$$= k[\sin(A+B)\cos(A-B) + \sin C\cos C]$$

$$= k[\sin(\pi - C)\cos(A-B) + \sin C\cos(\pi - (A+B))]$$

$$= k[\sin C\cos(A-B) - \sin C\cos(A+B)]$$

$$= k[\sin C(\cos(A-B) - \cos(A+B))]$$

$$= k\sin C[2\sin A\sin B]$$

$$= 2\sin C(k\sin A)\sin B$$

$$= 2a\sin B\sin C$$

Sine and Cosine Formulae and their Applications Ex-10.2 Q12

We know that by cosine rule

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\Rightarrow 2bc\cos A = b^2 + c^2 - a^2$$

$$\Rightarrow a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Rightarrow a^2 = b^2 + c^2 - 2bc\left(2\cos^2\frac{A}{2} - 1\right)$$

$$\Rightarrow a^2 = b^2 + c^2 + 2bc - 4bc\cos^2\frac{A}{2}$$

$$\Rightarrow a^2 = (b+c)^2 - 4bc\cos^2\frac{A}{2}$$

Sine and Cosine Formulae and their Applications Ex-10.2 Q13

$$4\left(bc\cos^2\frac{A}{2}+ca\cos^2\frac{B}{2}+ab\cos^2\frac{C}{2}\right)=(a+b+c)^2$$

$LHS,$

$$4\left(bc\cos^2\frac{A}{2}+ca\cos^2\frac{B}{2}+ab\cos^2\frac{C}{2}\right)$$

$$=2\left(bc.2\cos^2\frac{A}{2}+ca.2\cos^2\frac{B}{2}+ab.2\cos^2\frac{C}{2}\right)$$

$$=2(bc.(1-\cos A)+ca.(1-\cos B)+ab.(1-\cos C))$$

$$=2bc-2bc\cos A+2ca-2ca\cos B+2ab-2ab\cos C$$

$$=2bc-2bc\frac{b^2+c^2-a^2}{2bc}+2ca-2ca\frac{a^2+c^2-b^2}{2ca}+2ab$$

$$-2ab\frac{b^2+a^2-c^2}{2ab}[\cos \text{rule}]$$

$$=2bc-b^2-c^2+a^2+2ca-a^2-c^2+b^2+2ab-b^2-a^2+c^2$$

$$=a^2+b^2+c^2+2ab+2bc+2ca$$

$$=(a+b+c)^2=RHS$$

Sine and Cosine Formulae and their Applications Ex-10.2 Q14

$$\sin^3 A\cos(B-C)+\sin^3 B.\cos(C-A)+\sin^3 C.\cos(A-B)$$

$$=\sin^2 A\sin A\cos(B-C)+\sin^2 B.\sin B\cos(C-A)+\sin^2 C.\sin C.\cos(A-B)$$

$$=\sin^2 A\sin(\pi-(B+C))\cos(B-C)+\sin^2 B.\sin(\pi-(A+C)).\cos(C-A)$$

$$+\sin^2 C.\sin(\pi-(A+B)).\cos(A-B)$$

$$=\sin^2 A\sin(B+C)\cos(B-C)+\sin^2 B.\sin(C+A).\cos(C-A)$$

$$+\sin^2 C.\sin(A+B).\cos(A-B)$$

$$=\sin^2 A.(\sin 2B+\sin 2C)+\sin^2 B.(\sin 2C+\sin 2A)+\sin^2 C.(\sin 2A+\sin 2B)$$

$$=\sin^2 A.(2\sin B.\cos B+2\sin C\cos C)+\sin^2 B.(2\sin C\cos C+2\sin A\cos A)$$

$$+\sin^2 C.(2\sin A\cos A+2\sin B\cos B)$$

$$=\sin^2 A.(2\sin B\cos B+2\sin C\cos C)+\sin^2 B.(2\sin C\cos C+2\sin A\cos A)$$

$$+\sin^2 C.(2\sin A\cos A+2\sin B\cos B)$$

$$=\sin^2 A.2\sin B\cos B+\sin^2 A.2\sin C\cos C+\sin^2 B.2\sin C\cos C$$

$$+\sin^2 B.2\sin A\cos A+\sin^2 C.2\sin A\cos A+\sin^2 C.2\sin B\cos B$$

$$=k^2a^2\,2kb\cos B+k^2a^2.2kc\cos C+k^2b^2.2ka\cos C$$

$$+k^2b^2.2ka\cos A+k^2c^2.2ka\cos A+k^2c^2.2kb\cos B$$

$$=k^3ab(a\cos B+b\cos A)+k^3ac(a\cos C+c\cos A)+k^3bc(c\cos B+b\cos C)$$

$$=k^3abc+k^3acb+k^3bca$$

$$=k^3 3abc$$

$$=3(k\sin A.k\sin B.k\sin C)$$

$$=3abc=RHS$$

*********** END **********