Polynomials Ex 2.3 Q2

Answer :

(i). Given $g(t)=t^2-3$

$f(t)=2t^4+3t^3-2t^2-9t-12$

Here, degree $(f(t))=4$ and

Degree $(g(t))=2$

Therefore, quotient $q(t)$ is of degree $4-2=2$

Remainder $r(t)$ is of degree 1 or less

Let $q(t)=at^2+bt+c$ and

$r(t)=pt+q$

Using division algorithm, we have

$f(t)=g(t)+q(t)+r(t)$

$2t^4+3t^3-2t^2-9t-12=(at^2+bt+c)(t^2-3)+pt+q$

$2t^4+3t^3-2t^2-9t-12=at^4+bt^3+ct^2-3at^2-3bt-3c+pt+q$

$2t^4+3t^3-2t^2-9t-12=at^4+bt^3-t^2(3a-c)-t(3b-p)-3c+q$

Equating co-efficient of various powers of t, we get

On equating the co-efficient of t^4

$2t^4=at^4$

$2\cancel{t^4}=a\cancel{t^4}$

$2=a$

On equating the co-efficient of t^3

$3t^3 = bt^3$

$3\cancel{t^3} = b\cancel{t^3}$

$3 = b$

On equating the co-efficient of t^2

$2 = 3a - c$

Substituting $a = 2$, we get

$2 = 3 \times 2 - c$

$2 = 6 - c$

$2 - 6 = -c$

$\cancel{-}4 = \cancel{-}c$

$c = 4$

On equating the co-efficient of t

$9 = 3b - p$

Substituting $b = 3$, we get

$9 = 3 \times 3 - p$

$9 = 9 - p$

$9 - 9 = -p$

$0 = -p$

$p = 0$

On equating constant term

$-12 = -3c + q$

Substituting $c = 4$, we get

$$-12 = 3 \times 4 + q$$

$$-12 = -12 + q$$

$$-12 + 12 = +q$$

$$0 = q$$

Quotient $q(t) = at^2 + bt + c$

$= 2t^2 + 3t + 4$

Remainder $r(t) = pt + q$

$= 0t + 0$

$= 0$

Clearly, $r(t) = 0$

Hence, $g(t)$ is a factor of $f(t)$.

(ii) Given

$f(x) = x^5 - 4x^3 + x^2 + 3x + 1$

$g(x) = x^3 - 3x + 1$

Here, Degree $(f(x)) = 5$ and

Degree $(g(x)) = 3$

Therefore, quotient $q(x)$ is of degree $5 - 3 = 2$

Remainder $r(x)$ is of degree1

Let $q(x) = ax^2 + bx + c$ and

$r(x) = px + q$

********** END **********