



Exercise 7A

Question 15

(i) To prove $\sin^6 \theta + \cos^6 \theta = 1 - 3 \sin^2 \theta \cos^2 \theta$

We know, $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

put $a = \sin^2 \theta, b = \cos^2 \theta$

$$\begin{aligned}\therefore \sin^6 \theta + \cos^6 \theta &= (\sin^2 \theta + \cos^2 \theta)^3 - 3 \sin^2 \theta \cos^2 \theta (\sin^2 \theta + \cos^2 \theta) \\ &= 1 - 3 \sin^2 \theta \cos^2 \theta = \text{RHS}\end{aligned}$$

$\therefore \text{LHS} = \text{RHS}$

$$\begin{aligned}\text{(ii) LHS} &= \sin^2 \theta + \cos^4 \theta = 1 - \cos^2 \theta + \cos^4 \theta \\ &= 1 - \cos^2 \theta (1 - \cos^2 \theta) \\ &= 1 - \cos^2 \theta \sin^2 \theta\end{aligned}$$

$$\begin{aligned}\text{RHS} &= \cos^2 \theta + \sin^4 \theta = 1 - \sin^2 \theta + \sin^4 \theta \\ &= 1 - \sin^2 \theta (1 - \sin^2 \theta) = (1 - \sin^2 \theta \cos^2 \theta)\end{aligned}$$

$\therefore \text{LHS} = \text{RHS}$

$$\begin{aligned}\text{(iii) } \operatorname{cosec}^4 \theta - \operatorname{cosec}^2 \theta \\ \text{LHS} &= \operatorname{cosec}^2 \theta (\operatorname{cosec}^2 \theta - 1) \\ &= (1 + \cot^2 \theta) \cot^2 \theta \\ &= \cot^2 \theta + \cot^4 \theta = \text{RHS}\end{aligned}$$

$\therefore \text{LHS} = \text{RHS}$

Question 16

$$(i) \text{ RHS} = (\sec \theta - \tan \theta)^2 = \left(\frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} \right)^2$$

$$= \frac{(1 - \sin \theta)^2}{\cos^2 \theta}$$

$$= \frac{(1 - \sin \theta)^2}{1 - \sin^2 \theta}$$

$$= \frac{(1 - \sin \theta)^2}{(1 - \sin \theta)(1 + \sin \theta)}$$

$$= \frac{1 - \sin \theta}{1 + \sin \theta} = \text{LHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

$$(ii) \text{ RHS} = (\operatorname{cosec} \theta + \cot \theta)^2$$

$$= \left(\frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta} \right)^2$$

$$= \frac{(1 + \cos \theta)^2}{\sin^2 \theta}$$

$$= \frac{(1 + \cos \theta)^2}{1 - \cos^2 \theta}$$

$$= \frac{(1 + \cos \theta)^2}{(1 + \cos \theta)(1 - \cos \theta)}$$

$$= \frac{1 + \cos \theta}{1 - \cos \theta} = \text{LHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

***** END *****