Arithematic Progressions Ex 19.5 Q5

(i) If $(a-c)^2 = 4(a-b)(b-c)$

Then,

$$a^2+c^2-2ac = 4\{ab-b^2-ac+bc\}$$

$\Rightarrow \quad a^2+c^2 4b^2+2ac-4ab-4bc = 0$

$\Rightarrow \quad (a+c-2b)^2 = 0 \qquad \left[\text{Using } (a+b+c)^2 = a^2+b^2+c^2+2ab+2ac+2bc\right]$

$\therefore \quad a+c-2b = 0$

or $\quad a+c = 2b$

and since,

a, b, c are in A.P $\qquad$ [Given]

$a+c = 2b$

Hence proved.

$$(a-c)^2 = 4(a-b)(b-c)$$

(ii) If $a^2+c^2+4ac = 2(ab+bc+ca)$

Then,

$$a^2+c^2+2ac-2ab-2bc = 0$$

or $\quad (a+c-b)^2 - b^2 = 0 \qquad \left[\because (a+b+c)^2 = a^2+b^2+c^2+2ab+2ac+2bc\right]$

or $\quad b = a+c-b$

or $\quad 2b = a+c$

$$b = \frac{a+c}{2}$$

and since,

a, b, c are in A.P

$$b = \frac{a+c}{2}$$

Thus, $a^2+c^2+4ac = 2(ab+bc+ca)$Hence proved.

(iii) If $a^3+c^3+6abc = 8b^3$

or $\quad a^3+c^3-(2b)^3+6abc = 0$

or $\quad a^3+(-2b)^3+c^3+3\times a\times(-2b)\times c = 0$

$\therefore \quad (a-2b+c) = 0 \qquad \left[\begin{array}{l}\because x^3+y^3+z^3+3xyz = 0\\ \text{or if } x+y+z = 0\end{array}\right]$

or $\quad a+c = 2b$

$a-b = c-b$

and since, a, b, c are in A.P

Thus, $a-b = c-b$

Hence proved.$a^3+c^3+6abc = 8b^3$

Arithematic Progressions Ex 19.5 Q6

Here,

$$a\left(\frac{1}{b}+\frac{1}{c}\right),\ b\left(\frac{1}{c}+\frac{1}{a}\right),\ c\left(\frac{1}{a}+\frac{1}{b}\right) \text{ are in A.P.}$$

$$\Rightarrow \quad a\left(\frac{1}{b}+\frac{1}{c}\right)+1,\ b\left(\frac{1}{c}+\frac{1}{a}\right)+1,\ c\left(\frac{1}{a}+\frac{1}{b}\right)+1 \text{ are in A.P.}$$

$$\Rightarrow \quad \left(\frac{ac+ab+bc}{bc}\right),\ \left(\frac{ab+bc+ac}{ac}\right),\ \left(\frac{cb+ac+ab}{ab}\right) \text{ are in A.P.}$$

$$\Rightarrow \quad \frac{1}{bc},\ \frac{1}{ac},\ \frac{1}{ab} \text{ are in A.P.}$$

$$\Rightarrow \quad \frac{abc}{bc},\ \frac{abc}{ac},\ \frac{abc}{ab} \text{ are in A.P.}$$

$$\Rightarrow \quad a,\ b,\ c \text{ are in A.P.}$$

Arithematic Progressions Ex 19.5 Q7

x, y and z are in AP.
Let d be the common difference then,
$y = x+d$ and $z = x+2d$

To show $x^2 + xy + y^2$, $z^2 + zx + x^2$ and $y^2 + yz + z^2$ are consecutive terms of an A.P., it is enough to show that,
$$\left(z^2 + zx + x^2\right) - \left(x^2 + xy + y^2\right) = \left(y^2 + yz + z^2\right) - \left(z^2 + zx + x^2\right)$$

$$\begin{aligned}
\text{LHS} &= \left(z^2 + zx + x^2\right) - \left(x^2 + xy + y^2\right)\\
&= z^2 + zx - xy - y^2\\
&= \left(x+2d\right)^2 + \left(x+2d\right)x - x\left(x+d\right) - \left(x+d\right)^2\\
&= x^2 + 4xd + 4d^2 + x^2 + 2xd - x^2 - xd - x^2 - 2xd - d^2\\
&= 3xd + 3d^2
\end{aligned}$$

$$\begin{aligned}
\text{RHS} &= \left(y^2 + yz + z^2\right) - \left(z^2 + zx + x^2\right)\\
&= y^2 + yz - zx - x^2\\
&= \left(x+d\right)^2 + \left(x+d\right)\left(x+2d\right) - \left(x+2d\right)x - x^2\\
&= x^2 + 2dx + d^2 + x^2 + 2dx + xd + 2d^2 - x^2 - 2dx - x^2\\
&= 3xd + 3d^2
\end{aligned}$$

$\therefore$ LHS = RHS
$\therefore$ $x^2 + xy + y^2$, $z^2 + zx + x^2$ and $y^2 + yz + z^2$ are consecutive terms of an A.P.

*********** END **********