Indefinite Integrals Ex 19.8 Q11

Let $I = \int \sqrt{\frac{1-\sin 2x}{1+\sin 2x}}dx$ then,

$$I = \int \sqrt{\frac{1-\cos\left(\frac{\pi}{2}-2x\right)}{1+\cos\left(\frac{\pi}{2}-2x\right)}}dx$$

$$= \int \sqrt{\frac{2\sin^2\left(\frac{\pi}{4}-x\right)}{2\cos^2\left(\frac{\pi}{4}-x\right)}}dx$$

$$= \int \sqrt{\tan^2\left(\frac{\pi}{4}-x\right)}dx$$

$$= \int \tan\left(\frac{\pi}{4}-x\right)dx$$

$$= \log\left|\cos\left(\frac{\pi}{4}-x\right)\right| + c$$

Indefinite Integrals Ex 19.8 Q12

Let $I = \int \frac{e^{3x}}{e^{3x}+1} dx \quad -----(i)$

Let $e^{3x} + 1 = t$, then,

$d\left(e^{3x} + 1\right) = dt$

$\Rightarrow \quad 3e^{3x} dx = dt$

$\Rightarrow \quad dx = \frac{dt}{3e^{3x}}$

Putting $e^{3x} + 1 = t$ and $dx = \frac{dt}{3e^{3x}}$ in equation (i), we get

$I = \int \frac{e^{3x}}{t} \times \frac{dt}{3e^{3x}}$

$= \frac{1}{3} \int \frac{dt}{t}$

$= \frac{1}{3} \log|t| + c$

$= \frac{1}{3} \log\left|3e^{3x} + 1\right| + c$

$\therefore \quad = \frac{1}{3} \log\left|3e^{3x} + 1\right| + c$

Indefinite Integrals Ex 19.8 Q13

Let $I = \int \frac{\sec x \tan x}{3\sec x + 5} dx \quad -----(i)$

Let $3\sec x + 5 = t$, then,

$\Rightarrow \quad d\left(3\sec x + 5\right) = dt$

$\Rightarrow \quad 3\sec x \tan x \, dx = dt$

$\Rightarrow \quad dx = \frac{dt}{3\sec x \tan x}$

Putting $3\sec x \tan x dx = t$ and $dx = \frac{dt}{3\sec x \tan x}$ in equation (i), we get

$I = \int \frac{\sec x \tan x}{t} \times \frac{dt}{3\sec x \tan x}$

$= \frac{1}{3} \int \frac{1}{t} dt$

$= \frac{1}{3} \log|t| + c$

$= \frac{1}{3} \log\left|3\sec x + 5\right| + c$

Indefinite Integrals Ex 19.8 Q14

Let $I = \int \frac{1-\cot x}{1+\cot x} dx$ then,

$$I = \int \frac{1 - \frac{\cos x}{\sin x}}{1 + \frac{\cos x}{\sin x}} dx$$

$$= \int \frac{\frac{\sin x - \cos x}{\sin x}}{\frac{\sin x + \cos x}{\sin x}} dx$$

$\Rightarrow \quad I = \int \frac{\sin x - \cos x}{\sin x + \cos x} dx \; ----(\text{i})$

Let $\sin x + \cos x = t.$ then,

$d(\sin x + \cos x) = dt$

$\Rightarrow \quad (\cos x - \sin x) dx = dt$

$\Rightarrow \quad -(\sin x - \cos x) dx = dt$

$\Rightarrow \quad dx = -\frac{dt}{\sin x - \cos x}$

Putting $\sin x + \cos x = t$ and $dx = -\frac{dt}{\sin x - \cos x}$ in equation (i),we get,

$$I = \int \frac{\sin x - \cos x}{t} \times \frac{-dt}{\sin x - \cos x}$$

$$= \int \frac{-dt}{t}$$

$$= -\log|t| + c$$

$$= -\log|\sin x + \cos x| + c$$

Indefinite Integrals Ex 19.8 Q15

Let $I = \int \frac{\sec x \operatorname{cosec} x}{\log(\tan x)} dx$ then,

Let $\log(\tan x) = t$ then,

$d[\log(\tan x)] = dt$

$\Rightarrow \quad \sec x \operatorname{cosec} x \, dx = dt \qquad \left[\because \quad \frac{d}{dx}(\log \tan x) = \sec x \operatorname{cosec} x\right]$

$\Rightarrow \quad dx = \frac{dt}{\sec x \operatorname{cosec} x}$

Putting $\log(\tan x) = t$ and $dx = \frac{dt}{\sec x \operatorname{cosec} x}$ in equation (i),we get,

$$I = \int \frac{\sec x \operatorname{cosec} x}{t} \times \frac{dt}{\sec x \operatorname{cosec} x}$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + c$$

$$= \log|\log \tan x| + c$$

*********** END **********