Trigonometric Identities Ex 6.1 Q64

Answer :

We have to prove $\dfrac{\sin A}{\sec A+\tan A-1}+\dfrac{\cos A}{\operatorname{cosec}A+\cot A-1}=1$

We know that, $\sin^2 A+\cos^2 A=1$

So,

$$\begin{aligned}
&\frac{\sin A}{\sec A+\tan A-1}+\frac{\cos A}{\operatorname{cosec}A+\cot A-1}\\
&=\frac{\sin A}{\dfrac{1}{\cos A}+\dfrac{\sin A}{\cos A}-1}+\frac{\cos A}{\dfrac{1}{\sin A}+\dfrac{\cos A}{\sin A}-1}\\
&=\frac{\sin A}{\dfrac{1+\sin A-\cos A}{\cos A}}+\frac{\cos A}{\dfrac{1+\cos A-\sin A}{\sin A}}\\
&=\frac{\sin A\cos A}{1+\sin A-\cos A}+\frac{\sin A\cos A}{1+\cos A-\sin A}\\
&=\frac{\sin A\cos A(1+\cos A-\sin A)+\sin A\cos A(1+\sin A-\cos A)}{(1+\sin A-\cos A)(1+\cos A-\sin A)}\\
&=\frac{\sin A\cos A(1+\cos A-\sin A+1+\sin A-\cos A)}{\{1+(\sin A-\cos A)\}\{1-(\sin A-\cos A)\}}\\
&=\frac{2\sin A\cos A}{1-(\sin A-\cos A)^2}\\
&=\frac{2\sin A\cos A}{1-(\sin^2 A-2\sin A\cos A+\cos^2 A)}\\
&=\frac{2\sin A\cos A}{1-(\sin^2 A+\cos^2 A-2\sin A\cos A)}\\
&=\frac{2\sin A\cos A}{1-(1-2\sin A\cos A)}\\
&=\frac{2\sin A\cos A}{1-1+2\sin A\cos A}\\
&=\frac{2\sin A\cos A}{2\sin A\cos A}\\
&=1
\end{aligned}$$

Hence proved.

Trigonometric Identities Ex 6.1 Q65

Answer :

We have to prove $\frac{\tan A}{(1+\tan^2 A)^2}+\frac{\cot A}{(1+\cot^2 A)^2}=\sin A\cos A$

We know that, $\sin^2 A+\cos^2 A=1$.

So,

$$\frac{\tan A}{(1+\tan^2 A)^2}+\frac{\cot A}{(1+\cot^2 A)^2}$$
$$=\frac{\tan A}{(\sec^2 A)^2}+\frac{\cot A}{(\operatorname{cosec}^2 A)^2}$$
$$=\frac{\tan A}{\sec^4 A}+\frac{\cot A}{\operatorname{cosec}^4 A}$$
$$=\frac{\frac{\sin A}{\cos A}}{\frac{1}{\cos^4 A}}+\frac{\frac{\cos A}{\sin A}}{\frac{1}{\sin^4 A}}$$
$$=\frac{\sin A\cos^4 A}{\cos A}+\frac{\cos A\sin^4 A}{\sin A}$$
$$=\sin A\cos^3 A+\cos A\sin^3 A$$

$$=\sin A\cos A(\cos^2 A+\sin^2 A)$$
$$=\sin A\cos A$$

Hence proved.

*********** END **********