Chapter 9 Continuity Ex 9.2 Q5

We have given that $f(x)$ is continuous on $[0.\infty]$

$\therefore$ $f(x)$ is continuous at $x = 1$ and $x = \sqrt{2}$

$\therefore$ At $x = 1$, LHL = RHL = $f(1)$ .....(A)

$f(1) = a$ .....(1)

$$\text{LHL} = \lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} \frac{(1-h)^2}{a} = \frac{1}{a}$$

Using (A) we get,

$$a = \frac{1}{a} \quad \Rightarrow a^2 = 1 \Rightarrow \quad a = \pm 1$$

At $x = \sqrt{2}$ LHL = RHL = $f(\sqrt{2})$ .....(B)

$$f(\sqrt{2}) = \frac{2b^2 - 4b}{(\sqrt{2})^2} = \frac{2b^2 - 4b}{2} = b^2 - 2b \qquad ......(2)$$

$$\text{LHL} = \lim_{x \to \sqrt{2}^-} f(x) = \lim_{h \to 0} f(\sqrt{2} - h) = \lim_{h \to 0} a = a.$$

So, using (B), we get,

$b^2 - 2b = a$

For $a = 1$, $\quad b^2 - 2b - 1 = 0$

$$\Rightarrow b = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$$

For $a = -1$ $\quad b^2 - 2b + 1 = 0$

$\Rightarrow (b-1)^2 = 0 \quad \Rightarrow b = 1$

Thus, $a = -1$, $\quad b = 1$ or $a = 1$, $b = 1 \pm \sqrt{2}$

Chapter 9 Continuity Ex 9.2 Q6

Since, $f(x)$ is continuous on $[0,\pi]$

$f(x)$ is continuous at $x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$

At $x = \frac{\pi}{4}$,

LHL = RHL = $f\left(\frac{\pi}{4}\right)$.....(A)

Now, $f\left(\frac{\pi}{4}\right) = 2\frac{\pi}{4}.\cot\left(\frac{\pi}{4}\right) + b = \frac{\pi}{2}.1 + b = \frac{\pi}{2} + b$ ....(1)

LHL = $\lim_{x \to \frac{\pi}{4}^-} f(x) = \lim_{h \to 0} f\left(\frac{\pi}{4} - h\right) = \lim_{h \to 0}\left(\frac{\pi}{4} - h\right) + a\sqrt{2}\sin\left(\frac{\pi}{4} - h\right) = \frac{\pi}{4} + a\sqrt{2}.\frac{1}{\sqrt{2}} = \frac{\pi}{4} + a$

Thus, using (A)

$\frac{1}{2} + b = \frac{\pi}{4} + a$

$a - b = \frac{\pi}{4}$.....(B)

At $x = \frac{\pi}{2}$

LHL = RHL = $f\left(\frac{\pi}{2}\right)$.....(C)

Now, $f\left(\frac{\pi}{2}\right) = a\cos 2.\frac{\pi}{2} - b\sin\frac{\pi}{2} = -a - b$....(2)

LHL = $\lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{h \to 0} f\left(\frac{\pi}{2} - h\right)$

$$= \lim_{h \to 0} 2\left(\frac{\pi}{2} - h\right)\cot\left(\frac{\pi}{2} - h\right) + b = \pi \times 0 + b = b$$

using (C), we get,

$-a - b = b \quad \Rightarrow \quad 2b = -a \quad \Rightarrow \quad b = \frac{-a}{2}$

from (B), $a + \frac{a}{2} = \frac{\pi}{4}$

$$\Rightarrow \qquad \frac{3}{2}a = \frac{\pi}{4}$$

$$\Rightarrow \qquad a = \frac{\pi}{6}$$

and $b = \frac{-a}{2} = \frac{-\pi}{12}$

Thus, $a = \frac{\pi}{6}$, $\quad b = \frac{-\pi}{12}$

Chapter 9 Continuity Ex 9.2 Q7

It is given that the $f(x)$ is continuous on $[0,8]$

$f(x)$ is continuous at $x = 2$ and $x = 4$.

Now, At $x = 2$

LHL = RHL = $f(2) \ldots (A)$

$f(2) = 3 \times 2 + 2 = 8 \ldots (1)$

LHL = $\lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h) = \lim_{h \to 0} (2-h)^2 + a(2-h) + b = 4 + 2a + b$

from (A)

$4 + 2a + b = 8$

$2a + b = 4 \ldots (B)$

Now, At $x = 4$

LHL = RHL = $f(4) \ldots (C)$

$f(4) = 3 \times 4 + 2 = 14 \ldots (2)$

RHL = $\lim_{x \to 4^+} f(x) = \lim_{h \to 0} f(4+h) = \lim_{h \to 0} 2a(4+h) + 5b = 8a + 5b$

From (C), we get,

$8a + 5b = 14 \ldots (D)$

Solving (B) and (D), we get,

$a = 3$ and $b = -2$

Chapter 9 Continuity Ex 9.2 Q8

The function will be continuous on $\left[0, \frac{\pi}{2}\right]$ if it is continuous at every point in $\left[0, \frac{\pi}{2}\right]$

Let us consider the point $x = \frac{\pi}{4}$,

We must have,

LHL = RHL = $f\left(\frac{\pi}{4}\right) \ldots (A)$

$$\text{LHL} = \lim_{x \to \frac{\pi}{4}^-} f(x) = \lim_{h \to 0} f\left(\frac{\pi}{4} - h\right) = \lim_{h \to 0} \frac{\tan\left(\frac{\pi}{4} - \frac{\pi}{4} + h\right)}{\cot + 2\left(\frac{\pi}{4} - h\right)} = \lim_{h \to 0} \frac{\tan h}{\tan 2h} \qquad \left[\because \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta\right]$$

$$= \lim_{h \to 0} \frac{\frac{\tan h}{h}}{\frac{\tan 2h}{h}} = \frac{1}{2}$$

Thus, using (A) we get,

$f\left(\frac{\pi}{4}\right) = \frac{1}{2}$

Hence, $f(x)$ will be continuous on $\left[0, \frac{\pi}{2}\right]$ if $f\left(\frac{\pi}{4}\right) = \frac{1}{2}$.

Chapter 9 Continuity Ex 9.2 Q9

When $x < 2$, we have

$f(x) = 2x - 1$, which is a polynomial of degree 1.

So, $f(x)$ is continuous for $x < 2$.

When $x > 2$, we have

$f(x) = \dfrac{3x}{2}$, which is again a polynomial of degree 1.

So, $f(x)$ is continuous for $x > 2$.

Now, consider the point $x = 2$

$$f(2) = \frac{3 \times 2}{2} = 3$$

$$\text{LHL} = \lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h) = \lim_{h \to 0} 2(2-h) - 1 = 3$$

$$\text{RHL} = \lim_{x \to 2^+} f(x) = \lim_{h \to 0} f(2+h) = \lim_{h \to 0} \frac{3(2+h)}{2} = 3$$

$$\text{LHL} = \text{RHL} = f(2) = 3$$

Thus, $f(x)$ is continuous at $x = 2$

Hence, $f(x)$ is continuous every where.

*********** END **********