Mathematical Induction Ex 12.2 Q7

Let $P(n): \frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\ldots+\frac{1}{(3n-2)(3n+1)}=\frac{n}{3n+1}$

Put $n=1$

$$P(1): \frac{1}{1.4}=\frac{1}{4}$$
$$\frac{1}{4}=\frac{1}{4}$$

$\Rightarrow \quad P(n)$ is true for $n=1$

Let $P(n)$ is true for $n=k$, so

$$\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\ldots+\frac{1}{(3k-2)(3k+1)}=\frac{k}{3k+1} \qquad ---(1)$$

We have to show that,

$$\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\ldots+\frac{1}{(3k-2)(3k+1)}+\frac{1}{(3k+1)(3k+4)}=\frac{(k+1)}{(3k+4)}$$

Now,

$$\left\{\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\ldots+\frac{1}{(3k-2)(3k+1)}\right\}+\frac{1}{(3k+1)(3k+4)}$$

Now,

$$\left\{\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\ldots+\frac{1}{(3k-2)(3k+1)}\right\}+\frac{1}{(3k+1)(3k+4)}$$

$$=\frac{k}{(3k+1)}+\frac{1}{(3k+1)(3k+4)}$$

$$=\frac{1}{(3k+1)}\left[\frac{k}{1}+\frac{1}{(3k+4)}\right]$$

$$=\frac{1}{(3k+1)}\left[\frac{k(3k+4)+1}{(3k+4)}\right]$$

$$=\frac{1}{(3k+1)}\left[\frac{3k^2+4k+1}{(3k+4)}\right]$$

$$=\frac{1}{(3k+1)}\frac{(3k^2+3k+k+1)}{(3k+4)}$$

$$=\frac{3k(k+1)+(k+1)}{(3k+1)(3k+4)}$$

$$=\frac{(k+1)(3k+1)}{(3k+1)(3k+4)}$$

$$=\frac{(k+1)}{(3k+4)}$$

$\Rightarrow$ $P(n)$ is true for $n=k+1$

$\Rightarrow$ $P(n)$ is true for all $n\in N$ by PMI

Mathematical Induction Ex 12.2 Q8

Let $P(n):\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\ldots+\frac{1}{(2n+1)(2n+3)}=\frac{n}{3(2n+3)}$

Put $n=1$

$$\frac{1}{3.5}=\frac{1}{3(5)}$$

$$\frac{1}{15}=\frac{1}{15}$$

$\Rightarrow$ $P(n)$ is true for $n=1$

Let $P(n)$ is true for $n=k$, so

$$\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\ldots+\frac{1}{(2k+1)(2k+3)}=\frac{k}{3(2k+3)} \qquad ---(1)$$

We have to show that,

$$\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\ldots+\frac{1}{(2k+1)(2k+3)}+\frac{1}{(2k+3)(2k+5)}=\frac{(k+1)}{3(2k+5)}$$

Now,

$$\left\{\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\ldots+\frac{1}{(2k+1)(2k+3)}\right\}+\frac{1}{(2k+3)(2k+5)}$$

$$=\frac{k}{3(2k+3)}+\frac{1}{(2k+3)(2k+5)} \qquad \text{[Using equation (1)]}$$

$$= \frac{1}{(2k+3)}\left[\frac{k}{3} + \frac{1}{(2k+5)}\right]$$

$$= \frac{1}{(2k+3)}\left[\frac{k(2k+5)+3}{(2k+5)}\right]$$

$$= \frac{1}{(2k+3)}\left[\frac{2k^2+5k+3}{(2k+5)}\right]$$

$$= \frac{1}{(2k+3)}\left[\frac{2k^2+2k+3k+3}{(2k+5)}\right]$$

$$= \frac{1}{(2k+3)}\left[\frac{2k(k+1)+3(k+1)}{(2k+5)}\right]$$

$$= \frac{1}{(2k+3)}\left[\frac{(k+1)(2k+3)}{(2k+5)}\right]$$

$$= \frac{(k+1)}{2k+5}$$

$\Rightarrow$ $P(n)$ is true for $n = k+1$

$\Rightarrow$ $P(n)$ is true for all $n \in N$ by *PMI*

*********** END **********