Question 5:

Check whether the relation R in **R** defined as R = $\{(a, b): a \le b^3\}$ is reflexive, symmetric or transitive.

Answer

R = $\{(a, b): a \le b^3\}$

It is observed that $\left(\frac{1}{2}, \frac{1}{2}\right) \notin \mathrm{R}$ as $\frac{1}{2} > \left(\frac{1}{2}\right)^3 = \frac{1}{8}$.

∴ R is not reflexive.

Now,

$(1, 2) \in \mathrm{R}$ (as $1 < 2^3 = 8$)

But,

$(2, 1) \notin \mathrm{R}$ (as $2^3 > 1$)

∴ R is not symmetric.

We have $\left(3, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{6}{5}\right) \in \mathrm{R}$ as $3 < \left(\frac{3}{2}\right)^3$ and $\frac{3}{2} < \left(\frac{6}{5}\right)^3$.

But $\left(3, \frac{6}{5}\right) \notin \mathrm{R}$ as $3 > \left(\frac{6}{5}\right)^3$.

∴ R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

Question 6:

Show that the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} is symmetric but neither reflexive nor transitive.

Answer

Let *A* = {1, 2, 3}.

A relation R on *A* is defined as R = {(1, 2), (2, 1)}.

It is seen that (1, 1), (2, 2), (3, 3) ∉R.

∴ R is not reflexive.

Now, as (1, 2) ∈ R and (2, 1) ∈ R, then R is symmetric.

Now, (1, 2) and (2, 1) ∈ R

However,

(1, 1) ∉ R

∴ R is not transitive.

Hence, R is symmetric but neither reflexive nor transitive.

Question 7:

Show that the relation R in the set *A* of all the books in a library of a college, given by R = {(*x*, *y*): *x* and *y* have same number of pages} is an equivalence relation.

Answer

Set *A* is the set of all books in the library of a college.

R = {*x*, *y*): *x* and *y* have the same number of pages}

Now, R is reflexive since (*x*, *x*) ∈ R as *x* and *x* has the same number of pages.

Let (*x*, *y*) ∈ R ⇒ *x* and *y* have the same number of pages.

⇒ *y* and *x* have the same number of pages.

⇒ (*y*, *x*) ∈ R

∴R is symmetric.

Now, let (*x*, *y*) ∈R and (*y*, *z*) ∈ R.

⇒ *x* and *y* and have the same number of pages and *y* and *z* have the same number of pages.

⇒ *x* and *z* have the same number of pages.

⇒ (*x*, *z*) ∈ R

∴R is transitive.

Hence, R is an equivalence relation.

Question 8:

Show that the relation R in the set *A* = {1, 2, 3, 4, 5} given by $\mathrm{R} = \{(a, b): |a-b| \text{ is even}\}$, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of 2, 4}.

Answer

A = {1, 2, 3, 4, 5}

$A = \{1, 2, 3, 4, 5\}$

$\mathrm{R} = \{(a, b): |a-b| \text{ is even}\}$

It is clear that for any element $a \in A$, we have $|a-a| = 0$ (which is even).

∴R is reflexive.

Let $(a, b) \in \mathrm{R}$.

$\Rightarrow |a-b|$ is even.

$\Rightarrow |-(a-b)| = |b-a|$ is also even.

$\Rightarrow (b, a) \in \mathrm{R}$

∴R is symmetric.

Now, let $(a, b) \in \mathrm{R}$ and $(b, c) \in \mathrm{R}$.

$\Rightarrow |a-b|$ is even and $|b-c|$ is even.

$\Rightarrow (a-b)$ is even and $(b-c)$ is even.

$\Rightarrow (a-c) = (a-b)+(b-c)$ is even. [Sum of two even integers is even]

$\Rightarrow |a-c|$ is even.

$\Rightarrow (a, c) \in \mathrm{R}$

∴R is transitive.

Hence, R is an equivalence relation.

Now, all elements of the set {1, 2, 3} are related to each other as all the elements of this subset are odd. Thus, the modulus of the difference between any two elements will be even.

Similarly, all elements of the set {2, 4} are related to each other as all the elements of this subset are even.

Also, no element of the subset {1, 3, 5} can be related to any element of {2, 4} as all elements of {1, 3, 5} are odd and all elements of {2, 4} are even. Thus, the modulus of the difference between the two elements (from each of these two subsets) will not be even.

Question 9:

Show that each of the relation R in the set $\mathrm{A} = \{x \in \mathbf{Z}: 0 \le x \le 12\}$, given by

(i) $\mathrm{R} = \{(a, b): |a-b| \text{ is a multiple of } 4\}$

(ii) $\mathrm{R} = \{(a, b): a = b\}$

is an equivalence relation. Find the set of all elements related to 1 in each case.

Answer

$A = \{x \in \mathbf{Z}: 0 \le x \le 12\} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

(i) $\mathrm{R} = \{(a, b): |a-b| \text{ is a multiple of } 4\}$

For any element $a \in$A, we have $(a, a) \in$ R as $|a-a| = 0$ is a multiple of 4.

∴R is reflexive.

Now, let $(a, b) \in$ R ⇒ $|a-b|$ is a multiple of 4.

$\Rightarrow |-(a-b)| = |b-a|$ is a multiple of 4.

$\Rightarrow (b, a) \in \mathrm{R}$

∴R is symmetric.

Now, let (a, b), $(b, c) \in$ R.

$\Rightarrow |a-b|$ is a multiple of 4 and $|b-c|$ is a multiple of 4.

$\Rightarrow (a-b)$ is a multiple of 4 and $(b-c)$ is a multiple of 4.

$\Rightarrow (a-c) = (a-b)+(b-c)$ is a multiple of 4.

$\Rightarrow |a-c|$ is a multiple of 4.

$\Rightarrow (a, c) \in \mathrm{R}$

∴ R is transitive.

Hence, R is an equivalence relation.

The set of elements related to 1 is {1, 5, 9} since

$|1-1| = 0$ is a multiple of 4,

$|5-1| = 4$ is a multiple of 4, and

$|9-1| = 8$ is a multiple of 4.

(ii) R = $\{(a, b): a = b\}$

For any element $a \in$A, we have $(a, a) \in$ R, since $a = a$.

∴R is reflexive.

Now, let $(a, b) \in$ R.

$\Rightarrow a = b$

$\Rightarrow b = a$

$\Rightarrow (b, a) \in \mathrm{R}$

∴R is symmetric.

Now, let $(a, b) \in$ R and $(b, c) \in$ R.

$\Rightarrow a = b$ and $b = c$

$\Rightarrow a = c$

$\Rightarrow (a, c) \in$ R

$\therefore$ R is transitive.

Hence, R is an equivalence relation.

The elements in R that are related to 1 will be those elements from set A which are equal to 1.

Hence, the set of elements related to 1 is {1}.

Question 10:

Given an example of a relation. Which is

(i) Symmetric but neither reflexive nor transitive.

(ii) Transitive but neither reflexive nor symmetric.

(iii) Reflexive and symmetric but not transitive.

(iv) Reflexive and transitive but not symmetric.

(v) Symmetric and transitive but not reflexive.

Answer

(i) Let $A = \{5, 6, 7\}$.

Define a relation R on A as R = {(5, 6), (6, 5)}.

Relation R is not reflexive as (5, 5), (6, 6), (7, 7) $\notin$ R.

Now, as (5, 6) $\in$ R and also (6, 5) $\in$ R, R is symmetric.

$\Rightarrow$(5, 6), (6, 5) $\in$ R, but (5, 5) $\notin$ R

$\therefore$R is not transitive.

Hence, relation R is symmetric but not reflexive or transitive.

(ii)Consider a relation R in **R** defined as:

R = $\{(a, b): a < b\}$

For any $a \in$ R, we have $(a, a) \notin$ R since a cannot be strictly less than a itself. In fact, $a = a$.

$\therefore$ R is not reflexive.

Now,

(1, 2) $\in$ R (as 1 < 2)

But, 2 is not less than 1.

$\therefore$ (2, 1) $\notin$ R

$\therefore$ R is not symmetric.

Now, let $(a, b), (b, c) \in$ R.

$\Rightarrow a < b$ and $b < c$

$\Rightarrow a < c$

$\Rightarrow (a, c) \in$ R

$\therefore$R is transitive.

Hence, relation R is transitive but not reflexive and symmetric.

(iii)Let $A = \{4, 6, 8\}$.

Define a relation R on A as:

$A = \{(4, 4), (6, 6), (8, 8), (4, 6), (6, 4), (6, 8), (8, 6)\}$

Relation R is reflexive since for every $a \in A$, $(a, a) \in$R i.e., (4, 4), (6, 6), (8, 8)} $\in$ R.

Relation R is symmetric since $(a, b) \in$ R $\Rightarrow (b, a) \in$ R for all $a, b \in$ R.

Relation R is not transitive since (4, 6), (6, 8) $\in$ R, but (4, 8) $\notin$ R.

Hence, relation R is reflexive and symmetric but not transitive.

(iv) Define a relation R in **R** as:

R = $\{a, b): a^3 \geq b^3\}$

Clearly $(a, a) \in$ R as $a^3 = a^3$.

$\therefore$R is reflexive.

Now,

(2, 1) $\in$ R (as $2^3 \geq 1^3$)

But,

(1, 2) $\notin$ R (as $1^3 < 2^3$)

$\therefore$ R is not symmetric.

Now,

Let $(a, b), (b, c) \in$ R.

$\Rightarrow a^3 \geq b^3$ and $b^3 \geq c^3$

$\Rightarrow a^3 \geq c^3$

$\Rightarrow (a, c) \in$ R

$\therefore$R is transitive.

Hence, relation R is reflexive and transitive but not symmetric.

(v) Let $A = \{-5, -6\}$.

Define a relation R on A as:

R = {(−5, −6), (−6, −5), (−5, −5)}

Relation R is not reflexive as (−6, −6) $\notin$ R.

Relation R is symmetric as (−5, −6) $\in$ R and (−6, −5}$\in$R.

It is seen that (−5, −6), (−6, −5) $\in$ R. Also, (−5, −5) $\in$ R.

$\therefore$The relation R is transitive.

Hence, relation R is symmetric and transitive but not reflexive.

*********** END **********