



Differentiation Ex 11.4 Q26

Here,

$$\sin(xy) + \frac{y}{x} = x^2 - y^2$$

Differentiating with respect to x ,

$$\Rightarrow \frac{d}{dx}(\sin xy) + \frac{d}{dx}\left(\frac{y}{x}\right) = \frac{d}{dx}(x^2) - \frac{d}{dx}(y^2)$$

$$\Rightarrow \cos(xy) \frac{d}{dx}(xy) + \left[\frac{x \frac{dy}{dx} - y \frac{d}{dx}(x)}{x^2} \right] = 2x - 2y \frac{dy}{dx} \quad \left[\begin{array}{l} \text{Using chain rule, quotient rule,} \\ \text{product rule} \end{array} \right]$$

$$\Rightarrow \cos(xy) \left[x \frac{dy}{dx} + y \frac{d}{dx}(x) \right] + \left[\frac{x \frac{dy}{dx} - y(1)}{x^2} \right] = 2x - 2y \frac{dy}{dx}$$

$$\Rightarrow \cos(xy) \left[x \frac{dy}{dx} + y(1) \right] + \frac{1}{x^2} \left(x \frac{dy}{dx} - y \right) = 2x - 2y \frac{dy}{dx}$$

$$\Rightarrow x \cos(xy) \frac{dy}{dx} + y \cos(xy) + \frac{1}{x} \frac{dy}{dx} - \frac{y}{x^2} = 2x - 2y \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} \left[x \cos(xy) + \frac{1}{x} + 2y \right] = \frac{y}{x^2} - y \cos(xy) + 2x$$

$$\Rightarrow \frac{dy}{dx} \left[\frac{x^2 \cos(xy) + 1 + 2xy}{x} \right] = \frac{1}{x^2} (y - x^2 y \cos xy + 2x^3)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x^3 + y - x^2 y \cos(xy)}{x(x^2 \cos xy + 1 + 2xy)}$$

Differentiation Ex 11.4 Q27

Here,

$$\sqrt{y+x} + \sqrt{y-x} = c$$

Differentiating with respect to x ,

$$\begin{aligned} \Rightarrow \quad \frac{d}{dx}(\sqrt{y+x}) + \frac{d}{dx}\sqrt{y-x} &= \frac{d}{dx}(c) \\ \Rightarrow \quad \frac{1}{2\sqrt{y+x}} \frac{d}{dx}(y+x) + \frac{1}{2\sqrt{y-x}} \frac{d}{dx}(y-x) &= 0 \end{aligned}$$

Using chain rule

$$\begin{aligned} \Rightarrow \quad \frac{1}{2\sqrt{y+x}} \left[\frac{dy}{dx} + 1 \right] + \frac{1}{2\sqrt{y-x}} \left[\frac{dy}{dx} - 1 \right] &= 0 \\ \Rightarrow \quad \frac{dy}{dx} \left(\frac{1}{2\sqrt{y+x}} \right) + \frac{dy}{dx} \left(\frac{1}{2\sqrt{y-x}} \right) &= \frac{1}{2\sqrt{y-x}} - \frac{1}{2\sqrt{y+x}} \\ \Rightarrow \quad \frac{dy}{dx} \times \frac{1}{2} \left[\frac{1}{\sqrt{y+x}} + \frac{1}{\sqrt{y-x}} \right] &= \frac{1}{2} \left[\frac{\sqrt{y+x} - \sqrt{y-x}}{\sqrt{y-x}\sqrt{y+x}} \right] \\ \Rightarrow \quad \frac{dy}{dx} \left[\frac{\sqrt{y-x} + \sqrt{y+x}}{\sqrt{y+x}\sqrt{y-x}} \right] &= \left[\frac{\sqrt{y+x} - \sqrt{y-x}}{\sqrt{y-x}\sqrt{y+x}} \right] \\ \Rightarrow \quad \frac{dy}{dx} = \frac{\sqrt{y+x} - \sqrt{y-x}}{\sqrt{y-x} + \sqrt{y+x}} \times \frac{(\sqrt{y+x} - \sqrt{y-x})}{(\sqrt{y+x} - \sqrt{y-x})} \end{aligned}$$

[rationalizing the denominator]

$$\begin{aligned} \Rightarrow \quad \frac{dy}{dx} &= \frac{(y+x) + (y-x) - 2\sqrt{y+x}\sqrt{y-x}}{y+x - y+x} \\ \Rightarrow \quad \frac{dy}{dx} &= \frac{2y - 2\sqrt{y^2 - x^2}}{2x} \\ \Rightarrow \quad \frac{dy}{dx} &= \frac{2y}{2x} - \frac{2\sqrt{y^2 - x^2}}{2x} \\ \Rightarrow \quad \frac{dy}{dx} &= \frac{y}{x} - \sqrt{\frac{y^2 - x^2}{x^2}} \\ \frac{dy}{dx} &= \frac{y}{x} - \sqrt{\frac{y^2}{x^2} - 1} \end{aligned}$$

Differentiation Ex 11.4 Q28

Here,

$$\tan(x+y) + \tan(x-y) = 1$$

Differentiating with respect to x ,

$$\begin{aligned} \Rightarrow \quad \frac{d}{dx} \tan(x+y) + \frac{d}{dx} \tan(x-y) &= \frac{d}{dx}(1) \\ \Rightarrow \quad \sec^2(x+y) \frac{d}{dx}(x+y) + \sec^2(x-y) \frac{d}{dx}(x-y) &= 0 && \text{[Using chain rule]} \\ \Rightarrow \quad \sec^2(x+y) \left[1 + \frac{dy}{dx} \right] + \sec^2(x-y) \left[1 - \frac{dy}{dx} \right] &= 0 \\ \Rightarrow \quad \sec^2(x+y) \frac{dy}{dx} - \sec^2(x-y) \frac{dy}{dx} &= -[\sec^2(x+y) + \sec^2(x-y)] \\ \Rightarrow \quad \frac{dy}{dx} [\sec^2(x+y) - \sec^2(x-y)] &= -[\sec^2(x+y) + \sec^2(x-y)] \\ \Rightarrow \quad \frac{dy}{dx} = \frac{\sec^2(x+y) + \sec^2(x-y)}{\sec^2(x-y) - \sec^2(x+y)} \end{aligned}$$

Differentiation Ex 11.4 Q29

Here,

$$e^x + e^y = e^{x+y}$$

Differentiating with respect to x using chain rule,

$$\begin{aligned} \Rightarrow \frac{d}{dx}(e^x) + \frac{d}{dx}e^y &= \frac{d}{dx}(e^{x+y}) \\ \Rightarrow e^x + e^y \frac{dy}{dx} &= e^{x+y} \frac{d}{dx}(x+y) \\ \Rightarrow e^x + e^y \frac{dy}{dx} &= e^{x+y} \left[1 + \frac{dy}{dx}\right] \\ \Rightarrow e^y \frac{dy}{dx} - e^{x+y} \frac{dy}{dx} &= e^{x+y} - e^x \\ \Rightarrow \frac{dy}{dx}(e^y - e^{x+y}) &= e^{x+y} - e^x \\ \Rightarrow \frac{dy}{dx} &= \left(\frac{e^x \times e^y - e^x}{e^y - e^x \times e^y}\right) \\ \Rightarrow \frac{dy}{dx} &= \frac{e^x(e^y - 1)}{e^y(1 - e^x)} \\ \Rightarrow \frac{dy}{dx} &= -\frac{e^x(e^y - 1)}{e^y(e^x - 1)} \end{aligned}$$

Differentiation Ex 11.4 Q30

It is given that, $\cos y = x \cos(a+y)$

$$\begin{aligned} \therefore \frac{d}{dx}[\cos y] &= \frac{d}{dx}[x \cos(a+y)] \\ \Rightarrow -\sin y \frac{dy}{dx} &= \cos(a+y) \cdot \frac{d}{dx}(x) + x \cdot \frac{d}{dx}[\cos(a+y)] \\ \Rightarrow -\sin y \frac{dy}{dx} &= \cos(a+y) + x \cdot [-\sin(a+y)] \frac{dy}{dx} \\ \Rightarrow [x \sin(a+y) - \sin y] \frac{dy}{dx} &= \cos(a+y) \quad \dots(1) \end{aligned}$$

$$\text{Since } \cos y = x \cos(a+y), x = \frac{\cos y}{\cos(a+y)}$$

Then, equation (1) reduces to

$$\begin{aligned} \left[\frac{\cos y}{\cos(a+y)} \cdot \sin(a+y) - \sin y \right] \frac{dy}{dx} &= \cos(a+y) \\ \Rightarrow [\cos y \cdot \sin(a+y) - \sin y \cdot \cos(a+y)] \cdot \frac{dy}{dx} &= \cos^2(a+y) \\ \Rightarrow \sin(a+y-y) \frac{dy}{dx} &= \cos^2(a+b) \\ \Rightarrow \frac{dy}{dx} &= \frac{\cos^2(a+b)}{\sin a} \end{aligned}$$

Hence, proved.

***** END *****

