Exercise 10D

Question 12:
Let the two consecutive positive integers be x, x + 1

$$\text{Then, } x^2+(x+1)^2=365$$
$$\Rightarrow x^2+x^2+1+2x=365$$
$$\Rightarrow 2x^2+2x+1-365=0$$
$$\Rightarrow 2x^2+2x-364=0$$
$$\Rightarrow x^2+x-182=0$$
$$\Rightarrow x^2+14x-13x-182=0$$
$$\Rightarrow x(x+14)-13(x+14)=0$$
$$\Rightarrow (x+14)(x-13)=0$$
$$\Rightarrow x+14=0 \quad \text{or} \quad x-13=0$$
$$x=-14 \quad \text{or} \quad x=13$$

But, -14 is not a positive integer
Hence, the required numbers are 13, 14.

Question 13:
Let x, y be the two natural numbers and x > y
∴ $x^2 - y^2 = 45$ ------(1)
Also, square of smaller number = 4 × larger number
⇒ $y^2 = 4x$ ---------(2)
Putting y^2 value of from (1), we get

$$x^2-4x=45$$
$$x^2-4x-45=0$$
$$\Rightarrow x^2-9x+5x-45=0 \quad \text{or} \quad x(x-9)+5(x-9)=0$$
$$\Rightarrow (x-9)(x+5)=0 \Rightarrow \quad x=9,-5$$
$$\text{But} \quad x\neq -5 \quad \therefore x=9$$
$$\text{from}(2), \quad y^2=4x=4\times 9=36$$
$$\therefore y=6$$

Thus, the two required numbers are 9 and 6.

Question 14:
Let the required number be x and y, hen

$$x^2 + y^2 = 25(x + y) - - - (1)$$

$$x^2 + y^2 = 50(x - y) - - - (2)$$

$\Rightarrow 25(x + y) = 50(x - y) \Rightarrow x + y = 2(x - y)$

$\Rightarrow x = 3y$

putting $x = 3y$ in (1), we get

$9y^2 + y^2 = 100y \Rightarrow 10y^2 - 100y = 0$

$10y(y - 10) = 0$

$\Rightarrow y = 10$

Hence, $x = 30$ and $y = 10$

Question 15:
Let the smaller part and larger part be x, 16 - x
Then,

$$\Rightarrow 2x^2 - (16 - x)^2 = 164$$

$$\Rightarrow 2x^2 - \left(256 + x^2 - 32x\right) = 164$$

$$\Rightarrow 2x^2 - 256 - x^2 + 32x = 164$$

$$\Rightarrow x^2 + 32x - 256 - 164 = 0$$

$$\Rightarrow x^2 + 32x - 420 = 0$$

$$\Rightarrow x^2 + 42x - 10x - 420 = 0$$

$$\Rightarrow x(x + 42) - 10(x + 42) = 0$$

$$(x + 42)(x - 10) = 0$$

$$x + 42 = 0 \quad \text{or} \quad x - 10 = 0$$

$$x = -42 \quad \text{or} \quad x = 10$$

-42 is not a positive part
Hence, the larger and smaller parts are 10, 6 respectively.

Question 16:
Let the numerator and denominator be x, x + 3
Then,

$$\frac{x}{(x+3)}+\frac{(x+3)}{x}=2\frac{9}{10}\Rightarrow\frac{x^2+(x+3)^2}{x(x+3)}=\frac{29}{10}$$

$$\Rightarrow\frac{x^2+x^2+9+6x}{x^2+3x}=\frac{29}{10}\Rightarrow 20x^2+90+60x=29x^2+87x$$

$$\Rightarrow 20x^2-29x^2+60x-87x+90=0$$

$$\Rightarrow -9x^2-27x+90=0$$

$$\Rightarrow x^2+3x-10=0$$

$$\Rightarrow x^2+(5x-2x)-10=0$$

$$\Rightarrow x(x+5)-2(x+5)=0$$

$$\Rightarrow (x+5)(x-2)=0$$

$$\Rightarrow x+5=0 \quad \text{or} \quad x-2=0$$

$$x=-5 \quad \text{or} \quad x=2$$

Hence, numerator and denominator are 2 and 5 respectively and fraction is 2/5.

********** END **********