Trigonometric Ratios of Compound Angles Ex 7.1 Q25

We have,

$$\tan x + \tan\left(x+\frac{\pi}{3}\right) + \tan\left(x+\frac{2\pi}{3}\right) = 3$$

$$\Rightarrow \quad \tan x + \left[\frac{\tan x + \tan\frac{\pi}{3}}{1-\tan x\tan\frac{\pi}{3}}\right] + \left[\frac{\tan x + \tan\left(\frac{2\pi}{3}\right)}{1-\tan x\tan\frac{2\pi}{3}}\right] = 3$$

$$\Rightarrow \quad \tan x + \left[\frac{\tan x + \sqrt{3}}{1-\sqrt{3}\tan x}\right] + \left[\frac{\tan x + \tan\left(\frac{\pi}{2}+\frac{\pi}{3}\right)}{1-\tan x\tan\left(\frac{\pi}{2}+\frac{\pi}{3}\right)}\right] = 3$$

$$\Rightarrow \quad \tan x + \frac{\tan x + \sqrt{3}}{1-\sqrt{3}\tan x} + \frac{\tan x - \cot\frac{\pi}{3}}{1+\tan x\cot\frac{\pi}{3}} = 3 \qquad \left[\because \tan\theta \text{ is negative in second quadrant}\right]$$

$$\Rightarrow \quad \tan x + \frac{\tan x + \sqrt{3}}{1-\sqrt{3}\tan x} + \frac{\tan x - \sqrt{3}}{1+\sqrt{3}\tan x} = 3$$

$$\Rightarrow \quad \tan x + \frac{\left(\tan x + \sqrt{3}\right)\left(1+\sqrt{3}\tan x\right) + \left(\tan x - \sqrt{3}\right)\left(1-\sqrt{3}\tan x\right)}{\left(1-\sqrt{3}\tan x\right)\left(1+\sqrt{3}\tan x\right)} = 3$$

$$\Rightarrow \quad \tan x + \frac{\tan x + \sqrt{3}\tan^2 x + \sqrt{3} + 3\tan x + \tan x - \sqrt{3}\tan^2 x - \sqrt{3} + 3\tan x}{1-\left(\sqrt{3}\tan x\right)^2} = 3$$

$$\Rightarrow \quad \tan x + \frac{8\tan x}{1-3\tan^2 x} = 3$$

$$\Rightarrow \quad \frac{\tan x\left(1-3\tan^2 x\right) + 8\tan x}{1-3\tan^2 x} = 3$$

$$\Rightarrow \quad \frac{\tan x - 3\tan^3 x + 8\tan x}{1-3\tan^2 x} = 3$$

$$\Rightarrow \quad \frac{9\tan x - 3\tan^3 x}{1-3\tan^2 x} = 3$$

$$\Rightarrow \quad \frac{3\left(3\tan x - \tan^3 x\right)}{1-3\tan^2 x} = 3$$

$$\Rightarrow \quad \frac{3\tan x - \tan^3 x}{1-3\tan^2 x} = 1$$

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q26

We have,

$$\sin(\alpha + \beta) = 1$$

$$\Rightarrow \quad \sin(\alpha + \beta) = \sin\frac{\pi}{2}$$

$$\Rightarrow \quad \alpha + \beta = \frac{\pi}{2} \qquad ---(i)$$

and, $\sin(\alpha - \beta) = \frac{1}{2}$

$$\Rightarrow \quad \sin(\alpha - \beta) = \sin\frac{\pi}{6}$$

$$\Rightarrow \quad \alpha - \beta = \frac{\pi}{6} \qquad ---(ii)$$

Adding equations (i) and (ii), we get

$$2\alpha = \frac{\pi}{2} + \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

$$\Rightarrow \quad \alpha = \frac{\pi}{3}$$

Putting $\alpha = \frac{\pi}{3}$ in equation (i), we get

$$\frac{\pi}{3} + \beta = \frac{\pi}{2}$$

$$\Rightarrow \quad \beta = \frac{\pi}{2} - \frac{\pi}{3}$$

$$\Rightarrow \quad \beta = \frac{3\pi - 2\pi}{6}$$

$$= \frac{\pi}{6}$$

$$\Rightarrow \quad \beta = \frac{\pi}{6}$$

Now, $\tan(\alpha + 2\beta) = \tan\left(\frac{\pi}{3} + 2 \times \frac{\pi}{6}\right)$

$= \tan\left(\frac{\pi}{3} + \frac{\pi}{3}\right)$

$= \tan\frac{2\pi}{3}$

$= \tan\left(\frac{\pi}{2} + \frac{\pi}{6}\right)$

$= -\cot\frac{\pi}{6}$ $\left[\because \tan\theta \text{ is negative in second quadrant}\right]$

$= -\sqrt{3}$

$\therefore$ $t(\alpha + 2\beta) = -\sqrt{3}$

and, $\tan(2\alpha + \beta) = \tan\left(2 \times \frac{\pi}{3} + \frac{\pi}{6}\right)$

$= \tan\left(\frac{2\pi}{3} + \frac{\pi}{6}\right)$

$= \tan\left(\frac{4\pi + \pi}{6}\right)$

$= \tan\left(\frac{5\pi}{6}\right)$

$= \tan\left(\frac{\pi}{2} + \frac{\pi}{3}\right)$

$= -\cot\frac{\pi}{3}$ $\left[\because \tan\theta \text{ is negative in second quadrant}\right]$

$= -\frac{1}{\sqrt{3}}$

$\therefore$ $\tan(2\alpha + \beta) = \frac{-1}{\sqrt{3}}$

Trigonometric Ratios of Compound Angles Ex 7.1 Q27

We have,

$6\cos\theta + 8\sin\theta = 9$ ---(i)

$\Rightarrow$ $8\sin\theta = 9 - 6\cos\theta$

$\Rightarrow$ $(8\sin\theta)^2 = (9 - 6\cos\theta)^2$ $[\because \text{Squaring both sides}]$

$\Rightarrow$ $64\sin^2\theta = 81 + 36\cos^2\theta - 108\cos\theta$

$\Rightarrow$ $64\sin^2\theta = 81 + 36\cos^2\theta - 108\cos\theta$

$\Rightarrow$ $64(1 - \cos^2\theta) = 81 + 36\cos^2\theta - 108\cos\theta$

$\Rightarrow$ $64 - 64\cos^2\theta = 81 + 36\cos^2\theta - 108\cos\theta$

$\Rightarrow$ $36\cos^2\theta + 64\cos^2\theta - 108\cos\theta + 81 - 64 = 0$

$\Rightarrow$ $100\cos^2\theta - 108\cos\theta + 17 = 0$ ---(ii)

Since α, β are roots of equation (ii).

Therefore, $\cos\alpha$ and $\cos\beta$ are roots of equation (ii)

$\therefore$ $\cos\alpha + \cos\beta = \frac{17}{100}$ ---(iii)

Again, $6\cos\theta + 8\sin\theta = 9$

$\Rightarrow$ $6\cos\theta = 9 - 8\sin\theta$

$\Rightarrow$ $(6\cos\theta)^2 = (9 - 8\sin\theta)^2$ $[\because \text{Squaring both sides}]$

$\Rightarrow$ $36\cos^2\theta = 81 + 64\sin^2\theta - 144\sin\theta$

$\Rightarrow$ $36(1 - \sin^2\theta) = 81 + 64\sin^2\theta - 144\cos\theta$

$\Rightarrow$ $36 - 36\sin^2\theta = 81 + 64\sin^2\theta - 144\sin\theta$

$\Rightarrow$ $64\sin^2\theta + 36\sin^2\theta - 144\sin\theta + 81 - 36 = 0$

$\Rightarrow$ $100\sin^2\theta - 144\sin\theta + 45 = 0$ ---(iv)

$$\therefore \quad \sin\alpha \times \sin\beta = \frac{45}{100} \qquad ---(v)$$

Now, $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$$= \frac{17}{100} - \frac{45}{100} \qquad \left[\text{Using equation (iii) and (v)}\right]$$

$$= -\frac{28}{100}$$

$$= -\frac{7}{25}$$

Now, $\sin(\alpha+\beta) = \sqrt{1-(\cos\theta)^2}$

$$= \sqrt{1-\left(-\frac{7}{25}\right)^2}$$

$$= \sqrt{1-\frac{49}{625}}$$

$$= \sqrt{\frac{625-49}{625}}$$

$$= \sqrt{\frac{576}{625}}$$

$$= \frac{24}{25}$$

$$\therefore \quad \sin(\alpha+\beta) = \frac{24}{25}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q28

$\sin(\alpha+\beta) = \sqrt{1-\cos^2(\alpha+\beta)}$

$$b^2+a^2 = (\cos\alpha+\cos\beta)^2 + (\sin\alpha+\sin\beta)^2$$

$$\Rightarrow \quad b^2+a^2 = \left(\cos^2\alpha+\sin^2\alpha\right) + \left(\cos^2\beta+\sin^2\beta\right) + 2\left(\cos\alpha\cos\beta+\sin\alpha\sin\beta\right)$$

$$\Rightarrow \quad b^2+a^2 = 1+1+2\cos(\alpha-\beta) = 2+2\cos(\alpha-\beta)$$

and, $b^2-a^2 = (\cos\alpha+\cos\beta)^2 - (\sin\alpha+\sin\beta)^2$

$$b^2-a^2 = \cos^2\alpha+\cos^2\beta-\sin^2\alpha-\sin^2\beta+2\left(\cos\alpha\cos\beta-\sin\alpha\sin\beta\right)$$

$$\Rightarrow \quad b^2-a^2 = \left(\cos^2\alpha-\sin^2\beta\right) + \left(\cos^2\beta-\sin^2\alpha\right) + 2\cos(\alpha+\beta)$$

$$\Rightarrow \quad b^2-a^2 = \cos(\alpha+\beta)\cos(\alpha-\beta) + \cos(\beta+\alpha)\cos(\beta-\alpha) + 2\cos(\alpha+\beta)$$

$$\Rightarrow \quad b^2-a^2 = 2\cos(\alpha+\beta)\cos(\alpha-\beta) + 2\cos(\alpha+\beta)$$

$$\left[\because \cos(\beta-\alpha) = \cos\{-(\alpha-\beta)\} = \cos(\alpha-\beta)\right]$$

$$\Rightarrow \quad b^2-a^2 = \cos(\alpha+\beta)\{2\cos(\alpha-\beta)+2\}$$

$$\Rightarrow \quad b^2-a^2 = \cos(\alpha+\beta)\left(b^2+a^2\right) \qquad \left[\text{Using (i)}\right]$$

Thus, $b^2-a^2 = \left(b^2+a^2\right)\cos(\alpha+\beta)$

$$\Rightarrow \quad \cos(\alpha+\beta) = \frac{b^2-a^2}{b^2+a^2}$$

$$\Rightarrow \quad \sin(\alpha+\beta) = \sqrt{1-\left(\frac{b^2-a^2}{b^2-a^2}\right)^2} = \sqrt{\frac{4a^2b^2}{\left(a^2+b^2\right)^2}} = \frac{2ab}{a^2+b^2}$$

$$b^2 + a^2 = (\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2$$

$$\Rightarrow \quad b^2 + a^2 = \left(\cos^2\alpha + \sin^2\alpha\right) + \left(\cos^2\beta + \sin^2\beta\right) + 2\left(\cos\alpha\cos\beta + \sin\alpha\sin\beta\right)$$

$$\Rightarrow \quad b^2 + a^2 = 1 + 1 + 2\cos(\alpha - \beta) = 2 + 2\cos(\alpha - \beta)$$

and, $b^2 - a^2 = (\cos\alpha + \cos\beta)^2 - (\sin\alpha + \sin\beta)^2$

$$b^2 - a^2 = \cos^2\alpha + \cos^2\beta - \sin^2\alpha - \sin^2\beta + 2\left(\cos\alpha\cos\beta - \sin\alpha\sin\beta\right)$$

$$\Rightarrow \quad b^2 - a^2 = \left(\cos^2\alpha - \sin^2\beta\right) + \left(\cos^2\beta - \sin^2\alpha\right) + 2\cos(\alpha + \beta)$$

$$\Rightarrow \quad b^2 - a^2 = \cos(\alpha + \beta)\cos(\alpha - \beta) + \cos(\beta + \alpha)\cos(\beta - \alpha) + 2\cos(\alpha + \beta)$$

$$\Rightarrow \quad b^2 - a^2 = 2\cos(\alpha + \beta)\cos(\alpha - \beta) + 2\cos(\alpha + \beta)$$

$$\left[\because \cos(\beta - \alpha) = \cos\{-(\alpha - \beta)\} = \cos(\alpha - \beta)\right]$$

$$\Rightarrow \quad b^2 - a^2 = \cos(\alpha + \beta)\{2\cos(\alpha - \beta) + 2\}$$

$$\Rightarrow \quad b^2 - a^2 = \cos(\alpha + \beta)\left(b^2 + a^2\right) \qquad [\text{Using (i)}]$$

Thus, $b^2 - a^2 = \left(b^2 + a^2\right)\cos(\alpha + \beta)$

$$\Rightarrow \quad \cos(\alpha + \beta) = \frac{b^2 - a^2}{b^2 + a^2}$$

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