



Hence, $\theta = \frac{\pi}{3}$ and the components of \vec{a} are $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right)$.

Question 4:

Show that

$$(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$$

Answer

$$\begin{aligned} & (\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) \\ &= (\vec{a} - \vec{b}) \times \vec{a} + (\vec{a} - \vec{b}) \times \vec{b} \quad [\text{By distributivity of vector product over addition}] \\ &= \vec{a} \times \vec{a} - \vec{b} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{b} \quad [\text{Again, by distributivity of vector product over addition}] \\ &= \vec{0} + \vec{a} \times \vec{b} + \vec{a} \times \vec{b} - \vec{0} \\ &= 2\vec{a} \times \vec{b} \end{aligned}$$

Question 5:

Find λ and μ if $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0}$.

Answer

$$\begin{aligned} & (2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0} \\ & \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix} = 0\hat{i} + 0\hat{j} + 0\hat{k} \\ & \Rightarrow \hat{i}(6\mu - 27\lambda) - \hat{j}(2\mu - 27) + \hat{k}(2\lambda - 6) = 0\hat{i} + 0\hat{j} + 0\hat{k} \end{aligned}$$

On comparing the corresponding components, we have:

$$6\mu - 27\lambda = 0$$

$$2\mu - 27 = 0$$

$$2\lambda - 6 = 0$$

Now,

$$2\lambda - 6 = 0 \Rightarrow \lambda = 3$$

$$2\mu - 27 = 0 \Rightarrow \mu = \frac{27}{2}$$

Hence, $\lambda = 3$ and $\mu = \frac{27}{2}$.

Question 6:

Given that $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \times \vec{b} = \vec{0}$. What can you conclude about the vectors \vec{a} and \vec{b} ?

Answer

$$\vec{a} \cdot \vec{b} = 0$$

Then,

$$\text{(i) Either } |\vec{a}| = 0 \text{ or } |\vec{b}| = 0, \text{ or } \vec{a} \perp \vec{b} \text{ (in case } \vec{a} \text{ and } \vec{b} \text{ are non-zero)}$$

$$\vec{a} \times \vec{b} = \vec{0}$$

$$\text{(ii) Either } |\vec{a}| = 0 \text{ or } |\vec{b}| = 0, \text{ or } \vec{a} \parallel \vec{b} \text{ (in case } \vec{a} \text{ and } \vec{b} \text{ are non-zero)}$$

But, \vec{a} and \vec{b} cannot be perpendicular and parallel simultaneously.

$$\text{Hence, } |\vec{a}| = 0 \text{ or } |\vec{b}| = 0.$$

Question 7:

Let the vectors \vec{a} , \vec{b} , \vec{c} given as $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, $c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. Then show

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

that

Answer

We have,

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}, \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$$

$$\vec{b} + \vec{c} = (b_1 + c_1)\hat{i} + (b_2 + c_2)\hat{j} + (b_3 + c_3)\hat{k}$$

$$(b + \vec{c}) = (b_1 + c_1)i + (b_2 + c_2)j + (b_3 + c_3)k$$

$$\begin{aligned} \text{Now, } \vec{a} \times (\vec{b} + \vec{c}) &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 + c_1 & b_2 + c_2 & b_3 + c_3 \end{vmatrix} \\ &= \hat{i}[a_2(b_3 + c_3) - a_3(b_2 + c_2)] - \hat{j}[a_1(b_3 + c_3) - a_3(b_1 + c_1)] + \hat{k}[a_1(b_2 + c_2) - a_2(b_1 + c_1)] \\ &= \hat{i}[a_2b_3 + a_2c_3 - a_3b_2 - a_3c_2] + \hat{j}[-a_1b_3 - a_1c_3 + a_3b_1 + a_3c_1] + \hat{k}[a_1b_2 + a_1c_2 - a_2b_1 - a_2c_1] \quad \dots(1) \end{aligned}$$

$$\begin{aligned} \vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} \\ &= \hat{i}[a_2b_3 - a_3b_2] + \hat{j}[b_1a_3 - a_1b_3] + \hat{k}[a_1b_2 - a_2b_1] \quad (2) \end{aligned}$$

$$\begin{aligned} \vec{a} \times \vec{c} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \\ &= \hat{i}[a_2c_3 - a_3c_2] + \hat{j}[a_3c_1 - a_1c_3] + \hat{k}[a_1c_2 - a_2c_1] \quad (3) \end{aligned}$$

On adding (2) and (3), we get:

$$\begin{aligned} (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c}) &= \hat{i}[a_2b_3 + a_2c_3 - a_3b_2 - a_3c_2] + \hat{j}[b_1a_3 + a_3c_1 - a_1b_3 - a_1c_3] \\ &+ \hat{k}[a_1b_2 + a_1c_2 - a_2b_1 - a_2c_1] \quad (4) \end{aligned}$$

Now, from (1) and (4), we have:

$$\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$$

Hence, the given result is proved.

Question 8:

If either $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, then $\vec{a} \times \vec{b} = \vec{0}$. Is the converse true? Justify your answer with an example.

Answer

Take any parallel non-zero vectors so that $\vec{a} \times \vec{b} = \vec{0}$.

Let $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{b} = 4\hat{i} + 6\hat{j} + 8\hat{k}$.

Then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 4 & 6 & 8 \end{vmatrix} = \hat{i}(24 - 24) - \hat{j}(16 - 16) + \hat{k}(12 - 12) = 0\hat{i} + 0\hat{j} + 0\hat{k} = \vec{0}$$

It can now be observed that:

$$|\vec{a}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$$

$$\therefore \vec{a} \neq \vec{0}$$

$$|\vec{b}| = \sqrt{4^2 + 6^2 + 8^2} = \sqrt{116}$$

$$\therefore \vec{b} \neq \vec{0}$$

Hence, the converse of the given statement need not be true.

Question 9:

Find the area of the triangle with vertices A (1, 1, 2), B (2, 3, 5) and C (1, 5, 5).

Answer

The vertices of triangle ABC are given as A (1, 1, 2), B (2, 3, 5), and C (1, 5, 5).

The adjacent sides \overrightarrow{AB} and \overrightarrow{BC} of ΔABC are given as:

$$\overrightarrow{AB} = (2-1)\hat{i} + (3-1)\hat{j} + (5-2)\hat{k} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\overrightarrow{BC} = (1-2)\hat{i} + (5-3)\hat{j} + (5-5)\hat{k} = -\hat{i} + 2\hat{j}$$

$$\text{Area of } \Delta ABC = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{BC}|$$

$$\overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 2 & 0 \end{vmatrix} = \hat{i}(-6) - \hat{j}(3) + \hat{k}(2+2) = -6\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\therefore |\overrightarrow{AB} \times \overrightarrow{BC}| = \sqrt{(-6)^2 + (-3)^2 + 4^2} = \sqrt{36 + 9 + 16} = \sqrt{61}$$

$$\text{Hence, the area of } \Delta ABC \text{ is } \frac{\sqrt{61}}{2} \text{ square units.}$$

Question 10:

Find the area of the parallelogram whose adjacent sides are determined by the vector

$$\vec{a} = \hat{i} - \hat{j} + 3\hat{k} \text{ and } \vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}.$$

Answer

The area of the parallelogram whose adjacent sides are \vec{a} and \vec{b} is $|\vec{a} \times \vec{b}|$.

Adjacent sides are given as:

$$\vec{a} = \hat{i} - \hat{j} + 3\hat{k} \text{ and } \vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$$

$$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix} = \hat{i}(-1+21) - \hat{j}(1-6) + \hat{k}(-7+2) = 20\hat{i} + 5\hat{j} - 5\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{20^2 + 5^2 + 5^2} = \sqrt{400 + 25 + 25} = 15\sqrt{2}$$

Hence, the area of the given parallelogram is $15\sqrt{2}$ square units.

Question 11:

Let the vectors \vec{a} and \vec{b} be such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$, then $\vec{a} \times \vec{b}$ is a unit vector, if the angle between \vec{a} and \vec{b} is

- (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

Answer

It is given that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$.

We know that $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\hat{n}$, where \hat{n} is a unit vector perpendicular to both \vec{a} and \vec{b} and θ is the angle between \vec{a} and \vec{b} .

Now, $\vec{a} \times \vec{b}$ is a unit vector if $|\vec{a} \times \vec{b}| = 1$.

$$\begin{aligned} |\vec{a} \times \vec{b}| &= 1 \\ \Rightarrow |\vec{a}||\vec{b}|\sin\theta &= 1 \\ \Rightarrow |\vec{a}||\vec{b}|\sin\theta &= 1 \\ \Rightarrow 3 \times \frac{\sqrt{2}}{3} \times \sin\theta &= 1 \\ \Rightarrow \sin\theta &= \frac{1}{\sqrt{2}} \\ \Rightarrow \theta &= \frac{\pi}{4} \end{aligned}$$

Hence, $\vec{a} \times \vec{b}$ is a unit vector if the angle between \vec{a} and \vec{b} is $\frac{\pi}{4}$.

The correct answer is B.

Question 12:

Area of a rectangle having vertices A, B, C, and D with position vectors

$$-\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}, \hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}, \hat{i} - \frac{1}{2}\hat{j} + 4\hat{k} \text{ and } -\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k} \text{ respectively is}$$

- (A) $\frac{1}{2}$ (B) 1
(C) 2 (D) 4

Answer

The position vectors of vertices A, B, C, and D of rectangle ABCD are given as:

$$\vec{OA} = -\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}, \vec{OB} = \hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}, \vec{OC} = \hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}, \vec{OD} = -\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$$

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