Functions Ex 2.5 Q8

It is given that $f(x)=\frac{(4x+3)}{(6x-4)},\ x\neq\frac{2}{3}$.

$$(fof)(x)=f(f(x))=f\left(\frac{4x+3}{6x-4}\right)$$

$$=\frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4}=\frac{16x+12+18x-12}{24x+18-24x+16}=\frac{34x}{34}=x$$

Therefore, $fof(x)=x$, for all $x\neq\frac{2}{3}$.

$\Rightarrow fof=I$

Hence, the given function *f* is invertible and the inverse of *f* is *f* itself.

Functions Ex 2.5 Q9

f: $\mathbf{R}_+ \to [-5, \infty)$ is given as $f(x) = 9x^2 + 6x - 5$.

Let *y* be an arbitrary element of $[-5, \infty)$.

Let $y = 9x^2 + 6x - 5$.

$$\Rightarrow y=(3x+1)^2-1-5=(3x+1)^2-6$$

$$\Rightarrow (3x+1)^2=y+6$$

$$\Rightarrow 3x+1=\sqrt{y+6} \qquad [\text{as } y\geq -5 \Rightarrow y+6>0]$$

$$\Rightarrow x=\frac{\sqrt{y+6}-1}{3}$$

Therefore, *f* is onto, thereby range $f = [-5, \infty)$.

Let us define g: $[-5, \infty) \to \mathbf{R}_+$ as $g(y)=\frac{\sqrt{y+6}-1}{3}$.

We now have:

$$(gof)(x)=g(f(x))=g(9x^2+6x-5)$$

$$=g\left((3x+1)^2-6\right)$$

$$=\frac{\sqrt{(3x+1)^2-6+6}-1}{3}$$

$$=\frac{3x+1-1}{3}=x$$

$$\text{And,} (fog)(y) = f(g(y)) = f\left(\frac{\sqrt{y+6}-1}{3}\right)$$

$$= \left[3\left(\frac{\sqrt{y+6}-1}{3}\right)+1\right]^2 - 6$$

$$= \left(\sqrt{y+6}\right)^2 - 6 = y + 6 - 6 = y$$

Therefore, $gof = I_R$ and $fog = I_{(-5, \infty)}$

Hence, f is invertible and the inverse of f is given by

$$f^{-1}(y) = g(y) = \frac{\sqrt{y+6}-1}{3}.$$

Functions Ex 2.5 Q10

$f : R \to R$ be a function defined by

$f(x) = x^3 - 3$

Injectivity:

let $f(x_1) = f(x_2)$

$\Rightarrow \quad x_1^3 - 3 = x_2^3 - 3$

$\Rightarrow \quad x_1^3 = x_2^3$

$\Rightarrow \quad x_1 = x_2$

$\Rightarrow \quad f$ in one-one

Surjectivity: let $y \in R$ be arbitrary such that

$f(x) = y$

$\Rightarrow \quad x^3 - 3 - y = 0$

We know that an equation of odd degree must have atleast one real solution.

let $x = \alpha$ be that solution

$\therefore \quad \alpha^3 - 3 = y$

$\Rightarrow \quad f(\alpha) = y$

so, for each $y \in R$ in co-domain there exist $\alpha \in R$ in domain

$\Rightarrow \quad f$ in onto

Thus, f in one-one and onto, so

f^{-1} exists

Now,

$\because \quad f(x) = x^3 - 3 = y$

$\Rightarrow \quad x^3 = 3 + y$

$\Rightarrow \quad x = \sqrt[3]{3+y}$

$\Rightarrow \quad f^{-1}(x) = \sqrt[3]{3+x}$

Thus, $f^{-1} : R \to R$ be the inverse function defined by $f^{-1}(x) = (x+3)^{1/3}$

finally,

$f^{-1}(24) = (24+3)^{1/3} = 3$

$f^{-1}(5) = (5+3)^{1/3} = 2$

Functions Ex 2.5 Q11

We have,

$f : R \to R$ in a function defined by

$f(x) = x^3 + 4$

Injectivity: let $f(x_1) = f(x_2)$ for $x_1 x_2 \in R$

$\Rightarrow \quad x_1^3 + 4 = x_2^3 + 4$

$\Rightarrow \quad x_1^3 = x_2^3$

$\Rightarrow \quad x_1 = x_2$

$\Rightarrow \quad f$ in one-one

Surjectivity: let $y \in R$ be artritrary such that

$f(x) = y$

$\Rightarrow \quad x^3 + 4 = y$

$\Rightarrow \quad x^3 + 4 - y = 0$

We know that an odd degree equation must have a real root.

$\Rightarrow \quad \alpha^3 + 4 = y \Rightarrow f(\alpha) = y$

$\Rightarrow \quad f$ in onto

Since f in one-one and onto

$\Rightarrow \quad f$ in bijective

finally.

$f(x) = y$

$\Rightarrow \quad x^3 + 4 = y$

$\Rightarrow \quad x^3 = y - 4$

$\Rightarrow \quad x = (y - 4)^{1/3}$

$\therefore \quad f^{-1}(x) = (x - 4)^{1/3}$

$\therefore \quad f^{-1}(3) = (3 - 4)^{1/3} = -1$

*********** END **********