Question 5:

If area of triangle is 35 square units with vertices (2, −6), (5, 4), and (k, 4). Then k is

A. 12 **B.** −2 **C.** −12, −2 **D.** 12, −2

Answer

Answer: D

The area of the triangle with vertices (2, −6), (5, 4), and (k, 4) is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[2(4-4)+6(5-k)+1(20-4k)\right]$$

$$= \frac{1}{2}\left[30-6k+20-4k\right]$$

$$= \frac{1}{2}\left[50-10k\right]$$

$$= 25-5k$$

It is given that the area of the triangle is ±35.

Therefore, we have:

$$\Rightarrow 25-5k = \pm 35$$

$$\Rightarrow 5(5-k) = \pm 35$$

$$\Rightarrow 5-k = \pm 7$$

When $5 - k = -7$, $k = 5 + 7 = 12$.

When $5 - k = 7$, $k = 5 - 7 = -2$.

Hence, $k = 12, -2$.

The correct answer is D.

Exercise 4.4

Question 1:

Write Minors and Cofactors of the elements of following determinants:

(i) $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$ (ii) $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

Answer

(i) The given determinant is $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$.

Minor of element a_{ij} is M_{ij}.

$\therefore M_{11}$ = minor of element $a_{11} = 3$

M_{12} = minor of element $a_{12} = 0$

M_{21} = minor of element $a_{21} = -4$

M_{22} = minor of element $a_{22} = 2$

Cofactor of a_{ij} is $A_{ij} = (-1)^{i+j} M_{ij}$.

$\therefore A_{11} = (-1)^{1+1} M_{11} = (-1)^2 (3) = 3$

$A_{12} = (-1)^{1+2} M_{12} = (-1)^3 (0) = 0$

$A_{21} = (-1)^{2+1} M_{21} = (-1)^3 (-4) = 4$

$A_{22} = (-1)^{2+2} M_{22} = (-1)^4 (2) = 2$

(ii) The given determinant is $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$.

Minor of element a_{ij} is M_{ij}.

$\therefore M_{11}$ = minor of element $a_{11} = d$

M_{12} = minor of element $a_{12} = b$

M_{21} = minor of element $a_{21} = c$

M_{22} = minor of element $a_{22} = a$

Cofactor of a_{ij} is $A_{ij} = (-1)^{i+j} M_{ij}$.

$\therefore A_{11} = (-1)^{1+1} M_{11} = (-1)^2 (d) = d$

$A_{12} = (-1)^{1+2} M_{12} = (-1)^3 (b) = -b$
$A_{21} = (-1)^{2+1} M_{21} = (-1)^3 (c) = -c$
$A_{22} = (-1)^{2+2} M_{22} = (-1)^4 (a) = a$

Question 2:

(i) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ (ii) $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$

Answer

(i) The given determinant is $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$.

By the definition of minors and cofactors, we have:

M_{11} = minor of a_{11}= $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

M_{12} = minor of a_{12}= $\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$

M_{13} = minor of a_{13} = $\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$

M_{21} = minor of a_{21} = $\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$

M_{22} = minor of a_{22} = $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

M_{23} = minor of a_{23} = $\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

M_{31} = minor of a_{31}= $\begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$

M_{32} = minor of a_{32} = $\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

M_{33} = minor of a_{33} = $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

A_{11} = cofactor of a_{11}= $(-1)^{1+1} M_{11} = 1$
A_{12} = cofactor of a_{12} = $(-1)^{1+2} M_{12} = 0$
A_{13} = cofactor of a_{13} = $(-1)^{1+3} M_{13} = 0$
A_{21} = cofactor of a_{21} = $(-1)^{2+1} M_{21} = 0$
A_{22} = cofactor of a_{22} = $(-1)^{2+2} M_{22} = 1$
A_{23} = cofactor of a_{23} = $(-1)^{2+3} M_{23} = 0$
A_{31} = cofactor of a_{31} = $(-1)^{3+1} M_{31} = 0$
A_{32} = cofactor of a_{32} = $(-1)^{3+2} M_{32} = 0$
A_{33} = cofactor of a_{33} = $(-1)^{3+3} M_{33} = 1$

(ii) The given determinant is $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$.

By definition of minors and cofactors, we have:

M_{11} = minor of a_{11}= $\begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = 10 + 1 = 11$

M_{12} = minor of a_{12}= $\begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = 6 - 0 = 6$

M_{13} = minor of a_{13} = $\begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$

M_{21} = minor of a_{21} = $\begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} = 0 - 4 = -4$

M_{22} = minor of a_{22} = $\begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2$

M_{23} = minor of a_{23} = $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$

M_{31} = minor of a_{31}= $\begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix} = 0 - 20 = -20$

M_{32} = minor of a_{32} = $\begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix} = -1 - 12 = -13$

M_{33} = minor of a_{33} = $\begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 5 - 0 = 5$

A_{11} = cofactor of a_{11}= $(-1)^{1+1} M_{11} = 11$

A_{12} = cofactor of $a_{12} = (-1)^{1+2} M_{12} = -6$

A_{13} = cofactor of $a_{13} = (-1)^{1+3} M_{13} = 3$

A_{21} = cofactor of $a_{21} = (-1)^{2+1} M_{21} = 4$

A_{22} = cofactor of $a_{22} = (-1)^{2+2} M_{22} = 2$

A_{23} = cofactor of $a_{23} = (-1)^{2+3} M_{23} = -1$

A_{31} = cofactor of $a_{31} = (-1)^{3+1} M_{31} = -20$

A_{32} = cofactor of $a_{32} = (-1)^{3+2} M_{32} = 13$

A_{33} = cofactor of $a_{33} = (-1)^{3+3} M_{33} = 5$

Question 3:

Using Cofactors of elements of second row, evaluate $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$.

Answer

The given determinant is $\begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$.

We have:

$M_{21} = \begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} = 9 - 16 = -7$

$\therefore A_{21}$ = cofactor of $a_{21} = (-1)^{2+1} M_{21} = 7$

*********** END **********