

Binomial Theorem Ex 18.1 Q5(ii)

We have,

$$\begin{aligned} & \left(102\right)^5 = \left(100 + 2\right)^5 \\ &= {}^5C_0 \times 100^5 + {}^5C_1 \times 100^4 \times 2 + {}^5C_2 \times 100^3 \times 2^2 + {}^5C_3 \times 100^2 \times 2^3 + {}^5C_4 \times 100 \times 2^4 + {}^5C_5 \times 2^5 \\ &= 100^5 + 5 \times 100^4 \times 2 + 10 \times 100^3 \times 2^2 + 10 \times 100^2 \times 2^3 + 5 \times 100 \times 2^4 + 2^5 \\ &= 100000000000 + 1000000000 + 40000000 + 800000 + 800 + 32 \\ &= 11040808032 \end{aligned}$$

: (102)⁵ = 11040808032

Binomial Theorem Ex 18.1 Q5(iii)

We have,

$$\begin{aligned} & \left(101\right)^4 = \left(100+1\right)^4 \\ &= \, ^4C_0 \times 100^4 + \, ^4C_1 \times 100^3 + \, ^4C_2 \times 100^2 + \, ^4C_3 \times 100 + \, ^4C_4 \\ &= 100^4 + 4 \times 100^3 + 6 \times 100^2 + 4 \times 100 + 1 \\ &= 1000000000 + 40000000 + 600000 + 4000 + 1 \\ &= 104060401 \end{aligned}$$

Binomial Theorem Ex 18.1 Q5(iv)

We have,

$$\begin{pmatrix} 98 \end{pmatrix}^5 = \begin{pmatrix} 100 - 2 \end{pmatrix}^5 \\ = {}^5C_0 \times 100^5 + {}^5C_1 \times 100^4 \times (-2) + {}^5C_2 \times 100^3 \times (-2)^2 + {}^5C_3 \times 100^2 \times (-2)^3 + {}^5C_4 \times 100 \times (-2)^4 + {}^5C_5 \times (-2)^5 \\ = {}^5C_0 \times 100^5 - {}^5C_1 \times 100^4 \times 2 + {}^5C_2 \times 100^3 \times 4 - {}^5C_3 \times 100^2 \times 8 + {}^5C_4 \times 100 \times 16 - {}^5C_5 \times 32 \\ = 100^5 - 10 \times 100^4 + 40 \times 100^3 - 80 \times 100^2 + 80 \times 100 - 32 \\ = 10000000000 - 1000000000 + 400000000 - 800000 + 8000 - 32 \\ = 10040008000 - 10000800032 \\ = 9039207968$$

(98)⁵ = 9039207968

Binomial Theorem Ex 18.1 Q6

$$2^{3n} - 7n - 1$$

$$= 2^{3(n)} - 7(n) - 1$$

$$= 8^{n} - 7n - 1$$

$$= (1+7)^{n} - 7n - 1$$

$$= {\binom{n}{C_0} + {\binom{n}{C_1}}(7)^1 + {\binom{n}{C_2}}(7)^2 + \dots {\binom{n}{C_n}}(7)^n - 7n - 1}$$

$$= (1+7n+49^{n}C_2 + \dots + 49(7)^{n-2}) - 7n - 1$$

$$= 49 {\binom{n}{C_2} + \dots + 7^{n-2}}$$

$\therefore 2^{3n} - 7n - 1$ is divisible by 49

Hence, proved

Binomial Theorem Ex 18.1 Q7

$$\begin{split} &3^{2n+2}-8n-9\\ &=3^{2(n+1)}-8n-9\\ &=9^{n+1}-8n-9\\ &=(1+8)^{n+1}-8n-9\\ &=\binom{n+1}{0}+\binom{n+1}{1}-8n-9\\ &=\binom{n+1}{1}C_0+\binom{n+1}{1}C_18^1+\binom{n+1}{1}C_28^2+\dots+\binom{n+1}{1}C_{n+1}8^{n+1}-8n-9\\ &=\left(1+8(n+1)+64^{n+1}C_2+\dots+64(8)^{n-1}\right)-8n-9\\ &=64\binom{n+1}{1}C_2+\dots+8^{n-1} \end{split}$$

Thus, $3^{2n+2} - 8n - 9$ is divisible by 64.

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