



**Question 15:**

Solve  $\sin(\tan^{-1} x)$ ,  $|x| < 1$  is equal to

- (A)  $\frac{x}{\sqrt{1-x^2}}$  (B)  $\frac{1}{\sqrt{1-x^2}}$  (C)  $\frac{1}{\sqrt{1+x^2}}$  (D)  $\frac{x}{\sqrt{1+x^2}}$

Answer

$$\tan y = x \Rightarrow \sin y = \frac{x}{\sqrt{1+x^2}}.$$

Let  $\tan^{-1} x = y$ . Then,

$$\therefore y = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) \Rightarrow \tan^{-1} x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

$$\therefore \sin(\tan^{-1} x) = \sin\left(\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right) = \frac{x}{\sqrt{1+x^2}}$$

The correct answer is D.

**Question 16:**

Solve  $\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$ , then  $x$  is equal to

- (A) 0,  $\frac{1}{2}$  (B) 1,  $\frac{1}{2}$  (C) 0 (D)  $\frac{1}{2}$

Answer

$$\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$$

$$\Rightarrow -2\sin^{-1} x = \frac{\pi}{2} - \sin^{-1}(1-x)$$

$$\Rightarrow -2\sin^{-1} x = \cos^{-1}(1-x) \quad \dots(1)$$

$$\text{Let } \sin^{-1} x = \theta \Rightarrow \sin \theta = x \Rightarrow \cos \theta = \sqrt{1-x^2}.$$

$$\therefore \theta = \cos^{-1}(\sqrt{1-x^2})$$

$$\therefore \sin^{-1} x = \cos^{-1}(\sqrt{1-x^2})$$

Therefore, from equation (1), we have

$$-2\cos^{-1}(\sqrt{1-x^2}) = \cos^{-1}(1-x)$$

Put  $x = \sin y$ . Then, we have:

$$-2\cos^{-1}(\sqrt{1-\sin^2 y}) = \cos^{-1}(1-\sin y)$$

$$\Rightarrow -2\cos^{-1}(\cos y) = \cos^{-1}(1-\sin y)$$

$$\Rightarrow -2y = \cos^{-1}(1-\sin y)$$

$$\Rightarrow 1-\sin y = \cos(-2y) = \cos 2y$$

$$\Rightarrow 1-\sin y = 1-2\sin^2 y$$

$$\Rightarrow 2\sin^2 y - \sin y = 0$$

$$\Rightarrow \sin y(2\sin y - 1) = 0$$

$$\Rightarrow \sin y = 0 \text{ or } \frac{1}{2}$$

$$\therefore x = 0 \text{ or } x = \frac{1}{2}$$

But, when  $x = \frac{1}{2}$ , it can be observed that:

$$\text{L.H.S.} = \sin^{-1}\left(1-\frac{1}{2}\right) - 2\sin^{-1}\frac{1}{2}$$

$$= \sin^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\frac{1}{2}$$

$$= -\sin^{-1}\frac{1}{2}$$

$$= -\frac{\pi}{6} \neq \frac{\pi}{2} \neq \text{R.H.S.}$$

$\therefore x = \frac{1}{2}$  is not the solution of the given equation.

Thus,  $x = 0$ .

Hence, the correct answer is **C**.

**Question 17:**

Solve  $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\frac{x-y}{x+y}$  is equal to

(A)  $\frac{\pi}{2}$  (B)  $\frac{\pi}{3}$  (C)  $\frac{\pi}{4}$  (D)  $\frac{-3\pi}{4}$

Answer

$$\begin{aligned} & \tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\frac{x-y}{x+y} \\ &= \tan^{-1}\left[\frac{\frac{x}{y} - \frac{x-y}{x+y}}{1 + \left(\frac{x}{y}\right)\left(\frac{x-y}{x+y}\right)}\right] \qquad \left[\tan^{-1}y - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}\right] \\ &= \tan^{-1}\left[\frac{\frac{x(x+y) - y(x-y)}{y(x+y)}}{\frac{y(x+y) + x(x-y)}{y(x+y)}}\right] \\ &= \tan^{-1}\left(\frac{x^2 + xy - xy + y^2}{xy + y^2 + x^2 - xy}\right) \\ &= \tan^{-1}\left(\frac{x^2 + y^2}{x^2 + y^2}\right) = \tan^{-1}1 = \frac{\pi}{4} \end{aligned}$$

Hence, the correct answer is **C**.

\*\*\*\*\* END \*\*\*\*\*