Polynomials Ex 2.3 Q5

Answer :

We know that if $x=\alpha$ is a zero of a polynomial, then $(x-\alpha)$ is a factor of $f(x)$.

Since $-\sqrt{3}$ and $\sqrt{3}$ are zeros of $f(x)$.

Therefore

$$\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)=x^2+\sqrt{3}x-\sqrt{3}x-3$$

$$=x^2+\cancel{\sqrt{3}x}-\cancel{\sqrt{3}x}-3$$

$$=x^2-3$$

x^2-3 is a factor of $f(x)$. Now, we divide $f(x)=x^4-3x^3-x^2+9x-6$ by $g(x)=x^2-3$ to find the other zeros of $f(x)$.

$$\begin{array}{r|l}
 & x^2-3x+2 \\
\hline
x^2-3 & +\cancel{x^4}-3x^3-x^2+9x-6 \\
 & \underset{-}{+}\cancel{x^4}\underset{+}{-}0x^3\underset{+}{-}3x^2 \\
\hline
 & -\cancel{3x^3}+2x^2+\cancel{9x} \\
 & \underset{+}{-}\cancel{3x^3}\underset{-}{+}0\quad\underset{-}{+}\cancel{9x} \\
\hline
 & \cancel{2x^2}-\cancel{6} \\
 & \underset{-}{+}\cancel{2x^2}\underset{+}{-}\cancel{6} \\
\hline
 & 0
\end{array}$$

By using division algorithm we have, $f(x)=g(x)\times q(x)+r(x)$.

$$x^4-3x^3-x^2+9x-6=\left(x^2-3\right)\left(x^2-3x+2\right)+0$$

$$x^4-3x^3-x^2+9x-6=\left(x^2-3\right)\left(x^2-2x-1x+2\right)$$

$$x^4-3x^3-x^2+9x-6=\left(x^2-3\right)\left[x(x-2)-1(x-2)\right]$$

$$x^4-3x^3-x^2+9x-6=\left(x^2-3\right)\left[(x-1)(x-2)\right]$$

$$x^4-3x^3-x^2+9x-6=\left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right)(x-1)(x-2)$$

Hence, the zeros of the given polynomials are $\boxed{-\sqrt{3},+\sqrt{3},+1,+2}$.

Polynomials Ex 2.3 Q6

Answer :

Since $-\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$ are two zeros of $f(x)$. Therefore,

$$=\left(x-\sqrt{\frac{3}{2}}\right)\left(x+\sqrt{\frac{3}{2}}\right)$$

$$=\left(x^2-\frac{3}{2}\right)$$

$=\frac{1}{2}\left(2x^2-3\right)$ is a factor of $f(x)$.

Also $2x^2-3$ is a factor of $f(x)$.

Let us now divide $f(x)$ by $2x^2-3$. We have,

$$\begin{array}{r|l}
 & 1x^2-1x-2 \\
\hline
2x^2-3 & +\cancel{2x^4}-2x^3-7x^2+3x+6 \\
 & \underline{+\cancel{2x^4}+0\quad-3x^2} \\
 & \quad -\cancel{2x^3}-4x^2+\cancel{3x} \\
 & \quad \underline{-\cancel{2x^3}-0\quad+\cancel{3x}} \\
 & \qquad -\cancel{4x^2}+\cancel{6} \\
 & \qquad \underline{-\cancel{4x^2}+\cancel{6}} \\
 & \qquad\quad 0
\end{array}$$

By using division algorithm we have, $f(x)=g(x)\times q(x)+r(x)$

$$2x^4-2x^3-7x^2+3x+6=\left(2x^2-3\right)\left(x^2-x-2\right)+0$$

$$2x^4-2x^3-7x^2+3x+6=\left(\sqrt{2}x+\sqrt{3}\right)\left(\sqrt{2}x-\sqrt{3}\right)\left(x^2+1x-2x-2\right)$$

$$2x^4-2x^3-7x^2+3x+6=\left(\sqrt{2}x+\sqrt{3}\right)\left(\sqrt{2}x-\sqrt{3}\right)\left[x(x+1)-2(x+1)\right]$$

$$2x^4-2x^3-7x^2+3x+6=\left(\sqrt{2}x+\sqrt{3}\right)\left(\sqrt{2}x-\sqrt{3}\right)(x-2)(x+1)$$

Hence, The zeros of $f(x)$ are $\boxed{-\sqrt{\frac{3}{2}},\sqrt{\frac{3}{2}},2,-1}$.

*********** END **********