EXERCISE 12.1

Question 1:

A point is on the x-axis. What are its y-coordinates and z-coordinates?

Ans:

If a point is on the x-axis, then its y-coordinates and z-coordinates are zero.

Question 2:

A point is in the XZ-plane. What can you say about its y-coordinate?

Ans:

If a point is in the XZ plane, then its y-coordinate is zero.

Question 3:

Name the octants in which the following points lie:

(1, 2, 3), (4, −2, 3), (4, −2, −5), (4, 2, −5), (−4, 2, −5), (−4, 2, 5),

(−3, −1, 6), (2, −4, −7)

Ans:

The x-coordinate, y-coordinate, and z-coordinate of point (1, 2, 3) are all positive. Therefore, this point lies in octant **I**.

The x-coordinate, y-coordinate, and z-coordinate of point (4, −2, 3) are positive, negative, and positive respectively. Therefore, this point lies in octant **IV**.

The x-coordinate, y-coordinate, and z-coordinate of point (4, −2, −5) are positive, negative, and negative respectively. Therefore, this point lies in octant **VIII**.

The x-coordinate, y-coordinate, and z-coordinate of point (4, 2, −5) are positive, positive, and negative respectively. Therefore, this point lies in octant **V**.

The x-coordinate, y-coordinate, and z-coordinate of point (−4, 2, −5) are negative, positive, and negative respectively. Therefore, this point lies in octant **VI**.

The x-coordinate, y-coordinate, and z-coordinate of point (−4, 2, 5) are negative, positive, and positive respectively. Therefore, this point lies in octant **II**.

The x-coordinate, y-coordinate, and z-coordinate of point (−3, −1, 6) are negative, negative, and positive respectively. Therefore, this point lies in octant **III**.

The x-coordinate, y-coordinate, and z-coordinate of point (2, −4, −7) are positive, negative, and negative respectively. Therefore, this point lies in octant **VIII**.

Question 4:

Fill in the blanks:

Ans:

(i) The x-axis and y-axis taken together determine a plane known as $\underline{\text{XY}-\text{plane}}$.

(ii) The coordinates of points in the XY-plane are of the form $\underline{(x, y, 0)}$.

(iii) Coordinate planes divide the space into $\underline{\text{eight}}$ octants.

EXERCISE 12.2

Question 1:

Find the distance between the following pairs of points:

(i) (2, 3, 5) and (4, 3, 1) (ii) (−3, 7, 2) and (2, 4, −1)

(iii) (−1, 3, −4) and (1, −3, 4) (iv) (2, −1, 3) and (−2, 1, 3)

Ans:

The distance between points P(x_1, y_1, z_1) and P(x_2, y_2, z_2) is given by

$$\mathrm{PQ}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

(i) Distance between points (2, 3, 5) and (4, 3, 1)

$$=\sqrt{(4-2)^2+(3-3)^2+(1-5)^2}$$
$$=\sqrt{(2)^2+(0)^2+(-4)^2}$$
$$=\sqrt{4+16}$$
$$=\sqrt{20}$$
$$=2\sqrt{5}$$

(ii) Distance between points (−3, 7, 2) and (2, 4, −1)

$$=\sqrt{(2+3)^2+(4-7)^2+(-1-2)^2}$$
$$=\sqrt{(5)^2+(-3)^2+(-3)^2}$$
$$=\sqrt{25+9+9}$$
$$=\sqrt{43}$$

(iii) Distance between points (−1, 3, −4) and (1, −3, 4)

$$=\sqrt{(1+1)^2+(-3-3)^2+(4+4)^2}$$
$$=\sqrt{(2)^2+(-6)^3+(8)^2}$$
$$=\sqrt{4+36+64}=\sqrt{104}=2\sqrt{26}$$

(iv) Distance between points (2, −1, 3) and (−2, 1, 3)

$$=\sqrt{(-2-2)^2+(1+1)^2+(3-3)^2}$$
$$=\sqrt{(-4)^2+(2)^2+(0)^2}$$
$$=\sqrt{16+4}$$
$$=\sqrt{20}$$
$$=2\sqrt{5}$$

Question 2:

Show that the points (−2, 3, 5), (1, 2, 3) and (7, 0, −1) are collinear.

Ans:

Let points (−2, 3, 5), (1, 2, 3), and (7, 0, −1) be denoted by P, Q, and R respectively.

Points P, Q, and R are collinear if they lie on a line.

$$\begin{aligned} PQ &= \sqrt{(1+2)^2+(2-3)^2+(3-5)^2} \\ &= \sqrt{(3)^2+(-1)^2+(-2)^2} \\ &= \sqrt{9+1+4} \\ &= \sqrt{14} \end{aligned}$$

$$\begin{aligned} QR &= \sqrt{(7-1)^2+(0-2)^2+(-1-3)^2} \\ &= \sqrt{(6)^2+(-2)^2+(-4)^2} \\ &= \sqrt{36+4+16} \\ &= \sqrt{56} \\ &= 2\sqrt{14} \end{aligned}$$

$$\begin{aligned} PR &= \sqrt{(7+2)^2+(0-3)^2+(-1-5)^2} \\ &= \sqrt{(9)^2+(-3)^2+(-6)^2} \\ &= \sqrt{81+9+36} \\ &= \sqrt{126} \\ &= 3\sqrt{14} \end{aligned}$$

Here, PQ + QR $= \sqrt{14}+2\sqrt{14}=3\sqrt{14}$ = PR

Hence, points P(−2, 3, 5), Q(1, 2, 3), and R(7, 0, −1) are collinear.

Question 3:

Verify the following:

(i) (0, 7, −10), (1, 6, −6) and (4, 9, −6) are the vertices of an isosceles triangle.

(ii) (0, 7, 10), (−1, 6, 6) and (−4, 9, 6) are the vertices of a right angled triangle.

(iii) (−1, 2, 1), (1, −2, 5), (4, −7, 8) and (2, −3, 4) are the vertices of a parallelogram.

Ans:

(i) Let points (0, 7, −10), (1, 6, −6), and (4, 9, −6) be denoted by A, B, and C respectively.

$$
\begin{aligned}
AB &= \sqrt{(1-0)^2+(6-7)^2+(-6+10)^2} \\
&= \sqrt{(1)^2+(-1)^2+(4)^2} \\
&= \sqrt{1+1+16} \\
&= \sqrt{18} \\
&= 3\sqrt{2}
\end{aligned}
$$

$$
\begin{aligned}
BC &= \sqrt{(4-1)^2+(9-6)^2+(-6+6)^2} \\
&= \sqrt{(3)^2+(3)^2} \\
&= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}
\end{aligned}
$$

$$
\begin{aligned}
CA &= \sqrt{(0-4)^2+(7-9)^2+(-10+6)^2} \\
&= \sqrt{(-4)^2+(-2)^2+(-4)^2} \\
&= \sqrt{16+4+16} = \sqrt{36} = 6
\end{aligned}
$$

Here, AB = BC ≠ CA

Thus, the given points are the vertices of an isosceles triangle.

(i) Let (0, 7, 10), (−1, 6, 6), and (−4, 9, 6) be denoted by A, B, and C respectively.

$$
\begin{aligned}
AB &= \sqrt{(-1-0)^2+(6-7)^2+(6-10)^2} \\
&= \sqrt{(-1)^2+(-1)^2+(-4)^2} \\
&= \sqrt{1+1+16} = \sqrt{18} \\
&= 3\sqrt{2}
\end{aligned}
$$

$$
\begin{aligned}
BC &= \sqrt{(-4+1)^2+(9-6)^2+(6-6)^2} \\
&= \sqrt{(-3)^2+(3)^2+(0)^2} \\
&= \sqrt{9+9} = \sqrt{18} \\
&= 3\sqrt{2}
\end{aligned}
$$

$$
\begin{aligned}
CA &= \sqrt{(0+4)^2+(7-9)^2+(10-6)^2} \\
&= \sqrt{(4)^2+(-2)^2+(4)^2} \\
&= \sqrt{16+4+16} \\
&= \sqrt{36} \\
&= 6
\end{aligned}
$$

$$\text{Now, } AB^2+BC^2 = \left(3\sqrt{2}\right)^2+\left(3\sqrt{2}\right)^2 = 18+18 = 36 = AC^2$$

Therefore, by Pythagoras theorem, ABC is a right triangle.

Hence, the given points are the vertices of a right-angled triangle.

(ii) Let (−1, 2, 1), (1, −2, 5), (4, −7, 8), and (2, −3, 4) be denoted by A, B, C, and D respectively.

$$
\begin{aligned}
AB &= \sqrt{(1+1)^2+(-2-2)^2+(5-1)^2} \\
&= \sqrt{4+16+16} \\
&= \sqrt{36} \\
&= 6
\end{aligned}
$$

$$
\begin{aligned}
BC &= \sqrt{(4-1)^2+(-7+2)^2+(8-5)^2} \\
&= \sqrt{9+25+9} = \sqrt{43}
\end{aligned}
$$

$$\begin{aligned}CD &= \sqrt{(2-4)^2+(-3+7)^2+(4-8)^2}\\ &= \sqrt{4+16+16}\\ &= \sqrt{36}\\ &= 6\end{aligned}$$

$$\begin{aligned}DA &= \sqrt{(-1-2)^2+(2+3)^2+(1-4)^2}\\ &= \sqrt{9+25+9} = \sqrt{43}\end{aligned}$$

Here, AB = CD = 6, BC = AD = $\sqrt{43}$

Hence, the opposite sides of quadrilateral ABCD, whose vertices are taken in order, are equal.

Therefore, ABCD is a parallelogram.

Hence, the given points are the vertices of a parallelogram.

Question 4:

Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, −1).

Ans:

Let P (x, y, z) be the point that is equidistant from points A(1, 2, 3) and B(3, 2, −1).

Accordingly, PA = PB

$$\Rightarrow PA^2 = PB^2$$

$$\Rightarrow (x-1)^2+(y-2)^2+(z-3)^2 = (x-3)^2+(y-2)^2+(z+1)^2$$

$x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 + 2z + 1$

$-2x - 4y - 6z + 14 = -6x - 4y + 2z + 14$

$-2x - 6z + 6x - 2z = 0$

$4x - 8z = 0$

$x - 2z = 0$

Thus, the required equation is $x - 2z = 0$.

Question 5:

Find the equation of the set of points P, the sum of whose distances from A (4, 0, 0) and B (−4, 0, 0) is equal to 10.

Ans:

Let the coordinates of P be (x, y, z).

The coordinates of points A and B are (4, 0, 0) and (−4, 0, 0) respectively.

It is given that PA + PB = 10.

$$\Rightarrow \sqrt{(x-4)^2+y^2+z^2}+\sqrt{(x+4)^2+y^2+z^2} = 10$$

$$\Rightarrow \sqrt{(x-4)^2+y^2+z^2} = 10-\sqrt{(x+4)^2+y^2+z^2}$$

On squaring both sides, we obtain

$$\Rightarrow (x-4)^2+y^2+z^2 = 100-20\sqrt{(x+4)^2+y^2+z^2}+(x+4)^2+y^2+z^2$$

$$\Rightarrow x^2-8x+16+y^2+z^2 = 100-20\sqrt{x^2+8x+16+y^2+z^2}+x^2+8x+16+y^2+z^2$$

$$\Rightarrow 20\sqrt{x^2+8x+16+y^2+z^2} = 100+16x$$

$$\Rightarrow 5\sqrt{x^2+8x+16+y^2+z^2} = (25+4x)$$

On squaring both sides again, we obtain

$25(x^2 + 8x + 16 + y^2 + z^2) = 625 + 16x^2 + 200x$

$25x^2 + 200x + 400 + 25y^2 + 25z^2 = 625 + 16x^2 + 200x$

$9x^2 + 25y^2 + 25z^2 - 225 = 0$

Thus, the required equation is $9x^2 + 25y^2 + 25z^2 - 225 = 0$.

*********** END **********