Definite Integrals Ex 20.2 Q48

Let $\cos x = t$

Differentiating w.r.t. x, we get

$-\sin x dx = dt$

When $x = 0 \Rightarrow t = 1$

$x = \pi \Rightarrow t = -1$

Now,

$$\int_0^\pi \sin^3 x \left(1 + 2\cos x\right)\left(1 + \cos x\right)^2 dx$$

$$= \int_0^\pi \sin^2 x \left(1 + 2\cos x\right)\left(1 + \cos x\right)^2 . \sin x dx$$

$$= -\int_{-1}^{1} \left(1 - t^2\right)\left(1 + 2t\right)\left(1 + t\right)^2 dt \qquad \left[\sin^2 x = 1 - \cos^2 x\right]$$

$$= \int_{-1}^{1} \left(1 + 2t - t^2 - 2t^3\right)\left(1 + t^2 + 2t\right) dt$$

$$= \int_{-1}^{1} \left(1 - t^2 + 2t + 2t + 2t^3 + 4t^2 - t^2 - t^4 - 2t^3 - 2t^5 - 4t^4\right) dt$$

$$= \int_{-1}^{1} \left(1 + 4t + 4t^2 - 2t^3 - 5t^4 - 2t^5\right) dt$$

$$= \left[t + 2t^2 + \frac{4}{3}t^3 - \frac{t^4}{2} - t^5 - \frac{t6}{3}\right]_{-1}^{1}$$

$$= \left[2 + 0 + \frac{8}{3} - 0 - 2 - 0\right] = \frac{8}{3}$$

$$\therefore \int_0^\pi \sin^3 x \left(1 + 2\cos x\right)\left(1 + \cos x\right)^2 dx = \frac{8}{3}$$

Definite Integrals Ex 20.2 Q49

$$I = \int_0^{\frac{\pi}{2}} 2\sin x\cos x\tan^{-1}(\sin x)dx$$

Let $t = \sin x$

$$dt = \cos x dx$$

$$x = 0, t = 0$$

$$x = \frac{\pi}{2}, t = 1$$

$$I = \int_0^1 2t\tan^{-1}(t)dt$$

$$= 2\left[\frac{1}{2}t^2\tan^{-1}t - \frac{t}{2} + \frac{1}{2}\tan^{-1}t\right]_0^1$$

$$= 2\left[\frac{\pi}{4} - \frac{1}{2}\right]$$

$$= \frac{\pi}{2} - 1$$

$$\therefore I = \int_0^{\frac{\pi}{2}} 2\sin x\cos x\tan^{-1}(\sin x)dx = \frac{\pi}{2} - 1$$

Definite Integrals Ex 20.2 Q50

Let $\sin x = t$

Differentiating w.r.t. x, we get

$\cos x\, dx = dt$

Now,

$x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$\therefore \int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x)dx = 2\int_0^1 t\tan^{-1}t\,dt$ $\quad [\because \sin 2x = 2\sin x\cos x]$

Using by parts

$$= 2\left\{\tan^{-1}t\int t\,dt - \int\left(\int t\,dt\right)\frac{d\tan^{-1}t}{dt}dt\right\}$$

$$= 2\left\{\frac{t^2}{2}\tan^{-1} - \frac{1}{2}\int\frac{t^2}{1+t^2}dt\right\}$$

$$= 2\left\{\frac{t^2}{2}\tan^{-1}t - \frac{1}{2}\left(\int dt - \int\frac{dt}{1+t^2}\right)\right\}$$

$$= 2\left[\frac{t^2}{2}\tan^{-1}t - \frac{1}{2}\left(t - \tan^{-1}t\right)\right]_0^1$$

$$= 2\left\{\frac{1}{2}\frac{\pi}{4} - \frac{1}{2}\left(1 - \frac{\pi}{4}\right)\right\}$$

$$= 2\left\{\frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8}\right\}$$

$$= 2\left(\frac{\pi}{4} - \frac{1}{2}\right) = \frac{\pi}{2} - 1$$

$$\therefore \int_0^{\frac{\pi}{2}} \sin 2x\tan^{-1}(\sin x)dx = \frac{\pi}{2} - 1$$

Definite Integrals Ex 20.2 Q51

We have,

$$\int_0^1 \left(\cos^{-1} x\right)^2 dx = \left(\cos^{-1} x\right)^2 \int_0^1 dx - \int_0^1 \left(\int dx\right) \frac{d\left(\cos^{-1} x\right)^2}{dx} dx$$

$$= \left[x\left(\cos^{-1} x\right)^2\right]_0^1 + \int_0^1 \frac{x.2\cos^{-1}}{\sqrt{1-x^2}} dx$$

Now,

Let $\cos^{-1} x = t \Rightarrow -\frac{1}{\sqrt{1-x^2}} dx = dt$

When $x = 0 \Rightarrow t = \frac{\pi}{2}$

$x = 1 \Rightarrow t = 0$

$$\therefore \int_0^1 \frac{2x\cos^{-1} x}{\sqrt{1-x^2}} dx = -2\int_{\frac{\pi}{2}}^{0} t\cos t\, dt = 2\int_0^{\frac{\pi}{2}} t\cos t\, dt$$

$$= 2\left[t\int \cos t\, dt - \int \left(\cos t\, dt\right)\frac{dt}{dt}.dt\right]_0^{\frac{\pi}{2}}$$

$$= 2\left[t\sin t - \int \sin t\, dt\right]_0^{\frac{\pi}{2}}$$

$$= 2\left[t\sin t + \cos t\right]_0^{\frac{\pi}{2}}$$

$$= 2\left[\frac{\pi}{2} - 1\right]$$

$$\int_0^1 \left(\cos^{-1} x\right)^2 dx = \left[x\left(\cos^{-1} x\right)^2\right]_0^1 + \int_0^1 \frac{x.2\cos^{-1}}{\sqrt{1-x^2}} dx = \left[x\left(\cos^{-1} x\right)^2\right]_0^1 + 2\left(\frac{\pi}{2} - 1\right)$$

$$= 0 - 0 + 2\left(\frac{\pi}{2} - 1\right)$$

$$= \left(\pi - 2\right)$$

$$\therefore \int_0^1 \left(\cos^{-1} x\right)^2 dx = \left(\pi - 2\right)$$

*********** END **********