

Mathematical Induction Ex 12.2 Q1

Let 
$$P(n): 1+2+3+--+n = \frac{n(n+1)}{2}$$

For 
$$n = 1$$
,

LHS of 
$$P(n) = 1$$

RHS of P(n) = 
$$\frac{1(1+1)}{2}1 = 1$$

Since, LHS = RHS

$$\Rightarrow$$
 P(n) is true for  $n = 1$ 

Let P(n) be true for n = k, so

$$1 + 2 + 3 + - - - + k = \frac{k(k+1)}{2}$$
 --- (1)

Now

$$(1+2+3+---+k)+(k+1)$$
=\frac{k(k+1)}{2}+(k+1)
=(k+1)\left(\frac{k}{2}+1\right)
=\frac{(k+1)(k+2)}{2}

$$=\frac{(k+1)[(k+1)+1]}{2}$$

$$\Rightarrow$$
  $P(n)$  is true for  $n = k + 1$ 

$$\Rightarrow$$
  $P(n)$  is true for all  $n \in N$ 

So, by the principle of mathematical induction

$$P(n): 1+2+3+--+n = \frac{n(n+1)}{2}$$
 is true for all  $n \in N$ 

Mathematical Induction Ex 12.2 Q2

Let 
$$P(n): 1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$$

For n = 1

$$P(1): 1 = \frac{1(1+1)(2+1)}{6}$$
  
1 = 1

 $\Rightarrow$  P(n) is true for n = 1Let P(n) is true for n = k, so

$$P(k): 1^{2} + 2^{2} + 3^{2} + \dots + k^{2} = \frac{k(k+1)(2k+1)}{6}$$
 --- (1)

We have to show that P(n) is true for n = k + 1

$$\Rightarrow 1^2 + 2^2 + 3^2 + \dots + k^2 + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$$
  
So,  $1^2 + 2^2 + 3^2 + \dots + k^2 + (k+1)^2$ 

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^{2}$$
 [Using equation (1)]
$$= (k+1) \left[ \frac{2k^{2} + k}{6} + \frac{(k+1)}{1} \right]$$

$$= (k+1) \left[ \frac{2k^{2} + k + 6k + 6}{6} \right]$$

$$= (k+1) \left[ \frac{2k^{2} + 7k + 6}{6} \right]$$

$$= (k+1) \left[ \frac{2k^{2} + 4k + 3k + 6}{6} \right]$$

$$= (k+1) \left[ \frac{2k(k+2) + 3(k+2)}{6} \right]$$

$$=\frac{\left(k+1\right)\left(2k+3\right)\left(k+2\right)}{6}$$

 $\Rightarrow$  P(n) is true for n = k + 1

 $\Rightarrow$  P(n) is true for all  $n \in N$  by PMI

Mathematical Induction Ex 12.2 Q3

Let 
$$P(n): 1+3+3^2+...+3^{n-1}=\frac{3^n-1}{2}$$

For n = 1

$$P(1): 1 = \frac{3^{1} - 1}{2}$$

$$1 = 1$$

 $\Rightarrow$  P(n) is true for n = 1Let P(n) is true for n = k

We have to show P(n) is true for n = k + 1

i.e. 
$$1 + 3 + 3^2 + \ldots + 3^k = \frac{3^{k+1} - 1}{2}$$

Now,

$$\left\{1+3+3^2+\dots+3^{k-1}\right\}+3^{k+1-1}$$

$$= \frac{3k-1}{2} + 3^{k}$$
 [Using equation (1)]
$$= \frac{3^{k} - 1 + 2 \cdot 3^{k}}{2}$$
$$= \frac{3 \cdot 3^{k} - 1}{2}$$

$$= \frac{3^{k+1} - 1}{2}$$

 $\Rightarrow$  P(n) is true for n = k + 1

 $\Rightarrow$  P(n) is true for all  $n \in N$  by PMI

Mathematical Induction Ex 12.2 Q4

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