Q2 : Differentiate the function with respect to *x*.

$$\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$$

Answer :

$$\text{Let } y=\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$$

Taking logarithm on both the sides, we obtain

$$\log y=\log\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$$

$$\Rightarrow \log y=\frac{1}{2}\log\left[\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}\right]$$

$$\Rightarrow \log y=\frac{1}{2}\left[\log\{(x-1)(x-2)\}-\log\{(x-3)(x-4)(x-5)\}\right]$$

$$\Rightarrow \log y=\frac{1}{2}\left[\log(x-1)+\log(x-2)-\log(x-3)-\log(x-4)-\log(x-5)\right]$$

Differentiating both sides with respect to *x*, we obtain

$$\frac{1}{y}\frac{dy}{dx}=\frac{1}{2}\left[\begin{array}{l}\frac{1}{x-1}\cdot\frac{d}{dx}(x-1)+\frac{1}{x-2}\cdot\frac{d}{dx}(x-2)-\frac{1}{x-3}\cdot\frac{d}{dx}(x-3)\\ -\frac{1}{x-4}\cdot\frac{d}{dx}(x-4)-\frac{1}{x-5}\cdot\frac{d}{dx}(x-5)\end{array}\right]$$

$$\Rightarrow \frac{dy}{dx}=\frac{y}{2}\left(\frac{1}{x-1}+\frac{1}{x-2}-\frac{1}{x-3}-\frac{1}{x-4}-\frac{1}{x-5}\right)$$

$$\therefore \frac{dy}{dx}=\frac{1}{2}\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}\left[\frac{1}{x-1}+\frac{1}{x-2}-\frac{1}{x-3}-\frac{1}{x-4}-\frac{1}{x-5}\right]$$

Answer needs Correction? Click Here

Q3 : Differentiate the function with respect to *x*.

$$(\log x)^{\cos x}$$

Answer :

$$\text{Let } y=(\log x)^{\cos x}$$

Taking logarithm on both the sides, we obtain

$$\log y=\cos x\cdot\log(\log x)$$

Differentiating both sides with respect to *x*, we obtain

$$\frac{1}{y}\cdot\frac{dy}{dx}=\frac{d}{dx}(\cos x)\times\log(\log x)+\cos x\times\frac{d}{dx}\left[\log(\log x)\right]$$

$$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx}=-\sin x\log(\log x)+\cos x\times\frac{1}{\log x}\cdot\frac{d}{dx}(\log x)$$

$$\Rightarrow \frac{dy}{dx}=y\left[-\sin x\log(\log x)+\frac{\cos x}{\log x}\times\frac{1}{x}\right]$$

$$\therefore \frac{dy}{dx}=(\log x)^{\cos x}\left[\frac{\cos x}{x\log x}-\sin x\log(\log x)\right]$$

Answer needs Correction? Click Here

Q4 : Differentiate the function with respect to *x*.

$$x^x-2^{\sin x}$$

Answer :

$\text{Let } y=x^x-2^{\sin x}$

$\text{Also, let } x^x=u \text{ and } 2^{\sin x}=v$

$\therefore y=u-v$

$$\Rightarrow \frac{dy}{dx}=\frac{du}{dx}-\frac{dv}{dx}$$

$u = x^x$

Taking logarithm on both the sides, we obtain

$\log u=x\log x$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{u}\frac{du}{dx}=\left[\frac{d}{dx}(x)\times\log x+x\times\frac{d}{dx}(\log x)\right]$$
$$\Rightarrow\frac{du}{dx}=u\left[1\times\log x+x\times\frac{1}{x}\right]$$
$$\Rightarrow\frac{du}{dx}=x^x(\log x+1)$$
$$\Rightarrow\frac{du}{dx}=x^x(1+\log x)$$

$v = 2^{\sin x}$

Taking logarithm on both the sides with respect to x, we obtain

$$\log v=\sin x\cdot\log 2$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v}\cdot\frac{dv}{dx}=\log 2\cdot\frac{d}{dx}(\sin x)$$
$$\Rightarrow\frac{dv}{dx}=v\log 2\cos x$$
$$\Rightarrow\frac{dv}{dx}=2^{\sin x}\cos x\log 2$$
$$\therefore\frac{dy}{dx}=x^x(1+\log x)-2^{\sin x}\cos x\log 2$$

Answer needs Correction? **Click Here**

Q5 : Differentiate the function with respect to *x*.

$$(x+3)^2.(x+4)^3.(x+5)^4$$

Answer :

Let $y=(x+3)^2.(x+4)^3.(x+5)^4$

Taking logarithm on both the sides, we obtain

$$\log y=\log(x+3)^2+\log(x+4)^3+\log(x+5)^4$$
$$\Rightarrow\log y=2\log(x+3)+3\log(x+4)+4\log(x+5)$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{y}\cdot\frac{dy}{dx}=2\cdot\frac{1}{x+3}\cdot\frac{d}{dx}(x+3)+3\cdot\frac{1}{x+4}\cdot\frac{d}{dx}(x+4)+4\cdot\frac{1}{x+5}\cdot\frac{d}{dx}(x+5)$$
$$\Rightarrow\frac{dy}{dx}=y\left[\frac{2}{x+3}+\frac{3}{x+4}+\frac{4}{x+5}\right]$$
$$\Rightarrow\frac{dy}{dx}=(x+3)^2(x+4)^3(x+5)^4\cdot\left[\frac{2}{x+3}+\frac{3}{x+4}+\frac{4}{x+5}\right]$$
$$\Rightarrow\frac{dy}{dx}=(x+3)^2(x+4)^3(x+5)^4\cdot\left[\frac{2(x+4)(x+5)+3(x+3)(x+5)+4(x+3)(x+4)}{(x+3)(x+4)(x+5)}\right]$$
$$\Rightarrow\frac{dy}{dx}=(x+3)(x+4)^2(x+5)^3\cdot\left[2(x^2+9x+20)+3(x^2+8x+15)+4(x^2+7x+12)\right]$$
$$\therefore\frac{dy}{dx}=(x+3)(x+4)^2(x+5)^3(9x^2+70x+133)$$

Answer needs Correction? **Click Here**

Q6 : Differentiate the function with respect to *x*.

$$(x+3)^2.(x+4)^3.(x+5)^4$$

Answer :

Let $y=(x+3)^2.(x+4)^3.(x+5)^4$

Taking logarithm on both the sides, we obtain

$$\log y=\log(x+3)^2+\log(x+4)^3+\log(x+5)^4$$
$$\Rightarrow\log y=2\log(x+3)+3\log(x+4)+4\log(x+5)$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{y}\cdot\frac{dy}{dx}=2\cdot\frac{1}{x+3}\cdot\frac{d}{dx}(x+3)+3\cdot\frac{1}{x+4}\cdot\frac{d}{dx}(x+4)+4\cdot\frac{1}{x+5}\cdot\frac{d}{dx}(x+5)$$
$$\Rightarrow\frac{dy}{dx}=y\left[\frac{2}{x+3}+\frac{3}{x+4}+\frac{4}{x+5}\right]$$
$$\Rightarrow\frac{dy}{dx}=(x+3)^2(x+4)^3(x+5)^4\cdot\left[\frac{2}{x+3}+\frac{3}{x+4}+\frac{4}{x+5}\right]$$
$$\Rightarrow\frac{dy}{dx}=(x+3)^2(x+4)^3(x+5)^4\cdot\left[\frac{2(x+4)(x+5)+3(x+3)(x+5)+4(x+3)(x+4)}{(x+3)(x+4)(x+5)}\right]$$
$$\Rightarrow\frac{dy}{dx}=(x+3)(x+4)^2(x+5)^3\cdot\left[2(x^2+9x+20)+3(x^2+8x+15)+4(x^2+7x+12)\right]$$
$$\therefore\frac{dy}{dx}=(x+3)(x+4)^2(x+5)^3(9x^2+70x+133)$$

Answer needs Correction? **Click Here**

Q7 : Differentiate the function with respect to *x*.

$(\log x)^x + x^{\log x}$

Answer :

Let $y = (\log x)^x + x^{\log x}$

Also, let $u = (\log x)^x$ and $v = x^{\log x}$

$\therefore y = u + v$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ...(1)

$u = (\log x)^x$

$\Rightarrow \log u = \log\left[(\log x)^x\right]$

$\Rightarrow \log u = x\log(\log x)$

Differentiating both sides with respect to x, we obtain

$\frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(x)\times\log(\log x) + x\cdot\frac{d}{dx}\left[\log(\log x)\right]$

$\Rightarrow \frac{du}{dx} = u\left[1\times\log(\log x) + x\cdot\frac{1}{\log x}\cdot\frac{d}{dx}(\log x)\right]$

$\Rightarrow \frac{du}{dx} = (\log x)^x\left[\log(\log x) + \frac{x}{\log x}\cdot\frac{1}{x}\right]$

$\Rightarrow \frac{du}{dx} = (\log x)^x\left[\log(\log x) + \frac{1}{\log x}\right]$

$\Rightarrow \frac{du}{dx} = (\log x)^x\left[\frac{\log(\log x)\cdot\log x + 1}{\log x}\right]$

$\Rightarrow \frac{du}{dx} = (\log x)^{x-1}\left[1 + \log x.\log(\log x)\right]$ ...(2)

$v = x^{\log x}$

$\Rightarrow \log v = \log\left(x^{\log x}\right)$

$\Rightarrow \log v = \log x \log x = (\log x)^2$

Differentiating both sides with respect to x, we obtain

$\frac{1}{v}\cdot\frac{dv}{dx} = \frac{d}{dx}\left[(\log x)^2\right]$

$\Rightarrow \frac{1}{v}.\frac{dv}{dx} = 2(\log x)\cdot\frac{d}{dx}(\log x)$

$\Rightarrow \frac{dv}{dx} = 2v(\log x)\cdot\frac{1}{x}$

$\Rightarrow \frac{dv}{dx} = 2x^{\log x}\frac{\log x}{x}$

$\Rightarrow \frac{dv}{dx} = 2x^{\log x-1}\cdot\log x$ ...(3)

Therefore, from (1), (2), and (3), we obtain

$\frac{dy}{dx} = (\log x)^{x-1}\left[1 + \log x.\log(\log x)\right] + 2x^{\log x-1}\cdot\log x$

Answer needs Correction? **Click Here**

Q8 : Differentiate the function with respect to *x*.

$(\sin x)^x + \sin^{-1}\sqrt{x}$

Answer :

Let $y = (\sin x)^x + \sin^{-1}\sqrt{x}$

Also, let $u = (\sin x)^x$ and $v = \sin^{-1}\sqrt{x}$

$\therefore y = u + v$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ...(1)

$u = (\sin x)^x$

$\Rightarrow \log u = \log(\sin x)^x$

$\Rightarrow \log u = x\log(\sin x)$

Differentiating both sides with respect to x, we obtain

$\Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(x)\times\log(\sin x) + x\times\frac{d}{dx}\left[\log(\sin x)\right]$

$\Rightarrow \frac{du}{dx} = u\left[1\cdot\log(\sin x) + x\cdot\frac{1}{\sin x}\cdot\frac{d}{dx}(\sin x)\right]$

$\Rightarrow \frac{du}{dx} = (\sin x)^x\left[\log(\sin x) + \frac{x}{\sin x}\cdot\cos x\right]$

$\Rightarrow \frac{du}{dx} = (\sin x)^x(x\cot x + \log\sin x)$ ...(2)

$v = \sin^{-1}\sqrt{x}$

Differentiating both sides with respect to x, we obtain

$\frac{dv}{dx} = \frac{1}{\sqrt{1-(\sqrt{x})^2}}\cdot\frac{d}{dx}(\sqrt{x})$

$\Rightarrow \frac{dv}{dx} = \frac{1}{\sqrt{1-x}}\cdot\frac{1}{2\sqrt{x}}$

$\Rightarrow \frac{dv}{dx} = \frac{1}{\ldots}$ ...(3)

$$\frac{d}{dx} \quad 2\sqrt{x-x^2}$$

Therefore, from (1), (2), and (3), we obtain

$$\frac{dy}{dx}=(\sin x)^x(x\cot x+\log\sin x)+\frac{1}{2\sqrt{x-x^2}}$$

Answer needs Correction? **Click Here**

Q9 : Differentiate the function with respect to *x*.

$$x^{\sin x}+(\sin x)^{\cos x}$$

Answer :

$$\text{Let } y=x^{\sin x}+(\sin x)^{\cos x}$$
$$\text{Also, let } u=x^{\sin x} \text{ and } v=(\sin x)^{\cos x}$$
$$\therefore y=u+v$$
$$\Rightarrow \frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx} \quad ...(1)$$

$$u=x^{\sin x}$$
$$\Rightarrow \log u=\log\left(x^{\sin x}\right)$$
$$\Rightarrow \log u=\sin x\log x$$

Differentiating both sides with respect to *x*, we obtain

$$\frac{1}{u}\frac{du}{dx}=\frac{d}{dx}(\sin x)\cdot\log x+\sin x\cdot\frac{d}{dx}(\log x)$$
$$\Rightarrow \frac{du}{dx}=u\left[\cos x\log x+\sin x\cdot\frac{1}{x}\right]$$
$$\Rightarrow \frac{du}{dx}=x^{\sin x}\left[\cos x\log x+\frac{\sin x}{x}\right] \quad ...(2)$$

$$v=(\sin x)^{\cos x}$$
$$\Rightarrow \log v=\log(\sin x)^{\cos x}$$
$$\Rightarrow \log v=\cos x\log(\sin x)$$

Differentiating both sides with respect to *x*, we obtain

$$\frac{1}{v}\frac{dv}{dx}=\frac{d}{dx}(\cos x)\times\log(\sin x)+\cos x\times\frac{d}{dx}\left[\log(\sin x)\right]$$
$$\Rightarrow \frac{dv}{dx}=v\left[-\sin x.\log(\sin x)+\cos x\cdot\frac{1}{\sin x}\cdot\frac{d}{dx}(\sin x)\right]$$
$$\Rightarrow \frac{dv}{dx}=(\sin x)^{\cos x}\left[-\sin x\log\sin x+\frac{\cos x}{\sin x}\cos x\right]$$
$$\Rightarrow \frac{dv}{dx}=(\sin x)^{\cos x}\left[-\sin x\log\sin x+\cot x\cos x\right]$$
$$\Rightarrow \frac{dv}{dx}=(\sin x)^{\cos x}\left[\cot x\cos x-\sin x\log\sin x\right] \quad ...(3)$$

From (1), (2), and (3), we obtain

$$\frac{dy}{dx}=x^{\sin x}\left(\cos x\log x+\frac{\sin x}{x}\right)+(\sin x)^{\cos x}\left[\cos x\cot x-\sin x\log\sin x\right]$$

Answer needs Correction? **Click Here**

Q10 : Differentiate the function with respect to *x*.

$$x^{x\cos x}+\frac{x^2+1}{x^2-1}$$

Answer :

$$\text{Let } y=x^{x\cos x}+\frac{x^2+1}{x^2-1}$$
$$\text{Also, let } u=x^{x\cos x} \text{ and } v=\frac{x^2+1}{x^2-1}$$
$$\therefore y=u+v$$
$$\Rightarrow \frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx} \quad ...(1)$$

$$u=x^{x\cos x}$$
$$\Rightarrow \log u=\log\left(x^{x\cos x}\right)$$
$$\Rightarrow \log u=x\cos x\log x$$

Differentiating both sides with respect to *x*, we obtain

$$\frac{1}{u}\frac{du}{dx}=\frac{d}{dx}(x)\cdot\cos x\cdot\log x+x\cdot\frac{d}{dx}(\cos x)\cdot\log x+x\cos x\cdot\frac{d}{dx}(\log x)$$
$$\Rightarrow \frac{du}{dx}=u\left[1\cdot\cos x\cdot\log x+x\cdot(-\sin x)\log x+x\cos x\cdot\frac{1}{x}\right]$$
$$\Rightarrow \frac{du}{dx}=x^{x\cos x}(\cos x\log x-x\sin x\log x+\cos x)$$
$$\Rightarrow \frac{du}{dx}=x^{x\cos x}\left[\cos x(1+\log x)-x\sin x\log x\right] \quad ...(2)$$

$$v=\frac{x^2+1}{x^2-1}$$
$$\Rightarrow \log v=\log(x^2+1)-\log(x^2-1)$$

Differentiating both sides with respect to *x*, we obtain

$$\frac{1}{v}\frac{dv}{dx}=\frac{2x}{x^2+1}-\frac{2x}{x^2-1}$$
$$\Rightarrow \frac{dv}{dx}=v\left[\frac{2x\left(x^2-1\right)-2x\left(x^2+1\right)}{\left(x^2+1\right)\left(x^2-1\right)}\right]$$
$$\Rightarrow \frac{dv}{dx}=\frac{x^2+1}{x^2-1}\times\left[\frac{-4x}{\left(x^2+1\right)\left(x^2-1\right)}\right]$$
$$\Rightarrow \frac{dv}{dx}=\frac{-4x}{\left(x^2-1\right)^2} \qquad ...(3)$$

From (1), (2), and (3), we obtain

$$\frac{dy}{dx}=x^{x\cos x}\left[\cos x\left(1+\log x\right)-x\sin x\log x\right]-\frac{4x}{\left(x^2-1\right)^2}$$

Answer needs Correction? **Click Here**

Q11 : Differentiate the function with respect to *x*.

$$\left(x\cos x\right)^x+\left(x\sin x\right)^{\frac{1}{x}}$$

Answer :

Let $y=\left(x\cos x\right)^x+\left(x\sin x\right)^{\frac{1}{x}}$

Also, let $u=\left(x\cos x\right)^x$ and $v=\left(x\sin x\right)^{\frac{1}{x}}$

$\therefore\ y=u+v$

$$\Rightarrow \frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx} \qquad ...(1)$$

$$u=\left(x\cos x\right)^x$$
$$\Rightarrow \log u=\log\left(x\cos x\right)^x$$
$$\Rightarrow \log u=x\log\left(x\cos x\right)$$
$$\Rightarrow \log u=x\left[\log x+\log\cos x\right]$$
$$\Rightarrow \log u=x\log x+x\log\cos x$$

Differentiating both sides with respect to *x*, we obtain

$$\frac{1}{u}\frac{du}{dx}=\frac{d}{dx}\left(x\log x\right)+\frac{d}{dx}\left(x\log\cos x\right)$$
$$\Rightarrow \frac{du}{dx}=u\left[\left\{\log x\cdot\frac{d}{dx}(x)+x\cdot\frac{d}{dx}(\log x)\right\}+\left\{\log\cos x\cdot\frac{d}{dx}(x)+x\cdot\frac{d}{dx}(\log\cos x)\right\}\right]$$
$$\Rightarrow \frac{du}{dx}=\left(x\cos x\right)^x\left[\left(\log x\cdot 1+x\cdot\frac{1}{x}\right)+\left\{\log\cos x\cdot 1+x\cdot\frac{1}{\cos x}\cdot\frac{d}{dx}(\cos x)\right\}\right]$$
$$\Rightarrow \frac{du}{dx}=\left(x\cos x\right)^x\left[\left(\log x+1\right)+\left\{\log\cos x+\frac{x}{\cos x}\cdot(-\sin x)\right\}\right]$$
$$\Rightarrow \frac{du}{dx}=\left(x\cos x\right)^x\left[\left(1+\log x\right)+\left(\log\cos x-x\tan x\right)\right]$$
$$\Rightarrow \frac{du}{dx}=\left(x\cos x\right)^x\left[1-x\tan x+\left(\log x+\log\cos x\right)\right]$$
$$\Rightarrow \frac{du}{dx}=\left(x\cos x\right)^x\left[1-x\tan x+\log\left(x\cos x\right)\right] \qquad ...(2)$$

$$v=\left(x\sin x\right)^{\frac{1}{x}}$$
$$\Rightarrow \log v=\log\left(x\sin x\right)^{\frac{1}{x}}$$
$$\Rightarrow \log v=\frac{1}{x}\log\left(x\sin x\right)$$
$$\Rightarrow \log v=\frac{1}{x}\left(\log x+\log\sin x\right)$$
$$\Rightarrow \log v=\frac{1}{x}\log x+\frac{1}{x}\log\sin x$$

Differentiating both sides with respect to *x*, we obtain

$$\frac{1}{v}\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{x}\log x\right)+\frac{d}{dx}\left[\frac{1}{x}\log\left(\sin x\right)\right]$$
$$\Rightarrow \frac{1}{v}\frac{dv}{dx}=\left[\log x\cdot\frac{d}{dx}\left(\frac{1}{x}\right)+\frac{1}{x}\cdot\frac{d}{dx}(\log x)\right]+\left[\log\left(\sin x\right)\cdot\frac{d}{dx}\left(\frac{1}{x}\right)+\frac{1}{x}\cdot\frac{d}{dx}\left\{\log\left(\sin x\right)\right\}\right]$$
$$\Rightarrow \frac{1}{v}\frac{dv}{dx}=\left[\log x\cdot\left(-\frac{1}{x^2}\right)+\frac{1}{x}\cdot\frac{1}{x}\right]+\left[\log\left(\sin x\right)\cdot\left(-\frac{1}{x^2}\right)+\frac{1}{x}\cdot\frac{1}{\sin x}\cdot\frac{d}{dx}(\sin x)\right]$$
$$\Rightarrow \frac{1}{v}\frac{dv}{dx}=\frac{1}{x^2}\left(1-\log x\right)+\left[-\frac{\log\left(\sin x\right)}{x^2}+\frac{1}{x\sin x}\cdot\cos x\right]$$
$$\Rightarrow \frac{dv}{dx}=\left(x\sin x\right)^{\frac{1}{x}}\left[\frac{1-\log x}{x^2}+\frac{-\log\left(\sin x\right)+x\cot x}{x^2}\right]$$
$$\Rightarrow \frac{dv}{dx}=\left(x\sin x\right)^{\frac{1}{x}}\left[\frac{1-\log x-\log\left(\sin x\right)+x\cot x}{x^2}\right]$$
$$\Rightarrow \frac{dv}{dx}=\left(x\sin x\right)^{\frac{1}{x}}\left[\frac{1-\log\left(x\sin x\right)+x\cot x}{x^2}\right] \qquad ...(3)$$

From (1), (2), and (3), we obtain

$$\frac{dy}{dx}=\left(x\cos x\right)^x\left[1-x\tan x+\log\left(x\cos x\right)\right]+\left(x\sin x\right)^{\frac{1}{x}}\left[\frac{x\cot x+1-\log\left(x\sin x\right)}{x^2}\right]$$

Answer needs Correction? **Click Here**

Q12 : Find $\frac{dy}{dx}$ of function.

$x^y + y^x = 1$

Answer :

The given function is $x^y + y^x = 1$

Let $x^y = u$ and $y^x = v$

Then, the function becomes $u + v = 1$

$$\therefore \frac{du}{dx} + \frac{dv}{dx} = 0 \quad \text{...(1)}$$

$$u = x^y$$
$$\Rightarrow \log u = \log\left(x^y\right)$$
$$\Rightarrow \log u = y \log x$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{u}\frac{du}{dx} = \log x \frac{dy}{dx} + y \cdot \frac{d}{dx}(\log x)$$
$$\Rightarrow \frac{du}{dx} = u\left[\log x \frac{dy}{dx} + y \cdot \frac{1}{x}\right]$$
$$\Rightarrow \frac{du}{dx} = x^y\left(\log x \frac{dy}{dx} + \frac{y}{x}\right) \quad \text{...(2)}$$

$$v = y^x$$
$$\Rightarrow \log v = \log\left(y^x\right)$$
$$\Rightarrow \log v = x \log y$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v} \cdot \frac{dv}{dx} = \log y \cdot \frac{d}{dx}(x) + x \cdot \frac{d}{dx}(\log y)$$
$$\Rightarrow \frac{dv}{dx} = v\left(\log y \cdot 1 + x \cdot \frac{1}{y} \cdot \frac{dy}{dx}\right)$$
$$\Rightarrow \frac{dv}{dx} = y^x\left(\log y + \frac{x}{y}\frac{dy}{dx}\right) \quad \text{...(3)}$$

From (1), (2), and (3), we obtain

$$x^y\left(\log x \frac{dy}{dx} + \frac{y}{x}\right) + y^x\left(\log y + \frac{x}{y}\frac{dy}{dx}\right) = 0$$
$$\Rightarrow \left(x^y \log x + x y^{x-1}\right)\frac{dy}{dx} = -\left(y x^{y-1} + y^x \log y\right)$$
$$\therefore \frac{dy}{dx} = -\frac{y x^{y-1} + y^x \log y}{x^y \log x + x y^{x-1}}$$

Answer needs Correction? Click Here

Q13 : Find $\frac{dy}{dx}$ of function.

$y^x = x^y$

Answer :

The given function is $y^x = x^y$

Taking logarithm on both the sides, we obtain

$$x \log y = y \log x$$

Differentiating both sides with respect to x, we obtain

$$\log y \cdot \frac{d}{dx}(x) + x \cdot \frac{d}{dx}(\log y) = \log x \cdot \frac{d}{dx}(y) + y \cdot \frac{d}{dx}(\log x)$$
$$\Rightarrow \log y \cdot 1 + x \cdot \frac{1}{y} \cdot \frac{dy}{dx} = \log x \cdot \frac{dy}{dx} + y \cdot \frac{1}{x}$$
$$\Rightarrow \log y + \frac{x}{y}\frac{dy}{dx} = \log x \frac{dy}{dx} + \frac{y}{x}$$
$$\Rightarrow \left(\frac{x}{y} - \log x\right)\frac{dy}{dx} = \frac{y}{x} - \log y$$
$$\Rightarrow \left(\frac{x - y \log x}{y}\right)\frac{dy}{dx} = \frac{y - x \log y}{x}$$
$$\therefore \frac{dy}{dx} = \frac{y}{x}\left(\frac{y - x \log y}{x - y \log x}\right)$$

Answer needs Correction? Click Here

Q14 : Find $\frac{dy}{dx}$ of function.

$(\cos x)^y = (\cos y)^x$

Answer :

The given function is $(\cos x)^y = (\cos y)^x$

Taking logarithm on both the sides, we obtain

$y \log \cos x = x \log \cos y$

Differentiating both sides, we obtain

$$\log \cos x \cdot \frac{dy}{dx} + y \cdot \frac{d}{dx}(\log \cos x) = \log \cos y \cdot \frac{d}{dx}(x) + x \cdot \frac{d}{dx}(\log \cos y)$$
$$\Rightarrow \log \cos x \frac{dy}{dx} + y \cdot \frac{1}{\cos x} \cdot \frac{d}{dx}(\cos x) = \log \cos y \cdot 1 + x \cdot \frac{1}{\cos y} \cdot \frac{d}{dx}(\cos y)$$
$$\Rightarrow \log \cos x \frac{dy}{dx} + \frac{y}{\cos x} \cdot (-\sin x) = \log \cos y + \frac{x}{\cos y}(-\sin y) \cdot \frac{dy}{dx}$$
$$\Rightarrow \log \cos x \frac{dy}{dx} - y \tan x = \log \cos y - x \tan y \frac{dy}{dx}$$
$$\Rightarrow (\log \cos x + x \tan y)\frac{dy}{dx} = y \tan x + \log \cos y$$
$$\therefore \frac{dy}{dx} = \frac{y \tan x + \log \cos y}{x \tan y + \log \cos x}$$

Answer needs Correction? **Click Here**

Q15 : Find $\frac{dy}{dx}$ of function.

$xy = e^{(x-y)}$

Answer :

The given function is $xy = e^{(x-y)}$

Taking logarithm on both the sides, we obtain

$$\log(xy) = \log(e^{x-y})$$
$$\Rightarrow \log x + \log y = (x-y)\log e$$
$$\Rightarrow \log x + \log y = (x-y) \times 1$$
$$\Rightarrow \log x + \log y = x - y$$

Differentiating both sides with respect to *x*, we obtain

$$\frac{d}{dx}(\log x) + \frac{d}{dx}(\log y) = \frac{d}{dx}(x) - \frac{dy}{dx}$$
$$\Rightarrow \frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 1 - \frac{dy}{dx}$$
$$\Rightarrow \left(1 + \frac{1}{y}\right)\frac{dy}{dx} = 1 - \frac{1}{x}$$
$$\Rightarrow \left(\frac{y+1}{y}\right)\frac{dy}{dx} = \frac{x-1}{x}$$
$$\therefore \frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}$$

Answer needs Correction? **Click Here**

Q16 : Find the derivative of the function given by $f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$ and hence find $f'(1)$.

Answer :

The given relationship is $f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$

Taking logarithm on both the sides, we obtain

$$\log f(x) = \log(1+x) + \log(1+x^2) + \log(1+x^4) + \log(1+x^8)$$

Differentiating both sides with respect to *x*, we obtain

$$\frac{1}{f(x)} \cdot \frac{d}{dx}[f(x)] = \frac{d}{dx}\log(1+x) + \frac{d}{dx}\log(1+x^2) + \frac{d}{dx}\log(1+x^4) + \frac{d}{dx}\log(1+x^8)$$
$$\Rightarrow \frac{1}{f(x)} \cdot f'(x) = \frac{1}{1+x} \cdot \frac{d}{dx}(1+x) + \frac{1}{1+x^2} \cdot \frac{d}{dx}(1+x^2) + \frac{1}{1+x^4} \cdot \frac{d}{dx}(1+x^4) + \frac{1}{1+x^8} \cdot \frac{d}{dx}(1+x^8)$$
$$\Rightarrow f'(x) = f(x)\left[\frac{1}{1+x} + \frac{1}{1+x^2} . 2x + \frac{1}{1+x^4} \cdot 4x^3 + \frac{1}{1+x^8} \cdot 8x^7\right]$$
$$\therefore f'(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)\left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8}\right]$$
$$\text{Hence, } f'(1) = (1+1)(1+1^2)(1+1^4)(1+1^8)\left[\frac{1}{1+1} + \frac{2 \times 1}{1+1^2} + \frac{4 \times 1^3}{1+1^4} + \frac{8 \times 1^7}{1+1^8}\right]$$
$$= 2 \times 2 \times 2 \times 2\left[\frac{1}{2} + \frac{2}{2} + \frac{4}{2} + \frac{8}{2}\right]$$
$$= 16 \times \left(\frac{1+2+4+8}{2}\right)$$
$$= 16 \times \frac{15}{2} = 120$$

Answer needs Correction? **Click Here**

Q17 : Differentiate $(x^5 - 5x + 8)(x^3 + 7x + 9)$ in three ways mentioned below

(i) By using product rule.

(ii) By expanding the product to obtain a single polynomial.

(iii By logarithmic differentiation.

Do they all give the same answer?

Do they all give the same answer?

Answer :

Let $y=\left(x^5-5x+8\right)\left(x^3+7x+9\right)$

(i)

Let $x^2-5x+8=u$ and $x^3+7x+9=v$

$\therefore y=uv$

$\Rightarrow \frac{dy}{dx}=\frac{du}{dx}\cdot v+u\cdot\frac{dv}{dx}$ (By using product rule)

$\Rightarrow \frac{dy}{dx}=\frac{d}{dx}\left(x^2-5x+8\right)\cdot\left(x^3+7x+9\right)+\left(x^2-5x+8\right)\cdot\frac{d}{dx}\left(x^3+7x+9\right)$

$\Rightarrow \frac{dy}{dx}=(2x-5)\left(x^3+7x+9\right)+\left(x^2-5x+8\right)\left(3x^2+7\right)$

$\Rightarrow \frac{dy}{dx}=2x\left(x^3+7x+9\right)-5\left(x^3+7x+9\right)+x^2\left(3x^2+7\right)-5x\left(3x^2+7\right)+8\left(3x^2+7\right)$

$\Rightarrow \frac{dy}{dx}=\left(2x^4+14x^2+18x\right)-5x^3-35x-45+\left(3x^4+7x^2\right)-15x^3-35x+24x^2+56$

$\therefore \frac{dy}{dx}=5x^4-20x^3+45x^2-52x+11$

(ii)

$y=\left(x^2-5x+8\right)\left(x^3+7x+9\right)$

$=x^2\left(x^3+7x+9\right)-5x\left(x^3+7x+9\right)+8\left(x^3+7x+9\right)$

$=x^5+7x^3+9x^2-5x^4-35x^2-45x+8x^3+56x+72$

$=x^5-5x^4+15x^3-26x^2+11x+72$

$\therefore \frac{dy}{dx}=\frac{d}{dx}\left(x^5-5x^4+15x^3-26x^2+11x+72\right)$

$=\frac{d}{dx}\left(x^5\right)-5\frac{d}{dx}\left(x^4\right)+15\frac{d}{dx}\left(x^3\right)-26\frac{d}{dx}\left(x^2\right)+11\frac{d}{dx}(x)+\frac{d}{dx}(72)$

$=5x^4-5\times4x^3+15\times3x^2-26\times2x+11\times1+0$

$=5x^4-20x^3+45x^2-52x+11$

(iii) $y=\left(x^2-5x+8\right)\left(x^3+7x+9\right)$

Taking logarithm on both the sides, we obtain

$\log y=\log\left(x^2-5x+8\right)+\log\left(x^3+7x+9\right)$

Differentiating both sides with respect to x, we obtain

$\frac{1}{y}\frac{dy}{dx}=\frac{d}{dx}\log\left(x^2-5x+8\right)+\frac{d}{dx}\log\left(x^3+7x+9\right)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx}=\frac{1}{x^2-5x+8}\cdot\frac{d}{dx}\left(x^2-5x+8\right)+\frac{1}{x^3+7x+9}\cdot\frac{d}{dx}\left(x^3+7x+9\right)$

$\Rightarrow \frac{dy}{dx}=y\left[\frac{1}{x^2-5x+8}\times(2x-5)+\frac{1}{x^3+7x+9}\times\left(3x^2+7\right)\right]$

$\Rightarrow \frac{dy}{dx}=\left(x^2-5x+8\right)\left(x^3+7x+9\right)\left[\frac{2x-5}{x^2-5x+8}+\frac{3x^2+7}{x^3+7x+9}\right]$

$\Rightarrow \frac{dy}{dx}=\left(x^2-5x+8\right)\left(x^3+7x+9\right)\left[\frac{(2x-5)\left(x^3+7x+9\right)+\left(3x^2+7\right)\left(x^2-5x+8\right)}{\left(x^2-5x+8\right)\left(x^3+7x+9\right)}\right]$

$\Rightarrow \frac{dy}{dx}=2x\left(x^3+7x+9\right)-5\left(x^3+7x+9\right)+3x^2\left(x^2-5x+8\right)+7\left(x^2-5x+8\right)$

$\Rightarrow \frac{dy}{dx}=\left(2x^4+14x^2+18x\right)-5x^3-35x-45+\left(3x^4-15x^3+24x^2\right)+\left(7x^2-35x+56\right)$

$\Rightarrow \frac{dy}{dx}=5x^4-20x^3+45x^2-52x+11$

From the above three observations, it can be concluded that all the results of $\frac{dy}{dx}$ are same.

Q18 : If u, v and w are functions of x, then show that

$$\frac{d}{dx}(u.v.w)=\frac{du}{dx}v.w+u.\frac{dv}{dx}.w+u.v.\frac{dw}{dx}$$

in two ways-first by repeated application of product rule, second by logarithmic differentiation.

Answer :

Let $y=u.v.w=u.(v.w)$

By applying product rule, we obtain

$\frac{dy}{dx}=\frac{du}{dx}.(v.w)+u.\frac{d}{dx}(v\cdot w)$

$\Rightarrow \frac{dy}{dx}=\frac{du}{dx}v\cdot w+u\left[\frac{dv}{dx}\cdot w+v\cdot\frac{dw}{dx}\right]$ (Again applying product rule)

$\Rightarrow \frac{dy}{dx}=\frac{du}{dx}\cdot v\cdot w+u\cdot\frac{dv}{dx}\cdot w+u\cdot v\cdot\frac{dw}{dx}$

By taking logarithm on both sides of the equation $y=u.v.w$, we obtain

$\log y=\log u+\log v+\log w$

Differentiating both sides with respect to x, we obtain

$\frac{1}{y}\cdot\frac{dy}{dx}=\frac{d}{dx}(\log u)+\frac{d}{dx}(\log v)+\frac{d}{dx}(\log w)$

$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx}=\frac{1}{u}\frac{du}{dx}+\frac{1}{v}\frac{dv}{dx}+\frac{1}{w}\frac{dw}{dx}$

$$y \quad dx \quad u \quad dx \quad v \quad dx \quad w \quad dx$$

$$\Rightarrow \frac{dy}{dx} = y\left(\frac{1}{u}\frac{du}{dx} + \frac{1}{v}\frac{dv}{dx} + \frac{1}{w}\frac{dw}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx} = u.v.w.\left(\frac{1}{u}\frac{du}{dx} + \frac{1}{v}\frac{dv}{dx} + \frac{1}{w}\frac{dw}{dx}\right)$$

$$\therefore \frac{dy}{dx} = \frac{du}{dx}\cdot v\cdot w + u\cdot\frac{dv}{dx}\cdot w + u\cdot v\cdot\frac{dw}{dx}$$

*********** END **********