Mean Value Theorems Ex 15.2 Q1(xiii)

Here,

$$f(x) = x\sqrt{x^2-4} \text{ on } [2,4]$$

$f(x)$ is continuous at it attains a unique value for each $x \in [2,4]$ and

$$f'(x) = \frac{2x}{2\sqrt{x^2-4}}$$

$$f'(x) = \frac{x}{\sqrt{x^2-4}}$$

$\Rightarrow$ $f'(x)$ exists for each $x \in (2,4)$

$\Rightarrow$ $f(x)$ is differentiable in $(2,4)$, so

Lagrange's mean value theorem is applicable, so there exist a $c \in (2,4)$ such that

$$f'(c) = \frac{f(4)-f(2)}{4-2}$$

$$\Rightarrow \quad \frac{c}{\sqrt{c^2-4}} = \frac{\sqrt{12}-0}{2}$$

Squarintg both the sides,

$$\Rightarrow \quad \frac{c^2}{c^2-4} = \frac{12}{4}$$

$$\Rightarrow \quad 4c^2 = 12c^2 - 48$$

$$\Rightarrow \quad 8c^2 = 48$$

$$\Rightarrow \quad c^2 = 6$$

$$\Rightarrow \quad c = \sqrt{6} \in (2,4)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(xiv)

Here,

$$f(x) = x^2 + x - 1 \text{ on } [0,4]$$

$f(x)$ is polynomial, so it is continuous is $[0,4]$ and differentiable in $(0,4)$
as every polynomial is continuous and differentiable every where. So,
Lagrange's mean value theorem is applicable, so there exists a point $c \in [0,4]$ such that

$$f'(c) = \frac{f(4)-f(0)}{4-0}$$

$$\Rightarrow \quad 2c+1 = \frac{\left((4)^2+4-1\right)-(0-1)}{4}$$

$$\Rightarrow \quad 2c+1 = \frac{19+1}{4}$$

$$\Rightarrow \quad 2c+1 = 5$$

$$\Rightarrow \quad c = 2 \in (0,4)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(xv)

Here,

$$f(x) = \sin x - \sin 2x - x \text{ on } [0, \pi]$$

We know that $\sin x$ and polynomial is continuous and differentiable every where so, $f(x)$ is continuous in $[0, \pi]$ and differentiable in $[0, \pi]$. So, Lagrange's mean value theorem is applicable. So, there exist a point $c \in (0, \pi)$ such that

$$f'(c) = \frac{f(\pi) - f(0)}{\pi - 0}$$

$$\Rightarrow \quad \cos c - 2\cos 2c - 1 = \frac{(\sin\pi - \sin 2\pi - \pi) - (0)}{\pi}$$

$$\Rightarrow \quad \cos c - 2\cos 2c = -1 + 1$$

$$\Rightarrow \quad \cos c - 2\left(2\cos^2 c - 1\right) = 0$$

$$\Rightarrow \quad 4\cos^2 c - \cos c - 2 = 0$$

$$\Rightarrow \quad \cos c - \frac{-(-1) \pm \sqrt{1 - 4 \times 4 \times (-2)}}{8}$$

$$\Rightarrow \quad \cos c = \frac{1 \pm \sqrt{33}}{8}$$

$$\Rightarrow \quad c = \cos^{-1}\left(\frac{1 \pm \sqrt{33}}{8}\right) \in (0, \pi)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(xvi)

The given function is $f(x) = x^3 - 5x^2 - 3x$, f being a polynomial function, is continuous in [1,3] and is differentiable in [1,3] whose derivative is $3x^2 - 10x - 3$.

$$f(1) = 1^3 - 5(1)^2 - 3(1) = -7$$

$$f(3) = 3^3 - 5(3)^2 - 3(3) = 27 - 45 - 9 = -27$$

$$\therefore \frac{f(b) - f(a)}{b - a} = \frac{f(3) - f(1)}{3 - 1} = \frac{-27 + 7}{2} = -10$$

Mean value theorem states that there is a point $c(1,3)$ such that $f'(c) = 3c^2 - 10c - 3$

$$f'(c) = -10$$

$$3c^2 - 10c - 3 = -10$$

$$3c^2 - 10c + 7 = 0$$

$$3c^2 - 3c - 7c + 7 = 0$$

$$c = \frac{7}{3}, \text{ where } c = \frac{7}{3} \in (1,3)$$

Hence, Mean value theorem is verified for the given function.

*********** END **********