



Trigonometric Ratios Ex 5.3 Q6

Answer :

(i) We have to prove: $\tan\left(\frac{C+A}{2}\right) = \cot\frac{B}{2}$

Since we know that in triangle ABC

$$A + B + C = 180$$

$$\Rightarrow C + A = 180^\circ - B$$

$$\Rightarrow \frac{C+A}{2} = 90^\circ - \frac{B}{2}$$

$$\Rightarrow \tan\frac{C+A}{2} = \tan\left(90^\circ - \frac{B}{2}\right)$$

$$\Rightarrow \boxed{\tan\frac{C+A}{2} = \cot\frac{B}{2}}$$

Proved

(ii) We have to prove: $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$

Since we know that in triangle ABC

$$A + B + C = 180$$

$$\Rightarrow B + C = 180^\circ - A$$

$$\Rightarrow \frac{B+C}{2} = 90^\circ - \frac{A}{2}$$

$$\Rightarrow \sin\frac{B+C}{2} = \sin\left(90^\circ - \frac{A}{2}\right)$$

$$\Rightarrow \boxed{\sin\frac{B+C}{2} = \cos\frac{A}{2}}$$

Proved

Trigonometric Ratios Ex 5.3 Q7

Answer :

We are asked to find the value of $\tan 20^\circ \tan 35^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ$

(i) Therefore

$$\begin{aligned}\tan 20^\circ \tan 35^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ \\&= \tan(90^\circ - 70^\circ) \tan(90^\circ - 55^\circ) \tan 45^\circ \tan 55^\circ \tan 70^\circ \\&= \cot 70^\circ \cot 55^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ \\&= (\tan 70^\circ \cot 70^\circ) (\tan 55^\circ \cot 55^\circ) \tan 45^\circ \\&= 1 \times 1 \times 1 \\&= \boxed{1}\end{aligned}$$

Proved

(ii) We will simplify the left hand side

$$\begin{aligned}\sin 48^\circ \cdot \sec 48^\circ + \cos 48^\circ \cdot \operatorname{cosec} 42^\circ &= \sin 48^\circ \cdot \sec(90^\circ - 48^\circ) + \cos 48^\circ \cdot \operatorname{cosec}(90^\circ - 48^\circ) \\&= \sin 48^\circ \cdot \cos 48^\circ + \cos 48^\circ \cdot \sin 48^\circ \\&= 1 + 1 \\&= \boxed{2}\end{aligned}$$

Proved

(iii) We have, $\frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2 \cos 70^\circ \cdot \operatorname{cosec} 20^\circ = 0$

So we will calculate left hand side

$$\begin{aligned}\frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2 \cos 70^\circ \cdot \operatorname{cosec} 20^\circ &= \frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\cos 70^\circ}{\sin 20^\circ} - 2 \cos 70^\circ \cdot \operatorname{cosec}(90^\circ - 70^\circ) \\&= \frac{\sin(90^\circ - 20^\circ)}{\cos 20^\circ} + \frac{\cos(90^\circ - 20^\circ)}{\sin 20^\circ} - 2 \cos 70^\circ \cdot \sec 70^\circ \\&= \frac{\cos 20^\circ}{\cos 20^\circ} + \frac{\sin 20^\circ}{\sin 20^\circ} - 2 \times 1 \\&= 1 + 1 - 2 \\&= 2 - 2 \\&= 0\end{aligned}$$

Proved

(iv) We have $\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \cdot \operatorname{cosec} 31^\circ = 2$

We will simplify the left hand side

$$\begin{aligned}\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \cdot \operatorname{cosec} 31^\circ &= \frac{\cos(90^\circ - 10^\circ)}{\sin 10^\circ} + \cos 59^\circ \cdot \operatorname{cosec}(90^\circ - 59^\circ) \\&= \frac{\sin 10^\circ}{\sin 10^\circ} + \cos 59^\circ \cdot \sec 59^\circ \\&= 1 + 1 \\&= \boxed{2}\end{aligned}$$

Proved

***** END *****