Trigonometric Identities Ex 6.1 Q87

Answer :

Given:

$$x = a\sec\theta\cos\phi$$

$$\Rightarrow \frac{x}{a} = \sec\theta\cos\phi \qquad \ldots\ldots(1)$$

$$y = b\sec\theta\sin\phi$$

$$\Rightarrow \frac{y}{b} = \sec\theta\sin\phi \qquad \ldots\ldots(2)$$

$$\Rightarrow \frac{z}{c} = \tan\theta \qquad \ldots\ldots(3)$$

We have to prove that $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$.

Squaring the above equations and then subtracting the third from the sum of the first two, we have

$$\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2-\left(\frac{z}{c}\right)^2=(\sec\theta\cos\phi)^2+(\sec\theta\sin\phi)^2-(\tan\theta)^2$$

$$\Rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=\sec^2\theta\cos^2\phi+\sec^2\theta\sin^2\phi-\tan^2\theta$$

$$\Rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=(\sec^2\theta\cos^2\phi+\sec^2\theta\sin^2\phi)-\tan^2\theta$$

$$\Rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=\sec^2\theta(\cos^2\phi+\sin^2\phi)-\tan^2\theta$$

$$\Rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=\sec^2\theta(1)-\tan^2\theta$$

$$\Rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=\sec^2\theta-\tan^2\theta$$

$$\Rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$

Hence proved.

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