Indefinite Integrals Ex 19.9 Q40

$$\frac{(x+1)(x+\log x)^2}{x}=\left(\frac{x+1}{x}\right)(x+\log x)^2=\left(1+\frac{1}{x}\right)(x+\log x)^2$$

Let $(x+\log x)=t$

$$\Rightarrow \left(1+\frac{1}{x}\right)dx=dt$$

$$\Rightarrow \int\left(1+\frac{1}{x}\right)(x+\log x)^2\,dx=\int t^2\,dt$$

$$=\frac{t^3}{3}+C$$

$$=\frac{1}{3}(x+\log x)^3+C$$

Indefinite Integrals Ex 19.9 Q41

Let $\quad I=\int \tan x\sec^2 x\sqrt{1-\tan^2 x}\,dx - - - - - (i)$

Let $\quad 1-\tan^2 x=t$ then,

$d\left(1-\tan^2 x\right)=dt$

$\Rightarrow \quad -2\tan x\sec^2 x\,dx=dt$

$\Rightarrow \quad \tan x\sec^2 x\,dx=\frac{-dt}{2}$

Putting $1-\tan^2 x=t$ and $\tan x\sec^2 x\,dx=-\frac{dt}{2}$ in equation (i),

we get

$$I=\int\sqrt{t}\times\frac{-dt}{2}$$

$$=-\frac{1}{2}\int t^{\frac{1}{2}}dt$$

$$=-\frac{1}{2}\times\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+c$$

$$=-\frac{1}{3}t^{\frac{3}{2}}+c$$

$$\therefore \quad I=-\frac{1}{3}\left[1-\tan^2 x\right]^{\frac{3}{2}}+c$$

Indefinite Integrals Ex 19.9 Q42

Let $\quad I = \int \log x \frac{\sin\left(1+(\log x)^2\right)}{x} dx ------ (i)$

Let $\quad 1+(\log x)^2 = t$ then,

$d\left(1+(\log x)^2\right) = dt$

$\Rightarrow \quad 2\log x \frac{1}{x} dx = dt$

$\Rightarrow \quad \frac{\log x}{x} dx = \frac{dt}{2}$

Putting $1+(\log x)^2 = t$ and $\frac{\log x}{x} dx = \frac{dt}{2}$ in equation (i),

we get

$$I = \int \sin t \times \frac{dt}{2}$$

$$= \frac{1}{2} \int \sin t \, dt$$

$$\therefore \quad I = -\frac{1}{2} \cos t + c$$

$$= -\frac{1}{2} \cos\left[1+(\log x)^2\right] + c$$

$$\therefore \quad I = -\frac{1}{2} \cos\left[1+(\log x)^2\right] + c$$

Indefinite Integrals Ex 19.9 Q43

Let $\quad I = \int \frac{1}{x^2} \cos^2\left(\frac{1}{x}\right) dx - - - - - (i)$

Let $\quad \frac{1}{x} = t$ then,

$$d\left(\frac{1}{x}\right) = dt$$

$$\Rightarrow \quad \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \quad \frac{1}{x^2} dx = -dt$$

Putting $\frac{1}{x} = t$ and $\frac{1}{x^2} dx = -dt$ in equation (i), we get

$$I = \int \cos^2 t \, (-dt)$$

$$= -\int \cos^2 t \, dt$$

$$= -\int \frac{\cos^2 2t + 1}{2} dt$$

$$= -\frac{1}{2} \int \cos 2t \, dt - \frac{1}{2} \int dt$$

$$= -\frac{1}{2} \times \frac{\sin 2t}{2} - \frac{1}{2} t + c$$

$$\therefore \quad I = -\frac{1}{4} \sin 2t - \frac{1}{2} t + c$$

$$= -\frac{1}{4} \sin 2 \times \frac{1}{x} - \frac{1}{2} \times \frac{1}{x} + c$$

$$\therefore \quad I = -\frac{1}{4} \sin\left(\frac{2}{x}\right) - \frac{1}{2}\left(\frac{1}{x}\right) + c$$

Indefinite Integrals Ex 19.9 Q44

Let $\quad I = \int \sec^4 x \tan x \, dx - - - - (i)$

Let $\quad \tan x = t \quad$ then,

$d(\tan x) = dt$

$\Rightarrow \quad \sec^2 x \, dx = dt$

$\Rightarrow \quad dx = \dfrac{dt}{\sec^2 x}$

Putting $\tan x = t$ and $dx = \dfrac{dt}{\sec^2 x}$ in equation (i),

we get

$$
\begin{aligned}
I &= \int \sec^4 x \tan x \frac{dt}{\sec^2 x} \\
&= \int \sec^2 x \, t dt \\
&= \int \left(1 + \tan^2 x\right) t \, dt \\
&= \int \left(1 + t^2\right) t \, dt \\
&= \int \left(t + t^3\right) dt \\
&= \frac{t^2}{2} + \frac{t^4}{4} + c \\
&= \frac{\tan^2 x}{2} + \frac{\tan^4 x}{4} + c
\end{aligned}
$$

$\therefore \quad I = \dfrac{1}{2} \tan^2 x + \dfrac{1}{4} \tan^4 x + c$

*********** END **********