

Indefinite Integrals Ex 19.22 Q9

Let
$$I = \int \frac{1}{\cos x \left(\sin x + 2\cos x\right)} dx$$
$$= \int \frac{1}{\sin x \cos x + 2\cos^2 x} dx$$

Diving numerator and denominator by $\cos^2 x$,

$$I = \int \frac{\sec^2 x}{\tan x + 2} dx$$

Let
$$2 + \tan x = t$$

 $\sec^2 x \, dx = dt$
 $I = \int \frac{dt}{t}$
 $= \log |t| + c$

$$I = \log |2 + \tan x| + c$$

Indefinite Integrals Ex 19.22 Q10

Let
$$I = \int \frac{1}{\sin^2 x + \sin 2x} dx$$
$$= \int \frac{1}{\sin^2 x + 2\sin x \cos x} dx$$

Diving numerator and denominator by $\cos^2 x$,

$$I = \int \frac{\sec^2 x}{\tan^2 x + 2 \tan x} dx$$

Let
$$\tan x = t$$
$$\sec^2 x \, dx = dt$$
$$= \int \frac{dt}{t^2 + 2t + (1)^2 - (1)^2}$$
$$= \int \frac{dt}{(t+1)^2 - (1)^2}$$
$$= \frac{1}{2} \log \left| \frac{t+1-1}{t+1+1} \right| + c$$
$$= \frac{1}{2} \log \left| \frac{t}{t+2} \right| + c$$

$$I = \frac{1}{2} \log \left| \frac{\tan x}{\tan x + 2} \right| + c$$

Indefinite Integrals Ex 19.22 Q11

Let
$$I = \int \frac{1}{\cos 2x + 3\sin^2 x} dx$$
$$= \int \frac{1}{2\cos^2 x - 1 + 3\sin^2 x} dx$$

Diving numerator and denominator by $\cos^2 x$,

$$I = \int \frac{\sec^2 x}{2 - \sec^2 x + 3\tan^2 x} dx$$

$$= \int \frac{\sec^2 x}{2 - \left(1 + \tan^2 x\right)^2 + 3\tan^2 x} dx$$

$$= \int \frac{\sec^2 x}{2 - 1 - \tan^2 x + 3\tan^2 x} dx$$

$$= \int \frac{dt}{1 + 2\tan^2 x}$$

Let
$$\sqrt{2} \tan x = t$$

$$\sqrt{2} \sec^2 x \, dx = dt$$

$$I = \frac{1}{\sqrt{2}} \int \frac{1}{1+t^2}$$

$$= \frac{1}{\sqrt{2}} \tan^{-1} t + c$$

$$I = \frac{1}{\sqrt{2}} \tan^{-1} \left(\sqrt{2} \tan x \right) + c$$

********* END *******