Areas of Parallelograms and Triangles Ex 15.3 Q23

Answer :

Given:

(a) ΔABC and Δ BDE are two equilateral triangles

(b) D is the midpoint of BC

(c) AE intersect BC in F.

To prove:

(i) $\text{ar}(\Delta BDE) = \frac{1}{4}\text{ar}(\Delta ABC)$

(ii) $\text{ar}(\Delta BDE) = \frac{1}{2}\text{ar}(\Delta BAE)$

(iii) $\text{ar}(\Delta BFE) = \text{ar}(\Delta AFD)$

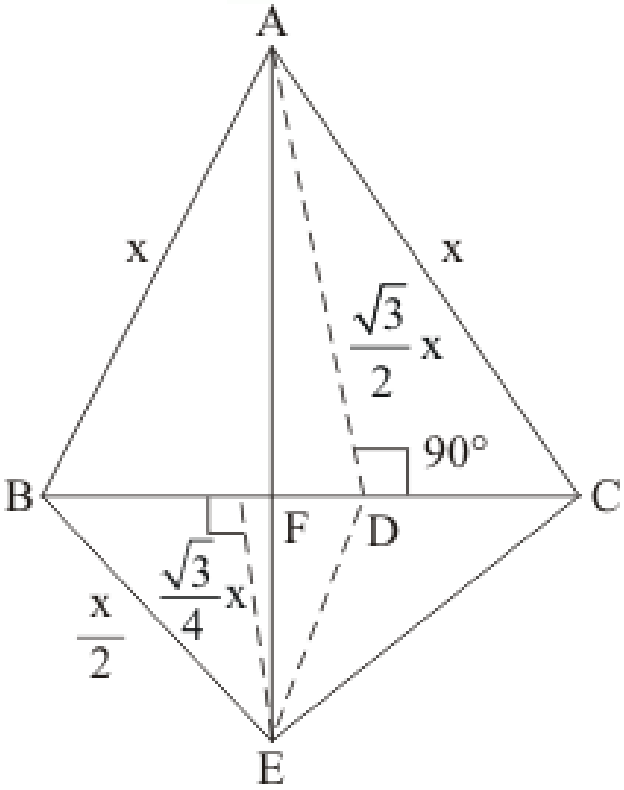
(iv) $\text{ar}(\Delta ABC) = 2\text{ar}(\Delta BEC)$

(v) $\text{ar}(\Delta FED) = \frac{1}{8}\text{ar}(\Delta AFC)$

(vi) $\text{ar}(\Delta BFE) = 2\text{ar}(\Delta EFD)$

Proof: Let AB = BC = CA = x cm.

Then BD = $\frac{x}{2}$ = DE = BE



(i) We have

$$ar(\Delta ABC) = \frac{\sqrt{3}}{4}x^2$$

$$ar(\Delta BDE) = \frac{\sqrt{3}}{4}\left(\frac{x}{2}\right)^2$$

$$ar(\Delta BDE) = \frac{1}{4}\left(\frac{\sqrt{3}}{4}\right)x^2$$

$$\boxed{ar(\Delta BDE) = \frac{1}{4}(\Delta ABC)}$$

(ii) We Know that ΔABC and ΔBED are equilateral triangles

$\Rightarrow \angle ACB = \angle DBE = 60^\circ$

$\Rightarrow BE \| AC$

$\Rightarrow ar(\Delta BAE) = ar(\Delta BEC)$

$ar(\Delta BAE) = 2(\Delta BDE)$ [$\because$ ED is a median of ΔEBC]

$$\boxed{ar(\Delta BDE) = \frac{1}{2}(\Delta BAE)}$$

(iii) We Know that ΔABC and ΔBED are equilateral triangles

$\Rightarrow \angle ACB = \angle BDE = 60^\circ$

$\Rightarrow \angle ACB = \angle BDE$

$\Rightarrow AB \| DE$

$\Rightarrow ar(\Delta BED) = ar(\Delta AED)$

$\Rightarrow ar(\Delta BED) - ar(\Delta EFD) = ar(\Delta AED) - ar(\Delta EFD)$

$\Rightarrow \boxed{ar(\Delta BED) = ar(\Delta AED)}$

(iv) Since ED is a median of Δ BEC

$\Rightarrow ar(\Delta BEC) = 2ar(\Delta BDE)$

$\Rightarrow ar(\Delta BEC) = 2 \times \frac{1}{4} ar(\Delta ABC) \quad$ [from (i)]

$\Rightarrow ar(\Delta BEC) = \frac{1}{2}(\Delta ABC)$

$\Rightarrow \boxed{ar(\Delta ABC) = 2(\Delta BEC)}$

(v) We basically want to find out FD. Let FD = y

Since triangle BED and triangle DEA are on the same base and between same parallels ED and BE respectively. So

$ar(\Delta DEA) = ar(\Delta BED)$

$ar(\Delta EFD) + ar(\Delta AFD) = ar(\Delta BED)$

Since altitude of altitude of any equilateral triangle having side x is $\frac{\sqrt{3}}{2}x$

$\Rightarrow \frac{1}{2} y \times \frac{\sqrt{3}x}{4} + \frac{1}{2} y \times \frac{\sqrt{3}x}{2} = \frac{\sqrt{3}}{4} \times \frac{x^2}{4}$

$\Rightarrow 2y + 4y = x$

$\Rightarrow y = \frac{1}{6}x$

So

$ar(\Delta EFD) = \frac{1}{2} y \times \frac{\sqrt{3}}{4} x$

$= \frac{\sqrt{3}}{8} \times x \times \frac{x}{6}$

$ar(\Delta EFD) = \frac{1}{8} \frac{\sqrt{3}}{6} x^2$ ...... (1)

Now

$ar(\Delta AFC) = \frac{1}{2}\left(y + \frac{x}{2}\right) \times \frac{\sqrt{3}}{2} x$

$$ar(\Delta AFC) = \frac{\sqrt{3}}{6}x^2 \quad \ldots\ldots \ (2)$$

From (1) and (2) we get

$$ar(\Delta EFD) = \frac{1}{8}ar(\Delta AFC)$$

(vi) Now we know y in terms of x. So

$$ar(\Delta BFE) = \frac{1}{2}\left(\frac{x}{2} - y\right) \times \frac{\sqrt{3}}{4}x$$

$$= \frac{\sqrt{3}}{8}x \times \frac{x}{3}$$

$$= \frac{\sqrt{3}x^2}{24} \quad \ldots\ldots\ (3)$$

$$ar(\Delta EFD) = \frac{1}{2}y \times \frac{\sqrt{3}}{4}x$$

$$= \frac{1}{2}\frac{1}{6}x \times \frac{\sqrt{3}x}{4}$$

$$= \frac{1}{2}\frac{\sqrt{3}x^2}{24} \quad \ldots\ldots \ (4)$$

From (3) and (4) we get

$$\boxed{ar(\Delta BFE) = 2ar(\Delta EFD)}$$

*********** END **********