



Differentials Errors and Approximation Ex14.1 Q13

Let r be the radius of the sphere and Δr be the error in measuring the radius.

Then,

$$r = 9 \text{ m and } \Delta r = 0.03 \text{ m}$$

Now, the surface area of the sphere (S) is given by,

$$S = 4\pi r^2$$

$$\therefore \frac{dS}{dr} = 8\pi r$$

$$\begin{aligned}\therefore dS &= \left(\frac{dS}{dr} \right) \Delta r \\ &= (8\pi r) \Delta r \\ &= 8\pi(9)(0.03) \text{ m}^2 \\ &= 2.16\pi \text{ m}^2\end{aligned}$$

Hence, the approximate error in calculating the surface area is $2.16\pi \text{ m}^2$.

Differentials Errors and Approximation Ex14.1 Q14

The surface area of a cube (S) of side x is given by $S = 6x^2$.

$$\begin{aligned}\therefore \frac{dS}{dx} &= \left(\frac{dS}{dx} \right) \Delta x \\ &= (12x) \Delta x \\ &= (12x)(0.01x) \quad \text{[as 1% of } x \text{ is } 0.01x\text{]} \\ &= 0.12x^2\end{aligned}$$

Hence, the approximate change in the surface area of the cube is $0.12x^2 \text{ m}^2$.

Differentials Errors and Approximation Ex14.1 Q15

Let r be the radius of the sphere and Δr be the error in measuring the radius.

Then,

$$r = 7 \text{ m and } \Delta r = 0.02 \text{ m}$$

Now, the volume V of the sphere is given by,

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ \therefore \frac{dV}{dr} &= 4\pi r^2 \\ \therefore dV &= \left(\frac{dV}{dr}\right)\Delta r \\ &= (4\pi r^2)\Delta r \\ &= 4\pi(7)^2(0.02) \text{ m}^3 = 3.92\pi \text{ m}^3 \end{aligned}$$

Hence, the approximate error in calculating the volume is $3.92 \pi \text{ m}^3$.

Differentials Errors and Approximation Ex14.1 Q16

The volume of a cube (V) of side x is given by $V = x^3$.

$$\begin{aligned} \therefore dV &= \left(\frac{dV}{dx}\right)\Delta x \\ &= (3x^2)\Delta x \\ &= (3x^2)(0.01x) \quad \text{[as 1% of } x \text{ is } 0.01x\text{]} \\ &= 0.03x^3 \end{aligned}$$

Hence, the approximate change in the volume of the cube is $0.03x^3 \text{ m}^3$.

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