Functions Ex2.2 Q8

Let $f : R \to R$ and $g : R \to R$ are defined as

$$f(x) = x + 1 \text{ and } g(x) = x - 1$$

Now,

$$f \circ g(x) = f(g(x)) = f(x - 1) = x - 1 + 1$$
$$= x = I_R \,\ldots\ldots\ldots\,(\text{i})$$

Again,

$$f \circ g(x) = f(g(x)) = g(x + 1) = x + 1 - 1$$
$$= x = I_R \,\ldots\ldots\ldots\,(\text{ii})$$

from (i) & (ii)

$$f \circ g = g \circ f = I_R$$

Functions Ex2.2 Q9

We have, $f : N \to Z_0$, $g : Z_0 \to Q$ and

$h : Q \to R$

Also, $f(x) = 2x$, $g(x) = \dfrac{1}{x}$ and $h(x) = e^x$

Now, $f : N \to Z_0$ and $h \circ g : Z_0 \to R$

$\therefore\ (h \circ g) \circ f : N \to R$

also, $g \circ f : N \to Q$ and $h : Q \to R$

$\therefore\ h \circ (g \circ f) : N \to R$

Thus, $(h \circ g) \circ f$ and $h \circ (g \circ f)$ exist and are function from N to set R.

Finally. $(h \circ g) \circ f\ (x) = (h \circ g)(f(x)) = (h \circ g)(2x)$

$$= h\left(\tfrac{1}{2x}\right)$$
$$= e^{1/2x}$$

now, $h \circ (g \circ f)(x) = h \circ (g(2x)) = h\left(\tfrac{1}{2x}\right)$

$$= e^{1/2x}$$

Hence, associativity verified.

Functions Ex2.2 Q10

We have,

$$h\circ(g\circ f)(x) = h(g\circ f(x)) = h(g(f(x)))$$
$$= h(g(2x)) = h(3(2x)+4)$$
$$= h(6x+4) = \sin(6x+4) \quad \forall x \in \mathbf{N}$$

$$((h\circ g)\circ f)(x) = (h\circ g)(f(x)) = (h\circ g)(2x)$$
$$= h(g(2x)) = = h(3(2x)+4)$$
$$= h(6x+4) = \sin(6x+4) \quad \forall x \in \mathbf{N}$$

This shows, $h\circ(g\circ f) = (h\circ g)\circ f$

Functions Ex2.2 Q11

Define f: **N** → **N** by, $f(x) = x + 1$

And, g: **N** → **N** by,

$$g(x) = \begin{cases} x-1 \text{ if } x>1 \\ 1 \quad\ \ \text{ if } x=1 \end{cases}$$

We first show that f is not onto.

For this, consider element 1 in co-domain **N**. It is clear that this element is not an image of any of the elements in domain **N**.

Therefore, f is not onto.

Now, *gof*: **N** → **N** is defined by,

Functions Ex2.2 Q12

Define f: **N** → **Z** as $f(x) = x$ and g: **Z** → **Z** as $g(x) = |x|$.

We first show that g is not injective.

It can be observed that:

$g(-1) = |-1| = 1$

$g(1) = |1| = 1$

Therefore, $g(-1) = g(1)$, but $-1 \neq 1$.

Therefore, g is not injective.

Now, *gof*: **N** → **Z** is defined as $gof(x) = g(f(x)) = g(x) = |x|$.

Let x, $y \in$ **N** such that $gof(x) = gof(y)$.

$\Rightarrow |x| = |y|$

Since x and $y \in$ **N,** both are positive.

$\therefore |x| = |y| \Rightarrow x = y$

Hence, *gof* is injective

Functions Ex2.2 Q13

We have, $f : A \to B$ and $g : B \to C$ are one-one functions

Now we have to prove : $g\circ f : A \to C$ in one-one

let $x, y \in A$ such that

$$g\circ f(x) = g\circ f(y)$$

$\Rightarrow \quad g(f(x)) = g(f(y))$

$\Rightarrow \quad f(x) = f(y) \qquad [\because g \text{ in one-one}]$

$\Rightarrow \quad x = y \qquad [\because f \text{ in one-one}]$

$\therefore \quad g\circ f$ is one-one function

Functions Ex2.2 Q14

We have, $f : A \to B$ and $g : B \to C$ are onto functions.

Now, we need to prove: $g \circ f : A \to C$ in onto.

let $y \in C$, then

$$g \circ f(x) = y$$

$$\Rightarrow \quad g(f(x)) = y \text{ ....(i)}$$

Since g is onto, for each element in C, then exists a preimage in B.

$$\therefore \quad g(x) = y \text{ ....(ii)}$$

From (i) & (ii)

$$f(x) = \alpha.$$

Since f is onto, for each element in B there exists a preimage in A

$$\therefore f(x) = \alpha \text{ ....(iii)}$$

From (ii) and (iii) we can conclude that for each $y \in C$, there exists a pre image in A such that $g \circ f(x) = y$

$\therefore$ $g \circ f$ is onto

*********** END **********