



Exercise 5.4 : Solutions of Questions on Page Number : 174

Q1 : Differentiate the following w.r.t. x :

$$\frac{e^x}{\sin x}$$

Answer :

$$\text{Let } y = \frac{e^x}{\sin x}$$

By using the quotient rule, we obtain

$$\begin{aligned} \frac{dy}{dx} &= \frac{\sin x \cdot \frac{d}{dx}(e^x) - e^x \cdot \frac{d}{dx}(\sin x)}{\sin^2 x} \\ &= \frac{\sin x \cdot (e^x) - e^x \cdot (\cos x)}{\sin^2 x} \\ &= \frac{e^x (\sin x - \cos x)}{\sin^2 x}, x \neq n\pi, n \in \mathbb{Z} \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q2 : Differentiate the following w.r.t. x :

$$e^{\sin^{-1} x}$$

Answer :

$$\text{Let } y = e^{\sin^{-1} x}$$

By using the chain rule, we obtain

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx}(e^{\sin^{-1} x}) \\ \Rightarrow \frac{dy}{dx} &= e^{\sin^{-1} x} \cdot \frac{d}{dx}(\sin^{-1} x) \\ &= e^{\sin^{-1} x} \cdot \frac{1}{\sqrt{1-x^2}} \\ &= \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}} \\ \therefore \frac{dy}{dx} &= \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}}, x \in (-1, 1) \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q3 : Differentiate the following w.r.t. x :

$$e^{x^3}$$

Answer :

$$\text{Let } y = e^{x^3}$$

By using the chain rule, we obtain

$$\frac{dy}{dx} = \frac{d}{dx}(e^{x^3}) = e^{x^3} \cdot \frac{d}{dx}(x^3) = e^{x^3} \cdot 3x^2 = 3x^2 e^{x^3}$$

Answer needs Correction? [Click Here](#)

Q4 : Differentiate the following w.r.t. x :

$$\sin(\tan^{-1} e^{-x})$$

Answer :

$$\text{Let } y = \sin(\tan^{-1} e^{-x})$$

By using the chain rule, we obtain

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx}[\sin(\tan^{-1} e^{-x})] \\ &= \cos(\tan^{-1} e^{-x}) \cdot \frac{d}{dx}(\tan^{-1} e^{-x}) \\ &= \cos(\tan^{-1} e^{-x}) \cdot \frac{1}{1+(e^{-x})^2} \cdot \frac{d}{dx}(e^{-x}) \\ &= \frac{\cos(\tan^{-1} e^{-x})}{1+e^{-2x}} \cdot e^{-x} \cdot \frac{d}{dx}(-x) \end{aligned}$$

$$\begin{aligned}
 & \frac{1+e^{-2x}}{e^{-x} \cos(\tan^{-1} e^{-x})} dx \\
 &= \frac{e^{-x} \cos(\tan^{-1} e^{-x})}{1+e^{-2x}} \times (-1) \\
 &= \frac{-e^{-x} \cos(\tan^{-1} e^{-x})}{1+e^{-2x}}
 \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q5 : Differentiate the following w.r.t. x :

$$\log(\cos e^x)$$

Answer :

$$\text{Let } y = \log(\cos e^x)$$

By using the chain rule, we obtain

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{d}{dx} [\log(\cos e^x)] \\
 &= \frac{1}{\cos e^x} \cdot \frac{d}{dx} (\cos e^x) \\
 &= \frac{1}{\cos e^x} \cdot (-\sin e^x) \cdot \frac{d}{dx} (e^x) \\
 &= \frac{-\sin e^x}{\cos e^x} \cdot e^x \\
 &= -\tan e^x, e^x \neq (2n+1)\frac{\pi}{2}, n \in \mathbb{N}
 \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q6 : Differentiate the following w.r.t. x :

$$e^x + e^{x^2} + \dots + e^{x^n}$$

Answer :

$$\begin{aligned}
 & \frac{d}{dx} (e^x + e^{x^2} + \dots + e^{x^n}) \\
 &= \frac{d}{dx} (e^x) + \frac{d}{dx} (e^{x^2}) + \frac{d}{dx} (e^{x^3}) + \frac{d}{dx} (e^{x^4}) + \frac{d}{dx} (e^{x^n}) \\
 &= e^x + \left[e^{x^2} \times \frac{d}{dx} (x^2) \right] + \left[e^{x^3} \cdot \frac{d}{dx} (x^3) \right] + \left[e^{x^4} \cdot \frac{d}{dx} (x^4) \right] + \left[e^{x^n} \cdot \frac{d}{dx} (x^n) \right] \\
 &= e^x + (e^{x^2} \times 2x) + (e^{x^3} \times 3x^2) + (e^{x^4} \times 4x^3) + (e^{x^n} \times nx^{n-1}) \\
 &= e^x + 2xe^{x^2} + 3x^2e^{x^3} + 4x^3e^{x^4} + 5x^4e^{x^5}
 \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q7 : Differentiate the following w.r.t. x :

$$\sqrt{e^{\sqrt{x}}}, x > 0$$

Answer :

$$\text{Let } y = \sqrt{e^{\sqrt{x}}}$$

$$\text{Then, } y^2 = e^{\sqrt{x}}$$

By differentiating this relationship with respect to x , we obtain

$$\begin{aligned}
 y^2 &= e^{\sqrt{x}} \\
 \Rightarrow 2y \frac{dy}{dx} &= e^{\sqrt{x}} \frac{d}{dx} (\sqrt{x}) \quad [\text{By applying the chain rule}] \\
 \Rightarrow 2y \frac{dy}{dx} &= e^{\sqrt{x}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{x}} \\
 \Rightarrow \frac{dy}{dx} &= \frac{e^{\sqrt{x}}}{4y\sqrt{x}} \\
 \Rightarrow \frac{dy}{dx} &= \frac{e^{\sqrt{x}}}{4\sqrt{e^{\sqrt{x}}} \sqrt{x}} \\
 \Rightarrow \frac{dy}{dx} &= \frac{e^{\sqrt{x}}}{4\sqrt{x}e^{\frac{\sqrt{x}}{2}}}, x > 0
 \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q8 : Differentiate the following w.r.t. x :

$$\log(\log x), x > 1$$

Answer :

$$\text{Let } y = \log(\log x)$$

By using the chain rule, we obtain

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{d}{dx} [\log(\log x)] \\
 &= \frac{1}{\log x} \cdot \frac{d}{dx} (\log x) \\
 &= \frac{1}{\log x} \cdot \frac{1}{x}
 \end{aligned}$$

$$= -\frac{1}{x \log x}, x > 1$$

Answer needs Correction? [Click Here](#)

Q9 : Differentiate the following w.r.t. x :

$$\frac{\cos x}{\log x}, x > 0$$

Answer :

$$\text{Let } y = \frac{\cos x}{\log x}$$

By using the quotient rule, we obtain

$$\begin{aligned} \frac{dy}{dx} &= \frac{\frac{d}{dx}(\cos x) \times \log x - \cos x \times \frac{d}{dx}(\log x)}{(\log x)^2} \\ &= \frac{-\sin x \log x - \cos x \times \frac{1}{x}}{(\log x)^2} \\ &= \frac{-[x \log x \cdot \sin x + \cos x]}{x (\log x)^2}, x > 0 \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q10 : Differentiate the following w.r.t. x :

$$\cos(\log x + e^x), x > 0$$

Answer :

$$\text{Let } y = \cos(\log x + e^x)$$

By using the chain rule, we obtain

$$\begin{aligned} \frac{dy}{dx} &= -\sin(\log x + e^x) \cdot \frac{d}{dx}(\log x + e^x) \\ &= -\sin(\log x + e^x) \cdot \left[\frac{d}{dx}(\log x) + \frac{d}{dx}(e^x) \right] \\ &= -\sin(\log x + e^x) \cdot \left(\frac{1}{x} + e^x \right) \\ &= -\left(\frac{1}{x} + e^x \right) \sin(\log x + e^x), x > 0 \end{aligned}$$

Answer needs Correction? [Click Here](#)

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