



$$\begin{aligned} \nu &= \frac{c}{\lambda} \\ &= \frac{3 \times 10^8}{9.7 \times 10^{-8}} \approx 3.1 \times 10^{15} \text{ Hz} \end{aligned}$$

Hence, the wavelength of the photon is 97 nm while the frequency is 3.1×10^{15} Hz.

Question 12.7:

(a) Using the Bohr's model calculate the speed of the electron in a hydrogen atom in the $n = 1, 2$, and 3 levels. (b) Calculate the orbital period in each of these levels.

Answer

(a) Let v_1 be the orbital speed of the electron in a hydrogen atom in the ground state level, $n_1 = 1$. For charge (e) of an electron, v_1 is given by the relation,

$$v_1 = \frac{e^2}{n_1 4\pi \epsilon_0 \left(\frac{h}{2\pi}\right)} = \frac{e^2}{2 \epsilon_0 h}$$

Where,

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\epsilon_0 = \text{Permittivity of free space} = 8.85 \times 10^{-12} \text{ N}^{-1} \text{ C}^2 \text{ m}^{-2}$$

$$h = \text{Planck's constant} = 6.62 \times 10^{-34} \text{ Js}$$

$$\begin{aligned} \therefore v_1 &= \frac{(1.6 \times 10^{-19})^2}{2 \times 8.85 \times 10^{-12} \times 6.62 \times 10^{-34}} \\ &= 0.0218 \times 10^8 = 2.18 \times 10^6 \text{ m/s} \end{aligned}$$

For level $n_2 = 2$, we can write the relation for the corresponding orbital speed as:

$$\begin{aligned} v_2 &= \frac{e^2}{n_2 2 \epsilon_0 h} \\ &= \frac{(1.6 \times 10^{-19})^2}{2 \times 2 \times 8.85 \times 10^{-12} \times 6.62 \times 10^{-34}} \\ &= 1.09 \times 10^6 \text{ m/s} \end{aligned}$$

And, for $n_3 = 3$, we can write the relation for the corresponding orbital speed as:

$$\begin{aligned} v_3 &= \frac{e^2}{n_3 2 \epsilon_0 h} \\ &= \frac{(1.6 \times 10^{-19})^2}{3 \times 2 \times 8.85 \times 10^{-12} \times 6.62 \times 10^{-34}} \\ &= 7.27 \times 10^5 \text{ m/s} \end{aligned}$$

Hence, the speed of the electron in a hydrogen atom in $n = 1$, $n=2$, and $n=3$ is 2.18×10^6 m/s, 1.09×10^6 m/s, 7.27×10^5 m/s respectively.

(b) Let T_1 be the orbital period of the electron when it is in level $n_1 = 1$.

Orbital period is related to orbital speed as:

$$T_1 = \frac{2\pi r_1}{v_1}$$

Where,

r_1 = Radius of the orbit

$$= \frac{n_1^2 h^2 \epsilon_0}{\pi m e^2}$$

h = Planck's constant = 6.62×10^{-34} Js

e = Charge on an electron = 1.6×10^{-19} C

ϵ_0 = Permittivity of free space = $8.85 \times 10^{-12} \text{ N}^{-1} \text{ C}^2 \text{ m}^{-2}$

m = Mass of an electron = 9.1×10^{-31} kg

$$\begin{aligned} \therefore T_1 &= \frac{2\pi r_1}{v_1} \\ &= \frac{2\pi \times (1)^2 \times (6.62 \times 10^{-34})^2 \times 8.85 \times 10^{-12}}{2.18 \times 10^6 \times \pi \times 9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^2} \\ &= 15.27 \times 10^{-17} = 1.527 \times 10^{-16} \text{ s} \end{aligned}$$

For level $n_2 = 2$, we can write the period as:

$$T_2 = \frac{2\pi r_2}{v_2}$$

Where,

r_2 = Radius of the electron in $n_2 = 2$

$$= \frac{(n_2)^2 h^2 \epsilon_0}{\pi m e^2}$$

$$\begin{aligned} \therefore T_2 &= \frac{2\pi r_2}{v_2} \\ &= \frac{2\pi \times (2)^2 \times (6.62 \times 10^{-34})^2 \times 8.85 \times 10^{-12}}{1.09 \times 10^6 \times \pi \times 9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^2} \\ &= 1.22 \times 10^{-15} \text{ s} \end{aligned}$$

And, for level $n_3 = 3$, we can write the period as:

$$T_3 = \frac{2\pi r_3}{v_3}$$

Where,

r_3 = Radius of the electron in $n_3 = 3$

$$= \frac{(n_3)^2 h^2 \epsilon_0}{\pi m e^2}$$

$$\begin{aligned} \therefore T_3 &= \frac{2\pi r_3}{v_3} \\ &= \frac{2\pi \times (3)^2 \times (6.62 \times 10^{-34})^2 \times 8.85 \times 10^{-12}}{7.27 \times 10^5 \times \pi \times 9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^2} \\ &= 4.12 \times 10^{-15} \text{ s} \end{aligned}$$

Hence, the orbital period in each of these levels is 1.52×10^{-16} s, 1.22×10^{-15} s, and 4.12×10^{-15} s respectively.

Question 12.8:

The radius of the innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. What are the radii of the $n = 2$ and $n = 3$ orbits?

Answer

The radius of the innermost orbit of a hydrogen atom, $r_1 = 5.3 \times 10^{-11} \text{ m}$.

Let r_2 be the radius of the orbit at $n = 2$. It is related to the radius of the innermost orbit as:

$$\begin{aligned} r_2 &= (n)^2 r_1 \\ &= 4 \times 5.3 \times 10^{-11} = 2.12 \times 10^{-10} \text{ m} \end{aligned}$$

For $n = 3$, we can write the corresponding electron radius as:

$$\begin{aligned} r_3 &= (n)^2 r_1 \\ &= 9 \times 5.3 \times 10^{-11} = 4.77 \times 10^{-10} \text{ m} \end{aligned}$$

Hence, the radii of an electron for $n = 2$ and $n = 3$ orbits are $2.12 \times 10^{-10} \text{ m}$ and $4.77 \times 10^{-10} \text{ m}$ respectively.

Question 12.9:

A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature.

What series of wavelengths will be emitted?

Answer

It is given that the energy of the electron beam used to bombard gaseous hydrogen at room temperature is 12.5 eV. Also, the energy of the gaseous hydrogen in its ground state at room temperature is -13.6 eV .

When gaseous hydrogen is bombarded with an electron beam, the energy of the gaseous hydrogen becomes $-13.6 + 12.5 \text{ eV}$ i.e., -1.1 eV .

Orbital energy is related to orbit level (n) as:

$$E = \frac{-13.6}{(n)^2} \text{ eV}$$

$$\text{For } n = 3, \quad E = \frac{-13.6}{9} = -1.5 \text{ eV}$$

This energy is approximately equal to the energy of gaseous hydrogen. It can be concluded that the electron has jumped from $n = 1$ to $n = 3$ level.

During its de-excitation, the electrons can jump from $n = 3$ to $n = 1$ directly, which forms a line of the Lyman series of the hydrogen spectrum.

We have the relation for wave number for Lyman series as:

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