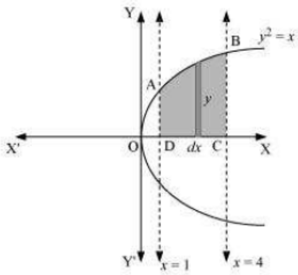
Exercise 8.1

Question 1:

Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$, $x = 4$ and the x-axis.

Answer



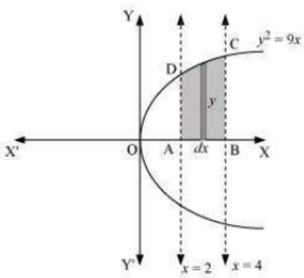
The area of the region bounded by the curve, $y^2 = x$, the lines, $x = 1$ and $x = 4$, and the x-axis is the area ABCD.

$$\begin{aligned}
\text{Area of ABCD} &= \int_1^4 y\,dx \\
&= \int_1^4 \sqrt{x}\,dx \\
&= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4 \\
&= \frac{2}{3}\left[(4)^{\frac{3}{2}} - (1)^{\frac{3}{2}}\right] \\
&= \frac{2}{3}[8-1] \\
&= \frac{14}{3} \text{ units}
\end{aligned}$$

Question 2:

Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the x-axis in the first quadrant.

Answer



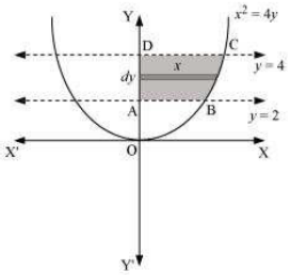
The area of the region bounded by the curve, $y^2 = 9x$, $x = 2$, and $x = 4$, and the x-axis is the area ABCD.

$$\begin{aligned}
\text{Area of ABCD} &= \int_2^4 y\,dx \\
&= \int_2^4 3\sqrt{x}\,dx \\
&= 3\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_2^4 \\
&= 2\left[x^{\frac{3}{2}}\right]_2^4 \\
&= 2\left[(4)^{\frac{3}{2}} - (2)^{\frac{3}{2}}\right] \\
&= 2\left[8 - 2\sqrt{2}\right] \\
&= \left(16 - 4\sqrt{2}\right) \text{ units}
\end{aligned}$$

Question 3:

Find the area of the region bounded by $x^2 = 4y$, $y = 2$, $y = 4$ and the y-axis in the first quadrant.

Answer



The area of the region bounded by the curve, $x^2 = 4y$, $y = 2$, and $y = 4$, and the y-axis is the area ABCD.

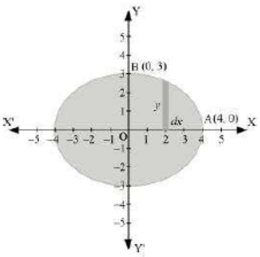
$$\begin{aligned}
\text{Area of ABCD} &= \int_2^4 x\,dy \\
&= \int_2^4 2\sqrt{y}\,dy \\
&= 2\int_2^4 \sqrt{y}\,dy \\
&= 2\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_2^4 \\
&= \frac{4}{3}\left[(4)^{\frac{3}{2}} - (2)^{\frac{3}{2}}\right] \\
&= \frac{4}{3}\left[8 - 2\sqrt{2}\right] \\
&= \left(\frac{32 - 8\sqrt{2}}{3}\right) \text{ units}
\end{aligned}$$

Question 4:

Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Answer

The given equation of the ellipse, $\frac{x^2}{16} + \frac{y^2}{9} = 1$, can be represented as



It can be observed that the ellipse is symmetrical about x-axis and y-axis.

∴ Area bounded by ellipse = 4 × Area of OAB

$$\begin{aligned}
\text{Area of OAB} &= \int_0^4 y\,dx \\
&= \int_0^4 3\sqrt{1 - \frac{x^2}{16}}\,dx \\
&= \frac{3}{4}\int_0^4 \sqrt{16 - x^2}\,dx \\
&= \frac{3}{4}\left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4 \\
&= \frac{3}{4}\left[2\sqrt{16-16} + 8\sin^{-1}(1) - 0 - 8\sin^{-1}(0)\right] \\
&= \frac{3}{4}\left[\frac{8\pi}{2}\right] \\
&= \frac{3}{4}[4\pi] \\
&= 3\pi
\end{aligned}$$

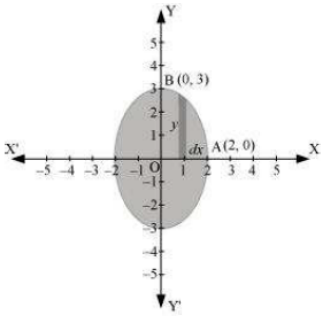
Therefore, area bounded by the ellipse = 4 × 3π = 12π units

Question 5:

Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$

Answer

The given equation of the ellipse can be represented as



$$\frac{x^2}{4}+\frac{y^2}{9}=1$$

$$\Rightarrow y=3\sqrt{1-\frac{x^2}{4}} \qquad ...(1)$$

It can be observed that the ellipse is symmetrical about *x*-axis and *y*-axis.

∴ Area bounded by ellipse = 4 × Area OAB

$$\therefore \text{Area of OAB}=\int_0^2 y\,dx$$

$$=\int_0^2 3\sqrt{1-\frac{x^2}{4}}dx \qquad \text{[Using (1)]}$$

$$=\frac{3}{2}\int_0^2\sqrt{4-x^2}\,dx$$

$$=\frac{3}{2}\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$$

$$=\frac{3}{2}\left[\frac{2\pi}{2}\right]$$

$$=\frac{3\pi}{2}$$

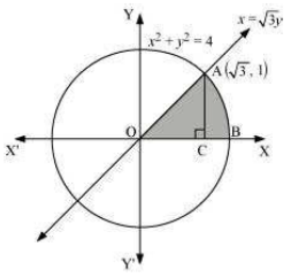
Therefore, area bounded by the ellipse = $4\times\frac{3\pi}{2}=6\pi$ units

Question 6:

Find the area of the region in the first quadrant enclosed by *x*-axis, line $x=\sqrt{3}y$ and the circle $x^2+y^2=4$

Answer

The area of the region bounded by the circle, $x^2+y^2=4, x=\sqrt{3}y$, and the *x*-axis is the area OAB.



The point of intersection of the line and the circle in the first quadrant is $\left(\sqrt{3},1\right)$.

Area OAB = Area ΔOCA + Area ACB

Area of OAC $=\frac{1}{2}\times OC\times AC=\frac{1}{2}\times\sqrt{3}\times 1=\frac{\sqrt{3}}{2} \qquad ...(1)$

Area of ABC $=\int_{\sqrt{3}}^2 y\,dx$

$$=\int_{\sqrt{3}}^2\sqrt{4-x^2}\,dx$$

$$=\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{\sqrt{3}}^2$$

$$=\left[2\times\frac{\pi}{2}-\frac{\sqrt{3}}{2}\sqrt{4-3}-2\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$$

$$=\left[\pi-\frac{\sqrt{3}\pi}{2}-2\left(\frac{}{3}\right)\right]$$

$$=\left[\pi-\frac{\sqrt{3}}{2}-\frac{2\pi}{3}\right]$$

$$=\left[\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right] \qquad ...(2)$$

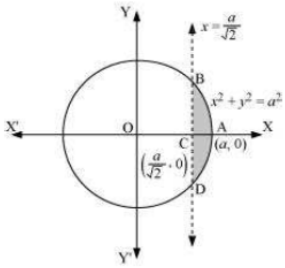
Therefore, area enclosed by x-axis, the line $x = \sqrt{3}y$, and the circle $x^2 + y^2 = 4$ in the first quadrant $= \frac{\sqrt{3}\pi}{2} + \frac{\ }{3} - \frac{3\sqrt{\ }\pi}{2} = \frac{\ }{3}$ units

Question 7:

Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$

Answer

The area of the smaller part of the circle, $x^2 + y^2 = a^2$, cut off by the line, $x = \frac{a}{\sqrt{2}}$, is the area ABCDA.



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