Indefinite Integrals Ex 19.9 Q5

Let $\quad I = \int \sqrt[3]{\cos^2 x}\, \sin x\, dx$ - - - - - (i)

Let $\cos x = t$ then,

$d(\cos x) = dt$

$\Rightarrow \quad -\sin x\, dx = dt$

$\Rightarrow \quad dx = -\dfrac{dt}{\sin x}$

Putting $\cos x = t$ and $dx = -\dfrac{dt}{\sin x}$ in equation (i), we get

$$I = \int \sqrt[3]{t^2}\, \sin x \times \frac{-dt}{\sin x}$$

$$= -\int t^{\frac{2}{3}} \sin x\, \frac{dt}{\sin x}$$

$$= -\int t^{\frac{2}{3}}\, dt$$

$$= -\frac{3}{5} \times \frac{t^{\frac{5}{3}}}{} + c$$

$$= -\frac{3}{5}(\cos x)^{\frac{5}{3}} + c$$

$\therefore \quad I = -\dfrac{3}{5}(\cos x)^{\frac{5}{3}} + c$

Indefinite Integrals Ex 19.9 Q6

Let $\quad I = \int \frac{e^x}{\left(1+e^x\right)^2} dx$ - - - - - (i)

Let $1+e^x = t$ then,

$d\left(1+e^x\right) = dt$

$\Rightarrow \quad e^x\, dx = dt$

$\Rightarrow \quad dx = \frac{dt}{e^x}$

Putting $1+e^x = t$ and $dx = \frac{dt}{e^x}$ in equation (i), we get

$$I = \int \frac{e^x}{t^2} \times \frac{dt}{e^x}$$
$$= \int \frac{dt}{t^2}$$
$$= \int t^{-2}\, dt$$
$$= -t^{-1} + c$$
$$= -\frac{1}{t} + c$$
$$= -\frac{1}{1+e^x} + c$$

$\therefore \quad I = -\frac{1}{1+e^x} + c$

Indefinite Integrals Ex 19.9 Q7

Let $\quad I = \int \cot^3 x \operatorname{cosec}^2 x\, dx$ - - - - - (i)

Let $\cot x = t$ then,

$d\left(\cot x\right) = dt$

$\Rightarrow \quad -\operatorname{cosec}^2 x dx = dt$

$\Rightarrow \quad dx = -\frac{dt}{\operatorname{cosec}^2 x}$

Putting $\cot x = t$ and $dx = -\frac{dt}{\operatorname{cosec}^2 x}$ in equation (i), we get

$$I = \int t^3 \operatorname{cosec}^2 x \times \frac{-dt}{\operatorname{cosec}^2 x}$$
$$= -\int t^3 dt$$
$$= -\frac{t^4}{4} + c$$
$$= -\frac{\cot^4 x}{4} + c$$

$\therefore \quad I = -\frac{\cot^4 x}{4} + c$

Indefinite Integrals Ex 19.9 Q8

Let $I = \int \frac{\left\{e^{\sin^{-1} x}\right\}^2}{\sqrt{1-x^2}} dx$ - - - - - (i)

Let $\sin^{-1} x = t$ then,

$d\left(\sin^{-1} x\right) = dt$

$\Rightarrow \frac{1}{\sqrt{1-x^2}} dx = dt$

$\Rightarrow dx = \sqrt{1-x^2}\, dt$

Putting $\sin^{-1} x = t$ and $dx = \sqrt{1-x^2} dt$ in equation (i), we get

$$I = \int \frac{\left(e^t\right)^2}{\sqrt{1-x^2}} \times \sqrt{1-x^2} dt$$
$$= \int e^{2t} dt$$
$$= \frac{e^{2t}}{2} + c$$
$$= \frac{e^{2\sin^{-1} x}}{2} + c$$

$\therefore \quad I = \frac{\left\{e^{\sin^{-1} x}\right\}^2}{2} + c$

Indefinite Integrals Ex 19.9 Q9

Let $I = \int \frac{1+\sin x}{\sqrt{x-\cos x}} dx$ - - - - - (i)

Let $x - \cos x = t$ then,

$d\left(x - \cos x\right) = dt$

$\Rightarrow \left[1 - \left(-\sin x\right)\right] dx = dt$

$\Rightarrow \left(1 + \sin x\right) dx = dt$

Putting $x - \cos x = t$ and $\left(1 + \sin x\right) dx = dt$ in equation (i), we get

$$I = \int \frac{dt}{\sqrt{t}}$$
$$= \int t^{\frac{-1}{2}} dt$$
$$= 2t^{\frac{1}{2}} + c$$
$$= 2\left(x - \cos x\right)^{\frac{1}{2}} + c$$

$\therefore \quad I = 2\sqrt{x - \cos x} + c$

********** END **********