Permutations Ex 16.3 Q12

We have,

$$\text{LHS} = 1.P(1,1) + 2.\ P(2,2) + 3.\ P(3,3) \ldots\ldots + n.\ P(n,n)$$

$$= 1.1 + 2.2! + 3.3! \ldots\ldots\ldots n.n! \qquad [\because\ P(n,n) = n!]$$

$$= \sum_{r=1}^{n} r.r!$$

$$= \sum_{r=1}^{n} [(r+1)r! - r!]$$

$$= \sum_{r=1}^{n} [(r+1)! - r!] \qquad [\because\ (r+1)r! = (r+1)!]$$

$$= [(2! - !) + (3! - 2!) + (4! - 3!) \ldots\ldots\ldots + (n+1)! - n!]$$

$$= (n+1)! - 1!$$

$$= {}^{n+1}P_{n+1} - 1! \qquad [\because\ {}^{n}P_{n} = n!]$$

$$= P(n+1, n+1) - 1$$

$\Rightarrow$ LHS = RHS

Hence proved.

Permutations Ex 16.3 Q13

We have,

$$P(15, r-1) = P(16, r-2) = 3 : 4$$

$$\Rightarrow \quad \frac{P(15, r-1)}{P(16, r-2)} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{\dfrac{15!}{[15-(r-1)]!}}{\dfrac{16!}{[16-(r-2)]!}} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{\dfrac{15!}{[16-r]!}}{\dfrac{16!}{[18-r]!}} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{15!}{(16-r)!} \times \frac{(18-r)!}{16!} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{15! \times (18-r)(17-r)(16-r)!}{(16-r)! \times 16 \times 15!} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{(18-r)(17-r)}{16} = \frac{3}{4}$$

$$\Rightarrow \quad 306 - 18r - 17r + r^2 = \frac{3}{4} \times 16$$

$$\Rightarrow \quad r^2 - 35r + 306 = 12$$
$$\Rightarrow \quad r^2 - 35r + 306 - 12 = 0$$
$$\Rightarrow \quad r^2 - 35r + 294 = 0$$
$$\Rightarrow \quad r^2 - 21r - 14r + 294 = 0$$
$$\Rightarrow \quad r(r-21) - 14(r-21) = 0$$
$$\Rightarrow \quad (r-21)(r-14) = 0$$
$$\Rightarrow \quad r - 14 = 0 \qquad [\because r = 21 \neq 0]$$
$$\Rightarrow \quad r = 14$$

Hence, $r = 14$

Permutations Ex 16.3 Q14

We have,

$$^{n+5}P_{n+1} = \frac{11(n-1)}{2}\ {}^{n+3}P_{n}$$

$$\Rightarrow \quad \frac{(n+5)!}{[n+5-(n+1)]!} = \frac{11(n-1)}{2} \times \frac{(n+3)!}{[n+3-n]!}$$

$$\Rightarrow \quad \frac{(n+5)!}{[n+5-n-1]!} = \frac{11(n-1)}{2} \times \frac{(n+3)!}{3!}$$

$$\Rightarrow \quad \frac{(n+5)!}{4!} = \frac{11(n-1)}{2} \times \frac{(n+3)!}{3!}$$

$$\Rightarrow \quad \frac{(n+5)(n+4)(n+3)!}{4!} = \frac{11(n-1)}{2} \times \frac{(n+3)!}{3!}$$

$$\Rightarrow \quad \frac{(n+5)(n+4)}{4 \times 3!} = \frac{11(n-1)}{2 \times 3!}$$

$$\Rightarrow \quad (n+5)(n+4) = \frac{11(n-1) \times 4}{2}$$

$$\Rightarrow \quad (n+5)(n+4) = 22(n-1)$$

$$\Rightarrow \quad n^2 + 4n + 5n + 20 = 22n - 22$$

$$\Rightarrow \quad n^2 + 9n - 22n + 20 + 22 = 0$$

$$\Rightarrow \quad n^2 - 13n + 42 = 0$$

$$\Rightarrow \quad n^2 - 6n - 7n + 42 = 0$$

$$\Rightarrow \quad n(n-6) - 7(n-6) = 0$$

$$\Rightarrow \quad n = 6 \quad \text{or,} \quad n = 7$$

Hence, $n = 6$ or,7

*********** END **********