Indefinite Integrals Ex 19.21 Q1

Let $I=\int\frac{x}{\sqrt{x^2+6x+10}}dx$

Let $x=\lambda\frac{d}{dx}\left(x^2+6x+10\right)+\mu$

$=\lambda\left(2x+6\right)+\mu$

$x=\left(2\lambda\right)x+6\lambda+\mu$

Comparing the coefficients of like powers of x,

$2\lambda=1 \quad\Rightarrow\quad \lambda=\frac{1}{2}$

$6\lambda+\mu=0 \quad\Rightarrow\quad 6\left(\frac{1}{2}\right)+\mu=0$

$\mu=-3$

so, $I_1=\int\frac{\frac{1}{2}\left(2x+6\right)-3}{\sqrt{x^2+6x+10}}dx$

$=\frac{1}{2}\int\frac{\left(2x+6\right)}{\sqrt{x^2+6x+10}}dx-3\int\frac{1}{\sqrt{x^2+2x\left(3\right)+\left(3\right)^2-\left(3\right)^2+10}}dx$

$I_1=\frac{1}{2}\int\frac{2x+6}{\sqrt{x^2+6x+10}}dx-3\int\frac{1}{\sqrt{\left(x+3\right)^2+\left(1\right)^2}}dx$

$I_1=\frac{1}{2}\left(2\sqrt{x^2+6x+10}\right)-3\log\left|x+3+\sqrt{\left(x+3\right)^2+1}\right|+c$ $\left[\text{since, }\int\frac{1}{\sqrt{x}}dx=2\sqrt{x}+c,\ \int\frac{1}{\sqrt{x^2+a^2}}dx=\log\left|x+\sqrt{x^2+a^2}\right|+c\right]$

$I=\sqrt{x^2+6x+10}-3\log\left|x+3+\sqrt{x^2+6x+10}\right|+c$

Indefinite Integrals Ex 19.21 Q2

Let $I=\int\frac{2x+1}{\sqrt{x^2+2x-1}}dx$

Let $2x+1=\lambda\frac{d}{dx}\left(x^2+2x-1\right)+\mu$

$=\lambda\left(2x+2\right)+\mu$

$2x+1=\left(2\lambda\right)x+2\lambda+\mu$

Comparing the coefficients of like powers of x,

$2\lambda=2 \quad\Rightarrow\quad \lambda=1$

$2\lambda+\mu=1 \quad\Rightarrow\quad 2\left(1\right)+\mu=1$

$\mu=-1$

so, $I=\int\frac{\left(2x+2\right)-1}{\sqrt{x^2+2x-1}}dx$

$=\int\frac{\left(2x+2\right)}{\sqrt{x^2+2x-1}}dx-\int\frac{1}{\sqrt{x^2+2x+\left(1\right)^2-\left(1\right)^2-1}}dx$

$I=\int\frac{2x+2}{\sqrt{x^2+2x-1}}dx-\int\frac{1}{\sqrt{\left(x+1\right)^2-\left(\sqrt{2}\right)^2}}dx$

$I=\left(2\sqrt{x^2+2x-1}\right)-\log\left|\left(x+1\right)+\sqrt{\left(x+1\right)^2-\left(\sqrt{2}\right)^2}\right|+c$ $\left[\text{since, }\int\frac{1}{\sqrt{x}}dx=2\sqrt{x}+c,\ \int\frac{1}{\sqrt{x^2-a^2}}dx=\log\left|x+\sqrt{x^2-a^2}\right|+c\right]$

$I=2\sqrt{x^2+2x-1}-\log\left|x+1+\sqrt{x^2+2x-1}\right|+c$

Indefinite Integrals Ex 19.21 Q3

Let $I = \int \frac{x+1}{\sqrt{4+5x-x^2}}dx$

Let $x+1 = \lambda \frac{d}{dx}\left(4+5x-x^2\right)+\mu$

$= \lambda\left(5-2x\right)+\mu$

$x = \left(-2\lambda\right)x + 5\lambda + \mu$

Comparing the coefficients of like powers of x,

$-2\lambda = 1 \quad \Rightarrow \quad \lambda = -\frac{1}{2}$

$5\lambda + \mu = 1 \quad \Rightarrow \quad 5\left(-\frac{1}{2}\right)+\mu = 1$

$\mu = \frac{7}{2}$

so, $I = \int \frac{-\frac{1}{2}\left(5-2x\right)+\frac{7}{2}}{\sqrt{4+5x-x^2}}dx$

$= -\frac{1}{2}\int \frac{\left(5-2x\right)}{\sqrt{4+5x-x^2}}dx + \frac{7}{2}\int \frac{1}{\sqrt{-\left[x^2-5x-4\right]}}dx$

$I = -\frac{1}{2}\int \frac{5-2x}{\sqrt{4+5x-x^2}}dx + \frac{7}{2}\int \frac{1}{\sqrt{-\left[x^2-2x\left(\frac{5}{2}\right)+\left(\frac{5}{2}\right)^2-\left(\frac{5}{2}\right)^2-4\right]}}dx$

$I = -\frac{1}{2}\int \frac{5-2x}{\sqrt{4+5x-x^2}}dx + \frac{7}{2}\int \frac{1}{\sqrt{-\left[\left(x-\frac{5}{2}\right)^2-\left(\frac{\sqrt{41}}{2}\right)^2\right]}}dx$

$I = -\frac{1}{2}\int \frac{5-2x}{\sqrt{4+5x-x^2}}dx + \frac{7}{2}\int \frac{1}{\sqrt{\left[\left(\frac{\sqrt{41}}{2}\right)^2-\left(x-\frac{5}{2}\right)^2\right]}}dx$

$I = -\frac{1}{2}\left(2\sqrt{4+5x-x^2}\right)+\frac{7}{2}\sin^{-1}\left(\frac{x-\frac{5}{2}}{\frac{\sqrt{41}}{2}}\right)+c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)+c\right]$

$I = -\sqrt{4+5x-x^2}+\frac{7}{2}\sin^{-1}\left(\frac{2x-5}{\sqrt{41}}\right)+c$

Indefinite Integrals Ex 19.21 Q4

Let $I = \int \frac{6x-5}{\sqrt{3x^2-5x+1}}dx$

Let $3x^2-5x+1 = t$

$\left(6x-5\right)dx = dt$

$I = \int \frac{dt}{\sqrt{t}}$

$= 2\sqrt{t}+c$

$I = 2\sqrt{3x^2-5x+1}+c$

Indefinite Integrals Ex 19.21 Q5

Let $I = \int \frac{3x+1}{\sqrt{5-2x-x^2}}dx$

Let $3x+1 = \lambda \frac{d}{dx}\left(5-2x-x^2\right)+\mu$

$= \lambda\left(-2-2x\right)+\mu$

$3x+1 = \left(-2\lambda\right)x - 2\lambda + \mu$

Comparing the coefficients of like powers of x,

$-2\lambda = 3 \quad \Rightarrow \quad \lambda = -\frac{3}{2}$

$-2\lambda + \mu = 1 \quad \Rightarrow \quad -2\left(-\frac{3}{2}\right)+\mu = 1$

$\mu = -2$

so, $I = \int \frac{-\frac{3}{2}\left(-2-2x\right)-2}{\sqrt{5-2x-x^2}}dx$

$= -\frac{3}{2}\int \frac{\left(-2-2x\right)}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{-\left[x^2+2x-5\right]}}dx$

$I = -\frac{3}{2}\int \frac{\left(-2-2x\right)}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{-\left[x^2+2x+\left(1\right)^2-\left(1\right)^2-5\right]}}dx$

$I = -\frac{3}{2}\int \frac{\left(-2-2x\right)}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{-\left[\left(x+1\right)^2-\left(\sqrt{6}\right)^2\right]}}dx$

$I = -\frac{3}{2}\int \frac{\left(-2-2x\right)}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{\left[\left(\sqrt{6}\right)^2-\left(x+1\right)^2\right]}}dx$

$I = -\frac{3}{2}\times 2\sqrt{5-2x-x^2} - 2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right)+c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)+c\right]$

$I = -3\sqrt{5-2x-x^2} - 2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right)+c$

*********** END **********