Trigonometric Ratios of multiple and Sub-multiple Angles Ex 9.1 Q 11

LHS,

$$(\cos\lambda+\cos\beta)^2+(\sin\lambda+\sin\beta)^2$$
$$=\cos^2\lambda+\cos^2\beta+2\cos\lambda\cos\beta+\sin^2\lambda+\sin^2\beta+2\sin\lambda+\sin\beta$$
$$=(\cos^2\lambda+\sin^2\lambda)+(\cos^2\beta+\sin^2\beta)+2(\cos\lambda\cos\beta+\sin\lambda\sin\beta)$$
$$=1+1+2\cos(\lambda-\beta)$$
$$=2+2\cos(\lambda-\beta)$$
$$=2(1+\cos(\lambda-\beta))$$
$$=2.2\cos^2\left(\frac{\lambda-\beta}{2}\right)$$
$$=4\cos^2\left(\frac{\lambda-\beta}{2}\right)$$
$$=\text{RHS}$$

Trigonometric Ratios of multiple and Sub-multiple Angles Ex 9.1 Q 12

LHS,

$$\sin^2\left(\frac{\pi}{8}+\frac{A}{2}\right)-\sin^2\left(\frac{\pi}{8}-\frac{A}{2}\right)$$
$$=\left[\sin\left(\frac{\pi}{8}+\frac{A}{2}\right)+\sin\left(\frac{\pi}{8}-\frac{A}{2}\right)\right]\left[\sin\left(\frac{\pi}{8}+\frac{A}{2}\right)-\sin\left(\frac{\pi}{8}-\frac{A}{2}\right)\right]$$
$$=\left[\sin\frac{\pi}{8}.\cos\frac{A}{2}+\cos\frac{\pi}{8}.\sin\frac{A}{2}+\sin\frac{\pi}{8}.\cos\frac{A}{2}-\cos\frac{\pi}{8}.\sin\frac{A}{2}\right]$$
$$=\left[\sin\frac{\pi}{8}.\cos\frac{A}{2}+\cos\frac{\pi}{8}.\sin\frac{A}{2}-\sin\frac{\pi}{8}.\cos\frac{A}{2}+\cos\frac{\pi}{8}.\sin\frac{A}{2}\right]$$
$$=\left(2\sin\frac{\pi}{8}.\cos\frac{A}{2}\right)\left(2\cos\frac{\pi}{8}.\sin\frac{A}{2}\right)$$
$$=2\sin\frac{\pi}{8}.\cos\frac{\pi}{2}\quad 2\sin\frac{A}{2}.\cos\frac{A}{2}$$
$$=\sin 2.\frac{\pi}{8}.\ \sin 2.\frac{A}{2}$$
$$=\sin\frac{\pi}{4}.\ \sin A$$
$$=\frac{1}{\sqrt{2}}\sin A$$
$$=\text{RHS}$$

Trigonometric Ratios of multiple and Sub-multiple Angles Ex 9.1 Q 13

LHS,

$$1+\cos^2 2\theta$$
$$=1+(\cos^2\theta-\sin^2\theta)^2 \qquad [\because \cos 2\theta=\cos^2\theta-\sin^2\theta]$$
$$=1+\cos^4\theta+\sin^4\theta-2\sin^2\theta.\cos^2\theta$$
$$=(\sin^2\theta+\cos^2\theta)^2+\cos^4\theta+\sin^4\theta-2\sin^2\theta.\cos^2\theta \qquad [\because \sin^2\theta+\cos^2\theta=1]$$
$$=\sin^4\theta+\cos^4\theta+2\sin^2\theta\cos^2\theta+\cos^4\theta+\sin^4\theta-2\sin^2\theta.\cos^2\theta$$
$$=2(\cos^4\theta+\sin^4\theta)$$
$$=\text{RHS}$$

Trigonometric Ratios of multiple and Sub-multiple Angles Ex 9.1 Q 14

$$\cos^3 2\theta + 3\cos 2\theta = 4\left(\cos^6\theta - \sin^6\theta\right)$$

$$\begin{aligned}
\text{RHS} &= 4\left[\left(\cos^2\theta\right)^3 - \left(\sin^2\theta\right)^3\right] \\
&= 4\left(\cos^2\theta - \sin^2\theta\right)\left[\cos^4\theta + \sin^4\theta + \sin^2\theta\cos^2\theta\right] \\
&= 4\cos^2\theta\left[\left(\cos^2\theta - \sin^2\theta\right)^2 + 2\sin^2\theta\cos^2\theta + \sin^2\theta\cos^2\theta\right] \\
&= 4\cos^2\theta\left[\cos^2 2\theta + 3\sin^2\theta\cos^2\theta\right] \\
&= 4\cos^2\theta\left[\cos^2 2\theta + 3\left(\frac{1-\cos^2\theta}{2}\right)\left(\frac{1+\cos^2\theta}{2}\right)\right] \\
&= 4\cos^2\theta\left[\cos^2 2\theta + \frac{3}{4}\left(1-\cos^2 2\theta\right)\right] \\
&= \cos^2\theta\left[4\cos^2 2\theta + 3 - 3\cos^2 2\theta\right] \\
&= \cos^2\theta\left[\cos^2 2\theta + 3\right] \\
&= \cos^3 2\theta + 3\cos^2\theta \\
&= \text{LHS}
\end{aligned}$$

LHS = RHS

Trigonometric Ratios of multiple and Sub-multiple Angles Ex 9.1 Q 15

$\text{LHS} = \left(\sin 3A + \sin A\right)\sin A\left(\cos 3A - \cos A\right)\cos A$

$\Rightarrow \quad 2\sin 2A.\ \cos A.\sin A + \left(-2\sin 2A.\ \sin A\cos A\right)$ $\quad \left[\begin{array}{l}\because \sin C + \sin D = 2\sin\frac{C+D}{2}.\cos\frac{C-D}{2} \\ \cos C - \cos D = -2\sin\frac{C+D}{2}.\ \sin\frac{C-D}{2}\end{array}\right]$

$\Rightarrow \quad 2\sin 2A\cos A.\sin A - 2\sin 2A\cos A.\sin A$

$\Rightarrow \quad 0 = \text{RHS}$

*********** END **********