Indefinite Integrals Ex 19.30 Q51

To evaluate the integral follow the steps:

$$\int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx$$

Let $1-\sin x = t$ and

$$-\cos x dx = dt$$

Therefore

$$-\int \frac{dt}{t(1+t)} = -\int \left(\frac{1}{t} - \frac{1}{t+1}\right) dt$$

$$= \ln|(t+1)| - \ln|t| + c$$

$$= \ln\left|\frac{t+1}{t}\right| + c$$

$$= \ln\left|\frac{2-\sin x}{1-\sin x}\right| + c$$

Indefinite Integrals Ex 19.30 Q52

Let $\frac{2x+1}{(x-2)(x-3)} = \frac{A}{(x-2)} + \frac{B}{(x-3)}$

$$\Rightarrow \quad 2x+1 = A(x-3) + B(x-2)$$

$$= (A+B)x + (-3A-2B)$$

Equating similar terms, we get,

$A+B = 2$, and $-3A-2B = 1$

Thus,

$$I = -5\int \frac{dx}{x-2} + 7\int \frac{dx}{x-3}$$

$$= -5\log|x-2| + 7\log|x-3| + c$$

$$I = \log\left|\frac{(x-3)^7}{(x-2)^5}\right| + c$$

Indefinite Integrals Ex 19.30 Q53

Let $x^2 = y$

Then $\frac{1}{(y+1)(y+2)} = \frac{A}{y+1} + \frac{B}{y+2}$

$$\Rightarrow \quad 1 = A(y+2) + B(y+1)$$
$$= (A+B)y + (2A+B)$$

Equating similar terms, we get,

$A + B = 0$, and $2A + B = 1$

Solving, we get,

Thus,

$$I = \int \frac{dx}{x^2+1} - \int \frac{dx}{x^2+2}$$

$$I = \tan^{-1} x - \frac{1}{\sqrt{2}} \tan^{-1} \frac{x}{\sqrt{2}} + c$$

Indefinite Integrals Ex 19.30 Q54

To evaluate the integral follow the steps:

$$\int \frac{1}{x(x^4-1)} dx$$

Let $\frac{1}{x(x^4-1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1} + \frac{D}{x^2+1}$

$$1 = A(x+1)(x-1)(x^2+1) + Bx(x-1)(x^2+1) + Cx(x+1)(x^2+1) + Dx(x+1)(x-1)$$

For $x=0 \quad A=-1,$

For $x=1 \quad C=\frac{1}{4}$

For $x=-1\, B=\frac{1}{4}$

For $x=2 \quad D=\frac{1}{4}$

Therefore

$$\int \frac{1}{x(x^4-1)} dx = -\int \frac{1}{x} dx + \frac{1}{4}\int \frac{dx}{x+1} + \frac{1}{4}\int \frac{dx}{x-1} + \frac{1}{4}\int \frac{dx}{x^2+1}$$

$$= -\ln|x| + \frac{1}{4}\ln|(x+1)| + \frac{1}{4}\ln|(x-1)| + \frac{1}{4}\ln|(x^2+1)| + c$$

$$= \frac{1}{4}\ln\left|\frac{x^4-1}{x^4}\right| + c$$

Indefinite Integrals Ex 19.30 Q55

To evaluate the integral follow the steps:

$$\int \frac{1}{\left(x^4-1\right)}dx$$

Let $\dfrac{1}{\left(x^4-1\right)}=\dfrac{A}{x+1}+\dfrac{B}{x-1}+\dfrac{C}{x^2+1}$

$$1=A(x-1)\left(x^2+1\right)+B(x+1)\left(x^2+1\right)+C(x+1)(x-1)$$

For $x=1 \quad B=\dfrac{1}{4}$

For $x=-1\ A=-\dfrac{1}{4}$

For $x=0 \quad C=-\dfrac{1}{2}$,

Therefore

$$\begin{aligned}\int \frac{1}{\left(x^4-1\right)}dx &= -\frac{1}{4}\int\frac{dx}{x+1}+\frac{1}{4}\int\frac{dx}{x-1}-\frac{1}{2}\int\frac{dx}{x^2+1}\\ &= -\frac{1}{4}\ln\left|(x+1)\right|+\frac{1}{4}\ln\left|(x-1)\right|-\frac{1}{2}\tan^{-1}x+c\\ &= \frac{1}{4}\ln\left|\frac{x-1}{x+1}\right|-\frac{1}{2}\tan^{-1}x+c\end{aligned}$$

*********** END **********