Differentiation Ex 11.5 Q34

Here,

$$x^{16}y^{9} = \left(x^2 + y\right)^{17}$$

Taking log on both the sides,

$$\log\left(x^{16} \times y^{9}\right) = \log\left(x^2 + y\right)^{17} \qquad \left[\text{Since, } \log(AB) = \log A + \log B, \log a^{b} = b\log a\right]$$

$$16\log x + 9\log y = 17\log\left(x^2 + y\right)$$

Differentiating it with respect to x using chain rule,

$$16\frac{d}{dx}(\log x) + 9\frac{d}{dx}(\log y) = 17\frac{d}{dx}\log\left(x^2 + y\right)$$

$$\frac{16}{x} + \frac{9}{y}\frac{dy}{dx} = 17\frac{1}{\left(x^2 + y\right)}\frac{d}{dx}\left(x^2 + y\right)$$

$$\frac{16}{x} + \frac{9}{y}\frac{dy}{dx} = \frac{17}{x^2 + y}\left[2x + \frac{dy}{dx}\right]$$

$$\frac{9}{y}\frac{dy}{dx} - \frac{17}{\left(x^2 + y\right)}\frac{dy}{dx} = \left(\frac{34x}{x^2 + y}\right) - \frac{16}{x}$$

$$\frac{dy}{dx}\left[\frac{9}{y} - \frac{17}{\left(x^2 + y\right)}\right] = \frac{34x^2 - 16x^2 - 16y}{x\left(x^2 + y\right)}$$

$$\frac{dy}{dx}\left[\frac{9x^2 + 9y - 17y}{y\left(x^2 + y\right)}\right] = \frac{18x^2 - 16y}{x\left(x^2 + y\right)}$$

$$\frac{dy}{dx} = \frac{y}{x}\left(\frac{2\left(9x^2 - 8y\right)}{9x^2 - 8y}\right)$$

$$\frac{dy}{dx} = \frac{2y}{x}$$

$$x\frac{dy}{dx} = 2y$$

Differentiation Ex 11.5 Q35

Here,

$$y = \sin\left(x^x\right) \quad \text{---(i)}$$

Let $u = x^x$ ---(ii)

Taking log on both the sides,

$$\log u = \log x^x$$
$$\log u = x \log x$$

Differentiating it with respect to x,

$$\frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(x \log x)$$
$$= x\frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x)$$
$$= x\left(\frac{1}{x}\right) + \log x\,(1)$$
$$\frac{1}{u}\frac{du}{dx} = 1 + \log x$$
$$\frac{du}{dx} = u(1 + \log x)$$
$$\frac{du}{dx} = x^x(1 + \log x) \quad \text{---(iii)} \quad \left[\text{Using equation (ii)}\right]$$

Now, using equation (ii) in equation (i),

$$y = \sin u$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}(\sin u)$$
$$= \cos u \frac{du}{dx}$$

Using equation (ii) and (iii),

$$\frac{dy}{dx} = \cos\left(x^x\right) \times x^x(1 + \log x)$$

Differentiation Ex 11.5 Q36

Here,

$$x^x + y^x = 1$$
$$e^{\log x^x} + e^{\log y^x} = 1$$
$$e^{x\log x} + e^{x\log y} = 1 \quad \left[\text{Since, } e^{\log a} = a, \log a^b = b\log a\right]$$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{d}{dx}\left(e^{x\log x}\right) + \frac{d}{dx}\left(e^{x\log y}\right) = \frac{d}{dx}(1)$$
$$e^{x\log x}\frac{d}{dx}(x\log x) + e^{x\log y}\frac{d}{dx}(x\log y) = 0$$
$$e^{\log x^x}\left[x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x)\right] + e^{\log y^x}\left[x\frac{d}{dx}(\log y) + \log y\frac{d}{dx}(x)\right] = 0$$
$$x^x\left[x\left(\frac{1}{x}\right) + \log x\,(1)\right] + y^x\left[x\left(\frac{1}{y}\right)\frac{dy}{dx} + \log y\,(1)\right] = 0$$
$$x^x\left[1 + \log x\right] + y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) = 0$$
$$y^x \times \frac{x}{y}\frac{dy}{dx} = -\left[x^x(1 + \log x) + y^x\log y\right]$$
$$\left(xy^{x-1}\right)\frac{dy}{dx} = -\left[x^x(1 + \log x) + y^x\log y\right]$$
$$\frac{dy}{dx} = -\left[\frac{x^x(1 + \log x) + y^x\log y}{xy^{x-1}}\right]$$

Differentiation Ex 11.5 Q37

Here,

$$x^y \times y^x = 1$$

Taking on both sides,

$$\log\left(x^y \times y^x\right) = \log(1)$$

$$y = \log x + x \log y = \log 1 \qquad \left[\text{Since, } \log(AB) = \log A + \log B, \log a^b = b \log a\right]$$

Differentiating it with respect to x using product rule,

$$\frac{d}{dx}(y \log x) + \frac{d}{dx}(x \log y) = \frac{d}{dx}(\log 1)$$

$$\left[y \frac{d}{dx}(\log x) + \log x \frac{dy}{dx}\right] + \left[x \frac{d}{dx}(\log y) + \log y \frac{d}{dx}(x)\right] = 0$$

$$\left[y\left(\frac{1}{x}\right) + \log x \frac{dy}{dx}\right] + \left[x\left(\frac{1}{y}\frac{dy}{dx}\right) + \log y\ (1)\right] = 0$$

$$\frac{y}{x} + \log x \frac{dy}{dx} + \frac{x}{y}\frac{dy}{dx} + \log y = 0$$

$$\frac{dy}{dx}\left(\log x + \frac{x}{y}\right) = -\left[\log y + \frac{y}{x}\right]$$

$$\frac{dy}{dx}\left[\frac{y \log x + x}{y}\right] = -\left[\frac{x \log y + y}{x}\right]$$

$$\frac{dy}{dx} = -\frac{y}{x}\left[\frac{x \log y + y}{y \log x + x}\right]$$

Differentiation Ex 11.5 Q38

Here,

$$x^y + y^x = (x+y)^{x+y}$$

$$e^{\log x^y} + e^{\log y^x} = e^{\log(x+y)^{(x+y)}}$$

$$e^{y \log x} + e^{x \log y} = e^{(x+y)\log(x+y)} \qquad \left[\text{Since, } e^{\log a} = a, \log a^b = b \log a\right]$$

Differentiating it with respect to x using chain rule, product rule,

$$\Rightarrow \quad \frac{d}{dx}\left(e^{y\log x}\right) + \frac{d}{dx}\left(e^{x \log y}\right) = \frac{d}{dx} e^{(x+y)\log(x+y)}$$

$$\Rightarrow \quad e^{y \log x}\left[y \frac{d}{dx}(\log x) + \log x \frac{dy}{dx}\right] + e^{x \log y}\left[x \frac{d}{dx}\log y + \log y \frac{d}{dx}(x)\right] = e^{(x+y)\log(x+y)} \frac{d}{dx}\left[(x+y)\log(x+y)\right]$$

$$\Rightarrow \quad e^{\log x^y}\left[y\left(\frac{1}{x}\right) + \log x \frac{dy}{dx}\right] + e^{\log x}\left[\frac{x}{y}\frac{dy}{dx} + \log y\ (1)\right] = e^{\log(x+y)^{(x+y)}}\left[(x+y)\frac{d}{dx}\log(x+y) + \log(x+y) \frac{d}{dx}(x+y)\right]$$

$$\Rightarrow \quad x^y\left[\frac{y}{x} + \log x \frac{dy}{dx}\right] + y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right] = (x+y)^{(x+y)}\left[(x+y)\frac{1}{(x+y)}\frac{d}{dx}(x+y) + \log(x+y)\left(1 + \frac{dy}{dx}\right)\right]$$

$$\Rightarrow \quad x^y \times \frac{y}{x} + x^y \log x \frac{dy}{dx} + y^x \times \frac{x}{y}\frac{dy}{dx} + y^x \log y = (x+y)^{(x+y)}\left[1 \times \left(1 + \frac{dy}{dx}\right) + \log(x+y)\left(1 + \frac{dy}{dx}\right)\right]$$

$$\Rightarrow \quad x^{y-1} \times y + x^y \log x \frac{dy}{dx} + y^{x-1} \times x \frac{dy}{dx} + y^x \log y = (x+y)^{(x+y)} + (x+y)^{(x+y)}\frac{dy}{dx} + (x+y)^{(x+y)}\log(x+y) + (x+y)^{(x+y)}\log(x+y)\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx}\left[x^y \log x + xy^{x-1} - (x+y)^{(x+y)}\left(1 + \log(x+y)\right)\right] = (x+y)^{(x+y)}\left(1 + \log(x+y)\right) - x^{y-1} \times y - y^x \log y$$

$$\Rightarrow \quad \frac{dy}{dx} = \left[\frac{(x+y)^{(x+y)}\left(1 + \log(x+y)\right) - x^{y-1} \times y - y^x \log y}{x^y \log x + xy^{x-1} - (x+y)^{(x+y)}\left(1 + \log(x+y)\right)}\right]$$

*********** END **********