Co-Ordinate Geometry Ex 14.2 Q26

Answer :

The distance *d* between two points (x_1, y_1) and (x_2, y_2) is given by the formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

In a rhombus all the sides are equal in length. And the area '*A*' of a rhombus is given as

$$A = \frac{1}{2}(\text{Product of both diagonals})$$

Here the four points are *A*(−3,2), *B*(−5,−5), *C*(2,−3) and *D*(4,4)

First let us check if all the four sides are equal.

$$AB = \sqrt{(-3+5)^2 + (2+5)^2}$$
$$= \sqrt{(2)^2 + (7)^2}$$
$$= \sqrt{4+49}$$
$$AB = \sqrt{53}$$

$$BC = \sqrt{(-5-2)^2 + (-5+3)^2}$$
$$= \sqrt{(-7)^2 + (-2)^2}$$
$$= \sqrt{49+4}$$
$$BC = \sqrt{53}$$

$$CD = \sqrt{(2-4)^2 + (-3-4)^2}$$
$$= \sqrt{(-2)^2 + (-7)^2}$$
$$= \sqrt{4+49}$$
$$CD = \sqrt{53}$$

$$AD = \sqrt{(-3-4)^2 + (2-4)^2}$$
$$= \sqrt{(-7)^2 + (-2)^2}$$
$$= \sqrt{49+4}$$
$$AD = \sqrt{53}$$

Here, we see that all the sides are equal, so it has to be a rhombus.

Hence we have proved that the quadrilateral formed by the given four vertices is a $\boxed{\text{rhombus}}$.

Now let us find out the lengths of the diagonals of the rhombus.

$$AC = \sqrt{(-3-2)^2 + (2+3)^2}$$
$$= \sqrt{(-5)^2 + (5)^2}$$
$$= \sqrt{25+25}$$
$$= \sqrt{50}$$
$$AC = 5\sqrt{2}$$

$$BD = \sqrt{(-5-4)^2 + (-5-4)^2}$$
$$= \sqrt{(-9)^2 + (-9)^2}$$
$$= \sqrt{81+81}$$
$$= \sqrt{162}$$
$$BD = 9\sqrt{2}$$

Now using these values in the formula for the area of a rhombus we have,

$$A = \frac{(5\sqrt{2})(9\sqrt{2})}{2}$$
$$= \frac{(5)(9)(2)}{2}$$
$$A = 45$$

Thus the area of the given rhombus is $\boxed{45 \text{ square units}}$.

Co-Ordinate Geometry Ex 14.2 Q27

Answer :

The distance *d* between two points (x_1, y_1) and (x_2, y_2) is given by the formula

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

The circumcentre of a triangle is the point which is equidistant from each of the three vertices of the triangle.

Here the three vertices of the triangle are given to be *A*(3,0), *B*(−1,−6) and *C*(4,−1)

Let the circumcentre of the triangle be represented by the point *R*(*x*, *y*).

So we have $AR = BR = CR$

$AR = \sqrt{(3-x)^2 + (-y)^2}$

$BR = \sqrt{(-1-x)^2 + (-6-y)^2}$

$CR = \sqrt{(4-x)^2 + (-1-y)^2}$

Equating the first pair of these equations we have,

$$AR = BR$$

$$\sqrt{(3-x)^2 + (-y)^2} = \sqrt{(-1-x)^2 + (-6-y)^2}$$

Squaring on both sides of the equation we have,

$$(3-x)^2 + (-y)^2 = (-1-x)^2 + (-6-y)^2$$
$$9 + x^2 - 6x + y^2 = 1 + x^2 + 2x + 36 + y^2 + 12y$$
$$8x + 12y = -28$$
$$2x + 3y = -7$$

Equating another pair of the equations we have,

$$AR = CR$$

$$\sqrt{(3-x)^2 + (-y)^2} = \sqrt{(4-x)^2 + (-1-y)^2}$$

Squaring on both sides of the equation we have,

$$(3-x)^2 + (-y)^2 = (4-x)^2 + (-1-y)^2$$
$$9 + x^2 - 6x + y^2 = 16 + x^2 - 8x + 1 + y^2 + 2y$$
$$2x - 2y = 8$$
$$x - y = 4$$

Now we have two equations for '*x*' and '*y*', which are

$$2x + 3y = -7$$
$$x - y = 4$$

From the second equation we have $y = x - 4$. Substituting this value of '*y*' in the first equation we have,

$$2x + 3(x-4) = -7$$
$$2x + 3x - 12 = -7$$
$$5x = 5$$
$$x = 1$$

Therefore the value of '*y*' is,

$$y = x - 4$$
$$= 1 - 4$$
$$y = -3$$

Hence the co-ordinates of the circumcentre of the triangle with the given vertices are $\boxed{(1, -3)}$.

The length of the circumradius can be found out substituting the values of '*x*' and '*y*' in '*AR*'

$$AR = \sqrt{(3-x)^2 + (-y)^2}$$
$$= \sqrt{(3-1)^2 + (3)^2}$$
$$= \sqrt{(2)^2 + (3)^2}$$
$$= \sqrt{4+9}$$
$$AR = \sqrt{13}$$

Thus the circumradius of the given triangle is $\boxed{\sqrt{13} \text{ units}}$.

*********** END **********