



Indefinite Integrals Ex 19.9 Q10

$$\text{Let } I = \int \frac{1}{\sqrt{1-x^2} (\sin^{-1} x)^2} dx \text{ ----- (i)}$$

$$\begin{aligned} \text{Let } \sin^{-1} x &= t & \text{then,} \\ d(\sin^{-1} x) &= dt \end{aligned}$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} dx = dt$$

Putting  $\sin^{-1} x = t$  and  $\frac{1}{\sqrt{1-x^2}} dx = dt$  in equation (i), we get

$$\begin{aligned} I &= \int \frac{dt}{t^2} \\ &= \int t^{-2} dt \\ &= -1t^{-1} + C \\ &= \frac{-1}{t} + C \\ &= \frac{-1}{\sin^{-1} x} + C \end{aligned}$$

$$\therefore I = \frac{-1}{\sin^{-1} x} + C$$

Indefinite Integrals Ex 19.9 Q11

$$\text{Let } I = \int \frac{\cot x}{\sqrt{\sin x}} dx \text{ ----- (i)}$$

$$\begin{aligned} \text{Let } \sin x &= t & \text{then,} \\ d(\sin x) &= dt \end{aligned}$$

$$\Rightarrow \cos x \, dx = dt$$

$$\begin{aligned} \text{Now, } I &= \int \frac{\cot x}{\sqrt{\sin x}} dx \\ &= \int \frac{\cos x}{\sin x \sqrt{\sin x}} dx \\ &= \int \frac{\cos x}{(\sin x)^{\frac{3}{2}}} dx \end{aligned}$$

$$\Rightarrow \int \frac{\cos x}{(\sin x)^{\frac{3}{2}}} dx \text{ ----- (ii)}$$

Putting  $\sin x = t$  and  $\cos x \, dx = dt$  in equation (ii), we get

$$\begin{aligned} I &= \int \frac{dt}{t^{\frac{3}{2}}} \\ &= \int t^{-\frac{3}{2}} dt \\ &= -2t^{-\frac{1}{2}} + C \\ &= \frac{-2}{\sqrt{t}} + C \\ &= \frac{-2}{\sqrt{\sin x}} + C \end{aligned}$$

$$\therefore I = \frac{-2}{\sqrt{\sin x}} + C$$

Indefinite Integrals Ex 19.9 Q12

$$\text{Let } I = \int \frac{\tan x}{\sqrt{\cos x}} dx$$

$$\begin{aligned} \therefore I &= \int \frac{\sin x}{\cos x \sqrt{\cos x}} dx \\ &= \int \frac{\sin x}{(\cos x)^{\frac{3}{2}}} dx \end{aligned}$$

$$\Rightarrow I = \int \frac{\sin x}{(\cos x)^{\frac{3}{2}}} dx \text{ ----- (i)}$$

$$\begin{aligned} \text{Let } \cos x &= t \quad \text{then,} \\ d(\cos x) &= dt \end{aligned}$$

$$\Rightarrow -\sin x dx = dt$$

$$\Rightarrow \sin x dx = -dt$$

Putting  $\cos x = t$  and  $\sin x dx = -dt$  in equation (i), we get

$$\begin{aligned} I &= \int \frac{-dt}{t^{\frac{3}{2}}} \\ &= -\int t^{-\frac{3}{2}} dt \\ &= -\left[ -2t^{-\frac{1}{2}} \right] + C \\ &= \frac{2}{t^{\frac{1}{2}}} + C \\ &= \frac{2}{\sqrt{\cos x}} + C \end{aligned}$$

$$\therefore I = \frac{2}{\sqrt{\cos x}} + C$$

Indefinite Integrals Ex 19.9 Q13

$$\text{Let } I = \int \frac{\cos^3 x}{\sqrt{\sin x}} dx$$

$$\begin{aligned} \therefore I &= \int \frac{\cos^2 x \cos x}{\sqrt{\sin x}} dx \\ &= \int \frac{(1 - \sin^2 x) \cos x}{\sqrt{\sin x}} dx \end{aligned}$$

$$\therefore I = \int \frac{(1 - \sin^2 x)}{\sqrt{\sin x}} \cos x dx \text{ ----- (i)}$$

$$\begin{aligned} \text{Let } \sin x &= t \quad \text{then,} \\ d(\sin x) &= dt \end{aligned}$$

$$\Rightarrow \cos x dx = dt$$

Putting  $\sin x = t$  and  $\cos x dx = dt$  in equation (i), we get

$$\begin{aligned} I &= \int \frac{1 - t^2}{\sqrt{t}} dt \\ &= \int \left( t^{\frac{-1}{2}} - t^2 \times t^{\frac{-1}{2}} \right) dt \\ &= \int \left( t^{\frac{-1}{2}} - t^{\frac{3}{2}} \right) dt \\ &= 2t^{\frac{1}{2}} - \frac{2}{5}t^{\frac{5}{2}} + c \\ \Rightarrow I &= 2(\sin x)^{\frac{1}{2}} - \frac{2}{5}(\sin x)^{\frac{5}{2}} + c \end{aligned}$$

$$\therefore I = 2\sqrt{\sin x} - \frac{2}{5}(\sin x)^{\frac{5}{2}} + c$$

Indefinite Integrals Ex 19.9 Q14

$$\text{Let } I = \int \frac{\sin^3 x}{\sqrt{\cos x}} dx$$

$$\therefore I = \int \frac{\sin^2 x \sin x}{\sqrt{\cos x}} dx$$

$$\Rightarrow I = \int \frac{(1 - \cos^2 x)}{\sqrt{\cos x}} \sin x dx \text{ ----- (i)}$$

$$\begin{aligned} \text{Let } \cos x &= t & \text{then,} \\ d(\cos x) &= dt \end{aligned}$$

$$\Rightarrow -\sin x dx = dt$$

$$\Rightarrow \sin x dx = -dt$$

Putting  $\cos x = t$  and  $\sin x dx = -dt$  in equation (i), we get

$$I = \int \frac{(1 - t^2)}{\sqrt{t}} \times -dt$$

$$= \int \frac{t^2 - 1}{\sqrt{t}} dt$$

$$= \int \left( \frac{t^2}{t^{\frac{1}{2}}} - \frac{1}{t^{\frac{1}{2}}} \right) dt$$

$$= \int \left( t^{2-\frac{1}{2}} - t^{-\frac{1}{2}} \right) dt$$

$$= \int \left( t^{\frac{3}{2}} - t^{-\frac{1}{2}} \right) dt$$

$$= \frac{2}{5} t^{\frac{5}{2}} - 2t^{\frac{1}{2}} + c$$

$$\therefore I = \frac{2}{5} \cos^{\frac{5}{2}} x - 2 \cos^{\frac{1}{2}} x + c$$

$$\therefore I = \frac{2}{5} \cos^{\frac{5}{2}} x - 2\sqrt{\cos x} + c$$

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