Binary Operations Ex 3.2 Q1

We have,

$$a * b = l.c.m.(a, b) \text{ for all } a, b \in N$$

(1)

Now,

$$2 * 4 = l.c.m\ (2,4) = 4$$
$$3 * 5 = l.c.m\ (3,5) = 15$$
$$1 * 6 = l.c.m\ (1,6) = 6$$

(ii)

Commutativity:

Let $a, b \in N$ then,

$$a * b = l.c.m(a, b)$$
$$= l.c.m(b, a)$$
$$= b * a$$

$\Rightarrow \quad a * b = b * a$

$\therefore \quad *$ is commutative on N.

Associativity:

Let $a, b, c \in N$ then,

$$(a * b) * c = l.c.m(a, b) * c$$
$$= l.c.m(a, b, c) \quad ---(i)$$

and, $$a * (b * c) = a * l.c.m(b, c)$$
$$= l.c.m(a, b, c) \quad ---(ii)$$

From (i) and (ii)

$$(a * b) * c = a * (b * c)$$

$\therefore \quad *$ is associative on N.

Binary Operations Ex 3.2 Q2

(i) Clearly, by definition $a * b = 1 = b * a$, $\forall a, b \in \mathbf{N}$

Also, $(a * b) * c = (1 * c) = 1$

and $a * (b * c) = (a * 1) = 1 \qquad \forall a, b, c \in \mathbf{N}$

Hence, $\mathbf{N}$ is both associative and commutative.

(ii) $a * b = \dfrac{a+b}{2} = \dfrac{b+a}{2} = b * a$,

which shows $*$ is commutative.

Further, $(a * b) * c = \left(\dfrac{a+b}{2}\right) * c = \dfrac{\left(\dfrac{a+b}{2}\right) + c}{2} = \dfrac{a+b+2c}{4}$

$a * (b * c) = a * \left(\dfrac{b+c}{2}\right) = \dfrac{a + \left(\dfrac{b+c}{2}\right)}{2} = \dfrac{2a+b+c}{2} \neq \dfrac{a+b+2c}{4}$

Hence, $*$ is not associative.

Binary Operations Ex 3.2 Q3

We have, binary operator $*$ defined on A and is given by

$a * b = b$ for all $a, b \in A$

Commutativity: Let $a, b \in A$, then

$a * b = b \neq a = b * a$

$\Rightarrow \qquad a * b \neq b * a$

$\therefore \qquad$ '$*$' is not commutative on A.

Associativity: Let $a, b, c \in A$, then

$(a * b) * c = b * c = c \qquad ---(i)$

and, $\quad a * (b * c) = a * c = c \qquad ---(ii)$

From (i) and (ii)

$(a * b) * c = a * (b * c)$

$\Rightarrow \qquad$ '$*$' is associative on A.

*********** END **********