Exercise 7B

Question 4:

$$(\sec\theta + \tan\theta) = m, (\sec\theta - \tan\theta) = n$$

$$\text{LHS} = mn = (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)$$

$$= \sec^2\theta - \tan^2\theta = 1 = \text{RHS}$$

∴ LHS = RHS

Question 5:

$$(\operatorname{cosec}\theta + \cot\theta) = m,\ (\operatorname{cosec}\theta - \cot\theta) = n$$

$$\text{LHS} = mn = (\operatorname{cosec}\theta + \cot\theta) \times (\operatorname{cosec}\theta - \cot\theta)$$

$$= \operatorname{cosec}^2\theta - \cot^2\theta = 1 = \text{RHS}$$

∴ LHS = RHS

Question 6:

$$x = a\cos^3\theta, y = b\sin^3\theta$$

$$\text{LHS} = \left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = \left(\frac{a\cos^3\theta}{a}\right)^{\frac{2}{3}} + \left(\frac{b\sin^3\theta}{b}\right)^{\frac{2}{3}}$$

$$= (\cos^3\theta)^{\frac{2}{3}} + (\sin^3\theta)^{\frac{2}{3}} = (\cos\theta)^{3\times\frac{2}{3}} + (\sin\theta)^{3\times\frac{2}{3}}$$

$$= \cos^2\theta + \sin^2\theta = 1 = \text{RHS}$$

∴ LHS = RHS

Question 7:

$(\tan\theta + \sin\theta) = m \quad \text{and} \quad (\tan\theta - \sin\theta) = n$

$$
\begin{aligned}
\text{LHS} &= \left(m^2 - n^2\right)^2 \\
&= \left[(\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2\right]^2 \\
&= [4\tan\theta\sin\theta]^2 \quad \left[\because (a+b)^2 - (a-b)^2 = 4ab\right] \\
&= 16\tan^2\theta\sin^2\theta \quad -----(1)
\end{aligned}
$$

$$
\begin{aligned}
\text{RHS} &= 16mn = 16(\tan\theta + \sin\theta)(\tan\theta - \sin\theta) \\
&= 16\left(\tan^2\theta - \sin^2\theta\right) = 16\left(\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta\right) \\
&= 16\left(\frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}\right) \\
&= 16\,\frac{\sin^2\theta\left(1 - \cos^2\theta\right)}{\cos^2\theta} \quad \left[\because 1 - \cos^2\theta = \sin^2\theta\right] \\
&= 16\,\frac{\sin^2\theta}{\cos^2\theta} \times \sin^2\theta \\
\text{RHS} &= 16\tan^2\sin^2\theta \quad ------(2)
\end{aligned}
$$

$\therefore$ LHS = RHS

*********** END **********