Combinations Ex 17.1 Q14

If ${}^{n}C_{r} = {}^{n}C_{p}$

then $r + P = n$

$\therefore\ 16 = r + r + 2$

$r = 7$

then ${}^{r}C_{4} = {}^{7}C_{4}$ $\quad (\because\ r = 7)$

$\Rightarrow \dfrac{7!}{4!(7-4)!} \quad \left(\because\ {}^{n}C_{r} = \dfrac{n!}{r!(n-r)!}\right)$

$\Rightarrow \dfrac{7 \times 5 \times 6}{3 \times 2}$

$= 35$

Combinations Ex 17.1 Q15

${}^{20}C_{5} + \sum\limits_{r=2}^{5} {}^{25-r}C_{4}$

$\Rightarrow \left({}^{20}C_{5} + {}^{20}C_{4}\right) + {}^{21}C_{4} + {}^{22}C_{4} + {}^{23}C_{4}$

$\Rightarrow \left({}^{21}C_{5} + {}^{21}C_{4}\right) + {}^{22}C_{4} + {}^{23}C_{4} \quad \left(\because\ {}^{n}C_{r-1} + {}^{n}C_{r} = {}^{n+1}C_{r}\right)$

$\Rightarrow \left({}^{22}C_{5} + {}^{22}C_{4}\right) + {}^{23}C_{4} \quad \left(\because\ {}^{n}C_{r-1} + {}^{n}C_{r} = {}^{n+1}C_{r}\right)$

$\Rightarrow {}^{23}C_{5} + {}^{23}C_{4} \quad \left(\because\ {}^{n}C_{r-1} + {}^{n}C_{r} = {}^{n+1}C_{r}\right)$

$\Rightarrow {}^{24}C_{5}$

$\Rightarrow 42504$

Combinations Ex 17.1 Q16

$$\text{Product} = \left[(2n+1)(2n+3)(2n+5)\ldots(2n+r)\right]$$

$$= \frac{(2n)!\left[(2n+1)(2n+3)\ldots(2n+r)\right]}{(2n)!}$$

$$= \frac{(2n)\left[(2n-1)(2n-2)\ldots4.2(2n+1)(2n+3)\right]}{(2n)!}$$

$$= \frac{(2n+r)!}{(2n)!}$$

Hence $r = 2n$

$$= \frac{(2n+2n)!}{2n}$$

$$= \frac{(4n)!}{(2n)!}$$

$$= (2n)!$$

Combinations Ex 17.1 Q17

L.H.S, $= {}^{2n}C_n + {}^{2n}C_{n-1}$

$$\frac{2n!}{n!\,n!} + \frac{2n!}{(n-1)!(n-1)!}$$

$$= (2n)!\left[\frac{1}{n(n-1)!(n)(n-1)!} + \frac{1}{(n-1)!(n-1)!}\right]$$

$$= \frac{(2n)!}{(n-1)!(n-1)!}\left[\frac{1+n^2}{n^2}\right] \ldots\ldots\ldots\ldots(i)$$

$${}^{2n+2}C_{n+1} = \frac{(2n+2)!}{(n+1)!(n+1)!}$$

$$= \frac{(2n+2)(2n+1)(2n)!}{n(n+1)(n-1)!(n+1)n(n-1)!} \ldots\ldots\ldots(ii)$$

$$\Rightarrow \quad \frac{(2n)!}{(n-1)!(n-1)!} \times \frac{(n+1)^2(n)^2(n-1)!(n-1)!}{(2n+2)(2n+1)(2n)!} \times \left(\frac{1+n^2}{n^2}\right)$$

$$= \frac{(n+1)^2\left(1+n^2\right)}{(2n+2)(2n+1)} = \frac{(n+1)(n+1)\left(n^2+1\right)}{2(n+1)(2n+1)}$$

$$= \frac{(n+1)\left(n^2+1\right)}{(2n+1)} \times \frac{1}{2}$$

*********** END **********