Exercise 7A

Question 24

$$(1-\sin\theta+\cos\theta)^2=(1-\sin\theta)^2+\cos^2\theta+2\cos\theta(1-\sin\theta)$$

$$\begin{aligned}&=1+\sin^2\theta-2\sin\theta+\cos^2\theta+2\cos\theta-2\cos\theta\sin\theta\\&=1+1-2\sin\theta+2\cos\theta-2\cos\theta\sin\theta\\&=2(1-\sin\theta)+2\cos\theta(1-\sin\theta)\\&=(1-\sin\theta)(2+2\cos\theta)\\&=2(1-\sin\theta)(1+\cos\theta)=\text{RHS}\end{aligned}$$

$\therefore$ LHS = RHS

Question 25

$$\text{LHS}=\frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)}=\frac{1+\cos\theta-(1-\cos^2\theta)}{\sin\theta(1+\cos\theta)}$$

$$=\frac{1+\cos\theta-1+\cos^2\theta}{\sin\theta(1+\cos\theta)}$$

$$=\frac{\cos\theta(1+\cos\theta)}{\sin\theta(1+\cos\theta)}=\frac{\cos\theta}{\sin\theta}=\cot\theta=\text{RHS}$$

$\therefore$ LHS = RHS

Question 26

(i)

$$\text{LHS} = \frac{(\operatorname{cosec}\theta + \cot\theta)}{(\operatorname{cosec}\theta - \cot\theta)} \times \frac{(\operatorname{cosec}\theta + \text{Cot}\theta)}{(\operatorname{cosec}\theta + \cot\theta)}$$

$$= \frac{(\operatorname{cosec}\theta + \cot\theta)^2}{(\operatorname{cosec}^2\theta - \cot^2\theta)} = (\operatorname{cosec}\theta + \cot\theta)^2$$

Further,

$$(\operatorname{cosec}\theta + \cot\theta)^2 = \operatorname{cosec}^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta\cot\theta$$

$$= 1 + \cot^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta\cot\theta$$

$$= 1 + 2\cot^2\theta + 2\operatorname{cosec}\theta\cot\theta$$

$\therefore$ LHS = RHS

(ii)

$$\text{LHS} = \frac{(\sec\theta + \tan\theta)}{(\sec\theta - \tan\theta)} \times \frac{(\sec\theta + \tan\theta)}{(\sec\theta + \tan\theta)}$$

$$= \frac{(\sec\theta + \tan\theta)^2}{(\sec^2\theta - \tan^2\theta)} = (\sec\theta + \tan\theta)^2$$

Further,

$$(\sec\theta + \tan\theta)^2 = \sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta$$

$$= 1 + \tan^2\theta + \tan^2\theta + 2\sec\theta\tan\theta$$

$$= 1 + 2\tan^2\theta + 2\sec\theta\tan\theta = \text{RHS}$$

$\therefore$ LHS = RHS

********** END **********