Chapter 10 Differentiability Ex 10.1 Q8

$$f(x)=\begin{cases} x^2+3x+a, & \text{if } x\le 1 \\ bx+2 \quad , & \text{if } x>1 \end{cases}$$

$$(\text{LHD at } x=1)=\lim_{x\to 1^-}\frac{f(x)-f(1)}{x-1}$$
$$=\lim_{h\to 0}\frac{f(1-h)-f(1)}{1-h-1}$$
$$=\lim_{h\to 0}\frac{\left[(1-h)^2+3(1-h)+a\right]-\left[1+3+a\right]}{-h}$$
$$=\lim_{h\to 0}\frac{h^2-5h}{-h}$$
$$=-5$$

$$(\text{RHD at } x=1)=\lim_{x\to 1^+}\frac{f(x)-f(1)}{x-1}$$
$$=\lim_{h\to 0}\frac{f(1+h)-f(1)}{1+h-1}$$
$$=\lim_{h\to 0}\frac{\left[b(1+h)+2\right]-(b+2)}{h}$$
$$=\lim_{h\to 0}\frac{b+bh+2-b-2}{h}$$
$$=b$$

Since $f(x)$ is differentiable, so

$$(\text{LHD at } x=1)=(\text{RHD at } x=1)$$
$$5=b$$
$$f(1)=1+3+a$$
$$=4+a$$

LHL $=\lim_{x\to 1^{-1}} f(x)$
$$=\lim_{h\to 0} f(1-h)$$
$$=\lim_{h\to 0}(1-h)^2+3(1-h)+a$$
$$=4+a$$

RHL $=\lim_{x\to 1^+} f(x)$
$$=\lim_{h\to 0} f(1+h)$$
$$=\lim_{h\to 0} b(1+b)+2$$
$$=b+2$$

Chapter 10 Differentiability Ex 10.1 Q9

$$f(x) = \begin{cases} |2x-3|\,[x], & x \geq 1 \\ \sin\left(\frac{\pi x}{2}\right) & , \quad x < 1 \end{cases}$$

$$f(x) = \begin{cases} (2x-3)[x]\ , & x \geq \frac{3}{2} \\ -(2x-3) & , \quad 1 \leq x \leq \frac{3}{2} \\ \sin\left(\frac{\pi x}{2}\right) & , \quad x < 1 \end{cases}$$

For continuity at $x = 1$

$f(1) = -(2.1-3) = 1$

LHL $= \lim_{x \to 1^{-1}} f(x)$

$= \lim_{h \to 0} f(1-h)$

$= \lim_{h \to 0} \sin\left(\frac{\pi(1-h)}{2}\right)$

$= \sin\frac{\pi}{2}$

$= 1$

RHL $= \lim_{x \to 0^{+}} f(x)$

$= \lim_{h \to 0} f(1+h)$

$= \lim_{h \to 0} -\left(2(1+h)-3\right)$

$= -1(-1)$

$= 1$

$$\text{LHL} = f(1) = \text{RHL}$$

So, $f(x)$ is continuous at $x = 1$

For differentiablility at $x = 1$

$$(\text{LHD at } x = 1) = \lim_{x \to 0^-} \frac{f(x) - f(1)}{x - 1}$$

$$= \lim_{h \to 0} \frac{f(1-h) - 1}{1 - h - 1}$$

$$= \lim_{h \to 0} \frac{\sin\left(\frac{\pi(1-h)}{2}\right) - 1}{-h}$$

$$= \lim_{h \to 0} \frac{\sin\left(\frac{\pi}{2} - \frac{\pi}{2}h\right) - 1}{-h}$$

$$= \lim_{h \to 0} \frac{\cos\left(\frac{\pi}{2}h\right) - 1}{\frac{-h}{2}}$$

$$= \lim_{h \to 0} \frac{2\sin^2\left(\frac{\pi}{4}h\right)}{h} \times \frac{\left(\frac{\pi}{4}h\right)^2}{\left(\frac{\pi}{4}h\right)^2}$$

$$= 0$$

$$(\text{RHD at } x = 1) = \lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1}$$

$$= \lim_{h \to 0} \frac{f(1+h) - f(1)}{1 + h - 1}$$

$$= \lim_{h \to 0} \frac{-[2(1+h) - 3] - 1}{h}$$

$$= \lim_{h \to 0} \frac{-2 - 2h + 3 - 1}{h}$$

$$= \lim_{h \to 0} \frac{-2h}{h}$$

$$= -2$$

$(\text{LHD at } x = 1) \neq (\text{RHD at } x = 1)$

$\therefore$ $f(x)$ is continuous but differentiable at $x = 1$.

*********** END **********