Inverse Trigonometric Functions Ex 4.1 Q2.

$$\text{Let } \cos^{-1}\left(\frac{1}{2}\right)=x. \text{ Then, } \cos x=\frac{1}{2}=\cos\left(\frac{\pi}{3}\right).$$

$$\therefore \cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$$

$$\text{Let } \sin^{-1}\left(\frac{1}{2}\right)=y. \text{ Then, } \sin y=\frac{1}{2}=\sin\left(\frac{\pi}{6}\right).$$

$$\therefore \sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$$

$$\therefore \cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}+\frac{2\pi}{6}=\frac{\pi}{3}+\frac{\pi}{3}=\frac{2\pi}{3}$$

$$\therefore \sin^{-1}\frac{1}{2}=\frac{\pi}{6}$$

$$\therefore \tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]=\tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right]$$

$$=\tan^{-1}\left[2\cos\frac{\pi}{3}\right]=\tan^{-1}\left[2\times\frac{1}{2}\right]$$

$$=\tan^{-1}1=\frac{\pi}{4}$$

Concept Insight:

Solve the innermost bracket first, so first find the principal value of $\sin^{-1}(1/2)$

$$\text{Let } \tan^{-1}(1)=x. \text{ Then, } \tan x=1=\tan\frac{\pi}{4}.$$

$$\therefore \tan^{-1}(1)=\frac{\pi}{4}$$

$$\text{Let } \cos^{-1}\left(-\frac{1}{2}\right)=y. \text{ Then, } \cos y=-\frac{1}{2}=-\cos\left(\frac{\pi}{3}\right)=\cos\left(\pi-\frac{\pi}{3}\right)=\cos\left(\frac{2\pi}{3}\right).$$

$$\therefore \cos^{-1}\left(-\frac{1}{2}\right)=\frac{2\pi}{3}$$

$$\text{Let } \sin^{-1}\left(-\frac{1}{2}\right)=z. \text{ Then, } \sin z=-\frac{1}{2}=-\sin\left(\frac{\pi}{6}\right)=\sin\left(-\frac{\pi}{6}\right).$$

$$\therefore \sin^{-1}\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$$

$$\therefore \tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$$

$$=\frac{\pi}{4}+\frac{2\pi}{3}-\frac{\pi}{6}$$

$$=\frac{3\pi+8\pi-2\pi}{12}=\frac{9\pi}{12}=\frac{3\pi}{4}$$

$\tan^{-1}\left(\sqrt{3}\right)$ = Angle in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is $\sqrt{3}$

$= \frac{\pi}{3}$

$\sec^{-1}(-2)$ = An angle in $[0, \pi] - \left\{\frac{\pi}{2}\right\}$ whose secant is (-2)

$= \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

$\operatorname{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$ = An angle in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ whose cosecant is $\left(\frac{2}{\sqrt{3}}\right)$

$= \frac{\pi}{3}$

Hence,

$$\tan^{-1}\sqrt{3} - \sec^{-1}(-2) + \operatorname{cosec}^{-1}\frac{2}{\sqrt{3}}$$

$$= \frac{\pi}{3} - \frac{2\pi}{3} + \frac{\pi}{3}$$

$$= 0$$

$$\therefore \tan^{-1}\sqrt{3} - \sec^{-1}\left(-\sqrt{2}\right) + \operatorname{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right) = 0$$

*********** END **********