Trigonometric Ratios Ex 5.1 Q18

Answer:

Given: $\sec\theta = \dfrac{5}{4}$ ......(1)

To find the value of $\dfrac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta}$

Now we know that $\sec\theta = \dfrac{1}{\cos\theta}$

Therefore,

$\cos\theta = \dfrac{1}{\sec\theta}$

Therefore from equation (1)

$\cos\theta = \dfrac{1}{\dfrac{5}{4}}$

$\cos\theta = \dfrac{4}{5}$ ...... (2)

Also, we know that $\cos^2\theta + \sin^2\theta = 1$

Therefore,

$\sin^2\theta = 1 - \cos^2\theta$

$\sin\theta = \sqrt{1 - \cos^2\theta}$

Substituting the value of $\cos\theta$ from equation (2)

We get,

$$\sin\theta = \sqrt{1 - \left(\frac{4}{5}\right)^2}$$

$$= \sqrt{1 - \frac{4^2}{5^2}}$$

$$= \sqrt{1 - \frac{16}{25}}$$

$$= \sqrt{\frac{25 - 16}{25}}$$

$$= \sqrt{\frac{9}{25}}$$

$$= \frac{3}{5}$$

Therefore

$$\sin\theta = \frac{3}{5} \quad \ldots\ldots (3)$$

Also, we know that $sec^2\theta = 1 + \tan^2\theta$.

Therefore,

$$\tan^2\theta = sec^2\theta - 1$$

Therefore

$$\tan^2\theta = \left(\frac{5}{4}\right)^2 - 1$$

$$= \frac{25}{16} - 1$$

$$= \frac{9}{16}$$

Therefore,

$$\tan\theta = \sqrt{\frac{9}{16}}$$

$$= \frac{3}{4}$$

Therefore,

$$\tan\theta = \frac{3}{4} \text{ ...... (4)}$$

Also $\cot\theta = \frac{1}{\tan\theta}$

Therefore, from equation (4)

We get,

$$\cot\theta = \frac{1}{\frac{3}{4}}$$

$$\cot\theta = \frac{4}{3} \text{ ...... (5)}$$

Substituting the value of $\cos\theta$, $\sin\theta$, $\cot\theta$ and $\tan\theta$ from equation (2) (3) (4) and (5) respectively in the expression below

$$\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta}$$

We get,

$$\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta} = \frac{\frac{3}{5} - 2\left(\frac{4}{5}\right)}{\frac{3}{4} - \frac{4}{3}}$$

$$= \frac{\frac{3}{5} - \frac{8}{5}}{\frac{(3\times3) - (4\times4)}{4\times3}}$$

$$= \frac{\frac{3}{5} - \frac{8}{5}}{\frac{(3\times3) - (4\times4)}{4\times3}}$$

$$= \frac{\frac{3-8}{5}}{\frac{9-16}{4\times3}}$$

$$= \frac{\frac{-5}{5}}{\frac{-7}{12}}$$

$$= \frac{12}{7}$$

Therefore, $\dfrac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta} = \dfrac{12}{7}$

********** END *********