Algebraic Identities Ex 4.3 Q15

Answer :

In the given problem, we have to find the value of $x^2+\frac{1}{x^2}, x^3+\frac{1}{x^3}, x^4+\frac{1}{x^4}$

Given $x+\frac{1}{x}=3$

We shall use the identity $(x+y)^2=x^2+y^2+2xy$

Here putting $x+\frac{1}{x}=3$,

$$\left(x+\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}+2\times x\times\frac{1}{x}$$

$$(3)^2=x^2+\frac{1}{x^2}+2\times\cancel{x}\times\frac{1}{\cancel{x}}$$

$$9=x^2+\frac{1}{x^2}+2$$

$$9-2=x^2+\frac{1}{x^2}$$

$$7=x^2+\frac{1}{x^2}$$

Again squaring on both sides we get,

$$\left(x^2+\frac{1}{x^2}\right)^2=(7)^2$$

We shall use the identity $(x+y)^2=x^2+y^2+2xy$

$$\left(x^2+\frac{1}{x^2}\right)^2=x^4+\frac{1}{x^4}+2\times x^2\times\frac{1}{x^2}$$

$$(7)^2=x^4+\frac{1}{x^4}+2\times \cancel{x^2}\times\frac{1}{\cancel{x^2}}$$

$$49=x^4+\frac{1}{x^4}+2$$

$$49-2=x^4+\frac{1}{x^4}$$

$$47=x^4+\frac{1}{x^4}$$

Again cubing on both sides we get,

$$\left(x+\frac{1}{x}\right)^3=(3)^3$$

We shall use identity $(a+b)^3=a^3+b^3+3ab(a+b)$

$$\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3\times x\times\frac{1}{x}\left(x+\frac{1}{x}\right)$$

$$(3)^3=x^3+\frac{1}{x^3}+3\times\cancel{x}\times\frac{1}{\cancel{x}}\times 3$$

$$27=x^3+\frac{1}{x^3}+9$$

$$27-9=x^3+\frac{1}{x^3}$$

$$18=x^3+\frac{1}{x^3}$$

Hence the value of $x^2+\frac{1}{x^2},x^3+\frac{1}{x^3},x^4+\frac{1}{x^4}$ is $\boxed{7,18,47}$ respectively.

*********** END **********