



Trigonometric Equations Ex 11.1 Q4(vi)

We have,

$$\begin{aligned}
 \cos \theta \cdot \cos 2\theta \cdot \cos 3\theta &= \frac{1}{4} \\
 \Rightarrow 2 \cos \theta \cdot \cos 3\theta \cdot \cos 2\theta &= \frac{1}{2} \\
 \Rightarrow (\cos 4\theta + \cos 2\theta) \cos 2\theta &= \frac{1}{2} \\
 \Rightarrow (2 \cos^2 2\theta - 1 + \cos 2\theta) \cos 2\theta &= \frac{1}{2} \\
 \Rightarrow 2 \cos^3 2\theta + \cos^2 2\theta - \cos 2\theta &= \frac{1}{2} \\
 \Rightarrow 4 \cos^2 2\theta + 2 \cos^2 2\theta - 2 \cos 2\theta - 1 &= 0 \\
 \Rightarrow 2 \cos^2 2\theta (2 \cos \theta + 1) - 1 (2 \cos 2\theta + 1) &= 0 \\
 \Rightarrow (2 \cos^2 2\theta - 1) (2 \cos 2\theta + 1) &= 0
 \end{aligned}$$

either

$$\begin{aligned}
 2 \cos^2 2\theta - 1 &= 0 & \text{or } \Rightarrow 2 \cos 2\theta + 1 &= 0 \\
 \Rightarrow \cos 4\theta &= 0 & \text{or } \Rightarrow \cos 2\theta &= -\frac{1}{2} \\
 \Rightarrow 4\theta &= (2n+1) \frac{\pi}{2} & \text{or } \Rightarrow \cos 2\theta &= \cos 2 \frac{\pi}{3} \\
 \Rightarrow \theta &= (2n+1) \frac{\pi}{8} & \text{or } \Rightarrow 2\theta &= 2m\pi \pm 2 \frac{\pi}{3} \\
 & & \Rightarrow \theta &= m\pi \pm \frac{\pi}{3}
 \end{aligned}$$

Thus,

$$\theta = (2n+1) \frac{\pi}{8} \quad \text{or } \theta = m\pi \pm \frac{\pi}{3}, m, n \in \mathbb{Z}$$

Trigonometric Equations Ex 11.1 Q4(v)

We have,

$$\cos \theta + \sin \theta = \cos 2\theta + \sin 2\theta$$

$$\begin{aligned}
 \Rightarrow \cos \theta - \cos 2\theta &= \sin 2\theta - \sin \theta \\
 \Rightarrow 2 \sin \frac{3\theta}{2} \cdot \sin \frac{\theta}{2} &= 2 \cos \frac{3\theta}{2} \cdot \sin \frac{\theta}{2} \\
 \Rightarrow 2 \sin \frac{\theta}{2} \left(\sin \frac{3\theta}{2} - \cos \frac{3\theta}{2} \right) &= 0 \\
 \Rightarrow 2 \sin \frac{\theta}{2} \left(\sin \frac{3\theta}{2} - \cos \frac{3\theta}{2} \right) &= 0
 \end{aligned}$$

either

$$\begin{aligned}
 \sin \frac{\theta}{2} &= 0 & \text{or } \sin \frac{3\theta}{2} - \cos \frac{3\theta}{2} &= 0 \\
 \Rightarrow \frac{\theta}{2} &= n\pi, n \in \mathbb{Z} & \text{or } \tan \frac{3\theta}{2} &= 1 = \tan \frac{\pi}{4} \\
 \Rightarrow \theta &= 2n\pi, n \in \mathbb{Z} & \text{or } \frac{3\theta}{2} &= n\pi + \frac{\pi}{4} \\
 & & \text{or } \theta &= 2n \frac{\pi}{3} + \frac{\pi}{3.2}, n \in \mathbb{Z}
 \end{aligned}$$

Thus,

$$\Rightarrow \theta = 2n\pi \quad \text{or } 2n \frac{\pi}{3} + \frac{\pi}{6}, n \in \mathbb{Z}$$

Trigonometric Equations Ex 11.1 Q4(vi)

We have,

$$\begin{aligned}
 & \sin \theta + \sin 2\theta + \sin 3\theta = 0 \\
 \Rightarrow & \sin 2\theta + 2 \sin 2\theta \cdot \cos \theta = 0 \\
 \Rightarrow & \sin 2\theta + (1 + 2 \cos \theta) = 0 \\
 \Rightarrow & \text{either} \\
 & \sin 2\theta = 0 \quad \text{or} \quad 1 + 2 \cos \theta = 0 \\
 \Rightarrow & 2\theta = n\pi, n \in \mathbb{Z} \quad \text{or} \quad \cos \theta = -\frac{1}{2} = \cos \left(\pi - \frac{\pi}{3} \right) \\
 \Rightarrow & \theta = \frac{n\pi}{2}, n \in \mathbb{Z} \quad \text{or} \quad \theta = 2m\pi \pm \frac{2\pi}{3}, m \in \mathbb{Z}
 \end{aligned}$$

Thus,

$$\theta = \frac{n\pi}{2}, n \in \mathbb{Z} \quad \text{or} \quad \theta = 2m\pi \pm \frac{2\pi}{3}, m \in \mathbb{Z}$$

Trigonometric Equations Ex 11.1 Q4(vii)

Given , $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$

$$(\sin 4x + \sin 2x) + (\sin 3x + \sin x) = 0$$

Using , $(\sin A + \sin B)$ formula \Rightarrow

$$2 \sin \left[\frac{(4x+2x)}{2} \right] \cos \left[\frac{4x-2x}{2} \right] + 2 \sin \left[\frac{(3x+x)}{2} \right] \cos \left[\frac{(3x-x)}{2} \right] = 0$$

$$2 \sin 3x \cos x + 2 \sin 2x \cos x = 0$$

$$2 \cos x (\sin 3x + \sin 2x) = 0$$

$$2 \cos x \left(2 \sin \left[\frac{(3x+2x)}{2} \right] \cos \left[\frac{(3x-2x)}{2} \right] \right) = 0$$

$$4 \cos x \sin \frac{5x}{2} \cos \frac{x}{2} = 0$$

$$\cos x = 0 ; \sin \frac{5x}{2} = 0 ; \cos \frac{x}{2} = 0$$

$$x = \frac{(2n+1)\pi}{2} ; \frac{5x}{2} = m\pi ; \frac{x}{2} = \frac{(2r+1)\pi}{2}$$

$$x = \frac{(2n+1)\pi}{2} ; x = \frac{2m\pi}{5} ; x = (2r+1)\pi, m, r, n \in \mathbb{Z}$$

***** END *****