Differentiability Ex 10.2 Q1

Here, $f(x) = x^2$ is a polynomial function so, it is differentiable at $x = 2$.

$$
\begin{aligned}
f'(2) &= \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} \\
&= \lim_{h\to 0} \frac{(2+h)^2 - (2)^2}{h} \\
&= \lim_{h\to 0} \frac{4 + h^2 + 4h - 4}{h} \\
&= \lim_{h\to 0} \frac{4h + h^2}{h} \\
&= \lim_{h\to 0} (4+h) \\
&= 4
\end{aligned}
$$

$\therefore \quad f'(2) = 4$

Chapter 10 Differentiability Ex 10.2 Q2

$f(x) = x^2 - 4x + 7$ is a polynomial function, So it is differentiable everywhere.

$$
\begin{aligned}
f'(5) &= \lim_{h\to 0} \frac{f(5+h) - f(5)}{h} \\
&= \lim_{h\to 0} \frac{\left\{(5+h)^2 - 4(5+h) + 7\right\} - [25 - 20 + 7]}{h} \\
&= \lim_{h\to 0} \frac{h^2 + 25 + 10h - 20 - 4h + 7 - 12}{h} \\
&= \lim_{h\to 0} \frac{h^2 + 6h}{h} \\
&= \lim_{h\to 0} (h+6) \\
&= 6
\end{aligned}
$$

$$
\begin{aligned}
f'\left(\frac{7}{2}\right) &= \lim_{h\to 0} \frac{f\left(\frac{7}{2}+h\right) - f\left(\frac{7}{2}\right)}{h} \\
&= \lim_{h\to 0} \frac{\left[\left(\frac{7}{2}+h\right)^2 - 4\left(\frac{7}{2}+h\right) + 7\right] - \left[\left(\frac{7}{2}\right)^2 - 4\left(\frac{7}{2}\right) + 7\right]}{h} \\
&= \lim_{h\to 0} \frac{\left[\frac{49}{2} + h^2 + 7h - 14 - 4h + 7\right] - \left[\frac{49}{4} - 14 + 7\right]}{h} \\
&= \lim_{h\to 0} \frac{\frac{49}{4} + h^2 + 7h - 14 - 4h + 7 - \frac{49}{4} + 14 - 7}{h} \\
&= \lim_{h\to 0} \frac{h^2 + 3h}{h} \\
&= \lim_{h\to 0} (h+3) \\
&= 3
\end{aligned}
$$

Now,

$$
\begin{aligned}
f'(5) &= 6 \\
&= 2(3) \\
f'(5) &= 2f'\left(\frac{7}{2}\right)
\end{aligned}
$$

Chapter 10 Differentiability Ex 10.2 Q3

We know that, $f(x) = 2x^3 - 9x^2 + 12x + 9$ is a polynomial function. So, it is differentiable every where. For $x = 1$

$$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$
$$= \lim_{h\to 0} \frac{\left[2(1+h)^3 - 9(1+h)^2 + 12(1+h) + 9\right] - \left[2 - 9 + 12 + 9\right]}{h}$$
$$= \lim_{h\to 0} \frac{\left[2\left(1+h^3+3h^2+3h\right) - 9\left(1+h^2+2h\right) + 12 + 12h + 9\right] - [14]}{h}$$
$$= \lim_{h\to 0} \frac{\left[2 + 2h^3 + 6h^2 + 6h - 9 - 9h^2 - 18h + 12 + 12h + 9 - 14\right]}{h}$$
$$= \lim_{h\to 0} \frac{2h^3 - 3h^2}{h}$$
$$= \lim_{h\to 0} \frac{h^2(2h-3)}{h}$$
$$= \lim_{h\to 0} h(2h-3)$$
$$f'(1) = 0 \qquad \text{---(i)}$$

For $x = 2$

$$f'(2) = \lim_{h\to 0} \frac{\left[2(2+h)^3 - 9(2+h)^2 + 12(12+h) + 9\right] - \left[16 - 36 + 24 + 9\right]}{h}$$
$$= \lim_{h\to 0} \frac{\left[2\left(8+h^3+12h+6h^2\right) - 9\left(4+h^2+4h\right) + 24 + 12h + 9\right] - [13]}{h}$$
$$= \lim_{h\to 0} \frac{\left[16 + 2h^3 + 24h + 12h^2 - 36 - 9h^2 - 36h + 33 + 12h - 13\right]}{h}$$
$$= \lim_{h\to 0} \frac{2h^3 + 3h^2}{h}$$
$$= \lim_{h\to 0} \frac{h^2(2h+3)}{h}$$
$$= \lim_{h\to 0} h(2h+3)$$
$$f'(2) = 0 \qquad \text{---(ii)}$$

From equation (i) and (ii),

$$f'(1) = f'(2)$$

Chapter 10 Differentiability Ex 10.2 Q4

$$\Phi(x) = \lambda x^2 + 7x - 4 \text{ and } \Phi'(5) = 97$$
$$\Phi'(5) = \lim_{h\to 0} \frac{\left[\lambda(5+h)^2 + 7(5+h) - 4\right] - \left[25\lambda + 35 - 4\right]}{h}$$
$$97 = \lim_{h\to 0} \frac{\lambda\left(25 + h^2 + 10h\right) + 35 + 7h - 4 - 25\lambda - 35 + 4}{h}$$
$$= \lim_{h\to 0} \frac{25\lambda + \lambda h^2 + 10\lambda h - 25\lambda + 7h}{h}$$
$$= \lim_{h\to 0} \frac{\lambda h^2 + h(10\lambda + 7)}{h}$$
$$= \lim_{h\to 0} \frac{f(\lambda h + 10\lambda + 7)}{h}$$
$$97 = 10\lambda + 7$$
$$10\lambda = 97 + 7$$
$$\lambda = \frac{90}{10}$$
$$\lambda = 9$$

*********** END **********