Functions Ex 2.1 Q5(i)

$f : N \rightarrow N$ given by $f(x) = x^2$

let $\quad x_1 = x_2 \quad$ for $\quad x_1, x_2 \in N$

$\Rightarrow \quad x_1^2 = x_2^2 \quad \Rightarrow \quad f(x_1) = f(x_2)$

$\therefore \quad f$ in one-one.

Surjectivity: Since f takes only square value like 1,4,9,16......

so, non-perfect square values in N (co-domain) do not have pre image in domain N.

Thus, f is not onto.

Functions Ex 2.1 Q5(ii)

$f : Z \rightarrow Z$ given by $f(x) = x^2$

Injectivity: let $x_1 \ \& - x_1 \in Z$

$\Rightarrow \quad x_1 \neq -x_1$

$\Rightarrow \quad x_1^2 = (-x_1)^2 \quad \Rightarrow \quad f(x_1) = f(-x_1)$

$\Rightarrow \quad f$ is not one-one.

Surjective: Again, f takes only square values 1,4,9,16,...

So, no non-perfect square values in Z have a pre image in domain Z.

$\therefore \quad f$ is not onto.

Functions Ex 2.1 Q5(iii)

$f : N \rightarrow N$, given by $f(x) = x^3$

Injectivity: let $y, x \in N$ such that

$\quad x = y$

$\Rightarrow \quad x^3 = y^3$

$\Rightarrow \quad f(x) = f(y)$

$\therefore \quad f$ is one-one

Surjective:

$\because \ f$ attain only cubic number like 1,8,27,64,...

So, no non-cubic values of N (co-domain) have pre image in N (Domain)

$\therefore \quad f$ is not onto.

Functions Ex 2.1 Q5(iv)

$f : Z \rightarrow Z \quad$ given by $\ f(x) = x^3$

Injectivity: let $x, y \in Z$ such that

$\quad x = y$

$\Rightarrow \quad x^3 = y^3$

$\Rightarrow \quad f(x) = f(y)$

$\Rightarrow \quad f(x) = f(y)$

$\Rightarrow \quad f$ is one-one.

Surjective: Since f attains only cubic values like $\pm 1, \pm 8, \pm 27, \ldots \ldots \ldots$

so, no non-cubic values of Z (co-domain) have pre image in Z (domain)

$\therefore \quad f$ is not onto.

Functions Ex 2.1 Q5(v)

$f : R \to R$ given by $f(x) = |x|$

Injectivity: let $x, y \in R$ such that
     $x = y$ but if $y = -x$
$\Rightarrow$ $|x| = |y|$ $\Rightarrow$ $|y| = |-x| = x$
$\therefore$ f is not one-one.

Surjective: Since f attains only positive values, for negative real numbers in R, there is no pre-image in domain R.

$\therefore$ f is not onto.

*********** END *********