Chapter 9 Continuity Ex 9.2 Q4(i)

We have given that the function is continuous at $x = 0$

$\therefore$ LHL = RHL = $f(0)$ ....(1)

LHL = $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \dfrac{\sin(-2h)}{5(-h)} = \lim_{h \to 0} \dfrac{-\sin 2h}{-5h} = \lim_{h \to 0} \dfrac{\sin 2h}{2h} \times \dfrac{2h}{5h} = \dfrac{2}{5}$

$f(0) = 3k$

So, using (1) we get,

$\dfrac{2}{5} = 3k$

$k = \dfrac{2}{15}$

Chapter 9 Continuity Ex 9.2 Q4(ii)

It is given that the function is continuous

$\therefore$ LHL = RHL = $f(2)$ ....(1)

LHL = $\lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h) = \lim_{h \to 0} k(2-h) + 5$ $= 2k + 5$

RHL = $\lim_{x \to 2^+} f(x) = \lim_{h \to 0} f(2+h) = \lim_{h \to 0} (2+h) - 1 = 1$

Thus, using (1), we get,

$2k + 5 = 1$

$k = -2$

Chapter 9 Continuity Ex 9.2 Q4(iii)

It is given that the function is continuous

LHL = RHL = $f(0)$....(1)

LHL = $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} k\left((-h)^2 + 3(-h)\right) = \lim_{h \to 0} k\left(h^2 - 3h\right) = 0$

$f(0) = \cos 2 \times 0 = \cos 0^\circ = 1$

LHL $\neq f(0)$

Hence, no value of k can make f continuous

Chapter 9 Continuity Ex 9.2 Q4(iv)

First check the continuity of the function at $x = 3$

$f(3) = 2$ .....(A)

RHL = $\lim_{x \to 3^+} f(x) = \lim_{h \to 0} f(3+h) = \lim_{h \to 0} a(3+h) + b$ $= 3a + b$.....(B)

$\therefore f(x)$ will be continuous at $x = 3$ if $3a + b = 2$.....(1)

Now, check the continuity at $x = 5$

$f(5) = 9$ ....(C)

LHL = $\lim_{x \to 5^-} f(x) = \lim_{h \to 0} f(5-h) = \lim_{h \to 0} a(5-h) + b = 5a + b$

$f(x)$ will be continuous at $x = 5$ if $5a + b = 9$....(2)

Solving (1) & (2), we get

$a = \dfrac{7}{2}$ and $b = \dfrac{-17}{2}$

Chapter 9 Continuity Ex 9.2 Q4(v)
It is given that the function is continuous

At $x = -1$
$f(-1) = 4$
RHL $= \lim_{x \to -1^+} f(x) = \lim_{h \to 0} f(-1+h) = \lim_{h \to 0} a(-1+h)^2 + b = a + b$

Since, $f(x)$ is continuous at $x = -1$
$\therefore \ a + b = 4$ ....(A)

Now, at $x = 0$,
$f(0) = \cos 0^\circ = 1$
LHL $= \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} a(-h)^2 + b = b$
Since, $f(x)$ is continuous at $x = 0$
$\therefore \ f(0) = LHL$
$\Rightarrow b = 1$
$\therefore$ from (A)
$a = 3$
Thus, $a = 3, \ b = 1$

Chapter 9 Continuity Ex 9.2 Q4(vi)
It is given that the function is continuous.
At $x = 0$

LHL $= \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \dfrac{\sqrt{1-Ph} - \sqrt{1+Ph}}{-h} = \lim_{h \to 0} \dfrac{\left(\sqrt{1-Ph} - \sqrt{1+Ph}\right)}{-h} \times \dfrac{\left(\sqrt{1-Ph} + \sqrt{1+Ph}\right)}{\left(\sqrt{1-Ph} + \sqrt{1+Ph}\right)}$

$= \lim_{h \to 0} \dfrac{(1-Ph) - (1+Ph)}{-h\left(\sqrt{1-Ph} + \sqrt{1+Ph}\right)} = \dfrac{2P}{2} = P$

RHL $= \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \dfrac{2h+1}{h-2} = \dfrac{-1}{2}$

Since, $f(x)$ is continuous so,
$P = \dfrac{-1}{2}$

Chapter 9 Continuity Ex 9.2 Q4(vii)

The given function f is $f(x) = \begin{cases} 5, & \text{if } x \le 2 \\ ax+b, & \text{if } 2 < x < 10 \\ 21, & \text{if } x \ge 10 \end{cases}$

It is evident that the given function f is defined at all points of the real line.

If f is a continuous function, then f is continuous at all real numbers.

In particular, f is continuous at $x = 2$ and $x = 10$

Since f is continuous at $x = 2$, we obtain

$\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^+} f(x) = f(2)$

$\Rightarrow \lim\limits_{x \to 2^-} (5) = \lim\limits_{x \to 2^+} (ax+b) = 5$

$\Rightarrow 5 = 2a+b = 5$

$\Rightarrow 2a+b = 5 \qquad \ldots(1)$

Since f is continuous at $x = 10$, we obtain

$\lim\limits_{x \to 10^-} f(x) = \lim\limits_{x \to 10^+} f(x) = f(10)$

$\Rightarrow \lim\limits_{x \to 10^-} (ax+b) = \lim\limits_{x \to 10^+} (21) = 21$

$\Rightarrow 10a+b = 21 = 21$

$\Rightarrow 10a+b = 21 \qquad \ldots(2)$

On subtracting equation (1) from equation (2), we obtain

$8a = 16$

$a = 2$

By putting $a = 2$ in equation (1), we obtain

$2 \times 2 + b = 5$

$4 + b = 5$

$b = 1$

Therefore, the values of a and b for which f is a continuous function are 2 and 1 respectively.

Chapter 9 Continuity Ex 9.2 Q4(viii)

Since the function is continuous at $x = \dfrac{\pi}{2}$ therefore

LHL of $f(x)$ at $x = \dfrac{\pi}{2}$ is

$$= \lim_{x \to \frac{\pi}{2}^-} f(x)$$

$$= \lim_{h \to 0} f\left(h - \frac{\pi}{2}\right)$$

$$= \lim_{h \to 0} \frac{k \cos\left(h - \dfrac{\pi}{2}\right)}{\pi - 2\left(h - \dfrac{\pi}{2}\right)}$$

$$= \lim_{h \to 0} \frac{k \sinh}{2\pi - 2h}$$

$$= \frac{k}{2} \lim_{h \to 0} \frac{\sin(\pi - h)}{(\pi - h)}$$

$$= \frac{k}{2}$$

Again

$$f\left(\frac{\pi}{2}\right) = 3$$

Hence

$$LHL = f\left(\frac{\pi}{3}\right)$$

$$\frac{k}{2} = 3$$

$$k = 6$$

********** END *********