



Differentiation Ex 11.8 Q4(ii)

Let  $u = \sin^{-1} \sqrt{1-x^2}$

Put  $x = \cos \theta$ , so,

$$u = \sin^{-1} \sqrt{1 - \cos^2 \theta}$$

$$u = \sin^{-1} (\sin \theta) \quad \text{---(i)}$$

And,  $v = \cos^{-1} x \quad \text{---(ii)}$

Here,

$$x \in (-1, 0)$$

$$\Rightarrow \cos \theta \in (-1, 0)$$

$$\Rightarrow \theta \in \left( \frac{\pi}{2}, \pi \right)$$

So, from equation (i),

$$u = \pi - \theta$$

$$\left[ \text{Since, } \sin^{-1} (\sin \theta) = \pi - \theta, \theta \in \left( \frac{\pi}{2}, \frac{3\pi}{2} \right) \right]$$

$$u = \pi - \cos^{-1} x$$

$$[\text{Since, } x = \cos \theta]$$

Differentiating it with respect to  $x$ ,

$$\frac{du}{dx} = 0 - \left( \frac{-1}{\sqrt{1-x^2}} \right)$$

$$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}} \quad \text{---(v)}$$

And, from equation (ii),

$$v = \cos^{-1} x$$

Differentiating it with respect to  $x$ ,

$$\frac{dv}{dx} = \frac{-1}{\sqrt{1-x^2}} \quad \text{---(vi)}$$

Dividing equation (v) by (vi)

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{1}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{-1}$$

$$\frac{du}{dv} = -1$$

Differentiation Ex 11.8 Q5(i)

Let  $u = \sin^{-1} \left( 4x\sqrt{1-4x^2} \right)$

Put  $2x = \cos \theta$ , so

$$u = \sin^{-1} \left( 2 \times \cos \theta \sqrt{1 - \cos^2 \theta} \right)$$

$$= \sin^{-1} (2 \cos \theta \sin \theta)$$

$$u = \sin^{-1} (\sin 2\theta) \quad \text{---(i)}$$

Let  $v = \sqrt{1-4x^2} \quad \text{---(ii)}$

Here,

$$x \in \left( -\frac{1}{2}, -\frac{1}{2\sqrt{2}} \right)$$

$$\Rightarrow 2x \in \left( -1, -\frac{1}{\sqrt{2}} \right)$$

$$\Rightarrow \theta \in \left( \frac{3\pi}{4}, \pi \right)$$

So, from equation (i),

$$u = \pi - 2\theta \quad \left[ \text{Since, } \sin^{-1} (\sin \theta) = \pi - \theta \text{ if } \theta \in \left[ \frac{\pi}{2}, \frac{3\pi}{2} \right] \right]$$

$$u = \pi - 2 \cos^{-1} (2x) \quad [\text{Since, } 2x = \cos \theta]$$

Differentiating it with respect to  $x$  using chain rule,

$$\frac{du}{dx} = 0 - 2 \left( \frac{-1}{\sqrt{1-(2x)^2}} \right) \frac{d}{dx} (2x)$$

$$= \frac{2}{\sqrt{1-4x^2}} (2)$$

$$\frac{du}{dx} = \frac{4}{\sqrt{1-4x^2}} \quad \text{---(vi)}$$

From equation (iv)

$$\frac{dv}{dx} = \frac{-4x}{\sqrt{1-4x^2}}$$

but,  $x \in \left( -\frac{1}{2}, -\frac{1}{2\sqrt{2}} \right)$

$$\frac{dv}{dx} = \frac{-4(-x)}{\sqrt{1-4(-x)^2}}$$

$$\frac{dv}{dx} = \frac{4x}{\sqrt{1-4x^2}} \quad \text{---(vii)}$$

Differentiating equation (ii) with respect to  $x$  using chain rule,

$$\frac{dv}{dx} = \frac{1}{2\sqrt{1-4x^2}} \frac{d}{dx} (1-4x^2)$$

$$= \frac{1}{2\sqrt{1-4x^2}} (-8x)$$

$$\frac{dv}{dx} = \frac{-4x}{\sqrt{1-4x^2}} \quad \text{---(iv)}$$

Divide equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{4}{\sqrt{1-4x^2}} \times \frac{\sqrt{1-4x^2}}{-4x}$$

$$\frac{du}{dv} = -\frac{1}{x}$$

Differentiation Ex 11.8 Q5(ii)

$$\text{Let } u = \sin^{-1} \left( 4x\sqrt{1-4x^2} \right)$$

$$\text{Put } 2x = \cos \theta, \text{ so}$$

$$u = \sin^{-1} \left( 2 \times \cos \theta \sqrt{1 - \cos^2 \theta} \right) \\ = \sin^{-1} (2 \cos \theta \sin \theta)$$

$$u = \sin^{-1} (\sin 2\theta) \quad \text{---(i)}$$

$$\text{Let } v = \sqrt{1-4x^2} \quad \text{---(ii)}$$

Here,

$$x \in \left( \frac{1}{2\sqrt{2}}, \frac{1}{2} \right)$$

$$\Rightarrow 2x \in \left( \frac{1}{\sqrt{2}}, 1 \right)$$

$$\Rightarrow \cos \theta \in \left( \frac{1}{\sqrt{2}}, 1 \right)$$

$$\Rightarrow \theta \in \left( 0, \frac{\pi}{4} \right)$$

So, from equation (i)

$$u = 2\theta \quad \left[ \text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right] \right]$$

$$u = 2 \cos^{-1} (2x) \quad [\text{Since, } 2x = \cos \theta]$$

Differentiate it with respect to  $x$  using chain rule,

$$\frac{du}{dx} = 2 \left( \frac{-1}{\sqrt{1-(2x)^2}} \right) \frac{d}{dx} (2x)$$

$$= \left( \frac{-2}{\sqrt{1-4x^2}} (2) \right)$$

$$\frac{du}{dx} = \frac{-4}{\sqrt{1-4x^2}} \quad \text{---(v)}$$

Dividing equation (v) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{-4}{\sqrt{1-4x^2}} \times \frac{\sqrt{1-4x^2}}{-4x}$$

$$\frac{du}{dv} = \frac{1}{x}$$

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