Maxima and Minima 18.5 Q23

Here, $ABCD$ is a rectangle with width $AB = x$ cm and length $AD = y$ cm.

The rectangle is rotated about AD. Let v be the volume of the cylinder so formed.

$\therefore \quad v = \pi r^2 y \qquad$ ---(i)

Again,

$\quad$ Perimeter of $ABCD = 2(l + b) = 2(x + y) \qquad$ ---(ii)

$\Rightarrow \quad 36 = 2(x + y)$

$\Rightarrow \quad y = 18 - x \qquad$ ---(iii)

From (i) and (ii), we get

$$v - \pi r^2 (18 - x) = \pi \left(18x^2 - x^3\right)$$

$$\Rightarrow \quad \frac{dv}{dx} = \pi \left(36x - 3x^2\right)$$

For maxima or minima, we have,

$$\frac{dv}{dx} = 0$$

$$\Rightarrow \quad \pi \left(36x - 3x^2\right) = 0$$

$$\Rightarrow \quad 3\pi \left(12x - x^2\right) = 0$$

$$\Rightarrow \quad x(12 - x) = 0$$

$\Rightarrow \quad x = 0$ (Not possible) or 12

$\therefore \quad x = 12$ cm

From (iii)

$\quad y = 18 - 12 = 6$ cm

Now,

$$\frac{d^2v}{dx^2} = \pi(36 - 6x)$$

At $\quad (x = 12, y = 6)\ \frac{d^2v}{dx^2} = \pi(36 - 72) = -36\pi < 0$

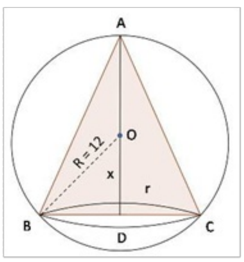
$\therefore \quad (x = 12, y = 6)$ is the point of local maxima,

Hence,

The dimension of rectangle, which wiout maximum value, when revolved about one of its side is width = 12 cm and length = 6 cm.

Maxima and Minima 18.5 Q24

Let r and h be the radius of the base of cone and height of the cone respectively.



Let $OD = x$

It is abvious that the axis of cone must be along the diameter of shpere for maximum volume of cone.

Now,

$$\text{In } \Delta BOD,\ BD = \sqrt{R^2 - x^2}$$
$$= \sqrt{144 - x^2}$$
$$AD = AO + OD = R + x = 12 + x$$
$$v = \text{volume of cone} = \frac{1}{3}\pi r^2 h$$
$$\Rightarrow \quad v = \frac{1}{3}\pi BD^2 \times AD$$
$$= \frac{1}{3}\pi\left(144 - x^2\right)(2 + x)$$
$$= \frac{1}{3}\pi\left(1728 + 144x - 12x^2 - x^3\right)$$
$$\therefore \quad \frac{dv}{dx} = \frac{1}{3}\pi\left(144 - 24x - 3x^2\right)$$

For maximum and minimum of v.

$$\frac{dv}{dx} = 0$$
$$\Rightarrow \quad \frac{1}{3}\pi\left(144 - 24x - 3x^2\right) = 0$$
$$\Rightarrow \quad x = -12, 4$$
$$x = -12 \text{ is not possible}$$
$$\therefore \quad x = 4$$

Now,

$$\frac{d^2v}{dx^2} = \frac{\pi}{3}(-24 - 6x)$$
$$\text{At} \quad x = 4, \frac{d^2v}{dx^2} = -2\pi(4 + x)$$
$$= -2\pi \times 8 = -16\pi < 0$$
$$\therefore \quad x = 4 \text{ is point of local maxima.}$$

Hence,

Height of cone of maximum volume $= R + x$

$= 12 + 4$

$= 16$ cm.

Maxima and Minima 18.5 Q25

We have, a closed cylinder whose volume $v = 2156$ cm^3

Let r and h be the radius and the height of the cylinder. Then,

$$\therefore \quad v = \pi r^2 h = 2156 \qquad \text{---(i)}$$

Total surface area = $S = 2\pi rh + 2\pi r^2$

$$\Rightarrow \quad S = 2\pi r(h + r) \qquad \text{---(ii)}$$

From (i) and (ii)

$$S = \frac{2156 \times 2}{r} + 2\pi r^2$$
$$\therefore \quad \frac{ds}{dr} = -\frac{4312}{r^2} + 4\pi r$$

For maximum and minimum

$$\frac{ds}{dr} = 0$$
$$\Rightarrow \quad \frac{-4312 + 4\pi r^3}{r^2} = 0$$
$$\Rightarrow \quad r^3 = \frac{4312}{4\pi}$$
$$\Rightarrow \quad r = 7$$

Now,

$$\frac{d^2s}{dr^2} = \frac{8624}{r^3} + 4\pi > 0 \text{ for } r = 7.$$
$$\therefore \quad r = 7 \text{ is the point of local minima}$$

Hence,

The total surface area of closed cylinder will be munimum at $r = 7$ cm.

*********** END **********