

## Exercise 7.10: Solutions of Questions on Page Number: 340

Q1: 
$$\int_0^1 \frac{x}{x^2+1} dx$$

## Answer:

$$\int_{0}^{1} \frac{x}{x^{2}+1} dx$$

 $\int_0^1 \frac{x}{x^2 + 1} dx$ Let  $x^2 + 1 = t \implies 2x dx = dt$ 

When x = 0, t = 1 and when x = 1, t = 2

$$\therefore \int_0^1 \frac{x}{x^2 + 1} dx = \frac{1}{2} \int_1^2 \frac{dt}{t}$$
$$= \frac{1}{2} \left[ \log |t| \right]_1^2$$
$$= \frac{1}{2} \left[ \log 2 - \log 1 \right]$$
$$= \frac{1}{2} \log 2$$

## Answer needs Correction? Click Here

Q2: 
$$\int_0^1 \frac{x}{x^2 + 1} dx$$

# Answer:

$$\int_{0}^{1} \frac{x}{x^{2}+1} dx$$

 $\int_0^1 \frac{x}{x^2 + 1} dx$ Let  $x^2 + 1 = t \implies 2x dx = dt$ 

When x = 0, t = 1 and when x = 1, t = 2

$$\therefore \int_0^1 \frac{x}{x^2 + 1} dx = \frac{1}{2} \int_1^2 \frac{dt}{t}$$
$$= \frac{1}{2} \left[ \log |t| \right]_1^2$$
$$= \frac{1}{2} \left[ \log 2 - \log 1 \right]$$
$$= \frac{1}{2} \log 2$$

# Answer needs Correction? Click Here

Q3: 
$$\int_0^{\frac{\pi}{2}} \sqrt{\sin\phi} \cos^5\phi d\phi$$

## Answer:

Let 
$$I = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi} \cos^5\phi \, d\phi = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi} \cos^4\phi \cos\phi \, d\phi$$

Also, let  $\sin \phi = t \Rightarrow \cos \phi \, d\phi = dt$ 

When 
$$\phi = 0$$
,  $t = 0$  and when  $\phi = \frac{\pi}{2}$ ,  $t = 1$ 

$$\therefore I = \int_{0}^{1} \sqrt{t} \left(1 - t^{2}\right)^{2} dt$$

$$= \int_{0}^{1} t^{\frac{1}{2}} \left(1 + t^{4} - 2t^{2}\right) dt$$

$$= \int_{0}^{1} \left[t^{\frac{1}{2}} + t^{\frac{9}{2}} - 2t^{\frac{5}{2}}\right] dt$$

$$= \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{11}{2}}}{\frac{11}{2}} - \frac{2t^{\frac{7}{2}}}{\frac{7}{2}}\right]_{0}^{1}$$

$$= \frac{2}{3} + \frac{2}{11} - \frac{4}{7}$$

$$= \frac{154 + 42 - 132}{231}$$

$$= \frac{64}{231}$$

# Answer needs Correction? Click Here

Q4: 
$$\int_0^{\infty} \sqrt{\sin\phi} \cos^5\phi d\phi$$

#### Answer

Let 
$$I = \int_{0}^{\frac{\pi}{2}} \sqrt{\sin\phi} \cos^5\phi \, d\phi = \int_{0}^{\frac{\pi}{2}} \sqrt{\sin\phi} \cos^4\phi \cos\phi \, d\phi$$

Also, let  $\sin \phi = t \Rightarrow \cos \phi \, d\phi = dt$ 

When 
$$\phi = 0$$
,  $t = 0$  and when  $\phi = \frac{\pi}{2}$ ,  $t = 1$ 

when 
$$\phi = 0$$
,  $t = 0$  and when  $\phi = \frac{1}{2}$ ,  $t =$ 

## Answer needs Correction? Click Here

Q5: 
$$\int_0^1 \sin^{-1} \left( \frac{2x}{1+x^2} \right) dx$$

#### Answer:

Let 
$$I = \int_0^1 \sin^{-1} \left( \frac{2x}{1+x^2} \right) dx$$

Also, let  $x = \tan \theta \Rightarrow dx = \sec^2 \theta d\theta$ 

When 
$$x = 0$$
,  $\theta = 0$  and when  $x = 1$ ,  $\theta = \frac{\pi}{4}$ 

$$I = \int_0^{\pi} \sin^{-1} \left( \frac{2 \tan \theta}{1 + \tan^2 \theta} \right) \sec^2 \theta \, d\theta$$

$$= \int_0^{\frac{\pi}{4}} \sin^{-1} (\sin 2\theta) \sec^2 \theta \, d\theta$$

$$= \int_0^{\pi} 2\theta \cdot \sec^2 \theta \, d\theta$$

$$=2\int_0^{\pi}\theta\cdot\sec^2\theta\,d\theta$$

Taking heta as first function and  $\sec^2 \theta$  as second function and integrating by parts, we obtain

$$\begin{split} I &= 2 \Bigg[ \theta \int \sec^2 \theta \, d\theta - \int \left\{ \left( \frac{d}{dx} \theta \right) \int \sec^2 \theta \, d\theta \right\} d\theta \Bigg]_0^{\frac{\pi}{4}} \\ &= 2 \Big[ \theta \tan \theta - \int \tan \theta \, d\theta \Bigg]_0^{\frac{\pi}{4}} \\ &= 2 \Big[ \theta \tan \theta + \log \left| \cos \theta \right| \right]_0^{\frac{\pi}{4}} \\ &= 2 \Big[ \frac{\pi}{4} \tan \frac{\pi}{4} + \log \left| \cos \frac{\pi}{4} \right| - \log \left| \cos \theta \right| \Bigg] \\ &= 2 \Big[ \frac{\pi}{4} + \log \left( \frac{1}{\sqrt{2}} \right) - \log 1 \Bigg] \\ &= 2 \Big[ \frac{\pi}{4} - \frac{1}{2} \log 2 \Bigg] \\ &= \frac{\pi}{2} - \log 2 \end{split}$$

# Answer needs Correction? Click Here

Q6: 
$$\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$$

## Answer:

Let 
$$I = \int_0^1 \sin^{-1} \left( \frac{2x}{1+x^2} \right) dx$$

Also, let  $x = \tan \theta \Rightarrow dx = \sec^2 \theta d\theta$ 

When x = 0,  $\theta = 0$  and when x = 1,  $\theta = \frac{\pi}{4}$ 

$$I = \int_0^{\frac{\pi}{4}} \sin^{-1} \left( \frac{2 \tan \theta}{1 + \tan^2 \theta} \right) \sec^2 \theta \, d\theta$$

$$= \int_0^{\pi} \sin^{-1} (\sin 2\theta) \sec^2 \theta \, d\theta$$

$$= \int_{0}^{\frac{\pi}{4}} 2\theta \cdot \sec^2 \theta \, d\theta$$

$$=2\int_0^{\pi}\theta\cdot\sec^2\theta\,d\theta$$

Taking $\theta$ as first function and  $\sec^2\theta$  as second function and integrating by parts, we obtain

$$I = 2 \left[ \theta \int \sec^2 \theta \, d\theta - \int \left\{ \left( \frac{d}{dx} \theta \right) \int \sec^2 \theta \, d\theta \right\} d\theta \right]_0^{\frac{\pi}{4}}$$

$$= 2\left[\theta \tan \theta - \int \tan \theta \, d\theta\right]_0^{\frac{\pi}{4}}$$

$$= 2\left[\theta \tan \theta + \log|\cos \theta|\right]_0^{\frac{\pi}{4}}$$

$$= 2\left[\frac{\pi}{4} \tan \frac{\pi}{4} + \log\left|\cos \frac{\pi}{4}\right| - \log|\cos 0|\right]$$

$$= 2\left[\frac{\pi}{4} + \log\left(\frac{1}{\sqrt{2}}\right) - \log 1\right]$$

$$= 2\left[\frac{\pi}{4} - \frac{1}{2}\log 2\right]$$

$$= \frac{\pi}{2} - \log 2$$

Answer needs Correction? Click Here

Q7: 
$$\int_0^2 x \sqrt{x+2} \, \left( \text{Put } x + 2 = t^2 \right)$$

#### Answer:

$$\int_{0}^{2} x \sqrt{x+2} dx$$

Let 
$$x + 2 = t^2 \Rightarrow dx = 2tdt$$

When x = 0,  $t = \sqrt{2}$  and when x = 2, t = 2

$$\therefore \int_{0}^{2} x \sqrt{x+2} dx = \int_{\sqrt{2}}^{2} (t^{2}-2) \sqrt{t^{2}} 2t dt$$

$$= 2 \int_{\sqrt{2}}^{2} (t^{2}-2)^{2} dt$$

$$= 2 \int_{\sqrt{2}}^{2} (t^{4}-2t^{2}) dt$$

$$= 2 \left[ \frac{t^{5}}{5} - \frac{2t^{3}}{3} \right]_{\sqrt{2}}^{2}$$

$$= 2 \left[ \frac{32}{5} - \frac{16}{3} - \frac{4\sqrt{2}}{5} + \frac{4\sqrt{2}}{3} \right]$$

$$= 2 \left[ \frac{96 - 80 - 12\sqrt{2} + 20\sqrt{2}}{15} \right]$$

$$= 2 \left[ \frac{16 + 8\sqrt{2}}{15} \right]$$

$$= \frac{16(2 + \sqrt{2})}{15}$$

$$= \frac{16\sqrt{2}(\sqrt{2} + 1)}{15}$$

Answer needs Correction? Click Here

Q8: 
$$\int_0^2 x \sqrt{x+2} \, \left( \text{Put } x + 2 = t^2 \right)$$

#### Answer:

$$\int_{0}^{2} x \sqrt{x+2} dx$$

Let 
$$x + 2 = t^2 \Rightarrow dx = 2tdt$$

When x = 0,  $t = \sqrt{2}$  and when x = 2, t = 2

$$\therefore \int_{0}^{2} x \sqrt{x+2} dx = \int_{\sqrt{2}}^{2} (t^{2}-2) \sqrt{t^{2}} 2t dt$$

$$= 2 \int_{\sqrt{2}}^{2} (t^{2}-2)^{2} dt$$

$$= 2 \int_{\sqrt{2}}^{2} (t^{4}-2t^{2}) dt$$

$$= 2 \left[ \frac{t^{5}}{5} - \frac{2t^{3}}{3} \right]_{\sqrt{2}}^{2}$$

$$= 2 \left[ \frac{32}{5} - \frac{16}{3} - \frac{4\sqrt{2}}{5} + \frac{4\sqrt{2}}{3} \right]$$

$$= 2 \left[ \frac{96 - 80 - 12\sqrt{2} + 20\sqrt{2}}{15} \right]$$

$$= 2 \left[ \frac{16 + 8\sqrt{2}}{15} \right]$$

$$= \frac{16(2 + \sqrt{2})}{15}$$

$$= \frac{16\sqrt{2}(\sqrt{2} + 1)}{15}$$

Answer needs Correction? Click Here

$$Q9: \int_0^x \frac{\sin x}{1+\cos^2 x} dx$$

## Answer:

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1 + \cos^2 x} dx$$

Let  $\cos x = t \Rightarrow -\sin x \, dx = dt$ 

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When x = 0, t = 1 and when  $x = \frac{\pi}{2}$ , t = 0

$$\Rightarrow \int_0^{\pi} \frac{\sin x}{1 + \cos^2 x} dx = -\int_0^0 \frac{dt}{1 + t^2}$$

$$= -\left[ \tan^{-1} t \right]_1^0$$

$$= -\left[ \tan^{-1} 0 - \tan^{-1} 1 \right]$$

$$= -\left[ -\frac{\pi}{4} \right]$$

$$= \frac{\pi}{4}$$

# Answer needs Correction? Click Here

Q10: 
$$\int_{0}^{x} \frac{\sin x}{1 + \cos^{2} x} dx$$

## Answer:

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1 + \cos^2 x} dx$$

Let 
$$\cos x = t \Rightarrow -\sin x \, dx = dt$$

When x = 0, t = 1 and when  $x = \frac{\pi}{2}$ , t = 0

$$\Rightarrow \int_0^{\pi} \frac{\sin x}{1 + \cos^2 x} dx = -\int_0^0 \frac{dt}{1 + t^2}$$

$$= -\left[\tan^{-1} t\right]_0^0$$

$$= -\left[\tan^{-1} 0 - \tan^{-1} 1\right]$$

$$= -\left[-\frac{\pi}{4}\right]$$

$$= \frac{\pi}{4}$$

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