



Definite Integrals Ex 20.2 Q53

$$\begin{aligned}
 & \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{1 + \cos x}}{(1 - \cos x)^{\frac{3}{2}}} dx \\
 &= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{2 \cos^2 \frac{x}{2}}}{\left(2 \sin^2 \frac{x}{2}\right)^{\frac{3}{2}}} dx & \left[\begin{array}{l} \because 1 + \cos x = 2 \cos^2 \frac{x}{2} \\ 1 - \cos x = 2 \sin^2 \frac{x}{2} \end{array} \right] \\
 &= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{2} \cos \frac{x}{2}}{2\sqrt{2} \sin^3 \frac{x}{2}} dx \\
 &= \frac{1}{2} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cot \frac{x}{2} \operatorname{cosec}^2 \frac{x}{2} dx & \left[\begin{array}{l} \because \operatorname{cosec}^2 \frac{x}{2} = \frac{1}{\sin^2 \frac{x}{2}} \\ \cot \frac{x}{2} = \frac{\cos \frac{x}{2}}{\sin \frac{x}{2}} \end{array} \right]
 \end{aligned}$$

$$\text{Let } \cot \frac{x}{2} = t$$

Differentiating w.r.t. x , we get

$$\frac{-1}{2} \operatorname{cosec}^2 \frac{x}{2} = dt$$

$$\text{Now, } x = \frac{\pi}{3} \Rightarrow t = \sqrt{3}$$

$$x = \frac{\pi}{2} \Rightarrow t = 1$$

$$\begin{aligned}
 \therefore \frac{1}{2} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cot \frac{x}{2} \operatorname{cosec}^2 \frac{x}{2} dx &= - \int_{\sqrt{3}}^1 t dt = - \left[\frac{t^2}{2} \right]_{\sqrt{3}}^1 = \frac{-1}{2} [1 - 3] \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 \therefore \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\sqrt{1 + \cos x}}{(1 - \cos x)^{\frac{3}{2}}} dx &= 1 \\
 &= 1
 \end{aligned}$$

Definite Integrals Ex 20.2 Q54

Substitute $x^2 = a^2 \cos 2\theta$

Differentiating w.r.t. x , we get

$$2x dx = -2a^2 \sin 2\theta d\theta$$

$$\text{Now, } x = 0 \Rightarrow \theta = \frac{\pi}{4}$$

$$x = a \Rightarrow \theta = 0$$

$$\therefore \int_0^a x \sqrt{\frac{a^2 - x^2}{a^2 + x^2}} dx = \int_{\frac{\pi}{4}}^0 \sqrt{\frac{a^2 (1 - \cos 2\theta)}{a^2 - (1 - \cos 2\theta)}} (-a^2 \sin 2\theta) d\theta$$

$$= -a^2 \int_{\frac{\pi}{4}}^0 \frac{\sin \theta}{\cos \theta} \sin 2\theta d\theta$$

$$= a^2 \int_0^{\frac{\pi}{4}} 2 \sin^2 \theta d\theta$$

$$= a^2 \int_0^{\frac{\pi}{4}} (1 - \cos 2\theta) d\theta$$

$$= a^2 \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{\frac{\pi}{4}}$$

$$= a^2 \left[\frac{\pi}{4} - \frac{1}{2} \right]$$

$$\therefore \int_0^a x \sqrt{\frac{a^2 - x^2}{a^2 + x^2}} dx = a^2 \left[\frac{\pi}{4} - \frac{1}{2} \right]$$

Definite Integrals Ex 20.2 Q55

Let $x = a \cos 2\theta$

Differentiating w.r.t. x , we get

$$dx = -2a \sin 2\theta$$

$$\text{Now, } x = -a \Rightarrow \theta = \frac{\pi}{2}$$

$$x = a \Rightarrow \theta = 0$$

$$\therefore \int_{-a}^a \sqrt{\frac{a-x}{a+x}} dx = \int_{\frac{\pi}{2}}^0 \sqrt{\frac{a(1-\cos 2\theta)}{a(1+\cos 2\theta)}} (-2 \sin 2\theta) d\theta$$

$$= 2a \int_0^{\frac{\pi}{2}} \frac{\sin \theta}{\cos \theta} \cdot \sin 2\theta d\theta$$

$$= 2a \int_0^{\frac{\pi}{2}} \frac{\sin \theta \cdot 2 \sin \theta \cos \theta}{\cos \theta} d\theta$$

$$= 4a \int_0^{\frac{\pi}{2}} \sin^2 \theta d\theta$$

$$= 2a \int_0^{\frac{\pi}{2}} (1 - \cos 2\theta) d\theta$$

$$= 2a \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{\frac{\pi}{2}}$$

$$= 2a \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{\frac{\pi}{2}}$$

$$= 2a \left[\frac{\pi}{2} - 0 - 0 + 0 \right] = \pi a$$

$$\therefore \int_{-a}^a \sqrt{\frac{a-x}{a+x}} dx = \pi a$$

$$\left[\begin{array}{l} \therefore 1 - \cos 2\theta = 2 \sin^2 \theta \\ 1 + \cos 2\theta = 2 \cos^2 \theta \\ -\int_a^b f(x) dx = \int_b^a f(x) dx \end{array} \right]$$

Definite Integrals Ex 20.2 Q56

Let $\cos x = t$

Differentiating w.r.t. x , we get

$$-\sin x dx = dt$$

Now, $x = 0 \Rightarrow t = 1$

$$x = \frac{\pi}{2} \Rightarrow t = 0$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x dx}{\cos^2 x + 3 \cos x + 2}$$

$$= - \int_1^0 \frac{tdt}{t^2 + 3t + 2}$$

$$= \int_0^1 \frac{tdt}{(t+2)(t+1)}$$

$$\left[\because - \int_a^b f(x) = \int_b^a f(x) \right]$$

$$= \int_0^1 \left(-\frac{1}{t+1} + \frac{2}{t+2} \right) dt \quad [\text{Applying partial fraction}]$$

$$= \left[-\log|1+t| + 2\log|t+2| \right]_0^1$$

$$= -\log 2 + 2\log 3 + 0 - 2\log 2$$

$$= 2\log 3 - 3\log 2$$

$$= \log \frac{9}{8}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x dx}{\cos^2 x + 3 \cos x + 2} = \log \frac{9}{8}$$

***** END *****