

Adjoint and Inverse of Matrix Ex 7.2 Q9

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

Now,
$$A = IA$$

$$\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$$

Applying
$$R_1 \rightarrow \frac{1}{3}R_1$$

$$\begin{bmatrix} 1 & -1 & \frac{4}{3} \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$$

Applying $R_2 \rightarrow R_2 - 2R_1$

$$\begin{bmatrix} 1 & -1 & \frac{4}{3} \\ 0 & -1 & \frac{4}{3} \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ \frac{-2}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . A$$

Applying
$$R_2 \rightarrow (-1)R_2$$

$$\begin{bmatrix} 1 & -1 & \frac{4}{3} \end{bmatrix} \begin{bmatrix} \frac{1}{3} & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & \frac{-4}{3} \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . A$$

Applying $R_1 \rightarrow R_1 + R_2$, $R_3 \rightarrow R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{-4}{3} \\ 0 & 0 & -\frac{1}{3} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ \frac{2}{3} & -1 & 0 \\ \frac{2}{3} & -1 & 1 \end{bmatrix} . A$$

Applying $R_3 \rightarrow (-3)R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{-4}{3} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ \frac{2}{3} & -1 & 0 \\ -2 & 3 & -3 \end{bmatrix} . A$$

$$R_2 \to R_2 + \frac{4}{3}R_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} . A$$

$$I = B.A$$

Hence, B is the inv. of A.

Adjoint and Inverse of Matrix Ex 7.2 Q10

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

$$A = I A$$
[1 2 0] [1 0 0]

$$\begin{vmatrix} 2 & 3 & -1 \\ 1 & -1 & 3 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} A$$

Applying
$$R_2 \rightarrow R_2 - 2R_1$$
, $R_3 \rightarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & -1 \\ 0 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}.A$$

Applying $R_2 \rightarrow (-1)R_2$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix} . A$$

Applying $R_1 \rightarrow R_1 - 2R_2$, $R_3 \rightarrow R_3 + 3R_2$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 6 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 0 \\ 2 & -1 & 0 \\ 5 & -3 & 1 \end{bmatrix}.A$$

Applying $R_3 \to \frac{R_3}{6}$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 0 \\ 2 & -1 & 0 \\ \frac{5}{6} & \frac{-1}{2} & \frac{1}{6} \end{bmatrix}.A$$

Applying $R_1 \rightarrow R_1 + 2R_3$, $R_2 \rightarrow R_2 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-4}{3} & 1 & \frac{1}{3} \\ \frac{7}{6} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{5}{6} & \frac{-1}{2} & \frac{1}{6} \end{bmatrix} .A$$

$$\begin{bmatrix} -4 & 1 & \frac{1}{3} \\ \frac{7}{7} & 1 & \frac{1}{1} \end{bmatrix}$$

Hence,
$$A^{-1} = \begin{bmatrix} \frac{7}{6} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{5}{6} & \frac{-1}{2} & \frac{1}{6} \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.2 Q11

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 4 \\ 3 & 1 & 1 \end{bmatrix}$$

Applying
$$R_1 \rightarrow \frac{R_1}{2}$$

$$\begin{bmatrix} -1 & 3 \end{bmatrix}$$

$$\begin{vmatrix} 1 & \frac{-1}{2} & \frac{3}{2} \\ 1 & 2 & 4 \\ 3 & 1 & 1 \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & A \\ 0 & 0 & 1 \end{vmatrix}$$

Applying
$$R_2 \rightarrow R_2 - R_1$$
, $R_3 \rightarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & \frac{-1}{2} & \frac{3}{2} \\ 0 & \frac{5}{2} & \frac{5}{2} \\ 0 & \frac{5}{2} & \frac{-7}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{-1}{2} & 1 & 0 \\ \frac{-3}{2} & 0 & 1 \end{bmatrix} . A$$

Applying
$$R_2 \rightarrow \left(\frac{2}{5}\right) R_2$$

$$\begin{bmatrix} 1 & \frac{-1}{2} & \frac{3}{2} \\ 0 & 1 & 1 \\ 0 & \frac{5}{2} & \frac{-7}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{-1}{5} & \frac{2}{5} & 0 \end{bmatrix} . A$$

Applying
$$R_1 \to R_1 + \frac{1}{2}R_2$$
, $R_3 \to R_3 - \frac{5}{2}R_2$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & -6 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} & 0 \\ \frac{-1}{5} & \frac{2}{5} & 0 \\ -1 & -1 & 1 \end{bmatrix} . A$$

Applying
$$R_3 \rightarrow \frac{R_3}{-6}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} & 0 \\ \frac{-1}{5} & \frac{2}{5} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{bmatrix} . A$$

Applying $R_2 \rightarrow R_2 - R_3$, $R_1 \rightarrow R_1 - 2R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{15} & \frac{-2}{15} & \frac{-1}{3} \\ \frac{-11}{30} & \frac{7}{30} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} \end{bmatrix} . A \qquad \left[\because I = A^{-1}.A \right]$$

Ans.

Adjoint and Inverse of Matrix Ex 7.2 Q12

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

$$A = I A
 \begin{bmatrix}
 1 & 1 & 2 \\
 3 & 1 & 1 \\
 2 & 3 & 1
 \end{bmatrix} = \begin{bmatrix}
 1 & 0 & 0 \\
 0 & 1 & 0 \\
 0 & 0 & 1
 \end{bmatrix} .A$$

Applying
$$R_2 \rightarrow R_2 - 3R_1$$
, $R_3 \rightarrow R_3 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} . A$$

Applying
$$R_2 \to \frac{R_2}{(-2)}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{3}{2} & \frac{-1}{2} & 0 \\ -2 & 0 & 1 \end{bmatrix} . A$$

Applying $R_1 \rightarrow R_1 - R_2$, $R_3 \rightarrow R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{-11}{2} \end{bmatrix} = \begin{bmatrix} \frac{-1}{2} & \frac{1}{2} & 0 \\ \frac{3}{2} & \frac{-1}{2} & 0 \\ \frac{-7}{2} & \frac{1}{2} & \frac{-2}{11} \end{bmatrix} . A$$

Applying $R_3 \to R_3$. $\left(\frac{-2}{11}\right)$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-1}{2} & \frac{1}{2} & 0 \\ \frac{3}{2} & \frac{-1}{2} & 0 \\ \frac{7}{11} & \frac{-1}{11} & \frac{-2}{11} \end{bmatrix} . A$$

Applying $R_1 \to R_1 + \frac{1}{2}R_3$, $R_2 \to R_2 - \frac{5}{2}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-2}{11} & \frac{5}{11} & \frac{-1}{11} \\ \frac{-1}{11} & \frac{-3}{11} & \frac{5}{11} \\ \frac{7}{11} & \frac{-1}{11} & \frac{-2}{11} \end{bmatrix} . A$$

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