



### Mean Value Theorems Ex 15.2 Q1(ix)

Here,

$$f(x) = \sqrt{25 - x^2} \text{ on } [-3, 4]$$

Given function is continuous as it has unique value for each  $x \in [-3, 4]$  and

$$f'(x) = \frac{-2x}{2\sqrt{25 - x^2}}$$

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So,  $f'(x)$  exists for all values for  $x \in (-3, 4)$  so,  $f(x)$  is differentiable in  $(-3, 4)$ . So, Lagrange's mean value theorem is applicable. Thus, there exists a point  $c \in (-3, 4)$  such that

$$\begin{aligned} f'(c) &= \frac{f(4) - f(-3)}{4 - (-3)} \\ \Rightarrow \frac{-2c}{2\sqrt{25 - c^2}} &= \frac{\sqrt{9} - \sqrt{16}}{7} \\ \Rightarrow -7c &= -\sqrt{25 - c^2} \end{aligned}$$

Squaring both the sides,

$$49c^2 = 25 - c^2$$

$$c^2 = \frac{1}{2}$$

$$c = \pm \frac{1}{\sqrt{2}} \in (-3, 4)$$

Hence, Lagrange's mean value theorem is verified.

### Mean Value Theorems Ex 15.2 Q1(x)

Here,

$$f(x) = \tan^{-1} x \text{ on } [0, 1]$$

We know that,  $\tan^{-1} x$  has unique value in  $[0, 1]$  so, it is continuous in  $[0, 1]$

$$f'(x) = \frac{1}{1 + x^2}$$

So,  $f'(x)$  exists for each  $x \in (0, 1)$

So,  $f'(x)$  is differentiable in  $(0, 1)$ , thus Lagrange's mean value theorem is applicable, so there exist a point  $c \in (0, 1)$  such that

$$\begin{aligned} f'(c) &= \frac{f(1) - f(0)}{1 - 0} \\ \Rightarrow \frac{1}{1 + c^2} &= \frac{\tan^{-1}(1) - \tan^{-1}(0)}{1} \\ \Rightarrow \frac{1}{1 + c^2} &= \frac{\frac{\pi}{4} - 0}{1} \\ \Rightarrow \frac{4}{\pi} &= 1 + c^2 \\ \Rightarrow c &= \sqrt{\frac{4}{\pi} - 1} \end{aligned}$$

Hence, Lagrange's mean value theorem is verified.

### Mean Value Theorems Ex 15.2 Q1(xi)

Here,

$$f(x) = x + \frac{1}{x} \text{ on } [1, 3]$$

$f(x)$  attains a unique value for each  $x \in [1, 3]$ , so it is continuous

$$f'(x) = 1 - \frac{1}{x^2} \text{ is defined for each } x \in (1, 3)$$

$\Rightarrow f(x)$  is differentiable in  $(1, 3)$ , so Lagrange's mean value theorem is applicable, so there exist a point  $c \in (1, 3)$  such that

$$\begin{aligned} f'(c) &= \frac{f(3) - f(1)}{3 - 1} \\ \Rightarrow 1 - \frac{1}{c^2} &= \frac{\left(3 + \frac{1}{3} - (1 + 1)\right)}{2} \\ \Rightarrow 1 - \frac{1}{c^2} &= \frac{\frac{10}{3} - 2}{2} \\ \Rightarrow 1 - \frac{1}{c^2} &= \frac{4}{3 \times 2} \\ \Rightarrow 1 - \frac{2}{3} &= \frac{1}{c^2} \\ \Rightarrow c^2 &= 3 \\ \Rightarrow c &= \sqrt{3} \in (1, 3) \end{aligned}$$

So, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(xii)

Here,

$$f(x) = x(x+4)^2 \text{ on } [0, 4]$$

We know that every polynomial function is continuous and differentiable everywhere, so,  $f(x)$  is continuous in  $[0, 4]$  and differentiable in  $(0, 4)$ , so, Lagrange's mean value theorem is applicable, thus there exist a point  $c \in (0, 4)$  such that

$$\begin{aligned} f'(c) &= \frac{f(4) - f(0)}{4 - 0} \\ \Rightarrow 3c^2 + 16c + 16 &= \frac{4 \times (8)^2 - 0}{4} \\ \Rightarrow 3c^2 + 16c + 16 &= 64 \\ \Rightarrow 3c^2 + 16c - 48 &= 0 \\ \Rightarrow c &= \frac{-16 \pm \sqrt{256 + 576}}{6} \\ \Rightarrow &= \frac{-16 \pm \sqrt{832}}{6} \\ \Rightarrow &= \frac{-16 \pm 8\sqrt{13}}{6} \\ \Rightarrow c &= \frac{-8 \pm 4\sqrt{13}}{3} \\ c &= \frac{-8 + 4\sqrt{13}}{3} \in (0, 4) \end{aligned}$$

Hence, Lagrange's mean value theorem is verified.

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