Definite Integrals Ex 20.4A Q4

We know

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

Hence

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin(\frac{\Pi}{2}-x)}}{\sqrt{\sin(\frac{\Pi}{2}-x)}+\sqrt{\cos(\frac{\Pi}{2}-x)}}dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

If

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

Then

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

Hence

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1dx$$

$$2I = \frac{\Pi}{6}$$

$$I = \frac{\Pi}{12}$$

Definite Integrals Ex 20.4A Q5

We know

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

Hence

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x}dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2(-x)}{1+e^{-x}}dx$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x}dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^{-x}}dx$$

If

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x}dx$$

Then

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^{-x}}dx$$

So

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x} + \frac{\tan^2 x}{1+e^{-x}}dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x} + \frac{\tan^2 x}{1+e^{-x}}dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x} + \frac{e^x \tan^2 x}{1+e^x}dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x + e^x \tan^2 x}{1+e^x}dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{(1+e^x)\tan^2 x}{1+e^x}dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x + e^x \tan^2 x}{1+e^x} dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{(1+e^x)\tan^2 x}{1+e^x} dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x dx$$

$$I = \frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x dx$$

We know

If f(x) is even

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$$

If f(x) is odd

$$\int_{-a}^{a} f(x)dx = 0$$

Here

$$f(x) = \tan^2 x$$

f(x) is even, hence

$$I = \int_{0}^{\frac{\pi}{4}} \tan^2 x dx$$

$$I = \int_{0}^{\frac{\pi}{4}} \sec^2 x - 1 dx$$

$$I = \{\tan x - x\}_0^{\frac{\pi}{4}}$$

$$I = 1 - \frac{\pi}{4}$$

Note: Answer given in the book is incorrect.

Definite Integrals Ex 20.4A Q6

We know

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

Hence

$$\int_{-a}^{a} \frac{1}{1+a^x}dx = \int_{-a}^{a} \frac{1}{1+a^{-x}}dx$$

If

$$I = \int_{-a}^{a} \frac{1}{1+a^x}dx$$

Then

$$I = \int_{-a}^{a} \frac{1}{1+a^{-x}}dx$$

So

$$2I = \int_{-a}^{a} \frac{1}{1+a^x} + \frac{1}{1+a^{-x}}dx$$

$$2I = \int_{-a}^{a} \frac{1}{1+a^x} + \frac{a^x}{1+a^x}dx$$

$$2I = \int_{-a}^{a} 1dx$$

$$2I = 2a$$

$$I = a$$

*********** END **********