Exercise 4.6

Question 1:

Examine the consistency of the system of equations.

$x + 2y = 2$

$2x + 3y = 3$

Answer

The given system of equations is:

$x + 2y = 2$

$2x + 3y = 3$

The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$$

Now,

$$|A| = 1(3) - 2(2) = 3 - 4 = -1 \neq 0$$

$\therefore$ A is non-singular.

Therefore, A^{-1} exists.

Hence, the given system of equations is consistent.

Question 2:

Examine the consistency of the system of equations.

$2x - y = 5$

$x + y = 4$

Answer

The given system of equations is:

$2x - y = 5$

$x + y = 4$

The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}.$$

Now,

$$|A| = 2(1) - (-1)(1) = 2 + 1 = 3 \neq 0$$

$\therefore$ A is non-singular.

Therefore, A^{-1} exists.

Hence, the given system of equations is consistent.

Question 3:

Examine the consistency of the system of equations.

$x + 3y = 5$

$2x + 6y = 8$

Answer

The given system of equations is:

$x + 3y = 5$

$2x + 6y = 8$

The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}.$$

Now,

$$|A| = 1(6) - 3(2) = 6 - 6 = 0$$

$\therefore$ A is a singular matrix.

Now,

$$(adjA)=\begin{bmatrix}6 & -3\\ -2 & 1\end{bmatrix}$$

$$(adjA)B=\begin{bmatrix}6 & -3\\ -2 & 1\end{bmatrix}\begin{bmatrix}5\\ 8\end{bmatrix}=\begin{bmatrix}30-24\\ -10+8\end{bmatrix}=\begin{bmatrix}6\\ -2\end{bmatrix}\neq O$$

Thus, the solution of the given system of equations does not exist. Hence, the system of equations is inconsistent.

Question 4:

Examine the consistency of the system of equations.

$x + y + z = 1$

$2x + 3y + 2z = 2$

$ax + ay + 2az = 4$

Answer

The given system of equations is:

$x + y + z = 1$

$2x + 3y + 2z = 2$

$ax + ay + 2az = 4$

This system of equations can be written in the form $AX = B$, where

$$A=\begin{bmatrix}1 & 1 & 1\\ 2 & 3 & 2\\ a & a & 2a\end{bmatrix},\ X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}\text{ and }B=\begin{bmatrix}1\\ 2\\ 4\end{bmatrix}.$$

Now,

$$|A|=1(6a-2a)-1(4a-2a)+1(2a-3a)$$
$$=4a-2a-a=4a-3a=a\neq 0$$

$\therefore$ A is non-singular.

Therefore, A^{-1} exists.

Hence, the given system of equations is consistent.

Question 5:

Examine the consistency of the system of equations.

$3x - y - 2z = 2$

$2y - z = -1$

$3x - 5y = 3$

Answer

The given system of equations is:

$3x - y - 2z = 2$

$2y - z = -1$

$3x - 5y = 3$

This system of equations can be written in the form of $AX = B$, where

$$A=\begin{bmatrix}3 & -1 & -2\\ 0 & 2 & -1\\ 3 & -5 & 0\end{bmatrix},\ X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}\text{ and }B=\begin{bmatrix}2\\ -1\\ 3\end{bmatrix}.$$

Now,

$$|A|=3(0-5)-0+3(1+4)=-15+15=0$$

$\therefore$ A is a singular matrix.

Now,

$$(adjA)=\begin{bmatrix}-5 & 10 & 5\\ -3 & 6 & 3\\ -6 & 12 & 6\end{bmatrix}$$

$$\therefore(adjA)B=\begin{bmatrix}-5 & 10 & 5\\ -3 & 6 & 3\\ -6 & 12 & 6\end{bmatrix}\begin{bmatrix}2\\ -1\\ 3\end{bmatrix}=\begin{bmatrix}-10-10+15\\ -6-6+9\\ -12-12+18\end{bmatrix}=\begin{bmatrix}-5\\ -3\\ -6\end{bmatrix}\neq O$$

Thus, the solution of the given system of equations does not exist. Hence, the system of equations is inconsistent.

Question 6:

Examine the consistency of the system of equations.

$5x - y + 4z = 5$

$2x + 3y + 5z = 2$

$5x - 2y + 6z = -1$

Answer

The given system of equations is:

$5x - y + 4z = 5$

$2x + 3y + 5z = 2$

$5x - 2y + 6z = -1$

This system of equations can be written in the form of $AX = B$, where

$A=\begin{bmatrix}5 & -1 & 4\\2 & 3 & 5\\5 & -2 & 6\end{bmatrix}$, $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B=\begin{bmatrix}5\\2\\-1\end{bmatrix}$.

Now,

$$\begin{aligned}|A| &= 5(18+10)+1(12-25)+4(-4-15)\\&=5(28)+1(-13)+4(-19)\\&=140-13-76\\&=51\neq 0\end{aligned}$$

$\therefore$ A is non-singular.

Therefore, A^{-1} exists.

Hence, the given system of equations is consistent.

Question 7:

Solve system of linear equations, using matrix method.

$5x+2y=4$

$7x+3y=5$

Answer

The given system of equations can be written in the form of $AX = B$, where

$A=\begin{bmatrix}5 & 2\\7 & 3\end{bmatrix}$, $X=\begin{bmatrix}x\\y\end{bmatrix}$ and $B=\begin{bmatrix}4\\5\end{bmatrix}$.

Now, $|A|=15-14=1\neq 0$.

Thus, A is non-singular. Therefore, its inverse exists.

Now,

$$A^{-1}=\frac{1}{|A|}(adjA)$$

$$\therefore A^{-1}=\begin{bmatrix}3 & -2\\-7 & 5\end{bmatrix}$$

$$\therefore X=A^{-1}B=\begin{bmatrix}3 & -2\\-7 & 5\end{bmatrix}\begin{bmatrix}4\\5\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}12-10\\-28+25\end{bmatrix}=\begin{bmatrix}2\\-3\end{bmatrix}$$

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