Binary Operations Ex 3.1 Q1(i)

We have,

$$a * b = a^b \text{ for all } a, b \in N$$

Let $a \in N$ and $b \in N$

$$\Rightarrow \quad a^b \in N$$
$$\Rightarrow \quad a * b \in N$$

The operation $*$ defines a binary operation on $N$

Binary Operations Ex 3.1 Q1(ii)

We have,

$$a \circ b = a^b \text{ for all } a, b \in Z$$

Let $a \in Z$ and $b \in Z$

$$\Rightarrow \quad a^b \notin Z \qquad \Rightarrow a \circ b \notin Z$$

For example, if $\quad a = 2, \quad b = -2$

$$\Rightarrow \quad a^b = 2^{-2} = \frac{1}{4} \notin Z$$

$\therefore$ The operation '$\circ$' does not define a binary operation on $Z$.

Binary Operations Ex 3.1 Q1(iii)

We have,

$$a * b = a + b - 2 \text{ for all } a, b \in N$$

Let $a \in N$ and $b \in N$

Then, $a + b - 2 \notin N$ for all $a, b \in N$

$$\Rightarrow \quad a * b \notin N$$

For example $\quad a = 1, \; b = 1$

$$\Rightarrow \quad a + b - 2 = O \notin N$$

$\therefore$ The operation $*$ does not define a binary operation on $N$

Binary Operations Ex 3.1 Q1(iv)

We have,

$$S = \{1, 2, 3, 4, 5\}$$

and, $a \times_6 b$ = Remainder when $ab$ is divided by 6

Let $a \in S$ and $b \in S$

$\Rightarrow$ $a \times_6 b \notin S$ for all $a, b \in S$

For example, $a = 2,\ b = 3$

$\Rightarrow$ $2 \times_6 3$ = Remainder when 6 is divided by 6 = $0 \notin S$

$\therefore$ $\times_6$ does not define a binary oparation on $S$

Binary Operations Ex 3.1 Q1(v)

We have,

$$S = \{0, 1, 2, 3, 4, 5\}$$

and, $a +_6 b = \begin{cases} a + b; & \text{if } a + b < 6 \\ a + b - 6; & \text{if } a + b \geq 6 \end{cases}$

Let $a \in S$ and $b \in S$ such that $a + b < 6$

Then $a +_6 b = a + b \in S$ $\quad [\because\ a + b < 6 = 0, 1, 2, 3, 4, 5]$

Let $a \in S$ and $b \in S$ such that $a + b > 6$

Then $a +_6 b = a + b - 6 \in S$ $[\because \text{if } a + b \geq 6 \text{ then } a + b - 6 \geq 0 = 0, 1, 2, 3, 4, 5]$

$\therefore$ $a +_6 b \in S$ for $a, b \in S$

$\therefore$ $+_6$ defines a binary oparation on $S$

Binary Operations Ex 3.1 Q1(vi)

We have,

$$a \circ b = a^b + b^a \text{ for all } a, b \in N$$

Let $a \in N$ and $b \in N$

$\Rightarrow$ $a^b \in N$ and $b^a \in N$

$\Rightarrow$ $a^b + b^a \in N$

$\Rightarrow$ $a \circ b \in N$

Thus, the operation '$\circ$' defines a binary relation on $N$

Binary Operations Ex 3.1 Q1(vii)

We have,

$$a * b = \frac{a - 1}{b + 1} \text{ for all } a, b \in Q$$

Let $a \in Q$ and $b \in Q$

Then $\frac{a - 1}{b + 1} \notin Q$ for $b = -1$

$\Rightarrow$ $a * b \notin Q$ for all $a, b \in Q$

Thus, the operation $*$ does not define a binary operation on $Q$

********** END **********