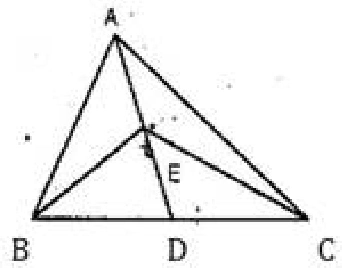
NCERT solutions for class-9 maths Areas of Parallelograms and Triangles Ex-9.3

Q1. In figure, E is any point on median AD of a Δ ABC. Show that ar (Δ ABE) = ar (Δ ACE).



Ans. In ΔABC, AD is a median.

ar (ΔABD) = ar (ΔACD) ....(i)

[$\because$ Median divides a Δ into two Δs of equal area]

Again in ΔEBC, ED is a median

ar (ΔEBD) = ar (ΔECD) ....(ii)

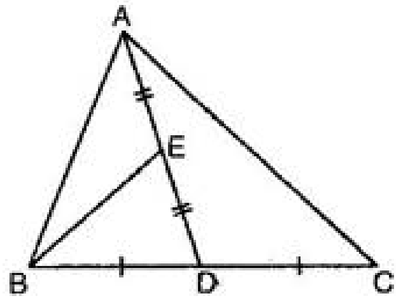
Subtracting eq. (ii) from (i),

ar (ΔABD) – ar (ΔEBD) = ar (ΔACD) – ar (ΔECD)

$\Rightarrow$ ar (ΔABE) = ar (ΔACE)

Q2. In a triangle ABC, E is the mid-point of median AD. Show that ar (BED) = $\frac{1}{4}$ ar (ABC).

Ans. Given: A ΔABC, AD is the median and E is the mid-point of median AD.



To prove: ar (ΔBED) = $\frac{1}{4}$ ar (ΔABC)

Proof: In ΔABC, AD is the median.

$\therefore$ ar (ΔABD) = ar (ΔADC)

[$\because$ Median divides a Δ into two Δs of equal area]

$\Rightarrow$ ar (ΔABD) = $\frac{1}{2}$ ar (ABC) ....(i)

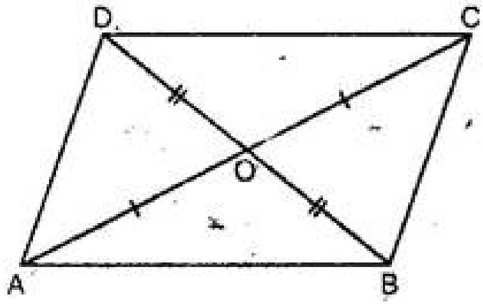
In ΔABD, BE is the median.

$\therefore$ ar (ΔBED) = ar (ΔBAE)

$\Rightarrow$ ar (ΔBED) = $\frac{1}{2}$ ar (ΔABD)

$\Rightarrow$ ar (ΔBED) = $\frac{1}{2} \times \frac{1}{2}$ ar (ΔABC) = $\frac{1}{4}$ ar (Δ ABC)

Q3. Show that the diagonals of a parallelogram divide it into four triangles of equal area.



Ans. Let parallelogram be ABCD and its diagonals AC and BD intersect each other at O.

In ΔABC and ΔADC,

AB = DC [Opposite sides of a parallelogram]

BC = AD [Opposite sides of a parallelogram]

And AC = AC [Common]

$\therefore$ ΔABC $\cong$ ΔCDA [By SSS congruency]

Since, diagonals of a parallelogram bisect each other.

$\therefore$ O is the mid-point of bisection.

Now in ΔADC, DO is the median.

$\therefore$ ar (ΔAOD) = ar (ΔCOD) ....(i)

[Median divides a triangle into two equal areas]

Similarly, in ΔABC, OB is the median.

$\therefore$ ar (ΔAOB) = ar (ΔBOC) ....(ii)

And in ΔAOB and ΔAOD, AO is the median.

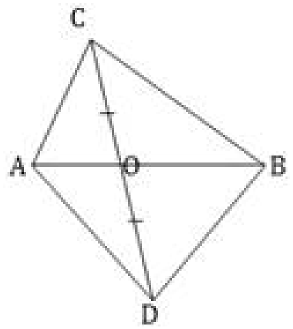
$\therefore$ ar (ΔAOB) = ar (ΔAOD) ....(iii)

From eq. (i), (ii) and (iii),

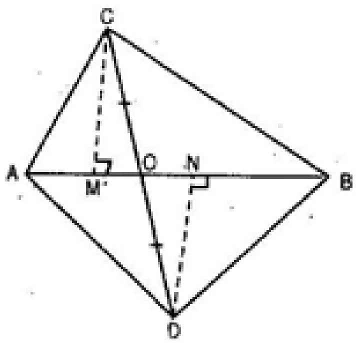
ar (ΔAOB) = ar (ΔAOD) = ar (ΔBOC) = ar (ΔCOD)

Thus diagonals of parallelogram divide it into four triangles of equal area.

Q4. In figure, ABC and ABD are two triangles on the same base AB. If line-segment CD is bisected by AB at O, show that ar (ABC) = ar (ABD).



Ans. Draw CM $\perp$ AB and DN $\perp$ AB.



In ΔCMO and ΔDNO,

$\angle$ CMO = $\angle$ DNO = 90° [By construction]

$\angle$ COM = $\angle$ DON [Vertically opposite]

OC = OD [Given]

$\therefore$ ΔCMO $\cong$ ΔDNO [By ASA congruency]

$\because$ AM = DN [By CPCT] ......(i)

Now ar (ΔABC) = $\frac{1}{2} \times AB \times CM$ ....(ii)

ar (ΔADB) = $\frac{1}{2} \times AB \times DN$ ....(iii)

Using eq. (i) and (iii),

ar (ΔADB) = $\frac{1}{2} \times AB \times CM$ ....(iv)

From eq. (ii) and (iv),

ar (ΔABC) = ar (ΔADB)

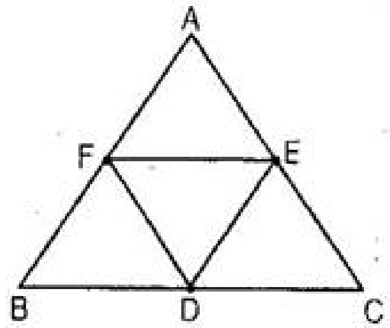
Q5. D, E and F are respectively the mid-points of the sides BC, CA and AB of a ΔABC. Show that:

(i) BDEF is a parallelogram.

(ii) ar (DEF) = $\frac{1}{4}$ ar (ABC)

(iii) ar (BDEF) = $\frac{1}{2}$ ar (ABC)

Ans. (i) F is the mid-point of AB and E is the mid-point of AC.



$\therefore$ FE $\parallel$ BC and FE = $\frac{1}{2}$ BD

[$\because$ Line joining the mid-points of two sides of a triangle is parallel to the third and half of it]

$\Rightarrow$ FE $\parallel$ BD [BD is the part of BC]

And FE = BD

Also, D is the mid-point of BC.

$\therefore$ BD = $\frac{1}{2}$ BC

And FE $\parallel$ BC and FE = BD

Again E is the mid-point of AC and D is the mid-point of BC.

$\therefore$ DE $\parallel$ AB and DE = $\frac{1}{2}$ AB

$\Rightarrow$ DE $\parallel$ AB [BF is the part of AB]

And DE = BF

Again F is the mid-point of AB.

$\therefore$ BF = $\frac{1}{2}$ AB

But DE = $\frac{1}{2}$ AB

$\therefore$ DE = BF

Now we have FE $\parallel$ BD and DE $\parallel$ BF

And FE = BD and DE = BF

Therefore, BDEF is a parallelogram.

(ii) BDEF is a parallelogram.

[diagonals of parallelogram divides it in two triangles of equal area]

DCEF is also parallelogram.

$\therefore$ ar (ΔDEF) = ar (ΔDEC) ....(ii)

Also, AEDF is also parallelogram.

$\therefore$ ar (ΔAFE) = ar (ΔDEF) ....(iii)

From eq. (i), (ii) and (iii),

ar (ΔDEF) = ar (ΔBDF) = ar (ΔDEC) = ar (Δ AFE) ....(iv)

Now, ar (ΔABC) = ar (ΔDEF) + ar (ΔBDF) + ar (ΔDEC) + ar (ΔAFE) ....(v)

$\Rightarrow$ ar (ΔABC) = ar (ΔDEF) + ar (ΔDEF) + ar (Δ DEF) + ar (ΔDEF)

[Using (iv) & (v)]

$\Rightarrow$ ar (ΔABC) = $4 \times$ ar (ΔDEF)

$\Rightarrow$ ar (ΔDEF) = $\frac{1}{4}$ ar (ΔABC)

(iii) ar ($\parallel$gm BDEF) = ar (ΔBDF) + ar (ΔDEF) = ar (ΔDEF) + ar (ΔDEF) [Using (iv)]

$\Rightarrow$ ar ($\parallel$gm BDEF) = 2 ar (ΔDEF)

$\Rightarrow$ ar ($\parallel$gm BDEF) = $2 \times \frac{1}{4}$ ar (ΔABC)

$\Rightarrow$ ar ($\parallel$gm BDEF) = $\frac{1}{2}$ ar (ΔABC)

*********** END **********