Arithmetic Progressions Ex 9.5 Q32

Answer :

(i) Here, we have an A.P. whose n^{th} term (a_n), first term (a) and common difference (d) are given. We need to find the number of terms (n) and the sum of first n terms (S_n).

Here,

First term (a) = 5

Last term (a_n) = 50

Common difference (d) = 3

So here we will find the value of n using the formula, $a_n = a+(n-1)d$

So, substituting the values in the above mentioned formula

$$50 = 5+(n-1)3$$

$$50 = 5+3n-3$$

$$50 = 2+3n$$

$$3n = 50-2$$

Further simplifying for n,

$$3n = 48$$

$$n = \frac{48}{3}$$

$$n = 16$$

Now, here we can find the sum of the n terms of the given A.P., using the formula,

$$S_n = \left(\frac{n}{2}\right)(a+l)$$

Where, a = the first term

l = the last term

So, for the given A.P, on substituting the values in the formula for the sum of n terms of an A.P., we get,

$$S_{16} = \left(\frac{16}{2}\right)[5+50]$$

$$= 8(55)$$

$$= 440$$

Therefore, for the given A.P $\boxed{n = 16 \text{ and } S_{16} = 440}$

(ii) Here, we have an A.P. whose n^{th} term (a_n), sum of first n terms (S_n) and common difference (d) are given. We need to find the number of terms (n) and the first term (a).

Here,

Last term (a_n) = 4

Common difference (d) = 2

Sum of n terms (S_n) = −14

So here we will find the value of n using the formula, $a_n = a+(n-1)d$

So, substituting the values in the above mentioned formula

$$4 = a+(n-1)2$$

$$4 = a+2n-2$$

$$4+2 = a+2n$$

$$n = \frac{6-a}{2} \qquad \ldots(1)$$

Now, here the sum of the n terms is given by the formula,

$$S_n = \left(\frac{n}{2}\right)(a+l)$$

Where, a = the first term

l = the last term

So, for the given A.P, on substituting the values in the formula for the sum of n terms of an A.P., we get,

$$-14 = \left(\frac{n}{2}\right)[a+4]$$

$$-14(2) = n(a+4)$$

$$n = \frac{-28}{a+4} \qquad \ldots(2)$$

Equating (1) and (2), we get,

$$\frac{6-a}{2} = \frac{-28}{a+4}$$

$$(6-a)(a+4) = -28(2)$$

$$6a - a^2 + 24 - 4a = -56$$

$$-a^2 + 2a + 24 + 56 = 0$$

So, we get the following quadratic equation,

$$-a^2 + 2a + 80 = 0$$

$$a^2 - 2a - 80 = 0$$

Further, solving it for a by splitting the middle term,

$$a^2 - 2a - 80 = 0$$

$$a^2 - 10a + 8a - 80 = 0$$

$$a(a-10) + 8(a-10) = 0$$

$$(a-10)(a+8) = 0$$

So, we get,

$$a - 10 = 0$$

$$a = 10$$

Or

$$a + 8 = 0$$

$$a = -8$$

Substituting, $a = 10$ in (1),

$$n = \frac{6-10}{2}$$

$$n = \frac{-4}{2}$$

$$n = -2$$

Here, we get n as negative, which is not possible. So, we take $a = -8$,

$$n = \frac{6-(-8)}{2}$$

$$n = \frac{6+8}{2}$$

********** END **********