



Indefinite Integrals Ex 19.25 Q40

$$\text{Let } I = \int \frac{x^2 \tan^{-1} x}{1+x^2} dx$$

$$\text{Let } \tan^{-1} x = t$$

$$x = \tan t$$

$$\frac{1}{1+x^2} dx = dt$$

$$I = \int t \tan^2 t dt$$

$$= \int t (\sec^2 t - 1) dt$$

$$= \int (t \sec^2 t - t) dt$$

$$= \int t \sec^2 t dt - \int t dt$$

$$= \left[ t \int \sec^2 t dt - \int (1) \sec^2 t dt \right] dt - \frac{t^2}{2}$$

$$= \left[ t \tan t - \int \tan t dt \right] - \frac{t^2}{2}$$

$$= t \tan t - \log \sec t - \frac{t^2}{2} + c$$

$$= x \tan^{-1} x - \log \sqrt{1+x^2} - \frac{\tan^2 x}{2} + c$$

$$I = x \tan^{-1} x - \frac{1}{2} \log |1+x^2| - \frac{\tan^2 x}{2} + c$$

Indefinite Integrals Ex 19.25 Q41

$$\text{Let } I = \int \cos^{-1} (4x^3 - 3x) dx$$

$$\text{Let } x = \cos \theta$$

$$dx = -\sin \theta d\theta$$

$$I = -\int \cos^{-1} (4 \cos^3 \theta - 3 \cos \theta) \sin \theta d\theta$$

$$= -\int \cos^{-1} (\cos 3\theta) \sin \theta d\theta$$

$$= -\int 3\theta \sin \theta d\theta$$

$$= -3 \left[ \theta \int \sin \theta d\theta - \int (1) \sin \theta d\theta \right] d\theta$$

$$= -3 \left[ -\theta \cos \theta + \int \cos \theta d\theta \right]$$

$$= 3\theta \cos \theta - 3 \sin \theta + c$$

$$I = 3x \cos^{-1} x - 3\sqrt{1-x^2} + c$$

Indefinite Integrals Ex 19.25 Q42

$$\text{Let } I = \int \cos^{-1} \left( \frac{1-x^2}{1+x^2} \right) dx$$

$$\text{Let } x = \tan t$$

$$dx = \sec^2 t dt$$

$$I = \int \cos^{-1} \left( \frac{1 - \tan^2 t}{1 + \tan^2 t} \right) \sec^2 t dt$$

$$= \int \cos^{-1} (\cos 2t) \sec^2 t dt$$

$$= \int 2t \sec^2 t dt$$

$$= 2 \left[ t \int \sec^2 t dt - \int (1) \sec^2 t dt \right] dt$$

$$= 2 \left[ t \tan^2 t - \int \tan t dt \right]$$

$$= 2 \left[ t \tan^2 t - \log \sec t \right] + c$$

$$= 2 \left[ x \tan^{-1} x - \log \sqrt{1+x^2} \right] + c$$

$$I = 2x \tan^{-1} x - \log |1+x^2| + c$$

Indefinite Integrals Ex 19.25 Q43

$$\text{Let } I = \int \tan^{-1} \left( \frac{2x}{1-x^2} \right) dx$$

$$\text{Let } x = \tan \theta$$

$$dx = \sec^2 \theta d\theta$$

$$I = \int \tan^{-1} \left( \frac{2 \tan \theta}{1 - \tan^2 \theta} \right) \sec^2 \theta d\theta$$

$$= \int \tan^{-1} (\tan 2\theta) \sec^2 \theta d\theta$$

$$= \int 2\theta \sec^2 \theta d\theta$$

$$= 2 \left[ \theta \int \sec^2 \theta d\theta - \int (1) \sec^2 \theta d\theta \right] d\theta$$

$$= 2 \left[ \theta \tan \theta - \int \tan \theta d\theta \right]$$

$$= 2 \left[ \theta \tan \theta - \log \sec \theta \right] + c$$

$$= 2 \left[ x \tan^{-1} x - \log \sqrt{1+x^2} \right] + c$$

$$I = 2x \tan^{-1} x - \log |1+x^2| + c$$

Indefinite Integrals Ex 19.25 Q44

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$$\begin{aligned}
 \text{Let } I &= \int (x+1) \log x \, dx \\
 &= \log x \int (x+1) \, dx - \int \left( \frac{1}{x} \int (x+1) \, dx \right) dx \\
 &= \left( \frac{x^2}{2} + x \right) \log x - \int \frac{1}{x} \left( \frac{x^2}{2} + x \right) dx \\
 &= \left( \frac{x^2}{2} + x \right) \log x - \frac{1}{2} \int x \, dx - \int dx \\
 &= \left( x + \frac{x^2}{2} \right) \log x - \frac{1}{2} \times \frac{x^2}{2} - x + c \\
 \\ 
 I &= \left( x + \frac{x^2}{2} \right) \log x - \left( \frac{x^2}{4} + x \right) + c
 \end{aligned}$$

Indefinite Integrals Ex 19.25 Q45

$$\begin{aligned}
 \text{Let } I &= \int x^2 \tan^{-1} x \, dx \\
 &= \tan^{-1} x \int x^2 \, dx - \int \left( \frac{1}{1+x^2} \int x^2 \, dx \right) \\
 &= \tan^{-1} x \left( \frac{x^3}{3} \right) - \frac{1}{3} \int \frac{x^3}{1+x^2} \, dx \\
 &= \frac{1}{3} x^3 \tan^{-1} x - \frac{1}{3} \int \left( x - \frac{x}{1+x^2} \right) dx \\
 &= \frac{1}{3} x^3 \tan^{-1} x - \frac{1}{3} \times \frac{x^2}{2} + \frac{1}{3} \int \frac{x}{1+x^2} \, dx \\
 \\ 
 I &= \frac{1}{3} x^3 \tan^{-1} x - \frac{1}{6} x^2 + \frac{1}{6} \log |1+x^2| + c
 \end{aligned}$$

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