

Sine and Cosine Formulae and their Applications Ex-10.2 Q15

Let
$$\frac{b+c}{12} = \frac{c+a}{13} = \frac{a+b}{15} = \lambda \text{ (say)}$$

 $b+c = 12\lambda, c+a = 13\lambda, a+b = 15\lambda$
 $(b+c+c+a+a+b) = 12\lambda + 13\lambda + 15\lambda$
 $2(a+b+c) = 40\lambda$
 $a+b+c = 20\lambda$
 $b+c = 12\lambda \text{ and } a+b+c = 20\lambda \Rightarrow a = 8\lambda$
 $c+a = 13\lambda \text{ and } a+b+c = 20\lambda \Rightarrow b = 7\lambda$
 $a+b = 15\lambda \text{ and } a+b+c = 20\lambda \Rightarrow c = 5\lambda$
 $\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{49\lambda^2+25\lambda^2-64\lambda^2}{70\lambda^2} = \frac{1}{7}$
 $\cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{64\lambda^2+25\lambda^2-49\lambda^2}{80\lambda^2} = \frac{1}{2}$
 $\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{64\lambda^2+49\lambda^2-25\lambda^2}{112\lambda^2} = \frac{11}{14}$
 $\cos A : \cos B : \cos C = \frac{1}{7} : \frac{1}{2} : \frac{11}{14} = 2 : 7 : 11$

Sine and Cosine Formulae and their Applications Ex-10.2 Q16

We have,
$$\angle B = 60^\circ$$

$$\cos B = \frac{1}{2} \Rightarrow \frac{a^2 + c^2 - b^2}{2ac} = \frac{1}{2}$$

$$\Rightarrow a^2 + c^2 - b^2 = ac$$

$$\Rightarrow a^2 + c^2 - ac = b^2 \qquad \dots \dots (i)$$

$$(a+b+c)(a-b+c) = 3ca$$

$$a^2 - ab + ac + ab - b^2 + bc + ac - bc + c^2 = 3ac$$

$$a^2 + c^2 - b^2 + 2ac - 3ac = 0$$

$$a^2 + c^2 - ac = b^2$$
which is given.

Sine and Cosine Formulae and their Applications Ex-10.2 Q17 Consider the given equation:

$$\cos^2 A + \cos^2 B + \cos^2 C = 1$$

 $\Rightarrow 1 - \sin^2 A + 1 - \sin^2 B + 1 - \sin^2 C = 1$
 $\Rightarrow 3 - \sin^2 A + 1 - \sin^2 B + 1 - \sin^2 C = 1$

Sine and Cosine Formulae and their Applications Ex-10.2 Q18

Let
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$$
. Then, $\sin A = ka$, $\sin B = kb$, $\sin C = kc$

$$Now, \cos C = \frac{\sin A}{2\sin B}$$

 $2\sin B\cos C = \sin A$

$$2\left(\frac{a^2+b^2-c^2}{2ab}\right)kb = ka$$

$$a^2 + b^2 - c^2 = a^2$$

$$b^2 = c^2$$

$$b = c$$

△ABCisisosceles.

Sine and Cosine Formulae and their Applications Ex-10.2 Q19

Let P and Q be the position of two ships at the end of 3 hours.

Then.

 $OP = 3 \times 24 = 72 km \text{ and } OQ = 3 \times 32 = 96 km$

Using cosine formula in ΔOPQ , we get

$$PQ^2 = OP^2 + OQ^2 - 2OP \times OQ\cos 90^\circ$$

$$PQ^2 = 72^2 + 96^2 - 2 \times 72 \times 96\cos 90^\circ$$

 $PQ^2 = 14400$

$$PQ = 120 \, km$$

