Maxima and Minima 18.3 Q1(v)

We have,

$$f(x) = xe^x$$

$$\therefore \quad f'(x) = e^x + xe^x = e^x(x+1)$$

$$f''(x) = e^x(x+1) + e^x$$

$$= e^x(x+2)$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad e^x(x+1) = 0$$

$$\Rightarrow \quad x = -1$$

Now,

$$f''(-1) = e^{-1} = \frac{1}{e} > 0$$

$\therefore$ $x = -1$ is point of local minima

Hence,

$$\text{local min value} = f(-1) = \frac{-1}{e}.$$

Maxima and Minima 18.3 Q1(vi)

We have,

$$f(x) = \frac{x}{2} + \frac{2}{x}, \; x > 0$$

$$\therefore \qquad f'(x) = \frac{1}{2} - \frac{2}{x^2}$$

and, $$f''(x) = \frac{4}{x^3}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \qquad \frac{1}{2} - \frac{2}{x^2} = 0$$

$$\Rightarrow \qquad \frac{x^2 - 4}{2x^2} = 0$$

$$\Rightarrow \qquad x = 2, -2$$

Now,

$$f''(2) = \frac{1}{2} > 0$$

$\therefore$ $x = 2$ is point of minima

We will not consider $x = -2$ as $x > 0$

$\therefore$ local min value $= f(2) = 2$.

Maxima and Minima 18.3 Q1(vii)

We have,

$$f(x) = (x+1)(x+2)^{\frac{1}{3}},\ x \geq -2$$

$$\therefore \quad f'(x) = (x+2)^{\frac{1}{3}} + \frac{1}{3}(x+1)(x+2)^{\frac{-2}{3}}$$

$$= (x+2)^{\frac{-2}{3}}\left(x+2+\frac{1}{3}(x+1)\right)$$

$$= \frac{1}{3}(x+2)^{\frac{-2}{3}}(4x+7)$$

and, $f''(x) = -\frac{2}{9}(x+2)^{\frac{-5}{3}}(4x+7) + \frac{1}{3}(x+2)^{\frac{-2}{3}} \times 4$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad \frac{1}{3}(x+2)^{\frac{-2}{3}}(4x+7) = 0$$

$$\Rightarrow \quad x = -\frac{7}{4}$$

Now,

$$f''\left(-\frac{7}{4}\right) = \frac{4}{3}\left(-\frac{7}{4}+2\right)^{\frac{-2}{3}}$$

$$\therefore \quad x = \frac{-7}{4} \text{ is point of minima}$$

$$\therefore \quad \text{local min value} = f\left(\frac{-7}{4}\right) = \frac{-3}{4^{\frac{4}{3}}}.$$

Maxima and Minima 18.3 Q1(viii)

We have,

$$f(x) = x\sqrt{32 - x^2}, -5 \le x \le 5$$

$$\therefore \quad f'(x) = \sqrt{32 - x^2} + \frac{x}{2\sqrt{32 - x^2}} \times (-2x)$$

$$= \frac{2\left(32 - x^2\right) - 2x^2}{2\sqrt{32 - x^2}}$$

$$= \frac{64 - 4x^2}{2\sqrt{32 - x^2}}$$

and, $$f''(x) = \frac{2\sqrt{32 - x^2} \times (-8x) \frac{-2\left(64 - 4x^2\right)}{2\sqrt{32 - x^2}} \times (-2x)}{4\left(32 - x^2\right)}$$

$$= \frac{-4\left(32 - x^2\right) \times 8x + 4x\left(64 - x^2\right)}{8\left(32 - x^2\right)^{\frac{3}{2}}}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad \frac{4\left(16 - x^2\right)}{2\sqrt{32 - x^2}} = 0$$

$$\Rightarrow \quad x = \pm 4$$

Now,

$$f''(4) = \frac{4 \times 4(64 - 16 - 8 \times 32 + 8 \times 16)}{8(32 - 16)^{\frac{3}{2}}} < 0$$

$\therefore \quad x = 4$ is point of maxima

*********** END **********