Functions Ex 3.4 Q5

We have,

$$f(x) = \log_e(1-x)$$

and $g(x) = [x]$

$f(x) = \log_e(1-x)$ is defined, if $1 - x > 0$

$\Rightarrow \quad 1 > x$

$\Rightarrow \quad x < 1$

$\Rightarrow \quad x \in (-\infty, 1)$

$\therefore$ Domain$(f) = (-\infty, 1)$

$g(x) = [x]$ is defined for all $x \in R$

$\therefore$ Domain$(g) = R$

$\therefore$ Domain$(f) \cap R$ Domain$(g) = (-\infty, 1) \cap R$

$= (-\infty, 1)$

(i) $f + g : (-\infty, 1) \to R$ defined by $(f+g)(x) = f(x) + g(x)$

$= \log_e(1-x) + [x]$

(ii) $fg : (-\infty, 1) \to R$ defined by $(fg)(x) = f(x) \times g(x)$

$= \log_e(1-x) \times [x]$

$= [x] \log_e(1-x)$

(iii) $g(x) = [x]$

$\therefore \quad [x] = 0$

$\Rightarrow \quad x \in (0, 1)$

So, domain$\left(\frac{f}{g}\right) =$ domain$(f) \cap$ domain$(g) - \{x : g(x) = 0\}$

$= (-\infty, 0)$

$\therefore \quad \frac{f}{g} : (-\infty, 0) \to R$ defined by $\left(\frac{f}{g}\right)(x) = \frac{\log_e(1-x)}{[x]}$

(iv) We have,

$$f(x) = \log_e(1-x)$$

$$\Rightarrow \quad \frac{1}{f(x)} = \frac{1}{\log_e(1-x)}$$

$\therefore \quad \frac{1}{f(x)}$ is defined if $\log_e(1-x)$ is defined and $\log_e(1-x) \neq 0$

$\Rightarrow \quad 1-x > 0$ and $1-x \neq 0$

$\Rightarrow \quad x < 1$ and $x \neq 0$

$\Rightarrow \quad x \in (-\infty, 0) \cup (0, 1)$

$\therefore \quad \text{domain}\left(\frac{g}{f}\right) = (-\infty, 0) \cup (0, 1)$

$\frac{g}{f} : (-\infty, 0) \cup (0, 1) \to R$ defined by $\left(\frac{g}{f}\right)(x) = \frac{[x]}{\log_e(1-x)}$

Now,

$$\begin{aligned}(f+g)(-1) &= f(-1) + g(-1) \\ &= \log_e(1-(-1)) + [-1] \\ &= \log_e 2 - 1\end{aligned}$$

$\Rightarrow \quad (f+g)(-1) = \log_e 2 - 1$

(v) $fg(0) = \log_e(1-0) \times [0]$
$= 0$

(vi) $\left(\frac{f}{g}\right)\left(\frac{1}{2}\right) =$ does not exist

(vii) $\left(\frac{g}{f}\right)\left(\frac{1}{2}\right) = \frac{\left[\frac{1}{2}\right]}{\log_e\left(1-\frac{1}{2}\right)} = 0$

Functions Ex 3.4 Q6

We have,

$$f(x) = \sqrt{x+1}, \; g(x) = \frac{1}{x}$$

and $\quad h(x) = 2x^2 - 3$

Clearly, $f(x)$ is defined for $x + 1 \geq 0$

$\Rightarrow \quad x \geq -1$

$\Rightarrow \quad x \in [-1, \infty]$

$\therefore$ Domain$(f) = [-1, \infty]$

$g(x)$ is defined for $x \neq 0$

$\Rightarrow \quad x \in R - \{0\}$

and, $\quad h(x)$ is defined for all $x \in R$

$\therefore$ Domain$(f) \cap$ Domain$(g) \cap$ Domain$(h) = [-1, \infty] - \{0\}$

Clearly,

$2f + g - h : [-1, \infty] - \{0\} \to R$ is given by

$$\begin{aligned}(2f+g-h)(x) &= 2f(x) + g(x) - h(x) \\ &= 2\sqrt{x+1} + \frac{1}{x} - 2x^2 + 3\end{aligned}$$

$$\begin{aligned}\therefore \quad (2f+g-h)(1) &= 2\sqrt{1+1} + \frac{1}{1} - 2 \times (1)^2 + 3 \\ &= 2\sqrt{2} + 1 - 2 + 3 \\ &= 2\sqrt{2} + 4 - 2 \\ &= 2\sqrt{2} + 2\end{aligned}$$

and, $\quad (2f+g-h)(0)$ does not exist, it is not lies in the domain $x \in [-1, \infty] - \{0\}$.

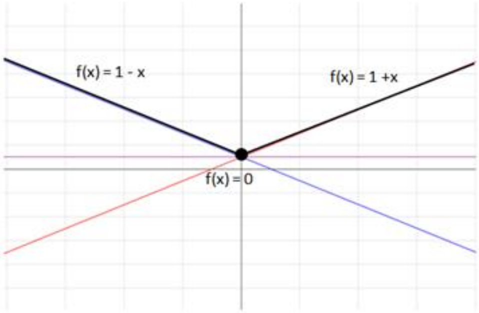
Functions Ex 3.4 Q7

Let,

$$y = f(x) = \begin{cases} 1 - x, & x < 0 \\ 1, & x = 0 \\ x + 1, & x > 0 \end{cases}$$

The graph of f(x) for x <0 is the part of the line y = 1-x that lies to the left of origin.
The graph of f(x) for x >0 is the part of the line y = 1+x that lies to the right of origin.
For x = 0, the graph of f(x) represents the point (0,1)

The graph of f(x) is shown below.



*********** END **********