Definite Integrals Ex 20.4A Q14

$$\text{Let } I = \int_0^7 \frac{\sqrt[3]{x}}{\sqrt[3]{x} + \sqrt[3]{7-x}} dx \qquad --(\text{i})$$

We know that $\int_0^a f(x) = \int_0^a f(a-x)$

Hence,

$$I = \int_0^7 \frac{\sqrt[3]{7-x}}{\sqrt[3]{7-x} + \sqrt[3]{x}} dx \qquad --(\text{ii})$$

Adding (i) & (ii)

$$2I = \int_0^7 \frac{\sqrt[3]{x}}{\sqrt[3]{x} + \sqrt[3]{7-x}} dx + \int_0^7 \frac{\sqrt[3]{7-x}}{\sqrt[3]{7-x} + \sqrt[3]{x}} dx$$

$$2I = \int_0^7 \frac{\sqrt[3]{x} + \sqrt[3]{7-x}}{\sqrt[3]{x} + \sqrt[3]{7-x}} dx$$

$$2I = \int_0^7 dx$$

$$2I = [x]_0^7$$

$$I = \frac{7}{2}$$

Definite Integrals Ex 20.4A Q15

$$\text{Let } I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{\tan x}}\,dx$$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \qquad --(\text{i})$$

We know that $\int_a^b f(x) = \int_a^b f(a+b-x)\,dx$

Hence,

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}\,dx$$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \qquad --(\text{ii})$$

Adding (i) & (ii)

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}+\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} dx$$

$$2I = [x]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$I = \frac{\pi}{12}$$

Definite Integrals Ex 20.4A Q16

$$I = \int_a^b x f(x)\,dx$$

$$I = \int_a^b (a+b-x) f(a+b-x)\,dx$$

$$I = \int_a^b (a+b-x) f(x)\,dx \ldots\ldots\ldots [\because f(a+b-x) = f(x)]$$

$$I = \int_a^b (a+b) f(x)\,dx - \int_a^b f(x)\,dx$$

$$I = (a+b)\int_a^b f(x)\,dx - I$$

$$2I = (a+b)\int_a^b f(x)\,dx$$

$$I = \frac{(a+b)}{2}\int_a^b f(x)\,dx$$

$$\therefore \int_a^b x f(x)\,dx = \frac{(a+b)}{2}\int_a^b f(x)\,dx$$

*********** END **********