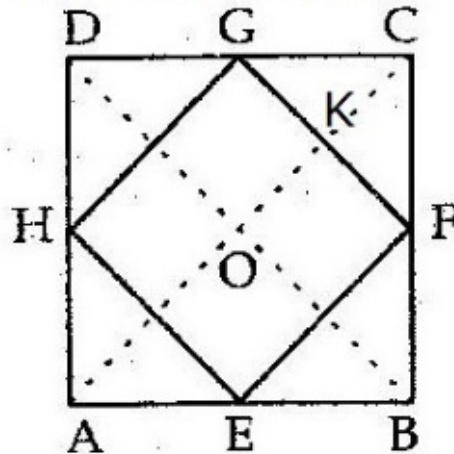




Exercise 9C

Question 11:

Given : ABCD is a square in which E, F, G and H are the mid points of AB, BC, CD and AD, respectively.
The mid points are joined together.



To prove : EFGH is a square.

Construction : Join AC and BD

Proof :

Midpoint Theorem: The line segment joining the midpoints of any two sides of a triangle is parallel to the third side and equal to half of it.

In $\triangle ABC$

E and F are the mid - points and by the Mid points Theorem , we have

$$EF \parallel AC \text{ and } EF = \frac{1}{2} AC$$

Similarly, in $\triangle ADC$,

H and G are the midpoints and by the Mid points Theorem , we have

$$HG \parallel AC \text{ and } HG = \frac{1}{2} AC$$

Thus, we have,

$$EF \parallel HG \text{ and } EF = HG = \frac{1}{2} AC \dots\dots(1)$$

In $\triangle BAD$,

H and E are the midpoints and by the Mid points Theorem , we have,

$$HE \parallel BD \text{ and } HE = \frac{1}{2} BD$$

In $\triangle BCD$,

G and F are the midpoints and by the Mid points Theorem , we have,

$$GF \parallel BD \text{ and } GF = \frac{1}{2} BD$$

Thus, we have,

$$HE \parallel GF \text{ and } HE = GF = \frac{1}{2} BD \dots (2)$$

The diagonals of a square are equal.

$$\Rightarrow AC = BD \dots (3)$$

From (1), (2) and (3), we have

$GF \parallel BD$ and $HE \parallel GF$.

Also, we have $EF = GF = GH = HE$

So, EFGH is a rhombus

Now, as diagonals of a square are equal and intersect at right angles.

$$\text{So, } \angle DOC = 90^\circ$$

In a parallelogram the sum of adjacent angles is 180° .

$$\text{So, } \angle DOC + \angle GKO = 180^\circ$$

$$\Rightarrow \angle GKO = 180^\circ - 90^\circ = 90^\circ$$

But $\angle GKO = \angle EFG$ [Corresponding angles]

$$\therefore \angle EFG = 90^\circ$$

\therefore EFGH is a square.

***** END *****