Algebra of Matrices Ex 5.3 Q21

Given, $\quad w$ is a complex cube root of unity,

$$\left(\begin{bmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{bmatrix} + \begin{bmatrix} w & w^2 & 1 \\ w^2 & 1 & w \\ w & w^2 & 1 \end{bmatrix}\right)\begin{bmatrix} 1 \\ w \\ w^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+w & w+w^2 & w^2+1 \\ w+w^2 & w^2+1 & 1+w \\ w^2+w & 1+w^2 & w+1 \end{bmatrix}\begin{bmatrix} 1 \\ w \\ w^2 \end{bmatrix}$$

$$= \begin{bmatrix} -w^2 & -1 & -w \\ -1 & -w & -w^2 \\ -1 & -w & -w^2 \end{bmatrix}\begin{bmatrix} 1 \\ w \\ w^2 \end{bmatrix} \qquad \left\{\begin{matrix} \text{Since } 1+w+w^2 = 0 \\ \text{and } w^3 = 1 \end{matrix}\right\} \qquad \text{---(i)}$$

$$= \begin{bmatrix} -w^2 - w - w^3 \\ -1 - w^2 - w^4 \\ -1 - w^2 - w^2 \end{bmatrix}$$

$$= \begin{bmatrix} -w\left(1+w+w^2\right) \\ -1 - w^2 - w^3 w \\ -1 - w^2 - w^3 w \end{bmatrix}$$

$$= \begin{bmatrix} -w.0 \\ -1 - w^2 - w \\ -1 - w^2 - w \end{bmatrix} \qquad \{\text{using reason (i)}\}$$

$$= \begin{bmatrix} 0 \\ -\left(1+w+w^2\right) \\ -\left(1+w+w^2\right) \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -(0) \\ -(0) \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q22

Given, $\quad A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$

$$A^2 = A, A$$

$$= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3-5 & -6-12+15 & -10-15+20 \\ -2-4+5 & 3+16-15 & 5+20-20 \\ 2+3-4 & -3-12+12 & -5-15+16 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$= A$$

Hence,

$$A^2 = A$$

Algebra of Matrices Ex 5.3 Q23

Given, $$A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$$

$$A^2 = A.A$$

$$= \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix} \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 16 - 3 - 12 & -4 + 0 + 4 & -16 + 4 + 12 \\ 12 + 0 - 12 & -3 + 0 + 4 & -12 + 0 + 12 \\ 12 - 3 - 9 & -3 + 0 + 3 & -12 + 4 + 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= I_3$$

Hence,

$$A^2 = I_3$$

*********** END **********