

Exercise 7.7: Solutions of Questions on Page Number: 330

Q1: $\sqrt{4-x^2}$

Answer:

Let
$$I = \int \sqrt{4 - x^2} dx = \int \sqrt{(2)^2 - (x)^2} dx$$

It is known that,
$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

$$I = \frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2} + C$$
$$= \frac{x}{2}\sqrt{4 - x^2} + 2\sin^{-1}\frac{x}{2} + C$$

Answer needs Correction? Click Here

Q2: $\sqrt{1-4x^2}$

Answer:

Let
$$I = \int \sqrt{1 - 4x^2} dx = \int \sqrt{(1)^2 - (2x)^2} dx$$

Let $2x = t \implies 2 dx = dt$
 $\therefore I = \frac{1}{2} \int \sqrt{(1)^2 - (t)^2} dt$

It is known that,
$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

$$\Rightarrow I = \frac{1}{2} \left[\frac{t}{2} \sqrt{1 - t^2} + \frac{1}{2} \sin^{-1} t \right] + C$$

$$= \frac{t}{4} \sqrt{1 - t^2} + \frac{1}{4} \sin^{-1} t + C$$

$$= \frac{2x}{4} \sqrt{1 - 4x^2} + \frac{1}{4} \sin^{-1} 2x + C$$

$$= \frac{x}{2} \sqrt{1 - 4x^2} + \frac{1}{4} \sin^{-1} 2x + C$$

Answer needs Correction? Click Here

Q3:
$$\sqrt{x^2 + 4x + 6}$$

Answer:

Let
$$I = \int \sqrt{x^2 + 4x + 6} \, dx$$

$$= \int \sqrt{x^2 + 4x + 4 + 2} \, dx$$

$$= \int \sqrt{(x^2 + 4x + 4) + 2} \, dx$$

$$= \int \sqrt{(x + 2)^2 + (\sqrt{2})^2} \, dx$$

It is known that,
$$\int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log |x + \sqrt{x^2 + a^2}| + C$$

$$I = \frac{(x+2)}{2} \sqrt{x^2 + 4x + 6} + \frac{2}{2} \log |(x+2) + \sqrt{x^2 + 4x + 6}| + C$$

$$= \frac{(x+2)}{2} \sqrt{x^2 + 4x + 6} + \log |(x+2) + \sqrt{x^2 + 4x + 6}| + C$$

Answer needs Correction? Click Here

Q4:
$$\sqrt{x^2 + 4x + 1}$$

Answer:

Let
$$I = \int \sqrt{x^2 + 4x + 1} \, dx$$

= $\int \sqrt{(x^2 + 4x + 4) - 3} \, dx$
= $\int \sqrt{(x + 2)^2 - (\sqrt{3})^2} \, dx$

It is known that,
$$\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + C$$

$$\therefore I = \frac{(x+2)}{2} \sqrt{x^2 + 4x + 1} - \frac{3}{2} \log |(x+2) + \sqrt{x^2 + 4x + 1}| + C$$

Answer needs Correction? Click Here

Q5: $\sqrt{1-4x-x^2}$

Answer:

Let
$$I = \int \sqrt{1 - 4x - x^2} dx$$

$$= \int \sqrt{1 - (x^2 + 4x + 4 - 4)} dx$$

$$= \int \sqrt{1 + 4 - (x + 2)^2} dx$$

$$= \int \sqrt{(\sqrt{5})^2 - (x + 2)^2} dx$$

It is known that,
$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

$$\therefore I = \frac{(x+2)}{2} \sqrt{1 - 4x - x^2} + \frac{5}{2} \sin^{-1} \left(\frac{x+2}{\sqrt{5}} \right) + C$$

Answer needs Correction? Click Here

Q6: $\sqrt{x^2+4x-5}$

Answer:

Let
$$I = \int \sqrt{x^2 + 4x - 5} \, dx$$

= $\int \sqrt{(x^2 + 4x + 4) - 9} \, dx$
= $\int \sqrt{(x + 2)^2 - (3)^2} \, dx$

It is known that,
$$\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + C$$

$$\therefore I = \frac{(x+2)}{2} \sqrt{x^2 + 4x - 5} - \frac{9}{2} \log |(x+2) + \sqrt{x^2 + 4x - 5}| + C$$

Answer needs Correction? Click Here

Q7: $\sqrt{1+3x-x^2}$

Answer:

Let
$$I = \int \sqrt{1 + 3x - x^2} dx$$

$$= \int \sqrt{1 - \left(x^2 - 3x + \frac{9}{4} - \frac{9}{4}\right)} dx$$

$$= \int \sqrt{\left(1 + \frac{9}{4}\right) - \left(x - \frac{3}{2}\right)^2} dx$$

$$= \int \sqrt{\left(\frac{\sqrt{13}}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2} dx$$

It is known that,
$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

$$\therefore I = \frac{x - \frac{3}{2}}{2} \sqrt{1 + 3x - x^2} + \frac{13}{4 \times 2} \sin^{-1} \left(\frac{x - \frac{3}{2}}{\frac{\sqrt{13}}{2}} \right) + C$$
$$= \frac{2x - 3}{4} \sqrt{1 + 3x - x^2} + \frac{13}{8} \sin^{-1} \left(\frac{2x - 3}{\sqrt{13}} \right) + C$$

Answer needs Correction? Click Here

Q8: $\sqrt{x^2 + 3x}$

Answer:

Let
$$I = \int \sqrt{x^2 + 3x} \, dx$$

$$= \int \sqrt{x^2 + 3x + \frac{9}{4} - \frac{9}{4}} \, dx$$

$$= \int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2} \, dx$$

It is known that,
$$\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log \left| x + \sqrt{x^2 - a^2} \right| + C$$

$$\therefore I = \frac{\left(x + \frac{3}{2}\right)}{2} \sqrt{x^2 + 3x} - \frac{\frac{9}{4}}{2} \log \left| \left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x} \right| + C$$
$$= \frac{\left(2x + 3\right)}{4} \sqrt{x^2 + 3x} - \frac{9}{8} \log \left| \left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x} \right| + C$$

Q9:
$$\sqrt{1+\frac{x^2}{9}}$$

Answer:

Let
$$I = \int \sqrt{1 + \frac{x^2}{9}} dx = \frac{1}{3} \int \sqrt{9 + x^2} dx = \frac{1}{3} \int \sqrt{(3)^2 + x^2} dx$$

It is known that,
$$\int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log |x + \sqrt{x^2 + a^2}| + C$$

$$\therefore I = \frac{1}{3} \left[\frac{x}{2} \sqrt{x^2 + 9} + \frac{9}{2} \log |x + \sqrt{x^2 + 9}| \right] + C$$
$$= \frac{x}{6} \sqrt{x^2 + 9} + \frac{3}{2} \log |x + \sqrt{x^2 + 9}| + C$$

Answer needs Correction? Click Here

Q10: $\int \sqrt{1+x^2} dx$ is equal to

A.
$$\frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\log|x+\sqrt{1+x^2}| + C$$

B.
$$\frac{2}{3}(1+x^2)^{\frac{2}{3}}+C$$

C.
$$\frac{2}{3}x(1+x^2)^{\frac{3}{2}}+C$$

D.
$$\frac{x^2}{2}\sqrt{1+x^2} + \frac{1}{2}x^2 \log |x + \sqrt{1+x^2}| + C$$

Answer

It is known that,
$$\int \sqrt{a^2 + x^2} dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \log |x + \sqrt{x^2 + a^2}| + C$$

$$\therefore \int \sqrt{1+x^2} \, dx = \frac{x}{2} \sqrt{1+x^2} + \frac{1}{2} \log \left| x + \sqrt{1+x^2} \right| + C$$

Hence, the correct answer is A.

Answer needs Correction? Click Here

Q11: $\int \sqrt{x^2-8x+7} dx$ is equal to

A.
$$\frac{1}{2}(x-4)\sqrt{x^2-8x+7}+9\log\left|x-4+\sqrt{x^2-8x+7}\right|+C$$

B.
$$\frac{1}{2}(x+4)\sqrt{x^2-8x+7}+9\log|x+4+\sqrt{x^2-8x+7}|+C$$

C.
$$\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-3\sqrt{2}\log|x-4+\sqrt{x^2-8x+7}|+C$$

D.
$$\frac{1}{2}(x-4)\sqrt{x^2-8x+7} - \frac{9}{2}\log|x-4+\sqrt{x^2-8x+7}| + C$$

Answer:

Let
$$I = \int \sqrt{x^2 - 8x + 7} \, dx$$

= $\int \sqrt{(x^2 - 8x + 16) - 9} \, dx$
= $\int \sqrt{(x - 4)^2 - (3)^2} \, dx$

It is known that, $\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + C$

$$\therefore I = \frac{(x-4)}{2} \sqrt{x^2 - 8x + 7} - \frac{9}{2} \log |(x-4) + \sqrt{x^2 - 8x + 7}| + C$$

Hence, the correct answer is D.

Answer needs Correction? Click Here