Differentiation Ex 11.8 Q14

Let $u = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$

$$= \tan^{-1}\left(\frac{\frac{\cos^2 x}{2} - \frac{\sin^2 x}{2}}{\frac{\cos^2 x}{2} + \frac{\sin^2 x}{2} + \frac{2\sin x}{2}\frac{\cos x}{2}}\right)$$

$$= \tan^{-1}\left(\frac{\left(\frac{\cos x}{2} + \frac{\sin x}{2}\right)\left(\frac{\cos x}{2} - \frac{\sin x}{2}\right)}{\left(\frac{\cos x}{2} + \frac{\sin x}{2}\right)^2}\right)$$

$$= \tan^{-1}\left(\frac{\frac{\cos x}{2} - \frac{\sin x}{2}}{\frac{\cos x}{2} + \frac{\sin x}{2}}\right)$$

$$= \tan^{-1}\left[\frac{\frac{\frac{\cos x}{2}}{\frac{\cos x}{2}} - \frac{\frac{\sin x}{2}}{\frac{\cos x}{2}}}{\frac{\frac{\cos x}{2}}{\frac{\cos x}{2}} + \frac{\frac{\sin x}{2}}{\frac{\cos x}{2}}}\right]$$

$$= \tan^{-1}\left[\frac{1 - \frac{\tan x}{2}}{1 + \frac{\tan x}{2}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{\tan \pi}{4} - \frac{\tan x}{2}}{1 + \frac{\tan \pi}{4} \times \frac{\tan x}{2}}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right]$$

$$u = \frac{\pi}{4} - \frac{x}{2}$$

Differentiating it with respect to $x$,

$$\frac{du}{dx} = 0 - \left(\frac{1}{2}\right)$$

$$\frac{du}{dx} = -\frac{1}{2} \qquad \text{---(i)}$$

Let $v = \sec^{-1} x$

Differentiating it with respect to $x$,

$$\frac{dv}{dx} = \frac{1}{x\sqrt{x^2-1}} \qquad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = -\frac{1}{2} \times \frac{x\sqrt{x^2-1}}{1}$$

$$\frac{du}{dv} = \frac{-x\sqrt{x^2-1}}{2}$$

Differentiation Ex 11.8 Q15

Let $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$, so

$u = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$u = \sin^{-1}(\sin 2\theta)$ ---(i)

Let $v = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$= \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$

$v = \tan^{-1}(\tan 2\theta)$ ---(ii)

Here, $-1 < x < 1$

$\Rightarrow \quad -1 < \tan\theta < 1$

$\Rightarrow \quad -\frac{\pi}{4} < \theta < \frac{\pi}{4}$

So, from equation (i),

$u = 2\theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$

$u = 2\tan^{-1}x$

Differentiating it with respect to $x$,

$$\frac{du}{dx} = \frac{2}{\left(1+x^2\right)} \qquad \text{---(iii)}$$

From equation (ii),

$v = 2\theta \qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$

$v = 2\tan^{-1}x$

Differentiating it with respect to $x$,

$$\frac{dv}{dx} = \frac{2}{1+x^2} \qquad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{1+x^2} \times \frac{1+x^2}{2}$$

$$\frac{du}{dv} = 1$$

********** END **********