



Binomial Theorem Ex 18.1 Q8

$$\begin{aligned}
 & 3^{2n} - 26n - 1 \\
 &= (3^2)^n - 26n - 1 \\
 &= 27^n - 26n - 1 \\
 &= (1 + 26)^n - 26n - 1 \\
 &= \left({}^nC_0 + {}^nC_1(26)^1 + {}^nC_2(26)^2 + \dots + {}^nC_n(26)^n \right) - 26n - 1 \\
 &= (1 + 26n + 676 {}^nC_2 + \dots + 676(26)^{n-2}) - 26n - 1 \\
 &= 676 \left({}^nC_2 + \dots + (26)^{n-2} \right)
 \end{aligned}$$

$\therefore 3^{2n} - 26n - 1$ is divisible for $n \in \mathbb{N}$.

Hence, proved

Binomial Theorem Ex 18.1 Q9

We have,

$$\begin{aligned}
 (1.1)^{10000} &= (1 + 0.1)^{10000} \\
 &= {}^{10000}C_0 + {}^{10000}C_1(0.1) + {}^{10000}C_2(0.1)^2 + \dots + {}^{10000}C_{10000}(0.1)^{10000} \\
 &= 1 + 10000 \times (0.1) + \text{other positive terms} \\
 &= 1 + 1000 + \text{other positive terms} \\
 &= 1001 + \text{other positive terms} > 1000
 \end{aligned}$$

$$\therefore (1.1)^{10000} > 1000$$

Binomial Theorem Ex 18.1 Q10

$$\begin{aligned}
 (1.2)^{4000} &= (1 + 0.2)^{4000} \\
 &= {}^{4000}C_0(0.2)^0(1)^{4000} + {}^{4000}C_1 \times (0.2)^1 \times 1^{3999} + \dots + {}^{4000}C_{4000}(0.2)^{4000}1^0 \\
 &= 1 + 4000 \times 0.2 \times 1 + \dots + (0.2)^{4000} \\
 &= 1 + 800 + \dots + (0.2)^{4000}
 \end{aligned}$$

Here, we clearly observe $(1.2)^{4000}$ is less than 801 thus, $(1.2)^{4000} < 800$.

Binomial Theorem Ex 18.1 Q11

$$\begin{aligned}
 (1.01)^{10} + (1 - 0.01)^{10} &= (1 + 0.01)^{10} + (1 - 0.01)^{10} \\
 &= \left({}^{10}C_1 + {}^{10}C_2 \frac{1}{10^2} + {}^{10}C_3 \frac{1}{10^3} \dots + {}^{10}C_{10} \frac{1}{10^{10}} \right) + \left({}^{10}C_1 - {}^{10}C_2 \frac{1}{10^2} + {}^{10}C_3 \frac{1}{10^3} - {}^{10}C_4 \frac{1}{10^4} + \dots \right) \\
 &= 2 \left({}^{10}C_1 - {}^{10}C_3 \frac{1}{10^3} + {}^{10}C_5 \frac{1}{10^5} - {}^{10}C_7 \frac{1}{10^7} + {}^{10}C_9 \frac{1}{10^9} \right) \\
 &= 2 \left(10 + \frac{10!}{3!7!} \frac{1}{1000} + \frac{10!}{5!5!} \frac{1}{(10)^5} + \frac{10!}{7!3!} \times \frac{1}{10^7} + \frac{10!}{9!1!} \frac{1}{10^9} \right) \\
 &= 2 \left(10 + \frac{9 \times 8}{3 \times 2 \times 1000} + \frac{9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 10^5} + \frac{9 \times 8}{3 \times 2 \times 10^7} + \frac{1}{10^8} \right) \\
 &= 2.0090042
 \end{aligned}$$

Binomial Theorem Ex 18.1 Q12

$$\begin{aligned}
2^{4n+4} - 15n - 16 &= 2^{4(n+1)} - 15n - 15 - 1 \\
&= (16)^{(n+1)} - 15(n+1) - 1 \\
&= (1+15)^{n+1} - 15(n+1) - 1 \\
&= \left[{}^{n+1}C_0 + {}^{n+1}C_1(15) + {}^{n+1}C_2(15)^2 + \dots + {}^{n+1}C_{n+1}(15)^{n+1} \right] - 15(n+1) - 1 \\
&= \left[1 + 15(n+1) + {}^{n+1}C_2(15)^2 + \dots + {}^{n+1}C_{n+1}(15)^{n+1} \right] - 15(n+1) - 1 \\
&= 225 \left[{}^{n+1}C_2 + \dots + {}^{n+1}C_{n+1}(15)^{n-1} \right] \\
&= 225 \times \text{natural number}
\end{aligned}$$

***** END *****