Chapter 10 Differentiability Ex 10.2 Q5

$f(x) = x^3 + 7x^2 + 8x - 9$ is a polynomial function. So, it is differentiable every where.

$$f'(4) = \lim_{h \to 0} \frac{f(4+h) - h(4)}{h}$$

$$= \lim_{h \to 0} \frac{\left[(4+h)^3 + 7(4+h)^2 + 8(4+h) - 9\right] - \left[64 + 112 + 32 - 9\right]}{h}$$

$$= \lim_{h \to 0} \frac{\left[64 + h^3 + 48h + 12h^2 + 112 + 7h^2 + 56h + 32 + 8h - 9\right] - \left[210 - 9\right]}{h}$$

$$= \lim_{h \to 0} \frac{h^3 + 19h^2 + 112h + 210 - 9 - 210 + 9}{h}$$

$$= \lim_{h \to 0} \frac{h^3 + 19h^2 + 112h}{h}$$

$$= \lim_{h \to 0} \frac{h\left(h^2 + 19h + 112\right)}{h}$$

$$f'(4) = 112$$

Chapter 10 Differentiability Ex 10.2 Q6

$$f(x) = mx + c$$

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{f(h) - h(0)}{h}$$

$$= \lim_{h \to 0} \frac{(mh + c) - (m \times 0 + c)}{h}$$

$$= \lim_{h \to 0} \frac{mh + c - c}{h}$$

$$= \lim_{h \to 0} \frac{mh}{h}$$

$$= m$$

$$f'(0) = m$$

Chapter 10 Differentiability Ex 10.2 Q7

$$f(x) = \begin{cases} 2x+3, \text{ if} -3 \le x < -2 \\ x+1, \text{ if} -2 \le x < 0 \\ x+2, \text{ if } 0 \le x \le 1 \end{cases}$$

We know that polynomial funtiona are continuous and differentiable everywhere.
So f(x) is differentiable on $x \in [-3,2)$, $x \in (-2,0)$ and $x \in (0,1]$.
We need to check the differentiability at x = -2 and x = 0

Differentiability at x = -2

$$(\text{LHD at } x = -2) = \lim_{x\to -2^-} \frac{f(x)-f(-2)}{x-(-2)} = \lim_{x\to -2^-} \frac{2x+3+1}{x+2} = \lim_{x\to -2^-} \frac{2(x+2)}{x+2} = 2$$

$$(\text{RHD at } x = -2) = \lim_{x\to -2^+} \frac{f(x)-f(-2)}{x-(-2)} = \lim_{x\to -2^+} \frac{x+1+1}{x+2} = \lim_{x\to -2^+} \frac{x+2}{x+2} = 1$$

$\therefore$ (LHD at x = -2) $\neq$ (RHD at x = -2)
So, $f(x)$ is not differentiable at x = -2.

Differentiability at x = 0

$$(\text{LHD at } x = 0) = \lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^-} \frac{x+1-2}{x} = \lim_{x\to 0^-} \frac{x-1}{x} \to \infty$$

$$(\text{RHD at } x = 0) = \lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^+} \frac{x+2-2}{x} = \lim_{x\to 0^+} \frac{x}{x} = 1$$

$\therefore$ (LHD at x = 0) $\neq$ (RHD at x = 0)
So, $f(x)$ is not differentiable at x = 0.

Chapter 10 Differentiability Ex 10.2 Q8

We know that, modulus function

$f(x) = |x|$ is continuous but bot differentiable at $x = 0$,

So,

$f(x) = |x| + |x-1| + |x-2| + |x-3| + |x-4|$ is continuous but not differentiable

$x = 0, 1, 2, 3, 4$.

*********** END **********