



Indefinite Integrals Ex 19.27 Q1

$$\text{Let } I = \int e^{ax} \cos bx dx$$

Integrating by parts,

$$\begin{aligned} I &= e^{ax} \frac{\sin bx}{b} - a \int e^{ax} \frac{\sin bx}{b} dx \\ &= \frac{1}{b} e^{ax} \sin bx - \frac{a}{b} \int e^{ax} \sin bx dx \\ &= \frac{1}{b} e^{ax} \sin bx - \frac{a}{b} \left[-e^{ax} \frac{\cos bx}{b} + \int a e^{ax} \frac{\cos bx}{b} dx \right] \\ &= \frac{1}{b} e^{ax} \sin bx + \frac{a}{b^2} e^{ax} \cos bx - \frac{a^2}{b^2} \int e^{ax} \cos bx dx \end{aligned}$$

$$\Rightarrow I = \frac{e^{ax}}{b^2} [b \sin bx + a \cos bx] - \frac{a^2}{b^2} I + c$$

$$\Rightarrow I \cdot \left\{ \frac{a^2 + b^2}{b^2} \right\} = \frac{e^{ax}}{b^2} [b \cos bx + a \cos bx] + c$$

Thus,

$$I = \frac{e^{ax}}{a^2 + b^2} [a \cos bx + a \cos bx] + c$$

Indefinite Integrals Ex 19.27 Q2

$$\text{Let } I = \int e^{ax} \sin(bx + c) dx$$

$$\begin{aligned} \Rightarrow & -e^{ax} \frac{\cos(bx + c)}{b} + \int a e^{ax} \frac{\cos(bx + c)}{b} dx \\ &= -\frac{1}{b} e^{ax} \cos(bx + c) + \frac{a}{b} \int e^{ax} \cos(bx + c) dx \\ &= -\frac{1}{b} e^{ax} \cos(bx + c) + \frac{a}{b} \left[\int e^{ax} \frac{\sin(bx + c)}{b} - \int a e^{ax} \frac{\sin(bx + c)}{b} dx \right] + c_1 \\ &= \frac{e^{ax}}{b^2} \{a \sin(bx + c) - b \cos(bx + c)\} - \frac{a^2}{b^2} \int e^{ax} \sin(bx + c) dx + c_1 \end{aligned}$$

$$\Rightarrow I = \frac{e^{ax}}{b^2} \{a \sin(bx + c) - b \cos(bx + c)\} - \frac{a^2}{b^2} I + c_1$$

$$\Rightarrow I = \left\{ \frac{a^2 + b^2}{b^2} \right\} - \frac{e^{ax}}{b^2} \{a \sin(bx + c) - b \cos(bx + c)\} + c_1$$

$$\Rightarrow I = \frac{e^{ax}}{a^2 + b^2} \{a \sin(bx + c) - b \cos(bx + c)\} + c_1$$

Indefinite Integrals Ex 19.27 Q3

Let $\log x = t$

$$\Rightarrow \frac{1}{x} dx = dt$$

$$\Rightarrow dx = x dt$$

$$\Rightarrow dx = e^t dt$$

$$\therefore I = \int \cos(\log x) dx = \int e^t \cos t dt$$

We know that

$$\int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2} \{a \cos bx + b \sin bx\} + c$$

Here $a = 1$, $b = 1$

$$\text{So, } I = \frac{e^t}{2} \{\cos t + \sin t\} + c$$

Hence,

$$I = \int \cos(\log x) dx = \frac{e^{\log x}}{2} \{\cos(\log x) + \sin(\log x)\} + c$$

$$\Rightarrow I = \frac{x}{2} \{\cos(\log x) + \sin(\log x)\} + c$$

Indefinite Integrals Ex 19.27 Q4

$$\text{Let } I = \int e^{2x} \cos(3x + 4) dx$$

Integrating by parts

$$\begin{aligned} I &= e^{2x} \frac{\sin(3x + 4)}{3} - \int 2e^{2x} \frac{\sin(3x + 4)}{3} dx \\ &= \frac{1}{3} e^{2x} \sin(3x + 4) - \frac{2}{3} \int e^{2x} \sin(3x + 4) dx \\ &= \frac{1}{3} e^{2x} \sin(3x + 4) - \frac{2}{3} \left[-e^{2x} \frac{\cos(3x + 4)}{3} + \int 2e^{2x} \frac{\cos(3x + 4)}{3} dx \right] + c \\ I &= \frac{e^{2x}}{9} \{2 \cos(3x + 4) + 3 \sin(3x + 4)\} + c \end{aligned}$$

Hence,

$$I = \frac{e^{2x}}{13} \{2 \cos(3x + 4) + 3 \sin(3x + 4)\} + c$$

***** END *****