



**Exercise 9.4 : Solutions of Questions on Page Number : 395**

**Q1 :**  $\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$

**Answer :**

The given differential equation is:

$$\begin{aligned}\frac{dy}{dx} &= \frac{1 - \cos x}{1 + \cos x} \\ \Rightarrow \frac{dy}{dx} &= \frac{2 \sin^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}} = \tan^2 \frac{x}{2} \\ \Rightarrow \frac{dy}{dx} &= \left( \sec^2 \frac{x}{2} - 1 \right)\end{aligned}$$

Separating the variables, we get:

$$dy = \left( \sec^2 \frac{x}{2} - 1 \right) dx$$

Now, integrating both sides of this equation, we get:

$$\begin{aligned}\int dy &= \int \left( \sec^2 \frac{x}{2} - 1 \right) dx = \int \sec^2 \frac{x}{2} dx - \int dx \\ \Rightarrow y &= 2 \tan \frac{x}{2} - x + C\end{aligned}$$

This is the required general solution of the given differential equation.

Answer needs Correction? [Click Here](#)

**Q2 :**  $\frac{dy}{dx} = \sqrt{4 - y^2} \quad (-2 < y < 2)$

**Answer :**

The given differential equation is:

$$\begin{aligned}\frac{dy}{dx} &= \sqrt{4 - y^2} \\ \Rightarrow \frac{dy}{\sqrt{4 - y^2}} &= dx\end{aligned}$$

Now, integrating both sides of this equation, we get:

$$\begin{aligned}\int \frac{dy}{\sqrt{4 - y^2}} &= \int dx \\ \Rightarrow \sin^{-1} \frac{y}{2} &= x + C \\ \Rightarrow \frac{y}{2} &= \sin(x + C) \\ \Rightarrow y &= 2 \sin(x + C)\end{aligned}$$

This is the required general solution of the given differential equation.

Answer needs Correction? [Click Here](#)

**Q3 :**  $\frac{dy}{dx} + y = 1 \quad (y \neq 1)$

**Answer :**

The given differential equation is:

$$\begin{aligned}\frac{dy}{dx} + y &= 1 \\ \Rightarrow dy + y \, dx &= dx \\ \Rightarrow dy &= (1 - y) \, dx\end{aligned}$$

Separating the variables, we get:

$$\Rightarrow \frac{dy}{1 - y} = dx$$

Now, integrating both sides, we get:

$$\begin{aligned}\int \frac{dy}{1 - y} &= \int dx \\ \Rightarrow \log(1 - y) &= x + \log C \\ \Rightarrow -\log C - \log(1 - y) &= x \\ \Rightarrow \log C(1 - y) &= -x\end{aligned}$$

$$\begin{aligned}\Rightarrow C(1-y) &= e^{-x} \\ \Rightarrow 1-y &= \frac{1}{C} e^{-x} \\ \Rightarrow y &= 1 - \frac{1}{C} e^{-x} \\ \Rightarrow y &= 1 + Ae^{-x} \text{ (where } A = -\frac{1}{C}\text{)}\end{aligned}$$

This is the required general solution of the given differential equation.

Answer needs Correction? [Click Here](#)

**Q4 :**  $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

**Answer :**

The given differential equation is:

$$\begin{aligned}\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy &= 0 \\ \Rightarrow \frac{\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy}{\tan x \tan y} &= 0 \\ \Rightarrow \frac{\sec^2 x}{\tan x} dx + \frac{\sec^2 y}{\tan y} dy &= 0 \\ \Rightarrow \frac{\sec^2 x}{\tan x} dx &= -\frac{\sec^2 y}{\tan y} dy\end{aligned}$$

Integrating both sides of this equation, we get:

$$\int \frac{\sec^2 x}{\tan x} dx = - \int \frac{\sec^2 y}{\tan y} dy \quad \dots(1)$$

Let  $\tan x = t$ .

$$\begin{aligned}\therefore \frac{d}{dx}(\tan x) &= \frac{dt}{dx} \\ \Rightarrow \sec^2 x &= \frac{dt}{dx} \\ \Rightarrow \sec^2 x \, dx &= dt \\ \text{Now, } \int \frac{\sec^2 x}{\tan x} dx &= \int \frac{1}{t} dt. \\ &= \log t \\ &= \log(\tan x)\end{aligned}$$

$$\text{Similarly, } \int \frac{\sec^2 y}{\tan y} dy = \log(\tan y).$$

Substituting these values in equation (1), we get:

$$\begin{aligned}\log(\tan x) &= -\log(\tan y) + \log C \\ \Rightarrow \log(\tan x) &= \log\left(\frac{C}{\tan y}\right) \\ \Rightarrow \tan x &= \frac{C}{\tan y} \\ \Rightarrow \tan x \tan y &= C\end{aligned}$$

This is the required general solution of the given differential equation.

Answer needs Correction? [Click Here](#)

**Q5 :**  $(e^x + e^{-x}) dy - (e^x - e^{-x}) dx = 0$

**Answer :**

The given differential equation is:

$$\begin{aligned}(e^x + e^{-x}) dy - (e^x - e^{-x}) dx &= 0 \\ \Rightarrow (e^x + e^{-x}) dy &= (e^x - e^{-x}) dx \\ \Rightarrow dy &= \left[ \frac{e^x - e^{-x}}{e^x + e^{-x}} \right] dx\end{aligned}$$

Integrating both sides of this equation, we get:

$$\begin{aligned}\int dy &= \int \left[ \frac{e^x - e^{-x}}{e^x + e^{-x}} \right] dx + C \\ \Rightarrow y &= \int \left[ \frac{e^x - e^{-x}}{e^x + e^{-x}} \right] dx + C \quad \dots(1)\end{aligned}$$

$$\text{Let } (e^x + e^{-x}) = t.$$

Differentiating both sides with respect to  $x$ , we get:

$$\begin{aligned}\frac{d}{dx}(e^x + e^{-x}) &= \frac{dt}{dx} \\ \Rightarrow e^x - e^{-x} &= \frac{dt}{dx} \\ \Rightarrow (e^x - e^{-x}) dx &= dt\end{aligned}$$

Substituting this value in equation (1), we get:

$$\begin{aligned}y &= \int \frac{1}{t} dt + C \\ \Rightarrow y &= \log(t) + C \\ \Rightarrow y &= \log(e^x + e^{-x}) + C\end{aligned}$$

This is the required general solution of the given differential equation.

Answer needs Correction? [Click Here](#)

Q6:  $\frac{dy}{dx} = (1+x^2)(1+y^2)$

Answer :

The given differential equation is:

$$\frac{dy}{dx} = (1+x^2)(1+y^2)$$

$$\Rightarrow \frac{dy}{1+y^2} = (1+x^2)dx$$

Integrating both sides of this equation, we get:

$$\int \frac{dy}{1+y^2} = \int (1+x^2)dx$$

$$\Rightarrow \tan^{-1} y = \int dx + \int x^2 dx$$

$$\Rightarrow \tan^{-1} y = x + \frac{x^3}{3} + C$$

This is the required general solution of the given differential equation.

Answer needs Correction? [Click Here](#)

Q7:  $y \log y dx - x dy = 0$

Answer :

The given differential equation is:

$$y \log y dx - x dy = 0$$

$$\Rightarrow y \log y dx = x dy$$

$$\Rightarrow \frac{dy}{y \log y} = \frac{dx}{x}$$

Integrating both sides, we get:

$$\int \frac{dy}{y \log y} = \int \frac{dx}{x} \quad \dots(1)$$

Let  $\log y = t$ .

$$\therefore \frac{d}{dy}(\log y) = \frac{dt}{dy}$$

$$\Rightarrow \frac{1}{y} = \frac{dt}{dy}$$

$$\Rightarrow \frac{1}{y} dy = dt$$

Substituting this value in equation (1), we get:

$$\int \frac{dt}{t} = \int \frac{dx}{x}$$

$$\Rightarrow \log t = \log x + \log C$$

$$\Rightarrow \log(\log y) = \log Cx$$

$$\Rightarrow \log y = Cx$$

$$\Rightarrow y = e^{Cx}$$

This is the required general solution of the given differential equation.

Answer needs Correction? [Click Here](#)

Q8:  $x^5 \frac{dy}{dx} = -y^5$

Answer :

The given differential equation is:

$$x^5 \frac{dy}{dx} = -y^5$$

$$\Rightarrow \frac{dy}{y^5} = -\frac{dx}{x^5}$$

$$\Rightarrow \frac{dx}{x^5} + \frac{dy}{y^5} = 0$$

Integrating both sides, we get:

$$\int \frac{dx}{x^5} + \int \frac{dy}{y^5} = k \quad (\text{where } k \text{ is any constant})$$

$$\Rightarrow \int x^{-5} dx + \int y^{-5} dy = k$$

$$\Rightarrow \frac{x^{-4}}{-4} + \frac{y^{-4}}{-4} = k$$

$$\Rightarrow x^{-4} + y^{-4} = -4k$$

$$\Rightarrow x^{-4} + y^{-4} = C \quad (C = -4k)$$

This is the required general solution of the given differential equation.

Answer needs Correction? [Click Here](#)

Q9:  $\frac{dy}{dx} = \sin^{-1} x$

**Answer :**

The given differential equation is:

$$\begin{aligned}\frac{dy}{dx} &= \sin^{-1} x \\ \Rightarrow dy &= \sin^{-1} x \, dx\end{aligned}$$

Integrating both sides, we get:

$$\begin{aligned}\int dy &= \int \sin^{-1} x \, dx \\ \Rightarrow y &= \int (\sin^{-1} x \cdot 1) \, dx \\ \Rightarrow y &= \sin^{-1} x \cdot \int (1) \, dx - \int \left[ \left( \frac{d}{dx} (\sin^{-1} x) \right) \cdot \int (1) \, dx \right] dx \\ \Rightarrow y &= \sin^{-1} x \cdot x - \int \left( \frac{1}{\sqrt{1-x^2}} \cdot x \right) dx \\ \Rightarrow y &= x \sin^{-1} x + \int \frac{-x}{\sqrt{1-x^2}} dx \quad \dots(1)\end{aligned}$$

Let  $1-x^2 = t$ .

$$\begin{aligned}\Rightarrow \frac{d}{dx} (1-x^2) &= \frac{dt}{dx} \\ \Rightarrow -2x &= \frac{dt}{dx} \\ \Rightarrow x \, dx &= -\frac{1}{2} dt\end{aligned}$$

Substituting this value in equation (1), we get:

$$\begin{aligned}y &= x \sin^{-1} x + \int \frac{1}{2\sqrt{t}} dt \\ \Rightarrow y &= x \sin^{-1} x + \frac{1}{2} \cdot \int (t)^{-\frac{1}{2}} dt \\ \Rightarrow y &= x \sin^{-1} x + \frac{1}{2} \cdot \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C \\ \Rightarrow y &= x \sin^{-1} x + \sqrt{t} + C \\ \Rightarrow y &= x \sin^{-1} x + \sqrt{1-x^2} + C\end{aligned}$$

This is the required general solution of the given differential equation.

Answer needs Correction? [Click Here](#)

---

**Q10 :**  $e^x \tan y \, dx + (1-e^x) \sec^2 y \, dy = 0$

**Answer :**

The given differential equation is:

$$\begin{aligned}e^x \tan y \, dx + (1-e^x) \sec^2 y \, dy &= 0 \\ (1-e^x) \sec^2 y \, dy &= -e^x \tan y \, dx\end{aligned}$$

Separating the variables, we get:

$$\frac{\sec^2 y}{\tan y} \, dy = \frac{-e^x}{1-e^x} \, dx$$

Integrating both sides, we get:

$$\int \frac{\sec^2 y}{\tan y} \, dy = \int \frac{-e^x}{1-e^x} \, dx \quad \dots(1)$$

Let  $\tan y = u$ .

$$\begin{aligned}\Rightarrow \frac{d}{dy} (\tan y) &= \frac{du}{dy} \\ \Rightarrow \sec^2 y &= \frac{du}{dy} \\ \Rightarrow \sec^2 y \, dy &= du \\ \therefore \int \frac{\sec^2 y}{\tan y} \, dy &= \int \frac{du}{u} = \log u = \log (\tan y)\end{aligned}$$

Now, let  $1-e^x = t$ .

$$\begin{aligned}\therefore \frac{d}{dx} (1-e^x) &= \frac{dt}{dx} \\ \Rightarrow -e^x &= \frac{dt}{dx} \\ \Rightarrow -e^x \, dx &= dt \\ \Rightarrow \int \frac{-e^x}{1-e^x} \, dx &= \int \frac{dt}{t} = \log t = \log (1-e^x)\end{aligned}$$

Substituting the values of  $\int \frac{\sec^2 y}{\tan y} \, dy$  and  $\int \frac{-e^x}{1-e^x} \, dx$  in equation (1), we get:

$$\begin{aligned}\Rightarrow \log (\tan y) &= \log (1-e^x) + \log C \\ \Rightarrow \log (\tan y) &= \log [C(1-e^x)] \\ \Rightarrow \tan y &= C(1-e^x)\end{aligned}$$

This is the required general solution of the given differential equation.

Answer needs Correction? [Click Here](#)

---

**Q11 :**  $(x^3 + x^2 + x + 1) \frac{dy}{dx} = 2x^2 + x$ ;  $y = 1$  when  $x = 0$

**Answer :**

The given differential equation is:

$$\begin{aligned}(x^3 + x^2 + x + 1) \frac{dy}{dx} &= 2x^2 + x \\ \Rightarrow \frac{dy}{dx} &= \frac{2x^2 + x}{(x^3 + x^2 + x + 1)} \\ \Rightarrow dy &= \frac{2x^2 + x}{(x+1)(x^2+1)} dx\end{aligned}$$

Integrating both sides, we get:

$$\int dy = \int \frac{2x^2 + x}{(x+1)(x^2+1)} dx \quad \dots(1)$$

$$\text{Let } \frac{2x^2 + x}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}. \quad \dots(2)$$

$$\begin{aligned}\Rightarrow \frac{2x^2 + x}{(x+1)(x^2+1)} &= \frac{Ax^2 + A + (Bx+C)(x+1)}{(x+1)(x^2+1)} \\ \Rightarrow 2x^2 + x &= Ax^2 + A + Bx^2 + Bx + Cx + C \\ \Rightarrow 2x^2 + x &= (A+B)x^2 + (B+C)x + (A+C)\end{aligned}$$

Comparing the coefficients of  $x^2$  and  $x$ , we get:

$$A + B = 2$$

$$B + C = 1$$

$$A + C = 0$$

Solving these equations, we get:

$$A = \frac{1}{2}, B = \frac{3}{2} \text{ and } C = -\frac{1}{2}$$

Substituting the values of A, B, and C in equation (2), we get:

$$\frac{2x^2 + x}{(x+1)(x^2+1)} = \frac{1}{2} \cdot \frac{1}{(x+1)} + \frac{1}{2} \frac{(3x-1)}{(x^2+1)}$$

Therefore, equation (1) becomes:

$$\begin{aligned}\int dy &= \frac{1}{2} \int \frac{1}{x+1} dx + \frac{1}{2} \int \frac{3x-1}{x^2+1} dx \\ \Rightarrow y &= \frac{1}{2} \log(x+1) + \frac{3}{2} \int \frac{x}{x^2+1} dx - \frac{1}{2} \int \frac{1}{x^2+1} dx \\ \Rightarrow y &= \frac{1}{2} \log(x+1) + \frac{3}{4} \cdot \int \frac{2x}{x^2+1} dx - \frac{1}{2} \tan^{-1} x + C \\ \Rightarrow y &= \frac{1}{2} \log(x+1) + \frac{3}{4} \log(x^2+1) - \frac{1}{2} \tan^{-1} x + C \\ \Rightarrow y &= \frac{1}{4} \left[ 2 \log(x+1) + 3 \log(x^2+1) \right] - \frac{1}{2} \tan^{-1} x + C \\ \Rightarrow y &= \frac{1}{4} \left[ (x+1)^2 (x^2+1)^3 \right] - \frac{1}{2} \tan^{-1} x + C \quad \dots(3)\end{aligned}$$

Now,  $y = 1$  when  $x = 0$ .

$$\Rightarrow 1 = \frac{1}{4} \log(1) - \frac{1}{2} \tan^{-1} 0 + C$$

$$\Rightarrow 1 = \frac{1}{4} \times 0 - \frac{1}{2} \times 0 + C$$

$$\Rightarrow C = 1$$

Substituting  $C = 1$  in equation (3), we get:

$$y = \frac{1}{4} \left[ \log(x+1)^2 (x^2+1)^3 \right] - \frac{1}{2} \tan^{-1} x + 1$$

Answer needs Correction? [Click Here](#)

**Q12 :**  $x(x^2 - 1) \frac{dy}{dx} = 1$ ;  $y = 0$  when  $x = 2$

**Answer :**

$$\begin{aligned}x(x^2 - 1) \frac{dy}{dx} &= 1 \\ \Rightarrow dy &= \frac{dx}{x(x^2 - 1)} \\ \Rightarrow dy &= \frac{1}{x(x-1)(x+1)} dx\end{aligned}$$

Integrating both sides, we get:

$$\int dy = \int \frac{1}{x(x-1)(x+1)} dx \quad \dots(1)$$

$$\text{Let } \frac{1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}. \quad \dots(2)$$

$$\begin{aligned}\Rightarrow \frac{1}{x(x-1)(x+1)} &= \frac{A(x-1)(x+1) + Bx(x+1) + Cx(x-1)}{x(x-1)(x+1)} \\ &= \frac{(A+B+C)x^2 + (B-C)x - A}{x(x-1)(x+1)}\end{aligned}$$

Comparing the coefficients of  $x^2$ ,  $x$ , and constant, we get:

$$A = -1$$

$$B - C = 0$$

$$A + B + C = 0$$

Solving these equations, we get  $B = \frac{1}{2}$  and  $C = \frac{1}{2}$ .

Putting these equations, we get  $x = 2$  and  $y = 2$ .

Substituting the values of  $A$ ,  $B$ , and  $C$  in equation (2), we get:

$$\frac{1}{x(x-1)(x+1)} = \frac{-1}{x} + \frac{1}{2(x-1)} + \frac{1}{2(x+1)}$$

Therefore, equation (1) becomes:

$$\begin{aligned} \int dy &= -\int \frac{1}{x} dx + \frac{1}{2} \int \frac{1}{x-1} dx + \frac{1}{2} \int \frac{1}{x+1} dx \\ \Rightarrow y &= -\log x + \frac{1}{2} \log(x-1) + \frac{1}{2} \log(x+1) + \log k \\ \Rightarrow y &= \frac{1}{2} \log \left[ \frac{k^2 (x-1)(x+1)}{x^2} \right] \quad \dots(3) \end{aligned}$$

Now,  $y = 0$  when  $x = 2$ .

$$\begin{aligned} \Rightarrow 0 &= \frac{1}{2} \log \left[ \frac{k^2 (2-1)(2+1)}{4} \right] \\ \Rightarrow \log \left( \frac{3k^2}{4} \right) &= 0 \\ \Rightarrow \frac{3k^2}{4} &= 1 \\ \Rightarrow 3k^2 &= 4 \\ \Rightarrow k^2 &= \frac{4}{3} \end{aligned}$$

Substituting the value of  $k^2$  in equation (3), we get:

$$\begin{aligned} y &= \frac{1}{2} \log \left[ \frac{4(x-1)(x+1)}{3x^2} \right] \\ y &= \frac{1}{2} \log \left[ \frac{4(x^2-1)}{3x^2} \right] \end{aligned}$$

Answer needs Correction? [Click Here](#)

**Q13 :**  $\cos \left( \frac{dy}{dx} \right) = a (a \in R); y = 1 \text{ when } x = 0$

**Answer :**

$$\begin{aligned} \cos \left( \frac{dy}{dx} \right) &= a \\ \Rightarrow \frac{dy}{dx} &= \cos^{-1} a \\ \Rightarrow dy &= \cos^{-1} a \, dx \end{aligned}$$

Integrating both sides, we get:

$$\begin{aligned} \int dy &= \cos^{-1} a \int dx \\ \Rightarrow y &= \cos^{-1} a \cdot x + C \\ \Rightarrow y &= x \cos^{-1} a + C \quad \dots(1) \end{aligned}$$

Now,  $y = 1$  when  $x = 0$ .

$$\begin{aligned} \Rightarrow 1 &= 0 \cdot \cos^{-1} a + C \\ \Rightarrow C &= 1 \end{aligned}$$

Substituting  $C = 1$  in equation (1), we get:

$$\begin{aligned} y &= x \cos^{-1} a + 1 \\ \Rightarrow \frac{y-1}{x} &= \cos^{-1} a \\ \Rightarrow \cos \left( \frac{y-1}{x} \right) &= a \end{aligned}$$

\*\*\*\*\* END \*\*\*\*\*