



**Exercise 7.4 : Solutions of Questions on Page Number : 315**

**Q1 :**  $\frac{3x^2}{x^6+1}$

**Answer :**

Let  $x^3 = t$

$\therefore 3x^2 dx = dt$

$$\begin{aligned}\Rightarrow \int \frac{3x^2}{x^6+1} dx &= \int \frac{dt}{t^2+1} \\ &= \tan^{-1} t + C \\ &= \tan^{-1}(x^3) + C\end{aligned}$$

Answer needs Correction? [Click Here](#)

**Q2 :**  $\frac{1}{\sqrt{1+4x^2}}$

**Answer :**

Let  $2x = t$

$\therefore 2dx = dt$

$$\begin{aligned}\Rightarrow \int \frac{1}{\sqrt{1+4x^2}} dx &= \frac{1}{2} \int \frac{dt}{\sqrt{1+t^2}} \\ &= \frac{1}{2} \left[ \log \left| t + \sqrt{t^2+1} \right| \right] + C \quad \left[ \int \frac{1}{\sqrt{x^2+a^2}} dt = \log \left| x + \sqrt{x^2+a^2} \right| \right] \\ &= \frac{1}{2} \log \left| 2x + \sqrt{4x^2+1} \right| + C\end{aligned}$$

Answer needs Correction? [Click Here](#)

**Q3 :**  $\frac{1}{\sqrt{(2-x)^2+1}}$

**Answer :**

Let  $2-x = t$

$\Rightarrow -dx = dt$

$$\begin{aligned}\Rightarrow \int \frac{1}{\sqrt{(2-x)^2+1}} dx &= - \int \frac{1}{\sqrt{t^2+1}} dt \\ &= - \log \left| t + \sqrt{t^2+1} \right| + C \quad \left[ \int \frac{1}{\sqrt{x^2+a^2}} dt = \log \left| x + \sqrt{x^2+a^2} \right| \right] \\ &= - \log \left| 2-x + \sqrt{(2-x)^2+1} \right| + C \\ &= \log \left| \frac{1}{(2-x) + \sqrt{x^2-4x+5}} \right| + C\end{aligned}$$

Answer needs Correction? [Click Here](#)

**Q4 :**  $\frac{1}{\sqrt{9-25x^2}}$

**Answer :**

Let  $5x = t$

$\therefore 5dx = dt$

$$\begin{aligned}\Rightarrow \int \frac{1}{\sqrt{9-25x^2}} dx &= \frac{1}{5} \int \frac{1}{\sqrt{9-t^2}} dt \\ &= \frac{1}{5} \int \frac{1}{\sqrt{3^2-t^2}} dt \\ &= \frac{1}{5} \sin^{-1} \left( \frac{t}{3} \right) + C \\ &= \frac{1}{5} \sin^{-1} \left( \frac{5x}{3} \right) + C\end{aligned}$$

Answer needs Correction? [Click Here](#)

Q5:  $\frac{3x}{1+2x^4}$

Answer :

Let  $\sqrt{2}x^2 = t$

$\therefore 2\sqrt{2}x \, dx = dt$

$$\begin{aligned}\Rightarrow \int \frac{3x}{1+2x^4} dx &= \frac{3}{2\sqrt{2}} \int \frac{dt}{1+t^2} \\ &= \frac{3}{2\sqrt{2}} \left[ \tan^{-1} t \right] + C \\ &= \frac{3}{2\sqrt{2}} \tan^{-1} (\sqrt{2}x^2) + C\end{aligned}$$

Answer needs Correction? [Click Here](#)

Q6:  $\frac{x^2}{1-x^6}$

Answer :

Let  $x^3 = t$

$\therefore 3x^2 \, dx = dt$

$$\begin{aligned}\Rightarrow \int \frac{x^2}{1-x^6} dx &= \frac{1}{3} \int \frac{dt}{1-t^2} \\ &= \frac{1}{3} \left[ \frac{1}{2} \log \left| \frac{1+t}{1-t} \right| \right] + C \\ &= \frac{1}{6} \log \left| \frac{1+x^3}{1-x^3} \right| + C\end{aligned}$$

Answer needs Correction? [Click Here](#)

Q7:  $\frac{x-1}{\sqrt{x^2-1}}$

Answer :

$$\int \frac{x-1}{\sqrt{x^2-1}} \, dx = \int \frac{x}{\sqrt{x^2-1}} \, dx - \int \frac{1}{\sqrt{x^2-1}} \, dx \quad \dots(1)$$

For  $\int \frac{x}{\sqrt{x^2-1}} \, dx$ , let  $x^2-1 = t \Rightarrow 2x \, dx = dt$

$$\begin{aligned}\therefore \int \frac{x}{\sqrt{x^2-1}} \, dx &= \frac{1}{2} \int \frac{dt}{\sqrt{t}} \\ &= \frac{1}{2} \int t^{-\frac{1}{2}} \, dt \\ &= \frac{1}{2} \left[ 2t^{\frac{1}{2}} \right] \\ &= \sqrt{t} \\ &= \sqrt{x^2-1}\end{aligned}$$

From (1), we obtain

$$\begin{aligned}\int \frac{x-1}{\sqrt{x^2-1}} \, dx &= \int \frac{x}{\sqrt{x^2-1}} \, dx - \int \frac{1}{\sqrt{x^2-1}} \, dx \quad \left[ \int \frac{1}{\sqrt{x^2-a^2}} \, dx = \log \left| x + \sqrt{x^2-a^2} \right| \right] \\ &= \sqrt{x^2-1} - \log \left| x + \sqrt{x^2-1} \right| + C\end{aligned}$$

Answer needs Correction? [Click Here](#)

Q8:  $\frac{x^2}{\sqrt{x^6+a^6}}$

Answer :

Let  $x^3 = t$

$\Rightarrow 3x^2 \, dx = dt$

$$\begin{aligned}\therefore \int \frac{x^2}{\sqrt{x^6+a^6}} \, dx &= \frac{1}{3} \int \frac{dt}{\sqrt{t^2+(a^3)^2}} \\ &= \frac{1}{3} \log \left| t + \sqrt{t^2+a^6} \right| + C \\ &= \frac{1}{3} \log \left| x^3 + \sqrt{x^6+a^6} \right| + C\end{aligned}$$

Answer needs Correction? [Click Here](#)

Q9:  $\frac{\sec^2 x}{\sqrt{\tan^3 x + 4}}$

Answer :

Let  $\tan x = t$

$\therefore \sec^2 x \, dx = dt$

$$\begin{aligned}
 \Rightarrow \int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx &= \int \frac{dt}{\sqrt{t^2 + 2^2}} \\
 &= \log \left| t + \sqrt{t^2 + 4} \right| + C \\
 &= \log \left| \tan x + \sqrt{\tan^2 x + 4} \right| + C
 \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q10 :  $\frac{1}{\sqrt{x^2 + 2x + 2}}$

Answer :

$$\begin{aligned}
 \int \frac{1}{\sqrt{x^2 + 2x + 2}} dx &= \int \frac{1}{\sqrt{(x+1)^2 + (1)^2}} dx \\
 \text{Let } x+1 &= t \\
 \therefore dx &= dt \\
 \Rightarrow \int \frac{1}{\sqrt{x^2 + 2x + 2}} dx &= \int \frac{1}{\sqrt{t^2 + 1}} dt \\
 &= \log \left| t + \sqrt{t^2 + 1} \right| + C \\
 &= \log \left| (x+1) + \sqrt{(x+1)^2 + 1} \right| + C \\
 &= \log \left| (x+1) + \sqrt{x^2 + 2x + 2} \right| + C
 \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q11 :

Answer :

Answer needs Correction? [Click Here](#)

Q12 :  $\frac{1}{\sqrt{7-6x-x^2}}$

Answer :

$$\begin{aligned}
 7-6x-x^2 &\text{ can be written as } 7-(x^2+6x+9-9). \\
 \text{Therefore,} \\
 7-(x^2+6x+9-9) \\
 &= 16-(x^2+6x+9) \\
 &= 16-(x+3)^2 \\
 &= (4)^2-(x+3)^2 \\
 \therefore \int \frac{1}{\sqrt{7-6x-x^2}} dx &= \int \frac{1}{\sqrt{(4)^2-(x+3)^2}} dx \\
 \text{Let } x+3 &= t \\
 \Rightarrow dx &= dt \\
 \Rightarrow \int \frac{1}{\sqrt{(4)^2-(x+3)^2}} dx &= \int \frac{1}{\sqrt{(4)^2-(t)^2}} dt \\
 &= \sin^{-1} \left( \frac{t}{4} \right) + C \\
 &= \sin^{-1} \left( \frac{x+3}{4} \right) + C
 \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q13 :  $\frac{1}{\sqrt{(x-1)(x-2)}}$

Answer :

$$\begin{aligned}
 (x-1)(x-2) &\text{ can be written as } x^2-3x+2. \\
 \text{Therefore,} \\
 x^2-3x+2 \\
 &= x^2-3x+\frac{9}{4}-\frac{9}{4}+2 \\
 &= \left(x-\frac{3}{2}\right)^2-\frac{1}{4} \\
 &= \left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2 \\
 \therefore \int \frac{1}{\sqrt{(x-1)(x-2)}} dx &= \int \frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}} dx \\
 \text{Let } x-\frac{3}{2} &= t \\
 \therefore dx &= dt \\
 \Rightarrow \int \frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}} dx &= \int \frac{1}{\sqrt{t^2-\left(\frac{1}{2}\right)^2}} dt
 \end{aligned}$$

$$\begin{aligned}
 \sqrt{\left(x - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2} &= \log \left| t + \sqrt{t^2 - \left(\frac{1}{2}\right)^2} \right| + C \\
 &= \log \left| \left(x - \frac{3}{2}\right) + \sqrt{x^2 - 3x + 2} \right| + C
 \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q14:  $\frac{1}{\sqrt{8+3x-x^2}}$

Answer :

$8+3x-x^2$  can be written as  $8 - \left(x^2 - 3x + \frac{9}{4} - \frac{9}{4}\right)$ .

Therefore,

$$\begin{aligned}
 &8 - \left(x^2 - 3x + \frac{9}{4} - \frac{9}{4}\right) \\
 &= \frac{41}{4} - \left(x - \frac{3}{2}\right)^2 \\
 &\Rightarrow \int \frac{1}{\sqrt{8+3x-x^2}} dx = \int \frac{1}{\sqrt{\frac{41}{4} - \left(x - \frac{3}{2}\right)^2}} dx
 \end{aligned}$$

Let  $x - \frac{3}{2} = t$

$\therefore dx = dt$

$$\begin{aligned}
 \Rightarrow \int \frac{1}{\sqrt{\frac{41}{4} - \left(x - \frac{3}{2}\right)^2}} dx &= \int \frac{1}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2 - t^2}} dt \\
 &= \sin^{-1} \left( \frac{t}{\frac{\sqrt{41}}{2}} \right) + C \\
 &= \sin^{-1} \left( \frac{x - \frac{3}{2}}{\frac{\sqrt{41}}{2}} \right) + C \\
 &= \sin^{-1} \left( \frac{2x-3}{\sqrt{41}} \right) + C
 \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q15:  $\frac{1}{\sqrt{(x-a)(x-b)}}$

Answer :

$(x-a)(x-b)$  can be written as  $x^2 - (a+b)x + ab$ .

Therefore,

$$\begin{aligned}
 &x^2 - (a+b)x + ab \\
 &= x^2 - (a+b)x + \frac{(a+b)^2}{4} - \frac{(a+b)^2}{4} + ab \\
 &= \left[ x - \frac{(a+b)}{2} \right]^2 - \frac{(a-b)^2}{4} \\
 &\Rightarrow \int \frac{1}{\sqrt{(x-a)(x-b)}} dx = \int \frac{1}{\sqrt{\left\{ x - \frac{(a+b)}{2} \right\}^2 - \left( \frac{a-b}{2} \right)^2}} dx
 \end{aligned}$$

Let  $x - \frac{(a+b)}{2} = t$

$\therefore dx = dt$

$$\begin{aligned}
 \Rightarrow \int \frac{1}{\sqrt{\left\{ x - \frac{(a+b)}{2} \right\}^2 - \left( \frac{a-b}{2} \right)^2}} dx &= \int \frac{1}{\sqrt{t^2 - \left( \frac{a-b}{2} \right)^2}} dt \\
 &= \log \left| t + \sqrt{t^2 - \left( \frac{a-b}{2} \right)^2} \right| + C \\
 &= \log \left| \left\{ x - \frac{(a+b)}{2} \right\} + \sqrt{(x-a)(x-b)} \right| + C
 \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q16:  $\frac{4x+1}{\sqrt{2x^2+x-3}}$

Answer :

Let  $4x+1 = A \frac{d}{dx}(2x^2+x-3) + B$

$\Rightarrow 4x+1 = A(4x+1) + B$

$\Rightarrow 4x+1 = 4Ax + A + B$

Equating the coefficients of  $x$  and constant term on both sides, we obtain

$4A = 4 \Rightarrow A = 1$

$$A + B = 1 \Rightarrow B = 0$$

$$\text{Let } 2x^2 + x - 3 = t$$

$$\therefore (4x + 1) dx = dt$$

$$\begin{aligned} \Rightarrow \int \frac{4x+1}{\sqrt{2x^2+x-3}} dx &= \int \frac{1}{\sqrt{t}} dt \\ &= 2\sqrt{t} + C \\ &= 2\sqrt{2x^2+x-3} + C \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q17:  $\frac{x+2}{\sqrt{x^2-1}}$

Answer :

$$\text{Let } x+2 = A \frac{d}{dx}(x^2-1) + B \quad \dots(1)$$

$$\Rightarrow x+2 = A(2x) + B$$

Equating the coefficients of  $x$  and constant term on both sides, we obtain

$$2A = 1 \Rightarrow A = \frac{1}{2}$$

$$B = 2$$

From (1), we obtain

$$(x+2) = \frac{1}{2}(2x) + 2$$

$$\begin{aligned} \text{Then, } \int \frac{x+2}{\sqrt{x^2-1}} dx &= \int \frac{\frac{1}{2}(2x) + 2}{\sqrt{x^2-1}} dx \\ &= \frac{1}{2} \int \frac{2x}{\sqrt{x^2-1}} dx + \int \frac{2}{\sqrt{x^2-1}} dx \quad \dots(2) \end{aligned}$$

$$\ln \frac{1}{2} \int \frac{2x}{\sqrt{x^2-1}} dx, \text{ let } x^2-1 = t \Rightarrow 2x dx = dt$$

$$\begin{aligned} \frac{1}{2} \int \frac{2x}{\sqrt{x^2-1}} dx &= \frac{1}{2} \int \frac{dt}{\sqrt{t}} \\ &= \frac{1}{2} [2\sqrt{t}] \\ &= \sqrt{t} \\ &= \sqrt{x^2-1} \end{aligned}$$

$$\text{Then, } \int \frac{2}{\sqrt{x^2-1}} dx = 2 \int \frac{1}{\sqrt{x^2-1}} dx = 2 \log |x + \sqrt{x^2-1}|$$

From equation (2), we obtain

$$\int \frac{x+2}{\sqrt{x^2-1}} dx = \sqrt{x^2-1} + 2 \log |x + \sqrt{x^2-1}| + C$$

Answer needs Correction? [Click Here](#)

Q18:  $\frac{6x+7}{\sqrt{(x-5)(x-4)}}$

Answer :

$$\frac{6x+7}{\sqrt{(x-5)(x-4)}} = \frac{6x+7}{\sqrt{x^2-9x+20}}$$

$$\text{Let } 6x+7 = A \frac{d}{dx}(x^2-9x+20) + B$$

$$\Rightarrow 6x+7 = A(2x-9) + B$$

Equating the coefficients of  $x$  and constant term, we obtain

$$2A = 6 \Rightarrow A = 3$$

$$-9A + B = 7 \Rightarrow B = 34$$

$$\therefore 6x+7 = 3(2x-9) + 34$$

$$\begin{aligned} \int \frac{6x+7}{\sqrt{x^2-9x+20}} &= \int \frac{3(2x-9) + 34}{\sqrt{x^2-9x+20}} dx \\ &= 3 \int \frac{2x-9}{\sqrt{x^2-9x+20}} dx + 34 \int \frac{1}{\sqrt{x^2-9x+20}} dx \end{aligned}$$

$$\text{Let } I_1 = \int \frac{2x-9}{\sqrt{x^2-9x+20}} dx \text{ and } I_2 = \int \frac{1}{\sqrt{x^2-9x+20}} dx$$

$$\therefore \int \frac{6x+7}{\sqrt{x^2-9x+20}} = 3I_1 + 34I_2 \quad \dots(1)$$

Then,

$$I_1 = \int \frac{2x-9}{\sqrt{x^2-9x+20}} dx$$

$$\text{Let } x^2-9x+20 = t$$

$$\Rightarrow (2x-9) dx = dt$$

$$\Rightarrow I_1 = \int \frac{dt}{\sqrt{t}}$$

$$I_1 = 2\sqrt{t}$$

$$I_1 = 2\sqrt{x^2-9x+20} \quad \dots(2)$$

$$\text{and } I_2 = \int \frac{1}{\sqrt{x^2-9x+20}} dx$$

$$\sqrt{x^2 - 9x + 20}$$

$x^2 - 9x + 20$  can be written as  $x^2 - 9x + 20 + \frac{81}{4} - \frac{81}{4}$ .

Therefore,

$$x^2 - 9x + 20 + \frac{81}{4} - \frac{81}{4}$$

$$= \left(x - \frac{9}{2}\right)^2 - \frac{1}{4}$$

$$= \left(x - \frac{9}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$\Rightarrow I_2 = \int \frac{1}{\sqrt{\left(x - \frac{9}{2}\right)^2 - \left(\frac{1}{2}\right)^2}} dx$$

$$I_2 = \log \left| \left(x - \frac{9}{2}\right) + \sqrt{x^2 - 9x + 20} \right| \quad \dots(3)$$

Substituting equations (2) and (3) in (1), we obtain

$$\begin{aligned} \int \frac{6x+7}{\sqrt{x^2-9x+20}} dx &= 3 \left[ 2\sqrt{x^2-9x+20} \right] + 34 \log \left[ \left(x - \frac{9}{2}\right) + \sqrt{x^2-9x+20} \right] + C \\ &= 6\sqrt{x^2-9x+20} + 34 \log \left[ \left(x - \frac{9}{2}\right) + \sqrt{x^2-9x+20} \right] + C \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q19 :  $\frac{x+2}{\sqrt{4x-x^2}}$

Answer :

$$\text{Let } x+2 = A \frac{d}{dx} (4x-x^2) + B$$

$$\Rightarrow x+2 = A(4-2x) + B$$

Equating the coefficients of  $x$  and constant term on both sides, we obtain

$$-2A = 1 \Rightarrow A = -\frac{1}{2}$$

$$4A + B = 2 \Rightarrow B = 4$$

$$\Rightarrow (x+2) = -\frac{1}{2}(4-2x) + 4$$

$$\begin{aligned} \therefore \int \frac{x+2}{\sqrt{4x-x^2}} dx &= \int \frac{-\frac{1}{2}(4-2x) + 4}{\sqrt{4x-x^2}} dx \\ &= -\frac{1}{2} \int \frac{4-2x}{\sqrt{4x-x^2}} dx + 4 \int \frac{1}{\sqrt{4x-x^2}} dx \end{aligned}$$

$$\text{Let } I_1 = \int \frac{4-2x}{\sqrt{4x-x^2}} dx \text{ and } I_2 = \int \frac{1}{\sqrt{4x-x^2}} dx$$

$$\therefore \int \frac{x+2}{\sqrt{4x-x^2}} dx = -\frac{1}{2} I_1 + 4 I_2 \quad \dots(1)$$

$$\text{Then, } I_1 = \int \frac{4-2x}{\sqrt{4x-x^2}} dx$$

$$\text{Let } 4x-x^2 = t$$

$$\Rightarrow (4-2x) dx = dt$$

$$\Rightarrow I_1 = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} = 2\sqrt{4x-x^2} \quad \dots(2)$$

$$I_2 = \int \frac{1}{\sqrt{4x-x^2}} dx$$

$$\Rightarrow 4x-x^2 = -(4x+x^2)$$

$$= (-4x+x^2+4-4)$$

$$= 4 - (x-2)^2$$

$$= (2)^2 - (x-2)^2$$

$$\therefore I_2 = \int \frac{1}{\sqrt{(2)^2 - (x-2)^2}} dx = \sin^{-1} \left( \frac{x-2}{2} \right) \quad \dots(3)$$

Using equations (2) and (3) in (1), we obtain

$$\begin{aligned} \int \frac{x+2}{\sqrt{4x-x^2}} dx &= -\frac{1}{2} (2\sqrt{4x-x^2}) + 4 \sin^{-1} \left( \frac{x-2}{2} \right) + C \\ &= -\sqrt{4x-x^2} + 4 \sin^{-1} \left( \frac{x-2}{2} \right) + C \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q20 :  $\frac{x+2}{\sqrt{x^2+2x+3}}$

Answer :

$$\begin{aligned} \int \frac{(x+2)}{\sqrt{x^2+2x+3}} dx &= \frac{1}{2} \int \frac{2(x+2)}{\sqrt{x^2+2x+3}} dx \\ &= \frac{1}{2} \int \frac{2x+4}{\sqrt{x^2+2x+3}} dx \\ &= \frac{1}{2} \int \frac{2x+2}{\sqrt{x^2+2x+3}} dx + \frac{1}{2} \int \frac{2}{\sqrt{x^2+2x+3}} dx \end{aligned}$$

$$= \frac{1}{2} \int \frac{2x+2}{\sqrt{x^2+2x+3}} dx + \int \frac{1}{\sqrt{x^2+2x+3}} dx$$

Let  $I_1 = \int \frac{2x+2}{\sqrt{x^2+2x+3}} dx$  and  $I_2 = \int \frac{1}{\sqrt{x^2+2x+3}} dx$

$$\therefore \int \frac{x+2}{\sqrt{x^2+2x+3}} dx = \frac{1}{2} I_1 + I_2 \quad \dots(1)$$

Then,  $I_1 = \int \frac{2x+2}{\sqrt{x^2+2x+3}} dx$

$$\text{Let } x^2 + 2x + 3 = t$$

$$\Rightarrow (2x+2) dx = dt$$

$$I_1 = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} = 2\sqrt{x^2+2x+3} \quad \dots(2)$$

$$I_2 = \int \frac{1}{\sqrt{x^2+2x+3}} dx$$

$$\Rightarrow x^2 + 2x + 3 = x^2 + 2x + 1 + 2 = (x+1)^2 + (\sqrt{2})^2$$

$$\therefore I_2 = \int \frac{1}{\sqrt{(x+1)^2 + (\sqrt{2})^2}} dx = \log \left| (x+1) + \sqrt{x^2+2x+3} \right| \quad \dots(3)$$

Using equations (2) and (3) in (1), we obtain

$$\int \frac{x+2}{\sqrt{x^2+2x+3}} dx = \frac{1}{2} \left[ 2\sqrt{x^2+2x+3} \right] + \log \left| (x+1) + \sqrt{x^2+2x+3} \right| + C$$

$$= \sqrt{x^2+2x+3} + \log \left| (x+1) + \sqrt{x^2+2x+3} \right| + C$$

Answer needs Correction? [Click Here](#)

Q21 :  $\frac{x+3}{x^2-2x-5}$

Answer :

$$\text{Let } (x+3) = A \frac{d}{dx} (x^2 - 2x - 5) + B$$

$$(x+3) = A(2x-2) + B$$

Equating the coefficients of  $x$  and constant term on both sides, we obtain

$$2A = 1 \Rightarrow A = \frac{1}{2}$$

$$-2A + B = 3 \Rightarrow B = 4$$

$$\therefore (x+3) = \frac{1}{2}(2x-2) + 4$$

$$\Rightarrow \int \frac{x+3}{x^2-2x-5} dx = \int \frac{\frac{1}{2}(2x-2) + 4}{x^2-2x-5} dx$$

$$= \frac{1}{2} \int \frac{2x-2}{x^2-2x-5} dx + 4 \int \frac{1}{x^2-2x-5} dx$$

$$\text{Let } I_1 = \int \frac{2x-2}{x^2-2x-5} dx \text{ and } I_2 = \int \frac{1}{x^2-2x-5} dx$$

$$\therefore \int \frac{x+3}{(x^2-2x-5)} dx = \frac{1}{2} I_1 + 4I_2 \quad \dots(1)$$

$$\text{Then, } I_1 = \int \frac{2x-2}{x^2-2x-5} dx$$

$$\text{Let } x^2 - 2x - 5 = t$$

$$\Rightarrow (2x-2) dx = dt$$

$$\Rightarrow I_1 = \int \frac{dt}{t} = \log |t| = \log |x^2 - 2x - 5| \quad \dots(2)$$

$$I_2 = \int \frac{1}{x^2-2x-5} dx$$

$$= \int \frac{1}{(x^2-2x+1)-6} dx$$

$$= \int \frac{1}{(x-1)^2 + (\sqrt{6})^2} dx$$

$$= \frac{1}{2\sqrt{6}} \log \left( \frac{x-1-\sqrt{6}}{x-1+\sqrt{6}} \right) \quad \dots(3)$$

Substituting (2) and (3) in (1), we obtain

$$\int \frac{x+3}{x^2-2x-5} dx = \frac{1}{2} \log |x^2 - 2x - 5| + \frac{4}{2\sqrt{6}} \log \left| \frac{x-1-\sqrt{6}}{x-1+\sqrt{6}} \right| + C$$

$$= \frac{1}{2} \log |x^2 - 2x - 5| + \frac{2}{\sqrt{6}} \log \left| \frac{x-1-\sqrt{6}}{x-1+\sqrt{6}} \right| + C$$

Answer needs Correction? [Click Here](#)

Q22 :  $\frac{5x+3}{\sqrt{x^2+4x+10}}$

Answer :

$$\text{Let } 5x+3 = A \frac{d}{dx} (x^2 + 4x + 10) + B$$

$$\Rightarrow 5x+3 = A(2x+4) + B$$

Equating the coefficients of  $x$  and constant term, we obtain

$$\begin{aligned}
 2A &= 5 \Rightarrow A = \frac{5}{2} \\
 4A + B &= 3 \Rightarrow B = -7 \\
 \therefore 5x + 3 &= \frac{5}{2}(2x + 4) - 7 \\
 \Rightarrow \int \frac{5x + 3}{\sqrt{x^2 + 4x + 10}} dx &= \int \frac{\frac{5}{2}(2x + 4) - 7}{\sqrt{x^2 + 4x + 10}} dx \\
 &= \frac{5}{2} \int \frac{2x + 4}{\sqrt{x^2 + 4x + 10}} dx - 7 \int \frac{1}{\sqrt{x^2 + 4x + 10}} dx \\
 \text{Let } I_1 &= \int \frac{2x + 4}{\sqrt{x^2 + 4x + 10}} dx \text{ and } I_2 = \int \frac{1}{\sqrt{x^2 + 4x + 10}} dx \\
 \therefore \int \frac{5x + 3}{\sqrt{x^2 + 4x + 10}} dx &= \frac{5}{2} I_1 - 7 I_2 \quad \dots(1) \\
 \text{Then, } I_1 &= \int \frac{2x + 4}{\sqrt{x^2 + 4x + 10}} dx
 \end{aligned}$$

$$\begin{aligned}
 \text{Let } x^2 + 4x + 10 &= t \\
 \therefore (2x + 4) dx &= dt \\
 \Rightarrow I_1 &= \int \frac{dt}{t} = 2\sqrt{t} = 2\sqrt{x^2 + 4x + 10} \quad \dots(2) \\
 I_2 &= \int \frac{1}{\sqrt{x^2 + 4x + 10}} dx \\
 &= \int \frac{1}{\sqrt{(x^2 + 4x + 4) + 6}} dx \\
 &= \int \frac{1}{(x + 2)^2 + (\sqrt{6})^2} dx \\
 &= \log |(x + 2)\sqrt{x^2 + 4x + 10}| \quad \dots(3)
 \end{aligned}$$

Using equations (2) and (3) in (1), we obtain

$$\begin{aligned}
 \int \frac{5x + 3}{\sqrt{x^2 + 4x + 10}} dx &= \frac{5}{2} \left[ 2\sqrt{x^2 + 4x + 10} \right] - 7 \log |(x + 2) + \sqrt{x^2 + 4x + 10}| + C \\
 &= 5\sqrt{x^2 + 4x + 10} - 7 \log |(x + 2) + \sqrt{x^2 + 4x + 10}| + C
 \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q23 :  $\int \frac{dx}{x^2 + 2x + 2}$  equals

- A.  $x \tan^{-1}(x + 1) + C$
- B.  $\tan^{-1}(x + 1) + C$
- C.  $(x + 1) \tan^{-1} x + C$
- D.  $\tan^{-1} x + C$

Answer :

$$\begin{aligned}
 \int \frac{dx}{x^2 + 2x + 2} &= \int \frac{dx}{(x^2 + 2x + 1) + 1} \\
 &= \int \frac{1}{(x + 1)^2 + (1)^2} dx \\
 &= \left[ \tan^{-1}(x + 1) \right] + C
 \end{aligned}$$

Hence, the correct answer is B.

Answer needs Correction? [Click Here](#)

Q24 :  $\int \frac{dx}{\sqrt{9x - 4x^2}}$  equals

- A.  $\frac{1}{9} \sin^{-1} \left( \frac{9x - 8}{8} \right) + C$
- B.  $\frac{1}{2} \sin^{-1} \left( \frac{8x - 9}{9} \right) + C$
- C.  $\frac{1}{3} \sin^{-1} \left( \frac{9x - 8}{8} \right) + C$
- D.  $\frac{1}{2} \sin^{-1} \left( \frac{9x - 8}{9} \right) + C$

Answer :

$$\begin{aligned}
 \int \frac{dx}{\sqrt{9x - 4x^2}} &= \int \frac{1}{\sqrt{-4 \left( x^2 - \frac{9}{4}x \right)}} dx \\
 &= \int \frac{1}{-4 \left( x^2 - \frac{9}{4}x + \frac{81}{64} - \frac{81}{64} \right)} dx \\
 &= \int \frac{1}{\sqrt{-4 \left[ \left( x - \frac{9}{8} \right)^2 - \left( \frac{9}{8} \right)^2 \right]}} dx \\
 &= \frac{1}{4} \int \frac{1}{\sqrt{\left( x - \frac{9}{8} \right)^2 - \left( \frac{9}{8} \right)^2}} dx
 \end{aligned}$$



$$\begin{aligned}
 &= \frac{1}{2} \int \sqrt{\left(\frac{9}{8}\right)^2 - \left(x - \frac{9}{8}\right)^2} dx \\
 &= \frac{1}{2} \left[ \sin^{-1} \left( \frac{x - \frac{9}{8}}{\frac{9}{8}} \right) \right] + C \qquad \left( \int \frac{dy}{\sqrt{a^2 - y^2}} = \sin^{-1} \frac{y}{a} + C \right) \\
 &= \frac{1}{2} \sin^{-1} \left( \frac{8x - 9}{9} \right) + C
 \end{aligned}$$

Hence, the correct answer is B.

\*\*\*\*\* END \*\*\*\*\*