Exercise 2.3

Q 3. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2-3, 2t^4+3t^3-2t^2-9t-12$

(ii) $x^2+3x+1, 3x^4+5x^3-7x^2+2x+2$

(iii) $x^3-3x+1, x^5-4x^3+x^2+3x+1$

Answer :

(i) $t^2-3,\ 2t^4+3t^3-2t^2-9t-12$

$t^2-3 = t^2+0.t-3$

$$
\begin{array}{r|l}
 & \quad 2t^2+3t+4 \\
\hline
t^2+0.t-3 & 2t^4+3t^3-2t^2-9t-12 \\
 & 2t^4+0.t^3-6t^2 \\
 & - \quad - \quad + \\
\cline{2-2}
 & \quad 3t^3+4t^2-9t-12 \\
 & \quad 3t^3+0.t^2-9t \\
 & \quad - \quad - \quad + \\
\cline{2-2}
 & \qquad 4t^2+0.t-12 \\
 & \qquad 4t^2+0.t-12 \\
 & \qquad - \quad - \quad + \\
\cline{2-2}
 & \qquad\qquad 0 \\
\end{array}
$$

Since the remainder is 0,

Hence, t^2-3 is a factor of $2t^4+3t^3-2t^2-9t-12$.

(ii) $x^2+3x+1,\ 3x^4+5x^3-7x^2+2x+2$

$$
\begin{array}{r|l}
 & 3x^2-4x+2 \\
\hline
x^2+3x+1 & 3x^4+5x^3-7x^2+2x+2 \\
 & 3x^4+9x^3+3x^2 \\
 & -\quad\ -\quad\ - \\
\cline{2-2}
 & \quad -4x^3-10x^2+2x+2 \\
 & \quad -4x^3-12x^2-4x \\
 & \quad +\quad\ +\quad\ + \\
\cline{2-2}
 & \qquad\qquad 2x^2+6x+2 \\
 & \qquad\qquad 2x^2+6x+2 \\
\cline{2-2}
 & \qquad\qquad\qquad 0 \\
\cline{2-2}
\end{array}
$$

Since the remainder is 0,

Hence, x^2+3x+1 is a factor of $3x^4+5x^3-7x^2+2x+2$.

(iii) $x^3-3x+1,\ x^5-4x^3+x^2+3x+1$

$$
\begin{array}{r|l}
 & \quad\quad x^2-1 \\ \hline
x^3-3x+1 & x^5-4x^3+x^2+3x+1 \\
 & x^5-3x^3+x^2 \\
 & - \quad\ + \quad\ - \\ \hline
 & \quad -x^3 \quad\quad +3x+1 \\
 & \quad -x^3 \quad\quad +3x-1 \\
 & \quad + \quad\quad\quad - \quad + \\ \hline
 & \quad\quad\quad\quad\quad\quad\quad 2 \\ \hline
\end{array}
$$

Since the remainder $\neq 0$,

Hence, x^3-3x+1 is not a factor of $x^5-4x^3+x^2+3x+1$.

Q 4. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, - 7, - 14 respectively.

Answer :

Let the polynomial be $ax^3 + bx^2 + cx + d$ and the zeroes be $\alpha, \beta, \text{ and } \gamma$.

It is given that

$$\alpha + \beta + \gamma = \frac{2}{1} = \frac{-b}{a}$$
$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{-7}{1} = \frac{c}{a}$$
$$\alpha\beta\gamma = \frac{-14}{1} = \frac{-d}{a}$$

If a = 1, then b = - 2, c = - 7, d = 14

Hence, the polynomial is $x^3 - 2x^2 - 7x + 14$.

Q 5. Obtain all other zeroes of $3x^4+6x^3-2x^2-10x-5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Answer :

$p(x)=3x^4+6x^3-2x^2-10x-5$

Since the two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$,

$\therefore \left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)=\left(x^2-\frac{5}{3}\right)$ is a factor of $3x^4+6x^3-2x^2-10x-5$.

Therefore, we divide the given polynomial by $x^2-\frac{5}{3}$.

$$
\begin{array}{r|l}
 & 3x^2+6x+3 \\
\hline
x^2+0.x-\frac{5}{3} & 3x^4+6x^3-2x^2-10x-5 \\
 & 3x^4+0x^3-5x^2 \\
 & -\quad -\quad + \\
\hline
 & 6x^3+3x^2-10x-5 \\
 & 6x^3+0x^2-10x \\
 & -\quad -\quad + \\
\hline
 & 3x^2+0x-5 \\
 & 3x^2+0x-5 \\
 & -\quad -\quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

$$
\begin{aligned}
3x^4+6x^3-2x^2-10x-5 &= \left(x^2-\frac{5}{3}\right)\left(3x^2+6x+3\right) \\
&= 3\left(x^2-\frac{5}{3}\right)\left(x^2+2x+1\right)
\end{aligned}
$$

We factorize x^2+2x+1

$=(x+1)^2$

Therefore, its zero is given by $x + 1 = 0$

$x = -1$

As it has the term $(x+1)^2$, therefore, there will be 2 zeroes at $x = -1$.

*********** END **********