Functions Ex 2.1 Q11

We have $f : R \to R$ given by $f(x) = 4x^3 + 7$

Let $x, y \in R$ such that

$$f(a) = f(b)$$

$$4a^3 + 7 = 4b^3 + 7$$

$$a = b$$

f is one-one.

Now let $y \in R$ be arbitrary, then

$$f(x) = y$$

$$4x^3 + 7 = y$$

$$x = (y - 7)^{\frac{1}{3}} \in R$$

f is onto.

Hence the function is a bijection

Functions Ex 2.1 Q12

We have $f : R \to R$ given by $f(x) = e^x$

let $x, y \in R$, such that

$$f(x) = f(y)$$
$$\Rightarrow \quad e^x = e^y$$
$$\Rightarrow \quad e^{x-y} = 1 = e^0$$
$$\Rightarrow \quad x - y = 0$$
$$\Rightarrow \quad x = y$$

$\therefore$ f is one-one

clearly range of $f = (0, \infty) \neq R$

$\therefore$ f is not onto

When co-domain in replaced by R_0^+ $i.e, (0, \infty)$ then f becomes an onto function.

Functions Ex 2.1 Q13

We have $f : R_0^+ \to R$ given by $f(x) = log_a x : a > 0$

let $x, y \in R_0^+$, such that

$$f(x) = f(y)$$
$$\Rightarrow \quad log_a x = log_a y$$
$$\Rightarrow \quad log_a^x \left(x/y\right) = 0$$
$$\Rightarrow \quad x/y = 1$$
$$\Rightarrow \quad x = y$$

$\therefore$ f is one-one

Now, let $y \in R$ be arbitrany, then

$$f(x) = y$$
$$\Rightarrow \quad log_a x = y \quad \Rightarrow \quad x = a^y \in R_0^+ \qquad \left[\because \ a > 0 \Rightarrow a^y > 0\right]$$

Thus, for all $y \in R$, there exist $x = a^y$ such that $f(x) = y$

$\therefore$ f is onto

$\because$ f is one-one and onto $\quad \therefore$ f is bijective

Functions Ex 2.1 Q14

Since f is one-one, three elements of {1, 2, 3} must be taken to 3 different elements of the co-domain {1, 2, 3} under f.

Hence, f has to be onto.

Functions Ex 2.1 Q15

Suppose f is not one-one.

Then, there exists two elements, say 1 and 2 in the domain whose image in the co-domain is same.

Also, the image of 3 under f can be only one element.

Therefore, the range set can have at most two elements of the co-domain {1, 2, 3}

i.e f is not an onto function, a contradiction.

Hence, f must be one-one.

Functions Ex 2.1 Q16

Onto functions from the set $\{1, 2, 3, \ldots, n\}$ to itself is simply a permutation on n symbols $1, 2, \ldots, n$.

Thus, the total number of onto maps from $\{1, 2, \ldots, n\}$ to itself is the same as the total number of permutations on n symbols $1, 2, \ldots, n$, which is $n!$.

*********** END **********