Areas Related to Circles Ex 15.3 Q5

Answer :

We know that the area of minor segment of angle θ in a circle of radius r is,

$$A=\left\{\frac{\pi\theta}{360°}-\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\}r^2$$

It is given that,

$r=14$ cm

$\theta=60°$

Substituting these values in above formula

$$A=\left\{\frac{3.14\times 60°}{360°}-\sin\frac{60°}{2}\cos\frac{60°}{2}\right\}\times 14\times 14$$

$$=\left\{\frac{3.14}{6}-\sin 30°\cos 30°\right\}\times 196$$

$$=\frac{3.14\times 196}{6}-\frac{1}{2}\times\frac{\sqrt{3}}{2}\times 196$$

$$=102.573-84.868$$

$$\boxed{A=17.70\text{ cm}^2}$$

Areas Related to Circles Ex 15.3 Q6

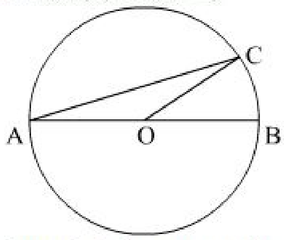
Answer :

We know that the area of minor segment of angle θ in a circle of radius r is,

$$A=\left\{\frac{\pi\theta}{360°}-\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\}r^2.$$

It is given that, $\angle BOC=\theta$

So, $\angle AOC=180°-\theta$



Area, A of minor segment cutoff by AC at angle $\angle AOC=180-\theta$

$$A=\left\{\frac{\pi(180°-\theta)}{360°}-\sin\frac{(180°-\theta)}{2}\cos\frac{(180°-\theta)}{2}\right\}r^2$$

Now, since $\sin(90°-\alpha)=\sin\alpha$ and $\cos(90°-\alpha)=\cos\alpha$

$$A=\left\{\frac{\pi(180°-\theta)}{360°}-\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\}r^2$$

We know that the area of sector of a circle of radius r at an angle θ is

$$A' = \frac{\theta}{360°} \times \pi r^2$$

So, the area of sector BOC, $A' = \frac{\theta}{360°} \times \pi r^2$

It is given that,

Area of minor segment cutoff by AC = $2 \times$ Area of sector BOC

$$\left\{\frac{\pi(180°-\theta)}{360°} - \cos\frac{\theta}{2}\sin\frac{\theta}{2}\right\} r^2 = \frac{2\theta}{360°} \times \pi r^2$$

$$\frac{\pi(180°-\theta)}{360°} - \cos\frac{\theta}{2}\sin\frac{\theta}{2} = \frac{2\pi\theta}{360°}$$

$$\frac{\pi 180°}{360°} - \frac{\pi\theta}{360°} - \cos\frac{\theta}{2}\sin\frac{\theta}{2} = \frac{2\pi\theta}{360°}$$

$$\cos\frac{\theta}{2}\sin\frac{\theta}{2} = \frac{\pi 180°}{360°} - \frac{\pi\theta}{360°} - \frac{2\pi\theta}{360°}$$

$$\cos\frac{\theta}{2}\sin\frac{\theta}{2} = \frac{\pi}{2} - \frac{3\pi\theta}{360°}$$

$$\cos\frac{\theta}{2}\sin\frac{\theta}{2} = \frac{\pi}{2} - \frac{\pi\theta}{120°}$$

$$\cos\frac{\theta}{2}\sin\frac{\theta}{2} = \pi\left(\frac{1}{2} - \frac{\theta}{120°}\right)$$

Areas Related to Circles Ex 15.3 Q7

Answer :

We know that the area of circle and area of minor segment of angle θ in a circle of radius r is given by,

$A' = \pi r^2$ and $A = \left\{\frac{\pi\theta}{360°} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} r^2$ respectively.

It is given that,

$$\text{Area of minor segment} = \frac{1}{8} \times \text{area of circle}$$

$$\left\{\frac{\pi\theta}{360°} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} r^2 = \frac{\pi r^2}{8}$$

$$\left\{\frac{\pi\theta}{360°} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\} \times 8 = \pi$$

$$\frac{8\pi\theta}{360°} - 8\sin\frac{\theta}{2}\cos\frac{\theta}{2} = \pi$$

$$\frac{\pi\theta}{45°} - 8\sin\frac{\theta}{2}\cos\frac{\theta}{2} = \pi$$

$$\boxed{\frac{\pi\theta}{45°} = 8\sin\frac{\theta}{2}\cos\frac{\theta}{2} + \pi}$$

*********** END **********