Derivatives as a Rate Measurer Ex 13.2 Q16(i)

Here,

$$2\frac{d(\sin\theta)}{dt} = \frac{d\theta}{dt}$$

$$2 \times \cos\theta\frac{d\theta}{dt} = \frac{d\theta}{dt}$$

$$2\cos\theta = 1$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}.$$

Derivatives as a Rate Measurer Ex 13.2 Q16(ii)

$$\frac{d\theta}{dt} = -2\frac{d}{dt}(\cos\theta)$$

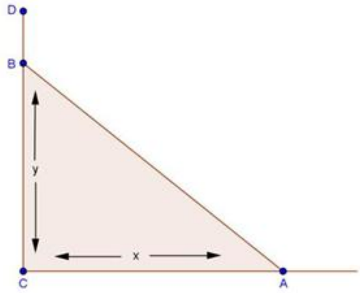
$$\frac{d\theta}{dt} = -2(-\sin\theta)\frac{d\theta}{dt}$$

$$1 = 2\sin\theta$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}$$

Derivatives as a Rate Measurer Ex 13.2 Q17



Let CD be the wall and AB is the ladder

Here, $AB = 6$ meter and $\left(\frac{dx}{dt}\right)_{x=4} = 0.5$ m/sec.

From figure,

$$AB^2 = x^2 + y^2$$
$$(6)^2 = x^2 + y^2$$
$$36 = x^2 + y^2$$

Differentiating it with respect to t,

$$0 = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$
$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt} \qquad \text{---(i)}$$
$$\left(\frac{dy}{dt}\right)_{x=4} = \frac{4(0.5)}{\sqrt{36 - x^2}}$$
$$= -\frac{2}{\sqrt{36 - 16}}$$
$$= -\frac{2}{2\sqrt{5}}$$
$$= -\frac{1}{\sqrt{5}} \text{ m/sec.}$$

So, ladder top is sliding at the rate of $\frac{1}{\sqrt{5}}$ m/sec.

Now, to find x when $\frac{dx}{dt} = -\frac{dy}{dt}$

From equation (i),

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$
$$-\frac{dx}{dt} = -\frac{x}{y}\frac{dx}{dt}$$
$$x = y$$

Now,

$$36 = x^2 + y^2$$
$$36 = x^2 + x^2$$
$$2x^2 = 36$$
$$x^2 = 18$$
$$x = 3\sqrt{2} \text{ m}$$

When foot and top are moving at the same rate, foot of wall is $3\sqrt{2}$ meters away from the wall

*********** END **********