



Indefinite Integrals Ex 19.30 Q17

$$\text{Let } I = \int \frac{1}{(x-1)(x+1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+2}$$

$$\Rightarrow 1 = A(x+1)(x+2) + B(x-1)(x+2) + C(x^2-1)$$

Put  $x = 1$

$$\Rightarrow 1 = 6A \Rightarrow A = \frac{1}{6}$$

Put  $x = -1$

$$\Rightarrow 1 = -2B \Rightarrow B = -\frac{1}{2}$$

Put  $x = -2$

$$\Rightarrow 1 = 3C \Rightarrow C = \frac{1}{3}$$

So,

$$I = \frac{1}{6} \int \frac{dx}{x-1} - \frac{1}{2} \int \frac{dx}{x+1} + \frac{1}{3} \int \frac{dx}{x+2}$$

$$I = \frac{1}{6} \log|x-1| - \frac{1}{2} \log|x+1| + \frac{1}{3} \log|x+2| + c$$

Indefinite Integrals Ex 19.30 Q18

Consider the integral

$$I = \int \frac{x^2}{(x^2 + 4)(x^2 + 9)} dx$$

Now let us separate the fraction  $\frac{x^2}{(x^2 + 4)(x^2 + 9)}$

through partial fractions.

Substitute  $x^2 = t$

$$\frac{x^2}{(x^2 + 4)(x^2 + 9)} = \frac{t}{(t + 4)(t + 9)}$$

$$\Rightarrow \frac{t}{(t + 4)(t + 9)} = \frac{A}{t + 4} + \frac{B}{t + 9}$$

$$\Rightarrow \frac{t}{(t + 4)(t + 9)} = \frac{A(t + 9) + B(t + 4)}{(t + 4)(t + 9)}$$

$$\Rightarrow t = A(t + 9) + B(t + 4)$$

$$\Rightarrow t = At + 9A + Bt + 4B$$

Comparing the coefficients, we have,

$$A + B = 1 \text{ and } 9A + 4B = 0$$

$$\Rightarrow A = -\frac{4}{5} \text{ and } B = \frac{9}{5}$$

$$\Rightarrow \frac{x^2}{(x^2 + 4)(x^2 + 9)} = -\frac{4}{5(t + 4)} + \frac{9}{5(t + 9)}$$

$$\Rightarrow \frac{x^2}{(x^2 + 4)(x^2 + 9)} = -\frac{4}{5(x^2 + 4)} + \frac{9}{5(x^2 + 9)}$$

Thus, we have,

$$I = \int \frac{x^2}{(x^2 + 4)(x^2 + 9)} dx$$

$$= \int \left[ -\frac{4}{5(x^2 + 4)} + \frac{9}{5(x^2 + 9)} \right] dx$$

$$= -\int \frac{4dx}{5(x^2 + 4)} + \int \frac{9dx}{5(x^2 + 9)}$$

$$= -\frac{4}{5} \int \frac{dx}{(x^2 + 4)} + \frac{9}{5} \int \frac{dx}{(x^2 + 9)}$$

$$= -\frac{4}{5} \times \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + \frac{9}{5} \times \frac{1}{3} \tan^{-1}\left(\frac{x}{3}\right) + C$$

$$= -\frac{2}{5} \tan^{-1}\left(\frac{x}{2}\right) + \frac{3}{5} \tan^{-1}\left(\frac{x}{3}\right) + C$$

$$\text{Let } \int \frac{5x^2 - 1}{x(x-1)(x+1)} dx = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$

$$\Rightarrow 5x^2 - 1 = A(x^2 - 1) + B(x+1)x + C(x-1)x$$

$$\text{Put } x = 0$$

$$\Rightarrow -1 = -A \Rightarrow A = 1$$

$$\text{Put } x = +1$$

$$\Rightarrow 4 = 2B \Rightarrow B = 2$$

$$\text{Put } x = -1$$

$$\Rightarrow 4 = 2C \Rightarrow C = 2$$

So,

$$\begin{aligned} I &= \int \frac{dx}{x} + \int \frac{2dx}{x-1} + \int \frac{2dx}{x+1} \\ &= \log|x| + 2\log|x-1| + 2\log|x+1| + c \end{aligned}$$

$$I = \log \left| x(x^2-1)^2 \right|$$

Indefinite Integrals Ex 19.30 Q20

$$\text{Let } I = \int \frac{x^2 + 6x - 8}{x^3 - 4x} dx$$

$$\Rightarrow I = \int \frac{x^2 + 6x - 8}{x(x+2)(x-2)} dx$$

Now,

$$\text{Let } \frac{x^2 + 6x - 8}{x(x+2)(x-2)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-2}$$

$$\Rightarrow x^2 + 6x - 8 = A(x^2 - 4) + B(x-2)x + C(x+2)x$$

Put  $x = 0$

$$\Rightarrow -8 = -4A \Rightarrow A = 2$$

Put  $x = -2$

$$\Rightarrow -16 = 8B \Rightarrow B = -2$$

Put  $x = 2$

$$\Rightarrow 8 = 8C \Rightarrow C = 1$$

Thus,

$$\begin{aligned} I &= \int \frac{2dx}{x} - \int \frac{2dx}{x+2} + \int \frac{dx}{x-2} \\ &= 2 \log|x| - 2 \log|x+2| + \log|x-2| + c \end{aligned}$$

$$\therefore I = \log \left| \frac{x^2(x-2)}{(x+2)^2} \right| + c$$

Indefinite Integrals Ex 19.30 Q21

$$\text{Let } \int \frac{x^2 + 1}{(2x + 1)(x^2 - 1)} = \frac{A}{2x + 1} + \frac{Bx + C}{x^2 - 1}$$

$$\begin{aligned} \Rightarrow x^2 + 1 &= A(x^2 - 1) + (Bx + C)(2x + 1) \\ &= (A + 2B)x^2 + (B + 2C)x + (-A + C) \end{aligned}$$

Equating similar terms, we get,

$$A + 2B = 1, B + 2C = 0 \text{ and } -A + C = 1$$

Solving we get,

$$A = -\frac{5}{3} \quad B = \frac{4}{3} \quad C = -\frac{2}{3}$$

Thus,

$$\begin{aligned} I &= -\frac{5}{3} \int \frac{dx}{2x + 1} + \int \frac{\frac{4}{3}x - \frac{2}{3}}{x^2 - 1} dx \\ &= -\frac{5}{3} \int \frac{dx}{2x + 1} + \frac{2}{3} \int \frac{2x dx}{x^2 - 1} - \frac{2}{3} \int \frac{dx}{x^2 - 1} \\ &= -\frac{5}{3} \int \frac{dx}{2x + 1} + \frac{2}{3} \int \frac{2x - 1}{(x + 1)(x - 1)} dx \\ &= -\frac{5}{3} \int \frac{dx}{2x + 1} + \frac{2}{3} \left( \int \left( \frac{\frac{3}{2}}{(x + 1)} + \frac{\frac{1}{2}}{(x - 1)} \right) dx \right) \end{aligned}$$

$$I = -\frac{5}{6} \log|2x + 1| + \log|x + 1| + \frac{1}{3} \log|x - 1| + c$$

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