CHAPTER 23

HEAT AND TEMPERATURE

23.1 HOT AND COLD BODIES

When we rub our hands for some time, they become warm. When a block slides on a rough surface, it becomes warm. Press against a rapidly spinning wheel. The wheel slows down and becomes warm. While going on a bicycle, touch the road with your shoe. The bicycle slows down and the shoe becomes warm. When two vehicles collide with each other during an accident, they become very hot. When an aeroplane crashes, it becomes so hot that it catches fire.

In each of these examples, mechanical energy is lost and the bodies in question become hot. Where does the mechanical energy vanish? It goes into the internal energy of the bodies. We conclude that the cold bodies absorb energy to become hot. In other words, a hot body has more internal energy than an otherwise identical cold body.

When a hot body is kept in contact with a cold body, the cold body warms up and the hot body cools down. The internal energy of the hot body decreases and the internal energy of the cold body increases. Thus, energy is transferred from the hot body to the cold body when they are placed in contact. Notice that no mechanical work is done during this transfer of energy (neglect any change in volume of the body). This is because there are no displacements involved. This is different from the case when we lift a ball vertically and the energy of the ball–earth system increases or when a compressed spring relaxes and a block attached to its end speeds up. In the case of lifting the ball, we do some work on the ball and the energy is increased by that amount. In the case of spring–block example, the spring does some work and the kinetic energy of the block increases.

The transfer of energy from a hot body to a cold body is a nonmechanical process. The energy that is transferred from one body to the other, without any mechanical work involved, is called *heat*.

23.2 ZEROTH LAW OF THERMODYNAMICS

Two bodies are said to be in *thermal equilibrium* if no transfer of heat takes place when they are placed in contact. We can now state the Zeroth law of thermodynamics as follows:

If two bodies A and B are in thermal equilibrium and A and C are also in thermal equilibrium then B and C are also in thermal equilibrium.

It is a matter of observation and experience that is described in the Zeroth law. It should not be taken as obvious. For example, if two persons A and B know each other and A and C know each other, it is not necessary that B and C know each other.

The Zeroth law allows us to introduce the concept of temperature to measure the hotness or coldness of a body. All bodies in thermal equilibrium are assigned equal temperature. A hotter body is assigned higher temperature than a colder body. Thus, the temperatures of two bodies decide the direction of heat-flow when the two bodies are put in contact. Heat flows from the body at higher temperature to the body at lower temperature.

23.3 DEFINING SCALE OF TEMPERATURE: MERCURY AND RESISTANCE THERMOMETERS

We are now in a position to say whether two given bodies are at the same temperature or not. If they are not at the same temperature, we also know which is at higher temperature and which is at lower temperature. Our next task is to define a scale of temperature so that we can give numerical value to the temperature of a body. To do this, we can choose a substance and look for a measurable property of the substance which monotonically changes with temperature. The temperature can then be defined as a chosen function of this property. As an example, take a mass of mercury in a glass bulb terminating in a long capillary. The length of the mercury column in the capillary changes with temperature. Each length corresponds to a particular temperature of the mercury. How can we assign a numerical value corresponding to an observed length of the mercury column ?

Figure 23.1

The earlier method was to choose two fixed points of temperature which can be easily reproduced in laboratory. The temperature of melting ice at 1 atm (called ice point) and the temperature of boiling water at 1 atm (called steam point) are often chosen as the fixed points. We arbitrarily assign a temperature t_1 to the ice point and t_2 to the steam point. Suppose the length of the mercury column is l_1 when the bulb is kept in melting ice and it is l_2 when the bulb is kept in boiling water. Thus, a length l_1 of mercury column means that the temperature of the bulb is t_1 and a length l_2 of the column means that the temperature is t_2. The temperature corresponding to any length l may be defined by assuming a linear relation between l and t,

$$t = al + b. \qquad \ldots (23.1)$$

A change of one degree in temperature will mean a change of $\frac{l_2 - l_1}{t_2 - t_1}$ in the length of mercury column. Thus, we can graduate the length of the capillary directly in degrees.

The centigrade system assumes ice point at 0°C and the steam point at 100°C. If the length of the mercury column between its values for 0°C and 100°C is divided equally in 100 parts, each part will correspond to a change of 1°C.

Let l_0 and l_{100} denote the lengths of the mercury column at 0°C and 100°C respectively. From equation (23.1),

$$0 = al_0 + b$$

$$\text{and } 100 = al_{100} + b$$

$$\text{giving } b = -al_0$$

$$\text{and } a = \frac{100}{l_{100} - l_0}.$$

Putting in equation (23.1), the temperature corresponding to a length l is given by

$$t = \frac{l - l_0}{l_{100} - l_0} \times 100 \text{ degree}. \qquad \ldots (23.2)$$

To measure the temperature of a body, the bulb containing mercury is kept in contact with the body and sufficient time is allowed so that the mercury comes to thermal equilibrium with the body. The temperature of the body is then the same as that of the mercury. The length of the column then gives the temperature according to equation (23.2).

Another popular system known as Fahrenheit system assumes 32°F for the ice point and 212°F for the steam point. A change of 1°F means $\frac{1}{180}$ of the interval between the steam point and the ice point. The average temperature of a normal human body is around 98°F. The conversion formula from centigrade to Fahrenheit scale is

$$F = 32 + \frac{9}{5}C.$$

The expansion of mercury is just one thermometric property that can be used to define a temperature scale and prepare thermometers. There may be many others. Electric resistance of a metal wire increases monotonically with temperature and may be used to define a temperature scale. If R_0 and R_{100} denote the resistances of a metal wire at ice point and steam point respectively, we can define temperature t corresponding to the resistance R_t as

$$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100 \text{ degree}. \qquad \ldots (23.3)$$

The temperatures of the ice point and the steam point are chosen to be 0°C and 100°C as in centigrade scale. A platinum wire is often used to construct a thermometer based on this scale. Such a thermometer is called platinum resistance thermometer and the temperature scale is called the *platinum scale.*

Platinum Resistance Thermometer

The platinum resistance thermometer works on the principle of *Wheatstone bridge* used to measure a resistance. In a Wheatstone bridge, four resistances P, Q, R and X are joined in a loop as shown in figure (23.2a). A galvanometer and a battery are also joined as shown. If

$$\frac{P}{Q} = \frac{R}{X},$$

there is no deflection in the galvanometer and the bridge is called balanced. If the condition is not fulfilled, there is a deflection.

Figure (23.2b) represents the arrangement for a platinum resistance thermometer. A thin platinum wire is coiled on a mica base and placed in a glass tube. Two connecting wires YY going through the

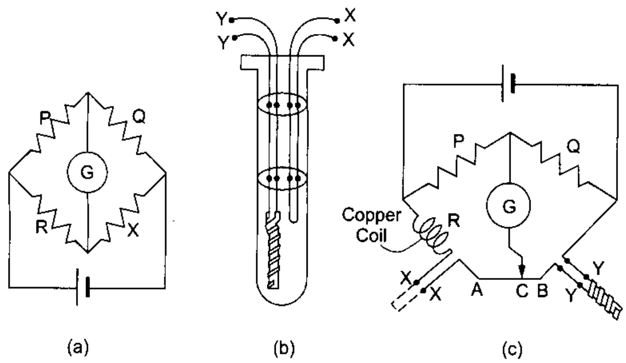

Figure 23.2

ebonite lid of the tube are connected to the platinum coil. A similar copper wire *XX*, called compensating wire, also goes into the tube as shown in the figure. A Wheatstone bridge is arranged as shown in figure (23.2c). Two equal resistances *P* and *Q* are connected in two arms of the bridge. A copper coil having resistance roughly equal to that of the platinum coil is connected in the third arm through the compensating wire *XX*. The platinum coil is inserted in the fourth arm of the bridge by connecting the points *Y*, *Y* in that arm. The end *C* of the wire connected to the galvanometer can slide on a uniform wire *AB* of length *l*. The end *A* of this wire is connected to the wire *XX* and the end *B* is connected to the wire *YY*. Thus, the copper coil, the compensating wire and the wire *AC* are in the third arm and the platinum coil, the connecting wire *YY* and the wire *CB* are in the fourth arm of the bridge. The test tube containing the platinum coil is immersed in the bath of which we want to measure the temperature. The end *C* is slid on *AB* till the deflection in the galvanometer becomes zero. Let $AC = x$.

Suppose the resistance of the copper coil connected in the third arm is R, that of the compensating wire is R_c and that of the wire AB is r. The resistance of the connecting wire YY is the same as R_c and that of the platinum coil is R_t. The resistance of the wire $AC = \frac{r}{l}x$ and that of the wire $CB = \frac{r}{l}(l - x)$. The net resistance in the third arm is $R + R_c + \frac{r}{l}x$ and that in the fourth arm is $R_t + R_c + \frac{r}{l}(l - x)$. For no deflection in the galvanometer,

$$\frac{P}{Q} = \frac{R + R_c + \frac{r}{l}x}{R_t + R_c + \frac{r}{l}(l - x)}.$$

As $P = Q$, we get

$$R_t + \frac{r}{l}(l - x) = R + \frac{r}{l}x$$

or, $$R_t = (R - r) + \frac{2r}{l}x.$$

If x_0, x_{100} and x_t are the values of x at the ice point, steam point and the temperature t respectively,

$$R_{100} = (R - r) + \frac{2r}{l}x_{100}$$

$$R_0 = (R - r) + \frac{2r}{l}x_0$$

and $$R_t = (R - r) + \frac{2r}{l}x_t.$$

This gives

$$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100 = \frac{x_t - x_0}{x_{100} - x_0} \times 100.$$

Absolute Scale and Ideal Gas Scale

Equation (23.3) tells us that the change in resistance ΔR when the temperature increases by one degree ($\Delta t = 1°$) is

$$\Delta R = \frac{R_{100} - R_0}{100}$$

and is the same for all temperatures. This means that the resistance increases uniformly as the temperature increases. Note that this conclusion follows from the particular definition (23.3) of temperature and is not the property of platinum or the other metal used to form the resistance. Similarly, it follows that the expansion of mercury is uniform on the mercury scale defined by (23.2). If we measure the change in resistance of a platinum wire against the temperature measured on mercury scale, we shall find that the resistance varies slowly at lower temperatures and slightly more rapidly at higher temperatures. Similarly, if the expansion of mercury is measured against the temperature defined on platinum scale (23.3), we shall find that mercury does not expand at uniform rate as the temperature varies.

Thus, the two scales of temperature do not agree with each other. They are forced to agree at ice point and steam point but for other physical states the readings of the two thermometers will be different. In general, the scale depends on the properties of the thermometric substance used to define the scale.

It is possible to define an absolute scale of temperature which does not depend on the thermometric substance and its properties. We now define the ideal gas temperature scale which happens to be identical to the absolute temperature scale.

23.4 CONSTANT VOLUME GAS THERMOMETER

A gas enclosed in a container has a definite volume and a definite pressure in any given state. If the volume is kept constant, the pressure of a gas increases monotonically with increasing temperature. This property of a gas may be used to construct a thermometer. Figure (23.3) shows a schematic diagram of a constant volume gas thermometer.

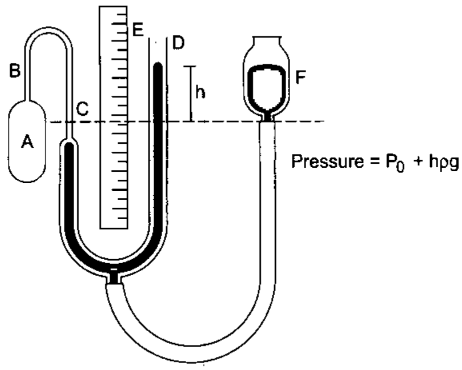

Figure 23.3

A mass of gas is enclosed in a bulb A connected to a capillary BC. The capillary is connected to the manometer CD which contains mercury. The other end of the manometer is open to atmosphere. A vertical metre scale E is fixed in such a way that the height of the mercury column in the tube D can easily be measured. The mercury in the manometer is connected to the mercury reservoir F through a rubber tube.

The capillary BC has a fixed mark at C. By raising or lowering the reservoir F, the mercury level in the left part of the manometer is maintained at C. This ensures that the volume of the gas enclosed in the bulb A (and the capillary BC) remains constant. The pressure of the gas is equal to the atmospheric pressure plus the pressure due to the difference of the mercury columns in the manometer. Thus, $p = p_0 + h\rho g$, where p_0 = atmospheric pressure, h = difference of mercury levels in the manometer tube, ρ = density of mercury and g = acceleration due to gravity.

If the temperature of the bulb is increased and its volume is kept constant by adjusting the height of the reservoir F, the pressure of the gas $p = p_0 + h\rho g$ increases. Thus, a temperature scale may be defined by choosing some suitable function of this pressure. Let us assume that the temperature is proportional to the pressure, i.e.,

$$T = cp \, , \qquad \ldots \text{ (23.4)}$$

where c is some constant.

In addition, the temperature of triple point of water is assigned a value 273·16 K. (Triple point is a state in which ice, water and water vapour can stay together in equilibrium.) The unit is called a kelvin and is denoted by the symbol K. To get the value of the constant c in equation (23.4), we can put the bulb A in a triple point cell and measure the pressure p_{tr} of the gas. From equation (23.4),

$$273{\cdot}16 \text{ K} = cp_{tr}$$

or,

$$c = \frac{273{\cdot}16 \text{ K}}{p_{tr}} .$$

The temperature of the gas when the pressure is p is obtained by putting this value of c in equation (23.4). It is

$$T = \frac{p}{p_{tr}} \times 273{\cdot}16 \text{ K}. \qquad \ldots \text{ (23.5)}$$

To use the thermometer, we must first determine the pressure of the gas p_{tr} at the triple point. This is a fixed value for the thermometer and is used in any measurement. To measure the temperature of a bath of extended volume, the bulb A is dipped in the bath. Sufficient time is allowed so that the gas in the bulb comes to thermal equilibrium with the bath. The reservoir F is adjusted to bring the volume of the gas to its original value and the pressure p of the gas is measured with the manometer. The temperature T on the gas scale is then obtained from equation (23.5).

One can also define a centigrade scale with gas thermometers. Suppose the pressure of the gas is p_0 when the bulb A is placed in melting ice (ice point) and it is p_{100} when the bulb is placed in a steam bath (steam point). We assign 0°C to the temperature of the ice point and 100°C to the steam point. The temperature t corresponding to a pressure p of the gas is defined by

$$t = \frac{p - p_0}{p_{100} - p_0} \times 100°\text{C}. \qquad \ldots \text{ (23.6)}$$

The constant volume gas thermometer allows several errors in the temperature measurement. The main sources of error are the following:

(a) The space in the capillary tube BC generally remains out of the heat bath in which the bulb A is placed. The gas in BC is, therefore, not at the same temperature as the gas in A.

(b) The volume of the glass bulb changes slightly with temperature allowing the volume of the gas to change.

Example 23.1

The pressure of air in the bulb of a constant volume gas thermometer is 73 cm *of mercury at* 0°C, 100·3 cm *of mercury at* 100°C *and* 77·8 cm *of mercury at room temperature. Find the room temperature in centigrades.*

Solution : We have $t = \dfrac{p - p_0}{p_{100} - p_0} \times 100°\text{C}$

$$= \frac{77{\cdot}8 - 73}{100{\cdot}3 - 73} \times 100°\text{C} = 17°\text{C}.$$

23.5 IDEAL GAS TEMPERATURE SCALE

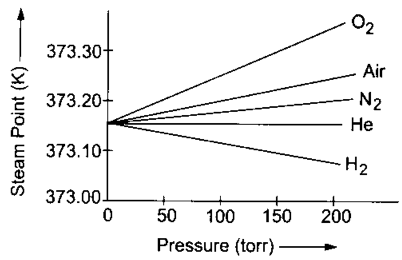

Figure 23.4

The temperature scale defined by equation (23.5) depends slightly on the gas used and its amount present in the thermometer. Figure (23.4) shows the temperature of a steam bath (steam point) as measured by placing different gases in different amounts. On the horizontal axis, we have plotted the pressure p_{tr} of the gas in the thermometer at the triple point of water. This is almost proportional to the mass of the gas present. From the figure, we see that although different gas thermometers differ in their measurement of steam point but the difference decreases as the amount of the gas and hence p_{tr} decreases. In the limit $p_{tr} \to 0$, all the different gas thermometers give the same value 373·15 K for steam point. So we define a temperature scale by the equation

$$T = \lim_{p_{tr} \to 0} \frac{p}{p_{tr}} \times 273{\cdot}16 \text{ K}. \qquad \dots (23.7)$$

When we use small amount of gas in a gas thermometer, the scale (23.5) is almost identical to (23.7).

The temperature scale defined by (23.7) is called *ideal gas temperature scale* and is independent of the gas chosen. However, it may depend on the properties of gases in general. As mentioned above, it is possible to define an *absolute temperature scale* which does not depend on any property of any substance. Such a temperature scale has been defined and is in use. The unit of this temperature is called kelvin and is abbreviated as K. The ideal gas temperature scale (23.7) happens to be identical with the absolute scale and hence we have used K to denote the unit.

23.6 CELSIUS TEMPERATURE SCALE

The temperature of the ice point on the ideal gas scale is 273·15 K and of the steam point is T = 373·15 K. The interval between the two is 100 K. The centigrade scale discussed earlier has 0°C for the ice point and 100°C for the steam point. However, dividing the temperature interval into 100 parts needs a thermometric substance like mercury in glass or resistance of platinum wire etc. Thus, the scales are different for different thermometers. The use of the term "centrigrade scale" is now replaced by "Celsius scale". Celsius scale is defined to have the ice point at 0°C and the steam point at 100°C. The size of a degree in Celsius scale is defined to be the same as the size of a degree in the ideal gas scale. The Celsius scale is shifted from the ideal gas scale by −273·15. If θ denotes the Celsius temperature and T denotes the kelvin temperature,

$$\theta = T - 273{\cdot}15 \text{ K}. \qquad \dots (23.8)$$

The mercury centigrade scale described earlier is quite close to the Celsius scale.

23.7 IDEAL GAS EQUATION

We have seen that the pressure of all gases changes with temperature in a similar fashion for low pressures. Many of the properties of gases are common at low pressures (and high temperatures, that is, far above their condensation point). The pressure, volume and the temperature in kelvin of such a gas obey the equation

$$pV = nRT, \qquad \dots (23.9)$$

where n is the amount of the gas in number of moles and R is a universal constant having value 8·314 J K^{-1} mol^{-1}. The constant R is called the gas constant. Equation (23.9) is known as *ideal gas equation*. A gas obeying this equation is called an *ideal gas*.

23.8 CALLENDAR'S COMPENSATED CONSTANT PRESSURE THERMOMETER

In constant volume gas thermometer, the gas in the capillary tube connecting the bulb and the manometer remains outside the heat bath. The temperature of this part is different from the main bulk of the gas and this introduces some error. Callender's compensated constant pressure thermometer avoids this problem by a special design.

Figure (23.5) shows a schematic diagram of the Callender's thermometer. An ideal gas is filled in a bulb A connected to a manometer M and another bulb B through a capillary cd. The bulb B is filled with mercury and is graduated in volume. Mercury may be

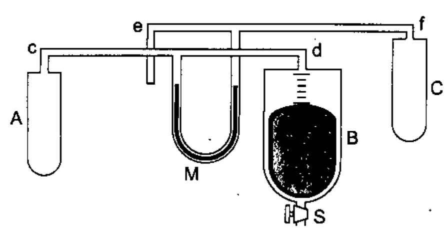

Figure 23.5

taken out from the bulb by opening a stopcock fitted in the lower part of B. The volume of the mercury taken out may be measured with the help of the graduations on B. The other arm of the manometer is connected to a capillary tube ef and a bulb C. The capillary ef and the capillary cd have equal volumes. The bulb A and the bulb C also have equal volumes.

The same ideal gas (as filled in A) is filled in the bulb C. The amount of gas in the bulb C (and the capillary ef) is kept the same as the amount in the bulb A (and the capillary cd). When all the parts of the thermometer are at the same temperature and the bulb B is completely filled with mercury, the levels of the liquid in the two arms of the manometer are equal.

To measure the temperature of a heat bath (at temperature larger than the ice point), the bulb A is placed in the heat bath and the bulbs B and C are placed in melting-ice baths. The pressure in the bulb A becomes more than the pressure in the bulb C and the levels in the manometer tubes become different. Mercury is taken out from B till the levels in the manometer tubes become equal. When mercury is taken out, the total volume of the gas in bulb A, capillary cd and bulb B increases. This decreases the pressure on this side. When sufficient amount of mercury is taken out, the pressure becomes equal to the pressure in the bulb C and hence, the levels in the manometer tubes again become equal.

Suppose,

the volume of A = volume of $C = V$,

volume of capillary cd = volume of capillary $ef = v_0$,

volume of the mercury taken out $= v'$,

temperature of the heat bath $= T$,

temperature of the ice bath $= T_0$,

and the temperature of cd and $ef = T'$.

Using $pV = nRT$, the number of moles in the bulb A, the capillary cd and the bulb B

$$= \frac{pV}{RT} + \frac{pv_0}{RT'} + \frac{pv'}{RT_0}$$

and, the number of moles in the bulb C and the capillary ef

$$= \frac{pV}{RT_0} + \frac{pv_0}{RT'}.$$

As equal amounts of the gas are filled on the two sides,

$$\frac{pV}{RT} + \frac{pv_0}{RT'} + \frac{pv'}{RT_0} = \frac{pV}{RT_0} + \frac{pv_0}{RT'}$$

or, $$\frac{V}{T} = \frac{V}{T_0} - \frac{v'}{T_0}$$

or, $$T = \frac{V}{V - v'} T_0. \quad \ldots (23.10)$$

All the quantities on the right side are known and hence, the temperature of the bath is obtained.

23.9 ADIABATIC AND DIATHERMIC WALLS

We have seen that heat flows from a high-temperature body to a low-temperature body when they are put in contact. There are certain materials which resist the flow of heat through them. When two bodies at different temperatures are separated by such a material, the heat-flow is very slow. We assume an idealised wall or separator which does not allow any heat-flow through it. The bodies on the two sides of such a wall may remain at different temperatures. Such a wall is called an *adiabatic wall.*

Opposite is the concept of a *diathermic wall* which allows heat transfer through it rapidly. If two bodies at different temperatures are separated by such a wall, their temperatures will become equal in a short time.

23.10 THERMAL EXPANSION

If the temperature of a body increases, in general, its size also increases. We used this expansion property to define a temperature scale. Now, we have an absolute scale of temperature independent of any property of any susbstance. We can study the thermal expansion of a body as a function of temperature. Consider a rod at temperature T and suppose its length at this temperature is L. As the temperature is changed to $T + \Delta T$, the length is changed to $L + \Delta L$. We define *average coefficient of linear expansion* in the temperature range ΔT as

$$\bar{\alpha} = \frac{1}{L}\frac{\Delta L}{\Delta T}.$$

The *coefficient of linear expansion* at temperature T is limit of average coefficient as $\Delta T \to 0$, i.e.,

$$\alpha = \lim_{\Delta T \to 0} \frac{1}{L}\frac{\Delta L}{\Delta T} = \frac{1}{L}\frac{dL}{dT}.$$

Suppose the length of a rod is L_0 at 0°C and L_θ at temperature θ measured in Celsius. If α is small and constant over the given temperature interval,

$$\alpha = \frac{L_\theta - L_0}{L_0 \theta}$$

or, $$L_\theta = L_0(1 + \alpha\theta). \quad \ldots (23.11)$$

This equation is widely used in solving problems.

The coefficient of volume expansion γ is defined in a similar way. If V is the volume of a body at temperature T, the *coefficient of volume expansion* at temperature T is

$$\gamma = \frac{1}{V}\frac{dV}{dT}.$$

It is also known as coefficient of cubical expansion.

If V_0 and V_θ denote the volumes at 0°C and θ (measured in Celsius) respectively and γ is small and constant over the given temperature range, we have

$$V_\theta = V_0(1 + \gamma\theta). \quad \ldots \text{ (23.12)}$$

It is easy to show that $\gamma = 3\alpha$.

The change in volume of water as temperature increases is slightly complicated. Figure (23.6) shows the volume of 1 g of water as the temperature increases from 0°C to 10°C. The volume of water decreases from about $1{\cdot}00013\ \text{cm}^3$ to $1{\cdot}00000\ \text{cm}^3$ as the temperature increases from 0°C to 4°C. This means γ is negative in this temperature range. The volume again increases as the temperature is increased further from 4°C. The density (mass/volume) of water is thus maximum at 4°C.

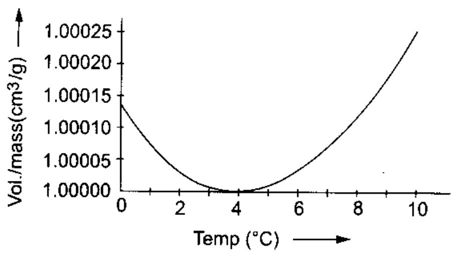

Figure 23.6

The anomalous expansion of water has a favourable effect for animals living in water. Since the density of water is maximum at 4°C, water at the bottom of lakes remain at 4°C in winter even if that at the surface freezes. This allows marine animals to remain alive and move near the bottom.

The study of expansion of a liquid presents another difficulty due to the expansion of the container. Consider a liquid kept in a flask with a graduated stem. As the temperature is increased, the volume of the flask expands faster than the liquid in the beginning and the level of liquid in the stem goes down as if the liquid has contracted. As the temperature of the liquid increases, the volume of the liquid increases and rises in the stem. The apparent increase in the volume is equal to the real increase in the volume of the liquid minus the increase in the volume of the container.

Worked Out Examples

1. *The pressure of the gas in a constant volume gas thermometer at steam point (373·15 K) is $1{\cdot}50 \times 10^4$ Pa. What will be the pressure at the triple point of water ?*

Solution : The temperature in kelvin is defined as

$$T = \frac{p}{p_{tr}} \times 273{\cdot}16\ \text{K}.$$

Thus,

$$373{\cdot}15 = \frac{1{\cdot}50 \times 10^4\ \text{Pa}}{p_{tr}} \times 273{\cdot}16$$

or,

$$p_{tr} = 1{\cdot}50 \times 10^4\ \text{Pa} \times \frac{273{\cdot}16}{373{\cdot}15}$$

$$= 1{\cdot}10 \times 10^4\ \text{Pa}.$$

2. *The pressure of air in the bulb of a constant volume gas thermometer at* 0°C *and* 100°C *are* 73·00 cm *and* 100 cm *of mercury respectively. Calculate the pressure at the room temperature* 20°C.

Solution : The room temperature on the scale measured by the thermometer is

$$t = \frac{p_t - p_0}{p_{100} - p_0} \times 100°\text{C}.$$

Thus,

$$20°\text{C} = \frac{p_t - 73{\cdot}00\ \text{cm of Hg}}{100\ \text{cm of Hg} - 73{\cdot}00\ \text{cm of Hg}} \times 100°\text{C}$$

or, $p_t = 78{\cdot}4$ cm of mercury.

3. *The pressure of the gas in a constant volume gas thermometer is* 80 cm *of mercury in melting ice at* 1 atm. *When the bulb is placed in a liquid, the pressure becomes* 160 cm *of mercury. Find the temperature of the liquid.*

Solution : For an ideal gas at constant volume,

$$\frac{T_1}{T_2} = \frac{p_1}{p_2}$$

or,

$$T_2 = \frac{p_2}{p_1} T_1.$$

The temperature of melting ice at 1 atm is 273·15 K. Thus, the temperature of the liquid is

$$T_2 = \frac{160}{80} \times 273{\cdot}15\ \text{K} = 546{\cdot}30\ \text{K}.$$

4. *In a constant volume gas thermometer, the pressure of the working gas is measured by the difference in the levels of mercury in the two arms of a U-tube connected to the gas at one end. When the bulb is placed at the room temperature* 27·0°C, *the mercury column in the arm open to atmosphere stands* 5·00 cm *above the level of mercury in the other arm. When the bulb is placed in a hot liquid, the difference of mercury levels becomes* 45·0 cm. *Calculate the temperature of the liquid. (Atmospheric pressure* = 75·0 cm of mercury.)

Solution : The pressure of the gas = atmospheric pressure + the pressure due to the difference in mercury levels.

At 27°C, the pressure is 75 cm + 5 cm = 80 cm of mercury. At the liquid temperature, the pressure is 75 cm + 45 cm = 120 cm of mercury. Using $T_2 = \frac{P_2}{P_1} T_1$, the temperature of the liquid is

$$T = \frac{120}{80} \times (27 \cdot 0 + 273 \cdot 15) \text{ K} = 450 \cdot 22 \text{ K}.$$

$$= 177 \cdot 07°\text{C} \approx 177°\text{C}.$$

5. *The resistances of a platinum resistance thermometer at the ice point, the steam point and the boiling point of sulphur are* 2·50, 3·50 *and* 6·50 Ω *respectively. Find the boiling point of sulphur on the platinum scale. The ice point and the steam point measure* 0° *and* 100° *respectively.*

Solution : The temperature on the platinum scale is defined as

$$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100°.$$

The boiling point of sulphur on this scale is

$$t = \frac{6 \cdot 50 - 2 \cdot 50}{3 \cdot 50 - 2 \cdot 50} \times 100° = 400°.$$

6. *A platinum resistance thermometer reads* 0° *and* 100° *at the ice point and the boiling point of water respectively. The resistance of a platinum wire varies with Celsius temperature* θ *as* $R_t = R_0 (1 + \alpha\theta + \beta\theta^2)$, *where* $\alpha = 3 \cdot 8 \times 10^{-3}\,°\text{C}^{-1}$ *and* $\beta = -5 \cdot 6 \times 10^{-7}\,°\text{C}^{-2}$. *What will be the reading of this thermometer if it is placed in a liquid bath maintained at* 50°C ?

Solution : The resistances of the wire in the thermometer at 100°C and 50°C are

$$R_{100} = R_0 \left[1 + \alpha \times 100°\text{C} + \beta \times (100°\text{C})^2\right]$$

and, $$R_{50} = R_0 \left[1 + \alpha \times 50°\text{C} + \beta \times (50°\text{C})^2\right].$$

The temperature t measured on the platinum thermometer is given by

$$t = \frac{R_{50} - R_0}{R_{100} - R_0} \times 100°$$

$$= \frac{\alpha \times 50°\text{C} + \beta \times (50°\text{C})^2}{\alpha \times 100°\text{C} + \beta \times (100°\text{C})^2} \times 100°$$

$$= 50 \cdot 4°.$$

7. *A platinum resistance thermometer is constructed which reads* 0° *at ice point and* 100° *at steam point. Let* t_p *denote the temperature on this scale and let* t *denote the temperature on a mercury thermometer scale. The resistance of the platinum coil varies with* t *as* $R_t = R_0 (1 + \alpha t + \beta t^2)$. *Derive an expression for the resistance as a function of* t_p.

Solution : Let R_{t_p} denote the resistance of the coil at the platinum scale temperature t_p. Then

$$t_p = \frac{R_{t_p} - R_0}{R_{100} - R_0} \times 100$$

or, $$R_{t_p} = \frac{t_p}{100}\left(R_{100} - R_0\right) + R_0$$

$$= \frac{t_p}{100}\left[R_0\left\{1 + \alpha \times 100 + \beta \times (100)^2\right\} - R_0\right] + R_0$$

$$= \frac{t_p}{100}\left[\alpha \times 100 + \beta \times (100)^2\right] R_0 + R_0$$

$$= R_0\left[1 + \left\{\alpha \times 100 + \beta \times (100)^2\right\}\frac{t_p}{100}\right]$$

$$= R_0\left[1 + \alpha t_p + \beta \times (100)\, t_p\right].$$

Only numerical values of α and β are to be used.

8. *An iron rod of length* 50 cm *is joined at an end to an aluminium rod of length* 100 cm. *All measurements refer to* 20°C. *Find the length of the composite system at* 100°C *and its average coefficient of linear expansion. The coefficient of linear expansion of iron and aluminium are* $12 \times 10^{-6}\,°\text{C}^{-1}$ *and* $24 \times 10^{-6}\,°\text{C}^{-1}$ *respectively.*

Solution : The length of the iron rod at 100°C is

$$l_1 = (50 \text{ cm}) \left[1 + (12 \times 10^{-6}\,°\text{C}^{-1})(100°\text{C} - 20°\text{C})\right]$$

$$= 50 \cdot 048 \text{ cm}.$$

The length of the aluminium rod at 100°C is

$$l_2 = (100 \text{ cm}) \left[1 + (24 \times 10^{-6}\,°\text{C}^{-1})(100°\text{C} - 20°\text{C})\right]$$

$$= 100 \cdot 192 \text{ cm}.$$

The length of the composite system at 100°C is

$$50 \cdot 048 \text{ cm} + 100 \cdot 192 \text{ cm} = 150 \cdot 24 \text{ cm}.$$

The length of the composite system at 20°C is 150 cm. So, the average coefficient of linear expansion of the composite rod is

$$\alpha = \frac{0 \cdot 24 \text{ cm}}{150 \text{ cm} \times (100°\text{C} - 20°\text{C})}$$

$$= 20 \times 10^{-6}\,°\text{C}^{-1}.$$

9. *An iron ring measuring* 15·00 cm *in diameter is to be shrunk on a pulley which is* 15·05 cm *in diameter. All measurements refer to the room temperature* 20°C. *To what minimum temperature should the ring be heated to make the job possible ? Calculate the strain developed in the ring when it comes to the room temperature. Coefficient of linear expansion of iron* $= 12 \times 10^{-6}\,°\text{C}^{-1}$.

Solution : The ring should be heated to increase its diameter from 15·00 cm to 15·05 cm.

Using $$l_2 = l_1 (1 + \alpha\, \Delta\theta),$$

$$= \frac{0{\cdot}05 \text{ cm}}{15{\cdot}00 \text{ cm} \times 12 \times 10^{-6}\ °C^{-1}}$$
$$= 278°C$$

The temperature $= 20°C + 278°C = 298°C$.

The strain developed $= \dfrac{l_2 - l_1}{l_1} = 3{\cdot}33 \times 10^{-3}$.

10. *A pendulum clock consists of an iron rod connected to a small, heavy bob. If it is designed to keep correct time at* 20°C, *how fast or slow will it go in* 24 hours *at* 40°C ? *Coefficient of linear expansion of iron* $= 1.2 \times 10^{-5}\ °C^{-1}$.

Solution : The time period at temperature θ is

$$T = 2\pi\sqrt{l_\theta / g}$$
$$= 2\pi\sqrt{l_0(1+\alpha\theta)/g}$$
$$= 2\pi\sqrt{l_0/g}\,(1+\alpha\theta)^{1/2}$$
$$\approx T_0\left(1 + \frac{1}{2}\alpha\theta\right).$$

Thus, $$T_{20} = T_0\left[1 + \frac{1}{2}\alpha(20°C)\right]$$

and, $$T_{40} = T_0\left[1 + \frac{1}{2}\alpha(40°C)\right]$$

or, $$\frac{T_{40}}{T_{20}} = [1 + (20°C)\alpha]\,[1 + (10°C)\alpha]^{-1}$$
$$\approx [1 + (20°C)\alpha]\,[1 - (10°C)\alpha]$$
$$\approx 1 + (10°C)\,\alpha$$

or, $$\frac{T_{40} - T_{20}}{T_{20}} = (10°C)\,\alpha = 1{\cdot}2 \times 10^{-4}. \quad \text{... (i)}$$

This is fractional loss of time. As the temperature increases, the time period also increases. Thus, the clock goes slow. The time lost in 24 hours is, by (i),

$$\Delta t = (24 \text{ hours})\,(1{\cdot}2 \times 10^{-4}) = 10{\cdot}4 \text{ s}.$$

11. *A pendulum clock having copper rod keeps correct time at* 20°C. *It gains* 15 seconds *per day if cooled to* 0°C. *Calculate the coefficient of linear expansion of copper.*

Solution : The time period at temperature θ is

$$T = 2\pi\sqrt{l_\theta / g}$$
$$\approx T_0\left(1 + \frac{1}{2}\alpha\theta\right)$$

Thus, $$T_{20} = T_0\,[1 + \alpha\,(10°C)]$$

or, $$\frac{(T_{20} - T_0)}{T_0} = \alpha\,(10°C). \quad \text{... (i)}$$

T_{20} is the correct time period. The period at 0°C is smaller so that the clock runs fast. Equation (i) gives approximately the fractional gain in time. The time gained in 24 hours is

$$\Delta T = (24 \text{ hours})\,[(10°C)\alpha]$$

or, $$15 \text{ s} = (24 \text{ hours})\,[(10°C)\alpha]$$

or, $$\alpha = \frac{15 \text{ s}}{(24 \text{ hours})\,(10°C)}$$
$$= 1{\cdot}7 \times 10^{-5}\ °C^{-1}.$$

12. *A piece of metal weighs* 46 g *in air and* 30 g *in a liquid of density* $1{\cdot}24 \times 10^3 \text{ kg m}^{-3}$ *kept at* 27°C. *When the temperature of the liquid is raised to* 42°C, *the metal piece weighs* 30·5 g. *The density of the liquid at* 42°C *is* $1{\cdot}20 \times 10^3 \text{ kg m}^{-3}$. *Calculate the coefficient of linear expansion of the metal.*

Solution : Let the volume of the metal piece be V_0 at 27°C and V_θ at 42°C. The density of the liquid at 27°C is $\rho_0 = 1{\cdot}24 \times 10^3 \text{ kg m}^{-3}$ and the density of the liquid at 42°C is $\rho_\theta = 1{\cdot}20 \times 10^3 \text{ kg m}^{-3}$.

The weight of the liquid displaced = apparent loss in the weight of the metal piece when dipped in the liquid. Thus,

$$V_0\,\rho_0 = 46 \text{ g} - 30 \text{ g} = 16 \text{ g}$$

and, $$V_\theta\,\rho_\theta = 46 \text{ g} - 30{\cdot}5 \text{ g} = 15{\cdot}5 \text{ g}.$$

Thus,

$$\frac{V_\theta}{V_0} = \frac{\rho_0}{\rho_\theta} \times \frac{15{\cdot}5}{16}$$

or, $$1 + 3\,\alpha\Delta\theta = \frac{1{\cdot}24 \times 10^3 \times 15{\cdot}5}{1{\cdot}20 \times 10^3 \times 16}$$

or, $1 + 3\alpha(42°C - 27°C) = 1{\cdot}00104$

or, $$\alpha = 2{\cdot}3 \times 10^{-5}\ °C^{-1}.$$

13. *A sphere of diameter* 7·0 cm *and mass* 266·5 g *floats in a bath of liquid. As the temperature is raised, the sphere begins to sink at a temperature of* 35°C. *If the density of the liquid is* $1{\cdot}527 \text{ g cm}^{-3}$ *at* 0°C, *find the coeffiecient of cubical expansion of the liquid. Neglect the expansion of the sphere.*

Solution : It is given that the expansion of the sphere is negligible as compared to the expansion of the liquid. At 0°C, the density of the liquid is $\rho_0 = 1{\cdot}527 \text{ g cm}^{-3}$. At 35°C, the density of the liquid equals the density of the sphere. Thus,

$$\rho_{35} = \frac{266{\cdot}5 \text{ g}}{\frac{4}{3}\pi\,(3{\cdot}5 \text{ cm})^3}$$
$$= 1{\cdot}484 \text{ g cm}^{-3}$$

We have $$\frac{\rho_\theta}{\rho_0} = \frac{V_0}{V_\theta} = \frac{1}{(1+\gamma\theta)}$$

or, $$\rho_\theta = \frac{\rho_0}{1+\gamma\theta}.$$

Thus, $$\gamma = \frac{\rho_0 - \rho_{35}}{\rho_{35}\,(35°C)}$$

$$= \frac{(1{\cdot}527 - 1{\cdot}484)\ \text{g cm}^{-3}}{(1{\cdot}484\ \text{g cm}^{-3})\ (35°\text{C})}$$
$$= 8{\cdot}28 \times 10^{-4}\ °\text{C}^{-1}.$$

14. *An iron rod and a copper rod lie side by side. As the temperature is changed, the difference in the lengths of the rods remains constant at a value of* 10 cm. *Find the lengths at* 0°C. *Coefficients of linear expansion of iron and copper are* $1{\cdot}1 \times 10^{-5}\ °\text{C}^{-1}$ *and* $1{\cdot}7 \times 10^{-5}\ °\text{C}^{-1}$ *respectively.*

Solution : Suppose the length of the iron rod at 0°C is l_{i0} and the length of the copper rod at 0°C is l_{c0}. The lengths at temperature θ are

$$l_{i\theta} = l_{i0}\,(1 + \alpha_i \theta) \qquad \text{... (i)}$$

and
$$l_{c\theta} = l_{c0}\,(1 + \alpha_c \theta). \qquad \text{... (ii)}$$

Subtracting,
$$l_{i\theta} - l_{c\theta} = (l_{i0} - l_{c0}) + (l_{i0}\,\alpha_i - l_{c0}\,\alpha_c)\,\theta. \qquad \text{... (iii)}$$

Now,
$$l_{i\theta} - l_{c\theta} = l_{i0} - l_{c0}\ (= 10\ \text{cm}).$$

Thus, from (iii), $l_{i0}\,\alpha_i = l_{c0}\,\alpha_c$

or,
$$\frac{l_{i0}}{l_{c0}} = \frac{\alpha_c}{\alpha_i}$$

or,
$$\frac{l_{i0}}{l_{i0} - l_{c0}} = \frac{\alpha_c}{\alpha_c - \alpha_i}$$
$$= \frac{1{\cdot}7 \times 10^{-5}\ °\text{C}^{-1}}{0{\cdot}6 \times 10^{-5}\ °\text{C}^{-1}} = \frac{17}{6}.$$

This shows that $l_{i0} - l_{c0}$ is positive. Its value is 10 cm as given in the question.

Hence,
$$l_{i0} = \frac{17}{6} \times (l_{i0} - l_{c0})$$
$$= \frac{17}{6} \times 10\ \text{cm} = 28{\cdot}3\ \text{cm}$$

and
$$l_{c0} = l_{i0} - 10\ \text{cm} = 18{\cdot}3\ \text{cm}.$$

15. *A uniform steel wire of cross-sectional area* $0{\cdot}20\ \text{mm}^2$ *is held fixed by clamping its two ends. Find the extra force exerted by each clamp on the wire if the wire is cooled from* 100°C *to* 0°C. *Young's modulus of steel* $= 2.0 \times 10^{11}\ \text{N m}^{-2}$. *Coefficient of linear expansion of steel* $= 1{\cdot}2 \times 10^{-5}\ °\text{C}^{-1}$.

Solution : Let us assume that the tension is zero at 100°C so that l_θ is the natural length of the wire at 100°C. As the wire cools down, its natural length decreases to l_0. As the wire is fixed at the clamps, its length remains the same as the length at 100°C. Thus, the extension of the wire over its natural length at 0°C is

$$l_\theta - l_0 = l_0\,(1 + \alpha\theta) - l_0 = l_0 \alpha\theta.$$

The strain developed is $\dfrac{l_\theta - l_0}{l_\theta} \approx \dfrac{l_\theta - l_0}{l_0} = \alpha\theta.$

The stress developed $= Y \times \text{strain} = Y\alpha\theta.$

The tension in the wire at 0°C is

$$T = \text{stress} \times \text{area}$$
$$= Y\alpha t \times 0{\cdot}20\ \text{mm}^2$$
$$= (2{\cdot}0 \times 10^{11}\ \text{N m}^{-2}) \times (1{\cdot}2 \times 10^{-5}\ °\text{C}^{-1}) \times 100°\text{C} \times (0{\cdot}20 \times 10^{-6}\ \text{m}^{-2})$$
$$= 48\ \text{N}.$$

This is equal to the extra force exerted by each clamp.

16. *A glass vessel of volume* $100\ \text{cm}^{-3}$ *is filled with mercury and is heated from* 25°C *to* 75°C. *What volume of mercury will overflow ? Coefficient of linear expansion of glass* $= 1.8 \times 10^{-6}\ °\text{C}^{-1}$ *and coefficient of volume expansion of mercury is* $1.8 \times 10^{-4}\ °\text{C}^{-1}$.

Solution : The volume of mercury at 25°C is

$$V_0 = 100\ \text{cm}^{-3}.$$

The coefficient of volume expansion of mercury

$$\gamma_L = 1{\cdot}8 \times 10^{-4}\ °\text{C}^{-1}.$$

The coefficient of volume expansion of glass

$$\gamma_S = 3 \times 1{\cdot}8 \times 10^{-6}\ °\text{C}^{-1}$$
$$= 5{\cdot}4 \times 10^{-6}\ °\text{C}^{-1}.$$

Thus, the volume of mercury at 75°C is

$$V_{L\theta} = V_0 (1 + \gamma_L\,\Delta\theta)$$

and the volume of the vessel at 75°C is

$$V_{S\theta} = V_0\,(1 + \gamma_S\,\Delta\theta).$$

The volume of mercury overflown

$$= V_{L\theta} - V_{S\theta} = V_0\,(\gamma_L - \gamma_S)\,\Delta\theta \qquad \text{... (i)}$$
$$= (100\ \text{cm}^{-3})\,(1{\cdot}8 \times 10^{-4} - 5{\cdot}4 \times 10^{-6})/°\text{C} \times (50°\text{C})$$
$$= 0{\cdot}87\ \text{cm}^3.$$

Note that $\gamma_a = (\gamma_L - \gamma_S)$ acts as the effective coefficient of expansion of the liquid with respect to the solid. The expansion of mercury 'as seen from the glass' can be written as

$$V_\theta - V_0 = V_0\,\gamma_a\theta$$

or,
$$V_\theta = V_0 (1 + \gamma_a\theta).$$

The constant γ_a is called the 'apparent coefficient of expansion' of the liquid with respect to the solid.

17. *A barometer reads* 75·0 cm *on a steel scale. The room temperature is* 30°C. *The scale is correctly graduated for* 0°C. *The coefficient of linear expansion of steel is* $\alpha = 1{\cdot}2 \times 10^{-5}\ °\text{C}^{-1}$ *and the coefficient of volume expansion of mercury is* $\gamma = 1{\cdot}8 \times 10^{-4}\ °\text{C}^{-1}$. *Find the correct atmospheric pressure.*

Solution : The 75 cm length of steel at 0°C will become l_θ at 30°C where,

$$l_\theta = (75\ \text{cm})\,[1 + \alpha\,(30°\text{C})]. \qquad \text{... (i)}$$

The length of mercury column at 30°C is l_θ. Suppose the length of the mercury column, if it were at 0°C, is l_0. Then,

$$l_\theta = l_0\,[1 + \frac{1}{3}\gamma\,(30°C)]. \qquad \dots \text{(ii)}$$

By (i) and (ii),

$$l_0[1 + \frac{1}{3}\gamma\,(30°C)] = 75\text{ cm}[1 + \alpha(30°C)]$$

or,
$$l_0 = 75\text{ cm}\frac{[1 + \alpha(30°C)]}{[1 + \frac{1}{3}\gamma\,(30°C)]}$$

$$\approx 75\text{ cm}\,[1 + (\alpha - \frac{\gamma}{3})\,(30°C)]$$

$$= 74{\cdot}89\text{ cm}.$$

□

QUESTIONS FOR SHORT ANSWER

1. If two bodies are in thermal equilibrium in one frame, will they be in thermal equilibrium in all frames ?
2. Does the temperature of a body depend on the frame from which it is observed ?
3. It is heard sometimes that mercury is used in defining the temperature scale because it expands uniformly with the temperature. If the temperature scale is not yet defined, is it logical to say that a substance expands uniformly with the temperature ?
4. In defining the ideal gas temperature scale, it is assumed that the pressure of the gas at constant volume is proportional to the temperature T. How can we verify whether this is true or not ? Are we using the kinetic theory of gases ? Are we using the experimental result that the pressure is proportional to temperature ?
5. Can the bulb of a thermometer be made of an adiabatic wall ?
6. Why do marine animals live deep inside a lake when the surface of the lake freezes ?
7. The length of a brass rod is found to be smaller on a hot summer day than on a cold winter day as measured by the same aluminium scale. Do we conclude that brass shrinks on heating ?
8. If mercury and glass had equal coefficient of volume expansion, could we make a mercury thermometer in a glass tube ?
9. The density of water at 4°C is supposed to be 1000 kg m^{-3}. Is it same at the sea level and at a high altitude ?
10. A tightly closed metal lid of a glass bottle can be opened more easily if it is put in hot water for some time. Explain.
11. If an automobile engine is overheated, it is cooled by putting water on it. It is advised that the water should be put slowly with engine running. Explain the reason.
12. Is it possible for two bodies to be in thermal equilibrium if they are not in contact ?
13. A spherical shell is heated. The volume changes according to the equation $V_\theta = V_0\,(1 + \gamma\theta)$. Does the volume refer to the volume enclosed by the shell or the volume of the material making up the shell ?

OBJECTIVE I

1. A system X is neither in thermal equilibrium with Y nor with Z. The systems Y and Z
   (a) must be in thermal equilibrium
   (b) cannot be in thermal equilibrium
   (c) may be in thermal equilibrium.
2. Which of the curves in figure (23-Q1) represents the relation between Celsius and Fahrenheit temperatures ?

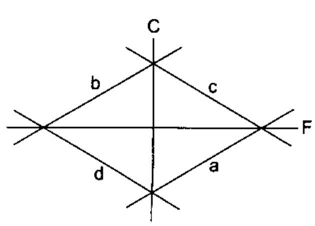

Figure 23-Q1

3. Which of the following pairs may give equal numerical values of the temperature of a body ?
   (a) Fahrenheit and kelvin (b) Celsius and kelvin
   (c) Kelvin and platinum
4. For a constant volume gas thermometer, one should fill the gas at
   (a) low temperature and low pressure
   (b) low temperature and high pressure
   (c) high temperature and low pressure
   (d) high temperature and high pressure.
5. Consider the following statements.
   (A) The coefficient of linear expansion has dimension K^{-1}.
   (B) The coefficient of volume expansion has dimension K^{-1}.
   (a) A and B are both correct.
   (b) A is correct but B is wrong.

(c) B is correct but A is wrong.
(d) A and B are both wrong.

6. A metal sheet with a circular hole is heated. The hole
(a) gets larger (b) gets smaller
(c) remains of the same size (d) gets deformed.

7. Two identical rectangular strips, one of copper and the other of steel, are rivetted together to form a bimetallic strip ($\alpha_{copper} > \alpha_{steel}$). On heating, this strip will
(a) remain straight
(b) bend with copper on convex side
(c) bend with steel on convex side
(d) get twisted.

8. If the temperature of a uniform rod is slightly increased by Δt, its moment of inertia I about a perpendicular bisector increases by
(a) zero (b) $\alpha I \Delta t$ (c) $2\alpha I \Delta t$ (d) $3\alpha I \Delta t$.

9. If the temperature of a uniform rod is slightly increased by Δt, its moment of inertia I about a line parallel to itself will increase by
(a) zero (b) $\alpha I \Delta t$ (c) $2\alpha I \Delta t$ (d) $3\alpha I \Delta t$.

10. The temperature of water at the surface of a deep lake is 2°C. The temperature expected at the bottom is
(a) 0°C (b) 2°C (c) 4°C (d) 6°C.

11. An aluminium sphere is dipped into water at 10°C. If the temperature is increased, the force of buoyancy
(a) will increase (b) will decrease
(c) will remain constant
(d) may increase or decrease depending on the radius of the sphere.

OBJECTIVE II

1. A spinning wheel is brought in contact with an identical wheel spinning at identical speed. The wheels slow down under the action of friction. Which of the following energies of the first wheel decrease ?
(a) Kinetic (b) Total (c) Mechanical (d) Internal

2. A spinning wheel A is brought in contact with another wheel B initially at rest. Because of the friction at contact, the second wheel also starts spinning. Which of the following energies of the wheel B increase ?
(a) Kinetic (b) Total (c) Mechanical (d) Internal

3. A body A is placed on a railway platform and an identical body B in a moving train. Which of the following energies of B are greater than those of A as seen from the ground ?
(a) Kinetic (b) Total (c) Mechanical (d) Internal

4. In which of the following pairs of temperature scales, the size of a degree is identical ?
(a) Mercury scale and ideal gas scale
(b) Celsius scale and mercury scale
(c) Celsius scale and ideal gas scale
(d) Ideal gas scale and absolute scale

5. A solid object is placed in water contained in an adiabatic container for some time. The temperature of water falls during the period and there is no appreciable change in the shape of the object. The temperature of the solid object
(a) must have increased (b) must have decreased
(c) may have increased
(d) may have remained constant.

6. As the temperature is increased, the time period of a pendulum
(a) increases proportionately with temperature
(b) increases (c) decreases
(d) remains constant.

EXERCISES

1. The steam point and the ice point of a mercury thermometer are marked as 80° and 20°. What will be the temperature in centigrade mercury scale when this thermometer reads 32° ?

2. A constant volume thermometer registers a pressure of $1{\cdot}500 \times 10^4$ Pa at the triple point of water and a pressure of $2{\cdot}050 \times 10^4$ Pa at the normal boiling point. What is the temperature at the normal boiling point ?

3. A gas thermometer measures the temperature from the variation of pressure of a sample of gas. If the pressure measured at the melting point of lead is 2·20 times the pressure measured at the triple point of water, find the melting point of lead.

4. The pressure measured by a constant volume gas thermometer is 40 kPa at the triple point of water. What will be the pressure measured at the boiling point of water (100°C) ?

5. The pressure of the gas in a constant volume gas thermometer is 70 kPa at the ice point. Find the pressure at the steam point.

6. The pressures of the gas in a constant volume gas thermometer are 80 cm, 90 cm and 100 cm of mercury at the ice point, the steam point and in a heated wax bath respectively. Find the temperature of the wax bath.

7. In a Callender's compensated constant pressure air thermometer, the volume of the bulb is 1800 cc. When the bulb is kept immersed in a vessel, 200 cc of mercury has to be poured out. Calculate the temperature of the vessel.

8. A platinum resistance thermometer reads 0° when its resistance is 80 Ω and 100° when its resistance is 90 Ω. Find the temperature at the platinum scale at which the resistance is 86 Ω.

9. A resistance thermometer reads $R = 20{\cdot}0\ \Omega$, $27{\cdot}5\ \Omega$, and $50{\cdot}0\ \Omega$ at the ice point (0°C), the steam point (100°C) and the zinc point (420°C) respectively. Assuming that the resistance varies with temperature as $R_\theta = R_0(1 + \alpha\theta + \beta\theta^2)$, find the values of R_0, α and β. Here θ represents the temperature on Celsius scale.

10. A concrete slab has a length of 10 m on a winter night when the temperature is 0°C. Find the length of the slab on a summer day when the temperature is 35°C. The coefficient of linear expansion of concrete is $1{\cdot}0 \times 10^{-5}\ °C^{-1}$.

11. A metre scale made of steel is calibrated at 20°C to give correct reading. Find the distance between 50 cm mark and 51 cm mark if the scale is used at 10°C. Coefficient of linear expansion of steel is $1{\cdot}1 \times 10^{-5}\ °C^{-1}$.

12. A railway track (made of iron) is laid in winter when the average temperature is 18°C. The track consists of sections of 12·0 m placed one after the other. How much gap should be left between two such sections so that there is no compression during summer when the maximum temperature goes to 48°C? Coefficient of linear expansion of iron $= 11 \times 10^{-6}\ °C^{-1}$.

13. A circular hole of diameter 2·00 cm is made in an aluminium plate at 0°C. What will be the diameter at 100°C? α for aluminium $= 2{\cdot}3 \times 10^{-5}\ °C^{-1}$.

14. Two metre scales, one of steel and the other of aluminium, agree at 20°C. Calculate the ratio aluminium-centimetre/steel-centimetre at (a) 0°C, (b) 40°C and (c) 100°C. α for steel $= 1{\cdot}1 \times 10^{-5}\ °C^{-1}$ and for aluminium $= 2{\cdot}3 \times 10^{-5}\ °C^{-1}$.

15. A metre scale is made up of steel and measures correct length at 16°C. What will be the percentage error if this scale is used (a) on a summer day when the temperature is 46°C and (b) on a winter day when the temperature is 6°C? Coefficient of linear expansion of steel $= 11 \times 10^{-6}\ °C^{-1}$.

16. A metre scale made of steel reads accurately at 20°C. In a sensitive experiment, distances accurate up to 0·055 mm in 1 m are required. Find the range of temperature in which the experiment can be performed with this metre scale. Coefficient of linear expansion of steel $= 11 \times 10^{-6}\ °C^{-1}$.

17. The density of water at 0°C is $0{\cdot}998\ g\ cm^{-3}$ and at 4°C is $1{\cdot}000\ g\ cm^{-3}$. Calculate the average coefficient of volume expansion of water in the temperature range 0 to 4°C.

18. Find the ratio of the lengths of an iron rod and an aluminium rod for which the difference in the lengths is independent of temperature. Coefficients of linear expansion of iron and aluminium are $12 \times 10^{-6}\ °C^{-1}$ and $23 \times 10^{-6}\ °C^{-1}$ respectively.

19. A pendulum clock gives correct time at 20°C at a place where $g = 9{\cdot}800\ m\ s^{-2}$. The pendulum consists of a light steel rod connected to a heavy ball. It is taken to a different place where $g = 9{\cdot}788\ m\ s^{-2}$. At what temperature will it give correct time? Coefficient of linear expansion of steel $= 12 \times 10^{-6}\ °C^{-1}$.

20. An aluminium plate fixed in a horizontal position has a hole of diameter 2·000 cm. A steel sphere of diameter 2·005 cm rests on this hole. All the lengths refer to a temperature of 10°C. The temperature of the entire system is slowly increased. At what temperature will the ball fall down? Coefficient of linear expansion of aluminium is $23 \times 10^{-6}\ °C^{-1}$ and that of steel is $11 \times 10^{-6}\ °C^{-1}$.

21. A glass window is to be fit in an aluminium frame. The temperature on the working day is 40°C and the glass window measures exactly 20 cm × 30 cm. What should be the size of the aluminium frame so that there is no stress on the glass in winter even if the temperature drops to 0°C? Coefficients of linear expansion for glass and aluminium are $9{\cdot}0 \times 10^{-6}\ °C^{-1}$ and $24 \times 10^{-6}\ °C^{-1}$ respectively.

22. The volume of a glass vessel is 1000 cc at 20°C. What volume of mercury should be poured into it at this temperature so that the volume of the remaining space does not change with temperature? Coefficients of cubical expansion of mercury and glass are $1{\cdot}8 \times 10^{-4}\ °C^{-1}$ and $9{\cdot}0 \times 10^{-6}\ °C^{-1}$ respectively.

23. An aluminium can of cylindrical shape contains $500\ cm^3$ of water. The area of the inner cross section of the can is $125\ cm^2$. All measurements refer to 10°C. Find the rise in the water level if the temperature increases to 80°C. The coefficient of linear expansion of aluminium $= 23 \times 10^{-6}\ °C^{-1}$ and the average coefficient of volume expansion of water $= 3.2 \times 10^{-4}\ °C^{-1}$ respectively.

24. A glass vessel measures exactly 10 cm × 10 cm × 10 cm at 0°C. It is filled completely with mercury at this temperature. When the temperature is raised to 10°C, $1{\cdot}6\ cm^3$ of mercury overflows. Calculate the coefficient of volume expansion of mercury. Coefficient of linear expansion of glass $= 6{\cdot}5 \times 10^{-6}\ °C^{-1}$.

25. The densities of wood and benzene at 0°C are $880\ kg\ m^{-3}$ and $900\ kg\ m^{-3}$ respectively. The coefficients of volume expansion are $1{\cdot}2 \times 10^{-3}\ °C^{-1}$ for wood and $1{\cdot}5 \times 10^{-3}\ °C^{-1}$ for benzene. At what temperature will a piece of wood just sink in benzene?

26. A steel rod of length 1 m rests on a smooth horizontal base. If it is heated from 0°C to 100°C, what is the longitudinal strain developed?

27. A steel rod is clamped at its two ends and rests on a fixed horizontal base. The rod is unstrained at 20°C. Find the longitudinal strain developed in the rod if the temperature rises to 50°C. Coefficient of linear expansion of steel $= 1{\cdot}2 \times 10^{-5}\ °C^{-1}$.

28. A steel wire of cross-sectional area $0{\cdot}5\ mm^2$ is held between two fixed supports. If the wire is just taut at 20°C, determine the tension when the temperature falls to 0°C. Coefficient of linear expansion of steel is $1{\cdot}2 \times 10^{-5}\ °C^{-1}$ and its Young's modulus is $2{\cdot}0 \times 10^{11}\ N\ m^{-2}$.

29. A steel rod is rigidly clamped at its two ends. The rod is under zero tension at 20°C. If the temperature rises to 100°C, what force will the rod exert on one of the clamps? Area of cross section of the rod = $2{\cdot}00\ mm^2$. Coefficient of linear expansion of steel = $12.0 \times 10^{-6}\ °C^{-1}$ and Young's modulus of steel = $2{\cdot}00 \times 10^{11}\ N\ m^{-2}$.

30. Two steel rods and an aluminium rod of equal length l_o and equal cross section are joined rigidly at their ends as shown in the figure below. All the rods are in a state of zero tension at 0°C. Find the length of the system when the temperature is raised to θ. Coefficient of linear expansion of aluminium and steel are α_a and α_s respectively. Young's modulus of aluminium is Y_a and of steel is Y_s.

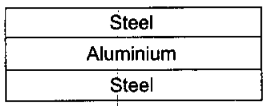

Figure 23-E1

31. A steel ball initially at a pressure of $1{\cdot}0 \times 10^5$ Pa is heated from 20°C to 120°C keeping its volume constant. Find the pressure inside the ball. Coefficient of linear expansion of steel = $12 \times 10^{-6}\ °C^{-1}$ and bulk modulus of steel = $1{\cdot}6 \times 10^{11}\ N\ m^{-2}$.

32. Show that moment of inertia of a solid body of any shape changes with temperature as $I = I_0(1 + 2\alpha\theta)$, where I_0 is the moment of inertia at 0°C and α is the coefficient of linear expansion of the solid.

33. A torsional pendulum consists of a solid disc connected to a thin wire $(\alpha = 2{\cdot}4 \times 10^{-5}\ °C^{-1})$ at its centre. Find the percentage change in the time period between peak winter (5°C) and peak summer (45°C).

34. A circular disc made of iron is rotated about its axis at a constant velocity ω. Calculate the percentage change in the linear speed of a particle of the rim as the disc is slowly heated from 20°C to 50°C keeping the angular velocity constant. Coefficient of linear expansion of iron = $1{\cdot}2 \times 10^{-5}\ °C^{-1}$.

□

ANSWERS

OBJECTIVE I

1. (c) 2. (a) 3. (a) 4. (c) 5. (a) 6. (a)
7. (b) 8. (c) 9. (c) 10. (c) 11. (b)

OBJECTIVE II

1. (a), (c) 2. all 3. (a), (b), (c)
4. (c), (d) 5. (a) 6. (b)

EXERCISES

1. 20°C
2. 373·3 K
3. 601 K
4. 55 kPa
5. 96 kPa
6. 200°C
7. 307 K
8. 60°
9. 20·0 Ω, $3{\cdot}8 \times 10^{-3}\ °C^{-1}$, $-5{\cdot}6 \times 10^{-7}\ °C^{-2}$
10. 10·0035 m
11. 0·99989 cm
12. 0·4 cm
13. 2·0046 cm
14. (a) 0·99977 (b) 1·00025 (c) 1·00096
15. (a) –0·033% (b) 0·011%
16. 15°C to 25°C
17. $-5 \times 10^{-4}\ °C^{-1}$
18. 23 : 12
19. – 82°C
20. 219°C
21. 20·012 cm × 30·018 cm
22. 50 cc
23. 0·089 cm
24. $1{\cdot}8 \times 10^{-4}\ °C^{-1}$
25. 83°C
26. zero
27. $-3{\cdot}6 \times 10^{-4}$
28. 24 N
29. 384 N
30. $l_o\left[1 + \frac{\alpha_a Y_a + 2\alpha_s Y_s}{Y_a + 2Y_s}\theta\right]$
31. $5{\cdot}8 \times 10^8$ Pa
33. $9{\cdot}6 \times 10^{-2}$
34. $3{\cdot}6 \times 10^{-2}$

□