EXERCISE 5.1

Question 1:

Express the given complex number in the form $a + ib$: $(5i)\left(-\frac{3}{5}i\right)$

Ans:

$$(5i)\left(\frac{-3}{5}i\right) = -5 \times \frac{3}{5} \times i \times i$$
$$= -3i^2$$
$$= -3(-1) \qquad \left[i^2 = -1\right]$$
$$= 3$$

Question 2:

Express the given complex number in the form $a + ib$: $i^9 + i^{19}$

Ans:

$$i^9 + i^{19} = i^{4\times2+1} + i^{4\times4+3}$$
$$= \left(i^4\right)^2 \cdot i + \left(i^4\right)^4 \cdot i^3$$
$$= 1 \times i + 1 \times (-i) \qquad \left[i^4 = 1,\ i^3 = -i\right]$$
$$= i + (-i)$$
$$= 0$$

Question 3:

Express the given complex number in the form $a + ib$: i^{-39}

Ans:

$$i^{-39} = i^{-4\times 9-3} = \left(i^4\right)^{-9} \cdot i^{-3}$$

$$= (1)^{-9} \cdot i^{-3} \qquad \left[i^4 = 1\right]$$

$$= \frac{1}{i^3} = \frac{1}{-i} \qquad \left[i^3 = -i\right]$$

$$= \frac{-1}{i} \times \frac{i}{i}$$

$$= \frac{-i}{i^2} = \frac{-i}{-1} = i \qquad \left[i^2 = -1\right]$$

Question 4:

Express the given complex number in the form $a + ib$: $3(7 + i7) + i(7 + i7)$

Ans:

$$3(7+i7)+i(7+i7) = 21+21i+7i+7i^2$$

$$= 21+28i+7\times(-1) \qquad \left[\because i^2 = -1\right]$$

$$= 14+28i$$

Question 5:

Express the given complex number in the form $a + ib$: $(1 - i) - (-1 + i6)$

Ans:

$$(1-i)-(-1+i6) = 1-i+1-6i$$

$$= 2-7i$$

Question 6:

Express the given complex number in the form $a + ib$: $\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$

Ans:

$$\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$$

$$=\frac{1}{5}+\frac{2}{5}i-4-\frac{5}{2}i$$

$$=\left(\frac{1}{5}-4\right)+i\left(\frac{2}{5}-\frac{5}{2}\right)$$

$$=\frac{-19}{5}+i\left(\frac{-21}{10}\right)$$

$$=\frac{-19}{5}-\frac{21}{10}i$$

Question 7:

Express the given complex number in the form a + ib: $\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)$

Ans:

$$\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(\frac{-4}{3}+i\right)$$

$$=\frac{1}{3}+\frac{7}{3}i+4+\frac{1}{3}i+\frac{4}{3}-i$$

$$=\left(\frac{1}{3}+4+\frac{4}{3}\right)+i\left(\frac{7}{3}+\frac{1}{3}-1\right)$$

$$=\frac{17}{3}+i\frac{5}{3}$$

Question 8:

Express the given complex number in the form $a + ib$: $(1 - i)^4$

Ans:

$$(1-i)^4=\left[(1-i)^2\right]^2$$

$$=\left[1^2+i^2-2i\right]^2$$

$$=[1-1-2i]^2$$

$$=(-2i)^2$$

$$=(-2i)\times(-2i)$$

$$=4i^2=-4 \qquad \left[i^2=-1\right]$$

Question 9:

Express the given complex number in the form $a + ib$: $\left(\frac{1}{3}+3i\right)^3$

Ans:

$$\left(\frac{1}{3}+3i\right)^3=\left(\frac{1}{3}\right)^3+(3i)^3+3\left(\frac{1}{3}\right)(3i)\left(\frac{1}{3}+3i\right)$$

$$=\frac{1}{27}+27i^3+3i\left(\frac{1}{3}+3i\right)$$

$$=\frac{1}{27}+27(-i)+i+9i^2 \qquad \left[i^3=-i\right]$$

$$=\frac{1}{27}-27i+i-9 \qquad \left[i^2=-1\right]$$

$$=\left(\frac{1}{27}-9\right)+i(-27+1)$$

$$=\frac{-242}{27}-26i$$

Question 10:

Express the given complex number in the form $a+ib$: $\left(-2-\frac{1}{3}i\right)^3$

Ans:

$$\left(-2-\frac{1}{3}i\right)^3=(-1)^3\left(2+\frac{1}{3}i\right)^3$$

$$=-\left[2^3+\left(\frac{i}{3}\right)^3+3(2)\left(\frac{i}{3}\right)\left(2+\frac{i}{3}\right)\right]$$

$$=-\left[8+\frac{i^3}{27}+2i\left(2+\frac{i}{3}\right)\right]$$

$$=-\left[8-\frac{i}{27}+4i+\frac{2i^2}{3}\right] \qquad \left[i^3=-i\right]$$

$$=-\left[8-\frac{i}{27}+4i-\frac{2}{3}\right] \qquad \left[i^2=-1\right]$$

$$=-\left[\frac{22}{3}+\frac{107i}{27}\right]$$

$$=-\frac{22}{3}-\frac{107}{27}i$$

Question 11:

Find the multiplicative inverse of the complex number $4-3i$

Ans:

Let $z = 4 - 3i$

Then, $\bar{z} = 4 + 3i$ and $|z|^2 = 4^2 + (-3)^2 = 16 + 9 = 25$

Therefore, the multiplicative inverse of $4 - 3i$ is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{4+3i}{25} = \frac{4}{25} + \frac{3}{25}i$$

Question 12:

Find the multiplicative inverse of the complex number $\sqrt{5} + 3i$

Ans:

Let $z = \sqrt{5} + 3i$

Then, $\bar{z} = \sqrt{5} - 3i$ and $|z|^2 = \left(\sqrt{5}\right)^2 + 3^2 = 5 + 9 = 14$

Therefore, the multiplicative inverse of $\sqrt{5} + 3i$ is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{\sqrt{5} - 3i}{14} = \frac{\sqrt{5}}{14} - \frac{3i}{14}$$

Question 13:

Find the multiplicative inverse of the complex number $-i$

Ans:

Let $z = -i$

Then, $\bar{z} = i$ and $|z|^2 = 1^2 = 1$

Therefore, the multiplicative inverse of $-i$ is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{i}{1} = i$$

Question 14:

Express the following expression in the form of $a + ib$.

$$\frac{\left(3 + i\sqrt{5}\right)\left(3 - i\sqrt{5}\right)}{\left(\sqrt{3} + \sqrt{2}i\right) - \left(\sqrt{3} - i\sqrt{2}\right)}$$

Ans:

$$\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}i\right)-\left(\sqrt{3}-i\sqrt{2}\right)}$$

$$=\frac{(3)^2-\left(i\sqrt{5}\right)^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+\sqrt{2}i} \qquad \left[(a+b)(a-b)=a^2-b^2\right]$$

$$=\frac{9-5i^2}{2\sqrt{2}i}$$

$$=\frac{9-5(-1)}{2\sqrt{2}i} \qquad \left[i^2=-1\right]$$

$$=\frac{9+5}{2\sqrt{2}i}\times\frac{i}{i}$$

$$=\frac{14i}{2\sqrt{2}i^2}$$

$$=\frac{14i}{2\sqrt{2}(-1)}$$

$$=\frac{-7i}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$=\frac{-7\sqrt{2}i}{2}$$

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