Tangents and Normals Ex 16.1 Q1(v)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

$x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$

Now,

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-a\sin\theta}{a(1 - \cos\theta)}$$

$\therefore$ Slope of tangent of $\theta = -\frac{\pi}{2}$

$$\left(\frac{dy}{dx}\right)_{\theta = -\frac{\pi}{2}} = \frac{-a\sin\left(-\frac{\pi}{2}\right)}{a\left(1 - \cos\left(-\frac{\pi}{2}\right)\right)}$$

$$= \frac{a}{a(1 - 0)} = 1$$

Also, the slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} = -1$$

Tangents and Normals Ex 16.1 Q1(vi)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

$x = a\cos^3\theta,\ y = a\sin^3\theta$

$\therefore \quad \frac{dx}{d\theta} = 3a\cos^2\theta \times (-\sin\theta) = -3a\sin\theta \times \cos^2\theta$

and $\quad \frac{dy}{dx} = 3a\sin^2\theta \times \cos\theta$

Now,

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3a\sin^2\theta \times \cos\theta}{-3a\sin\theta \times \cos^2\theta}$$

$$= -\tan\theta$$

$\therefore \quad$ Slope of tangent at $\theta = \frac{\pi}{4}$ is

$$\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{4}} = 1$$

Also, the slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} = -1$$

Tangents and Normals Ex 16.1 Q1(vii)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

$x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$

$$\therefore \qquad \frac{dx}{d\theta} = a(1 - \cos\theta), \quad \frac{dy}{d\theta} = a(0 + \sin\theta) = a\sin\theta$$

Now,

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\sin\theta}{a(1 - \cos\theta)}$$

Now, the slope of tangent at $\theta = \frac{\pi}{2}$ is

$$\left(\frac{dy}{dx}\right)_{\theta = \frac{\pi}{2}} = \frac{a\sin\frac{\pi}{2}}{a\left(1 - \cos\frac{\pi}{2}\right)} = \frac{a}{a} = 1$$

And, the slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = -1$$

Tangents and Normals Ex 16.1 Q1(viii)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad \text{---(B)}$$

$y = (\sin 2x + \cot x + 2)^2$

$$\therefore \qquad \frac{dy}{dx} = 2(\sin 2x + \cot x + 2)\left(2\cos 2x - \operatorname{cosec}^2 x\right)$$

$\therefore$ Slope of tangent of $x = \frac{\pi}{2}$ is

$$\left(\frac{dy}{dx}\right)_{x = \frac{\pi}{2}} = 2\left(\sin\pi + \cos\frac{\pi}{2} + 2\right)\left(2\cos\pi - \operatorname{cosec}^2\frac{\pi}{2}\right)$$

$$= 2(0 + 0 + 2)(-2 - 1)$$

$$= -12$$

$\therefore$ Slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{1}{12}$$

*********** END **********