Co-Ordinate Geometry Ex 14.5 Q8

Answer :

GIVEN: The area of triangle is 5.Two of its vertices are (2, 1) and (3, −2). The third vertex lies on *y* = *x*+3

TO FIND: The third vertex.

PROOF: Let the third vertex be (*x*, *y*)

We know area of triangle formed by three points $(x_1, y_1), (x_2, y_2)\, and\, (x_3, y_3)$ is given by

$$\Delta = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Now

Taking three points(*x*, *y*), (2, 1) and (3, −2)

$$\Delta = \frac{1}{2}|(x - 4 + 3y) - (2y + 3 - 2x)|$$

$$\Delta = \frac{1}{2}|3x + y - 7|$$

$$5 = \frac{1}{2}|3x + y - 7|$$

$$\pm 10 = 3x + y - 7$$

$$10 = 3x + y - 7 \quad \text{or} -10 = 3x + y - 7$$

$$0 = 3x + y - 17 \quad \ldots\ldots(1) \text{or } 0 = 3x + y + 3 \quad \ldots\ldots(2)$$

Also it is given the third vertex lies on *y* = *x*+3

Substituting the value in equation (1) and (2) we get

$\pm 10 = 3x + y - 7$

$10 = 3x + y - 7$

$0 = 3x + y - 17 \qquad \ldots\ldots(1)$

$0 = 3x + (x + 3) - 17$

$x = \frac{7}{2}$

Again subsituting the value of x in equation 1we get

$0 = 3x + y - 17 \qquad \ldots\ldots(1)$

$0 = 3\left(\frac{7}{2}\right) + y - 17$

$y = \frac{13}{2}$

Hence $\left(\frac{7}{2}, \frac{13}{2}\right)$

Similiarly

$-10 = 3x + y - 7$

$0 = 3x + y + 3 \qquad \ldots\ldots(2)$

$0 = 3x + (x + 3)$

$x = \frac{-3}{2}$

Again subsituting the value of x in equation 2 we get

$0 = 3x + y + 3 \qquad \ldots\ldots(2)$

$0 = 3\left(\frac{-3}{2}\right) + y + 3$

$y = \frac{3}{2}$

Hence $\left(\frac{-3}{2}, \frac{3}{2}\right)$

Hence the coordinates of $\left(\frac{7}{2}, \frac{13}{2}\right)$ and $\left(\frac{-3}{2}, \frac{3}{2}\right)$

Co-Ordinate Geometry Ex 14.5 Q9

Answer :

GIVEN: If $a \neq b \neq c$

TO PROVE: that the points $(a,a^2),(b,b^2),(c,c^2)$, can never be collinear.

PROOF:

We know three points $(x_1,y_1),(x_2,y_2)$ and (x_3,y_3) are collinear when

$$\frac{1}{2}\left|\,x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|=0$$

Now taking three points $(a,a^2),(b,b^2),(c,c^2)$,

$$\begin{aligned}\text{Area} &= \frac{1}{2}\left|a(b^2-c^2)+b(c^2-a^2)+c(a^2-b^2)\right| \\ &= \frac{1}{2}\left|ab^2-ac^2+bc^2-ba^2+ca^2-cb^2\right| \\ &= \frac{1}{2}\left|(a^2c-a^2b)+(ab^2-ac^2)+(bc^2-b^2c)\right| \\ &= \frac{1}{2}\left|(-a^2(b-c))+(a(b^2-c^2))-(bc(b-c))\right| \\ &= \frac{1}{2}\left|(b-c)(-a^2)+(a(b+c))-bc\right| \\ &= \frac{1}{2}\left|(b-c)(-a^2)+ab+ac-bc\right| \\ &= \frac{1}{2}\left|(b-c)(-a)(a-b)+c(a-b)\right|\end{aligned}$$

$$=\frac{1}{2}\left|(b-c)(a-b)(c-a)\right|$$

Also it is given that

$a \neq b \neq c$

Hence area of triangle made by these points is never zero. Hence given points are never collinear.

*********** END **********