



Functions Ex 3.2 Q6

We have,

$$f(x) = \begin{cases} x^2, & \text{when } x < 0 \\ x, & \text{when } 0 \leq x < 1 \\ \frac{1}{x}, & \text{when } x \geq 1 \end{cases}$$

$$(a) f(1/2) = \frac{1}{2}$$

$$(b) f(-2) = (-2)^2 = 4$$

$$(c) f(1) = \frac{1}{1} = 1$$

$$(d) f(\sqrt{3}) = \frac{1}{\sqrt{3}}$$

$$(e) f(\sqrt{-3}) = \text{does not exist because } \sqrt{-3} \notin \text{domain}(f).$$

Functions Ex 3.2 Q7

We have,

$$f(x) = x^3 - \frac{1}{x^3} \quad \text{---(i)}$$

Now,

$$f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^3 - \frac{1}{\left(\frac{1}{x}\right)^3}$$

$$= \frac{1}{x^3} - \frac{1}{\frac{1}{x^3}}$$

$$\Rightarrow f\left(\frac{1}{x}\right) = \frac{1}{x^3} - x^3 \quad \text{---(ii)}$$

Adding equation (i) and equation (ii), we get

$$\begin{aligned} f(x) + f\left(\frac{1}{x}\right) &= \left(x^3 - \frac{1}{x^3}\right) + \left(\frac{1}{x^3} - x^3\right) \\ &= x^3 - \frac{1}{x^3} + \frac{1}{x^3} - x^3 \\ &= 0 \end{aligned}$$

$$\therefore f(x) + f\left(\frac{1}{x}\right) = 0 \quad \text{Hence, proved.}$$

Functions Ex 3.2 Q8

We have,

$$f(x) = \frac{2x}{1+x^2}$$

Now,

$$\begin{aligned} f(\tan \theta) &= \frac{2(\tan \theta)}{1 + \tan^2 \theta} \\ &= \sin 2\theta \end{aligned}$$

$$\left[ \because \sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta} \right]$$

$\therefore f(\tan \theta) = \sin 2\theta$  Hence, proved.

Functions Ex 3.2 Q9

i.   $\frac{x-1}{x+1}$

$$f\left(\frac{1}{x}\right) = \frac{\frac{1}{x} - 1}{\frac{1}{x} + 1} = \frac{\frac{1-x}{x}}{\frac{1+x}{x}} = \frac{1-x}{1+x} = -f(x)$$

ii.  $f(x) = \frac{x-1}{x+1}$

$$f\left(-\frac{1}{x}\right) = \frac{-\frac{1}{x} - 1}{-\frac{1}{x} + 1} = \frac{\frac{-1-x}{x}}{\frac{-1+x}{x}} = \frac{-1-x}{-1+x} = -\frac{1}{\frac{1+x}{x-1}} = -\frac{1}{f(x)}$$

Functions Ex 3.2 Q10

We have,

$$f(x) = (a - x^n)^{1/n}, \quad a > 0$$

Now,

$$\begin{aligned} f(f(x)) &= f(a - x^n)^{1/n} \\ &= \left[ a - \left\{ (a - x^n)^{1/n} \right\}^n \right]^{1/n} \\ &= \left[ a - (a - x^n) \right]^{1/n} \\ &= \left[ a - a + x^n \right]^{1/n} \\ &= (x^n)^{1/n} \\ &= (x)^{n \times \frac{1}{n}} \\ &= x \end{aligned}$$

$\therefore f(f(x)) = x$  Hence, proved.

Functions Ex 3.2 Q11

We have,

$$af(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 5 \quad \text{--- (i)}$$

$$\Rightarrow \quad af\left(\frac{1}{x}\right) + bf(x) = \frac{1}{\frac{1}{x}} - 5 \\ = x - 5$$

$$\Rightarrow \quad af\left(\frac{1}{x}\right) + bf(x) = x - 5 \quad \text{--- (ii)}$$

Adding equations (i) and (ii), we get

$$af(x) + bf(x) + bf\left(\frac{1}{x}\right) + af\left(\frac{1}{x}\right) = \frac{1}{x} - 5 + x - 5$$

$$\Rightarrow \quad (a+b)f(x) + f\left(\frac{1}{x}\right)(a+b) = \frac{1}{x} + x - 10$$

$$\Rightarrow \quad f(x) + f\left(\frac{1}{x}\right) = \frac{1}{a+b} \left[ \frac{1}{x} + x - 10 \right] \quad \text{--- (iii)}$$

Subtracting equation (ii) from equation (i), we get

$$af(x) - bf(x) + bf\left(\frac{1}{x}\right) - af\left(\frac{1}{x}\right) = \frac{1}{x} - 5 - x + 5$$

$$\Rightarrow \quad (a-b)f(x) - f\left(\frac{1}{x}\right)(a-b) = \frac{1}{x} - x$$

$$\Rightarrow \quad f(x) - f\left(\frac{1}{x}\right) = \frac{1}{a-b} \left[ \frac{1}{x} - x \right]$$

Adding equations (iii) and (iv), we get

$$2f(x) = \frac{1}{a+b} \left[ \frac{1}{x} + x - 10 \right] + \frac{1}{a-b} \left[ \frac{1}{x} - x \right]$$

$$\Rightarrow \quad 2f(x) = \frac{(a-b) \left[ \frac{1}{x} + x - 10 \right] + (a+b) \left[ \frac{1}{x} - x \right]}{(a+b)(a-b)}$$

$$\Rightarrow \quad 2f(x) = \frac{\frac{a}{x} + ax - 10a - \frac{b}{x} - bx + 10b + \frac{a}{x} - ax + \frac{b}{x} - bx}{a^2 - b^2}$$

$$\Rightarrow \quad 2f(x) = \frac{\frac{2a}{x} - 10a + 10b - 2bx}{a^2 - b^2}$$

$$\Rightarrow \quad f(x) = \frac{1}{a^2 - b^2} \times \frac{1}{2} \left[ \frac{2a}{x} - 10a + 10b - 2bx \right] \\ = \frac{1}{a^2 - b^2} \left[ \frac{a}{x} - 5a + 5b - bx \right]$$

$$\therefore \quad f(x) = \frac{1}{a^2 - b^2} \left[ \frac{a}{x} - bx - 5a + 5b \right] \\ = \frac{1}{a^2 - b^2} \left[ \frac{a}{x} - bx \right] - \frac{5(a-b)}{a^2 - b^2} \\ = \frac{1}{a^2 - b^2} \left[ \frac{a}{x} - bx \right] - \frac{5(a-b)}{(a-b)(a+b)} \\ = \frac{1}{a^2 - b^2} \left[ \frac{a}{x} - bx \right] - \frac{5}{a+b}$$

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