Transformation Formulae Ex 8.2 Q 8(i)

We have,

$$\text{LHS} = \frac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A}$$

$$= \frac{(\sin 5A + \sin A) + \sin 3A}{(\cos 5A + \cos A) + \cos 3A}$$

$$= \frac{2\sin\left(\frac{5A + A}{2}\right)\cos\left(\frac{5A - A}{2}\right) + \sin 3A}{2\cos\left(\frac{5A + A}{2}\right)\cos\left(\frac{5A - A}{2}\right) + \cos 3A}$$

$$= \frac{2\sin 3A\cos 2A + \sin 3A}{2\cos 3A\cos 2A + \cos 3A}$$

$$= \frac{\sin 3A\,(2\cos 2A + 1)}{\cos 3A\,(2\cos 2A + 1)}$$

$$= \frac{\sin 3A}{\cos 3A}$$

$$= \tan 3A$$

$$= \text{RHS}$$

$$\therefore \frac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A} = \tan 3A$$ Hence proved.

Transformation Formulae Ex 8.2 Q 8(ii)

We have,

$$\text{LHS} = \frac{\cos 3A + 2\cos 5A + \cos 7A}{\cos A + 2\cos 3A + \cos 5A}$$

$$= \frac{(\cos 7A + \cos 3A) + 2\cos 5A}{(\cos 5A + \cos A) + 2\cos 3A}$$

$$= \frac{2\cos\left(\frac{7A + 3A}{2}\right)\cos\left(\frac{7A - 3A}{2}\right) + 2\cos 5A}{2\cos\left(\frac{5A + A}{2}\right)\cos\left(\frac{5A - A}{2}\right) + \cos 3A}$$

$$= \frac{2\cos 5A\cos 2A + 2\cos 5A}{2\cos 3A\cos 2A + 2\cos 3A}$$

$$= \frac{2\cos 5A\,(\cos 2A + 1)}{2\cos 3A\,(\cos 2A + 1)}$$

$$= \frac{\cos 5A}{\cos 3A}$$

$$= \text{RHS}$$

$$\therefore \frac{\cos 3A + 2\cos 5A + \cos 7A}{\cos A + 2\cos 3A + \cos 5A} = \frac{\cos 5A}{\cos 3A}$$ Hence proved.

Transformation Formulae Ex 8.2 Q 8(iii)

We have,

$$\text{LHS} = \frac{\cos 4A + \cos 3A + \cos 2A}{\sin 4A + \sin 3A + \sin 2A}$$

$$= \frac{(\cos 4A + \cos 2A) + \cos 3A}{(\sin 4A + \sin 2A) + \sin 3A}$$

$$= \frac{2\cos\left(\frac{4A+2A}{2}\right)\cos\left(\frac{4A-2A}{2}\right) + \cos 3A}{2\sin\left(\frac{4A+2A}{2}\right)\cos\left(\frac{4A-2A}{2}\right) + \sin 3A}$$

$$= \frac{2\cos 3A \cos A + \cos 3A}{2\sin 3A \cos A + \sin 3A}$$

$$= \frac{\cos 3A(2\cos A + 1)}{\sin 3A(2\cos A + 1)}$$

$$= \frac{\cos 3A}{\sin A}$$

$$= \cot 3A$$

$$= \text{RHS}$$

$\therefore \quad \dfrac{\cos 4A + \cos 3A + \cos 2A}{\sin 4A + \sin 3A + \sin 2A} = \cot 3A$ Hence proved.

Transformation Formulae Ex 8.2 Q 8(iv)

We have,

$$\text{LHS} = \frac{\sin 3A + \sin 5A + \sin 7A + \sin 9A}{\cos 3A + \cos 5A + \cos 7A + \cos 9A}$$

$$= \frac{(\sin 9A + \sin 3A) + (\sin 7A + \sin 5A)}{(\cos 9A + \cos 3A) + (\cos 7A + \cos 5A)}$$

$$= \frac{2\sin\left(\frac{9A+3A}{2}\right)\cos\left(\frac{9A-3A}{2}\right) + 2\sin\left(\frac{7A+5A}{2}\right)\cos\left(\frac{7A-5A}{2}\right)}{2\cos\left(\frac{9A+3A}{2}\right)\cos\left(\frac{9A-3A}{2}\right) + 2\cos\left(\frac{7A+5A}{2}\right)\cos\left(\frac{7A-5A}{2}\right)}$$

$$= \frac{2\sin 6A \cos 3A + 2\sin 6A \cos A}{2\cos 6A \cos 3A + 2\cos 6A \cos A}$$

$$= \frac{2\sin 6A(\cos 3A + \cos A)}{2\cos 6A(\cos 3A + \cos A)}$$

$$= \frac{\sin 6A}{\cos 6A}$$

$$= \tan 6A$$

$$= \text{RHS}$$

$\therefore \quad \dfrac{\sin 3A + \sin 5A + \sin 7A + \sin 9A}{\cos 3A + \cos 5A + \cos 7A + \cos 9A} = \tan 6A$ Hence proved.

Transformation Formulae Ex 8.2 Q 8(v)

We have,

$$\text{LHS} = \frac{\sin 5A - \sin 7A + \sin 8A - \sin 4A}{\cos 4A + \cos 7A - \cos 5A - \cos 8A}$$

$$= \frac{-(\sin 7A - \sin 5A) + (\sin 8A - \sin 4A)}{-(\cos 7A - \cos 5A) - (\cos 8A - \cos 4A)}$$

$$= \frac{-\left[2\sin\left(\frac{7A-5A}{2}\right)\cos\left(\frac{7A+5A}{2}\right)\right] + \left[2\sin\left(\frac{8A-4A}{2}\right)\cos\left(\frac{8A+4A}{2}\right)\right]}{-2\sin\left(\frac{7A+5A}{2}\right)\sin\left(\frac{7A-5A}{2}\right) - \left[-2\sin\left(\frac{8A+4A}{2}\right)\sin\left(\frac{8A-4A}{2}\right)\right]}$$

$$= \frac{-2\sin A\cos 6A + 2\sin 2A\cos 6A}{-2\sin 6A\sin A + 2\sin 6A\sin 2A}$$

$$= \frac{2\cos 6A[-\sin A + \sin 2A]}{2\sin 6A[-\sin A + \sin 2A]}$$

$$= \frac{\cos 6A}{\sin 6A}$$

$$= \cot 6A$$

= RHS

$$\therefore \quad \frac{\sin 5A - \sin 7A + \sin 8A - \sin 4A}{\cos 4A + \cos 7A - \cos 5A - \cos 8A} = \cot 6A$$ Hence proved.

Transformation Formulae Ex 8.2 Q 8(vi)

We have,

$$\text{LHS} = \frac{\sin 5A\cos 2A - \sin 6A\cos A}{\sin A\sin 2A - \cos 2A\cos 3A}$$

$$= \frac{2(\sin 5A\cos 2A - \sin 6A\cos A)}{2(\sin A\sin 2A - \cos 2A\cos 3A)}$$

$$= \frac{2\sin 5A\cos 2A - 2\sin 6A\cos A}{2\sin A\sin 2A - 2\cos 2A\cos 3A}$$

$$= \frac{\sin(5A+2A) + \sin(5A-2A) - [\sin(6A+A) + \sin(6A-A)]}{\cos(2A-A) - \cos(2A+A) - [\cos(3A+2A) + \cos(3A-2A)]}$$

$$= \frac{\sin 7A + \sin 3A - \sin 7A - \sin 5A}{\cos A - \cos 3A - \cos 5A - \cos A}$$

$$= \frac{\sin 3a - \sin 5A}{-\cos 3A - \cos 5A}$$

$$= \frac{-(\sin 5A - \sin 3A)}{-(\cos 5A + \cos 3A)}$$

$$= \frac{\sin 5A - \sin 3A}{\cos 5A + \cos 3A}$$

$$= \frac{2\sin\left(\frac{5A-3A}{2}\right)\cos\left(\frac{5A+3A}{2}\right)}{2\cos\left(\frac{5A+3A}{2}\right)\cos\left(\frac{5A-3A}{2}\right)}$$

$$= \frac{\sin A\cos 4A}{\cos 4A\cos A}$$

$$= \frac{\sin A}{\cos A}$$

$$= \tan A$$

= RHS

$$\therefore \quad \frac{\sin 5A\cos 2A - \sin 6A\cos A}{\sin A\sin 2A - \cos 2A\cos 3A} = \tan A$$ Hence proved.

Transformation Formulae Ex 8.2 Q 8(vii)

We have,

$$\text{LHS} = \frac{\sin 11A \sin A + \sin 7A \sin 3A}{\cos 11A \sin A + \cos 7A \sin 3A}$$

$$= \frac{2(\sin 11A \sin A + \sin 7A \sin 3A)}{2(\cos 11A \sin A + \cos 7A \sin 3A)}$$

$$= \frac{2\sin 11A \sin A + 2\sin 7A \sin 3A}{2\cos 11A \sin A + 2\cos 7A \sin 3A}$$

$$= \frac{\cos(11A - A) - \cos(11A + A) + \cos(7A - 3A) - \cos(7A + 3A)}{\sin(11A + A) - \sin(11A - A) + \sin(7A + 3A) - \sin(7A - 3A)}$$

$$= \frac{\cos 10A - \cos 12A + \cos 4A - \cos 10A}{\sin 12A - \sin 10A + \sin 10A - \sin 4A}$$

$$= \frac{-(\cos 12A - \cos 4A)}{\sin 12A - \sin 4A}$$

$$= \frac{-\left[-2\sin\left(\frac{12A + 4A}{2}\right)\sin\left(\frac{12A - 4A}{2}\right)\right]}{2\sin\left(\frac{12A - 4A}{2}\right)\cos\left(\frac{12A + 4A}{2}\right)}$$

$$= \frac{2\sin 8A \sin 4A}{2\sin 4A \cos 8A}$$

$$= \frac{\sin 8A}{\cos 8A}$$

$$= \tan 8A$$

$$= \text{RHS}$$

$\therefore \frac{\sin 11A \sin A + \sin 7A \sin 3A}{\cos 11A \sin A + \cos 7A \sin 3A} = \tan 8A$ Hence proved.

Transformation Formulae Ex 8.2 Q 8(viii)

$$\text{LHS} = \frac{\sin 3A \cos 4A - \sin A \cos 2A}{\sin 4A \sin A + \cos 6A \cos A}$$

$$= \frac{2(\sin 3A \cos 4A - \sin A \cos 2A)}{2(\sin 4A \sin A + \cos 6A \cos A)}$$

$$= \frac{2\sin 3A \cos 4A - 2\sin A \cos 2A}{2\sin 4A \sin A + 2\cos 6A \cos A}$$

$$= \frac{\sin(4A+3A) - \sin(4A-3A) - [\sin(2A+A) - \sin(2A-A)]}{\cos(4A-A) - \cos(4A+A) + \cos(6A+A) + \cos(6A-A)}$$

$$= \frac{\sin(7A) - \sin(A) - \sin(3A) + \sin(A)}{\cos(3A) - \cos(5A) + \cos(7A) + \cos(5A)}$$

$$= \frac{\sin(7A) - \sin(3A)}{\cos(3A) + \cos(7A)}$$

$$= \frac{2\sin\left(\frac{7A-3A}{2}\right)\cos\left(\frac{7A+3A}{2}\right)}{2\cos\left(\frac{7A+3A}{2}\right)\cos\left(\frac{7A-3A}{2}\right)}$$

$$= \frac{\sin 2A}{\cos 2A}$$

$$= \tan 2A$$

$$= RHS$$

Transformation Formulae Ex 8.2 Q 8(ix)

We have,

$$\text{LHS} = \frac{\sin A \sin 2A + \sin 3A \sin 6A}{\sin A \cos 2A + \sin 3A \cos 6A}$$

$$= \frac{2[\sin A \sin 2A + \sin 3A \sin 6A]}{2[\sin A \cos 2A + \sin 3A \cos 6A]}$$

$$= \frac{2 \sin 2A \sin A + 2 \sin 6A \sin 3A}{2 \cos 2A \sin A + 2 \cos 6A \sin 3A}$$

$$= \frac{\cos(2A - A) - \cos(2A + A) + \cos(6A - 3A) - \cos(6A + 3A)}{\sin(2A + A) - \sin(2A - A) + \sin(6A + 3A) - \sin(6A - 3A)}$$

$$= \frac{\cos A - \cos 3A + \cos 3A - \cos 9A}{\sin 3A - \sin A + \sin 9A - \sin 3A}$$

$$= \frac{\cos A - \cos 9A}{\sin 9A - \sin A}$$

$$= \frac{-[\cos 9A - \cos A]}{\sin 9A - \sin A}$$

$$= \frac{-\left(-2 \sin\left(\frac{9A + A}{2}\right) \times \sin\left(\frac{9A - A}{2}\right)\right)}{2 \sin\left(\frac{9A - A}{2}\right) \times \cos\left(\frac{9A + A}{2}\right)}$$

$$= \frac{\sin 5A \sin 4A}{\sin 4A \cos 5A}$$

$$= \tan 5A$$

$$= \text{RHS}$$

$$\therefore \frac{\sin A \sin 2A + \sin 3A \sin 6A}{\sin A \cos 2A + \sin 3A \cos 6A} = \tan 5A$$ Hence proved.

Transformation Formulae Ex 8.2 Q 8(x)

We have,

$$\text{LHS} = \frac{\sin A + 2 \sin 3A + \sin 5A}{\sin 3A + 2 \sin 5A + \sin 7A}$$

$$= \frac{\sin 5A + \sin A + 2 \sin 3A}{\sin 7A + \sin 3A + 2 \sin 5A}$$

$$= \frac{2 \sin\left(\frac{5A + A}{2}\right) \cos\left(\frac{5A - A}{2}\right) + 2 \sin 3A}{2 \sin\left(\frac{7A + 3A}{2}\right) \cos\left(\frac{7A - 3A}{2}\right) + 2 \sin 5A}$$

$$= \frac{2 \sin 3A \cos 2A + 2 \sin 3A}{2 \sin 5A \cos 2A + 2 \sin 5A}$$

$$= \frac{2 \sin 3A (\cos 2A + 1)}{2 \sin 5A (\cos 2A + 1)}$$

$$= \frac{\sin 3A}{\sin 5A}$$

$$= \text{RHS}$$

$$\therefore \frac{\sin A + 2 \sin 3A + \sin 5A}{\sin 3A + 2 \sin 5A + \sin 7A} = \frac{\sin 3A}{\sin 5A}$$ Hence proved.

Transformation Formulae Ex 8.2 Q 8(xi)

We have,

$$\text{LHS} = \frac{\sin(\theta+\phi) - 2\sin\theta + \sin(\theta-\phi)}{\cos(\theta+\phi) - 2\cos\theta + \cos(\theta-\phi)}$$

$$= \frac{\sin(\theta+\phi) + \sin(\theta-\phi) - 2\sin\theta}{\cos(\theta+\phi) + \cos(\theta-\phi) - 2\cos\theta}$$

$$= \frac{2\sin\left[\frac{(\theta+\phi)+(\theta-\phi)}{2}\right]\cos\left[\frac{(\theta+\phi)-(\theta-\phi)}{2}\right] - 2\sin\theta}{2\cos\left[\frac{(\theta+\phi)+(\theta-\phi)}{2}\right]\cos\left[\frac{(\theta+\phi)-(\theta-\phi)}{2}\right] - 2\cos\theta}$$

$$= \frac{2\sin(\theta)\cos(\phi) - 2\sin\theta}{2\cos(\theta)\cos(\phi) - 2\cos\theta}$$

$$= \frac{2\sin\theta(\cos\phi - 1)}{2\cos\theta(\cos\phi - 1)}$$

$$= \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$= \text{RHS}$$

$$\therefore \frac{\sin(\theta+\phi) - 2\sin\theta + \sin(\theta-\phi)}{\cos(\theta+\phi) - 2\cos\theta + \cos(\theta-\phi)} = \tan\theta$$ Hence proved.

*********** END **********