Exercise 7A

Question 32

$$\frac{\cot^2\theta(\sec\theta-1)}{(1+\sin\theta)}\times\frac{(\sec\theta+1)}{(\sec\theta+1)}$$

$$=\frac{\cot^2\theta\left(\sec^2\theta-1\right)}{(1+\sin\theta)(\sec\theta+1)}=\frac{\cot^2\theta\tan^2\theta}{(1+\sin\theta)(\sec\theta+1)}$$

$$=\frac{\frac{1}{\tan^2\theta}\times\tan^2\theta}{(1+\sin\theta)(\sec\theta+1)}$$

$$=\frac{1}{(1+\sin\theta)(\sec\theta+1)}\times\frac{(1-\sin\theta)}{(1-\sin\theta)}$$

$$=\frac{(1-\sin\theta)}{\left(1-\sin^2\theta\right)(\sec\theta+1)}=\frac{(1-\sin\theta)}{\cos^2\theta(1+\sec\theta)}$$

$$=\frac{\sec^2\theta(1-\sin\theta)}{(1+\sec\theta)}$$

$\therefore$ LHS = RHS

Question 33

$$\text{LHS} = \left[\frac{1}{\left(\sec^2\theta - \cos^2\theta\right)} + \frac{1}{\left(\operatorname{cosec}^2\theta - \sin^2\theta\right)}\right] \times \sin^2\theta\cos^2\theta$$

$$= \left[\frac{1}{\frac{1}{\cos^2\theta} - \cos^2\theta} + \frac{1}{\frac{1}{\sin^2\theta} - \sin^2\theta} + \right] \times \sin^2\theta\cos^2\theta$$

$$= \left[\frac{\sin^2\theta\cos^2\theta \times \cos^2\theta}{\left(1 - \cos^4\theta\right)} + \frac{\sin^2\theta\cos^2\theta\sin^2\theta}{1 - \sin^4\theta}\right]$$

$$= \left[\frac{\sin^2\theta \times \cos^4\theta}{\left(1 + \cos^2\theta\right)\left(1 - \cos^2\theta\right)} + \frac{\sin^4\theta\cos^2\theta}{\left(1 - \sin^2\theta\right)\left(1 + \sin^2\theta\right)}\right]$$

$$= \left[\frac{\cos^4\theta}{\left(1 + \cos^2\theta\right)} + \frac{\sin^4\theta}{1 + \sin^2\theta}\right]$$

$$= \frac{\cos^4\theta + \cos^4\theta\sin^2\theta + \sin^4\theta + \sin^4\theta\cos^2\theta}{\left(1 + \cos^2\theta\right)\left(1 + \sin^2\theta\right)}$$

$$= \frac{\cos^4\theta + \sin^4\theta + \cos^2\theta\sin^2\theta\left(\cos^2\theta + \sin^2\theta\right)}{1 + \sin^2\theta + \cos^2\theta + \cos^2\theta\sin^2\theta}$$

$$\left[\because a^2 + b^2 = (a + b)^2 - 2ab\right]$$

$$= \frac{\left(\cos^2\theta + \sin^2\theta\right)^2 - 2\cos^2\theta\sin^2\theta + \cos^2\theta\sin^2\theta}{2 + \cos^2\theta\sin^2\theta}$$

$$= \frac{1 - \cos^2\theta\sin^2\theta}{2 + \cos^2\theta\sin^2\theta} = \text{RHS}$$

$\therefore$ LHS = RHS

*********** END **********