## Trigonometric Identities Ex 6.1 Q84

**Answer :**

Given: $\cos\theta+\cos^2\theta=1$

We have to prove $\sin^{12}\theta+3\sin^{10}\theta+3\sin^8\theta+\sin^6\theta+2\sin^4\theta+2\sin^2\theta-2=1$

From the given equation, we have

$$\cos\theta+\cos^2\theta=1$$
$$\Rightarrow \quad \cos\theta=1-\cos^2\theta$$
$$\Rightarrow \quad \cos\theta=\sin^2\theta$$
$$\Rightarrow \quad \sin^2\theta=\cos\theta$$

Therefore, we have

$$\begin{aligned}
&\sin^{12}\theta+3\sin^{10}\theta+3\sin^8\theta+\sin^6\theta+2\sin^4\theta+2\sin^2\theta-2\\
&=(\sin^{12}\theta+3\sin^{10}\theta+3\sin^8\theta+\sin^6\theta)+(2\sin^4\theta+2\sin^2\theta)-2\\
&=\{(\sin^4\theta)^3+3(\sin^4\theta)^2\sin^2\theta+3\sin^4\theta(\sin^2\theta)^2+(\sin^2\theta)^3\}+2(\sin^4\theta+\sin^2\theta)-2\\
&=(\sin^4\theta+\sin^2\theta)^3+2(\sin^4\theta+\sin^2\theta)-2\\
&=(\cos^2\theta+\cos\theta)^3+2(\cos^2\theta+\cos\theta)-2\\
&=(1)^3+2(1)-2\\
&=1
\end{aligned}$$

Hence proved.

## Trigonometric Identities Ex 6.1 Q85

**Answer :**

Given: $(1+\cos\alpha)(1+\cos\beta)(1+\cos\gamma)=(1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma)$

Let us assume that

$$(1+\cos\alpha)(1+\cos\beta)(1+\cos\gamma)=(1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma)=L$$

We know that, $\sin^2\theta+\cos^2\theta=1$

Then, we have

$$L\times L=(1+\cos\alpha)(1+\cos\beta)(1+\cos\gamma)\times(1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma)$$
$$\Rightarrow L^2=\{(1+\cos\alpha)(1-\cos\alpha)\}\{(1+\cos\beta)(1-\cos\beta)\}\{(1+\cos\gamma)(1-\cos\gamma)\}$$
$$\Rightarrow L^2=(1-\cos^2\alpha)(1-\cos^2\beta)(1-\cos^2\gamma)$$
$$\Rightarrow L^2=\sin^2\alpha\sin^2\beta\sin^2\gamma$$
$$\Rightarrow L=\pm\sin\alpha\sin\beta\sin\gamma$$

Therefore, we have

$$(1+\cos\alpha)(1+\cos\beta)(1+\cos\gamma)=(1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma)=\pm\sin\alpha\sin\beta\sin\gamma$$

Taking the expression with the positive sign, we have

$$(1+\cos\alpha)(1+\cos\beta)(1+\cos\gamma)=(1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma)=\sin\alpha\sin\beta\sin\gamma$$

*********** END **********