Arithematic Progressions Ex 19.5 Q5

(i) If $(a-c)^2 = 4(a-b)(b-c)$

Then,

$$a^2 + c^2 - 2ac = 4\{ab - b^2 - ac + bc\}$$

$\Rightarrow \quad a^2 + c^2 4b^2 + 2ac - 4ab - 4bc = 0$

$\Rightarrow \quad (a + c - 2b)^2 = 0 \qquad$ [Using $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$]

$\therefore \quad a + c - 2b = 0$

or $\quad a + c = 2b$

and since,

a, b, c are in A.P $\qquad$ [Given]

$a + c = 2b$

Hence proved.

$$(a-c)^2 = 4(a-b)(b-c)$$

(ii) If $a^2 + c^2 + 4ac = 2(ab + bc + ca)$

Then,

$$a^2 + c^2 + 2ac - 2ab - 2bc = 0$$

or $\quad (a + c - b)^2 - b^2 = 0 \qquad$ [$\because (a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$]

or $\quad b = a + c - b$

or $\quad 2b = a + c$

$$b = \frac{a+c}{2}$$

and since,

a, b, c are in A.P

$$b = \frac{a+c}{2}$$

Thus, $a^2 + c^2 + 4ac = 2(ab + bc + ca)$Hence proved.

(iii) If $a^3 + c^3 + 6abc = 8b^3$

or $\quad a^3 + c^3 - (2b)^3 + 6abc = 0$

or $\quad a^3 + (-2b)^3 + c^3 + 3 \times a \times (-2b) \times c = 0$

$\therefore \quad (a - 2b + c) = 0 \qquad$ [$\because x^3 + y^3 + z^3 + 3xyz = 0$ or if $x + y + z = 0$]

or $\quad a + c = 2b$

$a - b = c - b$

and since, a, b, c are in A.P

Thus, $a - b = c - b$

Hence proved. $a^3 + c^3 + 6abc = 8b^3$

Arithematic Progressions Ex 19.5 Q6

Here,

$$a\left(\frac{1}{b} + \frac{1}{c}\right),\ b\left(\frac{1}{c} + \frac{1}{a}\right),\ c\left(\frac{1}{a} + \frac{1}{b}\right) \text{ are in A.P.}$$

$$\Rightarrow \quad a\left(\frac{1}{b} + \frac{1}{c}\right) + 1,\ b\left(\frac{1}{c} + \frac{1}{a}\right) + 1,\ c\left(\frac{1}{a} + \frac{1}{b}\right) + 1 \text{ are in A.P.}$$

$$\Rightarrow \quad \left(\frac{ac + ab + bc}{bc}\right),\ \left(\frac{ab + bc + ac}{ac}\right),\ \left(\frac{cb + ac + ab}{ab}\right) \text{ are in A.P.}$$

$$\Rightarrow \quad \frac{1}{bc},\ \frac{1}{ac},\ \frac{1}{ab} \text{ are in A.P.}$$

$$\Rightarrow \quad \frac{abc}{bc},\ \frac{abc}{ac},\ \frac{abc}{ab} \text{ are in A.P.}$$

$$\Rightarrow \quad a,\ b,\ c \text{ are in A.P.}$$

Arithematic Progressions Ex 19.5 Q7

x, y and z are in AP.
Let d be the common difference then,
$y = x+d$ and $z = x+2d$

To show $x^2 + xy + y^2$, $z^2 + zx + x^2$ and $y^2 + yz + z^2$ are consecutive terms of an A.P., it is enough to show that,
$$\left(z^2 + zx + x^2\right) - \left(x^2 + xy + y^2\right) = \left(y^2 + yz + z^2\right) - \left(z^2 + zx + x^2\right)$$

$$\begin{aligned}
\text{LHS} &= \left(z^2 + zx + x^2\right) - \left(x^2 + xy + y^2\right) \\
&= z^2 + zx - xy - y^2 \\
&= (x+2d)^2 + (x+2d)x - x(x+d) - (x+d)^2 \\
&= x^2 + 4xd + 4d^2 + x^2 + 2xd - x^2 - xd - x^2 - 2xd - d^2 \\
&= 3xd + 3d^2
\end{aligned}$$

$$\begin{aligned}
\text{RHS} &= \left(y^2 + yz + z^2\right) - \left(z^2 + zx + x^2\right) \\
&= y^2 + yz - zx - x^2 \\
&= (x+d)^2 + (x+d)(x+2d) - (x+2d)x - x^2 \\
&= x^2 + 2dx + d^2 + x^2 + 2dx + xd + 2d^2 - x^2 - 2dx - x^2 \\
&= 3xd + 3d^2
\end{aligned}$$

$\therefore$ LHS = RHS
$\therefore$ $x^2 + xy + y^2$, $z^2 + zx + x^2$ and $y^2 + yz + z^2$ are consecutive terms of an A.P.

*********** END **********