CHAPTER 47

THE SPECIAL THEORY OF RELATIVITY

Special theory of relativity stems from two bold postulates put forth by Albert Einstein—regarded by many as the greatest scientific mind of all times. The ideas put forward by special theory of relativity are fascinating and to the first-time learners they seem to go against their everyday experiences—what they think is 'common sense'. However, in our brief discussion here, we shall see that the whole of special theory of relativity is *logically deduced* from Einstein's postulates. In this chapter, we shall use only the inertial frames of reference and we shall frequently use the word *frame* for *inertial frame.*

47.1 THE PRINCIPLE OF RELATIVITY

The principle of relativity is not a new concept for us. We have seen that all frames of reference that move with uniform velocities with respect to an inertial frame are themselves inertial. Newton's laws of motion are valid in the same form in all such frames. Standing on a railway platform, you can drop a stone in such a way that it hits your left foot. If you repeat the same experiment in a train moving smoothly with a uniform velocity, the stone will again strike your left foot. One cannot distinguish between two inertial frames by repeating the same experiment in the two frames. No experiment done inside the train can tell whether the train is at rest at a platform or is moving at 120 km h^{-1} with respect to the platform provided there are no jerks, the train does not speed up or speed down and it does not bend.

This is the principle of relativity. There is no preferred inertial frame. All frames are equivalent. The motion between two frames is relative—you can choose any of the frames and call it at rest and the other in motion.

We can understand this on the basis of Newton's laws of motion. These laws have the same form in all inertial frames. Whether you measure acceleration, force and mass on the platform or on the train, force always equals mass times acceleration. As the results of experiments are governed by Newton's law, identical experiments will give identical results irrespective of the frame involved. But Newton's laws govern only the experiments of mechanics ! Can we do an experiment related to electricity inside a train and tell if the train is moving or is at rest ?

47.2 ARE MAXWELL'S LAWS INDEPENDENT OF FRAME ?

Maxwell's laws tell us that electromagnetic waves propagate in vacuum with a speed $c = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} \approx 3 \times 10^{8}\ \mathrm{m\ s^{-1}}$. Light is an electromagnetic wave. If Maxwell's laws are valid in the same form in all inertial frames, light must travel with the same speed $c = 3 \times 10^{8}\ \mathrm{m\ s^{-1}}$ in all such frames. Experiments show that this is true. Figure (47.1) shows a representative situation when two observers A and B look at a light pulse W.

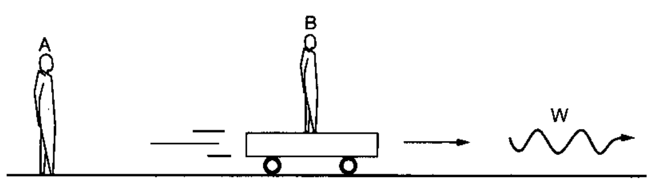

Figure 47.1

Suppose B moves away from A at a speed $u = c/2$ towards the right. The light pulse W moves away from A at a speed c towards the right, and W also moves away from B at the same speed c towards the right. Suppose at $t = 0$; A, B and W were at the same place. A notes down the distances of B and W from himself as a function of time and B notes down the distances of A and W from himself as a function of time. If you collect their diaries the next day, you will find something as given below.

Diary of A:

t	AB	AW
0	0	0
1 s	$1{\cdot}5 \times 10^{8}$ m	3×10^{8} m
2 s	3×10^{8} m	6×10^{8} m
3 s	$4{\cdot}5 \times 10^{8}$ m	9×10^{8} m

Diary of B:

t	BA	BW
0	0	0
1 s	$1{\cdot}5\times10^{8}$ m	3×10^{8} m
2 s	3×10^{8} m	6×10^{8} m
3 s	$4{\cdot}5\times10^{8}$ m	9×10^{8} m

It seems quite reasonable that $AB=BA$ at each t. But we also find that $AW=BW$ at each t even though $AB\neq0$. Do we not have $AW=AB+BW$? The two diaries suggest that something is wrong somewhere. May be, $t=1$ s of A is not the same as $t=1$ s of B. Then AW, AB and BW are measured at different instants. Or may be, 1 m of A is not the same as 1 m of B so that AW and BW are measured in different units.

If Maxwell's equations are valid in the same form for both A and B, something is fishy here. Either the clocks of A and B do not run at the same rate or the metre sticks used by A and B are not of equal length, or both. There are two possibilities.

1. Maxwell's equations have the same form in all inertial frames. Our understanding of clocks and metre sticks, i.e., of time and length has to be revised.

2. Maxwell's equations have different forms in different inertial frames.

If the second option is correct, the experiments of electricity, magnetism and optics should behave differently in different inertial frames. The experiments of mechanics could not distinguish between a train resting at a platform and a train moving uniformly with respect to the platform. That was because Newton's laws had the same form in the two frames. But now the experiments of electricity, magnetism and optics done inside a train should be able to tell whether the train is at rest or it is moving and if it is moving, with what velocity.

Experiments show that this is not true. Even experiments of electricity, magnetism and optics done inside a train cannot tell whether it is moving or not with respect to a platform. A very sophisticated experiment of this kind was designed by Michelson and Morley which is by any standard one of the greatest experiments in physics till date. The experiment attempted to measure the earth's velocity with respect to the imagined ether frame. However, it failed and created history.

We have to accept the first option as we cannot accept the second. The speed of electromagnetic wave must be the same in all inertial frames. Einstein put these ideas in the form of two postulates known as the *postulates of special relativity*.

Postulate 1: *The laws of nature have identical form in all inertial frames.*

Postulate 2: *The speed of light in vacuum has the same value c in all inertial frames.*

47.3 KINEMATICAL CONSEQUENCES

We have seen that the postulates of special relativity require that we must revise our concepts of length and time. If we construct two identical metre sticks, put one on a railway platform and another on a moving train, they behave differently. Similarly, if we construct two identical clocks, use one in the platform frame and the other in the train frame and measure the time interval between the occurrences of two events, the results may be different. Let us investigate these phenomena in detail.

(A) A Rod Moving Perpendicular to its Length

Let us imagine a hypothetical experiment as follows. Construct two identical rods L_1 and L_2. Place them together and verify that the ends match against each other, i.e., they are of equal length. Put some red paint at the ends of L_1 and blue paint at the ends of L_2. Separate L_1 from L_2 by moving it perpendicular to its length. Now move L_1 towards L_2 with a uniform velocity v and look from the frame of reference of L_2, that is, from the L_2-frame (figure 47.2a).

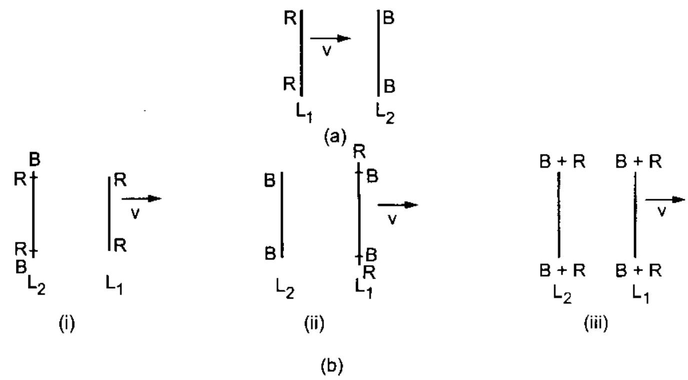

Figure 47.2

L_1 is moving, L_2 is at rest. Are the two rods still equal in length or is the moving rod shorter or is the moving rod longer? This can be checked by our experiment. As the rod L_1 passes over L_2, there are three possibilities (figure 47.2b).

(i) There are red marks on L_2 near the ends. This means that *the moving rod shrinks in its length.*

(ii) There are blue marks on L_1 near the ends. This means that *the moving rod extends in its length.*

(iii) Red and blue intermix at all the four ends. This means that *the moving rod has the same length as the stationary rod.*

Suppose option (i) is correct, i.e., L_1 leaves red marks on L_2 as it passes over L_2. This means that the *moving rod shrinks*. Now the same experiment may be observed by a person fixed with L_1, that is, from the L_1-frame. For this observer, L_1 is at rest, L_2 comes from the right and passes over L_1. Since L_2 gets red paint on it during the crossing, the observer concludes that the *moving rod extended in its length*. But by the principle of relativity you can choose any of the two rods to be at rest and the other moving. Laws of nature should have the same form in the L_1-frame and the L_2-frame. Thus, option (i) is wrong. Similarly, option (ii) is wrong. We conclude that *a rod's length remains unchanged when it is moved perpendicular to its length.*

(B) Moving Clocks (Time Dilation)

Consider another hypothetical experiment. Take a rod of length L as measured by an observer fixed with the rod and suppose that there are two mirrors fixed at the ends. Suppose a light pulse is reflected back and forth by the mirrors. Let us find the time interval between successive reflections from the mirror M_1. Let us call the first reflection 'event E_1' and the next reflection 'event E_2'. (The word 'event' also has a specialised meaning in the mathematical theory of relativity but we are using the literal meaning only.)

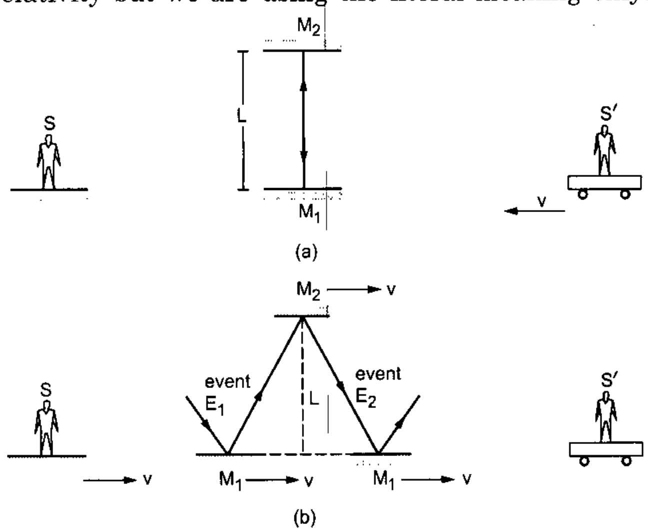

Figure 47.3

Figure (47.3a) shows the situation from a frame S in which the mirrors are at rest. The rod connecting the mirrors is not shown for clarity. The light pulse travels a distance $2L$ between successive reflections from M_1. As the speed of light is c, the time elapsed between these reflections is $\Delta t = \frac{2L}{c}$.

Now consider another frame S' moving with respect to the frame S with a speed v towards the left. From this frame, the rod and the mirrors are moving towards the right with a velocity v. The mirror M_1 at the time of the second reflection is at a place different from where it was at the time of the first reflection (figure 47.3b). If the time interval between these reflections is $\Delta t'$, the simple geometry of figure (47.3b) shows that the light pulse travels a distance

$$2\sqrt{L^2 + \left(\frac{v\Delta t'}{2}\right)^2}$$

between these reflections. Note that the length of the rod is unaltered as it moves in a direction perpendicular to its length. As the speed of light is c, the time interval between the successive reflections from M_1, is

$$\Delta t' = \frac{2}{c}\sqrt{L^2 + \left(\frac{v\Delta t'}{2}\right)^2}$$

or, $$\left(\frac{c\Delta t'}{2}\right)^2 = L^2 + \left(\frac{v\Delta t'}{2}\right)^2$$

or, $$(c^2 - v^2)\left(\frac{\Delta t'}{2}\right)^2 = L^2$$

or, $$\Delta t' = \frac{2L/c}{\sqrt{1 - v^2/c^2}}$$

or, $$\Delta t' = \frac{\Delta t}{\sqrt{1 - v^2/c^2}} = \gamma\,\Delta t \qquad \dots (47.1)$$

where $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$.

We shall use this factor again and again and hence a symbol γ is assigned to it. Note that γ is greater than 1. The time interval between the occurrences of the same two events is different as measured from different frames. In frame S, both E_1 and E_2 occur at the same place. The time interval measured from such a frame where the two events occur at the same place is called *proper time interval*. The time interval measured by a frame where the events occur at different places is called *improper time interval*. Here Δt is proper and $\Delta t'$ is improper time interval. According to equation (47.1),

The proper time interval between the occurrences of two events is smaller than the improper time interval by the factor γ.

This phenomenon is called *time dilation*.

The apparatus described above may be treated as a clock. Each reflection from the mirror M_1 can be thought of as a *tick* of the clock. We shall call it a *light-beam clock*. We see that when the clock is stationary with respect to the observer, it ticks at an interval $2L/c$ and when it moves with respect to the observer, it ticks at an interval $\gamma\,(2L/c)$. Thus,

A moving clock runs slower than a stationary clock by a factor of γ.

Remember, any of the two clocks may be taken to be stationary. Suppose A and B are two clocks moving with respect to each other. As seen from the frame of A, B runs slower and as seen from the frame of B, A runs slower.

Proper time interval

The concept of proper and improper time interval is valid for any two events and not only for two ticks of a clock.

Consider two events E_1 and E_2. Suppose, E_1 occurs at $x = 0$ at a time $t = 0$ and E_2 occurs at $x = L$ at a time t as seen from a frame S (figure 47.4).

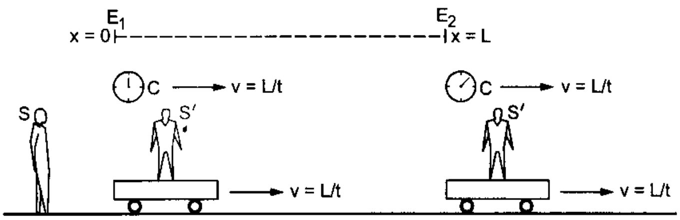

Figure 47.4

Suppose a clock C is at $x = 0$ at $t = 0$ and moves along the x-axis with a speed L/t. At time t, this clock will be at $x = L$. So this same clock is present at both the events E_1 and E_2.

Consider a frame S' moving along the x-axis at a speed L/t with respect to S. In this frame, the clock is at rest and both the events are measured on the same clock. In other words, both the events take place at the *same place in S'*.

As seen from S, the time interval between the events is t. The clock C is moving with respect to S and hence runs slower by a factor γ. The time interval between the events as measured by this clock is $t' = t/\gamma$. This is also the time interval between E_1 and E_2 in frame S' and hence is the proper time interval.

In the frame S', the events can be recorded by a single clock. All other frames where two clocks are needed to record the events, give improper time intervals. The proper time interval is smaller than an improper time interval by a factor of γ.

Example 47.1

A person in a train moving at a speed $3 \times 10^7 \text{ m s}^{-1}$ *sleeps at* 10·00 p.m. *by his watch and gets up at* 4·00 a.m. *How long did he sleep according to the clocks at the stations ?*

Solution : The time interval measured by the watch is the proper time interval because the events, 'sleeping' and 'getting up', are recorded by the single clock (the watch). The clocks at the stations represent the ground frame and in this frame he sleeps at one place and gets up at another place. Thus, the time interval measured by the station clocks is improper time interval and is more than the proper time interval.

The duration of his sleep in the ground frame is

$$\Delta t' = \gamma \Delta t = \frac{\Delta t}{\sqrt{1 - v^2/c^2}} = \frac{6 \text{ h}}{\sqrt{1 - \left(\frac{3 \times 10^7 \text{ m s}^{-1}}{3 \times 10^8 \text{ m s}^{-1}}\right)^2}}$$

$$= 6 \text{ h} \sqrt{\frac{100}{99}} = 6 \text{ hours } 1{\cdot}8 \text{ minutes}.$$

The speed of the train in this example is hypothetical. A typical fast train today runs at about 300 km h^{-1}. Repeat the exercise with such a train.

(C) A Rod Moving Parallel to its Length (Length Contraction)

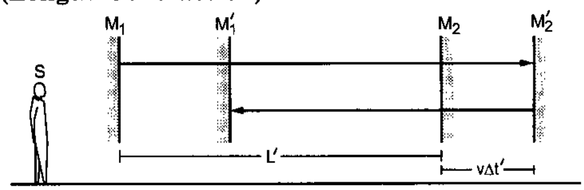

Figure 47.5

Consider the light-beam clock that we discussed. Suppose it is moved at a velocity v along its length (figure 47.5) with respect to an observer. As the rod in the light-beam clock is now moving parallel to its length, we do not know whether the rod retains its length or not. Suppose the length of the rod is L' in the frame S. Consider a light pulse reflected from M_1 and moving towards M_2. Now, M_2 is itself moving with velocity v in the same direction. Suppose that the pulse strikes M_2 at the position M_2' and that it has taken a time $\Delta t'$ to go from the position M_1 to M_2'. The mirror M_2 has moved ahead a distance $v\Delta t_1'$ so that the pulse has moved a distance $L' + v\Delta t_1'$ before striking M_2. But the speed of light pulse is c so that it must travel a distance $c\Delta t_1'$ in time $\Delta t_1'$. Thus,

$$c\Delta t_1' = L' + v\Delta t_1'$$

or, $$\Delta t_1' = \frac{L'}{c - v}.$$

Similarly, the time taken by the pulse in its return journey from M_2 to M_1 (it strikes M_1 at the position M_1') is

$$\Delta t_2' = \frac{L'}{c + v}.$$

The total time elapsed between successive reflections from M_1 is, therefore,

$$\Delta t' = \Delta t_1' + \Delta t_2' = \frac{L'}{c - v} + \frac{L'}{c + v}$$

$$= \frac{2L'c}{c^2 - v^2}. \qquad \ldots \text{ (i)}$$

But $\Delta t'$ is the improper time interval between the two reflections as they occur at different places. An observer, stationary with respect to the rod of the light-beam block, measures this interval to be $\Delta t = 2L/c$ which is the proper time interval between the same two events. Thus, from the equation (47.1),

$$\Delta t' = (\Delta t)\gamma = \left(\frac{2L}{c}\right)\gamma. \quad \ldots \text{(ii)}$$

Using (i) and (ii),

$$\frac{2L'c}{c^2 - v^2} = \frac{2L}{c\sqrt{1 - v^2/c^2}}$$

or, $$\frac{L'}{(1 - v^2/c^2)} = \frac{L}{\sqrt{1 - v^2/c^2}}$$

or, $$L' = L\sqrt{1 - v^2/c^2} = L/\gamma. \quad \ldots (47.2)$$

The length of a rod is contracted by a factor of γ if it moves parallel to its length. The length measured by an observer at rest with respect to the rod is called its *rest length* or *proper length*. Thus,

The length of a rod moving parallel to itself is shorter than its rest length by the factor γ. This phenomenon is called *length contraction*.

Example 47.2

The passenger of example 47.1 slept with his head towards the engine and feet towards the guard's coach. If he measured 6 ft *in the train frame, how tall is he in the ground frame ?*

Solution : In the ground frame, the passenger is moving with a velocity $c/10$. His length is thus contracted. The length measured in the train frame is the rest length of the passenger as the passenger is at rest in the train. Thus, his length in the ground frame is

$$L' = 6 \text{ ft} \sqrt{1 - \left(\frac{1}{10}\right)^2} = 6 \text{ ft}\sqrt{\frac{99}{100}} = 5 \text{ feet } 11{\cdot}6 \text{ inches.}$$

(D) Which Event Occurred Earlier ?

A very important result of special relativity that often surprises beginners is that the concept of simultaneity and ordering of events depends on frame. It is possible that an event E_1 occurs before another event E_2 in one frame but after E_2 in some other frame.

Suppose, a long box of rest length L, having two doors D_1 and D_2 at the ends, lies on the ground (figure 47.6a). At the middle point of the box, there is a light source which can be switched on or off rapidly. Suppose the mechanism is such that when a light pulse strikes a door, the door opens. There are two trains T_1 and T_2, the first moving towards the left and the other towards the right with respect to the ground, both at speed v. Figure (47.6a) shows the situation from the ground frame.

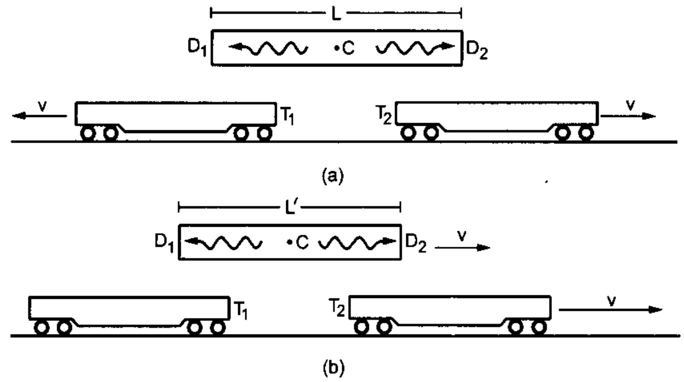

Figure 47.6

Suppose the light source at C is switched on at $t = 0$. Light pulses travel towards D_1 and D_2, finally striking and opening them. Which of the two doors opened first ? In the ground frame, D_1 and D_2 do not move and C is at the middle point. Both the light pulses travel with the same speed c and cover equal distances $L/2$ before striking the doors. Both the doors were opened at $t = \frac{L}{2c}$, i.e., the two events are simultaneous in the ground frame.

Now analyse the situation from the train T_1. The scene from T_1 is shown in figure (47.6b). The box is moving towards the right at a speed v. The length of the box is $L' = L\sqrt{1 - v^2/c^2}$. Consider the light pulse moving towards D_1. The door D_1 is coming towards the pulse with a velocity v. If the time taken by the pulse to reach D_1 is $\Delta t_1'$, the door has moved a distance $v\Delta t_1'$ and the pulse had to travel a distance $L'/2 - v\Delta t_1'$. Thus,

$$c\Delta t_1' = \frac{L'}{2} - v\Delta t_1'$$

or $$\Delta t_1' = \frac{L'}{2(c + v)}. \quad \ldots (47.3)$$

Similarly, the time taken by the pulse to reach D_2 is

$$\Delta t_2' = \frac{L'}{2(c - v)}$$

or, $$\Delta t_2' - \Delta t_1' = \frac{L'}{2}\left(\frac{1}{c - v} - \frac{1}{c + v}\right) = \frac{L'v}{c^2(1 - v^2/c^2)} = \frac{Lv}{c^2\sqrt{(1 - v^2/c^2)}}$$

or, $$\Delta t_2' - \Delta t_1' = \frac{Lv}{c^2}\gamma. \quad \ldots (47.4)$$

As $\Delta t_2' > \Delta t_1'$, the door D_2 opens after D_1 in the frame of T_1. Similar analysis from T_2 shows that D_2 opens before D_1 in the frame of T_2.

Which of the door opened first? The answer depends on the frame. In the ground frame both opened together, in the frame of the train T_1 the door D_1 opened before D_2 and in the frame of the train T_2 the door D_2 opened before D_1.

Example 47.3

Suppose the rest length of the box in figure (47.6) is 30 light seconds. The train T_1 travels at a speed of 0·8c. Find the time elapsed between opening of D_1 and D_2 in the frame of T_1.

Solution : The box moves in the frame of T_1 with a speed of $0{\cdot}8c$ so that

$$\gamma = \frac{1}{\sqrt{1-(0{\cdot}8)^2}} = \frac{1}{0{\cdot}6}.$$

In this frame, D_2 opens after D_1. The time elapsed between the openings of the doors is

$$\frac{Lv}{c^2}\gamma = \frac{(30 \text{ light seconds}) \times (0{\cdot}8c)}{c^2 \times 0{\cdot}6}$$

$$= \frac{(30 \text{ s})c \times (0{\cdot}8c)}{c^2 \times 0{\cdot}6} = 40 \text{ s}.$$

(E) Are the Clocks Synchronized ?

How do we synchronize two clocks separated from each other? The readings of the two clocks must be the same at the same instant. One way of doing this is to place a light source at the middle point between the clocks and send light pulses simultaneously towards the clocks. The time $t = 0$ on a clock may be set at the arrival of the pulse at the clock. As the light pulses travel at identical speeds and the source is placed exactly midway between the clocks, this process ensures that the clocks simultaneously read $t = 0$.

Once again consider the situation of figure (47.6a) redrawn in figure (47.7). Suppose the train T_1 is very long and a series of clocks are kept on it along its length. Suppose there are clocks fixed in the box at the doors D_1 and D_2 and the hands are set at zero as the light pulses reach the doors. These clocks are synchronized in the ground frame. The doors open simultaneously in the ground frame and hence the clocks simultaneously read $t = 0$.

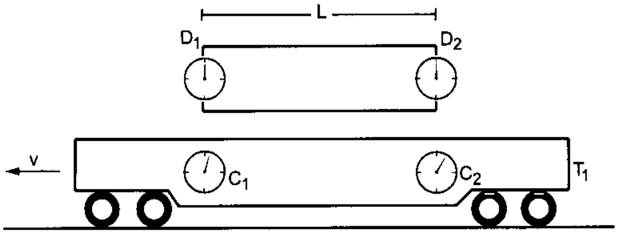

Figure 47.7

The clocks in T_1 are synchronized by the same procedure performed in the train. As the door D_1 opens, some clock C_1 on the train will be opposite to D_1 and it will have some reading, say t'_1. Similarly, as the door D_2 opens, there will be some clock C_2 on the train opposite to D_2 and it will have a reading t'_2.

We have seen that (equation 47.4) according to these clocks on T_1, door D_2 opens after D_1 and the time lag is

$$t'_2 - t'_1 = \frac{Lv}{c^2}\gamma. \qquad \text{... (i)}$$

In ground frame, the doors open at the same instant $t = 0$. So the clock C_1 reads t'_1 at the same instant when C_2 reads t'_2. So in the ground frame, the train clocks are out of synchronization. The clock C_2, that is at the rear, is ahead of the clock C_1, that is at the front, by an amount

$$\Delta t = \frac{Lv}{c^2}\gamma. \qquad \text{... (47.5)}$$

Here L is the separation between the doors D_1 and D_2 as measured from the ground frame. It is also the separation between C_1 and C_2 as measured from the ground frame. But the length C_1C_2 measured from the ground frame is the moving length as C_1 and C_2 are moving with respect to the ground. The distance between C_1 and C_2 in the train frame, i.e., the rest length C_1C_2, will be larger than the moving length. Thus, the rest separation of the clock C_1C_2 is $L_0 = L\gamma$ and equation (47.5) can be reframed as

$$\Delta t = \frac{L_0 v}{c^2}. \qquad \text{... (47.6)}$$

The clocks of a moving frame are out of synchronization. The clock at the rear leads the one at the front by $L_0 v/c^2$, where L_0 is the rest separation between the clocks, and v is the speed of the moving frame.

Remember that the first postulate asserts that you can call any inertial frame at rest and the conclusions above are valid for all frames. In the previous example, we can very well take the train T_1 as the rest frame and then the clocks of the ground frame will be out of synchronization.

47.4 DYNAMICS AT LARGE VELOCITY

When velocities comparable to the velocity of light are involved, Newton's second law of motion, $\vec{F} = m\vec{a}$, does not adequately govern the dynamics. We shall not deduce the correct laws but state them.

The linear momentum $\vec{p}$ of a particle is defined as

$$\vec{p} = m_0 \gamma \vec{v} \qquad \text{... (47.7)}$$

where m_0 is the mass of the particle as we know it in Newtonian mechanics. The quantity

$$m = m_0 \gamma = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

is called the *moving mass* of the particle when it moves at a speed v with respect to the observer. Thus, the mass of the particle is different for different observers. The mass m_0, measured by an observer at rest with respect to the particle, is called its *rest mass.*

With equation (47.7) as the definition of momentum, the law of dynamics is

$$\frac{d\vec{p}}{dt} = \vec{F}. \qquad \text{... (47.8)}$$

Equation (47.8) leads to the law of conservation of relativistic momentum. If no force acts on a particle, its momentum remains constant.

Example 47.4

A particle is kept at rest at the origin. A constant force $\vec{F}$ starts acting on it at $t = 0$. Find the speed of the particle at time t.

Solution :

The equation of motion is,

$$\frac{d\vec{p}}{dt} = \vec{F}.$$

As the particle starts from rest and the force is always in the same direction, the motion will be along this direction only. Thus, we can write

$$\frac{dp}{dt} = F$$

or, $$\int_0^p dp = \int_0^t F\,dt$$

or, $$p = Ft$$

or, $$\frac{m_0 v}{\sqrt{1 - v^2/c^2}} = Ft$$

or, $$m_0^2 v^2 = F^2 t^2 - \frac{F^2 t^2}{c^2} v^2$$

or, $$v^2 \left(m_0^2 + \frac{F^2 t^2}{c^2} \right) = F^2 t^2$$

or, $$v = \frac{Ftc}{\sqrt{m_0^2 c^2 + F^2 t^2}}.$$

Note from example (47.4) that however large t may be, v can never exceed c. No matter how long you apply a force, the speed of a particle will be less than the speed c.

47.5 ENERGY AND MOMENTUM

According to relativistic dynamics, matter is a condensed form of energy. The energy E equivalent to a mass m is given by the equation

$$E = mc^2. \qquad \text{... (47.9)}$$

Thus, matter can be converted into energy and energy into matter. If work is done on a particle, energy is supplied to it. Its energy increases and hence the mass increases. Energy and mass are names for one and the same physical quantity in this viewpoint. When a particle is at rest, its mass is m_0 which is called its rest mass. The energy concentrated in it is, therefore, $E_0 = m_0 c^2$. This is called the *rest mass energy* of the particle. If the particle moves at a speed v, its mass changes to

$$m = m_0 \gamma = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

and the total energy in it becomes $E = mc^2$

$$= m_0 c^2 \left(1 - \frac{v^2}{c^2} \right)^{-1/2}$$

$$= m_0 c^2 \left(1 + \frac{v^2}{2c^2} + \frac{1}{2} \cdot \frac{3}{4} \frac{v^4}{c^4} + \cdots \right)$$

$$= m_0 c^2 + \frac{1}{2} m_0 v^2 + \ldots \qquad \text{... (i)}$$

The extra energy $(mc^2 - m_0 c^2)$ is called the *kinetic energy.* If $v << c$, the higher order terms in (i) are negligible and hence the kinetic energy is $K = \frac{1}{2} m_0 v^2$ as usual. Combining the equations

$$p = mv = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$

and $$E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}},$$

one can deduce that

$$E^2 = m_0^2 c^4 + p^2 c^2. \qquad \text{... (47.10)}$$

For particles having zero rest mass like photons, $m_0 = 0$ and hence from equation (47.10),

$$E = pc$$

or, $$p = E/c. \qquad \text{... (47.11)}$$

This result has already been used in previous chapters for photons.

Example 47.5

If a mass of 3·6 g is fully converted into energy, how many kilowatt hour of electrical energy will be obtained ?

Solution :

The energy obtained is

$E = mc^2 = (3{\cdot}6 \times 10^{-3}\ \text{kg})\ (3 \times 10^8\ \text{m s}^{-1})^2 = 32{\cdot}4 \times 10^{13}\ \text{J}.$

Now 1 kilowatt hour $= 10^3\ \text{J s}^{-1} \times 3600\ \text{s} = 3{\cdot}6 \times 10^6\ \text{J}.$

Thus, $E = \dfrac{32{\cdot}4 \times 10^{13}}{3{\cdot}6 \times 10^6}\ \text{kWh} = 9 \times 10^7\ \text{kWh}.$

47.6 THE ULTIMATE SPEED

We saw in example (47.4) that even if we continue to apply a force on a particle for a long time, its speed cannot exceed c. This is a very general result in special relativity. In fact, no information can be sent with a speed greater than c. If we assume that information can be sent with a speed greater than c, it turns out that we shall have frames in which a bullet will hit the bird before it is actually fired, a dog can die before it is born and so on. If the *effect* cannot precede its *cause* in any frame, then c is the ultimate speed for any material particle or information.

Scientists have worked out the mathematics for a world in which all the particles are moving with respect to each other with speeds greater than c. Such a world can exist without violating the postulates of relativity but these particles can never be slowed down to a speed less than c. The particles of this hypothetical world cannot interact with ours and in that world, effect will always precede its cause. Such particles are named *tachyons* and a group of physicists is working to explore the possibility of the actual existence of such particles. These large speeds and the unworldly results remind us of several stories from Indian Scriptures and no wonder the idea of tachyons was mooted by an Indian scientist E.C.G. Sudarshan.

47.7 TWIN PARADOX

As the postulates of special relativity lead to results which contradict 'common sense', a number of interesting paradoxes have been floated. We shall describe one of the most famous paradoxes of relativity—the *twin paradox*. Consider the twins Ram and Balram living happily on the earth. Ram decides to make a trip to a distant planet P, which is at rest with respect to the earth, and come back. He boards a spaceship S_1, going towards the planet with a uniform velocity. When he reaches the planet, he jumps from the spaceship S_1 to another spaceship S_2 which is going towards the earth. When he reaches the earth, he jumps out and meets his brother Balram.

As Ram returns from his trip and stands next to Balram, do they have equal age ? Or is Ram younger than Balram or is he older than Balram ?

To keep the calculations simple, let us assume the following data:

Distance between the earth and the planet = 8 light-years,

speed of S_1 with respect to earth = 0·8c, and

speed of S_2 with respect to earth = 0·8c.

When we said that the distance between the earth and the planet P is 8 light-years, was it clear to you that this length is the length as measured from the earth frame ?

First, let us analyse the events from the point of view of Balram who is on the earth. For him, both the spaceships move at a speed 0·8c. So,

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{0 \cdot 6}.$$

When Ram is on S_1, he is moving and all his clocks run slower because of time dilation. His heartbeat, pulse beat, etc., represent clocks in themselves and they all run slower. Balram calculates that Ram will take 8 light-year/0·8c = 10 years to reach the planet P. But during all these 10 years, time is passing slowly on S_1 and the clocks will read only 10 years × 0·6 = 6 years in this period. The number of breaths taken by Ram corresponds to 6 years only.

Ram jumped into S_2 for the return journey. This spaceship is also moving at 0·8c and for Balram, time passes slowly on S_2 as well. Although 10 years passed on the earth during Ram's return journey, on the spaceship the journey was clocked at 6 years. Thus, Ram has aged only 12 years whereas Balram has aged 20 years during this expedition. Ram has become younger than Balram by 8 years. This difference in aging is real in the sense that Ram shows lesser signs of aging like he has lesser white hairs than his brother.

The observation of Balram is quite consistent with the special theory of relativity. Such experiments are indeed performed in laboratories with radioactive particles. Particles are accelerated to large speeds and are kept at these speeds for quite some time by magnetic fields. These particles with large speeds have longer lives than their counterparts kept at rest in the laboratory.

The paradox arises when we analyse the events from the point of veiw of Ram. When he is in the spaceship S_1, to him the distance between the earth and the planet is not 8 light-years. The earth and the planet P are moving with respect to Ram and hence he is measuring contracted length. The separation is, therefore, 8 light-years × 0·6 = 4·8 light-years. As the planet is approaching Ram at 0·8c, the time taken by the planet to reach Ram is 4·8 light-year/0·8c = 6 years. So according to Ram's clock, he jumped from S_1 to S_2 6 years after getting into S_1. Once he is on S_2, the earth and the planet are again moving with the same speed 0·8c. Again, the earth is 4·8 light-years from the planet and is approaching at 0·8c. It takes 6 years for the earth to reach Ram. Thus, according to Ram's clock, he was out for 12 years from the earth, the same result as Balram had expected.

But how about Ram's calculation of Balram's age ? When Ram is on S_1, the earth is going away from him with a speed $0{\cdot}8c$. Ram will find that the time on the earth is passing slower by a factor of 0·6 so that Balram is aging slower than he is. The same is true when he is on S_2. During this period also, Balram is moving (towards Ram) with a speed $0{\cdot}8c$ and hence time is passing slowly for Balram. As 12 years passes on Ram's clock, he calculates that Balram's clocks have advanced only by 12 years $\times 0{\cdot}6 = 7{\cdot}2$ years in this period. According to this analysis, Ram should find that Balram is $12 - 7{\cdot}2 = 4{\cdot}8$ years younger than him.

This is the paradox. According to Ram, Balram's clocks are running slow and according to Balram, Ram's clocks are running slow. Each thinks the other is younger. Where lies the fallacy ?

The fallacy lies in the fact that Ram has changed frames whereas Balram has stayed in an inertial frame. Thus, the roles of the twins are not symmetrical. The ordering of events are different in different frames and Ram must take that into account when he changes frames. Suppose Ram gets into the spaceship S_1 when his clock reads zero. So does Balram's clock. What is the reading of the planet's clock at this instant ? According to Balram, it is zero because both the earth and the planet are at rest and the clocks are synchronized in his frame. But that is not so in S_1. As Ram gets into S_1, he may have the following conversation with the captain of the ship.

Captain: Welcome aboard S_1. I saw you on the earth, coming towards us. Your jump to board this ship was perfect. Where are you going ?

Ram: Thank you. I am going to the planet P. How far is it from here and how long will it take for the planet to come to us ?

Captain: Planet P is 4·8 light-years from us at the moment. It is coming towards us at a speed of $0{\cdot}8c$ so it will take 4·8 light-years/$0{\cdot}8c$ = 6 years for the planet P to reach us.

Ram: Well, the clocks on the earth and the planet are running a bit slower than ours. I have been taught that moving clocks run slow by a factor of γ. This factor is 1/0·6 for these clocks. So they will advance by 6 years $\times 0{\cdot}6 = 3{\cdot}6$ years by the time the planet reaches us.

Captain: Yes, both the clocks will advance by 3·6 years by the time you jump on the planet P.

Ram: The earth-clock was reading $t = 0$ as we passed the earth. This means when I jump on the planet P the clocks on the earth and the planet will be reading 3·6 years.

Captain: Here you are mistaken. Don't you remember that the planet's clock is not synchronized with the earth's clock ? The planet's clock is at the rear end, and hence is running 6·4 years ahead of the earth's clock. At the instant the earth's clock was reading zero, the planet's clock was reading 6·4 years. As the planet reaches us, both the clocks will advance by 3·6 years. So when you jump out of S_1, the earth's clock will be reading 3·6 years but the planet's clock will be reading 10 years.

Ram understands the logic. In the earth's frame, the two clocks read zero simultaneously. But in S_1-frame, the event "planet's clock reading zero" occurred several years before "earth's clock reading zero". Six years pass in S_1 and Ram finds that the planet P has reached him. He finds another spaceship S_2 which is heading towards the earth. Ram jumps onto S_2. In the process he looks at the planet's clock and finds that it is reading 10 years as calculated by him on S_1. On S_2, he starts talking to the commander of the ship.

Commander: Welcome to S_2. How long will you be with us ?

Ram: Thank you. I am going to Earth. Earth is at present 4·8 light-years from here and is coming towards us with a speed of $0{\cdot}8c$. So I will be with you for 6 years. The captain of S_1 told me that the earth's clock is reading 3·6 years at this moment whereas the planet's clock reads 10 years. There is a difference of 6·4 years in the reading because the two clocks are not synchronized. Also ....

Commander: Sorry for interrupting you, but you are mistaken. It is true that the earth's clock and the planet's clock are not synchronized as they are moving past us. Also the difference in the readings of the two clocks is 6·4 years. But the planet's clock is at the front and the earth's clock is at the rear. It is the earth's clock that is leading by 6·4 years. At the moment the planet's clock reads 10 years and hence the earth's clock must be reading 16·4 years.

Ram: Hmm... you are right. In S_1, the earth was at the front and its clock lagged behind the planet's clock. But in S_2 it is the other way round. Indeed the earth's clock reads 16·4 years whereas the planet's clock reads 10 years.

Commander: That's right. The earth's clock is reading 16·4 years at present. It will advance by another 3·6 years during the 6 years you will be with us. So it will be reading 20 years when the earth reaches you.

We see that the paradox is resolved.

Worked Out Examples

1. *A hypothetical train moving with a speed of 0·6c passes by the platform of a small station without being slowed down. The observers on the platform note that the length of the train is just equal to the length of the platform which is 200 m. (a) Find the rest length of the train. (b) Find the length of the platform as measured by the observers in the train.*

Solution : (a) The length L' of the train at a speed $0{\cdot}6c$ is 200 m. If the rest length is L,

$$L' = L\sqrt{1 - v^2/c^2}$$

or,
$$L = \frac{L'}{\sqrt{1 - v^2/c^2}} = \frac{200\ \text{m}}{\sqrt{1-(0{\cdot}6)^2}}$$
$$= 250\ \text{m}.$$

(b) The rest length of the platform is 200 m. For the observers in the train, the platform is moving at a speed of $0{\cdot}6\,c$. The length as measured by the observers in the train is, therefore,

$$L' = 200\ \text{m}\sqrt{1-(0{\cdot}6)^2} = 160\ \text{m}.$$

2. *Unstable pions are produced as a beam in a nuclear reaction experiment. The pions leave the target at a speed of 0·995c. The intensity of the beam reduces to half its original value as the beam travels a distance of 39 m. Find the half-life of pions (a) in the laboratory frame, (b) in their rest frame.*

Solution : (a) The intensity of the pion beam reduces to half its original value in one half-life. The half-life of the pions as measured in the laboratory is

$$t_{1/2} = \frac{39\ \text{m}}{0{\cdot}995\,c} = \frac{39\ \text{m}}{0{\cdot}995\times 3\times 10^8\ \text{m s}^{-1}}$$
$$= 1{\cdot}3\times 10^{-7}\ \text{s}.$$

(b) The events—a pion leaving the target and its decaying—occur at the same place in the pion-frame. Thus, the time measured in the pion-frame is the proper time and is the smallest. It is equal to

$$t'_{1/2} = t_{1/2}\sqrt{1-v^2/c^2} = (1{\cdot}3\times 10^{-7}\ \text{s})\sqrt{1-(0{\cdot}995)^2}$$
$$= 1{\cdot}3\times 10^{-8}\ \text{s}.$$

3. *Two events A and B occur at places separated by 10^6 km, B occurring 5 s after A. (a) Find the velocity of a frame in which these events occur at the same place. (b) What is the time interval between the events in this frame?*

Solution :

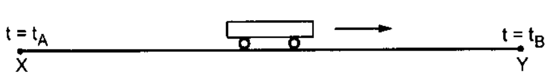

Figure 47-W1

(a) Suppose the events A and B occur at points X and Y at times t_A and t_B where $t_B = t_A + 5$ s. Consider a small train which is at the point X when the event A occurs. Suppose, this same train moves towards Y and reaches the point Y when the event B occurs. Thus, the events A and B occur at the same place in the train frame. This frame moves 10^6 km in 5 s as seen from the original frame. Thus, the velocity of the train frame is

$$v = \frac{10^6\ \text{km}}{5\ \text{s}} = 2\times 10^8\ \text{m s}^{-1}.$$

(b) As the events A and B occur at the same place in the train frame, the time interval between the events measured in this frame is the proper interval. Thus, this time interval is

$$= (5\ \text{s})\sqrt{1-v^2/c^2} = (5\ \text{s})\sqrt{1-\left(\frac{2}{3}\right)^2}$$
$$= 3{\cdot}7\ \text{s}.$$

4. *A satellite orbits the earth near its surface. By what amount does the satellite's clock fall behind the earth's clock in one revolution? Assume that nonrelativistic analysis can be made to compute the speed of the satellite and only the time dilation is to be taken into account for calculation of clock speeds.*

Solution : The speed of the satellite may be obtained from the equation,

$$\frac{GMm}{R^2} = \frac{mv^2}{R}$$

or,
$$v = \sqrt{\frac{GM}{R}}$$
$$= \left[\frac{(6{\cdot}67\times 10^{-11}\ \text{N m}^2\text{kg}^{-2})\,(6\times 10^{24}\ \text{kg})}{6400\times 10^3\ \text{m}}\right]^{1/2}$$
$$= 7910\ \text{m s}^{-1}. \qquad \dots \text{(i)}$$

Thus,
$$v/c = \frac{7910}{3\times 10^8} = 2{\cdot}637\times 10^{-5}$$

or,
$$\sqrt{1-\left(\frac{v}{c}\right)^2} = [1 - 6{\cdot}95\times 10^{-10}]^{1/2}$$
$$\approx 1 - 3{\cdot}48\times 10^{-10}.$$

The time taken by the satellite to complete one revolution is

$$T = \frac{2\pi R}{v} = \frac{6{\cdot}28\times 6400\times 10^3\ \text{m}}{7910\ \text{m s}^{-1}} = 5080\ \text{s}.$$

The clock on the satellite will slow down as observed from the earth. If the time elapsed on the satellite's clock is t as the satellite completes one revolution (this is proper time and 5080 s is improper time),

$$t = (1 - 3{\cdot}48\times 10^{-10})\times (5080\ \text{s})$$

or,
$$\frac{t}{5080\ \text{s}} = 1 - 3{\cdot}48\times 10^{-10}$$

or, $$\frac{(t - 5080 \text{ s})}{5080 \text{ s}} = -3.48 \times 10^{-10}$$

or, $$(t - 5080 \text{ s}) = -1.77 \times 10^{-6} \text{ s}.$$

The satellite's clock falls behind by 1.77×10^{-6} s in one revolution.

5. *The radius of our galaxy is about* 3×10^{20} m. *With what speed should a person travel so that he can reach from the centre of the galaxy to its edge in* 20 *years of his lifetime ?*

Solution : Let the speed of the person be v. As seen by the person, the edge of the galaxy is coming towards him at a speed v. In 20 years (as measured by the person), the edge moves (20 y) v and reaches the person. The radius of the galaxy as measured by the person is, therefore, (20 y)v. The rest length of the radius of the galaxy is 3×10^{20} m. Thus,

$$(20 \text{ y})v = (3 \times 10^{20} \text{ m}) \sqrt{1 - v^2/c^2}$$

or, $$(6.312 \times 10^{8} \text{ s})^2 v^2 = (9 \times 10^{40} \text{ m}^2)(1 - v^2/c^2).$$

Solving this,

$$v = 0.9999996\, c.$$

6. *Find the speed at which the mass of an electron is double of its rest mass.*

Solution : The mass of an electron at speed v is

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

where m_0 is its rest mass. If $m = 2\, m_0$,

$$2 = \frac{1}{\sqrt{1 - v^2/c^2}}$$

or, $$1 - \frac{v^2}{c^2} = \frac{1}{4}$$

or, $$v = \frac{\sqrt{3}}{2} c = 2.598 \times 10^{8} \text{ m s}^{-1}.$$

7. *Calculate the increase in mass when a body of rest mass* 1 kg *is lifted up through* 1 m *near the earth's surface.*

Solution : The increase in energy $= mgh$

$$= (1 \text{ kg})(9.8 \text{ m s}^{-2})(1 \text{ m}) = 9.8 \text{ J}.$$

The increase in mass $= \frac{9.8 \text{ J}}{c^2}$

$$= 1.11 \times 10^{-16} \text{ kg}.$$

8. *A body of rest mass* m_0 *collides perfectly inelastically at a speed of* 0.8c *with another body of equal rest mass kept at rest. Calculate the common speed of the bodies after the collision and the rest mass of the combined body.*

Solution : The linear momentum of the first body

$$= \frac{m_0 v}{\sqrt{1 - v^2/c^2}} = \frac{m_0 \times 0.8c}{0.6}$$

$$= \frac{4}{3} m_0 c.$$

This should be the total linear momentum after the collision. If the rest mass of the combined body is M_0 and it moves at speed v',

$$\frac{M_0 v'}{\sqrt{1 - v'^2/c^2}} = \frac{4}{3} m_0 c. \qquad \text{... (i)}$$

The energy before the collison is

$$\frac{m_0}{\sqrt{1 - v^2/c^2}} c^2 + m_0 c^2 = m_0 c^2 \left(\frac{1}{0.6} + 1\right)$$

$$= \frac{8}{3} m_0 c^2.$$

The energy after the collision is

$$\frac{M_0 c^2}{\sqrt{1 - v'^2/c^2}}.$$

Thus, $$\frac{M_0 c^2}{\sqrt{1 - v'^2/c^2}} = \frac{8}{3} m_0 c^2. \qquad \text{... (ii)}$$

Dividing (i) by (ii),

$$\frac{v'}{c^2} = \frac{1}{2c} \quad \text{or,} \quad v' = \frac{c}{2}.$$

Putting this value of v' in (ii),

$$M_0 = \frac{8}{3} m_0 \sqrt{1 - \frac{1}{4}}$$

or, $$M_0 = 2.309\, m_0.$$

The rest mass of the combined body is greater than the sum of the rest masses of the individual bodies.

□

QUESTIONS FOR SHORT ANSWER

1. The speed of light in glass is 2.0×10^{8} m s^{-1}. Does it violate the second postulate of special relativity ?
2. A uniformly moving train passes by a long platform. Consider the events 'engine crossing the beginning of the platform' and 'engine crossing the end of the platform'. Which frame (train frame or the platform frame) is the proper frame for the pair of events ?

3. An object may be regarded to be at rest or in motion depending on the frame of reference chosen to view the object. Because of length contraction it would mean that the same rod may have two different lengths depending on the state of the observer. Is this true ?

4. Mass of a particle depends on its speed. Does the attraction of the earth on the particle also depend on the particle's speed ?

5. A person travelling in a fast spaceship measures the distance between the earth and the moon. Is it the same, smaller or larger than the value quoted in this book ?

OBJECTIVE I

1. The magnitude of linear momentum of a particle moving at a relativistic speed v is proportional to
   (a) v (b) $1 - v^2/c^2$
   (c) $\sqrt{1 - v^2/c^2}$ (d) none of these.

2. As the speed of a particle increases, its rest mass
   (a) increases (b) decreases
   (c) remains the same (d) changes.

3. An experimenter measures the length of a rod. Initially the experimenter and the rod are at rest with respect to the lab. Consider the following statements.
   (A) If the rod starts moving parallel to its length but the observer stays at rest, the measured length will be reduced.
   (B) If the rod stays at rest but the observer starts moving parallel to the measured length of the rod, the length will be reduced.
   (a) A is true but B is false. (b) B is true but A is false.
   (c) Both A and B are true. (d) Both A and B are false.

4. An experimenter measures the length of a rod. In the cases listed, all motions are with respect to the lab and parallel to the length of the rod. In which of the cases the measured length will be minimum ?
   (a) The rod and the experimenter move with the same speed v in the same direction.
   (b) The rod and the experimenter move with the same speed v in opposite directions.
   (c) The rod moves at speed v but the experimenter stays at rest.
   (d) The rod stays at rest but the experimenter moves with the speed v.

5. If the speed of a particle moving at a relativistic speed is doubled, its linear momentum will
   (a) become double (b) become more than double
   (c) remain equal (d) become less than double.

6. If a constant force acts on a particle, its acceleration will
   (a) remain constant (b) gradually decrease
   (c) gradually increase (d) be undefined.

7. A charged particle is projected at a very high speed perpendicular to a uniform magnetic field. The particle will
   (a) move along a circle
   (b) move along a curve with increasing radius of curvature
   (c) move along a curve with decreasing radius of curvature
   (d) move along a straight line.

OBJECTIVE II

1. Mark the correct statements:
   (a) Equations of special relativity are not applicable for small speeds.
   (b) Equations of special relativity are applicable for all speeds.
   (c) Nonrelativistic equations give exact result for small speeds.
   (d) Nonrelativistic equations never give exact result.

2. If the speed of a rod moving at a relativistic speed parallel to its length is doubled,
   (a) the length will become half of the original value
   (b) the mass will become double of the original value
   (c) the length will decrease
   (d) the mass will increase.

3. Two events take place simultaneously at points A and B as seen in the lab frame. They also occur simultaneously in a frame moving with respect to the lab in a direction
   (a) parallel to AB (b) perpendicular to AB
   (c) making an angle of 45° with AB
   (d) making an angle of 135° with AB.

4. Which of the following quantities related to an electron has a finite upper limit ?
   (a) Mass (b) Momentum (c) Speed (d) Kinetic energy

5. A rod of rest length L moves at a relativistic speed. Let $L' = L/\gamma$. Its length
   (a) must be equal to L' (b) may be equal to L
   (c) may be more than L' but less than L
   (d) may be more than L.

6. When a rod moves at a relativistic speed v, its mass
   (a) must incrase by a factor of γ
   (b) may remain unchanged
   (c) may increase by a factor other than γ
   (d) may decrease.

EXERCISES

1. The *guru* of a *yogi* lives in a Himalyan cave, 1000 km away from the house of the yogi. The yogi claims that whenever he thinks about his guru, the guru immediately knows about it. Calculate the minimum possible time interval between the yogi thinking about the guru and the guru knowing about it.
2. A suitcase kept on a shop's rack is measured 50 cm × 25 cm × 10 cm by the shop's owner. A traveller takes this suitcase in a train moving with velocity $0{\cdot}6c$. If the suitcase is placed with its length along the train's velocity, find the dimensions measured by (a) the traveller and (b) a ground observer.
3. The length of a rod is exactly 1 m when measured at rest. What will be its length when it moves at a speed of (a) $3\times10^{5}\ \mathrm{m\ s^{-1}}$, (b) $3\times10^{6}\ \mathrm{m\ s^{-1}}$ and (c) $3\times10^{7}\ \mathrm{m\ s^{-1}}$?
4. A person standing on a platform finds that a train moving with velocity $0{\cdot}6c$ takes one second to pass by him. Find (a) the length of the train as seen by the person and (b) the rest length of the train.
5. An aeroplane travels over a rectangular field 100 m × 50 m, parallel to its length. What should be the speed of the plane so that the field becomes square in the plane frame?
6. The rest distance between Patna and Delhi is 1000 km. A nonstop train travels at $360\ \mathrm{km\ h^{-1}}$. (a) What is the distance between Patna and Delhi in the train frame? (b) How much time elapses in the train frame between Patna and Delhi?
7. A person travels by a car at a speed of $180\ \mathrm{km\ h^{-1}}$. It takes exactly 10 hours by his wristwatch to go from the station A to the station B. (a) What is the rest distance between the two stations? (b) How much time is taken in the road frame by the car to go from the station A to the station B?
8. A person travels on a spaceship moving at a speed of $5c/13$. (a) Find the time interval calculated by him between the consecutive birthday celebrations of his friend on the earth. (b) Find the time interval calculated by the friend on the earth between the consecutive birthday celebrations of the traveller.
9. According to the station clocks, two babies are born at the same instant, one in Howrah and other in Delhi. (a) Who is elder in the frame of 2301 Up Rajdhani Express going from Howrah to Delhi? (b) Who is elder in the frame of 2302 Dn Rajdhani Express going from Delhi to Howrah.
10. Two babies are born in a moving train, one in the compartment adjacent to the engine and other in the compartment adjacent to the guard. According to the train frame, the babies are born at the same instant of time. Who is elder according to the ground frame?
11. Suppose Swarglok (heaven) is in constant motion at a speed of $0{\cdot}9999c$ with respect to the earth. According to the earth's frame, how much time passes on the earth before one day passes on Swarglok?
12. If a person lives on the average 100 years in his rest frame, how long does he live in the earth frame if he spends all his life on a spaceship going at 60% of the speed of light.
13. An electric bulb, connected to a make and break power supply, switches off and on every second in its rest frame. What is the frequency of its switching off and on as seen from a spaceship travelling at a speed $0{\cdot}8c$?
14. A person travelling by a car moving at $100\ \mathrm{km\ h^{-1}}$ finds that his wristwatch agrees with the clock on a tower A. By what amount will his wristwatch lag or lead the clock on another tower B, 1000 km (in the earth's frame) from the tower A when the car reaches there?
15. At what speed the volume of an object shrinks to half its rest value?
16. A particular particle created in a nuclear reactor leaves a 1 cm track before decaying. Assuming that the particle moved at $0{\cdot}995c$, calculate the life of the particle (a) in the lab frame and (b) in the frame of the particle.
17. By what fraction does the mass of a spring change when it is compressed by 1 cm? The mass of the spring is 200 g at its natural length and the spring constant is $500\ \mathrm{N\ m^{-1}}$.
18. Find the increase in mass when 1 kg of water is heated from 0°C to 100°C. Specific heat capacity of water = $4200\ \mathrm{J\ kg^{-1}K^{-1}}$.
19. Find the loss in the mass of 1 mole of an ideal monatomic gas kept in a rigid container as it cools down by 10°C. The gas constant $R = 8{\cdot}3\ \mathrm{J\ K^{-1}mol^{-1}}$.
20. By what fraction does the mass of a boy increase when he starts running at a speed of $12\ \mathrm{km\ h^{-1}}$?
21. A 100 W bulb together with its power supply is suspended from a sensitive balance. Find the change in the mass recorded after the bulb remains on for 1 year.
22. The energy from the sun reaches just outside the earth's atmosphere at a rate of $1400\ \mathrm{W\ m^{-2}}$. The distance between the sun and the earth is $1{\cdot}5\times10^{11}$ m. (a) Calculate the rate at which the sun is losing its mass. (b) How long will the sun last assuming a constant decay at this rate? The present mass of the sun is 2×10^{30} kg.
23. An electron and a positron moving at small speeds collide and annihilate each other. Find the energy of the resulting gamma photon.
24. Find the mass, the kinetic energy and the momentum of an electron moving at $0{\cdot}8c$.
25. Through what potential difference should an electron be accelerated to give it a speed of (a) $0{\cdot}6c$, (b) $0{\cdot}9c$ and (c) $0{\cdot}99c$?
26. Find the speed of an electron with kinetic energy (a) 1 eV, (b) 10 keV and (c) 10 MeV.
27. What is the kinetic energy of an electron in electronvolts with mass equal to double its rest mass?
28. Find the speed at which the kinetic energy of a particle will differ by 1% from its nonrelativistic value $\frac{1}{2}m_0v^2$.

□

ANSWERS

OBJECTIVE I

1. (d) 2. (c) 3. (c) 4. (b) 5. (b) 6. (b)
7. (b)

OBJECTIVE II

1. (b), (d) 2. (c), (d) 3. (b)
4. (c) 5. (b), (c) 6. (a)

EXERCISES

1. 1/300 s
2. (a) 50 cm × 25 cm × 10 cm (b) 40 cm × 25 cm × 10 cm
3. (a) 0·9999995 m (b) 0·99995 m (c) 0·995 m
4. (a) $1{\cdot}8 \times 10^{8}$ m (b) $2{\cdot}25 \times 10^{8}$ m
5. 0·866c
6. (a) 56 nm less than 1000 km
   (b) 0·56 ns less than $\frac{500}{3}$ min
7. (a) 25 nm more than 1800 km
   (b) 0·5 ns more than 10 hours
8. $\frac{13}{12}$ y in both cases
9. (a) Delhi baby is elder (b) Howrah baby is elder
10. the baby adjacent to the guard is elder
11. 70·7 days
12. 125 y
13. $0{\cdot}6\ s^{-1}$
14. will lag by 0·154 ns
15. $\frac{\sqrt{3}c}{2}$
16. (a) 33·5 ps (b) 3·35 ps
17. $1{\cdot}4 \times 10^{-18}$
18. $4{\cdot}7 \times 10^{-12}$ kg
19. $1{\cdot}38 \times 10^{-15}$ kg
20. $6{\cdot}17 \times 10^{-17}$
21. $3{\cdot}5 \times 10^{-8}$ kg
22. (a) $4{\cdot}4 \times 10^{9}\ kg\ s^{-1}$ (b) $1{\cdot}44 \times 10^{13}$ y
23. 1·02 MeV
24. $15{\cdot}2 \times 10^{-31}$ kg, $5{\cdot}5 \times 10^{-14}$ J, $3{\cdot}65 \times 10^{-22}\ kg\ m\ s^{-1}$
25. (a) 128 kV (b) 661 kV (c) 3·1 MV
26. (a) $5{\cdot}92 \times 10^{5}\ m\ s^{-1}$ (b) $5{\cdot}85 \times 10^{7}\ m\ s^{-1}$
    (c) $2{\cdot}996 \times 10^{8}\ m\ s^{-1}$
27. 511 keV
28. $3{\cdot}46 \times 10^{7}\ m\ s^{-1}$

□