



Trigonometric Ratios Ex 5.1 Q14

Answer :

Given: $\cos \theta = \frac{12}{13}$ (1)

To show that $\sin \theta (1 - \tan \theta) = \frac{35}{156}$

Now, we know that $\cos \theta = \frac{\text{Base side adjacent to } \angle \theta}{\text{Hypotenuse}}$ (2)

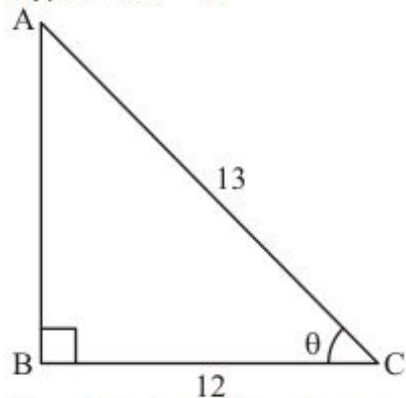
Therefore, by comparing equation (1) and (2)

We get,

Base side adjacent to $\angle \theta = 12$

And

Hypotenuse = 13



Therefore from above figure

Base side $BC = 12$

Hypotenuse $AC = 13$

Side AB is unknown and it can be determined by using Pythagoras theorem

Therefore by applying Pythagoras theorem

We get,

$$AC^2 = AB^2 + BC^2$$

Therefore by substituting the values of known sides

We get,

$$13^2 = AB^2 + 12^2$$

Therefore,

$$AB^2 = 13^2 - 12^2$$

$$AB^2 = 169 - 144$$

$$AB^2 = 25$$

$$AB = \sqrt{25}$$

Therefore,

$$AB = 5 \dots\dots (3)$$

Now, we know that

$$\sin \theta = \frac{\text{Perpendicular side opposite to } \angle \theta}{\text{Hypotenuse}}$$

Now from figure (a)

We get,

$$\sin \theta = \frac{AB}{AC}$$

Therefore,

$$\sin \theta = \frac{5}{13} \dots\dots (4)$$

Now, we know that

$$\tan \theta = \frac{\text{Perpendicular side opposite to } \angle \theta}{\text{Base side adjacent to } \angle \theta}$$

Now from figure (a)

We get,

$$\tan \theta = \frac{AB}{BC}$$

Therefore,

$$\tan \theta = \frac{5}{12} \dots\dots (5)$$

Now L.H.S. of the equation to be proved is as follows

$$L.H.S. = \sin \theta (1 - \tan \theta) \dots\dots (6)$$

Substituting the value of $\sin \theta$ and $\tan \theta$ from equation (4) and (5) respectively

We get,

$$L.H.S. = \frac{5}{13} \left(1 - \frac{5}{12} \right)$$

Taking L.C.M inside the bracket

We get,

$$L.H.S. = \frac{5}{13} \left(\frac{1 \times 12}{1 \times 12} - \frac{5}{12} \right)$$

Therefore,

$$L.H.S. = \frac{5}{13} \left(\frac{12-5}{12} \right)$$

$$L.H.S. = \frac{5}{13} \left(\frac{7}{12} \right)$$

Now, by opening the bracket and simplifying

We get,

$$L.H.S. = \frac{5 \times 7}{13 \times 12}$$

$$L.H.S. = \frac{35}{156} \dots\dots (7)$$

From equation (6) and (7) , it can be shown that

$$\sin \theta (1 - \tan \theta) = \frac{35}{156}$$

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