$$= \frac{-b}{a}$$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$= \frac{c}{a}$$

We have, $\alpha^2\beta + \alpha\beta^2$

By taking common factor $\alpha\beta$ we get, $= 2\beta(\alpha+\beta)$

By substituting $\alpha+\beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$ we get ,

$$= \frac{c}{a}\left(\frac{-b}{a}\right)$$

$$= \frac{-cb}{a^2}$$

Hence the value of $\alpha^2\beta + \alpha\beta^2$ is $\boxed{\frac{-cb}{a^2}}$.

(v) Given α and β are the zeros of the quadratic polynomial $f(x) = ax^2 + bx + c$

$$\alpha + \beta = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$= \frac{-b}{a}$$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$= \frac{c}{a}$$

We have,

$$\alpha^4 + \beta^4 = \left(\alpha^2 + \beta^2\right)^2 - 2\alpha^2\beta^2$$

$$\alpha^4 + \beta^4 = \left[\left(\alpha + \beta\right)^2 - 2\alpha\beta\right]^2 - 2\left(\alpha\beta\right)^2$$

By substituting $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$ we get ,

$$\alpha^4 + \beta^4 = \left[\left(\frac{-b}{a}\right)^2 - 2 \times \frac{c}{a}\right]^2 - 2\left(\frac{c}{a}\right)^2$$

$$\alpha^4 + \beta^4 = \left[\frac{b^2}{a^2} - \frac{2c}{a}\right]^2 - 2\left(\frac{c}{a}\right)^2$$

By taking least common factor we get

$$\alpha^4 + \beta^4 = \left[\left(\frac{-b}{a}\right)^2 - 2 \times \left(\frac{c}{a}\right)\right]^2 - 2 \times \left(\frac{c}{a}\right)^2$$

$$= \left[\frac{b^2}{a^2} - \frac{2c}{a}\right]^2 - 2 \times \left(\frac{c}{a}\right)^2$$

$$= \left[\frac{b^2 - 2ac}{a^2}\right]^2 - 2 \times \frac{c^2}{a^2}$$

$$= \frac{\left(b^2 - 2ac\right)^2}{a^4} - 2 \times \frac{c^2}{a^2}$$

$$= \frac{\left(b^2 - 2ac\right)^2 - 2c^2a^2}{a^4}$$

Hence the value of $\alpha^4 + \beta^4$ is $\frac{\left(b^2-2ac\right)^2-2c^2a^2}{a^4}$.

(*vi*) Since α and β are the zeros of the quadratic polynomial $f(x) = ax^2 + bx + c$

$$\alpha + \beta = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$= \frac{-b}{a}$$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$= \frac{c}{a}$$

We have, $\frac{1}{a\alpha + b} + \frac{1}{a\beta + b}$

$$\frac{1}{a\alpha + b} + \frac{1}{a\beta + b} = \frac{a\beta + b + a\alpha + b}{(a\alpha + b)(a\beta + b)}$$

$$\frac{1}{a\alpha + b} + \frac{1}{a\beta + b} = \frac{a(\alpha + \beta) + 2b}{a^2 \times \alpha\beta + ab\beta + ab\alpha + b^2}$$

$$\frac{1}{a\alpha + b} + \frac{1}{a\beta + b} = \frac{a(\alpha + \beta) + 2b}{a^2 \times \alpha\beta + ab(\alpha + \beta) + b^2}$$

By substituting $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$ we get ,

$$\frac{1}{a\alpha+b}+\frac{1}{a\beta+b}=\frac{a\times\frac{-b}{a}+2b}{a^2\times\frac{c}{a}+ab\times\frac{-b}{a}+b^2}$$

$$\frac{1}{a\alpha+b}+\frac{1}{a\beta+b}=\frac{\cancel{a}\times\frac{-b}{\cancel{a}}+2b}{a\cancel{2}^{1}\times\frac{c}{\cancel{a}}+\cancel{a}b\times\frac{-b}{\cancel{a}}+b^2}$$

$$\frac{1}{a\alpha+b}+\frac{1}{a\beta+b}=\frac{-b+2b}{a\times c-b^2+b^2}$$

$$\frac{1}{a\alpha+b}+\frac{1}{a\beta+b}=\frac{b}{ac-\cancel{b^2}+\cancel{b^2}}$$

$$\frac{1}{a\alpha+b}+\frac{1}{a\beta+b}=\frac{b}{ac}$$

Hence, the value of $\frac{1}{a\alpha+b}+\frac{1}{a\beta+b}$ is $\boxed{\frac{b}{ac}}$.

(*vii*) Since α and β are the zeros of the quadratic polynomial $f(x)=ax^2+bx+c$

$$\alpha+\beta=\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$=\frac{-b}{a}$$

$$\alpha\beta=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$=\frac{c}{a}$$

We have, $\frac{\beta}{a\alpha+b}+\frac{\alpha}{a\beta+b}$

$$\frac{\beta}{a\alpha+b}+\frac{\alpha}{a\beta+b}=\frac{\beta(a\beta+b)+\alpha(a\alpha+b)}{(a\alpha+b)(a\beta+b)}$$

$$\frac{\beta}{a\alpha+b}+\frac{\alpha}{a\beta+b}=\frac{a\beta^2+\beta b+a\alpha^2+b\alpha}{a^2\times\alpha\beta+ab\beta+ab\alpha+b^2}$$

$$\frac{\beta}{a\alpha+b}+\frac{\alpha}{a\beta+b}=\frac{a\beta^2+a\alpha^2+b\alpha+\beta b}{a^2\times\alpha\beta+ab(\alpha+\beta)+b^2}$$

$$\frac{\beta}{a\alpha+b}+\frac{\alpha}{a\beta+b}=\frac{a(\beta^2+\alpha^2)+b(\alpha+\beta)}{a^2\times\alpha\beta+ab(\alpha+\beta)+b^2}$$

$$\frac{\beta}{a\alpha+b}+\frac{\alpha}{a\beta+b}=\frac{a\left((\alpha+\beta)^2-2\alpha\beta\right)+b(\alpha+\beta)}{a^2\times\alpha\beta+ab(\alpha+\beta)+b^2}$$

By substituting $\alpha+\beta=\frac{-b}{a}$ and $\alpha\beta=\frac{c}{a}$ we get ,

$$\frac{\beta}{a\alpha+b}+\frac{\alpha}{a\beta+b}=\frac{a\times\left[\left(\frac{-b}{a}\right)^2-2\times\frac{c}{a}\right]+b\left(\frac{-b}{a}\right)}{\cancel{a^2}\times\frac{c}{\cancel{a}}+\cancel{a}b\times\frac{-b}{\cancel{a}}+b^2}$$

*********** END **********