Definite Integrals Ex 20.1 Q55

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \sin^3 x \, dx$$

$$I = \int_0^{\frac{\pi}{2}} \sin^2 x \cdot \sin x \, dx$$

$$= \int_0^{\frac{\pi}{2}} \left(1 - \cos^2 x\right) \sin x \, dx$$

$$= \int_0^{\frac{\pi}{2}} \sin x \, dx - \int_0^{\frac{\pi}{2}} \cos^2 x \cdot \sin x \, dx$$

$$= \left[-\cos x\right]_0^{\frac{\pi}{2}} + \left[\frac{\cos^3 x}{3}\right]_0^{\frac{\pi}{2}}$$

$$= 1 + \frac{1}{3}\left[-1\right] = 1 - \frac{1}{3} = \frac{2}{3}$$

Hence, the given result is proved.

Definite Integrals Ex 20.1 Q56

$$\text{Let } I = \int_0^{\pi} \left(\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2}\right) dx$$

$$= -\int_0^{\pi} \left(\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}\right) dx$$

$$= -\int_0^{\pi} \cos x \, dx$$

$$\int \cos x \, dx = \sin x = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}(\pi) - \mathrm{F}(0)$$

$$= \sin\pi - \sin 0$$

$$= 0$$

Definite Integrals Ex 20.1 Q57

$$\int_1^2\left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}dx$$

Let $2x = t \Rightarrow 2dx = dt$

When $x = 1$, $t = 2$ and when $x = 2$, $t = 4$

$$\therefore \int_1^2\left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}dx = \frac{1}{2}\int_2^4\left(\frac{2}{t}-\frac{2}{t^2}\right)e^t dt$$

$$= \int_2^4\left(\frac{1}{t}-\frac{1}{t^2}\right)e^t dt$$

Let $\frac{1}{t} = f(t)$

Then, $f'(t) = -\frac{1}{t^2}$

$$\Rightarrow \int_2^4\left(\frac{1}{t}-\frac{1}{t^2}\right)e^t dt = \int_2^4 e^t\left[f(t)+f'(t)\right]dt$$

$$= \left[e^t f(t)\right]_2^4$$

$$= \left[e^t \cdot \frac{2}{t}\right]_2^4$$

$$= \left[\frac{e^t}{t}\right]_2^4$$

$$= \frac{e^4}{4}-\frac{e^2}{2}$$

$$= \frac{e^4-2e^2}{4}$$

Definite Integrals Ex 20.1 Q58

$$\int_1^2 \frac{1}{\sqrt{(x-1)(2-x)}}\,dx$$

$$= \int_1^2 \frac{1}{\sqrt{-\left(x-\frac{3}{2}\right)^2 + \left(\frac{1}{4}\right)}}\,dx$$

$$= \int_1^2 \frac{1}{\sqrt{\left(\frac{1}{2}\right)^2 - \left(x-\frac{3}{2}\right)^2}}\,dx$$

$$= \left[\sin^{-1}(2x-3)\right]_1^2$$

$$= \sin^{-1}(1) - \sin^{-1}(-1)$$

$$= \pi$$

*********** END **********