



EXERCISE 12.3

Question 1:

Find the coordinates of the point which divides the line segment joining the points $(-2, 3, 5)$ and $(1, -4, 6)$ in the ratio (i) $2:3$ internally, (ii) $2:3$ externally.

Ans:

(i) The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) internally in the ratio $m:n$ are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n} \right).$$

Let R (x, y, z) be the point that divides the line segment joining points $(-2, 3, 5)$ and $(1, -4, 6)$ internally in the ratio $2:3$

$$x = \frac{2(1) + 3(-2)}{2+3}, y = \frac{2(-4) + 3(3)}{2+3}, \text{ and } z = \frac{2(6) + 3(5)}{2+3}$$

$$\text{i.e., } x = \frac{-4}{5}, y = \frac{1}{5}, \text{ and } z = \frac{27}{5}$$

Thus, the coordinates of the required point are $\left(-\frac{4}{5}, \frac{1}{5}, \frac{27}{5} \right)$.

(ii) The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) externally in the ratio $m:n$ are

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n} \right).$$

Let R (x, y, z) be the point that divides the line segment joining points $(-2, 3, 5)$ and $(1, -4, 6)$ externally in the ratio $2:3$

$$x = \frac{2(1) - 3(-2)}{2-3}, y = \frac{2(-4) - 3(3)}{2-3}, \text{ and } z = \frac{2(6) - 3(5)}{2-3}$$

$$\text{i.e., } x = -8, y = 17, \text{ and } z = 3$$

Thus, the coordinates of the required point are $(-8, 17, 3)$.

Question 2:

Given that P $(3, 2, -4)$, Q $(5, 4, -6)$ and R $(9, 8, -10)$ are collinear. Find the ratio in which Q divides PR.

Ans:

Let point Q $(5, 4, -6)$ divide the line segment joining points P $(3, 2, -4)$ and R $(9, 8, -10)$ in the ratio $k:1$.

Therefore, by section formula,

$$(5, 4, -6) = \left(\frac{k(9) + 3}{k+1}, \frac{k(8) + 2}{k+1}, \frac{k(-10) - 4}{k+1} \right)$$

$$\Rightarrow \frac{9k+3}{k+1} = 5$$

$$\Rightarrow 9k+3 = 5k+5$$

$$\Rightarrow 4k = 2$$

$$\Rightarrow k = \frac{2}{4} = \frac{1}{2}$$

Thus, point Q divides PR in the ratio $1:2$.

Question 3:

Find the ratio in which the YZ-plane divides the line segment formed by joining the points $(-2, 4, 7)$ and $(3, -5, 8)$.

Ans:

Let the YZ plane divide the line segment joining points $(-2, 4, 7)$ and $(3, -5, 8)$ in the ratio $k:1$.

Hence, by section formula, the coordinates of point of intersection are given

$$\text{by } \left(\frac{k(3) - 2}{k+1}, \frac{k(-5) + 4}{k+1}, \frac{k(8) + 7}{k+1} \right)$$

On the YZ plane, the x -coordinate of any point is zero.

$$\frac{3k - 2}{k+1} = 0$$

$$\Rightarrow 3k - 2 = 0$$

$$\Rightarrow k = \frac{2}{3}$$

Thus, the YZ plane divides the line segment formed by joining the given points in the ratio $2:3$.

Question 4:

Using section formula, show that the points A $(2, -3, 4)$, B $(-1, 2, 1)$ and C $\left(0, \frac{1}{3}, 2\right)$

are collinear.

Ans:

The given points are A $(2, -3, 4)$, B $(-1, 2, 1)$, and C $\left(0, \frac{1}{3}, 2\right)$.

Let P be a point that divides AB in the ratio $k:1$.

Hence, by section formula, the coordinates of P are given by

$$\left(\frac{k(-1) + 2}{k+1}, \frac{k(2) - 3}{k+1}, \frac{k(1) + 4}{k+1} \right)$$

Now, we find the value of k at which point P coincides with point C.

By taking $\frac{-k+2}{k+1} = 0$, we obtain $k = 2$.

For $k = 2$, the coordinates of point P are $\left(0, \frac{1}{3}, 2\right)$.

i.e., C $\left(0, \frac{1}{3}, 2\right)$ is a point that divides AB externally in the ratio $2:1$ and is the same as point P.

Hence, points A, B, and C are collinear.

Question 5:

Find the coordinates of the points which trisect the line segment joining the points P $(4, 2, -6)$ and Q $(10, -16, 6)$.

Ans:

Let A and B be the points that trisect the line segment joining points P $(4, 2, -6)$ and Q $(10, -16, 6)$



Point A divides PQ in the ratio $1:2$. Therefore, by section formula, the coordinates of point A are given by

$$\left(\frac{1(10) + 2(4)}{1+2}, \frac{1(-16) + 2(2)}{1+2}, \frac{1(6) + 2(-6)}{1+2} \right) = (6, -4, -2)$$

Point B divides PQ in the ratio $2:1$. Therefore, by section formula, the coordinates of point B are given by

$$\left(\frac{2(10) + 1(4)}{2+1}, \frac{2(-16) + 1(2)}{2+1}, \frac{2(6) - 1(6)}{2+1} \right) = (8, -10, 2)$$

Thus, $(6, -4, -2)$ and $(8, -10, 2)$ are the points that trisect the line segment joining points P $(4, 2, -6)$ and Q $(10, -16, 6)$.

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