NCERT MISCELLANEOUS SOLUTIONS

Question 1:

Evaluate:

$$\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^{3}$$

Ans:

$$\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$$

$$=\left[i^{4\times4+2}+\frac{1}{i^{4\times6+1}}\right]^3$$

$$=\left[\left(i^4\right)^4\cdot i^2+\frac{1}{\left(i^4\right)^6\cdot i}\right]^3$$

$$=\left[i^2+\frac{1}{i}\right]^3 \qquad \left[i^4=1\right]$$

$$=\left[-1+\frac{1}{i}\times\frac{i}{i}\right]^3 \qquad \left[i^2=-1\right]$$

$$=\left[-1+\frac{i}{i^2}\right]^3$$

$$=\left[-1-i\right]^3$$

$$=\left(-1\right)^3\left[1+i\right]^3$$

$$=-\left[1^3+i^3+3\cdot1\cdot i\left(1+i\right)\right]$$

$$=-\left[1+i^3+3i+3i^2\right]$$

$$=-\left[1-i+3i-3\right]$$

$$=-\left[-2+2i\right]$$

$$=2-2i$$

Question 2:

For any two complex numbers z_1 and z_2, prove that

$\text{Re}\,(z_1z_2) = \text{Re}\,z_1\,\text{Re}\,z_2 - \text{Im}\,z_1\,\text{Im}\,z_2$

Ans:

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$$\therefore z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2)$$
$$= x_1(x_2 + iy_2) + iy_1(x_2 + iy_2)$$
$$= x_1x_2 + ix_1y_2 + iy_1x_2 + i^2 y_1y_2$$
$$= x_1x_2 + ix_1y_2 + iy_1x_2 - y_1y_2 \qquad [i^2 = -1]$$
$$= (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2)$$
$$\Rightarrow \operatorname{Re}(z_1z_2) = x_1x_2 - y_1y_2$$
$$\Rightarrow \operatorname{Re}(z_1z_2) = \operatorname{Re} z_1 \operatorname{Re} z_2 - \operatorname{Im} z_1 \operatorname{Im} z_2$$

Hence, proved.

Question 3:

Reduce $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ to the standard form.

Ans:

$$\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right) = \left[\frac{(1+i)-2(1-4i)}{(1-4i)(1+i)}\right]\left[\frac{3-4i}{5+i}\right]$$
$$= \left[\frac{1+i-2+8i}{1+i-4i-4i^2}\right]\left[\frac{3-4i}{5+i}\right] = \left[\frac{-1+9i}{5-3i}\right]\left[\frac{3-4i}{5+i}\right]$$
$$= \left[\frac{-3+4i+27i-36i^2}{25+5i-15i-3i^2}\right] = \frac{33+31i}{28-10i} = \frac{33+31i}{2(14-5i)}$$
$$= \frac{(33+31i)}{2(14-5i)} \times \frac{(14+5i)}{(14+5i)} \qquad [\text{On multiplying numerator and denominator by } (14+5i)]$$
$$= \frac{462+165i+434i+155i^2}{2\left[(14)^2-(5i)^2\right]} = \frac{307+599i}{2(196-25i^2)}$$
$$= \frac{307+599i}{2(221)} = \frac{307+599i}{442} = \frac{307}{442} + \frac{599i}{442}$$

This is the required standard form.

Question 4:

If $x - iy = \sqrt{\frac{a-ib}{c-id}}$ prove that $\left(x^2+y^2\right)^2 = \frac{a^2+b^2}{c^2+d^2}$.

Ans:

$$x - iy = \sqrt{\frac{a - ib}{c - id}}$$

$$= \sqrt{\frac{a - ib}{c - id} \times \frac{c + id}{c + id}} \left[\text{On multiplying numerator and denominator by } (c + id)\right]$$

$$= \sqrt{\frac{(ac + bd) + i(ad - bc)}{c^2 + d^2}}$$

$$\therefore (x - iy)^2 = \frac{(ac + bd) + i(ad - bc)}{c^2 + d^2}$$

$$\Rightarrow x^2 - y^2 - 2ixy = \frac{(ac + bd) + i(ad - bc)}{c^2 + d^2}$$

On comparing real and imaginary parts, we obtain

$$x^2 - y^2 = \frac{ac + bd}{c^2 + d^2}, \quad -2xy = \frac{ad - bc}{c^2 + d^2} \qquad (1)$$

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$$

$$= \left(\frac{ac + bd}{c^2 + d^2}\right)^2 + \left(\frac{ad - bc}{c^2 + d^2}\right)^2 \qquad [\text{Using } (1)]$$

$$= \frac{a^2c^2 + b^2d^2 + 2acbd + a^2d^2 + b^2c^2 - 2adbc}{(c^2 + d^2)^2}$$

$$= \frac{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2}{(c^2 + d^2)^2}$$

$$= \frac{a^2(c^2 + d^2) + b^2(c^2 + d^2)}{(c^2 + d^2)^2}$$

$$= \frac{(c^2 + d^2)(a^2 + b^2)}{(c^2 + d^2)^2}$$

$$= \frac{a^2 + b^2}{c^2 + d^2}$$

Hence, proved.

Question 5:

Convert the following in the polar form:

(i) $\dfrac{1 + 7i}{(2 - i)^2}$, (ii) $\dfrac{1 + 3i}{1 - 2i}$

Ans:

(i) Here, $z = \frac{1+7i}{(2-i)^2}$

$$= \frac{1+7i}{(2-i)^2} = \frac{1+7i}{4+i^2-4i} = \frac{1+7i}{4-1-4i}$$

$$= \frac{1+7i}{3-4i} \times \frac{3+4i}{3+4i} = \frac{3+4i+21i+28i^2}{3^2+4^2}$$

$$= \frac{3+4i+21i-28}{3^2+4^2} = \frac{-25+25i}{25}$$

$$= -1+i$$

Let $r\cos\theta = -1$ and $r\sin\theta = 1$

On squaring and adding, we obtain

$r^2(\cos^2\theta + \sin^2\theta) = 1 + 1$

$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) = 2$

$\Rightarrow r^2 = 2$ $\quad [\cos^2\theta + \sin^2\theta = 1]$

$\Rightarrow r = \sqrt{2}$ $\quad$ [Conventionally, $r > 0$]

$\therefore \sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = 1$

$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}}$

$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ $\quad$ [As θ lies in II quadrant]

$\therefore z = r\cos\theta + i\,r\sin\theta$

$$= \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

This is the required polar form.

(ii) Here, $z = \frac{1+3i}{1-2i}$

$$= \frac{1+3i}{1-2i} \times \frac{1+2i}{1+2i}$$
$$= \frac{1+2i+3i-6}{1+4}$$
$$= \frac{-5+5i}{5} = -1+i$$

Let $r\cos\theta = -1$ and $r\sin\theta = 1$

On squaring and adding, we obtain

$r^2(\cos^2\theta + \sin^2\theta) = 1 + 1$
$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) = 2$

$\Rightarrow r^2 = 2$ $\quad [\cos^2\theta + \sin^2\theta = 1]$

$\Rightarrow r = \sqrt{2}$ $\quad$ [Conventionally, $r > 0$]

$\therefore \sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = 1$

$\Rightarrow \cos\theta = \frac{-1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}}$

$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ $\quad$ [As θ lies in II quadrant]

$\therefore z = r\cos\theta + i\, r\sin\theta$

$$= \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

This is the required polar form.

Question 6:

Solve the equation $3x^2 - 4x + \frac{20}{3} = 0$

Ans:

The given quadratic equation is $3x^2 - 4x + \frac{20}{3} = 0$

This equation can also be written as $9x^2 - 12x + 20 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 9$, $b = -12$, and $c = 20$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = (-12)^2 - 4 \times 9 \times 20 = 144 - 720 = -576$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-(-12) \pm \sqrt{-576}}{2 \times 9} = \frac{12 \pm \sqrt{576}\, i}{18} \qquad \left[\sqrt{-1} = i\right]$$
$$= \frac{12 \pm 24i}{18} = \frac{6(2 \pm 4i)}{18} = \frac{2 \pm 4i}{3} = \frac{2}{3} \pm \frac{4}{3} i$$

Question 7:

Solve the equation $x^2 - 2x + \frac{3}{2} = 0$

Ans:

The given quadratic equation is $x^2 - 2x + \frac{3}{2} = 0$

This equation can also be written as $2x^2 - 4x + 3 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2$, $b = -4$, and $c = 3$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = (-4)^2 - 4 \times 2 \times 3 = 16 - 24 = -8$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-(-4) \pm \sqrt{-8}}{2 \times 2} = \frac{4 \pm 2\sqrt{2}\, i}{4} \qquad \left[\sqrt{-1} = i\right]$$
$$= \frac{2 \pm \sqrt{2}\, i}{2} = 1 \pm \frac{\sqrt{2}}{2} i$$

Question 8:

Solve the equation $27x^2 - 10x + 1 = 0$

Ans:

The given quadratic equation is $27x^2 - 10x + 1 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$a = 27$, $b = -10$, and $c = 1$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = (-10)^2 - 4 \times 27 \times 1 = 100 - 108 = -8$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-(-10) \pm \sqrt{-8}}{2 \times 27} = \frac{10 \pm 2\sqrt{2}\,i}{54} \qquad \left[\sqrt{-1} = i\right]$$

$$= \frac{5 \pm \sqrt{2}\,i}{27} = \frac{5}{27} \pm \frac{\sqrt{2}}{27} i$$

Question 9:

Solve the equation $21x^2 - 28x + 10 = 0$

Ans:

The given quadratic equation is $21x^2 - 28x + 10 = 0$

On comparing the given equation with $ax^2 + bx + c = 0$, we obtain

$a = 21$, $b = -28$, and $c = 10$

Therefore, the discriminant of the given equation is

$D = b^2 - 4ac = (-28)^2 - 4 \times 21 \times 10 = 784 - 840 = -56$

Therefore, the required solutions are

$$\frac{-b \pm \sqrt{D}}{2a} = \frac{-(-28) \pm \sqrt{-56}}{2 \times 21} = \frac{28 \pm \sqrt{56}\,i}{42}$$

$$= \frac{28 \pm 2\sqrt{14}\,i}{42} = \frac{28}{42} \pm \frac{2\sqrt{14}}{42} i = \frac{2}{3} \pm \frac{\sqrt{14}}{21} i$$

Question 10:

If $z_1 = 2 - i$, $z_2 = 1 + i$, find $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right|$.

Ans:

$z_1 = 2 - i,\ z_2 = 1 + i$

$$\therefore \left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right| = \left|\frac{(2-i)+(1+i)+1}{(2-i)-(1+i)+1}\right|$$

$$= \left|\frac{4}{2-2i}\right| = \left|\frac{4}{2(1-i)}\right|$$

$$= \left|\frac{2}{1-i} \times \frac{1+i}{1+i}\right| = \left|\frac{2(1+i)}{1^2 - i^2}\right|$$

$$= \left|\frac{2(1+i)}{1+1}\right| \qquad \left[i^2 = -1\right]$$

$$= \left|\frac{2(1+i)}{2}\right|$$

$$= |1+i| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

Thus, the value of $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + 1}\right|$ is $\sqrt{2}$.

Question 11:

If $a + ib = \frac{(x+i)^2}{2x^2+1}$, prove that $a^2 + b^2 = \frac{\left(x^2+1\right)^2}{(2x+1)^2}$

Ans:

$$a+ib=\frac{(x+i)^2}{2x^2+1}$$
$$=\frac{x^2+i^2+2xi}{2x^2+1}$$
$$=\frac{x^2-1+i2x}{2x^2+1}$$
$$=\frac{x^2-1}{2x^2+1}+i\left(\frac{2x}{2x^2+1}\right)$$

On comparing real and imaginary parts, we obtain

$$a=\frac{x^2-1}{2x^2+1} \text{ and } b=\frac{2x}{2x^2+1}$$
$$\therefore a^2+b^2=\left(\frac{x^2-1}{2x^2+1}\right)^2+\left(\frac{2x}{2x^2+1}\right)^2$$
$$=\frac{x^4+1-2x^2+4x^2}{(2x+1)^2}$$
$$=\frac{x^4+1+2x^2}{(2x^2+1)^2}$$
$$=\frac{(x^2+1)^2}{(2x^2+1)^2}$$
$$\therefore a^2+b^2=\frac{(x^2+1)^2}{(2x^2+1)^2}$$

Hence, proved.

Question 12:

Let $z_1=2-i,\ z_2=-2+i$. Find

(i) $\operatorname{Re}\left(\frac{z_1 z_2}{\bar{z}_1}\right)$, (ii) $\operatorname{Im}\left(\frac{1}{z_1\bar{z}_1}\right)$

Ans:

$z_1 = 2 - i,\ z_2 = -2 + i$

(i) $z_1 z_2 = (2-i)(-2+i) = -4 + 2i + 2i - i^2 = -4 + 4i - (-1) = -3 + 4i$

$\bar{z}_1 = 2 + i$

$\therefore \dfrac{z_1 z_2}{\bar{z}_1} = \dfrac{-3+4i}{2+i}$

On multiplying numerator and denominator by $(2 - i)$, we obtain

$$\frac{z_1 z_2}{\bar{z}_1} = \frac{(-3+4i)(2-i)}{(2+i)(2-i)} = \frac{-6+3i+8i-4i^2}{2^2+1^2} = \frac{-6+11i-4(-1)}{2^2+1^2}$$
$$= \frac{-2+11i}{5} = \frac{-2}{5} + \frac{11}{5}i$$

On comparing real parts, we obtain

$$\mathrm{Re}\left(\frac{z_1 z_2}{\bar{z}_1}\right) = \frac{-2}{5}$$

(ii) $\dfrac{1}{z_1 \bar{z}_1} = \dfrac{1}{(2-i)(2+i)} = \dfrac{1}{(2)^2 + (1)^2} = \dfrac{1}{5}$

On comparing imaginary parts, we obtain

$$\mathrm{Im}\left(\frac{1}{z_1 \bar{z}_1}\right) = 0$$

Question 13:

Find the modulus and argument of the complex number $\dfrac{1+2i}{1-3i}$.

Ans:

Let $z=\frac{1+2i}{1-3i}$, then

$$z=\frac{1+2i}{1-3i}\times\frac{1+3i}{1+3i}=\frac{1+3i+2i+6i^2}{1^2+3^2}=\frac{1+5i+6(-1)}{1+9}$$
$$=\frac{-5+5i}{10}=\frac{-5}{10}+\frac{5i}{10}=\frac{-1}{2}+\frac{1}{2}i$$

Let $z=r\cos\theta+ir\sin\theta$

i.e., $r\cos\theta=\frac{-1}{2}$ and $r\sin\theta=\frac{1}{2}$

On squaring and adding, we obtain

$$r^2\left(\cos^2\theta+\sin^2\theta\right)=\left(\frac{-1}{2}\right)^2+\left(\frac{1}{2}\right)^2$$
$$\Rightarrow r^2=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$$
$$\Rightarrow r=\frac{1}{\sqrt{2}} \qquad [\text{Conventionally, } r>0]$$
$$\therefore \frac{1}{\sqrt{2}}\cos\theta=\frac{-1}{2} \text{ and } \frac{1}{\sqrt{2}}\sin\theta=\frac{1}{2}$$
$$\Rightarrow \cos\theta=\frac{-1}{\sqrt{2}} \text{ and } \sin\theta=\frac{1}{\sqrt{2}}$$
$$\therefore \theta=\pi-\frac{\pi}{4}=\frac{3\pi}{4} \qquad [\text{As } \theta \text{ lies in the II quadrant}]$$

Therefore, the modulus and argument of the given complex number are $\frac{1}{\sqrt{2}}$ and $\frac{3\pi}{4}$ respectively.

Question 14:

Find the real numbers x and y if $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$.

Ans:

Let $z=(x-iy)(3+5i)$

$$z=3x+5xi-3yi-5yi^2=3x+5xi-3yi+5y=(3x+5y)+i(5x-3y)$$
$$\therefore \bar{z}=(3x+5y)-i(5x-3y)$$

It is given that, $\bar{z}=-6-24i$

$$\therefore (3x+5y)-i(5x-3y)=-6-24i$$

Equating real and imaginary parts, we obtain

$$3x+5y=-6 \qquad \ldots \text{(i)}$$
$$5x-3y=24 \qquad \ldots \text{(ii)}$$

Multiplying equation (i) by 3 and equation (ii) by 5 and then adding them, we obtain

$$9x+15y=-18$$
$$25x-15y=120$$
$$34x \quad =102$$
$$\therefore x=\frac{102}{34}=3$$

Putting the value of x in equation (i), we obtain

$$3(3)+5y=-6$$
$$\Rightarrow 5y=-6-9=-15$$
$$\Rightarrow y=-3$$

Thus, the values of x and y are 3 and -3 respectively.

Question 15:

Find the modulus of $\frac{1+i}{1-i}-\frac{1-i}{1+i}$.

Ans:

$$\frac{1+i}{1-i}-\frac{1-i}{1+i}=\frac{(1+i)^2-(1-i)^2}{(1-i)(1+i)}$$

$$=\frac{1+i^2+2i-1-i^2+2i}{1^2+1^2}$$

$$=\frac{4i}{2}=2i$$

$$\therefore\left|\frac{1+i}{1-i}-\frac{1-i}{1+i}\right|=|2i|=\sqrt{2^2}=2$$

Question 16:

If $(x+iy)^3=u+iv$, then show that $\frac{u}{x}+\frac{v}{y}=4\left(x^2-y^2\right)$.

Ans:

$$(x+iy)^3 = u+iv$$

$$\Rightarrow x^3 + (iy)^3 + 3\cdot x\cdot iy(x+iy) = u+iv$$

$$\Rightarrow x^3 + i^3y^3 + 3x^2yi + 3xy^2i^2 = u+iv$$

$$\Rightarrow x^3 - iy^3 + 3x^2yi - 3xy^2 = u+iv$$

$$\Rightarrow (x^3 - 3xy^2) + i(3x^2y - y^3) = u+iv$$

On equating real and imaginary parts, we obtain

$$u = x^3 - 3xy^2,\ v = 3x^2y - y^3$$

$$\therefore \frac{u}{x} + \frac{v}{y} = \frac{x^3 - 3xy^2}{x} + \frac{3x^2y - y^3}{y}$$

$$= \frac{x(x^2 - 3y^2)}{x} + \frac{y(3x^2 - y^2)}{y}$$

$$= x^2 - 3y^2 + 3x^2 - y^2$$

$$= 4x^2 - 4y^2$$

$$= 4(x^2 - y^2)$$

$$\therefore \frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$$

Hence, proved.

Question 17:

If α and β are different complex numbers with $|\beta| = 1$, then find $\left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right|$.

Ans:

Let $\alpha = a + ib$ and $\beta = x + iy$

It is given that, $|\beta| = 1$

$$\therefore \sqrt{x^2 + y^2} = 1$$
$$\Rightarrow x^2 + y^2 = 1 \qquad \text{... (i)}$$

$$\left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right| = \left|\frac{(x+iy)-(a+ib)}{1-(a-ib)(x+iy)}\right|$$
$$= \left|\frac{(x-a)+i(y-b)}{1-(ax+aiy-ibx+by)}\right|$$
$$= \left|\frac{(x-a)+i(y-b)}{(1-ax-by)+i(bx-ay)}\right|$$
$$= \frac{|(x-a)+i(y-b)|}{|(1-ax-by)+i(bx-ay)|} \qquad \left[\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$$
$$= \frac{\sqrt{(x-a)^2+(y-b)^2}}{\sqrt{(1-ax-by)^2+(bx-ay)^2}}$$
$$= \frac{\sqrt{x^2+a^2-2ax+y^2+b^2-2by}}{\sqrt{1+a^2x^2+b^2y^2-2ax+2abxy-2by+b^2x^2+a^2y^2-2abxy}}$$
$$= \frac{\sqrt{(x^2+y^2)+a^2+b^2-2ax-2by}}{\sqrt{1+a^2(x^2+y^2)+b^2(y^2+x^2)-2ax-2by}}$$
$$= \frac{\sqrt{1+a^2+b^2-2ax-2by}}{\sqrt{1+a^2+b^2-2ax-2by}} \qquad [\text{Using } (1)]$$
$$= 1$$
$$\therefore \left|\frac{\beta - \alpha}{1 - \bar{\alpha}\beta}\right| = 1$$

Question 18:

Find the number of non-zero integral solutions of the equation $|1-i|^x = 2^x$.

Ans:

$$|1-i|^x = 2^x$$
$$\Rightarrow \left(\sqrt{1^2+(-1)^2}\right)^x = 2^x$$
$$\Rightarrow \left(\sqrt{2}\right)^x = 2^x$$
$$\Rightarrow 2^{\frac{x}{2}} = 2^x$$
$$\Rightarrow \frac{x}{2} = x$$
$$\Rightarrow x = 2x$$
$$\Rightarrow 2x - x = 0$$
$$\Rightarrow x = 0$$

Thus, 0 is the only integral solution of the given equation. Therefore, the number of non-zero integral solutions of the given equation is 0.

Question 19:

If $(a + ib)(c + id)(e + if)(g + ih) = A + iB$, then show that

$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$.

Ans:

$(a+ib)(c+id)(e+if)(g+ih) = A+iB$

$\therefore |(a+ib)(c+id)(e+if)(g+ih)| = |A+iB|$

$\Rightarrow |(a+ib)| \times |(c+id)| \times |(e+if)| \times |(g+ih)| = |A+iB| \qquad [|z_1 z_2| = |z_1||z_2|]$

$\Rightarrow \sqrt{a^2+b^2} \times \sqrt{c^2+d^2} \times \sqrt{e^2+f^2} \times \sqrt{g^2+h^2} = \sqrt{A^2+B^2}$

On squaring both sides, we obtain

$(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = A^2+B^2$

Hence, proved.

Question 20:

If $\left(\frac{1+i}{1-i}\right)^m = 1$, then find the least positive integral value of m.

Ans:

$$\left(\frac{1+i}{1-i}\right)^m = 1$$

$$\Rightarrow \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^m = 1$$

$$\Rightarrow \left(\frac{(1+i)^2}{1^2+1^2}\right)^m = 1$$

$$\Rightarrow \left(\frac{1^2+i^2+2i}{2}\right)^m = 1$$

$$\Rightarrow \left(\frac{1-1+2i}{2}\right)^m = 1$$

$$\Rightarrow \left(\frac{2i}{2}\right)^m = 1$$

$$\Rightarrow i^m = 1$$

$\therefore m = 4k$, where k is some integer.

Therefore, the least positive integer is 1.

Thus, the least positive integral value of m is 4 (= 4 × 1).

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