

Differentiation Ex 11.3 Q26

Let
$$y = \tan^{-1} \left(\frac{\sqrt{x} + \sqrt{a}}{1 - \sqrt{xa}} \right)$$

 $y = \tan^{-1} \sqrt{x} + \tan^{-1} \sqrt{a}$

Since,
$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x + y}{1 - xy}$$

Differentiating it with respect to x using chain rule,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \left(\tan^{-1} \sqrt{x} \right) + \frac{d}{dx} \left(\tan^{-1} \sqrt{a} \right) \\ &= \frac{1}{1 + \left(\sqrt{x} \right)^2} \frac{d}{dx} \left(\sqrt{x} \right) + 0 \\ &= \left(\frac{1}{1 + x} \right) \left(\frac{1}{2\sqrt{x}} \right) \end{aligned}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}\left(1+x\right)}.$$

Differentiation Ex 11.3 Q27

Differentiation Ex II.3 Q27

Let
$$y = \tan^{-1} \left[\frac{a + b \tan x}{b - a \tan x} \right]$$

$$= \tan^{-1} \left[\frac{\frac{a + b \tan x}{b}}{\frac{b - a \tan x}{b}} \right]$$

$$= \tan^{-1} \left[\frac{\frac{a}{b} + \tan x}{1 + \frac{a}{b} \tan x} \right]$$

$$= \tan^{-1} \left[\frac{\tan \left(\tan^{-1} \frac{a}{b} \right) + \tan x}{1 - \tan \left(\tan^{-1} \frac{a}{b} \right) + \tan x} \right]$$

$$= \tan^{-1} \left[\tan \left(\tan^{-1} \frac{a}{b} + x \right) \right]$$

$$y = \tan^{-1} \left(\frac{a}{b} \right) + x$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = 0 + 1$$
$$\frac{dy}{dx} = 1.$$

Differentiation Ex 11.3 Q28

Let
$$y = \tan^{-1}\left(\frac{a+bx}{b-ax}\right)$$

$$= \tan^{-1}\left(\frac{\frac{a+bx}{b}}{\frac{b-ax}{b}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{a}{b} + \frac{bx}{b}}{\frac{b-ax}{a}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{a}{b} + x}{1-\left(\frac{a}{b}\right)x}\right)$$

$$y = \tan^{-1}\left(\frac{\frac{a}{b} + x}{1-\left(\frac{a}{b}\right)x}\right)$$
Since, $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 0 + \frac{1}{1 + x^2}$$
$$\frac{dy}{dx} = \frac{1}{1 + x^2}.$$

Differentiation Ex 11.3 Q29

Let
$$y = \tan^{-1}\left(\frac{x-a}{x+a}\right)$$

$$= \tan^{-1}\left(\frac{\frac{x-a}{x}}{\frac{x}{x+a}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{x}{x}-\frac{a}{x}}{\frac{x}{x}+\frac{a}{x}}\right)$$

$$= \tan^{-1}\left(\frac{1-\frac{a}{x}}{1+1\times\frac{a}{x}}\right)$$

$$y = \tan^{-1}\left(1\right) - \tan^{-1}\left(\frac{a}{x}\right)$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = 0 - \frac{1}{1 + \left(\frac{a}{x}\right)^2} \frac{d}{dx} \left(\frac{a}{x}\right)$$
$$= -\frac{x^2}{x^2 + a^2} \left(\frac{-a}{x^2}\right)$$

$$\frac{dy}{dx} = \frac{a}{a^2 + x^2}.$$

Differentiation Ex 11.3 Q30

Let
$$y = \tan^{-1} \left(\frac{x}{1 + 6x^2} \right)$$

 $= \tan^{-1} \left(\frac{3x - 2x}{1 + (3x)(2x)} \right)$
 $y = \tan^{-1} 3x - \tan^{-1} 2x$ [Since, $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \left(\frac{x - y}{1 + xy} \right)$]

Differentiating it with respect to \boldsymbol{x} using chain rule,

$$\frac{dy}{dx} = \frac{1}{1 + (3x)^2} \frac{d}{dx} (3x) - \frac{1}{1 + (2x)^2} \frac{d}{dx} (2x)$$
$$= \frac{1}{1 + 9x^2} (3) - \frac{1}{1 + 4x^2} (2)$$

$$\frac{dy}{dx} = \frac{3}{1 + 9x^2} - \frac{2}{1 + 4x^2}.$$