Exercise 3D

Question 19:

$kx + 3y - (2k + 1) = 0$

$2(k + 1)x + 9y - (7k + 1) = 0$

These are of the form

$$a_1x + b_1y + c_1 = 0,\ a_2x + b_2y + c_2 = 0$$

where, $a_1 = k,\ b_1 = 3,\ c_1 = -(2k + 1)$

$$a_2 = 2(k + 1),\ b_2 = 9,\ c_2 = -(7k + 1)$$

For infinitely many solutions, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

This hold only when

$$\frac{k}{2(k+1)} = \frac{3}{9} = \frac{-(2k+1)}{-(7k+1)}$$

$$\Rightarrow \frac{k}{2(k+1)} = \frac{1}{3} = \frac{2k+1}{7k+1}$$

Now, the following cases arise

Case – (1)

$$\frac{k}{2(k+1)} = \frac{1}{3} \text{ [Taking I and II]}$$

$$\Rightarrow 2(k+1) = 3k \Rightarrow 2k + 2 = 3k$$

$$\Rightarrow \quad k = 2$$

Case (2)

$$\frac{1}{3} = \frac{2k+1}{7k+1} \text{[taking II and III]}$$

$$7k + 1 = 6k + 3 \Rightarrow 7k - 6k = 3 - 1$$

$$k = 2$$

Case (3)

$$\frac{k}{2(k+1)} = \frac{2k+1}{7k+1} \text{[taking I and III]}$$

$$k(7k+1) = 2(2k+1)(k+1)$$

$$\Rightarrow 7k^2 + k = 2(2k^2 + 2k + k + 1)$$

$$7k^2 + k = 2(2k^2 + 3k + 1)$$

$$7k^2 + k = 4k^2 + 6k + 2$$

$$7k^2 - 4k^2 + k - 6k - 2 = 0$$

$$3k^2 - 5k - 2 = 0$$

$$3k^2 - (6k - 1k) - 2 = 0$$

$$3k(k-2) + 1(k-2) = 0$$

$$(k-2)(3k+1) = 0$$

$$k = 2 \text{ or } k = \frac{-1}{3}$$

Thus, k = 2, is the common value for which there are infinitely many solutions

Question 20:

$5x + 2y - 2k = 0$

$2(k + 1)x + ky - (3k + 4) = 0$

These are of the form

$$a_1x + b_1y + c_1 = 0, \; a_2x + b_2y + c_2 = 0$$

where, $a_1 = 5, b_1 = 2, c_1 = -2k$

$$a_2 = 2(k + 1), \; b_2 = k, \; c_2 = -(3k + 4)$$

For infinitely many solutions, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

These hold only when

$$\frac{5}{2(k + 1)} = \frac{2}{k} = \frac{-2k}{-(3k + 4)}$$

$$\Rightarrow \frac{5}{2(k + 1)} = \frac{2}{k} = \frac{2k}{(3k + 4)}$$

Case I

$$\frac{5}{2(k + 1)} = \frac{2}{k} \left[\because \text{taking I and II}\right]$$

$$\Rightarrow 5k = 4(k + 1) \Rightarrow 5k = 4k + 4$$

$$k = 4$$

Case (2)

$$\frac{2}{k} = \frac{2k}{(3k+4)} \quad [\because \text{ taking II and III}]$$

$$2(3k+4) = 2k^2 \Rightarrow 6k + 8 = 2k^2$$

$$\Rightarrow 2k^2 - 6k - 8 = 0$$

$$2(k^2 - 3k - 4) = 0$$

$$k^2 - 3k - 4 = 0$$

$$k^2 - 4k + k - 4 = 0$$

$$k(k-4) + 1(k-4) = 0$$

$$(k-4)(k+1) = 0$$

$$(k-4) = 0 \text{ or } k + 1 = 0$$

$$k = 4 \text{ or } k = -1$$

Case (3)

$$\frac{5}{2(k+1)} = \frac{2k}{(3k+4)} \quad [\text{taking I and III}]$$

$$\Rightarrow 15k + 20 = 4k^2 = 4k$$

$$\Rightarrow 4k^2 + 4k - 15k - 20 = 0$$

$$4k^2 - 11k - 20 = 0$$

$$4k^2 - 16k + 5k - 20 = 0$$

$$4k(k-4) + 5(k-4) = 0$$

$$(k-4)(4k+5) = 0 \Rightarrow k = 4 \text{ or } k = \frac{-5}{4}$$

Thus, k = 4 is a common value for which there are infinitely by many solutions.

*********** END **********