Differentiation Ex 11.3 Q42

Here, $y = \cos^{-1}(2x) + 2\cos^{-1}\sqrt{1-4x^2}$

Put $2x = \cos\theta$, so

$$y = \cos^{-1}(\cos\theta) + 2\cos^{-1}\sqrt{1-\cos^2\theta}$$
$$= \cos^{-1}(\cos\theta) + 2\cos^{-1}(\sin\theta)$$
$$= \cos^{-1}(\cos\theta) + 2\cos^{-1}\left(\cos\left(\frac{\pi}{2} - \theta\right)\right) \qquad \text{---(i)}$$

Here, $0 < x < \frac{1}{2}$

$\Rightarrow \quad 0 < 2x < 1$

$\Rightarrow \quad 0 < \cos\theta < 1$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{2}$

and

$\Rightarrow \quad 0 > -\theta > -\frac{\pi}{2}$

$\Rightarrow \quad \frac{\pi}{2} > \left(\frac{\pi}{2} - \theta\right) > 0$

So, from equation (i),

$$y = \theta + 2\left(\frac{\pi}{2} - \theta\right) \qquad \left[\text{Since, } \cos^{-1}(\cos(\theta)) = \theta, \text{ if } \theta \in [0, \pi]\right]$$
$$= \theta + \pi - 2\theta$$
$$y = \pi - \theta$$
$$y = \pi - \cos^{-1}(2x) \qquad [\text{Since, } 2x = \cos\theta]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = 0 - \left[\frac{-1}{\sqrt{1-(2x)^2}}\right]\frac{d}{dx}(2x)$$
$$= \frac{1}{\sqrt{1-4x^2}}(2)$$
$$\frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}.$$

Differentiation Ex 11.3 Q43

Here, $\frac{d}{dx}\left[\tan^{-1}(a+bx)\right] = 1 \text{ at } x = 0$

So, using chain rule,

$$\left[\left\{\frac{1}{1+(a+bx)^2}\right\}\frac{d}{dx}(a+bx)\right]_{x=0} = 1$$

$$\left[\frac{1}{1+(a+bx)^2} \times (b)\right]_{x=0} = 1$$

$\Rightarrow \quad \dfrac{b}{1+(a+0)^2} = 1$

$\Rightarrow \quad b = 1 + a^2.$

Differentiation Ex 11.3 Q44

Here, $y = \cos^{-1}(2x) + 2\cos^{-1}\sqrt{1-4x^2}$

Put $2x = \cos\theta$, so,

$$y = \cos^{-1}(\cos\theta) + 2\cos^{-1}\sqrt{1-\cos^2\theta}$$

$$= \cos^{-1}(\cos\theta) + 2\cos^{-1}(\sin\theta)$$

$$y = \cos^{-1}(\cos\theta) + 2\cos^{-1}\left(\cos\left(\frac{\pi}{2}-\theta\right)\right) \quad \text{---(i)}$$

Now, $-\frac{1}{2} < x < 0$

$\Rightarrow \quad -1 < 2x < 0$

$\Rightarrow \quad -1 < \cos\theta < 0$

$\Rightarrow \quad \frac{\pi}{2} < \theta < \pi$

And

$\Rightarrow \quad -\frac{\pi}{2} > -\theta > -\pi$

$\Rightarrow \quad \left(\frac{\pi}{2}-\frac{\pi}{2}\right) > \left(\frac{\pi}{2}-\theta\right) > \left(\frac{\pi}{2}-\pi\right)$

$\Rightarrow \quad 0 > \left(\frac{\pi}{2}-\theta\right) > -\frac{\pi}{2}$

So, from equation (i),

$$y = \theta + 2\left[-\left(\frac{\pi}{2}-\theta\right)\right] \qquad \left[\begin{array}{l}\text{Since, } \cos^{-1}\cos(\theta) = \theta, \text{if } \theta \in [0,\pi] \\ \qquad \cos^{-1}\cos(\theta) = -\theta, \text{ if } \theta \in [-\pi, 0]\end{array}\right]$$

$$y = \theta - 2\times\frac{\pi}{2} + 2\theta$$

$$y = -\pi + 3\theta$$

$$y = -\pi + 3\cos^{-1}(2x) \qquad [\text{Since, } 2x = \cos\theta]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = 0 + 3\left(\frac{-1}{\sqrt{1-(2x)^2}}\right)\frac{d}{dx}(2x)$$

$$= \frac{-3}{\sqrt{1-4x^2}}(2)$$

$$\frac{dy}{dx} = -\frac{6}{\sqrt{1-4x^2}}$$

*********** END **********