Continuity Ex 9.1 Q41

It is given that function is continuous at $x = 0$. then,

LHL = RHL = $f(0)$ .... (1)

Now,

$f(0) = 2 \cdot 0 + k = k$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} -2(-h)^2 + k = k$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} 2\left(h^2\right) + k = k$$

Thus, the function will be continuous for any $k \in R$.

Continuity Ex 9.1 Q42

The given function f is $f(x) = \begin{cases} \lambda\left(x^2 - 2x\right), & \text{if } x \le 0 \\ 4x + 1, & \text{if } x > 0 \end{cases}$

If f is continuous at $x = 0$, then

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$$
$$\Rightarrow \lim_{x \to 0^-} \lambda\left(x^2 - 2x\right) = \lim_{x \to 0^+} (4x + 1) = \lambda\left(0^2 - 2 \times 0\right)$$
$$\Rightarrow \lambda\left(0^2 - 2 \times 0\right) = 4 \times 0 + 1 = 0$$
$$\Rightarrow 0 = 1 = 0, \text{ which is not possible}$$

Therefore, there is no value of λ for which f is continuous at $x = 0$

At $x = 1$,

$f(1) = 4x + 1 = 4 \times 1 + 1 = 5$

$$\lim_{x \to 1} (4x + 1) = 4 \times 1 + 1 = 5$$
$$\therefore \lim_{x \to 1} f(x) = f(1)$$

Therefore, for any values of λ, f is continuous at $x = 1$

Continuity Ex 9.1 Q43

The function will be continuous at $x = 2$

if LHL = RHL = $f(2)$ .... (1)

Now,

$f(2) = k$

LHL = $\lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h) = \lim_{h \to 0} 2(2-h) + 1 = 5.$

Thus, using (1) we get,

$k = 5$

Continuity Ex 9.1 Q44

It is given that the function is continuous at $x = \frac{\pi}{2}$

$\therefore$ LHL = RHL = $f\left(\frac{\pi}{2}\right)$ .....(1)

Now,

$f\left(\frac{\pi}{2}\right) = a$

$$\text{LHL} = \lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{h \to 0} f\left(\frac{\pi}{2} - h\right) = \lim_{h \to 0} \frac{1 - \sin^3\left(\frac{\pi}{2} - h\right)}{3\cos^2\left(\frac{\pi}{2} - h\right)} = \lim_{h \to 0} \frac{1 - \cos^3 h}{3\sin^2 h}$$

$$= \lim_{h \to 0} \frac{(1 - \cos h)(1 + \cos^2 h + \cos h)}{3\sin^2 h}$$

$$= \lim_{h \to 0} \frac{2\sin^2 \frac{h}{2}(1 + \cos^2 h + \cos h)}{3\sin^2 h}$$

$$= \lim_{h \to 0} \frac{2\left(\frac{\sin \frac{h}{2}}{\frac{h}{2}}\right)^2 \times \frac{h^2}{4} \cdot (1 + \cos^2 h + \cos h)}{3\left(\frac{\sin h}{h}\right)^2 . h^2}$$

$$= \lim_{h \to 0} \frac{2 . \frac{1}{4}(1 + \cos^2 h + \cos h)}{3} = \frac{1}{2}$$

$$\text{RHL} = \lim_{x \to \frac{\pi}{2}^+} f(x) = \lim_{h \to 0} f\left(\frac{\pi}{2} + h\right) = \lim_{h \to 0} \frac{b\left(1 - \sin\left(\frac{\pi}{2} + h\right)\right)}{\left(\pi - 2\left(\frac{\pi}{2} + h\right)\right)^2} = \lim_{h \to 0} \frac{b(1 - \cos h)}{(\pi - \pi - 2h)^2}$$

$$= \lim_{h \to 0} \frac{b . 2\sin^2 \frac{h}{2}}{(2h)^2}$$

$$= \lim_{h \to 0} \frac{b}{2}\left(\frac{\sin \frac{h}{2}}{\frac{h}{2}}\right)^2 \times \frac{1}{4}$$

$$= \lim_{h \to 0} \frac{b}{8} = \frac{b}{8}$$

Thus, using (1) we get,

$a = \frac{1}{2}$

And

$\frac{b}{8} = \frac{1}{2} \Rightarrow b = 4$

Thus, $a = \frac{1}{2}$ and $b = 4$

*********** END **********