Tangents and Normals Ex 16.2 Q5(iv)

We know that the equation of tangent and normal to any curve at the point $(x_1, y_1)$ is

$$y - y_1 = m(x - x_1) \qquad \text{---(A) Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{---(B) Normal}$$

Where $m$ is slope.

$x = a\sec t, \quad y = b\tan t, \quad t = t$

$$\therefore \qquad \frac{dx}{dt} = a\sec t \times \tan t$$

and

$$\frac{dy}{dt} = b\sec^2 t$$

$$\therefore \qquad \text{Slope } m = \frac{dy}{dx} = \frac{b\sec^2 t}{a\sec t \times \tan t}$$

$$= \frac{b}{a}\operatorname{cosec} t$$

From (A)

Equatin of tangent

$$(y - b\tan t) = \frac{b}{a}\operatorname{cosec} t\,(x - a\sec t)$$

$$\Rightarrow \qquad bx\operatorname{cosec} t - ay = ab\operatorname{cosec} t \times \sec t - ab\tan t$$

$$= \frac{ab\left[1 - \sin^2 t\right]}{\sin t \times \cos t}$$

$$= \frac{ab\cos t}{\sin t}$$

$$\Rightarrow \qquad bx\sec t - ay\tan t = ab$$

From (B)

Equation of normal is

$$(y - b\tan t) = \frac{-a\sin t}{b}(x - a\sec t)$$

$$\Rightarrow \qquad ax\sin t + by = a^2\tan t + b^2\tan t$$

$$\Rightarrow \qquad ax\cos t + by\cot t = a^2 + b^2$$

Tangents and Normals Ex 16.2 Q5(v)

We know that the equation of tangent and normal to any curve at the point $(x_1, y_1)$ is

$$y - y_1 = m(x - x_1) \qquad \text{---(A) Tangent}$$
$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{---(B) Normal}$$

Where $m$ is slope.

$$x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$$
$$\frac{dx}{d\theta} = a(1 + \cos\theta), \frac{dy}{d\theta} = a\sin\theta$$

$$\therefore \qquad \text{Slope } m = \frac{dy}{dx} = \frac{\sin\theta}{1 + \cos\theta} = \frac{\frac{2\sin\theta}{2} \times \frac{\cos\theta}{2}}{\frac{2\cos^2\theta}{2}}$$
$$= \frac{\tan\theta}{2}$$

Now,

From (A)

Equation of tangent

$$y - a(1 - \cos\theta) = \frac{\tan\theta}{2}\left(x - a(\theta + \sin\theta)\right)$$
$$\Rightarrow \qquad \frac{x\tan\theta}{2} - y = a(\theta + \sin\theta)\frac{\tan\theta}{2} - a(1 - \cos\theta)$$

From (B)

Equation of normal is

$$y - a(1 - \cos\theta) = \frac{-\cot\theta}{2}\left(x - a(\theta + \sin\theta)\right)$$

$$\Rightarrow \qquad (y - 2a)\frac{\tan\theta}{2} + x - a\theta = 0$$

Tangents and Normals Ex 16.2 Q5(vi)

$x = 3\cos\theta - \cos^3\theta,\ y = 3\sin\theta - \sin^3\theta$

$\Rightarrow \frac{dx}{d\theta} = -3\sin\theta + 3\cos^2\theta\sin\theta$ and $\frac{dy}{d\theta} = 3\cos\theta - 3\sin^2\theta\cos\theta$

$\Rightarrow \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3\cos\theta - 3\sin^2\theta\cos\theta}{-3\sin\theta + 3\cos^2\theta\sin\theta} = \frac{\cos\theta(1 - \sin^2\theta)}{-\sin\theta(1 - \cos^2\theta)} = \frac{\cos^3\theta}{-\sin^3\theta} = -\tan^3\theta$

So equation of the tangent at $\theta$ is

$y - 3\sin\theta + \sin^3\theta = -\tan^3\theta(x - 3\cos\theta + \cos^3\theta)$

$\Rightarrow 4(y\cos^3\theta - x\sin^3\theta) = 3\sin 4\theta$

So equation of normal at $\theta$ is

$y - 3\sin\theta + \sin^3\theta = \frac{1}{\tan^3\theta}(x - 3\cos\theta + \cos^3\theta)$

$\Rightarrow y\cos^3\theta - x\cos^3\theta = 3\sin^4\theta - \sin^6\theta - 3\cos^4\theta + \cos^6\theta$

$\Rightarrow y\sin^3\theta - x\cos^3\theta = 3\sin^4\theta - \sin^6\theta - 3\cos^4\theta + \cos^6\theta$

Tangents and Normals Ex 16.2 Q6

The given equation of curve is

$$x^2 + 2y^2 - 4x - 6y + 8 = 0 \qquad \text{---(i) at } x = 2$$

Differentiating with respect to $x$, we get

$$2x + 4y\frac{dy}{dx} - 4 - 6\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx}[4y - 6] = 4 - 2x$$

$$\therefore \quad \frac{dy}{dx} = \frac{2 - x}{2y - 3}$$

Now,

From (i) at $x = 2$

$$4 + 2y^2 - 8 - 6y + 8 = 0$$

$$\Rightarrow \quad 2y^2 - 6y + 4 = 0$$

$$\Rightarrow \quad y^2 - 3y + 2 = 0$$

$$\Rightarrow \quad (y - 2)(y - 1) = 0$$

$$\Rightarrow \quad y = 2, 1$$

Thus,

$$\text{Slope } m_1 = \left(\frac{dy}{dx}\right)_{(2,2)} = 0$$

$$m_2 = \left(\frac{dy}{dx}\right)_{(2,1)} = 0$$

Thus, the equation of normal is

$$(y - y_1) = \frac{-1}{0}(x - 2)$$

$$\Rightarrow \quad x = 2$$

Tangents and Normals Ex 16.2 Q7

The equation of the given curve is $ay^2 = x^3$.

On differentiating with respect to $x$, we have:

$$2ay\frac{dy}{dx} = 3x^2$$
$$\Rightarrow \frac{dy}{dx} = \frac{3x^2}{2ay}$$

The slope of a tangent to the curve at $(x_0, y_0)$ is $\left.\frac{dy}{dx}\right]_{(x_0, y_0)}$.

$\Rightarrow$ The slope of the tangent to the given curve at $(am^2, am^3)$ is

$$\left.\frac{dy}{dx}\right]_{(am^2,\ am^3)} = \frac{3\left(am^2\right)^2}{2a\left(am^3\right)} = \frac{3a^2m^4}{2a^2m^3} = \frac{3m}{2}.$$

$\therefore$ Slope of normal at $(am^2, am^3)$

$$= \frac{-1}{\text{slope of the tangent at } \left(am^2,\ am^3\right)} = \frac{-2}{3m}$$

Hence, the equation of the normal at $(am^2, am^3)$ is given by,

$$y - am^3 = \frac{-2}{3m}\left(x - am^2\right)$$

$$\Rightarrow 3my - 3am^4 = -2x + 2am^2$$
$$\Rightarrow 2x + 3my - am^2\left(2 + 3m^2\right) = 0$$

*********** END **********