Differentiation Ex 11.2 Q39

Let $\quad y = \dfrac{2^x \cos x}{\left(x^2+3\right)^2}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{2^x \cos x}{\left(x^2+3\right)^2}\right]$$

$$= \left[\frac{\left(x^2+3\right)^2 \frac{d}{dx}\left(2^x \cos x\right) - \left(2^x \cos x\right)\frac{d}{dx}\left(x^2+3\right)^2}{\left[\left(x^2+3\right)^2\right]^2}\right]$$

[Using quotient rule, product rule and chain rule]

$$= \left[\frac{\left(x^2+3\right)^2\left[2^x \frac{d}{dx}\cos x + \cos x \frac{d}{dx}2^x\right] - \left(2^x \cos x\right)2\left(x^2+3\right)\frac{d}{dx}\left(x^2+3\right)}{\left(x^2+3\right)^4}\right]$$

$$= \left[\frac{\left(x^2+3\right)^2\left[-2^x \sin x + \cos x\, 2^x \log 2\right] - 2\left(2^x \cos x\right)\left(x^2+3\right)(2x)}{\left(x^2+3\right)^4}\right]$$

$$= \left[\frac{2^x\left(x^2+3\right)\left[\left(x^2+3\right)\{\cos x \log 2 - \sin x\}\right] - 4x \cos x}{\left(x^2+3\right)^4}\right]$$

$$= \frac{2^x}{\left(x^2 3\right)^2}\left[\cos x \log 2 - \sin x - \frac{4x \cos x}{\left(x^2+3\right)}\right]$$

So,

$$\frac{d}{dx}\left(\frac{2^x \cos x}{\left(x^2+3\right)^2}\right) = \frac{2^x}{\left(x^2+3\right)^2}\left[\cos x \log 2 - \sin x - \frac{4x \cos x}{\left(x^2+3\right)}\right].$$

Differentiation Ex 11.2 Q40

Let $\quad y = x \sin 2x + 5^x + k^k + \left(\tan^2 x\right)^3$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[x \sin 2x + 5^x + k^k + \left(\tan^6 x\right)\right]$$

$$= \frac{d}{dx}(x \sin 2x) + \frac{d}{dx}\left(5^x\right) + \frac{d}{dx}\left(k^k\right) + \frac{d}{dx}\left(\tan^6 x\right)$$

$$= \left[x \frac{d}{dx}(\sin 2x) + \sin 2x \frac{d}{dx}(x)\right] + 5^x \log 5 + 0 + 6\tan^5 x \frac{d}{dx}(\tan x)$$

[Using product rule and chain rule]

$$= \left[x \cos 2x \frac{d}{dx}(2x) + \sin 2x\right] + 5^x \log 5 + 6\tan^5 x \sec^2 x$$

$$= 2x \cos 2x + \sin 2x + 5^x \log 5 + 6\tan^5 x \sec^2 x$$

so,

$$\frac{d}{dx}\left(x \sin 2x + 5^x + k^k + \left(\tan^2 x\right)^3\right) = 2x \cos 2x + \sin 2x + 5^x \log 5 + 6\tan^5 x \sec^2 x.$$

Differentiation Ex 11.2 Q41

Let $\quad y = \log(3x+2) - x^2\log(2x-1)$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\log(3x+2) - x^2\log(2x-1)\right]$$
$$= \frac{d}{dx}\log(3x+2) - \frac{d}{dx}\left(x^2\log(2x-1)\right)$$
$$= \frac{1}{(3x+2)}\frac{d}{dx}(3x+2) - \left[x^2\frac{d}{dx}\log(2x-1) + \log(2x-1)\frac{d}{dx}\left(x^2\right)\right]$$

[Using product rule and chain rule]

$$= \frac{3}{3x+2} - \left[x^2 \times \frac{1}{(2x-1)}\frac{d}{dx}(2x-1) + \log(2x-1)\times 2x\right]$$
$$= \frac{3}{3x+2} - \frac{2x^2}{(2x-1)} - 2x\log(2x-1)$$

So,

$$\frac{d}{dx}\left(\log(3x+2) - x^2\log(2x-1)\right) = \frac{3}{3x+2} - \frac{2x^2}{(2x-1)} - 2x\log(2x-1).$$

Differentiation Ex 11.2 Q42

Let $\quad y = \frac{3x^2\sin x}{\sqrt{7-x^2}}$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{3x^2\sin x}{\left(7-x^2\right)^{\frac{1}{2}}}\right)$$
$$= \frac{\left(7-x^2\right)^{\frac{1}{2}} \times \frac{d}{dx}\left(3x^2\sin x\right) - 3x^2\sin x\frac{d}{dx}\left(7-x^2\right)^{\frac{1}{2}}}{\left[\left(7-x^2\right)^{\frac{1}{2}}\right]^2}$$

[Using quotient rule, chain and product rule]

$$= \left[\frac{\left(7-x^2\right)^{\frac{1}{2}} \times 3 \times \left[x^2\frac{d}{dx}\sin x + \sin x\frac{d}{dx}x^2\right] - 3x^2\sin x \times \frac{1}{2}\left(7-x^2\right)^{\frac{-1}{2}}\frac{d}{dx}\left(7-x^2\right)}{\left(7-x^2\right)}\right]$$
$$= \left[\frac{\left(7-x^2\right)^{\frac{1}{2}} 3\left(x^2\cos x + 2x\sin x\right) - 3x^2\sin x \times \frac{1}{2}\left(7-x^2\right)^{\frac{-1}{2}}(-2x)}{\left(7-x^2\right)}\right]$$
$$= \left[\frac{\left(7-x^2\right)^{\frac{1}{2}} \times 3\left(x^2\cos x + 2x\sin x\right)}{\left(7-x^2\right)} + \frac{3x^3\sin x\left(7-x^2\right)^{\frac{-1}{2}}}{\left(7-x^2\right)}\right]$$
$$= \left[\frac{6x\sin x + 3x^2\cos x}{\sqrt{\left(7-x^2\right)}} + \frac{3x^3\sin x}{\left(7-x^2\right)^{\frac{3}{2}}}\right]$$

So,

$$\frac{d}{dx}\left(\frac{3x^2\sin x}{\sqrt{7-x^2}}\right) = \left[\frac{6x\sin x + 3x^2\cos x}{\sqrt{7-x^2}} + \frac{3x^3\sin x}{\left(7-x^2\right)^{\frac{3}{2}}}\right].$$

Differentiation Ex 11.2 Q43

Let $\quad y = \sin^2\left[\log(2x+3)\right]$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left[\sin^2\left(\log(2x+3)\right)\right]$$

$$= 2\sin\left(\log(2x+3)\right)\frac{d}{dx}\sin\left(\log(2x+3)\right) \quad \text{Using chain rule}$$

$$= 2\sin\left(\log(2x+3)\right)\cos\left(\log(2x+3)\right)\frac{d}{dx}\log(2x+3)$$

$$= \sin\left(2\log(2x+3)\right)\times\frac{1}{(2x+3)}\frac{d}{dx}(2x+3)$$

$$\left[\text{Since, } 2\sin A\cos A = \sin^2 A\right]$$

$$= \sin\left(2\log(2x+3)\right)\times\frac{2}{(2x+3)}$$

So,

$$\frac{d}{dx}\left(\sin^2\log(2x+3)\right) = \sin\left(2\log(2x+3)\right)\times\frac{2}{(2x+3)}.$$

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