Real Numbers Ex 1.5 Q1

Answer :

Let us assume that $\sqrt{p}$ is rational .Then, there exist positive co primes a and b such that

$$\sqrt{p} = \frac{a}{b}$$

$$p = \left(\frac{a}{b}\right)^2$$

$$\Rightarrow \quad p = \frac{a^2}{b^2}$$

$$\Rightarrow \quad pb^2 = a^2$$

$$\Rightarrow \quad pb^2 = a^2$$

$$\Rightarrow \quad p|a^2$$

$$\Rightarrow \quad p|a$$

$\Rightarrow \quad a = pc$ for some positive integer c

$$\Rightarrow \quad b^2 p = a^2$$

$$\Rightarrow \quad b^2 p = p^2 c^2 \left(\because a = pc\right)$$

$$\Rightarrow \quad p|b^2 \left(\text{since } p|c^2 p\right)$$

$$\Rightarrow \quad p|b$$

$$\Rightarrow p|a \text{ and } p|b$$

This contradicts the fact that a and b are co primes

Hence $\sqrt{p}$ is irrational

Real Numbers Ex 1.5 Q2

Answer :

(i) Let us assume that $\frac{1}{\sqrt{2}}$ is rational .Then , there exist positive co primes a and b such that

$$\frac{1}{\sqrt{2}} = \frac{a}{b}$$

$$\frac{1}{\sqrt{2}} = \left(\frac{a}{b}\right)^2$$

$$\Rightarrow \frac{1}{2} = \frac{a^2}{b^2}$$

$$\Rightarrow \quad b^2 = 2a^2$$

$$\Rightarrow \quad 2|b^2 \left(\because 2|2a^2\right)$$

$$\Rightarrow \quad 2|b$$

$\Rightarrow \quad b = 2c$ for some positive integer c

$$\Rightarrow \quad 2a^2 = b^2$$

$$\Rightarrow \quad 2a^2 = 4c^2 \left(\because a = pc\right)$$

$$\Rightarrow \quad a^2 = 2c^2$$

$$\Rightarrow \quad 2|a^2 \left(\because 2|2c^2\right)$$

$$\Rightarrow \quad 2|a$$

$$\Rightarrow 2|a \text{ and } 2|b$$

This contradicts the fact that a and b are co primes.

$\boxed{\text{Hence } \frac{1}{\sqrt{2}} \text{ is irrational}}$

(ii) Let us assume that $7\sqrt{5}$ is rational .Then , there exist positive co primes a and b such that

$$7\sqrt{5} = \frac{a}{b}$$

$$\sqrt{5} = \frac{a}{7b}$$

We know that $\sqrt{5}$ is an irrational number

Here we see that $\sqrt{5}$ is a rational number which is a contradiction

$\boxed{\text{Hence } 7\sqrt{5} \text{ is irrational}}$

(iii) Let us assume that $6+\sqrt{2}$ is rational. Then , there exist positive co primes a and b such that

$$6+\sqrt{2} = \frac{a}{b}$$

$$\sqrt{2} = \frac{a}{b} - 6$$

$$\sqrt{2} = \frac{a-6b}{b}$$

Here we see that $\sqrt{2}$ is a rational number which is a contradiction as we know that $\sqrt{2}$ is an irrational number

$\boxed{\text{Hence } 6+\sqrt{2} \text{ is irrational}}$

(iv) Let us assume that $3-\sqrt{5}$ is rational .Then, there exist positive co primes a and b such that

$$3-\sqrt{5} = \frac{a}{b}$$

$$\sqrt{5} = 3 - \frac{a}{b}$$

$$\sqrt{5} = \frac{3b-a}{b}$$

Here we see that $\sqrt{5}$ is a rational number which is a contradiction as we know that $\sqrt{5}$ is an irrational number

$\boxed{\text{Hence } 3-\sqrt{5} \text{ is irrational}}$

*********** END **********