Differentiation Ex 11.4 Q26

Here,

$$\sin(xy) + \frac{y}{x} = x^2 - y^2$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{d}{dx}(\sin xy) + \frac{d}{dx}\left(\frac{y}{x}\right) = \frac{d}{dx}\left(x^2\right) - \frac{d}{dx}\left(y^2\right)$$

$$\Rightarrow \quad \cos(xy)\frac{d}{dx}(xy) + \left[\frac{x\frac{dy}{dx} - y\frac{d}{dx}(x)}{x^2}\right] = 2x - 2y\frac{dy}{dx} \qquad \left[\begin{array}{l}\text{Using chain rule, quotient ruel,}\\ \text{product rule}\end{array}\right]$$

$$\Rightarrow \quad \cos(xy)\left[x\frac{dy}{dx} + y\frac{d}{dx}(x)\right] + \left[\frac{x\frac{dy}{dx} - y(1)}{x^2}\right] = 2x - 2y\frac{dy}{dx}$$

$$\Rightarrow \quad \cos(xy)\left[x\frac{dy}{dx} + y(1)\right] + \frac{1}{x^2}\left(x\frac{dy}{dx} - y\right) = 2x - 2y\frac{dy}{dx}$$

$$\Rightarrow \quad x\cos(xy)\frac{dy}{dx} + y\cos(xy) + \frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2} = 2x - 2y\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx}\left[x\cos(xy) + \frac{1}{x} + 2y\right] = \frac{y}{x^2} - y\cos(xy) + 2x$$

$$\Rightarrow \quad \frac{dy}{dx}\left[\frac{x^2\cos(xy) + 1 + 2xy}{x}\right] = \frac{1}{x^2}\left(y - x^2 y\cos xy + 2x^3\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2x^3 + y - x^2 y\cos(xy)}{x\left(x^2\cos xy + 1 + 2xy\right)}$$

Differentiation Ex 11.4 Q27

Here,

$$\sqrt{y+x}+\sqrt{y-x}=c$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{d}{dx}\left(\sqrt{y+x}\right)+\frac{d}{dx}\sqrt{y-x}=\frac{d}{dx}(c)$$

$$\Rightarrow \quad \frac{1}{2\sqrt{y+x}}\frac{d}{dx}(y+x)+\frac{1}{2\sqrt{y-x}}\frac{d}{dx}(y-x)=0$$

Using chain rule

$$\Rightarrow \quad \frac{1}{2\sqrt{y+x}}\left[\frac{dy}{dx}+1\right]+\frac{1}{2\sqrt{y-x}}\left[\frac{dy}{dx}-1\right]=0$$

$$\Rightarrow \quad \frac{dy}{dx}\left(\frac{1}{2\sqrt{y+x}}\right)+\frac{dy}{dx}\left(\frac{1}{2\sqrt{y-x}}\right)=\frac{1}{2\sqrt{y-x}}-\frac{1}{2\sqrt{y+x}}$$

$$\Rightarrow \quad \frac{dy}{dx}\times\frac{1}{2}\left[\frac{1}{\sqrt{y+x}}+\frac{1}{\sqrt{y-x}}\right]=\frac{1}{2}\left[\frac{\sqrt{y+x}-\sqrt{y-x}}{\sqrt{y-x}\sqrt{y+x}}\right]$$

$$\Rightarrow \quad \frac{dy}{dx}\left[\frac{\sqrt{y-x}+\sqrt{y+x}}{\sqrt{y+x}\sqrt{y-x}}\right]=\left[\frac{\sqrt{y+x}-\sqrt{y-x}}{\sqrt{y-x}\sqrt{y+x}}\right]$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{\sqrt{y+x}-\sqrt{y-x}}{\sqrt{y-x}+\sqrt{y+x}}\times\frac{\left(\sqrt{y+x}-\sqrt{y-x}\right)}{\left(\sqrt{y+x}-\sqrt{y-x}\right)}$$

[rationalizing the denominator]

$$\Rightarrow \quad \frac{dy}{dx}=\frac{(y+x)+(y-x)-2\sqrt{y+x}\sqrt{y-x}}{y+x-y+x}$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{2y-2\sqrt{y^2-x^2}}{2x}$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{2y}{2x}-\frac{2\sqrt{y^2-x^2}}{2x}$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{y}{x}-\sqrt{\frac{y^2-x^2}{x^2}}$$

$$\frac{dy}{dx}=\frac{y}{x}-\sqrt{\frac{y^2}{x^2}-1}$$

Differentiation Ex 11.4 Q28

Here,

$$\tan(x+y)+\tan(x-y)=1$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{d}{dx}\tan(x+y)+\frac{d}{dx}\tan(x-y)=\frac{d}{dx}(1)$$

$$\Rightarrow \quad \sec^2(x+y)\frac{d}{dx}(x+y)+\sec^2(x-y)\frac{d}{dx}(x-y)=0 \qquad \text{[Using chain rule]}$$

$$\Rightarrow \quad \sec^2(x+y)\left[1+\frac{dy}{dx}\right]+\sec^2(x-y)\left[1-\frac{dy}{dx}\right]=0$$

$$\Rightarrow \quad \sec^2(x+y)\frac{dy}{dx}-\sec^2(x-y)\frac{dy}{dx}=-\left[\sec^2(x+y)+\sec^2(x-y)\right]$$

$$\Rightarrow \quad \frac{dy}{dx}\left[\sec^2(x+y)-\sec^2(x-y)\right]=-\left[\sec^2(x+y)+\sec^2(x-y)\right]$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{\sec^2(x+y)+\sec^2(x-y)}{\sec^2(x-y)-\sec^2(x+y)}$$

Differentiation Ex 11.4 Q29

Here,

$$e^x + e^y = e^{x+y}$$

Differentiating with respect to x using chain rule,

$$\Rightarrow \quad \frac{d}{dx}\left(e^x\right) + \frac{d}{dx}e^y = \frac{d}{dx}\left(e^{x+y}\right)$$

$$\Rightarrow \quad e^x + e^y\frac{dy}{dx} = e^{x+y}\frac{d}{dx}(x+y)$$

$$\Rightarrow \quad e^x + e^y\frac{dy}{dx} = e^{x+y}\left[1 + \frac{dy}{dx}\right]$$

$$\Rightarrow \quad e^y\frac{dy}{dx} - e^{x+y}\frac{dy}{dx} = e^{x+y} - e^x$$

$$\Rightarrow \quad \frac{dy}{dx}\left(e^y - e^{x+y}\right) = e^{x+y} - e^x$$

$$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{e^x \times e^y - e^x}{e^y - e^x \times e^y}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{e^x\left(e^y - 1\right)}{e^y\left(1 - e^x\right)}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{e^x\left(e^y - 1\right)}{e^y\left(e^x - 1\right)}$$

Differentiation Ex 11.4 Q30

It is given that, $\cos y = x\cos(a+y)$

$$\therefore \frac{d}{dx}[\cos y] = \frac{d}{dx}[x\cos(a+y)]$$

$$\Rightarrow -\sin y\frac{dy}{dx} = \cos(a+y)\cdot\frac{d}{dx}(x) + x\cdot\frac{d}{dx}[\cos(a+y)]$$

$$\Rightarrow -\sin y\frac{dy}{dx} = \cos(a+y) + x\cdot[-\sin(a+y)]\frac{dy}{dx}$$

$$\Rightarrow [x\sin(a+y) - \sin y]\frac{dy}{dx} = \cos(a+y) \qquad ...(1)$$

Since $\cos y = x\cos(a+y)$, $x = \dfrac{\cos y}{\cos(a+y)}$

Then, equation (1) reduces to

$$\left[\frac{\cos y}{\cos(a+y)}\cdot\sin(a+y) - \sin y\right]\frac{dy}{dx} = \cos(a+y)$$

$$\Rightarrow [\cos y\cdot\sin(a+y) - \sin y\cdot\cos(a+y)]\cdot\frac{dy}{dx} = \cos^2(a+y)$$

$$\Rightarrow \sin(a+y-y)\frac{dy}{dx} = \cos^2(a+b)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos^2(a+b)}{\sin a}$$

Hence, proved.

*********** END **********