Co-Ordinate Geometry Ex 14.5 Q3

Answer :

GIVEN: The four vertices of quadrilateral are (1, 2), (−5, 6), (7, −4) and D (k, −2) taken in order. If the area of the quadrilateral is zero

TO FIND: value of k

PROOF: Let four vertices of quadrilateral are A (1, 2) and B (−5, 6) and C (7, −4) and D (k, −2)

We know area of triangle formed by three points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) is given by

$$\Delta = \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

$$\Delta = \frac{1}{2} \left| (x_1 y_2 + x_2 y_3 + x_3 y_1) - (x_1 y_3 + x_2 y_1 + x_3 y_2) \right|$$

Now Area of ΔABC

Taking three points when A (1, 2) and B (−5, 6) and C (7, −4)

$$\text{Area}(\Delta ABC)$$

$$= \frac{1}{2} \left| \{6 + 20 + 14\} - \{-10 + 42 - 4\} \right|$$

$$= \frac{1}{2} \left| \{40\} - \{28\} \right|$$

$$= \frac{1}{2} \left| \{12\} \right|$$

$$\boxed{\text{Area}(\Delta ABC) = 6 \text{ sq. units}}$$

Also,

Now Area of ΔACD

Taking three points when A (1, 2) and C (7, −4) and D (k, −2)

$$\text{Area}(\Delta ACD)$$

$$= \frac{1}{2} \left| \{-4 - 14 + 2k\} - \{14 - 4k - 2\} \right|$$

$$= \frac{1}{2} \left| \{2k - 18\} - \{12 - 4k\} \right|$$

$$= \frac{1}{2} \left| \{6k - 30\} \right|$$

$$= \left| \{3k - 15\} \right|$$

Hence

$$\text{Area}(ABCD) = \text{Area}(\Delta ABC) + \text{Area}(\Delta ACD)$$

$$0 = 6 + 3k - 15 \text{ (substituting the values)}$$

$$\boxed{k = 3}$$

Co-Ordinate Geometry Ex 14.5 Q4

Answer :

GIVEN: The vertices of triangle ABC are A (−2, 1) and B (5, 4) and C (2, −3)

TO FIND: The area of triangle ABC and length if the altitude through A

PROOF: We know area of triangle formed by three points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) is given by

$$\Delta = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Now Area of ΔABC

Taking three points A (−2, 1) and B (5, 4) and C(2, −3)

$$\text{Area} (\Delta ABC) = \frac{1}{2} |\{-8 - 15 + 2\} - \{5 + 8 + 6\}|$$

$$= \frac{1}{2} |\{-21\} - \{19\}|$$

$$= \frac{1}{2} |\{-40\}|$$

$$= \frac{1}{2} (40)$$

$$= 20$$

We have

$$BC = \sqrt{(5-2)^2 + (4+3)^2}$$

$$BC = \sqrt{(3)^2 + (7)^2}$$

$$BC = \sqrt{9 + 49}$$

$$BC = \sqrt{58}$$

Now,

$$\text{Area} (\Delta ABC) = \frac{1}{2} \times BC \times \text{length of altitude through A}$$

$$20 = \frac{1}{2} \times \sqrt{58} \times \text{length of altitude through A}$$

$$\boxed{\text{length of altitude through A} = \frac{40}{\sqrt{58}}}$$

Co-Ordinate Geometry Ex 14.5 Q5

Answer :

The formula for the area 'A' encompassed by three points (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by the formula,

$$A = \frac{1}{2} \begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_2 - x_3 & y_2 - y_3 \end{vmatrix}$$

$$A = \frac{1}{2} \{(x_1 - x_2)(y_2 - y_3) - (x_2 - x_3)(y_1 - y_2)\}$$

If three points are collinear the area encompassed by them is equal to 0.

The three given points are A(2,5), B(4,6) and C(8,8). Substituting these values in the earlier mentioned formula we have,

$$A = \frac{1}{2} \begin{vmatrix} 2-4 & 5-6 \\ 4-8 & 6-8 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} -2 & -1 \\ -4 & -2 \end{vmatrix}$$

$$= \frac{1}{2} |(-2)(-2) - (-1)(-4)|$$

$$= \frac{1}{2} |4 - 4|$$

$$A = 0$$

Since the area enclosed by the three points is equal to 0, the three points need to be $\boxed{\text{collinear}}$.

The three given points are $A(1,-1)$, $B(2,1)$ and $C(4,5)$. Substituting these values in the earlier mentioned formula we have,

$$A = \frac{1}{2}\begin{vmatrix} 1-2 & -1-1 \\ 2-4 & 1-5 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} -1 & -2 \\ -2 & -4 \end{vmatrix}$$

$$= \frac{1}{2}\left|(-1)(-4)-(-2)(-2)\right|$$

$$= \frac{1}{2}\left|4-4\right|$$

$$A = 0$$

Since the area enclosed by the three points is equal to 0, the three points need to be $\boxed{\text{collinear}}$.

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