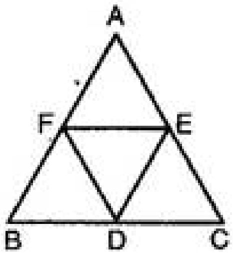
$\therefore$ DF $\parallel$ CA $\Rightarrow$ DE $\parallel$ AE ....(ii)



From (i) and (ii), we can say that AFDE is a parallelogram.

Similarly, BDEF is a parallelogram.

Now, in Δs DEF and ABC, we have

$\angle$ FDE = $\angle$A[opposite angles of $\parallel$gm AFDE]

And $\angle$DEF = $\angle$B[opposite angles of $\parallel$gm BDEF]

$\therefore$ By AA-criterion of similarity, we have ΔDEF ~ ΔABC

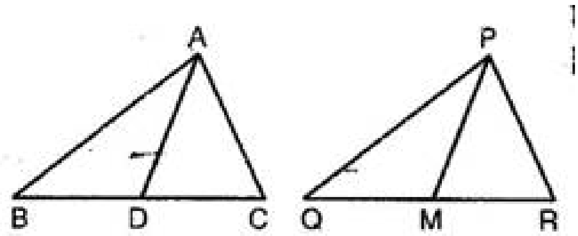
$$\Rightarrow \frac{\text{Area }(\Delta DEF)}{\text{Area }(\Delta ABC)} = \frac{DE^2}{AB^2} = \frac{\left(\frac{1}{2}AB\right)^2}{AB^2} \quad \frac{1}{4}$$

[$\because$ DE = $\frac{1}{2}$AB]

Hence, Area (ΔDEF): Area (ΔABC) = 1 : 4

6. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Ans. Given: $\Delta ABC \sim \Delta PQR$, AD and PM are the medians of Δs ABC and PQR respectively.



To Prove: $\frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta PQR)} = \frac{AD^2}{PM^2}$

Proof: Since $\Delta ABC \sim \Delta PQR$

$\therefore \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta PQR)} = \frac{AB^2}{PQ^2}$ ....(1)

But, $\frac{AB}{PQ} = \frac{AD}{PM}$ ....(2)

$\therefore$ From eq. (1) and (2), we have,

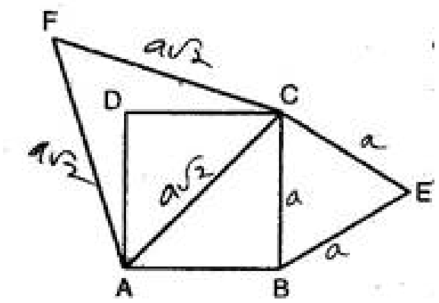
$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)} = \frac{AD^2}{PM^2}$$

7. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of the diagonals.

Tick the correct answer and justify:

Ans. Given: A square ABCD,

Equilateral Δs BCE and ACF have been drawn on side BC and the diagonal AC respectively.



To Prove: Area (Δ BCE) = $\frac{1}{2}$ Area (ΔACF)

Proof: $\Delta BCE \sim \Delta ACF$

[Being equilateral so similar by AAA criterion of similarity]

$$\Rightarrow \frac{\text{Area}(\Delta BCE)}{\text{Area}(\Delta ACF)} = \frac{BC^2}{AC^2}$$

$$\Rightarrow \frac{\text{Area }(\Delta BCE)}{\text{Area }(\Delta ACF)} = \frac{BC^2}{\left(\sqrt{2}BC\right)^2}$$

[$\because$ Diagonal = $\sqrt{2}$ side $\Rightarrow$ AC = $\sqrt{2}$ BC]

$$\Rightarrow \frac{\text{Area }(\Delta BCE)}{\text{Area }(\Delta ACF)} = \frac{1}{2}$$

8. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is:

(A) 2: 1

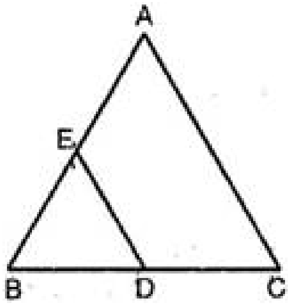
(B) 1: 2

(C) 4: 1

(D) 1: 4

Ans. (C) Since ΔABC and ΔBDE are equilateral, they are equiangular and hence,

$\Delta ABC \sim \Delta BDE$



$$\Rightarrow \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta BDE)} = \frac{BC^2}{BD^2}$$

$$\Rightarrow \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta BDE)} = \frac{(2BD)^2}{BD^2}$$

[$\because$ D is the mid-point of BC]

$$\Rightarrow \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta BDE)} = \frac{4}{1}$$

$\therefore$ (C) is the correct answer.

9. Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio:

(A) 2: 3

(B) 4: 9

(C) 81: 16

(D) 16: 81

Ans. (D) Since the ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides. Therefore,

$$\text{Ratio of areas} = \frac{(4)^2}{(9)^2} = \frac{16}{81}$$

$\therefore$ (D) is the correct answer.

*********** END **********