Differentiation Ex 11.5 Q58

Consider the given function, $(x-y)e^{\frac{x}{x-y}}=a.$

We need to prove that $y\frac{dy}{dx}+x=2y.$

Differentiating the given equation w.r.t. 'x' we get

$$(x-y)\left[e^{\frac{x}{x-y}}\left(\frac{(x-y)-x\left(1-\frac{dy}{dx}\right)}{(x-y)^2}\right)\right]+e^{\frac{x}{x-y}}\left(1-\frac{dy}{dx}\right)=0$$

$$\Rightarrow\frac{(x-y)-x\left(1-\frac{dy}{dx}\right)}{(x-y)}+\left(1-\frac{dy}{dx}\right)=0$$

$$\Rightarrow\left(1-\frac{dy}{dx}\right)\left(1-\frac{x}{x-y}\right)+1=0$$

$$\Rightarrow\left(1-\frac{dy}{dx}\right)\left(\frac{-y}{x-y}\right)+1=0$$

$$\Rightarrow -y+y\frac{dy}{dx}+x-y=0$$

$$\Rightarrow y\frac{dy}{dx}+x=2y$$

Differentiation Ex 11.5 Q59

$$x=e^{x/y}$$

$$\log x=\frac{x}{y}........(i)$$

$$y=\frac{x}{\log x}$$

$$\frac{dy}{dx}=\frac{\log x\frac{d}{dx}(x)-x\frac{d}{dx}(\log x)}{(\log x)^2}$$

$$\frac{dy}{dx}=\frac{\log x-x\,x\frac{1}{x}}{(\log x)^2}$$

$$\frac{dy}{dx}=\frac{\log x-1}{(\log x)^2}$$

$$\frac{dy}{dx}=\frac{\frac{x}{y}-1}{(\log x)^2}......[\text{from (i)}]$$

$$\frac{dy}{dx}=\frac{x-y}{y(\log x)^2}$$

$$\frac{dy}{dx}=\frac{x-y}{x(\log x)}......[\text{from (i)}]$$

Differentiation Ex 11.5 Q60

$$y = x^{\tan x} + \sqrt{\frac{x^2+1}{2}}$$

$$y = e^{\tan x \log x} + e^{\frac{1}{2}\log\left(\frac{x^2+1}{2}\right)}$$

$$\frac{dy}{dx} = e^{\tan x \log x}\frac{d}{dx}(\tan x \log x) + e^{\frac{1}{2}\log\left(\frac{x^2+1}{2}\right)}\frac{d}{dx}\left(\frac{1}{2}\log\left(\frac{x^2+1}{2}\right)\right)$$

$$\frac{dy}{dx} = x^{\tan x}\left[\frac{\tan x}{x} + \sec^2 x \log x\right] + \sqrt{\frac{x^2+1}{2}}\left(\frac{1}{2} \times \frac{2}{x^2+1} \times (x)\right)$$

$$\frac{dy}{dx} = x^{\tan x}\left[\frac{\tan x}{x} + \sec^2 x \log x\right] + \sqrt{\frac{x^2+1}{2}}\left(\frac{x}{x^2+1}\right)$$

$$\frac{dy}{dx} = x^{\tan x}\left[\frac{\tan x}{x} + \sec^2 x \log x\right] + \frac{x}{\sqrt{2(x^2+1)}}$$

Differentiation Ex 11.5 Q61

$$y = 1 + \frac{\alpha}{\left(\frac{1}{x}-\alpha\right)} + \frac{\beta/x}{\left(\frac{1}{x}-\alpha\right)\left(\frac{1}{x}-\beta\right)} + \frac{\gamma/x^2}{\left(\frac{1}{x}-\alpha\right)\left(\frac{1}{x}-\beta\right)\left(\frac{1}{x}-\gamma\right)}$$

$$\left\{\begin{array}{l} \text{Using the theorem,} \\ \text{If } y = 1 + \dfrac{ax^2}{(x-a)(x-b)(x-c)} + \dfrac{bx}{(x-b)(x-c)} + \dfrac{c}{(x-c)} \text{ then,} \\ \dfrac{dy}{dx} = \dfrac{y}{x}\left\{\dfrac{a}{a-x} + \dfrac{b}{b-x} + \dfrac{c}{c-x}\right\} \end{array}\right\}$$

Here we have $\frac{1}{x}$ instead of x.

So using above theorem we get,

$$\frac{dy}{dx} = \frac{\alpha}{\left(\frac{1}{x}-\alpha\right)} + \frac{\beta}{\left(\frac{1}{x}-\beta\right)} + \frac{\gamma}{\left(\frac{1}{x}-\gamma\right)}$$

*********** END **********