$$A(adjA) = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}\begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -12+12 & -6+6 \\ 24-24 & 12-12 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Also, $(adjA)A = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

$$= \begin{bmatrix} -12+12 & -18+18 \\ 8-8 & 12-12 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence, $A(adjA) = (adjA)A = |A|I$.

Question 4:

Verify $A(adj\,A) = (adj\,A)\,A = |A|I$.

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

Answer

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$|A| = 1(0-0)+1(9+2)+2(0-0) = 11$$

$$\therefore |A|I = 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

Now,

$$A_{11} = 0, A_{12} = -(9+2) = -11, A_{13} = 0$$

$$A_{21} = -(-3-0) = 3, A_{22} = 3-2 = 1, A_{23} = -(0+1) = -1$$

$$A_{31} = 2-0 = 2, A_{32} = -(-2-6) = 8, A_{33} = 0+3 = 3$$

$$\therefore adjA = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

Now,

$$A(adjA) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+11+0 & 3-1-2 & 2-8+6 \\ 0+0+0 & 9+0+2 & 6+0-6 \\ 0+0+0 & 3+0-3 & 2+0+9 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

Also,

$$(adjA)\cdot A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+9+2 & 0+0+0 & 0-6+6 \\ -11+3+8 & 11+0+0 & -22-2+24 \\ 0-3+3 & 0+0+0 & 0+2+9 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

Hence, $A(adjA) = (adjA)A = |A|I$.

Question 6:

Find the inverse of each of the matrices (if it exists).

$$\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$$

Answer

Let $A = \begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$.

we have,

$|A| = -2 + 15 = 13$

Now,

$A_{11} = 2, A_{12} = 3, A_{21} = -5, A_{22} = -1$

$\therefore adjA = \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|}adjA = \frac{1}{13}\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$

Question 7:

Find the inverse of each of the matrices (if it exists).

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

Answer

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$.

We have,

$|A| = 1(10-0) - 2(0-0) + 3(0-0) = 10$

Now,

$A_{11} = 10-0 = 10, A_{12} = -(0-0) = 0, A_{13} = 0-0 = 0$

$A_{21} = -(10-0) = -10, A_{22} = 5-0 = 5, A_{23} = -(0-0) = 0$

$A_{31} = 8-6 = 2, A_{32} = -(4-0) = -4, A_{33} = 2-0 = 2$

$\therefore adjA = \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|}adjA = \frac{1}{10}\begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$

Question 8:

Find the inverse of each of the matrices (if it exists).

$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$

Answer

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$.

We have,

$|A| = 1(-3-0) - 0 + 0 = -3$

Now,

$A_{11} = -3-0 = -3, A_{12} = -(-3-0) = 3, A_{13} = 6-15 = -9$

$A_{21} = -(0-0) = 0, A_{22} = -1-0 = -1, A_{23} = -(2-0) = -2$

$A_{31} = 0-0 = 0, A_{32} = -(0-0) = 0, A_{33} = 3-0 = 3$

$\therefore adjA = \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|}adjA = -\frac{1}{3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$

Question 9:

Find the inverse of each of the matrices (if it exists).

$\begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$

Answer

Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$.

We have,

$|A| = 2(-1-0) - 1(4-0) + 3(8-7)$

$= 2(-1) - 1(4) + 3(1)$

$= -2 - 4 + 3$

$= -3$

Now,

$A_{11} = -1-0 = -1, A_{12} = -(4-0) = -4, A_{13} = 8-7 = 1$

$A_{21} = -(1-6) = 5, A_{22} = 2+21 = 23, A_{23} = -(4+7) = -11$

$A_{31} = 0+3 = 3, A_{32} = -(0-12) = 12, A_{33} = -2-4 = -6$

$\begin{bmatrix} -1 & 5 & 3 \end{bmatrix}$

$$\therefore adj A = \begin{bmatrix} -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} adj A = -\frac{1}{3} \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$$

Question 10:

Find the inverse of each of the matrices (if it exists).

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}.$$

Answer

Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$.

By expanding along C_1, we have:

$|A| = 1(8-6) - 0 + 3(3-4) = 2 - 3 = -1$

Now,

$A_{11} = 8 - 6 = 2, A_{12} = -(0+9) = -9, A_{13} = 0 - 6 = -6$

$A_{21} = -(-4+4) = 0, A_{22} = 4 - 6 = -2, A_{23} = -(-2+3) = -1$

$A_{31} = 3 - 4 = -1, A_{32} = -(-3-0) = 3, A_{33} = 2 - 0 = 2$

********** END *********