On equating the co-efficient of $x^2$

$$a+7b+2c=-62$$

Substituting $a=5$ and $b=-9$, we get

$$5+7\times-9+2c=-62$$
$$5-63+2c=-62$$
$$2c=-62+63-5$$
$$2c=-4$$
$$c=\frac{-4}{2}$$
$$c=-2$$

On equating the co-efficient of $x$

$$b+7c+p=30$$

Substituting $b=-9$ and $c=-2$ ,we get

$$-9+7\times-2+p=30$$
$$-9-14+p=30$$
$$-23+p=30$$
$$p=30+23$$
$$p=53$$

On equating constant term, we get

$$c+q=-3$$

Substituting c=-2, we get

$$-2+q=-3$$
$$q=-3+2$$
$$q=-1$$

Therefore, quotient $q(x) = ax^2 + bx + c$

$= 5x^2 - 9x - 2$

Remainder $r(x) = px + q$

$= 53x - 1$

Hence, the quotient and remainder are $\boxed{q(x) = 5x^2 - 9x - 2}$ and $\boxed{r(x) = 53x - 1}$.

$(iii)$ we have

$f(x) = 4x^3 + 8x + 8x^2 + 7$

$g(x) = 2x^2 - x + 1$

Here, Degree $(f(x)) = 3$ and

Degree $(g(x)) = 2$

Therefore, quotient $q(x)$ is of degree $3 - 2 = 1$ and

Remainder $r(x)$ is of degree less than $2$

Let $q(x) = ax + b$ and

$r(x) = cx + d$

Using division algorithm, we have

$f(x) = g(x) \times q(x) + r(x)$

$4x^3 + 8x^2 + 8x + 7 = (2x^2 - x + 1)(ax + b) + cx + d$

$4x^3 + 8x^2 + 8x + 7 = 2ax^3 - ax^2 + ax + 2bx^2 - xb + b + cx + d$

$4x^3 + 8x^2 + 8x + 7 = 2ax^3 - ax^2 + 2bx^2 + ax - xb + cx + b + d$

$4x^3 + 8x^2 + 8x + 7 = 2ax^3 + x^2(-a + 2b) + x(a - b + c) + b + d$

Equating the co-efficient of various Powers of $x$ on both sides, we get

On equating the co-efficient of $x^3$

$2a = 4$

$a = \frac{4}{2}$

$a = 2$

On equating the co-efficient of $x^2$

$8 = -a + 2b$

Substituting $a = 2$ we get

$8 = -2 + 2b$

$8 + 2 = 2b$

$10 = 2b$

$\frac{10}{2} = b$

$5 = b$

On equating the co-efficient of $x$

$a - b + c = 8$

Substituting $a = 2$ and $b = 5$ we get

$2 - 5 + c = 8$

$-3 + c = 8$

$c = 8 + 3$

$c = 11$

*********** END **********