Exercise 5.6 : Solutions of Questions on Page Number : 181

Q1 : If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{dy}{dx}$.

$x = 2at^2,\ y = at^4$

Answer :

The given equations are $x = 2at^2$ and $y = at^4$

Then, $\frac{dx}{dt} = \frac{d}{dt}(2at^2) = 2a \cdot \frac{d}{dt}(t^2) = 2a \cdot 2t = 4at$

$\frac{dy}{dt} = \frac{d}{dt}(at^4) = a \cdot \frac{d}{dt}(t^4) = a \cdot 4 \cdot t^3 = 4at^3$

$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{4at^3}{4at} = t^2$

Answer needs Correction? **Click Here**

Q2 : If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{dy}{dx}$.

$x = a \cos \theta,\ y = b \cos \theta$

Answer :

The given equations are $x = a \cos \theta$ and $y = b \cos \theta$

Then, $\frac{dx}{d\theta} = \frac{d}{d\theta}(a \cos \theta) = a(-\sin \theta) = -a \sin \theta$

$\frac{dy}{d\theta} = \frac{d}{d\theta}(b \cos \theta) = b(-\sin \theta) = -b \sin \theta$

$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{-b \sin \theta}{-a \sin \theta} = \frac{b}{a}$

Answer needs Correction? **Click Here**

Q3 : If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{dy}{dx}$.

$x = \sin t,\ y = \cos 2t$

Answer :

The given equations are $x = \sin t$ and $y = \cos 2t$

Then, $\frac{dx}{dt} = \frac{d}{dt}(\sin t) = \cos t$

$\frac{dy}{dt} = \frac{d}{dt}(\cos 2t) = -\sin 2t \cdot \frac{d}{dt}(2t) = -2 \sin 2t$

$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{-2 \sin 2t}{\cos t} = \frac{-2 \cdot 2 \sin t \cos t}{\cos t} = -4 \sin t$

Answer needs Correction? **Click Here**

Q4 : If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{dy}{dx}$.

$x = 4t,\ y = \frac{4}{t}$

Answer :

The given equations are $x = 4t$ and $y = \frac{4}{t}$

The given equations are $x = 4t$ and $y = \frac{4}{t}$

$$\frac{dx}{dt} = \frac{d}{dt}(4t) = 4$$

$$\frac{dy}{dt} = \frac{d}{dt}\left(\frac{4}{t}\right) = 4 \cdot \frac{d}{dt}\left(\frac{1}{t}\right) = 4 \cdot \left(\frac{-1}{t^2}\right) = \frac{-4}{t^2}$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{\left(\frac{-4}{t^2}\right)}{4} = \frac{-1}{t^2}$$

Q5 : If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{dy}{dx}$.

$x = \cos\theta - \cos 2\theta,\; y = \sin\theta - \sin 2\theta$

Answer :

The given equations are $x = \cos\theta - \cos 2\theta$ and $y = \sin\theta - \sin 2\theta$

Then, $\dfrac{dx}{d\theta} = \dfrac{d}{d\theta}(\cos\theta - \cos 2\theta) = \dfrac{d}{d\theta}(\cos\theta) - \dfrac{d}{d\theta}(\cos 2\theta)$

$\qquad = -\sin\theta - (-2\sin 2\theta) = 2\sin 2\theta - \sin\theta$

$\dfrac{dy}{d\theta} = \dfrac{d}{d\theta}(\sin\theta - \sin 2\theta) = \dfrac{d}{d\theta}(\sin\theta) - \dfrac{d}{d\theta}(\sin 2\theta)$

$\qquad = \cos\theta - 2\cos 2\theta$

$\therefore \dfrac{dy}{dx} = \dfrac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \dfrac{\cos\theta - 2\cos 2\theta}{2\sin 2\theta - \sin\theta}$

Q6 : If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{dy}{dx}$.

$x = a(\theta - \sin\theta),\; y = a(1 + \cos\theta)$

Answer :

The given equations are $x = a(\theta - \sin\theta)$ and $y = a(1 + \cos\theta)$

Then, $\dfrac{dx}{d\theta} = a\left[\dfrac{d}{d\theta}(\theta) - \dfrac{d}{d\theta}(\sin\theta)\right] = a(1 - \cos\theta)$

$\dfrac{dy}{d\theta} = a\left[\dfrac{d}{d\theta}(1) + \dfrac{d}{d\theta}(\cos\theta)\right] = a[0 + (-\sin\theta)] = -a\sin\theta$

$\therefore \dfrac{dy}{dx} = \dfrac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \dfrac{-a\sin\theta}{a(1-\cos\theta)} = \dfrac{-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} = \dfrac{-\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} = -\cot\frac{\theta}{2}$

Q7 : If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{dy}{dx}$.

$x = \dfrac{\sin^3 t}{\sqrt{\cos 2t}},\; y = \dfrac{\cos^3 t}{\sqrt{\cos 2t}}$

Answer :

The given equations are $x = \dfrac{\sin^3 t}{\sqrt{\cos 2t}}$ and $y = \dfrac{\cos^3 t}{\sqrt{\cos 2t}}$

Then, $\dfrac{dx}{dt} = \dfrac{d}{dt}\left[\dfrac{\sin^3 t}{\sqrt{\cos 2t}}\right]$

$\quad = \dfrac{\sqrt{\cos 2t}\cdot\frac{d}{dt}(\sin^3 t) - \sin^3 t\cdot\frac{d}{dt}\sqrt{\cos 2t}}{\cos 2t}$

$\quad = \dfrac{\sqrt{\cos 2t}\cdot 3\sin^2 t\cdot\frac{d}{dt}(\sin t) - \sin^3 t\times\frac{1}{2\sqrt{\cos 2t}}\cdot\frac{d}{dt}(\cos 2t)}{\cos 2t}$

$\quad = \dfrac{3\sqrt{\cos 2t}\cdot\sin^2 t\cos t - \frac{\sin^3 t}{2\sqrt{\cos 2t}}\cdot(-2\sin 2t)}{\cos 2t}$

$\quad = \dfrac{3\cos 2t\sin^2 t\cos t + \sin^3 t\sin 2t}{\cos 2t\sqrt{\cos 2t}}$

$\dfrac{dy}{dt} = \dfrac{d}{dt}\left[\dfrac{\cos^3 t}{\sqrt{\cos 2t}}\right]$

$\quad = \dfrac{\sqrt{\cos 2t}\cdot\frac{d}{dt}(\cos^3 t) - \cos^3 t\cdot\frac{d}{dt}(\sqrt{\cos 2t})}{\cos 2t}$

$\quad = \dfrac{\sqrt{\cos 2t}\cdot 3\cos^2 t\cdot\frac{d}{dt}(\cos t) - \cos^3 t\cdot\frac{1}{\sqrt{}}\cdot\frac{d}{dt}(\cos 2t)}{\ }$

$$= \frac{\cdots \quad \frac{dt \, \smallsmile \quad }{2\sqrt{\cos 2t} \quad dt \, \smallsmile \quad }}{\cos 2t}$$

$$= \frac{3\sqrt{\cos 2t}.\cos^2 t\left(-\sin t\right) - \cos^3 t \cdot \dfrac{1}{2\sqrt{\cos 2t}} \cdot \left(-2\sin 2t\right)}{\cos 2t}$$

$$= \frac{-3\cos 2t.\cos^2 t.\sin t + \cos^3 t \sin 2t}{\cos 2t \cdot \sqrt{\cos 2t}}$$

$$\therefore \frac{dy}{dx} = \frac{\left(\dfrac{dy}{dt}\right)}{\left(\dfrac{dx}{dt}\right)} = \frac{-3\cos 2t.\cos^2 t.\sin t + \cos^3 t \sin 2t}{3\cos 2t \sin^2 t \cos t + \sin^3 t \sin 2t}$$

$$= \frac{-3\cos 2t.\cos^2 t.\sin t + \cos^3 t\left(2\sin t \cos t\right)}{3\cos 2t \sin^2 t \cos t + \sin^3 t\left(2\sin t \cos t\right)}$$

$$= \frac{\sin t \cos t\left[-3\cos 2t.\cos t + 2\cos^3 t\right]}{\sin t \cos t\left[3\cos 2t \sin t + 2\sin^3 t\right]}$$

$$= \frac{\left[-3\left(2\cos^2 t - 1\right)\cos t + 2\cos^3 t\right]}{\left[3\left(1 - 2\sin^2 t\right)\sin t + 2\sin^3 t\right]} \qquad \begin{bmatrix} \cos 2t = \left(2\cos^2 t - 1\right), \\ \cos 2t = \left(1 - 2\sin^2 t\right) \end{bmatrix}$$

$$= \frac{-4\cos^3 t + 3\cos t}{3\sin t - 4\sin^3 t}$$

$$= \frac{-\cos 3t}{\sin 3t} \qquad \begin{bmatrix} \cos 3t = 4\cos^3 t - 3\cos t, \\ \sin 3t = 3\sin t - 4\sin^3 t \end{bmatrix}$$

$$= -\cot 3t$$

Q8 : If x and y are connected parametrically by the equation, without eliminating the parameter, find $\dfrac{dy}{dx}$.

$$x = a\left(\cos t + \log\tan\frac{t}{2}\right), \; y = a\sin t$$

Answer :

The given equations are $x = a\left(\cos t + \log\tan\dfrac{t}{2}\right)$ and $y = a\sin t$

$$\text{Then,} \; \frac{dx}{dt} = a \cdot \left[\frac{d}{dt}(\cos t) + \frac{d}{dt}\left(\log\tan\frac{t}{2}\right)\right]$$

$$= a\left[-\sin t + \frac{1}{\tan\dfrac{t}{2}} \cdot \frac{d}{dt}\left(\tan\frac{t}{2}\right)\right]$$

$$= a\left[-\sin t + \cot\frac{t}{2} \cdot \sec^2\frac{t}{2} \cdot \frac{d}{dt}\left(\frac{t}{2}\right)\right]$$

$$= a\left[-\sin t + \frac{\cos\dfrac{t}{2}}{\sin\dfrac{t}{2}} \times \frac{1}{\cos^2\dfrac{t}{2}} \times \frac{1}{2}\right]$$

$$= a\left[-\sin t + \frac{1}{2\sin\dfrac{t}{2}\cos\dfrac{t}{2}}\right]$$

$$= a\left(-\sin t + \frac{1}{\sin t}\right)$$

$$= a\left(\frac{-\sin^2 t + 1}{\sin t}\right)$$

$$= a\frac{\cos^2 t}{\sin t}$$

$$\frac{dy}{dt} = a\frac{d}{dt}(\sin t) = a\cos t$$

$$\therefore \frac{dy}{dx} = \frac{\left(\dfrac{dy}{dt}\right)}{\left(\dfrac{dx}{dt}\right)} = \frac{a\cos t}{a\dfrac{\cos^2 t}{\sin t}} = \frac{\sin t}{\cos t} = \tan t$$

Q9 : If x and y are connected parametrically by the equation, without eliminating the parameter, find $\dfrac{dy}{dx}$.

$$x = a\sec\theta, \; y = b\tan\theta$$

Answer :

The given equations are $x = a\sec\theta$ and $y = b\tan\theta$

$$\text{Then,} \; \frac{dx}{d\theta} = a \cdot \frac{d}{d\theta}(\sec\theta) = a\sec\theta\tan\theta$$

$$\frac{dy}{d\theta} = b \cdot \frac{d}{d\theta}(\tan\theta) = b\sec^2\theta$$

$$\therefore \frac{dy}{dx} = \frac{\left(\dfrac{dy}{d\theta}\right)}{\left(\dfrac{dx}{d\theta}\right)} = \frac{b\sec^2\theta}{a\sec\theta\tan\theta} = \frac{b}{a}\sec\theta\cot\theta = \frac{b\cos\theta}{a\cos\theta\sin\theta} = \frac{b}{a} \times \frac{1}{\sin\theta} = \frac{b}{a}\csc\theta$$

Q10 : If x and y are connected parametrically by the equation, without eliminating the parameter, find $\dfrac{dy}{dx}$.

$x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$

Answer :

The given equations are $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$

Then, $\dfrac{dx}{d\theta} = a\left[\dfrac{d}{d\theta}\cos\theta + \dfrac{d}{d\theta}(\theta\sin\theta)\right] = a\left[-\sin\theta + \theta\dfrac{d}{d\theta}(\sin\theta) + \sin\theta\dfrac{d}{d\theta}(\theta)\right]$

$\qquad\qquad = a[-\sin\theta + \theta\cos\theta + \sin\theta] = a\theta\cos\theta$

$\dfrac{dy}{d\theta} = a\left[\dfrac{d}{d\theta}(\sin\theta) - \dfrac{d}{d\theta}(\theta\cos\theta)\right] = a\left[\cos\theta - \left\{\theta\dfrac{d}{d\theta}(\cos\theta) + \cos\theta\cdot\dfrac{d}{d\theta}(\theta)\right\}\right]$

$\qquad\qquad\qquad = a[\cos\theta + \theta\sin\theta - \cos\theta]$

$\qquad\qquad\qquad = a\theta\sin\theta$

$\therefore \dfrac{dy}{dx} = \dfrac{\left(\dfrac{dy}{d\theta}\right)}{\left(\dfrac{dx}{d\theta}\right)} = \dfrac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta$

Q11 : If $x = \sqrt{a^{\sin^{-1}t}}$, $y = \sqrt{a^{\cos^{-1}t}}$, show that $\dfrac{dy}{dx} = -\dfrac{y}{x}$

Answer :

The given equations are $x = \sqrt{a^{\sin^{-1}t}}$ and $y = \sqrt{a^{\cos^{-1}t}}$

$x = \sqrt{a^{\sin^{-1}t}}$ and $y = \sqrt{a^{\cos^{-1}t}}$

$\Rightarrow x = \left(a^{\sin^{-1}t}\right)^{\frac{1}{2}}$ and $y = \left(a^{\cos^{-1}t}\right)^{\frac{1}{2}}$

$\Rightarrow x = a^{\frac{1}{2}\sin^{-1}t}$ and $y = a^{\frac{1}{2}\cos^{-1}t}$

Consider $x = a^{\frac{1}{2}\sin^{-1}t}$

Taking logarithm on both the sides, we obtain

$\log x = \dfrac{1}{2}\sin^{-1}t \log a$

$\therefore \dfrac{1}{x}\cdot\dfrac{dx}{dt} = \dfrac{1}{2}\log a\cdot\dfrac{d}{dt}(\sin^{-1}t)$

$\Rightarrow \dfrac{dx}{dt} = \dfrac{x}{2}\log a\cdot\dfrac{1}{\sqrt{1-t^2}}$

$\Rightarrow \dfrac{dx}{dt} = \dfrac{x\log a}{2\sqrt{1-t^2}}$

Then, consider $y = a^{\frac{1}{2}\cos^{-1}t}$

Taking logarithm on both the sides, we obtain

$\log y = \dfrac{1}{2}\cos^{-1}t \log a$

$\therefore \dfrac{1}{y}\cdot\dfrac{dy}{dx} = \dfrac{1}{2}\log a\cdot\dfrac{d}{dt}(\cos^{-1}t)$

$\Rightarrow \dfrac{dy}{dt} = \dfrac{y\log a}{2}\cdot\left(\dfrac{-1}{\sqrt{1-t^2}}\right)$

$\Rightarrow \dfrac{dy}{dt} = \dfrac{-y\log a}{2\sqrt{1-t^2}}$

$\therefore \dfrac{dy}{dx} = \dfrac{\left(\dfrac{dy}{dt}\right)}{\left(\dfrac{dx}{dt}\right)} = \dfrac{\left(\dfrac{-y\log a}{2\sqrt{1-t^2}}\right)}{\left(\dfrac{x\log a}{2\sqrt{1-t^2}}\right)} = -\dfrac{y}{x}$

Hence, proved.

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