Exercise 7.5 : Solutions of Questions on Page Number : **322**

Q1 : $\dfrac{x}{(x+1)(x+2)}$

Answer :

Let $\dfrac{x}{(x+1)(x+2)} = \dfrac{A}{(x+1)} + \dfrac{B}{(x+2)}$

$\Rightarrow x = A(x+2) + B(x+1)$

Equating the coefficients of x and constant term, we obtain

$A + B = 1$

$2A + B = 0$

On solving, we obtain

$A = -1$ and $B = 2$

$$\therefore \frac{x}{(x+1)(x+2)} = \frac{-1}{(x+1)} + \frac{2}{(x+2)}$$

$$\Rightarrow \int \frac{x}{(x+1)(x+2)}dx = \int \frac{-1}{(x+1)} + \frac{2}{(x+2)}dx$$

$$= -\log|x+1| + 2\log|x+2| + C$$

$$= \log(x+2)^2 - \log|x+1| + C$$

$$= \log\frac{(x+2)^2}{(x+1)} + C$$

Answer needs Correction? **Click Here**

Q2 : $\dfrac{1}{x^2-9}$

Answer :

Let $\dfrac{1}{(x+3)(x-3)} = \dfrac{A}{(x+3)} + \dfrac{B}{(x-3)}$

$1 = A(x-3) + B(x+3)$

Equating the coefficients of x and constant term, we obtain

$A + B = 0$

$-3A + 3B = 1$

On solving, we obtain

$A = -\dfrac{1}{6}$ and $B = \dfrac{1}{6}$

$$\therefore \frac{1}{(x+3)(x-3)} = \frac{-1}{6(x+3)} + \frac{1}{6(x-3)}$$

$$\Rightarrow \int \frac{1}{(x^2-9)}dx = \int\left(\frac{-1}{6(x+3)} + \frac{1}{6(x-3)}\right)dx$$

$$= -\frac{1}{6}\log|x+3| + \frac{1}{6}\log|x-3| + C$$

$$= \frac{1}{6}\log\left|\frac{(x-3)}{(x+3)}\right| + C$$

Answer needs Correction? **Click Here**

Q3 : $\dfrac{3x-1}{(x-1)(x-2)(x-3)}$

Answer :

Let $\dfrac{3x-1}{(x-1)(x-2)(x-3)} = \dfrac{A}{(x-1)} + \dfrac{B}{(x-2)} + \dfrac{C}{(x-3)}$

$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2) \qquad \ldots(1)$

Substituting $x = 1$, 2, and 3 respectively in equation (1), we obtain

$A = 1$, $B = -5$, and $C = 4$

$$\therefore \frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{1}{(x-1)} - \frac{5}{(x-2)} + \frac{4}{(x-3)}$$

$$\Rightarrow \int \frac{\ldots}{(x-1)(x-2)(x-3)}dx = \int\left\{\frac{\ldots}{(x-1)} - \frac{\ldots}{(x-2)} + \frac{\ldots}{(x-3)}\right\}dx$$
$$= \log|x-1| - 5\log|x-2| + 4\log|x-3| + C$$

Answer needs Correction? **Click Here**

Q4 : $\frac{x}{(x-1)(x-2)(x-3)}$

Answer :

Let $\frac{x}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

$$x = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2) \qquad \ldots(1)$$

Substituting x = 1, 2, and 3 respectively in equation (1), we obtain $A = \frac{1}{2}$, $B = -2$, and $C = \frac{3}{2}$

$$\therefore \frac{x}{(x-1)(x-2)(x-3)} = \frac{1}{2(x-1)} - \frac{2}{(x-2)} + \frac{3}{2(x-3)}$$
$$\Rightarrow \int \frac{x}{(x-1)(x-2)(x-3)}dx = \int\left\{\frac{1}{2(x-1)} - \frac{2}{(x-2)} + \frac{3}{2(x-3)}\right\}dx$$
$$= \frac{1}{2}\log|x-1| - 2\log|x-2| + \frac{3}{2}\log|x-3| + C$$

Answer needs Correction? **Click Here**

Q5 : $\frac{2x}{x^2+3x+2}$

Answer :

Let $\frac{2x}{x^2+3x+2} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$

$$2x = A(x+2) + B(x+1) \qquad \ldots(1)$$

Substituting x = - 1 and - 2 in equation (1), we obtain

A = - 2 and B = 4

$$\therefore \frac{2x}{(x+1)(x+2)} = \frac{-2}{(x+1)} + \frac{4}{(x+2)}$$
$$\Rightarrow \int \frac{2x}{(x+1)(x+2)}dx = \int\left\{\frac{4}{(x+2)} - \frac{2}{(x+1)}\right\}dx$$
$$= 4\log|x+2| - 2\log|x+1| + C$$

Answer needs Correction? **Click Here**

Q6 : $\frac{1-x^2}{x(1-2x)}$

Answer :

It can be seen that the given integrand is not a proper fraction.

Therefore, on dividing $(1 - x^2)$ by $x(1 - 2x)$, we obtain

$$\frac{1-x^2}{x(1-2x)} = \frac{1}{2} + \frac{1}{2}\left(\frac{2-x}{x(1-2x)}\right)$$

Let $\frac{2-x}{x(1-2x)} = \frac{A}{x} + \frac{B}{(1-2x)}$

$$\Rightarrow (2-x) = A(1-2x) + Bx \qquad \ldots(1)$$

Substituting x = 0 and $\frac{1}{2}$ in equation (1), we obtain

A = 2 and B = 3

$$\therefore \frac{2-x}{x(1-2x)} = \frac{2}{x} + \frac{3}{1-2x}$$

Substituting in equation (1), we obtain

$$\frac{1-x^2}{x(1-2x)} = \frac{1}{2} + \frac{1}{2}\left\{\frac{2}{x} + \frac{3}{(1-2x)}\right\}$$
$$\Rightarrow \int \frac{1-x^2}{x(1-2x)}dx = \int\left\{\frac{1}{2} + \frac{1}{2}\left(\frac{2}{x} + \frac{3}{1-2x}\right)\right\}dx$$
$$= \frac{x}{2} + \log|x| + \frac{3}{2(-2)}\log|1-2x| + C$$
$$= \frac{x}{2} + \log|x| - \frac{3}{4}\log|1-2x| + C$$

Answer needs Correction? **Click Here**

Q7 : $\frac{x}{(x^2+1)(x-1)}$

Answer :

Let $\frac{x}{\ldots} = \frac{Ax+B}{\ldots} + \frac{C}{\ldots}$

Let $\frac{}{(x^2+1)(x-1)} = \frac{}{(x^2+1)} + \frac{}{(x-1)}$

$$x = (Ax+B)(x-1)+C(x^2+1)$$
$$x = Ax^2 - Ax + Bx - B + Cx^2 + C$$

Equating the coefficients of x^2, x, and constant term, we obtain

$A + C = 0$

$-A + B = 1$

$-B + C = 0$

On solving these equations, we obtain

$A = -\frac{1}{2}, B = \frac{1}{2}, \text{ and } C = \frac{1}{2}$

From equation (1), we obtain

$$\therefore \frac{x}{(x^2+1)(x-1)} = \frac{\left(-\frac{1}{2}x + \frac{1}{2}\right)}{x^2+1} + \frac{\frac{1}{2}}{(x-1)}$$
$$\Rightarrow \int \frac{x}{(x^2+1)(x-1)} = -\frac{1}{2}\int \frac{x}{x^2+1}dx + \frac{1}{2}\int \frac{1}{x^2+1}dx + \frac{1}{2}\int \frac{1}{x-1}dx$$
$$= -\frac{1}{4}\int \frac{2x}{x^2+1}dx + \frac{1}{2}\tan^{-1}x + \frac{1}{2}\log|x-1| + C$$

Consider $\int \frac{2x}{x^2+1}dx$, let $(x^2+1) = t \Rightarrow 2x\,dx = dt$

$$\Rightarrow \int \frac{2x}{x^2+1}dx = \int \frac{dt}{t} = \log|t| = \log|x^2+1|$$

$$\therefore \int \frac{x}{(x^2+1)(x-1)} = -\frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1}x + \frac{1}{2}\log|x-1| + C$$
$$= \frac{1}{2}\log|x-1| - \frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1}x + C$$

Answer needs Correction? Click Here

Q8 : $\frac{x}{(x-1)^2(x+2)}$

Answer :

Let $\frac{x}{(x-1)^2(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$

$$x = A(x-1)(x+2) + B(x+2) + C(x-1)^2$$

Substituting $x = 1$, we obtain

$B = \frac{1}{3}$

Equating the coefficients of x^2 and constant term, we obtain

$A + C = 0$

$-2A + 2B + C = 0$

On solving, we obtain

$A = \frac{2}{9}$ and $C = \frac{-2}{9}$

$$\therefore \frac{x}{(x-1)^2(x+2)} = \frac{2}{9(x-1)} + \frac{1}{3(x-1)^2} - \frac{2}{9(x+2)}$$
$$\Rightarrow \int \frac{x}{(x-1)^2(x+2)}dx = \frac{2}{9}\int \frac{1}{(x-1)}dx + \frac{1}{3}\int \frac{1}{(x-1)^2}dx - \frac{2}{9}\int \frac{1}{(x+2)}dx$$
$$= \frac{2}{9}\log|x-1| + \frac{1}{3}\left(\frac{-1}{x-1}\right) - \frac{2}{9}\log|x+2| + C$$
$$= \frac{2}{9}\log\left|\frac{x-1}{x+2}\right| - \frac{1}{3(x-1)} + C$$

Answer needs Correction? Click Here

Q9 : $\frac{3x+5}{x^3-x^2-x+1}$

Answer :

$$\frac{3x+5}{x^3-x^2-x+1} = \frac{3x+5}{(x-1)^2(x+1)}$$

Let $\frac{3x+5}{(x-1)^2(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)}$

$$3x+5 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$$
$$3x+5 = A(x^2-1) + B(x+1) + C(x^2+1-2x) \quad \text{...(1)}$$

Substituting $x = 1$ in equation (1), we obtain

$B = 4$

Equating the coefficients of x^2 and x, we obtain

$A + C = 0$

$B - 2C = 3$

On solving, we obtain

On solving, we obtain

$A=-\frac{1}{2}$ and $C=\frac{1}{2}$

$$\therefore \frac{3x+5}{(x-1)^2(x+1)}=\frac{-1}{2(x-1)}+\frac{4}{(x-1)^2}+\frac{1}{2(x+1)}$$

$$\Rightarrow \int\frac{3x+5}{(x-1)^2(x+1)}dx=-\frac{1}{2}\int\frac{1}{x-1}dx+4\int\frac{1}{(x-1)^2}dx+\frac{1}{2}\int\frac{1}{(x+1)}dx$$

$$=-\frac{1}{2}\log|x-1|+4\left(\frac{-1}{x-1}\right)+\frac{1}{2}\log|x+1|+C$$

$$=\frac{1}{2}\log\left|\frac{x+1}{x-1}\right|-\frac{4}{(x-1)}+C$$

Answer needs Correction? **Click Here**

Q10 : $\frac{2x-3}{(x^2-1)(2x+3)}$

Answer :

$$\frac{2x-3}{(x^2-1)(2x+3)}=\frac{2x-3}{(x+1)(x-1)(2x+3)}$$

Let $\frac{2x-3}{(x+1)(x-1)(2x+3)}=\frac{A}{(x+1)}+\frac{B}{(x-1)}+\frac{C}{(2x+3)}$

$$\Rightarrow(2x-3)=A(x-1)(2x+3)+B(x+1)(2x+3)+C(x+1)(x-1)$$

$$\Rightarrow(2x-3)=A(2x^2+x-3)+B(2x^2+5x+3)+C(x^2-1)$$

$$\Rightarrow(2x-3)=(2A+2B+C)x^2+(A+5B)x+(-3A+3B-C)$$

Equating the coefficients of x^2 and x, we obtain

$B=-\frac{1}{10}$, $A=\frac{5}{2}$, and $C=-\frac{24}{5}$

$$\therefore \frac{2x-3}{(x+1)(x-1)(2x+3)}=\frac{5}{2(x+1)}-\frac{1}{10(x-1)}-\frac{24}{5(2x+3)}$$

$$\Rightarrow \int\frac{2x-3}{(x^2-1)(2x+3)}dx=\frac{5}{2}\int\frac{1}{(x+1)}dx-\frac{1}{10}\int\frac{1}{x-1}dx-\frac{24}{5}\int\frac{1}{(2x+3)}dx$$

$$=\frac{5}{2}\log|x+1|-\frac{1}{10}\log|x-1|-\frac{24}{5\times2}\log|2x+3|$$

$$=\frac{5}{2}\log|x+1|-\frac{1}{10}\log|x-1|-\frac{12}{5}\log|2x+3|+C$$

Answer needs Correction? **Click Here**

Q11 : $\frac{5x}{(x+1)(x^2-4)}$

Answer :

$$\frac{5x}{(x+1)(x^2-4)}=\frac{5x}{(x+1)(x+2)(x-2)}$$

Let $\frac{5x}{(x+1)(x+2)(x-2)}=\frac{A}{(x+1)}+\frac{B}{(x+2)}+\frac{C}{(x-2)}$

$$5x=A(x+2)(x-2)+B(x+1)(x-2)+C(x+1)(x+2) \quad ...(1)$$

Substituting x = - 1, - 2, and 2 respectively in equation (1), we obtain

$A=\frac{5}{3}$, $B=-\frac{5}{2}$, and $C=\frac{5}{6}$

$$\therefore \frac{5x}{(x+1)(x+2)(x-2)}=\frac{5}{3(x+1)}-\frac{5}{2(x+2)}+\frac{5}{6(x-2)}$$

$$\Rightarrow \int\frac{5x}{(x+1)(x^2-4)}dx=\frac{5}{3}\int\frac{1}{(x+1)}dx-\frac{5}{2}\int\frac{1}{(x+2)}dx+\frac{5}{6}\int\frac{1}{(x-2)}dx$$

$$=\frac{5}{3}\log|x+1|-\frac{5}{2}\log|x+2|+\frac{5}{6}\log|x-2|+C$$

Answer needs Correction? **Click Here**

Q12 : $\frac{x^3+x+1}{x^2-1}$

Answer :

It can be seen that the given integrand is not a proper fraction.

Therefore, on dividing (x^3+x+1) by x^2-1, we obtain

$$\frac{x^3+x+1}{x^2-1}=x+\frac{2x+1}{x^2-1}$$

Let $\frac{2x+1}{x^2-1}=\frac{A}{(x+1)}+\frac{B}{(x-1)}$

$$2x+1=A(x-1)+B(x+1) \quad ...(1)$$

Substituting x = 1 and - 1 in equation (1), we obtain

$A=\frac{1}{2}$ and $B=\frac{3}{2}$

$$\therefore \frac{x^3+x+1}{x^2-1}=x+\frac{1}{2(x+1)}+\frac{3}{2(x-1)}$$

$$\Rightarrow \int \frac{x^3+x+1}{x^2-1}dx=\int x\,dx+\frac{1}{2}\int \frac{1}{(x+1)}dx+\frac{3}{2}\int \frac{1}{(x-1)}dx$$

$$=\frac{x^2}{2}+\frac{1}{2}\log|x+1|+\frac{3}{2}\log|x-1|+C$$

Answer needs Correction? **Click Here**

Q13 : $\dfrac{2}{(1-x)(1+x^2)}$

Answer :

$$\text{Let } \frac{2}{(1-x)(1+x^2)}=\frac{A}{(1-x)}+\frac{Bx+C}{(1+x^2)}$$

$$2=A(1+x^2)+(Bx+C)(1-x)$$

$$2=A+Ax^2+Bx-Bx^2+C-Cx$$

Equating the coefficient of x^2, x, and constant term, we obtain

$A - B = 0$

$B - C = 0$

$A + C = 2$

On solving these equations, we obtain

$A = 1$, $B = 1$, and $C = 1$

$$\therefore \frac{2}{(1-x)(1+x^2)}=\frac{1}{1-x}+\frac{x+1}{1+x^2}$$

$$\Rightarrow \int \frac{2}{(1-x)(1+x^2)}dx=\int \frac{1}{1-x}dx+\int \frac{x}{1+x^2}dx+\int \frac{1}{1+x^2}dx$$

$$=-\int \frac{1}{x-1}dx+\frac{1}{2}\int \frac{2x}{1+x^2}dx+\int \frac{1}{1+x^2}dx$$

$$=-\log|x-1|+\frac{1}{2}\log|1+x^2|+\tan^{-1}x+C$$

Answer needs Correction? **Click Here**

Q14 : $\dfrac{3x-1}{(x+2)^2}$

Answer :

$$\text{Let } \frac{3x-1}{(x+2)^2}=\frac{A}{(x+2)}+\frac{B}{(x+2)^2}$$

$$\Rightarrow 3x-1=A(x+2)+B$$

Equating the coefficient of x and constant term, we obtain

$A = 3$

$2A + B = -1 \Rightarrow B = -7$

$$\therefore \frac{3x-1}{(x+2)^2}=\frac{3}{(x+2)}-\frac{7}{(x+2)^2}$$

$$\Rightarrow \int \frac{3x-1}{(x+2)^2}dx=3\int \frac{1}{(x+2)}dx-7\int \frac{x}{(x+2)^2}dx$$

$$=3\log|x+2|-7\left(\frac{-1}{(x+2)}\right)+C$$

$$=3\log|x+2|+\frac{7}{(x+2)}+C$$

Answer needs Correction? **Click Here**

Q15 : $\dfrac{1}{x^4-1}$

Answer :

$$\frac{1}{(x^4-1)}=\frac{1}{(x^2-1)(x^2+1)}=\frac{1}{(x+1)(x-1)(1+x^2)}$$

$$\text{Let } \frac{1}{(x+1)(x-1)(1+x^2)}=\frac{A}{(x+1)}+\frac{B}{(x-1)}+\frac{Cx+D}{(x^2+1)}$$

$$1=A(x-1)(x^2+1)+B(x+1)(x^2+1)+(Cx+D)(x^2-1)$$

$$1=A(x^3+x-x^2-1)+B(x^3+x+x^2+1)+Cx^3+Dx^2-Cx-D$$

$$1=(A+B+C)x^3+(-A+B+D)x^2+(A+B-C)x+(-A+B-D)$$

Equating the coefficient of x^3, x^2, x, and constant term, we obtain

$A+B+C=0$

$-A+B+D=0$

$A+B-C=0$

$-A+B-D=1$

On solving these equations, we obtain

$$A=-\frac{1}{4}, B=\frac{1}{4}, C=0, \text{ and } D=-\frac{1}{2}$$

$$\therefore \frac{1}{x^4-1}=\frac{-1}{4(x+1)}+\frac{1}{4(x-1)}-\frac{1}{2(x^2+1)}$$

$$\Rightarrow \int \frac{1}{x^4-1}dx=-\frac{1}{4}\log|x-1|+\frac{1}{4}\log|x-1|-\frac{1}{2}\tan^{-1}x+C$$

$$=\frac{1}{4}\log\left|\frac{x-1}{x+1}\right|-\frac{1}{2}\tan^{-1}x+C$$

Answer needs Correction? **Click Here**

Q16 : $\frac{1}{x(x^n+1)}$ [Hint: multiply numerator and denominator by x^{n-1} and put $x^n = t$]

Answer :

$$\frac{1}{x(x^n+1)}$$

Multiplying numerator and denominator by x^{n-1}, we obtain

$$\frac{1}{x(x^n+1)}=\frac{x^{n-1}}{x^{n-1}x(x^n+1)}=\frac{x^{n-1}}{x^n(x^n+1)}$$

Let $x^n=t \Rightarrow x^{n-1}dx=dt$

$$\therefore \int \frac{1}{x(x^n+1)}dx=\int \frac{x^{n-1}}{x^n(x^n+1)}dx=\frac{1}{n}\int \frac{1}{t(t+1)}dt$$

$$\text{Let } \frac{1}{t(t+1)}=\frac{A}{t}+\frac{B}{(t+1)}$$

$$1=A(1+t)+Bt \qquad \ldots(1)$$

Substituting $t = 0, -1$ in equation (1), we obtain

$A = 1$ and $B = -1$

$$\therefore \frac{1}{t(t+1)}=\frac{1}{t}-\frac{1}{(1+t)}$$

$$\Rightarrow \int \frac{1}{x(x^n+1)}dx=\frac{1}{n}\int\left\{\frac{1}{t}-\frac{1}{(t+1)}\right\}dx$$

$$=\frac{1}{n}\left[\log|t|-\log|t+1|\right]+C$$

$$=-\frac{1}{n}\left[\log|x^n|-\log|x^n+1|\right]+C$$

$$=\frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right|+C$$

Answer needs Correction? **Click Here**

Q17 : $\frac{\cos x}{(1-\sin x)(2-\sin x)}$ [Hint: Put $\sin x = t$]

Answer :

$$\frac{\cos x}{(1-\sin x)(2-\sin x)}$$

Let $\sin x=t \Rightarrow \cos x\,dx=dt$

$$\therefore \int \frac{\cos x}{(1-\sin x)(2-\sin x)}dx=\int \frac{dt}{(1-t)(2-t)}$$

$$\text{Let } \frac{1}{(1-t)(2-t)}=\frac{A}{(1-t)}+\frac{B}{(2-t)}$$

$$1=A(2-t)+B(1-t) \qquad \ldots(1)$$

Substituting $t = 2$ and then $t = 1$ in equation (1), we obtain

$A = 1$ and $B = -1$

$$\therefore \frac{1}{(1-t)(2-t)}=\frac{1}{(1-t)}-\frac{1}{(2-t)}$$

$$\Rightarrow \int \frac{\cos x}{(1-\sin x)(2-\sin x)}dx=\int\left\{\frac{1}{1-t}-\frac{1}{(2-t)}\right\}dt$$

$$=-\log|1-t|+\log|2-t|+C$$

$$=\log\left|\frac{2-t}{1-t}\right|+C$$

$$=\log\left|\frac{2-\sin x}{1-\sin x}\right|+C$$

Answer needs Correction? **Click Here**

Q18 : $\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$

Answer :

$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}=1-\frac{(4x^2+10)}{(x^2+3)(x^2+4)}$$

$$\text{Let } \frac{4x^2+10}{(x^2+3)(x^2+4)}=\frac{Ax+B}{(x^2+3)}+\frac{Cx+D}{(x^2+4)}$$

$$4x^2+10=(Ax+B)(x^2+4)+(Cx+D)(x^2+3)$$

$$4x^2+10 = Ax^3+4Ax+Bx^2+4B+Cx^3+3Cx+Dx^2+3D$$
$$4x^2+10 = (A+C)x^3+(B+D)x^2+(4A+3C)x+(4B+3D)$$

Equating the coefficients of x^3, x^2, x, and constant term, we obtain

$A + C = 0$

$B + D = 4$

$4A + 3C = 0$

$4B + 3D = 10$

On solving these equations, we obtain

$A = 0$, $B = -2$, $C = 0$, and $D = 6$

$$\therefore \frac{4x^2+10}{(x^2+3)(x^2+4)} = \frac{-2}{(x^2+3)} + \frac{6}{(x^2+4)}$$

$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = 1 - \left(\frac{-2}{(x^2+3)} + \frac{6}{(x^2+4)}\right)$$

$$\Rightarrow \int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}dx = \int\left\{1+\frac{2}{(x^2+3)} - \frac{6}{(x^2+4)}\right\}dx$$
$$= \int\left\{1+\frac{2}{x^2+(\sqrt{3})^2} - \frac{6}{x^2+2^2}\right\}$$
$$= x + 2\left(\frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}\right) - 6\left(\frac{1}{2}\tan^{-1}\frac{x}{2}\right) + C$$
$$= x + \frac{2}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} - 3\tan^{-1}\frac{x}{2} + C$$

Answer needs Correction? **Click Here**

Q19 : $\dfrac{2x}{(x^2+1)(x^2+3)}$

Answer :

$$\frac{2x}{(x^2+1)(x^2+3)}$$

Let $x^2 = t \Rightarrow 2x\,dx = dt$

$$\therefore \int \frac{2x}{(x^2+1)(x^2+3)}dx = \int \frac{dt}{(t+1)(t+3)} \qquad ...(1)$$

$$\text{Let } \frac{1}{(t+1)(t+3)} = \frac{A}{(t+1)} + \frac{B}{(t+3)}$$
$$1 = A(t+3) + B(t+1) \qquad ...(1)$$

Substituting $t = -3$ and $t = -1$ in equation (1), we obtain

$$A = \frac{1}{2} \text{ and } B = -\frac{1}{2}$$

$$\therefore \frac{1}{(t+1)(t+3)} = \frac{1}{2(t+1)} - \frac{1}{2(t+3)}$$

$$\Rightarrow \int \frac{2x}{(x^2+1)(x^2+3)}dx = \int\left\{\frac{1}{2(t+1)} - \frac{1}{2(t+3)}\right\}dt$$
$$= \frac{1}{2}\log|(t+1)| - \frac{1}{2}\log|t+3| + C$$
$$= \frac{1}{2}\log\left|\frac{t+1}{t+3}\right| + C$$
$$= \frac{1}{2}\log\left|\frac{x^2+1}{x^2+3}\right| + C$$

Answer needs Correction? **Click Here**

Q20 : $\dfrac{1}{x(x^4-1)}$

Answer :

$$\frac{1}{x(x^4-1)}$$

Multiplying numerator and denominator by x^3, we obtain

$$\frac{1}{x(x^4-1)} = \frac{x^3}{x^4(x^4-1)}$$
$$\therefore \int \frac{1}{x(x^4-1)}dx = \int \frac{x^3}{x^4(x^4-1)}dx$$

Let $x^4 = t \Rightarrow 4x^3dx = dt$

$$\therefore \int \frac{1}{x(x^4-1)}dx = \frac{1}{4}\int \frac{dt}{t(t-1)}$$

$$\text{Let } \frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{(t-1)}$$
$$1 = A(t-1) + Bt \qquad ...(1)$$

Substituting $t = 0$ and 1 in (1), we obtain

A = - 1 and B = 1

$$\Rightarrow \frac{1}{t(t+1)} = \frac{-1}{t} + \frac{1}{t-1}$$

$$\Rightarrow \int \frac{1}{x(x^4-1)}dx = \frac{1}{4}\int\left\{\frac{-1}{t} + \frac{1}{t-1}\right\}dt$$
$$= \frac{1}{4}\left[-\log|t| + \log|t-1|\right] + C$$
$$= \frac{1}{4}\log\left|\frac{t-1}{t}\right| + C$$
$$= \frac{1}{4}\log\left|\frac{x^4-1}{x^4}\right| + C$$

Answer needs Correction? **Click Here**

Q21 : $\frac{1}{(e^x-1)}$ [Hint: Put $e^x = t$]

Answer :

$$\frac{1}{(e^x-1)}$$

Let $e^x = t \Rightarrow e^x\, dx = dt$

$$\Rightarrow \int \frac{1}{e^x-1}dx = \int \frac{1}{t-1} \times \frac{dt}{t} = \int \frac{1}{t(t-1)}dt$$

$$\text{Let } \frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1}$$
$$1 = A(t-1) + Bt \qquad ...(1)$$

Substituting t = 1 and t = 0 in equation (1), we obtain

A = - 1 and B = 1

$$\therefore \frac{1}{t(t-1)} = \frac{-1}{t} + \frac{1}{t-1}$$

$$\Rightarrow \int \frac{1}{t(t-1)}dt = \log\left|\frac{t-1}{t}\right| + C$$
$$= \log\left|\frac{e^x-1}{e^x}\right| + C$$

Answer needs Correction? **Click Here**

Q22 : $\int \frac{x\,dx}{(x-1)(x-2)}$ equals

A. $\log\left|\frac{(x-1)^2}{x-2}\right| + C$

B. $\log\left|\frac{(x-2)^2}{x-1}\right| + C$

C. $\log\left|\left(\frac{x-1}{x-2}\right)^2\right| + C$

D. $\log|(x-1)(x-2)| + C$

Answer :

$$\text{Let } \frac{x}{(x-1)(x-2)} = \frac{A}{(x-1)} + \frac{B}{(x-2)}$$
$$x = A(x-2) + B(x-1) \qquad ...(1)$$

Substituting x = 1 and 2 in (1), we obtain

A = - 1 and B = 2

$$\therefore \frac{x}{(x-1)(x-2)} = -\frac{1}{(x-1)} + \frac{2}{(x-2)}$$

$$\Rightarrow \int \frac{x}{(x-1)(x-2)}dx = \int\left\{\frac{-1}{(x-1)} + \frac{2}{(x-2)}\right\}dx$$
$$= -\log|x-1| + 2\log|x-2| + C$$
$$= \log\left|\frac{(x-2)^2}{x-1}\right| + C$$

Hence, the correct answer is B.

Answer needs Correction? **Click Here**

Q23 : $\int \frac{dx}{x(x^2+1)}$ equals

A. $\log|x| - \frac{1}{2}\log(x^2+1) + C$

B. $\log|x| + \frac{1}{2}\log(x^2+1) + C$

C. $-\log|x| + \frac{1}{2}\log(x^2+1) + C$

D. $\frac{1}{2}\log|x|+\log(x^2+1)+C$

Answer :

Let $\frac{1}{x(x^2+1)}=\frac{A}{x}+\frac{Bx+C}{x^2+1}$

$1=A(x^2+1)+(Bx+C)x$

Equating the coefficients of x^2, x, and constant term, we obtain

$A + B = 0$

$C = 0$

$A = 1$

On solving these equations, we obtain

$A = 1$, $B = -1$, and $C = 0$

$$\therefore \frac{1}{x(x^2+1)}=\frac{1}{x}+\frac{-x}{x^2+1}$$

$$\Rightarrow \int\frac{1}{x(x^2+1)}dx=\int\left\{\frac{1}{x}-\frac{x}{x^2+1}\right\}dx$$

$$=\log|x|-\frac{1}{2}\log|x^2+1|+C$$

Hence, the correct answer is A.

*********** END **********