Solution of Simultaneous Linear Equations Ex 8.2 Q1

$2x - y + z = 0$
$3x + 2y - z = 0$
$x + 4y + 3z = 0$

The systm can be written as

$$\begin{bmatrix} 2 & -1 & 1 \\ 3 & 2 & -1 \\ 1 & 4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
$$A \quad X = 0$$

Now $|A| = 2(10) + 1(10) + 1(10)$
$= 40$
$\neq 0$

Since $|A| \neq 0$, hence $x = y = z = 0$ is the only solution of this homogeneous system.

Solution of Simultaneous Linear Equations Ex 8.2 Q2

$$2x - y + 2z = 0$$
$$5x + 3y - z = 0$$
$$x + 5y - 5z = 0$$

$$\begin{bmatrix} 2 & -1 & 2 \\ 5 & 3 & -1 \\ 1 & 5 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{or } A \quad x = 0$$

$$|A| = 2(-10) + 1(-24) + 2(22)$$
$$= -20 - 24 + 44$$
$$= 0$$

Hence, the system has infinite solutions.

Let $z = k$

$$2x - y = -2k$$
$$5x + 3y = k$$

$$\begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2k \\ k \end{bmatrix}$$

$$A \quad x = B$$

$|A| = 6 + 5 = 11 \neq 0$ so A^{-1} exist

$$\text{Now } adj\ A = \begin{bmatrix} 3 & -5 \\ 1 & 2 \end{bmatrix}' = \begin{bmatrix} 3 & 1 \\ -5 & 2 \end{bmatrix}$$

$$x = A^{-1}.B = \frac{1}{|A|}(adj\ A)B = \frac{1}{11}\begin{bmatrix} 3 & 1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} -2k \\ k \end{bmatrix} = \begin{bmatrix} \frac{-5k}{11} \\ \frac{12k}{11} \end{bmatrix}$$

Hence, $x = \frac{-5k}{11}$, $y = \frac{12k}{11}$, $z = k$

Solution of Simultaneous Linear Equations Ex 8.2 Q3

$$3x - y + 2z = 0$$
$$4x + 3y + 3z = 0$$
$$5x + 7y + 4z = 0$$

$$|A| = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 3 & 3 \\ 5 & 7 & 4 \end{bmatrix}$$

$$= B(-9) + 1(1) + 2(13) = -27 + 1 + 26 = -27 + 27$$
$$= 0$$

Hence, it has infinite solutions.

Let $z = k$

$$3x - y = -2k$$
$$4x + 3y = -3k$$

or $\begin{bmatrix} 3 & -1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2k \\ -3k \end{bmatrix}$

or $A \quad x = B$

$|A| = 9 + 4 = 13 \neq 0$ hence A^{-1} exists

$$adj\ A = \begin{bmatrix} 3 & -4 \\ 1 & 3 \end{bmatrix}' = \begin{bmatrix} 3 & 1 \\ -4 & 3 \end{bmatrix}$$

Now $x = A^{-1}B = \frac{1}{|A|}(adj\ A)B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{13}\begin{bmatrix} 3 & 1 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} -2k \\ -3k \end{bmatrix} = \frac{1}{13}\begin{bmatrix} -9k \\ -k \end{bmatrix}$$

Hence, $x = \frac{-9k}{13}$, $y = \frac{-k}{13}$, $z = k$

Solution of Simultaneous Linear Equations Ex 8.2 Q4

$$x + y - 6z = 0$$
$$x - y + 2z = 0$$
$$-3x + y + 2z = 0$$

$$\text{Hence, } |A| = \begin{bmatrix} 1 & 1 & -6 \\ 1 & -1 & 2 \\ -3 & 1 & 2 \end{bmatrix}$$

$$= 1(-4) - 1(8) - 6(-2)$$
$$= -4 - 8 + 12$$
$$= 0$$

Hence, the system has infinite solutions.

Let $z = k$

$$x + y = 6k$$
$$x - y = -2k$$

or $\begin{bmatrix} 1 & +1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6k \\ -2k \end{bmatrix}$

or $A \quad X = B$

$|A| = -1 - 1 = -2 \neq 0$ hence A^{-1} exists.

$$adj\ A = \begin{bmatrix} -1 & -1 \\ -1 & +1 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{|A|}(adj\ A)B = \frac{1}{-2}\begin{bmatrix} -1 & -1 \\ -1 & +1 \end{bmatrix}\begin{bmatrix} 6k \\ -2k \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} -6k + 2k \\ -6k - 2k \end{bmatrix} = \left(\frac{1}{-2}\right)\begin{bmatrix} -4k \\ -8k \end{bmatrix} = \frac{-1}{2}\begin{bmatrix} -4k \\ -8k \end{bmatrix} = \begin{bmatrix} 2k \\ 4k \end{bmatrix}$$

Hence, $x = 2k,\ y = 4k, z = k$

*********** END **********