Trigonometric Identities Ex 6.1 Q62

Answer :

We have to prove $(\sec A - \operatorname{cosec} A)(1+\tan A+\cot A) = \tan A \sec A - \cot A \operatorname{cosec} A$

We know that, $\sin^2 A + \cos^2 A = 1$

So,

$$
\begin{aligned}
(\sec A - \operatorname{cosec} A)(1+\tan A+\cot A) &= \left(\frac{1}{\cos A} - \frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}\right) \\
&= \left(\frac{\sin A - \cos A}{\sin A \cos A}\right)\left(\frac{\sin A \cos A + \sin^2 A + \cos^2 A}{\sin A \cos A}\right) \\
&= \left(\frac{\sin A - \cos A}{\sin A \cos A}\right)\left(\frac{\sin A \cos A + 1}{\sin A \cos A}\right) \\
&= \frac{(\sin A - \cos A)(\sin A \cos A + 1)}{\sin^2 A \cos^2 A} \\
&= \frac{\sin^2 A \cos A + \sin A - \cos^2 A \sin A - \cos A}{\sin^2 A \cos^2 A} \\
&= \frac{(\sin^2 A \cos A - \cos A) + (\sin A - \cos^2 A \sin A)}{\sin^2 A \cos^2 A} \\
&= \frac{\cos A(\sin^2 A - 1) + \sin A(1 - \cos^2 A)}{\sin^2 A \cos^2 A} \\
&= \frac{\cos A(-\cos^2 A) + \sin A(\sin^2 A)}{\sin^2 A \cos^2 A} \\
&= \frac{-\cos^3 A + \sin^3 A}{\sin^2 A \cos^2 A} \\
&= \frac{\sin^3 A - \cos^3 A}{\sin^2 A \cos^2 A} \\
&= \frac{\sin^3 A}{\sin^2 A \cos^2 A} - \frac{\cos^3 A}{\sin^2 A \cos^2 A} \\
&= \frac{\sin A}{\cos^2 A} - \frac{\cos A}{\sin^2 A} \\
&= \frac{\sin A}{\cos A}\frac{1}{\cos A} - \frac{\cos A}{\sin A}\frac{1}{\sin A} \\
&= \tan A \sec A - \cot A \operatorname{cosec} A
\end{aligned}
$$

Hence proved.

Trigonometric Identities Ex 6.1 Q63

Answer :

We have to prove $\frac{\cos A\operatorname{cosec}A-\sin A\sec A}{\cos A+\sin A}=\operatorname{cosec}A-\sec A$

So,

$$\begin{aligned}\frac{\cos A\operatorname{cosec}A-\sin A\sec A}{\cos A+\sin A}&=\frac{\cos A\frac{1}{\sin A}-\sin A\frac{1}{\cos A}}{\cos A+\sin A}\\&=\frac{\frac{\cos^2 A-\sin^2 A}{\sin A\cos A}}{\cos A+\sin A}\\&=\frac{\cos^2 A-\sin^2 A}{\sin A\cos A(\cos A+\sin A)}\\&=\frac{(\cos A-\sin A)(\cos A+\sin A)}{\sin A\cos A(\cos A+\sin A)}\\&=\frac{\cos A-\sin A}{\sin A\cos A}\\&=\frac{\cos A}{\sin A\cos A}-\frac{\sin A}{\sin A\cos A}\\&=\frac{1}{\sin A}-\frac{1}{\cos A}\\&=\operatorname{cosec}A-\sec A\end{aligned}$$

Hence proved.

********** END **********