Determinants Ex 6.1 Q3

Since $|AB| = |A| \times |B|$

Hence $|A|^2 = |A| \times |A| \qquad ---(1)$

Now let $A = \begin{vmatrix} 2 & 3 & 7 \\ 13 & 17 & 5 \\ 15 & 20 & 12 \end{vmatrix}$

Expanding along the first column, we get

$$|A| = 2\begin{vmatrix} 17 & 5 \\ 20 & 12 \end{vmatrix} - 3\begin{vmatrix} 13 & 5 \\ 15 & 12 \end{vmatrix} + 7\begin{vmatrix} 13 & 17 \\ 15 & 20 \end{vmatrix}$$

$$= 2(204 - 100) - 3(156 - 75) + 7(260 - 255)$$

$$= 2(104) - 3(81) + 7(5)$$

$$= 208 - 243 + 35$$

$$= 243 - 243$$

$$= 0$$

Hence from eq. (1)

$|A|^2 = |A| \times |A| = 0 \times 0 = 0$

Determinants Ex 6.1 Q4

Evaluating the given determinant

$\sin 10^\circ \times \cos 80^\circ + \cos 10^\circ \sin 80^\circ$

$= \sin(10^\circ + 80^\circ) \qquad [\because \sin A \cos B + \cos A \sin B = \sin(A + B)]$

$= \sin 90^\circ$

$= 1$

Hence proved

Determinants Ex 6.1 Q5

We will evaluate the given determinant
(i) Along the first row
(ii) Along the first column

(i) Along the first row

$$
\begin{aligned}
|A| &= 2\begin{vmatrix} 1 & -2 \\ 4 & 1 \end{vmatrix} - 3\begin{vmatrix} 7 & -2 \\ -3 & 1 \end{vmatrix} - 5\begin{vmatrix} 7 & 1 \\ -3 & 4 \end{vmatrix} \\
&= 2(1+8) - 3(7-6) - 5(28+3) \\
&= 2(9) - 3(1) - 5(31) \\
&= 18 - 3 - 155 = -140
\end{aligned}
$$

(ii) Along the first column

$$
\begin{aligned}
|A| &= 2\begin{vmatrix} 1 & -2 \\ 4 & 1 \end{vmatrix} - 7\begin{vmatrix} 3 & -5 \\ 4 & 1 \end{vmatrix} - 3\begin{vmatrix} 3 & -5 \\ 1 & -2 \end{vmatrix} \\
&= 2(1+8) - 7(3+20) - 3(-6+5) \\
&= 18 - 7(23) - 3(-1) \\
&= 18 - 161 + 3 \\
&= 21 - 161 \\
&= -140
\end{aligned}
$$

We can see, the answer is same with both the methods.

Determinants Ex 6.1 Q6

$$
\Delta = \begin{vmatrix} 0 & \sin\alpha & -\cos\alpha \\ -\sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{vmatrix}
$$

$$
\begin{aligned}
&= -\sin\alpha(-\sin\beta\cos\alpha) - \cos\alpha(\sin\alpha\sin\beta) \\
&= 0
\end{aligned}
$$

Determinants Ex 6.1 Q7

$$
\Delta = \begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}
$$

Expanding along C_3, we have:

$$
\begin{aligned}
\Delta &= -\sin\alpha\left(-\sin\alpha\sin^2\beta - \cos^2\beta\sin\alpha\right) + \cos\alpha\left(\cos\alpha\cos^2\beta + \cos\alpha\sin^2\beta\right) \\
&= \sin^2\alpha\left(\sin^2\beta + \cos^2\beta\right) + \cos^2\alpha\left(\cos^2\beta + \sin^2\beta\right) \\
&= \sin^2\alpha(1) + \cos^2\alpha(1) \\
&= 1
\end{aligned}
$$

*********** END **********