Trigonometric Ratios of multiple and Sub-multiple Angles Ex 9.1 Q 39

We have,

$$2\tan\frac{\alpha}{2} = \tan\frac{\beta}{2}$$

$$\Rightarrow \quad \frac{\tan\frac{\alpha}{2}}{\tan\frac{\beta}{2}} = \frac{1}{2}$$

Let $\tan\frac{\alpha}{2} = K$ and $\tan\frac{\beta}{2} = 2K$

Then,

$$\cos\alpha = \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \frac{1-K^2}{1+K^2} \quad \ldots\ldots\ldots\ldots\ldots (A)$$

Also,

$$\frac{3+5\cos\beta}{5+3\cos\beta} = \frac{3+5\left(\dfrac{1-\tan^2\frac{\beta}{2}}{1+\tan^2\frac{\beta}{2}}\right)}{5+3\left(\dfrac{1-\tan^2\frac{\beta}{2}}{1+\tan^2\frac{\beta}{2}}\right)}$$

$$= \frac{3+5\left(\dfrac{1-4K^2}{1+4K^2}\right)}{5+3\left(\dfrac{1-4K^2}{1+4K^2}\right)}$$

$$= \frac{8-8K^2}{8+8K^2} = \frac{1-K^2}{1+K^2} \quad \ldots\ldots\ldots\ldots\ldots (B)$$

form (A) & (B)

$$\cos\alpha = \frac{3+5\cos\beta}{5+3\cos\beta}$$

Trigonometric Ratios of multiple and Sub-multiple Angles Ex 9.1 Q40

We have,

$$\cos\theta = \frac{\cos\alpha + \cos\beta}{1+\cos\alpha.\cos\beta}$$

Now,

$$\cos\theta = \frac{1-\tan^2\theta/2}{1+\tan^2\theta/2}$$

$$\Rightarrow \quad \frac{1-\tan^2\theta/2}{1+\tan^2\theta/2} = \frac{\cos\alpha + \cos\beta}{1+\cos\alpha\cos\beta}$$

by componende and dividendo, we get

$$\frac{\left(1-\tan^2\theta/2\right)+\left(1+\tan^2\theta/2\right)}{\left(1-\tan^2\theta/2\right)-\left(1+\tan^2\theta/2\right)} = \frac{1+\cos\alpha\cos\beta+\cos\alpha+\cos\beta}{-\left(1+\cos\alpha\cos\beta-\cos\alpha-\cos\beta\right)}$$

$$\Rightarrow \quad \frac{2}{2\tan^2\theta/2} = \frac{\left(1+\cos\alpha\right)\left(1+\cos\beta\right)}{\left(1-\cos\alpha\right)\left(1-\cos\beta\right)}$$

$$\Rightarrow \quad \tan^2\theta/2 = \frac{\left(1-\cos\alpha\right)\left(1-\cos\beta\right)}{\left(1+\cos\alpha\right)\left(1+\cos\beta\right)}$$

$$= \frac{2\sin^2\alpha/2 . 2\sin^2\beta/2}{2\cos^2\alpha/2 . 2\cos^2\beta/2}$$

$$\Rightarrow \quad \tan\theta/2 = \pm\tan\alpha/2 . \tan\beta/2$$

Trigonometric Ratios of multiple and Sub-multiple Angles Ex 9.1 Q41

We have,

$$\sec\left(\theta+\alpha\right)+\sec\left(\theta-\alpha\right) = 2\sec\theta.$$

$$\Rightarrow \quad \frac{1}{\cos\theta.\cos\alpha - \sin\theta\sin\alpha} + \frac{1}{\cos\theta.\cos\alpha + \sin\theta\sin\alpha} = \frac{2}{\cos\theta}$$

$$\Rightarrow \quad \frac{2\cos\theta\cos\alpha}{\cos^2\theta\cos^2\alpha - \sin^2\theta\sin^2\alpha} = \frac{2}{\cos\theta}$$

$$\Rightarrow \quad \frac{\cos\theta\cos\alpha}{\cos^2\theta\cos^2\alpha - \left(1-\cos^2\theta\right)\sin^2\alpha} = \frac{1}{\cos\theta}$$

$$\Rightarrow \quad \cos^2\theta\cos\alpha = \cos^2\theta\left(\cos^2\alpha+\sin^2\alpha\right)-\sin^2\alpha$$

$$\Rightarrow \quad \cos^2\theta\left(1-\cos\alpha\right) = \sin^2\alpha$$

$$\Rightarrow \quad \cos^2\theta = \frac{\sin^2\alpha}{2\sin^2\alpha/2}$$

$$= \frac{4\sin^2\alpha/2 . \cos^2\alpha/2}{2\sin^2\alpha/2}$$

$$\Rightarrow \quad \cos\theta = \pm\sqrt{2}\ \cos\alpha/2$$

Trigonometric Ratios of multiple and Sub-multiple Angles Ex 9.1 Q42

We have,

$$\cos\alpha + \cos\beta = \frac{1}{3} \text{ and } \sin\alpha + \sin\beta = 1/4$$

Squaring and adding, we get

$$\left(\cos^2\alpha + \cos^2\beta + 2\cos\alpha\ \cos\beta\right) + \left(\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta\right) = \frac{1}{9} + \frac{1}{16}$$

$$\Rightarrow \quad 1 + 1 + 2\left(\cos\alpha\ \cos\beta + \sin\alpha\sin\beta\right) = \frac{25}{144}$$

$$\Rightarrow \quad 2\cos\left(\alpha - \beta\right) = \frac{25}{144} - 2 = \frac{-263}{144}$$

$$\Rightarrow \quad \cos\left(\alpha - \beta\right) = \frac{-263}{288}$$

Now,

$$\cos\left(\frac{\alpha - \beta}{2}\right) = \sqrt{\frac{1 + \cos\left(\alpha - \beta\right)}{2}}$$

$$= \sqrt{\frac{1 - \frac{263}{288}}{2}} = \sqrt{\frac{25}{576}}$$

$$= \pm\frac{5}{24}$$

$$\therefore \quad \cos\left(\frac{\alpha - \beta}{2}\right) = \pm\frac{5}{24}$$

*********** END **********