



Algebra of Matrices Ex 5.3 Q57 pending

$$A^{k+1} = \begin{bmatrix} a^{k+1} & \frac{b(a^{k+1}-1)}{a-1} \\ 0 & 1 \end{bmatrix}$$

So,

A^n is true for $n = k + 1$ whenever it is true $n = k$.

Hence, by principle of mathematical induction A^n is true for all positive integer n .

Algebra of Matrices Ex 5.3 Q58

Given,

$$A = \begin{bmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{bmatrix}$$

To show that,

$$A^n = \begin{bmatrix} \cos n\theta & i \sin n\theta \\ i \sin n\theta & \cos n\theta \end{bmatrix} \text{ for all } n \in \mathbb{N}.$$

Put $n = 1$

$$A^1 = \begin{bmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{bmatrix}$$

So,

A^n is true for $n = 1$

Let, A^n is true for $n = k$, so

$$A^k = \begin{bmatrix} \cos k\theta & i \sin k\theta \\ i \sin k\theta & \cos k\theta \end{bmatrix} \quad \text{---(i)}$$

Now, we have to show that,

$$A^{k+1} = \begin{bmatrix} \cos(k+1)\theta & i \sin(k+1)\theta \\ i \sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$$

Now, $A^{k+1} = A^k \times A$

$$\begin{aligned} &= \begin{bmatrix} \cos k\theta & i \sin k\theta \\ i \sin k\theta & \cos k\theta \end{bmatrix} \begin{bmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{bmatrix} \\ &= \begin{bmatrix} \cos k\theta \cos \theta + i^2 \sin k\theta \sin \theta & i^2 \cos k\theta \sin \theta + i \sin k\theta \cos \theta \\ i \sin k\theta \cos \theta + i \cos k\theta \sin \theta & i^2 \sin k\theta \sin \theta + \cos \theta \cos k\theta \end{bmatrix} \\ &= \begin{bmatrix} \cos k\theta \cos \theta - \sin k\theta \sin \theta & i(\cos k\theta \sin \theta + \sin k\theta \cos \theta) \\ i(\sin k\theta \cos \theta + \cos k\theta \sin \theta) & \cos k\theta \cos \theta - \sin k\theta \sin \theta \end{bmatrix} \\ &= \begin{bmatrix} \cos(k+1)\theta & i \sin(k+1)\theta \\ i \sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix} \end{aligned}$$

So, A^n is true for $n = k + 1$ whenever it is true for $n = k$.

Hence, By principle of mathematical induction A^n is true for all positive integer.

Algebra of Matrices Ex 5.3 Q59

Given,

$$A = \begin{bmatrix} \cos \alpha + \sin \alpha & \sqrt{2} \sin \alpha \\ -\sqrt{2} \sin \alpha & \cos \alpha - \sin \alpha \end{bmatrix}$$

To prove $P(n)$: $A^n = \begin{bmatrix} \cos n\alpha + \sin n\alpha & \sqrt{2} \sin n\alpha \\ -\sqrt{2} \sin n\alpha & \cos n\alpha - \sin n\alpha \end{bmatrix}$ we use mathematical induction.

Step 1: To show $P(1)$ is true.

A^n is true for $n = 1$

Step 2: Let, $P(k)$ be true, so

$$A^k = \begin{bmatrix} \cos k\alpha + \sin k\alpha & \sqrt{2} \sin k\alpha \\ -\sqrt{2} \sin k\alpha & \cos k\alpha - \sin k\alpha \end{bmatrix} \quad \text{---(i)}$$

Step 3: Let, $P(k)$ is true.

Now, we have to show that

$$A^{k+1} = \begin{bmatrix} \cos(k+1)\alpha + \sin(k+1)\alpha & \sqrt{2} \sin(k+1)\alpha \\ -\sqrt{2} \sin(k+1)\alpha & \cos(k+1)\alpha - \sin(k+1)\alpha \end{bmatrix}$$

Now,

$$\begin{aligned} A^{k+1} &= A^k \times A \\ &= \begin{bmatrix} \cos k\alpha + \sin k\alpha & \sqrt{2} \sin k\alpha \\ -\sqrt{2} \sin k\alpha & \cos k\alpha - \sin k\alpha \end{bmatrix} \begin{bmatrix} \cos \alpha + \sin \alpha & \sqrt{2} \sin \alpha \\ -\sqrt{2} \sin \alpha & \cos \alpha - \sin \alpha \end{bmatrix} \\ &= \begin{bmatrix} (\cos k\alpha + \sin k\alpha)(\cos \alpha + \sin \alpha) - 2 \sin \alpha \sin k\alpha & (\cos k\alpha + \sin k\alpha)\sqrt{2} \sin \alpha + \sqrt{2} \sin k\alpha (\cos \alpha - \sin \alpha) \\ (\cos \alpha + \sin \alpha)(-\sqrt{2} \sin k\alpha) - \sqrt{2} \sin \alpha (\cos k\alpha - \sin k\alpha) & -2 \sin k\alpha \sin \alpha + (\cos k\alpha - \sin k\alpha)(\cos \alpha - \sin \alpha) \end{bmatrix} \\ &= \begin{bmatrix} \cos k\alpha \cos \alpha + \sin k\alpha \cos \alpha + \cos k\alpha \sin \alpha + \sin \alpha \sin k\alpha - 2 \sin \alpha \sin k\alpha & \sqrt{2} \cos k\alpha \sin \alpha + \sqrt{2} \sin \alpha \sin k\alpha + \sqrt{2} \sin k\alpha \cos \alpha - \sqrt{2} \sin k\alpha \sin \alpha \\ -\sqrt{2} \cos \alpha \sin \alpha - \sqrt{2} \sin \alpha \sin k\alpha - \sqrt{2} \sin \alpha & -2 \sin k\alpha \sin \alpha + \cos k\alpha \cos \alpha - \cos \alpha \cos k\alpha + \sqrt{2} \sin \alpha \sin k\alpha \sin \alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos \alpha \cos k\alpha + \sin \alpha \sin k\alpha & \sqrt{2} (\sin k\alpha \cos \alpha + \cos k\alpha \sin \alpha) \\ \sin \alpha \cos k\alpha + \sin k\alpha \cos \alpha & \cos k\alpha \cos \alpha - \sin k\alpha \sin \alpha - (\sin k\alpha \cos \alpha + \sin \alpha \cos k\alpha) \end{bmatrix} \\ &= \begin{bmatrix} \cos(k+1)\alpha + \sin(k+1)\alpha & \sqrt{2} \sin(k+1)\alpha \\ -\sqrt{2} \sin(k+1)\alpha & \cos(k+1)\alpha - \sin(k+1)\alpha \end{bmatrix} \end{aligned}$$

So, $P(k+1)$ is true whenever $P(k)$ is true.

Hence, by principle of mathematical induction $P(n)$ is true for all positive integer.

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