Question 10. 21. A tank with a square base of area 1.0 m^2 is divided by a vertical partition in the middle. The bottom of the partition has a small-hinged door of area 20 cm^2. The tank is filled with water in one compartment, and an acid (of relative density 1.7) in the other, both to a height of 4.0 m. Compute the force necessary to keep the door close.

Answer:

Pressure difference across the door

$$= (4 \times 1700 \times 9.8 - 4 \times 1000 \times 9.8) \text{ Pa}$$

$$(6.664 \times 10^4 - 3.92 \times 10^4) \text{ Pa} = 2.774 \times 10^4 \text{ Pa}$$

$$\text{Force on the door} = \text{Pressure difference} \times \text{Area of door}$$

$$= 2.774 \times 10^4 \times 20 \times 10^{-4} \text{ N}$$

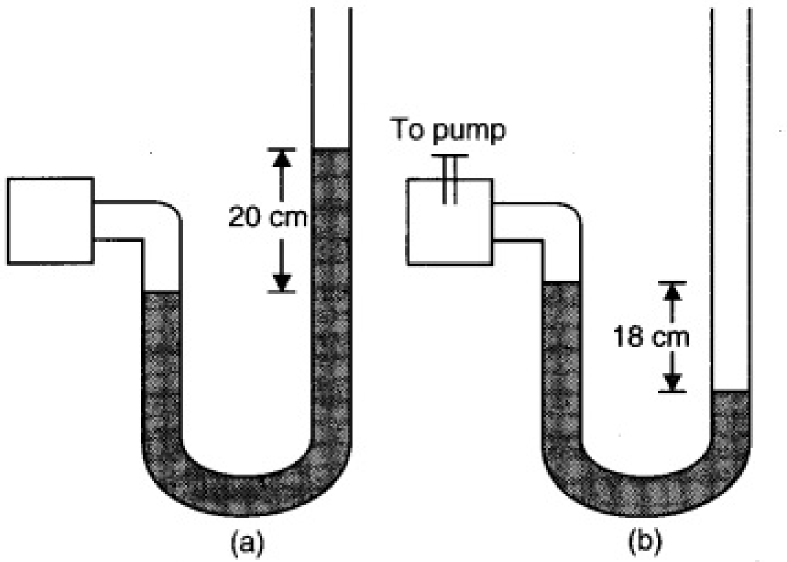
$$= 54.88 \text{ N} = \mathbf{55\ N.}$$

Note. Base area does not affect the answer.

Question 10. 22. A manometer reads the pressure of a gas in an enclosure as shown in Fig. (a) When a pump removes some of the gas, the manometer reads as in Fig. (b). The liquid used in the manometers is mercury and the atmospheric pressure is 76 cm of mercury.

(a) Give the absolute and gauge pressure of the gas in the enclosure for cases (a) and (b), in units of cm of mercury.

(b) How would the levels change in case (b) if 13.6 cm of water (immiscible with mercury) is poured into the right limb of 1 the manometer? Ignore the small change in the volume of the gas.



Answer:

The atmospheric pressure, $P = 76$ cm of mercury

(a) From figure (a),

Pressure head, $h = 20$ cm of mercury

$\therefore$ Absolute pressure $= p + h = 76 + 20 = 96$ cm of mercury

Also, Gauge pressure $= h = 20$ cm of mercury

From figure (b),

pressure head, $h = -18$ cm of mercury

$\therefore$ Absolute pressure $= p + h = 76 + (-18) = 58$ cm of mercury

Also, Gauge pressure $= h = -18$ cm of mercury

(b) When 13.6 cm of water is poured into the right limb of the manometer of figure (b), then, using the relation:

$$\text{Pressure} = \rho g h = \rho' g' h'$$

We get $h' = \dfrac{\rho h}{\rho'} = \dfrac{1 \times 13.6}{13.6} = 1$ cm of mercury [ρ' = density of mercury]

Therefore, pressure at the point B,

$$p_B = P + h' = 76 + 1 = 77 \text{ cm of mercury}$$

If h'' is the difference in the mercury levels in the two limbs, then taking $P_A = P_B$

$\Rightarrow$ $58 + h'' = 77 \Rightarrow h'' = 77 - 58 = 19$ cm of mercury.

Question 10. 23. Two vessels have the same base area but different shapes. The first vessel takes twice the volume of water that the second vessel requires to fill up to a particular common height. Is the force exerted by the water on the base of the vessel the same in the two cases? If so, why do the vessels filled with water to that same height give different readings on a weighing scale?
Answer: Pressure (and therefore force) on the two equal base areas are identical. But force is exerted by water on the sides of the vessels also, which has a non-zero vertical component when sides of the vessel are not perfectly normal to the base. This net vertical component of force by water on the sides of the vessel is greater for the first vessel than the second. Hence, the vessels weigh different even when the force on the base is the same in the two cases.

Question 10. 24. During blood transfusion, the needle is inserted in a vein where the gauge pressure is 2000 Pa. At what height must the blood container be placed so that blood may just enter the vein? Given: density of whole blood = 1.06×10^3 kg m^{-3}.
Answer: $h = P/\rho g = 200/(1.06 \times 10^3 \times 9.8) = 0.1925$ m
The blood may just enter the vein if the height at which the blood container be kept must be slightly greater than 0.1925 m i.e„ 0.2 m.

Question 10. 25. In deriving Bernoulli's equation, we equated the work done on the fluid in the tube to its change in the potential and kinetic energy, (a) What is the largest average velocity of blood flow in an artery of diameter 2×10^{-3} m if the flow must remain laminar? (b) Do the dissipative forces become more important as the fluid velocity increases? Discuss qualitatively.
Answer: (a) If dissipative forces are present, then some forces in liquid flow due to pressure difference is spent against dissipative forces, due to which the pressure drop becomes large.
(b) The dissipative forces become more important with increasing flow velocity, because of turbulence.

Question 10. 26. (a) What is the largest average velocity of blood flow in an artery of radius 2×10^3 m if the flow must remain laminar?
(b) What is the corresponding flow rate? Take viscosity of blood to be 2.084×10^{-3} Pa-s. Density of blood is 1.06×10^3 kg/m^3 .
Answer:

Here, $r = 2 \times 10^{-3}$ m; $D = 2r = 2 \times 2 \times 10^{-3} = 4 \times 10^{-3}$ m;

$\eta = 2.084 \times 10^{-3}$ Pa–s; $\rho = 1.06 \times 10^3$ kg m^{-3}.

For flow to be laminar, $N_R = 2000$

(a) Now, $v_c = \dfrac{N_R \eta}{\rho D} = \dfrac{2000 \times (2.084 \times 10^{-3})}{(1.06 \times 10^3) \times (4 \times 10^{-3})} = 0.98$ m/s.

(b) Volume flowing per second $= \pi r^2 v_c = \dfrac{22}{7} \times (2 \times 10^{-3})^2 \times 0.98 =$ **1.23×10^{-5} m^{3}s^{-1}**.

Question 10. 27. A plane is in level flight at constant speed and each of its wings has an area of 25 m^2. If the speed of the air is 180 km/h over the lower wing and 234 km/h over the upper wing surface, determine the plane's mass. (Take air density to be 1 kg/m^3), g = 9.8 m/s^2.

Answer:

Here speed of air over lower wing, $v_1 = 180$ km/h $= 180 \times \dfrac{5}{18} = 50$ ms^{-1}

Speed over the upper wing, $v_2 = 234$ km/h $= 234 \times \dfrac{5}{18} = 65$ ms^{-1}

$\therefore$ Pressure difference, $P_1 - P_2 = \dfrac{1}{2}\rho\,(v_2^2 - v_1^2) = \dfrac{1}{2} \times 1\,(65^2 - 50^2) = 862.5$ Pa

$\therefore$ Net upward force, $F = (P_1 - P_2)A$

This upward force balances the weight of the plane.

$$\therefore \quad mg = F = (P_1 - P_2)A \qquad [A = 25 \times 2 = 50 \text{ m}^2]$$

$$\therefore \quad m = \frac{(P_1 - P_2)\,A}{g} = \frac{862.5 \times 50}{9.8} = 4400 \text{ N.}$$

Question 10. 28. In Millikan's oil drop experiment, what is the terminal speed of an uncharged drop of radius 2.0 x 10^{-5} m and density 1.2 x 10^3 kg m^{-3}. Take the viscosity of air at the temperature of the experiment to be 1.8 x 10^{-5} Pa-s. How much is the viscous force on the drop at that speed? Neglect buoyancy of the drop due to air.

Answer: Here radius of drop, r = 2.0 x 10^{-5} m, density of drop, p = 1.2 x 10^3 kg/m^3, viscosity of air TI = 1.8 x 10^{-5} Pa-s.

Neglecting upward thrust due to air, we find that terminal speed is

$$v_T = \frac{2}{9}\frac{r^2 \rho g}{\eta} = \frac{2 \times (2.0 \times 10^{-5})^2 \times (1.2 \times 10^3) \times 9.8}{9 \times (1.8 \times 10^{-5})}$$

$$= 5.81 \times 10^{-2} \text{ ms}^{-1} \quad \text{or} \quad 5.81 \text{ cm s}^{-1}$$

Viscous force at this speed,

$$F = 6\pi\eta r v = 6 \times 3.14 \times (1.8 \times 10^{-5}) \times (2.0 \times 10^{-5}) \times (5.81 \times 10^{-2})$$

$$= 3.94 \times 10^{-10} \text{ N.}$$

Question 10. 29. Mercury has an angle of contact equal to 140° with soda-lime glass. A narrow tube of radius 1.0 mm made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside? Surface tension of mercury at the temperature of the experiment is 0.465 Nm^{-2}. Density of mercury = 13.6 x 10 kg m^{-3}

Answer:

Radius of tube, $r = 1.00 \text{ mm} = 10^{-3}$ m

Surface tension of mercury, $\sigma = 0.465 \text{ Nm}^{-1}$

Density of mercury, $\sigma = 13.6 \times 10^3 \text{ kg m}^{-3}$

Angle of contact, $\theta = 140°$

$$\therefore \quad h = \frac{2\sigma \cos\theta}{r\rho g} = \frac{2 \times 0.465 \times \cos 140°}{10^{-3} \times 13.6 \times 10^3 \times 9.8}$$

$$= \frac{2 \times 0.465 \times (-0.7660)}{10^{-3} \times 13.6 \times 10^3 \times 9.8}$$

$$= -5.34 \times 10^{-3} \text{ m} = \mathbf{-5.34 \text{ mm}}$$

Negative sign shows that the mercury level is depressed in the tube.

Question 10. 30. Two narrow bores of diameters 3.0 mm and 6.0 mm are joined together to form a U-tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water at the temperature of the experiment is 7.3×10^{-2} Nm^{-2}. Take the angle of contact to be zero and density of water to be 1.0×10^3 kg m^{-3}($g = 9.8$ ms^{-2}).

Answer: Let rx be the radius of one bore and r_2 be the radius of second bore of the U-tube. The, if h1 and h_2 are the heights of water on two sides, then

$$h_1 = \frac{2S\cos\theta}{r_1\rho g} \quad \text{and} \quad h_2 = \frac{2S\cos\theta}{r_2\rho g}$$

On subtraction, we get

$$h_1 - h_2 = \frac{2S\cos\theta}{r_1\rho g} - \frac{2S\cos\theta}{r_2\rho g} = \frac{2S\cos\theta}{\rho g}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Here, $S = 7.3 \times 10^{-2} \text{ Nm}^{-1}, \quad \theta = 0, \quad \rho = 1.0 \times 10^3 \text{ kg m}^{-3},$

$g = 9.8 \text{ ms}^{-2}, \quad r_1 = \frac{3}{2} \text{min} = 1.5 \times 10^{-3} \text{ m} \quad \text{and} \quad r_2 = \frac{6}{2} \text{mm}$

$= 3 \times 10^{-3}$ m

$$\therefore \quad h_1 - h_2 = \frac{2 \times 7.3 \times 10^{-2} \times \cos\theta}{1 \times 10^3 \times 9.8}\left[\frac{1}{1.5 \times 10^{-3}} - \frac{1}{3 \times 10^{-3}}\right]$$

$$= 1.49 \times 10^{-5} \times \frac{1}{3 \times 10^{-3}} \approx 4.97 \times 10^{-3} \text{ m} = 4.97 \text{ mm}$$

Question 10. 31. (a) It is known that density p of air decreases with height y as

$$\rho = \rho_0 e^{-y/y_0}$$

where $\rho_0 = 1.25$ kg m^{-3} is the density at sea level, and y_0 is a constant. This density variation is called the law of atmospheres. Obtain this law assuming that the temperature of atmosphere remains a constant (isothermal conditions). Also assume that the value of g remains constant.

(b) A large He balloon of volume 1425 m^3 is used to lift a payload of 400 kg. Assume that the balloon maintains constant radius as it rises. How high does it rise?[Take $y_0 = 8000$ m and $\rho_{He} = 0.18$ kg m^{-3}].

Answer: (a) We know that rate of decrease of density p of air is directly proportional to the height y. It is given as $d\rho/dy = -\rho/y_0$ where y is a constant of proportionality and -ve sign signifies that density is decreasing with increase in height. On integration, we get

$$\int_{\rho_0}^{\rho} \frac{d\rho}{\rho} = -\int_0^y \frac{1}{y_0} dy$$

$$\Rightarrow \quad [\log \rho]_{\rho_0}^{\rho} = -\left[\frac{y}{y_0}\right]_0^y, \quad \text{where, } \rho_0 = \text{density of air at sea level } i.e., \quad y = 0$$

$$\text{or} \quad \log_e \frac{\rho}{\rho_0} = -\frac{y}{y_0} \quad \text{or} \quad \rho = \rho_0 e^{-\frac{y}{y_0}}.$$

Here dimensions and units of constant y_0 are same as of y.

(*b*) Here volume of He balloon, $V = 1425 \text{ m}^3$, mass of payload, $m = 400$ kg

$y_0 = 8000$ m, density of He $\rho_{He} = 0.18 \text{ kgm}^{-3}$

$$\text{Mean density of balloon, } \rho = \frac{\text{Total mass of balloon}}{\text{Volume}} = \frac{m + V \cdot \rho_{He}}{V} \text{ Pa}$$

$$= \frac{400 + 1425 \times 0.18}{1425} = 0.4608 = 0.46 \text{ kgm}^{-3}$$

As density of air at sea level $\rho_0 = 1.25 \text{ kg m}^{-3}$. The balloon will rise up to a height y where density of air = density of balloon $\rho = 0.46 \text{ kgm}^{-3}$

$$\text{As} \quad \rho = \rho_0 e^{-\frac{y}{y_0}} \quad \text{or} \quad \frac{\rho_0}{\rho} = e^{\frac{y_0}{y}}$$

$$\therefore \quad \log_e\left(\frac{\rho_0}{\rho}\right) = \frac{y_0}{y} \quad \text{or} \quad y = \frac{y_0}{\log_e\left(\frac{\rho_0}{\rho}\right)} = \frac{8000}{\log_e\left(\frac{1.25}{0.46}\right)}$$

$$= 8002 \text{ m} \quad \text{or} \quad 8.0 \text{ km}.$$

*********** END **********