



Indefinite Integrals Ex 19.22 Q5

$$\text{Let } I = \int \frac{1}{1 + 3 \sin^2 x} dx$$

Dividing numerator and denominator by  $\cos^2 x$

$$\begin{aligned} I &= \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\cos^2 x} + \frac{3 \sin^2 x}{\cos^2 x}} dx \\ &= \int \frac{\sec^2 x}{\sec^2 x + 3 \tan^2 x} dx \\ &= \int \frac{\sec^2 x}{1 + \tan^2 x + 3 \tan^2 x} dx \\ &= \int \frac{\sec^2 x}{1 + 4 \tan^2 x} dx \\ &= \int \frac{\sec^2 x}{1 + (2 \tan x)^2} dx \end{aligned}$$

$$\begin{aligned} \text{Let } 2 \tan x &= t \\ 2 \sec^2 x dx &= dt \\ I &= \frac{1}{2} \int \frac{dt}{1 + t^2} \\ &= \frac{1}{2} \tan^{-1} t + c \end{aligned}$$

$$I = \frac{1}{2} \tan^{-1} (2 \tan x) + c$$

Indefinite Integrals Ex 19.22 Q6

Let  $I = \int \frac{1}{3 + 2 \cos^2 x} dx$

Dividing numerator and denominator by  $\cos^2 x$

$$I = \int \frac{\frac{1}{\cos^2 x}}{\frac{3}{\cos^2 x} + \frac{2 \cos^2 x}{\cos^2 x}} dx$$

$$= \int \frac{\sec^2 x}{3 \sec^2 x + 2} dx$$

$$= \int \frac{\sec^2 x}{3(1 + \tan^2 x) + 2} dx$$

$$= \int \frac{\sec^2 x}{3 + 3 \tan^2 x + 2} dx$$

$$= \int \frac{\sec^2 x}{5 + 3 \tan^2 x} dx$$

Let  $\sqrt{3} \tan x = t$

$$\sqrt{3} \sec^2 x dx = dt$$

$$I = \frac{1}{\sqrt{3}} \int \frac{dt}{(\sqrt{5})^2 + t^2}$$

$$= \frac{1}{\sqrt{3} \times \sqrt{5}} \tan^{-1} \left( \frac{t}{\sqrt{5}} \right) + c$$

$$I = \frac{1}{\sqrt{15}} \tan^{-1} \left( \frac{\sqrt{3} \tan x}{\sqrt{5}} \right) + c$$

Indefinite Integrals Ex 19.22 Q7

$$\begin{aligned}
 \text{Let } I &= \int \frac{1}{(\sin x - 2 \cos x)(2 \sin x + \cos x)} dx \\
 &= \int \frac{1}{2 \sin^2 x + \sin x \cos x - 4 \sin x \cos x - 2 \cos^2 x} dx \\
 &= \int \frac{1}{2 \sin^2 x - 3 \sin x \cos x - 2 \cos^2 x} dx
 \end{aligned}$$

Dividing numerator and denominator by  $\cos^2 x$

$$I = \int \frac{\sec^2 x}{2 \tan^2 x - 3 \tan x - 2} dx$$

$$\text{Let } \tan x = t$$

$$\sec^2 x dx = dt$$

$$I = \int \frac{dt}{2t^2 - 3t - 2}$$

$$= \frac{1}{2} \int \frac{dt}{t^2 - \frac{3}{2}t - 1}$$

$$= \frac{1}{2} \int \frac{dt}{t^2 - 2t\left(\frac{3}{4}\right) + \left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2 - 1}$$

$$I = \frac{1}{2} \int \frac{dt}{\left(t - \frac{3}{4}\right)^2 - \left(\frac{5}{4}\right)^2}$$

$$= \frac{1}{2} \times \frac{1}{2\left(\frac{5}{4}\right)} \log \left| \frac{t - \frac{3}{4} - \frac{5}{4}}{t - \frac{3}{4} + \frac{5}{4}} \right| + c$$

$$= \frac{1}{5} \log \left| \frac{t - 2}{2t + 1} \right| + c$$

$$I = \frac{1}{5} \log \left| \frac{\tan x - 2}{\tan x + 1} \right| + c$$

Indefinite Integrals Ex 19.22 Q8

$$\text{Let } I = \int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$$

Dividing numerator and denominator by  $\cos^4 x$

$$I = \int \frac{2 \tan x \sec^2 x}{\tan^4 x + 1} dx$$

$$\text{Let } \tan^2 x = t$$

$$2 \tan x \sec^2 x dx = dt$$

$$I = \int \frac{dt}{t^2 + 1}$$

$$= \tan^{-1} t + c$$

$$I = \tan^{-1}(\tan^2 x) + c$$

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