$$\lambda' = \frac{h}{\sqrt{2K'm_n}}$$

Where,

$m_n = 1.66 \times 10^{-27}$ kg
$h = 6.6 \times 10^{-34}$ Js
$K' = 6.75 \times 10^{-21}$ J

$$\therefore \lambda' = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 6.21 \times 10^{-21} \times 1.66 \times 10^{-27}}} = 1.46 \times 10^{-10} \text{ m} = 0.146 \text{ nm}$$

Therefore, the de Broglie wavelength of the neutron is 0.146 nm.

Question 11.18:

Show that the wavelength of electromagnetic radiation is equal to the de Broglie wavelength of its quantum (photon).

Answer

The momentum of a photon having energy ($h\nu$) is given as:

$$p = \frac{h\nu}{c} = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{p} \qquad \dots (i)$$

Where,

λ = Wavelength of the electromagnetic radiation

c = Speed of light

h = Planck's constant

De Broglie wavelength of the photon is given as:

$$\lambda = \frac{h}{mv}$$

But $p = mv$

$$\therefore \lambda = \frac{h}{p} \qquad \ldots (ii)$$

Where,

m = Mass of the photon

v = Velocity of the photon

Hence, it can be inferred from equations (*i*) and (*ii*) that the wavelength of the electromagnetic radiation is equal to the de Broglie wavelength of the photon.

Question 11.19:

What is the de Broglie wavelength of a nitrogen molecule in air at 300 K? Assume that the molecule is moving with the root-mean square speed of molecules at this temperature. (Atomic mass of nitrogen = 14.0076 u)

Answer

Temperature of the nitrogen molecule, T = 300 K

Atomic mass of nitrogen = 14.0076 u

Hence, mass of the nitrogen molecule, m = 2 × 14.0076 = 28.0152 u

But 1 u = 1.66×10^{-27} kg

$\therefore m = 28.0152 \times 1.66 \times 10^{-27}$ kg

Planck's constant, $h = 6.63 \times 10^{-34}$ Js

Boltzmann constant, $k = 1.38 \times 10^{-23}$ J K^{-1}

We have the expression that relates mean kinetic energy $\left(\frac{3}{2}kT\right)$ of the nitrogen molecule with the root mean square speed (v_{rms}) as:

$$\frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$$

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

Hence, the de Broglie wavelength of the nitrogen molecule is given as:

$$\lambda = \frac{h}{mv_{rms}} = \frac{h}{\sqrt{3mkT}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 28.0152 \times 1.66 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}}$$

$$= 0.028 \times 10^{-9} \text{ m}$$
$$= 0.028 \text{ nm}$$

Therefore, the de Broglie wavelength of the nitrogen molecule is 0.028 nm.

Question 11.20:

(a) Estimate the speed with which electrons emitted from a heated emitter of an evacuated tube impinge on the collector maintained at a potential difference of 500 V with respect to the emitter. Ignore the small initial speeds of the electrons. The *specific charge* of the electron, i.e., its e/m is given to be 1.76×10^{11} C kg^{-1}.

(b) Use the same formula you employ in (a) to obtain electron speed for an collector potential of 10 MV. Do you see what is wrong? In what way is the formula to be modified?

Answer

(a) Potential difference across the evacuated tube, V = 500 V

Specific charge of an electron, $e/m = 1.76 \times 10^{11}$ C kg^{-1}

The speed of each emitted electron is given by the relation for kinetic energy as:

$$KE = \frac{1}{2}mv^2 = eV$$

$$\therefore v = \left(\frac{2eV}{m}\right)^{\frac{1}{2}} = \left(2V \times \frac{e}{m}\right)^{\frac{1}{2}}$$

$$= \left(2 \times 500 \times 1.76 \times 10^{11}\right)^{\frac{1}{2}} = 1.327 \times 10^7 \text{ m/s}$$

Therefore, the speed of each emitted electron is 1.327×10^7 m/s.

(b) Potential of the anode, V = 10 MV = 10 × 10^6 V

The speed of each electron is given as:

$$v = \left(2V\frac{e}{m}\right)^{\frac{1}{2}}$$

$$= \left(2 \times 10^7 \times 1.76 \times 10^{11}\right)^{\frac{1}{2}}$$
$$= 1.88 \times 10^9 \text{ m/s}$$

This result is wrong because nothing can move faster than light. In the above formula, the expression ($mv^2/2$) for energy can only be used in the non-relativistic limit, i.e., for $v << c$.

For very high speed problems, relativistic equations must be considered for solving them. In the relativistic limit, the total energy is given as:

$E = mc^2$

Where,

m = Relativistic mass

$$= m_0\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}$$

m_0 = Mass of the particle at rest

Kinetic energy is given as:

$K = mc^2 - m_0c^2$

Question 11.21:

(a) A monoenergetic electron beam with electron speed of 5.20 × 10^6 m s^{-1} is subject to a magnetic field of 1.30 × 10^{-4} T normal to the beam velocity. What is the radius of the circle traced by the beam, given e/m for electron equals 1.76 × 10^{11} C kg^{-1}.

(b) Is the formula you employ in (a) valid for calculating radius of the path of a 20 MeV electron beam? If not, in what way is it modified?

[**Note:** Exercises 11.20(b) and 11.21(b) take you to relativistic mechanics which is beyond the scope of this book. They have been inserted here simply to emphasise the point that the formulas you use in part (a) of the exercises are not valid at very high speeds or energies. See answers at the end to know what 'very high speed or energy' means.]

Answer

(a) Speed of an electron, v = 5.20 × 10^6 m/s

Magnetic field experienced by the electron, B = 1.30 × 10^{-4} T

Specific charge of an electron, e/m = 1.76 × 10^{11} C kg^{-1}

Where,

e = Charge on the electron = 1.6 × 10^{-19} C

m = Mass of the electron = 9.1 × 10^{-31} kg^{-1}

The force exerted on the electron is given as:

$$F = e\left|\vec{v} \times \vec{B}\right|$$
$$= evB\sin\theta$$

θ = Angle between the magnetic field and the beam velocity

The magnetic field is normal to the direction of beam.

$\therefore \theta = 90°$

$$F = evB \qquad \ldots(1)$$

The beam traces a circular path of radius, r. It is the magnetic field, due to its bending nature, that provides the centripetal force $\left(F = \frac{mv^2}{r}\right)$ for the beam.

Hence, equation (1) reduces to:

$$evB = \frac{mv^2}{r}$$

$$\therefore r = \frac{mv}{eB} = \frac{v}{\left(\frac{e}{m}\right)B}$$

$$= \frac{5.20 \times 10^6}{\left(1.76 \times 10^{11}\right) \times 1.30 \times 10^{-4}} = 0.227 \text{ m} = 22.7 \text{ cm}$$

Therefore, the radius of the circular path is 22.7 cm.

(b) Energy of the electron beam, E = 20 MeV $= 20 \times 10^6 \times 1.6 \times 10^{-19}$ J

The energy of the electron is given as:

$$E = \frac{1}{2}mv^2$$

$$\therefore v = \left(\frac{2E}{m}\right)^{\frac{1}{2}}$$

$$= \sqrt{\frac{2 \times 20 \times 10^6 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} = 2.652 \times 10^9 \text{ m/s}$$

This result is incorrect because nothing can move faster than light. In the above formula, the expression ($mv^2/2$) for energy can only be used in the non-relativistic limit, i.e., for $v << c$

When very high speeds are concerned, the relativistic domain comes into consideration. In the relativistic domain, mass is given as:

$$m = m_0\left[1 - \frac{v^2}{c^2}\right]^{\frac{1}{2}}$$

Where,

m_0 = Mass of the particle at rest

Hence, the radius of the circular path is given as:

$$r = mv / eB$$

$$= \frac{m_0 v}{eB\sqrt{\frac{c^2 - v^2}{c^2}}}$$

Question 11.22:

An electron gun with its collector at a potential of 100 V fires out electrons in a spherical bulb containing hydrogen gas at low pressure (~10^{-2} mm of Hg). A magnetic field of 2.83×10^{-4} T curves the path of the electrons in a circular orbit of radius 12.0 cm. (The path can be viewed because the gas ions in the path focus the beam by attracting electrons, and emitting light by electron capture; this method is known as the 'fine beam tube' method. Determine e/m from the data.

Answer

Potential of an anode, V = 100 V

Magnetic field experienced by the electrons, $B = 2.83 \times 10^{-4}$ T

Radius of the circular orbit r = 12.0 cm = 12.0×10^{-2} m

Mass of each electron = m

Charge on each electron = e

Velocity of each electron = v

The energy of each electron is equal to its kinetic energy, i.e.,

$$\frac{1}{2}mv^2 = eV$$

$$v^2 = \frac{2eV}{m} \quad \text{... (1)}$$

It is the magnetic field, due to its bending nature, that provides the centripetal force $\left(F = \frac{mv^2}{r}\right)$ for the beam. Hence, we can write:

Centripetal force = Magnetic force

$$\frac{mv^2}{r} = evB$$

$$eB = \frac{mv}{r}$$

$$v = \frac{eBr}{m} \quad \text{... (2)}$$

Putting the value of v in equation (1), we get:

$$\frac{2eV}{m} = \frac{e^2B^2r^2}{m^2}$$

$$\frac{e}{m} = \frac{2V}{B^2r^2}$$

$$= \frac{2 \times 100}{\left(2.83 \times 10^{-4}\right)^2 \times \left(12 \times 10^{-2}\right)^2} = 1.73 \times 10^{11} \text{ C kg}^{-1}$$

Therefore, the specific charge ratio (e/m) is $1.73 \times 10^{11} \text{ C kg}^{-1}$.

Question 11.23:

(a) An X-ray tube produces a continuous spectrum of radiation with its short wavelengt end at 0.45 Å. What is the maximum energy of a photon in the radiation?

(b) From your answer to (a), guess what order of accelerating voltage (for electrons) is required in such a tube?

Answer

(a) Wavelength produced by an X-ray tube, $\lambda = 0.45 \text{ Å} = 0.45 \times 10^{-10} \text{ m}$

Planck's constant, $h = 6.626 \times 10^{-34}$ Js

Speed of light, $c = 3 \times 10^8$ m/s

The maximum energy of a photon is given as:

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{0.45 \times 10^{-10} \times 1.6 \times 10^{19}}$$

$$= 27.6 \times 10^3 \text{ eV} = 27.6 \text{ keV}$$

Therefore, the maximum energy of an X-ray photon is 27.6 keV.

(b) Accelerating voltage provides energy to the electrons for producing X-rays. To get an X-ray of 27.6 keV, the incident electrons must possess at least 27.6 keV of kinetic electric energy. Hence, an accelerating voltage of the order of 30 keV is required for producing X-rays.

Question 11.24:

In an accelerator experiment on high-energy collisions of electrons with positrons, a certain event is interpreted as annihilation of an electron-positron pair of total energy 10.2 BeV into two γ-rays of equal energy. What is the wavelength associated with each γ-ray? (1BeV = 10^9 eV)

Answer

Total energy of two γ-rays:

E = 10. 2 BeV

= 10.2 × 10^9 eV

= 10.2 × 10^9 × 1.6 × 10^{-10} J

Hence, the energy of each γ-ray:

$$E' = \frac{E}{2}$$

$$= \frac{10.2 \times 1.6 \times 10^{-10}}{2} = 8.16 \times 10^{-10} \text{ J}$$

Planck's constant, $h = 6.626 \times 10^{-34}$ Js

Speed of light, $c = 3 \times 10^8$ m/s

Energy is related to wavelength as:

$$E' = \frac{hc}{\lambda}$$

$$\therefore \lambda = \frac{hc}{E'}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{8.16 \times 10^{-10}} = 2.436 \times 10^{-16} \text{ m}$$

Therefore, the wavelength associated with each γ-ray is 2.436×10^{-16} m.

*********** END **********