Differentiation Ex 11.8 Q4(ii)

Let $\quad u = \sin^{-1}\sqrt{1-x^2}$

Put $\quad x = \cos\theta$, so,

$$u = \sin^{-1}\sqrt{1-\cos^2\theta}$$

$$u = \sin^{-1}(\sin\theta) \qquad \text{---(i)}$$

And, $\quad v = \cos^{-1}x \qquad \text{---(ii)}$

Here,

$$x \in (-1, 0)$$

$$\Rightarrow \quad \cos\theta \in (-,10)$$

$$\Rightarrow \quad \theta \in \left(\frac{\pi}{2}, \pi\right)$$

So, from equation (i),

$$u = \pi - \theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \pi - \theta, \theta \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)\right]$$

$$u = \pi - \cos^{-1}x \qquad [\text{Since, } x = \cos\theta]$$

Differentiating it with respect to $x$,

$$\frac{du}{dx} = 0 - \left(\frac{-1}{\sqrt{1-x^2}}\right)$$

$$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}} \qquad \text{---(v)}$$

And, from equation (ii),

$$v = \cos^{-1}x$$

Differentiating it with respect to $x$,

$$\frac{dv}{dx} = \frac{-1}{\sqrt{1-x^2}} \qquad \text{---(vi)}$$

Dividing equation (v) by (vi)

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{1}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{-1}$$

$$\frac{du}{dv} = -1$$

Differentiation Ex 11.8 Q5(i)

Let $u = \sin^{-1}\left(4x\sqrt{1-4x^2}\right)$

Put $2x = \cos\theta$, so

$$u = \sin^{-1}\left(2 \times \cos\theta\sqrt{1-\cos^2\theta}\right)$$
$$= \sin^{-1}(2\cos\theta\sin\theta)$$
$$u = \sin^{-1}(\sin 2\theta) \qquad \text{---(i)}$$

Let $v = \sqrt{1-4x^2}$ ---(ii)

Here,

$$x \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}}\right)$$
$$\Rightarrow \quad 2x \in \left(-1, -\frac{1}{\sqrt{2}}\right)$$
$$\Rightarrow \quad \theta \in \left(\frac{3\pi}{4}, \pi\right)$$

So, from equation (i),

$$u = \pi - 2\theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \pi - \theta \text{ if } \theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]\right]$$
$$u = \pi - 2\cos^{-1}(2x) \qquad [\text{Since, } 2x = \cos\theta]$$

Differentiating it with respect to $x$ using chain rule,

$$\frac{du}{dx} = 0 - 2\left(\frac{-1}{\sqrt{1-(2x)^2}}\right)\frac{d}{dx}(2x)$$
$$= \frac{2}{\sqrt{1-4x^2}}(2)$$
$$\frac{du}{dx} = \frac{4}{\sqrt{1-4x^2}} \qquad \text{---(vi)}$$

From equation (iv)

$$\frac{dv}{dx} = \frac{-4x}{\sqrt{1-4x^2}}$$

but, $x \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}}\right)$

$$\frac{dv}{dx} = \frac{-4(-x)}{\sqrt{1-4(-x)^2}}$$
$$\frac{dv}{dx} = \frac{4x}{\sqrt{1-4x^2}} \qquad \text{---(vii)}$$

Differentiating equation (ii) with respect to $x$ using chain rule,

$$\frac{dv}{dx} = \frac{1}{2\sqrt{1-4x^2}}\frac{d}{dx}\left(1-4x^2\right)$$
$$= \frac{1}{2\sqrt{1-4x^2}}(-8x)$$
$$\frac{dv}{dx} = \frac{-4x}{\sqrt{1-4x^2}} \qquad \text{---(iv)}$$

Divide equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{4}{\sqrt{1-4x^2}} \times \frac{\sqrt{1-4x^2}}{-4x}$$

$$\frac{du}{dv} = -\frac{1}{x}$$

## Differentiation Ex 11.8 Q5(ii)

Let $\quad u = \sin^{-1}\left(4x\sqrt{1-4x^2}\right)$

Put $\quad 2x = \cos\theta$, so

$$u = \sin^{-1}\left(2 \times \cos\theta\sqrt{1-\cos^2\theta}\right)$$

$$= \sin^{-1}\left(2\cos\theta\sin\theta\right)$$

$$u = \sin^{-1}\left(\sin 2\theta\right) \qquad \text{---(i)}$$

Let $\quad v = \sqrt{1-4x^2} \qquad \text{---(ii)}$

Here,

$$x \in \left(\frac{1}{2\sqrt{2}}, \frac{1}{2}\right)$$

$$\Rightarrow \quad 2x \in \left(\frac{1}{2\sqrt{2}}, 1\right)$$

$$\Rightarrow \quad \cos\theta \in \left(\frac{1}{\sqrt{2}}, 1\right)$$

$$\Rightarrow \quad \theta \in \left(0, \frac{\pi}{4}\right)$$

So, from equation (i)

$$u = 2\theta \qquad \left[\text{Since, } \sin^{-1}\left(\sin\theta\right) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$u = 2\cos^{-1}\left(2x\right) \qquad \left[\text{Since, } 2x = \cos\theta\right]$$

Differentiate it with respect to $x$ using chain rule,

$$\frac{du}{dx} = 2\left(\frac{-1}{\sqrt{1-(2x)^2}}\right)\frac{d}{dx}(2x)$$

$$= \left(\frac{-2}{\sqrt{1-4x^2}}(2)\right)$$

$$\frac{du}{dx} = \frac{-4}{\sqrt{1-4x^2}} \qquad \text{---(v)}$$

Dividing equation (v) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{-4}{\sqrt{1-4x^2}} \times \frac{\sqrt{1-4x^2}}{-4x}$$

$$\frac{du}{dv} = \frac{1}{x}$$

*********** END **********