Complex Numbers Ex 13.4 Q1(i)

The polar form of a complex number $z = x + iy$, is given by $z = |z|(\cos\theta + i\sin\theta)$

where,

$|z| = \sqrt{x^2 + y^2}$ and

$\arg(z) = \theta = \tan^{-1}\left(\frac{b}{a}\right)$

let $z = 1 + i$

$$|z| = \sqrt{1^2 + 1^2}$$
$$= \sqrt{2}$$

$\because x, y > 0$, so θ lies in first quadrant

Now,

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$
$$= \tan^{-1}\left(\frac{1}{1}\right) \qquad [\because a = 1 \text{ and } b = 1]$$
$$= \tan^{-1}(1)$$
$$= \tan^{-1}\left(\frac{\tan\pi}{4}\right) \qquad \left(\because \frac{\tan\pi}{4} = 1\right)$$
$$= \frac{\pi}{4} \qquad \left(\because \tan^{-1}(\tan x) = x\right)$$

$$\Rightarrow \arg(z) = \frac{\pi}{4}$$

Polar form of $1 + i$ is given by $z = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

Complex Numbers Ex 13.4 Q1(ii)

The polar form of a complex number $z = x + iy$, is given by $z = |z|(\cos\theta + i\sin\theta)$

where,

$|z| = \sqrt{x^2 + y^2}$ and

$\arg(z) = \theta = \tan^{-1}\left(\frac{b}{a}\right)$

let $z = \sqrt{3} + i$

$$
\begin{aligned}
|z| &= \sqrt{\left(\sqrt{3}\right)^2 + (1)^2} \\
&= \sqrt{3+1} \\
&= \sqrt{4} \\
&= 2
\end{aligned}
$$

$\because x = \sqrt{3} > 0$ & $y = 1 > 0$,

$\therefore$ θ lies in first quadrant

Hence

$$
\begin{aligned}
\theta = \arg(z) &= \tan^{-1}\left(\frac{y}{x}\right) \\
&= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \\
&= \tan^{-1}\left(\frac{\tan\pi}{6}\right) \\
&= \tan^{-1}\left(\because \tan^{-1}(\tan x) = x\right)
\end{aligned}
$$

polar form is given by $z = |z|(\cos\theta + i\sin\theta)$

i.e $z = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

*********** END **********