Indefinite Integrals Ex 19.26 Q22

Let $I = \int \left\{ \tan(\log x) + \sec^2(\log x) \right\} dx$

Let $\log x = z$

$\Rightarrow \quad x = e^z$

$\Rightarrow \quad dx = e^z dz$

$\therefore \quad I = \int \left\{ \tan z + \sec^2 z \right\} e^z dz$

Here, $f(z) = \tan z$ and $f'(z) = \sec^2 z$

And we know that

$\int e^{ax} \left(af(x) + f'(x) \right) dx = e^{ax} f(x) + c$

$\therefore \quad \int e^z \left\{ \tan z + \sec^2 z \right\} dz = e^z \tan z + c$

$\therefore \quad I = x \tan(\log x) + c$

Indefinite Integrals Ex 19.26 Q23

Let $I = \int \frac{e^x (x-4)}{(x-2)^3} dx$

$= \int e^x \left\{ \frac{(x-2)-2}{(x-2)^3} \right\} dx$

$= \int e^x \left\{ \frac{1}{(x-2)^2} - \frac{2}{(x-2)^3} \right\} dx$

Here, $f(x) = \frac{1}{(x-2)^2}$ and $f'(x) = \frac{-2}{(x-2)^3}$

And we know that,

$\int e^{ax} \left(af(x) + f'(x) \right) dx = e^{ax} f(x) + c$

$\therefore \quad \int e^x \left\{ \frac{1}{(x-2)^2} - \frac{2}{(x-2)^3} \right\} dx = \frac{e^x}{(x-2)^2} + c$

$\therefore \quad I = \frac{e^x}{(x-2)^2} + c$

Indefinite Integrals Ex 19.26 Q24

Let $I = \int e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right)dx$

We have, $\cos 2x = 1-2\sin^2 x$

$I = \int e^{2x}\left(\frac{1-\sin 2x}{1-\left(1-2\sin^2 x\right)}\right)dx$

$= \int e^{2x}\left(\frac{1-\sin 2x}{2\sin^2 x}\right)dx$

$= \int e^{2x}\left(\frac{\operatorname{cosec}^2 x}{2}-\frac{2\sin x\cos x}{2\sin^2 x}\right)dx$

$= \int e^{2x}\left(\frac{\operatorname{cosec}^2 x}{2}-\frac{\cos x}{\sin x}\right)dx$

$= \int e^{2x}\left(\frac{\operatorname{cosec}^2 x}{2}-\cot x\right)dx$

$= \frac{1}{2}\int e^{2x}\operatorname{cosec}^2 x dx - \int e^{2x}\cot x dx$

That is

$I = I_1 + I_2$, where, $I_1 = \frac{1}{2}\int e^{2x}\operatorname{cosec}^2 x dx$ and $I_2 = -\int e^{2x}\cot x dx$

Consider $I_1 = \frac{1}{2}\int e^{2x}\operatorname{cosec}^2 x dx$

Take e^{2x} as the first function and $\operatorname{cosec}^2 x$ as the second function.

So, $u = e^{2x}$; $du = 2e^{2x}dx$

and

$\int \operatorname{cosec}^2 x dx = \int dv$

$\Rightarrow v = -\cot x$

$I_1 = \frac{1}{2}\left[e^{2x}(-\cot x) - \int(-\cot x)2e^{2x}dx\right]$

$\Rightarrow I_1 = \frac{1}{2}\left[e^{2x}(-\cot x) + 2\int \cot x e^{2x}dx\right]$

$\Rightarrow I_1 = \frac{1}{2}\left[e^{2x}(-\cot x)\right] + \int \cot x e^{2x}dx$

Thus,

$I = \frac{1}{2}\left[e^{2x}(-\cot x)\right] + \int \cot x e^{2x}dx - \int e^{2x}\cot x dx$

$= I = \frac{1}{2}\left[e^{2x}(-\cot x)\right] + C$

*********** END **********