

Exercise 4D

Question 17:

Given: D is the midpoint of side BC, AE \perp BC, BC = α , AC = b, AB = c, ED = x, AD = p and AE = h

In $\triangle AEC$, $\angle AEC = 90^{\circ}$

$$AD^2 = 2AE^2 + ED^2$$
 (by Pythagoras theorem)

$$\Rightarrow$$
 p² = h² + x²

(i) In $\triangle AEC$, $\angle AEC = 90^{\circ}$

$$b^2 = h^2 + \left(x + \frac{a}{2}\right)^2 = \left(h^2 + x^2\right) + ax + \frac{a^2}{4}$$

$$b^2 = p^2 + ax + \frac{a^2}{4} - - - - (1)$$

(ii) In $\triangle ABE$, $\angle ABE = 90^{\circ}$

 $AB^2 = AE^2 + BE^2$ (by pythagoras theorem)

$$\Rightarrow c^{2} = h^{2} + \left(\frac{a}{2} - x\right)^{2} - - - - (2)$$

$$= \left(h^{2} + x^{2}\right) - ax + \frac{a^{2}}{4}$$

$$= p^{2} - ax + \frac{a^{2}}{4}$$

Hence,
$$c^2 = p^2 - ax + \frac{a^2}{4}$$

(iii) Adding (1) and (2), we get

$$b^2 + c^2 = p^2 + ax + \frac{a^2}{4} + p^2 - ax + \frac{a^2}{4}$$

$$(b^2 + c^2) = 2p^2 + \frac{1}{2}a^2$$

(iv) Subtracting (2) from (1), we get

$$b^{2} - c^{2} = p^{2} + ax + \frac{a^{2}}{4} - \left(p^{2} - ax + \frac{a^{2}}{4}\right)$$
$$= p^{2} + ax + \frac{a^{2}}{4} - p^{2} + ax - \frac{a^{2}}{4}$$
$$\left(b^{2} - c^{2}\right) = 2ax$$

******* END ********