



Indefinite Integrals Ex 19.32 Q1

$$\text{Let } I = \int \frac{1}{(x-1)\sqrt{x+2}} dx$$

$$\text{Let } x+2 = t^2$$

$$\begin{aligned} \therefore I &= \int \frac{2t dt}{(t^2-3)t} \\ &= 2 \int \frac{dt}{t^2-3} \\ &= \frac{2}{2\sqrt{3}} \log \left| \frac{t-\sqrt{3}}{t+\sqrt{3}} \right| + c \end{aligned}$$

Thus,

$$I = \frac{1}{\sqrt{3}} \log \left| \frac{\sqrt{x-2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}} \right| + c$$

Indefinite Integrals Ex 19.32 Q2

$$\text{Let } I = \int \frac{1}{(x-1)\sqrt{2x+3}} dx$$

$$\text{Let } I = \int \frac{1}{(x-1)\sqrt{2x+3}} dx$$

$$\text{Let } 2x+3 = t^2$$

$$\Rightarrow 2dx = t dt$$

$$\begin{aligned} \therefore I &= \int \frac{t dt}{\left(\frac{t^2-3}{2}-1\right)t} \\ &= 2 \int \frac{dt}{t^2-5} \\ &= \frac{2}{2\sqrt{5}} \log \left| \frac{t-\sqrt{5}}{t+\sqrt{5}} \right| + c \end{aligned}$$

Thus,

$$I = \frac{1}{\sqrt{5}} \log \left| \frac{\sqrt{2x+3}-\sqrt{5}}{\sqrt{2x+3}+\sqrt{5}} \right| + c$$

Indefinite Integrals Ex 19.32 Q3

$$\begin{aligned}
 \text{Let } I &= \int \frac{1}{(x-1)\sqrt{x+2}} dx \\
 I &= \int \frac{(x-1)+2}{(x-1)\sqrt{x+2}} dx \\
 I &= \int \frac{dx}{\sqrt{x+2}} + 2 \int \frac{dx}{(x-1)\sqrt{x+2}} \quad \text{---(A)}
 \end{aligned}$$

Now,

$$\int \frac{dx}{\sqrt{x+2}} = 2\sqrt{x+2} + c_1$$

and,

$$\int \frac{dx}{(x-1)\sqrt{x+2}}$$

$$\text{Let } x+2 = t^2$$

$$\Rightarrow dx = 2t dt$$

$$\begin{aligned}
 \therefore \int \frac{dx}{(x-1)\sqrt{x+2}} &= 2 \int \frac{t dt}{(t^2-3)t} = 2 \int \frac{dt}{t^2-3} \\
 &= \frac{2 \times 1}{2\sqrt{3}} \log \left| \frac{t-\sqrt{3}}{t+\sqrt{3}} \right| + c_2 \\
 &= \frac{1}{\sqrt{3}} \log \left| \frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}} \right| + c_2
 \end{aligned}$$

Thus, from (A),

$$I = 2\sqrt{x+2} + c_1 + \frac{2}{\sqrt{3}} \log \left| \frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}} \right| + c_2$$

Hence,

$$I = 2\sqrt{x+2} + \frac{2}{\sqrt{3}} \log \left| \frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}} \right| + c$$

Indefinite Integrals Ex 19.32 Q4

$$\begin{aligned}
 \text{Let } I &= \int \frac{x^2}{(x-1)\sqrt{x+2}} dx \\
 &= \int \frac{(x^2-1)+1}{(x-1)\sqrt{x+2}} dx \\
 &= \int \frac{(x+1)(x-1)}{(x-1)\sqrt{x+2}} dx + \int \frac{dx}{(x-1)\sqrt{x+2}} \\
 &= \int \frac{(x+1)}{\sqrt{x+2}} dx + \int \frac{dx}{(x-1)\sqrt{x+2}} \\
 &= \int \frac{(x+2)-1}{\sqrt{x+2}} dx + \int \frac{dx}{(x-1)\sqrt{x+2}} \\
 I &= \int \sqrt{x+2} dx - \int \frac{dx}{\sqrt{x+2}} + \int \frac{dx}{(x-1)\sqrt{x+2}} \quad \text{---(A)}
 \end{aligned}$$

Now,

$$\int \sqrt{x+2} dx = \frac{2}{3} (x+2)^{\frac{3}{2}} + c_1$$

and,

$$\int \frac{dx}{\sqrt{x+2}} = 2\sqrt{x+2} + c_2$$

$$\int \frac{dx}{(x-1)\sqrt{x+2}}$$

$$\text{Let } x+2 = t^2$$

$$\Rightarrow dx = 2t dt$$

$$\therefore 2 \int \frac{t dt}{(t^2-3)t} = 2 \int \frac{dt}{t^2-3}$$

$$= \frac{2}{2\sqrt{3}} \log \left| \frac{t-\sqrt{3}}{t+\sqrt{3}} \right| + c_3$$

$$\therefore \int \frac{dx}{(x-1)\sqrt{x+2}} = \frac{1}{\sqrt{3}} \log \left| \frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}} \right| + c_3$$

Thus, from (A)

$$I = \frac{2}{3} (x+2)^{\frac{3}{2}} - 2\sqrt{x+2} + \frac{1}{\sqrt{3}} \log \left| \frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{x+2}+\sqrt{3}} \right| + c \quad [\text{when } c = c_1 + c_2 + c_3]$$

Indefinite Integrals Ex 19.32 Q5

$$\begin{aligned}
 \text{Let } I &= \int \frac{x}{(x-3)\sqrt{x+1}} dx \\
 &= \int \frac{(x-3)+3}{(x-3)\sqrt{x+1}} dx \\
 I &= \int \frac{dx}{\sqrt{x+1}} + 3 \int \frac{dx}{(x-3)\sqrt{x+1}} \quad \text{---(A)}
 \end{aligned}$$

Now,

$$\int \frac{dx}{\sqrt{x+1}} = 2\sqrt{x+1} + c_1$$

and,

$$\int \frac{dx}{\sqrt{x+2}} = 2\sqrt{x+2} + c_2$$

$$\int \frac{dx}{(x-3)\sqrt{x+1}}$$

$$\text{Let } x+1 = t^2$$

$$\Rightarrow dx = 2t dt$$

$$\begin{aligned}
 \therefore \int \frac{dx}{(x-3)\sqrt{x+1}} &= 2 \int \frac{t dt}{(t^2-4)t} \\
 &= 2 \left| \frac{dt}{t^2-4} \right| \\
 &= \frac{2}{2 \times 2} \log \left| \frac{t-2}{t+2} \right| + c_2
 \end{aligned}$$

$$\therefore \int \frac{dx}{(x-3)\sqrt{x+1}} = \frac{1}{2} \log \left| \frac{\sqrt{x+1}-2}{\sqrt{x+1}+2} \right| + c_2$$

Thus, from (A)

$$I = 2\sqrt{x+1} + \frac{3}{2} \log \left| \frac{\sqrt{x+1}-2}{\sqrt{x+1}+2} \right| + c \quad [\text{when } c = c_1 + c_2]$$

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