Exercise 5.1 : Solutions of Questions on Page Number : **159**

Q1 : Prove that the function $f(x)=5x-3$ is continuous at $x=0$, at $x=-3$ and at $x=5$.

Answer :

The given function is $f(x)=5x-3$

At $x=0$, $f(0)=5\times 0-3=3$

$\lim\limits_{x\to 0} f(x)=\lim\limits_{x\to 0}(5x-3)=5\times 0-3=-3$

$\therefore \lim\limits_{x\to 0} f(x)=f(0)$

Therefore, *f* is continuous at *x*= 0

At $x=-3$, $f(-3)=5\times(-3)-3=-18$

$\lim\limits_{x\to -3} f(x)=\lim\limits_{x\to -3}(5x-3)=5\times(-3)-3=-18$

$\therefore \lim\limits_{x\to -3} f(x)=f(-3)$

Therefore, *f* is continuous at *x*= - 3

At $x=5$, $f(x)=f(5)=5\times 5-3=25-3=22$

$\lim\limits_{x\to 5} f(x)=\lim\limits_{x\to 5}(5x-3)=5\times 5-3=22$

$\therefore \lim\limits_{x\to 5} f(x)=f(5)$

Therefore, *f* is continuous at *x*= 5

Answer needs Correction? **Click Here**

Q2 : Examine the continuity of the function $f(x)=2x^2-1$ at $x=3$.

Answer :

The given function is $f(x)=2x^2-1$

At $x=3$, $f(x)=f(3)=2\times 3^2-1=17$

$\lim\limits_{x\to 3} f(x)=\lim\limits_{x\to 3}(2x^2-1)=2\times 3^2-1=17$

$\therefore \lim\limits_{x\to 3} f(x)=f(3)$

Thus, *f* is continuous at *x*= 3

Answer needs Correction? **Click Here**

Q3 : Examine the following functions for continuity.

(a) $f(x)=x-5$ (b) $f(x)=\dfrac{1}{x-5}, x\neq 5$

(c) $f(x)=\dfrac{x^2-25}{x+5}, x\neq -5$ (d) $f(x)=|x-5|$

Answer :

(a) The given function is $f(x)=x-5$

It is evident that *f* is defined at every real number *k* and its value at *k* is *k* - 5.

It is also observed that, $\lim\limits_{x\to k} f(x)=\lim\limits_{x\to k}(x-5)=k-5=f(k)$

$\therefore \lim\limits_{x\to k} f(x)=f(k)$

Hence, *f* is continuous at every real number and therefore, it is a continuous function.

(b) The given function is $f(x)=\dfrac{1}{x-5}, x\neq 5$

For any real number *k* ≠ 5, we obtain

$\lim\limits_{x\to k} f(x)=\lim\limits_{x\to k}\dfrac{1}{x-5}=\dfrac{1}{k-5}$

Also, $f(k)=\dfrac{1}{k-5}$ $\quad$ (As $k\neq 5$)

$\therefore \lim\limits_{x\to k} f(x)=f(k)$

Hence, *f* is continuous at every point in the domain of *f* and therefore, it is a continuous function.

(c) The given function is $f(x)=\dfrac{x^2-25}{x+5}, x\neq -5$

For any real number *c* ≠ - 5, we obtain

$$\lim_{x\to c} f(x) = \lim_{x\to c} \frac{x^2-25}{x+5} = \lim_{x\to c} \frac{(x+5)(x-5)}{x+5} = \lim_{x\to c}(x-5) = (c-5)$$

$$\text{Also, } f(c) = \frac{(c+5)(c-5)}{c+5} = (c-5) \quad (\text{as } c \neq -5)$$

$$\therefore \lim_{x\to c} f(x) = f(c)$$

Hence, *f* is continuous at every point in the domain of *f* and therefore, it is a continuous function.

(d) The given function is $f(x) = |x-5| = \begin{cases} 5-x, & \text{if } x < 5 \\ x-5, & \text{if } x \geq 5 \end{cases}$

This function *f* is defined at all points of the real line.

Let *c* be a point on a real line. Then, *c* < 5 or *c* = 5 or *c* > 5

Case I: *c* < 5

Then, *f*(*c*) = 5 - *c*

$$\lim_{x\to c} f(x) = \lim_{x\to c}(5-x) = 5-c$$

$$\therefore \lim_{x\to c} f(x) = f(c)$$

Therefore, *f* is continuous at all real numbers less than 5.

Case II : *c* = 5

Then, $f(c) = f(5) = (5-5) = 0$

$$\lim_{x\to 5^-} f(x) = \lim_{x\to 5}(5-x) = (5-5) = 0$$

$$\lim_{x\to 5^+} f(x) = \lim_{x\to 5}(x-5) = 0$$

$$\therefore \lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = f(c)$$

Therefore, *f* is continuous at *x* = 5

Case III: *c* > 5

$$\text{Then, } f(c) = f(5) = c-5$$

$$\lim_{x\to c} f(x) = \lim_{x\to c}(x-5) = c-5$$

$$\therefore \lim_{x\to c} f(x) = f(c)$$

Answer needs Correction? Click Here

Q4 : Prove that the function $f(x) = x^n$ is continuous at *x* = *n*, where *n* is a positive integer.

Answer :

The given function is *f*(*x*) = x^n

It is evident that *f* is defined at all positive integers, *n*, and its value at *n* is n^n.

$$\text{Then, } \lim_{x\to n} f(n) = \lim_{x\to n}(x^n) = n^n$$

$$\therefore \lim_{x\to n} f(x) = f(n)$$

Therefore, *f* is continuous at *n*, where *n* is a positive integer.

Answer needs Correction? Click Here

Q5 : Is the function *f* defined by

$$f(x) = \begin{cases} x, & \text{if } x \leq 1 \\ 5, & \text{if } x > 1 \end{cases}$$

continuous at *x* = 0? At *x* = 1? At *x* = 2?

Answer :

The given function *f* is $f(x) = \begin{cases} x, & \text{if } x \leq 1 \\ 5, & \text{if } x > 1 \end{cases}$

At *x* = 0,

It is evident that *f* is defined at 0 and its value at 0 is 0.

$$\text{Then, } \lim_{x\to 0} f(x) = \lim_{x\to 0} x = 0$$

$$\therefore \lim_{x\to 0} f(x) = f(0)$$

Therefore, *f* is continuous at *x* = 0

At *x* = 1,

f is defined at 1 and its value at 1 is 1.

The left hand limit of *f* at *x* = 1 is,

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x = 1$$

The right hand limit of *f* at *x* = 1 is,

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(5) = 5$$

$$\therefore \lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$$

Therefore, *f* is not continuous at *x* = 1

At *x* = 2,

f is defined at 2 and its value at 2 is 5.

$$\text{Then, } \lim_{x\to 2} f(x) = \lim_{x\to 2}(5) = 5$$

$\therefore \lim_{x \to 2} f(x) = f(2)$

Therefore, *f* is continuous at *x* = 2

Answer needs Correction? Click Here

Q6 : Find all points of discontinuity of *f*, where *f* is defined by

$$f(x) = \begin{cases} 2x+3, & \text{if } x \le 2 \\ 2x-3, & \text{if } x > 2 \end{cases}$$

Answer :

The given function *f* is $f(x) = \begin{cases} 2x+3, & \text{if } x \le 2 \\ 2x-3, & \text{if } x > 2 \end{cases}$

It is evident that the given function *f* is defined at all the points of the real line.

Let *c* be a point on the real line. Then, three cases arise.

(i) $c < 2$

(ii) $c > 2$

(iii) $c = 2$

Case (i) $c < 2$

$$\text{Then, } f(c) = 2c+3$$
$$\lim_{x \to c} f(x) = \lim_{x \to c} (2x+3) = 2c+3$$
$$\therefore \lim_{x \to c} f(x) = f(c)$$

Therefore, *f* is continuous at all points *x*, such that $x < 2$

Case (ii) $c > 2$

$$\text{Then, } f(c) = 2c-3$$
$$\lim_{x \to c} f(x) = \lim_{x \to c} (2x-3) = 2c-3$$
$$\therefore \lim_{x \to c} f(x) = f(c)$$

Therefore, *f* is continuous at all points *x*, such that $x > 2$

Case (iii) $c = 2$

Then, the left hand limit of *f* at $x = 2$ is,

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (2x+3) = 2 \times 2 + 3 = 7$$

The right hand limit of *f* at $x = 2$ is,

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (2x-3) = 2 \times 2 - 3 = 1$$

It is observed that the left and right hand limit of *f* at $x = 2$ do not coincide.

Therefore, *f* is not continuous at $x = 2$

Hence, $x = 2$ is the only point of discontinuity of *f*.

Answer needs Correction? Click Here

Q7 : Find all points of discontinuity of *f*, where *f* is defined by

$$f(x) = \begin{cases} |x|+3, \text{if } x \le -3 \\ -2x, \text{if } -3 < x < 3 \\ 6x+2, \text{if } x \ge 3 \end{cases}$$

Answer :

The given function *f* is $f(x) = \begin{cases} |x|+3 = -x+3, & \text{if } x \le -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x+2, & \text{if } x \ge 3 \end{cases}$

The given function *f* is defined at all the points of the real line.

Let *c* be a point on the real line.

Case I:

$$\text{If } c < -3, \text{ then } f(c) = -c+3$$
$$\lim_{x \to c} f(x) = \lim_{x \to c} (-x+3) = -c+3$$
$$\therefore \lim_{x \to c} f(x) = f(c)$$

Therefore, *f* is continuous at all points *x*, such that $x < -3$

Case II:

$$\text{If } c = -3, \text{ then } f(-3) = -(-3)+3 = 6$$

$$\lim_{x \to -3^-} f(x) = \lim_{x \to -3^-} (-x+3) = -(-3)+3 = 6$$
$$\lim_{x \to -3^+} f(x) = \lim_{x \to -3^+} (-2x) = -2 \times (-3) = 6$$
$$\therefore \lim_{x \to -3} f(x) = f(-3)$$

Therefore, *f* is continuous at $x = -3$

Case III:

$$\text{If } -3 < c < 3, \text{ then } f(c) = -2c \text{ and } \lim_{x \to c} f(x) = \lim_{x \to c} (-2x) = -2c$$

$$\therefore \lim_{x \to c} f(x) = f(c)$$

Therefore, f is continuous in (- 3, 3).

Case IV:

If $c = 3$, then the left hand limit of f at $x = 3$ is,

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}(-2x) = -2\times 3 = -6$$

The right hand limit of f at $x = 3$ is,

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+}(6x+2) = 6\times 3+2 = 20$$

It is observed that the left and right hand limit of f at $x = 3$ do not coincide.

Therefore, f is not continuous at $x = 3$

Case V:

$$\text{If } c>3, \text{ then } f(c) = 6c+2 \text{ and } \lim_{x\to c} f(x) = \lim_{x\to c}(6x+2) = 6c+2$$

$$\therefore \lim_{x\to c} f(x) = f(c)$$

Therefore, f is continuous at all points x, such that $x > 3$

Hence, $x = 3$ is the only point of discontinuity of f.

Answer needs Correction? Click Here

Q8 : Find all points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} \frac{|x|}{x} & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

Answer :

The given function f is $f(x) = \begin{cases} \frac{|x|}{x} & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

It is known that, $x<0 \Rightarrow |x| = -x$ and $x>0 \Rightarrow |x| = x$

Therefore, the given function can be rewritten as

$$f(x) = \begin{cases} \frac{|x|}{x} = \frac{-x}{x} = -1 & \text{if } x<0 \\ 0, & \text{if } x = 0 \\ \frac{|x|}{x} = \frac{x}{x} = 1, & \text{if } x>0 \end{cases}$$

The given function f is defined at all the points of the real line.

Let c be a point on the real line.

Case I:

$$\text{If } c<0, \text{ then } f(c) = -1$$

$$\lim_{x\to c} f(x) = \lim_{x\to c}(-1) = -1$$

$$\therefore \lim_{x\to c} f(x) = f(c)$$

Therefore, f is continuous at all points $x < 0$

Case II:

If $c = 0$, then the left hand limit of f at $x = 0$ is,

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(-1) = -1$$

The right hand limit of f at $x = 0$ is,

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(1) = 1$$

It is observed that the left and right hand limit of f at $x = 0$ do not coincide.

Therefore, f is not continuous at $x = 0$

Case III:

$$\text{If } c>0, \text{ then } f(c) = 1$$

$$\lim_{x\to c} f(x) = \lim_{x\to c}(1) = 1$$

$$\therefore \lim_{x\to c} f(x) = f(c)$$

Therefore, f is continuous at all points x, such that $x > 0$

Hence, $x = 0$ is the only point of discontinuity of f.

Answer needs Correction? Click Here

Q9 : Find all points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} \frac{x}{|x|}, & \text{if } x<0 \\ -1, & \text{if } x \geq 0 \end{cases}$$

Answer :

The given function f is $f(x) = \begin{cases} \frac{x}{|x|}, & \text{if } x<0 \\ -1, & \text{if } x \geq 0 \end{cases}$

It is known that, $x<0 \Rightarrow |x| = -x$

Therefore, the given function can be rewritten as

$$f(x)=\begin{cases}\frac{x}{|x|}=\frac{x}{-x}=-1, & \text{if } x<0\\ -1, & \text{if } x\geq 0\end{cases}$$

$$\Rightarrow f(x)=-1 \text{ for all } x\in\mathbf{R}$$

Let c be any real number. Then, $\lim_{x\to c} f(x)=\lim_{x\to c}(-1)=-1$

Also, $f(c)=-1=\lim_{x\to c} f(x)$

Therefore, the given function is a continuous function.

Hence, the given function has no point of discontinuity.

Answer needs Correction? **Click Here**

Q10 : Find all points of discontinuity of *f*, where *f* is defined by

$$f(x)=\begin{cases}x+1, & \text{if } x\geq 1\\ x^2+1, & \text{if } x<1\end{cases}$$

Answer :

The given function *f* is $f(x)=\begin{cases}x+1, & \text{if } x\geq 1\\ x^2+1, & \text{if } x<1\end{cases}$

The given function *f* is defined at all the points of the real line.

Let *c* be a point on the real line.

Case I:

If $c<1$, then $f(c)=c^2+1$ and $\lim_{x\to c} f(x)=\lim_{x\to c}(x^2+1)=c^2+1$

$\therefore \lim_{x\to c} f(x)=f(c)$

Therefore, *f* is continuous at all points *x*, such that *x*< 1

Case II:

If $c=1$, then $f(c)=f(1)=1+1=2$

The left hand limit of *f* at *x* = 1 is,

$\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-}(x^2+1)=1^2+1=2$

The right hand limit of *f* at *x* = 1 is,

$\lim_{x\to 1^+} f(x)=\lim_{x\to 1^+}(x+1)=1+1=2$

$\therefore \lim_{x\to 1} f(x)=f(1)$

Therefore, *f* is continuous at *x*= 1

Case III:

If $c>1$, then $f(c)=c+1$

$\lim_{x\to c} f(x)=\lim_{x\to c}(x+1)=c+1$

$\therefore \lim_{x\to c} f(x)=f(c)$

Therefore, *f* is continuous at all points *x*, such that *x*> 1

Hence,the given function *f* has no point of discontinuity.

Answer needs Correction? **Click Here**

Q11 : Find all points of discontinuity of *f*, where *f* is defined by

$$f(x)=\begin{cases}x^3-3, & \text{if } x\leq 2\\ x^2+1, & \text{if } x>2\end{cases}$$

Answer :

The given function *f* is $f(x)=\begin{cases}x^3-3, & \text{if } x\leq 2\\ x^2+1, & \text{if } x>2\end{cases}$

The given function *f* is defined at all the points of the real line.

Let *c* be a point on the real line.

Case I:

If $c<2$, then $f(c)=c^3-3$ and $\lim_{x\to c} f(x)=\lim_{x\to c}(x^3-3)=c^3-3$

$\therefore \lim_{x\to c} f(x)=f(c)$

Therefore, *f* is continuous at all points *x*, such that *x*< 2

Case II:

If $c=2$, then $f(c)=f(2)=2^3-3=5$

$\lim_{x\to 2^-} f(x)=\lim_{x\to 2^-}(x^3-3)=2^3-3=5$

$\lim_{x\to 2^+} f(x)=\lim_{x\to 2^+}(x^2+1)=2^2+1=5$

$\therefore \lim_{x\to 2} f(x)=f(2)$

Therefore, *f* is continuous at *x*= 2

Case III:

If $c>2$, then $f(c)=c^2+1$

$\lim\limits_{x \to c} f(x) = \lim\limits_{x \to c}(x^2 + 1) = c^2 + 1$

$\therefore \lim\limits_{x \to c} f(x) = f(c)$

Therefore, *f* is continuous at all points *x*, such that $x > 2$

Thus, the given function *f* is continuous at every point on the real line.

Hence, *f* has no point of discontinuity.

Answer needs Correction? Click Here

Q12 : Find all points of discontinuity of *f*, where *f* is defined by

$$f(x) = \begin{cases} x^{10} - 1, & \text{if } x \le 1 \\ x^2, & \text{if } x > 1 \end{cases}$$

Answer :

The given function *f* is $f(x) = \begin{cases} x^{10} - 1, & \text{if } x \le 1 \\ x^2, & \text{if } x > 1 \end{cases}$

The given function *f* is defined at all the points of the real line.

Let *c* be a point on the real line.

Case I:

If $c < 1$, then $f(c) = c^{10} - 1$ and $\lim\limits_{x \to c} f(x) = \lim\limits_{x \to c}(x^{10} - 1) = c^{10} - 1$

$\therefore \lim\limits_{x \to c} f(x) = f(c)$

Therefore, *f* is continuous at all points *x*, such that $x < 1$

Case II:

If $c = 1$, then the left hand limit of *f* at $x = 1$ is,

$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-}(x^{10} - 1) = 1^{10} - 1 = 1 - 1 = 0$

The right hand limit of *f* at $x = 1$ is,

$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+}(x^2) = 1^2 = 1$

It is observed that the left and right hand limit of *f* at $x = 1$ do not coincide.

Therefore, *f* is not continuous at $x = 1$

Case III:

If $c > 1$, then $f(c) = c^2$

$\lim\limits_{x \to c} f(x) = \lim\limits_{x \to c}(x^2) = c^2$

$\therefore \lim\limits_{x \to c} f(x) = f(c)$

Therefore, *f* is continuous at all points *x*, such that $x > 1$

Thus, from the above observation, it can be concluded that $x = 1$ is the only point of discontinuity of *f*.

Answer needs Correction? Click Here

Q13 : Is the function defined by

$$f(x) = \begin{cases} x + 5, & \text{if } x \le 1 \\ x - 5, & \text{if } x > 1 \end{cases}$$

a continuous function?

Answer :

The given function is $f(x) = \begin{cases} x + 5, & \text{if } x \le 1 \\ x - 5, & \text{if } x > 1 \end{cases}$

The given function *f* is defined at all the points of the real line.

Let *c* be a point on the real line.

Case I:

If $c < 1$, then $f(c) = c + 5$ and $\lim\limits_{x \to c} f(x) = \lim\limits_{x \to c}(x + 5) = c + 5$

$\therefore \lim\limits_{x \to c} f(x) = f(c)$

Therefore, *f* is continuous at all points *x*, such that $x < 1$

Case II:

If $c = 1$, then $f(1) = 1 + 5 = 6$

The left hand limit of *f* at $x = 1$ is,

$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-}(x + 5) = 1 + 5 = 6$

The right hand limit of *f* at $x = 1$ is,

$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+}(x - 5) = 1 - 5 = -4$

It is observed that the left and right hand limit of *f* at $x = 1$ do not coincide.

Therefore, *f* is not continuous at $x = 1$

Case III:

If $c > 1$, then $f(c) = c - 5$ and $\lim\limits_{x \to c} f(x) = \lim\limits_{x \to c}(x - 5) = c - 5$

$\therefore \lim_{x \to c} f(x) = f(c)$

Therefore, *f* is continuous at all points *x*, such that *x*> 1

Thus, from the above observation, it can be concluded that *x*= 1 is the only point of discontinuity of *f*.

Answer needs Correction? **Click Here**

Q14 : Discuss the continuity of the function *f*, where *f* is defined by

$$f(x) = \begin{cases} 3, & \text{if } 0 \le x \le 1 \\ 4, & \text{if } 1 < x < 3 \\ 5, & \text{if } 3 \le x \le 10 \end{cases}$$

Answer :

The given function is $f(x) = \begin{cases} 3, & \text{if } 0 \le x \le 1 \\ 4, & \text{if } 1 < x < 3 \\ 5, & \text{if } 3 \le x \le 10 \end{cases}$

The given function is defined at all points of the interval [0, 10].

Let *c* be a point in the interval [0, 10].

Case I:

$\text{If } 0 \le c < 1, \text{ then } f(c) = 3 \text{ and } \lim_{x \to c} f(x) = \lim_{x \to c} (3) = 3$

$\therefore \lim_{x \to c} f(x) = f(c)$

Therefore, *f* is continuous in the interval [0, 1).

Case II:

$\text{If } c = 1, \text{ then } f(3) = 3$

The left hand limit of *f* at *x* = 1 is,

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (3) = 3$

Theright hand limit of *f*at *x* = 1 is,

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (4) = 4$

It is observed that the left and right hand limits of *f* at *x*= 1 do not coincide.

Therefore, *f* is not continuous at *x*= 1

Case III:

$\text{If } 1 < c < 3, \text{ then } f(c) = 4 \text{ and } \lim_{x \to c} f(x) = \lim_{x \to c} (4) = 4$

$\therefore \lim_{x \to c} f(x) = f(c)$

Therefore, *f* is continuous at all points of the interval (1, 3).

Case IV:

$\text{If } c = 3, \text{ then } f(c) = 5$

The left hand limit of *f* at *x* = 3 is,

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (4) = 4$

The right hand limit of *f*at *x* = 3 is,

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (5) = 5$

It is observed that the left and right hand limits of *f* at *x*= 3 do not coincide.

Therefore, *f* is not continuous at *x*= 3

Case V:

$\text{If } 3 < c \le 10, \text{ then } f(c) = 5 \text{ and } \lim_{x \to c} f(x) = \lim_{x \to c} (5) = 5$

$\lim_{x \to c} f(x) = f(c)$

Therefore, *f* is continuous at all points of the interval (3, 10].

Hence, *f* is not continuous at *x* = 1 and *x* = 3

Answer needs Correction? **Click Here**

Q15 : Discuss the continuity of the function *f*, where *f* is defined by

$$f(x) = \begin{cases} 2x, & \text{if } x < 0 \\ 0, & \text{if } 0 \le x \le 1 \\ 4x, & \text{if } x > 1 \end{cases}$$

Answer :

The given function is $f(x) = \begin{cases} 2x, & \text{if } x < 0 \\ 0, & \text{if } 0 \le x \le 1 \\ 4x, & \text{if } x > 1 \end{cases}$

The given function is defined at all points of the real line.

Let *c* be a point on the real line.

Case I:

$\text{If } c < 0, \text{ then } f(c) = 2c$

$\lim_{x \to c} f(x) = \lim_{x \to c} (2x) = 2c$

$\therefore \lim\limits_{x \to c} f(x) = f(c)$

Therefore, *f* is continuous at all points *x*, such that $x < 0$

Case II:

If $c = 0$, then $f(c) = f(0) = 0$

The left hand limit of *f* at $x = 0$ is,

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} (2x) = 2 \times 0 = 0$

The right hand limit of *f* at $x = 0$ is,

$\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} (0) = 0$

$\therefore \lim\limits_{x \to 0} f(x) = f(0)$

Therefore, *f* is continuous at $x = 0$

Case III:

If $0 < c < 1$, then $f(x) = 0$ and $\lim\limits_{x \to c} f(x) = \lim\limits_{x \to c} (0) = 0$

$\therefore \lim\limits_{x \to c} f(x) = f(c)$

Therefore, *f* is continuous at all points of the interval (0, 1).

Case IV:

If $c = 1$, then $f(c) = f(1) = 0$

Theleft hand limit of *f* at $x = 1$ is,

$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} (0) = 0$

The right hand limit of *f* at $x = 1$ is,

$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} (4x) = 4 \times 1 = 4$

It is observed that the left and right hand limits of *f* at $x = 1$ do not coincide.

Therefore, *f* is not continuous at $x = 1$

Case V:

If $c < 1$, then $f(c) = 4c$ and $\lim\limits_{x \to c} f(x) = \lim\limits_{x \to c} (4x) = 4c$

$\therefore \lim\limits_{x \to c} f(x) = f(c)$

Therefore, *f* is continuous at all points *x*, such that $x > 1$

Hence, *f* is not continuous only at $x = 1$

Answer needs Correction? **Click Here**

Q16 : Discuss the continuity of the function *f*, where *f* is defined by

$$f(x) = \begin{cases} -2, & \text{if } x \le -1 \\ 2x, & \text{if } -1 < x \le 1 \\ 2, & \text{if } x > 1 \end{cases}$$

Answer :

The given function *f* is $f(x) = \begin{cases} -2, & \text{if } x \le -1 \\ 2x, & \text{if } -1 < x \le 1 \\ 2, & \text{if } x > 1 \end{cases}$

The given function is defined at all points of the real line.

Let *c* be a point on the real line.

Case I:

If $c < -1$, then $f(c) = -2$ and $\lim\limits_{x \to c} f(x) = \lim\limits_{x \to c} (-2) = -2$

$\therefore \lim\limits_{x \to c} f(x) = f(c)$

Therefore, *f* is continuous at all points *x*, such that $x < -1$

Case II:

If $c = -1$, then $f(c) = f(-1) = -2$

The left hand limit of *f* at $x = -1$ is,

$\lim\limits_{x \to -1^-} f(x) = \lim\limits_{x \to -1^-} (-2) = -2$

The right hand limit of *f* at $x = -1$ is,

$\lim\limits_{x \to -1^+} f(x) = \lim\limits_{x \to -1^+} (2x) = 2 \times (-1) = -2$

$\therefore \lim\limits_{x \to -1} f(x) = f(-1)$

Therefore, *f* is continuous at $x = -1$

Case III:

If $-1 < c < 1$, then $f(c) = 2c$

$\lim\limits_{x \to c} f(x) = \lim\limits_{x \to c} (2x) = 2c$

$\therefore \lim\limits_{x \to c} f(x) = f(c)$

Therefore, *f* is continuous at all points of the interval (- 1, 1).

Case IV:

If $c = 1$, then $f(c) = f(1) = 2 \times 1 = 2$

The left hand limit of *f* at $x = 1$ is,

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (2x) = 2 \times 1 = 2$$

The right hand limit of *f* at $x = 1$ is,

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 2 = 2$$

$$\therefore \lim_{x \to 1} f(x) = f(c)$$

Therefore, *f* is continuous at $x = 2$

Case V:

If $c > 1$, then $f(c) = 2$ and $\lim_{x \to c} f(x) = \lim_{x \to c} (2) = 2$

$$\lim_{x \to c} f(x) = f(c)$$

Therefore, *f* is continuous at all points *x*, such that $x > 1$

Thus, from the above observations, it can be concluded that *f* is continuous at all points of the real line.

Answer needs Correction? **Click Here**

*********** END **********