



Differentials Errors and Approximation Ex14.1 Q9(xxii)

$$\text{Let } x = \frac{16}{81}, \quad x + \Delta x = \frac{17}{81}$$

$$\begin{aligned}\Delta x &= \frac{17}{81} - \frac{16}{81} \\ &= \frac{1}{81}\end{aligned}$$

$$\text{Let } y = x^{\frac{1}{4}}$$

$$\frac{dy}{dx} = \frac{1}{4x^{\frac{3}{4}}}$$

$$\begin{aligned}\left(\frac{dy}{dx}\right)_{x=\frac{16}{81}} &= \frac{1}{4\left(\frac{16}{81}\right)^{\frac{3}{4}}} \\ &= \frac{27}{32} \\ &= 0.84375\end{aligned}$$

$$\begin{aligned}\Delta y &= \left(\frac{dy}{dx}\right)_{x=\frac{16}{81}} \times (\Delta x) \\ &= (0.84375) \left(\frac{1}{81}\right) \\ &= 0.01041\end{aligned}$$

$$\begin{aligned}
 \left(\frac{17}{81}\right)^{\frac{1}{4}} &= y + \Delta y \\
 &= \left(\frac{16}{81}\right)^{\frac{1}{4}} + 0.01041 \\
 &= 0.6666 + 0.01041
 \end{aligned}$$

$$\left(\frac{17}{81}\right)^{\frac{1}{4}} = 0.67707$$

Differentials Errors and Approximation Ex14.1 Q9(xxiii)

$$\begin{aligned}
 \text{Let } x &= 32, \quad x + \Delta x = 33 \\
 \Delta x &= 33 - 32 \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 \text{Let } y &= x^{\frac{1}{5}} \\
 \frac{dy}{dx} &= \frac{1}{5x^{\frac{4}{5}}} \\
 \left(\frac{dy}{dx}\right)_{x=32} &= \frac{1}{5(32)^{\frac{4}{5}}} \\
 &= \frac{1}{80} \\
 &= 0.0125
 \end{aligned}$$

$$\therefore \Delta y = \left(\frac{dy}{dx}\right)_{x=32} \times (\Delta x)$$

$$\begin{aligned} & \left( \frac{dx}{dy} \right)_{x=32} \\ &= (0.0125)(1) \\ \Delta y &= 0.0125 \end{aligned}$$

$$\begin{aligned} (33)^{\frac{1}{5}} &= y + \Delta y \\ &= x^{\frac{1}{5}} + 0.0125 \\ &= (32)^{\frac{1}{5}} + 0.0125 \end{aligned}$$

$$(33)^{\frac{1}{5}} = 2.0125$$

$$\begin{aligned}\text{Let } x &= 36, \quad x + \Delta x = 36.6 \\ \Delta x &= 36.6 - 36 \\ &= 0.6\end{aligned}$$

$$\begin{aligned}\text{Let } y &= \sqrt{x} \\ \frac{dy}{dx} &= \frac{1}{2\sqrt{x}} \\ \left(\frac{dy}{dx}\right)_{x=36} &= \frac{1}{2\sqrt{36}} \\ &= \frac{1}{12} \\ &= 0.0833\end{aligned}$$

$$\begin{aligned}\therefore \Delta y &= \left(\frac{dy}{dx}\right)_{x=36} \times (\Delta x) \\ &= (0.0833)(0.6) \\ &= 0.04998\end{aligned}$$

$$\begin{aligned}\sqrt{36.6} &= y + \Delta y \\ &= \sqrt{x} + 0.04998 \\ &= \sqrt{36} + 0.04998\end{aligned}$$

$$\sqrt{36.6} = 6.04998$$

Differentials Errors and Approximation Ex14.1 Q9(xxv)

$$\begin{aligned}\text{Let } x &= 27, \quad x + \Delta x = 25 \\ \Delta x &= 25 - 27 \\ &= -2\end{aligned}$$

$$\text{Let } y = x^{\frac{1}{3}}$$

$$\frac{dy}{dx} = \frac{1}{3x^{\frac{2}{3}}}$$

$$\left(\frac{dy}{dx}\right)_{x=27} = \frac{1}{3(27)^{\frac{2}{3}}}$$

$$= \frac{1}{27}$$

$$= 0.037$$

$$\therefore \Delta y = \left(\frac{dy}{dx}\right)_{x=27} \times (\Delta x)$$

$$= (0.037)(-2)$$

$$= -0.074$$

$$(25)^{\frac{1}{3}} = y + \Delta y$$

$$= x^{\frac{1}{3}} + (-0.074)$$

$$= (27)^{\frac{1}{3}} - 0.074$$

$$= 3 - 0.074$$

$$(25)^{\frac{1}{3}} = 2.926$$

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