



Quadratic Equations Ex 8.5 Q3

Answer :

(i) We have been given,

$$\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}, x \neq 2, 4$$

Now we solve the above equation as follows,

$$\begin{aligned}\frac{(x-1)(x-4) + (x-3)(x-2)}{(x-2)(x-4)} &= \frac{10}{3} \\ \frac{x^2 - 5x + 4 + x^2 - 5x + 6}{x^2 - 6x + 8} &= \frac{10}{3} \\ 6x^2 - 30x + 30 &= 10x^2 - 60x + 80 \\ 4x^2 - 30x + 50 &= 0 \\ 2x^2 - 15x + 25 &= 0\end{aligned}$$

Now we also know that for an equation $ax^2 + bx + c = 0$, the discriminant is given by the following equation:

$$D = b^2 - 4ac$$

Now, according to the equation given to us, we have, $a = 2$, $b = -15$ and $c = 25$.

Therefore, the discriminant is given as,

$$\begin{aligned}D &= (-15)^2 - 4(2)(25) \\ &= 225 - 200 \\ &= 25\end{aligned}$$

Now, the roots of an equation is given by the following equation,

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

Therefore, the roots of the equation are given as follows,

$$\begin{aligned}x &= \frac{-(-15) \pm \sqrt{25}}{2(2)} \\ &= \frac{15 \pm 5}{4}\end{aligned}$$

Now we solve both cases for the two values of x . So, we have,

$$\begin{aligned}x &= \frac{15+5}{4} \\ &= 5\end{aligned}$$

Also,

$$\begin{aligned}x &= \frac{15-5}{4} \\ &= \frac{5}{2}\end{aligned}$$

Therefore, the value of $x = 5, \frac{5}{2}$

(ii) We have been given,

$$\frac{1}{x} - \frac{1}{x-2} = 3, x \neq 0, 2$$

Now we solve the above equation as follows,

$$\frac{(x-2)-x}{(x-2)(x)} = 3$$

$$\frac{-2}{x^2-2x} = 3$$

$$-2 = 3x^2 - 6x$$

$$3x^2 - 6x + 2 = 0$$

Now we also know that for an equation $ax^2 + bx + c = 0$, the discriminant is given by the following equation:

$$D = b^2 - 4ac$$

Now, according to the equation given to us, we have, $a = 3$, $b = -6$ and $c = 2$.

Therefore, the discriminant is given as,

$$\begin{aligned} D &= (-6)^2 - 4(3)(2) \\ &= 36 - 24 \\ &= 12 \end{aligned}$$

Now, the roots of an equation is given by the following equation,

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

Therefore, the roots of the equation are given as follows,

$$\begin{aligned} x &= \frac{-(-6) \pm \sqrt{12}}{2(3)} \\ &= \frac{6 \pm 2\sqrt{3}}{6} \\ &= \frac{3 \pm \sqrt{3}}{3} \end{aligned}$$

Now we solve both cases for the two values of x . So, we have,

$$x = \frac{3 + \sqrt{3}}{3}$$

Also,

$$x = \frac{3 - \sqrt{3}}{3}$$

Therefore, the value of $x = \frac{3 + \sqrt{3}}{3}, \frac{3 - \sqrt{3}}{3}$

(iii) We have been given,

$$x + \frac{1}{x} = 3, x \neq 0$$

Now, we solve the equation as follows:

$$\frac{x^2 + 1}{x} = 3$$

$$x^2 + 1 = 3x$$

$$x^2 - 3x + 1 = 0$$

Now we also know that for an equation $ax^2 + bx + c = 0$, the discriminant is given by the following equation:

$$D = b^2 - 4ac$$

Now, according to the equation given to us, we have, $a = 1$, $b = -3$ and $c = 1$.

Therefore, the discriminant is given as,

$$\begin{aligned} D &= (-3)^2 - 4(1)(1) \\ &= 9 - 4 \\ &= 5 \end{aligned}$$

Now, the roots of an equation is given by the following equation,

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

Therefore, the roots of the equation are given as follows,

$$x = \frac{-(-3) \pm \sqrt{5}}{2(1)}$$

$$= \frac{3 \pm \sqrt{5}}{2}$$

Now we solve both cases for the two values of x . So, we have,

$$x = \frac{3 + \sqrt{5}}{2}$$

Also,

$$x = \frac{3 - \sqrt{5}}{2}$$

Therefore, the value of $x = \frac{3 + \sqrt{5}}{2}, \frac{3 - \sqrt{5}}{2}$

(iv) We have been given,

$$\frac{16}{x} - 1 = \frac{15}{x+1}, x \neq 0, -1$$

Now we solve the above equation as follows,

$$\frac{16-x}{x} = \frac{15}{x+1}$$

$$\Rightarrow (16-x)(x+1) = 15x$$

$$\Rightarrow 16x + 16 - x^2 - x = 15x$$

$$\Rightarrow 15x + 16 - x^2 - 15x = 0$$

$$\Rightarrow 16 - x^2 = 0$$

$$\Rightarrow x^2 - 16 = 0$$

Now we also know that for an equation $ax^2 + bx + c = 0$, the discriminant is given by the following equation:

$$D = b^2 - 4ac$$

Now, according to the equation given to us, we have, $a = 1$, $b = 0$ and $c = -16$.

Therefore, the discriminant is given as,

$$D = (0)^2 - 4(1)(-16) \\ = 64$$

Now, the roots of an equation is given by the following equation,

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

Therefore, the roots of the equation are given as follows,

$$x = \frac{-0 \pm \sqrt{64}}{2(1)} \\ = \frac{\pm 8}{2} \\ = \pm 4$$

Therefore, the value of $x = \pm 4$.

***** END *****