Exercise 1C

Questions 3:

(i) If possible, let $\sqrt{6}$ be rational and let its simplest form be $\frac{a}{b}$ then, a and b are integers having no common factor other than 1, and $b \neq 0$.

Now, $\sqrt{6} = \frac{a}{b} \Rightarrow 6 = \frac{a^2}{b^2}$ [on squaring both sides]

$\Rightarrow \quad 6b^2 = a^2 \quad .....(1)$

$\Rightarrow 6$ divides $a^2$ $\quad [\because 6 \text{ divides } 6b^2]$

$\Rightarrow 6$ divides a

Let a = 6c for some integer c

Putting a = 6c in (1), we get

$6b^2 = 36c^2 \Rightarrow b^2 = 6c^2$

$\Rightarrow 6$ divides $b^2$ $\quad [\because 6 \text{ divides } 6c^2]$

$\Rightarrow 6$ divides b $\quad [\because 6 \text{ divides } b^2 = 6 \text{ divides } b]$

Thus, 6 is a common factor of a and b

But, this contradicts the fact that a and b have no common factor other than 1

The contradiction arises by assuming that $\sqrt{6}$ is rational.

Hence $\sqrt{6}$ is irrational.

(ii) If possible let $2 - \sqrt{3}$ is rational

$\Rightarrow 2 - \left(2 - \sqrt{3}\right)$ is rational

[$\because$ difference of two rationals is rational]

$\therefore \sqrt{3}$ is rational

This contradicts the fact $\sqrt{3}$ is irrational

Since the contradiction arises by assuming $2 - \sqrt{3}$ rational.

Hence, $2 - \sqrt{3}$ is irrational.

(iii) If possible let $3+\sqrt{2}$ is rational

$\Rightarrow \left(3+\sqrt{2}\right)-3=\sqrt{2}$ is rational

[$\because$ difference of two rational is rational]

$\therefore$ $\sqrt{2}$ is rational

This contradicts the fact that $\sqrt{2}$ is irrational

Since the contradiction arises by assuming that $3+\sqrt{2}$ is rational.

Hence $3+\sqrt{2}$ is irrational.

(iv) If possible, let $2+\sqrt{5}$ is rational.

$\Rightarrow \left(2+\sqrt{5}\right)-2=\sqrt{5}$ is rational

[$\because$ difference of two rational is rational]

$\therefore$ $\sqrt{5}$ is rational.

This contradicts the fact that $\sqrt{5}$ is irrational

Since, the contradiction arises by assuming $2+\sqrt{5}$ is rational.

Hence, $2+\sqrt{5}$ is irrational.

(v) If possible, let $5+3\sqrt{2}$ is rational

Now, $\left(5+3\sqrt{2}\right)-5=3\sqrt{2}$ is rational

[$\because$ Difference of two rational is rational]

Also, $\frac{1}{3}\times 3\sqrt{2}=\sqrt{2}$ is rational

[$\because$ Product of two rational is rational]

$\therefore$ $\sqrt{2}$ is rational.

This contradicts the fact that $\sqrt{2}$ is irrational.

Since, the contradiction arises by assuming that $5+3\sqrt{2}$ is irrational.

Hence, $5+3\sqrt{2}$ is irrational

(vi) If possible, let $3\sqrt{7}$ be rational.

Let its simplest form be $3\sqrt{7}=\frac{a}{b}$, where a and b are positive integers having no common factor other than 1, then

$3\sqrt{7}=\frac{a}{b}\Rightarrow$

$\sqrt{7}=\frac{a}{3b}$ - - - - - (2)

Since, a and 3b are non –integers, so $\frac{a}{3b}$ is rational.

Thus, from (2), its follows that $\sqrt{7}$ is rational.

This contradicts the fact that $\sqrt{7}$ is irrational.

The contradiction arises by assuming that $3\sqrt{7}$ is rational.

Hence, $3\sqrt{7}$ is irrational.

(vii) $\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3}{5}.\sqrt{5}$ -------(3)

If possible, let $\frac{3}{\sqrt{5}}$ be rational.

Then, from (3), it follows that $\frac{3}{5}.\sqrt{5}$ is rational

Let $\frac{3}{5}\sqrt{5} = \frac{a}{b}$, where a and b are non-zero integers having no common factor other than 1.

Now,

$\frac{3\sqrt{5}}{5} = \frac{a}{b} \Rightarrow$

$\sqrt{5} = \frac{5a}{3b}$ - - - - - (4)

But, 3a and 5b are non-zero integers.

$\therefore$ $\frac{5a}{3b}$ is rational.

Thus, from (4), it follows that $\sqrt{5}$ is rational.

This contradicts the fact that $\sqrt{5}$ is irrational.

The contradiction arises by assuming that $\frac{3}{\sqrt{5}}$ is rational.

Hence $\frac{3}{\sqrt{5}}$ is irrational.

(viii) If possible, let $2 - 3\sqrt{5}$ is rational.

$\Rightarrow \left(2 - 3\sqrt{5}\right) - 2 = -3\sqrt{5}$ is rational.

[$\because$ Difference of two rational is rational]

$\Rightarrow \left(-\frac{1}{3}\right) \times \left(-3\sqrt{5}\right) = \sqrt{5}$ is rational.

[$\because$ Product of two rationals is rational]

This contradicts that fact that $\sqrt{5}$ is irrational.

Since, the contradiction arises by assuming $2 - 3\sqrt{5}$ is rational.

Hence, $2 - 3\sqrt{5}$ is irrational.

(ix) If possible, let $\left(\sqrt{3} + \sqrt{5}\right)$ be rational

Let $\sqrt{3} + \sqrt{5} = a$, where a is rational.

$\therefore$ $\sqrt{3} = a - \sqrt{5}$

Squaring both sides, we get

$$3 = \left(a - \sqrt{5}\right)^2 = a^2 + 5 - 2a\sqrt{5}$$

$$\Rightarrow \quad a^2 + 2 - 2a\sqrt{5} = 0$$

$$\therefore \quad \sqrt{5} = \frac{a^2 + 2}{2a} \quad - - - - - (5)$$

But, $\frac{a^2 + 2}{2a}$ is a rational number.

Thus from (5), $\sqrt{5}$ is rational.

This contradicts the fact that $\sqrt{5}$ is irrational.

Since, the contradiction arises by assuming $\left(\sqrt{3} + \sqrt{5}\right)$ is rational.

Hence $\left(\sqrt{3} + \sqrt{5}\right)$ is irrational.

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