Binomial Theorem Ex 18.2 Q27

We are given,

$$T_3 = a,\ \ T_4 = b,\ \ T_5 = c,\ \ T_6 = d$$

We have to prove that

$$\frac{b^2 - ac}{c^2 - bd} = \frac{5a}{3c}$$

$$\Rightarrow \quad \frac{b^2 - ac}{a} = \frac{5}{3}\left[\frac{c^2 - bd}{c}\right]$$

$$\Rightarrow \quad \frac{1}{b}\left[\frac{b^2 - ac}{a}\right] = \frac{5}{3}\left[\frac{c^2 - bd}{bc}\right]$$

$$\Rightarrow \quad \frac{b}{a} - \frac{c}{b} = \frac{5}{3}\left[\frac{c}{b} - \frac{d}{c}\right] \qquad \text{---(i)}$$

Now we know,

$$a = {}^nC_2 x^{n-2}\alpha^2$$

$$b = {}^nC_3 x^{n-3}\alpha^3$$

$$c = {}^nC_4 x^{n-4}\alpha^4$$

$$d = {}^nC_5 x^{n-5}\alpha^5$$

Putting these values in equation (i), we get

$$\frac{{}^nC_3 x^{n-3}\alpha^3}{{}^nC_2 x^{n-2}\alpha^2} - \frac{{}^nC_4 x^{n-4}\alpha^4}{{}^nC_3 x^{n-3}\alpha^3} = \frac{5}{3}\left[\frac{{}^nC_4 x^{n-4}\alpha^4}{{}^nC_3 x^{n-3}\alpha^3} - \frac{{}^nC_5 x^{n-5}\alpha^5}{{}^nC_4 x^{n-4}\alpha^4}\right]$$

$$\Rightarrow \quad \left[\frac{{}^nC_3}{{}^nC_2} - \frac{{}^nC_4}{{}^nC_3}\right]\frac{\alpha}{x} = \frac{5\alpha}{3x}\left[\frac{{}^nC_4}{{}^nC_3} - \frac{{}^nC_5}{{}^nC_4}\right]$$

We know that,

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n - r + 1}{r}$$

$\therefore$ The given equation above becomes,

$$\left[\frac{n-2}{3} - \frac{n-3}{4}\right] = \frac{5}{3}\left[\frac{n-3}{4} - \frac{n-4}{5}\right]$$

$$\Rightarrow \quad \frac{4n - 8 - 3n + 9}{3 \times 4} = \frac{5n - 15 - 4n + 16}{3 \times 4}$$

$$\Rightarrow \quad \frac{n+1}{12} = \frac{n+1}{12}$$

Which is true.

Hence proved.

Binomial Theorem Ex 18.2 Q28

Suppose the binomial is $(x+\alpha)^n$

We are given,

$$T_6 = a,\ T_7 = b,\ T_8 = c,\ T_9 = d$$

We have to prove that

$$\frac{b^2 - ac}{c^2 - bd} = \frac{4a}{3c}$$

$$\Rightarrow \quad \frac{b^2 - ac}{a} = \frac{4}{3}\left[\frac{c^2 - bd}{c}\right]$$

$$\Rightarrow \quad \frac{1}{b}\left[\frac{b^2 - ac}{a}\right] = \frac{4}{3}\left[\frac{c^2 - bd}{bc}\right]$$

$$\Rightarrow \quad \frac{b}{a} - \frac{c}{b} = \frac{4}{3}\left[\frac{c}{b} - \frac{d}{c}\right] \qquad ---(i)$$

Now we know,

$$a = {}^nC_5 x^{n-5}\alpha^5$$
$$b = {}^nC_6 x^{n-6}\alpha^6$$
$$c = {}^nC_7 x^{n-7}\alpha^7$$
$$d = {}^nC_8 x^{n-8}\alpha^8$$

Putting these values in equation (i), we get

$$\frac{{}^nC_6 x^{n-6}\alpha^6}{{}^nC_5 x^{n-5}\alpha^5} - \frac{{}^nC_7 x^{n-7}\alpha^7 v}{{}^nC_6 x^{n-6}\alpha^6} = \frac{4}{3}\left[\frac{{}^nC_7 x^{n-7}\alpha^7}{{}^nC_6 x^{n-6}\alpha^6} - \frac{{}^nC_8 x^{n-8}\alpha^8}{{}^nC_7 x^{n-7}\alpha^7}\right]$$

$$\Rightarrow \quad \left[\frac{{}^nC_6}{{}^nC_5} - \frac{{}^nC_7}{{}^nC_6}\right]\frac{\alpha}{x} = \frac{4\alpha}{3x}\left[\frac{{}^nC_7}{{}^nC_6} - \frac{{}^nC_8}{{}^nC_7}\right]$$

We know that,

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$$

$\therefore$ The given equation above becomes,

$$\left[\frac{n-5}{6} - \frac{n-6}{7}\right] = \frac{4}{3}\left[\frac{n-6}{7} - \frac{n-7}{8}\right]$$

$$\Rightarrow \quad \frac{7n - 35 - 6n + 36}{6 \times 7} = \frac{8n - 48 - 7n + 49}{3 \times 7 \times 2}$$

$$\Rightarrow \quad \frac{n+1}{42} = \frac{n+1}{42}$$

Which is true.

Hence proved.

Binomial Theorem Ex 18.2 Q29

We have,

$$(1+x)^n$$

Let the three consecutive terms are T_r, T_{r+1} and T_{r+2}

$\therefore$ Coefficients of rth term $= {}^nC_{r-1} = 76$

Coefficients of $(r+1)$th term $= {}^nC_{r+1-1} = {}^nC_r = 95$

and, Coefficients of $(r+2)$th term $= {}^nC_{r+2-1} = {}^nC_{r+1} = 76$

Now,

$$\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{76}{95}$$

$$\Rightarrow \quad \frac{n-(r+1)+1}{r+1} = \frac{76}{95} \qquad \left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}\right]$$

$$\Rightarrow \quad \frac{n-r-1}{r+1} = \frac{4}{5}$$

$$\Rightarrow \quad \frac{n-r}{r+1} = \frac{4}{5}$$

$$\Rightarrow \quad 5n - 5r = 4r + 4$$

$$\Rightarrow \quad 5n - 5r - 4r = 4$$

$$\Rightarrow \quad 5n - 9r = 4 \qquad ---(i)$$

and,

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{95}{76}$$

$$\Rightarrow \quad \frac{n-r+1}{r} = \frac{5}{4}$$

$$\Rightarrow \quad 4n - 4r + 4 = 5r$$

$$\Rightarrow \quad 4n - 9r = -4 \qquad ---(ii)$$

Subtracting equation (ii) from (i), we get

$$n = 4 + 4$$

$$\Rightarrow \quad n = 8$$

********** END *********