Definite Integrals Ex 20.1 Q43

We have,

$$\int_0^4 \frac{dx}{\sqrt{4x - x^2}}$$

$$= \int_0^4 \frac{dx}{\sqrt{4 - 4 + 4x - x^2}} \qquad \left[\text{Add and subtract 4 in denominator}\right]$$

$$= \int_0^4 \frac{dx}{\sqrt{4 - \left(x^2 - 4x + 4\right)}}$$

$$= \int_0^4 \frac{dx}{\sqrt{(2)^2 - (x-2)^2}}$$

$$= \left[\sin^{-1}\left(\frac{x-2}{2}\right)\right]_0^4 \qquad \left[\because \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a}\right]$$

$$= \sin^{-1}(1) - \sin^{-1}(-1)$$

$$= \frac{\pi}{2} - \left(-\frac{\pi}{2}\right)$$

$$= \frac{2\pi}{2} = \pi$$

$$\therefore \int_0^4 \frac{dx}{\sqrt{4x - x^2}} = \pi$$

Definite Integrals Ex 20.1 Q44

$$\int_{-1}^{1} \frac{dx}{x^2 + 2x + 5} = \int_{-1}^{1} \frac{dx}{\left(x^2 + 2x + 1\right) + 4} = \int_{-1}^{1} \frac{dx}{(x+1)^2 + (2)^2}$$

Let $x + 1 = t \Rightarrow dx = dt$

When $x = -1$, $t = 0$ and when $x = 1$, $t = 2$

$$\therefore \int_{-1}^{1} \frac{dx}{(x+1)^2 + (2)^2} = \int_0^2 \frac{dt}{t^2 + 2^2}$$

$$= \left[\frac{1}{2}\tan^{-1}\frac{t}{2}\right]_0^2$$

$$= \frac{1}{2}\tan^{-1}1 - \frac{1}{2}\tan^{-1}0$$

$$= \frac{1}{2}\left(\frac{\pi}{4}\right) = \frac{\pi}{8}$$

Definite Integrals Ex 20.1 Q45

We have,

$$\int_1^4 \frac{x^2+x}{\sqrt{2x+1}}\,dx$$

Let $2x+1=t^2$

$\Rightarrow 2dx = 2t\,dt$

Now,

$x=1 \Rightarrow t=\sqrt{3}$

$x=4 \Rightarrow t=3$

$$\therefore \int_1^4 \frac{x^2+x}{\sqrt{2x+1}}\,dx = \int_{\sqrt{3}}^3 \frac{\left(\frac{t^2-1}{2}\right)^2+\left(\frac{t^2-1}{2}\right)}{t}\,t\,dt$$

$$= \frac{1}{4}\int_{\sqrt{3}}^3 \left(t^4-2t^2+1+2t^2-2\right)dt$$

$$= \frac{1}{4}\int_{\sqrt{3}}^3 t^4-1$$

$$= \frac{1}{4}\left[\frac{t^5}{5}-t\right]_{\sqrt{3}}^3$$

$$= \frac{1}{4}\left[\frac{243-9\sqrt{3}}{5}-3+\sqrt{3}\right]$$

$$= \frac{1}{4}\left[\frac{228}{5}-\sqrt{3}\,(4)\right]$$

$$= \frac{57-\sqrt{3}}{5}$$

$$\therefore \int_1^4 \frac{x^2+x}{\sqrt{2x+1}}\,dx = \frac{57-\sqrt{3}}{5}$$

Definite Integrals Ex 20.1 Q46

We have,

$$\int_0^1 x(1-x)^5\,dx$$

Expanding $(1-x)^5$ by Binomial theorem

$$\therefore\ (1-x)^5 = 1^5 + {}^5C_1(-x) + {}^5C_2(-x)^2 + {}^5C_3(-x)^3 + {}^5C_4(-x)^4 + {}^5C_5(-x)^5$$
$$= 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$$

$$= \int_0^1 x\left(1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5\right)dx$$

$$= \left[\frac{x^2}{2} - \frac{5x^3}{3} + \frac{10x^4}{4} - \frac{10x^5}{5} + \frac{5x^6}{6} - \frac{x^7}{7}\right]_0^1$$

$$= \frac{1}{2} - \frac{5}{3} + \frac{10}{4} - \frac{10}{5} + \frac{5}{6} - \frac{1}{7}$$

$$= \frac{1}{42}$$

$$\therefore\ \int_0^1 x(1-x)^5\,dx = \frac{1}{42}$$

*********** END **********