



Trigonometric Identities Ex 6.1 Q83

Answer :

(i) We have,

$$\begin{aligned}\frac{\sqrt{\sec \theta - 1}}{\sqrt{\sec \theta + 1}} + \frac{\sqrt{\sec \theta + 1}}{\sqrt{\sec \theta - 1}} &= \frac{\sqrt{\sec \theta - 1}}{\sqrt{\sec \theta + 1}} + \frac{\sqrt{\sec \theta + 1}}{\sqrt{\sec \theta - 1}} \\&= \frac{\sqrt{\sec \theta - 1} \sqrt{\sec \theta - 1} + \sqrt{\sec \theta + 1} \sqrt{\sec \theta + 1}}{\sqrt{\sec \theta + 1} \sqrt{\sec \theta - 1}} \\&= \frac{(\sqrt{\sec \theta - 1})^2 + (\sqrt{\sec \theta + 1})^2}{\sqrt{(\sec \theta - 1)(\sec \theta + 1)}} \\&= \frac{\sec \theta - 1 + \sec \theta + 1}{\sqrt{\sec^2 \theta - 1}} \\&= \frac{2 \sec \theta}{\sqrt{\tan^2 \theta}} \\&= \frac{2 \sec \theta}{\tan \theta} \\&= \frac{2 \frac{1}{\cos \theta}}{\frac{\sin \theta}{\cos \theta}} \\&= 2 \frac{1}{\sin \theta} \\&= 2 \operatorname{cosec} \theta\end{aligned}$$

(ii) We have,

$$\begin{aligned}
 \sqrt{\frac{1+\sin \theta}{1-\sin \theta}} + \sqrt{\frac{1-\sin \theta}{1+\sin \theta}} &= \frac{\sqrt{1+\sin \theta}}{\sqrt{1-\sin \theta}} + \frac{\sqrt{1-\sin \theta}}{\sqrt{1+\sin \theta}} \\
 &= \frac{\sqrt{1+\sin \theta}\sqrt{1+\sin \theta} + \sqrt{1-\sin \theta}\sqrt{1-\sin \theta}}{\sqrt{1-\sin \theta}\sqrt{1+\sin \theta}} \\
 &= \frac{(\sqrt{1+\sin \theta})^2 + (\sqrt{1-\sin \theta})^2}{\sqrt{(1-\sin \theta)(1+\sin \theta)}} \\
 &= \frac{1+\sin \theta + 1-\sin \theta}{\sqrt{1-\sin^2 \theta}} \\
 &= \frac{2}{\sqrt{\cos^2 \theta}} \\
 &= \frac{2}{\cos \theta} \\
 &= 2 \sec \theta
 \end{aligned}$$

(iii) We have,

$$\begin{aligned}
 \sqrt{\frac{1+\cos \theta}{1-\cos \theta}} + \sqrt{\frac{1-\cos \theta}{1+\cos \theta}} &= \frac{\sqrt{1+\cos \theta}}{\sqrt{1-\cos \theta}} + \frac{\sqrt{1-\cos \theta}}{\sqrt{1+\cos \theta}} \\
 &= \frac{\sqrt{1+\cos \theta}\sqrt{1+\cos \theta} + \sqrt{1-\cos \theta}\sqrt{1-\cos \theta}}{\sqrt{1-\cos \theta}\sqrt{1+\cos \theta}} \\
 &= \frac{(\sqrt{1+\cos \theta})^2 + (\sqrt{1-\cos \theta})^2}{\sqrt{(1-\cos \theta)(1+\cos \theta)}} \\
 &= \frac{1+\cos \theta + 1-\cos \theta}{\sqrt{1-\cos^2 \theta}} \\
 &= \frac{2}{\sqrt{\sin^2 \theta}} \\
 &= \frac{2}{\sin \theta} \\
 &= 2 \operatorname{cosec} \theta
 \end{aligned}$$

(iv) We have,

$$\begin{aligned}
 \frac{\sec \theta - 1}{\sec \theta + 1} &= \frac{\frac{1}{\cos \theta} - 1}{\frac{1}{\cos \theta} + 1} \\
 &= \frac{\frac{1-\cos \theta}{\cos \theta}}{\frac{1+\cos \theta}{\cos \theta}} \\
 &= \frac{1-\cos \theta}{1+\cos \theta}
 \end{aligned}$$

Multiplying both the numerator and the denominator by $(1+\cos \theta)$, we have

$$\begin{aligned}
 \frac{\sec \theta - 1}{\sec \theta + 1} &= \frac{(1-\cos \theta)(1+\cos \theta)}{(1+\cos \theta)(1+\cos \theta)} \\
 &= \frac{(1-\cos^2 \theta)}{(1+\cos \theta)^2}
 \end{aligned}$$

(v) We have,

$$\frac{\sin \theta + 1 - \cos \theta}{\cos \theta - 1 + \sin \theta} = \frac{\sin \theta + (1 - \cos \theta)}{\sin \theta - (1 - \cos \theta)}$$

Multiplying both the numerator and the denominator by $\sin \theta + (1 - \cos \theta)$, we have

$$\frac{\sin \theta + 1 - \cos \theta}{\cos \theta - 1 + \sin \theta} = \frac{\{\sin \theta + (1 - \cos \theta)\} \{\sin \theta + (1 - \cos \theta)\}}{\{\sin \theta - (1 - \cos \theta)\} \{\sin \theta + (1 - \cos \theta)\}}$$

$$\begin{aligned}
&= \frac{\{\sin \theta + (1 - \cos \theta)\}^2}{\{\sin^2 \theta - (1 - \cos \theta)^2\}} \\
&= \frac{\sin^2 \theta + 2 \sin \theta (1 - \cos \theta) + (1 - \cos \theta)^2}{\sin^2 \theta - (1 - 2 \cos \theta + \cos^2 \theta)} \\
&= \frac{\sin^2 \theta + 2 \sin \theta - 2 \sin \theta \cos \theta + (1 - 2 \cos \theta + \cos^2 \theta)}{\sin^2 \theta - (\sin^2 \theta + \cos^2 \theta - 2 \cos \theta + \cos^2 \theta)} \\
&= \frac{(\sin^2 \theta + \cos^2 \theta) + 2 \sin \theta - 2 \sin \theta \cos \theta + 1 - 2 \cos \theta}{-2 \cos^2 \theta + 2 \cos \theta} \\
&= \frac{1 + 2 \sin \theta - 2 \sin \theta \cos \theta + 1 - 2 \cos \theta}{-2 \cos^2 \theta + 2 \cos \theta} \\
&= \frac{2 + 2 \sin \theta - 2 \sin \theta \cos \theta - 2 \cos \theta}{2 \cos \theta (1 - \cos \theta)} \\
&= \frac{2(1 + \sin \theta) - 2 \cos \theta (\sin \theta + 1)}{2 \cos \theta (\cos \theta - 1)} \\
&= \frac{(1 + \sin \theta)(2 - 2 \cos \theta)}{2 \cos \theta (1 - \cos \theta)} \\
&= \frac{2(1 + \sin \theta)(1 - \cos \theta)}{2 \cos \theta (1 - \cos \theta)} \\
&= \frac{1 + \sin \theta}{\cos \theta}
\end{aligned}$$

***** END *****