

Definite Integrals Ex 20.4A Q1

We know

$$\int_{0}^{2\pi} f(x)dx = \int_{0}^{2\pi} f(2\Pi - x)dx$$

Hence

$$\int_{0}^{2\Pi} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}} dx = \int_{0}^{2\Pi} \frac{e^{\sin(2\Pi - x)}}{e^{\sin(2\Pi - x)} + e^{-\sin(2\Pi - x)}} dx$$

We know

$$\sin(2\Pi - x) = -\sin x$$

$$\int_{0}^{2\pi} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}} dx = \int_{0}^{2\pi} \frac{e^{-\sin x}}{e^{-\sin x} + e^{\sin x}} dx$$

lf

$$I = \int_{0}^{2\pi} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}} dx$$

Then also

$$I = \int_{0}^{2\pi} \frac{e^{-\sin x}}{e^{-\sin x} + e^{\sin x}} dx$$

Hence

$$2I = \int_{0}^{2\pi} \frac{e^{-\sin x}}{e^{-\sin x} + e^{\sin x}} dx + \int_{0}^{2\pi} \frac{e^{\sin x}}{e^{-\sin x} + e^{\sin x}} dx$$

$$2I = \int_{0}^{2\pi i} \frac{e^{-\sin x}}{e^{-\sin x} + e^{\sin x}} + \frac{e^{\sin x}}{e^{-\sin x} + e^{\sin x}} dx$$

$$2I = \int_{0}^{2\pi} dx$$

$$2I = 2\Pi$$

$$I = \Pi$$

Definite Integrals Ex 20.4A Q2

We know
$$\int_{0}^{2\pi} f(x)dx = \int_{0}^{2\pi} f(2\Pi - x)dx$$
Hence
$$\int_{0}^{2\pi} \log(\sec x + \tan x)dx = \int_{0}^{2\pi} \log(\sec(2\Pi - x) + \tan(2\Pi - x))dx$$

$$\int_{0}^{2\pi} \log(\sec x + \tan x)dx = \int_{0}^{2\pi} \log(\sec x - \tan x)dx$$
If
$$I = \int_{0}^{2\pi} \log(\sec x + \tan x)dx$$
Then
$$I = \int_{0}^{2\pi} \log(\sec x - \tan x)dx$$

$$2I = \int_{0}^{2\pi} \log(\sec x + \tan x)dx + \int_{0}^{2\pi} \log(\sec x - \tan x)dx$$

$$2I = \int_{0}^{2\pi} \log(\sec x + \tan x) + \log(\sec x - \tan x)dx$$

$$2I = \int_{0}^{2\pi} \log(\sec^2 x - \tan^2 x)dx$$

$$2I = \int_{0}^{2\pi} \log(\sec^2 x - \tan^2 x)dx$$

$$2I = \int_{0}^{2\pi} \log(\sin^2 x - \sin^2 x)dx$$

Definite Integrals Ex 20.4A Q3

We know
$$\int_{a}^{b} f(x)dx = \int_{a}^{b} f(a+b-x)dx$$
Hence
$$\frac{\pi}{\frac{3}{6}} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan(\frac{\Pi}{2} - x)}}{\sqrt{\tan(\frac{\Pi}{2} - x)} + \sqrt{\cot(\frac{\Pi}{2} - x)}} dx$$

$$\frac{\pi}{\frac{3}{6}} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cot x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx$$
If
$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx$$
Then
$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx$$
So
$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cot x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\sqrt{\tan x}} dx + \sqrt{\cot x} dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1 dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1 dx$$

$$2I = \frac{\pi}{6}$$

$$I = \frac{\Pi}{12}$$

******* END *******