Higher Order Derivatives Ex 12.1 Q34

It is given that, $y = e^{a\cos^{-1}x}$

Taking logarithm on both the sides, we obtain

$\log y = a\cos^{-1}x \log e$

$\log y = a\cos^{-1}x$

Differentiating both sides with respect to x, we obtain

$\dfrac{1}{y}\dfrac{dy}{dx} = a \times \dfrac{-1}{\sqrt{1-x^2}}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{-ay}{\sqrt{1-x^2}}$

By squaring both the sides, we obtain

$\left(\dfrac{dy}{dx}\right)^2 = \dfrac{a^2 y^2}{1-x^2}$

$\Rightarrow (1-x^2)\left(\dfrac{dy}{dx}\right)^2 = a^2 y^2$

$(1-x^2)\left(\dfrac{dy}{dx}\right)^2 = a^2 y^2$

Again differentiating both sides with respect to x, we obtain

$\left(\dfrac{dy}{dx}\right)^2 \dfrac{d}{dx}(1-x^2) + (1-x^2) \times \dfrac{d}{dx}\left[\left(\dfrac{dy}{dx}\right)^2\right] = a^2 \dfrac{d}{dx}(y^2)$

$\Rightarrow \left(\dfrac{dy}{dx}\right)^2 (-2x) + (1-x^2) \times 2\dfrac{dy}{dx}.\dfrac{d^2y}{dx^2} = a^2.2y.\dfrac{dy}{dx}$

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$\Rightarrow -x\dfrac{dy}{dx} + (1-x^2)\dfrac{d^2y}{dx^2} = a^2.y \qquad \left[\dfrac{dy}{dx} \neq 0\right]$

$\Rightarrow (1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - a^2 y = 0$

Hence, proved.

Higher Order Derivatives Ex 12.1 Q35

It is given that, $y = 500e^{7x} + 600e^{-7x}$

Then,

$$\frac{dy}{dx} = 500 \cdot \frac{d}{dx}\left(e^{7x}\right) + 600 \cdot \frac{d}{dx}\left(e^{-7x}\right)$$

$$= 500 \cdot e^{7x} \cdot \frac{d}{dx}(7x) + 600 \cdot e^{-7x} \cdot \frac{d}{dx}(-7x)$$

$$= 3500e^{7x} - 4200e^{-7x}$$

$$\therefore \frac{d^2y}{dx^2} = 3500 \cdot \frac{d}{dx}\left(e^{7x}\right) - 4200 \cdot \frac{d}{dx}\left(e^{-7x}\right)$$

$$= 3500 \cdot e^{7x} \cdot \frac{d}{dx}(7x) - 4200 \cdot e^{-7x} \cdot \frac{d}{dx}(-7x)$$

$$= 7 \times 3500 \cdot e^{7x} + 7 \times 4200 \cdot e^{-7x}$$

$$= 49 \times 500 e^{7x} + 49 \times 600 e^{-7x}$$

$$= 49\left(500e^{7x} + 600e^{-7x}\right)$$

$$= 49y$$

Hence, proved

Higher Order Derivatives Ex 12.1 Q36

$y = 2\cos t - \cos 2t; \quad y = 2\sin t - \sin 2t$

differentiating w.r.t. t

$\Rightarrow \quad \frac{dy}{dt} = 2(-\sin t) - 2(-\sin 2t); \quad \frac{dy}{dt} = 2\cos t - 2\cos 2t$

dividing (2) by (1)

$\Rightarrow \quad \frac{dy}{dx} = \frac{2(\cos t - \cos 2t)}{2(\sin 2t - \sin t)}$

differentiating w.r.t. t

$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{dt} = \frac{(\sin 2t - \sin t)(-\sin t + 2\sin 2t) - (\cos t - \cos 2t)(2\cos 2t - \cos t)}{(\sin 2t - \sin t)^2}$ ....(3)

dividing (3) by (1)

$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{(\sin 2t - \sin t)(2\sin 2t - \sin t) - (\cos t - \cos 2t)(2\cos 2t - \cos t)}{2(\sin 2t - \sin t)^3}$

Putting $t = \pi/2$

$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{(0-1)(0-1) - (0-(-1))(2(-1)-0)}{2(0-1)^3} = \frac{1+2}{-2} = \frac{-3}{2}$

Higher Order Derivatives Ex 12.1 Q37

$x = 4z^2 + 5$ $\qquad\qquad$ $y = 6z^2 + 72 + 3$

differentiating both w.r.t. z

$\Rightarrow \qquad \dfrac{dx}{dz} = 8z + 0 \qquad\qquad \dfrac{dy}{dz} = 12z + 7$

$\Rightarrow \qquad \dfrac{dx}{dz} = \dfrac{12z + 7}{8z} = \dfrac{12z}{8z} + \dfrac{7}{8z}$

differentiating w.r.t. z

$\Rightarrow \qquad \dfrac{d\left(\dfrac{dy}{dx}\right)}{dz} = 0 + \dfrac{7}{8}\left(\dfrac{-1}{z^2}\right) \qquad\qquad \dots\dots (3)$

dividing (3) by (1)

$\Rightarrow \qquad \dfrac{d^2y}{dx^2} = \dfrac{-7}{8z^2 \times 8z} = \dfrac{-7}{64z^3}$

********** END *********