Trigonometric Ratios of multiple and Sub-multiple Angles Ex 9.1 Q 43

We have,

$$\sin\alpha = \frac{4}{5} \quad \& \quad \cos\beta = \frac{5}{13} \qquad \Rightarrow \cos\alpha = \frac{3}{5} \ \& \ \sin\beta = \frac{12}{13}$$

$$\therefore \cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha.\sin\beta$$

$$= \frac{3}{5}\cdot\frac{5}{13} + \frac{4}{5}\cdot\frac{12}{13}$$

$$= \frac{15}{65} + \frac{48}{65} = \frac{63}{65}$$

Now,

$$\cos\left(\frac{\alpha-\beta}{2}\right) = \sqrt{\frac{1+\cos(\alpha-\beta)}{2}}$$

$$= \sqrt{\frac{1+\frac{63}{65}}{2}}$$

$$= \sqrt{\frac{128}{65\times 2}} = \sqrt{\frac{64}{65}}$$

$$= \pm\frac{8}{\sqrt{65}}$$

$$\therefore \cos\left(\frac{\alpha-\beta}{2}\right) = \frac{8}{\sqrt{65}}$$

Trigonometric Ratios of multiple and Sub-multiple Angles Ex 9.1 Q44

$$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$Sin2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

substitute these values in the given equation, it reduces to

$$a(1-\tan^2\theta)+b(2\tan\theta)=c(1+\tan^2\theta)$$

$$(c+a)\tan^2\theta + 2b\tan\theta + c - a = 0$$

As α and β are roots

$$\text{sum of the roots, } \tan\alpha + \tan\beta = \frac{2b}{c+a}$$

$$\text{Product of roots, } \tan\alpha\ \tan\beta = \frac{c-a}{c+a}$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\ \tan\beta} = \frac{2b}{c+a-c+a} = \frac{b}{a}$$

Trigonometric Ratios of multiple and Sub-multiple Angles Ex 9.1 Q 45

$\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$

squaring on both sides gives

$\cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta = \sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta$

Bring square terms on one side, we get

$\cos 2\alpha + \cos 2\beta = -2(-\sin\alpha\sin\beta + \cos\alpha\cos\beta) = -2\cos(\alpha+\beta)$

*********** END **********