



**Exercise 9.3 : Solutions of Questions on Page Number : 391**

**Q1 :**  $\frac{x}{a} + \frac{y}{b} = 1$

**Answer :**

$$\frac{x}{a} + \frac{y}{b} = 1$$

Differentiating both sides of the given equation with respect to  $x$ , we get:

$$\begin{aligned}\frac{1}{a} + \frac{1}{b} \frac{dy}{dx} &= 0 \\ \Rightarrow \frac{1}{a} + \frac{1}{b} y' &= 0\end{aligned}$$

Again, differentiating both sides with respect to  $x$ , we get:

$$\begin{aligned}0 + \frac{1}{b} y'' &= 0 \\ \Rightarrow \frac{1}{b} y'' &= 0 \\ \Rightarrow y'' &= 0\end{aligned}$$

Hence, the required differential equation of the given curve is  $y'' = 0$ .

Answer needs Correction? [Click Here](#)

**Q2 :**  $y^2 = a(b^2 - x^2)$

**Answer :**

$$y^2 = a(b^2 - x^2)$$

Differentiating both sides with respect to  $x$ , we get:

$$\begin{aligned}2y \frac{dy}{dx} &= a(-2x) \\ \Rightarrow 2yy' &= -2ax \\ \Rightarrow yy' &= -ax \quad \dots(1)\end{aligned}$$

Again, differentiating both sides with respect to  $x$ , we get:

$$\begin{aligned}y' \cdot y' + yy'' &= -a \\ \Rightarrow (y')^2 + yy'' &= -a \quad \dots(2)\end{aligned}$$

Dividing equation (2) by equation (1), we get:

$$\begin{aligned}\frac{(y')^2 + yy''}{yy'} &= \frac{-a}{-ax} \\ \Rightarrow xy'' + x(y')^2 - yy'' &= 0\end{aligned}$$

This is the required differential equation of the given curve.

Answer needs Correction? [Click Here](#)

**Q3 :**  $y = ae^{3x} + be^{-2x}$

**Answer :**

$$y = ae^{3x} + be^{-2x} \quad \dots(1)$$

Differentiating both sides with respect to  $x$ , we get:

$$y' = 3ae^{3x} - 2be^{-2x} \quad \dots(2)$$

Again, differentiating both sides with respect to  $x$ , we get:

$$y'' = 9ae^{3x} + 4be^{-2x} \quad \dots(3)$$

Multiplying equation (1) with (2) and then adding it to equation (2), we get:

$$\begin{aligned}(2ae^{3x} + 2be^{-2x}) + (3ae^{3x} - 2be^{-2x}) &= 2y + y' \\ \Rightarrow 5ae^{3x} &= 2y + y' \\ \Rightarrow ae^{3x} &= \frac{2y + y'}{5}\end{aligned}$$

Now, multiplying equation (1) with 3 and subtracting equation (2) from it, we get:

$$\begin{aligned}(3ae^{3x} + 3be^{-2x}) - (3ae^{3x} - 2be^{-2x}) &= 3y - y' \\ \Rightarrow 5be^{-2x} &= 3y - y' \\ \Rightarrow be^{-2x} &= \frac{3y - y'}{5}\end{aligned}$$

Substituting the values of  $ae^{3x}$  and  $be^{-2x}$  in equation (3), we get:

$$\begin{aligned} y'' &= 9 \cdot \frac{(2y + y')}{5} + 4 \cdot \frac{(3y - y')}{5} \\ \Rightarrow y'' &= \frac{18y + 9y'}{5} + \frac{12y - 4y'}{5} \\ \Rightarrow y'' &= \frac{30y + 5y'}{5} \\ \Rightarrow y'' &= 6y + y' \\ \Rightarrow y'' - y' - 6y &= 0 \end{aligned}$$

This is the required differential equation of the given curve.

Answer needs Correction? [Click Here](#)

**Q4 :**  $y = e^{2x} (a + bx)$

**Answer :**

$$y = e^{2x} (a + bx) \quad \dots(1)$$

Differentiating both sides with respect to  $x$ , we get:

$$\begin{aligned} y' &= 2e^{2x} (a + bx) + e^{2x} \cdot b \\ \Rightarrow y' &= e^{2x} (2a + 2bx + b) \quad \dots(2) \end{aligned}$$

Multiplying equation (1) with equation (2) and then subtracting it from equation (2), we get:

$$\begin{aligned} y' - 2y &= e^{2x} (2a + 2bx + b) - e^{2x} (2a + 2bx) \\ \Rightarrow y' - 2y &= be^{2x} \quad \dots(3) \end{aligned}$$

Differentiating both sides with respect to  $x$ , we get:

$$y'' - 2y' = 2be^{2x} \quad \dots(4)$$

Dividing equation (4) by equation (3), we get:

$$\begin{aligned} \frac{y'' - 2y'}{y' - 2y} &= 2 \\ \Rightarrow y'' - 2y' &= 2y' - 4y \\ \Rightarrow y'' - 4y' + 4y &= 0 \end{aligned}$$

This is the required differential equation of the given curve.

Answer needs Correction? [Click Here](#)

**Q5 :**  $y = e^x (a \cos x + b \sin x)$

**Answer :**

$$y = e^x (a \cos x + b \sin x) \quad \dots(1)$$

Differentiating both sides with respect to  $x$ , we get:

$$\begin{aligned} y' &= e^x (a \cos x + b \sin x) + e^x (-a \sin x + b \cos x) \\ \Rightarrow y' &= e^x [(a + b) \cos x - (a - b) \sin x] \quad \dots(2) \end{aligned}$$

Again, differentiating with respect to  $x$ , we get:

$$\begin{aligned} y'' &= e^x [(a + b) \cos x - (a - b) \sin x] + e^x [-(a + b) \sin x - (a - b) \cos x] \\ y'' &= e^x [2b \cos x - 2a \sin x] \\ y'' &= 2e^x (b \cos x - a \sin x) \\ \Rightarrow \frac{y''}{2} &= e^x (b \cos x - a \sin x) \quad \dots(3) \end{aligned}$$

Adding equations (1) and (3), we get:

$$\begin{aligned} y + \frac{y''}{2} &= e^x [(a + b) \cos x - (a - b) \sin x] \\ \Rightarrow y + \frac{y''}{2} &= y' \\ \Rightarrow 2y + y'' &= 2y' \\ \Rightarrow y'' - 2y' + 2y &= 0 \end{aligned}$$

This is the required differential equation of the given curve.

Answer needs Correction? [Click Here](#)

**Q6 :** Form the differential equation of the family of circles touching the  $y$ -axis at the origin.

**Answer :**

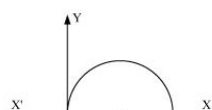
The centre of the circle touching the  $y$ -axis at origin lies on the  $x$ -axis.

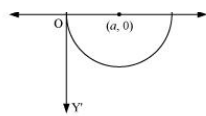
Let  $(a, 0)$  be the centre of the circle.

Since it touches the  $y$ -axis at origin, its radius is  $a$ .

Now, the equation of the circle with centre  $(a, 0)$  and radius  $(a)$  is

$$\begin{aligned} (x - a)^2 + y^2 &= a^2 \\ \Rightarrow x^2 + y^2 &= 2ax \quad \dots(1) \end{aligned}$$





Differentiating equation (1) with respect to  $x$ , we get:

$$2x + 2yy' = 2a$$

$$\Rightarrow x + yy' = a$$

Now, on substituting the value of  $a$  in equation (1), we get:

$$x^2 + y^2 = 2(x + yy')x$$

$$\Rightarrow x^2 + y^2 = 2x^2 + 2xyy'$$

$$\Rightarrow 2xyy' + x^2 = y^2$$

This is the required differential equation.

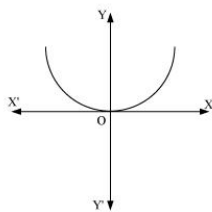
[Answer needs Correction? Click Here](#)

**Q7 : Form the differential equation of the family of parabolas having vertex at origin and axis along positive  $y$ -axis.**

**Answer :**

The equation of the parabola having the vertex at origin and the axis along the positive  $y$ -axis is:

$$x^2 = 4ay \quad \dots(1)$$



Differentiating equation (1) with respect to  $x$ , we get:

$$2x = 4ay' \quad \dots(2)$$

Dividing equation (2) by equation (1), we get:

$$\frac{2x}{x^2} = \frac{4ay'}{4ay}$$

$$\Rightarrow \frac{2}{x} = \frac{y'}{y}$$

$$\Rightarrow xy' = 2y$$

$$\Rightarrow xy' - 2y = 0$$

This is the required differential equation.

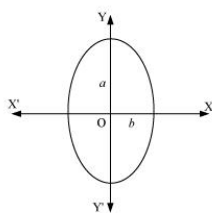
[Answer needs Correction? Click Here](#)

**Q8 : Form the differential equation of the family of ellipses having foci on  $y$ -axis and centre at origin.**

**Answer :**

The equation of the family of ellipses having foci on the  $y$ -axis and the centre at origin is as follows:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad \dots(1)$$



Differentiating equation (1) with respect to  $x$ , we get:

$$\frac{2x}{b^2} + \frac{2yy'}{a^2} = 0$$

$$\Rightarrow \frac{x}{b^2} + \frac{yy'}{a^2} = 0 \quad \dots(2)$$

Again, differentiating with respect to  $x$ , we get:

$$\frac{1}{b^2} + \frac{y' \cdot y' + y \cdot y''}{a^2} = 0$$

$$\Rightarrow \frac{1}{b^2} + \frac{1}{a^2}(y'^2 + yy'') = 0$$

$$\Rightarrow \frac{1}{b^2} = -\frac{1}{a^2}(y'^2 + yy'')$$

Substituting this value in equation (2), we get:

$$x \left[ -\frac{1}{a^2}((y')^2 + yy'') \right] + \frac{yy'}{a^2} = 0$$

$$\Rightarrow -x((y')^2 + yy'') + yy' = 0$$

$$\Rightarrow xy'' + x(y')^2 - yy' = 0$$

This is the required differential equation.

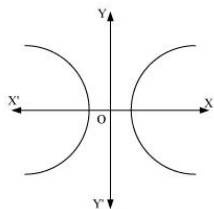
Answer needs Correction? [Click Here](#)

**Q9 : Form the differential equation of the family of hyperbolas having foci on x-axis and centre at origin.**

**Answer :**

The equation of the family of hyperbolas with the centre at origin and foci along the x-axis is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \dots(1)$$



Differentiating both sides of equation (1) with respect to  $x$ , we get:

$$\begin{aligned} \frac{2x}{a^2} - \frac{2yy'}{b^2} &= 0 \\ \Rightarrow \frac{x}{a^2} - \frac{yy'}{b^2} &= 0 \quad \dots(2) \end{aligned}$$

Again, differentiating both sides with respect to  $x$ , we get:

$$\begin{aligned} \frac{1}{a^2} - \frac{y' \cdot y' + yy''}{b^2} &= 0 \\ \Rightarrow \frac{1}{a^2} &= \frac{1}{b^2} ((y')^2 + yy'') \end{aligned}$$

Substituting the value of  $\frac{1}{a^2}$  in equation (2), we get:

$$\begin{aligned} \frac{x}{b^2} ((y')^2 + yy'') - \frac{yy'}{b^2} &= 0 \\ \Rightarrow x((y')^2 + yy'') - yy' &= 0 \\ \Rightarrow xyy'' + x((y')^2) - yy' &= 0 \end{aligned}$$

This is the required differential equation.

Answer needs Correction? [Click Here](#)

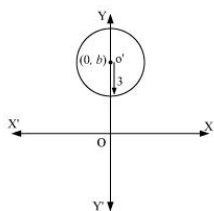
**Q10 : Form the differential equation of the family of circles having centre on y-axis and radius 3 units.**

**Answer :**

Let the centre of the circle on y-axis be  $(0, b)$ .

The differential equation of the family of circles with centre at  $(0, b)$  and radius 3 is as follows:

$$\begin{aligned} x^2 + (y-b)^2 &= 3^2 \\ \Rightarrow x^2 + (y-b)^2 &= 9 \quad \dots(1) \end{aligned}$$



Differentiating equation (1) with respect to  $x$ , we get:

$$\begin{aligned} 2x + 2(y-b) \cdot y' &= 0 \\ \Rightarrow (y-b) \cdot y' &= -x \\ \Rightarrow y-b &= \frac{-x}{y'} \end{aligned}$$

Substituting the value of  $(y-b)$  in equation (1), we get:

$$\begin{aligned} x^2 + \left( \frac{-x}{y'} \right)^2 &= 9 \\ \Rightarrow x^2 \left[ 1 + \frac{1}{(y')^2} \right] &= 9 \\ \Rightarrow x^2 ((y')^2 + 1) &= 9(y')^2 \\ \Rightarrow (x^2 - 9)(y')^2 + x^2 &= 0 \end{aligned}$$

This is the required differential equation.

Answer needs Correction? [Click Here](#)

**Q11 : Which of the following differential equations has  $y = c_1 e^x + c_2 e^{-x}$  as the general solution?**

→

- A.  $\frac{d^2 y}{dx^2} + y = 0$
- B.  $\frac{d^2 y}{dx^2} - y = 0$
- C.  $\frac{d^2 y}{dx^2} + 1 = 0$
- D.  $\frac{d^2 y}{dx^2} - 1 = 0$

**Answer :**

The given equation is:

$$y = c_1 e^x + c_2 e^{-x} \quad \dots(1)$$

Differentiating with respect to  $x$ , we get:

$$\frac{dy}{dx} = c_1 e^x - c_2 e^{-x}$$

Again, differentiating with respect to  $x$ , we get:

$$\frac{d^2 y}{dx^2} = c_1 e^x + c_2 e^{-x}$$

$$\Rightarrow \frac{d^2 y}{dx^2} = y$$

$$\Rightarrow \frac{d^2 y}{dx^2} - y = 0$$

This is the required differential equation of the given equation of curve.

Hence, the correct answer is B.

Answer needs Correction? [Click Here](#)

**Q12 :** Which of the following differential equation has  $y = x$  as one of its particular solution?

- A.  $\frac{d^2 y}{dx^2} - x^2 \frac{dy}{dx} + xy = x$
- B.  $\frac{d^2 y}{dx^2} + x \frac{dy}{dx} + xy = x$
- C.  $\frac{d^2 y}{dx^2} - x^2 \frac{dy}{dx} + xy = 0$
- D.  $\frac{d^2 y}{dx^2} + x \frac{dy}{dx} + xy = 0$

**Answer :**

The given equation of curve is  $y = x$ .

Differentiating with respect to  $x$ , we get:

$$\frac{dy}{dx} = 1 \quad \dots(1)$$

Again, differentiating with respect to  $x$ , we get:

$$\frac{d^2 y}{dx^2} = 0 \quad \dots(2)$$

Now, on substituting the values of  $y$ ,  $\frac{d^2 y}{dx^2}$ , and  $\frac{dy}{dx}$  from equation (1) and (2) in each of the given alternatives, we find that only the differential equation given in alternative C is correct.

$$\begin{aligned} \frac{d^2 y}{dx^2} - x^2 \frac{dy}{dx} + xy &= 0 - x^2 \cdot 1 + x \cdot x \\ &= -x^2 + x^2 \\ &= 0 \end{aligned}$$

Hence, the correct answer is C.

Answer needs Correction? [Click Here](#)

\*\*\*\*\* END \*\*\*\*\*