Q11 : $\int_0^2 \frac{dx}{x+4-x^2}$

Answer :

$$\int_0^2 \frac{dx}{x+4-x^2} = \int_0^2 \frac{dx}{-\left(x^2-x-4\right)}$$

$$= \int_0^2 \frac{dx}{-\left(x^2-x+\frac{1}{4}-\frac{1}{4}-4\right)}$$

$$= \int_0^2 \frac{dx}{-\left[\left(x-\frac{1}{2}\right)^2-\frac{17}{4}\right]}$$

$$= \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}$$

Let $x-\frac{1}{2}=t \Rightarrow dx = dt$

When $x=0$, $t=-\frac{1}{2}$ and when $x=2$, $t=\frac{3}{2}$

$$\therefore \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2} = \int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{dt}{\left(\frac{\sqrt{17}}{2}\right)^2-t^2}$$

$$= \left[\frac{1}{2\left(\frac{\sqrt{17}}{2}\right)} \log \frac{\frac{\sqrt{17}}{2}+t}{\frac{\sqrt{17}}{2}-t}\right]_{-\frac{1}{2}}^{\frac{3}{2}}$$

$$= \frac{1}{\sqrt{17}}\left[\log \frac{\frac{\sqrt{17}}{2}+\frac{3}{2}}{\frac{\sqrt{17}}{2}-\frac{3}{2}} - \frac{\log \frac{\sqrt{17}}{2}-\frac{1}{2}}{\log \frac{\sqrt{17}}{2}+\frac{1}{2}}\right]$$

$$= \frac{1}{\sqrt{17}}\left[\log \frac{\sqrt{17}+3}{\sqrt{17}-3} - \log \frac{\sqrt{17}-1}{\sqrt{17}+1}\right]$$

$$= \frac{1}{\sqrt{17}} \log \frac{\sqrt{17}+3}{\sqrt{17}-3} \times \frac{\sqrt{17}+1}{\sqrt{17}-1}$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{17+3+4\sqrt{17}}{17+3-4\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{20+4\sqrt{17}}{20-4\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{5+\sqrt{17}}{5-\sqrt{17}}\right)$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{\left(5+\sqrt{17}\right)\left(5+\sqrt{17}\right)}{25-17}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{25+17+10\sqrt{17}}{8}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{42+10\sqrt{17}}{8}\right)$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{21+5\sqrt{17}}{4}\right)$$

Answer needs Correction? **Click Here**

Q12 : $\int_0^2 \frac{dx}{x+4-x^2}$

Answer :

$$\int_0^2 \frac{dx}{x+4-x^2} = \int_0^2 \frac{dx}{-\left(x^2-x-4\right)}$$

$$= \int_0^2 \frac{dx}{-\left(x^2-x+\frac{1}{4}-\frac{1}{4}-4\right)}$$

$$= \int_0^2 \frac{dx}{-\left[\left(x-\frac{1}{2}\right)^2-\frac{17}{4}\right]}$$

$$= \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2}$$

Let $x - \frac{1}{2} = t \Rightarrow dx = dt$

When $x = 0$, $t = -\frac{1}{2}$ and when $x = 2$, $t = \frac{3}{2}$

$$\therefore \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2} = \int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{dt}{\left(\frac{\sqrt{17}}{2}\right)^2 - t^2}$$

$$= \left[\frac{1}{2\left(\frac{\sqrt{17}}{2}\right)} \log \frac{\frac{\sqrt{17}}{2} + t}{\frac{\sqrt{17}}{2} - t} \right]_{-\frac{1}{2}}^{\frac{3}{2}}$$

$$= \frac{1}{\sqrt{17}} \left[\log \frac{\frac{\sqrt{17}}{2} + \frac{3}{2}}{\frac{\sqrt{17}}{2} - \frac{3}{2}} - \log \frac{\frac{\sqrt{17}}{2} - \frac{1}{2}}{\frac{\sqrt{17}}{2} + \frac{1}{2}} \right]$$

$$= \frac{1}{\sqrt{17}} \left[\log \frac{\sqrt{17}+3}{\sqrt{17}-3} - \log \frac{\sqrt{17}-1}{\sqrt{17}+1} \right]$$

$$= \frac{1}{\sqrt{17}} \log \frac{\sqrt{17}+3}{\sqrt{17}-3} \times \frac{\sqrt{17}+1}{\sqrt{17}-1}$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{17+3+4\sqrt{17}}{17+3-4\sqrt{17}} \right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{20+4\sqrt{17}}{20-4\sqrt{17}} \right]$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{5+\sqrt{17}}{5-\sqrt{17}} \right)$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{\left(5+\sqrt{17}\right)\left(5+\sqrt{17}\right)}{25-17} \right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{25+17+10\sqrt{17}}{8} \right]$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{42+10\sqrt{17}}{8} \right)$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{21+5\sqrt{17}}{4} \right)$$

Answer needs Correction? **Click Here**

Q13 : $\int_{-1}^{1} \frac{dx}{x^2+2x+5}$

Answer :

$$\int_{-1}^{1} \frac{dx}{x^2+2x+5} = \int_{-1}^{1} \frac{dx}{\left(x^2+2x+1\right)+4} = \int_{-1}^{1} \frac{dx}{(x+1)^2+(2)^2}$$

Let $x + 1 = t \Rightarrow dx = dt$

When $x = -1$, $t = 0$ and when $x = 1$, $t = 2$

$$\therefore \int_{-1}^{1} \frac{dx}{(x+1)^2+(2)^2} = \int_0^2 \frac{dt}{t^2+2^2}$$

$$= \left[\frac{1}{2} \tan^{-1} \frac{t}{2} \right]_0^2$$

$$= \frac{1}{2} \tan^{-1} 1 - \frac{1}{2} \tan^{-1} 0$$

$$= \frac{1}{2}\left(\frac{\pi}{4}\right) = \frac{\pi}{8}$$

Answer needs Correction? **Click Here**

Q14 : $\int_{-1}^{1} \frac{dx}{x^2+2x+5}$

Answer :

$$\int_{-1}^{1} \frac{dx}{x^2+2x+5} = \int_{-1}^{1} \frac{dx}{\left(x^2+2x+1\right)+4} = \int_{-1}^{1} \frac{dx}{(x+1)^2+(2)^2}$$

Let $x + 1 = t \Rightarrow dx = dt$

When $x = -1$, $t = 0$ and when $x = 1$, $t = 2$

$$\therefore \int_{-1}^{1} \frac{dx}{(x+1)^2+(2)^2} = \int_0^2 \frac{dt}{t^2+2^2}$$

$$= \left[\frac{1}{2} \tan^{-1} \frac{t}{2} \right]_0^2$$

$$= \frac{1}{2} \tan^{-1} 1 - \frac{1}{2} \tan^{-1} 0$$

$$= \frac{1}{2}\left(\frac{\pi}{4}\right) = \frac{\pi}{8}$$

Answer needs Correction? **Click Here**

Q15 : $\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx$

Answer :

$\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx$

Let $2x = t \Rightarrow 2dx = dt$

When $x = 1$, $t = 2$ and when $x = 2$, $t = 4$

$$\therefore \int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx = \frac{1}{2} \int_2^4 \left(\frac{2}{t} - \frac{2}{t^2}\right) e^t dt$$
$$= \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2}\right) e^t dt$$

Let $\frac{1}{t} = f(t)$

Then, $f'(t) = -\frac{1}{t^2}$

$$\Rightarrow \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2}\right) e^t dt = \int_2^4 e^t \left[f(t) + f'(t)\right] dt$$
$$= \left[e^t f(t)\right]_2^4$$
$$= \left[e^t \cdot \frac{2}{t}\right]_2^4$$
$$= \left[\frac{e^t}{t}\right]_2^4$$
$$= \frac{e^4}{4} - \frac{e^2}{2}$$
$$= \frac{e^2\left(e^2 - 2\right)}{4}$$

Answer needs Correction? **Click Here**

Q16 : $\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx$

Answer :

$\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx$

Let $2x = t \Rightarrow 2dx = dt$

When $x = 1$, $t = 2$ and when $x = 2$, $t = 4$

$$\therefore \int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx = \frac{1}{2} \int_2^4 \left(\frac{2}{t} - \frac{2}{t^2}\right) e^t dt$$
$$= \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2}\right) e^t dt$$

Let $\frac{1}{t} = f(t)$

Then, $f'(t) = -\frac{1}{t^2}$

$$\Rightarrow \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2}\right) e^t dt = \int_2^4 e^t \left[f(t) + f'(t)\right] dt$$
$$= \left[e^t f(t)\right]_2^4$$
$$= \left[e^t \cdot \frac{2}{t}\right]_2^4$$
$$= \left[\frac{e^t}{t}\right]_2^4$$
$$= \frac{e^4}{4} - \frac{e^2}{2}$$
$$= \frac{e^2\left(e^2 - 2\right)}{4}$$

Answer needs Correction? **Click Here**

Q17 : The value of the integral $\int_{\frac{1}{3}}^{1} \frac{\left(x - x^3\right)^{\frac{1}{3}}}{x^4} dx$ is

A. 6

B. 0

C. 3

D. 4

Answer :

Let $I = \int_{\frac{1}{3}}^{1} \frac{\left(x - x^3\right)^{\frac{1}{3}}}{x^4} dx$

Also, let $x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$

When $x = \frac{1}{3}$, $\theta = \sin^{-1}\left(\frac{1}{3}\right)$ and when $x = 1$, $\theta = \frac{\pi}{2}$

$$\Rightarrow I = \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{\left(\sin\theta - \sin^3\theta\right)^{\frac{1}{3}}}{\sin^4\theta} \cos\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\sin\theta)^{\frac{1}{3}}\left(1 - \sin^2\theta\right)^{\frac{1}{3}}}{\sin^4\theta} \cos\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\sin\theta)^{\frac{1}{3}}(\cos\theta)^{\frac{2}{3}}}{\sin^4\theta} \cos\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\sin\theta)^{\frac{1}{3}}(\cos\theta)^{\frac{2}{3}}}{\sin^2\theta \sin^2\theta} \cos\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\cos\theta)^{\frac{5}{3}}}{(\sin\theta)^{\frac{5}{3}}} \operatorname{cosec}^2\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} (\cot\theta)^{\frac{5}{3}} \operatorname{cosec}^2\theta\, d\theta$$

Let cotθ = $t \Rightarrow$ - cosec2θ $d\theta$= dt

When $\theta = \sin^{-1}\left(\frac{1}{3}\right)$, $t = 2\sqrt{2}$ and when $\theta = \frac{\pi}{2}$, $t = 0$

$$\therefore I = -\int_{2\sqrt{2}}^{0} (t)^{\frac{5}{3}} dt$$

$$= -\left[\frac{3}{8}(t)^{\frac{8}{3}}\right]_{2\sqrt{2}}^{0}$$

$$= -\frac{3}{8}\left[(t)^{\frac{8}{3}}\right]_{2\sqrt{2}}^{0}$$

$$= -\frac{3}{8}\left[-\left(2\sqrt{2}\right)^{\frac{8}{3}}\right]$$

$$= \frac{3}{8}\left[\left(\sqrt{8}\right)^{\frac{8}{3}}\right]$$

$$= \frac{3}{8}\left[(8)^{\frac{4}{3}}\right]$$

$$= \frac{3}{8}[16]$$

$$= 3 \times 2$$

$$= 6$$

Hence, the correct answer is A.

Answer needs Correction? **Click Here**

Q18 : The value of the integral $\int_{\frac{1}{3}}^{1} \frac{\left(x - x^3\right)^{\frac{1}{3}}}{x^4} dx$ is

A. 6

B. 0

C. 3

D. 4

Answer :

Let $I = \int_{\frac{1}{3}}^{1} \frac{\left(x - x^3\right)^{\frac{1}{3}}}{x^4} dx$

Also, let $x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$

When $x = \frac{1}{3}$, $\theta = \sin^{-1}\left(\frac{1}{3}\right)$ and when $x = 1$, $\theta = \frac{\pi}{2}$

$$\Rightarrow I = \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{\left(\sin\theta - \sin^3\theta\right)^{\frac{1}{3}}}{\sin^4\theta} \cos\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\sin\theta)^{\frac{1}{3}}\left(1 - \sin^2\theta\right)^{\frac{1}{3}}}{\sin^4\theta} \cos\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\sin\theta)^{\frac{1}{3}}(\cos\theta)^{\frac{2}{3}}}{\sin^4\theta} \cos\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\sin\theta)^{\frac{1}{3}}(\cos\theta)^{\frac{2}{3}}}{\sin^2\theta \sin^2\theta} \cos\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \frac{(\cos\theta)^{\frac{5}{3}}}{(\sin\theta)^{\frac{5}{3}}} \operatorname{cosec}^2\theta\, d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} (\cot\theta)^{\frac{5}{3}} \operatorname{cosec}^2\theta\, d\theta$$

Let cotθ = $t \Rightarrow$ - cosec2θ $d\theta$= dt

When $\theta = \sin^{-1}\left(\frac{1}{3}\right)$, $t = 2\sqrt{2}$ and when $\theta = \frac{\pi}{2}$, $t = 0$

$$\therefore I = -\int_{2\sqrt{2}}^{0} (t)^{\frac{5}{3}} dt$$

$$= -\left[\frac{3}{8}(t)^{\frac{8}{3}}\right]^{0}$$

$$
\begin{aligned}
&= -\frac{3}{8}\left[(t)^{\frac{8}{3}}\right]_{2\sqrt{2}}^{0} \\
&= -\frac{3}{8}\left[-\left(2\sqrt{2}\right)^{\frac{8}{3}}\right] \\
&= \frac{3}{8}\left[\left(\sqrt{8}\right)^{\frac{8}{3}}\right] \\
&= \frac{3}{8}\left[(8)^{\frac{4}{3}}\right] \\
&= \frac{3}{8}[16] \\
&= 3\times 2 \\
&= 6
\end{aligned}
$$

Hence, the correct answer is A.

Answer needs Correction? Click Here

Q19 : If $f(x)=\int_0^x t\sin t\,dt$, then $f'(x)$ is

A. $\cos x + x\sin x$

B. $x\sin x$

C. $x\cos x$

D. $\sin x + x\cos x$

Answer :

$f(x)=\int_0^x t\sin t\,dt$

Integrating by parts, we obtain

$$
\begin{aligned}
f(x) &= t\int_0^x \sin t\,dt - \int_0^x\left\{\left(\frac{d}{dt}t\right)\int \sin t\,dt\right\}dt \\
&= \left[t(-\cos t)\right]_0^x - \int_0^x(-\cos t)\,dt \\
&= \left[-t\cos t + \sin t\right]_0^x \\
&= -x\cos x + \sin x
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow f'(x) &= -\left[\left\{x(-\sin x)\right\} + \cos x\right] + \cos x \\
&= x\sin x - \cos x + \cos x \\
&= x\sin x
\end{aligned}
$$

Hence, the correct answer is B.

Answer needs Correction? Click Here

Q20 : If $f(x)=\int_0^x t\sin t\,dt$, then $f'(x)$ is

A. $\cos x + x\sin x$

B. $x\sin x$

C. $x\cos x$

D. $\sin x + x\cos x$

Answer :

$f(x)=\int_0^x t\sin t\,dt$

Integrating by parts, we obtain

$$
\begin{aligned}
f(x) &= t\int_0^x \sin t\,dt - \int_0^x\left\{\left(\frac{d}{dt}t\right)\int \sin t\,dt\right\}dt \\
&= \left[t(-\cos t)\right]_0^x - \int_0^x(-\cos t)\,dt \\
&= \left[-t\cos t + \sin t\right]_0^x \\
&= -x\cos x + \sin x
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow f'(x) &= -\left[\left\{x(-\sin x)\right\} + \cos x\right] + \cos x \\
&= x\sin x - \cos x + \cos x \\
&= x\sin x
\end{aligned}
$$

Hence, the correct answer is B.

Answer needs Correction? Click Here

*********** END **********