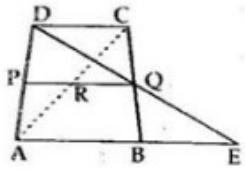
Exercise 9C

Question 3:

Given : ABCD is trapezium in which AB || DC

P and Q are the mid – points of AD and BC. DQ is joined and produced and AB is also produced and so that they meet at E.

AC cuts PQ at R.



To prove :

(i) DQ = QE

(ii) PR || AB

(iii) AR = RC

Proof :

(i) Consider the triangles ΔQCD and ΔQBE

$\angle DQC = \angle BQE$ [vertically opposite angles]

CQ = BQ [$\because$ Q is the midpoint of BC]

$\angle QDC = \angle QEB$ [AE || DC, BC is a transversal, and thus alternate angles are equal]

Thus, by Angle-Side-Angle criterion of congruence, we have

$\Delta QCD \cong \Delta QEB$ [by ASA]

The corresponding parts of the congruent triangles are equal.

Thus, DQ = QE [by c.p.c.t]

(ii) Midpoint Theorem: The line segment joining the midpoints of any two sides of a triangle is parallel to the third side and equal to half of it.

Thus by the midpoint Theorem, PQ || AE.

AB is a part of AE and hence, we have PQ || AB

Since the intercepts made by the lines AB, PQ and DC on AD

Since PQ || AB || DC

So, PR which is part of PQ is also parallel to AB

$\therefore$ PR || AB || DC

(iii) Intercept Theorem: If there are three parallel lines and the intercepts made by them on one transversal are equal then the intercept on any other transversal are also equal.

The three lines PR, AB and DC are cut by AC and AD.

So, by intercept Theorem, AR=RC

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