Indefinite Integrals Ex 19.25 Q8

Let $I = \int x^2 \cos 2x \, dx$

Using integration by parts,

$$I = x^2 \int \cos 2x \, dx - \int \left(2x \int \cos 2x \, dx\right) dx$$

$$= x^2 \frac{\sin 2x}{2} - 2\int x \left(\frac{\sin 2x}{2}\right) dx$$

$$= \frac{1}{2} x^2 \sin 2x - \int x \sin 2x \, dx$$

$$= \frac{1}{2} x^2 \sin 2x - \left[x \int \sin 2x \, dx - \int \left(1 \int \sin 2x \, dx\right) dx\right]$$

$$= \frac{1}{2} x^2 \sin 2x - \left[x \left(\frac{-\cos 2x}{2}\right) - \int \left(-\frac{\cos 2x}{2}\right) dx\right]$$

$$= \frac{1}{2} x^2 \sin 2x + \frac{x}{2} \cos 2x - \frac{1}{2} \int (\cos 2x) \, dx$$

$$I = \frac{1}{2} x^2 \sin 2x + \frac{x}{2} \cos 2x - \frac{1}{4} \sin 2x + c$$

Indefinite Integrals Ex 19.25 Q9

Let $I = \int x \sin 2x \, dx$

Using integration by parts,

$$I = x \int \sin 2x \, dx - \int \left(1 \int \sin 2x \, dx\right) dx$$

$$= x \left(-\frac{\cos 2x}{2}\right) - \int \left(-\frac{\cos 2x}{2}\right) dx$$

$$= -\frac{x}{2} \cos 2x + \frac{1}{2} \int \cos 2x \, dx$$

$$= -\frac{x}{2} \cos 2x + \frac{1}{2} \frac{\sin 2x}{2} + c$$

$$I = -\frac{x}{2} \cos 2x + \frac{1}{4} \sin 2x + c$$

Indefinite Integrals Ex 19.25 Q10

Let $I = \int \frac{\log(\log x)}{x} dx$

$= \int \left(\frac{1}{x}\right)(\log(\log x))\, dx$

Using integration by parts,

$I = \log \log x \int \frac{1}{x} dx - \int \left(\frac{1}{x \log x} \int \frac{1}{x} dx\right) dx$

$= \log x \times \log(\log x) - \int \left(\frac{1}{x \log x} \log x\right) dx$

$= \log x \times \log(\log x) - \int \frac{1}{x} dx$

$= \log x \times \log(\log x) - \log x + c$

$I = \log x\,(\log\log x - 1) + c$

Indefinite Integrals Ex 19.25 Q11

Let $I = \int x^2 \cos x\, dx$

Using integration by parts,

$I = x^2 \int \cos x\, dx - \int (2x \int \cos x\, dx) dx$

$= x^2 \sin x - 2 \int x \sin x\, dx$

$= x^2 \sin x - 2\left[x \int \sin x\, dx - \int (1 \int \sin x\, dx) dx\right]$

$= x^2 \sin x - 2\left[x(-\cos x) - \int (-\cos x) dx\right]$

$= x^2 \sin x + 2x \cos x - 2 \int (\cos x) dx$

$I = x^2 \sin x + 2x \cos x - 2 \sin x + c$

Indefinite Integrals Ex 19.25 Q12

Let $I = \int x \operatorname{cosec}^2 x\, dx$

Using integration by parts,

$I = x \int \operatorname{cosec}^2 x\, dx - \int \left(\int \operatorname{cosec}^2 x\, dx\right) dx$

$= -x \cot x + \int \cot x\, dx$

$= -x \cot x + \log|\sin x| + c$

Indefinite Integrals Ex 19.25 Q13

Let $I = \int x\cos^2 x\,dx$

Using integration by parts,

$$I = x\int \cos^2 x\,dx - \int\left(1\int \cos^2 x\,dx\right)dx$$

$$= x\int\left(\frac{\cos 2x+1}{2}\right)dx - \int\left(\int\left(\frac{1+\cos 2x}{2}\right)dx\right)dx$$

$$= \frac{x}{2}\left[\frac{\sin 2x}{2}+x\right] - \frac{1}{2}\int\left(x+\frac{\sin 2x}{2}\right)dx$$

$$= \frac{x}{4}\sin 2x + \frac{x^2}{2} - \frac{1}{2}\times\frac{x^2}{2} - \frac{1}{4}\left(-\frac{\cos 2x}{2}\right)+c$$

$$I = \frac{x}{4}\sin 2x + \frac{x^2}{4} + \frac{1}{8}\cos 2x + c$$

Indefinite Integrals Ex 19.25 Q14

Let $I = \int x^n \log x\,dx$

Using integration by parts,

$$I = \log x\int x^2\,dx - \int\left(\frac{1}{x}\int x^2\,dx\right)dx$$

$$= \frac{x^{n+1}}{n+1}\log x - \int\left(\frac{1}{x}\times\frac{x^{n+1}}{n+1}\right)dx$$

$$= \frac{x^{n+1}}{n+1}\log x - \int\left(\frac{x^n}{n+1}\right)dx$$

$$I = \frac{x^{n+1}}{n+1}\log x - \frac{1}{(n+1)^2}x^{n+1} + c$$

********** END **********