Differentials Errors and Approximation Ex14.1 Q5

Let x be the radius of sphere,

$$\Delta x = 0.1\% \text{ of } x$$
$$\Delta x = 0.001x$$

Now,

Let y = volume of sphere

$$y = \frac{4}{3}\pi x^3$$
$$\frac{dy}{dx} = 4\pi x^2$$
$$\Delta y = \left(\frac{dy}{dx}\right) \times \Delta x$$
$$= 4\pi x^2 \times 0.001x$$
$$= \frac{4}{3}\pi x^3 (0.003)$$
$$= \frac{0.3}{100} \times y$$
$$\Delta y = 0.3\% \text{ of } y$$

So, percentage error in volume of error = 0.3%.

Differentials Errors and Approximation Ex14.1 Q6

Given, $\Delta v = -\frac{1}{2}\%$

$= -0.5\%$

$\Delta v = -0.005$

Here,

$$pv^{1.4} = k$$

Taking log on both the sides,

$$\log\left(pv^{1.4}\right) = \log k$$

$$\log p + 1.4\log v = \log k$$

Differentiate it with respect to v,

$$\frac{1}{p}\frac{dp}{dv} + \frac{1.4}{v} = 0$$

$$\frac{dp}{dv} = -\frac{1.4}{v}p$$

$$\Delta p = \left(\frac{dp}{dv}\right)\Delta v$$

$$= -\frac{1.4p}{v} \times (-0.005)$$

$$\Delta p = \frac{1.4p(0.005)}{v}$$

$$\Delta p \text{ in } \% = \frac{\Delta p}{p} \times 100$$

$$= \frac{1.4p(0.005)}{p} \times 100$$

$$= 0.7\%$$

So, percentage error in $p = 0.7\%$.

Let h be the height of the cone, and α be the semivertide angle.

Here vertgide angle α is fixed.

$$\Delta h = k\% \text{ of } h$$
$$= \frac{k}{100} \times h$$
$$\Delta h = (0.0k)h$$

(i)
$$A = \pi r (r + l)$$
$$= \pi \left(r^2 + rl\right)$$
$$= \pi \left(r^2\right) + r\sqrt{h^2 + r^2} \qquad \left[\text{Since, in a cone } l^2 = h^2 + r^2\right]$$

$$r = h \tan\alpha \qquad [\text{from figure}]$$
$$A = \pi\left[h^2 \tan^2\alpha + h \tan\alpha \sqrt{h^2 + h^2 \tan^2\alpha}\right]$$
$$= \pi\left[h^2 \tan^2\alpha + h \tan\alpha \sqrt{h^2\left(1 + \tan^2\alpha\right)}\right]$$
$$= \pi\left[h^2 \tan^2\alpha + h \tan\alpha \times h \sec\alpha\right]$$
$$= \pi h^2 \left[\frac{\sin^2\alpha}{\cos^2\alpha} + \frac{\sin\alpha}{\cos^2\alpha}\right]$$
$$A = \pi h^2 \frac{\sin\alpha\,(\sin\alpha + 1)}{\cos^2\alpha}$$

Differentiating with respect to h as α is fixed.

$$\frac{dA}{dh} = 2\pi h \frac{\sin\alpha\,(\sin\alpha + 1)}{\cos^2\alpha}$$

So,

$$\Delta A = \frac{dA}{dh} \times \Delta h$$
$$\Delta a = \frac{2\pi h\,(0.0kh)\sin\alpha\,(\sin\alpha + 1)}{\cos^2\alpha}$$
$$\Delta A \text{ in \% of } A = \frac{2\pi h\,(0.0kh)\sin\alpha\,(\sin\alpha + 1)}{\cos^2\alpha} \times \frac{100}{A}$$
$$= \frac{2\pi k h^2 \times \sin\alpha\,(\sin\alpha + 1)}{\cos^2\alpha} \times \frac{\cos^2\alpha}{\pi h^2 \sin\alpha\,(\sin\alpha + 1)}$$
$$= 2k\%$$

So, percentage increase in area $= 2k\%$.

(ii)

Let v = volume of cone

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi\left(h\tan\alpha\right)^2 h$$

$$v = \frac{\pi}{3}\tan^2\alpha h^2$$

Differentiating it with respect to h treating α as constant,

$$\frac{dv}{dh} = \pi\tan^2\alpha \times h^2$$

$$\Delta v = \left(\frac{dv}{dh}\right)\Delta h$$

$$= \pi\tan^2\alpha h^2 \times \left(0.0kh\right)$$

$$\Delta v = 0.0k\pi h^3\tan^2\alpha$$

Percentage increase in $v = \frac{\Delta v \times 100}{v}$

$$= \frac{0.0k\pi h^3\tan^2\alpha \times 100}{\frac{\pi}{3}\tan^2\alpha \times h^3}$$

$$= 3k\%$$

So, percentage increase in volume = $3k\%$.

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