Indefinite Integrals Ex 19.26 Q1

Let $I = \int e^x \left(\cos x - \sin x\right) dx$

$= \int e^x \cos x dx - \int e^x \sin x dx$

Integrating by parts

$$= e^x \cos x - \int e^x \left(\frac{d}{dx}\cos x\right) dx - \int e^x \sin x dx$$

$$= e^x \cos x + \int e^x \sin x dx - \int e^x \sin x dx$$

$$= e^x \cos x + c$$

$$\therefore \quad \int e^x \left(\cos x - \sin x\right) dx = e^x \cos x + c$$

Indefinite Integrals Ex 19.26 Q2

$I = \int e^x \left(x^{-2} - 2x^{-3}\right) dx$

$= \int e^x x^{-2} dx - 2\int e^x x^{-3} dx$

Integrating by parts

$$= e^x x^{-2} - \int e^x \left(\frac{d}{dx}\left(x^{-2}\right)\right) dx - 2\int e^x x^{-3} dx$$

$$= e^x x^{-2} + 2\int e^x x^{-3} dx - 2\int e^x x^{-3} dx$$

$$= \frac{e^x}{x^2} + c$$

$$\int e^x \left(\frac{1}{x^2} - \frac{2}{x^3}\right) dx = \frac{e^x}{x^2} + c$$

Indefinite Integrals Ex 19.26 Q3

$$e^x\left(\frac{1+\sin x}{1+\cos x}\right)$$

$$=e^x\left(\frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$$

$$=\frac{e^x\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}}$$

$$=\frac{1}{2}e^x\cdot\left(\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\cos\frac{x}{2}}\right)^2$$

$$=\frac{1}{2}e^x\left[\tan\frac{x}{2}+1\right]^2$$

$$=\frac{1}{2}e^2\left(1+\tan\frac{x}{2}\right)^2$$

$$=\frac{1}{2}e^x\left[1+\tan^2\frac{x}{2}+2\tan\frac{x}{2}\right]$$

$$=\frac{1}{2}e^x\left[\sec^2\frac{x}{2}+2\tan\frac{x}{2}\right]$$

$$\frac{e^x(1+\sin x)dx}{(1+\cos x)}=e^x\left[\frac{1}{2}\sec^2\frac{x}{2}+\tan\frac{x}{2}\right] \qquad ...(1)$$

Let $\tan\frac{x}{2}=f(x)\Rightarrow f'(x)=\frac{1}{2}\sec^2\frac{x}{2}$

It is known that, $\int e^x\{f(x)+f'(x)\}dx=e^x f(x)+\mathrm{C}$

From equation (1), we obtain

$$\int\frac{e^x(1+\sin x)}{(1+\cos x)}dx=e^x\tan\frac{x}{2}+\mathrm{C}$$

Indefinite Integrals Ex 19.26 Q4

Let $I = \int e^x\left(\cot x - \operatorname{cosec}^2 x\right)dx$

$$= \int e^x \cot x dx - \int e^x \operatorname{cosec}^2 x dx$$

Integrating by parts

$$= e^x \cot x - \int e^x\left(\frac{d}{dx}\cot x\right)dx - \int e^x \operatorname{cosec}^2 x dx$$

$$= e^x \cot x + \int e^x \operatorname{cosec}^2 x dx - \int e^x \operatorname{cosec}^2 x dx$$

$$= e^x \cot x + c$$

$$\int e^x\left(\cot x - \operatorname{cosec}^2 x\right)dx = e^x \cot x + c$$

*********** END **********