Trigonometric Identities Ex 6.1 Q68

Answer :

We have prove that

$$(1+\cot A+\tan A)(\sin A-\cos A)=\frac{\sec A}{\operatorname{cosec}^2 A}-\frac{\operatorname{cosec} A}{\sec^2 A}=\sin A\tan A-\cot A\cos A$$

We know that, $\sin^2 A+\cos^2 A=1$

So,

$$\begin{aligned}
&(1+\cot A+\tan A)(\sin A-\cos A)\\
&=\left(1+\frac{\cos A}{\sin A}+\frac{\sin A}{\cos A}\right)(\sin A-\cos A)\\
&=\left(\frac{\sin A\cos A+\cos^2 A+\sin^2 A}{\sin A\cos A}\right)(\sin A-\cos A)\\
&=\left(\frac{\sin A\cos A+1}{\sin A\cos A}\right)(\sin A-\cos A)\\
&=\frac{(\sin A-\cos A)(\sin A\cos A+1)}{\sin A\cos A}\\
&=\frac{\sin^2 A\cos A+\sin A-\cos^2 A\sin A-\cos A}{\sin A\cos A}\\
&=\frac{(\sin^2 A\cos A-\cos A)+(\sin A-\cos^2 A\sin A)}{\sin A\cos A}
\end{aligned}$$

$$= \frac{\cos A(\sin^2 A - 1) + \sin A(1 - \cos^2 A)}{\sin A \cos A}$$

$$= \frac{\cos A(-\cos^2 A) + \sin A(\sin^2 A)}{\sin A \cos A}$$

$$= \frac{-\cos^3 A + \sin^3 A}{\sin A \cos A}$$

$$= \frac{\sin^3 A - \cos^3 A}{\sin A \cos A}$$

$$= \frac{\sin^3 A}{\sin A \cos A} - \frac{\cos^3 A}{\sin A \cos A}$$

$$= \frac{\sin^2 A}{\cos A} - \frac{\cos^2 A}{\sin A}$$

$$= \frac{\sin A}{\cos A}\sin A - \frac{\cos A}{\sin A}\cos A$$

$$= \tan A \sin A - \cot A \cos A$$

$$= \sin A \tan A - \cos A \cot A$$

Now,

$$\frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A} = \frac{\frac{1}{\cos A}}{\frac{1}{\sin^2 A}} - \frac{\frac{1}{\sin A}}{\frac{1}{\cos^2 A}}$$

$$= \frac{\sin^2 A}{\cos A} - \frac{\cos^2 A}{\sin A}$$

$$= \sin A \frac{\sin A}{\cos A} - \cos A \frac{\cos A}{\sin A}$$

$$= \sin A \tan A - \cos A \cot A$$

Hence proved.

*********** END **********