



Geometric Progressions Ex 20.5 Q 12

a, b, c are in G.P.

$$a, b = ar, c = ar^2$$

$$a(b^2 + c^2) = c(a^2 + b^2)$$

$$a(a^2r^2 + a^2r^4) = ar^2(a^2 + a^2r^2)$$

$$a^3r^2(1 + r^2) = a^3r^2(1 + r^2)$$

$$\text{LHS} = \text{RHS}$$

a, b, c are in G.P.

$$a, b = ar, c = ar^2$$

$$\begin{aligned} \text{LHS} &= a^2b^2c^2\left(\frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3}\right) \\ &= a^2 \times a^2r^2 \times a^2r^4\left(\frac{1}{a^3} + \frac{1}{a^3r^3} + \frac{1}{a^3r^6}\right) \\ &= a^6r^6\left(\frac{r^6 + r^3 + 1}{a^3r^6}\right) \\ &= a^3(r^6 + r^3 + 1) \\ &= a^3 + a^3r^3 + a^3r^6 \\ &= a^3 + (ar)^3 + (ar^2)^3 \\ &= a^3 + b^3 + c^3 \\ &= \text{RHS} \\ \therefore \quad \text{LHS} &= \text{RHS} \end{aligned}$$

a, b, c are in G.P.

$$a, b = ar, c = ar^2$$

$$\begin{aligned}\text{LHS} &= \frac{(a+b+c)^2}{a^2+b^2+c^2} \\&= \frac{(a+ar+ar^2)^2}{a^2+a^2r^2+a^2r^4} \\&= \frac{a^2(1+r+r^2)^2}{a^2(1+r^2+r^4)} \\&= \frac{a^2(1+r+r^2)^2}{a^2[(1+2r^2+r^4)-r^2]} \\&= \frac{a^2(1+r+r^2)^2}{a^2[(1+r^2-r)(1+r^2+r)]} \\&= \frac{a(1+r+r^2)}{a(1+r^2-r)} \\&= \frac{a+ar+ar^2}{a+ar^2-ar} \\&= \frac{a+b+c}{a-b+c} \\&= \text{RHS} \\ \therefore \quad \text{LHS} &= \text{RHS}\end{aligned}$$

a, b, c are in G.P.

$$a, b = ar, c = ar^2$$

$$\begin{aligned}\text{LHS} &= \frac{1}{a^2-b^2} + \frac{1}{b^2} \\&= \frac{1}{a^2-a^2r^2} + \frac{1}{a^2r^2} \\&= \frac{1}{a^2} \left[\frac{1}{1-r^2} + \frac{1}{r^2} \right] \\&= \frac{1}{a^2} \left[\frac{r^2+1-r^2}{(1-r^2)r^2} \right] \\&= \frac{1}{a^2} \left[\frac{1}{r^2-r^4} \right] \\&= \frac{1}{(ar)^2 - (ar^2)^2} \\&= \frac{1}{b^2-c^2} \\&= \text{RHS} \\ \therefore \quad \text{LHS} &= \text{RHS}\end{aligned}$$

a, b, c are in G.P.

$$a, b = ar, c = ar^2$$

$$\begin{aligned}\text{LHS} &= (a + 2b + 2c)(a - 2b + 2c) \\&= (a + 2ar + 2ar^2)(a - 2ar + 2ar^2) \\&= a^2(1 + 2r + 2r^2)(1 - 2r + 2r^2) \\&= a^2 \left[(1 + 2r^2)^2 - (2r)^2 \right] \\&= a^2 [1 + 4r^4 + 4r^2 - 4r^2] \\&= a^2 [1 + 4r^4] \\&= a^2 + 4(ar^2)^2 \\&= a^2 + 4c^2 \\&= \text{RHS} \\ \therefore \quad \text{LHS} &= \text{RHS}\end{aligned}$$

***** END *****