Transformation Formulae Ex 8.2 Q 16.

We have,

$$y \sin\phi = x \sin(2\theta + \phi)$$

$$\Rightarrow \quad \frac{\sin\phi}{\sin(2\theta + \phi)} = \frac{x}{y} \qquad \qquad \text{--- (i)}$$

Now,

$$\frac{\sin\phi}{\sin(2\theta + \phi)} = \frac{x}{y}$$

$$\Rightarrow \quad \frac{\sin\phi}{\sin(2\theta + \phi)} + 1 = \frac{x}{y} + 1$$

$$\Rightarrow \quad \frac{\sin\phi + \sin(2\theta + \phi)}{\sin(2\theta + \phi)} = \frac{x + y}{y} \qquad \qquad \text{--- (ii)}$$

Again,

$$\frac{\sin\phi}{\sin(2\theta + \phi)} = \frac{x}{y} \qquad \qquad \left[\text{By equation (i)}\right]$$

$$\Rightarrow \quad \frac{\sin\phi}{\sin(2\theta + \phi)} - 1 = \frac{x}{y} - 1$$

$$\Rightarrow \quad \frac{\sin\phi - \sin(2\theta + \phi)}{\sin(2\theta + \phi)} = \frac{x - y}{y} \qquad \qquad \text{--- (iii)}$$

Dividing equation (ii) by equation (iii), we get

$$\frac{\sin\phi + \sin(2\theta + \phi)}{\sin\phi - \sin(2\theta + \phi)} = \frac{x + y}{x - y}$$

$$\Rightarrow \quad \frac{2\sin\left(\dfrac{\phi + 2\theta + \phi}{2}\right)\cos\left(\dfrac{\phi - 2\theta - \phi}{2}\right)}{2\sin\left(\dfrac{\phi - 2\theta - \phi}{2}\right)\cos\left(\dfrac{\phi + 2\theta + \phi}{2}\right)} = \frac{x + y}{x - y}$$

$$\Rightarrow \quad \frac{\sin(\theta + \phi)\cos(\theta - \phi)}{\sin(-\theta)\cos(\theta + \phi)} = \frac{x + y}{x - y}$$

$$\Rightarrow \quad \frac{\sin(\theta + \phi)\cos(\theta)}{\cos(\theta + \phi)\left[-\sin(\theta)\right]} = \frac{x + y}{x - y}$$

$$\Rightarrow \quad \frac{-\cot(\theta)}{\cot(\theta + \phi)} = \frac{x + y}{x - y}$$

$$\Rightarrow \quad -(x - y)\cot\theta = (x + y)\cot(\theta + \phi)$$

$$\Rightarrow \quad (y - x)\cot\theta = (x + y)\cot(\theta + \phi)$$

$$\Rightarrow \quad (x + y)\cot(\theta + \phi) = (y - x)\cot\theta \qquad \qquad \text{Hence proved.}$$

Transformation Formulae Ex 8.2 Q 17.

We have,

$$\cos(A+B)\sin(C-D) = \cos(A-B)\sin(C+D)$$

$$\Rightarrow \quad \frac{\cos(A+B)}{\cos(A-B)} = \frac{\sin(C+D)}{\sin(C-D)} \qquad \text{---(i)}$$

Now,

$$\frac{\cos(A+B)}{\cos(A-B)} = \frac{\sin(C+D)}{\sin(C-D)}$$

$$\Rightarrow \quad \frac{\cos(A+B)}{\cos(A-B)} + 1 = \frac{\sin(C+D)}{\sin(C-D)} + 1$$

$$\Rightarrow \quad \frac{\cos(A+B) + \cos(A-B)}{\cos(A-B)} = \frac{\sin(C+D) + \sin(C-D)}{\sin(C-D)} \qquad \text{---(ii)}$$

Again,

$$\frac{\cos(A+B)}{\cos(A-B)} = \frac{\sin(C+D)}{\sin(C-D)} \qquad \left[\text{By equation (i)}\right]$$

$$\Rightarrow \quad \frac{\cos(A+B)}{\cos(A-B)} - 1 = \frac{\sin(C+D)}{\sin(C-D)} - 1$$

$$\Rightarrow \quad \frac{\cos(A+B) - \cos(A-B)}{\cos(A-B)} = \frac{\sin(C+D) - \sin(C-D)}{\sin(C-D)} \qquad \text{---(iii)}$$

Dividing equation (ii) by equation (iii), we get

$$\frac{\cos(A+B) + \cos(A-B)}{\cos(A+B) - \cos(A-B)} = \frac{\sin(C+D) + \sin(C-D)}{\sin(C+D) - \sin(C-D)}$$

$$\Rightarrow \quad \frac{2\cos\left\{\frac{A+B+A-B}{2}\right\}\cos\left\{\frac{A+B-A+B}{2}\right\}}{-2\sin\left\{\frac{A+B+A-B}{2}\right\}\sin\left\{\frac{A+B-A+B}{2}\right\}} = \frac{2\sin\left\{\frac{C+D+C-D}{2}\right\}\cos\left\{\frac{C+D-C+D}{2}\right\}}{2\sin\left\{\frac{C+D-C+D}{2}\right\}\cos\left\{\frac{C+D+C-D}{2}\right\}}$$

$$\Rightarrow \quad \frac{\cos A \cos B}{-\sin A \sin B} = \frac{\sin C \cos D}{\sin D \cos C}$$

$$\Rightarrow \quad \frac{1}{-\tan A \tan B} = \frac{\sin C \cos D}{\cos C \sin D}$$

$$\Rightarrow \quad \frac{-1}{\tan A \tan B} = \frac{\tan C}{\tan D}$$

$$\Rightarrow \quad -\tan D = \tan A \tan B \tan C$$

$$\Rightarrow \quad \tan A \tan B \tan C = -\tan D$$

$$\Rightarrow \quad \tan A \tan B \tan C + \tan D = 0 \qquad \text{Hence proved.}$$

Transformation Formulae Ex 8.2 Q 18.

$$\text{Given } x\cos\theta = y\cos\left(\theta + \frac{2\pi}{3}\right) = z\cos\left(\theta + \frac{4\pi}{3}\right) = k\,(\text{say})$$

$$x = \frac{k}{\cos\theta}$$

$$y = \frac{k}{\cos\left(\theta + \frac{2\pi}{3}\right)}$$

$$z = \frac{k}{\cos\left(\theta + \frac{4\pi}{3}\right)}$$

$$xy + yz + zx = k^2\left[\frac{1}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)} + \frac{1}{\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)} + \frac{1}{\cos\left(\theta + \frac{4\pi}{3}\right)\cos\theta}\right]$$

$$= k^2\left[\frac{\cos\left(\theta + \frac{4\pi}{3}\right) + \cos\theta + \cos\left(\theta + \frac{2\pi}{3}\right)}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)}\right]$$

$$= k^2\left[\frac{\cos\theta\cos\frac{4\pi}{3} - \sin\theta\sin\frac{4\pi}{3} + \cos\theta + \cos\theta\cos\frac{2\pi}{3} - \sin\theta\sin\frac{2\pi}{3}}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)}\right]$$

$$= k^2\left[\frac{\cos\theta\left(\frac{-1}{2}\right) - \sin\theta\left(\frac{-\sqrt{3}}{2}\right) + \cos\theta + \cos\theta\left(\frac{-1}{2}\right) - \sin\theta\left(\frac{\sqrt{3}}{2}\right)}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)}\right]$$

$$= k^2\left[\frac{-\cos\theta + \sin\theta\left(\frac{\sqrt{3}}{2}\right) + \cos\theta + -\sin\theta\left(\frac{\sqrt{3}}{2}\right)}{\cos\theta\cos\left(\theta + \frac{2\pi}{3}\right)\cos\left(\theta + \frac{4\pi}{3}\right)}\right]$$

$$= 0$$

Hence Proved

Transformation Formulae Ex 8.2 Q19

Given that $m \sin\theta = n \sin(\theta + 2a)$,

We need to prove that $\tan(\theta + a) = \dfrac{m+n}{m-n} \tan a$

$m \sin\theta = n \sin(\theta + 2a)$

$\Rightarrow \dfrac{\sin(\theta + 2a)}{\sin\theta} = \dfrac{m}{n}$

Using Componendo − Dividendo, we have,

$\Rightarrow \dfrac{\sin(\theta + 2a) + \sin\theta}{\sin(\theta + 2a) - \sin\theta} = \dfrac{m+n}{m-n}$ ....(1)

We know that,

$\sin C + \sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

and

$\sin C - \sin D = 2\cos\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

Applying the above formulae in equation (1), we have,

$\dfrac{2\sin\dfrac{\theta+2a+\theta}{2}\cos\dfrac{\theta+2a-\theta}{2}}{2\cos\dfrac{\theta+2a+\theta}{2}\sin\dfrac{\theta+2a-\theta}{2}} = \dfrac{m+n}{m-n}$

$\Rightarrow \dfrac{2\sin(\theta + a)\cos a}{2\cos(\theta + a)\sin a} = \dfrac{m+n}{m-n}$

$\Rightarrow \dfrac{\tan(\theta + a)}{\tan a} = \dfrac{m+n}{m-n}$

$\Rightarrow \tan(\theta + a) = \dfrac{m+n}{m-n} \times \tan a$

Hence proved.

********** END *********