



Trigonometric Ratios Ex 5.1 Q22

Answer :

Given: $\sin \theta = \frac{3}{5}$ (1)

To find the value of $\frac{\cos \theta - \frac{1}{\tan \theta}}{2 \cot \theta}$

Now, we know the following trigonometric identity

$$\cos^2 \theta + \sin^2 \theta = 1$$

Therefore, by substituting the value of $\sin \theta$ from equation (1) ,

We get,

$$\cos^2 \theta + \left(\frac{3}{5}\right)^2 = 1$$

Therefore,

$$\begin{aligned}\cos^2 \theta &= 1 - \left(\frac{3}{5}\right)^2 \\ &= 1 - \frac{(3)^2}{(5)^2} \\ &= 1 - \frac{9}{25}\end{aligned}$$

Now by taking L.C.M

We get,

$$\begin{aligned}\cos^2 \theta &= \frac{25-9}{25} \\ &= \frac{16}{25}\end{aligned}$$

Therefore by taking square root on both sides

We get,

$$\begin{aligned}\cos \theta &= \sqrt{\frac{16}{25}} \\ &= \frac{\sqrt{16}}{\sqrt{25}} \\ &= \frac{4}{5}\end{aligned}$$

Therefore,

$$\cos \theta = \frac{4}{5} \text{ (2)}$$

Now, we know that

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

Therefore by substituting the value of $\sin \theta$ and $\cos \theta$ from equation (1) and (2) respectively

We get,

$$\tan \theta = \frac{\frac{3}{5}}{\frac{4}{5}}$$

$$\begin{aligned}\tan \theta &= \frac{3}{5} \times \frac{5}{4} \\ &= \frac{3}{4} \\ \tan \theta &= \frac{3}{4} \dots\dots (3)\end{aligned}$$

Also, we know that

$$\cot \theta = \frac{1}{\tan \theta}$$

Therefore from equation (4) ,

We get,

$$\begin{aligned}\cot \theta &= \frac{1}{\frac{3}{4}} \\ &= \frac{4}{3}\end{aligned}$$

Therefore,

$$\cot \theta = \frac{4}{3} \dots\dots (4)$$

Now, by substituting the value of $\cos \theta$, $\tan \theta$ and $\cot \theta$ from equation (2) , (3) and (4) respectively in the expression below

$$\frac{\cos \theta - \frac{1}{\tan \theta}}{2 \cot \theta}$$

We get,

$$\begin{aligned}\frac{\cos \theta - \frac{1}{\tan \theta}}{2 \cot \theta} &= \frac{\frac{4}{5} - \frac{1}{3}}{2 \times \frac{4}{3}} \\ &= \frac{\frac{4}{5} - \frac{4}{3}}{2 \times 4} \\ &= \frac{\frac{4 \times 3}{5 \times 3} - \frac{4 \times 5}{3 \times 5}}{\frac{8}{3}}\end{aligned}$$

$$\begin{aligned}\frac{\cos \theta - \frac{1}{\tan \theta}}{2 \cot \theta} &= \frac{\frac{12}{15} - \frac{20}{15}}{\frac{8}{3}} \\ &= \frac{\frac{-8}{15}}{\frac{8}{3}}\end{aligned}$$

$$\begin{aligned}&= \frac{-8}{15} \times \frac{3}{8} \\ &= \frac{-1}{5}\end{aligned}$$

Therefore,
$$\frac{\cos \theta - \frac{1}{\tan \theta}}{2 \cot \theta} = \frac{-1}{5}$$

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