Mathematical Induction Ex 12.2 Q24

Let $P(n): n(n+1)(n+5)$ is a multiple of 3 for all $n \in N$

For $n = 1$

$1.(1+1)(1+5)$

$= (2)(6)$

$= 12$

it is a multiple of 3

Let $P(n)$ is true for $n = k$

$k(k+1)(k+5)$ is a multiple of 3

$$k(k+1)(k+5) = 3\lambda \qquad ---(1)$$

We have to show that,

$(k+1)[(k+1)+1][(k+1)+5]$ is a multiple of 3

$(k+1)[(k+1)+1][(k+1)+5] = 3\mu$

Now,

$$(k+1)(k+2)[(k+1)+5]$$
$$= [k(k+1)+2(k+1)][(k+5)+1]$$
$$= k(k+1)(k+5)+k(k+1)+2(k+1)(k+5)+2(k+1)$$
$$= 3\lambda+k^2+k+2(k^2+6k+5)+2k+2 \qquad [\text{Using equation } (1)]$$
$$= 3\lambda+k^2+k+2k^2+12k+10+2k+2$$
$$= 3\lambda+3k^2+15k+12$$
$$= 3(\lambda+k^2+5k+4)$$
$$= 3\mu$$

$\Rightarrow \quad P(n)$ is true for $n = k+1$

$\Rightarrow \quad P(n)$ is true for all $n \in N$ by PMI

Mathematical Induction Ex 12.2 Q25

Let $P(n): 7^{2n} + 2^{3n-3}.3^{n-1}$ is divisible by 25

For $n = 1$

$7^2 + 2^0.3^0$

$= 49 + 1$

$= 50$

it is divisible of 25

$\Rightarrow \quad P(n)$ is true for $n = 1$

Let $P(n)$ is true for $n = k$,

$7^{2k} + 2^{3k-3}.3^{k-1}$ is divisible by 25

$\Rightarrow \quad 7^{2k} + 2^{3k-3}.3^{k-1} = 25\lambda \qquad\qquad ---(1)$

We have to show that,

$7^{2(k+1)} + 2^{3k}3^k$ is divisible by 25

$7^{2(k+1)} + 2^{3k}.3^k = 25\mu$

Now,

$7^{2(k+1)} + 2^{3k}.3^k$

$= 7^{2k}.7^2 + 2^{3k}.3^k$

$= \left(25\lambda - 2^{3k-3}.3^{k-1}\right)49 + 2^{3k}.3k \qquad$ [Using equation (1)]

$= 25\lambda.49 - \frac{2^{3k}}{8}.\frac{3^k}{3}.49 + 2^{3k}.3^k$

$= 24.25.49\lambda - 2^{3k}.3^k.49 + 24.2^{3k}.3^k$

$= 24.25.49\lambda - 25.2^{3k}.3^k$

$= 25\left(24.49\lambda - 2^{3k}.3^k\right)$

$= 25\mu$

$\Rightarrow \quad P(n)$ is true for $n = k + 1$

$\Rightarrow \quad P(n)$ is true for all $n \in N$ by PMI

Mathematical Induction Ex 12.2 Q26

Let $P(n): 2.7^n + 3.5^n - 5$ is divisible by 24

For $n = 1$
$2.7 + 3.5 - 5$

$= 24$

it is divisible of 24
$\Rightarrow \quad P(n)$ is true for $n = 1$
Let $P(n)$ is true for $n = k$, so

$2.7^k + 3.5^k - 5$ is divisible by 24
$2.7^k + 3.5^k - 5 = 24\lambda \qquad ---(1)$

We have to show that,

$2.7^{(k+1)} + 3.5^{(k+1)} - 5$

$= 2.7^k.7 + 3.5^k.5 - 5$
$= (24\lambda - 3.5^k + 5)7 + 15.5^k - 5$
$= 24.7\lambda - 21.5^k + 35 + 15.5^k - 5$
$= 24.7\lambda - 6.5^k + 30$
$= 24.7\lambda - 6(5^k - 5)$
$= 24.7\lambda - 6.(20\nu) \qquad$ [Since $5^k - 5$ is multiple of 20]
$= 24(7\lambda - 5\nu)$

$= 24\mu$

$\Rightarrow \quad P(n)$ is true for $n = k + 1$
$\Rightarrow \quad P(n)$ is true for all $n \in N$ by PMI

*********** END **********