



Exponents of Real Numbers Ex 2.1 Q1

Answer :

We have to simplify the following, assuming that x, y, z are positive real numbers

(i) Given $(\sqrt{x^{-3}})^5$

As x is positive real number then we have

$$\begin{aligned}(\sqrt{x^{-3}})^5 &= \left(\sqrt{\frac{1}{x^3}}\right)^5 \\&= \left(\frac{\sqrt{1}}{\sqrt{x^3}}\right)^5 \\&= \left(\frac{1}{x^{3 \times \frac{1}{2}}}\right)^5 \\&= \left(\frac{1}{x^{\frac{3}{2}}}\right)^5 \\(\sqrt{x^{-3}})^5 &= \left(\frac{1^5}{x^{\frac{3}{2} \times 5}}\right) \\&= \frac{1}{x^{\frac{15}{2}}}\end{aligned}$$

Hence the simplified value of $\left(\sqrt{x^{-3}}\right)^5$ is $\boxed{\frac{1}{x^{\frac{15}{2}}}}$

(ii) Given $\sqrt{x^3 y^{-2}}$

As x and y are positive real numbers then we can write

$$\begin{aligned}\sqrt{x^3 y^{-2}} &= \left(x^3 y^{-2}\right)^{\frac{1}{2}} \\ &= \left(x^{3 \times \frac{1}{2}} \times y^{-2 \times \frac{1}{2}}\right) \\ &= \left(x^{\frac{3}{2}} \times y^{-\frac{2 \times 1}{2}}\right) \\ &= \left(x^{\frac{3}{2}} y^{-1}\right)\end{aligned}$$

By using law of rational exponents $a^{-n} = \frac{1}{a^n}$ we have

$$\begin{aligned}\sqrt{x^3 y^{-2}} &= x^{\frac{3}{2}} \times \frac{1}{y} \\ &= \frac{x^{\frac{3}{2}}}{y}\end{aligned}$$

Hence the simplified value of $\sqrt{x^3 y^{-2}}$ is $\boxed{\frac{x^{\frac{3}{2}}}{y}}$

(iii) Given $\left(x^{\frac{-2}{3}} y^{\frac{-1}{2}}\right)^2$

As x and y are positive real numbers then we have

$$\left(x^{\frac{-2}{3}} y^{\frac{-1}{2}}\right)^2 = \left(x^{\frac{-2}{3}} \times x^{\frac{-2}{3}} \times y^{\frac{-1}{2}} \times y^{\frac{-1}{2}}\right)$$

By using law of rational exponents $a^{-n} = \frac{1}{a^n}$ we have

$$\begin{aligned}\left(x^{\frac{-2}{3}} y^{\frac{-1}{2}}\right)^2 &= \frac{1}{x^{\frac{2}{3}}} \times \frac{1}{x^{\frac{2}{3}}} \times \frac{1}{y^{\frac{1}{2}}} \times \frac{1}{y^{\frac{1}{2}}} \\ \Rightarrow \left(x^{\frac{-2}{3}} y^{\frac{-1}{2}}\right)^2 &= \frac{1}{x^{\frac{2}{3}} \times x^{\frac{2}{3}}} \times \frac{1}{y^{\frac{1}{2}} \times y^{\frac{1}{2}}}\end{aligned}$$

By using law of rational exponents $a^m \times a^n = a^{m+n}$ we have

$$\begin{aligned}
 \left(x^{-\frac{2}{3}}y^{-\frac{1}{2}}\right)^2 &= \frac{1}{x^{\frac{2}{3}+\frac{2}{3}}} \times \frac{1}{y^{\frac{1}{2}+\frac{1}{2}}} \\
 &= \frac{1}{x^{\frac{4}{3}}} \times \frac{1}{y^{\frac{2}{2}}} = \frac{1}{x^{\frac{4}{3}}} \times \frac{1}{y} \\
 &= \frac{1}{x^{\frac{4}{3}}y}
 \end{aligned}$$

Hence the simplified value of $\left(x^{-\frac{2}{3}}y^{-\frac{1}{2}}\right)^2$ is $\boxed{\frac{1}{x^{\frac{4}{3}}y}}$.

$$\begin{aligned}
 \text{(iv)} \quad & (\sqrt{x})^{-\frac{2}{3}} \sqrt{y^4} \div \sqrt{xy^{-\frac{1}{2}}} \\
 &= \left(x^{\frac{1}{2}}\right)^{-\frac{2}{3}} (y^4)^{\frac{1}{2}} \div \left(x \times y^{-\frac{1}{2}}\right)^{\frac{1}{2}} \\
 &= \frac{x^{\frac{1}{2} \times -\frac{2}{3}} \times y^{4 \times \frac{1}{2}}}{x^{\frac{1}{2}} \times y^{-\frac{1}{2} \times \frac{1}{2}}} \\
 &= \frac{x^{-\frac{1}{3}} \times y^2}{x^{\frac{1}{2}} \times y^{-\frac{1}{4}}}
 \end{aligned}$$

by using the law of rational exponents, $a^m \div a^n = a^{m-n}$, we have

$$\begin{aligned}
 & x^{-\frac{1}{3}-\frac{1}{2}} \times y^{2+\frac{1}{4}} \\
 &= x^{-\frac{5}{6}} \times y^{\frac{9}{4}} \\
 &= \frac{y^{\frac{9}{4}}}{x^{\frac{5}{6}}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(v).} \quad & \sqrt[5]{243 \ x^{10} \ y^5 \ z^{10}} \\
 &= (243 \times x^{10} \times y^5 \times z^{10})^{\frac{1}{5}} \\
 &= (243)^{\frac{1}{5}} \times (x^{10})^{\frac{1}{5}} \times (y^5)^{\frac{1}{5}} \times (z^{10})^{\frac{1}{5}} \\
 &= (3^5)^{\frac{1}{5}} \times x^{10 \times \frac{1}{5}} \times y^{5 \times \frac{1}{5}} \times z^{10 \times \frac{1}{5}} \\
 &= 3 \times x^2 \times y \times z^2 \\
 &= 3x^2yz^2
 \end{aligned}$$

$$\text{(vi)} \quad \left(\frac{x^{-4}}{y^{-10}} \right)^{\frac{5}{4}}$$

$$= \frac{(x^{-4})^{\frac{5}{4}}}{(y^{-10})^{\frac{5}{4}}}$$

$$= \frac{x^{-4 \times \frac{5}{4}}}{y^{-10 \times \frac{5}{4}}}$$

$$= \frac{x^{-5}}{y^{-\frac{25}{2}}}$$

$$= \frac{y^{\frac{25}{2}}}{x^5}$$

***** END *****