Mathematical Induction Ex 12.2 Q12

Let $1.3+2.4+3.5+\ldots+n(n+2)=\frac{1}{6}n(n+1)(2n+7)$

For $n=1$

$$1.3=\frac{1}{6}.1.(2)(9)$$
$$3=3$$

$\Rightarrow$ $P(n)$ is true for $n=1$

Let $P(n)$ is true for $n=k$, so

$$1.3+2.4+3.5+\ldots+k(k+2)=\frac{1}{6}k(k+1)(2k+7) \qquad ---(1)$$

We have to show that,

$$1.3+2.4+3.5+\ldots+k(k+2)+(k+1)(k+3)=\frac{(k+1)}{6}(k+2)(2k+9)$$

Now,

$$\{1.3+2.4+3.5+\ldots+k(k+2)\}+(k+1)(k+3)$$

$$=\frac{1}{6}k(k+1)(2k+7)+(k+1)(k+3) \qquad [\text{Using equation }(1)]$$
$$=(k+1)\left[\frac{k(2k+7)}{6}+\frac{k+3}{1}\right]$$
$$=(k+1)\left[\frac{2k^2+7k+6k+18}{6}\right]$$
$$=(k+1)\left(\frac{2k^2+13k+18}{6}\right)$$
$$=(k+1)\left[\frac{2k^2+4k+9k+18}{6}\right]$$
$$=(k+1)\left[\frac{2k(k+2)+9(k+2)}{6}\right]$$
$$=(k+1)\left[\frac{(2k+9)(k+2)}{6}\right]$$

$$=\frac{1}{6}(k+1)(k+2)(2k+9)$$

$\Rightarrow$ $P(n)$ is true for $n=k+1$

$\Rightarrow$ $P(n)$ is true for all $n\in N$ by PMI

Mathematical Induction Ex 12.2 Q13

Let $P(n): 1.3 + 3.5 + 5.7 + \ldots + (2n-1)(2n+1) = \dfrac{n(4n^2 + 6n - 1)}{3}$

For $n = 1$

$$1.3 = \frac{1(4 + 6 - 1)}{3}$$

$$3 = 3$$

$\Rightarrow$ $P(n)$ is true for $n = 1$

Let $P(n)$ is true for $n = k$, so

$$1.3 + 3.5 + 5.7 + \ldots + (2k-1)(2k+1) = \frac{k(4k^2 + 6k - 1)}{3} \qquad ---(1)$$

We have to show that,

$$1.3 + 3.5 + 5.7 + \ldots + (2k-1)(2k+1) + (2k+1)(2k+3) = \frac{(k+1)\left[4(k+1)^2 + 6(k+1) - 1\right]}{3}$$

Now,

$$\{1.3 + 3.5 + 5.7 + \ldots + (2k-1)(2k+1)\} + (2k+1)(2k+3)$$

$$= \frac{k(4k^2 + 6k - 1)}{3} + (2k+1)(2k+3) \qquad [\text{Using equation } (1)]$$

$$= \frac{k(4k^2 + 6k - 1) + 3(4k^2 + 6k + 2k + 3)}{3}$$

$$= \frac{4k^3 + 6k^2 - k + 12k^2 + 18k + 6k + 9}{3}$$

$$= \frac{4k^3 + 18k^2 + 23k + 9}{3}$$

$$= \frac{4k^3 + 4k^2 + 14k^2 + 14k + 9k + 9}{3}$$

$$= \frac{(k+1)(4k^2 + 8k + 4 + 6k + 6 - 1)}{3}$$

$$= \frac{(k+1)\left[4(k+1)^2 + 6(k+1) - 1\right]}{3}$$

$\Rightarrow$ $P(n)$ is true for $n = k + 1$

$\Rightarrow$ $P(n)$ is true for all $n \in N$ by PMI

Mathematical Induction Ex 12.2 Q14

Let $P(n): 1.2 + 2.3 + 3.4 + \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$

For $n = 1$

$$1.2 = \frac{1(1+1)(1+2)}{3}$$

$$2 = 2$$

$\Rightarrow$ $P(n)$ is true for $n = 1$

Let $P(n)$ is true for $n = k$

$$\Rightarrow \quad 1.2 + 2.3 + 3.4 + \ldots + k(k+1) = \frac{k(k+1)(k+2)}{3} \qquad ---(1)$$

We have to show that,

$$1.2 + 2.3 + 3.4 + \ldots + k(k+1) + (k+1)(k+2) = \frac{(k+1)(k+2)(k+3)}{3}$$

Now,

$$\{1.2 + 2.3 + 3.4 + \ldots + k(k+1)\} + (k+1)(k+2)$$

$$= \frac{k(k+1)(k+2)}{3} + \frac{(k+1)(k+2)}{1}$$

$$= (k+1)(k+2)\left[\frac{k}{3} + 1\right]$$

$$= \frac{(k+1)(k+2)(k+3)}{3}$$

$\Rightarrow$ $P(n)$ is true for $n = k+1$

$\Rightarrow$ $P(n)$ is true for all $n \in N$ by PMI

Mathematical Induction Ex 12.2 Q15

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