Definite Integrals Ex 20.4B Q41

We have,

$$I = \int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_a^{2a} f(x)\,dx$$

$$I = \int_0^{a} f(x)\,dx + I_1$$

Let $2a - t = x$ then $dx = -dt$

$t = a, \; x = a$

$t = 2a \quad x = 0$

$$I_1 = \int_0^{2a} f(x) = \int_a^{0} f(2a - t)(-dt)$$

$$= -\int_a^{0} f(2a - t)\,dt$$

$$I_1 = \int_0^{a} f(2a - t)\,dt = \int_0^{a} f(2a - x)\,dx$$

$$I = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a - x)\,dx$$

$$I = \int_0^{a} f(x)\,dx - \int_0^{a} f(x) \qquad \left[\because f(2a - x) = -f(x)\right]$$

$$I = 0$$

Hence,

$$\int_0^{2a} f(x)\,dx = 0$$

Definite Integrals Ex 20.4B Q42

(i) We have,

$$I = \int_{-a}^{a} f\left(x^2\right)dx$$

Clearly $f\left(x^2\right)$ is an even function.

So,

$$\int_{-a}^{a} f(t) = 2\int_0^{a} f(t)$$

$$I = 2\int_0^{a} f\left(x^2\right)dx$$

(ii) We have,

$$I = \int_{-a}^{a} x f\left(x^2\right)dx$$

Clearly, $xf\left(x^2\right)$ is odd function.

So, $I = 0$

$$\therefore \int_{-a}^{a} x f\left(x^2\right)dx = 0$$

Definite Integrals Ex 20.4B Q43

We have from LHS,

$$I = \int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_a^{2a} f(x)\,dx \qquad \ldots(i)$$

Let $x = 2a - t$, then $dx = -dt$

$x = a \Rightarrow t = a$, and $x = 2a \Rightarrow t = 0$

$$\therefore \int_0^{2a} f(x)\,dx = -\int_a^{0} f(2a-t)\,dt$$

$$\Rightarrow \int_0^{2a} f(x)\,dx = \int_0^{a} f(2a-t)\,dt$$

$$\Rightarrow \int_0^{2a} f(x)\,dx = \int_0^{a} f(2a-x)\,dx$$

Substituting $\int_0^{2a} f(x)\,dx = \int_0^{a} f(2a-x)\,dx$ in (i)

we get,

$$\int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a-x)\,dx$$

$$\Rightarrow \int_0^{2a} f(x)\,dx = \int_0^{a} \{f(x) + f(2a-x)\}\,dx$$

*********** END **********