



Definite Integrals Ex 20.4B Q29

$$\begin{aligned}
 I &= \int_{-\pi}^{\pi} \frac{2x(1 + \sin x)}{1 + \cos^2 x} dx \\
 I &= \int_{-\pi}^{\pi} \frac{2x}{1 + \cos^2 x} dx + \int_{-\pi}^{\pi} \frac{2x \sin x}{1 + \cos^2 x} dx \\
 I &= 0 + \int_{-\pi}^{\pi} \frac{2x \sin x}{1 + \cos^2 x} dx, \dots \dots \dots \left[\because \frac{2x}{1 + \cos^2 x} \text{ is an odd function} \right] \\
 I &= 2 \int_0^{\pi} \frac{2x \sin x}{1 + \cos^2 x} dx, \dots \dots \dots \left[\because \frac{2x \sin x}{1 + \cos^2 x} \text{ is an even function} \right] \\
 I &= 4 \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx \\
 I &= 2\pi \int_0^{\pi} \frac{\sin x}{1 + \cos^2 x} dx, \dots \dots \dots \left[\because \int_0^a xf(x) dx = \frac{a}{2} \int_0^a f(x) dx \right]
 \end{aligned}$$

Put $\cos x = t$ then $-\sin x dx = dt$

$$\begin{aligned}
 I &= -2\pi \int_1^{-1} \frac{1}{1 + t^2} dt \\
 I &= -2\pi [\tan^{-1} t]_1^{-1} \\
 I &= \pi^2
 \end{aligned}$$

Definite Integrals Ex 20.4B Q30

$$\begin{aligned}
 I &= \int_{-\pi}^{\pi} \log \left(\frac{a - \sin \theta}{a + \sin \theta} \right) d\theta \\
 \text{Let } f(\theta) &= \log \left(\frac{a - \sin \theta}{a + \sin \theta} \right) \\
 f(-\theta) &= \log \left(\frac{a - \sin(-\theta)}{a + \sin(-\theta)} \right) = -\log \left(\frac{a - \sin \theta}{a + \sin \theta} \right) = -f(\theta) \\
 \therefore f(\theta) &= \log \left(\frac{a - \sin \theta}{a + \sin \theta} \right) \text{ is an odd function.} \\
 \therefore I &= \int_{-\pi}^{\pi} \log \left(\frac{a - \sin \theta}{a + \sin \theta} \right) d\theta = 0
 \end{aligned}$$

Definite Integrals Ex 20.4B Q31

$$I = \int_{-2}^2 \frac{3x^3 + 2|x| + 1}{x^2 + |x| + 1} dx$$

$$I = \int_{-2}^2 \frac{3x^3}{x^2 + |x| + 1} dx + \int_{-2}^2 \frac{2|x| + 1}{x^2 + |x| + 1} dx$$

$$I = 0 + \int_{-2}^2 \frac{2|x| + 1}{x^2 + |x| + 1} dx \dots \dots \dots \left[\because \frac{3x^3}{x^2 + |x| + 1} \text{ is an odd function} \right]$$

$$I = 2 \int_0^2 \frac{2|x| + 1}{x^2 + |x| + 1} dx \dots \dots \dots \left[\because \frac{2|x| + 1}{x^2 + |x| + 1} \text{ is an even function} \right]$$

$$I = 2 \left[\log(x^2 + |x| + 1) \right]_0^2$$

$$I = 2 [\log(4 + 2 + 1) - \log(1)]$$

$$I = 2 \log_e (7)$$

Definite Integrals Ex 20.4B Q32

$$I = \int_{-3\pi/2}^{-\pi/2} \{ \sin^2(3\pi + x) + (\pi + x)^3 \} dx$$

Substitute $\pi + x = u$ then $dx = du$

$$I = \int_{-\pi/2}^{\pi/2} \{ \sin^2(2\pi + u) + (u)^3 \} du$$

$$I = \int_{-\pi/2}^{\pi/2} \{ \sin^2(u) + (u)^3 \} du$$

$$I = \left[\frac{1}{2} \left(u - \frac{1}{2} \sin(2u) \right) + \frac{u^4}{4} \right]_{-\pi/2}^{\pi/2}$$

$$I = \frac{\pi}{2}$$

***** END *****