



Exercise 10C

Question 9:

$$x^2 - x + 2 = 0$$

The given equation is

This is of form $ax^2+bx+c=0$, when

$a = 1, b = -1, c = 2$

$$\therefore D = b^2 - 4ac = (-1)^2 - 4 \times 1 \times 2 = 1 - 8 = -7 < 0$$

So the equation has no real roots

Question 10:

$$kx^2 + 2x + 1 = 0$$

The given equation is

This is of the form $ax^2+bx+c=0$, where $a = k, b = 2, c = 1$

$$\therefore D = b^2 - 4ac = (2)^2 - 4 \times k \times 1 = 4 - 4k$$

The given equation will have real and distinct roots if

Now,

$$D > 0 \Rightarrow 4 - 4k > 0 \Rightarrow 4k < 4 \Rightarrow k < 1$$

Question 11:

$$2x^2 + px + 8 = 0$$

The given equation is

This is of the form $ax^2+bx+c=0$, where

$a = 2, b = p, c = 8$

$$\therefore D = b^2 - 4ac = [(p)^2 - 4 \times 2 \times 8] = p^2 - 64$$

The given equation will have real roots if $D \geq 0$

$$\begin{aligned} \text{Now, } D \geq 0 &\Rightarrow p^2 - 64 \geq 0 \Rightarrow p^2 \geq 64 \\ &\Rightarrow p \geq 8 \text{ or } p \leq -8 \end{aligned}$$

Question 12:

The given equation is

$$(\alpha - 12)x^2 + 2(\alpha - 12)x + 2 = 0$$

This is of the form $ax^2 + bx + c = 0$,

where $a = (\alpha - 12), b = 2(\alpha - 12), c = 2$

$$\begin{aligned}
 \therefore D &= 2^2 \times (\alpha - 12)^2 - 4 \times 2 \times (\alpha - 12) \\
 &= 4(\alpha - 12)^2 - 8(\alpha - 12) \\
 &= 4(\alpha - 12)[(\alpha - 12) - 2] \\
 &= 4\{(\alpha - 12)(\alpha - 14)\}
 \end{aligned}$$

For real and equal roots, we must have $D = 0$

Now, $D = 0$

$$4(\alpha - 12)(\alpha - 14) = 0$$

$$\Rightarrow (\alpha - 14) = 0, (\alpha - 12) = 0$$

But $\alpha \neq 12$ $\{\because$ the equation will not be a quadratic one if $\alpha = 12\}$

Hence $\alpha = 14$

Question 13:

The given equation is

$$(1 + m^2)x^2 + 2mcx + (c^2 - a^2) = 0$$

This is of the form $ax^2 + bx + c = 0$,

where $a = (1 + m^2)$, $b = 2mc$, $c = (c^2 - a^2)$

$$\begin{aligned}
 \therefore D &= b^2 - 4ac = (2mc)^2 - 4(1 + m^2)(c^2 - a^2) \\
 \Rightarrow D &= 4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2) \\
 \Rightarrow D &= 4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4m^2a^2 \\
 \Rightarrow D &= 4m^2a^2 + 4a^2 - 4c^2
 \end{aligned}$$

For real and equal roots, we must have $D = 0$

$$\text{Now, } D = 0 \Rightarrow 4(m^2a^2 + a^2 - c^2) = 0$$

$$\Rightarrow m^2a^2 + a^2 - c^2 = 0$$

$$\Rightarrow m^2a^2 + a^2 = c^2$$

$$\Rightarrow a^2(m^2 + 1) = c^2$$

$$\Rightarrow c^2 = a^2(m^2 + 1) \quad \text{proved}$$

Question 14:

The given equation is

$$(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$$

This is the form of $ax^2 + bx + c = 0$

$$a = (c^2 - ab), b = -2(a^2 - bc), c = (b^2 - ac)$$

$$\therefore D = B^2 - 4AC$$

$$\begin{aligned} &= (-2)^2 \times (a^2 - bc)^2 - 4 \times (c^2 - ab) \times (b^2 - ac) \\ &= 4 \times (a^4 + b^2c^2 - 2a^2bc) - 4 \times (c^2b^2 - ac^3 - ab^3 + a^2bc) \\ &= 4a^4 + 4b^2c^2 - 8a^2bc - 4c^2b^2 + 4ac^3 + 4ab^3 - 4a^2bc \\ &= 4a^4 - 12a^2bc + 4ac^3 + 4ab^3 \\ &= 4a(a^3 - 3abc + c^3 + b^3) \end{aligned}$$

For real and equal roots, we must have $D = 0$

$$\text{Now, } D = 0 \Rightarrow 4a(a^3 - 3abc + c^3 + b^3) = 0$$

$$\Rightarrow \text{Either, } a^3 - 3abc + c^3 + b^3 = 0 \text{ or } a = 0$$

$$\text{Either } a = 0 \text{ or } a^3 + b^3 + c^3 = 3abc \text{ proved}$$

***** END *****