Trigonometric Identities Ex 6.1 Q17

Answer:

We have to prove $(\sec\theta + \cos\theta)(\sec\theta - \cos\theta) = \tan^2\theta + \sin^2\theta$

We know that,

$\sin^2\theta + \cos^2\theta = 1$,

$\sec^2\theta - \tan^2\theta = 1$

$$(\sec\theta + \cos\theta)(\sec\theta - \cos\theta) = \sec^2\theta - \cos^2\theta$$
$$= (1 + \tan^2\theta) - (1 - \sin^2\theta)$$
$$= 1 + \tan^2\theta - 1 + \sin^2\theta$$
$$= \tan^2\theta + \sin^2\theta$$

Trigonometric Identities Ex 6.1 Q18

Answer:

We have to prove $\sec A(1 - \sin A)(\sec A + \tan A) = 1$

We know that, $\sec^2 A - \tan^2 A = 1$

So,

$$\sec A(1 - \sin A)(\sec A + \tan A) = \{\sec A(1 - \sin A)\}(\sec A + \tan A)$$
$$= (\sec A - \sec A \sin A)(\sec A + \tan A)$$
$$= \left(\sec A - \frac{1}{\cos A}\sin A\right)(\sec A + \tan A)$$
$$= \left(\sec A - \frac{\sin A}{\cos A}\right)(\sec A + \tan A)$$
$$= (\sec A - \tan A)(\sec A + \tan A)$$
$$= \sec^2 A - \tan^2 A$$
$$= 1$$

Trigonometric Identities Ex 6.1 Q9

Answer :

We have to prove $(\operatorname{cosec}A - \sin A)(\sec A - \cos A)(\tan A + \cot A) = 1$

We know that, $\sin^2 A + \cos^2 A = 1$

So,

$(\operatorname{cosec}A - \sin A)(\sec A - \cos A)(\tan A + \cot A)$

$= \left(\dfrac{1}{\sin A} - \sin A\right)\left(\dfrac{1}{\cos A} - \cos A\right)\left(\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}\right)$

$= \left(\dfrac{1 - \sin^2 A}{\sin A}\right)\left(\dfrac{1 - \cos^2 A}{\cos A}\right)\left(\dfrac{\sin^2 A + \cos^2 A}{\sin A \cos A}\right)$

$= \left(\dfrac{\cos^2 A}{\sin A}\right)\left(\dfrac{\sin^2 A}{\cos A}\right)\left(\dfrac{1}{\sin A \cos A}\right)$

$= \dfrac{\sin^2 A \cos^2 A}{\sin^2 A \cos^2 A}$

$= 1$

Trigonometric Identities Ex 6.1 Q20

Answer :

We have to prove $\tan^2 \theta - \sin^2 \theta = \tan^2 \theta \sin^2 \theta$

We know that, $\sin^2 \theta + \cos^2 \theta = 1$

So,

$$\tan^2 \theta - \sin^2 \theta = \dfrac{\sin^2 \theta}{\cos^2 \theta} - \sin^2 \theta$$

$$= \dfrac{\sin^2 \theta - \sin^2 \theta \cos^2 \theta}{\cos^2 \theta}$$

$$= \dfrac{\sin^2 \theta(1 - \cos^2 \theta)}{\cos^2 \theta}$$

$$= \dfrac{\sin^2 \theta \sin^2 \theta}{\cos^2 \theta}$$

$$= \dfrac{\sin^2 \theta}{\cos^2 \theta} \sin^2 \theta$$

$$= \tan^2 \theta \sin^2 \theta$$

********** END *********