Indefinite Integrals Ex 19.30 Q6

Let $I = \int \frac{x^2}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$

$\Rightarrow \quad x^2 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

Put $x = 1$

$\Rightarrow \quad 1 = 2A \Rightarrow \quad A = \frac{1}{2}$

Put $x = 2$

$\Rightarrow \quad 4 = -B \quad \Rightarrow \quad B = -4$

Put $x = 3$

$\Rightarrow \quad 9 = 2C \Rightarrow \quad C = \frac{9}{2}$

Thus,

$$I = \int \frac{x^2}{(x-1)(x-2)(x-3)} dx = \frac{1}{2}\int \frac{dx}{x-1} - 4\int \frac{dx}{x-2} + \frac{9}{2}\int \frac{dx}{x-3}$$

$$= \frac{1}{2}\log|x-1| - 4\log|x-2| + \frac{9}{2}\log|x-3| + c$$

Hence,

$$I = \frac{1}{2}\log|x-1| - 4\log|x-2| + \frac{9}{2}\log|x-3| + c$$

Indefinite Integrals Ex 19.30 Q7

$$\frac{5x}{(x+1)(x^2-4)}=\frac{5x}{(x+1)(x+2)(x-2)}$$

Let $\frac{5x}{(x+1)(x+2)(x-2)}=\frac{A}{(x+1)}+\frac{B}{(x+2)}+\frac{C}{(x-2)}$

$$5x=A(x+2)(x-2)+B(x+1)(x-2)+C(x+1)(x+2) \qquad ...(1)$$

Substituting $x = -1, -2$, and 2 respectively in equation (1), we obtain

$$A=\frac{5}{3},\ B=-\frac{5}{2},\text{ and } C=\frac{5}{6}$$

$$\therefore \frac{5x}{(x+1)(x+2)(x-2)}=\frac{5}{3(x+1)}-\frac{5}{2(x+2)}+\frac{5}{6(x-2)}$$

$$\Rightarrow \int\frac{5x}{(x+1)(x^2-4)}dx=\frac{5}{3}\int\frac{1}{(x+1)}dx-\frac{5}{2}\int\frac{1}{(x+2)}dx+\frac{5}{6}\int\frac{1}{(x-2)}dx$$

$$=\frac{5}{3}\log|x+1|-\frac{5}{2}\log|x+2|+\frac{5}{6}\log|x-2|+C$$

Indefinite Integrals Ex 19.30 Q8

Let $I=\int\frac{x^2+1}{x(x^2-1)}dx=\int\frac{x^2+1}{x(x+1)(x-1)}dx$

Let $\frac{x^2+1}{x(x+1)(x-1)}=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{x-1}$

$\Rightarrow \quad x^2+1=A(x+1)(x-1)+B.x(x-1)+Cx(x+1)$

Put $x = 0$

$\Rightarrow \quad 1=-A \Rightarrow \quad A=-1$

Put $x = -1$

$\Rightarrow \quad 2=2B \Rightarrow \quad B=1$

Put $x = 1$

$\Rightarrow \quad 2=2C \Rightarrow \quad C=1$

Thus,

$$I=-\int\frac{dx}{x}+\int\frac{dx}{x+1}+\int\frac{dx}{x-1}$$

$$=-\log|x|+\log|x+1|+\log|x-1|+c$$

$$I=\log\left|\frac{x^2-1}{x}\right|+c$$

Indefinite Integrals Ex 19.30 Q9

Let $I = \int \frac{2x-3}{(x^2-1)(2x+3)} dx = \int \frac{2x-3}{(x+1)(x-1)(2x+3)} dx$

Let $\frac{2x-3}{(x+1)(x-1)(2x+3)} = \frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{C}{(2x+3)}$

$\Rightarrow \quad 2x-3 = A(x-1)(2x+3) + B(x+1)(2x+3) + C(x^2-1)$

Put $x = -1$

$\Rightarrow \quad -5 = -2A \quad \Rightarrow \quad A = \frac{5}{2}$

Put $x = 1$

$\Rightarrow \quad -1 = 10B \quad \Rightarrow \quad B = -\frac{1}{10}$

Put $x = -\frac{3}{2}$

$\Rightarrow \quad -6 = \frac{5}{4}C \quad \Rightarrow \quad C = -\frac{24}{5}$

Thus,

$I = \frac{5}{2}\int \frac{dx}{x+1} - \frac{1}{10}\int \frac{dx}{x-1} - \frac{24}{5}\int \frac{dx}{2x+3}$

$= \frac{5}{2}\log|x+1| - \frac{1}{10}\log|x-1| - \frac{24}{5}.\frac{1}{2}\log|2x+3| + c$

Hence,

$I = \frac{5}{2}\log|x+1| - \frac{1}{10}\log|x-1| - \frac{12}{5}\log|2x+3| + c$

Indefinite Integrals Ex 19.30 Q10

Let $I = \int \frac{x^3}{(x-1)(x-2)(x-3)} dx$

$$= \int 1 + \frac{6x^2 - 9x + 6}{(x-1)(x-2)(x-3)} dx$$

Let $\frac{6x^2 - 11x + 6}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$

$\Rightarrow \quad 6x^2 - 11x + 6 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

Put $x = 1$

$\Rightarrow \quad 1 = 2A \Rightarrow \quad A = \frac{1}{2}$

Put $x = 2$

$\Rightarrow \quad 8 = -B \quad \Rightarrow \quad B = -8$

Put $x = 3$

$\Rightarrow \quad 27 = 2C \quad \Rightarrow \quad C = \frac{27}{2}$

Thus,

$$I = \int dx + \frac{1}{2}\int \frac{dx}{x-1} - 8\int \frac{dx}{x-2} + \frac{27}{2}\int \frac{dx}{x-3}$$

$$= x + \frac{1}{2}\log|x-1| - 8\log|x-2| + \frac{27}{2}\log|x-3| + c$$

Hence,

$$I = x + \frac{1}{2}\log|x-1| - 8\log|x-2| + \frac{27}{2}\log|x-3| + c$$

********** END **********