Trigonometric Ratios Ex 5.1 Q32

Answer :

Given:

$\operatorname{cosec} A = 2$ ...... (1)

To find:

$$\frac{1}{\tan A} + \frac{\sin A}{1+\cos A}$$

Now we know $\operatorname{cosec} A$ is defined as below

$$\operatorname{cosec} A = \frac{1}{\sin A}$$

Therefore,

$$\sin A = \frac{1}{\operatorname{cosec} A}$$

Now by substituting the value of $\cos ec A$ from equation (1)

We get,

$\sin A = \frac{1}{2}$ ...... (2)

Now by substituting the value of $\sin A$ in the following identity of trigonometry

$$\sin^2 A + \cos^2 A = 1$$

We get,

$$\cos^2 A = 1 - \sin^2 A$$

$$= 1 - \left(\frac{1}{2}\right)^2$$

$$= 1 - \frac{1}{4}$$

Now by taking L.C.M we get

$$\cos^2 A = \frac{4-1}{4}$$

$$= \frac{3}{4}$$

Now by taking square root on both sides

We get,

$$\cos A = \sqrt{\frac{3}{4}}$$

$$= \frac{\sqrt{3}}{\sqrt{4}}$$

$$= \frac{\sqrt{3}}{2}$$

Therefore,

$\cos A = \frac{\sqrt{3}}{2}$ ...... (3)

Now $\tan A$ is defined as follows

$$\tan A = \frac{\sin A}{\cos A}$$

Now by substituting the value of $\sin A$ and $\cos A$ from equation (2) and (3) respectively we get,

$$\tan A = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$
$$= \frac{1}{2} \times \frac{2}{\sqrt{3}}$$
$$= \frac{1}{\sqrt{3}}$$

Therefore,

$$\tan A = \frac{1}{\sqrt{3}} \quad \ldots\ldots (4)$$

Now by substituting the value of $\sin A$, $\cos A$ and $\tan A$ from equation (2), (3) and (4) respectively we get,

$$\frac{1}{\tan A} + \frac{\sin A}{1+\cos A} = \frac{1}{\frac{1}{\sqrt{3}}} + \frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}}$$
$$= \frac{\sqrt{3}}{1} + \frac{1}{2\left(1+\frac{\sqrt{3}}{2}\right)}$$

Now by taking L.C.M we get

$$\frac{1}{\tan A} + \frac{\sin A}{1+\cos A} = \frac{\sqrt{3}}{1} + \frac{1}{2\left(\frac{2+\sqrt{3}}{2}\right)}$$

Now 2 gets cancelled and we get

$$\frac{1}{\tan A} + \frac{\sin A}{1+\cos A} = \frac{\sqrt{3}}{1} + \frac{1}{\left(2+\sqrt{3}\right)}$$

Now by taking L.C.M, we get,

$$\frac{1}{\tan A} + \frac{\sin A}{1+\cos A} = \frac{\sqrt{3}\times\left(2+\sqrt{3}\right)}{1\times\left(2+\sqrt{3}\right)} + \frac{1}{\left(2+\sqrt{3}\right)}$$
$$= \frac{\sqrt{3}\times\left(2+\sqrt{3}\right)+1}{\left(2+\sqrt{3}\right)}$$

Now by opening the brackets in the numerator

We get,

$$\frac{1}{\tan A} + \frac{\sin A}{1+\cos A} = \frac{2\sqrt{3}+\sqrt{3}\sqrt{3}+1}{\left(2+\sqrt{3}\right)}$$

Since $\sqrt{3}\sqrt{3} = 3$

Therefore,

$$\frac{1}{\tan A} + \frac{\sin A}{1+\cos A} = \frac{2\sqrt{3}+3+1}{(2+\sqrt{3})}$$

$$= \frac{2\sqrt{3}+4}{(2+\sqrt{3})}$$

Now by taking 2 common

We get,

$$\frac{1}{\tan A} + \frac{\sin A}{1+\cos A} = \frac{2(\sqrt{3}+2)}{(2+\sqrt{3})}$$

$$= \frac{2(2+\sqrt{3})}{(2+\sqrt{3})}$$

Now as $(2+\sqrt{3})$ is present in both numerator as well as denominator, it gets cancelled

Therefore,

$$\frac{1}{\tan A} + \frac{\sin A}{1+\cos A} = 2$$

*********** END **********