Question 2.13:

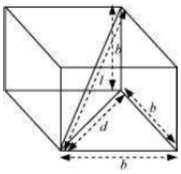
A cube of side *b* has a charge *q* at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube.

Answer

Length of the side of a cube = *b*

Charge at each of its vertices = *q*

A cube of side *b* is shown in the following figure.



d = Diagonal of one of the six faces of the cube

$$d^2 = \sqrt{b^2 + b^2} = \sqrt{2b^2}$$

$$d = b\sqrt{2}$$

l = Length of the diagonal of the cube

$$l^2 = \sqrt{d^2 + b^2}$$

$$= \sqrt{\left(\sqrt{2}b\right)^2 + b^2} = \sqrt{2b^2 + b^2} = \sqrt{3b^2}$$

$$l = b\sqrt{3}$$

$r = \frac{l}{2} = \frac{b\sqrt{3}}{2}$ is the distance between the centre of the cube and one of the eight vertices

The electric potential (V) at the centre of the cube is due to the presence of eight charges at the vertices.

$$V = \frac{8q}{4\pi\epsilon_0}$$

$$= \frac{8q}{4\pi\epsilon_0\left(b\frac{\sqrt{3}}{2}\right)}$$

$$= \frac{4q}{\sqrt{3}\pi\epsilon_0 b}$$

Therefore, the potential at the centre of the cube is $\frac{4q}{\sqrt{3}\pi\epsilon_0 b}$.

The electric field at the centre of the cube, due to the eight charges, gets cancelled. This is because the charges are distributed symmetrically with respect to the centre of the cube. Hence, the electric field is zero at the centre.

Question 2.14:

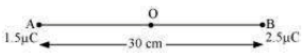
Two tiny spheres carrying charges 1.5 μC and 2.5 μC are located 30 cm apart. Find the potential and electric field:

(a) at the mid-point of the line joining the two charges, and

(b) at a point 10 cm from this midpoint in a plane normal to the line and passing through the mid-point.

Answer

Two charges placed at points A and B are represented in the given figure. O is the mid-point of the line joining the two charges.



Magnitude of charge located at A, q_1 = 1.5 μC

Magnitude of charge located at B, q_2 = 2.5 μC

Distance between the two charges, d = 30 cm = 0.3 m

(a) Let V_1 and E_1 are the electric potential and electric field respectively at O.

V_1 = Potential due to charge at A + Potential due to charge at B

$$V_1 = \frac{q_1}{4\pi\epsilon_0\left(\frac{d}{2}\right)} + \frac{q_2}{4\pi\epsilon_0\left(\frac{d}{2}\right)} = \frac{1}{4\pi\epsilon_0\left(\frac{d}{2}\right)}(q_1 + q_2)$$

Where,

ϵ_0 = Permittivity of free space

$$\frac{1}{4\pi \in_0} = 9\times10^9\ \mathrm{NC^2\ m^{-2}}$$

$$\therefore V_1 = \frac{9\times10^9\times10^{-6}}{\left(\frac{0.30}{2}\right)}(2.5+1.5) = 2.4\times10^5\ \mathrm{V}$$

E_1 = Electric field due to q_2 − Electric field due to q_1

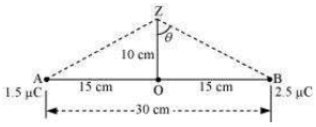
$$= \frac{q_2}{4\pi \in_0 \left(\frac{d}{2}\right)^2} - \frac{q_1}{4\pi \in_0 \left(\frac{d}{2}\right)^2}$$

$$= \frac{9\times10^9}{\left(\frac{0.30}{2}\right)^2}\times10^6\times(2.5-1.5)$$

$$= 4\times10^5\ \mathrm{V\ m^{-1}}$$

Therefore, the potential at mid-point is 2.4×10^5 V and the electric field at mid-point is 4×10^5 V $\mathrm{m^{-1}}$. The field is directed from the larger charge to the smaller charge.

(b) Consider a point Z such that normal distanceOZ = 10 cm = 0.1 m, as shown in the following figure.



V_2 and E_2 are the electric potential and electric field respectively at Z.

It can be observed from the figure that distance,

$$\mathrm{BZ} = \mathrm{AZ} = \sqrt{(0.1)^2 + (0.15)^2} = 0.18\ \mathrm{m}$$

V_2= Electric potential due to A + Electric Potential due to B

$$= \frac{q_1}{4\pi \in_0 (\mathrm{AZ})} + \frac{q_1}{4\pi \in_0 (\mathrm{BZ})}$$

$$= \frac{9\times10^9\times10^{-6}}{0.18}(1.5+2.5)$$

$$= 2\times10^5\ \mathrm{V}$$

Electric field due to q at Z,

$$E_\mathrm{A} = \frac{q_1}{4\pi \in_0 (\mathrm{AZ})^2}$$

$$= \frac{9\times10^9\times1.5\times10^{-6}}{(0.18)^2}$$

$$= 0.416\times10^6\ \mathrm{V/m}$$

Electric field due to q_2 at Z,

$$E_\mathrm{B} = \frac{q_2}{4\pi \in_0 (\mathrm{BZ})^2}$$

$$= \frac{9\times10^9\times2.5\times10^{-6}}{(0.18)^2}$$

$$= 0.69\times10^6\ \mathrm{V\ m^{-1}}$$

The resultant field intensity at Z,

$$E = \sqrt{E_\mathrm{A}^2 + E_\mathrm{B}^2 + 2E_\mathrm{A}E_\mathrm{B}\cos 2\theta}$$

Where, $2\theta$is the angle, $\angle AZ$ B

From the figure, we obtain

$$\cos\theta = \frac{0.10}{0.18} = \frac{5}{9} = 0.5556$$

$$\theta = \cos^{-1}0.5556 = 56.25$$

$$\therefore 2\theta = 112.5^\circ$$

$$\cos 2\theta = -0.38$$

$$E = \sqrt{(0.416\times10^6)^2 \times (0.69\times10^6)^2 + 2\times0.416\times0.69\times10^{12}\times(-0.38)}$$

$$= 6.6\times10^5\ \mathrm{V\ m^{-1}}$$

Therefore, the potential at a point 10 cm (perpendicular to the mid-point) is 2.0×10^5 V and electric field is 6.6×10^5 V $\mathrm{m^{-1}}$.

Question 2.15:

A spherical conducting shell of inner radius $r1$ and outer radius $r2$ has a charge Q.

(a) A charge q is placed at the centre of the shell. What is the surface charge density on the inner and outer surfaces of the shell?

(b) Is the electric field inside a cavity (with no charge) zero, even if the shell is not spherical, but has any irregular shape? Explain.

Answer

(a) Charge placed at the centre of a shell is $+q$. Hence, a charge of magnitude $-q$ will be induced to the inner surface of the shell. Therefore, total charge on the inner surface of the shell is $-q$.

Surface charge density at the inner surface of the shell is given by the relation,

$$\sigma_1 = \frac{\text{Total charge}}{\text{Inner surface area}} = \frac{-q}{4\pi r_1^2} \quad \text{... } (i)$$

A charge of $+q$ is induced on the outer surface of the shell. A charge of magnitude Q is placed on the outer surface of the shell. Therefore, total charge on the outer surface of the shell is $Q + q$. Surface charge density at the outer surface of the shell,

$$\sigma_2 = \frac{\text{Total charge}}{\text{Outer surface area}} = \frac{Q+q}{4\pi r_2^2} \quad \text{... } (ii)$$

(b) Yes

The electric field intensity inside a cavity is zero, even if the shell is not spherical and has any irregular shape. Take a closed loop such that a part of it is inside the cavity along a field line while the rest is inside the conductor. Net work done by the field in carrying a test charge over a closed loop is zero because the field inside the conductor is zero. Hence, electric field is zero, whatever is the shape.

Question 2.16:

(a) Show that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another given by

$$\left(\vec{E_2} - \vec{E_1}\right).\hat{n} = \frac{\sigma}{\epsilon_0}$$

Where $\hat{n}$ is a unit vector normal to the surface at a point and σ is the surface charge density at that point. (The direction of $\hat{n}$ is from side 1 to side 2.) Hence show that just outside a conductor, the electric field is σ $\hat{n}/\epsilon_0$

(b) Show that the tangential component of electrostatic field is continuous from one side of a charged surface to another. [Hint: For (a), use Gauss's law. For, (b) use the fact that work done by electrostatic field on a closed loop is zero.]

Answer

(a) Electric field on one side of a charged body is E_1 and electric field on the other side of the same body is E_2. If infinite plane charged body has a uniform thickness, then electric field due to one surface of the charged body is given by,

$$\vec{E_1} = -\frac{\sigma}{2\epsilon_0}\hat{n} \quad \text{... } (i)$$

Where,

$\hat{n}$ = Unit vector normal to the surface at a point

σ = Surface charge density at that point

Electric field due to the other surface of the charged body,

$$\vec{E_2} = -\frac{\sigma}{2\epsilon_0}\hat{n} \quad \text{... } (ii)$$

Electric field at any point due to the two surfaces,

$$\vec{E_2} - \vec{E_1} = \frac{\sigma}{2\epsilon_0}\hat{n} + \frac{\sigma}{2\epsilon_0}\hat{n} = \frac{\sigma}{\epsilon_0}\hat{n}$$

$$\left(\vec{E_2} - \vec{E_1}\right).\hat{n} = \frac{\sigma}{\epsilon_0} \quad \text{... } (iii)$$

Since inside a closed conductor, $\vec{E_1}$ = 0,

$$\therefore \vec{E} = \vec{E_2} = -\frac{\sigma}{2\epsilon_0}\hat{n}$$

Therefore, the electric field just outside the conductor is $\frac{\sigma}{\epsilon_0}\hat{n}$.

(b) When a charged particle is moved from one point to the other on a closed loop, the work done by the electrostatic field is zero. Hence, the tangential component of electrostatic field is continuous from one side of a charged surface to the other.

Question 2.17:

A long charged cylinder of linear charged density λ is surrounded by a hollow co-axial conducting cylinder. What is the electric field in the space between the two cylinders?

Answer

Charge density of the long charged cylinder of length L and radius r is λ.

Another cylinder of same length surrounds the pervious cylinder. The radius of this cylinder is R.

Let E be the electric field produced in the space between the two cylinders.

Electric flux through the Gaussian surface is given by Gauss's theorem as,

$$\phi = E\left(2\pi d\right)L$$

Where, d = Distance of a point from the common axis of the cylinders

Let q be the total charge on the cylinder.

It can be written as

$$\therefore \phi = E\left(2\pi dL\right) = \frac{q}{\epsilon_\circ}$$

Where,

q = Charge on the inner sphere of the outer cylinder

ϵ_0 = Permittivity of free space

$$E(2\pi dL) = \frac{\lambda L}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi \epsilon_0 d}$$

Therefore, the electric field in the space between the two cylinders is $\frac{\lambda}{2\pi \epsilon_0 d}$.

Question 2.18:

In a hydrogen atom, the electron and proton are bound at a distance of about 0.53 Å:

(a) Estimate the potential energy of the system in eV, taking the zero of the potential energy at infinite separation of the electron from proton.

(b) What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (a)?

(c) What are the answers to (a) and (b) above if the zero of potential energy is taken at 1.06 Å separation?

Answer

The distance between electron-proton of a hydrogen atom, $d = 0.53\ \text{Å}$

Charge on an electron, $q_1 = -1.6 \times 10^{-19}$ C

Charge on a proton, $q_2 = +1.6 \times 10^{-19}$ C

(a) Potential at infinity is zero.

Potential energy of the system, *p-e* = Potential energy at infinity − Potential energy at distance, *d*

$$= 0 - \frac{q_1 q_2}{4\pi \epsilon_0 d}$$

Where,

ϵ_0 is the permittivity of free space

$$\frac{1}{4\pi \epsilon_0} = 9\times10^9\ \text{Nm}^2\ \text{C}^{-2}$$

$$\therefore \text{Potential energy} = 0 - \frac{9\times10^9 \times (1.6\times10^{-19})^2}{0.53\times10^{10}} = -43.7\times10^{-19}\ \text{J}$$

Since $1.6\times10^{-19}\ \text{J} = 1\ \text{eV}$,

$$\therefore \text{Potential energy} = -43.7\times10^{-19} = \frac{-43.7\times10^{-19}}{1.6\times10^{-19}} = -27.2\ \text{eV}$$

Therefore, the potential energy of the system is −27.2 eV.

(b) Kinetic energy is half of the magnitude of potential energy.

$$\text{Kinetic energy} = \frac{1}{2}\times(-27.2) = 13.6\ \text{eV}$$

Total energy = 13.6 − 27.2 = 13.6 eV

Therefore, the minimum work required to free the electron is 13.6 eV.

(c) When zero of potential energy is taken, $d_1 = 1.06\ \text{Å}$

∴Potential energy of the system = Potential energy at d_1 − Potential energy at d

$$= \frac{q_1 q_2}{4\pi \epsilon_0 d_1} - 27.2\ \text{eV}$$

$$= \frac{9\times10^9 \times (1.6\times10^{-19})^2}{1.06\times10^{-10}} - 27.2\ \text{eV}$$

$$= 21.73\times10^{-19}\ \text{J} - 27.2\ \text{eV}$$

$$= 13.58\ \text{eV} - 27.2\ \text{eV}$$

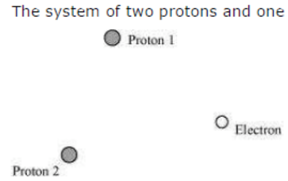
$$= -13.6\ \text{eV}$$

Question 2.19:

If one of the two electrons of a H_2 molecule is removed, we get a hydrogen molecular ion H_2^+. In the ground state of an H_2^+, the two protons are separated by roughly 1.5 Å, and the electron is roughly 1 Å from each proton. Determine the potential energy of the system. Specify your choice of the zero of potential energy.

Answer

The system of two protons and one electron is represented in the given figure.



Charge on proton 1, $q_1 = 1.6 \times 10^{-19}$ C

Charge on proton 2, $q_2 = 1.6 \times 10^{-19}$ C

Charge on electron, $q_3 = -1.6 \times 10^{-19}$ C

Charge on electron, $q_3 = -1.6 \times 10^{-19}$ C
Distance between protons 1 and 2, $d_1 = 1.5 \times 10^{-10}$ m
Distance between proton 1 and electron, $d_2 = 1 \times 10^{-10}$ m
Distance between proton 2 and electron, $d_3 = 1 \times 10^{-10}$ m
The potential energy at infinity is zero.
Potential energy of the system,

$$V = \frac{q_1 q_2}{4\pi \epsilon_0 d_1} + \frac{q_2 q_3}{4\pi \epsilon_0 d_3} + \frac{q_3 q_1}{4\pi \epsilon_0 d_2}$$

Substituting $\frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$, we obtain

$$V = \frac{9 \times 10^9 \times 10^{-19} \times 10^{-19}}{10^{-10}} \left[-(16)^2 + \frac{(1.6)^2}{1.5} + -(1.6)^2 \right]$$

$$= -30.7 \times 10^{-19} \text{J}$$
$$= -19.2 \text{ eV}$$

Therefore, the potential energy of the system is −19.2 eV.

Question 2.20:

Two charged conducting spheres of radii *a* and *b* are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions.

Answer

Let *a* be the radius of a sphere A, Q_A be the charge on the sphere, and C_A be the capacitance of the sphere. Let *b* be the radius of a sphere B, Q_B be the charge on the sphere, and C_B be the capacitance of the sphere. Since the two spheres are connected with a wire, their potential (*V*) will become equal.

Let E_A be the electric field of sphere A and E_B be the electric field of sphere B. Therefore, their ratio,

$$\frac{E_A}{E_B} = \frac{Q_A}{4\pi \epsilon_0 \times a_2} \times \frac{b^2 \times 4\pi \epsilon_0}{Q_B}$$

$$\frac{E_A}{E_B} = \frac{Q_A}{Q_B} \times \frac{b^2}{a^2} \quad \text{... (1)}$$

However, $\frac{Q_A}{Q_B} = \frac{C_A V}{C_B V}$

And, $\frac{C_A}{C_B} = \frac{a}{b}$

$$\therefore \frac{Q_A}{Q_B} = \frac{a}{b} \quad \text{... (2)}$$

Putting the value of (2) in (1), we obtain

$$\therefore \frac{E_A}{E_B} = \frac{a}{b}\frac{b^2}{a^2} = \frac{b}{a}$$

Therefore, the ratio of electric fields at the surface is $\frac{b}{a}$.

Question 2.21:

Two charges $-q$ and $+q$ are located at points (0, 0, − *a*) and (0, 0, *a*), respectively.

(a) What is the electrostatic potential at the points?

(b) Obtain the dependence of potential on the distance *r* of a point from the origin when $r/a >> 1$.

(c) How much work is done in moving a small test charge from the point (5, 0, 0) to (−7, 0, 0) along the *x*-axis? Does the answer change if the path of the test charge between the same points is not along the *x*-axis?

Answer

(a) Zero at both the points

Charge − *q* is located at (0, 0, − *a*) and charge + *q* is located at (0, 0, *a*). Hence, they form a dipole. Point (0, 0, *z*) is on the axis of this dipole and point (*x*, *y*, 0) is normal to the axis of the dipole. Hence, electrostatic potential at point (*x*, *y*, 0) is zero.

Electrostatic potential at point (0, 0, *z*) is given by,

*********** END **********