Trigonometric Ratios of Compound Angles Ex 7.2 Q2

Let $f(\theta) = \sqrt{3}\sin\theta - \cos\theta$

Multiplying and dividing by $\sqrt{\left(\sqrt{3}\right)^2 + (-1)^2}$, we get

$$f(\theta) = \sqrt{\left(\sqrt{3}\right)^2 + (-1)^2}\left[\frac{\sqrt{3}\sin\theta}{\sqrt{\left(\sqrt{3}\right)^2 + (-1)^2}} - \frac{\cos\theta}{\sqrt{\left(\sqrt{3}\right)^2 + (-1)^2}}\right]$$

$$= \sqrt{3+1}\left[\frac{\sqrt{3}\sin\theta}{\sqrt{3+1}} - \frac{\cos\theta}{\sqrt{3+1}}\right]$$

$$\Rightarrow \quad f(\theta) = 2\left[\frac{\sqrt{3}\sin\theta}{2} - \frac{\cos\theta}{2}\right] \qquad ---(i)$$

$$\Rightarrow \quad f(\theta) = 2\left[\frac{\sqrt{3}}{2}\times\sin\theta - \frac{1}{2}\times\cos\theta\right]$$

$$= 2\left[\cos\frac{\pi}{6}\times\sin\theta - \sin\frac{\pi}{6}\times\cos\theta\right]$$

$$= 2\left[\sin\theta\times\cos\frac{\pi}{6} - \cos\theta\times\sin\frac{\pi}{6}\right]$$

$$= 2\sin\left(\theta - \frac{\pi}{6}\right) \qquad \left[\because \sin(A-B) = \sin A\cos B - \cos A\sin B\right]$$

$$\Rightarrow \quad f(\theta) = 2\sin\left(\theta - \frac{\pi}{6}\right)$$

Again,

$$f(\theta) = 2\left[\frac{\sqrt{3}}{2}\sin\theta - \frac{\cos\theta}{2}\right]$$

$$= -2\left[\frac{1}{2}\times\cos\theta - \frac{\sqrt{3}}{2}\times\sin\theta\right]$$

$$= -2\left[\cos\frac{\pi}{3}\times\cos\theta - \sin\frac{\pi}{3}\times\sin\theta\right]$$

$$= -2\cos\left(\frac{\pi}{3} + \theta\right)$$

Let $f(\theta) = \cos\theta - \sin\theta$

Multiplying and dividing by $\sqrt{1^2+1^2}$, we get

$$f(\theta) = \sqrt{1^2+1^2}\left[\frac{\cos\theta}{\sqrt{1^2+1^2}} - \frac{\sin\theta}{\sqrt{1^2+1^2}}\right]$$

$$\Rightarrow \quad f(\theta) = \sqrt{2}\left[\frac{\cos\theta}{\sqrt{2}} - \frac{\sin\theta}{\sqrt{2}}\right] \qquad ---(i)$$

Now, $f(\theta) = \sqrt{2}\left[\frac{1}{\sqrt{2}}\times\cos\theta - \frac{1}{\sqrt{2}}\times\sin\theta\right]$

$$= \sqrt{2}\left[\sin\frac{\pi}{4}\times\cos\theta - \cos\frac{\pi}{4}\times\sin\theta\right]$$

$$= \sqrt{2}\sin\left(\frac{\pi}{4}-\theta\right) \qquad [\because \sin(A-B) = \sin A\cos B - \cos A\sin B]$$

$$\Rightarrow \quad f(\theta) = \sqrt{2}\sin\left(\frac{\pi}{4}-\theta\right)$$

Again,

$$f(\theta) = \sqrt{2}\left[\frac{1}{\sqrt{2}}\times\cos\theta - \frac{1}{\sqrt{2}}\times\sin\theta\right]$$

$$= \sqrt{2}\left[\cos\frac{\pi}{4}\times\cos\theta - \sin\frac{\pi}{4}\times\sin\theta\right]$$

$$= \sqrt{2}\cos\left(\frac{\pi}{4}+\theta\right) \qquad [\because \cos(A+B) = \cos A\cos B - \sin A\sin B]$$

$$\Rightarrow \quad f(\theta) = \sqrt{2}\cos\left(\frac{\pi}{4}+\theta\right)$$

Let $f(\theta) = 24\cos\theta + 7\sin\theta$

Multiplying and dividing by $\sqrt{(24)^2+(7)^2}$, we get

$$f(\theta) = \sqrt{(24)^2+7^2}\left[\frac{24\cos\theta}{\sqrt{24^2+7^2}} + \frac{7\sin\theta}{\sqrt{24^2+7^2}}\right]$$

$$= \sqrt{576+49}\left[\frac{24\cos\theta}{\sqrt{576+49}} + \frac{7\sin\theta}{\sqrt{576+49}}\right]$$

$$= \sqrt{625}\left[\frac{24\cos\theta}{\sqrt{625}} + \frac{7\sin\theta}{\sqrt{625}}\right]$$

$$= 25\left[\frac{24}{25}\times\cos\theta + \frac{7}{25}\times\sin\theta\right]$$

$$\Rightarrow \quad f(\theta) = 25\left[\frac{24}{25}\times\cos\theta + \frac{7}{25}\times\sin\theta\right] \qquad ---(i)$$

Now, $f(\theta) = 25\left[\frac{24}{25}\times\cos\theta + \frac{7}{25}\times\sin\theta\right]$

$$= 25[\sin\alpha\times\cos\theta + \cos\alpha\times\sin\theta]$$

where $\sin\alpha = \frac{24}{25}$ and $\cos\alpha = \frac{7}{25}$

$$\Rightarrow \quad f(\theta) = 25\sin(\alpha+\theta), \text{ where } \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{24}{7}$$

Again,

$$f(\theta) = 25\left[\frac{24}{25}\times\cos\theta + \frac{7}{25}\times\sin\theta\right]$$

$$= 25[\cos\alpha\times\cos\theta + \sin\alpha\times\sin\theta], \text{ where } \cos\alpha = \frac{24}{25} \text{ and } \sin\alpha = \frac{7}{25}$$

$$\Rightarrow \quad f(\theta) = 25\cos(\alpha-\theta), \text{ where } \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{7}{24}$$

*********** END **********