Rationalisation Ex 3.2 Q3

Answer :

(i) We know that rationalization factor for $3+\sqrt{2}$ is $3-\sqrt{2}$. We will multiply numerator and denominator of the given expression $\frac{1}{3+\sqrt{2}}$ by $3-\sqrt{2}$, to get

$$\begin{aligned}\frac{1}{3+\sqrt{2}}\times\frac{3-\sqrt{2}}{3-\sqrt{2}}&=\frac{3-\sqrt{2}}{3^2-\left(\sqrt{2}\right)^2}\\&=\frac{3-\sqrt{2}}{9-2}\\&=\frac{3-\sqrt{2}}{7}\end{aligned}$$

Hence the given expression is simplified with rational denominator to $\boxed{\frac{3-\sqrt{2}}{7}}$.

(ii) We know that rationalization factor for $\sqrt{6}-\sqrt{5}$ is $\sqrt{6}+\sqrt{5}$. We will multiply numerator and denominator of the given expression $\frac{1}{\sqrt{6}-\sqrt{5}}$ by $\sqrt{6}+\sqrt{5}$, to get

$$\begin{aligned}\frac{1}{\sqrt{6}-\sqrt{5}}\times\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}&=\frac{\sqrt{6}+\sqrt{5}}{\left(\sqrt{6}\right)^2-\left(\sqrt{5}\right)^2}\\&=\frac{\sqrt{6}+\sqrt{5}}{6-5}\\&=\frac{\sqrt{6}+\sqrt{5}}{1}\\&=\sqrt{6}+\sqrt{5}\end{aligned}$$

Hence the given expression is simplified with rational denominator to $\boxed{\sqrt{6}+\sqrt{5}}$.

(iii) We know that rationalization factor for $\sqrt{41}-5$ is $\sqrt{41}+5$. We will multiply numerator and denominator of the given expression $\frac{16}{\sqrt{41}-5}$ by $\sqrt{41}+5$, to get

$$\begin{aligned}\frac{16}{\sqrt{41}-5}\times\frac{\sqrt{41}+5}{\sqrt{41}+5}&=\frac{16\left(\sqrt{41}+5\right)}{\left(\sqrt{41}\right)^2-(5)^2}\\&=\frac{16\left(\sqrt{41}+5\right)}{41-25}\\&=\frac{16\left(\sqrt{41}+5\right)}{16}\\&=\sqrt{41}+5\end{aligned}$$

Hence the given expression is simplified with rational denominator to $\boxed{\sqrt{41}+5}$.

(iv) We know that rationalization factor for $5\sqrt{3}-3\sqrt{5}$ is $5\sqrt{3}+3\sqrt{5}$. We will multiply numerator and denominator of the given expression $\frac{30}{5\sqrt{3}-3\sqrt{5}}$ by $5\sqrt{3}+3\sqrt{5}$, to get

$$\begin{aligned}\frac{30}{5\sqrt{3}-3\sqrt{5}}\times\frac{5\sqrt{3}+3\sqrt{5}}{5\sqrt{3}+3\sqrt{5}}&=\frac{30\times5\times\sqrt{3}+30\times3\times\sqrt{5}}{\left(5\sqrt{3}\right)^2-\left(3\sqrt{5}\right)^2}\\&=\frac{30\times5\times\sqrt{3}+30\times3\times\sqrt{5}}{25\times3-9\times5}\\&=\frac{30\times5\times\sqrt{3}+30\times3\times\sqrt{5}}{75-45}\\&=\frac{30\times5\times\sqrt{3}+30\times3\times\sqrt{5}}{30}\\&=5\sqrt{3}+3\sqrt{5}\end{aligned}$$

Hence the given expression is simplified with rational denominator to $\boxed{5\sqrt{3}+3\sqrt{5}}$.

(v) We know that rationalization factor for $2\sqrt{5}-\sqrt{3}$ is $2\sqrt{5}+\sqrt{3}$. We will multiply numerator and denominator of the given expression $\frac{1}{2\sqrt{5}-\sqrt{3}}$ by $2\sqrt{5}+\sqrt{3}$, to get

$$\begin{aligned}\frac{1}{2\sqrt{5}-\sqrt{3}}\times\frac{2\sqrt{5}+\sqrt{3}}{2\sqrt{5}+\sqrt{3}} &= \frac{2\sqrt{5}+\sqrt{3}}{\left(2\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}\\ &= \frac{2\sqrt{5}+\sqrt{3}}{4\times5-3}\\ &= \frac{2\sqrt{5}+\sqrt{3}}{20-3}\\ &= \frac{2\sqrt{5}+\sqrt{3}}{17}\\ &= 5\sqrt{3}+3\sqrt{5}\end{aligned}$$

Hence the given expression is simplified with rational denominator to $\boxed{\frac{2\sqrt{5}+\sqrt{3}}{17}}$.

(vi) We know that rationalization factor for $2\sqrt{2}-\sqrt{3}$ is $2\sqrt{2}+\sqrt{3}$. We will multiply numerator and denominator of the given expression $\frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}}$ by $2\sqrt{2}+\sqrt{3}$, to get

$$\begin{aligned}\frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}}\times\frac{2\sqrt{2}+\sqrt{3}}{2\sqrt{2}+\sqrt{3}} &= \frac{2\times\sqrt{3}\times\sqrt{2}+\sqrt{3}\times\sqrt{3}+2\sqrt{2}+\sqrt{3}}{\left(2\sqrt{2}\right)^2-\left(\sqrt{3}\right)^2}\\ &= \frac{2\sqrt{3\times2}+3+2\sqrt{2}+\sqrt{3}}{4\times2-3}\\ &= \frac{2\sqrt{6}+3+2\sqrt{2}+\sqrt{3}}{8-3}\\ &= \frac{2\sqrt{6}+3+2\sqrt{2}+\sqrt{3}}{5}\\ &= \frac{2\sqrt{6}+3+2\sqrt{2}+\sqrt{3}}{5}\end{aligned}$$

Hence the given expression is simplified with rational denominator to $\boxed{\frac{2\sqrt{6}+3+2\sqrt{2}+\sqrt{3}}{5}}$.

(vii) We know that rationalization factor for $6+4\sqrt{2}$ is $6-4\sqrt{2}$. We will multiply numerator and denominator of the given expression $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$ by $6-4\sqrt{2}$, to get

$$\begin{aligned}\frac{6-4\sqrt{2}}{6+4\sqrt{2}}\times\frac{6-4\sqrt{2}}{6-4\sqrt{2}} &= \frac{6^2+\left(4\sqrt{2}\right)^2-2\times6\times4\sqrt{2}}{\left(6\right)^2-\left(4\sqrt{2}\right)^2}\\ &= \frac{36+16\times2-48\sqrt{2}}{36-16\times2}\\ &= \frac{36+32-48\sqrt{2}}{36-32}\\ &= \frac{68-48\sqrt{2}}{4}\\ &= 17-12\sqrt{2}\end{aligned}$$

Hence the given expression is simplified with rational denominator to $\boxed{17-12\sqrt{2}}$.

(viii) We know that rationalization factor for $2\sqrt{5}-3$ is $2\sqrt{5}+3$. We will multiply numerator and denominator of the given expression $\frac{3\sqrt{2}+1}{2\sqrt{5}-3}$ by $2\sqrt{5}+3$, to get

$$\begin{aligned}\frac{3\sqrt{2}+1}{2\sqrt{5}-3}\times\frac{2\sqrt{5}+3}{2\sqrt{5}+3} &= \frac{3\sqrt{2}\times2\sqrt{5}+3\times3\sqrt{2}+2\sqrt{5}+3}{\left(2\sqrt{5}\right)^2-\left(3\right)^2}\\ &= \frac{3\times2\times\sqrt{2}\times\sqrt{5}+3\times3\sqrt{2}+2\sqrt{5}+3}{4\times5-9}\\ &= \frac{6\sqrt{2\times5}+9\sqrt{2}+2\sqrt{5}+3}{4\times5-9}\\ &= \frac{6\sqrt{10}+9\sqrt{2}+2\sqrt{5}+3}{11}\end{aligned}$$

Hence the given expression is simplified with rational denominator to $\boxed{\frac{6\sqrt{10}+9\sqrt{2}+2\sqrt{5}+3}{11}}$.

(ix) We know that rationalization factor for $\sqrt{a^2+b^2}+a$ is $\sqrt{a^2+b^2}-a$. We will multiply numerator and denominator of the given expression $\frac{b^2}{\sqrt{a^2+b^2}+a}$ by $\sqrt{a^2+b^2}-a$, to get

$$\frac{b^2}{\sqrt{a^2+b^2}+a}\times\frac{\sqrt{a^2+b^2}-a}{\sqrt{a^2+b^2}-a}=\frac{b^2\left(\sqrt{a^2+b^2}-a\right)}{\left(\sqrt{a^2+b^2}\right)^2-a^2}$$

$$=\frac{b^2\left(\sqrt{a^2+b^2}-a\right)}{a^2+b^2-a^2}$$

$$=\frac{b^2\left(\sqrt{a^2+b^2}-a\right)}{b^2}$$

$$=\sqrt{a^2+b^2}-a$$

Hence the given expression is simplified with rational denominator to $\boxed{\sqrt{a^2+b^2}-a}$.

********** END **********