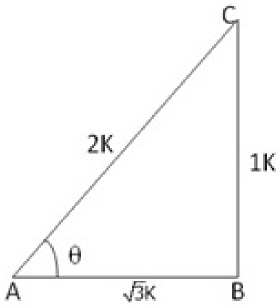
Question 9

Given: $cosec\theta = \frac{AC}{BC} = \frac{2}{1}$

Let AC = 2k and BC = 1k,

Where k is positive

Let us draw a ΔABC in which $\angle B = 90^0$ and $\angle BAC = \theta$



By Pythagoras theorem, we have

$$AC^2 = (AB)^2 + (BC)^2$$

$$\Rightarrow (AB)^2 = (AC)^2 - (BC)^2$$

$$= \left[(2k)^2 - (1k)^2\right] = \left(4k^2 - 1k^2\right) = 3k^2$$

$$\Rightarrow (AB) = \sqrt{3}k$$

$$\sin\theta = \frac{BC}{AC} = \frac{1k}{2k} = \frac{1}{2}$$

$$\cos\theta = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \left(\frac{\sqrt{3}}{2} \times \frac{2}{1}\right) = \sqrt{3}$$

$$\Rightarrow \left[\cot\theta + \frac{\sin\theta}{1 + \cos\theta}\right] = \left[\sqrt{3} + \frac{\frac{1}{2}}{1 + \frac{\sqrt{3}}{2}}\right]$$

$$= \left(\sqrt{3} + \frac{1}{2 + \sqrt{3}}\right) = \left(\frac{2\sqrt{3} + 3 + 1}{2 + \sqrt{3}}\right)$$

$$= \left(\frac{2\sqrt{3} + 4}{2 + \sqrt{3}}\right) = 2\left(\frac{\sqrt{3} + 2}{2 + \sqrt{3}}\right) = 2$$

Hence, $\left[\cot\theta + \frac{\sin\theta}{1 + \cos\theta}\right] = 2$

*********** END **********