Continuity Ex 9.1 Q31

We are given that the function is continuous at $x = 2$

$\therefore$ LHL = RHL = $f(2)$ .....(1)

Now,

$f(2) = k$ ....(A)

$$\text{LHL} = \lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h) = \lim_{h \to 0} \frac{2^{(2-h)+2} - 16}{4^{(2-h)} - 16} = \lim_{h \to 0} \frac{2^{4-h} - 16}{4^{2-h} - 16}$$

$$= \lim_{h \to 0} \frac{2^4 . 2^{-h} - 16}{4^2 . 4^{-h} - 16}$$

$$= \lim_{h \to 0} \frac{16 . 2^{-h} - 16}{16 . 4^{-h} - 16}$$

$$= \lim_{h \to 0} \frac{16\left(2^{-h} - 1\right)}{16\left(4^{-h} - 1\right)}$$

$$= \lim_{h \to 0} \frac{2^{-h} - 1}{\left(2^{-h}\right) - 1^2} \qquad \left[\because 2^{-2h} = \left(2^{-h}\right)^2 = 4^{-h}\right]$$

$$= \lim_{h \to 0} \frac{2^{-h} - 1}{\left(2^{-h} - 1\right)\left(2^{-h} + 1\right)} = \frac{1}{2} \quad ......(B)$$

$\therefore$ Using (1) from (A) & (B)

$k = \frac{1}{2}$

Continuity Ex 9.1 Q33

We know that a function is said to be continuous at $x = \pi$ if

LHL = RHL = value of the function at $x = \pi$ .....(1)

$$\text{LHL} = \lim_{x \to \pi^-} f(x) = \lim_{h \to 0} f(\pi - h) = \lim_{h \to 0} \frac{1 - \cos 7(\pi - h - \pi)}{5\left((\pi - h) - \pi\right)^2} = \lim_{h \to 0} \frac{1 - \cos 7h}{5h^2}$$

$$= \lim_{h \to 0} \frac{2\sin^2 \frac{7}{2}h}{5h^2}$$

$$= \lim_{h \to 0} \frac{2}{5}\left(\frac{\sin \frac{7}{2}h}{\frac{7}{2}h}\right)^2 \times \left(\frac{7}{2}\right)^2$$

$$= \frac{2}{5} \times \frac{49}{4} = \frac{49}{10} \quad ....(B)$$

Thus, using (1) we get,

$f(\pi) = \frac{49}{10}$

Continuity Ex 9.1 Q34

It is given that the function is continuous at $x = 0$

$\therefore$ LHL = RHL = $f(0)$....(1)

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{2(-h) + 3\sin(-h)}{3(-h) + 2\sin(-h)} = \lim_{h \to 0} \frac{-2h - 3\sin h}{-3h - 2\sin h}$$

$$= \lim_{h \to 0} \frac{\frac{2h + 3\sin h}{h}}{\frac{3h + 2\sin h}{h}}$$

$$= \lim_{h \to 0} \frac{2 + 3\frac{\sin h}{h}}{3 + 2\frac{\sin h}{h}} = \frac{2+3}{3+2} = 1 \qquad \left[\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1\right]$$

Using (1) we get,

$f(0) = 1$

Continuity Ex 9.1 Q35

It is given that the function is continuous at $x = 0$.

LHL = RHL = $f(0)$ .... (1)

$f(0) = k$ ....(A)

LHL = $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{1 - \cos 4(-h)}{8(-h)^2} = \lim_{h \to 0} \frac{1 - \cos 4h}{8h^2} = \lim_{h \to 0} \frac{2\sin^2 2h}{8h^2} = \lim_{h \to 0} \left(\frac{\sin 2h}{2h}\right)^2 = 1$

Thus, using (1) we get,

$k = 1$

*********** END **********