Mean Value Theorems Ex 15.2 Q2

Here,

$$f(x) = |x| \text{ on } [-1, 1]$$

$$f(x) = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

For differentiability at $x = 0$

$$\text{LHD} = \lim_{x \to 0^-} \frac{f(0-h) - f(0)}{-h}$$

$$= \lim_{h \to 0} \frac{-(0-h) - 0}{-h}$$

$$= \lim_{h \to 0} \frac{h}{-h}$$

$$\text{LHD} = -1$$

$$\text{RHD} = \lim_{x \to 0^+} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{(0+h) - 0}{h}$$

$$= \lim_{h \to 0} \frac{h}{h}$$

$$= 1$$

$\therefore$ LHD $\neq$ RHD

$\Rightarrow$ $f(x)$ is not differentiable at $x = 0 \in (-1, 1)$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q3

Here,

$$f(x) = \frac{1}{x} \text{ on } [-1, 1]$$

$$f'(x) = -\frac{1}{x^2}$$

$\Rightarrow$ $f'(x)$ doesnot exist at $x = 0 \in (-1, 1)$

$\Rightarrow$ $f(x)$ is not differentiable in $(-1, 1)$

Hence, LMVT is verified

Mean Value Theorems Ex 15.2 Q4

Here,

$$f(x)=\frac{1}{4x-1}, x\in[1,4]$$

f(x) attain unique value for each $x\in[1,4]$, so f(x) is continuous in [1, 4].

$$f'(x)=-\frac{4}{(4x-1)^2}$$

$\Rightarrow f'(x)$ exists for each $x\in(1, 4)$

$\Rightarrow f'(x)$ is differentiable in$(1, 4)$

So, Lagranges mean value theroem is applicable.

So, there exist a point $c\in(1, 4)$ such that,

$$f'(c)=\frac{f(4)-f(1)}{4-1}$$

$$\Rightarrow -\frac{4}{(4x-1)^2}=\frac{\frac{1}{15}-\frac{1}{3}}{3}$$

$$\Rightarrow -\frac{4}{(4x-1)^2}=-\frac{4}{45}$$

$$\Rightarrow (4x-1)^2=45$$

$$\Rightarrow 4x-1=\pm 3\sqrt{5}$$

$$\Rightarrow x=\frac{3\sqrt{5}+1}{4}\in[1,4]$$

*********** END **********