Definite Integrals Ex 20.1 Q1

We know that $\int x^n dx = \dfrac{x^{n+1}}{n+1} + C$

Now,

$$\int_4^9 \frac{1}{\sqrt{x}} dx$$

$$= \left[\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]_4^9$$

$$= \left[\frac{\sqrt{x}}{\frac{1}{2}}\right]_4^9$$

$$= 2[\sqrt{9} - \sqrt{4}]$$

$$= 2[3 - 2]$$

$$= 2$$

$$\therefore \int_4^9 \frac{1}{\sqrt{x}} dx = 2$$

Definite Integrals Ex 20.1 Q2

We know that $\int \dfrac{dx}{x} = \log x + C$

Now,

$$\int_{-2}^3 \frac{1}{x+7} dx$$

$$= \left[\log(x+7)\right]_{-2}^3$$

$$= \left[\log 10 - \log 5\right]_{-2}^3$$

$$= \log\frac{10}{5} \qquad \left[\because \log a - \log b = \log\frac{a}{b}\right]$$

$$= \log 2$$

$$\therefore \int_{-2}^3 \frac{1}{x+7} dx = \log 2$$

Definite Integrals Ex 20.1 Q3

Let $x = \sin\theta$

$\Rightarrow dx = \cos\theta d\theta$

Now,

$x = 0 \Rightarrow \theta = 0$

$x = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

$$\therefore \int_0^{\frac{\pi}{6}} \frac{1}{\sqrt{1-\sin^2\theta}} \cos\theta \, d\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{\cos\theta \, d\theta}{\cos\theta}$$

$$= \int_0^{\frac{\pi}{6}} d\theta$$

$$= \left[\theta\right]_0^{\frac{\pi}{6}}$$

$$= \left[\frac{\pi}{6} - 0\right]$$

$$= \frac{\pi}{6}$$

$$\therefore \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} = \frac{\pi}{6}$$

Definite Integrals Ex 20.1 Q4

We have,

$$I = \int_0^1 \frac{1}{1+x^2}\,dx$$

$$= \left[\tan^{-1} x\right]_0^1$$

$$= \left[\tan^{-1} 1 - \tan^{-1} 0\right]$$

$$= \left[\frac{\pi}{4} - 0\right] \qquad \left[\because \tan^{-1} 1 = \frac{\pi}{4}\right]$$

$$= \frac{\pi}{4}$$

$$\therefore \int_0^1 \frac{1}{1+x^2}\,dx = \frac{\pi}{4}$$

*********** END **********