Complex Numbers Ex 13.2 Q23

Let $z = x + iy$

$|z| = z + 1 + 2i$

$\Rightarrow |x + iy| = x + iy + 1 + 2i$

$\Rightarrow \sqrt{x^2 + y^2} = (x+1) + i(y+2)$

$\Rightarrow x^2 + y^2 = (x+1)^2 + 2i(x+1)(y+2) - (y+2)^2$ [Squaring both sides]

$\Rightarrow x^2 + y^2 = x^2 + 2x + 1 + 2i(xy + 2x + y + 2) - (y^2 + 4y + 4)$

$\Rightarrow 2y^2 - 2x + 4y + 4 = 2i(xy + 2x + y + 2)$

$\Rightarrow y^2 - x + 2y + 2 = i(xy + 2x + y + 2)$

$\Rightarrow (y^2 - x + 2y + 2) - i(xy + 2x + y + 2) = 0$

Comparing we get,

$(xy + 2x + y + 2) = 0$

$\Rightarrow (x+1)(y+2) = 0$

$\Rightarrow x = -1 \,\&\, y = -2$

Also, $(y^2 - x + 2y + 2) = 0$

Taking $x = -1$, $(y^2 - (-1) + 2y + 2) = 0$

$\Rightarrow (y^2 + 2y + 3) = 0$

Doesnot have a solution since roots will be imaginary

Taking $y = -2$, $(4 - x - 4 + 2) = 0$

$\Rightarrow x = 2$

$\therefore z = x + iy = 2 - 2i$

Complex Numbers Ex 13.2 Q24

$(1+i)^{2n} = (1-i)^{2n}$

$\Rightarrow \left(\frac{1+i}{1-i}\right)^{2n} = 1$

$\Rightarrow \left(\frac{(1+i)(1+i)}{(1-i)(1+i)}\right)^{2n} = 1$ [Rationalizing the denominator]

$\Rightarrow \left(\frac{1+2i-1}{1+1}\right)^{2n} = 1$

$\Rightarrow \left(\frac{2i}{2}\right)^{2n} = 1$

$\Rightarrow i^{2n} = 1$

$\therefore n = 2$

Complex Numbers Ex 13.2 Q25

$$|z_1 + z_2 + z_3| = \left|\frac{z_1\overline{z_1}}{\overline{z_1}} + \frac{z_2\overline{z_2}}{\overline{z_2}} + \frac{z_3\overline{z_3}}{\overline{z_3}}\right|$$

$$= \left|\frac{|z_1|^2}{\overline{z_1}} + \frac{|z_2|^2}{\overline{z_2}} + \frac{|z_3|^2}{\overline{z_3}}\right|$$

$$= \frac{\left|\frac{1}{\overline{z_1}} + \frac{1}{\overline{z_2}} + \frac{1}{\overline{z_3}}\right|}{\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|} \ldots\ldots\ldots\ldots \left[\because |z_1| = |z_2| = |z_3| = 1\right]$$

$$= \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|$$

$$= 1$$

Complex Numbers Ex 13.2 Q26

Let $z = x + iy$

$$z^2 = (x + iy)^2 = x^2 - y^2 + 2xyi$$

$$|z|^2 = z\overline{z} = (x + iy)(x - iy) = x^2 + y^2$$

$$z^2 + |z|^2 = 0$$

$$x^2 - y^2 + 2xyi + x^2 + y^2 = 0$$

$$2x^2 + 2xyi = 0$$

$\Rightarrow 2x^2 = 0$ and $2xy = 0$

$\Rightarrow x = 0$ and $y \in R$

$\therefore\ z = 0 + iy$ where $y \in R$

*********** END **********