Indefinite Integrals Ex 19.19 Q5

Let $I = \int \frac{x^2}{x^2+7x+10}dx$

$$= \int \left\{1 - \frac{7x+10}{x^2+7x+10}\right\}dx$$

$$I = x - \int \frac{7x+10}{x^2+7x+10}dx + c_1 \text{ ---- (i)}$$

Let $I_1 = \int \frac{7x+10}{x^2+7x+10}dx$

Let $7x+10 = \lambda \frac{d}{dx}\left(x^2+7x+10\right) + \mu$

$$= \lambda\left(2x+7\right) + \mu$$

$$7x+10 = \left(2\lambda\right)x + 7\lambda + \mu$$

Comparing the coefficients of like powers of x,

$$7 = 2\lambda \quad \Rightarrow \quad \lambda = \frac{7}{2}$$

$$7\lambda + \mu = 10 \quad \Rightarrow \quad 7\left(\frac{7}{2}\right) + \mu = 10$$

$$\mu = -\frac{29}{2}$$

so, $I = \int \frac{\frac{1}{6}\left(6x-4\right) - \frac{1}{3}}{3x^2-4x+3}dx$

$$I = \frac{1}{6}\int \frac{6x-4}{3x^2-4x+3}dx - \frac{1}{9}\int \frac{1}{x^2 - \frac{4}{3}x + 1}dx$$

$$I = \frac{1}{6}\int \frac{6x-4}{3x^2-4x+3}dx - \frac{1}{9}\int \frac{1}{x^2 - 2x\left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^2 + \left(2\right)^2}dx$$

$$I = \frac{1}{6}\int \frac{6x-4}{3x^2-4x+3}dx - \frac{1}{9}\int \frac{1}{\left(x-\frac{2}{3}\right)^2 + \left(\frac{\sqrt{5}}{3}\right)^2}dx$$

$$= \frac{1}{6}\log\left|3x^2-4x+3\right| - \frac{1}{9} \times \frac{1}{\left(\frac{\sqrt{5}}{3}\right)}\tan^{-1}\left(\frac{x-\frac{2}{3}}{\frac{\sqrt{5}}{3}}\right) + c \quad \left[\text{since, } \int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \frac{1}{6}\log\left|3x^2-4x+3\right| - \frac{\sqrt{5}}{15}\tan^{-1}\left(\frac{3x-2}{\sqrt{5}}\right) + c$$

Indefinite Integrals Ex 19.19 Q6

We need to evaluate the integral $\int \frac{2x}{2+x-x^2}dx$

write the numerator in the following form

$$2x = \lambda\left\{\frac{d}{dx}\left(2+x-x^2\right)\right\}+\mu$$

i.e. $2x = \lambda\{-2x+1\}+\mu$

Equating the coefficients will give the values of λ,μ

$\lambda = -1, \mu = 1$

$$\int \frac{2x}{2+x-x^2}dx = \int \frac{\lambda\{-2x+1\}+\mu}{2+x-x^2}dx$$

$$= \int \frac{-1\{-2x+1\}+1}{2+x-x^2}dx$$

$$= \int \frac{-1\{-2x+1\}}{2+x-x^2}dx + \frac{1}{2+x-x^2}dx$$

$$= -\log\left|2+x-x^2\right| + \int \frac{1}{2+x-x^2}dx$$

$$= -\log\left|2+x-x^2\right| - \int \frac{1}{\left(x^2-x-2\right)}dx$$

$$= -\log\left|2+x-x^2\right| - \int \frac{1}{\left(x^2-x+\frac{1}{4}-2-\frac{1}{4}\right)}dx$$

$$= -\log\left|2+x-x^2\right| - \int \frac{1}{\left(x^2-x+\frac{1}{4}-\frac{9}{4}\right)}dx$$

$$= -\log\left|2+x-x^2\right| - \int \frac{1}{\left(\left(x-\frac{1}{2}\right)^2-\left(\frac{3}{2}\right)^2\right)}dx$$

$$= -\log\left|2+x-x^2\right| - \frac{1}{3}\log\left|\frac{\left(x-\frac{1}{2}\right)-\left(\frac{3}{2}\right)}{\left(x-\frac{1}{2}\right)+\left(\frac{3}{2}\right)}\right| + C$$

$$= -\log\left|2+x-x^2\right| - \frac{1}{3}\log\left|\frac{(x-2)}{(x+1)}\right| + C$$

Indefinite Integrals Ex 19.19 Q7

Let $I = \int \frac{1-3x}{3x^2+4x+2}dx$

Let $1-3x = \lambda \frac{d}{dx}\left(3x^2+4x+2\right)+\mu$

$= \lambda\left(6x+4\right)+\mu$

$1-3x = \left(6\lambda\right)x+\left(4\lambda+\mu\right)$

Comparing the coefficients of like powers of x,

$6\lambda = -3 \quad \Rightarrow \quad \lambda = -\frac{1}{2}$

$4\lambda+\mu = 1 \quad \Rightarrow \quad 4\left(-\frac{1}{2}\right)+\mu = 1$

$\mu = 3$

so, $I = \int \frac{-\frac{1}{2}\left(6x+4\right)+3}{3x^2+4x+2}dx$

$I = -\frac{1}{2}\int \frac{6x+4}{3x^2+4x+2}dx + 3\int \frac{1}{3x^2+4x+2}dx$

$I = -\frac{1}{2}\int \frac{6x+4}{3x^2+4x+2}dx + \frac{3}{3}\int \frac{1}{x^2+\frac{4}{3}x+\frac{2}{3}}dx$

$I = -\frac{1}{2}\int \frac{6x+4}{3x^2+4x+2}dx + \int \frac{1}{x^2+2x\left(\frac{2}{3}\right)+\left(\frac{2}{3}\right)^2-\left(\frac{2}{3}\right)^2+\frac{2}{3}}dx$

$= -\frac{1}{2}\int \frac{6x+4}{3x^2+4x+2}dx + \int \frac{1}{\left(x+\frac{2}{3}\right)^2+\frac{2}{9}}dx$

$I = -\frac{1}{2}\int \frac{6x+4}{3x^2+4x+2}dx + \int \frac{1}{\left(x+\frac{2}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2}dx$

$= -\frac{1}{2}\log\left|3x^2+4x+2\right| + \frac{3}{\sqrt{2}}\tan^{-1}\left(\frac{x+\frac{2}{3}}{\frac{\sqrt{2}}{3}}\right)+c \qquad \left[\text{since, } \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$

$I = -\frac{1}{2}\log\left|3x^2+4x+2\right| + \frac{3}{\sqrt{2}}\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)+c$

Indefinite Integrals Ex 19.19 Q8

Let $I = \int \frac{2x+5}{x^2-x-2}dx$

Let $2x+5 = \lambda \frac{d}{dx}\left(x^2-x-2\right)+\mu$

$= \lambda\left(2x-1\right)+\mu$

$2x+5 = \left(2\lambda\right)x-\lambda+\mu$

Comparing the coefficients of like powers of x,

$2\lambda = 2 \quad \Rightarrow \quad \lambda = 1$

$-\lambda+\mu = 5 \quad \Rightarrow \quad -1+\mu = 5$

$\mu = 6$

so, $I = \int \frac{\left(2x-1\right)+6}{x^2-x-2}dx$

$I = \int \frac{\left(2x-1\right)}{x^2-x-2}dx + 6\int \frac{1}{x^2-2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2-2}dx$

$I = \int \frac{2x-1}{x^2-x-2}dx + 6\int \frac{1}{\left(x-\frac{1}{2}\right)^2-\frac{9}{4}}dx$

$I = \int \frac{2x-1}{x^2-x-2}dx + 6\int \frac{1}{\left(x-\frac{1}{2}\right)^2-\left(\frac{3}{2}\right)^2}dx$

$I = \log\left|x^2-x-2\right| + \frac{6}{2\left(\frac{3}{2}\right)}\log\left|\frac{x-\frac{1}{2}-\frac{3}{2}}{x-\frac{1}{2}+\frac{3}{2}}\right|+c \qquad \left[\text{since, } \int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c\right]$

$I = \log\left|x^2-x-2\right| + 2\log\left|\frac{x-2}{x+1}\right|+c$

Indefinite Integrals Ex 19.19 Q9

Let $I = \int \frac{ax^3 + bx}{x^4 + c^2} dx$

Let $ax^3 + bx = \lambda \frac{d}{dx}\left(x^4 + c^2\right) + \mu$

$$ax^3 + bx = \lambda\left(4x^3\right) + \mu$$

Comparing the coefficients of like powers c

$$4\lambda = a \quad \Rightarrow \quad \lambda = \frac{a}{4}$$

$$\mu = 0 \quad \Rightarrow \quad \mu = 0$$

so, $\quad I = \int \frac{\frac{a}{4}\left(4x^3\right) + bx}{x^4 + c^2} dx$

$$I = \frac{a}{4}\int \frac{4x^3}{x^4 + c^2} dx + b\int \frac{x}{\left(x^2\right)^2 + c^2} dx$$

$$I = \frac{a}{4}\int \frac{4x^3}{x^4 + c^2} dx + \frac{b}{2}\int \frac{2x}{\left(x^2\right)^2 + c^2} dx$$

$$= \frac{a}{4}\log\left|x^4 + c^2\right| + \frac{b}{2} I_1 - - - - (i)$$

Now,

$$I_1 = \int \frac{2x}{\left(x^2\right)^2 + c^2} dx$$

Put $x^2 = t$

$\Rightarrow \quad 2x\, dx = dt$

$$I_1 = \int \frac{1}{(t)^2 + c^2} dx$$

$$= \frac{1}{c}\tan^{-1}\left(\frac{t}{c}\right) + c_1$$

$$I_1 = \frac{1}{c}\tan^{-1}\left(\frac{x^2}{c}\right) + c_1 - - - - - (ii)$$

Using equation (ii) in equation (i),

$$I = \frac{a}{4}\log\left|x^4 + c^2\right| + \frac{b}{2c}\tan^{-1}\left(\frac{x^2}{c}\right) + k$$

K = Integration constant

*********** END **********