



Differentiation Ex 11.5 Q6

Let $y = (\log x)^{\cos x}$ --- (i)

Taking \log on both the sides,

$$\begin{aligned}\log y &= \log(\log x)^{\cos x} \\ \log y &= \cos x \log(\log x) \quad \quad \quad [\text{Since, } \log a^b = b \log a]\end{aligned}$$

Differentiating with respect to x , using product rule, chain rule,

$$\begin{aligned}\frac{1}{y} \frac{dy}{dx} &= \cos x \frac{d}{dx} \log(\log x) + \log \log x \frac{d}{dx} (\cos x) \\ &= \frac{\cos x}{\log x} \frac{d}{dx} (\log x) + \log \log x \times (-\sin x) \\ \frac{1}{y} \frac{dy}{dx} &= \frac{\cos x}{\log x} \times \left(\frac{1}{x}\right) - \sin x \log \log x \\ \frac{dy}{dx} &= y \left[\frac{\cos x}{x \log x} - \sin x \log \log x \right] \\ \frac{dy}{dx} &= (\log x)^{\cos x} \left[\frac{\cos x}{x \log x} - \sin x \log \log x \right] \quad \quad \quad [\text{Using equation (i)}]\end{aligned}$$

Differentiation Ex 11.5 Q7

Let $y = (\sin x)^{\cos x}$ --- (i)

Taking \log on both the sides,

$$\begin{aligned}\log y &= \log(\sin x)^{\cos x} \\ \log y &= \cos x \log \sin x \quad \quad \quad [\text{Since, } \log a^b = b \log a]\end{aligned}$$

Differentiating with respect to x , using product rule, chain rule,

$$\begin{aligned}\frac{1}{y} \frac{dy}{dx} &= \cos x \frac{d}{dx} \log \sin x + \log \sin x \frac{d}{dx} \cos x \\ &= \cos x \frac{1}{\sin x} \frac{d}{dx} (\sin x) + \log \sin x (-\sin x) \\ \frac{1}{y} \frac{dy}{dx} &= \frac{\cos x}{\sin x} (\cos x) - \sin x \log \sin x \\ \frac{dy}{dx} &= y [\cos x \cot x - \sin x \log \sin x] \\ \frac{dy}{dx} &= (\sin x)^{\cos x} [\cos x \cot x - \sin x \log \sin x]\end{aligned}$$

Differentiation Ex 11.5 Q8

$$\text{Let } y = e^{x \log x}$$

$$\Rightarrow y = e^{\log x^x}$$

$$[\text{Since, } \log a^b = b \log a]$$

$$\Rightarrow y = x^x$$

--- (i)

$$[\text{Since, } e^{\log a} = a]$$

Taking log both the sides,

$$\log y = \log x^x$$

$$\log y = x \log x$$

Differentiating with respect to x , using product rule,

$$\frac{1}{y} \frac{dy}{dx} = x \frac{d}{dx} (\log x) + \log x \frac{d}{dx} (x)$$

$$= x \left(\frac{1}{x} \right) + \log x (1)$$

$$\frac{1}{y} \frac{dy}{dx} = 1 + \log x$$

$$\frac{dy}{dx} = y [1 + \log x]$$

$$\frac{dy}{dx} = x^x (1 + \log x)$$

[Using equation (i)]

Differentiation Ex 11.5 Q9

$$\text{Let } y = (\sin x)^{\log x} \quad \text{--- (i)}$$

Taking log on both the sides,

$$\log y = \log (\sin x)^{\log x}$$

$$\log y = \log x \log (\sin x)$$

[Using $\log a^b = b \log a$]

Differentiating with respect to x , using product rule and chain rule,

$$\frac{1}{y} \frac{dy}{dx} = \log x \frac{d}{dx} (\log \sin x) + \log \sin x \frac{d}{dx} (\log x)$$

$$= \log x \left(\frac{1}{\sin x} \right) \frac{d}{dx} (\sin x) + \log \sin x \left(\frac{1}{x} \right)$$

$$= \frac{\log x}{\sin x} \times \cos x + \frac{\log \sin x}{x}$$

$$\frac{1}{y} \frac{dy}{dx} = \log x \cot x + \frac{\log \sin x}{x}$$

$$\frac{dy}{dx} = y \left[\log x \cot x + \frac{\log \sin x}{x} \right]$$

$$\frac{dy}{dx} = (\sin x)^{\log x} \left[\log x \cot x + \frac{\log \sin x}{x} \right]$$

[Using equation (i)]

Differentiation Ex 11.5 Q10

$$\text{Let } y = 10^{\log \sin x} \quad \text{--- (i)}$$

Taking log on both the sides,

$$\log y = \log 10^{\log \sin x}$$

$$\log y = \log \sin x \log 10$$

[Since, $\log a^b = b \log a$]

Differentiating with respect to x , using chain rule,

$$\frac{1}{y} \frac{dy}{dx} = \log 10 \frac{d}{dx} (\log \sin x)$$

$$= \log 10 \left(\frac{1}{\sin x} \right) \frac{d}{dx} (\sin x)$$

$$\frac{1}{y} \frac{dy}{dx} = \log 10 \left(\frac{1}{\sin x} \right) (\cos x)$$

$$\frac{dy}{dx} = y [\log 10 \cot x]$$

$$\frac{dy}{dx} = 10^{\log \sin x} [\log 10 \times \cot x]$$

[Using equation (i)]

***** END *****