Definite Integrals Ex 20.4B Q25

We have,

$$I = \int_{-1}^{1} \log\left(\frac{2-x}{2+x}\right) dx$$

Since, $\log\left\{\frac{2-(-x)}{2+(-x)}\right\} = -\log\left(\frac{2-x}{2+x}\right)$ $\therefore$ This is an odd function.

Hence,

$I = 0$

Definite Integrals Ex 20.4B Q26

We have,

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x \, dx$$

$\sin^2 x$ is even function.

Hence,

$$I = 2\int_0^{\frac{\pi}{4}} \sin^2 x \, dx = 2\int_0^{\frac{\pi}{4}} \left(\frac{1-\cos 2x}{2}\right) dx = \frac{2}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{4}} = \frac{1}{2}\left[\frac{2\pi}{4} - \sin\frac{\pi}{2} - 0 + \sin 0\right]$$

$$= \frac{1}{2}\left[\frac{2\pi}{4} - 1\right]$$

$$= \frac{\pi}{4} - \frac{1}{2}$$

$$\therefore \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x \, dx = \frac{\pi}{4} - \frac{1}{2}$$

Definite Integrals Ex 20.4B Q27

$$I = \int_0^{\pi} \log(1 - \cos x)\,dx$$
$$= \int_0^{\pi} \log\left(2\sin^2\frac{x}{2}\right)dx$$
$$= \int_0^{\pi} \log 2\,dx + \int_0^{\pi} \log\sin^2\frac{x}{2}\,dx$$
$$= \int_0^{\pi} \log 2\,dx + 2\int_0^{\pi} \log\sin\frac{x}{2}\,dx$$
$$I = \log 2[x]_0^{\pi} + 4\int_0^{\frac{\pi}{2}} \log\sin t\,dt \qquad \left[\text{Put} \quad t = \frac{x}{2} \Rightarrow dt = \frac{1}{2}dx\right]$$

$$I = \pi\log 2 + 4I_1 \qquad \ldots(\text{i})$$
$$I_1 = \int_0^{\frac{\pi}{2}} \log\sin t\,dt \qquad \ldots(\text{ii})$$
$$= \int_0^{\frac{\pi}{2}} \log\cos t\,dt \qquad \ldots(\text{iii})$$

Adding (ii) & (iii) we get

$$2I_1 = \int_0^{\frac{\pi}{2}} \log\sin t.\cos t\,dt = \int_0^{\frac{\pi}{2}} \log\left(\frac{\sin 2t}{2}\right)dt = \int_0^{\frac{\pi}{2}} \log\sin 2t\,dt - \frac{\pi}{2}\log 2$$

We know the property $\int_a^b f(x) = \int_a^b f(t)$

$$2I_1 = I_1 - \frac{\pi}{2}\log 2$$
$$\Rightarrow I_1 = -\frac{\pi}{2}\log 2 \qquad \ldots(\text{iv})$$

Putting the value from (iv) to (i)

$$I = \pi\log 2 + 4\left(-\frac{\pi}{2}\log 2\right) = \pi\log 2 - 2\pi\log 2 = -\pi\log 2$$
$$I = -\pi\log 2$$

Definite Integrals Ex 20.4B Q28

We have,

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \log\left(\frac{2 - \sin x}{2 + \sin x}\right)dx$$

Let $f(x) = \log\left(\frac{2 - \sin x}{2 + \sin x}\right)$

Then,

$$f(-x) = \log\left(\frac{2 - \sin(-x)}{2 + \sin(-x)}\right) = -\log\left(\frac{2 - \sin x}{2 + \sin x}\right) = -f(x)$$

Thus, f(x) is an odd function.

$$\therefore I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \log\left(\frac{2 - \sinx}{2 + \sin x}\right)dx = 0$$

*********** END **********