



Algebraic Identities Ex 4.1 Q12

Answer :

(i) In the given problem, we have to find product of $\left(\frac{1}{2}a - 3b\right)\left(3b + \frac{1}{2}a\right)\left(\frac{1}{4}a^2 + 9b^2\right)$

We have been given $\left(\frac{1}{2}a - 3b\right)\left(3b + \frac{1}{2}a\right)\left(\frac{1}{4}a^2 + 9b^2\right)$

On rearranging we get, $\left(\frac{1}{2}a - 3b\right)\left(\frac{1}{2}a + 3b\right)\left(\frac{1}{4}a^2 + 9b^2\right)$

We shall use the identity $(x - y)(x + y) = x^2 - y^2$

By substituting $x = \frac{1}{2}a$, $y = 3b$ we get,

$$\begin{aligned}\left(\frac{1}{2}a - 3b\right)\left(3b + \frac{1}{2}a\right)\left(\frac{1}{4}a^2 + 9b^2\right) &= \left(\frac{1}{2}a\right)^2 - (3b)^2 \left(\frac{1}{4}a^2 + 9b^2\right) \\ &= \left(\frac{1}{4}a^2 - 9b^2\right)\left(\frac{1}{4}a^2 + 9b^2\right)\end{aligned}$$

We shall use the identity $(x - y)(x + y) = x^2 - y^2$

$$\begin{aligned}\left(\frac{1}{2}a - 3b\right)\left(3b + \frac{1}{2}a\right)\left(\frac{1}{4}a^2 + 9b^2\right) &= \left(\frac{1}{4}a^2\right)^2 - (9b^2)^2 \\ &= \frac{1}{16}a^4 - 81b^4\end{aligned}$$

Hence the value of $\left(\frac{1}{2}a - 3b\right)\left(3b + \frac{1}{2}a\right)\left(\frac{1}{4}a^2 + 9b^2\right)$ is $\boxed{\frac{1}{16}a^4 - 81b^4}$

(ii) In the given problem, we have to find product of $\left(m + \frac{n}{7}\right)^3\left(m - \frac{n}{7}\right)$

We have been given $\left(m + \frac{n}{7}\right)^3\left(m - \frac{n}{7}\right)$

On rearranging we get

$$\left(m + \frac{n}{7}\right)^3\left(m - \frac{n}{7}\right) = \left(m + \frac{n}{7}\right)^2\left(m + \frac{n}{7}\right)\left(m - \frac{n}{7}\right)$$

We shall use the identity $(x - y)(x + y) = x^2 - y^2$

By substituting $x = m$, $y = \frac{n}{7}$, we get ,

$$\begin{aligned}\left(m + \frac{n}{7}\right)^3\left(m - \frac{n}{7}\right) &= \left(m + \frac{n}{7}\right)^2\left(m^2 - \left(\frac{n}{7}\right)^2\right) \\ &= \left(m + \frac{n}{7}\right)^2\left(m^2 - \frac{n^2}{49}\right)\end{aligned}$$

Hence the value of $\left(m + \frac{n}{7}\right)^3\left(m - \frac{n}{7}\right)$ is $\boxed{\left(m + \frac{n}{7}\right)^2\left(m^2 - \frac{n^2}{49}\right)}$

(iii) In the given problem, we have to find product of $\left(\frac{x}{2} - \frac{2}{5}\right)\left(\frac{2}{5} - \frac{x}{2}\right) - x^2 + 2x$

On rearranging we get

$$\left(\frac{x}{2} - \frac{2}{5}\right)\left(\frac{2}{5} - \frac{x}{2}\right) - x^2 + 2x = \left(\frac{x}{2} - \frac{2}{5}\right)\left[-\left(\frac{x}{2} - \frac{2}{5}\right)\right] - x^2 + 2x$$

$$\Rightarrow \left(\frac{x}{2} - \frac{2}{5}\right)\left(\frac{2}{5} - \frac{x}{2}\right) - x^2 + 2x = -\left(\frac{x}{2} - \frac{2}{5}\right)^2 - x^2 + 2x$$

We shall use the identity $(x - y)^2 = x^2 - 2xy + y^2$

By substituting $x = \frac{x}{2}$, $y = \frac{2}{5}$

$$\begin{aligned} \left(\frac{x}{2} - \frac{2}{5}\right)\left(\frac{2}{5} - \frac{x}{2}\right) - x^2 + 2x &= \left(\frac{x^2}{4} + \frac{4}{25} - \frac{2 \times x}{2} \times \frac{2}{5}\right) - x^2 + 2x \\ &= -\left(\frac{x^2}{4} + \frac{4}{25} - \frac{2x}{5}\right) - x^2 + 2x \\ &= -\frac{x^2}{4} - \frac{4}{25} + \frac{2x}{5} - x^2 + 2x \\ &= \left[-\frac{x^2}{4} - x^2\right] - \frac{4}{25} + \left[\frac{2x}{5} + 2x\right] \\ &= \left[-\frac{x^2}{4} - \frac{x^2 \times 4}{1 \times 4}\right] - \frac{4}{25} + \left[\frac{2x}{5} + \frac{2x \times 5}{1 \times 5}\right] \\ &= \left[-\frac{x^2}{4} - \frac{4x^2}{4}\right] - \frac{4}{25} + \left[\frac{2x}{5} + \frac{10x}{5}\right] \\ &= \left[\frac{-x^2 - 4x^2}{4}\right] - \frac{4}{25} + \left[\frac{2x + 10x}{5}\right] \\ &= \frac{-5x^2}{4} - \frac{4}{25} + \frac{12x}{5} \end{aligned}$$

Hence the value of $\left(\frac{x}{2} - \frac{2}{5}\right)\left(\frac{x}{2} + \frac{2}{5}\right) - x^2 + 2x$ is $\boxed{\frac{-5x^2}{4} - \frac{4}{25} + \frac{12x}{5}}$

(iv) In the given problem, we have to find product of $(x^2 + x - 2)(x^2 - x + 2)$

On rearranging we get

$$(x^2 + x - 2)(x^2 - x + 2) = [x^2 + (x - 2)][x^2 - (x - 2)]$$

We shall use the identity $(x - y)(x + y) = x^2 - y^2$

$$\begin{aligned} (x^2 + x - 2)(x^2 - x + 2) &= [(x^2)^2 - (x - 2)^2] \\ &= x^4 - (x^2 - 2 \times 2 \times x + 2^2) \\ &= x^4 - (x^2 - 4x + 4) \\ &= x^4 - x^2 + 4x - 4 \end{aligned}$$

Hence the value of $(x^2 + x - 2)(x^2 - x + 2)$ is $\boxed{x^4 - x^2 + 4x - 4}$

(v) In the given problem, we have to find product of $(x^3 - 3x^2 - x)(x^2 - 3x + 1)$

Taking x as common factor $= x(x^2 - 3x - 1)(x^2 - 3x + 1)$

$$\begin{aligned} (x^3 - 3x^2 - x)(x^2 - 3x + 1) &= [x(x^2 - 3x - 1)(x^2 - 3x + 1)] \\ &= x\left[\{(x^2 - 3x) - 1\}\{(x^2 - 3x) + 1\}\right] \end{aligned}$$

We shall use the identity $(x - y)(x + y) = x^2 - y^2$

$$\begin{aligned} (x^3 - 3x^2 - x)(x^2 - 3x + 1) &= x\left[(x^2 - 3x)^2 - 1^2\right] \\ &= x(x^4 - 6x^3 + 9x^2 - 1) \\ &= x^5 - 6x^4 + 9x^3 - x \end{aligned}$$

Hence the value of $(x^3 - 3x^2 - x)(x^2 - 3x + 1)$ is $\boxed{x^5 - 6x^4 + 9x^3 - x}$

(vi) In the given problem, we have to find product of $(2x^4 - 4x^2 + 1)(2x^4 - 4x^2 - 1)$

On rearranging we get $([2x^4 - 4x^2] + 1)([2x^4 - 4x^2] - 1)$

We shall use the identity $(x - y)(x + y) = x^2 - y^2$

$$\begin{aligned}(2x^4 - 4x^2 + 1)(2x^4 - 4x^2 - 1) &= [2x^4 - 4x^2]^2 - 1^2 \\ &= [4x^8 + 16x^4 - 2 \times 2x^4 \times 4x^2 - 1] \\ &= 4x^8 + 16x^4 - 16x^6 - 1\end{aligned}$$

Hence the value of $(2x^4 - 4x^2 + 1)(2x^4 - 4x^2 - 1)$ is $\boxed{4x^8 + 16x^4 - 16x^6 - 1}$.

Answer :

In the given problem, we have to prove $a^2 + b^2 + c^2 - ab - bc - ca$ is always non negative for all a, b, c that is we have to prove that $a^2 + b^2 + c^2 - ab - bc - ca \geq 0$

Consider,

$$\begin{aligned}a^2 + b^2 + c^2 - ab - bc - ca \\ a^2 + b^2 + c^2 - ab - bc - ca &= \frac{1}{2}(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca) \\ &= \frac{1}{2}[(a^2 - 2ab + b^2) + (b^2 - 2bc + c^2) + (c^2 - 2ac + a^2)] \\ &= \frac{1}{2}[(a - b)^2 + (b - c)^2 + (c - a)^2]\end{aligned}$$

$$\Rightarrow a^2 + b^2 + c^2 - ab - bc - ca \geq 0$$

Hence $\boxed{(a - b)^2 + (b - c)^2 + (c - a)^2}$ is always non negative for all a, b, c .

Note: Square of all negative numbers is always positive or non negative.

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