Differentiation Ex 11.2 Q71

Here, $y = e^x + e^{-x}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^x + e^{-x}\right)$$
$$= \frac{d}{dx}e^x + \frac{d}{dx}e^{-x}$$
$$= e^x + e^{-x}\frac{d}{dx}(-x) \quad \text{[Using chian rule]}$$
$$= e^x + e^{-x}(-1)$$
$$= \left(e^x - e^{-x}\right)$$
$$= \sqrt{\left(e^x + e^{-x}\right)^2 - 4e^x \times e^{-x}} \quad \left[\text{Since } (a-b) = \sqrt{(a+b)^2 - 4ab}\right]$$
$$= \sqrt{y^2 - 4} \quad \left[\text{Since } e^x + e^{-x} = y\right]$$

So,

$$\frac{dy}{dx} = \sqrt{y^2 - 4}.$$

Differentiation Ex 11.2 Q72

Given, $y = \sqrt{a^2 - x^2}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{a^2 - x^2}\right)$$
$$= \frac{1}{2\sqrt{a^2 - x^2}}\frac{d}{dx}\left(a^2 - x^2\right) \quad \text{[Using chain rule]}$$
$$= \frac{1}{2\sqrt{a^2 - x^2}}(-2x)$$
$$= \frac{-x}{\sqrt{a^2 - x^2}}$$
$$\Rightarrow \quad \frac{dy}{dx} = \frac{-x}{y} \quad \left[\text{Since } \sqrt{a^2 - x^2} = y\right]$$
$$\Rightarrow \quad y\frac{dy}{dx} = -x$$

$$y\frac{dy}{dx} + x = 0$$

Differentiation Ex 11.2 Q73

Here, $xy = 4$

$\Rightarrow \quad y = \frac{4}{x}$

Differentiate with respect to x,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(\frac{4}{x}\right)\\ &= 4\frac{d}{dx}(x^{-1})\\ &= 4\left(-1 \times x^{-1-1}\right)\\ &= 4\left(-\frac{1}{x^2}\right)\\ &= \frac{-4}{x^2}\\ &= -\frac{4}{\left(\frac{x}{y}\right)^2} \qquad \left[\text{Since } x = \frac{4}{y}\right]\\ &= -\frac{4y^2}{16}\end{aligned}$$

$$\frac{dy}{dx} = -\frac{y^2}{4}$$

$\Rightarrow \quad 4\frac{dy}{dx} = -y^2$

$\Rightarrow \quad 4\frac{dy}{dx} = 3y^2 - 4y^2$

$\Rightarrow \quad 4\frac{dy}{dx} + 4y^2 = 3y^2$

$\Rightarrow \quad 4\left(\frac{dy}{dx} + y^2\right) = 3y^2$

Dividing both the sides by x,

$\Rightarrow \quad \frac{4}{x}\left(\frac{dy}{dx} + y^2\right) = \frac{3y^2}{x}$

$\Rightarrow \quad y\left(\frac{dy}{dx} + y^2\right) = \frac{3y^2}{x} \qquad \left[\text{Since } \frac{4}{x} = y\right]$

$\Rightarrow \quad x\left(\frac{dy}{dx} + y^2\right) = \frac{3y^2}{y}$

$\Rightarrow \quad x\left(\frac{dy}{dx} + y^2\right) = 3y.$

Differentiation Ex 11.2 Q74

$$\frac{d}{dx}\left\{\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right\}=\sqrt{a^2-x^2}$$

LHS $= \frac{d}{dx}\left\{\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right\}$

$= \frac{d}{dx}\left(\frac{x}{2}\sqrt{a^2-x^2}\right)+\frac{d}{dx}\left(\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right)$

$= \frac{1}{2}\left[x\frac{d}{dx}\sqrt{a^2-x^2}+\sqrt{a^2-x^2}\frac{d}{dx}(x)\right]+\frac{a^2}{2}\times\frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}\times\frac{d}{dx}\left(\frac{x}{a}\right)$

[Using product rule, chain rule]

$= \frac{1}{2}\left[x\times\frac{1}{2\sqrt{a^2-x^2}}\frac{d}{dx}\left(a^2-x^2\right)+\sqrt{a^2-x^2}\right]+\left(\frac{a^2}{2}\right)\times\frac{1}{\sqrt{\frac{a^2-x^2}{a^2}}}\times\left(\frac{1}{a}\right)$

$= \frac{1}{2}\left[\frac{x(-2x)}{2\sqrt{a^2-x^2}}+\sqrt{a^2-x^2}\right]+\left(\frac{a^2}{2}\right)\frac{a}{\sqrt{a^2-x^2}}\times\left(\frac{1}{a}\right)$

$= \frac{1}{2}\left[\frac{-2x^2+2\left(a^2-x^2\right)}{2\sqrt{a^2-x^2}}\right]+\frac{a^2}{2\sqrt{a^2-x^2}}$

$= \frac{1}{2}\left[\frac{2\left(a^2-2x^2\right)}{2\sqrt{a^2-x^2}}\right]+\frac{a^2}{2\sqrt{a^2-x^2}}$

$= \frac{a^2-2x^2}{2\sqrt{a^2-x^2}}+\frac{a^2}{2\sqrt{a^2-x^2}}$

$= \frac{a^2-2x^2+a^2}{2\sqrt{a^2-x^2}}$

$= \frac{2a^2-2x^2}{2\sqrt{a^2-x^2}}$

$= \frac{2\left(a^2-x^2\right)}{2\sqrt{a^2-x^2}}$

$= \frac{\left(a^2-x^2\right)}{\sqrt{a^2-x^2}}$

$= \sqrt{a^2-x^2}$

= RHS

********** END **********