



Maxima and Minima 18.3 Q2(i)

$$f(x) = (x - 1)(x - 2)^2$$

$$\begin{aligned}\therefore f'(x) &= (x - 2)^2 + 2(x - 1)(x - 2) \\ &= (x - 2)(x - 2 + 2x - 2) \\ &= (x - 2)(3x - 4)\end{aligned}$$

$$f''(x) = (3x - 4) + 3(x - 2)$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow (x - 2)(3x - 4) = 0$$

$$\Rightarrow x = 2, \frac{4}{3}$$

Now,

$$f''(2) > 0$$

$\therefore x = 2$  is local minima

$$f''\left(\frac{4}{3}\right) = -2 < 0$$

$\therefore x = \frac{4}{3}$  is point of local maxima

$$\therefore \text{local max value} = f\left(\frac{4}{3}\right) = \frac{4}{27}$$

$$\text{local min value} = f(2) = 0.$$

Maxima and Minima 18.3 Q2(ii)

$$f(x) = x\sqrt{1-x}$$

$$\begin{aligned}\therefore f'(x) &= \sqrt{1-x} + \frac{x}{2\sqrt{1-x}}(-1) \\ &= \frac{2(1-x) - x}{2\sqrt{1-x}} \\ &= \frac{2-3x}{2\sqrt{1-x}} \\ f''(x) &= \frac{2\sqrt{1-x}(-3) + \frac{(2-3x)}{\sqrt{1-x}}}{4(1-x)}\end{aligned}$$

For maximum and minimum,

$$f'(x) = 0$$

$$\Rightarrow \frac{2-3x}{2\sqrt{1-x}} = 0$$

$$\Rightarrow x = \frac{2}{3}$$

Now,

$$f''\left(\frac{2}{3}\right) < 0$$

$$\therefore x = \frac{2}{3} \text{ is point of maxima}$$

$$\therefore \text{local max value} = f\left(\frac{2}{3}\right) = \frac{2}{3\sqrt{3}}.$$

Maxima and Minima 18.3 Q2(iii)

$$\begin{aligned}
 f(x) &= -(x-1)^3(x+1)^2 \\
 \therefore f'(x) &= -3(x-1)^2(x+1)^2 - 2(x-1)^3(x+1) \\
 &= -(x-1)^2(x+1)(3x+3+2x-2) \\
 &= -(x-1)^2(x+1)(5x+1) \\
 \therefore f''(x) &= -2(x-1)(x+1)(5x+1) - (x-1)^2(5x+1) - 5(x-1)^2(x+1)
 \end{aligned}$$

For maximum and minimum value,

$$\begin{aligned}
 f'(x) &= 0 \\
 \Rightarrow -(x-1)^2(x+1)(5x+1) &= 0 \\
 \Rightarrow x &= 1, -1, -\frac{1}{5}
 \end{aligned}$$

Now,

$$\begin{aligned}
 f''(1) &= 0 \\
 \therefore x = 1 &\text{ is inflection point} \\
 f''(-1) &= -4 \times -4 = 16 > 0 \\
 \therefore x = -1 &\text{ is point of minima} \\
 f''\left(-\frac{1}{5}\right) &= -5\left(\frac{36}{25}\right) \times \frac{4}{5} = \frac{-144}{25} < 0 \\
 \therefore x = -\frac{1}{5} &\text{ is point of maxima}
 \end{aligned}$$

Hence,

$$\begin{aligned}
 \text{local max value} &= f\left(-\frac{1}{5}\right) = \frac{3456}{3125} \\
 \text{local min value} &= f(-1) = 0.
 \end{aligned}$$

Maxima and Minima 18.3 Q3

We have,

$$\begin{aligned}
 y &= a \log x + bx^2 + x \\
 \therefore \frac{dy}{dx} &= \frac{a}{x} + 2bx + 1 \\
 \text{and } \frac{d^2y}{dx^2} &= \frac{-a}{x^2} + 2b
 \end{aligned}$$

For maximum and minimum value,

$$\begin{aligned}
 \frac{dy}{dx} &= 0 \\
 \Rightarrow \frac{a}{x} + 2bx + 1 &= 0
 \end{aligned}$$

Given that extreme value exist at  $x = 1, 2$

$$\Rightarrow a + 2b = -1 \quad \text{--- (i)}$$

$$\begin{aligned}
 \frac{a}{2} + 4b &= -1 \\
 \Rightarrow a + 8b &= -2 \quad \text{--- (ii)}
 \end{aligned}$$

Solving (i) and (ii), we get

$$a = \frac{-2}{3}, \quad b = \frac{-1}{6}.$$

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