

Indefinite Integrals Ex 19.31 Q1

$$I = \int \frac{x^2 + 1}{x^4 + x^2 + 1} dx$$
$$= \int \frac{1 + \frac{1}{x^2}}{x^2 + 1 + \frac{1}{x^2}} dx$$

Dividing numerator and denominator by \varkappa^2

$$= \int \frac{\left(1 + \frac{1}{x^2}\right)}{\left(x - \frac{1}{x}\right)^2 + 3} dx$$
Let $x - \frac{1}{x} = t \Rightarrow \left(1 + \frac{1}{x^2}\right) dx = dt$

$$\therefore I = \int \frac{dt}{t^2 + 3}$$

$$= \frac{1}{\sqrt{3}} tan^{-1} \left(\frac{t}{\sqrt{3}}\right) + c$$

$$I = \frac{1}{\sqrt{3}} tan^{-1} \left(\frac{x^2 - 1}{\sqrt{3}x} \right) + c$$

Indefinite Integrals Ex 19.31 Q2

$$\int \sqrt{\cot \theta} d\theta$$
Let $\cot \theta = x^2$

$$\Rightarrow -\cos ec^2 \theta d\theta = 2x dx$$

$$\Rightarrow d\theta = \frac{-2x}{\cos ec^2 \theta} dx$$

$$= \frac{-2x}{1 + \cot^2 \theta} dx$$

$$= \frac{-2x}{1 + x^4} dx$$

$$\therefore I = -\int \frac{2x^2}{1 + x^4} dx$$

$$= -\int \frac{2}{1 + x^4} dx$$

Dividing numerator and denominator by x^2

$$= -\int \frac{1 + \frac{1}{x^2} + 1 - \frac{1}{x^2}}{x^2 + \frac{1}{x^2}} dx$$

$$= -\int \frac{\left(1 + \frac{1}{x^2}\right) dx}{\left(x - \frac{1}{x}\right)^2 + 2} - \int \frac{\left(1 - \frac{1}{x^2}\right) dx}{\left(x + \frac{1}{x}\right)^2 - 2}$$
Let $x - \frac{1}{x} = t \Rightarrow \left(1 + \frac{1}{x^2}\right) dx = dt$
and $x + \frac{1}{x} = z \Rightarrow \left(1 - \frac{1}{x^2}\right) dx = dz$

$$\Rightarrow I = -\int \frac{dt}{t^2 + 2} - \int \frac{dz}{z^2 - 2}$$

$$= -\frac{1}{\sqrt{2}} tan^{-1} \left(\frac{t}{\sqrt{2}}\right) - \frac{1}{2\sqrt{2}} log \left|\frac{z - \sqrt{2}}{z + \sqrt{2}}\right| + c$$

$$= -\frac{1}{\sqrt{2}} tan^{-1} \left(\frac{x^2 - 1}{\sqrt{2}x}\right) - \frac{1}{2\sqrt{2}} log \left|\frac{x^2 + 1 - \sqrt{2}x}{x^2 + 1 + \sqrt{2}x}\right| + c$$

$$I = -\frac{1}{\sqrt{2}} tan^{-1} \left(\frac{\cot \theta - 1}{\sqrt{2}\cot \theta}\right) - \frac{1}{2\sqrt{2}} log \left|\frac{\cot \theta + 1 - \sqrt{2}\cot \theta}{\cot \theta + 1 - \sqrt{2}\cot \theta}\right| + c$$

Indefinite Integrals Ex 19.31 Q3

Let
$$I = \int \frac{x^2 + 9}{x^4 + 81} dx$$

Dividing numerator and denominator by x^2

$$I = \int \frac{1 + \frac{9}{x^2}}{x^2 + \frac{81}{x^2}} dx$$

$$= \int \frac{1 + \frac{9}{x^2}}{\left(x - \frac{9}{x}\right)^2 + 18} dx$$
Let $\left(x - \frac{9}{x}\right) = t \Rightarrow \left(1 + \frac{9}{x^2}\right) dx = dt$

$$\therefore I = \int \frac{dt}{t^2 + 18}$$

$$\Rightarrow I = \frac{1}{3\sqrt{2}} tan^{-1} \left(\frac{t}{3\sqrt{2}}\right) + c$$
Thus,
$$I = \frac{1}{3\sqrt{2}} tan^{-1} \left(\frac{x^2 - 9}{3\sqrt{2}x}\right) + c$$

Indefinite Integrals Ex 19.31 Q4

Let
$$I = \int \frac{1}{x^4 + x^2 + 1} dx$$

Dividing numerator and denominator by x^2

$$I = \int \frac{\frac{1}{x^2}}{x^2 + 1 + \frac{1}{x^2}} dx$$

$$= \frac{1}{2} \int \frac{1 + \frac{1}{x^2} - 1 + \frac{1}{x^2}}{x^2 + 1 + \frac{1}{x^2}} dx$$

$$= \frac{1}{2} \left\{ \int \frac{\left(1 + \frac{1}{x^2}\right) dx}{\left(x - \frac{1}{x}\right)^2 + 3} - \int \frac{\left(1 - \frac{1}{x^2}\right)}{\left(x + \frac{1}{x}\right)^2 - 1} dx \right\}$$
Let $\left(x - \frac{1}{x}\right) = t$

$$\Rightarrow \left(1 + \frac{1}{x^2}\right) dx = dt$$
and $\left(x + \frac{1}{x}\right) = z$

$$\Rightarrow \left(1 - \frac{1}{x^2}\right) dx = dz$$

$$\therefore I = \frac{1}{2} \int \frac{dt}{t^2 + 3} - \frac{1}{2} \int \frac{dz}{z^2 - 1}$$

$$\Rightarrow I = \frac{1}{2\sqrt{3}} \tan^{-1} \left(\frac{t}{\sqrt{3}}\right) - \frac{1}{4} \log \left|\frac{z - 1}{z + 1}\right| + c$$

$$\Rightarrow I = \frac{1}{2\sqrt{3}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{3}x}\right) - \frac{1}{4} \log \left|\frac{x^2 + 1 - x}{x^2 + 1 + x}\right| + c$$

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