Polynomials Ex 2.3 Q2

Answer :

(i). Given $g(t) = t^2 - 3$

$f(t) = 2t^4 + 3t^3 - 2t^2 - 9t - 12$

Here, degree $(f(t)) = 4$ and

Degree $(g(t)) = 2$

Therefore, quotient $q(t)$ is of degree $4 - 2 = 2$

Remainder $r(t)$ is of degree 1 or less

Let $q(t) = at^2 + bt + c$ and

$r(t) = pt + q$

Using division algorithm, we have

$$f(t) = g(t) + q(t) + r(t)$$

$$2t^4 + 3t^3 - 2t^2 - 9t - 12 = (at^2 + bt + c)(t^2 - 3) + pt + q$$

$$2t^4 + 3t^3 - 2t^2 - 9t - 12 = at^4 + bt^3 + ct^2 - 3at^2 - 3bt - 3c + pt + q$$

$$2t^4 + 3t^3 - 2t^2 - 9t - 12 = at^4 + bt^3 - t^2(3a - c) - t(3b - p) - 3c + q$$

Equating co-efficient of various powers of t, we get

On equating the co-efficient of t^4

$$2t^4 = at^4$$

$$2\cancel{t^4} = a\cancel{t^4}$$

$$2 = a$$

On equating the co-efficient of t^3

$3t^3 = bt^3$

$3\cancel{t^3} = b\cancel{t^3}$

$3 = b$

On equating the co-efficient of t^2

$2 = 3a - c$

Substituting $a = 2$, we get

$2 = 3 \times 2 - c$

$2 = 6 - c$

$2 - 6 = -c$

$\cancel{-}4 = \cancel{-}c$

$c = 4$

On equating the co-efficient of t

$9 = 3b - p$

Substituting $b = 3$, we get

$9 = 3 \times 3 - p$

$9 = 9 - p$

$9 - 9 = -p$

$0 = -p$

$p = 0$

On equating constant term

$-12 = -3c + q$

Substituting $c=4$, we get

$$-12=3\times4+q$$

$$-12=-12+q$$

$$-12+12=+q$$

$$0=q$$

Quotient $q(t)=at^2+bt+c$

$=2t^2+3t+4$

Remainder $r(t)=pt+q$

$=0t+0$

$=0$

Clearly, $r(t)=0$

Hence, $g(t)$ is a factor of $f(t)$.

(ii) Given

$f(x)=x^5-4x^3+x^2+3x+1$

$g(x)=x^3-3x+1$

Here, Degree $(f(x))=5$ and

Degree $(g(x))=3$

Therefore, quotient $q(x)$ is of degree $5-3=2$

Remainder $r(x)$ is of degree1

Let $q(x)=ax^2+bx+c$ and

$r(x)=px+q$

*********** END **********