Mean Value Theorems Ex 15.1 Q1(i)

$f(x) = 3 + (x-2)^{\frac{2}{3}}$ on $[1, 3]$

Differentiating it with respect to x,

$$f'(x) = \frac{2}{3} \times \frac{1}{(x-2)^{\frac{1}{3}}}$$

Clearly, $\lim\limits_{x \to 2} = \dfrac{2}{3} \times \dfrac{1}{(x-2)^{\frac{1}{3}}}$

Thus, $f(x)$ is not differentiable at $x = 2 \in (1, 3)$

Hene, Rolle's theorem is not applicable for $f(x)$ in $x \in [1, 3]$.

Mean Value Theorems Ex 15.1 Q1(ii)

Here, $f(x) = [x]$ and $x \in [-1, 1]$, at $n = 1$

$$\begin{aligned}
\text{LHL} &= \lim_{x \to (1-h)} [x] \\
&= \lim_{h \to 0} [1 - h] \\
&= 0 \\
\text{RHL} &= \lim_{x \to (1+h)} [x] \\
&= \lim_{h \to 0} [1 + h] \\
&= 1 \\
&\text{LHL} \neq RHL
\end{aligned}$$

So, $f(x)$ is not continuous at $1 \in [-1, 1]$

Hence, rolle's theorem is not applicable on $f(x)$ in $[-1, 1]$.

Mean Value Theorems Ex 15.1 Q1(iii)

Here, $f(x) = \sin\left(\frac{1}{x}\right)$, $x \in [-1,1]$, at $n = 0$

$$\begin{aligned}
\text{LHS} &= \lim_{x\to(0-h)} \sin\left(\frac{1}{x}\right) \\
&= \lim_{h\to 0} \sin\left(\frac{1}{0-h}\right) \\
&= \lim_{h\to 0} \sin\left(\frac{-1}{h}\right) \\
&= -\lim_{h\to 0} \sin\left(\frac{1}{h}\right) \\
&= -k \qquad \left[\text{Let } \lim_{h\to 0} \sin\left(\frac{1}{h}\right) = k \text{ as } k \in [-1,1]\right]
\end{aligned}$$

$$\begin{aligned}
\text{RHS} &= \lim_{x\to(0+h)} \sin\left(\frac{1}{x}\right) \\
&= \lim_{h\to 0} \sin\left(\frac{1}{h}\right) \\
&= k
\end{aligned}$$

$\Rightarrow$ LHS $\neq$ RHS

$\Rightarrow$ $f(x)$ is not continuous at $n = 0$

So, rolle's theorem is not applicable on $f(x)$ in $[-1,1]$

Mean Value Theorems Ex 15.1 Q1(iv)

Here, $f(x) = 2x^2 - 5x + 3$ on $[1,3]$

$f(x)$ is continuous in $[1,3]$ and $f(x)$ is differentiable is $(1,3)$ since it is a polynomial function.

Now,

$$\begin{aligned}
f(x) &= 2x^2 - 5x + 3 \\
f(1) &= 3(1)^2 - 5(1) + 3 \\
&= 2 - 5 + 3 \\
f(1) &= 0 \qquad \text{---(i)} \\
f(3) &= 2(3)^2 - 5(3) + 3 \\
&= 18 - 15 + 3 \\
f(3) &= 6 \qquad \text{---(ii)}
\end{aligned}$$

From equation (i) and (ii),

$$f(1) \neq f(3)$$

So, rolle's theorem is not applicable on $f(x)$ in $[1,3]$.

*********** END **********