



Definite Integrals Ex 20.4B Q1

We have,

$$\frac{1}{1 + \tan x} = \frac{1}{1 + \frac{\sin x}{\cos x}} = \frac{\cos x}{\cos x + \sin x}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{dx}{1 + \tan x} = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx$$

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx \quad \text{--- (I)}$$

So,

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2} - x\right)}{\cos\left(\frac{\pi}{2} - x\right) + \sin\left(\frac{\pi}{2} - x\right)} dx \quad \left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x}{\cos x + \sin x} dx \quad \text{--- (II)}$$

Hence, adding (I) & (II)

$$2I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx + \int_0^{\frac{\pi}{2}} \frac{\sin x}{\cos x + \sin x} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos x + \sin x}{\cos x + \sin x} dx$$

$$= \int_0^{\frac{\pi}{2}} dx$$

$$2I = [x]_0^{\frac{\pi}{2}}$$

$$2I = \left[\frac{\pi}{2} - 0 \right] \Rightarrow I = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q2

We have,

$$\frac{1}{1 + \cot x} = \frac{1}{1 + \frac{\cos x}{\sin x}} = \frac{\sin x}{\sin x + \cos x}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{1}{1 + \cot x} dx = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx$$

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx \quad \text{--- (I)}$$

So,

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx \quad \left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x} dx \quad \text{--- (II)}$$

Adding (I) & (II)

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx + \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$2I = \int_0^{\frac{\pi}{2}} dx$$

$$= [x]_0^{\frac{\pi}{2}}$$

$$2I = \left[\frac{\pi}{2} - 0 \right]$$

$$I = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q3

We have,

$$\frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} = \frac{\frac{\sqrt{\cos x}}{\sqrt{\sin x}}}{\frac{\sqrt{\cos x}}{\sqrt{\sin x}} + \frac{\sqrt{\sin x}}{\sqrt{\cos x}}} = \frac{\frac{\sqrt{\cos x}}{\sqrt{\sin x}}}{\frac{\cos x + \sin x}{\sqrt{\sin x} \sqrt{\cos x}}} = \frac{\sqrt{\cos x}}{\sqrt{\sin x}} \times \frac{\sqrt{\sin x} \sqrt{\cos x}}{\cos x + \sin x} = \frac{\cos x}{\cos x + \sin x}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx$$

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx \quad \dots (I)$$

So,

$$\begin{aligned} I &= \int_0^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2} - x\right)}{\cos\left(\frac{\pi}{2} - x\right) + \sin\left(\frac{\pi}{2} - x\right)} dx & \left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right] \\ &= \int_0^{\frac{\pi}{2}} \frac{\sin x}{\cos x + \sin x} dx & \dots (II) \end{aligned}$$

Adding (I) & (II)

$$\begin{aligned} 2I &= \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx + \int_0^{\frac{\pi}{2}} \frac{\sin x}{\cos x + \sin x} dx \\ 2I &= \int_0^{\frac{\pi}{2}} \frac{\cos x + \sin x}{\cos x + \sin x} dx \\ 2I &= \int_0^{\frac{\pi}{2}} 1 dx \\ 2I &= [x]_0^{\frac{\pi}{2}} \\ 2I &= \left[\frac{\pi}{2} - 0 \right] \\ I &= \frac{\pi}{4} \end{aligned}$$

Definite Integrals Ex 20.4B Q4

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \quad \dots (1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right) + \cos^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)} dx \quad \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx \right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \quad \dots (2)$$

Adding (1) and (2), we obtain

$$\begin{aligned} 2I &= \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \\ \Rightarrow 2I &= \int_0^{\frac{\pi}{2}} 1 dx \\ \Rightarrow 2I &= [x]_0^{\frac{\pi}{2}} \\ \Rightarrow 2I &= \frac{\pi}{2} \\ \Rightarrow I &= \frac{\pi}{4} \end{aligned}$$

***** END *****

