Trigonometric Ratios Ex 5.1 Q20

Answer :

Given: $\tan\theta = \frac{12}{13}$ .....(1)

To find the value of $\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$

Now, we know the following trigonometric identity

$\text{cosec}^2\theta = 1 + \tan^2\theta$

Therefore, by substituting the value of $\tan\theta$ from equation (1) ,

We get,

$$\text{cosec}^2\theta = 1 + \left(\frac{12}{13}\right)^2$$

$$= 1 + \frac{(12)^2}{(13)^2}$$

$$= 1 + \frac{144}{169}$$

By taking L.C.M. on the R.H.S,

We get,

$$\text{cosec}^2\theta = \frac{169 + 144}{169}$$

$$= \frac{313}{169}$$

Therefore

$$\text{cosec}\,\theta = \sqrt{\frac{313}{169}}$$

$$= \frac{\sqrt{313}}{13}$$

Therefore

$$\text{cosec}\,\theta = \frac{\sqrt{313}}{13} \quad \ldots\ldots (2)$$

13

Now, we know that

$$\text{cosec}\,\theta = \frac{1}{\sin\theta}$$

Therefore,

$$\sin\theta = \frac{1}{\text{cosec}\,\theta}$$

$$\sin\theta = \frac{1}{\frac{\sqrt{313}}{13}}$$

$$= \frac{13}{\sqrt{313}}$$

Therefore

$$\sin\theta = \frac{13}{\sqrt{313}} \quad \ldots\ldots (3)$$

Now, we know the following trigonometric identity

$$\cos^2\theta + \sin^2\theta = 1$$

Therefore,

$$\cos^2\theta = 1 - \sin^2\theta$$

Now by substituting the value of $\sin\theta$ from equation (3)

We get,

$$\cos^2\theta = 1 - \left(\frac{13}{\sqrt{313}}\right)^2$$

$$= 1 - \frac{(13)^2}{\left(\sqrt{313}\right)^2}$$

$$= 1 - \frac{169}{313}$$

Therefore, by taking L.C.M on R.H.S

We get,

$$\cos^2\theta = \frac{313 - 169}{313}$$

$$= \frac{144}{313}$$

Now, by taking square root on both sides

We get,

$$\cos\theta = \sqrt{\frac{144}{313}}$$

$$= \frac{12}{\sqrt{313}}$$

Therefore,

$$\cos\theta = \frac{12}{\sqrt{313}} \quad \ldots\ldots (4)$$

Substituting the value of $\sin\theta$ and $\cos\theta$ from equation (3) and (4) respectively in the expression below

$$\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$$

Therefore,

$$\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = \frac{2\times\frac{13}{\sqrt{313}}\times\frac{12}{\sqrt{313}}}{\left(\frac{13}{\sqrt{313}}\right)^2 - \left(\frac{12}{\sqrt{313}}\right)^2}$$

$$= \frac{\frac{2\times13\times12}{313}}{\frac{(13)^2}{313} - \frac{(12)^2}{313}}$$

$$= \frac{\frac{312}{313}}{\frac{169}{313} - \frac{144}{313}}$$

$$= \frac{\frac{312}{313}}{\frac{169-144}{313}}$$

$$= \frac{\frac{312}{313}}{\frac{25}{313}}$$

$$= \frac{312}{25}$$

Therefore, $\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = \frac{312}{25}$

********** END **********