Differentials Errors and Approximation Ex14.1 Q10

Let $x = 2$ and $\Delta x = 0.01$. Then, we have:

$f(2.01) = f(x + \Delta x) = 4(x + \Delta x)^2 + 5(x + \Delta x) + 2$

Now, $\Delta y = f(x + \Delta x) - f(x)$

$\therefore f(x + \Delta x) = f(x) + \Delta y$

$\underset{\approx}{\approx} f(x) + f'(x) \cdot \Delta x$ (as $dx = \Delta x$)

$\Rightarrow f(2.01) \approx (4x^2 + 5x + 2) + (8x + 5)\Delta x$

$= \left[4(2)^2 + 5(2) + 2 \right] + \left[8(2) + 5 \right](0.01)$ [as $x = 2$, $\Delta x = 0.01$]

$= (16 + 10 + 2) + (16 + 5)(0.01)$

$= 28 + (21)(0.01)$

$= 28 + 0.21$

$= 28.21$

Hence, the approximate value of $f(2.01)$ is 28.21.

Differentials Errors and Approximation Ex14.1 Q11

Let $x = 5$ and $\Delta x = 0.001$. Then, we have:

$f(5.001) = f(x + \Delta x) = (x + \Delta x)^3 - 7(x + \Delta x)^2 + 15$

Now, $\Delta y = f(x + \Delta x) - f(x)$

$\therefore f(x + \Delta x) = f(x) + \Delta y$

$\approx f(x) + f'(x) \cdot \Delta x$ (as $dx = \Delta x$)

$\Rightarrow f(5.001) \approx (x^3 - 7x^2 + 15) + (3x^2 - 14x)\Delta x$

$= \left[(5)^3 - 7(5)^2 + 15 \right] + \left[3(5)^2 - 14(5) \right](0.001)$ [$x = 5, \Delta x = 0.001$]

$= (125 - 175 + 15) + (75 - 70)(0.001)$

$= -35 + (5)(0.001)$

$= -35 + 0.005$

$= -34.995$

Hence, the approximate value of $f(5.001)$ is -34.995.

Differentials Errors and Approximation Ex14.1 Q12

Let $x = 1000,\ x + \Delta x = 1005$

$\Delta x = 1005 - 1000$

$= 5$

Let $y = \log_{10} x$

$$\frac{dy}{dx} = \frac{\log_e x}{\log_e 10} \qquad \left[\because \log_a b = \frac{\log_e b}{\log_e a} \right]$$

$$\frac{dy}{dx} = \frac{1}{x \log_e 10}$$

$$\left(\frac{dy}{dx} \right)_{x=1000} = \frac{\log_{10} e}{1000} \qquad \left[\because \log_a b = \frac{1}{\log_b a} \right]$$

$$= \frac{0.4343}{1000}$$

$$= (0.0004343)$$

$\therefore$ $\Delta y = \left(\dfrac{dy}{dx} \right)_{x=1000} \times (\Delta x)$

$$= (0.0004343)(5)$$

$$= 0.0021715$$

$\log_{10} 1005 = y + \Delta y$

$$= \log_{10} x + 0.0021715$$

$$= \log_{10} 1000 + 0.0021715$$

$$= \log_{10} 10^3 + 0.0021715$$

$$= 3 \log_{10} 10 + 0.0021715$$

$\log_{10} 1005 = 3.0021715$

********** END *********