

NCERT MISCELLANEOUS SOLUTIONS

Question-1

The relation
$$f$$
 is defined by $f(x) = \begin{cases} x^2, & 0 \le x \le 3 \\ 3x, & 3 \le x \le 10 \end{cases}$

The relation
$$g$$
 is defined by $g(x) = \begin{cases} x^2, & 0 \le x \le 2\\ 3x, & 2 \le x \le 10 \end{cases}$

Show that f is a function and g is not a function

Ans.

The relation f is defined as $f(x) = \begin{cases} x^2, & 0 \le x \le 3\\ 3x, & 3 \le x \le 10 \end{cases}$

It is observed that for

$$0 \le x < 3, f(x) = x^2$$

$$3 < x \le 10, f(x) = 3x$$

Also, at
$$x = 3$$
, $f(x) = 3^2 = 9$ or $f(x) = 3 \times 3 = 9$

i.e., at
$$x = 3$$
, $f(x) = 9$

Therefore, for $0 \le x \le 10$, the images of f(x) are unique.

Thus, the given relation is a function.

The relation g is defined as
$$g(x) = \begin{cases} x^2, & 0 \le x \le 2\\ 3x, & 2 \le x \le 10 \end{cases}$$

It can be observed that for x = 2, $g(x) = 2^2 = 4$ and $g(x) = 3 \times 2 = 6$

Hence, element 2 of the domain of the relation g corresponds to two different images i.e., 4 and 6. Hence, this relation is not a function.

Question-2

If
$$f(x) = x^2$$
, find $\frac{f(1.1) - f(1)}{(1.1-1)}$

Ans.

$$f(x) = x^2$$

$$\therefore \frac{f(1.1) - f(1)}{(1.1 - 1)} = \frac{(1.1)^2 - (1)^2}{(1.1 - 1)} = \frac{1.21 - 1}{0.1} = \frac{0.21}{0.1} = 2.1$$

Ouestion-3

Find the domain of the function $f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$

Ans.

The given function is $f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$.

$$f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12} = \frac{x^2 + 2x + 1}{(x - 6)(x - 2)}$$

It can be seen that function f is defined for all real numbers except at x = 6 and x = 2.

Hence, the domain of f is $\mathbf{R} - \{2, 6\}$.

Question-4

Find the domain and the range of the real function f defined by $f(x) = \sqrt{(x-1)}$.

Ans.

The given real function is $f(x) = \sqrt{x-1}$

It can be seen that $\sqrt{x-1}$ is defined for $(x-1) \ge 0$.

i.e.,
$$f(x) = \sqrt{(x-1)}$$
 is defined for $x \ge 1$.

Therefore, the domain of f is the set of all real numbers greater than or equal to 1 i.e., the domain of $f = [1, \infty)$.

As
$$x \ge 1 \Rightarrow (x-1) \ge 0 \Rightarrow \sqrt{x-1} \ge 0$$

Therefore, the range of f is the set of all real numbers greater than or equal to 0 i.e., the range of $f = [0, \infty)$.

Question-5

Find the domain and the range of the real function f defined by f(x) = |x - 1|.

Ans.

The given real function is f(x) = |x - 1|.

It is clear that |x-1| is defined for all real numbers.

 \therefore Domain of $f = \mathbf{R}$

Also, for $x \in \mathbb{R}$, |x-1| assumes all real numbers.

Hence, the range of f is the set of all non-negative real numbers

Question-6

Let
$$f = \left\{ \left(x, \frac{x^2}{1 + x^2} \right) : x \in \mathbf{R} \right\}$$
 be a function from \mathbf{R} into \mathbf{R} . Determine the range of f .

Ans.

Let
$$f = \left\{ \left(x, \frac{x^2}{1+x^2} \right) : x \in \mathbf{R} \right\}$$
 be a function from \mathbf{R} into \mathbf{R} . Determine the range of f .

Ouestion-7

Let $f,g:\mathbf{R}\to\mathbf{R}$ be defined, respectively by f(x)=x+1, g(x)=2x-3. Find f+g, f-g and $\frac{f}{g}$.

Ans.

$$f, g: \mathbf{R} \to \mathbf{R}$$
 is defined as $f(x) = x + 1$, $g(x) = 2x - 3$

$$(f+g)(x) = f(x) + g(x) = (x+1) + (2x-3) = 3x-2$$

$$\therefore (f+g)(x) = 3x - 2$$

$$(f-g)(x) = f(x) - g(x) = (x+1) - (2x-3) = x+1-2x+3 = -x+4$$

$$\therefore (f-g)(x) = -x + 4$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0, x \in \mathbf{R}$$

$$\therefore \left(\frac{f}{g}\right)(x) = \frac{x+1}{2x-3}, \ 2x-3 \neq 0 \text{ or } 2x \neq 3$$

$$\left(\frac{f}{g}\right)(x) = \frac{x+1}{2x-3}, x \neq \frac{3}{2}$$

Question-8

Let $f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$ be a function from \mathbf{Z} to \mathbf{Z} defined by f(x) = ax + b, for some integers a, b. Determine a, b.

Ans

$$f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$$

$$f(x) = ax + b$$

$$(1,1)\in f$$

$$\Rightarrow f(1) = 1$$

$$\Rightarrow a \times 1 + b = 1$$

$$\Rightarrow a + b = 1$$

$$(0,-1)\in f$$

$$\Rightarrow f(0) = -1$$

$$\Rightarrow a \times 0 + b = -1$$

$$\Rightarrow b = -1$$

On substituting b = -1 in a + b = 1, we obtain $a + (-1) = 1 \Rightarrow a = 1 + 1 = 2$.

Thus, the respective values of a and b are 2 and -1.

Question-9

Let R be a relation from **N** to **N** defined by R = $\{(a, b): a, b \in \mathbf{N} \text{ and } a = b^2\}$. Are the following true?

(i)
$$(a, a) \in \mathbb{R}$$
, for all $a \in \mathbb{N}$ (ii) $(a, b) \in \mathbb{R}$, implies $(b, a) \in \mathbb{R}$

(iii)
$$(a, b) \in R$$
, $(b, c) \in R$ implies $(a, c) \in R$.

Justify your answer in each case.

Ans.

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R = \{(a, b): a, b \in \mathbb{N} \text{ and } a = b^2\}
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(i) It can be seen that $2 \in \mathbb{N}$; however, $2 \neq 2^2 = 4$.

Therefore, the statement " $(a, a) \in \mathbb{R}$, for all $a \in \mathbb{N}$ " is not true.

(ii) It can be seen that $(9, 3) \in \mathbb{N}$ because $9, 3 \in \mathbb{N}$ and $9 = 3^2$.

Now, $3 \neq 9^2 = 81$; therefore, $(3, 9) \notin \mathbf{N}$

Therefore, the statement " $(a, b) \in \mathbb{R}$, implies $(b, a) \in \mathbb{R}$ " is not true.

(iii) It can be seen that $(9, 3) \in R$, $(16, 4) \in R$ because 9, 3, 16, $4 \in N$ and $9 = 3^2$ and $16 = 4^2$.

Now, $9 \neq 4^2 = 16$; therefore, $(9, 4) \notin \mathbf{N}$

Therefore, the statement " $(a, b) \in R$, $(b, c) \in R$ implies $(a, c) \in R$ " is not true

Question-10

Let $A = \{1, 2, 3, 4\}$, $B = \{1, 5, 9, 11, 15, 16\}$ and $f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$. Are the following true?

(i) f is a relation from A to B (ii) f is a function from A to B.

Justify your answer in each case

Ans.

 $A = \{1, 2, 3, 4\}$ and $B = \{1, 5, 9, 11, 15, 16\}$

 $\begin{array}{l} :: A \times B = \{(1,1),(1,5),(1,9),(1,11),(1,15),(1,16),(2,1),(2,5),(2,9),(2,11),(2,15),(2,16),(3,1),(3,5),(3,9),(3,11),(3,15),(3,16),(4,1),(4,5),(4,9),(4,11),(4,15),(4,16)\} \end{array}$

It is given that $f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$

(i) A relation from a non-empty set A to a non-empty set B is a subset of the Cartesian product A \times B.

It is observed that f is a subset of $A \times B$.

Thus, f is a relation from A to B

(ii) Since the same first element i.e., 2 corresponds to two different images i.e., 9 and 11, relation fis not a function.

Question-11

Let f be the subset of $\mathbf{Z} \times \mathbf{Z}$ defined by $f = \{(ab, a + b): a, b \in \mathbf{Z}\}$. Is f a function from \mathbf{Z} to \mathbf{Z} : justify your answer

Ans.

The relation f is defined as $f = \{(ab, a+b): a, b \in \mathbb{Z}\}$

We know that a relation f from a set A to a set B is said to be a function if every element of set A has unique images in set B.

Since 2, 6,
$$-2$$
, $-6 \in \mathbb{Z}$, $(2 \times 6, 2 + 6)$, $(-2 \times -6, -2 + (-6)) \in f$

i.e., (12, 8), (12,
$$-8$$
) $\in f$

It can be seen that the same first element i.e., 12 corresponds to two different images i.e., 8 and -8. Thus, relation f is not a function.

Question-12

Let $A = \{9, 10, 11, 12, 13\}$ and let $f: A \rightarrow \mathbf{N}$ be defined by f(n) = the highest prime factor of n. Find the range of f.

Ans.

$$A = \{9, 10, 11, 12, 13\}$$

 $f: A \rightarrow N$ is defined as

f(n) = The highest prime factor of n

Prime factor of 9 = 3

Prime factors of 10 = 2, 5

Prime factor of 11 = 11

Prime factors of 12 = 2, 3

Prime factor of 13 = 13

 $\therefore f(9) =$ The highest prime factor of 9 = 3

f(10) = The highest prime factor of 10 = 5

f(11) = The highest prime factor of 11 = 11

f(12) = The highest prime factor of 12 = 3

f(13) = The highest prime factor of 13 = 13

The range of f is the set of all f(n), where $n \in A$.

:.Range of $f = \{3, 5, 11, 13\}$

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