



$$\Rightarrow \lambda = \frac{4}{6} = \frac{2}{3}$$

Hence, point B divides AC in the ratio 2:3.

Question 9:

Find the position vector of a point R which divides the line joining two points P and Q

whose position vectors are $(2\vec{a} + \vec{b})$ and $(\vec{a} - 3\vec{b})$ externally in the ratio 1: 2. Also, show that P is the mid point of the line segment RQ.

Answer

It is given that $\vec{OP} = 2\vec{a} + \vec{b}$, $\vec{OQ} = \vec{a} - 3\vec{b}$.

It is given that point R divides a line segment joining two points P and Q externally in the ratio 1: 2. Then, on using the section formula, we get:

$$\vec{OR} = \frac{2(2\vec{a} + \vec{b}) - (\vec{a} - 3\vec{b})}{2-1} = \frac{4\vec{a} + 2\vec{b} - \vec{a} + 3\vec{b}}{1} = 3\vec{a} + 5\vec{b}$$

Therefore, the position vector of point R is $3\vec{a} + 5\vec{b}$.

$$\text{Position vector of the mid-point of RQ} = \frac{\vec{OQ} + \vec{OR}}{2}$$

$$= \frac{(\vec{a} - 3\vec{b}) + (3\vec{a} + 5\vec{b})}{2}$$

$$= 2\vec{a} + \vec{b}$$

$$= \vec{OP}$$

Hence, P is the mid-point of the line segment RQ.

Question 10:

The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$.

Find the unit vector parallel to its diagonal. Also, find its area.

Answer

Adjacent sides of a parallelogram are given as: $\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} - 3\hat{k}$

Then, the diagonal of a parallelogram is given by $\vec{a} + \vec{b}$.

$$\vec{a} + \vec{b} = (2+1)\hat{i} + (-4-2)\hat{j} + (5-3)\hat{k} = 3\hat{i} - 6\hat{j} + 2\hat{k}$$

Thus, the unit vector parallel to the diagonal is

$$\frac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|} = \frac{3\hat{i} - 6\hat{j} + 2\hat{k}}{\sqrt{3^2 + (-6)^2 + 2^2}} = \frac{3\hat{i} - 6\hat{j} + 2\hat{k}}{\sqrt{9+36+4}} = \frac{3\hat{i} - 6\hat{j} + 2\hat{k}}{7} = \frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k}.$$

$$\therefore \text{Area of parallelogram ABCD} = |\vec{a} \times \vec{b}|$$

$$\begin{aligned} \vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 5 \\ 1 & -2 & -3 \end{vmatrix} \\ &= \hat{i}(12+10) - \hat{j}(-6-5) + \hat{k}(-4+4) \\ &= 22\hat{i} + 11\hat{j} \\ &= 11(2\hat{i} + \hat{j}) \end{aligned}$$

$$\therefore |\vec{a} \times \vec{b}| = 11\sqrt{2^2 + 1^2} = 11\sqrt{5}$$

Hence, the area of the parallelogram is $11\sqrt{5}$ square units.

Question 11:

Show that the direction cosines of a vector equally inclined to the axes OX, OY and OZ

$$\text{are } \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}.$$

Answer

Let a vector be equally inclined to axes OX, OY, and OZ at angle α .

Then, the direction cosines of the vector are $\cos \alpha$, $\cos \alpha$, and $\cos \alpha$.

Now,

$$\cos^2 \alpha + \cos^2 \alpha + \cos^2 \alpha = 1$$

$$\Rightarrow 3\cos^2 \alpha = 1$$

$$\Rightarrow \cos \alpha = \frac{1}{\sqrt{3}}$$

Hence, the direction cosines of the vector which are equally inclined to the axes

$$\text{are } \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}.$$

Question 12:

Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector \vec{d} which is perpendicular to both \vec{a} and \vec{b} , and $\vec{c} \cdot \vec{d} = 15$.

Answer

$$\text{Let } \vec{d} = d_1\hat{i} + d_2\hat{j} + d_3\hat{k}.$$

Since \vec{d} is perpendicular to both \vec{a} and \vec{b} , we have:

$$\vec{d} \cdot \vec{a} = 0$$

$$\Rightarrow d_1 + 4d_2 + 2d_3 = 0 \quad \dots(i)$$

And,

$$\vec{d} \cdot \vec{b} = 0$$

$$\Rightarrow 3d_1 - 2d_2 + 7d_3 = 0 \quad \dots(ii)$$

Also, it is given that:

$$\vec{c} \cdot \vec{d} = 15$$

$$\Rightarrow 2d_1 - d_2 + 4d_3 = 15 \quad \dots(iii)$$

On solving (i), (ii), and (iii), we get:

$$d_1 = \frac{160}{3}, d_2 = -\frac{5}{3} \text{ and } d_3 = -\frac{70}{3}$$

$$\therefore \vec{d} = \frac{160}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{70}{3}\hat{k} = \frac{1}{3}(160\hat{i} - 5\hat{j} - 70\hat{k})$$

$$\text{Hence, the required vector is } \frac{1}{3}(160\hat{i} - 5\hat{j} - 70\hat{k}).$$

Question 13:

The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ .

Answer

$$\begin{aligned} & (2\hat{i} + 4\hat{j} - 5\hat{k}) + (\lambda\hat{i} + 2\hat{j} + 3\hat{k}) \\ &= (2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k} \end{aligned}$$

Therefore, unit vector along $(2\hat{i} + 4\hat{j} - 5\hat{k}) + (\lambda\hat{i} + 2\hat{j} + 3\hat{k})$ is given as:

$$\frac{(2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2 + \lambda)^2 + 6^2 + (-2)^2}} = \frac{(2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{4 + 4\lambda + \lambda^2 + 36 + 4}} = \frac{(2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{\lambda^2 + 4\lambda + 44}}$$

Scalar product of $(\hat{i} + \hat{j} + \hat{k})$ with this unit vector is 1.

$$\Rightarrow (\hat{i} + \hat{j} + \hat{k}) \cdot \frac{(2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{\lambda^2 + 4\lambda + 44}} = 1$$

$$\Rightarrow \frac{(2 + \lambda) + 6 - 2}{\sqrt{\lambda^2 + 4\lambda + 44}} = 1$$

$$\Rightarrow \sqrt{\lambda^2 + 4\lambda + 44} = \lambda + 6$$

$$\Rightarrow \lambda^2 + 4\lambda + 44 = (\lambda + 6)^2$$

$$\Rightarrow \lambda^2 + 4\lambda + 44 = \lambda^2 + 12\lambda + 36$$

$$\Rightarrow 8\lambda = 8$$

$$\Rightarrow \lambda = 1$$

Hence, the value of λ is 1.

Question 14:

If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to \vec{a}, \vec{b} and \vec{c} .

Answer

Since \vec{a}, \vec{b} , and \vec{c} are mutually perpendicular vectors, we have

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0.$$

It is given that:

$$|\vec{a}| = |\vec{b}| = |\vec{c}|$$

Let vector $\vec{a} + \vec{b} + \vec{c}$ be inclined to \vec{a}, \vec{b} , and \vec{c} at angles θ_1, θ_2 , and θ_3 respectively.

Then, we have:

$$\begin{aligned}\cos\theta_1 &= \frac{(\vec{a} + \vec{b} + \vec{c}) \cdot \vec{a}}{|\vec{a} + \vec{b} + \vec{c}| |\vec{a}|} = \frac{\vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{a} + \vec{c} \cdot \vec{a}}{|\vec{a} + \vec{b} + \vec{c}| |\vec{a}|} \\ &= \frac{|\vec{a}|^2}{|\vec{a} + \vec{b} + \vec{c}| |\vec{a}|} \quad [\vec{b} \cdot \vec{a} = \vec{c} \cdot \vec{a} = 0] \\ &= \frac{|\vec{a}|}{|\vec{a} + \vec{b} + \vec{c}|}\end{aligned}$$

$$\begin{aligned}\cos\theta_2 &= \frac{(\vec{a} + \vec{b} + \vec{c}) \cdot \vec{b}}{|\vec{a} + \vec{b} + \vec{c}| |\vec{b}|} = \frac{\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{b}}{|\vec{a} + \vec{b} + \vec{c}| |\vec{b}|} \\ &= \frac{|\vec{b}|^2}{|\vec{a} + \vec{b} + \vec{c}| |\vec{b}|} \quad [\vec{a} \cdot \vec{b} = \vec{c} \cdot \vec{b} = 0] \\ &= \frac{|\vec{b}|}{|\vec{a} + \vec{b} + \vec{c}|}\end{aligned}$$

$$\begin{aligned}\cos\theta_3 &= \frac{(\vec{a} + \vec{b} + \vec{c}) \cdot \vec{c}}{|\vec{a} + \vec{b} + \vec{c}| |\vec{c}|} = \frac{\vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{c}}{|\vec{a} + \vec{b} + \vec{c}| |\vec{c}|} \\ &= \frac{|\vec{c}|^2}{|\vec{a} + \vec{b} + \vec{c}| |\vec{c}|} \quad [\vec{a} \cdot \vec{c} = \vec{b} \cdot \vec{c} = 0] \\ &= \frac{|\vec{c}|}{|\vec{a} + \vec{b} + \vec{c}|}\end{aligned}$$

Now, as $|\vec{a}| = |\vec{b}| = |\vec{c}|$, $\cos\theta_1 = \cos\theta_2 = \cos\theta_3$.

$$\therefore \theta_1 = \theta_2 = \theta_3$$

Hence, the vector $(\vec{a} + \vec{b} + \vec{c})$ is equally inclined to \vec{a}, \vec{b} , and \vec{c} .

Question 15:

Prove that $(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = |\vec{a}|^2 + |\vec{b}|^2$, if and only if \vec{a}, \vec{b} are perpendicular,

given $\vec{a} \neq \vec{0}, \vec{b} \neq \vec{0}$.

Answer

$$\begin{aligned}(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) &= |\vec{a}|^2 + |\vec{b}|^2 \\ \Leftrightarrow \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} &= |\vec{a}|^2 + |\vec{b}|^2 \quad [\text{Distributivity of scalar products over addition}] \\ \Leftrightarrow |\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2 &= |\vec{a}|^2 + |\vec{b}|^2 \quad [\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} \text{ (Scalar product is commutative)}] \\ \Leftrightarrow 2\vec{a} \cdot \vec{b} &= 0 \\ \Leftrightarrow \vec{a} \cdot \vec{b} &= 0 \\ \therefore \vec{a} \text{ and } \vec{b} &\text{ are perpendicular.} \quad [\vec{a} \neq \vec{0}, \vec{b} \neq \vec{0} \text{ (Given)}]\end{aligned}$$

Question 16:

If θ is the angle between two vectors \vec{a} and \vec{b} , then $\vec{a} \cdot \vec{b} \geq 0$ only when

$$(A) \quad 0 < \theta < \frac{\pi}{2} \quad (B) \quad 0 \leq \theta \leq \frac{\pi}{2}$$

$$(C) \quad 0 < \theta < \pi \quad (D) \quad 0 \leq \theta \leq \pi$$

Answer

Let θ be the angle between two vectors \vec{a} and \vec{b} .

Then, without loss of generality, \vec{a} and \vec{b} are non-zero vectors so

that $|\vec{a}|$ and $|\vec{b}|$ are positive.

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