Geometric Progressions Ex 20.3 Q 4

$5+55+555+\ldots n$ terms

Taking 5 common from each term.

$$5[1+11+111+\ldots n \text{ terms}]$$

Dividing and multiplying by 9

$$=\frac{5}{9}[9+99+999+\ldots n \text{ terms}]$$

$$=\frac{5}{9}\left[(10-1)+\left(10^2-1\right)+\left(10^3-1\right)+\ldots n \text{ terms}\right]$$

$$=\frac{5}{9}\left[\left(10+10^2+10^3+\ldots n \text{ terms}\right)-n\right] \text{ this is G.P.}$$

So, $S_n=\dfrac{a\left(r^n-1\right)}{r-1}$

$a=10,\ r=10,\ n=n$

$$=\frac{5}{9}\left[\frac{10\left(10^n-1\right)}{10-1}-n\right]$$

$$=\frac{5}{9\times 9}\left(10^{n+1}-10-9n\right)$$

$$=\frac{5}{81}\left(10^{n+1}-9n-10\right)$$

Now we have

$$7+77+777+\cdots \text{ to } n \text{ terms}=7[1+11+111+\cdots \text{ to } n \text{ terms}]$$

$$=\frac{7}{9}[9+99+999+\cdots \text{ to } n \text{ terms}]$$

$$=\frac{7}{9}\left[(10-1)+\left(10^2-1\right)+\left(10^3-1\right)+\cdots \text{ to } n \text{ terms}\right]$$

$$=\frac{7}{9}\left[10+10^2+10^3+\cdots \text{ to } n \text{ terms}\right]-\frac{7}{9}(1+1+1+\cdots\cdots \text{ to } n \text{ terms})$$

$$=\frac{7}{9}\cdot\frac{10\left(10^n-1\right)}{10-1}-\frac{7n}{9}$$

$$=\frac{7}{81}\left(10^{n+1}-9n-10\right)$$

$9+99+999+\ldots n$ term

This can be written as

$$=(10-1)+(100-1)+(1000-1)+\ldots n \text{ term}$$

$$=\left(10+10^2+10^3+\ldots n \text{ term}\right)-n$$

$$\Rightarrow \quad S_n=\frac{a\left(r^n-1\right)}{r-1},\ a=10,\ r=10,\ n=n$$

$$=\frac{10\left(10^n-1\right)}{10-1}-n$$

$$=\frac{10}{9}\left(10^n-1\right)-n$$

$$=\frac{1}{9}\left[10^{n+1}-10-9n\right]$$

$$=\frac{1}{9}\left[10^{n+1}-9n-10\right]$$

$0.5 + 0.55 + 0.555 + \&..$ to n

$=5\times0.1+5\times0.11+5\times0.111+.....$

$$=\frac{5}{9}\left\{\frac{9}{10}+\frac{99}{100}+\frac{999}{1000}+.....+-\right\}$$

$$=\frac{5}{9}\{(1-\frac{1}{10})+(1-\frac{1}{100})+.....+\ \}$$

$$=\frac{5}{9}\left\{n-\left(\frac{1}{10}+\frac{1}{10^2}+.....+\frac{1}{10^n}\right)\right\}$$

$$=\frac{5}{9}\left[n-\frac{1}{10}\frac{\left\{1-(\frac{1}{10})^n\right\}}{\left(1-\frac{1}{10}\right)}\right]$$

$$=\frac{5}{9}\left[n-\frac{1}{9}(1-\frac{1}{10^n})\right]$$

$0.6+ 0.66 + 0.666 + \&..$ to n

$=6\times0.1+6\times0.11+6\times0.111+.....$

$$=\frac{6}{9}\left\{\frac{9}{10}+\frac{99}{100}+\frac{999}{1000}+.....+-\right\}$$

$$=\frac{6}{9}\{(1-\frac{1}{10})+(1-\frac{1}{100})+.....+\ \}$$

$$=\frac{6}{9}\left\{n-\left(\frac{1}{10}+\frac{1}{10^2}+.....+\frac{1}{10^n}\right)\right\}$$

$$=\frac{6}{9}\left[n-\frac{1}{10}\frac{\left\{1-(\frac{1}{10})^n\right\}}{\left(1-\frac{1}{10}\right)}\right]$$

$$=\frac{6}{9}\left[n-\frac{1}{9}(1-\frac{1}{10^n})\right]$$

********** END **********