Trigonometric Identities Ex 6.1 Q73

Answer :

We have to prove $\tan^2 A\sec^2 B-\sec^2 A\tan^2 B=\tan^2 A-\tan^2 B$

We know that, $\sec^2 A-\tan^2 A=1$

So,

$$\begin{aligned}\tan^2 A\sec^2 B-\sec^2 A\tan^2 B&=\tan^2 A(1+\tan^2 B)-(1+\tan^2 A)\tan^2 B\\&=\tan^2 A+\tan^2 A\tan^2 B-\tan^2 B-\tan^2 A\tan^2 B\\&=\tan^2 A-\tan^2 B\end{aligned}$$

Hence proved.

Trigonometric Identities Ex 6.1 Q74

Answer :

Given that,

$x=a\sec\theta+b\tan\theta,$

$y=a\tan\theta+b\sec\theta$

We have to prove $x^2-y^2=a^2-b^2$

We know that, $\sec^2\theta-\tan^2\theta=1$

So,

$$\begin{aligned}&x^2-y^2\\&=(a\sec\theta+b\tan\theta)^2-(a\tan\theta+b\sec\theta)^2\\&=(a^2\sec^2\theta+2ab\sec\theta\tan\theta+b^2\tan^2\theta)-(a^2\tan^2\theta+2ab\sec\theta\tan\theta+b^2\sec^2\theta)\\&=a^2(\sec^2\theta-\tan^2\theta)-b^2(\sec^2\theta-\tan^2\theta)\\&=a^2-b^2\end{aligned}$$

Hence proved.

Trigonometric Identities Ex 6.1 Q75

Answer :

Given that,

$$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1 \qquad \ldots\ldots(1)$$

$$\frac{x}{a}\sin\theta-\frac{y}{b}\cos\theta=1 \qquad \ldots\ldots(2)$$

We have to prove $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$

We know that, $\sin^2\theta+\cos^2\theta=1$

Squaring and then adding the above two equations, we have

$$\begin{aligned}&\left(\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta\right)^2+\left(\frac{x}{a}\sin\theta-\frac{y}{b}\cos\theta\right)^2=1+1\\&\Rightarrow\left(\frac{x^2}{a^2}\cos^2\theta+2\frac{xy}{ab}\sin\theta\cos\theta+\frac{y^2}{b^2}\sin^2\theta\right)+\left(\frac{x^2}{a^2}\sin^2\theta-2\frac{xy}{ab}\sin\theta\cos\theta+\frac{y^2}{b^2}\cos^2\theta\right)=2\\&\Rightarrow\frac{x^2}{a^2}(\cos^2\theta+\sin^2\theta)+\frac{y^2}{b^2}(\sin^2\theta+\cos^2\theta)=2\\&\Rightarrow\frac{x^2}{a^2}+\frac{y^2}{b^2}=2\end{aligned}$$

*********** END **********