Half life of ${}^{90}_{38}\mathrm{Sr}$, $t_{1/2}$ = 28 years

= 28 × 365 × 24 × 60 × 60

= 8.83 × 10^8 s

Mass of the isotope, m = 15 mg

90 g of ${}^{90}_{38}\mathrm{Sr}$ atom contains 6.023 × 10^{23} (Avogadro's number) atoms.

Question 13.12:

Find the Q-value and the kinetic energy of the emitted α-particle in the α-decay of (a) ${}^{226}_{88}\mathrm{Ra}$ and (b) ${}^{220}_{86}\mathrm{Rn}$.

Given $m\left({}^{226}_{88}\mathrm{Ra}\right)$ = 226.02540 u, $m\left({}^{222}_{86}\mathrm{Rn}\right)$ = 222.01750 u,

$m\left({}^{220}_{86}\mathrm{Rn}\right)$ = 220.01137 u, $m\left({}^{216}_{84}\mathrm{Po}\right)$ = 216.00189 u.

Answer

(a) Alpha particle decay of ${}^{226}_{88}\mathrm{Ra}$ emits a helium nucleus. As a result, its mass number reduces to (226 − 4) 222 and its atomic number reduces to (88 − 2) 86. This is shown in the following nuclear reaction.

$${}^{226}_{88}\mathrm{Ra} \longrightarrow {}^{222}_{86}\mathrm{Ra} + {}^{4}_{2}\mathrm{He}$$

Q-value of

emitted α-particle = (Sum of initial mass − Sum of final mass) c^2

Where,

c = Speed of light

It is given that:

$$m\left({}^{226}_{88}\mathrm{Ra}\right) = 226.02540 \text{ u}$$

$$m\left({}^{222}_{86}\mathrm{Rn}\right) = 222.01750 \text{ u}$$

$$m\left({}^{4}_{2}\mathrm{He}\right) = 4.002603 \text{ u}$$

Q-value = [226.02540 − (222.01750 + 4.002603)] u c^2

= 0.005297 u c^2

But 1 u = 931.5 MeV/c^2

∴Q = 0.005297 × 931.5 ≈ 4.94 MeV

Kinetic energy of the α-particle $= \left(\dfrac{\text{Mass number after decay}}{\text{Mass number before decay}}\right) \times Q$

$$= \frac{222}{226} \times 4.94 = 4.85 \text{ MeV}$$

(b) Alpha particle decay of $\left({}^{220}_{86}\mathrm{Rn}\right)$ is shown by the following nuclear reaction.

$${}^{220}_{86}\mathrm{Rn} \longrightarrow {}^{216}_{84}\mathrm{Po} + {}^{4}_{2}\mathrm{He}$$

It is given that:

Mass of $\left({}^{220}_{86}\mathrm{Rn}\right)$ = 220.01137 u

Mass of $\left({}^{216}_{84}\mathrm{Po}\right)$ = 216.00189 u

∴Q-value = $\left[220.01137 - (216.00189 + 4.00260)\right] \times 931.5$

≈ 641 MeV

Kinetic energy of the α-particle $= \left(\dfrac{220-4}{220}\right) \times 6.41$

= 6.29 MeV

Question 13.13:

Question 13.13:

The radionuclide ^{11}C decays according to

$$^{11}_{6}\text{C} \rightarrow {}^{11}_{5}\text{B} + e^{+} + \nu: \quad T_{1/2} = 20.3 \text{ min}$$

The maximum energy of the emitted positron is 0.960 MeV.

Given the mass values:

$$m\left({}^{11}_{6}\text{C}\right) = 11.011434 \text{ u and } m\left({}^{11}_{6}\text{B}\right) = 11.009305 \text{ u},$$

calculate Q and compare it with the maximum energy of the positron emitted

Answer

The given nuclear reaction is:

$$^{11}_{6}\text{C} \rightarrow {}^{11}_{5}\text{B} + e^{+} + \nu$$

Half life of $^{11}_{6}$C nuclei, $T_{1/2} = 20.3$ min

Atomic mass of $m\left({}^{11}_{6}\text{C}\right)$ = 11.011434 u

Atomic mass of $m\left({}^{11}_{6}\text{B}\right) = 11.009305$ u

Maximum energy possessed by the emitted positron = 0.960 MeV

The change in the Q-value (ΔQ) of the nuclear masses of the $^{11}_{6}\text{C}$ nucleus is given as:

$$\Delta Q = \left[m'\left({}_{6}\text{C}^{11}\right) - \left[m'\left({}^{11}_{5}\text{B}\right) + m_e\right]\right]c^2 \qquad \ldots (1)$$

Where,

m_e = Mass of an electron or positron = 0.000548 u

c = Speed of light

m' = Respective nuclear masses

If atomic masses are used instead of nuclear masses, then we have to add 6 m_e in the case of ^{11}C and 5 m_e in the case of ^{11}B.

Hence, equation (1) reduces to:

$$\Delta Q = \left[m\left({}_{6}\text{C}^{11}\right) - m\left({}^{11}_{5}\text{B}\right) - 2m_e\right]c^2$$

Here, $m\left({}_{6}\text{C}^{11}\right)$ and $m\left({}^{11}_{5}\text{B}\right)$ are the atomic masses.

$\therefore \Delta Q$ = [11.011434 − 11.009305 − 2 × 0.000548] c^2

= (0.001033 c^2) u

But 1 u = 931.5 Mev/c^2

$\therefore \Delta Q$ = 0.001033 × 931.5 ≈ 0.962 MeV

The value of Q is almost comparable to the maximum energy of the emitted positron.

Question 13.14:

The nucleus $^{23}_{10}\text{Ne}$ decays by β^{-} emission. Write down the β decay equation and determine the maximum kinetic energy of the electrons emitted. Given that:

$m\left({}^{23}_{10}\text{Ne}\right)$ = 22.994466 u

$m\left({}^{23}_{11}\text{Na}\right)$ = 22.989770 u.

Answer

In β^{-} emission, the number of protons increases by 1, and one electron and an antineutrino are emitted from the parent nucleus.

β^{-} emission of the nucleus $^{23}_{10}\text{Ne}$ is given as:

$$^{23}_{10}\text{Ne} \rightarrow {}^{23}_{11}\text{Na} + e^{-} + \bar{\nu} + Q$$

It is given that:

Atomic mass of $m\left({}^{23}_{10}\text{Ne}\right)$ = 22.994466 u

Atomic mass of $m\left({}^{23}_{11}\text{Na}\right)$ = 22.989770 u

Mass of an electron, m_e = 0.000548 u

Q-value of the given reaction is given as:

$$Q = \left[m\left({}^{23}_{10}\text{Ne}\right) - \left[m\left({}^{23}_{11}\text{Na}\right) + m_e\right]\right]c^2$$

There are 10 electrons in $^{23}_{10}\text{Ne}$ and 11 electrons in $^{23}_{11}\text{Na}$. Hence, the mass of the electron is cancelled in the Q-value equation.

$$\therefore Q = [22.994466 - 22.989770]c^2$$

$$= \left(0.004696\ c^2\right) \text{ u}$$

But 1 u = 931.5 MeV/c^2

$\therefore Q = 0.004696 \times 931.5 = 4.374$ MeV

The daughter nucleus is too heavy as compared to e^- and $\bar{\nu}$. Hence, it carries negligible energy. The kinetic energy of the antineutrino is nearly zero. Hence, the maximum kinetic energy of the emitted electrons is almost equal to the *Q*-value, i.e., 4.374 MeV.

Question 13.15:

The *Q* value of a nuclear reaction *A* + *b* → *C* + *d* is defined by

$Q = [\ m_A + m_b - m_C - m_d]c^2$ where the masses refer to the respective nuclei. Determine from the given data the *Q*-value of the following reactions and state whether the reactions are exothermic or endothermic.

(i) ${}^1_1\mathrm{H} + {}^3_1\mathrm{H} \rightarrow {}^2_1\mathrm{H} + {}^2_1\mathrm{H}$

(ii) ${}^{12}_6\mathrm{C} + {}^{12}_6\mathrm{C} \rightarrow {}^{20}_{10}\mathrm{Ne} + {}^4_2\mathrm{He}$

Atomic masses are given to be

$m\left({}^2_1\mathrm{H}\right) = 2.014102$ u

$m\left({}^3_1\mathrm{H}\right) = 3.016049$ u

$m\left({}^{12}_6\mathrm{C}\right) = 12.000000$ u

$m\left({}^{20}_{10}\mathrm{Ne}\right) = 19.992439$ u

Answer

(i) The given nuclear reaction is:

${}_1\mathrm{H}^1 + {}^3_1\mathrm{H} \rightarrow {}^2_1\mathrm{H} + {}^2_1\mathrm{H}$

It is given that:

Atomic mass $m\left({}^1_1\mathrm{H}\right) = 1.007825$ u

Atomic mass $m\left({}^3_1\mathrm{H}\right) = 3.016049$ u

Atomic mass $m\left({}^2_1\mathrm{H}\right) = 2.014102$ u

According to the question, the *Q*-value of the reaction can be written as:

$$Q = \left[m\left({}^1_1\mathrm{H}\right) + m\left({}^3_1\mathrm{H}\right) - 2m\left({}^2_1\mathrm{H}\right)\right]c^2$$

$$= \left[1.007825 + 3.016049 - 2 \times 2.014102\right]c^2$$

$$Q = \left(-0.00433\ c^2\right)\ \mathrm{u}$$

But 1 u = 931.5 MeV/c^2

$\therefore Q = -0.00433 \times 931.5 = -4.0334$ MeV

The negativeQ-value of the reaction shows that the reaction is endothermic.

(ii) The given nuclear reaction is:

${}^{12}_6\mathrm{C} + {}^{12}_6\mathrm{C} \rightarrow {}^{20}_{10}\mathrm{Ne} + {}^4_2\mathrm{He}$

It is given that:

Atomic mass of $m\left({}^{12}_6\mathrm{C}\right) = 12.0$ u

Atomic mass of $m\left({}^{20}_{10}\mathrm{Ne}\right) = 19.992439$ u

Atomic mass of $m\left({}^4_2\mathrm{He}\right) = 4.002603$ u

The *Q*-value of this reaction is given as:

$$Q = \left[2m\left({}^{12}_6\mathrm{C}\right) - m\left({}^{20}_{10}\mathrm{Ne}\right) - m\left({}^4_2\mathrm{He}\right)\right]c^2$$

*********** END **********