Indefinite Integrals Ex 19.13 Q6

Let $I = \int \frac{x}{\sqrt{4 - x^4}} dx$

Let $x^2 = t$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad x\,dx = \frac{dt}{2}$

$I = \frac{1}{2}\int \frac{dt}{\sqrt{(2)^2 - t^2}}$

$= \frac{1}{2}\sin^{-1}\left(\frac{t}{2}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = \frac{1}{2}\sin^{-1}\left(\frac{x^2}{2}\right) + c$

Indefinite Integrals Ex 19.13 Q7

Let $I = \int \frac{1}{x\sqrt{4 - 9(\log x)^2}} dx$

Let $3\log x = t$

$\Rightarrow \quad \frac{3}{x} dx = dt$

$\Rightarrow \quad \frac{1}{x} dx = \frac{dt}{3}$

$I = \frac{1}{3}\int \frac{dt}{\sqrt{(2)^2 - t^2}}$

$= \frac{1}{3}\sin^{-1}\left(\frac{t}{2}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = \frac{1}{3}\sin^{-1}\left(\frac{3\log x}{2}\right) + c$

Indefinite Integrals Ex 19.13 Q8

Let $I = \int \frac{\sin 8x}{\sqrt{9 + (\sin 4x)^4}} dx$

Let $\sin^2 4x = t$

$\Rightarrow \quad 2\sin 4x.\cos 4x\,(4)dx = dt$

$\Rightarrow \quad 4\sin 8x\,dx = dt$

$\Rightarrow \quad \sin 8x\,dx = \frac{dt}{4}$

$I = \frac{1}{4}\int \frac{dt}{\sqrt{(3)^2 + t^2}}$

$= \frac{1}{4}\log\left|t + \sqrt{(3)^2 + t^2}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 + x^2}} dx = \log\left|x + \sqrt{a^2 + x^2}\right| + c\right]$

$I = \frac{1}{4}\log\left|\sin^2 4x + \sqrt{9 + \sin^4 4x}\right| + c$

Indefinite Integrals Ex 19.13 Q9

Let $I = \int \frac{\cos 2x}{\sqrt{\sin^2 2x + 8}} dx$

Let $\sin 2x = t$

$\Rightarrow \quad 2\cos 2x dx = dt$

$\Rightarrow \quad \cos 2x\, dx = \frac{dt}{2}$

$$I = \frac{1}{2}\int \frac{dt}{\sqrt{t^2 + \left(2\sqrt{2}\right)^2}}$$

$$= \frac{1}{2}\log\left|t + \sqrt{t^2 + \left(2\sqrt{2}\right)^2}\right| + c \qquad \left[\text{Since} \quad \int \frac{1}{\sqrt{a^2 + x^2}} dx = \log\left|x + \sqrt{x^2 + a^2}\right| + c\right]$$

$$I = \frac{1}{2}\log\left|\sin 2x + \sqrt{\sin^2 2x + 8}\right| + c$$

********** END **********