



## EXERCISE.8.1

Question-1

Expand the expression  $(1 - 2x)^5$

Ans.

By using Binomial Theorem, the expression  $(1 - 2x)^5$  can be expanded as

$$\begin{aligned}
 (1 - 2x)^5 &= {}^5C_0(1)^5 - {}^5C_1(1)^4(2x) + {}^5C_2(1)^3(2x)^2 - {}^5C_3(1)^2(2x)^3 + {}^5C_4(1)^1(2x)^4 - {}^5C_5(2x)^5 \\
 &= 1 - 5(2x) + 10(4x^2) - 10(8x^3) + 5(16x^4) - (32x^5) \\
 &= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5
 \end{aligned}$$

Question-2

Expand the expression  $\left(\frac{2}{x} - \frac{x}{2}\right)^5$

Ans.

By using Binomial Theorem, the expression  $\left(\frac{2}{x} - \frac{x}{2}\right)^5$  can be expanded as

$$\begin{aligned}
 \left(\frac{2}{x} - \frac{x}{2}\right)^5 &= {}^5C_0\left(\frac{2}{x}\right)^5 - {}^5C_1\left(\frac{2}{x}\right)^4\left(\frac{x}{2}\right) + {}^5C_2\left(\frac{2}{x}\right)^3\left(\frac{x}{2}\right)^2 \\
 &\quad - {}^5C_3\left(\frac{2}{x}\right)^2\left(\frac{x}{2}\right)^3 + {}^5C_4\left(\frac{2}{x}\right)\left(\frac{x}{2}\right)^4 - {}^5C_5\left(\frac{x}{2}\right)^5 \\
 &= \frac{32}{x^5} - 5\left(\frac{16}{x^4}\right)\left(\frac{x}{2}\right) + 10\left(\frac{8}{x^3}\right)\left(\frac{x^2}{4}\right) - 10\left(\frac{4}{x^2}\right)\left(\frac{x^3}{8}\right) + 5\left(\frac{2}{x}\right)\left(\frac{x^4}{16}\right) - \frac{x^5}{32} \\
 &= \frac{32}{x^5} - \frac{40}{x^3} + \frac{20}{x} - 5x + \frac{5}{8}x^3 - \frac{x^5}{32}
 \end{aligned}$$

Question-3

Expand the expression  $(2x - 3)^6$

Ans.

By using Binomial Theorem, the expression  $(2x - 3)^6$  can be expanded as

$$\begin{aligned}
 (2x-3)^6 &= {}^6C_0(2x)^6 - {}^6C_1(2x)^5(3) + {}^6C_2(2x)^4(3)^2 - {}^6C_3(2x)^3(3)^3 \\
 &\quad + {}^6C_4(2x)^2(3)^4 - {}^6C_5(2x)(3)^5 + {}^6C_6(3)^6 \\
 &= 64x^6 - 6(32x^5)(3) + 15(16x^4)(9) - 20(8x^3)(27) \\
 &\quad + 15(4x^2)(81) - 6(2x)(243) + 729 \\
 &= 64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729
 \end{aligned}$$

Question-4

Expand the expression  $\left(\frac{x}{3} + \frac{1}{x}\right)^5$

Ans.

By using Binomial Theorem, the expression  $\left(\frac{x}{3} + \frac{1}{x}\right)^5$  can be expanded as

$$\begin{aligned}
 \left(\frac{x}{3} + \frac{1}{x}\right)^5 &= {}^5C_0\left(\frac{x}{3}\right)^5 + {}^5C_1\left(\frac{x}{3}\right)^4\left(\frac{1}{x}\right) + {}^5C_2\left(\frac{x}{3}\right)^3\left(\frac{1}{x}\right)^2 \\
 &\quad + {}^5C_3\left(\frac{x}{3}\right)^2\left(\frac{1}{x}\right)^3 + {}^5C_4\left(\frac{x}{3}\right)\left(\frac{1}{x}\right)^4 + {}^5C_5\left(\frac{1}{x}\right)^5 \\
 &= \frac{x^5}{243} + 5\left(\frac{x^4}{81}\right)\left(\frac{1}{x}\right) + 10\left(\frac{x^3}{27}\right)\left(\frac{1}{x^2}\right) + 10\left(\frac{x^2}{9}\right)\left(\frac{1}{x^3}\right) + 5\left(\frac{x}{3}\right)\left(\frac{1}{x^4}\right) + \frac{1}{x^5} \\
 &= \frac{x^5}{243} + \frac{5x^3}{81} + \frac{10x}{27} + \frac{10}{9x} + \frac{5}{3x^3} + \frac{1}{x^5}
 \end{aligned}$$

Question-5

Expand  $\left(x + \frac{1}{x}\right)^6$

Ans.

By using Binomial Theorem, the expression  $\left(x + \frac{1}{x}\right)^6$  can be expanded as

$$\begin{aligned}\left(x + \frac{1}{x}\right)^6 &= {}^6C_0(x)^6 + {}^6C_1(x)^5\left(\frac{1}{x}\right) + {}^6C_2(x)^4\left(\frac{1}{x}\right)^2 \\ &\quad + {}^6C_3(x)^3\left(\frac{1}{x}\right)^3 + {}^6C_4(x)^2\left(\frac{1}{x}\right)^4 + {}^6C_5(x)\left(\frac{1}{x}\right)^5 + {}^6C_6\left(\frac{1}{x}\right)^6 \\ &= x^6 + 6(x)^5\left(\frac{1}{x}\right) + 15(x)^4\left(\frac{1}{x^2}\right) + 20(x)^3\left(\frac{1}{x^3}\right) + 15(x)^2\left(\frac{1}{x^4}\right) + 6(x)\left(\frac{1}{x^5}\right) + \frac{1}{x^6} \\ &= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}\end{aligned}$$

Question-6

Using Binomial Theorem, evaluate  $(96)^3$

Ans.

96 can be expressed as the sum or difference of two numbers whose powers are easier to calculate and then, binomial theorem can be applied.

It can be written that,  $96 = 100 - 4$

$$\begin{aligned}\therefore (96)^3 &= (100 - 4)^3 \\ &= {}^3C_0(100)^3 - {}^3C_1(100)^2(4) + {}^3C_2(100)(4)^2 - {}^3C_3(4)^3 \\ &= (100)^3 - 3(100)^2(4) + 3(100)(4)^2 - (4)^3 \\ &= 1000000 - 120000 + 4800 - 64 \\ &= 884736\end{aligned}$$

Question-7

Using Binomial Theorem, evaluate  $(102)^5$

Ans.

102 can be expressed as the sum or difference of two numbers whose powers are easier to calculate and then, Binomial Theorem can be applied.

It can be written that,  $102 = 100 + 2$

$$\begin{aligned}\therefore (102)^5 &= (100 + 2)^5 \\ &= {}^5C_0(100)^5 + {}^5C_1(100)^4(2) + {}^5C_2(100)^3(2)^2 + {}^5C_3(100)^2(2)^3 \\ &\quad + {}^5C_4(100)(2)^4 + {}^5C_5(2)^5 \\ &= (100)^5 + 5(100)^4(2) + 10(100)^3(2)^2 + 10(100)^2(2)^3 + 5(100)(2)^4 + (2)^5 \\ &= 10000000000 + 1000000000 + 40000000 + 800000 + 8000 + 32 \\ &= 11040808032\end{aligned}$$

Question-8

Using Binomial Theorem, evaluate  $(101)^4$

Ans.

101 can be expressed as the sum or difference of two numbers whose powers are easier to calculate and then, Binomial Theorem can be applied.

It can be written that,  $101 = 100 + 1$

$$\begin{aligned}\therefore (101)^4 &= (100+1)^4 \\ &= {}^4C_0(100)^4 + {}^4C_1(100)^3(1) + {}^4C_2(100)^2(1)^2 + {}^4C_3(100)(1)^3 + {}^4C_4(1)^4 \\ &= (100)^4 + 4(100)^3 + 6(100)^2 + 4(100) + (1)^4 \\ &= 100000000 + 4000000 + 60000 + 400 + 1 \\ &= 104060401\end{aligned}$$

Question-9

Using Binomial Theorem, evaluate  $(99)^5$

Ans.

99 can be written as the sum or difference of two numbers whose powers are easier to calculate and then, Binomial Theorem can be applied.

It can be written that,  $99 = 100 - 1$

$$\begin{aligned}\therefore (99)^5 &= (100-1)^5 \\ &= {}^5C_0(100)^5 - {}^5C_1(100)^4(1) + {}^5C_2(100)^3(1)^2 - {}^5C_3(100)^2(1)^3 \\ &\quad + {}^5C_4(100)(1)^4 - {}^5C_5(1)^5 \\ &= (100)^5 - 5(100)^4 + 10(100)^3 - 10(100)^2 + 5(100) - 1 \\ &= 10000000000 - 5000000000 + 100000000 - 1000000 + 500 - 1 \\ &= 10010000500 - 500100001 \\ &= 9509900499\end{aligned}$$

Question-10

Using Binomial Theorem, indicate which number is larger  $(1.1)^{10000}$  or 1000.

Ans.

By splitting 1.1 and then applying Binomial Theorem, the first few terms of  $(1.1)^{10000}$  can be obtained as

$$\begin{aligned}(1.1)^{10000} &= (1+0.1)^{10000} \\ &= {}^{10000}C_0 + {}^{10000}C_1(1.1) + \text{Other positive terms} \\ &= 1 + 10000 \times 1.1 + \text{Other positive terms} \\ &= 1 + 11000 + \text{Other positive terms} \\ &> 1000\end{aligned}$$

Hence,  $(1.1)^{10000} > 1000$

Question-11

Find  $(a+b)^4 - (a-b)^4$ . Hence, evaluate  $(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4$ .

Ans.

Using Binomial Theorem, the expressions,  $(a + b)^4$  and  $(a - b)^4$ , can be expanded as

$$\begin{aligned}(a+b)^4 &= {}^4C_0a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_3ab^3 + {}^4C_4b^4 \\(a-b)^4 &= {}^4C_0a^4 - {}^4C_1a^3b + {}^4C_2a^2b^2 - {}^4C_3ab^3 + {}^4C_4b^4 \\ \therefore (a+b)^4 - (a-b)^4 &= {}^4C_0a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_3ab^3 + {}^4C_4b^4 \\ &\quad - [{}^4C_0a^4 - {}^4C_1a^3b + {}^4C_2a^2b^2 - {}^4C_3ab^3 + {}^4C_4b^4] \\ &= 2({}^4C_1a^3b + {}^4C_3ab^3) = 2(4a^3b + 4ab^3) \\ &= 8ab(a^2 + b^2)\end{aligned}$$

By putting  $a = \sqrt{3}$  and  $b = \sqrt{2}$ , we obtain

$$\begin{aligned}(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4 &= 8(\sqrt{3})(\sqrt{2})\{(\sqrt{3})^2 + (\sqrt{2})^2\} \\ &= 8(\sqrt{6})\{3 + 2\} = 40\sqrt{6}\end{aligned}$$

#### Question-12

Find  $(x + 1)^6 + (x - 1)^6$ . Hence or otherwise evaluate  $(\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6$ .

Ans.

Using Binomial Theorem, the expressions,  $(x + 1)^6$  and  $(x - 1)^6$ , can be expanded as

$$\begin{aligned}(x+1)^6 &= {}^6C_0x^6 + {}^6C_1x^5 + {}^6C_2x^4 + {}^6C_3x^3 + {}^6C_4x^2 + {}^6C_5x + {}^6C_6 \\(x-1)^6 &= {}^6C_0x^6 - {}^6C_1x^5 + {}^6C_2x^4 - {}^6C_3x^3 + {}^6C_4x^2 - {}^6C_5x + {}^6C_6 \\ \therefore (x+1)^6 + (x-1)^6 &= 2[{}^6C_0x^6 + {}^6C_2x^4 + {}^6C_4x^2 + {}^6C_6] \\ &= 2[x^6 + 15x^4 + 15x^2 + 1]\end{aligned}$$

By putting  $x = \sqrt{2}$ , we obtain

$$\begin{aligned}(\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6 &= 2\left[(\sqrt{2})^6 + 15(\sqrt{2})^4 + 15(\sqrt{2})^2 + 1\right] \\ &= 2(8 + 15 \times 4 + 15 \times 2 + 1) \\ &= 2(8 + 60 + 30 + 1) \\ &= 2(99) = 198\end{aligned}$$

#### Question-13

Show that  $9^{n+1} - 8n - 9$  is divisible by 64, whenever  $n$  is a positive integer.

Ans.

In order to show that  $9^{n+1} - 8n - 9$  is divisible by 64, it has to be proved that,

$$9^{n+1} - 8n - 9 = 64k, \text{ where } k \text{ is some natural number}$$

By Binomial Theorem,

$$(1+a)^m = {}^mC_0 + {}^mC_1a + {}^mC_2a^2 + \dots + {}^mC_ma^m$$

For  $a = 8$  and  $m = n + 1$ , we obtain

$$\begin{aligned}(1+8)^{n+1} &= {}^{n+1}C_0 + {}^{n+1}C_1(8) + {}^{n+1}C_2(8)^2 + \dots + {}^{n+1}C_{n+1}(8)^{n+1} \\ \Rightarrow 9^{n+1} &= 1 + (n+1)(8) + 8^2 \left[ {}^{n+1}C_2 + {}^{n+1}C_3 \times 8 + \dots + {}^{n+1}C_{n+1}(8)^{n-1} \right] \\ \Rightarrow 9^{n+1} &= 9 + 8n + 64 \left[ {}^{n+1}C_2 + {}^{n+1}C_3 \times 8 + \dots + {}^{n+1}C_{n+1}(8)^{n-1} \right] \\ \Rightarrow 9^{n+1} - 8n - 9 &= 64k, \text{ where } k = {}^{n+1}C_2 + {}^{n+1}C_3 \times 8 + \dots + {}^{n+1}C_{n+1}(8)^{n-1} \text{ is a natural number}\end{aligned}$$

Thus,  $9^{n+1} - 8n - 9$  is divisible by 64, whenever  $n$  is a positive integer.

#### Question-14

Prove that  $\sum_{r=0}^n 3^r {}^nC_r = 4^n$ .

Ans.

By Binomial Theorem,

$$\sum_{r=0}^n {}^nC_r a^{n-r} b^r = (a+b)^n$$

By putting  $b = 3$  and  $a = 1$  in the above equation, we obtain

$$\begin{aligned} \sum_{r=0}^n {}^nC_r (1)^{n-r} (3)^r &= (1+3)^n \\ \Rightarrow \sum_{r=0}^n 3^r {}^nC_r &= 4^n \end{aligned}$$

Hence, proved.

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