Real Numbers Ex 1.5 Q3

Answer :

Let us assume that $\sqrt{5}+\sqrt{3}$ is rational .Then, there exist positive co primes a and b such that

$$\sqrt{5}+\sqrt{3} = \frac{a}{b}$$

$$\sqrt{5} = \frac{a}{b} - \sqrt{3}$$

$$\left(\sqrt{5}\right)^2 = \left(\frac{a}{b} - \sqrt{3}\right)^2$$

$$5 = \left(\frac{a}{b}\right)^2 - \frac{2a\sqrt{3}}{b} + 3$$

$$\Rightarrow \quad 5-3 = \left(\frac{a}{b}\right)^2 - \frac{2a\sqrt{3}}{b}$$

$$\Rightarrow \quad 2 = \left(\frac{a}{b}\right)^2 - \frac{2a\sqrt{3}}{b}$$

$$\Rightarrow \quad \left(\frac{a}{b}\right)^2 - 2 = \frac{2a\sqrt{3}}{b}$$

$$\Rightarrow \quad \frac{a^2-2b^2}{b^2} = \frac{2a\sqrt{3}}{b}$$

$$\Rightarrow \quad \left(\frac{a^2-2b^2}{b^2}\right)\left(\frac{b}{2a}\right) = \sqrt{3}$$

$$\Rightarrow \quad \sqrt{3} = \left(\frac{a^2-2b^2}{2ab}\right)$$

Here we see that $\sqrt{3}$ is a rational number which is a contradiction as we know that $\sqrt{3}$ is an irrational number.

$\boxed{\text{Hence } \sqrt{5}+\sqrt{3} \text{ is irrational}}$

Real Numbers Ex 1.5 Q4

Answer :

Let us assume that $\sqrt{p}+\sqrt{q}$ is rational. Then, there exist positive co primes a and b such that

$$\sqrt{p}+\sqrt{q} = \frac{a}{b}$$

$$\sqrt{p} = \frac{a}{b} - \sqrt{q}$$

$$\left(\sqrt{p}\right)^2 = \left(\frac{a}{b} - \sqrt{q}\right)^2$$

$$p = \left(\frac{a}{b}\right)^2 - \frac{2a\sqrt{q}}{b} + q$$

$$p-q = \left(\frac{a}{b}\right)^2 - \frac{2a\sqrt{q}}{b}$$

$$p-q = \left(\frac{a}{b}\right)^2 - \frac{2a\sqrt{q}}{b}$$

$$\left(\frac{a}{b}\right)^2 - (p-q) = \frac{2a\sqrt{q}}{b}$$

$$\frac{a^2-b^2(p-q)}{b^2}=\frac{2a\sqrt{q}}{b}$$

$$\left(\frac{a^2-b^2(p-q)}{b^2}\right)\left(\frac{b}{2a}\right)=\sqrt{q}$$

$$\sqrt{q}=\left(\frac{a^2-b^2(p-q)}{2ab}\right)$$

Here we see that $\sqrt{q}$ is a rational number which is a contradiction as we know that $\sqrt{q}$ is an irrational number

$\boxed{\text{Hence } \sqrt{p}+\sqrt{q} \text{ is irrational}}$

Real Numbers Ex 1.5 Q5

Answer :

Let us assume that $\sqrt{3}+\sqrt{4}$ is rational .Then, there exist positive co primes a and b such that

$$\sqrt{3}+\sqrt{4}=\frac{a}{b}$$

$$\sqrt{4}=\frac{a}{b}-\sqrt{3}$$

$$\left(\sqrt{4}\right)^2=\left(\frac{a}{b}-\sqrt{3}\right)^2$$

$$4=\left(\frac{a}{b}\right)^2-\frac{2a\sqrt{3}}{b}+3$$

$$4-3=\left(\frac{a}{b}\right)^2-\frac{2a\sqrt{3}}{b}$$

$$1=\left(\frac{a}{b}\right)^2-\frac{2a\sqrt{3}}{b}$$

$$\left(\frac{a}{b}\right)^2-1=\frac{2a\sqrt{3}}{b}$$

$$\frac{a^2-b^2}{b^2}=\frac{2a\sqrt{3}}{b}$$

$$\left(\frac{a^2-b^2}{b^2}\right)\left(\frac{b}{2a}\right)=\sqrt{3}$$

$$\sqrt{3}=\left(\frac{a^2-b^2}{2ab}\right)$$

Here we see that $\sqrt{3}$ is a rational number which is a contradiction as we know that $\sqrt{3}$ is an irrational number

$\boxed{\text{Hence } \sqrt{3}+\sqrt{4} \text{ is irrational}}$

*********** END **********