Functions Ex 2.1 Q6

Given, $f : A \to B$ is injective such that range$(f) = \{a\}$

We know that in injective map different elements have different images.

$\therefore$ A has only one element.

Functions Ex 2.1 Q7

A = **R** − {3}, B = **R** − {1}

f: A → B is defined as $f(x) = \left(\dfrac{x-2}{x-3}\right)$.

Let $x,\ y \in$ A such that $f(x) = f(y)$.

$$\Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$$

$$\Rightarrow (x-2)(y-3) = (y-2)(x-3)$$

$$\Rightarrow xy - 3x - 2y + 6 = xy - 3y - 2x + 6$$

$$\Rightarrow -3x - 2y = -3y - 2x$$

$$\Rightarrow 3x - 2x = 3y - 2y$$

$$\Rightarrow x = y$$

Therefore, f is one-one.

Let $y \in B = \mathbf{R} - \{1\}$.

Then, $y \neq 1$.

The function f is onto if there exists $x \in A$ such that $f(x) = y$.

Now,

$$f(x) = y$$

$$\Rightarrow \frac{x-2}{x-3} = y$$

$$\Rightarrow x - 2 = xy - 3y$$

$$\Rightarrow x(1-y) = -3y + 2$$

$$\Rightarrow x = \frac{2-3y}{1-y} \in A \qquad [y \neq 1]$$

Thus, for any $y \in B$, there exists $\frac{2-3y}{1-y} \in A$ such that

$$f\left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right) - 2}{\left(\frac{2-3y}{1-y}\right) - 3} = \frac{2-3y-2+2y}{2-3y-3+3y} = \frac{-y}{-1} = y.$$

$\therefore f$ is onto.

Hence, function f is one-one and onto.

Functions Ex 2.1 Q8

We have $f : R \to R$ given by $f(x) = x - [x]$

Now,

check for injectivity:

$\because f(x) = x - [x] \Rightarrow f(x) = 0$ for $x \in Z$

$\therefore$ Range of $f = [0, 1] \neq R$

$\therefore f$ is not one-one, where as many-one

Again, Range of $f = [0, 1] \neq R$

$\therefore f$ is an into function

*********** END **********