Indefinite Integrals Ex 19.9 Q25

Let $\quad I = \int \frac{1+\cos x}{(x+\sin x)^3}\,dx$ - - - - - (i)

Let $\quad x + \sin x = t \quad$ then,

$d(x+\sin x) = dt$

$\Rightarrow \quad (1+\cos x)\,dx = dt$

Putting $x + \sin x = t$ and $(1+\cos x)dx = dt$ in equation (i), we get

$$I = \int \frac{dt}{t^3}$$
$$= \int t^{-3}dt$$
$$= \frac{t^{-2}}{-2} + c$$
$$= -\frac{1}{2t^2} + c$$
$$= \frac{-1}{2(x+\sin x)^2} + c$$

$$\therefore \quad I = \frac{-1}{2(x+\sin x)^2} + c$$

Indefinite Integrals Ex 19.9 Q26

$$\frac{\cos x - \sin x}{1+\sin 2x} = \frac{\cos x - \sin x}{(\sin^2 x + \cos^2 x) + 2\sin x\cos x}$$

$$\left[\sin^2 x + \cos^2 x = 1;\ \ \sin 2x = 2\sin x\cos x\right]$$

$$= \frac{\cos x - \sin x}{(\sin x + \cos x)^2}$$

Let $\sin x + \cos x = t$

$\therefore (\cos x - \sin x)\,dx = dt$

$$\Rightarrow \int \frac{\cos x - \sin x}{1+\sin 2x}dx = \int \frac{\cos x - \sin x}{(\sin x+\cos x)^2}dx$$
$$= \int \frac{dt}{t^2}$$
$$= \int t^{-2}dt$$
$$= -t^{-1} + C$$
$$= -\frac{1}{t} + C$$
$$= \frac{-1}{\sin x + \cos x} + C$$

Indefinite Integrals Ex 19.9 Q27

Let $\quad I = \int \frac{\sin 2x}{(a+b\cos 2x)^2}dx - - - - - (i)$

Let $\quad a+b\cos 2x = t \qquad$ then,
$d(a+b\cos 2x) = dt$

$\Rightarrow \quad b(-2\sin 2x)dx = dt$

$\Rightarrow \quad \sin 2x\, dx = -\frac{dt}{2b}$

Putting $a+b\cos 2x = t$ and $\sin 2x\, dx = -\frac{dt}{2b}$ in equation (i), we get

$$I = \int \frac{1}{t^2} \times \frac{-dt}{2b}$$
$$= \frac{-1}{2b}\int t^{-2}dt$$
$$= -\frac{1}{2b}\left(-1t^{-1}\right)+c$$
$$= \frac{1}{2bt}+c$$
$$= \frac{1}{2b(a+b\cos 2x)}+c$$

$\therefore \quad I = \frac{1}{2b(a+b\cos 2x)}+c$

Indefinite Integrals Ex 19.9 Q28

Let $\quad I = \int \frac{\log x^2}{x}dx - - - - - (i)$

Let $\quad \log x = t \qquad$ then,
$d(\log x) = dt$

$\Rightarrow \quad \frac{1}{x}dx = dt$

$\Rightarrow \quad \frac{dx}{x} = dt$

Now, $\quad I = \int \frac{\log x^2}{x}dx$
$$= \int \frac{2\log x}{x}dx$$
$$= 2\int \frac{\log x}{x}dx - - - - (ii)$$

Putting $\log x = t$ and $\frac{dx}{x} = dt$ in equation (ii), we get

$$I = 2\int t\, dt$$
$$= \frac{2t^2}{2}+c$$
$$= t^2+c$$

$\therefore \quad I = (\log x)^2 + c$

Indefinite Integrals Ex 19.9 Q29

Let $\quad I = \int \frac{\sin x}{(1+\cos x)^2} dx - - - - - (i)$

Let $\quad 1+\cos x = t \quad$ then,

$d(1+\cos x) = dt$

$\Rightarrow \quad -\sin x \, dx = dt$

$\Rightarrow \quad \sin x \, dx = -dt$

Putting $1+\cos x = t$ and $\sin dx = -dt$ in equation (ii), we get

$$I = \int \frac{-dt}{t^2}$$
$$= -\int t^{-2} dt$$
$$= -\left(-1t^{-1}\right) + c$$
$$= \frac{1}{t} + c$$
$$= \frac{1}{1+\cos x} + c$$

$$\therefore \quad I = \frac{1}{1+\cos x} + c$$

*********** END **********