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L.H.S.,

$$\begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix}$$

$$= \begin{vmatrix} x+\lambda & 2x & 2x \\ 2x & x+\lambda & 2x \\ 2x & 2x & x+\lambda \end{vmatrix} [C_1 \to C_1 - C_3, C_2 \to C_2 - C_3]$$

$$= \begin{vmatrix} \lambda - x & 0 & 2x \\ 0 & \lambda - x & 2x \\ x-\lambda & x-\lambda & x+\lambda \end{vmatrix}$$

$$= (\lambda - x)(\lambda - x)\begin{vmatrix} 1 & 0 & 2x \\ 0 & 1 & 2x \\ -1 & -1 & x+\lambda \end{vmatrix}$$

$$= (\lambda - x)^2\begin{vmatrix} 1 & 0 & 2x \\ 0 & 1 & 2x \\ -1 & -1 & x+\lambda \end{vmatrix}$$

$$= (\lambda - x)^2[1(x+\lambda) + 2x + 2x(0+1)]$$

$$= (\lambda - x)^2[x+\lambda+2x+2x]$$

$$= (\lambda - x)^2[5x+\lambda]$$

= R.H.S

Hence Proved

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$$LHS = \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

Apply $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 5x+4 & 2x & 2x \\ 5x+4 & x+4 & 2x \\ 5x+4 & 2x & x+4 \end{vmatrix}$$

$$= (5x+4)\begin{vmatrix} 1 & 2x & 2x \\ 1 & x+4 & 2x \\ 1 & 2x & x+4 \end{vmatrix}$$

$$= (5x+4)\begin{vmatrix} 1 & 2x & 2x \\ 0 & -x+4 & 0 \\ 0 & 0 & -x+4 \end{vmatrix}$$

$$= (5x+4)(4-x)^2\begin{vmatrix} 1 & 2x & 2x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= (5x+4)(4-x)^2$$

$$= RHS$$

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$$\text{Let } \Delta = \begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$

$$\Delta = \begin{vmatrix} y & -x & y-x \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$$

Applying $R_1 \to R_1 - R_3$

$$\Delta = \begin{vmatrix} 0 & -2x & -2x \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$$

$$\Delta = 2x\left[z(x+y) - xy\right] - 2x\left[zx - y(z+x)\right]$$

$$\Delta = 2x\left[zx + zy - xy - zx + yz + yx\right]$$

$$\Delta = 4xyz$$

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$$\begin{vmatrix} -a\left(b^2+c^2-a^2\right) & 2b^3 & 2c^3 \\ 2a^3 & -b\left(c^2+a^2-b^2\right) & 2c^3 \\ 2a^3 & 2b^3 & -c\left(a^2+b^2-c^2\right) \end{vmatrix} = abc\left(a^2+b^2+c^2\right)^3$$

$$\text{LHS} = \begin{vmatrix} -a\left(b^2+c^2-a^2\right) & 2b^3 & 2c^3 \\ 2a^3 & -b\left(c^2+a^2-b^2\right) & 2c^3 \\ 2a^3 & 2b^3 & -c\left(a^2+b^2-c^2\right) \end{vmatrix}$$

Take a, b and c common from C_1, C_2 and C_3 respectively.

$$= abc\begin{vmatrix} -\left(b^2+c^2-a^2\right) & 2b^2 & 2c^2 \\ 2a^2 & -\left(c^2+a^2-b^2\right) & 2c^2 \\ 2a^2 & 2b^2 & -\left(a^2+b^2-c^2\right) \end{vmatrix}$$

Apply: $R_1 \to R_1 - R_3, R_2 \to R_2 - R_3$

$$= abc\begin{vmatrix} -\left(b^2+c^2-a^2\right)-2a^2 & 0 & 2c^2+\left(a^2+b^2-c^2\right) \\ 0 & -\left(c^2+a^2-b^2\right)-2b^2 & 2c^2+\left(a^2+b^2-c^2\right) \\ 2a^2 & 2b^2 & -\left(a^2+b^2-c^2\right) \end{vmatrix}$$

$$= abc\begin{vmatrix} -\left(b^2+c^2+a^2\right) & 0 & \left(a^2+b^2+c^2\right) \\ 0 & -\left(c^2+a^2+b^2\right) & \left(a^2+b^2+c^2\right) \\ 2a^2 & 2b^2 & -\left(a^2+b^2-c^2\right) \end{vmatrix}$$

$$= abc\left(b^2+c^2+a^2\right)^2\begin{vmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2a^2 & 2b^2 & -\left(a^2+b^2-c^2\right) \end{vmatrix}$$

$$= abc\left(b^2+c^2+a^2\right)^2\begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 1 \\ 2a^2 & 2b^2 & -\left(a^2+b^2-c^2\right)+2a^2 \end{vmatrix}$$

$$= abc\left(b^2+c^2+a^2\right)^2\begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 1 \\ 2a^2 & 2b^2 & -b^2+c^2+a^2 \end{vmatrix}$$

$$= -abc\left(b^2+c^2+a^2\right)^2\left[(-1)\left(-b^2+c^2+a^2\right) - (1)\left(2b^2\right)\right]$$

$$abc\left(a^2+b^2+c^2\right)^3$$

$$= RHS$$

*********** END **********