Trigonometric Functions Ex 5.1 Q6

$$\text{LHS} = \frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$$

$$= \frac{(\sin A / \cos A)}{\left(1-\frac{\cos A}{\sin A}\right)} + \frac{(\cos A / \sin A)}{1-\frac{\sin A}{\cos A}}$$

$$= \frac{\sin A}{\cos A\frac{(\sin A-\cos A)}{\sin A}} + \frac{\cos A}{\sin A\frac{(\cos A-\sin A)}{\cos A}}$$

$$= \frac{\sin^2 A}{\cos A(\sin A-\cos A)} + \frac{\cos^2 A}{\sin A(\cos A-\sin A)}$$

$$= \frac{\sin^3 A-\cos^3 A}{\cos A\sin A(\sin A-\cos A)}$$

$$= \frac{(\sin A-\cos A)(\sin^2 A+\cos^2 A+\sin A\cos A)}{\cos A\sin A(\sin A-\cos A)} \qquad \left[\text{Using } a^3-b^3=(a-b)(a^2+b^2+ab)\right]$$

$$= \frac{1+\sin A\cos A}{\sin A\cos A} \qquad \left[\because \sin^2 A+\cos^2 A=1\right]$$

$$= \frac{1}{\sin A\cos A} + \frac{\sin A\cos A}{\sin A\cos A}$$

$$= \sec A\operatorname{cosec} A + 1 \qquad \left[\because \frac{1}{\cos A}=\sec A, \frac{1}{\sin A}=\operatorname{cosec} A\right]$$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q7

$$\text{LHS} = \frac{\sin^3 A+\cos^3 A}{\sin A+\cos A} + \frac{\sin^3 A-\cos^3 A}{\sin A-\cos A}$$

$$= \frac{(\sin A+\cos A)(\sin^2 A+\cos^2 A-\sin A\cos A)}{(\sin A+\cos A)} + \frac{(\sin A-\cos A)(\sin^2 A+\cos^2 A+\sin A\cos A)}{\sin A-\cos A}$$

$$\left(\text{Using } a^3+b^3=(a+b)(a^2+b^2-ab) \text{ and } a^3-b^3=(a-b)(a^2+b^2+ab)\right)$$

$$= (1-\sin A\cos A)+(1+\sin A\cos A) \left(\because \sin^2 A+\cos^2 A=1\right)$$

$= 2$

= RHS

Trigonometric Functions Ex 5.1 Q8

$$\text{LHS} = (\sec A\sec B+\tan A\tan B)^2-(\sec A\tan B+\tan A\sec B)^2$$

$$= (\sec A\sec B)^2+(\tan A\tan B)^2+2\sec A\sec B\tan A\tan B$$

$$-\left((\sec A\tan B)^2+(\tan A\sec B)^2+2\sec A\tan B\tan A\sec B\right) \qquad \left[\text{Using}(a+b)^2=a^2+b^2+2ab\right]$$

$$= \sec^2 A\sec^2 B+\tan^2 A\tan^2 B+2\sec A\sec B\tan A\tan B$$

$$-\sec^2 A\tan^2 B-\tan^2 A\sec^2 B-2\sec A\sec B\tan A\tan B \left[\text{Using } (ab)^2=a^2b^2\right]$$

$$= \sec^2 A\sec^2 B-\sec^2 A\tan^2 B+\tan^2 A\tan^2 B-\tan^2 A\sec^2 B$$

$$= \sec^2 A(\sec^2 B-\tan^2 B)+\tan^2 A(\tan^2 B-\sec^2 B)$$

$$= \sec^2 A\,1-\tan^2 A\,1 \qquad \left[\begin{array}{l}\because \sec^2\theta=1+\tan^2\theta\\ \Rightarrow \sec^2\theta-\tan^2\theta=1\end{array}\right]$$

$$= 1+\tan^2 A-\tan^2 A$$

$= 1$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q9

$$\text{RHS} = \frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}$$

$$= \frac{((1+\cos\theta)+\sin\theta)}{(1+\cos\theta)-\sin\theta} \times \frac{((1+\cos\theta)+\sin\theta)}{(1+\cos\theta+\sin\theta)}$$

$$= \frac{((1+\cos\theta)+\sin\theta)^2}{(1+\cos\theta)^2-\sin^2\theta} \qquad \left(\begin{array}{r} \text{Using } (a+b)(a+b)=(a+b)^2 \\ \&\ (a+b)(a-b)=a^2b^2 \end{array}\right)$$

$$= \frac{(1+\cos\theta)^2+\sin^2\theta+2\sin\theta(1+\cos\theta)}{1+\cos^2\theta+2\cos\theta-\sin^2\theta} \qquad \left(\text{Using } (a+b)^2=a^2+b^2+2ab\right)$$

$$= \frac{1+\cos^2\theta+2.1\cos\theta+\sin^2\theta+2\sin\theta(1+\cos\theta)}{1+\cos^2+2\cos\theta-(1-\cos^2\theta)} \qquad \left(\text{Using } \sin^2\theta=1-\cos^2\theta\right)$$

$$= \frac{1+1+2\cos\theta+2\sin\theta(1+\cos\theta)}{1-1+\cos^2\theta+\cos^2\theta+2\cos\theta} \qquad \left(\text{Using } \sin^2\theta+\cos^2\theta=1\right)$$

$$= \frac{2+2\cos\theta+2\sin\theta(1+\cos\theta)}{2\cos^2\theta+2\cos\theta}$$

$$= \frac{2(1+\cos\theta)+2\sin\theta(1+\cos\theta)}{2\cos\theta(\cos\theta+1)}$$

$$= \frac{(1+\cos\theta)(2+2\sin\theta)}{2\cos\theta(1+\cos\theta)}$$

$$= \frac{1+\sin\theta}{\cos\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$$

$$= \frac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$$

$$= \frac{\cos\theta}{1-\sin\theta}$$

Trigonometric Functions Ex 5.1 Q10

$$\text{LHS} = \frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1+\cot^2\theta}$$

$$= \frac{\sin^3\theta}{\cos^3\theta\left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)} + \frac{\cos^3\theta}{\sin^3\theta\left(1+\frac{\cos^2\theta}{\sin^2\theta}\right)} \qquad \left(\begin{array}{l} \because\ \tan\theta = \frac{\sin\theta}{\cos\theta} \\ \text{and } \cot\theta = \frac{\cos\theta}{\sin\theta} \end{array}\right)$$

$$= \frac{\sin^3\theta\cos^2\theta}{\cos^3\theta(\cos^2\theta+\sin^2\theta)} + \frac{\cos^3\theta\sin^2\theta}{\sin^3\theta(\sin^2\theta+\cos^2\theta)}$$

$$= \frac{\sin^3\theta}{\cos\theta} + \frac{\cos^3\theta}{\sin\theta} \qquad (\because\ \cos^2\theta+\sin^2\theta=1)$$

$$= \frac{\sin^4\theta+\cos^4\theta}{\sin\theta\cos\theta}$$

$$= \frac{(\sin^2\theta)^2+(\cos^2\theta)^2+2\sin^2\theta\cos^2\theta-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta} \qquad (\text{adding and subtracting } 2\sin^2\theta\cos^2\theta)$$

$$= \frac{(\sin^2\theta+\cos^2\theta)^2-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1^2-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta} \qquad (\because\ \sin^2\theta+\cos^2\theta=1)$$

$$= \frac{1-2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$$

= RHS

Proved

*********** END **********