Rationalisation Ex 3.2 Q6

Answer :

(i) We know that rationalization factor for $\sqrt{3}+1$ is $\sqrt{3}-1$. We will multiply numerator and denominator of the given expression $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ by $\sqrt{3}-1$, to get

$$\begin{aligned}\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1} &= \frac{\left(\sqrt{3}\right)^2+(1)^2-2\times\sqrt{3}\times 1}{\left(\sqrt{3}\right)^2-(1)^2}\\ &= \frac{3+1-2\sqrt{3}}{3-1}\\ &= \frac{4-2\sqrt{3}}{2}\\ &= 2-\sqrt{3}\end{aligned}$$

On equating rational and irrational terms, we get

$$\begin{aligned}a-b\sqrt{3} &= 2-\sqrt{3}\\ &= 2-1\sqrt{3}\end{aligned}$$

Hence, we get $\boxed{a=2, b=1}$.

(ii) We know that rationalization factor for $2+\sqrt{2}$ is $2-\sqrt{2}$. We will multiply numerator and denominator of the given expression $\frac{4+\sqrt{2}}{2+\sqrt{2}}$ by $2-\sqrt{2}$, to get

$$\begin{aligned}\frac{4+\sqrt{2}}{2+\sqrt{2}}\times\frac{2-\sqrt{2}}{2-\sqrt{2}} &= \frac{4\times 2-4\times\sqrt{2}+2\times\sqrt{2}-\left(\sqrt{2}\right)^2}{(2)^2-\left(\sqrt{2}\right)^2}\\ &= \frac{8-4\sqrt{2}+2\sqrt{2}-2}{4-2}\\ &= \frac{6-2\sqrt{2}}{2}\\ &= 3-\sqrt{2}\end{aligned}$$

On equating rational and irrational terms, we get

$$a-\sqrt{b}=3-\sqrt{2}$$

Hence, we get $\boxed{a=3, b=2}$.

(iii) We know that rationalization factor for $3-\sqrt{2}$ is $3+\sqrt{2}$. We will multiply numerator and denominator of the given expression $\frac{3+\sqrt{2}}{3-\sqrt{2}}$ by $3+\sqrt{2}$, to get

$$\begin{aligned}\frac{3+\sqrt{2}}{3-\sqrt{2}}\times\frac{3+\sqrt{2}}{3+\sqrt{2}} &= \frac{(3)^2+\left(\sqrt{2}\right)^2+2\times 3\times\sqrt{2}}{(3)^2-\left(\sqrt{2}\right)^2}\\ &= \frac{9+2+6\sqrt{2}}{9-2}\\ &= \frac{11+6\sqrt{2}}{7}\\ &= \frac{11}{7}+\frac{6}{7}\sqrt{2}\end{aligned}$$

On equating rational and irrational terms, we get

$$a+b\sqrt{2}=\frac{11}{7}+\frac{6}{7}\sqrt{2}$$

Hence, we get $\boxed{a=\frac{11}{7}, b=\frac{6}{7}}$.

(iv) We know that rationalization factor for $7+4\sqrt{3}$ is $7-4\sqrt{3}$. We will multiply numerator and denominator of the given expression $\frac{5+3\sqrt{3}}{7+4\sqrt{3}}$ by $7-4\sqrt{3}$, to get

$$\begin{aligned}\frac{5+3\sqrt{3}}{7+4\sqrt{3}}\times\frac{7-4\sqrt{3}}{7-4\sqrt{3}} &= \frac{5\times7-5\times4\times\sqrt{3}+3\times7\times\sqrt{3}-3\times4\times\left(\sqrt{3}\right)^2}{(7)^2-\left(4\sqrt{3}\right)^2}\\ &= \frac{35-20\sqrt{3}+21\sqrt{3}-36}{49-48}\\ &= \frac{\sqrt{3}-1}{1}\\ &= \sqrt{3}-1\end{aligned}$$

On equating rational and irrational terms, we get

$$\begin{aligned}a+b\sqrt{3} &= \sqrt{3}-1\\ &= -1+1\sqrt{3}\end{aligned}$$

Hence, we get $\boxed{a=-1, b=1}$.

(v) We know that rationalization factor for $\sqrt{11}+\sqrt{7}$ is $\sqrt{11}-\sqrt{7}$. We will multiply numerator and denominator of the given expression $\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}}$ by $\sqrt{11}-\sqrt{7}$, to get

$$\begin{aligned}\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}}\times\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}-\sqrt{7}} &= \frac{\left(\sqrt{11}\right)^2+\left(\sqrt{7}\right)^2-2\times\sqrt{11}\times\sqrt{7}}{\left(\sqrt{11}\right)^2-\left(\sqrt{7}\right)^2}\\ &= \frac{11+7-2\sqrt{77}}{11-7}\\ &= \frac{18-2\sqrt{77}}{4}\\ &= \frac{9}{2}-\frac{1}{2}\sqrt{77}\end{aligned}$$

On equating rational and irrational terms, we get

$$a-b\sqrt{77}=\frac{9}{2}-\frac{1}{2}\sqrt{77}$$

Hence, we get $\boxed{a=\frac{9}{2}, b=\frac{1}{2}}$.

(vi) We know that rationalization factor for $4-3\sqrt{5}$ is $4+3\sqrt{5}$. We will multiply numerator and denominator of the given expression $\frac{4+3\sqrt{5}}{4-3\sqrt{5}}$ by $4+3\sqrt{5}$, to get

$$\begin{aligned}\frac{4+3\sqrt{5}}{4-3\sqrt{5}}\times\frac{4+3\sqrt{5}}{4+3\sqrt{5}} &= \frac{(4)^2+\left(3\sqrt{5}\right)^2+2\times4\times3\sqrt{5}}{(4)^2-\left(3\sqrt{5}\right)^2}\\ &= \frac{16+45+24\sqrt{5}}{16-45}\\ &= \frac{61+24\sqrt{5}}{-29}\\ &= -\frac{61}{29}-\frac{24}{29}\sqrt{5}\end{aligned}$$

On equating rational and irrational terms, we get

$$a+b\sqrt{5}=-\frac{61}{29}-\frac{24}{29}\sqrt{5}$$

Hence, we get $\boxed{a=-\frac{61}{29}, b=-\frac{24}{29}}$.

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