Exercise 7.6 : Solutions of Questions on Page Number : **327**

Q1 : $x\sin x$

Answer :

Let $I = \int x\sin x\,dx$

Taking x as first function and sin x as second function and integrating by parts, we obtain

$$I = x\int \sin x\ dx - \int\left\{\left(\frac{d}{dx}x\right)\int \sin x\ dx\right\}dx$$
$$= x(-\cos x) - \int 1\cdot(-\cos x)\,dx$$
$$= -x\cos x + \sin x + C$$

Answer needs Correction? **Click Here**

Q2 : $x\sin 3x$

Answer :

Let $I = \int x\sin 3x\,dx$

Taking x as first function and sin $3x$ as second function and integrating by parts, we obtain

$$I = x\int \sin 3x\ dx - \int\left\{\left(\frac{d}{dx}x\right)\int \sin 3x\ dx\right\}$$
$$= x\left(\frac{-\cos 3x}{3}\right) - \int 1\cdot\left(\frac{-\cos 3x}{3}\right)dx$$
$$= \frac{-x\cos 3x}{3} + \frac{1}{3}\int \cos 3x\ dx$$
$$= \frac{-x\cos 3x}{3} + \frac{1}{9}\sin 3x + C$$

Answer needs Correction? **Click Here**

Q3 : $x^2 e^x$

Answer :

Let $I = \int x^2 e^x dx$

Taking x^2 as first function and e^x as second function and integrating by parts, we obtain

$$I = x^2\int e^x\,dx - \int\left\{\left(\frac{d}{dx}x^2\right)\int e^x dx\right\}dx$$
$$= x^2 e^x - \int 2x\cdot e^x dx$$
$$= x^2 e^x - 2\int x\cdot e^x dx$$

Again integrating by parts, we obtain

$$= x^2 e^x - 2\left[x\cdot\int e^x dx - \int\left\{\left(\frac{d}{dx}x\right)\cdot\int e^x dx\right\}dx\right]$$
$$= x^2 e^x - 2\left[xe^x - \int e^x dx\right]$$
$$= x^2 e^x - 2\left[xe^x - e^x\right]$$
$$= x^2 e^x - 2xe^x + 2e^x + C$$
$$= e^x\left(x^2 - 2x + 2\right) + C$$

Answer needs Correction? **Click Here**

Q4 : $x\log x$

Answer :

Let $I = \int x\log x dx$

Taking log x as first function and x as second function and integrating by parts, we obtain

$$I = \log x\int x\ dx - \int\left\{\left(\frac{d}{dx}\log x\right)\int x\ dx\right\}dx$$
$$= \log x\cdot\frac{x^2}{2} - \int\frac{1}{x}\cdot\frac{x^2}{2}dx$$
$$= \frac{x^2\log x}{2} - \int\frac{x}{2}dx$$
$$= \frac{x^2\log x}{2} - \frac{x^2}{4} + C$$

2 4

Q5 : $x \log 2x$

Answer :

Let $I = \int x \log 2x dx$

Taking $\log 2x$ as first function and x as second function and integrating by parts, we obtain

$$I = \log 2x \int x\, dx - \int \left\{ \left(\frac{d}{dx} 2\log x \right) \int x\, dx \right\} dx$$
$$= \log 2x \cdot \frac{x^2}{2} - \int \frac{2}{2x} \cdot \frac{x^2}{2} dx$$
$$= \frac{x^2 \log 2x}{2} - \int \frac{x}{2} dx$$
$$= \frac{x^2 \log 2x}{2} - \frac{x^2}{4} + \mathrm{C}$$

Q6 : $x^2 \log x$

Answer :

Let $I = \int x^2 \log x\, dx$

Taking $\log x$ as first function and x^2 as second function and integrating by parts, we obtain

$$I = \log x \int x^2\, dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x^2 dx \right\} dx$$
$$= \log x \left(\frac{x^3}{3} \right) - \int \frac{1}{x} \cdot \frac{x^3}{3} dx$$
$$= \frac{x^3 \log x}{3} - \int \frac{x^2}{3} dx$$
$$= \frac{x^3 \log x}{3} - \frac{x^3}{9} + \mathrm{C}$$

Q7 : $x \sin^{-1} x$

Answer :

Let $I = \int x \sin^{-1} x\, dx$

Taking $\sin^{-1} x$ as first function and x as second function and integrating by parts, we obtain

$$I = \sin^{-1} x \int x\, dx - \int \left\{ \left(\frac{d}{dx} \sin^{-1} x \right) \int x\, dx \right\} dx$$
$$= \sin^{-1} x \left(\frac{x^2}{2} \right) - \int \frac{1}{\sqrt{1-x^2}} \cdot \frac{x^2}{2} dx$$
$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \frac{-x^2}{\sqrt{1-x^2}} dx$$
$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left\{ \frac{1-x^2}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} \right\} dx$$
$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left\{ \sqrt{1-x^2} - \frac{1}{\sqrt{1-x^2}} \right\} dx$$
$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left\{ \int \sqrt{1-x^2} dx - \int \frac{1}{\sqrt{1-x^2}} dx \right\}$$
$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left\{ \frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x - \sin^{-1} x \right\} + \mathrm{C}$$
$$= \frac{x^2 \sin^{-1} x}{2} + \frac{x}{4} \sqrt{1-x^2} + \frac{1}{4} \sin^{-1} x - \frac{1}{2} \sin^{-1} x + \mathrm{C}$$
$$= \frac{1}{4} \left(2x^2 - 1 \right) \sin^{-1} x + \frac{x}{4} \sqrt{1-x^2} + \mathrm{C}$$

Q8 : $x \tan^{-1} x$

Answer :

Let $I = \int x \tan^{-1} x\, dx$

Taking $\tan^{-1} x$ as first function and x as second function and integrating by parts, we obtain

$$I = \tan^{-1} x \int x\, dx - \int \left\{ \left(\frac{d}{dx} \tan^{-1} x \right) \int x\, dx \right\} dx$$
$$= \tan^{-1} x \left(\frac{x^2}{2} \right) - \int \frac{1}{1+x^2} \cdot \frac{x^2}{2} dx$$
$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \frac{x^2}{1+x^2} dx$$
$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \left(\frac{x^2+1}{1+x^2} - \frac{1}{1+x^2} \right) dx$$
$$= \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int \left(1 - \frac{1}{\ldots} \right) dx$$

$$2 \qquad 2\int\left(\quad 1+x^2 \right)$$

$$=\frac{x^2\tan^{-1}x}{2}-\frac{1}{2}\left(x-\tan^{-1}x\right)+C$$

$$=\frac{x^2}{2}\tan^{-1}x-\frac{x}{2}+\frac{1}{2}\tan^{-1}x+C$$

Answer needs Correction? **Click Here**

Q9 : $x\cos^{-1}x$

Answer :

Let $I=\int x\cos^{-1}x\,dx$

Taking $\cos^{-1}x$ as first function and x as second function and integrating by parts, we obtain

$$I=\cos^{-1}x\int x\,dx-\int\left\{\left(\frac{d}{dx}\cos^{-1}x\right)\int x\,dx\right\}dx$$

$$=\cos^{-1}x\frac{x^2}{2}-\int\frac{-1}{\sqrt{1-x^2}}\cdot\frac{x^2}{2}dx$$

$$=\frac{x^2\cos^{-1}x}{2}-\frac{1}{2}\int\frac{1-x^2-1}{\sqrt{1-x^2}}dx$$

$$=\frac{x^2\cos^{-1}x}{2}-\frac{1}{2}\int\left\{\sqrt{1-x^2}+\left(\frac{-1}{\sqrt{1-x^2}}\right)\right\}dx$$

$$=\frac{x^2\cos^{-1}x}{2}-\frac{1}{2}\int\sqrt{1-x^2}\,dx-\frac{1}{2}\int\left(\frac{-1}{\sqrt{1-x^2}}\right)dx$$

$$=\frac{x^2\cos^{-1}x}{2}-\frac{1}{2}I_1-\frac{1}{2}\cos^{-1}x \qquad \ldots(1)$$

where, $I_1=\int\sqrt{1-x^2}\,dx$

$$\Rightarrow I_1=x\sqrt{1-x^2}-\int\frac{d}{dx}\sqrt{1-x^2}\int x\,dx$$

$$\Rightarrow I_1=x\sqrt{1-x^2}-\int\frac{-2x}{2\sqrt{1-x^2}}.x\,dx$$

$$\Rightarrow I_1=x\sqrt{1-x^2}-\int\frac{-x^2}{\sqrt{1-x^2}}dx$$

$$\Rightarrow I_1=x\sqrt{1-x^2}-\int\frac{1-x^2-1}{\sqrt{1-x^2}}dx$$

$$\Rightarrow I_1=x\sqrt{1-x^2}-\left\{\int\sqrt{1-x^2}\,dx+\int\frac{-dx}{\sqrt{1-x^2}}\right\}$$

$$\Rightarrow I_1=x\sqrt{1-x^2}-\left\{I_1+\cos^{-1}x\right\}$$

$$\Rightarrow 2I_1=x\sqrt{1-x^2}-\cos^{-1}x$$

$$\therefore I_1=\frac{x}{2}\sqrt{1-x^2}-\frac{1}{2}\cos^{-1}x$$

Substituting in (1), we obtain

$$I=\frac{x\cos^{-1}x}{2}-\frac{1}{2}\left(\frac{x}{2}\sqrt{1-x^2}-\frac{1}{2}\cos^{-1}x\right)-\frac{1}{2}\cos^{-1}x$$

$$=\frac{\left(2x^2-1\right)}{4}\cos^{-1}x-\frac{x}{4}\sqrt{1-x^2}+C$$

Answer needs Correction? **Click Here**

Q10 : $\left(\sin^{-1}x\right)^2$

Answer :

Let $I=\int\left(\sin^{-1}x\right)^2\cdot 1\,dx$

Taking $\left(\sin^{-1}x\right)^2$ as first function and 1 as second function and integrating by parts, we obtain

$$I=\left(\sin^{-1}x\right)\int 1\,dx-\int\left\{\frac{d}{dx}\left(\sin^{-1}x\right)^2\cdot\int 1\cdot dx\right\}dx$$

$$=\left(\sin^{-1}x\right)^2\cdot x-\int\frac{2\sin^{-1}x}{\sqrt{1-x^2}}\cdot x\,dx$$

$$=x\left(\sin^{-1}x\right)^2+\int\sin^{-1}x\cdot\left(\frac{-2x}{\sqrt{1-x^2}}\right)dx$$

$$=x\left(\sin^{-1}x\right)^2+\left[\sin^{-1}x\int\frac{-2x}{\sqrt{1-x^2}}dx-\int\left\{\left(\frac{d}{dx}\sin^{-1}x\right)\int\frac{-2x}{\sqrt{1-x^2}}dx\right\}dx\right]$$

$$=x\left(\sin^{-1}x\right)^2+\left[\sin^{-1}x\cdot 2\sqrt{1-x^2}-\int\frac{1}{\sqrt{1-x^2}}\cdot 2\sqrt{1-x^2}\,dx\right]$$

$$=x\left(\sin^{-1}x\right)^2+2\sqrt{1-x^2}\sin^{-1}x-\int 2\,dx$$

$$=x\left(\sin^{-1}x\right)^2+2\sqrt{1-x^2}\sin^{-1}x-2x+C$$

Answer needs Correction? **Click Here**

Q11 : $\dfrac{x\cos^{-1}x}{\sqrt{1-x^2}}$

Answer :

Let $I=\int\frac{x\cos^{-1}x}{\sqrt{1-x^2}}dx$

$$I=\frac{-1}{2}\int\frac{-2x}{\sqrt{1-x^2}}\cdot\cos^{-1}x\,dx$$

Taking $\cos^{-1}x$ as first function and $\left(\frac{-2x}{\sqrt{1-x^2}}\right)$ as second function and integrating by parts, we obtain

$$I=\frac{-1}{2}\left[\cos^{-1}x\int\frac{-2x}{\sqrt{1-x^2}}dx-\int\left\{\left(\frac{d}{dx}\cos^{-1}x\right)\int\frac{-2x}{\sqrt{1-x^2}}dx\right\}dx\right]$$
$$=\frac{-1}{2}\left[\cos^{-1}x\cdot2\sqrt{1-x^2}-\int\frac{-1}{\sqrt{1-x^2}}\cdot2\sqrt{1-x^2}\,dx\right]$$
$$=\frac{-1}{2}\left[2\sqrt{1-x^2}\cos^{-1}x+\int2\,dx\right]$$
$$=\frac{-1}{2}\left[2\sqrt{1-x^2}\cos^{-1}x+2x\right]+C$$
$$=-\left[\sqrt{1-x^2}\cos^{-1}x+x\right]+C$$

Answer needs Correction? **Click Here**

Q12 : $x\sec^2 x$

Answer :

Let $I=\int x\sec^2 x\,dx$

Taking x as first function and $\sec^2 x$ as second function and integrating by parts, we obtain

$$I=x\int\sec^2x\,dx-\int\left\{\left\{\frac{d}{dx}x\right\}\int\sec^2x\,dx\right\}dx$$
$$=x\tan x-\int1\cdot\tan x\,dx$$
$$=x\tan x+\log|\cos x|+C$$

Answer needs Correction? **Click Here**

Q13 : $\tan^{-1}x$

Answer :

Let $I=\int1\cdot\tan^{-1}x\,dx$

Taking $\tan^{-1}x$ as first function and 1 as second function and integrating by parts, we obtain

$$I=\tan^{-1}x\int1dx-\int\left\{\left(\frac{d}{dx}\tan^{-1}x\right)\int1\cdot dx\right\}dx$$
$$=\tan^{-1}x\cdot x-\int\frac{1}{1+x^2}\cdot x\,dx$$
$$=x\tan^{-1}x-\frac{1}{2}\int\frac{2x}{1+x^2}dx$$
$$=x\tan^{-1}x-\frac{1}{2}\log|1+x^2|+C$$
$$=x\tan^{-1}x-\frac{1}{2}\log\left(1+x^2\right)+C$$

Answer needs Correction? **Click Here**

Q14 : $x(\log x)^2$

Answer :

$$I=\int x(\log x)^2\,dx$$

Taking $(\log x)^2$ as first function and x as second function and integrating by parts, we obtain

Answer needs Correction? **Click Here**

Q15 : $\left(x^2+1\right)\log x$

Answer :

Let $I=\int\left(x^2+1\right)\log x\,dx=\int x^2\log x\,dx+\int\log x\,dx$

Let $I = I_1 + I_2$ ... (1)

Where, $I_1=\int x^2\log x\,dx$ and $I_2=\int\log x\,dx$

$$I_1=\int x^2\log x\,dx$$

Taking $\log x$ as first function and x^2 as second function and integrating by parts, we obtain

$$I_1=\log x-\int x^2dx-\int\left\{\left(\frac{d}{dx}\log x\right)\int x^2dx\right\}dx$$
$$=\log x\cdot\frac{x^3}{3}-\int\frac{1}{x}\cdot\frac{x^3}{3}dx$$
$$=\frac{x^3}{3}\log x-\frac{1}{3}\left(\int x^2dx\right)$$
$$=\frac{x^3}{3}\log x-\frac{x^3}{9}+C_1\qquad\ldots(2)$$

$$I_2=\int\log x\,dx$$

Taking log x as first function and 1 as second function and integrating by parts, we obtain

$$I_2 = \log x \int 1 \cdot dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int 1 \cdot dx \right\}$$
$$= \log x \cdot x - \int \frac{1}{x} \cdot x dx$$
$$= x \log x - \int 1 dx$$
$$= x \log x - x + C_2 \qquad \ldots (3)$$

Using equations (2) and (3) in (1), we obtain

$$I = \frac{x^3}{3} \log x - \frac{x^3}{9} + C_1 + x \log x - x + C_2$$
$$= \frac{x^3}{3} \log x - \frac{x^3}{9} + x \log x - x + (C_1 + C_2)$$
$$= \left(\frac{x^3}{3} + x \right) \log x - \frac{x^3}{9} - x + C$$

Answer needs Correction? **Click Here**

Q16 : $e^x (\sin x + \cos x)$

Answer :

Let $I = \int e^x (\sin x + \cos x) dx$

Let $f(x) = \sin x$

$\Rightarrow f'(x) = \cos x$

$\therefore I = \int e^x \{ f(x) + f'(x) \} dx$

It is known that, $\int e^x \{ f(x) + f'(x) \} dx = e^x f(x) + C$

$\therefore I = e^x \sin x + C$

Answer needs Correction? **Click Here**

Q17 : $\dfrac{xe^x}{(1+x)^2}$

Answer :

Let $I = \int \frac{xe^x}{(1+x)^2} dx = \int e^x \left\{ \frac{x}{(1+x)^2} \right\} dx$

$$= \int e^x \left\{ \frac{1+x-1}{(1+x)^2} \right\} dx$$

$$= \int e^x \left\{ \frac{1}{1+x} - \frac{1}{(1+x)^2} \right\} dx$$

Let $f(x) = \frac{1}{1+x} \Rightarrow f'(x) = \frac{-1}{(1+x)^2}$

$$\Rightarrow \int \frac{xe^x}{(1+x)^2} dx = \int e^x \{ f(x) + f'(x) \} dx$$

It is known that, $\int e^x \{ f(x) + f'(x) \} dx = e^x f(x) + C$

$$\therefore \int \frac{xe^x}{(1+x)^2} dx = \frac{e^x}{1+x} + C$$

Answer needs Correction? **Click Here**

Q18 : $e^x \left(\dfrac{1+\sin x}{1+\cos x} \right)$

Answer :

$$e^x \left(\frac{1+\sin x}{1+\cos x} \right)$$
$$= e^x \left(\frac{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \right)$$
$$= \frac{e^x \left(\sin \frac{x}{2} + \cos \frac{x}{2} \right)^2}{2 \cos^2 \frac{x}{2}}$$
$$= \frac{1}{2} e^x \cdot \left(\frac{\sin \frac{x}{2} + \cos \frac{x}{2}}{\cos \frac{x}{2}} \right)^2$$
$$= \frac{1}{2} e^x \left[\tan \frac{x}{2} + 1 \right]^2$$
$$= \frac{1}{2} e^2 \left(1 + \tan \frac{x}{2} \right)^2$$
$$= \frac{1}{2} e^x \left[1 + \tan^2 \frac{x}{2} + 2 \tan \frac{x}{2} \right]$$

$$=\frac{1}{2}e^{x}\left[\sec^{2}\frac{x}{2}+2\tan\frac{x}{2}\right]$$

$$\frac{e^{x}(1+\sin x)dx}{(1+\cos x)}=e^{x}\left[\frac{1}{2}\sec^{2}\frac{x}{2}+\tan\frac{x}{2}\right] \quad ...(1)$$

Let $\tan\frac{x}{2}=f(x) \Rightarrow f'(x)=\frac{1}{2}\sec^{2}\frac{x}{2}$

It is known that, $\int e^{x}\{f(x)+f'(x)\}dx=e^{x}f(x)+C$

From equation (1), we obtain

$$\int\frac{e^{x}(1+\sin x)}{(1+\cos x)}dx=e^{x}\tan\frac{x}{2}+C$$

Answer needs Correction? **Click Here**

Q19 : $e^{x}\left(\frac{1}{x}-\frac{1}{x^{2}}\right)$

Answer :

Let $I=\int e^{x}\left[\frac{1}{x}-\frac{1}{x^{2}}\right]dx$

Also, let $\frac{1}{x}=f(x) \Rightarrow f'(x)=\frac{-1}{x^{2}}$

It is known that, $\int e^{x}\{f(x)+f'(x)\}dx=e^{x}f(x)+C$

$$\therefore I=\frac{e^{x}}{x}+C$$

Answer needs Correction? **Click Here**

Q20 : $\frac{(x-3)e^{x}}{(x-1)^{3}}$

Answer :

$$\int e^{x}\left\{\frac{x-3}{(x-1)^{3}}\right\}dx=\int e^{x}\left\{\frac{x-1-2}{(x-1)^{3}}\right\}dx$$

$$=\int e^{x}\left\{\frac{1}{(x-1)^{2}}-\frac{2}{(x-1)^{3}}\right\}dx$$

Let $f(x)=\frac{1}{(x-1)^{2}} \Rightarrow f'(x)=\frac{-2}{(x-1)^{3}}$

It is known that, $\int e^{x}\{f(x)+f'(x)\}dx=e^{x}f(x)+C$

$$\therefore \int e^{x}\left\{\frac{(x-3)}{(x-1)^{2}}\right\}dx=\frac{e^{x}}{(x-1)^{2}}+C$$

Answer needs Correction? **Click Here**

Q21 : $e^{2x}\sin x$

Answer :

Let $I=\int e^{2x}\sin x\,dx \quad ...(1)$

Integrating by parts, we obtain

$$I=\sin x\int e^{2x}dx-\int\left\{\left(\frac{d}{dx}\sin x\right)\int e^{2x}dx\right\}dx$$

$$\Rightarrow I=\sin x\cdot\frac{e^{2x}}{2}-\int\cos x\cdot\frac{e^{2x}}{2}dx$$

$$\Rightarrow I=\frac{e^{2x}\sin x}{2}-\frac{1}{2}\int e^{2x}\cos x\,dx$$

Again integrating by parts, we obtain

$$I=\frac{e^{2x}\cdot\sin x}{2}-\frac{1}{2}\left[\cos x\int e^{2x}dx-\int\left\{\left(\frac{d}{dx}\cos x\right)\int e^{2x}dx\right\}dx\right]$$

$$\Rightarrow I=\frac{e^{2x}\sin x}{2}-\frac{1}{2}\left[\cos x\cdot\frac{e^{2x}}{2}-\int(-\sin x)\frac{e^{2x}}{2}dx\right]$$

$$\Rightarrow I=\frac{e^{2x}\cdot\sin x}{2}-\frac{1}{2}\left[\frac{e^{2x}\cos x}{2}+\frac{1}{2}\int e^{2x}\sin x dx\right]$$

$$\Rightarrow I=\frac{e^{2x}\sin x}{2}-\frac{e^{2x}\cos x}{4}-\frac{1}{4}I \quad \text{[From (1)]}$$

$$\Rightarrow I+\frac{1}{4}I=\frac{e^{2x}\cdot\sin x}{2}-\frac{e^{2x}\cos x}{4}$$

$$\Rightarrow \frac{5}{4}I=\frac{e^{2x}\sin x}{2}-\frac{e^{2x}\cos x}{4}$$

$$\Rightarrow I=\frac{4}{5}\left[\frac{e^{2x}\sin x}{2}-\frac{e^{2x}\cos x}{4}\right]+C$$

$$\Rightarrow I=\frac{e^{2x}}{5}[2\sin x-\cos x]+C$$

Answer needs Correction? **Click Here**

Q22 : $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Answer :

Let $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

$$\therefore \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta$$

$$\Rightarrow \int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx = \int 2\theta\cdot\sec^2\theta\, d\theta = 2\int \theta\cdot\sec^2\theta\, d\theta$$

Integrating by parts, we obtain

$$2\left[\theta\cdot\int\sec^2\theta d\theta - \int\left\{\left(\frac{d}{d\theta}\theta\right)\int\sec^2\theta d\theta\right\}d\theta\right]$$
$$= 2\left[\theta\cdot\tan\theta - \int\tan\theta d\theta\right]$$
$$= 2\left[\theta\tan\theta + \log|\cos\theta|\right] + C$$
$$= 2\left[x\tan^{-1}x + \log\left|\frac{1}{\sqrt{1+x^2}}\right|\right] + C$$
$$= 2x\tan^{-1}x + 2\log(1+x^2)^{-\frac{1}{2}} + C$$
$$= 2x\tan^{-1}x + 2\left[-\frac{1}{2}\log(1+x^2)\right] + C$$
$$= 2x\tan^{-1}x - \log(1+x^2) + C$$

Answer needs Correction? Click Here

Q23 : $\int x^2 e^{x^3} dx$ equals

(A) $\frac{1}{3}e^{x^3} + C$ (B) $\frac{1}{3}e^{x^2} + C$

(C) $\frac{1}{2}e^{x^3} + C$ (D) $\frac{1}{3}e^{x^2} + C$

Answer :

Let $I = \int x^2 e^{x^3} dx$

Also, let $x^3 = t \Rightarrow 3x^2 dx = dt$

$$\Rightarrow I = \frac{1}{3}\int e^t dt$$
$$= \frac{1}{3}(e^t) + C$$
$$= \frac{1}{3}e^{x^3} + C$$

Hence, the correct answer is A.

Answer needs Correction? Click Here

Q24 : $\int e^x \sec x(1+\tan x)dx$ equals

(A) $e^x \cos x + C$ (B) $e^x \sec x + C$

(C) $e^x \sin x + C$ (D) $e^x \tan x + C$

Answer :

$\int e^x \sec x(1+\tan x)dx$

Let $I = \int e^x \sec x(1+\tan x)dx = \int e^x(\sec x + \sec x\tan x)dx$

Also, let $\sec x = f(x) \Rightarrow \sec x\tan x = f'(x)$

It is known that, $\int e^x\{f(x)+f'(x)\}dx = e^x f(x) + C$

$\therefore I = e^x \sec x + C$

Hence, the correct answer is B.

Answer needs Correction? Click Here

*********** END **********