Mean Value Theorems Ex 15.1 Q3(iv)

Here,

$f(x) = e^x \times \sin x \quad$ on $[0, \pi]$

We know that since and expential function are continuous and differentiable every where so, $f(x)$ is continuous is $[0, \pi]$ and differentiable is $(0, \pi)$.

Now,

$f(0) = e^0 \sin 0 = 0$

$f(\pi) = e^\pi \sin \pi = 0$

$\Rightarrow \quad f(0) = f(\pi)$

So, Rolle's theorem is applicable, so there must exist a point $c \in (0, \pi)$ such that $f'(c) = 0$.

Now,

$f(x) = e^x \sin x$

$f'(x) = e^x \cos x + e^x \sin x$

Now, $\quad f'(c) = 0$

$e^c (\cos c + \sin c) = 0$

$\Rightarrow \quad e^c = 0$ or $\cos c = -\sin c$

$\Rightarrow \quad e^c = 0$ gives no value of c or $\tan c = -1$

$\Rightarrow \quad \tan c = \tan\left(\pi - \frac{\pi}{4}\right)$

$c = \frac{3\pi}{4} \in (0, \pi)$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(v)

Here,

$$f(x) = e^x \cos x \text{ on } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

We know that expontial and cosine function are continuous and differentiable every where so, $f(x)$ is continuous is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and differentiable is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Now,

$$f\left(-\frac{\pi}{2}\right) = e^{\frac{\pi}{2}} \cos\left(-\frac{\pi}{2}\right) = 0$$

$$f\left(\frac{\pi}{2}\right) = e^{\frac{\pi}{2}} \cos\left(\frac{\pi}{2}\right) = 0$$

$$\Rightarrow \quad f\left(-\frac{\pi}{2}\right) = f\left(\frac{\pi}{2}\right)$$

So, Rolle's theorem is applicable, so there must exist a point $c \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $f'(c) = 0$.

Now,

$$f(x) = e^x \cos x$$

$$f'(x) = -e^x \sin x + e^x \cos x$$

So, $\quad f'(c) = 0$

$$e^c(-\sin c + \cos c) = 0$$

$\Rightarrow \quad e^c = 0$ gives no value of c

$$\Rightarrow \quad -\sin c + \cos c = 0$$

$$\Rightarrow \quad \tan c = 1$$

$$\Rightarrow \quad c = \frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(vi)

Here,

$$f(x) = \cos 2x \text{ on } [0, \pi]$$

We know that, cosine function is contiuous and differentiable every where, so $f(x)$ is continuous is $[0, \pi]$ and differentiable is $(0, \pi)$.

Now,

$$f(0) = \cos 0 = 1$$

$$f(\pi) = \cos(2\pi) = 1$$

$$\Rightarrow \quad f(0) = f(\pi)$$

So, Rolle's theorem is applicable, so there must exist a point $c \in (0, \pi)$ such that $f'(c) = 0$.

Now,

$$f(x) = \cos 2x$$

$$f'(x) = -2\sin 2x$$

So, $\quad f'(c) = 0$

$$\Rightarrow \quad -2\sin 2c = 0$$

$$\Rightarrow \quad \sin 2c = 0$$

$$\Rightarrow \quad 2c = 0 \quad \text{or} \quad 2c = \pi$$

$$\Rightarrow \quad c = 0 \quad \text{or} \quad c = \frac{\pi}{2} \in (0, \pi)$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(vii)

$f(x) = \frac{\sin x}{e^x}$ *on* $x \in [0, \pi]$

We know that, exponential and sine both functions are continuous and differentiable every where, so $f(x)$ *is continuous is* $[0, \pi]$ *and differentiable is* $[0, \pi]$

Now,

$$f(0) = \frac{\sin 0}{e^0} = 0$$

$$f(\pi) = \frac{\sin \pi}{e^\pi} = 0$$

$\Rightarrow f(0) = f(\pi)$

Since Rolle's theorem applicable, therefore there must exist a point $c \in [0, \pi]$ such that $f'(c) = 0$

Now,

$$f(x) = \frac{\sin x}{e^x}$$

$$\Rightarrow f'(x) = \frac{e^x(\cos x) - e^x(\sin x)}{(e^x)^2}$$

Now,

$$f'(c) = 0$$

$$\Rightarrow e^c(\cos c - \sin c) = 0$$

$$\Rightarrow e^c \neq 0 \text{ and } \cos c - \sin c = 0$$

$$\Rightarrow \tan c = 1$$

$$c = \frac{\pi}{4} \in [0, \pi]$$

Hence, Rolle's theorem is verified.

*********** END **********