Indefinite Integrals Ex 19.25 Q46

Let $I = \int \left(e^{\log x} + \sin x\right) \cos x \, dx$

$= \int \left(x + \sin x\right) \cos x \, dx$

$= \int x \cos x \, dx + \int \sin x \cos x \, dx$

$= \left[x \int \cos x \, dx - \int \left(1 \int \cos x \, dx\right) dx\right] + \frac{1}{2} \int \sin 2x \, dx$

$= \left[x \sin x - \int \sin x \, dx\right] + \frac{1}{2}\left(-\frac{\cos 2x}{2}\right) + c$

$$I = x \sin x + \cos x - \frac{1}{4} \cos 2x + c$$

$$I = x \sin x + \cos x - \frac{1}{4}\left[1 - 2\sin^2 x\right] + c$$

$$I = x \sin x + \cos x - \frac{1}{4} + \frac{1}{2}\sin^2 x + c$$

$$I = x \sin x + \cos x + \frac{1}{2}\sin^2 x + c - \frac{1}{4}$$

$$I = x \sin x + \cos x + \frac{1}{2}\sin^2 x + k, \text{ where } k = c - \frac{1}{4}$$

Indefinite Integrals Ex 19.25 Q47

Let $I = \int \frac{\left(x \tan^{-1} x\right)}{\left(1 + x^2\right)^{\frac{3}{2}}} dx$

Let $\tan^{-1} x = t$

$\frac{1}{1 + x^2} dx = dt$

$I = \int \frac{t \tan t}{\sqrt{1 + \tan^2 t}} dt$

$= \int \frac{t \tan t}{\sec t} dt$

$= \int t \frac{\sin t}{\cos t} \cos t \, dt$

$= \int t \sin t \, dt$

$= \left[t \int \sin t dt - \int \left(1 \int \sin t \, dt\right) dt\right]$

$= \left[-t \cos t + \int \cos t \, dt\right]$

$= \left[-t \cos t + \sin t\right] + c$

$$I = -\frac{\tan^{-1} x}{\sqrt{1 + x^2}} + \frac{x}{\sqrt{1 + x^2}} + c$$

Indefinite Integrals Ex 19.25 Q48

Let $\quad I = \int \tan^{-1}\left(\sqrt{x}\right)dx$

Let $\quad x = t^2$

$dx = 2t\,dt$

$I = \int 2t\tan^{-1} t\,dt$

$= 2\left[\tan^{-1} t \int t\,dt - \int\left(\frac{1}{1+t^2}\int t\,dt\right)dt\right]$

$= 2\left[\frac{t^2}{2}\tan^{-1} t - \int\frac{t^2}{2\left(1+t^2\right)}dt\right]$

$= t^2\tan^{-1} t - \int\frac{t^2+1-1}{1+t^2}dt$

$= t^2\tan^{-1} t - \int\left(1-\frac{1}{1+t^2}\right)dt$

$= t^2\tan^{-1} t - t + \tan^{-1} t + c$

$= \left(t^2+1\right)\tan^{-1} t - t + c$

$I = \left(x+1\right)\tan^{-1}\sqrt{x} - \sqrt{x} + c$

Indefinite Integrals Ex 19.25 Q49

$\int x^3\tan^{-1} x\,dx =$

$\int x^3\tan^{-1} x\,dx = \tan^{-1} x\int x^3 dx - \left(\int\frac{d\tan^{-1} x}{dx}\left(\int x^3 dx\right)dx\right)$

$= \tan^{-1} x\,\frac{x^4}{4} - \left(\int\frac{1}{1+x^2}\left(\frac{x^4}{4}\right)dx\right)$

$= \tan^{-1} x\,\frac{x^4}{4} - \left(\int\frac{1}{1+x^2}\left(\frac{x^4}{4}\right)dx\right)$

$\int\frac{1}{1+x^2}\left(\frac{x^4}{4}\right)dx = \frac{1}{4}\left[\int\frac{1}{1+x^2}dx + (x^2-1)dx\right]$

$\int\frac{1}{1+x^2}\left(\frac{x^4}{4}\right)dx = \frac{1}{4}\left[\tan^{-1} x + \frac{x^3}{3} - x\right]$

$\int x^3\tan^{-1} x\,dx = \frac{x^4}{4}\tan^{-1} x - \frac{1}{4}\left[\tan^{-1} x + \frac{x^3}{3} - x\right] + C$

Indefinite Integrals Ex 19.25 Q50

Let $\quad I = \int x \sin x \cos 2x \, dx$

$$= \frac{1}{2}\int x\left(2\sin x\cos 2x\right)dx$$

$$= \frac{1}{2}\int x\left(\sin\left(x+2x\right)-\sin\left(2x-x\right)\right)dx$$

$$= \frac{1}{2}\int x\left(\sin 3x-\sin x\right)dx$$

$$= \frac{1}{2}\left[x\int\left(\sin 3x-\sin x\right)dx-\int\left(1\int\left(\sin 3x-\sin x\right)dx\right)dx\right]$$

$$= \frac{1}{2}\left[x\left(\frac{-\cos 3x}{3}+\cos x\right)-\int\left(-\frac{\cos 3x}{3}+\cos x\right)dx\right]$$

$$I = \frac{1}{2}\left[-x\,\frac{\cos 3x}{3}+x\cos x+\frac{1}{9}\sin 3x-\sin x\right]+c$$

*********** END **********