Polynomials Ex 2.2 Q1

Answer :

We have,

$$f(x) = 2x^3 + 2x^2 - 5x + 2$$

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$$

$$f\left(\frac{1}{2}\right) = \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2$$

$$f\left(\frac{1}{2}\right) = 0$$

$$f(1) = 2(1)^3 + (1)^2 - 5(1) + 2$$

$$f(1) = 2 + 1 - 5 + 2$$

$$f(1) = 0$$

$$f(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$$

$$f(-2) = -16 + 4 + 10 + 2$$

$$f(-2) = 0$$

So, $\frac{1}{2}, 1$ and 2 are the zeros of polynomial p(x)

Let $\alpha = \frac{1}{2}, \beta = 1$ and $\gamma = -2$. Then

$$\alpha + \beta + \gamma = \frac{1}{2} + 1 - 2$$

$$= \frac{1}{2} + \frac{1 \times 2}{1 \times 2} - \frac{2 \times 2}{1 \times 2}$$

$$= \frac{1 + 2 - 4}{2}$$

$$= \frac{-1}{2}$$

From $f(x) = 2x^3 + 2x^2 - 5x + 2$

$$\alpha + \beta + \gamma = \frac{-\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$$

$$\alpha + \beta + \gamma = -\left(\frac{1}{2}\right)$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{1}{2}(1) + 1(-2) - 2\left(\frac{1}{2}\right)$$

$$= \frac{1}{2} - 2 - \frac{2}{2}$$

Taking least common factor we get,

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{1}{2} - \frac{2 \times 2}{1 \times 2} - \frac{2}{2}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{1 - 4 - 2}{2}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{1 - 6}{2}$$

$\alpha\beta + \beta\gamma + \gamma\alpha = \dfrac{-5}{2}$

From $f(x) = 2x^3 + 2x^2 - 5x + 2$

$\alpha\beta + \beta\gamma + \gamma\alpha = \dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$

$\alpha\beta + \beta\gamma + \gamma\alpha = \dfrac{-5}{2}$

$\alpha\beta\gamma = \dfrac{1}{2} \times 1 \times (-2)$

$\alpha\beta\gamma = \dfrac{\not{2}}{\not{2}}$

$\alpha\beta\gamma = -1$

From $f(x) = 2x^3 + 2x^2 - 5x + 2$

$\alpha\beta\gamma = \dfrac{-\text{Constant term}}{\text{Coefficient of } x^2}$

$\alpha\beta\gamma = -\left(\dfrac{\not{2}}{\not{2}}\right)$

$\alpha\beta\gamma = -1$

Hence, it is verified that the numbers given along side of the cubic polynomials are their zeros and also verified the relationship between the zeros and coefficients

(ii) We have,

$g(x) = x^3 - 4x^2 + 5x - 2$

$g(2) = (2)^3 - 4(2)^2 + 5(2) - 2$

$g(2) = 8 - 16 + 10 - 2$

$g(2) = 0$

$g(1) = (1)^3 - 4(1)^2 + 5(1) - 2$

$\quad = 1 - 4 + 5 - 2$

$\quad = 0$

$g(1) = (1)^3 - 4(1)^2 + 5(1) - 2$

$\quad = 1 - 4 + 5 - 2$

$\quad = 0$

So 2, 1 and 1 are the zeros of the polynomial g(x)

Let $\alpha = 2, \beta = 1$ and $\lambda = 1$. Then,

$\alpha + \beta + \gamma = 2 + 1 + 1$

$\alpha + \beta + \gamma = 4$

From $g(x) = x^3 - 4x^2 + 5x - 2$

$\alpha + \beta + \gamma = \dfrac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

$\alpha + \beta + \gamma = \dfrac{4}{1}$

$\alpha + \beta + \gamma = 4$

$\alpha\beta + \beta\gamma + \gamma\alpha = 2(1) + 1(1) + 1(2)$

$\alpha\beta + \beta\gamma + \gamma\alpha = 2 + 1 + 2$

$\alpha\beta + \beta\gamma + \gamma\alpha = 5$

From $g(x) = x^3 - 4x^2 + 5x - 2$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{5}{1}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 5$$

$$\alpha\beta\gamma = 2 \times 1 \times 1$$

$$\alpha\beta\gamma = 2$$

From $g(x) = x^3 - 4x^2 + 5x - 2$

$$\alpha\beta\gamma = \frac{-\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\alpha\beta\gamma = -\left(\frac{-2}{1}\right)$$

$$\alpha\beta\gamma = 2$$

Hence, it is verified that the numbers given along side of the cubic polynomials are their zeros and also verified the relationship between the zeros and coefficients.

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