Q1. In figure, A, B, C are three points on a circle with centre O such that $\angle BOC = 30°$; $\angle AOB = 60°$. If D is a point on the circle other than the arc ABC, find $\angle ADC$.
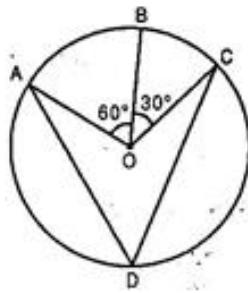
Ans. $\angle AOC = \angle AOB + \angle BOC$

$\Rightarrow \angle AOC = 60° + 30° = 90°$

Now $\angle AOC = 2\angle ADC$

[∵ Angled subtended by an arc, at the centre of the circle is double the angle subtended by the same arc at any point in the remaining part of the circle]

$\Rightarrow \angle ADC = \dfrac{1}{2} \angle AOC$

$\Rightarrow \angle ADC = \dfrac{1}{2} \times 90° = 45°$

Q2. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord on a point on the minor arc and also at a point on the major arc.

Ans. Let AB be the minor arc of circle.
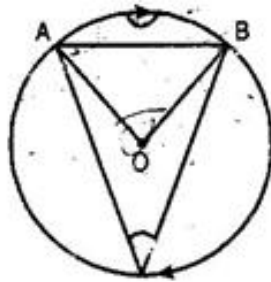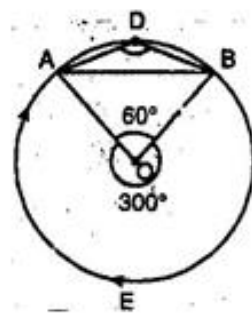

∴ Chord AB = Radius OA = Radius OB

∴ $\triangle$ AOB is an equilateral triangle.

$\Rightarrow \angle$ AOB = 60°

Now $m\overarc{AB} + m\overarc{BA} = 360°$

$\Rightarrow \angle$ AOB + $\angle$ BOA = 360°

$\Rightarrow$ 60°+ $\angle$ BOA = 360°

$\Rightarrow \angle\,\text{BOA} = 360° - 60° = 300°$

D is a point in the minor arc.

$\therefore m\overset{\frown}{\text{BA}} = 2\angle\,\text{BDA}$

$\Rightarrow \angle\,\text{BOA} = 2\angle\,\text{BDA}$

$\Rightarrow \angle\,\text{BDA} = \dfrac{1}{2}\,\angle\,\text{BOA} = \dfrac{1}{2} \times 300°$

$\Rightarrow \angle\,\text{BDA} = 150°$
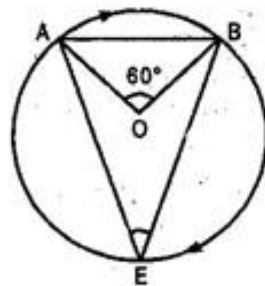
Thus angle subtended by major arc, $\overset{\frown}{\text{BA}}$ at any point D in the minor arc is $150°$.

Let E be a point in the major arc $\overset{\frown}{\text{BA}}$.

$\therefore m\overset{\frown}{\text{AB}} = 2\angle\,\text{AEB}$

$\Rightarrow \angle\,\text{AOB} = 2\angle\,\text{AEB}$

$\Rightarrow \angle\,\text{AEB} = \dfrac{1}{2}\,\angle\,\text{AOB}$

$\Rightarrow \angle\,\text{AEB} = \dfrac{1}{2} \times 60° = 30°$

Q3. In figure, $\angle PQR = 100°$ where P, Q, R are points on a circle with centre O. Find $\angle OPR$.
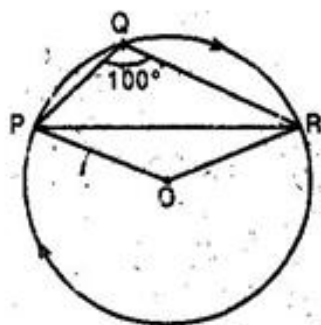
Ans. In the figure, Q is a point in the minor arc $\overparen{PQR}$.

$\therefore m\overparen{RP} = 2\angle PQR$

$\Rightarrow \angle ROP = 2\angle PQR$

$\Rightarrow \angle ROP = 2 \times 100° = 200°$

Now $m\overparen{PR} + m\overparen{RP} = 360°$

$\Rightarrow \angle POR + \angle ROP = 360°$

$\Rightarrow \angle POR + 200° = 360°$

$\Rightarrow \angle POR = 360° - 200° = 160°.....$(i)

Now $\triangle$ OPR is an isosceles triangle.

$\therefore$ OP = OR [radii of the circle]

$\Rightarrow \angle OPR = \angle ORP$ [angles opposite to equal sides are equal] .....(ii)

Now in isosceles triangle OPR,
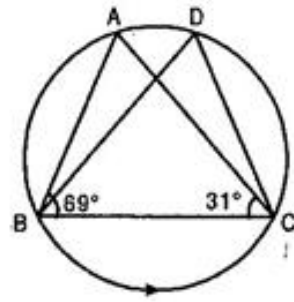
$\angle OPR + \angle ORP + \angle POR = 180°$

$\Rightarrow \angle OPR + \angle ORP + 160° = 180°$

$\Rightarrow 2\angle OPR = 180° - 160°$ [Using (i) & (ii)]

$\Rightarrow 2\angle OPR = 20°$

$\Rightarrow \angle OPR = 10°$

Q4. In figure, $\angle ABC = 69°$ $\angle ACB = 31°$ find $\angle$ BDC.

Ans. In triangle ABC,

$\angle BAC + \angle ABC + \angle ACB = 180°$

$\Rightarrow \angle BAC + 69° + 31° = 180°$

$\Rightarrow \angle BAC = 180° - 69° - 31°$
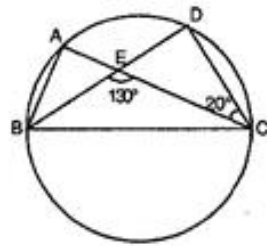
$\Rightarrow \angle BAC = 80°$........(i)

Since, A and D are the points in the same segment of the circle.

$\therefore \angle BDC = \angle BAC$

[Angles subtended by the same arc at any points in the alternate segment of a circle are equal]

$\Rightarrow \angle BDC = 80°$[Using (i)]

Q5. In figure, A, B, C, D are four points on a circle. AC and BD intersect at a point E such that $\angle$ BEC = 130°and

$\angle$ **ECD =20°Find** $\angle$ **BAC.**

Ans. Given: $\angle$ BEC = $130°$ and $\angle$ ECD = $20°$

$\angle$ DEC = $180° - \angle$ BEC = $180° - 130° = 50°$ [Linear pair]

Now in $\triangle$DEC,

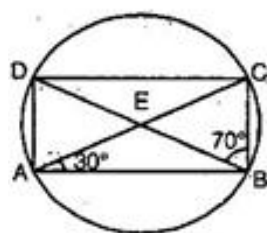$\angle$ DEC + $\angle$ DCE + $\angle$ EDC = $180°$ [Angle sum property]

$\Rightarrow 50° + 20° + \angle$ EDC = $180°$

$\Rightarrow \angle$ EDC = $110°$

$\Rightarrow \angle$ BAC = $\angle$ EDC = $110°$ [Angles in same segment]

Q6. ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. $\angle$ DBC = 70°, $\angle$ BAC is $30°$ find $\angle$ BCD. Further if AB = BC, find $\angle$ ECD.

Ans. For chord CD

$\angle BCD = 80°$ $\angle CBD = \angle CAD$ (Angles in same segment)

$\angle CAD = 70°$

$\angle BAD = \angle BAC + \angle CAD$

$= 30° + 70° = 100°$

$\angle BCD + \angle BAD = 180°$ (Opposite angles of a cyclic quadrilateral)

$\angle BCD + 100 = 180°$

$\angle BCD = 80°$
$In \triangle ABC$

AB = BC (given)
$\therefore \angle BCA = \angle CAB$ (Angles opposite to equal sides of a triangle)

$\angle BCA = 30°$
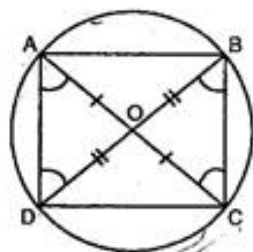
We have $\angle BCD = 80°$ $\angle BCA + \angle ACD = 80°$

$30° + \angle ACD = 80°$

$\angle ACD = 50°$

$\angle ECD = 50°$

Q7. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Ans. Let ABCD a cyclic quadrilateral having diagonals as BD and AC intersecting each other at point O.

$$\angle BAD = \frac{1}{2} \angle BOD = \frac{180°}{2} = 90°$$

(Consider BD as a chord) $\angle BCD + \angle BAD = 180°$ (Cyclic quadrilateral)

$$\angle BCD = 180° - 90° = 90°$$

$$\angle ADC = \frac{1}{2} \angle AOC = \frac{180°}{2} = 90°$$

(Considering AC as a chord)

$\angle ADC + \angle ABC = 180°$ (Cyclic quadrilateral)

$$90° + \angle ABC = 180°$$

$$\angle ABC = 90°$$

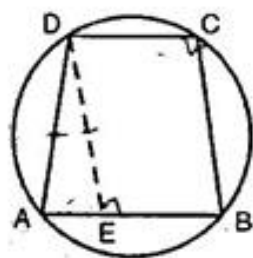Here, each interior angle of cyclic quadrilateral is of $90°$. Hence it is a rectangle.

Q8. If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

Ans. Given: A trapezium ABCD in which AB∥ CD and AD = BC.

To prove: The points A, B, C, D are concyclic.

Construction: Draw DE $\parallel$ CB.

Proof: Since DE $\parallel$ CB and EB $\parallel$ DC.

∴ EBCD is a parallelogram.

∴ DE = CB and $\angle$ DEB = $\angle$ DCB

Now AD = BC and DA = DE

$\Rightarrow \angle$ DAE = $\angle$ DEB

But $\angle$ DEA + $\angle$ DEB = $180°$
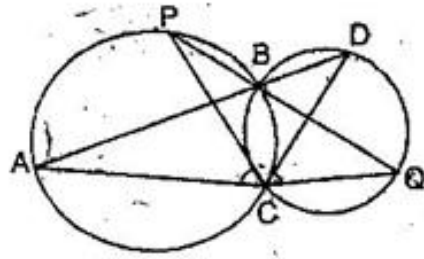
$\Rightarrow \angle$ DAE + $\angle$ DCB = $180°$

[∵ $\angle$ DEA = $\angle$ DAE and $\angle$ DEB = $\angle$ DCB]

$\Rightarrow \angle$ DAB + $\angle$ DCB = $180°$

$\Rightarrow \angle$ A + $\angle$ C = $180°$

Hence, ABCD is a cyclic trapezium.

Q9. Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D, P, Q respectively (see figure). Prove that $\angle$ ACP = $\angle$ QCD.

Ans. In triangles ACD and QCP,

$\angle$ A = $\angle$ P and $\angle$ Q = $\angle$ D [Angles in same segment]

$\therefore$ $\angle$ ACD = $\angle$ QCP [Third angles] ....(i)

Subtracting $\angle$ PCD from both the sides of eq. (i), we get,

$\angle$ ACD – $\angle$ PCD = $\angle$ QCP – $\angle$ PCD

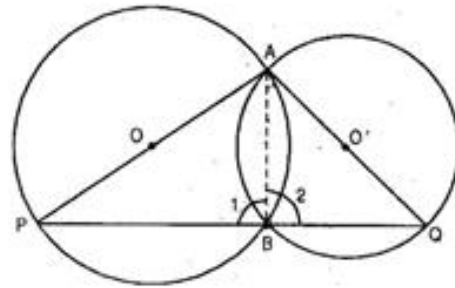$\Rightarrow$ $\angle$ ACPO = $\angle$ QCD

Hence proved.

Q10. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Ans. Given: Two circles intersect each other at points A and B. AP and AQ be their respective diameters.

To prove: Point B lies on the third side PQ.

Construction: Join A and B.

Proof: AP is a diameter.

$\therefore \angle 1 = 90°$

[Angle in semicircle]

Also AQ is a diameter.

$\therefore \angle 2 = 90°$
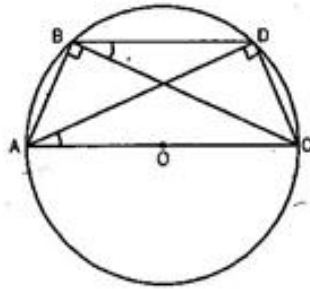
[Angle in semicircle]

$\angle 1 + \angle 2 = 90° + 90°$

$\Rightarrow \angle PBQ = 180°$

$\Rightarrow PBQ$ is a line.

Thus point B. i.e. point of intersection of these circles lies on the third side i.e., on PQ.

Q11. ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle$ CAD = $\angle$ ABD.

Ans. We have ABC and ADC two right triangles, right angled at B and D respectively.

$\Rightarrow \angle$ ABC = ADC [Each 90°]

If we draw a circle with AC (the common hypotenuse) as diameter, this circle will definitely passes through of an arc AC, Because B and D are the points in the alternate segment of an arc AC.

Now we have $\overset{\frown}{CD}$ subtending $\angle$ CBD and $\angle$ CAD in the same segment.

$\therefore \angle$ CAD = $\angle$ CBD

Hence proved.