Chapter 9 Continuity Ex 9.2 Q1

When $x < 0$, we have, $f(x) = \frac{\sin x}{x}$

We know that $\sin x$ and the identity function x both are everywhere continuous.

So, the quotient function $\frac{\sin x}{x} = f(x)$ is continuous for $x < 0$

When $x > 0$, we have $f(x) = x + 1$, which being a polynomial, is continuous for $x > 0$

Let us now consider $x = 0$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{\sin(-h)}{-h} = \lim_{h \to 0} \frac{-\sin h}{-h} = 1$$

$$RHL = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{\sin h}{h} = 1$$

$f(0) = 0 + 1 = 1$

Thus, $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0) = 1$

$\therefore f(x)$ is contiunuos at $x = 0$

Hence, $f(x)$ is contiunuos everywhere.

Chapter 9 Continuity Ex 9.2 Q2

When $x \neq 0$,

$$f(x) = \frac{x}{|x|} = \begin{cases} \frac{-x}{x} = -1 \ ; \ x < 0 \\ \frac{x}{|x|} = 1 \ ; x > 0 \end{cases}$$

So, $f(x)$ is a constant function when $x \neq 0$

hence, is continuous for all $x < 0$ and $x > 0$

Now,

Consider the point $x = 0$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{-h}{|-h|} = -1$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{h}{|h|} = 1$$

So, LHL $\neq$ RHL

Hence, function is discontinuous at $x = 0$

*********** END **********