Functions Ex 2.5 Q12

Given that $f(x) = 2x$ *and* $g(x) = x + 2$.

We need to prove that f *and* g *are bijective maps.*

Let $x, y \in Q$.

Consider $f(x) = f(y)$

$\Rightarrow 2x = 2y$

$\Rightarrow x = y$

$\Rightarrow f$ *is one – one.*

Let y *be an arbitrary element of* Q *such that* $f(x) = y$

Then $f(x) = y = 2x \Rightarrow x = \dfrac{y}{2}$

Thus, for any $y \in Q$, *there exists* $x = \dfrac{y}{2} \in Q$ *such that,*

$f(x) = f\left(\dfrac{y}{2}\right) = 2\dfrac{y}{2} = y$

So $f : Q \to Q$ *is a bijection and hence invertible.*

Let f^{-1} *denote the inverse of* f.

Thus, $f^{-1}(x) = \dfrac{x}{2} \ldots(1)$

Let $x, y \in Q$.

Consider $g(x) = g(y)$

$\Rightarrow x + 2 = y + 2$

$\Rightarrow x = y$

$\Rightarrow g$ *is one – one.*

Let y *be an arbitrary element of* Q *such that* $g(x) = y$

Then $g(x) = y = x + 2 \Rightarrow x = y - 2$

Thus, for any $y \in Q$, *there exists* $x = y - 2$, $y \in Q$ *such that,*

$g(x) = g(y - 2) = y - 2 + 2 = y$

So $g : Q \to Q$ *is a bijection and hence invertible.*

Let g^{-1} *denote the inverse of* g.

Thus, $g^{-1}(x) = x - 2 \ldots(2)$

$Now\ consider\ g \circ f = g[f(x)] = g(2x) = 2x + 2$

$Thus,\ (g \circ f)^{-1} = \frac{x-2}{2} \ldots(3)$

$From\ (1)\ and\ (2),\ we\ have$

$f^{-1} \circ g^{-1} = f^{-1}[g^{-1}(x)] = f^{-1}[x-2] = \frac{x-2}{2} \ldots(4)$

$From\ (3)\ and\ (4),\ it\ is\ clear\ that$

$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

Functions Ex 2.5 Q13

$$Given\ that\ f(x) = \frac{x-2}{x-3};$$

$$Let\ f(x) = y;$$

$$\Rightarrow y = \frac{x-2}{x-3}$$

$$Interchange\ x\ and\ y;$$

$$\Rightarrow x = \frac{y-2}{y-3}$$

$$\Rightarrow (y-3)x = y-2$$

$$\Rightarrow xy - 3x = y - 2$$

$$\Rightarrow xy - y = 3x - 2$$

$$\Rightarrow y(x-1) = 3x - 2$$

$$\Rightarrow y = \frac{3x-2}{x-1}$$

$$\Rightarrow f^{-1}(x) = \frac{3x-2}{x-1}$$

Functions Ex 2.5 Q14

$f : R^+ \to [-9, \infty)$ given by $f(x) = 5x^2 + 6x - 9$

For any $x,\ y \in R^+$

$f(x) = f(y)$

$\Rightarrow 5x^2 + 6x - 9 = 5y^2 + 6y - 9$

$\Rightarrow 5(x^2 - y^2) + 6(x - y) = 0$

$\Rightarrow (x - y)[5(x + y) + 6] = 0$

$\Rightarrow x - y = 0 \qquad [\because 5(x + y) + 6 \neq 0 \text{ as } x, y \in R^+]$

$\Rightarrow x = y$

So, f is an injection.

Let y be an arbitrary element of $[-9, \infty)$.

$f(x) = y$

$\Rightarrow 5x^2 + 6x - 9 = y$

$\Rightarrow 25x^2 + 30x - 45 = 5y$

$\Rightarrow 25x^2 + 30x + 9 - 54 = 5y$

$\Rightarrow (5x + 3)^2 = 5y + 54$

$\Rightarrow (5x + 3) = \sqrt{5y + 54}$

$\Rightarrow x = \dfrac{\sqrt{5y + 54} - 3}{5}$

Now, $y \in [-9, \infty)$

$\Rightarrow y \geq -9$

$\Rightarrow 5y + 54 \geq 9$

$\Rightarrow \sqrt{5y + 54} \geq 3$

$\Rightarrow \sqrt{5y + 54} - 3 \geq 0$

$\Rightarrow \dfrac{\sqrt{5y + 54} - 3}{5} \geq 0$

$\Rightarrow x \geq 0 \Rightarrow x \in R^+$

Thus, for every $y \in [-9, \infty)$ there exist $x = \dfrac{\sqrt{5y + 54} - 3}{5} \in R^+$ such that $f(x) = y$.

So, $f : R^+ \to [-9, \infty)$ is onto.

Thus, $f : R^+ \to [-9, \infty)$ is a bijection and hence invertible.

Let f^{-1} denote the inverse of f.

Then,

$(fof^{-1})(y) = y$ for all $y \in [-9, \infty)$

$f(f^{-1}(y)) = y$ for all $y \in [-9, \infty)$

$\Rightarrow 5\{f^{-1}(y)\}^2 + 6\{f^{-1}(y)\} - 9 = y$ for all $y \in [-9, \infty)$

$\Rightarrow 25\{f^{-1}(y)\}^2 + 30\{f^{-1}(y)\} - 45 = 5y$ for all $y \in [-9, \infty)$

$\Rightarrow 25\{f^{-1}(y)\}^2 + 30\{f^{-1}(y)\} + 9 = 5y + 54$ for all $y \in [-9, \infty)$

$\Rightarrow \{5f^{-1}(y) + 3\}^2 = 5y + 54$ for all $y \in [-9, \infty)$

$\Rightarrow 5f^{-1}(y) + 3 = \sqrt{5y + 54}$ for all $y \in [-9, \infty)$

$\Rightarrow f^{-1}(y) \dfrac{\sqrt{5y + 54} - 3}{5}$

*********** END **********