Differentiation Ex 11.5 Q44

Here,

$$e^{y} = y^{x}$$

Taking log on both the sides,

$$\log e^{y} = \log y^{x}$$

$$y \log e = x \log y \qquad \left[\text{Since, } \log a^{b} = b \log a, \log_{e} e = 1\right]$$

$$y = x \log y \qquad \text{---(i)}$$

Differentiating it with respect to x using product rule,

$$\frac{dy}{dx} = \frac{d}{dx}(x \log y)$$

$$= x \frac{dy}{dx}(\log y) + \log y \frac{d}{dx}(x)$$

$$\frac{dy}{dx} = \frac{x}{y}\frac{dy}{dx} + \log y\ (1)$$

$$\frac{dy}{dx}\left(1 - \frac{x}{y}\right) = \log y$$

$$\frac{dy}{dx}\left(\frac{y - x}{y}\right) = \log y$$

$$\frac{dy}{dx} = \frac{y \log y}{y - x}$$

$$\frac{dy}{dx} = \frac{y \log y}{\left(y - \frac{y}{\log y}\right)} \qquad \left[\text{Since, using equation (i)}\right]$$

$$= \frac{y \log y\ x \log y}{y \log y - y}$$

$$= \frac{y (\log y)^{2}}{y (\log y - 1)}$$

$$\frac{dy}{dx} = \frac{(\log y)^{2}}{(\log y - 1)}$$

Differentiation Ex 11.5 Q45

Here,

$$e^{x+y} - x = 0$$

$$e^{x+y} = x \qquad \text{---(i)}$$

Differentiating it with respect to x using chain rule,

$$\frac{d}{dx}\left(e^{x+y}\right) = \frac{d}{dx}(x)$$

$$e^{x+y}\frac{d}{dx}(x + y) = 1$$

$$x\left[1 + \frac{dy}{dx}\right] = 1 \qquad \left[\text{Using equatoin (i)}\right]$$

$$1 + \frac{dy}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{x} - 1$$

$$\frac{dy}{dx} = \frac{1 - x}{x}$$

Differentiation Ex 11.5 Q46

Here $y = x\sin(a+y)$

Differentiating it with respect to x using the chain rule and product rule,

$$\frac{dy}{dx} = x\frac{d}{dx}\sin(a+y) + \sin(a+y)\frac{dx}{dx}$$

$$\frac{dy}{dx} = x\cos(a+y)\frac{dy}{dx} + \sin(a+y)$$

$$(1 - x\cos(a+y))\frac{dy}{dx} = \sin(a+y)$$

$$\frac{dy}{dx} = \frac{\sin(a+y)}{(1 - x\cos(a+y))}$$

$$\frac{dy}{dx} = \frac{\sin(a+y)}{\left(1 - \frac{y}{\sin(a+y)}\cos(a+y)\right)} \qquad \left[\text{Since } \frac{y}{\sin(a+y)} = x\right]$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin(a+y) - y\cos(a+y)}$$

Differentiation Ex 11.5 Q47

Here $x\sin(a+y) + \sin a\cos(a+y) = 0$

Differentiating it with respect to x using the chain rule and product rule,

$$\frac{d}{dx}\left[x\sin(a+y) + \sin a\cos(a+y)\right] = 0$$

$$x\frac{d}{dx}\sin(a+y) + \sin(a+y)\frac{dx}{dx} + \sin a\frac{d}{dx}\cos(a+y) + \cos(a+y)\frac{d}{dx}\sin a = 0$$

$$x\cos(a+y)\left(0 + \frac{dy}{dx}\right) + \sin(a+y) + \sin a\left(-\sin(a+y)\frac{dy}{dx}\right) + 0 = 0$$

$$\left[x\cos(a+y) - \sin a\sin(a+y)\right]\frac{dy}{dx} + \sin(a+y) = 0$$

$$\frac{dy}{dx} = -\frac{\sin(a+y)}{x\cos(a+y) - \sin a\sin(a+y)}$$

$$\frac{dy}{dx} = \frac{-\sin(a+y)}{\left(-\frac{\sin a\cos(a+y)}{\sin(a+y)}\right)\cos(a+y) - \sin a\sin(a+y)} \qquad \left[\text{Since } x = -\frac{\sin a\cos(a+y)}{\sin(a+y)}\right]$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{(\sin a)\cos^2(a+y) + \sin a\sin^2(a+y)}$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{(\sin a)\left[\cos^2(a+y) + \sin^2(a+y)\right]}$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{(\sin a)} \qquad \left[\text{Since } \cos^2(a+y) + \sin^2(a+y) = 1\right]$$

Differentiation Ex 11.5 Q48

Here,

$$(\sin x)^y = x + y$$

Taking log on both the sides,

$$\log(\sin x)^y = \log(x+y)$$

$$y\log(\sin x) = \log(x+y) \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating it with respect to x using chain rule, product rule,

$$\frac{d}{dx}(y\log(\sin x)) = \frac{d}{dx}\log(x+y)$$

$$y\frac{d}{dx}\log\sin x + \log\sin x\frac{dy}{dx} = \frac{1}{x+y}\frac{d}{dx}(x+y)$$

$$\frac{y}{\sin x}\frac{d}{dx}(\sin x) + \log\sin x\frac{dy}{dx} = \frac{1}{(x+y)}\left[1 + \frac{dy}{dx}\right]$$

$$\frac{y(\cos x)}{(\sin x)} + \log\sin x\frac{dy}{dx} = \frac{1}{(x+y)} + \frac{1}{(x+y)}\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(\log\sin x - \frac{1}{x+y}\right) = \frac{1}{(x+y)} - y\cot x$$

$$\frac{dy}{dx}\left(\frac{(x+y)\log\sin x - 1}{(x+y)}\right) = \left(\frac{1 - y(x+y)\cot x}{x+y}\right)$$

$$\frac{dy}{dx} = \left(\frac{1 - y(x+y)\cot x}{(x+y)\log\sin x - 1}\right)$$

*********** END **********