Geometric Progressions Ex 20.4 Q6.

$a = 1$

$$a_n = a_{n+1} + a_{n+2} + a_{n+3} + \ldots\ldots\ldots$$
$$ar^{n-1} = ar^n + ar^{n+1} + ar^{n+2} + \ldots\ldots\ldots$$
$$ar^{n-1} = ar^n\left(1 + r + r^2 + \ldots\ldots\infty\right)$$
$$1 = r\left(\frac{1}{1-r}\right)$$
$$1 - r = r$$
$$1 = 2r$$
$$r = \frac{1}{2}$$

G.P. is $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\ldots\ldots$

Geometric Progressions Ex 20.4 Q 7

$a + ar = 5$

$a(1+r) = 5 - - - - - (1)$

$$a_n = 3\left(a_{n+1} + a_{n+2} + a_{n+3} + \ldots\ldots\ldots\right)$$
$$ar^{n-1} = 3\left(ar^n + ar^{n+1} + ar^{n+2} + \ldots\ldots\ldots\right)$$
$$ar^{n-1} = 3ar^n\left(1 + r + r^2 + \ldots\ldots\infty\right)$$
$$1 = 3r\left(\frac{1}{1-r}\right)$$
$$1 - r = 3r$$
$$1 = 4r$$
$$r = \frac{1}{4}$$
$$a(1+r) = 5$$
$$a\left(\frac{5}{4}\right) = 5$$
$$a = 4$$

G.P. is $4, 1, \frac{1}{4}, \frac{1}{16}, \ldots\ldots\ldots$

Geometric Progressions Ex 20.4 Q8

$$0.125125125\ldots\ldots\ldots = 0.\overline{125}$$
$$= 0.125 + 0.000125 + 0.000000125 + \ldots\ldots$$
$$= \frac{125}{10^3} + \frac{125}{10^6} + \frac{125}{10^9} + \ldots\ldots$$
$$= \frac{125}{10^3}\left(1 + \frac{1}{10^3} + \frac{1}{10^6} + \ldots\ldots\right)$$
$$= \frac{125}{10^3}\left(\frac{1}{1 - \frac{1}{1000}}\right)$$
$$= \frac{125}{1000}\left(\frac{1000}{999}\right)$$
$$0.125125125\ldots\ldots\ldots = \frac{125}{999}$$

Geometric Progressions Ex 20.4 Q 9

$$0.4\overline{23} = 0.4 + 0.0232323\ldots\ldots\ldots$$
$$= 0.4 + 0.023 + 0.00023 + \ldots\ldots\ldots$$
$$= 0.4 + \frac{23}{10^3} + \frac{23}{10^5} + \ldots\ldots\ldots$$
$$= 0.4 + \frac{23}{10^3}\left(1 + \frac{1}{10^2} + \frac{1}{10^4} + \ldots\ldots\right)$$
$$= 0.4 + \frac{23}{1000}\left(\frac{1}{1 - \frac{1}{100}}\right)$$
$$= 0.4 + \frac{23}{1000}\left(\frac{100}{99}\right)$$
$$= \frac{4}{10} + \frac{23}{990}$$
$$= \frac{396 + 23}{990}$$
$$0.4\overline{23} = \frac{419}{990}$$

Geometric Progressions Ex 20.4 Q 10

Let a be first term and r be common ratio of G.P. Here,

$$\frac{a_n}{(a_{n+1} + a_{n+2} + \ldots\infty)} = \frac{ar^{n-1}}{ar^n + ar^{n+1} + \ldots}$$
$$= \frac{ar^{n-1}}{ar^n\left(1 + r + r^2 + \ldots\infty\right)}$$
$$= \frac{ar^{n-1}}{ar^n\left(\frac{1}{1-r}\right)}$$
$$= \left(\frac{1-r}{r}\right)$$

Since r is a constant, so

$$\left(\frac{a_n}{a_{n+1} + a_{n+2} + \ldots\infty}\right) = k \text{ (constant)}$$

Such that $k = \left(\frac{1-r}{r}\right)$

********** END **********