



Sine and Cosine Formulae and their Applications Ex-10.1 Q9

$$\sin\left(\frac{B-C}{2}\right) = \frac{b-c}{a} \cos\frac{A}{2}$$

$$\text{Let } a = k \sin A, b = k \sin B, c = k \sin C$$

*RHS*

$$\begin{aligned} & \frac{b-c}{a} \cos\frac{A}{2} \\ &= \frac{k \sin B - k \sin C}{k \sin A} \cdot \cos\frac{A}{2} \\ &= \frac{\sin B - \sin C}{\sin A} \cdot \cos\frac{A}{2} \\ &= \frac{2 \cos\frac{B+C}{2} \cdot \sin\frac{B-C}{2}}{2 \sin\frac{A}{2} \cdot \cos\frac{A}{2}} \cos\frac{A}{2} \\ &= \frac{\cos\frac{\pi-A}{2} \sin\frac{B-C}{2}}{\sin\frac{A}{2}} \\ &= \frac{\sin\frac{A}{2} \sin\frac{B-C}{2}}{\sin\frac{A}{2}} = \sin\frac{B-C}{2} = \text{RHS} \end{aligned}$$

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$$\text{let } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

LHS,

$$\begin{aligned} & \frac{a^2 - c^2}{b^2} \\ &= \frac{k^2 \sin^2 A - k^2 \sin^2 C}{k^2 \sin^2 B} \\ &= \frac{k^2 (\sin^2 A - \sin^2 C)}{k^2 \sin^2 B} \\ &= \frac{(\sin^2 A - \sin^2 C)}{\sin^2 (\pi - (A + C))} \\ &= \frac{\sin(A + C) \sin(A - C)}{\sin^2(A + C)} \\ &= \frac{\sin(A - C)}{\sin(A + C)} = RHS \end{aligned}$$

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$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$$

RHS,

$$\begin{aligned} & a \sin(B - C) \\ &= a \sin B \cdot \cos C - a \sin C \cdot \cos B \\ &= a(bk) \cdot \left( \frac{a^2 + b^2 - c^2}{2ab} \right) - a(c k) \cdot \left( \frac{a^2 + c^2 - b^2}{2ac} \right) \\ &= k \cdot \frac{(a^2 + b^2 - c^2)}{2} - k \cdot \frac{(a^2 + c^2 - b^2)}{2} \\ &= 2k \cdot \frac{(b^2 - c^2)}{2} \\ &= b \cdot (kb) - c \cdot (kc) \\ &= b(\sin B) - c(\sin C) \end{aligned}$$

LHS

Sine and Cosine Formulae and their Applications Ex-10.1 Q12

$$a^2 \sin(B - C) = (b^2 - c^2) \sin A$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$$

*LHS,*

$$a^2 \sin(B - C)$$

$$= a^2 \{ \sin B \cos C - \sin C \cos B \}$$

$$= a^2 k b \cdot \frac{a^2 + b^2 - c^2}{2ab} - a^2 k c \cdot \frac{a^2 + c^2 - b^2}{2ac} \quad [\text{Using cos rule and sine rule}]$$

$$= a^2 k \cdot \frac{a^2 + b^2 - c^2}{2a} - a^2 k \cdot \frac{a^2 + c^2 - b^2}{2a}$$

$$= a^2 k \cdot \left( \frac{a^2 + b^2 - c^2 - a^2 - c^2 + b^2}{2a} \right)$$

$$= a^2 k \cdot \left( \frac{2b^2 - 2c^2}{2a} \right)$$

$$= ak \cdot (b^2 - c^2)$$

$$= \sin A (b^2 - c^2) = RHS$$

Hence Proved

\*\*\*\*\* END \*\*\*\*\*