CHAPTER 43

BOHR'S MODEL AND PHYSICS OF THE ATOM

43.1 EARLY ATOMIC MODELS

The idea that all matter is made of very small indivisible particles is very old. It has taken a long time, intelligent reasoning and classic experiments to cover the journey from this idea to the present day atomic models.

We can start our discussion with the mention of English scientist Robert Boyle (1627–1691) who studied the expansion and compression of air. The fact that air can be compressed or expanded, tells that air is made of tiny particles with lot of empty space between the particles. When air is compressed, these particles get closer to each other, reducing the empty space. We mention Robert Boyle here, because, with him atomism entered a new phase, from mere reasoning to experimental observations. The smallest unit of an element, which carries all the properties of the element is called an atom. Experiments on discharge tube, measurement of e/m by Thomson, etc., established the existence of negatively charged electrons in the atoms. And then started the search for the structure of the positive charge inside an atom because the matter as a whole is electrically neutral.

Thomson's Model of the Atom

Thomson suggested in 1898 that the atom is a positively charged solid sphere and electrons are embedded in it in sufficient number so as to make the atom electrically neutral. One can compare Thomson's atom to a birthday cake in which cherries are embedded. This model was quite attractive as it could explain several observations available at that time. It could explain why only negatively charged particles are emitted when a metal is heated and never the positively charged particles. It could also explain the formation of ions and ionic compounds of chemistry.

Lenard's Suggestion

Lenard had noted that cathode rays could pass through materials of small thickness almost undeviated. If the atoms were solid spheres, most of the electrons in the cathode rays would hit them and would not be able to go ahead in the forward direction. Lenard, therefore, suggested in 1903 that the atom must have a lot of empty space in it. He proposed that the atom is made of electrons and similar tiny particles carrying positive charge. But then, the question was, why on heating a metal, these tiny positively charged particles were not ejected ?

Rutherford's Model of the Atom

Thomson's model and Lenard's model, both had certain advantages and disadvantages. Thomson's model made the positive charge immovable by assuming it to be spread over the total volume of the atom. On the other hand, electrons were tiny particles and could be ejected on heating a metal. But the almost free passage of cathode rays through an atom was not consistent with Thomson's model. For that, the atom should have a lot of empty space as suggested by Lenard. So, the positive charge should be in the form of tiny particles occupying a very small volume, yet these particles should not be able to come out on heating.

It was Ernest Rutherford who solved the problem by doing a series of experiments from 1906 to 1911 on alpha particle scattering.

In these experiments, a beam of alpha particles was bombarded on a thin gold foil and their deflections were studied (figure 43.1). Most of the alpha particles passed through the gold foil either undeviated or with a small deviation. This was expected because an alpha particle is a heavy particle and will brush aside any tiny particle coming in its way. However, some of the alpha particles were deflected by large angles.

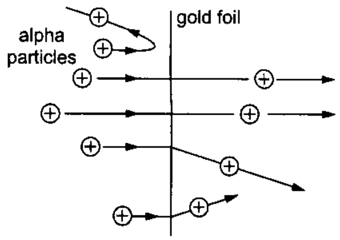

Figure 43.1

Rutherford found that some of the alpha particles, about one in 8000, were deflected by more than 90°, i.e., they were turned back by the foil.

This was interesting. When 8000 alpha particles could go through the gold atoms undeflected, why then one was forced to turn back. The alpha particle itself is about 7350 times heavier than the electron. So neither an electron, nor a similar positively charged particle could cause a large scale deflection of an alpha particle. The alpha particle must have encountered a very heavy particle in its path, a particle with mass of the order of the mass of the atom itself. Also, thousands of alpha particles go undeviated or almost undeviated. So this heavy mass in the atom should occupy a very small volume so that the atom may contain lot of empty space.

From the pattern of the scattering of alpha particles, Rutherford made quantitative analysis. He found that the heavy particle from which an alpha particle suffered large deflection, had a positive charge and virtually all the mass of the atom was concentrated in it. Its size was also estimated from the same experiment. The linear size was found to be about 10 fermi (1 fermi = 1 femtometre = 10^{-15} m) which was about 10^{-5} of the size of the linear atom. As the volume is proportional to the cube of the linear size, the volume of this positively charged particle was only about 10^{-15} of the volume of the atom.

Based on these observations, Rutherford proposed the model of *nuclear atom* which remains accepted to a large extent even today. According to this model, the atom contains a positively charged tiny particle at its centre called the *nucleus* of the atom. This nucleus contains almost all the mass of the atom. Outside this nucleus, there are electrons which move around it at some separation. The space between the nucleus and the electrons is empty and determines the size of the atom. The amount of the positive charge on the nucleus is exactly equal to the total amount of negative charges on all the electrons of the atom. Figure (43.2) shows schematic representations of an atom in Thomson's model and Rutherford's model.

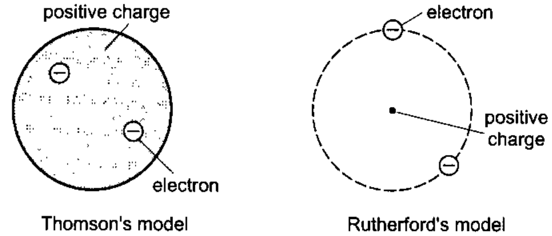

Figure 43.2

So, Rutherford's model explains the charge neutrality and the large empty space inside the atom as suggested by Lenard. It also explains why only negatively charged particles are ejected easily by an atom. It is so because the positively charged particle (nucleus) is so heavy that when an atom gets energy from heating or otherwise, this particle is hardly affected.

The movement of electrons around the nucleus was a necessary part of Rutherford's model. If the electrons were at rest, they would fall into the nucleus because of Coulomb attraction. If the electrons move in circular orbits, the Coulomb force will only change the direction of velocity providing the necessary centripetal force. This electronic motion, however, created difficulties for Rutherford's model as we shall now study.

43.2 HYDROGEN SPECTRA

When a material body is heated, it emits electromagnetic radiation. The radiation may consist of various components having different wavelengths. When the filament of an electric bulb is heated, it gives white light and all wavelengths in the visible range are present in the emitted radiation. If the emitted light is passed through a prism, components of different wavelengths deviate by different amounts and we get a continuous spectrum.

If hydrogen gas enclosed in a sealed tube is heated to high temperatures, it emits radiation. If this radiation is passed through a prism, components of different wavelengths are deviated by different amounts and thus we get the hydrogen spectrum. The most striking feature in this spectrum is that only some sharply defined, discrete wavelengths exist in the emitted radiation. For example, light of wavelength 656·3 nm is observed and then light of wavelength 486·1 nm is observed. Hydrogen atoms do not emit any radiation between 656·3 nm and 486·1 nm.

A hydrogen sample also emits radiation with wavelengths less than those in the visible range and also with wavelengths larger than those in the visible range. Figure (43.3) shows a schematic arrangement of the wavelengths present in a hydrogen spectrum. We see that the lines may be grouped in separate series. In each series, the separation between the consecutive wavelengths decreases as we go from higher wavelength to lower wavelength. In fact, the wavelengths in each series approach a limiting value known as the *series limit*. Thus, we have indicated the Lyman series (ultraviolet region), Balmer series

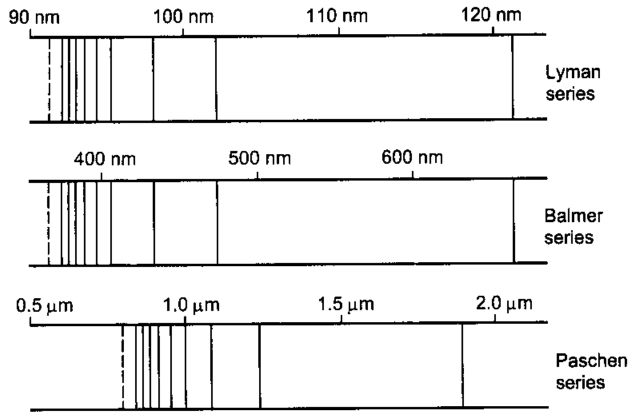

Figure 43.3

(visible region), Paschen series (infrared region), etc., in the figure.

The wavelengths nicely fit in the equation

$$\frac{1}{\lambda} = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \qquad \ldots (43.1)$$

where $R \approx 1{\cdot}09737 \times 10^7 \text{ m}^{-1}$ and n and m are integers with $m > n$. Lyman series can be reproduced by setting $n = 1$ and varying m from 2 onwards, Balmer series by setting $n = 2$ with $m > 2$, Paschen series by setting $n = 3$ with $m > 3$, etc.

It is said that John Jacob Balmer (1825–1898), a Swiss schoolteacher, was fond of playing with numbers. Once he complained to his physicist friend that he was getting bored as he had no numbers to play with. The friend gave him four wavelengths 656·3, 486·1, 434·1 and 420·2 nm of hydrogen spectrum and asked if Balmer could find a relation amongst them. And Balmer soon came out with his formula

$$\lambda = \frac{364{\cdot}56\, m^2}{m^2 - 4}, \text{ where } m = 3, 4, 5, 6$$

which was later put in more convenient form (equation 43.1) by Rydberg.

43.3 DIFFICULTIES WITH RUTHERFORD'S MODEL

The sharply defined, discrete wavelengths in hydrogen spectra posed a serious puzzle before physicists.

A hydrogen atom consists of an electron and a nucleus containing just a proton. The important question is why the electron does not hit the proton due to Coulomb attraction. In Rutherford's model, we assume that the electron revolves round the proton and the Coulomb force provides the necessary centripetal force to keep it moving in circular orbit. From the point of view of mechanics, a revolving electron in an atom is a satisfactory picture. But Maxwell's equations of electromagnetism show that any accelerated electron must continuously emit electromagnetic radiation. The revolving electron should, therefore, always emit radiation at all temperatures. The wavelength of the radiation should be related to the frequency of revolution. If the radiation is continuously emitted, the energy is spent and the radius of the circle should gradually decrease and the electron should finally fall into the proton. Also, the frequency of revolution changes continuously as the energy is spent, and so, the electron should emit radiation of continuously varying wavelength during the period of its motion.

The actual observations are quite different. At room temperature or below, hydrogen is very stable; it neither emits radiation nor does the electron collapse into the proton. When extra energy is supplied through heat or electric discharge, radiation is emitted, but the wavelengths are sharply defined as given by equation (43.1). These sharply defined wavelengths may be taken as the fingerprints of the element (hydrogen). Be it Calcutta, Delhi, Madras, Hyderabad, New York, London or Canberra, sun or upper atmosphere, hydrogen always emits only these fixed wavelengths. Such observations could not be explained by classical concepts and something new was about to take birth.

43.4 BOHR'S MODEL

In 1913, Niels Bohr, a great name in physics, suggested that the puzzle of hydrogen spectra may be solved if we make the following assumptions.

Bohr's Postulates

(a) The electron revolves round the nucleus in circular orbits.

(b) The orbit of the electron around the nucleus can take only some special values of radius. In these orbits of special radii, the electron does not radiate energy as expected from Maxwell's laws. These orbits are called *stationary orbits*.

(c) The energy of the atom has a definite value in a given stationary orbit. The electron can jump from one stationary orbit to other. If it jumps from an orbit of higher energy E_2 to an orbit of lower energy E_1, it emits a photon of radiation. The energy of the photon is $E_2 - E_1$. The wavelength of the emitted radiation is given by the Einstein–Planck equation

$$E_2 - E_1 = h\nu = \frac{hc}{\lambda}.$$

The electron can also absorb energy from some source and jump from a lower energy orbit to a higher energy orbit.

(d) In stationary orbits, the angular momentum l of the electron about the nucleus is an integral multiple of the Planck constant h divided by 2π,

$$l = n\frac{h}{2\pi}.$$

This last assumption is called *Bohr's quantization rule* and the assumptions (a) to (d) are known as *Bohr's postulates.*

Energy of a Hydrogen Atom

Let us now use the above postulates to find the allowed energies of the atom for different allowed orbits of the electron. The theory developed is applicable to hydrogen atoms, and ions having just one electron. Thus, it is valid for He^{+}, Li^{++}, Be^{+++}, etc. These ions are often called *hydrogen-like ions.* Let us assume that the nucleus has a positive charge Ze (i.e., there are Z protons in it) and an electron moves with a constant speed v along a circle of radius r with the centre at the nucleus. The force acting on the electron is that due to Coulomb attraction and is equal to

$$F = \frac{Ze^2}{4\pi\varepsilon_0 r^2}.$$

The acceleration of the electron is towards the centre and has a magnitude v^2/r. If m is the mass of the electron, from Newton's law,

$$\frac{Ze^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}$$

or,

$$r = \frac{Ze^2}{4\pi\varepsilon_0 mv^2}. \qquad \text{... (i)}$$

Also, from Bohr's quantization rule, the angular momentum is

$$mvr = n\frac{h}{2\pi} \qquad \text{... (ii)}$$

where n is a positive integer.

Eliminating r from (i) and (ii), we get

$$v = \frac{Ze^2}{2\varepsilon_0 hn}. \qquad \text{... (43.2)}$$

Substituting this in (ii),

$$r = \frac{\varepsilon_0 h^2 n^2}{\pi m Ze^2}. \qquad \text{... (43.3)}$$

We see that the allowed radii are proportional to n^2. For each value of n, we have an allowed orbit. For $n = 1$, we have the first orbit (smallest radius), for $n = 2$, we have the second orbit and so on.

From equation (43.2), the kinetic energy of the electron in the nth orbit is

$$K = \frac{1}{2}mv^2 = \frac{mZ^2e^4}{8\varepsilon_0^2 h^2 n^2}. \qquad \text{... (43.4)}$$

The potential energy of the atom is

$$V = -\frac{Ze^2}{4\pi\varepsilon_0 r} = -\frac{mZ^2e^4}{4\varepsilon_0^2 h^2 n^2}. \qquad \text{... (43.5)}$$

We have taken the potential energy to be zero when the nucleus and the electron are widely separated.

The total energy of the atom is

$$E = K + V$$

$$= -\frac{mZ^2e^4}{8\varepsilon_0^2 h^2 n^2}. \qquad \text{... (43.6)}$$

Equations (43.2) through (43.6) give various parameters of the atom when the electron is in the nth orbit. The atom is also said to be in the *nth energy state* in this case. In deriving the expression for the total energy E, we have considered the kinetic energy of the electron and the potential energy of the electron–nucleus pair. It is assumed that the acceleration of the nucleus is negligible on account of its large mass.

Radii of different orbits

From equation (43.3), the radius of the smallest circle allowed to the electron is ($n = 1$)

$$r_1 = \frac{\varepsilon_0 h^2}{\pi m Ze^2}.$$

For hydrogen, $Z = 1$ and putting the values of other constants we get $r_1 = 53$ picometre ($1\ \text{pm} = 10^{-12}\ \text{m}$) or 0·053 nm. This length is called the *Bohr radius* and is a convenient unit for measuring lengths in atomic physics. It is generally denoted by the symbol a_0.

The second allowed radius is $4a_0$, third is $9a_0$ and so on. In general, the radius of the nth orbit is

$$r_n = n^2 a_0.$$

For a hydrogen-like ion with Z protons in the nucleus,

$$r_n = \frac{n^2 a_0}{Z}. \qquad \text{... (43.7)}$$

Ground and excited states

From equation (43.6), the total energy of the atom in the state $n = 1$ is

$$E_1 = -\frac{mZ^2e^4}{8\varepsilon_0^2 h^2}.$$

For hydrogen atom, $Z = 1$ and putting the values of the constants, $E_1 = -13{\cdot}6$ eV. This is the energy when the electron revolves in the smallest allowed orbit $r = a_0$, i.e., the one with radius 0·053 nm. We also see from equation (43.6) that the energy of the atom in the nth energy state is proportional to $\frac{1}{n^2}$. Thus,

$$E_n = \frac{E_1}{n^2} = -\frac{13{\cdot}6\text{ eV}}{n^2}. \qquad \dots (43.8)$$

The energy in the state $n = 2$ is $E_2 = E_1/4 = -3{\cdot}4$ eV. In the state $n = 3$, it is $E_1/9 = -1{\cdot}5$ eV, etc. The lowest energy corresponds to the smallest circle. Note that the energy is negative and hence a larger magnitude means lower energy. The zero of energy corresponds to the state where the electron and the nucleus are widely separated. Figure (43.4) shows schematically the allowed orbits together with the energies of the atom. It also displays the allowed energies separately.

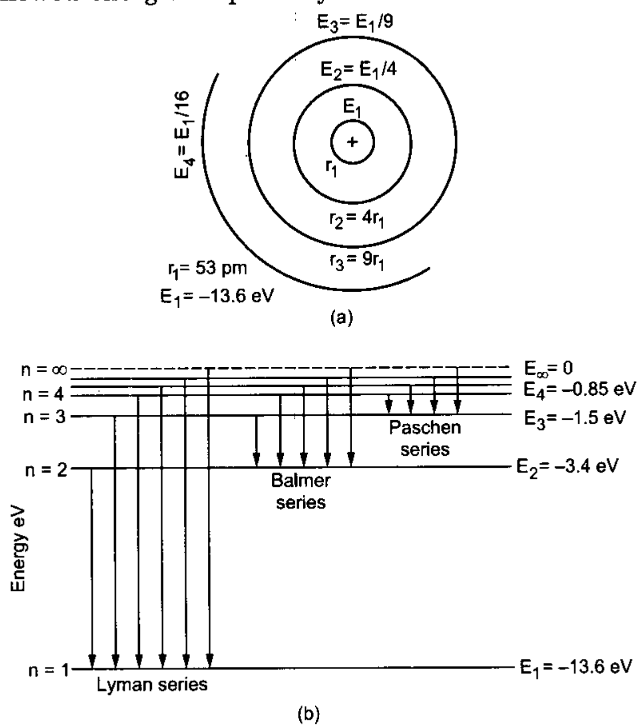

Figure 43.4

The state of an atom with the lowest energy is called its *ground state*. The states with higher energies are called *excited states*. Thus, the energy of a hydrogen atom in the ground state is $-13{\cdot}6$ eV and in the first excited state $-3{\cdot}4$ eV.

Hydrogen Spectra

We can now explain why hydrogen gas kept in a flask at room temperature does not emit radiation. This is because almost all the atoms are in the ground state and there are no orbits of lower energy to which an electron can jump. Hence, the atoms cannot emit any radiation. When energy is given in the form of heat or by electric discharge or by some other means, some of the electrons jump to the higher energy orbits $n = 2$, $n = 3$, etc. These electrons then jump back to lower energy orbits. The atoms radiate energy in the process. This explains why the atoms radiate only when they are heated or given energy in some other form.

If an electron makes a jump from the mth orbit to the nth orbit ($m > n$), the energy of the atom changes from E_m to E_n. This extra energy $E_m - E_n$ is emitted as a photon of electromagnetic radiation. The corresponding wavelength is given by

$$\frac{1}{\lambda} = \frac{E_m - E_n}{hc}$$

$$= \frac{mZ^2e^4}{8\varepsilon_0^2 h^3 c}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

$$= RZ^2\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \qquad \dots (43.9)$$

where $R = \frac{me^4}{8\varepsilon_0^2 h^3 c}$ is called the *Rydberg constant.*

Putting the values of different constants, the Rydberg constant R comes out to be $1{\cdot}0973 \times 10^7\text{ m}^{-1}$ and equation (43.9) is in excellent agreement with the experimental formula (43.1). In terms of the Rydberg constant, the energy of the atom in the nth state is $E = \frac{-RhcZ^2}{n^2}$. Quite often, the energy of the atom is mentioned in unit of rydberg. An energy of 1 rydberg means $-13{\cdot}6$ eV. It is useful to remember that $Rhc = 13{\cdot}6$ eV.

Example 43.1

Calculate the energy of a He$^+$ *ion in its first excited state.*

Solution :

The energy is $E_n = \frac{-RhcZ^2}{n^2} = -\frac{(13{\cdot}6\text{ eV})Z^2}{n^2}$

For a He$^+$ ion, $Z = 2$ and for the first excited state, $n = 2$ so that the energy of He$^+$ ion in the first excited state is $-13{\cdot}6$ eV.

Example 43.2

Calculate the wavelength of radiation emitted when He$^+$ *makes a transition from the state $n = 3$ to the state $n = 2$.*

Solution :

The wavelength λ is given by

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

$$= 4R\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5}{9}R$$

or, $$\lambda = \frac{9}{5R} = \frac{9}{5 \times 1{\cdot}09737 \times 10^7\text{ m}^{-1}} = 164{\cdot}0\text{ nm}.$$

Series structure

If a hydrogen atom makes transition from the state $n = 2$ to the state $n = 1$, the wavelength of the emitted radiation is given by

$$\frac{1}{\lambda} = R\left(1 - \frac{1}{4}\right) \text{ or, } \lambda = 121{\cdot}6 \text{ nm.}$$

If it makes transition from the state $n = \infty$ to the state $n = 1$, the wavelength emitted is given by

$$\frac{1}{\lambda} = R(1 - 0) \text{ or, } \lambda = 91{\cdot}2 \text{ nm.}$$

Thus, all the transitions ending at $n = 1$ correspond to wavelengths grouped between 121·6 nm and 91·2 nm. These lines constitute the *Lyman series.*

Similarly, transitions from higher states to $n = 2$ lead to emission of radiation with wavelengths between 656·3 nm and 365·0 nm. These wavelengths fall in the visible region and constitute the *Balmer series.* The transitions from higher states to $n = 3$ give rise to *Paschen series* with wavelengths between 1875 nm and 822 nm, and similarly for other series. This explains the grouping of wavelengths in different series as shown in figure (43.3).

Ionization potential

What happens if we supply more than 13·6 eV to a hydrogen atom in its ground state ? The total energy is then positive. The equations deduced above are not applicable in this case. In fact, a total energy of zero corresponds to electron and nucleus separated by an infinite distance. In this case, the electron is not bound to the nucleus and is free to move anywhere. The atom is said to be ionized, i.e., its electron has been detatched from the nucleus. Positive energy means that the atom is ionized and the electron is moving independently with some kinetic energy.

The minimum energy needed to ionize an atom is called *ionization energy.* The potential difference through which an electron should be accelerated to acquire this much energy is called *ionization potential.* Thus, ionization energy of hydrogen atom in ground state is 13·6 eV and ionization potential is 13·6 V.

Binding energy

Binding energy of a system is defined as the energy released when its constituents are brought from infinity to form the system. It may also be defined as the energy needed to separate its constituents to large distances. If an electron and a proton are initially at rest and brought from large distances to form a hydrogen atom, 13·6 eV energy will be released. The binding energy of a hydrogen atom is, therefore, 13·6 eV, same as its ionization energy.

Excitation potential

The energy needed to take the atom from its ground state to an excited state is called the *excitation energy* of that excited state. The hydrogen atom in ground state needs 10·2 eV to go into the first excited state. Thus, the excitation energy of hydrogen atom in the first excited state is 10·2 eV. The potential through which an electron should be accelerated to acquire this much of energy is called the *excitation potential.* Thus, the excitation potential of hydrogen atom in first excited state is 10·2 V.

Example 43.3

The excitation energy of a hydrogen-like ion in its first excited state is 40·8 eV. Find the energy needed to remove the electron from the ion.

Solution :

The excitation energy in the first excited state is

$$E = RhcZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$
$$= (13{\cdot}6 \text{ eV}) \times Z^2 \times \frac{3}{4}.$$

Equating this to 40·8 eV, we get $Z = 2$. So, the ion in question is He^+. The energy of the ion in the ground state is

$$E = -\frac{RhcZ^2}{1^2} = -4 \times (13{\cdot}6 \text{ eV})$$
$$= -54{\cdot}4 \text{ eV.}$$

Thus 54·4 eV is required to remove the electron from the ion.

43.5 LIMITATIONS OF BOHR'S MODEL

Bohr's model was a great success at a time when the physicists were struggling hard to understand the discrete wavelengths in hydrogen spectra. Even today the model is very popular among beginners and nonphysicists, who can 'visualise' the inside of the atom as electrons going in circles around the nucleus. However, the model did not go too far. It could not be extended for atoms or ions having more than one electron. Even helium spectrum was beyond the scope of the Bohr's model. As technology improved and the wavelengths were measured with greater accuracy, deviations were observed even in the case of hydrogen spectral lines. Thus, at least seven components having slightly different wavelengths are revealed in what was previously known as the 656·3 nm line. On the theoretical side also, the model is not quite consistent with the physics in totality. Bohr's postulates look more like a patch on Maxwell's electromagnetism. Maxwell's theory is not replaced or refuted but it is arbitrarily assumed that in certain orbits, electrons get the licence to disobey the laws of electromagnetism and are allowed not to radiate energy.

43.6 THE WAVE FUNCTION OF AN ELECTRON

Physicists now have a mathematically and logically sound theory in the name of *quantum mechanics* which describes the spectra in a much better way. A very brief introduction to this theory is given below.

We have seen in previous chapters that to understand the behaviour of light, we must use the wave picture (the electric field $\vec{E}$) as well as the particle picture (the photon). The energy of a particular 'photon' is related to the 'wavelength' of the $\vec{E}$ wave. Light going in x-direction is represented by the wave function

$$E(x, t) = E_0 \sin(kx - \omega t).$$

In general, if light can go in any direction, the wave function is

$$\vec{E}(\vec{r},t) = \vec{E}_0 \sin(\vec{k}\cdot\vec{r} - \omega t). \quad \ldots \text{(i)}$$

If $|\vec{E}|^2$ at a certain point $\vec{r}$ is large, the intensity of light is high and we say that the 'density of photons' at that position is high. Suppose the intensity is so low that we expect only a single photon in a large volume. Even this weak light is represented by a wave given by (i) with $|\vec{E}_0|^2$ having a small value. Where is this photon at time t? We can't assign a unique position to the photon because $\vec{E}$ is spread over a large space, and wherever $\vec{E} \neq 0$ there is light. But if we put an instrument to detect the photon, we shall not detect a part of photon here and a part there. The whole photon is detected at just one point. The probability of finding the photon is more where $|\vec{E}(\vec{r}, t)|^2$ is large. To know something about the 'photon', we have to get the wave function $\vec{E}(\vec{r}, t)$ of light and correlate the wave properties with the particle properties. The wave function $\vec{E}(\vec{r}, t)$ satisfies Maxwell's equations. Similar is the case with electrons. An electron also has a wave character as well as a particle character. Its wave function is $\psi(\vec{r}, t)$ which may be obtained by solving *Schrodinger's wave equation*. The particle properties of the electron must be understood through this wave function $\psi(\vec{r}, t)$. The wave function varies continuously in space and may be extended over a large part of space at a given instant. This does not mean that the electron is spread over that large part. If we put an instrument to detect the electron at a point, we shall either detect a whole electron or none. But where will this electron be found ? The answer is again hidden in $\psi(\vec{r}, t)$. Wherever $\psi \neq 0$, there is a chance to find the electron. Greater the value of $|\psi(\vec{r}, t)|^2$, greater is the probability of detecting the electron there. Not only the information about the electron's position but information about all the properties including energy is also contained in the wave function $\psi(\vec{r}, t)$.

43.7 QUANTUM MECHANICS OF THE HYDROGEN ATOM

The wave function $\psi(\vec{r}, t)$ of the electron and the possible energies E of a hydrogen atom or a hydrogen-like ion are obtained from the Schrodinger's equation

$$\frac{-h^2}{8\pi^2 m}\left[\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right] - \frac{Ze^2\psi}{4\pi\varepsilon_0 r} = E\psi. \quad \ldots (43.10)$$

Here (x,y,z) refers to a point with the nucleus as the origin and r is the distance of this point from the nucleus. E refers to energy. The constant Z is the number of protons in the nucleus. For hydrogen, we have to put $Z = 1$. There are infinite number of functions $\psi(\vec{r})$ which satisfy equation (43.10). These functions, which are solutions of equation (43.10), may be characterised in terms of three parameters n, l and m_l. With each solution ψ_{nlm_l}, there is associated a unique value of the energy E of the atom or the ion. The energy E corresponding to the wave function ψ_{nlm_l} depends only on n and may be written as

$$E_n = -\frac{mZ^2e^4}{8\varepsilon_0^2 h^2 n^2}. \quad \ldots (43.11)$$

These energies happen to be identical with the allowed energies calculated in Bohr's model. This explains the success of Bohr's model in quantitatively obtaining the wavelengths in a hydrogen spectrum. For each n there are n values of l, namely $l = 0, 1, 2, \ldots, n-1$ and for each l there are $2l+1$ values of m_l namely $m_l = -l, -l+1, -l+2, \ldots, l-1, l$. The parameter n is called the *principal quantum number,* l the *orbital angular momentum quantum number* and m_l the *magnetic quantum number.*

The lowest possible energy for the hydrogen atom is $-Rhc = -13{\cdot}6$ eV and the wave function of the electron in this 'ground state' is

$$\psi(\vec{r}) = \psi_{100} = \sqrt{\frac{Z^3}{\pi a_0^2}}\, e^{-r/a_0}. \quad \ldots (43.12)$$

In Bohr's model, we say that the electron moves in a circular orbit of radius $a_0 = 0{\cdot}053$ nm in the ground state. In quantum mechanics, the very idea of orbit is invalid. In ground state, the wave function of the electron is given by equation (43.12). At any instant this wave function is spread over large distances in space, and wherever $\psi \neq 0$, the presence of electron may be felt. However, if the electron is detected by some experiment, it will be detected at one single

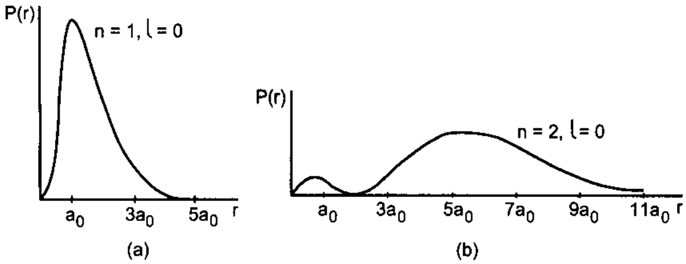

Figure 43.5

position only. The probability of finding the electron in a small volume dV is $|\psi(\vec{r})|^2 dV$. One can calculate the probability $P(r)dr$ of finding the electron at a distance between r and $r + dr$ from the nucleus. The function $P(r)$ is called linear probability density. In ground state, given by equation (43.12), $P(r)$ comes out to be

$$P(r) = \frac{4}{a_0^3} r^2 e^{-2r/a_0}.$$

A plot of $P(r)$ versus r is shown in figure (43.5a). Note that $P(r)$ is maximum at $r = a_0$. This means that the electron is more likely to be found near $r = a_0$ than at any other distance.

It may be satisfying that at least the probability of finding the electron is maximum at the same radial distance from the nucleus where the Bohr's model assigns the electron to be. However, even this cannot be stretched too far. The linear probability density $P(r)$ for $n = 2$, $l = 0$, $m = 0$ is plotted in figure (43.5b) which has two maxima, one near $r = a_0$ and the other near $r = 5{\cdot}4\ a_0$. In Bohr's model, all $n = 2$ electrons should be at $r = 4\ a_0$.

43.8 NOMENCLATURE IN ATOMIC PHYSICS

We have neglected the *spin* of the electron in the discussion so far. A very interesting property of electrons is that each electron has a permanent angular momentum whose component along any given direction is $\frac{h}{4\pi}$ or $-\frac{h}{4\pi}\cdot$ This angular momentum is different from the angular momentum resulting from the motion of the electron and is known as the *spin angular momentum* of the electron. The complete wave function of an electron also has a part depending on the state of the spin. The spin part of the wave function is characterized by a spin quantum number m_s which can take values $m_s = +1/2$ or $-1/2$. A wave function is thus characterised by n, l, m_l and m_s. A particular wave function described by particular values of n, l, m_l, m_s corresponds to a *quantum state*. For $n = 1$, we have $l = 0$ and $m_l = 0$. But m_s can be $+1/2$ or $-1/2$. So there are two quantum states corresponding to $n = 1$. For $n = 2$ there are 8 quantum states, for $n = 3$ there are 18 quantum states and so on. In general, there are $2n^2$ quantum states corresponding to a particular n. The quantum states corresponding to a particular n are together called a *major shell*. The major shell corresponding to $n = 1$ is called K shell, corresponding to $n = 2$ is called L shell, corresponding to $n = 3$ is called M shell, etc.

A very interesting and important law of nature is that *there cannot be more than one electron in any quantum state.* This is known as *Pauli exclusion principle.* Thus, a K shell can contain a maximum of 2 electrons, an L shell can contain a maximum of 8 electrons, an M shell can contain a maximum of 18 electrons and so on.

It is customary to use the symbols s, p, d, f, etc., to denote the values of the orbital angular momentum quantum number l. These symbols correspond to $l = 0$, 1, 2, 3, etc., respectively. The quantum states corresponding to a given principal quantum number n and a given orbital angular momentum quantum number l form what we call a *subshell.* Thus $n = 1$, $l = 0$ is called 1s subshell. Similarly $n = 2$, $l = 0$ is called 2s subshell, $n = 2$, $l = 1$ is called 2p subshell and so on. For atoms having more than one electron also, the concept of n, l, m_l, m_s is valid. The energy then depends on n as well as on l. Thus 1s, 2s, 2p, etc., also designate the *energy levels.* For an atom having many electrons, the quantum states are, in general, gradually filled from lower energy to higher energy to form the ground state of the atom.

We have seen that electrons obey Pauli exclusion principle. Apart from electrons, there are other particles which obey this principle. Protons and neutrons also obey this principle. Any particle that obeys Pauli exclusion principle, is called a *fermion.* Electrons, protons, neutrons are all fermions.

43.9 LASER

When an atom jumps from a higher energy state to a lower energy state, it emits a photon of light. In an ordinary source of light, atoms emit photons independently of each other. As a result, different photons have different phases and the light as a whole becomes incoherent. Also, the energy of transition differs slightly from photon to photon so that the wavelength is not uniquely defined. There is a spread $\Delta\lambda$ in the wavelength λ. The direction of light is also different for different transitions so that we do not get a strictly parallel beam of light.

LASER (Light Amplification by Stimulated Emission of Radiation) is a process by which we get a light beam which is coherent, highly monochromatic and almost perfectly parallel. The word 'laser' is also used for the light beam obtained by this process. All the photons in the light beam, emitted by different atoms at different instants, are in phase. The spread $\Delta\lambda$ in wavelength is very small, of the order of 10^{-6} nm which is about 1000 times smaller than the spread in the usual ^{86}Kr light. A beam of laser can go to the moon and return to the earth without much loss of intensity. This shows that laser may be obtained as an almost perfectly parallel beam.

To understand the process involved in laser, we have to first discuss *stimulated emission.*

Stimulated Emission

Consider an atom which has an allowed state at energy E_1 and another allowed state at a higher energy E_2. Suppose the atom is in the lower energy state E_1. If a photon of light having energy $E_2 - E_1$ is incident on this atom, the atom may absorb the photon and jump to the higher energy state E_2 (figure 43.6a). This process is called *stimulated absorption* of light photon. The incident photon has stimulated the atom to absorb the energy.

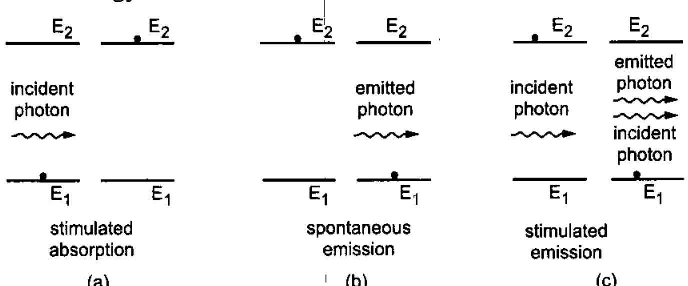

Figure 43.6

Now, suppose the atom is in the higher energy state E_2. If we just leave the atom there, it will eventually come down to the lower state by emitting a photon having energy $E_2 - E_1$ (figure 43.6b). This process is called *spontaneous emission*. Typically, an atom stays for about 10 ns in an excited state. The average time for which an atom stays in an excited state is called the *lifetime* of that state. There are atoms which have certain excited states having a lifetime of the order of a millisecond, i.e., about 10^5 times longer than the usual lifetimes. Such states are called *metastable states.*

Finally, suppose the atom is in the higher energy state E_2 and a photon having energy $(E_2 - E_1)$ is incident on it (figure 43-6c). The incident photon interacts with the atom and may cause the atom to come down to the lower energy state. A fresh photon is emitted in the process. This process is different from spontaneous emission in which the atom jumps to the lower energy state on its own. In the present case, the incident photon has 'stimulated' the atom to make the jump.

When an atom emits a photon due to its interaction with a photon incident on it, the process is called stimulated emission. The emitted photon has exactly the same energy, phase and direction as the incident photon.

Basic Process of Laser

The basic scheme to get light amplification by stimulated emission is as follows.

A system is chosen which has a metastable state having an energy E_2 (figure 43.7). There is another allowed energy E_1 which is less than E_2. The system may be a gas or a liquid in a cylindrical tube or a solid in the shape of a cylindrical rod. Suppose, by some technique, the number of atoms in the metastable state E_2 is made much larger than that in E_1. Suppose a photon of light of energy $E_2 - E_1$ is incident on one of the atoms in the metastable state. This atom drops to the state E_1 emitting a photon in the same phase, energy and direction as the first one. These two photons interact with two more atoms in the state E_2 and so on. So the number of photons keeps on increasing. All these photons have the same phase, the same energy and the same direction. So the amplification of light is achieved.

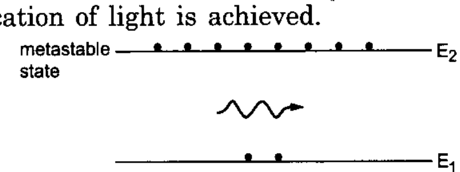

Figure 43.7

In this scheme, two arrangements are necessary. Firstly, the metastable state with energy E_2 must all the time have larger number of atoms than the number in the lower energy state. If the lower energy state has a larger number of atoms, these atoms will absorb a sizable number of photons to go up in energy. This way the stimulated emission will be weakened and the amplification will not be possible. When a higher energy state has more number of atoms than a lower energy state has, we say that *population inversion* has taken place. This is because, normally, the population in the lower energy state is higher. To sustain laser action, we need an arrangement which ensures population inversion between the states E_1 and E_2. The metastable state should continue to get atoms and the atoms should be continuously removed from the lower energy state E_1. This process is called *pumping.*

Secondly, the photons emitted due to stimulating action should stimulate other atoms to emit more photons. This means, the stimulated photons should spend enough time in the system, interacting with the atoms. To achieve this, two mirrors are fixed at the ends of the cylindrical region containing the lasing material. The mirrors reflect the photons back and forth to keep them inside the region for a long time. One of the mirrors is made slightly transmitting so that a small fraction, say 1%, of the light comes out of the region. This is the laser which becomes available to us for use.

Any photon travelling in a direction not parallel to the axis of the cylindrical region is thrown out from the sides after few reflections. The photons moving parallel to the axis remain in the region for long time and hence only the light along the axis is amplified. This explains why the laser light is highly directional.

Let us now discuss a He–Ne laser which is most widely used in classroom demonstrations.

He–Ne Laser

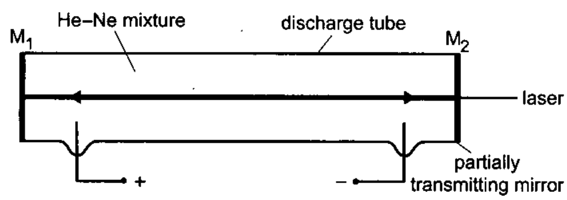

Figure 43.8

A schematic design of the system is shown in figure (43.8). A mixture of helium (about 90%) and neon (about 10%) at low pressures is taken in a cylindrical glass tube. Two parallel mirrors M_1 and M_2 are fixed at the ends. One of the two mirrors, M_2 in the figure, is slightly transmitting and laser light comes out of it. The tube contains two electrodes which are connected to a high-voltage power supply so that a large electric field is established in the tube.

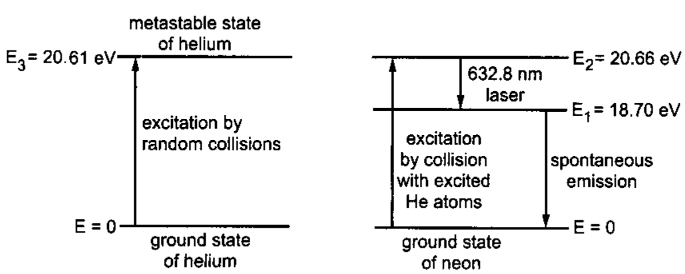

Figure 43.9

The relevant energy levels of helium and neon are shown in figure (43.9). Lasing action takes place between the state at energy $E_2 = 20{\cdot}66$ eV and the state at energy $E_1 = 18{\cdot}70$ eV of neon atoms. Helium has a metastable state at $E_3 = 20{\cdot}61$ eV which happens to be close to the level E_2 of neon. Helium is used to pump the neon atoms to the state E_2 from where they may come down to the state E_1 by stimulated emission. The energy difference is

$$E_2 - E_1 = 1{\cdot}96 \text{ eV}$$

so that the wavelength of He–Ne laser is

$$\lambda = \frac{hc}{E_2 - E_1} = 632{\cdot}8 \text{ nm}.$$

Working

When the power supply is switched on and the electric field is established, some of the atoms of the mixture get ionized. The electrons freed by these atoms are accelerated by the high electric field. These electrons collide with helium atoms to take them to the metastable state at energy E_3. Such an excited atom collides with a neon atom and transfers the extra energy to it. As a result, the helium atom comes back to its ground state and the neon atom is excited to the state at energy E_2. This process takes place continuously so that the neon atoms are continuously pumped to the state at energy E_2 to keep the population of this state large.

Stimulated emission takes place between the states at energies E_2 and E_1. As the state at energy E_1 has a small lifetime, of the order of 10 ns, these atoms readily jump to the still lower states. This way the population of the state at energy E_1 is always very small. Thus, population inversion between E_2 and E_1 is achieved and maintained.

Laser light comes out of the partially transmitting mirror.

Note that the higher energy state E_2 of neon is not itself metastable. But the metastable state of helium accumulates atoms at higher energy which take neon atoms to the level E_2 by means of collisions.

Uses of Laser

Laser was invented in 1960. Since then, laser technology has greatly advanced and now lasers have widespread use in industry, scientific research, surgery, etc.

Because of the near-perfect parallel and monochromatic character, a laser beam can be focused by a converging lens to a very small spot. This results in very high intensity over that tiny spot. It can, therefore, be used for very accurate microsurgery where a very small area is to be treated. Lasers in infrared region are used to burn away cervical tumours. These lasers are also used for cutting tissues. Lasers are used to spot-weld detached retina with great accuracy. Because of the high intensity, lasers are used to drill sharp holes in metals and diamond. In garment industry, lasers are used to cut many layers (say 50 layers at a time) of cloth without frayed edges.

Lasers are widely used to send telephone signals over long distances through optical fibres. They are also used in nuclear fusion research which is likely to be the ultimate source of energy for us.

Because of its directional properties, lasers are used in surveying. Another use of laser is to align tools and equipment in industry and scientific research. Laser light is sent to the moon from where it is reflected back to the earth without much loss of intensity. Thus, points on the moon's surface may be monitored from the earth using lasers. Laser has numerous military applications.

An interesting application of lasers is to produce holograms, which record a 3 D image of an object. When the hologram is viewed, again with a laser, the same 3 D perception is achieved as it is with the actual object.

In compact disc (CD) audio systems, a laser beam is used in place of the phonographic needle. Sound is recorded on the compact disc using digital electronic techniques. This results in great compression of the sound data and a very large number of songs, speeches etc. can be stored on a CD which is much smaller than a traditional record. Also, the playback of the music is more 'true' than traditional systems and almost without any distortion. Using lasers, video images can also be stored on discs which can be played back using a laser disc player and a TV. Since the combination of digital electronic techniques and CD allows us to store a large amount of data in a small volume, books of large volume like dictionaries and encyclopedias, are now available on CDs. This technique is now being used in computers for data retrieval and storage. Lasers are used in laser printers. The present book was also prepared with the help of a laser printer. Incredible new applications are being created everyday using lasers.

Worked Out Examples

First Bohr radius $a_0 = 53$ pm, energy of hydrogen atom in ground state $= -13{\cdot}6$ eV, Planck constant $h = 4{\cdot}14 \times 10^{-15}$ eVs, speed of light $= 3 \times 10^{8}$ m s^{-1}.

---

1. *Find the radius of* Li^{++} *ions in its ground state assuming Bohr's model to be valid.*

Solution : For hydrogen-like ions, the radius of the nth orbit is

$$a_n = \frac{n^2 a_0}{Z}.$$

For Li^{++}, $Z = 3$ and in ground state $n = 1$. The radius is

$$a_1 = \frac{53\ \text{pm}}{3} \approx 18\ \text{pm}.$$

2. *A particular hydrogen-like ion emits radiation of frequency* $2{\cdot}467 \times 10^{15}$ Hz *when it makes transition from* $n = 2$ *to* $n = 1$. *What will be the frequency of the radiation emitted in a transition from* $n = 3$ *to* $n = 1$?

Solution : The frequency of radiation emitted is given by

$$\nu = \frac{c}{\lambda} = K\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right).$$

Thus, $2{\cdot}467 \times 10^{15}\ \text{Hz} = K\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$

or, $$K = \frac{4}{3} \times 2{\cdot}467 \times 10^{15}\ \text{Hz}.$$

The frequency of the radiation emitted in the transition $n = 3$ to $n = 1$ is

$$\nu' = K\left[\frac{1}{1^2} - \frac{1}{3^2}\right]$$

$$= \frac{8}{9}K = \frac{8}{9} \times \frac{4}{3} \times 2{\cdot}467 \times 10^{15}\ \text{Hz}$$

$$= 2{\cdot}92 \times 10^{15}\ \text{Hz}.$$

3. *Calculate the two highest wavelengths of the radiation emitted when hydrogen atoms make transitions from higher states to* $n = 2$ *states.*

Solution : The highest wavelength corresponds to the lowest energy of transition. This will be the case for the transition $n = 3$ to $n = 2$. The second highest wavelength corresponds to the transition $n = 4$ to $n = 2$.

The energy of the state n is $E_n = \frac{E_1}{n^2}$.

Thus, $$E_2 = -\frac{13{\cdot}6\ \text{eV}}{4} = -3{\cdot}4\ \text{eV}$$

$$E_3 = -\frac{13{\cdot}6\ \text{eV}}{9} = -1{\cdot}5\ \text{eV}$$

and $$E_4 = -\frac{13{\cdot}6\ \text{eV}}{16} = -0{\cdot}85\ \text{eV}.$$

The highest wavelength is $\lambda_1 = \frac{hc}{\Delta E}$

$$= \frac{1242\ \text{eVnm}}{(3{\cdot}4\ \text{eV} - 1{\cdot}5\ \text{eV})} = 654\ \text{nm}.$$

The second highest wavelength is

$$\lambda_2 = \frac{1242\ \text{eVnm}}{(3{\cdot}4\ \text{eV} - 0{\cdot}85\ \text{eV})} = 487\ \text{nm}.$$

4. *What is the wavelength of the radiation emitted when the electron in a hydrogen atom jumps from* $n = \infty$ *to* $n = 2$?

Solution : The energy of $n = 2$ state is

$$E_2 = \frac{-13{\cdot}6\ \text{eV}}{4} = -3{\cdot}4\ \text{eV}.$$

The energy of $n = \infty$ state is zero.

The wavelength emitted in the given transition is

$$\lambda = \frac{hc}{\Delta E}$$

$$= \frac{1242 \text{ eV nm}}{3{\cdot}4 \text{ eV}} = 365 \text{ nm}.$$

5. *(a) Find the wavelength of the radiation required to excite the electron in Li^{++} from the first to the third Bohr orbit. (b) How many spectral lines are observed in the emission spectrum of the above excited system ?*

Solution : (a) The energy in the first orbit $= E_1 = Z^2 E_0$ where $E_0 = -13{\cdot}6$ eV is the energy of a hydrogen atom in ground state. Thus for Li^{++},

$$E_1 = 9E_0 = 9 \times (-13{\cdot}6 \text{ eV}).$$

The energy in the third orbit is

$$E_3 = \frac{E_1}{n^2} = \frac{E_1}{9} = -13{\cdot}6 \text{ eV}.$$

Thus, $\quad E_3 - E_1 = 8 \times 13{\cdot}6 \text{ eV} = 108{\cdot}8 \text{ eV}.$

The wavelength of radiation required to excite Li^{++} from the first orbit to the third orbit is given by

$$\frac{hc}{\lambda} = E_3 - E_1$$

or,

$$\lambda = \frac{hc}{E_3 - E_1}$$

$$= \frac{1242 \text{ eV nm}}{108{\cdot}8 \text{ eV}} \approx 11{\cdot}4 \text{ nm}.$$

(b) The spectral lines emitted are due to the transitions $n = 3 \rightarrow n = 2$, $n = 3 \rightarrow n = 1$ and $n = 2 \rightarrow n = 1$. Thus, there will be three spectral lines in the spectrum.

6. *Find the wavelengths present in the radiation emitted when hydrogen atoms excited to $n = 3$ states return to their ground states.*

Solution : A hydrogen atom may return directly to the ground state or it may go to $n = 2$ and from there to the ground state. Thus, wavelengths corresponding to $n = 3 \rightarrow n = 1$, $n = 3 \rightarrow n = 2$ and $n = 2 \rightarrow n = 1$ are present in the radiation.

The energies in $n = 1$, 2 and 3 states are

$$E_1 = -13{\cdot}6 \text{ eV}$$

$$E_2 = -\frac{13{\cdot}6}{4} \text{ eV} = -3{\cdot}4 \text{ eV}$$

and

$$E_3 = -\frac{13{\cdot}6}{9} \text{ eV} = -1{\cdot}5 \text{ eV}.$$

The wavelength emitted in the transition $n = 3$ to the ground state is

$$\lambda = \frac{hc}{\Delta E}$$

$$= \frac{1242 \text{ eV nm}}{13{\cdot}6 \text{ eV} - 1{\cdot}5 \text{ eV}} = 103 \text{ nm}.$$

Similarly, the wavelength emitted in the transition $n = 3$ to $n = 2$ is 654 nm and that emitted in the transition $n = 2$ to $n = 1$ is 122 nm. The wavelengths present in the radiation are, therefore, 103 nm, 122 nm and 654 nm.

7. *How many different wavelengths may be observed in the spectrum from a hydrogen sample if the atoms are excited to states with principal quantum number n ?*

Solution : From the nth state, the atom may go to $(n-1)$th state, ... , 2nd state or 1st state. So there are $(n-1)$ possible transitions starting from the nth state. The atoms reaching $(n-1)$th state may make $(n-2)$ different transitions. Similarly for other lower states. The total number of possible transitions is

$$(n-1) + (n-2) + (n-3) + \dots 2 + 1$$

$$= \frac{n(n-1)}{2}.$$

8. *Monochromatic radiation of wavelength λ is incident on a hydrogen sample in ground state. Hydrogen atoms absorb a fraction of light and subsequently emit radiation of six different wavelengths. Find the value of λ.*

Solution : As the hydrogen atoms emit radiation of six different wavelengths, some of them must have been excited to $n = 4$. The energy in $n = 4$ state is

$$E_4 = \frac{E_1}{4^2} = -\frac{13{\cdot}6 \text{ eV}}{16} = -0{\cdot}85 \text{ eV}.$$

The energy needed to take a hydrogen atom from its ground state to $n = 4$ is

$$13{\cdot}6 \text{ eV} - 0{\cdot}85 \text{ eV} = 12{\cdot}75 \text{ eV}.$$

The photons of the incident radiation should have $12{\cdot}75$ eV of energy. So

$$\frac{hc}{\lambda} = 12{\cdot}75 \text{ eV}$$

or,

$$\lambda = \frac{hc}{12{\cdot}75 \text{ eV}}$$

$$= \frac{1242 \text{ eV nm}}{12{\cdot}75 \text{ eV}} = 97{\cdot}5 \text{ nm}.$$

9. *The energy needed to detach the electron of a hydrogen-like ion in ground state is 4 rydberg. (a) What is the wavelength of the radiation emitted when the electron jumps from the first excited state to the ground state ? (b) What is the radius of the first orbit for this atom ?*

Solution : (a) In energy units, 1 rydberg $= 13{\cdot}6$ eV. The energy needed to detach the electron is $4 \times 13{\cdot}6$ eV. The energy in the ground state is, therefore, $E_1 = -4 \times 13{\cdot}6$ eV. The energy of the first excited state $(n = 2)$ is $E_2 = \frac{E_1}{4} = -13{\cdot}6$ eV. The energy difference is $E_2 - E_1 = 3 \times 13{\cdot}6 \text{ eV} = 40{\cdot}8$ eV. The wavelength of the radiation emitted is

$$\lambda = \frac{hc}{\Delta E}$$

$$= \frac{1242 \text{ eV nm}}{40.8 \text{ eV}} = 30.4 \text{ nm}.$$

(b) The energy of a hydrogen-like ion in ground state is $E = Z^2 E_0$ where Z = atomic number and $E_0 = -13.6$ eV. Thus, $Z = 2$. The radius of the first orbit is $\frac{a_0}{Z}$ where $a_0 = 53$ pm. Thus,

$$r = \frac{53 \text{ pm}}{2} = 26.5 \text{ pm}.$$

10. *A hydrogen sample is prepared in a particular excited state A. Photons of energy 2.55 eV get absorbed into the sample to take some of the electrons to a further excited state B. Find the quantum numbers of the states A and B.*

Solution : The allowed energies of hydrogen atoms are

$$E_1 = -13.6 \text{ eV}$$

$$E_2 = -3.4 \text{ eV}$$

$$E_3 = -1.5 \text{ eV}$$

$$E_4 = -0.85 \text{ eV}$$

$$E_5 = -0.54 \text{ eV}.$$

We see that a difference of 2.55 eV can only be absorbed in transition $n = 2$ to $n = 4$. So the state A has quantum number 2 and the state B has quantum number 4.

11. *(a) Find the maximum wavelength λ_0 of light which can ionize a hydrogen atom in its ground state. (b) Light of wavelength λ_0 is incident on a hydrogen atom which is in its first excited state. Find the kinetic energy of the electron coming out.*

Solution : (a) To ionize a hydrogen atom in ground state, a minimum of 13.6 eV energy should be given to it. A photon of light should have this much of energy in order to ionize a hydrogen atom. Thus,

$$\frac{hc}{\lambda_0} = 13.6 \text{ eV}$$

or, $$\lambda_0 = \frac{1242 \text{ eV nm}}{13.6 \text{ eV}} = 91.3 \text{ nm}.$$

(b) The energy of the hydrogen atom in its first excited state is $-\frac{13.6 \text{ eV}}{4} = -3.4$ eV. Thus, 3.4 eV of energy is needed to take the electron out of the atom. The energy of a photon of the light of wavelength λ_0 is 13.6 eV. Thus, the electron coming out will have a kinetic energy 13.6 eV − 3.4 eV = 10.2 eV.

12. *Derive an expression for the magnetic field at the site of the nucleus in a hydrogen atom due to the circular motion of the electron. Assume that the atom is in its ground state and give the answer in terms of fundamental constants.*

Solution : We have

$$\frac{mv^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

or, $$v^2 r = \frac{e^2}{4\pi\varepsilon_0 m}. \quad \text{... (i)}$$

From Bohr's quantization rule, in ground state,

$$vr = \frac{h}{2\pi m}. \quad \text{... (ii)}$$

From (i) and (ii),

$$v = \frac{e^2}{2\varepsilon_0 h} \quad \text{... (iii)}$$

and $$r = \frac{\varepsilon_0 h^2}{\pi m e^2}. \quad \text{... (iv)}$$

As the electron moves along a circle, it crosses any point on the circle $\frac{v}{2\pi r}$ times per unit time. The charge crossing per unit time, that is the current, is $i = \frac{ev}{2\pi r}$. The magnetic field at the centre due to this circular current is

$$B = \frac{\mu_0 i}{2r} = \frac{\mu_0 ev}{4\pi r^2}.$$

From (iii) and (iv),

$$B = \frac{\mu_0 e}{4\pi} \frac{e^2}{2\varepsilon_0 h} \times \frac{\pi^2 m^2 e^4}{\varepsilon_0^2 h^4}$$

$$= \frac{\mu_0 e^7 \pi m^2}{8\varepsilon_0^3 h^5}.$$

13. *A lithium atom has three electrons. Assume the following simple picture of the atom. Two electrons move close to the nucleus making up a spherical cloud around it and the third moves outside this cloud in a circular orbit. Bohr's model can be used for the motion of this third electron but $n = 1$ states are not available to it. Calculate the ionization energy of lithium in ground state using the above picture.*

Solution : In this picture, the third electron moves in the field of a total charge $+3e - 2e = +e$. Thus, the energies are the same as that of hydrogen atoms. The lowest energy is

$$E_2 = \frac{E_1}{4} = \frac{-13.6 \text{ eV}}{4} = -3.4 \text{ eV}.$$

Thus, the ionization energy of the atom in this picture is 3.4 eV.

14. *A particle known as μ-meson, has a charge equal to that of an electron and mass 208 times the mass of the electron. It moves in a circular orbit around a nucleus of charge $+3e$. Take the mass of the nucleus to be infinite. Assuming that the Bohr's model is applicable to this system, (a) derive an expression for the radius of the nth Bohr orbit, (b) find the value of n for which the radius of the orbit is approximately the same as that of* *the first Bohr orbit for a hydrogen atom and (c) find the wavelength of the radiation emitted when the μ-meson jumps from the third orbit to the first orbit.*

Solution : (a) We have,

$$\frac{mv^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$$

or,
$$v^2 r = \frac{Ze^2}{4\pi\varepsilon_0 m} \cdot \qquad \ldots \text{(i)}$$

The quantization rule is $vr = \dfrac{nh}{2\pi m} \cdot$

The radius is $r = \dfrac{(vr)^2}{v^2 r} = \dfrac{n^2 h^2}{4\pi^2 m^2} \dfrac{4\pi\varepsilon_0 m}{Ze^2}$

$$= \frac{n^2 h^2 \varepsilon_0}{Z\pi m e^2} \cdot \qquad \ldots \text{(ii)}$$

For the given system, $Z = 3$ and $m = 208\ m_e$.

Thus
$$r_\mu = \frac{n^2 h^2 \varepsilon_0}{624\pi m_e e^2} \cdot$$

(b) From (ii), the radius of the first Bohr orbit for the hydrogen atom is

$$r_h = \frac{h^2 \varepsilon_0}{\pi m_e e^2} \cdot$$

For $r_\mu = r_h$,

$$\frac{n^2 h^2 \varepsilon_0}{624\pi m_e e^2} = \frac{h^2 \varepsilon_0}{\pi m_e e^2}$$

or,
$$n^2 = 624$$

or,
$$n \approx 25.$$

(c) From (i), the kinetic energy of the atom is

$$\frac{mv^2}{2} = \frac{Ze^2}{8\pi\varepsilon_0 r}$$

and the potential energy is $-\dfrac{Ze^2}{4\pi\varepsilon_0 r} \cdot$

The total energy is $E_n = -\dfrac{Ze^2}{8\pi\varepsilon_0 r} \cdot$

Using (ii),

$$E_n = -\frac{Z^2 \pi m e^4}{8\pi\varepsilon_0^2 n^2 h^2} = -\frac{9 \times 208\ m_e e^4}{8\varepsilon_0^2 n^2 h^2}$$

$$= \frac{1872}{n^2}\left(-\frac{m_e e^4}{8\varepsilon_0^2 h^2}\right) \cdot \qquad \ldots \text{(iii)}$$

But $\left(-\dfrac{m_e e^4}{8\varepsilon_0^2 h^2}\right)$ is the ground state energy of hydrogen atom and hence is equal to $-13{\cdot}6$ eV.

From (iii), $E_n = -\dfrac{1872}{n^2} \times 13{\cdot}6 \text{ eV} = \dfrac{-25459{\cdot}2 \text{ eV}}{n^2} \cdot$

Thus, $E_1 = -25459{\cdot}2$ eV and $E_3 = \dfrac{E_1}{9} = -2828{\cdot}8$ eV. The energy difference is $E_3 - E_1 = 22630{\cdot}4$ eV.

The wavelength emitted is

$$\lambda = \frac{hc}{\Delta E}$$

$$= \frac{1242 \text{ eV nm}}{22630{\cdot}4 \text{ eV}} = 55 \text{ pm}.$$

15. *Find the wavelengths in a hydrogen spectrum between the range* 500 nm *to* 700 nm.

Solution : The energy of a photon of wavelength 500 nm is

$$\frac{hc}{\lambda} = \frac{1242 \text{ eV nm}}{500 \text{ nm}} = 2{\cdot}44 \text{ eV}.$$

The energy of a photon of wavelength 700 nm is

$$\frac{hc}{\lambda} = \frac{1242 \text{ eV nm}}{700 \text{ nm}} = 1{\cdot}77 \text{ eV}.$$

The energy difference between the states involved in the transition should, therefore, be between 1·77 eV and 2·44 eV.

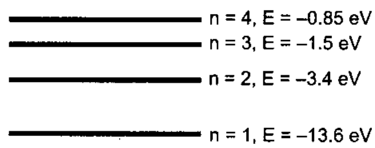

Figure 43-W1

Figure (43-W1) shows some of the energies of hydrogen states. It is clear that only those transitions which end at $n = 2$ may emit photons of energy between 1·77 eV and 2·44 eV. Out of these only $n = 3 \rightarrow n = 2$ falls in the proper range. The energy of the photon emitted in the transition $n = 3$ to $n = 2$ is $\Delta E = (3{\cdot}4 - 1{\cdot}5)$ eV $= 1{\cdot}9$ eV. The wavelength is

$$\lambda = \frac{hc}{\Delta E}$$

$$= \frac{1242 \text{ eV nm}}{1{\cdot}9 \text{ eV}} = 654 \text{ nm}.$$

16. *A beam of ultraviolet radiation having wavelength between* 100 nm *and* 200 nm *is incident on a sample of atomic hydrogen gas. Assuming that the atoms are in ground state, which wavelengths will have low intensity in the transmitted beam? If the energy of a photon is equal to the difference between the energies of an excited state and the ground state, it has large probability of being absorbed by an atom in the ground state.*

Solution : The energy of a photon corresponding to $\lambda = 100$ nm is

$$\frac{1242 \text{ eV nm}}{100 \text{ nm}} = 12{\cdot}42 \text{ eV}$$

and that corresponding to λ = 200 nm is 6·21 eV.

The energy needed to take the atom from the ground state to the first excited state is

$$E_2 - E_1 = 13{\cdot}6 \text{ eV} - 3{\cdot}4 \text{ eV} = 10{\cdot}2 \text{ eV},$$

to the second excited state is

$$E_3 - E_1 = 13{\cdot}6 \text{ eV} - 1{\cdot}5 \text{ eV} = 12{\cdot}1 \text{ eV},$$

to the third excited state is

$$E_4 - E_1 = 13{\cdot}6 \text{ eV} - 0{\cdot}85 \text{ eV} = 12{\cdot}75 \text{ eV, etc.}$$

Thus, 10·2 eV photons and 12·1 eV photons have large probability of being absorbed from the given range 6·21 eV to 12·42 eV. The corresponding wavelengths are

$$\lambda_1 = \frac{1242 \text{ eV nm}}{10{\cdot}2 \text{ eV}} = 122 \text{ nm}$$

and $$\lambda_2 = \frac{1242 \text{ eV nm}}{12{\cdot}1 \text{ eV}} = 103 \text{ nm}.$$

These wavelengths will have low intensity in the transmitted beam.

17. *A neutron moving with speed v makes a head-on collision with a hydrogen atom in ground state kept at rest. Find the minimum kinetic energy of the neutron for which inelastic (completely or partially) collision may take place. The mass of neutron ≈ mass of hydrogen* $= 1{\cdot}67 \times 10^{-27}$ kg.

Solution : Suppose the neutron and the hydrogen atom move at speeds v_1 and v_2 after the collision. The collision will be inelastic if a part of the kinetic energy is used to excite the atom. Suppose an energy ΔE is used in this way. Using conservation of linear momentum and energy,

$$mv = mv_1 + mv_2 \qquad \text{... (i)}$$

and $$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \Delta E \qquad \text{... (ii)}$$

From (i), $$v^2 = v_1^2 + v_2^2 + 2v_1v_2.$$

From (ii), $$v^2 = v_1^2 + v_2^2 + \frac{2\Delta E}{m}.$$

Thus, $$2v_1v_2 = \frac{2\Delta E}{m}.$$

Hence, $$(v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2 = v^2 - \frac{4\Delta E}{m}.$$

As $v_1 - v_2$ must be real,

$$v^2 - \frac{4\Delta E}{m} \geq 0$$

or, $$\frac{1}{2}mv^2 > 2\Delta E.$$

The minimum energy that can be absorbed by the hydrogen atom in ground state to go in an excited state is 10·2 eV. Thus, the minimum kinetic energy of the neutron needed for an inelastic collision is

$$\frac{1}{2}mv_{min}^2 = 2 \times 10{\cdot}2 \text{ eV} = 20{\cdot}4 \text{ eV}.$$

18. *Light corresponding to the transition n = 4 to n = 2 in hydrogen atoms falls on cesium metal (work function = 1·9 eV). Find the maximum kinetic energy of the photoelectrons emitted.*

Solution : The energy of the photons emitted in transition $n = 4$ to $n = 2$ is

$$h\nu = 13{\cdot}6 \text{ eV}\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = 2{\cdot}55 \text{ eV}.$$

The maximum kinetic energy of the photoelectrons is

$$= 2{\cdot}55 \text{ eV} - 1{\cdot}9 \text{ eV} = 0{\cdot}65 \text{ eV}.$$

19. *A small particle of mass m moves in such a way that the potential energy* $U = \frac{1}{2}m^2\omega^2r^2$ *where ω is a constant and r is the distance of the particle from the origin. Assuming Bohr's model of quantization of angular momentum and circular orbits, show that radius of the nth allowed orbit is proportional to √n.*

Solution : The force at a distance r is

$$F = -\frac{dU}{dr} = -m\omega^2 r. \qquad \text{... (i)}$$

Suppose the particle moves along a circle of radius r. The net force on it should be mv^2/r along the radius. Comparing with (i),

$$\frac{mv^2}{r} = m\omega^2 r$$

or, $$v = \omega r. \qquad \text{... (ii)}$$

The quantization of angular momentum gives

$$mvr = \frac{nh}{2\pi}$$

or, $$v = \frac{nh}{2\pi mr}. \qquad \text{... (iii)}$$

From (ii) and (iii),

$$r = \left(\frac{nh}{2\pi m\omega}\right)^{1/2}.$$

Thus, the radius of the nth orbit is proportional to $\sqrt{n}$.

□

QUESTIONS FOR SHORT ANSWER

1. How many wavelengths are emitted by atomic hydrogen in visible range (380 nm–780 nm)? In the range 50 nm to 100 nm?
2. The first excited energy of a He^+ ion is the same as the ground state energy of hydrogen. Is it always true that one of the energies of any hydrogen-like ion will be the same as the ground state energy of a hydrogen atom?
3. Which wavelengths will be emitted by a sample of atomic hydrogen gas (in ground state) if electrons of energy 12·2 eV collide with the atoms of the gas?
4. When white radiation is passed through a sample of hydrogen gas at room temperature, absorption lines are observed in Lyman series only. Explain.

5. Balmer series was observed and analysed before the other series. Can you suggest a reason for such an order ?
6. What will be the energy corresponding to the first excited state of a hydrogen atom if the potential energy of the atom is taken to be 10 eV when the electron is widely separated from the proton ? Can we still write $E_n = E_1/n^2$, or $r_n = a_0 n^2$?
7. The difference in the frequencies of series limit of Lyman series and Balmer series is equal to the frequency of the first line of the Lyman series. Explain.
8. The numerical value of ionization energy in eV equals the ionization potential in volts. Does the equality hold if these quantities are measured in some other units ?
9. We have stimulated emission and spontaneous emission. Do we also have stimulated absorption and spontaneous absorption ?
10. An atom is in its excited state. Does the probability of its coming to ground state depend on whether the radiation is already present or not ? If yes, does it also depend on the wavelength of the radiation present ?

OBJECTIVE I

1. The minimum orbital angular momentum of the electron in a hydrogen atom is
(a) h (b) $h/2$ (c) $h/2\pi$ (d) h/λ.
2. Three photons coming from excited atomic-hydrogen sample are picked up. Their energies are 12·1 eV, 10·2 eV and 1·9 eV. These photons must come from
(a) a single atom (b) two atoms
(c) three atoms (d) either two atoms or three atoms.
3. Suppose, the electron in a hydrogen atom makes transition from $n = 3$ to $n = 2$ in 10^{-8} s. The order of the torque acting on the electron in this period, using the relation between torque and angular momentum as discussed in the chapter on rotational mechanics is
(a) 10^{-34} N m (b) 10^{-24} N m
(c) 10^{-42} N m (d) 10^{-8} N m.
4. In which of the following transitions will the wavelength be minimum ?
(a) $n = 5$ to $n = 4$ (b) $n = 4$ to $n = 3$
(c) $n = 3$ to $n = 2$ (d) $n = 2$ to $n = 1$.
5. In which of the following systems will the radius of the first orbit ($n = 1$) be minimum ?
(a) Hydrogen atom (b) Deuterium atom
(c) Singly ionized helium (d) Doubly ionized lithium.
6. In which of the following systems will the wavelength corresponding to $n = 2$ to $n = 1$ be minimum ?
(a) Hydrogen atom (b) Deuterium atom
(c) Singly ionized helium (d) Doubly ionized lithium.
7. Which of the following curves may represent the speed of the electron in a hydrogen atom as a function of the principal quantum number n ?

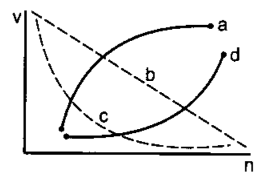

Figure 43-Q1

8. As one considers orbits with higher values of n in a hydrogen atom, the electric potential energy of the atom
(a) decreases (b) increases
(c) remains the same (d) does not increase.
9. The energy of an atom (or ion) in its ground state is –54·4 eV. It may be
(a) hydrogen (b) deuterium (c) He^+ (d) Li^{++}.
10. The radius of the shortest orbit in a one-electron system is 18 pm. It may be
(a) hydrogen (b) deuterium (c) He^+ (d) Li^{++}.
11. A hydrogen atom in ground state absorbs 10·2 eV of energy. The orbital angular momentum of the electron is increased by
(a) $1{\cdot}05 \times 10^{-34}$ J s (b) $2{\cdot}11 \times 10^{-34}$ J s
(c) $3{\cdot}16 \times 10^{-34}$ J s (d) $4{\cdot}22 \times 10^{-34}$ J s.
12. Which of the following parameters are the same for all hydrogen-like atoms and ions in their ground states ?
(a) Radius of the orbit (b) Speed of the electron
(c) Energy of the atom
(d) Orbital angular momentum of the electron
13. In a laser tube, all the photons
(a) have same wavelength (b) have same energy
(c) move in same direction (d) move with same speed.

OBJECTIVE II

1. In a laboratory experiment on emission from atomic hydrogen in a discharge tube, only a small number of lines are observed whereas a large number of lines are present in the hydrogen spectrum of a star. This is because in a laboratory
(a) the amount of hydrogen taken is much smaller than that present in the star
(b) the temperature of hydrogen is much smaller than that of the star
(c) the pressure of hydrogen is much smaller than that of the star
(d) the gravitational pull is much smaller than that in the star.

2. An electron with kinetic energy 5 eV is incident on a hydrogen atom in its ground state. The collision
(a) must be elastic
(b) may be partially elastic
(c) must be completely inelastic
(d) may be completely inelastic.

3. Which of the following products in a hydrogen atom are independent of the principal quantum number n? The symbols have their usual meanings.
(a) vn (b) Er (c) En (d) vr

4. Let A_n be the area enclosed by the nth orbit in a hydrogen atom. The graph of $\ln (A_n/A_1)$ against $\ln(n)$
(a) will pass through the origin
(b) will be a straight line with slope 4
(c) will be a monotonically increasing nonlinear curve
(d) will be a circle.

5. Ionization energy of a hydrogen-like ion A is greater than that of another hydrogen-like ion B. Let r, u, E and L represent the radius of the orbit, speed of the electron, energy of the atom and orbital angular momentum of the electron respectively. In ground state
(a) $r_A > r_B$ (b) $u_A > u_B$ (c) $E_A > E_B$ (d) $L_A > L_B$.

6. When a photon stimulates the emission of another photon, the two photons have
(a) same energy
(b) same direction
(c) same phase
(d) same wavelength.

EXERCISES

Planck constant $h = 6{\cdot}63 \times 10^{-34}$ Js $= 4{\cdot}14 \times 10^{-15}$ eVs, first Bohr radius of hydrogen $a_0 = 53$ pm, energy of hydrogen atom in ground state $= -13{\cdot}6$ eV, Rydberg's constant $= 1{\cdot}097 \times 10^{7}\ \mathrm{m}^{-1}$.

---

1. The Bohr radius is given by $a_0 = \frac{\varepsilon_0 h^2}{\pi m e^2}$. Verify that the RHS has dimensions of length.

2. Find the wavelength of the radiation emitted by hydrogen in the transitions (a) $n = 3$ to $n = 2$, (b) $n = 5$ to $n = 4$ and (c) $n = 10$ to $n = 9$.

3. Calculate the smallest wavelength of radiation that may be emitted by (a) hydrogen, (b) He^{+} and (c) Li^{++}.

4. Evaluate Rydberg constant by putting the values of the fundamental constants in its expression.

5. Find the binding energy of a hydrogen atom in the state $n = 2$.

6. Find the radius and energy of a He^{+} ion in the states (a) $n = 1$, (b) $n = 4$ and (c) $n = 10$.

7. A hydrogen atom emits ultraviolet radiation of wavelength 102·5 nm. What are the quantum numbers of the states involved in the transition?

8. (a) Find the first excitation potential of He^{+} ion. (b) Find the ionization potential of Li^{++} ion.

9. A group of hydrogen atoms are prepared in $n = 4$ states. List the wavelengths that are emitted as the atoms make transitions and return to $n = 2$ states.

10. A positive ion having just one electron ejects it if a photon of wavelength 228 Å or less is absorbed by it. Identify the ion.

11. Find the maximum Coulomb force that can act on the electron due to the nucleus in a hydrogen atom.

12. A hydrogen atom in a state having a binding energy of 0·85 eV makes transition to a state with excitation energy 10·2 eV. (a) Identify the quantum numbers n of the upper and the lower energy states involved in the transition. (b) Find the wavelength of the emitted radiation.

13. Whenever a photon is emitted by hydrogen in Balmer series, it is followed by another photon in Lyman series. What wavelength does this latter photon correspond to?

14. A hydrogen atom in state $n = 6$ makes two successive transitions and reaches the ground state. In the first transition a photon of 1·13 eV is emitted. (a) Find the energy of the photon emitted in the second transition. (b) What is the value of n in the intermediate state?

15. What is the energy of a hydrogen atom in the first excited state if the potential energy is taken to be zero in the ground state?

16. A hot gas emits radiation of wavelengths 46·0 nm, 82·8 nm and 103·5 nm only. Assume that the atoms have only two excited states and the difference between consecutive energy levels decreases as energy is increased. Taking the energy of the highest energy state to be zero, find the energies of the ground state and the first excited state.

17. A gas of hydrogen-like ions is prepared in a particular excited state A. It emits photons having wavelength equal to the wavelength of the first line of the Lyman series together with photons of five other wavelengths. Identify the gas and find the principal quantum number of the state A.

18. Find the maximum angular speed of the electron of a hydrogen atom in a stationary orbit.

19. A spectroscopic instrument can resolve two nearby wavelengths λ and $\lambda + \Delta\lambda$ if $\lambda/\Delta\lambda$ is smaller than 8000. This is used to study the spectral lines of the Balmer series of hydrogen. Approximately how many lines will be resolved by the instrument?

20. Suppose, in certain conditions only those transitions are allowed to hydrogen atoms in which the principal quantum number n changes by 2. (a) Find the smallest wavelength emitted by hydrogen. (b) List the wavelengths emitted by hydrogen in the visible range (380 nm to 780 nm).

21. According to Maxwell's theory of electrodynamics, an electron going in a circle should emit radiation of frequency equal to its frequency of revolution. What

should be the wavelength of the radiation emitted by a hydrogen atom in ground state if this rule is followed?

22. The average kinetic energy of molecules in a gas at temperature T is 1·5 kT. Find the temperature at which the average kinetic energy of the molecules of hydrogen equals the binding energy of its atoms. Will hydrogen remain in molecular form at this temperature? Take $k = 8{\cdot}62 \times 10^{-5}$ eV K^{-1}.

23. Find the temperature at which the average thermal kinetic energy is equal to the energy needed to take a hydrogen atom from its ground state to $n = 3$ state. Hydrogen can now emit red light of wavelength 653·1 nm. Because of Maxwellian distribution of speeds, a hydrogen sample emits red light at temperatures much lower than that obtained from this problem. Assume that hydrogen molecules dissociate into atoms.

24. Average lifetime of a hydrogen atom excited to $n = 2$ state is 10^{-8} s. Find the number of revolutions made by the electron on the average before it jumps to the ground state.

25. Calculate the magnetic dipole moment corresponding to the motion of the electron in the ground state of a hydrogen atom.

26. Show that the ratio of the magnetic dipole moment to the angular momentum ($l = mvr$) is a universal constant for hydrogen-like atoms and ions. Find its value.

27. A beam of light having wavelengths distributed uniformly between 450 nm to 550 nm passes through a sample of hydrogen gas. Which wavelength will have the least intensity in the transmitted beam?

28. Radiation coming from transitions $n = 2$ to $n = 1$ of hydrogen atoms falls on helium ions in $n = 1$ and $n = 2$ states. What are the possible transitions of helium ions as they absorb energy from the radiation?

29. A hydrogen atom in ground state absorbs a photon of ultraviolet radiation of wavelength 50 nm. Assuming that the entire photon energy is taken up by the electron, with what kinetic energy will the electron be ejected?

30. A parallel beam of light of wavelength 100 nm passes through a sample of atomic hydrogen gas in ground state. (a) Assume that when a photon supplies some of its energy to a hydrogen atom, the rest of the energy appears as another photon moving in the same direction as the incident photon. Neglecting the light emitted by the excited hydrogen atoms in the direction of the incident beam, what wavelengths may be observed in the transmitted beam? (b) A radiation detector is placed near the gas to detect radiation coming perpendicular to the incident beam. Find the wavelengths of radiation that may be detected by the detector.

31. A beam of monochromatic light of wavelength λ ejects photoelectrons from a cesium surface ($\Phi = 1{\cdot}9$ eV). These photoelectrons are made to collide with hydrogen atoms in ground state. Find the maximum value of λ for which (a) hydrogen atoms may be ionized, (b) hydrogen atoms may get excited from the ground state to the first excited state and (c) the excited hydrogen atoms may emit visible light.

32. Electrons are emitted from an electron gun at almost zero velocity and are accelerated by an electric field E through a distance of 1·0 m. The electrons are now scattered by an atomic hydrogen sample in ground state. What should be the minimum value of E so that red light of wavelength 656·3 nm may be emitted by the hydrogen?

33. A neutron having kinetic energy 12·5 eV collides with a hydrogen atom at rest. Nelgect the difference in mass between the neutron and the hydrogen atom and assume that the neutron does not leave its line of motion. Find the possible kinetic energies of the neutron after the event.

34. A hydrogen atom moving at speed v collides with another hydrogen atom kept at rest. Find the minimum value of v for which one of the atoms may get ionized. The mass of a hydrogen atom $= 1{\cdot}67 \times 10^{-27}$ kg.

35. A neutron moving with a speed v strikes a hydrogen atom in ground state moving towards it with the same speed. Find the minimum speed of the neutron for which inelastic (completely or partially) collision may take place. The mass of neutron $\approx$ mass of hydrogen $= 1{\cdot}67 \times 10^{-27}$ kg.

36. When a photon is emitted by a hydrogen atom, the photon carries a momentum with it. (a) Calculate the momentum carried by the photon when a hydrogen atom emits light of wavelength 656·3 nm. (b) With what speed does the atom recoil during this transition? Take the mass of the hydrogen atom $= 1{\cdot}67 \times 10^{-27}$ kg. (c) Find the kinetic energy of recoil of the atom.

37. When a photon is emitted from an atom, the atom recoils. The kinetic energy of recoil and the energy of the photon come from the difference in energies between the states involved in the transition. Suppose, a hydrogen atom changes its state from $n = 3$ to $n = 2$. Calculate the fractional change in the wavelength of light emitted, due to the recoil.

38. The light emitted in the transition $n = 3$ to $n = 2$ in hydrogen is called H_α light. Find the maximum work function a metal can have so that H_α light can emit photoelectrons from it.

39. Light from Balmer series of hydrogen is able to eject photoelectrons from a metal. What can be the maximum work function of the metal?

40. Radiation from hydrogen discharge tube falls on a cesium plate. Find the maximum possible kinetic energy of the photoelectrons. Work function of cesium is 1·9 eV.

41. A filter transmits only the radiation of wavelength greater than 440 nm. Radiation from a hydrogen-discharge tube goes through such a filter and is incident on a metal of work function 2·0 eV. Find the stopping potential which can stop the photoelectrons.

42. The earth revolves round the sun due to gravitational attraction. Suppose that the sun and the earth are point particles with their existing masses and that Bohr's quantization rule for angular momentum is valid in the case of gravitation. (a) Calculate the minimum radius the earth can have for its orbit. (b) What is the value of the principal quantum number n for the present radius ? Mass of the earth $= 6.0 \times 10^{24}$ kg, mass of the sun $= 2.0 \times 10^{30}$ kg, earth–sun distance $= 1.5 \times 10^{11}$ m.

43. Consider a neutron and an electron bound to each other due to gravitational force. Assuming Bohr's quantization rule for angular momentum to be valid in this case, derive an expression for the energy of the neutron–electron system.

44. A uniform magnetic field B exists in a region. An electron projected perpendicular to the field goes in a circle. Assuming Bohr's quantization rule for angular momentum, calculate (a) the smallest possible radius of the electron (b) the radius of the nth orbit and (c) the minimum possible speed of the electron.

45. Suppose in an imaginary world the angular momentum is quantized to be even integral multiples of $h/2\pi$. What is the longest possible wavelength emitted by hydrogen atoms in visible range in such a world according to Bohr's model ?

46. Consider an excited hydrogen atom in state n moving with a velocity $v (v << c)$. It emits a photon in the direction of its motion and changes its state to a lower state m. Apply momentum and energy conservation principles to calculate the frequency ν of the emitted radiation. Compare this with the frequency ν_0 emitted if the atom were at rest.

□

ANSWERS

OBJECTIVE I

1. (c)	2. (d)	3. (b)	4. (d)	5. (d)	6. (d)
7. (c)	8. (b)	9. (c)	10. (d)	11. (a)	12. (d)
13. (d)					

OBJECTIVE II

1. (b)	2. (a)	3. (a), (b)	4. (a), (b)
5. (b)	6. all		

EXERCISES

2. (a) 654 nm (b) 4050 nm (c) 38860 nm
3. (a) 91 nm (b) 23 nm (c) 10 nm
4. $1.097 \times 10^{7}\ \mathrm{m}^{-1}$
5. 3·4 eV
6. (a) 0·265 A, –54·4 eV (b) 4·24 A, –3·4 eV (c) 26·5 A, –0·544 eV
7. 1 and 3
8. (a) 40·8 V (b) 122·4 V
9. 487 nm, 654 nm, 1910 nm
10. He^{+}
11. 8.2×10^{-8} N
12. (a) 4, 2 (b) 487 nm
13. 122 nm
14. 12·1 eV, 3
15. 23·8 eV
16. –27 eV, –12 eV
17. He^{+}, 4
18. $4.1 \times 10^{16}\ \mathrm{rad\ s^{-1}}$
19. 38
20. (a) 103 nm (b) 487 nm
21. 45·7 nm
22. 1.05×10^{5} K
23. 9.4×10^{4} K
24. 8.2×10^{6}
25. $9.2 \times 10^{-24}\ \mathrm{A\ m^{-2}}$
26. $\frac{e}{2m} = 8.8 \times 10^{10}\ \mathrm{C\ kg^{-1}}$
27. 487 nm
28. $n = 2$ to $n = 3$ and $n = 2$ to $n = 4$
29. 11·24 eV
30. (a) 100 nm, 560 nm, 3880 nm (b) 103 nm, 121 nm, 654 nm
31. (a) 80 nm (b) 102 nm (c) 89 nm
32. $12.1\ \mathrm{Vm^{-1}}$
33. zero
34. $7.2 \times 10^{4}\ \mathrm{m\ s^{-1}}$
35. $3.13 \times 10^{4}\ \mathrm{m\ s^{-1}}$

36. (a) $1{\cdot}0 \times 10^{27}$ kg m s^{-1} (b) $0{\cdot}6$ m s^{-1}
(c) $1{\cdot}9 \times 10^{-9}$ eV

37. 10^{-9}

38. 1·9 eV

39. 3·4 eV

40. 11·7 eV

41. 0·55 V

42. (a) $2{\cdot}3 \times 10^{-138}$ m (b) $2{\cdot}5 \times 10^{74}$

43. $-\dfrac{2\pi^2 G^2 m_n^2 m_e^3}{2h^2 n^2}$

44. (a) $\sqrt{\dfrac{h}{2\pi eB}}$ (b) $\sqrt{\dfrac{nh}{2\pi eB}}$ (c) $\sqrt{\dfrac{heB}{2\pi m^2}}$

45. 487 nm

46. $\nu = \nu_0\left(1 + \dfrac{v}{c}\right)$

□