Indefinite Integrals Ex 19.30 Q61

$$f(x) = \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$$

Now,

$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$$

$$= \frac{x^4+3x^2+2}{x^4+7x^2+12}$$

$$= \frac{(x^4+7x^2+12)-4x^2-10}{x^4+7x^2+12}$$

$$= 1-\frac{4x^2+10}{x^4+7x^2+12}$$

Now,

$$\frac{4x^2+10}{x^4+7x^2+12} = \frac{4x^2+10}{(x^2+3)(x^2+4)}$$

Let $$\frac{4x^2+10}{(x^2+3)(x^2+4)} = \frac{Ax+B}{x^2+3}+\frac{Cx+D}{x^2+4}$$

$$\Rightarrow \quad 4x^2+10 = (Ax+B)(x^2+4)+(Cx+D)(x^2+3)$$

Let $x = 0$, we get

$$10 = 4B+3D \qquad ---(i)$$

If $x = 1$, we get

$$14 = 5(A+B)+4(C+D) = 5A+5B+4C+4D \qquad ---(ii)$$

if $x = -1$, we get

$$14 = 5(-A+B)+4(-C+D) = -5A+5B-4C+4D \qquad ---(iii)$$

Applying (ii) and (iii), we get

$$28 = 10B+8D$$

$$\Rightarrow \quad 14 = 5B+4D \qquad ---(iv)$$

From (i), we get

$$10 = 4B+3D \qquad ---(i)$$

Multiplying equation (iv) by 3 and (i) by 4 and subtracting, we get

$$42-40 = 15B-16B$$

$$\Rightarrow \quad 2 = -B$$

or $$B = -2 \qquad ---(v)$$

Putting value of B in (i), we get

$$10 = 4(-2)+3D$$

$$\frac{10+8}{3} = D$$

$$\Rightarrow \quad D = 6 \qquad ---(vi)$$

Comparing coefficients of x^3 in

$4x^2 + 10 = (Ax + B)(x^2 + 4) + (Cx + 4)(x^2 + 3)$, we get,

$0 = A + C$ ---(vii)

Comparing coefficients of x, we get

$0 = 4A + 3C$ ---(viii)

$\Rightarrow \quad A = C = 0$

$\therefore \quad f(x) = 1 - \frac{(-2)}{x^2 + 3} - \frac{6}{x^2 + 4}$

$= 1 + \frac{2}{x^2 + 3} - \frac{6}{x^2 + 4}$

$\therefore \quad \int f(x)\,dx = \int 1 + \frac{2}{x^2 + 3} - \frac{6}{x^2 + 4}\,dx$

$= x + \frac{2}{\sqrt{3}} \tan^{-1} x \frac{x}{\sqrt{3}} - 3 \tan^{-1} \frac{x}{2} + c$

Indefinite Integrals Ex 19.30 Q62

Let $x^2 = y$

$$\therefore \quad \frac{4x^4 + 3}{(x^2 + 2)(x^2 + 3)(x^2 + 4)} = \frac{4y^2 + 3}{(y + 2)(y + 3)(y + 4)}$$

Now,

Let $\frac{4y^2 + 3}{(y + 2)(y + 3)(y + 4)} = \frac{A}{y + 2} + \frac{B}{y + 3} + \frac{C}{y + 4}$

$$\Rightarrow \quad 4y^2 + 3 = A(y + 3)(y + 4) + B = (y + 2)(y + 4) + C(y + 2)(y + 3)$$
$$= (A + B + C)y^2 + (7A + 6A + 5C)y + 12A + 8B + 6C$$

Equating similar terms,

$A + B + C = 4, \; 7A + 6A + 5C = 0, \; 12A + 8B + 6C = 3$

Solving, we get

$A = \frac{19}{2}, \; B = -39, \; C = \frac{67}{2}$

Thus,

$$I = \frac{19}{2} \int \frac{dx}{x^2 + 2} + (-39) \int \frac{dx}{x^2 + 3} + \frac{67}{2} \int \frac{dx}{x^2 + 4}$$

$$\Rightarrow \quad I = \frac{19}{2\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) - \frac{39}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + \frac{67}{4} \tan^{-1}\left(\frac{x}{2}\right) + c$$

Hence,

$$I = \frac{19}{2\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) - \frac{39}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + \frac{67}{4} \tan^{-1}\left(\frac{x}{2}\right) + c$$

Indefinite Integrals Ex 19.30 Q63

$$\frac{x^4}{(x-1)(x^2+1)} = \frac{x^4}{x^3-x^2+x-1}$$

$$= \frac{x(x^3-x^2+x-1)+1(x^3-x^2+x-1)+1}{(x^3-x^2+x-1)}$$

$$= x+1+\frac{1}{(x-1)(x^2+1)}$$

Now, suppose

$$\frac{1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

$$\Rightarrow \quad 1 = A(x^2+1) + (Bx+C)(x-1)$$

Put $x = 1$

$$1 = 2A$$

$$\Rightarrow \quad A = \frac{1}{2}$$

Put $x = 0$

$$1 = A - C$$

$$\Rightarrow \quad C = A - 1 = -\frac{1}{2}$$

Put $x = -1$

$$1 = 2A + 2B - 2C = 2(A-C) + 2B$$

$$\Rightarrow \quad 1 = 2 + 2B$$

$$\Rightarrow \quad 2B = -1$$

$$\Rightarrow \quad B = -\frac{1}{2}$$

$$\therefore \quad \int \frac{x^4}{(x-1)(x^2+1)}dx = \int x dx + \int 1 dx + \frac{1}{2}\int \frac{1}{x-1}dx - \frac{1}{2}\int \frac{x+1}{x^2+1}dx$$

$$= \frac{x^2}{2} + x + \frac{1}{2}\log|x-1| - \frac{1}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1}x + C$$

*********** END **********