Tangents and Normals Ex 16.1 Q16

The given equations of curve and the line are

$$y = x^2 - 4x + 5 \qquad \text{---(i)}$$
$$2y + x = 7 \qquad \text{---(ii)}$$

Slope of the tangent to (i) is

$$m_1 = \frac{dy}{dx} = 2x - 4 \qquad \text{---(iii)}$$

Slope of the line (ii) is

$$m_2 = \frac{dy}{dx} = \frac{-1}{2} \qquad \text{---(iv)}$$

We have given that slope of (i) and (ii) are perpendicular to each other.

$$\therefore \quad m_1 \times m_2 = -1$$
$$\Rightarrow \quad (2x - 4)\left(\frac{-1}{2}\right) = -1$$
$$\Rightarrow \quad -2x + 4 = -2$$
$$\Rightarrow \quad x = 3$$

From (i)

$$y = 2$$

Thus, the required point is $(3, 2)$.

Tangents and Normals Ex 16.1 Q17

Differentiating $\frac{x^2}{4} + \frac{y^2}{25} = 1$ with respect to x, we get

$$\frac{x}{2} + \frac{2y}{25} \cdot \frac{dy}{dx} = 0$$

or

$$\frac{dy}{dx} = \frac{-25}{4} \cdot \frac{x}{y}$$

(i) Now, the tangent is parallel to the x – axis if the slope of the tangent is zero.

$$\therefore \quad \frac{-25}{4} \cdot \frac{x}{y} = 0$$

This is possible if $x = 0$.

Then $\frac{x^2}{4} + \frac{y^2}{25} = 1$ for $x = 0$ gives $y^2 = 25$

$$\therefore \quad y = \pm 5$$

Thus, the points at which the tangents are parallel to the x – axis are $(0, 5)$ and $(0, -5)$.

(ii) Now, the tangent is parallel to the y – axis if the slope of the normal is zero.

$$\therefore \quad \frac{4y}{25x} = 0$$

This is possible if $y = 0$.

Then $\frac{x^2}{4} + \frac{y^2}{25} = 1$ for $y = 0$ gives $x^2 = 4$

$$\therefore \quad x = \pm 2$$

Thus, the points at which the tangents are parallel to the y – axis are $(2, 0)$ and $(-2, 0)$.

Tangents and Normals Ex 16.1 Q18

The equation of the given curve is $x^2 + y^2 - 2x - 3 = 0$.

On differentiating with respect to x, we have:

$$2x + 2y\frac{dy}{dx} - 2 = 0$$
$$\Rightarrow y\frac{dy}{dx} = 1 - x$$
$$\Rightarrow \frac{dy}{dx} = \frac{1-x}{y}$$

Now, the tangents are parallel to the x-axis if the slope of the tangent is 0.

$$\therefore \frac{1-x}{y} = 0 \Rightarrow 1 - x = 0 \Rightarrow x = 1$$

But, $x^2 + y^2 - 2x - 3 = 0$ for $x = 1$.

$$\Rightarrow y^2 = 4 \Rightarrow y = \pm 2$$

Hence, the points at which the tangents are parallel to the x-axis are $(1, 2)$ and $(1, -2)$

(b)
Now, the tangents are parallel to the x-axis if the slope of the tangents is 0

$$\frac{y}{1-x} = 0$$
$$y = 0$$

But,

$$x^2 + y^2 - 2x - 3 = 0 \text{ for } y = 0$$
$$x^2 - 2x - 3 = 0$$
$$x = -1, 3$$

Hence, the points at which the tangents are parallel to the y-axis are,
$(-1,0), (3,0)$

*********** END **********