NCERT solutions for class 9 maths chapter 8 quadrilaterals Ex 8.1

Q1. The angles of a quadrilateral are in the ratio 3: 5: 9: 13. Find all angles of the quadrilateral.

Ans. Let in quadrilateral ABCD, $\angle A = 3x$, $\angle B = 5x$, $\angle C = 9x$ and $\angle D = 13x$.

Since, sum of all the angles of a quadrilateral = $360°$

$\therefore \angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow 3x + 5x + 9x + 13x = 360°$

$\Rightarrow 30x = 360°$

$\Rightarrow x = 12°$

Now $\angle A = 3x = 3 \times 12 = 36°$

$\angle B = 5x = 5 \times 12 = 60°$

$\angle C = 9x = 9 \times 12 = 108°$

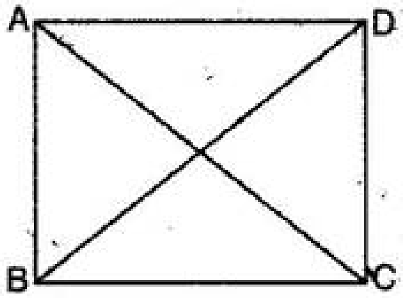
And $\angle D = 13x = 13 \times 12 = 156°$

Hence angles of given quadrilateral are $36°, 60°, 108°$ and $156°$.

Q2. If the diagonals of a parallelogram are equal, show that it is a rectangle.

Ans. Given: ABCD is a parallelogram with diagonal AC = diagonal BD

To prove: ABCD is a rectangle.



Proof: In triangles ABC and ABD,

AB = AB[Common]

AC = BD[Given]

AD = BC[opp. Sides of a $||$ gm]

$\therefore$ ΔABC $\cong$ Δ BAD [By SSS congruency]

$\Rightarrow$ $\angle$DAB = $\angle$ CBA [By C.P.C.T.] ....(i)

But $\angle$ DAB + $\angle$ CBA = 180° ....(ii)

[$\because$ AD$||$BC and AB cuts them, the sum of the interior angles of the same side of transversal is 180°]

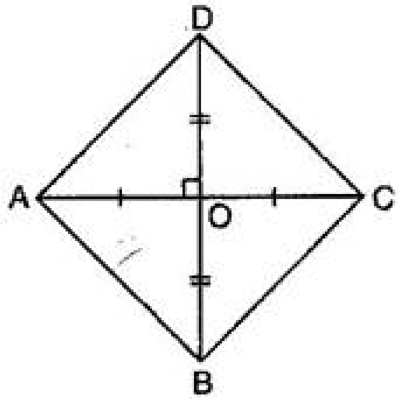
From eq. (i) and (ii),

$\angle$ DAB = $\angle$ CBA = 90°

Hence ABCD is a rectangle.

Q3. Show that is diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Ans. Given: Let ABCD is a quadrilateral.



Let its diagonal AC and BD bisect each other at right angle at point O.

$\therefore$ OA = OC, OB = OD

And $\angle$ AOB = $\angle$ BOC = $\angle$ COD = $\angle$ AOD = 90°

To prove: ABCD is a rhombus.

Proof: In ΔAOD and ΔBOC,

OA = OC[Given]

$\angle$ AOD = $\angle$ BOC [Given]

OB = OD[Given]

$\therefore$ Δ AOD $\cong$ Δ COB [By SAS congruency]

$\Rightarrow$ AD = CB[By C.P.C.T.] ....(i)

Again, In ΔAOB and ΔCOD,

OA = OC[Given]

$\angle$ AOB = $\angle$ COD [Given]

OB = OD[Given]

$\therefore$ Δ AOB $\cong$ Δ COD [By SAS congruency]

$\Rightarrow$ AD = CB [By C.P.C.T.] ....(ii)

Now In Δ AOD and Δ BOC,

OA = OC[Given]

$\angle$ AOB = $\angle$ BOC [Given]

OB = OB[Common]

$\therefore$ Δ AOB $\cong$ Δ COB [By SAS congruency]

$\Rightarrow$ AB = BC[By C.P.C.T.] ....(iii)

From eq. (i), (ii) and (iii),

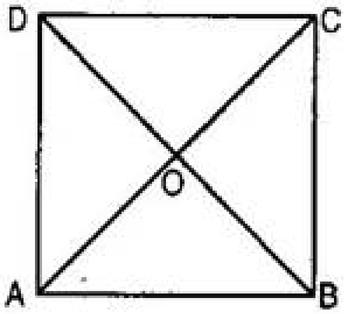
AD = BC = CD = AB

And the diagonals of quadrilateral ABCD bisect each other at right angle.

Therefore, ABCD is a rhombus.

Q4. Show that the diagonals of a square are equal and bisect each other at right angles.

Ans. Given: ABCD is a square. AC and BD are its diagonals bisect each other at point O.



To prove: AC = BD and AC $\perp$ BD at point O.

Proof: In triangles ABC and BAD,

AB = AB[Common]

$\angle$ ABC = $\angle$ BAD = 90°

BC = AD[Sides of a square]

$\therefore$ Δ ABC $\cong$ Δ BAD [By SAS congruency]

$\Rightarrow$ AC = BD[By C.P.C.T.]Hence proved.

Now in triangles AOB and AOD,

AO = AO[Common]

AB = AD[Sides of a square]

OB = OD[Diagonals of a square bisect each other]

$\therefore$ Δ AOB $\cong$ Δ AOD [By SSS congruency]

$\angle$ AOB = $\angle$ AOD [By C.P.C.T.]

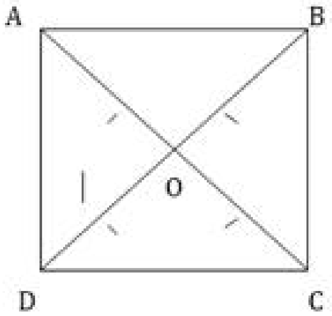
But $\angle$ AOB + $\angle$ AOD = 180° [Linear pair]

$\therefore$ $\angle$ AOB = $\angle$ AOD = 90°

$\Rightarrow$ OA $\perp$ BD or AC $\perp$ BD Hence proved.

Q5. Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Ans. Let ABCD be a quadrilateral in which equal diagonals AC and BD bisect each other at right angle at point O.



We haveAC = BD and OA = OC..........(i)

And OB = OD..........(ii)

Now OA + OC = OB + OD

$\Rightarrow$ OC + OC = OB + OB [Using (i) & (ii)]

$\Rightarrow$ 2OC = 2OB

$\Rightarrow$ OC = OB..........(iii)

From eq. (i), (ii) and (iii), we get, OA = OB = OC = OD ....(iv)

Now in ΔAOB and ΔCOD,

OA = OD [proved]

$\angle$AOB = $\angle$COD[vertically opposite angles]

OB = OC [proved]

$\therefore$ ΔAOB $\cong$ ΔDOC[By SAS congruency]

$\Rightarrow$ AB = DC [By C.P.C.T.]..........(v)

Similarly, ΔBOC $\cong$ ΔAOD [By SAS congruency]

$\Rightarrow$ BC = AD [By C.P.C.T.]..........(vi)

From eq. (v) and (vi), it is concluded that ABCD is a parallelogram because opposite sides of a quadrilateral are equal.

Now in ΔABC and ΔBAD,

AB = BA [Common]

BC = AD [proved above]

AC = BD [Given]

$\therefore$ ΔABC $\cong$ ΔBAD[By SSS congruency]

$\Rightarrow$ $\angle$ABC = $\angle$BAD[By C.P.C.T.]..........(vii)

But $\angle$ABC + $\angle$BAD = 180° [ABCD is a parallelogram]..........(viii)

$\therefore$ AD $\parallel$ BC and AB is a transversal.

$\Rightarrow \angle ABC + \angle ABC = 180^\circ$ [Using eq. (vii) and (viii)]

$\Rightarrow 2\angle ABC = 180^\circ \Rightarrow \angle ABC = 90^\circ$

$\therefore \angle ABC = \angle BAD = 90^\circ$ ....(ix)

Opposite angles of a parallelogram are equal.

But $\angle ABC = \angle BAD =$

$\therefore \angle ABC = \angle ADC = 90^\circ$ ....(x)

$\therefore \angle BAD = \angle BDC = 90^\circ$ ....(xi)

From eq. (x) and (xi), we get

$\angle ABC = \angle ADC = \angle BAD = \angle BDC = 90^\circ$ ....(xii)

Now in ΔAOB and ΔBOC,

OA = OC [Given]

$\angle AOB = \angle BOC = 90^\circ$ [Given]

OB = OB [Common]

$\therefore \Delta AOB \cong \Delta COB$[By SAS congruency]

$\Rightarrow AB = BC$..........(xiii)

From eq. (v), (vi) and (xiii), we get,

AB = BC = CD = AD ....(xiv)

Now, from eq. (xii) and (xiv), we have a quadrilateral whose equal diagonals bisect each other at right angle.

Also sides are equal make an angle of 90° with each other.

$\therefore$ ABCD is a square.

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