Differentiation Ex 11.3 Q1

Let $y = \cos^{-1}\left\{2x\sqrt{1-x^2}\right\}$

Put $\quad x = \cos\theta$

$$y = \cos^{-1}\left\{2\cos\theta\sqrt{1-\cos^2\theta}\right\}$$

$$= \cos^{-1}\{2\cos\theta\sin\theta\}$$

$$y = \cos^{-1}\{\sin 2\theta\} \qquad \left[\text{Since } \sin 2\theta = 2\sin\theta\cos\theta, \sin^2\theta + \cos^2\theta = 1\right]$$

$$y = \cos^{-1}\left[\cos\left(\frac{\pi}{2} - \theta\right)\right] \qquad \text{---(i)}$$

Now,

$$\frac{1}{\sqrt{2}} < x < 1$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} < \cos\theta < 1$$

$$\Rightarrow \quad 0 < \theta < \frac{\pi}{4}$$

$$\Rightarrow \quad 0 < 2\theta < \frac{\pi}{2}$$

$$\Rightarrow \quad 0 > -2\theta > -\frac{\pi}{2}$$

$$\Rightarrow \quad \frac{\pi}{2} > \left(\frac{\pi}{2} - 2\theta\right) > 0$$

Hence, from equation (i),

$$y = \frac{\pi}{2} - 2\theta \qquad \left[\text{Since } \cos^{-1}(\cos\theta) = \theta, \text{if } \theta \in [0, \pi]\right]$$

$$y = \frac{\pi}{2} - 2\cos^{-1}x \qquad [\text{Since } x = \cos\theta]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{2}\right) - 2\frac{d}{dx}\left(\cos^{-1}x\right)$$

$$= 0 - 2\left(\frac{-1}{\sqrt{1-x^2}}\right)$$

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q2

Let $y = \cos^{-1}\left\{\sqrt{\frac{1+x}{2}}\right\}$

Put $x = \cos 2\theta$

$$y = \cos^{-1}\left\{\sqrt{\frac{1+\cos 2\theta}{2}}\right\}$$

$$= \cos^{-1}\left\{\sqrt{\frac{2\cos^2\theta}{2}}\right\}$$

$$y = \cos^{-1}\{\cos\theta\} \qquad \text{---(i)}$$

Here, $-1 < x < 1$

$\rightarrow \quad -1 < \cos 2\theta < 1$

$\Rightarrow \quad 0 < 2\theta < \pi$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{2}$

So, from equation (i),

$$y = \theta \qquad \left[\text{Since } \cos^{-1}(\cos\theta) = \theta \text{ if } \theta \in [0, \pi]\right]$$

$$y = \frac{1}{2}\cos^{-1} x \qquad [\text{Since } x = \cos 2\theta]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = -\frac{1}{2\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q3

Let $y = \sin^{-1}\left\{\sqrt{\frac{1-x}{2}}\right\}$

Let $x = \cos 2\theta$

$$y = \sin^{-1}\left\{\sqrt{\left(\frac{1-\cos 2\theta}{2}\right)}\right\}$$

$$= \sin^{-1}\left\{\sqrt{\frac{2\sin^2\theta}{2}}\right\}$$

$$y = \sin^{-1}(\sin\theta) \qquad \text{---(i)}$$

Here, $0 < x < 1$

$\Rightarrow \quad 0 < \cos 2\theta < 1$

$\Rightarrow \quad 0 < 2\theta < \frac{\pi}{2}$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{4}$

so, from equation (i),

$$y = \theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$y = \frac{1}{2}\cos^{-1} x \qquad [\text{Since, } x = \cos 2\theta]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = -\frac{1}{2\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q4

Let $\quad y = \sin^{-1}\left\{\sqrt{1-x^2}\right\}$

Let $\quad x = \cos\theta$

$$y = \sin^{-1}\left\{\sqrt{1-\cos^2\theta}\right\}$$

$$y = \sin^{-1}(\sin\theta) \qquad \text{---(i)}$$

Here, $\quad 0 < x < 1$

$\Rightarrow \quad 0 < \cos\theta < 1$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{2}$

From equatoin (i),

$$y = \theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$y = \cos^{-1} x \qquad [\text{Since } x = \cos\theta]$$

Differentiating with respect to x,

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q5

Let $\quad y = \tan^{-1}\left\{\frac{x}{\sqrt{a^2-x^2}}\right\}$

Let $\quad x = a\sin\theta$

$$y = \tan^{-1}\left\{\frac{a\sin\theta}{\sqrt{a^2 - a^2\sin^2\theta}}\right\}$$

$$y = \tan^{-1}\left\{\frac{a\sin\theta}{\sqrt{a^2\left(1-\sin^2\theta\right)}}\right\}$$

$$y = \tan^{-1}\left\{\frac{a\sin\theta}{a\cos\theta}\right\}$$

$$y = \tan^{-1}(\tan\theta) \qquad \text{---(i)}$$

Here, $\quad -a < x < a$

$\Rightarrow \quad -1 < \frac{x}{a} < 1$

$\Rightarrow \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

From equatoin (i),

$$y = \theta \qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$y = \sin^{-1}\left(\frac{x}{a}\right) \qquad [\text{Since } x = a\sin\theta]$$

Differentiating it with respect to x,

Using chain rule,

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}\frac{d}{dx}\left(\frac{x}{a}\right)$$

$$= \frac{a}{\sqrt{a^2-x^2}} \times \left(\frac{1}{a}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{a^2-x^2}}.$$

*********** END **********