Trigonometric Identities Ex 6.1 Q21

Answer :

We have to prove $(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta)=1$

We know that,

$\sin^2\theta+\cos^2\theta=1,$

$\sec^2\theta-\tan^2\theta=1$

So,

$$\begin{aligned}(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta) &= (1+\tan^2\theta)\{(1-\sin\theta)(1+\sin\theta)\}\\ &= (1+\tan^2\theta)(1-\sin^2\theta)\\ &= \sec^2\theta\cos^2\theta\\ &= \frac{1}{\cos^2\theta}\cos^2\theta\\ &= 1\end{aligned}$$

Trigonometric Identities Ex 6.1 Q22

Answer :

We have to prove $\sin^2 A\cot^2 A+\cos^2 A\tan^2 A=1$

We know that, $\sin^2 A+\cos^2 A=1$

So,

$$\begin{aligned}\sin^2 A\cot^2 A+\cos^2 A\tan^2 A &= \sin^2 A\frac{\cos^2 A}{\sin^2 A}+\cos^2 A\frac{\sin^2 A}{\cos^2 A}\\ &= \cos^2 A+\sin^2 A\\ &= 1\end{aligned}$$

Trigonometric Identities Ex 6.1 Q23

Answer :

(i) We have to prove $\cot\theta - \tan\theta = \dfrac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$

We know that, $\sin^2\theta + \cos^2\theta = 1$

So,

$$\begin{aligned}
\cot\theta - \tan\theta &= \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} \\
&= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta} \\
&= \frac{\cos^2\theta - (1-\cos^2\theta)}{\sin\theta\cos\theta} \\
&= \frac{\cos^2\theta - 1 + \cos^2\theta}{\sin\theta\cos\theta} \\
&= \frac{2\cos^2\theta - 1}{\sin\theta\cos\theta}
\end{aligned}$$

(ii) We have to prove $\tan\theta - \cot\theta = \dfrac{2\sin^2\theta - 1}{\sin\theta\cos\theta}$

We know that, $\sin^2\theta + \cos^2\theta = 1$

So,

$$\begin{aligned}
\tan\theta - \cot\theta &= \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta} \\
&= \frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta} \\
&= \frac{\sin^2\theta - (1-\sin^2\theta)}{\sin\theta\cos\theta} \\
&= \frac{\sin^2\theta - 1 + \sin^2\theta}{\sin\theta\cos\theta} \\
&= \frac{2\sin^2\theta - 1}{\sin\theta\cos\theta}
\end{aligned}$$

*********** END **********