Exercise 1E

Question 14:

For rationalising the denominator of a number, we multiply its numerator and denominator by its rationalising factor.

If a and b are integers, then

$\left(a+\sqrt{b}\right)$ and $\left(a-\sqrt{b}\right)$ are rationalising factor of each other,

as $\left(a+\sqrt{b}\right)\left(a-\sqrt{b}\right)=\left(a^2-b\right)$, which is rational.

Let us rationalise the denominator of the first term on the Left hand side.

We have,

$$
\begin{aligned}
\frac{\sqrt{5}-1}{\sqrt{5}+1} &= \frac{\sqrt{5}-1}{\sqrt{5}+1}\times\frac{\sqrt{5}-1}{\sqrt{5}-1}\\
&= \frac{\left(\sqrt{5}-1\right)^2}{\left(\sqrt{5}\right)^2-(1)^2}\\
&= \frac{\left(\sqrt{5}\right)^2-2\left(\sqrt{5}\right)(1)+1}{5-1}\\
&= \frac{5-2\sqrt{5}+1}{4}=\frac{6-2\sqrt{5}}{4}\text{.....(1)}
\end{aligned}
$$

Now consider the denominator of the second term on the left hand side:

$$
\begin{aligned}
\frac{\sqrt{5}+1}{\sqrt{5}-1} &= \frac{\sqrt{5}+1}{\sqrt{5}-1}\times\frac{\sqrt{5}+1}{\sqrt{5}+1}\\
&= \frac{\left(\sqrt{5}+1\right)^2}{\left(\sqrt{5^2}\right)-(1)^2}\\
&= \frac{\left(\sqrt{5}\right)^2+2\left(\sqrt{5}\right)(1)+(1)^2}{5-1}\\
&= \frac{5+2\sqrt{5}+1}{4}=\frac{6+2\sqrt{5}}{4}\text{.......(2)}
\end{aligned}
$$

Adding equations (1) and (2), we have

$$
\begin{aligned}
\therefore \frac{\sqrt{5}-1}{\sqrt{5}+1}+\frac{\sqrt{5}+1}{\sqrt{5}-1} &= \frac{6-2\sqrt{5}}{4}+\frac{6+2\sqrt{5}}{4}\\
&= \frac{6-2\sqrt{5}+6+2\sqrt{5}}{4}=\frac{12}{4}=3.
\end{aligned}
$$

*********** END **********