Exercise 5.3

$\Rightarrow 1272 = 18n + 8n^2 - 8n$

$\Rightarrow 8n^2 + 10n - 1272 = 0$

$\Rightarrow 4n^2 + 5n - 636 = 0$

Comparing equation $4n^2 + 5n - 636 = 0$ with general form $an^2 + bn + c = 0$, we get

$a = 4$, $b = 5$ and $c = -636$

Applying quadratic formula, $n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and putting values of a, b and c, we get

$n = \frac{-5 \pm \sqrt{5^2 - 4(4)(-636)}}{8}$

$\Rightarrow n = \frac{-5 \pm \sqrt{25 + 10176}}{8}$

$\Rightarrow n = \frac{-5 \pm 101}{8}$

$\Rightarrow n = \frac{96}{8}, \frac{-106}{8} = 12, -\frac{106}{8}$

We discard negative value of n here because n cannot be in negative, n can only be a positive integer.

Therefore, $n = 12$

Therefore, 12 terms of the given sequence make sum equal to 636.

5. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Ans. First term = a = 5, Last term = l = 45, $S_n = 400$

Applying formula, $S_n = \frac{n}{2}[a + l]$ to find sum of n terms of AP, we get

$400 = \frac{n}{2}[5 + 45]$

$\Rightarrow \frac{400}{50} = \frac{n}{2} \Rightarrow n = 16$

Applying formula, $S_n = \frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP and putting value of n, we get

$$400 = \frac{16}{2}[10+(16-1)d]$$

$\Rightarrow 400 = 8\ (10 + 15d)$

$\Rightarrow 400 = 80 + 120d$

$\Rightarrow 320 = 120d$

$\Rightarrow d = \frac{320}{120} = \frac{8}{3}$

6. The first and the last terms of an AP are 17 and 350 respectively. If, the common difference is 9, how many terms are there and what is their sum?

Ans. First term = a = 17, Last term = l = 350 and Common difference = d = 9

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression, we get

$350 = 17 + (n - 1)\ (9)$

$\Rightarrow 350 = 17 + 9n - 9$

$\Rightarrow 342 = 9n \Rightarrow n = 38$

Applying formula, $S_n = \frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP and putting value of n, we get

$$S_{38} = \frac{38}{2}[34+(38-1)9]$$

$\Rightarrow S_{38} = 19\ (34 + 333) = 6973$

Therefore, there are 38 terms and their sum is equal to 6973.

7. Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.

Ans. It is given that 22nd term is equal to 149

$\Rightarrow a_{22} = 149$

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression, we get

$149 = a + (22 - 1)(7)$

$\Rightarrow 149 = a + 147 \Rightarrow a = 2$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP and putting value of a, we get

$S_{22} = \frac{22}{2}[4 + (22-1)7]$

$\Rightarrow S_{22} = 11(4 + 147)$

$\Rightarrow S_{22} = 1661$

Therefore, sum of first 22 terms of AP is equal to 1661.

8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Ans. It is given that second and third term of AP are 14 and 18 respectively.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression, we get

$14 = a + (2 - 1)d$

$\Rightarrow 14 = a + d \ldots (1)$

And, $18 = a + (3 - 1)d$

$\Rightarrow 18 = a + 2d \ldots (2)$

These are equations consisting of two variables.

Using equation (1), we get, $a = 14 - d$

Putting value of a in equation (2), we get

$18 = 14 - d + 2d$

$\Rightarrow d = 4$

Therefore, common difference $d = 4$

Putting value of d in equation (1), we get

$18 = a + 2\ (4)$

$\Rightarrow a = 10$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$S_{51} = \frac{51}{2}[20 + (51-1)d] = \frac{51}{2}(20+200) = \frac{51}{2} \times 220 = 51 \times 110 = 5610$$

Therefore, sum of first 51 terms of an AP is equal to 5610.

9. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Ans. It is given that sum of first 7 terms of an AP is equal to 49 and sum of first 17 terms is equal to 289.

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$49 = \frac{7}{2}[2a + (7-1)d]$ 49

$\Rightarrow 98 = 7\ (2a + 6d)$

$\Rightarrow 7 = a + 3d \Rightarrow a = 7 - 3d$ ... (1)

And, $289 = \frac{17}{2}[2a + (17-1)d]$

$\Rightarrow 578 = 17\ (2a + 16d)$

$\Rightarrow 34 = 2a + 16d$

$\Rightarrow 17 = a + 8d$

Putting equation (1) in the above equation, we get

$17 = 7 - 3d + 8d$

$\Rightarrow 10 = 5d \Rightarrow d = 2$

Putting value of d in equation (1), we get

$a = 7 - 3d = 7 - 3\ (2) = 7 - 6 = 1$

Again applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$S_n = \frac{n}{2}[2(1) + (n-1)2]$$

$$\Rightarrow S_n = \frac{n}{2}[2 + 2n - 2]$$

$$\Rightarrow S_n = \frac{n}{2} \times 2n \Rightarrow S_n = n^2$$

Therefore, sum of n terms of AP is equal to n^2.

10. Show that $a_1, a_2 \ldots a_n$ form an AP where a_n is defined as below:

(i) $a_n = 3 + 4n$

(ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

Ans. (i) We need to show that $a_1, a_2 \ldots a_n$ form an AP where $a_n = 3 + 4n$

Let us calculate values of $a_1, a_2, a_3 \ldots$ using $a_n = 3 + 4n$

$a_1 = 3 + 4(1) = 3 + 4 = 7 a_2$

$= 3 + 4(2) = 3 + 8 = 11$

$a_3 = 3 + 4(3) = 3 + 12 = 15 a_4$

$= 3 + 4(4) = 3 + 16 = 19$

So, the sequence is of the form 7, 11, 15, 19 ...

Let us check difference between consecutive terms of this sequence.

$11 - 7 = 4$, $15 - 11 = 4$, $19 - 15 = 4$

Therefore, the difference between consecutive terms is constant which means terms $a_1, a_2 \ldots a_n$ form an AP.

We have sequence 7, 11, 15, 19 ...

First term = a = 7 and Common difference = d = 4