



Exercise 6.4 : Solutions of Questions on Page Number : 216

Q1 : 1. Using differentials, find the approximate value of each of the following up to 3 places of decimal

(i) $\sqrt{25.3}$ (ii) $\sqrt{49.5}$ (iii) $\sqrt{0.6}$

(iv) $(0.009)^{\frac{1}{3}}$ (v) $(0.999)^{\frac{1}{10}}$ (vi) $(15)^{\frac{1}{4}}$

(vii) $(26)^{\frac{1}{3}}$ (viii) $(255)^{\frac{1}{4}}$ (ix) $(82)^{\frac{1}{4}}$

(x) $(401)^{\frac{1}{2}}$

Answer :

(i) $\sqrt{25.3}$

Consider $y = \sqrt{x}$. Let $x = 25$ and $\Delta x = 0.3$.

Then,

$$\Delta y = \sqrt{x + \Delta x} - \sqrt{x} = \sqrt{25.3} - \sqrt{25} = \sqrt{25.3} - 5$$

$$\Rightarrow \sqrt{25.3} = \Delta y + 5$$

Now, dy is approximately equal to Δy and is given by,

$$dy = \left(\frac{dy}{dx} \right) \Delta x = \frac{1}{2\sqrt{x}} (0.3) \quad \left[\text{as } y = \sqrt{x} \right]$$

$$= \frac{1}{2\sqrt{25}} (0.3) = 0.03$$

Hence, the approximate value of $\sqrt{25.3}$ is $0.03 + 5 = 5.03$.

(ii) $\sqrt{49.5}$

Consider $y = \sqrt{x}$. Let $x = 49$ and $\Delta x = 0.5$.

Then,

$$\Delta y = \sqrt{x + \Delta x} - \sqrt{x} = \sqrt{49.5} - \sqrt{49} = \sqrt{49.5} - 7$$

$$\Rightarrow \sqrt{49.5} = 7 + \Delta y$$

Now, dy is approximately equal to Δy and is given by,

$$dy = \left(\frac{dy}{dx} \right) \Delta x = \frac{1}{2\sqrt{x}} (0.5) \quad \left[\text{as } y = \sqrt{x} \right]$$

$$= \frac{1}{2\sqrt{49}} (0.5) = \frac{1}{14} (0.5) = 0.035$$

Hence, the approximate value of $\sqrt{49.5}$ is $7 + 0.035 = 7.035$.

(iii) $\sqrt{0.6}$

Consider $y = \sqrt{x}$. Let $x = 1$ and $\Delta x = -0.4$.

Then,

$$\Delta y = \sqrt{x + \Delta x} - \sqrt{x} = \sqrt{0.6} - 1$$

$$\Rightarrow \sqrt{0.6} = 1 + \Delta y$$

Now, dy is approximately equal to Δy and is given by,

$$dy = \left(\frac{dy}{dx} \right) \Delta x = \frac{1}{2\sqrt{x}} (\Delta x) \quad \left[\text{as } y = \sqrt{x} \right]$$

$$= \frac{1}{2} (-0.4) = -0.2$$

Hence, the approximate value of $\sqrt{0.6}$ is $1 + (-0.2) = 1 - 0.2 = 0.8$.

(iv) $(0.009)^{\frac{1}{3}}$

Consider $y = x^{\frac{1}{3}}$. Let $x = 0.008$ and $\Delta x = 0.001$.

Then,

$$\Delta y = (x + \Delta x)^{\frac{1}{3}} - (x)^{\frac{1}{3}} = (0.009)^{\frac{1}{3}} - (0.008)^{\frac{1}{3}} = (0.009)^{\frac{1}{3}} - 0.2$$

$$\Rightarrow (0.009)^{\frac{1}{3}} = 0.2 + \Delta y$$

Now, dy is approximately equal to Δy and is given by,

$$dy = \left(\frac{dy}{dx} \right) \Delta x = \frac{1}{3(x)^{\frac{2}{3}}} (\Delta x) \quad \left[\text{as } y = x^{\frac{1}{3}} \right]$$

$$= \frac{1}{3 \times 0.04} (0.001) = \frac{0.001}{0.12} = 0.008$$

Hence, the approximate value of $(0.009)^{\frac{1}{3}}$ is $0.2 + 0.008 = 0.208$.

(v) $(0.999)^{\frac{1}{10}}$

Answer needs Correction? [Click Here](#)

Q2 : Find the approximate value of $f(2.01)$, where $f(x) = 4x^2 + 5x + 2$

Answer :

Let $x = 2$ and $\Delta x = 0.01$. Then, we have:

$$f(2.01) = f(x + \Delta x) = 4(x + \Delta x)^2 + 5(x + \Delta x) + 2$$

$$\text{Now, } \Delta y = f(x + \Delta x) - f(x)$$

$$\therefore f(x + \Delta x) = f(x) + \Delta y$$

$$\approx f(x) + f'(x) \cdot \Delta x \quad (\text{as } dx = \Delta x)$$

$$\begin{aligned} \Rightarrow f(2.01) &\approx (4x^2 + 5x + 2) + (8x + 5)\Delta x \\ &= [4(2)^2 + 5(2) + 2] + [8(2) + 5](0.01) \quad [x = 2, \Delta x = 0.01] \\ &= (16 + 10 + 2) + (16 + 5)(0.01) \\ &= 28 + (21)(0.01) \\ &= 28 + 0.21 \\ &= 28.21 \end{aligned}$$

Hence, the approximate value of $f(2.01)$ is 28.21.

Answer needs Correction? [Click Here](#)

Q3 : Find the approximate value of $f(5.001)$, where $f(x) = x^3 - 7x^2 + 15$.

Answer :

Let $x = 5$ and $\Delta x = 0.001$. Then, we have:

$$f(5.001) = f(x + \Delta x) = (x + \Delta x)^3 - 7(x + \Delta x)^2 + 15$$

$$\text{Now, } \Delta y = f(x + \Delta x) - f(x)$$

$$\therefore f(x + \Delta x) = f(x) + \Delta y$$

$$\approx f(x) + f'(x) \cdot \Delta x \quad (\text{as } dx = \Delta x)$$

$$\begin{aligned} \Rightarrow f(5.001) &\approx (x^3 - 7x^2 + 15) + (3x^2 - 14x)\Delta x \\ &= [(5)^3 - 7(5)^2 + 15] + [3(5)^2 - 14(5)](0.001) \quad [x = 5, \Delta x = 0.001] \\ &= (125 - 175 + 15) + (75 - 70)(0.001) \\ &= -35 + (5)(0.001) \\ &= -35 + 0.005 \\ &= -34.995 \end{aligned}$$

Hence, the approximate value of $f(5.001)$ is - 34.995.

Answer needs Correction? [Click Here](#)

Q4 : Find the approximate change in the volume V of a cube of side x metres caused by increasing side by 1%.

Answer :

The volume of a cube (V) of side x is given by $V = x^3$.

$$\begin{aligned} \therefore dV &= \left(\frac{dV}{dx} \right) \Delta x \\ &= (3x^2) \Delta x \\ &= (3x^2)(0.01x) \quad [\text{as 1\% of } x \text{ is } 0.01x] \\ &= 0.03x^3 \end{aligned}$$

Hence, the approximate change in the volume of the cube is $0.03x^3 \text{ m}^3$.

Answer needs Correction? [Click Here](#)

Q5 : Find the approximate change in the surface area of a cube of side x metres caused by decreasing the side by 1%

Answer :

The surface area of a cube (S) of side x is given by $S = 6x^2$.

$$\begin{aligned} \therefore \frac{dS}{dx} &= \left(\frac{dS}{dx} \right) \Delta x \\ &= (12x) \Delta x \\ &= (12x)(0.01x) \quad [\text{as 1\% of } x \text{ is } 0.01x] \\ &= 0.12x^2 \end{aligned}$$

Hence, the approximate change in the surface area of the cube is $0.12x^2 \text{ m}^2$.

Answer needs Correction? [Click Here](#)

Q6 : If the radius of a sphere is measured as 7 m with an error of 0.02m, then find the approximate error in calculating its volume.

Answer :

Let r be the radius of the sphere and Δr be the error in measuring the radius.

Then,

$$r = 7 \text{ m and } \Delta r = 0.02 \text{ m}$$

Now, the volume V of the sphere is given by,

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ \therefore \frac{dV}{dr} &= 4\pi r^2 \\ \therefore dV &= \left(\frac{dV}{dr}\right)\Delta r \\ &= (4\pi r^2)\Delta r \\ &= 4\pi(7)^2(0.02) \text{ m}^3 = 3.92\pi \text{ m}^3 \end{aligned}$$

Hence, the approximate error in calculating the volume is $3.92\pi \text{ m}^3$.

Answer needs Correction? [Click Here](#)

Q7 : If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating in surface area.

Answer :

Let r be the radius of the sphere and Δr be the error in measuring the radius.

Then,

$$r = 9 \text{ m and } \Delta r = 0.03 \text{ m}$$

Now, the surface area of the sphere (S) is given by,

$$\begin{aligned} S &= 4\pi r^2 \\ \therefore \frac{dS}{dr} &= 8\pi r \\ \therefore dS &= \left(\frac{dS}{dr}\right)\Delta r \\ &= (8\pi r)\Delta r \\ &= 8\pi(9)(0.03) \text{ m}^2 \\ &= 2.16\pi \text{ m}^2 \end{aligned}$$

Hence, the approximate error in calculating the surface area is $2.16\pi \text{ m}^2$.

Answer needs Correction? [Click Here](#)

Q8 : If $f(x) = 3x^2 + 15x + 5$, then the approximate value of $f(3.02)$ is

A. 47.66 B. 57.66 C. 67.66 D. 77.66

Answer :

Let $x = 3$ and $\Delta x = 0.02$. Then, we have:

$$\begin{aligned} f(3.02) &= f(x + \Delta x) = 3(x + \Delta x)^2 + 15(x + \Delta x) + 5 \\ \text{Now, } \Delta y &= f(x + \Delta x) - f(x) \\ \Rightarrow f(x + \Delta x) &= f(x) + \Delta y \\ &\approx f(x) + f'(x)\Delta x \quad (\text{As } dx = \Delta x) \\ \Rightarrow f(3.02) &\approx (3x^2 + 15x + 5) + (6x + 15)\Delta x \\ &= [3(3)^2 + 15(3) + 5] + [6(3) + 15](0.02) \quad [\text{As } x = 3, \Delta x = 0.02] \\ &= (27 + 45 + 5) + (18 + 15)(0.02) \\ &= 77 + (33)(0.02) \\ &= 77 + 0.66 \\ &= 77.66 \end{aligned}$$

Hence, the approximate value of $f(3.02)$ is 77.66.

The correct answer is D.

Answer needs Correction? [Click Here](#)

Q9 : The approximate change in the volume of a cube of side x metres caused by increasing the side by 3% is

A. $0.06x^3 \text{ m}^3$ B. $0.6x^3 \text{ m}^3$ C. $0.09x^3 \text{ m}^3$ D. $0.9x^3 \text{ m}^3$

Answer :

The volume of a cube (V) of side x is given by $V = x^3$.

$$\begin{aligned} \therefore dV &= \left(\frac{dV}{dx}\right)\Delta x \\ &= (3x^2)\Delta x \\ &= (3x^2)(0.03x) \quad [\text{As 3\% of } x \text{ is } 0.03x] \\ &= 0.09x^3 \text{ m}^3 \end{aligned}$$

Hence, the approximate change in the volume of the cube is $0.09x^3 \text{ m}^3$.

The correct answer is C.

Answer needs Correction? [Click Here](#)

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