



Higher Order Derivatives Ex 12.1 Q38

$$y = \log(1 + \cos x)$$

differentiating w.r.t. x

$$\Rightarrow \frac{dy}{dx} = \frac{1}{1 + \cos x} \times -\sin x = \frac{-\sin x}{1 + \cos x}$$

differentiating w.r.t. x

$$\Rightarrow \frac{d^2y}{dx^2} = - \left[\frac{(1 + \cos x) \cos x - \sin x (-\sin x)}{(1 + \cos x)^2} \right]$$

$$\Rightarrow \frac{d^2y}{dx^2} = - \left[\frac{\cos x + \cos^2 x + \sin^2 x}{(1 + \cos x)^2} \right] = - \left[\frac{1 + \cos x}{(1 + \cos x)^2} \right] = \frac{-1}{1 + \cos x}$$

differentiating w.r.t. x

$$\Rightarrow \frac{d^3y}{dx^3} = - \left[\frac{+1}{(1 + \cos x)^2} \times -\sin x \right] = - \left(\frac{-\sin x}{(1 + \cos x)^2} \right) \times \left(\frac{-1}{1 + \cos x} \right) = - \frac{dy}{dx} \cdot \frac{d^2y}{dx^2}$$

$$\Rightarrow \frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} \cdot \frac{dy}{dx} = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q39

$$y = \sin(\log x)$$

$$\Rightarrow \frac{dy}{dx} = \cos(\log x) \times \frac{1}{x}$$

$$\Rightarrow x \frac{dy}{dx} = \cos(\log x)$$

$$\Rightarrow x \frac{d^2y}{dx^2} + \frac{dy}{dx} = -\sin(\log x) \times \frac{1}{x}$$

$$\Rightarrow x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} = -y$$

$$\Rightarrow x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q40

Given $y = 3e^{2x} + 2e^{3x}$

Then, $\frac{dy}{dx} = 6e^{2x} + 6e^{3x} = 6(e^{2x} + e^{3x})$

$\therefore \frac{d^2y}{dx^2} = 12e^{2x} + 18e^{3x} = 6(2e^{2x} + 3e^{3x})$

Hence,

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 6(2e^{2x} + 3e^{3x}) - 30(e^{2x} + e^{3x}) + 6(3e^{2x} + 2e^{3x})$$

$$= 0$$

Higher Order Derivatives Ex 12.1 Q41

$$y = (\cot^{-1} x)^2$$

differentiating w.r.t. x

$$\Rightarrow \frac{dy}{dx} = y_1 = 2 \cot^{-1} x \frac{-1}{1+x^2}$$

$$= \frac{-2 \cot^{-1} x}{1+x^2} \text{ (chain rule)}$$

$$\Rightarrow (1+x^2) \frac{dy}{dx} = -2 \cot^{-1} x$$

differentiating w.r.t. x

$$\Rightarrow (1+x^2) y_2 + 2xy_1 = +2 \left(\frac{-1}{1+x^2} \right)$$

(multiplication rule on LHS)

$$\Rightarrow (1+x^2)^2 y_2 + 2x(1+x^2) y_1 = 2$$

Hence proved!

***** END *****