Tangents and Normals Ex 16.3 Q6

$xy = 4$

$\Rightarrow x = \frac{4}{y}$......(i)

$x^2 + y^2 = 8$......(ii)

Substituting eq (i) in (ii) we get,

$x^2 + y^2 = 8$

$\Rightarrow \left(\frac{4}{y}\right)^2 + y^2 = 8$

$\Rightarrow 16 + y^4 = 8y^2$

$\Rightarrow y^4 - 8y^2 + 16 = 0$

$\Rightarrow (y^2 - 4)^2 = 0$

$\Rightarrow y^2 = 4$

$\Rightarrow y = \pm 2$

From(i) when $y = 2$,we get $x = 2$ and when $y = -2$,we get $x = -2$

Thus the two curves intersect at $(2, 2)$ and $(-2, 2)$.

Differnentiating (i) wrt x, we get

$y + x\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$

Differnentiating (i) wrt x, we get

$y + x\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$

Differnentiating (ii) wrt x, we get

$2x + 2y\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{x}{y}$

At $(2, 2)$

$\left(\frac{dy}{dx}\right)_{C_1} = -1$

$\left(\frac{dy}{dx}\right)_{C_2} = -1$

Clearly $\left(\frac{dy}{dx}\right)_{C_1} = \left(\frac{dy}{dx}\right)_{C_2}$ at $(2, 2)$

So given two curves touch each other at $(2, 2)$.

Simillarly, it can be seen that two curves touch each other at $(-2, -2)$.

Tangents and Normals Ex 16.3 Q7

$y^2 = 4x......(i)$

$x^2 + y^2 - 6x + 1 = 0.......(ii)$

Differnentiating (i) wrt x, we get

$$2y\frac{dy}{dx} = 4$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{y}$$

Differnentiating (ii) wrt x, we get

$$2x + 2y\frac{dy}{dx} - 6 + 0 = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{3-x}{y}$$

At (1, 2)

$$\left(\frac{dy}{dx}\right)_{c_1} = \frac{2}{2} = 1$$

$$\left(\frac{dy}{dx}\right)_{c_2} = \frac{3-1}{2} = \frac{2}{2} = 1$$

Clearly $\left(\frac{dy}{dx}\right)_{c_1} = \left(\frac{dy}{dx}\right)_{c_2}$ at (1, 2)

So given two curves touch each other at (1, 2).

*********** END **********