



Indefinite Integrals Ex 19.23 Q1

$$\text{Let } I = \int \frac{1}{5 + 4 \cos x} dx$$

$$\text{Put } \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$I = \int \frac{1}{5 + 4 \left( \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right)} dx$$

$$= \int \frac{1 + \tan^2 \frac{x}{2}}{5 \left( 1 + \tan^2 \frac{x}{2} \right) + 4 \left( 1 - \tan^2 \frac{x}{2} \right)} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{5 + 5 \tan^2 \frac{x}{2} + 4 - 4 \tan^2 \frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{9 + \tan^2 \frac{x}{2}} dx$$

$$\text{Let } \tan \frac{x}{2} = t$$

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

$$= \int \frac{2dt}{(3)^2 + t^2}$$

$$= 2 \times \frac{1}{3} \tan^{-1}(t) + c$$

$$I = \frac{2}{3} \tan^{-1} \left( \frac{\tan \frac{x}{2}}{3} \right) + c$$

Indefinite Integrals Ex 19.23 Q2

$$\text{Let } I = \int \frac{1}{5 - 4 \sin x} dx$$

$$\text{Put } \sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$= \int \frac{1}{5 - 4 \left( \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right)} dx$$

$$= \int \frac{1 + \tan^2 \frac{x}{2}}{5 \left( 1 + \tan^2 \frac{x}{2} \right) - 4 \left( 2 \tan \frac{x}{2} \right)} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{5 + 5 \tan^2 \frac{x}{2} - 8 \tan \frac{x}{2}} dx$$

$$\text{Let } \tan \frac{x}{2} = t$$

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

$$= \int \frac{2dt}{5t^2 - 8t + 5}$$

$$= \frac{2}{5} \int \frac{dt}{t^2 - \frac{8}{5}t + 1}$$

$$= \frac{2}{5} \int \frac{dt}{t^2 - 2t \left( \frac{4}{5} \right) + \left( \frac{4}{5} \right)^2 - \left( \frac{4}{5} \right)^2 + 1}$$

$$I = \frac{2}{5} \int \frac{dt}{\left( t - \frac{4}{5} \right)^2 + \left( \frac{3}{5} \right)^2}$$

$$= \frac{2}{5} \times \frac{1}{\frac{3}{5}} \tan^{-1} \left( \frac{t - \frac{4}{5}}{\frac{3}{5}} \right) + c$$

$$= \frac{2}{3} \tan^{-1} \left( \frac{5t - 4}{3} \right) + c$$

$$I = \frac{2}{3} \tan^{-1} \left( \frac{5 \tan \frac{x}{2} - 4}{3} \right) + c$$

Indefinite Integrals Ex 19.23 Q3

$$\text{Let } I = \int \frac{1}{1 - 2 \sin x} dx$$

$$\text{Put } \sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$I = \int \frac{1}{1 - 2 \left( \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right)} dx$$

$$= \int \frac{1 + \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2} - 4 \tan \frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{\tan^2 \frac{x}{2} - 4 \tan \frac{x}{2} + 1} dx$$

$$\text{Let } \tan \frac{x}{2} = t$$

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

$$I = \int \frac{2dt}{t^2 - 4t + 1}$$

$$= \int \frac{2dt}{t^2 - 2t(2) + (2)^2 - (2)^2 + 1}$$

$$I = 2 \int \frac{dt}{(t - 2)^2 + 3}$$

$$= 2 \int \frac{dt}{(t - 2)^2 + (\sqrt{3})^2}$$

$$= 2 \times \frac{1}{2\sqrt{3}} \log \left| \frac{t - 2 - \sqrt{3}}{t - 2 + \sqrt{3}} \right| + c$$

$$I = \frac{1}{\sqrt{3}} \log \left| \frac{\tan \frac{x}{2} - 2 - \sqrt{3}}{\tan \frac{x}{2} - 2 + \sqrt{3}} \right| + c$$

Indefinite Integrals Ex 19.23 Q4

$$\text{Let } I = \int \frac{1}{4 \cos x - 1} dx$$

$$\text{Put } \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$I = \int \frac{1}{4 \left( \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right) - 1} dx$$

$$= \int \frac{1 + \tan^2 \frac{x}{2}}{4 \left( 1 - \tan^2 \frac{x}{2} \right) - \left( 1 + \tan^2 \frac{x}{2} \right)} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{4 - 4 \tan^2 \frac{x}{2} - 1 - \tan^2 \frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{3 - 5 \tan^2 \frac{x}{2}} dx$$

$$\text{Let } \sqrt{5} \tan \frac{x}{2} = t$$

$$\frac{\sqrt{5}}{2} \sec^2 \frac{x}{2} dt = dt$$

$$I = \int \frac{dt}{(\sqrt{3})^2 - t^2}$$

$$I = \frac{1}{\sqrt{15}} \log \left| \frac{\sqrt{3} + \sqrt{5} \tan \frac{x}{2}}{\sqrt{3} - \sqrt{5} \tan \frac{x}{2}} \right| + c$$

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