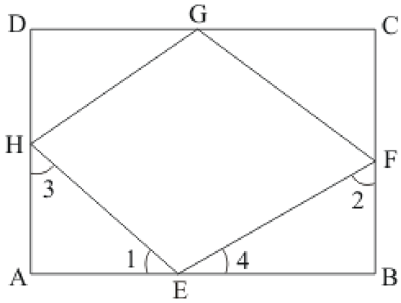
Quadrilaterals Ex 14.3 Q7

Answer :

Square *ABCD* is given:



E, F, G and *H* are the points on *AB, BC, CD* and *DA* respectively, such that :

$AE = BF = CG = DH$

We need to prove that *EFGH* is a square.

Say, $AE = BF = CG = DH = x$

As sides of a square are equal. Then, we can also say that:

$BE = CF = DG = AH = y$

In ΔAEH and ΔBFE ,we have:

$AE = BF$ (Given)

$\angle A = \angle B$ (Each equal to 90°)

$BE = AH$ (Each equal to *y*)

By SAS Congruence criteria, we have:

$\Delta AEH \cong \Delta BFE$

Therefore, EH = EF

Similarly, EF= FG, FG= HG and HG= HE

Thus, HE=EF=FG=HG

Also,

$\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

But,

$\angle 1 + \angle 3 = 90^0$ and $\angle 2 + \angle 4 = 90^0$

Therefore,

$$\angle 1 + \angle 3 + \angle 2 + \angle 4 = 90^0 + 90^0$$
$$\angle 1 + \angle 4 + \angle 1 + \angle 4 = 180^0$$
$$2(\angle 1 + \angle 4) = 180^0$$
$$\angle 1 + \angle 4 = 90^0$$

i.e., $\angle HEF = 90^0$

Similarly,

$\angle F = 90^0$

$\angle G = 90^0$

$\angle H = 90^0$

Thus, *EFGH* is a square.

Hence proved.

*********** END **********