



Therefore, 100 kg of fertiliser F_1 and 80 kg of fertilizer F_2 should be used to minimize the cost. The minimum cost is Rs 1000.

Question 11:

The corner points of the feasible region determined by the following system of linear inequalities:

$2x + y \leq 10, x + 3y \leq 15, x, y \geq 0$ are $(0, 0), (5, 0), (3, 4)$ and $(0, 5)$. Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of Z occurs at both $(3, 4)$ and $(0, 5)$ is (A) $p = q$ (B) $p = 2q$ (C) $p = 3q$ (D) $q = 3p$

Answer

The maximum value of Z is unique.

It is given that the maximum value of Z occurs at two points, $(3, 4)$ and $(0, 5)$.

$$\therefore \text{Value of } Z \text{ at } (3, 4) = \text{Value of } Z \text{ at } (0, 5)$$

$$\Rightarrow p(3) + q(4) = p(0) + q(5)$$

$$\Rightarrow 3p + 4q = 5q$$

$$\Rightarrow q = 3p$$

Hence, the correct answer is D.

Miscellaneous Solutions

Question 1:

Refer to Example 9. How many packets of each food should be used to maximize the amount of vitamin A in the diet? What is the maximum amount of vitamin A in the diet?

Answer

Let the diet contain x and y packets of foods P and Q respectively. Therefore,

$$x \geq 0 \text{ and } y \geq 0$$

The mathematical formulation of the given problem is as follows.

$$\text{Maximize } z = 6x + 3y \dots (1)$$

subject to the constraints,

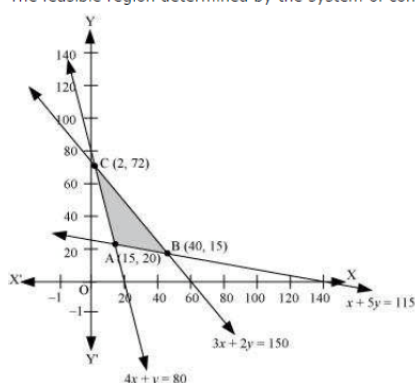
$$4x + y \geq 80 \dots (2)$$

$$x + 5y \geq 115 \dots (3)$$

$$3x + 2y \leq 150 \dots (4)$$

$$x, y \geq 0 \dots (5)$$

The feasible region determined by the system of constraints is as follows.



The corner points of the feasible region are A (15, 20), B (40, 15), and C (2, 72).

The values of z at these corner points are as follows.

Corner point	$z = 6x + 3y$	
A(15, 20)	150	
B(40, 15)	285	→ Maximum
C(2, 72)	228	

Thus, the maximum value of z is 285 at (40, 15).

Therefore, to maximize the amount of vitamin A in the diet, 40 packets of food P and 15 packets of food Q should be used. The maximum amount of vitamin A in the diet is 285 units.

Question 2:

Question 2:

A farmer mixes two brands P and Q of cattle feed. Brand P, costing Rs 250 per bag contains 3 units of nutritional element A, 2.5 units of element B and 2 units of element C. Brand Q costing Rs 200 per bag contains 1.5 units of nutritional elements A, 11.25 units of element B, and 3 units of element C. The minimum requirements of nutrients A, B and C are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag? What is the minimum cost of the mixture per bag?

Answer

Let the farmer mix x bags of brand P and y bags of brand Q.

The given information can be compiled in a table as follows.

	Vitamin A (units/kg)	Vitamin B (units/kg)	Cost (Rs/kg)
Food P	3	5	60
Food Q	4	2	80
Requirement (units/kg)	18	45	

The given problem can be formulated as follows.

Minimize $z = 250x + 200y$... (1)

subject to the constraints,

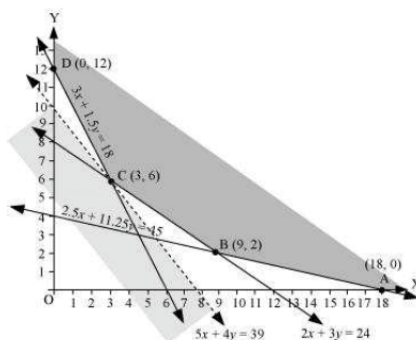
$$3x + 1.5y \geq 18 \quad \dots(2)$$

$$2.5x + 11.25y \geq 45 \quad \dots(3)$$

$$2x + 3y \geq 24 \quad \dots(4)$$

$$x, y \geq 0 \quad \dots(5)$$

The feasible region determined by the system of constraints is as follows.



The corner points of the feasible region are A (18, 0), B (9, 2), C (3, 6), and D (0, 12).

The values of z at these corner points are as follows.

Corner point	$z = 250x + 200y$	
A (18, 0)	4500	
B (9, 2)	2650	
C (3, 6)	1950	→ Minimum
D (0, 12)	2400	

As the feasible region is unbounded, therefore, 1950 may or may not be the minimum value of z .

For this, we draw a graph of the inequality, $250x + 200y < 1950$ or $5x + 4y < 39$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $5x + 4y < 39$

Therefore, the minimum value of z is 2000 at (3, 6).

Thus, 3 bags of brand P and 6 bags of brand Q should be used in the mixture to minimize the cost to Rs 1950.

Question 3:

A dietician wishes to mix together two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin content of one kg food is given below:

Food	Vitamin A	Vitamin B	Vitamin C
X	1	2	3
Y	2	2	1

One kg of food X costs Rs 16 and one kg of food Y costs Rs 20. Find the least cost of the mixture which will produce the required diet?

Answer

Let the mixture contain x kg of food X and y kg of food Y.

The mathematical formulation of the given problem is as follows.

Minimize $z = 16x + 20y \dots (1)$

subject to the constraints,

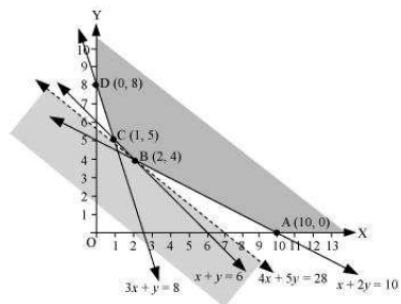
$$x + 2y \geq 10 \dots (2)$$

$$x + y \geq 6 \dots (3)$$

$$3x + y \geq 8 \dots (4)$$

$$x, y \geq 0 \dots (5)$$

The feasible region determined by the system of constraints is as follows.



The corner points of the feasible region are A (10, 0), B (2, 4), C (1, 5), and D (0, 8).

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