



$$\therefore \text{adj}A = \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj}A = - \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

Question 11:

Find the inverse of each of the matrices (if it exists).

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & \sin \alpha & -\cos \alpha \end{bmatrix}$$

Answer

$$\text{Let } A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & \sin \alpha & -\cos \alpha \end{bmatrix}.$$

We have,

$$|A| = 1(-\cos^2 \alpha - \sin^2 \alpha) = -(\cos^2 \alpha + \sin^2 \alpha) = -1$$

Now,

$$A_{11} = -\cos^2 \alpha - \sin^2 \alpha = -1, A_{12} = 0, A_{13} = 0$$

$$A_{21} = 0, A_{22} = -\cos \alpha, A_{23} = -\sin \alpha$$

$$A_{31} = 0, A_{32} = -\sin \alpha, A_{33} = \cos \alpha$$

$$\therefore \text{adj}A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos \alpha & -\sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \cdot \text{adj}A = - \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos \alpha & -\sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & \sin \alpha & -\cos \alpha \end{bmatrix}$$

Question 12:

$$\text{Let } A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}. \text{ Verify that } (AB)^{-1} = B^{-1}A^{-1}$$

Answer

$$\text{Let } A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}.$$

We have,

$$|A| = 15 - 14 = 1$$

Now,

$$A_{11} = 5, A_{12} = -2, A_{21} = -7, A_{22} = 3$$

$$\therefore \text{adj}A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \cdot \text{adj}A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$\text{Now, let } B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}.$$

We have,

$$|B| = 54 - 56 = -2$$

$$\therefore \text{adj}B = \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$$

$$\therefore B^{-1} = \frac{1}{|B|} \text{adj}B = -\frac{1}{2} \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix} = \begin{bmatrix} -\frac{9}{2} & 4 \\ \frac{7}{2} & -3 \end{bmatrix}$$

Now,

$$\begin{bmatrix} -\frac{9}{2} & 4 \end{bmatrix}$$

$$\begin{aligned}
 B^{-1}A^{-1} &= \begin{bmatrix} 2 & 7 \\ \frac{7}{2} & -3 \end{bmatrix} \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} -\frac{45}{2}-8 & \frac{63}{2}+12 \\ \frac{35}{2}+6 & -\frac{49}{2}-9 \end{bmatrix} = \begin{bmatrix} -\frac{61}{2} & \frac{87}{2} \\ \frac{47}{2} & -\frac{67}{2} \end{bmatrix} \quad \dots(1)
 \end{aligned}$$

Then,

$$\begin{aligned}
 AB &= \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix} \\
 &= \begin{bmatrix} 18+49 & 24+63 \\ 12+35 & 16+45 \end{bmatrix} \\
 &= \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}
 \end{aligned}$$

Therefore, we have $|AB| = 67 \times 61 - 87 \times 47 = 4087 - 4089 = -2$.

Also,

$$\begin{aligned}
 \text{adj}(AB) &= \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix} \\
 \therefore (AB)^{-1} &= \frac{1}{|AB|} \text{adj}(AB) = -\frac{1}{2} \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix} \\
 &= \begin{bmatrix} -\frac{61}{2} & \frac{87}{2} \\ \frac{47}{2} & -\frac{67}{2} \end{bmatrix} \quad \dots(2)
 \end{aligned}$$

From (1) and (2), we have:

$$(AB)^{-1} = B^{-1}A^{-1}$$

Hence, the given result is proved.

Question 13:

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}, \text{ show that } A^2 - 5A + 7I = O. \text{ Hence find } A^{-1}.$$

Answer

$$\begin{aligned}
 A &= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \\
 A^2 &= A \cdot A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} \\
 \therefore A^2 - 5A + 7I &= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\
 &= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}
 \end{aligned}$$

Hence, $A^2 - 5A + 7I = O$.

$$\therefore A \cdot A - 5A = -7I$$

$$\Rightarrow A \cdot A(A^{-1}) - 5AA^{-1} = -7IA^{-1} \quad [\text{Post-multiplying by } A^{-1} \text{ as } |A| \neq 0]$$

$$\Rightarrow A(AA^{-1}) - 5I = -7A^{-1}$$

$$\Rightarrow AI - 5I = -7A^{-1}$$

$$\Rightarrow A^{-1} = -\frac{1}{7}(A - 5I)$$

$$\Rightarrow A^{-1} = \frac{1}{7}(5I - A)$$

$$= \frac{1}{7} \left(\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \right) = \frac{1}{7} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{7} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

Question 14:

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}, \text{ find the numbers } a \text{ and } b \text{ such that } A^2 + aA + bI = O.$$

Answer

$$\begin{aligned}
 A &= \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \\
 \therefore A^2 &= \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 9+2 & 6+2 \\ 3+1 & 2+1 \end{bmatrix} = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}
 \end{aligned}$$

Now,

$$A^2 + aA + bI = O$$

$$\Rightarrow (AA)A^{-1} + aAA^{-1} + bIA^{-1} = O \quad \left[\text{Post-multiplying by } A^{-1} \text{ as } |A| \neq 0 \right]$$

$$\Rightarrow A(AA^{-1}) + aI + b(AA^{-1}) = O$$

$$\Rightarrow AI + aI + bA^{-1} = O$$

$$\Rightarrow A + aI = -bA^{-1}$$

$$\Rightarrow A^{-1} = -\frac{1}{b}(A + aI)$$

Now,

$$A^{-1} = \frac{1}{|A|} \text{adj}A = \frac{1}{1} \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$$

We have:

$$\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} = -\frac{1}{b} \left(\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \right) = -\frac{1}{b} \begin{bmatrix} 3+a & 2 \\ 1 & 1+a \end{bmatrix} = \begin{bmatrix} \frac{-3-a}{b} & -\frac{2}{b} \\ -\frac{1}{b} & \frac{-1-a}{b} \end{bmatrix}$$

Comparing the corresponding elements of the two matrices, we have:

$$-\frac{1}{b} = -1 \Rightarrow b = 1$$

$$\frac{-3-a}{b} = 1 \Rightarrow -3-a = 1 \Rightarrow a = -4$$

Hence, -4 and 1 are the required values of a and b respectively.

Question 15:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

For the matrix A^{-1} , show that $A^3 - 6A^2 + 5A + 11I = O$. Hence,

Answer

$$\begin{aligned} A &= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} \\ A^2 &= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 1+1+2 & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} \end{aligned}$$

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