Indefinite Integrals Ex 19.20 Q6

Let $I = \int \frac{x^2+x+1}{x^2-x+1}dx$

$$= \int \left[1 + \frac{2x}{x^2-x+1}\right]dx$$

$$I = x + \int \frac{2x}{x^2-x+1}dx + c_1 \text{ - - - - (i)}$$

Let $I = \int \frac{2x}{x^2-x+1}dx$

Let $2x = \lambda \frac{d}{dx}\left(x^2-x+1\right) + \mu$

$$= \lambda\left(2x-1\right) + \mu$$

$$2x = \left(2\lambda\right)x - \lambda + \mu$$

Comparing the coefficients of like powers of x,

$$2 = 2\lambda \quad \Rightarrow \quad \lambda = 1$$

$$-\lambda + \mu = 0 \quad \Rightarrow \quad -1 + \mu = 0$$

$$\mu = 1$$

so, $I_1 = \int \frac{\left(2x-1\right)+1}{x^2-x+1}dx$

$$= \int \frac{\left(2x-1\right)}{x^2-x+1}dx + \int \frac{1}{x^2 - 2x\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1}dx$$

$$I_1 = \int \frac{2x-1}{x^2-x+1}dx + \int \frac{1}{\left(x-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}dx$$

$$= \log\left|x^2-x+1\right| + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c_2 \qquad \left[\text{since, } \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$= \log\left|x^2-x+1\right| + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + c_2 \text{ - - - - - - (ii)}$$

Using equation (i) and (ii)

$$I = x + \log\left|x^2-x+1\right| + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + c$$

Indefinite Integrals Ex 19.20 Q7

Let $I = \int \frac{(x-1)^2}{x^2+2x+2}dx$

$= \int \frac{x^2-2x+1}{x^2+2x+2}dx$

$= \int \left[1 - \frac{4x+1}{x^2+2x+2}\right]dx$

$I = x - \int \frac{4x+1}{x^2+2x+2}dx + c_1 ----(i)$

Let $I = \int \frac{4x+1}{x^2+2x+2}dx$

Let $4x+1 = \lambda \frac{d}{dx}(x^2+2x+2) + \mu$

$= \lambda(2x+2) + \mu$

$= (2\lambda)x + (2\lambda + \mu)$

Comparing the coefficients of like powers of x,

$4 = 2\lambda \quad \Rightarrow \quad \lambda = 2$

$2\lambda + \mu = 1 \quad \Rightarrow \quad 2(2) + \mu = 1$

$\mu = -3$

so, $I_1 = \int \frac{2(2x+2)-3}{x^2+2x+2}dx$

$= 2\int \frac{(2x+2)}{x^2+2x+2}dx - 3\int \frac{1}{x^2-2x+(1)^2-(1)^2+2}dx$

$I_1 = 2\int \frac{2x+2}{x^2+2x+2}dx - 3\int \frac{1}{(x+1)^2+(1)^2}dx$

$I_1 = 2\log\left|x^2+2x+2\right| - 3\tan^{-1}(x+1) + c_2 ----(ii)$ $\left[\text{since, } \int \frac{1}{x^2+1}dx = \tan^{-1}x + c\right]$

Using equation (i) and (ii)

$I = x - 2\log\left|x^2+2x+2\right| + 3\tan^{-1}(x+1) + c$

Indefinite Integrals Ex 19.20 Q8

Let $I = \int \frac{x^3+x^2+2x+1}{x^2-x+1}dx$

$= \int \left[x + 2 + \frac{3x-1}{x^2-x+1}\right]dx$

$I = \frac{x^2}{2} + 2x + \int \frac{3x-1}{x^2-x+1}dx + c_1 ----(i)$

Let $I_1 = \int \frac{3x-1}{x^2-x+1}dx$

Let $3x-1 = \lambda \frac{d}{dx}(x^2-x+1) + \mu$

$= \lambda(2x-1) + \mu$

$3x-1 = (2\lambda)x - \lambda + \mu$

Comparing the coefficients of like powers of x,

$3 = 2\lambda \quad \Rightarrow \quad \lambda = \frac{3}{2}$

$-\lambda + \mu = -1 \quad \Rightarrow \quad -\left(\frac{3}{2}\right) + \mu = -1$

$\mu = \frac{1}{2}$

so, $I_1 = \int \frac{\frac{3}{2}(2x-1)+\frac{1}{2}}{x^2-x+1}dx$

$= \frac{3}{2}\int \frac{(2x-1)}{x^2-x+1}dx + \frac{1}{2}\int \frac{1}{x^2-2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}dx$

$I_1 = \frac{3}{2}\int \frac{2x-1}{x^2-x+1}dx + \frac{1}{2}\int \frac{1}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dx$

$I_1 = \frac{3}{2}\log\left|x^2-x+1\right| + \frac{1}{2} \times \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c_2$ $\left[\text{since, } \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c\right]$

$I_1 = \frac{3}{2}\log\left|x^2-x+1\right| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + c_2 ---(ii)$

Using equation (i) and (ii)

$I = \frac{x^2}{2} + 2x + \frac{3}{2}\log\left|x^2-x+1\right| + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + c$

Indefinite Integrals Ex 19.20 Q9

$$\text{Let } I = \int \frac{x^2\left(x^4+4\right)}{\left(x^2+4\right)}dx$$

$$= \int \frac{x^6+4x^2}{\left(x^2+4\right)}dx$$

$$= \int \left[x^4 - 4x^2 + 20 - \frac{80}{x^2+4}\right]dx$$

$$I = \frac{x^5}{5} - \frac{4x^3}{3} + 20x - 80\int \frac{1}{x^2+4}dx + c_1 - - - - (i)$$

$$\text{Let } I_1 = \int \frac{1}{x^2+4}dx$$

$$= \int \frac{1}{x^2+(2)^2}dx$$

$$I_1 = \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + c_2 - - - (ii) \quad \left[\text{Since } \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$$

Using equation (i) and (ii)

$$I = \frac{x^5}{5} - \frac{4x^3}{3} + 20x - \frac{80}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$$

$$I = \frac{x^5}{5} - \frac{4x^3}{3} + 20x - 40\tan^{-1}\left(\frac{x}{2}\right) + c$$

Indefinite Integrals Ex 19.20 Q10

$$\text{Let } I = \int \frac{x^2}{x^2+6x+12}dx$$

$$= \int \left[1 - \frac{6x+12}{x^2+6x+12}\right]dx$$

$$= x - \int \frac{6x+12}{x^2+6x+12}dx + c_1 - - - - (i)$$

$$\text{Let } I_1 = \int \frac{6x+12}{x^2+6x+12}dx$$

$$\text{Let } 6x+12 = \lambda \frac{d}{dx}\left(x^2+6x+12\right) + \mu$$

$$= \lambda\left(2x+6\right) + \mu$$

$$6x+12 = \left(2\lambda\right)x + 6\lambda + \mu$$

Comparing the coefficients of like powers of x,

$$6 = 2\lambda \quad \Rightarrow \quad \lambda = 3$$

$$6\lambda + \mu = 12 \quad \Rightarrow \quad 6\left(3\right) + \mu = 12$$

$$\mu = -6$$

$$\text{so,} \quad I_1 = \int \frac{3\left(2x+6\right)-6}{x^2+6x+12}dx$$

$$= 3\int \frac{\left(2x+6\right)}{x^2+6x+12}dx - 6\int \frac{1}{x^2+2x\left(3\right)+\left(3\right)^2-\left(3\right)^2+12}dx$$

$$I_1 = 3\int \frac{2x+6}{x^2+6x+12}dx + 6\int \frac{1}{\left(x+3\right)^2+\left(\sqrt{3}\right)^2}dx$$

$$I_1 = 3\log\left|x^2+6x+12\right| + \frac{6}{\sqrt{3}}\tan^{-1}\left(\frac{x+3}{\sqrt{3}}\right) + c_2 \qquad \left[\text{since, } \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$$

$$I_1 = 3\log\left|x^2+6x+12\right| + 2\sqrt{3}\tan^{-1}\left(\frac{x+3}{\sqrt{3}}\right) + c_2 - - - (ii)$$

Using equation (i) and (ii)

$$I = x - 3\log\left|x^2+6x+12\right| + 2\sqrt{3}\tan^{-1}\left(\frac{x+3}{\sqrt{3}}\right) + c$$

*********** END **********