

Transformation Formulae Ex 8.1 Q6(i)

We have,

LHS =
$$\sin A \sin (B - C) + \sin B \sin (C - A) + \sin C \sin (A - B)$$

= $\frac{1}{2} [2 \sin A \sin (B - C) + 2 \sin B \sin (C - A) + 2 \sin C \sin (A - B)]$
= $\frac{1}{2} [\cos (A - B + C) - \cos (A + B - C) + \cos (B - C + A) - \cos (B + C - A)]$
+ $\cos (C - A + B) - \cos (C + A - B)]$
= $\frac{1}{2} [\cos (A - B + C) - \cos (A - B + C) - \cos (A + B - C) + \cos (A + B - C)]$
- $\cos (B + C - A) + \cos (B + C - A)$
= $\frac{1}{2} \times 0$
= 0
= RHS

:: sin A sin (B - C) + sin B sin (C - A) + sin C sin (A - B) = 0 Hence proved.

Transformation Formulae Ex 8.1 Q6(ii)

We have,
LHS =
$$sin(B-C)\cos(A-D) + sin(C-A)\cos(B-D) + sin(A-B)\cos(C-D)$$

= $\frac{1}{2}[2sin(B-C)\cos(A-D) + 2sin(C-A)\cos(B-D) + 2sin(A-B)\cos(C-D)]$
= $\frac{1}{2}[sin(B-C+A-D) + sin(B-C-A+D) + sin(C-A+B-D) + sin(C-A-B+D)]$
+ $sin(A-B+C-D) + sin(A-B-C+D)$
= $\frac{1}{2}[sin(A+B-C-D) + sin(B+D-C-A) + sin(B+C-A-D) + sin(C+D-A-B)]$
+ $sin(A+C-B-D) + sin(A+D-B-C)$
= $\frac{1}{2}[sin(A+B-C-D) + sin(B+D-C-A) + sin(-(A+D-B-C)) + sin(-(A+B-C-D))]$
+ $sin(-(B+D-A-C)) + sin(A+D-B-C)$
= $\frac{1}{2}[sin(A+B-C-D) + sin(B+D-C-A) - sin(A+D-B-C) - sin(A+D-B-C)]$
= $\frac{1}{2}[sin(A+B-C-D) + sin(B+D-C-A) - sin(A+D-B-C) - sin(A+D-B-C)]$
= $\frac{1}{2}[sin(A+B-C-D) + sin(B+D-C-A) - sin(A+D-B-C) - sin(A+D-B-C)]$
= $\frac{1}{2}$ \tag{\text{\text{\$\chincless{1}}}} \sin(-\theta) = -sin\theta}{\text{\$\chincless{1}}} \sin(-\theta) = -sin\theta}{\text{\$\chincless{1}}} \sin(-\theta) = -sin\theta]

Hence proved.

Transformation Formulae Ex 8.1 Q7

 $:: \quad sin\left(B-C\right)cos\left(A-D\right)+sin\left(C-A\right)cos\left(B-D\right)+sin\left(A-B\right)cos\left(C-D\right)=0$

We have,

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LHS = tan \theta tan (60^{\circ} - \theta) tan (60^{\circ} + \theta)
                        sin\theta sin(60^{\circ} - \theta) sin(60^{\circ} + \theta)
                  = \frac{\cos \theta \cos (60^{\circ} - \theta) \cos (60^{\circ} + \theta)}{\cos (60^{\circ} + \theta)}
                          2 sin θ sin (60° - θ) sin (60° + θ)
                   2cos θcos (60° - θ) cos (60° + θ)
                       sin \theta [2 sin (60^{\circ} - \theta) sin (60^{\circ} + \theta)]
                  \cos \theta \left[ 2\cos \left( 60^{\circ} - \theta \right) \cos \left( 60^{\circ} + \theta \right) \right]
                       \sin \theta \left[\cos \left\{ (60^{\circ} - \theta) - (60^{\circ} + \theta) \right\} - \cos \left\{ (60^{\circ} - \theta) + (60^{\circ} + \theta) \right\} \right]
                  = \frac{\cos \theta \left[\cos \left\{ (60^{\circ} - \theta) + (60^{\circ} + \theta) \right\} + \cos \left\{ (60^{\circ} - \theta) - (60^{\circ} + \theta) \right\} \right]}{\cos \theta \left[\cos \left\{ (60^{\circ} - \theta) + (60^{\circ} + \theta) \right\} \right]}
                       sin \theta [cos (-2\theta) - cos 120^{\circ}]
                   cos θ [cos 120° + cos (-2θ)]
                       sin θ[cos 2θ - cos 120°]
                                                                                                                                                             [\because \cos(-\theta) = \cos\theta]
                   cos 0 [cos 120° + cos 20]
                  \sin \theta [\cos 2\theta - \cos (90^{\circ} + 30^{\circ})]
\cos \theta [\cos (90^{\circ} + 30^{\circ}) + \cos 2\theta]
                  =\frac{\sin\theta[\cos 2\theta + \sin 30^{\circ}]}{\cos\theta[-\sin 30^{\circ} + \cos 2\theta]}
                                                                                                                                                            [∵ ∞s is negative in IInd quadrant]
                       \sin\theta \left[\cos 2\theta + \frac{1}{2}\right]
                 \cos \theta \left[ \frac{-1}{2} + \cos 2\theta \right]
                          \sin \theta \cos 2\theta + \frac{1}{2} \sin \theta
                      \frac{2}{2}\cos\theta + \cos\theta\cos 2\theta
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Transformation Formulae Ex 8.1 Q8

Let
$$y = \cos \alpha \cos \beta$$
 then,

$$y = \frac{1}{2} \left[\cos (\alpha + \beta) + \cos (\alpha - \beta) \right]$$

$$= \frac{1}{2} \left[\cos 90^{\circ} + \cos (\alpha - \beta) \right]$$

$$= \frac{1}{2} \left[0 + \cos (\alpha - \beta) \right]$$

$$= \frac{1}{2} \left[0 + \cos (\alpha - \beta) \right]$$

$$= \frac{1}{2} \cos (\alpha - \beta)$$

$$\Rightarrow y = \frac{1}{2} \cos (\alpha - \beta)$$
Now,
$$-1 \le \cos (\alpha - \beta) \le 1$$

$$\Rightarrow \frac{-1}{2} \le \frac{1}{2} \cos (\alpha - \beta) \le \frac{1}{2}$$

$$\Rightarrow \frac{-1}{2} \le y \le \frac{1}{2}$$

$$\Rightarrow \frac{-1}{2} \le \cos \alpha \cos \beta \le \frac{1}{2}$$

Hence, the maximum values of $\cos\!\alpha\cos\beta$ is $\frac{1}{2}.$

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