



Indefinite Integrals Ex 19.17 Q1

$$\begin{aligned}\text{Let } I &= \int \frac{1}{\sqrt{2x - x^2}} dx \\&= \int \frac{1}{\sqrt{-(x^2 - 2x)}} dx \\&= \int \frac{1}{\sqrt{-(x^2 - 2x(1) + 1^2 - 1^2)}} dx \\&= \int \frac{1}{\sqrt{-(x-1)^2 - 1}} dx \\&= \int \frac{1}{\sqrt{1 - (x-1)^2}} dx\end{aligned}$$

$$\text{Let } (x-1) = t$$

$$\Rightarrow dx = dt$$

$$\text{so, } I = \int \frac{1}{\sqrt{1-t^2}} dt$$

$$= \sin^{-1} t + c \quad \left[\text{Since } \int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c \right]$$

$$I = \sin^{-1} (x-1) + c$$

Indefinite Integrals Ex 19.17 Q2

$8+3x-x^2$ can be written as $8-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)$.

Therefore,

$$8-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)$$

$$=\frac{41}{4}-\left(x-\frac{3}{2}\right)^2$$

$$\Rightarrow \int \frac{1}{\sqrt{8+3x-x^2}} dx = \int \frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}} dx$$

$$\text{Let } x-\frac{3}{2}=t$$

$$\therefore dx = dt$$

$$\Rightarrow \int \frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}} dx = \int \frac{1}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2-t^2}} dt$$

$$= \sin^{-1} \left(\frac{t}{\frac{\sqrt{41}}{2}} \right) + C$$

$$= \sin^{-1} \left(\frac{x-\frac{3}{2}}{\frac{\sqrt{41}}{2}} \right) + C$$

$$= \sin^{-1} \left(\frac{2x-3}{\sqrt{41}} \right) + C$$

Indefinite Integrals Ex 19.17 Q3

$$\begin{aligned}
 \text{Let } I &= \int \frac{1}{\sqrt{5-4x-2x^2}} dx \\
 &= \int \frac{1}{\sqrt{-2\left[x^2+2x-\frac{5}{2}\right]}} dx \\
 &= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{-\left[x^2+2x+(1)^2-(1)^2-\frac{5}{2}\right]}} dx \\
 &= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{-\left[(x+1)^2-\frac{7}{2}\right]}} dx \\
 &= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\frac{7}{2}-(x+1)^2}} dx
 \end{aligned}$$

$$\text{Let } (x+1) = t$$

$$\Rightarrow dx = dt$$

$$\begin{aligned}
 \text{so, } I &= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\left(\sqrt{\frac{7}{2}}\right)^2 - t^2}} dt \\
 &= \frac{1}{\sqrt{2}} \sin^{-1} \left(\frac{t}{\sqrt{\frac{7}{2}}} \right) + c \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + c \right]
 \end{aligned}$$

$$I = \frac{1}{\sqrt{2}} \sin^{-1} \left(\sqrt{\frac{2}{7}} \times (x+1) \right) + c$$

Indefinite Integrals Ex 19.17 Q4

$$\begin{aligned}
 \text{Let } I &= \int \frac{1}{\sqrt{3x^2+5x+7}} dx \\
 &= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{x^2+\frac{5}{3}x+\frac{7}{3}}} dx \\
 &= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{x^2+2x\left(\frac{5}{6}\right)+\left(\frac{5}{6}\right)^2-\left(\frac{5}{6}\right)^2+\frac{7}{3}}} dx \\
 &= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{\left(x+\frac{5}{6}\right)^2-\frac{59}{36}}} dx
 \end{aligned}$$

$$\text{Let } \left(x+\frac{5}{6}\right) = t$$

$$\Rightarrow dx = dt$$

$$\begin{aligned}
 I &= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{t^2-\left(\frac{\sqrt{59}}{6}\right)^2}} dt \\
 &= \frac{1}{\sqrt{3}} \log \left| t + \sqrt{t^2-\left(\frac{\sqrt{59}}{6}\right)^2} \right| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{x^2-a^2}} dx = \log \left| x + \sqrt{x^2-a^2} \right| + c \right] \\
 I &= \frac{1}{\sqrt{3}} \log \left| x + \frac{5}{6} + \sqrt{\left(x+\frac{5}{6}\right)^2-\left(\frac{\sqrt{59}}{6}\right)^2} \right| + c
 \end{aligned}$$

$$I = \frac{1}{\sqrt{3}} \log \left| x + \frac{5}{6} + \sqrt{x^2+\frac{5x}{3}+\frac{7}{3}} \right| + c$$

***** END *****