

Adjoint and Inverse of Matrix Ex 7.1 Q11

$$A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} \quad \therefore |A| = 1 \neq 0 \text{ and } \text{adj } A = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$
$$\therefore A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{1} \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & 7 \\ 7 & 9 \end{bmatrix} \quad \therefore |B| = -2 \neq 0 \text{ and } \text{adj} B = \begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}$$
$$\therefore B^{-1} = \frac{\text{adj} B}{|B|} = \frac{1}{(-2)} \begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}$$

Now,
$$(AB)^{-1} = B^{-1}.A^{-1}$$

 $(AB)^{-1} = \frac{1}{(-2)} \begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix} \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$
 $(AB)^{-1} = -\frac{1}{2} \begin{bmatrix} 94 & -39 \\ -82 & 34 \end{bmatrix}$

$$(AB)^{-1} = \begin{bmatrix} -47 & \frac{39}{2} \\ 41 & -17 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q12

$$A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} \therefore |A| = 2 \neq 0$$

$$adj A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{2} \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

To show:
$$2A^{-1} = 9I - A$$

LHS:
$$2A^{-1} = 2.\frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

RHS:
$$9I - A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

Therefore,
$$2A^{-1} = 9I - A$$

Adjoint and Inverse of Matrix Ex 7.1 Q13

$$A = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$$
 $\therefore |A| = -6 \text{ and } \text{adj } A = \begin{bmatrix} 1 & -5 \\ -2 & 4 \end{bmatrix}$ $\therefore A^{-1} = \frac{1}{-6} \begin{bmatrix} 1 & -5 \\ -2 & 4 \end{bmatrix}$

To show:
$$A - 3I = 2(I + 3A^{-1})$$

$$\therefore \qquad \text{LHS} = A - 3I = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 2 & -2 \end{bmatrix}$$

RHS:
$$2(I+3A^{-1}) = 2I+2.3.A^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + 2.3.\frac{1}{6} \begin{bmatrix} -1 & 5 \\ 2 & -4 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 5 \\ 2 & -4 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 5 \\ 2 & -2 \end{bmatrix}$$

$$A - 3I = 2(I + 3A^{-1})$$

Adjoint and Inverse of Matrix Ex 7.1 Q14

$$A = \begin{bmatrix} a & b \\ c & \frac{1+bc}{a} \end{bmatrix}$$
$$\Rightarrow |A| = (1+bc) - bc = 1 \neq 0$$

$$A^{-1} = \frac{1}{|A|} \cdot adjA = \frac{1}{1} \begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$$

Now
$$aA^{-1} = \left(a^2 + bc + 1\right)I - aA$$

$$LHS: aA^{-1} = a\begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} 1+bc & -ab \\ -ac & a^2 \end{bmatrix}$$

$$RHS: \left(a^2+bc+1\right)I - aA = \begin{bmatrix} a^2+bc+1 & 0 \\ 0 & a^2+bc+1 \end{bmatrix} - \begin{bmatrix} a^2 & ab \\ ac & 1+bc \end{bmatrix} = \begin{bmatrix} 1+bc & -ab \\ -ac & a^2 \end{bmatrix}$$

Since, LH.S = RHS

Hence, proved

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