A. π

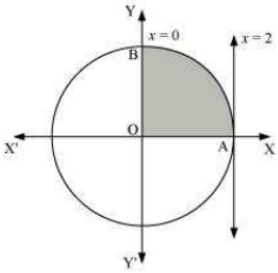
B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

Answer

The area bounded by the circle and the lines, $x = 0$ and $x = 2$, in the first quadrant is represented as



$$\therefore \text{Area OAB} = \int_0^2 y\,dx$$

$$= \int_0^2 \sqrt{4-x^2}\,dx$$

$$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$$

$$= 2\left(\frac{\pi}{2}\right)$$

$$= \pi \text{ units}$$

Thus, the correct answer is A.

Question 13:

Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is

A. 2

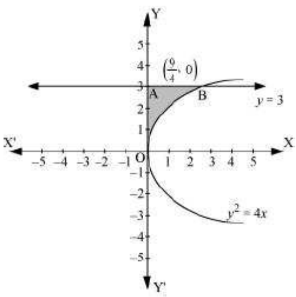
B. $\frac{9}{4}$

C. $\frac{9}{3}$

D. $\frac{9}{2}$

Answer

The area bounded by the curve, $y^2 = 4x$, y-axis, and $y = 3$ is represented as



$$\therefore \text{Area OAB} = \int_0^3 x\,dy$$

$$= \int_0^3 \frac{y^2}{4}dy$$

$$= \frac{1}{4}\left[\frac{y^3}{3}\right]_0^3$$

$$= \frac{1}{12}(27)$$

$$= \frac{9}{4} \text{ units}$$

Thus, the correct answer is B.

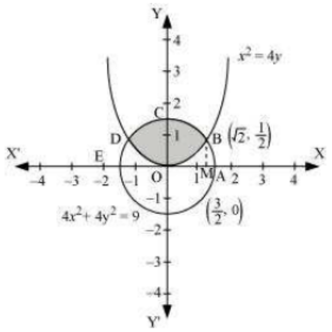
Exercise 8.2

Question 1:

Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$

Answer

The required area is represented by the shaded area OBCDO.



Solving the given equation of circle, $4x^2 + 4y^2 = 9$, and parabola, $x^2 = 4y$, we obtain the point of intersection as $B\left(\sqrt{2}, \frac{1}{2}\right)$ and $D\left(-\sqrt{2}, \frac{1}{2}\right)$.

It can be observed that the required area is symmetrical about y-axis.

∴ Area OBCDO = 2 × Area OBCO

We draw BM perpendicular to OA.

Therefore, the coordinates of M are $\left(\sqrt{2}, 0\right)$.

Therefore, Area OBCO = Area OMBCO – Area OMBO

$$= \int_0^{\sqrt{2}} \sqrt{\frac{\left(9-4x^2\right)}{4}}dx - \int_0^{\sqrt{2}} \sqrt{\frac{x^2}{4}}dx$$

$$= \frac{1}{2}\int_0^{\sqrt{2}} \sqrt{9-4x^2}\,dx - \frac{1}{4}\int_0^{\sqrt{2}} x^2 dx$$

$$= \frac{1}{4}\left[x\sqrt{9-4x^2} + \frac{9}{2}\sin^{-1}\frac{2x}{3}\right]_0^{\sqrt{2}} - \frac{1}{4}\left[\frac{x^3}{3}\right]_0^{\sqrt{2}}$$

$$= \frac{1}{4}\left[\sqrt{2}\sqrt{9-8} + \frac{9}{2}\sin^{-1}\frac{2\sqrt{2}}{3}\right] - \frac{1}{12}\left(\sqrt{2}\right)^3$$

$$= \frac{\sqrt{2}}{4} + \frac{9}{8}\sin^{-1}\frac{2\sqrt{2}}{3} - \frac{\sqrt{2}}{6}$$

$$= \frac{\sqrt{2}}{12} + \frac{9}{8}\sin^{-1}\frac{2\sqrt{2}}{3}$$

$$= \frac{1}{2}\left(\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right)$$

Therefore, the required area OBCDO is

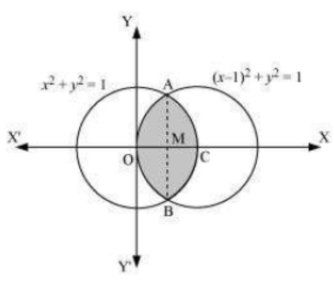
$$\left(2 \times \frac{1}{2}\left[\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right]\right) = \left[\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right] \text{ units}$$

Question 2:

Find the area bounded by curves $(x - 1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$

Answer

The area bounded by the curves, $(x - 1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$, is represented by the shaded area as



On solving the equations, $(x-1)^2+y^2=1$ and $x^2+y^2=1$, we obtain the point of intersection as A $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ and B $\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$.

It can be observed that the required area is symmetrical about x-axis.

∴ Area OBCAO = 2 × Area OCAO

We join AB, which intersects OC at M, such that AM is perpendicular to OC.

The coordinates of M are $\left(\frac{1}{2},0\right)$.

$$\Rightarrow \text{Area } OCAO = \text{Area } OMAO + \text{Area } MCAM$$

$$=\left[\int_0^{\frac{1}{2}}\sqrt{1-(x-1)^2}\,dx+\int_{\frac{1}{2}}^{1}\sqrt{1-x^2}\,dx\right]$$

$$=\left[\frac{x-1}{2}\sqrt{1-(x-1)^2}+\frac{1}{2}\sin^{-1}(x-1)\right]_0^{\frac{1}{2}}+\left[\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x\right]_{\frac{1}{2}}^{1}$$

$$=\left[-\frac{1}{4}\sqrt{1-\left(-\frac{1}{2}\right)^2}+\frac{1}{2}\sin^{-1}\left(\frac{1}{2}-1\right)-\frac{1}{2}\sin^{-1}(-1)\right]+\left[\frac{1}{2}\sin^{-1}(1)-\frac{1}{4}\sqrt{1-\left(\frac{1}{2}\right)^2}-\frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right)\right]$$

$$=\left[-\frac{\sqrt{3}}{8}+\frac{1}{2}\left(-\frac{\pi}{6}\right)-\frac{1}{2}\left(-\frac{\pi}{2}\right)\right]+\left[\frac{1}{2}\left(\frac{\pi}{2}\right)-\frac{\sqrt{3}}{8}-\frac{1}{2}\left(\frac{\pi}{6}\right)\right]$$

$$=\left[-\frac{\sqrt{3}}{4}-\frac{\pi}{12}+\frac{\pi}{4}+\frac{\pi}{4}-\frac{\pi}{12}\right]$$

$$=\left[-\frac{\sqrt{3}}{4}-\frac{\pi}{6}+\frac{\pi}{2}\right]$$

$$=\left[\frac{2\pi}{6}-\frac{\sqrt{3}}{4}\right]$$

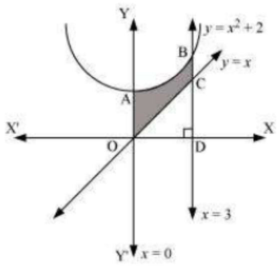
Therefore, required area OBCAO = $2\times\left(\frac{2\pi}{6}-\frac{\sqrt{3}}{4}\right)=\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)$ units

Question 3:

Find the area of the region bounded by the curves $y=x^2+2$, $y=x$, $x=0$ and $x=3$

Answer

The area bounded by the curves, $y=x^2+2$, $y=x$, $x=0$, and $x=3$, is represented by the shaded area OCBAO as



Then, Area OCBAO = Area ODBAO – Area ODCO

$$=\int_0^3\left(x^2+2\right)dx-\int_0^3 x\,dx$$

$$=\left[\frac{x^3}{3}+2x\right]_0^3-\left[\frac{x^2}{2}\right]_0^3$$

$$=[9+6]-\left[\frac{9}{2}\right]$$

$$=15-\frac{9}{2}$$

$$=\frac{21}{2}\text{ units}$$

Question 4:

Using integration finds the area of the region bounded by the triangle whose vertices are (–1, 0), (1, 3) and (3, 2).

Answer

BL and CM are drawn perpendicular to x-axis.

It can be observed in the following figure that,

Area (ΔACB) = Area (ALBA) + Area (BLMCB) – Area (AMCA) … (1)

*********** END **********