



Indefinite Integrals Ex 19.10 Q5

$$\text{Let } I = \int (2x^2 + 3) \sqrt{x+2} dx$$

Substituting $x + 2 = t$ and $dx = dt$, we get

$$\begin{aligned} I &= \int [2(t-2)^2 + 3] \sqrt{t} dt \\ &= \int [2(t^2 + 4 - 4t) + 3] \sqrt{t} dt \\ &= \int [2t^2 + 8 - 8t + 3] \sqrt{t} dt \\ &= \int \left(2t^{\frac{5}{2}} + 11t^{\frac{1}{2}} - 8t^{\frac{3}{2}} \right) dt \\ &= \frac{4}{7} t^{\frac{7}{2}} + \frac{22}{3} t^{\frac{3}{2}} - \frac{16}{5} t^{\frac{5}{2}} + c \\ &= \frac{4}{7} (x+2)^{\frac{7}{2}} - \frac{16}{5} (x+2)^{\frac{5}{2}} + \frac{22}{3} (x+2)^{\frac{3}{2}} + c \end{aligned}$$

$$\therefore I = \frac{4}{7} (x+2)^{\frac{7}{2}} - \frac{16}{5} (x+2)^{\frac{5}{2}} + \frac{22}{3} (x+2)^{\frac{3}{2}} + c$$

Indefinite Integrals Ex 19.10 Q6

$$\text{Let } I = \int \frac{x^2 + 3x + 1}{(x+1)^2} dx$$

Substituting $x + 1 = t$ and $dx = dt$, we get

$$\begin{aligned} I &= \int \frac{(t-1)^2 + 3(t-1) + 1}{t^2} dt \\ &= \int \frac{t^2 + 1 - 2t + 3t - 3 + 1}{t^2} dt \\ &= \int \frac{t^2 + t - 1}{t^2} dt \\ &= \int \left(\frac{t^2}{t^2} + \frac{t}{t^2} - \frac{1}{t^2} \right) dt \\ &= \int \left(1 + \frac{1}{t} - t^{-2} \right) dt \\ &= t + \log|t| + t^{-1} + c \\ &= t + \log|t| + \frac{1}{t} + c \\ &= (x+1) + \log|x+1| + \frac{1}{x+1} + c \end{aligned}$$

Indefinite Integrals Ex 19.10 Q7

$$\text{Let } I = \int \frac{x^2}{\sqrt{1-x}} dx$$

Substituting $1-x = t$ and $dx = -dt$, we get

$$\begin{aligned} I &= \int \frac{(1-t)^2}{\sqrt{t}} \times -dt \\ &= -\int \frac{1+t^2-2t}{\sqrt{t}} \times dt \\ &= -\int \left(t^{-\frac{1}{2}} + t^{\frac{3}{2}} - 2t^{\frac{1}{2}} \right) dt \\ &= -\left[2t^{\frac{1}{2}} + \frac{2}{5}t^{\frac{5}{2}} - \frac{4}{3}t^{\frac{3}{2}} \right] + C \\ &= -\left[\frac{30t^{\frac{1}{2}} + 6t^{\frac{5}{2}} - 20t^{\frac{3}{2}}}{15} \right] + C \\ &= -\frac{2t^{\frac{1}{2}}}{15} [15 + 3t^2 - 10t] + C \\ &= -\frac{2}{15} \sqrt{1-x} [15 + 3(1-x)^2 - 10(1-x)] + C \\ &= -\frac{2}{15} \sqrt{1-x} (15 + 3(1+x^2-2x) - 10 + 10x) + C \\ &= -\frac{2}{15} \sqrt{1-x} (5 + 3 + 3x^2 - 6x + 10x) + C \\ &= -\frac{2}{15} \sqrt{1-x} (3x^2 + 4x + 8) + C \\ &= -\frac{2}{15} (3x^2 + 4x + 8) \sqrt{1-x} + C \end{aligned}$$

$$\therefore I = -\frac{2}{15} (3x^2 + 4x + 8) \sqrt{1-x} + C$$

Indefinite Integrals Ex 19.10 Q8

$$\text{Let } I = \int x(1-x)^{23} dx$$

Substituting $1-x = t$ and $dx = -dt$, we get

$$\begin{aligned} I &= -\int (1-t)t^{23} dt \\ &= -\int (t^{23} - t^{24}) dt \\ &= -\int \left(\frac{t^{24}}{24} - \frac{t^{25}}{25} \right) + C \\ &= \frac{t^{25}}{25} - \frac{t^{24}}{24} + C \\ &= \frac{(1-x)^{25}}{25} - \frac{(1-x)^{24}}{24} + C \end{aligned}$$

$$\begin{aligned} \therefore I &= \frac{(1-x)^{25}}{25} - \frac{(1-x)^{24}}{24} + C. \\ &= \frac{1}{600} (1-x)^{24} [24(1-x) - 25] + C \\ &= \frac{1}{600} (1-x)^{24} [24 - 24x - 25] + C \\ &= \frac{1}{600} (1-x)^{24} [-1 - 24x] + C \\ &= \frac{1}{600} (1-x)^{24} \times -[1 + 24x] + C \\ &= -\frac{1}{600} (1-x)^{24} (1 + 24x) + C \end{aligned}$$

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