Differentiation Ex 11.7 Q20

Here,

$$x = \left(t + \frac{1}{t}\right)^a$$

Differentiating it with respect to t using chain rule,

$$\frac{dx}{dt} = \frac{d}{dt}\left(\left(t + \frac{1}{t}\right)^a\right)$$

$$= a\left(t + \frac{1}{t}\right)^{a-1} \frac{d}{dt}\left(t + \frac{1}{t}\right)$$

$$\frac{dx}{dt} = a\left(t + \frac{1}{t}\right)^{1-1}\left(1 - \frac{1}{t^2}\right) \qquad \text{---(i)}$$

And, $y = a^{\left(t + \frac{1}{t}\right)}$

Differentiating it with respect to t using chain rule,

$$\frac{dy}{dt} = \frac{d}{dt}\left(a^{\left(t+\frac{1}{t}\right)}\right)$$

$$= a^{\left(t+\frac{1}{t}\right)} \times \log a \frac{d}{dt}\left(t + \frac{1}{t}\right)$$

$$\frac{dy}{dt} = a^{\left(t+\frac{1}{t}\right)} \times \log a\left(1 - \frac{1}{t^2}\right) \qquad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a^{\left(t+\frac{1}{t}\right)} \times \log a\left(1 - \frac{1}{t^2}\right)}{a\left(t + \frac{1}{t}\right)^{a-1}\left(1 - \frac{1}{t^2}\right)}$$

$$\frac{dy}{dx} = \frac{a^{\left(t+\frac{1}{t}\right)} \times \log a}{a\left(t + \frac{1}{t}\right)^{a-1}}$$

Differentiation Ex 11.7 Q21

Here,

$$x = a\left(\frac{1+t^2}{1-t^2}\right)$$

Differentiating it with respect to t using chain rule,

$$\frac{dx}{dt} = a\left[\frac{\left(1+t^2\right)\frac{d}{dt}\left(1+t^2\right) - \left(1+t^2\right)\frac{d}{dt}\left(1-t^2\right)}{\left(1-t^2\right)^2}\right]$$

$$= a\left[\frac{\left(1-t^2\right)(2t) - \left(1+t^2\right)(-2t)}{\left(1-t^2\right)^2}\right]$$

$$= a\left[\frac{2t - 2t^2 + 2t + 2t^3}{\left(1-t^2\right)^2}\right]$$

$$\frac{dy}{dt} = \frac{4at}{\left(1-t^2\right)^2} \qquad \text{---(i)}$$

And, $y = \frac{2t}{1-t^2}$

Differentiating it with respect to t using quotient rule,

$$\frac{dy}{dt} = 2\left[\frac{\left(1-t^2\right)\frac{d}{dt}(t) - t\frac{d}{dt}\left(1-t^2\right)}{\left(1-t^2\right)^2}\right]$$

$$= 2\left[\frac{\left(1-t^2\right)(1) - t(-2t)}{\left(1-t^2\right)^2}\right]$$

$$= 2\left[\frac{1-t^2+2t^2}{\left(1-t^2\right)^2}\right]$$

$$\frac{dy}{dt} = \frac{2\left(1+t^2\right)}{\left(1-t^2\right)} \qquad \text{---(ii)}$$

Differentiation Ex 11.7 Q22

It is given that, $y=12(1-\cos t),\ x=10(t-\sin t)$

$$\therefore \frac{dx}{dt}=\frac{d}{dt}\left[10(t-\sin t)\right]=10\cdot\frac{d}{dt}(t-\sin t)=10(1-\cos t)$$

$$\frac{dy}{dt}=\frac{d}{dt}\left[12(1-\cos t)\right]=12\cdot\frac{d}{dt}(1-\cos t)=12\cdot\left[0-(-\sin t)\right]=12\sin t$$

$$\therefore \frac{dy}{dx}=\frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}=\frac{12\sin t}{10(1-\cos t)}=\frac{12\cdot 2\sin\frac{t}{2}\cdot\cos\frac{t}{2}}{10\cdot 2\sin^2\frac{t}{2}}=\frac{6}{5}\cot\frac{t}{2}$$

Differentiation Ex 11.7 Q23

Here $x=a(\theta-\sin\theta)$ and $y=a(1+\cos\theta)$

Then,

$$\frac{dx}{d\theta}=\frac{d}{d\theta}\left[a(\theta-\sin\theta)\right]=a(1-\cos\theta)$$

$$\frac{dy}{d\theta}=\frac{d}{d\theta}\left[a(1+\cos\theta)\right]=a(-\sin\theta)$$

$$\therefore \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\left.\frac{-a\sin\theta}{a(1-\cos\theta)}\right|_{\theta=\frac{\pi}{3}}=-\frac{\sin\frac{\pi}{3}}{1-\cos\frac{\pi}{3}}=\frac{\frac{\sqrt{3}}{2}}{1-\frac{1}{2}}=-\sqrt{3}$$

Differentiation Ex 11.7 Q24

Consider the given functions,

$x=a\sin 2t\ (1+\cos 2t)$ and $y=b\cos 2t\ (1-\cos 2t)$

Rewriting the above function, we have,

$x=a\sin 2t+\frac{a}{2}\sin 4t$

Differentiating the above function w.r.t. 't', we have,

$$\frac{dx}{dt}=2a\cos 2t+2a\cos 4t\ldots(1)$$

$y=b\cos 2t\ (1-\cos 2t)$

$y=b\cos 2t-b\cos^2 2t$

$$\frac{dy}{dt}=-2b\sin 2t+2b\cos 2t\sin 2t=-2b\sin 2t+b\sin 4t\ldots(2)$$

From (1) and (2),

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{-2b\sin 2t+b\sin 4t}{2a\cos 2t+2a\cos 4t}$$

$$\therefore \left.\frac{dy}{dx}\right|_{\pi/4}=\left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_{t=\pi/4}=\frac{-2b}{-2a}=\frac{b}{a}$$

*********** END **********