Continuity Ex 9.1 Q27

We have given that the function is continuous at $x = 0$

$\therefore$ LHL = RHL = $f(0)$.....(1)

$f(0) = \dfrac{1}{2}$

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{1-\cos k(-h)}{-h\sin(-h)} = \lim_{h\to 0}\frac{1-\cos kh}{+h\sin h}$$

$$= \lim_{h\to 0}\frac{2\sin^2\frac{kh}{2}}{h.2\sin\frac{h}{2}.\cos\frac{h}{2}}$$

$$= \lim_{h\to 0}\left(\frac{\sin\frac{kh}{2}}{\frac{kh}{2}}\right)^2 \times \frac{\frac{k^2h^2}{4}}{\frac{\sin\frac{h}{2}}{\frac{h}{2}}\times\frac{h}{2}}\cdot\frac{1}{h}$$

$$= \lim_{h\to 0}\left(\frac{\sin\frac{kh}{2}}{\frac{kh}{2}}\right)^2 \cdot \frac{\frac{k^2}{4}}{\frac{\sin\frac{h}{2}}{\frac{h}{2}}\cdot\frac{1}{2}}$$

$$= \frac{k^2}{2}$$

$\therefore$ Using (1) we get,

$\dfrac{k^2}{2} = \dfrac{1}{2} \Rightarrow k = \pm 1$

Continuity Ex 9.1 Q28

We have given that the function is continuous at $x = 4$

$\therefore$ LHL = RHL = $f(4)$....(1)

$f(4) = a + b$.....(A)

$$\text{LHL} = \lim_{x\to 4^-} f(x) = \lim_{h\to 0} f(4-h) = \lim_{h\to 0}\frac{(4-h)-4}{|(4-h)-4|} + a = \lim_{h\to 0}\frac{-h}{h} + a = a - 1 \qquad ....(B)$$

$$\text{RHL} = \lim_{x\to 4^+} f(x) = \lim_{h\to 0} f(4+h) = \lim_{h\to 0}\frac{(4+h)-4}{|(4+h)-4|} + b = \lim_{h\to 0}\frac{h}{h} + b = b + 1 \qquad ....(C)$$

$\therefore$ from (1)

$a - 1 = b + 1 \Rightarrow a - b = 2$ .....(D)

from (A) and (B)

$a + b = a - 1 \Rightarrow b = -1$

from (A) and (C)

$a + b = b + 1 \Rightarrow a = 1$

Thus, $a = 1$ and $b = -1$

Continuity Ex 9.1 Q29

We have given that the function is continuous at $x = 0$

$\therefore$ LHL = RHL = $f(0)$.....(1)

$f(0) = k$

$$\text{LHL} = \lim_{x \to 0} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{\sin 2(0-h)}{-h} = \lim_{h \to 0} \frac{-\sin 2h}{-h} = 2$$

$\therefore$ using (1), we get $k = 2$

Continuity Ex 9.1 Q30

We know that a function is continuous at $x = 0$ if,

LHL = RHL = $f(0)$....(1)

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{\log\left(1-\frac{h}{a}\right) - \log\left(1+\frac{h}{b}\right)}{(-h)} = \lim_{h \to 0} \frac{\log\left(1+\left(\frac{-h}{a}\right)\right)}{\left(\frac{-h}{a}\right) \times a} + \frac{\log\left(1+\frac{h}{b}\right)}{h}$$

$$= \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$$

from (1),

$f(0) = \frac{a+b}{ab}$

*********** END **********