



Consider a natural number (n) in co-domain \mathbf{N} .

Case **I**: n is odd

$\therefore n = 2r + 1$ for some $r \in \mathbf{N}$. Then, there exists $4r + 1 \in \mathbf{N}$ such that

$$f(4r+1) = \frac{4r+1+1}{2} = 2r+1$$

Case **II**: n is even

$\therefore n = 2r$ for some $r \in \mathbf{N}$. Then, there exists $4r \in \mathbf{N}$ such that $f(4r) = \frac{4r}{2} = 2r$.

$\therefore f$ is onto.

Hence, f is not a bijective function.

Question 10:

Let $A = \mathbf{R} - \{3\}$ and $B = \mathbf{R} - \{1\}$. Consider the function $f: A \rightarrow B$ defined by

$$f(x) = \left(\frac{x-2}{x-3} \right). \text{ Is } f \text{ one-one and onto? Justify your answer.}$$

Answer

$$A = \mathbf{R} - \{3\}, B = \mathbf{R} - \{1\}$$

$$f: A \rightarrow B \text{ is defined as } f(x) = \left(\frac{x-2}{x-3} \right).$$

$$\text{Let } x, y \in A \text{ such that } f(x) = f(y).$$

$$\Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$$

$$\Rightarrow (x-2)(y-3) = (y-2)(x-3)$$

$$\Rightarrow xy - 3x - 2y + 6 = xy - 3y - 2x + 6$$

$$\Rightarrow -3x - 2y = -3y - 2x$$

$$\Rightarrow 3x - 2x = 3y - 2y$$

$$\Rightarrow x = y$$

$\therefore f$ is one-one.

Let $y \in B = \mathbf{R} - \{1\}$. Then, $y \neq 1$.

The function f is onto if there exists $x \in A$ such that $f(x) = y$.

Now,

$$f(x) = y$$

$$\Rightarrow \frac{x-2}{x-3} = y$$

$$\Rightarrow x-2 = xy-3y$$

$$\Rightarrow x(1-y) = -3y+2$$

$$\Rightarrow x = \frac{2-3y}{1-y} \in A \quad [y \neq 1]$$

Thus, for any $y \in B$, there exists $\frac{2-3y}{1-y} \in A$ such that

$$f\left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right)-2}{\left(\frac{2-3y}{1-y}\right)-3} = \frac{2-3y-2+2y}{2-3y-3+3y} = \frac{-y}{-1} = y.$$

$\therefore f$ is onto.

Hence, function f is one-one and onto.

Question 11:

Let $f: \mathbf{R} \rightarrow \mathbf{R}$ be defined as $f(x) = x^4$. Choose the correct answer.

(A) f is one-one onto (B) f is many-one onto

(C) f is one-one but not onto (D) f is neither one-one nor onto

Answer

$f: \mathbf{R} \rightarrow \mathbf{R}$ is defined as $f(x) = x^4$.

Let $x, y \in \mathbf{R}$ such that $f(x) = f(y)$.

$$\Rightarrow x^4 = y^4$$

$$\Rightarrow x = \pm y$$

$\therefore f(x_1) = f(x_2)$ does not imply that $x_1 = x_2$.

For instance,

$$f(1) = f(-1) = 1$$

$\therefore f$ is not one-one.

Consider an element 2 in co-domain \mathbf{R} . It is clear that there does not exist any x in domain \mathbf{R} such that $f(x) = 2$.

$\therefore f$ is not onto.

Hence, function f is neither one-one nor onto.

The correct answer is D.

Question 12:

Let $f: \mathbf{R} \rightarrow \mathbf{R}$ be defined as $f(x) = 3x$. Choose the correct answer.

(A) f is one-one onto (B) f is many-one onto

(C) f is one-one but not onto (D) f is neither one-one nor onto

Answer

$f: \mathbf{R} \rightarrow \mathbf{R}$ is defined as $f(x) = 3x$.

Let $x, y \in \mathbf{R}$ such that $f(x) = f(y)$.

$$\Rightarrow 3x = 3y$$

$$\Rightarrow x = y$$

$\therefore f$ is one-one.

Also, for any real number (y) in co-domain \mathbf{R} , there exists $\frac{y}{3}$ in \mathbf{R} such that

$$f\left(\frac{y}{3}\right) = 3\left(\frac{y}{3}\right) = y$$

$\therefore f$ is onto.

Hence, function f is one-one and onto.

The correct answer is A.

Exercise 1.3

Question 1:

Let $f: \{1, 3, 4\} \rightarrow \{1, 2, 5\}$ and $g: \{1, 2, 5\} \rightarrow \{1, 3\}$ be given by $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(1, 3), (2, 3), (5, 1)\}$. Write down gof .

Answer

The functions $f: \{1, 3, 4\} \rightarrow \{1, 2, 5\}$ and $g: \{1, 2, 5\} \rightarrow \{1, 3\}$ are defined as

$f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(1, 3), (2, 3), (5, 1)\}$.

$$gof(1) = g(f(1)) = g(2) = 3 \quad [f(1) = 2 \text{ and } g(2) = 3]$$

$$gof(3) = g(f(3)) = g(5) = 1 \quad [f(3) = 5 \text{ and } g(5) = 1]$$

$$gof(4) = g(f(4)) = g(1) = 3 \quad [f(4) = 1 \text{ and } g(1) = 3]$$

$$\therefore gof = \{(1, 3), (3, 1), (4, 3)\}$$

Question 2:

Let f, g and h be functions from \mathbf{R} to \mathbf{R} . Show that

$$(f + g)oh = foh + goh$$

$$(f \cdot g)oh = (foh) \cdot (goh)$$

Answer

To prove:

$$(f + g)oh = foh + goh$$

Consider:

$$((f + g)oh)(x)$$

$$= (f + g)(h(x))$$

$$= f(h(x)) + g(h(x))$$

$$= (foh)(x) + (goh)(x)$$

$$= \{(foh) + (goh)\}(x)$$

$$\therefore ((f + g)oh)(x) = \{(foh) + (goh)\}(x) \quad \forall x \in \mathbf{R}$$

Hence, $(f + g)oh = foh + goh$.

To prove:

$$(f \cdot g)oh = (foh) \cdot (goh)$$

Consider:

$$((f \cdot g)oh)(x)$$

$$= (f \cdot g)(h(x))$$

$$= f(h(x)) \cdot g(h(x))$$

$$= (foh)(x) \cdot (goh)(x)$$

$$= \{(foh) \cdot (goh)\}(x)$$

$$\therefore ((f \cdot g)oh)(x) = \{(foh) \cdot (goh)\}(x) \quad \forall x \in \mathbf{R}$$

Hence, $(f \cdot g)oh = (foh) \cdot (goh)$.

Question 3:

Find gof and fog , if

$$(i) \quad f(x) = |x| \text{ and } g(x) = |5x - 2|$$

$$(ii) \quad f(x) = 8x^3 \text{ and } g(x) = x^{\frac{1}{3}}$$

Answer

$$(i) \quad f(x) = |x| \text{ and } g(x) = |5x - 2|$$

$$\therefore (g \circ f)(x) = g(f(x)) = g(|x|) = |5|x| - 2|$$

$$(f \circ g)(x) = f(g(x)) = f(|5x - 2|) = ||5x - 2|| = |5x - 2|$$

$$(ii) \quad f(x) = 8x^3 \text{ and } g(x) = x^{\frac{1}{3}}$$

$$\therefore (g \circ f)(x) = g(f(x)) = g(8x^3) = (8x^3)^{\frac{1}{3}} = 2x$$

$$(f \circ g)(x) = f(g(x)) = f\left(x^{\frac{1}{3}}\right) = 8\left(x^{\frac{1}{3}}\right)^3 = 8x$$

Question 4:

$$f(x) = \frac{(4x+3)}{(6x-4)}, x \neq \frac{2}{3}$$

If , show that $f \circ f(x) = x$, for all $x \neq \frac{2}{3}$. What is the inverse of f ?

Answer

$$f(x) = \frac{(4x+3)}{(6x-4)}, x \neq \frac{2}{3}$$

It is given that

$$(f \circ f)(x) = f(f(x)) = f\left(\frac{4x+3}{6x-4}\right)$$

$$= \frac{4\left(\frac{4x+3}{6x-4}\right) + 3}{6\left(\frac{4x+3}{6x-4}\right) - 4} = \frac{16x+12+18x-12}{24x+18-24x+16} = \frac{34x}{34} = x$$

$$\text{Therefore, } f \circ f(x) = x, \text{ for all } x \neq \frac{2}{3}.$$

$$\Rightarrow f \circ f = I$$

Hence, the given function f is invertible and the inverse of f is f itself.

Question 5:

State with reason whether following functions have inverse

(i) $f: \{1, 2, 3, 4\} \rightarrow \{10\}$ with

$$f = \{(1, 10), (2, 10), (3, 10), (4, 10)\}$$

(ii) $g: \{5, 6, 7, 8\} \rightarrow \{1, 2, 3, 4\}$ with

$$g = \{(5, 4), (6, 3), (7, 4), (8, 2)\}$$

(iii) $h: \{2, 3, 4, 5\} \rightarrow \{7, 9, 11, 13\}$ with

$$h = \{(2, 7), (3, 9), (4, 11), (5, 13)\}$$

Answer

***** END *****