



Geometric Progressions Ex 20.3 Q 2

$$0.15 + 0.015 + 0.0015 + \dots \text{upto 8 terms}$$

$$= 15(0.1 + 0.01 + 0.001 + \dots \text{upto 8 terms})$$

$$= 15\left(\frac{1}{10} + \frac{1}{100} + \dots\right)$$

$$r = \frac{1}{10}, a = \frac{1}{10}$$

$$Sum = 15 \left(\frac{\frac{1}{10} \left(1 - \frac{1}{10^8}\right)}{1 - \frac{1}{10}} \right)$$

$$= \frac{5}{3} \left(1 - \frac{1}{10^8}\right)$$

Here the first term of the series is $a = \sqrt{2}$ and the common ratio is $r = \frac{\frac{1}{\sqrt{2}}}{\sqrt{2}} = \frac{1}{2}$

Thus the sum of the G.P up to 8th terms is:

$$S_8 = \frac{a(1-r^8)}{1-r} = \frac{\sqrt{2} \left(1 - \left(\frac{1}{2}\right)^8\right)}{1 - \frac{1}{2}} = 2\sqrt{2} \left(1 - \frac{1}{256}\right) = \frac{255\sqrt{2}}{128}$$

$$\frac{2}{9} - \frac{1}{3} + \frac{1}{2} - \frac{3}{4} + \dots \text{ to 5 terms.}$$

$$a = \frac{2}{9}, r = \frac{-1}{\frac{3}{2}} = \frac{-1}{3} \times \frac{9}{2} = \frac{-3}{2}, n = 5$$

$$S_5 = a \frac{(1-r^5)}{1-r}$$

$$= \frac{2}{9} \frac{\left(1 - \left(\frac{-3}{2}\right)^5\right)}{1 - \left(\frac{-3}{2}\right)}$$

$$= \frac{2}{9} \frac{\left(1 + \frac{243}{32}\right)}{1 + \frac{3}{2}}$$

$$= \frac{2}{9} \frac{(275)}{32} \times \frac{2}{5}$$

$$= \frac{55}{72}$$

$$\begin{aligned}
& (x+y) + (x^2+xy+y^2) + (x^3+x^2y+xy^2+y^3) + \dots \\
&= \frac{1}{x-y} \left\{ (x^2-y^2) + (x^3-y^3) + \dots \text{to } \infty \right\} \dots \left[\because \frac{x^n-y^n}{x-y} = x^{n-1} + x^{n-2}y + \dots + y^{n-1} \right] \\
&= \frac{1}{x-y} \left\{ (x^2+x^3+\dots \text{to } \infty) - (y^2+y^3+\dots \text{to } \infty) \right\} \\
&= \frac{1}{x-y} \left\{ \frac{x^2}{1-x} - \frac{y^2}{1-y} \right\} \\
&= \frac{1}{x-y} \left\{ \frac{x^2 - x^2y - y^2 + xy^2}{(1-x)(1-y)} \right\} \\
&= \frac{x+y-xy}{(1-x)(1-y)}
\end{aligned}$$

The series can be written as:

$$3 \left(\frac{1}{5} + \frac{1}{5^3} + \frac{1}{5^5} + \dots \text{ } n \text{ terms} \right) + 4 \left(\frac{1}{5^2} + \frac{1}{5^4} + \frac{1}{5^6} + \dots \text{ } n \text{ terms} \right)$$

For the first part $a = \frac{1}{5}$ and the common ratio $r = \frac{1}{5^2} = \frac{1}{25}$

Thus the sum is:

$$\begin{aligned}
3 \left(\frac{1}{5} + \frac{1}{5^3} + \frac{1}{5^5} + \dots \text{ } n \text{ terms} \right) &= 3 \cdot \frac{\frac{1}{5} \left(1 - \left(\frac{1}{25} \right)^n \right)}{1 - \frac{1}{25}} \\
&= \frac{5}{8} \left(1 - \frac{1}{5^{2n}} \right)
\end{aligned}$$

For the second part $a = \frac{1}{25}$ and common ratio $r = \frac{1}{25}$ then

$$\begin{aligned}
4 \left(\frac{1}{5^2} + \frac{1}{5^4} + \frac{1}{5^6} + \dots \text{ } n \text{ terms} \right) &= 4 \cdot \frac{\frac{1}{25} \left(1 - \left(\frac{1}{25} \right)^n \right)}{1 - \frac{1}{25}} \\
&= \frac{1}{6} \left(1 - \frac{1}{5^{2n}} \right)
\end{aligned}$$

Thus the sum is:

$$\frac{3}{5} + \frac{4}{5^2} + \frac{3}{5^3} + \dots \text{ } 2n \text{ terms} = \frac{5}{8} \left(1 - \frac{1}{5^{2n}} \right) + \frac{1}{6} \left(1 - \frac{1}{5^{2n}} \right)$$

$$\frac{a}{1+i} + \frac{a}{(1+i)^2} + \frac{a}{(1+i)^3} + \dots + \frac{a}{(1+i)^n}$$

$$a = \frac{a}{1+i}, \quad r = \frac{\frac{a}{(1+i)^2}}{\frac{a}{1+i}} = \frac{1}{1+i}$$

$$S_n = a \frac{(1-r^n)}{1-r}$$

$$= \frac{a}{1+i} \frac{\left(1 - \left(\frac{1}{1+i} \right)^n \right)}{1 - \frac{1}{1+i}}$$

$$= \frac{a}{1+i} \times \frac{1+i}{(-i)} \left(1 - (1+i)^n \right)$$

$$= -ai \left(1 - (1+i)^n \right)$$

Re writing the sequence and sum we get,

$$\text{Sum} = 1 - a + a^2 - a^3 + a^4 - a^5 + \dots$$

Here, $r = -a$ and first term $= 1$

$$\text{Sum} = \frac{[1 - (-a)^n]}{1 + a}$$

Here the first term of the G.P is $a = x^3$ and the common ratio is $r = \frac{x^5}{x^3} = x^2$

Thus the sum of the G.P is:

$$x^3 + x^5 + x^7 + \dots \text{ to } n \text{ terms} = \frac{x^3((x^2)^n - 1)}{x^2 - 1} = \frac{x^3(x^{2n} - 1)}{x^2 - 1}$$

Here the first term of the G.P is $a = \sqrt{7}$ and the common ratio is $r = \frac{\sqrt{21}}{\sqrt{7}} = \sqrt{3}$

Thus the sum of the G.P is:

$$\sqrt{7} + \sqrt{21} + 3\sqrt{7} + \dots \text{ to } n \text{ terms} = \frac{\sqrt{7}((\sqrt{3})^n - 1)}{\sqrt{3} - 1} = \frac{\sqrt{7}(3^{\frac{n}{2}} - 1)}{\sqrt{3} - 1}$$

***** END *****