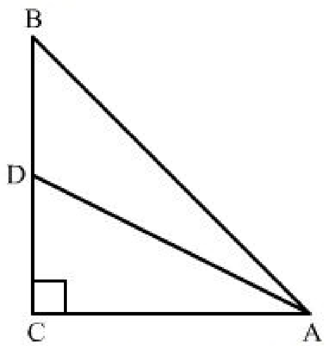
Triangles Ex 4.7 Q18

Answer :

ΔABC is a right-angled triangle with $\angle C = 90°$. D is the mid-point of BC.

We need to prove that $AB^2 = 4AD^2 - 3AC^2$.

Join AD.



Since D is the midpoint of the side BC, we get

BD = DC

$\therefore BC = 2DC$

Using Pythagoras theorem in triangles right angled triangle ABC

$AB^2 = AC^2 + BC^2$

$AB^2 = AC^2 + (2DC)^2$

$AB^2 = AC^2 + 4DC^2$ .....(1)

Again using Pythagoras theorem in the right angled triangle ADC

$AD^2 = AC^2 + DC^2$

$DC^2 = AD^2 - AC^2$ .....(2)

From (1) and (2), we get

$AB^2 = AC^2 + 4(AD^2 - AC^2)$

$AB^2 = AC^2 + 4AD^2 - 4AC^2$

$AB^2 = 4AD^2 - 3AC^2$

Hence, $AB^2 = 4AD^2 - 3AC^2$.

Triangles Ex 4.7 Q19

Answer :

(i) It is given that D is the midpoint of BC and $BC = a$.

Therefore, $BD = DC = \frac{a}{2}$ ....(1)

Using Pythagoras theorem in the right angled triangle AED,

$AD^2 = AE^2 + ED^2$ ....(2)

Let us substitute $AD = p$, $AE = h$ and $ED = x$ in equation (2), we get

$p^2 = h^2 + x^2$

Let us take another right angled triangle that is triangle AEC.

Using Pythagoras theorem,

$AC^2 = AE^2 + EC^2$ ....(3)

Let us substitute $AE = h$ and $EC = x + \frac{a}{2}$ in equation (3) we get,

Here we know that $DC = \frac{a}{2}$ and $ED = x$.

$EC = ED + DC$

Substituting $AC = b$, $DC = \frac{a}{2}$ and $ED = x$ we get $EC = \left(x + \frac{a}{2}\right)$

$b^2 = h^2 + \left(x + \frac{a}{2}\right)^2$

$b^2 = h^2 + x^2 + xa + \frac{a^2}{4}$ ....(4)

From equation (1) we can substitute $h^2 + x^2 = p^2$ in equation (4),

$b^2 = h^2 + x^2 + xa + \frac{a^2}{4}$ ....(5)

(ii) Using Pythagoras theorem in right angled triangle AEB,

$AB^2 = AE^2 + BE^2$ .... (6)

We know that AB = *c* and AE = *h* now we will find BE.

$BD = BE + ED$

Therefore, $BE = BD - ED$

We know that $BD = \frac{a}{2}$ and $ED = x$ substituting these values in $BE = BD - ED$ we get,

$BE = \frac{a}{2} - x$

Now we will substitute AB = c, AE = h and $BE = \frac{a}{2} - x$ in equation (6) we get,

$c^2 = h^2 + \left(\frac{a}{2} - x\right)^2$

$c^2 = h^2 + \frac{a^2}{4} - ax + x^2$ ....(7)

Let us rewrite the equation (7) as below,

$c^2 = h^2 + x^2 + \frac{a^2}{4} - ax$ ....(8)

From equation (1) we can substitute $h^2 + x^2 = p^2$ in equation (8),

$c^2 = p^2 + \frac{a^2}{4} - ax$

$c^2 = p^2 - ax + \frac{a^2}{4}$ ....(9)

(iii) Now we will add equations (5) and (9) as shown below,

$b^2 + c^2 = p^2 + xa + \frac{a^2}{4} + p^2 - ax + \frac{a^2}{4}$

$b^2 + c^2 = p^2 + \frac{a^2}{4} + p^2 + \frac{a^2}{4}$

$b^2 + c^2 = 2p^2 + \frac{a^2}{2}$

Therefore, $b^2 + c^2 = 2p^2 + \frac{a^2}{2}$.

*********** END **********