Definite Integrals Ex 20.4B Q21

$$\int_0^{\Pi} x\sin x\cos^2 x dx = \int_0^{\Pi} (\Pi - x)\sin(\Pi - x)\cos^2(\Pi - x) dx$$

$$\int_0^{\Pi} x\sin x\cos^2 x dx = \int_0^{\Pi} (\Pi - x)\sin x\cos^2 x dx$$

$$\int_0^{\Pi} x\sin x\cos^2 x dx = \int_0^{\Pi} \Pi\sin x\cos^2 x dx - \int_0^{\Pi} x\sin x\cos^2 x dx$$

$$2\int_0^{\Pi} x\sin x\cos^2 x dx = \int_0^{\Pi} \Pi\sin x\cos^2 x dx$$

$$\int_0^{\Pi} x\sin x\cos^2 x dx = \frac{\Pi}{2}\int_0^{\Pi} \sin x\cos^2 x dx$$

Now

$$\int_0^{\Pi} \sin x\cos^2 x dx$$

Let cosx=t

$$\sin x dx = -dt$$

$$-\int_1^{-1} t^2 dt$$

$$\int_{-1}^{1} t^2 dt$$

$$\left\{\frac{t^3}{3}\right\}_{-1}^{1}$$

$$\frac{2}{3}$$

$$\therefore \int_0^{\pi} x\sin x\cos^2 x dx = \frac{\pi}{2} \times \frac{2}{3} = \frac{\pi}{3}$$

Definite Integrals Ex 20.4B Q22

We have,

$$I = \int_0^{\frac{\pi}{2}} \frac{x \sin x . \cos x}{\sin^4 x + \cos^4 x} dx \qquad --(\text{i})$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2} - x\right) \cos x . \sin x}{\cos^4 x + \sin^4 x} dx \qquad --(\text{ii})$$

Adding (i) & (ii)

$$2I = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{\sin x . \cos x}{\cos^4 x + \sin^4 x} dx$$

$$2I = \frac{\pi}{4} \int_0^{\frac{\pi}{2}} \frac{2 \sin x . \cos x}{\cos^4 x + \sin^4 x} dx$$

Let $t = \sin^2 x$

$$\Rightarrow 2I = \frac{\pi}{4} \int_0^1 \frac{1}{(1-t)^2 + t^2} dt$$

$$\Rightarrow 2I = \frac{\pi}{8} \int_0^1 \frac{1}{\left(t - \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} dt$$

$$\Rightarrow 2I = \frac{\pi}{8} \times 2 \left[\tan^{-1}(2t-1)\right]_0^1$$

$$\Rightarrow I = \frac{\pi}{8}\left[\frac{\pi}{4} + \frac{\pi}{4}\right] = \frac{\pi^2}{16}$$

Definite Integrals Ex 20.4B Q23

Let $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 x \, dx$

$$f(-x) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 (-x) \, dx$$

$$= -\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 x \, dx$$

Here $f(x) = -f(+x)$

Hence $f(x)$ is odd function.

So,

$I = 0$

Definite Integrals Ex 20.4B Q24

We have,

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x\,dx = 2\int_0^{\frac{\pi}{2}} \sin^4 x\,dx \qquad \left[\because \sin^4 x \text{ is an even function}\right]$$

$$= 2\int_0^{\frac{\pi}{2}} \left(\sin^2 x\right)^2 dx$$

$$= 2\int_0^{\frac{\pi}{2}} \left(\frac{1-\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \left(1-\cos 2x\right)^2 dx$$

$$= \frac{1}{2}\left[\int_0^{\frac{\pi}{2}} \left(1+\cos^2 2x - 2\cos 2x\right) dx\right]$$

$$= \frac{1}{2}\left[\int_0^{\frac{\pi}{2}} \left(1-2\cos 2x + \frac{1+\cos 4x}{2}\right) dx\right]$$

$$= \frac{1}{4}\left[\int_0^{\frac{\pi}{2}} \left(3-4\cos 2x + \cos 4x\right) dx\right]$$

$$= \frac{1}{4}\left[3x - \frac{4\sin 2x}{2} + \frac{\sin 4x}{4}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{4}\left[\left\{\frac{3\pi}{2} - 2\sin\pi + \frac{1}{4}\sin 2\pi\right\} - \left\{0-0+0\right\}\right]$$

$$= \frac{1}{4}\left[\frac{3\pi}{2} - 0 + 0\right] = \frac{1}{4}\times\frac{3\pi}{2}$$

$$= \frac{3\pi}{8}$$

$$\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x\,dx = \frac{3\pi}{8}$$

*********** END **********