Continuity Ex 9.1 Q9

We want, to check the continuty of the function at $x = a$.

$$\text{LHL} = \lim_{x \to a^-} f(x) = \lim_{h \to 0} f(a-h) = \lim_{h \to 0} \frac{|a-h-a|}{a-h-a} = \lim_{h \to 0} \frac{h}{-h} = -1$$

$$\text{RHL} = \lim_{x \to a^+} f(x) = \lim_{h \to 0} f(a+h) = \lim_{h \to 0} \frac{|a+h-a|}{a+h-a} = \lim_{h \to 0} \frac{h}{h} = 1$$

Thus, LHL $\neq$ RHL

Hence, function is discontinuous at $x = a$. And the discontinuty is of first kind.

Continuity Ex 9.1 Q10(i)

We want, to check the continuity at $x = 0$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} |-h| \cos\left(\frac{1}{-h}\right) = \lim_{h \to 0} h \cos\left(\frac{1}{h}\right) = 0$$

$$\text{RHL} = \lim_{x \to 0^+} = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} |h| \cos\left(\frac{1}{h}\right) = 0$$

$$f(0) = 0$$

Thus, LHL = RHL = $f(0) = 0$

Hence, function is continuous at $x = 0$.

Continuity Ex 9.1 Q10(ii)

We want, to check the continuity at $x = 0$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} (-h)^2 \sin\left(\frac{1}{-h}\right) = 0$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} h^2 \sin\left(\frac{1}{h}\right) = 0$$

$$f(0) = 0$$

Thus, LHL = RHL = $f(0) = 0$

Hence, the function is continuous at $x = 0$.

Continuity Ex 9.1 Q10(iii)

We want, to check the continuity of the function at $x = a$.

$$\text{LHL} = \lim_{x \to a^-} f(x) = \lim_{h \to 0} f(a-h) = \lim_{h \to 0} (a-h-a)\sin\left(\frac{1}{a-h-a}\right) = \lim_{h \to 0} -h\sin\left(\frac{-1}{h}\right) = 0$$

$$\text{RHL} = \lim_{x \to a^+} f(x) = \lim_{h \to 0} f(a+h) = \lim_{h \to 0} (a+h-a)\sin\left(\frac{1}{a+h-a}\right) = \lim_{h \to 0} h\sin\left(\frac{1}{h}\right) = 0$$

$f(a) = 0$

Thus, LHL $\neq$ RHL $= f(a) = 0$
Hence, the function is continuous at $x = a$.

Continuity Ex 9.1 Q10(iv)

We want, to check the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{e^{-h}-1}{\log(1+2(-h))} = \lim_{h \to 0} \frac{e^{-h}-1}{\log(1-2h)} = DNE$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{e^{h}-1}{\log(1+2h)} = DNE$$

Thus, Both LHL and RHL do not exist

$\therefore$ Function is discontinuous and the discontinutiy is of IInd kind.

Continuity Ex 9.1 Q10(v)

We want, to check the continuity at $x = 1$

$$\text{LHL} = \lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} \frac{1-(1-h)^n}{1-(1-h)} = \lim_{h \to 0} \frac{1-\left[1-nh+\frac{n(n-1)}{2!}h^2+\dots\right]}{h}$$

$$= \lim_{h \to 0} n - \frac{n(n-1)}{2!}h + \dots$$

$$= n$$

$$\text{RHL} = \lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h) = \lim_{h \to 0} \frac{1-(1+h)^n}{1-(1+h)} = \lim_{h \to 0} \frac{1-\left[1+nh+\frac{n(n-1)}{2!}h^2+\dots\right]}{-h}$$

$$= \lim_{h \to 0} n + \frac{n(n-1)}{2!}h + \dots$$

$$= n$$

$f(1) = n-1$

Thus, LHL = RHL $\neq f(1)$
Hence, function is discontinuous at $x = 1$
This is removable discontinuity.

Continuity Ex 9.1 Q10(vi)

We want, to check the continuity at $x = 1$.

$$\text{LHL} = \lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} \frac{\left|(1-h)^2-1\right|}{(1-h)-1} \quad = \lim_{h \to 0} \frac{\left|h^2-2h\right|}{-h} = \lim_{h \to 0} -(h-2) = 2$$

$$\text{RHL} = \lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h) = \lim_{h \to 0} \frac{\left|(1+h)^2-1\right|}{1+h-1} \quad = \lim_{h \to 0} \frac{h^2+2h}{h} = 2$$

$f(1) = 2$

$\therefore$ LHL = RHL $= f(1) = 2$
Hence, function is continuous.

Continuity Ex 9.1 Q10(vii)

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{2(|-h|) + (-h)^2}{-h} = \lim_{h \to 0} \frac{2h + h^2}{-h} = -2$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{2 \times |h| + h^2}{h} = 2$$

Thus, LHL $\neq$ RHL
Function is not continuous at $x = 0$
This is discontinuity of I^{st} kind.

*********** END **********