Trigonometric Identities Ex 6.1 Q86

Answer :

Given: $\sin\theta + \cos\theta = x$

Squaring the given equation, we have

$$(\sin\theta + \cos\theta)^2 = x^2$$

$$\Rightarrow \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta = x^2$$

$$\Rightarrow (\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta = x^2$$

$$\Rightarrow 1 + 2\sin\theta\cos\theta = x^2$$

$$\Rightarrow 2\sin\theta\cos\theta = x^2 - 1$$

$$\Rightarrow \sin\theta\cos\theta = \frac{x^2-1}{2}$$

Squaring the last equation, we have

$$(\sin\theta\cos\theta)^2 = \frac{(x^2-1)^2}{4}$$

$$\Rightarrow \sin^2\theta\cos^2\theta = \frac{(x^2-1)^2}{4}$$

Therefore, we have

$$\begin{aligned}
\sin^6\theta + \cos^6\theta &= \left(\sin^2\theta\right)^3 + \left(\cos^2\theta\right)^3 \\
&= \left(\sin^2\theta + \cos^2\theta\right)^3 - 3\sin^2\theta\cos^2\theta\left(\sin^2\theta + \cos^2\theta\right) \\
&= (1)^3 - 3\frac{(x^2-1)^2}{4}(1) \\
&= 1 - 3\frac{(x^2-1)^2}{4} \\
&= \frac{4 - 3(x^2-1)^2}{4}
\end{aligned}$$

Hence proved.

*********** END **********