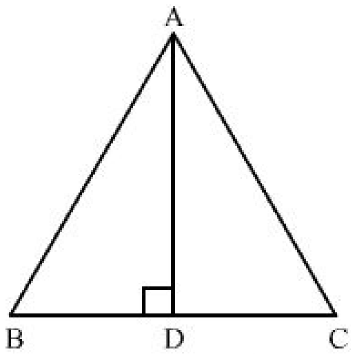
Triangles Ex 4.7 Q24

Answer :



We have to prove that $AD^2 = 3BD^2$.

In right angled ΔABD, using Pythagoras theorem we get,

$AB^2 = AD^2 + BD^2$ ....(1)

We know that in an equilateral triangle every altitude is also median.

Therefore, AD bisects BC.

Therefore, we have $BD = DC$

Since ΔABC is an equilateral triangle, $AB = BC = AC$

Therefore, we can write equation (1) as

$BC^2 = AD^2 + BD^2$ ....(2)

But $BC = 2BD$

Therefore, equation (2) becomes,

$(2BD)^2 = AD^2 + BD^2$

Simplifying the equation we get,

$4BD^2 - BD^2 = AD^2$

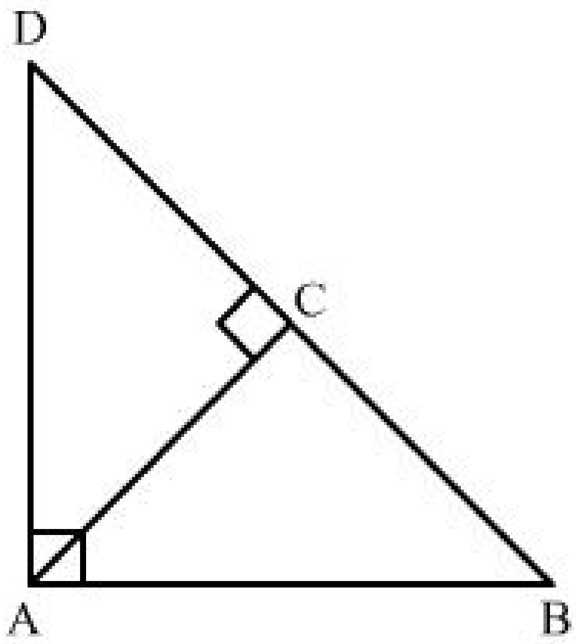
$3BD^2 = AD^2$

Therefore, $\boxed{AD^2 = 3BD^2}$.

Triangles Ex 4.7 Q25

Answer :

(i)



In ΔABD and ΔABC,

$\angle ACB = \angle A = 90^\circ$

$\angle B = \angle B$ (Common angle)

So, by AA criterion $\Delta ABD \sim \Delta CBA$

$$\therefore \frac{AB}{BC} = \frac{BD}{AB} = \frac{AD}{AC}$$

$$\therefore \frac{AB}{BC} = \frac{BD}{AB}$$

$$\therefore AB^2 = BD.BC \qquad .....(1)$$

(ii) In ΔABD and ΔACD,

$\angle C = \angle A = 90°$

$\angle D = \angle D$ (Common angle)

So, by AA criterion $\Delta ABD \sim \Delta CAD$

$\therefore \frac{AB}{AC} = \frac{BD}{AD} = \frac{AD}{CD}$

$\therefore \frac{BD}{AD} = \frac{AD}{CD}$

$\therefore AD^2 = BD.CD$ .....(2)

(iii) We have shown that ΔABD is similar to ΔCBA and ΔABD is similar to ΔCAD therefore, by the property of transitivity ΔCBA is similar to ΔCAD.

$\therefore \frac{BC}{AC} = \frac{AB}{AD} = \frac{AC}{CD}$

$\therefore \frac{BC}{AC} = \frac{AC}{CD}$

$\therefore AC^2 = BC.CD$ .....(3)

(iv) Now to obtained $AB^2/AC^2 = BD/DC$, we will divide equation (1) by equation (2) as shown below,

$\therefore \frac{AB^2}{AC^2} = \frac{BD.BC}{BC.CD}$

Canceling BC we get,

$\frac{AB^2}{AC^2} = \frac{BD}{CD}$

Therefore, $\boxed{\frac{AB^2}{AC^2} = \frac{BD}{CD}}$

*********** END **********