Indefinite Integrals Ex 19.5 Q1

Let $I = \int \frac{x+1}{\sqrt{2x+3}} dx$

Let $x + 1 = \lambda(2x+3) + \mu$

On equating the coefficients of like powers of x on bothy sides, we get

$$I = 2\lambda, \ 3\lambda + \mu = 1$$

$$\Rightarrow \quad \lambda = \frac{1}{2} \text{ and } 3 \times \frac{1}{2} + \mu = 1$$

$$\Rightarrow \quad \lambda = \frac{1}{2} \text{ and } \mu = \frac{-1}{2}$$

Replacing $x + 1$ by $\lambda(2x+3) + \mu$ in the given integral, we get

$$I = \int \frac{\lambda(2x+3) + \mu}{\sqrt{2x+3}} dx$$

$$= \int \frac{\lambda(2x+3)}{\sqrt{2x+3}} dx + \mu \int \frac{1}{\sqrt{2x+3}} dx$$

$$= \lambda \int (2x+3)^{\frac{1}{2}} dx + \mu \int (2x+3)^{\frac{-1}{2}} dx$$

$$= \lambda \frac{(2x+3)^{\frac{3}{2}}}{\frac{3}{2} \times 2} + \mu \frac{(2x+3)^{\frac{1}{2}}}{2 \times \frac{1}{2}} + c$$

$$= \frac{1}{2} \times \frac{(2x+3)^{\frac{3}{2}}}{3} + \left(\frac{-1}{2}\right) \times (2x+3)^{\frac{1}{2}} + c \qquad \left[\because \lambda = \frac{1}{2}, \mu = \frac{-1}{2}\right]$$

$$= \frac{(2x+3)^{\frac{3}{2}}}{6} - \frac{(2x+3)^{\frac{1}{2}}}{2} + c$$

$$\therefore \quad I = \frac{1}{6} \times (2x+3)^{\frac{3}{2}} - \frac{1}{2}(2x+3)^{\frac{1}{2}} + c.$$

Indefinite Integrals Ex 19.5 Q2

Let $I = \int x\sqrt{x+2} dx$. Then,

$$I = \int \{(x+2) - 2\} x + 2dx \qquad [\because x = (x+2) - 2]$$

$$\Rightarrow \quad I = f\left\{(x+2)^{\frac{3}{2}} - 2(x+2)^{\frac{1}{2}}\right\} dx$$

$$\Rightarrow \quad I = \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + c$$

Indefinite Integrals Ex 19.5 Q3

Let $I = \int \frac{x-1}{\sqrt{x+4}}dx$. Then,

$$\begin{aligned} I &= \int \frac{x+4-4-1}{\sqrt{x+4}} \\ &= \int \frac{x+4-5}{\sqrt{x+4}}dx \\ &= \int \frac{x+4}{\sqrt{x+4}}dx - 5\int \frac{1}{\sqrt{x+4}}dx \\ &= \int (x+4)^{\frac{1}{2}}dx - 5\int (x+4)^{\frac{-1}{2}}dx \\ &= \frac{(x+4)^{\frac{3}{2}}}{\frac{3}{2}} - 5\frac{(x+4)^{\frac{1}{2}}}{\frac{1}{2}} + c \\ &= \frac{2}{3}\times(x+4)^{\frac{3}{2}} - 10(x+4)^{\frac{1}{2}} + c \end{aligned}$$

$$\therefore \quad I = \frac{2}{3}\times(x+4)^{\frac{3}{2}} - 10(x+4)^{\frac{1}{2}} + c$$

Indefinite Integrals Ex 19.5 Q4

Let $I = \int (x+2)\sqrt{3x+5}dx$

Let $x+2 = \lambda(3x+5)+\mu$ on equating the coefficients of like powers of x on both sides, we get

$$3\lambda = 1 \quad \text{and} \quad 5\lambda + \mu = 2$$

$$\Rightarrow \quad \lambda = \frac{1}{3} \quad \text{and} \quad 5\times\frac{1}{3} + \mu = 2$$

$$\Rightarrow \quad \lambda = \frac{1}{3} \quad \text{and} \quad \mu = \frac{1}{3}$$

$$\begin{aligned} \therefore \quad I &= \int \{\lambda(3x+5)+\mu\}\sqrt{3x+5}dx \\ &= \lambda\int (3x+5)\sqrt{3x+5}dx + \mu\int \sqrt{3x+5} \\ &= \lambda\int (3x+5)^{\frac{3}{2}}dx + \mu\int (3x+5)^{\frac{1}{2}}dx \\ &= \lambda\times\frac{(3x+5)^{\frac{5}{2}}}{\frac{5}{2}\times 3} + \mu\frac{(3x+5)^{\frac{3}{2}}}{3\times\frac{3}{2}} + c \\ &= \frac{1}{3}\times\frac{2}{15}\times(3x+5)^{\frac{5}{2}} + \frac{1}{3}\times\frac{2}{9}(3x+5)^{\frac{3}{2}} + c \\ &= \frac{2}{45}\times(3x+5)^{\frac{5}{2}} + \frac{2}{27}\times(3x+5)^{\frac{3}{2}} + c \\ &= \frac{2}{9}\times(3x+5)^{\frac{3}{2}}\left[\frac{1}{5}\times(3x+5)^{1} + \frac{1}{3}\right] + c \\ &= \frac{2}{9}\times(3x+5)^{\frac{3}{2}}\left[\frac{3(3x+5)+5}{15}\right] + c \\ &= \frac{2}{9}\times(3x+5)^{\frac{3}{2}}\frac{(9x+15+5)}{15} + c \\ &= \frac{2}{135}\times(3x+5)^{\frac{3}{2}}(9x+20) + c \end{aligned}$$

$$\therefore \quad I = \frac{2}{135}\times(9x+20)(3x+5)^{\frac{3}{2}} + c.$$

Indefinite Integrals Ex 19.5 Q5

Let $I = \int \frac{2x+1}{\sqrt{3x+2}}dx$

Let $2x+1 = \lambda(3x+2)+\mu$ on equating the coefficients of like powers of x on both sides, we get

$$3\lambda = 2 \quad \text{and} \quad 2\lambda + \mu = 1$$

$$\Rightarrow \quad \lambda = \frac{2}{3} \quad \text{and} \quad 2 \times \frac{2}{3} + \mu = 1$$

$$\Rightarrow \quad \lambda = \frac{2}{3} \quad \text{and} \quad \mu = \frac{-1}{3}$$

$$\therefore \quad I = \int \frac{\lambda(3x+2)+\mu}{\sqrt{3x+2}}dx$$

$$= \lambda \int \frac{3x+2}{\sqrt{3x+2}}dx + \mu \int \frac{1}{\sqrt{3x+2}}dx$$

$$= \lambda \int (3x+2)^{\frac{1}{2}}dx + \mu \int (3x+2)^{\frac{-1}{2}}dx$$

$$= \lambda \times \frac{(3x+2)^{\frac{3}{2}}}{\frac{3}{2}\times 3} + \mu \frac{(3x+2)^{\frac{1}{2}}}{\frac{1}{2}\times 3} + c$$

$$= \frac{2}{3} \times \frac{2}{9} \times (3x+2)^{\frac{3}{2}} - \frac{1}{3} \times \frac{2}{3}(3x+2)^{\frac{1}{2}} + c$$

$$= \frac{4}{27} \times (3x+2)^{\frac{3}{2}} - \frac{2}{9} \times (3x+2)^{\frac{1}{2}} + c$$

$$= \frac{2}{9} \times \sqrt{3x+2}\left[\frac{2}{3} \times (3x+2) - 1\right] + c$$

$$= \frac{2}{9} \times \sqrt{3x+2}\left[\frac{6x+4-3}{3}\right] + c$$

$$= \frac{2}{27} \times \sqrt{3x+2}\,(6x+1) + c$$

$$\therefore \quad I = \frac{2}{27} \times (6x+1)\sqrt{3x+2} + c.$$

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