Indefinite Integrals Ex 19.23 Q5

Let $\quad I = \int \frac{1}{1 - \sin x + \cos x} dx$

Put $\quad \sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$I = \int \frac{1}{1 - \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} + \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}} dx$$

$$= \int \frac{1 + \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2} - 2 \tan \frac{x}{2} + 1 - \tan^2 \frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{2 - 2 \tan \frac{x}{2}} dx$$

Let $\quad \tan \frac{x}{2} = t$

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

$$= \frac{2}{2} \int \frac{dt}{1 - t}$$

$$= -\log |1 - t| + c$$

$$I = -\log \left| 1 - \tan \frac{x}{2} \right| + c$$

Indefinite Integrals Ex 19.23 Q6

Let $\quad I = \int \frac{1}{3+2\sin x+\cos x}dx$

Put $\quad \sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}, \qquad \cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$I = \int \frac{1}{3+2\left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)+\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx$$

$$= \int \frac{\left(1+\tan^2\frac{x}{2}\right)}{3\left(1+\tan^2\frac{x}{2}\right)+4\tan\frac{x}{2}+1-\tan^2\frac{x}{2}}dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{3+3\tan^2\frac{x}{2}+4\tan\frac{x}{2}+1-\tan^2\frac{x}{2}}dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{2\tan^2\frac{x}{2}+4\tan\frac{x}{2}+4}dx$$

Let $\quad \tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2}dx = dt$$

$$I = \frac{2}{2}\int\frac{dt}{t^2+2t+2}$$

$$= \int\frac{dt}{t^2+2t+1-1+2}$$

$$I = \int\frac{dt}{(t+1)^2+(1)^2}$$

$$= \tan^{-1}(t+1)+c$$

$$I = \tan^{-1}\left(\tan\frac{x}{2}+1\right)+c$$

Indefinite Integrals Ex 19.23 Q7

Let $\quad I = \int \frac{1}{13 + 3\cos x + 4\sin x}\,dx$

Put $\quad \sin x = \dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}, \qquad \cos x = \dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$I = \int \frac{1}{13 + 3\left(\dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right) + 4\left(\dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx$$

$$= \int \frac{\left(1+\tan^2\frac{x}{2}\right)}{13\left(1+\tan^2\frac{x}{2}\right) + 3 - 3\tan^2\frac{x}{2} + 8\tan\frac{x}{2}}dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{16 + 13\tan^2\frac{x}{2} - 3\tan^2\frac{x}{2} + 8\tan\frac{x}{2}}dx$$

Let $\quad \tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2}\,dx = dt$$

$$I = \int \frac{2dt}{16 + 10t^2 + 8t}$$

$$= \frac{2}{10}\int \frac{dt}{t^2 + \frac{4}{5}t + \frac{8}{5}}$$

$$I = \frac{1}{5}\int \frac{dt}{t^2 + 2t\left(\frac{2}{5}\right)^2 + \left(\frac{2}{5}\right)^2 - \left(\frac{2}{5}\right)^2 + \frac{8}{5}}$$

$$= \frac{1}{5}\int \frac{dt}{\left(t+\frac{2}{5}\right)^2 + \left(\frac{6}{5}\right)^2}$$

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$$= \frac{1}{5} \times \frac{1}{\left(\frac{6}{5}\right)}\tan^{-1}\left(\frac{t+\frac{2}{5}}{\frac{6}{5}}\right) + c$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{5t+2}{6}\right) + c$$

$$I = \frac{1}{6}\tan^{-1}\left(\frac{5\tan\frac{x}{2} + 2}{6}\right) + c$$

Indefinite Integrals Ex 19.23 Q8

Let $I = \int \frac{1}{\cos x - \sin x} dx$

Put $\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$I = \int \frac{1}{\left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right) - \left(\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right)} dx$$

$$= \int \frac{\left(1 + \tan^2 \frac{x}{2}\right)}{1 - \tan^2 \frac{x}{2} - 2 \tan \frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{1 - \tan^2 \frac{x}{2} - 2 \tan \frac{x}{2}} dx$$

Let $\tan \frac{x}{2} = t$

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

$$I = \int \frac{2dt}{1 - t^2 - 2t}$$

$$= -\int \frac{2dt}{t^2 + 2t - 1}$$

$$I = -\int \frac{2dt}{t^2 + 2t + 1 - 1 - 1}$$

$$I = -\int \frac{2dt}{(t+1)^2 - \left(\sqrt{2}\right)^2}$$

$$= \int \frac{2dt}{\left(\sqrt{2}\right)^2 - (t+1)^2}$$

$$= \frac{2}{2\sqrt{2}} \log \left| \frac{\sqrt{2} + t + 1}{\sqrt{2} - t - 1} \right| + c$$

$$I = \frac{1}{\sqrt{2}} \log \left| \frac{\sqrt{2} + \tan \frac{x}{2} + 1}{\sqrt{2} - \tan \frac{x}{2} - 1} \right| + c$$

*********** END **********