Adjoint and Inverse of Matrix Ex 7.1 Q28

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \text{ and } |A| = 3 + 6 - 8 = 1$$

$$\begin{array}{lll} C_{11} = 1 & C_{21} = -1 & C_{31} = 0 \\ C_{12} = -2 & C_{22} = 3 & C_{32} = -4 \\ C_{13} = -2 & C_{23} = +3 & C_{33} = -3 \end{array}$$

$$A^{-1} = \frac{1}{|A|}.adjA = \frac{1}{1}\begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} \qquad ---(1)$$

Now

$$A^2 = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A^3 = A^2.A = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} \qquad ---(2)$$

From (1) and (2)

$A^{-1} = A^3$

Hence proved.

Adjoint and Inverse of Matrix Ex 7.1 Q29

$$A=\begin{bmatrix}-1 & 2 & 0\\ -1 & 1 & 1\\ 0 & 1 & 0\end{bmatrix}$$

Expanding using 1st row, we get

$$|A| = -1\begin{vmatrix}1 & 1\\ 1 & 0\end{vmatrix} - 2\begin{vmatrix}-1 & 1\\ 0 & 0\end{vmatrix} + 0$$
$$= -1(0-1) - 2(0) + 0$$
$$= 1 - 0 + 0$$
$$|A| = 1$$

$$A^2 = AA = \begin{bmatrix}-1 & 2 & 0\\ -1 & 1 & 1\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}-1 & 2 & 0\\ -1 & 1 & 1\\ 0 & 1 & 0\end{bmatrix} = \begin{bmatrix}-1 & 0 & 2\\ 0 & 0 & 1\\ -1 & 1 & 2\end{bmatrix}$$

Cofactors of A are:

$C_{11} = -1$ $\quad$ $C_{21} = 0$ $\quad$ $C_{31} = 2$

$C_{12} = 0$ $\quad$ $C_{22} = 0$ $\quad$ $C_{32} = 1$

$C_{13} = -1$ $\quad$ $C_{23} = +1$ $\quad$ $C_{33} = 1$

$$\therefore \quad \text{adj}A = \begin{bmatrix}C_{11} & C_{12} & C_{13}\\ C_{21} & C_{22} & C_{23}\\ C_{31} & C_{32} & C_{33}\end{bmatrix}^T = \begin{bmatrix}-1 & 0 & -1\\ 0 & 0 & 1\\ 2 & 1 & 1\end{bmatrix}^T = \begin{bmatrix}-1 & 0 & 2\\ 0 & 0 & 1\\ -1 & 1 & 1\end{bmatrix}$$

Now, $\quad A^{-1} = \dfrac{1}{|A|}.\text{adj}A$

$$A^{-1} = \frac{1}{1}\begin{bmatrix}-1 & 0 & 2\\ 0 & 0 & 1\\ -1 & 1 & 1\end{bmatrix} = \begin{bmatrix}-1 & 0 & 2\\ 0 & 0 & 1\\ -1 & 1 & 1\end{bmatrix} = A^2$$

Hence, $A^2 = A^{-1}$

Adjoint and Inverse of Matrix Ex 7.1 Q30

$$\begin{bmatrix}5 & 4\\ 1 & 1\end{bmatrix}X = \begin{bmatrix}1 & -2\\ 1 & 3\end{bmatrix}$$

Let $A = \begin{bmatrix}5 & 4\\ 1 & 1\end{bmatrix}$ and $B = \begin{bmatrix}1 & -2\\ 1 & 3\end{bmatrix}$

So, $\quad AX = B$

or $\quad X = A^{-1}.B \qquad ---(i)$

$$|A| = 1 \neq 0$$

Cofactors of A are:

$$C_{11} = 1 \qquad C_{12} = -1$$
$$C_{21} = -4 \qquad C_{22} = 5$$

$$\therefore \qquad \text{adj}\, A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$$
$$= \begin{bmatrix} 1 & -1 \\ -4 & 5 \end{bmatrix}^T$$
$$= \begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}$$

Now, $\quad A^{-1} = \dfrac{1}{|A|}.\text{adj}\, A$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}$$

So from (i)

$$X = \begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$$
$$X = \begin{bmatrix} -3 & -14 \\ 4 & 17 \end{bmatrix}$$

Ans.

Adjoint and Inverse of Matrix Ex 7.1 Q31

$$X\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$$

Let $B = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$

So, $\quad XB = C$

$$XBB^{-1} = CB^{-1}$$

$$XI = C.B^{-1}$$

$$X = C.B^{-1} \qquad ---(\text{i})$$

Now, $\quad |B| = -7 \neq 0$

Cofactors of B are:

$$C_{11} = -2 \qquad C_{12} = 1$$

$$C_{21} = -3 \qquad C_{22} = 5$$

$$\therefore \qquad \operatorname{adj} A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$$

$$= \begin{bmatrix} -2 & 1 \\ -3 & 5 \end{bmatrix}^T$$

$$= \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|}.\operatorname{adj}(B)$$

$$= \frac{1}{(-7)} \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix} \ \frac{-1}{7} \begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$$

Now from (i)

$$X = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix} \cdot \frac{1}{7} \cdot \begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$$

$$= \frac{7}{7} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 \\ 1 & -2 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q32

Let $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$

$B = \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix}$

$C = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$

Then the given equation becomes

$A \times B = C$

$\Rightarrow \quad X = A^{-1}CB^{-1}$

Now $|A| = 35 - 14 = 21$

$|B| = -1 + 2 = 1$

$$A^{-1} = \frac{adj(A)}{|A|} = \frac{1}{21}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$B^{-1} = \frac{adj(B)}{|B|} = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$$

$$\therefore \quad X = A^{-1}CB^{-1} = \frac{1}{21}\begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -16 & 3 \\ 24 & -5 \end{bmatrix}$$

*********** END **********