Mathematical Induction Ex 12.2 Q9

Let $P(n): \frac{1}{3.7}+\frac{1}{7.11}+\frac{1}{11.15}+\ldots+\frac{1}{(4n-1)(4n+3)}=\frac{n}{3(4n+3)}$

For $n=1$

$\frac{1}{3.7}=\frac{1}{3(7)}$

$\frac{1}{21}=\frac{1}{21}$

$\Rightarrow \quad P(n)$ is true for $n=1$

Let $P(n)$ is true for $n=k$, so

$$\frac{1}{3.7}+\frac{1}{7.11}+\frac{1}{11.15}+\ldots+\frac{1}{(4n-1)(4n+3)}=\frac{k}{3(4k+3)} \qquad ---(1)$$

We have to show that,

$$\frac{1}{3.7}+\frac{1}{7.11}+\frac{1}{11.15}+\ldots+\frac{1}{(4k-1)(4k+3)}+\frac{1}{(4k+3)(4k+7)}=\frac{(k+1)}{3\quad(4k+7)}$$

Now,

$$\left\{\frac{1}{3.7}+\frac{1}{7.11}+\frac{1}{11.15}+\ldots+\frac{1}{(4k-1)(4k+3)}\right\}+\frac{1}{(4k+3)(4k+7)}$$

Now,

$$\left\{\frac{1}{3.7}+\frac{1}{7.11}+\frac{1}{11.15}+\ldots+\frac{1}{(4k-1)(4k+3)}\right\}+\frac{1}{(4k+3)(4k+7)}$$

$$=\frac{k}{3(4k+3)}+\frac{1}{(4k+3)(4k+7)}$$

$$=\frac{1}{(4k+3)}\left[\frac{k}{3}+\frac{1}{4k+7}\right]$$

$$=\frac{1}{(4k+3)}\left[\frac{k(4k+7)+3}{3(4k+7)}\right]$$

$$=\frac{1}{(4k+3)}\left[\frac{4k^2+7k+3}{3(4k+7)}\right]$$

$$=\frac{1}{(4k+3)}\left[\frac{4k^2+4k+3k+3}{3(4k+7)}\right]$$

$$=\frac{1}{(4k+3)}\left[\frac{4k(k+1)+3(k+1)}{3(4k+7)}\right]$$

$$=\frac{1}{(4k+3)}\left[\frac{(4k+3)(k+1)}{3(4k+7)}\right]$$

$$=\frac{(k+1)}{3(4k+7)}$$

$\Rightarrow \quad P(n)$ is true for $n=k+1$

$\Rightarrow \quad P(n)$ is true for all $n \in N$ by *PMI*

Mathematical Induction Ex 12.2 Q10

Let $P(n): 1.2 + 2.2^2 + 3.2^3 + \ldots + n.2^n = (n-1)2^{n+1} + 2$

For $n = 1$

$1.2 = 0.2^0 + 2$

$2 = 2$

$\Rightarrow \quad P(n)$ is true for $n = 1$

Let $P(n)$ is true for $n = k$, so

$$1.2 + 2.2^2 + 3.2^3 + \ldots + k.2^k = (k-1)2^{k+1} + 2 \qquad ---(1)$$

We have to show that,

$$\left\{1.2 + 2.2^2 + 3.2^3 + \ldots + k.2^k\right\} + (k+1)2^{k+1} = k2^{k+2} + 2$$

Now,

$$\left\{1.2 + 2.2^2 + 3.2^3 + \ldots + k.2^k\right\} + (k+1)2^{k+1}$$

$= \left[(k-1)2^{k+1} + 2\right] + (k+1)2^{k+1}$ [Using equation (1)]

$= (k-1)2^{k+1} + 2 + (k+1)2^{k+1}$

$= 2^{k+1}(k - 1 + k + 1) + 2$

$= 2^{k+1}.2k + 2$

$= k2^{k+2} + 2$

$\Rightarrow \quad P(n)$ is true for $n = k + 1$

$\Rightarrow \quad P(n)$ is true for all $n \in N$ by PMI

Mathematical Induction Ex 12.2 Q11

Let $P(n): 2+5+8+11+\ldots+(3n-1)=\frac{1}{2}n(3n+1)$

For $n=1$

$P(1)\ 2=\frac{1}{2}.1.(4)$

$2=2$

$\Rightarrow \quad P(n)$ is true for $n=1$

Let $P(n)$ is true for $n=k$, so

$$2+5+8+11+\ldots+(3k-1)=\frac{1}{2}k(3k+1) \qquad ---(1)$$

We have to show that,

$$2+5+8+11+\ldots+(3k-1)+(3k+2)=\frac{1}{2}(k+1)(3k+4)$$

Now,

$\{2+5+8+11+\ldots+(3k-1)\}+(3k+2)$

$$=\frac{1}{2}k(3k+1)+(3k+2)$$
$$=\frac{3k^2+k+2(3k+2)}{2}$$
$$=\frac{3k^2+k+6k+4}{2}$$
$$=\frac{3k^2+7k+4}{2}$$
$$=\frac{3k^2+3k+4k+k}{2}$$
$$=\frac{3k(k+1)+4(k+1)}{2}$$

$$=\frac{(k+1)(3k+4)}{2}$$

$\Rightarrow \quad P(n)$ is true for $n=k+1$

$\Rightarrow \quad P(n)$ is true for all $n \in N$ by PMI

*********** END **********