



Binomial Theorem Ex 18.2 Q19

Now, Coefficient of $(r+1)$ th term in the expansion of $(1+x)^{n+1} = {}^{n+1}C_r + {}^{n+1}C_{r+1}$

and, Coefficient of r th term in $(1+x)^n$ + Coefficient of $(r+1)$ th term in $(1+x)^n$

$$\begin{aligned}
 &= {}^nC_{r-1} + {}^nC_{r+1-1} \\
 &= {}^nC_{r-1} + {}^nC_r \\
 &= \frac{n!}{(n-r+1)! (r-1)!} + \frac{n!}{(n-r)! r!} \\
 &= \frac{n!}{(n-r+1)! (r-1)!} + \frac{n!}{(n-r)! r!} \\
 &= \frac{n!}{(n-r+1)(n-r)! (r-1)!} + \frac{n!}{(n-r)! r (r-1)!} \\
 &= \frac{n!}{(n-r)! (r-1)!} \left[\frac{1}{n-r+1} + \frac{1}{r} \right] \\
 &= \frac{n!}{(n-r)! (r-1)!} \left[\frac{r+n-r+1}{(n-r+1)r} \right] \\
 &= \frac{n!}{(n-r)! (r-1)!} \left[\frac{n+1}{(n-r+1)r} \right] \\
 &= \frac{n! (n+1)}{(n-r)! (n-r+1) (r-1)! r} \\
 &= \frac{(n+1)!}{(n-r+1)! r!} \\
 &= \frac{(n+1)!}{(n+1-r)! r!} \\
 &= {}^{n+1}C_r
 \end{aligned}$$

$$\therefore {}^{n+1}C_r = {}^nC_{r-1} + {}^nC_r$$

The coefficient of $(r+1)$ th term in the expansion of $(1+x)^{n+1}$ is equal to the sum of the coefficients of r th and $(r+1)$ th terms in the expansion of $(1+x)^n$.

Binomial Theorem Ex 18.2 Q20

We have,

$$\left(x + \frac{1}{x}\right)^{2n}$$

Let $(r+1)^{\text{th}}$ term be independent of x .

$$\begin{aligned} \therefore T_{r+1} &= {}^{2n}C_r (x)^{2n-r} \left(\frac{1}{x}\right)^r \\ &= {}^{2n}C_r (x)^{2n-r-r} \\ &= {}^{2n}C_r x^{2n-2r} \end{aligned}$$

If it is independent of x , we must have,

$$2n - 2r = 0$$

$$\Rightarrow 2n = 2r$$

$$\Rightarrow r = n$$

\therefore Term independent of $x = T_{n+1}$

Now,

$$\begin{aligned} T_{n+1} &= {}^{2n}C_n (x-1)^{2n-n} \left(\frac{1}{x}\right)^n \\ &= {}^{2n}C_n \\ &= \frac{(2n)!}{(2n-n)!n!} \\ &= \frac{(2n)!}{n!n!} \\ &= \frac{(2n)(2n-1)(2n-2)\dots 5 \times 4 \times 3 \times 2 \times 1}{n!n!} \\ &= \frac{\{1 \times 3 \times 5 \times \dots (2n-1)\} \{2 \times 4 \times 6 \times \dots 2n\}}{n!n!} \\ &= \frac{\{1 \times 3 \times 5 \times \dots (2n-1)\} \times 2^n \{1 \times 2 \times 3 \times \dots n\}}{n!n!} \\ &= \frac{\{1 \times 3 \times 5 \times \dots (2n-1)\} \times 2^n \times n!}{n!n!} \\ &= 2^n \times \frac{\{1 \times 3 \times 5 \times \dots (2n-1)\}}{n!} \end{aligned}$$

\therefore The term independent to $x = \frac{\{1 \times 3 \times 5 \times \dots (2n-1)\}}{n!} \times 2^n$ Hence proved.

Binomial Theorem Ex 18.2 Q21

We have,

$$(1+x)^n$$

Now,

$$\text{Coefficient of 5th term} = {}^nC_{5-1} = {}^nC_4$$

$$\text{Coefficient of 5th term} = {}^nC_{6-1} = {}^nC_5$$

$$\text{and, Coefficient of 5th term} = {}^nC_{7-1} = {}^nC_6$$

It is given that these coefficients are in A.P.

$$\therefore 2{}^nC_5 = {}^nC_4 + {}^nC_6$$

$$\Rightarrow 2 \left[\frac{n!}{(n-5)!5!} \right] = \frac{n!}{(n-4)!4!} + \frac{n!}{(n-6)!6!}$$

$$\Rightarrow \frac{2}{(n-5)!5!} = \frac{1}{(n-4)!4!} + \frac{1}{(n-6)!6!}$$

$$\Rightarrow \frac{2}{(n-5)(n-6)!5 \times 4!} = \frac{1}{(n-4)(n-5)(n-6)!4!} + \frac{1}{(n-6)!6 \times 5 \times 4!}$$

$$\Rightarrow \frac{2}{(n-5) \times 5} = \frac{1}{(n-4)(n-5)} + \frac{1}{6 \times 5}$$

$$\Rightarrow \frac{2}{5(n-5)} - \frac{1}{30} = \frac{1}{(n-4)(n-5)}$$

$$\Rightarrow \frac{12 - (n-5)}{30(n-5)} = \frac{1}{(n-4)(n-5)}$$

$$\Rightarrow \frac{12 - n + 5}{30} = \frac{1}{(n-4)(n-5)}$$

$$\Rightarrow \frac{17 - n}{30} = \frac{1}{n-4}$$

$$\Rightarrow 17n - 68 - n^2 + 4n = 30$$

$$\Rightarrow 21n - 68 - n^2 - 30 = 0$$

$$\Rightarrow 21n - n^2 - 98 = 0$$

$$\Rightarrow n^2 - 21n + 98 = 0$$

$$\Rightarrow n^2 - 7n - 14n + 98 = 0$$

$$\Rightarrow n(n-7) - 17(n-7) = 0$$

$$\Rightarrow (n-7)(n-14) = 0$$

$$\Rightarrow n = 7 \text{ or, } n = 14$$

Binomial Theorem Ex 18.2 Q22

We have,

$$(1+x)^{2n}$$

Now,

$$\text{Coefficient 2nd term} = {}^{2n}C_{2-1} = {}^{2n}C_1$$

$$\text{Coefficient 3rd term} = {}^{2n}C_{3-1} = {}^{2n}C_2$$

$$\text{and, Coefficient 4th term} = {}^{2n}C_{4-1} = {}^{2n}C_3$$

It is given that these coefficients are in A.P.

$$\therefore 2 {}^{2n}C_2 = {}^{2n}C_1 + {}^{2n}C_3$$

$$\Rightarrow 2 = \frac{{}^{2n}C_1}{{}^{2n}C_2} + \frac{{}^{2n}C_3}{{}^{2n}C_2}$$

$$\Rightarrow 2 = \frac{2}{2n-2+1} + \frac{2n-3+1}{3}$$

$$\left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r} \right]$$

$$\Rightarrow 2 = \frac{2}{2n-1} + \frac{2n-2}{3}$$

$$\Rightarrow 2 = \frac{6 + (2n-1)(2n-2)}{3(2n-1)}$$

$$\Rightarrow 6(2n-1) = 6 + 4n^2 - 4n - 2n + 2$$

$$\Rightarrow 12n - 6 = 8 + 4n^2 - 6n$$

$$\Rightarrow 4n^2 - 6n - 12n + 8 + 6 = 0$$

$$\Rightarrow 4n^2 - 18n + 14 = 0$$

$$\Rightarrow 2(2n^2 - 9n + 7) = 0$$

$$\Rightarrow 2n^2 - 9n + 7 = 0 \quad \text{Hence proved.}$$

***** END *****