Definite Integrals Ex 20.5 Q13

We have,

$$\int_a^b f(x)\,dx = \lim_{h \to 0} h\left[f(a) + f(a+h) + f(a+2h) + \cdots - f(a + (n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 1$, $b = 4$ and $f(x) = x^2 - x$

$$h = \frac{3}{n} \implies nh = 3$$

Thus, we have,

$$I = \int_1^4 \left(x^2 - x\right)\,dx$$

$$= \lim_{h \to 0} h\left[f(1) + f(1+h) + f(1+2h) + \cdots - f(1 + (n-1)h)\right]$$

$$= \lim_{h \to 0} h\left[\left(1^2 - 1\right) + \left\{(1+h)^2 - (1+h)\right\} + \left\{(1+2h)^2 - (1+h)\right\} + \cdots-\right]$$

$$= \lim_{h \to 0} h\left[0 + \left(h + h^2\right) + \left\{2h + (2h)^2\right\} + \cdots-\right]$$

$$= \lim_{h \to 0} h\left[h + \left(1 + 2 + 3 + \cdots (n-1)\right) + h^2\left\{\left(1 + 2^2 + 3^2 + \cdots (n-1)^2\right)\right\}\right]$$

$$\because h = \frac{3}{n} \text{ \& if } h \to 0 \implies n \to \infty$$

$$= \lim_{n \to \infty} \frac{3}{n}\left[\frac{3}{n}\frac{n(n-1)}{2} + \frac{9}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n \to \infty} \frac{9}{n^2}n^2\left(1 - \frac{1}{n}\right) + \frac{3}{2n^3}n^3\left(1 - \frac{1}{n}\right)\left(2 - \frac{1}{n}\right)$$

$$= \frac{9}{2} + 3 = \frac{27}{2}$$

$$\therefore \int_1^4 \left(x^2 - x\right)\,dx = \frac{27}{2}$$

Definite Integrals Ex 20.5 Q14

We have,

$$\int_a^b f(x)\,dx = \lim_{h \to 0} h\left[f(a) + f(a+h) + f(a+2h) + - - - - f(a + (n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 0$, $b = 1$ and $f(x) = 3x^2 + 5x$

$$h = \frac{1}{n} \quad \Rightarrow nh = 1$$

Thus, we have,

$$I = \int_0^1 \left(3x^2 + 5x\right)dx$$

$$= \lim_{h \to 0} h\left[f(0) + f(0+h) + f(0+2h) + - - - - f(0 + (n-1)h)\right]$$

$$= \lim_{h \to 0} h\left[\{0 + (3h^2 + 5h)\} + \{3(2h)^2 + 5(2h)\} + - - - - -\right]$$

$$= \lim_{h \to 0} h\left[\{3h^2(1 + 2^2 + 3^2 + - - - (n-1)^2)\} + 5h\{1 + 2 + 3 + - - - (n-1)\}\right]$$

$$\because h = \frac{1}{n} \quad \text{if } h \to 0 \Rightarrow n \to \infty$$

$$= \lim_{n \to \infty} \frac{1}{n}\left[\frac{3}{n^2}\frac{n(n-1)(2n-1)}{6} + \frac{5}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n \to \infty} \frac{3}{n^3}\frac{n^3\left(1 - \frac{1}{n}\right)\left(2 - \frac{1}{n}\right)}{6} + \frac{5}{2n^2}n^2\left(1 - \frac{1}{n}\right)$$

$$= \frac{3 \times 2}{6} + \frac{5}{2} = \frac{7}{2}$$

$$\therefore \int_0^1 \left(3x^2 + 5x\right)dx = \frac{7}{2}$$

Definite Integrals Ex 20.5 Q15

We have

$$\int_a^b f(x) = dx \lim_{k \to 0} h\left[f(a) + f(a+h) + f(a+2h) + ... + f(a+(n-1)h)\right]$$

Where $h = \dfrac{b-a}{n}$

Here

$$a = 0, b = 2 \text{ and } f(x) = e^x$$

Now

$$h = \frac{2}{n}$$

$$nh = 2$$

Thus, we have

$$I = \int_0^2 e^x dx$$

$$= \lim_{k \to 0} h\left[f(0) + f(h) + f(2h) + ... + f((n-1)h)\right]$$

$$= \lim_{k \to 0} h\left[1 + e^h + e^{2h} + ... + e^{(n-1)k}\right]$$

$$= \lim_{k \to 0} h\left\{\frac{(e^h)^n - 1}{e^h - 1}\right\}$$

$$= \lim_{k \to 0} h\left\{\frac{e^{nh} - 1}{e^h - 1}\right\}$$

$$= \lim_{k \to 0} h\left\{\frac{e^2 - 1}{e^h - 1}\right\} \quad [nh = 2]$$

$$= \lim_{k \to 0} \left\{\frac{e^2 - 1}{\frac{e^h - 1}{h}}\right\}$$

$$= e^2 - 1$$

Definite Integrals Ex 20.5 Q16

We have,

$$\int_a^b f(x)\,dx = \lim_{h \to 0} h\left[f(a) + f(a+h) + f(a+2h) + ---- f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here, $a = a$, $b = b$ and $f(x) = e^x$

$\therefore \quad h = \dfrac{b-a}{n} \implies nh = b-a$

Thus, we have,

$$I = \lim_{h \to 0} h\left[f(a) + f(a+h) + f(a+2h) + ---- f(a+(n-1)h)\right]$$

$$= \lim_{h \to 0} h\left[e^a + e^{a+h} + e^{a+2h} + ---- e^{a+(n-1)h}\right]$$

$$= \lim_{h \to 0} he^a\left[1 + e^h + e^{2h} + e^{3h} + ---- e^{(n-1)h}\right]$$

$$= \lim_{h \to 0} he^a\left[1 + e^h + \left(e^h\right)^2 + \left(e^h\right)^3 + ---- \left(e^h\right)^{n-1}\right]$$

$$= \lim_{h \to 0} he^a\left\{\frac{\left(e^h\right)^n - 1}{e^h - 1}\right\} \qquad\qquad \left[\because a + ar + ar^2 + ---- ar^{n-1} = a\left\{\frac{r^n - 1}{r - 1}\right\} \text{ if } r > 1\right]$$

$$= \lim_{h \to 0} he^a n\left\{\frac{e^{nh} - 1}{nh}\right\} \times \left(\frac{h}{e^{h-1}}\right) \qquad\qquad \left[\because \lim_{\theta \to 0} \frac{e^\theta - 1}{\theta} = 1 \ \& \ nh = b-a\right]$$

$$\therefore \lim_{h \to 0}\left(e^{b-a} - 1\right) = e^b - e^a$$

$$\therefore \int_a^b e^x\,dx = e^b - e^a$$

********** END **********