$$= -(AB - BA)$$

$$\therefore (AB - BA)' = -(AB - BA)$$

Thus, (*AB* − *BA*) is a skew-symmetric matrix.

**Question 12:**

If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then $A + A' = I$, if the value of α is

**A.** $\frac{\pi}{6}$ **B.** $\frac{\pi}{3}$

**C.** π **D.** $\frac{3\pi}{2}$

Answer

The correct answer is B.

$$A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$\Rightarrow A' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

Now, $A + A' = I$

$$\therefore \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Comparing the corresponding elements of the two matrices, we have:

$$2\cos\alpha = 1$$

$$\Rightarrow \cos\alpha = \frac{1}{2} = \cos\frac{\pi}{3}$$

$$\therefore \alpha = \frac{\pi}{3}$$

Exercise 3.4

**Question 1:**

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

Answer

$$\text{Let } A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

We know that *A* = *IA*

$$\therefore \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A \qquad (R_2 \to R_2 - 2R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A \qquad \left(R_2 \to \frac{1}{5} R_2\right)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A \qquad (R_1 \to R_1 + R_2)$$

$$\therefore A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

$$\begin{bmatrix} 5 & 5 \end{bmatrix}$$

**Question 2:**

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

Answer

$$\text{Let } A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A \qquad (R_1 \to R_1 - R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A \qquad (R_2 \to R_2 - R_1)$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

**Question 3:**

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$$

Answer

$$\text{Let } A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A \qquad (R_2 \to R_2 - 2R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} A \qquad (R_1 \to R_1 - 3R_2)$$

$$\therefore A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

**Question 4:**

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$$

Answer

$$\text{Let } A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{3}{2} \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A \qquad \left(R_1 \to \frac{1}{2}R_1\right)$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{3}{2} \\ 0 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{5}{2} & 1 \end{bmatrix} A \qquad (R_2 \to R_2 - 5R_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ -\frac{5}{2} & 1 \end{bmatrix} A \qquad (R_1 \to R_1 + 3R_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix} A \qquad (R_2 \to -2R_1)$$

$$\therefore A^{-1} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$$

**Question 5:**

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$$

Answer

Let $A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{1}{2} \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A \qquad \left(R_1 \to \frac{1}{2} R_1\right)$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{7}{2} & 1 \end{bmatrix} A \qquad \left(R_2 \to R_2 - 7R_1\right)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -\frac{7}{2} & 1 \end{bmatrix} A \qquad \left(R_1 \to R_1 - R_2\right)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} A \qquad \left(R_2 \to 2R_2\right)$$

$$\therefore A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$$

**Question 6:**

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

Answer

Let $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{5}{2} \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A \qquad \left(R_1 \to \frac{1}{2} R_1\right)$$

$$\Rightarrow \begin{bmatrix} 1 & \frac{5}{2} \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & 1 \end{bmatrix} A \qquad \left(R_2 \to R_2 - R_1\right)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -\frac{1}{2} & 1 \end{bmatrix} A \qquad \left(R_1 \to R_2 - 5R_2\right)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A \qquad \left(R_2 \to 2R_2\right)$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

*********** END **********