Differentiation Ex 11.3 Q6

Let $y = \sin^{-1}\left\{\frac{x}{\sqrt{x^2+a^2}}\right\}$

Put $x = a\tan\theta$

$$y = \sin^{-1}\left\{\frac{a\tan\theta}{\sqrt{a^2+\tan^2\theta+a^2}}\right\}$$

$$= \sin^{-1}\left\{\frac{a\tan\theta}{\sqrt{a^2\left(\tan^2\theta+1\right)}}\right\}$$

$$= \sin^{-1}\left\{\frac{a\tan\theta}{a\sec\theta}\right\}$$

$$= \sin^{-1}\{\sin\theta\}$$

$$= \theta$$

$$y = \tan^{-1}\left(\frac{x}{a}\right) \qquad [x = a\tan\theta]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{1+\left(\frac{x}{a}\right)^2}\frac{d}{dx}\left(\frac{x}{a}\right)$$

$$= \frac{a^2}{a^2+x^2}\times\left(\frac{1}{a}\right)$$

$$\frac{dy}{dx} = \frac{a}{a^2+x^2}.$$

Differentiation Ex 11.3 Q7

Let $y = \sin^{-1}\left\{2x^2 - 1\right\}$

Let $x = \cos\theta$

$$y = \sin^{-1}\left(2\cos^2\theta - 1\right)$$

$$= \sin^{-1}(\cos 2\theta)$$

$$y = \sin^{-1}\left\{\sin\left(\frac{\pi}{2} - 2\theta\right)\right\} \qquad \text{---(i)}$$

Here, $0 < x < 1$

$\Rightarrow \quad 0 < \cos\theta < 1$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{2}$

$\Rightarrow \quad 0 < 2\theta < \pi$

$\Rightarrow \quad 0 > -2\theta > -\pi$

$\Rightarrow \quad \frac{\pi}{2} > \left(\frac{\pi}{2} - 2\theta\right) > -\frac{\pi}{2}$

So, from equatoin (i),

$$y = \frac{\pi}{2} - 2\theta \qquad \left[\text{Since, } \sin^{-1}(\cos\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$y = \frac{\pi}{2} - 2\cos^{-1}x \qquad [\text{Since } x = \cos\theta]$$

$$\frac{dy}{dx} = 0 - 2\frac{d}{dx}\left(\cos^{-1}x\right)$$

$$= -2\left(-\frac{1}{\sqrt{1-x^2}}\right)$$

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q8

Let $y = \sin^{-1}\left\{1 - 2x^2\right\}$

Let $x = \sin\theta$, So,

$$y = \sin^{-1}\left(1 - 2\sin^2\theta\right)$$

$$= \sin^{-1}(\cos 2\theta)$$

$$y = \sin^{-1}\left\{\sin\left(\frac{\pi}{2} - 2\theta\right)\right\} \qquad \text{---(i)}$$

Here, $0 < x < 1$

$\Rightarrow \quad 0 < \sin\theta < 1$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{2}$

$\Rightarrow \quad 0 < 2\theta < \pi$

$\Rightarrow \quad 0 > -2\theta > -\pi$

$\Rightarrow \quad \frac{\pi}{2} > \left(\frac{\pi}{2} - 2\theta\right) > \frac{\pi}{2} - \pi$

$\Rightarrow \quad \frac{\pi}{2} > \left(\frac{\pi}{2} - 2\theta\right) > \left(-\frac{\pi}{2}\right)$

So, from equatoin (i),

$$y = \frac{\pi}{2} - 2\theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$y = \frac{\pi}{2} - 2\sin^{-1}x \qquad [\text{Since } x = \sin\theta]$$

Differentiating with respect to x,

$$\frac{dy}{dx} = 0 - 2\left(\frac{1}{\sqrt{1-x^2}}\right)$$

$$\frac{dy}{dx} = -\frac{2}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q9

Let $\quad y = \cos^{-1}\left\{\frac{x}{\sqrt{x^2+a^2}}\right\}$

Put $\quad x = a\cot\theta,$

$$y = \cos^{-1}\left\{\frac{a\cot\theta}{\sqrt{a^2\cot^2\theta + a^2}}\right\}$$

$$= \cos^{-1}\left\{\frac{a\cot\theta}{a\,\mathrm{cosec}\,\theta}\right\}$$

$$= \cos^{-1}\left\{\frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}\right\}$$

$$= \cos^{-1}(\cos\theta)$$

$$= \theta$$

$$y = \cot^{-1}\left(\frac{x}{a}\right) \qquad [\text{Since, } a\cot\theta = x]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{-1}{1+\left(\frac{x}{a}\right)^2}\frac{d}{dx}\left(\frac{x}{a}\right)$$

$$= \frac{-a^2}{a^2+x^2}\times\left(\frac{1}{a}\right)$$

$$\frac{dy}{dx} = \frac{-a}{a^2+x^2}.$$

Differentiation Ex 11.3 Q10

Let $\quad y = \sin^{-1}\left\{\frac{\sin x + \cos x}{\sqrt{2}}\right\}$

$$= \sin^{-1}\left\{\sin x\left(\frac{1}{\sqrt{2}}\right) + \cos x \times\left(\frac{1}{\sqrt{2}}\right)\right\}$$

$$= \sin^{-1}\left\{\sin x\cos\frac{\pi}{4} + \cos x \times \sin\frac{\pi}{4}\right\}$$

$$y = \sin^{-1}\left\{\sin\left(x+\frac{\pi}{4}\right)\right\}$$

Here, $\quad \frac{-3\pi}{4} < x < \frac{\pi}{4}$

$\Rightarrow \quad \left(\frac{-3\pi}{4}+\frac{\pi}{4}\right) \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]\right]$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 1 + 0$$

$$\frac{dy}{dx} = 1$$

********** END **********