



Q21 :  $\frac{2 + \sin 2x}{1 + \cos 2x} e^x$

Answer :

$$\begin{aligned} I &= \int \left( \frac{2 + \sin 2x}{1 + \cos 2x} \right) e^x \\ &= \int \left( \frac{2 + 2 \sin x \cos x}{2 \cos^2 x} \right) e^x \\ &= \int \left( \frac{1 + \sin x \cos x}{\cos^2 x} \right) e^x \\ &= \int (\sec^2 x + \tan x) e^x \end{aligned}$$

Let  $f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$

$$\begin{aligned} \therefore I &= \int (f(x) + f'(x)) e^x dx \\ &= e^x f(x) + C \\ &= e^x \tan x + C \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q22 :  $\frac{x^2 + x + 1}{(x+1)^2 (x+2)}$

Answer :

$$\begin{aligned} \text{Let } \frac{x^2 + x + 1}{(x+1)^2 (x+2)} &= \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+2)} \quad \dots(1) \\ \Rightarrow x^2 + x + 1 &= A(x+1)(x+2) + B(x+2) + C(x^2 + 2x + 1) \\ \Rightarrow x^2 + x + 1 &= A(x^2 + 3x + 2) + B(x+2) + C(x^2 + 2x + 1) \\ \Rightarrow x^2 + x + 1 &= (A+C)x^2 + (3A+B+2C)x + (2A+2B+C) \end{aligned}$$

Equating the coefficients of  $x^2$ ,  $x$ , and constant term, we obtain

$$A + C = 1$$

$$3A + B + 2C = 1$$

$$2A + 2B + C = 1$$

On solving these equations, we obtain

$$A = -2, B = 1, \text{ and } C = 3$$

From equation (1), we obtain

$$\begin{aligned} \frac{x^2 + x + 1}{(x+1)^2 (x+2)} &= \frac{-2}{(x+1)} + \frac{3}{(x+2)} + \frac{1}{(x+1)^2} \\ \int \frac{x^2 + x + 1}{(x+1)^2 (x+2)} dx &= -2 \int \frac{1}{x+1} dx + 3 \int \frac{1}{(x+2)} dx + \int \frac{1}{(x+1)^2} dx \\ &= -2 \log|x+1| + 3 \log|x+2| - \frac{1}{(x+1)} + C \end{aligned}$$

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Q23 :  $\tan^{-1} \sqrt{\frac{1-x}{1+x}}$

Answer :

$$I = \tan^{-1} \sqrt{\frac{1-x}{1+x}} dx$$

Let  $x = \cos \theta \Rightarrow dx = -\sin \theta d\theta$

$$I = \int \tan^{-1} \sqrt{\frac{1-\cos \theta}{1+\cos \theta}} (-\sin \theta d\theta)$$

$$= -\int \tan^{-1} \sqrt{\frac{2 \sin^2 \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2}}} \sin \theta d\theta$$

$$= -\int \tan^{-1} \tan \frac{\theta}{2} \sin \theta d\theta$$

$$= -\frac{1}{2} \int \theta \cdot \sin \theta d\theta$$

$$\begin{aligned}
&= -\frac{1}{2} \left[ \theta \cdot (-\cos \theta) - \int 1 \cdot (-\cos \theta) d\theta \right] \\
&= -\frac{1}{2} \left[ -\theta \cos \theta + \sin \theta \right] \\
&= +\frac{1}{2} \theta \cos \theta - \frac{1}{2} \sin \theta \\
&= \frac{1}{2} \cos^{-1} x \cdot x - \frac{1}{2} \sqrt{1-x^2} + C \\
&= \frac{x}{2} \cos^{-1} x - \frac{1}{2} \sqrt{1-x^2} + C \\
&= \frac{1}{2} \left( x \cos^{-1} x - \sqrt{1-x^2} \right) + C
\end{aligned}$$

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Q24 :  $\frac{\sqrt{x^2+1} \left[ \log(x^2+1) - 2 \log x \right]}{x^4}$

Answer :

$$\begin{aligned}
\frac{\sqrt{x^2+1} \left[ \log(x^2+1) - 2 \log x \right]}{x^4} &= \frac{\sqrt{x^2+1}}{x^4} \left[ \log(x^2+1) - \log x^2 \right] \\
&= \frac{\sqrt{x^2+1}}{x^4} \left[ \log \left( \frac{x^2+1}{x^2} \right) \right] \\
&= \frac{\sqrt{x^2+1}}{x^4} \log \left( 1 + \frac{1}{x^2} \right) \\
&= \frac{1}{x^3} \sqrt{\frac{x^2+1}{x^2}} \log \left( 1 + \frac{1}{x^2} \right) \\
&= \frac{1}{x^3} \sqrt{1 + \frac{1}{x^2}} \log \left( 1 + \frac{1}{x^2} \right)
\end{aligned}$$

Let  $1 + \frac{1}{x^2} = t \Rightarrow \frac{-2}{x^3} dx = dt$

$$\begin{aligned}
\therefore I &= \int \frac{1}{x^3} \sqrt{1 + \frac{1}{x^2}} \log \left( 1 + \frac{1}{x^2} \right) dx \\
&= -\frac{1}{2} \int \sqrt{t} \log t \, dt \\
&= -\frac{1}{2} \int t^{\frac{1}{2}} \cdot \log t \, dt
\end{aligned}$$

Integrating by parts, we obtain

$$\begin{aligned}
I &= -\frac{1}{2} \left[ \log t \cdot \int t^{\frac{1}{2}} dt - \left\{ \left( \frac{d}{dt} \log t \right) \int t^{\frac{1}{2}} dt \right\} dt \right] \\
&= -\frac{1}{2} \left[ \log t \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \int t^{\frac{1}{2}} \cdot \frac{t^{\frac{3}{2}}}{3} dt \right] \\
&= -\frac{1}{2} \left[ \frac{2}{3} t^{\frac{3}{2}} \log t - \frac{2}{3} \int t^{\frac{1}{2}} dt \right] \\
&= -\frac{1}{2} \left[ \frac{2}{3} t^{\frac{3}{2}} \log t - \frac{4}{9} t^{\frac{3}{2}} \right] \\
&= -\frac{1}{3} t^{\frac{3}{2}} \log t + \frac{2}{9} t^{\frac{3}{2}} \\
&= -\frac{1}{3} t^{\frac{3}{2}} \left[ \log t - \frac{2}{3} \right] \\
&= -\frac{1}{3} \left( 1 + \frac{1}{x^2} \right)^{\frac{3}{2}} \left[ \log \left( 1 + \frac{1}{x^2} \right) - \frac{2}{3} \right] + C
\end{aligned}$$

Answer needs Correction? [Click Here](#)

Q25 :  $\int_{\frac{\pi}{2}}^{\pi} e^x \left( \frac{1 - \sin x}{1 - \cos x} \right) dx$

Answer :

$$\begin{aligned}
I &= \int_{\frac{\pi}{2}}^{\pi} e^x \left( \frac{1 - \sin x}{1 - \cos x} \right) dx \\
&= \int_{\frac{\pi}{2}}^{\pi} e^x \left( \frac{1 - 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \sin^2 \frac{x}{2}} \right) dx \\
&= \int_{\frac{\pi}{2}}^{\pi} e^x \left( \frac{\operatorname{cosec}^2 \frac{x}{2}}{2} - \cot \frac{x}{2} \right) dx
\end{aligned}$$

Let  $f(x) = -\cot \frac{x}{2}$

$$\Rightarrow f'(x) = -\left( -\frac{1}{2} \operatorname{cosec}^2 \frac{x}{2} \right) = \frac{1}{2} \operatorname{cosec}^2 \frac{x}{2}$$

$$\begin{aligned}
\therefore I &= \int_{\frac{\pi}{2}}^{\pi} e^x \left( f(x) + f'(x) \right) dx \\
&= \left[ e^x \cdot f(x) \right]_{\frac{\pi}{2}}^{\pi} \\
&\quad \left[ -\cot \frac{x}{2} \right]
\end{aligned}$$

$$\begin{aligned}
 &= - \left[ e^x \cdot \cot \frac{x}{2} \right]_{\frac{\pi}{2}}^{\pi} \\
 &= - \left[ e^{\pi} \times \cot \frac{\pi}{2} - e^{\frac{\pi}{2}} \times \cot \frac{\pi}{4} \right] \\
 &= - \left[ e^{\pi} \times 0 - e^{\frac{\pi}{2}} \times 1 \right] \\
 &= e^{\frac{\pi}{2}}
 \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q26:  $\int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x} dx$

Answer :

$$\begin{aligned}
 \text{Let } I &= \int_0^{\frac{\pi}{4}} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x} dx \\
 &= \int_0^{\frac{\pi}{4}} \frac{(\sin x \cos x)}{\frac{\cos^4 x}{(\cos^4 x + \sin^4 x)}} dx \\
 &= \int_0^{\frac{\pi}{4}} \frac{\tan x \sec^2 x}{1 + \tan^4 x} dx \\
 \text{Let } \tan^2 x &= t \Rightarrow 2 \tan x \sec^2 x dx = dt
 \end{aligned}$$

When  $x = 0$ ,  $t = 0$  and when  $x = \frac{\pi}{4}$ ,  $t = 1$

$$\begin{aligned}
 \therefore I &= \frac{1}{2} \int_0^1 \frac{dt}{1+t^2} \\
 &= \frac{1}{2} \left[ \tan^{-1} t \right]_0^1 \\
 &= \frac{1}{2} \left[ \tan^{-1} 1 - \tan^{-1} 0 \right] \\
 &= \frac{1}{2} \left[ \frac{\pi}{4} \right] \\
 &= \frac{\pi}{8}
 \end{aligned}$$

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Q27:  $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x dx}{\cos^2 x + 4 \sin^2 x}$

Answer :

$$\begin{aligned}
 \text{Let } I &= \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 x + 4 \sin^2 x} dx \\
 \Rightarrow I &= \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 x + 4(1 - \cos^2 x)} dx \\
 \Rightarrow I &= \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 x + 4 - 4 \cos^2 x} dx \\
 \Rightarrow I &= \frac{-1}{3} \int_0^{\frac{\pi}{2}} \frac{4 - 3 \cos^2 x - 4}{4 - 3 \cos^2 x} dx \\
 \Rightarrow I &= \frac{-1}{3} \int_0^{\frac{\pi}{2}} \frac{4 - 3 \cos^2 x}{4 - 3 \cos^2 x} dx + \frac{1}{3} \int_0^{\frac{\pi}{2}} \frac{4}{4 - 3 \cos^2 x} dx \\
 \Rightarrow I &= \frac{-1}{3} \int_0^{\frac{\pi}{2}} 1 dx + \frac{1}{3} \int_0^{\frac{\pi}{2}} \frac{4 \sec^2 x}{4 \sec^2 x - 3} dx \\
 \Rightarrow I &= \frac{-1}{3} \left[ x \right]_0^{\frac{\pi}{2}} + \frac{1}{3} \int_0^{\frac{\pi}{2}} \frac{4 \sec^2 x}{4(1 + \tan^2 x) - 3} dx \\
 \Rightarrow I &= -\frac{\pi}{6} + \frac{2}{3} \int_0^{\frac{\pi}{2}} \frac{2 \sec^2 x}{1 + 4 \tan^2 x} dx \quad \dots(1)
 \end{aligned}$$

Consider,  $\int_0^{\frac{\pi}{2}} \frac{2 \sec^2 x}{1 + 4 \tan^2 x} dx$

Let  $2 \tan x = t \Rightarrow 2 \sec^2 x dx = dt$

When  $x = 0$ ,  $t = 0$  and when  $x = \frac{\pi}{2}$ ,  $t = \infty$

$$\begin{aligned}
 \Rightarrow \int_0^{\frac{\pi}{2}} \frac{2 \sec^2 x}{1 + 4 \tan^2 x} dx &= \int_0^{\infty} \frac{dt}{1+t^2} \\
 &= \left[ \tan^{-1} t \right]_0^{\infty} \\
 &= \left[ \tan^{-1}(\infty) - \tan^{-1}(0) \right] \\
 &= \frac{\pi}{2}
 \end{aligned}$$

Therefore, from (1), we obtain

$$I = -\frac{\pi}{6} + \frac{2}{3} \left[ \frac{\pi}{2} \right] = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$$

Answer needs Correction? [Click Here](#)

Q28:  $\int_0^{\frac{\pi}{4}} \sin x + \cos x dx$

$$\int_6^{\frac{\pi}{6}} \frac{1}{\sqrt{\sin 2x}} dx$$

Answer :

$$\begin{aligned} \text{Let } I &= \int_6^{\frac{\pi}{6}} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx \\ \Rightarrow I &= \int_6^{\frac{\pi}{6}} \frac{(\sin x + \cos x)}{\sqrt{-(-\sin 2x)}} dx \\ \Rightarrow I &= \int_6^{\frac{\pi}{6}} \frac{\sin x + \cos x}{\sqrt{-(-1 + 1 - 2 \sin x \cos x)}} dx \\ \Rightarrow I &= \int_6^{\frac{\pi}{6}} \frac{(\sin x + \cos x)}{\sqrt{1 - (\sin^2 x + \cos^2 x - 2 \sin x \cos x)}} dx \\ \Rightarrow I &= \int_6^{\frac{\pi}{6}} \frac{(\sin x + \cos x) dx}{\sqrt{1 - (\sin x - \cos x)^2}} \end{aligned}$$

$$\text{Let } (\sin x - \cos x) = t \Rightarrow (\sin x + \cos x) dx = dt$$

$$\text{When } x = \frac{\pi}{6}, t = \left( \frac{1 - \sqrt{3}}{2} \right) \text{ and when } x = \frac{\pi}{3}, t = \left( \frac{\sqrt{3} - 1}{2} \right)$$

$$\begin{aligned} I &= \int_{\frac{1 - \sqrt{3}}{2}}^{\frac{\sqrt{3} - 1}{2}} \frac{dt}{\sqrt{1 - t^2}} \\ \Rightarrow I &= \int_{\left( \frac{1 - \sqrt{3}}{2} \right)}^{\left( \frac{\sqrt{3} - 1}{2} \right)} \frac{dt}{\sqrt{1 - t^2}} \end{aligned}$$

As  $\frac{1}{\sqrt{1 - (-t)^2}} = \frac{1}{\sqrt{1 - t^2}}$ , therefore,  $\frac{1}{\sqrt{1 - t^2}}$  is an even function.

It is known that if  $f(x)$  is an even function, then  $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$

$$\begin{aligned} \Rightarrow I &= 2 \int_0^{\frac{\sqrt{3} - 1}{2}} \frac{dt}{\sqrt{1 - t^2}} \\ &= \left[ 2 \sin^{-1} t \right]_0^{\frac{\sqrt{3} - 1}{2}} \\ &= 2 \sin^{-1} \left( \frac{\sqrt{3} - 1}{2} \right) \end{aligned}$$

Answer needs Correction? [Click Here](#)

$$\text{Q29 : } \int_0^1 \frac{dx}{\sqrt{1+x} - \sqrt{x}}$$

Answer :

$$\begin{aligned} \text{Let } I &= \int_0^1 \frac{dx}{\sqrt{1+x} - \sqrt{x}} \\ I &= \int_0^1 \frac{1}{(\sqrt{1+x} - \sqrt{x})} \times \frac{(\sqrt{1+x} + \sqrt{x})}{(\sqrt{1+x} + \sqrt{x})} dx \\ &= \int_0^1 \frac{\sqrt{1+x} + \sqrt{x}}{1+x-x} dx \\ &= \int_0^1 \sqrt{1+x} dx + \int_0^1 \sqrt{x} dx \\ &= \left[ \frac{2}{3} (1+x)^{\frac{3}{2}} \right]_0^1 + \left[ \frac{2}{3} (x)^{\frac{3}{2}} \right]_0^1 \\ &= \frac{2}{3} \left[ (2)^{\frac{3}{2}} - 1 \right] + \frac{2}{3} [1] \\ &= \frac{2}{3} (2)^{\frac{3}{2}} \\ &= \frac{2 \cdot 2\sqrt{2}}{3} \\ &= \frac{4\sqrt{2}}{3} \end{aligned}$$

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$$\text{Q30 : } \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$$

Answer :

$$\begin{aligned} \text{Let } I &= \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx \\ \text{Also, let } \sin x - \cos x &= t \Rightarrow (\cos x + \sin x) dx = dt \\ \text{When } x &= 0, t = -1 \text{ and when } x = \frac{\pi}{4}, t = 0 \\ \Rightarrow (\sin x - \cos x)^2 &= t^2 \\ \Rightarrow \sin^2 x + \cos^2 x - 2 \sin x \cos x &= t^2 \\ \Rightarrow 1 - \sin 2x &= t^2 \\ \Rightarrow \sin 2x &= 1 - t^2 \\ \therefore I &= \int_{-1}^0 \frac{dt}{9 - (1 - t^2)} \end{aligned}$$

$$\begin{aligned}
 &= \int_{-1}^0 \frac{dt}{9+16-16t^2} \\
 &= \int_{-1}^0 \frac{dt}{25-16t^2} = \int_{-1}^0 \frac{dt}{(5)^2 - (4t)^2} \\
 &= \frac{1}{4} \left[ \frac{1}{2(5)} \log \left| \frac{5+4t}{5-4t} \right| \right]_{-1}^0 \\
 &= \frac{1}{40} \left[ \log(1) - \log \left| \frac{1}{9} \right| \right] \\
 &= \frac{1}{40} \log 9
 \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q31 :  $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) dx$

Answer :

Let  $I = \int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) dx = \int_0^{\frac{\pi}{2}} 2 \sin x \cos x \tan^{-1}(\sin x) dx$

Also, let  $\sin x = t \Rightarrow \cos x dx = dt$

When  $x = 0, t = 0$  and when  $x = \frac{\pi}{2}, t = 1$

$\Rightarrow I = 2 \int_0^1 t \tan^{-1}(t) dt \quad \dots(1)$

Consider  $\int t \cdot \tan^{-1} t dt = \tan^{-1} t \cdot \int t dt - \int \left\{ \frac{d}{dt} (\tan^{-1} t) \int t dt \right\} dt$

$$\begin{aligned}
 &= \tan^{-1} t \cdot \frac{t^2}{2} - \int \frac{1}{1+t^2} \cdot \frac{t^2}{2} dt \\
 &= \frac{t^2 \tan^{-1} t}{2} - \frac{1}{2} \int \frac{t^2 + 1 - 1}{1+t^2} dt \\
 &= \frac{t^2 \tan^{-1} t}{2} - \frac{1}{2} \int 1 dt + \frac{1}{2} \int \frac{1}{1+t^2} dt \\
 &= \frac{t^2 \tan^{-1} t}{2} - \frac{1}{2} \cdot t + \frac{1}{2} \tan^{-1} t
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow \int_0^1 t \cdot \tan^{-1} t dt &= \left[ \frac{t^2 \cdot \tan^{-1} t}{2} - \frac{t}{2} + \frac{1}{2} \tan^{-1} t \right]_0^1 \\
 &= \frac{1}{2} \left[ \frac{\pi}{4} - 1 + \frac{\pi}{4} \right] \\
 &= \frac{1}{2} \left[ \frac{\pi}{2} - 1 \right] = \frac{\pi}{4} - \frac{1}{2}
 \end{aligned}$$

From equation (1), we obtain

$I = 2 \left[ \frac{\pi}{4} - \frac{1}{2} \right] = \frac{\pi}{2} - 1$

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Q32 :  $\int \frac{x \tan x}{\sec x + \tan x} dx$

Answer :

Let  $I = \int \frac{x \tan x}{\sec x + \tan x} dx \quad \dots(1)$

$I = \int_0^{\pi} \left\{ \frac{(\pi-x) \tan(\pi-x)}{\sec(\pi-x) + \tan(\pi-x)} \right\} dx \quad \left( \int_0^a f(x) dx = \int_0^a f(a-x) dx \right)$

$\Rightarrow I = \int_0^{\pi} \left\{ \frac{-(\pi-x) \tan x}{-(\sec x + \tan x)} \right\} dx$

$\Rightarrow I = \int_0^{\pi} \frac{(\pi-x) \tan x}{\sec x + \tan x} dx \quad \dots(2)$

Adding (1) and (2), we obtain

$$\begin{aligned}
 2I &= \int_0^{\pi} \frac{\pi \tan x}{\sec x + \tan x} dx \\
 &\Rightarrow 2I = \pi \int_0^{\pi} \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}} dx \\
 &\Rightarrow 2I = \pi \int_0^{\pi} \frac{\sin x + 1 - 1}{1 + \sin x} dx \\
 &\Rightarrow 2I = \pi \int_0^{\pi} 1 dx - \pi \int_0^{\pi} \frac{1}{1 + \sin x} dx \\
 &\Rightarrow 2I = \pi [x]_0^{\pi} - \pi \int_0^{\pi} \frac{1 - \sin x}{\cos^2 x} dx \\
 &\Rightarrow 2I = \pi^2 - \pi \int_0^{\pi} (\sec^2 x - \tan x \sec x) dx \\
 &\Rightarrow 2I = \pi^2 - \pi [\tan x - \sec x]_0^{\pi} \\
 &\Rightarrow 2I = \pi^2 - \pi [\tan \pi - \sec \pi - \tan 0 + \sec 0] \\
 &\Rightarrow 2I = \pi^2 - \pi [0 - (-1) - 0 + 1] \\
 &\Rightarrow 2I = \pi^2 - 2\pi \\
 &\Rightarrow 2I = \pi(\pi - 2)
 \end{aligned}$$

$$\Rightarrow I = \frac{\pi}{2}(\pi - 2)$$

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**Q33:**  $\int_1^4 [|x-1| + |x-2| + |x-3|] dx$

**Answer :**

$$\text{Let } I = \int_1^4 [|x-1| + |x-2| + |x-3|] dx$$

$$\Rightarrow I = \int_1^4 |x-1| dx + \int_1^4 |x-2| dx + \int_1^4 |x-3| dx$$

$$I = I_1 + I_2 + I_3 \quad \dots(1)$$

$$\text{where, } I_1 = \int_1^4 |x-1| dx, I_2 = \int_1^4 |x-2| dx, \text{ and } I_3 = \int_1^4 |x-3| dx$$

$$I_1 = \int_1^4 |x-1| dx$$

$$(x-1) \geq 0 \text{ for } 1 \leq x \leq 4$$

$$\therefore I_1 = \int_1^4 (x-1) dx$$

$$\Rightarrow I_1 = \left[ \frac{x^2}{2} - x \right]_1^4$$

$$\Rightarrow I_1 = \left[ 8 - 4 - \frac{1}{2} + 1 \right] = \frac{9}{2} \quad \dots(2)$$

$$I_2 = \int_1^4 |x-2| dx$$

$$x-2 \geq 0 \text{ for } 2 \leq x \leq 4 \text{ and } x-2 \leq 0 \text{ for } 1 \leq x \leq 2$$

$$\therefore I_2 = \int_1^2 (2-x) dx + \int_2^4 (x-2) dx$$

$$\Rightarrow I_2 = \left[ 2x - \frac{x^2}{2} \right]_1^2 + \left[ \frac{x^2}{2} - 2x \right]_2^4$$

$$\Rightarrow I_2 = \left[ 4 - 2 - 2 + \frac{1}{2} \right] + \left[ 8 - 8 - 2 + 4 \right]$$

$$\Rightarrow I_2 = \frac{1}{2} + 2 = \frac{5}{2} \quad \dots(3)$$

$$I_3 = \int_1^4 |x-3| dx$$

$$x-3 \geq 0 \text{ for } 3 \leq x \leq 4 \text{ and } x-3 \leq 0 \text{ for } 1 \leq x \leq 3$$

$$\therefore I_3 = \int_1^3 (3-x) dx + \int_3^4 (x-3) dx$$

$$\Rightarrow I_3 = \left[ 3x - \frac{x^2}{2} \right]_1^3 + \left[ \frac{x^2}{2} - 3x \right]_3^4$$

$$\Rightarrow I_3 = \left[ 9 - \frac{9}{2} - 3 + \frac{1}{2} \right] + \left[ 8 - 12 - \frac{9}{2} + 9 \right]$$

$$\Rightarrow I_3 = \left[ 6 - 4 \right] + \left[ \frac{1}{2} \right] = \frac{5}{2} \quad \dots(4)$$

From equations (1), (2), (3), and (4), we obtain

$$I = \frac{9}{2} + \frac{5}{2} + \frac{5}{2} = \frac{19}{2}$$

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**Q34:**  $\int \frac{dx}{x^2(x+1)} = \frac{2}{3} + \log \frac{2}{3}$

**Answer :**

$$\text{Let } I = \int \frac{dx}{x^2(x+1)}$$

$$\text{Also, let } \frac{1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$

$$\Rightarrow 1 = Ax(x+1) + B(x+1) + C(x^2)$$

$$\Rightarrow 1 = Ax^2 + Ax + Bx + B + Cx^2$$

Equating the coefficients of  $x^2$ ,  $x$ , and constant term, we obtain

$$A + C = 0$$

$$A + B = 0$$

$$B = 1$$

On solving these equations, we obtain

$$A = -1, C = 1, \text{ and } B = 1$$

$$\therefore \frac{1}{x^2(x+1)} = -\frac{1}{x} + \frac{1}{x^2} + \frac{1}{(x+1)}$$

$$\Rightarrow I = \int \left\{ -\frac{1}{x} + \frac{1}{x^2} + \frac{1}{(x+1)} \right\} dx$$

$$= \left[ -\log x - \frac{1}{x} + \log(x+1) \right]_1^3$$

$$= \left[ \log \left( \frac{x+1}{x} \right) - \frac{1}{x} \right]_1^3$$

$$= \log \left( \frac{4}{2} \right) - \frac{1}{2} - \log \left( \frac{2}{1} \right) + 1$$

$$\begin{aligned}
 &= \log 4 - \log 3 - \log 2 + \frac{2}{3} \\
 &= \log 2 - \log 3 + \frac{2}{3} \\
 &= \log\left(\frac{2}{3}\right) + \frac{2}{3}
 \end{aligned}$$

Hence, the given result is proved.

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Q35 :  $\int_0^1 x e^x dx = 1$

Answer :

Let  $I = \int_0^1 x e^x dx$

Integrating by parts, we obtain

$$\begin{aligned}
 I &= x \int_0^1 e^x dx - \int_0^1 \left\{ \left( \frac{d}{dx}(x) \right) \int e^x dx \right\} dx \\
 &= [x e^x]_0^1 - \int_0^1 e^x dx \\
 &= [x e^x]_0^1 - [e^x]_0^1 \\
 &= e - e + 1 \\
 &= 1
 \end{aligned}$$

Hence, the given result is proved.

Answer needs Correction? [Click Here](#)

Q36 :  $\int_{-1}^1 x^{17} \cos^4 x dx = 0$

Answer :

Let  $I = \int_{-1}^1 x^{17} \cos^4 x dx$

Also, let  $f(x) = x^{17} \cos^4 x$

$$\Rightarrow f(-x) = (-x)^{17} \cos^4(-x) = -x^{17} \cos^4 x = -f(x)$$

Therefore,  $f(x)$  is an odd function.

It is known that if  $f(x)$  is an odd function, then  $\int_{-a}^a f(x) dx = 0$

$$\therefore I = \int_{-1}^1 x^{17} \cos^4 x dx = 0$$

Hence, the given result is proved.

Answer needs Correction? [Click Here](#)

Q37 :  $\int_0^{\frac{\pi}{2}} \sin^3 x dx = \frac{2}{3}$

Answer :

Let  $I = \int_0^{\frac{\pi}{2}} \sin^3 x dx$

$$I = \int_0^{\frac{\pi}{2}} \sin^2 x \cdot \sin x dx$$

$$= \int_0^{\frac{\pi}{2}} (1 - \cos^2 x) \sin x dx$$

$$= \int_0^{\frac{\pi}{2}} \sin x dx - \int_0^{\frac{\pi}{2}} \cos^2 x \cdot \sin x dx$$

$$= [-\cos x]_0^{\frac{\pi}{2}} + \left[ \frac{\cos^3 x}{3} \right]_0^{\frac{\pi}{2}}$$

$$= 1 + \frac{1}{3}[-1] = 1 - \frac{1}{3} = \frac{2}{3}$$

Hence, the given result is proved.

Answer needs Correction? [Click Here](#)

Q38 :  $\int_0^{\frac{\pi}{4}} 2 \tan^3 x dx = 1 - \log 2$

Answer :

Let  $I = \int_0^{\frac{\pi}{4}} 2 \tan^3 x dx$

$$I = 2 \int_0^{\frac{\pi}{4}} \tan^2 x \tan x dx = 2 \int_0^{\frac{\pi}{4}} (\sec^2 x - 1) \tan x dx$$

$$= 2 \int_0^{\frac{\pi}{4}} \sec^2 x \tan x dx - 2 \int_0^{\frac{\pi}{4}} \tan x dx$$

$$\left[ \tan^2 x \right]_0^{\frac{\pi}{4}} - \left[ \log |\sec x| \right]_0^{\frac{\pi}{4}}$$

$$\begin{aligned}
&= 2 \left[ \frac{\tan^{-1} x}{2} \right]_0^1 + 2 \left[ \log \cos x \right]_0^{\frac{\pi}{4}} \\
&= 1 + 2 \left[ \log \cos \frac{\pi}{4} - \log \cos 0 \right] \\
&= 1 + 2 \left[ \log \frac{1}{\sqrt{2}} - \log 1 \right] \\
&= 1 - \log 2 - \log 1 = 1 - \log 2
\end{aligned}$$

Hence, the given result is proved.

Answer needs Correction? [Click Here](#)

Q39 :  $\int_0^1 \sin^{-1} x \, dx = \frac{\pi}{2} - 1$

Answer :

$$\text{Let } I = \int_0^1 \sin^{-1} x \, dx$$

$$\Rightarrow I = \int_0^1 \sin^{-1} x \cdot 1 \cdot dx$$

Integrating by parts, we obtain

$$\begin{aligned}
I &= \left[ \sin^{-1} x \cdot x \right]_0^1 - \int_0^1 \frac{1}{\sqrt{1-x^2}} \cdot x \, dx \\
&= \left[ x \sin^{-1} x \right]_0^1 + \frac{1}{2} \int_0^1 \frac{(-2x)}{\sqrt{1-x^2}} \, dx
\end{aligned}$$

$$\text{Let } 1 - x^2 = t \Rightarrow -2x \, dx = dt$$

When  $x = 0$ ,  $t = 1$  and when  $x = 1$ ,  $t = 0$

$$\begin{aligned}
I &= \left[ x \sin^{-1} x \right]_0^1 + \frac{1}{2} \int_1^0 \frac{dt}{\sqrt{t}} \\
&= \left[ x \sin^{-1} x \right]_0^1 + \frac{1}{2} \left[ 2\sqrt{t} \right]_1^0 \\
&= \sin^{-1}(1) + \left[ -\sqrt{1} \right] \\
&= \frac{\pi}{2} - 1
\end{aligned}$$

Hence, the given result is proved.

Answer needs Correction? [Click Here](#)

Q40 : Evaluate  $\int_0^1 e^{2-3x} \, dx$  as a limit of a sum.

Answer :

$$\text{Let } I = \int_0^1 e^{2-3x} \, dx$$

It is known that,

$$\int_a^b f(x) \, dx = (b-a) \lim_{n \rightarrow \infty} \frac{1}{n} \left[ f(a) + f(a+h) + \dots + f(a+(n-1)h) \right]$$

$$\text{Where, } h = \frac{b-a}{n}$$

$$\text{Here, } a = 0, b = 1, \text{ and } f(x) = e^{2-3x}$$

$$\Rightarrow h = \frac{1-0}{n} = \frac{1}{n}$$

$$\begin{aligned}
\therefore \int_0^1 e^{2-3x} \, dx &= (1-0) \lim_{n \rightarrow \infty} \frac{1}{n} \left[ f(0) + f(0+h) + \dots + f(0+(n-1)h) \right] \\
&= \lim_{n \rightarrow \infty} \frac{1}{n} \left[ e^2 + e^{2-3h} + \dots + e^{2-3(n-1)h} \right]
\end{aligned}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \left[ e^2 \left\{ 1 + e^{-3h} + e^{-6h} + e^{-9h} + \dots + e^{-3(n-1)h} \right\} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \left[ e^2 \left\{ \frac{1 - (e^{-3h})^n}{1 - e^{-3h}} \right\} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \left[ e^2 \left\{ \frac{1 - e^{-\frac{3}{n}}}{1 - e^{-\frac{3}{n}}}} \right\} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \left[ \frac{e^2 (1 - e^{-3})}{1 - e^{-\frac{3}{n}}} \right]$$

$$= e^2 (e^{-3} - 1) \lim_{n \rightarrow \infty} \frac{1}{n} \left[ \frac{1}{e^{-\frac{3}{n}} - 1} \right]$$

$$= e^2 (e^{-3} - 1) \lim_{n \rightarrow \infty} \left( -\frac{1}{3} \right) \left[ \frac{-\frac{3}{n}}{e^{-\frac{3}{n}} - 1} \right]$$

$$= \frac{-e^2 (e^{-3} - 1)}{3} \lim_{n \rightarrow \infty} \left[ \frac{-\frac{3}{n}}{e^{-\frac{3}{n}} - 1} \right]$$

$$= \frac{-e^2 (e^{-3} - 1)}{3} (1) \left[ \lim_{n \rightarrow \infty} \frac{x}{e^x - 1} \right]$$

$$= -e^{-1} + e^2$$



$$= \frac{1}{3} \left( e^2 - \frac{1}{e} \right)$$

Answer needs Correction? [Click Here](#)

Q41 :  $\int \frac{dx}{e^x + e^{-x}}$  is equal to

- A.  $\tan^{-1}(e^x) + C$
- B.  $\tan^{-1}(e^{-x}) + C$
- C.  $\log(e^x - e^{-x}) + C$
- D.  $\log(e^x + e^{-x}) + C$

Answer :

$$\text{Let } I = \int \frac{dx}{e^x + e^{-x}} = \int \frac{e^x}{e^{2x} + 1} dx$$

$$\text{Also, let } e^x = t \Rightarrow e^x dx = dt$$

$$\begin{aligned} \therefore I &= \int \frac{dt}{1+t^2} \\ &= \tan^{-1} t + C \\ &= \tan^{-1}(e^x) + C \end{aligned}$$

Hence, the correct answer is A.

Answer needs Correction? [Click Here](#)

Q42 :  $\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$  is equal to

- A.  $\frac{-1}{\sin x + \cos x} + C$
- B.  $\log|\sin x + \cos x| + C$
- C.  $\log|\sin x - \cos x| + C$
- D.  $\frac{1}{(\sin x + \cos x)^2}$

Answer :

$$\text{Let } I = \frac{\cos 2x}{(\cos x + \sin x)^2}$$

$$\begin{aligned} I &= \int \frac{\cos^2 x - \sin^2 x}{(\cos x + \sin x)^3} dx \\ &= \int \frac{(\cos x + \sin x)(\cos x - \sin x)}{(\cos x + \sin x)^3} dx \\ &= \int \frac{\cos x - \sin x}{\cos x + \sin x} dx \end{aligned}$$

$$\text{Let } \cos x + \sin x = t \Rightarrow (\cos x - \sin x) dx = dt$$

$$\begin{aligned} \therefore I &= \int \frac{dt}{t} \\ &= \log|t| + C \\ &= \log|\cos x + \sin x| + C \end{aligned}$$

Hence, the correct answer is B.

Answer needs Correction? [Click Here](#)

Q43 : If  $f(a+b-x) = f(x)$ , then  $\int_a^b x f(x) dx$  is equal to

- A.  $\frac{a+b}{2} \int_a^b f(b-x) dx$
- B.  $\frac{a+b}{2} \int_a^b f(b+x) dx$
- C.  $\frac{b-a}{2} \int_a^b f(x) dx$
- D.  $\frac{a+b}{2} \int_a^b f(x) dx$

Answer :

$$\text{Let } I = \int_a^b x f(x) dx \quad \dots(1)$$

$$I = \int_a^b (a+b-x) f(a+b-x) dx \quad \left( \int_a^b f(x) dx = \int_a^b f(a+b-x) dx \right)$$

$$\Rightarrow I = \int_a^b (a+b-x) f(x) dx$$

$$\Rightarrow I = (a+b) \int_a^b f(x) dx - I \quad \left[ \text{Using (1)} \right]$$

$$\Rightarrow I + I = (a+b) \int_a^b f(x) dx$$

$$\Rightarrow 2I = (a+b) \int_a^b f(x) dx$$

$$\Rightarrow I = \left(\frac{a+b}{2}\right) \int_a^b f(x) dx$$

Hence, the correct answer is D.

Answer needs Correction? [Click Here](#)

Q44 : The value of  $\int_0^1 \tan^{-1} \left( \frac{2x-1}{1+x-x^2} \right) dx$  is

A. 1

B. 0

C. - 1

D.  $\frac{\pi}{4}$

Answer :

$$\text{Let } I = \int_0^1 \tan^{-1} \left( \frac{2x-1}{1+x-x^2} \right) dx$$

$$\Rightarrow I = \int_0^1 \tan^{-1} \left( \frac{x-(1-x)}{1+x(1-x)} \right) dx$$

$$\Rightarrow I = \int_0^1 \left[ \tan^{-1} x - \tan^{-1} (1-x) \right] dx \quad \dots(1)$$

$$\Rightarrow I = \int_0^1 \left[ \tan^{-1} (1-x) - \tan^{-1} (1-1+x) \right] dx$$

$$\Rightarrow I = \int_0^1 \left[ \tan^{-1} (1-x) - \tan^{-1} (x) \right] dx$$

$$\Rightarrow I = \int_0^1 \left[ \tan^{-1} (1-x) - \tan^{-1} (x) \right] dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^1 \left( \tan^{-1} x + \tan^{-1} (1-x) - \tan^{-1} (1-x) - \tan^{-1} x \right) dx$$

$$\Rightarrow 2I = 0$$

$$\Rightarrow I = 0$$

Hence, the correct answer is B.

\*\*\*\*\* END \*\*\*\*\*