



Trigonometric Functions Ex 5.1 Q11

$$\begin{aligned}
 \text{LHS} &= 1 - \frac{\sin^2 \theta}{1 + \cot \theta} - \frac{\cos^2 \theta}{1 + \tan \theta} \\
 &= 1 - \frac{\sin^2 \theta}{1 + \frac{\cos \theta}{\sin \theta}} - \frac{\cos^2 \theta}{1 + \frac{\sin \theta}{\cos \theta}} \left(\because \cot \theta = \frac{\cos \theta}{\sin \theta}, \tan \theta = \frac{\sin \theta}{\cos \theta} \right) \\
 &= 1 - \frac{\sin^2 \theta}{\frac{\sin \theta + \cos \theta}{\sin \theta}} - \frac{\cos^2 \theta}{\frac{\cos \theta + \sin \theta}{\cos \theta}} \\
 &= 1 - \frac{\sin^3 \theta}{\sin \theta + \cos \theta} - \frac{\cos^3 \theta}{\cos \theta + \sin \theta} \\
 &= \frac{\sin \theta + \cos \theta - (\sin^3 + \cos^3 \theta)}{\sin \theta + \cos \theta} \\
 &= \frac{\sin \theta + \cos \theta - (\sin \theta + \cos \theta) \{ \sin^2 \theta + \cos^2 \theta - \sin \theta \cos \theta \}}{\sin \theta + \cos \theta} \\
 &\quad \left(\text{Using } a^3 + b^3 = (a + b)(a^2 + b^2 - ab) \right) \\
 &= \frac{(\sin \theta + \cos \theta) \{ 1 - (1 - \sin \theta \cos \theta) \}}{\sin \theta + \cos \theta} \quad \left(\text{Using } \sin^2 \theta + \cos^2 \theta = 1 \right) \\
 &= \sin \theta \cos \theta \\
 &= \text{RHS} \\
 &\quad \text{Proved}
 \end{aligned}$$

Trigonometric Functions Ex 5.1 Q12

$$\begin{aligned}
\text{LHS} &= \left(\frac{1}{\sec^2 \theta - \cos^2 \theta} + \frac{1}{\cos \sec^2 \theta - \sin^2 \theta} \right) \sin^2 \theta \cos^2 \theta \\
&= \left(\frac{1}{\frac{1}{\cos^2 \theta} - \cos^2 \theta} + \frac{1}{\left(\frac{1}{\sin^2 \theta} - \sin^2 \theta \right)} \right) \sin^2 \theta \cos^2 \theta \\
&= \left(\frac{1}{\frac{1 - \cos^4 \theta}{\cos^2 \theta}} + \frac{1}{\frac{1 - \sin^4 \theta}{\sin^2 \theta}} \right) \sin^2 \theta \cos^2 \theta \\
&= \left(\frac{\cos^2 \theta}{(1 - \cos^2 \theta)(1 + \cos^2 \theta)} + \frac{\sin^2 \theta}{(1 - \sin^2 \theta)(1 + \sin^2 \theta)} \right) \sin^2 \theta \cos^2 \theta \\
&\quad \left(\begin{array}{l} \text{Using } 1 - a^4 = 1 - (a^2)^2 \\ \quad \quad \quad = (1 - a^2)(1 + a^2) \end{array} \right) \\
&= \left(\frac{\cos^2 \theta}{\sin^2 \theta (1 + \cos^2 \theta)} + \frac{\sin^2 \theta}{\cos^2 \theta (1 + \sin^2 \theta)} \right) \sin^2 \theta \cos^2 \theta \quad \left(\begin{array}{l} \text{Using } 1 - \cos^2 \theta = \sin^2 \theta \\ \quad \& \quad 1 - \sin^2 \theta = \cos^2 \theta \end{array} \right) \\
&= \left(\frac{\cos^4 \theta (1 + \sin^2 \theta) + \sin^4 \theta (1 + \cos^2 \theta)}{\sin^2 \theta \cos^2 \theta (1 + \cos^2 \theta)(1 + \sin^2 \theta)} \right) \sin^2 \theta \cos^2 \theta \\
&= \frac{\cos^4 \theta + \sin^2 \theta \cos^4 \theta + \sin^4 \theta + \cos^2 \theta \sin^4 \theta}{(1 + \cos^2 \theta)(1 + \sin^2 \theta)} \\
&= \frac{(\cos^2 \theta)^2 + (\sin^2 \theta)^2 + 2 \cos^2 \theta \sin^2 \theta - 2 \cos^2 \theta \sin^2 \theta + \sin^2 \theta \cos^4 \theta + \cos^2 \theta \sin^4 \theta}{(1 + \cos^2 \theta)(1 + \sin^2 \theta)} \\
&\quad \left(\text{adding and subtracting } 2 \cos^2 \theta \sin^2 \theta \right) \\
&= \frac{(\cos^2 \theta + \sin^2 \theta)^2 - 2 \cos^2 \theta \sin^2 \theta + \sin^2 \theta \cos^2 \theta (\cos^2 \theta + \sin^2 \theta)}{1 + \sin^2 \theta + \cos^2 \theta + \sin^2 \theta \cos^2 \theta} \\
&= \frac{1^2 - 2 \cos^2 \theta \sin^2 \theta + \sin^2 \theta \cos^2 \theta \cdot 1}{1 + 1 + \sin^2 \theta \cos^2 \theta} \\
&= \frac{1 - \sin^2 \theta \cos^2 \theta}{2 + \sin^2 \theta \cos^2 \theta} \\
&= \text{RHS} \\
&\quad \text{Proved}
\end{aligned}$$

Trigonometric Functions Ex 5.1 Q13

$$\begin{aligned}
\text{LHS} &= (1 + \tan \alpha \tan \beta)^2 + (\tan \alpha - \tan \beta)^2 \\
&= 1 + (\tan \alpha + \tan \beta)^2 + 2 \cdot 1 \tan \alpha \tan \beta + (\tan \alpha)^2 + (\tan \beta)^2 - 2 \tan \alpha \cdot \tan \beta \\
&\quad \left(\text{Using } (a + b)^2 = a^2 + b^2 + 2ab \text{ and } (a - b)^2 = a^2 + b^2 - 2ab \right) \\
&= 1 + \tan^2 \alpha + \tan^2 \beta + 2 \tan \alpha \tan \beta + \tan^2 \alpha + \tan^2 \beta - 2 \tan \alpha \tan \beta \\
&= 1 + \tan^2 \alpha + \tan^2 \alpha + \tan^2 \beta + \tan^2 \beta \\
&= \sec^2 \alpha + \tan^2 \beta (1 + \tan^2 \alpha) \quad \left(\because 1 + \tan^2 \alpha = \sec^2 \alpha \right) \\
&= \sec^2 \alpha + \tan^2 \beta \cdot \sec^2 \alpha \\
&= \sec^2 \alpha (1 + \tan^2 \beta) \\
&= \sec^2 \alpha \cdot \sec^2 \beta \quad \left(\because 1 + \tan^2 \beta = \sec^2 \beta \right) \\
&= \text{RHS} \\
&\quad \text{Proved}
\end{aligned}$$

Trigonometric Functions Ex 5.1 Q14

$$\begin{aligned}
\text{LHS} &= \frac{(1 + \cot \theta + \tan \theta)(\sin \theta - \cos \theta)}{\sec^3 \theta - \cos \sec^3 \theta} \\
&= \frac{\left(1 + \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta}\right)}{\left(\frac{1}{\cos^3 \theta} - \frac{1}{\sin^3 \theta}\right)} \qquad \left(\because \cot \theta = \frac{\cos \theta}{\sin \theta}, \tan \theta = \frac{\sin \theta}{\cos \theta}\right) \\
&\qquad \left(\sec \theta = \frac{1}{\cos \theta}, \cos \sec \theta = \frac{1}{\sin \theta}\right) \\
&= \left(\frac{1 + \frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cos \theta}}{\frac{\sin^3 \theta - \cos^3 \theta}{\cos^3 \theta \sin^3 \theta}}\right)(\sin \theta - \cos \theta) \\
&= \frac{(\sin \theta \cos \theta + 1) \sin^3 \theta \cos^3 \theta}{\sin \theta \cos \theta (\sin^3 \theta - \cos^3 \theta)} (\sin \theta - \cos \theta) \qquad (\because \sin^2 \theta + \cos^2 \theta = 1) \\
&= \frac{(1 + \sin \theta \cos \theta) \sin^2 \theta \cos^2 \theta (\sin \theta - \cos \theta)}{(\sin \theta - \cos \theta) (\sin^2 \theta + \cos^2 \theta + \sin \theta \cos \theta)} \qquad (\because a^3 - b^3 = (a - b)(a^2 + b^2 + ab)) \\
&= \frac{(1 + \sin \theta \cos \theta) \cdot \sin^2 \theta \cos^2 \theta}{(1 + \sin \theta \cos \theta)} \\
&= \sin^2 \theta \cos^2 \theta \\
&= \text{RHS} \\
&\quad \text{Proved}
\end{aligned}$$

Trigonometric Functions Ex 5.1 Q15

$$\begin{aligned}
\text{LHS} &= \frac{2 \sin \theta \cos \theta - \cos \theta}{1 - \sin \theta + \sin^2 \theta - \cos^2 \theta} \\
&= \frac{\cos \theta (2 \sin \theta - 1)}{1 - \cos^2 \theta + \sin^2 \theta - \sin \theta} \\
&= \frac{\cos \theta (2 \sin \theta - 1)}{\sin^2 \theta + \sin^2 \theta - \sin \theta} \qquad (\because 1 - \cos^2 \theta = \sin^2 \theta) \\
&= \frac{\cos \theta (2 \sin \theta - 1)}{2 \sin^2 \theta - \sin \theta} \\
&= \frac{\cos \theta (2 \sin \theta - 1)}{\sin \theta (2 \sin \theta - 1)} \\
&= \frac{\cos \theta}{\sin \theta} \\
&= \cot \theta \\
&= \text{RHS} \\
&\quad \text{Proved}
\end{aligned}$$

***** END *****