Functions Ex 2.5 Q 1.

i) $f : \{1,2,3,4\} \to \{10\}$ given by

$f\{(1,10),(2,10),(3,10),(4,10)\}$

clearly f is many-one function

$\Rightarrow$ f is not bijective

$\Rightarrow$ f is not invertible

ii) $g : \{5,6,7,8\} \to \{1,2,3,4\}$ given by

$g\{(5,4),(6,3),(7,4),(8,2)\}$

Since, 5 and 7 have same image 4

$\therefore$ g is not bijectible

$\Rightarrow$ g is not bijective

$\Rightarrow$ g is not invertible

iii) $h : \{2,3,4,5\} \to \{7,9,11,13\}$ given by

$h\{(2,7),(3,9),(4,11),(5,13)\}$

We can observe that different element of domain have defferent image is co-domain.

Functions Ex 2.5 Q2

$A = \{0,-1,-3,2\}$, $B = \{-9,-3,0,6\}$

$f : A \to B$ is defined by $f(x) = 3x$

Since different elements of A have different images in B.

$\therefore$ f is one-one

Again, each element in B has a preimage in A.

$\therefore$ f is onto

$\because$ f in one-one bijective

$\Rightarrow$ $f^{-1} : B \to A$ exists and is given by

$$f^{-1}(x) = \frac{x}{3}$$

$A = \{1,3,5,7,9\}$, $B = \{0,1,9,25,49,81\}$

$f : A \to B$ be a function defined by $f(x) = x^2$

Since different elements of A have different images in B.

$\therefore$ f is one-one

Again, $0 \in B$ does not have a preimage in A.

$\therefore$ f is not onto

Hence, f^{-1} does not exist.

Functions Ex 2.5 Q3

Given that $f:\{1,2,3\} \to \{a,b,c\}$ *and* $g:\{a,b,c\} \to \{apple,\ ball,\ cat\}$ *such that*
$f(1) = a,\ f(2) = b,\ f(3) = c,\ g(a) = apple,\ g(b) = ball$ *and* $g(c) = cat$
We need to prove that f, g *and* $g \circ f$ *are invertible.*
In order to prove that f *is invertiblem is is sufficient to show that*
$f:\{1,2,3\} \to \{a,b,c\}$ *is a bijection.*
f *is one – one:*
Each and every element of the set $\{1,2,3\}$ *is having an image in the set* $\{a,b,c\}$
Thus, f *is one – one.*
Obviously, the number of element of the sets $\{1,2,3\}$ *and* $\{a,b,c\}$ *are equal and hence*
f *is onto.*
Thus, the function f *is invertible.*
Similarly, let us observe for the function g:
g *is one – one:*
Each and every element of the set $\{a,b,c\}$ *is having an image in the set* $\{apple,\ ball,\ cat\}$
Thus, g *is one – one.*
Obviously, the number of element of the sets $\{a,b,c\}$ *and* $\{apple,\ ball,\ cat\}$ *are equal and hence*
g *is onto.*
Thus, the function g *is invertible.*

Now let us consider the function $g \circ f = g[f(x)]$
Each and every element of of the set $\{1,2,3\}$ *is having an image in the set*
$\{apple,\ ball,\ cat\}$.
Therefore, $g \circ f = \{(1, apple),\ (2, ball),\ (3, cat)\}$
Thus, $g \circ f$ *is one – one.*
Since the number of elemenets in the sets $\{1,2,3\}$ *and* $\{apple,\ ball,\ cat\}$ *are equal.*
Hence $g \circ f$ *is onto.*
Therefore, function $g \circ f$ *is invertible.*
Let us now find f^{-1}:
We have $f:\{1,2,3\} \to \{a,b,c\}$
Thus, $f^{-1}:\{a,b,c\} \to \{1,2,3\}$
$\Rightarrow f^{-1} = \{(a,1),(b,2),(c,3)\}$
Let us now find g^{-1}:
We have $g:\{a,b,c\} \to \{apple, ball, cat\}$
Thus, $g^{-1}:\{apple, ball, cat\} \to \{a,b,c\}$
$\Rightarrow g^{-1} = \{(apple,a),(ball,b),(cat,c)\}$
Let us now find $f^{-1} \circ g^{-1}$:
$\Rightarrow f^{-1} \circ g^{-1} = \{(apple,1),\ (ball,\ 2),\ (cat,3)\}$.....(1)
Also, let us find, $(g \circ f)^{-1}$:
$\Rightarrow (g \circ f)^{-1} = \{(apple,\ 1),\ (ball,2),\ (cat,3)\}$....(2)
From (1) *and* (2), *we have,*
$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

*********** END **********