Exercise 7.9 : Solutions of Questions on Page Number : **338**

Q1 : $\int_{-1}^{1}(x+1)dx$

Answer :

$$\text{Let } I=\int_{-1}^{1}(x+1)dx$$
$$\int(x+1)\,dx=\frac{x^2}{2}+x=\mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned}I&=\mathrm{F}(1)-\mathrm{F}(-1)\\&=\left(\frac{1}{2}+1\right)-\left(\frac{1}{2}-1\right)\\&=\frac{1}{2}+1-\frac{1}{2}+1\\&=2\end{aligned}$$

Answer needs Correction? **Click Here**

Q2 : $\int_{2}^{3}\frac{1}{x}dx$

Answer :

$$\text{Let } I=\int_{2}^{3}\frac{1}{x}\,dx$$
$$\int\frac{1}{x}dx=\log|x|=\mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned}I&=\mathrm{F}(3)-\mathrm{F}(2)\\&=\log|3|-\log|2|=\log\frac{3}{2}\end{aligned}$$

Answer needs Correction? **Click Here**

Q3 : $\int_{1}^{2}\left(4x^3-5x^2+6x+9\right)dx$

Answer :

$$\text{Let } I=\int_{1}^{2}\left(4x^3-5x^2+6x+9\right)dx$$
$$\begin{aligned}\int\left(4x^3-5x^2+6x+9\right)dx&=4\left(\frac{x^4}{4}\right)-5\left(\frac{x^3}{3}\right)+6\left(\frac{x^2}{2}\right)+9(x)\\&=x^4-\frac{5x^3}{3}+3x^2+9x=\mathrm{F}(x)\end{aligned}$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned}I&=\mathrm{F}(2)-\mathrm{F}(1)\\I&=\left\{2^4-\frac{5\cdot(2)^3}{3}+3(2)^2+9(2)\right\}-\left\{(1)^4-\frac{5(1)^3}{3}+3(1)^2+9(1)\right\}\\&=\left(16-\frac{40}{3}+12+18\right)-\left(1-\frac{5}{3}+3+9\right)\\&=16-\frac{40}{3}+12+18-1+\frac{5}{3}-3-9\\&=33-\frac{35}{3}\\&=\frac{99-35}{3}\\&=\frac{64}{3}\end{aligned}$$

Answer needs Correction? **Click Here**

Q4 : $\int_{0}^{\frac{\pi}{4}}\sin 2x\,dx$

Answer :

$$\text{Let } I=\int_{0}^{\frac{\pi}{4}}\sin 2x\,dx$$
$$\int\sin 2x\,dx=\left(\frac{-\cos 2x}{2}\right)=\mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}\left(\frac{\pi}{4}\right)-\mathrm{F}(0)$$
$$= -\frac{1}{2}\left[\cos 2\left(\frac{\pi}{4}\right)-\cos 0\right]$$
$$= -\frac{1}{2}\left[\cos\left(\frac{\pi}{2}\right)-\cos 0\right]$$
$$= -\frac{1}{2}[0-1]$$
$$= \frac{1}{2}$$

Answer needs Correction? **Click Here**

Q5 : $\int_0^{\frac{\pi}{2}} \cos 2x\, dx$

Answer :

Let $I = \int_0^{\frac{\pi}{2}} \cos 2x\, dx$

$$\int \cos 2x\, dx = \left(\frac{\sin 2x}{2}\right) = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}\left(\frac{\pi}{2}\right)-\mathrm{F}(0)$$
$$= \frac{1}{2}\left[\sin 2\left(\frac{\pi}{2}\right)-\sin 0\right]$$
$$= \frac{1}{2}[\sin\pi - \sin 0]$$
$$= \frac{1}{2}[0-0] = 0$$

Answer needs Correction? **Click Here**

Q6 : $\int_4^5 e^x dx$

Answer :

Let $I = \int_4^5 e^x dx$

$$\int e^x dx = e^x = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}(5)-\mathrm{F}(4)$$
$$= e^5 - e^4$$
$$= e^4(e-1)$$

Answer needs Correction? **Click Here**

Q7 : $\int_0^{\frac{\pi}{4}} \tan x\, dx$

Answer :

Let $I = \int_0^{\frac{\pi}{4}} \tan x\, dx$

$$\int \tan x\, dx = -\log|\cos x| = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}\left(\frac{\pi}{4}\right)-\mathrm{F}(0)$$
$$= -\log\left|\cos\frac{\pi}{4}\right| + \log|\cos 0|$$
$$= -\log\left|\frac{1}{\sqrt{2}}\right| + \log|1|$$
$$= -\log(2)^{-\frac{1}{2}}$$
$$= \frac{1}{2}\log 2$$

Answer needs Correction? **Click Here**

Q8 : $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x\ dx$

Answer :

Let $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x\, dx$

$$\int \operatorname{cosec} x\ dx = \log|\operatorname{cosec} x - \cot x| = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}\left(\frac{\pi}{4}\right)-\mathrm{F}\left(\frac{\pi}{6}\right)$$

$$\left(4\right) \quad \left(6\right)$$

$$= \log\left|\operatorname{cosec}\frac{\pi}{4} - \cot\frac{\pi}{4}\right| - \log\left|\operatorname{cosec}\frac{\pi}{6} - \cot\frac{\pi}{6}\right|$$
$$= \log\left|\sqrt{2} - 1\right| - \log\left|2 - \sqrt{3}\right|$$
$$= \log\left(\frac{\sqrt{2} - 1}{2 - \sqrt{3}}\right)$$

Answer needs Correction? **Click Here**

Q9 : $\int_0^1 \frac{dx}{\sqrt{1 - x^2}}$

Answer :

$$\text{Let } I = \int_0^1 \frac{dx}{\sqrt{1 - x^2}}$$
$$\int \frac{dx}{\sqrt{1 - x^2}} = \sin^{-1} x = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}(1) - \mathrm{F}(0)$$
$$= \sin^{-1}(1) - \sin^{-1}(0)$$
$$= \frac{\pi}{2} - 0$$
$$= \frac{\pi}{2}$$

Answer needs Correction? **Click Here**

Q10 : $\int_0^1 \frac{dx}{1 + x^2}$

Answer :

$$\text{Let } I = \int_0^1 \frac{dx}{1 + x^2}$$
$$\int \frac{dx}{1 + x^2} = \tan^{-1} x = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}(1) - \mathrm{F}(0)$$
$$= \tan^{-1}(1) - \tan^{-1}(0)$$
$$= \frac{\pi}{4}$$

Answer needs Correction? **Click Here**

Q11 : $\int_2^3 \frac{dx}{x^2 - 1}$

Answer :

$$\text{Let } I = \int_2^3 \frac{dx}{x^2 - 1}$$
$$\int \frac{dx}{x^2 - 1} = \frac{1}{2}\log\left|\frac{x - 1}{x + 1}\right| = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}(3) - \mathrm{F}(2)$$
$$= \frac{1}{2}\left[\log\left|\frac{3 - 1}{3 + 1}\right| - \log\left|\frac{2 - 1}{2 + 1}\right|\right]$$
$$= \frac{1}{2}\left[\log\left|\frac{2}{4}\right| - \log\left|\frac{1}{3}\right|\right]$$
$$= \frac{1}{2}\left[\log\frac{1}{2} - \log\frac{1}{3}\right]$$
$$= \frac{1}{2}\left[\log\frac{3}{2}\right]$$

Answer needs Correction? **Click Here**

Q12 : $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$

Answer :

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \cos^2 x \, dx$$
$$\int \cos^2 x \, dx = \int\left(\frac{1 + \cos 2x}{2}\right)dx = \frac{x}{2} + \frac{\sin 2x}{4} = \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \left[\mathrm{F}\left(\frac{\pi}{2}\right) - \mathrm{F}(0)\right]$$
$$= \frac{1}{2}\left[\left(\frac{\pi}{2} - \frac{\sin\pi}{2}\right) - \left(0 + \frac{\sin 0}{2}\right)\right]$$
$$= \frac{1}{2}\left[\frac{\pi}{2} + 0 - 0 - 0\right]$$

$$= \frac{\pi}{4}$$

Answer needs Correction? **Click Here**

Q13 : $\int_2^3 \frac{x\,dx}{x^2+1}$

Answer :

$$\text{Let } I = \int_2^3 \frac{x}{x^2+1}dx$$

$$\int \frac{x}{x^2+1}dx = \frac{1}{2}\int \frac{2x}{x^2+1}dx = \frac{1}{2}\log\left(1+x^2\right) = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= \mathrm{F}(3) - \mathrm{F}(2) \\ &= \frac{1}{2}\left[\log\left(1+(3)^2\right) - \log\left(1+(2)^2\right)\right] \\ &= \frac{1}{2}\left[\log(10) - \log(5)\right] \\ &= \frac{1}{2}\log\left(\frac{10}{5}\right) = \frac{1}{2}\log 2 \end{aligned}$$

Answer needs Correction? **Click Here**

Q14 : $\int_0^1 \frac{2x+3}{5x^2+1}dx$

Answer :

$$\text{Let } I = \int_0^1 \frac{2x+3}{5x^2+1}dx$$

$$\begin{aligned} \int \frac{2x+3}{5x^2+1}dx &= \frac{1}{5}\int \frac{5(2x+3)}{5x^2+1}dx \\ &= \frac{1}{5}\int \frac{10x+15}{5x^2+1}dx \\ &= \frac{1}{5}\int \frac{10x}{5x^2+1}dx + 3\int \frac{1}{5x^2+1}dx \\ &= \frac{1}{5}\int \frac{10x}{5x^2+1}dx + 3\int \frac{1}{5\left(x^2+\frac{1}{5}\right)}dx \\ &= \frac{1}{5}\log\left(5x^2+1\right) + \frac{3}{5}\cdot\frac{1}{\frac{1}{\sqrt{5}}}\tan^{-1}\frac{x}{\frac{1}{\sqrt{5}}} \\ &= \frac{1}{5}\log\left(5x^2+1\right) + \frac{3}{\sqrt{5}}\tan^{-1}\left(\sqrt{5}x\right) \\ &= \mathrm{F}(x) \end{aligned}$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= \mathrm{F}(1) - \mathrm{F}(0) \\ &= \left\{\frac{1}{5}\log(5+1) + \frac{3}{\sqrt{5}}\tan^{-1}\left(\sqrt{5}\right)\right\} - \left\{\frac{1}{5}\log(1) + \frac{3}{\sqrt{5}}\tan^{-1}(0)\right\} \\ &= \frac{1}{5}\log 6 + \frac{3}{\sqrt{5}}\tan^{-1}\sqrt{5} \end{aligned}$$

Answer needs Correction? **Click Here**

Q15 : $\int_0^1 xe^{x^2}dx$

Answer :

$$\text{Let } I = \int_0^1 xe^{x^2}dx$$

$$\text{Put } x^2 = t \Rightarrow 2x\,dx = dt$$

$$\text{As } x \to 0, t \to 0 \text{ and as } x \to 1, t \to 1,$$

$$\therefore I = \frac{1}{2}\int_0^1 e^t dt$$

$$\frac{1}{2}\int e^t dt = \frac{1}{2}e^t = \mathrm{F}(t)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} I &= \mathrm{F}(1) - \mathrm{F}(0) \\ &= \frac{1}{2}e - \frac{1}{2}e^0 \\ &= \frac{1}{2}(e-1) \end{aligned}$$

Answer needs Correction? **Click Here**

Q16 : $\int_0^1 \frac{5x^2}{x^2+4x+3}$

Answer :

Let $I = \int_1^2 \frac{5x^2}{x^2+4x+3}dx$

Dividing $5x^2$ by x^2+4x+3, we obtain

Dividing $5x^2$ by x^2+4x+3, we obtain

$$I=\int_1^2\left\{5-\frac{20x+15}{x^2+4x+3}\right\}dx$$

$$=\int_1^2 5dx-\int_1^2\frac{20x+15}{x^2+4x+3}dx$$

$$=\left[5x\right]_1^2-\int_1^2\frac{20x+15}{x^2+4x+3}dx$$

$$I=5-I_1,\text{ where } I=\int_1^2\frac{20x+15}{x^2+4x+3}dx \qquad ...(1)$$

Consider $I_1=\int_1^2\frac{20x+15}{x^2+4x+8}dx$

Let $20x+15=\mathrm{A}\frac{d}{dx}\left(x^2+4x+3\right)+\mathrm{B}$

$$=2\mathrm{A}x+\left(4\mathrm{A}+\mathrm{B}\right)$$

Equating the coefficients of x and constant term, we obtain

A = 10 and B = - 25

$$\Rightarrow I_1=10\int_1^2\frac{2x+4}{x^2+4x+3}dx-25\int_1^2\frac{dx}{x^2+4x+3}$$

Let $x^2+4x+3=t$

$\Rightarrow\left(2x+4\right)dx=dt$

$$\Rightarrow I_1=10\int\frac{dt}{t}-25\int\frac{dx}{\left(x+2\right)^2-1^2}$$

$$=10\log t-25\left[\frac{1}{2}\log\left(\frac{x+2-1}{x+2+1}\right)\right]$$

$$=\left[10\log\left(x^2+4x+3\right)\right]_1^2-25\left[\frac{1}{2}\log\left(\frac{x+1}{x+3}\right)\right]_1^2$$

$$=\left[10\log 15-10\log 8\right]-25\left[\frac{1}{2}\log\frac{3}{5}-\frac{1}{2}\log\frac{2}{4}\right]$$

$$=\left[10\log\left(5\times 3\right)-10\log\left(4\times 2\right)\right]-\frac{25}{2}\left[\log 3-\log 5-\log 2+\log 4\right]$$

$$=\left[10\log 5+10\log 3-10\log 4-10\log 2\right]-\frac{25}{2}\left[\log 3-\log 5-\log 2+\log 4\right]$$

$$=\left[10+\frac{25}{2}\right]\log 5+\left[-10-\frac{25}{2}\right]\log 4+\left[10-\frac{25}{2}\right]\log 3+\left[-10+\frac{25}{2}\right]\log 2$$

$$=\frac{45}{2}\log 5-\frac{45}{2}\log 4-\frac{5}{2}\log 3+\frac{5}{2}\log 2$$

$$=\frac{45}{2}\log\frac{5}{4}-\frac{5}{2}\log\frac{3}{2}$$

Substituting the value of I_1 in (1), we obtain

$$I=5-\left[\frac{45}{2}\log\frac{5}{4}-\frac{5}{2}\log\frac{3}{2}\right]$$

$$=5-\frac{5}{2}\left[9\log\frac{5}{4}-\log\frac{3}{2}\right]$$

Answer needs Correction? **Click Here**

Q17 : $\int_0^{\frac{\pi}{4}}\left(2\sec^2 x+x^3+2\right)dx$

Answer :

Let $I=\int_0^{\frac{\pi}{4}}\left(2\sec^2 x+x^3+2\right)dx$

$$\int\left(2\sec^2 x+x^3+2\right)dx=2\tan x+\frac{x^4}{4}+2x=\mathrm{F}\left(x\right)$$

By second fundamental theorem of calculus, we obtain

$$I=\mathrm{F}\left(\frac{\pi}{4}\right)-\mathrm{F}\left(0\right)$$

$$=\left\{\left(2\tan\frac{\pi}{4}+\frac{1}{4}\left(\frac{\pi}{4}\right)^4+2\left(\frac{\pi}{4}\right)\right)-\left(2\tan 0+0+0\right)\right\}$$

$$=2\tan\frac{\pi}{4}+\frac{\pi^4}{4^5}+\frac{\pi}{2}$$

$$=2+\frac{\pi}{2}+\frac{\pi^4}{1024}$$

Answer needs Correction? **Click Here**

Q18 : $\int_0^{\pi}\left(\sin^2\frac{x}{2}-\cos^2\frac{x}{2}\right)dx$

Answer :

Let $I=\int_0^{\pi}\left(\sin^2\frac{x}{2}-\cos^2\frac{x}{2}\right)dx$

$$=-\int_0^{\pi}\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)dx$$

$$=-\int_0^{\pi}\cos x\,dx$$

$$\int\cos x\,dx=\sin x=\mathrm{F}\left(x\right)$$

By second fundamental theorem of calculus, we obtain

$$I=\mathrm{F}\left(\pi\right)-\mathrm{F}\left(0\right)$$

$= \sin\pi - \sin 0$
$= 0$

Answer needs Correction? Click Here

Q19 : $\int_0^2 \frac{6x+3}{x^2+4}dx$

Answer :

Let $I = \int_0^2 \frac{6x+3}{x^2+4}dx$

$$\int \frac{6x+3}{x^2+4}dx = 3\int \frac{2x+1}{x^2+4}dx$$
$$= 3\int \frac{2x}{x^2+4}dx + 3\int \frac{1}{x^2+4}dx$$
$$= 3\log\left(x^2+4\right) + \frac{3}{2}\tan^{-1}\frac{x}{2} = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}(2) - \mathrm{F}(0)$$
$$= \left\{3\log\left(2^2+4\right) + \frac{3}{2}\tan^{-1}\left(\frac{2}{2}\right)\right\} - \left\{3\log(0+4) + \frac{3}{2}\tan^{-1}\left(\frac{0}{2}\right)\right\}$$
$$= 3\log 8 + \frac{3}{2}\tan^{-1}1 - 3\log 4 - \frac{3}{2}\tan^{-1}0$$
$$= 3\log 8 + \frac{3}{2}\left(\frac{\pi}{4}\right) - 3\log 4 - 0$$
$$= 3\log\left(\frac{8}{4}\right) + \frac{3\pi}{8}$$
$$= 3\log 2 + \frac{3\pi}{8}$$

Answer needs Correction? Click Here

Q20 : $\int_0^1 \left(xe^x + \sin\frac{\pi x}{4}\right)dx$

Answer :

Let $I = \int_0^1 \left(xe^x + \sin\frac{\pi x}{4}\right)dx$

$$\int \left(xe^x + \sin\frac{\pi x}{4}\right)dx = x\int e^x dx - \int\left\{\left(\frac{d}{dx}x\right)\int e^x dx\right\}dx + \left\{\frac{-\cos\frac{\pi x}{4}}{\frac{\pi}{4}}\right\}$$
$$= xe^x - \int e^x dx - \frac{4\pi}{\pi}\cos\frac{x}{4}$$
$$= xe^x - e^x - \frac{4\pi}{\pi}\cos\frac{x}{4}$$
$$= \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \mathrm{F}(1) - \mathrm{F}(0)$$
$$= \left(1.e^1 - e^1 - \frac{4}{\pi}\cos\frac{\pi}{4}\right) - \left(0.e^0 - e^0 - \frac{4}{\pi}\cos 0\right)$$
$$= e - e - \frac{4}{\pi}\left(\frac{1}{\sqrt{2}}\right) + 1 + \frac{4}{\pi}$$
$$= 1 + \frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}$$

Answer needs Correction? Click Here

Q21 : $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ equals

A. $\frac{\pi}{3}$

B. $\frac{2\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{12}$

Answer :

$$\int \frac{dx}{1+x^2} = \tan^{-1}x = \mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$\int_1^{\sqrt{3}} \frac{dx}{1+x^2} = \mathrm{F}\left(\sqrt{3}\right) - \mathrm{F}(1)$$
$$= \tan^{-1}\sqrt{3} - \tan^{-1}1$$
$$= \frac{\pi}{3} - \frac{\pi}{4}$$
$$= \frac{\pi}{12}$$

Hence, the correct answer is D.

Answer needs Correction? **Click Here**

Q22 : $\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$ equals

A. $\frac{\pi}{6}$

B. $\frac{\pi}{12}$

C. $\frac{\pi}{24}$

D. $\frac{\pi}{4}$

Answer :

$$\int \frac{dx}{4+9x^2} = \int \frac{dx}{(2)^2+(3x)^2}$$

Put $3x = t \Rightarrow 3dx = dt$

$$\begin{aligned} \therefore \int \frac{dx}{(2)^2+(3x)^2} &= \frac{1}{3}\int \frac{dt}{(2)^2+t^2} \\ &= \frac{1}{3}\left[\frac{1}{2}\tan^{-1}\frac{t}{2}\right] \\ &= \frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right) \\ &= \mathrm{F}(x) \end{aligned}$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned} \int_0^{\frac{2}{3}} \frac{dx}{4+9x^2} &= \mathrm{F}\left(\frac{2}{3}\right) - \mathrm{F}(0) \\ &= \frac{1}{6}\tan^{-1}\left(\frac{3}{2}\cdot\frac{2}{3}\right) - \frac{1}{6}\tan^{-1}0 \\ &= \frac{1}{6}\tan^{-1}1 - 0 \\ &= \frac{1}{6}\times\frac{\pi}{4} \\ &= \frac{\pi}{24} \end{aligned}$$

Hence, the correct answer is C.

Answer needs Correction? **Click Here**

*********** END **********