Trigonometric Identities Ex 6.1 Q52

Answer :

In the given question, we need to prove $\frac{\cos\theta}{\operatorname{cosec}\theta+1}+\frac{\cos\theta}{\operatorname{cosec}\theta-1}=2\tan\theta$

Using the identity $a^2-b^2=(a+b)(a-b)$, we get

$$\frac{\cos\theta}{\operatorname{cosec}\theta+1}+\frac{\cos\theta}{\operatorname{cosec}\theta-1}=\frac{\cos\theta(\operatorname{cosec}\theta-1)+\cos\theta(\operatorname{cosec}\theta+1)}{\operatorname{cosec}^2\theta-1}$$
$$=\frac{\cos\theta(\operatorname{cosec}\theta-1+\operatorname{cosec}\theta+1)}{\operatorname{cosec}^2\theta-1}$$

Further, using the property $1+\cot^2\theta=\operatorname{cosec}^2\theta$, we get

$$\frac{\cos\theta(\operatorname{cosec}\theta-1+\operatorname{cosec}\theta+1)}{\operatorname{cosec}^2\theta-1}=\frac{\cos\theta(2\operatorname{cosec}\theta)}{\cot^2\theta}$$
$$=\frac{(2\cos\theta)\left(\frac{1}{\sin\theta}\right)}{\left(\frac{\cos^2\theta}{\sin^2\theta}\right)}$$
$$=2\left(\frac{\cos\theta}{\sin\theta}\right)\left(\frac{\sin^2\theta}{\cos^2\theta}\right)$$
$$=2\frac{\sin\theta}{\cos\theta}$$
$$=2\tan\theta$$

Hence proved.

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Answer :

In the given question, we need to prove $\frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)}=\cot\theta$.

Using the property $\sin^2\theta+\cos^2\theta=1$, we get

So,

$$\frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)}$$
$$=\frac{1+\cos\theta-(1-\cos^2\theta)}{\sin\theta(1+\cos\theta)}$$
$$=\frac{\cos\theta+\cos^2\theta}{\sin\theta(1+\cos\theta)}$$

Solving further, we get

$$\frac{\cos\theta+\cos^2\theta}{\sin\theta(1+\cos\theta)}=\frac{\cos\theta(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$$
$$=\frac{\cos\theta}{\sin\theta}$$
$$=\cot\theta$$

Hence proved.

Answer :

In the given question, we need to prove $\frac{\tan^3\theta}{1+\tan^2\theta}+\frac{\cot^3\theta}{1+\cot^2\theta}=\sec\theta\operatorname{cosec}\theta-2\sin\theta\cos\theta$

Using the property $1+\tan^2\theta=\sec^2\theta$ and $1+\cot^2\theta=\operatorname{cosec}^2\theta$, we get

$$\frac{\tan^3\theta}{1+\tan^2\theta}+\frac{\cot^3\theta}{1+\cot^2\theta}=\frac{\tan^3\theta}{\sec^2\theta}+\frac{\cot^3\theta}{\operatorname{cosec}^2\theta}$$
$$=\frac{\left(\frac{\sin^3\theta}{\cos^3\theta}\right)}{\left(\frac{1}{\cos^2\theta}\right)}+\frac{\left(\frac{\cos^3\theta}{\sin^3\theta}\right)}{\left(\frac{1}{\sin^2\theta}\right)}$$

Taking the reciprocal of the denominator, we get

$$\frac{\left(\frac{\sin^3\theta}{\cos^3\theta}\right)}{\left(\frac{1}{\cos^2\theta}\right)}+\frac{\left(\frac{\cos^3\theta}{\sin^3\theta}\right)}{\left(\frac{1}{\sin^2\theta}\right)}=\left(\frac{\sin^3\theta}{\cos^3\theta}\times\frac{\cos^2\theta}{1}\right)+\left(\frac{\cos^3\theta}{\sin^3\theta}\times\frac{\sin^2\theta}{1}\right)$$

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$$=\frac{\sin^3\theta}{\cos\theta}+\frac{\cos^3\theta}{\sin\theta}$$
$$=\frac{\sin^4\theta+\cos^4\theta}{\cos\theta\sin\theta}$$
$$=\frac{\left(\sin^2\theta\right)^2+\left(\cos^2\theta\right)^2}{\cos\theta\sin\theta}$$

Further, using the identity $a^2+b^2=(a+b)^2-2ab$, we get

$$\frac{\left(\sin^2\theta\right)^2+\left(\cos^2\theta\right)^2}{\cos\theta\sin\theta}=\frac{\left(\sin^2\theta+\cos^2\theta\right)^2-2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$$
$$=\frac{1}{\cos\theta\sin\theta}-\frac{2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}\ (\text{using}\sin^2\theta+\cos^2\theta=1)$$
$$=\frac{1}{\cos\theta\sin\theta}-2\sin\theta\cos\theta$$
$$=\sec\theta\operatorname{cosec}\theta-2\sin\theta\cos\theta$$

Hence proved.

*********** END **********