Indefinite Integrals Ex 19.32 Q11

$$I = \int \frac{x}{\left(x^2+4\right)\sqrt{x^2+1}}\,dx$$

Let $x^2+1 = u^2$

$$\Rightarrow \qquad 2x\,dx = 2u\,du$$

$$\therefore \qquad I = \int \frac{u}{\left(u^2+3\right)u}\,du$$

$$= \int \frac{1}{u^2+3}\,du$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{u}{\sqrt{3}}\right) + C$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\left(\sqrt{\frac{x^2+1}{3}}\right) + C$$

Indefinite Integrals Ex 19.32 Q12

Let $I = \int \frac{1}{\left(1+x^2\right)\sqrt{1-x^2}}dx$

Let $x = \frac{1}{t}$

$\Rightarrow \quad dx = -\frac{1}{t^2}dt$

$$\therefore \quad I = \int \frac{-\frac{1}{t^2}dt}{\left(\frac{1}{t^2}+1\right)\sqrt{\left(1-\frac{1}{t^2}\right)}}$$

$$= -\int \frac{tdt}{\left(t^2+1\right)\sqrt{t^2-1}}$$

Let $t^2 - 1 = u^2$

$\Rightarrow \quad 2tdt = 2udu$

$$I = -\int \frac{udu}{\left(u^2+2\right)u}$$

$$= -\int \frac{du}{u^2+2}$$

$$= -\frac{1}{\sqrt{2}}tan^{-1}\left(\frac{u}{\sqrt{2}}\right)+c$$

$$= -\frac{1}{\sqrt{2}}tan^{-1}\left(\frac{\sqrt{t^2-1}}{\sqrt{2}}\right)+c$$

Thus,

$$I = -\frac{1}{\sqrt{2}}tan^{-1}\left|\sqrt{\frac{1-x^2}{2x^2}}\right|+c$$

Indefinite Integrals Ex 19.32 Q13

Let $\quad I = \int \frac{1}{\left(2x^2+3\right)\sqrt{x^2-4}}\,dx$

Let $x = \frac{1}{t}$

$\Rightarrow \qquad dx = -\frac{1}{t^2}\,dt$

$$\therefore \qquad I = \int \frac{-\frac{1}{t^2}\,dt}{\left(\frac{2}{t^2}+3\right)\sqrt{\left(\frac{1}{t^2}-4\right)}}$$

$$= -\int \frac{t\,dt}{\left(2+3t^2\right)\sqrt{1-4t^2}}$$

Let $1-4t^2 = u^2$

$\Rightarrow \qquad -8t\,dt = 2u\,du$

$$\therefore \qquad I = \frac{1}{4}\int \frac{u\,du}{\frac{\left(11-3u^2\right)}{4}u}$$

$$= \frac{1}{3}\int \frac{du}{\frac{11}{3}-u^2}$$

$$= \frac{1}{2\sqrt{33}}\log\left|\frac{u-\sqrt{\frac{11}{3}}}{u+\sqrt{\frac{11}{3}}}\right| + c$$

$$= \frac{1}{2\sqrt{33}}\log\left|\frac{\sqrt{1-4t^2}-\sqrt{\frac{11}{3}}}{\sqrt{1-4t^2}+\sqrt{\frac{11}{3}}} + c\right|$$

Hence,

$$I = \frac{1}{2\sqrt{33}}\log\left|\frac{\sqrt{11}x+\sqrt{3x^2-12}}{\sqrt{11}x-\sqrt{3x^2-12}}\right| + c$$

Indefinite Integrals Ex 19.32 Q14

$$I = \int \frac{x}{\left(x^2+4\right)\sqrt{x^2+9}}\,dx$$

Let $x^2 + 9 = u^2$

$$\Rightarrow \qquad 2x\,dx = 2u\,du$$

$$\therefore \qquad I = \int \frac{u}{\left(u^2-5\right)u}\,du$$

$$= \int \frac{du}{u^2-5}$$

$$= \frac{1}{2\sqrt{5}}\log\left(\frac{u-\sqrt{5}}{u+\sqrt{5}}\right) + C$$

$$= \frac{1}{2\sqrt{5}}\log\left(\frac{\sqrt{x^2+9}-\sqrt{5}}{\sqrt{x^2+9}+\sqrt{5}}\right) + C$$

*********** END **********