Question 15:

Solve $\sin\left(\tan^{-1}x\right),\ |x|<1$ is equal to

(A) $\frac{x}{\sqrt{1-x^2}}$ **(B)** $\frac{1}{\sqrt{1-x^2}}$ **(C)** $\frac{1}{\sqrt{1+x^2}}$ **(D)** $\frac{x}{\sqrt{1+x^2}}$

Answer

Let $\tan^{-1} x = y$. Then, $\tan y = x \Rightarrow \sin y = \frac{x}{\sqrt{1+x^2}}$.

$$\therefore y = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) \Rightarrow \tan^{-1}x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

$$\therefore \sin\left(\tan^{-1}x\right) = \sin\left(\sin^{-1}\frac{x}{\sqrt{1+x^2}}\right) = \frac{x}{\sqrt{1+x^2}}$$

The correct answer is D.

Question 16:

Solve $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$, then x is equal to

(A) $0, \frac{1}{2}$ **(B)** $1, \frac{1}{2}$ **(C)** 0 **(D)** $\frac{1}{2}$

Answer

$$\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$$

$$\Rightarrow -2\sin^{-1}x = \frac{\pi}{2} - \sin^{-1}(1-x)$$

$$\Rightarrow -2\sin^{-1}x = \cos^{-1}(1-x) \qquad ...(1)$$

Let $\sin^{-1}x = \theta \Rightarrow \sin\theta = x \Rightarrow \cos\theta = \sqrt{1-x^2}$.

$$\therefore \theta = \cos^{-1}\left(\sqrt{1-x^2}\right)$$

$$\therefore \sin^{-1}x = \cos^{-1}\left(\sqrt{1-x^2}\right)$$

Therefore, from equation (1), we have

$$-2\cos^{-1}\left(\sqrt{1-x^2}\right) = \cos^{-1}(1-x)$$

Put x = sin y. Then, we have:

$$-2\cos^{-1}\left(\sqrt{1-\sin^2 y}\right) = \cos^{-1}(1-\sin y)$$

$$\Rightarrow -2\cos^{-1}(\cos y) = \cos^{-1}(1-\sin y)$$

$$\Rightarrow -2y = \cos^{-1}(1-\sin y)$$

$$\Rightarrow 1-\sin y = \cos(-2y) = \cos 2y$$

$$\Rightarrow 1-\sin y = 1-2\sin^2 y$$

$$\Rightarrow 2\sin^2 y - \sin y = 0$$

$$\Rightarrow \sin y(2\sin y - 1) = 0$$

$$\Rightarrow \sin y = 0 \text{ or } \frac{1}{2}$$

$$\therefore x = 0 \text{ or } x = \frac{1}{2}$$

But, when $x = \frac{1}{2}$, it can be observed that:

$$\text{L.H.S.} = \sin^{-1}\left(1-\frac{1}{2}\right) - 2\sin^{-1}\frac{1}{2}$$

$$= \sin^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\frac{1}{2}$$

$$= -\sin^{-1}\frac{1}{2}$$

$$= -\frac{\pi}{6} \neq \frac{\pi}{2} \neq \text{R.H.S.}$$

$\therefore x = \frac{1}{2}$ is not the solution of the given equation.

Thus, x = 0.

Hence, the correct answer is **C**.

Question 17:

Solve $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\frac{x-y}{x+y}$ is equal to

(A) $\frac{\pi}{2}$ **(B).** $\frac{\pi}{3}$ **(C)** $\frac{\pi}{4}$ **(D)** $\frac{-3\pi}{4}$

Answer

$$\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\frac{x-y}{x+y}$$

$$=\tan^{-1}\left[\frac{\frac{x}{y}-\frac{x-y}{x+y}}{1+\left(\frac{x}{y}\right)\left(\frac{x-y}{x+y}\right)}\right] \qquad \left[\tan^{-1}y-\tan^{-1}y=\tan^{-1}\frac{x-y}{1+xy}\right]$$

$$=\tan^{-1}\left[\frac{\frac{x(x+y)-y(x-y)}{y(x+y)}}{\frac{y(x+y)+x(x-y)}{y(x+y)}}\right]$$

$$=\tan^{-1}\left(\frac{x^2+xy-xy+y^2}{xy+y^2+x^2-xy}\right)$$

$$=\tan^{-1}\left(\frac{x^2+y^2}{x^2+y^2}\right)=\tan^{-1}1=\frac{\pi}{4}$$

Hence, the correct answer is **C**.

*********** END **********