



Differentiation Ex 11.8 Q12

$$\text{Let } u = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Put $x = \tan\theta$, so

$$u = \tan^{-1}\left(\frac{2 \tan \theta}{1 - \tan^2 \theta}\right)$$

$$u = \tan^{-1}(\tan 2\theta) \quad \text{---(i)}$$

$$\text{Let } v = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$= \cos^{-1}\left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}\right)$$

$$v = \cos^{-1}(\cos 2\theta) \quad \text{---(ii)}$$

Here, $0 < x < 1$

$$\Rightarrow 0 < \tan \theta < 1$$

$$\Rightarrow 0 < \theta < \frac{\pi}{4}$$

So, from equation (i),

$$u = 2\theta \quad \left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \right]$$

$$u = 2 \tan^{-1} x \quad [\text{Since, } x = \tan \theta]$$

Differentiating it with respect to x ,

$$\frac{du}{dx} = \frac{2}{1+x^2} \quad \text{---(iii)}$$

From equation (ii),

$$v = \theta \quad \left[\text{Since, } \cos^{-1}(\cos \theta) = \theta, \text{ if } \theta \in [0, \pi] \right]$$

$$v = 2 \tan^{-1} x \quad [\text{Since, } x = \tan \theta]$$

Differentiating it with respect to x ,

$$\frac{dv}{dx} = \frac{2}{1+x^2} \quad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{1+x^2} \times \frac{1+x^2}{2}$$

$$\frac{du}{dv} = 1$$

Differentiation Ex 11.8 Q13

$$\begin{aligned}
 \text{Let } u &= \tan^{-1}\left(\frac{x-1}{x+1}\right) \\
 \text{Put } x &= \tan \theta, \text{ so} \\
 u &= \tan^{-1}\left(\frac{\tan \theta - 1}{\tan \theta + 1}\right) \\
 &= \tan^{-1}\left(\frac{\tan \theta - \frac{\tan \pi}{4}}{1 + \tan \theta \frac{\tan \pi}{4}}\right) \\
 u &= \tan^{-1}\left(\tan\left(\theta - \frac{\pi}{4}\right)\right) \quad \text{---(i)}
 \end{aligned}$$

$$\begin{aligned}
 \text{Here, } -\frac{1}{2} &< x < \frac{1}{2} \\
 \Rightarrow -\frac{1}{2} &< \tan \theta < \frac{1}{2} \\
 \Rightarrow -\tan^{-1}\left(\frac{1}{2}\right) &< \theta < \tan^{-1}\left(\frac{1}{2}\right)
 \end{aligned}$$

$$\begin{aligned}
 \text{So,} \\
 u &= \theta - \frac{\pi}{4} \quad \left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \right] \\
 u &= \tan^{-1} x - \frac{\pi}{4}
 \end{aligned}$$

Differentiating it with respect to x ,

$$\begin{aligned}
 \frac{du}{dx} &= \frac{1}{1+x^2} - 0 \\
 \frac{du}{dx} &= \frac{1}{1+x^2} \quad \text{---(ii)}
 \end{aligned}$$

And,

$$\begin{aligned}
 \text{Let } v &= \sin^{-1}(3x - 4x^3) \\
 \text{Put } x &= \sin \theta, \text{ so} \\
 v &= \sin^{-1}(3 \sin \theta - 4 \sin^3 \theta) \\
 v &= \sin^{-1}(\sin 3\theta) \quad \text{---(iii)}
 \end{aligned}$$

$$\begin{aligned}
 \text{Now, } -\frac{1}{2} &< x < \frac{1}{2} \\
 \Rightarrow -\frac{1}{2} &< \sin \theta < \frac{1}{2} \\
 \Rightarrow -\frac{1}{6} &< \theta < \frac{\pi}{6}
 \end{aligned}$$

So, from equation (iii),

$$\begin{aligned}
 v &= 3\theta \quad \left[\text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \right] \\
 v &= 3 \sin^{-1} x \quad [\text{Since, } x = \sin \theta]
 \end{aligned}$$

Differentiating it with respect to x ,

$$\frac{dv}{dx} = \frac{3}{\sqrt{1-x^2}} \quad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\begin{aligned}
 \frac{\frac{du}{dx}}{\frac{dv}{dx}} &= \frac{1}{1+x^2} \times \frac{\sqrt{1-x^2}}{3} \\
 \frac{du}{dv} &= \frac{\sqrt{1-x^2}}{3(1+x^2)}
 \end{aligned}$$

***** END *****

