



Definite Integrals Ex 20.2 Q5

$$\text{Let } a^2 + x^2 = t^2$$

Differentiating w.r.t. x , we get

$$2x dx = 2t dt$$

$$x dx = t dt$$

$$\text{Now, } x = 0 \Rightarrow t = 0$$

$$x = a \Rightarrow t = \sqrt{2}a$$

$$\begin{aligned}\therefore \int_0^a \frac{x dx}{\sqrt{a^2 + x^2}} &= \int_a^{\sqrt{2}a} \frac{t dt}{t} \\ &= \int_a^{\sqrt{2}a} dt \\ &= [t]_a^{\sqrt{2}a} \\ &= [\sqrt{2}a - a] \\ &= a(\sqrt{2} - 1)\end{aligned}$$

$$\therefore \int_0^a \frac{x}{\sqrt{a^2 + x^2}} dx = a(\sqrt{2} - 1)$$

Definite Integrals Ex 20.2 Q6

$$\text{Let } e^x = t$$

Differentiating w.r.t. x , we get

$$e^x dx = dt$$

$$\text{Now, } x = 0 \Rightarrow t = 1$$

$$x = 1 \Rightarrow t = e$$

$$\begin{aligned}\therefore \int_0^1 \frac{e^x}{1 + e^{2x}} dx &= \int_1^e \frac{dt}{1 + t^2} \\ &= [\tan^{-1} t]_1^e \\ &= [\tan^{-1} e - \tan^{-1} 1] \\ &= \tan^{-1} e - \frac{\pi}{4}\end{aligned}$$

$$\begin{aligned}\left[\because \int \frac{dt}{1 + t^2} = \tan^{-1} t \right] \\ \left[\because \tan \frac{\pi}{4} = 1 \right]\end{aligned}$$

$$\therefore \int_0^1 \frac{e^x}{1 + e^{2x}} dx = \tan^{-1} e - \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q7

Let $x^2 = t$

Differentiating w.r.t. x , we get

$$2x \, dx = dt$$

Now,

$$x = 0 \Rightarrow t = 0$$

$$x = 1 \Rightarrow t = 1$$

$$\begin{aligned}\therefore \int_0^1 x e^{x^2} dx &= \int_0^1 \frac{e^t dt}{2} \\ &= \frac{1}{2} \int_0^1 e^t dt \\ &= \frac{1}{2} [e^t]_0^1 \\ &= \frac{1}{2} [e^1 - e^0] \quad [\because e^0 = 1] \\ &= \frac{1}{2} (e - 1)\end{aligned}$$

$$\therefore \int_0^1 x e^{x^2} dx = \frac{1}{2} (e - 1)$$

Definite Integrals Ex 20.2 Q8

Let $\log x = t$

Differentiating w.r.t. x , we get

$$\frac{1}{x} dx = dt$$

Now,

$$x = 1 \Rightarrow t = 0$$

$$x = 3 \Rightarrow t = \log 3$$

$$\begin{aligned} & \int_1^3 \frac{\cos(\log x)}{x} dx \\ &= \int_0^{\log 3} \cos t \, dt \quad [\because \cos t = \sin t] \\ &= [\sin t]_0^{\log 3} \\ &= \sin(\log 3) - \sin 0 \\ &= \sin(\log 3) \end{aligned}$$

$$\int_1^3 \frac{\cos(\log x)}{x} dx = \sin(\log 3)$$

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