Exercise 2B

Question 16:

$\sqrt{3}$ and $-\sqrt{3}$ are the zeros of polynomial

$2x^4 - 3x^3 - 5x^2 + 9x - 3$

$\therefore \left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right) = x^2 - 3$ will divide the given polynomial completely

Dividing $2x^4 - 3x^3 - 5x^2 + 9x - 3$ by $x^2 - 3$, we get

$$
\begin{array}{r|l}
 & 2x^2 - 3x + 1 \\
\hline
x^2 - 3 & 2x^4 - 3x^3 - 5x^2 + 9x - 3 \\
 & 2x^4 \qquad\quad - 6x^2 \\
 & - \qquad\qquad + \\
\hline
 & \quad -3x^3 + x^2 + 9x \\
 & \quad -3x^3 \qquad + 9x \\
 & \quad + \qquad\quad - \\
\hline
 & \qquad\qquad x^2 \qquad - 3 \\
 & \qquad\qquad x^2 \qquad - 3 \\
 & \qquad\qquad - \qquad + \\
\hline
 & \qquad\qquad\quad 0
\end{array}
$$

$\therefore$ Quotient $q(x) = 2x^2 - 3x + 1 = 2x^2 - 2x - x + 1$

$= 2x(x - 1) - (x - 1) = (x - 1)(2x - 1)$

Other zeros of given polynomial are given by

$q(x) = 0 \Rightarrow (x - 1)(2x - 1) = 0 \quad \therefore x = 1, \frac{1}{2}$

Hence, zeros of given polynomial are $\sqrt{3}, -\sqrt{3}, 1, \frac{1}{2}$

Question 17:

Sum of $3+\sqrt{2}$ and $3-\sqrt{2} = (3+\sqrt{2})+(3-\sqrt{2}) = 6$

Product of $(3+\sqrt{2})$ and $(3-\sqrt{2})$

$$= (3+\sqrt{2})(3-\sqrt{2}) = 9-2 = 7$$

Polynomial whose zeros are $3+\sqrt{2}$ and $3-\sqrt{2}$ is

$x^2 - (\text{sum of zeros})x + (\text{product of zeros}) = x^2 - 6x + 7$

Dividing $p(x)$ by $x^2 - 6x + 7$

$$\begin{array}{r|l}
 & 2x^2 + x - 1 \\
\hline
x^2 - 6x + 7 & 2x^4 - 11x^3 + 7x^2 + 13x - 7 \\
 & 2x^4 - 12x^3 + 14x^2 \\
 & - \quad + \quad - \\
\hline
 & x^3 - 7x^2 + 13x \\
 & x^3 - 6x^2 + 7x \\
 & - \quad + \quad - \\
\hline
 & -x^2 + 6x - 7 \\
 & -x^2 + 6x - 7 \\
 & + \quad - \quad + \\
\hline
 & 0 \\
\hline
\end{array}$$

Quotient $= 2x^2 + x - 1$

$\therefore$ Other zeros of polynomial $p(x)$ are also the zeros of $q(x)$

$\therefore \quad q(x) = 2x^2 + x - 1 = 2x^2 + 2x - x - 1$

$$= 2x(x+1) - (x+1) = (x+1)(2x-1)$$

$q(x) = 0$

$\Rightarrow (x+1)(2x-1) = 0$

$\Rightarrow$ Either $x + 1 = 0$ or $2x - 1 = 0$

$\Rightarrow$ Either $x = -1$ or $x = \frac{1}{2}$

$\therefore$ The zeros of given polynomial $p(x)$ are

$\frac{1}{2}$, -1, $(3+\sqrt{2})$ and $(3-\sqrt{2})$

*********** END **********