Indefinite Integrals Ex 19.19 Q1

Let $I = \int \frac{x}{x^2+3x+2}dx$

Let $x = \lambda \frac{d}{dx}\left(x^2+3x+2\right)+\mu$

$= \lambda\left(2x+3\right)+\mu$

$x = \left(2\lambda\right)x+\left(3\lambda+\mu\right)$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \Rightarrow \quad \lambda = \frac{1}{2}$

$3\lambda+\mu = 0 \quad \Rightarrow \quad 3\left(\frac{1}{2}\right)+\mu = 0$

$\mu = -\frac{3}{2}$

so, $I = \int \frac{\frac{1}{2}\left(2x+3\right)-\frac{3}{2}}{x^2+3x+2}dx$

$$I = \frac{1}{2}\int\frac{2x+3}{x^2+3x+2}dx - \frac{3}{2}\int\frac{1}{x^2+3x+2}dx$$

$$= \frac{1}{2}\int\frac{2x+3}{x^2+3x+2}dx - \frac{3}{2}\int\frac{1}{x^2+2x\left(\frac{3}{2}\right)+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+2}dx$$

$$I = \frac{1}{2}\int\frac{2x+3}{x^2+3x+2}dx - \frac{3}{2}\int\frac{1}{\left(x+\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}dx$$

$$= \frac{1}{2}\log\left|x^2+3x+2\right| - \frac{3}{2}\times\frac{1}{2\left(\frac{1}{2}\right)}\log\left|\frac{x+\frac{3}{2}-\frac{1}{2}}{x+\frac{3}{2}+\frac{1}{2}}\right|+c \qquad \left[\text{since, } \int\frac{1}{a^2-x^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c\right]$$

$$I = \frac{1}{2}\log\left|x^2+3x+2\right| - \frac{3}{2}\log\left|\frac{x+1}{x+2}\right|+c$$

Indefinite Integrals Ex 19.19 Q2

Let $I = \int\frac{x+1}{x^2+x+3}dx$

Let $x+1 = \lambda\frac{d}{dx}\left(x^2+x+3\right)+\mu$

$x+1 = \lambda\left(2x+1\right)+\mu$

$x+1 = \left(2\lambda\right)x+\left(\lambda+\mu\right)$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \Rightarrow \quad \lambda = \frac{1}{2}$

$\lambda+\mu = 1 \quad \Rightarrow \quad \left(\frac{1}{2}\right)+\mu = 0$

$\mu = \frac{1}{2}$

so, $I = \int\frac{\frac{1}{2}\left(2x+1\right)+\frac{1}{2}}{x^2+x+3}dx$

$$I = \frac{1}{2}\int\frac{2x+1}{x^2+x+3}dx + \frac{1}{2}\int\frac{1}{x^2+2x\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+3}dx$$

$$I = \frac{1}{2}\int\frac{2x+1}{x^2+x+3}dx + \frac{1}{2}\int\frac{1}{\left(x+\frac{1}{2}\right)^2+\left(\frac{11}{4}\right)}dx$$

$$I = \frac{1}{2}\int\frac{2x+1}{x^2+x+3}dx + \frac{1}{2}\int\frac{1}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{11}}{2}\right)^2}dx$$

$$= \frac{1}{2}\log\left|x^2+x+3\right| + \frac{1}{2}\times\frac{1}{\left(\frac{\sqrt{11}}{2}\right)}\tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{11}}{2}}\right)+c \qquad \left[\text{since, } \int\frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$$

$$I = \frac{1}{2}\log\left|x^2+x+3\right| + \frac{1}{\sqrt{11}}\tan^{-1}\left(\frac{2x+1}{\sqrt{11}}\right)+c$$

Indefinite Integrals Ex 19.19 Q3

Let $I = \int \frac{x-3}{x^2+2x-4}dx$

Let $x - 3 = \lambda \frac{d}{dx}\left(x^2+2x-4\right)+\mu$

$= \lambda\left(2x+2\right)+\mu$

$x - 3 = \left(2\lambda\right)x + \left(2\lambda+\mu\right)$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \Rightarrow \quad \lambda = \frac{1}{2}$

$2\lambda + \mu = -3 \quad \Rightarrow \quad 2\left(\frac{1}{2}\right)+\mu = -3$

$\mu = -4$

so, $I = \int \frac{\frac{1}{2}\left(2x+2\right)-4}{x^2+2x-4}dx$

$I = \frac{1}{2}\int \frac{2x+2}{x^2+2x-4}dx - 4\int \frac{1}{x^2+2x+\left(1\right)^2-\left(1\right)^2-4}dx$

$I = \frac{1}{2}\int \frac{2x+2}{x^2+2x-4}dx - 4\int \frac{1}{\left(x+1\right)^2-\left(\sqrt{5}\right)}dx$

$I = \frac{1}{2}\log\left|x^2+2x-4\right| - 4\times\frac{1}{2\sqrt{5}}\log\left|\frac{x+1-\sqrt{5}}{x+1+\sqrt{5}}\right|+c \quad \left[\text{since, } \int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c\right]$

$I = \frac{1}{2}\log\left|x^2+2x-4\right| - \frac{2}{\sqrt{5}}\log\left|\frac{x+1-\sqrt{5}}{x+1+\sqrt{5}}\right|+c$

Indefinite Integrals Ex 19.19 Q4

Let $I = \int \frac{2x-3}{x^2+6x+13}dx$

Let $2x - 3 = \lambda \frac{d}{dx}\left(x^2+6x+13\right)+\mu$

$= \lambda\left(2x+6\right)+\mu$

$2x - 3 = \left(2\lambda\right)x + \left(6\lambda+\mu\right)$

Comparing the coefficients of like powers of x,

$2\lambda = 2 \Rightarrow \quad \lambda = 1$

$6\lambda + \mu = -3 \quad \Rightarrow \quad 6\left(1\right)+\mu = -3$

$\mu = -9$

so, $I = \int \frac{1\left(2x+6\right)-9}{x^2+6x+13}dx$

$I = \int \frac{2x+6}{x^2+6x+13}dx - 9\int \frac{1}{x^2+2x\left(3\right)+\left(3\right)^2-\left(3\right)^2+13}dx$

$I = \int \frac{2x+6}{x^2+6x+13}dx - 9\int \frac{1}{\left(x+3\right)^2+\left(2\right)^2}dx$

$= \log\left|x^2+6x+13\right| - 9\times\frac{1}{2}\tan^{-1}\left(\frac{x+3}{2}\right)+c \quad \left[\text{since, } \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$

$I = \log\left|x^2+6x+13\right| - \frac{9}{2}\tan^{-1}\left(\frac{x+3}{2}\right)+c$

*********** END **********