Exercise 2B

Question 1:

$$p(x) = x^3 - 2x^2 - 5x + 6$$

$$\therefore p(3) = (3)^3 - 2(3)^2 - 5(3) + 6$$
$$= 27 - 18 - 15 + 6 = 0$$

$$p(-2) = (-2)^3 - 2(-2)^2 - 5(-2) + 6$$
$$= -8 - 8 + 10 + 6 = 0$$

$$p(1) = (1)^3 - 2(1)^2 - 5(1) + 6$$
$$= 1 - 2 - 5 + 6 = 0$$

Thus, 3, -2, 1 are the zeros of $p(x) = x^3 - 2x^2 - 5x + 6$

$\therefore \alpha = 3, \beta = -2$ and $\gamma = 1$

Comparing the given polynomial with

$p(x) = ax^3 + bx^2 + cx + d,$

We get, $a = 1, b = -2, c = -5$ and $d = 6$

Now, $(\alpha + \beta + \gamma) = (3 - 2 + 1) = 2 = -\dfrac{b}{a}$

$$(\alpha\beta + \beta\gamma + \gamma\alpha) = [3 \times (-2) + (-2) \times 1 + (1) \times 3]$$
$$= (-6 - 2 + 3) = -5 = \frac{c}{a}$$

and $\alpha\beta\gamma = [3 \times (-2) \times 1] = -6 = \dfrac{-d}{a}$

Question 2:

$$p(x) = 3x^3 - 10x^2 - 27x + 10$$

$$\therefore \quad p(5) = 3(5)^3 - 10(5)^2 - 27(5) + 10$$

$$= 375 - 250 - 135 + 10 = 0$$

$$p(-2) = 3(-2)^3 - 10(-2)^2 - 27(-2) + 10$$

$$= -24 - 40 + 54 + 10 = 0$$

$$p\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right)^3 - 10\left(\frac{1}{3}\right)^2 - 27\left(\frac{1}{3}\right) + 10$$

$$= \frac{1}{9} - \frac{10}{9} - 9 + 10 = 0$$

$\therefore$ Then, $5, -2, \frac{1}{3}$ zeros of

$$p(x) = 3x^3 - 10x^2 - 27x + 10$$

$$\therefore \alpha = 5,\ \beta = -2,\ \gamma = \frac{1}{3}$$

Comparing the given polynomial with

$$p(x) = ax^3 + bx^2 + cx + d$$

We get $a = 3$, $b = -10$, $c = -27$ and $d = 10$

Now, $(\alpha + \beta + \gamma) = \left(5 - 2 + \frac{1}{3}\right) = \frac{10}{3} = \frac{-b}{a}$

$$(\alpha\beta + \beta\gamma + \gamma\alpha) = \left[5 \times (-2) + (-2) \times \frac{1}{3} + \left(\frac{1}{3}\right) \times 5\right]$$

$$= \left(-10 - \frac{2}{3} + \frac{5}{3}\right) = \frac{-27}{3} = \frac{c}{a}$$

and $\alpha\beta\gamma = \left[5 \times (-2) \times \frac{1}{3}\right] = \frac{-10}{3} = \frac{-d}{a}$

*********** END **********