Indefinite Integrals Ex 19.21 Q11

Let $I = \int \frac{x+1}{\sqrt{x^2+1}}dx$

Let $x+1 = \lambda \frac{d}{dx}\left(x^2+1\right) + \mu$

$x+1 = \lambda\left(2x\right) + \mu$

Comparing the coefficients of like powers of x,

$2\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$\Rightarrow \quad \mu = 1$

so, $I = \int \frac{\frac{1}{2}\left(2x\right)+1}{\sqrt{x^2+1}}dx$

$= \frac{1}{2}\int \frac{\left(2x\right)}{\sqrt{x^2+1}}dx + \int \frac{1}{\sqrt{x^2+1}}dx$

$I = \frac{1}{2}\times 2\sqrt{x^2+1} + \log\left|x+\sqrt{x^2+1}\right| + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{x^2+1}}dx = \log\left|x+\sqrt{x^2-a^2}\right| + c\right]$

$I = \sqrt{x^2+1} + \log\left|x+\sqrt{x^2+1}\right| + c$

Indefinite Integrals Ex 19.21 Q12

Let $I = \int \frac{2x+5}{\sqrt{x^2+2x+5}}dx$

Let $2x+5 = \lambda \frac{d}{dx}\left(x^2+2x+5\right) + \mu$

$= \lambda\left(2x+2\right) + \mu$

$2x+5 = \left(2\lambda\right)x + 2\lambda + \mu$

Comparing the coefficients of like powers of x,

$2\lambda = 2 \quad \Rightarrow \quad \lambda = 1$

$2\lambda + \mu = 5 \quad \Rightarrow \quad 2\left(1\right) + \mu = 5$

$\Rightarrow \quad \mu = 3$

so, $I = \int \frac{\left(2x+2\right)+3}{\sqrt{x^2+2x+5}}dx$

$= \int \frac{\left(2x+3\right)}{\sqrt{x^2+2x+5}}dx + 3\int \frac{1}{\sqrt{x^2+2x+\left(1\right)^2-\left(1\right)^2+5}}dx$

$I = \int \frac{2x+3}{\sqrt{x^2+2x+5}}dx + 3\int \frac{1}{\sqrt{\left(x+1\right)^2+\left(2\right)^2}}dx$

$I = 2\sqrt{x^2+2x+5} + 3\log\left|x+1+\sqrt{\left(x+1\right)^2+\left(2\right)^2}\right| + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{x^2+a^2}}dx = \log\left|x+\sqrt{x^2+a^2}\right| + c\right]$

$I = 2\sqrt{x^2+2x+5} + 3\log\left|x+1+\sqrt{x^2+2x+5}\right| + c$

Indefinite Integrals Ex 19.21 Q13

Let $I = \int \frac{3x+1}{\sqrt{5-2x-x^2}}dx$

Let $3x+1 = \lambda \frac{d}{dx}\left(5-2x-x^2\right) + \mu$

$= \lambda\left(-2-2x\right) + \mu$

$3x+1 = \left(-2\lambda\right)x - 2\lambda + \mu$

Comparing the coefficients of like powers of x,

$-2\lambda = 3 \quad \Rightarrow \quad \lambda = -\frac{3}{2}$

$-2\lambda + \mu = 1 \quad \Rightarrow \quad -2\left(-\frac{3}{2}\right) + \mu = 1$

$\Rightarrow \quad \mu = -2$

so, $I = \int \frac{-\frac{3}{2}\left(-2-2x\right)-2}{\sqrt{5-2x-x^2}}dx$

$= -\frac{3}{2}\int \frac{\left(-2-2x\right)}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{-\left[x^2+2x-5\right]}}dx$

$I = -\frac{3}{2}\int \frac{-2-2x}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{-\left[x^2+2x+\left(1\right)^2-\left(1\right)^2+5\right]}}dx$

$I = -\frac{3}{2}\int \frac{-2-2x}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{-\left[\left(x+1\right)^2-\left(\sqrt{6}\right)^2\right]}}dx$

$I = -\frac{3}{2}\int \frac{-2-2x}{\sqrt{5-2x-x^2}}dx - 2\int \frac{1}{\sqrt{\left(\sqrt{6}\right)^2-\left(x+1\right)^2}}dx$

$I = -\frac{3}{2}\times 2\sqrt{5-2x-x^2} - 2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = -3\sqrt{5-2x-x^2} - 2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right) + c$

Indefinite Integrals Ex 19.21 Q14

Let $I = \int \sqrt{\frac{1-x}{1+x}}dx$

$= \int \sqrt{\frac{1-x}{1+x} \times \frac{1-x}{1-x}}dx$

$= \int \frac{1-x}{\sqrt{1-x^2}}dx$

Let $1-x = \lambda \frac{d}{dx}\left(1-x^2\right) + \mu$

$= \lambda\left(-2x\right) + \mu$

$1-x = \left(-2\lambda\right)x + \mu$

Comparing the coefficients of like powers of x,

$-2\lambda = -1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$\Rightarrow \quad \mu = 1$

so, $I = \int \frac{\frac{1}{2}\left(-2x\right)+1}{\sqrt{1-x^2}}dx$

$= \frac{1}{2}\int \frac{\left(-2x\right)}{\sqrt{1-x^2}}dx + \int \frac{1}{\sqrt{1-x^2}}dx$

$I = \frac{1}{2} \times 2\sqrt{1-x^2} + \sin^{-1}x + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x} + c,\ \int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}\left(x\right) + c\right]$

$I = \sqrt{1-x^2} + \sin^{-1}x + c$

*********** END **********