



Differentiation Ex 11.2 Q16

$$\text{Let } y = \sqrt{\frac{1+\sin x}{1-\sin x}}$$

Differentiate it with respect to x ,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \left(\frac{1+\sin x}{1-\sin x} \right)^{\frac{1}{2}} \\ &= \frac{1}{2} \left(\frac{1+\sin x}{1-\sin x} \right)^{\frac{1}{2}-1} \frac{d}{dx} \left(\frac{1+\sin x}{1-\sin x} \right) \\ &= \frac{1}{2} \left(\frac{1+\sin x}{1-\sin x} \right)^{-\frac{1}{2}} \left[\frac{(1-\sin x)(\cos x) - (1+\sin x)(-\cos x)}{(1-\sin x)^2} \right] \\ &= \frac{1}{2} \left(\frac{1+\sin x}{1-\sin x} \right)^{-\frac{1}{2}} \left[\frac{\cos x - \cos x \sin x + \cos x + \sin x \cos x}{(1-\sin x)^2} \right] \\ &= \frac{1}{2} \times \frac{2 \cos x}{\sqrt{1+\sin x} (1-\sin x)^{\frac{3}{2}}} \\ &= \frac{\cos x}{\sqrt{1+\sin x} (1-\sin x)^{\frac{3}{2}}} \\ &= \frac{\cos x}{\sqrt{1+\sin x} \sqrt{1-\sin x} (1-\sin x)} \\ &= \frac{\cos x}{\sqrt{1-\sin^2 x} (1-\sin x)} \\ &= \frac{\cos x}{\cos x (1-\sin x)} \quad \left[\text{Using } 1 - \sin^2 x = \cos^2 x \right] \\ &= \frac{1}{(1-\sin x)} \times \frac{(1+\sin x)}{(1+\sin x)} \\ &= \frac{(1+\sin x)}{(1-\sin^2 x)} \\ &= \frac{1+\sin x}{\cos^2 x} \end{aligned}$$

$$\text{Thus, } \frac{dy}{dx} = \frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}$$

$$\Rightarrow \frac{dy}{dx} = \sec^2 x + \tan x \sec x$$

$$\Rightarrow \frac{dy}{dx} = \sec x [\tan x + \sec x]$$

Differentiation Ex 11.2 Q17

$$\text{Let } y = \sqrt{\frac{1-x^2}{1+x^2}}$$

$$y = \left(\frac{1-x^2}{1+x^2}\right)^{\frac{1}{2}}$$

Differentiate it with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\frac{1-x^2}{1+x^2}\right)^{\frac{1}{2}} \\ &= \frac{1}{2} \left(\frac{1-x^2}{1+x^2}\right)^{\frac{1}{2}-1} \frac{d}{dx} \left(\frac{1-x^2}{1+x^2}\right) && \text{[Using chain rule]} \\ &= \frac{1}{2} \left(\frac{1-x^2}{1+x^2}\right)^{-\frac{1}{2}} \left[\frac{(1+x^2) \frac{d}{dx}(1-x^2) - (1-x^2) \frac{d}{dx}(1+x^2)}{(1+x^2)^2} \right] && \text{[Using quotient rule]} \\ &= \frac{1}{2} \left(\frac{1+x^2}{1-x^2}\right)^{\frac{1}{2}} \left[\frac{(1+x^2)(-2x) - (1-x^2)(2x)}{(1+x^2)^2} \right] \\ &= \frac{1}{2} \left(\frac{1+x^2}{1-x^2}\right)^{\frac{1}{2}} \left[\frac{-2x - 2x^3 - 2x + 2x^3}{(1+x^2)^2} \right] \\ &= \frac{1}{2} \frac{-4x}{\sqrt{1-x^2} (1+x^2)^{\frac{3}{2}}}\end{aligned}$$

So,

$$\frac{d}{dx} \left(\sqrt{\frac{1-x^2}{1+x^2}} \right) = \frac{-2x}{\sqrt{1-x^2} (1+x^2)^{\frac{3}{2}}}.$$

Differentiation Ex 11.2 Q18

$$\text{Let } y = (\log \sin x)^2$$

Differentiate with respect to x ,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} (\log \sin x)^2 \\ &= 2 (\log \sin x) \frac{d}{dx} (\log \sin x) && \text{[Using chain rule]} \\ &= 2 (\log \sin x) \times \frac{1}{\sin x} \frac{d}{dx} (\log x) \\ &= 2 (\log \sin x) \times \frac{1}{\sin x} \times \frac{1}{x} \\ &= \frac{2 \log \sin x}{x \sin x}\end{aligned}$$

So,

$$\frac{d}{dx} (\log \sin x)^2 = \frac{2 \log \sin x}{x \sin x}$$

Differentiation Ex 11.2 Q19

$$\text{Let } y = \sqrt{\frac{1+x}{1-x}}$$

$$\Rightarrow y = \left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$$

Differentiate it with respect to x ,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \left(\frac{1+x}{1-x}\right)^{\frac{1}{2}} \\ &= \frac{1}{2} \left(\frac{1+x}{1-x}\right)^{\frac{1}{2}-1} \frac{d}{dx} \left(\frac{1+x}{1-x}\right) \end{aligned} \quad \text{[Using chain rule]}$$

$$= \frac{1}{2} \left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}} \left[\frac{(1-x) \frac{d}{dx}(1+x) - (1+x) \frac{d}{dx}(1-x)}{(1-x)^2} \right] \quad \text{[Using chain rule]}$$

$$= \frac{1}{2} \left(\frac{1-x}{1+x}\right)^{\frac{1}{2}} \left[\frac{(1-x)(1) - (1+x)(-1)}{(1-x)^2} \right]$$

$$= \frac{1}{2} \left(\frac{1-x}{1+x}\right)^{\frac{1}{2}} \left[\frac{1-x+1+x}{(1-x)^2} \right]$$

$$= \frac{1}{2} \left(\frac{1-x}{1+x}\right)^{\frac{1}{2}} \times \frac{2}{(1-x)^2}$$

$$= \frac{1}{\sqrt{1+x} (1-x)^{3/2}}$$

So,

$$\frac{d}{dx} \left(\sqrt{\frac{1+x}{1-x}} \right) = \frac{1}{\sqrt{1+x} (1-x)^{3/2}}$$

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