Functions Ex 2.5 Q15

We have given that

$f : R \to (-1, 1)$ defined by

$f(x) = \dfrac{10^x - 10^{-x}}{10^x + 10^{-x}}$ is invertible

let $f(x) = y$

$$\Rightarrow \quad \frac{10^x - 10^{-x}}{10^x + 10^{-x}} = y$$

$$\Rightarrow \quad \frac{10^{2x} - 1}{10^{2x} - 1} = y$$

$$\Rightarrow \quad 10^{2x} - 1 = y\left(10^{2x} + 1\right)$$

$$\Rightarrow \quad 10^{2x} - 10^{2x}y = y + 1$$

$$\Rightarrow \quad 10^{2x}(1 - y) = y + 1$$

$$\Rightarrow \quad 10^{2x} = \frac{y + 1}{1 - y}$$

$$\Rightarrow \quad 2x = log_{10}\left(\frac{1 + y}{1 - y}\right)$$

$$x = \frac{1}{2} log_{10}\left(\frac{1 + y}{1 - y}\right)$$

$$\therefore \quad f^{-1}(x) = \frac{1}{2} log_{10}\left(\frac{1 + x}{1 - x}\right)$$

Functions Ex 2.5 Q16

We have given that

$f : R \to (0, 2)$ defined by

$f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}} + 1$ is invertible.

let $f(x) = y$

$$\Rightarrow \quad \frac{e^x - e^{-x}}{e^x + e^{-x}} + 1 = y$$

$$\Rightarrow \quad \frac{2e^x}{e^x + e^{-x}} = y$$

$$\Rightarrow \quad \frac{2e^{2x}}{e^{2x} + 1} = y$$

$$\Rightarrow \quad 2e^{2x} = y\left(e^{2x} + 1\right)$$

$$\Rightarrow \quad e^{2x}(2 - y) = y$$

$$\Rightarrow \quad e^{2x} = \frac{y}{2 - y} \Rightarrow x = \frac{1}{2} \log_e \left(\frac{y}{2 - y}\right)$$

$$\Rightarrow \quad f^{-1}(x) = \frac{1}{2} \log_e \left(\frac{x}{2 - x}\right)$$

Functions Ex 2.5 Q17

Given: that

$f:[-1,\infty]\to[-1,\infty]$ is a function

given by $f(x)=(x+1)^2-1$

In order to show that f in invertible, we need to prove that f in bijective.

Injective: let $x, y \in [-1,\infty]$, Such that

$f(x)=f(y)$

$\Rightarrow \quad (x+1)^2-1=(y+1)^2-1$

$\Rightarrow \quad (x+1)^2=(y+1)^2$

$\Rightarrow \quad x+1=y+1 \qquad [x, y \in [-1,\infty]]$

$\Rightarrow \quad x=y$

$\Rightarrow \quad f$ is one-one

Surjectivity: let $y \in [-1,\infty]$ be arbitrary

such that $f(x)=y$

$\Rightarrow \quad (x+1)^2-1=y$

$= \quad (x+1)^2=y+1$

$\Rightarrow \quad x+1=\sqrt{y+1}$

$\Rightarrow \quad x=\sqrt{y+1}-1 \in [-1,\infty]$

So, for each $y \in [-1,\infty]$ (co-domain) there exist $x=\sqrt{y+1}-1 \in [-1,\infty]$ (domain)

$\therefore \quad f$ is onto

Thus, f is bijective $\Rightarrow$ $\quad f$ is invertible.

Now,

$f(x)=f^{-1}(x)$

$\Rightarrow \quad (x+1)^2-1=\sqrt{x+1}-1$

$\Rightarrow \quad (x+1)^2-\sqrt{x+1}=0$

$\Rightarrow \quad \sqrt{x+1}\left((x+1)^{3/2}-1\right)=0$

$\Rightarrow \quad \sqrt{x+1}=0$ or $(x+1)^{3/2}-1=0$

$\Rightarrow \quad x=-1$ or $x=0$

$\therefore \quad x=0,-1$

Hence, $S=\{0,-1\}$

Functions Ex 2.5 Q18

$A = \{x \in R : -1 \le x \le 1\}$ and $f : A \to A$, $g : A \to A$ are two functions defined by $f(x) = x^2$ and $g(x) = sin\left(\frac{\pi x}{2}\right)$

Here, $f : A \to A$ is defined by

$$f(x) = x^2$$

Clearly f in not injective, $\because f(1) = f(-1) = 1$

So, f is not bijective and hence not invertible.

Hence, f^{-1} does not exist

Now, $g : A \to A$ defined by

$$g(x) = sin\left(\frac{\pi x}{2}\right)$$

Injectivity: Let $x_1 = x_2$

$$\Rightarrow \quad \frac{\pi x_1}{2} = \frac{\pi x_2}{2}$$

$$\Rightarrow \quad sin\left(\frac{\pi x_1}{2}\right) = sin\left(\frac{\pi x_2}{2}\right) \qquad [\because -1 \le x \le 1]$$

$$\Rightarrow \quad g(x_1) = g(x_2)$$

$\Rightarrow$ g is one-one ....(i)

Surjectivity: let y be aribitrary such that

$$g(x) = y$$

$$\Rightarrow \quad sin\left(\frac{\pi x}{2}\right) = y$$

$$\Rightarrow \quad \frac{\pi x}{2} = sin^{-1} y$$

$$\Rightarrow \quad x = \frac{2}{\pi} sin^{-1} y = [-1, 1]$$

Thus, for each y in codomain, there exists x in domain, such that

$$g(x) = y$$

$\Rightarrow$ g is surjective ....(ii)

From (i) & (ii)

*********** END **********