Mathematical Induction Ex 12.2 Q18

Let $P(n): a+(a+d)+(a+2d)+\ldots+(a+(n-1)d)=\frac{n}{2}[2a+(n-1)d]$

For $n=1$

$$a=\frac{1}{2}[2a+(1-1)d]$$
$$a=a$$

$\Rightarrow \quad P(n)$ is true for $n=1$

Let $P(n)$ is true for $n=k$, so

$$a+(a+d)+(a+2d)+\ldots+(a+(k-1)d)=\frac{k}{2}[2a+(k-1)d] \qquad ---(1)$$

We have to show that,

$$a+(a+d)+(a+2d)+\ldots+(a+(k-1)d)+(a+(k)d)=\frac{(k+1)}{2}[2a+kd]$$

Now,

$$\{a+(a+d)+(a+2d)+\ldots+(a+(k-1)d)\}+(a+kd)$$

$$=\frac{k}{2}[2a+(k-1)d]+(a+kd) \qquad [\text{Using equation }(1)]$$
$$=\frac{2ka+k(k-1)d+2(a+kd)}{2}$$
$$=\frac{2ka+k^2d-kd+2a+2kd}{2}$$
$$=\frac{2ka+2a+k^2d+kd}{2}$$
$$=\frac{2a(k+1)+d(k^2+k)}{2}$$

$$=\frac{(k+1)}{2}[2a+kd]$$

$\Rightarrow \quad P(n)$ is true for $n=k+1$

$\Rightarrow \quad P(n)$ is true for all $n\in N$ by PMI

Mathematical Induction Ex 12.2 Q19

Let $P(n): \left(5^{2n} - 1\right)$ is divisible by 24

For $n = 1$

$5^2 - 1 = 24$

Which is divisible by 24

$\Rightarrow \quad P(n)$ is true for $n = 1$

Let $P(n)$ is true for $n = k$

$\Rightarrow \quad \left(5^{2k} - 1\right)$ is divisible by 24

$$\Rightarrow \quad 5^{2k} - 1 = 24\lambda \qquad ---(1)$$

We have to show that,

$\left(5^{2k} - 1\right)$ is divisible by 24

$5^{2(k+1)} - 1 = 24\mu$

Now,

$5^{2(k+1)} - 1$

$= 5^{2k}.5^2 - 1$

$= 25.5^{2k} - 1$

$$= 25\left(24\lambda + 1\right) - 1 \qquad \left[\text{Using equation } (1)\right]$$

$= 25.24\lambda + 24$

$= 24\left(25\lambda + 1\right)$

$= 24\mu$

$\Rightarrow \quad P(n)$ is true for $n = k + 1$

$\Rightarrow \quad P(n)$ is true for all $n \in N$ by PMI

Mathematical Induction Ex 12.2 Q20

Let $P(n): 3^{2n} + 7$ is divisible by 8

For $n = 1$

$3^2 + 7 = 16$

Which is divisible by 8

$\Rightarrow \quad P(n)$ is true for $n = 1$

Let $P(n)$ is true for $n = k$, so

$3^{2k} + 7$ is divisible by 8

$\Rightarrow \quad 3^{2k} + 7 = 8\lambda \qquad ---(1)$

We have to show that,

$3^{2(k+1)} + 7$ is divisible by 8

$3^{2(k+1)} + 7 = 8\mu$

Now,

$3^{2(k+1)} + 7$

$= 3^{2k}.3^2 + 7$

$= 9.3^{2k} + 7$

$= 9.(8\lambda - 7) + 7$

$= 72\lambda - 56$

$= 8(9\lambda - 7)$

$= 8\mu$

$\Rightarrow \quad P(n)$ is true for $n = k + 1$

$\Rightarrow \quad P(n)$ is true for all $n \in N$ by PMI

*********** END **********