Exercise 2D

Question 15:

Let $f(x) = x^3 - 3x^2 - 13x + 15$

Now, $x^2 + 2x - 3 = x^2 + 3x - x - 3$

$= x (x + 3) - 1 (x + 3)$

$= (x + 3) (x - 1)$

Thus, f(x) will be exactly divisible by $x^2 + 2x - 3 = (x + 3) (x - 1)$ if $(x + 3)$ and $(x - 1)$ are both factors of f(x), so by factor theorem, we should have $f(-3) = 0$ and $f(1) = 0$.

Now, $f(-3) = (-3)^3 - 3 (-3)^2 - 13 (-3) + 15$

$= -27 - 3 \times 9 + 39 + 15$

$= -27 - 27 + 39 + 15$

$= -54 + 54 = 0$

And, $f(1) = 1^3 - 3 \times 1^2 - 13 \times 1 + 15$

$= 1 - 3 - 13 + 15$

$= 16 - 16 = 0$

$\therefore f(-3) = 0$ and $f(1) = 0$

So, $x^2 + 2x - 3$ divides f(x) exactly.

Question 16:

Let $f(x) = (x^3 + ax^2 + bx + 6)$

Now, by remainder theorem, f(x) when divided by (x - 3) will leave a remainder as f(3).

So, $f(3) = 3^3 + a \times 3^2 + b \times 3 + 6 = 3$

$\Rightarrow 27 + 9a + 3b + 6 = 3$

$\Rightarrow 9a + 3b + 33 = 3$

$\Rightarrow 9a + 3b = 3 - 33$

$\Rightarrow 9a + 3b = -30$

$\Rightarrow 3a + b = -10$ ....(i)

Given that (x - 2) is a factor of f(x).

By the Factor Theorem, (x - a) will be a factor of f(x) if f(a) = 0 and therefore f(2) = 0.

$f(2) = 2^3 + a \times 2^2 + b \times 2 + 6 = 0$

$\Rightarrow 8 + 4a + 2b + 6 = 0$

$\Rightarrow 4a + 2b = -14$

$\Rightarrow 2a + b = -7$ ....(ii)

Subtracting (ii) from (i), we get,

$\Rightarrow a = -3$

Substituting the value of a = -3 in (i), we get,

$\Rightarrow 3(-3) + b = -10$

$\Rightarrow -9 + b = -10$

$\Rightarrow b = -10 + 9$

$\Rightarrow b = -1$

$\therefore a = -3$ and $b = -1$.

*********** END **********