



### Indefinite Integrals Ex 19.29 Q1

$$\text{Let } I = \int (x+1) \sqrt{x^2 - x + 1} dx \quad \dots (1)$$

$$\begin{aligned} \text{Let } x+1 &= \lambda \frac{d}{dx} (x^2 - x + 1) + \mu \\ &= \lambda (2x - 1) + \mu \end{aligned}$$

Equating similar terms, we get,

$$\begin{aligned} 2\lambda &= 1 \Rightarrow \lambda = \frac{1}{2} \\ -\lambda + \mu &= 1 \end{aligned}$$

$$\Rightarrow \mu = 1 + \lambda = 1 + \frac{1}{2} = \frac{3}{2} \therefore \mu = \frac{3}{2}$$

So,

$$\begin{aligned} I &= \int \left( \frac{1}{2} (2x - 1) + \frac{3}{2} \right) \sqrt{x^2 - x + 1} dx \\ &= \frac{1}{2} \int (2x - 1) \sqrt{x^2 - x + 1} dx + \frac{3}{2} \int \sqrt{x^2 - x + 1} dx \end{aligned}$$

$$\begin{aligned} \text{Let } x^2 - x + 1 &= t \\ \Rightarrow (2x - 1) dx &= dt \end{aligned}$$

$$= \frac{1}{2} \int \sqrt{t} dt + \frac{3}{2} \int \sqrt{\left(x - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx$$

$$= \frac{1}{2} \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{3}{2} \left\{ \frac{\left(x - \frac{1}{2}\right)}{2} \sqrt{x^2 - x + 1} + \frac{3}{8} \log \left| \left(x - \frac{1}{2}\right) + \sqrt{x^2 - x + 1} \right| \right\}$$

$$\Rightarrow I = \frac{1}{3} t^{\frac{3}{2}} + \frac{3}{8} (2x - 1) \sqrt{x^2 - x + 1} + \frac{9}{16} \log \left| \left(x - \frac{1}{2}\right) + \sqrt{x^2 - x + 1} \right| + c$$

Hence,

$$I = \frac{1}{3} (x^2 - x + 1)^{\frac{3}{2}} + \frac{3}{8} (2x - 1) \sqrt{x^2 - x + 1} + \frac{9}{16} \log \left| \left(x - \frac{1}{2}\right) + \sqrt{x^2 - x + 1} \right| + c$$

### Indefinite Integrals Ex 19.29 Q2

$$\text{Let } I = \int (x+1) \sqrt{2x^2+3} dx$$

$$\begin{aligned} \text{Let } x+1 &= \lambda \frac{d}{dx} (2x^2+3) + \mu \\ &= \lambda (4x) + \mu \end{aligned}$$

Equating similar terms, we get,

$$4\lambda = 1 \Rightarrow \lambda = \frac{1}{4}$$

$$\mu = 1$$

$$\therefore I = \int \frac{1}{4} (4x) \sqrt{2x^2+3} dx + \int 1 \cdot \sqrt{2x^2+3} dx$$

$$\begin{aligned} \text{Let } 2x^2+3 &= t \\ \Rightarrow 4x dx &= dt \end{aligned}$$

$$\begin{aligned} I &= \frac{1}{4} \int \sqrt{t} dt + \sqrt{2} \int \sqrt{x^2 + \frac{3}{2}} dx \\ &= \frac{1}{4} \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \sqrt{2} \left\{ \frac{x}{2} \sqrt{x^2 + \frac{3}{2}} + \frac{3}{4} \log \left| x + \sqrt{x^2 + \frac{3}{2}} \right| \right\} + c \end{aligned}$$

Hence,

$$I = \frac{1}{6} (2x^2+3)^{\frac{3}{2}} + \frac{x}{2} \sqrt{2x^2+3} + \frac{3}{2\sqrt{2}} \log \left| x + \sqrt{x^2 + \frac{3}{2}} \right| + c$$

Indefinite Integrals Ex 19.29 Q3

$$\text{Let } I = \int (2x - 5) \sqrt{2 + 3x - x^2} dx$$

$$\begin{aligned} \text{Let } 2x - 5 &= \lambda \frac{d}{dx} (2 + 3x - x^2) + \mu \\ &= \lambda (3 - 2x) + \mu \end{aligned}$$

Equating similar terms, we get,

$$\begin{aligned} -2\lambda &= 2 & \Rightarrow & \lambda = -1 \\ 3\lambda + \mu &= -5 & \Rightarrow & \mu = -5 - 3\lambda = -2 \end{aligned}$$

$$\therefore \mu = -2$$

So,

$$\begin{aligned} I &= \int (-1(3 - 2x) - 2) \sqrt{2 + 3x - x^2} dx \\ &= -\int (3 - 2x) \sqrt{2 + 3x - x^2} dx - 2 \int \sqrt{2 + 3x - x^2} dx \end{aligned}$$

$$\begin{aligned} \text{Let } 2 + 3x - x^2 &= t \\ \Rightarrow (3 - 2x) dx &= dt \end{aligned}$$

$$\begin{aligned} I &= -\int \sqrt{t} dt - 2 \int \sqrt{\frac{17}{4} - \left(\frac{9}{4} - 3x - x^2\right)} dx \\ &= -\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - 2 \int \sqrt{\left(\frac{\sqrt{17}}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2} dx \end{aligned}$$

$$\Rightarrow I = -\frac{2}{3} (2 + 3x - x^2)^{\frac{3}{2}} - 2 \left\{ \frac{\left(x - \frac{3}{2}\right)}{2} \sqrt{2 + 3x - x^2} + \frac{17}{8} \sin^{-1} \left( \frac{x - \frac{3}{2}}{\frac{\sqrt{17}}{2}} \right) \right\} + c$$

Hence,

$$I = -\frac{2}{3} (2 + 3x - x^2)^{\frac{3}{2}} - \frac{(2x - 3)}{2} \sqrt{2 + 3x - x^2} - \frac{17}{8} \sin^{-1} \left( \frac{2x - 3}{\sqrt{17}} \right) + c$$

Indefinite Integrals Ex 19.29 Q4

$$\text{Let } I = \int (x+2) \sqrt{x^2+x+1} dx$$

$$\begin{aligned} \text{Let } x+2 &= \lambda \frac{d}{dx} (x^2+x+1) + \mu \\ &= \lambda (2x+1) + \mu \end{aligned}$$

Equating similar terms, we get,

$$2\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$$

$$\lambda + \mu = 2 \quad \Rightarrow \quad \mu = 2 - \lambda = \frac{3}{2}$$

$$\therefore \quad \mu = \frac{3}{2}$$

$$\begin{aligned} \therefore \quad I &= \int \left( \frac{1}{2} (2x+1) + \frac{3}{2} \right) \sqrt{x^2+x+1} dx \\ &= \frac{1}{2} \int (2x+1) \sqrt{x^2+x+1} + \frac{3}{2} \int \sqrt{x^2+x+1} dx \end{aligned}$$

$$\begin{aligned} \text{Let } x^2+x+1 &= t \\ (2x+1) dx &= dt \end{aligned}$$

$$\begin{aligned} \therefore \quad I &= \frac{1}{2} \int \sqrt{t} dt + \frac{3}{2} \int \sqrt{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx \\ \Rightarrow \quad I &= \frac{1}{2} \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{3}{2} \left\{ \frac{\left(x + \frac{1}{2}\right)}{2} \sqrt{x^2+x+1} + \frac{3}{8} \log \left| \left(x + \frac{1}{2}\right) + \sqrt{x^2+x+1} \right| \right\} + c \end{aligned}$$

Hence,

$$I = \frac{1}{3} (x^2+x+1)^{\frac{3}{2}} + \frac{3}{8} (2x+1) \sqrt{x^2+x+1} + \frac{9}{16} \log \left| \left(x + \frac{1}{2}\right) + \sqrt{x^2+x+1} \right| + c$$

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