Relations Ex 1.1 Q7.

$R = \{(a, b): a \leq b^3\}$

It is observed that $\left(\frac{1}{2}, \frac{1}{2}\right) \notin R$ as $\frac{1}{2} > \left(\frac{1}{2}\right)^3 = \frac{1}{8}$.

Therefore, R is not reflexive.

Now, (1, 2) ∈ R (as $1 < 2^3 = 8$)

But, (2, 1) ∉ R (as $2^3 > 1$)

Therefore, R is not symmetric.

We have

$\left(3, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{6}{5}\right) \in R$ as $3 < \left(\frac{3}{2}\right)^3$ and $\frac{3}{2} < \left(\frac{6}{5}\right)^3$.

But $\left(3, \frac{6}{5}\right) \notin R$ as $3 > \left(\frac{6}{5}\right)^3$.

Therefore, R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

Relations Ex 1.1 Q8

Let A be a set.

Then $I_A = \{(a, a) ; a \in A\}$ is the identity relation on A.

Hence, every identity relation on a set is reflexive by definition.

Converse:

Let $A = \{(a, b, c)\}$ be a set.

Let $R = \{(a, a)(b, b)(c, c)(a, b)\}$ be a relation defined on A.

Clearly R is reflexive on set A, but it is not identity relation on set A as $(a, b) \in R$

Hence, a reflexive relation need not be identity relation.

Relations Ex 1.1 Q9

We have, $A = \{1, 2, 3, 4\}$

(i) $R = \{(1, 1)(2, 2)(3, 3)(4, 4)(1, 2)\}$ is a relation on set A which is reflexive, transitive but not symmetric

(ii) $R = \{(2, 3)(3, 2)\}$ is a relation on set A which is symmetric but neither reflexive nor transitive

(iii) $R = \{(1, 1)(2, 2)(3, 3)(4, 4)(1, 2)(2, 1)\}$ is a relation on set A which is reflexive, symmetric and transitive

Relations Ex 1.1 Q10

We have, $R = \{(x, y); x, y \in N, 2x + y = 41\}$

Then Domain of R is $x \in N$, such that

$2x + y = 41$

$\Rightarrow \quad x = \frac{41 - y}{2}$

Since $y \in N$, largest value that x can take corresponds to the smallest value that y can take.

$\therefore \quad x = \{1, 2, 3, \ldots\ldots\ldots\ldots 20\}$

Range of R is $y \in N$ such that

$2x + y = 41$

$\Rightarrow \quad y = 41 - 2x$

Since, $x = \{1, 2, 3, \ldots\ldots\ldots\ldots 20\}$

$\therefore \quad y = \{39, 37, 35, 33, \ldots\ldots\ldots\ldots 7, 5, 3, 1\}$

Since, $(2, 2) \notin R$, R is not reflexive.

Also, since $(1, 39) \in R$ but $(39, 1) \notin R$, R is not symmetric.

Finally, since, $(15, 11) \in R$ and $(11, 19) \in R$ but $(15, 19) \notin R$

$\therefore \quad R$ is not trasitive.

********** END **********