Chapter 6 Determinants Ex 6.2 Q45

$$\text{Let } \Delta = \begin{vmatrix} a^3 & 2 & a \\ b^3 & 2 & b \\ c^3 & 2 & c \end{vmatrix}$$

$$\Delta = 2\begin{vmatrix} a^3 & 1 & a \\ b^3 & 1 & b \\ c^3 & 1 & c \end{vmatrix}$$

$$\Delta = 2\left\{a^3(c-b) - 1\left(b^3c - bc^3\right) + a\left(b^3 - c^3\right)\right\}$$

$$\Delta = 2\left\{a^3(c-b) - bc(b-c)(b+c) + a(b-c)\left(b^2 + bc + c^2\right)\right\}$$

$$\Delta = 2(b\text{-}c)\left\{-a^3 - bc(b+c) + a\left(b^2 + bc + c^2\right)\right\}$$

$$\Delta = 2(a\text{-}b)(b\text{-}c)(c-a)(a+b+c)$$

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$$\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = -\begin{vmatrix} x & y & z \\ a & b & c \\ p & q & r \end{vmatrix} = (-1)^2\begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix} = \begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix}$$

$$= (-1)\begin{vmatrix} y & x & z \\ q & p & r \\ b & a & c \end{vmatrix}$$

$$= (-1)^2\begin{vmatrix} y & x & z \\ b & a & c \\ q & p & r \end{vmatrix}$$

Taking transpose, we get

$$= \begin{vmatrix} y & b & p \\ x & a & q \\ z & c & r \end{vmatrix}$$

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Consider the determinant $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$, where a, b, c are in A.P.

Let $\Delta = \begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$, we have,

$$\Delta = \begin{vmatrix} 3x+1+2+a & x+2 & x+a \\ 3x+2+3+b & x+3 & x+b \\ 3x+3+4+c & x+4 & x+c \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 3x+3+a & x+2 & x+a \\ 3x+5+b & x+3 & x+b \\ 3x+7+c & x+4 & x+c \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_2$, we have,

$$\Rightarrow \Delta = \begin{vmatrix} 3x+3+a & x+2 & x+a \\ 2+b-a & 1 & b-a \\ 2+c-b & 1 & c-b \end{vmatrix}$$

Since a, b and c are in arithmetic progression, we have

$b-a = c-b = k(say)$.

Thus,

$$\Delta = \begin{vmatrix} 3x+3+a & x+2 & x+a \\ 2+k & 1 & k \\ 2+k & 1 & k \end{vmatrix}$$

Since the second row and the third row are identical, we have

$\Delta = 0$

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Since, α, β, γ are in A.P, $2\beta = \alpha + \gamma$

$$LHS = \begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix}$$

$R_2 \to R_2 - \frac{R_1}{2} - \frac{R_3}{2}$

$$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ (x-2)-\frac{x-3}{2}-\frac{x-1}{2} & (x-3)-\frac{x-4}{2}-\frac{x-2}{2} & (x-\beta)-\frac{x-\alpha}{2}-\frac{x-\gamma}{2} \\ x-1 & x-2 & x-\gamma \end{vmatrix}$$

$$= \begin{vmatrix} x-3 & x-4 & x-\alpha \\ 0 & 0 & 0 \\ x-1 & x-2 & x-\gamma \end{vmatrix} \qquad [\because 2\beta = \alpha + \gamma]$$

$= 0$

*********** END **********