Geometric Progressions Ex 20.5 Q 15

a, b, c, d are in G.P.

$a,\ b = ar,\ c = ar^2,\ d = ar^3$

Now,

$$\left(b^2+c^2\right)^2 = \left(a^2+b^2\right)\left(c^2+d^2\right)$$

$$\left(a^2r^2+a^2r^4\right)^2 = \left(a^2+a^2r^2\right)\left(a^2r^4+a^2r^6\right)$$

$$a^4\left(r^2+r^4\right)^2 = a^2\left(1+r^2\right)a^2r^4\left(1+r^2\right)$$

$$a^4r^4\left(1+r^2\right)^2 = a^4r^4\left(1+r^2\right)^2$$

LHS = RHS

$\Rightarrow$ $\left(b^2+c^2\right)^2 = \left(a^2+b^2\right)\left(c^2+d^2\right)$

$\Rightarrow$ $\left(a^2+b^2\right),\ \left(b^2+c^2\right),\ \left(c^2+d^2\right)$ are in G.P.

a, b, c, d are in G.P.

$a,\ b = ar,\ c = ar^2,\ d = ar^3$

Now,

$$\left(b^2-c^2\right)^2 = \left(a^2-b^2\right)\left(c^2-d^2\right)$$

$$\left(a^2r^2-a^2r^4\right)^2 = \left(a^2-a^2r^2\right)\left(a^2r^4-a^2r^6\right)$$

$$a^4\left(r^2-r^4\right)^2 = a^2\left(1-r^2\right)a^2r^4\left(1-r^2\right)$$

$$a^4r^4\left(1-r^2\right)^2 = a^4r^4\left(1-r^2\right)^2$$

LHS = RHS

$\Rightarrow$ $\left(b^2-c^2\right)^2 = \left(a^2-b^2\right)\left(c^2-d^2\right)$

$\Rightarrow$ $\left(a^2-b^2\right),\ \left(b^2-c^2\right),\ \left(c^2-d^2\right)$ are in G.P.

a, b, c, d are in G.P.

$a,\ b = ar,\ c = ar^2,\ d = ar^3$

Now,

$$\left(\frac{1}{b^2+c^2}\right)^2 = \left(\frac{1}{a^2+b^2}\right)\left(\frac{1}{c^2+d^2}\right)$$

$$\left(\frac{1}{a^2r^2+a^2r^4}\right)^2 = \left(\frac{1}{a^2+a^2r^2}\right)\left(\frac{1}{a^2r^4+a^2r^6}\right)$$

$$\frac{1}{a^4\left(r^2+r^4\right)^2} = \frac{1}{a^2\left(1+r^2\right)} \times \frac{1}{a^2\left(r^4+r^6\right)}$$

$$\frac{1}{a^4r^4\left(1+r^2\right)^2} = \frac{1}{a^2r^4\left(1+r^2\right)\left(1+r^2\right)}$$

$$\frac{1}{a^4r^4\left(1+r^2\right)^2} = \frac{1}{a^2r^4\left(1+r^2\right)^2}$$

LHS = RHS

$\Rightarrow$ $\left(\frac{1}{b^2+c^2}\right)^2 = \left(\frac{1}{a^2+b^2}\right)\left(\frac{1}{c^2+d^2}\right)$

$\Rightarrow$ $\left(\frac{1}{a^2+b^2}\right),\ \left(\frac{1}{b^2+c^2}\right),\ \left(\frac{1}{c^2+d^2}\right)$ are in G.P.

a, b, c, d are in G.P.

$a,\ b = ar,\ c = ar^2,\ d = ar^3$

Now,

$$(ab+bc+cd)^2 = \left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right)$$

$$\left(a^2r + a^2r^3 + a^2r^5\right)^2 = \left(a^2 + a^2r^2 + a^2r^4\right)\left(a^2r^2 + a^2r^4 + a^2r^6\right)$$

$$a^4\left(r + r^3 + r^5\right)^2 = a^2\left(1 + r^2 + r^4\right)a^2r^2\left(1 + r^2 + r^4\right)$$

$$a^4r^2\left(1 + r^2 + r^4\right)^2 = a4r^2\left(1 + r^2 + r^4\right)^2$$

LHS = RHS

$\Rightarrow$ $(ab+bc+cd)^2 = \left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right)$

$\Rightarrow$ $\left(a^2+b^2+c^2\right),\ (ab+bc+cd),\ \left(b^2+c^2+d^2\right)$ are in G.P.

Geometric Progressions Ex 20.5 Q 16

$(a-b),\ (b-c),\ (c-a)$ are in G.P.

$$(b-c)^2 = (a-b)(c-a)$$

$$b^2 + c^2 - 2bc = ac - a^2 - bc + ab$$

$$b^2 + c^2 + a^2 = ac + bc + ab \qquad \text{---(i)}$$

Now,

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$= ac + bc + ab + 2ab + 2bc + 2ca$$

Using equation (i)

$$= 3ab + 3bc + 3ca$$

$$(a+b+c)^2 = 3(ab+bc+ca)$$

********** END **********