



### Co-Ordinate Geometry Ex 14.5 Q10

**Answer :**

GIVEN: four points A (6, 3), B (-3, 5) C (4, -2) and D(x, 3x) such that  $\frac{\Delta DBC}{\Delta ABC} = \frac{1}{2}$

TO FIND: the value of x

PROOF:

We know area of the triangles formed by three points  $(x_1, y_1), (x_2, y_2)$  and  $(x_3, y_3)$  is given by

$$\text{Area of triangle} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Now

Area of triangle DBC taking D(x, 3x), B (-3, 5), C (4, -2)

$$\Delta DBC \Rightarrow \frac{1}{2} |x(5 - (-2)) + (-3)((-2) - 3x) + (4)(3x - 5)|$$

$$\Delta DBC \Rightarrow \frac{1}{2} |7x + 6 + 9x + 12x - 20|$$

$$\Delta DBC \Rightarrow \frac{1}{2} |28x - 14|$$

$$\Delta DBC \Rightarrow \frac{1}{2} |14(2x - 1)|$$

$$\Delta DBC \Rightarrow |7(2x - 1)| \quad \dots\dots(1)$$

Area of triangle ABC taking, A (6, 3), B (-3, 5), C (4, -2)

$$\Rightarrow \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$\begin{aligned}
 &\Rightarrow \frac{1}{2} | 6(5 - (-2)) + (-3)((-2) - 3) + (4)(3 - 5) | \\
 &\Rightarrow \frac{1}{2} | 6(7) + (-3)(-5) + (4)(-2) | \\
 &\Rightarrow \frac{1}{2} | 42 + 15 - 8 | \\
 &\Rightarrow \frac{49}{2} \quad \dots\dots(2)
 \end{aligned}$$

Also it is given that

$$\frac{\Delta DBC}{\Delta ABC} = \frac{1}{2}$$

Substituting the values from (1) and (2) we get

$$\frac{\Delta DBC}{\Delta ABC} = \frac{1}{2}$$

$$\frac{\pm 7(2x-1)}{\frac{49}{2}} = \frac{1}{2}$$

$$\frac{2 \times 7(2x-1)}{49} = \frac{1}{2} \text{ or } \frac{-2 \times 7(2x-1)}{49} = \frac{1}{2}$$

$$(2x-1) = \frac{1}{2} \times \frac{7}{2} \text{ or } (-2x+1) = \frac{1}{2} \times \frac{7}{2}$$

$$2x = \frac{7}{4} + 1 \quad \text{or } 2x = \frac{7}{4} - 1$$

$$2x = \frac{11}{4} \quad \text{or } 2x = \frac{-3}{4}$$

$$\boxed{x = \frac{11}{8}} \text{ or } \boxed{x = \frac{-3}{8}}$$

**Answer :**

The formula for the area 'A' encompassed by three points  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$  is given by the formula,

$$A = \frac{1}{2} \begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_2 - x_3 & y_2 - y_3 \end{vmatrix}$$

$$A = \frac{1}{2} \{ (x_1 - x_2)(y_2 - y_3) - (x_2 - x_3)(y_1 - y_2) \}$$

If three points are collinear the area encompassed by them is equal to 0.

The three given points are  $A(a, 1)$ ,  $B(1, -1)$  and  $C(11, 4)$ . It is also said that they are collinear and hence the area enclosed by them should be 0.

$$A = \frac{1}{2} \begin{vmatrix} a - 1 & 1 + 1 \\ 1 - 11 & -1 - 4 \end{vmatrix}$$

$$A = \frac{1}{2} \begin{vmatrix} a - 1 & 2 \\ -10 & -5 \end{vmatrix}$$

$$0 = \frac{1}{2} \{ (a - 1)(-5) - (-10)(2) \}$$

$$0 = \frac{1}{2} \{ -5a + 5 + 20 \}$$

$$0 = -5a + 5 + 20$$

$$5a = 25$$

$$a = 5$$

Hence the value of 'a' for which the given points are collinear is  $\boxed{a = 5}$ .

### Co-Ordinate Geometry Ex 14.5 Q12

**Answer :**

The formula for the area 'A' encompassed by three points  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$  is given by the formula,

$$A = \frac{1}{2} \begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_2 - x_3 & y_2 - y_3 \end{vmatrix}$$

$$A = \frac{1}{2} \{ (x_1 - x_2)(y_2 - y_3) - (x_2 - x_3)(y_1 - y_2) \}$$

If three points are collinear the area encompassed by them is equal to 0.

The three given points are  $(a, b)$ ,  $(a_1, b_1)$  and  $(a - a_1, b - b_1)$ . If they are collinear then the area enclosed by them should be 0.

$$A = \frac{1}{2} \begin{vmatrix} a - a_1 & b - b_1 \\ a_1 - a + a_1 & b_1 - b + b_1 \end{vmatrix}$$

$$0 = \frac{1}{2} \{ (a - a_1)(2b_1 - b) - (2a_1 - a)(b - b_1) \}$$

$$0 = \frac{1}{2} \{ 2ab_1 - ab - 2a_1b_1 + a_1b - 2a_1b + 2a_1b_1 + ab - ab_1 \}$$

$$0 = 2ab_1 + a_1b - 2a_1b - ab_1$$

$$a_1b = ab_1$$

Hence we have proved that for the given conditions to be satisfied we need to have  $\boxed{a_1b = ab_1}$ .

\*\*\*\*\* END \*\*\*\*\*