

Trigonometric Ratios Ex 5.1 Q25

Answer:

Given:

$$\sec A = \frac{17}{8}$$
 (1)

To verify:

$$\frac{3 - 4\sin^2 A}{4\cos^2 A - 3} = \frac{3 - \tan^2 A}{1 - 3\tan^2 A} \dots (2)$$

Now we know that $\sec A = \frac{1}{\cos A}$

Therefore
$$\cos A = \frac{1}{\sec A}$$

Now, by substituting the value of $\sec A$ from equation (1) We get,

$$\cos A = \frac{1}{\frac{17}{8}}$$
$$= \frac{8}{17}$$

Therefore,

$$\cos A = \frac{8}{17}$$
 (3)

Now, we know the following trigonometric identity

$$\cos^2 A + \sin^2 A = 1$$

Therefore,

$$\sin^2 A = 1 - \cos^2 A$$

Now by substituting the value of $\cos A$ from equation (3) We get

$$\sin^2 A = 1 - \left(\frac{8}{17}\right)^2$$
$$= 1 - \frac{\left(8\right)^2}{\left(17\right)^2}$$
$$= 1 - \frac{64}{289}$$

Now by taking L.C.M

We get,

$$\sin^2 A = \frac{289 - 64}{289}$$
$$= \frac{225}{289}$$

Now, by taking square root on both sides

We get,

$$\sin A = \sqrt{\frac{225}{289}}$$

$$=\frac{\sqrt{225}}{\sqrt{289}}$$
$$=\frac{15}{17}$$

Therefore,

$$\sin A = \frac{15}{17}$$
 (4)

Now, we know that $\tan A = \frac{\sin A}{\cos A}$

Now by substituting the value of $\cos A$ and $\sin A$ from equation (3) and (4) respectively We get,

$$\tan A = \frac{\frac{15}{17}}{\frac{8}{17}}$$

$$= \frac{15}{17} \times \frac{17}{8}$$

$$= \frac{15}{8}$$

Therefore

$$\tan A = \frac{15}{8}$$
 (5)

Now from the expression of equation (2)

$$L.H.S = \frac{3 - 4\sin^2 A}{4\cos^2 A - 3}$$

Now by substituting the value of $\cos A$ and $\sin A$ from equation (3) and (4) We get,

L.H.S =
$$\frac{3 - 4\left(\frac{15}{17}\right)^2}{4\left(\frac{8}{17}\right)^2 - 3}$$

Therefore,

L.H.S =
$$\frac{3 - 4\left(\frac{225}{289}\right)}{4\left(\frac{64}{289}\right) - 3}$$
$$= \frac{3 - \frac{900}{289}}{\frac{256}{289} - 3}$$

Now by taking L.C.M of both numerator and denominator We get,

L.H.S =
$$\frac{\frac{3 \times 289}{1 \times 289} - \frac{900}{289}}{\frac{256}{289} - \frac{3 \times 289}{1 \times 289}}$$

$$= \frac{\frac{867}{289} - \frac{900}{289}}{\frac{256}{289} - \frac{867}{289}}$$

$$= \frac{\frac{867}{289} - \frac{900}{289}}{\frac{256}{289} - \frac{867}{289}}$$

$$= \frac{\frac{-33}{289}}{\frac{-611}{289}}$$

$$= \frac{33}{611}$$

Therefore,

$$\frac{3 - 4\sin^2 A}{4\cos^2 A - 3} = \frac{33}{611} \dots (6)$$

Now from the expression of equation (2)

$$R.H.S = \frac{3 - \tan^2 A}{1 - 3\tan^2 A}$$

Now by substituting the value of tan A from equation (5)

We get,

$$3-\left(\frac{15}{9}\right)^2$$

$$R.H.S. = \frac{\frac{6}{1 - 3\left(\frac{15}{8}\right)^2}}{1 - 3\left(\frac{15}{8}\right)^2}$$
$$= \frac{3 - \frac{225}{64}}{1 - \frac{3 \times 225}{64}}$$

Now by taking L.C.M We get,

$$R.H.S = \frac{\frac{3 \times 64}{1 \times 64} - \frac{225}{64}}{\frac{64 - 675}{64}}$$

$$= \frac{\frac{192}{64} - \frac{225}{64}}{\frac{-611}{64}}$$

$$= \frac{\frac{192 - 225}{64}}{\frac{-611}{64}}$$

Therefore

$$R.H.S = \frac{\frac{-33}{64}}{\frac{-611}{64}}$$
$$= \frac{-33}{64} \times \frac{64}{-611}$$
$$= \frac{33}{611}$$

Therefore,

$$\frac{3 - \tan^2 A}{1 - 3\tan^2 A} = \frac{33}{611} \dots (7)$$

Now by comparing equation (6) and (7) We get,

$$\frac{3 - 4\sin^2 A}{4\cos^2 A - 3} = \frac{3 - \tan^2 A}{1 - 3\tan^2 A}$$

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