

Trigonometric Ratios of Compound Angles Ex 7.1 Q17

We have,

$$8\theta = 6\theta + 2\theta$$

$$\Rightarrow$$
 tan 8 θ = tan (6 θ + 2 θ)

$$\Rightarrow \qquad \tan 8\theta = \frac{\tan 6\theta + \tan 2\theta}{1 - \tan 6\theta \tan 2\theta} \qquad \left[\because \tan \left(A + B \right) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \right]$$

 \Rightarrow tan 8 θ (1 - tan 6 θ tan 2 θ) = tan 6 θ + tan 2 θ

 \Rightarrow $\tan 8\theta - \tan 8\theta \tan 6\theta \tan 2\theta = \tan 6\theta + \tan 2\theta$

 \Rightarrow tan 8 θ - tan 6 θ - tan 2 θ = tan 8 θ tan 6 θ tan 2 θ

Hence proved.

We have,

$$\Rightarrow tan 45^{\circ} = tan (30^{\circ} + 15^{\circ})$$

$$\Rightarrow 1 = \frac{\tan 30^\circ + \tan 15^\circ}{1 - \tan 30^\circ \tan 15^\circ} \qquad \left[\because \tan \left(A + B \right) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \right]$$

⇒ 1 - tan30° tan15° = tan15° + tan30°

 \Rightarrow 1 = tan15° + tan30° + tan30° tan15°

 \Rightarrow tan 15° + tan 30° + tan 15° tan 30° = 1

Hence proved.

We have,

$$45^{\circ} = 9^{\circ} + 36^{\circ}$$

$$\Rightarrow$$
 tan 45° = tan (9° + 36°)

$$\Rightarrow 1 = \frac{\tan 9^\circ + \tan 36^\circ}{1 - \tan 9^\circ \tan 36^\circ} \qquad \left[\because \tan \left(A + B \right) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \right]$$

⇒ 1 - tan 9° tan 36° = tan 9° + tan 36°

 \Rightarrow 1 = tan 9° + tan 36° + tan 9° tan 36°

 \Rightarrow tan 9° + tan 36° + tan 9° tan 36° = 1

Hence proved.

We have,

$$13\theta = 9\theta + 4\theta$$

$$\Rightarrow \tan 13\theta = \tan (9\theta + 4\theta)$$

$$\Rightarrow \qquad \tan 13\theta = \frac{\tan 9\theta + \tan 4\theta}{1 - \tan 9\theta \tan 4\theta} \qquad \qquad \left[\because \tan \left(A + B \right) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \right]$$

 \Rightarrow tan 13 θ (1 - tan 9 θ tan 4 θ) = tan 9 θ + tan 4 θ

 \Rightarrow tan 13 θ - tan 13 θ tan 9 θ tan 4 θ = tan 9 θ + tan 4 θ

 \Rightarrow tan 13 θ - tan 9 θ - tan 4 θ = tan 13 θ tan 9 θ tan 4 θ

Hence proved.

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We have,

RHS =
$$\tan 3\theta \tan \theta$$

= $\tan (2\theta + \theta) \times \tan (2\theta - \theta)$
= $\left[\frac{\tan 2\theta + \tan \theta}{1 - \tan 2\theta \tan \theta}\right] \times \left[\frac{\tan 2\theta - \tan \theta}{1 + \tan 2\theta \tan \theta}\right]$
= $\frac{(\tan 2\theta + \tan \theta)(\tan 2\theta - \tan \theta)}{(1 - \tan 2\theta \tan \theta)(1 + \tan 2\theta \tan \theta)}$
= $\frac{\tan^2 2\theta - \tan^2 \theta}{1 - \tan^2 2\theta \tan^2 \theta}$ $\left[\because (a - b)(a + b) = a^2 - b^2\right]$
= LHS

: LHS = RHS

Hence proved

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$$\frac{\sin x \cdot \cos y + \sin y \cdot \cos x}{\sin x \cdot \cos y - \sin y \cdot \cos x} = \frac{a+b}{a-b}$$

$$\Rightarrow \frac{\sin x \cdot \cos y + \sin y \cdot \cos x}{\sin x \cdot \cos y + \sin y \cdot \cos x - \sin x \cdot \cos y - \sin y \cdot \cos x} = \frac{a+b+a-b}{a+b-a+b} [Using Componendo and Dividendo]$$

$$\Rightarrow \frac{2\sin x \cdot \cos y}{2\sin x \cdot \cos y} = \frac{2a}{2b}$$

$$\Rightarrow \frac{\tan x}{\tan y} = \frac{a}{b}$$
Hence Proved

******* END *******