



Q12 : A point on the hypotenuse of a triangle is at distance  $a$  and  $b$  from the sides of the triangle.

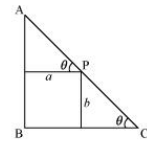
Show that the minimum length of the hypotenuse is  $\left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^{\frac{3}{2}}$

Answer :

Let  $\triangle ABC$  be right-angled at B. Let  $AB = x$  and  $BC = y$ .

Let P be a point on the hypotenuse of the triangle such that P is at a distance of  $a$  and  $b$  from the sides AB and BC respectively.

Let  $\angle C = \theta$ .



We have,

$$AC = \sqrt{x^2 + y^2}$$

Now,

$$PC = b \operatorname{cosec} \theta$$

$$\text{And, } AP = a \sec \theta$$

$$\therefore AC = AP + PC$$

$$\Rightarrow AC = b \operatorname{cosec} \theta + a \sec \theta \dots (1)$$

$$\therefore \frac{d(AC)}{d\theta} = -b \operatorname{cosec} \theta \cot \theta + a \sec \theta \tan \theta$$

$$\therefore \frac{d(AC)}{d\theta} = 0$$

$$\Rightarrow a \sec \theta \tan \theta = b \operatorname{cosec} \theta \cot \theta$$

$$\Rightarrow \frac{a}{\cos \theta} \cdot \frac{\sin \theta}{\cos \theta} = \frac{b}{\sin \theta} \cdot \frac{\cos \theta}{\sin \theta}$$

$$\Rightarrow a \sin^3 \theta = b \cos^3 \theta$$

$$\Rightarrow \left(a\right)^{\frac{1}{3}} \sin \theta = \left(b\right)^{\frac{1}{3}} \cos \theta$$

$$\Rightarrow \tan \theta = \left(\frac{b}{a}\right)^{\frac{1}{3}}$$

$$\therefore \sin \theta = \frac{\left(b\right)^{\frac{1}{3}}}{\sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}} \text{ and } \cos \theta = \frac{\left(a\right)^{\frac{1}{3}}}{\sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}} \dots (2)$$

It can be clearly shown that  $\frac{d^2(AC)}{d\theta^2} < 0$  when  $\tan \theta = \left(\frac{b}{a}\right)^{\frac{1}{3}}$ .

Therefore, by second derivative test, the length of the hypotenuse is the maximum when

$$\tan \theta = \left(\frac{b}{a}\right)^{\frac{1}{3}}$$

Now, when  $\tan \theta = \left(\frac{b}{a}\right)^{\frac{1}{3}}$ , we have:

$$\begin{aligned} AC &= \frac{b \sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}}{\frac{1}{b^{\frac{1}{3}}}} + \frac{a \sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}}}{\frac{1}{a^{\frac{1}{3}}}} && [\text{Using (1) and (2)}] \\ &= \sqrt{a^{\frac{2}{3}} + b^{\frac{2}{3}}} \left( b^{\frac{2}{3}} + a^{\frac{2}{3}} \right) \\ &= \left( a^{\frac{2}{3}} + b^{\frac{2}{3}} \right)^{\frac{3}{2}} \end{aligned}$$

Hence, the maximum length of the hypotenuses is  $\left(a^{\frac{2}{3}} + b^{\frac{2}{3}}\right)^{\frac{3}{2}}$ .

Answer needs Correction? [Click Here](#)

Q13 : Find the points at which the function  $f$  given by  $f(x) = (x-2)^4 (x+1)^3$  has

(i) local maxima (ii) local minima

(ii) point of inflexion

Answer :

Answer :

The given function is  $f(x) = (x-2)^4(x+1)^3$ .

$$\begin{aligned}\therefore f'(x) &= 4(x-2)^3(x+1)^3 + 3(x+1)^2(x-2)^4 \\ &= (x-2)^3(x+1)^2[4(x+1) + 3(x-2)] \\ &= (x-2)^3(x+1)^2(7x-2)\end{aligned}$$

Now,  $f'(x) = 0 \Rightarrow x = -1$  and  $x = \frac{2}{7}$  or  $x = 2$

Now, for values of  $x$  close to  $\frac{2}{7}$  and to the left of  $\frac{2}{7}$ ,  $f'(x) > 0$ . Also, for values of  $x$  close to  $\frac{2}{7}$  and to the right of  $\frac{2}{7}$ ,  $f'(x) < 0$ .

Thus,  $x = \frac{2}{7}$  is the point of local maxima.

Now, for values of  $x$  close to 2 and to the left of 2,  $f'(x) < 0$ . Also, for values of  $x$  close to 2 and to the right of 2,  $f'(x) > 0$ .

Thus,  $x = 2$  is the point of local minima.

Now, as the value of  $x$  varies through -1,  $f'(x)$  does not change its sign.

Thus,  $x = -1$  is the point of inflexion.

Answer needs Correction? [Click Here](#)

**Q14 :** Find the absolute maximum and minimum values of the function  $f$  given by

$$f(x) = \cos^2 x + \sin x, x \in [0, \pi]$$

Answer :

$$\begin{aligned}f(x) &= \cos^2 x + \sin x \\ f'(x) &= 2 \cos x(-\sin x) + \cos x \\ &= -2 \sin x \cos x + \cos x \\ \text{Now, } f'(x) &= 0 \\ \Rightarrow 2 \sin x \cos x &= \cos x \Rightarrow \cos x(2 \sin x - 1) = 0 \\ \Rightarrow \sin x &= \frac{1}{2} \text{ or } \cos x = 0 \\ \Rightarrow x &= \frac{\pi}{6}, \text{ or } \frac{\pi}{2} \text{ as } x \in [0, \pi]\end{aligned}$$

Now, evaluating the value of  $f$  at critical points  $x = \frac{\pi}{6}$  and  $x = \frac{\pi}{2}$  and at the end points of the interval  $[0, \pi]$  (i.e., at  $x = 0$  and  $x = \pi$ ), we have:

$$\begin{aligned}f\left(\frac{\pi}{6}\right) &= \cos^2 \frac{\pi}{6} + \sin \frac{\pi}{6} = \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{1}{2} = \frac{5}{4} \\ f(0) &= \cos^2 0 + \sin 0 = 1 + 0 = 1 \\ f(\pi) &= \cos^2 \pi + \sin \pi = (-1)^2 + 0 = 1 \\ f\left(\frac{\pi}{2}\right) &= \cos^2 \frac{\pi}{2} + \sin \frac{\pi}{2} = 0 + 1 = 1\end{aligned}$$

Hence, the absolute maximum value of  $f$  is  $\frac{5}{4}$  occurring at  $x = \frac{\pi}{6}$  and the absolute minimum value of  $f$  is 1 occurring at  $x = 0, \frac{\pi}{2}, \text{ and } \pi$ .

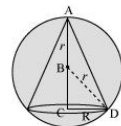
Answer needs Correction? [Click Here](#)

**Q15 :** Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius  $r$  is  $\frac{4r}{3}$ .

Answer :

A sphere of fixed radius ( $r$ ) is given.

Let  $R$  and  $h$  be the radius and the height of the cone respectively.



The volume ( $V$ ) of the cone is given by,

$$V = \frac{1}{3} \pi R^2 h$$

Now, from the right triangle BCD, we have:

$$BC = \sqrt{r^2 - R^2}$$

$$\therefore h = r + \sqrt{r^2 - R^2}$$

$$\therefore V = \frac{1}{3} \pi R^2 (r + \sqrt{r^2 - R^2}) = \frac{1}{3} \pi R^2 r + \frac{1}{3} \pi R^2 \sqrt{r^2 - R^2}$$

$$\begin{aligned}\therefore \frac{dV}{dR} &= \frac{2}{3} \pi R r + \frac{2}{3} \pi R \sqrt{r^2 - R^2} + \frac{\pi R^2}{3} \cdot \frac{(-2R)}{2\sqrt{r^2 - R^2}} \\ &= \frac{2}{3} \pi R r + \frac{2}{3} \pi R \sqrt{r^2 - R^2} - \frac{\pi R^3}{3\sqrt{r^2 - R^2}}\end{aligned}$$

$$\begin{aligned}
&= \frac{2}{3}\pi Rr + \frac{2\pi R(r^2 - R^2) - \pi R^3}{3\sqrt{r^2 - R^2}} \\
&= \frac{2}{3}\pi Rr + \frac{2\pi Rr^2 - 3\pi R^3}{3\sqrt{r^2 - R^2}} \\
\text{Now, } \frac{dV}{dR^2} &= 0 \\
\Rightarrow \frac{2\pi Rr}{3} &= \frac{3\pi R^3 - 2\pi Rr^2}{3\sqrt{r^2 - R^2}} \\
\Rightarrow 2r\sqrt{r^2 - R^2} &= 3R^2 - 2r^2 \\
\Rightarrow 4r^2(r^2 - R^2) &= (3R^2 - 2r^2)^2 \\
\Rightarrow 4r^4 - 4r^2R^2 &= 9R^4 + 4r^4 - 12R^2r^2 \\
\Rightarrow 9R^4 - 8r^2R^2 &= 0 \\
\Rightarrow 9R^2 &= 8r^2 \\
\Rightarrow R^2 &= \frac{8r^2}{9}
\end{aligned}$$

$$\begin{aligned}
\text{Now, } \frac{d^2V}{dR^2} &= \frac{2\pi r}{3} + \frac{3\sqrt{r^2 - R^2}(2\pi r^2 - 9\pi R^2) - (2\pi Rr^2 - 3\pi R^3)(-6R)}{9(r^2 - R^2)^{\frac{3}{2}}} \\
&= \frac{2\pi r}{3} + \frac{3\sqrt{r^2 - R^2}(2\pi r^2 - 9\pi R^2) + (2\pi Rr^2 - 3\pi R^3)(3R)}{9(r^2 - R^2)^{\frac{3}{2}}}
\end{aligned}$$

Now, when  $R^2 = \frac{8r^2}{9}$ , it can be shown that  $\frac{d^2V}{dR^2} < 0$ .

$\therefore$  The volume is the maximum when  $R^2 = \frac{8r^2}{9}$ .

When  $R^2 = \frac{8r^2}{9}$ , height of the cone  $= r + \sqrt{r^2 - \frac{8r^2}{9}} = r + \sqrt{\frac{r^2}{9}} = r + \frac{r}{3} = \frac{4r}{3}$ .

Hence, it can be seen that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius  $r$  is  $\frac{4r}{3}$ .

Answer needs Correction? [Click Here](#)

**Q16 :** Let  $f$  be a function defined on  $[a, b]$  such that  $f'(x) > 0$ , for all  $x \in (a, b)$ . Then prove that  $f$  is an increasing function on  $(a, b)$ .

**Answer :**

Let \_\_\_\_\_ such that \_\_\_\_\_.  
Consider the sub-interval [\_\_\_\_\_]

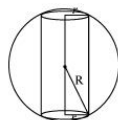
Answer needs Correction? [Click Here](#)

**Q17 :** Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius  $R$  is  $\frac{2R}{\sqrt{3}}$ . Also find the maximum volume.

**Answer :**

A sphere of fixed radius ( $R$ ) is given.

Let  $r$  and  $h$  be the radius and the height of the cylinder respectively.



From the given figure, we have  $h = 2\sqrt{R^2 - r^2}$ .

The volume ( $V$ ) of the cylinder is given by,

$$\begin{aligned}
V &= \pi r^2 h = 2\pi r^2 \sqrt{R^2 - r^2} \\
\therefore \frac{dV}{dr} &= 4\pi r \sqrt{R^2 - r^2} + \frac{2\pi r^2 (-2r)}{2\sqrt{R^2 - r^2}} \\
&= 4\pi r \sqrt{R^2 - r^2} - \frac{2\pi r^3}{\sqrt{R^2 - r^2}} \\
&= \frac{4\pi r(R^2 - r^2) - 2\pi r^3}{\sqrt{R^2 - r^2}} \\
&= \frac{4\pi rR^2 - 6\pi r^3}{\sqrt{R^2 - r^2}} \\
\text{Now, } \frac{dV}{dr} &= 0 \Rightarrow 4\pi rR^2 - 6\pi r^3 = 0 \\
\Rightarrow r^2 &= \frac{2R^2}{3} \\
\text{Now, } \frac{d^2V}{dr^2} &= \frac{\sqrt{R^2 - r^2}(4\pi R^2 - 18\pi r^2) - (4\pi rR^2 - 6\pi r^3)(-2r)}{(R^2 - r^2)^{\frac{3}{2}}} \\
&= \frac{(R^2 - r^2)(4\pi R^2 - 18\pi r^2) + r(4\pi rR^2 - 6\pi r^3)}{(R^2 - r^2)^{\frac{3}{2}}} \\
&= \frac{4\pi R^4 - 22\pi r^2 R^2 + 12\pi r^4 + 4\pi r^2 R^2}{(R^2 - r^2)^{\frac{3}{2}}}
\end{aligned}$$

Now, it can be observed that at  $r^2 = \frac{2R^2}{3}$ ,  $\frac{d^2V}{dr^2} < 0$ .

∴ The volume is the maximum when  $r^2 = \frac{2R^2}{3}$ .

When  $r^2 = \frac{2R^2}{3}$ , the height of the cylinder is  $2\sqrt{R^2 - \frac{2R^2}{3}} = 2\sqrt{\frac{R^2}{3}} = \frac{2R}{\sqrt{3}}$ .

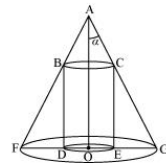
Hence, the volume of the cylinder is the maximum when the height of the cylinder is  $\frac{2R}{\sqrt{3}}$ .

Answer needs Correction? [Click Here](#)

**Q18 :** Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height  $h$  and semi vertical angle  $\alpha$  is one-third that of the cone and the greatest volume of cylinder is  $\frac{4}{27}\pi h^3 \tan^2 \alpha$ .

**Answer :**

The given right circular cone of fixed height ( $h$ ) and semi-vertical angle ( $\alpha$ ) can be drawn as:



Here, a cylinder of radius  $R$  and height  $H$  is inscribed in the cone.

Then,  $\angle GAO = \alpha$ ,  $OG = r$ ,  $OA = h$ ,  $OE = R$ , and  $CE = H$ .

We have,

$$r = h \tan \alpha$$

Now, since  $\triangle AOG$  is similar to  $\triangle CEG$ , we have:

$$\begin{aligned} \frac{AO}{OG} &= \frac{CE}{EG} \\ \Rightarrow \frac{h}{r} &= \frac{H}{r-R} \quad [EG = OG - OE] \\ \Rightarrow H &= \frac{h}{r}(r-R) = \frac{h}{h \tan \alpha}(h \tan \alpha - R) = \frac{1}{\tan \alpha}(h \tan \alpha - R) \end{aligned}$$

Now, the volume ( $V$ ) of the cylinder is given by,

$$V = \pi R^2 H = \frac{\pi R^2}{\tan \alpha}(h \tan \alpha - R) = \pi R^2 h - \frac{\pi R^3}{\tan \alpha}$$

$$\therefore \frac{dV}{dR} = 2\pi R h - \frac{3\pi R^2}{\tan \alpha}$$

$$\text{Now, } \frac{dV}{dR} = 0$$

$$\Rightarrow 2\pi R h = \frac{3\pi R^2}{\tan \alpha}$$

$$\Rightarrow 2h \tan \alpha = 3R$$

$$\Rightarrow R = \frac{2h}{3} \tan \alpha$$

$$\text{Now, } \frac{d^2V}{dR^2} = 2\pi h - \frac{6\pi R}{\tan \alpha}$$

And, for  $R = \frac{2h}{3} \tan \alpha$ , we have:

$$\frac{d^2V}{dR^2} = 2\pi h - \frac{6\pi}{\tan \alpha} \left( \frac{2h}{3} \tan \alpha \right) = 2\pi h - 4\pi h = -2\pi h < 0$$

∴ By second derivative test, the volume of the cylinder is the greatest when

$$R = \frac{2h}{3} \tan \alpha.$$

$$\text{When } R = \frac{2h}{3} \tan \alpha, H = \frac{1}{\tan \alpha} \left( h \tan \alpha - \frac{2h}{3} \tan \alpha \right) = \frac{1}{\tan \alpha} \left( \frac{h \tan \alpha}{3} \right) = \frac{h}{3}.$$

Thus, the height of the cylinder is one-third the height of the cone when the volume of the cylinder is the greatest.

Now, the maximum volume of the cylinder can be obtained as:

$$\pi \left( \frac{2h}{3} \tan \alpha \right)^2 \left( \frac{h}{3} \right) = \pi \left( \frac{4h^2}{9} \tan^2 \alpha \right) \left( \frac{h}{3} \right) = \frac{4}{27} \pi h^3 \tan^2 \alpha$$

Hence, the given result is proved.

Answer needs Correction? [Click Here](#)

**Q19 :** A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic mere per hour. Then the depth of the wheat is increasing at the rate of

- (A) 1 m/h (B) 0.1 m/h  
(C) 1.1 m/h (D) 0.5 m/h

**Answer :**

Let  $r$  be the radius of the cylinder.

Then, the volume of the cylinder is given by,

Then, volume ( $V$ ) of the cylinder is given by,

$$\begin{aligned} V &= \pi(\text{radius})^2 \times \text{height} \\ &= \pi(10)^2 h \quad (\text{radius} = 10 \text{ m}) \\ &= 100\pi h \end{aligned}$$

Differentiating with respect to time  $t$ , we have:

$$\frac{dV}{dt} = 100\pi \frac{dh}{dt}$$

The tank is being filled with wheat at the rate of 314 cubic metres per hour.

$$\therefore \frac{dV}{dt} = 314 \text{ m}^3/\text{h}$$

Thus, we have:

$$\begin{aligned} 314 &= 100\pi \frac{dh}{dt} \\ \Rightarrow \frac{dh}{dt} &= \frac{314}{100(3.14)} = \frac{314}{314} = 1 \end{aligned}$$

Hence, the depth of wheat is increasing at the rate of 1 m/h.

The correct answer is A.

Answer needs Correction? [Click Here](#)

Q20 : The slope of the tangent to the curve  $x = t^2 + 3t - 8$ ,  $y = 2t^2 - 2t - 5$  at the point (2, -1) is

(A)  $\frac{22}{7}$  (B)  $\frac{6}{7}$  (C)  $\frac{7}{6}$  (D)  $-\frac{6}{7}$

Answer :

The given curve is  $x = t^2 + 3t - 8$  and  $y = 2t^2 - 2t - 5$ .

$$\therefore \frac{dx}{dt} = 2t + 3 \text{ and } \frac{dy}{dt} = 4t - 2$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{4t - 2}{2t + 3}$$

The given point is (2, -1).

At  $x = 2$ , we have:

$$\begin{aligned} t^2 + 3t - 8 &= 2 \\ \Rightarrow t^2 + 3t - 10 &= 0 \\ \Rightarrow (t - 2)(t + 5) &= 0 \\ \Rightarrow t &= 2 \text{ or } t = -5 \end{aligned}$$

At  $y = -1$ , we have:

$$\begin{aligned} 2t^2 - 2t - 5 &= -1 \\ \Rightarrow 2t^2 - 2t - 4 &= 0 \\ \Rightarrow 2(t^2 - t - 2) &= 0 \\ \Rightarrow (t - 2)(t + 1) &= 0 \\ \Rightarrow t &= 2 \text{ or } t = -1 \end{aligned}$$

The common value of  $t$  is 2.

Hence, the slope of the tangent to the given curve at point (2, -1) is

$$\left. \frac{dy}{dx} \right|_{t=2} = \frac{4(2) - 2}{2(2) + 3} = \frac{8 - 2}{4 + 3} = \frac{6}{7}$$

The correct answer is B.

Answer needs Correction? [Click Here](#)

Q21 : The line  $y = mx + 1$  is a tangent to the curve  $y^2 = 4x$  if the value of  $m$  is

(A) 1 (B) 2 (C) 3 (D)  $\frac{1}{2}$

Answer :

The equation of the tangent to the given curve is  $y = mx + 1$ .

Now, substituting  $y = mx + 1$  in  $y^2 = 4x$ , we get:

$$\begin{aligned} \Rightarrow (mx + 1)^2 &= 4x \\ \Rightarrow m^2x^2 + 1 + 2mx - 4x &= 0 \\ \Rightarrow m^2x^2 + x(2m - 4) + 1 &= 0 \quad \dots(i) \end{aligned}$$

Since a tangent touches the curve at one point, the roots of equation (i) must be equal.

Therefore, we have:

$$\begin{aligned} \text{Discriminant} &= 0 \\ (2m - 4)^2 - 4(m^2)(1) &= 0 \\ \Rightarrow 4m^2 + 16 - 16m - 4m^2 &= 0 \\ \Rightarrow 16 - 16m &= 0 \\ \Rightarrow m &= 1 \end{aligned}$$

Hence, the required value of  $m$  is 1.

The correct answer is A.

Answer needs Correction? [Click Here](#)

Q22 : The normal at the point (1, 1) on the curve  $2y + x^2 = 3$  is

- (A)  $x + y = 0$  (B)  $x - y = 0$   
(C)  $x + y + 1 = 0$  (D)  $x - y = 1$

Answer :

The equation of the given curve is  $2y + x^2 = 3$ .

Differentiating with respect to  $x$ , we have:

$$\begin{aligned}\frac{2dy}{dx} + 2x &= 0 \\ \Rightarrow \frac{dy}{dx} &= -x \\ \therefore \left. \frac{dy}{dx} \right|_{(1,1)} &= -1\end{aligned}$$

The slope of the normal to the given curve at point (1, 1) is

$$\left. \frac{-1}{\frac{dy}{dx}} \right|_{(1,1)} = 1.$$

Hence, the equation of the normal to the given curve at (1, 1) is given as:

$$\begin{aligned}\Rightarrow y - 1 &= 1(x - 1) \\ \Rightarrow y - 1 &= x - 1 \\ \Rightarrow x - y &= 0\end{aligned}$$

The correct answer is B.

Answer needs Correction? [Click Here](#)

Q23 : The normal to the curve  $x^2 = 4y$  passing (1, 2) is

- (A)  $x + y = 3$  (B)  $x - y = 3$   
(C)  $x + y = 1$  (D)  $x - y = 1$

Answer :

The equation of the given curve is  $x^2 = 4y$ .

Differentiating with respect to  $x$ , we have:

$$\begin{aligned}2x &= 4 \cdot \frac{dy}{dx} \\ \Rightarrow \frac{dy}{dx} &= \frac{x}{2}\end{aligned}$$

The slope of the normal to the given curve at point  $(h, k)$  is given by,

$$\left. \frac{-1}{\frac{dy}{dx}} \right|_{(h,k)} = -\frac{2}{h}$$

$\therefore$  Equation of the normal at point  $(h, k)$  is given as:

$$y - k = \frac{-2}{h}(x - h)$$

Now, it is given that the normal passes through the point (1, 2).

Therefore, we have:

$$2 - k = \frac{-2}{h}(1 - h) \text{ or } k = 2 + \frac{2}{h}(1 - h) \quad \dots (i)$$

Since  $(h, k)$  lies on the curve  $x^2 = 4y$ , we have  $h^2 = 4k$ .

$$\Rightarrow k = \frac{h^2}{4}$$

From equation (i), we have:

$$\begin{aligned}\frac{h^2}{4} &= 2 + \frac{2}{h}(1 - h) \\ \Rightarrow \frac{h^3}{4} &= 2h + 2 - 2h = 2 \\ \Rightarrow h^3 &= 8 \\ \Rightarrow h &= 2 \\ \therefore k &= \frac{h^2}{4} \Rightarrow k = 1\end{aligned}$$

Hence, the equation of the normal is given as:

$$\begin{aligned}\Rightarrow y - 1 &= \frac{-2}{2}(x - 2) \\ \Rightarrow y - 1 &= -(x - 2) \\ \Rightarrow x + y &= 3\end{aligned}$$

The correct answer is A.

Answer needs Correction? [Click Here](#)

Q24 : The points on the curve  $9y^2 = x^3$ , where the normal to the curve makes equal intercepts with the axes are

- (A)  $\left(4, \pm \frac{8}{3}\right)$  (B)  $\left(4, -\frac{8}{3}\right)$   
(C)  $\left(4, \pm \frac{3}{4}\right)$  (D)  $\left(\pm 4, \frac{3}{4}\right)$

$$\therefore (x_1^3 - 8) = (x_1^3 - 8)$$

**Answer :**

The equation of the given curve is  $9y^2 = x^3$ .

Differentiating with respect to  $x$ , we have:

$$9(2y) \frac{dy}{dx} = 3x^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2}{6y}$$

The slope of the normal to the given curve at point  $(x_1, y_1)$  is

$$\left. \frac{dy}{dx} \right|_{(x_1, y_1)} = -\frac{6y_1}{x_1^2}.$$

$\therefore$  The equation of the normal to the curve at  $(x_1, y_1)$  is

$$y - y_1 = \frac{-6y_1}{x_1^2} (x - x_1).$$

$$\Rightarrow x_1^2 y - x_1^2 y_1 = -6xy_1 + 6x_1 y_1$$

$$\Rightarrow 6xy_1 + x_1^2 y = 6x_1 y_1 + x_1^2 y_1$$

$$\Rightarrow \frac{6xy_1}{6x_1 y_1 + x_1^3 y_1} + \frac{x_1^2 y}{6x_1 y_1 + x_1^3 y_1} = 1$$

$$\Rightarrow \frac{x}{x_1(6 + x_1)} + \frac{y}{y_1(6 + x_1)} = 1$$

It is given that the normal makes equal intercepts with the axes.

Therefore, We have:

$$\therefore \frac{x_1(6 + x_1)}{6} = \frac{y_1(6 + x_1)}{x_1}$$

$$\Rightarrow \frac{x_1}{6} = \frac{y_1}{x_1}$$

$$\Rightarrow x_1^2 = 6y_1 \quad \dots(i)$$

Also, the point  $(x_1, y_1)$  lies on the curve, so we have

$$9y_1^2 = x_1^3 \quad \dots(ii)$$

From (i) and (ii), we have:

$$9\left(\frac{x_1^2}{6}\right)^2 = x_1^3 \Rightarrow \frac{x_1^4}{4} = x_1^3 \Rightarrow x_1 = 4$$

From (ii), we have:

$$9y_1^2 = (4)^3 = 64$$

$$\Rightarrow y_1^2 = \frac{64}{9}$$

$$\Rightarrow y_1 = \pm \frac{8}{3}$$

Hence, the required points are  $\left(4, \pm \frac{8}{3}\right)$ .

The correct answer is A.

\*\*\*\*\* END \*\*\*\*\*