Polynomials Ex 2.3 Q7

Answer :

We know that,

$$f(x)=g(x)\times q(x)+r(x)$$
$$f(x)-r(x)=g(x)\times q(x)$$
$$f(x)+\{-r(x)\}=g(x)\times q(x)$$

Clearly , Right hand side is divisible by $g(x)$.

Therefore, Left hand side is also divisible by $g(x)$.Thus, if we add $-r(x)$ to $f(x)$, then the resulting polynomial is divisible by $g(x)$.

Let us now find the remainder when $f(x)$ is divided by $g(x)$.

$$\begin{array}{r|l} & x^2+1 \\ x^2+2x-3 & \cancel{x^4}+\cancel{2x^3}-2x^2+x-1 \\ & \underline{\cancel{x^4}+\cancel{2x^3}-3x^2} \\ & \qquad\qquad +\cancel{x^2}+x-1 \\ & \qquad\qquad \underline{+\cancel{x^2}+2x-3} \\ & \qquad\qquad\quad -x+2 \end{array}$$

Hence, we should add $-r(x)=x-2$ to $f(x)$ so that the resulting polynomial is divisible by $g(x)$.

. Polynomials Ex 2.3 Q8

Answer :

We know that Dividend = Quotient × Divisor + Remainder.
Dividend − Remainder = Quotient × Divisor.
Clearly, Right hand side of the above result is divisible by the divisor.
Therefore, left hand side is also divisible by the divisor.
Thus, if we subtract remainder from the dividend, then it will be exactly divisible by the divisor.
Dividing $x^4+2x^3-13x^2-12x+21$ by x^2-4x+3

$$\begin{array}{r|l} & x^2+6x+8 \\ x^2-4x+3 & +\cancel{x^4}+2x^3-13x^2-12x+21 \\ & \underline{+\cancel{x^4}-4x^3+3x^2} \\ & \quad +\cancel{6x^3}-16x^2-12 \\ & \quad \underline{+\cancel{6x^3}-24x^2+18x} \\ & \qquad\qquad +\cancel{8x^2}-30x+21 \\ & \qquad\qquad \underline{+\cancel{8x^2}-32x+24} \\ & \qquad\qquad\qquad +2x-3 \end{array}$$

Therefore, quotient $=x^2+6x+8$ and remainder $=(2x-3)$.

Thus, if we subtract the remainder $\boxed{2x-3}$ from $x^4+2x^3-13x^2-12x+21$, it will be divisible by x^2-4x+3.

Polynomials Ex 2.3 Q9

Answer :

We know that if $x=\alpha$ is a zero of a polynomial, then $x-\alpha$ is a factor of $f(x)$.

Since, 2 and -2 are zeros of $f(x)$.

Therefore

$(x+2)(x-2)=x^2-2^2$

$=x^2-4$

x^2-4 is a factor of $f(x)$. Now, we divide $x^4+x^3-34x^2-4x+120$ by $g(x)=x^2-4$ to find the other zeros of $f(x)$.

$$
\begin{array}{r|l}
 & x^2+x-30 \\
\hline
x^2-4 & +\cancel{x^4}+x^3-34x^2-4x+120 \\
 & \underline{\underset{-}{+}\cancel{x^4}+0\underset{-}{+}\ 4x^2} \\
 & \quad +\cancel{x^3}-30x^2-\cancel{4x} \\
 & \quad \underline{\underset{-}{+}\cancel{x^3}+\ 0\underset{+}{-}\cancel{4x}} \\
 & \qquad -\cancel{30x^2}+\cancel{120} \\
 & \qquad \underline{\underset{+}{-}\cancel{30x^2}\underset{-}{+}\cancel{120}} \\
 & \qquad\quad 0
\end{array}
$$

By using division algorithm we have $f(x)=g(x)\times q(x)-r(x)$

$x^4+x^3-34x^2-4x+120=(x^2-4)(x^2+x-30)-0$

$x^4+x^3-34x^2-4x+120=(x+2)(x-2)(x^2+6x-5x-30)$

$x^4+x^3-34x^2-4x+120=(x+2)(x-2)(x(x+6)-5(x+6))$

$x^4+x^3-34x^2-4x+120=(x+2)(x-2)(x+6)(x-5)$

Hence, the zeros of the given polynomial are $\boxed{-2,\ +2,\ -6, \text{ and } 5}$.

*********** END **********