



Trigonometric Ratios Ex 5.1 Q23

**Answer :**

Given:

$$\sec A = \frac{5}{4} \dots\dots (1)$$

To verify:

$$\frac{3 \sin A - 4 \sin^3 A}{4 \cos^3 A - 3 \cos A} = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A} \dots\dots (2)$$

Now we know that  $\sec A = \frac{1}{\cos A}$

Therefore  $\cos A = \frac{1}{\sec A}$

Now, by substituting the value of  $\sec A$  from equation (1)

We get,

$$\begin{aligned} \cos A &= \frac{1}{\frac{5}{4}} \\ &= \frac{4}{5} \end{aligned}$$

Therefore,

$$\cos A = \frac{4}{5} \dots\dots (3)$$

Now, we know the following trigonometric identity

$$\cos^2 A + \sin^2 A = 1$$

Therefore,

$$\sin^2 A = 1 - \cos^2 A$$

Now by substituting the value of  $\cos A$  from equation (3)

We get,

$$\begin{aligned}\sin^2 A &= 1 - \left(\frac{4}{5}\right)^2 \\ &= 1 - \frac{(4)^2}{(5)^2} \\ &= 1 - \frac{16}{25}\end{aligned}$$

Now by taking L.C.M

We get,

$$\begin{aligned}\sin^2 A &= \frac{25-16}{25} \\ &= \frac{9}{25}\end{aligned}$$

Now, by taking square root on both sides

We get,

$$\begin{aligned}\sin A &= \sqrt{\frac{9}{25}} \\ &= \frac{\sqrt{9}}{\sqrt{25}} \\ &= \frac{3}{5}\end{aligned}$$

Therefore,

$$\sin A = \frac{3}{5} \dots\dots (4)$$

Now, we know that  $\tan A = \frac{\sin A}{\cos A}$

Now by substituting the value of  $\sin A$  and  $\cos A$  from equation (3) and (4) respectively

We get,

$$\begin{aligned}\tan A &= \frac{\frac{3}{5}}{\frac{4}{5}} \\ &= \frac{3}{5} \times \frac{5}{4} \\ &= \frac{3}{4}\end{aligned}$$

Therefore

$$\tan A = \frac{3}{4} \dots\dots (5)$$

Now from the expression of equation (2)

$$\text{L.H.S} = \frac{3 \sin A - 4 \sin^3 A}{4 \cos^3 A - 3 \cos A}$$

Now by substituting the value of  $\cos A$  and  $\sin A$  from equation (3) and (4)

We get,

$$\text{L.H.S} = \frac{3\left(\frac{3}{5}\right) - 4\left(\frac{3}{5}\right)^3}{4\left(\frac{4}{5}\right)^3 - 3\left(\frac{4}{5}\right)}$$

Therefore,

$$\begin{aligned} \text{L.H.S} &= \frac{\frac{9}{5} - 4\left(\frac{27}{125}\right)}{4\left(\frac{64}{125}\right) - \frac{12}{5}} \\ &= \frac{\frac{9}{5} - \frac{108}{125}}{\frac{256}{125} - \frac{12}{5}} \end{aligned}$$

Now by taking L.C.M of both numerator and denominator

We get,

$$\begin{aligned} \text{L.H.S} &= \frac{\frac{9 \times 25}{125} - \frac{108}{125}}{\frac{256 \times 25}{125} - \frac{12 \times 25}{5 \times 25}} \\ &= \frac{\frac{225}{125} - \frac{108}{125}}{\frac{256}{125} - \frac{300}{125}} \\ &= \frac{\frac{225 - 108}{125}}{\frac{256 - 300}{125}} \\ &= \frac{117}{-44} \\ &= \frac{-117}{44} \end{aligned}$$

$$\frac{3 \sin A - 4 \sin^3 A}{4 \cos^3 A - 3 \cos A} = \frac{-117}{44} \dots\dots (6)$$

Now from the expression of equation (2)

$$\text{R.H.S} = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$$

Now by substituting the value of  $\tan A$  from equation (5)

We get,

$$\text{R H S} = \underline{\underline{3\left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^3}}$$

$$1 - 3\left(\frac{3}{4}\right)^2$$

$$= \frac{9}{4} - \frac{27}{16}$$

$$= \frac{4 \times 9 - 27}{16}$$

Now by taking L.C.M

We get,

$$\text{R.H.S} = \frac{9 \times 16}{4 \times 16} - \frac{27}{16}$$

$$= \frac{144 - 27}{16}$$

$$= \frac{117}{16}$$

$$= \frac{117}{16}$$

$$\begin{aligned}
 & \frac{117}{64} \\
 &= \frac{64}{-11} \\
 & \frac{16}{64} \\
 &= \frac{117}{64} \times \frac{16}{-11}
 \end{aligned}$$

Now,  $16 \times 4 = 64$

Therefore,

$$\begin{aligned}
 \text{R.H.S} &= \frac{117}{4} \times \frac{1}{-11} \\
 &= \frac{117 \times 1}{4 \times -11} \\
 &= \frac{117}{-44} \\
 &= \frac{-117}{44}
 \end{aligned}$$

Therefore,

$$\frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A} = \frac{-117}{44} \dots\dots (7)$$

Now by comparing equation (6) and (7)

$$\text{We get, } \frac{3 \sin A - 4 \sin^3 A}{4 \cos^3 A - 3 \cos A} = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$$

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