Chapter 6 Determinants Ex 6.4 Q29

Here,

$$D = \begin{vmatrix} 2 & 1 & -2 \\ 1 & -2 & 1 \\ 5 & -5 & 1 \end{vmatrix} = \begin{vmatrix} 12 & 9 & -2 \\ -4 & -3 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 1(-36+36) = 0$$

$$D_1 = \begin{vmatrix} 4 & 1 & -2 \\ -2 & -2 & 1 \\ -2 & -5 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 1 & -2 \\ 0 & -2 & 1 \\ 0 & -5 & 1 \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} 2 & 4 & -2 \\ 1 & -2 & 1 \\ 5 & -2 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 0 & -2 \\ 1 & 0 & 1 \\ 5 & 0 & 1 \end{vmatrix} = 0$$

$$D_3 = \begin{vmatrix} 2 & 1 & 4 \\ 1 & -2 & -2 \\ 5 & -5 & -2 \end{vmatrix} = \begin{vmatrix} 4 & -3 & 0 \\ 1 & -2 & -2 \\ 4 & -3 & 0 \end{vmatrix} = 2(-12+12) = 0$$

So, $D = D_1 = D_2 = D_3 = 0$

So, the given system is either inconsistent or has infinite solutions.

Consider the 2nd and 3rd equation, written as

$$x - 2y = -2 - z$$
$$5x - 5y = -2 - z$$

Then,

$$D = \begin{vmatrix} 1 & -2 \\ 5 & -5 \end{vmatrix} = -5 - (-10) = 5$$

$$D_1 = \begin{vmatrix} -2-z & -2 \\ -2-z & -5 \end{vmatrix} = (2+z)(5) - 2(2+z) = 3(2+z) = 6+3z$$

$$D_2 = \begin{vmatrix} 1 & -(2+z) \\ 5 & -(2+z) \end{vmatrix} = -(2+z) + 5(2+z) = 4(2+z) = 8+4z$$

$$\therefore \quad x = \frac{D_1}{D} = \frac{6+3z}{5}$$

$$y = \frac{D_2}{D} = \frac{8+4z}{5}$$

Let $z = k$, then

$x = \frac{6+3k}{5}$, $y = \frac{8+4k}{5}$, $z = k$ are the infinite solution of the given system of equations.

Chapter 6 Determinants Ex 6.4 Q30

Here,

$$D = \begin{vmatrix} 1 & -1 & 3 \\ 1 & 3 & -3 \\ 5 & 3 & 3 \end{vmatrix} = \begin{vmatrix} 2 & 2 & 0 \\ 1 & 3 & -3 \\ 6 & 6 & 0 \end{vmatrix} = 3(12-12) = 0$$

$$D_1 = \begin{vmatrix} 6 & -1 & 3 \\ -4 & 3 & -3 \\ 10 & 3 & 3 \end{vmatrix} = \begin{vmatrix} 2 & 2 & 0 \\ -4 & 3 & -3 \\ 6 & 6 & 0 \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} 1 & 6 & 3 \\ 1 & -4 & -3 \\ 5 & 10 & 3 \end{vmatrix} = \begin{vmatrix} 2 & 2 & 0 \\ 1 & -4 & -3 \\ 6 & 6 & 0 \end{vmatrix} = 0$$

$$D_3 = \begin{vmatrix} 1 & -1 & 6 \\ 1 & 3 & -4 \\ 5 & 3 & 10 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 6 \\ 0 & 4 & -10 \\ 0 & 8 & -20 \end{vmatrix} = 1(-80+80) = 0$$

So, $D = D_1 = D_2 = D_3 = 0$

So, the given system is either inconsistent or has infinite solutions.

Consider the first to equations, written as

$$x - y = 6 - 3z$$
$$x + 3y = -4 + 3z$$

Here,

$$D = \begin{vmatrix} 1 & -1 \\ 1 & 3 \end{vmatrix} = 3 + 1 = 4$$

$$D_1 = \begin{vmatrix} 6-3z & -1 \\ -4+3z & 3 \end{vmatrix} = 3(6-3z) + (-4+3z) = 14 - 6z$$

$$D_2 = \begin{vmatrix} 1 & 6-3z \\ 1 & -4+3z \end{vmatrix} = (-4+3z) - (6-3z) = -10 + 6z$$

$$\therefore \quad x = \frac{D_1}{D} = \frac{14-6z}{4} = \frac{7-3z}{2}$$

$$y = \frac{D_2}{D} = \frac{6\cancel{z}-10}{4} = \frac{3\cancel{z}-5}{2}$$

Let $z = k$, then

$x = \frac{7-3k}{2}$, $y = \frac{3k-5}{2}$, $z = k$ are the infinite solution of the given system of equations.

Chapter 6 Determinants Ex 6.4 Q31

Let the rates of commissions on items A, B and C be x, y and z respectively.
Then we can express the given model as a system of linear equations

$$90x + 100y + 20z = 800$$
$$130x + 50y + 40z = 900$$
$$60x + 100y + 30z = 850$$

We will solve this using the Cramer's rule.
Here,

$$D = \begin{vmatrix} 90 & 100 & 20 \\ 130 & 50 & 40 \\ 60 & 100 & 30 \end{vmatrix} = \begin{vmatrix} -170 & 0 & -60 \\ 130 & 50 & 40 \\ -200 & 0 & -50 \end{vmatrix} = 50(8500 - 12000) = -175000$$

$$D_1 = \begin{vmatrix} 800 & 100 & 20 \\ 900 & 50 & 40 \\ 850 & 100 & 30 \end{vmatrix} = \begin{vmatrix} -1000 & 0 & -60 \\ 900 & 50 & 40 \\ -950 & 0 & -50 \end{vmatrix} = 50(50000 - 57000) = -350000$$

$$D_2 = \begin{vmatrix} 90 & 800 & 20 \\ 130 & 900 & 40 \\ 60 & 850 & 30 \end{vmatrix} = \begin{vmatrix} 90 & 800 & 20 \\ -50 & -700 & 0 \\ -75 & -350 & 0 \end{vmatrix} = 20(17500 - 52500) = -700000$$

$$D_3 = \begin{vmatrix} 90 & 100 & 800 \\ 130 & 50 & 900 \\ 60 & 100 & 850 \end{vmatrix} = \begin{vmatrix} -170 & 0 & -1000 \\ 130 & 50 & 900 \\ -200 & 0 & -950 \end{vmatrix} = 50(161500 - 200000) = -1925000$$

$$\therefore \quad x = \frac{D_1}{D} = \frac{-350000}{-175000} = 2$$
$$y = \frac{D_2}{D} = \frac{-700000}{-175000} = 4$$
$$z = \frac{D_3}{D} = \frac{-1925000}{-175000} = 11$$

$\therefore$ The rates of commission of items A, B and C are 2%, 4% and 11% respectively.

Chapter 6 Determinants Ex 6.4 Q32

Expressing the given information as a system of linear equations we get

$$2x + 3y + 4z = 29$$
$$x + y + 2z = 13$$
$$3x + 2y + z = 16$$

Where x, y, z is the number of cars C_1, C_2 and C_3 produced.
We use Cramer's rule to solve this system.
Here,

$$D = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 1 & 2 \\ 3 & 2 & 1 \end{vmatrix} = \begin{vmatrix} -10 & -5 & 0 \\ -5 & -3 & 0 \\ 3 & 2 & 1 \end{vmatrix} = 1(30 - 25) = 5$$

$$D_1 = \begin{vmatrix} 29 & 3 & 4 \\ 13 & 1 & 2 \\ 16 & 2 & 1 \end{vmatrix} = \begin{vmatrix} -35 & -5 & 0 \\ -19 & -3 & 0 \\ 16 & 2 & 1 \end{vmatrix} = 1(105 - 95) = 10$$

$$D_2 = \begin{vmatrix} 0 & 29 & 4 \\ 1 & 13 & 2 \\ 3 & 16 & 1 \end{vmatrix} = \begin{vmatrix} -10 & -35 & 0 \\ -5 & -19 & 0 \\ 3 & 16 & 1 \end{vmatrix} = 1(190 - 175) = 15$$

$$D_3 = \begin{vmatrix} 2 & 3 & 29 \\ 1 & 1 & 13 \\ 3 & 2 & 16 \end{vmatrix} = \begin{vmatrix} -2 & 0 & 0 \\ 1 & 1 & 13 \\ 3 & 2 & 16 \end{vmatrix} = -2(16 - 26) = 20$$

$$\therefore \quad x = \frac{D_1}{D} = \frac{10}{5} = 2$$
$$y = \frac{D_2}{D} = \frac{15}{5} = 3$$
and
$$z = \frac{D_3}{D} = \frac{20}{5} = 4$$

Hence, the number of cars produced of type C_1, C_2 and C_3 are 2, 3 and 4 respectively.

*********** END **********