



Maxima and Minima 18.4 Q1(i)

The given function is $f(x) = 4x - \frac{1}{2}x^2$.

$$\therefore f'(x) = 4 - \frac{1}{2}(2x) = 4 - x$$

Now,

$$f'(x) = 0 \Rightarrow x = 4$$

Then, we evaluate the value of f at critical point $x = 4$ and at the end points of the interval $\left[-2, \frac{9}{2}\right]$.

$$f(4) = 4\left(4\right) - \frac{1}{2}(16) = 16 - 8 = 8$$

$$f(-2) = 4(-2) - \frac{1}{2}(4) = -8 - 2 = -10$$

$$f\left(\frac{9}{2}\right) = 4\left(\frac{9}{2}\right) - \frac{1}{2}\left(\frac{9}{2}\right)^2 = 18 - \frac{81}{8} = 18 - 10.125 = 7.875$$

Hence, we can conclude that the absolute maximum value of f on $\left[-2, \frac{9}{2}\right]$ is 8 occurring at $x = 4$

and the absolute minimum value of f on $\left[-2, \frac{9}{2}\right]$ is -10 occurring at $x = -2$.

Maxima and Minima 18.4 Q1(ii)

The given function is $f(x) = (x-1)^2 + 3$.

$$\therefore f'(x) = 2(x-1)$$

Now,

$$f'(x) = 0 \Rightarrow 2(x-1) = 0 \Rightarrow x = 1$$

Then, we evaluate the value of f at critical point $x = 1$ and at the end points of the interval $[-3, 1]$.

$$f(1) = (1-1)^2 + 3 = 0 + 3 = 3$$

$$f(-3) = (-3-1)^2 + 3 = 16 + 3 = 19$$

Hence, we can conclude that the absolute maximum value of f on $[-3, 1]$ is 19 occurring at $x = -3$ and the minimum value of f on $[-3, 1]$ is 3 occurring at $x = 1$.

Maxima and Minima 18.4 Q1(iii)

$$\text{Let } f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25.$$

$$\begin{aligned}\therefore f'(x) &= 12x^3 - 24x^2 + 24x - 48 \\ &= 12(x^3 - 2x^2 + 2x - 4) \\ &= 12[x^2(x-2) + 2(x-2)] \\ &= 12(x-2)(x^2+2)\end{aligned}$$

Now, $f'(x) = 0$ gives $x = 2$ or $x^2 + 2 = 0$ for which there are no real roots.

Therefore, we consider only $x = 2 \in [0, 3]$.

Now, we evaluate the value of f at critical point $x = 2$ and at the end points of the interval $[0, 3]$.

$$\begin{aligned}f(2) &= 3(16) - 8(8) + 12(4) - 48(2) + 25 \\ &= 48 - 64 + 48 - 96 + 25 \\ &= -39 \\ f(0) &= 3(0) - 8(0) + 12(0) - 48(0) + 25 \\ &= 25 \\ f(3) &= 3(81) - 8(27) + 12(9) - 48(3) + 25 \\ &= 243 - 216 + 108 - 144 + 25 = 16\end{aligned}$$

Hence, we can conclude that the absolute maximum value of f on $[0, 3]$ is 25 occurring at $x = 0$ and the absolute minimum value of f at $[0, 3]$ is -39 occurring at $x = 2$.

Maxima and Minima 18.4 Q1(iv)

$$f(x) = (x-2)\sqrt{x-1}$$

$$\Rightarrow f'(x) = \sqrt{x-1} + (x-2) \frac{1}{2\sqrt{x-1}}$$

$$\text{Put } f'(x) = 0$$

$$\Rightarrow \sqrt{x-1} + \frac{x-2}{2\sqrt{x-1}} = 0$$

$$\Rightarrow \frac{2(x-1) + (x-2)}{2\sqrt{x-1}} = 0$$

$$\Rightarrow \frac{3x-4}{2\sqrt{x-1}} = 0$$

$$\Rightarrow x = \frac{4}{3}$$

Now,

$$f(1) = 0$$

$$f\left(\frac{4}{3}\right) = \left(\frac{4}{3} - 2\right)\sqrt{\frac{4}{3} - 1} = \frac{4-6}{3\sqrt{3}} = \frac{-2}{3\sqrt{3}} = \frac{-2\sqrt{3}}{9}$$

$$f(9) = (9-2)\sqrt{9-1} = 7\sqrt{8} = 14\sqrt{2}$$

\therefore The absolute maximum value of $f(x)$ is $14\sqrt{2}$ at $x = 9$ and the absolute minimum value is $\frac{-2\sqrt{3}}{9}$ at $x = \frac{4}{3}$.

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