Trigonometric Identities Ex 6.1 Q49

Answer :

In the given question, we need to prove $\tan^2 A+\cot^2 A=\sec^2 A\operatorname{cosec}^2 A-2$

Now, using $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ and $\cot\theta=\dfrac{\cos\theta}{\sin\theta}$ in L.H.S, we get

$$\begin{aligned}\tan^2 A+\cot^2 A&=\frac{\sin^2 A}{\cos^2 A}+\frac{\cos^2 A}{\sin^2 A}\\&=\frac{\sin^4 A+\cos^4 A}{\cos^2 A\sin^2 A}\\&=\frac{(\sin^2 A)^2+(\cos^2 A)^2}{\cos^2 A\sin^2 A}\end{aligned}$$

Further, using the identity $a^2+b^2=(a+b)^2-2ab$, we get

$$\begin{aligned}\frac{(\sin^2 A)^2+(\cos^2 A)^2}{\cos^2 A\sin^2 A}&=\frac{(\sin^2 A+\cos^2 A)^2-2\sin^2 A\cos^2 A}{\sin^2 A\cos^2 A}\\&=\frac{(1)^2-2\sin^2 A\cos^2 A}{\sin^2 A\cos^2 A}\\&=\frac{1}{\sin^2 A\cos^2 A}-\frac{2\sin^2 A\cos^2 A}{\sin^2 A\cos^2 A}\\&=\operatorname{cosec}^2 A\sec^2 A-2\end{aligned}$$

Since L.H.S = R.H.S

Hence proved.

Trigonometric Identities Ex 6.1 Q50

Answer :

In the given question, we need to prove $\dfrac{1-\tan^2 A}{\cot^2 A-1}=\tan^2 A$

Now, using $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ and $\cot\theta=\dfrac{\cos\theta}{\sin\theta}$ in the L.H.S, we get

$$\begin{aligned}\frac{1-\tan^2 A}{\cot^2 A-1}&=\frac{1-\dfrac{\sin^2 A}{\cos^2 A}}{\dfrac{\cos^2 A}{\sin^2 A}-1}\\&=\frac{\dfrac{\cos^2 A-\sin^2 A}{\cos^2 A}}{\dfrac{\cos^2 A-\sin^2 A}{\sin^2 A}}\\&=\frac{\cos^2 A-\sin^2 A}{\cos^2 A}\times\frac{\sin^2 A}{\cos^2 A-\sin^2 A}\end{aligned}$$

Solving further, we get

$$\begin{aligned}\frac{\cos^2 A-\sin^2 A}{\cos^2 A}\times\frac{\sin^2 A}{\cos^2 A-\sin^2 A}&=\frac{\sin^2 A}{\cos^2 A}\\&=\tan^2 A\end{aligned}$$

Hence proved.

Trigonometric Identities Ex 6.1 Q51

Answer :

In the given question, we need to prove $1+\frac{\cot^2\theta}{1+\operatorname{cosec}\theta}=\operatorname{cosec}\theta$

Using $\cot\theta=\frac{\cos\theta}{\sin\theta}$ and $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$, we get

$$\begin{aligned}
1+\frac{\cot^2\theta}{1+\operatorname{cosec}\theta}&=\frac{1+\operatorname{cosec}\theta+\cot^2\theta}{1+\operatorname{cosec}\theta}\\
&=\frac{\left(1+\frac{1}{\sin\theta}+\frac{\cos^2\theta}{\sin^2\theta}\right)}{\left(1+\frac{1}{\sin\theta}\right)}\\
&=\frac{\left(\frac{\sin^2\theta+\sin\theta+\cos^2\theta}{\sin^2\theta}\right)}{\left(\frac{\sin\theta+1}{\sin\theta}\right)}
\end{aligned}$$

Further, using the property $\sin^2\theta+\cos^2\theta=1$, we get

$$\begin{aligned}
\frac{\left(\frac{\sin^2\theta+\sin\theta+\cos^2\theta}{\sin^2\theta}\right)}{\left(\frac{\sin\theta+1}{\sin\theta}\right)}&=\frac{\left(\frac{1+\sin\theta}{\sin^2\theta}\right)}{\left(\frac{\sin\theta+1}{\sin\theta}\right)}\\
&=\left(\frac{1+\sin\theta}{\sin^2\theta}\right)\left(\frac{\sin\theta}{1+\sin\theta}\right)\\
&=\frac{1}{\sin\theta}\\
&=\operatorname{cosec}\theta
\end{aligned}$$

Hence proved.

*********** END **********