



Indefinite Integrals Ex 19.24 Q8

$$\begin{aligned}\text{Let } I &= \int \frac{2 \tan x + 3}{3 \tan x + 4} dx \\ &= \int \frac{2 \sin x + 3 \cos x}{3 \sin x + 4 \cos x} dx\end{aligned}$$

$$\begin{aligned}\text{Let } 2 \sin x + 3 \cos x &= \lambda \frac{d}{dx} (3 \sin x + 4 \cos x) + \mu (3 \sin x + 4 \cos x) + v \\ 2 \sin x + 3 \cos x &= \lambda (3 \cos x - 4 \sin x) + \mu (3 \sin x + 4 \cos x) + v \\ 2 \sin x + 3 \cos x &= (3\lambda + 4\mu) \cos x + (-4\lambda + 3\mu) \sin x + v\end{aligned}$$

Comparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$$3\lambda + 4\mu = 3 \text{ --- (1)}$$

$$-4\lambda + 3\mu = 2 \text{ --- (2)}$$

$$v = 0$$

Solving the equation (1), (2) and (3),

$$\mu = \frac{18}{25}$$

$$\lambda = \frac{1}{25}$$

$$v = 0$$

$$I = \frac{1}{25} \int \frac{(3 \cos x - 4 \sin x)}{(3 \sin x + 4 \cos x)} dx + \frac{18}{25} \int dx$$

$$I = \frac{18}{25} x + \frac{1}{25} \log |3 \sin x + 4 \cos x| + c$$

Indefinite Integrals Ex 19.24 Q9

$$\begin{aligned}\text{Let } I &= \int \frac{1}{4 + 3 \tan x} dx \\ I &= \int \frac{\cos x}{4 \cos x + 3 \sin x} dx\end{aligned}$$

$$\begin{aligned}\text{Let } \cos x &= \lambda \frac{d}{dx} (4 \cos x + 3 \sin x) + \mu (4 \cos x + 3 \sin x) + v \\ \cos x &= \lambda (-4 \sin x + 3 \cos x) + \mu (4 \cos x + 3 \sin x) + v \\ \cos x &= (-4\lambda + 3\mu) \sin x + (3\lambda + 4\mu) \cos x + v\end{aligned}$$

Comparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$$-4\lambda + 3\mu = 0 \text{ --- (1)}$$

$$3\lambda + 4\mu = 1 \text{ --- (2)}$$

$$v = 0 \text{ --- (3)}$$

Solving the equation (1), (2) and (3),

$$\lambda = \frac{3}{25}$$

$$\mu = \frac{4}{25}$$

$$v = 0$$

$$I = \int \frac{3}{25} \frac{(-4 \sin x + 3 \cos x)}{(4 \cos x + 3 \sin x)} dx + \frac{4}{25} \int dx$$

$$I = \frac{3}{25} \log |4 \cos x + 3 \sin x| + \frac{4}{25} x + c$$

Indefinite Integrals Ex 19.24 Q10

$$\text{Let } I = \int \frac{8 \cot x + 1}{3 \cot x + 2} dx$$

$$I = \int \frac{8 \cos x + \sin x}{3 \cos x + 2 \sin x} dx$$

$$\text{Let } 8 \cos x + \sin x = \lambda \frac{d}{dx} (3 \cos x + 2 \sin x) + \mu (3 \cos x + 2 \sin x) + v$$

$$8 \cos x + \sin x = \lambda (-3 \sin x + 2 \cos x) + \mu (3 \cos x + 2 \sin x) + v$$

$$8 \cos x + \sin x = (-3\lambda + 2\mu) \sin x + (2\lambda + 3\mu) \cos x + v$$

Comparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$$2\lambda + 3\mu = 8 \text{ --- (1)}$$

$$-3\lambda + 2\mu = 1 \text{ --- (2)}$$

$$v = 0 \text{ --- (3)}$$

Solving equation (1), (2) and (3),

$$\lambda = 1$$

$$\mu = 2$$

$$v = 0$$

$$I = \int \frac{(-3 \sin x + 2 \cos x)}{(3 \cos x + 2 \sin x)} dx + 2 \int dx$$

$$I = \log|3 \cos x + 2 \sin x| + 2x + c$$

Indefinite Integrals Ex 19.24 Q11

$$\text{Let } I = \int \frac{4 \sin x + 5 \cos x}{5 \sin x + 4 \cos x} dx$$

$$\text{Let } 4 \sin x + 5 \cos x = \lambda \frac{d}{dx} (5 \sin x + 4 \cos x) + \mu (5 \sin x + 4 \cos x) + v$$

$$4 \sin x + 5 \cos x = \lambda (5 \cos x - 4 \sin x) + \mu (5 \sin x + 4 \cos x) + v$$

$$4 \sin x + 5 \cos x = (5\lambda + 4\mu) \cos x + (-4\lambda + 5\mu) \sin x + v$$

Comparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$$-4\lambda + 5\mu = 4 \text{ --- (1)}$$

$$5\lambda + 4\mu = 5 \text{ --- (2)}$$

$$v = 0 \text{ --- (3)}$$

Solving equation (1), (2) and (3),

$$\lambda = \frac{9}{41}$$

$$\mu = \frac{40}{41}$$

$$v = 0$$

Now,

$$I = \frac{40}{41} \int dx + \frac{9}{41} \int \frac{(5 \cos x - 4 \sin x)}{(5 \sin x + 4 \cos x)} dx$$

$$I = \frac{40}{41} x + \frac{9}{41} \log|5 \sin x + 4 \cos x| + c$$

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