CHAPTER 32

ELECTRIC CURRENT IN CONDUCTORS

32.1 ELECTRIC CURRENT AND CURRENT DENSITY

When there is a transfer of charge from one side of an area to the other, we say that there is an *electric current* through the area. If the moving charges are positive, the current is in the direction of motion. If they are negative, the current is opposite to the direction of motion. If a charge ΔQ crosses an area in time Δt, we define the average electric current through the area during this time as

$$\bar{i} = \frac{\Delta Q}{\Delta t}.$$

The current at time t is

$$i = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}. \quad \ldots (32.1)$$

Thus, electric current through an area is the rate of transfer of charge from one side of the area to the other. The SI unit of current is ampere. If one coulomb of charge crosses an area in one second, the current is one ampere. It is one of the seven base units accepted in SI.

We shall now define a vector quantity known as *electric current density* at a point. To define the current density at a point P, we draw a small area ΔS through P perpendicular to the flow of charges (figure 32.1a). If Δi be the current through the area ΔS, the average current density is

$$\bar{j} = \frac{\Delta i}{\Delta S}.$$

The current density at the point P is

$$j = \lim_{\Delta S \to 0} \frac{\Delta i}{\Delta S} = \frac{di}{dS}.$$

The direction of the current density is the same as the direction of the current. Thus, it is along the motion of the moving charges if the charges are positive and opposite to the motion of the charges if the charges are negative. If a current i is uniformly distributed over an area S and is perpendicular to it,

$$j = \frac{i}{S}. \quad \ldots (32.2)$$

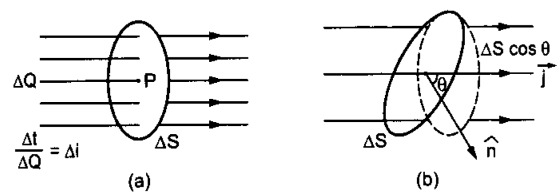


Figure 32.1

Now let us consider an area ΔS which is not necessarily perpendicular to the current (figure 32-1b). If the normal to the area makes an angle θ with the direction of the current, the current density is,

$$j = \frac{\Delta i}{\Delta S \cos\theta}$$

or, $$\Delta i = j \, \Delta S \cos\theta$$

where Δi is the current through ΔS. If $\vec{\Delta S}$ be the *area-vector* corresponding to the area ΔS, we have

$$\Delta i = \vec{j} \cdot \vec{\Delta S}.$$

For a finite area,

$$i = \int \vec{j} \cdot d\vec{S}. \quad \ldots (32.3)$$

Note carefully that an electric current has direction as well as magnitude but it is not a vector quantity. It does not add like vectors. The current density is a vector quantity.

Example 32.1

An electron beam has an aperture $1{\cdot}0 \text{ mm}^2$. A total of $6{\cdot}0 \times 10^{16}$ electrons go through any perpendicular cross section per second. Find (a) the current and (b) the current density in the beam.

Solution :

(a) The total charge crossing a perpendicular cross section in one second is

$$q = ne$$
$$= 6{\cdot}0 \times 10^{16} \times 1{\cdot}6 \times 10^{-19} \text{ C}$$
$$= 9{\cdot}6 \times 10^{-3} \text{ C}.$$

The current is

$$i = \frac{q}{t}$$

$$= \frac{9{\cdot}6 \times 10^{-3}\ \mathrm{C}}{1\ \mathrm{s}} = 9{\cdot}6 \times 10^{-3}\ \mathrm{A}.$$

As the charge is negative, the current is opposite to the direction of motion of the beam.

(b) The current density is

$$j = \frac{i}{S} = \frac{9{\cdot}6 \times 10^{-3}\ \mathrm{A}}{1{\cdot}0\ \mathrm{mm}^2} = 9{\cdot}6 \times 10^{3}\ \mathrm{A\ m^{-2}}.$$

Electric current can be obtained in a variety of ways. When a metal is heated to high temperatures, it emits electrons. These electrons travel in space. Considering any area perpendicular to their velocity, there is a current through it.

In many solutions, positive and negative ions wander. If the solution is placed in an electric field, the positive ions move (inside the solution) along the field and the negative ions move opposite to the field. Both movements contribute to a current in the direction of the field.

When a battery is connected across a capacitor, charges flow from the battery to the capacitor through the connecting wires. There is a current through any cross section of the wires as long as the charges keep going to the plates.

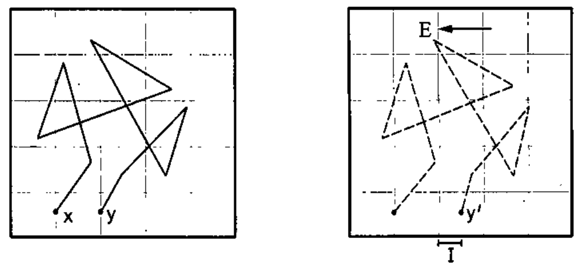
In this chapter, we shall study the electric current in a conductor when an electric field is established inside it.

32.2 DRIFT SPEED

A conductor contains a large number of loosely bound electrons which we call *free electrons* or *conduction electrons*. The remaining material is a collection of relatively heavy positive ions which we call *lattice*. These ions keep on vibrating about their mean positions. The average amplitude depends on the temperature. Occasionally, a free electron collides or interacts in some other fashion with the lattice. The speed and direction of the electron changes randomly at each such event. As a result, the electron moves in a zig-zag path. As there is a large number of free electrons moving in random directions, the number of electrons crossing an area ΔS from one side very nearly equals the number crossing from the other side in any given time interval. The electric current through the area is, therefore, zero.

When there is an electric field inside the conductor, a force acts on each electron in the direction opposite to the field. The electrons get biased in their random motion in favour of the force. As a result, the electrons drift slowly in this direction. At each collision, the electron starts afresh in a random direction with a random speed but gains an additional velocity v' due to the electric field. This velocity v' increases with time and suddenly becomes zero as the electron makes a collision with the lattice and starts afresh with a random velocity. As the time t between successive collisions is small, the electron slowly and steadily drifts opposite to the applied field (figure 32.2). If the electron drifts a distance l in a long time t, we define drift speed as

$$v_d = \frac{l}{t}.$$



(a) (b)

Figure 32.2

If τ be the average time between successive collisions, the distance drifted during this period is

$$l = \frac{1}{2}a(\tau)^2 = \frac{1}{2}\left(\frac{eE}{m}\right)(\tau)^2.$$

The drift speed is

$$v_d = \frac{l}{\tau} = \frac{1}{2}\left(\frac{eE}{m}\right)\tau.$$

It is proportional to the electric field E and to the average collision-time τ.

The random motion of free electrons does not contribute to the drift of these electrons. Also, the average collision-time is constant for a given material at a given temperature. We, therefore, make the following assumption for our present purpose of discussing electric current.

When no electric field exists in a conductor, the free electrons stay at rest ($v_d = 0$) and when a field E exists, they move with a constant velocity

$$v_d = \frac{e\tau}{2\,m}E = kE \qquad \text{... (32.4)}$$

opposite to the field. The constant k depends on the material of the conductor and its temperature.

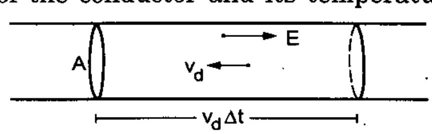


Figure 32.3

Let us now find the relation between the current density and the drift speed. Consider a cylindrical conductor of cross-sectional area A in which an electric field E exists. Consider a length $v_d \, \Delta t$ of the conductor (figure 32.3). The volume of this portion is $Av_d \Delta t$. If there are n free electrons per unit volume of the wire, the number of free electrons in this portion is $nAv_d \, \Delta t$. All these electrons cross the area A in time Δt. Thus, the charge crossing this area in time Δt is

$$\Delta Q = nAv_d \, \Delta t \, e$$

or, $$i = \frac{\Delta Q}{\Delta t} = nAv_d \, e$$

and $$j = \frac{i}{A} = nev_d. \qquad \dots (32.5)$$

Example 32.2

Calculate the drift speed of the electrons when 1 A of current exists in a copper wire of cross section 2 mm². The number of free electrons in 1 cm³ of copper is 8·5 × 10²².

Solution : We have

$$j = nev_d$$

or, $$v_d = \frac{j}{n\,e} = \frac{i}{A\,ne}$$

$$= \frac{1 \text{ A}}{(2 \times 10^{-6} \text{ m}^2)(8{\cdot}5 \times 10^{22} \times 10^{6} \text{ m}^{-3})(1{\cdot}6 \times 10^{-19} \text{ C})}$$

$$= 0{\cdot}036 \text{ mm s}^{-1}.$$

We see that the drift speed is indeed small.

32.3 OHM'S LAW

Using equations (32.4) and (32.5),

$$j = nev_d = \frac{ne^2 \tau}{2\,m} E$$

or, $$j = \sigma E \qquad \dots (32.6)$$

where σ depends only on the material of the conductor and its temperature. This constant is called the *electrical conductivity* of the material. Equation (32.6) is known as *Ohm's law.*

The *resistivity* of a material is defined as

$$\rho = \frac{1}{\sigma} \cdot \qquad \dots (32.7)$$

Ohm's law tells us that the conductivity (or resistivity) of a material is independent of the electric field existing in the material. This is valid for conductors over a wide range of field.

Suppose we have a conductor of length l and uniform cross sectional area A (figure 32.4a). Let us apply a potential difference V between the ends of the conductor. The electric field inside the conductor is $E = V/l$. If the current in the conductor is i, the current density is $j = \frac{i}{A} \cdot$ Ohm's law $j = \sigma E$ then becomes

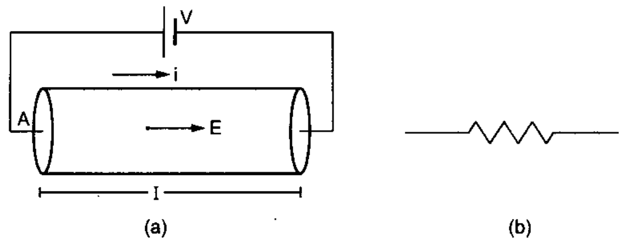


Figure 32.4

$$\frac{i}{A} = \sigma \frac{V}{l}$$

or, $$V = \frac{1}{\sigma} \frac{l}{A} i = \rho \frac{l}{A} i$$

or, $$V = R\,i \qquad \dots (32.8)$$

where $$R = \rho \frac{l}{A} \qquad \dots (32.9)$$

is called the *resistance* of the given conductor. The quantity $1/R$ is called *conductance.*

Equation (32.8) is another form of Ohm's law which is widely used in circuit analysis. The unit of resistance is called ohm and is denoted by the symbol Ω. An object of conducting material, having a resistance of desired value, is called a *resistor.* A resistor is represented by the symbol shown in figure (32.4b).

From equation (32.9), the unit of resistivity ρ is ohm metre, also written as Ωm. The unit of conductivity (σ) is (ohm m)$^{-1}$ written as mho m^{-1}.

Example 32.3

Calculate the resistance of an aluminium wire of length 50 cm and cross sectional area 2·0 mm². The resistivity of aluminium is $\rho = 2{\cdot}6 \times 10^{-8}$ Ωm.

Solution :

The resistance is $R = \rho \frac{l}{A}$

$$= \frac{(2{\cdot}6 \times 10^{-8} \, \Omega\text{m}) \times (0{\cdot}50 \text{ m})}{2 \times 10^{-6} \text{ m}^2} = 0{\cdot}0065 \, \Omega.$$

We arrived at Ohm's law (equation 32.6 or 32.8) by making several assumptions about the existence and behaviour of the free electrons. These assumptions are not valid for semiconductors, insulators, solutions, etc. Ohm's law cannot be applied in such cases.

Colour Code for Resistors

Resistors of different values are commercially available. To make a resistor, carbon with a suitable binding agent is molded into a cylinder. Wire leads are

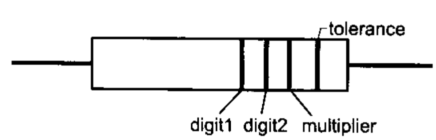


Figure 32.5

attached to this cylinder and the entire resistor is encased in a ceramic or plastic jacket. The two leads connect the resistor to a circuit. These resistors are widely used in electronic circuits such as those for radios, amplifiers, etc. The value of the resistance is indicated by four coloured-bands, marked on the surface of the cylinder (figure 32.5). The meanings of the four positions of the bands are shown in figure (32.5) and the meanings of different colours are given in table (32.1).

Table 32.1 : *Resistance codes (resistance given in ohm)*

Colour	Digit	Multiplier	Tolerance
Black	0	1	
Brown	1	10	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		0·1	5%
Silver		0·01	10%

For example, suppose the colours on the resistor shown in figure (32.5) are brown, yellow, green and gold as read from left to right. Using table (32.1), the resistance is

$$(14 \times 10^{5} \pm 5\%)\Omega = (1\cdot 4 \pm 0\cdot 07)\ \mathrm{M}\Omega.$$

Sometimes, the tolerance band is missing from the code so that there are only three bands. This means the tolerance is 20%.

32.4 TEMPERATURE DEPENDENCE OF RESISTIVITY

As the temperature of a conductor is increased, the thermal agitation increases and the collisions become more frequent. The average time τ between the successive collisions decreases and hence the drift speed decreases. Thus, the conductivity decreases and the resistivity increases as the temperature increases. For small temperature variations, we can write for most of the materials,

$$\rho(T) = \rho(T_0)\ [1 + \alpha(T - T_0)]$$

where $\rho(T)$ and $\rho(T_0)$ are resistivities at temperatures T and T_0 respectively and α is a constant for the given material. In fact, α depends to a small extent on the temperature. The constant α is called the *temperature coefficient of resistivity*. Table (32.2) lists the resistivity at room temperature and the average value of α for some materials.

Table 32.2 : *Resistivities of different materials*

Material	$\rho(\Omega\mathrm{m})$	$\alpha(\mathrm{K}^{-1})$
Silver	$1{\cdot}47 \times 10^{-8}$	0·0038
Copper	$1{\cdot}72 \times 10^{-8}$	0·0039
Gold	$2{\cdot}35 \times 10^{-8}$	0·0034
Aluminium	$2{\cdot}63 \times 10^{-8}$	0·0039
Tungsten	$5{\cdot}51 \times 10^{-8}$	0·0045
Nickel	$86{\cdot}84 \times 10^{-8}$	0·0060
Iron	$9{\cdot}71 \times 10^{-8}$	0·0050
Magnesium	44×10^{-8}	0·0000
Mercury	96×10^{-8}	0·0009
Nichrome	100×10^{-8}	0·0004
Silicon	640	– 0·075
Germanium	0·46	– 0·048
Fused quartz	$7{\cdot}5 \times 10^{17}$	

The resistance of a given conductor depends on its length and area of cross section besides the resistivity (equation 32.9). As temperature changes, the length and the area also change. But these changes are quite small and the factor l/A may be treated as constant. Then $R \propto \rho$ and hence

$$R(T) = R(T_0)\ [1 + \alpha(T - T_0)].$$

From table (32.2), we see that resistivity varies over a wide range. We have metals with resistivity of the order of $10^{-8}\ \Omega\mathrm{m}$. They are good conductors of electricity. Fused quartz has resistivity as high as $7{\cdot}5 \times 10^{17}\ \Omega\mathrm{m}$. This is an *insulator*. Then we have materials like silicon and germanium which have resistivity much smaller than that of insulators, but much larger than that of the metals. They are called *semiconductors.*

Thermistor

The temperature coefficient of resistivity is negative for semiconductors. This means that the resistivity decreases as we raise the temperature of such a material. The picture of a large number of free electrons colliding with each other and with the lattice is not adequate for semiconductors. The magnitude of the temperature coefficient of resistivity is often quite large for a semiconducting material. This fact is used to construct thermometers to detect small changes in temperatures. Such a device is called a *thermistor.* Thermistors are usually prepared from oxides of various metals such as nickel, iron, cobalt, copper, etc. These compounds are also semiconductors. A thermistor is usually enclosed in a capsule with epoxy surface. The thermistor is dipped in the bath whose temperature is to be measured. The circuit is completed by connecting a battery. The current through the thermistor is measured. If the temperature increases, the current also increases because of the decrease in resistivity. Thus, by noting the change in the current, one can find the change in temperature. A typical thermistor can easily measure a change in temperature of the order of 10^{-3} °C.

Superconductors

There are certain materials for which the resistivity suddenly becomes zero below a certain temperature. This temperature is called the *critical temperature* for this transition. The material in this state is called a *superconductor*. Above the critical temperature, the resistivity follows the trend of a normal metal (figure 32.6). This phenomenon was observed for mercury in 1911 by H Kamerlingh Onnes. The critical temperature for mercury is 4·2 K.

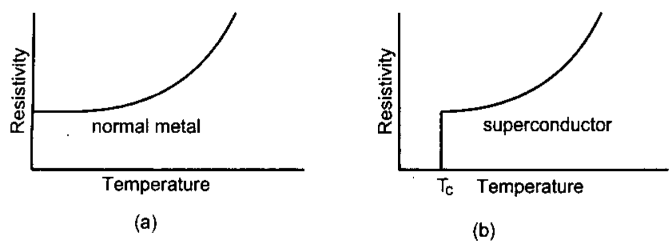


Figure 32.6

If an electric current is set up in a superconducting material, it can persist for long time without any applied emf. Steady currents have been observed for several years in superconducting loops without any observable decrease. Superconductors are used to construct very strong magnets. This has useful applications in material science research and high-energy particle physics. Possible applications of superconductors are ultrafast computer switches and transmission of electric power through superconducting power lines. However, the requirement of low temperature is posing difficulty. Scientists are putting great effort to construct compounds and alloys which would be superconducting at room temperature (300 K). Superconductivity at around 125 K has already been achieved and efforts are on to improve upon this.

32.5 BATTERY AND EMF

A battery is a device which maintains a potential difference between its two terminals A and B. Figure (32.7) shows a schematic diagram of a battery. Some internal mechanism exerts forces on the charges of the battery material. This force drives the positive charges of the battery material towards A and the negative charges of the battery material towards B. We show the force on a positive charge q as $\vec{F_b}$. As positive charge accumulates on A and negative charge on B, a potential difference develops and grows between A and B. An electric field $\vec{E}$ is developed in the battery material from A to B and exerts a force $\vec{F_e} = q\vec{E}$ on a charge q. The direction of this force is opposite to that of $\vec{F_b}$. In steady state, the charge accumulation on A and B is such that $F_b = F_e$. No further accumulation takes place.

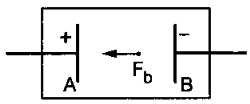


Figure 32.7

If a charge q is taken from the terminal B to the terminal A, the work done by the battery force F_b is $W = F_b d$ where d is the distance between A and B. The work done by the battery force per unit charge is

$$\mathcal{E} = \frac{W}{q} = \frac{F_b d}{q}. \quad \text{... (32.10)}$$

This quantity is called the *emf* of the battery. The full form of emf is *electromotive force.* The name is misleading in the sense that emf is not a force, it is work done/charge. We shall continue to denote this quantity by the short name emf. If nothing is connected externally between A and B,

$$F_b = F_e = qE$$

or, $$F_b d = qEd = qV,$$

where $V = Ed$ is the potential difference between the terminals. Thus,

$$\mathcal{E} = \frac{F_b d}{q} = V.$$

Thus, *the emf of a battery equals the potential difference between its terminals when the terminals are not connected externally.*

Potential difference and emf are two different quantities whose magnitudes may be equal in certain conditions. The emf is the work done per unit charge by the battery force F_b which is nonelectrostatic in nature. The potential difference originates from the electrostatic field created by the charges accumulated on the terminals of the battery.

A battery is often prepared by putting two rods or plates of different metals in a chemical solution. Such a battery, using chemical reactions to generate emf, is often called a *cell.*

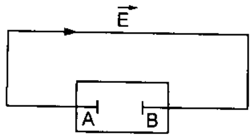


Figure 32.8

Now suppose the terminals of a battery are connected by a conducting wire as shown in figure (32.8). As the terminal A is at a higher potential than B, there is an electric field in the wire in the direction shown in the figure. The free electrons in the wire move in the opposite direction and enter the battery at the terminal A. Some electrons are withdrawn from the terminal B which enter the wire through the right end. Thus, the potential difference between A and B tends to decrease. If this potential difference decreases, the electrostatic force F_e inside the battery also decreases. The force F_b due to the battery mechanism remains the same. Thus, there is a net force on the positive charges of the battery material from B to A. The positive charges rush towards A and neutralise the effect of the electrons coming at A from the wire. Similarly, the negative charges rush towards B. Thus, the potential difference between A and B is maintained.

For calculation of current, motion of a positive charge in one direction is equivalent to the motion of a negative charge in opposite direction. Using this fact, we can describe the above situation by a simpler model. The positive terminal of the battery supplies positive charges to the wire. These charges are pushed through the wire by the electric field and they reach the negative terminal of the battery. The battery mechanism drives these charges back to the positive terminal against the electric field existing in the battery and the process continues. This maintains a steady current in the circuit.

Current can also be driven into a battery in the reverse direction. In such a case, positive charge enters the battery at the positive terminal, moves inside the battery to the negative terminal and leaves the battery from the negative terminal. Such a process is called *charging* of the battery. The more common process in which the positive charge comes out of the battery from the positive terminal is called *discharging* of the battery.

32.6 ENERGY TRANSFER IN AN ELECTRIC CIRCUIT

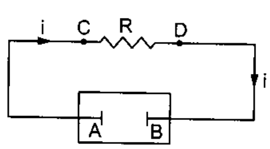


Figure 32.9

Figure (32.9) shows a simple circuit in which a resistor CD having a resistance R is connected to a battery of emf $\mathcal{E}$ through two connecting wires CA and DB. The connecting wires are assumed to have negligible resistance. This ensures that potential differences across AC and across BD are zero even when there is a current through them. The potential difference across the resistor is the same as that across the battery. If the current in the circuit is i, this potential difference is

$$V = V_A - V_B = V_C - V_D = iR.$$

Thermal Energy Produced in the Resistor

In time t, a charge $q = it$ goes through the circuit. As this charge moves from C to D, the electric potential energy decreases by

$$U = qV = (it)(iR) = i^2Rt. \quad \ldots (32.11)$$

This loss in electric potential energy appears as increased thermal energy of the resistor. Thus, a current i for a time t through a resistance R increases the thermal energy by i^2Rt. The power developed is

$$P = \frac{U}{t} = i^2R. \quad \ldots (32.12)$$

Using Ohm's law, this can also be written as

$$P = \frac{V^2}{R} = Vi.$$

Example 32.4

A resistor develops 400 J *of thermal energy in* 10 s *when a current of* 2 A *is passed through it. (a) Find its resistance. (b) If the current is increased to* 4 A, *what will be the energy developed in* 10 s.

Solution :

(a) Using $U = i^2Rt$,

$$400\ \text{J} = (2\ \text{A})^2 R (10\ \text{s})$$

or, $R = 10\ \Omega$.

(b) The thermal energy developed, when the current is 4 A, is

$$U = i^2Rt$$
$$= (4\ \text{A})^2 \times (10\ \Omega) \times (10\ \text{s}) = 1600\ \text{J}.$$

Internal Resistance of a Battery

As the charge $q = it$ goes through the battery from the negative terminal B to the positive terminal A, work is done by the nonelectrostatic battery force F_b. This work is $U_1 = q\mathcal{E} = \mathcal{E}it$. As the potential of A is higher than the potential of B by an amount V, the electric potential energy increases by an amount

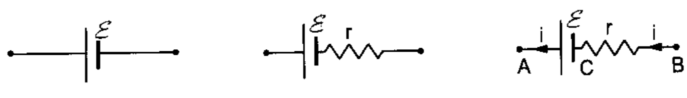
$$U_2 = V(it).$$

The remaining energy $U_1 - U_2$ appears as thermal energy of the battery material. The fraction appearing as thermal energy depends on the battery material and the battery mechanism. If no thermal energy is developed as the charge goes through the battery,

$$\mathcal{E} it = Vit$$

or, $$\mathcal{E} = V.$$

Such a battery is called an *ideal battery*. The potential difference between the terminals of an ideal battery remains equal to its emf even if there is a current through it. As discussed earlier, an ideal battery is denoted by the symbol shown in figure (32.10a). The potential difference between the facing parallel lines is $V = \mathcal{E}$, the longer line being at the higher potential.



(a) (b) (c)

Figure 32.10

A nonideal battery develops thermal energy as a current passes through it and the potential difference between the terminals is smaller than the emf. Such a battery may be represented by the symbol shown in figure (32.10b). This is a combination of an ideal battery of emf $\mathcal{E}$ and a resistance r. If there is a current i through the battery in the direction indicated in figure (32.10c), the potential difference between the terminals is

$$V_A - V_B = (V_A - V_C) - (V_B - V_C)$$
$$= \mathcal{E} - ir.$$

The thermal energy developed in time t is $i^2 rt$. The addition of a resistance r accounts for the difference between $\mathcal{E}$ and V as well as for the thermal energy developed in the battery. This resistance is called the *internal resistance* of the battery.

Example 32.5

A battery of emf 2·0 V *and internal resistance* 0·50 Ω *supplies a current of* 100 mA. *Find (a) the potential difference across the terminals of the battery and (b) the thermal energy developed in the battery in* 10 s.

Solution : The situation is the same as that shown in figure (32.10c).

(a) The potential difference across the terminals is

$$V_A - V_B = (V_A - V_C) - (V_B - V_C)$$
$$= \mathcal{E} - ir$$
$$= 2{\cdot}0 \text{ V} - (0{\cdot}100 \text{ A})(0{\cdot}50\ \Omega) = 1{\cdot}95 \text{ V}.$$

(b) The thermal energy developed in the battery is

$$U = i^2 rt = (0{\cdot}100 \text{ A})^2 (0{\cdot}50\ \Omega)(10 \text{ s}) = 0{\cdot}05 \text{ J}.$$

32.7 KIRCHHOFF'S LAWS

The Junction Law

The sum of all the currents directed towards a point in a circuit is equal to the sum of all the currents directed away from the point.

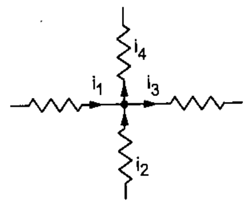


Figure 32.11

Thus, in figure (32.11),

$$i_1 + i_2 = i_3 + i_4. \qquad \dots \text{(i)}$$

If we take the current directed towards a point as positive and that directed away from the point as negative, we can restate the junction law as, *the algebraic sum of all the currents directed towards a point is zero.*

In figure (32.11), the currents directed towards the junction point are i_1, i_2, $-i_3$ and $-i_4$.

Thus,

$$i_1 + i_2 + (-i_3) + (-i_4) = 0$$

which is the same as (i).

The junction law follows from the fact that no point in a circuit keeps on accumulating charge or keeps on supplying charge. Charges pass through the point. So, the net charge coming towards the point should be equal to that going away from it in the same time.

The Loop Law

The algebraic sum of all the potential differences along a closed loop in a circuit is zero.

While using this rule, one starts from a point on the loop and goes along the loop, either clockwise or anticlockwise, to reach the same point again. Any potential *drop* encountered is taken to be positive and any potential *rise* is taken to be negative. The net

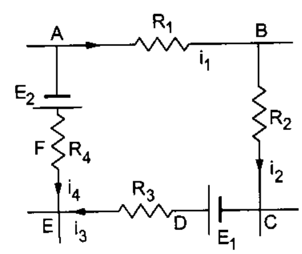


Figure 32.12

sum of all these potential differences should be zero. In figure (32.12), we show a loop *ABCDEFA* of a circuit. As we start from *A* and go along the loop clockwise to reach the same point *A*, we get the following potential differences:

$$V_A - V_B = i_1 R_1$$
$$V_B - V_C = i_2 R_2$$
$$V_C - V_D = -\mathcal{E}_1$$
$$V_D - V_E = i_3 R_3$$
$$V_E - V_F = -i_4 R_4$$
$$V_F - V_A = \mathcal{E}_2 .$$

Adding all these,

$$0 = i_1 R_1 + i_2 R_2 - \mathcal{E}_1 + i_3 R_3 - i_4 R_4 + \mathcal{E}_2 .$$

The loop law follows directly from the fact that electrostatic force is a conservative force and the work done by it in any closed path is zero.

32.8 COMBINATION OF RESISTORS IN SERIES AND PARALLEL

Several resistors may be combined to form a network. The combination should have two end points to connect it with a battery or other circuit elements. If a potential difference V is applied to the combination, it draws some current i. We define *equivalent resistance* of the combination as

$$R_{eq} = V/i.$$

This single resistance draws the same current as the given combination when the same potential difference is applied across the end points.

Series Combination

Two or more resistors are said to be connected in series if the same current passes through all the resistors.

Figure (32.13) shows three resistors having resistances R_1, R_2 and R_3 connected in series. The combination has two points P and N through which it can be connected to a battery or other circuit elements. Any current going through R_1 also goes through R_2 and R_3.

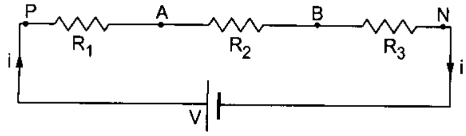


Figure 32.13

Suppose we apply a potential difference V between the points P and N. A current i passes through all the resistors. Using Kirchhoff's loop law for the loop *PABNP*,

$$iR_1 + iR_2 + iR_3 - V = 0$$

or, $$i = \frac{V}{R_1 + R_2 + R_3}$$

or, $$\frac{V}{i} = R_1 + R_2 + R_3.$$

Thus, the equivalent resistance is

$$R_{eq} = R_1 + R_2 + R_3.$$

This argument may be extended for any number of resistors connected in series.

$$R_{eq} = R_1 + R_2 + R_3 + \ldots \qquad \ldots (32.13)$$

Parallel Combination

Two or more resistors are said to be connected in parallel if the same potential difference exists across all the resistors.

Figure (32.14) shows three resistors having resistances R_1, R_2 and R_3 connected in parallel. The combination has two end points P and N. One end of each resistor is joined to P and other end to N. Thus, the potential difference across any resistor is the same.

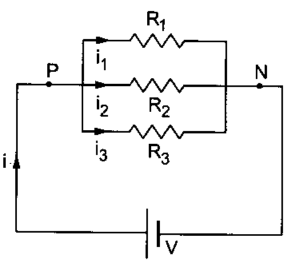


Figure 32.14

To find the equivalent resistance of the combination, let us apply a potential difference V between the points P and N. If the current through P is i, the equivalent resistance is

$$R_{eq} = V/i. \qquad \ldots \text{(i)}$$

The current i is divided at the junction P. Suppose a current i_1 goes through R_1, i_2 through R_2 and i_3 through R_3. These combine at N to give a total current i. Using Kirchhoff's junction law at P,

$$i = i_1 + i_2 + i_3. \qquad \ldots \text{(ii)}$$

The potential difference across each resistor is $V_P - V_N = V$. Using Ohm's law for the resistances R_1, R_2 and R_3 separately;

$$i_1 = \frac{V}{R_1},\ i_2 = \frac{V}{R_2} \text{ and } i_3 = \frac{V}{R_3}.$$

Adding the above three equations and using (ii),

$$i = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right).$$

Using (i),

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

The process may be generalised for any number of resistors connected in parallel, so that

$$\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\ldots \qquad \ldots (32.14)$$

For two resistors in parallel,

$$\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2} \text{ or, } R_{eq}=\frac{R_1R_2}{R_1+R_2}.$$

Note that the equivalent resistance is smaller than the smallest individual resistance.

Example 32.6

Find the equivalent resistance of the network shown in figure (32.15) between the points A and B.

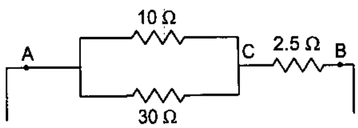


Figure 32.15

Solution : The 10 Ω resistor and the 30 Ω resistor are connected in parallel. The equivalent resistance between A and C is

$$R_1=\frac{(10\ \Omega)\ (30\ \Omega)}{10\ \Omega+30\ \Omega}=7{\cdot}5\ \Omega.$$

This is connected with 2·5 Ω in series. The equivalent resistance between A and B is $7{\cdot}5\ \Omega+2{\cdot}5\ \Omega=10\ \Omega$.

Division of Current in Resistors Joined in Parallel

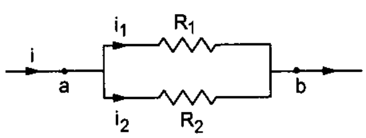


Figure 32.16

Consider the situation shown in figure (32.16). Using Ohm's law on resistors R_1 and R_2,

$$V_a-V_b=i_1R_1 \text{ and } V_a-V_b=i_2R_2.$$

Thus, $\quad i_1R_1=i_2R_2$

or, $$\frac{i_1}{i_2}=\frac{R_2}{R_1}. \qquad \ldots (i)$$

We see that the current is divided in resistors, connected in parallel, in inverse ratio of the resistances.

From (i), $$\frac{i_1}{i_1+i_2}=\frac{R_2}{R_1+R_2}$$

or, $$\frac{i_1}{i}=\frac{R_2}{R_1+R_2}$$

or, $$i_1=\frac{R_2}{R_1+R_2}\,i.$$

Similarly,

$$i_2=\frac{R_1}{R_1+R_2}\,i.$$

32.9 GROUPING OF BATTERIES

Series Connection

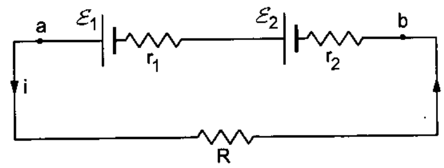


Figure 32.17

Suppose two batteries having emfs $\mathcal{E}_1$ and $\mathcal{E}_2$ and internal resistances r_1 and r_2 are connected in series as shown in figure (32.17). The points a and b act as the terminals of the combination. Suppose an external resistance R is connected across the combination. From Kirchhoff's loop law,

$$Ri+r_2i-\mathcal{E}_2+r_1i-\mathcal{E}_1=0$$

or, $$i=\frac{\mathcal{E}_1+\mathcal{E}_2}{R+(r_1+r_2)}=\frac{\mathcal{E}_0}{R+r_0}.$$

where i is the current through the resistance R.

We see that the combination acts as a battery of emf $\mathcal{E}_0=\mathcal{E}_1+\mathcal{E}_2$ having an internal resistance $r_0=r_1+r_2$.

If the polarity of one of the batteries is reversed, the equivalent emf will be $|\mathcal{E}_1-\mathcal{E}_2|$.

Parallel Connection

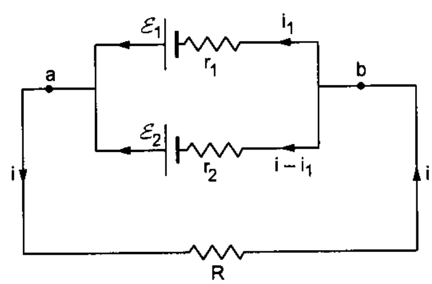


Figure 32.18

Now suppose the batteries are connected in parallel as shown in figure (32.18). The currents are also shown in the figure. Applying Kirchhoff's loop law in the loop containing $\mathcal{E}_1$, r_1 and R,

$$Ri+r_1i_1-\mathcal{E}_1=0. \qquad \ldots (i)$$

Similarly, applying Kirchhoff's law in the loop containing $\mathcal{E}_2, r_2$ and R,

$$Ri+r_2(i-i_1)-\mathcal{E}_2=0 \qquad \ldots (ii)$$

Multiply (i) by r_2, (ii) by r_1 and add. This gives,

$$iR(r_1 + r_2) + r_1 r_2 i - \mathcal{E}_1 r_2 - \mathcal{E}_2 r_1 = 0$$

$$\text{or,} \quad i = \frac{\mathcal{E}_1 r_2 + \mathcal{E}_2 r_1}{R(r_1 + r_2) + r_1 r_2} = \frac{\dfrac{\mathcal{E}_1 r_2 + \mathcal{E}_2 r_1}{r_1 + r_2}}{R + \dfrac{r_1 r_2}{r_1 + r_2}} = \frac{\mathcal{E}_0}{R + r_0}.$$

We see that the combination acts as a battery of emf

$$\mathcal{E}_0 = \frac{\mathcal{E}_1 r_2 + \mathcal{E}_2 r_1}{r_1 + r_2}$$

and internal resistance

$$r_0 = \frac{r_1 r_2}{r_1 + r_2}.$$

If $\mathcal{E}_1 = \mathcal{E}_2 = \mathcal{E}$ and $r_1 = r_2 = r$, $\mathcal{E}_0 = \mathcal{E}$ and $r_0 = r/2$.

32.10 WHEATSTONE BRIDGE

Wheatstone bridge is an arrangement of four resistances which can be used to measure one of them in terms of the rest.

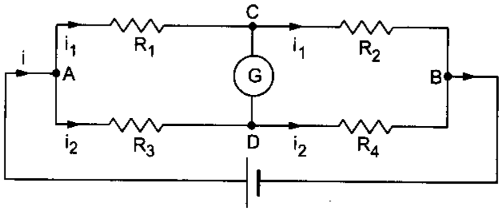


Figure 32.19

The arrangement is shown in figure (32.19). Four resistors with resistances R_1, R_2, R_3 and R_4 are connected to form a loop. There are four joints *A, B, C* and *D*. A battery is connected between two opposite joints *A* and *B* and a galvanometer is connected between the other two opposite joints *C* and *D*.

We shall discuss the construction and working of a galvanometer later. Here, we only state that a galvanometer has a needle which deflects when an electric current passes through the galvanometer. The needle deflects towards left if the current passes in one direction and towards right if the current is reversed.

The current i from the battery is divided at *A* in two parts. A part i_1 goes through R_1 and the rest i_2 goes through R_3. For a particular relation between the resistances, there is no current through the galvanometer. The Wheatstone bridge is then said to be *balanced*. In this case, the current in R_2 is the same as the current in R_1 and the current in R_4 is the same as that in R_3. As there is no current through the galvanometer, the potential difference across its terminals is zero. Thus,

$$V_C = V_D.$$

Applying Ohm's law to R_1 and R_2,

$$V_A - V_C = i_1 R_1$$

and $$V_C - V_B = i_1 R_2.$$

Thus, $$\frac{V_A - V_C}{V_C - V_B} = \frac{R_1}{R_2}. \quad \text{... (ii)}$$

Applying Ohm's law to R_3 and R_4,

$$V_A - V_D = i_2 R_3$$

and $$V_D - V_B = i_2 R_4.$$

Thus, $$\frac{V_A - V_D}{V_D - V_B} = \frac{R_3}{R_4}. \quad \text{... (iii)}$$

As $V_C = V_D$, left sides of (ii) and (iii) are equal. Thus,

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}. \quad \text{... (32.15)}$$

This is the condition for which a Wheatstone bridge is balanced.

To measure the resistance of a resistor, it is connected as one of the four resistors in the bridge. One of the other three should be a variable resistor. Let us suppose R_4 is the resistance to be measured and R_3 is the variable resistance. When the Wheatstone bridge is connected, in general, there will be a deflection in the galvanometer. The value of the variable resistance R_3 is adjusted so that the deflection in the galvanometer becomes zero. In this case, the bridge is balanced and from equation (32.15),

$$R_4 = \frac{R_2}{R_1} R_3.$$

Knowing R_1, R_2 and R_3, the value of R_4 is calculated.

Example 32.7

Find the value of R in figure (32.20) so that there is no current in the 50 Ω *resistor.*

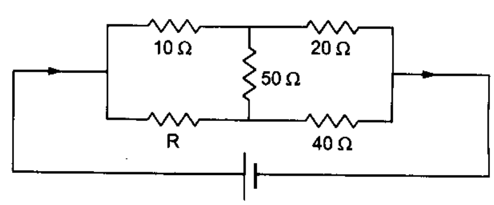


Figure 32.20

Solution :

This is a Wheatstone bridge with the galvanometer replaced by the 50 Ω resistor. There will be no current in the 50 Ω resistor if the bridge is balanced.

In this case,

$$\frac{10\ \Omega}{20\ \Omega} = \frac{R}{40\ \Omega}$$

or, $$R = 20\ \Omega.$$

32.11 AMMETER AND VOLTMETER

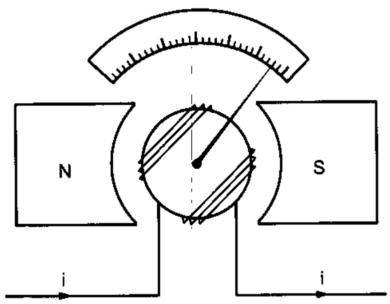


Figure 32.21

Ammeter is a device to measure an electric current and *voltmeter* is a device to measure a potential difference. In both the instruments there is a coil, suspended between the poles of a magnet. When a current is passed through the coil, it deflects. The angle of deflection is proportional to the current going through the coil. A needle is fixed to the coil. When the coil deflects, the needle moves on a graduated scale.

Ammeter

In an ammeter, a resistor having a small resistance is connected in parallel with the coil. This resistor is called the *shunt*. The current to be measured is passed through the ammeter by connecting it in series with the segment which carries the current. Plus and minus signs are marked near the terminals of the ammeter. The current should enter the ammeter through the terminal marked "plus". When no current passes through the ammeter, the needle stays at zero which is marked at the left extreme of the scale.

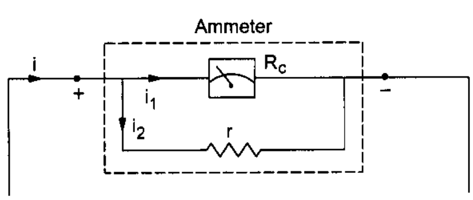


Figure 32.22

Suppose the coil has a resistance R_c and the small resistance connected in parallel (shunt) has a value r. When a current i is sent through the ammeter, the current gets divided in two parts. A part i_1 goes through the coil and the rest, $i - i_1$, through the shunt. As the potential difference across R_c is the same as that across r,

$$i_1 R_c = (i - i_1) r$$

or, $$i_1 = \frac{r}{R_c + r} i. \qquad \text{... (i)}$$

The deflection is proportional to i_1 and hence to i. The scale is graduated to read the value of i directly.

The equivalent resistance of an ammeter is given by

$$R_{eq} = \frac{R_c r}{R_c + r}.$$

When the ammeter is connected in a segment of a circuit, the resistance of the segment increases by this amount R_{eq}. This reduces the main current which we wish to measure. To minimise this error, the equivalent resistant R_{eq} should be small. This is one reason why the shunt having a small resistance r is connected in parallel to the coil. This makes R_{eq} small.

Galvanometer is very similar to an ammeter in construction. When no current passes through it, the needle stays in the middle of the graduated scale. This point is marked zero. Current can be passed through the galvanometer in either direction. The needle deflects accordingly towards left or towards right.

Example 32.8

The ammeter shown in figure (32.23) consists of a 480 Ω *coil connected in parallel to a* 20 Ω *shunt. Find the reading of the ammeter.*

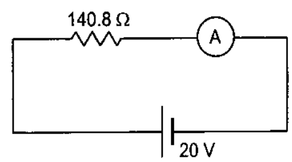


Figure 32.23

Solution : The equivalent resistance of the ammeter is

$$\frac{(480\ \Omega)\ (20\ \Omega)}{480\ \Omega + 20\ \Omega} = 19{\cdot}2\ \Omega.$$

The equivalent resistance of the circuit is

$$140{\cdot}8\ \Omega + 19{\cdot}2\ \Omega = 160\ \Omega\ .$$

The current is $i = \frac{20\ \text{V}}{160\ \Omega} = 0{\cdot}125\ \text{A}.$

This current goes through the ammeter and hence the reading of the ammeter is 0·125 A.

Voltmeter

In a voltmeter, a resistor having a high resistance R is connected in series with the coil. The end points (terminals) are connected to the points A and B between which the potential difference is to be measured. Plus and minus signs are marked on the terminals. The terminal marked "plus" should be connected to the point at higher potential. When no potential difference is applied between the terminals, the needle stays at zero which is marked at the left extreme of the scale. When the potential difference is applied, a current passes through the coil and the high resistance. If R_c be the resistance of the coil and V be the potential difference applied to the voltmeter, the current in the coil is

$$i = \frac{V}{R_c + R}.$$

The deflection is proportional to the current i and hence to V. The scale is graduated to read the potential difference directly.

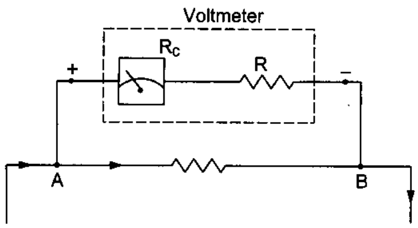


Figure 32.24

When the voltmeter is used in a circuit, its resistance $R_{eq} = R_c + R$ is connected in parallel to some element of the circuit. This changes the overall current in the circuit and hence, the potential difference to be measured is also changed. To minimise the error due to this, the equivalent resistance R_{eq} of the voltmeter should be large. (When a large resistance is connected in parallel to a small resistance, the equivalent resistance is only slightly less than the smaller one.) That is why, a large resistance R is added in series with the coil of a voltmeter.

32.12 STRETCHED-WIRE POTENTIOMETER

An ideal voltmeter which does not change the original potential difference, should have infinite resistance. But in the design described above, the resistance cannot be made infinite. *Potentiometer* is a device which does not draw any current from the given circuit and still measures the potential difference. Thus, it is equivalent to an ideal voltmeter.

The stretched-wire potentiometer consists of a long wire AB, usually 5 to 10 metres long, fixed on a wooden platform (figure 32.25). The wire has a uniform cross section. Usually, separate pieces of wire, each 1 m long, are fixed parallel to each other on the platform. The wires are joined to each other by thick copper strips so that the combination acts as a single wire of desired length (5 to 10 metres). The ends A and B are connected to a driving circuit consisting of a strong battery, a plug key and a rheostat. The driving circuit sends a constant current i through the wire AB. Thus, the potential gradually decreases from A to B. One end of a galvanometer is connected to a metal rod fixed on

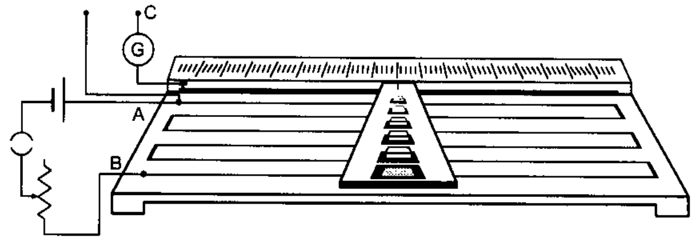


Figure 32.25

the wooden platform. A "jockey" may be slid on this metal rod and may touch the wire AB at any desired point. In this way the galvanometer gets connected to the point of AB which is touched by the jockey. The length of the wire between the end A and this point can be measured with the help of a metre scale fixed on the platform. The other end C of the galvanometer and the high-potential end A of the wire, form the two end points (terminals) of the potentiometer. These points are connected to the points between which the potential difference is to be measured.

Suppose, we have to measure the potential difference between the points a and b. Also let a be at a higher potential and b at a lower potential. The end A of the wire AB is connected to the point a and the end C of the galvanometer is connected to the point b. The circuit is represented schematically in figure (35.26).

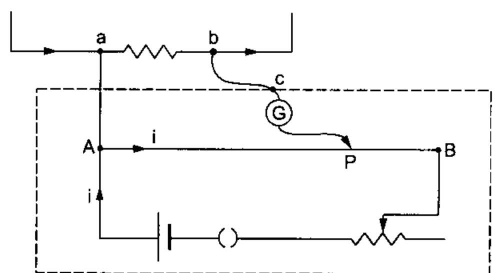


Figure 32.26

The connecting wire Aa has a negligible resistance and hence potentials of A and a are equal. Suppose, the potential drop across ab is smaller than the potential drop across AB. Then there will be a point P on AB which will have the same potential as b. If the jockey is slid to touch the wire at this point P, the potential difference across the galvanometer is zero and there will be no current through it. The process of measurement is to search for a point P so that there is no deflection in the galvanometer.

Suppose, the driving circuit sets up a potential difference V_0 between the ends A and B of the potentiometer wire. As the wire is uniform, the resistance of a piece of the wire is proportional to its length. Hence, the potential difference across a piece of wire is also proportional to its length. If $AB = L$ and

$AP = l$, the potential difference between the points A and P is

$$V = V_0 \frac{l}{L} \cdot \qquad \dots \text{(i)}$$

This is equal to the potential difference between a and b which we had to measure.

In order to get the value of the potential difference V, the total potential drop V_0 on AB must be known. One way to do this is to use a standard cell having known and constant emf in place of ab. If the emf of the standard cell is $\mathcal{E}$ and the potentiometer is balanced (no deflection in the galvanometer) when $AP = l_0$, we have, from (i),

$$\mathcal{E} = V_0 \frac{l_0}{L}$$

or,
$$V_0 = \frac{L}{l_0} \mathcal{E}.$$

The potential difference V between a and b is, from (i)

$$V = \frac{l}{l_0} \mathcal{E}.$$

This process of finding V_0 is called *calibration of the potentiometer*. Note that there is no current through the standard cell when the potentiometer is balanced during its calibration. Thus, the emf $\mathcal{E}$ equals the potential difference between its terminals.

Comparison of Emf's of Two Batteries

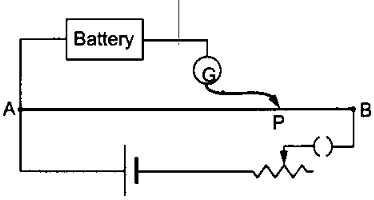


Figure 32.27

The driving circuit of the potentiometer is set up with a strong battery so that the potential difference V_0 across AB is larger than the emf of either battery. One of the batteries is connected between the positive end A and the galvanometer. The jockey is adjusted to touch the wire at a point P_1 so that there is no deflection in the galvanometer. The length $AP_1 = l_1$ is noted. Now, the first battery is replaced by the second and the length $AP_2 = l_2$ for the balance is noted. If the length $AB = L$, the emf of the first battery is, from (i) above,

$$\mathcal{E}_1 = \frac{l_1}{L} V_0$$

and that of the second battery is

$$\mathcal{E}_2 = \frac{l_2}{L} V_0.$$

Thus,
$$\frac{\mathcal{E}_1}{\mathcal{E}_2} = \frac{l_1}{l_2}.$$

Note that no calibration is needed in this case.

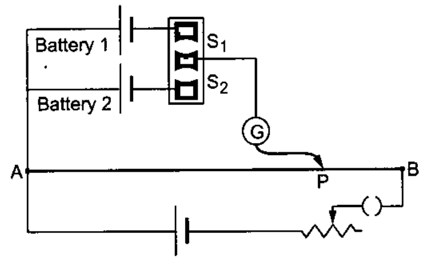


Figure 32.28

One can use a two-way key to connect both the batteries together as shown in figure (32.28). When the key is pressed in the plug S_1, the first battery is brought into the circuit. When the key is taken out from S_1 and pressed in the plug S_2, the second battery is brought into the circuit.

The value of emf of a battery can also be obtained by this same method by taking the other battery to be a standard cell. The emf of the standard cell is known and hence the emf of the given battery can be obtained.

Measurement of Internal Resistance of a Battery

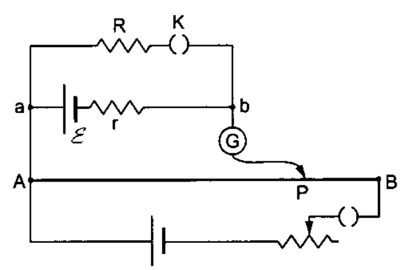


Figure 32.29

Figure (32.29) shows the arrangement for measuring the internal resistance of a battery. The emf of the battery is $\mathcal{E}$ and its internal resistance is r. A known resistance R is connected across the battery together with a plug key K. The potentiometer circuit is set up as usual. The plug key K is opened and the balance point P is searched on the wire AB so that there is no deflection in the galvanometer. As the key is open, there is no current through the resistance R. Hence, there is no current through the battery and the potential difference across the terminals a, b is the same as the emf $\mathcal{E}$ of the battery. If $AP = l$, we have

$$\mathcal{E} = \frac{l}{L} V_0 \qquad \dots \text{(i)}$$

with the symbols having their usual meanings.

Now the key K is closed and the new balance point P' is searched. There is a current

$$i = \frac{\mathcal{E}}{R+r} \text{ through the battery.}$$

The potential difference between a and b is

$$V_a - V_b = Ri = \frac{\mathcal{E}R}{R+r}.$$

If $AP' = l'$, we have

$$\frac{\mathcal{E}R}{R+r} = \frac{l'}{L} V_0. \qquad \dots \text{(ii)}$$

Dividing (ii) by (i),

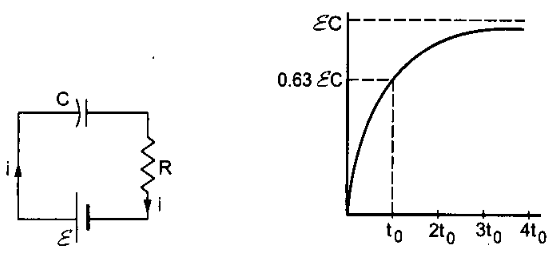
$$\frac{R}{R+r} = \frac{l'}{l}$$

or,
$$r = \frac{R(l-l')}{l'}.$$

32.13 CHARGING AND DISCHARGING OF CAPACITORS

Charging

When a capacitor is connected to a battery, positive charge appears on one plate and negative charge on the other. The potential difference between the plates ultimately becomes equal to the emf of the battery. The whole process takes some time and during this time there is an electric current through the connecting wires and the battery. Figure (32.30a) shows a typical connection. The resistance of the connecting wires and the internal resistance of the battery taken together is shown as the resistance R. The capacitor has capacitance C.



(a) (b)

Figure 32.30

Suppose, the battery is connected at $t = 0$. Suppose the charge on the capacitor and the current in the circuit are q and i respectively at time t. The potential drop on the capacitor is q/C and on the resistor it is Ri. Also, the charge deposited on the positive plate in time dt is

$$dq = i\,dt$$

so that
$$i = \frac{dq}{dt}.$$

Using Kirchhoff's loop law,

$$\frac{q}{C} + Ri - \mathcal{E} = 0$$

or,
$$Ri = \mathcal{E} - \frac{q}{C}$$

or,
$$R\frac{dq}{dt} = \frac{\mathcal{E}C - q}{C}$$

or,
$$\int_0^q \frac{dq}{\mathcal{E}C - q} = \int_0^t \frac{1}{CR}\,dt$$

or,
$$-\ln\frac{\mathcal{E}C - q}{\mathcal{E}C} = \frac{t}{CR}$$

or,
$$1 - \frac{q}{\mathcal{E}C} = e^{-t/CR}$$

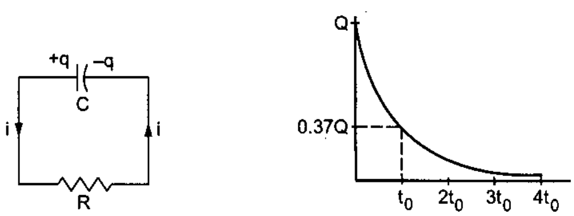
or,
$$q = \mathcal{E}C(1 - e^{-t/CR}). \qquad \dots (32.16)$$

This gives the charge on the capacitor at time t. As t increases, q also increases. The maximum charge is obtained, in principle, at $t = \infty$ and its value is $\mathcal{E}C$. The constant CR has dimensions of time and is called *time constant* of the circuit. In one time constant $\tau(= CR)$, the charge accumulated on the capacitor is

$$q = \mathcal{E}C\left(1 - \frac{1}{e}\right) = 0{\cdot}63\,\mathcal{E}C.$$

Thus, 63% of the maximum charge is deposited in one time constant. Figure (32.30b) shows a plot of q versus t.

Discharging



(a) (b)

Figure 32.31

If the plates of a charged capacitor are connected through a conducting wire, the capacitor gets discharged. Again there is a flow of charge through the wires and hence there is a current. Suppose a capacitor of capacitance C has a charge Q. At $t = 0$, the plates are connected through a resistance R (figure 32.31a). Let the charge on the capacitor be q and the current in the circuit be i at time t.

Using Kirchhoff's loop law,

$$\frac{q}{C} - Ri = 0.$$

Here $i = -\frac{dq}{dt}$ because the charge q decreases as time passes.

Thus, $$R\frac{dq}{dt} = -\frac{q}{C}.$$

or, $$\frac{dq}{q} = -\frac{1}{CR}dt$$

or, $$\int_Q^q \frac{dq}{q} = \int_0^t -\frac{1}{CR}dt$$

or, $$\ln\frac{q}{Q} = -\frac{t}{CR}$$

or, $$q = Q e^{-t/CR}. \qquad \ldots (32.17)$$

In principle, discharging is complete only at $t = \infty$. The constant CR is the time constant. At $t = CR$, the remaining charge is $q = \frac{Q}{e} = 0{\cdot}37\ Q$. Thus, 63% of the discharging is complete in one time constant. Figure (32.31b) shows the charge as a function of time.

Example 32.9

A capacitor of capacitance 100 μF *is charged by connecting it to a battery of emf* 12 V *and internal resistance* 2 Ω. *(a) Find the time constant of the circuit. (b) Find the time taken before* 99% *of the maximum charge is stored on the capacitor.*

Solution : The time constant is

$$\tau = CR = (100\ \mu F)\,(2\ \Omega) = 200\ \mu s.$$

The charge at time t is

$$q = \mathcal{E}C\,(1 - e^{-t/CR}).$$

Putting $$q = 0{\cdot}99\ \mathcal{E}C,$$

$$0{\cdot}99 = 1 - e^{-t/(200\ \mu s)}$$

or, $$-\frac{t}{200\ \mu s} = \ln(0{\cdot}01)$$

or, $$t = 920\ \mu s = 0{\cdot}92\ ms.$$

Example 32.10

The plates of a 50 μF *capacitor charged to* 400 μC *are connected through a resistance of* 1·0 kΩ. *Find the charge remaining on the capacitor* 1 s *after the connection is made.*

Solution : The time constant is

$$CR = (50\ \mu F)\,(1{\cdot}0\ k\Omega) = 50\ ms.$$

At $t = 1$ s, $t/CR = 1\ s/50\ ms = 20$.

The charge remaining on the capacitor is

$$q = Q e^{-t/CR}$$

$$= (400\ \mu C)\, e^{-20} = 8{\cdot}2 \times 10^{-7}\ \mu C.$$

We see that in a typical charging or discharging circuit, the time constant is of the order of a millisecond. Also, four to five time constants are sufficient for 99% of the charging or discharging. Thus, for practical purposes, we can assume that charging or discharging is complete in a fraction of a second.

32.14 ATMOSPHERIC ELECTRICITY

The earth and the atmosphere surrounding it show very interesting electric phenomena. The earth has a negative charge spread with approximately uniform density over its surface. The average surface charge density on the earth is little less than one nanocoulomb per square metre. There is a corresponding electric field of about 100 V m^{-1} in the atmosphere above the earth. This field is in the vertically downward direction. This means, if you look at a flat desert, the electric potential increases by about 100 V as you move up by 1 m. The potential keeps on increasing as one goes higher in atmosphere but the magnitude of the electric field gradually decreases. At about 50 km from the earth's surface, the field is negligible. The total potential difference between the earth's surface and the top of the atmosphere is about 400 kV.

The atmosphere contains a number of ions, both positively charged and negatively charged. The main source of these ions is *cosmic rays* which come from outside the earth, even from outside the solar system. These rays come down to the earth and ionize molecules in the air. Air contains dust particles which become charged by friction as they move through the air. This is another source of the presence of charged ions in air. Because of the electric field in the atmosphere, positive ions come down and negative ions go up. Thus, there is an electric current in the atmosphere. This current is about $3{\cdot}5 \times 10^{-12}$ A over a square metre area parallel to the earth's surface. When the total surface area of the earth is considered, 1800 A of current reaches the earth.

The density of ions increases with height over the earth's surface. Also, the density of air decreases and the ions can travel larger distances between collisions. Both these factors contribute to the fact that "conductivity of air" increases with altitude. At about 50 km above the earth's surface, the air becomes highly conducting. We can draw an equivalent picture by assuming that at about this height there is a perfectly conducting surface having a potential of 400 kV and current comes down from this surface to the earth.

If 1800 A of current flows towards the earth, the entire negative charge of the earth should get neutralised in about half an hour and the electric field in the atmosphere should reduce to zero. But it is not so. So, there must be some mechanism which brings negative charge back to the earth, so that the 400 kV potential difference is maintained. This situation is like that of a battery. The current provided by a battery discharges it. There is a source of emf which maintains the potential difference across the battery's terminals. So, what is the source that charges our atmospheric battery. The answer is *thunderstorms* and *lightning*.

Because of the difference in temperature and pressure between different parts of the atmosphere, air packets keep on moving in a rather systematic fashion. As the upper atmosphere is cool (temperature is around −10°C at a height of 3–4 km and −20°C at a height of 6–7 km), water vapour condenses to form small water droplets and tiny ice particles. A parcel of air with these droplets and ice particles forms a thunderstorm. A typical thunderstorm may have an average horizontal extension of about 7–8 km and a vertical extension of about 3 km. A matured thunderstorm is formed with its lower end at a height of about 3–4 km above the earth's surface and the upper end at about 6–7 km above the earth's surface.

The upper part of a thunderstorm contains excess positive charge and the lower part contains excess negative charge. The density of negative charge in the clouds in the lower part of the storm is very high. This negative charge creates a potential difference of 20 to 100 MV between these clouds and the earth. Note that

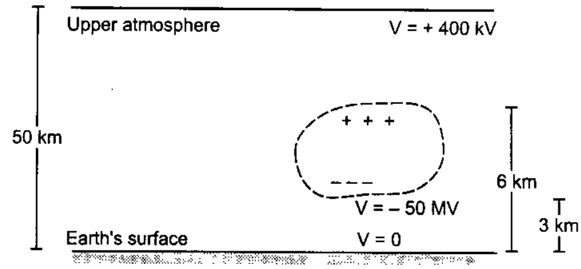


Figure 32.32

this potential difference is much larger than the 400 kV between the earth and the top of atmosphere and is opposite in sign. Figure (32.32) represents a typical situation.

The high electric field between the lower part of the storm and the earth is often sufficient for the dielectric breakdown of air and the air becomes conducting. Negative charge thus jumps from the cloud to the earth's surface. This phenomenon is called *lightning*. The positive charge in the upper part of the storm gradually moves up to enter the high-altitude (≈ 50 km) layer of high conductivity. In one lightning stroke, about 20 C of negative charge is deposited to the earth. It takes about 5 s for the clouds to regain the charge for the next lightning stroke. There are a number of thunderstorms everyday throughout the earth. They charge the atmospheric battery by supplying negative charge to the earth and positive charge to the upper atmosphere. In the area of clear weather, the battery is discharged by the movement of positive ions towards the earth and negative ions away from it (the 1800 A current discussed earlier).

Worked Out Examples

1. *An electron moves in a circle of radius* 10 cm *with a constant speed of* $4{\cdot}0 \times 10^{6}\ \mathrm{m\,s^{-1}}$. *Find the electric current at a point on the circle.*

Solution : Consider a point A on the circle. The electron crosses this point once in every revolution. In one revolution, the electron travels $2\pi \times (10\ \mathrm{cm})$ distance. Hence, the number of revolutions made by the electron in one second is

$$\frac{4{\cdot}0 \times 10^{6}\ \mathrm{m}}{20\pi \times 10^{-2}\ \mathrm{m}} = \frac{2}{\pi} \times 10^{7}.$$

The charge crossing the point A per second is

$$\frac{2}{\pi} \times 10^{7} \times 1{\cdot}6 \times 10^{-19}\ \mathrm{C} = \frac{3{\cdot}2}{\pi} \times 10^{-12}\ \mathrm{C}.$$

Thus, the electric current at this point is

$$\frac{3{\cdot}2}{\pi} \times 10^{-12}\ \mathrm{A} \approx 1{\cdot}0 \times 10^{-12}\ \mathrm{A}.$$

2. *A current of* $2{\cdot}0$ A *exists in a wire of cross sectional area* $1{\cdot}0\ \mathrm{mm}^{2}$. *If each cubic metre of the wire contains* $6{\cdot}0 \times 10^{28}$ *free electrons, find the drift speed.*

Solution : The current density in the wire is

$$j = \frac{i}{A} = \frac{2{\cdot}0\ \mathrm{A}}{1\ \mathrm{mm}^{2}} = 2{\cdot}0 \times 10^{6}\ \mathrm{A\,m^{-2}}.$$

The drift speed is

$$v = \frac{j}{ne} = \frac{2{\cdot}0 \times 10^{6}\ \mathrm{A\,m^{-2}}}{6{\cdot}0 \times 10^{28}\ \mathrm{m^{-3}} \times 1{\cdot}6 \times 10^{-19}\ \mathrm{C}}$$

$$= 2{\cdot}1 \times 10^{-4}\ \mathrm{m\,s^{-1}}.$$

3. *Find the resistance of a copper coil of total wire-length* 10 m *and area of cross section* 1·0 mm ^{2}. *What would be the resistance of a similar coil of aluminium? The resistivity of copper* $= 1{\cdot}7 \times 10^{-8}\ \Omega\ \text{m}$ *and that of aluminium* $= 2{\cdot}6 \times 10^{-8}\ \Omega\ \text{m}$.

Solution : The resistance of the copper coil is

$$\rho \frac{l}{A} = \frac{(1{\cdot}7 \times 10^{-8}\ \Omega\ \text{m}) \times 10\ \text{m}}{1{\cdot}0 \times 10^{-6}\ \text{m}^2} = 0{\cdot}17\ \Omega.$$

The resistance of the similar aluminium coil will be

$$\frac{(2{\cdot}6 \times 10^{-8}\ \Omega\ \text{m}) \times 10\ \text{m}}{1{\cdot}0 \times 10^{-6}\ \text{m}^2} = 0{\cdot}26\ \Omega.$$

4. *A parallel-plate capacitor has plates of area* 10 cm ^{2} *separated by a distance of* 1 mm. *It is filled with the dielectric mica and connected to a battery of emf* 6 volts. *Find the leakage current through the capacitor. Resistivity of mica* $= 1 \times 10^{13}\ \Omega\ \text{m}$.

Solution : The resistance of the mica between the two faces is

$$\rho \frac{l}{A} = \frac{(1 \times 10^{13}\ \Omega\ \text{m}) \times 10^{-3}\ \text{m}}{10{\cdot}0 \times 10^{-4}\ \text{m}^2}$$

$$= 1 \times 10^{13}\ \Omega.$$

The leakage current $= \dfrac{6\ \text{V}}{1 \times 10^{13}\ \Omega} = 6 \times 10^{-13}\ \text{A}.$

5. *Find the resistance of a hollow cylindrical conductor of length* 1·0 m *and inner and outer radii* 1·0 mm *and* 2·0 mm *respectively. The resistivity of the material is* $2{\cdot}0 \times 10^{-8}\ \Omega\ \text{m}$.

Solution : The area of cross section of the conductor through which the charges will flow is

$$A = \pi(2{\cdot}0\ \text{mm})^2 - \pi(1{\cdot}0\ \text{mm})^2$$

$$= 3{\cdot}0 \times \pi\ \text{mm}^2.$$

The resistance of the wire is, therefore,

$$R = \rho \frac{l}{A} = \frac{(2{\cdot}0 \times 10^{-8}\ \Omega\ \text{m}) \times 1{\cdot}0\ \text{m}}{3{\cdot}0 \times \pi \times 10^{-6}\ \text{m}^2} = 2{\cdot}1 \times 10^{-3}\ \Omega.$$

6. *A battery of emf* 2 V *and internal resistance* 0·5 Ω *is connected across a resistance of* 9·5 Ω. *How many electrons cross through a cross section of the resistance in* 1 *second?*

Solution : The current in the circuit is

$$i = \frac{2\ \text{V}}{9{\cdot}5\ \Omega + 0{\cdot}5\ \Omega} = 0{\cdot}2\ \text{A}.$$

Thus, a net transfer of 0·2 C per second takes place across any cross section in the circuit. The number of electrons crossing the section in 1 second is, therefore,

$$\frac{0{\cdot}2\ \text{C}}{1{\cdot}6 \times 10^{-19}\ \text{C}} = 0{\cdot}125 \times 10^{19} = 1{\cdot}25 \times 10^{18}.$$

7. *A battery of emf* 2·0 volts *and internal resistance* 0·10 Ω *is being charged with a current of* 5·0 A. *What is the potential difference between the terminals of the battery?*

Solution :

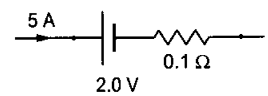


Figure 32-W1

As the battery is being charged, the current goes into the positive terminal as shown in figure (32-W1).

The potential drop across the internal resistance is

$$5{\cdot}0\ \text{A} \times 0{\cdot}10\ \Omega = 0{\cdot}50\ \text{V}.$$

Hence, the potential drop across the terminals will be

$$2{\cdot}0\ \text{V} + 0{\cdot}50\ \text{V} = 2{\cdot}5\ \text{V}.$$

8. *Figure (32-W2) shows n batteries connected to form a circuit. The resistances denote the internal resistances of the batteries which are related to the emf's as* $r_i = k\mathcal{E}_i$ *where k is a constant. The solid dots represent the terminals of the batteries. Find (a) the current through the circuit and (b) the potential difference between the terminals of the ith battery.*

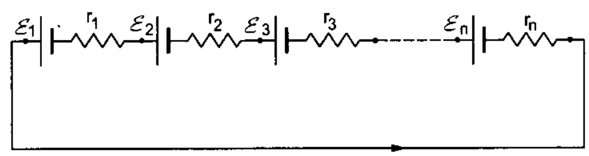


Figure 32-W2

Solution : (a) Suppose the current is i in the indicated direction. Applying Kirchoff's loop law,

$$\mathcal{E}_1 - ir_1 + \mathcal{E}_2 - ir_2 + \mathcal{E}_3 - ir_3 + \ldots + \mathcal{E}_n - ir_n = 0$$

or, $$i = \frac{\mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3 + \ldots + \mathcal{E}_n}{r_1 + r_2 + r_3 + \ldots + r_n}$$

$$= \frac{\mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3 + \ldots + \mathcal{E}_n}{k(\mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3 + \ldots + \mathcal{E}_n)} = \frac{1}{k}.$$

(b) The potential difference between the terminals of the ith battery is

$$\mathcal{E}_i - ir_i$$

$$= \mathcal{E}_i - \left(\frac{1}{k}\right)(k\mathcal{E}_i) = 0.$$

9. *A copper rod of length* 20 cm *and cross-sectional area* 2 mm ^{2} *is joined with a similar aluminium rod as shown in figure (32-W3). Find the resistance of the combination between the ends. Resistivity of copper* $= 1{\cdot}7 \times 10^{-8}\ \Omega\ \text{m}$ *and that of aluminium* $= 2{\cdot}6 \times 10^{-8}\ \Omega\ \text{m}$.

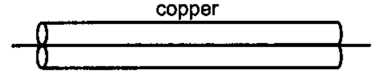


Figure 32-W3

Solution : The resistance of the copper rod

$$=\rho\frac{l}{A}=\frac{(1{\cdot}7\times10^{-8}\,\Omega\,\mathrm{m})\times(20\times10^{-2}\,\mathrm{m})}{2{\cdot}0\times10^{-6}\,\mathrm{m}^2}$$

$$=1{\cdot}7\times10^{-3}\,\Omega.$$

Similarly, the resistance of the aluminium rod

$$=2{\cdot}6\times10^{-3}\,\Omega.$$

These rods are joined in parallel so that the equivalent resistance R between the ends is given by

$$\frac{1}{R}=\frac{1}{1{\cdot}7\times10^{-3}\,\Omega}+\frac{1}{2{\cdot}6\times10^{-3}\,\Omega}$$

or, $$R=\frac{1{\cdot}7\times2{\cdot}6}{4{\cdot}3}\times10^{-3}\,\Omega\approx1{\cdot}0\ \mathrm{m}\Omega.$$

10. *A wire of resistance* 10 Ω *is bent to form a complete circle. Find its resistance between two diametrically opposite points.*

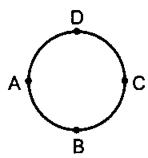


Figure 32-W4

Solution :

Let $ABCDA$ be the wire of resistance 10 Ω. We have to calculate the resistance of this loop between the diametrically opposite points A and C. The wires ADC and ABC will have resistances 5 Ω each. These two are joined in parallel between A and C. The equivalent resistance R between A and C is, therefore, given by

$$R=\frac{5\,\Omega\times5\,\Omega}{5\,\Omega+5\,\Omega}=2{\cdot}5\,\Omega.$$

11. *Find the currents in the different resistors shown in figure (32-W5).*

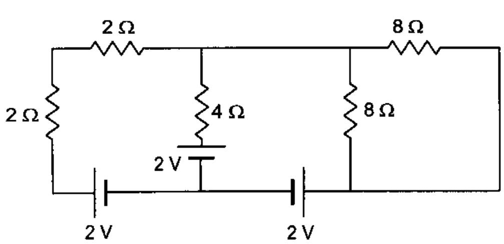


Figure 32-W5

Solution : The two 2 Ω resistors are in series so that their equivalent resistance is 4 Ω. The two 8 Ω resistors are in parallel and their equivalent resistance is also 4 Ω. The circuit may be redrawn as in figure (32-W6a).

Suppose the middle 4 Ω resistor is removed. The remaining circuit is redrawn in figure (32-W6b). It is easy to see that no current will go through any resistor. If we take the potential at b to be zero, the potential at

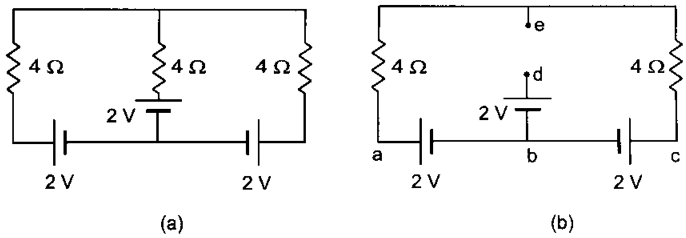


Figure 32-W6

d will be 2 V. The potential at a and c will also be 2 V. As there is no current in the 4 Ω resistors, the potential at e will also be 2 V. Thus, there is no potential difference between d and e. When a 4 Ω resistor is added between d and e, no current will be drawn into it and hence no change will occur in the remaining part of the circuit. This circuit is then the same as the given circuit. Thus, the current in all the resistors in the given circuit is zero.

12. *Find the current supplied by the battery in the circuit shown in figure (32-W7).*

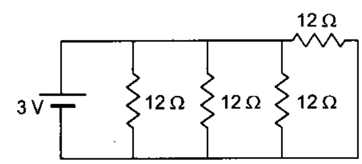


Figure 32-W7

Solution : All the resistors shown in the figure are connected in parallel between the terminals of the battery. The equivalent resistance R between the terminals is, therefore, given by

$$\frac{1}{R}=\frac{1}{12\,\Omega}+\frac{1}{12\,\Omega}+\frac{1}{12\,\Omega}+\frac{1}{12\,\Omega}$$

or, $$R=3\,\Omega.$$

The current supplied by the battery is

$$i=\frac{V}{R}=\frac{3\,\mathrm{V}}{3\,\Omega}=1\,\mathrm{A}.$$

13. *Find the equivalent resistance between the points a and b of the network shown in figure (32-W8).*

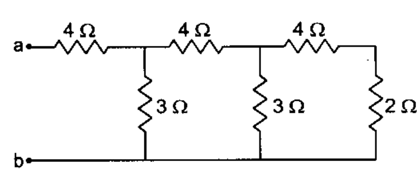


Figure 32-W8

Solution : The two resistors 4 Ω and 2 Ω at the right end are joined in series and may be replaced by a single resistor of 6 Ω. This 6 Ω is connected with the adjacent 3 Ω resistor in parallel. The equivalent resistance of these two is

$$\frac{6\,\Omega \times 3\,\Omega}{6\,\Omega + 3\,\Omega} = 2\,\Omega.$$

This is connected in series with the adjacent $4\,\Omega$ resistor giving an equivalent resistance of $6\,\Omega$ which is connected in parallel with the $3\,\Omega$ resistor. Their equivalent resistance is $2\,\Omega$ which is connected in series with the first $4\,\Omega$ resistor from left. Thus, the equivalent resistance between a and b is $6\,\Omega$.

14. *Find the effective resistance between the points A and B in figure (32-W9).*

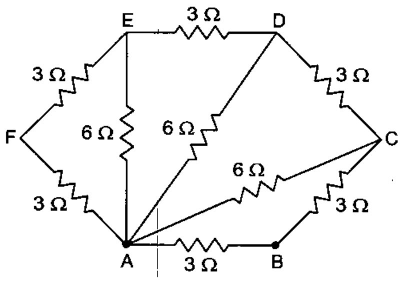


Figure 32-W9

Solution : The resistors AF and FE are in series. Their equivalent resistance is $3\,\Omega + 3\,\Omega = 6\,\Omega$. This is connected in parallel with AE. Their equivalent between A and E is, therefore,

$$\frac{6\,\Omega \times 6\,\Omega}{6\,\Omega + 6\,\Omega} = 3\,\Omega.$$

This $3\,\Omega$ resistance between A and E is in series with ED and the combination is in parallel with AD. Their equivalent between A and D is again $3\,\Omega$.

Similarly, the equivalent of this $3\,\Omega$, DC and AC is $3\,\Omega$. This $3\,\Omega$ is in series with CB and the combination is in parallel with AB. The equivalent resistance between A and B is, therefore,

$$\frac{6\,\Omega \times 3\,\Omega}{6\,\Omega + 3\,\Omega} = 2\,\Omega.$$

15. *Find the equivalent resistance of the network shown in figure (32-W10) between the points a and b when (a) the switch S is open and (b) the switch S is closed.*

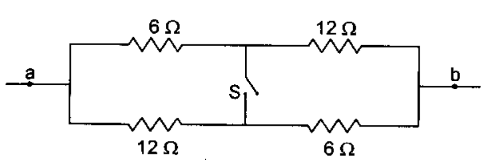


Figure 32-W10

Solution : (a) When the switch is open, $6\,\Omega$ and $12\,\Omega$ resistors on the upper line are in series giving an equivalent of $18\,\Omega$. Similarly, the resistors on the lower line have equivalent resistance $18\,\Omega$. These two $18\,\Omega$ resistances are connected in parallel between a and b so that the equivalent resistance is $9\,\Omega$.

(b) When the switch is closed, the $6\,\Omega$ and $12\,\Omega$ resistors on the left are in parallel giving an equivalent resistance of $4\,\Omega$. Similarly, the two resistors on the right half are equivalent to $4\,\Omega$. These two are connected in series between a and b so that the equivalent resistance is $8\,\Omega$.

16. *Each resistor shown in figure (32-W11) has a resistance of $10\,\Omega$ and the battery has an emf of 6 V. Find the current supplied by the battery.*

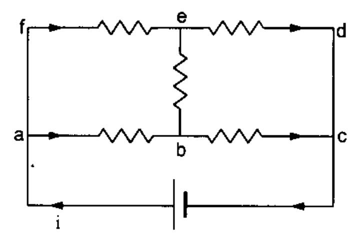


Figure 32-W11

Solution : Suppose a current i starts from the positive terminal of the battery. By symmetry, it divides equally in the resistors ab and fe, so that each of these carries a current $i/2$. The current going into the negative terminal is also i and by symmetry, equal currents should come from ed and bc. Thus, the current in ed is also $i/2$ and hence there will be no current in eb.

We have,

$$V_a - V_c = (V_a - V_b) + (V_b - V_c)$$

or, $$6\text{ V} = \frac{i}{2} \times 10\,\Omega + \frac{i}{2} \times 10\,\Omega$$

giving $$i = 0{\cdot}6\text{ A}.$$

This is a balanced Wheatstone bridge.

17. *Find the equivalent resistance of the network shown in figure (32-W12) between the points A and B.*

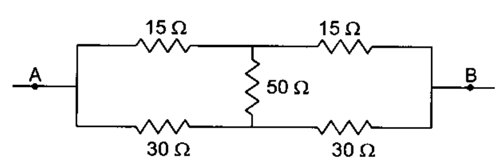
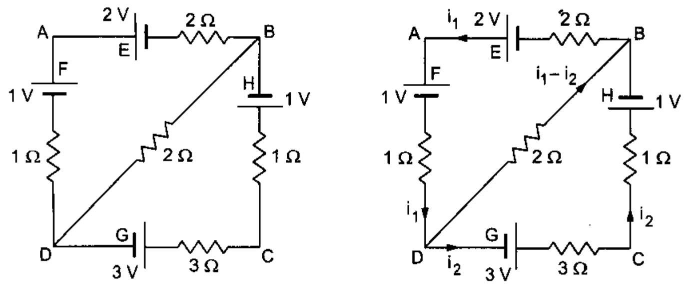


Figure 32-W12

Solution : Suppose an ideal battery of emf $\mathcal{E}$ is connected across the points A and B. The circuit is a Wheatstone bridge with the galvanometer replaced by a $50\,\Omega$ resistance. As the bridge is balanced $(R_1/R_2 = R_3/R_4)$, there will be no current through the $50\,\Omega$ resistance. We can just remove the $50\,\Omega$ resistance without changing any other current. The circuit is then equivalent to two resistances $30\,\Omega$ and $60\,\Omega$ connected in parallel. The equivalent resistance is

$$R = \frac{(30\,\Omega) \times (60\,\Omega)}{(30\,\Omega) + (60\,\Omega)} = 20\,\Omega.$$

18. *In the circuit shown in figure (32-W13a) E, F, G and H are cells of emf 2, 1, 3 and 1 V respectively. The resistances 2, 1, 3 and 1 Ω are their respective internal resistances. Calculate (a) the potential difference between B and D and (b) the potential differences across the terminals of each of the cells G and H.*



(a) (b)

Figure 32-W13

Solution : Suppose a current i_1 goes in the branch BAD and a current i_2 in the branch DCB. The current in DB will be $i_1 - i_2$ from the junction law. The circuit with the currents shown is redrawn in figure (32-W13b). Applying the loop law to $BADB$ we get,

$$(2\ \Omega)i_1 - 2\text{ V} + 1\text{ V} + (1\ \Omega)i_1 + (2\ \Omega)(i_1 - i_2) = 0$$

or, $\quad (5\ \Omega)i_1 - (2\ \Omega)i_2 = 1\text{ V}.$ ... (i)

Applying the same law to the loop $DCBD$, we get

$$-3\text{ V} + (3\ \Omega)i_2 + (1\ \Omega)i_2 + 1\text{ V} - (2\ \Omega)(i_1 - i_2) = 0$$

or, $\quad -(2\ \Omega)i_1 + (6\ \Omega)i_2 = 2\text{ V}.$ ... (ii)

From (i) and (ii),

$$i_1 = \frac{5}{13}\text{ A},\ i_2 = \frac{6}{13}\text{ A}$$

so that $i_1 - i_2 = -\frac{1}{13}\text{ A}.$

The current in BD is from B to D.

(a) $V_B - V_D = (2\ \Omega)\left(\frac{1}{13}\text{ A}\right) = \frac{2}{13}\text{ V}.$

(b) The potential difference across the cell G is

$$V_C - V_D = -(3\ \Omega)i_2 + 3\text{ V}$$

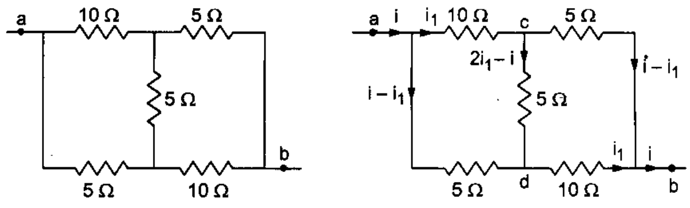
$$= \left(3\text{ V} - \frac{18}{13}\text{ V}\right) = \frac{21}{13}\text{ V}.$$

The potential difference across the cell H is

$$V_C - V_B = (1\ \Omega)i_2 + 1\text{ V} = (1\ \Omega)\left(\frac{6}{13}\text{ A}\right) + 1\text{ V} = \frac{19}{13}\text{ V}.$$

19. *Find the equivalent resistance between the points a and b of the circuit shown in figure (32-W14a).*

Solution : Suppose a current i enters the circuit at the point a, a part i_1 goes through the 10 Ω resistor and the rest $i - i_1$ through the 5 Ω resistor. By symmetry, the current i coming out from the point b will be composed of a part i_1 from the 10 Ω resistor and $i - i_1$ from the 5 Ω resistor. Applying Kirchhoff's junction law, we can find the current through the middle 5 Ω resistor. The current distribution is shown in figure (32-W14b).



(a) (b)

Figure 32-W14

We have

$$V_a - V_b = (V_a - V_c) + (V_c - V_b)$$
$$= (10\ \Omega)i_1 + (5\ \Omega)(i - i_1)$$
$$= (5\ \Omega)i + (5\ \Omega)i_1. \quad \text{... (i)}$$

Also, $\quad V_a - V_b = (V_a - V_c) + (V_c - V_d) + (V_d - V_b)$

$$= (10\ \Omega)i_1 + (5\ \Omega)(2i_1 - i) + (10\ \Omega)i_1$$
$$= -(5\ \Omega)i + (30\ \Omega)i_1 \quad \text{... (ii)}$$

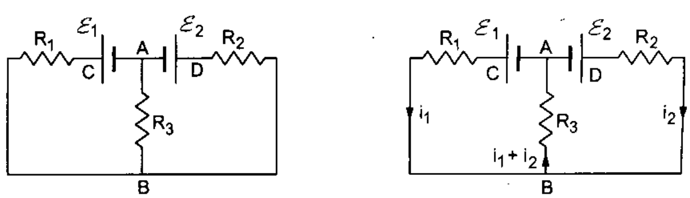
Multiplying (i) by 6 and subtracting (ii) from it, we eliminate i_1 and get,

$$5(V_a - V_b) = (35\ \Omega)i$$

or, $$\frac{V_a - V_b}{i} = 7\ \Omega.$$

Thus, the equivalent resistance between the points a and b is 7 Ω.

20. *Find the currents going through the three resistors R_1, R_2 and R_3 in the circuit of figure (32-W15a).*



(a) (b)

Figure 32-W15

Solution : Let us take the potential of the point A to be zero. The potential at C will be $\mathcal{E}_1$ and that at D will be $\mathcal{E}_2$. Let the potential at B be V. The currents through the three resistors are i_1, i_2 and $i_1 + i_2$ as shown in figure (32.15b). Note that the current directed towards B equals the current directed away from B.

Applying Ohm's law to the three resistors R_1, R_2 and R_3, we get

$$\mathcal{E}_1 - V = R_1 i_1 \quad \text{... (i)}$$

$$\mathcal{E}_2 - V = R_2 i_2 \quad \text{... (ii)}$$

and $\quad V - 0 = R_3(i_1 + i_2).$ ... (iii)

Adding (i) and (iii),

$$\mathcal{E}_1 = R_1 i_1 + R_3(i_1 + i_2)$$
$$= (R_1 + R_3)i_1 + R_3 i_2 \qquad \dots \text{(iv)}$$

and adding (ii) and (iii),

$$\mathcal{E}_2 = R_2 i_2 + R_3(i_1 + i_2)$$
$$= (R_2 + R_3)i_2 + R_3 i_1. \qquad \dots \text{(v)}$$

Equations (iv) and (v) may be directly written from Kirchhoff's loop law applied to the left half and the right half of the circuit.

Multiply (iv) by $(R_2 + R_3)$, (v) by R_3 and subtract to eliminate i_2. This gives

$$i_1 = \frac{\mathcal{E}_1(R_2 + R_3) - \mathcal{E}_2 R_3}{(R_1 + R_3)(R_2 + R_3) - R_3^2}$$
$$= \frac{\mathcal{E}_1(R_2 + R_3) - \mathcal{E}_2 R_3}{R_1R_2 + R_2R_3 + R_3R_1}.$$

Similarly, eliminating i_1 from (iv) and (v) we obtain,

$$i_2 = \frac{\mathcal{E}_2(R_1 + R_3) - \mathcal{E}_1 R_3}{R_1R_2 + R_2R_3 + R_3R_1}.$$

And so,

$$i_1 + i_2 = \frac{\mathcal{E}_1 R_2 + \mathcal{E}_2 R_1}{R_1R_2 + R_2R_3 + R_3R_1}.$$

21. *Find the equivalent resistance between the points a and c of the network shown in figure (32-W16a). Each resistance is equal to r.*

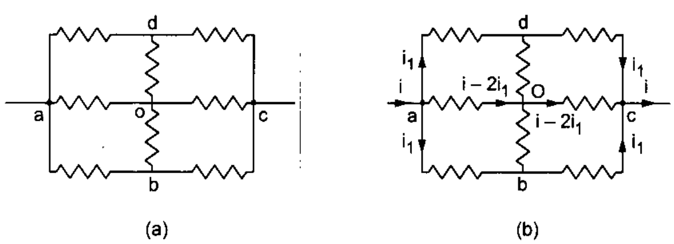


Figure 32-W16

Solution : Suppose a potential difference V is applied between a and c so that a current i enters at a and the same current leaves at c (figure 32-W16b). The current i divides in three parts at a. By symmetry, the part in ad and in ab will be equal. Let each of these currents be i_1. The current through ao is $i - 2i_1$. Similarly, currents from dc, bc and oc combine at c to give the total current i. Since the situation at c is equivalent to that at a, by symmetry, the currents in dc and bc will be i_1 and that in oc will be $i - 2i_1$.

Applying Kirchhoff's junction law at d, we see that the current in do is zero. Similarly, the current in ob is zero. We can remove do and ob for further analysis. It is then equivalent to three resistances, each of value $2r$, in parallel. The equivalent resistance is, therefore, $2r/3$.

22. *Twelve wires, each having resistance r, are joined to form a cube as shown in figure (32-W17). Find the equivalent resistance between the ends of a face diagonal such as a and c.*

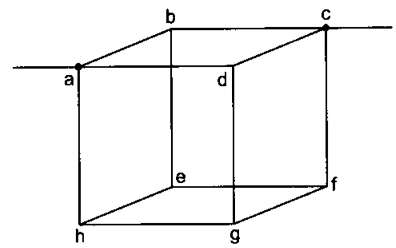


Figure 32-W17

Solution : Suppose a potential difference V is applied between the points a and c so that a current i enters at a and the same current leaves at c. The current distribution is shown in figure (32-W18a).

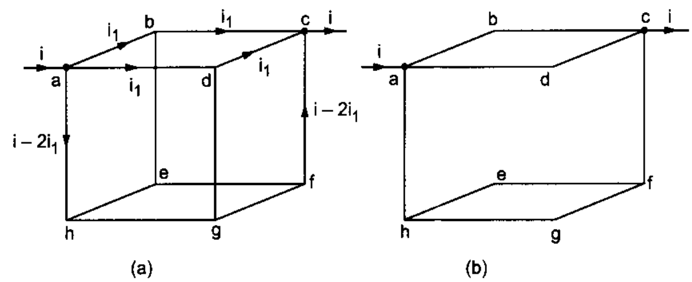


Figure 32-W18

By symmetry, the paths ad and ab are equivalent and hence will carry the same current i_1. The path ah will carry the remaining current $i - 2i_1$ (using Kirchhoff's junction law). Similarly at junction c, currents coming from dc and bc will be i_1 each and from fc will be $i - 2i_1$. Kirchhoff's junction law at b and d shows that currents through be and dg will be zero and hence may be ignored for further analysis. Omitting these two wires, the circuit is redrawn in figure (32-W18b).

The wire hef and hgf are joined in parallel and have equivalent resistance $\frac{(2r)(2r)}{(2r)+(2r)} = r$ between h and f. This is joined in series with ah and fc giving equivalent resistance $r + r + r = 3r$. This $3r$ is joined in parallel with adc $(2r)$ and abc $(2r)$ between a and c.

The equivalent resistance R between a and c is, therefore, given by

$$\frac{1}{R} = \frac{1}{3r} + \frac{1}{2r} + \frac{1}{2r},$$

giving
$$R = \frac{3}{4}r.$$

23. *Find the equivalent resistance of the circuit of the previous problem between the ends of an edge such as a and b in figure (32-W19a).*

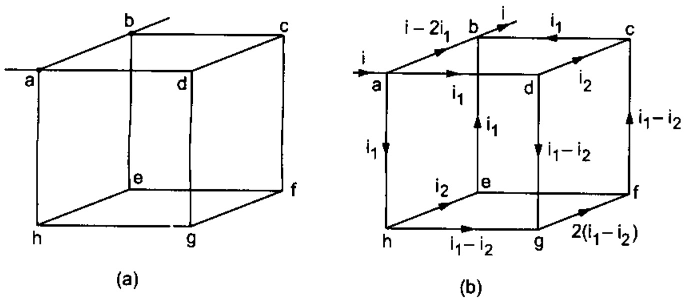


Figure 32-W19

Solution : Suppose a current i enters the circuit at the point a and the same current leaves the circuit at the point b. The current distribution is shown in figure (32-W19b). The paths through ad and ah are equivalent and carry equal current i_1. The current through ab is then $i-2i_1$.

The same distribution holds at the junction b. Currents in eb and cb are i_1 each. The current i_1 in ah is divided into a part i_2 in he and i_1-i_2 in hg. Similar is the division of current i_1 in ad into dc and dg. The rest of the currents may be written easily using Kirchhoff's junction law.

The potential difference V between a and b may be written from the paths ab, $aheb$ and $ahgfcb$ as

$$V=(i-2i_1)r$$

$$V=(i_1+i_2+i_1)r$$

and $$V=[i_1+(i_1-i_2)+2(i_1-i_2)+(i_1-i_2)+i_1]r$$

which may be written as

$$V=(i-2i_1)r$$

$$V=(2i_1+i_2)r$$

and $$V=(6i_1-4i_2)r.$$

Eliminating i_1 and i_2 from these equations,

$$\frac{V}{i}=\frac{7}{12}r$$

which is the equivalent resistance.

24. *Find the equivalent resistance between the points a and b of the infinite ladder shown in figure (32-W20a).*

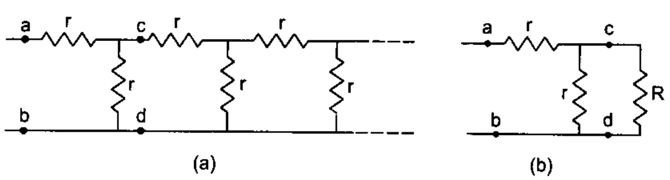


Figure 32-W20

Solution : Let the equivalent resistance between a and b be R. As the ladder is infinite, R is also the equivalent resistance of the ladder to the right of the points c and d. Thus, we can replace the part to the right of cd by a resistance R and redraw the circuit as in figure (32-W20b).

This gives

$$R=r+\frac{rR}{r+R}$$

or, $$rR+R^2=r^2+2rR$$

or, $$R^2-rR-r^2=0$$

or, $$R=\frac{r+\sqrt{r^2+4r^2}}{2}=\frac{1+\sqrt{5}}{2}r.$$

25. *Find the equivalent resistance of the network shown in figure (32-W21) between the points a and b.*

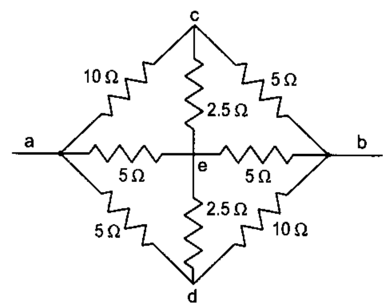


Figure 32-W21

Solution :

Suppose a current i enters the network at point a and the same current leaves it at point b. Suppose, the currents in ac, ad and ae are i_1, i_2 and i_3 respectively. Similar will be the distribution of current at b. The current i leaving at b is composed of i_1 from db, i_2 from cb and i_3 from eb. The situation is shown in figure (32-W22a).

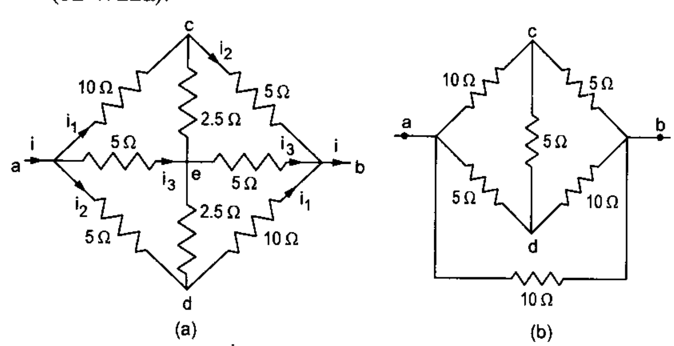


Figure 32-W22

As the current in ae is equal to that in eb, the current in ce will be equal to the current in ed from the junction law. If we assume that the branches ced and aeb do not physically touch at e, nothing will be changed in the current distribution. We can then represent the branch aeb by a single resistance of 10 Ω connected between a and b. Similarly, the branch ced may be replaced a single 5 Ω resistor between c and d. The circuit is redrawn in figure (32-W22b). This is same as the circuit in figure (32-W14a) connected in parallel with a resistance of 10 Ω. So the network is equivalent to a parallel combination of 7 Ω and 10 Ω resistor. The equivalent resistance of the whole network is, therefore,

$$R = \frac{(7\ \Omega) \times (10\ \Omega)}{7\ \Omega + 10\ \Omega} \approx 4{\cdot}1\ \Omega.$$

26. *(a) Find the current i supplied by the battery in the network shown in figure (32-W23) in steady state. (b) Find the charge on the capacitor.*

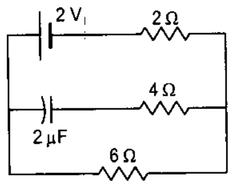


Figure 32-W23

Solution : (a) Once the capacitor is charged, no current will go through it and hence the current through the middle branch of the circuit is zero in steady state. The $4\ \Omega$ resistor will have no current in it and may be omitted for current analysis. The $2\ \Omega$ and $6\ \Omega$ resistors are, therefore, connected in series and hence

$$i = \frac{2\ \text{V}}{2\ \Omega + 6\ \Omega} = 0{\cdot}25\ \text{A}.$$

(b) The potential drop across the $6\ \Omega$ resistor is $6\ \Omega \times 0{\cdot}25\ \text{A} = 1{\cdot}5\ \text{V}$. As there is no current in the $4\ \Omega$ resistor, there is no potential drop across it. The potential difference across the capacitor is, therefore, $1{\cdot}5$ V. The charge on this capacitor is

$$Q = CV = 2\ \mu\text{F} \times 1{\cdot}5\ \text{V} = 3\ \mu\text{C}.$$

27. *A part of a circuit in steady state along with the currents flowing in the branches, the values of resistances, etc., is shown in figure (32-W24). Calculate the energy stored in the capacitor.*

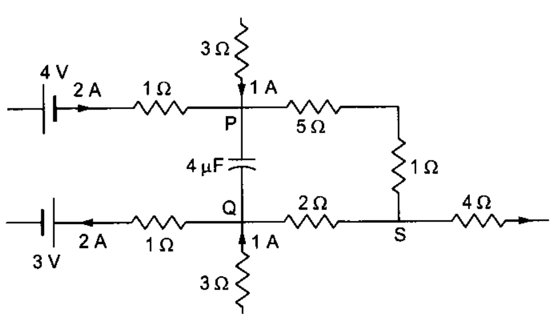


Figure 32-W24

Solution : To get the energy stored in the capacitor, we shall calculate the potential difference between the points P and Q. In steady state, there is no current in the capacitor branch. Applying Kirchhoff's junction law at P, the current in the $5\ \Omega - 1\ \Omega$ branch will be 3 A and hence

$$V_P - V_S = 6\ \Omega \times 3\ \text{A} = 18\ \text{V}.$$

Applying the same theorem at Q, the current in the $2\ \Omega$ resistor will be 1 A towards Q so that

$$V_S - V_Q = 2\ \Omega \times 1\ \text{A} = 2\ \text{V}.$$

Thus, $V_P - V_Q = (V_P - V_S) + (V_S - V_Q) = 20$ V.

The energy stored in the capacitor

$$= \frac{1}{2} CV^2 = \frac{1}{2} \times 4\ \mu\text{F} \times 400\ \text{V}^2$$

$$= 800\ \mu\text{J}.$$

28. *(a) Find the potential drops across the two resistors shown in figure (32-W25a). (b) A voltmeter of resistance 600 Ω is used to measure the potential drop across the 300 Ω resistor. What will be the measured potential drop ?*

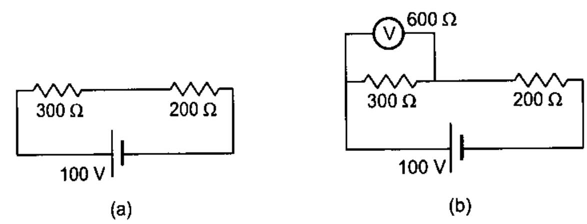


Figure 32-W25

Solution :

(a) The current in the circuit is $\frac{100\ \text{V}}{300\ \Omega + 200\ \Omega} = 0{\cdot}2$ A.

The potential drop across the $300\ \Omega$ resistor is

$$300\ \Omega \times 0{\cdot}2\ \text{A} = 60\ \text{V}.$$

Similarly, the drop across the $200\ \Omega$ resistor is 40 V.

(b) The equivalent resistance, when the voltmeter is connected across $300\ \Omega$, is (figure 32-W25b)

$$R = 200\ \Omega + \frac{600\ \Omega \times 300\ \Omega}{600\ \Omega + 300\ \Omega} = 400\ \Omega.$$

Thus, the main current from the battery is

$$i = \frac{100\ \text{V}}{400\ \Omega} = 0{\cdot}25\ \text{A}.$$

The potential drop across the $200\ \Omega$ resistor is, therefore, $200\ \Omega \times 0{\cdot}25\ \text{A} = 50$ V and that across $300\ \Omega$ is also 50 V. This is also the potential drop across the voltmeter and hence the reading of the voltmeter is 50 V.

29. *A galvanometer has a coil of resistance 100 Ω showing a full-scale deflection at 50 μA. What resistance should be added to use it as (a) a voltmeter of range 50 V (b) an ammeter of range 10 mA ?*

Solution : (a) When a potential difference of 50 V is applied across the voltmeter, full-scale deflection should take place. Thus, $50\ \mu$A should go through the coil. We

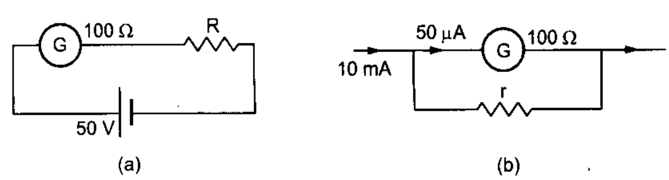


Figure 32-W26

add a resistance R in series with the given coil to achieve this (figure 32-W26a).

We have,

$$50\ \mu A = \frac{50\ V}{100\ \Omega + R}$$

or, $R = 10^6\ \Omega - 100\ \Omega \approx 10^6\ \Omega.$

(b) When a current of 10 mA is passed through the ammeter, 50 μA should go through the coil. We add a resistance r in parallel to the coil to achieve this (figure 32-W26b).

The current through the coil is

$$50\ \mu A = (10\ mA)\frac{r}{r + 100\ \Omega}$$

or, $r \approx 0{\cdot}5\ \Omega.$

30. *The electric field between the plates of a parallel-plate capacitor of capacitance 2·0 μF drops to one third of its initial value in 4·4 μs when the plates are connected by a thin wire. Find the resistance of the wire.*

Solution : The electric field between the plates is

$$E = \frac{Q}{A\varepsilon_0} = \frac{Q_0}{A\varepsilon_0}e^{-t/RC}$$

or, $E = E_0\, e^{-t/RC}.$

In the given problem, $E = \frac{1}{3}E_0$ at $t = 4{\cdot}4\ \mu s$.

Thus, $\frac{1}{3} = e^{-\frac{4\cdot4\ \mu s}{RC}}$

or, $\frac{4{\cdot}4\ \mu s}{RC} = \ln 3 = 1{\cdot}1$

or, $R = \frac{4{\cdot}4\ \mu s}{1{\cdot}1 \times 2{\cdot}0\ \mu F} = 2{\cdot}0\ \Omega.$

31. *A capacitor is connected to a 12 V battery through a resistance of 10 Ω. It is found that the potential difference across the capacitor rises to 4·0 V in 1 μs. Find the capacitance of the capacitor.*

Solution : The charge on the capacitor during charging is given by

$$Q = Q_0(1 - e^{-t/RC}).$$

Hence, the potential difference across the capacitor is

$$V = Q/C = Q_0/C(1 - e^{-t/RC}).$$

Here at $t = 1\ \mu s$, the potential difference is 4 V whereas the steady state potential difference is $Q_0/C = 12$ V. So,

$$4\ V = 12\ V(1 - e^{-t/RC})$$

or, $1 - e^{-t/RC} = \frac{1}{3}$

or, $e^{-t/RC} = \frac{2}{3}$

or, $\frac{t}{RC} = \ln\left(\frac{3}{2}\right) = 0{\cdot}405$

or, $RC = \frac{t}{0{\cdot}405} = \frac{1\ \mu s}{0{\cdot}405} = 2{\cdot}469\ \mu s$

or, $C = \frac{2{\cdot}469\ \mu s}{10\ \Omega} \approx 0{\cdot}25\ \mu F.$

32. *A capacitor charged to 50 V is discharged by connecting the two plates at $t = 0$. If the potential difference across the plates drops to 1·0 V at $t = 10$ ms, what will be the potential difference at $t = 20$ ms ?*

Solution : The potential difference at time t is given by

$$V = Q/C = (Q_0/C)\, e^{-t/RC}$$

or, $V = V_0 e^{-t/RC}.$

According to the given data,

$$1\ V = (50\ V)\, e^{-10\ ms/RC}$$

or, $e^{-10\ ms/RC} = \frac{1}{50}.$

The potential difference at t = 20 ms is

$$V = V_0 e^{-t/RC}$$
$$= (50\ V)\, e^{-20\ ms/RC} = (50\ V)\left(e^{-10\ ms/RC}\right)^2$$
$$= 0{\cdot}02\ V.$$

33. *A 5·0 μF capacitor having a charge of 20 μC is discharged through a wire of resistance 5·0 Ω. Find the heat dissipated in the wire between 25 to 50 μs after the connections are made.*

Solution : The charge on the capacitor at time t after the connections are made is

$$Q = Q_0\, e^{-t/RC}$$

or, $i = \frac{dQ}{dt} = -(Q_0/RC)\, e^{-t/RC}.$

Heat dissipated during the time t_1 to t_2 is

$$U = \int_{t_1}^{t_2} i^2 R\, dt$$
$$= \int_{t_1}^{t_2} \frac{Q_0^{\ 2}}{RC^2} e^{-2t/RC}\, dt$$
$$= \frac{Q_0^2}{2\,C}\left(e^{-\frac{2t_1}{RC}} - e^{-\frac{2t_2}{RC}}\right). \quad \text{... (i)}$$

The time constant RC is $5\ \Omega \times 5{\cdot}0\ \mu F = 25\ \mu s$.

Putting $t_1 = 25\ \mu s$, $t_2 = 50\ \mu s$ and other values in (i),

$$U = \frac{(20\ \mu C)^2}{2 \times 5{\cdot}0\ \mu F}(e^{-2} - e^{-4}) = 4{\cdot}7\ \mu J.$$

□

QUESTIONS FOR SHORT ANSWER

1. Suppose you have three resistors each of value 30 Ω. List all the different resistances you can obtain using them.
2. A proton beam is going from east to west. Is there an electric current? If yes, in what direction?
3. In an electrolyte, the positive ions move from left to right and the negative ions from right to left. Is there a net current? If yes, in what direction?
4. In a TV tube, the electrons are accelerated from the rear to the front. What is the direction of the current?
5. The drift speed is defined as $v_d = \Delta l/\Delta t$ where Δl is the distance drifted in a long time Δt. Why don't we define the drift speed as the limit of $\Delta l/\Delta t$ as $\Delta t \to 0$?
6. One of your friends argues that he has read in previous chapters that there can be no electric field inside a conductor. And hence there can be no current through it. What is the fallacy in this argument?
7. When a current is established in a wire, the free electrons drift in the direction opposite to the current. Does the number of free electrons in the wire continuously decrease?
8. A fan with copper winding in its motor consumes less power as compared to an otherwise similar fan having aluminium winding. Explain.
9. The thermal energy developed in a current-carrying resistor is given by $U = i^2Rt$ and also by $U = Vit$. Should we say that U is proportional to i^2 or to i?
10. Consider a circuit containing an ideal battery connected to a resistor. Do "work done by the battery" and "the thermal energy developed" represent two names of the same physical quantity?
11. Is work done by a battery always equal to the thermal energy developed in electrical circuits? What happens if a capacitor is connected in the circuit?
12. A nonideal battery is connected to a resistor. Is work done by the battery equal to the thermal energy developed in the resistor? Does your answer change if the battery is ideal?
13. Sometimes it is said that "heat is developed" in a resistance when there is an electric current in it. Recall that heat is defined as the energy being transferred due to the temperature difference. Is the statement under quotes technically correct?
14. We often say "a current is going through the wire". What goes through the wire, the charge or the current?
15. Would you prefer a voltmeter or a potentiometer to measure the emf of a battery?
16. Does a conductor become charged when a current is passed through it?
17. Can the potential difference across a battery be greater than its emf?

OBJECTIVE I

1. A metallic resistor is connected across a battery. If the number of collisions of the free electrons with the lattice is somehow decreased in the resistor (for example, by cooling it), the current will
   (a) increase (b) decrease
   (c) remain constant (d) become zero.
2. Two resistors A and B have resistances R_A and R_B respectively with $R_A < R_B$. The resistivities of their materials are ρ_A and ρ_B.
   (a) $\rho_A > \rho_B$ (b) $\rho_A = \rho_B$ (c) $\rho_A < \rho_B$
   (d) The information is not sufficient to find the relation between ρ_A and ρ_B.
3. The product of resistivity and conductivity of a cylindrical conductor depends on
   (a) temperature (b) material
   (c) area of cross section (d) none of these.
4. As the temperature of a metallic resistor is increased, the product of its resistivity and conductivity
   (a) increases (b) decreases
   (c) remains constant (d) may increase or decrease.
5. In an electric circuit containing a battery, the charge (assumed positive) inside the battery
   (a) always goes from the positive terminal to the negative terminal
   (b) may go from the positive terminal to the negative terminal
   (c) always goes from the negative terminal to the positive terminal
   (d) does not move.
6. A resistor of resistance R is connected to an ideal battery. If the value of R is decreased, the power dissipated in the resistor will
   (a) increase (b) decrease (c) remain unchanged.
7. A current passes through a resistor. Let K_1 and K_2 represent the average kinetic energy of the conduction electrons and the metal ions respectively.
   (a) $K_1 < K_2$ (b) $K_1 = K_2$
   (c) $K_1 > K_2$ (d) Any of these three may occur.
8. Two resistors R and $2R$ are connected in series in an electric circuit. The thermal energy developed in R and $2R$ are in the ratio
   (a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1.
9. Two resistances R and $2R$ are connected in parallel in an electric circuit. The thermal energy developed in R and $2R$ are in the ratio
   (a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1.
10. A uniform wire of resistance 50 Ω is cut into 5 equal parts. These parts are now connected in parallel. The

equivalent resistance of the combination is
(a) 2 Ω (b) 10 Ω (c) 250 Ω (d) 6250 Ω.

11. Consider the following two statements:
(A) Kirchhoff's junction law follows from conservation of charge.
(B) Kirchhoff's loop law follows from conservative nature of electric field.
(a) Both A and B are correct.
(b) A is correct but B is wrong.
(c) B is correct but A is wrong.
(d) Both A and B are wrong.

12. Two nonideal batteries are connected in series. Consider the following statements:
(A) The equivalent emf is larger than either of the two emfs.
(B) The equivalent internal resistance is smaller than either of the two internal resistances.
(a) Each of A and B is correct.
(b) A is correct but B is wrong.
(c) B is correct but A is wrong.
(d) Each of A and B is wrong.

13. Two nonideal batteries are connected in parallel. Consider the following statements:
(A) The equivalent emf is smaller than either of the two emfs.
(B) The equivalent internal resistance is smaller than either of the two internal resistances.
(a) Both A and B are correct.
(b) A is correct but B is wrong.
(c) B is correct but A is wrong.
(d) Both A and B are wrong.

14. The net resistance of an ammeter should be small to ensure that
(a) it does not get overheated
(b) it does not draw excessive current
(c) it can measure large currents
(d) it does not appreciably change the current to be measured.

15. The net resistance of a voltmeter should be large to ensure that
(a) it does not get overheated
(b) it does not draw excessive current
(c) it can measure large potential differences
(d) it does not appreciably change the potential difference to be measured.

16. Consider a capacitor-charging circuit. Let Q_1 be the charge given to the capacitor in a time interval of 10 ms and Q_2 be the charge given in the next time interval of 10 ms. Let 10 μC charge be deposited in a time interval t_1 and the next 10 μC charge is deposited in the next time interval t_2.
(a) $Q_1 > Q_2$, $t_1 > t_2$ (b) $Q_1 > Q_2$, $t_1 < t_2$
(c) $Q_1 < Q_2$, $t_1 > t_2$ (d) $Q_1 < Q_2$, $t_1 < t_2$

OBJECTIVE II

1. Electrons are emitted by a hot filament and are accelerated by an electric field as shown in figure (32-Q1). The two stops at the left ensure that the electron beam has a uniform cross-section.
(a) The speed of the electron is more at B than at A.
(b) The electric current is from left to right.
(c) The magnitude of the current is larger at B than at A.
(d) The current density is more at B than at A.

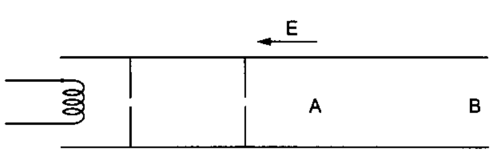


Figure 32-Q1

2. A capacitor with no dielectric is connected to a battery at $t = 0$. Consider a point A in the connecting wires and a point B in between the plates.
(a) There is no current through A.
(b) There is no current through B.
(c) There is a current through A as long as the charging is not complete.
(d) There is a current through B as long as the charging is not complete.

3. When no current is passed through a conductor,
(a) the free electrons do not move
(b) the average speed of a free electron over a large period of time is zero
(c) the average velocity of a free electron over a large period of time is zero
(d) the average of the velocities of all the free electrons at an instant is zero.

4. Which of the following quantities do not change when a resistor connected to a battery is heated due to the current ?
(a) Drift speed (b) Resistivity
(c) Resistance (d) Number of free electrons

5. As the temperature of a conductor increases, its resistivity and conductivity change. The ratio of resistivity to conductivity
(a) increases (b) decreases (c) remains constant
(d) may increase or decrease depending on the actual temperature.

6. A current passes through a wire of nonuniform cross-section. Which of the following quantities are independent of the cross section ?
(a) The charge crossing in a given time interval
(b) Drift speed
(c) Current density
(d) Free-electron density

7. Mark out the correct options.
(a) An ammeter should have small resistance.
(b) An ammeter should have large resistance.

(c) A voltmeter should have small resistance.
(d) A voltmeter should have large resistance.

8. A capacitor of capacitance 500 μF is connected to a battery through a 10 kΩ resistor. The charge stored on the capacitor in the first 5 s is larger than the charge stored in the next
(a) 5 s (b) 50 s (c) 500 s (d) 500 s

9. A capacitor C_1 of capacitance 1 μF and a capacitor C_2 of capacitance 2 μF are separately charged by a common battery for a long time. The two capacitors are then separately discharged through equal resistors. Both the discharge circuits are connected at $t = 0$.
(a) The current in each of the two discharging circuits is zero at $t = 0$.
(b) The currents in the two discharging circuits at $t = 0$ are equal but not zero.
(c) The currents in the two discharging circuits at $t = 0$ are unequal.
(d) C_1 loses 50% of its initial charge sooner than C_2 loses 50% of its initial charge.

EXERCISES

1. The amount of charge passed in time t through a cross-section of a wire is

$$Q(t) = At^2 + Bt + C.$$

(a) Write the dimensional formulae for A, B and C.
(b) If the numerical values of A, B and C are 5, 3 and 1 respectively in SI units, find the value of the current at $t = 5$ s.

2. An electron gun emits 2.0×10^{16} electrons per second. What electric current does this correspond to?

3. The electric current existing in a discharge tube is 2.0 μA. How much charge is transferred across a cross-section of the tube in 5 minutes?

4. The current through a wire depends on time as

$$i = i_0 + \alpha t,$$

where $i_0 = 10$ A and $\alpha = 4$ A s^{-1}. Find the charge crossed through a section of the wire in 10 seconds.

5. A current of 1.0 A exists in a copper wire of cross-section 1.0 mm^2. Assuming one free electron per atom calculate the drift speed of the free electrons in the wire. The density of copper is 9000 kg m^{-3}.

6. A wire of length 1 m and radius 0.1 mm has a resistance of 100 Ω. Find the resistivity of the material.

7. A uniform wire of resistance 100 Ω is melted and recast in a wire of length double that of the original. What would be the resistance of the wire?

8. Consider a wire of length 4 m and cross-sectional area 1 mm^2 carrying a current of 2 A. If each cubic metre of the material contains 10^{29} free electrons, find the average time taken by an electron to cross the length of the wire.

9. What length of a copper wire of cross-sectional area 0.01 mm^2 will be needed to prepare a resistance of 1 kΩ? Resistivity of copper $= 1.7 \times 10^{-8}$ Ω m.

10. Figure (32-E1) shows a conductor of length l having a circular cross section. The radius of cross section varies linearly from a to b. The resistivity of the material is ρ. Assuming that $b - a << l$, find the resistance of the conductor.

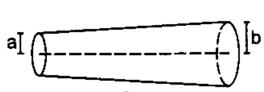


Figure 32-E1

11. A copper wire of radius 0.1 mm and resistance 1 kΩ is connected across a power supply of 20 V. (a) How many electrons are transferred per second between the supply and the wire at one end? (b) Write down the current density in the wire.

12. Calculate the electric field in a copper wire of cross-sectional area 2.0 mm^2 carrying a current of 1 A. The resistivity of copper $= 1.7 \times 10^{-8}$ Ω m.

13. A wire has a length of 2.0 m and a resistance of 5.0 Ω. Find the elecric field existing inside the wire if it carries a current of 10 A.

14. The resistances of an iron wire and a copper wire at 20°C are 3.9 Ω and 4.1 Ω respectively. At what temperature will the resistances be equal? Temperature coefficient of resistivity for iron is 5.0×10^{-3} K^{-1} and for copper it is 4.0×10^{-3} K^{-1}. Neglect any thermal expansion.

15. The current in a conductor and the potential difference across its ends are measured by an ammeter and a voltmeter. The meters draw negligible currents. The ammeter is accurate but the voltmeter has a zero error (that is, it does not read zero when no potential difference is applied). Calculate the zero error if the readings for two different conditions are 1.75 A, 14.4 V and 2.75 A, 22.4 V.

16. Figure (32-E2) shows an arrangement to measure the emf $\mathcal{E}$ and internal resistance r of a battery. The voltmeter has a very high resistance and the ammeter also has some resistance. The voltmeter reads 1.52 V when the switch S is open. When the switch is closed the voltmeter reading drops to 1.45 V and the ammeter reads 1.0 A. Find the emf and the internal resistance of the battery.

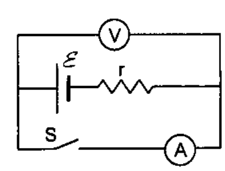


Figure 32-E2

17. The potential difference between the terminals of a battery of emf 6.0 V and internal resistance 1 Ω drops to 5.8 V when connected across an external resistor. Find the resistance of the external resistor.

18. The potential difference between the terminals of a 6·0 V battery is 7·2 V when it is being charged by a current of 2·0 A. What is the internal resistance of the battery ?

19. The internal resistance of an accumulator battery of emf 6 V is 10 Ω when it is fully discharged. As the battery gets charged up, its internal resistance decreases to 1 Ω. The battery in its completely discharged state is connected to a charger which maintains a constant potential difference of 9 V. Find the current through the battery (a) just after the connections are made and (b) after a long time when it is completely charged.

20. Find the value of i_1/i_2 in figure (32-E3) if (a) $R = 0·1\ \Omega$, (b) $R = 1\ \Omega$ (c) $R = 10\ \Omega$. Note from your answers that in order to get more current from a combination of two batteries they should be joined in parallel if the external resistance is small and in series if the external resistance is large as compared to the internal resistances.

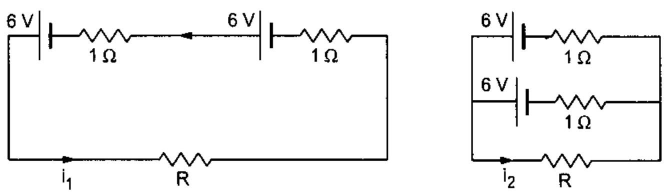


Figure 32-E3

21. Consider $N = n_1 n_2$ identical cells, each of emf $\mathcal{E}$ and internal resistance r. Suppose n_1 cells are joined in series to form a line and n_2 such lines are connected in parallel. The combination drives a current in an external resistance R. (a) Find the current in the external resistance. (b) Assuming that n_1 and n_2 can be continuously varied, find the relation between n_1, n_2, R and r for which the current in R is maximum.

22. A battery of emf 100 V and a resistor of resistance 10 kΩ are joined in series. This system is used as a source to supply current to an external resistance R. If R is not greater than 100 Ω, the current through it is constant up to two significant digits. Find its value. This is the basic principle of a *constant-current source.*

23. If the reading of ammeter A_1 in figure (32-E4) is 2·4 A, what will the ammeters A_2 and A_3 read ? Neglect the resistances of the ammeters.

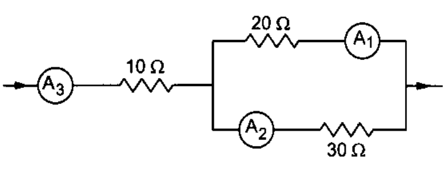


Figure 32-E4

24. The resistance of the rheostat shown in figure (32-E5) is 30 Ω. Neglecting the meter resistance, find the

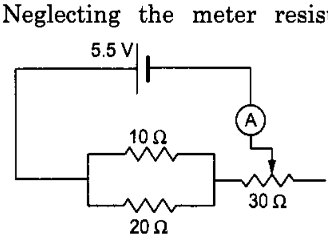


Figure 32-E5

minimum and maximum currents through the ammeter as the rheostat is varied.

25. Three bulbs, each having a resistance of 180 Ω, are connected in parallel to an ideal battery of emf 60 V. Find the current delivered by the battery when (a) all the bulbs are switched on, (b) two of the bulbs are switched on and (c) only one bulb is switched on.

26. Suppose you have three resistors of 20 Ω, 50 Ω and 100 Ω. What minimum and maximum resistances can you obtain from these resistors ?

27. A bulb is made using two filaments. A switch selects whether the filaments are used individually or in parallel. When used with a 15 V battery, the bulb can be operated at 5 W, 10 W or 15 W. What should be the resistances of the filaments ?

28. Figure (32-E6) shows a part of a circuit. If a current of 12 mA exists in the 5 kΩ resistor, find the currents in the other three resistors. What is the potential difference between the points A and B ?

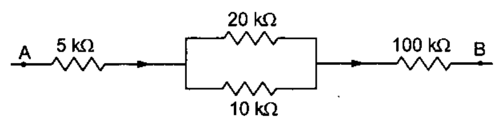


Figure 32-E6

29. An ideal battery sends a current of 5 A in a resistor. When another resistor of value 10 Ω is connected in parallel, the current through the battery is increased to 6 A. Find the resistance of the first resistor.

30. Find the equivalent resistance of the network shown in figure (32-E7) between the points a and b.

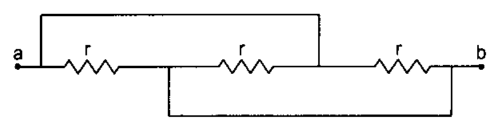


Figure 32-E7

31. A wire of resistance 15·0 Ω is bent to form a regular hexagon $ABCDEFA$. Find the equivalent resistance of the loop between the points (a) A and B, (b) A and C and (c) A and D.

32. Consider the circuit shown in figure (32-E8). Find the current through the 10 Ω resistor when the switch S is (a) open (b) closed.

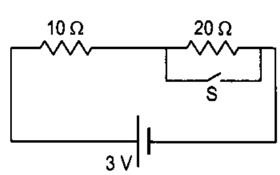


Figure 32-E8

33. Find the currents through the three resistors shown in figure (32-E9).

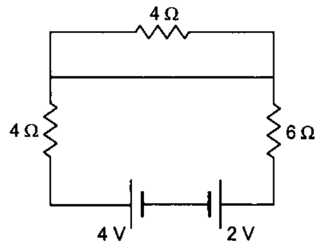


Figure 32-E9

34. Figure (32-E10) shows a part of an electric circuit. The potentials at the points a, b and c are 30 V, 12 V and 2 V respectively. Find the currents through the three resistors.

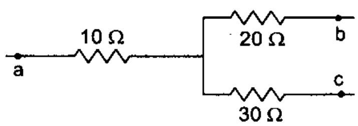


Figure 32-E10

35. Each of the resistors shown in figure (32-E11) has a resistance of 10 Ω and each of the batteries has an emf of 10 V. Find the currents through the resistors a and b in the two circuits.

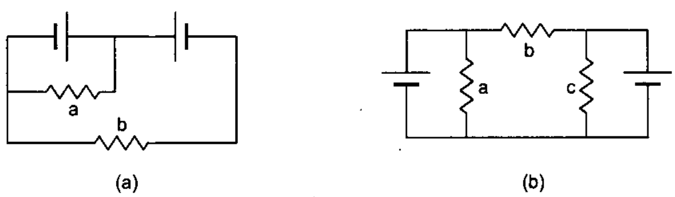


Figure 32-E11

36. Find the potential difference $V_a - V_b$ in the circuits shown in figure (32-E12).

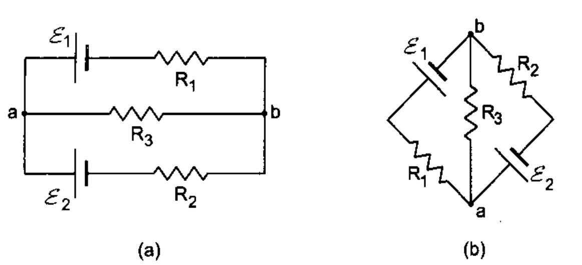


Figure 32-E12

37. In the circuit shown in figure (32-E13), $\mathcal{E}_1 = 3$ V, $\mathcal{E}_2 = 2$ V, $\mathcal{E}_3 = 1$ V and $r_1 = r_2 = r_3 = 1$ Ω. Find the potential difference between the points A and B and the current through each branch.

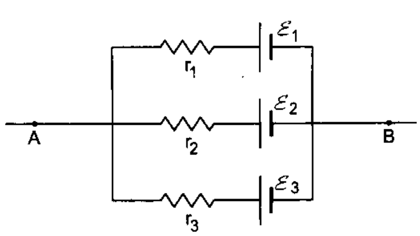


Figure 32-E13

38. Find the current through the 10 Ω resistor shown in figure (32-E14).

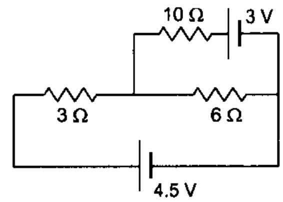


Figure 32-E14

39. Find the current in the three resistors shown in figure (32-E15).

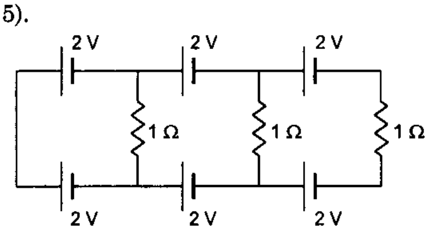


Figure 32-E15

40. What should be the value of R in figure (32-E16) for which the current in it is zero ?

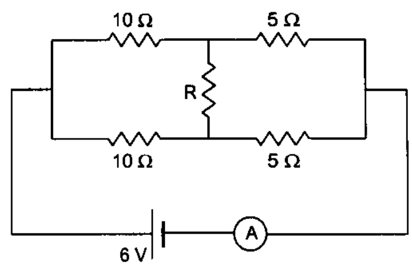


Figure 32-E16

41. Find the equivalent resistance of the circuits shown in figure (32-E17) between the points a and b. Each resistor has a resistance r.

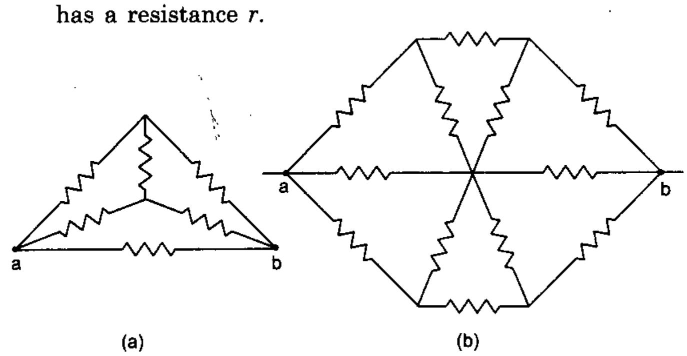


Figure 32-E17

42. Find the current measured by the ammeter in the circuit shown in figure (32-E18).

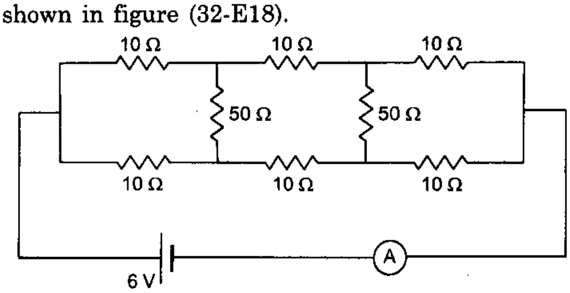


Figure 32-E18

43. Consider the circuit shown in figure (32-E19a). Find (a) the current in the circuit, (b) the potential drop across the 5 Ω resistor, (c) the potential drop across the 10 Ω resistor. (d) Answer the parts (a), (b) and (c) with reference to figure (32-E19b).

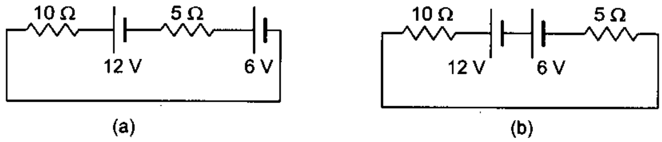


Figure 32-E19

44. Twelve wires, each having equal resistance r, are joined to form a cube as shown in figure (32-E20). Find the equivalent resistance between the diagonally opposite points a and f.

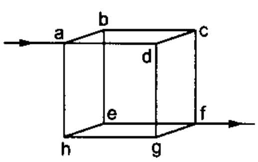


Figure 32-E20

45. Find the equivalent resistances of the networks shown in figure (32-E21) between the points a and b.

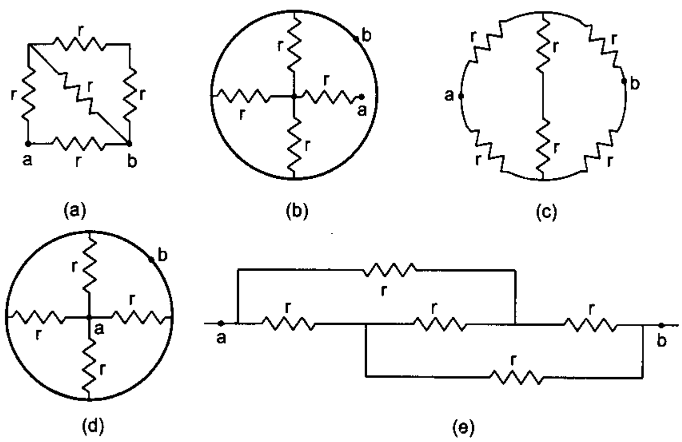


Figure 32-E21

46. An infinite ladder is constructed with 1 Ω and 2 Ω resistors as shown in figure (32-E22). (a) Find the effective resistance between the points A and B. (b) Find the current that passes through the 2 Ω resistor nearest to the battery.

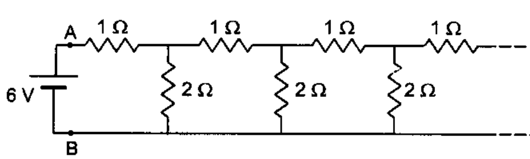


Figure 32-E22

47. The emf $\mathscr{E}$ and the internal resistance r of the battery shown in figure (32-E23) are 4·3 V and 1·0 Ω respectively. The external resistance R is 50 Ω. The resistances of the ammeter and voltmeter are 2·0 Ω and 200 Ω respectively. (a) Find the readings of the two meters. (b) The switch is thrown to the other side. What will be the readings of the two meters now ?

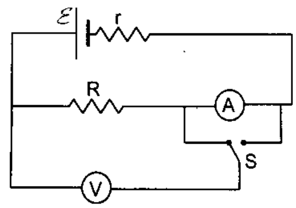


Figure 32-E23

48. A voltmeter of resistance 400 Ω is used to measure the potential difference across the 100 Ω resistor in the circuit shown in figure (32-E24). (a) What will be the reading of the voltmeter ? (b) What was the potential difference across 100 Ω before the voltmeter was connected ?

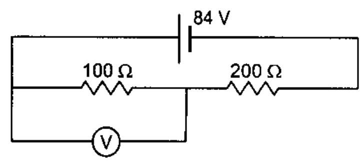


Figure 32-E24

49. The voltmeter shown in figure (32-E25) reads 18 V across the 50 Ω resistor. Find the resistance of the voltmeter.

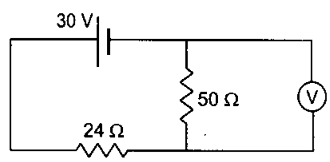


Figure 32-E25

50. A voltmeter consists of a 25 Ω coil connected in series with a 575 Ω resistor. The coil takes 10 mA for full scale deflection. What maximum potential difference can be measured on this voltmeter ?

51. An ammeter is to be constructed which can read currents up to 2·0 A. If the coil has a resistance of 25 Ω and takes 1 mA for full-scale deflection, what should be the resistance of the shunt used ?

52. A voltmeter coil has resistance 50·0 Ω and a resistor of 1·15 kΩ is connected in series. It can read potential differences upto 12 volts. If this same coil is used to construct an ammeter which can measure currents up to 2·0 A, what should be the resistance of the shunt used ?

53. The potentiometer wire AB shown in figure (32-E26) is 40 cm long. Where should the free end of the galvanometer be connected on AB so that the galvanometer may show zero deflection ?

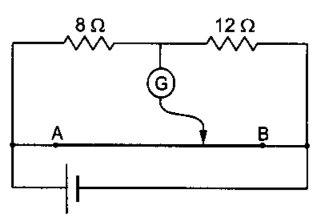


Figure 32-E26

54. The potentiometer wire AB shown in figure (32-E27) is 50 cm long. When $AD = 30$ cm, no deflection occurs in the galvanometer. Find R.

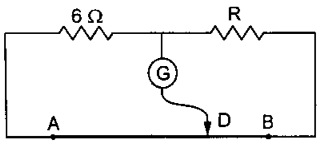


Figure 32-E27

55. A 6-volt battery of negligible internal resistance is connected across a uniform wire AB of length 100 cm. The positive terminal of another battery of emf 4 V and internal resistance 1 Ω is joined to the point A as shown in figure (32-E28). Take the potential at B to be zero. (a) What are the potentials at the points A and C ? (b) At which point D of the wire AB, the potential is equal to the potenial at C ? (c) If the points C and D are connected by a wire, what will be the current through it ? (d) If the 4 V battery is replaced by 7·5 V battery, what would be the answers of parts (a) and (b) ?

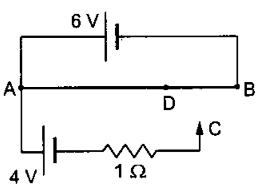


Figure 32-E28

56. Consider the potentiometer circuit arranged as in figure (32-E29). The potentiometer wire is 600 cm long. (a) At what distance from the point A should the jockey touch the wire to get zero deflection in the galvanometer? (b) If the jockey touches the wire at a distance of 560 cm from A, what will be the current in the galvanometer ?

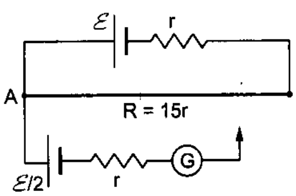


Figure 32-E29

57. Find the charge on the capacitor shown in figure (32-E30).

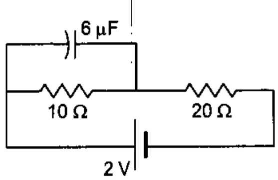


Figure 32-E30

58. (a) Find the current in the 20 Ω resistor shown in figure (32-E31). (b) If a capacitor of capacitance 4 μF is joined between the points A and B, what would be the electrostatic energy stored in it in steady state ?

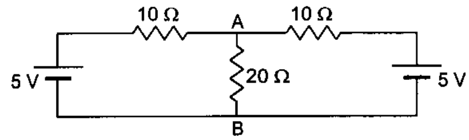


Figure 32-E31

59. Find the charges on the four capacitors of capacitances 1 μF, 2 μF, 3 μF and 4 μF shown in figure (32-E32).

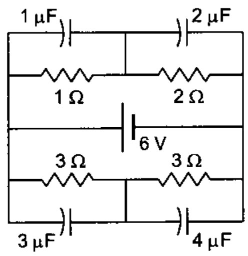


Figure 32-E32

60. Find the potential difference between the points A and B and between the points B and C of figure (32-E33) in steady state.

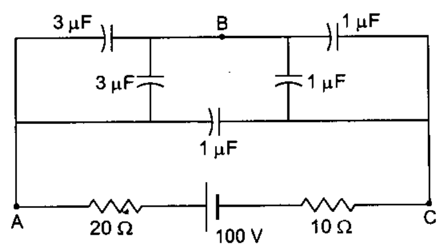


Figure 32-E33

61. A capacitance C, a resistance R and an emf $\mathcal{E}$ are connected in series at $t = 0$. What is the maximum value of (a) the potential difference across the resistor, (b) the current in the circuit, (c) the potential difference across the capacitor, (d) the energy stored in the capacitor, (e) the power delivered by the battery and (f) the power converted into heat.

62. A parallel-plate capacitor with plate area 20 cm^2 and plate separation 1·0 mm is connected to a battery. The resistance of the circuit is 10 kΩ. Find the time constant of the circuit.

63. A capacitor of capacitance 10 μF is connected to a battery of emf 2 V. It is found that it takes 50 ms for the charge on the capacitor to become 12·6 μC. Find the resistance of the circuit.

64. A 20 μF capacitor is joined to a battery of emf 6·0 V through a resistance of 100 Ω. Find the charge on the capacitor 2·0 ms after the connections are made.

65. The plates of a capacitor of capacitance 10 μF, charged to 60 μC, are joined together by a wire of resistance 10 Ω at $t = 0$. Find the charge on the capacitor in the circuit at (a) $t = 0$, (b) $t = 30$ μs, (c) $t = 120$ μs and (d) $t = 1·0$ ms.

66. A capacitor of capacitance 8·0 μF is connected to a battery of emf 6·0 V through a resistance of 24 Ω. Find

the current in the circuit (a) just after the connections are made and (b) one time constant after the connections are made.

67. A parallel-plate capacitor of plate area 40 cm^2 and separation between the plates 0·10 mm is connected to a battery of emf 2·0 V through a 16 Ω resistor. Find the electric field in the capacitor 10 ns after the connections are made.

68. A parallel-plate capacitor has plate area 20 cm^2, plate separation 1·0 mm and a dielectric slab of dielectric constant 5·0 filling up the space between the plates. This capacitor is joined to a battery of emf 6·0 V through a 100 kΩ resistor. Find the energy of the capacitor 8·9 μs after the connections are made.

69. A 100 μF capacitor is joined to a 24 V battery through a 1·0 MΩ resistor. Plot qualitative graphs (a) between current and time for the first 10 minutes and (b) between charge and time for the same period.

70. How many time constants will elapse before the current in a charging RC circuit drops to half of its initial value ? Answer the same question for a discharging RC circuit.

71. How many time constants will elapse before the charge on a capacitor falls to 0·1% of its maximum value in a discharging RC circuit ?

72. How many time constants will elapse before the energy stored in the capacitor reaches half of its equilibrium value in a charging RC circuit ?

73. How many time constants will elapse before the power delivered by the battery drops to half of its maximum value in an RC circuit ?

74. A capacitor of capacitance C is connected to a battery of emf $\mathscr{E}$ at $t = 0$ through a resistance R. Find the maximum rate at which energy is stored in the capacitor. When does the rate has this maximum value?

75. A capacitor of capacitance 12·0 μF is connected to a battery of emf 6·00 V and internal resistance 1·00 Ω through resistanceless leads. 12·0 μs after the connections are made, what will be (a) the current in the circuit, (b) the power delivered by the battery, (c) the power dissipated in heat and (d) the rate at which the energy stored in the capacitor is increasing.

76. A capacitance C charged to a potential difference V is discharged by connecting its plates through a resistance R. Find the heat dissipated in one time constant after the connections are made. Do this by calculating $\int i^2 R\, dt$ and also by finding the decrease in the energy stored in the capacitor.

77. By evaluating $\int i^2 R dt$, show that when a capacitor is charged by connecting it to a battery through a resistor, the energy dissipated as heat equals the energy stored in the capacitor.

78. A parallel-plate capacitor is filled with a dielectric material having resistivity ρ and dielectric constant K. The capacitor is charged and disconnected from the charging source. The capacitor is slowly discharged through the dielectric. Show that the time constant of the discharge is independent of all geometrical parameters like the plate area or separation between the plates. Find this time constant.

79. Find the charge on each of the capacitors 0·20 ms after the switch S is closed in figure (32-E34).

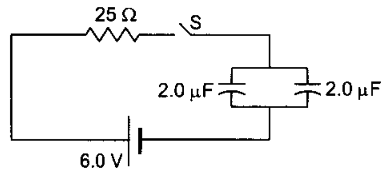


Figure 32-E34

80. The switch S shown in figure (32-E35) is kept closed for a long time and is then opened at $t = 0$. Find the current in the middle 10 Ω resistor at $t = 1·0$ ms.

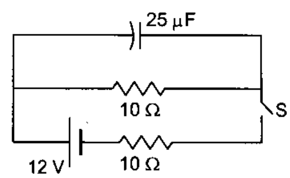


Figure 32-E35

81. A capacitor of capacitance 100 μF is connected across a battery of emf 6·0 V through a resistance of 20 kΩ for 4·0 s. The battery is then replaced by a thick wire. What will be the charge on the capacitor 4·0 s after the battery is disconnected ?

82. Consider the situation shown in figure (32-E36). The switch is closed at $t = 0$ when the capacitors are uncharged. Find the charge on the capacitor C_1 as a function of time t.

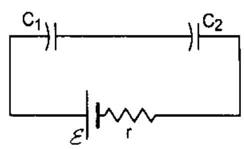


Figure 32-E36

83. A capacitor of capacitance C is given a charge Q. At $t = 0$, it is connected to an uncharged capacitor of equal capacitance through a resistance R. Find the charge on the second capacitor as a function of time.

84. A capacitor of capacitance C is given a charge Q. At $t = 0$, it is connected to an ideal battery of emf $\mathscr{E}$ through a resistance R. Find the charge on the capacitor at time t.

□

ANSWERS

OBJECTIVE I

1. (a)	2. (d)	3. (d)	4. (c)	5. (b)	6. (a)
7. (c)	8. (a)	9. (b)	10. (a)	11. (a)	12. (b)
13. (c)	14. (d)	15. (d)	16. (b)		

OBJECTIVE II

1. (a)	2. (b), (c)	3. (c), (d)
4. (d)	5. (a)	6. (a), (d)
7. (a), (d)	8. all	9. (b), (d)

EXERCISES

1. (a) IT^{-1}, I, IT (b) 53 A
2. 3.2×10^{-3} A
3. 6.0×10^{-4} C
4. 300 C
5. 0·074 mm s^{-1}
6. $\pi \times 10^{-6}$ Ω m
7. 400 Ω
8. 3.2×10^{4} s ≈ 8·9 hours
9. 0·6 km
10. $\frac{\rho l}{\pi ab}$
11. (a) 1.25×10^{17} (b) 6.37×10^{5} A/m^2
12. 8·5 mV m^{-1}
13. 25 V m^{-1}
14. 84·5°C
15. 0·4 V
16. 1·52 V, 0·07 Ω
17. 29 Ω
18. 0·6 Ω
19. (a) 0·3 A (b) 3 A
20. (a) 0·57 (b) 1 (c) 1·75
21. (a) $\frac{n_1 \mathcal{E}}{R + \frac{n_1 r}{n_2}}$ (b) $rn_1 = Rn_2$
22. 10 mA
23. 1·6 A, 4·0 A
24. 0·15 A, 0·83 A
25. (a) 1·0 A (b) 0·67 A (c) 0·33 A
26. 12·5 Ω, 170 Ω
27. 45 Ω, 22·5 Ω
28. 4 mA in 20 kΩ resistor, 8 mA in 10 kΩ resistor and 12 mA in 100 kΩ resistor, 1340 V
29. 2 Ω
30. $r/3$
31. (a) 2·08 Ω (b) 3·33 Ω (c) 3·75 Ω
32. (a) 0·1 A (b) 0·3 A
33. zero in the upper 4 Ω resistor and 0·2 A in the rest two
34. 1 A through 10 Ω, 0·4 Ω through 20 Ω and 0·6 A through 30 Ω
35. 1 A in a and zero in b in both the circuits
36. (a) $\frac{\frac{\mathcal{E}_1}{R_1} + \frac{\mathcal{E}_2}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$ (b) same as (a)
37. 2 V, $i_1 = 1$ A, $i_2 = 0$, $i_3 = -1$ A
38. zero
39. zero
40. any value of R will do
41. (a) $r/2$ (b) $4r/5$
42. 0·4 A
43. (a) 1·2 A (b) 6 V (c) 12 V (d) same as the parts (a), (b) and (c)
44. $\frac{5}{6} r$
45. (a) $\frac{5}{8} r$ (b) $\frac{4}{3} r$ (c) r (d) $\frac{r}{4}$ (e) r
46. (a) 2 Ω (b) 1·5 A
47. (a) 0·1 A, 4·0 V (b) 0·08 A, 4·2 V
48. (a) 24 V (b) 28 V
49. 130 Ω
50. 6 V
51. 1.25×10^{-2} Ω
52. 0·251 Ω
53. 16 cm from A
54. 4 Ω
55. (a) 6 V, 2 V (b) AD = 66·7 cm (c) zero (d) 6 V, – 1·5 V, no such point D exists.
56. (a) 320 cm (b) $\frac{3\mathcal{E}}{22r}$
57. 4 μC
58. (a) 0·2 A (b) 32 μJ
59. 2 μC, 8 μC, 9 μC and 12 μC
60. 25 V, 75 V
61. (a) $\mathcal{E}$ (b) $\frac{\mathcal{E}}{R}$ (c) $\mathcal{E}$ (d) $\frac{1}{2} C\mathcal{E}^2$ (e) $\frac{\mathcal{E}^2}{R}$ (f) $\frac{\mathcal{E}^2}{R}$
62. 0·18 μs
63. 5 kΩ
64. 76 μC
65. (a) 60 μC (b) 44 μC (c) 18 μC (d) 0·003 μC
66. (a) 0·25 A (b) 0·09 A
67. 1.7×10^{4} V m^{-1}
68. 6.3×10^{-10} J
70. 0·69 in both cases
71. 6·9

72. 1·23
73. 0·69
74. $\frac{\mathcal{E}^2}{4R}$, $CR \ln 2$
75. (a) 2·21 A (b) 13·2 W (c) 4·87 W (d) 8·37 W
76. $\frac{1}{2}(1 - 1/e^2)\,CV^2$
78. $\varepsilon_0 \rho K$
79. 10·37 μC
80. 11 mA
81. 70 μC
82. $q = \mathcal{E}C(1 - e^{-t/rc})$, where $C = \frac{C_1 C_2}{C_1 + C_2}$.
83. $\frac{Q}{2}(1 - e^{-2t/RC})$
84. $C\mathcal{E}(1 - e^{-t/CR}) + Q\,e^{-t/CR}$

□