Chapter 6 Determinants Ex 6.4 Q19

$$D = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$$c_2 \to c_2 - c_1, c_3 \to c_3 - c_1$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^2 & b^2-a^2 & c^2-a^2 \end{vmatrix}$$

Now taking $(b-a)$ from c_2, and $(c-a)$ c_3 common

$$= (b-a)(c-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & b+a & c+a \end{vmatrix}$$

Expanding along R_1

$= (b-a)(c-a)[c+a-b-a]$

$= (b-a)(c-a)(c-b)$

$= (a-b)(b-c)(c-a)$

$$\text{Again} \quad D_1 = -\begin{vmatrix} 1 & 1 & 1 \\ d & b & c \\ d^2 & b^2 & c^2 \end{vmatrix}$$

$$c_2 \to c_2 - c_1, c_3 \to c_3 - c_1$$

$$D_1 = -\begin{vmatrix} 1 & 0 & 0 \\ d & b-d & c-d \\ d^2 & b^2-d^2 & c^2-d^2 \end{vmatrix}$$

Taking $(b-d)$ common from c_2 and $(c-d)$ from c_3

$$-(b-d)(c-d)\begin{vmatrix} 1 & 0 & 0 \\ d & 1 & 1 \\ d^2 & b+d & c+d \end{vmatrix}$$

Expanding along R_1

$= -(b-d)(c-d)\left[1(c+d-b-d)\right]$

$= -(b-d)(c-d)(c-b)$

$= -(b-c)(c-d)(d-b)$

$$\text{Again}\quad D_2 = -\begin{vmatrix} 1 & 1 & 1 \\ a & d & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$$c_2 \to c_2 - c_1, c_3 \to c_3 - c_1$$

$$= -\begin{vmatrix} 1 & 0 & 0 \\ a & d-a & c-a \\ a^2 & d^2-a^2 & c^2-a^2 \end{vmatrix}$$

Taking $(d-a)$ *common from* c_2 *and* $(c-a)$ *from* c_3

$$= -(d-a)(c-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & d+a & c+a \end{vmatrix}$$

Expanding along R_1

$$= -(d-a)(c-a)\times 1[c+a-d-a]$$
$$= -(d-a)(c-a)(c-d)$$
$$= -(a-d)(d-c)(c-a)$$

$$\textit{Also}\quad D_3 = -\begin{vmatrix} 1 & 1 & 1 \\ a & b & d \\ a^2 & b^2 & d^2 \end{vmatrix}$$

$$c_2 \to c_2 - c_1, c_3 \to c_3 - c_1$$

$$= -\begin{vmatrix} 1 & 0 & 0 \\ a & b-a & d-a \\ a^2 & b^2-a^2 & d^2-a^2 \end{vmatrix}$$

Now, taking $(b-a)$ *common from* c_2 *and* $(d-a)$ *from* c_3

$$= -(b-a)(d-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & b+a & d+a \end{vmatrix}$$

Expanding along R_1

$$= -(b-a)(d-a)\times 1[d+a-b-a]$$
$$= -(b-a)(d-a)(d-b)$$
$$= -(a-b)(b-d)(d-a)$$

$$\text{Now}\quad x = \frac{D_1}{D} = -\frac{(b-c)(c-d)(d-b)}{(a-b)(b-c)(c-a)}$$

$$y = \frac{D_2}{D} = -\frac{(a-d)(d-c)(c-a)}{(a-b)(b-c)(c-a)}$$

$$z = \frac{D_3}{D} = -\frac{(a-b)(b-d)(d-a)}{(a-b)(b-c)(c-a)}$$

Here $D = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -2 & 2 & 2 \\ 2 & 1 & -2 & 2 \\ 3 & -1 & 3 & -3 \end{vmatrix}$

$$\therefore\ D = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & -3 & 1 & 1 \\ 2 & -1 & -4 & 0 \\ 3 & -4 & 0 & -6 \end{vmatrix} = 1\begin{vmatrix} -3 & 1 & 1 \\ -1 & -4 & 0 \\ -4 & 0 & -6 \end{vmatrix} \qquad \begin{bmatrix} C2 \to C2 - C1 \\ C3 \to C3 - C1 \\ C4 \to C4 - C1 \end{bmatrix}$$

$$\therefore \qquad D = \begin{vmatrix} 0 & 0 & 1 \\ -1 & -4 & 0 \\ -22 & 6 & -6 \end{vmatrix} \qquad \begin{bmatrix} C1 \to C1 + 3C3 \\ C2 \to C2 - C3 \end{bmatrix}$$

$$= 1(-6 - 88) = -94$$

$$D_1 = \begin{vmatrix} 2 & 1 & 1 & 1 \\ -6 & -2 & 2 & 2 \\ -5 & 1 & -2 & 2 \\ -3 & -1 & 3 & -3 \end{vmatrix} = 188$$

$$D_2 = \begin{vmatrix} 1 & 2 & 1 & 1 \\ 1 & -6 & 2 & 2 \\ 2 & -5 & -2 & 2 \\ 3 & -3 & 3 & -3 \end{vmatrix} = -282$$

$$D_3 = \begin{vmatrix} 1 & 1 & 2 & 1 \\ 1 & -2 & -6 & 2 \\ 2 & 1 & -5 & 2 \\ 3 & -1 & -3 & -3 \end{vmatrix} = -141$$

$$D_4 = \begin{vmatrix} 1 & 1 & 1 & 2 \\ 1 & -2 & 2 & -6 \\ 2 & 1 & -2 & -5 \\ 3 & -1 & 3 & -3 \end{vmatrix} = 47$$

Now $x = \frac{D_1}{D} = \frac{188}{-94} = -2$

$$y = \frac{D_2}{D} = \frac{-282}{-94} = 3$$

$$z = \frac{D_3}{D} = \frac{-141}{-94} = \frac{3}{2}$$

$$w = \frac{D_4}{D} = \frac{47}{-94} = -\frac{1}{2}$$

Hence $x = -2, y = 3, z = \frac{3}{2}, w = -\frac{1}{2}$

Chapter 6 Determinants Ex 6.4 Q21

Here $D = \begin{vmatrix} 2 & 0 & -3 & 1 \\ 1 & -1 & 0 & 2 \\ 0 & -3 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$

$$\therefore D = \begin{vmatrix} 2 & -2 & -5 & 1 \\ 1 & -2 & -1 & 2 \\ 0 & -3 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{vmatrix} = -1\begin{vmatrix} -2 & -5 & 1 \\ -2 & -1 & 2 \\ -3 & 1 & 1 \end{vmatrix} \qquad \begin{bmatrix} C2 \to C2 - C1 \\ C3 \to C3 - C1 \end{bmatrix}$$

$$\therefore \quad D = -1\begin{vmatrix} 1 & -6 & 1 \\ 4 & -3 & 2 \\ 0 & 0 & 1 \end{vmatrix} \qquad \begin{bmatrix} C1 \to C1 + 3C3 \\ C2 \to C2 - C3 \end{bmatrix}$$

$$= -1(-3+24) = -21$$

$$D_1 = \begin{vmatrix} 1 & 0 & -3 & 1 \\ 1 & -1 & 0 & 2 \\ 1 & -3 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = -21$$

$$D_2 = \begin{vmatrix} 2 & 1 & -3 & 1 \\ 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = -6$$

$$D_3 = \begin{vmatrix} 2 & 0 & 1 & 1 \\ 1 & -1 & 1 & 2 \\ 0 & -3 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix} = -6$$

$$D_4 = \begin{vmatrix} 2 & 0 & -3 & 1 \\ 1 & -1 & 0 & 1 \\ 0 & -3 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{vmatrix} = 3$$

Now $x = \frac{D_1}{D} = \frac{-21}{-21} = 1$

$$y = \frac{D_2}{D} = \frac{-6}{-21} = \frac{2}{7}$$

$$z = \frac{D_3}{D} = \frac{-6}{-21} = \frac{2}{7}$$

$$w = \frac{D_4}{D} = \frac{3}{-21} = -\frac{1}{7}$$

Chapter 6 Determinants Ex 6.4 Q22

$$\text{Let } D = \begin{vmatrix} 2 & -1 \\ 4 & -2 \end{vmatrix}$$

Expanding along R_1

$= -4 + 4 = 0$

$$\text{Also } D_1 = \begin{vmatrix} 5 & -1 \\ 7 & -2 \end{vmatrix} = -3$$

$$\text{Also } D_2 = \begin{vmatrix} 2 & 5 \\ 4 & 7 \end{vmatrix} = -6$$

And since $D = 0$ and D_1 and D_2 are non-zero, hence the given system of equations is inconsistent.

Hence proved.

Chapter 6 Determinants Ex 6.4 Q23

$$D = \begin{vmatrix} 3 & 1 \\ -6 & -2 \end{vmatrix} = -6 + 6 = 0$$

$$D_1 = \begin{vmatrix} 5 & 1 \\ 9 & -2 \end{vmatrix} = -10 - 9 = -19 \neq 0$$

Since $D = 0$ but $D_1 \neq 0$

Hence the given system of equations is inconsistent.

*********** END **********