

Definite Integrals Ex 20.2 Q32 We have,

$$\int_{0}^{1} x \tan^{-1} x \, dx = \tan^{-1} x \int_{0}^{1} x \, dx - \int_{0}^{1} \left( \int x \, dx \right) \frac{d}{dx} \left( \tan^{-1} x \right) dx$$

$$= \left[ \frac{x^{2}}{2} \tan^{-1} x \right]_{0}^{1} - \frac{1}{2} \int_{0}^{1} \frac{x^{2}}{1 + x^{2}} \, dx$$

$$= \left[ \frac{x^{2}}{2} \tan^{-1} x \right]_{0}^{1} - \frac{1}{2} \int_{0}^{1} \frac{1 + x^{2} - 1}{1 + x^{2}} \, dx$$

$$= \frac{1}{2} \left( \frac{\pi}{4} \right) - \frac{1}{2} \left[ \int_{0}^{1} dx - \int_{0}^{1} \frac{dx}{1 + x^{2}} \right]$$

$$= \frac{\pi}{8} - \frac{1}{2} \left[ x - \tan^{-1} x \right]_{0}^{1}$$

$$= \frac{\pi}{8} - \frac{1}{2} \left[ 1 - \frac{\pi}{4} \right]$$

$$= \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8}$$

$$\int_{0}^{1} x \tan^{-1} x dx = \frac{\pi}{4} - \frac{1}{2}$$

Definite Integrals Ex 20.2 Q33

Let 
$$I = \int \frac{1 - x^2}{x^4 + x^2 + 1} dx = -\int \frac{x^2 - 1}{x^4 + x^2 + 1} dx$$
.

Then,

$$I = -\int \frac{1 - \frac{1}{x^2}}{x^2 + 1 + \frac{1}{x^2}} dx$$
$$1 - \frac{1}{x^2}$$

 $\begin{bmatrix} \text{Dividing the numerator and} \\ \text{denominator by } x^2 \end{bmatrix}$ 

$$\Rightarrow I = -\int \frac{1 - \frac{1}{x^2}}{\left(x + \frac{1}{x}\right)^2 - 1^2} dx$$

Let, 
$$x + \frac{1}{x} = u$$
. Then,  $d\left(x + \frac{1}{x}\right) = du \Rightarrow \left(1 - \frac{1}{x^2}\right)dx = du$ 

$$I = -\int \frac{du}{u^2 - 1^2}$$

$$\Rightarrow I = -\frac{1}{2(1)} \log \left| \frac{u-1}{u+1} \right| + C$$

$$\Rightarrow I = -\frac{1}{2} \log \left| \frac{x + \frac{1}{x} - 1}{x + \frac{1}{x} + 1} \right| + C = -\frac{1}{2} \log \left| \frac{x^2 - x + 1}{x^2 + x + 1} \right| + C$$

$$\int_{0}^{1} \frac{1 - x^{2}}{x^{4} + x^{2} + 1} dx = \left[ -\frac{1}{2} \log \left| \frac{x^{2} - x + 1}{x^{2} + x + 1} \right| \right]_{0}^{1} = \left( -\frac{1}{2} \log \left| \frac{1}{3} \right| \right) - \left( -\frac{1}{2} \log \left| 1 \right| \right) = \log \sqrt{3}$$

$$= \log 3^{\frac{1}{2}}$$

$$= \frac{1}{2} \log 3$$

Let 
$$1 + x^2 = t$$
  
Differentiating w.r.t.  $x$ , we get  
 $2xdx = dt$ 

Now, 
$$x = 0 \Rightarrow t = 1$$
  
 $x = 1 \Rightarrow t = 2$ 

$$\int_{0}^{1} \frac{24x^{3}}{(1+x^{2})^{4}} dx = \int_{1}^{2} \frac{12(t-1)}{t^{4}} dt$$

$$= 12 \int_{1}^{2} \left(\frac{1}{t^{3}} - \frac{1}{t^{4}}\right) dt$$

$$= 12 \left[ -\frac{1}{2t^{2}} - \frac{1}{3t^{3}} \right]_{1}^{2}$$

$$= 12 \left[ -\frac{1}{8} + \frac{1}{24} + \frac{1}{2} - \frac{1}{3} \right]$$

$$= 12 \left[ \frac{-3+1+12-8}{24} \right]$$

$$= \frac{12 \times 2}{24} = 1$$

$$\int_{0}^{1} \frac{24x^{3}}{\left(1+x^{2}\right)^{4}} dx = 1$$

Definite Integrals Ex 20.2 Q35

Let 
$$x - 4 = t^3$$
  
Differentiating w.r.t.  $x$ , we get  $dx = 3t^2dt$ 

Now, 
$$x = 4 \Rightarrow t = 0$$
  
 $x = 12 \Rightarrow t = 2$ 

$$\int_{4}^{12} x (x - 4)^{\frac{1}{3}} dx = \int_{0}^{2} (t^{3} + 1)t \cdot 3t^{2} dt$$

$$= 3 \int_{0}^{2} (t^{6} + 4t^{3}) dt$$

$$= 3 \left[ \frac{t^{7}}{7} + t^{4} \right]_{0}^{2}$$

$$= 3 \left[ \frac{128}{7} + 16 \right]$$

$$= \frac{720}{7}$$

$$\int_{4}^{12} x \left(x - 4\right)^{\frac{1}{3}} dx = \frac{720}{7}$$

\*\*\*\*\*\*\*\*\* END \*\*\*\*\*\*\*