Adjoint and Inverse of Matrix Ex 7.1 Q33

Let $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$

$B = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$

Then the given equation can be written as

$A \times B = I$

$\Rightarrow \quad X = A^{-1}B^{-1}$

Now $|A| = 6 - 5 = 1$

$|B| = 10 - 9 = 1$

$$A^{-1} = \frac{adj\,(A)}{|A|} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

$$B^{-1} = \frac{adj\,(B)}{|B|} = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$$

$$\therefore \quad X = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 9 & -14 \\ -16 & 25 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q34

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

Now, $A^2 + 4A - 5I$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Also, $A^2 - 4A + 5I = O$

$$A^{-1}.AA - 4A^{-1}.A - 5A^{-1}.I = 0$$

$$A - 4I - 5A^{-1} = 0$$

$$A^{-1} = \frac{1}{5}\left[A - 4I\right]$$

$$= \frac{1}{5}\left\{\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}\right\}$$

$$= \frac{1}{5}\left(\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}\right) = \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q35

$$|A \text{ adj } A| = |A|^n$$

$$\begin{aligned}
\text{LHS} &= |A \text{ Adj } A| \\
&= |A| \cdot |\text{Adj } A| \\
&= |A| \cdot |A|^{n-1} \\
&= |A|^{n-1+1} \\
&= |A|^n \\
&= \text{RHS}
\end{aligned}$$

Adjoint and Inverse of Matrix Ex 7.1 Q36

Here

$$B^{-1} = \frac{1}{|B|} adj\,|B|$$

Co-factors of B are

$$C_{11} = 3 \qquad C_{21} = 2 \qquad C_{31} = 6$$
$$C_{12} = 1 \qquad C_{22} = 1 \qquad C_{32} = 2$$
$$C_{13} = 2 \qquad C_{23} = 2 \qquad C_{33} = 5$$

Therefore,

$$adj\,B = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$
$$= \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}^T$$
$$= \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

Therefore,

$$B^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

Now,

$$(AB)^{-1} = B^{-1}A^{-1}$$
$$= \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 61 & -24 & 22 \end{bmatrix}$$

********** END **********