Real Numbers Ex 1.1 Q4

Answer :

To Show: That any positive odd integer is of the form $6q + 1$ or $6q + 3$ or $6q + 5$ where q is any some integer.

Proof: Let a be any odd positive integer and $b = 6$.

Then, there exists integers q and r such that

$a = 6q + r$, $0 \le r < 6$ (by division algorithm)

$\Rightarrow a = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$

But $6q$ or $6q + 2$ or $6q + 4$ are even positive integers.

$\boxed{\text{So, } a = 6q+1 \text{ or } 6q+3 \text{ or } 6q+5}$

Hence it is proved that any positive odd integer is of the form $6q + 1$ or $6q + 3$ or $6q + 5$, where q is any some integer.

Real Numbers Ex 1.1 Q5

Answer :

To Prove: that the square of an positive integer is of the form $3m$ or $3m + 1$ but not of the form $3m + 2$.

Proof: Since positive integer n is of the form of $3q$, $3q + 1$ and $3q + 2$

If $n = 3q$

$$\Rightarrow n^2 = (3q)^2$$

$$\Rightarrow n^2 = 9q^2$$

$$\Rightarrow n^2 = 3(3q^2)$$

$$\Rightarrow n^2 = 3m\left(m = 3q^2\right)$$

If $n = 3q + 1$

Then, $n^2 = (3q + 1)^2$

$$\Rightarrow n^2 = (3q^2) + 6q + 1$$

$$\Rightarrow n^2 = 9q^2 + 6q + 1$$

$$\Rightarrow n^2 = 3q(3q+1)+1$$

$$\boxed{\Rightarrow n^2 = 3m+1\left(\text{where } m = (3q+2)\right)}$$

If $n = 3q + 2$

Then, $n^2 = (3q + 2)^2$

$$\Rightarrow n^2 = (3q^2)+12q+4$$

$$\Rightarrow n^2 = 9q^2 + 12q + 4$$

$$\Rightarrow n^2 = 3(3q+4q+1)+1$$

$$\boxed{\Rightarrow n^2 = 3m+1\left(\text{where } q = (3q+4q+1)\right)}$$

Hence n^2 integer is of the form $3m$, $3m + 1$ but not of the form $3m + 2$.

Real Numbers Ex 1.1 Q6

Answer :

To Prove: that the square of any positive integer is of the form $4q$ or $4q+1$ for some integer q.

Proof: Since positive integer n is of the form of $2q$ or $2q+1$

If $n = 2q$

Then, $n^2 = (2q)^2$

$\Rightarrow \quad n^2 = 4q^2$

$\Rightarrow \quad n^2 = 4m$ (where $m = q^2$)

If $n = 2q + 1$

Then, $n^2 = (2q+1)^2$

$\Rightarrow n^2 = (2q)^2 + 4q + 1$

$\Rightarrow n^2 = 4q^2 + 4q + 1$

$\Rightarrow n^2 = 4q(q+1)+1$

$\boxed{\Rightarrow n^2 = 4q+1}$ (where $m = q(q+1)$)

Hence it is proved that the square of any positive integer is of the form $4q$ or $4q+1$, for some integer q.

*********** END **********