coordinates, (2, −1, 2). The point is (−1, −5, −10).

The distance d between the points, (2, −1, 2) and (−1, −5, −10), is

$$d=\sqrt{(-1-2)^2+(-5+1)^2+(-10-2)^2}=\sqrt{9+16+144}=\sqrt{169}=13$$

Question 19:

Find the vector equation of the line passing through (1, 2, 3) and parallel to the planes $\vec{r}=\left(\hat{i}-\hat{j}+2\hat{k}\right)=5$ and $\vec{r}\cdot\left(3\hat{i}+\hat{j}+\hat{k}\right)=6$.

Answer

Let the required line be parallel to vector $\vec{b}$ given by,

$\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

The position vector of the point (1, 2, 3) is $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$

The equation of line passing through (1, 2, 3) and parallel to $\vec{b}$ is given by,

$\vec{r}=\vec{a}+\lambda\vec{b}$

$\Rightarrow\vec{r}\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(b_1\hat{i}+b_2\hat{j}+b_3\hat{k}\right)$ ...(1)

The equations of the given planes are

$\vec{r}\cdot\left(\hat{i}-\hat{j}+2\hat{k}\right)=5$ ...(2)

$\vec{r}\cdot\left(3\hat{i}+\hat{j}+\hat{k}\right)=6$ ...(3)

The line in equation (1) and plane in equation (2) are parallel. Therefore, the normal to the plane of equation (2) and the given line are perpendicular.

$\Rightarrow\left(\hat{i}-\hat{j}+2\hat{k}\right)\cdot\lambda\left(b_1\hat{i}+b_2\hat{j}+b_3\hat{k}\right)=0$

$\Rightarrow\lambda\left(b_1-b_2+2b_3\right)=0$

$\Rightarrow b_1-b_2+2b_3=0$ ...(4)

Similarly, $\left(3\hat{i}+\hat{j}+\hat{k}\right)\cdot\lambda\left(b_1\hat{i}+b_2\hat{j}+b_3\hat{k}\right)=0$

$\Rightarrow\lambda\left(3b_1+b_2+b_3\right)=0$

$\Rightarrow 3b_1+b_2+b_3=0$ ...(5)

From equations (4) and (5), we obtain

$$\frac{b_1}{(-1)\times1-1\times2}=\frac{b_2}{2\times3-1\times1}=\frac{b_3}{1\times1-3(-1)}$$

$$\Rightarrow\frac{b_1}{-3}=\frac{b_2}{5}=\frac{b_3}{4}$$

Therefore, the direction ratios of $\vec{b}$ are −3, 5, and 4.

$\therefore\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}=-3\hat{i}+5\hat{j}+4\hat{k}$

Substituting the value of $\vec{b}$ in equation (1), we obtain

$\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(-3\hat{i}+5\hat{j}+4\hat{k}\right)$

This is the equation of the required line.

Question 20:

Find the vector equation of the line passing through the point (1, 2, − 4) and perpendicular to the two lines: $\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$

Answer

Let the required line be parallel to the vector $\vec{b}$ given by, $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

The position vector of the point (1, 2, − 4) is $\vec{a}=\hat{i}+2\hat{j}-4\hat{k}$

The equation of the line passing through (1, 2, −4) and parallel to vector $\vec{b}$ is

$\vec{r}=\vec{a}+\lambda\vec{b}$

$\Rightarrow\vec{r}\left(\hat{i}+2\hat{j}-4\hat{k}\right)+\lambda\left(b_1\hat{i}+b_2\hat{j}+b_3\hat{k}\right)$ ...(1)

The equations of the lines are

$\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$ ...(2)

$\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$ ...(3)

Line (1) and line (2) are perpendicular to each other.

$\therefore 3b_1 - 16b_2 + 7b_3 = 0 \qquad ...(4)$

Also, line (1) and line (3) are perpendicular to each other.

$\therefore 3b_1 + 8b_2 - 5b_3 = 0 \qquad ...(5)$

From equations (4) and (5), we obtain

$$\frac{b_1}{(-16)(-5)-8\times 7} = \frac{b_2}{7\times 3-3(-5)} = \frac{b_3}{3\times 8-3(-16)}$$

$$\Rightarrow \frac{b_1}{24} = \frac{b_2}{36} = \frac{b_3}{72}$$

$$\Rightarrow \frac{b_1}{2} = \frac{b_2}{3} = \frac{b_3}{6}$$

$\therefore$Direction ratios of $\vec{b}$ are 2, 3, and 6.

$\therefore \vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

Substituting $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ in equation (1), we obtain

$\vec{r} = \left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$

This is the equation of the required line.

Question 21:

Prove that if a plane has the intercepts *a*, *b*, *c* and is at a distance of *P* units from the origin, then $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$

Answer

The equation of a plane having intercepts *a*, *b*, *c* with *x*, *y*, and *z* axes respectively is given by,

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \qquad ...(1)$

The distance (*p*) of the plane from the origin is given by,

$$p = \left| \frac{\frac{0}{a} + \frac{0}{b} + \frac{0}{c} - 1}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2}} \right|$$

$$\Rightarrow p = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}$$

$$\Rightarrow p^2 = \frac{1}{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}$$

$$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$$

Question 22:

Distance between the two planes: $2x + 3y + 4z = 4$ and $4x + 6y + 8z = 12$ is

(A)2 units (B)4 units (C)8 units

(D) $\frac{2}{\sqrt{29}}$ units

Answer

The equations of the planes are

$2x + 3y + 4z = 4 \qquad ...(1)$

$4x + 6y + 8z = 12$

$\Rightarrow 2x + 3y + 4z = 6 \qquad ...(2)$

It can be seen that the given planes are parallel.

It is known that the distance between two parallel planes, *ax* + *by* + *cz* = d_1 and *ax* + *by* + *cz* = d_2, is given by,

$$D = \left| \frac{d_2 - d_1}{\sqrt{a^2 + b^2 + c^2}} \right|$$

$$\Rightarrow D = \left| \frac{6-4}{\sqrt{(2)^2 + (3)^2 + (4)^2}} \right|$$

$$D = \frac{2}{\sqrt{29}}$$

Thus, the distance between the lines is $\frac{2}{\sqrt{29}}$ units.

Hence, the correct answer is D.

Question 23:

The planes: $2x - y + 4z = 5$ and $5x - 2.5y + 10z = 6$ are

(A) Perpendicular (B) Parallel (C) intersect y-axis

(C) passes through $\left(0,0,\frac{5}{4}\right)$

Answer

The equations of the planes are

$2x - y + 4z = 5$ ... (1)

$5x - 2.5y + 10z = 6$ ... (2)

It can be seen that,

$$\frac{a_1}{a_2} = \frac{2}{5}$$

$$\frac{b_1}{b_2} = \frac{-1}{-2.5} = \frac{2}{5}$$

$$\frac{c_1}{c_2} = \frac{4}{10} = \frac{2}{5}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore, the given planes are parallel.
Hence, the correct answer is B.

*********** END **********