Co-Ordinate Geometry Ex 14.2 Q24

Answer :

The distance *d* between two points (x_1, y_1) and (x_2, y_2) is given by the formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

It is said that *P*(0,2) is equidistant from both *A*(3,*k*) and *B*(*k*,5).

So, using the distance formula for both these pairs of points we have

$$AP = \sqrt{(3)^2 + (k-2)^2}$$

$$BP = \sqrt{(k)^2 + (3)^2}$$

Now since both these distances are given to be the same, let us equate both.

$$AP = BP$$

$$\sqrt{(3)^2 + (k-2)^2} = \sqrt{(k)^2 + (3)^2}$$

Squaring on both sides we have,

$$(3)^2 + (k-2)^2 = (k)^2 + (3)^2$$

$$9 + k^2 + 4 - 4k = k^2 + 9$$

$$4k = 4$$

$$k = 1$$

Hence the value of '*k*' for which the point '*P*' is equidistant from the other two given points is $\boxed{k = 1}$.

Co-Ordinate Geometry Ex 14.2 Q25

Answer :

The distance *d* between two points (x_1, y_1) and (x_2, y_2) is given by the formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

In a square all the sides are of equal length. The diagonals are also equal to each other. Also in a square the diagonal is equal to $\sqrt{2}$ times the side of the square.

Here let the two points which are said to be the opposite vertices of a diagonal of a square be *A*(5,4) and *C*(1,−6).

Let us find the distance between them which is the length of the diagonal of the square.

$$AC = \sqrt{(5-1)^2 + (4+6)^2}$$

$$= \sqrt{(4)^2 + (10)^2}$$

$$= \sqrt{16 + 100}$$

$$AC = 2\sqrt{29}$$

Now we know that in a square,

$$\text{Side of the square} = \frac{\text{Diagonal of the square}}{\sqrt{2}}$$

Substituting the value of the diagonal we found out earlier in this equation we have,

$$\text{Side of the square} = \frac{2\sqrt{29}}{\sqrt{2}}$$

$$\text{Side of the square} = \sqrt{58}$$

Now, a vertex of a square has to be at equal distances from each of its adjacent vertices.
Let $P(x, y)$ represent another vertex of the same square adjacent to both 'A' and 'C'.

$$AP=\sqrt{(5-x)^2+(4-y)^2}$$
$$CP=\sqrt{(1-x)^2+(-6-y)^2}$$

But these two are nothing but the sides of the square and need to be equal to each other.

$$AP=CP$$
$$\sqrt{(5-x)^2+(4-y)^2}=\sqrt{(1-x)^2+(-6-y)^2}$$

Squaring on both sides we have,

$$(5-x)^2+(4-y)^2=(1-x)^2+(-6-y)^2$$
$$25+x^2-10x+16+y^2-8y=1+x^2-2x+36+y^2+12y$$
$$8x+20y=4$$
$$2x+5y=1$$

From this we have, $x=\dfrac{1-5y}{2}$

Substituting this value of 'x' and the length of the side in the equation for 'AP' we have,

$$AP=\sqrt{(5-x)^2+(4-y)^2}$$
$$\sqrt{58}=\sqrt{(5-x)^2+(4-y)^2}$$

Squaring on both sides,

$$58=(5-x)^2+(4-y)^2$$
$$58=\left(5-\left(\frac{1-5y}{2}\right)\right)^2+(4-y)^2$$
$$58=\left(\frac{9+5y}{2}\right)^2+(4-y)^2$$
$$58=\frac{81+25y^2+90y}{4}+16+y^2-8y$$
$$232=81+25y^2+90y+64+4y^2-32y$$
$$87=29y^2+58y$$

We have a quadratic equation. Solving for the roots of the equation we have,

$$29y^2+58y-87=0$$
$$29y^2+87y-29y-87=0$$
$$29y(y+3)-29(y+3)=0$$
$$(y+3)(29y-29)=0$$
$$(y+3)(y-1)=0$$

The roots of this equation are −3 and 1.
Now we can find the respective values of 'x' by substituting the two values of 'y'

When $y=-3$

$$x=\frac{1-5(-3)}{2}$$
$$=\frac{1+15}{2}$$
$$x=8$$

When $y=1$

$$x=\frac{1-5(1)}{2}$$
$$=\frac{1-5}{2}$$
$$x=-2$$

Therefore the other two vertices of the square are $\boxed{(8,-3)\text{ and }(-2,1)}$.

*********** END **********