



Trigonometric Ratios Ex 5.1 Q21

Answer :

Given: $\cos \theta = \frac{3}{5}$ (1)

To find the value of $\frac{\sin \theta - \frac{1}{\tan \theta}}{2 \tan \theta}$

Now, we know the following trigonometric identity

$$\cos^2 \theta + \sin^2 \theta = 1$$

Therefore, by substituting the value of $\cos \theta$ from equation (1) ,

We get,

$$\left(\frac{3}{5}\right)^2 + \sin^2 \theta = 1$$

Therefore,

$$\sin^2 \theta = 1 - \left(\frac{3}{5}\right)^2$$

$$= 1 - \frac{(3)^2}{(5)^2}$$

$$= 1 - \frac{9}{25}$$

$$\sin^2 \theta = \frac{25-9}{25}$$

$$= \frac{16}{25}$$

Therefore by taking square root on both sides

We get,

$$\begin{aligned}\sin \theta &= \sqrt{\frac{16}{25}} \\ &= \frac{\sqrt{16}}{\sqrt{25}} \\ &= \frac{4}{5}\end{aligned}$$

Therefore,

$$\sin \theta = \frac{4}{5} \text{ (2)}$$

Now, we know that

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

Therefore by substituting the value of $\sin \theta$ and $\cos \theta$ from equation (2) and (1) respectively

We get,

$$\tan \theta = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3} \text{ (4)}$$

Now, by substituting the value of $\sin \theta$ and $\tan \theta$ from equation (2) and (4) respectively in the expression below

$$\frac{\sin \theta - \frac{1}{\tan \theta}}{2 \tan \theta}$$

We get,

$$\begin{aligned} \frac{\sin \theta - \frac{1}{\tan \theta}}{2 \tan \theta} &= \frac{\frac{4}{5} - \frac{1}{4}}{2 \times \frac{4}{3}} \\ &= \frac{\frac{4}{5} - \frac{1}{4}}{\frac{8}{3}} \\ &= \frac{\frac{4 \times 4}{5 \times 4} - \frac{3 \times 5}{4 \times 5}}{\frac{8}{3}} \end{aligned}$$

Therefore,

$$\frac{\sin \theta - \frac{1}{\tan \theta}}{2 \tan \theta} = \frac{\frac{16}{20} - \frac{15}{20}}{\frac{8}{3}}$$

$$\begin{aligned} &= \frac{\frac{1}{20}}{\frac{8}{3}} \\ &= \frac{3}{160} \end{aligned}$$

$$\text{Therefore, } \frac{\sin \theta - \frac{1}{\tan \theta}}{2 \tan \theta} = \frac{3}{160}$$

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