Some Applications of Trigonometry Ex 12.1 Q52

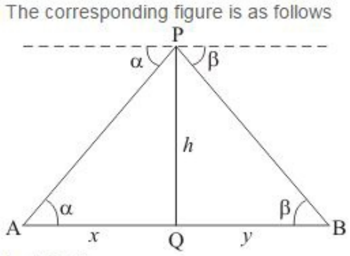
**Answer :**

Let $h$ be the height of aero plane $P$ above the road. And $A$ and $B$ be the two consecutive milestone, then $AB = 1$ mile. We have $\angle PAQ = \alpha$ and $\angle PBQ = \beta$.

We have to prove that

$$h = \frac{\tan\alpha \tan\beta}{\tan\alpha + \tan\beta}$$

The corresponding figure is as follows




In $\Delta PAQ$

$$\Rightarrow \quad \tan\alpha = \frac{PQ}{AQ}$$

$$\Rightarrow \quad \tan\alpha = \frac{h}{x}$$

$$\Rightarrow \quad x = \frac{h}{\tan\alpha}$$

$$\Rightarrow \quad x = h\cot\alpha$$

Again in $\Delta PBQ$

$$\Rightarrow \quad \tan\beta = \frac{PQ}{BQ}$$

$$\Rightarrow \quad \tan\beta = \frac{h}{y}$$

$$\Rightarrow \quad y = \frac{h}{\tan\beta}$$

$$\Rightarrow \quad y = h\cot\beta$$

Now,

$$\Rightarrow \quad AB = x + y$$

$$\Rightarrow \quad AB = h(\cot\alpha + \cot\beta)$$

$$\Rightarrow \quad AB = h\left(\frac{1}{\tan\alpha} + \frac{1}{\tan\beta}\right)$$

$$\Rightarrow \quad AB = h\left(\frac{\tan\alpha + \tan\beta}{\tan\alpha\tan\beta}\right)$$

Therefore $h = \dfrac{\tan\alpha\tan\beta}{\tan\alpha + \tan\beta}$ (since $AB = 1$)

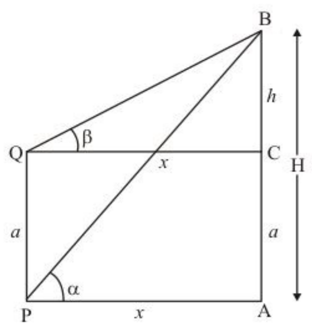
Hence height of aero plane is $\boxed{\dfrac{\tan\alpha\tan\beta}{\tan\alpha + \tan\beta}}$

Some Applications of Trigonometry Ex 12.1 Q53

**Answer :**

Let AB be the tower of height $H$ and PQ is a given post of height $a$ , $\alpha$ and $\beta$ are angles of elevation of top of tower $AB$ from $P$ and $Q$. Let PA = $x$. $PQ = a$ and BC = $h$.

The corresponding figure is as follows




In $\Delta QCB$,

$$\Rightarrow \quad \tan\beta = \frac{h}{x}$$

$$\Rightarrow \quad x = \frac{h}{\tan\beta}$$

Again in $\Delta$PAB,

$$\Rightarrow \qquad \tan\alpha = \frac{h+a}{x}$$

$$\Rightarrow \qquad \tan\alpha = \frac{(h+a)\tan\beta}{h}$$

$$\Rightarrow \qquad h\tan\alpha = (h+a)\tan\beta$$

$$\Rightarrow h(\tan\alpha - \tan\beta) = a\tan\beta$$

$$\Rightarrow \qquad h = \frac{a\tan\beta}{\tan\alpha - \tan\beta}$$

Now

$$\Rightarrow \qquad x = \frac{a\tan\beta}{(\tan\alpha - \tan\beta)\times\tan\beta}$$

$$\Rightarrow \qquad x = \frac{a}{\tan\alpha - \tan\beta}$$

$$\Rightarrow \qquad H = a + \frac{a\tan\beta}{\tan\alpha - \tan\beta}$$

$$\Rightarrow \qquad H = \frac{a\tan\alpha}{\tan\alpha - \tan\beta}$$

Hence required height is $\boxed{\dfrac{a\tan\alpha}{\tan\alpha - \tan\beta}}$. And distance is $\boxed{\dfrac{a}{\tan\alpha - \tan\beta}}$

Some Applications of Trigonometry Ex 12.1 Q54

**Answer :**

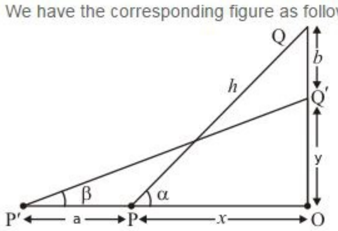
Let $PQ$ be the ladder such that its top $Q$ is on the wall $OQ$ and bottom $P$ is on the ground. The ladder is pulled away from the wall through a distance a, so that its top $Q$ slides and takes position $Q'$. So $PQ = P'Q'$

$\angle OPQ = \alpha$ And $\angle OP'Q' = \beta$. Let $PQ = h$

We have to prove that

$$\frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$$

We have the corresponding figure as follows




We use trigonometric ratios.

In $\Delta POQ$

$$\Rightarrow \qquad \sin\alpha = \frac{OQ}{PQ}$$

$$\Rightarrow \qquad \sin\alpha = \frac{b+y}{h}$$

And

$$\Rightarrow \qquad \cos\alpha = \frac{OP}{PQ}$$

$$\Rightarrow \qquad \cos\alpha = \frac{x}{h}$$

Again in $\Delta P'OQ'$

$$\Rightarrow \qquad \sin\beta = \frac{OQ'}{P'Q'}$$

$$\Rightarrow \qquad \sin\beta = \frac{y}{h}$$

And

$$\Rightarrow \qquad \cos\beta = \frac{OP'}{P'Q'}$$

$$\Rightarrow \qquad \cos\beta = \frac{a+x}{h}$$

Now,

$$\Rightarrow \sin\alpha - \sin\beta = \frac{b+y}{h} - \frac{y}{h}$$

$$\Rightarrow \sin\alpha - \sin\beta = \frac{b}{h}$$

And

$$\Rightarrow \cos\beta - \cos\alpha = \frac{a+x}{h} - \frac{x}{h}$$

$$\Rightarrow \cos\beta - \cos\alpha = \frac{a}{h}$$

So

$$\Rightarrow \frac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \frac{b}{a}$$

$$\Rightarrow \qquad \frac{a}{b} = \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$$

$$\Rightarrow \qquad \frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$$

Hence $\boxed{\dfrac{a}{b} = \dfrac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}}$.

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