Higher Order Derivatives Ex 12.1 Q45

$x=a\ (\cos t\ +t\sin t)$

$\frac{dx}{dt}=-a\sin t+at\cos t+a\sin t$

$=at\cos t$

$\frac{d^2x}{dt^2}=-at\sin t+a\cos t$

$y=a(\sin t-t\cos t)$

$\frac{dy}{dt}=a\cos t-a\cos t+at\sin t$

$=at\sin t$

$\frac{d^2y}{dt^2}=at\cos t+a\sin t$

$\frac{d^2y}{dx^2}=\frac{\frac{dx}{dt}\frac{d^2y}{dt^2}-\frac{dy}{dt}\frac{d^2x}{dt^2}}{\left(\frac{dx}{dt}\right)^3}$

$=\frac{at\cos t(at\cos t+a\sin t)-at\sin t(-at\sin t+a\cos t)}{(at\cos t)^3}$

$=\frac{a^2t^2\cos^2 t+a^2t\cos t\sin t+a^2t^2\sin^2 t-a^2t\sin t\cos t}{(at\cos t)^3}$

$=\frac{a^2t^2}{a^3t^3\cos^3 t}=\frac{1}{at\cos^3 t}$

$\left.\frac{d^2y}{dx^2}\right|_{t=\frac{\pi}{4}}=\frac{1}{a\times\frac{\pi}{4}\cos^3\frac{\pi}{4}}=\frac{8\sqrt{2}}{\pi a}$

Higher Order Derivatives Ex 12.1 Q46

$$x=a\left(\cos t+\log\tan\frac{t}{2}\right) \text{ and } y=a\sin t$$

$$\frac{dx}{dt}=-a\sin t+a\frac{1}{\tan\frac{t}{2}}\times\sec^2\frac{t}{2}\times\frac{1}{2}$$

$$=-a\sin t+a\frac{1}{2\sin\frac{t}{2}\cos\frac{t}{2}}$$

$$=-a\sin t+a\,\text{cosec}\,t$$

$$\frac{d^2x}{dt^2}=-a\cos t-a\,\text{cosec}\,t\cot t$$

$$\frac{dy}{dt}=a\cos t$$

$$\frac{d^2y}{dt^2}=-a\sin t$$

$$\frac{d^2y}{dx^2}=\frac{\frac{dx}{dt}\frac{d^2y}{dt^2}-\frac{dy}{dt}\frac{d^2x}{dt^2}}{\left(\frac{dx}{dt}\right)^3}$$

$$=\frac{(-a\sin t+a\,\text{cosec}\,t)(-a\sin t)-(a\cos t)(-a\cos t-a\,\text{cosec}\,t\cot t)}{(-a\sin t+a\,\text{cosec}\,t)^3}$$

$$=\frac{a^2\sin^2t+a^2\cos^2t-a^2+a^2\cot^2t}{\left(-a\sin t+\frac{a}{\sin t}\right)^3}$$

$$=\frac{a^2\cot^2t}{a^3\cos^6t}\times\sin^3t=\frac{1}{a}\times\frac{\sin t}{\cos^4t}$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=\frac{\pi}{3}}=\frac{1}{a}\times\frac{\sin\frac{\pi}{3}}{\cos^4\frac{\pi}{3}}=\frac{8\sqrt{3}}{a}$$

Higher Order Derivatives Ex 12.1 Q47

$$x = a\left(\cos 2t + 2t\sin 2t\right)$$

$$\frac{dx}{dt} = -2a\sin 2t + 2a\sin 2t + 4at\cos 2t = 4at\cos 2t$$

$$y = a\left(\sin 2t - 2t\cos 2t\right)$$

$$\frac{dy}{dt} = 2a\cos 2t - 2a\cos 2t + 4at\sin 2t = 4at\sin 2t$$

$$\frac{dy}{dx} = \tan 2t$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(\tan 2t)$$

$$\frac{d^2y}{dx^2} = \sec^2 2t\,\frac{d}{dx}(2t)$$

$$\frac{d^2y}{dx^2} = 2\sec^2 2t\,\frac{d}{dx}(t)$$

$$\frac{d^2y}{dx^2} = 2\sec^2 2t \times \frac{1}{4at\cos 2t}$$

$$\frac{d^2y}{dx^2} = \frac{1}{2a}\sec^3 2t$$

********** END **********