



## EXERCISE.16.2

### Question-1

A die is rolled. Let E be the event "die shows 4" and F be the event "die shows even number". Are E and F mutually exclusive?

Ans.

When a die is rolled, the sample space is given by

$$S = \{1, 2, 3, 4, 5, 6\}$$

Accordingly,  $E = \{4\}$  and  $F = \{2, 4, 6\}$

It is observed that  $E \cap F = \{4\} \neq \emptyset$

Therefore, E and F are not mutually exclusive events.

### Question-2

A die is thrown. Describe the following events:

(i) A: a number less than 7 (ii) B: a number greater than 7 (iii) C: a multiple of 3

(iv) D: a number less than 4 (v) E: an even number greater than 4 (vi) F: a number not less than 3

Also find  $A \cup B, A \cap B, B \cup C, E \cap F, D \cap E, A - C, D - E, E \cap F', F'$

Ans.

When a die is thrown, the sample space is given by  $S = \{1, 2, 3, 4, 5, 6\}$ .

Accordingly:

(i)  $A = \{1, 2, 3, 4, 5, 6\}$

(ii)  $B = \emptyset$

(iii)  $C = \{3, 6\}$

(iv)  $D = \{1, 2, 3\}$

(v)  $E = \{6\}$

(vi)  $F = \{3, 4, 5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$ ,  $A \cap B = \emptyset$

$B \cup C = \{3, 6\}$ ,  $E \cap F = \{6\}$

$D \cap E = \emptyset$ ,  $A - C = \{1, 2, 4, 5\}$

$D - E = \{1, 2, 3\}$ ,  $F' = \{1, 2\}$ ,  $E \cap F' = \emptyset$

### Question-3

An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events:

A: the sum is greater than 8, B: 2 occurs on either die

C: The sum is at least 7 and a multiple of 3.

Which pairs of these events are mutually exclusive?

Ans.

When a pair of dice is rolled, the sample space is given by

$$S = \{(x, y) : x, y = 1, 2, 3, 4, 5, 6\}$$

$$= \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{array} \right\}$$

Accordingly,

$$A = \{(3,6), (4,5), (4,6), (5,4), (5,5), (5,6), (6,3), (6,4), (6,5), (6,6)\}$$

$$B = \{(2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (1,2), (3,2), (4,2), (5,2), (6,2)\}$$

$$C = \{(3,6), (4,5), (5,4), (6,3), (6,6)\}$$

It is observed that

$$A \cap B = \Phi$$

$$B \cap C = \Phi$$

$$C \cap A = \{(3,6), (4,5), (5,4), (6,3), (6,6)\} \neq \phi$$

Hence, events A and B and events B and C are mutually exclusive

#### Question-4

Three coins are tossed once. Let A denote the event 'three heads show', B denote the event 'two heads and one tail show', C denote the event 'three tails show' and D denote the event 'a head shows on the first coin'. Which events are

- (i) mutually exclusive? (ii) simple? (iii) compound?

Ans.

When three coins are tossed, the sample space is given by

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Accordingly,

$$A = \{HHH\}$$

$$B = \{HHT, HTH, THH\}$$

$$C = \{TTT\}$$

$$D = \{HHH, HHT, HTH, HTT\}$$

We now observe that

$$A \cap B = \Phi, A \cap C = \Phi, A \cap D = \{HHH\} \neq \Phi$$

$$B \cap C = \Phi, B \cap D = \{HHT, \{HTH\} \neq \Phi$$

$$C \cap D = \Phi$$

(i) Event A and B; event A and C; event B and C; and event C and D are all mutually exclusive.

(ii) If an event has only one sample point of a sample space, it is called a simple event. Thus, A and C are simple events.

(iii) If an event has more than one sample point of a sample space, it is called a compound event. Thus, B and D are compound events.

### Question-5

Three coins are tossed. Describe

- (i) Two events which are mutually exclusive.
- (ii) Three events which are mutually exclusive and exhaustive.
- (iii) Two events, which are not mutually exclusive.
- (iv) Two events which are mutually exclusive but not exhaustive.
- (v) Three events which are mutually exclusive but not exhaustive.

Ans.

When three coins are tossed, the sample space is given by

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

(i) Two events that are mutually exclusive can be

A: getting no heads and B: getting no tails

This is because sets  $A = \{TTT\}$  and  $B = \{HHH\}$  are disjoint.

(ii) Three events that are mutually exclusive and exhaustive can be

A: getting no heads

B: getting exactly one head

C: getting at least two heads

i.e.,

$$A = \{TTT\}$$

$$B = \{HTT, THT, TTH\}$$

$$C = \{HHH, HHT, HTH, THH\}$$

This is because  $A \cap B = B \cap C = C \cap A = \Phi$  and  $A \cup B \cup C = S$

(iii) Two events that are not mutually exclusive can be

A: getting three heads

B: getting at least 2 heads

i.e.,

$$A = \{HHH\}$$

$$B = \{HHH, HHT, HTH, THH\}$$

This is because  $A \cap B = \{HHH\} \neq \Phi$

(iv) Two events which are mutually exclusive but not exhaustive can be

A: getting exactly one head

B: getting exactly one tail

That is

$$A = \{HTT, THT, TTH\}$$

$$B = \{HHT, HTH, THH\}$$

It is because,  $A \cap B = \Phi$ , but  $A \cup B \neq S$

(v) Three events that are mutually exclusive but not exhaustive can be

A: getting exactly three heads

B: getting one head and two tails

C: getting one tail and two heads

i.e.,

$$A = \{HHH\}$$

$$B = \{HTT, THT, TTH\}$$

$$C = \{HHT, HTH, THH\}$$

This is because  $A \cap B = B \cap C = C \cap A = \Phi$ , but  $A \cup B \cup C \neq S$

#### Question-6

Two dice are thrown. The events A, B and C are as follows:

A: getting an even number on the first die.

B: getting an odd number on the first die.

C: getting the sum of the numbers on the dice  $\leq 5$

Describe the events

(i)  $A'$  (ii) not B (iii) A or B

(iv) A and B (v) A but not C (vi) B or C

(vii) B and C (viii)  $A \cap B' \cap C'$

Ans.

When two dice are thrown, the sample space is given by

$$S = \{(x, y) : x, y = 1, 2, 3, 4, 5, 6\}$$

$$= \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{array} \right\}$$

Accordingly,

$$A = \left\{ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \right\}$$

$$B = \left\{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \right\}$$

$$C = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1)\}$$

$$(i) \quad A' = \left\{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \right\} = B$$

$$(ii) \quad \text{Not } B = B' = \left\{ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \right\} = A$$

(iii)

$$A \text{ or } B = A \cup B = \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{array} \right\} = S$$

$$(iv) \quad A \text{ and } B = A \cap B = \phi$$

$$(v) \quad A \text{ but not } C = A - C$$

$$= \left\{ (2,4), (2,5), (2,6), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \right\}$$

$$(vi) \quad B \text{ or } C = B \cup C$$

$$= \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), \\ (2,3), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), \\ (4,1), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \end{array} \right\}$$

$$(vii) \quad B \text{ and } C = B \cap C = \{(1,1), (1,2), (1,3), (1,4), (3,1), (3,2)\}$$

(viii)

$$C' = \left\{ \begin{array}{l} (1,5), (1,6), (2,4), (2,5), (2,6), (3,3), (3,4), (3,5), (3,6), (4,2), \\ (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{array} \right\}$$

$$\therefore A \cap B' \cap C' = A \cap A \cap C' = A \cap C'$$

$$= \left\{ (2,4), (2,5), (2,6), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \right\}$$

Question-7

Two dice are thrown. The events A, B and C are as follows:

A: getting an even number on the first die.

B: getting an odd number on the first die.

C: getting the sum of the numbers on the dice  $\leq 5$

State true or false: (give reason for your answer)

(i) A and B are mutually exclusive

(ii) A and B are mutually exclusive and exhaustive

(iii)  $A = B'$

(iv) A and C are mutually exclusive

(v) A and  $B'$  are mutually exclusive

(vi)  $A', B', C$  are mutually exclusive and exhaustive.

Ans.

$$A = \left\{ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), \right. \\ \left. (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \right\}$$
$$B = \left\{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (3,1), (3,2), (3,3), \right. \\ \left. (3,4), (3,5), (3,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \right\}$$
$$C = \left\{ (1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1) \right\}$$

(i) It is observed that  $A \cap B = \emptyset$

A and B are mutually exclusive.

Thus, the given statement is true.

(ii) It is observed that  $A \cap B = \emptyset$  and  $A \cup B = S$

A and B are mutually exclusive and exhaustive.

Thus, the given statement is true.

(iii) It is observed that

$$B' = \left\{ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), \right. \\ \left. (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \right\} = A$$

Thus, the given statement is true.

(iv) It is observed that  $A \cap C = \{(2, 1), (2, 2), (2, 3), (4, 1)\} \neq \emptyset$

A and C are not mutually exclusive.

Thus, the given statement is false.

(v)  $A \cap B' = A \cap A = A$

$\therefore A \cap B' \neq \emptyset$

A and  $B'$  are not mutually exclusive.

Thus, the given statement is false.

(vi) It is observed that  $A' \cup B' \cup C = S$ ;

However,  $B' \cap C = \{(2, 1), (2, 2), (2, 3), (4, 1)\} \neq \emptyset$

Therefore, events  $A', B'$  and C are not mutually exclusive and exhaustive.

Thus, the given statement is false.

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