Trigonometric Identities Ex 6.1 Q66

Answer :

We have to prove $\sec^4 A(1-\sin^4 A) - 2\tan^2 A = 1$

We know that, $\sin^2 A + \cos^2 A = 1$

So,

$$\sec^4 A(1-\sin^4 A) - 2\tan^2 A = \frac{1}{\cos^4 A}(1-\sin^4 A) - 2\frac{\sin^2 A}{\cos^2 A}$$

$$= \left(\frac{1}{\cos^4 A} - \frac{\sin^4 A}{\cos^4 A}\right) - 2\frac{\sin^2 A}{\cos^2 A}$$

$$= \left(\frac{1-\sin^4 A}{\cos^4 A}\right) - 2\frac{\sin^2 A}{\cos^2 A}$$

$$= \frac{(1-\sin^2 A)(1+\sin^2 A)}{\cos^4 A} - 2\frac{\sin^2 A}{\cos^2 A}$$

$$= \frac{\cos^2 A(1+\sin^2 A)}{\cos^4 A} - 2\frac{\sin^2 A}{\cos^2 A}$$

$$= \frac{1+\sin^2 A}{\cos^2 A} - 2\frac{\sin^2 A}{\cos^2 A}$$

$$= \frac{1+\sin^2 A - 2\sin^2 A}{\cos^2 A}$$

$$= \frac{1-\sin^2 A}{\cos^2 A}$$

$$= \frac{\cos^2 A}{\cos^2 A}$$

$$= 1$$

Hence proved.

Trigonometric Identities Ex 6.1 Q67

Answer :

We have to prove $\dfrac{\cot^2 A(\sec A - 1)}{1 + \sin A} = \sec^2 A\left(\dfrac{1 - \sin A}{1 + \sec A}\right)$.

We know that, $\sin^2 A + \cos^2 A = 1$

So,

$$\dfrac{\cot^2 A(\sec A - 1)}{1 + \sin A} = \sec^2 A\left(\dfrac{1 - \sin A}{1 + \sec A}\right)$$

$$= \dfrac{\dfrac{\cos^2 A}{\sin^2 A}\left(\dfrac{1}{\cos A} - 1\right)}{1 + \sin A}$$

$$= \dfrac{\dfrac{\cos^2 A}{\sin^2 A}\left(\dfrac{1 - \cos A}{\cos A}\right)}{1 + \sin A}$$

$$= \dfrac{\cos A(1 - \cos A)}{\sin^2 A(1 + \sin A)}$$

$$= \dfrac{\cos A(1 - \cos A)}{(1 - \cos^2 A)(1 + \sin A)}$$

$$= \dfrac{\cos A(1 - \cos A)}{(1 - \cos A)(1 + \cos A)(1 + \sin A)}$$

$$= \dfrac{\cos A}{(1 + \cos A)(1 + \sin A)}$$

$$= \dfrac{\dfrac{1}{\sec A}}{\left(1 + \dfrac{1}{\sec A}\right)(1 + \sin A)}$$

$$= \dfrac{\dfrac{1}{\sec A}}{\left(\dfrac{\sec A + 1}{\sec A}\right)(1 + \sin A)}$$

$$= \dfrac{1}{(\sec A + 1)(1 + \sin A)}$$

Multiplying both the numerator and denominator by $(1 - \sin A)$, we have

$$= \dfrac{(1 - \sin A)}{(\sec A + 1)(1 + \sin A)(1 - \sin A)}$$

$$= \dfrac{(1 - \sin A)}{(\sec A + 1)(1 - \sin^2 A)}$$

$$= \dfrac{(1 - \sin A)}{(\sec A + 1)\cos^2 A}$$

$$= \sec^2 A\dfrac{(1 - \sin A)}{(\sec A + 1)}$$

$$= \sec^2 A\left(\dfrac{1 - \sin A}{1 + \sec A}\right)$$

Hence proved.

********** END *********