



Trigonometric Ratios Ex 5.1 Q24

Answer :

Given:

$$\sin \theta = \frac{3}{4} \dots\dots (1)$$

To prove:

$$\sqrt{\frac{\operatorname{cosec}^2 \theta - \cot^2 \theta}{\sec^2 \theta - 1}} = \frac{\sqrt{7}}{3} \dots\dots (2)$$

By definition,

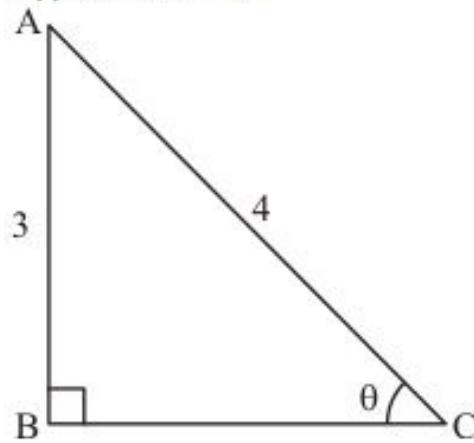
$$\sin A = \frac{\text{Perpendicular side opposite to } \angle A}{\text{Hypotenuse}} \dots\dots (3)$$

By Comparing (1) and (3)

We get,

Perpendicular side = 3 and

Hypotenuse = 4



Side BC is unknown.

So we find BC by applying Pythagoras theorem to right angled $\triangle ABC$,

Hence,

$$AC^2 = AB^2 + BC^2$$

Now we substitute the value of perpendicular side (AB) and hypotenuse (AC) and get the base side (BC)

Therefore,

$$4^2 = 3^2 + BC^2$$

$$BC^2 = 4^2 - 3^2$$

$$BC^2 = 16 - 9$$

$$BC^2 = 7$$

$$BC = \sqrt{7}$$

$$BC^2 = 4^2 - 3^2$$

$$BC^2 = 16 - 9$$

$$BC^2 = 7$$

$$BC = \sqrt{7}$$

Hence, Base side $BC = \sqrt{7}$ (3)

$$\text{Now, } \cos A = \frac{\text{Base}}{\text{Hypotenuse}}$$

Therefore from fig. a and equation (3)

$$\begin{aligned}\cos A &= \frac{BC}{AC} \\ &= \frac{\sqrt{7}}{4}\end{aligned}$$

Therefore,

$$\cos A = \frac{\sqrt{7}}{4} \text{ (4)}$$

$$\text{Now, } \operatorname{cosec} A = \frac{1}{\sin A}$$

Therefore from fig. a and equation (1) ,

$$\operatorname{cosec} A = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$$

$$\operatorname{cosec} A = \frac{4}{3} \text{ (5)}$$

$$\text{Now, } \sec A = \frac{1}{\cos A}$$

Therefore from fig. a and equation (4) ,

$$\sec A = \frac{4}{\sqrt{7}} \text{ (6)}$$

$$\text{Now, } \cot A = \frac{\cos A}{\sin A}$$

Therefore by substituting the values from equation (1) and (4) ,

We get,

$$\begin{aligned}\cot A &= \frac{\frac{\sqrt{7}}{3}}{\frac{4}{3}} \\ &= \frac{\sqrt{7}}{4} \times \frac{4}{3} \\ &= \frac{\sqrt{7}}{3}\end{aligned}$$

Therefore,

$$\cot A = \frac{\sqrt{7}}{3} \dots\dots (7)$$

Now by substituting the value of $\operatorname{cosec} A$, $\sec A$ and $\cot A$ from equation (5) ,(6) and (7) respectively in the L.H.S of expression (2) ,

We get,

$$\sqrt{\frac{\operatorname{cosec}^2 \theta - \cot^2 \theta}{\sec^2 \theta - 1}} = \sqrt{\frac{\left(\frac{4}{3}\right)^2 - \left(\frac{\sqrt{7}}{3}\right)^2}{\left(\frac{4}{\sqrt{7}}\right)^2 - 1}}$$

$$= \sqrt{\frac{\frac{(4)^2}{(3)^2} - \frac{(\sqrt{7})^2}{(3)^2}}{\frac{(4)^2}{(\sqrt{7})^2} - 1}}$$

$$= \sqrt{\frac{\frac{16}{9} - \frac{7}{9}}{\frac{16}{7} - 1}}$$

$$= \sqrt{\frac{\frac{16-7}{9}}{\frac{16-7}{7}}}$$

$$= \sqrt{\frac{\frac{9}{9}}{\frac{9}{7}}}$$

$$= \sqrt{\frac{9}{9} \times \frac{7}{9}}$$

Therefore,

$$\begin{aligned}\sqrt{\frac{\operatorname{cosec}^2 \theta - \cot^2 \theta}{\sec^2 \theta - 1}} &= \sqrt{\frac{7}{9}} \\ &= \frac{\sqrt{7}}{\sqrt{9}} \\ &= \frac{\sqrt{7}}{3}\end{aligned}$$

Hence it is proved that

$$\sqrt{\frac{\operatorname{cosec}^2 \theta - \cot^2 \theta}{\sec^2 \theta - 1}} = \frac{\sqrt{7}}{3}$$

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