Trigonometric Ratios Ex 5.1 Q35

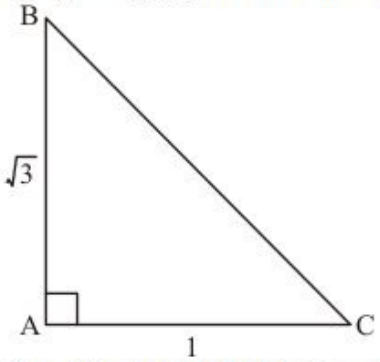
Answer :

Given:

$\tan C = \sqrt{3}$

To find: $\sin B \cos C + \cos B \sin C$

The given ΔABC is as shown in figure below



Side *BC* is unknown and can be found using Pythagoras theorem

Therefore,

$BC^2 = AB^2 + AC^2$

Now by substituting the value of known sides from figure (*a*)

We get,

$$\begin{aligned} BC^2 &= \left(\sqrt{3}\right)^2 + 1^2 \\ &= 3 + 1 \\ &= 4 \end{aligned}$$

Now by taking square root on both sides

We get,

$$BC = \sqrt{4}$$

$$= 2$$

Therefore Hypotenuse side $BC = 2$ ..... (1)

Now $\sin B = \dfrac{\text{Perpendicular side opposite to} \angle B}{\text{Hypotenuse}}$

Therefore,

$$\sin B = \frac{AC}{BC}$$

Now by substituting the values from equation (1) and figure (a)

We get,

$$\sin B = \frac{1}{2} \quad \text{..... (2)}$$

Now $\cos B = \dfrac{\text{Base side adjacent to} \angle B}{\text{Hypotenuse}}$

Therefore,

$$\cos B = \frac{AB}{BC}$$

Now by substituting the values from equation (1) and figure (a)

We get,

$$\cos B = \frac{\sqrt{3}}{2} \quad \text{..... (3)}$$

Now $\sin C = \dfrac{\text{Perpendicular side opposite to} \angle C}{\text{Hypotenuse}}$

Therefore,

$$\sin C = \frac{AB}{BC}$$

Now by substituting the values from equation (1) and figure (a)

We get,

$$\sin C = \frac{\sqrt{3}}{2} \quad \text{..... (4)}$$

Now by definition,

$$\tan C = \frac{\sin C}{\cos C}$$

Therefore,

$$\cos C = \frac{\sin C}{\tan C}$$

Now by substituting the value of $\sin C$ and $\tan C$ from equation (4) and given data respectively

We get,

$$\cos C = \frac{\frac{\sqrt{3}}{2}}{\sqrt{3}}$$

$$\cos C = \frac{\sqrt{3}}{2\sqrt{3}}$$

Now $\sqrt{3}$ gets cancelled as it is present in both numerator and denominator

Therefore,

$$\cos C = \frac{1}{2} \quad \text{..... (5)}$$

Now by substituting the value of $\sin B, \cos B, \sin C$ and $\cos C$ from equation (2) , (3) , (4) and (5) respectively in $\sin B \cos C + \cos B \sin C$

We get,

$$\sin B \cos C + \cos B \sin C = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4}$$

$$= \frac{4}{4}$$

$$= 1$$

$$\sin B \cos C + \cos B \sin C = 1$$

********** END **********