Co-Ordinate Geometry Ex 14.4 Q6

Answer :

Let $A(x_1, y_1)$; $B(x_2, y_2)$; $C(x_3, y_3)$ be the coordinates of the vertices of $\triangle ABC$.

Let us assume that centroid of the $\triangle ABC$ is at the origin G.

So, the coordinates of G are $G(0, 0)$.

Now, $\frac{x_1+x_2+x_3}{3} = 0$; $\frac{y_1+y_2+y_3}{3} = 0$

so, $x_1 + x_2 + x_3 = 0 \quad \ldots\ldots.(1)$

$y_1 + y_2 + y_3 = 0 \quad \ldots\ldots..(2)$

Squaring (1) and (2), we get

$x_1{}^2 + x_2{}^2 + x_3{}^2 + 2x_1x_2 + 2x_2x_3 + 2x_3x_1 = 0 \quad \ldots\ldots.(3)$

$y_1{}^2 + y_2{}^2 + y_3{}^2 + 2y_1y_2 + 2y_2y_3 + 2y_3y_1 = 0 \quad \ldots\ldots(4)$

$LHS = AB^2 + BC^2 + CA^2$

$= \left[\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}\right]^2 + \left[\sqrt{(x_3-x_2)^2 + (y_3-y_2)^2}\right]^2$

$+ \left[\sqrt{(x_3-x_1)^2 + (y_3-y_1)^2}\right]^2$

$= (x_2-x_1)^2 + (y_2-y_1)^2 + (x_3-x_2)^2 + (y_3-y_2)^2 + (x_3-x_1)^2 + (y_3-y_1)^2$

$= x_1{}^2 + x_2{}^2 - 2x_1x_2 + y_1{}^2 + y_2{}^2 - 2y_1y_2 + x_2{}^2 + x_3{}^2 - 2x_2x_3 + y_2{}^2 + y_3{}^2 - 2y_2y_3 + x_1{}^2$

$+ x_3{}^2 - 2x_1x_3 + y_1{}^2 + y_3{}^2 - 2y_1y_3$

$= 2(x_1{}^2 + x_2{}^2 + x_3{}^2) + 2(y_1{}^2 + y_2{}^2 + y_3{}^2) - (2x_1x_2 + 2x_2x_3 + 2x_3x_1)$

$- (2y_1y_2 + 2y_2y_3 + 2y_3y_1)$

$= 2(x_1{}^2 + x_2{}^2 + x_3{}^2) + 2(y_1{}^2 + y_2{}^2 + y_3{}^2) + (x_1{}^2 + x_2{}^2 + x_3{}^2) + (y_1{}^2 + y_2{}^2 + y_3{}^2)$

$= 3(x_1{}^2 + x_2{}^2 + x_3{}^2 + y_1{}^2 + y_2{}^2 + y_3{}^2)$

$RHS = 3\left(GA^2 + GB^2 + GC^2\right)$

$= 3\left[\left\{\sqrt{(x_1-0)^2 + (y_1-0)^2}\right\}^2 + \left\{\sqrt{(x_2-0)^2 + (y_2-0)^2}\right\}^2 + \left\{\sqrt{(x_3-0)^2 + (y_3-0)^2}\right.\right.$

$= 3(x_1{}^2 + x_2{}^2 + x_3{}^2 + y_1{}^2 + y_2{}^2 + y_3{}^2)$

Hence, $AB^2 + BC^2 + CA^2 = 3\left(GA^2 + GB^2 + GC^2\right)$

Co-Ordinate Geometry Ex 14.4 Q7

Answer :

Let ΔABC be ant triangle such that P (−2, 3); Q (4,−3) and R (4, 5) are the mid-points of the sides AB, BC, CA respectively.

We have to find the co-ordinates of the centroid of the triangle.

Let the vertices of the triangle be $A(x_1, y_1)$; $B(x_2, y_2)$; $C(x_3, y_3)$

In general to find the mid-point $P(x, y)$ of two points $A(x_1, y_1)$ and $B(x_2, y_2)$ we use section formula as,

$$P(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

So, co-ordinates of P,

$$(-2, 3) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Equate the *x* component on both the sides to get,

$x_1 + x_2 = -4 \quad \ldots\ldots (1)$

Similarly,

$y_1 + y_2 = 6 \quad \ldots\ldots (2)$

Similarly, co-ordinates of Q,

$$(4, -3) = \left(\frac{x_3 + x_2}{2}, \frac{y_3 + y_2}{2}\right)$$

Equate the *x* component on both the sides to get,

$x_3 + x_2 = 8 \quad \ldots\ldots (3)$

Similarly,

$y_3 + y_2 = -6$ …… (4)

Similarly, co-ordinates of R,

$$(4,5) = \left(\frac{x_3 + x_1}{2}, \frac{y_3 + y_1}{2}\right)$$

Equate the *x* component on both the sides to get,

$x_3 + x_1 = 8$ …… (5)

Similarly,

$y_3 + y_1 = 10$ …… (6)

Add equation (1) (3) and (5) to get,

$$2(x_1 + x_2 + x_3) = 12$$
$$x_1 + x_2 + x_3 = 6$$

Similarly, add equation (2) (4) and (6) to get,

$$2(y_1 + y_2 + y_3) = 10$$
$$y_1 + y_2 + y_3 = 5$$

We know that the co-ordinates of the centroid G of a triangle whose vertices are $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is-

$$G\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

So, centroid G of a triangle ΔABC is,

$$\boxed{G\left(2, \frac{5}{3}\right)}$$

*********** END **********