Differentiation Ex 11.8 Q16

Let $\quad u = \cos^{-1}\left(4x^3 - 3x\right)$

Put $\quad x = \cos\theta \Rightarrow \theta = \cos^{-1}x$, so

$$u = \cos^{-1}\left(4\cos^3\theta - 3\cos\theta\right)$$
$$u = \cos^{-1}(\cos 3\theta) \qquad \text{---(i)}$$

Let $\quad v = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

$$= \tan^{-1}\left(\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}\right)$$
$$= \tan^{-1}\left(\frac{\sin\theta}{\cot\theta}\right)$$
$$v = \tan^{-1}(\tan\theta) \qquad \text{---(ii)}$$

Here,

$$\frac{1}{2} < x < 1$$
$$\Rightarrow \quad \frac{1}{2} < \cos\theta < 1$$
$$\Rightarrow \quad 0 < \theta < \frac{\pi}{3}$$

So, from equation (i),

$$u = 3\theta \qquad \left[\text{Since, } \cos^{-1}(\cos\theta) = \theta, \text{ if } \theta \in [0, \pi]\right]$$
$$u = 3\cos^{-1}x$$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{-3}{\sqrt{1-x^2}} \qquad \text{---(iii)}$$

From equation (ii),

$$v = \theta \qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$$
$$v = \cos^{-1}x$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{-1}{\sqrt{1-x^2}} \qquad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \left(\frac{-3}{\sqrt{1-x^2}}\right)\left(-\frac{\sqrt{1-x^2}}{1}\right)$$

$$\frac{du}{dv} = 3$$

Differentiation Ex 11.8 Q17

Let $\quad u = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

Put $\quad x = \cos\theta \Rightarrow \theta = \sin^{-1}x$, so

$$u = \tan^{-1}\left(\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$$

$$u = \tan^{-1}(\tan\theta) \qquad \text{---(i)}$$

And,

Let $\quad v = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

$$v = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$$

$$= \sin^{-1}(2\sin\theta\cos\theta)$$

$$v = \sin^{-1}(\sin 2\theta) \qquad \text{---(ii)}$$

Here, $\quad -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

$\Rightarrow \quad -\frac{1}{\sqrt{2}} < \sin\theta < \frac{1}{\sqrt{2}}$

$\Rightarrow \quad -\frac{\pi}{4} < \theta < \frac{\pi}{4}$

So, from equation (i),

$$u = \theta \qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$$

$$u = \sin^{-1}x$$

*********** END **********