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Trigonometric Functions Ex 5.1 Q1

$$\begin{aligned}\text{LHS} &= \sec^4\theta - \sec^2\theta \\ &= \sec^2\theta(\sec^2\theta - 1) \\ &= (1+\tan^2\theta)\tan^2\theta \qquad [\because \sec^2\theta = 1+\tan^2\theta] \\ &= \tan^2\theta + \tan^4\theta \\ &= \tan^4\theta + \tan^2\theta \\ &= \text{RHS}\end{aligned}$$

LHS = RHS

Proved

Trigonometric Functions Ex 5.1 Q2

$$\begin{aligned}\text{LHS} &= \sin^6\theta + \cos^6\theta \\ &= (\sin^2\theta)^3 + (\cos^2\theta)^3 \\ &= (\sin^2\theta+\cos^2\theta)\left[(\sin^2\theta)^2 - \sin^2\theta\cos^2\theta + (\cos^2\theta)^2\right] \qquad (\because a^3+b^3 = (a+b)(a^2-ab+b^2)) \\ &= (\sin^2\theta)^2 + (\cos\ \theta)^2 + 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta)\end{aligned}$$

[adding and subtracting $2\sin^2\theta\cos^2\theta$ and using identity $\sin^2\theta+\cos^2\theta = 1$]

$$\begin{aligned} &= (\sin^2\theta+\cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta \\ &= (\sin^2\theta+\cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta \\ &= 1^2 - 3\sin^2\theta\cos^2\theta \qquad (\because \sin^2\theta+\cos^2\theta = 1) \\ &= 1 - 3\sin^2\theta\cos^2\theta \\ &= \text{RHS}\end{aligned}$$

$\therefore$ LHS = RHS

Proved

Trigonometric Functions Ex 5.1 Q3

$$\begin{aligned}\text{LHS} &= (\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) \\ &= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)\end{aligned}$$

$$\left[\because \operatorname{cosec}\theta = \frac{1}{\sin\theta},\ \sec\theta = \frac{1}{\cos\theta},\ \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right)$$

$$= \frac{\cos^2\theta.\sin^2\theta.1}{\sin^2\theta.\cos^2\theta} \left(\begin{aligned}&\because \sin^2\theta+\cos^2\theta = 1 \\ &\Rightarrow 1-\sin^2\theta = \cos^2\theta, \text{ and} \\ &\quad 1-\cos^2\theta = \sin^2\theta\end{aligned}\right)$$

= 1

= RHS

Proved

Trigonometric Functions Ex 5.1 Q4

$$\text{LHS} = cosec\theta(\sec\theta - 1) - \cot\theta(1-\cos\theta)$$

$$= \frac{1}{\sin\theta}\left(\frac{1}{\cos\theta} - 1\right) - \frac{\cos\theta}{\sin\theta}(1-\cos\theta) \qquad \left[\because cosec\theta = \frac{1}{\sin\theta}, \sec\theta = \frac{1}{\cos\theta}\cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$= \frac{(1-\cos\theta)}{\sin\theta\cos\theta} - \frac{\cos\theta(1-\cos\theta)}{\sin\theta}$$

$$= \frac{(1-\cos\theta) - \cos^2\theta(1-\cos\theta)}{\sin\theta\cos\theta}$$

$$= \frac{(1-\cos\theta)(1-\cos^2\theta)}{\sin\theta\cos\theta}$$

$$= \frac{(1-\cos\theta)\sin^2\theta}{\sin\theta\cos\theta} \quad (\because\ 1-\cos^2\theta = \sin^2\theta)$$

$$= (1-\cos\theta)\frac{\sin\theta}{\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta} - \sin\theta$$

$$= \tan\theta - \sin\theta \quad (\because \tan\theta = \sin\theta - \cos\theta)$$

= RHS

Proved

Trigonometric Functions Ex 5.1 Q5

$$\text{LHS} = \frac{1-\sin A\cos A}{\cos A(\sec A - cosec A)} \cdot \frac{\sin^2 A - \cos^2 A}{\sin^3 A + \cos^3 A}$$

$$= \frac{1-\sin A\cos A}{\cos A\left(\frac{1}{\cos A} - \frac{1}{\sin A}\right)} \cdot \frac{(\sin A + \cos A)(\sin A - \cos A)}{(\sin A + \cos A)(\sin^2 A + \cos^2 A - \sin A\cos A)}$$

$$\left[\begin{array}{l}\text{Using } a^2 - b^2 = (a-b)(a+b) \\ \quad \text{and } a^3 + b^3 = (a+b)(a^2+b^2-ab)\end{array}\right]$$

$$= \frac{(1-\sin A\cos A)}{\cos A\left(\frac{\sin A - \cos A}{\cos A\sin A}\right)} \cdot \frac{(\sin A - \cos A)}{(1-\sin A\cos A)} \quad (\because \sin^2 A + \cos^2 A = 1)$$

$$= \frac{\cos A\sin A}{\cos A}$$

$$= \sin A$$

= RHS

Proved

*********** END **********