$$\Rightarrow a = -7k,\; b = 8k,\; c = -3k$$

Substituting the values of a, b, and c in equation (1), we obtain

$$-7k(x+1)+8k(y-3)-3k(z-2)=0$$
$$\Rightarrow (-7x-7)+(8y-24)-3z+6=0$$
$$\Rightarrow -7x+8y-3z-25=0$$
$$\Rightarrow 7x-8y+3z+25=0$$

This is the required equation of the plane.

Question 14:

If the points (1, 1, p) and (−3, 0, 1) be equidistant from the plane

$$\vec{r}\cdot\left(3\hat{i}+4\hat{j}-12\hat{k}\right)+13=0$$, then find the value of p.

Answer

The position vector through the point (1, 1, p) is $\vec{a}_1 = \hat{i}+\hat{j}+p\hat{k}$

Similarly, the position vector through the point (−3, 0, 1) is

$$\vec{a}_2 = -4\hat{i}+\hat{k}$$

The equation of the given plane is $\vec{r}\cdot\left(3\hat{i}+4\hat{j}-12\hat{k}\right)+13=0$

It is known that the perpendicular distance between a point whose position vector is

$$D=\frac{\left|\vec{a}\cdot\vec{N}-d\right|}{\left|\vec{N}\right|}$$

$\vec{a}$ and the plane, $\vec{r}\cdot\vec{N}=d$, is given by,

Here, $\vec{N}=3\hat{i}+4\hat{j}-12\hat{k}$ and $d=-13$

Therefore, the distance between the point (1, 1, p) and the given plane is

$$D_1 = \frac{\left|\left(\hat{i}+\hat{j}+p\hat{k}\right)\cdot\left(3\hat{i}+4\hat{j}-12\hat{k}\right)+13\right|}{\left|3\hat{i}+4\hat{j}-12\hat{k}\right|}$$

$$\Rightarrow D_1 = \frac{\left|3+4-12p+13\right|}{\sqrt{3^2+4^2+(-12)^2}}$$

$$\Rightarrow D_1 = \frac{\left|20-12p\right|}{13} \qquad \ldots(1)$$

Similarly, the distance between the point (−3, 0, 1) and the given plane is

$$D_2 = \frac{\left|\left(-3\hat{i}+\hat{k}\right)\cdot\left(3\hat{i}+4\hat{j}-12\hat{k}\right)+13\right|}{\left|3\hat{i}+4\hat{j}-12\hat{k}\right|}$$

$$\Rightarrow D_2 = \frac{\left|-9-12+13\right|}{\sqrt{3^2+4^2+(-12)^2}}$$

$$\Rightarrow D_2 = \frac{8}{13} \qquad \ldots(2)$$

It is given that the distance between the required plane and the points, (1, 1, p) and (−3, 0, 1), is equal.

$$\therefore D_1 = D_2$$

$$\Rightarrow \frac{\left|20-12p\right|}{13} = \frac{8}{13}$$
$$\Rightarrow 20-12p=8 \text{ or } -(20-12p)=8$$
$$\Rightarrow 12p=12 \text{ or } 12p=28$$
$$\Rightarrow p=1 \text{ or } p=\frac{7}{3}$$

Question 15:

Find the equation of the plane passing through the line of intersection of the planes

$$\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=1 \quad \text{and} \quad \vec{r}\cdot\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0$$

and parallel to x-axis.

Answer

The given planes are

$$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$$

$$\Rightarrow \vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1 = 0$$

$$\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$$

The equation of any plane passing through the line of intersection of these planes is

$$[\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1] + \lambda [\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4] = 0$$

$$\vec{r} \cdot [(2\lambda + 1)\hat{i} + (3\lambda + 1)\hat{j} + (1 - \lambda)\hat{k}] + (4\lambda + 1) = 0 \qquad \text{...(1)}$$

Its direction ratios are $(2\lambda + 1)$, $(3\lambda + 1)$, and $(1 - \lambda)$.

The required plane is parallel to x-axis. Therefore, its normal is perpendicular to x-axis.

The direction ratios of x-axis are 1, 0, and 0.

$$\therefore 1.(2\lambda + 1) + 0(3\lambda + 1) + 0(1 - \lambda) = 0$$

$$\Rightarrow 2\lambda + 1 = 0$$

$$\Rightarrow \lambda = -\frac{1}{2}$$

Substituting $\lambda = -\frac{1}{2}$ in equation (1), we obtain

$$\Rightarrow \vec{r} \cdot \left[-\frac{1}{2}\hat{j} + \frac{3}{2}\hat{k} \right] + (-3) = 0$$

$$\Rightarrow \vec{r} \cdot (\hat{j} - 3\hat{k}) + 6 = 0$$

Therefore, its Cartesian equation is $y - 3z + 6 = 0$

This is the equation of the required plane.

Question 16:

If O be the origin and the coordinates of P be (1, 2, −3), then find the equation of the plane passing through P and perpendicular to OP.

Answer

The coordinates of the points, O and P, are (0, 0, 0) and (1, 2, −3) respectively.

Therefore, the direction ratios of OP are $(1 - 0) = 1$, $(2 - 0) = 2$, and $(-3 - 0) = -3$

It is known that the equation of the plane passing through the point (x_1, y_1, z_1) is

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$ where, a, b, and c are the direction ratios of normal.

Here, the direction ratios of normal are 1, 2, and −3 and the point P is (1, 2, −3).

Thus, the equation of the required plane is

$$1(x - 1) + 2(y - 2) - 3(z + 3) = 0$$

$$\Rightarrow x + 2y - 3z - 14 = 0$$

Question 17:

Find the equation of the plane which contains the line of intersection of the planes

$$\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0 \text{ , } \vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$$ and which is perpendicular to the plane

$$\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0 \text{ .}$$

Answer

The equations of the given planes are

$$\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0 \qquad \text{...(1)}$$

$$\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0 \qquad \text{...(2)}$$

The equation of the plane passing through the line intersection of the plane given in equation (1) and equation (2) is

$$[\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4] + \lambda [\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5] = 0$$

$$\vec{r} \cdot [(2\lambda + 1)\hat{i} + (\lambda + 2)\hat{j} + (3 - \lambda)\hat{k}] + (5\lambda - 4) = 0 \qquad \text{...(3)}$$

The plane in equation (3) is perpendicular to the plane, $\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$

$$\therefore 5(2\lambda + 1) + 3(\lambda + 2) - 6(3 - \lambda) = 0$$

$$\Rightarrow 19\lambda - 7 = 0$$

$$\Rightarrow \lambda = \frac{7}{19}$$

Substituting $\lambda = \frac{7}{19}$ in equation (3), we obtain

$$\Rightarrow \vec{r} \cdot \left[\frac{33}{19}\hat{i} + \frac{45}{19}\hat{j} + \frac{50}{19}\hat{k} \right] - \frac{41}{19} = 0$$

$$\Rightarrow \vec{r} \cdot (33\hat{i} + 45\hat{j} + 50\hat{k}) - 41 = 0 \qquad \text{...(4)}$$

This is the vector equation of the required plane.

The Cartesian equation of this plane can be obtained by substituting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in

The Cartesian equation of this plane can be obtained by substituting ~~~~~~~~~ in equation (3).

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\cdot\left(33\hat{i}+45\hat{j}+50\hat{k}\right)-41=0$$
$$\Rightarrow 33x+45y+50z-41=0$$

Question 18:

Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the line

$\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$ and the plane $\vec{r}\cdot\left(\hat{i}-\hat{j}+\hat{k}\right)=5$.

Answer

The equation of the given line is

$$\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right) \qquad ...(1)$$

The equation of the given plane is

$$\vec{r}\cdot\left(\hat{i}-\hat{j}+\hat{k}\right)=5 \qquad ...(2)$$

Substituting the value of $\vec{r}$ from equation (1) in equation (2), we obtain

$$\left[2\hat{i}-\hat{j}+2\hat{k}+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)\right]\cdot\left(\hat{i}-\hat{j}+\hat{k}\right)=5$$
$$\Rightarrow\left[(3\lambda+2)\hat{i}+(4\lambda-1)\hat{j}+(2\lambda+2)\hat{k}\right]\cdot\left(\hat{i}-\hat{j}+\hat{k}\right)=5$$
$$\Rightarrow(3\lambda+2)-(4\lambda-1)+(2\lambda+2)=5$$
$$\Rightarrow\lambda=0$$

Substituting this value in equation (1), we obtain the equation of the line as

$$\vec{r}=2\hat{i}-\hat{j}+2\hat{k}$$

This means that the position vector of the point of intersection of the line and the plane

is $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}$

This shows that the point of intersection of the given line and plane is given by the

********** END *********