$$=\begin{bmatrix}2-1 & 0+2 & 2+1\\3-2 & 0+4 & 3+2\\-1-1 & 0+2 & -1+1\end{bmatrix}=\begin{bmatrix}1 & 2 & 3\\1 & 4 & 5\\-2 & 2 & 0\end{bmatrix}$$

(vi) $\begin{bmatrix}3 & -1 & 3\\-1 & 0 & 2\end{bmatrix}\begin{bmatrix}2 & -3\\1 & 0\\3 & 1\end{bmatrix}$

$$=\begin{bmatrix}3(2)-1(1)+3(3) & 3(-3)-1(0)+3(1)\\-1(2)+0(1)+2(3) & -1(-3)+0(0)+2(1)\end{bmatrix}$$
$$=\begin{bmatrix}6-1+9 & -9-0+3\\-2+0+6 & 3+0+2\end{bmatrix}=\begin{bmatrix}14 & -6\\4 & 5\end{bmatrix}$$

Question 4:

If $A=\begin{bmatrix}1 & 2 & -3\\5 & 0 & 2\\1 & -1 & 1\end{bmatrix}$, $B=\begin{bmatrix}3 & -1 & 2\\4 & 2 & 5\\2 & 0 & 3\end{bmatrix}$, and $C=\begin{bmatrix}4 & 1 & 2\\0 & 3 & 2\\1 & -2 & 3\end{bmatrix}$, then compute $(A+B)$ and $(B-C)$. Also, verify that $A+(B-C)=(A+B)-C$

Answer

$$A+B=\begin{bmatrix}1 & 2 & -3\\5 & 0 & 2\\1 & -1 & 1\end{bmatrix}+\begin{bmatrix}3 & -1 & 2\\4 & 2 & 5\\2 & 0 & 3\end{bmatrix}$$
$$=\begin{bmatrix}1+3 & 2-1 & -3+2\\5+4 & 0+2 & 2+5\\1+2 & -1+0 & 1+3\end{bmatrix}=\begin{bmatrix}4 & 1 & -1\\9 & 2 & 7\\3 & -1 & 4\end{bmatrix}$$

$$B-C=\begin{bmatrix}3 & -1 & 2\\4 & 2 & 5\\2 & 0 & 3\end{bmatrix}-\begin{bmatrix}4 & 1 & 2\\0 & 3 & 2\\1 & -2 & 3\end{bmatrix}$$
$$=\begin{bmatrix}3-4 & -1-1 & 2-2\\4-0 & 2-3 & 5-2\\2-1 & 0-(-2) & 3-3\end{bmatrix}=\begin{bmatrix}-1 & -2 & 0\\4 & -1 & 3\\1 & 2 & 0\end{bmatrix}$$

$$A+(B-C)=\begin{bmatrix}1 & 2 & -3\\5 & 0 & 2\\1 & -1 & 1\end{bmatrix}+\begin{bmatrix}-1 & -2 & 0\\4 & -1 & 3\\1 & 2 & 0\end{bmatrix}$$
$$=\begin{bmatrix}1+(-1) & 2+(-2) & -3+0\\5+4 & 0+(-1) & 2+3\\1+1 & -1+2 & 1+0\end{bmatrix}=\begin{bmatrix}0 & 0 & -3\\9 & -1 & 5\\2 & 1 & 1\end{bmatrix}$$

$$(A+B)-C=\begin{bmatrix}4 & 1 & -1\\9 & 2 & 7\\3 & -1 & 4\end{bmatrix}-\begin{bmatrix}4 & 1 & 2\\0 & 3 & 2\\1 & -2 & 3\end{bmatrix}$$
$$=\begin{bmatrix}4-4 & 1-1 & -1-2\\9-0 & 2-3 & 7-2\\3-1 & -1-(-2) & 4-3\end{bmatrix}=\begin{bmatrix}0 & 0 & -3\\9 & -1 & 5\\2 & 1 & 1\end{bmatrix}$$

Hence, we have verified that $A+(B-C)=(A+B)-C$.

Question 5:

If $A=\begin{bmatrix}\frac{2}{3} & 1 & \frac{5}{3}\\\frac{1}{3} & \frac{2}{3} & \frac{4}{3}\\\frac{7}{3} & 2 & \frac{2}{3}\end{bmatrix}$ and $B=\begin{bmatrix}\frac{2}{5} & \frac{3}{5} & 1\\\frac{1}{5} & \frac{2}{5} & \frac{4}{5}\\\frac{7}{5} & \frac{6}{5} & \frac{2}{5}\end{bmatrix}$ then compute $3A-5B$.

Answer

$$\begin{bmatrix}\frac{2}{3} & 1 & \frac{5}{3}\end{bmatrix}\quad\begin{bmatrix}\frac{2}{5} & \frac{3}{5} & 1\end{bmatrix}$$

$$3A-5B=3\begin{bmatrix}\frac{1}{3} & \frac{2}{3} & \frac{4}{3}\\ \frac{7}{3} & 2 & \frac{2}{3}\end{bmatrix}-5\begin{bmatrix}\frac{1}{5} & \frac{2}{5} & \frac{4}{5}\\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5}\end{bmatrix}$$

$$=\begin{bmatrix}2 & 3 & 5\\ 1 & 2 & 4\\ 7 & 6 & 2\end{bmatrix}-\begin{bmatrix}2 & 3 & 5\\ 1 & 2 & 4\\ 7 & 6 & 2\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

Question 6:

Simplify $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$

Answer

$$\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ -\sin\theta\cos\theta & \cos^2\theta\end{bmatrix}+\begin{bmatrix}\sin^2\theta & -\sin\theta\cos\theta\\ \sin\theta\cos\theta & \sin^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta+\sin^2\theta & \cos\theta\sin\theta-\sin\theta\cos\theta\\ -\sin\theta\cos\theta+\sin\theta\cos\theta & \cos^2\theta+\sin^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \qquad \left(\because \cos^2\theta+\sin^2\theta=1\right)$$

Question 7:

Find X and Y, if

(i) $X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$ and $X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$

(ii) $2X+3Y=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$ and $3X+2Y=\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}$

Answer

(i)

$$X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix} \quad ...(1)$$

$$X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix} \quad ...(2)$$

Adding equations (1) and (2), we get:

$$2X=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}+\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}=\begin{bmatrix}7+3 & 0+0\\ 2+0 & 5+3\end{bmatrix}=\begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}$$

$$\therefore X=\frac{1}{2}\begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}=\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$$

$$\text{Now, } X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$$

$$\Rightarrow Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}-\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$$

$$\Rightarrow Y=\begin{bmatrix}7-5 & 0-0\\ 2-1 & 5-4\end{bmatrix}$$

$$\therefore Y=\begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}$$

(ii)

$$2X+3Y=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix} \quad ...(3)$$

$$3X+2Y=\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix} \quad ...(4)$$

Multiplying equation (3) with (2), we get:

$$2(2X+3Y)=2\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$$

$$\Rightarrow 4X+6Y=\begin{bmatrix}4 & 6\\ 8 & 0\end{bmatrix} \quad ...(5)$$

Multiplying equation (4) with (3), we get:

$$3(3X+2Y)=3\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}$$

$$\Rightarrow 9X+6Y=\begin{bmatrix}6 & -6\end{bmatrix} \quad ...(6)$$

$$\Rightarrow 9X+6Y=\begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix} \quad ...(6)$$

From (5) and (6), we have:

$$(4X+6Y)-(9X+6Y)=\begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix}-\begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix}$$

$$\Rightarrow -5X=\begin{bmatrix} 4-6 & 6-(-6) \\ 8-(-3) & 0-15 \end{bmatrix}=\begin{bmatrix} -2 & 12 \\ 11 & -15 \end{bmatrix}$$

$$\therefore X=-\frac{1}{5}\begin{bmatrix} -2 & 12 \\ 11 & -15 \end{bmatrix}=\begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$$

Now,

$$2X+3Y=\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow 2\begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}+3Y=\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix}+3Y=\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow 3Y=\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}-\begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix}$$

$$\Rightarrow 3Y=\begin{bmatrix} 2-\frac{4}{5} & 3+\frac{24}{5} \\ 4+\frac{22}{5} & 0-6 \end{bmatrix}=\begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix}$$

$$\therefore Y=\frac{1}{3}\begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix}=\begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

Question 8:

Find X, if $Y=\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X+Y=\begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

Answer

$$2X+Y=\begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow 2X+\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow 2X=\begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}-\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}=\begin{bmatrix} 1-3 & 0-2 \\ -3-1 & 2-4 \end{bmatrix}$$

$$\Rightarrow 2X=\begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$\therefore X=\frac{1}{2}\begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}=\begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$

*********** END **********