Transformation Formulae Ex 8.1 Q6(i)

We have,

$$\begin{aligned}
\text{LHS} &= \sin A \sin(B - C) + \sin B \sin(C - A) + \sin C \sin(A - B) \\
&= \frac{1}{2}\left[2 \sin A \sin(B - C) + 2 \sin B \sin(C - A) + 2 \sin C \sin(A - B)\right] \\
&= \frac{1}{2}\left[\begin{aligned}&\cos(A - B + C) - \cos(A + B - C) + \cos(B - C + A) - \cos(B + C - A) \\ &\qquad + \cos(C - A + B) - \cos(C + A - B)\end{aligned}\right] \\
&= \frac{1}{2}\left[\begin{aligned}&\cos(A - B + C) - \cos(A - B + C) - \cos(A + B - C) + \cos(A + B - C) \\ &\qquad - \cos(B + C - A) + \cos(B + C - A)\end{aligned}\right] \\
&= \frac{1}{2} \times 0 \\
&= 0 \\
&= \text{RHS}
\end{aligned}$$

$\therefore$ $\sin A \sin(B - C) + \sin B \sin(C - A) + \sin C \sin(A - B) = 0$ Hence proved.

Transformation Formulae Ex 8.1 Q6(ii)

We have,

$$\begin{aligned}
\text{LHS} &= \sin(B - C)\cos(A - D) + \sin(C - A)\cos(B - D) + \sin(A - B)\cos(C - D) \\
&= \frac{1}{2}\left[2\sin(B - C)\cos(A - D) + 2\sin(C - A)\cos(B - D) + 2\sin(A - B)\cos(C - D)\right] \\
&= \frac{1}{2}\left[\begin{aligned}&\sin(B - C + A - D) + \sin(B - C - A + D) + \sin(C - A + B - D) + \sin(C - A - B + D) \\ &\qquad + \sin(A - B + C - D) + \sin(A - B - C + D)\end{aligned}\right] \\
&= \frac{1}{2}\left[\begin{aligned}&\sin(A + B - C - D) + \sin(B + D - C - A) + \sin(B + C - A - D) + \sin(C + D - A - B) \\ &\qquad + \sin(A + C - B - D) + \sin(A + D - B - C)\end{aligned}\right] \\
&= \frac{1}{2}\left[\begin{aligned}&\sin(A + B - C - D) + \sin(B + D - C - A) + \sin\{-(A + D - B - C)\} + \sin\{-(A + B - C - D)\} \\ &\qquad + \sin\{-(B + D - A - C)\} + \sin(A + D - B - C)\end{aligned}\right] \\
&= \frac{1}{2}\left[\begin{aligned}&\sin(A + B - C - D) + \sin(B + D - C - A) - \sin(A + D - B - C) - \sin(A + B - C - D) \\ &\qquad - \sin(B + D - A - C) + \sin(A + D - B - C)\end{aligned}\right] \\
&= \frac{1}{2} \times 0 \qquad\qquad [\because \sin(-\theta) = -\sin\theta] \\
&= 0 \\
&= \text{RHS}
\end{aligned}$$

$\therefore$ $\sin(B - C)\cos(A - D) + \sin(C - A)\cos(B - D) + \sin(A - B)\cos(C - D) = 0$ Hence proved.

Transformation Formulae Ex 8.1 Q7

We have,

$$\begin{aligned}
\text{LHS} &= \tan\theta \tan(60° - \theta)\tan(60° + \theta) \\
&= \frac{\sin\theta \sin(60° - \theta)\sin(60° + \theta)}{\cos\theta \cos(60° - \theta)\cos(60° + \theta)} \\
&= \frac{2\sin\theta \sin(60° - \theta)\sin(60° + \theta)}{2\cos\theta \cos(60° - \theta)\cos(60° + \theta)} \\
&= \frac{\sin\theta\left[2\sin(60° - \theta)\sin(60° + \theta)\right]}{\cos\theta\left[2\cos(60° - \theta)\cos(60° + \theta)\right]} \\
&= \frac{\sin\theta\left[\cos\{(60° - \theta) - (60° + \theta)\} - \cos\{(60° - \theta) + (60° + \theta)\}\right]}{\cos\theta\left[\cos\{(60° - \theta) + (60° + \theta)\} + \cos\{(60° - \theta) - (60° + \theta)\}\right]} \\
&= \frac{\sin\theta\left[\cos(-2\theta) - \cos 120°\right]}{\cos\theta\left[\cos 120° + \cos(-2\theta)\right]} \\
&= \frac{\sin\theta\left[\cos 2\theta - \cos 120°\right]}{\cos\theta\left[\cos 120° + \cos 2\theta\right]} \qquad [\because \cos(-\theta) = \cos\theta] \\
&= \frac{\sin\theta\left[\cos 2\theta - \cos(90° + 30°)\right]}{\cos\theta\left[\cos(90° + 30°) + \cos 2\theta\right]} \\
&= \frac{\sin\theta\left[\cos 2\theta + \sin 30°\right]}{\cos\theta\left[-\sin 30° + \cos 2\theta\right]} \qquad [\because \cos \text{ is negative in IInd quadrant}] \\
&= \frac{\sin\theta\left[\cos 2\theta + \frac{1}{2}\right]}{\cos\theta\left[\frac{-1}{2} + \cos 2\theta\right]} \\
&= \frac{\sin\theta\cos 2\theta + \frac{1}{2}\sin\theta}{\frac{-1}{2}\cos\theta + \cos\theta\cos 2\theta}
\end{aligned}$$

Transformation Formulae Ex 8.1 Q8

Let $y = \cos\alpha \cos\beta$ then,

$$y = \frac{1}{2}(2\cos\alpha\cos\beta)$$

$$= \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$$

$$= \frac{1}{2}\left[\cos 90^\circ + \cos(\alpha-\beta)\right] \qquad [\because \alpha+\beta = 90^\circ]$$

$$= \frac{1}{2}\left[0 + \cos(\alpha-\beta)\right]$$

$$= \frac{1}{2}\cos(\alpha-\beta)$$

$$\Rightarrow \quad y = \frac{1}{2}\cos(\alpha-\beta)$$

Now,

$$-1 \le \cos(\alpha-\beta) \le 1$$

$$\Rightarrow \quad \frac{-1}{2} \le \frac{1}{2}\cos(\alpha-\beta) \le \frac{1}{2}$$

$$\Rightarrow \quad \frac{-1}{2} \le y \le \frac{1}{2}$$

$$\Rightarrow \quad \frac{-1}{2} \le \cos\alpha\cos\beta \le \frac{1}{2}$$

Hence, the maximum values of $\cos\alpha \cos\beta$ is $\frac{1}{2}$.

*********** END **********