



Definite Integrals Ex 20.2 Q48

Let  $\cos x = t$

Differentiating w.r.t.  $x$ , we get

$$-\sin x dx = dt$$

When  $x = 0 \Rightarrow t = 1$

$$x = \pi \Rightarrow t = -1$$

Now,

$$\begin{aligned} & \int_0^{\pi} \sin^3 x (1 + 2 \cos x) (1 + \cos x)^2 dx \\ &= \int_0^{\pi} \sin^2 x (1 + 2 \cos x) (1 + \cos x)^2 \cdot \sin x dx \\ &= - \int_{-1}^1 (1 - t^2) (1 + 2t) (1 + t)^2 dt \quad [\sin^2 x = 1 - \cos^2 x] \\ &= - \int_{-1}^1 (1 + 2t - t^2 - 2t^3) (1 + t^2 + 2t) dt \\ &= - \int_{-1}^1 (1 - t^2 + 2t + 2t + 2t^3 + 4t^2 - t^2 - t^4 - 2t^3 - 2t^5 - 4t^4) dt \\ &= - \int_{-1}^1 (1 + 4t + 4t^2 - 2t^3 - 5t^4 - 2t^5) dt \\ &= \left[ t + 2t^2 + \frac{4}{3}t^3 - \frac{t^4}{2} - t^5 - \frac{t^6}{3} \right]_{-1}^1 \\ &= \left[ 2 + 0 + \frac{8}{3} - 0 - 2 - 0 \right] = \frac{8}{3} \end{aligned}$$

$$\therefore \int_0^{\pi} \sin^3 x (1 + 2 \cos x) (1 + \cos x)^2 dx = \frac{8}{3}$$

Definite Integrals Ex 20.2 Q49

$$I = \int_0^{\frac{\pi}{2}} 2 \sin x \cos x \tan^{-1}(\sin x) dx$$

Let  $t = \sin x$

$$dt = \cos x dx$$

$$x = 0, t = 0$$

$$x = \frac{\pi}{2}, t = 1$$

$$I = \int_0^1 2t \tan^{-1}(t) dt$$

$$= 2 \left[ \frac{1}{2} t^2 \tan^{-1} t - \frac{t}{2} + \frac{1}{2} \tan^{-1} t \right]_0^1$$

$$= 2 \left[ \frac{\pi}{4} - \frac{1}{2} \right]$$

$$= \frac{\pi}{2} - 1$$

$$\therefore I = \int_0^{\frac{\pi}{2}} 2 \sin x \cos x \tan^{-1}(\sin x) dx = \frac{\pi}{2} - 1$$

Definite Integrals Ex 20.2 Q50

Let  $\sin x = t$

Differentiating w.r.t.  $x$ , we get

$$\cos x dx = dt$$

Now,

$$x = 0 \Rightarrow t = 0$$

$$x = \frac{\pi}{2} \Rightarrow t = 1$$

$$\therefore \int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) dx = 2 \int_0^1 t \tan^{-1} t dt \quad [\because \sin 2x = 2 \sin x \cos x]$$

Using by parts

$$= 2 \left\{ \tan^{-1} t \int t dt - \int \left( \int t dt \right) \frac{d \tan^{-1} t}{dt} dt \right\}$$

$$= 2 \left\{ \frac{t^2}{2} \tan^{-1} t - \frac{1}{2} \int \frac{t^2}{1+t^2} dt \right\}$$

$$= 2 \left\{ \frac{t^2}{2} \tan^{-1} t - \frac{1}{2} \left( \int dt - \int \frac{dt}{1+t^2} \right) \right\}$$

$$= 2 \left[ \frac{t^2}{2} \tan^{-1} t - \frac{1}{2} \left( t - \tan^{-1} t \right) \right]_0^1$$

$$= 2 \left\{ \frac{1}{2} \frac{\pi}{4} - \frac{1}{2} \left( 1 - \frac{\pi}{4} \right) \right\}$$

$$= 2 \left\{ \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8} \right\}$$

$$= 2 \left( \frac{\pi}{4} - \frac{1}{2} \right) = \frac{\pi}{2} - 1$$

$$\therefore \int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) dx = \frac{\pi}{2} - 1$$

Definite Integrals Ex 20.2 Q51

We have,

$$\begin{aligned}\int_0^1 (\cos^{-1} x)^2 dx &= (\cos^{-1} x)^2 \int_0^1 dx - \int_0^1 (dx) \frac{d(\cos^{-1} x)^2}{dx} dx \\ &= \left[ x (\cos^{-1} x)^2 \right]_0^1 + \int_0^1 \frac{x \cdot 2 \cos^{-1} x}{\sqrt{1-x^2}} dx\end{aligned}$$

Now,

$$\text{Let } \cos^{-1} x = t \Rightarrow -\frac{1}{\sqrt{1-x^2}} dx = dt$$

$$\begin{aligned}\text{When } x = 0 &\Rightarrow t = \frac{\pi}{2} \\ x = 1 &\Rightarrow t = 0\end{aligned}$$

$$\begin{aligned}\therefore \int_0^1 \frac{2x \cos^{-1} x}{\sqrt{1-x^2}} dx &= -2 \int_{\frac{\pi}{2}}^0 t \cos t dt = 2 \int_0^{\frac{\pi}{2}} t \cos t dt \\ &= 2 \left[ t \int \cos t dt - \int (\cos t) \frac{dt}{dt} dt \right]_0^{\frac{\pi}{2}} \\ &= 2 \left[ t \sin t - \int \sin t dt \right]_0^{\frac{\pi}{2}} \\ &= 2 \left[ t \sin t + \cos t \right]_0^{\frac{\pi}{2}} \\ &= 2 \left[ \frac{\pi}{2} - 1 \right]\end{aligned}$$

$$\begin{aligned}\int_0^1 (\cos^{-1} x)^2 dx &= \left[ x (\cos^{-1} x)^2 \right]_0^1 + \int_0^1 \frac{x \cdot 2 \cos^{-1} x}{\sqrt{1-x^2}} dx = \left[ x (\cos^{-1} x)^2 \right]_0^1 + 2 \left( \frac{\pi}{2} - 1 \right) \\ &= 0 - 0 + 2 \left( \frac{\pi}{2} - 1 \right) \\ &= (\pi - 2)\end{aligned}$$

$$\therefore \int_0^1 (\cos^{-1} x)^2 dx = (\pi - 2)$$

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