Question 3

(i) $\text{LHS} = \sin^2\theta + \dfrac{1}{1+\tan^2\theta}$ $\left[\because \left(1+\tan^2\theta\right) = \sec^2\theta\right]$

$= \sin^2\theta + \dfrac{1}{\sec^2\theta}$

$= \sin^2\theta + \cos^2\theta = 1 = \text{RHS}$

$\therefore \text{LHS} = \text{RHS}$

(ii) $\text{LHS} = \dfrac{1}{\left(1+\tan^2\theta\right)} + \dfrac{1}{\left(1+\cot^2\theta\right)}$

$= \dfrac{1}{\sec^2\theta} + \dfrac{1}{\cosec^2\theta}$ $\left[\begin{array}{l}\because\left(1+\tan^2\theta\right) = \sec^2 \text{ or}\\ \left(1+\cot^2\theta\right) = \cosec^2\theta\end{array}\right]$

$= \cos^2\theta + \sin^2\theta = 1 = \text{RHS}$

$\therefore \text{LHS} = \text{RHS}$

Question 4

(i) $\text{LHS} = \left(1+\cos\theta\right)\left(1-\cos\theta\right)\left(1+\cot^2\theta\right)$

$= \left(1-\cos^2\theta\right)\left(1+\cot^2\theta\right)$

$= \sin^2\theta \times \cosec^2\theta$

$\left[\begin{array}{l}\because \left(1-\cos^2\theta\right) = \sin^2\theta\\ \text{and } \left(1+\cot^2\theta\right) = \cosec^2\theta\end{array}\right]$

$= \sin^2\theta \times \dfrac{1}{\sin^2\theta} = 1 = \text{RHS}$

$\therefore \text{LHS} = \text{RHS}$

(ii) $\text{LHS} = \cosec\theta\left(1+\cos\theta\right)\left(\cosec\theta - \cot\theta\right)$

$= \left(\cosec\theta + \dfrac{\cos\theta}{\sin\theta}\right)\left(\cosec\theta - \cot\theta\right)$

$= \left(\cosec\theta + \cot\theta\right)\left(\cosec\theta - \cot\theta\right)$

$= \left(\cosec^2\theta - \cot^2\theta\right) = 1 = \text{RHS}$

$\therefore \text{LHS} = \text{RHS}$