Definite Integrals Ex 20.1 Q59

We have,

$$\int_0^k \frac{dx}{2+8x^2} = \frac{\pi}{16}$$

$$\Rightarrow \frac{1}{8}\int_0^k \frac{dx}{\left(\frac{1}{2}\right)^2 + x^2} = \frac{\pi}{16}$$

$$\Rightarrow \frac{1}{8}\left[2\tan^{-1}2x\right]_0^k = \frac{\pi}{16} \qquad \left[\because \int \frac{dx}{a^2-x^2} = 2\tan^{-1}\frac{x}{a}\right]$$

$$\Rightarrow \frac{1}{4}\left[\tan^{-1}2k - \tan^{-1}0\right] = \frac{\pi}{16}$$

$$\Rightarrow \tan^{-1}2k - 0 = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}2k = \frac{\pi}{4}$$

$$\Rightarrow 2k = 1$$

$$k = \frac{1}{2}$$

Definite Integrals Ex 20.1 Q60

We have,

$$\int_0^a 3x^2 dx = 8$$

$$\Rightarrow \left[x^3\right]_0^a = 8$$

$$\Rightarrow a^3 = 8$$

$$\Rightarrow a = 2$$

Definite Integrals Ex 20.1 Q61

$$\int_{\pi}^{\frac{3\pi}{2}} \sqrt{1-(1-2\sin^2 x)}dx$$

$$\int_{\pi}^{\frac{3\pi}{2}} \sqrt{2\sin^2 x}dx$$

$$\sqrt{2}\int_{\pi}^{\frac{3\pi}{2}} \sin x dx$$

$$\sqrt{2}(-\cos x)_{\pi}^{\frac{3\pi}{2}}$$

$$= \sqrt{2}$$

Definite Integrals Ex 20.1 Q62

$$I = \int_0^{2\pi} \sqrt{1+\sin\frac{x}{2}}\,dx$$

$$\Rightarrow I = \int_0^{2\pi} \sqrt{\sin^2\frac{x}{4} + \cos^2\frac{x}{4} + 2\sin\frac{x}{4}\cos\frac{x}{4}}\,dx$$

$$\Rightarrow I = \int_0^{2\pi} \sqrt{\left(\sin\frac{x}{4} + \cos\frac{x}{4}\right)^2}\,dx$$

$$\Rightarrow I = \int_0^{2\pi} \left(\sin\frac{x}{4} + \cos\frac{x}{4}\right)dx$$

$$\Rightarrow I = \left[\frac{-\cos\frac{x}{4}}{\frac{1}{4}} + \frac{\sin\frac{x}{4}}{\frac{1}{4}}\right]_0^{2\pi}$$

$$\Rightarrow I = 4(0+1+1-0)$$

$$\Rightarrow I = 8$$

*********** END **********