Thus, in order to obtain an annual total interest of Rs 1800, the trust fund should invest Rs 15000 in the first bond and the remaining Rs 15000 in the second bond.

(b) Let Rs *x* be invested in the first bond. Then, the sum of money invested in the second bond will be Rs (30000 − *x*).

Therefore, in order to obtain an annual total interest of Rs 2000, we have:

$$\begin{bmatrix} x & (30000-x) \end{bmatrix} \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 2000$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000-x)}{100} = 2000$$
$$\Rightarrow 5x + 210000 - 7x = 200000$$
$$\Rightarrow 210000 - 2x = 200000$$
$$\Rightarrow 2x = 210000 - 200000$$
$$\Rightarrow 2x = 10000$$
$$\Rightarrow x = 5000$$

Thus, in order to obtain an annual total interest of Rs 2000, the trust fund should invest Rs 5000 in the first bond and the remaining Rs 25000 in the second bond.

Question 20:

The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs 80, Rs 60 and Rs 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

Answer

The bookshop has 10 dozen chemistry books, 8 dozen physics books, and 10 dozen economics books.

The selling prices of a chemistry book, a physics book, and an economics book are respectively given as Rs 80, Rs 60, and Rs 40.

The total amount of money that will be received from the sale of all these books can be represented in the form of a matrix as:

$$12\begin{bmatrix} 10 & 8 & 10 \end{bmatrix} \begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}$$

$$= 12\left[10\times 80 + 8\times 60 + 10\times 40\right]$$
$$= 12(800 + 480 + 400)$$
$$= 12(1680)$$
$$= 20160$$

Thus, the bookshop will receive Rs 20160 from the sale of all these books.

Question 21:

Assume *X*, *Y*, *Z*, *W* and *P* are matrices of order $2\times n, 3\times k, 2\times p,\ n\times 3$, and $p\times k$ respectively. The restriction on *n*, *k* and *p* so that $PY + WY$ will be defined are:

A. *k* = 3, *p* = *n*

B. *k* is arbitrary, *p* = 2

C. *p* is arbitrary, *k* = 3

D. *k* = 2, *p* = 3

Answer

Matrices *P* and *Y* are of the orders *p* × *k* and 3 × *k* respectively.

Therefore, matrix *PY* will be defined if *k* = 3. Consequently, *PY* will be of the order *p* × *k*.

Matrices *W* and *Y* are of the orders *n* × 3 and 3 × *k* respectively.

Since the number of columns in *W* is equal to the number of rows in *Y*, matrix *WY* is well-defined and is of the order *n* × *k*.

Matrices *PY* and *WY* can be added only when their orders are the same.

However, *PY* is of the order *p* × *k* and *WY* is of the order *n* × *k*. Therefore, we must have *p* = *n*.

Thus, *k* = 3 and *p* = *n* are the restrictions on *n*, *k*, and *p* so that $PY + WY$ will be defined.

Question 22:

Assume *X*, *Y*, *Z*, *W* and *P* are matrices of order $2\times n, 3\times k, 2\times p, n\times 3$, and $p\times k$ respectively. If *n* = *p*, then the order of the matrix $7X-5Z$ is

A *p* × 2 **B** 2 × *n* **C** *n* × 3 **D** *p* × *n*

Answer

The correct answer is B.

Matrix *X* is of the order 2 × *n*.

Therefore, matrix 7*X* is also of the same order.

Matrix *Z* is of the order 2 × *p*, i.e., 2 × *n* [Since *n* = *p*]

Therefore, matrix 5*Z* is also of the same order.

Now, both the matrices 7*X* and 5*Z* are of the order 2 × *n*.

Thus, matrix 7*X* − 5*Z* is well-defined and is of the order 2 × *n*.

Exercise 3.3

Question 1:

Find the transpose of each of the following matrices:

(i) $\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ (iii) $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

Answer

(i) $\text{Let } A=\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}, \text{ then } A^{T}=\begin{bmatrix} 5 & \frac{1}{2} & -1 \end{bmatrix}$

(ii) $\text{Let } A=\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}, \text{ then } A^{T}=\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

(iii) $\text{Let } A=\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}, \text{ then } A^{T}=\begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$

Question 2:

If $A=\begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, then verify that

(i) $(A+B)'=A'+B'$

(ii) $(A-B)'=A'-B'$

Answer

We have:

$$A'=\begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix}, B'=\begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$$

(i)

$$A+B=\begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}+\begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}=\begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}$$

$$\therefore (A+B)'=\begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$$

$$A'+B'=\begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix}+\begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}=\begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$$

Hence, we have verified that $(A+B)'=A'+B'$

(ii)

$$A-B=\begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}-\begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}=\begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix}$$

$$\therefore (A-B)'=\begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$$

$$A' - B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$$

Hence, we have verified that $(A-B)' = A' - B'$.

Question 3:

If $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then verify that

(i) $(A+B)' = A' + B'$

(ii) $(A-B)' = A' - B'$

Answer

(i) It is known that $A = (A')'$

Therefore, we have:

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

$$B' = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 5 & 4 & 4 \end{bmatrix}$$

$$\therefore (A+B)' = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$$

$$A' + B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$$

Thus, we have verified that $(A+B)' = A' + B'$.

(ii)

$$A - B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix}$$

$$\therefore (A-B)' = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

$$A' - B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

Thus, we have verified that $(A-B)' = A' - B'$.

*********** END **********