Indefinite Integrals Ex 19.30 Q57

$$\text{Let } I = \int \frac{dx}{\cos x\,(5 - 4\sin x)}$$
$$= \int \frac{\cos x\,dx}{\cos^2 x\,(5 - 4\sin x)}$$
$$= \int \frac{\cos x\,dx}{\left(1 - \sin^2 x\right)(5 - 4\sin x)}$$

Let $\sin x = t \quad \Rightarrow \quad \cos x\,dx = dt$

$$\therefore \quad I = \int \frac{dt}{\left(1 - t^2\right)(5 - 4t)}$$

Now,

$$\text{Let } \frac{1}{\left(1 - t^2\right)(5 - 4t)} = \frac{A}{1 - t} + \frac{B}{1 + t} + \frac{C}{5 - 4t}$$

$$\Rightarrow \quad 1 = A(1 + t)(5 - 4t) + B(1 - t)(5 - 4t) + C\left(1 - t^2\right)$$

Put $t = 1$

$$\Rightarrow \quad 1 = 2A \Rightarrow \quad A = \frac{1}{2}$$

Put $t = -1$

$$\Rightarrow \quad 1 = 18B \quad \Rightarrow \quad B = \frac{1}{18}$$

Put $t = \frac{5}{4}$

$$\Rightarrow \quad 1 = -\frac{9C}{16} \Rightarrow \quad C = -\frac{16}{9}$$

Thus,

$$I = \frac{1}{2}\int \frac{dt}{1 - t} + \frac{1}{18}\int \frac{dt}{1 + t} - \frac{16}{9}\int \frac{dt}{5 - 4t}$$
$$= -\frac{1}{2}\log|1 - t| + \frac{1}{18}\log|1 + t| + \frac{4}{9}\log|5 - 4t| + c$$

Hence,

$$I = -\frac{1}{2}\log|1 - \sin x| + \frac{1}{18}\log|1 + \sin x| + \frac{4}{9}\log|5 - 4\sin x| + c$$

Indefinite Integrals Ex 19.30 Q58

Let $I = \int \frac{1}{\sin x\left(3+2\cos x\right)}dx$

$= \int \frac{\sin x dx}{\sin^2 x\left(3+2\cos x\right)}$

$= \int \frac{\sin x dx}{\left(1-\cos^2 x\right)\left(3+2\cos x\right)}$

Let $\cos x = t$

$\Rightarrow \quad -\sin x dx = dt$

$\therefore \quad I = \int \frac{dt}{\left(t^2-1\right)\left(3+2t\right)}$

Now,

Let $\frac{1}{\left(t^2-1\right)\left(3+2t\right)} = \frac{A}{t-1}+\frac{B}{t+1}+\frac{C}{3+2t}$

$\Rightarrow \quad 1 = A\left(t+1\right)\left(3+2t\right)+B\left(t-1\right)\left(3+2t\right)+C\left(t^2-1\right)$

Put $t = 1$

$\Rightarrow \quad 1 = 10A \Rightarrow \quad A = \frac{1}{10}$

Put $t = -1$

$\Rightarrow \quad 1 = -2B \quad \Rightarrow \quad B = -\frac{1}{2}$

Put $t = -\frac{3}{2}$

$\Rightarrow \quad 1 = \frac{5}{4}.C \Rightarrow \quad C = \frac{4}{5}$

Thus,

$I = \frac{1}{10}\int \frac{dt}{t-1} - \frac{1}{2}\int \frac{dt}{t+1} + \frac{5}{4}\int \frac{dt}{3+2t}$

$= \frac{1}{10}\log\left|t-1\right| - \frac{1}{2}\log\left|t+1\right| + \frac{2}{5}\log\left|3+2t\right| + c$

Hence,

$I = \frac{1}{10}\log\left|\cos x-1\right| - \frac{1}{2}\log\left|\cos x+1\right| + \frac{2}{5}\log\left|3+2\cos x\right| + c$

Indefinite Integrals Ex 19.30 Q59

$$\text{Let } I = \int \frac{1}{\sin x + \sin 2x} dx$$
$$= \int \frac{dx}{\sin x + 2\sin x \cos x}$$

$$= \int \frac{\sin x dx}{\left(1-\cos^2 x\right) + 2\left(1-\cos^2 x\right)\cos x}$$

Let $\cos x = t \quad \Rightarrow \qquad -\sin x dx = dt$

$$\therefore \qquad I = \int \frac{dt}{\left(t^2-1\right) + 2\left(t^2-1\right)t}$$
$$= \int \frac{dt}{\left(t^2-1\right)\left(1+2t\right)}$$

$$\text{Let } \int \frac{1}{\left(t^2-1\right)\left(1+2t\right)} = \frac{A}{t-1} + \frac{B}{t+1} + \frac{C}{1+2t}$$

$$\Rightarrow \qquad 1 = A\left(t+1\right)\left(1+2t\right) + B\left(t-1\right)\left(1+2t\right) + C\left(t^2-1\right)$$

Put $t = 1$

$$\Rightarrow \qquad 1 = 6A \Rightarrow \qquad A = \frac{1}{6}$$

Put $t = -1$

$$\Rightarrow \qquad 1 = 2B \;\Rightarrow \qquad B = \frac{1}{2}$$

Put $t = -\frac{1}{2}$

$$\Rightarrow \qquad 1 = -\frac{3}{4}C \Rightarrow \quad C = -\frac{4}{3}$$

Thus,

$$I = \frac{1}{6}\int \frac{dt}{t-1} + \frac{1}{2}\int \frac{dt}{t+1} - \frac{4}{3}\int \frac{dt}{1+2t}$$
$$= \frac{1}{6}\log\left|t-1\right| + \frac{1}{2}\log\left|t+1\right| - \frac{2}{3}\log\left|1+2t\right| + c$$

Hence,

$$I = \frac{1}{6}\log\left|\cos x - 1\right| + \frac{1}{2}\log\left|\cos x + 1\right| - \frac{2}{3}\log\left|1 + 2\cos x\right| + c$$

Indefinite Integrals Ex 19.30 Q60

$$\text{Let } I = \int \frac{x+1}{x\left(1+xe^x\right)}dx$$

$$= \int \frac{(x+1)\left(1+xe^x - xe^x\right)}{x\left(1+xe^x\right)}dx$$

$$= \int \frac{(x+1)\left(1+xe^x\right)}{x\left(1+xe^x\right)}dx - \int \frac{(x+1)\left(xe^x\right)}{x\left(1+xe^x\right)}dx$$

$$= \int \frac{(x+1)}{x}dx - \int \frac{e^x\,(x+1)}{1+xe^x}dx$$

$$= \int \frac{(x+1)e^x}{xe^x}dx - \int \frac{e^x\,(x+1)}{1+xe^x}dx$$

$$= \log\left|xe^x\right| - \log\left|1+xe^x\right| + c$$

$$\therefore \qquad I = \log\left|\frac{xe^x}{1+xe^x}\right| + c$$

*********** END **********