



### Maxima and Minima 18.5 Q19

Let  $r$  and  $h$  be the radius and the height (altitude) of the cone respectively.

Then, the volume ( $V$ ) of the cone is given as:

$$V = \frac{1}{3}\pi r^2 h \Rightarrow h = \frac{3V}{r^2}$$

The surface area ( $S$ ) of the cone is given by,

$S = \pi r l$  (where  $l$  is the slant height)

$$\begin{aligned} &= \pi r \sqrt{r^2 + h^2} \\ &= \pi r \sqrt{r^2 + \frac{9V^2}{r^4}} = \frac{r \sqrt{9r^6 + V^2}}{\pi r^2} \\ &= \frac{1}{r} \sqrt{\pi^2 r^6 + 9V^2} \end{aligned}$$

$$\begin{aligned} \therefore \frac{dS}{dr} &= \frac{r \cdot \frac{6\pi^2 r^5}{2\sqrt{\pi^2 r^6 + 9V^2}} - \sqrt{\pi^2 r^6 + 9V^2}}{r^2} \\ &= \frac{3\pi^2 r^6 - \pi^2 r^6 - 9V^2}{r^2 \sqrt{\pi^2 r^6 + 9V^2}} \\ &= \frac{2\pi^2 r^6 - 9V^2}{r^2 \sqrt{\pi^2 r^6 + 9V^2}} \end{aligned}$$

$$= \frac{2\pi^2 r^6 - 9V^2}{r^2 \sqrt{\pi^2 r^6 + 9V^2}}$$

$$\text{Now, } \frac{dS}{dr} = 0 \Rightarrow 2\pi^2 r^6 = 9V^2 \Rightarrow r^6 = \frac{9V^2}{2\pi^2}$$

Thus, it can be easily verified that when  $r^6 = \frac{9V^2}{2\pi^2}$ ,  $\frac{d^2S}{dr^2} > 0$ .

$\therefore$  By second derivative test, the surface area of the cone is the least when  $r^6 = \frac{9V^2}{2\pi^2}$ .

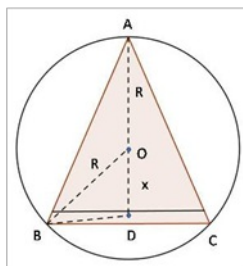
$$\text{When } r^6 = \frac{9V^2}{2\pi^2}, h = \frac{3V}{\pi r^2} = \frac{3}{\pi r^2} \left( \frac{2\pi^2 r^6}{9} \right)^{\frac{1}{2}} = \frac{3}{\pi r^2} \cdot \frac{\sqrt{2}\pi r^3}{3} = \sqrt{2}r.$$

Hence, for a given volume, the right circular cone of the least curved surface has an altitude equal to  $\sqrt{2}$  times the radius of the base.

### Maxima and Minima 18.5 Q20

We have a cone, which is inscribed in a sphere.

Let  $v$  be the volume of greatest cone  $ABC$ . It is obvious that, for maximum volume the axis of the cone must be along the diameter of sphere.



Let  $OD = x$  and  $AO = OB = R$

$$\Rightarrow BD = \sqrt{R^2 - x^2} \text{ and } AD = R + x$$

Now,

$$\begin{aligned} V &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \pi BD^2 \times AD \\ &= \frac{1}{3} \pi (R^2 - x^2) \times (R + x) \end{aligned}$$

$$\begin{aligned} \therefore \frac{dV}{dx} &= \frac{\pi}{3} [-2x(R + x) + R^2 - x^2] \\ &= \frac{\pi}{3} [R^2 - 2xR - 3x^2] \end{aligned}$$

For maximum and minimum

$$\begin{aligned} \frac{dV}{dx} &= 0 \\ \Rightarrow \frac{\pi}{3} [R^2 - 2xR - 3x^2] &= 0 \\ \Rightarrow \frac{\pi}{3} [(R - 3x)(R + x)] &= 0 \\ \Rightarrow R - 3x = 0 \text{ or } x = -R \\ \Rightarrow x = \frac{R}{3} &\quad \left[ \because x = -R \text{ is not possible as } x = -R \text{ will make the} \right. \\ &\quad \left. \text{altitude 0} \right] \end{aligned}$$

Now,

$$\begin{aligned} \frac{d^2V}{dx^2} &= \frac{\pi}{3} [-2R - 6x] \\ \text{At } x = \frac{R}{3}, \quad \frac{d^2V}{dx^2} &= \frac{\pi}{3} [-2R - 2R] \\ &= \frac{-4\pi R}{3} < 0 \end{aligned}$$

$$\therefore x = \frac{R}{3} \text{ is the point of local maxima.}$$

\*\*\*\*\* END \*\*\*\*\*