Definite Integrals Ex 20.4B Q5

$$\int_0^{\frac{\pi}{2}} \frac{\sin^n x}{\sin^n x + \cos^n x} dx$$

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin^n x}{\sin^n x + \cos^n x} dx \qquad --(\text{i})$$

So,

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^n\left(\frac{\pi}{2} - x\right)}{\sin^n\left(\frac{\pi}{2} - x\right) + \cos^n\left(\frac{\pi}{2} - x\right)} dx \qquad \left[\because \int_0^a f(x)dx = \int_0^a f(a - x)dx\right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos^n x}{\sin^n x + \cos^n x} \qquad --(\text{II})$$

Adding (I) & (II)

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^n x}{\sin^n x + \cos^n x} dx + \int_0^{\frac{\pi}{2}} \frac{\cos^n x}{\sin^n x + \cos^n x}$$

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^n + \cos^n x}{\sin^n + \cos^n x} dx$$

$$2I = \int_0^{\frac{\pi}{2}} dx$$

$$2I = [x]_0^{\frac{\pi}{2}}$$

$$2I = \left[\frac{\pi}{2} - 0\right]$$

$$I = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q6

We have,

$$\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\tan x}}\,dx = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$$

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \qquad --\text{(i)}$$

So

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}\,dx \qquad \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} \qquad --\text{(ii)}$$

Adding (i) & (ii)

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx + \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}+\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$$

$$2I = \int_0^{\frac{\pi}{2}} dx$$

$$2I = [x]_0^{\frac{\pi}{2}}$$

$$I = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q7

$$\text{Let } I = \int_0^a \frac{dx}{x + \sqrt{a^2 - x^2}}$$

Let $x = a\sin\theta$

$dx = a\cos\theta\, d\theta$

Now, $x = 0 \Rightarrow \theta = 0$

$x = a \Rightarrow \theta = \frac{\pi}{2}$

$$I = \int_0^{\frac{\pi}{2}} \frac{a\cos\theta\, d\theta}{a\sin\theta + a\cos\theta}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sin\theta + \cos\theta} \qquad --(\text{i})$$

So,

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2} - \theta\right)}{\sin\left(\frac{\pi}{2} - \theta\right) + \cos\left(\frac{\pi}{2} - \theta\right)} d\theta \qquad \left[\because \int_0^a f(x)dx = \int_0^a f(a - x)\,dx\right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin\theta}{\cos\theta + \sin\theta} \qquad --(\text{ii})$$

Adding (i) & (ii) we get

$$2I = \int_0^{\frac{\pi}{2}} \frac{\cos\theta + \sin\theta}{\cos\theta + \sin\theta} d\theta$$

$$2I = \int_0^{\frac{\pi}{2}} d\theta$$

$$2I = \frac{1}{2}[\theta]_0^{\frac{\pi}{2}}$$

$$I = \frac{\pi}{4}$$

Definite Integrals Ex 20.4B Q8

Put $\quad x = \tan\theta$

$\Rightarrow \quad dx = \sec^2\theta d\theta$

If $\quad x = 0,\ \theta = 0$

If $\quad x = \infty,\ \theta = \frac{\pi}{2}$

$\therefore \quad I = \int_0^\infty \frac{\log x}{1+x^2}dx$

$$= \int_0^{\frac{\pi}{2}} \frac{\log(\tan\theta)\sec^2\theta d\theta}{1+\tan^2\theta}$$

$$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \log(\tan\theta)d\theta \qquad ---(i)$$

$$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \log\tan\left(\frac{\pi}{2}-\theta\right)d\theta$$

$$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \log\cot(\theta)d\theta \qquad ---(ii)$$

Adding (i) and (ii), we get

$$2I = \int_0^{\frac{\pi}{2}} (\log\tan\theta + \log\cot\theta)d\theta$$

$$\Rightarrow \quad 2I = \int_0^{\frac{\pi}{2}} \log 1 \times dx = \int_0^{\frac{\pi}{2}} 0 \times dx = 0$$

$\Rightarrow \quad I = 0$

*********** END **********