Functions Ex 3.4 Q1

We have,

$$f(x) = x^3 + 1 \text{ and } g(x) = x + 1$$

Now,

$$f + g : R \to R \text{ given by } (f + g)(x) = x^3 + x + 2$$

$$f - g : R \to R \text{ given by } (f - g)(x) = x^3 + 1 - (x + 1)$$
$$= x^3 - x$$

$$cf : R \to R \text{ given by } (cf)(x) = c(x^3 + 1)$$

$$fg : R \to R \text{ given by } (fg)(x) = (x^3 + 1)(x + 1)$$
$$= x^4 + x^3 + x + 1$$

$$\frac{1}{f} : R - \{-1\} \to \text{R given by } \left(\frac{1}{f}\right)(x) = \frac{1}{x^3 + 1}$$

$$\frac{f}{g} : R - \{-1\} \to \text{R given by } \left(\frac{f}{g}\right)(x) = \frac{(x + 1)(x^2 - x + 1)}{x + 1}$$
$$= x^2 - x + 1$$

We have,

$$f(x) = \sqrt{x - 1} \text{ and } g(x) = \sqrt{x + 1}$$

Now,

$$f + g : (1, \infty) \to R \text{ defined by } (f + g)(x) = \sqrt{x - 1} + \sqrt{x + 1},$$

$$f - g : (1, \infty) \to R \text{ defined by } (f - g)(x) = \sqrt{x - 1} - \sqrt{x + 1},$$

$$cf : (1, \infty) \to R \text{ defined by } (cf)(x) = c\sqrt{x - 1},$$

$$fg : (1, \infty) \to R \text{ defined by } (fg)(x) = \left(\sqrt{x - 1}\right)\left(\sqrt{x + 1}\right)$$
$$= \sqrt{x^2 - 1}$$

$$\frac{1}{f} : (1, \infty) \to \text{R defined by } \left(\frac{1}{f}\right)(x) = \frac{1}{\sqrt{x - 1}}$$

$$\frac{f}{g} : (1, \infty) \to \text{R defined by } \left(\frac{f}{g}\right)(x) = \sqrt{\frac{x - 1}{x + 1}}$$

Functions Ex 3.4 Q2

We have,

$$f(x) = 2x + 5 \text{ and } g(x) = x^2 + x$$

We observe that $f(x) = 2x + 5$ is defined for all $x \in R$.

So, $\text{domain}(f) = R$

Clearly $g(x) = x^2 + x$ is defined for all $x \in R$

So, $\text{domain}(g) = R$

$\therefore$ $\text{Domain}(f) \cap \text{Domain}(g) = R$

(i) Clearly, $(f + g) : R \to R$ is given by

$$\begin{aligned} (f+g)(x) &= f(x) + g(x) \\ &= 2x + 5 + x^2 + x \\ &= x^2 + 3x + 5 \end{aligned}$$

$\text{Domain}(f + g) = R$

(ii) We find that $f - g : R \to R$ is defined as

$$\begin{aligned} (f-g)(x) &= f(x) - g(x) \\ &= 2x + 5 - (x^2 + x) \\ &= 2x + 5 - x^2 - x \\ &= -x^2 + x + 5 \end{aligned}$$

$\text{Domain}(f - g) = R$

(iii) We find that $fg : R \to R$ is given by

$$\begin{aligned} (fg)(x) &= f(x) \times g(x) \\ &= (2x + 5) \times (x^2 + x) \\ &= 2x^3 + 2x^2 + 5x^2 + 5x \\ &= 2x^3 + 7x^2 + 5x \end{aligned}$$

$\text{Domain}(fg) = R$

(iv) We have,

$g(x) = x^2 + x$

$\therefore$ $f(x) = 0 \Rightarrow x^2 + x = 0$

$\Rightarrow$ $x(x + 1) = 0$

$\Rightarrow$ $x = 0$ or, $x = -1$

So, $\text{domain}\left(\frac{f}{g}\right) = \text{domain}(f) \cap \text{domain}(g) - \{x : g(x) = 0\}$

$= R - \{-\phi, 0\}$

We find that, $\frac{f}{g} : R - \{-1, 0\} \to R$ is given by $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x + 5}{x^2 + x}$

$\text{Domain}\left(\frac{f}{g}\right) = R - \{-1, 0\}$

Functions Ex 3.4 Q3

We have,

$$f(x) = \begin{cases} -1, & -2 \le x \le 0 \\ x - 1, & 0 < x \le 2 \end{cases}$$

Now,

$$f(|x|) = |x| - 1, \text{ where } -2 \le x \le 2$$

and $$|f(x)| = \begin{cases} 1, & -2 \le x \le 0 \\ -(x-1), & 0 \le x \le 1 \\ (x-1), & 1 \le x \le 2 \end{cases}$$

$$\therefore \quad g(x) = f(|x|) + |f(x)|$$

$$= \begin{cases} -x & -2 \le x \le 0 \\ 0, & 0 < x < 1 \\ 2(x-1), & 1 \le x \le 2 \end{cases}$$

*********** END **********