Differentiation Ex 11.5 Q54

The given function is $xy = e^{(x-y)}$

Taking logarithm on both the sides, we obtain

$$\log(xy) = \log\left(e^{x-y}\right)$$
$$\Rightarrow \log x + \log y = (x-y)\log e$$
$$\Rightarrow \log x + \log y = (x-y)\times 1$$
$$\Rightarrow \log x + \log y = x - y$$

Differentiating both sides with respect to x, we obtain

$$\frac{d}{dx}(\log x) + \frac{d}{dx}(\log y) = \frac{d}{dx}(x) - \frac{dy}{dx}$$
$$\Rightarrow \frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 1 - \frac{dy}{dx}$$
$$\Rightarrow \left(1+\frac{1}{y}\right)\frac{dy}{dx} = 1 - \frac{1}{x}$$
$$\Rightarrow \left(\frac{y+1}{y}\right)\frac{dy}{dx} = \frac{x-1}{x}$$
$$\therefore \frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}$$

Differentiation Ex 11.5 Q55

Given that $y^x + x^y + x^x = a^b$.

Putting $u = y^x$, $v = x^y$ and $w = x^x$, we get $u + v + w = a^b$

Therefore $\frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} = 0$ ...(1)

Now, $u = y^x$. Taking logrithm on both sides, we have

$$\log u = x \log y$$

Differentiating both sides w.r.t. x, we have

$$\frac{1}{u}\cdot\frac{du}{dx} = x\frac{d}{dx}(\log y) + \log y\frac{d}{dx}(x)$$

$$= x\frac{1}{y}\cdot\frac{dy}{dx} + \log y.1$$

So $$\frac{du}{dx} = u\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) = y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right] \quad ...(2)$$

Also $v = x^y$

Taking logarithm on both sides, we have

$$\log v = y \log x$$

Differentiating both sides w.r.t. x, we have

$$\frac{1}{v}\cdot\frac{dv}{dx} = y\frac{d}{dx}(\log x) + \log x\frac{dy}{dx}$$

$$= y.\frac{1}{x} + \log x.\frac{dy}{dx}$$

So $$\frac{dv}{dx} = v\left[\frac{y}{x} + \log x\frac{dy}{dx}\right]$$

$$= x^y\left[\frac{y}{x} + \log x\frac{dy}{dx}\right] \quad ...(3)$$

Again $w = x^x$

Takinglogarithm on both sides, we have

$$\log w = x \log x.$$

Differentiating both sides w.r.t. x, we have

$$\frac{1}{w}\cdot\frac{dw}{dx} = x\frac{d}{dx}(\log x) + \log x.\frac{d}{dx}(x)$$

$$= x.\frac{1}{x} + \log x\ .\ 1$$

i.e. $$\frac{dw}{dx} = w(1 + \log x)$$

$$= x^x(1 + \log x) \quad ...(4)$$

From (1), (2), (3), (4), we have

$$y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) + x^y\left(\frac{y}{x} + \log x\frac{dy}{dx}\right) + x^x(1 + \log x) = 0$$

or $$\left(x.y^{x-1} + x^y\ .\log x\right)\frac{dy}{dx} = -x^x(1 + \log x) - y.x^{y-1} - y^x \log y$$

Therefore $$\frac{dy}{dx} = \frac{-\left[y^x \log y + y.x^{y-1} + x^x(1 + \log x)\right]}{x.y^{x-1} + x^y \log x}$$

Differentiation Ex 11.5 Q56

Here $(\cos x)^y = (\cos y)^x$

Taking log on both sides,

$$\log(\cos x)^y = \log(\cos y)^x$$

$$y\log\cos x = x\log\cos y$$

Differentiating it with respect to x using the chain rule and product rule,

$$\frac{d}{dx}(y\log\cos x) = \frac{d}{dx}(x\log\cos y)$$

$$y\frac{d}{dx}\log\cos x + \log\cos x\frac{dy}{dx} = x\frac{d}{dx}\log\cos y + \log\cos y\frac{d}{dx}x$$

$$y\frac{1}{\cos x}(-\sin x) + \log\cos x\frac{dy}{dx} = x\frac{1}{\cos y}(-\sin y)\frac{dy}{dx} + \log\cos y$$

$$\left(\log\cos x + \frac{x\sin y}{\cos y}\right)\frac{dy}{dx} = \log\cos y + y\frac{\sin y}{\cos y}$$

$$(\log\cos x + x\tan y)\frac{dy}{dx} = \log\cos y + y\tan y$$

$$\frac{dy}{dx} = \frac{\log\cos y + y\tan y}{(\log\cos x + x\tan y)}$$

Differentiation Ex 11.5 Q57

Consider the given function,

$\cos y = x \cos(a+y)$, where $\cos a \neq \pm 1$

Differentiating both sides w.r.t. 'x' we get

$$-\sin y \frac{dy}{dx} = x\left(-\sin(a+y)\frac{dy}{dx}\right) + \cos(a+y)$$

$$\Rightarrow \frac{dy}{dx}\left[x\sin(a+y) - \sin y\right] = \cos(a+y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos(a+y)}{x\sin(a+y) - \sin y}$$

Multiplying the numerator and the denominator by $\cos(a+y)$ on the R.H.S., we have,

$$\frac{dy}{dx} = \frac{\cos^2(a+y)}{x\cos(a+y)\sin(a+y) - \cos(a+y)\sin y}$$

$$= \frac{\cos^2(a+y)}{\cos y\sin(a+y) - \cos(a+y)\sin y} \quad \left[\because \cos y = x\cos(a+y),\ \text{given function}\right]$$

$$= \frac{\cos^2(a+y)}{\sin\left[(a+y) - y\right]} = \frac{\cos^2(a+y)}{\sin a}$$

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