



Combinations Ex 17.1 Q14

$$\begin{aligned} \text{If } {}^nC_r &= {}^nC_p \\ \text{then } r + p &= n \\ \therefore 16 &= r + r + 2 \\ r &= 7 \end{aligned}$$

$$\begin{aligned} \text{then } {}^rC_4 &= {}^7C_4 \quad (\because r = 7) \\ \Rightarrow \frac{7!}{4!(7-4)!} & \quad \left(\because {}^nC_r = \frac{n!}{r!(n-r)!} \right) \\ \Rightarrow \frac{7 \times 5 \times 6}{3 \times 2} \\ &= 35 \end{aligned}$$

Combinations Ex 17.1 Q15

$$\begin{aligned} & {}^{20}C_5 + \sum_{r=2}^5 {}^{25-r}C_4 \\ \Rightarrow & \left({}^{20}C_5 + {}^{20}C_4 \right) + {}^{21}C_4 + {}^{22}C_4 + {}^{23}C_4 \\ \Rightarrow & \left({}^{21}C_5 + {}^{21}C_4 \right) + {}^{22}C_4 + {}^{23}C_4 \quad \left(\because {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r \right) \\ \Rightarrow & \left({}^{22}C_5 + {}^{22}C_4 \right) + {}^{23}C_4 \quad \left(\because {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r \right) \\ \Rightarrow & {}^{23}C_5 + {}^{23}C_4 \quad \left(\because {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r \right) \\ \Rightarrow & {}^{24}C_5 \\ \Rightarrow & 42504 \end{aligned}$$

Combinations Ex 17.1 Q16

$$\begin{aligned}
\text{Product} &= [(2n+1)(2n+3)(2n+5)\dots(2n+r)] \\
&= \frac{(2n)![(2n+1)(2n+3)\dots(2n+r)]}{(2n)!} \\
&= \frac{(2n)[(2n-1)(2n-2)\dots 4.2(2n+1)(2n+3)]}{(2n)!} \\
&= \frac{(2n+r)!}{(2n)!}
\end{aligned}$$

Hence $r = 2n$

$$\begin{aligned}
&= \frac{(2n+2n)!}{2n} \\
&= \frac{(4n)!}{(2n)!} \\
&= (2n)!
\end{aligned}$$

Combinations Ex 17.1 Q17

$$\text{L.H.S.} = {}^{2n}C_n + {}^{2n}C_{n-1}$$

$$\begin{aligned}
&\frac{2n!}{n!n!} + \frac{2n!}{(n-1)!(n-1)!} \\
&= (2n)! \left[\frac{1}{n(n-1)!(n-1)!} + \frac{1}{(n-1)!(n-1)!} \right] \\
&= \frac{(2n)!}{(n-1)!(n-1)!} \left[\frac{1+n^2}{n^2} \right] \dots\dots\dots (i)
\end{aligned}$$

$$\begin{aligned}
{}^{2n+2}C_{n+1} &= \frac{(2n+2)!}{(n+1)!(n+1)!} \\
&= \frac{(2n+2)(2n+1)(2n)!}{n(n+1)(n-1)!(n+1)n(n-1)!} \dots\dots\dots (ii) \\
&= \frac{(2n)!}{(n-1)!(n-1)!} \times \frac{(n+1)^2(n)^2(n-1)!(n-1)!}{(2n+2)(2n+1)(2n)!} \times \left(\frac{1+n^2}{n^2} \right) \\
&= \frac{(n+1)^2(1+n^2)}{(2n+2)(2n+1)} = \frac{(n+1)(n+1)(n^2+1)}{2(n+1)(2n+1)} \\
&= \frac{(n+1)(n^2+1)}{(2n+1)} \times \frac{1}{2}
\end{aligned}$$

***** END *****