Trigonometric Identities Ex 6.1 Q58

Answer :

In the given question, we need to prove $\left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2=\frac{1-\cos\theta}{1+\cos\theta}$

Taking $\sin\theta$ common from the numerator and the denominator of the L.H.S, we get

$$\left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2=\left(\frac{(\sin\theta)(\operatorname{cosec}\theta+1-\cot\theta)}{(\sin\theta)(\operatorname{cosec}\theta+1+\cot\theta)}\right)^2$$
$$=\left(\frac{1+\operatorname{cosec}\theta-\cot\theta}{1+\operatorname{cosec}\theta+\cot\theta}\right)^2$$

Now, using the property $1+\cot^2\theta=\operatorname{cosec}^2\theta$, we get

$$\left(\frac{1+\operatorname{cosec}\theta-\cot\theta}{1+\operatorname{cosec}\theta+\cot\theta}\right)^2=\left(\frac{(\operatorname{cosec}^2\theta-\cot^2\theta)+\operatorname{cosec}\theta-\cot\theta}{1+\operatorname{cosec}\theta+\cot\theta}\right)^2$$

Using $a^2-b^2=(a+b)(a-b)$, we get

$$\left(\frac{(\operatorname{cosec}^2\theta-\cot^2\theta)+\operatorname{cosec}\theta-\cot\theta}{1+\operatorname{cosec}\theta+\cot\theta}\right)^2=\left(\frac{(\operatorname{cosec}\theta+\cot\theta)(\operatorname{cosec}\theta-\cot\theta)+(\operatorname{cosec}\theta-\cot\theta)}{1+\operatorname{cosec}\theta+\cot\theta}\right)^2$$

Taking $\operatorname{cosec}\theta-\cot\theta$ common from the numerator, we get

$$\left(\frac{(\operatorname{cosec}\theta+\cot\theta)(\operatorname{cosec}\theta-\cot\theta)+\operatorname{cosec}\theta-\cot\theta}{1+\operatorname{cosec}\theta+\cot\theta}\right)^2=\left(\frac{(\operatorname{cosec}\theta-\cot\theta)(\operatorname{cosec}\theta+\cot\theta+1)}{1+\operatorname{cosec}\theta+\cot\theta}\right)^2$$
$$=(\operatorname{cosec}\theta-\cot\theta)^2$$

Using $\cot\theta=\frac{\cos\theta}{\sin\theta}$ and $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$, we get

$$(\operatorname{cosec}\theta-\cot\theta)^2=\left(\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\right)^2$$
$$=\left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$
$$=\frac{(1-\cos\theta)^2}{\sin^2\theta}$$

Now, using the property $\sin^2\theta+\cos^2\theta=1$, we get

$$\frac{(1-\cos\theta)^2}{\sin^2\theta}=\frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$
$$=\frac{(1-\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)}$$
$$=\frac{1-\cos\theta}{1+\cos\theta}$$

Hence proved.

Trigonometric Identities Ex 6.1 Q59

Answer :

We have to prove $(\sec A+\tan A-1)(\sec A-\tan A+1)=2\tan A$

We know that, $\sec^2 A-\tan^2 A=1$

So, we have

$$\begin{aligned}(\sec A+\tan A-1)(\sec A-\tan A+1)&=\{\sec A+(\tan A-1)\}\{\sec A-(\tan A-1)\}\\&=\sec^2 A-(\tan A-1)^2\\&=\sec^2 A-(\tan^2 A-2\tan A+1)\\&=(\sec^2 A-\tan^2 A)+2\tan A-1\end{aligned}$$

So, we have

$$\begin{aligned}(\sec A+\tan A-1)(\sec A-\tan A+1)&=1+2\tan A-1\\&=2\tan A\end{aligned}$$

Hence proved.

*********** END **********