Polynomials Ex 2.1 Q15

Answer :

Since α and β are the zeros of the quadratic polynomial $f(x) = x^2 - 1$

The roots are $\alpha and \beta$

$$\alpha + \beta = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\alpha + \beta = \frac{0}{1}$$

$$\alpha + \beta = 0$$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\alpha\beta = \frac{-1}{1}$$

$$\alpha\beta = -1$$

Let S and P denote respectively the sum and product of zeros of the required polynomial. Then,

$$S = \frac{2\alpha}{\beta} + \frac{2\beta}{\alpha}$$

Taking least common factor we get,

$$S = \frac{2\alpha^2 + 2\beta^2}{\alpha\beta}$$

$$S = \frac{2\left(\alpha^2 + \beta^2\right)}{\alpha\beta}$$

$$S = \frac{2\left[(\alpha + \beta) - 2\alpha\beta\right]}{\alpha\beta}$$

$$S = \frac{2[0 - 2 \times -1]}{-1}$$

$$S = \frac{2[-2 \times -1]}{-1}$$

$$S = \frac{2 \times 2}{-1}$$

$$S = \frac{4}{-1}$$

$$S = -4$$

$$P = \frac{2\alpha}{\beta} \times \frac{2\beta}{\alpha}$$

$$P = \frac{2\cancel{\alpha}}{\cancel{\beta}} \times \frac{2\cancel{\beta}}{\cancel{\alpha}}$$

$$P = 4$$

Hence, the required polynomial $f(x)$ is given by,

$$f(x) = k\left(x^2 - Sx + p\right)$$

$$f(x) = k\left(x^2 - (-4)x + 4\right)$$

$$f(x) = k\left(x^2 + 4x + 4\right)$$

Hence, required equation is $\boxed{f(x) = k\left(x^2 + 4x + 4\right)}$ Where k is any non zero real number.

Polynomials Ex 2.1 Q16

Answer :

Since $\alpha and \beta$ are the zeros of the quadratic polynomial $f(x)=x^2-3x-2$

The roots are α and β

$$\alpha+\beta=\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\alpha+\beta=-\left(\frac{-3}{1}\right)$$

$$\alpha+\beta=-(-3)$$

$$\alpha+\beta=3$$

$$\alpha\beta=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\alpha\beta=\frac{-2}{1}$$

$$\alpha\beta=-2$$

Let S and P denote respectively the sum and the product of zero of the required polynomial . Then,

$$S=\frac{1}{2\alpha+\beta}+\frac{1}{2\beta+\alpha}$$

Taking least common factor then we have ,

$$S=\frac{1}{2\alpha+\beta}\times\frac{2\beta+\alpha}{2\beta+\alpha}+\frac{1}{2\beta+\alpha}\times\frac{2\alpha+\beta}{2\alpha+\beta}$$

$$S=\frac{2\beta+\alpha}{(2\alpha+\beta)(2\beta+\alpha)}+\frac{2\alpha+\beta}{(2\beta+\alpha)(2\alpha+\beta)}$$

$$S=\frac{2\beta+\alpha+2\alpha+\beta}{(2\alpha+\beta)(2\beta+\alpha)}$$

$$S=\frac{3\beta+3\alpha}{4\alpha\beta+2\beta^2+2\alpha^2+\beta\alpha}$$

$$S=\frac{3(\beta+\alpha)}{5\alpha\beta+2(\alpha^2+\beta^2)}$$

$$S=\frac{3(\beta+\alpha)}{5\alpha\beta+2\left[(\alpha+\beta)^2-2\alpha\beta\right]}$$

By substituting $\alpha+\beta=3$ and $\alpha\beta=-2$ we get ,

$$S=\frac{3(3)}{5(-2)+2\left[(3)^2-2\times-2\right]}$$

$$S=\frac{9}{-10+2(13)}$$

$$S=\frac{9}{-10+26}$$

$$S=\frac{9}{16}$$

$$P=\frac{1}{2\alpha+\beta}\times\frac{1}{2\beta+\alpha}$$

$$P=\frac{1}{(2\alpha+\beta)(2\beta+\alpha)}$$

$$P = \frac{1}{4\alpha\beta + 2\beta^2 + 2\alpha^2 + \beta\alpha}$$

$$P = \frac{1}{5\alpha\beta + 2\left(\alpha^2 + \beta^2\right)}$$

$$P = \frac{1}{5\alpha\beta + 2\left[\left(\alpha + \beta\right)^2 - 2\alpha\beta\right]}$$

By substituting $\alpha + \beta = 3$ and $\alpha\beta = -2$ we get ,

$$P = \frac{1}{5(-2) + 2\left[(3)^2 - 2 \times -2\right]}$$

$$P = \frac{1}{10 + 2[9 + 4]}$$

$$P = \frac{1}{10 + 2(13)}$$

$$P = \frac{1}{-10 + 26}$$

$$P = \frac{1}{16}$$

Hence ,the required polynomial $f(x)$ is given by

$$f(x) = k\left(x^2 - Sx + P\right)$$

$$f(x) = k\left(x^2 - \frac{9}{16}x + \frac{1}{16}\right)$$

Hence, the required equation is $\boxed{f(x) = k\left(x^2 - \frac{9}{16}x + \frac{1}{16}\right)}$ Where k is any non zero real number.

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