Adjoint and Inverse of Matrix Ex 7.2 Q13

$$A = \begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$$

Now, $\quad A = I\ A$

$$\begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$$

Applying $R_1 \to \frac{1}{2} R_1$

$$\begin{bmatrix} 1 & \frac{-1}{2} & 2 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$$

Applying $R_2 \to R_2 - 4R_1,\ R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & \frac{-1}{2} & 2 \\ 0 & 2 & -6 \\ 0 & \frac{-1}{2} & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -2 & 1 & 0 \\ \frac{-3}{2} & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to \frac{1}{2} . R_2$

$$\begin{bmatrix} 1 & \frac{-1}{2} & 2 \\ 0 & 1 & -3 \\ 0 & \frac{-1}{2} & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -1 & \frac{1}{2} & 0 \\ \frac{-3}{2} & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 + \frac{1}{2}R_2,\ R_3 \to R_3 + \frac{1}{2}R_2$

$$\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & -3 \\ 0 & 0 & \frac{-1}{2} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{4} & 0 \\ -1 & \frac{1}{2} & 0 \\ -2 & \frac{1}{4} & 1 \end{bmatrix}.A$$

Applying $R_3 \to (-2).R_3$

$$\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{4} & 0 \\ -1 & \frac{1}{2} & 0 \\ 4 & \frac{-1}{2} & -2 \end{bmatrix}.A$$

Applying $R_1 \to R_1 - \frac{1}{2}R_3,\ R_2 \to R_2 + 3R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & \frac{1}{2} & 1 \\ 11 & -1 & -6 \\ 4 & \frac{-1}{2} & -2 \end{bmatrix}.A$$

$$I = B.A$$

Hence, B is the inv. of A.

Adjoint and Inverse of Matrix Ex 7.2 Q14

$$A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

Now, $\quad A = I\,A$

$$\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.A$$

$$\begin{bmatrix} 0 & 4 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

Applying $R_1 \to \frac{1}{3}R_1$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{3} \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$$

Applying $R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{3} \\ 0 & 3 & \frac{2}{3} \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ \frac{-2}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}A$$

Applying $R_2 \to \frac{1}{3}R_2$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{3} \\ 0 & 1 & \frac{2}{9} \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ \frac{-2}{9} & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix}A$$

Applying $R_3 \to R_3 - 4R_2$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{3} \\ 0 & 1 & \frac{2}{9} \\ 0 & 0 & \frac{1}{9} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ \frac{-2}{9} & \frac{1}{3} & 0 \\ \frac{8}{9} & \frac{-4}{3} & 1 \end{bmatrix}A$$

Applying $R_3 \to 9.R_3$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{3} \\ 0 & 1 & \frac{2}{9} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ \frac{-2}{9} & \frac{1}{3} & 0 \\ 8 & -12 & 9 \end{bmatrix} A$$

Applying $R_1 \to R_1 + \frac{1}{3}R_3,\ R_2 \to R_2 - \frac{2}{9}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix}.A$$

or $\qquad I = B.A$

Hence, B is the inv. of A.

Adjoint and Inverse of Matrix Ex 7.2 Q15

Consider the given matrix:

$$Let\ A=\begin{bmatrix}1 & 3 & -2\\ -3 & 0 & 1\\ 2 & 1 & 0\end{bmatrix}$$

We know that $A = IA$

Thus, we have,

$$\begin{bmatrix}1 & 3 & -2\\ -3 & 0 & 1\\ 2 & 1 & 0\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}A$$

Applying $R_2 \to 3R_1 + R_2$ and $R_3 \to R_3 - 2R_1$, we have,

$$\begin{bmatrix}1 & 3 & -2\\ 0 & 9 & -5\\ 0 & -5 & 4\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 3 & 1 & 0\\ -2 & 0 & 1\end{bmatrix}A$$

Applying $R_1 \to R_1 - 3R_2$ and $R_3 \to R_3 + 5R_2$, we have,

$$\begin{bmatrix}1 & 0 & \frac{-1}{3}\\ 0 & 1 & \frac{-5}{9}\\ 0 & 0 & \frac{11}{9}\end{bmatrix}=\begin{bmatrix}0 & \frac{-1}{3} & 0\\ \frac{1}{3} & \frac{1}{9} & 0\\ \frac{-1}{3} & \frac{5}{9} & 1\end{bmatrix}A$$

Applying $R_1 \to \frac{R_1}{\frac{11}{9}}$ we have,

$$\begin{bmatrix}1 & 0 & \frac{-1}{3}\\ 0 & 1 & \frac{-5}{9}\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}0 & \frac{-1}{3} & 0\\ \frac{1}{3} & \frac{1}{9} & 0\\ \frac{-3}{11} & \frac{5}{11} & \frac{9}{11}\end{bmatrix}A$$

Applying $R_2 \to R_2 + \frac{5}{9}R_3$ and $R_1 \to R_1 + \frac{1}{3}R_3$, we have,

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}\frac{-1}{11} & \frac{-2}{11} & \frac{3}{11}\\ \frac{2}{11} & \frac{4}{11} & \frac{5}{11}\\ \frac{-3}{11} & \frac{5}{11} & \frac{9}{11}\end{bmatrix}A$$

$$\Rightarrow \textit{Inverse of the given matrix is}\begin{bmatrix}\frac{-1}{11} & \frac{-2}{11} & \frac{3}{11}\\ \frac{2}{11} & \frac{4}{11} & \frac{5}{11}\\ \frac{-3}{11} & \frac{5}{11} & \frac{9}{11}\end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.2 Q16

$$\textit{Consider the given matrix}\begin{bmatrix}-1 & 1 & 2\\ 1 & 2 & 3\end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 1 \end{bmatrix}$$

Let $A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

We know that $A = IA$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to (-1)R_1$, *we have*

$$\begin{bmatrix} 1 & -1 & -2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - 3R_1$, *we have*

$$\begin{bmatrix} 1 & -1 & -2 \\ 0 & 3 & 5 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to \dfrac{R_2}{3}$, *we have,*

$$\begin{bmatrix} 1 & -1 & -2 \\ 0 & 1 & \frac{5}{3} \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 + R_2$ *and* $R_3 \to R_3 - 4R_2$, *we have*

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{5}{3} & \frac{-4}{3} & 1 \end{bmatrix} A$$

Applying $R_3 \to \dfrac{R_3}{3}$, *we have*

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ 5 & -4 & 3 \end{bmatrix} A$$

Applying $R_1 \to R_1 + \dfrac{1}{3}R_3$, $R_2 \to R_2 - \dfrac{5}{3}R_3$, *we have,*

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} A$$

Thus, the inverse of the given matrix is $\begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$.

*********** END **********