Trigonometric Ratios Ex 5.1 Q26

Answer :

Given:

$\cot\theta = \frac{3}{4}$ ...... (1)

To prove:

$\sqrt{\frac{\sec\theta - \operatorname{cosec}\theta}{\sec\theta + \operatorname{cosec}\theta}} = \frac{1}{\sqrt{7}}$

Now we know $\tan\theta$ is defined as follows

$\cot\theta = \frac{\text{Base side adjacent to}\angle\theta}{\text{Perpendicular side opposite to}\angle\theta}$ ...... (2)

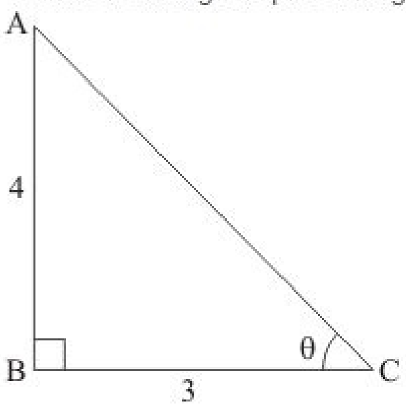
Now by comparing equation (1) and (2)

We get,

Base side adjacent to $\angle\theta$ = 3

Perpendicular side opposite to $\angle\theta$ = 4

Therefore triangle representing angle θ is as shown below



Side *AC* is unknown and can be found using Pythagoras theorem

Therefore,

$$AC^2 = AB^2 + BC^2$$

Now by substituting the value of known sides from figure

We get,

$$\begin{aligned} AC^2 &= 4^2 + 3^2 \\ &= 16 + 9 \\ &= 25 \end{aligned}$$

Now by taking square root on both sides

We get,

$$\begin{aligned} AC &= \sqrt{25} \\ &= 5 \end{aligned}$$

Therefore Hypotenuse side *AC* = 5 …… (3)

Now we know, $\sin\theta$ is defined as follows

$$\sin\theta = \frac{\text{Perpendicular side opposite to} \angle\theta}{\text{Hypotenuse}}$$

Therefore from figure (a) and equation (3)

We get,

$$\begin{aligned} \sin\theta &= \frac{AB}{AC} \\ &= \frac{4}{5} \end{aligned}$$

$$\sin\theta = \frac{4}{5} \text{ …… (4)}$$

Now we know $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

Therefore by substituting the value of $\sin\theta$ from equation (4)

We get,

$$\begin{aligned} \operatorname{cosec}\theta &= \frac{1}{\frac{4}{5}} \\ &= \frac{5}{4} \end{aligned}$$

Therefore,

$$\operatorname{cosec}\theta = \frac{5}{4} \text{ …… (5)}$$

Now we know, $\cos\theta$ is defined as follows

$$\cos\theta = \frac{\text{Base side adjacent to} \angle\theta}{\text{Hypotenuse}}$$

Therefore from figure (a) and equation (3)

We get,

$$\begin{aligned} \cos\theta &= \frac{BC}{AC} \\ &= \frac{3}{5} \end{aligned}$$

$$\cos\theta = \frac{3}{5} \text{ …… (6)}$$

Now we know $\sec\theta = \frac{1}{\cos\theta}$

Therefore by substituting the value of $\cos\theta$ from equation (6)

We get,

$$\sec\theta = \frac{1}{\frac{3}{5}}$$
$$= \frac{5}{3}$$

Therefore,

$$\sec\theta = \frac{5}{3} \quad \ldots\ldots (7)$$

Now, in expression $\sqrt{\frac{\sec\theta - \operatorname{cosec}\theta}{\sec\theta + \operatorname{cosec}\theta}}$, by substituting the value of $\operatorname{cosec}\theta$ and $\sec\theta$ from equation (6) and (7) respectively, we get,

$$\sqrt{\frac{\sec\theta - \operatorname{cosec}\theta}{\sec\theta + \operatorname{cosec}\theta}} = \sqrt{\frac{\frac{5}{3} - \frac{5}{4}}{\frac{5}{3} + \frac{5}{4}}}$$

L.C.M of 3 and 4 is 12

Now by taking L.C.M in above expression

We get,

$$\sqrt{\frac{\sec\theta - \operatorname{cosec}\theta}{\sec\theta + \operatorname{cosec}\theta}} = \sqrt{\frac{\frac{5\times 4}{3\times 4} - \frac{5\times 3}{4\times 3}}{\frac{5\times 4}{3\times 4} + \frac{5\times 3}{4\times 3}}}$$

$$= \sqrt{\frac{\frac{20}{12} - \frac{15}{12}}{\frac{20}{12} + \frac{15}{12}}}$$

$$= \sqrt{\frac{\frac{20-15}{12}}{\frac{20+15}{12}}}$$

$$= \sqrt{\frac{\frac{5}{12}}{\frac{35}{12}}}$$

$$= \sqrt{\frac{5}{12} \times \frac{12}{35}}$$

Now 12 gets cancelled and we get,

$$\sqrt{\frac{\sec\theta - \operatorname{cosec}\theta}{\sec\theta + \operatorname{cosec}\theta}} = \sqrt{\frac{5}{35}}$$

Now $35 = 5 \times 7$

Therefore,

$$\sqrt{\frac{\sec\theta - \operatorname{cosec}\theta}{\sec\theta + \operatorname{cosec}\theta}} = \sqrt{\frac{5}{5 \times 7}}$$

Now 5 gets cancelled and we get,

$$\sqrt{\frac{\sec\theta - \operatorname{cosec}\theta}{\sec\theta + \operatorname{cosec}\theta}} = \sqrt{\frac{1}{7}}$$

$$= \frac{1}{\sqrt{7}}$$

Therefore, it is proved that $\sqrt{\frac{\sec\theta - \operatorname{cosec}\theta}{\sec\theta + \operatorname{cosec}\theta}} = \frac{1}{\sqrt{7}}$

*********** END **********