Binary Operations Ex 3.2 Q4(i)

'$*$' is a binary operator on Z defined by $a*b = a+b+ab$ for all $a,b \in Z$.

Commutativity of '$*$':

Let $a,b \in Z$, then
$$a*b = a+b+ab = b+a+ba = b*a$$

$\therefore \qquad a*b = b*a$

Associative of '$*$':

Let $a,b \in Z$, then
$$(a*b)*c = (a+b+ab)*c = a+b+ab+c+ac+bc+abc$$
$$= a+b+c+ab+bc+ac+abc \qquad\qquad ---(i)$$

Again, $a*(b*c) = a*(b+c+bc)$
$$= a+b+c+bc+ab+ac+abc \qquad\qquad ---(ii)$$

From (i) & (ii), we get
$$(a*b)*c = a*(b*c)$$

$\therefore \qquad *$ is commutative and associative on Z

Binary Operations Ex 3.2 Q4(ii)

Commutative:

Let $a,b \in N$, then
$$a*b = 2^{ab} = 2^{ba} = b*a$$

$\therefore \qquad a*b = b*a$

$\therefore \qquad *$ is commutative on N

Associative:

Let $a,b,c \in N$, then
$$(a*b)*c = 2^{ab}*c = 2^{2^{ab}\cdot c} \qquad\qquad ---(i)$$

and, $\quad a*(b*c) = a*2^{bc} = 2^{a\cdot 2^{bc}} \qquad\qquad ---(ii)$

From (i) & (ii), we get
$$(a*b)*c \neq a*(b*c)$$

$\therefore \qquad *$ is not associative on N

Binary Operations Ex 3.2 Q4(iii)

Commutativity:

Let $a, b \in Q$, then
$$a * b = a - b \neq b - a = b * a$$

$\therefore \quad a * b \neq b * a$

$\Rightarrow \quad *$ is not commutative on Q

Associative:

Let $a, b, c \in Q$, then
$$(a * b) * c = (a - b) * c = a - b - c \qquad \qquad ---(i)$$

and, $\quad a * (b * c) = a * (b - c) = a - b + c \qquad \qquad ---(ii)$

From (i) & (ii), we get
$$(a * b) * c \neq a * (b * c)$$

$\therefore \quad *$ is not associative on Q

Binary Operations Ex 3.2 Q4(iv)

Commutative:

Let $a, b \in Q$, then
$$a \, e \, b = a^2 + b^2 = b^2 + a^2 = b \, e \, a$$

$\Rightarrow \quad a \, e \, b = b \, e \, a$

$\therefore \quad e$ is commutative on Q.

Associative:

Let $a, b, c \in Q$, then
$$(a \, e \, b) \, e \, c = \left(a^2 + b^2\right) \, e \, c = \left(a^2 + b^2\right)^2 + c^2 \qquad ---(i)$$

and, $\quad a \, e \, (b \, e \, c) = a \, e \, \left(a^2 + b^2\right) = a^2 + \left(b^2 + c^2\right)^2 \qquad ---(ii)$

From (i) & (ii),
$$(a \, e \, b) \, e \, c \neq a \, e \, (b \, e \, c)$$

$\therefore \quad e$ is not associative on Q.

Binary Operations Ex 3.2 Q4(v)

Binary operation 'o' defined on Q, given by $aob = \dfrac{ab}{2}$ for all $a, b \in Q$

Commutative:

Let $a, b \in Q$, then
$$a \circ b = \frac{ab}{2} = \frac{ba}{2} = b \circ a$$

$\Rightarrow \qquad a \circ b = b \circ a$

$\therefore \qquad$ o is commutative on Q.

Associativity:

Let $a, b, c \in Q$, then
$$(a \circ b) \circ c = \left(\frac{ab}{2}\right) \circ c = \frac{abc}{4} \qquad\qquad ---(i)$$
$$a \circ (b \circ c) = a \circ \left(\frac{bc}{2}\right) = \frac{abc}{4} \qquad\qquad ---(ii)$$

From (i) & (ii) we get
$$(a \circ b) \circ c = a \circ (b \circ c)$$

$\therefore \qquad$ 'o' is associative on Q.

Binary Operations Ex 3.2 Q4(vi)
Commutative:

Let $a, b \in Q$, then
$$a * b = ab^2 \neq ba^2 = b * a$$

$\Rightarrow \qquad a * b \neq b * a$

$\therefore \qquad *$ is not commutative on Q

Associativity:

Let $a, b, c \in Q$, then
$$(a * b) * c = ab^2 * c = ab^2c^2 \qquad\qquad ---(i)$$
$$\& \qquad a * (b * c) = a * bc^2 = a\left(bc^2\right)^2 \qquad\qquad ---(ii)$$

From (i) and (ii)
$$(a * b) * c \neq a * (b * c)$$

$\therefore \qquad *$ is not associative on Q

Binary Operations Ex 3.2 Q4(vii)

Commutativity:

Let $a, b \in Q$, then
$$a * b = a + ab \qquad \qquad ---(i)$$
$$b * a = b + ab \qquad \qquad ---(ii)$$

From (i) & (ii)
$$a * b \neq b * a$$

$\Rightarrow \qquad *$ is not commutative on Q

Associativity:

Let $a, b, c \in Q$, then
$$(a * b) * c = (a + ab) * c = a + ab + ac + abc \qquad \qquad ---(i)$$
$$a * (b * c) = a * (b + bc)$$
$$= a + ab + abc \qquad \qquad ---(ii)$$

From (i) and (ii)
$$(a * b) * c \neq a * (b * c)$$

$\Rightarrow \qquad *$ is not associative on Q

********** END *********