



Algebra of Matrices Ex 5.5 Q1

Given,

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

$$\begin{aligned} (A - A^T) &= \left(\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^T \right) \\ &= \left(\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} \right) \\ &= \begin{bmatrix} 2-2 & 3-4 \\ 4-3 & 5-5 \end{bmatrix} \end{aligned}$$

$$(A - A^T) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad \text{---(i)}$$

$$\begin{aligned} -(A - A^T)^T &= -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^T \\ &= -\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \end{aligned}$$

$$-(A - A^T)^T = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad \text{---(ii)}$$

From (i) and (ii),

$$(A - A^T) = -(A - A^T)^T$$

We know that, x is a skew symmetric matrix if $x = -x^T$

So, $(A - A^T)$ is skew symmetric.

Algebra of Matrices Ex 5.5 Q2

Given,

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

$$\begin{aligned} A - A^T &= \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}^T \\ &= \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 3-3 & -4-1 \\ 1+4 & -1+1 \end{bmatrix} \end{aligned}$$

$$A - A^T = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} \quad \text{--- (i)}$$

$$\begin{aligned} -(A - A^T)^T &= -\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}^T \\ &= -\begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix} \end{aligned}$$

$$-(A - A^T)^T = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} \quad \text{--- (ii)}$$

From equation (i) and (ii),

$$(A - A^T) = -(A - A^T)^T$$

We know that, x is skewsymmetric matrix if $x = -x^T$

So, $(A - A^T)$ is skewsymmetric matrix.

Algebra of Matrices Ex 5.5 Q3

Given,

$$A = \begin{bmatrix} 5 & 2 & x \\ y & z & -3 \\ 4 & t & -7 \end{bmatrix} \text{ is a symmetric matrix.}$$

We know that $A = [a_{ij}]_{m \times n}$ is a symmetric matrix if $a_{ij} = a_{ji}$

$$\text{So, } x = a_{13} = a_{31} = 4$$

$$y = a_{21} = a_{12} = 2$$

$$z = a_{22} = a_{22} = z$$

$$t = a_{32} = a_{23} = -3$$

Hence,

$x = 4, y = 2, t = -3$ and z can have any value.

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