It is known that $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta$.

$\therefore \vec{a}\cdot\vec{b}\geq 0$

$\Rightarrow |\vec{a}||\vec{b}|\cos\theta\geq 0$

$\Rightarrow \cos\theta\geq 0 \qquad \left[|\vec{a}| \text{ and } |\vec{b}| \text{ are positive}\right]$

$\Rightarrow 0\leq\theta\leq\frac{\pi}{2}$

Hence, $\vec{a}.\vec{b}\geq 0$ when $0\leq\theta\leq\frac{\pi}{2}$.

The correct answer is B.

Question 17:

Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a}+\vec{b}$ is a unit vector if

(A) $\theta=\frac{\pi}{4}$ (B) $\theta=\frac{\pi}{3}$ (C) $\theta=\frac{\pi}{2}$ (D) $\theta=\frac{2\pi}{3}$

Answer

Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ be the angle between them.

Then, $|\vec{a}|=|\vec{b}|=1$.

Now, $\vec{a}+\vec{b}$ is a unit vector if $|\vec{a}+\vec{b}|=1$.

$|\vec{a}+\vec{b}|=1$

$\Rightarrow (\vec{a}+\vec{b})^2=1$

$\Rightarrow (\vec{a}+\vec{b}).(\vec{a}+\vec{b})=1$

$\Rightarrow \vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{b}.\vec{a}+\vec{b}.\vec{b}=1$

$\Rightarrow |\vec{a}|^2+2\vec{a}.\vec{b}+|\vec{b}|^2=1$

$\Rightarrow 1^2+2|\vec{a}||\vec{b}|\cos\theta+1^2=1$

$\Rightarrow 1+2.1.1\cos\theta+1=1$

$\Rightarrow \cos\theta=-\frac{1}{2}$

$\Rightarrow \theta=\frac{2\pi}{3}$

Hence, $\vec{a}+\vec{b}$ is a unit vector if $\theta=\frac{2\pi}{3}$.

The correct answer is D.

Question 18:

The value of $\hat{i}.(\hat{j}\times\hat{k})+\hat{j}.(\hat{i}\times\hat{k})+\hat{k}.(\hat{i}\times\hat{j})$ is

(A) 0 (B) −1 (C) 1 (D) 3

Answer

$\hat{i}.(\hat{j}\times\hat{k})+\hat{j}.(\hat{i}\times\hat{k})+\hat{k}.(\hat{i}\times\hat{j})$

$=\hat{i}\cdot\hat{i}+\hat{j}\cdot(-\hat{j})+\hat{k}\cdot\hat{k}$

$=1-\hat{j}\cdot\hat{j}+1$

$=1-1+1$

$=1$

The correct answer is C.

Question 19:

If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$, then $|\vec{a}.\vec{b}|=|\vec{a}\times\vec{b}|$ when θ is equal to

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π

Answer

Let θ be the angle between two vectors $\vec{a}$ and $\vec{b}$.

Then, without loss of generality, $\vec{a}$ and $\vec{b}$ are non-zero vectors, so that $|\vec{a}|$ and $|\vec{b}|$ are positive.

$$|\vec{a}\cdot\vec{b}| = |\vec{a}\times\vec{b}|$$

$$\Rightarrow |\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|\sin\theta$$

$$\Rightarrow \cos\theta = \sin\theta \qquad \left[|\vec{a}| \text{ and } |\vec{b}| \text{ are positive}\right]$$

$$\Rightarrow \tan\theta = 1$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

Hence, $|\vec{a}.\vec{b}| = |\vec{a}\times\vec{b}|$ when θ is equal to $\frac{\pi}{4}$.

The correct answer is B.

********** END **********