



Q21 : $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \, dx$

Answer :

Let $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \, dx$

As $\sin^2(-x) = (\sin(-x))^2 = (-\sin x)^2 = \sin^2 x$, therefore, $\sin^2 x$ is an even function.

It is known that if $f(x)$ is an even function, then $\int_{-a}^a f(x) \, dx = 2 \int_0^a f(x) \, dx$

$$\begin{aligned} I &= 2 \int_0^{\frac{\pi}{2}} \sin^2 x \, dx \\ &= 2 \int_0^{\frac{\pi}{2}} \frac{1 - \cos 2x}{2} \, dx \\ &= \int_0^{\frac{\pi}{2}} (1 - \cos 2x) \, dx \\ &= \left[x - \frac{\sin 2x}{2} \right]_0^{\frac{\pi}{2}} \\ &= \frac{\pi}{2} \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q22 : $\int_0^{\pi} \frac{x \, dx}{1 + \sin x}$

Answer :

Let $I = \int_0^{\pi} \frac{x \, dx}{1 + \sin x} \quad \dots(1)$

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1 + \sin(\pi - x)} \, dx \quad \left(\int_0^a f(x) \, dx = \int_0^a f(a - x) \, dx \right)$$

$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1 + \sin x} \, dx \quad \dots(2)$

Adding (1) and (2), we obtain

$$\begin{aligned} 2I &= \int_0^{\pi} \frac{\pi}{1 + \sin x} \, dx \\ \Rightarrow 2I &= \pi \int_0^{\pi} \frac{(1 - \sin x)}{(1 + \sin x)(1 - \sin x)} \, dx \\ \Rightarrow 2I &= \pi \int_0^{\pi} \frac{1 - \sin x}{\cos^2 x} \, dx \\ \Rightarrow 2I &= \pi \int_0^{\pi} \{\sec^2 x - \tan x \sec x\} \, dx \\ \Rightarrow 2I &= \pi [\tan x - \sec x]_0^{\pi} \\ \Rightarrow 2I &= \pi [2] \\ \Rightarrow I &= \pi \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q23 : $\int_0^{\pi} \frac{x \, dx}{1 + \sin x}$

Answer :

Let $I = \int_0^{\pi} \frac{x \, dx}{1 + \sin x} \quad \dots(1)$

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1 + \sin(\pi - x)} \, dx \quad \left(\int_0^a f(x) \, dx = \int_0^a f(a - x) \, dx \right)$$

$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1 + \sin x} \, dx \quad \dots(2)$

Adding (1) and (2), we obtain

$$\begin{aligned} 2I &= \int_0^{\pi} \frac{\pi}{1 + \sin x} \, dx \\ \Rightarrow 2I &= \pi \int_0^{\pi} \frac{(1 - \sin x)}{(1 + \sin x)(1 - \sin x)} \, dx \\ \Rightarrow 2I &= \pi \int_0^{\pi} \frac{1 - \sin x}{\cos^2 x} \, dx \\ \Rightarrow 2I &= \pi \int_0^{\pi} \{\sec^2 x - \tan x \sec x\} \, dx \end{aligned}$$

$$\Rightarrow 2I = \pi \left[\tan x - \sec x \right]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \pi [2]$$

$$\Rightarrow I = \pi$$

Answer needs Correction? [Click Here](#)

Q24 : $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x \, dx$

Answer :

Let $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x \, dx$... (1)

As $\sin^7 (-x) = (\sin (-x))^7 = (-\sin x)^7 = -\sin^7 x$, therefore, $\sin^2 x$ is an odd function.

It is known that, if $f(x)$ is an odd function, then $\int_{-a}^a f(x) \, dx = 0$

$$\therefore I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x \, dx = 0$$

Answer needs Correction? [Click Here](#)

Q25 : $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x \, dx$

Answer :

Let $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x \, dx$... (1)

As $\sin^7 (-x) = (\sin (-x))^7 = (-\sin x)^7 = -\sin^7 x$, therefore, $\sin^2 x$ is an odd function.

It is known that, if $f(x)$ is an odd function, then $\int_{-a}^a f(x) \, dx = 0$

$$\therefore I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x \, dx = 0$$

Answer needs Correction? [Click Here](#)

Q26 : $\int_0^{2\pi} \cos^5 x \, dx$

Answer :

Let $I = \int_0^{2\pi} \cos^5 x \, dx$... (1)

$$\cos^5 (2\pi - x) = \cos^5 x$$

It is known that,

$$\begin{aligned} \int_0^{2a} f(x) \, dx &= 2 \int_0^a f(x) \, dx, \text{ if } f(2a-x) = f(x) \\ &= 0 \text{ if } f(2a-x) = -f(x) \end{aligned}$$

$$\therefore I = 2 \int_0^{\pi} \cos^5 x \, dx$$

$$\Rightarrow I = 2(0) = 0 \quad \left[\cos^5 (\pi - x) = -\cos^5 x \right]$$

Answer needs Correction? [Click Here](#)

Q27 : $\int_0^{2\pi} \cos^5 x \, dx$

Answer :

Let $I = \int_0^{2\pi} \cos^5 x \, dx$... (1)

$$\cos^5 (2\pi - x) = \cos^5 x$$

It is known that,

$$\begin{aligned} \int_0^{2a} f(x) \, dx &= 2 \int_0^a f(x) \, dx, \text{ if } f(2a-x) = f(x) \\ &= 0 \text{ if } f(2a-x) = -f(x) \end{aligned}$$

$$\therefore I = 2 \int_0^{\pi} \cos^5 x \, dx$$

$$\Rightarrow I = 2(0) = 0 \quad \left[\cos^5 (\pi - x) = -\cos^5 x \right]$$

Answer needs Correction? [Click Here](#)

Q28 : $\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} \, dx$

Answer :

Let $I = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} \, dx$... (1)

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{1}{1 + \sin x \cos x} dx \quad \dots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right) - \cos\left(\frac{\pi}{2} - x\right)}{1 + \sin\left(\frac{\pi}{2} - x\right) \cos\left(\frac{\pi}{2} - x\right)} dx \quad \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1 + \sin x \cos x} dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{0}{1 + \sin x \cos x} dx$$

$$\Rightarrow I = 0$$

Answer needs Correction? [Click Here](#)

Q29 : $\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$

Answer :

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx \quad \dots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right) - \cos\left(\frac{\pi}{2} - x\right)}{1 + \sin\left(\frac{\pi}{2} - x\right) \cos\left(\frac{\pi}{2} - x\right)} dx \quad \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1 + \sin x \cos x} dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{0}{1 + \sin x \cos x} dx$$

$$\Rightarrow I = 0$$

Answer needs Correction? [Click Here](#)

Q30 : $\int_0^{\pi} \log(1 + \cos x) dx$

Answer :

$$\text{Let } I = \int_0^{\pi} \log(1 + \cos x) dx \quad \dots(1)$$

$$\Rightarrow I = \int_0^{\pi} \log(1 + \cos(\pi - x)) dx \quad \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx\right)$$

$$\Rightarrow I = \int_0^{\pi} \log(1 - \cos x) dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\pi} \{\log(1 + \cos x) + \log(1 - \cos x)\} dx$$

$$\Rightarrow 2I = \int_0^{\pi} \log(1 - \cos^2 x) dx$$

$$\Rightarrow 2I = \int_0^{\pi} \log \sin^2 x dx$$

$$\Rightarrow 2I = 2 \int_0^{\pi} \log \sin x dx$$

$$\Rightarrow I = \int_0^{\pi} \log \sin x dx \quad \dots(3)$$

$$\sin(\pi - x) = \sin x$$

$$\therefore I = 2 \int_0^{\frac{\pi}{2}} \log \sin x dx \quad \dots(4)$$

$$\Rightarrow I = 2 \int_0^{\frac{\pi}{2}} \log \sin\left(\frac{\pi}{2} - x\right) dx = 2 \int_0^{\frac{\pi}{2}} \log \cos x dx \quad \dots(5)$$

Adding (4) and (5), we obtain

$$2I = 2 \int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x) dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x + \log 2 - \log 2) dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} (\log 2 \sin x \cos x - \log 2) dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log \sin 2x dx - \int_0^{\frac{\pi}{2}} \log 2 dx$$

$$\text{Let } 2x = t \Rightarrow 2 dx = dt$$

$$\text{When } x = 0, t = 0$$

$$\text{and when}$$

$$\therefore$$

Answer needs Correction? [Click Here](#)

Q31 : $\int_0^{\pi} \log(1 + \cos x) dx$

Answer :

$$\text{Let } I = \int_0^{\pi} \log(1 + \cos x) dx \quad \dots(1)$$

$$\Rightarrow I = \int_0^{\pi} \log(1 + \cos(\pi - x)) dx \quad \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx \right)$$

$$\Rightarrow I = \int_0^{\pi} \log(1 - \cos x) dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\pi} \{\log(1 + \cos x) + \log(1 - \cos x)\} dx$$

$$\Rightarrow 2I = \int_0^{\pi} \log(1 - \cos^2 x) dx$$

$$\Rightarrow 2I = \int_0^{\pi} \log \sin^2 x dx$$

$$\Rightarrow 2I = 2 \int_0^{\pi} \log \sin x dx$$

$$\Rightarrow I = \int_0^{\pi} \log \sin x dx \quad \dots(3)$$

$$\sin(\pi - x) = \sin x$$

$$\therefore I = 2 \int_0^{\frac{\pi}{2}} \log \sin x dx \quad \dots(4)$$

$$\Rightarrow I = 2 \int_0^{\frac{\pi}{2}} \log \sin\left(\frac{\pi}{2} - x\right) dx = 2 \int_0^{\frac{\pi}{2}} \log \cos x dx \quad \dots(5)$$

Adding (4) and (5), we obtain

$$2I = 2 \int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x) dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x + \log 2 - \log 2) dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} (\log 2 \sin x \cos x - \log 2) dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log \sin 2x dx - \int_0^{\frac{\pi}{2}} \log 2 dx$$

$$\text{Let } 2x = t \Rightarrow 2 dx = dt$$

$$\text{When } x = 0, t = 0$$

$$\text{and when}$$

$$\therefore$$

Answer needs Correction? [Click Here](#)

Q32 : $\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx$

Answer :

$$\text{Let } I = \int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx \quad \dots(1)$$

$$\text{It is known that, } \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx \right)$$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x} + \sqrt{x}} dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^a \frac{\sqrt{x} + \sqrt{a-x}}{\sqrt{x} + \sqrt{a-x}} dx$$

$$\Rightarrow 2I = \int_0^a 1 dx$$

$$\Rightarrow 2I = [x]_0^a$$

$$\Rightarrow 2I = a$$

$$\Rightarrow I = \frac{a}{2}$$

Answer needs Correction? [Click Here](#)

Q33 : $\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx$

Answer :

$$\text{Let } I = \int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a-x}} dx \quad \dots(1)$$

$$\text{It is known that, } \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx \right)$$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x} + \sqrt{x}} dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^a \frac{\sqrt{x} + \sqrt{a-x}}{\sqrt{x} + \sqrt{a-x}} dx$$

$$\Rightarrow 2I = \int_0^a 1 dx$$

$$\Rightarrow 2I = [x]_0^a$$

$$\Rightarrow 2I = a$$

$$\Rightarrow I = \frac{a}{2}$$

Answer needs Correction? [Click Here](#)

Q34 : $\int_0^4 |x-1| dx$

Answer :

$$I = \int_0^4 |x-1| dx$$

It can be seen that, $(x-1) \leq 0$ when $0 \leq x \leq 1$ and $(x-1) \geq 0$ when $1 \leq x \leq 4$

$$\begin{aligned} I &= \int_0^1 |x-1| dx + \int_1^4 |x-1| dx & \left(\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx \right) \\ &= \int_0^1 -(x-1) dx + \int_1^4 (x-1) dx \\ &= \left[x - \frac{x^2}{2} \right]_0^1 + \left[\frac{x^2}{2} - x \right]_1^4 \\ &= 1 - \frac{1}{2} + \frac{(4)^2}{2} - 4 - \frac{1}{2} + 1 \\ &= 1 - \frac{1}{2} + 8 - 4 - \frac{1}{2} + 1 \\ &= 5 \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q35 : $\int_0^4 |x-1| dx$

Answer :

$$I = \int_0^4 |x-1| dx$$

It can be seen that, $(x-1) \leq 0$ when $0 \leq x \leq 1$ and $(x-1) \geq 0$ when $1 \leq x \leq 4$

$$\begin{aligned} I &= \int_0^1 |x-1| dx + \int_1^4 |x-1| dx & \left(\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx \right) \\ &= \int_0^1 -(x-1) dx + \int_1^4 (x-1) dx \\ &= \left[x - \frac{x^2}{2} \right]_0^1 + \left[\frac{x^2}{2} - x \right]_1^4 \\ &= 1 - \frac{1}{2} + \frac{(4)^2}{2} - 4 - \frac{1}{2} + 1 \\ &= 1 - \frac{1}{2} + 8 - 4 - \frac{1}{2} + 1 \\ &= 5 \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q36 : Show that $\int_0^a f(x)g(x)dx = 2 \int_0^a f(x)dx$, if f and g are defined as $f(x) = f(a-x)$ and $g(x) + g(a-x) = 4$

Answer :

$$\begin{aligned} \text{Let } I &= \int_0^a f(x)g(x)dx & \dots(1) \\ \Rightarrow I &= \int_0^a f(a-x)g(a-x)dx & \left(\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx \right) \\ \Rightarrow I &= \int_0^a f(x)g(a-x)dx & \dots(2) \end{aligned}$$

Adding (1) and (2), we obtain

$$\begin{aligned} 2I &= \int_0^a \{f(x)g(x) + f(x)g(a-x)\} dx \\ \Rightarrow 2I &= \int_0^a f(x)\{g(x) + g(a-x)\} dx \\ \Rightarrow 2I &= \int_0^a f(x) \times 4 dx & [g(x) + g(a-x) = 4] \\ \Rightarrow I &= 2 \int_0^a f(x) dx \end{aligned}$$

Answer needs Correction? [Click Here](#)

Q37 : Show that $\int_0^a f(x)g(x)dx = 2 \int_0^a f(x)dx$, if f and g are defined as $f(x) = f(a-x)$ and $g(x) + g(a-x) = 4$

Answer :

$$\begin{aligned} \text{Let } I &= \int_0^a f(x)g(x)dx & \dots(1) \\ \Rightarrow I &= \int_0^a f(a-x)g(a-x)dx & \left(\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx \right) \\ \Rightarrow I &= \int_0^a f(x)g(a-x)dx & \dots(2) \end{aligned}$$

Adding (1) and (2), we obtain

$$\begin{aligned} 2I &= \int_0^a \{f(x)g(x) + f(x)g(a-x)\} dx \\ \Rightarrow 2I &= \int_0^a f(x)\{g(x) + g(a-x)\} dx \\ \Rightarrow 2I &= \int_0^a f(x) \times 4 dx & [g(x) + g(a-x) = 4] \\ \Rightarrow I &= 2 \int_0^a f(x) dx \end{aligned}$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} f(x) dx$$

Answer needs Correction? [Click Here](#)

Q38 : The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3 + x \cos x + \tan^5 x + 1) dx$ is

- A. 0
- B. 2
- C. π
- D. 1

Answer :

$$\begin{aligned} \text{Let } I &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3 + x \cos x + \tan^5 x + 1) dx \\ \Rightarrow I &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^3 dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \tan^5 x dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 \cdot dx \end{aligned}$$

It is known that if $f(x)$ is an even function, then $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$ and

if $f(x)$ is an odd function, then $\int_{-a}^a f(x) dx = 0$

$$\begin{aligned} I &= 0 + 0 + 0 + 2 \int_0^{\frac{\pi}{2}} 1 \cdot dx \\ &= 2 \left[x \right]_0^{\frac{\pi}{2}} \\ &= \frac{2\pi}{2} \\ &= \pi \end{aligned}$$

Hence, the correct answer is C.

Answer needs Correction? [Click Here](#)

Q39 : The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3 + x \cos x + \tan^5 x + 1) dx$ is

- A. 0
- B. 2
- C. π
- D. 1

Answer :

$$\begin{aligned} \text{Let } I &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^3 + x \cos x + \tan^5 x + 1) dx \\ \Rightarrow I &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^3 dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \tan^5 x dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 \cdot dx \end{aligned}$$

It is known that if $f(x)$ is an even function, then $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$ and

if $f(x)$ is an odd function, then $\int_{-a}^a f(x) dx = 0$

$$\begin{aligned} I &= 0 + 0 + 0 + 2 \int_0^{\frac{\pi}{2}} 1 \cdot dx \\ &= 2 \left[x \right]_0^{\frac{\pi}{2}} \\ &= \frac{2\pi}{2} \\ &= \pi \end{aligned}$$

Hence, the correct answer is C.

Answer needs Correction? [Click Here](#)

Q40 : The value of $\int_0^{\frac{\pi}{2}} \log \left(\frac{4+3\sin x}{4+3\cos x} \right) dx$ is

- A. 2
- B. $\frac{3}{4}$
- C. 0
- D. -2

Answer :

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \log \left(\frac{4+3\sin x}{4+3\cos x} \right) dx \quad \dots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log \left[\frac{4+3\sin \left(\frac{\pi}{2} - x \right)}{4+3\cos \left(\frac{\pi}{2} - x \right)} \right] dx \quad \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx \right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log \left(\frac{4+3\cos x}{4+3\sin x} \right) dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$\begin{aligned} 2I &= \int_0^{\frac{\pi}{2}} \left[\log \left(\frac{4+3\sin x}{4+3\cos x} \right) + \log \left(\frac{4+3\cos x}{4+3\sin x} \right) \right] dx \\ \Rightarrow 2I &= \int_0^{\frac{\pi}{2}} \log \left(\frac{4+3\sin x}{4+3\cos x} \times \frac{4+3\cos x}{4+3\sin x} \right) dx \\ \Rightarrow 2I &= \int_0^{\frac{\pi}{2}} \log 1 dx \\ \Rightarrow 2I &= \int_0^{\frac{\pi}{2}} 0 dx \\ \Rightarrow I &= 0 \end{aligned}$$

Hence, the correct answer is C.

Answer needs Correction? [Click Here](#)

Q41 : The value of $\int_0^{\frac{\pi}{2}} \log \left(\frac{4+3\sin x}{4+3\cos x} \right) dx$ is

- A. 2
- B. $\frac{3}{4}$
- C. 0
- D. -2

Answer :

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \log \left(\frac{4+3\sin x}{4+3\cos x} \right) dx \quad \dots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log \left[\frac{4+3\sin \left(\frac{\pi}{2} - x \right)}{4+3\cos \left(\frac{\pi}{2} - x \right)} \right] dx \quad \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx \right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log \left(\frac{4+3\cos x}{4+3\sin x} \right) dx \quad \dots(2)$$

Adding (1) and (2), we obtain

$$\begin{aligned} 2I &= \int_0^{\frac{\pi}{2}} \left[\log \left(\frac{4+3\sin x}{4+3\cos x} \right) + \log \left(\frac{4+3\cos x}{4+3\sin x} \right) \right] dx \\ \Rightarrow 2I &= \int_0^{\frac{\pi}{2}} \log \left(\frac{4+3\sin x}{4+3\cos x} \times \frac{4+3\cos x}{4+3\sin x} \right) dx \\ \Rightarrow 2I &= \int_0^{\frac{\pi}{2}} \log 1 dx \\ \Rightarrow 2I &= \int_0^{\frac{\pi}{2}} 0 dx \\ \Rightarrow I &= 0 \end{aligned}$$

Hence, the correct answer is C.

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