Relations Ex 1.2 Q9

(i) We have, L is the set of lines.

$R = \{(L_1, L_2): \text{ L}_1 \text{ is parallel to L}_2\}$ be a relation on L

Now,

Reflexivity: Let $L_1 \in L$

Since a line is always parallel to itself.

$\therefore \quad (L_1, L_2) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $L_1, L_2 \in L$ and $(L_1, L_2) \in R$

$\Rightarrow \quad L_1$ is parallel to L_2

$\Rightarrow \quad L_2$ is parallel to L_1

$\Rightarrow \quad (L_1, L_2) \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let $L_1, L_2 and L_3 \in L$ such that $(L_1, L_2) \in R$ and $(L_2, L_3) \in R$

$\Rightarrow \quad L_1$ is parallel to L_2 and L_2 is parallel to L_3

$\Rightarrow \quad L_1$ is parallel to L_3

$\Rightarrow \quad (L_1, L_3) \in R$

$\Rightarrow \quad R$ is transitive

Since, R is reflexive, symmetric and transitive, so R is an equivalence relation.

(ii) The set of lines parallel to the line $y = 2x + 4$ is

$y = 2x + c$ For all $c \in R$

Where R is the set of real numbers.

Relations Ex 1.2 Q10

R = {(P_1, P_2): P_1 and P2 have same the number of sides}

R is reflexive since (P_1, P_1) ∈ R as the same polygon has the same number of sides with itself.

Let (P_1, P_2) ∈ R.

⇒ P_1 and P_2 have the same number of sides.

⇒ P_2 and P_1 have the same number of sides.

⇒ (P_2, P_1) ∈ R

∴R is symmetric.

Now,

Let (P_1, P_2), (P_2, P3) ∈ R.

⇒ P_1 and P_2 have the same number of sides. Also, P_2 and P3 have the same number of sides.

⇒ P_1 and P3 have the same number of sides.

⇒ (P_1, P3) ∈ R

∴R is transitive.

Hence, R is an equivalence relation.

The elements in A related to the right-angled triangle (T) with sides 3, 4, and 5 are those polygons which have 3 sides (since T is a polygon with 3 sides).

Hence, the set of all elements in A related to triangle T is the set of all triangles.

Relations Ex 1.2 Q11

Let A be set of points on plane.
Let $R = \{(P, Q) : OP = OQ\}$ be a relation on A where O is the origin.

To prove R is an equivalence relation, we need to show that R is reflexive, symmetric and transitive on A.

Now,
Reflexivity: Let $p \in A$

Since $OP = OP \Rightarrow \quad (P, P) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $(P, Q) \in R$ for $P, Q \in A$

Then $OP = OQ$
$\Rightarrow \quad OQ = OP$
$\Rightarrow \quad (Q, P) \in R$
$\Rightarrow \quad R$ is symmetric

Transitive: Let $(P, Q) \in R$ and $(Q, S) \in R$

$\Rightarrow \quad OP = OQ$ and $OQ = OS$
$\Rightarrow \quad OP = OS$
$\Rightarrow \quad (P, S) \in R$

$\Rightarrow \quad R$ is transitive

Thus, R is an equivalence relation on A

Relations Ex 1.2 Q12

Given A={1,2,3,4,5,6,7} and R={(a,b):both a and b are either odd or even number}
Therefore,
R={(1,1),(1,3),(1,5),(1,6),(3,3),(3,5),(3,7),(5,5),(5,7),(7,7),(7,5),(7,3),(5,3),(6,1),(5,1),(3,1),
(2,2),(2,4),(2,6),(4,4),(4,6),(6,6),(6,4),(6,2),(4,2)}
Form the relation R it is seen that R is symmetric, reflecive and transitive also. Therefore R is an equivalent relation.
From the relation R it is seen that {1,3,5,7} are related with each other only and {2,4,6} are related with each other

*********** END **********