



### Trigonometric Ratios of Compound Angles Ex 7.1 Q25

We have,

$$\tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3$$

$$\Rightarrow \tan x + \left[ \frac{\tan x + \tan \frac{\pi}{3}}{1 - \tan x \tan \frac{\pi}{3}} \right] + \left[ \frac{\tan x + \tan\left(\frac{2\pi}{3}\right)}{1 - \tan x \tan \frac{2\pi}{3}} \right] = 3$$

$$\Rightarrow \tan x + \left[ \frac{\tan x + \sqrt{3}}{1 - \sqrt{3} \tan x} \right] + \left[ \frac{\tan x + \tan\left(\frac{\pi}{2} + \frac{\pi}{3}\right)}{1 - \tan x \tan\left(\frac{\pi}{2} + \frac{\pi}{3}\right)} \right] = 3$$

$$\Rightarrow \tan x + \frac{\tan x + \sqrt{3}}{1 - \sqrt{3} \tan x} + \frac{\tan x - \cot \frac{\pi}{3}}{1 + \tan x \cot \frac{\pi}{3}} = 3 \quad \left[ \because \tan \theta \text{ is negative in } \begin{matrix} \text{second quadrant} \end{matrix} \right]$$

$$\Rightarrow \tan x + \frac{\tan x + \sqrt{3}}{1 - \sqrt{3} \tan x} + \frac{\tan x - \sqrt{3}}{1 + \sqrt{3} \tan x} = 3$$

$$\Rightarrow \tan x + \frac{(\tan x + \sqrt{3})(1 + \sqrt{3} \tan x) + (\tan x - \sqrt{3})(1 - \sqrt{3} \tan x)}{(1 - \sqrt{3} \tan x)(1 + \sqrt{3} \tan x)} = 3$$

$$\Rightarrow \tan x + \frac{\tan x + \sqrt{3} \tan^2 x + \sqrt{3} + 3 \tan x + \tan x - \sqrt{3} \tan^2 x - \sqrt{3} + 3 \tan x}{1 - (\sqrt{3} \tan x)^2} = 3$$

$$\Rightarrow \tan x + \frac{8 \tan x}{1 - 3 \tan^2 x} = 3$$

$$\Rightarrow \frac{\tan x (1 - 3 \tan^2 x) + 8 \tan x}{1 - 3 \tan^2 x} = 3$$

$$\Rightarrow \frac{\tan x - 3 \tan^3 x + 8 \tan x}{1 - 3 \tan^2 x} = 3$$

$$\Rightarrow \frac{9 \tan x - 3 \tan^3 x}{1 - 3 \tan^2 x} = 3$$

$$\Rightarrow \frac{3(3 \tan x - \tan^3 x)}{1 - 3 \tan^2 x} = 3$$

$$\Rightarrow \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} = 1$$

Hence proved.

### Trigonometric Ratios of Compound Angles Ex 7.1 Q26

We have,

$$\sin(\alpha + \beta) = 1$$

$$\Rightarrow \sin(\alpha + \beta) = \sin \frac{\pi}{2}$$

$$\Rightarrow \alpha + \beta = \frac{\pi}{2} \quad \text{--- (i)}$$

$$\text{and, } \sin(\alpha - \beta) = \frac{1}{2}$$

$$\Rightarrow \sin(\alpha - \beta) = \sin \frac{\pi}{6}$$

$$\Rightarrow \alpha - \beta = \frac{\pi}{6} \quad \text{--- (ii)}$$

Adding equations (i) and (ii), we get

$$2\alpha = \frac{\pi}{2} + \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

$$\Rightarrow \alpha = \frac{\pi}{3}$$

Putting  $\alpha = \frac{\pi}{3}$  in equation (i), we get

$$\frac{\pi}{3} + \beta = \frac{\pi}{2}$$

$$\Rightarrow \beta = \frac{\pi}{2} - \frac{\pi}{3}$$

$$\Rightarrow \beta = \frac{3\pi - 2\pi}{6}$$

$$= \frac{\pi}{6}$$

$$\Rightarrow \beta = \frac{\pi}{6}$$

$$\begin{aligned}
\text{Now, } \tan(\alpha + 2\beta) &= \tan\left(\frac{\pi}{3} + 2 \times \frac{\pi}{6}\right) \\
&= \tan\left(\frac{\pi}{3} + \frac{\pi}{3}\right) \\
&= \tan\frac{2\pi}{3} \\
&= \tan\left(\frac{\pi}{2} + \frac{\pi}{6}\right) \\
&= -\cot\frac{\pi}{6} & \left[ \because \tan\theta \text{ is negative in} \right. \\
&= -\sqrt{3} & \left. \text{second quadrant} \right]
\end{aligned}$$

$$\therefore \tan(\alpha + 2\beta) = -\sqrt{3}$$

$$\begin{aligned}
\text{and, } \tan(2\alpha + \beta) &= \tan\left(2 \times \frac{\pi}{3} + \frac{\pi}{6}\right) \\
&= \tan\left(\frac{2\pi}{3} + \frac{\pi}{6}\right) \\
&= \tan\left(\frac{4\pi}{6} + \frac{\pi}{6}\right) \\
&= \tan\left(\frac{5\pi}{6}\right) \\
&= \tan\left(\frac{\pi}{2} + \frac{\pi}{6}\right) \\
&= -\cot\frac{\pi}{6} & \left[ \because \tan\theta \text{ is negative in} \right. \\
&= -\frac{1}{\sqrt{3}} & \left. \text{second quadrant} \right]
\end{aligned}$$

$$\therefore \tan(2\alpha + \beta) = \frac{-1}{\sqrt{3}}$$

#### Trigonometric Ratios of Compound Angles Ex 7.1 Q27

We have,

$$6 \cos \theta + 8 \sin \theta = 9 \quad \text{--- (i)}$$

$$\begin{aligned}
\Rightarrow 8 \sin \theta &= 9 - 6 \cos \theta \\
\Rightarrow (8 \sin \theta)^2 &= (9 - 6 \cos \theta)^2 & [\because \text{Squaring both sides}] \\
\Rightarrow 64 \sin^2 \theta &= 81 + 36 \cos^2 \theta - 108 \cos \theta \\
\Rightarrow 64 \sin^2 \theta &= 81 + 36 \cos^2 \theta - 108 \cos \theta \\
\Rightarrow 64(1 - \cos^2 \theta) &= 81 + 36 \cos^2 \theta - 108 \cos \theta \\
\Rightarrow 64 - 64 \cos^2 \theta &= 81 + 36 \cos^2 \theta - 108 \cos \theta \\
\Rightarrow 36 \cos^2 \theta + 64 \cos^2 \theta - 108 \cos \theta + 81 - 64 &= 0 \\
\Rightarrow 100 \cos^2 \theta - 108 \cos \theta + 17 &= 0 & \text{--- (ii)}
\end{aligned}$$

Since  $\alpha, \beta$  are roots of equation (ii).

Therefore,  $\cos \alpha$  and  $\cos \beta$  are roots of equation (ii)

$$\therefore \cos \alpha + \cos \beta = \frac{17}{100} \quad \text{--- (iii)}$$

Again,  $6 \cos \theta + 8 \sin \theta = 9$

$$\begin{aligned}
\Rightarrow 6 \cos \theta &= 9 - 8 \sin \theta \\
\Rightarrow (6 \cos \theta)^2 &= (9 - 8 \sin \theta)^2 & [\because \text{Squaring both sides}] \\
\Rightarrow 36 \cos^2 \theta &= 81 + 64 \sin^2 \theta - 144 \sin \theta \\
\Rightarrow 36(1 - \sin^2 \theta) &= 81 + 64 \sin^2 \theta - 144 \sin \theta \\
\Rightarrow 36 - 36 \sin^2 \theta &= 81 + 64 \sin^2 \theta - 144 \sin \theta \\
\Rightarrow 64 \sin^2 \theta + 36 \sin^2 \theta - 144 \sin \theta + 81 - 36 &= 0 \\
\Rightarrow 100 \sin^2 \theta - 144 \sin \theta + 45 &= 0 & \text{--- (iv)}
\end{aligned}$$

$$\therefore \sin \alpha \times \sin \beta = \frac{45}{100} \quad \text{--- (v)}$$

$$\begin{aligned} \text{Now, } \cos(\alpha + \beta) &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ &= \frac{17}{100} - \frac{45}{100} \quad \text{[Using equation (iii) and (v)]} \\ &= -\frac{28}{100} \\ &= -\frac{7}{25} \end{aligned}$$

$$\begin{aligned} \text{Now, } \sin(\alpha + \beta) &= \sqrt{1 - (\cos \theta)^2} \\ &= \sqrt{1 - \left(-\frac{7}{25}\right)^2} \\ &= \sqrt{1 - \frac{49}{625}} \\ &= \sqrt{\frac{625 - 49}{625}} \\ &= \sqrt{\frac{576}{625}} \\ &= \frac{24}{25} \end{aligned}$$

$$\therefore \sin(\alpha + \beta) = \frac{24}{25}$$

#### Trigonometric Ratios of Compound Angles Ex 7.1 Q28

$$\begin{aligned} \sin(\alpha + \beta) &= \sqrt{1 - \cos^2(\alpha + \beta)} \\ b^2 + a^2 &= (\cos \alpha + \cos \beta)^2 + (\sin \alpha + \sin \beta)^2 \\ \Rightarrow b^2 + a^2 &= (\cos^2 \alpha + \sin^2 \alpha) + (\cos^2 \beta + \sin^2 \beta) + 2(\cos \alpha \cos \beta + \sin \alpha \sin \beta) \\ \Rightarrow b^2 + a^2 &= 1 + 1 + 2 \cos(\alpha - \beta) = 2 + 2 \cos(\alpha - \beta) \\ \text{and, } b^2 - a^2 &= (\cos \alpha + \cos \beta)^2 - (\sin \alpha + \sin \beta)^2 \\ b^2 - a^2 &= \cos^2 \alpha + \cos^2 \beta - \sin^2 \alpha - \sin^2 \beta + 2(\cos \alpha \cos \beta - \sin \alpha \sin \beta) \\ \Rightarrow b^2 - a^2 &= (\cos^2 \alpha - \sin^2 \alpha) + (\cos^2 \beta - \sin^2 \beta) + 2 \cos(\alpha + \beta) \\ \Rightarrow b^2 - a^2 &= \cos(\alpha + \beta) \cos(\alpha - \beta) + \cos(\beta + \alpha) \cos(\beta - \alpha) + 2 \cos(\alpha + \beta) \\ \Rightarrow b^2 - a^2 &= 2 \cos(\alpha + \beta) \cos(\alpha - \beta) + 2 \cos(\alpha + \beta) \\ &\quad \left[ \because \cos(\beta - \alpha) = \cos\{-(\alpha - \beta)\} = \cos(\alpha - \beta) \right] \\ \Rightarrow b^2 - a^2 &= \cos(\alpha + \beta) \{2 \cos(\alpha - \beta) + 2\} \\ \Rightarrow b^2 - a^2 &= \cos(\alpha + \beta) (b^2 + a^2) \quad \text{[Using (i)]} \end{aligned}$$

$$\text{Thus, } b^2 - a^2 = (b^2 + a^2) \cos(\alpha + \beta)$$

$$\Rightarrow \cos(\alpha + \beta) = \frac{b^2 - a^2}{b^2 + a^2}$$

$$\Rightarrow \sin(\alpha + \beta) = \sqrt{1 - \left(\frac{b^2 - a^2}{b^2 + a^2}\right)^2} = \frac{\sqrt{4a^2b^2}}{\sqrt{(a^2 + b^2)^2}} = \frac{2ab}{a^2 + b^2}$$

$$\begin{aligned}
b^2 + a^2 &= (\cos \alpha + \cos \beta)^2 + (\sin \alpha + \sin \beta)^2 \\
\Rightarrow b^2 + a^2 &= (\cos^2 \alpha + \sin^2 \alpha) + (\cos^2 \beta + \sin^2 \beta) + 2(\cos \alpha \cos \beta + \sin \alpha \sin \beta) \\
\Rightarrow b^2 + a^2 &= 1 + 1 + 2 \cos(\alpha - \beta) = 2 + 2 \cos(\alpha - \beta)
\end{aligned}$$

$$\begin{aligned}
\text{and, } b^2 - a^2 &= (\cos \alpha + \cos \beta)^2 - (\sin \alpha + \sin \beta)^2 \\
b^2 - a^2 &= \cos^2 \alpha + \cos^2 \beta - \sin^2 \alpha - \sin^2 \beta + 2(\cos \alpha \cos \beta - \sin \alpha \sin \beta) \\
\Rightarrow b^2 - a^2 &= (\cos^2 \alpha - \sin^2 \beta) + (\cos^2 \beta - \sin^2 \alpha) + 2 \cos(\alpha + \beta) \\
\Rightarrow b^2 - a^2 &= \cos(\alpha + \beta) \cos(\alpha - \beta) + \cos(\beta + \alpha) \cos(\beta - \alpha) + 2 \cos(\alpha + \beta) \\
\Rightarrow b^2 - a^2 &= 2 \cos(\alpha + \beta) \cos(\alpha - \beta) + 2 \cos(\alpha + \beta) \\
&\quad \left[ \because \cos(\beta - \alpha) = \cos\{-(\alpha - \beta)\} = \cos(\alpha - \beta) \right] \\
\Rightarrow b^2 - a^2 &= \cos(\alpha + \beta) \{2 \cos(\alpha - \beta) + 2\} \\
\Rightarrow b^2 - a^2 &= \cos(\alpha + \beta) (b^2 + a^2) \quad \quad \quad [\text{Using (i)}]
\end{aligned}$$

$$\text{Thus, } b^2 - a^2 = (b^2 + a^2) \cos(\alpha + \beta)$$

$$\Rightarrow \cos(\alpha + \beta) = \frac{b^2 - a^2}{b^2 + a^2}$$

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