Indefinite Integrals Ex 19.31 Q5

Let $I = \int \frac{x^2 - 3x + 1}{x^4 + x^2 + 1} dx$

Dividing numerator and denominator by x^2

$$\therefore \quad I = \int \frac{1 - \frac{3}{x} + \frac{1}{x^2}}{x^2 + 1 + \frac{1}{x^2}} dx$$

$$= \int \frac{\left(1 + \frac{1}{x^2}\right) dx}{\left(x - \frac{1}{x}\right)^2 + 3} - \int \frac{3x}{x^4 + x^2 + 1} dx$$

Let $\left(x - \frac{1}{x}\right) = t$

$\Rightarrow \quad \left(1 + \frac{1}{x^2}\right) dx = dt \qquad$ [For Ist part]

Let $\quad x^2 = z$

$\Rightarrow \quad 2x dx = dz \qquad$ [For IInd part]

$$\therefore \quad I = \int \frac{dt}{t^2 + 3} - \frac{3}{2} \int \frac{dz}{z^2 + z + 1}$$

$$\Rightarrow \quad = \int \frac{dt}{t^2 + 3} - \frac{3}{2} \int \frac{dz}{\left(z + \frac{1}{2}\right)^2 + \frac{3}{4}}$$

$$\Rightarrow \quad = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{t}{\sqrt{3}}\right) - \frac{3}{2} \times \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{z + \frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c$$

$$\Rightarrow \quad I = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x^2 - 1}{\sqrt{3}x}\right) - \sqrt{3} \tan^{-1}\left(\frac{2z + 1}{\sqrt{3}}\right) + c$$

Hence,

$$I = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x^2 - 1}{\sqrt{3}x}\right) - \sqrt{3} \tan^{-1}\left(\frac{2x^2 + 1}{\sqrt{3}}\right) + c$$

Indefinite Integrals Ex 19.31 Q6

Let $I = \int \frac{x^2+1}{x^4-x^2+1} dx$

Dividing numerator and denominator by x^2

$$\therefore \quad I = \int \frac{\left(1+\frac{1}{x^2}\right)}{x^2-1+\frac{1}{x^2}} dx$$

$$= \int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+1}$$

Let $\left(x-\frac{1}{x}\right) = t$

$$\Rightarrow \quad \left(1+\frac{1}{x^2}\right)dx = dt$$

$$\Rightarrow \quad I = \int \frac{dt}{t^2+1}$$

$$= tan^{-1} t + c$$

$$\therefore \quad I = tan^{-1}\left(\frac{x^2-1}{x}\right) + c$$

Indefinite Integrals Ex 19.31 Q7

Let $I = \int \frac{x^2-1}{x^4+1} dx$

Dividing numerator and denominator by x^2

$$\therefore \quad I = \int \frac{\left(1-\frac{1}{x^2}\right)}{x^2+\frac{1}{x^2}} dx$$

$$= \int \frac{\left(1-\frac{1}{x^2}\right)}{\left(x+\frac{1}{x}\right)^2-2} dx$$

Let $\left(x+\frac{1}{x}\right) = t$

$$\Rightarrow \quad \left(1-\frac{1}{x^2}\right)dx = dt$$

$$\therefore \quad I = \int \frac{dt}{t^2-2}$$

$$= \frac{1}{2\sqrt{2}} log\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right| + c$$

So,

$$I = \frac{1}{2\sqrt{2}} log\left|\frac{x^2+1-\sqrt{2}x}{x^2+1+\sqrt{2}x}\right| + c$$

Indefinite Integrals Ex 19.31 Q8

Let $I = \int \frac{x^2+1}{x^4+7x^2+1}dx$

Dividing numerator and denominator by x^2

$$\therefore \qquad I = \int \frac{\left(1+\frac{1}{x^2}\right)}{x^2+7+\frac{1}{x^2}}dx$$

$$= \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x-\frac{1}{x}\right)^2+9}dx$$

Let $\left(x-\frac{1}{x}\right) = t$

$$\Rightarrow \qquad \left(1+\frac{1}{x^2}\right)dx = dt$$

$$\therefore \qquad I = \int \frac{dt}{t^2+9}$$

$$= \frac{1}{3}tan^{-1}\left|\frac{t}{3}\right|+c$$

Hence,

$$I = \frac{1}{3}tan^{-1}\left(\frac{x^2-1}{3x}\right)+c$$

*********** END **********