Definite Integrals Ex 20.5 Q21

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 0$, $b = 2$ and $f(x) = 3x^2 - 2$

$$\therefore \quad h = \frac{2}{n} \quad \Rightarrow \quad nh = 2$$

Thus, we have,

$$I = \int_0^2 \left(3x^2 - 2\right)dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(0+h)+f(0+2h)+----f\{0+(n-1)h\}\right]$$

$$= \lim_{h\to 0} h\left[-2+\left(3h^2-2\right)+\left\{3(2h)^2-2\right\}+----\right]$$

$$= \lim_{h\to 0} h\left[-2h+3h^2\left(1+2^2+3^2+----\right)\right]$$

$$\because h = \frac{2}{n} \quad \text{if } h\to 0 \Rightarrow n\to\infty$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[-2n+\frac{12}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} -4+\frac{4}{n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right) = -4+8 = 4$$

$$\therefore \quad \int_0^2 \left(3x^2-2\right)dx = 4$$

Definite Integrals Ex 20.5 Q22

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a=0,\ b=2$ and $f(x)=x^2+2$

$$\therefore\quad h=\frac{2}{n} \ \Rightarrow\ nh=2$$

Thus, we have,

$$I=\int_0^2 (x^2+2)\,dx$$

$$=\lim_{h\to 0} h\left[f(0)+f(0+h)+f(2h)+----f(0+(n-1)h)\right]$$

$$=\lim_{h\to 0} h\left[2+(h^2+2)+\{(2h)^2+2\}+----\right]$$

$$=\lim_{h\to 0} h\left[2h+h^2\left(1+2^2+3^2+----(n-1)^2\right]\right.$$

$$\because h=\frac{2}{n}\ \ \&\ \text{if } h\to 0 \Rightarrow n\to\infty$$

$$=\lim_{n\to\infty} \frac{2}{n}\left[2n+\frac{4}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$=\lim_{n\to\infty}\ 4+\frac{4}{3n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$=\ 4+\frac{8}{3}\ =\frac{20}{3}$$

$$\therefore\quad \int_0^2 (x^2+2)\,dx=\frac{20}{3}$$

Definite Integrals Ex 20.5 Q23

It is known that,

$$\int_a^b f(x)\,dx=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+\ldots+f(a+(n-1)h)\right], \text{ where } h=\frac{b-a}{n}$$

Here, $a=0$, $b=4$, and $f(x)=x+e^{2x}$

$$\therefore h=\frac{4-0}{n}=\frac{4}{n}$$

$$\Rightarrow \int_0^4 (x+e^{2x})\,dx=(4-0)\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f(h)+f(2h)+\ldots+f((n-1)h)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[(0+e^0)+(h+e^{2h})+(2h+e^{2\cdot2h})+\ldots+\left\{(n-1)h+e^{2(n-1)h}\right\}\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[1+(h+e^{2h})+(2h+e^{4h})+\ldots+\left\{(n-1)h+e^{2(n-1)h}\right\}\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\left\{h+2h+3h+\ldots+(n-1)h\right\}+\left(1+e^{2h}+e^{4h}+\ldots+e^{2(n-1)h}\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[h\left\{1+2+\ldots(n-1)\right\}+\left(\frac{e^{2hn}-1}{e^{2h}-1}\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\frac{(h(n-1)n)}{2}+\left(\frac{e^{2hn}-1}{e^{2h}-1}\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\frac{4}{n}\cdot\frac{(n-1)n}{2}+\left(\frac{e^8-1}{e^{\frac{8}{n}}-1}\right)\right]$$

$$=4(2)+4\lim_{n\to\infty}\frac{(e^8-1)}{\left(\frac{e^{\frac{8}{n}}-1}{\frac{8}{n}}\right)8}$$

$$=8+\frac{4\cdot(e^8-1)}{8} \qquad \left(\lim_{x\to0}\frac{e^x-1}{x}=1\right)$$

$$=8+\frac{e^8-1}{2}$$

$$=\frac{15+e^8}{2}$$

Definite Integrals Ex 20.5 Q24

We have,

$$\int_a^b f(x)\,dx=\lim_{h\to0}h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

where $h=\dfrac{b-a}{n}$

Thus, we have,

$$I=\int_0^2 (x^2+x)\,dx$$

$$=\lim_{h\to0}h\left[f(0)+f(0+h)+f(0+2h)+----f\{(n-1)h\}\right]$$

$$=\lim_{h\to0}h\left[0+(h^2+h)+\left\{(2h)^2+2h\right\}+----\right]$$

$$=\lim_{h\to0}h\left[\left\{h^2\left(1+2^2+3^2+----(n-1)^2\right\}+h\left\{1+2+3----(n-1)\right\}\right]$$

$$\because h=\frac{2}{n} \quad \& \text{ if } h\to0\Rightarrow n\to\infty$$

$$=\lim_{n\to\infty}\frac{2}{n}\left[\frac{4}{n^2}\frac{n(n-1)(2n-1)}{6}+\frac{2}{n}\frac{n(n-1)}{2}\right]$$

$$=\lim_{n\to\infty}\frac{4}{3n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+\frac{2}{n^2}n^2\left(1-\frac{1}{n}\right)$$

$$=\frac{8}{3}+2=\frac{14}{3}$$

$$\therefore \int_0^2 (x^2+x)\,dx=\frac{14}{3}$$

********** END *********