Mean Value Theorems Ex 15.1 Q2(viii)

Here, $f(x) = x^2 + 5x + 6$ on $[-3,-2]$

$f(x)$ is continuous is $[-3,-2]$ and $f(x)$ is differentiable is $(-3,-2)$ since it is a polynomial function.

Now,

$$f(x) = x^2 + 5x + 6$$

$$f(-3) = (-3)^2 + 5(-3) + 6$$

$$= 9 - 15 + 6$$

$$f(-3) = 0 \qquad \text{---(i)}$$

$$f(-2) = (-2)^2 + 5(-2) + 6$$

$$= 4 - 10 + 6$$

$$f(-2) = 20 \qquad \text{---(ii)}$$

From equation (i) and (ii),

$$f(-3) = f(-2)$$

So, Rolle's theorem is applicable is $[-3,-2]$, we have to show that $f'(c) = 0$ as $c \in (-3,-2)$.

Now,

$$f(x) = x^2 + 5x + 6$$

$$f'(x) = 2x + 5$$

$$\Rightarrow \quad f'(c) = 0$$

$$2c + 5 = 0$$

$$c = \frac{-5}{2} \in (-3,-2)$$

So, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(i)

Here,

$$f(x) = \cos 2\left(x - \frac{\pi}{4}\right) \text{ on } \left[0, \frac{\pi}{2}\right]$$

We know that cosine function is continuous and differentiable every where, so $f(x)$ is continuous is $\left[0, \frac{\pi}{2}\right]$ and differentiable is $\left[0, \frac{\pi}{2}\right]$.

Now,

$$f(0) = \cos 2\left(0 - \frac{\pi}{4}\right) = 0$$

$$f\left(\frac{\pi}{2}\right) = \cos 2\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = 0$$

$$\Rightarrow \quad f(0) = f\left(\frac{\pi}{2}\right)$$

So, Rolle's theorem is applicable.

Hence, there must exists a $c \in \left(0, \frac{\pi}{2}\right)$ such that $f'(c) = 0$.

Now,

$$f'(x) = -\sin 2\left(x - \frac{\pi}{4}\right) \times 2$$

$$f'(x) = -2\sin\left(2x - \frac{\pi}{2}\right)$$

$$\Rightarrow \quad -2\sin\left(2c - \frac{\pi}{2}\right) = 0$$

$$\Rightarrow \quad \sin\left(2c - \frac{\pi}{2}\right) = \sin 0$$

$$\Rightarrow \quad 2c - \frac{\pi}{2} = 0$$

$$c = \frac{\pi}{4} \in \left(0, \frac{\pi}{2}\right)$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(ii)

Here,

$$f(x) = \sin 2x \text{ on } \left[0, \frac{\pi}{2}\right]$$

We know that $\sin x$ is a continuous and differentiable every where. So, $f(x)$ is continuous in $\left[0, \frac{\pi}{2}\right]$ and differentiable is $\left(0, \frac{\pi}{2}\right)$.

Now,

$$f(0) = \sin 0 = 0$$

$$f\left(\frac{\pi}{2}\right) = \sin \pi = 0$$

$$\Rightarrow \quad f(0) = f\left(\frac{\pi}{2}\right)$$

So, Rolle's theorem is applicable, so, there must exist a $c \in \left(0, \frac{\pi}{2}\right)$ such that $f'(c) = 0$

Now,

$$f'(x) = 2\cos 2x$$

$$f'(c) = 2\cos 2c = 0$$

$$\Rightarrow \quad \cos 2c = 0$$

$$\Rightarrow \quad 2c = \frac{\pi}{2}$$

$$\Rightarrow \quad c = \frac{\pi}{4} \in \left(0, \frac{\pi}{4}\right)$$

Thus, Rolle's theorem verified.

Mean Value Theorems Ex 15.1 Q3(iii)

Here,

$$f(x) = \cos 2x \text{ on } \left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$$

We know that $\cos x$ is a continuous and differentiable every where. So,

$f(x)$ is continuous in $\left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$ and differentiable is $\left(\frac{-\pi}{4}, \frac{\pi}{4}\right)$.

Now, $f\left(-\frac{\pi}{4}\right) = \cos 2\left(-\frac{\pi}{4}\right) = \cos\left(-\frac{\pi}{2}\right) = 0$

$$f\left(\frac{\pi}{4}\right) = \cos 2\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{2}\right) = 0$$

$$\Rightarrow \quad f\left(-\frac{\pi}{4}\right) = f\left(\frac{\pi}{4}\right)$$

So, Rolle's theorem is applicable, so, there must exist a $c \in \left(0, \frac{\pi}{2}\right)$

such that $f'(c) = 0$

Now,

$$f'(x) = 2 \sin 2x$$

$$f'(c) = 2 \sin 2c = 0$$

$$\Rightarrow \quad \sin 2c = 0$$

$$\Rightarrow \quad 2c = 0$$

$$\Rightarrow \quad c = 0 \in \left(\frac{-\pi}{4}, \frac{\pi}{4}\right)$$

Thus, Rolle's theorem verified.

*********** END **********