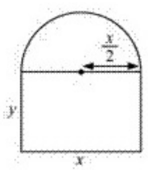
Maxima and Minima 18.5 Q15 Maxima and Minima 18.5 Q15

Radius of the semicircular opening $= \frac{x}{2}$



It is given that the perimeter of the window is 10 m.

$$\therefore x + 2y + \frac{\pi x}{2} = 10$$
$$\Rightarrow x\left(1 + \frac{\pi}{2}\right) + 2y = 10$$
$$\Rightarrow 2y = 10 - x\left(1 + \frac{\pi}{2}\right)$$
$$\Rightarrow y = 5 - x\left(\frac{1}{2} + \frac{\pi}{4}\right)$$

$\therefore$Area of the window (A) is given by,

$$A = xy + \frac{\pi}{2}\left(\frac{x}{2}\right)^2$$
$$= x\left[5 - x\left(\frac{1}{2} + \frac{\pi}{4}\right)\right] + \frac{\pi}{8}x^2$$
$$= 5x - x^2\left(\frac{1}{2} + \frac{\pi}{4}\right) + \frac{\pi}{8}x^2$$
$$\therefore \frac{dA}{dx} = 5 - 2x\left(\frac{1}{2} + \frac{\pi}{4}\right) + \frac{\pi}{4}x$$
$$= 5 - x\left(1 + \frac{\pi}{2}\right) + \frac{\pi}{4}x$$
$$\therefore \frac{d^2A}{dx^2} = -\left(1 + \frac{\pi}{2}\right) + \frac{\pi}{4} = -1 - \frac{\pi}{4}$$

Now, $\frac{dA}{dx}=0$

$$\Rightarrow 5-x\left(1+\frac{\pi}{2}\right)+\frac{\pi}{4}x=0$$

$$\Rightarrow 5-x-\frac{\pi}{4}x=0$$

$$\Rightarrow x\left(1+\frac{\pi}{4}\right)=5$$

$$\Rightarrow x=\frac{5}{\left(1+\frac{\pi}{4}\right)}=\frac{20}{\pi+4}$$

Thus, when $x=\frac{20}{\pi+4}$ then $\frac{d^2A}{dx^2}<0$.

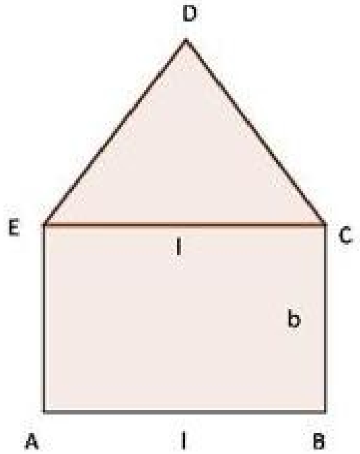
Therefore, by second derivative test, the area is the maximum when length $x=\frac{20}{\pi+4}$ m.

Now,

$$y=5-\frac{20}{\pi+4}\left(\frac{2+\pi}{4}\right)=5-\frac{5(2+\pi)}{\pi+4}=\frac{10}{\pi+4}\text{ m}$$

Hence, the required dimensions of the window to admit maximum light is given by length $=\frac{20}{\pi+4}$ m and breadth $=\frac{10}{\pi+4}$ m.

Maxima and Minima 18.5 Q16



The perimeter of the window = 12 m

$\Rightarrow \quad (l + 2b) + (l + l) = 12$

$\Rightarrow \quad 3l + 2b = 12 \qquad$ ----- (i)

Let S = Area of the rectangle + Area of the equilateral Δ

From (i),

$$S = l\left(\frac{12-3l}{2}\right) + \frac{\sqrt{3}}{4}l^2$$

$$\therefore \quad \frac{dS}{dl} = 6 - 3l + \frac{\sqrt{3}}{2}l = 6 - \sqrt{3}\left(\sqrt{3} - \frac{1}{2}\right)l$$

For maxima and minima,

$$\frac{dS}{dl} = 0$$

$$\Rightarrow \quad 6 - \sqrt{3}\left(\sqrt{3} - \frac{1}{2}\right)l = 0$$

$$\Rightarrow \quad l = \frac{6}{\sqrt{3}\left(\sqrt{3} - \frac{1}{2}\right)} = \frac{12}{6-\sqrt{3}}$$

Now, $\dfrac{d^2S}{dl^2} = -\sqrt{3}\left(\sqrt{3} - \dfrac{1}{2}\right) = -3 + \dfrac{\sqrt{3}}{2} < 0$

$\therefore \quad l = \dfrac{12}{6-\sqrt{3}}$ is the point of local maxima

From (i),

$$b = \frac{12-3l}{2} = \frac{12 - 3\left(\frac{12}{6-\sqrt{3}}\right)}{2} = \frac{24-6\sqrt{3}}{6-\sqrt{3}}$$

*********** END **********