Indefinite Integrals Ex 19.3 Q6

Let $I = \int \frac{1}{\sqrt{2x+3}+\sqrt{2x-3}}dx$. Then,

$$I = \int \frac{1}{\sqrt{2x+3}+\sqrt{2x-3}} \times \frac{\sqrt{2x+3}-\sqrt{2x-3}}{\sqrt{2x+3}-\sqrt{2x-3}} \times dx$$

$$= \int \frac{\sqrt{2x+3}-\sqrt{2x-3}}{\left(\sqrt{2x+3}\right)^2-\left(\sqrt{2x-3}\right)^2} \times dx$$

$$= \int \frac{\sqrt{2x+3}-\sqrt{2x-3}}{2x+3-2x+3} \times dx$$

$$= \frac{1}{6}\int (2x+3)^{\frac{1}{2}}dx - \frac{1}{6}\int (2x-3)^{\frac{1}{2}}dx$$

$$= \frac{1}{6} \times \frac{(2x+3)^{\frac{3}{2}}}{\frac{3}{2}\times 2} - \frac{1}{6}\frac{(2x-3)^{\frac{3}{2}}}{\frac{3}{2}\times 2} + c$$

$$= \frac{1}{18} \times (2x+3)^{\frac{3}{2}} - \frac{1}{18}(2x-3)^{\frac{3}{2}} + c$$

$$\therefore \quad I = \frac{1}{18}(2x+3)^{\frac{3}{2}} - \frac{1}{18}(2x-3)^{\frac{3}{2}} + c.$$

Indefinite Integrals Ex 19.3 Q7

Let $I = \int \frac{2x}{(2x+1)^2}dx$. Then,

$$I = \int \frac{2x+1-1}{(2x+1)^2} \times dx$$

$$= \int \frac{2x+1}{(2x+1)^2} \times dx - \int \frac{1}{(2x+1)^2} \times dx$$

$$= \int \frac{1}{2x+1} \times dx - \int (2x+1)^{-2} \times dx$$

$$= \frac{1}{2}\log|2x+1| - \frac{(2x+1)^{-1}}{-1\times 2} + c$$

$$= \frac{1}{2}\log|2x+1| + \frac{1}{2} \times \frac{1}{2x+1} + c$$

$$\therefore \quad I = \frac{1}{2}\log|2x+1| + \frac{1}{2(2x+1)} + c.$$

Indefinite Integrals Ex 19.3 Q8

Let $I = \int \frac{1}{\sqrt{x+a}+\sqrt{x+b}}dx$. Then,

$$I = \int \frac{1}{\sqrt{x+a}+\sqrt{x+b}} \times \frac{\sqrt{x+a}-\sqrt{x+b}}{\sqrt{x+a}-\sqrt{x+b}} \times dx$$
$$= \int \frac{\sqrt{x+a}-\sqrt{x+b}}{x+a-x-b} \times dx$$
$$= \int \frac{\sqrt{x+a}-\sqrt{x+b}}{a-b} \times dx$$
$$= \frac{1}{a-b}\left[\frac{2}{3}(x+a)^{\frac{3}{2}} - \frac{2}{3}(x+b)^{\frac{3}{2}}\right] + c$$
$$= \frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}}\right] + c$$

$$\therefore \quad I = \frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}}\right] + c.$$

Indefinite Integrals Ex 19.3 Q9

Let $I = \int \sin\sqrt{1+c\,os2x}\ dx$

$$I = \int \sin x \times \sqrt{2\cos^2 x} \times dx$$
$$= \int \sin x \times \sqrt{2} \times \cos x \times dx$$
$$= \sqrt{2}\int \sin x \times \cos x \times dx$$
$$= \frac{\sqrt{2}}{2}\int 2\sin x \times \cos x \times dx$$
$$= \frac{\sqrt{2}}{2}\int \sin 2x dx$$
$$= \frac{\sqrt{2}}{2} \times \frac{-\cos 2x}{2} + c$$
$$= \frac{-1}{2\sqrt{2}} \times \cos 2x + c$$

$$\therefore \quad I = \frac{-1}{2\sqrt{2}} \times \cos 2x + c$$

Indefinite Integrals Ex 19.3 Q10

Let $I = \int \frac{1+\cos x}{1-\cos x} dx$. Then,

$$I = \int \frac{2\cos^2 \frac{x}{2}}{2\sin^2 \frac{x}{2}} \times dx$$

$$= \int \frac{\cos^2 \frac{x}{2}}{\sin^2 \frac{x}{2}} \times dx$$

$$= \int \cot^2 \frac{x}{2} \times dx$$

$$= \int \left(\operatorname{cosec}^2 \frac{x}{2} - 1 \right) dx$$

$$= \frac{-\cot \frac{x}{2}}{\frac{1}{2}} - x + c$$

$$= -2\cot \frac{x}{2} - x + c$$

********** END **********