Differentiation Ex 11.8 Q5(iii)

Let $u = \sin^{-1}\left(4x\sqrt{1-4x^2}\right)$

Put $2x = \cos\theta$, so

$$u = \sin^{-1}\left(2 \times \cos\theta\sqrt{1-\cos^2\theta}\right)$$

$$= \sin^{-1}(2\cos\theta\sin\theta)$$

$$u = \sin^{-1}(\sin 2\theta) \quad \text{---(i)}$$

Let $v = \sqrt{1-4x^2}$ ---(ii)

Here,

$$x \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}}\right)$$

$$\Rightarrow \quad 2x \in \left(-1, -\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad \theta \in \left(\frac{3\pi}{4}, \pi\right)$$

So, from equation (i),

$$u = \pi - 2\theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \pi - \theta \text{ if } \theta \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]\right]$$

$$u = \pi - 2\cos^{-1}(2x) \qquad [\text{Since, } 2x = \cos\theta]$$

Differentiating it with respect to x using chain rule,

$$\frac{du}{dx} = 0 - 2\left(\frac{-1}{\sqrt{1-(2x)^2}}\right)\frac{d}{dx}(2x)$$

$$= \frac{2}{\sqrt{1-4x^2}}(2)$$

$$\frac{du}{dx} = \frac{4}{\sqrt{1-4x^2}} \quad \text{---(vi)}$$

From equation (iv)

$$\frac{dv}{dx} = \frac{-4x}{\sqrt{1-4x^2}}$$

but, $x \in \left(-\frac{1}{2}, -\frac{1}{2\sqrt{2}}\right)$

$$\frac{dv}{dx} = \frac{-4(-x)}{\sqrt{1-4(-x)^2}}$$

$$\frac{dv}{dx} = \frac{4x}{\sqrt{1-4x^2}} \quad \text{---(vii)}$$

Dividing equation (vi) by (vii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{4}{\sqrt{1-4x^2}} \times \frac{\sqrt{1-4x^2}}{4x}$$

$$\frac{du}{dv} = \frac{1}{x}$$

Differentiation Ex 11.8 Q6

Let $u = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

Put $x = \tan\theta$, so

$$u = \tan^{-1}\left(\sqrt{1+\tan^2\theta} - 1\right.$$

$$= \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{\frac{2\sin^2\theta}{2}}{\frac{2\sin\theta}{2}\frac{\cos\theta}{2}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{\sin\theta}{2}}{\frac{\cos\theta}{2}}\right)$$

$$u = \tan^{-1}\left(\frac{\tan\theta}{2}\right) \quad \text{---(i)}$$

And,

Let $v = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$$= \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$v = \sin^{-1}(\sin 2\theta) \quad \text{---(ii)}$$

Here,

$-1 < x < 1$

$\Rightarrow \quad -1 < \tan\theta < 1$

$\Rightarrow \quad -\frac{\pi}{4} < \theta < \frac{\pi}{4} \quad$ ---(A)

So, from equation (i),

$$u = \frac{\theta}{2} \qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$u = \frac{1}{2}\tan^{-1}x \qquad [\text{Since, } x = \tan\theta]$$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{1}{2}\left(\frac{1}{1+x^2}\right)$$

$$\frac{du}{dx} = \frac{1}{2(1+x^2)} \quad \text{---(iii)}$$

Now, from equation (ii) and (A)

$$v = 2\theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$v = 2\tan^{-1}x \qquad [\text{Since, } x = \tan\theta]$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = 2\left(\frac{1}{1+x^2}\right) \quad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{1}{2(1+x^2)} \times \frac{(1+x^2)}{2}$$

$$\frac{du}{dv} = \frac{1}{4}$$

*********** END **********