Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 26

$$\tan 82\frac{1^\circ}{2} = \tan\left(90 - 7\frac{1}{2}\right)^\circ$$

$$= \cot 7\frac{1^\circ}{2}$$

$$= \cot A \qquad \text{If } A = 7\frac{1^\circ}{2}$$

Now

$$\cot A = \frac{\cos A}{\sin A}$$

$$= \frac{2\cos^2 A}{2\sin A\cos A}$$

$$= \frac{1+\cos^2 A}{\sin^2 A}$$

$$\cot A = \frac{1+\cos 15}{\sin 15}$$

$$= \frac{1+\cos(45-30)}{\sin 15}$$

$$= \frac{1+\left(\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\times\frac{1}{2}\right)}{\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}\times\frac{1}{2}}$$

$$= \frac{2\sqrt{2}+(\sqrt{3}+1)}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{2\sqrt{2}(\sqrt{3}+1)+(\sqrt{3}+1)^2}{3-1}$$

$$= \frac{2\sqrt{6}+2\sqrt{2}+4+2\sqrt{3}}{2}$$

$$\cot A = \sqrt{6}+\sqrt{2}+2+\sqrt{3} \text{ ------ (1)}$$

$$= \sqrt{2}+2+\sqrt{6}+\sqrt{3}$$

$$= \sqrt{2}(1+\sqrt{2})+\sqrt{3}(\sqrt{2}+1)$$

$$\cot A = (\sqrt{2}+1)(\sqrt{2}+\sqrt{3}) \text{ ------- (2)}$$

From equation (1) and (2)

$$\tan 82\frac{1^\circ}{2} = \cot 7\frac{1^\circ}{2} = \sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$$

$$= (\sqrt{2}+1)(\sqrt{2}+\sqrt{3})$$

Trigonometric Ratios of multiple and Sub multiple Angles Ex 9.1 Q 27

We know that,

$$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$$

Put $A = 45°$,

$$\sin 22\frac{1}{2}° = \sqrt{\frac{1-\cos 45°}{2}} \qquad \left\{\text{since } \sin 22\frac{1}{2}°, \text{ is positive}\right\}$$

$$= \sqrt{\frac{1-\frac{1}{\sqrt{2}}}{2}}$$

$$\sin 22\frac{1}{2}° = \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$$

And

$$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$$

Put $A = 45°$

$$\cos 22\frac{1}{2}° = \sqrt{\frac{1+\cos 45°}{2}}$$

$$= \sqrt{\frac{1+\frac{1}{\sqrt{2}}}{2}}$$

$$\cos 22\frac{1}{2}° = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$$

Now,

$$\cot 22\frac{1}{2}° = \frac{\cos 22\frac{1}{2}°}{\sin 22\frac{1}{2}°}$$

$$= \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}} \times \frac{2\sqrt{2}}{\sqrt{2}-1}}$$

$$= \sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}}$$

Rationalizing denominator,

$$= \sqrt{\frac{\sqrt{2}+1 \times \sqrt{2}+1}{\sqrt{2}-1 \times \sqrt{2}+1}}$$

$$= \sqrt{\frac{\left(\sqrt{2}+1\right)^2}{2-1}}$$

$$\cot 22\frac{1}{2}° = \sqrt{2}+1$$

********** END **********