Trigonometric Ratios of Compound Angles Ex 7.1 Q16

We have,

$$\text{LHS} = \frac{\sin(A+B)+\sin(A-B)}{\cos(A+B)+\cos(A-B)}$$

$$= \frac{2\sin A\cos B}{2\cos A\cos B} \qquad \left[\begin{array}{l} \because \quad 2\sin A\cos B = \sin(A+B)+\sin(A-B) \\ \text{and, } 2\cos A\cos B = \cos(A+B)+\cos(A-B) \end{array}\right]$$

$$= \frac{\sin A}{\cos A}$$

$$= \tan A$$

$$= \text{RHS}$$

$\therefore$ LHS = RHS

Hence proved.

$$\text{LHS} = \frac{\sin(A-B)}{\cos A\cos B} + \frac{\sin(B-C)}{\cos B\cos C} + \frac{\sin(C-A)}{\cos C\cos A}$$

$$= \frac{\sin A\cos B - \cos A\sin B}{\cos A\cos B} + \frac{\sin B\cos C - \cos B\sin C}{\cos B\cos C} + \frac{\sin C\cos A - \cos C\sin A}{\cos C\cos A}$$

$$= \frac{\sin A\cos B}{\cos A\cos B} - \frac{\cos A\sin B}{\cos A\cos B} + \frac{\sin B\cos C}{\cos B\cos C} - \frac{\cos B\sin C}{\cos B\cos C} + \frac{\sin C\cos A}{\cos C\cos A} - \frac{\cos C\sin A}{\cos C\cos A}$$

$$= \tan A - \tan B + \tan B - \tan C + \tan C - \tan A$$

$$= 0$$

$$= \text{RHS}$$

$\therefore$ LHS = RHS

Hence proved.

We have,

$$\text{LHS} = \frac{\sin(A-B)}{\sin A\sin B} + \frac{\sin(B-C)}{\sin B\sin C} + \frac{\sin(C-A)}{\sin C\sin A}$$

$$= \frac{\sin A\cos B - \cos A\sin B}{\sin A\sin B} + \frac{\sin B\cos C - \cos B\sin C}{\sin B\sin C} + \frac{\sin C\cos A - \cos C\sin A}{\sin C\sin A}$$

$$= \frac{\sin A\cos B}{\sin A\sin B} - \frac{\cos A\sin B}{\sin A\sin B} + \frac{\sin B\cos C}{\sin B\sin C} - \frac{\cos B\sin C}{\sin B\sin C} + \frac{\sin C\cos A}{\sin C\sin A} - \frac{\cos C\sin A}{\sin C\sin A}$$

$$= \frac{\cos B}{\sin B} - \frac{\cos A}{\sin A} + \frac{\cos C}{\sin C} - \frac{\cos B}{\sin B} + \frac{\cos A}{\sin A} - \frac{\cos C}{\sin C}$$

$$= \cot B - \cot A + \cot C - \cot B + \cot A - \cot C \qquad \left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$$

$$= 0$$

$$= \text{RHS}$$

$\therefore$ LHS = RHS

Hence proved.

We have,

$$
\begin{aligned}
\text{RHS} &= \sin^2 A + \sin^2(A-B) - 2\sin A\cos B\sin(A-B) \\
&= \sin^2 A + \sin(A-B)\left[\sin(A-B) - 2\sin A\cos B\right] \\
&= \sin^2 A + \sin(A-B)\left[\sin A\cos B - \cos A\sin B - 2\sin A\cos B\right] \\
&= \sin^2 A + \sin(A-B)\left[-\sin A\cos B - \cos A\sin B\right] \\
&= \sin^2 A - \sin(A-B)\left(\sin A\cos B + \cos A\sin B\right) \\
&= \sin^2 A - \sin(A-B)\left(\sin(A+B)\right) \\
&= \sin^2 A - \sin(A-B)\sin(A+B) \\
&= \sin^2 A - \left(\sin^2 A - \sin^2 B\right) \qquad \left[\because \sin(A-B)\sin(A+B) = \sin^2 A - \sin^2 B\right] \\
&= \sin^2 A - \sin^2 A + \sin^2 B \\
&= \sin^2 B \\
&= \text{LHS}
\end{aligned}
$$

$\therefore$ LHS = RHS

Hence proved.

$$
\begin{aligned}
\text{RHS} &= \cos^2 A + \cos^2 B - 2\cos A\cos B\cos(A+B) \\
&= \cos^2 A + \left(1 - \sin^2 B\right) - 2\cos A\cos B\cos(A+B) \\
&= \left[\cos^2 A - \sin^2 B\right] - 2\cos A\cos B\cos(A+B) + 1 \\
&= \left[\cos(A+B)\cos(A-B)\right] - 2\cos A\cos B\cos(A+B) + 1 \\
&= \cos(A+B)\left[\cos(A-B) - 2\cos A\cos B\right] + 1 \\
&= \cos(A+B)\left[\cos A\cos B + \sin A\sin B - 2\cos A\cos B\right] + 1 \\
&= \cos(A+B)\left[-\cos A\cos B + \sin A\sin B\right] + 1 \\
&= -\cos(A+B)\left[\cos A\cos B - \sin A\sin B\right] + 1 \\
&= -\cos(A+B)\left[\cos(A+B)\right] + 1 \\
&= -\cos^2(A+B) + 1 \\
&= 1 - \cos^2(A+B) \\
&= \sin^2(A+B) \qquad \left[\sin^2\theta = 1 - \cos^2\theta\right] \\
&= \text{RHS}
\end{aligned}
$$

$\therefore$ LHS = RHS

Hence proved.

We have,

$$\text{LHS} = \frac{\tan(A+B)}{\cot(A-B)}$$

$$= \frac{\tan(A+B)}{\dfrac{1}{\tan(A-B)}} \qquad \left[\because \cot\theta = \frac{1}{\tan\theta}\right]$$

$$= \tan(A+B)\tan(A-B)$$

$$= \left[\frac{\tan A + \tan B}{1 - \tan A \tan B}\right]\left[\frac{\tan A - \tan B}{1 + \tan A \tan B}\right]$$

$$= \frac{(\tan A + \tan B)(\tan A - \tan B)}{(1 - \tan A \tan B)(1 + \tan A \tan B)}$$

$$= \frac{\tan^2 A - \tan^2 B}{1 - (\tan A \tan B)^2} \qquad \left[\because (a-b)(a+b) = a^2 - b^2\right]$$

$$= \frac{\tan^2 A - \tan^2 B}{1 - \tan^2 A \tan^2 B}$$

$$= \text{RHS}$$

$\therefore$ LHS = RHS

Hence proved.

*********** END **********