Squares and Square Roots Ex 3.2 Q9

Answer :

Observing the three numbers for right hand side of the equalities:
The first equality, whose biggest number on the LHS is 1, has 1, 1 and 1 as the three numbers.
The second equality, whose biggest number on the LHS is 2, has 2, 2 and 1 as the three numbers.
The third equality, whose biggest number on the LHS is 3, has 3, 3 and 1 as the three numbers.
The fourth equality, whose biggest number on the LHS is 4, has 4, 4 and 1 as the three numbers.
Hence, if the biggest number on the LHS is *n*, the three numbers on the RHS will be *n*, *n* and 1.
Using this property, we can calculate the sums for (i) and (ii) as follows:

(i) $1 + 2 + 3 + \ldots\ldots\ldots + 50 = \frac{1}{2} \times 50 \times (50 + 1) = 1275$

(ii) The sum can be expressed as the difference of the two sums as follows:

$31 + 32 + \ldots\ldots + 50 = (1 + 2 + 3 + \ldots\ldots + 50) - (1 + 2 + 3 + \ldots\ldots + 30)$

The result of the first bracket is exactly the same as in part (i).

$$1 + 2 + \ldots. + 50 = 1275$$

Then, the second bracket:

$$1 + 2 + \ldots\ldots + 30 = \frac{1}{2}(30 \times (30 + 1)) = 465$$

Finally, we have:

$$31 + 32 + \ldots. + 50 = 1275 - 465 = 810$$

Squares and Square Roots Ex 3.2 Q10

Answer :

Observing the six numbers on the RHS of the equalities:
The first equality, whose biggest number on the LHS is 1, has 1, 1, 1, 2, 1 and 1 as the six numbers.
The second equality, whose biggest number on the LHS is 2, has 2, 2, 1, 2, 2 and 1 as the six numbers.
The third equality, whose biggest number on the LHS is 3, has 3, 3, 1, 2, 3 and 1 as the six numbers.
The fourth equality, whose biggest number on the LHS is 4, has numbers 4, 4, 1, 2, 4 and 1 as the six numbers.
Note that the fourth number on the RHS is always 2 and the sixth number is always 1. The remaining numbers are equal to the biggest number on the LHS.
Hence, if the biggest number on the LHS is *n*, the six numbers on the RHS would be *n*, *n*, 1, 2, *n* and 1.
Using this property, we can calculate the sums for (i) and (ii) as follows:

(i) $1^2 + 2^2 + \ldots\ldots\ldots + 10^2 = \frac{1}{6} \times [10 \times (10 + 1) \times (2 \times 10 + 1)]$

$$= \frac{1}{6} \times [10 \times 11 \times 12] = 385.$$

(ii) The sum can be expressed as the difference of the two sums as follows:

$$5^2 + 6^2 + \ldots\ldots + 12^2 = \left(1^2 + 2^2 + \ldots\ldots + 12^2\right) - \left(1^2 + 2^2 + \ldots\ldots + 4^2\right)$$

The sum of the first bracket on the RHS:

$$\begin{aligned} 1^2 + 2^2 + \ldots\ldots + 12^2 &= \tfrac{1}{6}\left[12 \times (12+1) \times (2 \times 12 + 1)\right] \\ &= 650 \end{aligned}$$

The second bracket is:

$$\begin{aligned} & 1^2 + 2^2 + \ldots\ldots + 4^2 \\ &= \tfrac{1}{6} \times \left[4 \times (4+1) \times (2 \times 4 + 1)\right] \\ &= \tfrac{1}{6} \times 4 \times 5 \times 9 = 30 \end{aligned}$$

Finally, the wanted sum is:

$$\begin{aligned} & 5^2 + 6^2 + \ldots\ldots + 12^2 \\ &= \left(1^2 + 2^2 + \ldots\ldots + 12^2\right) - \left(1^2 + 2^2 + \ldots\ldots + 12^2\right) \\ &= 650 - 30 = 620 \end{aligned}$$

*********** END **********