



Exercise 11.1

Question 1:

If a line makes angles 90° , 135° , 45° with x , y and z -axes respectively, find its direction cosines.

Answer

Let direction cosines of the line be l , m , and n .

$$l = \cos 90^\circ = 0$$

$$m = \cos 135^\circ = -\frac{1}{\sqrt{2}}$$

$$n = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

Therefore, the direction cosines of the line are $0, -\frac{1}{\sqrt{2}}$, and $\frac{1}{\sqrt{2}}$.

Question 2:

Find the direction cosines of a line which makes equal angles with the coordinate axes.

Answer

Let the direction cosines of the line make an angle α with each of the coordinate axes.

$$\therefore l = \cos \alpha, m = \cos \alpha, n = \cos \alpha$$

$$l^2 + m^2 + n^2 = 1$$

$$\Rightarrow \cos^2 \alpha + \cos^2 \alpha + \cos^2 \alpha = 1$$

$$\Rightarrow 3 \cos^2 \alpha = 1$$

$$\Rightarrow \cos^2 \alpha = \frac{1}{3}$$

$$\Rightarrow \cos \alpha = \pm \frac{1}{\sqrt{3}}$$

Thus, the direction cosines of the line, which is equally inclined to the coordinate axes,

are $\pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}$, and $\pm \frac{1}{\sqrt{3}}$.

Question 3:

If a line has the direction ratios $-18, 12, -4$, then what are its direction cosines?

Answer

If a line has direction ratios of $-18, 12$, and -4 , then its direction cosines are

$$\frac{-18}{\sqrt{(-18)^2 + (12)^2 + (-4)^2}}, \frac{12}{\sqrt{(-18)^2 + (12)^2 + (-4)^2}}, \frac{-4}{\sqrt{(-18)^2 + (12)^2 + (-4)^2}}$$

$$\text{i.e., } \frac{-18}{22}, \frac{12}{22}, \frac{-4}{22}$$

$$\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$$

Thus, the direction cosines are $\frac{-9}{11}, \frac{6}{11}$, and $\frac{-2}{11}$.

Question 4:

Show that the points $(2, 3, 4)$, $(-1, -2, 1)$, $(5, 8, 7)$ are collinear.

Answer

The given points are A $(2, 3, 4)$, B $(-1, -2, 1)$, and C $(5, 8, 7)$.

It is known that the direction ratios of line joining the points, (x_1, y_1, z_1) and (x_2, y_2, z_2) , are given by, $x_2 - x_1, y_2 - y_1$, and $z_2 - z_1$.

The direction ratios of AB are $(-1 - 2)$, $(-2 - 3)$, and $(1 - 4)$ i.e., $-3, -5$, and -3 .

The direction ratios of BC are $(5 - (-1))$, $(8 - (-2))$, and $(7 - 1)$ i.e., $6, 10$, and 6 .

It can be seen that the direction ratios of BC are -2 times that of AB i.e., they are proportional.

Therefore, AB is parallel to BC. Since point B is common to both AB and BC, points A, B,

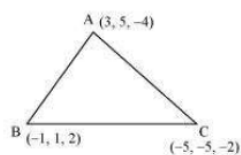
and C are collinear.

Question 5:

Find the direction cosines of the sides of the triangle whose vertices are $(3, 5, -4)$, $(-1, 1, 2)$ and $(-5, -5, -2)$

Answer

The vertices of $\triangle ABC$ are A $(3, 5, -4)$, B $(-1, 1, 2)$, and C $(-5, -5, -2)$.



The direction ratios of side AB are $(-1 - 3)$, $(1 - 5)$, and $(2 - (-4))$ i.e., -4 , -4 , and 6 .

$$\begin{aligned}\text{Then, } \sqrt{(-4)^2 + (-4)^2 + (6)^2} &= \sqrt{16 + 16 + 36} \\ &= \sqrt{68} \\ &= 2\sqrt{17}\end{aligned}$$

Therefore, the direction cosines of AB are

$$\begin{aligned}\frac{-4}{\sqrt{(-4)^2 + (-4)^2 + (6)^2}}, \frac{-4}{\sqrt{(-4)^2 + (-4)^2 + (6)^2}}, \frac{6}{\sqrt{(-4)^2 + (-4)^2 + (6)^2}} \\ \frac{-4}{2\sqrt{17}}, \frac{-4}{2\sqrt{17}}, \frac{6}{2\sqrt{17}} \\ \frac{-2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{3}{\sqrt{17}}\end{aligned}$$

The direction ratios of BC are $(-5 - (-1))$, $(-5 - 1)$, and $(-2 - 2)$ i.e., -4 , -6 , and -4 .

Therefore, the direction cosines of BC are

$$\begin{aligned}\frac{-4}{\sqrt{(-4)^2 + (-6)^2 + (-4)^2}}, \frac{-6}{\sqrt{(-4)^2 + (-6)^2 + (-4)^2}}, \frac{-4}{\sqrt{(-4)^2 + (-6)^2 + (-4)^2}} \\ \frac{-4}{2\sqrt{17}}, \frac{-6}{2\sqrt{17}}, \frac{-4}{2\sqrt{17}}\end{aligned}$$

The direction ratios of CA are $(-5 - 3)$, $(-5 - 5)$, and $(-2 - (-4))$ i.e., -8 , -10 , and 2 .

Therefore, the direction cosines of AC are

$$\begin{aligned}\frac{-8}{\sqrt{(-8)^2 + (10)^2 + (2)^2}}, \frac{-10}{\sqrt{(-8)^2 + (10)^2 + (2)^2}}, \frac{2}{\sqrt{(-8)^2 + (10)^2 + (2)^2}} \\ \frac{-8}{2\sqrt{42}}, \frac{-10}{2\sqrt{42}}, \frac{2}{2\sqrt{42}}\end{aligned}$$

Exercise 11.2

Question 1:

Show that the three lines with direction cosines

$$\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}; \frac{4}{13}, \frac{12}{13}, \frac{3}{13}; \frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$$

are mutually perpendicular.

Answer

Two lines with direction cosines, l_1, m_1, n_1 and l_2, m_2, n_2 , are perpendicular to each other, if $l_1l_2 + m_1m_2 + n_1n_2 = 0$

(i) For the lines with direction cosines, $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}$ and $\frac{4}{13}, \frac{12}{13}, \frac{3}{13}$, we obtain

$$\begin{aligned}l_1l_2 + m_1m_2 + n_1n_2 &= \frac{12}{13} \times \frac{4}{13} + \left(\frac{-3}{13}\right) \times \frac{12}{13} + \left(\frac{-4}{13}\right) \times \frac{3}{13} \\ &= \frac{48}{169} - \frac{36}{169} - \frac{12}{169} \\ &= 0\end{aligned}$$

Therefore, the lines are perpendicular.

(ii) For the lines with direction cosines, $\frac{4}{13}, \frac{12}{13}, \frac{3}{13}$ and $\frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$, we obtain

$$\begin{aligned}l_1l_2 + m_1m_2 + n_1n_2 &= \frac{4}{13} \times \frac{3}{13} + \frac{12}{13} \times \left(\frac{-4}{13}\right) + \frac{3}{13} \times \frac{12}{13} \\ &= \frac{12}{169} - \frac{48}{169} + \frac{36}{169} \\ &= 0\end{aligned}$$

Therefore, the lines are perpendicular.

(iii) For the lines with direction cosines, $\frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$ and $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}$, we obtain

$$\begin{aligned}l_1l_2 + m_1m_2 + n_1n_2 &= \left(\frac{3}{13}\right) \times \left(\frac{12}{13}\right) + \left(\frac{-4}{13}\right) \times \left(\frac{-3}{13}\right) + \left(\frac{12}{13}\right) \times \left(\frac{-4}{13}\right) \\ &= \frac{36}{169} + \frac{12}{169} - \frac{48}{169} \\ &= 0\end{aligned}$$

Therefore, the lines are perpendicular.

Thus, all the lines are mutually perpendicular.

Question 2:

Show that the line through the points $(1, -1, 2)$ $(3, 4, -2)$ is perpendicular to the line through the points $(0, 3, 2)$ and $(3, 5, 6)$.

Answer

Let AB be the line joining the points, $(1, -1, 2)$ and $(3, 4, -2)$, and CD be the line joining the points, $(0, 3, 2)$ and $(3, 5, 6)$.

The direction ratios, a_1, b_1, c_1 , of AB are $(3 - 1)$, $(4 - (-1))$, and $(-2 - 2)$ i.e., 2, 5, and -4.

The direction ratios, a_2, b_2, c_2 , of CD are $(3 - 0)$, $(5 - 3)$, and $(6 - 2)$ i.e., 3, 2, and 4.

AB and CD will be perpendicular to each other, if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$a_1a_2 + b_1b_2 + c_1c_2 = 2 \times 3 + 5 \times 2 + (-4) \times 4$$

$$= 6 + 10 - 16$$

$$= 0$$

Therefore, AB and CD are perpendicular to each other.

Question 3:

Show that the line through the points $(4, 7, 8)$ $(2, 3, 4)$ is parallel to the line through the points $(-1, -2, 1)$, $(1, 2, 5)$.

Answer

Let AB be the line through the points, $(4, 7, 8)$ and $(2, 3, 4)$, and CD be the line through the points, $(-1, -2, 1)$ and $(1, 2, 5)$.

The directions ratios, a_1, b_1, c_1 , of AB are $(2 - 4)$, $(3 - 7)$, and $(4 - 8)$ i.e., -2, -4, and -4.

The direction ratios, a_2, b_2, c_2 , of CD are $(1 - (-1))$, $(2 - (-2))$, and $(5 - 1)$ i.e., 2, 4, and 4.

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