Geometric Progressions Ex 20.6 Q 7

Given,

$$A.M = 25$$
$$G.M = 20$$

Now, $A.M = \dfrac{a+b}{2} = 25$

and, $G.M = \sqrt{ab} = 20$

$$a+b = 50,\ ab = 400$$

$$(a-b) = \sqrt{(a+b)^2 - 4ab}$$
$$= \sqrt{(50)^2 - 1600}$$
$$= \sqrt{2500 - 1600}$$
$$= \pm 30$$

$$a - b = \pm 30$$
$$\underline{a + b = 50}$$
$$2a = 80$$
$$a = 40$$

Also, $-2b = -20$

$$b = 10$$

$\therefore$ The numbers are 40, 10.

Geometric Progressions Ex 20.6 Q 8

A.M. between two numbers a and b $(a > b)$ is $\dfrac{a+b}{2}$

Also, geometric mean between 2 numbers is $\sqrt{ab}$

Given,

$$A.M = 2G.M$$

$$\Rightarrow \quad \frac{a+b}{2} = 2\sqrt{ab}$$

$$a+b = 4\sqrt{ab}$$

$$\frac{a+b}{2\sqrt{ab}} = \frac{2}{1}$$

$$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{2+1}{2-1} = \frac{3}{1} \qquad \text{[By componendo and dividendo]}$$

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\left(\sqrt{a}-\sqrt{b}\right)^2} = \frac{\left(\sqrt{3}\right)^2}{(1)^2}$$

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{3}}{1}$$

By componendo and dividendo, we get

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)+\left(\sqrt{a}-\sqrt{b}\right)}{\left(\sqrt{a}+\sqrt{b}\right)-\left(\sqrt{a}-\sqrt{b}\right)} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$\frac{a}{b} = \frac{\left(\sqrt{3}+1\right)^2}{\left(\sqrt{3}-1\right)^2} = \frac{3+1+2\sqrt{3}}{3+1-2\sqrt{3}}$$

$$= \frac{4+2\sqrt{3}}{4-2\sqrt{3}}$$

$$\frac{a}{b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$$

Thus, $a : b = \left(2+\sqrt{3}\right) : \left(2-\sqrt{3}\right)$.

Geometric Progressions Ex 20.6 Q 9

Let A.M = A between a and b

G.M = G_1 and G_2 between a and b

$\Rightarrow \quad A = \frac{a+b}{2}$

$a, G_1 G_2, b$ is G.P. with common ratio $r = \left(\frac{b}{a}\right)^{\frac{1}{3}}$

$$G_1 = ar = a\left(\frac{b}{a}\right)^{\frac{1}{3}}$$

$$G_2 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{3}} = a^{\frac{1}{3}}b^{\frac{2}{3}}$$

Now,

$$G_1{}^2 = a^2\left(\frac{b}{a}\right)^{\frac{2}{3}}$$

$$G_2{}^2 = a^{\frac{2}{3}}b^{\frac{4}{3}}$$

Then,

$$\frac{G^2{}_1}{G_2} + \frac{G^2{}_2}{G_1} = \frac{a^2\left(\frac{b}{a}\right)^{\frac{2}{3}}}{a^{\frac{1}{3}}b^{\frac{2}{3}}} + \frac{a^{\frac{2}{3}}b^{\frac{4}{3}}}{a^2\left(\frac{b}{a}\right)^{\frac{2}{3}}}$$

$$= a^{2-\frac{2}{3}-\frac{1}{3}}b^{\frac{2}{3}-\frac{2}{3}} + a^{\frac{2}{3}-2+\frac{2}{3}}b^{\frac{4}{3}-\frac{2}{3}}$$

$$= a^{\frac{3}{3}}b^0 + a^0 b$$

$$= a + b$$

$$= 2a$$

$$= \text{RHS}$$

Geometric Progressions Ex 20.6 Q 10

A.M. of root of quadratic equation is A.

G.M. of root of quadretic equation is G.

Then,

$$\frac{a+b}{2} = A, \; F = \sqrt{ab}$$

The equation having a and b as roots of quadratic equation is

$$x^2 - Sx + P = 0$$

$$x^2 - (a+b)x + ab = 0$$

$$x^2 - 2Ax + G^2 = 0$$

Geometric Progressions Ex 20.6 Q 11

Let a, b be the numbers.

$$a + b = 6\left(\text{G.M of } a, b\right)$$

$$a + b = 6\sqrt{ab}$$

$$\frac{a+b}{2\sqrt{ab}} = \frac{3}{1}$$

Applying components and dividens,

$$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{3+1}{3-1}$$

$$\left(\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}\right)^2 = \frac{4}{2}$$

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{2}}{1}$$

Again applying components and dividends,

$$\frac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$$\frac{2\sqrt{a}}{2\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$$\left(\frac{a}{b}\right) = \left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)^2$$

$$= \frac{2+1+2\sqrt{2}}{2+1-2\sqrt{2}}$$

$$= \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

$$a : b = \left(3+2\sqrt{2}\right) : \left(3-2\sqrt{2}\right)$$

Geometric Progressions Ex 20.6 Q 12

Let quadratic equation be $(x-\alpha)(x-\beta) = 0$

$$x^2 - (\alpha+\beta)x + \alpha\beta = 0$$

Roots are α, β

Here,

$$\frac{\alpha+\beta}{2} = 8, \quad \sqrt{\alpha\beta} = 5$$

$$\alpha+\beta = 16, \quad \alpha\beta = 25$$

$\therefore$ Required quadratic equation is,

$$x^2 - 16x + 25 = 0$$

Geometric Progressions Ex 20.6 Q 13

The AM and GM of a and b will be:

$$\frac{a+b}{2}=10 \Rightarrow a+b=20 \qquad \text{......(1)}$$

$$\sqrt{ab}=8 \Rightarrow ab=64$$

Now

$$a-b=\sqrt{(a+b)^2-4ab}$$

$$=\sqrt{20^2-4.64}$$

$$=\sqrt{400-256}$$

$$=\sqrt{144}$$

$$a-b=12 \qquad \text{......(2)}$$

Adding (1) and (2)

$$2a=32$$

$$a=16$$

From (1)

$$b=20-16=4$$

Thus the numbers are $a=16$ and $b=4$.

RD Sharma Class 11 Solutions

*********** END **********