Maxima and Minima Ex 18.2 Q9

$f(x) = \cos x, \quad 0 < x < \pi$

$\therefore \quad f'(x) = -\sin x$

For, the point of local maxima and minima,

$$f'(x) = 0$$
$$\Rightarrow \quad -\sin x = 0$$
$$\Rightarrow \quad x = 0, \text{ and } \pi$$

But, these two points lies outside the interval $(0, \pi)$

So, no local maxima and minima will exist in the interval $(0, \pi)$.

Maxima and Minima Ex 18.2 Q10

$\therefore \quad f'(x) = 2\cos 2x - 1$

For, the point of local maxima and minima,

$$f'(x) = 0$$
$$\Rightarrow \quad 2\cos 2x - 1 = 0$$
$$\Rightarrow \quad \cos 2x = \frac{1}{2} = \cos\frac{\pi}{3}$$
$$\Rightarrow \quad 2x = \frac{\pi}{3}, -\frac{\pi}{3}$$
$$\Rightarrow \quad x = \frac{\pi}{6}, -\frac{\pi}{6}$$

At $\quad x = -\frac{\pi}{6}$, $f'(x)$ changes from $-$ ve to $+$ ve

$\therefore \quad x = -\frac{\pi}{6}$ is point of local manima

At $\quad x = \frac{\pi}{6}$, $f'(x)$ changes from $+$ ve to $-$ ve

$\therefore \quad x = \frac{\pi}{6}$ is point of local maxima

Hence, local max value $= f\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$

local min value $= f\left(-\frac{\pi}{6}\right) = \frac{-\sqrt{3}}{2} + \frac{\pi}{6}$.

Maxima and Minima Ex 18.2 Q11

$$f(x) = 2\sin x - x, \; -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$

For checking the minima and maxima, we have

$$f'(x) = 2\cos x - 1 = 0$$

$$\Rightarrow \cos x = \frac{1}{2} = \cos\frac{\pi}{3}$$

$$\Rightarrow x = -\frac{\pi}{3}, \frac{\pi}{3}$$

At $x = -\frac{\pi}{3}$, *f(x) changes from* $-ve$ *to* $+ve$

$$\Rightarrow x = -\frac{\pi}{3} \text{ is point of local minima with value } = -\sqrt{3} - \frac{\pi}{3}$$

At $x = \frac{\pi}{3}$, *f(x) changes from* $+ve$ *to* $+ve$

$$\Rightarrow x = \frac{\pi}{3} \text{ is point of local maxima with value } = \sqrt{3} - \frac{\pi}{3}$$

Maxima and Minima Ex 18.2 Q12

$$\therefore f'(x) = \sqrt{1-x} + x \cdot \frac{1}{2\sqrt{1-x}}(-1) = \sqrt{1-x} - \frac{x}{2\sqrt{1-x}}$$

$$= \frac{2(1-x)-x}{2\sqrt{1-x}} = \frac{2-3x}{2\sqrt{1-x}}$$

$$f'(x) = 0 \Rightarrow \frac{2-3x}{2\sqrt{1-x}} = 0 \Rightarrow 2-3x = 0 \Rightarrow x = \frac{2}{3}$$

$$f''(x) = \frac{1}{2}\left[\frac{\sqrt{1-x}(-3) - (2-3x)\left(\frac{-1}{2\sqrt{1-x}}\right)}{1-x}\right]$$

$$= \frac{\sqrt{1-x}(-3) + (2-3x)\left(\frac{1}{2\sqrt{1-x}}\right)}{2(1-x)}$$

$$= \frac{-6(1-x) + (2-3x)}{4(1-x)^{\frac{3}{2}}}$$

$$= \frac{3x-4}{4(1-x)^{\frac{3}{2}}}$$

$$f''\left(\frac{2}{3}\right) = \frac{3\left(\frac{2}{3}\right) - 4}{4\left(1-\frac{2}{3}\right)^{\frac{3}{2}}} = \frac{2-4}{4\left(\frac{1}{3}\right)^{\frac{3}{2}}} = \frac{-1}{2\left(\frac{1}{3}\right)^{\frac{3}{2}}} < 0$$

Therefore, by second derivative test, $x = \frac{2}{3}$ is a point of local maxima and the local maximum value of f at $x = \frac{2}{3}$ is

$$f\left(\frac{2}{3}\right) = \frac{2}{3}\sqrt{1-\frac{2}{3}} = \frac{2}{3}\sqrt{\frac{1}{3}} = \frac{2}{3\sqrt{3}} = \frac{2\sqrt{3}}{9}.$$

*********** END **********