



Half life of $^{90}_{38}\text{Sr}$, $t_{1/2} = 28$ years

$$= 28 \times 365 \times 24 \times 60 \times 60$$

$$= 8.83 \times 10^8 \text{ s}$$

Mass of the isotope, $m = 15$ mg

90 g of $^{90}_{38}\text{Sr}$ atom contains 6.023×10^{23} (Avogadro's number) atoms.

Question 13.12:

Find the Q-value and the kinetic energy of the emitted α -particle in the α -decay of (a)

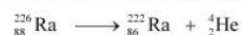
$^{226}_{88}\text{Ra}$ and (b) $^{220}_{86}\text{Rn}$.

Given $m(^{226}_{88}\text{Ra}) = 226.02540 \text{ u}$, $m(^{222}_{86}\text{Rn}) = 222.01750 \text{ u}$,

$m(^{220}_{86}\text{Rn}) = 220.01137 \text{ u}$, $m(^{216}_{84}\text{Po}) = 216.00189 \text{ u}$.

Answer

(a) Alpha particle decay of $^{226}_{88}\text{Ra}$ emits a helium nucleus. As a result, its mass number reduces to $(226 - 4)$ 222 and its atomic number reduces to $(88 - 2)$ 86. This is shown in the following nuclear reaction.



Q-value of

emitted α -particle = (Sum of initial mass – Sum of final mass) c^2

Where,

c = Speed of light

It is given that:

$$m(^{226}_{88}\text{Ra}) = 226.02540 \text{ u}$$

$$m(^{222}_{86}\text{Rn}) = 222.01750 \text{ u}$$

$$m(^4_2\text{He}) = 4.002603 \text{ u}$$

$$\text{Q-value} = [226.02540 - (222.01750 + 4.002603)] \text{ u } c^2$$

$$= 0.005297 \text{ u } c^2$$

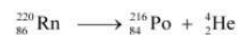
$$\text{But } 1 \text{ u} = 931.5 \text{ MeV}/c^2$$

$$\therefore Q = 0.005297 \times 931.5 \approx 4.94 \text{ MeV}$$

$$\text{Kinetic energy of the } \alpha\text{-particle} = \left(\frac{\text{Mass number after decay}}{\text{Mass number before decay}} \right) \times Q$$

$$= \frac{222}{226} \times 4.94 = 4.85 \text{ MeV}$$

(b) Alpha particle decay of $^{220}_{86}\text{Rn}$ is shown by the following nuclear reaction.



It is given that:

$$\text{Mass of } (^{220}_{86}\text{Rn}) = 220.01137 \text{ u}$$

$$\text{Mass of } (^{216}_{84}\text{Po}) = 216.00189 \text{ u}$$

$$\therefore \text{Q-value} = [220.01137 - (216.00189 + 4.00260)] \times 931.5$$

$$\approx 6.41 \text{ MeV}$$

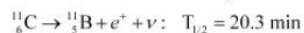
$$\text{Kinetic energy of the } \alpha\text{-particle} = \left(\frac{220 - 4}{220} \right) \times 6.41$$

$$= 6.29 \text{ MeV}$$

Question 13.13:

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The radionuclide ^{11}C decays according to



The maximum energy of the emitted positron is 0.960 MeV.

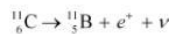
Given the mass values:

$$m(^{11}_6\text{C}) = 11.011434 \text{ u and } m(^{11}_5\text{B}) = 11.009305 \text{ u,}$$

calculate Q and compare it with the maximum energy of the positron emitted

Answer

The given nuclear reaction is:



Half life of $^{11}_6\text{C}$ nuclei, $T_{1/2} = 20.3 \text{ min}$

$$\text{Atomic mass of } m(^{11}_6\text{C}) = 11.011434 \text{ u}$$

$$\text{Atomic mass of } m(^{11}_5\text{B}) = 11.009305 \text{ u}$$

Maximum energy possessed by the emitted positron = 0.960 MeV

The change in the Q -value (ΔQ) of the nuclear masses of the $^{11}_6\text{C}$ nucleus is given as:

$$\Delta Q = [m(^{11}_6\text{C}) - [m(^{11}_5\text{B}) + m_e]]c^2 \quad \dots (1)$$

Where,

m_e = Mass of an electron or positron = 0.000548 u

c = Speed of light

m' = Respective nuclear masses

If atomic masses are used instead of nuclear masses, then we have to add 6 m_e in the

case of $^{11}_6\text{C}$ and 5 m_e in the case of $^{11}_5\text{B}$.

Hence, equation (1) reduces to:

$$\Delta Q = [m(^{11}_6\text{C}) - m(^{11}_5\text{B}) - 2m_e]c^2$$

Here, $m(^{11}_6\text{C})$ and $m(^{11}_5\text{B})$ are the atomic masses.

$$\therefore \Delta Q = [11.011434 - 11.009305 - 2 \times 0.000548] c^2$$

$$= (0.001033 c^2) \text{ u}$$

$$\text{But } 1 \text{ u} = 931.5 \text{ MeV}/c^2$$

$$\therefore \Delta Q = 0.001033 \times 931.5 \approx 0.962 \text{ MeV}$$

The value of Q is almost comparable to the maximum energy of the emitted positron.

Question 13.14:

The nucleus $^{23}_{10}\text{Ne}$ decays by β^- emission. Write down the β^- decay equation and determine the maximum kinetic energy of the electrons emitted. Given that:

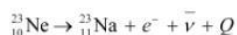
$$m(^{23}_{10}\text{Ne}) = 22.994466 \text{ u}$$

$$m(^{23}_{11}\text{Na}) = 22.989770 \text{ u.}$$

Answer

In β^- emission, the number of protons increases by 1, and one electron and an antineutrino are emitted from the parent nucleus.

β^- emission of the nucleus $^{23}_{10}\text{Ne}$ is given as:



It is given that:

$$\text{Atomic mass of } m(^{23}_{10}\text{Ne}) = 22.994466 \text{ u}$$

$$\text{Atomic mass of } m(^{23}_{11}\text{Na}) = 22.989770 \text{ u}$$

Mass of an electron, $m_e = 0.000548 \text{ u}$

Q -value of the given reaction is given as:

$$Q = [m(^{23}_{10}\text{Ne}) - [m(^{23}_{11}\text{Na}) + m_e]]c^2$$

There are 10 electrons in $^{23}_{10}\text{Ne}$ and 11 electrons in $^{23}_{11}\text{Na}$. Hence, the mass of the electron is cancelled in the Q -value equation.

$$\therefore Q = [22.994466 - 22.989770]c^2$$

$$= (0.004696 c^2) \text{ u}$$

But $1 \text{ u} = 931.5 \text{ MeV}/c^2$

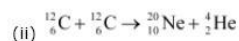
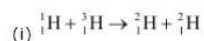
$\therefore Q = 0.004696 \times 931.5 = 4.374 \text{ MeV}$

The daughter nucleus is too heavy as compared to e^- and $\bar{\nu}$. Hence, it carries negligible energy. The kinetic energy of the antineutrino is nearly zero. Hence, the maximum kinetic energy of the emitted electrons is almost equal to the Q -value, i.e., 4.374 MeV.

Question 13.15:

The Q value of a nuclear reaction $A + b \rightarrow C + d$ is defined by

$Q = [m_A + m_b - m_C - m_d]c^2$ where the masses refer to the respective nuclei. Determine from the given data the Q -value of the following reactions and state whether the reactions are exothermic or endothermic.



Atomic masses are given to be

$m({}_1^2\text{H}) = 2.014102 \text{ u}$

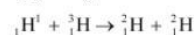
$m({}_1^3\text{H}) = 3.016049 \text{ u}$

$m({}_6^{12}\text{C}) = 12.000000 \text{ u}$

$m({}_{10}^{20}\text{Ne}) = 19.992439 \text{ u}$

Answer

(i) The given nuclear reaction is:



It is given that:

Atomic mass $m({}_1^1\text{H}) = 1.007825 \text{ u}$

Atomic mass $m({}_1^3\text{H}) = 3.016049 \text{ u}$

Atomic mass $m({}_1^2\text{H}) = 2.014102 \text{ u}$

According to the question, the Q -value of the reaction can be written as:

$$Q = [m({}_1^1\text{H}) + m({}_1^3\text{H}) - 2m({}_1^2\text{H})]c^2$$

$$= [1.007825 + 3.016049 - 2 \times 2.014102]c^2$$

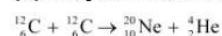
$$Q = (-0.00433 \text{ u})c^2$$

But $1 \text{ u} = 931.5 \text{ MeV}/c^2$

$\therefore Q = -0.00433 \times 931.5 = -4.0334 \text{ MeV}$

The negative Q -value of the reaction shows that the reaction is endothermic.

(ii) The given nuclear reaction is:



It is given that:

Atomic mass of $m({}_6^{12}\text{C}) = 12.0 \text{ u}$

Atomic mass of $m({}_{10}^{20}\text{Ne}) = 19.992439 \text{ u}$

Atomic mass of $m({}_2^4\text{He}) = 4.002603 \text{ u}$

The Q -value of this reaction is given as:

$$Q = [2m({}_6^{12}\text{C}) - m({}_{10}^{20}\text{Ne}) - m({}_2^4\text{He})]c^2$$

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