



Definite Integrals Ex 20.1 Q51

We have,

$$\begin{aligned}\int_0^{2\pi} e^{x/2} \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) dx &= \int_0^{2\pi} e^{x/2} \left(\sin \frac{x}{2} \cos \frac{\pi}{4} + \cos \frac{x}{2} \sin \frac{\pi}{4} \right) dx \\ &= \int_0^{2\pi} e^{x/2} \sin \frac{x}{2} \cdot \frac{1}{\sqrt{2}} dx + \int_0^{2\pi} e^{x/2} \cos \frac{x}{2} \cdot \frac{1}{\sqrt{2}} dx\end{aligned}$$

Expanding 1st part by parts, we get,

$$\begin{aligned}\int_0^{2\pi} e^{x/2} \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) dx &= \frac{1}{\sqrt{2}} \left\{ \sin \frac{x}{2} \int_0^{2\pi} e^{x/2} dx - \int_0^{2\pi} \left(\int_0^{2\pi} e^{x/2} dx \right) \cdot \frac{d\left(\sin \frac{x}{2}\right)}{dx} dx \right\} + \frac{1}{\sqrt{2}} \int_0^{2\pi} e^{x/2} \cdot \cos \frac{x}{2} dx \\ &= \frac{1}{\sqrt{2}} \left\{ \sin \frac{x}{2} \cdot 2e^{x/2} \right\}_0^{2\pi} - \frac{1}{\sqrt{2}} \int_0^{2\pi} e^{x/2} \cdot \frac{1}{2} \cos \frac{x}{2} dx + \frac{1}{\sqrt{2}} \int_0^{2\pi} e^{x/2} \cos \frac{x}{2} dx \\ &= \frac{1}{\sqrt{2}} \left\{ \sin \frac{x}{2} \cdot 2e^{x/2} \right\}_0^{2\pi} = \frac{1}{\sqrt{2}} \{0 - 0\} = 0\end{aligned}$$

$$\therefore \int_0^{2\pi} e^{x/2} \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) dx = 0$$

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$$\text{Let } I = \int_0^{2\pi} e^x \cos\left(\frac{\pi}{4} + \frac{x}{2}\right) dx = \left[\cos\left(\frac{\pi}{4} + \frac{x}{2}\right) e^x \right]_0^{2\pi} + \frac{1}{2} \int_0^{2\pi} e^x \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) dx$$

$$\Rightarrow I = \left[\cos\left(\frac{\pi}{4} + \frac{x}{2}\right) e^x \right]_0^{2\pi} + \frac{1}{2} \left[\sin\left(\frac{\pi}{4} + \frac{x}{2}\right) e^x \right]_0^{2\pi} - \frac{1}{2} \int_0^{2\pi} e^x \cos\left(\frac{\pi}{4} + \frac{x}{2}\right) dx$$

$$I = \left[\cos\left(\pi + \frac{\pi}{4}\right) e^{2\pi} - \cos \frac{\pi}{4} \right] + \frac{1}{2} \left[\sin\left(\pi + \frac{\pi}{4}\right) e^{2\pi} - \sin \frac{\pi}{4} \right] - \frac{1}{2} I$$

$$I = \left[-\cos \frac{\pi}{4} \cdot e^{2\pi} - \cos \frac{\pi}{4} \right] + \frac{1}{2} \left[-\sin \frac{\pi}{4} \cdot e^{2\pi} - \sin \frac{\pi}{4} \right] - \frac{I}{2}$$

$$\frac{5I}{4} = -\frac{1}{\sqrt{2}} (e^{2\pi} + 1) - \frac{1}{2} \cdot \frac{1}{\sqrt{2}} (e^{2\pi} + 1) = \frac{-3}{2\sqrt{2}} (e^{2\pi} + 1)$$

$$I = \frac{-3\sqrt{2}}{5} (e^{2\pi} + 1)$$

$$\therefore \int_0^{2\pi} e^x \cos\left(\frac{\pi}{4} + \frac{x}{2}\right) dx = \frac{-3\sqrt{2}}{5} (e^{2\pi} + 1)$$

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$$\begin{aligned}
\text{Let } I &= \int_0^1 \frac{dx}{\sqrt{1+x} - \sqrt{x}} \\
I &= \int_0^1 \frac{1}{\left(\sqrt{1+x} - \sqrt{x}\right)} \times \frac{\left(\sqrt{1+x} + \sqrt{x}\right)}{\left(\sqrt{1+x} + \sqrt{x}\right)} dx \\
&= \int_0^1 \frac{\sqrt{1+x} + \sqrt{x}}{1+x-x} dx \\
&= \int_0^1 \sqrt{1+x} dx + \int_0^1 \sqrt{x} dx \\
&= \left[\frac{2}{3} (1+x)^{\frac{3}{2}} \right]_0^1 + \left[\frac{2}{3} (x)^{\frac{3}{2}} \right]_0^1 \\
&= \frac{2}{3} \left[(2)^{\frac{3}{2}} - 1 \right] + \frac{2}{3} [1] \\
&= \frac{2}{3} (2)^{\frac{3}{2}} \\
&= \frac{2 \cdot 2\sqrt{2}}{3} \\
&= \frac{4\sqrt{2}}{3}
\end{aligned}$$

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$$\begin{aligned}
\int_1^2 \frac{x}{(x+1)(x+2)} dx &= -\int_1^2 \frac{1}{x+1} dx + \int_1^2 \frac{2}{x+2} dx \quad [\text{Using Partial Fraction}] \\
&= -\log(x+1) \Big|_1^2 + 2\log(x+2) \Big|_1^2 \\
&= -(\log 3 - \log 2) + 2(\log 4 - \log 3) \\
&= -3\log 3 + 5\log 2 \\
&= \log \frac{32}{27}
\end{aligned}$$

***** END *****