



Polynomials Ex 2.1 Q10

Answer :

Since α and β are the zeros of the quadratic polynomial $p(s) = 3s^2 - 6s + 4$

$$\alpha + \beta = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\alpha + \beta = \frac{-(-6)}{3}$$

$$\alpha + \beta = \frac{6}{3}$$

$$\alpha + \beta = 2$$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\alpha\beta = \frac{4}{3}$$

$$\text{We have, } \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$$

$$= \frac{\alpha^2 + \beta^2}{\alpha\beta} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} + 2\left(\frac{\alpha + \beta}{\alpha\beta}\right) + 3\alpha\beta$$

By substituting $\alpha + \beta = 2$ and $\alpha\beta = \frac{4}{3}$ we get ,

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta = \frac{(2)^2 - 2\left(\frac{4}{3}\right)}{\frac{4}{3}} + 2\frac{(2)}{\frac{4}{3}} + 3\left(\frac{4}{3}\right)$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta = \frac{4 - \frac{8}{3}}{\frac{4}{3}} + \frac{4}{\frac{4}{3}} + \frac{12}{3}$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta = \frac{\frac{4 \times 3}{4} - \frac{8}{3}}{\frac{4}{3}} + \frac{4}{\frac{4}{3}} + \frac{12}{3}$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta = \frac{\frac{12 - 8}{3}}{\frac{4}{3}} + \frac{4}{\frac{4}{3}} + \frac{12}{3}$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta = \frac{4}{\frac{4}{3}} + \frac{4}{\frac{4}{3}} + \frac{12}{3}$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta = \cancel{\frac{4}{\cancel{4}}} \times \cancel{\frac{3}{\cancel{4}}} + \frac{4 \times 3}{4} + \frac{12}{3}$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta = 1 + \frac{12}{4} + \frac{12}{3}$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta = \frac{1 \times 12}{1 \times 12} + \frac{12 \times 3}{4 \times 3} + \frac{12 \times 4}{3 \times 4}$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta = \frac{12 + 36 + 48}{12}$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta = \frac{48 + 48}{12}$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta = \frac{\cancel{96}^8}{\cancel{12}}$$

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta = 8$$

Hence, the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$ is 8

***** END *****