Maxima and Minima 18.5 Q10

ABC is a right angled triangle. Hypotenuse $h = AC = 5$ cm.

Let x and y one the other two side of the triangle.

$$\therefore \quad x^2 + y^2 = 25 \qquad \text{---(i)}$$

$$\therefore \quad \text{Area of } \Delta ABC = \frac{1}{2} BC \times AB$$

$$\Rightarrow \quad S = \frac{1}{2} xy \qquad \text{---(ii)}$$

From (i) and (ii)

$$S = \frac{1}{2} x \sqrt{25 - x}$$

$$\therefore \quad \frac{ds}{dx} = \frac{1}{2}\left[\sqrt{25 - x^2} - \frac{2x^2}{2\sqrt{25 - x^2}}\right]$$

$$= \frac{1}{2} \frac{\left[25 - x^2 - x^2\right]}{\sqrt{25 - x^2}}$$

$$= \frac{1}{2}\left[\frac{25 - 2x^2}{\sqrt{25 - x^2}}\right]$$

For maxima and minima,

$$\frac{ds}{dx} = 0$$

$$\Rightarrow \quad \frac{1}{2}\left[\frac{25 - 2x^2}{\sqrt{25 - x^2}}\right] = 0$$

$$\Rightarrow \quad x = 5\sqrt{2}$$

Now,

$$\frac{d^2s}{dx^2} = \frac{1}{2} \frac{\sqrt{25 - x^2} \times (-4x) + \frac{\left(25 - 2x^2\right)2x}{2\sqrt{25 - x^2}}}{\left(25 - x^2\right)}$$

$$\text{At} \quad x = \frac{5}{\sqrt{2}}, \quad \frac{d^2s}{dx^2} = \frac{1}{2} \frac{\left[-\frac{25}{\sqrt{2}} \times \frac{5}{\sqrt{2}} + 0\right]}{\frac{25}{2}}$$

$$= -\frac{5}{2} < 0$$

$$\therefore \quad x = \frac{5}{\sqrt{2}} \text{ is a point local maxima,}$$

Maxima and Minima 18.5 Q11

ABC is a given triangle with $AB = a, BC = b$ and $\angle ABC = \theta$.
AD in perpendicular to BC.

$\therefore \quad BD = a\sin\theta$

Now,

$$\text{Area of } \Delta ABC = \frac{1}{2} \times BC \times AD$$

$$\Rightarrow \quad A = \frac{1}{2} b \times a\sin\theta \qquad \text{---(i)}$$

$$\therefore \quad \frac{dA}{d\theta} = \frac{1}{2} ab\cos\theta$$

For maxima and minima,

$$\frac{dA}{d\theta} = 0$$

$$\Rightarrow \quad \frac{1}{2} ab\cos\theta = 0$$

$$\Rightarrow \quad \cos\theta = 0$$

$$\Rightarrow \quad \theta = \frac{\pi}{2}$$

Now,

$$\frac{d^2A}{d\theta^2} = -\frac{1}{2} ab\sin\theta$$

At $\quad \theta = \frac{\pi}{2}, \quad \frac{d^2A}{d\theta^2} = -\frac{1}{2} ab < 0$

$\therefore \quad \theta = \frac{\pi}{2}$ is point of local maxima

$\therefore$ Maximum area of $\Delta = \frac{1}{2} ab\sin\frac{\pi}{2} = \frac{1}{2} ab.$

Maxima and Minima 18.5 Q12

Let the side of the square to be cut off be x cm. Then, the length and the breadth of the box will be $(18 - 2x)$ cm each and the height of the box is x cm.

Therefore, the volume $V(x)$ of the box is given by,

$V(x) = x(18 - 2x)^2$

$$\begin{aligned}
\therefore V'(x) &= (18-2x)^2 - 4x(18-2x) \\
&= (18-2x)[18-2x-4x] \\
&= (18-2x)(18-6x) \\
&= 6\times 2(9-x)(3-x) \\
&= 12(9-x)(3-x)
\end{aligned}$$

$$\begin{aligned}
\text{And, } V''(x) &= 12[-(9-x)-(3-x)] \\
&= -12(9-x+3-x) \\
&= -12(12-2x) \\
&= -24(6-x)
\end{aligned}$$

Maximum volume is $V_{x=3} = 3\times(18-2\times 3)^2$

$\Rightarrow V = 3\times 12^2$

$\Rightarrow V = 3\times 144$

$\Rightarrow V = 432\ cm^3$

*********** END **********