



Trigonometric Ratios of multiple and Submultiple Angles Ex 9.3 Q 6

$$\begin{aligned}
 \cos \frac{5\pi}{15} &= \cos \frac{\pi}{3} = \frac{1}{2} \\
 \cos \frac{7\pi}{15} &= \cos \left(\pi - \frac{8\pi}{15} \right) \\
 \cos \frac{7\pi}{15} &= -\cos \frac{8\pi}{15}
 \end{aligned}$$

Now

$$\begin{aligned}
 \text{LHS} &= \cos \frac{\pi}{15} \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{3\pi}{15} \cos \frac{5\pi}{15} \cos \frac{6\pi}{15} \cos \frac{7\pi}{15} \\
 &= \left[\cos \frac{\pi}{15} \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \left(-\cos \frac{8\pi}{15} \right) \right] \left(\cos \frac{3\pi}{15} \cos \frac{6\pi}{15} \right) \frac{1}{2} \\
 &= -\frac{2^3}{2^4 \sin \frac{\pi}{15}} \left[2 \sin \frac{\pi}{15} \cos \frac{\pi}{15} \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \right] \\
 &\quad \times \frac{2}{8 \sin \frac{3\pi}{15}} \left(2 \sin \frac{3\pi}{15} \cos \frac{3\pi}{15} \cos \frac{6\pi}{15} \right) \\
 &= -\frac{2^3}{16 \sin \frac{\pi}{15}} \left[\sin \frac{2\pi}{15} \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \right] \times \frac{2}{8 \sin \frac{3\pi}{15}} \left(\sin \frac{6\pi}{15} \cos \frac{6\pi}{15} \right) \\
 &= -\frac{2^2}{16 \sin \frac{\pi}{15}} \left[2 \sin \frac{2\pi}{15} \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \right] \times \frac{1}{8 \sin \frac{3\pi}{15}} \left(2 \sin \frac{6\pi}{15} \cos \frac{6\pi}{15} \right) \\
 &= -\frac{2}{16 \sin \frac{\pi}{15}} \left[\sin \frac{8\pi}{15} \cos \frac{8\pi}{15} \right] \frac{\sin \frac{12\pi}{15}}{8 \sin \frac{3\pi}{15}} \\
 &= -\frac{1}{16 \sin \frac{\pi}{15}} \left(\sin \frac{16\pi}{15} \right) \frac{\sin \frac{12\pi}{15}}{8 \sin \frac{3\pi}{15}} \\
 &= -\frac{\sin \left(\pi + \frac{\pi}{15} \right)}{128 \sin \frac{\pi}{15}} \times \frac{\sin \left(\pi - \frac{3\pi}{15} \right)}{\sin \frac{3\pi}{15}} \\
 &= -\frac{\sin \frac{\pi}{15}}{128 \sin \frac{\pi}{15}} \times \frac{\sin \frac{3\pi}{15}}{\sin \frac{3\pi}{15}} \\
 &= \frac{1}{128}
 \end{aligned}$$

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.3 Q 7

$$\text{L.H.S} = \cos 6^\circ, \cos 42^\circ, \cos 66^\circ, \cos 78^\circ$$

$$= \frac{1}{4} (2 \cos 6^\circ, \cos 66^\circ) (2 \cos 42^\circ, \cos 78^\circ)$$

$$= \frac{1}{4} (\cos 72^\circ + \cos 60^\circ) (\cos 120^\circ + \cos 36^\circ)$$

$$= \frac{1}{4} \left(\sin 18^\circ + \frac{1}{2} \right) \left(-\frac{2}{2} + \frac{\sqrt{5}+1}{4} \right)$$

$$= \frac{1}{4} \left(\frac{\sqrt{5}-1}{4} + \frac{1}{2} \right) \left(\frac{\sqrt{5}+1}{4} - \frac{1}{2} \right)$$

$$= \frac{1}{4} \left(\frac{\sqrt{5}-1+2}{4} \right) \left(\frac{\sqrt{5}+1-2}{4} \right)$$

$$= \frac{1}{64} (\sqrt{5}+1)(\sqrt{5}-1)$$

$$= \frac{1}{64} (\sqrt{5})^2 - 1^2$$

$$= \frac{1}{64} (5-1)$$

$$= \frac{1}{16}$$

$$= \text{RHS}$$

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.3 Q 8

$$\text{L.H.S} = \sin 6^\circ, \sin 42^\circ, \sin 66^\circ, \sin 78^\circ$$

$$= \frac{1}{4} (2 \sin 6^\circ, \sin 66^\circ) (2 \sin 42^\circ, \sin 78^\circ)$$

$$= \frac{1}{4} (\cos 60^\circ - \cos 72^\circ) (\cos 36^\circ - \cos 120^\circ)$$

$$= \frac{1}{4} \left(\frac{1}{2} - \sin 18^\circ \right) \left(\frac{\sqrt{5}+1}{4} + \frac{1}{2} \right)$$

$$= \frac{1}{4} \left(\frac{1}{2} - \frac{\sqrt{5}-1}{4} \right) \left(\frac{\sqrt{5}+1}{4} + \frac{1}{2} \right)$$

$$= \frac{1}{4} \left(\frac{2-\sqrt{5}+1}{4} \right) \left(\frac{\sqrt{5}+1+2}{4} \right)$$

$$= \frac{1}{64} (3^2 - \sqrt{5}^2)$$

$$= \frac{1}{64} (9-5)$$

$$= \frac{1}{16}$$

$$= \text{RHS}$$

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.3 Q 9

$$\text{L.H.S} = \cos 36^\circ \cdot \cos 42^\circ \cdot \cos 60^\circ \cdot \cos 78^\circ$$

$$\begin{aligned} &= \frac{1}{2} \cos 36^\circ \cdot \cos 60^\circ \cdot (2 \cos 42^\circ \cdot \cos 78^\circ) \\ &= \frac{1}{2} \left(\frac{\sqrt{5} + 1}{4} \right) \cdot \frac{1}{2} (\cos 120^\circ + \cos 36^\circ) \\ &= \frac{(\sqrt{5} + 1)}{16} \left(\frac{-1}{2} + \frac{\sqrt{5} + 1}{4} \right) \\ &= \frac{(\sqrt{5} + 1)}{16} \left(\frac{-2 + \sqrt{5} + 1}{4} \right) \\ &= \frac{(\sqrt{5} + 1)(\sqrt{5} - 1)}{64} \\ &= \frac{5 - 1}{64} \\ &= \frac{1}{16} \\ &= \text{RHS} \end{aligned}$$

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.3 Q 10

L.H.S,

$$\begin{aligned} &\sin 36^\circ \cdot \sin 72^\circ \cdot \sin 108^\circ \cdot \sin 144^\circ \quad \left[\begin{array}{l} \because \sin 144^\circ = \sin (180^\circ - 36^\circ) = \sin 36^\circ \\ \text{and } \sin 108^\circ = \sin (180^\circ - 72^\circ) = \sin 72^\circ \end{array} \right] \\ &= \sin 36^\circ \cdot \sin 72^\circ \cdot \sin 72^\circ \cdot \sin 36^\circ \\ &= \frac{1}{4} (2 \sin 36^\circ \cdot \sin 72^\circ)^2 \\ &= \frac{1}{4} (2 \sin 36^\circ \cos 18^\circ)^2 \quad [\because \sin 72^\circ = \cos 18^\circ] \\ &= \frac{4}{4} \left(\frac{\sqrt{10 - 2\sqrt{5}}}{4} \cdot \frac{\sqrt{10 + 2\sqrt{5}}}{4} \right)^2 \\ &= \frac{1}{64} (10 - 2\sqrt{5})(10 + 2\sqrt{5}) \\ &= \frac{100 - 20}{64 \times 4} \\ &= \frac{80}{256} \\ &= \frac{5}{16} \\ &= \text{RHS} \end{aligned}$$

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