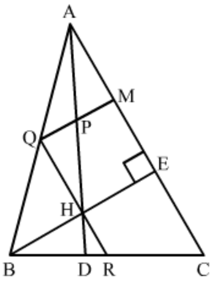
Quadrilaterals Ex 14.4 Q10

Answer :

ΔABC is given with $BE \perp AC$

AD is any line from A to BC intersecting BE in H.



P, Q and R respectively are the mid-points of AH, AB and BC.

We need to prove that $\angle PQR = 90^0$

Let us extend QP to meet AC at M.

In ΔABC, R and Q are the mid-points of BC and AB respectively.

Theorem states, the line segment joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.

Therefore, we get:

$QR \parallel AC$

$QH \parallel ME$ ...... (i)

Similarly, in ΔABH,

$QP \parallel BH$

$QM \parallel HE$ ...... (ii)

From (i) and (ii),we get:

$QM \parallel HE$ and $QH \parallel ME$

We get, $QHME$ is a parallelogram.

Also, $BE \perp AC$

Therefore, $QHME$ is a rectangle.

Thus, $\angle MQH = 90^0$

Or,

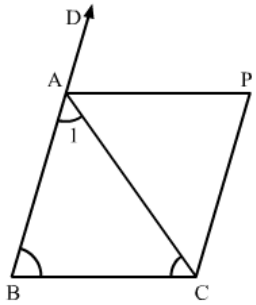
$$\boxed{\angle PQR = 90^0}$$

Hence proved.

Quadrilaterals Ex 14.4 Q11

Answer :

We have the following given figure:



We have $AB = AC$ and $CP \parallel BA$ and *AP* is the bisector of exterior angle $\angle CAD$ of ΔABC.

(i) We need to prove that $\angle PAC = \angle BCA$

In ΔABC,

We have $AB = AC$ (Given)

Thus, $\angle ABC = \angle BCA$ (Angles opposite to equal sides are equal)

By angle sum property of a triangle, we get:

$\angle 1 + \angle BCA + \angle ABC = 180^0$

$\angle 1 + \angle BCA + \angle BCA = 180^0$

$\angle 1 + 2\angle BCA = 180^0$ …… (i)

Now,

$\angle PAC = \angle PAD$ (*AP* is the bisector of exterior angle $\angle CAD$)

$\angle 1 + \angle PAC + \angle PAD = 180^0$ (Linear Pair)

$\angle 1 + \angle PAC + \angle PAC = 180^0$

$\angle 1 + 2\angle PAC = 180^0$ …… (ii)

From equation (i) and (ii), we get:

$\angle 1 + 2\angle BCA = \angle 1 + 2\angle PAC$

$\boxed{\angle BCA = \angle PAC}$

(ii) We need to prove that $ABCP$ is a parallelogram.

We have proved that $\angle PAC = \angle BCA$

This means, $AP \parallel BC$

Also it is given that $CP \parallel BA$

We know that a quadrilateral with opposite sides parallel is a parallelogram.

Therefore, $ABCP$ is a parallelogram.

*********** END **********