



Differentiation Ex 11.3 Q35

$$\text{Let } y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$$

$$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$\text{Put, } x = \tan \theta$$

$$y = \sin^{-1}\left(\frac{2 \tan \theta}{1 + \tan^2 \theta}\right) + \cos^{-1}\left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}\right)$$

$$y = \sin^{-1}(\sin 2\theta) + \cos^{-1}(\cos 2\theta) \quad \text{---(i)}$$

$$\text{Here, } 0 < x < 1$$

$$\Rightarrow 0 < \tan \theta < 1$$

$$\Rightarrow 0 < \theta < \frac{\pi}{4}$$

$$\Rightarrow 0 < (2\theta) < \frac{\pi}{2}$$

So, from equation (i),

$$y = 2\theta + 2\theta$$

$$y = 4\theta$$

$$y = 4 \tan^{-1} x$$

$$\left[\begin{array}{l} \text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ \cos^{-1}(\cos \theta) = \theta, \text{ if } \theta \in [0, \pi] \end{array} \right]$$

$$[\text{Since, } x = \tan \theta]$$

Differentiating it with respect to x ,

$$\frac{dy}{dx} = \frac{4}{1+x^2}.$$

Differentiation Ex 11.3 Q36

$$\text{Here, } y = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) + \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$$

$$\text{Put } x = \tan \theta$$

$$y = \sin^{-1}\left(\frac{\tan \theta}{\sqrt{1+\tan^2 \theta}}\right) + \cos^{-1}\left(\frac{1}{\sqrt{1+\tan^2 \theta}}\right)$$

$$= \sin^{-1}\left(\frac{\sin \theta}{\frac{\cos \theta}{\sec \theta}}\right) + \cos^{-1}\left(\frac{1}{\sec \theta}\right)$$

$$= \sin^{-1}\left(\frac{\sin \theta}{\frac{\cos \theta}{1}}\right) + \cos^{-1}(\cos \theta)$$

$$y = \sin^{-1}(\sin \theta) + \cos^{-1}(\cos \theta) \quad \text{---(i)}$$

$$\text{Here, } 0 < x < \infty$$

$$\Rightarrow 0 < \tan \theta < \infty$$

$$\Rightarrow 0 < \theta < \frac{\pi}{2}$$

So, from equation (i),

$$y = \theta + \theta$$

$$= 2\theta$$

$$y = 2 \tan^{-1} x$$

$$\left[\begin{array}{l} \text{Since, } \sin^{-1}(\sin \theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ \text{and } \cos^{-1}(\cos \theta) = \theta, \text{ if } \theta \in [0, \pi] \end{array} \right]$$

$$[\text{Since, } x = \tan \theta]$$

Differentiating it with respect to x ,

$$\frac{dy}{dx} = \frac{2}{1+x^2}.$$

Differentiation Ex 11.3 Q37

$$\text{Let } f(x) = \cos^{-1}(\sin x)$$

We observe that this function is defined for all real numbers.

$$f(x) = \cos^{-1}(\sin x)$$

$$= \cos^{-1}\left[\cos\left(\frac{\pi}{2} - x\right)\right] = \frac{\pi}{2} - x$$

$$\text{Thus, } f'(x) = \frac{d}{dx}\left(\frac{\pi}{2} - x\right) = -1$$

Let $y = \cot^{-1}\left(\frac{1-x}{1+x}\right)$

Put $x = \tan \theta$, so,

$$y = \cot^{-1}\left(\frac{1 - \tan \theta}{1 + \tan \theta}\right)$$

$$= \cot^{-1}\left(\frac{\tan \frac{\pi}{4} - \tan \theta}{1 + \tan \frac{\pi}{4} \tan \theta}\right)$$

$$= \cot^{-1}\left[\tan\left(\frac{\pi}{4} - \theta\right)\right]$$

$$= \cot^{-1}\left[\cot\left(\frac{\pi}{2} - \frac{\pi}{4} + \theta\right)\right]$$

$$= \frac{\pi}{4} + \theta$$

$$y = \frac{\pi}{4} + \tan^{-1} x \quad \text{[Since } x = \tan \theta \text{]}$$

Differentiating it with respect to x ,

$$\frac{dy}{dx} = 0 + \frac{1}{1+x^2}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}.$$

***** END *****