



Polynomials Ex 2.1 Q20

Answer :

If α and β are the zeros of the quadratic polynomial $f(x) = x^2 + px + q$

$$\alpha + \beta = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$= \frac{-p}{1}$$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$= \frac{q}{1}$$

$$= q$$

Let S and P denote respectively the sums and product of the zeros of the polynomial whose zeros are $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$. Then,

$$S = (\alpha + \beta)^2 + (\alpha - \beta)^2$$

$$S = \alpha^2 + \beta^2 + 2\alpha\beta + \alpha^2 + \beta^2 - 2\alpha\beta$$

$$S = 2[\alpha^2 + \beta^2]$$

$$S = 2[(\alpha + \beta)^2 - 2\alpha\beta]$$

$$S = 2(p^2 - 2q)$$

$$S = 2(p^2 - 2q)$$

$$S = 2(p^2 - 2q)$$

$$P = (\alpha + \beta)^2 (\alpha - \beta)^2$$

$$P = (\alpha^2 + \beta^2 + 2\alpha\beta)(\alpha^2 + \beta^2 - 2\alpha\beta)$$

$$P = ((\alpha + \beta)^2 - 2\alpha\beta + 2\alpha\beta)((\alpha + \beta)^2 - 2\alpha\beta - 2\alpha\beta)$$

$$P = (p)^2 ((p)^2 - 4q)$$

$$P = p^2 (p^2 - 4q)$$

The required polynomial of $f(x) = k(kx^2 - sx + p)$ is given by

$$f(x) = k\{x^2 - 2(p^2 - 2q)x + p^2(p^2 - 4q)\}$$

$$\boxed{f(x) = k\{x^2 - 2(p^2 - 2q)x + p^2(p^2 - 4q)\}}, \text{ where } k \text{ is any non-zero real number.}$$

***** END *****