Trigonometric Ratios Ex 5.1 Q18

Answer :

Given: $\sec\theta = \dfrac{5}{4}$ ......(1)

To find the value of $\dfrac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta}$

Now we know that $\sec\theta = \dfrac{1}{\cos\theta}$

Therefore,

$$\cos\theta = \dfrac{1}{\sec\theta}$$

Therefore from equation (1)

$$\cos\theta = \dfrac{1}{\dfrac{5}{4}}$$

$$\cos\theta = \dfrac{4}{5} \quad \ldots\ldots (2)$$

Also, we know that $\cos^2\theta + \sin^2\theta = 1$

Therefore,

$$\sin^2\theta = 1 - \cos^2\theta$$
$$\sin\theta = \sqrt{1 - \cos^2\theta}$$

Substituting the value of $\cos\theta$ from equation (2)

We get,

$$\sin\theta = \sqrt{1-\left(\frac{4}{5}\right)^2}$$

$$= \sqrt{1-\frac{4^2}{5^2}}$$

$$= \sqrt{1-\frac{16}{25}}$$

$$= \sqrt{\frac{25-16}{25}}$$

$$= \sqrt{\frac{9}{25}}$$

$$= \frac{3}{5}$$

Therefore

$$\sin\theta = \frac{3}{5} \quad \ldots\ldots (3)$$

Also, we know that $sec^2\theta = 1+\tan^2\theta$.

Therefore,

$$\tan^2\theta = sec^2\theta - 1$$

Therefore

$$\tan^2\theta = \left(\frac{5}{4}\right)^2 - 1$$

$$= \frac{25}{16} - 1$$

$= \frac{9}{16}$

Therefore,

$\tan\theta = \sqrt{\frac{9}{16}}$

$= \frac{3}{4}$

Therefore,

$\tan\theta = \frac{3}{4}$ ...... (4)

Also $\cot\theta = \frac{1}{\tan\theta}$

Therefore, from equation (4)

We get,

$\cot\theta = \frac{1}{\frac{3}{4}}$

$\cot\theta = \frac{4}{3}$ ...... (5)

Substituting the value of $\cos\theta$, $\sin\theta$, $\cot\theta$ and $\tan\theta$ from equation (2) (3) (4) and (5) respectively in the expression below

$\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta}$

We get,

$\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta} = \frac{\frac{3}{5} - 2\left(\frac{4}{5}\right)}{\frac{3}{4} - \frac{4}{3}}$

$= \frac{\frac{3}{5} - \frac{8}{5}}{\frac{(3\times3) - (4\times4)}{4\times3}}$

$$= \frac{\dfrac{3}{5} - \dfrac{8}{5}}{\dfrac{(3\times3) - (4\times4)}{4\times3}}$$

$$= \frac{\dfrac{3-8}{5}}{\dfrac{9-16}{4\times3}}$$

$$= \frac{\dfrac{-5}{5}}{\dfrac{-7}{12}}$$

$$= \frac{12}{7}$$

Therefore, $\dfrac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta} = \dfrac{12}{7}$

*********** END *********