Mathematical Induction Ex 12.2 Q30

Let $P(n)$ be the statement given by

$P(n): 1 + 2 + 2^2 + \cdots\cdots + 2^n = 2^{n+1} - 1$ for all $n \in N$.

Step I:

$P(1): 1 + 2^1 = 2^{1+1} - 1$

$\Rightarrow 1 + 2 = 4 - 1$

$\Rightarrow 3 = 3$

$\therefore P(1)$ is true.

Step II:

Let $P(m)$ is true. Then,

$1 + 2 + 2^2 + \cdots\cdots + 2^m = 2^{m+1} - 1 \ldots\ldots\ldots(i)$

We have to prove that $P(m+1)$ is true.

$$\begin{aligned}1 + 2 + 2^2 + \cdots\cdots + 2^{m+1} &= 1 + 2 + 2^2 + \cdots\cdots + 2^m + 2^{m+1} \\ &= \left(2^{m+1} - 1\right) + 2^{m+1} \ldots\ldots\ldots\ldots\ldots\ldots[\text{Using (i)}] \\ &= \left(2^{m+1} + 2^{m+1}\right) - 1 \\ &= 2 \times 2^{m+1} - 1 \\ &= 2^{m+2} - 1\end{aligned}$$

$\Rightarrow P(m+1)$ is true.

Hence by the principle of mathematical induction, the given result is true for all $n \in N$.

Mathematical Induction Ex 12.2 Q31

Let P(n) be the statement given by

$P(n): 7 + 77 + 777 + \cdots + \underbrace{777\ldots\ldots7}_{n-\text{digits}} = \frac{7}{81}\left[10^{n+1} - 9n - 10\right]$ for all $n \in N$.

Step I:

$P(1): 7 = \frac{7}{81}\left[10^{1+1} - 9(1) - 10\right]$

$\Rightarrow 7 = \frac{7}{81} \times (100 - 9 - 10)$

$\Rightarrow 7 = \frac{7}{81} \times 81$

$\Rightarrow 7 = 7 \times (1)$

$\therefore$ P(1) is true.

Step II:

Let P(m) is true. Then,

$7 + 77 + 777 + \cdots + \underbrace{777\ldots\ldots7}_{m-\text{digits}} = \frac{7}{81}\left[10^{m+1} - 9m - 10\right]$........(i)

We have to prove that P(m + 1) is true.

$7 + 77 + 777 + \cdots + \underbrace{777\ldots\ldots7}_{m+1-\text{digits}} = 7 + 77 + 777 + \cdots + \underbrace{777\ldots\ldots7}_{m-\text{digits}} + \underbrace{777\ldots\ldots7}_{m+1-\text{digits}}$

$= \frac{7}{81}\left[10^{m+1} - 9m - 10\right] + 7\left[\underbrace{1111\ldots\ldots1}_{m+1-\text{digits}}\right]$ [Using (i)]

$= \frac{7}{81}\left[10^{m+1} - 9m - 10\right] + \frac{7}{9}\left[\underbrace{9999\ldots\ldots9}_{m+1-\text{digits}}\right]$

$= \frac{7}{81}\left[10^{m+1} - 9m - 10\right] + \frac{7}{9}\left[10^{m+1} - 1\right]$

$= \frac{7}{81}\left[(1+9)10^{m+1} - 9m - 19\right]$

$= \frac{7}{81}\left[10 \times 10^{m+1} - 9(m+1) - 10\right]$

$= \frac{7}{81}\left[10^{m+2} - 9(m+1) - 10\right]$

$\Rightarrow$ P(m+1) is true.

Hence by the principle of mathematical induction, the given result is true for all $n \in N$.

Mathematical Induction Ex 12.2 Q32

Let $P(n): \frac{n^7}{7} + \frac{n^5}{5} + \frac{n^3}{3} + \frac{n^2}{2} - \frac{37}{210}n$ is a positive integer

For $n = 1$

$$\frac{1}{7} + \frac{1}{5} + \frac{1}{3} + \frac{1}{2} - \frac{37}{210}$$

$$= \frac{30 + 42 + 70 + 105 - 37}{210}$$

$$= \frac{247 - 37}{210}$$

It is a positive integer

$\Rightarrow$ $P(n)$ is true for $n = 1$

Let $P(n)$ is true for $n = k$,

$\frac{k^7}{7} + \frac{k^5}{5} + \frac{k^3}{3} + \frac{k^2}{2} - \frac{37}{210}k$ is positive integer

$\frac{k^7}{7} + \frac{k^5}{5} + \frac{k^3}{3} + \frac{k^2}{2} - \frac{37}{210}k = \lambda$

For $n = k + 1$,

$$\frac{(k+1)^7}{7} + \frac{(k+1)^5}{5} + \frac{(k+1)^3}{3} + \frac{(k+1)^2}{2} - \frac{37}{210}(k+1)$$

$$= \frac{1}{7}\left[k^7 + 7k^6 + 21k^5 + 35k^4 + 35k^3 + 21k^2 + 7k + 1\right] + \frac{1}{5}\left[k^5 + 5k^4 + 10k^3 + 10k^2 + 5k + 1\right] + \frac{1}{3}\left[k^3 + 3k^2 + 3k + 1\right] + \frac{1}{2}\left[k^2 + 2k + 1\right] - \frac{37k}{210} - \frac{37}{210}$$

$$= \left[\frac{k^7}{7} + \frac{k^5}{5} + \frac{k^3}{3} + \frac{k^2}{2} - \frac{37k}{210}\right] + \left[k^6 + 3k^5 + 5k^4 + 5k^3 + 3k^2 + k + \frac{1}{7} + k^4 + 2k^3 + 2k^2 + \frac{1}{5} + k^2 + k + \frac{1}{3} + k + \frac{1}{2} - \frac{37}{210}\right]$$

$$= \lambda + k^6 + 3k^5 + 6k^4 + 7k^3 + 6k^2 + 3k + \frac{1}{7} + \frac{1}{5} + \frac{1}{3} + \frac{1}{2} - \frac{37}{210}$$

$$= \lambda + k^6 + 3k^5 + 6k^4 + 7k^3 + 6k^2 + 3k + 1$$

= Positive integer

$\Rightarrow$ $P(n)$ is true for $n = k + 1$

$\Rightarrow$ $P(n)$ is true for all $n \in N$ by *PMI*

*********** END **********