



Real Numbers Ex 1.1 Q7

Answer :

To Prove: that the square of any positive integer is of the form $5q$ or $5q + 1$, $5q + 4$ for some integer q .

Proof: Since positive integer n is of the form of $5q$ or $5q + 1$, $5q + 4$

If $n = 5q$

$$\text{Then, } n^2 = (5q)^2$$

$$\Rightarrow n^2 = 25q^2$$

$$\Rightarrow n^2 = 5(5q)$$

$$\Rightarrow n^2 = 5m \text{ (where } m = 5q \text{)}$$

If $n = 5q + 1$

$$\text{Then, } n^2 = (5q + 1)^2$$

$$\Rightarrow n^2 = (5q)^2 + 10q + 1$$

$$\Rightarrow n^2 = 25q^2 + 10q + 1$$

$$\Rightarrow n^2 = 5q(5q + 2) + 1$$

$$\Rightarrow n^2 = 5m + 1 \text{ (where } m = q(5q + 2) \text{)}$$

If $n = 5q + 2$

$$\text{Then, } n^2 = (5q + 2)^2$$

$$\Rightarrow n^2 = (5q)^2 + 20q + 4$$

$$\Rightarrow n^2 = 25q^2 + 20q + 4$$

$$\Rightarrow n^2 = 5(5q^2 + 4q + 3) + 1$$

$$\Rightarrow n^2 = 5m + 1 \text{ (where } m = (5q^2 + 4q + 3) \text{)}$$

Hence it is proved that the square of a positive integer is of the form $5q$ or $5q + 1$, $5q + 4$ for some integer q .

Real Numbers Ex 1.1 Q8

Answer :

To Prove: that if a positive integer is of the form $6q + 5$ then it is of the form $3q + 2$ for some integer q , but not conversely.

Proof: Let $n = 6q + 5$

Since any positive integer n is of the form of $3k$ or $3k + 1$, $3k + 2$

If $q = 3k$

$$\text{Then, } n = 6q + 5$$

$$\Rightarrow n = 18k + 5 \text{ (} q = 3k \text{)}$$

$$\Rightarrow n = 3(6k + 1) + 2$$

$$\Rightarrow n = 3m + 2 \text{ (where } m = (6k + 1) \text{)}$$

If $q = 3k + 1$

$$\text{Then, } n = (6q + 5)$$

$$\Rightarrow n = (6(3k + 1) + 5) \text{ (} q = 3k + 1 \text{)}$$

$$\Rightarrow n = 18k + 6 + 5$$

$$\Rightarrow n = 18k + 11$$

$$\Rightarrow n = 3(6k + 3) + 2$$

$$\Rightarrow n = 3m + 2 \text{ (where } m = (6k + 3) \text{)}$$

If $q = 3k + 2$

$$\text{Then, } n = (6q + 5)$$

$$\Rightarrow n = (6(3k + 2) + 5) \text{ (} q = 3k + 2 \text{)}$$

$$\Rightarrow n = 18k + 12 + 5$$

$$\Rightarrow n = 18k + 17$$

$$\Rightarrow n = 3(6k + 5) + 2$$

$$\Rightarrow n = 3m + 2 \text{ (where } m = (6k + 5) \text{)}$$

Consider here 8 which is the form $3q + 2$ i.e. $3 \times 2 + 2$ but it can't be written in the form $6q + 5$.

Hence the converse is not true

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