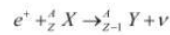




Question 13.22:

For the β^+ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the K-shell, is captured by the nucleus and a neutrino is emitted).



Show that if β^+ emission is energetically allowed, electron capture is necessarily allowed but not vice-versa.

Answer

Let the amount of energy released during the electron capture process be Q_1 . The nuclear reaction can be written as:



Let the amount of energy released during the positron capture process be Q_2 . The nuclear reaction can be written as:



$$m_N({}^A_Z X) = \text{Nuclear mass of } {}^A_Z X$$

$$m_N({}^A_{Z-1} Y) = \text{Nuclear mass of } {}^A_{Z-1} Y$$

$$m({}^A_Z X) = \text{Atomic mass of } {}^A_Z X$$

$$m({}^A_{Z-1} Y) = \text{Atomic mass of } {}^A_{Z-1} Y$$

$$m_e = \text{Mass of an electron}$$

$$c = \text{Speed of light}$$

Q -value of the electron capture reaction is given as:

$$\begin{aligned} Q_1 &= [m_N({}^A_Z X) + m_e - m_N({}^A_{Z-1} Y)]c^2 \\ &= [m({}^A_Z X) - Zm_e + m_e - m({}^A_{Z-1} Y) + (Z-1)m_e]c^2 \\ &= [m({}^A_Z X) - m({}^A_{Z-1} Y)]c^2 \quad \dots (3) \end{aligned}$$

Q -value of the positron capture reaction is given as:

$$\begin{aligned} Q_2 &= [m_N({}^A_Z X) - m_N({}^A_{Z-1} Y) - m_e]c^2 \\ &= [m({}^A_Z X) - Zm_e - m({}^A_{Z-1} Y) + (Z-1)m_e - m_e]c^2 \\ &= [m({}^A_Z X) - m({}^A_{Z-1} Y) - 2m_e]c^2 \quad \dots (4) \end{aligned}$$

It can be inferred that if $Q_2 > 0$, then $Q_1 > 0$; Also, if $Q_1 > 0$, it does not necessarily mean that $Q_2 > 0$.

Question 13.24:

The neutron separation energy is defined as the energy required to remove a neutron

from the nucleus. Obtain the neutron separation energies of the nuclei ${}^{41}_{20}\text{Ca}$ and

${}^{27}_{13}\text{Al}$ from the following data:

$$m({}^{40}_{20}\text{Ca}) = 39.962591 \text{ u}$$

$$m({}^{41}_{20}\text{Ca}) = 40.962278 \text{ u}$$

$$m({}^{26}_{13}\text{Al}) = 25.986895 \text{ u}$$

$$m({}^{27}_{13}\text{Al}) = 26.981541 \text{ u}$$

Answer

$$\text{For } {}^{41}_{20}\text{Ca: Separation energy} = 8.363007 \text{ MeV}$$

$$\text{For } {}^{27}_{13}\text{Al: Separation energy} = 13.059 \text{ MeV}$$

A neutron (${}_0^1\text{n}$) is removed from a ${}_{20}^{41}\text{Ca}$ nucleus. The corresponding nuclear reaction can be written as:



It is given that:

$$\text{Mass } m({}_{20}^{40}\text{Ca}) = 39.962591 \text{ u}$$

$$\text{Mass } m({}_{20}^{41}\text{Ca}) = 40.962278 \text{ u}$$

$$\text{Mass } m({}_0^1\text{n}) = 1.008665 \text{ u}$$

The mass defect of this reaction is given as:

$$\begin{aligned}\Delta m &= m({}_{20}^{40}\text{Ca}) + m({}_0^1\text{n}) - m({}_{20}^{41}\text{Ca}) \\ &= 39.962591 + 1.008665 - 40.962278 = 0.008978 \text{ u}\end{aligned}$$

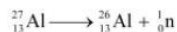
$$\text{But } 1 \text{ u} = 931.5 \text{ MeV}/c^2$$

$$\therefore \Delta m = 0.008978 \times 931.5 \text{ MeV}/c^2$$

Hence, the energy required for neutron removal is calculated as:

$$\begin{aligned}E &= \Delta mc^2 \\ &= 0.008978 \times 931.5 = 8.363007 \text{ MeV}\end{aligned}$$

For ${}_{13}^{27}\text{Al}$, the neutron removal reaction can be written as:



It is given that:

$$\text{Mass } m({}_{13}^{27}\text{Al}) = 26.981541 \text{ u}$$

$$\text{Mass } m({}_{13}^{26}\text{Al}) = 25.986895 \text{ u}$$

The mass defect of this reaction is given as:

$$\begin{aligned}\Delta m &= m({}_{13}^{26}\text{Al}) + m({}_0^1\text{n}) - m({}_{13}^{27}\text{Al}) \\ &= 25.986895 + 1.008665 - 26.981541 \\ &= 0.014019 \text{ u} \\ &= 0.014019 \times 931.5 \text{ MeV}/c^2\end{aligned}$$

Hence, the energy required for neutron removal is calculated as:

$$\begin{aligned}E &= \Delta mc^2 \\ &= 0.014019 \times 931.5 = 13.059 \text{ MeV}\end{aligned}$$

Question 13.25:

A source contains two phosphorous radio nuclides ${}_{15}^{32}\text{P}$ ($T_{1/2} = 14.3\text{d}$) and ${}_{15}^{33}\text{P}$ ($T_{1/2} = 25.3\text{d}$). Initially, 10% of the decays come from ${}_{15}^{33}\text{P}$. How long one must wait until 90% do so?

Answer

Half life of ${}_{15}^{32}\text{P}$, $T_{1/2} = 14.3$ days

Half life of ${}_{15}^{33}\text{P}$, $T'_{1/2} = 25.3$ days

${}_{15}^{33}\text{P}$ nucleus decay is 10% of the total amount of decay.

The source has initially 10% of ${}_{15}^{33}\text{P}$ nucleus and 90% of ${}_{15}^{32}\text{P}$ nucleus.

Suppose after t days, the source has 10% of ${}_{15}^{32}\text{P}$ nucleus and 90% of ${}_{15}^{33}\text{P}$ nucleus.

Initially:

Number of ${}_{15}^{33}\text{P}$ nucleus = N

Number of ${}_{15}^{32}\text{P}$ nucleus = $9N$

Finally:

Number of ${}_{15}^{33}\text{P}$ nucleus = $9N'$

Number of ${}_{15}^{32}\text{P}$ nucleus = N'

For ${}_{15}^{32}\text{P}$ nucleus, we can write the number ratio as:

$$\frac{N'}{9N} = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

$$N' = 9N(2)^{-\frac{t}{14.3}} \quad \dots (1)$$

For ${}_{15}^{33}\text{P}$, we can write the number ratio as:

For $\frac{1}{9}$, we can write the number as 2^{-x}

$$\frac{9N'}{N} = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

$$9N' = N(2)^{\frac{-t}{25.3}} \quad \dots (2)$$

On dividing equation (1) by equation (2), we get:

$$\frac{1}{9} = 9 \times 2^{\left(\frac{t}{25.3} - \frac{t}{14.3}\right)}$$

$$\frac{1}{81} = 2^{\left(\frac{-11t}{25.3 \times 14.3}\right)}$$

$$\log 1 - \log 81 = \frac{-11t}{25.3 \times 14.3} \log 2$$

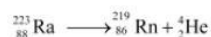
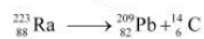
$$\frac{-11t}{25.3 \times 14.3} = \frac{0 - 1.908}{0.301}$$

$$t = \frac{25.3 \times 14.3 \times 1.908}{11 \times 0.301} \approx 208.5 \text{ days}$$

Hence, it will take about 208.5 days for 90% decay of $^{13}\text{P}^{33}$.

Question 13.26:

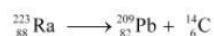
Under certain circumstances, a nucleus can decay by emitting a particle more massive than an α -particle. Consider the following decay processes:



Calculate the Q -values for these decays and determine that both are energetically allowed.

Answer

Take a $^{14}_6\text{C}$ emission nuclear reaction:



We know that:

$$\text{Mass of } ^{223}_{88}\text{Ra}, m_1 = 223.01850 \text{ u}$$

$$\text{Mass of } ^{209}_{82}\text{Pb}, m_2 = 208.98107 \text{ u}$$

$$\text{Mass of } ^{14}_6\text{C}, m_3 = 14.00324 \text{ u}$$

Hence, the Q -value of the reaction is given as:

$$\begin{aligned} Q &= (m_1 - m_2 - m_3) c^2 \\ &= (223.01850 - 208.98107 - 14.00324) c^2 \\ &= (0.03419 c^2) \text{ u} \end{aligned}$$

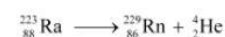
$$\text{But } 1 \text{ u} = 931.5 \text{ MeV}/c^2$$

$$\therefore Q = 0.03419 \times 931.5$$

$$= 31.848 \text{ MeV}$$

Hence, the Q -value of the nuclear reaction is 31.848 MeV. Since the value is positive, the reaction is energetically allowed.

Now take a ^4_2He emission nuclear reaction:



We know that:

$$\text{Mass of } ^{223}_{88}\text{Ra}, m_1 = 223.01850$$

$$\text{Mass of } ^{219}_{86}\text{Rn}, m_2 = 219.00948$$

$$\text{Mass of } ^4_2\text{He}, m_3 = 4.00260$$

***** END *****