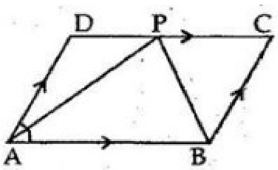
Exercise 9B

Question 3:

ABCD is a parallelogram in which DA=60° and bisectors of A and B meets DC at P.



(i) In a parallelogram, opposite angles are equal.

So, $\angle C = \angle A = 60°$

In a parallelogram the sum of all the four angles is 360°.

$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360°$

Now, $\angle B + \angle D = 360° - (\angle A + \angle C)$

$= 360° - (60° + 60°) = 240°$

$\therefore$ $2\angle B = 240°$ $[\because \angle B = \angle D]$

So, $\angle B = \angle D = \dfrac{240°}{2} = 120°$

Since AB $\parallel$ DP and AP is a transversal

So, $\angle APD = \angle PAB = \dfrac{60°}{2} = 30°$ ....(1)

[$\therefore$ alternate angles]

Also, AB $\parallel$ PC and BP is a transversal.

So, $\angle ABP = \angle CPB$

But, $\angle ABP = \dfrac{\angle B}{2} = \dfrac{120°}{2} = 60°$

$\therefore$ $\angle CPB = 60°$ ....(2)

Now, $\angle APD + \angle APB + \angle CPB = 180°$

[As DPC is a straight line]

$30° + \angle APB + 60° = 180°$

$\Rightarrow$ $\angle APB = 180° - 30° - 60° = 90°$

(ii) Since $\angle APD = 30°$ [from (1)]

and $\angle DAP = \dfrac{60°}{2} = 30°$

So, $\angle APD = \angle DAP$

Now in ΔAPD,

$\angle APD = \angle DAP$......(3)

$\therefore$ DP = AD [isosceles triangle, sides are equal]

As $\angle CPB = 60°$ [from (2)]

and $\angle C = 60°$

So, $\angle PBC = 180° - 60° - 60° = 60°$

Since all angles in the ΔPCB are equal,

it is an equilateral triangle.

$\therefore$ PB = PC = BC....(4)

(iii) $\angle DPA = \angle PAD$, [from(3)]

$\therefore$ DP = AD [isoscele striangle, sides are equal]

= BC [opposite sides are equal]

= PC [from (4)]

$= \dfrac{1}{2}$DC [$\because$ DP = PC $\Rightarrow$ P is the midpoint of DC]

$\therefore$ DC = 2AD.

*********** END **********