Trigonometric Ratios of multiple and Sub-multiple Angles Ex 9.1 Q36

$$\text{LHS} = \cos\frac{\pi}{65}.\cos\frac{2\pi}{65}.\cos\frac{4\pi}{65}.\cos\frac{8\pi}{15}\cos\frac{16\pi}{65}\cos\frac{32\pi}{65}$$

Divide and Multiply by $2\sin\frac{\pi}{65}$, we get

$$= \frac{2.\sin\frac{\pi}{65}}{2\sin\frac{\pi}{65}}.\cos\frac{\pi}{65}.\cos\frac{2\pi}{65}.\cos\frac{4\pi}{65}.\cos\frac{8\pi}{65}.\cos\frac{16\pi}{65}.\cos\frac{32\pi}{65}$$

$$= \frac{2.\sin\frac{2\pi}{65}}{2.2\sin\frac{\pi}{65}}.\cos\frac{2\pi}{65}.\cos\frac{4\pi}{65}.\cos\frac{8\pi}{65}.\cos\frac{16\pi}{65}.\cos\frac{32\pi}{65}$$

$$= \frac{2.\sin\frac{4\pi}{65}}{2.4\sin\frac{\pi}{65}}.\cos\frac{4\pi}{65}.\cos\frac{8\pi}{65}.\cos\frac{16\pi}{65}.\cos\frac{32\pi}{65}$$

$$= \frac{2.\sin\frac{8\pi}{65}}{2.8\sin\frac{\pi}{65}}\cos\frac{8\pi}{65}.\cos\frac{16\pi}{65}.\cos\frac{32\pi}{65}$$

$$= \frac{2.\sin\frac{16\pi}{65}}{2.16\sin\frac{\pi}{65}}\cos\frac{16\pi}{65}.\cos\frac{32\pi}{65}$$

$$= \frac{2.\sin\frac{32\pi}{65}}{2.32\sin\frac{\pi}{65}}\cos\frac{32\pi}{65}$$

$$= \frac{\sin\frac{64\pi}{65}}{64.\sin\frac{\pi}{65}}$$

$$= \frac{1}{64}.\frac{\sin\left(\pi - \frac{\pi}{65}\right)}{\sin\frac{\pi}{65}}$$

$$= \frac{1}{64}\frac{\sin\frac{\pi}{65}}{\sin\cancel{\pi/65}}$$

$$= \frac{1}{64}$$

$$= \text{RHS}$$

Trigonometric Ratios of multiple and Sub-multiple Angles Ex 9.1 Q 37

We have, $2\tan\alpha = 3\tan\beta$

$$\Rightarrow \frac{\tan\alpha}{\tan\beta} = \frac{3}{2}$$

Let $\tan\alpha = 3K$ and $\tan\beta = 2K$

Now, $\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha.\ \tan\beta} = \frac{3K-2K}{1+3K.2K} = \frac{K}{1+6K2}$ ...(A)

Also,

$$\frac{\sin 2\beta}{5-\cos 2\beta} = \frac{\frac{2\tan\beta}{1+\tan^2\beta}}{5-\left(\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)}$$

$$= \frac{\frac{2.2K}{1+4K^2}}{5-\left(\frac{1-4K^2}{1+4K^2}\right)}$$

$$= \frac{4K}{5+20K^2-1+4K^2}$$

$$= \frac{4K}{4+24K^2} = \frac{K}{1+6K^2} \quad \ldots(B)$$

form (A) & (B)

$$\tan(\alpha-\beta) = \frac{\sin 2\beta}{5-\cos 2\beta}$$

Trigonometric Ratios of multiple and Sub-multiple Angles Ex 9.1 Q 38(i)

We have,

$\sin\alpha + \sin\beta = a$ & $\cos\alpha + \cos\beta = b$ ....(A)

Squaring and adding, we get

$$\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta + \cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta = a^2+b^2$$

$\Rightarrow \quad 1+1+2(\sin\alpha\sin\beta + \cos\alpha\cos\beta) = a^2+b^2$

$\Rightarrow \quad 2(\sin\alpha\sin\beta + \cos\alpha\cos\beta) = a^2+b^2-2$

$\therefore \quad 2\cos(\alpha-\beta) = a^2+b^2-2$

Thus, $\cos(\alpha-\beta) = \frac{a^2+b^2-2}{2}$ ....(ii)

Again,

$\sin\alpha + \sin\beta = a \quad \Rightarrow \quad 2\sin\frac{\alpha+\beta}{2}.\ \cos\frac{\alpha-\beta}{2} = a$

$\cos\alpha + \cos\beta = b \quad \Rightarrow \quad 2\cos\frac{\alpha+\beta}{2}.\ \cos\frac{\alpha-\beta}{2} = b$

$\Rightarrow \quad \tan\frac{\alpha+\beta}{2} = \frac{a}{b}$ ....(B)

Now,

$$\sin(\alpha+\beta) = \frac{2\tan\frac{\alpha+\beta}{2}}{1+\tan^2\left(\frac{\alpha+\beta}{2}\right)}$$

$$= \frac{2\,a/b}{1+\frac{a^2}{b^2}} = \frac{2ab}{a^2+b^2}$$

Thus,

$$\sin(\alpha+\beta) = \frac{2ab}{a^2+b^2}$$

Trigonometric Ratios of multiple and Sub-multiple Angles Ex 9.1 Q 38(ii)

We have,

$\sin\alpha + \sin\beta = a \;\; \& \;\; \cos\alpha + \cos\beta = b$

Squaring and adding, we get

$$\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta + \cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta = a^2 + b^2$$

$\Rightarrow \quad 1 + 1 + 2\left(\sin\alpha\sin\beta + \cos\alpha\cos\beta\right) = a^2 + b^2$

$\Rightarrow \quad 2\left(\sin\alpha\sin\beta + \cos\alpha\cos\beta\right) = a^2 + b^2 - 2$

$\therefore \quad 2\cos\left(\alpha - \beta\right) = a^2 + b^2 - 2$

Thus, $\cos\left(\alpha - \beta\right) = \dfrac{a^2 + b^2 - 2}{2}$

*********** END **********