CHAPTER 35

MAGNETIC FIELD DUE TO A CURRENT

In the previous chapter, we defined magnetic field in terms of the force it exerts on a moving charge. In this chapter, we shall discuss how a magnetic field can be produced. A magnetic field can be produced by moving charges or electric currents. The basic equation governing the magnetic field due to a current distribution is the *Biot–Savart law.*

35.1 BIOT–SAVART LAW

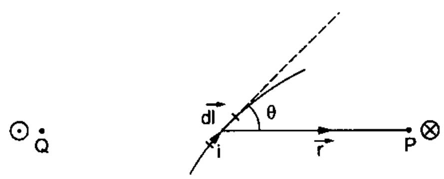

Figure 35.1

The magnetic field at a point P, due to a current element, is given by

$$d\vec{B} = \frac{1}{4\pi\varepsilon_0 c^2} i \frac{\vec{dl} \times \vec{r}}{r^3} \quad \text{... (35.1)}$$

where c is the speed of light, i is the current, $\vec{dl}$ is the length-vector of the current element and $\vec{r}$ is the vector joining the current element to the point P. The quantity $\frac{1}{\varepsilon_0 c^2}$ is written as μ_0 and is called the *permeability of vacuum.* Its value is $4\pi \times 10^{-7}$ T mA^{-1}. In terms of μ_0, equation (35.1) becomes

$$d\vec{B} = \frac{\mu_0}{4\pi} i \frac{\vec{dl} \times \vec{r}}{r^3} . \quad \text{... (35.2)}$$

This equation is the mathematical form of Biot–Savart Law.

The magnitude of the field is

$$dB = \frac{\mu_0}{4\pi} \frac{idl \sin\theta}{r^2} \quad \text{... (35.3)}$$

where θ is the angle between $\vec{dl}$ and $\vec{r}$. The direction of the field is perpendicular to the plane containing the current element and the point P according to the rules of cross product. If we place the stretched right-hand palm along $\vec{dl}$ in such a way that the fingers curl towards $\vec{r}$, the cross product $\vec{dl} \times \vec{r}$ is along the thumb. Usually, the plane of the diagram contains both $\vec{dl}$ and $\vec{r}$. The magnetic field $d\vec{B}$ is then perpendicular to the plane of the diagram, either going into the plane or coming out of the plane. As usual, we denote the direction going into the plane by an encircled cross and the direction coming out of the plane by an encircled dot. In figure (35.1), the magnetic field at the point P goes into the plane of the diagram and that at Q comes out of this plane.

Example 35.1

A wire placed along north–south direction carries a current of 10 A *from south to north. Find the magnetic field due to a* 1 cm *piece of wire at a point* 200 cm *north-east from the piece.*

Solution :

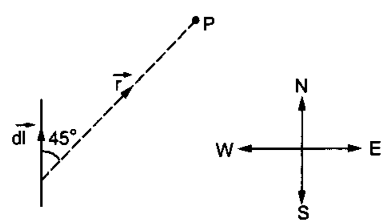

Figure 35.2

The situation is shown in figure (35.2). As the distance of P from the wire is much larger than the length of the wire, we can treat the wire as a small element. The magnetic field is given by

$$d\vec{B} = \frac{\mu_0}{4\pi} i \frac{\vec{dl} \times \vec{r}}{r^3}$$

or,
$$dB = \frac{\mu_0}{4\pi} i \frac{dl \sin\theta}{r^2}$$

$$= (10^{-7}\ \text{T mA}^{-1})\ (10\ \text{A}) \frac{(10^{-2}\ \text{m}) \sin 45°}{(2\ \text{m})^2}$$

$$= 1{\cdot}8 \times 10^{-9}\ \text{T}.$$

The direction of $\vec{B}$ is the same as that of $\vec{dl} \times \vec{r}$. From the figure, it is vertically downward.

35.2 MAGNETIC FIELD DUE TO CURRENT IN A STRAIGHT WIRE

Let MN (figure 35.3) be a portion of a straight wire carrying a current i. Let P be a point at a distance $OP = d$ from it. The point O is the foot of the perpendicular from P to the wire.

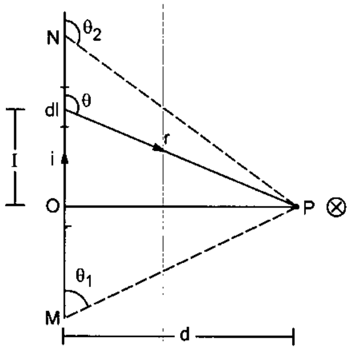

Figure 35.3

Let us consider an element dl of the wire at a distance l from the point O. The vector joining the element dl with the point P is $\vec{r}$. Let θ be the angle between $\vec{dl}$ and $\vec{r}$. The magnetic field at P due to the element is

$$dB = \frac{\mu_0}{4\pi} i \frac{dl \sin\theta}{r^2}. \quad \dots \text{(i)}$$

The direction of the field is determined by the vector $\vec{dl} \times \vec{r}$. It is perpendicular to the plane of the diagram and going into it. The direction of the field is the same for all elements of the wire and hence the net field due to the wire MN is obtained by integrating equation (i) under proper limits.

From the figure, $l = -d \cot\theta$

or, $\quad dl = d \operatorname{cosec}^2\theta\, d\theta.$

Also, $\quad r = d \operatorname{cosec}\theta.$

Putting in (i),

$$dB = \frac{\mu_0 i}{4\pi d} \sin\theta\, d\theta$$

or,
$$B = \frac{\mu_0 i}{4\pi d} [-\cos\theta]_{\theta_1}^{\theta_2}$$

$$= \frac{\mu_0 i}{4\pi d} (\cos\theta_1 - \cos\theta_2). \quad \dots (35.4)$$

Here θ_1 and θ_2 are the values of θ corresponding to the lower end and the upper end respectively.

Field on a Perpendicular Bisector

Suppose, the length $MN = a$ and the point P is on its perpendicular bisector. So,

$$OM = ON = a/2 \text{ and}$$

$$\cos\theta_1 = \frac{a/2}{\sqrt{\frac{a^2}{4} + d^2}} = \frac{a}{\sqrt{a^2 + 4d^2}}$$

and
$$\cos\theta_2 = -\frac{a}{\sqrt{a^2 + 4d^2}}.$$

Equation (35.4) then becomes

$$B = \frac{\mu_0 i}{4\pi d} \frac{2a}{\sqrt{a^2 + 4d^2}}$$

$$= \frac{\mu_0 ia}{2\pi d\sqrt{a^2 + 4d^2}}. \quad \dots (35.5)$$

Field due to a Long, Straight Wire

In this case $\theta_1 = 0$ and $\theta_2 = \pi$. From equation (35.4), the magnetic field is

$$B = \frac{\mu_0 i}{2\pi d}. \quad \dots (35.6)$$

The direction of the magnetic field at a point P due to a long, straight wire can be found by a slight variation in the right-hand thumb rule. If we stretch the thumb of the right hand along the long current and curl our fingers to pass through the point P, the direction of the fingers at P gives the direction of the magnetic field there.

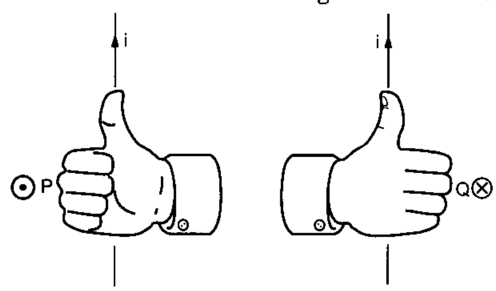

Figure 35.4

Example 35.2

Figure (35.5) shows two long, straight wires carrying electric currents in opposite directions. The separation between the wires is 5·0 cm. *Find the magnetic field at a point P midway between the wires.*

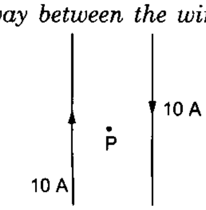

Figure 35.5

Solution : The right-hand thumb rule shows that the magnetic field at P due to each of the wires is perpendicular to the plane of the diagram and is going into it. The magnitude of the field due to each wire is

$$B = \frac{\mu_0 i}{2\pi d}$$

$$= \frac{(2\times10^{-7}\ \mathrm{T\,mA^{-1}})\,(10\ \mathrm{A})}{2{\cdot}5\times10^{-2}\ \mathrm{m}}$$

$$= 80\ \mu\mathrm{T}.$$

The net field due to both the wires is $2\times 80\ \mu\mathrm{T} = 160\ \mu\mathrm{T}$.

We can draw magnetic field lines on the pattern of electric field lines. A tangent to a magnetic field line gives the direction of the magnetic field existing at that point. For a long straight wire, the field lines are circles with their centres on the wire (figure 35.6).

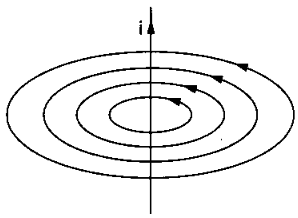

Figure 35.6

35.3 FORCE BETWEEN PARALLEL CURRENTS

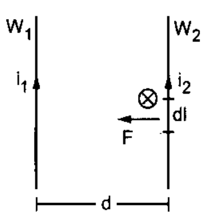

Figure 35.7

Consider two long wires W_1 and W_2 kept parallel to each other and carrying currents i_1 and i_2 respectively in the same direction (figure 35.7). The separation between the wires is d. Consider a small element dl of the wire W_2. The magnetic field at dl due to the wire W_1 is

$$B = \frac{\mu_0 i_1}{2\pi d}. \quad \text{... (i)}$$

The field due to the portions of the wire W_2, above and below dl, is zero. Thus, (i) gives the net field at dl. The direction of this field is perpendicular to the plane of the diagram and going into it. The magnetic force at the element dl is

$$d\vec{F} = i_2\, d\vec{l} \times \vec{B}$$

or, $$dF = i_2\, dl\, \frac{\mu_0 i_1}{2\pi d}.$$

The vector product $d\vec{l}\times\vec{B}$ has a direction towards the wire W_1. Thus, the length dl of wire W_2 is attracted towards the wire W_1. The force per unit length of the wire W_2 due to the wire W_1 is

$$\frac{dF}{dl} = \frac{\mu_0 i_1 i_2}{2\pi d}. \quad \text{... (35.7)}$$

If we take an element dl in the wire W_1 and calculate the magnetic force per unit length of wire W_1 due to W_2, it is again given by (35.7).

If the parallel wires carry currents in opposite directions, the wires repel each other.

Example 35.3

Two long, straight wires, each carrying an electric current of 5·0 A, *are kept parallel to each other at a separation of* 2·5 cm. *Find the magnitude of the magnetic force experienced by* 10 cm *of a wire.*

Solution : The field at the site of one wire due to the other is

$$B = \frac{\mu_0 i}{2\pi d} = \frac{(2\times10^{-7}\ \mathrm{T\,mA^{-1}})\,(5{\cdot}0\ \mathrm{A})}{2{\cdot}5\times10^{-2}\ \mathrm{m}} = 4{\cdot}0\times10^{-5}\ \mathrm{T}.$$

The force experienced by 10 cm of this wire due to the other is

$$F = i\,lB$$

$$= (5{\cdot}0\ \mathrm{A})\,(10\times10^{-2}\ \mathrm{m})\,(4{\cdot}0\times10^{-5}\ \mathrm{T})$$

$$= 2{\cdot}0\times10^{-5}\ \mathrm{N}.$$

Definition of Ampere

Consider two parallel wires separated by 1 m and carrying a current of 1 A each. Then $i_1 = i_2 = 1$ A and $d = 1$ m, so that from equation (35.7),

$$\frac{dF}{dl} = 2\times10^{-7}\ \mathrm{N\,m^{-1}}.$$

This is used to formally define the unit 'ampere' of electric current. *If two parallel, long wires, kept* 1 m *apart in vacuum, carry equal currents in the same direction and there is a force of attraction of* 2×10^{-7} newton per metre *of each wire, the current in each wire is said to be* 1 ampere.

35.4 FIELD DUE TO A CIRCULAR CURRENT

Field at the Centre

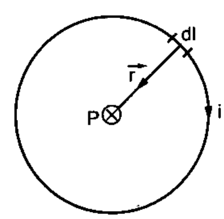

Figure 35.8

Consider a circular loop of radius a carrying a current i. We have to find the magnetic field due tc this current at the centre of the loop. Consider any small element dl of the wire (figure 35.8). The magnetic field at the centre O due to the current element $i\vec{dl}$ is

$$d\vec{B} = \frac{\mu_0}{4\pi} i \frac{\vec{dl} \times \vec{r}}{r^3}$$

where $\vec{r}$ is the vector joining the element to the centre O. The direction of this field is perpendicular to the plane of the diagram and is going into it. The magnitude is

$$dB = \frac{\mu_0}{4\pi} \frac{idl}{a^2}. \quad \ldots \text{(i)}$$

As the fields due to all such elements have the same direction, the net field is also in this direction. It can, therefore, be obtained by integrating (i) under proper limits. Thus,

$$B = \int dB = \int \frac{\mu_0 i}{4\pi a^2} dl$$

$$= \frac{\mu_0 i}{4\pi a^2} \int dl = \frac{\mu_0 i}{4\pi a^2} \times 2\pi a = \frac{\mu_0 i}{2a}.$$

The direction of the magnetic field at the centre of a circular wire can be obtained using the right-hand thumb rule. If the fingers are curled along the current, the stretched thumb will point towards the magnetic field (figure 35.9).

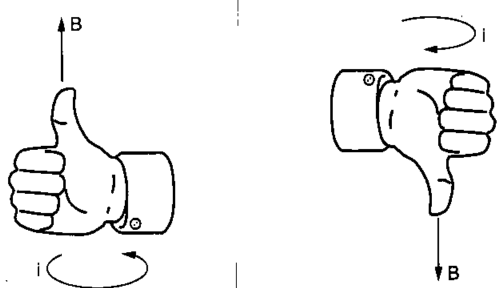

Figure 35.9

Another way to find the direction is to look into the loop along its axis. If the current is in anticlockwise direction, the magnetic field is towards the viewer. If the current is in clockwise direction, the field is away from the viewer.

In figure (35.8), the current is clockwise as seen by you. The magnetic field at the centre is away from you, i.e., is going into the plane of the diagram.

Example 35.4

A circular coil of radius 1·5 cm *carries a current of* 1·5 A. *If the coil has* 25 *turns, find the magnetic field at the centre.*

Solution : The magnetic field at the centre due to each turn is

$$\frac{\mu_0 i}{2a}.$$

The net field due to all 25 turns is

$$B = \frac{\mu_0 i n}{2a} = \frac{(2\pi \times 10^{-7}\ \mathrm{T\ mA^{-1}})\,(1{\cdot}5\ \mathrm{A}) \times 25}{1{\cdot}5 \times 10^{-2}\ \mathrm{m}}$$

$$= 1{\cdot}57 \times 10^{-3}\ \mathrm{T}.$$

Field at an Axial Point

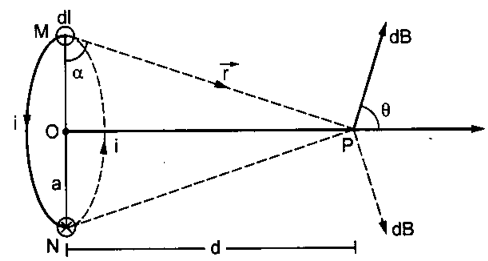

Figure 35.10

Consider a circular loop of radius a carrying a current i. We have to find the magnetic field at a point P on the axis of the loop at a distance d from its centre O. In figure (35.10), the loop is perpendicular to the plane of the figure while its axis is in the plane of the figure. The current comes out of the plane at M and goes into it at N. Consider a current element $i\vec{dl}$ of the wire at M. The vector joining the element to the point P is $\vec{r} = \vec{MP}$. The magnetic field at P due to this current element is

$$d\vec{B} = \frac{\mu_0}{4\pi} i \frac{\vec{dl} \times \vec{r}}{r^3}.$$

As $\vec{dl}$ is perpendicular to the plane of the figure, $\vec{dl} \times \vec{r}$ must be in the plane. The figure shows the direction of $d\vec{B}$ according to the rules of vector product. The magnitude of the field is

$$dB = \frac{\mu_0}{4\pi} \frac{i\,dl}{r^2} = \frac{\mu_0 i}{4\pi} \frac{dl}{a^2 + d^2}.$$

The component along the axis is

$$dB \cos\theta = \frac{\mu_0\, ia\, dl}{4\pi(a^2 + d^2)^{3/2}}. \quad \ldots \text{(i)}$$

Now, consider the diametrically opposite current element at N. The field due to this element will have the same magnitude dB and its direction will be along the dotted arrow shown in the figure. The two fields due to the elements at M and at N have a resultant along the axis of the loop. Dividing the loop in such pairs of diametrically opposite elements, we conclude that the resultant magnetic field at P must be along the axis. The resultant field at P can, therefore, be obtained by integrating the right-hand side of (i), i.e.,

$$B = \int \frac{\mu_0\, ia}{4\pi(a^2 + d^2)^{3/2}} dl$$

$$= \frac{\mu_0\, ia}{4\pi(a^2 + d^2)^{3/2}} \times 2\pi a$$

$$= \frac{\mu_0\, ia^2}{2(a^2 + d^2)^{3/2}} . \qquad \dots (35.8)$$

The right-hand thumb rule can be used to find the direction of the field.

Field at a point far away from the centre

If $d >> a$, equation (35.8) gives

$$B \approx \frac{\mu_0\, ia^2}{2\, d^3} = \frac{2\mu_0\, i(\pi a^2)}{4\pi\, d^3} . \qquad \dots (ii)$$

Now, πa^2 is the area of the loop and $i(\pi a^2)$ is its magnetic dipole moment μ. Using right-hand thumb rule, we see that the direction of the area-vector $\vec{A}$ and hence of the dipole moment is along the field $\vec{B}$. The magnetic field due to this small loop at an axial point is, therefore,

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{2\vec{\mu}}{d^3} . \qquad \dots (35.9)$$

One can compare this with the expression for the electric field due to an electric dipole at a point on the dipole-axis. It is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{2\vec{p}}{d^3} .$$

where $\vec{p}$ is the electric dipole moment.

The magnetic field at a point not on the axis is mathematically difficult to calculate. We show qualitatively in figure (35.11) the magnetic field lines due to a circular current which will give some idea of the field.

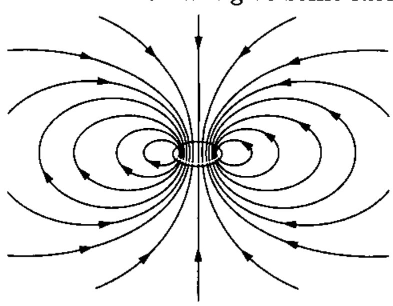

Figure 35.11

35.5 AMPERE'S LAW

Ampere's law gives another method to calculate the magnetic field due to a given current distribution. Consider any closed, plane curve (figure 35.12). Assign a sense to the curve by putting an arrow on the curve. Using the right-hand thumb rule, assign one side of the plane as positive and the other as negative. If you curl the fingers of the right hand along the arrow on the curve, the stretched thumb gives the positive side. The positive side can also be determined by looking into the loop along its axis. If the arrow on the loop is anticlockwise, the positive side is towards the viewer. If the arrow is clockwise, the positive side is away from the viewer. In figure (35.12), the positive side is going into the plane of the diagram.

Take a small length-element $\vec{dl}$ on the curve and let $\vec{B}$ be the resultant magnetic field at the position of $\vec{dl}$. Calculate the scalar product $\vec{B} \cdot \vec{dl}$ and integrate by varying $\vec{dl}$ on the closed curve. This integration is called *line integral* or *circulation* of $\vec{B}$ along the curve and is represented by the symbol

$$\oint \vec{B} \cdot \vec{dl} .$$

Now look at the currents crossing the area bounded by the curve. A current directed towards the positive side of the plane area is taken as positive and a current directed towards the negative side is taken as negative. Ampere's law then states:

The circulation $\oint \vec{B} \cdot \vec{dl}$ *of the resultant magnetic field along a closed, plane curve is equal to* μ_0 *times the total current crossing the area bounded by the closed curve provided the electric field inside the loop remains constant.* Thus,

$$\oint \vec{B} \cdot \vec{dl} = \mu_0\, i . \qquad \dots (35.10)$$

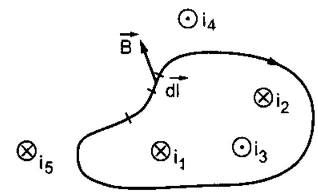

Figure 35.12

In figure (35.12), the positive side is going into the area of the diagram so that i_1 and i_2 are positive and i_3 is negative. Thus, the total current crossing the area is $i_1 + i_2 - i_3$. Any current outside the area is not included in writing the right-hand side of equation (35.10). The magnetic field $\vec{B}$ on the left-hand side is the resultant field due to all the currents existing anywhere.

Ampere's law may be derived from the Biot–Savart law and Biot–Savart law may be derived from the Ampere's law. Thus, the two are equivalent in scientific content. However, Ampere's law is more useful under certain symmetrical conditions. In such cases, the mathematics of finding the magnetic field becomes much simpler if we use the Ampere's law.

35.6 MAGNETIC FIELD AT A POINT DUE TO A LONG, STRAIGHT CURRENT

Figure (35.13a) shows a long, straight current i. We have to calculate the magnetic field at a point P

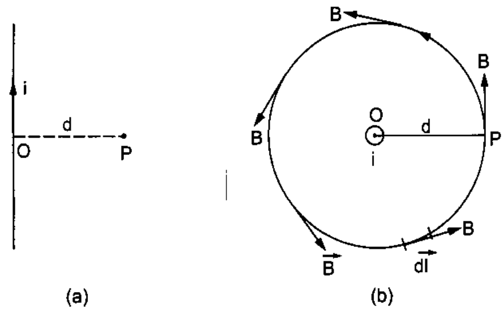

Figure 35.13

which is at a distance $OP = d$ from the wire. Figure (35.13b) shows the situation in the plane perpendicular to the wire and passing through P. The current is perpendicular to the plane of the diagram and is coming out of it.

Let us draw a circle passing through the point P and with the centre at O. We put an arrow to show the positive sense of the circle. The radius of the circle is $OP = d$. The magnetic field due to the long, straight current at any point on the circle is along the tangent as shown in the figure. Same is the direction of the length-element $\vec{dl}$ there. By symmetry, all points of the circle are equivalent and hence the magnitude of the magnetic field should be the same at all these points. The circulation of magnetic field along the circle is

$$\oint \vec{B} \cdot \vec{dl} = \oint B\, dl$$

$$= B \oint dl = B\, 2\pi d.$$

The current crossing the area bounded by the circle is i. Thus, from Ampere's law,

$$B\, 2\pi d = \mu_0\, i$$

or, $$B = \frac{\mu_0\, i}{2\pi d}.$$

We have already derived this equation from Biot–Savart law (equation 35.6).

35.7 SOLENOID

A solenoid is a wire wound closely in the form of a helix. The wire is coated with an insulating material so that although the adjacent turns physically touch each other, they are electrically insulated. Generally, the length of the solenoid is large as compared to the transverse dimension. For example, if the solenoid has circular turns, the length is large as compared to its radius. If it has rectangular turns, the length should be large as compared to the edges. The magnetic field due to a current-carrying solenoid can be easily pictured by examining the field due to a circular current. The field lines due to a circular current were drawn in figure (35.11). Figure (35.14a) shows the magnetic field lines due to a circular loop A carrying a current and also due to a similar loop B placed coaxially and carrying equal current. We see that at a point P, which is quite off the axis, the magnetic fields due to the two loops are in opposite directions. On the other hand, at a point Q which is on the axis or close to the axis, the two fields are in the same direction. Figure (35.14b) shows the resultant field lines due to the two loops. A solenoid may be thought of as a stack of such circular currents placed coaxially one after the other. Figure (35.14c) represents the field due to a loosely wound solenoid. The fields at an outside point due to the neighbouring loops oppose each other, whereas at an inside point, the fields are in the same direction. These tendencies to have zero field outside and a uniform field inside become more and more effective as the solenoid is more and more tightly wound. The magnetic field inside a very tightly wound, long solenoid is uniform everywhere and is zero outside it (figure 35.14d).

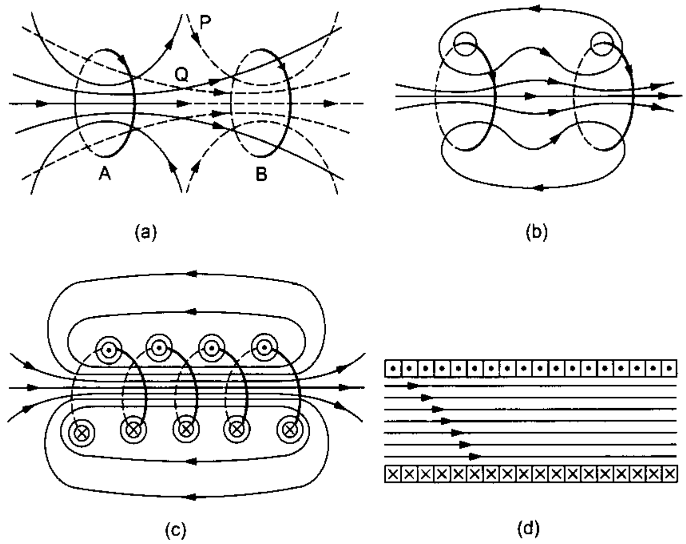

Figure 35.14

To calculate the magnetic field at a point P inside the solenoid, let us draw a rectangle $PQRS$ as shown in figure (35.15). The line PQ is parallel to the solenoid axis and hence parallel to the magnetic field $\vec{B}$ inside the solenoid. Thus,

$$\int_P^Q \vec{B} \cdot \vec{dl} = Bl.$$

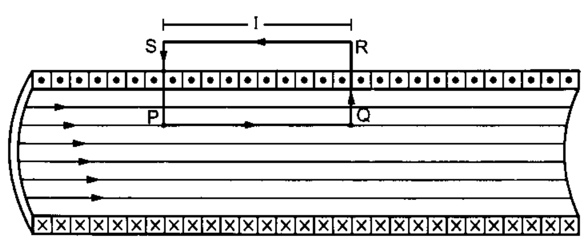

Figure 35.15

On the remaining three sides, $\vec{B}\cdot\vec{dl}$ is zero everywhere as $\vec{B}$ is either zero (outside the solenoid) or perpendicular to $\vec{dl}$ (inside the solenoid). Thus, the circulation of $\vec{B}$ along $PQRS$ is

$$\oint \vec{B}\cdot\vec{dl} = Bl.$$

Let n be the number of turns per unit length along the length of the solenoid. A total of nl turns cross the rectangle $PQRS$. Each turn carries a current i. So the net current crossing $PQRS = nli$.

Using Ampere's law,

$$\oint \vec{B}\cdot\vec{dl} = \mu_0\, nli$$

or, $$Bl = \mu_0\, nli$$

or, $$B = \mu_0\, ni. \qquad \dots (35.11)$$

Example 35.5

A long solenoid is formed by winding 20 turns cm^{-1}. *What current is necessary to produce a magnetic field of* 20 mT *inside the solenoid?*

Solution : The magnetic field inside the solenoid is

$$B = \mu_0\, ni$$

or, $$20\times10^{-3}\ \mathrm{T} = (4\pi\times10^{-7}\ \mathrm{T\ mA^{-1}})\times(20\times10^{2}\ \mathrm{m^{-1}})i$$

or, $$i = 8{\cdot}0\ \mathrm{A}.$$

Equation (35.11) is strictly valid only for an infinitely long solenoid. In practice, if the length of the solenoid is more than about five to six times the diameter and we are looking at the field in the middle region, we can use this equation.

35.8 TOROID

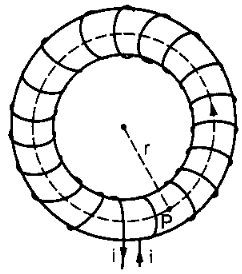

Figure 35.16

If a solenoid is bent in a circular shape and the ends are joined, we get a toroid. Alternatively, one can start with a nonconducting ring and wind a conducting wire closely on it. The magnetic field in such a toroid can be obtained by using Ampere's law.

Suppose, we have to find the field at a point P inside the toroid. Let the distance of P from the centre be r. Draw a circle through the point P and concentric with the toroid. By symmetry, the field will have equal magnitude at all points of this circle. Also, the field is everywhere tangential to the circle. Thus,

$$\oint \vec{B}\cdot\vec{dl} = \int B\,dl = B\int dl = 2\pi rB.$$

If the total number of turns is N, the current crossing the area bounded by the circle is Ni where i is the current in the toroid. Using Ampere's law on this circle,

$$\oint \vec{B}\cdot\vec{dl} = \mu_0\, Ni$$

or, $$2\pi rB = \mu_0\, Ni$$

or, $$B = \frac{\mu_0 Ni}{2\pi r}. \qquad \dots (35.12)$$

Worked Out Examples

1. *Two long wires a and b, carrying equal currents of* 10·0 A, *are placed parallel to each other with a separation of* 4·00 cm *between them as shown in figure (35-W1). Find the magnetic field B at each of the points P, Q and R.*

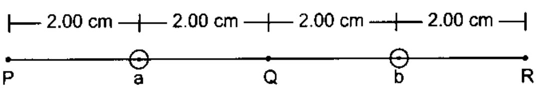

Figure 35-W1

Solution : The magnetic field at P due to the wire a has magnitude

$$B_1 = \frac{\mu_0 i}{2\pi d} = \frac{4\pi\times10^{-7}\ \mathrm{T\ mA^{-1}}\times10\ \mathrm{A}}{2\pi\times2\times10^{-2}\ \mathrm{m}}$$

$$= 1{\cdot}00\times10^{-4}\ \mathrm{T}.$$

Its direction will be perpendicular to the line shown and will point downward in the figure.

The field at this point due to the other wire has magnitude

$$B_2 = \frac{\mu_0 i}{2\pi d} = \frac{4\pi\times10^{-7}\ \mathrm{T\ mA^{-1}}\times10\ \mathrm{A}}{2\pi\times6\times10^{-2}\ \mathrm{m}}$$

$$= 0{\cdot}33\times10^{-4}\ \mathrm{T}.$$

Its direction will be the same as that of B_1. Thus, the resultant field will be $1{\cdot}33\times10^{-4}$ T also along the same direction.

Similarly, the resultant magnetic field at R will be $= 1{\cdot}33\times10^{-4}$ T along the direction pointing upward in the figure.

The magnetic field at point Q due to the two wires will have equal magnitudes but opposite directions and hence the resultant field will be zero.

2. *Two parallel wires P and Q placed at a separation $d = 6$ cm carry electric currents $i_1 = 5$ A and $i_2 = 2$ A in opposite directions as shown in figure (35-W2a). Find the point on the line PQ where the resultant magnetic field is zero.*

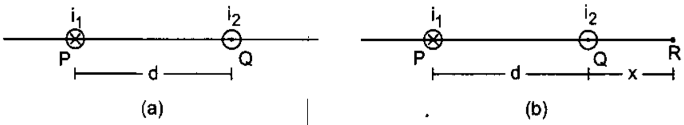

Figure 35-W2

Solution : At the desired point, the magnetic fields due to the two wires must have equal magnitude but opposite directions. The point should be either to the left of P or to the right of Q. As the wire Q has smaller current, the point should be closer to Q. Let this point R be at a distance x from Q (figure 35-W2b).

The magnetic field at R due to the current i_1 will have magnitude

$$B_1 = \frac{\mu_0 i_1}{2\pi(d+x)}$$

and will be directed downward in the plane of the figure. The field at the same point due to the current i_2 will be

$$B_2 = \frac{\mu_0 i_2}{2\pi x}$$

directed upward in the plane of the figure. If the resultant field at R is zero, we should have $B_1 = B_2$, so that

$$\frac{i_1}{d+x} = \frac{i_2}{x}$$

giving,

$$x = \frac{i_2 d}{i_1 - i_2} = \frac{(2\text{ A})\,(6\text{ cm})}{(3\text{ A})} = 4\text{ cm}.$$

3. *Two long, straight wires a and b are 2·0 m apart, perpendicular to the plane of the paper as shown in figure (35-W3). The wire a carries a current of 9·6 A directed into the plane of the figure. The magnetic field at the point p at a distance of 10/11 m from the wire b is zero. Find (a) the magnitude and direction of the current in b, (b) the magnitude of the magnetic field B at the point s and (c) the force per unit length on the wire b.*

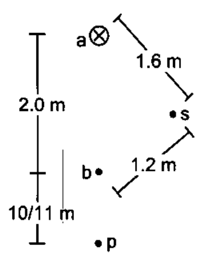

Figure 35-W3

Solution :

(a) For the magnetic field at p to be zero, the current in the wire b should be coming out of the plane of the figure so that the fields due to a and b may be opposite at p. The magnitudes of these fields should be equal, so that

$$\frac{\mu_0 (9\cdot6\text{ A})}{2\pi\left(2+\frac{10}{11}\right)\text{m}} = \frac{\mu_0 i}{2\pi\left(\frac{10}{11}\right)\text{m}}$$

or, $\quad i = 3\cdot0$ A.

(b) $\quad (ab)^2 = 4\text{ m}^2$

$(as)^2 = 2\cdot56\text{ m}^2$

and $\quad (bs)^2 = 1\cdot44\text{ m}^2$

so that $(ab)^2 = (as)^2 + (bs)^2$ and angle $asb = 90°$.

The field at s due to the wire a

$$= \frac{\mu_0 (9\cdot6\text{ A})}{2\pi \times 1\cdot6\text{ m}} = \frac{\mu_0}{2\pi} \times 6\text{ Am}^{-1}$$

and that due to the wire b

$$= \frac{\mu_0}{2\pi}\,\frac{3\text{ A}}{1\cdot2\text{ m}} = \frac{\mu_0}{2\pi} \times 2\cdot5\text{ Am}^{-1}.$$

These fields are at 90° to each other so that their resultant will have a magnitude

$$\sqrt{\left(\frac{\mu_0}{2\pi} \times 6\text{ Am}^{-1}\right)^2 + \left(\frac{\mu_0}{2\pi} \times 2\cdot5\text{ Am}^{-1}\right)^2}$$

$$= \frac{\mu_0}{2\pi}\sqrt{36 + 6\cdot25}\text{ Am}^{-1}$$

$$= 2 \times 10^{-7}\text{ T mA}^{-1} \times 6\cdot5\text{ Am}^{-1}$$

$$= 1\cdot3 \times 10^{-6}\text{ T}.$$

(c) The force per unit length on the wire b

$$= \frac{\mu_0 i_1 i_2}{2\pi d} = (2 \times 10^{-7}\text{ T mA}^{-1}) \times \frac{(9\cdot6\text{ A})\,(3\text{ A})}{2\cdot0\text{ m}}$$

$$= 2\cdot9 \times 10^{-6}\text{ N}.$$

4. *A current of 2·00 A exists in a square loop of edge 10·0 cm. Find the magnetic field B at the centre of the square loop.*

Solution : The magnetic fields at the centre due to the four sides will be equal in magnitude and direction. The field due to one side will be

$$B_1 = \frac{\mu_0\, i a}{2\pi d\sqrt{a^2 + 4d^2}}\cdot$$

Here, $a = 10$ cm and $d = a/2 = 5$ cm.

Thus,

$$B_1 = \frac{\mu_0\,(2\text{ A})}{2\pi(5\text{ cm})}\left[\frac{10\text{ cm}}{\sqrt{(10\text{ cm})^2 + 4(5\text{ cm})^2}}\right]$$

$$= 2 \times 10^{-7}\text{ T mA}^{-1} \times 2\text{ A} \times \frac{1}{5\sqrt{2}\text{ cm}}$$

$$= 5\cdot66 \times 10^{-6}\text{ T}.$$

Hence, the net field at the centre of the loop will be

$$4 \times 5{\cdot}66 \times 10^{-6}\ \text{T} = 22{\cdot}6 \times 10^{-6}\ \text{T}.$$

5. *Figure (35-W4) shows a square loop made from a uniform wire. Find the magnetic field at the centre of the square if a battery is connected between the points A and C.*

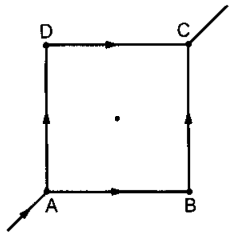

Figure 35-W4

Solution : The current will be equally divided at A. The fields at the centre due to the currents in the wires AB and DC will be equal in magnitude and opposite in direction. The resultant of these two fields will be zero. Similarly, the resultant of the fields due to the wires AD and BC will be zero. Hence, the net field at the centre will be zero.

6. *Two long wires, carrying currents i_1 and i_2, are placed perpendicular to each other in such a way that they just avoid a contact. Find the magnetic force on a small length dl of the second wire situated at a distance l from the first wire.*

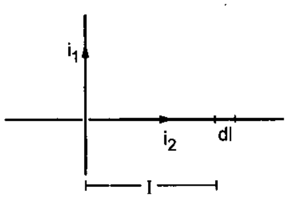

Figure 35-W5

Solution : The situation is shown in figure (35-W5). The magnetic field at the site of dl, due to the first wire is,

$$B = \frac{\mu_0 i_1}{2\pi l}.$$

This field is perpendicular to the plane of the figure going into it. The magnetic force on the length dl is,

$$dF = i_2\, dl\, B \sin 90°$$

$$= \frac{\mu_0 i_1 i_2 dl}{2\pi l}$$

This force is parallel to the current i_1.

7. *Figure (35-W6) shows a part of an electric circuit. ABCD is a rectangular loop made of uniform wire. The length* $AD = BC = 1$ cm. *The sides AB and DC are long as compared to the other two sides. Find the magnetic force per unit length acting on the wire DC due to the wire AB if the ammeter reads* 10 A.

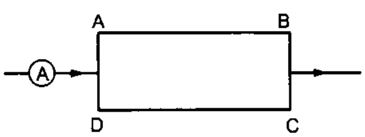

Figure 35-W6

Solution : By symmetry, each of the wires AB and DC carries a current of 5 A. As the separation between them is 1 cm, the magnetic force per unit length of DC is

$$\frac{dF}{dl} = \frac{\mu_0 i_1 i_2}{2\pi d}$$

$$= \frac{(2 \times 10^{-7}\ \text{T mA}^{-1})\,(5\ \text{A})\,(5\ \text{A})}{1 \times 10^{-2}\ \text{m}}$$

$$= 5 \times 10^{-4}\ \text{TA} = 5 \times 10^{-4}\ \text{N m}^{-1}.$$

8. *Figure (35-W7) shows a current loop having two circular arcs joined by two radial lines. Find the magnetic field B at the centre O.*

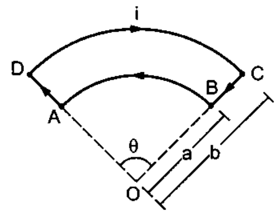

Figure 35-W7

Solution : As the point O is on the line AD, the magnetic field at O due to AD is zero. Similarly, the field at O due to BC is also zero. The field at the centre of a circular current loop is given by $B = \frac{\mu_0 i}{2a}$. The field due to the circular arc BA will be

$$B_1 = \left(\frac{\theta}{2\pi}\right)\left(\frac{\mu_0 i}{2a}\right).$$

The right-hand thumb rule shows that the field is coming out of the plane of the figure. The field due to the circular arc DC is

$$B_2 = \left(\frac{\theta}{2\pi}\right)\left(\frac{\mu_0 i}{2b}\right)$$

going into the plane of the figure. The resultant field at O is

$$B = B_1 - B_2 = \frac{\mu_0 i\theta(b - a)}{4\pi ab}$$

coming out of the plane.

9. *Find the magnetic field at the point P in figure (35-W8). The curved portion is a semicircle and the straight wires are long.*

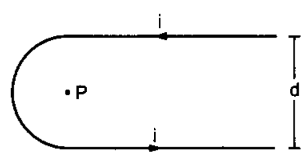

Figure 35-W8

Solution : The magnetic field at P due to any current element in the figure is perpendicular to the plane of the figure and coming out of it. The field due to the upper straight wire is

$$B_1 = \frac{1}{2} \times \frac{\mu_0 i}{2\pi\left(\frac{d}{2}\right)} = \frac{\mu_0 i}{2\pi d}.$$

Same is the field B_2 due to the lower straight wire. The field due to the semicircle of radius ($d/2$) is

$$B_3 = \frac{1}{2} \times \frac{\mu_0 i}{2\left(\frac{d}{2}\right)} = \frac{\mu_0 i}{2d}.$$

The net field is $B = B_1 + B_2 + B_3 = \frac{\mu_0 i}{2d}\left(1 + \frac{2}{\pi}\right)$.

10. *The magnetic field B due to a current-carrying circular loop of radius* 12 cm *at its centre is* 0.50×10^{-4} T. *Find the magnetic field due to this loop at a point on the axis at a distance of* 5.0 cm *from the centre.*

Solution : The magnetic field at the centre of a circular loop is

$$B_0 = \frac{\mu_0 i}{2a}$$

and that at an axial point is

$$B = \frac{\mu_0 i a^2}{2(a^2 + x^2)^{3/2}}.$$

Thus, $\frac{B}{B_0} = \frac{a^3}{(a^2 + x^2)^{3/2}}$

or, $$B = (0.50 \times 10^{-4}\ \text{T}) \times \frac{(12\ \text{cm})^3}{(144\ \text{cm}^2 + 25\ \text{cm}^2)^{3/2}}$$

$$= 3.9 \times 10^{-5}\ \text{T}.$$

11. *Consider a coaxial cable which consists of an inner wire of radius a surrounded by an outer shell of inner and outer radii b and c respectively. The inner wire carries an electric current i_0 and the outer shell carries an equal current in opposite direction. Find the magnetic field at a distance x from the axis where (a) $x < a$, (b) $a < x < b$ (c) $b < x < c$ and (d) $x > c$. Assume that the current density is uniform in the inner wire and also uniform in the outer shell.*

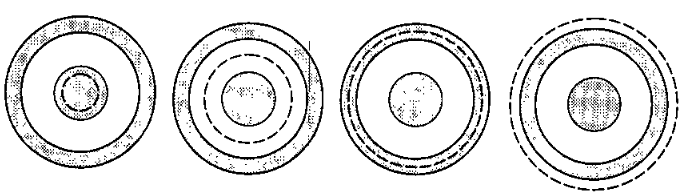

Figure 35-W9

Solution :

A cross section of the cable is shown in figure (35-W9). Draw a circle of radius x with the centre at the axis of the cable. The parts a, b, c and d of the figure correspond to the four parts of the problem. By symmetry, the magnetic field at each point of a circle will have the same magnitude and will be tangential to it. The circulation of B along this circle is, therefore,

$$\oint \vec{B} \cdot d\vec{l} = B\, 2\pi x$$

in each of the four parts of the figure.

(a) The current enclosed within the circle in part a is

$$\frac{i_0}{\pi a^2} \cdot \pi x^2 = \frac{i_0}{a^2} x^2.$$

Ampere's law $\oint \vec{B} \cdot d\vec{l} = \mu_0 i$ gives

$$B\, 2\pi x = \frac{\mu_0 i_0 x^2}{a^2} \quad \text{or,} \quad B = \frac{\mu_0 i_0 x}{2\pi a^2}.$$

The direction will be along the tangent to the circle.

(b) The current enclosed within the circle in part b is i_0 so that

$$B\, 2\pi x = \mu_0 i_0 \quad \text{or,} \quad B = \frac{\mu_0 i_0}{2\pi x}.$$

(c) The area of cross section of the outer shell is $\pi c^2 - \pi b^2$. The area of cross section of the outer shell within the circle in part c of the figure is $\pi x^2 - \pi b^2$. Thus, the current through this part is $\frac{i_0(x^2 - b^2)}{c^2 - b^2}$. This is in the opposite direction to the current i_0 in the inner wire. Thus, the net current enclosed by the circle is

$$i_0 - \frac{i_0(x^2 - b^2)}{c^2 - b^2} = \frac{i_0(c^2 - x^2)}{c^2 - b^2}.$$

From Ampere's law,

$$B\, 2\pi x = \frac{\mu_0 i_0(c^2 - x^2)}{c^2 - b^2}$$

or, $$B = \frac{\mu_0 i_0(c^2 - x^2)}{2\pi x(c^2 - b^2)}.$$

(d) The net current enclosed by the circle in part d of the figure is zero and hence

$$B\, 2\pi x = 0 \quad \text{or,} \quad B = 0.$$

12. *Figure (35-W10) shows a cross section of a large metal sheet carrying an electric current along its surface. The current in a strip of width dl is Kdl where K is a constant. Find the magnetic field at a point P at a distance x from the metal sheet.*

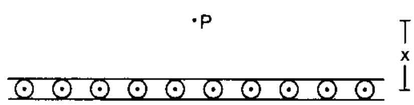

Figure 35-W10

Solution : Consider two strips A and C of the sheet situated symmetrically on the two sides of P (figure 35-W11a). The magnetic field at P due to the strip A is B_a perpendicular to AP and that due to the strip C is B_c perpendicular to CP. The resultant of these two is parallel to the width AC of the sheet. The field due to the whole sheet will also be in this direction. Suppose this field has magnitude B.

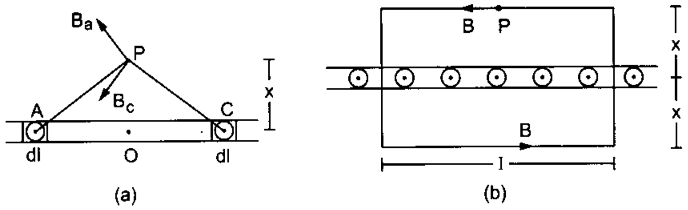

Figure 35-W11

The field on the opposite side of the sheet at the same distance will also be B but in opposite direction. Applying Ampere's law to the rectangle shown in figure (35-W11b),

$$2Bl = \mu_0 Kl$$

or, $$B = \frac{1}{2}\mu_0 K.$$

Note that it is independent of x.

13. *Consider the situation described in the previous example. A particle of mass m having a charge q is placed at a distance d from the metal sheet and is projected towards it. Find the maximum velocity of projection for which the particle does not hit the sheet.*

Solution : As the magnetic field is uniform and the particle is projected in a direction perpendicular to the field, it will describe a circular path. The particle will not hit the metal sheet if the radius of this circle is smaller than d. For the maximum velocity, the radius is just equal to d. Thus,

$$qvB = \frac{mv^2}{d}$$

or, $$qv\frac{\mu_0 K}{2} = \frac{mv^2}{d}$$

or, $$v = \frac{\mu_0 qKd}{2m}.$$

14. *Three identical long solenoids P, Q and R are connected to each other as shown in figure (35-W12). If the magnetic field at the centre of P is 2·0 T, what would be the field at the centre of Q? Assume that the field due to any solenoid is confined within the volume of that solenoid only.*

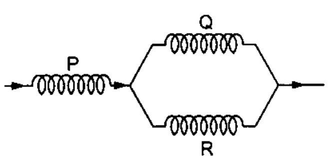

Figure 35-W12

Solution : As the solenoids are identical, the currents in Q and R will be the same and will be half the current in P. The magnetic field within a solenoid is given by $B = \mu_0 ni$. Hence the field in Q will be equal to the field in R and will be half the field in P, i.e., will be 1·0 T.

15. *A long, straight wire carries a current i. A particle having a positive charge q and mass m, kept at a distance x_0 from the wire is projected towards it with a speed v. Find the minimum separation between the wire and the particle.*

Solution :

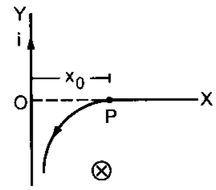

Figure 35-W13

Let the particle be initially at P (figure 35-W13). Take the wire as the y-axis and the foot of perpendicular from P to the wire as the origin. Take the line OP as the x-axis. We have, $OP = x_0$. The magnetic field B at any point to the right of the wire is along the negative z-axis. The magnetic force on the particle is, therefore, in the x–y plane. As there is no initial velocity along the z-axis, the motion will be in the x–y plane. Also, its speed remains unchanged. As the magnetic field is not uniform, the particle does not go along a circle.

The force at time t is $\vec{F} = q\vec{v} \times \vec{B}$

$$= q(\vec{i}\,v_x + \vec{j}\,v_y) \times \left(-\frac{\mu_0 i}{2\pi x}\vec{k}\right)$$

$$= \vec{j}\,qv_x\frac{\mu_0 i}{2\pi x} - \vec{i}\,qv_y\frac{\mu_0 i}{2\pi x}.$$

Thus $$a_x = \frac{F_x}{m} = -\frac{\mu_0 qi}{2\pi m}\frac{v_y}{x} = -\lambda\frac{v_y}{x} \quad \ldots \text{(i)}$$

where $$\lambda = \frac{\mu_0 qi}{2\pi m}.$$

Also, $$a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx}\frac{dx}{dt} = \frac{v_x dv_x}{dx}. \quad \ldots \text{(ii)}$$

As $$v_x^2 + v_y^2 = v^2,$$

$$2v_x\,dv_x + 2v_y\,dv_y = 0$$

giving $$v_x\,dv_x = -v_y\,dv_y. \quad \ldots \text{(iii)}$$

From (i), (ii) and (iii),

$$\frac{v_y\,dv_y}{dx} = \frac{\lambda v_y}{x}$$

or, $$\frac{dx}{x} = \frac{dv_y}{\lambda}.$$

Initially $x = x_0$ and $v_y = 0$. At minimum separation from the wire, $v_x = 0$ so that $v_y = -v$.

Thus $$\int_{x_0}^{x}\frac{dx}{x} = \int_0^{-v}\frac{dv_y}{\lambda}$$

or, $$\ln\frac{x}{x_0} = -\frac{v}{\lambda}$$

or, $$x = x_0 e^{-v/\lambda} = x_0 e^{-\frac{2\pi mv}{\mu_0 qi}}.$$

QUESTIONS FOR SHORT ANSWER

1. An electric current flows in a wire from north to south. What will be the direction of the magnetic field due to this wire at a point (a) east of the wire, (b) west of the wire, (c) vertically above the wire and (d) vertically below the wire?
2. The magnetic field due to a long straight wire has been derived in terms of μ_0, i and d. Express this in terms of ε_0, c, i and d.
3. You are facing a circular wire carrying an electric current. The current is clockwise as seen by you. Is the field at the centre coming towards you or going away from you?
4. In Ampere's law $\oint \vec{B} \cdot d\vec{l} = \mu_0 i$, the current outside the curve is not included on the right hand side. Does it mean that the magnetic field B calculated by using Ampere's law, gives the contribution of only the currents crossing the area bounded by the curve?
5. The magnetic field inside a tightly wound, long solenoid is $B = \mu_0 ni$. It suggests that the field does not depend on the total length of the solenoid, and hence if we add more loops at the ends of a solenoid the field should not increase. Explain qualitatively why the extra-added loops do not have a considerable effect on the field inside the solenoid.
6. A long, straight wire carries a current. Is Ampere's law valid for a loop that does not enclose the wire, or that encloses the wire but is not circular?
7. A straight wire carrying an electric current is placed along the axis of a uniformly charged ring. Will there be a magnetic force on the wire if the ring starts rotating about the wire? If yes, in which direction?
8. Two wires carrying equal currents i each, are placed perpendicular to each other, just avoiding a contact. If one wire is held fixed and the other is free to move under magnetic forces, what kind of motion will result?
9. Two proton beams going in the same direction repel each other whereas two wires carrying currents in the same direction attract each other. Explain.
10. In order to have a current in a long wire, it should be connected to a battery or some such device. Can we obtain the magnetic field due to a straight, long wire by using Ampere's law without mentioning this other part of the circuit?
11. Quite often, connecting wires carrying currents in opposite directions are twisted together in using electrical appliances. Explain how it avoids unwanted magnetic fields.
12. Two current-carrying wires may attract each other. In absence of other forces, the wires will move towards each other increasing the kinetic energy. Does it contradict the fact that the magnetic force cannot do any work and hence cannot increase the kinetic energy?

OBJECTIVE I

1. A vertical wire carries a current in upward direction. An electron beam sent horizontally towards the wire will be deflected
   (a) towards right (b) towards left
   (c) upwards (d) downwards.
2. A current-carrying, straight wire is kept along the axis of a circular loop carrying a current. The straight wire
   (a) will exert an inward force on the circular loop
   (b) will exert an outward force on the circular loop
   (c) will not exert any force on the circular loop
   (d) will exert a force on the circular loop parallel to itself.
3. A proton beam is going from north to south and an electron beam is going from south to north. Neglecting the earth's magnetic field, the electron beam will be deflected
   (a) towards the proton beam
   (b) away from the proton beam
   (c) upwards (d) downwards.
4. A circular loop is kept in that vertical plane which contains the north–south direction. It carries a current that is towards north at the topmost point. Let A be a point on the axis of the circle to the east of it and B a point on this axis to the west of it. The magnetic field due to the loop
   (a) is towards east at A and towards west at B
   (b) is towards west at A and towards east at B
   (c) is towards east at both A and B
   (d) is towards west at both A and B.
5. Consider the situation shown in figure (35-Q1). The straight wire is fixed but the loop can move under magnetic force. The loop will
   (a) remain stationary
   (b) move towards the wire
   (c) move away from the wire
   (d) rotate about the wire.

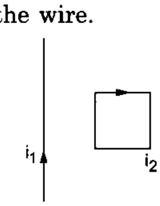

Figure 35-Q1

6. A charged particle is moved along a magnetic field line. The magnetic force on the particle is
   (a) along its velocity (b) opposite to its velocity
   (c) perpendicular to its velocity (d) zero.
7. A moving charge produces
   (a) electric field only (b) magnetic field only
   (c) both of them (d) none of them.

8. A particle is projected in a plane perpendicular to a uniform magnetic field. The area bounded by the path described by the particle is proportional to
(a) the velocity (b) the momentum
(c) the kinetic energy (d) none of these.

9. Two particles X and Y having equal charge, after being accelerated through the same potential difference enter a region of uniform magnetic field and describe circular paths of radii R_1 and R_2 respectively. The ratio of the mass of X to that of Y is
(a) $(R_1/R_2)^{1/2}$ (b) R_1/R_2 (c) $(R_1/R_2)^2$ (d) R_1R_2.

10. Two parallel wires carry currents of 20 A and 40 A in opposite directions. Another wire carrying a current antiparallel to 20 A is placed midway between the two wires. The magnetic force on it will be
(a) towards 20 A (b) towards 40 A (c) zero
(d) perpendicular to the plane of the currents.

11. Two parallel, long wires carry currents i_1 and i_2 with $i_1 > i_2$. When the currents are in the same direction, the magnetic field at a point midway between the wires is 10 μT. If the direction of i_2 is reversed, the field becomes 30 μT. The ratio i_1/i_2 is
(a) 4 (b) 3 (c) 2 (d) 1.

12. Consider a long, straight wire of cross-sectional area A carrying a current i. Let there be n free electrons per unit volume. An observer places himself on a trolley moving in the direction opposite to the current with a speed $v = \frac{i}{nAe}$ and separated from the wire by a distance r. The magnetic field seen by the observer is very nearly
(a) $\frac{\mu_0 i}{2\pi r}$ (b) zero (c) $\frac{\mu_0 i}{\pi r}$ (d) $\frac{2\mu_0 i}{\pi r}$.

OBJECTIVE II

1. The magnetic field at the origin due to a current element $i\,\vec{dl}$ placed at a position $\vec{r}$ is
(a) $\frac{\mu_0 i}{4\pi}\frac{\vec{dl}\times\vec{r}}{r^3}$ (b) $-\frac{\mu_0 i}{4\pi}\frac{\vec{r}\times\vec{dl}}{r^3}$
(c) $\frac{\mu_0 i}{4\pi}\frac{\vec{r}\times\vec{dl}}{r^3}$ (d) $-\frac{\mu_0 i}{4\pi}\frac{\vec{dl}\times\vec{r}}{r^3}$.

2. Consider three quantities $x = E/B$, $y = \sqrt{1/\mu_0\varepsilon_0}$ and $z = \frac{l}{CR}$. Here, l is the length of a wire, C is a capacitance and R is a resistance. All other symbols have standard meanings.
(a) x, y have the same dimensions.
(b) y, z have the same dimensions.
(c) z, x have the same dimensions.
(d) None of the three pairs have the same dimensions.

3. A long, straight wire carries a current along the z-axis. One can find two points in the x–y plane such that
(a) the magnetic fields are equal
(b) the directions of the magnetic fields are the same
(c) the magnitudes of the magnetic fields are equal
(d) the field at one point is opposite to that at the other point.

4. A long, straight wire of radius R carries a current distributed uniformly over its cross section. The magnitude of the magnetic field is
(a) maximum at the axis of the wire
(b) minimum at the axis of the wire
(c) maximum at the surface of the wire
(d) minimum at the surface of the wire.

5. A hollow tube is carrying an electric current along its length distributed uniformly over its surface. The magnetic field
(a) increases linearly from the axis to the surface
(b) is constant inside the tube
(c) is zero at the axis
(d) is zero just outside the tube.

6. In a coaxial, straight cable, the central conductor and the outer conductor carry equal currents in opposite directions. The magnetic field is zero
(a) outside the cable
(b) inside the inner conductor
(c) inside the outer conductor
(d) in between the two conductors.

7. A steady electric current is flowing through a cylindrical conductor.
(a) The electric field at the axis of the conductor is zero.
(b) The magnetic field at the axis of the conductor is zero.
(c) The electric field in the vicinity of the conductor is zero.
(d) The magnetic field in the vicinity of the conductor is zero.

EXERCISES

1. Using the formulae $\vec{F} = q\vec{v}\times\vec{B}$ and $B = \frac{\mu_0 i}{2\pi r}$, show that the SI units of the magnetic field B and the permeability constant μ_0 may be written as $\mathrm{N\,mA^{-1}}$ and $\mathrm{N\,A^{-2}}$ respectively.

2. A current of 10 A is established in a long wire along the positive z-axis. Find the magnetic field $\vec{B}$ at the point (1 m, 0, 0).

3. A copper wire of diameter 1·6 mm carries a current of 20 A. Find the maximum magnitude of the magnetic field $\vec{B}$ due to this current.
4. A transmission wire carries a current of 100 A. What would be the magnetic field B at a point on the road if the wire is 8 m above the road ?
5. A long, straight wire carrying a current of 1·0 A is placed horizontally in a uniform magnetic field $B = 1{\cdot}0 \times 10^{-5}$ T pointing vertically upward (figure 35-E1). Find the magnitude of the resultant magnetic field at the points P and Q, both situated at a distance of 2·0 cm from the wire in the same horizontal plane.

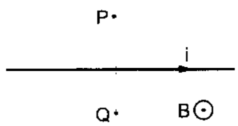

Figure 35-E1

6. A long, straight wire of radius r carries a current i and is placed horizontally in a uniform magnetic field B pointing vertically upward. The current is uniformly distributed over its cross section. (a) At what points will the resultant magnetic field have maximum magnitude ? What will be the maximum magnitude ? (b) What will be the minimum magnitude of the resultant magnetic field ?
7. A long, straight wire carrying a current of 30 A is placed in an external, uniform magnetic field of $4{\cdot}0 \times 10^{-4}$ T parallel to the current. Find the magnitude of the resultant magnetic field at a point 2·0 cm away from the wire.
8. A long, vertical wire carrying a current of 10 A in the upward direction is placed in a region where a horizontal magnetic field of magnitude $2{\cdot}0 \times 10^{-3}$ T exists from south to north. Find the point where the resultant magnetic field is zero.
9. Figure (35-E2) shows two parallel wires separated by a distance of 4·0 cm and carrying equal currents of 10 A along opposite directions. Find the magnitude of the magnetic field B at the points A_1, A_2, A_3 and A_4.

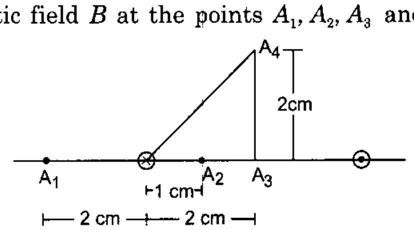

Figure 35-E2

10. Two parallel wires carry equal currents of 10 A along the same direction and are separated by a distance of 2·0 cm. Find the magnetic field at a point which is 2·0 cm away from each of these wires.
11. Two long, straight wires, each carrying a current of 5 A, are placed along the x- and y-axes respectively. The currents point along the positive directions of the axes. Find the magnetic fields at the points (a) (1 m, 1 m), (b) (– 1 m, 1 m), (c) (– 1 m, – 1 m) and (d) (1 m, – 1 m).
12. Four long, straight wires, each carrying a current of 5·0 A, are placed in a plane as shown in figure (35-E3). The points of intersection form a square of side 5·0 cm. (a) Find the magnetic field at the centre P of the square. (b) Q_1, Q_2, Q_3 and Q_4 are points situated on the diagonals of the square and at a distance from P that is equal to the length of the diagonal of the square. Find the magnetic fields at these points.

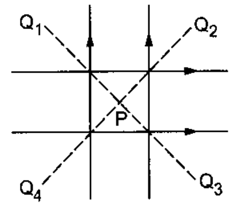

Figure 35-E3

13. Figure (35-E4) shows a long wire bent at the middle to form a right angle. Show that the magnitudes of the magnetic fields at the points P, Q, R and S are equal and find this magnitude.

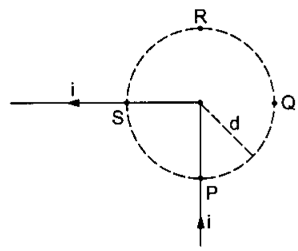

Figure 35-E4

14. Consider a straight piece of length x of a wire carrying a current i. Let P be a point on the perpendicular bisector of the piece, situated at a distance d from its middle point. Show that for $d >> x$, the magnetic field at P varies as $1/d^2$ whereas for $d << x$, it varies as $1/d$.
15. Consider a 10-cm long piece of a wire which carries a current of 10 A. Find the magnitude of the magnetic field due to the piece at a point which makes an equilateral triangle with the ends of the piece.
16. A long, straight wire carries a current i. Let B_1 be the magnetic field at a point P at a distance d from the wire. Consider a section of length l of this wire such that the point P lies on a perpendicular bisector of the section. Let B_2 be the magnetic field at this point due to this section only. Find the value of d/l so that B_2 differs from B_1 by 1%.
17. Figure (35-E5) shows a square loop $ABCD$ with edge-length a. The resistance of the wire ABC is r and that of ADC is $2r$. Find the magnetic field B at the centre of the loop assuming uniform wires.

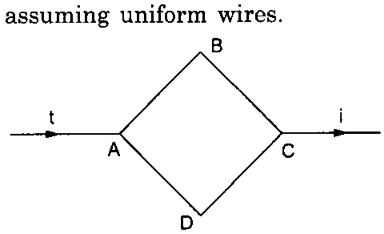

Figure 35-E5

18. Figure (35-E6) shows a square loop of edge a made of a uniform wire. A current i enters the loop at the point A and leaves it at the point C. Find the magnetic field at

the point P which is on the perpendicular bisector of AB at a distance $a/4$ from it.

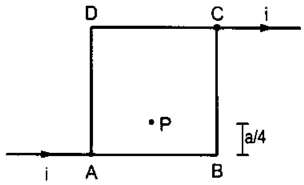

Figure 35-E6

19. Consider the situation described in the previous problem. Suppose the current i enters the loop at the point A and leaves it at the point B. Find the magnetic field at the centre of the loop.

20. The wire ABC shown in figure (35-E7) forms an equilateral triangle. Find the magnetic field B at the centre O of the triangle assuming the wire to be uniform.

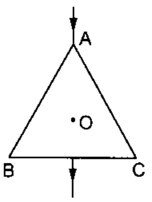

Figure 35-E7

21. A wire of length l is bent in the form of an equilateral triangle and carries an electric current i. (a) Find the magnetic field B at the centre. (b) If the wire is bent in the form of a square, what would be the value of B at the centre ?

22. A long wire carrying a current i is bent to form a plane angle α. Find the magnetic field B at a point on the bisector of this angle situated at a distance x from the vertex.

23. Find the magnetic field B at the centre of a rectangular loop of length l and width b, carrying a current i.

24. A regular polygon of n sides is formed by bending a wire of total length $2\pi r$ which carries a current i. (a) Find the magnetic field B at the centre of the polygon. (b) By letting $n \to \infty$, deduce the expression for the magnetic field at the centre of a circular current.

25. Each of the batteries shown in figure (35-E8) has an emf equal to 5 V. Show that the magnetic field B at the point P is zero for any set of values of the resistances.

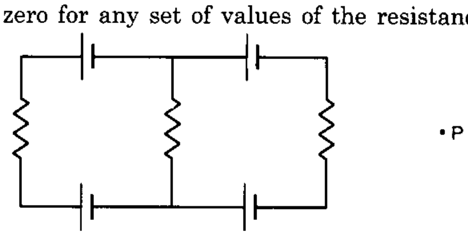

Figure 35-E8

26. A straight, long wire carries a current of 20 A. Another wire carrying equal current is placed parallel to it. If the force acting on a length of 10 cm of the second wire is $2{\cdot}0 \times 10^{-5}$ N, what is the separation between them ?

27. Three coplanar parallel wires, each carrying a current of 10 A along the same direction, are placed with a separation 5·0 cm between the consecutive ones. Find the magnitude of the magnetic force per unit length acting on the wires.

28. Two parallel wires separated by a distance of 10 cm carry currents of 10 A and 40 A along the same direction. Where should a third current be placed so that it experiences no magnetic force ?

29. Figure (35-E9) shows a part of an electric circuit. The wires AB, CD and EF are long and have identical resistances. The separation between the neighbouring wires is 1·0 cm. The wires AE and BF have negligible resistance and the ammeter reads 30 A. Calculate the magnetic force per unit length of AB and CD.

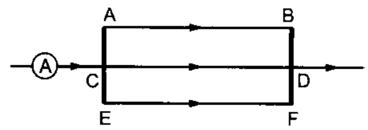

Figure 35-E9

30. A long, straight wire is fixed horizontally and carries a current of 50·0 A. A second wire having linear mass density $1{\cdot}0 \times 10^{-4}$ kg m^{-1} is placed parallel to and directly above this wire at a separation of 5·0 mm. What current should this second wire carry such that the magnetic repulsion can balance its weight ?

31. A square loop $PQRS$ carrying a current of 6·0 A is placed near a long wire carrying 10 A as shown in figure (35-E10). (a) Show that the magnetic force acting on the part PQ is equal and opposite to that on the part RS. (b) Find the magnetic force on the square loop.

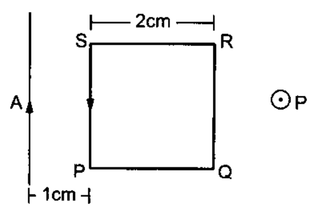

Figure 35-E10

32. A circular loop of one turn carries a current of 5·00 A. If the magnetic field B at the centre is 0·200 mT, find the radius of the loop.

33. A current-carrying circular coil of 100 turns and radius 5·0 cm produces a magnetic field of $6{\cdot}0 \times 10^{-5}$ T at its centre. Find the value of the current.

34. An electron makes 3×10^{5} revolutions per second in a circle of radius 0·5 angstrom. Find the magnetic field B at the centre of the circle.

35. A conducting circular loop of radius a is connected to two long, straight wires. The straight wires carry a current i as shown in figure (35-E11). Find the magnetic field B at the centre of the loop.

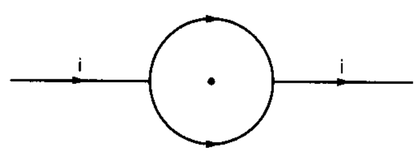

Figure 35-E11

36. Two circular coils of radii 5·0 cm and 10 cm carry equal currents of 2·0 A. The coils have 50 and 100 turns respectively and are placed in such a way that their planes as well as the centres coincide. Find the magnitude of the magnetic field B at the common centre of the coils if the currents in the coils are (a) in the same sense (b) in the opposite sense.

37. If the outer coil of the previous problem is rotated through 90° about a diameter, what would be the magnitude of the magnetic field B at the centre?

38. A circular loop of radius 20 cm carries a current of 10 A. An electron crosses the plane of the loop with a speed of $2 \cdot 0 \times 10^{6}$ m s^{-1}. The direction of motion makes an angle of 30° with the axis of the circle and passes through its centre. Find the magnitude of the magnetic force on the electron at the instant it crosses the plane.

39. A circular loop of radius R carries a current I. Another circular loop of radius r ($<< R$) carries a current i and is placed at the centre of the larger loop. The planes of the two circles are at right angle to each other. Find the torque acting on the smaller loop.

40. A circular loop of radius r carrying a current i is held at the centre of another circular loop of radius $R(>> r)$ carrying a current I. The plane of the smaller loop makes an angle of 30° with that of the larger loop. If the smaller loop is held fixed in this position by applying a single force at a point on its periphery, what would be the minimum magnitude of this force?

41. Find the magnetic field B due to a semicircular wire of radius 10·0 cm carrying a current of 5·0 A at its centre of curvature.

42. A piece of wire carrying a current of 6·00 A is bent in the form of a circular arc of radius 10·0 cm, and it subtends an angle of 120° at the centre. Find the magnetic field B due to this piece of wire at the centre.

43. A circular loop of radius r carries a current i. How should a long, straight wire carrying a current $4i$ be placed in the plane of the circle so that the magnetic field at the centre becomes zero?

44. A circular coil of 200 turns has a radius of 10 cm and carries a current of 2·0 A. (a) Find the magnitude of the magnetic field $\vec{B}$ at the centre of the coil. (b) At what distance from the centre along the axis of the coil will the field B drop to half its value at the centre? ($\sqrt[3]{4} = 1 \cdot 5874 \ldots$)

45. A circular loop of radius 4·0 cm is placed in a horizontal plane and carries an electric current of 5·0 A in the clockwise direction as seen from above. Find the magnetic field (a) at a point 3·0 cm above the centre of the loop (b) at a point 3·0 cm below the centre of the loop.

46. A charge of $3 \cdot 14 \times 10^{-6}$ C is distributed uniformly over a circular ring of radius 20·0 cm. The ring rotates about its axis with an angular velocity of 60.0 rad s^{-1}. Find the ratio of the electric field to the magnetic field at a point on the axis at a distance of 5·00 cm from the centre.

47. A thin but long, hollow, cylindrical tube of radius r carries a current i along its length. Find the magnitude of the magnetic field at a distance $r/2$ from the surface (a) inside the tube (b) outside the tube.

48. A long, cylindrical tube of inner and outer radii a and b carries a current i distributed uniformly over its cross section. Find the magnitude of the magnetic field at a point (a) just inside the tube (b) just outside the tube.

49. A long, cylindrical wire of radius b carries a current i distributed uniformly over its cross section. Find the magnitude of the magnetic field at a point inside the wire at a distance a from the axis.

50. A solid wire of radius 10 cm carries a current of 5·0 A distributed uniformly over its cross section. Find the magnetic field B at a point at a distance (a) 2 cm (b) 10 cm and (c) 20 cm away from the axis. Sketch a graph of B versus x for $0 < x < 20$ cm.

51. Sometimes we show an idealised magnetic field which is uniform in a given region and falls to zero abruptly. One such field is represented in figure (35-E12). Using Ampere's law over the path $PQRS$, show that such a field is not possible.

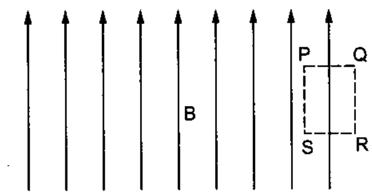

Figure 35-E12

52. Two large metal sheets carry surface currents as shown in figure (35-E13). The current through a strip of width dl is Kdl where K is a constant. Find the magnetic field at the points P, Q and R.

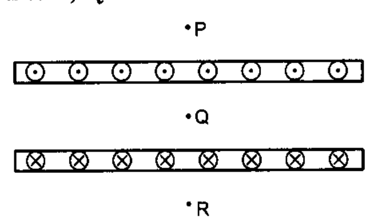

Figure 35-E13

53. Consider the situation of the previous problem. A particle having charge q and mass m is projected from the point Q in a direction going into the plane of the diagram. It is found to describe a circle of radius r between the two plates. Find the speed of the charged particle.

54. The magnetic field B inside a long solenoid, carrying a current of 5·00 A, is $3 \cdot 14 \times 10^{-2}$ T. Find the number of turns per unit length of the solenoid.

55. A long solenoid is fabricated by closely winding a wire of radius 0·5 mm over a cylindrical nonmagnetic frame so that the successive turns nearly touch each other. What would be the magnetic field B at the centre of the solenoid if it carries a current of 5 A?

56. A copper wire having resistance 0·01 ohm in each metre is used to wind a 400-turn solenoid of radius 1·0 cm and length 20 cm. Find the emf of a battery which when connected across the solenoid will cause a magnetic field of $1{\cdot}0 \times 10^{-2}$ T near the centre of the solenoid.

57. A tightly-wound solenoid of radius a and length l has n turns per unit length. It carries an electric current i. Consider a length dx of the solenoid at a distance x from one end. This contains $n\,dx$ turns and may be approximated as a circular current $i\,n\,dx$. (a) Write the magnetic field at the centre of the solenoid due to this circular current. Integrate this expression under proper limits to find the magnetic field at the centre of the solenoid. (b) Verify that if $l >> a$, the field tends to $B = \mu_0 ni$ and if $a >> l$, the field tends to $B = \frac{\mu_0 nil}{2a}$. Interpret these results.

58. A tightly-wound, long solenoid carries a current of 2·00 A. An electron is found to execute a uniform circular motion inside the solenoid with a frequency of $1{\cdot}00 \times 10^{8}$ rev s^{-1}. Find the number of turns per metre in the solenoid.

59. A tightly-wound, long solenoid has n turns per unit length, a radius r and carries a current i. A particle having charge q and mass m is projected from a point on the axis in a direction perpendicular to the axis. What can be the maximum speed for which the particle does not strike the solenoid ?

60. A tightly-wound, long solenoid is kept with its axis parallel to a large metal sheet carrying a surface current. The surface current through a width dl of the sheet is Kdl and the number of turns per unit length of the solenoid is n. The magnetic field near the centre of the solenoid is found to be zero. (a) Find the current in the solenoid. (b) If the solenoid is rotated to make its axis perpendicular to the metal sheet, what would be the magnitude of the magnetic field near its centre ?

61. A capacitor of capacitance 100 μF is connected to a battery of 20 volts for a long time and then disconnected from it. It is now connected across a long solenoid having 4000 turns per metre. It is found that the potential difference across the capacitor drops to 90% of its maximum value in 2·0 seconds. Estimate the average magnetic field produced at the centre of the solenoid during this period.

□

ANSWERS

OBJECTIVE I

1. (c)	2. (c)	3. (a)	4. (d)	5. (b)	6. (d)
7. (c)	8. (c)	9. (c)	10. (b)	11. (c)	12. (a)

OBJECTIVE II

1. (c), (d)	2. (a), (b), (c)	3. (b), (c), (d)
4. (b), (c)	5. (b), (c)	6. (a)
7. (b), (c)		

EXERCISES

2. 2 μT along the positive y-axis
3. 5·0 mT
4. 2·5 μT
5. 20 μT, zero
6. (a) looking along the current, at the leftmost points on the wire's surface, $B + \frac{\mu_0 i}{2\pi r}$

   (b) zero if $r \le \frac{\mu_0 i}{2\pi B}$, $B - \frac{\mu_0 i}{2\pi r}$ if $r > \frac{\mu_0 i}{2\pi B}$
7. 5×10^{-4} T
8. 1·0 mm west to the wire
9. (a) $0{\cdot}67 \times 10^{-4}$ T (b) $2{\cdot}7 \times 10^{-4}$ T (c) $2{\cdot}0 \times 10^{-4}$ T (d) $1{\cdot}0 \times 10^{-4}$ T
10. $1{\cdot}7 \times 10^{-4}$ T in a direction parallel to the plane of the wires and perpendicular to the wires
11. (a) zero (b) 2 μT along the z-axis (c) zero and (d) 2 μT along the negative z-axis
12. (a) zero (b) $Q_1 : 1{\cdot}1 \times 10^{-4}$ T, ⊙ , Q_2 : zero, $Q_3 : 1{\cdot}1 \times 10^{-4}$ T, ⊗, and Q_4 : zero
13. $\frac{\mu_0 i}{4\pi d}$
15. 11·5 μT
16. 0·07
17. $\frac{\sqrt{2}\,\mu_0 i}{3\pi a}$, ⊗
18. $\frac{2\,\mu_0 i}{\pi a}\left(\frac{1}{\sqrt{5}} - \frac{1}{3\sqrt{13}}\right)$
19. zero
20. zero
21. (a) $\frac{27\mu_0 i}{2\pi l}$ (b) $\frac{8\sqrt{2}\mu_0 i}{\pi l}$
22. $\frac{\mu_0 i}{2\pi x} \cot \frac{\alpha}{4}$
23. $\frac{2\mu_0 i\sqrt{l^2 + b^2}}{\pi l b}$
24. (a) $\frac{\mu_0 i n^2 \sin \frac{\pi}{n} \tan \frac{\pi}{n}}{2\pi^2 r}$

26. 40 cm

27. zero on the middle wire and $6 \cdot 0 \times 10^{-4}$ N towards the middle wire on each of the rest two

28. 2 cm from the 10 A current and 8 cm from the other

29. 3×10^{-3} N m^{-1}, downward zero

30. 0·49 A in opposite directiom

31. (b) $1 \cdot 6 \times 10^{-5}$ N towards right

32. 1·57 cm

33. 48 mA

34. 6×10^{-10} T

35. zero

36. (a) $8\pi \times 10^{-4}$ T (b) zero

37. 1·8 mT

38. $16\pi \times 10^{-19}$ N

39. $\dfrac{\mu_0 \pi i I r^2}{2R}$

40. $\dfrac{\mu_0 \pi i I r}{4R}$

41. $1 \cdot 6 \times 10^{-5}$ T

42. $1 \cdot 26 \times 10^{-5}$ T

43. at a distance of $4r/\pi$ from the centre in such a way that the direction of the current in it is opposite to that in the nearest part of the circular wire

44. (a) 2·51 mT (b) 7·66 cm

45. $4 \cdot 0 \times 10^{-5}$ T, downwards in both the cases

46. $1 \cdot 88 \times 10^{15}$ m s^{-1}

47. (a) zero (b) $\dfrac{\mu_0 i}{3\pi r}$

48. (a) zero (b) $\dfrac{\mu_0 i}{2\pi b}$

49. $\dfrac{\mu_0 i a}{2\pi b^2}$

50. (a) 2·0 μT (b) 10 μT (c) 5·0 μT

52. 0, $\mu_0 K$ towards right in the figure, 0

53. $\dfrac{\mu_0 K q r}{m}$

54. 5000 turns m^{-1}

55. $2\pi \times 10^{-3}$ T

56. 1 V

57. (a) $\dfrac{\mu_0 n i}{\sqrt{1 + \left(\dfrac{2a}{l}\right)^2}}$

58. 1420 turns m^{-1}

59. $\dfrac{\mu_0 q r n i}{2m}$

60. (a) $\dfrac{K}{2n}$ (b) $\dfrac{\mu_0 K}{\sqrt{2}}$

61. $16\pi \times 10^{-8}$ T

□