Relations Ex 1.1 Q11

No, it is not necessary that a relation which is symmetric and transitive is reflexive as well.

For Example,

Let $A = \{a, b, c\}$ be a set and

$R_2 = \{(a, a)\}$ is a relation defined on $A$.

Clearly,

$R_2$ is symmetric and transitive but not reflexive.

Relations Ex 1.1 Q12

It is given that an integer m is said to be relative to another integer n if m is a multiple of n.

In other words

$$R = \{(m, n); \quad m = kn, k \in z\}$$

Reflexivity: Let, m $\in z$

$\Rightarrow \quad m = 1.m$

$\Rightarrow \quad (m, m) \in R$

$\therefore \quad R$ is reflexive

Transitive: Let $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow \quad a = kb$ and $b = k'c$

$\Rightarrow \quad a = kk'c \qquad [\because \quad kk' \in z]$

$\Rightarrow \quad a = lc \qquad [\because \quad l = kk' \in z]$

$\Rightarrow \quad (a, c) \in R$

$\therefore \quad R$ is transitive

Symmetric: Let $(a, b) \in R$

$\Rightarrow \quad a = kb$

$\Rightarrow \quad b = \frac{1}{k}a \qquad$ but $\frac{1}{k} \notin z$ if $k \in z$

$\therefore \quad (b, a) \notin R$

$\therefore \quad R$ is not symmetric

Relations Ex 1.1 Q13

We have,
relation $R$ = " $\geq$ " on the set $R$ of all real numbers

Reflexivity: Let a $\in R$

$\Rightarrow \quad a \geq a$
$\Rightarrow \quad$ " $\geq$ " is reflexive

Symmetric: Let a,b $\in R$
such that a $\geq$ b $\not\Rightarrow$ b $\geq$ a

$\therefore \quad$ " $\geq$ " not symmetric

Transitivity: Let a,b,c $\in R$
and a $\geq b$ & b $\geq c$

$\Rightarrow \quad a \geq c$

$\Rightarrow \quad$ " $\geq$ " is transitive

Relations Ex 1.1 Q14

(i )Let $A = \{4, 6, 8\}$.
Define a relation R on A as:
$A = \{(4, 4), (6, 6), (8, 8), (4, 6), (6, 4), (6, 8), (8, 6)\}$
Relation R is reflexive since for every $a \in A$, $(a, a) \in$R i.e., $(4, 4), (6, 6), (8, 8)\} \in$ R.
Relation R is symmetric since $(a, b) \in$ R $\Rightarrow (b, a) \in$ R for all $a, b \in$ R.
Relation R is not transitive since $(4, 6), (6, 8) \in$ R, but $(4, 8) \notin$ R.
Hence, relation R is reflexive and symmetric but not transitive.

(ii ) Define a relation R in **R** as:
R = $\{a, b): a^3 \geq b^3\}$
Clearly $(a, a) \in$ R as $a^3 = a^3$. $a = a$.
Therefore, R is reflexive.
Now, $(2, 1) \in$ R (as $2^3 \geq 1^3$)
But, $(1, 2) \notin$ R (as $1^3 < 2^3$)
Therefore, R is not symmetric.
Now, Let $(a, b), (b, c) \in$ R.
$\Rightarrow a^3 \geq b^3$ and $b^3 \geq c^3$
$\Rightarrow a^3 \geq c^3$
$\Rightarrow (a, c) \in$ R
Therefore, R is transitive.
Hence, relation R is reflexive and transitive but not symmetric.
Hence, relation R is transitive but not reflexive and symmetric.

(iv)Let $A = \{5, 6, 7\}$.

Define a relation R on $A$ as R = $\{(5, 6), (6, 5)\}$.

Relation R is not reflexive as $(5, 5), (6, 6), (7, 7) \notin$ R.

Now, as $(5, 6) \in$ R and also $(6, 5) \in$ R, R is symmetric.

$\Rightarrow (5, 6), (6, 5) \in$ R, but $(5, 5) \notin$ R

Therefore, R is not transitive.

Hence, relation R is symmetric but not reflexive or transitive.

(v)Consider a relation R in **R** defined as:

R = $\{(a, b): a < b\}$

For any $a \in$ R, we have $(a, a) \notin$ R since $a$ cannot be strictly less than $a$ itself. In fact, $a = a$.

Therefore, R is not reflexive.

Now, $(1, 2) \in$ R (as $1 < 2$)

But, 2 is not less than 1.

Therefore, $(2, 1) \notin$ R

Therefore, R is not symmetric.

Now, let $(a, b), (b, c) \in$ R.

$\Rightarrow a < b$ and $b < c$

$\Rightarrow a < c$

$\Rightarrow (a, c) \in$ R

Therefore, R is transitive.

Hence, relation R is transitive but not reflexive and symmetric.

*********** END **********