



# Geometric Progressions Ex 20.5 Q 13

$a, b, c, d$  are in G.P.

$$a, b = ar, c = ar^2, d = ar^3$$

$$\frac{ab - cd}{b^2 - c^2} = \frac{a + c}{b}$$

$$\frac{a(ar) - (ar^2)(ar^3)}{a^2r^2 - a^2r^4} = \frac{a + ar^2}{ar}$$

$$\frac{a^2r - a^2r^5}{a^2r^2(1 - r^2)} = \frac{a(1 + r^2)}{ar}$$

$$\frac{a^2r(1 - r^4)}{a^2r^2(1 - r^2)} = \frac{a(1 + r^2)}{ar}$$

$$\frac{1 + r^2}{r} = \frac{1 + r^2}{r}$$

LHS = RHS

$a, b, c, d$  are in G.P.

$$a, b = ar, c = ar^2, d = ar^3$$

$$(a + b + c + d)^2 = (a + b)^2 + 2(b + c)^2 + (c + d)^2$$

$$\Rightarrow (a + ar + ar^2 + ar^3)^2 = (a + ar)^2 + 2(ar + ar^2)^2 + (ar^2 + ar^3)^2$$

$$\Rightarrow a^2(1 + r + r^2 + r^3)^2 = a^2[(1 + r)^2 + 2(r + r^2)^2 + (r^2 + r^3)^2]$$

$$\Rightarrow (1 + r + r^2 + r^3)^2 = 1 + r^2 + 2r + 2(r^2 + r^4 + 2r^3) + r^4 + r^6 + 2r^5$$

$$\Rightarrow (1 + r + r^2 + r^3 + r + r^2 + r^3 + r^4 + r^2 + r^3 + r^4 + r^5 + r^3 + r^4 + r^5 + r^6) = (1 + r^2 + 2r + 2r^2 + 2r^4 + 4r^3 + r^4 + r^6 + 2r^5)$$

$$\Rightarrow (r^6 + 2r^5 + 3r^4 + 4r^3 + 3r^2 + 2r + 1) = (r^6 + 2r^5 + 3r^4 + 4r^3 + 3r^2 + 2r + 1)$$

LHS = RHS

$a, b, c, d$  are in G.P.

$$a, b = ar, c = ar^2, d = ar^3$$

$$(b + c)(b + d) = (c + a)(c + d)$$

$$\Rightarrow (ar + ar^2)(ar + ar^3) = (ar^2 + a)(ar^2 + ar^3)$$

$$\Rightarrow a^2(r + r^2)(r + r^3) = a^2(r^2 + 1)(r^2 + r^3)$$

$$\Rightarrow r^2(1 + r)(1 + r^2) = r^2(1 + r^2)(1 + r)$$

$\therefore$  LHS = RHS

$a, b, c$  are in G.P.

$$\Rightarrow b^2 = ac \quad \text{---(i)}$$

$$(b^2)^2 = (ac)^2$$

$$(b^2)^2 = a^2c^2$$

$$\Rightarrow a^2, b^2, c^2 \text{ are in G.P.}$$

$a, b, c$  are in G.P.

$$a, b = ar, c = ar^2$$

$$(b^3)^2 = a^3 c^3$$

$$\left((ar)^3\right)^2 = a^3 (ar^2)^3$$

$$a^6 r^6 = a^3 (a^3 r^6)$$

$$a^6 r^6 = a^6 r^6$$

$$\text{LHS} = \text{RHS}$$

$$\Rightarrow (b^3)^2 = a^3 c^3$$

So,

$a^3, b^3, c^3$  are in G.P.

$a, b, c$  are in G.P.

$$a, b = ar, c = ar^2$$

$$(ab + bc)^2 = (a^2 + b^2)(b^2 + c^2)$$

$$(a \times ar + ar \times ar^2)^2 (a^2 + (ar)^2) ((ar)^2 + (ar^2)^2)$$

$$(a^2 r + a^2 r^3)^2 = (a^2 + a^2 r^2)(a^2 r^2 + a^2 r^4)$$

$$a^4 (r + r^3)^2 = a^4 (1 + r^2)(r^2 + r^4)$$

$$a^4 r^2 (1 + r^2)^2 = a^4 (1 + r^2) r^2 (1 + r^2)$$

$$a^4 r^2 (1 + r^2)^2 = a^4 r^2 (1 + r^2)^2$$

$$\text{LHS} = \text{RHS}$$

$$\Rightarrow (ab + bc)^2 = (a^2 + b^2)(b^2 + c^2)$$

$$\Rightarrow (a^2 + b^2), (ab + bc), (b^2 + c^2) \text{ are in G.P.}$$

\*\*\*\*\* END \*\*\*\*\*