



EXERCISE.13.1

Question-1

Evaluate the Given limit: $\lim_{x \rightarrow 3} x + 3$

Ans.

Evaluate the Given limit: $\lim_{x \rightarrow \pi} \left(x - \frac{22}{7} \right)$

Question-2

Evaluate the Given limit: $\lim_{x \rightarrow \pi} \left(x - \frac{22}{7} \right)$

Ans.

$$\lim_{x \rightarrow \pi} \left(x - \frac{22}{7} \right) = \left(\pi - \frac{22}{7} \right)$$

Question-3

Evaluate the Given limit: $\lim_{r \rightarrow 1} \pi r^2$

Ans.

$$\lim_{r \rightarrow 1} \pi r^2 = \pi (1)^2 = \pi$$

Question-4

Evaluate the Given limit: $\lim_{x \rightarrow 4} \frac{4x+3}{x-2}$

Ans.

$$\lim_{x \rightarrow 4} \frac{4x+3}{x-2} = \frac{4(4)+3}{4-2} = \frac{16+3}{2} = \frac{19}{2}$$

Question-5

Evaluate the Given limit: $\lim_{x \rightarrow -1} \frac{x^{10} + x^5 + 1}{x-1}$

Ans.

$$\lim_{x \rightarrow -1} \frac{x^{10} + x^5 + 1}{x-1} = \frac{(-1)^{10} + (-1)^5 + 1}{-1-1} = \frac{1-1+1}{-2} = -\frac{1}{2}$$

Question-6

Evaluate the Given limit: $\lim_{x \rightarrow 0} \frac{(x+1)^5 - 1}{x}$

Ans.

$$\lim_{x \rightarrow 0} \frac{(x+1)^5 - 1}{x}$$

Put $x + 1 = y$ so that $y \rightarrow 1$ as $x \rightarrow 0$.

$$\begin{aligned} \text{Accordingly, } \lim_{x \rightarrow 0} \frac{(x+1)^5 - 1}{x} &= \lim_{y \rightarrow 1} \frac{y^5 - 1}{y - 1} \\ &= \lim_{y \rightarrow 1} \frac{y^5 - 1^5}{y - 1} \\ &= 5 \cdot 1^{5-1} \\ &= 5 \end{aligned}$$

$$\left[\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} \right]$$

$$\therefore \lim_{x \rightarrow 0} \frac{(x+1)^5 - 1}{x} = 5$$

Question-7

Evaluate the Given limit: $\lim_{x \rightarrow 2} \frac{3x^2 - x - 10}{x^2 - 4}$

Ans.

At $x = 2$, the value of the given rational function takes the form $\frac{0}{0}$.

$$\begin{aligned}\therefore \lim_{x \rightarrow 2} \frac{3x^2 - x - 10}{x^2 - 4} &= \lim_{x \rightarrow 2} \frac{(x-2)(3x+5)}{(x-2)(x+2)} \\ &= \lim_{x \rightarrow 2} \frac{3x+5}{x+2} \\ &= \frac{3(2)+5}{2+2} \\ &= \frac{11}{4}\end{aligned}$$

Question-8

Evaluate the Given limit: $\lim_{x \rightarrow 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$

Ans.

At $x = 3$, the value of the given rational function takes the form $\frac{0}{0}$.

$$\begin{aligned}\therefore \lim_{x \rightarrow 3} \frac{x^4 - 81}{2x^2 - 5x - 3} &= \lim_{x \rightarrow 3} \frac{(x-3)(x+3)(x^2+9)}{(x-3)(2x+1)} \\ &= \lim_{x \rightarrow 3} \frac{(x+3)(x^2+9)}{2x+1} \\ &= \frac{(3+3)(3^2+9)}{2(3)+1} \\ &= \frac{6 \times 18}{7} \\ &= \frac{108}{7}\end{aligned}$$

Question-9

Evaluate the Given limit: $\lim_{x \rightarrow 0} \frac{ax + b}{cx + 1}$

Ans.

$$\lim_{x \rightarrow 0} \frac{ax + b}{cx + 1} = \frac{a(0) + b}{c(0) + 1} = b$$

Question-10

Evaluate the Given limit: $\lim_{z \rightarrow 1} \frac{z^{\frac{1}{3}} - 1}{z^6 - 1}$

Ans.

$$\lim_{z \rightarrow 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1}$$

At $z = 1$, the value of the given function takes the form $\frac{0}{0}$.

Put $z^{\frac{1}{6}} = x$ so that $z \rightarrow 1$ as $x \rightarrow 1$.

$$\begin{aligned} \text{Accordingly, } \lim_{z \rightarrow 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1} &= \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} \\ &= \lim_{x \rightarrow 1} \frac{x^2 - 1^2}{x - 1} \\ &= 2 \cdot 1^{2-1} \\ &= 2 \end{aligned} \quad \left[\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} \right]$$

$$\therefore \lim_{z \rightarrow 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1} = 2$$

Question-11

Evaluate the Given limit: $\lim_{x \rightarrow 1} \frac{ax^2 + bx + c}{cx^2 + bx + a}, a + b + c \neq 0$

Ans.

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{ax^2 + bx + c}{cx^2 + bx + a} &= \frac{a(1)^2 + b(1) + c}{c(1)^2 + b(1) + a} \\ &= \frac{a + b + c}{a + b + c} \\ &= 1 \end{aligned} \quad [a + b + c \neq 0]$$

Question-12

Evaluate the Given limit: $\lim_{x \rightarrow -2} \frac{\frac{1}{x} + \frac{1}{2}}{x + 2}$

Ans.

$$\lim_{x \rightarrow -2} \frac{\frac{1}{x} + \frac{1}{2}}{x+2}$$

At $x = -2$, the value of the given function takes the form $\frac{0}{0}$.

$$\begin{aligned}\text{Now, } \lim_{x \rightarrow -2} \frac{\frac{1}{x} + \frac{1}{2}}{x+2} &= \lim_{x \rightarrow -2} \frac{\left(\frac{2+x}{2x}\right)}{x+2} \\ &= \lim_{x \rightarrow -2} \frac{1}{2x} \\ &= \frac{1}{2(-2)} = -\frac{1}{4}\end{aligned}$$

Question-13

Evaluate the Given limit: $\lim_{x \rightarrow 0} \frac{\sin ax}{bx}$

Ans.

$$\lim_{x \rightarrow 0} \frac{\sin ax}{bx}$$

At $x = 0$, the value of the given function takes the form $\frac{0}{0}$.

$$\begin{aligned}\text{Now, } \lim_{x \rightarrow 0} \frac{\sin ax}{bx} &= \lim_{x \rightarrow 0} \frac{\sin ax}{ax} \times \frac{ax}{bx} \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin ax}{ax} \right) \times \left(\frac{a}{b} \right) \\ &= \frac{a}{b} \lim_{ax \rightarrow 0} \left(\frac{\sin ax}{ax} \right) \quad [x \rightarrow 0 \Rightarrow ax \rightarrow 0] \\ &= \frac{a}{b} \times 1 \quad \left[\lim_{y \rightarrow 0} \frac{\sin y}{y} = 1 \right] \\ &= \frac{a}{b}\end{aligned}$$

Question-14

Evaluate the Given limit: $\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx}$, $a, b \neq 0$

Ans.

$$\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx}, \quad a, b \neq 0$$

At $x = 0$, the value of the given function takes the form $\frac{0}{0}$.

$$\begin{aligned} \text{Now, } \lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx} &= \lim_{x \rightarrow 0} \frac{\left(\frac{\sin ax}{ax}\right) \times ax}{\left(\frac{\sin bx}{bx}\right) \times bx} \\ &= \left(\frac{a}{b}\right) \times \frac{\lim_{ax \rightarrow 0} \left(\frac{\sin ax}{ax}\right)}{\lim_{bx \rightarrow 0} \left(\frac{\sin bx}{bx}\right)} \quad \left[\begin{array}{l} x \rightarrow 0 \Rightarrow ax \rightarrow 0 \\ \text{and } x \rightarrow 0 \Rightarrow bx \rightarrow 0 \end{array} \right] \\ &= \left(\frac{a}{b}\right) \times \frac{1}{1} \quad \left[\lim_{y \rightarrow 0} \frac{\sin y}{y} = 1 \right] \\ &= \frac{a}{b} \end{aligned}$$

Question-15

Evaluate the Given limit: $\lim_{x \rightarrow \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$

Ans.

$$\lim_{x \rightarrow \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$$

It is seen that $x \rightarrow \pi \Rightarrow (\pi - x) \rightarrow 0$

$$\begin{aligned} \therefore \lim_{x \rightarrow \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)} &= \frac{1}{\pi} \lim_{(\pi - x) \rightarrow 0} \frac{\sin(\pi - x)}{(\pi - x)} \\ &= \frac{1}{\pi} \times 1 \quad \left[\lim_{y \rightarrow 0} \frac{\sin y}{y} = 1 \right] \\ &= \frac{1}{\pi} \end{aligned}$$

Question-16

Evaluate the given limit: $\lim_{x \rightarrow 0} \frac{\cos x}{\pi - x}$

Ans.

$$\lim_{x \rightarrow 0} \frac{\cos x}{\pi - x} = \frac{\cos 0}{\pi - 0} = \frac{1}{\pi}$$

Question-17

Evaluate the Given limit: $\lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\cos x - 1}$

Ans.

$$\lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\cos x - 1}$$

At $x = 0$, the value of the given function takes the form $\frac{0}{0}$.

Now,

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\cos x - 1} &= \lim_{x \rightarrow 0} \frac{1 - 2\sin^2 x - 1}{1 - 2\sin^2 \frac{x}{2} - 1} \quad \left[\cos x = 1 - 2\sin^2 \frac{x}{2} \right] \\ &= \lim_{x \rightarrow 0} \frac{\sin^2 x}{\sin^2 \frac{x}{2}} = \lim_{x \rightarrow 0} \frac{\left(\frac{\sin^2 x}{x^2} \right) \times x^2}{\left(\frac{\sin^2 \frac{x}{2}}{\left(\frac{x}{2} \right)^2} \right) \times \frac{x^2}{4}} \\ &= 4 \frac{\lim_{x \rightarrow 0} \left(\frac{\sin^2 x}{x^2} \right)}{\lim_{x \rightarrow 0} \left(\frac{\sin^2 \frac{x}{2}}{\left(\frac{x}{2} \right)^2} \right)} \\ &= 4 \frac{\left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right)^2}{\left(\lim_{\frac{x}{2} \rightarrow 0} \frac{\sin \frac{x}{2}}{\frac{x}{2}} \right)^2} \quad \left[x \rightarrow 0 \Rightarrow \frac{x}{2} \rightarrow 0 \right] \\ &= 4 \frac{1^2}{1^2} \quad \left[\lim_{y \rightarrow 0} \frac{\sin y}{y} = 1 \right] \\ &= 4 \end{aligned}$$

Question-18

Evaluate the Given limit: $\lim_{x \rightarrow 0} \frac{ax + x \cos x}{b \sin x}$

Ans.

$$\lim_{x \rightarrow 0} \frac{ax + x \cos x}{b \sin x}$$

At $x = 0$, the value of the given function takes the form $\frac{0}{0}$.

Now,

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{ax + x \cos x}{b \sin x} &= \frac{1}{b} \lim_{x \rightarrow 0} \frac{x(a + \cos x)}{\sin x} \\ &= \frac{1}{b} \lim_{x \rightarrow 0} \left(\frac{x}{\sin x} \right) \times \lim_{x \rightarrow 0} (a + \cos x) \\ &= \frac{1}{b} \times \frac{1}{\left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right)} \times \lim_{x \rightarrow 0} (a + \cos x) \\ &= \frac{1}{b} \times (a + \cos 0) \quad \left[\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right] \\ &= \frac{a+1}{b} \end{aligned}$$

Question-19

Evaluate the Given limit: $\lim_{x \rightarrow 0} x \sec x$

Ans.

$$\lim_{x \rightarrow 0} x \sec x = \lim_{x \rightarrow 0} \frac{x}{\cos x} = \frac{0}{\cos 0} = \frac{0}{1} = 0$$

Question-20

Evaluate the Given limit: $\lim_{x \rightarrow 0} \frac{\sin ax + bx}{ax + \sin bx}$ $a, b, a+b \neq 0$

Ans.

At $x = 0$, the value of the given function takes the form $\frac{0}{0}$.

Now,

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin ax + bx}{ax + \sin bx} &= \lim_{x \rightarrow 0} \frac{\left(\frac{\sin ax}{ax} \right) ax + bx}{ax + bx \left(\frac{\sin bx}{bx} \right)} \\ &= \frac{\left(\lim_{ax \rightarrow 0} \frac{\sin ax}{ax} \right) \times \lim_{x \rightarrow 0} (ax) + \lim_{x \rightarrow 0} bx}{\lim_{x \rightarrow 0} ax + \lim_{x \rightarrow 0} bx \left(\lim_{bx \rightarrow 0} \frac{\sin bx}{bx} \right)} \quad [\text{As } x \rightarrow 0 \Rightarrow ax \rightarrow 0 \text{ and } bx \rightarrow 0] \\ &= \frac{\lim_{x \rightarrow 0} (ax) + \lim_{x \rightarrow 0} bx}{\lim_{x \rightarrow 0} ax + \lim_{x \rightarrow 0} bx} \quad \left[\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right] \\ &= \frac{\lim_{x \rightarrow 0} (ax + bx)}{\lim_{x \rightarrow 0} (ax + bx)} \\ &= \lim_{x \rightarrow 0} (1) \\ &= 1 \end{aligned}$$

Question-21

Evaluate the Given limit: $\lim_{x \rightarrow 0} (\operatorname{cosec} x - \cot x)$

Ans.

At $x = 0$, the value of the given function takes the form $\infty - \infty$.

Now,

$$\begin{aligned} & \lim_{x \rightarrow 0} (\operatorname{cosec} x - \cot x) \\ &= \lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{\sin x} \right) \\ &= \lim_{x \rightarrow 0} \frac{\left(\frac{1 - \cos x}{x} \right)}{\left(\frac{\sin x}{x} \right)} \\ &= \frac{\lim_{x \rightarrow 0} \frac{1 - \cos x}{x}}{\lim_{x \rightarrow 0} \frac{\sin x}{x}} \\ &= \frac{0}{1} \quad \left[\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0 \text{ and } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right] \\ &= 0 \end{aligned}$$

Question-22

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$$

Ans.

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$$

At $x = \frac{\pi}{2}$, the value of the given function takes the form $\frac{0}{0}$.

Now, put $x - \frac{\pi}{2} = y$ so that $x \rightarrow \frac{\pi}{2}$, $y \rightarrow 0$.

$$\begin{aligned} \therefore \lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}} &= \lim_{y \rightarrow 0} \frac{\tan 2\left(y + \frac{\pi}{2}\right)}{y} \\ &= \lim_{y \rightarrow 0} \frac{\tan(\pi + 2y)}{y} \\ &= \lim_{y \rightarrow 0} \frac{\tan 2y}{y} \quad [\tan(\pi + 2y) = \tan 2y] \\ &= \lim_{y \rightarrow 0} \frac{\sin 2y}{y \cos 2y} \\ &= \lim_{y \rightarrow 0} \left(\frac{\sin 2y}{2y} \times \frac{2}{\cos 2y} \right) \\ &= \left(\lim_{2y \rightarrow 0} \frac{\sin 2y}{2y} \right) \times \lim_{y \rightarrow 0} \left(\frac{2}{\cos 2y} \right) \quad [y \rightarrow 0 \Rightarrow 2y \rightarrow 0] \\ &= 1 \times \frac{2}{\cos 0} \quad \left[\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right] \\ &= 1 \times \frac{2}{1} \\ &= 2 \end{aligned}$$

Question-23

Find $\lim_{x \rightarrow 0^-} f(x)$ and $\lim_{x \rightarrow 1} f(x)$, where $f(x) = \begin{cases} 2x+3, & x \leq 0 \\ 3(x+1), & x > 0 \end{cases}$

Ans.

The given function is

$$f(x) = \begin{cases} 2x+3, & x \leq 0 \\ 3(x+1), & x > 0 \end{cases}$$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0} [2x+3] = 2(0)+3 = 3$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0} 3(x+1) = 3(0+1) = 3$$

$$\therefore \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0} f(x) = 3$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1} 3(x+1) = 3(1+1) = 6$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1} 3(x+1) = 3(1+1) = 6$$

$$\therefore \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1} f(x) = 6$$

Question-24

Find $\lim_{x \rightarrow 1} f(x)$, where $f(x) = \begin{cases} x^2 - 1, & x \leq 1 \\ -x^2 - 1, & x > 1 \end{cases}$

Ans.

The given function is

$$f(x) = \begin{cases} x^2 - 1, & x \leq 1 \\ -x^2 - 1, & x > 1 \end{cases}$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} [x^2 - 1] = 1^2 - 1 = 1 - 1 = 0$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} [-x^2 - 1] = -1^2 - 1 = -1 - 1 = -2$$

It is observed that $\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$.

Hence, $\lim_{x \rightarrow 1} f(x)$ does not exist.

Question-25

Evaluate $\lim_{x \rightarrow 0} f(x)$, where $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Ans.

$$f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$\begin{aligned} \lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^-} \left[\frac{|x|}{x} \right] \\ &= \lim_{x \rightarrow 0^-} \left(\frac{-x}{x} \right) \quad \left[\text{When } x \text{ is negative, } |x| = -x \right] \\ &= \lim_{x \rightarrow 0^-} (-1) \\ &= -1 \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow 0^+} f(x) &= \lim_{x \rightarrow 0^+} \left[\frac{|x|}{x} \right] \\ &= \lim_{x \rightarrow 0^+} \left[\frac{x}{x} \right] \quad \left[\text{When } x \text{ is positive, } |x| = x \right] \\ &= \lim_{x \rightarrow 0^+} (1) \\ &= 1 \end{aligned}$$

It is observed that $\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$.

Hence, $\lim_{x \rightarrow 0} f(x)$ does not exist.

Question-26

$$\text{Find } \lim_{x \rightarrow 0} f(x), \text{ where } f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Ans.

The given function is

$$f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$\begin{aligned} \lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^-} \left[\frac{x}{|x|} \right] \\ &= \lim_{x \rightarrow 0} \left[\frac{x}{-x} \right] \quad \left[\text{When } x < 0, |x| = -x \right] \\ &= \lim_{x \rightarrow 0} (-1) \\ &= -1 \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow 0^+} f(x) &= \lim_{x \rightarrow 0^+} \left[\frac{x}{|x|} \right] \\ &= \lim_{x \rightarrow 0} \left[\frac{x}{x} \right] \quad \left[\text{When } x > 0, |x| = x \right] \\ &= \lim_{x \rightarrow 0} (1) \\ &= 1 \end{aligned}$$

It is observed that $\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$.

Hence, $\lim_{x \rightarrow 0} f(x)$ does not exist.

Question-27

$$\text{Find } \lim_{x \rightarrow 5} f(x), \text{ where } f(x) = |x| - 5$$

Ans.

The given function is $f(x) = |x| - 5$.

$$\begin{aligned}\lim_{x \rightarrow 5^-} f(x) &= \lim_{x \rightarrow 5^-} [|x| - 5] \\ &= \lim_{x \rightarrow 5^-} (x - 5) \quad \left[\text{When } x > 0, |x| = x \right] \\ &= 5 - 5 \\ &= 0\end{aligned}$$

$$\begin{aligned}\lim_{x \rightarrow 5^+} f(x) &= \lim_{x \rightarrow 5^+} [|x| - 5] \\ &= \lim_{x \rightarrow 5^+} (x - 5) \quad \left[\text{When } x > 0, |x| = x \right] \\ &= 5 - 5 \\ &= 0\end{aligned}$$

$$\therefore \lim_{x \rightarrow 5^-} f(x) = \lim_{x \rightarrow 5^+} f(x) = 0$$

$$\text{Hence, } \lim_{x \rightarrow 5} f(x) = 0$$

Question-28

$$\text{Suppose } f(x) = \begin{cases} a + bx, & x < 1 \\ 4, & x = 1 \\ b - ax, & x > 1 \end{cases} \text{ and if } \lim_{x \rightarrow 1} f(x) = f(1) \text{ what are possible values of } a \text{ and } b?$$

Ans.

The given function is

$$f(x) = \begin{cases} a + bx, & x < 1 \\ 4, & x = 1 \\ b - ax, & x > 1 \end{cases}$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (a + bx) = a + b$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (b - ax) = b - a$$

$$f(1) = 4$$

$$\text{It is given that } \lim_{x \rightarrow 1} f(x) = f(1).$$

$$\therefore \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1} f(x) = f(1)$$

$$\Rightarrow a + b = 4 \text{ and } b - a = 4$$

On solving these two equations, we obtain $a = 0$ and $b = 4$.

Thus, the respective possible values of a and b are 0 and 4.

Question-29

Let a_1, a_2, \dots, a_n be fixed real numbers and define a function

$$f(x) = (x - a_1)(x - a_2) \dots (x - a_n)$$

What is $\lim_{x \rightarrow a_1} f(x)$? For some $a \neq a_1, a_2, \dots, a_n$, compute $\lim_{x \rightarrow a} f(x)$

Ans.

The given function is $f(x) = (x - a_1)(x - a_2) \dots (x - a_n)$.

$$\begin{aligned}\lim_{x \rightarrow a_1} f(x) &= \lim_{x \rightarrow a_1} [(x - a_1)(x - a_2) \dots (x - a_n)] \\ &= \left[\lim_{x \rightarrow a_1} (x - a_1) \right] \left[\lim_{x \rightarrow a_1} (x - a_2) \right] \dots \left[\lim_{x \rightarrow a_1} (x - a_n) \right] \\ &= (a_1 - a_1)(a_1 - a_2) \dots (a_1 - a_n) = 0\end{aligned}$$

$$\therefore \lim_{x \rightarrow a_1} f(x) = 0$$

$$\begin{aligned}\text{Now, } \lim_{x \rightarrow a} f(x) &= \lim_{x \rightarrow a} [(x - a_1)(x - a_2) \dots (x - a_n)] \\ &= \left[\lim_{x \rightarrow a} (x - a_1) \right] \left[\lim_{x \rightarrow a} (x - a_2) \right] \dots \left[\lim_{x \rightarrow a} (x - a_n) \right] \\ &= (a - a_1)(a - a_2) \dots (a - a_n)\end{aligned}$$

$$\therefore \lim_{x \rightarrow a} f(x) = (a - a_1)(a - a_2) \dots (a - a_n)$$

Question-30

$$\text{If } f(x) = \begin{cases} |x| + 1, & x < 0 \\ 0, & x = 0 \\ |x| - 1, & x > 0 \end{cases}.$$

For what value (s) of a does $\lim_{x \rightarrow a} f(x)$ exists?

Ans.

The given function is

$$f(x) = \begin{cases} |x| + 1, & x < 0 \\ 0, & x = 0 \\ |x| - 1, & x > 0 \end{cases}$$

When $a = 0$,

$$\begin{aligned}\lim_{x \rightarrow 0^-} f(x) &= \lim_{x \rightarrow 0^-} (|x| + 1) \\ &= \lim_{x \rightarrow 0^-} (-x + 1) \quad \left[\text{If } x < 0, |x| = -x \right] \\ &= -0 + 1 \\ &= 1\end{aligned}$$

$$\begin{aligned}\lim_{x \rightarrow 0^+} f(x) &= \lim_{x \rightarrow 0^+} (|x| - 1) \\ &= \lim_{x \rightarrow 0^+} (x - 1) \quad \left[\text{If } x > 0, |x| = x \right] \\ &= 0 - 1 \\ &= -1\end{aligned}$$

Here, it is observed that $\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$.

$\therefore \lim_{x \rightarrow 0} f(x)$ does not exist.

When $a < 0$,

$$\begin{aligned}\lim_{x \rightarrow a^-} f(x) &= \lim_{x \rightarrow a^-} (|x| + 1) \\ &= \lim_{x \rightarrow a^-} (-x + 1) \quad [x < a < 0 \Rightarrow |x| = -x] \\ &= -a + 1\end{aligned}$$

$$\begin{aligned}\lim_{x \rightarrow a^+} f(x) &= \lim_{x \rightarrow a^+} (|x| + 1) \\ &= \lim_{x \rightarrow a^+} (-x + 1) \quad [a < x < 0 \Rightarrow |x| = -x] \\ &= -a + 1\end{aligned}$$

$$\therefore \lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = -a + 1$$

Thus, limit of $f(x)$ exists at $x = a$, where $a < 0$.

When $a > 0$

$$\begin{aligned}\lim_{x \rightarrow a^-} f(x) &= \lim_{x \rightarrow a^-} (|x| - 1) \\ &= \lim_{x \rightarrow a^-} (x - 1) \quad [0 < x < a \Rightarrow |x| = x] \\ &= a - 1\end{aligned}$$

$$\begin{aligned}\lim_{x \rightarrow a^+} f(x) &= \lim_{x \rightarrow a^+} (|x| - 1) \\ &= \lim_{x \rightarrow a^+} (x - 1) \quad [0 < a < x \Rightarrow |x| = x] \\ &= a - 1\end{aligned}$$

$$\therefore \lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = a - 1$$

Thus, limit of $f(x)$ exists at $x = a$, where $a > 0$.

Thus, $\lim_{x \rightarrow a} f(x)$ exists for all $a \neq 0$.

Question-31

If the function $f(x)$ satisfies $\lim_{x \rightarrow 1} \frac{f(x) - 2}{x^2 - 1} = \pi$, evaluate $\lim_{x \rightarrow 1} f(x)$.

Ans.

$$\begin{aligned}
& \lim_{x \rightarrow 1} \frac{f(x) - 2}{x^2 - 1} = \pi \\
& \Rightarrow \frac{\lim_{x \rightarrow 1} (f(x) - 2)}{\lim_{x \rightarrow 1} (x^2 - 1)} = \pi \\
& \Rightarrow \lim_{x \rightarrow 1} (f(x) - 2) = \pi \lim_{x \rightarrow 1} (x^2 - 1) \\
& \Rightarrow \lim_{x \rightarrow 1} (f(x) - 2) = \pi(1^2 - 1) \\
& \Rightarrow \lim_{x \rightarrow 1} (f(x) - 2) = 0 \\
& \Rightarrow \lim_{x \rightarrow 1} f(x) - \lim_{x \rightarrow 1} 2 = 0 \\
& \Rightarrow \lim_{x \rightarrow 1} f(x) - 2 = 0 \\
& \therefore \lim_{x \rightarrow 1} f(x) = 2
\end{aligned}$$

Question-32

$$\text{If } f(x) = \begin{cases} mx^2 + n, & x < 0 \\ nx + m, & 0 \leq x \leq 1 \\ nx^3 + m, & x > 1 \end{cases} \text{ . For what integers } m \text{ and } n \text{ does } \lim_{x \rightarrow 0} f(x) \text{ and}$$

$\lim_{x \rightarrow 1} f(x)$ exist?

Ans.

The given function is

$$f(x) = \begin{cases} mx^2 + n, & x < 0 \\ nx + m, & 0 \leq x \leq 1 \\ nx^3 + m, & x > 1 \end{cases}$$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (mx^2 + n)$$

$$= m(0)^2 + n$$

$$= n$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (nx + m)$$

$$= n(0) + m$$

$$= m.$$

Thus, $\lim_{x \rightarrow 0} f(x)$ exists if $m = n$.

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (nx + m)$$

$$= n(1) + m$$

$$= m + n$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (nx^3 + m)$$

$$= n(1)^3 + m$$

$$= m + n$$

$$\therefore \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1} f(x).$$

Thus, $\lim_{x \rightarrow 1} f(x)$ exists for any integral value of m and n .

***** END *****