Dipole moment of the system, $p = q \times dl = -10^{-7}$ C m

Rate of increase of electric field per unit length,

$$\frac{dE}{dl} = 10^{+5}\ \text{N C}^{-1}$$

Force (F) experienced by the system is given by the relation,

$F = qE$

$$F = q\frac{dE}{dl} \times dl$$

$$= p \times \frac{dE}{dl}$$

$= -10^{-7} \times 10^{-5}$

$= -10^{-2}$ N

The force is -10^{-2} N in the negative z-direction i.e., opposite to the direction of electric field. Hence, the angle between electric field and dipole moment is 180°.

Torque (τ) is given by the relation,

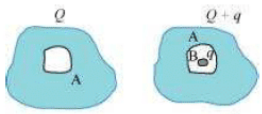
$\tau = pE \sin 180°$

$= 0$

Therefore, the torque experienced by the system is zero.

Question 1.28:

(a) A conductor A with a cavity as shown in Fig. 1.36(a) is given a charge Q. Show that the entire charge must appear on the outer surface of the conductor. (b) Another conductor B with charge q is inserted into the cavity keeping B insulated from A. Show that the total charge on the outside surface of A is $Q + q$ [Fig. 1.36(b)]. (c) A sensitive instrument is to be shielded from the strong electrostatic fields in its environment. Suggest a possible way.



(a) (b)

Answer:

(a) Let us consider a Gaussian surface that is lying wholly within a conductor and enclosing the cavity. The electric field intensity E inside the charged conductor is zero.

Let q is the charge inside the conductor and ϵ_0 is the permittivity of free space.

According to Gauss's law,

Flux, $\phi = \vec{E}.\vec{ds} = \frac{q}{\epsilon_0}$

Here, $E = 0$

$$\frac{q}{\epsilon_0} = 0$$

$\because \epsilon_0 \neq 0$

$\therefore q = 0$

Therefore, charge inside the conductor is zero.

The entire charge Q appears on the outer surface of the conductor.

(b) The outer surface of conductor A has a charge of amount Q. Another conductor B having charge $+q$ is kept inside conductor A and it is insulated from A. Hence, a charge of amount $-q$ will be induced in the inner surface of conductor A and $+q$ is induced on the outer surface of conductor A. Therefore, total charge on the outer surface of conductor A is $Q + q$.

(c) A sensitive instrument can be shielded from the strong electrostatic field in its environment by enclosing it fully inside a metallic surface. A closed metallic body acts as an electrostatic shield.

Question 1.29:

A hollow charged conductor has a tiny hole cut into its surface. Show that the electric

A hollow charged conductor has a tiny hole cut into its surface. Show that the electric field in the hole is $\left(\frac{\sigma}{2\epsilon_0}\right)\hat{n}$, where $\hat{n}$ is the unit vector in the outward normal direction, and σ is the surface charge density near the hole.

Answer:

Let us consider a conductor with a cavity or a hole. Electric field inside the cavity is zero.

Let *E* is the electric field just outside the conductor, *q* is the electric charge, σ is the charge density, and ϵ_0 is the permittivity of free space.

Charge $|q| = \vec{\sigma} \times \vec{ds}$

According to Gauss's law,

$$\text{Flux, } \phi = \vec{E}.\vec{ds} = \frac{|q|}{\epsilon_0}$$

$$Eds = \frac{\vec{\sigma} \times \vec{ds}}{\epsilon_0}$$

$$\therefore E = \frac{\sigma}{\epsilon_0}\hat{n}$$

Therefore, the electric field just outside the conductor is $\frac{\sigma}{\epsilon_0}\hat{n}$. This field is a superposition of field due to the cavity (E') and the field due to the rest of the charged conductor (E'). These fields are equal and opposite inside the conductor, and equal in magnitude and direction outside the conductor.

$$\therefore E' + E' = E$$

$$E' = \frac{E}{2}$$

$$= \frac{\sigma}{2\epsilon_0}\hat{n}$$

Therefore, the field due to the rest of the conductor is $\frac{\sigma}{\epsilon_0}\hat{n}$.

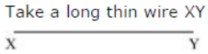
Hence, proved.

Question 1.30:

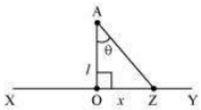
Obtain the formula for the electric field due to a long thin wire of uniform linear charge density *λ* without using Gauss's law. [*Hint:* Use Coulomb's law directly and evaluate the necessary integral.]

Answer:

Take a long thin wire XY (as shown in the figure) of uniform linear charge density λ.



Consider a point A at a perpendicular distance *l* from the mid-point O of the wire, as shown in the following figure.



Let *E* be the electric field at point A due to the wire, XY.

Consider a small length element *dx* on the wire section with OZ = *x*

Let *q* be the charge on this piece.

$$\therefore q = \lambda dx$$

Electric field due to the piece,

$$dE = \frac{1}{4\pi\epsilon_0}\frac{\lambda dx}{(\text{AZ})^2}$$

$$\text{However, } AZ = \sqrt{(l^2 + x^2)}$$

$$\therefore dE = \frac{\lambda dx}{4\pi\epsilon_0 (l^2 + x^2)}$$

The electric field is resolved into two rectangular components. $dE\cos\theta$ is the perpendicular component and $dE\sin\theta$ is the parallel component.

When the whole wire is considered, the component $dE\sin\theta$ is cancelled.

Only the perpendicular component $dE\cos\theta$ affects point A.

Hence, effective electric field at point A due to the element *dx* is dE_1.

$$\therefore dE_1 = \frac{\lambda dx \cos\theta}{4\pi\epsilon_0 (x^2 + l^2)} \quad \text{... (1)}$$

$$\text{In} \Delta \text{AZO},$$

$$\tan\theta = \frac{x}{l}$$

$$x = l\tan\theta \quad \text{... (2)}$$

On differentiating equation (2), we obtain

$$\frac{dx}{d\theta} = l\sin^2\theta$$

$$dx = l\sin^2\theta\, d\theta \qquad \dots(3)$$

From equation (2),

$$x^2 + l^2 = l^2 + l^2\tan^2\theta$$

$$\therefore l^2\left(1+\tan^2\theta\right) = l^2\sec^2\theta$$

$$x^2 + l^2 = l^2\sin^2\theta \qquad \dots(4)$$

Putting equations (3) and (4) in equation (1), we obtain

$$\therefore dE_1 = \frac{\lambda l\sec^2 d\theta}{4\pi\in_0 l^2\sec^2\theta}\times\cos\theta$$

$$\therefore dE_1 = \frac{\lambda\cos\theta d\theta}{4\pi\in_0 l} \qquad \dots(5)$$

The wire is so long that θ tends from $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$.

By integrating equation (5), we obtain the value of field E_1 as,

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dE_1 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\lambda}{4\pi\in_0 l}\cos\theta d\theta$$

$$E_1 = \frac{\lambda}{4\pi\in_0 l}\left[\sin\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{\lambda}{4\pi\in_0 l}\times 2$$

$$E_1 = \frac{\lambda}{2\pi\in_0 l}$$

Therefore, the electric field due to long wire is $\frac{\lambda}{2\pi\in_0 l}$.

Question 1.31:

It is now believed that protons and neutrons (which constitute nuclei of ordinary matter) are themselves built out of more elementary units called quarks. A proton and a neutron consist of three quarks each. Two types of quarks, the so called 'up' quark (denoted by u) of charge (+2/3) *e*, and the 'down' quark (denoted by d) of charge (−1/3) *e*, together with electrons build up ordinary matter. (Quarks of other types have also been found which give rise to different unusual varieties of matter.) Suggest a possible quark composition of a proton and neutron.

Answer:

A proton has three quarks. Let there be *n* up quarks in a proton, each having a charge of $+\frac{2}{3}e$.

Charge due to *n* up quarks $=\left(\frac{2}{3}e\right)n$

Number of down quarks in a proton = 3 − *n*

Each down quark has a charge of $-\frac{1}{3}e$.

Charge due to (3 − *n*) down quarks $=\left(-\frac{1}{3}e\right)(3-n)$

Total charge on a proton = + *e*

$$\therefore e = \left(\frac{2}{3}e\right)n + \left(-\frac{1}{3}e\right)(3-n)$$

$$e = \left(\frac{2ne}{3}\right) - e + \frac{ne}{3}$$

$$2e = ne$$

$$n = 2$$

Number of up quarks in a proton, *n* = 2

Number of down quarks in a proton = 3 − *n* = 3 − 2 = 1

Therefore, a proton can be represented as 'uud'.

A neutron also has three quarks. Let there be *n* up quarks in a neutron, each having a charge of $+\frac{3}{2}e$.

Charge on a neutron due to *n* up quarks $=\left(+\frac{3}{2}e\right)n$

Number of down quarks is 3 − *n*,each having a charge of $\left(-\frac{1}{3}\right)e$.

Charge on a neutron due to $(3-n)$ down quarks = $\left(-\frac{1}{3}e\right)(3-n)$

Total charge on a neutron = 0

Total charge on a neutron = 0

$$0 = \left(\frac{2}{3}e\right)n + \left(-\frac{1}{3}e\right)(3-n)$$

$$0 = \frac{2}{3}en - e + \frac{ne}{3}$$

$$e = ne$$

$$n = 1$$

Number of up quarks in a neutron, n = 1

Number of down quarks in a neutron = 3 − n = 2

Therefore, a neutron can be represented as 'udd'.

Question 1.32:

(a) Consider an arbitrary electrostatic field configuration. A small test charge is placed at a null point (i.e., where **E** = 0) of the configuration. Show that the equilibrium of the test charge is necessarily unstable.

(b) Verify this result for the simple configuration of two charges of the same magnitude and sign placed a certain distance apart.

Answer:

(a) Let the equilibrium of the test charge be stable. If a test charge is in equilibrium and displaced from its position in any direction, then it experiences a restoring force towards a null point, where the electric field is zero. All the field lines near the null point are directed inwards towards the null point. There is a net inward flux of electric field through a closed surface around the null point. According to Gauss's law, the flux of electric field through a surface, which is not enclosing any charge, is zero. Hence, the equilibrium of the test charge can be stable.

(b) Two charges of same magnitude and same sign are placed at a certain distance. The mid-point of the joining line of the charges is the null point. When a test charged is displaced along the line, it experiences a restoring force. If it is displaced normal to the joining line, then the net force takes it away from the null point. Hence, the charge is unstable because stability of equilibrium requires restoring force in all directions.

Question 1.33:

A particle of mass m and charge ($-q$) enters the region between the two charged plates initially moving along x-axis with speed vx (like particle 1 in Fig. 1.33). The length of plate is L and an uniform electric field E is maintained between the plates. Show that the vertical deflection of the particle at the far edge of the plate is $qEL^2/(2m v_x^2)$.

Compare this motion with motion of a projectile in gravitational field discussed in Section 4.10 of Class XI Textbook of Physics.

Answer:

Charge on a particle of mass m = $-q$

Velocity of the particle = v_x

Length of the plates = L

Magnitude of the uniform electric field between the plates = E

Mechanical force, F = Mass (m) × Acceleration (a)

$$a = \frac{F}{m}$$

However, electric force, $F = qE$

Therefore, acceleration, $a = \dfrac{qE}{m}$ ... (1)

Time taken by the particle to cross the field of length L is given by,

$$t = \frac{\text{Length of the plate}}{\text{Velocity of the particle}} = \frac{L}{v_x} \quad \text{... (2)}$$

In the vertical direction, initial velocity, u = 0

According to the third equation of motion, vertical deflection s of the particle can be obtained as,

$$s = ut + \frac{1}{2}at^2$$

$$s = 0 + \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{L}{v_x}\right)^2$$

$$s = \frac{qEL^2}{2mV_x^2} \quad \text{... (3)}$$

Hence, vertical deflection of the particle at the far edge of the plate is $qEL^2/\left(2mv_x^2\right)$. This is similar to the motion of horizontal projectiles under gravity.

Question 1.34:

Suppose that the particle in Exercise in 1.33 is an electron projected with velocity v_x= 2.0 × 10^6 m s^{-1}. If E between the plates separated by 0.5 cm is 9.1 × 10^2 N/C, where will the electron strike the upper plate? ($|e| = 1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg.)

Answer:

Velocity of the particle, $v_x = 2.0 \times 10^6$ m/s

Separation of the two plates, d = 0.5 cm = 0.005 m

Electric field between the two plates, $E = 9.1 \times 10^2$ N/C

Charge on an electron, $q = 1.6 \times 10^{-19}$ C

Mass of an electron, $m_e = 9.1 \times 10^{-31}$ kg

Let the electron strike the upper plate at the end of plate L, when deflection is s.

Therefore,

$$s = \frac{qEL^2}{2mv_x^2}$$

$$L = \sqrt{\frac{2dmv_x^2}{qE}}$$

$$= \sqrt{\frac{2 \times 0.005 \times 9.1 \times 10^{-31} \times \left(2.0 \times 10^6\right)^2}{1.6 \times 10^{-10} \times 9.1 \times 10^2}}$$

$$= \sqrt{0.025 \times 10^{-2}} = \sqrt{2.5 \times 10^{-4}}$$

$$= 1.6 \times 10^{-2} \text{ m}$$

$$= 1.6 \text{ cm}$$

Therefore, the electron will strike the upper plate after travelling 1.6 cm.

*********** END **********