Question 5. 31. A train runs along an un banked circular track of radius 30 m at a speed of 54 km/h. The mass of the train is 10^6 kg. What provides the centripetal force required for this purpose the engine or the rails? What is the angle of banking required to prevent wearing out of the rail?

Answer:

Here $r = 30$ m, $v = 54$ km/h $= 54 \times \frac{5}{18}$ m/s $= 15$ m/s, mass of train $m = 10^6$ kg.

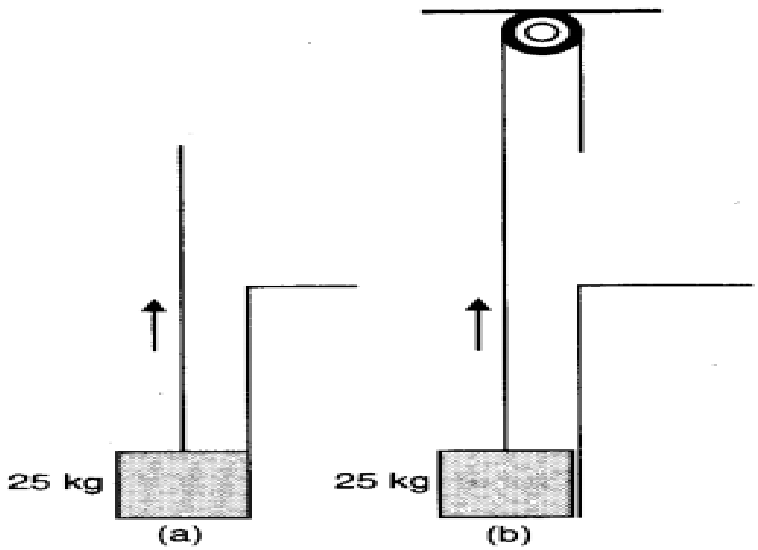
The centripetal force $F = \frac{mv^2}{r}$ for negotiating the circular track is provided by the force of lateral friction due to rails on the wheels of the train.

To prevent wearing out of rails, the angle of banking θ is given by

$$\tan\theta = \frac{v^2}{rg} = \frac{15 \times 15}{30 \times 10} = 0.75$$

$$\Rightarrow \quad \theta = \tan^{-1}(0.75) \simeq 37°.$$

Question 5. 32. A block of mass 25 kg is raised by a 50 kg man in two different ways as shown in Fig. What is the action on the floor by the man in the two cases? If the floor yields to a normal force of 700 N, which mode should the man adopt to lift the block without the floor yielding?



Answer:

In 1st case, man applies an upward force of 25 kg wt.r (same as the weight of the block). According to Newton's third law of motion, there will be a downward reaction on the floor.

The action on the floor by the man.

= 50 kg wt. + 25 kg wt. = 75 kg wt

= 75 kg x 10 m/s^2 = 750 N.

In case II, the man applies a downward force of 25 kg wt. According to Newton's third law, the reaction is in the upward direction.

In this case, action on the floor by the man

= 50 kg wt - 25 kg wt. = 25 kg wt.

= 25 kg x 10 m/s^2 = 250 N.

Therefore, the man should adopt the second method.

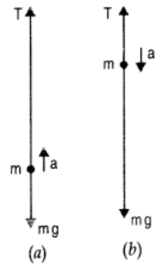
Question 5. 33. A monkey of mass 40 kg climbs on a rope (Fig.) which can stand a maximum tension of 600 N. In which of the

following cases will the rope break: the monkey
(a) climbs up with an acceleration of 6 ms^{-2}
(b) climbs down with an acceleration of 4 ms^{-2}
(c) climbs up with a uniform speed of 5 ms^{-1}
(d) falls down the rope nearly freely under gravity?
(Ignore the mass of the rope).
Answer:
(a) When the monkey climbs up with an acceleration a, then T - mg = ma
where T represents the tension (figure a).

$\therefore \quad T = mg + ma = m(g + a)$

or $\quad T = 40 \text{ kg } (10 + 6) \text{ ms}^{-2} = 640 \text{ N}$

But the rope can withstand a maximum tension of 600 N. So the rope will break.



(b) When the monkey is climbing down with an acceleration, then

$mg - T = ma \quad$ (Figure (b))

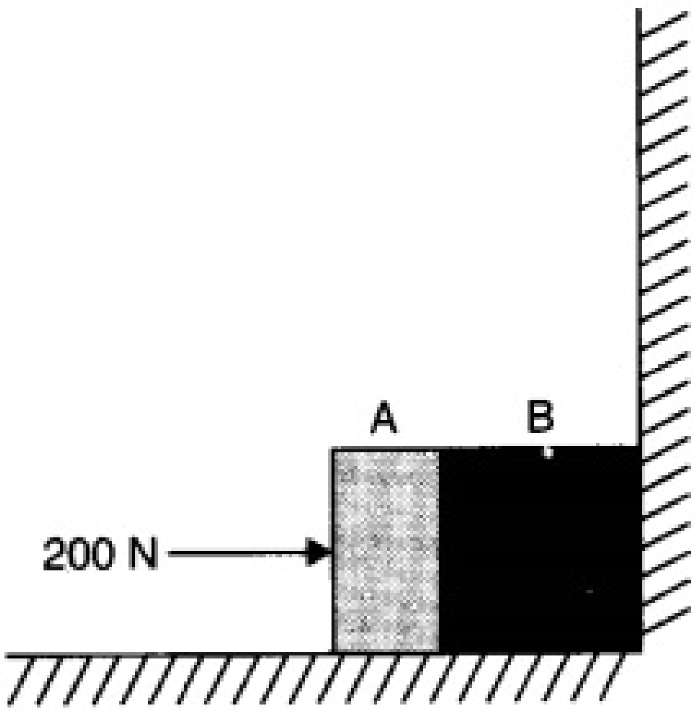
$\Rightarrow \quad T = mg - ma = m(g - a)$

or $\quad T = 40 \text{ kg} \times (10 - 4) \text{ ms}^{-2} = 240 \text{ N}$

The rope will not break.

(c) When the monkey climbs up with uniform speed, then
T mg = 40 kg x 10 ms^{-2} = 400 N The rope will hot break.
(d) When the monkey is falling freely, it would be a state of weightlessness. So, tension will be zero and the rope will not break.

Question 5. 34. Two bodies A and B of masses 5 kg and 10 kg in contact with each other rest on a table against a rigid wall (Fig.). The coefficient of friction between the bodies and the table is 0.15. A force of 200 N is applied horizontally to A. What are (a) the reaction of the partition (b) the action reaction forces between A and B? What happens when the wall is removed? Does the answer to change, when the bodies are in motion? Ignore the difference between μs and μk.



Answer:
(i) When the wall exists and blocks A and B are pushing the wall,

there can't be any motion i.e., blocks are at rest. Hence,
(a) reaction of the partition = - (force applied on A) = 200 N towards left.
(b) action reaction forces between A and B are 200 N each. A presses B towards right with an action force 200 N and B exerts a reaction force on A towards left having magnitude 200 N.
(ii) When the wall is removed, motion can take place such that net pushing force provides the acceleration to the block system. Hence, taking kinetic friction into account, we have

$$200 - \mu(m_1 + m_2)g = (m_1 + m_2)a$$

$$\Rightarrow \quad a = \frac{200 - \mu(m_1 + m_2)g}{(m_1 + m_2)} = \frac{200 - 0.15 \times (5 + 10) \times 10}{(5 + 10)}$$

$$= \frac{200 - 22.5}{15} = \frac{177.5}{15} = 11.8 \text{ ms}^{-2}$$

$\therefore$ If force exerted by A on B be F_{BA}, then considering equilibrium (or free body diagram) of only block A, we have

$$200 - f_{k_1} = m_1 a + F_{BA} \quad \text{or} \quad 200 - \mu m_1 g = m_1 a + F_{BA}$$

$$\Rightarrow \quad F_{BA} = 200 - \mu m_1 g - m_1 a = 200 - (0.15 \times 5 \times 10) - (5 \times 11.8)$$

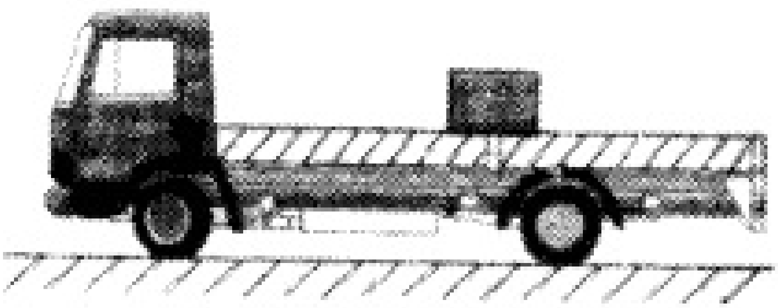
$$= 200 - 7.5 - 59$$

$$= 200 - 66.5 = 133.5 \text{ N} \simeq 1.3 \times 10^2 \text{ N towards right}$$

$\therefore$ Force exerted on A by B $F_{AB} = -F_{BA} = 1.3 \times 10^2$ N towards left.

Question 5. 35. A block of mass 15 kg is placed on a long trolley. The coefficient of static friction between the block and the trolley is 0.18. The trolley accelerates from rest with 0.5 ms^{-1} for 20 s and then moves with uniform velocity. Discuss the motion of the block as viewed by (a) a stationary observer on the ground, (b) an observer moving with the trolley.
Answer:
(a) Force experienced by block, F = ma = 15 x 0.5 = 7.5 N
Force of friction, F_f = p mg = 0.18 x 15 x 10 = 27 N. i.e., force experienced by block will be less than the friction.So the block will not move.
It will remain stationary w.r.t. trolley for a stationary observer on ground.
(b) The observer moving with trolley has an accelerated motion i.e., he forms non-inertial frame in which Newton's laws of motion are not applicable. The box will be at rest relative to the observer.

Question 5. 36. The rear side of a truck is open and a box of 40 kg mass is placed 5 m away from the open end as shown in Fig. The coefficient of friction between the box and the surface below it is 0.15. On a straight road, the truck starts from rest and accelerates with 2 ms^{-2}. At what distance from the starting point does the box fall off the truck? (Ignore the size of the box).



Answer:

Force experienced by box, $F = ma = 40 \times 2 = 80$ N
Frictional force $F_f = \mu mg = 0.15 \times 40 \times 10 = 60$ N.
Net force $= F - F_f = 80 - 60 = 20$ N.

Backward acceleration produced in the box, $a = \frac{20}{40}\left(\frac{\text{Net force}}{m}\right)$

$\Rightarrow \qquad a = 0.5 \text{ ms}^{-2}$

If t is time taken by the box to travel $s = 5$ metre and fall off the truck, then from

$$S = ut + \frac{1}{2}at^2$$

$$5 = 0 \times t + \frac{1}{2} \times 0.5\, t^2$$

$$t = \sqrt{\frac{5 \times 2}{0.5}} = 4.47 \text{ s.}$$

If the truck travels a distance x during this time, then again from

$$S = ut + \frac{1}{2}at^2$$

$$x = 0 \times 4.47 + \frac{1}{2} \times 2\,(4.47)^2 = 19.98 \text{ m.}$$

Question 5. 37. A disc revolves with a speed of 33 1/3 rpm and has a radius of 15 cm. Two coins are placed at 4 cm and 14 cm away from the centre of the record. If the coefficient of friction between the coins and record is 0.15, which of the coins will revolve with the record?

Answer: If the coin is to revolve with the record, then the force of friction must be enough to provide the necessary centripetal force.

$$\therefore \qquad mr\,\omega^2 \le \mu_s\, mg \quad \text{or} \quad r \le \frac{\mu_s\, mg}{m\,\omega^2} \quad \text{or} \quad r \le \frac{\mu_s\, g}{\omega^2}$$

$$\text{frequency} = 33\frac{1}{3} \text{ rpm} = \frac{100}{3} \text{ rpm} = \frac{100}{3 \times 60} \text{ rps}$$

The problems in which centripetal force is obtained from force of friction, start with the following equation:

$$m\,r\omega^2 \le \mu_s\, mg$$

$$\omega = 2\pi \times \frac{100}{3 \times 60} \text{ rad s}^{-1} = \frac{10}{9}\pi \text{ rad s}^{-1}$$

$$\frac{\mu_s\, g}{\omega^2} = \frac{0.15 \times 10}{\left(\frac{10}{9}\pi\right)^2} \text{ m} = 0.12 \text{ m} = 12 \text{ cm}$$

The condition ($r \le 12$ cm) is satisfied by the coin placed at 4 cm from the centre of the record. So, **the coin at 4 cm will revolve with the record.**

Question 5. 38. You may have seen in a circus a motorcyclist driving in vertical loops inside a 'death well' (a hollow spherical chamber with holes, so the spectators can watch from outside). Explain clearly why the motorcyclist does not drop down when he is at the uppermost point, with no support from below. What is the minimum speed required at the uppermost position to perform a vertical loop if the radius of the chamber is 25 m?

Answer: When the motorcyclist is at the highest point of the death-well, the normal reaction R on the motorcyclist by the ceiling of the chamber acts downwards. His weight mg also acts downwards. These two forces are balanced by the outward centrifugal force acting on him.

$$\therefore \qquad R + mg = \frac{mv^2}{r} \qquad \text{...(1)}$$

Here v is the speed of the motorcyclist and m is the mass of the motorcyclist (including the mass of the motor cycle). Because of the balancing of the forces, the motorcyclist does not fall down. The minimum speed required to perform a vertical loop is given by equation (1) when R = 0.

$$\therefore \qquad mg = \frac{mv^2_{\min}}{r} \quad \text{or} \quad v^2_{\min.} = gr$$

$$\text{or} \qquad v = \sqrt{gr} = \sqrt{10 \times 25} \text{ m s}^{-1} = 15.8 \text{ m s}^{-1}$$

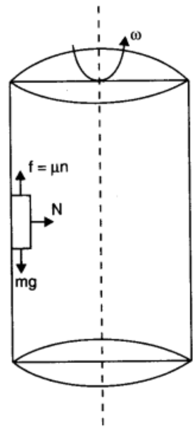
Question 5. 39. A 70 kg man stands in contact against the inner wall of a hollow cylindrical drum of radius 3 m rotating about its vertical axis with 200 rev/min. The coefficient of friction between the wall and his clothing is 0.15. What is the minimum rotational speed of the cylinder to enable the man to remain stuck to the wall (without falling) when the floor is suddenly removed?

Answer:

$R = 3$ m, $\omega = 200$ rev/min $= 2 \times \frac{22}{7} \times \frac{200}{60}$ rad/s

$$= \frac{440}{21} \text{ rad/s}$$

and $\mu = 0.15$



As shown in the figure, the normal reaction (N) of the wall on the man acts in the horizontal direction towards the axis of the cylinder while the force of friction (f) acts vertically upwards. The required centripetal force will be provided by the horizontal reaction N of the wall on the man, *i.e.,*

$$N = \frac{mv^2}{R} = m\omega^2 R$$

The frictional force f acting vertically upwards will be balanced by the weight of the man. Hence, the man remains stuck to the wall after the floor is removed if $mg \le$ limiting frictional force f_e (or μN)

or if $mg \le \mu\, m\omega^2 R$

or $g \le \mu\, \omega^2 R$

or $\mu\, \omega^2 R \ge g$ or $\omega \ge \frac{g}{R\mu}$

Hence, for minimum rotational speed of the cylinder

$$\omega^2 = \frac{g}{\mu R} = \frac{10}{0.15 \times 3} = 22.2$$

$\Rightarrow$ $\omega = \sqrt{22.2} = 4.7$ rad/s.

Question 5. 40.

A thin circular loop of radius R rotates about its vertical diameter with an angular frequency ω. Show that a small bead on the wire loop remains at its lowermost point for $\omega \le \sqrt{g/R}$. What is the angle made by the radius vector joining the centre to the bead with the vertical downward direction for $\omega = \sqrt{2g/R}$? Neglect friction.

Answer:

Let the radius vector joining the bead to the centre of the wire make an angle θ with the vertical downward direction. If N is normal reaction, then from fig,

$$mg = N\cos\theta \quad \text{...(i)}$$

$$mr\omega^2 = N\sin\theta \quad \text{...(ii)}$$

or $m(R\sin\theta)\,\omega^2 = N\sin\theta$

or $mR\omega^2 = N$

From equation (*i*), $mg = mR\omega^2\cos\theta$

or $$\cos\theta = \frac{g}{R\omega^2} \quad \text{...(iii)}$$

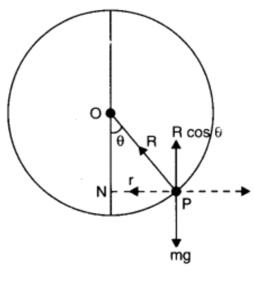
As $|\cos\theta| \le 1$, therefore bead will remain at its lowermost point for

$$\frac{g}{R\omega^2} \le 1 \quad \text{or} \quad \omega \le \sqrt{\frac{g}{R}}$$

When $\omega = \sqrt{\frac{2g}{R}}$ from equation (*iii*),

$$\cos\theta = \frac{g}{R}\left(\frac{R}{2g}\right) = \frac{1}{2}$$

$\therefore$ $\theta = 60°$.



*********** END **********