$$I' = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

Where,

ϕ = Phase difference between the two waves

For monochromatic light waves,

$$I_1 = I_2$$

$$\therefore I' = I_1 + I_1 + 2\sqrt{I_1 I_1}\cos\phi$$
$$= 2I_1 + 2I_1\cos\phi$$

Phase difference = $\frac{2\pi}{\lambda} \times \text{Path difference}$

Since path difference = λ,

Phase difference, $\phi = 2\pi$

$$\therefore I' = 2I_1 + 2I_1 = 4I_1$$

Given,

I' = K

$$\therefore I_1 = \frac{K}{4} \quad \dots (1)$$

When path difference $= \frac{\lambda}{3}$,

Phase difference, $\phi = \frac{2\pi}{3}$

Hence, resultant intensity, $I'_R = I_1 + I_1 + 2\sqrt{I_1 I_1}\cos\frac{2\pi}{3}$

$$= 2I_1 + 2I_1\left(-\frac{1}{2}\right) = I_1$$

Using equation (1), we can write:

$$I_R = I_1 = \frac{K}{4}$$

Hence, the intensity of light at a point where the path difference is $\frac{\lambda}{3}$ is $\frac{K}{4}$ units.

Question 10.6:

A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes in a Young's double-slit experiment.

(a) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 650 nm.

(b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

Answer

Wavelength of the light beam, $\lambda_1 = 650 \text{ nm}$

Wavelength of another light beam, $\lambda_2 = 520 \text{ nm}$

Distance of the slits from the screen = *D*

Distance between the two slits = *d*

(a) Distance of the n^{th} bright fringe on the screen from the central maximum is given by the relation,

$$x = n\lambda_1\left(\frac{D}{d}\right)$$

For third bright fringe, $n = 3$

$$\therefore x = 3\times 650\frac{D}{d} = 1950\left(\frac{D}{d}\right) \text{ nm}$$

(b) Let the n^{th} bright fringe due to wavelength λ_2 and $(n - 1)^{th}$ bright fringe due to wavelength λ_1 coincide on the screen. We can equate the conditions for bright fringes as:

$$n\lambda_2 = (n-1)\lambda_1$$
$$520n = 650n - 650$$
$$650 = 130n$$
$$\therefore n = 5$$

Hence, the least distance from the central maximum can be obtained by the relation:

$$x = n\lambda_2\frac{D}{d}$$
$$= 5\times 520\frac{D}{d} = 2600\frac{D}{d} \text{ nm}$$

Note: The value of *d* and *D* are not given in the question.

Question 10.7:

In a double-slit experiment the angular width of a fringe is found to be 0.2° on a screen placed 1 m away. The wavelength of light used is 600 nm. What will be the angular width of the fringe if the entire experimental apparatus is immersed in water? Take refractive index of water to be 4/3.

Answer

Distance of the screen from the slits, *D* = 1 m

Wavelength of light used, $\lambda_1 = 600 \text{ nm}$

Angular width of the fringe in $\text{air, } \theta_1 = 0.2°$

Angular width of the fringe in water = θ_2

Refractive index of water, $\mu = \frac{4}{3}$

Refractive index is related to angular width as:

$$\mu = \frac{\theta_1}{\theta_2}$$
$$\theta_2 = \frac{3}{4}\theta_1$$
$$= \frac{3}{4}\times 0.2 = 0.15$$

Therefore, the angular width of the fringe in water will reduce to 0.15°.

uestion 10.8:

What is the Brewster angle for air to glass transition? (Refractive index of glass = 1.5.)

Answer

Refractive index of glass, $\mu = 1.5$

Brewster angle = θ

Brewster angle is related to refractive index as:

$$\tan\theta = \mu$$
$$\theta = \tan^{-1}(1.5) = 56.31°$$

Therefore, the Brewster angle for air to glass transition is 56.31°.

Question 10.9:

Light of wavelength 5000 Å falls on a plane reflecting surface. What are the wavelength and frequency of the reflected light? For what angle of incidence is the reflected ray normal to the incident ray?

Answer

Wavelength of incident light, λ = 5000 Å = 5000×10^{-10} m

Speed of light, $c = 3 \times 10^8$ m

Frequency of incident light is given by the relation,

$$\nu = \frac{c}{\lambda}$$

$$= \frac{3\times10^8}{5000\times10^{-10}} = 6\times10^{14}\ \text{Hz}$$

The wavelength and frequency of incident light is the same as that of reflected ray. Hence, the wavelength of reflected light is 5000 Å and its frequency is 6×10^{14} Hz.

When reflected ray is normal to incident ray, the sum of the angle of incidence, $\angle i$ and angle of reflection, $\angle r$ is 90°.

According to the law of reflection, the angle of incidence is always equal to the angle of reflection. Hence, we can write the sum as:

$$\angle i + \angle r = 90$$

$$\angle i + \angle i = 90$$

$$\angle i = \frac{90}{2} = 45°$$

Therefore, the angle of incidence for the given condition is 45°.

Question 10.10:

Estimate the distance for which ray optics is good approximation for an aperture of 4 mm and wavelength 400 nm.

Answer

Fresnel's distance (Z_F) is the distance for which the ray optics is a good approximation. It is given by the relation,

*********** END **********