EXERCISE 5.2

Question 1:

Find the modulus and the argument of the complex number $z=-1-i\sqrt{3}$

Ans:

$z=-1-i\sqrt{3}$

Let $r\cos\theta=-1$ and $r\sin\theta=-\sqrt{3}$

On squaring and adding, we obtain

$$(r\cos\theta)^2+(r\sin\theta)^2=(-1)^2+(-\sqrt{3})^2$$
$$\Rightarrow r^2(\cos^2\theta+\sin^2\theta)=1+3$$
$$\Rightarrow r^2=4 \qquad [\cos^2\theta+\sin^2\theta=1]$$
$$\Rightarrow r=\sqrt{4}=2 \qquad [\text{Conventionally, } r>0]$$
$$\therefore \text{Modulus}=2$$
$$\therefore\ 2\cos\theta=-1 \text{ and } 2\sin\theta=-\sqrt{3}$$
$$\Rightarrow \cos\theta=\frac{-1}{2} \text{ and } \sin\theta=\frac{-\sqrt{3}}{2}$$

Since both the values of sin θ and cos θ are negative and sinθ and cosθ are negative in III quadrant,

$$\text{Argument}=-\left(\pi-\frac{\pi}{3}\right)=\frac{-2\pi}{3}$$

Thus, the modulus and argument of the complex number $-1-\sqrt{3}i$ are 2 and $\frac{-2\pi}{3}$ respectively.

Question 2:

Find the modulus and the argument of the complex number $z=-\sqrt{3}+i$

Ans:

$z=-\sqrt{3}+i$

Let $r\cos\theta=-\sqrt{3}$ and $r\sin\theta=1$

On squaring and adding, we obtain

$r^2\cos^2\theta+r^2\sin^2\theta=\left(-\sqrt{3}\right)^2+1^2$

$\Rightarrow r^2=3+1=4 \qquad \left[\cos^2\theta+\sin^2\theta=1\right]$

$\Rightarrow r=\sqrt{4}=2 \qquad$ [Conventionally, $r>0$]

$\therefore$ Modulus $=2$

$\therefore\ 2\cos\theta=-\sqrt{3}$ and $2\sin\theta=1$

$\Rightarrow \cos\theta=\frac{-\sqrt{3}}{2}$ and $\sin\theta=\frac{1}{2}$

$\therefore \theta=\pi-\frac{\pi}{6}=\frac{5\pi}{6} \qquad$ [As θ lies in the II quadrant]

Thus, the modulus and argument of the complex number $-\sqrt{3}+i$ are 2 and $\frac{5\pi}{6}$ respectively.

Question 3:

Convert the given complex number in polar form: 1 − *i*

Ans:

$1-i$

Let $r\cos\theta=1$ and $r\sin\theta=-1$

On squaring and adding, we obtain

$r^2\cos^2\theta+r^2\sin^2\theta=1^2+(-1)^2$

$\Rightarrow r^2\left(\cos^2\theta+\sin^2\theta\right)=1+1$

$\Rightarrow r^2=2$

$\Rightarrow r=\sqrt{2} \qquad$ [Conventionally, $r>0$]

$\therefore \sqrt{2}\cos\theta=1$ and $\sqrt{2}\sin\theta=-1$

$\Rightarrow \cos\theta=\frac{1}{\sqrt{2}}$ and $\sin\theta=-\frac{1}{\sqrt{2}}$

$\therefore \theta=-\frac{\pi}{4} \qquad$ [As θ lies in the IV quadrant]

$$\therefore 1-i=r\cos\theta+ir\sin\theta=\sqrt{2}\cos\left(-\frac{\pi}{4}\right)+i\sqrt{2}\sin\left(-\frac{\pi}{4}\right)=\sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right]$$

This is the required polar form.

Question 4:

Convert the given complex number in polar form: − 1 + *i*

Ans:

$-1+i$

Let $r\cos\theta=-1$ and $r\sin\theta=1$

On squaring and adding, we obtain

$$r^2\cos^2\theta+r^2\sin^2\theta=(-1)^2+1^2$$
$$\Rightarrow r^2\left(\cos^2\theta+\sin^2\theta\right)=1+1$$
$$\Rightarrow r^2=2$$
$$\Rightarrow r=\sqrt{2} \qquad [\text{Conventionally, } r>0]$$
$$\therefore \sqrt{2}\cos\theta=-1 \text{ and } \sqrt{2}\sin\theta=1$$
$$\Rightarrow \cos\theta=-\frac{1}{\sqrt{2}} \text{ and } \sin\theta=\frac{1}{\sqrt{2}}$$
$$\therefore \theta=\pi-\frac{\pi}{4}=\frac{3\pi}{4} \qquad [\text{As } \theta \text{ lies in the II quadrant}]$$

It can be written,

$$\therefore -1+i=r\cos\theta+ir\sin\theta=\sqrt{2}\cos\frac{3\pi}{4}+i\sqrt{2}\sin\frac{3\pi}{4}=\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$$

This is the required polar form.

Question 5:

Convert the given complex number in polar form: $-1-i$

Ans:

$-1-i$

Let $r\cos\theta=-1$ and $r\sin\theta=-1$

On squaring and adding, we obtain

$$r^2\cos^2\theta+r^2\sin^2\theta=(-1)^2+(-1)^2$$
$$\Rightarrow r^2\left(\cos^2\theta+\sin^2\theta\right)=1+1$$
$$\Rightarrow r^2=2$$
$$\Rightarrow r=\sqrt{2} \qquad [\text{Conventionally, } r>0]$$
$$\therefore \sqrt{2}\cos\theta=-1 \text{ and } \sqrt{2}\sin\theta=-1$$
$$\Rightarrow \cos\theta=-\frac{1}{\sqrt{2}} \text{ and } \sin\theta=-\frac{1}{\sqrt{2}}$$
$$\therefore \theta=-\left(\pi-\frac{\pi}{4}\right)=-\frac{3\pi}{4} \qquad [\text{As } \theta \text{ lies in the III quadrant}]$$

$$\therefore -1-i=r\cos\theta+ir\sin\theta=\sqrt{2}\cos\frac{-3\pi}{4}+i\sqrt{2}\sin\frac{-3\pi}{4}=\sqrt{2}\left(\cos\frac{-3\pi}{4}+i\sin\frac{-3\pi}{4}\right)$$

This is the required polar form.

Question 6:

Convert the given complex number in polar form: -3

Ans:

-3

Let $r \cos \theta = -3$ and $r \sin \theta = 0$

On squaring and adding, we obtain

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = (-3)^2$$
$$\Rightarrow r^2 \left(\cos^2 \theta + \sin^2 \theta\right) = 9$$
$$\Rightarrow r^2 = 9$$
$$\Rightarrow r = \sqrt{9} = 3 \qquad [\text{Conventionally, } r > 0]$$
$$\therefore 3\cos\theta = -3 \text{ and } 3\sin\theta = 0$$
$$\Rightarrow \cos\theta = -1 \text{ and } \sin\theta = 0$$
$$\therefore \theta = \pi$$

$$\therefore -3 = r\cos\theta + i r \sin\theta = 3\cos\pi + i3\sin\pi = 3(\cos\pi + i\sin\pi)$$

This is the required polar form.

Question 7:

Convert the given complex number in polar form: $\sqrt{3} + i$

Ans:

$\sqrt{3} + i$

Let $r \cos \theta = \sqrt{3}$ and $r \sin \theta = 1$

On squaring and adding, we obtain

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = \left(\sqrt{3}\right)^2 + 1^2$$
$$\Rightarrow r^2 \left(\cos^2 \theta + \sin^2 \theta\right) = 3 + 1$$
$$\Rightarrow r^2 = 4$$
$$\Rightarrow r = \sqrt{4} = 2 \qquad [\text{Conventionally, } r > 0]$$
$$\therefore 2\cos\theta = \sqrt{3} \text{ and } 2\sin\theta = 1$$
$$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \text{ and } \sin\theta = \frac{1}{2}$$
$$\therefore \theta = \frac{\pi}{6} \qquad [\text{As } \theta \text{ lies in the I quadrant}]$$

$$\therefore \sqrt{3} + i = r\cos\theta + i r\sin\theta = 2\cos\frac{\pi}{6} + i\,2\sin\frac{\pi}{6} = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

This is the required polar form.

Question 8:

Convert the given complex number in polar form: i

Ans:

i

Let $r\cos\theta = 0$ and $r\sin\,\theta = 1$

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = 0^2 + 1^2$$
$$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 1$$
$$\Rightarrow r^2 = 1$$
$$\Rightarrow r = \sqrt{1} = 1 \qquad \left[\text{Conventionally, } r > 0\right]$$
$$\therefore \cos\theta = 0 \text{ and } \sin\theta = 1$$
$$\therefore \theta = \frac{\pi}{2}$$

$$\therefore i = r\cos\theta + i\,r\sin\theta = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$$

This is the required polar form.

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