Definite Integrals Ex 20.2 Q44

Let $x^{\frac{2}{3}} = t$

Differentiating w.r.t. x, we get

$$\frac{3}{2}\sqrt{x}dx = dt$$

Now, $x = 0 \Rightarrow t = 0$

$$x = \pi^{\frac{2}{3}} \Rightarrow t = \pi$$

$$\therefore \int_0^{\pi^{\frac{2}{3}}} \sqrt{x} \cos^2 x^{\frac{3}{2}} dx$$

$$= \frac{2}{3}\int_0^{\pi} \cos^2 t \, dt$$

$$= \frac{1}{3}\int_0^{\pi} 1 + \cos 2t \, dt \qquad \left[\because 2\cos^2 t = t + \cos 2t\right]$$

$$= \frac{1}{3}\left[t + \frac{\sin 2t}{t}\right]_0^{\pi}$$

$$= \frac{1}{3}\left[\pi + 0 - 0 - 0\right] = \frac{\pi}{3}$$

$$\therefore \int_0^{\pi^{\frac{2}{3}}} \sqrt{x} \cos^2 x^{\frac{3}{2}} dx = \frac{\pi}{3}$$

Definite Integrals Ex 20.2 Q45

Let $1+\log x = t$

Differentiating w.r.t. x, we get

$$\frac{1}{x}dx = dt$$

When $x = 1 \Rightarrow t = 1$

$x = 2 \Rightarrow t = 1+\log 2$

$$\therefore \int_1^2 \frac{dx}{x\left(1+\log x\right)^2}$$

$$= \int_1^{1+\log 2} \frac{dt}{t^2}$$

$$= \left[-\frac{1}{t}\right]_1^{1+\log 2}$$

$$= 1 - \frac{1}{1+\log 2}$$

$$= \frac{\log 2}{1+\log 2}$$

$$\therefore \int_1^2 \frac{dx}{x\left(1+\log x\right)^2} = \frac{\log 2}{1+\log 2}$$

Definite Integrals Ex 20.2 Q46

We have,

$$\int_0^{\frac{\pi}{2}} \cos^5 x\, dx = \int_0^{\frac{\pi}{2}} \left(1 - \sin^2 x\right)^2 \cos x\, dx$$

Let $\sin x = t$

Differentiating w.r.t. x, we get

$\cos x dx = dt$

When $x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\int_0^{\frac{\pi}{2}} \left(1 - \sin^2 x\right)^2 \cos x\, dx$$

$$= \int_0^1 \left(1 - t^2\right)^2 dt$$

$$= \int_0^1 \left(1 - 2t^2 + t^4\right) dt$$

$$= \left[t - \frac{2}{3}t^3 + \frac{t^5}{5}\right]_0^1$$

$$= 1 - \frac{2}{3} + \frac{1}{5}$$

$$= \frac{8}{15}$$

$$\therefore \int_0^{\frac{\pi}{2}} \cos^5 x\, dx = \frac{8}{15}$$

Definite Integrals Ex 20.2 Q47

Let $I = \int \frac{\sqrt{x}}{\left(30 - x^{\frac{3}{2}}\right)}$ dx. We first find the anti derivative of the integrand.

Pu $30 - x^{\frac{3}{2}} = t$. Then $-\frac{3}{2}\sqrt{x}$ dx = dt or $\sqrt{x}$ dx = $-\frac{2}{3}$ dt

Thus, $\int \frac{\sqrt{x}}{\left(30 - x^{\frac{3}{2}}\right)^2} dx = -\frac{2}{3}\int \frac{dt}{t^2} = \frac{2}{3}\left[\frac{1}{t}\right] = \frac{2}{3}\left[\frac{1}{\left(30 - x^{\frac{3}{2}}\right)}\right] = f(x)$

Therefore, by the second fundamental theorem of calculus, we have

$$I = F(9) - F(4) = \frac{2}{3}\left[\frac{1}{\left(30 - x^{\frac{3}{2}}\right)}\right]_4^9$$

$$= \frac{2}{3}\left[\frac{1}{(30 - 27)} - \frac{1}{30 - 8}\right] = \frac{2}{3}\left[\frac{1}{3} - \frac{1}{22}\right] = \frac{19}{99}$$

*********** END **********