Differentiation Ex 11.5 Q1

Let $\quad y = x^{\frac{1}{x}} \qquad$ ---(i)

Taking log on both the sides,

$$\Rightarrow \quad \log y = \log x^{\frac{1}{x}}$$

$$\Rightarrow \quad \log y = \frac{1}{x}\log x \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\frac{d}{dx}(\log x) + \log x\frac{d}{dx}\left(x^{-1}\right) \qquad [\text{Using product rule}]$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \frac{1}{x} \times \frac{1}{x} + (\log x) \times \left(-\frac{1}{x^2}\right)$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2} - \frac{\log x}{x^2}$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \frac{(1-\log x)}{x^2}$$

$$\Rightarrow \quad \frac{dy}{dx} = y\left[\frac{1-\log x}{x^2}\right]$$

Put the value of y from equation (i),

$$\frac{dy}{dx} = x^{\frac{1}{x}}\left[\frac{1-\log x}{x}\right]$$

Differentiation Ex 11.5 Q2

Let $\quad y = x^{\sin x} \qquad$ ---(i)

Taking log on both the sides,

$$\log y = \log x^{\sin x}$$

$$\log y = \sin x \log x \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating with respect to x,

$$\frac{1}{y}\frac{dy}{dx} = \sin x\frac{d}{dx}\log x + \log x\frac{d}{dx}\sin x \qquad [\text{Using product rule}]$$

$$\frac{1}{y}\frac{dy}{dx} = \sin x\left(\frac{1}{x}\right) + \log x(\cos x)$$

$$\frac{dy}{dx} = y\left[\frac{\sin x}{x} + (\log x)(\cos x)\right]$$

Put the value of y,

$$\frac{dy}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + (\log x)(\cos x)\right]$$

Differentiation Ex 11.5 Q3

Let $\quad y = (1+\cos x)^x \qquad$ ---(i)

Taking log on both the sides,

$$\log y = \log(1+\cos x)^x$$
$$\log y = x\log(1+\cos x)$$

Differentiating with respect to x,

$$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}\log(1+\cos x) + \log(1+\cos x)\frac{d}{dx}(x) \qquad \text{[Using product rule and chain rule]}$$
$$\frac{1}{y}\frac{dy}{dx} = x\frac{1}{(1+\cos x)}\frac{d}{dx}(1+\cos x) + \log(1+\cos x)(1)$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{x}{(1+\cos x)}(0-\sin x) + \log(1+\cos x)$$
$$\frac{1}{y}\frac{dy}{dx} = \log(1+\cos x) - \frac{xsomx}{(1+\cos x)}$$
$$\frac{dy}{dx} = y\left[\log(1+\cos x) - \frac{x\sin x}{1+\cos x}\right]$$
$$\frac{dy}{dx} = (1+\cos x)^x\left[\log(1+\cos x) - \frac{x\sin x}{(1+\cos x)}\right] \qquad \text{[Using equation (i)]}$$

Differentiation Ex 11.5 Q4

Let $\quad y = x^{\cos^{-1}x} \qquad$ ---(i)

Taking log on both the sides,

$$\log y = \log x^{\cos^{-1}x}$$
$$\log y = \cos^{-1}x\log x \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating it with respect to x using product rule,

$$\frac{1}{y}\frac{dy}{dx} = \cos^{-1}x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}\left(\cos^{-1}x\right)$$
$$= \cos^{-1}x\left(\frac{1}{x}\right) + \log x\left(\frac{-1}{\sqrt{1-x^2}}\right)$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{\cos^{-1}x}{x} - \frac{\log x}{\sqrt{1-x^2}}$$
$$\frac{dy}{dx} = y\left[\frac{\cos^{-1}x}{x} - \frac{\log x}{\sqrt{1-x^2}}\right]$$
$$\frac{dy}{dx} = x^{\cos^{-1}x}\left[\frac{\cos^{-1}x}{x} - \frac{\log x}{\sqrt{1-x^2}}\right] \qquad \text{[Using equation (i)]}$$

Differentiation Ex 11.5 Q5

Let $\quad y = (\log x)^x \qquad$ ---(i)

Taking log on both the sides,

$$\log y = \log(\log x)^x$$
$$\log y = x\log(\log x) \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating with respect to x, using product rule, chain rule,

$$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}\log(\log x) + \log\log x\frac{d}{dx}(x)$$
$$= x\frac{1}{\log x}\frac{d}{dx}(\log x) + \log\log x\,(1)$$
$$= \frac{x}{\log x}\left(\frac{1}{x}\right) + \log\log x$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{\log x} + \log\log x$$
$$\frac{dy}{dx} = y\left[\frac{1}{\log x} + \log\log x\right]$$
$$\frac{dy}{dx} = (\log x)^x\left[\frac{1}{\log x} + \log\log x\right] \qquad \text{[Using equation (i)]}$$

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