Exercise 7A

Question 15

(i) To prove $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

We know, $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

put $a = \sin^2\theta, b = \cos^2\theta$

$\therefore \sin^6\theta + \cos^6\theta = \left(\sin^2\theta + \cos^2\theta\right)^3 - 3\sin^2\theta\cos^2\theta \times \left(\sin^2\theta + \cos^2\theta\right)$

$= 1 - 3\sin^2\theta\cos^2\theta = RHS$

$\therefore$ LHS = RHS

(ii) $LHS = \sin^2\theta + \cos^4\theta = 1 - \cos^2\theta + \cos^4\theta$

$= 1 - \cos^2\theta\left(1 - \cos^2\theta\right)$

$= 1 - \cos^2\theta\sin^2\theta$

$RHS = \cos^2\theta + \sin^4\theta = 1 - \sin^2\theta + \sin^4\theta$

$= 1 - \sin^2\theta\left(1 - \sin^2\theta\right) = \left(1 - \sin^2\theta\cos^2\theta\right)$

$\therefore$ LHS = RHS

(iii) $\text{cosec}^4\theta - \text{cosec}^2\theta$

$LHS = \text{cosec}^2\theta\left(\text{cosec}^2\theta - 1\right)$

$= \left(1 + \cot^2\theta\right)\cot^2\theta$

$= \cot^2\theta + \cot^4\theta = RHS$

$\therefore$ LHS = RHS

Question 16

(i) $\text{RHS} = (\sec\theta - \tan\theta)^2 = \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$

$= \frac{(1-\sin\theta)^2}{\cos^2\theta}$

$= \frac{(1-\sin\theta)^2}{1-\sin^2\theta}$

$= \frac{(1-\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}$

$= \frac{1-\sin\theta}{1+\sin\theta} = \text{LHS}$

$\therefore$ LHS = RHS

(ii) $\text{RHS} = (\text{cosec}\theta + \cot\theta)^2$

$= \left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)^2$

$= \frac{(1+\cos\theta)^2}{\sin^2\theta}$

$= \frac{(1+\cos\theta)^2}{1-\cos^2\theta}$

$= \frac{(1+\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)}$

$= \frac{1+\cos\theta}{1-\cos\theta} = \text{LHS}$

$\therefore$ LHS = RHS

********** END **********