

Indefinite Integrals Ex 19.23 Q9

Let 
$$I = \int \frac{1}{\sin x + \cos x} dx$$
Put 
$$\sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$I = \int \frac{1}{\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}} + \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$= \int \frac{1 + \tan^2 \frac{x}{2}}{2 \tan \frac{x}{2} + 1 - \tan^2 \frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{2 \tan \frac{x}{2} + 1 - \tan^2 \frac{x}{2}} dx$$
Let 
$$\tan \frac{x}{2} = t$$

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

$$I = \int \frac{2dt}{2t + 1 - t^2}$$

$$= -2\int \frac{dt}{t^2 - 2t - 1}$$

$$I = -2\int \frac{dt}{(t - 1)^2 - (\sqrt{2})^2}$$

$$= 2\int \frac{2dt}{(\sqrt{2})^2 - (t - 1)^2}$$

$$= \frac{2}{2\sqrt{2}} \log \left| \frac{\sqrt{2} + t - 1}{\sqrt{2} - t + 1} \right| + c$$

$$I = \frac{1}{\sqrt{2}} \log \left| \frac{\sqrt{2} + \tan \frac{x}{2} - 1}{\sqrt{2} - \tan \frac{x}{2} + 1} \right| + c$$

Indefinite Integrals Ex 19.23 Q10

Let 
$$I = \int \frac{1}{5 - 4\cos x} dx$$
  
Put  $\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$   
 $I = \int \frac{1}{5 - 4\left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right)} dx$   
 $= \int \frac{\left(1 + \tan^2 \frac{x}{2}\right)}{5 + 5\tan^2 \frac{x}{2} - 4 + 4\tan^2 \frac{x}{2}} dx$   
 $= \int \frac{\sec^2 \frac{x}{2}}{9\tan^2 \frac{x}{2} + 1} dx$   
Let  $3\tan \frac{x}{2} = t$   
 $\frac{3}{2}\sec^2 \frac{x}{2} dx = dt$   
 $I = \frac{2}{3}\int \frac{dt}{t^2 + 1}$   
 $= \frac{2}{3}\tan^{-1}(t) + c$   
 $I = \frac{2}{3}\tan^{-1}\left(3\tan \frac{x}{2}\right) + c$ 

Indefinite Integrals Ex 19.23 Q11

Let 
$$I = \int \frac{1}{2 + \sin x + \cos x} dx$$
  
Put  $\sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$   
 $I = \int \frac{1}{2 + \left(\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right) + \left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right)} dx$   
 $= \int \frac{1}{2 + 2 \tan^2 \frac{x}{2} + 2 \tan \frac{x}{2} + 1 - \tan^2 \frac{x}{2}} dx$   
 $= \int \frac{\sec^2 \frac{x}{2}}{\tan^2 \frac{x}{2} + 2 \tan \frac{x}{2} + 3} dx$   
Let  $\tan \frac{x}{2} = t$   
 $\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$   
 $I = \int \frac{2dt}{t^2 + 2t + 1 - 1 + 3}$   
 $I = 2\int \frac{dt}{(t + 1)^2 + (\sqrt{2})^2}$   
 $= \frac{2}{\sqrt{2}} \tan^{-1} \left(\frac{t + 1}{\sqrt{2}}\right) + c$   
 $I = \sqrt{2} \tan^{-1} \left(\frac{\tan \frac{x}{2} + 1}{\sqrt{2}}\right) + c$ 

Indefinite Integrals Ex 19.23 Q12

Let 
$$I = \int \frac{1}{\sin x + \sqrt{3} \cos x} dx$$

Put  $\sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$ 

$$I = \int \frac{1}{\left(\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right) + \sqrt{3} \left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right)}$$

$$= \int \frac{1}{2 \tan \frac{x}{2} + \sqrt{3} - \sqrt{3} \tan^2 \frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{2 \tan \frac{x}{2} + \sqrt{3} - \sqrt{3} \tan^2 \frac{x}{2}} dx$$

Let  $\tan \frac{x}{2} = t$ 

$$\frac{1}{2} \sec^2 \frac{x}{2} dx = dt$$

$$I = \int \frac{2dt}{2t + \sqrt{3} - \sqrt{3}t^2}$$

$$= -\frac{2}{\sqrt{3}} \int \frac{dt}{t^2 - \frac{2}{\sqrt{3}}t + \left(\frac{1}{\sqrt{3}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2 - 1}$$

$$I = -\frac{2}{\sqrt{3}} \int \frac{dt}{\left(t - \frac{1}{\sqrt{3}}\right)^2 - \left(\frac{2}{\sqrt{3}}\right)^2}$$

$$= \frac{2}{\sqrt{3}} \times \frac{1}{2\left(\frac{2}{\sqrt{3}}\right)} \log \frac{\frac{2}{\sqrt{3}} + t + \frac{1}{\sqrt{3}}}{\frac{2}{\sqrt{3}} - t + \frac{1}{\sqrt{3}}} + c$$

$$I = \frac{1}{2} \log \frac{\sqrt{3}t + 1}{3 - \sqrt{3}t} + c$$

$$I = \frac{1}{2} \log \frac{1 + \sqrt{3} \tan \frac{x}{2}}{3 - \sqrt{3} \tan \frac{x}{2}} + c$$

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