Higher Order Derivatives Ex 12.1 Q11

$x = a\cos\theta$

differentiating w.r.t. θ

$$\Rightarrow \quad \frac{dy}{d\theta} = -a\sin\theta \quad .....(1)$$

$$\Rightarrow \quad \frac{dy}{d\theta} = b\cos\theta \quad .....(2)$$

Dividing (2) by (1)

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-b\cos\theta}{a\sin\theta} \quad ......(3)$$

differentiating (3) w.r.t. θ

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \frac{-b}{a}\left\{\frac{\sin\theta(-\sin\theta) - \cos\theta(\cos\theta)}{\sin^2\theta}\right\} = \frac{b}{a}\frac{\left(\sin^2\theta + \cos^2\theta\right)}{\sin^2\theta} = \frac{b}{a\sin^2\theta} \quad ......(4)$$

Dividing (4) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-b}{a^2\sin^3\theta} \times \frac{b^3}{b^3}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

Higher Order Derivatives Ex 12.1 Q12

$x = a\left(1 - \cos^3\theta\right); \quad y = a\sin^3\theta$

differentiating both w.r.t. θ

$$\Rightarrow \quad \frac{dx}{d\theta} = a\left(0 - 3\cos^2\theta(-\sin\theta)\right); \quad \frac{dy}{d\theta} = a\left(3\sin^2\theta \times \cos\theta\right) ......(2)$$

$$\Rightarrow \quad \frac{dy}{d\theta} = 3a\sin\theta\cos^2\theta; \quad \frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{3a\sin^2\theta\cos\theta}{3a\sin\theta\cos^2\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

Differentiating w.r.t. θ

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \sec^2\theta \quad ....(3)$$

Dividing (3) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{\sec^2\theta}{3a\sin\theta\cos^2\theta}$$

Putting $\theta = \pi/6$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{\frac{2}{\sqrt{3}} \times \frac{2}{\sqrt{3}}}{3a \times \frac{1}{2} \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}} = \frac{2^5}{3a \times \left(\sqrt{3}\right)^4} = \frac{32}{27a}$$

Higher Order Derivatives Ex 12.1 Q13

$x = a(\theta + \sin\theta)$; $y = a(1 + \cos\theta)$

differentiating both w.r.t.θ

$$\Rightarrow \quad \frac{dx}{d\theta} = a(1 + \cos\theta); \quad (1)$$

$$\Rightarrow \quad \frac{dy}{d\theta} = a(0 - \sin\theta) \quad (2)$$

Dividing (2) by (1)

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{-a\sin\theta}{a(1 + \cos\theta)}$$

Differentiating w.r.t.θ

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = -\left\{\frac{(1 + \cos\theta)(\cos\theta) - (\sin\theta)(0 - \sin\theta)}{(1 + \cos\theta)^2}\right\} = -\left\{\frac{\cos\theta + \cos^2\theta + \sin^2\theta}{(1 + \cos\theta)^2}\right\}$$

$$= -\left\{\frac{\cos\theta + 1}{(\cos + 1)^2}\right\}$$

$$= \frac{-1}{1 + \cos\theta} \quad \ldots\ldots(3)$$

dividing (3) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-1 \times a}{a(1 + \cos\theta)^2 \times a} = \frac{-a}{y^2}$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q14

$x = a(\theta - \sin\theta);\ y = a(1 + \cos\theta)$

Diiferentiating the above functions with respect to θ, *we get*,

$$\frac{dx}{d\theta} = a(1 - \cos\theta) \quad ...(1)$$

$$\frac{dy}{d\theta} = a(-\sin\theta) \quad ...(2)$$

Dividing equation (2) *by* (1), *we have*,

$$\frac{dy}{dx} = \frac{a(-\sin\theta)}{a(1 - \cos\theta)} = \frac{-\sin\theta}{1 - \cos\theta}$$

Differentiating with respect to θ, *we have*,

$$\frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \frac{(1 - \cos\theta)(-\cos\theta) + \sin\theta(\sin\theta)}{(1 - \cos\theta)^2}$$

$$= \frac{-\cos\theta + \cos^2\theta + \sin^2\theta}{(1 - \cos\theta)^2}$$

$$= \frac{1 - \cos\theta}{(1 - \cos\theta)^2}$$

$$\frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \frac{1}{1 - \cos\theta} \ ...(3)$$

Dividing equation (3) *by* (1), *we have*,

$$\frac{d^2y}{dx^2} = \frac{1}{1 - \cos\theta} \times \frac{1}{a(1 - \cos\theta)}$$

$$= \frac{1}{a(1 - \cos\theta)^2}$$

$$= \frac{1}{a\left(2\sin^2\frac{\theta}{2}\right)^2}$$

$$= \frac{1}{4a\sin^4\left(\frac{\theta}{2}\right)}$$

$$= \frac{1}{4a}\operatorname{cosec}^4\left(\frac{\theta}{2}\right)$$

*********** END **********