CHAPTER 30

GAUSS'S LAW

Gauss's law is one of the fundamental laws of physics. It relates the electric field to the charge distribution which has produced this field. In section (30.1) we define the flux of an electric field and in the next section we discuss the concept of a solid angle. These will be needed to state and understand Gauss's law.

30.1 FLUX OF AN ELECTRIC FIELD THROUGH A SURFACE

Consider a hypothetical plane surface of area ΔS and suppose a uniform electric field $\vec{E}$ exists in the space (figure 30.1). Draw a line perpendicular to the surface and call one side of it, the positive normal to the surface. Suppose, the electric field $\vec{E}$ makes an angle θ with the positive normal.

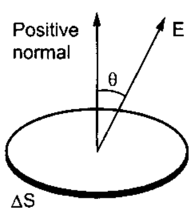


Figure 30.1

The quantity

$$\Delta\Phi = E\,\Delta S\cos\theta$$

is called *the flux of the electric field through the chosen surface*. If we draw a vector of magnitude ΔS along the positive normal, it is called the *area-vector* $\Delta\vec{S}$ corresponding to the area ΔS. One can then write

$$\Delta\Phi = \vec{E}\cdot\Delta\vec{S}.$$

Remember, the direction of the area-vector is always along the normal to the corresponding surface. If the field $\vec{E}$ is perpendicular to the surface, it is parallel to the area-vector. If $\vec{E}$ is along the positive normal, $\theta = 0$, and $\Delta\Phi = E\,\Delta S$. If it is opposite to the positive normal, $\theta = \pi$, and $\Delta\Phi = -E\,\Delta S$. If the electric field is parallel to the surface, $\theta = \pi/2$, and $\Delta\Phi = 0$.

Flux is a scalar quantity and may be added using the rules of scalar addition. Thus, if the surface ΔS has two parts ΔS_1 and ΔS_2, the flux through ΔS equals the flux through ΔS_1 plus the flux through ΔS_2. This gives us a clue to define the flux through surfaces which are not plane, as well as the flux when the field is not uniform. We divide the given surface into smaller parts so that each part is approximately plane and the variation of electric field over each part can be neglected. We calculate the flux through each part separately, using the relation $\Delta\Phi = \vec{E}\cdot\Delta\vec{S}$ and then add the flux through all the parts. Using the techniques of integration, the flux is

$$\Phi = \int \vec{E}\cdot d\vec{S}$$

where integration has to be performed over the entire surface through which the flux is required.

The surface under consideration may be a closed one, enclosing a volume, such as a spherical surface. A hemispherical surface is an open surface. A cylindrical surface is also open. A cylindrical surface plus two plane surfaces perpendicular to the axis enclose a volume and these three taken together form a closed surface. When flux through a closed surface is required, we use a small circular sign on the integration symbol;

$$\Phi = \oint \vec{E}\cdot d\vec{S}.$$

It is customary to take the outward normal as positive in this case.

Example 30.1

A square frame of edge 10 cm *is placed with its positive normal making an angle of* 60° *with a uniform electric field of* 20 V m^{-1}. *Find the flux of the electric field through the surface bounded by the frame.*

Solution : The surface considered is plane and the electric field is uniform (figure 30.2). Hence, the flux is

$$\Delta\Phi = \vec{E}\cdot\Delta\vec{S}$$
$$= E\,\Delta S\cos 60°$$

$$= (20 \text{ V m}^{-1}) (0{\cdot}01 \text{ m}^2) \left(\frac{1}{2}\right) = 0{\cdot}1 \text{ Vm}.$$

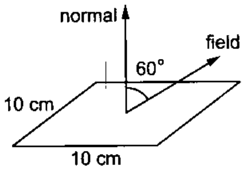


Figure 30.2

Example 30.2

A charge q is placed at the centre of a sphere. Taking outward normal as positive, find the flux of the electric field through the surface of the sphere due to the enclosed charge.

Solution :

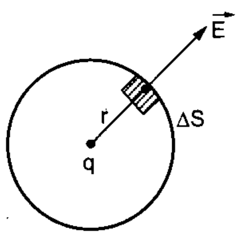


Figure 30.3

Let us take a small element ΔS on the surface of the sphere (Figure 30.3). The electric field here is radially outward and has the magnitude

$$\frac{q}{4\pi\varepsilon_0 r^2},$$

where r is the radius of the sphere. As the positive normal is also outward, $\theta = 0$ and the flux through this part is

$$\Delta\Phi = \vec{E} \cdot \Delta\vec{S} = \frac{q}{4\pi\varepsilon_0 r^2} \Delta S.$$

Summing over all the parts of the spherical surface,

$$\Phi = \sum \Delta\Phi = \frac{q}{4\pi\varepsilon_0 r^2} \sum \Delta S = \frac{q}{4\pi\varepsilon_0 r^2} 4\pi r^2 = \frac{q}{\varepsilon_0}.$$

Example 30.3

A uniform electric field exists in space. Find the flux of this field through a cylindrical surface with the axis parallel to the field.

Solution :

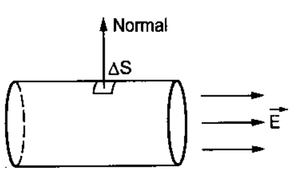


Figure 30.4

Consider figure (30.4) and take a small area ΔS on the cylindrical surface. The normal to this area will be perpendicular to the axis of the cylinder. But the electric field is parallel to the axis and hence

$$\Delta\Phi = \vec{E} \cdot \Delta\vec{S} = E \, \Delta S \cos(\pi/2) = 0.$$

This is true for each small part of the cylindrical surface. Summing over the entire surface, the total flux is zero.

30.2 SOLID ANGLE

Solid angle is a generalisation of the plane angle. In figure (30.5a) we show a plane curve AB. The end points A and B are joined to the point O. We say that the curve AB subtends an *angle* or a *plane angle* at O. An angle is formed at O by the two lines OA and OB passing through O.

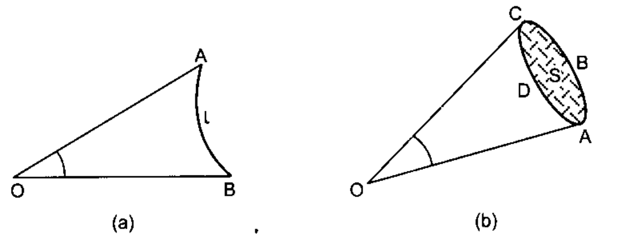


Figure 30.5

To construct a solid angle, we start with a surface S (figure 30.5b) and join all the points on the periphery such as A, B, C, D, etc., with the given point O. We then say that a *solid angle* is formed at O and that the surface S has subtended the solid angle. The solid angle is formed by the lines joining the points on the periphery with O. The whole figure looks like a cone. As a typical example, think of the paper containers used by Moongfaliwalas.

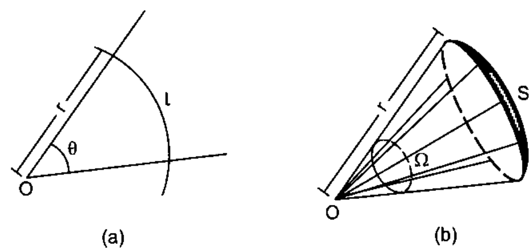


Figure 30.6

How do we measure a solid angle ? Let us consider how do we measure a plane angle. See figure (30.6a). We draw a circle of any radius r with the centre at O and measure the length l of the arc intercepted by the angle. The angle θ is then defined as $\theta = l/r$. In order to measure a solid angle at the point O (figure 30.6b), we draw a sphere of any radius r with O as the centre and measure the area S of the part of the sphere intercepted by the cone. The solid angle Ω is then defined as

$$\Omega = S/r^2.$$

Note that this definition makes the solid angle a dimensionless quantity. It is independent of the radius of the sphere drawn.

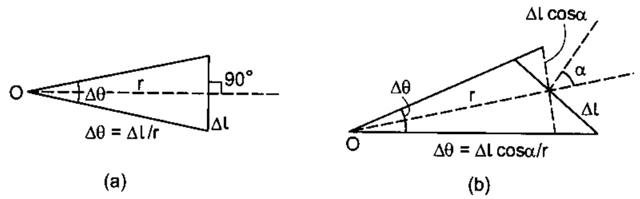


Figure 30.7

Next, consider a plane angle subtended at a point O by a small line segment Δl (figure 30.7a). Suppose, the line joining O to the middle point of Δl is perpendicular to Δl. As the segment is small, we can approximately write

$$\Delta\theta = \frac{\Delta l}{r} \cdot$$

As Δl gets smaller, the approximation becomes better. Now suppose, the line joining O to Δl is not perpendiclar to Δl (figure 30.7b). Suppose, this line makes an angle α with the perpendicular to Δl. The angle subtended by Δl at O is

$$\Delta\theta = \frac{\Delta l \cos\alpha}{r} \cdot$$

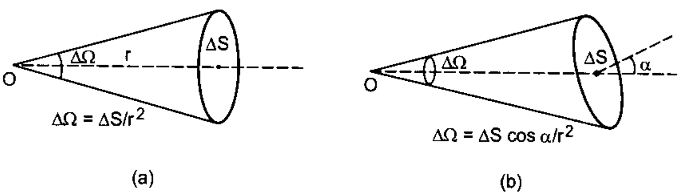


Figure 30.8

Similarly, if a small plane area ΔS (figure 30.8a) subtends a solid angle $\Delta\Omega$ at O in such a way that the line joining O to ΔS is normal to ΔS, we can write $\Delta\Omega = \Delta S/r^2$. But if the line joining O to ΔS makes an angle α with the normal to ΔS (figure 30.8b), we should write

$$\Delta\Omega = \frac{\Delta S \cos\alpha}{r^2} \cdot$$

A complete circle subtends an angle

$$\theta = \frac{l}{r} = \frac{2\pi r}{r} = 2\pi$$

at the centre. In fact, any closed curve subtends an angle 2π at any of the internal points. Similarly, a complete sphere subtends a solid angle

$$\Omega = \frac{S}{r^2} = \frac{4\pi r^2}{r^2} = 4\pi$$

at the centre. Also, any closed surface subtends a solid angle 4π at any internal point.

How much is the angle subtended by a closed plane curve at an external point ?

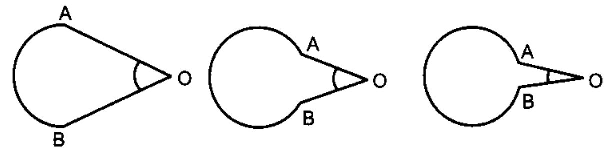


Figure 30.9

See figure (30.9). As we gradually close the curve, the angle finally diminishes to zero. A closed curve subtends zero angle at an external point. Similarly, a closed surface subtends zero solid angle at an external point.

30.3 GAUSS'S LAW AND ITS DERIVATION FROM COULOMB'S LAW

The statement of the Gauss's law may be written as follows:

The flux of the net electric field through a closed surface equals the net charge enclosed by the surface divided by ε_0. In symbols,

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\varepsilon_0} \qquad \text{... (30.1)}$$

where q_{in} is the net charge enclosed by the surface through which the flux is calculated.

It should be carefully noted that the electric field on the left-hand side of equation (30.1) is the resultant electric field due to all the charges existing in the space, whereas, the charge appearing on the right-hand side includes only those which are inside the closed surface.

Gauss's law is taken as a fundamental law of nature, a law whose validity is shown by experiments. However, historically Coulomb's law was discovered before Gauss's law and it is possible to derive Gauss's law from Coulomb's law.

Proof of Gauss's Law (Assuming Coulomb's Law)

Flux due to an internal charge

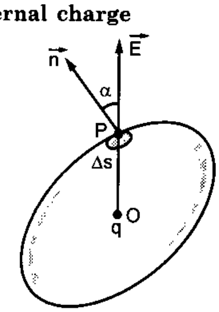


Figure 30.10

Suppose a charge q is placed at a point O inside a "closed" surface (figure 30.10). Take a point P on the surface and consider a small area ΔS on the surface around P. Let $OP = r$. The electric field at P due to the charge q is

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

along the line OP. Suppose this line OP makes an angle α with the outward normal to ΔS. The flux of the electric field through ΔS is

$$\Delta\Phi = \vec{E}\cdot\Delta\vec{S} = E\,\Delta S\cos\alpha$$

$$= \frac{q}{4\pi\varepsilon_0 r^2}\Delta S\cos\alpha$$

$$= \frac{q}{4\pi\varepsilon_0}\Delta\Omega$$

where $\Delta\Omega = \frac{\Delta S\cos\alpha}{r^2}$ is the solid angle subtended by ΔS at O. The flux through the entire surface is

$$\Phi = \Sigma\frac{q}{4\pi\varepsilon_0}\Delta\Omega = \frac{q}{4\pi\varepsilon_0}\Sigma\Delta\Omega.$$

The sum over $\Delta\Omega$ is the total solid angle subtended by the closed surface at the internal point O and hence is equal to 4π.

The total flux of the electric field due to the internal charge q through the closed surface is, therefore,

$$\Phi = \frac{q}{4\pi\varepsilon_0}4\pi = \frac{q}{\varepsilon_0}\cdot \quad \ldots \text{(i)}$$

Flux due to an external charge

Now, suppose a charge q is placed at a point O outside the closed surface. The flux of the electric field due to q through the small area ΔS is again

$$\Delta\Phi = \frac{q}{4\pi\varepsilon_0}\frac{\Delta S\cos\alpha}{r^2} = \frac{q}{4\pi\varepsilon_0}\Delta\Omega.$$

When we sum over all the small area elements of the closed surface we get $\Sigma\Delta\Omega = 0$ as this is the total solid angle subtended by the closed surface at an external point. Hence,

$$\Phi = 0. \quad \ldots \text{(ii)}$$

Flux due to a combination of charges

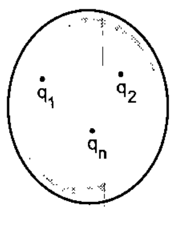


Figure 30.11

Finally, consider a general situation (figure 30.11) where charges $q_1, q_2, \ldots, q_n$ are inside a closed surface and charges $Q_1, Q_2, \ldots, Q_N$ are outside it. The resultant electric field at any point is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots + \vec{E}_n + \vec{E'}_1 + \vec{E'}_2 + \ldots + \vec{E'}_N$$

where E_i and E'_i are the fields due to q_i and Q_i respectively. Thus, the flux of the resultant electric field through the closed surface is

$$\Phi = \oint\vec{E}\cdot d\vec{S} = \oint\vec{E}_1\cdot d\vec{S} + \oint\vec{E}_2\cdot d\vec{S} + \ldots + \oint\vec{E}_n\cdot d\vec{S}$$
$$+ \oint\vec{E'}_1\cdot d\vec{S} + \oint\vec{E'}_2\cdot d\vec{S} + \ldots + \oint\vec{E'}_N\cdot d\vec{S}. \quad \ldots \text{(iii)}$$

Now, $\oint\vec{E}_i\cdot d\vec{S}$ is the flux of the electric field due to the charge q_i only. As this charge is inside the closed surface, from (i), it is equal to q_i/ε_0. Also, $\oint\vec{E'}_i\cdot d\vec{S}$ is the flux of the electric field due to the charge Q_i which is outside the closed surface. This flux is, therefore, zero from (ii). Using these results in (iii),

$$\Phi = \frac{q_1}{\varepsilon_0} + \frac{q_2}{\varepsilon_0} + \ldots + \frac{q_n}{\varepsilon_0} + 0 + \ldots + 0$$

or, $$\Phi = \frac{1}{\varepsilon_0}\Sigma q_i$$

or, $$\oint\vec{E}\cdot d\vec{S} = \frac{q_{in}}{\varepsilon_0}\cdot$$

This completes the derivation of Gauss's law (equation 30.1).

We once again emphasise that the electric field appearing in the Gauss's law is the resultant electric field due to all the charges present inside as well as outside the given closed surface. On the other hand, the charge q_{in} appearing in the law is only the charge contained within the closed surface. The contribution of the charges outside the closed surface in producing the flux is zero. A surface on which Gauss's law is applied, is sometimes called the *Gaussian surface.*

Example 30.4

A charge Q is distributed uniformly on a ring of radius r. A sphere of equal radius r is constructed with its centre at the periphery of the ring (figure 30.12). Find the flux of the electric field through the surface of the sphere.

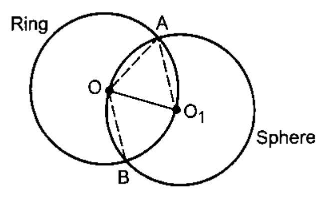


Figure 30.12

Solution : From the geometry of the figure, $OA = OO_1$ and $O_1A = O_1O$. Thus, OAO_1 is an equilateral triangle. Hence $\angle AOO_1 = 60°$ or $\angle AOB = 120°$.

The arc AO_1B of the ring subtends an angle 120° at the centre O. Thus, one third of the ring is inside the sphere.

The charge enclosed by the sphere $= \frac{Q}{3}$. From Gauss's law, the flux of the electric field through the surface of the sphere is $\frac{Q}{3\,\varepsilon_0}$.

30.4 APPLICATIONS OF GAUSS'S LAW

(A) Charged Conductor

As discussed earlier, an electric conductor has a large number of free electrons and when placed in an electric field, these electrons redistribute themselves to make the field zero at all the points inside the conductor.

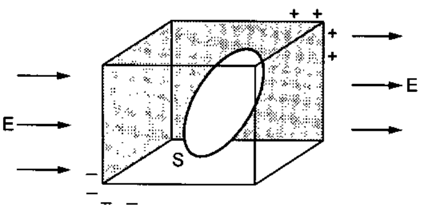


Figure 30.13

Consider a charged conductor placed in an electric field (figure 30.13). We assume that the redistribution of free electrons (if any) is complete. Draw a closed surface S going through the interior points only. As the electric field at all the points of this surface is zero (they are interior points), the flux $\oint \vec{E}\cdot d\vec{S}$ is also zero. But from Gauss's law, this equals the charge contained inside the surface divided by ε_0. Hence, the charge enclosed by the surface is zero. This shows that any volume completely inside a conductor is electrically neutral. If a charge is injected anywhere in the conductor, it must come over to the surface of the conductor so that the interior is always charge free. Also, if the conductor has a cavity, the charge must come over to the outer surface.

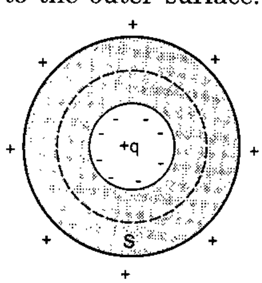


Figure 30.14

However, if a charge is placed within the cavity as in figure (30.14), the inner surface cannot be charge free. Taking the Gaussian surface S as shown, $\vec{E} = 0$ at all the points of this surface and hence $\oint \vec{E}\cdot d\vec{S} = 0$. This ensures that the charge contained in S is zero and if a charge $+q$ is placed in the cavity, there must be a charge $-q$ on the inner surface of the conductor. If the conductor is neutral, i.e., no charge is placed on it, a charge $+q$ will appear on the outer surface.

If there is a cavity in the conductor and no charge is placed in the cavity, the electric field at all the points in the cavity is zero. This can be proved using a little more advanced mathematics.

(B) Electric Field due to a Uniformly Charged Sphere

Suppose a total charge Q is uniformly distributed in a spherical volume of radius R and we are required to find the electric field at a point P which is at a distance r from the centre of the charge distribution.

Field at an outside point

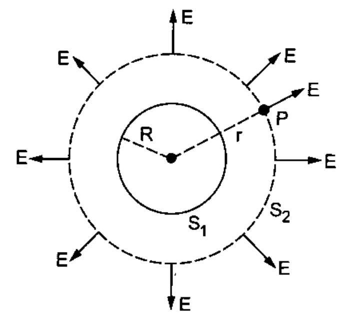


Figure 30.15

For a point P outside the charge distribution (figure 30.15), we have $r > R$. Draw a spherical surface passing through the point P and concentric with the charge distribution. Take this to be the Gaussian surface. The electric field is radial by symmetry and if Q is positive the field is outward. Also, its magnitude at all the points of the Gaussian surface must be equal. Let this magnitude be E. This is also the magnitude of the field at P. As the field $\vec{E}$ is normal to the surface element everywhere, $\vec{E}\cdot d\vec{S} = E\,dS$ for each element. The flux of the electric field through this closed surface is

$$\Phi = \oint \vec{E}\cdot d\vec{S}$$

$$= \oint E\,dS = E \oint dS = E\,4\pi r^2.$$

This should be equal to the charge contained inside the Gaussian surface divided by ε_0. As the entire charge Q is contained inside the Gaussian surface, we get

$$E\,4\pi r^2 = Q/\varepsilon_0$$

or, $$E = \frac{Q}{4\pi\varepsilon_0 r^2}. \qquad \dots (30.2)$$

The electric field due to a uniformly charged sphere at a point outside it, is identical with the field due to an equal point charge placed at the centre.

Notice the use of the argument of symmetry. All the points of the sphere through P are equivalent. No point on this surface has any special property which a different point does not have. That is why we could say that the field has the same magnitude E at all these points. Also, the field is radial at all the points. We have wisely chosen the Gaussian surface which has these properties. We could then easily evaluate the flux $\oint \vec{E} \cdot d\vec{S} = E\, 4\pi r^2$.

Field at an internal point

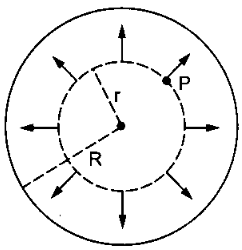


Figure 30.16

Suppose, we wish to find the electric field at a point P inside the spherical charge distribution (figure 30.16). We draw a spherical surface passing through P and concentric with the given charge distribution. The radius of this sphere will be r. All the points of this sphere are equivalent. By symmetry, the field is radial at all the points of this surface and has a constant magnitude E. The flux through this spherical surface is

$$\Phi = \oint \vec{E} \cdot d\vec{S}$$

$$= \oint E\, dS = E \oint dS = E\, 4\pi r^2. \quad \ldots \text{(i)}$$

Let us now calculate the total charge contained inside this spherical surface. As the charge is uniformly distributed within the given spherical volume, the charge per unit volume is $\frac{Q}{\frac{4}{3}\pi R^3}$. The volume enclosed by the Gaussian surface, through which the flux is calculated, is $\frac{4}{3}\pi r^3$. Hence, the charge enclosed is

$$\frac{Q}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi r^3 = \frac{Q r^3}{R^3}.$$

Using Gauss's law and (i),

$$E\, 4\pi r^2 = \frac{Qr^3}{\varepsilon_0 R^3}$$

or, $$E = \frac{Qr}{4\pi\varepsilon_0 R^3}. \quad \ldots (30.3)$$

The electric field due to a uniformly charged sphere at an internal point is $\frac{Qr}{4\pi\varepsilon_0 R^3}$ in radial direction.

At the centre, $r = 0$ and hence $E = 0$. This is clear from the symmetry arguments as well. At the centre, all directions are equivalent. If the electric field is not zero, what can be its direction ? You cannot choose a unique direction. The field has to be zero. It is proportional to the distance r from the centre for the internal points. Equations (30.2) and (30.3) give the same value of the field at the surface, where $r = R$.

(C) Electric Field due to a Linear

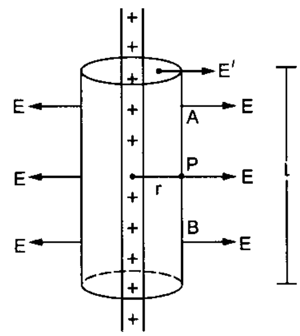


Figure 30.17

Charge Distribution

Consider a long line charge with a linear charge density (that is, charge per unit length) λ. We have to calculate the electric field at a point P which is at a distance r from the line charge (figure 30.17). What can be the direction of the electric field at P? Can it be along PA ? If yes, then why not along PB ? PA and PB are equivalent to each other. In fact, the only unique direction through P is along the perpendicular to the line charge. The electric field must be along this direction. If the charge is positive, the field will be outward.

Now, we construct a Gaussian surface. We draw a cylinder of length l passing through P and coaxial with the line charge. Let us close the cylinder with two plane surfaces perpendicular to the line charge. The curved surface of the cylinder together with the two plane parallel surfaces constitutes a closed surface as shown in figure (30.17). We use this surface as the Gaussian surface.

All the points on the curved part of this Gaussian surface are at the same perpendicular distance from the line charge. All these points are equivalent. The electric field at all these points will have the same magnitude E as that at P. Also, the direction of the field at any point on the curved surface is normal to the line and hence normal to the cylindrical surface element there. The flux through the curved part is, therefore,

$$\int \vec{E}\cdot d\vec{S} = \int E\, dS = E \int dS = E\, 2\pi r l.$$

Now, consider the flat parts of the Gaussian surface, that is, the lids of the cylinder. The electric field at any point is perpendicular to the line charge. The normal to any element on these plane surfaces is parallel to the line charge. Hence, the field and the area-vector make an angle of 90° with each other so that $\int \vec{E}\cdot d\vec{S} = 0$ on these parts. The total flux through the closed Gaussian surface is, therefore,

$$\oint \vec{E}\cdot d\vec{S} = E\, 2\pi r l. \qquad \ldots \text{ (i)}$$

The charge enclosed in the Gaussian surface is λl as a length l of the line charge is inside the closed surface. Using Gauss's law and (i),

$$E\, 2\pi r l = (\lambda l)/\varepsilon_0$$

or, $$E = \frac{\lambda}{2\pi\varepsilon_0 r}\cdot \qquad \ldots \text{ (30.4)}$$

This is the field at a distance r from the line. It is directed away from the line if the charge is positive and towards the line if the charge is negative.

(D) Electric Field due to a Plane Sheet of Charge

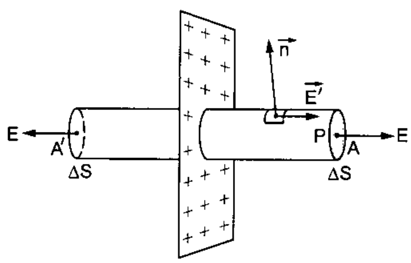


Figure 30.18

Consider a large plane sheet of charge with surface charge density (charge per unit area) σ. We have to find the electric field E at a point P in front of the sheet (Figure 30.18). What can be the direction of the electric field at P? The only unique direction we can identify is along the perpendicular to the plane. The field must be along this line. If the charge is positive, the field is away from the plane. Same is true for all points near the plane provided the sheet is large and the charge density is uniform. If these conditions are not fulfilled, the argument of symmetry may fail.

To calculate the field E at P, we choose a Gaussian surface as follows. Draw a plane surface A passing through P and parallel to the charge sheet. Draw a cylinder with this surface as a cross section and extend it to the other side of the plane charge sheet. Close the cylinder on the other side by a cross section A' such that A and A' are equidistant from the sheet. Also, A and A' have equal area ΔS. The cylinder together with its cross sectional areas forms a closed surface and we apply Gauss's law to this surface.

The electric field at all the points of A has the same magnitude E. The direction is along the positive normal to A. Thus, the flux of the electric field through A is

$$\Phi = \vec{E}\cdot\Delta\vec{S} = E\,\Delta S.$$

Note that the two sides of the charge sheet are equivalent in all respect. As A and A' are equidistant from the sheet, the electric field at any point of A' is also equal to E and is along the positive normal (that is, the outward normal) to A'. Hence, the flux of the electric field through A' is also $E\,\Delta S$. At the points on the curved surface, the field and the outward normal make an angle of 90° with each other and hence $\vec{E}\cdot\Delta\vec{S} = 0$. The total flux through the closed surface is

$$\Phi = \oint \vec{E}\cdot d\vec{S} = E\,\Delta S + E\,\Delta S + 0 = 2\,E\,\Delta S.$$

The area of the sheet enclosed in the cylinder is ΔS. The charge contained in the cylinder is, therefore, $\sigma\,\Delta S$. Hence from Gauss's law,

$$2\,E\Delta S = \frac{\sigma\,\Delta S}{\varepsilon_0}$$

or, $$E = \frac{\sigma}{2\varepsilon_0}\cdot \qquad \ldots \text{ (30.5)}$$

We see that the field is uniform and does not depend on the distance from the charge sheet. This is true as long as the sheet is large as compared to its distance from P.

(E) Electric Field near a Charged Conducting Surface

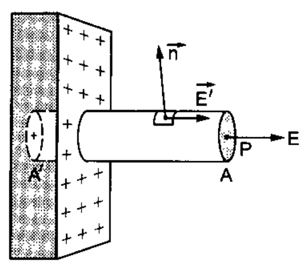


Figure 30.19

In figure (30.19), we show a large, plane conducting sheet. The surface on right has a uniform surface charge density σ. We have to find the electric field at a point P near this surface and outside the conductor. As we know, the conducting surface is an equipotential surface and the electric field near the surface is perpendicular to the surface. For positive charge on the surface, the field is away from the surface. To find the electric field, we construct a Gaussian surface as follows. Take a small plane surface A passing through the point P and parallel to the given conducting surface. Draw a cylinder with A as a cross section and terminate it with another plane surface A' parallel to A and lying in the interior of the conducting sheet.

If the electric field at P is E, the flux through the plane surface A is

$$\Phi = E\,\Delta S,$$

where ΔS is the area of A. At the curved parts of the cylinder, the electric field is either zero (inside the conductor) or is parallel to the curved surface (outside the conductor). The field $\vec{E}$ and the area-vector $\Delta\vec{S}$ are perpendicular to each other making $\vec{E}\cdot\Delta\vec{S} = 0$ at these outside points. The flux on the curved part is, therefore, zero. Also, the flux on A' is zero as the field inside the conductor is zero.

The total flux through the Gaussian surface constructed is, therefore, $E\,\Delta S$. The charge enclosed inside the closed surface is $\sigma\,\Delta S$ and hence from Gauss's law,

$$E\,\Delta S = \frac{\sigma\,\Delta S}{\varepsilon_0}$$

or, $$E = \frac{\sigma}{\varepsilon_0} \cdot \qquad \ldots (30.6)$$

The electric field near a charged conducting surface is σ/ε_0 and it is normal to the surface.

Compare this result with the field due to a plane sheet of charge of surface density σ (equation 30.5). The field E had a magnitude $\sigma/(2\varepsilon_0)$ in that case. Apparently, for a conductor also we have a plane sheet of charge of the same density but the field derived is σ/ε_0 and not $\sigma/(2\varepsilon_0)$. Consider the conducting sheet shown in figure (30.20). In fact, the field due to the charge on the right surface is indeed $\frac{\sigma}{2\varepsilon_0}$ at P. Where does the extra $\frac{\sigma}{2\varepsilon_0}$ field come from ?

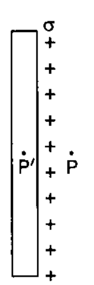


Figure 30.20

Consider a point P' inside the sheet as shown in figure (30.20). The electric field at this point due to the charge sheet on the right surface of the conductor is $\sigma/(2\varepsilon_0)$ towards left. But P' is a point inside the conductor and hence the field here must be zero. This means that the charge distribution shown in figure (30.20) is not complete as it does not ensure zero field inside the conductor. Apart from the surface charge of density σ shown in the figure, there must be other charges nearby. These other nearby charges must create a field at P' towards right so that the resultant field at P' becomes zero. These other charges also create a field $\sigma/(2\varepsilon_0)$ towards right at P which adds to the field due to the surface charge shown in figure (30.20). Thus, the field at P becomes

$$\frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

towards right. As Gauss's law gives the net field, equation (30.6) gives $E = \sigma/\varepsilon_0$ which is the actual field due to all the charges and not the field due to the surface charge only. As examples, we give in figure (30.21) some of the possible complete charge distributions which ensure zero field inside the conductor.

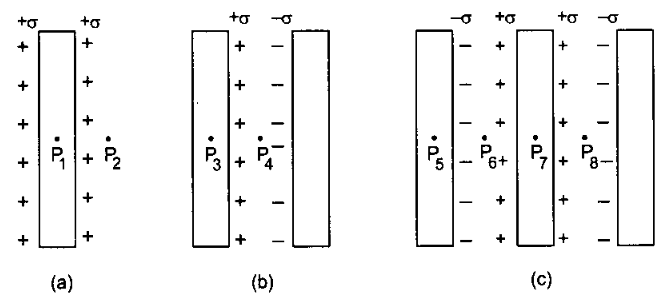


Figure 30.21

Calculate the electric field at the points indicated using the formula $E = \sigma/(2\varepsilon_0)$ for each charged surface. Verify that the field at each of the points P_1, P_3, P_5 and P_7 is zero and at each of the points P_2, P_4, P_6 and P_8 is σ/ε_0.

Note that electric field changes discontinuously at the surface of a conductor. Just outside the conductor it is σ/ε_0 and inside the conductor it is zero. In fact, the field gradually decreases from σ/ε_0 to zero in a small thickness of about 4–5 atomic layers at the surface. When we say 'the surface of a conductor' we actually mean this small thickness.

30.5 SPHERICAL CHARGE DISTRIBUTIONS

We have seen that the electric field due to a uniformly charged sphere at an external point is the same as that due to an equal point charge placed at the centre. Similar result was obtained for the gravitational field due to a uniform sphere. This similarity is expected because the Coulomb force

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

and the gravitational force

$$F = \frac{Gm_1m_2}{r^2}$$

have similar mathematical structure.

Many of the results derived for gravitational field, potential and potential energy may, therefore, be used for the corresponding electrical quantities. We state some of the useful results for a spherical charge distribution of radius R.

(a) The electric field due to a uniformly charged, thin spherical shell at an external point is the same as that due to an equal point charge placed at the centre of the shell, $E = Q/(4\pi\varepsilon_0 r^2)$.

(b) The electric field due to a uniformly charged thin spherical shell at an internal point is zero.

(c) The electric field due to a uniformly charged sphere at an external point is the same as that due to an equal point charge placed at the centre of the sphere.

(d) The electric field due to a uniformly charged sphere at an internal point is proportional to the distance of the point from the centre of the sphere. Thus, it is zero at the centre and increases linearly as one moves out towards the surface.

(e) The electric potential due to a uniformly charged, thin spherical shell at an external point is the same as that due to an equal point charge placed at the centre, $V = Q/(4\pi\varepsilon_0 r)$.

(f) The electric potential due to a uniformly charged, thin spherical shell at an internal point is the same everywhere and is equal to that at the surface, $V = Q/(4\pi\varepsilon_0 R)$.

(g) The electric potential due to a uniformly charged sphere at an external point is the same as that due to an equal point charge placed at the centre, $V = Q/(4\pi\varepsilon_0 r)$.

Electric Potential Energy of a Uniformly Charged Sphere

Consider a uniformly charged sphere of radius R having a total charge Q. The electric potential energy of this sphere is equal to the work done in bringing the charges from infinity to assemble the sphere. Let us assume that at some instant, charge is assembled up to a radius x. In the next step, we bring some charge from infinity and put it on this sphere to increase the radius from x to $x + dx$. The entire sphere is assembled as x varies from 0 to R.

The charge density is

$$\rho = \frac{3Q}{4\pi R^3}.$$

When the radius of the sphere is x, the charge contained in it is,

$$q = \frac{4}{3}\pi x^3 \rho = \frac{Q}{R^3}x^3.$$

The potential at the surface is

$$V = \frac{q}{4\pi\varepsilon_0 x} = \frac{Q}{4\pi\varepsilon_0 R^3}x^2.$$

The charge needed to increase the radius from x to $x + dx$ is

$$dq = (4\pi x^2 dx)\rho = \frac{3Q}{R^3}x^2 dx.$$

The work done in bringing the charge dq from infinity to the surface of the sphere of radius x is

$$dW = V(dq) = \frac{3Q^2}{4\pi\varepsilon_0 R^6}x^4 dx.$$

The total work done in assembling the charged sphere of radius R is

$$W = \frac{3Q^2}{4\pi\varepsilon_0 R^6}\int_0^R x^4 dx = \frac{3Q^2}{20\pi\varepsilon_0 R}.$$

This is the electric potential energy of the charged sphere.

Electric Potential Energy of a Uniformly Charged, Thin Spherical Shell

Consider a uniformly charged, thin spherical shell of radius R having a total charge Q. The electric potential energy is equal to the work done in bringing charges from infinity and put them on the shell. Suppose at some instant, a charge q is placed on the shell. The potential at the surface is

$$V = \frac{q}{4\pi\varepsilon_0 R}.$$

The work done in bringing a charge dq from infinity to this shell is

$$dW = V(dq) = \frac{q\,dq}{4\pi\varepsilon_0 R}.$$

The total work done in assembling the charge on the shell is

$$W = \int_0^Q \frac{q dq}{4\pi\varepsilon_0 R} = \frac{Q^2}{8\pi\varepsilon_0 R}.$$

This is the electric potential energy of the charged spherical shell.

30.6 EARTHING A CONDUCTOR

The earth is a good conductor of electricity. If we assume that the earth is uncharged, its potential will be zero. In fact, the earth's surface has a negative charge of about 1 nC m^{-2} and hence is at a constant potential V. All conductors which are not given any external charge, are also very nearly at the same potential. In turns out that for many practical calculations, we can ignore the charge on the earth. The potential of the earth can then be taken as the same as that of a point far away from all charges, i.e., at infinity. So, the potential of the earth is often taken to be zero. Also, if a small quantity of charge is given to the earth or is taken away from it, the potential does not change by any appreciable extent. This is because of the large size of the earth.

If a conductor is connected to the earth, the potential of the conductor becomes equal to that of the earth, i.e., zero. If the conductor was at some other potential, charges will flow from it to the earth or from the earth to it to bring its potential to zero.

When a conductor is connected to the earth, the conductor is said to be *earthed* or *grounded*. Figure (30.22a) shows the symbol for earthing.

Suppose a spherical conductor of radius R is given a charge Q. The charge will be distributed uniformly on the surface. So it is equivalent to a uniformly charged, thin spherical shell. Its potential will, therefore, become $Q/(4\pi\varepsilon_0 R)$. If this conductor is connected to the earth, the charge Q will be transferred to the earth so that the potential will become zero.

Next suppose, a charge $+Q$ is placed at the centre of a spherical conducting shell. A charge $-Q$ will appear on its inner surface and $+Q$ on its outer surface (figure 30.22b). The potential of the sphere due to the charge at the centre and that due to the charge at the inner surface are $\frac{Q}{4\pi\varepsilon_0 R}$ and $\frac{-Q}{4\pi\varepsilon_0 R}$ respectively. The potential due to the charge on the outer surface is $\frac{Q}{4\pi\varepsilon_0 R}\cdot$ The net potential of the sphere is, therefore, $\frac{Q}{4\pi\varepsilon_0 R}\cdot$ If this sphere is now connected to the earth (figure 30.22c), the charge Q on the outer surface flows to the earth and the potential of the sphere becomes zero.

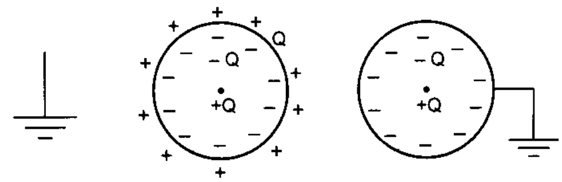


Figure 30.22

Earthing a conductor is a technical job. A thick metal plate is buried deep into the earth and wires are drawn from this plate. The electric wiring in our houses has three wires: live, neutral and earth. The live and neutral wires carry electric currents which come from the power station. The earth wire is connected to the metal plate buried in the earth. The metallic bodies of electric appliances such as electric iron, refrigerator, etc. are connected to the earth wire. This ensures that the metallic body remains at zero potential while an appliance is being used. If by any fault, the live wire touches the metallic body, charge flows to the earth and the potential of the metallic body remains zero. If it is not connected to the earth, the user may get an electric shock.

Worked Out Examples

1. *A uniform electric field of magnitude* $E = 100\ \mathrm{N\ C^{-1}}$ *exists in the space in x-direction. Calculate the flux of this field through a plane square area of edge* 10 cm *placed in the y–z plane. Take the normal along the positive x-axis to be positive.*

Solution : The flux $\Phi = \int E\cos\theta\, dS$. As the normal to the area points along the electric field, $\theta = 0$. Also, E is uniform, so

$$\Phi = E\,\Delta S$$

$$= (100\ \mathrm{N\ C^{-1}})\,(0\cdot 10\ \mathrm{m})^2 = 1\cdot 0\ \mathrm{N\ m^2 C^{-1}}\cdot$$

2. *A large plane charge sheet having surface charge density* $\sigma = 2\cdot 0\times 10^{-6}\ \mathrm{C\ m^{-2}}$ *lies in the x–y plane. Find the flux of the electric field through a circular area of radius* 1 cm *lying completely in the region where x, y, z are all positive and with its normal making an angle of* 60° *with the z-axis.*

Solution : The electric field near the plane charge sheet is $E = \sigma/2\varepsilon_0$ in the direction away from the sheet. At the given area, the field is along the z-axis.

The area $= \pi r^2 = 3\cdot 14\times 1\ \mathrm{cm}^2 = 3\cdot 14\times 10^{-4}\ \mathrm{m}^2$.

The angle between the normal to the area and the field is 60°.

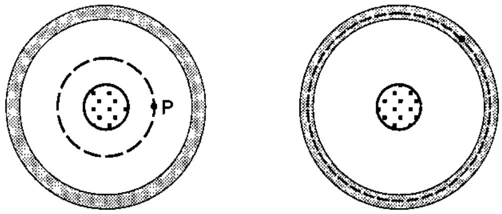
Hence, the flux $= \vec{E}\cdot\Delta\vec{S} = E\,\Delta S\cos\theta = \frac{\sigma}{2\varepsilon_0}\pi r^2\cos 60°$

$$= \frac{2\cdot 0\times 10^{-6}\ \mathrm{C\ m^{-2}}}{2\times 8\cdot 85\times 10^{-12}\ \mathrm{C^2 N\ m^{-2}}}\times(3\cdot 14\times 10^{-4}\ \mathrm{m}^2)\,\frac{1}{2}$$

$$= 17\cdot 5\ \mathrm{N\ m^2\ C^{-1}}.$$

3. *A charge of* 4×10^{-8} C *is distributed uniformly on the surface of a sphere of radius* 1 cm. *It is covered by a concentric, hollow conducting sphere of radius* 5 cm. *(a) Find the electric field at a point* 2 cm *away from the centre. (b) A charge of* 6×10^{-8} C *is placed on the hollow sphere. Find the surface charge density on the outer surface of the hollow sphere.*

Solution :



(a) (b)

Figure 30-W1

(a) Let us consider figure (30-W1a). Suppose, we have to find the field at the point P. Draw a concentric spherical surface through P. All the points on this surface are equivalent and by symmetry, the field at all these points will be equal in magnitude and radial in direction.

The flux through this surface $= \oint \vec{E} \cdot d\vec{S}$

$$= \oint E\, dS = E \oint dS$$

$$= 4\pi x^2 E,$$

where $\quad x = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}.$

From Gauss's law, this flux is equal to the charge q contained inside the surface divided by ε_0. Thus,

$$4\pi x^2 E = q/\varepsilon_0$$

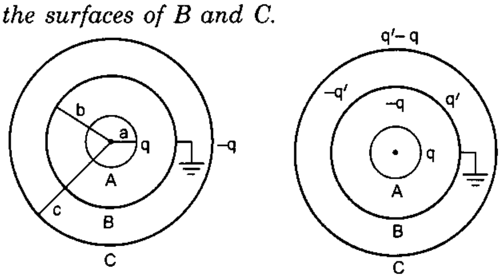
or, $$E = \frac{q}{4\pi\varepsilon_0 x^2}$$

$$= (9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}) \times \frac{4 \times 10^{-8} \text{ C}}{4 \times 10^{-4} \text{ m}^2}$$

$$= 9 \times 10^5 \text{ N C}^{-1}.$$

(b) See figure (30-W1b). Take a Gaussian surface through the material of the hollow sphere. As the electric field in a conducting material is zero, the flux $\oint \vec{E} \cdot d\vec{S}$ through this Gaussian surface is zero. Using Gauss's law, the total charge enclosed must be zero. Hence, the charge on the inner surface of the hollow sphere is -4×10^{-8} C. But the total charge given to this hollow sphere is 6×10^{-8} C. Hence, the charge on the outer surface will be 10×10^{-8} C.

4. *Figure (30-W2a) shows three concentric thin spherical shells A, B and C of radii a, b and c respectively. The shells A and C are given charges q and –q respectively and the shell B is earthed. Find the charges appearing on the surfaces of B and C.*



(a) (b)

Figure 30-W2

Solution :

As shown in the previous worked out example, the inner surface of B must have a charge $-q$ from the Gauss's law. Suppose, the outer surface of B has a charge q'. The inner surface of C must have a charge $-q'$ from the Gauss's law. As the net charge on C must be $-q$, its outer surface should have a charge $q' - q$. The charge distribution is shown in figure (30-W2b).

The potential at B due to the charge q on A

$$= \frac{q}{4\pi\varepsilon_0 b},$$

due to the charge $-q$ on the inner surface of B

$$= \frac{-q}{4\pi\varepsilon_0 b},$$

due to the charge q' on the outer surface of B

$$= \frac{q'}{4\pi\varepsilon_0 b},$$

due to the charge $-q'$, on the inner surface of C

$$= \frac{-q'}{4\pi\varepsilon_0 c}$$

and due to the charge $q' - q$ on the outer surface of C

$$= \frac{q' - q}{4\pi\varepsilon_0 c}.$$

The net potential is

$$V_B = \frac{q'}{4\pi\varepsilon_0 b} - \frac{q}{4\pi\varepsilon_0 c}.$$

This should be zero as the shell B is earthed. Thus,

$$q' = \frac{b}{c} q.$$

The charges on various surfaces are as shown in figure (30-W3).

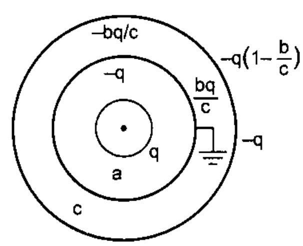


Figure 30-W3

5. *An electric dipole consists of charges* $\pm 2{\cdot}0 \times 10^{-8}$ C *separated by a distance of* $2{\cdot}0 \times 10^{-3}$ m. *It is placed near a long line charge of linear charge density* $4{\cdot}0 \times 10^{-4}$ C m^{-1} *as shown in figure (30-W4), such that the negative charge is at a distance of* 2·0 cm *from the line charge. Find the force acting on the dipole.*

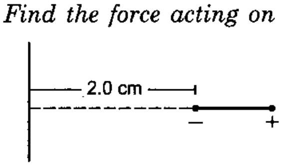


Figure 30-W4

Solution : The electric field at a distance r from the line charge of linear density λ is given by

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}.$$

Hence, the field at the negative charge is

$$E_1 = \frac{(4.0 \times 10^{-4}\ \mathrm{C\ m^{-1}})\,(2 \times 9 \times 10^{9}\ \mathrm{N\ m^2\ C^{-2}})}{0.02\ \mathrm{m}}$$

$$= 3.6 \times 10^{8}\ \mathrm{N\ C^{-1}}.$$

The force on the negative charge is

$$F_1 = (3.6 \times 10^{8}\ \mathrm{N\ C^{-1}})\,(2.0 \times 10^{-8}\ \mathrm{C}) = 7.2\ \mathrm{N}$$

towards the line charge.

Similarly, the field at the positive charge, i.e., at $r = 0.022$ m is

$$E_2 = 3.3 \times 10^{8}\ \mathrm{N\ C^{-1}}.$$

The force on the positive charge is

$$F_2 = (3.3 \times 10^{8}\ \mathrm{N\ C^{-1}}) \times (2.0 \times 10^{-8}\ \mathrm{C})$$

$$= 6.6\ \mathrm{N}\ \text{ away from the line charge}.$$

Hence, the net force on the dipole $= (7.2 - 6.6)$ N

$= 0.6$ N towards the line charge.

6. *The electric field in a region is radially outward with magnitude $E = Ar$. Find the charge contained in a sphere of radius a centred at the origin. Take $A = 100\ \mathrm{V\ m^{-2}}$ and $a = 20.0$ cm.*

Solution : The electric field at the surface of the sphere is Aa and being radial it is along the outward normal. The flux of the electric field is, therefore,

$$\Phi = \oint E\, dS \cos\theta = Aa(4\pi a^2).$$

The charge contained in the sphere is, from Gauss's law,

$$Q_{inside} = \varepsilon_0 \Phi = 4\pi\varepsilon_0 A a^3$$

$$= \left(\frac{1}{9 \times 10^{9}}\ \mathrm{C^2\ N^{-1}\ m^{-2}}\right)(100\ \mathrm{V\ m^{-2}})\ (0.20\ \mathrm{m})^3$$

$$= 8.89 \times 10^{-11}\ \mathrm{C}.$$

7. *A particle of mass 5×10^{-6} g is kept over a large horizontal sheet of charge of density $4.0 \times 10^{-6}\ \mathrm{C\ m^{-2}}$ (figure 30-W5). What charge should be given to this particle so that if released, it does not fall down? How many electrons are to be removed to give this charge? How much mass is decreased due to the removal of these electrons?*

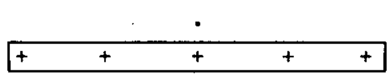


Figure 30-W5

Solution : The electric field in front of the sheet is

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{4.0 \times 10^{-6}\ \mathrm{C\ m^{-2}}}{2 \times 8.85 \times 10^{-12}\ \mathrm{C^2\ N^{-1}\ m^{-2}}}$$

$$= 2.26 \times 10^{5}\ \mathrm{N\ C^{-1}}.$$

If a charge q is given to the particle, the electric force qE acts in the upward direction. It will balance the weight of the particle if

$$q \times 2.26 \times 10^{5}\ \mathrm{N\ C^{-1}} = 5 \times 10^{-9}\ \mathrm{kg} \times 9.8\ \mathrm{m\ s^{-2}}$$

or,
$$q = \frac{4.9 \times 10^{-8}}{2.26 \times 10^{5}}\ \mathrm{C}$$

$$= 2.21 \times 10^{-13}\ \mathrm{C}.$$

The charge on one electron is 1.6×10^{-19} C. The number of electrons to be removed

$$= \frac{2.21 \times 10^{-13}\ \mathrm{C}}{1.6 \times 10^{-19}\ \mathrm{C}} = 1.4 \times 10^{6}.$$

Mass decreased due to the removal of these electrons

$$= 1.4 \times 10^{6} \times 9.1 \times 10^{-31}\ \mathrm{kg}$$

$$= 1.3 \times 10^{-24}\ \mathrm{kg}.$$

8. *Two conducting plates A and B are placed parallel to each other. A is given a charge Q_1 and B a charge Q_2. Find the distribution of charges on the four surfaces.*

Solution : Consider a Gaussian surface as shown in figure (30-W6a). Two faces of this closed surface lie completely inside the conductor where the electric field is zero. The flux through these faces is, therefore, zero. The other parts of the closed surface which are outside the conductor are parallel to the electric field and hence the flux on these parts is also zero. The total flux of the electric field through the closed surface is, therefore, zero. From Gauss's law, the total charge inside this closed surface should be zero. The charge on the inner surface of A should be equal and opposite to that on the inner surface of B.

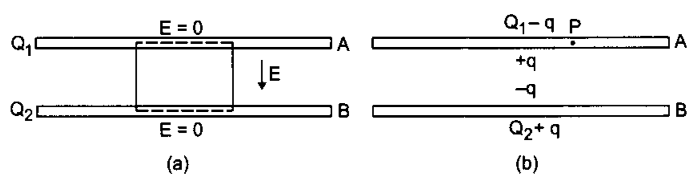


Figure 30-W6

The distribution should be like the one shown in figure (30-W6b). To find the value of q, consider the field at a point P inside the plate A. Suppose, the surface area of the plate (one side) is A. Using the equation $E = \sigma/(2\varepsilon_0)$, the electric field at P

due to the charge $Q_1 - q = \dfrac{Q_1 - q}{2A\varepsilon_0}$ (downward),

due to the charge $+q = \dfrac{q}{2A\varepsilon_0}$ (upward),

due to the charge $-q = \dfrac{q}{2A\varepsilon_0}$ (downward), and due to the charge $Q_2 + q = \dfrac{Q_2 + q}{2A\varepsilon_0}$ (upward).

The net electric field at P due to all the four charged surfaces is (in the downward direction)

$$\frac{Q_1 - q}{2A\varepsilon_0} - \frac{q}{2A\varepsilon_0} + \frac{q}{2A\varepsilon_0} - \frac{Q_2 + q}{2A\varepsilon_0}.$$

As the point P is inside the conductor, this field should be zero. Hence,

$$Q_1 - q - Q_2 - q = 0$$

or, $$q = \frac{Q_1 - Q_2}{2}. \quad \text{... (i)}$$

Thus, $$Q_1 - q = \frac{Q_1 + Q_2}{2} \quad \text{... (ii)}$$

and $$Q_2 + q = \frac{Q_1 + Q_2}{2}.$$

Using these equations, the distribution shown in the figure (30-W6) can be redrawn as in figure (30-W7).

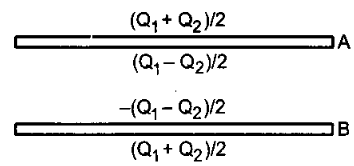


Figure 30-W7

This result is a special case of the following result. When charged conducting plates are placed parallel to each other, the two outermost surfaces get equal charges and the facing surfaces get equal and opposite charges.

□

QUESTIONS FOR SHORT ANSWER

1. A small plane area is rotated in an electric field. In which orientation of the area is the flux of electric field through the area maximum ? In which orientation is it zero ?
2. A circular ring of radius r made of a nonconducting material is placed with its axis parallel to a uniform electric field. The ring is rotated about a diameter through 180°. Does the flux of electric field change ? If yes, does it decrease or increase ?
3. A charge Q is uniformly distributed on a thin spherical shell. What is the field at the centre of the shell ? If a point charge is brought close to the shell, will the field at the centre change ? Does your answer depend on whether the shell is conducting or nonconducting ?
4. A spherical shell made of plastic, contains a charge Q distributed uniformly over its surface. What is the electric field inside the shell ? If the shell is hammered to deshape it without altering the charge, will the field inside be changed ? What happens if the shell is made of a metal ?
5. A point charge q is placed in a cavity in a metal block. If a charge Q is brought outside the metal, will the charge q feel an electric force ?
6. A rubber balloon is given a charge Q distributed uniformly over its surface. Is the field inside the balloon zero everywhere if the balloon does not have a spherical surface ?
7. It is said that any charge given to a conductor comes to its surface. Should all the protons come to the surface ? Should all the electrons come to the surface ? Should all the free electrons come to the surface ?

OBJECTIVE I

1. A charge Q is uniformly distributed over a large plastic plate. The electric field at a point P close to the centre of the plate is $10\ \text{V m}^{-1}$. If the plastic plate is replaced by a copper plate of the same geometrical dimensions and carrying the same charge Q, the electric field at the point P will become
   (a) zero (b) $5\ \text{V m}^{-1}$ (c) $10\ \text{V m}^{-1}$ (d) $20\ \text{V m}^{-1}$.
2. A metallic particle having no net charge is placed near a finite metal plate carrying a positive charge. The electric force on the particle will be
   (a) towards the plate (b) away from the plate
   (c) parallel to the plate (d) zero.
3. A thin, metallic spherical shell contains a charge Q on it. A point charge q is placed at the centre of the shell and another charge q_1 is placed outside it as shown in figure (30-Q1). All the three charges are positive. The force on the charge at the centre is

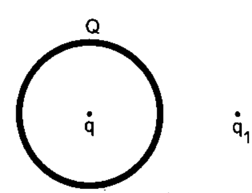


Figure 30-Q1

(a) towards left (b) towards right
(c) upward (d) zero.

4. Consider the situation of the previous problem. The force on the central charge due to the shell is
(a) towards left (b) towards right
(c) upward (d) zero.

5. Electric charges are distributed in a small volume. The flux of the electric field through a spherical surface of radius 10 cm surrounding the total charge is 25 V m. The flux over a concentric sphere of radius 20 cm will be
(a) 25 V m (b) 50 V m (c) 100 V m (d) 200 V m.

6. Figure (30-Q2a) shows an imaginary cube of edge $L/2$. A uniformly charged rod of length L moves towards left at a small but constant speed v. At $t = 0$, the left end just touches the centre of the face of the cube opposite it. Which of the graphs shown in figure (30-Q2b) represents the flux of the electric field through the cube as the rod goes through it ?

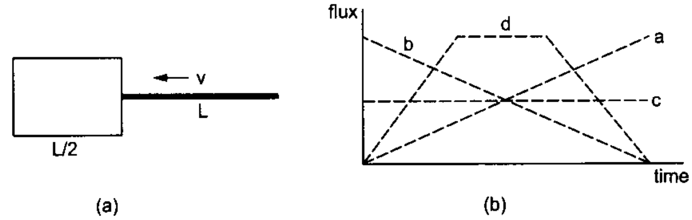


Figure 30-Q2

7. A charge q is placed at the centre of the open end of a cylindrical vessel (figure 30-Q3). The flux of the electric field through the surface of the vessel is
(a) zero (b) q/ε_0 (c) $q/2\varepsilon_0$ (d) $2q/\varepsilon_0$.

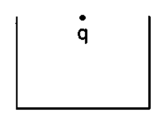


Figure 30-Q3

OBJECTIVE II

1. Mark the correct options:
(a) Gauss's law is valid only for symmetrical charge distributions.
(b) Gauss's law is valid only for charges placed in vacuum.
(c) The electric field calculated by Gauss's law is the field due to the charges inside the Gaussian surface.
(d) The flux of the electric field through a closed surface due to all the charges is equal to the flux due to the charges enclosed by the surface.

2. A positive point charge Q is brought near an isolated metal cube.
(a) The cube becomes negatively charged.
(b) The cube becomes positively charged.
(c) The interior becomes positively charged and the surface becomes negatively charged.
(d) The interior remains charge free and the surface gets nonuniform charge distribution.

3. A large nonconducting sheet M is given a uniform charge density. Two uncharged small metal rods A and B are placed near the sheet as shown in figure (30-Q4).
(a) M attracts A. (b) M attracts B.
(c) A attracts B. (d) B attracts A.

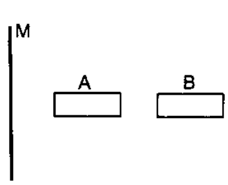


Figure 30-Q4

4. If the flux of the electric field through a closed surface is zero,
(a) the electric field must be zero everywhere on the surface
(b) the electric field may be zero everywhere on the surface
(c) the charge inside the surface must be zero
(d) the charge in the vicinity of the surface must be zero.

5. An electric dipole is placed at the centre of a sphere. Mark the correct options:
(a) The flux of the electric field through the sphere is zero.
(b) The electric field is zero at every point of the sphere.
(c) The electric field is not zero anywhere on the sphere.
(d) The electric field is zero on a circle on the sphere.

6. Figure (30-Q5) shows a charge q placed at the centre of a hemisphere. A second charge Q is placed at one of the positions A, B, C and D. In which position(s) of this second charge, the flux of the electric field through the hemisphere remains unchanged ?
(a) A (b) B (c) C (d) D.

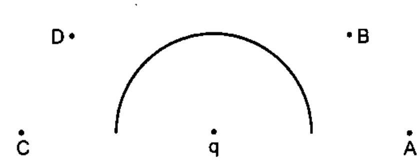


Figure 30-Q5

7. A closed surface S is constructed around a conducting wire connected to a battery and a switch (figure 30-Q6). As the switch is closed, the free electrons in the wire start moving along the wire. In any time interval, the number of electrons entering the closed surface S is equal to the number of electrons leaving it. On closing

the switch, the flux of the electric field through the closed surface
(a) is increased (b) is decreased
(c) remains unchanged (d) remains zero.

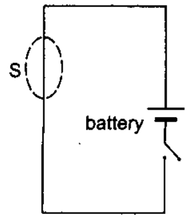


Figure 30-Q6

8. Figure (30-Q7) shows a closed surface which intersects a conducting sphere. If a positive charge is placed at the point P, the flux of the electric field through the closed surface
(a) will remain zero (b) will become positive
(c) will become negative (d) will become undefined.

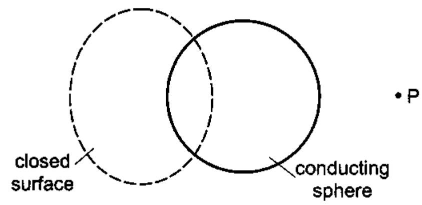


Figure 30-Q7

EXERCISES

1. The electric field in a region is given by $\vec{E} = \frac{3}{5} E_0 \vec{i} + \frac{4}{5} E_0 \vec{j}$ with $E_0 = 2{\cdot}0 \times 10^3$ N C^{-1}. Find the flux of this field through a rectangular surface of area $0{\cdot}2$ m^2 parallel to the y–z plane.

2. A charge Q is uniformly distributed over a rod of length l. Consider a hypothetical cube of edge l with the centre of the cube at one end of the rod. Find the minimum possible flux of the electric field through the entire surface of the cube.

3. Show that there can be no net charge in a region in which the electric field is uniform at all points.

4. The electric field in a region is given by $\vec{E} = \frac{E_0 x}{l} \vec{i}$. Find the charge contained inside a cubical volume bounded by the surfaces $x = 0$, $x = a$, $y = 0$, $y = a$, $z = 0$ and $z = a$. Take $E_0 = 5 \times 10^3$ N C^{-1}, $l = 2$ cm and $a = 1$ cm.

5. A charge Q is placed at the centre of a cube. Find the flux of the electric field through the six surfaces of the cube.

6. A charge Q is placed at a distance $a/2$ above the centre of a horizontal, square surface of edge a as shown in figure (30-E1). Find the flux of the electric field through the square surface.

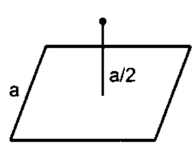


Figure 30-E1

7. Find the flux of the electric field through a spherical surface of radius R due to a charge of 10^{-7} C at the centre and another equal charge at a point $2R$ away from the centre (figure 30-E2).

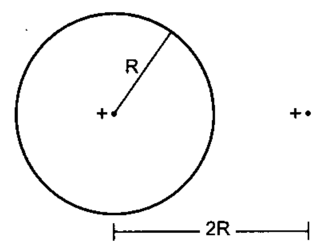


Figure 30-E2

8. A charge Q is placed at the centre of an imaginary hemispherical surface. Using symmetry arguments and the Gauss's law, find the flux of the electric field due to this charge through the surface of the hemisphere (figure 30-E3).

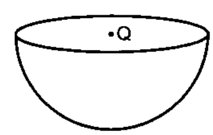


Figure 30-E3

9. A spherical volume contains a uniformly distributed charge of density $2{\cdot}0 \times 10^{-4}$ C m^{-3}. Find the electric field at a point inside the volume at a distance $4{\cdot}0$ cm from the centre.

10. The radius of a gold nucleus ($Z = 79$) is about $7{\cdot}0 \times 10^{-15}$ m. Assume that the positive charge is distributed uniformly throughout the nuclear volume. Find the strength of the electric field at (a) the surface of the nucleus and (b) at the middle point of a radius. Remembering that gold is a conductor, is it justified to assume that the positive charge is uniformly distributed over the entire volume of the nucleus and does not come to the outer surface ?

11. A charge Q is distributed uniformly within the material of a hollow sphere of inner and outer radii r_1 and r_2 (figure 30-E4). Find the electric field at a point P a

distance x away from the centre for $r_1 < x < r_2$. Draw a rough graph showing the electric field as a function of x for $0 < x < 2r_2$ (figure 30-E4).

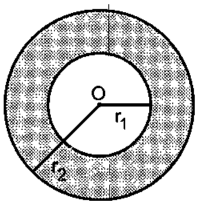


Figure 30-E4

12. A charge Q is placed at the centre of an uncharged, hollow metallic sphere of radius a. (a) Find the surface charge density on the inner surface and on the outer surface. (b) If a charge q is put on the sphere, what would be the surface charge densities on the inner and the outer surfaces? (c) Find the electric field inside the sphere at a distance x from the centre in the situations (a) and (b).

13. Consider the following very rough model of a beryllium atom. The nucleus has four protons and four neutrons confined to a small volume of radius 10^{-15} m. The two $1s$ electrons make a spherical charge cloud at an average distance of $1{\cdot}3 \times 10^{-11}$ m from the nucleus, whereas the two $2s$ electrons make another spherical cloud at an average distance of $5{\cdot}2 \times 10^{-11}$ m from the nucleus. Find the electric field at (a) a point just inside the $1s$ cloud and (b) a point just inside the $2s$ cloud.

14. Find the magnitude of the electric field at a point 4 cm away from a line charge of density $2 \times 10^{-6}\ \mathrm{C\,m^{-1}}$.

15. A long cylindrical wire carries a positive charge of linear density $2{\cdot}0 \times 10^{-8}\ \mathrm{C\,m^{-1}}$. An electron revolves around it in a circular path under the influence of the attractive electrostatic force. Find the kinetic energy of the electron. Note that it is independent of the radius.

16. A long cylindrical volume contains a uniformly distributed charge of density ρ. Find the electric field at a point P inside the cylindrical volume at a distance x from its axis (figure 30-E5)

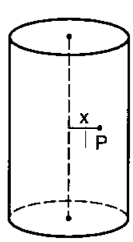


Figure 30-E5

17. A nonconducting sheet of large surface area and thickness d contains uniform charge distribution of density ρ. Find the electric field at a point P inside the plate, at a distance x from the central plane. Draw a qualitative graph of E against x for $0 < x < d$.

18. A charged particle having a charge of $-2{\cdot}0 \times 10^{-6}$ C is placed close to a nonconducting plate having a surface charge density $4{\cdot}0 \times 10^{-6}\ \mathrm{C\,m^{-2}}$. Find the force of attraction between the particle and the plate.

19. One end of a 10 cm long silk thread is fixed to a large vertical surface of a charged nonconducting plate and the other end is fastened to a small ball having a mass of 10 g and a charge of $4{\cdot}0 \times 10^{-6}$ C. In equilibrium, the thread makes an angle of 60° with the vertical. Find the surface charge density on the plate.

20. Consider the situation of the previous problem. (a) Find the tension in the string in equilibrium. (b) Suppose the ball is slightly pushed aside and released. Find the time period of the small oscillations.

21. Two large conducting plates are placed parallel to each other with a separation of 2·00 cm between them. An electron starting from rest near one of the plates reaches the other plate in 2·00 microseconds. Find the surface charge density on the inner surfaces.

22. Two large conducting plates are placed parallel to each other and they carry equal and opposite charges with surface density σ as shown in figure (30-E6). Find the electric field (a) at the left of the plates, (b) in between the plates and (c) at the right of the plates.

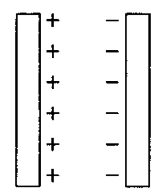


Figure 30-E6

23. Two conducting plates X and Y, each having large surface area A (on one side), are placed parallel to each other as shown in figure (30-E7). The plate X is given a charge Q whereas the other is neutral. Find (a) the surface charge density at the inner surface of the plate X, (b) the electric field at a point to the left of the plates, (c) the electric field at a point in between the plates and (d) the electric field at a point to the right of the plates.

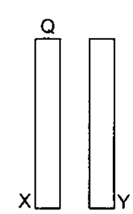


Figure 30-E7

24. Three identical metal plates with large surface areas are kept parallel to each other as shown in figure (30-E8). The leftmost plate is given a charge Q, the rightmost a charge $-2Q$ and the middle one remains neutral. Find the charge appearing on the outer surface of the rightmost plate.

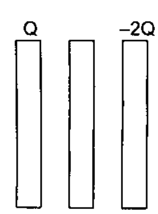


Figure 30-E8

ANSWERS

OBJECTIVE I

1. (c) 2. (a) 3. (d) 4. (b) 5. (a) 6. (d)
7. (c)

OBJECTIVE II

1. (d) 2. (d) 3. all
4. (b), (c) 5. (a), (c) 6. (a), (c)
7. (c), (d) 8. (b)

EXERCISES

1. $240\ \text{N m}^2\ \text{C}^{-1}$
2. $Q/(2\varepsilon_0)$
4. $2{\cdot}2 \times 10^{-12}$ C
5. Q/ε_0
6. $Q/(6\varepsilon_0)$
7. $1{\cdot}1 \times 10^4\ \text{N m}^{-2}\ \text{C}^{-1}$
8. $Q/(2\varepsilon_0)$
9. $3{\cdot}0 \times 10^5\ \text{N C}^{-1}$
10. (a) $2{\cdot}32 \times 10^{21}\ \text{N C}^{-1}$ (b) $1{\cdot}16 \times 10^{21}\ \text{N C}^{-1}$
11. $\dfrac{Q(x^3 - r_1^3)}{4\pi\varepsilon_0 x^2 (r_2^3 - r_1^3)}$
12. (a) $-\dfrac{Q}{4\pi a^2}, \dfrac{Q}{4\pi a^2}$ (b) $-\dfrac{Q}{4\pi a^2}, \dfrac{Q+q}{4\pi a^2}$
    (c) $\dfrac{Q}{4\pi\varepsilon_0 x^2}$ in both situations
13. (a) $3{\cdot}4 \times 10^{13}\ \text{N C}^{-1}$ (b) $1{\cdot}1 \times 10^{12}\ \text{N C}^{-1}$
14. $9 \times 10^5\ \text{N C}^{-1}$
15. $2{\cdot}88 \times 10^{-17}$ J
16. $\rho x/(2\varepsilon_0)$
17. $\rho x/\varepsilon_0$
18. 0·45 N
19. $7{\cdot}5 \times 10^{-7}\ \text{C m}^{-2}$
20. (a) 0·20 N (b) 0·45 s
21. $0{\cdot}505 \times 10^{-12}\ \text{C m}^{-2}$
22. (a) zero (b) σ/ε_0 (c) zero
23. (a) $\dfrac{Q}{2A}$ (b) $\dfrac{Q}{2A\varepsilon_0}$ towards left (c) $\dfrac{Q}{2A\varepsilon_0}$ towards right
    (d) $\dfrac{Q}{2A\varepsilon_0}$ towards right
24. $-Q/2$

□