



Differentials Errors and Approximation Ex14.1 Q8

Let error in radius ( $r$ ) =  $x\%$  of  $r$

$$\Delta r = 0.0x r$$

Let  $v$  = volume of sphere

$$v = \frac{4}{3} \pi r^3$$

Differentiating it with respect to  $r$ ,

$$\frac{dv}{dr} = 4\pi r^2$$

So,

$$\begin{aligned} \Delta v &= \left( \frac{dv}{dr} \right) \times \Delta r \\ &= (4\pi r^2) (0.0x) r \\ \Delta v &= 0.0x \times 4\pi r^3 \end{aligned}$$

$$\begin{aligned} \text{Percentage of error in volume} &= \frac{\Delta v \times 100}{v} \\ &= \frac{(0.0x) 4\pi r^3 \times 100}{\frac{4}{3} \pi r^3} \\ &= 3x\% \end{aligned}$$

Percentage of error in volume = 3 (percentage of error in radius).

Differentials Errors and Approximation Ex14.1 Q9(i)

$$\text{Let } x = 25, x + \Delta x = 25.02$$

$$\Delta x = 25.02 - 25$$

$$\Delta x = 0.02$$

$$\text{Let } y = \sqrt{x}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\left(\frac{dy}{dx}\right)_{x=25} = \frac{1}{2\sqrt{25}}$$

$$\left(\frac{dy}{dx}\right)_{x=25} = \frac{1}{10}$$

Now,

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=25} \times \Delta x$$

$$= \frac{1}{10}(0.02)$$

$$\Delta y = 0.002$$

$$\sqrt{25.02} = y + \Delta y$$

$$= \sqrt{25} + 0.002$$

$$= 5 + 0.002$$

$$\sqrt{25.02} = 5.002$$

Differentials Errors and Approximation Ex14.1 Q9(ii)

$$\text{Let } x = 0.008, x + \Delta x = 0.009$$

$$\Delta x = 0.009 - 0.008$$

$$\Delta x = 0.001$$

$$\begin{aligned}
 \text{Let } y &= x^{\frac{1}{3}} \\
 \frac{dy}{dx} &= \frac{1}{3x^{\frac{2}{3}}} \\
 \left(\frac{dy}{dx}\right)_{x=0.008} &= \frac{1}{3(0.008)^{\frac{2}{3}}} \\
 &= \frac{1}{3(0.04)} \\
 &= \frac{100}{12} \\
 &= 0.8333
 \end{aligned}$$

So,

$$\begin{aligned}
 \Delta y &= \left(\frac{dy}{dx}\right)_{x=0.008} \times \Delta x \\
 &= (0.8333)(0.001) \\
 \Delta y &= 0.0008333 \\
 (0.009)^{\frac{1}{3}} &= y + \Delta y \\
 &= (x)^{\frac{1}{3}} + 0.0008333 \\
 &= (0.008)^{\frac{1}{3}} + 0.0008333 \\
 &= 0.52 + 0.0008333
 \end{aligned}$$

$$(0.009)^{\frac{1}{3}} = 0.208333$$

$$\begin{aligned}\text{Let } x &= 0.008, \quad x + \Delta x = 0.007 \\ \Delta x &= 0.007 - 0.008 \\ \Delta x &= -0.001\end{aligned}$$

$$\begin{aligned}\text{Let } y &= x^{\frac{1}{3}} \\ \frac{dy}{dx} &= \frac{1}{3(x)^{\frac{2}{3}}} \\ \left(\frac{dy}{dx}\right)_{x=0.008} &= \frac{1}{3(0.008)^{\frac{2}{3}}} \\ &= \frac{100}{12} \\ \left(\frac{dy}{dx}\right)_{x=0.008} &= 8.333\end{aligned}$$

$$\begin{aligned}\Delta y &= \left(\frac{dy}{dx}\right)_{x=0.008} \times \Delta x \\ &= (8.333)(-0.001) \\ \Delta y &= -0.008333\end{aligned}$$

$$\begin{aligned}(0.007)^{\frac{1}{3}} &= y + \Delta y \\ &= x^{\frac{1}{3}} - 0.008333 \\ &= (0.008)^{\frac{1}{3}} - 0.008333 \\ &= 0.2 - 0.008333\end{aligned}$$

$$(0.007)^{\frac{1}{3}} = 0.191667$$

\*\*\*\*\* END \*\*\*\*\*

