Indefinite Integrals Ex 19.25 Q15

$$\int \frac{\log x}{x^n}dx = \int (\log x)\left(\frac{1}{x^n}\right)dx$$

by integration by parts

$$\int (\log x)\left(\frac{1}{x^n}\right)dx = \log x \int \left(\frac{1}{x^n}\right)dx - \int \left(\frac{d(\log x)}{dx}\right)\left(\int \left(\frac{1}{x^n}\right)dx\right)dx$$

$$= \log x\left(\frac{x^{1-n}}{1-n}\right) - \int \frac{1}{x}\left(\frac{x^{1-n}}{1-n}\right)dx = \log x\left(\frac{x^{1-n}}{1-n}\right) - \int \left(\frac{x^{-n}}{1-n}\right)dx$$

$$= \log x\left(\frac{x^{1-n}}{1-n}\right) - \left(\frac{1}{1-n}\right)\left(\frac{x^{1-n}}{1-n}\right) = \log x\left(\frac{x^{1-n}}{1-n}\right) - \left(\frac{x^{1-n}}{[1-n]^2}\right) + C$$

Indefinite Integrals Ex 19.25 Q16

Let $\quad I = \int x^2 \sin^2 x\, dx$

$$= \int x^2\left(\frac{1-\cos 2x}{2}\right)dx$$

$$= \int \frac{x^2}{2}dx - \int \left(\frac{x^2\cos 2x}{2}\right)dx$$

$$= \frac{x^3}{6} - \frac{1}{2}\left[\int x^2 \cos 2x\, dx\right]$$

$$= \frac{x^3}{6} - \frac{1}{2}\left[x^2\int \cos 2x\, dx - \int \left(2x\int \cos 2x\, dx\right)dx\right]$$

$$= \frac{x^3}{6} - \frac{1}{2}\left(x^2\frac{\sin 2x}{2}\right) + \frac{1}{2}\times 2\int \left(x\,\frac{\sin 2x}{2}\right)dx$$

$$= \frac{x^3}{6} - \frac{1}{4}x^2\sin 2x + \frac{1}{2}\left[x\int \sin 2x\, dx - \int \left(1\int \sin 2x\, dx\right)dx\right]$$

$$= \frac{x^3}{6} - \frac{1}{4}x^2\sin 2x + \frac{1}{2}\left[x\left(-\frac{\cos 2x}{2}\right) - \int \left(-\frac{\cos 2x}{2}\right)dx\right]$$

$$= \frac{x^3}{6} - \frac{1}{4}x^2\sin 2x - \frac{1}{4}x\cos 2x + \frac{1}{4}\frac{\sin 2x}{2} + c$$

$$I = \frac{x^3}{6} - \frac{1}{4}x^2\sin 2x - \frac{1}{4}x\cos 2x + \frac{1}{8}\sin 2x + c$$

Indefinite Integrals Ex 19.25 Q17

Let $\quad I = \int 2x^3 e^{x^2} x\, dx$

Let $\quad x^2 = t$

$2x\, dx = dt$

$I = \int t \times e^t dt$

Using integration by parts,

$= t\int e^t dt - \int \left(1 \times \int e^t dt\right) dt$

$= te^t - \int e^t dt$

$= te^t - e^t + c$

$= e^t \left(t - 1\right) + c$

$I = e^{x^2} \left(x^2 - 1\right) + c$

Indefinite Integrals Ex 19.25 Q18

Let $\quad I = \int x^3 \cos x^2\, dx$

Let $\quad x^2 = t$

$2x\, dx = dt$

$I = \frac{1}{2} \int t \cos t dt$

Using integration by parts,

$= \frac{1}{2}\left[t\int \cos t\, dt - \int \left(1 \times \int \cos t dt\right) dt\right]$

$= \frac{1}{2}\left[t \times \sin t - \int \sin t\, dt\right]$

$= \frac{1}{2}\left[t \sin t + \cos t\right] + c$

$I = \frac{1}{2}\left[x^2 \sin x^2 + \cos x^2\right] + c$

Indefinite Integrals Ex 19.25 Q19

Let $\quad I = \int x \sin x \cos x\, dx$

$= \int \frac{x}{2}\left(2 \sin x \cos x\right) dx$

$= \frac{1}{2} \int x \sin 2x\, dx$

Using integration by parts,

$= \frac{1}{2}\left[x\int \sin 2x\, dx - \int \left(1 \times \int \sin 2x\, dx\right) dx\right]$

$= \frac{1}{2}\left[x\left(\frac{-\cos 2x}{2}\right) - \int \left(\frac{-\cos 2x}{2}\right) dx\right]$

$= -\frac{1}{4} x \cos 2x + \frac{1}{4} \int \cos 2x\, dx$

$I = -\frac{1}{4} x \cos 2x + \frac{1}{8} \sin 2x + c$

Indefinite Integrals Ex 19.25 Q20

Let $\quad I = \int \sin x \left(\log \cos x\right) dx$

Let $\quad \cos x = t$

$-\sin x \, dx = dt$

$I = -\int \log t \, dt$

$= -\int 1 \times \log t \, dt$

Using integration by parts,

$$= -\left[\log t \int dt - \int \left(\frac{1}{t} \times \int dt\right) dt\right]$$

$$= -\left[t \log t - \int \frac{1}{t} \times t \, dt\right]$$

$$= -\left[t \log t - \int dt\right]$$

$$= -\left[t \log t - t + c_1\right]$$

$$= t\left(1 - t \log t\right) + c$$

$$I = \cos x \left(1 - \log \cos x\right) + c$$

Indefinite Integrals Ex 19.25 Q21

Let $\quad I = \int \left(\log x\right)^2 x \, dx$

Using integration by parts,

$$= \left(\log x\right)^2 \int x \, dx - \int \left(2\left(\log x\right)\left(\frac{1}{x}\right) \int x \, dx\right) dx$$

$$= \frac{x^2}{2}\left(\log x\right)^2 - 2\int \left(\log x\right)\left(\frac{1}{x}\right)\left(\frac{x^2}{2}\right) dx$$

$$= \frac{x^2}{2}\left(\log x\right)^2 - \int x \left(\log x\right) dx$$

$$= \frac{x^2}{2}\left(\log x\right)^2 - \left[\log x \int x \, dx - \int \left(\frac{1}{x} \int x \, dx\right) dx\right]$$

$$= \frac{x^2}{2}\left(\log x\right)^2 - \left[\frac{x^2}{2} \log x - \int \left(\frac{1}{x} \times \frac{x^2}{2}\right) dx\right]$$

$$= \frac{x^2}{2}\left(\log x\right)^2 - \frac{x^2}{2} \log x + \frac{1}{2} \int x \, dx$$

$$= \frac{x^2}{2}\left(\log x\right)^2 - \frac{x^2}{2} \log x + \frac{1}{4} x^2 + c$$

$$I = \frac{x^2}{2}\left[\left(\log x\right)^2 - \log x + \frac{1}{2}\right] + c$$

*********** END **********