Indefinite Integrals Ex 19.28 Q6

Let $I = \int e^x \sqrt{e^{2x}+1}dx$

Let $e^x = t$

$\Rightarrow \quad e^x dx = dt$

$\therefore \quad I = \int \sqrt{t^2+1^2}dt$

$= \frac{t}{2}\sqrt{t^2+1} + \frac{1}{2}\log\left|t+\sqrt{t^2+1}\right| + c$

$\therefore \quad I = \frac{e^x}{2}\sqrt{e^{2x}+1} + \frac{1}{2}\log\left|e^x+\sqrt{e^{2x}+1}\right| + c$

Indefinite Integrals Ex 19.28 Q7

Let $I = \int \sqrt{3^2-x^2}$

We know that,

$\int \sqrt{a^2-x^2} = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$

$\therefore \quad I = \frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3} + c$

Indefinite Integrals Ex 19.28 Q8

Let $I = \int \sqrt{16x^2+25}dx$

$= \int \sqrt{(4x)^2+5^2}dx$

$= 4\int \sqrt{x^2+\left(\frac{5}{4}\right)^2}dx$

$= 4\left\{\frac{x}{2}\sqrt{x^2+\left(\frac{5}{4}\right)^2} + \frac{\left(\frac{5}{4}\right)^2}{2}\log\left|x+\sqrt{x^2+\left(\frac{5}{4}\right)^2}\right| + c\right\}$

$\therefore \quad I = 2x\sqrt{x^2+\frac{25}{16}} + \frac{25}{8}\log\left|x+\sqrt{x^2+\frac{25}{16}}\right| + c$

Indefinite Integrals Ex 19.28 Q9

Let $I = \int \sqrt{4x^2 - 5}dx$

$$= 2\int \sqrt{x^2 - \left(\frac{\sqrt{5}}{2}\right)^2}dx$$

$$= 2\left\{\frac{x}{2}\sqrt{x^2 - \frac{5}{4}} - \frac{5}{8}\log\left|x + \sqrt{x^2 - \frac{5}{4}}\right|\right. + c$$

$$\therefore \quad I = x\sqrt{x^2 - \frac{5}{4}} - \frac{5}{4}\log\left|x + \sqrt{x^2 - \frac{5}{4}}\right| + c$$

Indefinite Integrals Ex 19.28 Q10

Let $I = \int \sqrt{2x^2 + 3x + 4}dx$

$$= \sqrt{2}\int \sqrt{x^2 + \frac{3}{2}x + 2}dx$$

$$= \sqrt{2}\int \sqrt{x^2 + \frac{3}{2}x + \frac{9}{16} + \frac{23}{16}}dx$$

$$= \sqrt{2}\int \sqrt{\left(x + \frac{3}{4}\right)^2 + \left(\frac{\sqrt{23}}{4}\right)^2}dx$$

$$= \sqrt{2}\left\{\frac{\left(x + \frac{3}{4}\right)}{2}\sqrt{x^2 + \frac{3}{2}x + 2} + \frac{23}{32}.\log\left|\left(x + \frac{3}{4}\right) + \sqrt{x^2 + \frac{3}{2}x + 2}\right| + c\right\}$$

$$\therefore \quad I = \frac{4x + 3}{8}\sqrt{2x^2 + 3x + 4} + \frac{23\sqrt{2}}{32}.\log\left|\left(x + \frac{3}{4}\right) + \sqrt{x^2 + \frac{3}{2}x + 2}\right| + c$$

Indefinite Integrals Ex 19.28 Q11

Let $I = \int \sqrt{3 - 2x - 2x^2}dx$

$$= \sqrt{2}\int \sqrt{\frac{3}{2} - x - x^2}dx$$

$$= \sqrt{2}\int \sqrt{\frac{7}{4} - \left(\frac{1}{4} + x + x^2\right)}dx \qquad \left[\text{Adding and subtracting } \frac{1}{4}\right]$$

$$= \sqrt{2}\int \sqrt{\left(\frac{\sqrt{7}}{2}\right)^2 - \left(x + \frac{1}{2}\right)^2}dx$$

$$= \sqrt{2}\left\{\frac{x + \frac{1}{2}}{2}\sqrt{\frac{3}{2} - x - x^2} + \frac{7}{8}\sin^{-1}\left(\frac{x + \frac{1}{2}}{\frac{\sqrt{7}}{2}}\right) + c\right.$$

$$\therefore \quad I = \frac{2x + 1}{4}\sqrt{3 - 2x - 2x^2} + \frac{7\sqrt{2}}{8}\sin^{-1}\left(\frac{2x + 1}{\sqrt{7}}\right) + c$$

********** END **********