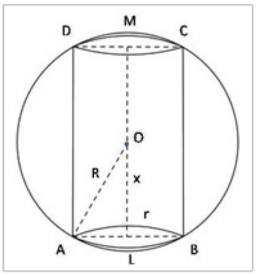
Maxima and Minima 18.5 Q26

Let r be the radius of the base of the cylinder and h be the height of the cylinder.

$\therefore \quad LM = h.$



Let $\quad R = 5\sqrt{3}$ cm be the radius of the sphere.

It is obvious, that for maximum volume of cylinder $ABCD$, the axis of cylinder must be along the diameter of sphere.

Let $\quad OL = x$

$\therefore \quad h = 2x$

Now,

$$\text{In } \Delta AOL,\ AL = \sqrt{AO^2 - OL^2}$$
$$= \sqrt{75 - x^2}$$

Now,

$$v = \text{volume of cylinder} = \pi r^2 h$$
$$\Rightarrow \quad v = \pi AL^2 \times ML$$
$$= \pi\left(75 - x^2\right) \times 2x$$

For maxima and minima of v, we must have,

$$\frac{dv}{dx} = \pi\left[150 - 6x^2\right] = 0$$
$$\Rightarrow \quad x = 5 \text{ cm}$$

Also, $\quad \dfrac{d^2v}{dx^2} = -12\pi x$

At $\quad x = 5,\ \dfrac{d^2v}{dx^2} = -60\pi x < 0$

$\therefore \quad x = 5$ is point of local maxima.

Hence,

The maximum volume of cylinder is $= \pi\left(75 - 25\right) \times 10 = 500\pi \text{ cm}^3$.

Maxima and Minima 18.5 Q27

Let x and y be two positive numbers with

$$x^2 + y^2 = r^2 \qquad \text{---(i)}$$

Let $S = x + y \qquad \text{---(ii)}$

$\therefore \quad S = x + \sqrt{r^2 - x^2} \qquad$ from (ii)

$\therefore \quad \dfrac{dS}{dx} = 1 - \dfrac{x}{\sqrt{r^2 - x^2}}$

For maxima and minima,

$$\frac{dS}{dx} = 0$$

$\Rightarrow \quad 1 - \dfrac{x}{\sqrt{r^2 - x^2}} = 0$

$\Rightarrow \quad x = \sqrt{r^2 - x^2}$

$\Rightarrow \quad 2x^2 = r^2$

$\Rightarrow \quad x = \dfrac{r}{\sqrt{2}}, \dfrac{-r}{\sqrt{2}}$

$\therefore \quad x$ & y are positive numbers

$\therefore \quad x = \dfrac{r}{\sqrt{2}}$

Also, $\dfrac{d^2S}{dx^2} = \dfrac{-\left(\sqrt{r^2 - x^2} + \dfrac{x^2}{\sqrt{r^2 - x^2}}\right)}{r^2 - x^2}$

At, $x = \dfrac{r}{\sqrt{2}}, \dfrac{d^2S}{dx^2} = -\left[\dfrac{\dfrac{r}{\sqrt{2}} + \dfrac{\frac{r^2}{2}}{\frac{r}{\sqrt{2}}}}{\dfrac{r^2}{2}}\right] < 0$

Since $\dfrac{d^2S}{dx^2} < 0$, *the sum is largest when* $x = y = \dfrac{r}{\sqrt{2}}$

Maxima and Minima 18.5 Q28

The given equation of parabola is

$$x^2 = 4y \qquad \text{---(i)}$$

Let $P(x, y)$ be the nearest point on (i) from the point $A(0, 5)$

Let S be the square of the distance of P from A.

$$\therefore \qquad S = x^2 + (y-5)^2 \qquad \text{---(ii)}$$

From (i),

$$S = 4y + (y-5)^2$$

$$\Rightarrow \qquad \frac{dS}{dy} = 4 + 2(y-5)$$

For maxima or minima, we have

$$\frac{dS}{dy} = 0$$

$$\Rightarrow \qquad 4 + 2(y-5) = 0$$

$$\Rightarrow \qquad 2y = 6$$

$$\Rightarrow \qquad y = 3$$

From (i)

$$x^2 = 12$$

$$\therefore \qquad x = \pm 2\sqrt{3}$$

$$\Rightarrow \qquad P = \left(2\sqrt{3}, 3\right) \text{ and } P' = \left(-2\sqrt{3}, 3\right)$$

Now,

$$\frac{d^2S}{dy^2} = 2 > 0$$

$\therefore$ P and P' are the point of local minima

Hence, the nearest points are $P\left(2\sqrt{3}, 3\right)$ and $P'\left(-2\sqrt{3}, 3\right)$.

*********** END **********