Definite Integrals Ex 20.2 Q60

$$\int \sec^2 x \frac{\tan^2 x}{\tan^6 x + 2\tan^3 x + 1} dx$$

$$u = \tan x \rightarrow \frac{du}{dx} = \sec^2 x$$

$$\int \frac{u^2}{u^6 + 2u^3 + 1} du$$

$$v = u^3 \rightarrow \frac{dv}{du} = 3u^2$$

$$\frac{1}{3}\int \frac{1}{v^2 + 2v + 1} dv$$

$$\frac{1}{3}\int \frac{1}{(v+1)^2} dv$$

$$-\frac{1}{3(v+1)}$$

$$-\frac{1}{3(u^3+1)}$$

$$-\frac{1}{3(\tan^3 x + 1)}$$

$$\left\{-\frac{1}{3(\tan^3 x + 1)}\right\}_0^{\frac{\pi}{4}}$$

$$\left\{-\frac{1}{6} + \frac{1}{3}\right\}$$

$$\frac{1}{6}$$

Definite Integrals Ex 20.2 Q61

$$\int_0^{\frac{\pi}{2}} \sqrt{\cos x(1-\cos^2 x)}\tan^2 x\cos^2 x dx$$

$$\int_0^{\frac{\pi}{2}} \sqrt{\cos x\sin^2 x}\sin^2 x dx$$

$$\int_0^{\frac{\pi}{2}} \sqrt{\cos x}\sin^3 x dx$$

$$\cos x = t \rightarrow -\sin x = \frac{dt}{dx}$$

$$-\int_1^0 \sqrt{t}(1-t^2)dt$$

$$\int_0^1 (\sqrt{t}-t^{\frac{5}{2}})dt$$

$$\left\{\frac{2t^{\frac{3}{2}}}{3}-\frac{2t^{\frac{7}{2}}}{7}\right\}_0^1$$

$$\frac{2}{3}-\frac{2}{7}$$

$$\frac{8}{21}$$

Definite Integrals Ex 20.2 Q62

$$I = \int_0^{\pi/2} \frac{\cos x}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^n} dx$$

$$I = \int_0^{\pi/2} \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^n} dx$$

$$I = \int_0^{\pi/2} \frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^{n-1}} dx$$

Let $\cos\frac{x}{2} + \sin\frac{x}{2} = t$

$$\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right) dx = 2dt$$

$x = 0 \Rightarrow t = 1$ and $x = \frac{\pi}{2} \Rightarrow t = \sqrt{2}$

$$I = \int_1^{\sqrt{2}} \frac{2}{(t)^{n-1}} dt$$

$$I = \left[\frac{2t^{-n+2}}{-n+2}\right]_1^{\sqrt{2}}$$

$$I = \frac{2}{2-n}\left[\left(\sqrt{2}\right)^{2-n} - 1\right]$$

$$I = \frac{2}{2-n}\left[2^{1-\frac{n}{2}} - 1\right]$$

*********** END **********