Geometric Progressions Ex 20.4 Q 2

$$9^{\frac{1}{3}} \times 9^{\frac{1}{9}} \times 9^{\frac{1}{27}} \ldots \infty$$

$$= 9^{\left(\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+000\right)}$$

$$= 9^{\left(\frac{\frac{1}{3}}{1-\frac{1}{3}}\right)} \qquad \left[\text{Using } S_\infty = \frac{a}{1-r}\right]$$

$$= 9^{\left(\frac{1}{3}\times\frac{3}{2}\right)}$$

$$= 9^{\frac{1}{2}}$$

$$= 3$$

So,

$$9^{\frac{1}{3}} \times 9^{\frac{1}{9}} \times 9^{\frac{1}{27}} \ldots \infty = 3$$

Geometric Progressions Ex 20.4 Q 3

$$2^{\frac{1}{4}}.4^{\frac{1}{8}}.8^{\frac{1}{16}}.16^{\frac{1}{32}}\ldots\ldots\infty$$

$$= 2^{\frac{1}{4}}.2^{\frac{2}{8}}.2^{\frac{3}{16}}.2^{\frac{4}{32}}\ldots\ldots\infty$$

$$= \left(\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{4}{32}+\ldots\ldots\infty\right)$$

$$= 2$$

$$= 2^5 ------ (1)$$

$$S = \frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{4}{32}+\ldots\ldots\infty$$

$$S = \left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots\ldots\infty\right)2$$

$$\frac{S}{2} = \frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots\ldots\infty$$

$$= \frac{\frac{1}{4}}{1-\frac{1}{2}}$$

$$= \frac{1}{4}\times\frac{2}{1}$$

$$S = \frac{1}{2}$$

$$S = 1$$

Thus $\quad 2^{\frac{1}{4}}.4^{\frac{1}{8}}.8^{\frac{1}{16}}.16^{\frac{1}{32}}\ldots\ldots = 2^1 = 2$

Geometric Progressions Ex 20.4 Q4

$$S_p = 1 + r^p + r^{2p} + \ldots + \infty$$

$$S_p = \frac{1}{1 - r^p}$$

$$s_p = 1 - r^p + r^{2p} + \ldots + \infty$$

$$s_p = \frac{1}{1 + r^p}$$

Now,

$$S_p + s_p = \frac{1}{1 - r^p} + \frac{1}{1 + r^p}$$

$$= \frac{2}{1 - r^{2p}}$$

$$S_p + s_p = 2 \times S_{2p}$$

Geometric Progressions Ex 20.4 Q5.

Here,

$$a = 4$$

$$A_3 - a_5 = \frac{31}{81}$$

$$ar^2 - ar^4 = \frac{32}{81}$$

$$r^2 4\left(1 - r^2\right) = \frac{32}{81}$$

$$r^2\left(1 - r^2\right) = \frac{8}{81}$$

Let $r^2 = A$

$$A\left(1 - A\right) = \frac{8}{81}$$

$$A - A^2 = \frac{8}{81}$$

$$81A - 81A^2 = 8$$

$$81A^2 - 81A + 8 = 0$$

$$A = \frac{81 \pm \sqrt{(81)^2 - 4 \times 81 \times 8}}{81 \times 2}$$

$$= \frac{81 \pm \sqrt{6561 - 2592}}{162}$$

$$= \frac{81 \pm \sqrt{3969}}{162}$$

$$= \frac{81 \pm 63}{162}$$

$$= \frac{81 + 63}{162} \text{ or } \frac{81 - 63}{162}$$

$$= \frac{144}{162} \text{ or } \frac{18}{162}$$

$$r^2 = \frac{8}{9} \text{ or } \frac{1}{9}$$

$$r = \pm\frac{2\sqrt{2}}{3} \text{ or } \pm\frac{1}{3}$$

Since it is a decreasing G.P.

$$r = \frac{2\sqrt{2}}{3}, \frac{1}{3}$$

$$S_\infty = \frac{4}{1 - \frac{2\sqrt{2}}{3}} \text{ and } S_\infty = \frac{4}{1 - \frac{1}{3}}$$

$$S_\infty = \frac{12}{3 - 2\sqrt{2}}, 6$$

*********** END **********