Trigonometric Identities Ex 6.1 Q76

Answer :

Given that,

$$\operatorname{cosec}\theta - \sin\theta = a^3 \qquad \ldots\ldots(1)$$

$$\sec\theta - \cos\theta = b^3 \qquad \ldots\ldots(2)$$

We have to prove $a^2b^2(a^2+b^2)=1$

We know that $\sin^2\theta + \cos^2\theta = 1$

Now from the first equation, we have

$$\operatorname{cosec}\theta - \sin\theta = a^3$$

$$\Rightarrow \frac{1}{\sin\theta} - \sin\theta = a^3$$

$$\Rightarrow \quad \frac{1-\sin^2\theta}{\sin\theta} = a^3$$

$$\Rightarrow \quad \frac{\cos^2\theta}{\sin\theta} = a^3$$

$$\Rightarrow \quad a = \frac{\cos^{2/3}\theta}{\sin^{1/3}\theta}$$

Again from the second equation, we have

$$\sec\theta - \cos\theta = b^3$$

$$\Rightarrow \frac{1}{\cos\theta} - \cos\theta = b^3$$

$$\Rightarrow \quad \frac{1-\cos^2\theta}{\cos\theta} = b^3$$

$$\Rightarrow \quad \frac{\sin^2\theta}{\cos\theta} = b^3$$

$$\Rightarrow \quad b = \frac{\sin^{2/3}\theta}{\cos^{1/3}\theta}$$

Therefore, we have

$$a^2b^2\left(a^2+b^2\right) = \frac{\cos^{4/3}\theta}{\sin^{2/3}\theta}\frac{\sin^{4/3}\theta}{\cos^{2/3}\theta}\left(\frac{\cos^{4/3}\theta}{\sin^{2/3}\theta} + \frac{\sin^{4/3}\theta}{\cos^{2/3}\theta}\right)$$

$$= \sin^{2/3}\theta\cos^{2/3}\theta\left(\frac{\cos^{4/3}\theta}{\sin^{2/3}\theta} + \frac{\sin^{4/3}\theta}{\cos^{2/3}\theta}\right)$$

$$= \cos^{2/3}\theta\cos^{4/3}\theta + \sin^{2/3}\theta\sin^{4/3}\theta$$

$$= \cos^2\theta + \sin^2\theta$$

$$= 1$$

Hence proved.

Trigonometric Identities Ex 6.1 Q77

Answer :

Given that,

$$a\cos^3\theta + 3a\cos\theta\sin^2\theta = m,$$

$$a\sin^3\theta + 3a\cos^2\theta\sin\theta = n$$

We have to prove $(m+n)^{\frac{2}{3}} + (m-n)^{\frac{2}{3}} = 2a^{\frac{2}{3}}$

Adding both the equations, we get

$$m+n = a\cos^3\theta + 3a\cos\theta\sin^2\theta + a\sin^3\theta + 3a\cos^2\theta\sin\theta$$

$$= a(\cos^3\theta + 3\cos^2\theta\sin\theta + 3\cos\theta\sin^2\theta + \sin^3\theta)$$

$$= a(\cos\theta + \sin\theta)^3$$

Also.

$$
\begin{aligned}
m-n &= a\cos^3\theta + 3a\cos\theta\sin^2\theta - (a\sin^3\theta + 3a\cos^2\theta\sin\theta) \\
&= a(\cos^3\theta - 3\cos^2\theta\sin\theta + 3\cos\theta\sin^2\theta - \sin^3\theta) \\
&= a(\cos\theta - \sin\theta)^3
\end{aligned}
$$

Therefore, we have

$$
\begin{aligned}
&(m+n)^{2/3} + (m-n)^{2/3} \\
&\quad = a^{2/3}(\cos\theta + \sin\theta)^2 + a^{2/3}(\cos\theta - \sin\theta)^2 \\
&\quad = a^{2/3}\{(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2\} \\
&\quad = a^{2/3}\{(\cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta) + (\cos^2\theta - 2\cos\theta\sin\theta + \sin^2\theta)\} \\
&\quad = a^{2/3}\{(\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta) + (\cos^2\theta + \sin^2\theta - 2\cos\theta\sin\theta)\} \\
&\quad = a^{2/3}\{(1 + 2\cos\theta\sin\theta) + (1 - 2\cos\theta\sin\theta)\} \\
&\quad = a^{2/3}(1 + 2\cos\theta\sin\theta + 1 - 2\cos\theta\sin\theta) \\
&\quad = 2a^{2/3}
\end{aligned}
$$

Hence proved.

*********** END **********