Indefinite Integrals Ex 19.20 Q1

Let $I = \int \frac{x^2 + x + 1}{x^2 - x} dx$

$$= \int \left[1 + \frac{2x + 1}{x^2 - x}\right] dx$$

$$= x + \int \frac{2x + 1}{x^2 - x} dx + c_1 - - - - (i)$$

$$I_1 = \int \frac{2x + 1}{x^2 - x} dx$$

Let $2x + 1 = \lambda \frac{d}{dx}\left(x^2 - x\right) + \mu$

$$= \lambda\left(2x - 1\right) + \mu$$

$$2x + 1 = \left(2\lambda\right)x - \lambda + \mu$$

Comparing the coefficients of like powers of x,

$2 = 2\lambda \quad \Rightarrow \quad \lambda = 1$

$-\lambda + \mu = 1 \quad \Rightarrow \quad \mu = 2$

so, $I_1 = \int \frac{\left(2x - 1\right) + 2}{x^2 - x} dx$

$$I = \int \frac{2x - 1}{x^2 - x} dx + 2\int \frac{1}{x^2 - x} dx$$

$$I = \int \frac{2x - 1}{x^2 - x} dx + 2\int \frac{1}{x^2 - 2x\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx$$

$$= \int \frac{2x - 1}{x^2 - x} dx + 2\int \frac{1}{\left(x - \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} dx$$

$$I = \log\left|x^2 - x\right| + 2 \times \frac{1}{2\left(\frac{1}{2}\right)} \log\left|\frac{x - \frac{1}{2} - \frac{1}{2}}{x - \frac{1}{2} + \frac{1}{2}}\right| + c_1 \quad \left[\text{since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a}\log\left|\frac{x - a}{x + a}\right| + c\right]$$

$$I_1 = \log\left|x^2 - x\right| + 2\log\left|\frac{x - 1}{x}\right| + c_2 - - - - (ii)$$

Using equation (i) and (ii)

$$I = x + \log\left|x^2 - x\right| + 2\log\left|\frac{x-1}{x}\right| + c$$

Let $2x + 1 = \lambda \frac{d}{dx}\left(x^2 - x\right) + \mu$

$$= \lambda\left(2x - 1\right) + \mu$$

$$2x + 1 = \left(2\lambda\right)x - \lambda + \mu$$

Comparing the coefficients of like powers of x,

$$2 = 2\lambda \qquad \Rightarrow \qquad \lambda = 1$$

$$-\lambda + \mu = 1 \qquad \Rightarrow \qquad \mu = 2$$

so, $\quad I_1 = \int \frac{\left(2x-1\right)+2}{x^2 - x}dx$

$$I = \int \frac{2x-1}{x^2-x}dx + 2\int \frac{1}{x^2-x}dx$$

$$I = \int \frac{2x-1}{x^2-x}dx + 2\int \frac{1}{x^2 - 2x\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}dx$$

$$= \int \frac{2x-1}{x^2-x}dx + 2\int \frac{1}{\left(x - \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}dx$$

$$I = \log\left|x^2 - x\right| + 2 \times \frac{1}{2\left(\frac{1}{2}\right)}\log\left|\frac{x - \frac{1}{2} - \frac{1}{2}}{x - \frac{1}{2} + \frac{1}{2}}\right| + c_1 \quad \left[\text{since, } \int \frac{1}{x^2 - a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c\right]$$

$$I_1 = \log\left|x^2 - x\right| + 2\log\left|\frac{x-1}{x}\right| + c_2 \text{ - - - - (ii)}$$

Using equation (i) and (ii)

$$I = x + \log\left|x^2 - x\right| + 2\log\left|\frac{x-1}{x}\right| + c$$

Indefinite Integrals Ex 19.20 Q2

Let $I = \int \frac{x^2 + x - 1}{x^2 + x - 6}dx$

$$= \int\left[1 + \frac{5}{x^2 + x - 6}\right]dx$$

$$I = x + \int \frac{5}{x^2 + x - 6}dx + c_1 \text{ - - - - (i)}$$

Let $\quad I_1 = 5\int \frac{1}{x^2 + x - 6}dx$

$$= 5\int \frac{1}{x^2 + 2x\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 6}dx$$

$$= 5\int \frac{1}{\left(x + \frac{1}{2}\right)^2 - \left(\frac{5}{2}\right)^2}dx$$

$$= 5\frac{1}{2\left(\frac{5}{2}\right)}\log\left|\frac{x + \frac{1}{2} - \frac{5}{2}}{x + \frac{1}{2} + \frac{5}{2}}\right| + c_2 \qquad \left[\text{since, } \int \frac{1}{x^2 - a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c\right]$$

$$I_1 = \log\left|\frac{x-2}{x+3}\right| + c_2 \text{ - - - - (ii)}$$

Using equation (i) and (ii)

$$I = x + \log\left|\frac{x-2}{x+3}\right| + c$$

Indefinite Integrals Ex 19.20 Q3

Let $I=\int \frac{1-x^2}{x(1-2x)}dx$

$$=\int \frac{1-x^2}{x-2x^2}dx$$

$$=\int \frac{x^2-1}{2x^2-x}dx$$

$$=\int \left[\frac{1}{2}+\frac{\frac{x}{2}-1}{2x^2-x}\right]dx$$

$$I=\frac{1}{2}x+\int \frac{\frac{x}{2}-1}{2x^2-x}dx+c_1 \text{ ----(i)}$$

Let $I_1=\int \frac{\frac{x}{2}-1}{2x^2-x}dx$

Let $\frac{x}{2}-1=\lambda\frac{d}{dx}\left(2x^2-x\right)+\mu$

$$=\lambda(4x-1)+\mu$$

$$\frac{x}{2}-1=(4\lambda)x-\lambda+\mu$$

Comparing the coefficients of like powers of x,

$$\frac{1}{2}=4\lambda \quad\Rightarrow\quad \lambda=\frac{1}{8}$$

$$-\lambda+\mu=-1 \quad\Rightarrow\quad -\left(\frac{1}{8}\right)+\mu=-1$$

$$\mu=-\frac{7}{8}$$

so, $I_1=\int \frac{\frac{1}{8}(4x-1)-\frac{7}{8}}{2x^2-x}dx$

$$I=\frac{1}{8}\int \frac{4x-1}{2x^2-x}dx-\frac{7}{8}\int \frac{1}{2\left(x^2-\frac{x}{2}\right)}dx$$

$$I=\frac{1}{8}\int \frac{4x-1}{2x^2-x}dx-\frac{7}{16}\int \frac{1}{x^2-2x\left(\frac{1}{4}\right)+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2}dx$$

$$=\frac{1}{8}\int \frac{4x-1}{2x^2-x}dx-\frac{7}{16}\int \frac{1}{\left(x-\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2}dx$$

$$I_1=\frac{1}{8}\log\left|2x^2-x\right|-\frac{7}{16}\times\frac{1}{2\left(\frac{1}{4}\right)}\log\left|\frac{x-\frac{1}{4}-\frac{1}{4}}{x-\frac{1}{4}+\frac{1}{4}}\right|+c_2 \quad \left[\text{since, } \int \frac{1}{x^2-a^2}dx=\frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c\right]$$

$$I=\frac{1}{8}\log|x|+\frac{1}{8}\log|2x-1|-\frac{7}{8}\log|1\text{-}2x|+\frac{7}{8}\log 2+\frac{7}{8}\log|x|+c_2$$

$$I_1=\log|x|-\frac{3}{4}\log|1\text{-}2x|+c_3 \text{ ----(ii)} \quad \left[\text{say, } c_3=c_2+\frac{7}{8}\log 2\right]$$

Using equation (i) and (ii)

$$I=\frac{1}{2}x+\log|x|-\frac{3}{4}\log|1\text{-}2x|+c$$

Indefinite Integrals Ex 19.20 Q4

Here the integrand $\frac{x^2+1}{x^2-5x+6}$ isnot proper rational function, so we divide x^2+1 by x^2-5x+6 and find tha

$$\frac{x^2+1}{x^2-5x+6}=1+\frac{5x-5}{x^2-5x+6}=1+\frac{5x-5}{(x-2)(x-3)}$$

Let $\frac{5x-5}{(x-2)(x-3)}=\frac{A}{x-2}+\frac{B}{x-3}$

So that

Equating the coefficients of x and constant terms on both sides, we get $A+B=5$ and $3A+2B=5$. Solving the we get $A=-5$ and $B=10$

Thus, $\frac{x^2+1}{x^2-5x+6}=1-\frac{5}{x-2}+\frac{10}{x-3}$

Therefore, $\int\frac{x^2+1}{(x+1)^2(x+3)}dx=\int dx-5\int\frac{1}{x-2}dx+10\int\frac{x}{x-3}$

$$=x-5\log|x-2|+10\log|x-3|+C.$$

Indefinite Integrals Ex 19.20 Q5

Let $I = \int \frac{x^2}{x^2 + 7x + 10} dx$

$= \int \left\{1 - \frac{7x + 10}{x^2 + 7x + 10}\right\} dx$

$I = x - \int \frac{7x + 10}{x^2 + 7x + 10} dx + c_1 ----(i)$

Let $I_1 = \int \frac{7x + 10}{x^2 + 7x + 10} dx$

Let $7x + 10 = \lambda \frac{d}{dx}\left(x^2 + 7x + 10\right) + \mu$

$= \lambda\left(2x + 7\right) + \mu$

$7x + 10 = \left(2\lambda\right)x + 7\lambda + \mu$

Comparing the coefficients of like powers of x,

$7 = 2\lambda \quad \Rightarrow \quad \lambda = \frac{7}{2}$

$7\lambda + \mu = 10 \quad \Rightarrow \quad 7\left(\frac{7}{2}\right) + \mu = 10$

$\mu = -\frac{29}{2}$

so, $I_1 = \int \frac{\frac{7}{2}\left(2x + 7\right) - \frac{29}{2}}{x^2 + 7x + 10} dx$

$= \frac{7}{2}\int \frac{\left(2x + 7\right)}{x^2 + 7x + 10} dx - \frac{29}{2}\int \frac{1}{x^2 + 2x\left(\frac{7}{2}\right) + \left(\frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 + 10} dx$

$I_1 = \frac{7}{2}\int \frac{2x + 7}{x^2 + 7x + 10} dx - \frac{29}{2}\int \frac{1}{\left(x + \frac{7}{2}\right)^2 - \left(\frac{3}{2}\right)^2} dx$

$I_1 = \frac{2}{7}\log\left|x^2 + 7x + 10\right| - \frac{29}{2} \times \frac{1}{2\left(\frac{3}{2}\right)}\log\left|\frac{x + \frac{7}{2} - \frac{3}{2}}{x + \frac{7}{2} + \frac{3}{2}}\right| + c_2 \qquad \left[\text{since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a}\log\left|\frac{x - a}{x + a}\right| + c\right]$

$= \frac{7}{2}\log\left|x^2 + 7x + 10\right| - \frac{29}{6}\log\left|\frac{x + 2}{x + 5}\right| + c_2 ------(ii)$

Using equation (i) and (ii)

$I = x - \frac{7}{2}\log\left|x^2 + 7x + 10\right| + \frac{29}{6}\log\left|\frac{x + 2}{x + 5}\right| + c$

*********** END **********