



Indefinite Integrals Ex 19.18 Q14

$$\text{Let } I = \int \frac{1}{\sqrt{(1-x^2)[9+(\sin^{-1}x)^2]}} dx$$

$$\text{Let } \sin^{-1}x = t$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} dx = dt$$

$$I = \int \frac{dt}{\sqrt{(3)^2 + t^2}}$$

$$= \log|t + \sqrt{9+t^2}| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2+x^2}} dx = \log|x + \sqrt{a^2+x^2}| + c \right]$$

$$I = \log|\sin^{-1}x + \sqrt{9+(\sin^{-1}x)^2}| + c$$

Indefinite Integrals Ex 19.18 Q15

$$\text{Let } I = \int \frac{\cos x}{\sqrt{\sin^2 x - 2 \sin x - 3}} dx$$

$$\text{Let } \sin x = t$$

$$\Rightarrow \cos x dx = dt$$

$$= \int \frac{dt}{\sqrt{t^2 - 2t - 3}}$$

$$= \int \frac{dt}{\sqrt{t^2 - 2t + (1)^2 - (1)^2 - 3}}$$

$$= \int \frac{dt}{\sqrt{(t-1)^2 - (2)^2}}$$

$$\text{Let } t-1 = u$$

$$\Rightarrow dt = du$$

$$I = \int \frac{du}{\sqrt{u^2 - (2)^2}}$$

$$= \log|u + \sqrt{u^2 - 4}| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log|x + \sqrt{x^2 - a^2}| + c \right]$$

$$= \log|t-1 + \sqrt{(t-1)^2 - 4}| + c$$

$$I = \log|\sin x - 1 + \sqrt{\sin^2 x - 2 \sin x - 3}| + c$$

Indefinite Integrals Ex 19.18 Q16

$$\begin{aligned}
 \text{Let } I &= \int \sqrt{\operatorname{cosec} x - 1} dx \\
 &= \int \sqrt{\frac{1 - \sin x}{\sin x}} dx \\
 &= \int \sqrt{\frac{(1 - \sin x) + (1 + \sin x)}{\sin x (1 + \sin x)}} dx \\
 &= \int \sqrt{\frac{\cos^2 x}{\sin^2 x + \sin x}} dx \\
 &= \int \frac{\cos x}{\sqrt{\sin^2 x + \sin x}} dx
 \end{aligned}$$

$$\begin{aligned}
 \text{Let } \sin x &= t \\
 \Rightarrow \cos x \, dx &= dt \\
 &= \int \frac{dt}{\sqrt{t^2 + t}} \\
 &= \int \frac{dt}{\sqrt{t^2 + 2t\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}} \\
 &= \int \frac{dt}{\sqrt{\left(t + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}
 \end{aligned}$$

$$\begin{aligned}
 \text{Let, } t + \frac{1}{2} &= u \\
 \Rightarrow dt &= du \\
 &= \int \frac{du}{\sqrt{u^2 - \left(\frac{1}{2}\right)^2}} \\
 &= \log \left| u + \sqrt{u^2 - \left(\frac{1}{2}\right)^2} \right| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log \left| x + \sqrt{x^2 - a^2} \right| + c \right] \\
 &= \log \left| \left(t + \frac{1}{2}\right) + \sqrt{\left(t + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} \right| + c
 \end{aligned}$$

$$I = \log \left| \sin x + \frac{1}{2} + \sqrt{\sin^2 x + \sin x} \right| + c$$

Indefinite Integrals Ex 19.18 Q17

To evaluate the following integral follow the steps:

$$\int \frac{\sin x - \cos x}{\sqrt{\sin 2x}} dx = \int \frac{\sin x - \cos x}{\sqrt{(\sin x + \cos x)^2 - 1}} dx$$

Let $\sin x + \cos x = t$ therefore $(\cos x - \sin x) dx = dt$

Now

$$\begin{aligned}
 \int \frac{\sin x - \cos x}{\sqrt{(\sin x + \cos x)^2 - 1}} dx &= - \int \frac{dt}{\sqrt{t^2 - 1}} \\
 &= -\ln \left| t + \sqrt{t^2 - 1} \right| + c \\
 &= -\ln \left| \sin x + \cos x + \sqrt{\sin 2x} \right| + c
 \end{aligned}$$

***** END *****