Question 1:

$$A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \dfrac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$$

In the matrix , write:

(i) The order of the matrix (ii) The number of elements,

(iii) Write the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$

Answer

(i) In the given matrix, the number of rows is 3 and the number of columns is 4.

Therefore, the order of the matrix is 3 × 4.

(ii) Since the order of the matrix is 3 × 4, there are 3 × 4 = 12 elements in it.

(iii) a_{13} = 19, a_{21} = 35, a_{33} = −5, a_{24} = 12, a_{23} = $\dfrac{5}{2}$

Question 2:

If a matrix has 24 elements, what are the possible order it can have? What, if it has 13 elements?

Answer

We know that if a matrix is of the order $m \times n$, it has mn elements. Thus, to find all the possible orders of a matrix having 24 elements, we have to find all the ordered pairs of natural numbers whose product is 24.

The ordered pairs are: (1, 24), (24, 1), (2, 12), (12, 2), (3, 8), (8, 3), (4, 6), and (6, 4)

Hence, the possible orders of a matrix having 24 elements are:

1 × 24, 24 × 1, 2 × 12, 12 × 2, 3 × 8, 8 × 3, 4 × 6, and 6 × 4

(1, 13) and (13, 1) are the ordered pairs of natural numbers whose product is 13.

Hence, the possible orders of a matrix having 13 elements are 1 × 13 and 13 × 1.

Question 3:

If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?

Answer

We know that if a matrix is of the order $m \times n$, it has mn elements. Thus, to find all the possible orders of a matrix having 18 elements, we have to find all the ordered pairs of natural numbers whose product is 18.

The ordered pairs are: (1, 18), (18, 1), (2, 9), (9, 2), (3, 6,), and (6, 3)

Hence, the possible orders of a matrix having 18 elements are:

1 × 18, 18 × 1, 2 × 9, 9 × 2, 3 × 6, and 6 × 3

(1, 5) and (5, 1) are the ordered pairs of natural numbers whose product is 5.

Hence, the possible orders of a matrix having 5 elements are 1 × 5 and 5 × 1.

Question 5:

Construct a 3 × 4 matrix, whose elements are given by

(i) $a_{ij} = \dfrac{1}{2}\left| -3i + j \right|$ (ii) $a_{ij} = 2i - j$

Answer

In general, a 3 × 4 matrix is given by $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$

(i) $a_{ij} = \dfrac{1}{2}\left| -3i + j \right|, i = 1, 2, 3$ and $j = 1, 2, 3, 4$

$\therefore a_{11} = \dfrac{1}{2}\left| -3 \times 1 + 1 \right| = \dfrac{1}{2}\left| -3 + 1 \right| = \dfrac{1}{2}\left| -2 \right| = \dfrac{2}{2} = 1$

$a_{21} = \dfrac{1}{2}\left| -3 \times 2 + 1 \right| = \dfrac{1}{2}\left| -6 + 1 \right| = \dfrac{1}{2}\left| -5 \right| = \dfrac{5}{2}$

$$a_{31} = \frac{1}{2}|-3 \times 3 + 1| = \frac{1}{2}|-9 + 1| = \frac{1}{2}|-8| = \frac{8}{2} = 4$$

$$a_{12} = \frac{1}{2}|-3 \times 1 + 2| = \frac{1}{2}|-3 + 2| = \frac{1}{2}|-1| = \frac{1}{2}$$
$$a_{22} = \frac{1}{2}|-3 \times 2 + 2| = \frac{1}{2}|-6 + 2| = \frac{1}{2}|-4| = \frac{4}{2} = 2$$
$$a_{32} = \frac{1}{2}|-3 \times 3 + 2| = \frac{1}{2}|-9 + 2| = \frac{1}{2}|-7| = \frac{7}{2}$$

$$a_{13} = \frac{1}{2}|-3 \times 1 + 3| = \frac{1}{2}|-3 + 3| = 0$$
$$a_{23} = \frac{1}{2}|-3 \times 2 + 3| = \frac{1}{2}|-6 + 3| = \frac{1}{2}|-3| = \frac{3}{2}$$
$$a_{33} = \frac{1}{2}|-3 \times 3 + 3| = \frac{1}{2}|-9 + 3| = \frac{1}{2}|-6| = \frac{6}{2} = 3$$

$$a_{14} = \frac{1}{2}|-3 \times 1 + 4| = \frac{1}{2}|-3 + 4| = \frac{1}{2}|1| = \frac{1}{2}$$
$$a_{24} = \frac{1}{2}|-3 \times 2 + 4| = \frac{1}{2}|-6 + 4| = \frac{1}{2}|-2| = \frac{2}{2} = 1$$
$$a_{34} = \frac{1}{2}|-3 \times 3 + 4| = \frac{1}{2}|-9 + 4| = \frac{1}{2}|-5| = \frac{5}{2}$$

$$A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$$

Therefore, the required matrix is

(ii) $a_{ij} = 2i - j$, $i = 1,2,3$ and $j = 1,2,3,4$

$$\therefore a_{11} = 2 \times 1 - 1 = 2 - 1 = 1$$
$$a_{21} = 2 \times 2 - 1 = 4 - 1 = 3$$
$$a_{31} = 2 \times 3 - 1 = 6 - 1 = 5$$

$$a_{12} = 2 \times 1 - 2 = 2 - 2 = 0$$
$$a_{22} = 2 \times 2 - 2 = 4 - 2 = 2$$
$$a_{32} = 2 \times 3 - 2 = 6 - 2 = 4$$

$$a_{13} = 2 \times 1 - 3 = 2 - 3 = -1$$
$$a_{23} = 2 \times 2 - 3 = 4 - 3 = 1$$
$$a_{33} = 2 \times 3 - 3 = 6 - 3 = 3$$

$$a_{14} = 2 \times 1 - 4 = 2 - 4 = -2$$
$$a_{24} = 2 \times 2 - 4 = 4 - 4 = 0$$
$$a_{34} = 2 \times 3 - 4 = 6 - 4 = 2$$

$$A = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$$

Therefore, the required matrix is

Question 6:

Find the value of x, y, and z from the following equation:

(i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$
(ii) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

(iii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Answer

(i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

As the given matrices are equal, their corresponding elements are also equal.
Comparing the corresponding elements, we get:

$x = 1$, $y = 4$, and $z = 3$

(ii) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

As the given matrices are equal, their corresponding elements are also equal.
Comparing the corresponding elements, we get:

$x + y = 6$, $xy = 8$, $5 + z = 5$

Now, $5 + z = 5 \Rightarrow z = 0$

We know that:

$(x - y)^2 = (x + y)^2 - 4xy$

$\Rightarrow (x - y)^2 = 36 - 32 = 4$

$\Rightarrow x - y = \pm 2$

Now, when $x - y = 2$ and $x + y = 6$, we get $x = 4$ and $y = 2$

When $x - y = -2$ and $x + y = 6$, we get $x = 2$ and $y = 4$

$\therefore x = 4$, $y = 2$, and $z = 0$ or $x = 2$, $y = 4$, and $z = 0$

(iii)
$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

As the two matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

$x + y + z = 9$ ... (1)

$x + z = 5$ ... (2)

$y + z = 7$ ... (3)

From (1) and (2), we have:

$y + 5 = 9$

$\Rightarrow y = 4$

Then, from (3), we have:

$4 + z = 7$

$\Rightarrow z = 3$

$\therefore x + z = 5$

$\Rightarrow x = 2$

$\therefore x = 2$, $y = 4$, and $z = 3$

Question 7:

Find the value of a, b, c, and d from the equation:

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

Answer

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

As the two matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

$a - b = -1$ ... (1)

$2a - b = 0$ ... (2)

$2a + c = 5$ ... (3)

$3c + d = 13$ ... (4)

From (2), we have:

$b = 2a$

Then, from (1), we have:

$a - 2a = -1$

$\Rightarrow a = 1$

$\Rightarrow b = 2$

Now, from (3), we have:

$2 \times 1 + c = 5$

$\Rightarrow c = 3$

From (4) we have:

$3 \times 3 + d = 13$

$\Rightarrow 9 + d = 13 \Rightarrow d = 4$

$\therefore a = 1$, $b = 2$, $c = 3$, and $d = 4$

Question 8:

$A = \left[a_{ij} \right]_{m \times n}$ is a square matrix, if

(A) $m < n$

(B) $m > n$

(C) $m = n$

(D) None of these

Answer

The correct answer is C.

It is known that a given matrix is said to be a square matrix if the number of rows is equal to the number of columns.

Therefore, $A = \left[a_{ij} \right]_{m \times n}$ is a square matrix, if $m = n$.

********** END *********