Mean Value Theorems Ex 15.1 Q3(viii)

Here,

$$f(x) = \sin 3x \text{ on } [0, \pi]$$

We know that, sine function is continuous and differentiable every where. So, $f(x)$ is continuous is $(0, \pi)$ and differentiable is $(0, \pi)$.

Now,

$$f(0) = \sin 0 = 0$$

$$f(\pi) = \sin 3\pi = 0$$

$$\Rightarrow \quad f(0) = f(\pi)$$

So, Rolle's theorem is applicable, so there must exists a point $c \in (0, \pi)$ such that $f'(c) = 0$.

Now,

$$f(x) = \sin 3x$$

$$f'(x) = 3\cos 3x$$

Now,

$$f'(c) = 0$$

$$\Rightarrow \quad 3\cos 3x = 0$$

$$\Rightarrow \quad \cos 3x = 0$$

$$\Rightarrow \quad 3x = \frac{\pi}{2}$$

$$\Rightarrow \quad x = \frac{\pi}{6} \in (0, \pi)$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(ix)

Here,

$$f(x) = e^{1-x^2} \text{ on } [-1, 1]$$

We know that, exponential function is continuous and differentiable every where. So, $f(x)$ is continuous is $[-1, 1]$ and differentiable is $(-1, 1)$.

Now,

$$f(-1) = e^{1-1} = 1$$

$$f(1) = e^{1-1} = 1$$

$$\Rightarrow \quad f(-1) = 1$$

So, Rolle's theorem is applicable, so there must exist a point $c \in (-1, 1)$ such that $f'(c) = 0$.

Now,

$$f(x) = e^{1-x^2}$$

$$f'(x) = e^{1-x^2}(-2x)$$

Now,

$$f'(c) = 0$$

$$-2ce^{1-c^2} = 0$$

$$\Rightarrow \quad c = 0 \quad \text{or} \quad e^{1-c^2} = 0$$

$$\Rightarrow \quad c = 0 \in (-1, 1)$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(x)

Here,

$$f(x) = \log\left(x^2+2\right) - \log 3 \text{ on } [-1,1]$$

We know that, logarithmic function is continuous and differentiable is its domain, so $f(x)$ is continuous is $[-1,1]$ and differentiable is $(-1,1)$.

Now,

$$f(-1) = \log(1+2) - \log 3 = 0$$

$$f(1) = \log(1+2) - \log 3 = 0$$

$$\Rightarrow \quad f(-1) = f(1)$$

So, Rolle's theorem is applicable, so there must exist a point $c \in (-1,1)$ such that $f'(c) = 0$.

Now,

$$f(x) = \log\left(x^2+2\right) - \log 3$$

$$f'(x) = \frac{(2x)}{x^2+2}$$

Now,

$$f'(c) = 0$$

$$\frac{2c}{c^2+2} = 0$$

$$\Rightarrow \quad c = 0 \in (-1,1)$$

Hence, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q3(xi)

Here,

$$f(x) = \sin x + \cos x \text{ on } \left[0,\frac{\pi}{2}\right]$$

We know that $\sin x$ and $\cos x$ are continuous and differentiable every where, so $f(x)$ is continuous is $\left[0,\frac{\pi}{2}\right]$ and differentiable is $\left(0,\frac{\pi}{2}\right)$.

Now,

$$f(0) = \sin 0 + \cos c0 = 1$$

$$f\left(\frac{\pi}{2}\right) = \frac{\sin\pi}{2} + \frac{\cos\pi}{2} = 1$$

$$\Rightarrow \quad f(0) = f\left(\frac{\pi}{2}\right)$$

So, Rolle's theorem is applicable, so there must exist a point $c \in \left(0,\frac{\pi}{2}\right)$ such that $f'(c) = 0$.

Now,

$$f(x) = \sin x + \cos x$$

$$f'(x) = \cos x - \sin x$$

Now,

$$f'(c) = 0$$

$$\cos c - \sin c = 0$$

$$\Rightarrow \quad \tan c = 1$$

$$\Rightarrow \quad c = \frac{\pi}{4} \in \left(0,\frac{\pi}{2}\right)$$

Hence, Rolle's theorem is verified.

*********** END **********