

And,
$$\frac{1}{4\pi \in_0} = 9 \times 10^9 \, \text{N m}^2 \, \, \text{C}^{-2}$$

The numerical value of the taken quantity will be:

$$4\pi \in_{0} \times \frac{\left(\frac{h}{2\pi}\right)^{2}}{m_{e}e^{2}}$$

$$= \frac{1}{9 \times 10^{9}} \times \frac{\left(\frac{6.63 \times 10^{-34}}{2 \times 3.14}\right)^{2}}{9.1 \times 10^{-31} \times \left(1.6 \times 10^{-19}\right)^{2}}$$

$$=0.53\times10^{-10} m$$

Hence, the value of the quantity taken is of the order of the atomic size.

Question 12.15:

The total energy of an electron in the first excited state of the hydrogen atom is about $-3.4~{\rm eV}.$

- (a) What is the kinetic energy of the electron in this state?
- (b) What is the potential energy of the electron in this state?
- (c) Which of the answers above would change if the choice of the zero of potential energy is changed?

Answer

(a) Total energy of the electron, E = -3.4 eV

Kinetic energy of the electron is equal to the negative of the total energy.

$$\Rightarrow K = -E$$
$$= -(-3.4) = +3.4 \text{ eV}$$

Hence, the kinetic energy of the electron in the given state is +3.4 eV.

(b) Potential energy (U) of the electron is equal to the negative of twice of its kinetic energy.

$$\Rightarrow U = -2 K$$
$$= -2 \times 3.4 = -6.8 \text{ eV}$$

Hence, the potential energy of the electron in the given state is - 6.8 eV.

(c) The potential energy of a system depends on the reference point taken. Here, the potential energy of the reference point is taken as zero. If the reference point is changed, then the value of the potential energy of the system also changes. Since total energy is the sum of kinetic and potential energies, total energy of the system will also change.

Question 12.16:

If Bohr's quantisation postulate (angular momentum = $nh/2\pi$) is a basic law of nature, it should be equally valid for the case of planetary motion also. Why then do we never speak of quantisation of orbits of planets around the sun?

Answer

We never speak of quantization of orbits of planets around the Sun because the angular momentum associated with planetary motion is largely relative to the value of Planck's constant (h). The angular momentum of the Earth in its orbit is of the order of $10^{70}h$. This leads to a very high value of quantum levels n of the order of 10^{70} . For large values of n, successive energies and angular momenta are relatively very small. Hence, the quantum levels for planetary motion are considered continuous.

Question 12.17:

Obtain the first Bohr's radius and the ground state energy of a muonic hydrogen atom [i.e., an atom in which a negatively charged muon (μ^-) of mass about 207m_e orbits around a proton].

Answer

Mass of a negatively charged muon, $\, m_{\mu} = 207 m_{e} \,$

According to Bohr's model,

$$r_e \propto \left(\frac{1}{m_e}\right)$$

And, energy of a ground state electronic hydrogen atom, $^{E_{\mathrm{e}} \propto m_{\mathrm{e}}.}$

Also, energy of a ground state muonic hydrogen atom, $E_n \propto m_n$.

We have the value of the first Bohr orbit, $r_e = 0.53 \ {\rm A} = 0.53 \times 10^{-10} \ {\rm m}$

Let r_{μ} be the radius of muonic hydrogen atom.

At equilibrium, we can write the relation as:

$$\begin{split} & m_{\mu}r_{\mu} = m_{e}r_{e} \\ & 207m_{e} \times r_{\mu} = m_{e}r_{e} \\ & \therefore r_{\mu} = \frac{0.53 \times 10^{-10}}{207} = 2.56 \times 10^{-13} \text{ m} \end{split}$$

Hence, the value of the first Bohr radius of a muonic hydrogen atom is $2.56 \times 10^{-13} \text{ m}.$

We have,

$$E_e = -13.6 \text{ eV}$$

Take the ratio of these energies as:

$$\begin{aligned} \frac{E_e}{E_{\mu}} &= \frac{m_e}{m_{\mu}} = \frac{m_e}{207m_e} \\ E_{\mu} &= 207E_e \\ &= 207 \times (-13.6) = -2.81 \text{ keV} \end{aligned}$$

Hence, the ground state energy of a $muonic\ hydrogen\ atom$ is $-2.81\ keV$.