



Differentiation Ex 11.4 Q11

Given,

$$\sin xy + \cos(x+y) = 1$$

Differentiating with respect to x ,

$$\begin{aligned} \frac{d}{dx} \sin xy + \frac{d}{dx} \cos(x+y) &= \frac{d}{dx} (1) \\ \Rightarrow \cos xy \frac{d}{dx} (xy) - \sin(x+y) \frac{d}{dx} (x+y) &= 0 \quad \text{[Using chain rule and product rule]} \\ \Rightarrow \cos(xy) \left[x \frac{dy}{dx} + y \frac{d}{dx} (x) \right] - \sin(x+y) \left[1 + \frac{dy}{dx} \right] &= 0 \\ \Rightarrow \cos(xy) \left[x \frac{dy}{dx} + y(1) \right] - \sin(x+y) + \sin(x+y) \frac{dy}{dx} &= 0 \\ \Rightarrow x \cos(xy) \frac{dy}{dx} + y \cos(xy) - \sin(x+y) + \sin(x+y) \frac{dy}{dx} &= 0 \\ \Rightarrow [x \cos(xy) + \sin(x+y)] \frac{dy}{dx} &= [\sin(x+y) - y \cos xy] \\ \Rightarrow \frac{dy}{dx} &= \left[\frac{\sin(x+y) - y \cos xy}{x \cos xy + \sin(x+y)} \right] \end{aligned}$$

Differentiation Ex 11.4 Q12

Here,

$$\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$$

Let $x = \sin A, y = \sin B$, so

$$\begin{aligned} \Rightarrow \sqrt{1-\sin^2 A} + \sqrt{1-\sin^2 B} &= a(\sin A - \sin B) \\ \Rightarrow \cos A + \cos B &= a(\sin A - \sin B) \quad \text{[Since } 1 - \sin^2 \theta = \cos^2 \theta \text{]} \\ \Rightarrow a &= \frac{\cos A + \cos B}{\sin A - \sin B} \\ \Rightarrow a &= \frac{2 \cos \frac{A+B}{2} \times \cos \frac{A-B}{2}}{2 \cos \frac{A+B}{2} \times \sin \frac{A-B}{2}} \quad \left[\begin{array}{l} \text{Since, } \sin A - \sin B = 2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right) \\ \cos A + \cos B = 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right) \end{array} \right] \\ \Rightarrow a &= \cot \left(\frac{A-B}{2} \right) \\ \Rightarrow \cot^{-1} a &= \frac{A-B}{2} \\ \Rightarrow 2 \cot^{-1} a &= A-B \\ \Rightarrow 2 \cot^{-1} a &= \sin^{-1} x - \sin^{-1} y \quad \text{[Since } x = \sin A, y = \sin B \text{]} \end{aligned}$$

Differentiating with respect to x ,

$$\begin{aligned} \frac{d}{dx} (2 \cot^{-1} a) &= \frac{d}{dx} (\sin^{-1} x) - \frac{d}{dx} (\sin^{-1} y) \\ \Rightarrow 0 &= \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} \\ \Rightarrow \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} &= \frac{1}{\sqrt{1-x^2}} \\ \Rightarrow \frac{dy}{dx} &= \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} \\ \frac{dy}{dx} &= \sqrt{\frac{1-y^2}{1-x^2}} \end{aligned}$$

Differentiation Ex 11.4 Q13

Here,

$$y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$$

Let $x = \sin A, y = \sin B$

$$\Rightarrow \sin B \sqrt{1-\sin^2 A} + \sin A \sqrt{1-\sin^2 B} = 1$$

$$\Rightarrow \sin B \cos A + \sin A \cos B = 1$$

$$\left[\begin{array}{l} \text{since } 1 - \sin^2 \theta = \cos^2 \theta \text{ and} \\ \sin(x+y) = \sin x \cos y + \cos x \sin y \end{array} \right]$$

$$\Rightarrow \sin(A+B) = 1$$

$$\Rightarrow A+B = \sin^{-1}(1)$$

$$\Rightarrow \sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$$

$$[\text{Since } x = \sin A, y = \sin B]$$

Differentiating with respect to x ,

$$\Rightarrow \frac{d}{dx}(\sin^{-1} x) + \frac{d}{dx}(\sin^{-1} y) = \frac{d}{dx}\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$$

Differentiation Ex 11.4 Q14

Here,

$$xy = 1 \quad \text{---(i)}$$

Differentiating with respect to x ,

$$\frac{d}{dx}(xy) = \frac{d}{dx}(1)$$

$$\Rightarrow x \frac{dy}{dx} + y \frac{d}{dx}(x) = 0$$

[Using product rule]

$$\Rightarrow x \frac{dy}{dx} + y(1) = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

$$\left[\text{Put } x = \frac{1}{y} \text{ from equation (i)} \right]$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y}{\frac{1}{y}}$$

$$\Rightarrow \frac{dy}{dx} = -y^2$$

$$\Rightarrow \frac{dy}{dx} + y^2 = 0$$

Differentiation Ex 11.4 Q15

Here,

$$xy^2 = 1$$

Differentiating with respect to x ,

$$\frac{d}{dx}(xy^2) = \frac{d}{dx}(1)$$

$$\Rightarrow x \frac{d}{dx}(y^2) + y^2 \frac{d}{dx}(x) = 0 \quad \text{[Using product rule]}$$

$$\Rightarrow x(2y) \frac{dy}{dx} + y^2(1) = 0$$

$$\Rightarrow 2xy \frac{dy}{dx} = -y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y^2}{2xy}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{2x}$$

Put $x = \frac{1}{y^2}$ from equation (i)

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{2\left(\frac{1}{y^2}\right)}$$

$$\Rightarrow 2 \frac{dy}{dx} = -y^3$$

$$2 \frac{dy}{dx} + y^3 = 0$$

***** END *****