



Final current  $I_2 = 20 \text{ A}$

Change in current,  $dI = I_2 - I_1 = 20 - 0 = 20 \text{ A}$

Time taken for the change,  $t = 0.5 \text{ s}$

$$\text{Induced emf, } e = \frac{d\phi}{dt} \quad \dots (1)$$

Where  $d\phi$  is the change in the flux linkage with the coil.

Emf is related with mutual inductance as:

$$e = \mu \frac{dI}{dt} \quad \dots (2)$$

Equating equations (1) and (2), we get

$$\begin{aligned} \frac{d\phi}{dt} &= \mu \frac{dI}{dt} \\ d\phi &= 1.5 \times (20) \\ &= 30 \text{ Wb} \end{aligned}$$

Hence, the change in the flux linkage is 30 Wb.

#### Question 6.10:

A jet plane is travelling towards west at a speed of 1800 km/h. What is the voltage difference developed between the ends of the wing having a span of 25 m, if the Earth's magnetic field at the location has a magnitude of  $5 \times 10^{-4} \text{ T}$  and the dip angle is  $30^\circ$ .

Answer

Speed of the jet plane,  $v = 1800 \text{ km/h} = 500 \text{ m/s}$

Wing span of jet plane,  $l = 25 \text{ m}$

Earth's magnetic field strength,  $B = 5.0 \times 10^{-4} \text{ T}$

Angle of dip,  $\delta = 30^\circ$

Vertical component of Earth's magnetic field,

$$\begin{aligned} B_V &= B \sin \delta \\ &= 5 \times 10^{-4} \sin 30^\circ \\ &= 2.5 \times 10^{-4} \text{ T} \end{aligned}$$

Voltage difference between the ends of the wing can be calculated as:

$$\begin{aligned} e &= (B_V) \times l \times v \\ &= 2.5 \times 10^{-4} \times 25 \times 500 \\ &= 3.125 \text{ V} \end{aligned}$$

Hence, the voltage difference developed between the ends of the wings is 3.125 V.

#### Question 6.11:

Suppose the loop in Exercise 6.4 is stationary but the current feeding the electromagnet that produces the magnetic field is gradually reduced so that the field decreases from its initial value of 0.3 T at the rate of  $0.02 \text{ T s}^{-1}$ . If the cut is joined and the loop has a resistance of  $1.6 \Omega$  how much power is dissipated by the loop as heat? What is the source of this power?

Answer

Sides of the rectangular loop are 8 cm and 2 cm.

Hence, area of the rectangular wire loop,

$$\begin{aligned} A &= \text{length} \times \text{width} \\ &= 8 \times 2 = 16 \text{ cm}^2 \\ &= 16 \times 10^{-4} \text{ m}^2 \end{aligned}$$

Initial value of the magnetic field,  $B = 0.3 \text{ T}$

$$\text{Rate of decrease of the magnetic field, } \frac{dB}{dt} = 0.02 \text{ T/s}$$

Emf developed in the loop is given as:

$$e = \frac{d\phi}{dt}$$

Where,

$$\begin{aligned} d\phi &= \text{Change in flux through the loop area} \\ &= AB \end{aligned}$$

$$\begin{aligned} \therefore e &= \frac{d(AB)}{dt} = A \frac{dB}{dt} \\ &= 16 \times 10^{-4} \times 0.02 = 0.32 \times 10^{-4} \text{ V} \end{aligned}$$

Resistance of the loop,  $R = 1.6 \Omega$

The current induced in the loop is given as:

$$i = \frac{e}{R} \\ = \frac{0.32 \times 10^{-4}}{1.6} = 2 \times 10^{-5} \text{ A}$$

Power dissipated in the loop in the form of heat is given as:

$$P = i^2 R \\ = (2 \times 10^{-5})^2 \times 1.6 \\ = 6.4 \times 10^{-10} \text{ W}$$

The source of this heat loss is an external agent, which is responsible for changing the magnetic field with time.

#### Question 6.12:

A square loop of side 12 cm with its sides parallel to X and Y axes is moved with a velocity of  $8 \text{ cm s}^{-1}$  in the positive x-direction in an environment containing a magnetic field in the positive z-direction. The field is neither uniform in space nor constant in time. It has a gradient of  $10^{-3} \text{ T cm}^{-1}$  along the negative x-direction (that is it increases by  $10^{-3} \text{ T cm}^{-1}$  as one moves in the negative x-direction), and it is decreasing in time at the rate of  $10^{-3} \text{ T s}^{-1}$ . Determine the direction and magnitude of the induced current in the loop if its resistance is  $4.50 \text{ m}\Omega$ .

Answer

Side of the square loop,  $s = 12 \text{ cm} = 0.12 \text{ m}$

Area of the square loop,  $A = 0.12 \times 0.12 = 0.0144 \text{ m}^2$

Velocity of the loop,  $v = 8 \text{ cm/s} = 0.08 \text{ m/s}$

Gradient of the magnetic field along negative x-direction,

$$\frac{dB}{dx} = 10^{-3} \text{ T cm}^{-1} = 10^{-1} \text{ T m}^{-1}$$

And, rate of decrease of the magnetic field,

$$\frac{dB}{dt} = 10^{-3} \text{ T s}^{-1}$$

Resistance of the loop,  $R = 4.5 \text{ m}\Omega = 4.5 \times 10^{-3} \Omega$

Rate of change of the magnetic flux due to the motion of the loop in a non-uniform magnetic field is given as:

$$\frac{d\phi}{dt} = A \times \frac{dB}{dx} \times v \\ = 144 \times 10^{-4} \text{ m}^2 \times 10^{-1} \times 0.08 \\ = 11.52 \times 10^{-5} \text{ T m}^2 \text{ s}^{-1}$$

Rate of change of the flux due to explicit time variation in field  $B$  is given as:

$$\frac{d\phi'}{dt} = A \times \frac{dB}{dt} \\ = 144 \times 10^{-4} \times 10^{-3} \\ = 1.44 \times 10^{-5} \text{ T m}^2 \text{ s}^{-1}$$

Since the rate of change of the flux is the induced *emf*, the total induced *emf* in the loop can be calculated as:

$$e = 1.44 \times 10^{-5} + 11.52 \times 10^{-5} \\ = 12.96 \times 10^{-5} \text{ V}$$

$$\therefore \text{Induced current, } i = \frac{e}{R}$$

$$= \frac{12.96 \times 10^{-5}}{4.5 \times 10^{-3}}$$

$$i = 2.88 \times 10^{-2} \text{ A}$$

Hence, the direction of the induced current is such that there is an increase in the flux through the loop along positive z-direction.

#### Question 6.13:

It is desired to measure the magnitude of field between the poles of a powerful loud speaker magnet. A small flat search coil of area  $2 \text{ cm}^2$  with 25 closely wound turns, is positioned normal to the field direction, and then quickly snatched out of the field region. Equivalently, one can give it a quick  $90^\circ$  turn to bring its plane parallel to the field direction. The total charge flown in the coil (measured by a ballistic galvanometer connected to coil) is  $7.5 \text{ mC}$ . The combined resistance of the coil and the galvanometer is  $0.50 \Omega$ . Estimate the field strength of magnet.

Answer

Area of the small flat search coil,  $A = 2 \text{ cm}^2 = 2 \times 10^{-4} \text{ m}^2$

Number of turns on the coil,  $N = 25$

Total charge flowing in the coil,  $Q = 7.5 \text{ mC} = 7.5 \times 10^{-3} \text{ C}$

Total resistance of the coil and galvanometer,  $R = 0.50 \Omega$

Induced current in the coil,

$$I = \frac{\text{Induced emf } (e)}{R} \quad \dots (1)$$

Induced emf is given as:

$$e = -N \frac{d\phi}{dt} \quad \dots (2)$$

Where,

$d\phi$  = Change in flux

Combining equations (1) and (2), we get

$$I = -\frac{N}{R} \frac{d\phi}{dt}$$

$$Idt = -\frac{N}{R} d\phi \quad \dots (3)$$

Initial flux through the coil,  $\phi_i = BA$

Where,

$B$  = Magnetic field strength

Final flux through the coil,  $\phi_f = 0$

Integrating equation (3) on both sides, we have

$$\int Idt = -\frac{N}{R} \int_{\phi_i}^{\phi_f} d\phi$$

But total charge,  $Q = \int Idt$ .

$$\therefore Q = \frac{-N}{R} (\phi_f - \phi_i) = \frac{-N}{R} (-\phi_i) = +\frac{N\phi_i}{R}$$

$$Q = \frac{NBA}{R}$$

$$\therefore B = \frac{QR}{NA}$$

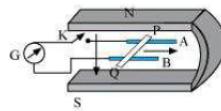
$$= \frac{7.5 \times 10^{-3} \times 0.5}{25 \times 10^{-4}} = 0.75 \text{ T}$$

Hence, the field strength of the magnet is 0.75 T.

#### Question 6.14:

Figure 6.20 shows a metal rod PQ resting on the smooth rails AB and positioned between the poles of a permanent magnet. The rails, the rod, and the magnetic field are in three mutual perpendicular directions. A galvanometer G connects the rails through a switch K. Length of the rod = 15 cm,  $B = 0.50 \text{ T}$ , resistance of the closed loop containing the rod =  $9.0 \text{ m}\Omega$ . Assume the field to be uniform.

(a) Suppose K is open and the rod is moved with a speed of  $12 \text{ cm s}^{-1}$  in the direction shown. Give the polarity and magnitude of the induced emf.



(b) Is there an excess charge built up at the ends of the rods when K is open? What if K is closed?

(c) With K open and the rod moving uniformly, there is *no net force* on the electrons in the rod PQ even though they do experience magnetic force due to the motion of the rod. Explain.

(d) What is the retarding force on the rod when K is closed?

(e) How much power is required (by an external agent) to keep the rod moving at the same speed ( $=12 \text{ cm s}^{-1}$ ) when K is closed? How much power is required when K is open?

(f) How much power is dissipated as heat in the closed circuit? What is the source of this power?

(g) What is the induced emf in the moving rod if the magnetic field is parallel to the rails instead of being perpendicular?

Answer

Length of the rod,  $l = 15 \text{ cm} = 0.15 \text{ m}$

Magnetic field strength,  $B = 0.50 \text{ T}$

Resistance of the closed loop,  $R = 9 \text{ m}\Omega = 9 \times 10^{-3} \Omega$

(a) Induced emf =  $9 \text{ mV}$ ; polarity of the induced emf is such that end P shows positive while end Q shows negative ends.

Speed of the rod,  $v = 12 \text{ cm/s} = 0.12 \text{ m/s}$

Induced emf is given as:

$$e = Bvl$$

$$= 0.5 \times 0.12 \times 0.15$$

$$= 9 \times 10^{-3} \text{ V}$$

$$= 9 \text{ mV}$$

The polarity of the induced emf is such that end P shows positive while end Q shows negative ends.

(b) Yes; when key K is closed, excess charge is maintained by the continuous flow of current.

When key K is open, there is excess charge built up at both ends of the rods.

When key K is closed, excess charge is maintained by the continuous flow of current.

(c) Magnetic force is cancelled by the electric force set-up due to the excess charge of opposite nature at both ends of the rod.

There is no net force on the electrons in rod PQ when key K is open and the rod is moving uniformly. This is because magnetic force is cancelled by the electric force set-up due to the excess charge of opposite nature at both ends of the rods.

**(d)** Retarding force exerted on the rod,  $F = IBl$

Where,

$I$  = Current flowing through the rod

$$= \frac{e}{R} = \frac{9 \times 10^{-3}}{9 \times 10^{-3}} = 1 \text{ A}$$

$$\therefore F = 1 \times 0.5 \times 0.15$$

$$= 75 \times 10^{-3} \text{ N}$$

\*\*\*\*\* END \*\*\*\*\*