Exercise 1E

Question 16:

Given, $x = \left(4 - \sqrt{15}\right)$

Then,

$$\left(x + \frac{1}{x}\right) = \left(4 - \sqrt{15} + \frac{1}{4 - \sqrt{15}}\right)$$

$$= \left(4 - \sqrt{15} + \frac{1}{4 - \sqrt{15}} \times \frac{4 + \sqrt{15}}{4 + \sqrt{15}}\right) \text{ [rationalisation]}$$

$$= \left(4 - \sqrt{15} + \frac{4 + \sqrt{15}}{(4)^2 - \left(\sqrt{15}\right)^2}\right)$$

$$= \left(4 - \sqrt{15} + \frac{4 + \sqrt{15}}{16 - 15}\right)$$

$$= \left(4 - \sqrt{15} + \frac{4 + \sqrt{15}}{1}\right)$$

$$= 4 - \sqrt{15} + 4 + \sqrt{15} = 8.$$

Question 17:

Given, $x = (2 + \sqrt{3})$

$$\therefore \frac{1}{x} = \frac{1}{2 + \sqrt{3}} = \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}} \text{ [rationalising the denominator]}$$

$$= \frac{2 - \sqrt{3}}{(2)^2 - \left(\sqrt{3}\right)^2}$$

$$= \frac{2 - \sqrt{3}}{4 - 3} = \frac{\left(2 - \sqrt{3}\right)}{1} = \left(2 - \sqrt{3}\right)$$

$$\therefore \left(x + \frac{1}{x}\right) = \left(2 + \sqrt{3}\right) + \left(2 - \sqrt{3}\right) = 4$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 4^2 = 16$$

$$\Rightarrow x^2 + \frac{1}{x^2} + 2 \times x \times \frac{1}{x} = 16$$

$$\Rightarrow x^2 + \frac{1}{x^2} = (16 - 2) = 14$$

$$\therefore \left(x^2 + \frac{1}{x^2}\right) = 14.$$

Question 18:

L.H.S =

$$\frac{1}{(3-\sqrt{8})} - \frac{1}{(\sqrt{8}-\sqrt{7})} + \frac{1}{(\sqrt{7}-\sqrt{6})} - \frac{1}{(\sqrt{6}-\sqrt{5})} + \frac{1}{(\sqrt{5}-2)}$$

$$= \frac{3+\sqrt{8}}{(3-\sqrt{8})(3+\sqrt{8})} - \frac{\sqrt{8}+\sqrt{7}}{(\sqrt{8}-\sqrt{7})(\sqrt{8}+\sqrt{7})} + \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6})}$$

$$- \frac{\sqrt{6}+\sqrt{5}}{(\sqrt{6}-\sqrt{5})(\sqrt{6}+\sqrt{5})} + \frac{\sqrt{5}+2}{(\sqrt{5}-2)(\sqrt{5}+2)}$$

$$= \frac{3+\sqrt{8}}{9-8} - \frac{\sqrt{8}+\sqrt{7}}{8-7} + \frac{\sqrt{7}+\sqrt{6}}{7-6} - \frac{\sqrt{6}+\sqrt{5}}{6-5} + \frac{\sqrt{5}+2}{5-4}$$

$$= 3+\sqrt{8}-\sqrt{8}-\sqrt{7}+\sqrt{7}+\sqrt{6}-\sqrt{6}-\sqrt{5}+\sqrt{5}+2$$

$$= 3+2 = 5 = \text{R.H.S}$$

$$\therefore \quad \text{L.H.S} = \text{R.H.S}$$

********** END **********