Indefinite Integrals Ex 19.9 Q5

Let $\quad I = \int \sqrt[3]{\cos^2 x}\, \sin x\, dx \text{ - - - - - (i)}$

Let $\cos x = t$ then,

$d(\cos x) = dt$

$\Rightarrow \quad -\sin x\, dx = dt$

$\Rightarrow \quad dx = -\dfrac{dt}{\sin x}$

Putting $\cos x = t$ and $dx = -\dfrac{dt}{\sin x}$ in equation (i), we get

$$I = \int \sqrt[3]{t^2} \sin x \times \frac{-dt}{\sin x}$$

$$= -\int t^{\frac{2}{3}} \sin x \frac{dt}{\sin x}$$

$$= -\int t^{\frac{2}{3}} dt$$

$$= -\frac{3}{5} \times \frac{t^{\frac{5}{3}}}{} + c$$

$$= -\frac{3}{5} (\cos x)^{\frac{5}{3}} + c$$

$$\therefore \quad I = -\frac{3}{5} (\cos x)^{\frac{5}{3}} + c$$

Indefinite Integrals Ex 19.9 Q6

Let $I = \int \frac{e^x}{\left(1+e^x\right)^2} dx \text{ - - - - - } (i)$

Let $1+e^x = t$ then,

$d\left(1+e^x\right) = dt$

$\Rightarrow \quad e^x\, dx = dt$

$\Rightarrow \quad dx = \frac{dt}{e^x}$

Putting $1+e^x = t$ and $dx = \frac{dt}{e^x}$ in equation (i), we get

$$I = \int \frac{e^x}{t^2} \times \frac{dt}{e^x}$$

$$= \int \frac{dt}{t^2}$$

$$= \int t^{-2}\, dt$$

$$= -t^{-1} + c$$

$$= -\frac{1}{t} + c$$

$$= -\frac{1}{1+e^x} + c$$

$\therefore \quad I = -\frac{1}{1+e^x} + c$

Indefinite Integrals Ex 19.9 Q7

Let $I = \int \cot^3 x \operatorname{cosec}^2 x\, dx \text{ - - - - - } (i)$

Let $\cot x = t$ then,

$d(\cot x) = dt$

$\Rightarrow \quad -\operatorname{cosec}^2 x dx = dt$

$\Rightarrow \quad dx = -\frac{dt}{\operatorname{cosec}^2 x}$

Putting $\cot x = t$ and $dx = -\frac{dt}{\operatorname{cosec}^2 x}$ in equation (i), we get

$$I = \int t^3 \operatorname{cosec}^2 x \times \frac{-dt}{\operatorname{cosec}^2 x}$$

$$= -\int t^3 dt$$

$$= -\frac{t^4}{4} + c$$

$$= -\frac{\cot^4 x}{4} + c$$

$\therefore \quad I = -\frac{\cot^4 x}{4} + c$

Indefinite Integrals Ex 19.9 Q8

Let $\quad I = \int \frac{\left\{e^{\sin^{-1}x}\right\}^2}{\sqrt{1-x^2}}\,dx \text{ - - - - - } (i)$

Let $\sin^{-1}x = t$ then,

$d\left(\sin^{-1}x\right) = dt$

$\Rightarrow \quad \frac{1}{\sqrt{1-x^2}}\,dx = dt$

$\Rightarrow \quad dx = \sqrt{1-x^2}\,dt$

Putting $\sin^{-1}x = t$ and $dx = \sqrt{1-x^2}dt$ in equation (i), we get

$$I = \int \frac{\left(e^t\right)^2}{\sqrt{1-x^2}} \times \sqrt{1-x^2}dt$$
$$= \int e^{2t}dt$$
$$= \frac{e^{2t}}{2} + c$$
$$= \frac{e^{2\sin^{-1}x}}{2} + c$$

$$\therefore \quad I = \frac{\left\{e^{\sin^{-1}x}\right\}^2}{2} + c$$

Indefinite Integrals Ex 19.9 Q9

Let $\quad I = \int \frac{1+\sin x}{\sqrt{x-\cos x}}\,dx \text{ - - - - - } (i)$

Let $x - \cos x = t$ then,

$d\left(x-\cos x\right) = dt$

$\Rightarrow \quad \left[1-\left(-\sin x\right)\right]dx = dt$

$\Rightarrow \quad \left(1+\sin x\right)dx = dt$

Putting $x - \cos x = t$ and $\left(1+\sin x\right)dx = dt$ in equation (i), we get

$$I = \int \frac{dt}{\sqrt{t}}$$
$$= \int t^{\frac{-1}{2}}dt$$
$$= 2t^{\frac{1}{2}} + c$$
$$= 2\left(x-\cos x\right)^{\frac{1}{2}} + c$$

$$\therefore \quad I = 2\sqrt{x-\cos x} + c$$

*********** END **********