Tangents and Normals Ex 16.3 Q1(i)

We know that angle of intersection of two curves is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad \text{---(A)}$$

Where m_1 and m_2 are slopes of curves.

The given equations are

$$y^2 = x \qquad \text{---(i)}$$

$$x^2 = y \qquad \text{---(ii)}$$

$$m_1 = \frac{dy}{dx} = \frac{1}{2y}$$

$$m_2 = \frac{dy}{dx} = 2x$$

Solving (i) and (ii)

$$x^4 - x = 0 \quad \Rightarrow \quad x\left(x^3 - 1\right) = 0$$

and $\quad y = 0, 1$

$$\therefore \quad m_1 = \frac{1}{2}, \infty \quad \text{and} \quad m_2 = 0 \text{ or } 2$$

$$\therefore \quad \tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \frac{3}{4}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{3}{4}\right)$$

and $\quad \tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \infty$

$$\theta = \frac{\pi}{2}$$

Tangents and Normals Ex 16.3 Q1(ii)

We know that angle of intersection of two curves is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad ---(A)$$

Where m_1 and m_2 are slopes of curves.

$y = x^2 \qquad ---(i)$

$x^2 + y^2 = 20 \qquad ---(ii)$

Solving (i) and (ii)

$y + y^2 = 20$

$\Rightarrow \quad y^2 + y - 20 = 0$

$\Rightarrow \quad (y + 5)(y - 4) = 0$

$\Rightarrow \quad y = -5, 4$

$\therefore \quad x = \sqrt{-5}, \pm 2$

$\therefore$ Points are $P = (2, 4)$, $Q = (-2, 4)$

Now,

Slope m_1 for (i)

$m_1 = 2x = 4$

Slope m_2 for (ii)

$$m_2 = \frac{dy}{dx} = \frac{-x}{y} = \frac{-1}{2}$$

Now,

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{\frac{-1}{2} - 4}{1 - \frac{1}{2} \times 4}\right|$$

$$= \frac{9}{2}$$

$\therefore \quad \theta = \tan^{-1}\frac{9}{2}$

Tangents and Normals Ex 16.3 Q1(iii)

We know that angle of intersection of two curves is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad \text{---(A)}$$

Where m_1 and m_2 are slopes of curves.

$$2y^2 = x^3 \qquad \text{---(i)}$$
$$y^2 = 32x \qquad \text{---(ii)}$$

Solving (i) and (ii)

$$x^3 = 64x$$
$$\Rightarrow \quad x\left(x^2 - 64\right) = 0$$
$$\Rightarrow \quad x\left(x+8\right)\left(x-8\right) = 0$$
$$\Rightarrow \quad x = 0, -8, 8$$

$$\therefore \quad y = 0, -, 16$$

$$\therefore \quad P = \left(0, 0\right),\ Q = \left(8, 16\right)$$

Now,

$$m_1 = \frac{dy}{dx} = \frac{3x^2}{4y} = 0 \text{ or } 3$$
$$m_2 = \frac{dy}{dx} = \frac{32}{2y} = \infty \text{ or } 1$$

From (A)

$$\tan\theta = \left|\frac{\infty - 0}{10}\right| = \infty \Rightarrow \theta = \frac{\pi}{2}$$
$$\text{and} \quad \tan\theta = \left|\frac{3-1}{13} = \frac{2}{4} = \frac{1}{2}\right|$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{1}{2}\right)$$

Thus,

$$\theta = \frac{\pi}{2} \text{ and } \tan^{-1}\left(\frac{1}{2}\right)$$

Tangents and Normals Ex 16.3 Q1(iv)

We have,

$$x^2 + y^2 - 4x - 1 = 0 \qquad \text{---(i)}$$

and $$x^2 + y^2 - 2y - 9 = 0 \qquad \text{---(ii)}$$

Equation (i) can be written as

$$(x-2)^2 + y^2 - 5 = 0 \qquad \text{---(iii)}$$

Subtracting (ii) from (i), we get

$$-4x + 2y + 8 = 0$$

$$\Rightarrow \quad y = 2x - 4$$

Substituting in (iii), we get

$$(x-2)^2 + (2x-4)^2 - 5 = 0$$

$$\Rightarrow \quad (x-2)^2 + 4(x-2)^2 - 5 = 0$$

$$\Rightarrow \quad (x-2)^2 = 1$$

$$\Rightarrow \quad x - 2 = 1,\ x - 2 = -1$$

$$\Rightarrow \quad x = 3 \text{ or } x = 1$$

$$\therefore \quad y = 2(3) - 4 = 2 \text{ or } y = -2$$

$\therefore$ The points of intersection of the two curves are $(3,2)$ and $(-1,-2)$

Differentiation (i) and (ii), w.r.t x we get

$$2x + 2y\frac{dy}{dx} - 4 = 0 \qquad \text{---(iv)}$$

and $$2x + 2y\frac{dy}{dx} - 2\frac{dy}{dx} = 0 \qquad \text{---(v)}$$

$\therefore$ At $(3,2)$, from equation (iv) we have,

$$\left(\frac{dy}{dx}\right)_{C_1} = \frac{4 - 2(3)}{2(2)} = \frac{-1}{2}$$

$$\left(\frac{dy}{dx}\right)_{C_2} = \frac{-2(3)}{(2 \times 2 - 3)} = \frac{-6}{2} = -3$$

$\therefore$ If φ is the angle between the curves

Then,

$$\tan\varphi = \frac{\left(\frac{dy}{dx}\right)_{C_1} - \left(\frac{dy}{dx}\right)_{C_2}}{1 + \left(\frac{dy}{dx}\right)_{C_1}\left(\frac{dy}{dx}\right)_{C_2}}$$

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