Exercise 10B

Question 22:

$9x^2 - 4 = 0$

Comparing it with $ax^2 + bx + c = 0$

a = 9, b = 0, c = -4

$\therefore D = \left(b^2 - 4ac\right) = \left[(0)^2 - \left(4 \times 9 \times (-4)\right)\right] = 144 > 0$

Hence, the given equation has real roots, given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{0 + \sqrt{144}}{2 \times 9} = \frac{12}{2 \times 9} = \frac{2}{3}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{0 - \sqrt{144}}{2 \times 9} = \frac{-12}{2 \times 9} = \frac{-2}{3}$$

Hence, $\frac{2}{3}$ and $\frac{-2}{3}$ are the roots of the given equation

Question 23:

The given equation is $3a^2x^2 + 10x - 8\sqrt{3} = 0$

Comparing it with $Ax^2 + Bx + C = 0$, we get

$A = 3a^2, B = 8ab, C = 4b^2$

$\therefore D = \left(b^2 - 4ac\right) = \left[(8ab)^2 - 4 \times 3a^2 \times 4b^2\right] = \left[64a^2b^2 - 48a^2b^2\right]$

$= 16a^2b^2 > 0$

Hence, the given equation has real roots, given by

$$\alpha = \frac{-B + \sqrt{D}}{2A} = \frac{-8ab + \sqrt{16a^2b^2}}{2 \times 3a^2} = \frac{-4ab}{2 \times 3a^2} = \frac{-2b}{3a}$$

$$\beta = \frac{-B - \sqrt{D}}{2A} = \frac{-8ab - \sqrt{16a^2b^2}}{2 \times 3a^2} = \frac{-12ab}{2 \times 3a^2} = \frac{-2b}{a}$$

Hence, $\frac{-2b}{3a}$ and $\frac{-2b}{a}$ are the roots of the given equation

Question 24:

The given equation is $p^2x^2 + \left(p^2 - q^2\right)x - q^2 = 0$

Comparing it with $ax^2 + bx + c = 0$

$a = p^2,\ b = \left(p^2 - q^2\right),\ c = -q^2$

$\therefore D = \left(b^2 - 4ac\right) = \left[\left(p^2 - q^2\right)^2 - 4 \times p^2 \times \left(-q^2\right)\right]$

$= q^4 + p^4 - 2p^2q^2 + 4p^2q^2 \Rightarrow p^4 + q^4 + 2p^2q^2$

$\Rightarrow \left(p^2\right)^2 + \left(q^2\right)^2 + 2 \times p^2 \times q^2 \Rightarrow \left(p^2 + q^2\right)^2 > 0$

hence, the given equation has real roots, given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-\left(p^2 - q^2\right) + \left(p^2 + q^2\right)}{2 \times p^2} = \frac{2q^2}{2p^2} = \frac{q^2}{p^2}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-\left(p^2 - q^2\right) - \left(p^2 + q^2\right)}{2 \times p^2} = \frac{-2p^2}{2p^2} = -1$$

Hence, $\frac{q^2}{p^2}$ and -1 are the roots of the given equation

Question 25:

The given equation is $x^2 - 2ax + \left(a^2 - b^2\right) = 0$

Comparing it with $Ax^2 + Bx + C = 0$

$A = 1,\ B = -2a,\ C = \left(a^2 - b^2\right)$

$\therefore D = \left(B^2 - 4AC\right) = \left[(-2a)^2 - 4 \times (1) \times \left(a^2 - b^2\right)\right]$

$= \left[4a^2 - 4a^2 + 4b^2\right] = 4b^2 \geq 0$

$$\alpha = \frac{-B + \sqrt{D}}{2A} = \frac{2a + \sqrt{4b^2}}{2 \times 1} = \frac{2a + 2b}{2} = \frac{2(a+b)}{2} = a + b$$

$$\beta = \frac{-B - \sqrt{D}}{2A} = \frac{2a - \sqrt{4b^2}}{2 \times 1} = \frac{2a - 2b}{2} = \frac{2(a-b)}{2} = a - b$$

Hence, (a +b) and (a - b) are the roots of the given equation

Question 26:

The given equation is $abx^2 + (b^2 - ac)x - bc = 0$

Comparing it with $Ax^2 + Bx + C = 0$

$$A = ab,\ B = (b^2 - ac),\ \ C = -bc$$

$$\therefore D = B^2 - 4AC = (b^2 - ac)^2 - 4 \times ab \times (-bc)$$

$$\Rightarrow b^4 + a^2c^2 - 2b^2ac + 4ab^2c$$

$$\Rightarrow b^4 + a^2c^2 + 2b^2ac = (b^2 + ac)^2 \geq 0$$

Hence, the given equation has real roots, given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(b^2 - ac) + \sqrt{(b^2 + ac)^2}}{2 \times ab}$$

$$= \frac{-b^2 + ac + b^2 + ac}{2ab}$$

$$= \frac{2ac}{2ab} = \frac{c}{b}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(b^2 - ac) - (b^2 + ac)}{2 \times ab}$$

$$= \frac{-b^2 + ac - b^2 - ac}{2ab}$$

$$= \frac{-2b^2}{2ab} = \frac{-b}{a}$$

Hence, $\frac{c}{b}$ and $\frac{-b}{a}$ are the roots of the given equation

Question 27:

The given equation is $12abx^2 - \left(9a^2 - 8b^2\right)x - 6ab = 0$

Comparing it with $Ax^2 + Bx + C = 0$

$$A = 12ab,\ \ B = -\left(9a^2 - 8b^2\right) = 8b^2 - 9a^2, C = -6ab$$

$$\therefore D = B^2 - 4AC = \left(8b^2 - 9a^2\right)^2 - 4 \times 12ab \times (-6ab)$$

$$= 64b^4 + 81a^4 - 144a^2b^2 + 288a^2b^2$$

$$\Rightarrow 64b^2 + 81a^4 + 144a^2b^2 \Rightarrow \left(8b^2\right)^2 + \left(9a^2\right)^2 + 2 \times 8b^2 \times 9a^2$$

$$\Rightarrow \left(8b^2 + 9a^2\right)^2 \geq 0$$

Hence, the given equation has real roots, given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{9a^2 - 8b^2 + 8b^2 + 9a^2}{2 \times 12ab} = \frac{18a^2}{2 \times 12ab} = \frac{3a}{4b}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{\left(9a^2 - 8b^2\right) - \left(8b^2 + 9a^2\right)}{2 \times 12ab} = \frac{-16a^2}{2 \times 12ab} = \frac{-2b}{3a}$$

Hence, $\frac{3a}{4b}$ and $\frac{-2b}{3a}$ are the roots of given equation

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