Definite Integrals Ex 20.4B Q17

Let $I = \int_0^{\pi} x \cos^2 x \, dx$

$$I = \int_0^{\pi} (\pi - x) \cos^2 (\pi - x) \, dx \qquad \left[\because \int_0^a f(x) dx = \int_0^a f(a-x) \, dx\right]$$

$$I = \pi \int_0^{\pi} \cos^2 x \, dx - \int_0^{\pi} x \cos^2 x \, dx$$

$$2I = \pi \int_0^{\pi} \cos^2 x \, dx$$

$$= \pi \int_0^{\pi} \left(\frac{1 + \cos 2x}{2}\right) dx \qquad \text{Since } \cos^2 x = \frac{1 + \cos 2x}{2}$$

$$= \frac{\pi}{2} \int_0^{\pi} (1 + \cos 2x) \, dx$$

$$= \frac{\pi}{2} \left[x + \left(-\frac{\sin 2x}{2}\right)\right]_0^{\pi}$$

$$\therefore 2I = \frac{\pi}{2}\left[\pi - \frac{\sin 2\pi}{2} - 0 + \frac{\sin 0}{2}\right]$$

$$\Rightarrow 2I = \frac{\pi}{2}[\pi - 0 - 0 + 0]$$

$$I = \frac{\pi^2}{4}$$

Definite Integrals Ex 20.4B Q18

$$I = \int_{\pi/6}^{\pi/3} \frac{1}{1 + \cot^{3/2} x} dx$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sin^{3/2}\left(\frac{\pi}{2} - x\right)}{\sin^{3/2}\left(\frac{\pi}{2} - x\right) + \cos^{3/2}\left(\frac{\pi}{2} - x\right)} dx = \int_{\pi/6}^{\pi/3} \frac{\cos^{3/2}(x)}{\cos^{3/2}(x) + \sin^{3/2}(x)} dx$$

$$\therefore 2I = \int_{\pi/6}^{\pi/3} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx + \int_{\pi/6}^{\pi/3} \frac{\cos^{3/2}(x)}{\cos^{3/2}(x) + \sin^{3/2}(x)} dx$$

$$2I = \int_{\pi/6}^{\pi/3} \frac{\sin^{3/2} x + \cos^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx$$

$$I = \frac{1}{2} \int_{\pi/6}^{\pi/3} dx$$

$$I = \frac{\pi}{12}$$

Definite Integrals Ex 20.4B Q19

$$I=\int_0^{\frac{\pi}{2}}\frac{\tan^7 x}{\tan^7 x+\cot^7 x}dx$$

$$I=\int_0^{\frac{\pi}{2}}\frac{\tan^7\left(\frac{\Pi}{2}-x\right)}{\tan^7\left(\frac{\Pi}{2}-x\right)+\cot^7\left(\frac{\Pi}{2}-x\right)}dx$$

$$I=\int_0^{\frac{\pi}{2}}\frac{\cot^7 x}{\tan^7 x+\cot^7 x}dx$$

Hence

$$2I=\int_0^{\frac{\pi}{2}}\frac{\tan^7 x}{\tan^7 x+\cot^7 x}+\frac{\cot^7 x}{\tan^7 x+\cot^7 x}dx$$

$$2I=\int_0^{\frac{\pi}{2}}1dx$$

$$2I=\frac{\Pi}{2}$$

$$I=\frac{\Pi}{4}$$

Definite Integrals Ex 20.4B Q20

$$I=\int_2^8\frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}dx$$

$$I=\int_2^8\frac{\sqrt{10-(8+2-x)}}{\sqrt{(8+2-x)}+\sqrt{10-(8+2-x)}}dx$$

$$I=\int_2^8\frac{\sqrt{x}}{\sqrt{x}+\sqrt{10-x}}dx$$

$$2I=\int_2^8\frac{\sqrt{x}}{\sqrt{x}+\sqrt{10-x}}+\frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}dx$$

$$2I=\int_2^8 1dx$$

$$2I=6$$

$$I=3$$

*********** END **********