Algebra of Matrices Ex 5.3 Q49

Let, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Given,

$$A\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} a+b & -2a+4b \\ c+d & -2c+4d \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$$a + b = 6 \quad \text{---(i)}$$

$$-2a + 4b = 0 \quad \text{---(ii)}$$

$$c + d = 0 \quad \text{---(iii)}$$

$$-2c + 4d = 6 \quad \text{---(iv)}$$

Solving equation (i) and (ii)

$$4a + 4b = 24$$

$$\begin{array}{l} -2a + 4b = 0 \\ \;(+)\quad\; (-) \\ \hline 6a \qquad\quad = 24 \end{array}$$

$$\Rightarrow \quad a = \frac{24}{6}$$

$$a = 4$$

Put $a = 4$ in equation (i)

$$a + b = 6$$
$$4 + b = 6$$
$$b = 6 - 4$$
$$b = 2$$

Solving equation (iii) and (iv)

$$\begin{array}{r} 2c + 2d = 0 \\ \underline{-2c + 4d = 6} \\ 6d = 6 \end{array}$$
$$d = \frac{6}{6}$$
$$d = 1$$

Put $d = 1$ in equation (iii)

$$c + d = 0$$
$$c = -1$$

Hence,

$$A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q50

Given,

$$A = \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$$

$$A^2 = A \times A$$

$$= \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

$$A^4 = A^2 \times A^2$$

$$= 0 \times 0$$

$$= 0$$

$$A^{16} = A^4 \times A^4$$

$$= 0 \times 0$$

$$= 0$$

So,

A^{16} is a nill matrix

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Solving the LHS of the given equation we have ,

$$\Rightarrow \quad A + B = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 0 & -x + 1 \\ x + 1 & 0 \end{bmatrix}$$

$$(A + B)^2 = \begin{bmatrix} 0 & -x + 1 \\ x + 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & -x + 1 \\ x + 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -x + 1 \\ x + 1 & 0 \end{bmatrix}$$

$$(A + B)^2 = \begin{bmatrix} (0 \times 0) + ((-x + 1) \times (x + 1)) & (0 \times (-x + 1)) + ((-x + 1) \times 0) \\ ((x + 1) \times 0) + (0 \times (x + 1)) & ((x + 1) \times (-x + 1)) + (0 \times 0) \end{bmatrix}$$

$$(A + B)^2 = \begin{bmatrix} 1 - x^2 & 0 \\ 0 & 1 - x^2 \end{bmatrix}.$$

Solving the RHS we get,

$$\Rightarrow \quad A^2 + B^2 = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}\begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 + B^2 = \begin{bmatrix} (0 \times 0) + ((-x) \times (x)) & (0 \times (-x)) + ((-x) \times 0) \\ ((x) \times 0) + (0 \times (x)) & ((x) \times (-x)) + (0 \times 0) \end{bmatrix} + \begin{bmatrix} (0 \times 0) + (1 \times 1) & (0 \times 1) + (1 \times 0) \\ (1 \times 0) + (0 \times 1) & (1 \times 1) + (0 \times 0) \end{bmatrix}$$

$$A^2 + B^2 = \begin{bmatrix} -x^2 & 0 \\ 0 & -x^2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^2 + B^2 = \begin{bmatrix} 1 - x^2 & 0 \\ 0 & 1 - x^2 \end{bmatrix}$$

Subsituting the value of $x^2 = -1$ in the LHS and RHS above,

$$\Rightarrow \quad (A + B)^2 = \begin{bmatrix} 1 - x^2 & 0 \\ 0 & 1 - x^2 \end{bmatrix} = \begin{bmatrix} 1+1 & 0 \\ 0 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \text{ and}$$

$$A^2 + B^2 = \begin{bmatrix} 1 - x^2 & 0 \\ 0 & 1 - x^2 \end{bmatrix} = \begin{bmatrix} 1+1 & 0 \\ 0 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \quad (A + B)^2 = A^2 + B^2.$$

*********** END **********