Differentiation Ex 11.4 Q1

Given,

$$xy = c^2$$

Differentiate with respect to x,

$$\frac{d}{dx}(xy) = \frac{d}{dx}(c^2)$$

$$\Rightarrow \quad x\frac{dy}{dx} + y\frac{d}{dx}(x) = 0 \qquad \text{[Using product rule]}$$

$$\Rightarrow \quad x\frac{dy}{dx} + y = 0$$

$$\Rightarrow \quad x\frac{dy}{dx} = -y$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{y}{x}$$

Differentiation Ex 11.4 Q2

Here, $y^3 - 3xy^2 = x^3 + 3x^2y$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{d}{dx}(y^3) - \frac{d}{dx}(3xy^2) = \frac{d}{dx}(x^3) + \frac{d}{dx}(3x^2y)$$

$$\Rightarrow \quad 3y^2\frac{dy}{dx} - 3\left[x\frac{d}{dx}y^2\frac{d}{dx}(x)\right] = 3x^2 + 3\left[x^2\frac{d}{dx}(y) + y\frac{d}{dx}(x^2)\right] \qquad \text{[Using product rule]}$$

$$\Rightarrow \quad 3y^2\frac{dy}{dx} - 3\left[x(2y)\frac{dy}{dx} + y^2\right] = 3x^2 + 3\left[x^2\frac{dy}{dx} + y(2x)\right]$$

$$\Rightarrow \quad 3y^2\frac{dy}{dx} - 6xy\frac{dy}{dx} - 3y^2 = 3x^2 + 3x^2\frac{dy}{dx} + 6xy$$

$$\Rightarrow \quad 3y^2\frac{dy}{dx} - 6xy\frac{dy}{dx} - 3x^2\frac{dy}{dx} = 3x^2 + 6xy + 3y^2$$

$$\Rightarrow \quad 3\frac{dy}{dx}(y^2 - 2xy - x^2) = 3(x^2 + 2xy + y^2)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{3(x+y)^2}{3(y^2 - 2xy - x^2)}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{(x+y)^2}{y^2 - 2xy - x^2}$$

Differentiation Ex 11.4 Q3

Here, $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

Differentiate it with respect to x,

$$\frac{d}{dx}\left(x^{\frac{2}{3}}\right) + \frac{d}{dx}\left(y^{\frac{2}{3}}\right) = \frac{d}{dx}\left(a^{\frac{2}{3}}\right)$$

$$\Rightarrow \quad \frac{2}{3}x^{\left(\frac{2}{3}-1\right)} + \frac{2}{3}y^{\left(\frac{2}{3}-1\right)}\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{2}{3}x^{\frac{-1}{3}} + \frac{2}{3}y^{\frac{-1}{3}}\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{2}{3}y^{\frac{-1}{3}}\frac{dy}{dx} = -\frac{2}{3}x^{\frac{-1}{3}}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{2}{3}x^{\frac{-1}{3}} \times \frac{3}{2y^{\frac{-1}{3}}}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{x^{\frac{-1}{3}}}{y^{\frac{-1}{3}}}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{y^{\frac{1}{3}}}{x^{\frac{1}{3}}}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$$

Differentiation Ex 11.4 Q4

Given, $4x + 3y = \log(4x - 3y)$

Differentiating with respect to x,

$$\frac{d}{dx}(4x) + \frac{d}{dx}(3y) = \frac{d}{dx}\left(\log(4x - 3y)\right)$$

$$\Rightarrow \quad 4 + 3\frac{dy}{dx} = \frac{1}{(4x - 3y)}\frac{d}{dx}(4x - 3y) \qquad \text{[Using chain rule]}$$

$$\Rightarrow \quad 4 + 3\frac{dy}{dx} = \frac{1}{(4x - 3y)}\left(4 - 3\frac{dy}{dx}\right)$$

$$\Rightarrow \quad 4 + 3\frac{dy}{dx} = \frac{4}{(4x - 3y)} - \frac{3}{(4x - 3y)}\frac{dy}{dx}$$

$$\Rightarrow \quad 3\frac{dy}{dx} + \frac{3}{(4x - 3y)}\frac{dy}{dx} = \frac{4}{(4x - 3y)} - 4$$

$$\Rightarrow \quad 3\frac{dy}{dx}\left(1 + \frac{1}{(4x - 3y)}\right) = 4\left(\frac{1}{(4x - 3y)} - 1\right)$$

$$\Rightarrow \quad 3\frac{dy}{dx}\left[\frac{4x - 3y + 1}{(4x - 3y)}\right] = 4\left[\frac{1 - 4x + 3y}{(4x - 3y)}\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{4}{3}\left[\frac{1 - 4x + 3y}{(4x - 3y)}\right]\left[\frac{4x - 3y}{4x - 3y + 1}\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{4}{3}\left(\frac{1 - 4x + 3y}{4x - 3y + 1}\right)$$

Differentiation Ex 11.4 Q5

Given,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Differentiating with respect to x,

$$\frac{d}{dx}\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)=\frac{d}{dx}(1)$$

$$\Rightarrow \quad \frac{d}{dx}\left(\frac{x^2}{a^2}\right)+\frac{d}{dx}\left(\frac{y^2}{b^2}\right)=0$$

$$\Rightarrow \quad \frac{1}{a^2}(2x)+\frac{1}{b^2}(2y)\frac{dy}{dx}=0$$

$$\Rightarrow \quad \frac{2y}{b^2}\frac{dy}{dx}=-\frac{2x}{a^2}$$

$$\Rightarrow \quad \frac{dy}{dx}=-\left(\frac{2x}{a^2}\right)\left(\frac{b^2}{2y}\right)$$

$$\Rightarrow \quad \frac{dy}{dx}=-\frac{b^2x}{a^2y}$$

********** END **********