Higher Order Derivatives Ex 12.1 Q42

We know that, $\dfrac{d}{dx}\left(\cosec^{-1}x\right) = \dfrac{-1}{|x|\sqrt{x^2-1}}$

Let $y = \cosec^{-1}x$

$\dfrac{dy}{dx} = \dfrac{-1}{|x|\sqrt{x^2-1}}$

Since $x > 1$, $|x| = x$

Thus,

$\dfrac{dy}{dx} = \dfrac{-1}{x\sqrt{x^2-1}}$ ...(1)

Differentiating the above function with respect to x, we have,

$\dfrac{d^2y}{dx^2} = \dfrac{x\dfrac{2x}{2\sqrt{x^2-1}} + \sqrt{x^2-1}}{x^2(x^2-1)}$

$= \dfrac{\dfrac{x^2}{\sqrt{x^2-1}} + \sqrt{x^2-1}}{x^2(x^2-1)}$

$= \dfrac{x^2 + x^2 - 1}{x^2(x^2-1)^{\frac{3}{2}}}$

$= \dfrac{2x^2 - 1}{x^2(x^2-1)^{\frac{3}{2}}}$

Thus, $x(x^2-1)\dfrac{d^2y}{dx^2} = \dfrac{2x^2-1}{x\sqrt{x^2-1}}$ ...(2)

Similarly, from (1), we have

$(2x^2-1)\dfrac{dy}{dx} = \dfrac{-2x^2+1}{x\sqrt{x^2-1}}$ ...(3)

Thus, from (2) and (3), we have,

$x(x^2-1)\dfrac{d^2y}{dx^2} + (2x^2-1)\dfrac{dy}{dx} = \dfrac{2x^2-1}{x\sqrt{x^2-1}} + \left(\dfrac{-2x^2+1}{x\sqrt{x^2-1}}\right) = 0$

Hence proved.

Higher Order Derivatives Ex 12.1 Q43

Given that, $x = \cos t + \log \tan \dfrac{t}{2}$, $y = \sin t$

Differentiating with respect to t, we have,

$$\frac{dx}{dt} = -\sin t + \frac{1}{\tan\dfrac{t}{2}} \times \sec^2\frac{t}{2} \times \frac{1}{2}$$

$$= -\sin t + \frac{1}{\dfrac{\sin\dfrac{t}{2}}{\cos\dfrac{t}{2}}} \times \frac{1}{\cos^2\dfrac{t}{2}} \times \frac{1}{2}$$

$$= -\sin t + \frac{1}{\dfrac{\sin\dfrac{t}{2}}{\cos\dfrac{t}{2}}} \times \frac{1}{\cos^2\dfrac{t}{2}} \times \frac{1}{2}$$

$$= -\sin t + \frac{1}{2\sin\dfrac{t}{2}\cos\dfrac{t}{2}}$$

$$= -\sin t + \frac{1}{\sin t}$$

$$= \frac{1 - \sin^2 t}{\sin t}$$

$$= \frac{\cos^2 t}{\sin t}$$

$$= \cos t \times \cot t$$

Now find the value of $\dfrac{dy}{dt}$:

$$\frac{dy}{dt} = \cos t$$

Thus, $\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx} = \cos t \times \dfrac{1}{\cos t \times \cot t}$

$$\Rightarrow \frac{dy}{dx} = \tan t$$

Since $\dfrac{dy}{dt} = \cos t$, we have $\dfrac{d^2 y}{dt^2} = -\sin t$

At $t = \dfrac{\pi}{4}$, $\left(\dfrac{d^2 y}{dt^2}\right)_{t=\frac{\pi}{4}} = -\sin\left(\dfrac{\pi}{4}\right) = \dfrac{-1}{\sqrt{2}}$

$$\frac{d^2y}{dx^2} = \frac{\dfrac{d}{dt}\left(\dfrac{dy}{dx}\right)}{\dfrac{dx}{dt}}$$

$$= \frac{\dfrac{d}{dt}(\tan t)}{\cos t \times \cot t}$$

$$= \frac{\sec^2 t}{\cos t \times \cot t}$$

$$= \frac{\sec^2 t}{\cos t \times \dfrac{\cos t}{\sin t}}$$

$$= \frac{\sec^2 t}{\cos^2 t} \times \sin t$$

$$= \sec^4 t \times \sin t$$

$$Thus,\ \left(\frac{d^2y}{dx^2}\right)_{t=\frac{\pi}{4}} = \sec^4\left(\frac{\pi}{4}\right) \times \sin\frac{\pi}{4} = 2$$

Higher Order Derivatives Ex 12.1 Q44

$$x = a\sin t \text{ and } y = a\left(\cos t + \log\tan\frac{t}{2}\right)$$

$$\frac{dx}{dt} = a\cos t$$

$$\frac{d^2x}{dt^2} = -a\sin t$$

$$\frac{dy}{dt} = -a\sin t + a\frac{1}{\tan\dfrac{t}{2}} \times \sec^2\frac{t}{2} \times \frac{1}{2}$$

$$= -a\sin t + a\frac{1}{2\sin\dfrac{t}{2}\cos\dfrac{t}{2}}$$

$$= -a\sin t + a\cosec t$$

$$\frac{d^2y}{dt^2} = -a\cos t - a\cosec t \cot t$$

$$\frac{d^2y}{dx^2} = \frac{\dfrac{dx}{dt}\dfrac{d^2y}{dt^2} - \dfrac{dy}{dt}\dfrac{d^2x}{dt^2}}{\left(\dfrac{dx}{dt}\right)^3}$$

$$= \frac{a\cos t\left(-a\cos t - a\cosec t \cot t\right) - \left(-a\sin t + a\cosec t\right)\left(-a\sin t\right)}{\left(a\cos t\right)^3}$$

$$= \frac{-a^2\cos^2 t - a^2\cot^2 t - a^2\sin^2 t + a^2}{a^3\cos^3 t}$$

$$= \frac{-a^2\cos^2 t - a^2\sin^2 t - a^2\cot^2 t + a^2}{a^3\cos^3 t}$$

$$= \frac{-a^2\left(\cos^2 t + \sin^2 t\right) - a^2\cot^2 t + a^2}{a^3\cos^3 t}$$

$$= -\frac{1}{a\sin^2 t\cos t}$$

********** END *********