Complex Numbers Ex 13.2 Q4(iii)

$$\text{let } z = 4 - 3i$$

$$\text{Then } z^{-1} = \frac{4}{4^2 + (-3)^2} - \frac{(-3)}{4^2 + (-3)^2}$$

$$= \frac{4}{16+9} + \frac{3}{16+9}i$$

$$= \frac{4}{25} + \frac{3}{25}i$$

Complex Numbers Ex 13.2 Q4(iv)

$$\text{let } z = \sqrt{5} + 3i$$

$$\text{Then } z^{-1} = \frac{\sqrt{5}}{\left(\sqrt{5}\right)^2 + (3)^2} - \frac{3}{\left(\sqrt{5}\right)^2 + (3)^2}i$$

$$= \frac{\sqrt{5}}{5+9} - \frac{3}{5+9}i$$

$$= \frac{\sqrt{5}}{14} - \frac{3}{14}i$$

Complex Numbers Ex 13.2 Q5

If $z = x + iy$ then $|z| = \sqrt{x^2 + y^2}$

We have,

$$z_1 = 2 - i, z_2 = 1 + i$$

$$z_1 + z_2 = 2 - i + 1 + i$$
$$= 3$$

And $$z_1 - z_2 = 2 - i - 1 - i$$
$$= 1 - 2i$$

$$\frac{z_1 + z_2 + 1}{z_1 - z_2 + i} = \frac{3+1}{1 - 2i + i}$$
$$= \frac{4}{1-i}$$
$$= \frac{4}{1-i} \times \frac{1+i}{1+i}$$
$$= \frac{4(1+i)}{1^2 + 1^2}$$
$$= \frac{4(1+i)}{2}$$
$$= 2(1+i)$$

$$\therefore \left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right| = |2(1+i)|$$
$$= |2||1+i| \qquad (\because |z_1 z_2| = |z_1| \times |z_2|)$$
$$= 2 \times \sqrt{1^2 + 1^2}$$
$$= 2 \times \sqrt{2}$$
$$= 2\sqrt{2}$$

Complex Numbers Ex 13.2 Q6

(i)

$$\frac{z_1 z_2}{z_1} = \frac{z_1 z_2}{z_1} \times \frac{z_1}{z_1} \qquad \text{(rationalising the denominator)}$$

$$= \frac{(z_1)^2 z_2}{z_1 z_1}$$

$$= \frac{(2-i)^2(-2+i)}{|z_1|^2} \qquad \left(\because z\bar{z} = |z|^2\right)$$

$$= \frac{\left(2^2 + i^2 - 2 \times 2 \times i\right)(-2+i)}{|2-i|^2}$$

$$= \frac{(4 - 1 - 4i)(-2+i)}{2^2 + (-1)^2}$$

$$= \frac{(3-4i)(-2+i)}{4+i}$$

$$= 3(-2+i) - 4i(-2+i)$$

$$= \frac{-6 + 3i + 8i + 4}{5}$$

$$= \frac{-2 + 11i}{5}$$

$$\therefore \operatorname{Re}\left(\frac{z_1 z_2}{z_1}\right) = \operatorname{Re}\left(\frac{-2}{5} + \frac{11}{5} i\right)$$

$$= \frac{-2}{5}$$

(ii)

$$\frac{1}{z_1 \overline{z_1}} = \frac{1}{|z_1|^2}$$

$$= \frac{1}{|2-i|^2}$$

$$= \frac{1}{2^2 + (-1)^2}$$

$$= \frac{1}{4+1}$$

$$= \frac{1}{5}, \text{ which is purely real}$$

$$\therefore \operatorname{Im}\left(\frac{1}{z_1 z_1}\right) = 0$$

*********** END **********