Differentiation Ex 11.3 Q21

Let $f(x) = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$

This function is defined for all real numbers where $\cos x \neq 1$

i.e at all odd multiples of π

$$f(x) = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$$

$$= \tan^{-1}\left[\frac{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}{2\cos^2\left(\frac{x}{2}\right)}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{x}{2}\right)\right] = \frac{x}{2}$$

Thus, $f'(x) = \frac{d}{dx}\left(\frac{x}{2}\right) = \frac{1}{2}$

Differentiation Ex 11.3 Q22

Let $\quad y = \sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$

Put $\quad x = \cot\theta$

$$y = \sin^{-1}\left(\frac{1}{\sqrt{1+\cot^2\theta}}\right)$$

$$= \sin^{-1}\left(\frac{1}{\sqrt{\operatorname{cosec}^2\theta}}\right)$$

$$= \sin^{-1}(\sin\theta)$$

$$= \theta$$

$$y = \cot^{-1}x \qquad [\text{Since, } \cot\theta = x]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = -\frac{1}{(1+x^2)}.$$

Differentiation Ex 11.3 Q23

Let $y = \cos^{-1}\left(\frac{1-x^{2n}}{1+x^{2n}}\right)$

Put $x^n = \tan\theta, so,$

$$y = \cos^{-1}\left(\frac{1-\left(x^n\right)^2}{1+\left(x^n\right)^2}\right)$$

$$= \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$y = \cos^{-1}\left(\cos 2\theta\right) \qquad ---(i)$$

Here, $0 < x < \infty$

$\Rightarrow \quad 0 < x^n < \infty$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{2}$

$\Rightarrow \quad 0 < \left(2\theta\right) < \pi$

So, from equation (i),

$$y = 2\theta \qquad \left[\text{Sicne, } \cos^{-1}\left(\cos\theta\right) = \theta, \text{ if } \theta \in \left[0, \pi\right]\right]$$

$$y = 2\tan^{-1}\left(x^n\right)$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = 2\left(\frac{1}{1+\left(x^n\right)^2}\right)\frac{d}{dx}\left(x^n\right)$$

$$= \frac{2}{1+x^{2n}} \times \left(nx^{n-1}\right)$$

$$\frac{dy}{dx} = \frac{2nx^{n-1}}{1+x^{2n}}.$$

Differentiation Ex 11.3 Q24

Let $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right) + \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

$$= \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) \qquad \left[\text{Since, } \sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right)\right]$$

$$y = \frac{\pi}{2} \qquad \left[\text{Since, } \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 0.$$

Differentiation Ex 11.3 Q25

Let $y = \tan^{-1}\left(\frac{a+x}{1-ax}\right)$

$$y = \tan^{-1}a + \tan^{-1}x \qquad \left[\text{Since, } \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\tan^{-1}a\right) + \frac{d}{dx}\left(\tan^{-}x\right)$$

$$= 0 + \frac{1}{1+x^2}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}.$$

*********** END **********