Higher Order Derivatives Ex 12.1 Q50

$y = Ae^{-kt}\cos(pt+c)$

differentiating w.r.t. t

$$\Rightarrow \quad \frac{dy}{dt} = A\left\{e^{-kt}\left(-\sin(pt+c)\times p\right) + \left(\cos(pt+c)\right)\left(-re^{-kt}\right)\right\}$$

$$\Rightarrow \quad -Ape^{-kt}\sin(pt+c) - kAe^{-kt}\cos(pt+c)$$

$$\Rightarrow \quad \frac{dy}{dt} = -Ape^{-kt}\sin(pt+c) - ky$$

differentiating w.r.t. t

$$\Rightarrow \quad \frac{d^2y}{dt^2} = -Ap\left\{e^{-kt}\left(\cos(pt+c)\times p\right) + \left(\sin(pt+c)\right)\left(e^{-kt}\times -R\right) - ky^1\right\}$$

$$= -p^2y + Apke^{-kt}\sin(pt+c) - ky^1$$

Adding & subtracting ky^1 on RHS

$$\Rightarrow \quad \frac{d^2y}{dt^2} = +Apke^{-kt}\sin(pt+c) - p^2y - 2ky^1 + ky^1$$

$$\frac{d^2y}{dt^2} = Apke^{-kt}\sin(pt+c) - p^2y - 2ky^1 - kApe^{-kt}\sin(pt+c) - k^2y$$

$$\Rightarrow \quad \frac{d^2y}{dt^2} = -\left(p^2+k^2\right)y - 2k\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{d^2y}{dt^2} + 2k\frac{dy}{dt} + n^2y = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q51

$y = x^n\{a\cos(\log x) + b\sin(\log x)\}$

$y = ax^n\cos(\log x) + bx^n\sin(\log x)$

$$\frac{dy}{dx} = anx^{n-1}\cos(\log x) - ax^{n-1}\sin(\log x) + bnx^{n-1}\sin(\log x) + bx^{n-1}\cos(\log x)$$

$$\frac{dy}{dx} = x^{n-1}\cos(\log x)(na+b) + x^{n-1}\sin(\log x)(bn-a)$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(x^{n-1}\cos(\log x)(na+b) + x^{n-1}\sin(\log x)(bn-a)\right)$$

$$\frac{d^2y}{dx^2} = (na+b)\left[(n-1)x^{n-2}\cos(\log x) - x^{n-2}\sin(\log x)\right] + (bn-a)\left[(n-1)x^{n-2}\sin(\log x) + x^{n-2}\cos(\log x)\right]$$

$$\frac{d^2y}{dx^2} = (na+b)x^{n-2}\left[(n-1)\cos(\log x) - \sin(\log x)\right] + (bn-a)x^{n-2}\left[(n-1)\sin(\log x) + \cos(\log x)\right]$$

$$x^2\frac{d^2y}{dx^2} + (1-2n)\frac{dy}{dx} + \left(1+n^2\right)y$$

$$= (na+b)x^n\left[(n-1)\cos(\log x) - \sin(\log x)\right] + (bn-a)x^n\left[(n-1)\sin(\log x) + \cos(\log x)\right]$$

$$+ (1-2n)x^{n-1}\cos(\log x)(na+b) + (1-2n)x^{n-1}\sin(\log x)(bn-a)$$

$$+ a\left(1+n^2\right)x^n\cos(\log x) + b\left(1+n^2\right)x^n\sin(\log x)$$

$$= 0$$

Higher Order Derivatives Ex 12.1 Q52

$$y = a\left\{x+\sqrt{x^2+1}\right\}^n + b\left\{x-\sqrt{x^2+1}\right\}^{-n},$$

$$\frac{dy}{dx} = na\left\{x+\sqrt{x^2+1}\right\}^{n-1}\left[1+x\left(x^2+1\right)^{-\frac{1}{2}}\right] - nb\left\{x-\sqrt{x^2+1}\right\}^{-n-1}\left[1-x\left(x^2+1\right)^{-\frac{1}{2}}\right]$$

$$\frac{dy}{dx} = \frac{na}{\sqrt{x^2+1}}\left\{x+\sqrt{x^2+1}\right\}^n + \frac{nb}{\sqrt{x^2+1}}\left\{x-\sqrt{x^2+1}\right\}^{-n}$$

$$\frac{dy}{dx} = \frac{n}{\sqrt{x^2+1}}\left[a\left\{x+\sqrt{x^2+1}\right\}^n + b\left\{x-\sqrt{x^2+1}\right\}^{-n}\right]$$

$$x\frac{dy}{dx} = \frac{nx}{\sqrt{x^2+1}}y$$

$$\frac{d^2y}{dx^2} = \frac{nx}{\sqrt{x^2+1}}\frac{dy}{dx} + y\left[\frac{\sqrt{x^2+1} - x^2\left(x^2+1\right)^{-\frac{1}{2}}}{x^2+1}\right]$$

$$\frac{d^2y}{dx^2} = \frac{n^2x^2}{x^2+1} + y\left[\frac{1}{\left(x^2+1\right)\sqrt{x^2+1}}\right]$$

$$\frac{d^2y}{dx^2} = \frac{n^2x^2\left(\sqrt{x^2+1}\right)+y}{\left(x^2+1\right)\sqrt{x^2+1}}$$

$$\left(x^2-1\right)\frac{d^2y}{dx^2} = \frac{n^2x^4\left(\sqrt{x^2+1}\right)+x^2y}{\left(x^2+1\right)\sqrt{x^2+1}} - \frac{n^2x^2\left(\sqrt{x^2+1}\right)+y}{\left(x^2+1\right)\sqrt{x^2+1}}$$

Now

$$\left(x^2-1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - ny$$

$$= \frac{n^2x^4\left(\sqrt{x^2+1}\right)+x^2y}{\left(x^2+1\right)\sqrt{x^2+1}} - \frac{n^2x^2\left(\sqrt{x^2+1}\right)+y}{\left(x^2+1\right)\sqrt{x^2+1}} + \frac{nx}{\sqrt{x^2+1}}y - ny$$

$$= 0$$

*********** END **********