Let $A=\begin{bmatrix}1 & 3 & -2\\ -3 & 0 & -5\\ 2 & 5 & 0\end{bmatrix}$

We know that $A = IA$

$$\therefore \begin{bmatrix}1 & 3 & -2\\ -3 & 0 & -5\\ 2 & 5 & 0\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}A$$

Applying $R_2 \to R_2 + 3R_1$ and $R_3 \to R_3 - 2R_1$, we have:

$$\begin{bmatrix}1 & 3 & -2\\ 0 & 9 & -11\\ 0 & -1 & 4\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 3 & 1 & 0\\ -2 & 0 & 1\end{bmatrix}A$$

Applying $R_1 \to R_1 + 3R_3$ and $R_2 \to R_2 + 8R_3$, we have:

$$\begin{bmatrix}1 & 0 & 10\\ 0 & 1 & 21\\ 0 & -1 & 4\end{bmatrix}=\begin{bmatrix}-5 & 0 & 3\\ -13 & 1 & 8\\ -2 & 0 & 1\end{bmatrix}A$$

Applying $R_3 \to R_3 + R_2$, we have:

$$\begin{bmatrix}1 & 0 & 10\\ 0 & 1 & 21\\ 0 & 0 & 25\end{bmatrix}=\begin{bmatrix}-5 & 0 & 3\\ -13 & 1 & 8\\ -15 & 1 & 9\end{bmatrix}A$$

Applying $R_3 \to \frac{1}{25}R_3$, we have:

$$\begin{bmatrix}1 & 0 & 10\\ 0 & 1 & 21\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}-5 & 0 & 3\\ -13 & 1 & 8\\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25}\end{bmatrix}A$$

Applying $R_1 \to R_1 - 10R_3$, and $R_2 \to R_2 - 21R_3$, we have:

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & -\frac{2}{5} & -\frac{3}{5}\\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25}\\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25}\end{bmatrix}A$$

$$\therefore A^{-1}=\begin{bmatrix}1 & -\frac{2}{5} & -\frac{3}{5}\\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25}\\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25}\end{bmatrix}$$

Question 17:

Find the inverse of each of the matrices, if it exists.

$$\begin{bmatrix}2 & 0 & -1\\ 5 & 1 & 0\\ 0 & 1 & 3\end{bmatrix}$$

Answer

Let $A=\begin{bmatrix}2 & 0 & -1\\ 5 & 1 & 0\\ 0 & 1 & 3\end{bmatrix}$

We know that $A = IA$

$$\therefore \begin{bmatrix}2 & 0 & -1\\ 5 & 1 & 0\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}A$$

$$\begin{bmatrix} 0 & 1 & 3 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

Applying $R_1 \to \frac{1}{2}R_1$, we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 5R_1$, we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \to R_3 - R_2$, we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ \frac{5}{2} & -1 & 1 \end{bmatrix} A$$

Applying $R_3 \to 2R_3$, we have:

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 5 & -2 & 2 \end{bmatrix} A$$

Applying $R_1 \to R_1 + \frac{1}{2}R_3$, and $R_2 \to R_2 - \frac{5}{2}R_3$, we have:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

Question 18:

Matrices *A* and *B* will be inverse of each other only if

A. $AB = BA$

C. $AB = 0$, $BA = I$

B. $AB = BA = 0$

D. $AB = BA = I$

Answer

Answer: D

We know that if *A* is a square matrix of order *m*, and if there exists another square matrix *B* of the same order *m*, such that $AB = BA = I$, then *B* is said to be the inverse of *A*. In this case, it is clear that *A* is the inverse of *B*.

Thus, matrices *A* and *B* will be inverses of each other only if $AB = BA = I$.

Miscellaneous Solutions

Question 1:

Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, show that $(aI + bA)^n = a^n I + na^{n-1} bA$, where I is the identity matrix of order 2 and $n \in \mathbf{N}$

Answer

It is given that $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

To show: $\quad P(n) : (aI + bA)^n = a^n I + na^{n-1} bA,\ n \in \mathbf{N}$

We shall prove the result by using the principle of mathematical induction.

For $n = 1$, we have:

$P(1) : (aI + bA) = aI + ba^0 A = aI + bA$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

That is,

$$P(k):(aI+bA)^k = a^kI + ka^{k-1}bA$$

Now, we prove that the result is true for $n = k + 1$.

Consider

$$\begin{aligned}(aI+bA)^{k+1} &= (aI+bA)^k(aI+bA)\\ &= (a^kI+ka^{k-1}bA)(aI+bA)\\ &= a^{k+1}I+ka^kbAI+a^kbIA+ka^{k-1}b^2A^2\\ &= a^{k+1}I+(k+1)a^kbA+ka^{k-1}b^2A^2 \qquad ...(1)\end{aligned}$$

$$\text{Now, } A^2 = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix} = O$$

From (1), we have:

$$\begin{aligned}(aI+bA)^{k+1} &= a^{k+1}I+(k+1)a^kbA+O\\ &= a^{k+1}I+(k+1)a^kbA\end{aligned}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have:

$$(aI+bA)^n = a^nI + na^{n-1}bA \text{ where } A = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix},\ n \in \mathbf{N}$$

Question 2:

If $A = \begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$, prove that $A^n = \begin{bmatrix}3^{n-1} & 3^{n-1} & 3^{n-1}\\ 3^{n-1} & 3^{n-1} & 3^{n-1}\\ 3^{n-1} & 3^{n-1} & 3^{n-1}\end{bmatrix},\ n \in \mathbf{N}$

Answer

It is given that $A = \begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$

$$\text{To show:} \quad P(n): A^n = \begin{bmatrix}3^{n-1} & 3^{n-1} & 3^{n-1}\\ 3^{n-1} & 3^{n-1} & 3^{n-1}\\ 3^{n-1} & 3^{n-1} & 3^{n-1}\end{bmatrix},\ n \in \mathbf{N}$$

We shall prove the result by using the principle of mathematical induction.

For $n = 1$, we have:

$$P(1): \begin{bmatrix}3^{1-1} & 3^{1-1} & 3^{1-1}\\ 3^{1-1} & 3^{1-1} & 3^{1-1}\\ 3^{1-1} & 3^{1-1} & 3^{1-1}\end{bmatrix} = \begin{bmatrix}3^0 & 3^0 & 3^0\\ 3^0 & 3^0 & 3^0\\ 3^0 & 3^0 & 3^0\end{bmatrix} = \begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix} = A$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

That is $P(k): A^k = \begin{bmatrix}3^{k-1} & 3^{k-1} & 3^{k-1}\\ 3^{k-1} & 3^{k-1} & 3^{k-1}\\ 3^{k-1} & 3^{k-1} & 3^{k-1}\end{bmatrix}$

*********** END **********