Algebraic Identities Ex 4.2 Q2

Answer :

In the given problem, we have to simplify the expressions

(i) Given $(a+b+c)^2+(a-b+c)^2$

By using identity $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$

Hence the equation becomes

$$(a+b+c)^2+(a-b+c)=\left[a^2+b^2+c^2+2ab+2bc+2ca\right]+\left[a^2+(-b)^2+c^2+2a(-b)+2(-b)(c)+2ca\right]$$
$$=a^2+b^2+c^2+2ab+2bc+2ca+a^2+b^2+c^2-2ab-2bc+2ca$$
$$=a^2+a^2+b^2+b^2+c^2+c^2+\cancel{2ab}-\cancel{2ab}+\cancel{2bc}-\cancel{2bc}+2ca+2ca$$
$$=2a^2+2b^2+2c^2+4ca$$

Taking 2 as common factor we get

$$=2\left(a^2+b^2+c^2+2ca\right)$$

Hence the simplified value of $(a+b+c)^2+(a-b+c)^2$ is $\boxed{2\left(a^2+b^2+c^2+2ca\right)}$

(ii) Given $(a+b+c)^2-(a-b+c)^2$

By using identity $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$

Hence the equation becomes

$$(a+b+c)^2-(a-b+c)=\left[a^2+b^2+c^2+2ab+2bc+2ca\right]-\left[a^2+(-b)^2+c^2+2a(-b)+2(-b)(c)+2c\right.$$
$$=a^2+b^2+c^2+2ab+2bc+2ca-a^2-b^2-c^2+2ab+2bc-2ca$$
$$=\cancel{a^2}-\cancel{a^2}+\cancel{b^2}-\cancel{b^2}+\cancel{c^2}-\cancel{c^2}+2ab+2ab+2bc+2bc+\cancel{2ca}-\cancel{2ca}$$
$$=4ab+4bc$$

Taking 4 as common factor we get

$$=4(ab+bc)$$

Hence the simplified value of $(a+b+c)^2-(a-b+c)^2$ is $\boxed{4(ab+bc)}$.

(iii) Given $(a+b+c)^2+(a-b+c)^2+(a+b-c)^2$

By using identity $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$, we have

$$(a+b+c)^2+(a-b+c)^2+(a+b-c)^2$$
$$=\left[a^2+b^2+c^2+2ab+2bc+2ca\right]+\left[a^2+(-b)^2+c^2+2a(-b)+2(-b)(c)+2ca\right]$$
$$+\left[a^2+(-b)^2+c^2+2ab+2b(-c)+2(-c)a\right]$$
$$=a^2+b^2+c^2+2ab+2bc+2ca+a^2+b^2+c^2-2ab-2bc+2ca+a^2+b^2+c^2$$
$$+2ab-2bc-2ca$$
$$(a+b+c)^2+(a-b+c)^2+(a+b-c)^2$$
$$=a^2+a^2+a^2+b^2+b^2+b^2+c^2+c^2+c^2+2ab+\cancel{2ab}-\cancel{2ab}-2bc+2\cancel{bc}-\cancel{2bc}$$
$$+2ca+\cancel{2ca}-\cancel{2ca}$$
$$=3a^2+3b^2+3c^2+2ab-2bc+2ca$$

Taking 3 as a common factor we get

$$(a+b+c)^2+(a-b+c)^2+(a+b-c)^2=3(a^2+b^2+c^2)+2ab-2bc+2ca$$

Hence the value of $(a+b+c)^2+(a-b+c)^2+(a+b-c)^2$ is

$\boxed{3(a^2+b^2+c^2)+2ab-2bc+2ca}$.

(iv) Given $(2x+p-c)^2-(2x-p+c)^2$

By using identity $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$, we get

$$\begin{aligned}
&(2x+p-c)^2-(2x-p+c)^2\\
&=(2x)^2+(p)^2+(-c)^2+2(2x)(p)+2(p)(-c)+2(-c)(2x)\\
&\quad-\left[(2x)^2+(-p)^2+(c)^2+2(2x)(-p)+2(-p)(c)+2(c)(2x)\right]\\
&=4x^2+p^2+c^2+4xp-2cp-4cx-\left[4x^2+p^2+c^2-4xp-2cp+4cx\right]
\end{aligned}$$

By cancelling the opposite terms, we get

$$\begin{aligned}
(2x+p-c)^2-(2x-p+c)^2&=\cancel{4x^2}+\cancel{p^2}+\cancel{c^2}+4xp-\cancel{2cp}-4cx-\cancel{4x^2}-\cancel{p^2}-\cancel{c^2}+4xp\\
&\quad+\cancel{2cp}-4cx\\
&=4xp+4xp-4cx-4cx\\
&=8xp-8cx
\end{aligned}$$

Taking $8x$ as common a factor we get,

$$(2x+p-c)^2-(2x-p+c)^2=8x(p-c)$$

Hence the value of $(2x+p-c)^2-(2x-p+c)^2$ is $\boxed{8x(p-c)}$

(v) We have $(x^2+y^2-z^2)-(x^2-y^2+z^2)^2$

Using formula $(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$, we get

$$\begin{aligned}
&(x^2+y^2-z^2)-(x^2-y^2+z^2)^2\\
&=(x^2)^2+(y^2)^2+(-z^2)^2+2(x^2)(y^2)+2(y^2)(-z^2)+2(-z^2)(x^2)\\
&\quad-\left[(x^2)^2+(-y^2)^2+(z^2)^2+2(x^2)(-y^2)+2(-y^2)(z^2)+2(z^2)(x^2)\right]\\
&=x^4+y^4+z^4+2x^2y^2-2y^2z^2-2z^2x^2\\
&\quad-\left[x^4+y^4+z^4-2x^2y^2-2y^2z^2+2z^2x^2\right]
\end{aligned}$$

By canceling the opposite terms, we get

$$\begin{aligned}
(x^2+y^2-z^2)^2-(x^2+y^2+z^2)^2&=\cancel{x^4}+\cancel{y^4}+\cancel{z^4}+2x^2y^2-\cancel{2y^2z^2}-2z^2x^2\\
&\quad\cancel{-x^4}-\cancel{y^4}-\cancel{z^4}+2x^2y^2+\cancel{2y^2z^2}-2z^2x^2\\
&=4x^2y^2-4z^2x^2
\end{aligned}$$

Taking $4x^2$ as common factor we get

$$(x^2+y^2-z^2)^2-(x^2+y^2+z^2)^2=4x^2(y^2-z^2)$$

Hence the value of $(x^2+y^2-z^2)^2-(x^2+y^2+z^2)^2$ is $\boxed{4x^2(y^2-z^2)}$.

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