Question 7. 21. A solid cylinder rolls up an inclined plane of angle of inclination 30°. At the bottom of the inclined plane the centre of mass of the cylinder has a speed of 5 m/s.
(a) How far will the cylinder go up the plane?
(b) How long will it take to return to the bottom?
Answer:
Here, $\theta$= 30°, v = 5 m/ s
Let the cylinder go up the plane up to a height h.
From 1/2 $mv^2$ +1/2$IW^2$ = mgh

$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2 = mgh$$

$$\frac{3}{4}mv^2 = mgh$$

$$h = \frac{3v^2}{4g} = \frac{3 \times 5^2}{4 \times 9.8} = 1.913 \text{ m}$$

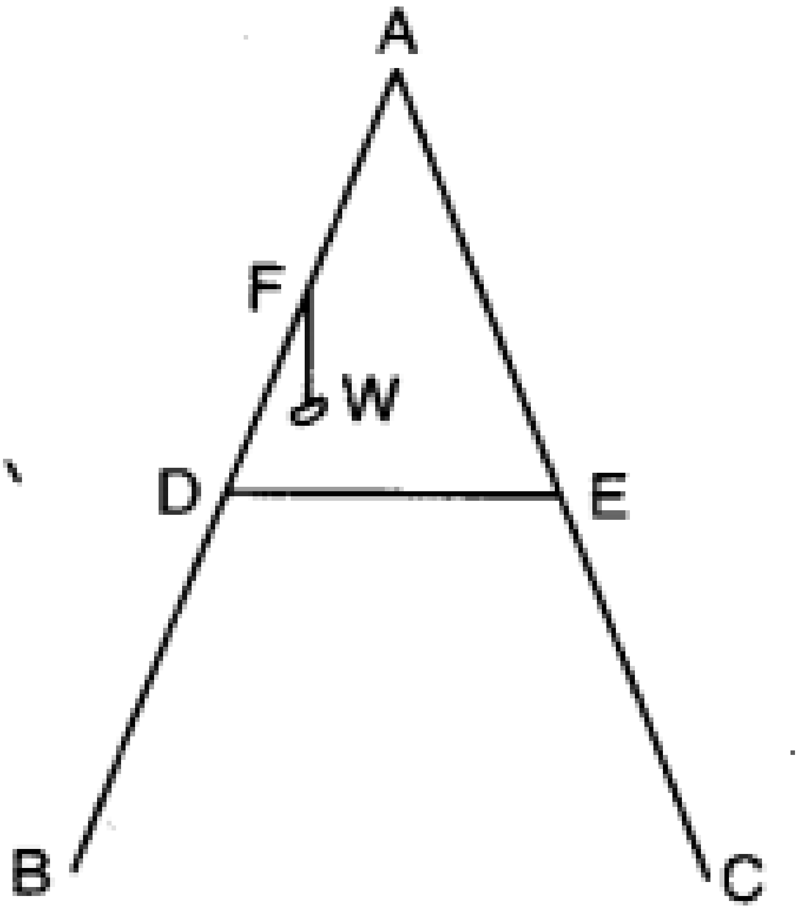
If $s$ is the distance up the inclined plane, then as

$$\sin\theta = \frac{h}{s}, \quad s = \frac{h}{\sin\theta} = \frac{1.913}{\sin 30°} \; 3.856 \text{ m}$$

Time taken to return to the bottom

$$t = \sqrt{\frac{2s\left(1 + \frac{k^2}{r^2}\right)}{g\sin\theta}} = \sqrt{\frac{2 \times 3.826\left(1 + \frac{1}{2}\right)}{9.8 \sin 30°}} = 1.53\text{s.}$$

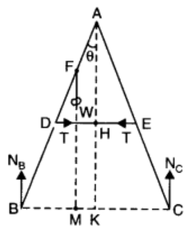
Question 7. 22. As shown in Fig. the two sides of a step ladder BA and CA are 1.6 m long and hinged at A. A rope DE, 0.5 m is tied halfway up. A weight 40 kg is suspended from a point F, 1.2 m from B along the ladder BA. Assuming the floor to be friction less and neglecting the weight of the ladder, find the tension in the rope and forces exerted by the floor on the ladder. (Take g = 9.8 $m^2$ )(Hint: Consider the equilibrium of each side of the ladder separately.)




Answer: The forces acting on the ladder are shown in Fig. 7.14.
Here, IV = 40 kg = 40 x 9.8 N = 392 N, AB = AC = 1.6 m, BD = 1/2 x 1.6 m = 0.8 m,
BF = 1.2 m and DE 0.5 m,

In the Fig. $\Delta ADE$ and $\Delta ABC$ are similar triangles, hence

$$BC = DE \times \frac{AB}{AD} = \frac{0.5 \times 1.6}{0.8} = 1.0 \text{ m}$$




Now, considering equilibrium at point $B$, $\Sigma\tau = 0$

$$\therefore \quad W \times (MB) = N_C \times (CB) \quad ...(i)$$

But
$$MB = \frac{KB \times BF}{BA} = \frac{0.5 \times 1.2}{1.6} = 0.375 \text{ m}$$

Substituting this value in ($i$), we get

$$\therefore \quad N_C = \frac{W \times (MB)}{(CB)} = \frac{392 \times 0.375}{1} = 147 \text{ N}$$

Again considering equilibrium at point $C$ in similar manner, we have

$$W \times (MC) = N_B \times (BC)$$

$$\therefore \quad N_B = \frac{W \times (MC)}{(BC)} = \frac{W \times (BC - BM)}{(BC)},$$

= (392 x (1-0.375))/1=245 N
Now, it can be easily shown that tension in the string T = $N_B$ - $N_C$ = 245 - 147 = 98 N.

Question 7. 23. A man stands on a rotating platform, with his arms stretched horizontally holding a 5 kg weight in each hand. The angular speed of the platform is 30 revolutions per minutes. The man then brings his arms close to his body with the distance of each weight from the axis changing from 90 cm to 20 cm. The moment of inertia of the man together with the platform may be taken to be constant and equal to 7.6 kg $m^2$.(a) What is his new angular speed? (Neglect friction)(b) Is kinetic energy conserved in the process? If not, from where does the change come about?
Answer:
Here, $I_1$= 7.6 + 2 x 5 $(0.9)^2$ = 15.7 kg $m^2$
$w_1$ = 30 rpm

$I_2 = 7.6 + 2 \times 5\ (0.2)2 = 8.0$ kg $m^2$
$w_2$ = ?
According to the principle of conservation of angular momentum,
$I_2 w_2 = I_1 w_1$
$w_2 = I_1/I_2\ \ w_1 = 15.7 \times 30 / 8.0 = 58.88$ rpm
No, kinetic energy is not conserved in the process. In fact, as moment of inertia decreases, K.E. of rotation increases. This change comes about as work is done by the man in bringing his arms closer to his body.

Question 7. 24. A bullet of mass 10 g and speed 500 m/s is fired into a door and gets embedded exactly at the centre of the door. The door is 1.0 m wide and weighs 12 kg. It is hinged at one end and rotates about a vertical axis practically without friction. Find the angular speed of the door just after the bullet embeds into it.(Hint: The moment of inertia of the door about the vertical axis at one end is $ML^2/3$.)
Answer:
Angular momentum imparted by the bullet, L = mv x r
$= (10 \times 10^{-3}) \times 500 \times 1/2 = 2.5$
Also, $I = ML^2/3 = 12 \times (1.0)^2/3 = 4$ kg $m^2$
Since L = Iw
w = L/I = 2.5/4 = 0.625 rad / s

Question 7. 25. Two discs of moments of inertia $I_1$ and $I_2$ about their respective axes (normal to the disc and passing through the centre), and rotating with angular speed $w_1$ and $w_2$ are brought into contact face to face with their axes of rotation coincident, (a) What is the angular speed of the two-disc system? (b) Show that the kinetic energy of the combined system is less than the sum of the initial kinetic energies of the two discs. How do you account for this loss in energy? Take $w_1$ not equal to $w_2$.
Answer: (a) Let $I_1$ and $I_2$ be the moments of inertia of two discs having angular speeds $w_1$, and $w_2$ respectively. When they are brought in contact, the moment of inertia of the two-disc system will be $I_1 + I_2$. Let the system now have an angular speed w. From the law of conservation of angular momentum, we know that

$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\,\omega$$

∴ The angular speed of the two-disc system,

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

(b) The sum of kinetic energies of the two discs before coming in contact,

$$k_1 = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2$$

The final kinetic energy of the two-disc system,

$$k_2 = \frac{1}{2}(I_1 + I_2)\,\omega^2$$

$$= \frac{1}{2}(I_1 + I_2) \times \left(\frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}\right)^2$$

$$= \frac{1}{2}\left(\frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}\right)^2$$

Now, $$k_1 - k_2 = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 - \frac{1}{2}\left(\frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}\right)^2$$

$$= \frac{1}{2(I_1 + I_2)} \times \left[(I_1\omega_1^2 + I_2\omega_2^2)(I_1 + I_2) - (I_1^2\omega_1^2 + I_2^2\omega_2^2 + 2I_1I_2\,\omega_1\omega_2)\right]$$

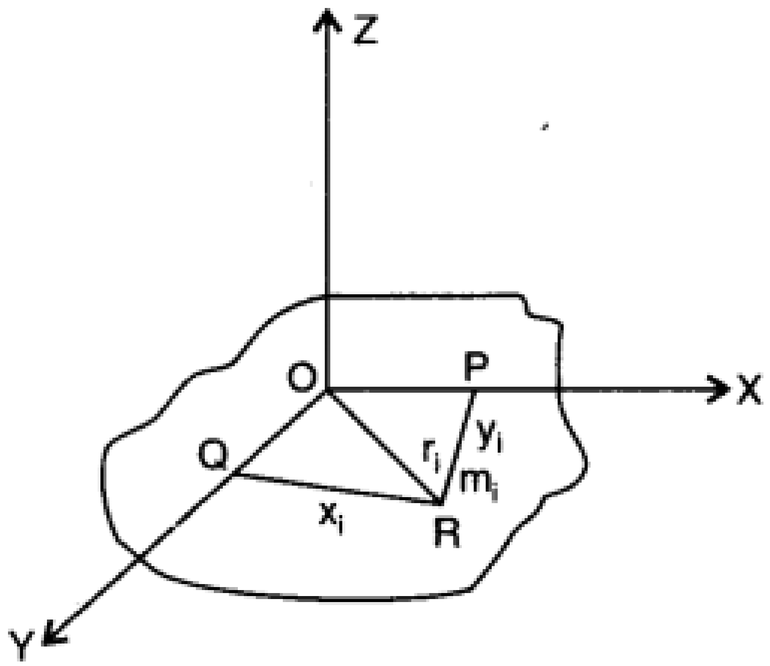
$$= \frac{1}{2(I_1 + I_2)} \times [I_1^2\omega_1^2 + I_2^2\omega_2^2 + I_1I_2\omega_1^2 + I_1I_2\,\omega_2^2 - I_1^2\omega_1^2 - I_2^2\omega_2^2 - 2I_1I_2\omega_1\omega_2]$$

$$= \frac{1}{2(I_1 + I_2)}[I_1I_2(\omega_1^2 + \omega_2^2 - 2\omega_1\omega_2)]$$

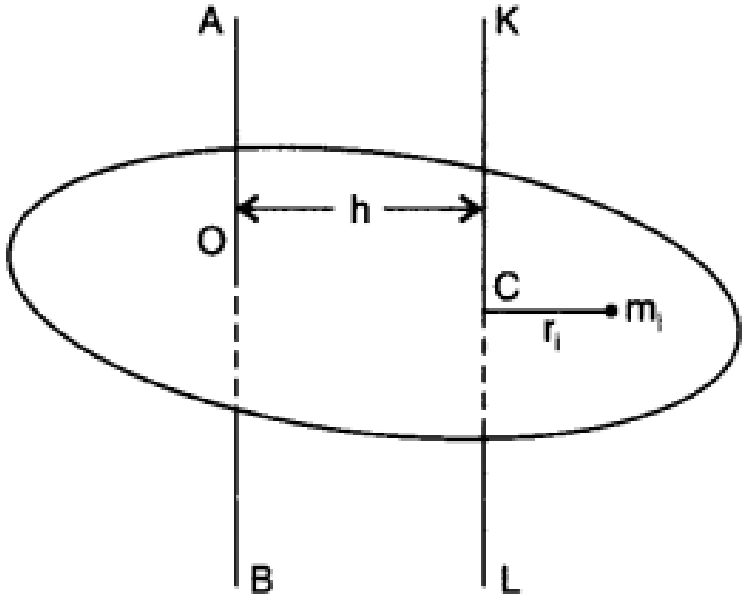
$$= \frac{I_1I_2}{2(I_1 + I_2)}(\omega_1 - \omega_2)^2$$

Now, $(w_1 - w_2)^2$ will be positive whether w1 is greater or smaller than $w_2$.
Also, $I_1I_2/2(I_1 + I_2)$ is also positive because $I_1$ and $I_2$ are positive.
Thus, $k_1 - k_2$ is a positive quantity.
$\therefore k_1 = k_2$ + a positive quantity or $k_1 > k_2$
$\therefore$ The kinetic energy of the combined system ( $k_2$) is less than the sum of the kinetic energies of the two dies.The loss of energy on combining the two discs is due to the energy being used up because of the frictional forces between the surfaces of the two discs. These forces, in fact, bring about a common angular speed of the two discs on combining.

Question 7. 26. (a) Prove the theorem of perpendicular axes.
Hint: Square of the distance of a point (x, y) in the x-y plane from an axis through the origin perpendicular to the plane is $x^2 + y^2$]
(b) Prove the theorem of parallel axes.
Hint: If the centre of mass of chosen the origin [$\Sigma m_ir_i = 0$]
Answer:
(a) The theorem of perpendicular axes: According to this theorem, the moment of inertia of a plane lamina (i.e., a two dimensional body of any shape/size) about any axis OZ perpendicular to the plane of the lamina is equal to sum of the moments of inertia of the lamina about any two mutually perpendicular axes OX and OY in the plane of lamina, meeting at a point where the given axis OZ passes through the lamina. Suppose at the point 'R' m{ particle is situated moment of inertia about Z axis of lamina
= moment of inertia of body about r-axis
= moment of inertia of body about y-axis.




(b) Theorem of parallel axes: According to this theorem, moment of inertia of a rigid body about any axis AB is equal to moment of inertia of the body about another axis KL passing through centre of mass C of the body in a direction parallel to AB, plus the product of total mass M of the body and square of the perpendicular distance between the two parallel axes. If h is perpendicular distance between the axes AB and KL, then Suppose rigid body is made up of n particles m1, m2, .... mn, mn at perpendicular distances $r_1$, $r_2$, $r_i$.... $r_n$. respectively from the axis KL passing through centre of mass C of the body.
If h is the perpendicular distance of the particle of mass m{ from KL, then




The perpendicular distance of $i^{th}$ particle from the axis

$$AB = (r_i + n)$$

or
$$I_{AB} = \sum_i m_i (r_i + h)^2$$
$$= \sum_i m_i (r_i^2 + h^2 + 2r_i h)$$
$$= \sum_i m_i r_i^2 + \sum_i m_i h^2 + 2h \sum_i m_i r_i \quad ...(ii)$$

As the body is balanced about the centre of mass, the algebraic sum of the moments of the weights of all particles about an axis passing through C must be zero.

$$\sum_i (m_i g) r_i = 0 \quad \text{or} \quad g \sum_i m_i r_i$$

or
$$\sum_i m_i r_i = 0 \quad ...(iii)$$

From equation (*ii*), we have

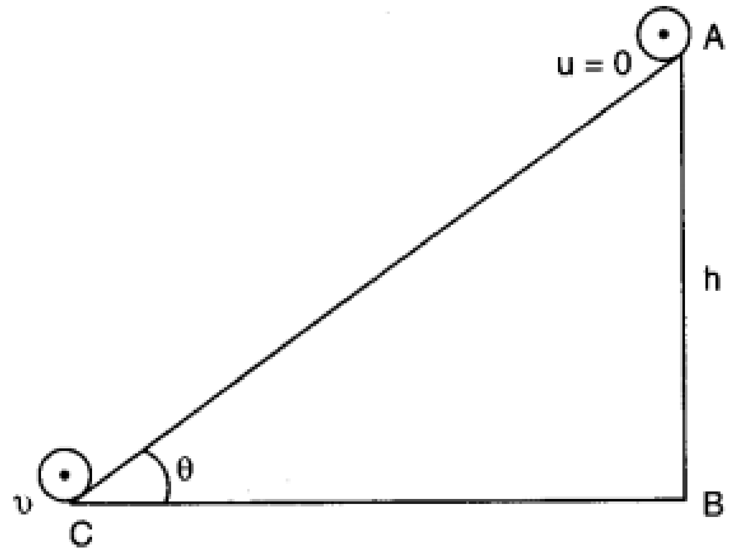
$$I_{AB} = \sum_i m_i r_i^2 + \left(\sum m_i\right) h^2 + 0$$

or
$$I_{AB} = I_{KL} + Mh^2$$

where
$$I_{KL} = \sum_i m_i r_i^2 \quad \text{and} \quad M = \sum m_i$$

Question 7. 27. Prove the result that the velocity v of translation of a rolling body (like a ring, disc, cylinder or sphere) at the bottom of an inclined plane of a height h is given by,$v^2=2gh/(1+k^2/R^2)$ using dynamical consideration (i.e., by consideration of forces and torques). Note k is the radius of gyration of the body about its symmetry axis, and R is the radius of the body. The body starts from rest at the top of the plane.

Answer: Let a rolling body ($I = Mk^2$) rolls down an inclined plane with an initial velocity u = 0; When it reaches the bottom of inclined plane, let its linear velocity be v. Then from conservation of mechanical energy, we have Loss in P.E. = Gain in translational K.E. + Gain in rotational K.E.



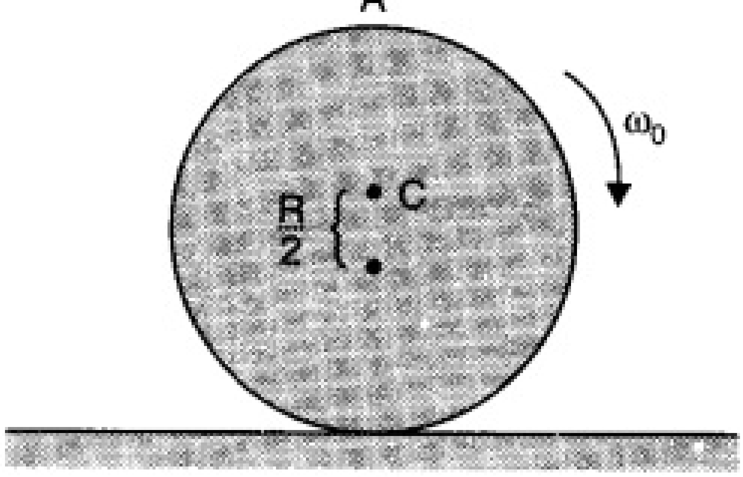

$$\therefore \qquad Mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}(mk^2)\left(\frac{v^2}{R^2}\right)$$

$$\therefore \qquad Mgh = \frac{1}{2}mv^2\left(1 + \frac{k^2}{R^2}\right)$$

$$\Rightarrow \qquad v^2 = \frac{2gh}{\left(1 + \frac{k^2}{R^2}\right)}.$$

Question 7. 28. A disc rotating about its axis with angular speed $w_o$ is placed lightly (without any translational push) on a perfectly friction less table. The radius of the disc is R. What are the linear velocities of the points A, B and C on the disc shown in Fig.? Will the disc roll in the direction indicated?




Answer:

Since $v = r\omega$,

For point, $A$, $v_A = R\omega_0$ in the direction of arrow.

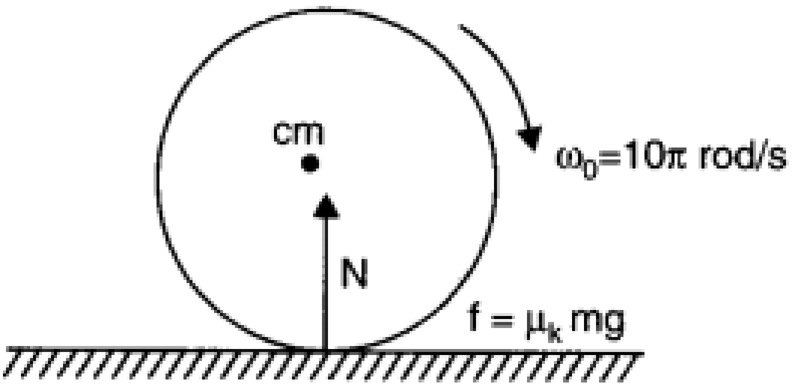
For point, $B$, $v_B = R\omega_0$ in the opposite direction of arrow.

For point, $C$, $v_C = \frac{R}{2}\omega_0$ in the direction of arrow.

The disc will not roll in the given direction because friction is necessary for the same.

Question 7. 29. Explain why friction is necessaiy to make the disc roll (refer to Q. 28) in the direction indicated.
(a) Give the direction of frictional force at B, and the sense of frictional torque, before perfect rolling begins.
(b) What is the force of friction after perfect rolling begins?
Answer: To roll a disc, we require a torque, which can be provided only by a tangential force. As force of friction is the only tangential force in this case, it is necessary.
(a) As frictional force at B opposes the velocity of point B, which is to the left, the frictional force must be to the right. The sense of frictional torque will be perpendicular to the plane of the disc and outwards.
(b) As frictional force at B decreases the velocity of the point of contact B with the surface, the perfect rolling begins only when velocity of point B becomes zero. Also, force of friction would become zero at this stage.

Question 7. 30. A solid disc and a ring, both of radius 10 cm are placed on a horizontal table simultaneously, with initial angular speed equal to 10π rad/s. Which of two will start to roll earlier? The coefficient of kinetic friction is $\upsilon_k = 0.2$.
Answer: When a disc or ring starts rotatory motion on a horizontal surface, initial translational velocity of centre of mass is zero.




The frictional force causes the centre of mass to accelerate linearly but frictional torque causes angular retardation. As force of normal reaction N = mg, hence frictional force f = $\upsilon_k$ N = $\upsilon_k$ mg.
For linear motion f = $\upsilon_k$ . mg = ma ----------(i)

and for rotational motion, $\tau = f \cdot R = \mu_k\, mg \cdot R = -I\alpha$ ...(*ii*)

Let perfect rolling motion starts at time $t$, when $v = R\omega$

From (*i*) $\quad a = \mu_k \cdot g$

$\therefore \quad v = u + at = 0 + \mu_k \cdot g \cdot t$ ...(*iii*)

From (*ii*) $\quad \alpha = -\dfrac{\mu_k \cdot mgR}{I} = -\dfrac{\mu_k \cdot mgR}{mK^2} = -\dfrac{\mu_k \cdot gR}{K^2}$

$\therefore \quad \omega = \omega_0 + \alpha t = \omega_0 - \dfrac{\mu_k \cdot gR}{K^2}t$ ...(*iv*)

Since $\quad v = R\omega$, hence $\mu_k \cdot g \cdot t = R\left[\omega_0 - \mu_k \cdot \dfrac{gR}{K^2}t\right]$

$\Rightarrow \quad t^2 = \dfrac{R\omega_0}{\mu_k \cdot g\left(1 + \dfrac{R^2}{K^2}\right)}$

For disc, $\quad K^2 = \dfrac{R^2}{2}$, hence $t = \dfrac{\omega_0 R}{\mu_k \cdot g\left(1 + \dfrac{R^2}{R^2/2}\right)} = \dfrac{\omega_0 R}{3\mu_k \cdot g}$

For ring, $\quad K^2 = R^2$, hence $t = \dfrac{\omega_0 R}{\mu_k \cdot g\left(1 + \dfrac{R^2}{R^2}\right)} = \dfrac{\omega_0 R}{2\mu_k \cdot g}$

Thus, it is clear that disc will start to roll earlier. The actual time at which disc starts rolling will be

$$t = \frac{\omega_0 R}{2\mu_k \cdot g} = \frac{(10\pi) \times (0.1)}{3 \times (0.2) \times 9.8} = 0.538.$$

*********** END **********