



Complex Numbers Ex 13.4 Q3(ii)

Let $z = \tan \alpha - i$

$\tan \alpha$ is periodic function with period π

So, let us take α lying in the interval $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$.

Case - I : When $\alpha \in \left[0, \frac{\pi}{2}\right)$

$$|z| = \sqrt{\tan^2 \alpha + 1} = \sqrt{\sec^2 \alpha} = |\sec \alpha| = \sec \alpha$$

Let β be acute angle given by $\tan \beta = \frac{|\operatorname{Im}(z)|}{|\operatorname{Re}(z)|}$.

$$\tan \beta = \frac{1}{|\tan \alpha|} = |\cot \alpha| = \cot \alpha = \tan \left(\frac{\pi}{2} - \alpha\right)$$

$$\Rightarrow \beta = \frac{\pi}{2} - \alpha$$

As z is represented by a point in fourth quadrant.

$$\therefore \arg(z) = -\beta = \alpha - \frac{\pi}{2}.$$

So polar form of z is $\sec \alpha \left(\cos \left(\alpha - \frac{\pi}{2} \right) + i \sin \left(\alpha - \frac{\pi}{2} \right) \right)$

Case - II : When $\alpha \in \left(\frac{\pi}{2}, \pi\right]$

$$|z| = \sqrt{\tan^2 \alpha + 1} = \sqrt{\sec^2 \alpha} = |\sec \alpha| = -\sec \alpha$$

Let β be acute angle given by $\tan \beta = \frac{|\operatorname{Im}(z)|}{|\operatorname{Re}(z)|}$.

$$\tan \beta = \frac{1}{|\tan \alpha|} = |\cot \alpha| = -\cot \alpha = \tan \left(\alpha - \frac{\pi}{2} \right)$$

$$\Rightarrow \beta = \alpha - \frac{\pi}{2}$$

As z is represented by a point in third quadrant.

$$\therefore \arg(z) = \pi + \beta = \frac{\pi}{2} + \alpha.$$

So polar form of z is $-\sec \alpha \left(\cos \left(\frac{\pi}{2} + \alpha \right) + i \sin \left(\frac{\pi}{2} + \alpha \right) \right)$.

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Let $z = (1 - \sin \alpha) + i \cos \alpha$.

Since sine and cosine are periodic functions with period 2π .

So, let us take α lying in the interval $[0, 2\pi]$.

Now, $z = (1 - \sin \alpha) + i \cos \alpha$

$$\Rightarrow |z| = \sqrt{(1 - \sin \alpha)^2 + \cos^2 \alpha} = \sqrt{2 - 2\sin \alpha} = \sqrt{2}\sqrt{1 - \sin \alpha}$$

$$\Rightarrow |z| = \sqrt{2} \sqrt{\left(\cos \frac{\alpha}{2} - \sin \frac{\alpha}{2}\right)^2} = \sqrt{2} \left| \cos \frac{\alpha}{2} - \sin \frac{\alpha}{2} \right|$$

Let β be acute angle given by $\tan \beta = \frac{|\operatorname{Im}(z)|}{|\operatorname{Re}(z)|}$.

$$\tan \beta = \frac{|\cos \alpha|}{|1 - \sin \alpha|} = \frac{|\cos \alpha|}{|1 - \sin \alpha|} = \frac{\left| \cos^2 \frac{\alpha}{2} - \sin^2 \frac{\alpha}{2} \right|}{\left| \left(\cos \frac{\alpha}{2} - \sin \frac{\alpha}{2} \right)^2 \right|} = \frac{\left| \cos \frac{\alpha}{2} + \sin \frac{\alpha}{2} \right|}{\left| \cos \frac{\alpha}{2} - \sin \frac{\alpha}{2} \right|}$$

$$\Rightarrow \tan \beta = \frac{\left| 1 + \tan \frac{\alpha}{2} \right|}{\left| 1 - \tan \frac{\alpha}{2} \right|} = \left| \tan \left(\frac{\pi}{4} + \frac{\alpha}{2} \right) \right|$$

Following cases arise:

Case I: When $0 \leq \alpha < \frac{\pi}{2}$

$$\cos \frac{\alpha}{2} > \sin \frac{\alpha}{2} \text{ and } \frac{\pi}{4} + \frac{\alpha}{2} \in \left[\frac{\pi}{4}, \frac{\pi}{2} \right)$$

$$\therefore |z| = \sqrt{2} \left(\cos \frac{\alpha}{2} - \sin \frac{\alpha}{2} \right)$$

$$\text{and, } \tan \beta = \left| \tan \left(\frac{\pi}{4} + \frac{\alpha}{2} \right) \right| = \tan \left(\frac{\pi}{4} + \frac{\alpha}{2} \right) \Rightarrow \beta = \frac{\pi}{4} + \frac{\alpha}{2}$$

Clearly, z lies in the first quadrant.

$$\therefore \arg(z) = \frac{\pi}{4} + \frac{\alpha}{2}$$

$$\text{So polar form of } z \text{ is } \sqrt{2} \left(\cos \frac{\alpha}{2} - \sin \frac{\alpha}{2} \right) \left(\cos \left(\frac{\pi}{4} + \frac{\alpha}{2} \right) + i \sin \left(\frac{\pi}{4} + \frac{\alpha}{2} \right) \right)$$

Case II: When $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$

$$\cos \frac{\alpha}{2} < \sin \frac{\alpha}{2} \text{ and } \frac{\pi}{4} + \frac{\alpha}{2} \in \left(\frac{\pi}{2}, \pi\right)$$

$$\therefore |z| = \sqrt{2} \left| \cos \frac{\alpha}{2} - \sin \frac{\alpha}{2} \right| = -\sqrt{2} \left(\cos \frac{\alpha}{2} - \sin \frac{\alpha}{2} \right)$$

$$\text{and, } \tan \beta = \left| \tan \left(\frac{\pi}{4} + \frac{\alpha}{2} \right) \right| = -\tan \left(\frac{\pi}{4} + \frac{\alpha}{2} \right) = \tan \left\{ \pi - \left(\frac{\pi}{4} + \frac{\alpha}{2} \right) \right\} = \tan \left(\frac{3\pi}{4} - \frac{\alpha}{2} \right)$$

$$\Rightarrow \beta = \frac{3\pi}{4} - \frac{\alpha}{2}$$

Since $1 - \sin \alpha > 0$ and $\cos \alpha < 0$.

Clearly, z lies in the fourth quadrant.

$$\therefore \arg(z) = -\beta = \frac{\alpha}{2} - \frac{3\pi}{4}$$

$$\text{So polar form of } z \text{ is } -\sqrt{2} \left(\cos \frac{\alpha}{2} - \sin \frac{\alpha}{2} \right) \left(\cos \left(\frac{\alpha}{2} - \frac{3\pi}{4} \right) + i \sin \left(\frac{\alpha}{2} - \frac{3\pi}{4} \right) \right)$$

Case III: When $\frac{3\pi}{2} < \alpha < 2\pi$

$$\cos \frac{\alpha}{2} < \sin \frac{\alpha}{2} \text{ and } \frac{\pi}{4} + \frac{\alpha}{2} \in \left(\pi, \frac{5\pi}{4}\right)$$

$$\therefore |z| = \sqrt{2} \left| \cos \frac{\alpha}{2} - \sin \frac{\alpha}{2} \right| = -\sqrt{2} \left(\cos \frac{\alpha}{2} - \sin \frac{\alpha}{2} \right)$$

$$\text{and, } \tan \beta = \left| \tan \left(\frac{\pi}{4} + \frac{\alpha}{2} \right) \right| = \tan \left(\frac{\pi}{4} + \frac{\alpha}{2} \right) = -\tan \left\{ \pi - \left(\frac{\pi}{4} + \frac{\alpha}{2} \right) \right\} = \tan \left(\frac{\alpha}{2} - \frac{3\pi}{4} \right)$$

$$\Rightarrow \beta = \frac{\alpha}{2} - \frac{3\pi}{4}$$

Clearly, $\operatorname{Re}(z) < 0$ and $\operatorname{Im}(z) > 0$.

So, z lies in the first quadrant.

$$\therefore \arg(z) = \beta = \frac{\alpha}{2} - \frac{3\pi}{4}$$

$$\text{So polar form of } z \text{ is } -\sqrt{2} \left(\cos \frac{\alpha}{2} - \sin \frac{\alpha}{2} \right) \left(\cos \left(\frac{\alpha}{2} - \frac{3\pi}{4} \right) + i \sin \left(\frac{\alpha}{2} - \frac{3\pi}{4} \right) \right).$$

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$$\text{Let } z = \frac{1-i}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}} = \frac{1-i}{\frac{1}{2} + i \frac{\sqrt{3}}{2}} = \frac{2-2i}{1+i\sqrt{3}} = \frac{(2-2i)(1-i\sqrt{3})}{(1+i\sqrt{3})(1-i\sqrt{3})} =$$

$$\frac{(2-2\sqrt{3})-i(2\sqrt{3}+2)}{4} = \frac{(1-\sqrt{3})}{2} - i \frac{(\sqrt{3}+1)}{2}$$

$$|z| = \sqrt{\frac{(1-\sqrt{3})^2}{4} + \frac{(\sqrt{3}+1)^2}{4}} = \sqrt{\frac{8}{4}} = \sqrt{2}$$

$$\text{Let } \beta \text{ be acute angle given by } \tan \beta = \frac{|\operatorname{Im}(z)|}{|\operatorname{Re}(z)|}.$$

$$\tan \beta = \frac{\left| -\frac{(\sqrt{3}+1)}{2} \right|}{\left| \frac{(1-\sqrt{3})}{2} \right|} = \frac{\left| -(\sqrt{3}+1) \right|}{\left| (1-\sqrt{3}) \right|} = \left| 2+\sqrt{3} \right| = \tan \left(\frac{7\pi}{12} \right)$$

$$\Rightarrow \beta = \frac{7\pi}{12}$$

Z is represented by a point in second quadrant.

So polar form of z is $\sqrt{2} \left(\cos \frac{7\pi}{12} - i \sin \frac{7\pi}{12} \right)$.

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