



Algebra of Matrices Ex 5.4 Q7

$$\text{Given that } A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}.$$

We need to find $A^T - B^T$.

$$\text{Given that, } B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$\Rightarrow B^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

Let us find $A^T - B^T$:

$$A^T - B^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$\Rightarrow A^T - B^T = \begin{bmatrix} 3+1 & 4-1 \\ -1-2 & 2-2 \\ 0-1 & 1-3 \end{bmatrix}$$

$$\Rightarrow A^T - B^T = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

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(i)

$$A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$$

$$\therefore A' = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

$$\begin{aligned} A'A &= \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \\ &= \begin{bmatrix} (\cos \alpha)(\cos \alpha) + (-\sin \alpha)(-\sin \alpha) & (\cos \alpha)(\sin \alpha) + (-\sin \alpha)(\cos \alpha) \\ (\sin \alpha)(\cos \alpha) + (\cos \alpha)(-\sin \alpha) & (\sin \alpha)(\sin \alpha) + (\cos \alpha)(\cos \alpha) \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & \sin \alpha \cos \alpha - \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha - \sin \alpha \cos \alpha & \sin^2 \alpha + \cos^2 \alpha \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \end{aligned}$$

Hence, we have verified that $A'A = I$.

Algebra of Matrices Ex 5.4 Q9

(ii)

$$A = \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix}$$

$$\therefore A' = \begin{bmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix}$$

$$\begin{aligned} A'A &= \begin{bmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix} \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix} \\ &= \begin{bmatrix} (\sin \alpha)(\sin \alpha) + (-\cos \alpha)(-\cos \alpha) & (\sin \alpha)(\cos \alpha) + (-\cos \alpha)(\sin \alpha) \\ (\cos \alpha)(\sin \alpha) + (\sin \alpha)(-\cos \alpha) & (\cos \alpha)(\cos \alpha) + (\sin \alpha)(\sin \alpha) \end{bmatrix} \\ &= \begin{bmatrix} \sin^2 \alpha + \cos^2 \alpha & \sin \alpha \cos \alpha - \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha - \sin \alpha \cos \alpha & \cos^2 \alpha + \sin^2 \alpha \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \end{aligned}$$

Hence, we have verified that $A'A = I$.

Algebra of Matrices Ex 5.4 Q10

Given,

l_i, m_i, n_i are direction cosines of three mutually perpendicular vectors

$$\Rightarrow \left. \begin{aligned} l_1 l_2 + m_1 m_2 + n_1 n_2 &= 0 \\ l_2 l_3 + m_2 m_3 + n_2 n_3 &= 0 \\ l_1 l_3 + m_1 m_3 + n_1 n_3 &= 0 \end{aligned} \right\} \quad \text{---(A)}$$

And,

$$\left. \begin{aligned} l_1^2 + m_1^2 + n_1^2 &= 1 \\ l_2^2 + m_2^2 + n_2^2 &= 1 \\ l_3^2 + m_3^2 + n_3^2 &= 1 \end{aligned} \right\} \quad \text{---(B)}$$

Given,

$$\begin{aligned} A &= \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \\ AA^T &= \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}^T \\ &= \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix} \\ &= \begin{bmatrix} l_1^2 + m_1^2 + n_1^2 & l_1 l_2 + m_1 m_2 + n_1 n_2 & l_1 l_3 + m_1 m_3 + n_1 n_3 \\ l_1 l_2 + m_1 m_2 + n_1 n_2 & l_2^2 + m_2^2 + n_2^2 & l_2 l_3 + m_2 m_3 + n_2 n_3 \\ l_1 l_3 + m_1 m_3 + n_1 n_3 & l_2 l_3 + m_2 m_3 + n_2 n_3 & l_3^2 + m_3^2 + n_3^2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{(Using (A) and (B))} \\ &= I \end{aligned}$$

Hence,

$$AA^T = I$$

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