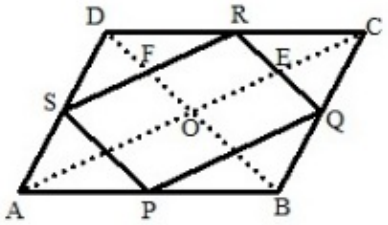
Exercise 9C

Question 10:

Given : ABCD is a rhombus in which P,Q,R and S are the mid – points of AB, BC, CD and DA respectively.



To Prove :PQRS is a rectangle.

Construction : Join AC and BD.

Proof :

Midpoint Theorem: The line segment joining the midpoints of any two sides of a triangle is parallel to the third side and equal to half of it.

In ΔABC

P and Q are the mid points of AB and BC.

So by Mid point Theorem,

$PQ \parallel AC$ and $PQ = \frac{1}{2}AC$

Similarly, from ΔADC,

$RS \parallel AC$ and $RS = \frac{1}{2}AC$

$\Rightarrow PQ \parallel RS$ and $PQ = RS = \frac{1}{2}AC$.....(1)

Now, in ΔBAD,

P and S are the mid – points of AB and AD.

So by Mid – point Theorem, we have

$PS \parallel BD$ and $PS = \frac{1}{2}DB$

Similarly, from ΔBCD,

$RQ \parallel BD$ and $RQ = \frac{1}{2}DB$

$\Rightarrow PS \parallel RQ$ and $PS = RQ = \frac{1}{2}DB$.....(2)

From (1) and (2), we have

PQRS is a parallelogram as its opposite sides are parallel.

We know , that in a rhombus, diagonals intersects at right angles.

$\therefore \quad \angle EOF = 90^0$

Now, $RQ \parallel DB$

$\Rightarrow \quad RE \parallel FO$

Also, $SR \parallel AC$

$\Rightarrow \quad FR \parallel OE$

$\therefore$ OERF is a parallelogram.

In a parallelogram, opposite angles are equal.

So, $\angle FRE = \angle EOF = 90^0$

Thus, PQRS is a parallelogram with $\angle R = 90^0$

Hence, PQRS is a rectangle.

********** END **********