Indefinite Integrals Ex 19.5 Q6

Let $I = \int \frac{3x+5}{\sqrt{7x+9}} dx$

Let $3x + 5 = \lambda(7x+9) + \mu$ on equating the coefficients of like powers of x on both sides, we get

$$7\lambda = 3 \quad \text{and} \quad 9\lambda + \mu = 5$$

$$\Rightarrow \quad \lambda = \frac{3}{7} \quad \text{and} \quad 9 \times \frac{3}{7} + \mu = 5$$

$$\Rightarrow \quad \lambda = \frac{3}{7} \quad \text{and} \quad \mu = \frac{8}{7}$$

$$\therefore \quad I = \int \frac{\lambda(7x+9)+\mu}{\sqrt{7x+9}} dx$$

$$= \lambda \int \frac{7x+9}{\sqrt{7x+9}} dx + \mu \int \frac{1}{\sqrt{7x+9}} dx$$

$$= \lambda \int (7x+9)^{\frac{1}{2}} dx + \mu \int (7x+9)^{\frac{-1}{2}} dx$$

$$= \lambda \times \frac{(7x+9)^{\frac{3}{2}}}{\frac{3}{2}\times 7} + \mu \frac{(7x+9)^{\frac{1}{2}}}{\frac{1}{2}\times 7} + c$$

$$= \frac{3}{7} \times \frac{2}{21} \times (7x+9)^{\frac{3}{2}} + \frac{8}{7} \times \frac{2}{7}(7x+9)^{\frac{1}{2}} + c$$

$$= \frac{2}{49} \times (7x+9)^{\frac{3}{2}} + \frac{16}{49} \times (7x+9)^{\frac{1}{2}} + c$$

$$= \frac{2}{49} \times (7x+9)^{\frac{1}{2}} [7x+9+8] + c$$

$$= \frac{2}{49} \times (7x+9)^{\frac{1}{2}} [7x+17] + c$$

$$= \frac{2}{49} \times (7x+17)\sqrt{7x+9} + c$$

Indefinite Integrals Ex 19.5 Q7

Let $I = \int \frac{x}{\sqrt{x+4}}dx$. Then,

$$I = \int \frac{x+4-4}{\sqrt{x+4}}dx$$

$$= \int \frac{x+4}{\sqrt{x+4}}dx - 4\int \frac{1}{\sqrt{x+4}}dx$$

$$= \int (x+4)^{\frac{1}{2}}dx - 4\int (x+4)^{\frac{-1}{2}}dx$$

$$= \frac{(x+4)^{\frac{3}{2}}}{\frac{3}{2}} - 4\frac{(x+4)^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= \frac{2(x+4)^{\frac{3}{2}}}{3} - 8(x+4)^{\frac{1}{2}} + c$$

$$= 2(x+4)^{\frac{1}{2}}\left[\frac{1}{3}(x+4) - 4\right] + c$$

$$= 2(x+4)^{\frac{1}{2}}\left[\frac{(x+4)-12}{3}\right] + c$$

$$= \frac{2}{3}(x+4)^{\frac{1}{2}}[x-8] + c$$

$$\therefore \qquad I = \frac{2}{3} \times (x-8)\sqrt{x+4} + c.$$

Indefinite Integrals Ex 19.5 Q8

Let $I = \int \frac{2-3x}{\sqrt{1+3x}} \times dx$. Then,

$$I = \int \frac{2-3x-1+1}{\sqrt{1+3x}} \times dx$$

$$= \int \frac{-3x-1+3}{\sqrt{1+3x}} \times dx$$

$$= \int -\frac{(3x+1)}{\sqrt{1+3x}} \times dx + 3\int \frac{1}{\sqrt{1+3x}} dx$$

$$= -1\int \frac{1+3x}{\sqrt{1+3x}} \times dx + 3\int \frac{1}{\sqrt{1+3x}} dx$$

$$= -1\int (1+3x)^{\frac{1}{2}} dx + 3\int (1+3x)^{\frac{-1}{2}} dx$$

$$= -1 \times \frac{(1+3x)^{\frac{3}{2}}}{\frac{3}{2} \times 3} + 3 \times \frac{(1+3x)^{\frac{1}{2}}}{\frac{1}{2} \times 3} + c$$

$$= -\frac{2}{9} \times (1+3x)^{\frac{3}{2}} + 2(1+3x)^{\frac{1}{2}} + c$$

$$= 2(1+3x)^{\frac{1}{2}} \left[-\frac{1}{9}(1+3x)^{1} + 1\right] + c$$

$$= 2(1+3x)^{\frac{1}{2}} \left[\frac{-1-3x+9}{9}\right] + c$$

$$= 2(1+3x)^{\frac{1}{2}} \left[\frac{8-3x}{9}\right] + c$$

$$= \frac{2}{9}\sqrt{1+3x}\,(8-3x) + c$$

$$\therefore \quad I = \frac{2}{9}(8-9x)\sqrt{1+3x} + c$$

Indefinite Integrals Ex 19.5 Q9

Let $I = \int 5x + 3\sqrt{2x - 1}\ dx$

Let $5x + 3 = \lambda\left(2x - 1\right) + \mu$ comparing both sides, we get

$$2\lambda = 5 \quad \text{and} \quad -\lambda + \mu = 3$$

$$\Rightarrow \quad \lambda = \frac{5}{2} \quad \text{and} \quad \frac{-5}{2} + \mu = 3$$

$$\Rightarrow \quad \lambda = \frac{5}{2} \quad \text{and} \quad \mu = \frac{11}{2}$$

$$\therefore \quad I = \int\left\{\lambda\left(2x - 1\right) + \mu\right\}\sqrt{2x - 1}dx$$

$$= \lambda\int\left(2x - 1\right)\sqrt{2x - 1}dx + \mu\int\sqrt{2x - 1}dx$$

$$= \lambda\int\left(2x - 1\right)^{\frac{3}{2}}dx + \mu\int\left(2x - 1\right)^{\frac{1}{2}}dx$$

$$= \lambda\frac{\left(2x - 1\right)^{\frac{5}{2}}}{\frac{5}{2}\times 2} + \mu\frac{\left(2x - 1\right)^{\frac{3}{2}}}{\frac{3}{2}\times 2}$$

$$= \lambda\frac{\left(2x - 1\right)^{\frac{5}{2}}}{5} + \mu\frac{\left(2x - 1\right)^{\frac{3}{2}}}{3} + c$$

$$= \frac{5}{2}\times\frac{\left(2x - 1\right)^{\frac{5}{2}}}{5} + \frac{11}{2}\times\frac{\left(2x - 1\right)^{\frac{3}{2}}}{3} + c$$

$$= \frac{\left(2x - 1\right)^{\frac{5}{2}}}{2} + \frac{11}{6}\times\left(2x - 1\right)^{\frac{3}{2}} + c$$

$$= \frac{1}{2}\left(2x - 1\right)^{\frac{3}{2}}\left[\left(2x - 1\right) + \frac{11}{3}\right] + c$$

$$= \frac{1}{2}\times\left(2x - 1\right)^{\frac{3}{2}}\left[\frac{6x + 8}{3}\right] + c$$

$$= \frac{1}{2}\times\left(2x - 1\right)^{\frac{3}{2}}\times 2\frac{\left(3x + 4\right)}{3} + c$$

$$= \left(2x - 1\right)^{\frac{3}{2}}\times\frac{\left(3x + 4\right)}{3} + c$$

$$= \frac{1}{3}\times\left(3x + 4\right)\left(2x - 1\right)^{\frac{3}{2}} + c$$

$$\therefore \quad I = \frac{1}{3}\times\left(3x + 4\right)\left(2x - 1\right)^{\frac{3}{2}} + c.$$

*********** END **********