



Sine and Cosine Formulae and their Applications Ex-10.2 Q10

In any $\triangle ABC$, we have

$$a = b \cos C + c \cos B$$

$$b = c \cos A + a \cos C$$

$$c = a \cos B + b \cos A$$

Therefore,

$$\begin{aligned} L.H.S &= a(\cos B + \cos C - 1) + b(\cos C + \cos A - 1) + c(\cos A + \cos B - 1) \\ &= a \cos B + a \cos C - a + b \cos C + b \cos A - b + c \cos A + c \cos B - c \\ &= c - b \cos A + a \cos C - a + a - c \cos B + b \cos A - b + b - a \cos C + c \cos B - c \\ &= 0 \\ &= R.H.S \end{aligned}$$

Hence proved.

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By sine rule we have

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$k \sin A = a, k \sin B = b, k \sin C = c$$

$$\begin{aligned} a \cos A + b \cos B + c \cos C &= k \sin A \cos A + k \sin B \cos B + k \sin C \cos C \\ &= \left(\frac{1}{2}\right)k[2 \sin A \cos A + 2 \sin B \cos B + 2 \sin C \cos C] \\ &= \left(\frac{1}{2}\right)k[\sin 2A + \sin 2B + \sin 2C] \\ &= k[\sin(A+B) \cos(A-B) + \sin C \cos C] \\ &= k[\sin(\pi - C) \cos(A-B) + \sin C \cos(\pi - (A+B))] \\ &= k[\sin C \cos(A-B) - \sin C \cos(A+B)] \\ &= k[\sin C(\cos(A-B) - \cos(A+B))] \\ &= k \sin C[2 \sin A \sin B] \\ &= 2 \sin C(k \sin A) \sin B \\ &= 2a \sin B \sin C \end{aligned}$$

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We know that by cosine rule

$$\begin{aligned} \cos A &= \frac{b^2 + c^2 - a^2}{2bc} \\ \Rightarrow 2bc \cos A &= b^2 + c^2 - a^2 \\ \Rightarrow a^2 &= b^2 + c^2 - 2bc \cos A \\ \Rightarrow a^2 &= b^2 + c^2 - 2bc \left(2 \cos^2 \frac{A}{2} - 1\right) \\ \Rightarrow a^2 &= b^2 + c^2 + 2bc - 4bc \cos^2 \frac{A}{2} \\ \Rightarrow a^2 &= (b+c)^2 - 4bc \cos^2 \frac{A}{2} \end{aligned}$$

Sine and Cosine Formulae and their Applications Ex-10.2 Q13

$$4\left(bc\cos^2\frac{A}{2}+ca\cos^2\frac{B}{2}+ab\cos^2\frac{C}{2}\right)=(a+b+c)^2$$

LHS,

$$\begin{aligned} & 4\left(bc\cos^2\frac{A}{2}+ca\cos^2\frac{B}{2}+ab\cos^2\frac{C}{2}\right) \\ &= 2\left(bc.2\cos^2\frac{A}{2}+ca.2\cos^2\frac{B}{2}+ab.2\cos^2\frac{C}{2}\right) \\ &= 2(bc.(1-\cos A)+ca.(1-\cos B)+ab.(1-\cos C)) \\ &= 2bc-2bc\cos A+2ca-2ca\cos B+2ab-2ab\cos C \\ &= 2bc-2bc\frac{b^2+c^2-a^2}{2bc}+2ca-2ca\frac{a^2+c^2-b^2}{2ca}+2ab \\ &\quad -2ab\frac{b^2+a^2-c^2}{2ab}[\cos\text{ rule}] \\ &= 2bc-b^2-c^2+a^2+2ca-a^2-c^2+b^2+2ab-b^2-a^2+c^2 \\ &= a^2+b^2+c^2+2ab+2bc+2ca \\ &= (a+b+c)^2 = RHS \end{aligned}$$

Sine and Cosine Formulae and their Applications Ex-10.2 Q14

$$\begin{aligned} & \sin^3 A \cos(B-C) + \sin^3 B \cos(C-A) + \sin^3 C \cos(A-B) \\ &= \sin^2 A \sin A \cos(B-C) + \sin^2 B \sin B \cos(C-A) + \sin^2 C \sin C \cos(A-B) \\ &= \sin^2 A \sin(\pi-(B+C)) \cos(B-C) + \sin^2 B \sin(\pi-(A+C)) \cos(C-A) \\ &\quad + \sin^2 C \sin(\pi-(A+B)) \cos(A-B) \\ &= \sin^2 A \sin(B+C) \cos(B-C) + \sin^2 B \sin(C+A) \cos(C-A) \\ &\quad + \sin^2 C \sin(A+B) \cos(A-B) \\ &= \sin^2 A (\sin 2B + \sin 2C) + \sin^2 B (\sin 2C + \sin 2A) + \sin^2 C (\sin 2A + \sin 2B) \\ &= \sin^2 A (2 \sin B \cos B + 2 \sin C \cos C) + \sin^2 B (2 \sin C \cos C + 2 \sin A \cos A) \\ &\quad + \sin^2 C (2 \sin A \cos A + 2 \sin B \cos B) \\ &= \sin^2 A (2 \sin B \cos B + 2 \sin C \cos C) + \sin^2 B (2 \sin C \cos C + 2 \sin A \cos A) \\ &\quad + \sin^2 C (2 \sin A \cos A + 2 \sin B \cos B) \\ &= \sin^2 A 2 \sin B \cos B + \sin^2 A 2 \sin C \cos C + \sin^2 B 2 \sin C \cos C \\ &\quad + \sin^2 B 2 \sin A \cos A + \sin^2 C 2 \sin A \cos A + \sin^2 C 2 \sin B \cos B \\ &= k^2 a^2 2kb \cos B + k^2 a^2 2kc \cos C + k^2 b^2 2ka \cos C \\ &\quad + k^2 b^2 2ka \cos A + k^2 c^2 2ka \cos A + k^2 c^2 2kb \cos B \\ &= k^3 ab(a \cos B + b \cos A) + k^3 ac(a \cos C + c \cos A) + k^3 bc(c \cos B + b \cos C) \\ &= k^3 abc + k^3 acb + k^3 bca \\ &= k^3 3abc \\ &= 3(k \sin A k \sin B k \sin C) \\ &= 3abc = RHS \end{aligned}$$

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