Tangents and Normals Ex 16.1 Q1(ix)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad\qquad \text{---(A)}$$

And the slope of the normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{-1}{f'(x)} \qquad\qquad \text{---(B)}$$

$x^2 + 3y + y^2 = 5$

Differentiating with respect to x, we get

$$2x + 3\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx}(3 + 2y) = -2x$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-2x}{3 + 2y}$$

So, the slope of tangent at $(1,1)$ is

$$\frac{dy}{dx} = \frac{-2.1}{3 + 2.1} = \frac{-2}{5}$$

The slope of normal is

$$\frac{-1}{\frac{dy}{dx}} = \frac{5}{2}$$

Tangents and Normals Ex 16.1 Q1(x)

We know that the slope of the tangent to the curve $y = f(x)$ is

$$\frac{dy}{dx} = f'(x) \qquad\qquad\qquad ---(A)$$

And the slope of the normal is

$$\frac{-1}{\dfrac{dy}{dx}} = \frac{-1}{f'(x)} \qquad\qquad ---(B)$$

$xy = 6$

Differentiating with respect to x, we get

$$y + x\frac{dy}{dx} = 0$$

$$\Rightarrow \qquad \frac{dy}{dx} = \frac{-y}{x}$$

∴ Slope of tangent at $(1,6)$ is

$$\frac{dy}{dx} = -6 \text{ and}$$

Slope of normal is

$$\frac{-1}{\dfrac{dy}{dx}} = \frac{1}{6}$$

Tangents and Normals Ex 16.1 Q2

Differentiating with respect to x, we get

$$y + x\frac{dy}{dx} + a + b\frac{dy}{dx} = 0$$

$$\Rightarrow \qquad \frac{dy}{dx}(x + b) = -(a + y)$$

$$\Rightarrow \qquad \frac{dy}{dx} = \frac{-(a+y)}{x+b}$$

∴ Slope of tangent $= \left(\dfrac{dy}{dx}\right)_{x=1,\; y=1} = \dfrac{-(a+1)}{b+1} = 2$ \qquad [given]

$$\Rightarrow \qquad -(a+1) = 2b + 2$$

$$\Rightarrow \qquad 2b + a = -3 \qquad\qquad ---(i)$$

Also, $(1,1)$ lies on the curve, so $x = 1$, $y = 1$ satisfies the equation

$$xy + ax + by = 2$$

$$\Rightarrow \qquad 1 + a + b = 2$$

$$\Rightarrow \qquad a + b = 1 \qquad\qquad ---(ii)$$

Solving (i) and (ii), we get

$$a = 5,\; b = -4$$

Tangents and Normals Ex 16.1 Q3

We have,

$$y = x^3 + ax + b \qquad \text{---(i)}$$
$$x - y + 5 = 0 \qquad \text{---(ii)}$$

Now,

Point $(1, -6)$ lies on (i), so,

$$-6 = 1 + a + b$$
$$\Rightarrow \quad a + b = -7 \qquad \text{---(iii)}$$

Also,

Slope of tangent to (i) is

$$\frac{dy}{dx} = 3x^2 + a$$
$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(1,-6)} = 3 + a$$

And slope of tangent to (ii) is

$$\frac{dy}{dx} = 1$$

According to the question slope of (i) and (ii) are parallel

$$\therefore \quad 3 + a = 1$$
$$\Rightarrow \quad a = -2$$

From (iii)

$$b = -5$$

********** END *********