**Q1 :** $\dfrac{1}{x - x^3}$

**Answer :**

$$\frac{1}{x - x^3} = \frac{1}{x(1 - x^2)} = \frac{1}{x(1-x)(1+x)}$$

Let $\dfrac{1}{x(1-x)(1+x)} = \dfrac{A}{x} + \dfrac{B}{(1-x)} + \dfrac{C}{1+x}$     ...(1)

$\Rightarrow 1 = A(1 - x^2) + Bx(1+x) + Cx(1-x)$

$\Rightarrow 1 = A - Ax^2 + Bx + Bx^2 + Cx - Cx^2$

Equating the coefficients of $x^2$, $x$, and constant term, we obtain

$-A + B - C = 0$

$B + C = 0$

$A = 1$

On solving these equations, we obtain

$A = 1, B = \dfrac{1}{2}$, and $C = -\dfrac{1}{2}$

From equation (1), we obtain

$$\frac{1}{x(1-x)(1+x)} = \frac{1}{x} + \frac{1}{2(1-x)} - \frac{1}{2(1+x)}$$

$$\Rightarrow \int \frac{1}{x(1-x)(1+x)}dx = \int \frac{1}{x}dx + \frac{1}{2}\int \frac{1}{1-x}dx - \frac{1}{2}\int \frac{1}{1+x}dx$$

$$= \log|x| - \frac{1}{2}\log|(1-x)| - \frac{1}{2}\log|(1+x)|$$

$$= \log|x| - \log\left|(1-x)^{\frac{1}{2}}\right| - \log\left|(1+x)^{\frac{1}{2}}\right|$$

$$= \log\left|\frac{x}{(1-x)^{\frac{1}{2}}(1+x)^{\frac{1}{2}}}\right| + C$$

$$= \log\left|\left(\frac{x^2}{1-x^2}\right)^{\frac{1}{2}}\right| + C$$

$$= \frac{1}{2}\log\left|\frac{x^2}{1-x^2}\right| + C$$

**Q2 :** $\dfrac{1}{\sqrt{x+a} + \sqrt{(x+b)}}$

**Answer :**

$$\frac{1}{\sqrt{x+a} + \sqrt{x+b}} = \frac{1}{\sqrt{x+a} + \sqrt{x+b}} \times \frac{\sqrt{x+a} - \sqrt{x+b}}{\sqrt{x+a} - \sqrt{x+b}}$$

$$= \frac{\sqrt{x+a} - \sqrt{x+b}}{(x+a) - (x+b)}$$

$$= \frac{\left(\sqrt{x+a} - \sqrt{x+b}\right)}{a - b}$$

$$\Rightarrow \int \frac{1}{\sqrt{x+a} - \sqrt{x+b}}dx = \frac{1}{a-b}\int \left(\sqrt{x+a} - \sqrt{x+b}\right)dx$$

$$= \frac{1}{(a-b)}\left[\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x+b)^{\frac{3}{2}}}{\frac{3}{2}}\right]$$

$$= \frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}}\right] + C$$

**Q3 :** $\dfrac{1}{x\sqrt{ax - x^2}}$   [Hint: Put $x = \dfrac{a}{t}$]

**Answer :**

$$\frac{1}{x\sqrt{ax-x^2}}$$

Let $x = \frac{a}{t} \Rightarrow dx = -\frac{a}{t^2}dt$

$$\Rightarrow \int \frac{1}{x\sqrt{ax-x^2}}dx = \int \frac{1}{\frac{a}{t}\sqrt{a\cdot\frac{a}{t}-\left(\frac{a}{t}\right)^2}}\left(-\frac{a}{t^2}dt\right)$$

$$= -\int \frac{1}{at}\cdot\frac{1}{\sqrt{\frac{1}{t}-\frac{1}{t^2}}}dt$$

$$= -\frac{1}{a}\int \frac{1}{\sqrt{\frac{t^2}{t}-\frac{t^2}{t^2}}}dt$$

$$= -\frac{1}{a}\int \frac{1}{\sqrt{t-1}}dt$$

$$= -\frac{1}{a}\left[2\sqrt{t-1}\right]+C$$

$$= -\frac{1}{a}\left[2\sqrt{\frac{a}{x}-1}\right]+C$$

$$= -\frac{2}{a}\left(\frac{\sqrt{a-x}}{\sqrt{x}}\right)+C$$

$$= -\frac{2}{a}\left(\sqrt{\frac{a-x}{x}}\right)+C$$

---

**Q4 :** $\dfrac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}$

**Answer :**

$$\frac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}$$

Multiplying and dividing by $x^{-3}$, we obtain

$$\frac{x^{-3}}{x^2\cdot x^{-3}\left(x^4+1\right)^{\frac{3}{4}}} = \frac{x^{-3}\left(x^4+1\right)^{\frac{-3}{4}}}{x^2\cdot x^{-3}}$$

$$= \frac{\left(x^4+1\right)^{\frac{-3}{4}}}{x^5\cdot\left(x^4\right)^{\frac{3}{4}}}$$

$$= \frac{1}{x^5}\left(\frac{x^4+1}{x^4}\right)^{\frac{-3}{4}}$$

$$= \frac{1}{x^5}\left(1+\frac{1}{x^4}\right)^{\frac{-3}{4}}$$

Let $\frac{1}{x^4} = t \Rightarrow -\frac{4}{x^5}dx = dt \Rightarrow \frac{1}{x^5}dx = -\frac{dt}{4}$

$$\therefore \int \frac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}dx = \int \frac{1}{x^5}\left(1+\frac{1}{x^4}\right)^{\frac{-3}{4}}dx$$

$$= -\frac{1}{4}\int(1+t)^{\frac{-3}{4}}dt$$

$$= -\frac{1}{4}\left[\frac{(1+t)^{\frac{1}{4}}}{\frac{1}{4}}\right]+C$$

$$= -\frac{1}{4}\frac{\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}}{\frac{1}{4}}+C$$

$$= -\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}+C$$

---

**Q5 :** $\dfrac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}$ $\left[\text{Hint:}\dfrac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}} = \dfrac{1}{x^{\frac{1}{3}}\left(1+x^{\frac{1}{6}}\right)}\text{ Put }x=t^6\right]$

**Answer :**

$$\frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}} = \frac{1}{x^{\frac{1}{3}}\left(1+x^{\frac{1}{6}}\right)}$$

Let $x = t^6 \Rightarrow dx = 6t^5 dt$

$$\therefore \int \frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}dx = \int \frac{1}{x^{\frac{1}{3}}\left(1+x^{\frac{1}{6}}\right)}dx$$

$$= \int \frac{6t^5}{t^2(1+t)}dt$$

$$= 6\int \frac{t^3}{(1+t)}dt$$

On dividing, we obtain

$$\int \frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}dx = 6\int \left\{(t^2-t+1)-\frac{1}{1+t}\right\}dt$$

$$= 6\left[\left(\frac{t^3}{3}\right)-\left(\frac{t^2}{2}\right)+t-\log|1+t|\right]$$

$$= 2x^{\frac{1}{2}}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\log\left(1+x^{\frac{1}{6}}\right)+C$$

$$= 2\sqrt{x}-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\log\left(1+x^{\frac{1}{6}}\right)+C$$

---

**Q6 :** $\dfrac{5x}{(x+1)(x^2+9)}$

**Answer :**

Let $\dfrac{5x}{(x+1)(x^2+9)} = \dfrac{A}{(x+1)}+\dfrac{Bx+C}{(x^2+9)}$    ...(1)

$$\Rightarrow 5x = A(x^2+9)+(Bx+C)(x+1)$$

$$\Rightarrow 5x = Ax^2+9A+Bx^2+Bx+Cx+C$$

Equating the coefficients of $x^2$, $x$, and constant term, we obtain

$A + B = 0$

$B + C = 5$

$9A + C = 0$

On solving these equations, we obtain

$A = -\dfrac{1}{2}, B = \dfrac{1}{2}$, and $C = \dfrac{9}{2}$

From equation (1), we obtain

$$\frac{5x}{(x+1)(x^2+9)} = \frac{-1}{2(x+1)}+\frac{\dfrac{x}{2}+\dfrac{9}{2}}{(x^2+9)}$$

$$\int \frac{5x}{(x+1)(x^2+9)}dx = \int \left\{\frac{-1}{2(x+1)}+\frac{(x+9)}{2(x^2+9)}\right\}dx$$

$$= -\frac{1}{2}\log|x+1|+\frac{1}{2}\int \frac{x}{x^2+9}dx+\frac{9}{2}\int \frac{1}{x^2+9}dx$$

$$= -\frac{1}{2}\log|x+1|+\frac{1}{4}\int \frac{2x}{x^2+9}dx+\frac{9}{2}\int \frac{1}{x^2+9}dx$$

$$= -\frac{1}{2}\log|x+1|+\frac{1}{4}\log|x^2+9|+\frac{9}{2}\cdot\frac{1}{3}\tan^{-1}\frac{x}{3}$$

$$= -\frac{1}{2}\log|x+1|+\frac{1}{4}\log(x^2+9)+\frac{3}{2}\tan^{-1}\frac{x}{3}+C$$

---

**Q7 :** $\dfrac{\sin x}{\sin(x-a)}$

**Answer :**

$$\frac{\sin x}{\sin(x-a)}$$

Let $x - a = t \Rightarrow dx = dt$

$$\int \frac{\sin x}{\sin(x-a)}dx = \int \frac{\sin(t+a)}{\sin t}dt$$

$$= \int \frac{\sin t\cos a+\cos t\sin a}{\sin t}dt$$

$$= \int (\cos a+\cot t\sin a)dt$$

$$= t\cos a+\sin a\log|\sin t|+C_1$$

$$= (x-a)\cos a+\sin a\log|\sin(x-a)|+C_1$$

$$= x\cos a+\sin a\log|\sin(x-a)|-a\cos a+C_1$$

$$= \sin a\log|\sin(x-a)|+x\cos a+C$$

---

**Q8 :** $\dfrac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}}$

**Answer :**

$$\frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}}=\frac{e^{4\log x}\left(e^{\log x}-1\right)}{e^{2\log x}\left(e^{\log x}-1\right)}$$

$$=e^{2\log x}$$

$$=e^{\log x^2}$$

$$=x^2$$

$$\therefore \int\frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}}dx=\int x^2 dx=\frac{x^3}{3}+C$$

Answer needs Correction? **Click Here**

---

**Q9 :** $\dfrac{\cos x}{\sqrt{4-\sin^2 x}}$

**Answer :**

$$\frac{\cos x}{\sqrt{4-\sin^2 x}}$$

Let $\sin x = t \Rightarrow \cos x\, dx = dt$

$$\Rightarrow \int\frac{\cos x}{\sqrt{4-\sin^2 x}}dx=\int\frac{dt}{\sqrt{(2)^2-(t)^2}}$$

$$=\sin^{-1}\left(\frac{t}{2}\right)+C$$

$$=\sin^{-1}\left(\frac{\sin x}{2}\right)+C$$

Answer needs Correction? **Click Here**

---

**Q10 :** $\dfrac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}$

**Answer :**

$$\frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}=\frac{\left(\sin^4 x+\cos^4 x\right)\left(\sin^4 x-\cos^4 x\right)}{\sin^2 x+\cos^2 x-\sin^2 x\cos^2 x-\sin^2 x\cos^2 x}$$

$$=\frac{\left(\sin^4 x+\cos^4 x\right)\left(\sin^2 x+\cos^2 x\right)\left(\sin^2 x-\cos^2 x\right)}{\left(\sin^2 x-\sin^2 x\cos^2 x\right)+\left(\cos^2 x-\sin^2 x\cos^2 x\right)}$$

$$=\frac{\left(\sin^4 x+\cos^4 x\right)\left(\sin^2 x-\cos^2 x\right)}{\sin^2 x\left(1-\cos^2 x\right)+\cos^2 x\left(1-\sin^2 x\right)}$$

$$=\frac{-\left(\sin^4 x+\cos^4 x\right)\left(\cos^2 x-\sin^2 x\right)}{\left(\sin^4 x+\cos^4 x\right)}$$

$$=-\cos 2x$$

$$\therefore \int\frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}dx=\int-\cos 2x\, dx=-\frac{\sin 2x}{2}+C$$

Answer needs Correction? **Click Here**

---

**Q11 :** $\dfrac{1}{\cos(x+a)\cos(x+b)}$

**Answer :**

$$\frac{1}{\cos(x+a)\cos(x+b)}$$

Multiplying and dividing by $\sin(a-b)$, we obtain

$$\frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x+a)\cos(x+b)}\right]$$

$$=\frac{1}{\sin(a-b)}\left[\frac{\sin\left[(x+a)-(x+b)\right]}{\cos(x+a)\cos(x+b)}\right]$$

$$=\frac{1}{\sin(a-b)}\left[\frac{\sin(x+a)\cdot\cos(x+b)-\cos(x+a)\sin(x+b)}{\cos(x+a)\cos(x+b)}\right]$$

$$=\frac{1}{\sin(a-b)}\left[\frac{\sin(x+a)}{\cos(x+a)}-\frac{\sin(x+b)}{\cos(x+b)}\right]$$

$$=\frac{1}{\sin(a-b)}\left[\tan(x+a)-\tan(x+b)\right]$$

$$\int\frac{1}{\cos(x+a)\cos(x+b)}dx=\frac{1}{\sin(a-b)}\int\left[\tan(x+a)-\tan(x+b)\right]dx$$

$$=\frac{1}{\sin(a-b)}\left[-\log|\cos(x+a)|+\log|\cos(x+b)|\right]+C$$

$$=\frac{1}{\sin(a-b)}\log\left|\frac{\cos(x+b)}{\cos(x+a)}\right|+C$$

Answer needs Correction? **Click Here**

**Q12 :** $\dfrac{x^3}{\sqrt{1-x^8}}$

**Answer :**

$\dfrac{x^3}{\sqrt{1-x^8}}$

Let $x^4 = t \Rightarrow 4x^3\ dx = dt$

$$\Rightarrow \int \dfrac{x^3}{\sqrt{1-x^8}}\,dx = \dfrac{1}{4}\int \dfrac{dt}{\sqrt{1-t^2}}$$
$$= \dfrac{1}{4}\sin^{-1}t + C$$
$$= \dfrac{1}{4}\sin^{-1}\left(x^4\right) + C$$

Answer needs Correction? **Click Here**

---

**Q13 :** $\dfrac{e^x}{\left(1+e^x\right)\left(2+e^x\right)}$

**Answer :**

$\dfrac{e^x}{\left(1+e^x\right)\left(2+e^x\right)}$

Let $e^x = t \Rightarrow e^x\ dx = dt$

$$\Rightarrow \int \dfrac{e^x}{\left(1+e^x\right)\left(2+e^x\right)}\,dx = \int \dfrac{dt}{(t+1)(t+2)}$$
$$= \int \left[\dfrac{1}{(t+1)} - \dfrac{1}{(t+2)}\right]dt$$
$$= \log|t+1| - \log|t+2| + C$$
$$= \log\left|\dfrac{t+1}{t+2}\right| + C$$
$$= \log\left|\dfrac{1+e^x}{2+e^x}\right| + C$$

Answer needs Correction? **Click Here**

---

**Q14 :** $\dfrac{1}{\left(x^2+1\right)\left(x^2+4\right)}$

**Answer :**

$$\therefore \dfrac{1}{\left(x^2+1\right)\left(x^2+4\right)} = \dfrac{Ax+B}{\left(x^2+1\right)} + \dfrac{Cx+D}{\left(x^2+4\right)}$$
$$\Rightarrow 1 = (Ax+B)\left(x^2+4\right) + (Cx+D)\left(x^2+1\right)$$
$$\Rightarrow 1 = Ax^3 + 4Ax + Bx^2 + 4B + Cx^3 + Cx + Dx^2 + D$$

Equating the coefficients of $x^3$, $x^2$, $x$, and constant term, we obtain

$A + C = 0$

$B + D = 0$

$4A + C = 0$

$4B + D = 1$

On solving these equations, we obtain

$A = 0$, $B = \dfrac{1}{3}$, $C = 0$, and $D = -\dfrac{1}{3}$

From equation (1), we obtain

$$\dfrac{1}{\left(x^2+1\right)\left(x^2+4\right)} = \dfrac{1}{3\left(x^2+1\right)} - \dfrac{1}{3\left(x^2+4\right)}$$
$$\int \dfrac{1}{\left(x^2+1\right)\left(x^2+4\right)}\,dx = \dfrac{1}{3}\int \dfrac{1}{x^2+1}\,dx - \dfrac{1}{3}\int \dfrac{1}{x^2+4}\,dx$$
$$= \dfrac{1}{3}\tan^{-1}x - \dfrac{1}{3}\cdot\dfrac{1}{2}\tan^{-1}\dfrac{x}{2} + C$$
$$= \dfrac{1}{3}\tan^{-1}x - \dfrac{1}{6}\tan^{-1}\dfrac{x}{2} + C$$

Answer needs Correction? **Click Here**

---

**Q15 :** $\cos^3 x\, e^{\log \sin x}$

**Answer :**

$\cos^3 x\, e^{\log \sin x} = \cos^3 x \times \sin x$

Let $\cos x = t \Rightarrow -\sin x\ dx = dt$

$$\Rightarrow \int \cos^3 x \, e^{\log \sin x} dx = \int \cos^3 x \sin x \, dx$$
$$= -\int t \cdot dt$$
$$= -\frac{t^4}{4} + C$$
$$= -\frac{\cos^4 x}{4} + C$$

---

**Q16 :** $e^{3\log x} \left(x^4 + 1\right)^{-1}$

**Answer :**

$$e^{3\log x} \left(x^4 + 1\right)^{-1} = e^{\log x^3} \left(x^4 + 1\right)^{-1} = \frac{x^3}{\left(x^4 + 1\right)}$$

Let $x^4 + 1 = t \Rightarrow 4x^3 dx = dt$

$$\Rightarrow \int e^{3\log x} \left(x^4 + 1\right)^{-1} dx = \int \frac{x^3}{\left(x^4 + 1\right)} dx$$
$$= \frac{1}{4} \int \frac{dt}{t}$$
$$= \frac{1}{4} \log|t| + C$$
$$= \frac{1}{4} \log|x^4 + 1| + C$$
$$= \frac{1}{4} \log\left(x^4 + 1\right) + C$$

---

**Q17 :** $f'(ax + b)\left[f(ax + b)\right]^n$

**Answer :**

$$f'(ax + b)\left[f(ax + b)\right]^n$$
Let $f(ax + b) = t \Rightarrow af'(ax + b) dx = dt$

$$\Rightarrow \int f'(ax + b)\left[f(ax + b)\right]^n dx = \frac{1}{a} \int t^n dt$$
$$= \frac{1}{a}\left[\frac{t^{n+1}}{n+1}\right]$$
$$= \frac{1}{a(n+1)}\left(f(ax + b)\right)^{n+1} + C$$

---

**Q18 :** $\dfrac{1}{\sqrt{\sin^3 x \sin(x + \alpha)}}$

**Answer :**

$$\frac{1}{\sqrt{\sin^3 x \sin(x + \alpha)}} = \frac{1}{\sqrt{\sin^3 x (\sin x \cos \alpha + \cos x \sin \alpha)}}$$
$$= \frac{1}{\sqrt{\sin^4 x \cos \alpha + \sin^3 x \cos x \sin \alpha}}$$
$$= \frac{1}{\sin^2 x \sqrt{\cos \alpha + \cot x \sin \alpha}}$$
$$= \frac{\csc^2 x}{\sqrt{\cos \alpha + \cot x \sin \alpha}}$$

Let $\cos \alpha + \cot x \sin \alpha = t \Rightarrow -\csc^2 x \sin \alpha \, dx = dt$

$$\therefore \int \frac{1}{\sin^3 x \sin(x + \alpha)} dx = \int \frac{\csc^2 x}{\sqrt{\cos \alpha + \cot x \sin \alpha}} dx$$
$$= \frac{-1}{\sin \alpha} \int \frac{dt}{\sqrt{t}}$$
$$= \frac{-1}{\sin \alpha}\left[2\sqrt{t}\right] + C$$
$$= \frac{-1}{\sin \alpha}\left[2\sqrt{\cos \alpha + \cot x \sin \alpha}\right] + C$$
$$= \frac{-2}{\sin \alpha}\sqrt{\cos \alpha + \frac{\cos x \sin \alpha}{\sin x}} + C$$
$$= \frac{-2}{\sin \alpha}\sqrt{\frac{\sin x \cos \alpha + \cos x \sin \alpha}{\sin x}} + C$$
$$= -\frac{2}{\sin \alpha}\sqrt{\frac{\sin(x + \alpha)}{\sin x}} + C$$

---

**Q19 :** $\dfrac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}, x \in [0,1]$

Answer :

Q20 : $\sqrt{\dfrac{1-\sqrt{x}}{1+\sqrt{x}}}$

Answer :

$$I = \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$$

Let $x = \cos^2\theta \Rightarrow dx = -2\sin\theta\cos\theta\,d\theta$

$$I = \int\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\left(-2\sin\theta\cos\theta\right)d\theta$$

$$= -\int\sqrt{\frac{2\sin^2\dfrac{\theta}{2}}{2\cos^2\dfrac{\theta}{2}}}\,\sin 2\theta\,d\theta$$

$$= -\int\tan\frac{\theta}{2}\cdot 2\sin\theta\cos\theta\,d\theta$$

$$= -2\int\frac{\sin\dfrac{\theta}{2}}{\cos\dfrac{\theta}{2}}\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)\cos\theta\,d\theta$$

$$= -4\int\sin^2\frac{\theta}{2}\cos\theta\,d\theta$$

$$= -4\int\sin^2\frac{\theta}{2}\cdot\left(2\cos^2\frac{\theta}{2}-1\right)d\theta$$

$$= -4\int\left(2\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}\right)d\theta$$

$$= -8\int\sin^2\frac{\theta}{2}\cdot\cos^2\frac{\theta}{2}\,d\theta + 4\int\sin^2\frac{\theta}{2}\,d\theta$$

$$= -2\int\sin^2\theta\,d\theta + 4\int\sin^2\frac{\theta}{2}\,d\theta$$

$$= -2\int\left(\frac{1-\cos 2\theta}{2}\right)d\theta + 4\int\frac{1-\cos\theta}{2}\,d\theta$$

$$= -2\left[\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right] + 4\left[\frac{\theta}{2}-\frac{\sin\theta}{2}\right] + C$$

$$= -\theta + \frac{\sin 2\theta}{2} + 2\theta - 2\sin\theta + C$$

$$= \theta + \frac{\sin 2\theta}{2} - 2\sin\theta + C$$

$$= \theta + \frac{2\sin\theta\cos\theta}{2} - 2\sin\theta + C$$

$$= \theta + \sqrt{1-\cos^2\theta}\cdot\cos\theta - 2\sqrt{1-\cos^2\theta} + C$$

$$= \cos^{-1}\sqrt{x} + \sqrt{1-x}\cdot\sqrt{x} - 2\sqrt{1-x} + C$$

$$= -2\sqrt{1-x} + \cos^{-1}\sqrt{x} + \sqrt{x(1-x)} + C$$

$$= -2\sqrt{1-x} + \cos^{-1}\sqrt{x} + \sqrt{x-x^2} + C$$

********** END *********