$$\tan\phi = \frac{\omega L}{R}$$
$$= \frac{2\pi \times 50 \times 0.5}{100} = 1.57$$
$$\phi = 57.5^\circ = \frac{57.5\pi}{180} \text{ rad}$$
$$\omega t = \frac{57.5\pi}{180}$$
$$t = \frac{57.5}{180 \times 2\pi \times 50}$$
$$= 3.19 \times 10^{-3} \text{s}$$
$$= 3.2 \text{ ms}$$

Hence, the time lag between maximum voltage and maximum current is 3.2 ms.

Question 7.14:

Obtain the answers (a) to (b) in Exercise 7.13 if the circuit is connected to a high frequency supply (240 V, 10 kHz). Hence, explain the statement that at very high frequency, an inductor in a circuit nearly amounts to an open circuit. How does an inductor behave in a dc circuit after the steady state?

Answer

Inductance of the inductor, L = 0.5 Hz

Resistance of the resistor, R = 100 Ω

Potential of the supply voltages, V = 240 V

Frequency of the supply, ν = 10 kHz = 10^4 Hz

Angular frequency, $\omega = 2\pi\nu = 2\pi \times 10^4$ rad/s

(a) Peak voltage, $V_0 = \sqrt{2} \times V = 240\sqrt{2}$ V

Maximum current, $I_0 = \frac{V_0}{\sqrt{R^2 + \omega^2 L^2}}$

$$= \frac{240\sqrt{2}}{\sqrt{(100)^2 + (2\pi \times 10^4)^2 \times (0.50)^2}} = 1.1 \times 10^{-2} \text{ A}$$

(b) For phase difference Φ, we have the relation:

$$\tan\phi = \frac{\omega L}{R}$$
$$= \frac{2\pi \times 10^4 \times 0.5}{100} = 100\pi$$
$$\phi = 89.82^\circ = \frac{89.82\pi}{180} \text{ rad}$$
$$\omega t = \frac{89.82\pi}{180}$$
$$t = \frac{89.82\pi}{180 \times 2\pi \times 10^4} = 25 \mu\text{s}$$

It can be observed that I_0 is very small in this case. Hence, at high frequencies, the inductor amounts to an open circuit.

In a dc circuit, after a steady state is achieved, ω = 0. Hence, inductor L behaves like a pure conducting object.

Question 7.15:

A 100 μF capacitor in series with a 40 Ω resistance is connected to a 110 V, 60 Hz supply.

(a) What is the maximum current in the circuit?

(b) What is the time lag between the current maximum and the voltage maximum?

Answer

Capacitance of the capacitor, C = 100 μF = 100 × 10^{-6} F

Resistance of the resistor, R = 40 Ω

Supply voltage, V = 110 V

(a) Frequency of oscillations, ν = 60 Hz

Angular frequency, $\omega = 2\pi\nu = 2\pi \times 60$ rad/s

For a RC circuit, we have the relation for impedance as:

$$Z = R^2 + \frac{1}{\omega^2 C^2}$$

Peak voltage, V_0 = $V\sqrt{2} = 110\sqrt{2}$ V

Maximum current is given as:

$$I_0 = \frac{V_0}{Z}$$

$$= \frac{V_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

$$= \frac{110\sqrt{2}}{\sqrt{(40)^2 + \frac{1}{(120\pi)^2 \times (10^{-4})^2}}}$$

$$= \frac{110\sqrt{2}}{\sqrt{1600 + \frac{10^8}{(120\pi)^2}}} = 3.24 \text{ A}$$

(b) In a capacitor circuit, the voltage lags behind the current by a phase angle ofΦ. This angle is given by the relation:

$$\therefore \tan\phi = \frac{\frac{1}{\omega C}}{R} = \frac{1}{\omega C R}$$

$$= \frac{1}{120\pi \times 10^{-4} \times 40} = 0.6635$$

$$\phi = \tan^{-1}(0.6635) = 33.56°$$

$$= \frac{33.56\pi}{180} \text{ rad}$$

$$\therefore \text{Time lag } = \frac{\phi}{\omega}$$

$$= \frac{33.56\pi}{180 \times 120\pi} = 1.55 \times 10^{-3} \text{s} = 1.55 \text{ ms}$$

Hence, the time lag between maximum current and maximum voltage is 1.55 ms.

Question 7.16:

Obtain the answers to (a) and (b) in Exercise 7.15 if the circuit is connected to a 110 V, 12 kHz supply? Hence, explain the statement that a capacitor is a conductor at very high frequencies. Compare this behaviour with that of a capacitor in a dc circuit after the steady state.

Answer

Capacitance of the capacitor, C = 100 μF = 100 × 10^{-6} F

Resistance of the resistor, R = 40 Ω

Supply voltage, V = 110 V

Frequency of the supply, ν = 12 kHz = 12 × 10^3 Hz

Angular Frequency, ω = 2 πν= 2 × π × 12 × $10^3$03

= 24π × 10^3 rad/s

Peak voltage, $V_0 = V\sqrt{2} = 110\sqrt{2}$ V

Maximum current, $$I_0 = \frac{V_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

$$= \frac{110\sqrt{2}}{\sqrt{(40)^2 + \frac{1}{(24\pi \times 10^3 \times 100 \times 10^{-6})^2}}}$$

$$= \frac{110\sqrt{2}}{\sqrt{1600 + \left(\frac{10}{24\pi}\right)^2}} = 3.9 \text{ A}$$

For an RC circuit, the voltage lags behind the current by a phase angle of Φ given as:

$$\tan\phi = \frac{\frac{1}{\omega C}}{R} = \frac{1}{\omega C R}$$

$$= \frac{1}{24\pi \times 10^3 \times 100 \times 10^{-6} \times 40}$$

$$\tan\phi = \frac{1}{96\pi}$$

$$\therefore \phi = 0.2°$$

$$= \frac{0.2\pi}{180} \text{ rad}$$

$$\therefore \text{Time lag } = \frac{\phi}{\omega}$$

$$= \frac{0.2\pi}{180 \times 24\pi \times 10^3} = 1.55 \times 10^{-3} \text{s} = 0.04 \mu\text{s}$$

Hence, Φ tends to become zero at high frequencies. At a high frequency, capacitor C acts as a conductor.

In a dc circuit, after the steady state is achieved, $\omega = 0$. Hence, capacitor C amounts to an open circuit.

Question 7.17:

Keeping the source frequency equal to the resonating frequency of the series *LCR* circuit, if the three elements, *L*, *C* and *R* are arranged in parallel, show that the total current in the parallel *LCR* circuit is minimum at this frequency. Obtain the current rms value in each branch of the circuit for the elements and source specified in Exercise 7.11 for this frequency.

Answer

An inductor (*L*), a capacitor (*C*), and a resistor (*R*) is connected in parallel with each other in a circuit where,

$L = 5.0$ H

$C = 80\ \mu F = 80 \times 10^{-6}$ F

$R = 40\ \Omega$

Potential of the voltage source, $V = 230$ V

Impedance (Z) of the given parallel *LCR* circuit is given as:

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2}$$

Where,

ω = Angular frequency

At resonance, $\frac{1}{\omega L} - \omega C = 0$

$$\therefore \omega = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = 50 \text{ rad/s}$$

Hence, the magnitude of *Z* is the maximum at 50 rad/s. As a result, the total current is minimum.

Rms current flowing through inductor L is given as:

$$I_L = \frac{V}{\omega L}$$

$$= \frac{230}{50 \times 5} = 0.92 \text{ A}$$

Rms current flowing through capacitor C is given as:

$$I_C = \frac{V}{\frac{1}{\omega C}} = \omega C V$$

$$= 50 \times 80 \times 10^{-6} \times 230 = 0.92 \text{ A}$$

Rms current flowing through resistor R is given as:

$$I_R = \frac{V}{R}$$

$$= \frac{230}{40} = 5.75 \text{ A}$$

Question 7.18:

A circuit containing a 80 mH inductor and a 60 μF capacitor in series is connected to a 230 V, 50 Hz supply. The resistance of the circuit is negligible.

(a) Obtain the current amplitude and rms values.

(b) Obtain the rms values of potential drops across each element.

(c) What is the average power transferred to the inductor?

(d) What is the average power transferred to the capacitor?

(e) What is the total average power absorbed by the circuit? ['Average' implies 'averaged over one cycle'.]

Answer

Inductance, $L = 80$ mH $= 80 \times 10^{-3}$ H

Capacitance, $C = 60\ \mu F = 60 \times 10^{-6}$ F

Supply voltage, $V = 230$ V

Frequency, $\nu = 50$ Hz

Angular frequency, $\omega = 2\pi\nu = 100\ \pi$ rad/s

Peak voltage, $V_0 = V\sqrt{2} = 230\sqrt{2}$ V

*********** END **********