Trigonometric Identities Ex 6.1 Q78

Answer :

Given:

$$x = a\cos^3\theta$$

$$\Rightarrow \frac{x}{a} = \cos^3\theta,$$

$$x = b\sin^3\theta$$

$$\Rightarrow \frac{y}{b} = \sin^3\theta$$

We have to prove $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$

We know that $\sin^2\theta + \cos^2\theta = 1$

So, we have

$$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = \left(\cos^3\theta\right)^{2/3} + \left(\sin^3\theta\right)^{2/3}$$

$$\Rightarrow \quad \left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = \cos^2\theta + \sin^2\theta$$

$$\Rightarrow \quad \left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$$

Hence proved.

Trigonometric Identities Ex 6.1 Q79

Answer :

Given: $3\sin\theta + 5\cos\theta = 5$

We have to prove that 5sin θ – 3cos θ = ±3.

We know that, $\sin^2\theta + \cos^2\theta = 1$

Squaring the given equation, we have

$$(3\sin\theta + 5\cos\theta)^2 = (5)^2$$

$$\Rightarrow \quad 9\sin^2\theta + 2\times 3\sin\theta \times 5\cos\theta + 25\cos^2\theta = 25$$

$$\Rightarrow \quad 9(1-\cos^2\theta) + 2\times 3\sin\theta \times 5\cos\theta + 25(1-\sin^2\theta) = 25$$

$$\Rightarrow \quad 9 - 9\cos^2\theta + 2\times 3\sin\theta \times 5\cos\theta + 25 - 25\sin^2\theta = 25$$

$$\Rightarrow \quad 34 - (9\cos^2\theta - 2\times 3\sin\theta \times 5\cos\theta + 25\sin^2\theta) = 25$$

$$\Rightarrow \quad -(25\sin^2\theta - 2\times 5\sin\theta \times 3\cos\theta + 9\cos^2\theta) = -9$$

$$\Rightarrow \quad (25\sin^2\theta - 2\times 5\sin\theta \times 3\cos\theta + 9\cos^2\theta) = 9$$

$$\Rightarrow \quad (5\sin\theta - 3\cos\theta)^2 = 9$$

$$\Rightarrow \quad 5\sin\theta - 3\cos\theta = \pm 3$$

Hence proved.

Trigonometric Identities Ex 6.1 Q80

Answer :

Given:

$a\cos\theta + b\sin\theta = m,$

$a\sin\theta - b\cos\theta = n$

We have to prove $a^2 + b^2 = m^2 + n^2$

We know that, $\sin^2\theta + \cos^2\theta = 1$

Now, squaring and adding the two equations, we get

$$(a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2 = m^2 + n^2$$

$$\Rightarrow (a^2\cos^2\theta + 2ab\sin\theta\cos\theta + b^2\sin^2\theta) + (a^2\sin^2\theta - 2ab\sin\theta\cos\theta + b^2\cos^2\theta) = 2$$

$$\Rightarrow a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta) = m^2 + n^2$$

$$\Rightarrow a^2 + b^2 = m^2 + n^2$$

Hence proved.

*********** END **********