Higher Order Derivatives Ex 12.1 Q7

$$y = \frac{\log x}{x}$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x\left(\frac{1}{x}\right) - (\log x)(i)}{x^2}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1 - \log x}{x^2}$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{x^2\left(-\frac{1}{x}\right) - (1 - \log x)(2x)}{x^4}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-x - 2x + 2x \log x}{x^4} = \frac{x(2\log x - 3)}{x^4}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{2\log x - 3}{x^3}$$

Higher Order Derivatives Ex 12.1 Q8

$x = a\sec\theta \qquad y = b\tan\theta$

differentiating both w.r.t.θ

$$\Rightarrow \qquad \frac{dx}{d\theta} = a\sec\theta\tan\theta \qquad \ldots\ldots\ldots(1)$$

$$\Rightarrow \qquad \frac{dy}{d\theta} = b\sec^2\theta \qquad \ldots\ldots(2)$$

Dividing (2) by (1)

$$\Rightarrow \qquad \frac{dy}{dx} = \frac{b\sec^2\theta}{a\sec\theta\tan\theta} \qquad \ldots.(3)$$

Differentiating (3) w.r.t.θ

$$\Rightarrow \qquad \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \frac{b}{a}\left[\frac{\tan\theta\left(\sec\theta\tan\theta\right) - \sec\theta\left(\sec^2\theta\right)}{\tan^2\theta}\right]$$

$$\Rightarrow \qquad \frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \frac{b}{a}\left[\frac{\sec\theta\left(\tan^2\theta\right) - \sec^2\theta}{\tan^2\theta}\right] \qquad \ldots\ldots(4)$$

Dividing (4) by (1)

$$\Rightarrow \qquad \frac{d^2y}{dx^2} = \frac{b\sec\theta\left(\tan^2\theta - \sec^2\theta\right)}{a \times a\sec\theta\tan\theta \times \tan^2\theta}$$

Multiplying & dividing RHS by b^3

$$\Rightarrow \qquad \frac{d^2y}{dx^2} = \frac{-b^4}{a^2 \times b^3\tan^3\theta}$$

$$\Rightarrow \qquad \frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

Higher Order Derivatives Ex 12.1 Q9

It is given that, $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$

$$\therefore \frac{dx}{dt} = a \cdot \frac{d}{dt}(\cos t + t\sin t)$$

$$= a\left[-\sin t + \sin t \cdot \frac{d}{dt}(t) + t \cdot \frac{d}{dt}(\sin t)\right]$$

$$= a\left[-\sin t + \sin t + t\cos t\right] = at\cos t$$

$$\frac{dy}{dt} = a \cdot \frac{d}{dt}(\sin t - t\cos t)$$

$$= a\left[\cos t - \left\{\cos t \cdot \frac{d}{dt}(t) + t \cdot \frac{d}{dt}(\cos t)\right\}\right]$$

$$= a\left[\cos t - \{\cos t - t\sin t\}\right] = at\sin t$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{at\sin t}{at\cos t} = \tan t$$

Then, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(\tan t) = \sec^2 t \cdot \frac{dt}{dx}$

$$= \sec^2 t \cdot \frac{1}{at\cos t} \qquad \left[\frac{dx}{dt} = at\cos t \Rightarrow \frac{dt}{dx} = \frac{1}{at\cos t}\right]$$

$$= \frac{\sec^3 t}{at}, 0 < t < \frac{\pi}{2}$$

Higher Order Derivatives Ex 12.1 Q10

$y = e^x \cos x$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = e^x(-\sin x) + e^x \cos x = e^x(\cos x - \sin x)$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = e^x(-\cos x - \sin x) + e^x(\cos x - \sin x)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -2e^x \sin x$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 2e^x \cos\left(x + \frac{\pi}{2}\right)$$

********** END **********