Transformation Formulae Ex 8.2 Q9(i)

We have,

$$\begin{aligned}
\text{LHS} &= \sin\alpha + \sin\beta + \sin\gamma - \sin(\alpha+\beta+\gamma)\\
&= (\sin\alpha + \sin\beta) + (\sin\gamma - \sin(\alpha+\beta+\gamma))\\
&= 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) + 2\sin\left(\frac{\gamma-(\alpha+\beta+\gamma)}{2}\right)\cos\left(\frac{\gamma+\alpha+\beta+\gamma}{2}\right)\\
&= 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) + 2\sin\left(\frac{-\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta+2\gamma}{2}\right)\\
&= 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) - 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha+\beta+2\gamma}{2}\right)\\
&= 2\sin\left(\frac{\alpha+\beta}{2}\right)\left[\cos\left(\frac{\alpha-\beta}{2}\right) - \cos\left(\frac{\alpha+\beta+2\gamma}{2}\right)\right]\\
&= 2\sin\left(\frac{\alpha+\beta}{2}\right)\left[-2\sin\frac{\left[\frac{\alpha-\beta}{2}+\frac{\alpha+\beta+2\gamma}{2}\right]}{2}\sin\frac{\left[\frac{\alpha-\beta}{2}-\frac{\alpha+\beta+2\gamma}{2}\right]}{2}\right]\\
&= 2\sin\left(\frac{\alpha+\beta}{2}\right)\left[-2\sin\left[\frac{2\alpha+2\gamma}{2\times 2}\right]\sin\left[\frac{-2\beta-2\gamma}{2\times 2}\right]\right]\\
&= -4\sin\left(\frac{\alpha+\beta}{2}\right)\left[\sin\left(\frac{\alpha+\gamma}{2}\right)\sin\left[\frac{-(\beta+\gamma)}{2}\right]\right]\\
&= 4\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha+\gamma}{2}\right)\sin\left(\frac{\beta+\gamma}{2}\right)\\
&= 4\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\beta+\gamma}{2}\right)\sin\left(\frac{\alpha+\gamma}{2}\right)\\
&= \text{RHS}
\end{aligned}$$

$\therefore\ \sin\alpha + \sin\beta + \sin\gamma - \sin(\alpha+\beta+\gamma) = 4\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\beta+\gamma}{2}\right)\sin\left(\frac{\alpha+\gamma}{2}\right)$ Hence proved.

Transformation Formulae Ex 8.2 Q9(ii)

We have,

$$\begin{aligned}
\text{LHS} &= \cos(A+B+C) + \cos(A-B+C) + \cos(A+B-C) + \cos(-A+B+C)\\
&= [\cos(A+B+C) + \cos(A-B+C)] + [\cos(A+B-C) + \cos(-A+B+C)]\\
&= 2\cos\left\{\frac{A+B+C+A-B+C}{2}\right\}\cos\left\{\frac{A+B+C-A+B-C}{2}\right\} + 2\left\{\begin{array}{l}\cos\left\{\frac{A+B-C-A+B+C}{2}\right\}\\ \cos\left\{\frac{A+B-C+A-B-C}{2}\right\}\end{array}\right\}\\
&= 2\cos\left\{\frac{2A+2C}{2}\right\}\cos\left\{\frac{2B}{2}\right\} + 2\cos\left\{\frac{2B}{2}\right\}\cos\left\{\frac{2A-2C}{2}\right\}\\
&= 2\cos(A+C)\cos(B) + 2\cos(B)\cos(A-C)\\
&= 2\cos(B)[\cos(A+C) + \cos(A-C)]\\
&= 2\cos(B)\left[2\cos\left(\frac{A+C+A-C}{2}\right)\cos\left(\frac{A+C-A+C}{2}\right)\right]\\
&= 2\cos(B)[2\cos A\cos C]\\
&= 4\cos A\cos B\cos C.
\end{aligned}$$

Transformation Formulae Ex 8.2 Q10

We have,

$$\cos A + \cos B = \frac{1}{2}$$

and, $\sin A + \sin B = \frac{1}{4}$

Now,

$$\frac{\sin A + \sin B}{\cos A + \cos B} = \frac{\frac{1}{4}}{\frac{1}{2}} \qquad [\text{On dividing}]$$

$$\Rightarrow \quad \frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{\sin\left(\frac{A+B}{2}\right)}{\cos\left(\frac{A+B}{2}\right)} = \frac{1}{2}$$

$$\Rightarrow \quad \tan\left(\frac{A+B}{2}\right) = \frac{1}{2}$$ Hence proved.

Transformation Formulae Ex 8.2 Q 11.

We have,

$$\operatorname{cosec} A + \sec A = \operatorname{cosec} B + \sec B$$

$$\Rightarrow \quad \sec A - \sec B = \operatorname{cosec} B - \operatorname{cosec} A$$

$$\Rightarrow \quad \frac{1}{\cos A} - \frac{1}{\cos B} = \frac{1}{\sin B} - \frac{1}{\sin A}$$

$$\Rightarrow \quad \frac{\cos B - \cos A}{\cos A \cos B} = \frac{\sin A - \sin B}{\sin A \sin B}$$

$$\Rightarrow \quad \frac{\sin A \sin B}{\cos A \cos B} = \frac{\sin A - \sin B}{\cos B - \cos A}$$

$$\Rightarrow \quad \tan A \tan B = \frac{2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)}{-2\sin\left(\frac{B-A}{2}\right)\sin\left(\frac{B+A}{2}\right)}$$

$$\Rightarrow \quad \tan A \tan B = \frac{-\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)}{-\sin\left(\frac{A-B}{2}\right)\sin\left(\frac{A+B}{2}\right)} \qquad [\because \sin(-\theta) = -\sin\theta]$$

$$\Rightarrow \quad \tan A \tan B = \cot\left(\frac{A+B}{2}\right)$$ Hence proved.

*********** END **********