Q1 : Differentiate the following w.r.t. x:

$$\frac{e^x}{\sin x}$$

Answer :

Let $y = \frac{e^x}{\sin x}$

By using the quotient rule, we obtain

$$\frac{dy}{dx} = \frac{\sin x \frac{d}{dx}(e^x) - e^x \frac{d}{dx}(\sin x)}{\sin^2 x}$$

$$= \frac{\sin x.(e^x) - e^x \cdot (\cos x)}{\sin^2 x}$$

$$= \frac{e^x(\sin x - \cos x)}{\sin^2 x}, x \neq n\pi, n \in \mathbf{Z}$$

Answer needs Correction? **Click Here**

Q2 : Differentiate the following w.r.t. x:

$$e^{\sin^{-1} x}$$

Answer :

Let $y = e^{\sin^{-1} x}$

By using the chain rule, we obtain

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\sin^{-1} x}\right)$$

$$\Rightarrow \frac{dy}{dx} = e^{\sin^{-1} x} \cdot \frac{d}{dx}\left(\sin^{-1} x\right)$$

$$= e^{\sin^{-1} x} \cdot \frac{1}{\sqrt{1-x^2}}$$

$$= \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}}$$

$$\therefore \frac{dy}{dx} = \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}}, x \in (-1,1)$$

Answer needs Correction? **Click Here**

Q3 : Differentiate the following w.r.t. x:

$$e^{x^3}$$

Answer :

Let $y = e^{x^3}$

By using the chain rule, we obtain

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{x^3}\right) = e^{x^3} \cdot \frac{d}{dx}\left(x^3\right) = e^{x^3} \cdot 3x^2 = 3x^2 e^{x^3}$$

Answer needs Correction? **Click Here**

Q4 : Differentiate the following w.r.t. x:

$$\sin\left(\tan^{-1} e^{-x}\right)$$

Answer :

Let $y = \sin\left(\tan^{-1} e^{-x}\right)$

By using the chain rule, we obtain

$$\frac{dy}{dx} = \frac{d}{dx}\left[\sin\left(\tan^{-1} e^{-x}\right)\right]$$

$$= \cos\left(\tan^{-1} e^{-x}\right) \cdot \frac{d}{dx}\left(\tan^{-1} e^{-x}\right)$$

$$= \cos\left(\tan^{-1} e^{-x}\right) \cdot \frac{1}{1+\left(e^{-x}\right)^2} \cdot \frac{d}{dx}\left(e^{-x}\right)$$

$$= \frac{\cos\left(\tan^{-1} e^{-x}\right)}{} \cdot e^{-x} \cdot \frac{d}{dx}(-x)$$

$$1 + e^{-2x} \qquad dx$$

$$= \frac{e^{-x} \cos\left(\tan^{-1} e^{-x}\right)}{1 + e^{-2x}} \times (-1)$$

$$= \frac{-e^{-x} \cos\left(\tan^{-1} e^{-x}\right)}{1 + e^{-2x}}$$

Q5 : Differentiate the following w.r.t. x:

$$\log\left(\cos e^x\right)$$

Answer :

Let $y = \log\left(\cos e^x\right)$

By using the chain rule, we obtain

$$\frac{dy}{dx} = \frac{d}{dx}\left[\log\left(\cos e^x\right)\right]$$

$$= \frac{1}{\cos e^x} \cdot \frac{d}{dx}\left(\cos e^x\right)$$

$$= \frac{1}{\cos e^x} \cdot \left(-\sin e^x\right) \cdot \frac{d}{dx}\left(e^x\right)$$

$$= \frac{-\sin e^x}{\cos e^x} \cdot e^x$$

$$= -e^x \tan e^x, e^x \neq (2n+1)\frac{\pi}{2}, n \in \mathbf{N}$$

Q6 : Differentiate the following w.r.t. x:

$$e^x + e^{x^2} + \ldots + e^{x^5}$$

Answer :

$$\frac{d}{dx}\left(e^x + e^{x^2} + \ldots + e^{x^5}\right)$$

$$= \frac{d}{dx}\left(e^x\right) + \frac{d}{dx}\left(e^{x^2}\right) + \frac{d}{dx}\left(e^{x^3}\right) + \frac{d}{dx}\left(e^{x^4}\right) + \frac{d}{dx}\left(e^{x^5}\right)$$

$$= e^x + \left[e^{x^2} \times \frac{d}{dx}\left(x^2\right)\right] + \left[e^{x^3} \cdot \frac{d}{dx}\left(x^3\right)\right] + \left[e^{x^4} \cdot \frac{d}{dx}\left(x^4\right)\right] + \left[e^{x^5} \cdot \frac{d}{dx}\left(x^5\right)\right]$$

$$= e^x + \left(e^{x^2} \times 2x\right) + \left(e^{x^3} \times 3x^2\right) + \left(e^{x^4} \times 4x^3\right) + \left(e^{x^5} \times 5x^4\right)$$

$$= e^x + 2xe^{x^2} + 3x^2e^{x^3} + 4x^3e^{x^4} + 5x^4e^{x^5}$$

Q7 : Differentiate the following w.r.t. x:

$$\sqrt{e^{\sqrt{x}}}, x > 0$$

Answer :

Let $y = \sqrt{e^{\sqrt{x}}}$

Then, $y^2 = e^{\sqrt{x}}$

By differentiating this relationship with respect to x, we obtain

$$y^2 = e^{\sqrt{x}}$$

$$\Rightarrow 2y\frac{dy}{dx} = e^{\sqrt{x}} \frac{d}{dx}\left(\sqrt{x}\right) \qquad \left[\text{By applying the chain rule}\right]$$

$$\Rightarrow 2y\frac{dy}{dx} = e^{\sqrt{x}} \frac{1}{2} \cdot \frac{1}{\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{\sqrt{x}}}{4y\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{\sqrt{x}}}{4\sqrt{e^{\sqrt{x}}}\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{\sqrt{x}}}{4\sqrt{xe^{\sqrt{x}}}}, x > 0$$

Q8 : Differentiate the following w.r.t. x:

$$\log\left(\log x\right), x > 1$$

Answer :

Let $y = \log\left(\log x\right)$

By using the chain rule, we obtain

$$\frac{dy}{dx} = \frac{d}{dx}\left[\log\left(\log x\right)\right]$$

$$= \frac{1}{\log x} \cdot \frac{d}{dx}\left(\log x\right)$$

$$= \frac{1}{\log x} \cdot \frac{1}{x}$$

$$= \frac{1}{x \log x}, \ x > 1$$

Q9 : Differentiate the following w.r.t. *x*:

$$\frac{\cos x}{\log x}, x > 0$$

Answer :

Let $y = \dfrac{\cos x}{\log x}$

By using the quotient rule, we obtain

$$\frac{dy}{dx} = \frac{\dfrac{d}{dx}(\cos x) \times \log x - \cos x \times \dfrac{d}{dx}(\log x)}{(\log x)^2}$$

$$= \frac{-\sin x \log x - \cos x \times \dfrac{1}{x}}{(\log x)^2}$$

$$= \frac{-[x \log x . \sin x + \cos x]}{x(\log x)^2}, x > 0$$

Q10 : Differentiate the following w.r.t. *x*:

$$\cos\left(\log x + e^x\right), x > 0$$

Answer :

Let $y = \cos\left(\log x + e^x\right)$

By using the chain rule, we obtain

$$\frac{dy}{dx} = -\sin\left(\log x + e^x\right) \cdot \frac{d}{dx}\left(\log x + e^x\right)$$

$$= -\sin\left(\log x + e^x\right) \cdot \left[\frac{d}{dx}(\log x) + \frac{d}{dx}(e^x)\right]$$

$$= -\sin\left(\log x + e^x\right) \cdot \left(\frac{1}{x} + e^x\right)$$

$$= -\left(\frac{1}{x} + e^x\right)\sin\left(\log x + e^x\right), x > 0$$

********** END *********