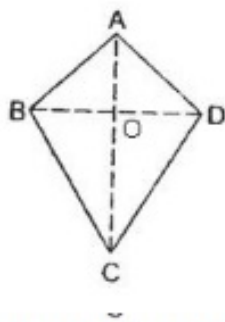Exercise 5A

Question 29:

Given: a quarilateral ABCD in which AB=AD and BC=DC

To Prove: (i) AC bisects ∠A and ∠C
(ii) AC ⊥ BD and AC bisects BD
Proof: In ∆ABC and ∆ADC,we have

$$AB=AD \qquad \text{[Given]}$$
$$BC=DC \qquad \text{[Given]}$$
$$AC=AC \qquad \text{[Common]}$$

Thus by Side-Side-Side criterion of congruence,

$$\triangle ABC \cong \triangle ADC \qquad \text{[By SSS]}$$

The corresponding parts of the congruent triangles are equal.

So, $\qquad$ ∠BAC =∠DAC $\qquad$ [C.P.C.T]
⇒ $\qquad\qquad$ ∠BAO =∠DAO $\qquad$ .....(1)

It means that AC bisects ∠BAD, that is ∠A

Also, $\qquad$ ∠BCA=∠DCA $\qquad$ [C.P.C.T]

It means that AC bisects ∠BCD, that is ∠C

(ii)

Now in ∆ABO and ∆ADO

$$AB =AD \qquad \text{[Given]}$$
$$∠BAO =∠DAO \qquad \text{[from (1)]}$$
$$AO =AO \qquad \text{[Common]}$$

Thus, by Side-Angle-Side criterion of congruence, we have

$$\triangle ABO \cong \triangle ADO \qquad \text{[By SAS]}$$

The corresponding parts of the congruent triangles are equal.

∴ $\qquad$ ∠BOA =∠DOA
But $\qquad$ ∠BOA+∠DOA = 180°
Or $\qquad$ 2∠BOA = 180°

⇒ $\qquad\qquad\qquad$ $∠BOA = \dfrac{180°}{2} = 90°$

Also, as $\qquad$ ∆ABO ≅ ∆ADO
So, $\qquad\qquad$ BO = OD
which means that AC bisects BD.

********** END *********