Mean Value Theorems Ex 15.1 Q10

By Rolle's Theorem, for a function $f:[a,\ b]\to\mathbf{R}$, if

(a) f is continuous on $[a, b]$

(b) f is differentiable on (a, b)

(c) $f(a) = f(b)$

then, there exists some $c \in (a, b)$ such that $f'(c)=0$

Therefore, Rolle's Theorem is not applicable to those functions that do not satisfy any of the three conditions of the hypothesis.

(i) $f(x)=[x]$ for $x\in[5,\ 9]$

It is evident that the given function $f(x)$ is not continuous at every integral point.

In particular, $f(x)$ is not continuous at $x = 5$ and $x = 9$

$f(x)$ is not continuous in $[5, 9]$.

Also, $f(5)=[5]=5$ and $f(9)=[9]=9$

$\therefore f(5)\neq f(9)$

The differentiability of f in $(5, 9)$ is checked as follows.

Let n be an integer such that $n \in (5, 9)$.

The left hand limit of f at $x=n$ is,

$$\lim_{h\to 0^-}\frac{f(n+h)-f(n)}{h}=\lim_{h\to 0^-}\frac{[n+h]-[n]}{h}=\lim_{h\to 0^-}\frac{n-1-n}{h}=\lim_{h\to 0^-}\frac{-1}{h}=\infty$$

The right hand limit of f at $x=n$ is,

$$\lim_{h\to 0^+}\frac{f(n+h)-f(n)}{h}=\lim_{h\to 0^+}\frac{[n+h]-[n]}{h}=\lim_{h\to 0^+}\frac{n-n}{h}=\lim_{h\to 0^+}0=0$$

Since the left and right hand limits of f at $x = n$ are not equal, f is not differentiable at $x = n$

f is not differentiable in $(5, 9)$.

It is observed that f does not satisfy all the conditions of the hypothesis of Rolle's Theorem.

Hence, Rolle's Theorem is not applicable for $f(x)=[x]$ for $x\in[5,\ 9]$.

(ii) $f(x)=[x]$ for $x\in[-2,\ 2]$

It is evident that the given function $f(x)$ is not continuous at every integral point.

In particular, $f(x)$ is not continuous at $x=-2$ and $x=2$

$f(x)$ is not continuous in $[-2, 2]$.

Also, $f(-2)=[-2]=-2$ and $f(2)=[2]=2$

$\therefore f(-2)\neq f(2)$

The differentiability of f in $(-2, 2)$ is checked as follows.

Let n be an integer such that $n\in(-2, 2)$.

The left hand limit of f at $x=n$ is,

$$\lim_{h\to 0^-}\frac{f(n+h)-f(n)}{h}=\lim_{h\to 0^-}\frac{[n+h]-[n]}{h}=\lim_{h\to 0^-}\frac{n-1-n}{h}=\lim_{h\to 0^-}\frac{-1}{h}=\infty$$

The right hand limit of f at $x=n$ is,

$$\lim_{h\to 0^+}\frac{f(n+h)-f(n)}{h}=\lim_{h\to 0^+}\frac{[n+h]-[n]}{h}=\lim_{h\to 0^+}\frac{n-n}{h}=\lim_{h\to 0^+}0=0$$

Since the left and right hand limits of f at $x=n$ are not equal, f is not differentiable at $x=n$

f is not differentiable in $(-2, 2)$.

It is observed that f does not satisfy all the conditions of the hypothesis of Rolle's Theorem.

Hence, Rolle's Theorem is not applicable for $f(x)=[x]$ for $x\in[-2,\ 2]$.

Mean Value Theorems Ex 15.1 Q11

It is given that the Rolle's Theorem holds for the function $f(x)=x^3+bx^2+cx,\ x\in[1,2]$ at the point $x=\frac{4}{3}$.

We need to find the values of b and c.

$f(x)=x^3+bx^2+cx$

Since it satisfies the rolle's theorem, we have,

$f(1)=f(2)$

$\Rightarrow 1^3+b\times 1^2+c\times 1=2^3+b\times 2^2+c\times 2$

$\Rightarrow 1+b+c=8+4b+2c$

$\Rightarrow 3b+c=-7\ldots(1)$

Differentiating the given function, we have,

$f'(x)=3x^2+2bx+c$

$f'\left(\frac{4}{3}\right)=3\times\left(\frac{4}{3}\right)^2+2b\times\left(\frac{4}{3}\right)+c$

$\Rightarrow 0=\frac{16}{3}+\frac{8b}{3}+c\ldots(2)$

Solving the equations (1) and (2), we have,

b = −5 and c = 8

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