$= O = \text{R.H.S.}$

$\therefore A^2 - 5A + 7I = O$

Question 9:

Find x, if
$$\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

Answer

We have:

$$\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} x+0-2 & 0-10+0 & 2x-5-3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow \left[x(x-2) - 40 + 2x - 8\right] = O$$
$$\Rightarrow \left[x^2 - 2x - 40 + 2x - 8\right] = [0]$$
$$\Rightarrow \left[x^2 - 48\right] = [0]$$

$$\therefore x^2 - 48 = 0$$
$$\Rightarrow x^2 = 48$$
$$\Rightarrow x = \pm 4\sqrt{3}$$

Question 10:

A manufacturer produces three products x, y, z which he sells in two markets.

Annual sales are indicated below:

Market	**Products**		
I	10000	2000	18000
II	6000	20000	8000

(a) If unit sale prices of x, y and z are Rs 2.50, Rs 1.50 and Rs 1.00, respectively, find the total revenue in each market with the help of matrix algebra.

(b) If the unit costs of the above three commodities are Rs 2.00, Rs 1.00 and 50 paise respectively. Find the gross profit.

Answer

(a) The unit sale prices of x, y, and z are respectively given as Rs 2.50, Rs 1.50, and Rs 1.00.

Consequently, the total revenue in market **I** can be represented in the form of a matrix as:

$$\begin{bmatrix} 10000 & 2000 & 18000 \end{bmatrix} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$$
$$= 10000 \times 2.50 + 2000 \times 1.50 + 18000 \times 1.00$$
$$= 25000 + 3000 + 18000$$
$$= 46000$$

The total revenue in market **II** can be represented in the form of a matrix as:

$$\begin{bmatrix} 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$$
$$= 6000 \times 2.50 + 20000 \times 1.50 + 8000 \times 1.00$$
$$= 15000 + 30000 + 8000$$
$$= 53000$$

Therefore, the total revenue in market **I** isRs 46000 and the same in market **II** isRs 53000.

(b) The unit cost prices of x, y, and z are respectively given as Rs 2.00, Rs 1.00, and 50 paise.

Consequently, the total cost prices of all the products in market **I** can be represented in the form of a matrix as:

$$\begin{bmatrix}10000 & 2000 & 18000\end{bmatrix}\begin{bmatrix}2.00\\1.00\\0.50\end{bmatrix}$$
$$=10000\times 2.00+2000\times 1.00+18000\times 0.50$$
$$=20000+2000+9000$$
$$=31000$$

Since the total revenue in market **I** isRs 46000, the gross profit in this marketis (Rs 46000 − Rs 31000) Rs 15000.

The total cost prices of all the products in market **II** can be represented in the form of a matrix as:

$$\begin{bmatrix}6000 & 20000 & 8000\end{bmatrix}\begin{bmatrix}2.00\\1.00\\0.50\end{bmatrix}$$
$$=6000\times 2.00+20000\times 1.00+8000\times 0.50$$
$$=12000+20000+4000$$
$$=\text{Rs }36000$$

Since the total revenue in market **II** isRs 53000, the gross profit in this market is (Rs 53000 − Rs 36000) Rs 17000.

Question 11:

Find the matrix X so that $X\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$

Answer

It is given that:

$$X\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$$

The matrix given on the R.H.S. of the equation is a 2 × 3 matrix and the one given on the L.H.S. of the equation is a 2 × 3 matrix. Therefore, X has to be a 2 × 2 matrix.

Now, let $X=\begin{bmatrix}a & c\\b & d\end{bmatrix}$

Therefore, we have:

$$\begin{bmatrix}a & c\\b & d\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$$
$$\Rightarrow\begin{bmatrix}a+4c & 2a+5c & 3a+6c\\b+4d & 2b+5d & 3b+6d\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$$

Equating the corresponding elements of the two matrices, we have:

$$a+4c=-7,\quad 2a+5c=-8,\quad 3a+6c=-9$$
$$b+4d=2,\quad 2b+5d=4,\quad 3b+6d=6$$

$$\text{Now, } a+4c=-7\Rightarrow a=-7-4c$$

$$\therefore 2a+5c=-8\Rightarrow -14-8c+5c=-8$$
$$\Rightarrow -3c=6$$
$$\Rightarrow c=-2$$

$$\therefore a=-7-4(-2)=-7+8=1$$

$$\text{Now, } b+4d=2\Rightarrow b=2-4d$$

$$\therefore 2b+5d=4\Rightarrow 4-8d+5d=4$$
$$\Rightarrow -3d=0$$
$$\Rightarrow d=0$$

$$\therefore b=2-4(0)=2$$

Thus, a = 1, b = 2, c = −2, d = 0

Hence, the required matrix X is $\begin{bmatrix}1 & -2\\2 & 0\end{bmatrix}$.

Question 12:

If A and B are square matrices of the same order such that AB = BA, then prove by induction that $AB^n=B^nA$. Further, prove that $(AB)^n=A^nB^n$ for all $n\in\mathbf{N}$

Answer

A and B are square matrices of the same order such that AB = BA.

To prove: $\quad \text{P}(n): AB^n=B^nA,\ n\in\mathbf{N}$

For $n = 1$, we have:

$P(1): AB = BA \quad$ [Given]

$\Rightarrow AB^1 = B^1 A$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

$P(k): AB^k = B^k A \quad ...(1)$

Now, we prove that the result is true for $n = k + 1$.

$$\begin{aligned}
AB^{k+1} &= AB^k \cdot B \\
&= \left(B^k A\right)B && [\text{By (1)}] \\
&= B^k (AB) && [\text{Associative law}] \\
&= B^k (BA) && [AB = BA \text{ (Given)}] \\
&= \left(B^k B\right)A && [\text{Associative law}] \\
&= B^{k+1} A
\end{aligned}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have $AB^n = B^n A,\ n \in \mathbf{N}$.

Now, we prove that $(AB)^n = A^n B^n$ for all $n \in \mathbf{N}$

For $n = 1$, we have:

$(AB)^1 = A^1 B^1 = AB$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

$(AB)^k = A^k B^k \quad ...(2)$

Now, we prove that the result is true for $n = k + 1$.

$$\begin{aligned}
(AB)^{k+1} &= (AB)^k \cdot (AB) \\
&= \left(A^k B^k\right) \cdot (AB) && [\text{By (2)}] \\
&= A^k \left(B^k A\right) B && [\text{Associative law}] \\
&= A^k \left(AB^k\right) B && [AB^n = B^n A \text{ for all } n \in \mathbf{N}] \\
&= \left(A^k A\right) \cdot \left(B^k B\right) && [\text{Associative law}] \\
&= A^{k+1} B^{k+1}
\end{aligned}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have $(AB)^n = A^n B^n$, for all natural numbers.

Question 13:

Choose the correct answer in the following questions:

If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$ then

A. $1 + \alpha^2 + \beta\gamma = 0$

B. $1 - \alpha^2 + \beta\gamma = 0$

*********** END **********