



Exercise 10D

Question 28:

Let the speed of the Deccan Queen = x kmph

The, speed of other train = $(x - 20)$ kmph

Then, time taken by Deccan Queen = $192/x$ hours

Time taken by other train = $192/(x-20)$ hours

Difference of time taken by two trains is $48/60 = 4/5$ h

$$\begin{aligned}\frac{192}{x-20} - \frac{192}{x} &= \frac{4}{5} \Rightarrow \frac{1}{x-20} - \frac{1}{x} = \frac{1}{240} \\ \Rightarrow \frac{x - x + 20}{x^2 - 20x} &= \frac{1}{240} \\ \Rightarrow x^2 - 20x - 4800 &= 0 \\ \Rightarrow x^2 - 80x + 60x - 4800 &= 0 \\ \Rightarrow x(x - 80) + 60(x - 80) &= 0 \\ \Rightarrow (x - 80)(x + 60) &= 0 \\ \Rightarrow x = 80 \text{ or } x = -60 \\ \therefore x = 80 \text{ [}\because \text{Speed cannot be negative]}\end{aligned}$$

Hence, speed of Deccan Queen = 80 km/h.

Question 29:

Let the speed of the stream be = x km/h

Speed of boat in still waters = 9 km/h

Speed of boat down stream = $9 + x$

time taken by boat to go 15 km downstream = $15/(9+x)$ hours

Speed of boat upstream = $9 - x$

time taken by boat to go 15 km of stream = $15/(9-x)$ hours

$$\text{Total time} = \frac{15}{9+x} + \frac{15}{9-x} = 3\frac{45}{60} = 3\frac{3}{4} = \frac{15}{4}$$

Dividing by 15

$$\begin{aligned}\frac{1}{9+x} + \frac{1}{9-x} &= \frac{1}{4} \text{ or } \frac{9+x+9-x}{(9+x)(9-x)} = \frac{1}{4} \\ \Rightarrow \frac{18}{81-x^2} &= \frac{1}{4} \text{ or } 81-x^2 = 72 \\ \therefore x^2 &= 81-72 = 9 \quad \therefore x = \pm 3 \\ \text{But } x &\neq -3 \\ \therefore \text{Speed of stream} &= 3 \text{ km / h}\end{aligned}$$

Question 30:

Let the speed of stream be x km/h

Speed of boat in still stream = 18 km/h

Speed of boat up the stream = $18 - x$ km/h

Time taken by boat to go up the stream 24 km = $24/(18+x)$ hours

Time taken by boat to go down the stream = $24/(18-x)$ hours

Time taken by the boat to go up the stream is 1 hour more than the time taken down the stream

$$\frac{24}{18-x} - \frac{24}{18+x} = 1 \Rightarrow \frac{1}{18-x} - \frac{1}{18+x} = \frac{1}{24}$$

$$\text{or } \frac{18+x-(18-x)}{(18+x)(18-x)} = \frac{1}{24} \quad \text{or } \frac{2x}{324-x^2} = \frac{1}{24}$$

$$\Rightarrow 324 - x^2 = 48x \quad \text{or } x^2 + 48x - 324 = 0$$

$$\Rightarrow x^2 + 54x - 6x - 324 = 0$$

$$\Rightarrow x(x+54) - 6(x+54) = 0 \quad \text{or } (x+54)(x-6) = 0$$

$$\therefore x+54 = 0 \quad \text{or } x-6 = 0$$

$$\Rightarrow x = -54 \quad \text{which is not true}$$

$$\therefore x-6 = 0 \quad \therefore x = 6$$

Speed of the stream = 6 km/h.

Question 31:

Let the speed of the stream be x kmph

Then the speed of boat down stream = $(8 + x)$ kmph

And the speed of boat upstream = $(8 - x)$ kmph

Time taken to cover 15 km upstream = $15/(8-x)$ hours

Time taken to cover 22 km downstream = $22/(8+x)$ hours

Total time taken = 5 hours

$$\therefore \frac{15}{8-x} + \frac{22}{8+x} = 5$$

$$\therefore \frac{15(8+x) + 22(8-x)}{(8+x)(8-x)} = 5$$

$$\Rightarrow \frac{120 + 15x + 176 - 22x}{64 - x^2} = 5$$

$$\Rightarrow 296 - 7x = 320 - 5x^2$$

$$5x^2 - 7x - 24 = 0$$

$$5x^2 - 15x + 8x - 24 = 0$$

$$\Rightarrow 5x(x-3) + 8(x-3) = 0$$

$$\Rightarrow (5x+8)(x-3) = 0$$

$$x = \frac{-5}{8} \quad \text{or } x = 3$$

$$\Rightarrow x = 3 \quad (\because \text{speed cannot be negative})$$

Hence, the speed of stream is 3 kmph.

Question 32:

Let the speed of the boat in still water be x kmph, then

Speed of boat downstream = $(x + 2)$ km/h

And the speed of boat upstream = $(x - 2)$ kmph

Time taken to cover 8 km downstream = $8/(x+2)$ hours

Time taken to cover 8 km upstream = $8/(x-2)$ hours

Total time taken = $5/3$ hours

$$\frac{8}{(x+2)} + \frac{8}{(x-2)} = \frac{5}{3} \Rightarrow \frac{1}{x+2} + \frac{1}{x-2} = \frac{5}{24}$$

$$\Rightarrow \frac{x-2+x+2}{(x+2)(x-2)} = \frac{5}{24} \Rightarrow \frac{2x}{x^2-4} = \frac{5}{24}$$

$$\Rightarrow 5x^2 - 20 - 48x = 0$$

$$\Rightarrow 5x^2 - 48x - 20 = 0$$

$$\Rightarrow 5x^2 - 50x + 2x - 20 = 0$$

$$\Rightarrow 5x(x-10) + 2(x-10) = 0$$

$$\Rightarrow (x-10)(5x+2) = 0$$

$$\Rightarrow x = 10 \text{ or } x = \frac{-2}{5}$$

$$\Rightarrow x = 10 \text{ (speed cannot be negative)}$$

Then speed of the boat in still water is 10 kmph.

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