Indefinite Integrals Ex 19.8 Q41

Let $\quad I = \int \frac{\sec^2 x}{\tan x + 2} dx \text{ - - - - - - - } (i)$

Let $\tan x + 2 = t$ then,

$d(\tan x + 2) = dt$

$\Rightarrow \quad \sec^2 x dx = dt$

$\Rightarrow \quad dx = \frac{1}{\sec^2 x} dt$

Putting $\tan x + 2 = t$ and $dx = \frac{dt}{\sec^2 x}$ in equation (i), we get,

$I = \int \frac{\sec^2 x}{t} \times \frac{1}{\sec^2 x} dt$

$= \int \frac{dt}{t}$

$= \log|t| + c$

$= \log|\tan x + 2| + c$

$\Rightarrow \quad I = \log|\tan x + 2| + c$

Indefinite Integrals Ex 19.8 Q42

Let $\quad I = \int \frac{2\cos 2x + \sec^2 x}{\sin 2x + \tan x - 5} dx \text{ - - - - - - - } (i)$

Let $\sin 2x + \tan x - 5 = t$ then,

$d(\sin 2x + \tan x - 5) = dt$

$\Rightarrow \quad (2\cos 2x + \sec^2 x) dx = dt$

$\Rightarrow \quad dx = \frac{1}{2\cos 2x + \sec^2 x} dt$

Putting $\sin 2x + \tan x - 5 = t$ and $dx = \frac{dt}{2\cos 2x + \sec^2 x}$ in equation (i), we get,

$I = \int \frac{2\cos 2x + \sec^2 x}{t} \times \frac{1}{2\cos 2x + \sec^2 x} dt$

$= \int \frac{dt}{t}$

$= \log|t| + c$

$= \log|\sin 2x + \tan x - 5| + c$

$\therefore \quad I = \log|\sin 2x + \tan x - 5| + c$

Indefinite Integrals Ex 19.8 Q43

Let $\quad I = \int \frac{\sin 2x}{\sin 5x \sin 3x} dx \quad$ then,

$$I = \int \frac{\sin(5x - 3x)}{\sin 5x \sin 3x} dx$$
$$= \int \frac{\sin 5x \cos 3x - \cos 5x \sin 3x}{\sin 5x \sin 3x} dx$$
$$= \int \frac{\sin 5x \cos 3x}{\sin 5x \sin 3x} dx - \int \frac{\cos 5x \sin 3x}{\sin 5x \sin 3x} dx$$
$$= \int \frac{\cos 3x}{\sin 3x} dx - \int \frac{\cos 5x}{\sin 5x} dx$$
$$= \int \cot 3x \, dx - \int \cot 5x \, dx$$
$$= \frac{1}{3} \log|\sin 3x| - \frac{1}{5} \log|\sin 5x| + c$$

$$\therefore \quad I = \frac{1}{3} \log|\sin 3x| - \frac{1}{5} \log|\sin 5x| + c$$

Indefinite Integrals Ex 19.8 Q44

Let $\quad I = \int \frac{1 + \cot x}{x + \log \sin x} dx \text{ - - - - - (i)}$

Let $x + \log \sin x = t \quad$ then,
$d(x + \log \sin x) = dt$

$$\Rightarrow \quad (1 + \cot x) dx = dt \qquad \left[\because \quad \frac{d}{dx}(\log \sin x) = \cot x\right]$$
$$\Rightarrow \quad dx = \frac{dt}{1 + \cot x}$$

Putting $x + \log \sin x = t$ and $dx = \frac{dt}{1 + \cot x}$ in equation (i), we get

$$I = \int \frac{1 + \cot x}{t} \times \frac{dt}{1 + \cot x}$$
$$= \int \frac{dt}{t}$$
$$= \log|t| + c$$
$$= \log|x + \log \sin x| + c$$

$$\therefore \quad I = \log|x + \log \sin x| + c$$

Indefinite Integrals Ex 19.8 Q45

Let $\quad I = \int \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)} dx - - - - - (i)$

Let $\sqrt{x}+1 = t$ then,

$d\left(\sqrt{x}+1\right) = dt$

$\Rightarrow \quad \frac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow \quad dx = 2\sqrt{x}\, dt$

Putting $\sqrt{x}+1 = t$ and $dx = 2\sqrt{x}\, dt$ in equation (i), we get

$$I = \int \frac{1}{\sqrt{x}\, t} \times 2\sqrt{x}\, dt$$

$$= 2\int \frac{dt}{t}$$

$$= 2\log|t| + c$$

$$= 2\log\left|\sqrt{x}+1\right| + c$$

$\therefore \quad I = 2\log\left|\sqrt{x}+1\right| + c$

*********** END **********