$$\therefore \sin^{-1}\frac{3}{5}=\tan^{-1}\frac{3}{4} \quad \text{...(i)}$$

$$\text{Now, } \cot^{-1}\frac{3}{2}=\tan^{-1}\frac{2}{3} \quad \text{...(ii)} \qquad \left[\tan^{-1}\frac{1}{x}=\cot^{-1}x\right]$$

$$\text{Hence, } \tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$$

$$=\tan\left(\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{2}{3}\right) \qquad [\text{Using (i) and (ii)}]$$

$$=\tan\left(\tan^{-1}\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\cdot\frac{2}{3}}\right) \qquad \left[\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}\right]$$

$$=\tan\left(\tan^{-1}\frac{9+8}{12-6}\right)$$

$$=\tan\left(\tan^{-1}\frac{17}{6}\right)=\frac{17}{6}$$

Question 19:

Find the values of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is equal to

(A) $\frac{7\pi}{6}$ **(B)** $\frac{5\pi}{6}$ **(C)** $\frac{\pi}{3}$ **(D)** $\frac{\pi}{6}$

Answer

We know that $\cos^{-1}(\cos x) = x$ if $x\in[0,\pi]$, which is the principal value branch of $\cos^{-1}x$.

Here, $\frac{7\pi}{6}\notin x\in[0,\pi]$.

Now, $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ can be written as:

$$\cos^{-1}\left(\cos\frac{7\pi}{6}\right)=\cos^{-1}\left(\cos\frac{-7\pi}{6}\right)=\cos^{-1}\left[\cos\left(2\pi-\frac{7\pi}{6}\right)\right] \qquad [\cos(2\pi+x)=\cos x]$$

$$=\cos^{-1}\left[\cos\frac{5\pi}{6}\right] \text{ where } \frac{5\pi}{6}\in[0,\pi]$$

$$\therefore \cos^{-1}\left(\cos\frac{7\pi}{6}\right)=\cos^{-1}\left(\cos\frac{5\pi}{6}\right)=\frac{5\pi}{6}$$

The correct answer is B.

Question 20:

Find the values of $\sin\left(\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right)$ is equal to

(A) $\frac{1}{2}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{4}$ **(D)** 1

Answer

Let $\sin^{-1}\left(\frac{-1}{2}\right)=x$. Then, $\sin x=\frac{-1}{2}=-\sin\frac{\pi}{6}=\sin\left(\frac{-\pi}{6}\right)$.

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$.

$$\therefore \sin^{-1}\left(\frac{-1}{2}\right)=\frac{-\pi}{6}$$

$$\therefore \sin\left(\frac{\pi}{3}-\sin^{-1}\left(\frac{-1}{2}\right)\right)=\sin\left(\frac{\pi}{3}+\frac{\pi}{6}\right)=\sin\left(\frac{3\pi}{6}\right)=\sin\left(\frac{\pi}{2}\right)=1$$

The correct answer is D.

Miscellaneous Solutions

Question 1:

$\cos^{-1}\left(\cos\frac{13\pi}{\ldots}\right)$

Find the value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

Answer

We know that $\cos^{-1}(\cos x) = x$ if $x \in [0, \pi]$, which is the principal value branch of $\cos^{-1}x$.

Here, $\frac{13\pi}{6} \notin [0, \pi]$.

Now, $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ can be written as:

$$\cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \cos^{-1}\left[\cos\left(2\pi + \frac{\pi}{6}\right)\right] = \cos^{-1}\left[\cos\left(\frac{\pi}{6}\right)\right], \text{ where } \frac{\pi}{6} \in [0, \pi].$$

$$\therefore \cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \cos^{-1}\left[\cos\left(\frac{\pi}{6}\right)\right] = \frac{\pi}{6}$$

Question 2:

Find the value of $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

Answer

We know that $\tan^{-1}(\tan x) = x$ if $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, which is the principal value branch of $\tan^{-1}x$.

Here, $\frac{7\pi}{6} \notin \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Now, $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$ can be written as:

$$\tan^{-1}\left(\tan\frac{7\pi}{6}\right) = \tan^{-1}\left[\tan\left(2\pi - \frac{5\pi}{6}\right)\right] \qquad \left[\tan(2\pi - x) = -\tan x\right]$$

$$= \tan^{-1}\left[-\tan\left(\frac{5\pi}{6}\right)\right] = \tan^{-1}\left[\tan\left(-\frac{5\pi}{6}\right)\right] = \tan^{-1}\left[\tan\left(\pi - \frac{5\pi}{6}\right)\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{6}\right)\right], \text{ where } \frac{\pi}{6} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\therefore \tan^{-1}\left(\tan\frac{7\pi}{6}\right) = \tan^{-1}\left(\tan\frac{\pi}{6}\right) = \frac{\pi}{6}$$

Question 3:

Prove $2\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{24}{7}$

Answer

Let $\sin^{-1}\frac{3}{5} = x$. Then, $\sin x = \frac{3}{5}$.

$$\Rightarrow \cos x = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$$

$$\therefore \tan x = \frac{3}{4}$$

$$\therefore x = \tan^{-1}\frac{3}{4} \Rightarrow \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{4}$$

Now, we have:

$$\text{L.H.S.} = 2\sin^{-1}\frac{3}{5} = 2\tan^{-1}\frac{3}{4}$$

$$= \tan^{-1}\left(\frac{2 \times \frac{3}{4}}{1 - \left(\frac{3}{4}\right)^2}\right) \qquad \left[2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$$

$$= \tan^{-1}\left(\frac{\frac{3}{2}}{\frac{16-9}{16}}\right) = \tan^{-1}\left(\frac{3}{2} \times \frac{16}{7}\right)$$

$$= \tan^{-1}\frac{24}{7} = \text{R.H.S.}$$

Question 4:

Prove $\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$

Answer

Let $\sin^{-1}\frac{8}{17} = x$. Then, $\sin x = \frac{8}{17} \Rightarrow \cos x = \sqrt{1 - \left(\frac{8}{17}\right)^2} = \sqrt{\frac{225}{289}} = \frac{15}{17}$.

$$\therefore \tan x = \frac{8}{15} \Rightarrow x = \tan^{-1}\frac{8}{15}$$

$$\therefore \sin^{-1}\frac{8}{17} = \tan^{-1}\frac{8}{15} \qquad ...(1)$$

$$\text{Now, let} \sin^{-1}\frac{3}{5} = y. \text{ Then, } \sin y = \frac{3}{5} \Rightarrow \cos y = \sqrt{1-\left(\frac{3}{5}\right)^2} = \sqrt{\frac{16}{25}} = \frac{4}{5}.$$

$$\therefore \tan y = \frac{3}{4} \Rightarrow y = \tan^{-1}\frac{3}{4}$$

$$\therefore \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{4} \qquad ...(2)$$

Now, we have:

$$\text{L.H.S.} = \sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5}$$

$$= \tan^{-1}\frac{8}{15} + \tan^{-1}\frac{3}{4} \qquad \left[\text{Using (1) and (2)}\right]$$

$$= \tan^{-1}\frac{\frac{8}{15}+\frac{3}{4}}{1-\frac{8}{15}\times\frac{3}{4}}$$

$$= \tan^{-1}\left(\frac{32+45}{60-24}\right) \qquad \left[\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$$

$$= \tan^{-1}\frac{77}{36} = \text{R.H.S.}$$

Question 5:

Prove $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$

Answer

$$\text{Let } \cos^{-1}\frac{4}{5} = x. \text{ Then, } \cos x = \frac{4}{5} \Rightarrow \sin x = \sqrt{1-\left(\frac{4}{5}\right)^2} = \frac{3}{5}.$$

$$\therefore \tan x = \frac{3}{4} \Rightarrow x = \tan^{-1}\frac{3}{4}$$

$$\therefore \cos^{-1}\frac{4}{5} = \tan^{-1}\frac{3}{4} \qquad ...(1)$$

$$\text{Now, let} \cos^{-1}\frac{12}{13} = y. \text{ Then, } \cos y = \frac{12}{13} \Rightarrow \sin y = \frac{5}{13}.$$

$$\therefore \tan y = \frac{5}{12} \Rightarrow y = \tan^{-1}\frac{5}{12}$$

$$\therefore \cos^{-1}\frac{12}{13} = \tan^{-1}\frac{5}{12} \qquad ...(2)$$

$$\text{Let} \cos^{-1}\frac{33}{65} = z. \text{ Then, } \cos z = \frac{33}{65} \Rightarrow \sin z = \frac{56}{65}.$$

$$\therefore \tan z = \frac{56}{33} \Rightarrow z = \tan^{-1}\frac{56}{33}$$

$$\therefore \cos^{-1}\frac{33}{65} = \tan^{-1}\frac{56}{33} \qquad ...(3)$$

Now, we will prove that:

$$\text{L.H.S.} = \cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13}$$

$$= \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{5}{12} \qquad \left[\text{Using (1) and (2)}\right]$$

$$= \tan^{-1}\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\cdot\frac{5}{12}} \qquad \left[\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$$

$$= \tan^{-1}\frac{36+20}{48-15}$$

$$= \tan^{-1}\frac{56}{33}$$

$$= \tan^{-1}\frac{56}{33} \qquad \left[\text{by (3)}\right]$$

$$= \text{R.H.S.}$$

*********** END **********