



Differentiation Ex 11.7 Q25

Consider the given functions,

$$x = \cos t (3 - 2\cos^2 t)$$

$$x = 3\cos t - 2\cos^3 t$$

$$\frac{dx}{dt} = -3\sin t + 6\cos^2 t \sin t \dots (1)$$

$$y = \sin t (3 - 2\sin^2 t)$$

$$y = 3\sin t - 2\sin^3 t$$

$$\frac{dy}{dt} = 3\cos t - 6\sin^2 t \cos t \dots (2)$$

$$\frac{dy}{dx} = \left(\frac{dy}{dt}\right) / \left(\frac{dx}{dt}\right) \dots [\text{From equations (1) and (2)}]$$

$$= \frac{3\cos t - 6\sin^2 t \cos t}{-3\sin t + 6\cos^2 t \sin t}$$

$$= \frac{3\cos t (1 - 2\sin^2 t)}{3\sin t (2\cos^2 t - 1)}$$

$$= \cot t \frac{(1 - 2(1 - \cos^2 t))}{(2\cos^2 t - 1)}$$

$$= \cot t$$

$$\left. \frac{dy}{dx} \right|_{\pi/4} = \cot \frac{\pi}{4} = 1$$

Differentiation Ex 11.7 Q26

$$x = \frac{1 + \log t}{t^2}, y = \frac{3 + 2\log t}{t}$$

$$\frac{dx}{dt} = \frac{t^2 \left(\frac{1}{t}\right) - (1 + \log t)(2t)}{t^4} = \frac{t - 2t - 2t \log t}{t^4} = \frac{-2\log t - 1}{t^3}$$

$$\frac{dy}{dt} = \frac{t \left(\frac{2}{t}\right) - (3 + 2\log t)(1)}{t^2} = \frac{2 - 3 - 2\log t}{t^2} = \frac{-2\log t - 1}{t^2}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{-2\log t - 1}{t^2}}{\frac{-2\log t - 1}{t^3}} = t$$

Differentiation Ex 11.7 Q27

$$x = 3 \sin t - \sin 3t, y = 3 \cos t - \cos 3t$$

$$\frac{dx}{dt} = 3 \cos t - 3 \cos 3t$$

$$\frac{dy}{dt} = -3 \sin t + 3 \sin 3t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-3 \sin t + 3 \sin 3t}{3 \cos t - 3 \cos 3t}$$

$$\text{When } t = \frac{\pi}{3}$$

$$\frac{dy}{dx} = \frac{-3 \sin\left(\frac{\pi}{3}\right) + 3 \sin(\pi)}{3 \cos\left(\frac{\pi}{3}\right) - 3 \cos(\pi)} = \frac{-3 \times \frac{\sqrt{3}}{2} + 0}{3 \times \frac{1}{2} - 3(-1)} = -\frac{1}{\sqrt{3}}$$

Differentiation Ex 11.7 Q28

$$\sin x = \frac{2t}{1+t^2}, \tan y = \frac{2t}{1-t^2}$$

$$\Rightarrow x = \sin^{-1}\left(\frac{2t}{1+t^2}\right) \text{ and } y = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$$

$$\frac{dx}{dt} = \frac{1}{\sqrt{1 - \left(\frac{2t}{1+t^2}\right)^2}} \times \frac{2(1+t^2) - (2t)(2t)}{(1+t^2)^2}$$

$$\frac{dx}{dt} = \frac{2}{(1+t^2)}$$

$$\frac{dy}{dt} = \frac{1}{\left(\frac{2t}{1-t^2}\right)^2 + 1} \times \frac{2(1-t^2) - (2t)(-2t)}{(1-t^2)^2}$$

$$\frac{dy}{dt} = \frac{2}{(1+t^2)}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{2}{(1+t^2)}}{\frac{2}{(1+t^2)}} = 1$$

***** END *****