Tangents and Normals Ex 16.3 Q1(v)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad ---(i)$$

$$x^2 + y^2 = ab \qquad ---(ii)$$

From (ii), we get

$$y^2 = ab - x^2$$

$\therefore$ From (i), we get

$$\frac{x^2}{a^2} + \frac{ab - x^2}{b^2} = 1$$

$$\Rightarrow \quad b^2x^2 + a^3b - a^2x^2 = a^2b^2$$

$$\Rightarrow \quad \left(b^2 - a^2\right)x^2 = a^2b^2 - a^3b$$

$$\Rightarrow \quad x^2 = \frac{a^2b^2 - a^3b}{b^2 - a^2}$$

$$= \frac{a^2b\,(b - a)}{(b - a)\,(b + a)}$$

$$= \frac{a^2b}{b + a}$$

$$\therefore \quad x = \pm\sqrt{\frac{a^2b}{a + b}}$$

$$\therefore \quad y^2 = ab - x^2 = ab - \frac{a^2b}{a + b}$$

$$= \frac{a^2b + ab^2 - a^2b}{a + b} = \frac{ab^2}{a + b}$$

Differentiating (i) and (ii) w.r.t x we get

$$\frac{2x}{a^2} + \frac{2y}{b^2} \times \left(\frac{dy}{dx}\right)_{C_1} = 0$$

and $$2x + 2y\left(\frac{dy}{dx}\right)_{C_2} = 0$$

$$\therefore \quad \left(\frac{dy}{dx}\right)_{C_1} = \frac{-x}{a^2} \times \frac{b^2}{y} = \frac{-b^2x}{a^2y}$$

$$\left(\frac{dy}{\ }\right) = \frac{-x}{\ }$$

$$\left(\frac{dx}{dy}\right)_{C_2} \quad y$$

At $\left(\pm\sqrt{\frac{a^2b}{a+b}}, \pm\sqrt{\frac{ab^2}{a+b}}\right)$ we get

$$\left(\frac{dy}{dx}\right)_{C_1} = \frac{-b^2}{a^2}\sqrt{\frac{a}{b}} = \frac{-b^2\sqrt{a}}{a^2\sqrt{b}}$$

$$\left(\frac{dy}{dx}\right)_{C_2} = -\sqrt{\frac{a}{b}}$$

Tangents and Normals Ex 16.3 Q1(vi)

We know that angle of intersection of two curves is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right| \qquad \text{---(A)}$$

Where m_1 and m_2 are slopes of curves.

$$x^2 + 4y^2 = 8 \qquad \text{---(i)}$$

$$x^2 - 2y^2 = 2 \qquad \text{---(ii)}$$

Solving (i) and (ii)

$$6y^2 = 6 \Rightarrow \quad y = \pm 1$$

$$\therefore \quad x^2 = 2 + 2 \Rightarrow \quad x = \pm 2$$

$\therefore$ Point of intersection are

$$P = (2,1) \text{ and } (-2,-1)$$

Now,

Slope m_1 for (i)

$$8y\frac{dy}{dx} = -2x \Rightarrow \frac{dy}{dx} = -\frac{x}{4y}$$

$$\therefore \quad m_1 = \frac{1}{2}$$

Slope m_2 for (ii)

$$4y\frac{dy}{dx} = 2x \Rightarrow \frac{dy}{dx} = \frac{x}{2y}$$

$$\therefore \quad m_2 = 1$$

From (A)

$$\tan\theta = \left|\frac{1 - \frac{1}{2}}{1 + 1 \times \frac{1}{2}}\right| = \frac{1}{3}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{1}{3}\right)$$

Tangents and Normals Ex 16.3 Q1(vii)

We know that angle of intersection of two curves is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad \text{---(A)}$$

Where m_1 and m_2 are slopes of curves.

$$x^2 = 27y \qquad \text{---(i)}$$
$$y^2 = 8x \qquad \text{---(ii)}$$

Solving (i) and (ii) are

$$\frac{y^4}{64} = 27y$$
$$\Rightarrow \quad y\left(y^3 - 27 \times 64\right) = 0$$
$$\Rightarrow \quad y = 0 \text{ or } 12$$

$$\therefore \quad x = 0 \text{ or } 18$$

$\therefore$ Points or intersection is $(0,0)$ and $(18,12)$

Now,

Slope of (i)

$$m_1 = \frac{2x}{27} = \frac{12}{9} = \frac{4}{3}$$

Slope of (ii)

$$m_2 = \frac{8}{2y} = \frac{8}{24} = \frac{1}{3}$$

From (A)

$$\tan\theta = \left|\frac{\frac{4}{3} - \frac{1}{3}}{1 + \frac{4}{3} \times \frac{1}{3}}\right| = \frac{9}{13}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{9}{13}\right)$$

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