Exercise 10D

Question 1:
Let two numbers be x and 8 - x

Sum of their reciprocals $= \dfrac{1}{x} + \dfrac{1}{8-x} = \dfrac{8}{15}$

$\Rightarrow \dfrac{8-x+x}{x(8-x)} = \dfrac{8}{15}$ or $\dfrac{8}{x(8-x)} = \dfrac{8}{15}$

$\Rightarrow \quad x(8-x) = 15$ or $8x - x^2 = 15$

$\quad x^2 - 8x + 15 = 0$ or $x^2 - 5x - 3x + 15 = 0$

$\Rightarrow x(x-5) - 3(x-5) = 0$ or $(x-5)(x-3) = 0$

$\therefore \quad x = 5, 3$

Thus, two numbers are 3 and 5.

Question 2:
Let the two numbers are x and x + 4

Difference of their reciprocals $= \dfrac{1}{x} - \dfrac{1}{x+4} = \dfrac{4}{21}$

$\therefore \dfrac{x+4-x}{x(x+4)} = \dfrac{4}{21}$ or $\dfrac{4}{x(x+4)} = \dfrac{4}{21}$

$\Rightarrow x(x+4) = 21$ or $x^2 + 4x - 21 = 0$

$\Rightarrow x^2 + 7x - 3x - 21 = 0$ or $x(x+7) - 3(x+7) = 0$

$\Rightarrow (x+7)(x-3) = 0$

$\therefore \quad x = 3$ or -7

But, $x \neq -7$

Hence, the two required numbers are 3 and 7.

Question 3:
Let the required number be x and (18- x)

Then, $\dfrac{1}{x} + \dfrac{1}{18-x} = \dfrac{1}{4}$

$\Rightarrow \dfrac{18 - x + x}{18x - x^2} = \dfrac{1}{4}$

$\Rightarrow 18 - x^2 = 72$

$\Rightarrow x^2 - 18x + 72 = 0$

$\Rightarrow x^2 - 12x - 6x + 72 = 0$

$\Rightarrow x(x - 12) - 6(x - 12) = 0$

$\Rightarrow (x - 12)(x - 6) = 0$

x = 12, x = 6
hence, the required numbers are 12 and 6.

Question 4:
Let the number be x and y
x - y = 5 -----(1)
Difference of reciprocal

$$\dfrac{1}{y} - \dfrac{1}{x} = \dfrac{1}{10}$$

or $\dfrac{x-y}{xy} = \dfrac{1}{10}$ ------(2)

Put x - y = 5 in (2)

$\dfrac{5}{xy} = \dfrac{1}{10}$ ∴ xy = 50 (3)

from (1), x = y + 5, put value of x in (3)

(y + 5)y = 50   or $y^2 + 5y - 50 = 0$

$\Rightarrow y^2 + 10y - 5y - 50 = 0$ $\Rightarrow y(y + 10) - 5(y + 10) = 0$

or (y + 10)(y - 5) = 0

∴ y + 10 = 0 or y = -10

   y - 5 = 0 or y = 5

When y = -10, from (1)

x = y + 5 = -10 + 5 = -5

∴ x = -5, y = -10

when y = 5,   x + 5 = 10

∴ x = 10, y = 5

Hence, the two numbers are 10 and 5.

Question 5:
Let the number be x

Then, $x + \dfrac{1}{x} = 3\dfrac{41}{80} \Rightarrow \dfrac{x^2 + 1}{x} = \dfrac{281}{80}$

$\Rightarrow 80x^2 + 80 = 281x$

$\Rightarrow 80x^2 - 281x + 80 = 0$

$\Rightarrow x = \dfrac{281 \pm \sqrt{78961 - 25600}}{160} = \dfrac{281 \pm \sqrt{53361}}{160}$

$\Rightarrow \dfrac{(281 \pm 231)}{160}$

$\Rightarrow x = \dfrac{(281 + 231)}{160} = \dfrac{512}{160} = \dfrac{16}{5}$ or $x = \dfrac{(281 - 231)}{160} = \dfrac{50}{160} = \dfrac{5}{16}$

Hence, the required number is 5/16 or 16/5.

Question 6:
Let the required number be x and (57 - x), then

$x(57 - x) = 782 \Rightarrow x^2 - 57x + 782 = 0$

$\Rightarrow \dfrac{57 \pm \sqrt{(57)^2 - 4 \times 1 \times 782}}{2} = \dfrac{57 \pm \sqrt{(3249 - 3128)}}{2}$

$\Rightarrow x = \dfrac{57 \pm \sqrt{121}}{2}$

$x = \dfrac{(57 + 11)}{2} = \dfrac{68}{2} = 34$ or $x = \dfrac{(57 - 11)}{2} = \dfrac{46}{2} = 23$

The required numbers are 34 and 23.

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