Mathematical Induction Ex 12.2 Q1

Let $P(n): 1+2+3+---+n = \frac{n(n+1)}{2}$

For $n = 1$,

LHS of P(n) = 1

RHS of P(n) $= \frac{1(1+1)}{2} 1 = 1$

Since, LHS = RHS

$\Rightarrow$ $P(n)$ is true for $n = 1$

Let $P(n)$ be true for $n = k$, so

$$1+2+3+---+k = \frac{k(k+1)}{2} \qquad ---(1)$$

Now

$$(1+2+3+---+k)+(k+1)$$

$$= \frac{k(k+1)}{2}+(k+1)$$

$$= (k+1)\left(\frac{k}{2}+1\right)$$

$$= \frac{(k+1)(k+2)}{2}$$

$$= \frac{(k+1)[(k+1)+1]}{2}$$

$\Rightarrow$ $P(n)$ is true for $n = k+1$

$\Rightarrow$ $P(n)$ is true for all $n \in N$

So, by the principle of mathematical induction

$P(n): 1+2+3+---+n = \frac{n(n+1)}{2}$ is true for all $n \in N$

Mathematical Induction Ex 12.2 Q2

Let $P(n): 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

For $n = 1$

$$P(1): 1 = \frac{1(1+1)(2+1)}{6}$$
$$1 = 1$$

$\Rightarrow \quad P(n)$ is true for $n = 1$

Let $P(n)$ is true for $n = k$, so

$$P(k): 1^2 + 2^2 + 3^2 + \ldots + k^2 = \frac{k(k+1)(2k+1)}{6} \qquad ---(1)$$

We have to show that $P(n)$ is true for $n = k + 1$

$$\Rightarrow \quad 1^2 + 2^2 + 3^2 + \ldots + k^2 + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$$

So, $1^2 + 2^2 + 3^2 + \ldots + k^2 + (k+1)^2$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2 \qquad \left[\text{Using equation } (1)\right]$$
$$= (k+1)\left[\frac{2k^2+k}{6} + \frac{(k+1)}{1}\right]$$
$$= (k+1)\left[\frac{2k^2+k+6k+6}{6}\right]$$
$$= (k+1)\left[\frac{2k^2+7k+6}{6}\right]$$
$$= (k+1)\left[\frac{2k^2+4k+3k+6}{6}\right]$$
$$= (k+1)\left[\frac{2k(k+2)+3(k+2)}{6}\right]$$
$$= \frac{(k+1)(2k+3)(k+2)}{6}$$

$\Rightarrow \quad P(n)$ is true for $n = k + 1$

$\Rightarrow \quad P(n)$ is true for all $n \in N$ by PMI

Mathematical Induction Ex 12.2 Q3

Let $P(n): 1+3+3^2+\ldots+3^{n-1} = \frac{3^n-1}{2}$

For $n=1$

$P(1): 1 = \frac{3^1-1}{2}$

$1 = 1$

$\Rightarrow \quad P(n)$ is true for $n=1$

Let $P(n)$ is true for $n=k$

$$1+3+3^2+\ldots+3^{k-1} = \frac{3^k-1}{2} \qquad ---(1)$$

We have to show $P(n)$ is true for $n=k+1$

i.e. $1+3+3^2+\ldots+3^k = \frac{3^{k+1}-1}{2}$

Now,

$\left\{1+3+3^2+\ldots+3^{k-1}\right\}+3^{k+1-1}$

$= \frac{3k-1}{2}+3^k \qquad$ [Using equation (1)]

$= \frac{3^k-1+2.3^k}{2}$

$= \frac{3.3^k-1}{2}$

$= \frac{3^{k+1}-1}{2}$

$\Rightarrow \quad P(n)$ is true for $n=k+1$

$\Rightarrow \quad P(n)$ is true for all $n \in N$ by PMI

Mathematical Induction Ex 12.2 Q4

*********** END **********