CHAPTER 16

SOUND WAVES

16.1 THE NATURE AND PROPAGATION OF SOUND WAVES

Sound is produced in a material medium by a vibrating source. As the vibrating source moves forward, it compresses the medium past it, increasing the density locally. This part of the medium compresses the layer next to it by collisions. The compression travels in the medium at a speed which depends on the elastic and inertia properties of the medium. As the source moves back, it drags the medium and produces a rarefaction in the layer. The layer next to it is then dragged back and thus the rarefaction pulse passes forward. In this way, compression and rarefaction pulses are produced which travel in the medium.

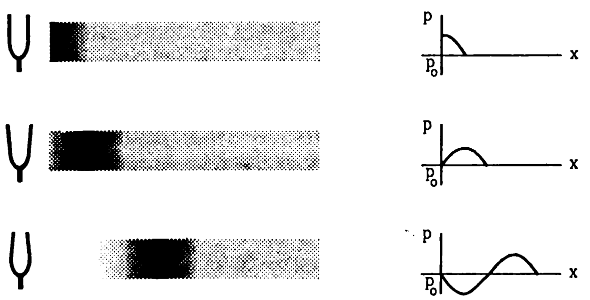

Figure 16.1

Figure (16.1) describes a typical case of propagation of sound waves. A tuning fork is vibrated in air. The prongs vibrate in simple harmonic motion. When the fork moves towards right, the layer next to it is compressed and consequently, the density is increased. The increase in density and hence, in pressure, is related to the velocity of the prong. The compression so produced travels in air towards right at the wave speed v. The velocity of the prong changes during the forward motion, being maximum at the mean position and zero at the extreme end. A compression wave pulse of length $vT/2$ is thus produced during the half period $T/2$ of forward motion. The prong now returns towards left and drags the air with it. The density and the pressure of the layer close to it go below the normal level, a rarefaction pulse is thus produced. During this half period of backward motion of the prong, a rarefaction pulse of length $vT/2$ is produced. As the prong continues executing its simple harmonic motion, a series of alternate compression and rarefaction pulses are produced which travel down the air.

As the prong vibrates in simple harmonic motion, the pressure variations in the layer close to the prong also change in a simple harmonic fashion. The increase in pressure above its normal value may, therefore, be written as

$$\delta P = P - P_0 = \delta P_0 \sin \omega t,$$

where δP_0 is the maximum increase in pressure above its normal value. As this disturbance travels towards right with the speed v (the wave speed and not the particle speed), the equation for the excess pressure at any point x at any time t is given by

$$\delta P = \delta P_0 \sin \omega (t - x/v).$$

This is the equation of a wave travelling in x-direction with velocity v. The excess pressure oscillates between $+\delta P_0$ and $-\delta P_0$. The frequency of this wave is $\nu = \omega/(2\pi)$ and is equal to the frequency of vibration of the source. Henceforth, we shall use the symbol p for the excess pressure developed above the equilibrium pressure and p_0 for the maximum change in pressure. The wave equation is then

$$p = p_0 \sin \omega (t - x/v). \qquad \text{... (16.1)}$$

Sound waves constitute alternate compression and rarefaction pulses travelling in the medium. However, sound is audible only if the frequency of alteration of pressure is between 20 Hz to 20,000 Hz. These limits are subjective and may vary slightly from person to person. An average human ear is not able to detect disturbance in the medium if the frequency is outside this range. Electronic detectors can detect waves of lower and higher frequencies as well. A dog can hear sound of frequency upto about 50 kHz and a bat upto about 100 kHz. The waves with frequency below the audible range are called *infrasonic* waves and the waves with frequency above the audible range are called *ultrasonic*.

Example 16.1

A wave of wavelength 0·60 cm *is produced in air and it travels at a speed of* 300 m/s. *Will it be audible?*

Solution : From the relation $v = \nu\lambda$, the frequency of the wave is

$$\nu = \frac{v}{\lambda} = \frac{300\ \text{m/s}}{0{\cdot}60\times 10^{-2}\ \text{m}} = 50000\ \text{Hz}.$$

This is much above the audible range. It is an ultrasonic wave and will not be audible.

The disturbance produced by a source of sound is not always a sine wave. A pure sine wave has a unique frequency but a disturbance of other waveform may have many frequency components in it. For example, when we clap our hands, a pulse of disturbance is created which travels in the air. This pulse does not have the shape of a sine wave. However, it can be obtained by superposition of a large number of sine waves of different frequencies and amplitudes. We then say that the clapping sound has all these frequency components in it.

The compression and rarefaction in a sound wave is caused due to the back and forth motion of the particles of the medium. This motion is along the direction of propagation of sound and hence the sound waves are longitudinal.

All directions, perpendicular to the direction of propagation, are equivalent and hence, a sound wave can not be polarized. If we make a slit on a cardboard and place it in the path of the sound, rotating the cardboard in its plane will produce no effect on the intensity of sound on the other side.

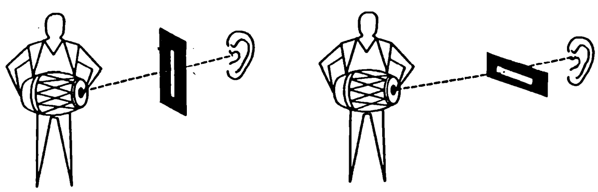

Figure 16.2

Wavefront

The sound produced at some point by a vibrating source travels in all directions in the medium if the medium is extended. The sound waves are, in general, three dimensional waves. For a small source, we have spherical layers of the medium on which the pressure at various elements have the same phase at a given instant. The compression, produced by the source at say $t = 0$, reaches the spherical surface of radius $r = vt$ at time t and the pressure at all the points on this sphere is maximum at this instant. A half time-period later, the pressure at all the points on this sphere is reduced to minimum. The surface through the points, having the same phase of disturbance, is called a *wavefront*. For a homogeneous and isotropic medium, the wavefronts are normal to the direction of propagation.

For a point source placed in a homogeneous and isotropic medium, the wavefronts are spherical and the wave is called a *spherical wave*. If sound is produced by vibrating a large plane sheet, the disturbance produced in front of the sheet will have the same phase on a plane parallel to the sheet. The wavefronts are then planes (neglecting the end effects) and the direction of propagation is perpendicular to these planes. Such waves are called *plane waves*. The wavefront can have several other shapes. In this chapter, we shall mostly consider sound waves travelling in a fixed direction i.e., plane waves. However, most of the results will be applicable to other waves also.

16.2 DISPLACEMENT WAVE AND PRESSURE WAVE

A longitudinal wave in a fluid (liquid or gas) can be described either in terms of the longitudinal displacement suffered by the particles of the medium or in terms of the excess pressure generated due to the compression or rarefaction. Let us see how the two representations are related to each other.

Consider a wave going in the x-direction in a fluid. Suppose that at a time t, the particle at the undisturbed position x suffers a displacement s in the x-direction. The wave can then be described by the equation

$$s = s_0 \sin \omega(t - x/v). \quad \text{... (i)}$$

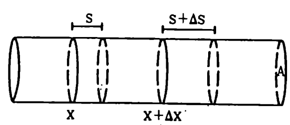

Figure 16.3

Consider the element of the material which is contained within x and $x + \Delta x$ (figure 16.3) in the undisturbed state. Considering a cross-sectional area A, the volume of the element in the undisturbed state is $A\,\Delta x$ and its mass is $\rho A\,\Delta x$. As the wave passes, the ends at x and $x + \Delta x$ are displaced by amounts s and $s + \Delta s$ according to equation (i) above. The increase in volume of this element at time t is

$$\Delta V = A\,\Delta s$$
$$= A\,s_0(-\,\omega/v)\cos \omega(t - x/v)\Delta x,$$

where Δs has been obtained by differentiating equation (i) with respect to x. The element is, therefore, under a volume strain.

$$\frac{\Delta V}{V} = \frac{-A\, s_0 \omega \cos \omega(t - x/v) \Delta x}{vA\, \Delta x}$$

$$= \frac{-s_0 \omega}{v} \cos \omega(t - x/v).$$

The corresponding stress i.e., the excess pressure developed in the element at x at time t is,

$$p = B\left(\frac{-\Delta V}{V}\right),$$

where B is the bulk modulus of the material. Thus,

$$p = B\frac{s_0 \omega}{v} \cos \omega(t - x/v). \quad \text{... (ii)}$$

Comparing with (i), we see that the pressure amplitude p_0 and the displacement amplitude s_0 are related as

$$p_0 = \frac{B\,\omega}{v} s_0 = Bk\, s_0, \quad \text{... (16.2)}$$

where k is the wave number. Also, we see from (i) and (ii) that the pressure wave differs in phase by $\pi/2$ from the displacement wave. The pressure maxima occur where the displacement is zero and displacement maxima occur where the pressure is at its normal level.

The fact that, displacement is zero where the pressure-change is maximum and vice versa, puts the two descriptions on different footings. The human ear or an electronic detector responds to the change in pressure and not to the displacement in a straight forward way. Suppose two audio speakers are driven by the same amplifier and are placed facing each other (figure 16.4). A detector is placed midway between them.

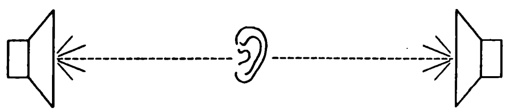

Figure 16.4

The displacement of the air particles near the detector will be zero as the two sources drive these particles in opposite directions. However, both send compression waves and rarefaction waves together. As a result, pressure increases at the detector simultaneously due to both the sources. Accordingly, the pressure amplitude will be doubled, although the displacement remains zero here. A detector detects maximum intensity in such a condition. Thus, the description in terms of pressure wave is more appropriate than the description in terms of the displacement wave as far as sound properties are concerned.

Example 16.2

A sound wave of wavelength 40 cm *travels in air. If the difference between the maximum and minimum pressures at a given point is* $1{\cdot}0 \times 10^{-3}\,\mathrm{N/m^2}$, *find the amplitude of vibration of the particles of the medium. The bulk modulus of air is* $1{\cdot}4 \times 10^{5}\,\mathrm{N/m^2}$.

Solution : The pressure amplitude is

$$p_0 = \frac{1{\cdot}0 \times 10^{-3}\,\mathrm{N/m^2}}{2} = 0{\cdot}5 \times 10^{-3}\,\mathrm{N/m^2}.$$

The displacement amplitude s_0 is given by

$$p_0 = B\,k\,s_0$$

or, $$s_0 = \frac{p_0}{B\,k} = \frac{p_0\,\lambda}{2\pi B}$$

$$= \frac{0{\cdot}5 \times 10^{-3}\,\mathrm{N/m^2} \times (40 \times 10^{-2}\,\mathrm{m})}{2 \times 3{\cdot}14 \times 1{\cdot}4 \times 10^{5}\,\mathrm{N/m^2}}$$

$$= 2{\cdot}2 \times 10^{-10}\,\mathrm{m}.$$

16.3 SPEED OF A SOUND WAVE IN A MATERIAL MEDIUM

Consider again a sound wave going in x-direction in a fluid whose particles are displaced according to the equation

$$s = s_0 \sin \omega(t - x/v). \quad \text{... (i)}$$

The pressure varies according to the equation

$$p = \frac{B\,s_0 \omega}{v} \cos \omega(t - x/v). \quad \text{... (ii)}$$

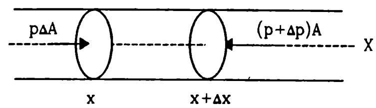

Figure 16.5

Consider the element of the fluid which is contained between the positions x and $x + \Delta x$ in the undisturbed state (figure 16.5). The excess pressure at time t at the end x is p and at $x + \Delta x$ it is $p + \Delta p$. Taking a cross-sectional area A, the force on the element from the left is pA and from the right it is $(p + \Delta p)A$. The resultant force on the element at time t is

$$\Delta F = Ap - A(p + \Delta p) = -A\,\Delta p$$

$$= -A\frac{B\,s_0\,\omega}{v}(\omega/v)\,\Delta x \sin \omega(t - x/v)$$

$$= -A\frac{B\,s_0\,\omega^2}{v^2} \sin \omega(t - x/v)\,\Delta x.$$

The change in pressure Δp between x and $x + \Delta x$ is obtained by differentiating equation (ii) with respect to x. If ρ is the normal density of the fluid, the mass of the element considered is $\rho A \Delta x$. Using Newton's second law of motion, the acceleration of the element is given by

$$a = \frac{\Delta F}{\rho A \Delta x} = -\frac{B s_0 \omega^2}{\rho v^2} \sin \omega(t - x/v). \quad \dots \text{(iii)}$$

However, the acceleration can also be obtained from equation (i). It is

$$a = \frac{\partial^2 s}{\partial t^2}$$

or, $$a = -\omega^2 s_0 \sin \omega(t - x/v). \quad \dots \text{(iv)}$$

Comparing (iii) and (iv),

$$\frac{B s_0 \omega^2}{\rho v^2} = \omega^2 s_0$$

or, $$v = \sqrt{B/\rho}. \quad \dots (16.3)$$

We see that the velocity of a longitudinal wave in a medium depends on its elastic properties and inertia properties as was the case with the waves on a string.

Sound Waves in Solids

Sound waves can travel in solids just like they can travel in fluids. The speed of longitudinal sound waves in a solid rod can be shown to be

$$v = \sqrt{Y/\rho}, \quad \dots (16.4)$$

where Y is the Young's modulus of the solid and ρ its density. For extended solids, the speed is a more complicated function of bulk modulus and shear modulus. Table (16.1) gives the speed of sound in some common materials.

Table 16.1

Medium	Speed m/s	Medium	Speed m/s
Air (dry 0°C)	332	Copper	3810
Hydrogen	1330	Aluminium	5000
Water	1486	Steel	5200

16.4 SPEED OF SOUND IN A GAS : NEWTON'S FORMULA AND LAPLACE'S CORRECTION

The speed of sound in a gas can be expressed in terms of its pressure and density. The derivation uses some of the properties of gases that we shall study in another chapter. We summarise these properties below.

(a) For a given mass of an ideal gas, the pressure, volume and the temperature are related as $\frac{PV}{T}$ = constant. If the temperature remains constant (called an isothermal process), the pressure and the volume of a given mass of a gas satisfy PV = constant.

Here T is the absolute temperature of the gas. This is known as *Boyle's law*.

(b) If no heat is supplied to a given mass of a gas (called an adiabatic process), its pressure and volume satisfy

$$PV^{\gamma} = \text{constant};$$

where γ is a constant for the given gas. It is, in fact, the ratio C_p/C_v of two specific heat capacities of the gas.

Newton suggested a theoretical expression for the velocity of sound wave in a gaseous medium. He assumed that when a sound wave propagates through a gas, the temperature variations in the layers of compression and rarefaction are negligible. The logic perhaps was that the layers are in contact with wider mass of the gas so that by exchanging heat with the surrounding the temperature of the layer will remain equal to that of the surrounding. Hence, the conditions are isothermal and Boyle's law will be applicable.

Thus, $$PV = \text{constant}$$

or, $$P\Delta V + V\Delta P = 0$$

or, $$B = -\frac{\Delta P}{\Delta V/V} = P. \quad \dots \text{(i)}$$

Using this result in equation (16.3), the speed of sound in the gas is given by

$$v = \sqrt{P/\rho}. \quad \dots (16.5)$$

The density of air at temperature 0°C and pressure 76 cm of mercury column is $\rho = 1{\cdot}293\ \text{kg/m}^3$. This temperature and pressure is called standard temperature and pressure and is written as STP. According to equation (16.5), the speed of sound in air at this temperature and pressure should be 280 m/s. This value is somewhat smaller than the measured speed of sound which is about 332 m/s.

Laplace suggested that the compression or rarefaction takes place too rapidly and the gas element being compressed or rarefied does not get enough time to exchange heat with the surroundings. Thus, it is an adiabatic process and one should use the equation

$$PV^{\gamma} = \text{constant}.$$

Taking logarithms,

$$\ln P + \gamma \ln V = \text{constant}.$$

Taking differentials,

$$\frac{\Delta P}{P} + \gamma \frac{\Delta V}{V} = 0$$

or, $$B = -\frac{\Delta P}{\Delta V/V} = \gamma P.$$

Thus, the speed of sound is $v = \sqrt{\dfrac{\gamma P}{\rho}}$ ... (16.6)

For air, $\gamma = 1{\cdot}4$ and putting values of P and ρ as before, equation (16.6) gives the speed of sound in air at STP to be 332 m/s which is quite close to the observed value.

16.5 EFFECT OF PRESSURE, TEMPERATURE AND HUMIDITY ON THE SPEED OF SOUND IN AIR

We have stated that for an ideal gas, the pressure, volume and temperature of a given mass satisfy

$$\frac{PV}{T} = \text{constant.}$$

As the density of a given mass is inversely proportional to its volume, the above equation may also be written as

$$\frac{P}{\rho} = cT,$$

where c is a constant. The speed of sound is

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma cT}. \qquad (16.7)$$

Thus, if pressure is changed but the temperature is kept constant, the density varies proportionally and P/ρ remains constant. The speed of sound is not affected by the change in pressure provided the temperature is kept constant.

If the temperature of air is changed then the speed of sound is also changed.

From equation (16.7),

$$v \propto \sqrt{T}.$$

At STP, the temperature is 0°C or 273 K. If the speed of sound at 0°C is v_0, its value at the room temperature T (in kelvin) will satisfy

$$\frac{v}{v_0} = \sqrt{\frac{T}{273}} = \sqrt{\frac{273 + t}{273}},$$

where t is the temperature in °C. This may be approximated as

$$\frac{v}{v_0} = \left(1 + \frac{t}{273}\right)^{\frac{1}{2}} \approx 1 + \frac{t}{546}$$

or, $$v = v_0\left(1 + \frac{t}{546}\right).$$

The density of water vapour is less than dry air at the same pressure. Thus, the density of moist air is less than that of dry air. As a result, the speed of sound increases with increasing humidity.

16.6 INTENSITY OF SOUND WAVES

As a wave travels in a medium, energy is transported from one part of the space to another part. The intensity of a sound wave is defined as the average energy crossing a unit cross-sectional area perpendicular to the direction of propagation of the wave in unit time. It may also be stated as the average power transmitted across a unit cross-sectional area perpendicular to the direction of propagation.

The loudness of sound that we feel is mainly related to the intensity of sound. It also depends on the frequency to some extent.

Consider again a sound wave travelling along the x-direction. Let the equations for the displacement of the particles and the excess pressure developed by the wave be given by

$$\left.\begin{aligned} s &= s_0 \sin \omega(t - x/v) \\ \text{and} \quad p &= p_0 \cos \omega(t - x/v) \end{aligned}\right| \quad \text{... (i)}$$

where $$p_0 = \frac{B\,\omega\, s_0}{v}.$$

Consider a cross-section of area A perpendicular to the x-direction. The medium to the left to it exerts a force pA on the medium to the right along the X-axis. The points of application of this force move longitudinally, that is along the force, with a speed $\frac{\partial s}{\partial t}$. Thus, the power W, transmitted by the wave from left to right across the cross-section considered, is

$$W = (pA)\frac{\partial s}{\partial t}.$$

By (i),

$$W = A\,p_0 \cos \omega(t - x/v)\, \omega s_0 \cos \omega(t - x/v)$$

$$= \frac{A\,\omega^2 s_0^2 B}{v} \cos^2 \omega(t - x/v).$$

The average of $\cos^2 \omega(t - x/v)$ over a complete cycle or over a long time is 1/2. The intensity I, which is equal to the average power transmitted across unit cross-sectional area is thus,

$$I = \frac{1}{2}\frac{\omega^2 s_0^2 B}{v} = \frac{2\pi^2 B}{v} s_0^2 \nu^2.$$

Using equation (16.2),

$$I = \frac{p_0^2 v}{2B}. \qquad \text{... (16.8)}$$

As $B = \rho v^2$, the intensity can also be written as

$$I = \frac{v}{2\rho v^2} p_0^2 = \frac{p_0^2}{2\rho v}. \qquad \text{... (16.9)}$$

We see that the intensity is proportional to the square of the pressure amplitude p_0.

Example 16.3

The pressure amplitude in a sound wave from a radio receiver is $2{\cdot}0\times10^{-2}$ N/m^2 *and the intensity at a point is* $5{\cdot}0\times10^{-7}$ W/m^2. *If by turning the "volume" knob the pressure amplitude is increased to* $2{\cdot}5\times10^{-2}$ N/m^2, *evaluate the intensity.*

Solution : The intensity is proportional to the square of the pressure amplitude.

Thus, $$\frac{I'}{I}=\left(\frac{p'_0}{p_0}\right)^2$$

or, $$I'=\left(\frac{p'_0}{p_0}\right)^2 I=\left(\frac{2{\cdot}5}{2{\cdot}0}\right)^2\times5{\cdot}0\times10^{-7}\text{ W/m}^2$$
$$=7{\cdot}8\times10^{-7}\text{ W/m}^2.$$

16.7 APPEARANCE OF SOUND TO HUMAN EAR

The appearance of sound to a human ear is characterised by three parameters (a) *pitch* (b) *loudness* and (c) *quality*. All the three are subjective description of sound though they are related to objectively defined quantities. Pitch is related to frequency, loudness is related to intensity and quality is related to the waveform of the sound wave.

Pitch and Frequency

Pitch of a sound is that sensation by which we differentiate a buffalo voice, a male voice and a female voice. We say that a buffalo voice is of low pitch, a male voice has higher pitch and a female voice has (generally) still higher pitch. This sensation primarily depends on the dominant frequency present in the sound. Higher the frequency, higher will be the pitch and vice versa. The dominant frequency of a buffalo voice is smaller than that of a male voice which in turn is smaller than that of a female voice.

Loudness and Intensity

The loudness that we sense is related to the intensity of sound though it is not directly proportional to it. Our perception of loudness is better correlated with the *sound level* measured in decibels (abbreviated as dB) and defined as follows.

$$\beta=10\log_{10}\left(\frac{I}{I_0}\right), \qquad \ldots (16.10)$$

where I is the intensity of the sound and I_0 is a constant reference intensity 10^{-12} W/m^2. The reference intensity represents roughly the minimum intensity that is just audible at intermediate frequencies. For $I=I_0$, the sound level $\beta=0$. Table (16.2) shows the approximate sound levels of some of the sounds commonly encountered.

Table 16.2 : *Sound Levels*

Minimum audible sound	≈0 dB
Whispering (at 1 m)	10 dB
Normal talk (at 1 m)	60 dB
Maximum tolerable sound	120 dB

Example 16.4

If the intensity is increased by a factor of 20, by how many decibels is the sound level increased?

Solution : Let the initial intensity be I and the sound level be β_1. When the intensity is increased to 20 I, the level increases to β_2.

Then $\beta_1=10\log(I/I_0)$

and $\beta_2=10\log(20\,I/I_0)$.

Thus, $$\beta_2-\beta_1=10\log(20\,I/I)$$
$$=10\log 20$$
$$=13\text{ dB}.$$

Quality and Waveform

A sound generated by a source may contain a number of frequency components in it. Different frequency components have different amplitudes and superposition of them results in the actual waveform. The appearance of sound depends on this waveform apart from the dominant frequency and intensity. Figure (16.6) shows waveforms for a tuning fork, a clarinet and a cornet playing the same note (fundamental frequency = 440 Hz) with equal loudness.

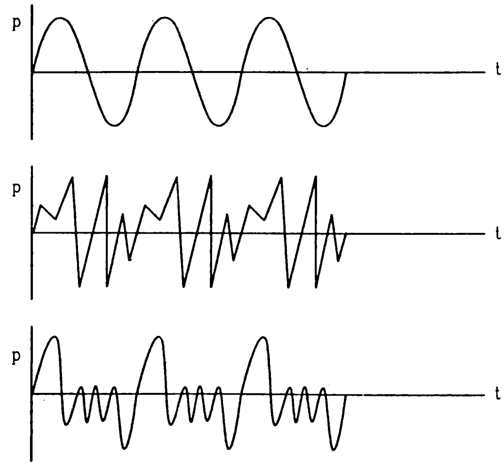

Figure 16.6

We differentiate between the sound from a tabla and that from a mridang by saying that they have different *quality*. A musical sound has certain well-defined frequencies which have considerable amplitude. These frequencies are generally harmonics of a fundamental frequency. Such a sound is particularly pleasant to the ear. On the other hand, a noise has frequencies that do not bear any well-defined relationship among themselves.

16.8 INTERFERENCE OF SOUND WAVES

The principle of superposition introduced in the previous chapter is valid for sound waves as well. If two or more waves pass through the same region of a medium, the resultant disturbance is equal to the sum of the disturbances produced by individual waves. Depending on the phase difference, the waves can interfere constructively or destructively leading to a corresponding increase or decrease in the resultant intensity. While discussing the interference of two sound waves, it is advised that the waves be expressed in terms of pressure change. The resultant change in pressure is the algebraic sum of the changes in pressure due to the individual waves. Thus, one should not add the displacement vectors so as to obtain the resultant displacement wave.

Figure (16.7) shows two tuning forks S_1 and S_2, placed side by side, which vibrate with equal frequency and equal amplitude. The point P is situated at a distance x from S_1 and $x + \Delta x$ from S_2.

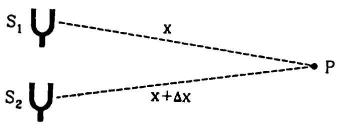

Figure 16.7

The forks may be set into vibration with a phase difference δ_0. In case of tuning forks, the phase difference δ_0 remains constant in time. Two sources whose phase difference remains constant in time are called *coherent sources.* If there were two drum beaters beating the drums independently, the sources would have been *incoherent.*

Suppose the two forks are vibrating in phase so that $\delta_0 = 0$. Also, let p_{01} and p_{02} be the amplitudes of the waves from S_1 and S_2 respectively. Let us examine the resultant change in pressure at a point P. The pressure-change at A due to the two waves are described by

$$p_1 = p_{01} \sin(kx - \omega t)$$

$$p_2 = p_{02} \sin[k(x + \Delta x) - \omega t]$$

$$= p_{02} \sin[(kx - \omega t) + \delta],$$

where $$\delta = k\,\Delta x = \frac{2\pi\,\Delta x}{\lambda} \qquad \ldots (16.11)$$

is the phase difference between the two waves reaching P. These equations are identical to those discussed in chapter 15, section 15.7. The resultant wave at P is given by

$$p = p_0 \sin[(kx - \omega t) + \varepsilon],$$

where $$p_0^2 = p_{01}^2 + p_{02}^2 + 2\,p_{01}\,p_{02} \cos\delta,$$

and $$\tan \varepsilon = \frac{p_{02} \sin\delta}{p_{01} + p_{02} \cos\delta}.$$

The resultant amplitude is maximum when $\delta = 2n\pi$ and is minimum when $\delta = (2n + 1)\pi$ where n is an integer. These are correspondingly the conditions for constructive and destructive interference

$$\left.\begin{array}{ll} \delta = 2n\pi & \text{constructive interference} \\ \delta = (2n+1)\pi & \text{destructive interference.} \end{array}\right| \quad \ldots (16.12)$$

Using equation (16.11) i.e., $\delta = \frac{2\pi}{\lambda}\Delta x$, these conditions may be written in terms of the path difference as

$$\left.\begin{array}{ll} \Delta x = n\lambda & \text{(constructive)} \\ \Delta x = (n + 1/2)\lambda & \text{(destructive).} \end{array}\right| \quad \ldots (16.13)$$

At constructive interference,

$$p_0 = p_{01} + p_{02}$$

and at destructive interference,

$$p_0 = |\, p_{01} - p_{02} \,|.$$

Suppose $p_{01} = p_{02}$ and $\Delta x = \lambda/2$. The resultant pressure amplitude of the disturbance is zero and no sound is detected at such a point P. If $\Delta x = \lambda$, the amplitude is doubled. The intensity of a wave is proportional to the square of the amplitude and hence, at the points of constructive interference, the resultant intensity of sound is four times the intensity due to an individual source. This is a characteristic property of wave motion. Two sources can cancel the effects of each other or they can reinforce the effect.

If the sources have an initial phase difference δ_0 between them, the waves reaching P at time t are represented by

$$p = p_{01} \sin[kx - \omega t]$$

and $$p = p_{02} \sin[k(x + \Delta x) - \omega t + \delta_0].$$

The phase difference between these waves is

$$\delta = \delta_0 + k\,\Delta x = \delta_0 + \frac{2\pi\,\Delta x}{\lambda}.$$

The interference maxima and minima occur according to equation (16.12).

For incoherent sources, δ_0 is not constant and varies rapidly and randomly with time. At any point P, sometimes constructive and sometimes destructive interference takes place. If the intensity due to each source is I, the resultant intensity rapidly and randomly changes between zero and $4I$ so that the average observable intensity is $2I$. No interference effect is, therefore, observed. For observable interference, the sources must be coherent.

One way to obtain a pair of coherent sources is to obtain two sound waves from the same source by dividing the original wave along two different paths and then combining them. The two waves then differ in phase only because of different paths travelled.

A popular demonstration of interference of sound is given by the Quinke's apparatus (figure 16.8).

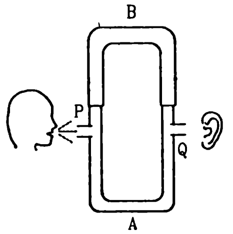

Figure 16.8

Sound produced near the end P travels down in P and is divided in two parts, one going through the tube A and the other through B. They recombine and the resultant sound moves along Q to reach the listener (which may be an electronic detector to do a quantitative analysis). The tube B can be slid in and out to change the path length of one of the waves. If the sound is produced at a unique frequency (a tuning fork may be used for it), the wavelength $\lambda\,(= v/\nu)$ has a single value. The intensity at Q will be a maximum or a minimum depending on whether the difference in path lengths is an integral multiple of λ or a half integral multiple. Thus, when the tube B is slowly pulled out the intensity at Q oscillates.

Phase Difference and Path Difference

From equation (16.11) we see that if a wave travels an extra distance Δx with respect to the other wave, a phase difference

$$\delta = \frac{\omega}{v}\Delta x = \frac{2\pi\,\Delta x}{\lambda}$$

is introduced between them.

Note carefully that this is the phase difference introduced due to the different path lengths covered by the waves from their origin. Any initial difference of phase that may exist between the sources must be added to it so as to get the actual phase difference.

Example 16.5

Two sound waves, originating from the same source, travel along different paths in air and then meet at a point. If the source vibrates at a frequency of 1·0 kHz *and one path is* 83 cm *longer than the other, what will be the nature of interference? The speed of sound in air is* 332 m/s.

Solution : The wavelength of sound wave is $\lambda = \frac{v}{\nu}$

$$= \frac{332\ \mathrm{m/s}}{1\cdot0\times10^{3}\ \mathrm{Hz}} = 0\cdot332\ \mathrm{m}.$$

The phase difference between the waves arriving at the point of observation is

$$\delta = \frac{2\pi}{\lambda}\Delta x = 2\pi\times\frac{0\cdot83\ \mathrm{m}}{0\cdot332\ \mathrm{m}} = 2\pi\times2\cdot5 = 5\pi.$$

As this is an odd multiple of π, the waves interfere destructively.

Reflection of Sound Waves

When there exists a discontinuity in the medium, the wave gets reflected. When a sound wave gets reflected from a rigid boundary, the particles at the boundary are unable to vibrate. Thus, a reflected wave is generated which interferes with the oncoming wave to produce zero displacement at the rigid boundary. At these points (zero displacement), the pressure variation is maximum. Thus, a reflected pressure wave has the same phase as the incident wave. That means, a compression pulse reflects as a compression pulse and a rarefaction pulse reflects as a rarefaction pulse.

A sound wave is also reflected if it encounters a low pressure region. A practical example is when a sound wave travels in a narrow open tube. When the wave reaches an open end, it gets reflected. The force on the particles there due to the outside air is quite small and hence, the particles vibrate with increased amplitude. As a result, the pressure there remains at the average value. Thus, the reflected pressure wave interferes destructively with the oncoming wave. There is a phase change of π in the pressure wave when it is reflected by an open end. That means, a compression pulse reflects as a rarefaction pulse and vice versa.

16.9 STANDING LONGITUDINAL WAVES AND VIBRATIONS OF AIR COLUMNS

If two longitudinal waves of the same frequency and amplitude travel through a medium in the opposite directions, standing waves are produced. If the equations of the two waves are written as

$$p_1 = p_0 \sin\omega(t - x/v)$$

and $$p_2 = p_0 \sin\omega(t + x/v),$$

the resultant wave is by the principle of superposition,

$$\begin{aligned} p &= p_1 + p_2 \\ &= 2\,p_0\cos(\omega x/v)\sin\omega t \\ &= 2\,p_0\cos kx\sin\omega t. \end{aligned}$$

This equation is similar to the equation obtained in chapter 15 for standing waves on a string. Hence, all the characteristics of standing waves on a string are also present in longitudinal standing waves. At different points in the medium, the pressure amplitudes have different magnitudes. In particular, at certain points the pressure remains permanently at its average value, they are called the *pressure nodes* and midway between the nodes, there are *pressure antinodes* where the amplitude is maximum. The separation between two consecutive nodes or between two consecutive antinodes is $\lambda/2$. It may be noted that a pressure node is a displacement antinode and a pressure antinode is a displacement node.

Standing waves can be produced in air columns trapped in tubes of cylindrical shape. Organ pipes are such vibrating air columns.

(A) Closed Organ Pipe

A closed organ pipe is a cylindrical tube having an air column with one end closed. Sound waves are sent in by a source vibrating near the open end. An ingoing pressure wave gets reflected from the fixed end. This inverted wave is again reflected at the open end. After two reflections, it moves towards the fixed end and interferes with the new wave sent by the source in that direction. The twice reflected wave has travelled an extra distance of $2l$ causing a phase advance of $\frac{2\pi}{\lambda}\cdot 2l = \frac{4\pi l}{\lambda}$ with respect to the new wave sent in by the source. Also, the twice reflected wave suffered a phase change of π at the open end. The phase difference between the two waves is then $\delta = \frac{4\pi l}{\lambda} + \pi$. The waves interfere constructively if $\delta = 2n\pi$

or, $$\frac{4\pi l}{\lambda} + \pi = 2n\pi$$

or, $$l = (2n - 1)\frac{\lambda}{4}$$

where $n = 1, 2, 3, \ldots$. This may also be written as

$$l = (2n + 1)\frac{\lambda}{4}, \qquad \ldots (16.14)$$

where $n = 0, 1, 2, \ldots$

In such a case, the amplitude goes on increasing until the energy dissipated through various damping effects equals the fresh input of energy from the source. Such waves exist in both the directions and they interfere to give standing waves of large amplitudes in the tube. The fixed end is always a pressure antinode (or displacement node) and the open end is a pressure node (or displacement antinode). In fact, the pressure node is not exactly at the open end because the air outside does exert some force on the air in the tube. We shall neglect this end correction for the time being.

The condition for having standing waves in a closed organ pipe is given by equation (16.14),

$$l = (2n + 1)\frac{\lambda}{4}.$$

The frequency ν is thus given by

$$\nu = \frac{v}{\lambda} = (2n + 1)\frac{v}{4l}. \qquad \ldots (16.15)$$

We see that there are certain discrete frequencies with which standing waves can be set up in a closed organ pipe. These frequencies are called *natural frequencies, normal frequencies or resonant frequencies.*

Figure (16.9) shows the variation of excess pressure and displacement of particles in a closed organ pipe for the first three resonant frequencies.

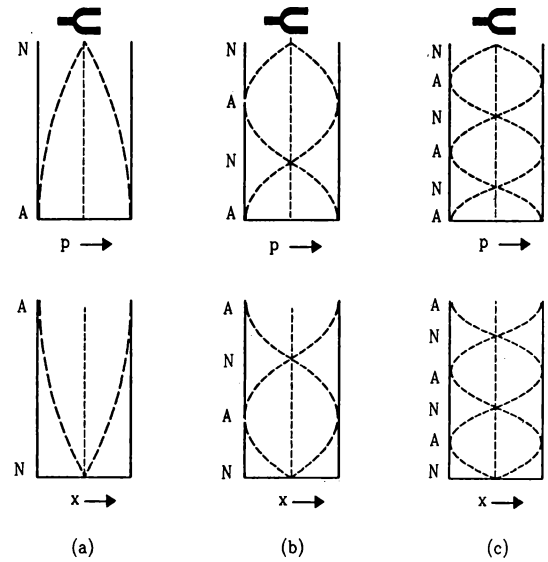

Figure 16.9

The minimum allowed frequency is obtained by putting $n = 0$ in equation (16.15). This is called the fundamental frequency ν_0 of the tube. We have

$$\nu_0 = \frac{v}{4l}. \qquad \ldots (16.16)$$

By equation (16.14), $l = \lambda/4$ in this case. A pressure antinode is formed at the closed end and a node is formed at the open end. There are no other nodes or antinodes in between. The air column is said to vibrate in its fundamental mode (figure 16.9a).

Putting $n = 1$ in equation (16.15), we get the first overtone frequency

$$\nu_1 = \frac{3v}{4l} = 3\,\nu_0.$$

By equation (16.14), the length of the tube in this mode is

$$l = 3\lambda/4.$$

In the first overtone mode of vibration (figure 16.9b), there is an antinode and a node in the air column apart from those at the ends.

In the second overtone, there are two nodes and two antinodes in the column apart from the ends (figure 16.10c). In this mode, $l = 5\,\lambda/4$ and the frequency is

$$\nu_2 = \frac{5v}{4l} = 5\,\nu_0 .$$

The higher overtones can be described in a similar way.

Thus, an air column of length l fixed at one end can vibrate with frequencies $(2n+1)\,\nu_0$ where $\nu_0 = \frac{v}{4l}$ and n is an integer. We see that all the overtone frequencies are harmonics (i.e., integral multiple) of the fundamental frequency but all the harmonics of fundamental frequency are not the allowed frequencies. Only the odd harmonics of the fundamental are the allowed frequencies.

(B) Open Organ Pipe

An open organ pipe is again a cylindrical tube containing an air column open at both ends. A source of sound near one of the ends sends the waves in the pipe (figure 16.11). The wave is reflected by the other open end and travels towards the source. It suffers second reflection at the open end near the source and then interferes with the new wave sent by the source. The twice reflected wave is ahead of the new wave coming in by a path difference $2l$. The phase difference is $\delta = \frac{2\pi}{\lambda}\,2l = \frac{4\pi l}{\lambda}$. Constructive interference takes place if

$$\delta = 2n\pi$$

$$l = n\lambda/2 \qquad (16.17)$$

where n = 1, 2, 3,...

The amplitude then keeps on growing. Waves moving in both the directions with large amplitudes are established and finally standing waves are set up. As already discussed, in this case the energy lost by various damping effects equals the energy input from the source. The frequencies with which a standing wave can be set up in an open organ pipe are

$$\nu = \frac{v}{\lambda} = \frac{nv}{2l} \qquad \ldots (16.18)$$

Figure (16.10) shows the variation of excess pressure and displacement in the open organ pipe for the first three resonant frequencies. The minimum frequency ν_0 is obtained by putting $n = 1$ in equation (16.18). Thus,

$$\nu_0 = \frac{v}{2l} \cdot \qquad \ldots (16.19)$$

This corresponds to the fundamental mode of vibration. By equation (16.18), $l = \lambda/2$ in the fundamental mode of vibration (figure 16.10a).

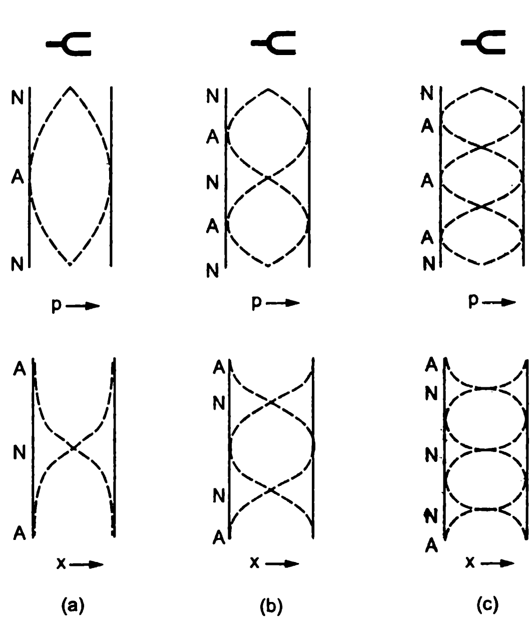

Figure 16.10

As the air at both ends is free to vibrate (neglecting the effect of the pressure of the air outside the pipe), pressure nodes are formed at these points. There is an antinode in between these nodes.

If the source vibrates at the frequency $\nu_0 = \frac{v}{2l}$, it will set up the column in the fundamental mode of vibration.

Putting $n = 2$ in equation (16.18), we get the frequency of the first overtone mode as

$$\nu_1 = 2\,\frac{v}{2l} = 2\,\nu_0 .$$

The length of the tube is, by equation (16.17), $l = \lambda$. There are two pressure antinodes and one node apart from the nodes at the two ends. In the nth overtone, there are $(n+1)$ pressure antinodes. The frequency of the nth overtone is given by

$$\nu_n = \frac{nv}{2l} = n\,\nu_0 .$$

The overtone frequencies are $\frac{2v}{2l}$, $\frac{3v}{2l}$ ⋯ etc. All the overtone frequencies are harmonics of the fundamental and all the harmonics of the fundamental are allowed overtone frequencies. There are no missing harmonics in an open organ pipe. The quality of sound from an open organ pipe is, therefore, richer than that from a closed organ pipe in which all the even harmonics of the fundamental are missing.

Example 16.6

An air column is constructed by fitting a movable piston in a long cylindrical tube. Longitudinal waves are sent in the tube by a tuning fork of frequency 416 Hz. *How* *far from the open end should the piston be so that the air column in the tube may vibrate in its first overtone ? Speed of sound in air is* 333 m/s.

Solution : The piston provides the closed end of the column and an antinode of pressure is formed there. At the open end, a pressure node is formed. In the first overtone there is one more node and an antinode in the column as shown in figure (16.11). The length of the tube should then be $3\lambda/4$.

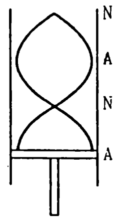

Figure 16.11

The wavelength is $\lambda = \frac{v}{\nu} = \frac{333 \text{ m/s}}{416 \text{ s}^{-1}} = 0{\cdot}800 \text{ m}$.

Thus, the length of the tube is

$$\frac{3\lambda}{4} = \frac{3 \times 0{\cdot}800 \text{ m}}{4} = 60{\cdot}0 \text{ cm}.$$

16.10 DETERMINATION OF SPEED OF SOUND IN AIR

(a) Resonance Column Method

Figure (16.12) shows schematically the diagram of a simple apparatus used in laboratories to measure the speed of sound in air. A long cylindrical glass tube (say about 1 m) is fixed on a vertical wooden frame. It is also called a resonance tube. A rubber tube connects the lower end of this glass tube to a vessel which can slide vertically on the same wooden frame. A meter scale is fitted parallel to and close to the glass tube.

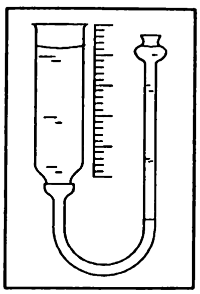

Figure 16.12

The vessel contains water which also goes in the resonance tube through the rubber tube. The level of water in the resonance tube is same as that in the vessel. Thus, by sliding the vessel up and down, one can change the water level in the resonance tube.

A tuning fork (frequency > 256 Hz if the tube is 1 m long) is vibrated by hitting it on a rubber pad and is held near the open end of the tube in such a way that the prongs vibrate parallel to the length of the tube. Longitudinal waves are then sent in the tube.

The water level in the tube is initially kept high. The tuning fork is vibrated and kept close to the open end, and the loudness of sound coming from the tube is estimated. The vessel is brought down a little to decrease the water level in the resonance tube. The tuning fork is again vibrated, kept close to the open end and the loudness of the sound coming from the tube is estimated. The process is repeated until the water level corresponding to the maximum loudness is located. Fine adjustments of water level are made to locate accurately the level corresponding to the maximum loudness. The length of the air column is read on the scale attached. In this case, the air column vibrates in resonance with the tuning fork. The minimum length of the air column for which the resonance takes place corresponds to the fundamental mode of vibration. A pressure antinode is formed at the water surface (which is the closed end of the air column) and a pressure node is formed near the open end. In fact, the node is formed slightly above the open end (end correction) because of the air-pressure from outside.

Thus, for the first resonance the length l_1 of the air column in the resonance tube is given by

$$l_1 + d = \frac{\lambda}{4}, \qquad \dots \text{(i)}$$

where d is the end correction.

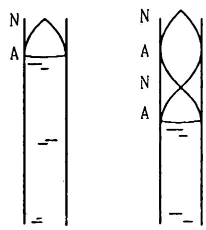

Figure 16.13

The length of the air column is increased to a little less than three times of l. The water level is adjusted so that the loudness of the sound coming from the tube becomes maximum again. The length of the air column is noted on the scale. In this second resonance the air column vibrates in the first overtone. There is one node and one antinode in between the ends of the column. The length l_2 of the column is given by

$$l_2 + d = 3\lambda/4. \qquad \dots \text{(ii)}$$

By (i) and (ii),

$$(l_2 - l_1) = \lambda/2. \quad \text{or,} \quad \lambda = 2(l_2 - l_1).$$

The frequency of the wave is same as the frequency of the tuning fork. Thus, the speed of sound in the air of the laboratory is

$$v = \nu\lambda = 2(l_2 - l_1)\,\nu. \qquad \dots (16.20)$$

(b) Kundt's Tube Method of Determining the Speed of Sound in a Gas

The resonance column method described above can be used to find the speed of sound in air only, as the tube is open at the end to the atmosphere. In the Kundt's method, a gas is enclosed in a long cylindrical tube closed at both ends, one by a disc D and the other by a movable piston (figure 16.14). A metal rod is welded with the disc and is clamped exactly at the middle point. The length of the tube can be varied by moving the movable piston. Some powder is sprinkled inside the tube along its length.

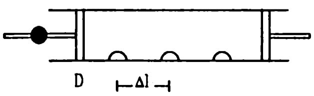

Figure 16.14

The rod is set into longitudinal vibrations electronically or by rubbing it with some rosined cloth or otherwise. If the length of the gas column is such that one of its resonant frequencies is equal to the frequency of the longitudinal vibration of the rod, standing waves are formed in the gas. The powder particles at the displacement antinodes fly apart due to the violent disturbance there, whereas the powder at the displacement nodes remain undisturbed because the particles here do not vibrate. Thus, the powder which was initially dispersed along the whole length of the tube collects in heaps at the displacement nodes. By measuring the separation Δl between the successive heaps, one can find the wavelength of sound in the enclosed gas

$$\lambda = 2\Delta l.$$

The length of the gas column is adjusted by moving the movable piston such that the gas resonates with the disc and the wavelength λ is obtained. If the frequency of the longitudinal vibration of the rod is ν, the speed of sound in the gas is given by

$$v = \nu\lambda = 2\,\Delta l\,\nu. \qquad \dots \text{(i)}$$

If the frequency of the longitudinal vibrations in the rod is not known, the experiment is repeated with air filled in the tube. The length between the heaps of the powder, $\Delta l'$ is measured. The speed of sound in air is then

$$v' = 2\,\Delta l'\nu. \qquad \dots \text{(ii)}$$

By (i) and (ii),

$$\frac{v}{v'} = \frac{\Delta l}{\Delta l'}$$

or,
$$v = v'\frac{\Delta l}{\Delta l'}.$$

Knowing the speed v' of sound in air, one can find the speed in the experimental gas.

Kundt's tube method can also be used to measure the speed of sound in a solid. Air at normal pressure is filled in the tube. The speed of sound in air is supposed to be known. The rod attached to the disc is clamped at the middle and is set into longitudinal vibration. The rod behaves like an open organ pipe as the two ends are free to vibrate. Assuming that it vibrates in its fundamental mode of vibration, the clamped point is a pressure antinode and the two ends of the rod are pressure nodes. Thus, the wavelength of sound in the rod is $\lambda = 2l$. If the powder piles up at successive distances Δl and the speed of sound in air is v_a then $v_a = 2\,\Delta l\,\nu$. Also, if v be the speed of sound in the rod, $v = \nu\lambda = 2\,\nu l$.

Thus,
$$\frac{v}{v_a} = \frac{l}{\Delta l} \quad \text{or,} \quad v = \frac{l}{\Delta l}v_a.$$

As the speed v_a of sound in air is known, measurements of l and Δl give the speed of sound in the material of the rod.

16.11 BEATS

So far we have considered superposition of two sound waves of equal frequency. Let us now consider two sound waves having equal amplitudes and travelling in a medium in the same direction but having slightly different frequencies. The equations of the two waves are given by

$$p_1 = p_0 \sin \omega_1(t - x/v)$$
$$p_2 = p_0 \sin \omega_2(t - x/v),$$

where we have chosen the two waves to be in phase at $x = 0, t = 0$. The speed of sound wave does not depend on its frequency and hence, same wave speed v is used for both the equations. The angular frequencies ω_1 and ω_2 only slightly differ so that the difference $|\omega_1 - \omega_2|$ is small as compared to ω_1 or ω_2. By the principle of superposition, the resultant change in pressure is

$$\begin{aligned} p &= p_1 + p_2 \\ &= p_0[\sin \omega_1(t - x/v) + \sin \omega_2(t - x/v)] \\ &= 2\,p_0 \cos\left[\frac{\omega_1 - \omega_2}{2}(t - x/v)\right] \sin\left[\frac{\omega_1 + \omega_2}{2}(t - x/v)\right]. \end{aligned}$$

Writing $|\omega_1 - \omega_2| = \Delta\omega$, and $\dfrac{\omega_1 + \omega_2}{2} = \omega, \qquad \dots \text{(i)}$

the resultant change in pressure is

$$p = 2\,p_0 \cos\frac{\Delta\omega}{2}(t - x/v)\sin\omega(t - x/v) \quad \ldots \ (16.21)$$

$$= A\sin\omega(t - x/v),$$

where $A = 2\,p_0 \cos\frac{\Delta\omega}{2}(t - x/v)$. ... (16.22)

As $\omega >> \frac{\Delta\omega}{2}$, the term A varies slowly with time as compared to $\sin\omega(t - x/v)$. Thus, we can interpret this equation by saying that the resultant disturbance is a wave of angular frequency $\omega = (\omega_1 + \omega_2)/2$ whose amplitude varies with time and is given by equation (16.22). As a negative value of amplitude has no meaning, the amplitude is, in fact, $|A|$.

Let us concentrate our attention to a particular position x and look for the pressure-change there as a function of time. Equations (16.21) and (16.22) tell that the pressure oscillates back and forth with a frequency equal to the average frequency ω of the two waves but the amplitude of pressure variation itself changes periodically between $2p_0$ and zero.

Figure (16.15) shows the plots of

$$A = 2\,p_0\cos\frac{\Delta\omega}{2}(t - x/v),\ B = \sin\omega(t - x/v)$$

and their product

$$p = 2\,p_0\cos\frac{\Delta\omega}{2}(t - x/v)\sin\omega(t - x/v)$$

as a function of time for a fixed x.

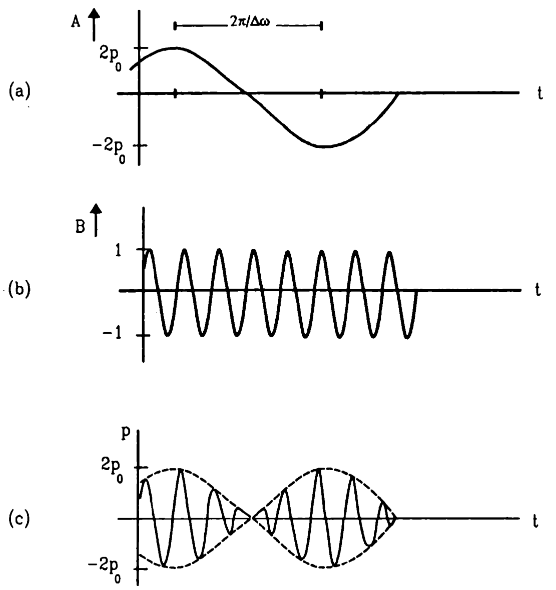

Figure 16.15

Variation of Intensity at a Point

The amplitude $|A|$ oscillates between 0 to $2\,p_0$ with a frequency which is double of the frequency of A. This is because as A oscillates from $2\,p_0$ to $-2\,p_0$ (i.e., half the oscillation), the amplitude $|A|$ covers full oscillation from $2\,p_0$ to zero to $2\,p_0$ (figure 16.16).

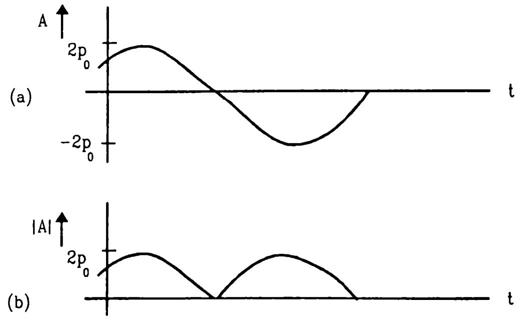

Figure 16.16

By equation (16.22), the frequency of variation of A is $\frac{\Delta\omega/2}{2\pi} = \frac{\Delta\omega}{4\pi}$. Thus, the frequency of amplitude variation is

$$\nu' = 2 \times \frac{\Delta\omega}{4\pi} = \frac{|\,\omega_1 - \omega_2\,|}{2\pi} = |\,\nu_1 - \nu_2\,|,$$

where ν_1 and ν_2 are the frequencies of the original waves. Notice that we have written this frequency as $|\,\nu_1 - \nu_2\,|$. It is the difference between ν_1 and ν_2 with which the amplitude oscillates. The intensity is proportional to the square of the pressure amplitude and it also varies periodically with frequency $|\,\nu_1 - \nu_2\,|$. This phenomenon of periodic variation of intensity of sound when two sound waves of slightly different frequencies interfere, is called *beats*. Specifically, one cycle of maximum intensity and minimum intensity is counted as one beat, so that the frequency of beats is $|\,\nu_1 - \nu_2\,|$.

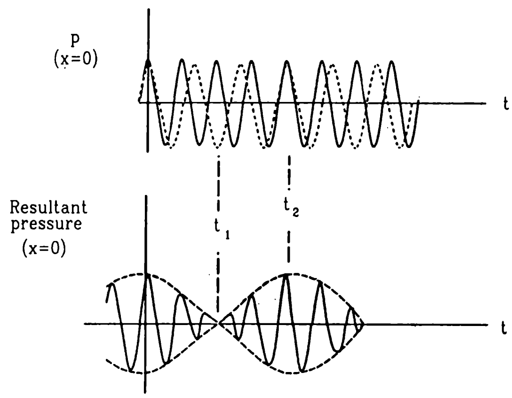

Figure 16.17

In figure (16.17), we explain the formation of beats graphically. The plots in part (a) show the pressure variation due to individual waves at a fixed position $x = 0$ as a function of time. At $t = 0$, the two waves produce pressure-changes in phase. But as the frequencies and hence the time periods are slightly different, the phase difference grows and at a time t_1, the two pressure-changes become out of phase. The resultant becomes zero at this time. The phase difference increases further and at time t_2, it becomes 2π. The pressure-changes are again in phase. The superposition of the two waves (figure 16.17) gives the resultant pressure change as a function of time which is identical in shape to the part (c) of figure (16.15).

The phenomenon of beats can be observed by taking two tuning forks of the same frequency and putting some wax on the prongs of one of the forks. Loading with wax decreases the frequency of a tuning fork a little. When these two forks are vibrated and kept side by side, the listener can recognise the periodic variation of loudness of the resulting sound. The number of beats per second equals the difference in frequency.

For beats to be audible, the frequency $|\nu_1 - \nu_2|$ should not be very large. An average human ear cannot distinguish the variation of intensity if the variation is more than 16 times per second. Thus, the difference of the component frequencies should be less than 16 Hz for the beats to be heard.

Example 16.7

A tuning fork A of frequency 384 Hz *gives* 6 beats *in* 2 seconds *when sounded with another tuning fork B. What could be the frequency of B?*

Solution : The frequency of beats is $|\nu_1 - \nu_2|$, which is 3 Hz according to the problem. The frequency of the tuning fork B is either 3 Hz more or 3 Hz less than the frequency of A. Thus, it could be either 381 Hz or 387 Hz.

16.12 DIFFRACTION

When waves are originated by a vibrating source, they spread in the medium. If the medium is homogeneous and isotropic, the waves from a point source have spherical wavefronts, the rays going in all directions. Far from the source, the wavefronts are nearly planes. The shape of the wavefront is changed when the wave meets an obstacle or an opening in its path. This leads to bending of the wave around the edges. For example, if a small cardboard is placed between a source of sound and a listener, the sound beyond the cardboard is not completely stopped, rather the waves bend at the edges of the cardboard to reach the listener. If a plane wave is passed through a small hole (an opening in a large obstacle), spherical waves are obtained on the other side as if the hole itself is a source sending waves in all directions (it is not a real source, no backward spherical waves are observed). Such bending of waves from an obstacle or an opening is called *diffraction.*

Diffraction is a characteristic property of wave motion and all kinds of waves exhibit diffraction.

The diffraction effects are appreciable when the dimensions of openings or the obstacles are comparable or smaller than the wavelength of the wave. If the opening or the obstacle is large compared to the wavelength, the diffraction effects are almost negligible.

The frequency of audible sound ranges from about 20 Hz to 20 kHz. Velocity of sound in air is around 332 m/s. The wavelength ($\lambda = v/\nu$) of audible sound in air thus ranges from 16 m to 1·6 cm. Quite often, the wavelength of sound is much larger than the obstacles or openings and diffraction is prominently displayed.

16.13 DOPPLER EFFECT

When a tuning fork is vibrated in air, sound waves travel from the fork. An observer stationed at some distance x from the fork receives the sound as the wave disturbs the air near his ear and the pressure varies between a maximum and a minimum. The pitch of the sound heard by the observer depends on how many times the pressure near the ear oscillates per unit time. Each time the fork moves forward it sends a compression pulse and each time it moves backward it sends a rarefaction pulse. Suppose the source and the observer are both at rest with respect to the medium. Each compression or rarefaction pulse, sent by the tuning fork, takes same time to reach the air near the ear. Thus, the pressure near the ear oscillates as many times as the fork oscillates in a given interval. The frequency observed is then equal to the frequency of the source.

However, if the source or the observer or both, move with respect to the medium, the frequency observed may be different from the frequency of the source. This apparent change in frequency of the wave due to motion of the source or the observer is called *Doppler Effect.*

Observer Stationary and Source Moving

Now suppose the observer is at rest with respect to the medium and the source moves towards the observer at a speed u which is less than the wave speed v.

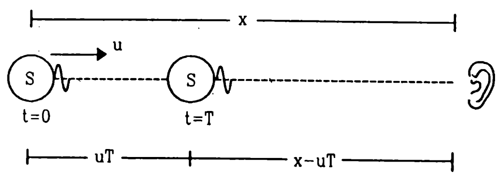

Figure 16.18

If the frequency of vibration of the source is ν_0, it sends compression pulses at a regular interval of $T = 1/\nu_0$. Suppose the separation between the source and the observer is x (figure 16.20) when a compression pulse is emitted at $t = 0$. The next compression pulse will be emitted after a time T. The source will travel a distance uT in this time and hence this second compression wave is emitted from a place which is at a distance $x - uT$ from the observer. The first pulse takes a time x/v to reach the observer whereas the next one takes $\frac{x - uT}{v}$.

Thus, the first compression wave reaches the observer at $t_1 = x/v$ and the next compression wave reaches at $t_2 = T + \frac{x - uT}{v}$. The time interval between the consecutive compression pulses detected by the observer is, therefore,

$$T' = t_2 - t_1$$

$$= T + \frac{x - uT}{v} - \frac{x}{v} = \left(1 - \frac{u}{v}\right)T = \frac{v - u}{v}T.$$

The apparent frequency of the sound as experienced by the observer is

$$\nu' = \frac{1}{T'},$$

or,
$$\nu' = \frac{v}{v - u}\nu_0. \qquad \ldots (16.23)$$

Similarly, if the source recedes from the observer at a speed u, the apparent frequency will be

$$\nu' = \frac{v}{v + u}\nu_0. \qquad \ldots (16.24)$$

Source Stationary and Observer Moving

Next, consider the case when the source remains stationary with respect to the medium and the observer approaches the source with a speed u.

As the source remains stationary in the medium, compression pulses are emitted at regular interval T from the same point in the medium. These pulses travel down the medium with a speed v and at any instant the separation between two consecutive compression pulses is $\lambda = vT$ (figure 16.19)

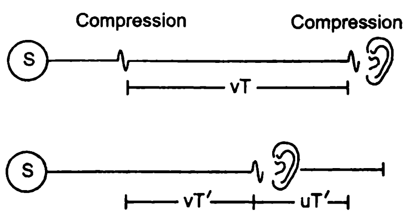

Figure 16.19

When the observer receives a compression pulse, the next compression pulse (towards the source) is a distance vT away from it. This second compression pulse moves towards the observer at a speed v and the observer moves towards it at a speed u. As a result, the observer will receive this second compression wave a time T' after receiving the first one where

$$T' = \frac{vT}{v + u}.$$

The apparent frequency of sound experienced by the observer is then $\nu' = \frac{1}{T'}$,

or,
$$\nu' = \frac{v + u}{v}\nu_0. \qquad \ldots (16.25)$$

Note that, in this case, it is not the same part of air that gives the sensation of pressure variation to the ear at frequency ν'. The pressure in any part of the air still oscillates with a frequency ν_0 but the observer moves in the medium to detect the pressure of some other part which reaches its maximum a little earlier. Similarly, if the source is stationary in the medium and the observer recedes from it at a speed u, the apparent frequency will be

$$\nu' = \frac{v - u}{v}\nu. \qquad \ldots (16.26)$$

The equation (16.23) through (16.26) may be generalised as

$$\nu = \frac{v + u_o}{v - u_s}\nu_0 \qquad \ldots (16.27)$$

where,

v = speed of sound in the medium.

u_o = speed of the observer with respect to the medium, considered positive when it moves towards the source and negative when it moves away from the source

and u_s = speed of the source with respect to the medium, considered positive when it moves towards the observer and negative when it moves away from the observer.

It should be carefully noted that the speeds u_s and u_o are to be written with respect to the medium carrying the sound. If the medium itself is moving with respect to the given frame of reference, appropriate calculations must be made to obtain the speeds of the source and the observer with respect to the medium.

Example 16.8

A sound detector is placed on a railway platform. A train, approaching the platform at a speed of 36 km/h, *sounds its whistle. The detector detects* 12·0 kHz *as the most dominant frequency in the whistle. If the train stops at* *the platform and sounds the whistle, what would be the most dominant frequency detected? The speed of sound in air is* 340 m/s.

Solution : Here the observer (detector) is at rest with respect to the medium (air). Suppose the dominant frequency as emitted by the train is ν_0. When the train is at rest at the platform, the detector will detect the dominant frequency as ν_0. When this same train was approaching the observer, the frequency detected was,

$$\nu' = \frac{v}{v - u_s}\nu_0$$

or, $$\nu_0 = \frac{v - u_s}{v}\nu' = \left(1 - \frac{u_s}{v}\right)\nu'.$$

The speed of the source is

$$u_s = 36\ \text{km/h} = \frac{36 \times 10^3\ \text{m}}{3600\ \text{s}} = 10\ \text{m/s}.$$

Thus, $$\nu_0 = \left(1 - \frac{10}{340}\right) \times 12{\cdot}0\ \text{kHz}$$
$$= 11{\cdot}6\ \text{kHz}.$$

We have derived the equations for Doppler shift in frequency assuming that the motion of the source or the observer is along the line joining the two. If the motion is along some other direction, the component of the velocity along the line joining the source and the observer should be used for u_s and u_o.

It is helpful to remember that the apparent frequency is larger than the actual, if the separation between the source and the observer is decreasing and is smaller if the separation is increasing. If you are standing on a platform and a whistling train passes by, you can easily notice the change in the pitch of the whistle. When the train is approaching you, the pitch is higher. As it passes through, there is a sudden fall in the pitch.

Change in Wavelength

If the source moves with respect to the medium, the wavelength becomes different from the wavelength observed when there is no relative motion between the source and the medium. The formula for apparent wavelength may be derived immediately from the relation $\lambda = v/\nu$ and equation (16.23). It is

$$\lambda' = \frac{v - u}{v}\lambda. \qquad \text{... (16.28)}$$

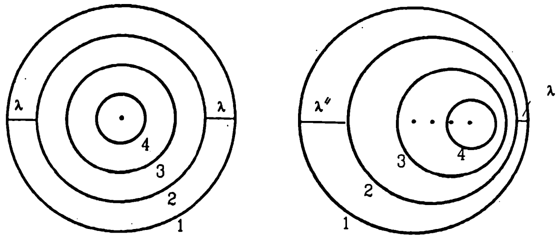

Figure 16.20

(Figure 16.20) shows qualitatively the change in wavelength as the source moves through the medium. The circles represent the region of space where the pressure is maximum. They are wavefronts separated by a wavelength. We can say that these wavefronts representing the pressure-maxima originate from the source and spread in all directions with a speed v.

Labels 1, 2, 3, 4 show wavefronts emitted successively at regular interval $T = 1/\nu$ from the source. Each wavefront will have its centre at the position where the source was situated while emitting the wavefront. The radius of a preceding wavefront will exceed the next one by an amount $\lambda = vt$.

When the source stays stationary, all the wavefronts are originated from the same position of the source and the separation between the successive wavefronts is equal to the difference in their radii. However, when the source moves, the centres of the wavefronts differ in position, so that the spacing between the successive wavefronts decreases along the direction of motion and increases on the opposite side. This separation being the new wavelength, the wavelength changes due to the motion of the source.

16.14 SONIC BOOMS

In the discussion of the Doppler effect, we considered only subsonic velocities for the source and the observer, that is, $u_s < v$ and $u_o < v$. What happens when the source moves through the medium at a speed u_s greater than the wave speed v? A supersonic plane travels in air with a speed greater than the speed of sound in air. It sends a cracking sound called sonic boom which can break glass dishes, window panes and even cause damage to buildings. Let us extend figure (16.20) for the case where a tiny source moves through air with a speed $u_s > v$. The wavefronts are drawn for the pressure maxima. The spherical wavefronts intersect over the surface of a cone with the apex at the source. Because of constructive interference of a large number of waves arriving at the same instant on the surface of the cone, pressure waves of very large amplitude are sent with the conical wavefront. Such waves are one variety of shock waves.

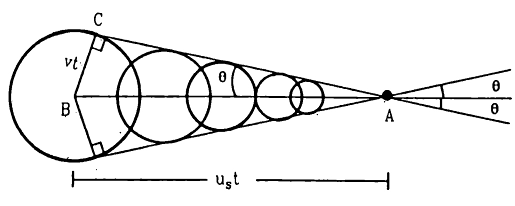

Figure 16.21

From the triangle ABC, the semivertical angle θ of the cone is given by

$$\sin\theta = \frac{vt}{u_s t} = \frac{v}{u_s}.$$

The ratio $\frac{u_s}{v}$ is called the "Mach Number".

As the tiny source moves, it drags the cone with it. When an observer on ground is intercepted by the cone surface, the boom is heard. There is a common misconception that the boom is produced at the instant the speed of the plane crosses the speed of sound and once it achieves the supersonic speed it sends no further shock wave. The sonic boom is not a one time affair that occurs when the speed just exceeds the speed of sound. As long as the plane moves with a supersonic speed, it continues to send the boom.

16.15 MUSICAL SCALE

A musical scale is a sequence of frequencies which have a particularly pleasing effect on the human ear. A widely used musical scale, called diatonic scale, has eight frequencies covering an octave. Each frequency is called a *note.* Table (16.3) gives these frequencies together with their Indian and Western names with the lowest of the octave at 256 Hz.

Table 16.3

Symbol	Indian name	Western name	Frequency Hz
C	Sa	Do	256
D	Re	Re	288
E	Ga	Mi	320
F	Ma	Fa	$341\frac{1}{3}$
G	Pa	Sol	384
A	Dha	La	$426\frac{2}{3}$
B	Ni	Ti	480
C_1	Sa	Do	512

16.16 ACOUSTICS OF BUILDINGS

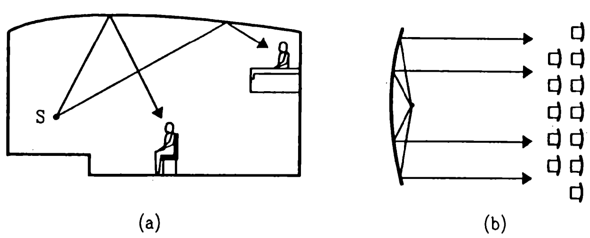

Figure 16.22

While designing an auditorium for speech or musical concerts, one has to take proper care for the absorption and reflection of sound. If these factors are poorly considered, a listener in the auditorium will not be able to clearly hear the sound. To have the intensity of sound almost uniform in the hall, the walls and the ceilings may be curved in proper fashion. Figure (16.22a) shows a curved ceiling used to make sound audible uniformly at the balcony seats and the seats on the floor. Figure (16.22b) shows the use of a parabolic wall to make sound uniform across the width of the hall. Reflection of sound is helpful in maintaining a good loudness level throughout the hall. However, it also has several unwanted effects. Sound can reach a listener directly from the source as well as after reflection from a wall or the ceiling. This leads to *echo* which is heard after an interval of hearing the first sound. This echo interferes with the next sound signal affecting the clarity.

Another effect of multiple reflection is the *reverberation.* A listener hears the direct sound, sound coming after one reflection, after two reflections and so on. The time interval between the successive arrival of the same sound signal keeps on decreasing. Also, the intensity of the signal gradually decreases.

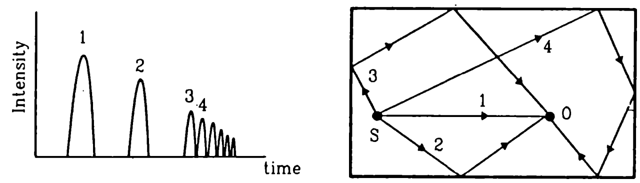

Figure 16.23

Figure (16.23) shows **a typical** situation. After sometime the signals coming from multiple reflections are so close that they form an almost continuous sound of decreasing intensity. This part of the sound is called *reverberant sound.* The time for which the reverberation persists is a major factor in the acoustics of halls. Quantitatively, the time taken by the reverberant sound to decrease its intensity by a factor of 10^6 is called the *reverberation time.* If the reverberation time is too large, it disturbs the listener. A sound signal from the source may not be clearly heard due to the presence of reverberation of the previous signal. There are certain materials which absorb sound very effectively. Reverberation may be decreased by fixing such materials on the walls, ceiling, floor, furnitures etc. However, this process decreases the overall intensity level in the hall. Also in musical concerts, some amount of reverberation adds to the quality of music. An auditorium with a very small reverberation time is called *acoustically dead.* Thus, one has to make a compromise. The reverberation time should not be very large, otherwise unpleasant echos will seriously affect clarity. On the other hand it should not be too small, otherwise intesity and quality will be seriously affected.

Electrical amplifying systems are often used in large auditorium. If a loudspeaker is kept near the back or at the side walls, a listener may hear the sound from the speaker earlier than the sound from the stage. Loudspeakers are, therefore, placed with proper inclinations and electronic delays are installed so that sound from the stage and from the loudspeaker reach a listener almost simultaneously. Another problem with electrical amplifying systems is *ringing*. The amplifying system may pick up sound from the loudspeaker to again amplify it. This gives a very unpleasant whistling sound.

One also has to avoid any noise coming from outside the auditorium or from different equipment inside the auditorium. Sound of fans, exhausts, airconditioners etc. often create annoyance to the listener.

Worked Out Examples

1. *An ultrasound signal of frequency* 50 kHz *is sent vertically into sea water. The signal gets reflected from the ocean bed and returns to the surface* 0·80 s *after it was emitted. The speed of sound in sea water is* 1500 m/s. *(a) Find the depth of the sea. (b) What is the wavelength of this signal in water?*

Solution : (a) Let the depth of the sea be d. The total distance travelled by the signal is $2d$. By the question,

$$2d = (1500 \text{ m/s})(0{\cdot}8 \text{ s}) = 1200 \text{ m}$$

or, $$d = 600 \text{ m}.$$

(b) Using the equation $v = \nu\lambda$,

$$\lambda = \frac{v}{\nu} = \frac{1500 \text{ m/s}}{50 \times 10^{3} \text{ s}^{-1}} = 3{\cdot}0 \text{ cm}.$$

2. *An aeroplane is going towards east at a speed of* 510 km/h *at a height of* 2000 m. *At a certain instant, the sound of the plane heard by a ground observer appears to come from a point vertically above him. Where is the plane at this instant? Speed of sound in air* = 340 m/s.

Solution :

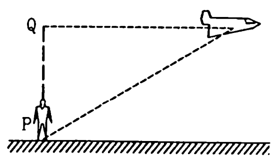

Figure 16-W1

The sound reaching the ground observer P, was emitted by the plane when it was at the point Q vertically above his head. The time taken by the sound to reach the observer is

$$t = \frac{2000 \text{ m}}{340 \text{ m/s}} = \frac{100}{17} \text{ s}.$$

The distance moved by the plane during this period is

$$d = (510 \text{ km/h})\left(\frac{100}{17} \text{ s}\right)$$

$$= \frac{30 \times 10^{5}}{3600} \text{ m} = 833 \text{ m}.$$

Thus, the plane will be 833 m ahead of the observer on its line of motion when he hears the sound coming vertically to him.

3. *The equation of a sound wave in air is given by*

$$p = (0{\cdot}01 \text{ N/m}^{2}) \sin[(1000 \text{ s}^{-1})\, t - (3{\cdot}0 \text{ m}^{-1})\, x]$$

(a) Find the frequency, wavelength and the speed of sound wave in air. (b) If the equilibrium pressure of air is $1{\cdot}0 \times 10^{5}$ N/m², *what are the maximum and minimum pressures at a point as the wave passes through that point?*

Solution : (a) Comparing with the standard form of a travelling wave

$$p = p_0 \sin[\omega(t - x/v)]$$

we see that $\omega = 1000 \text{ s}^{-1}$. The frequency is

$$\nu = \frac{\omega}{2\pi} = \frac{1000}{2\pi} \text{ Hz} = 160 \text{ Hz}.$$

Also from the same comparison, $\omega/v = 3{\cdot}0 \text{ m}^{-1}$

or, $$v = \frac{\omega}{3{\cdot}0 \text{ m}^{-1}} = \frac{1000 \text{ s}^{-1}}{3{\cdot}0 \text{ m}^{-1}}$$

$$\approx 330 \text{ m/s}.$$

The wavelength is $\lambda = \frac{v}{\nu} = \frac{330 \text{ m/s}}{160 \text{ Hz}} = 2{\cdot}1 \text{ m}.$

(b) The pressure amplitude is $p_0 = 0{\cdot}01 \text{ N/m}^{2}$. Hence, the maximum and minimum pressures at a point in the wave motion will be $(1{\cdot}01 \times 10^{5} \pm 0{\cdot}01) \text{ N/m}^{2}$.

4. *A sound wave of frequency* 10 kHz *is travelling in air with a speed of* 340 m/s. *Find the minimum separation between two points where the phase difference is* 60°.

Solution : The wavelength of the wave is

$$\lambda = \frac{v}{\nu} = \frac{340 \text{ m/s}}{10 \times 10^{3} \text{ s}^{-1}} = 3{\cdot}4 \text{ cm}.$$

The wave number is $k = \frac{2\pi}{\lambda} = \frac{2\pi}{3{\cdot}4} \text{ cm}^{-1}$.

The phase of the wave is $(kx - \omega t)$. At any given instant, the phase difference between two points at a separation d is kd. If this phase difference is 60° i.e., $\pi/3$ radian;

$$\frac{\pi}{3} = \left(\frac{2\pi}{3{\cdot}4} \text{ cm}^{-1}\right) d \text{ or } d = \frac{3{\cdot}4}{6} \text{ cm} = 0{\cdot}57 \text{ cm}.$$

5. *On a winter day sound travels* 336 metres *in one second. Find the atmospheric temperature. Speed of sound at* 0°C = 332 m/s.

Solution : The speed of sound is proportional to the square root of the absolute temperature.

The speed of sound at 0°C or 273 K is 332 m/s. If the atmospheric temperature is T,

$$\frac{336\ \text{m/s}}{332\ \text{m/s}} = \sqrt{\frac{T}{273\ \text{K}}}$$

or, $$T = \left(\frac{336}{332}\right)^2 \times 273\ \text{K} = 280\ \text{K}$$

or, $$t = 7°\text{C}.$$

6. *The constant* γ *for oxygen as well as for hydrogen is* 1·40. *If the speed of sound in oxygen is* 470 m/s, *what will be the speed in hydrogen at the same temperature and pressure?*

Solution : The speed of sound in a gas is given by $v = \sqrt{\frac{\gamma P}{\rho}}$. At STP, 22·4 litres of oxygen has a mass of 32 g whereas the same volume of hydrogen has a mass of 2 g. Thus, the density of oxygen is 16 times the density of hydrogen at the same temperature and pressure. As γ is same for both the gases,

$$\frac{v\ (\text{hydrogen})}{v\ (\text{oxygen})} = \sqrt{\frac{\rho(\text{oxygen})}{\rho\ (\text{hydrogen})}}$$

or, $$v(\text{hydrogen}) = 4\ v\ (\text{oxygen})$$

$$= 4 \times 470\ \text{m/s} = 1880\ \text{m/s}.$$

7. *A microphone of cross-sectional area* 0·80 cm^2 *is placed in front of a small speaker emitting* 3·0 W *of sound output. If the distance between the microphone and the speaker is* 2·0 m, *how much energy falls on the microphone in* 5·0 s?

Solution : The energy emitted by the speaker in one second is 3·0 J. Let us consider a sphere of radius 2·0 m centred at the speaker. The energy 3·0 J falls normally on the total surface of this sphere in one second. The energy falling on the area 0·8 cm^2 of the microphone in one second

$$= \frac{0{\cdot}8\ \text{cm}^2}{4\pi\ (2{\cdot}0\ \text{m})^2} \times 3{\cdot}0\ \text{J} = 4{\cdot}8 \times 10^{-6}\ \text{J}.$$

The energy falling on the microphone in 5·0 s is

$$4{\cdot}8 \times 10^{-6}\ \text{J} \times 5 = 24\ \mu\text{J}.$$

8. *Find the amplitude of vibration of the particles of air through which a sound wave of intensity* $2{\cdot}0 \times 10^{-6}$ W/m^2 *and frequency* 1·0 kHz *is passing. Density of air* = 1·2 kg/m^3 *and speed of sound in air* = 330 m/s.

Solution : The relation between the intensity of sound and the displacement amplitude is

$$I = 2\pi^2 s_0^2 \nu^2 \rho_0 v$$

or, $$s_0^2 = \frac{I}{2\pi^2 \nu^2 \rho_0 v}$$

$$= \frac{2{\cdot}0 \times 10^{-6}\ \text{W/m}^2}{2\pi^2 \times (1{\cdot}0 \times 10^6\ \text{s}^{-2}) \times (1{\cdot}2\ \text{kg/m}^3) \times (330\ \text{m/s})}$$

$$= 2{\cdot}53 \times 10^{-16}\ \text{m}^2$$

or, $$s_0 = 1{\cdot}6 \times 10^{-8}\ \text{m}.$$

9. *The sound level at a point is increased by* 30 dB. *By what factor is the pressure amplitude increased?*

Solution : The sound level in dB is

$$\beta = 10 \log_{10}\left(\frac{I}{I_0}\right).$$

If β_1 and β_2 are the sound levels and I_1 and I_2 are the intensities in the two cases,

$$\beta_2 - \beta_1 = 10\left[\log_{10}\left(\frac{I_2}{I_0}\right) - \log_{10}\left(\frac{I_1}{I_0}\right)\right]$$

or, $$30 = 10 \log_{10}\left(\frac{I_2}{I_1}\right)$$

or, $$\frac{I_2}{I_1} = 10^3.$$

As the intensity is proportional to the square of the pressure amplitude, we have $\frac{p_2}{p_1} = \sqrt{\frac{I_2}{I_1}} = \sqrt{1000} \approx 32.$

10. *Figure (16-W2) shows a tube structure in which a sound signal is sent from one end and is received at the other end. The semicircular part has a radius of* 20·0 cm. *The frequency of the sound source can be varied electronically between* 1000 *and* 4000 Hz. *Find the frequencies at which maxima of intensity are detected. The speed of sound in air* = 340 m/s.

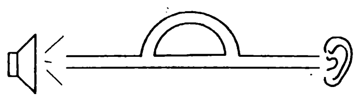

Figure 16-W2

Solution : The sound wave bifurcates at the junction of the straight and the semicircular parts. The wave through the straight part travels a distance $l_1 = 2 \times 20$ cm and the wave through the curved part travels a distance $l_2 = \pi\ 20$ cm = 62·8 cm before they meet again and travel to the receiver. The path difference between the two waves received is, therefore,

$$\Delta l = l_2 - l_1 = 62{\cdot}8\ \text{cm} - 40\ \text{cm} = 22{\cdot}8\ \text{cm} = 0{\cdot}228\ \text{m}.$$

The wavelength of either wave is $\frac{v}{\nu} = \frac{340 \text{ m/s}}{\nu}$. For constructive interference, $\Delta l = n\lambda$, where n is an integer.

or, $$0{\cdot}228 \text{ m} = n \frac{340 \text{ m/s}}{\nu}$$

or, $$\nu = n \frac{340 \text{ m/s}}{0{\cdot}228 \text{ m}} = n\, 1491{\cdot}2 \text{ Hz} \approx n\, 1490 \text{ Hz}.$$

Thus, the frequencies within the specified range which cause maxima of intensity are 1490 Hz and 2980 Hz.

11. *A source emitting sound of frequency* 180 Hz *is placed in front of a wall at a distance of* 2 m *from it. A detector is also placed in front of the wall at the same distance from it. Find the minimum distance between the source and the detector for which the detector detects a maximum of sound. Speed of sound in air* = 360 m/s.

Solution :

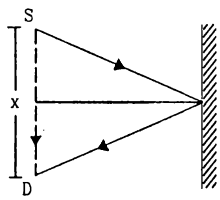

Figure 16-W3

The situation is shown in figure (16-W3). Suppose the detector is placed at a distance of x meter from the source. The direct wave received from the source travels a distance of x meter. The wave reaching the detector after reflection from the wall has travelled a distance of $2[(2)^2 + x^2/4]^{1/2}$ meter. The path difference between the two waves is

$$\Delta = \left\{ 2\left[(2)^2 + \frac{x^2}{4} \right]^{1/2} - x \right\} \text{ meter.}$$

Constructive interference will take place when $\Delta = \lambda, 2\lambda, \dots$ The minimum distance x for a maximum corresponds to

$$\Delta = \lambda. \qquad \text{... (i)}$$

The wavelength is $\lambda = \frac{v}{\nu} = \frac{360 \text{ m/s}}{180 \text{ s}^{-1}} = 2 \text{ m}.$

Thus, by (i), $$2\left[(2)^2 + \frac{x^2}{4} \right]^{1/2} - x = 2$$

or, $$\left[4 + \frac{x^2}{4} \right]^{1/2} = 1 + \frac{x}{2}$$

or, $$4 + \frac{x^2}{4} = 1 + \frac{x^2}{4} + x$$

or, $$3 = x.$$

Thus, the detector should be placed at a distance of 3 m from the source. Note that there is no abrupt phase-change.

12. *A tuning fork vibrates at* 264 Hz. *Find the length of the shortest closed organ pipe that will resonate with the tuning fork. Speed of sound in air is* 350 m/s.

Solution : The resonant frequency of a closed organ pipe of length l is $\frac{nv}{4l}$, where n is a positive odd integer and v is the speed of sound in air. To resonate with the given tuning fork,

$$\frac{nv}{4l} = 264 \text{ s}^{-1}$$

or, $$l = \frac{n \times 350 \text{ m/s}}{4 \times 264 \text{ s}^{-1}}.$$

For l to be minimum, $n = 1$ so that

$$l_{min} = \frac{350}{4 \times 264} \text{ m} = 33 \text{ cm}.$$

13. *The fundamental frequency of a closed organ pipe is equal to the first overtone frequency of an open organ pipe. If the length of the open pipe is* 60 cm, *what is the length of the closed pipe ?*

Solution : The fundamental frequency of a closed organ pipe is $\frac{v}{4l_1}$. For an open pipe, the fundamental frequency is $\frac{v}{2l_2}$ and the first overtone is $\frac{2v}{2l_2} = \frac{v}{l_2}$. Here l_1 is the length of the closed pipe and $l_2 = 60$ cm is the length of the open pipe, We have,

$$\frac{v}{4l_1} = \frac{v}{60 \text{ cm}}$$

$$l_1 = \frac{1}{4} \times 60 \text{ cm} = 15 \text{ cm}.$$

14. *A tuning fork vibrating at frequency* 800 Hz *produces resonance in a resonance column tube. The upper end is open and the lower end is closed by the water surface which can be varied. Successive resonances are observed at lengths* 9·75 cm, 31·25 cm *and* 52·75 cm. *Calculate the speed of sound in air from these data.*

Solution : For the tube open at one end, the resonance frequencies are $\frac{nv}{4l}$, where n is a positive odd integer. If the tuning fork has a frequency ν and l_1, l_2, l_3 are the successive lengths of the tube in resonance with it, we have

$$\frac{nv}{4\,l_1} = \nu$$

$$\frac{(n+2)\,v}{4\,l_2} = \nu$$

$$\frac{(n+4)\,v}{4\,l_3} = \nu$$

giving $$l_3 - l_2 = l_2 - l_1 = \frac{2v}{4\nu} = \frac{v}{2\nu}.$$

By the question, $l_3 - l_2 = (52{\cdot}75 - 31{\cdot}25)$ cm = $21{\cdot}50$ cm

and $l_2 - l_1 = (31{\cdot}25 - 9{\cdot}75)$ cm = $21{\cdot}50$ cm.

Thus, $$\frac{v}{2\nu} = 21{\cdot}50 \text{ cm}$$

or, $v = 2\nu \times 21{\cdot}50 \text{ cm} = 2 \times 800 \text{ s}^{-1} \times 21{\cdot}50 \text{ cm} = 344 \text{ m/s}.$

15. *A certain organ pipe resonates in its fundamental mode at a frequency of* 500 Hz *in air. What will be the fundamental frequency if the air is replaced by hydrogen at the same temperature? The density of air is* $1{\cdot}20$ kg/m^3 *and that of hydrogen is* $0{\cdot}089$ kg/m^3.

Solution : Suppose the speed of sound in hydrogen is v_h and that in air is v_a. The fundamental frequency of an organ pipe is proportional to the speed of sound in the gas contained in it. If the fundamental frequency with hydrogen in the tube is ν, we have

$$\frac{\nu}{500 \text{ Hz}} = \frac{v_h}{v_a} = \sqrt{\frac{\rho_a}{\rho_h}} = \sqrt{\frac{1{\cdot}2}{0{\cdot}089}} = 3{\cdot}67$$

or, $$\nu = 3{\cdot}67 \times 500 \text{ Hz} \approx 1840 \text{ Hz}.$$

16. *An aluminium rod having a length of* $90{\cdot}0$ cm *is clamped at its middle point and is set into longitudinal vibrations by stroking it with a rosined cloth. Assume that the rod vibrates in its fundamental mode of vibration. The density of aluminium is* 2600 kg/m^3 *and its Young's modulus is* $7{\cdot}80 \times 10^{10}$ N/m^2. *Find (a) the speed of sound in aluminium, (b) the wavelength of sound waves produced in the rod, (c) the frequency of the sound produced and (d) the wavelength of the sound produced in air. Take the speed of sound in air to be* 340 m/s.

Solution : (a) The speed of sound in aluminium is

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{7{\cdot}80 \times 10^{10} \text{ N/m}^2}{2600 \text{ kg/m}^3}}$$
$$= 5480 \text{ m/s}.$$

(b) Since the rod is clamped at the middle, the middle point is a pressure antinode. The free ends of the rod are the nodes. As the rod vibrates in its fundamental mode, there are no other nodes or antinodes. The length of the rod, which is also the distance between the successive nodes, is, therefore, equal to half the wavelength. Thus, the wavelength of sound in the aluminium rod is

$$\lambda = 2\,l = 180 \text{ cm}.$$

(c) The frequency of the sound produced which is also equal to the frequency of vibration of the rod is

$$\nu = \frac{v}{\lambda} = \frac{5480 \text{ m/s}}{180 \text{ cm}} = 3050 \text{ Hz}.$$

(d) The wavelength of sound in air is

$$\lambda = \frac{v}{\nu} = \frac{340 \text{ m/s}}{3050 \text{ Hz}} = 11{\cdot}1 \text{ cm}.$$

17. *The string of a violin emits a note of* 440 Hz *at its correct tension. The string is bit taut and produces* 4 *beats per second with a tuning fork of frequency* 440 Hz. *Find the frequency of the note emitted by this taut string.*

Solution : The frequency of vibration of a string increases with increase in the tension. Thus, the note emitted by the string will be a little more than 440 Hz. As it produces 4 beats per second with the 440 Hz tuning fork, the frequency will be 444 Hz.

18. *A siren is fitted on a car going towards a vertical wall at a speed of* 36 km/h. *A person standing on the ground, behind the car, listens to the siren sound coming directly from the source as well as that coming after reflection from the wall. Calculate the apparent frequency of the wave (a) coming directly from the siren to the person and (b) coming after reflection. Take the speed of sound to be* 340 m/s.

Solution :

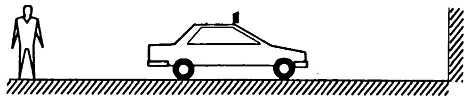

Figure 16-W4

The speed of the car is 36 km/h = 10 m/s.

(a) Here the observer is at rest with respect to the medium and the source is going away from the observer. The apparent frequency heard by the observer is, therefore,

$$\nu' = \frac{v}{v + u_s}\,\nu$$
$$= \frac{340}{340 + 10} \times 500 \text{ Hz} = 486 \text{ Hz}.$$

(b) The frequency received by the wall is

$$\nu'' = \frac{v}{v - u_s}\,\nu = \frac{340}{340 - 10} \times 500 \text{ Hz} = 515 \text{ Hz}.$$

The wall reflects this sound without changing the frequency. Thus, the frequency of the reflected wave as heard by the ground observer is 515 Hz.

19. *Two trains are moving towards each other at speeds of* 72 km/h *and* 54 km/h *relative to the ground. The first train sounds a whistle of frequency* 600 Hz. *Find the frequency of the whistle as heard by a passenger in the second train (a) before the trains meet and (b) after the trains have crossed each other. The speed of sound in air is* 340 m/s.

Solution : The speed of the first train = 72 km/h = 20 m/s and that of the second = 54 km/h = 15 m/s.

(a) Here both the source and the observer move with respect to the medium. Before the trains meet, the source is going towards the observer and the observer is also going towards the source. The apparent frequency heard by the observer will be

$$\nu' = \frac{v + u_o}{v - u_s}\nu$$

$$= \frac{340 + 15}{340 - 20} \times 600\ \text{Hz} = 666\ \text{Hz}.$$

(b) After the trains have crossed each other, the source goes away from the observer and the observer goes away from the source. The frequency heard by the observer is, therefore,

$$\nu'' = \frac{v - u_o}{v + u_s}\nu$$

$$= \frac{340 - 15}{340 + 20} \times 600\ \text{Hz} = 542\ \text{Hz}.$$

20. *A person going away from a factory on his scooter at a speed of* 36 km/h *listens to the siren of the factory. If the main frequency of the siren is* 600 Hz *and a wind is blowing along the direction of the scooter at* 36 km/h, *find the main frequency as heard by the person.*

Solution : The speed of sound in still air is 340 m/s. Let us work from the frame of reference of the air. As both the observer and the wind are moving at the same speed along the same direction with respect to the ground, the observer is at rest with respect to the medium. The source (the siren) is moving with respect to the wind at a speed of 36 km/h i.e., 10 m/s. As the source is going away from the observer who is at rest with respect to the medium, the frequency heard is

$$\nu' = \frac{v}{v + u_s}\nu = \frac{340}{340 + 10} \times 600\ \text{Hz} = 583\ \text{Hz}.$$

21. *A source and a detector move away from each other, each with a speed of* 10 m/s *with respect to the ground with no wind. If the detector detects a frequency* 1950 Hz *of the sound coming from the source, what is the original frequency of the source? Speed of sound in air* = 340 m/s.

Solution : If the original frequency of the source is ν, the apparent frequency heard by the observer is

$$\nu' = \frac{v - u_o}{v + u_s}\nu,$$

where u_o is the speed of the observer going away from the source and u_s is the speed of the source going away from the observer. By the question,

$$1950\ \text{Hz} = \frac{340 - 10}{340 + 10}\nu$$

or, $$\nu = \frac{35}{33} \times 1950\ \text{Hz} = 2070\ \text{Hz}.$$

22. *The driver of a car approaching a vertical wall notices that the frequency of his car's horn changes from* 440 Hz *to* 480 Hz *when it gets reflected from the wall. Find the speed of the car if that of the sound is* 330 m/s.

Solution :

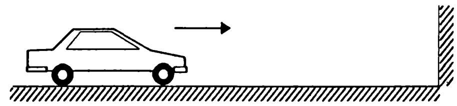

Figure 16-W5

Suppose, the car is going towards the wall at a speed u. The wall is stationary with respect to the air and the horn is going towards it. If the frequency of the horn is ν, that received by the wall is

$$\nu' = \frac{v}{v - u}\nu.$$

The wall reflects this sound without changing the frequency. Thus, the wall becomes the source of frequency ν' and the car-driver is the listener. The wall (which acts as the source now) is at rest with respect to the air and the car (which is the observer now) is going towards the wall at speed u. The frequency heard by the car-driver for this reflected wave is, therefore,

$$\nu'' = \frac{v + u}{v}\nu'$$

$$= \frac{v + u}{v} \cdot \frac{v}{v - u}\nu$$

$$= \frac{v + u}{v - u}\nu.$$

Putting the values,

$$480 = \frac{v + u}{v - u}440$$

or, $$\frac{v + u}{v - u} = \frac{48}{44}$$

or, $$\frac{u}{v} = \frac{4}{92}$$

or, $$u = \frac{4}{92} \times 330\ \text{m/s} = 14{\cdot}3\ \text{m/s} \approx 52\ \text{km/h}.$$

23. *A train approaching a railway crossing at a speed of* 120 km/h *sounds a short whistle at frequency* 640 Hz *when it is* 300 m *away from the crossing. The speed of sound in air is* 340 m/s. *What will be the frequency heard by a person standing on a road perpendicular to the track through the crossing at a distance of* 400 m *from the crossing?*

Solution :

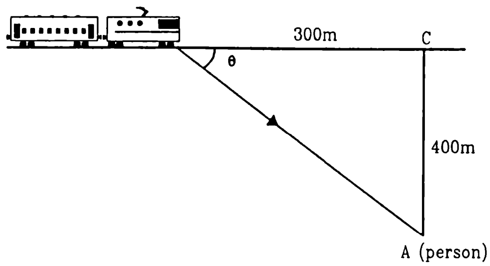

Figure 16-W6