



Exercise 7A

Question 5

$$\begin{aligned} \text{(i) LHS} &= 1 + \frac{\cot^2 \theta}{(1 + \operatorname{cosec} \theta)} \\ &= 1 + \frac{\operatorname{cosec}^2 \theta - 1}{(1 + \operatorname{cosec} \theta)} \\ &= \frac{1 + \operatorname{cosec} \theta + \operatorname{cosec}^2 \theta - 1^2}{(1 + \operatorname{cosec} \theta)} \\ &= \frac{(1 + \operatorname{cosec} \theta) + (\operatorname{cosec} \theta + 1)(\operatorname{cosec} \theta - 1)}{(1 + \operatorname{cosec} \theta)} \\ &= \frac{(1 + \operatorname{cosec} \theta)[1 + (\operatorname{cosec} \theta - 1)]}{(1 + \operatorname{cosec} \theta)} \\ &= 1 + \operatorname{cosec} \theta - 1 \\ &= \operatorname{cosec} \theta = \text{RHS} \end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

$$\begin{aligned} \text{(ii) LHS} &= 1 + \frac{\tan^2 \theta}{(1 + \sec \theta)} = \frac{1 + \sec \theta + \tan^2 \theta}{(1 + \sec \theta)} \\ &= \frac{\sec^2 \theta + \sec \theta}{(1 + \sec \theta)} \quad \left[\because (1 + \tan^2 \theta) = \sec^2 \theta \right] \\ &= \frac{\sec \theta (1 + \sec \theta)}{(1 + \sec \theta)} = \sec \theta = \text{RHS} \end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

Question 6

$$\begin{aligned}
 \text{(i) LHS} &= \sec \theta (1 - \sin \theta) (\sec \theta + \tan \theta) \\
 &= \left[\sec \theta - \frac{\sin \theta}{\cos \theta} \right] \times (\sec \theta + \tan \theta) \\
 &= (\sec \theta - \tan \theta) (\sec \theta + \tan \theta) \\
 &= (\sec^2 \theta - \tan^2 \theta) = 1 = \text{RHS}
 \end{aligned}$$

$\therefore \text{LHS} = \text{RHS}$

$$\begin{aligned}
 \text{(ii) LHS} &= \sin \theta (1 + \tan \theta) + \cos \theta (1 + \cot \theta) \\
 &= \left(\sin \theta + \frac{\sin^2 \theta}{\cos \theta} \right) + \cos \theta \left(1 + \frac{\cos \theta}{\sin \theta} \right) \\
 &= \left(\frac{\sin \theta \cos \theta + \sin^2 \theta}{\cos \theta} \right) + \left(\cos \theta + \frac{\cos^2 \theta}{\sin \theta} \right) \\
 &= \left(\frac{\sin \theta \cos \theta + \sin^2 \theta}{\cos \theta} \right) + \left(\frac{\cos \theta \sin \theta + \cos^2 \theta}{\sin \theta} \right) \\
 &= \frac{\sin \theta \cos \theta}{\cos \theta} + \frac{\sin^2 \theta}{\cos \theta} + \frac{\cos \theta \sin \theta}{\sin \theta} + \frac{\cos^2 \theta}{\sin \theta} \\
 &= \frac{\sin \theta \cos \theta}{\cos \theta} + \frac{\cos \theta \sin \theta}{\sin \theta} + \frac{\sin^2 \theta}{\cos \theta} + \frac{\cos^2 \theta}{\sin \theta} \\
 &= \sin \theta \cos \theta \left(\frac{\sin \theta + \cos \theta}{\cos \theta \sin \theta} \right) + \frac{\sin^3 \theta + \cos^3 \theta}{\cos \theta \sin \theta} \\
 &= \sin \theta \cos \theta \left(\frac{\sin \theta + \cos \theta}{\cos \theta \sin \theta} \right) \\
 &\quad + \frac{(\sin \theta + \cos \theta)(\sin^2 \theta - \sin \theta \cos \theta + \cos^2 \theta)}{\cos \theta \sin \theta} \\
 &\quad \left[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2) \right] \\
 &= (\sin \theta + \cos \theta) \left[\frac{\sin \theta \cos \theta}{\cos \theta \sin \theta} + \frac{(1 - \sin \theta \cos \theta)}{\cos \theta \sin \theta} \right] \\
 &= (\sin \theta + \cos \theta) \left[\frac{\sin \cos \theta + 1 - \sin \theta \cos \theta}{\cos \theta \sin \theta} \right] \\
 &= (\sin \theta + \cos \theta) \left(\frac{1}{\cos \theta \sin \theta} \right) \\
 &= \frac{\sin}{\cos \theta \sin \theta} + \frac{\cos \theta}{\cos \theta \sin \theta} = \sec \theta + \csc \theta = \text{RHS}
 \end{aligned}$$

$\therefore \text{LHS} = \text{RHS}$

***** END *****