

LAWS OF THERMODYNAMICS

26.1 THE FIRST LAW OF THERMODYNAMICS

We have seen that heat is just a form of energy. A system can be given energy either by supplying heat to it (by placing it in contact with a hotter object) or by doing mechanical work on it. Consider an ideal gas in a cylindrical container fitted with a piston (figure 26.1). Suppose the piston is fixed in its position and the walls of the cylinder are kept at a temperature higher than that of the gas. The gas molecules strike the wall and rebound. The average kinetic energy of a wall molecule is larger than the average kinetic energy of a gas molecule. Thus, on collision, the gas molecules receive energy from the wall molecules. This increased kinetic energy is shared by other molecules of the gas and in this way the total internal energy of the gas increases.

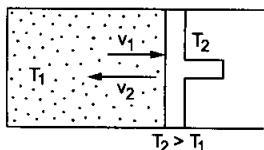


Figure 26.1

Next, consider the same initial situation but now the walls are at the same temperature as the gas. Suppose the piston is pushed slowly to compress the gas. As a gas molecule collides with the piston coming towards it, the speed of the molecule increases on collision (assuming elastic collision, $v_2 = v_1 + 2u$ in figure 26.2). This way the internal energy of the molecules increases as the piston is pushed in.

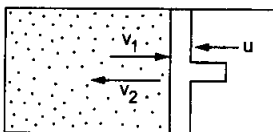


Figure 26.2

We see that the total internal energy of the gas may be increased because of the temperature

difference between the walls and the gas (heat transfer) or because of the motion of the piston (work done on the gas).

In a general situation both modes of energy transfer may go together. As an example, consider a gas kept in a cylindrical can fitted with a movable piston. If the can is put on a hot stove, heat is supplied by the hot bottom to the gas and the piston is pushed out to some distance. As the piston moves out, work is done by the gas on it and the gas loses this much amount of energy. Thus, the gas gains energy as heat is supplied to it and it loses energy as work is done by it.

Suppose, in a process, an amount ΔQ of heat is given to the gas and an amount ΔW of work is done by it. The total energy of the gas must increase by $\Delta Q - \Delta W$. As a result, the entire gas together with its container may start moving (systematic motion) or the internal energy (random motion of the molecules) of the gas may increase. If the energy does not appear as a systematic motion of the gas then this net energy $\Delta Q - \Delta W$ must go in the form of its internal energy. If we denote the change in internal energy by ΔU , we get

$$\Delta U = \Delta Q - \Delta W$$

or,

$$\Delta Q = \Delta U + \Delta W. \quad \dots (26.1)$$

Equation (26.1) is the statement of the *first law of thermodynamics*. In an ideal monatomic gas, the internal energy of the gas is simply translational kinetic energy of all its molecules. In general, the internal energy may get contributions from the vibrational kinetic energy of molecules, rotational kinetic energy of molecules as well as from the potential energy corresponding to the molecular forces. Equation (26.1) represents a statement of conservation of energy and is applicable to any system, however complicated.

Example 26.1

A gas is contained in a vessel fitted with a movable piston. The container is placed on a hot stove. A total of

100 cal of heat is given to the gas and the gas does 40 J of work in the expansion resulting from heating. Calculate the increase in internal energy in the process.

Solution : Heat given to the gas is $\Delta Q = 100 \text{ cal} = 418 \text{ J}$.

Work done by the gas is $\Delta W = 40 \text{ J}$.

The increase in internal energy is

$$\begin{aligned}\Delta U &= \Delta Q - \Delta W \\ &= 418 \text{ J} - 40 \text{ J} = 378 \text{ J}.\end{aligned}$$

First law of thermodynamics may be viewed from different angles. Equation (26.1) tells us that if we take a system from an initial state i to a final state f by several different processes, $\Delta Q - \Delta W$ should be identical in all the processes. This is because $\Delta Q - \Delta W = \Delta U = U_f - U_i$ depends only on the end states i and f . Both ΔQ and ΔW may be different in different processes, but $\Delta Q - \Delta W$ is the same for all the processes taking the system from i to f . Thus, we do not write $\Delta Q = Q_f - Q_i$ or we do not write $\Delta W = W_f - W_i$, but we do write $\Delta U = U_f - U_i$. The first law may be taken as a statement that there exists an internal energy function U that has a fixed value in a given state.

It should be remembered that when work is done by the system, ΔW is positive. If work is done on the system, ΔW is negative. When heat is given to the system, ΔQ is positive. If heat is given by the system, ΔQ is negative. A positive ΔW decreases the internal energy and a positive ΔQ increases it.

26.2 WORK DONE BY A GAS

Consider a gas contained in a cylinder of cross-sectional area A fitted with a movable piston. Let the pressure of the gas be p . The force exerted by the gas on the piston is pA in outward direction. Suppose the gas expands a little and the piston is pushed out by a small distance Δx . The work done by the gas on the piston is

$$\Delta W = (pA) (\Delta x) = p \Delta V,$$

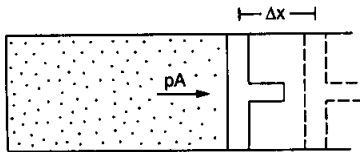


Figure 26.3

where $\Delta V = A \Delta x$ is the change in the volume of the gas. For a finite change of volume from V_1 to V_2 , the pressure may not be constant. We can divide the whole process of expansion in small steps and add the work done in each step. Thus, the total work done by the gas in the process is

$$W = \int_{V_1}^{V_2} p dV. \quad \dots (26.2)$$

If we show the process in a p - V diagram, the work done is equal to the area bounded by the p - V curve, the V -axis and the ordinates $V = V_1$ and $V = V_2$.

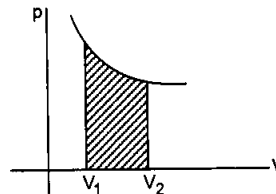


Figure 26.4

Equation (26.2) is derived for a cylindrical vessel only for mathematical simplicity. It is equally true for any shape of the vessel. It is also true for the expansion of solids and liquids or even in phase-changes.

Example 26.2

Calculate the work done by a gas as it is taken from the state a to b , b to c and c to a as shown in figure (26.5).

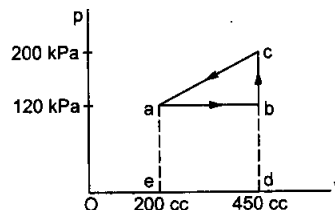


Figure 26.5

Solution : The work done by the gas in the process a to b is the area of $abde$. This is

$$\begin{aligned}W_{ab} &= (120 \text{ kPa}) (250 \text{ cc}) \\ &= 120 \times 10^3 \times 250 \times 10^{-6} \text{ J} = 30 \text{ J}.\end{aligned}$$

In the process b to c the volume remains constant and the work done is zero.

In the process c to a the gas is compressed. The volume is decreased and the work done by the gas is negative. The magnitude is equal to the area of $caed$. This area is $cab + baed$

$$\begin{aligned}&= \frac{1}{2} (80 \text{ kPa}) (250 \text{ cc}) + 30 \text{ J} \\ &= 10 \text{ J} + 30 \text{ J} = 40 \text{ J}.\end{aligned}$$

Thus, the work done in the process c to a is -40 J .

Work Done in an Isothermal Process on an Ideal Gas

Suppose an ideal gas has initial pressure, volume and temperature as p_1 , V_1 and T respectively. In a process, the temperature is kept constant and its pressure and volume are changed from p_1 , V_1 to p_2 , V_2 . The work done by the gas is

$$W = \int_{V_1}^{V_2} p \, dV.$$

As $pV = nRT$, we have $p = \frac{nRT}{V}$.

$$\begin{aligned} \text{Thus, } W &= \int_{V_1}^{V_2} \frac{nRT}{V} dV \\ &= nRT \int_{V_1}^{V_2} \frac{dV}{V} \\ &= nRT \ln \left(\frac{V_2}{V_1} \right). \end{aligned} \quad \dots (26.3)$$

Work Done in an Isobaric Process

Suppose the pressure of a system remains constant at a value p and the volume changes from V_1 to V_2 . The work done by the system is

$$\begin{aligned} W &= \int_{V_1}^{V_2} p \, dV \\ &= p \int_{V_1}^{V_2} dV = p(V_2 - V_1). \end{aligned}$$

Work Done in an Isochoric Process

In an isochoric process the volume remains constant and no work is done by the system.

26.3 HEAT ENGINES

We have seen that when mechanical work is done on a system, its internal energy increases (remember, we assume that the system does not have any systematic motion). The reverse process in which mechanical work is obtained at the expense of internal energy is also possible. Heat engines are devices to perform this task. The basic activity of a heat engine is shown in figure (26.6). It takes some heat from bodies at higher temperature, converts a part of it into the mechanical work and delivers the rest to bodies at lower temperature.

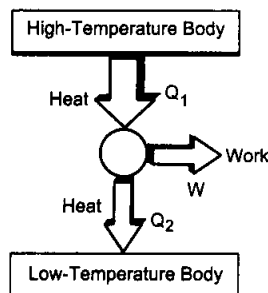


Figure 26.6

The substance inside the engine comes back to the original state. A process in which the final state of a system is the same as its initial state, is called a *cyclic process*. An engine works in cyclic process.

Efficiency

Suppose an engine takes an amount Q_1 of heat from high-temperature bodies, converts a part W of it into work and rejects an amount Q_2 of heat to low-temperature bodies. If the final state of the substance inside the engine is the same as the initial state, there is no change in its internal energy. By first law of thermodynamics, $W = Q_1 - Q_2$.

The efficiency of the engine is defined as

$$\begin{aligned} \eta &= \frac{\text{work done by the engine}}{\text{heat supplied to it}} \\ &= \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}. \end{aligned} \quad \dots (26.4)$$

We now describe some of the heat engines in use.

Steam Engine

A steam engine takes heat from steam and converts a part of it into mechanical motion of a piston which is then used to move heavy objects, such as trains. It is said that James Watt got the idea of steam engine while watching the lid of a kettle being pushed by the steam produced in the kettle in his kitchen. There have been a number of important changes in the design of steam engines, but the essential features remain the same. The main parts of a steam engine are shown in figure (26.7a).

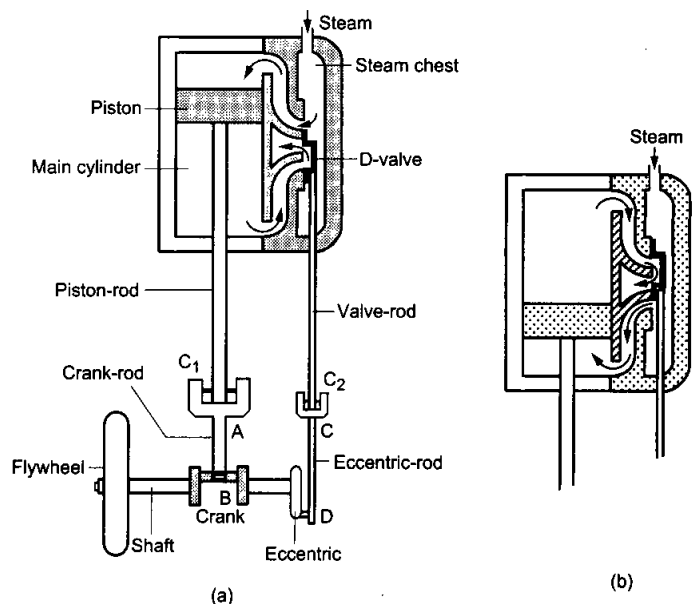


Figure 26.7

Description

It consists of a stout cylindrical vessel known as the *main cylinder*. A tight-fitting, movable piston separates the cylinder into two parts: upper part and lower part. The piston is connected to a rod, known as the *piston-rod* which comes out of the main cylinder.

Adjacent to the main cylinder, there is another stout chamber known as the *steam chest*. The steam chest and the cylinder have a common wall in which there are three holes. The upper and the lower holes open in the upper and the lower parts of the cylinder. The middle hole is connected to a *condenser* (not shown in the figure) through a pipe. Condenser is just a vessel in which the steam cools down and condenses.

The steam chest contains a valve of the shape of an open box. It slides on the surface of the common wall between the steam chest and the main cylinder. Together with the wall, the valve forms the shape of "D" and hence it is called a *D-Valve*. Its size is such that at any time it covers either the upper or the lower hole and leaves the other open. The middle hole is always covered. In the situation shown in the figure, the upper hole is open and the middle and the lower holes are covered. The valve is connected to a *valve-rod* which comes out of the steam chest.

A *flywheel* is connected to a *crankshaft*. A *crank* is fixed to this shaft. If a force is applied to the crank in such a way that the force does not intersect the axis of the shaft, a torque acts on the shaft and it rotates. The flywheel and the crank also rotate with it.

A circular disc called *eccentric* is also connected to the shaft. The axis of the disc does not coincide with the axis of the shaft.

The piston-rod is connected to the crank by another rod *AB* known as the *crank-rod*, which is hinged with the piston-rod at the crosshead C_1 . If the piston moves down, the crosshead C_1 also moves down. The crank has to rotate so as to move the end *B* of the crank-rod, farther. Similarly, if the piston-rod moves up, the crank rotates so as to move the end *B* of the crank-rod closer.

Similarly, the valve-rod is connected to the eccentric by another rod *CD* known as the *eccentric-rod*, which is hinged with the valve-rod at the crosshead C_2 . When the eccentric disc rotates, the crosshead C_2 has to move up and down. Correspondingly, the D-valve slides up and down. The arrangement is such that when the piston moves down, the D-valve moves up and when the piston moves up, the D-valve moves down.

Working

Water is boiled in a big boiler and the steam so prepared is allowed to go into the steam chest. The flow of steam into the steam chest is controlled by a valve.

In the position shown in figure (26.7a), the piston is near the top end of the cylinder and the D-valve is near its lowest position. Steam is forced into the upper part of the cylinder through the upper hole. The piston is pushed down and the steam in the lower part passes through the lower hole and then through the middle hole to the condenser. As the piston comes down, the crank rotates the shaft and the D-valve slides up. When the piston reaches near the bottom end, the D-valve closes the upper hole and opens the lower hole (figure 26.7b). Thus, steam is forced into the lower part of the cylinder which pushes the piston up. The steam in the upper part passes through the upper hole and then through the middle hole to the condenser. The D-valve slides down. This process is repeated continuously. The piston thus keeps on moving up and down rotating the shaft and the flywheel. The kinetic energy of the piston, flywheel and any part connected to the engine comes from the internal energy of the steam. Thus, a part of the internal energy is converted into mechanical energy.

The efficiency of a steam engine is often measured as the ratio of the mechanical work obtained to the heat that could be produced in burning the fuel (to produce steam in this case). The efficiency of a steam engine is typically of the order of 3 to 10%.

Internal Combustion Engine

In a steam engine, there is a separate furnace to boil water. There are engines in which there is no separate furnace and heat is produced in the main cylinder itself. Such engines are called *internal combustion engines*. We will describe two types of internal combustion engines: *petrol engine* and *diesel engine*.

Petrol Engine

This engine was designed by Otto in 1876 and hence is also called an *Otto engine*.

In this, petrol from a tank goes to a chamber known as the *carburettor*, in the form of jets. In the carburettor, the petrol is mixed with proper amount of air and the mixture is allowed to go into the main cylinder shown schematically in figure (26.8). The cylinder is made of steel and is fitted with a movable piston just fitting in the cylinder. Two valves V_1 and V_2 are fixed at the top end of the cylinder. The valve V_1 is used for inlet into the cylinder and V_2 is used for outlet from the cylinder. Valves are opened

and closed at proper times. The piston is rigidly connected to a piston-rod which is connected to a crankshaft much like that in a steam engine.

A spark plug is placed in the main cylinder and is used to produce electric sparks. These sparks burn the petrol-air mixture.

The working of the engine may be described in four steps known as four strokes. Figure (26.8) is used to explain these strokes.

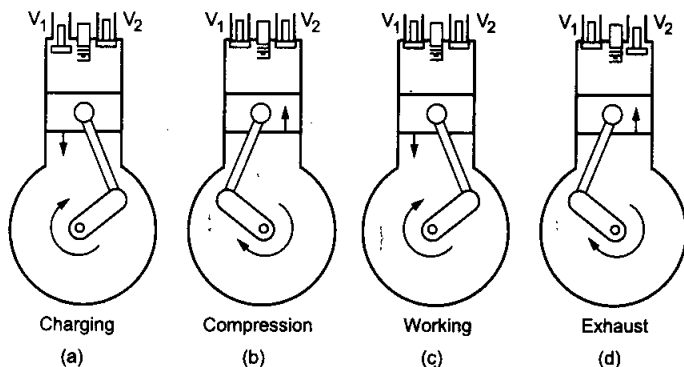


Figure 26.8

(a) Charging Stroke

The valve V_1 is opened and the mixture of petrol vapour and air enters into the cylinder. The piston goes down.

(b) Compression Stroke

Both the inlet and the outlet valves are closed and the piston moves up in the cylinder. This compresses the mixture to a high pressure and the temperature increases to about 500°C .

(c) Working Stroke

The spark plug produces a spark at the end of the compression stroke. Both the valves are closed. The fuel mixture ignites. The temperature increases to about 2000°C and the pressure to about 15 atm. The piston is pushed down and this rotates the crankshaft and the flywheel connected to it. This stroke provides a large amount of mechanical energy and, therefore, is called the working stroke.

(d) Exhaust Stroke

In this stroke, the valve V_2 is opened and the burnt gases are flushed out. The piston moves in and the cycle is completed.

Diesel Engine

In a diesel engine, diesel is used as a fuel. Its construction and working is similar to that of a petrol engine.

The main action takes place in a cylinder fitted with an inlet valve, an outlet valve and a valve that allows the fuel to come into the cylinder. No spark plug is used in it. The inlet valve is opened and air is sucked in due to the forward motion of the piston. When the piston moves backward, the inlet valve is closed and the air gets compressed. Due to the compression, the temperature rises to about 1000°C and the pressure to about 36 atm. The fuel valve is opened at this moment and fuel is injected into the cylinder. The fuel ignites readily due to the high temperature. The piston is pushed forward with a great force. This is the working stroke of the engine in which large amount of mechanical energy is obtained. At the end of this stroke the outlet valve opens and the burnt gases are expelled by the backward motion of the piston.

Internal combustion engines have better efficiency than steam engines. They occupy small space and are used with scooters, motorcycles, etc.

26.4 THE SECOND LAW OF THERMODYNAMICS

When a body at 100°C is kept in contact with a similar body at 0°C , heat flows from the hotter body to the colder body and both come to 50°C . Is the reverse process possible? That is, if we put two similar bodies both at 50°C in contact, can heat flow from one body to the other so that one body reaches 0°C and the other 100°C ? A block moving at a speed v_0 on a rough table eventually stops and the table and the block warm up. The kinetic energy of the block appears as the internal energy of the table and the block. Can the reverse process be possible? That is, we heat the block and the table and put the block on the table. Can the bodies cool down and the block start sliding with speed v_0 on the table converting the internal energy into kinetic energy? Consider a container with rigid walls divided in two parts by a partition having a valve. A gas is put in one part and vacuum is created in the other part. The valve is now opened. The gas eventually occupies both the parts of the container. Is the reverse process possible? We put the gas distributed in both the parts with the valve open. Can the gas go into one part evacuating the other part all by itself?

The answer to all these questions is NO. Of course, the first law of thermodynamics is not violated in any of these proposed reverse processes. The energy is conserved in the direct process as well as in the reverse process. Still the reverse process is not possible. There must be a law of nature other than the first law which decides, whether a given process, allowed by the first law, will actually take place or not. This law is the second law of thermodynamics. This law may be stated

in various ways. We give here one statement in terms of working of heat engines. We know that a heat engine takes Q_1 amount of heat energy from a hot body, converts a part of it into mechanical work and rejects the rest amount Q_2 to a cold body. The efficiency of the engine is $1 - Q_2/Q_1$. The efficiency would be 1, that is, 100% if $Q_2 = 0$. Such an engine would not need any "low-temperature body" to which it needs to reject heat. Hence, it needs only one body at a single temperature, from which it will take heat and convert it completely into mechanical work. This temperature can even be the temperature of the surrounding and hence we will not have to burn any fuel to prepare steam or gases at high temperature to run the engine. A scooter could be run by an engine taking heat from the body of the scooter without needing any petrol. A ship could be run by an engine taking heat from the ocean. However, all attempts to construct such a 100% efficient engine failed. In fact, it is not possible to have such an engine and this is one form of the second law of thermodynamics stated more precisely as follows:

It is not possible to design a heat engine which works in cyclic process and whose only result is to take heat from a body at a single temperature and convert it completely into mechanical work.

This statement of the second law is called the *Kelvin-Planck statement*.

One can convert mechanical work completely into heat but one cannot convert heat completely into mechanical work. In this respect, heat and work are not equivalent. We shall now study some other aspects of the second law of thermodynamics.

26.5 REVERSIBLE AND IRREVERSIBLE PROCESSES

Consider a sample of an ideal gas kept in an enclosure. The state of the gas is described by specifying its pressure p , volume V and temperature T . If these parameters can be uniquely specified at a time, we say that the gas is in *thermodynamic equilibrium*. If we put the enclosure on a hot stove, the temperature of various parts of the gas will be different and we will not be able to specify a unique temperature of the gas. The gas is not in thermodynamic equilibrium in such a case.

When we perform a process on a given system, its state is, in general, changed. Suppose the initial state of the system is described by the values p_1, V_1, T_1 and the final state by p_2, V_2, T_2 . If the process is performed in such a way that at any instant during the process, the system is very nearly in thermodynamic equilibrium, the process is called *quasi-static*. This means, we can specify the parameters p, V, T uniquely at any instant during such a process.

Actual processes are not quasi-static. To change the pressure of a gas, we can move a piston inside the enclosure. The gas near the piston is acted upon by greater force as compared to the gas away from the piston. The pressure of the gas may not be uniform everywhere while the piston is moving. However, we can move the piston very slowly to make the process as close to quasi-static as we wish. Thus, a *quasi-static process is an idealised process in which all changes take place infinitely slowly*.

A quasi-static process on a gas can be represented by a curve on a p - V diagram (or a p - T or a V - T diagram). This is because at any instant we have a unique value of p and a unique value of V . Suppose the curve in figure (26.9) shows such a quasi-static process taking the system from an initial state i to a final state f . Let AB be any arbitrary small part of this process. Suppose in this part the gas takes an amount ΔQ of heat from its surrounding and performs an amount ΔW of work on the surrounding. It may be possible to design a reverse quasi-static process which takes the system from the state f to the state i satisfying the following conditions:

(a) the reverse process is represented by the same curve as the direct process, with the arrow inverted,

(b) in the part BA , the system gives an amount ΔQ of heat to the surrounding and an amount ΔW of work is performed on the system.

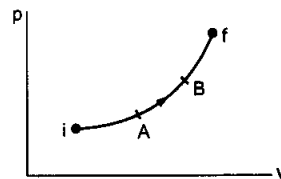


Figure 26.9

If such a reverse process is possible, the original process is called *reversible process*. In the direct process the system has passed through certain equilibrium states in a sequence. When the process is reversed, the system passes through the same states in the reverse sequence. Also, in any small part of the reverse process, it returns the same amount of heat to the surrounding as was taken during the corresponding part in the direct process. Similarly, any work done by the system in the direct process is compensated by the equal work done on the system in the corresponding reverse process.

A process can be reversible if it satisfies two conditions. The process must be quasi-static and it should be nondissipative. This means, friction, viscosity, etc., should be completely absent.

All processes described in this and the following chapters will be assumed to be reversible unless stated otherwise.

Reversible Cycle

We know that if the state of a system at the end of a process is the same as the state of the system at the beginning, the process is called a cyclic process. If all parts of a cyclic process are reversible, it is called a *reversible cycle*.

26.6 ENTROPY

Like pressure, volume, temperature, internal energy, etc., we have another thermodynamic variable of a system, named *entropy*. In a given equilibrium state, the system has a definite value of entropy. If the system has a temperature T (in absolute scale) and a small amount of heat ΔQ is given to it, we define the change in the entropy of the system as

$$\Delta S = \frac{\Delta Q}{T} \quad \dots (26.5)$$

In general, the temperature of the system may change during a process. If the process is reversible, the change in entropy is defined as

$$S_f - S_i = \int_i^f \frac{\Delta Q}{T} \quad \dots (26.6)$$

In an adiabatic reversible process, no heat is given to the system. The entropy of the system remains constant in such a process.

Entropy is related to the *disorder* in the system. Thus, if all the molecules in a given sample of a gas are made to move in the same direction with the same velocity, the entropy will be smaller than that in the actual situation in which the molecules move randomly in all directions.

An interesting fact about entropy is that it is not a conserved quantity. More interesting is the fact that entropy can be created but cannot be destroyed. Once some entropy is created in a process, the universe has to carry the burden of that entropy for ever. The second law of thermodynamics may be stated in terms of entropy as follows:

It is not possible to have a process in which the entropy of an isolated system is decreased.

26.7 CARNOT ENGINE

The French scientist N L Sadi Carnot, in 1824, suggested an idealised engine which we call *Carnot engine* and which has an intimate relation with the second law of thermodynamics. To understand the principle, let us consider an ideal gas taken in a cylinder. The bottom of the cylinder is diathermic whereas rest of it is adiabatic. An adiabatic piston is fitted into the cylinder. Also, suppose we have two

large bodies, one at a constant high temperature T_1 and the other at a lower temperature T_2 .

Figure (26.10a) shows the basic process of a Carnot engine on a p - V diagram. The other parts of the figure represent the process schematically. Suppose, the cylinder is kept in contact with the high-temperature body at temperature T_1 in a compressed state. This state is represented by the point a in the p - V diagram. The gas is isothermally expanded to a state b (figure 26.10b). Work is done by the gas and Q_1 amount of heat is supplied to it by the body at temperature T_1 . The cylinder is now kept on an adiabatic platform and the gas is allowed to expand further to the state c (figure 26.10c). This is an adiabatic expansion and the temperature falls from T_1 to T_2 . Work is done by the gas. At this stage, the cylinder is put in contact with the lower temperature body at temperature T_2 . It is isothermally compressed to a state d (figure 26.10d). Work is done on the gas and the gas rejects an amount Q_2 of heat to the body at the lower temperature T_2 . Finally, it is kept on the adiabatic platform and is further compressed to reach the state a where the temperature is T_1 (figure 26.10e).

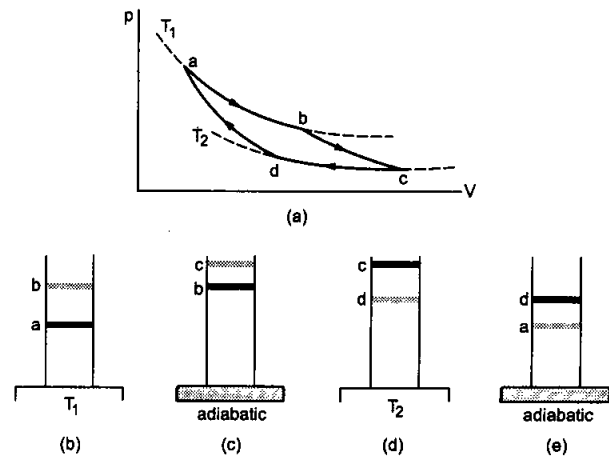


Figure 26.10

The process represented by $abcd$ in figure (26.10a) is a cyclic process. If the piston is frictionless and is always moved very slowly, the process is a reversible cyclic process.

Efficiency of a Carnot Engine

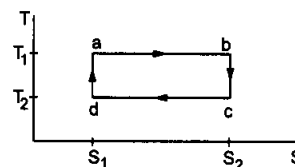


Figure 26.11

The basic process of a Carnot engine, described above, is again shown in figure (26.11) in a T - S (temperature-entropy) diagram. The points a , b , c and

d represent the same states as in figure (26.10a). Let the entropy in state a be S_1 . An amount Q_1 of heat is supplied to the system in the isothermal process ab at the temperature T_1 . The entropy increases in this part as heat is supplied to the system. Also, by definition,

$$S_2 - S_1 = \frac{Q_1}{T_1} \quad \dots (i)$$

The entropy remains constant in the part bc as it describes an adiabatic process. So the entropy in state c is S_2 . In the part cd , the system gives a heat Q_2 at the lower temperature T_2 and its entropy is decreased. The part da represents an adiabatic process and the entropy remains constant. As the entropy in state a is S_1 , the entropy in state d is also S_1 . Using the definition of change in entropy for the process cd ,

$$S_1 - S_2 = \frac{-Q_2}{T_2} \quad \dots (ii)$$

From (i) and (ii),

$$\begin{aligned} \frac{Q_1}{T_1} &= \frac{Q_2}{T_2} \\ \text{or,} \quad \frac{Q_2}{Q_1} &= \frac{T_2}{T_1} \end{aligned}$$

The efficiency of the engine is

$$\begin{aligned} \eta &= \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} \\ &= 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} \quad \dots (26.7) \end{aligned}$$

Thus, the efficiency of the engine depends only on the temperatures of the hot and cold bodies between which the engine works.

Carnot's Theorem

Carnot engine is a reversible engine. It can be proved from the second law of thermodynamics that:

All reversible engines operating between the same two temperatures have equal efficiency and no engine operating between the same two temperatures can have an efficiency greater than this.

This theorem is called *Carnot's theorem*. It is a consequence of the second law and puts a theoretical limit $\eta = 1 - \frac{T_2}{T_1}$ to the maximum efficiency of heat engines.

Refrigerator or Heat Pump

A heat engine takes heat from a hot body, converts part of it into work and rejects the rest to a cold body. The reverse operation is done by a *refrigerator* also known as a *heat pump*. It takes an amount Q_2 of heat from a cold body, an amount W of work is done on it by the surrounding and the total energy $Q_1 = Q_2 + W$ is supplied to a hot body in the form of heat. Thus, heat is passed from the cold body to the hot body. Figure (26.12) shows the process schematically. If the heat is taken at a single low temperature T_2 , it is rejected at a single high temperature T_1 and all the parts of the process are carried out reversibly, we get a *Carnot refrigerator*. If the operating temperatures are fixed, a Carnot refrigerator needs minimum amount of work done to extract a given amount Q_2 of heat from the colder body.

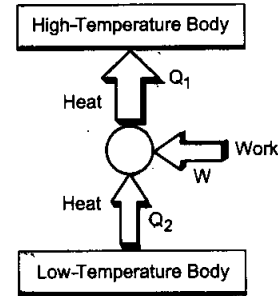


Figure 26.12

In this case,

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$\text{or,} \quad \frac{Q_2 + W}{Q_2} = \frac{T_1}{T_2}$$

$$\text{or,} \quad W = Q_2 \left(\frac{T_1}{T_2} - 1 \right)$$

A minimum of this much work has to be done by the surrounding, if we wish to transfer heat Q_2 from the low-temperature body to the high-temperature body. This leads to another statement of second law of thermodynamics as follows:

It is not possible to design a refrigerator which works in cyclic process and whose only result is to transfer heat from a body to a hotter body.

This is known as the *Claussius statement* of the second law.

Worked Out Examples

1. A sample of an ideal gas is taken through the cyclic process *abca* (figure 26-W1). It absorbs 50 J of heat during the part *ab*, no heat during *bc* and rejects 70 J of heat during *ca*. 40 J of work is done on the gas during the part *bc*. (a) Find the internal energy of the gas at *b* and *c* if it is 1500 J at *a*. (b) Calculate the work done by the gas during the part *ca*.

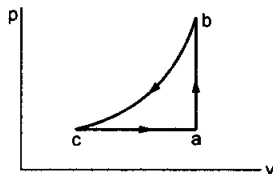


Figure 26-W1

Solution : (a) In the part *ab* the volume remains constant. Thus, the work done by the gas is zero. The heat absorbed by the gas is 50 J. The increase in internal energy from *a* to *b* is

$$\Delta U = \Delta Q = 50 \text{ J.}$$

As the internal energy is 1500 J at *a*, it will be 1550 J at *b*. In the part *bc*, the work done by the gas is $\Delta W = -40 \text{ J}$ and no heat is given to the system. The increase in internal energy from *b* to *c* is

$$\Delta U = -\Delta W = 40 \text{ J.}$$

As the internal energy is 1550 J at *b*, it will be 1590 J at *c*.

(b) The change in internal energy from *c* to *a* is

$$\Delta U = 1500 \text{ J} - 1590 \text{ J} = -90 \text{ J.}$$

The heat given to the system is $\Delta Q = -70 \text{ J}$.

Using $\Delta Q = \Delta U + \Delta W$,

$$\begin{aligned} \Delta W &= \Delta Q - \Delta U \\ &= -70 \text{ J} + 90 \text{ J} = 20 \text{ J.} \end{aligned}$$

2. A thermodynamic system is taken through the cycle *abcd* (figure 26-W2). (a) Calculate the work done by the gas during the parts *ab*, *bc*, *cd* and *da*. (b) Find the total heat rejected by the gas during the process.

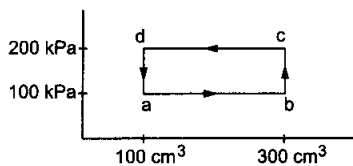


Figure 26-W2

Solution : (a) The work done during the part *ab*,

$$\begin{aligned} &= \int_a^b p dV = (100 \text{ kPa}) \int_{100}^{300} dV \\ &= (100 \text{ kPa}) (300 \text{ cm}^3 - 100 \text{ cm}^3) \end{aligned}$$

$$= 20 \text{ J.}$$

The work done during *bc* is zero as the volume does not change. The work done during *cd*

$$\begin{aligned} &= \int_c^d p dV = (200 \text{ kPa}) (100 \text{ cm}^3 - 300 \text{ cm}^3) \\ &= -40 \text{ J.} \end{aligned}$$

The work done during *da* is zero as the volume does not change.

(b) The total work done by the system during the cycle *abcd*

$$\Delta W = 20 \text{ J} - 40 \text{ J} = -20 \text{ J.}$$

The change in internal energy $\Delta U = 0$ as the initial state is the same as the final state. Thus $\Delta Q = \Delta U + \Delta W = -20 \text{ J}$. So the system rejects 20 J of heat during the cycle.

3. Calculate the increase in internal energy of 1 kg of water at 100°C when it is converted into steam at the same temperature and at 1 atm (100 kPa). The density of water and steam are 1000 kg m^{-3} and 0.6 kg m^{-3} respectively. The latent heat of vaporization of water is $2.25 \times 10^6 \text{ J kg}^{-1}$.

Solution : The volume of 1 kg of water

$$= \frac{1}{1000} \text{ m}^3 \text{ and of 1 kg of steam} = \frac{1}{0.6} \text{ m}^3.$$

The increase in volume

$$\begin{aligned} &= \frac{1}{0.6} \text{ m}^3 - \frac{1}{1000} \text{ m}^3 \\ &= (1.7 - 0.001) \text{ m}^3 \approx 1.7 \text{ m}^3. \end{aligned}$$

The work done by the system is $p\Delta V$

$$\begin{aligned} &= (100 \text{ kPa}) (1.7 \text{ m}^3) \\ &= 1.7 \times 10^5 \text{ J.} \end{aligned}$$

The heat given to convert 1 kg of water into steam

$$= 2.25 \times 10^6 \text{ J.}$$

The change in internal energy is

$$\begin{aligned} \Delta U &= \Delta Q - \Delta W \\ &= 2.25 \times 10^6 \text{ J} - 1.7 \times 10^5 \text{ J} \\ &= 2.08 \times 10^6 \text{ J.} \end{aligned}$$

4. The internal energy of a monatomic ideal gas is $1.5 nRT$. One mole of helium is kept in a cylinder of cross section 8.5 cm^2 . The cylinder is closed by a light frictionless piston. The gas is heated slowly in a process during which a total of 42 J heat is given to the gas. If the temperature rises through 2°C , find the distance moved by the piston. Atmospheric pressure = 100 kPa.

Solution : The change in internal energy of the gas is

$$\begin{aligned}\Delta U &= 1.5 nR (\Delta T) \\ &= 1.5 (1 \text{ mol}) (8.3 \text{ J K}^{-1} \text{ mol}^{-1}) (2 \text{ K}) \\ &= 24.9 \text{ J}.\end{aligned}$$

The heat given to the gas = 42 J.

The work done by the gas is

$$\begin{aligned}\Delta W &= \Delta Q - \Delta U \\ &= 42 \text{ J} - 24.9 \text{ J} = 17.1 \text{ J}.\end{aligned}$$

If the distance moved by the piston is x , the work done is

$$\Delta W = (100 \text{ kPa}) (8.5 \text{ cm}^2) x.$$

Thus,

$$(10^5 \text{ N m}^{-2}) (8.5 \times 10^{-4} \text{ m}^2) x = 17.1 \text{ J}$$

or,

$$x = 0.2 \text{ m} = 20 \text{ cm}.$$

5. A steam engine intakes 100 g of steam at 100°C per minute and cools it down to 20°C. Calculate the heat rejected by the steam engine per minute. Latent heat of vaporization of steam = 540 cal g⁻¹.

Solution : Heat rejected during the condensation of steam in one minute

$$= (100 \text{ g}) \times (540 \text{ cal g}^{-1}) = 5.4 \times 10^4 \text{ cal}.$$

Heat rejected during the cooling of water

$$\begin{aligned}&= (100 \text{ g}) \times (1 \text{ cal g}^{-1} \text{ } ^\circ\text{C}^{-1}) (100^\circ\text{C} - 20^\circ\text{C}) \\ &= 8000 \text{ cal}.\end{aligned}$$

Thus, heat rejected by the engine per minute

$$\begin{aligned}&= 5.4 \times 10^4 \text{ cal} + 0.8 \times 10^4 \text{ cal} \\ &= 6.2 \times 10^4 \text{ cal}.\end{aligned}$$

6. Figure (26-W3) shows a process ABCA performed on an ideal gas. Find the net heat given to the system during the process.

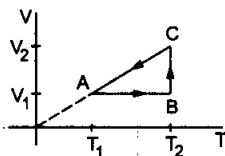


Figure 26-W3

Solution : As the process is cyclic, the change in internal energy is zero. The heat given to the system is then equal to the work done by it.

The work done in part AB is $W_1 = 0$ as the volume remains constant. The part BC represents an isothermal process so that the work done by the gas during this part is

$$W_2 = nR T_2 \ln(V_2/V_1).$$

During the part CA,

$$V \propto T.$$

So, V/T is constant and hence,

$$p = \frac{nRT}{V} \text{ is constant.}$$

The work done by the gas during the part CA is

$$\begin{aligned}W_3 &= p (V_1 - V_2) \\ &= nRT_1 - nRT_2 \\ &= -nR (T_2 - T_1).\end{aligned}$$

The net work done by the gas in the process ABCA is

$$W = W_1 + W_2 + W_3 = nR [T_2 \ln \frac{V_2}{V_1} - (T_2 - T_1)].$$

The same amount of heat is given to the gas.

7. Consider the cyclic process ABCA on a sample of 2.0 mol of an ideal gas as shown in figure (26-W4). The temperatures of the gas at A and B are 300 K and 500 K respectively. A total of 1200 J heat is withdrawn from the sample in the process. Find the work done by the gas in part BC. Take $R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$.

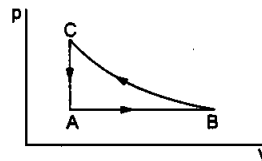


Figure 26-W4

Solution : The change in internal energy during the cyclic process is zero. Hence, the heat supplied to the gas is equal to the work done by it. Hence,

$$W_{AB} + W_{BC} + W_{CA} = -1200 \text{ J.} \quad \dots (i)$$

The work done during the process AB is

$$\begin{aligned}W_{AB} &= p_A (V_B - V_A) \\ &= nR (T_B - T_A) \\ &= (2.0 \text{ mol}) (8.3 \text{ J K}^{-1} \text{ mol}^{-1}) (200 \text{ K}) \\ &= 3320 \text{ J}.\end{aligned}$$

The work done by the gas during the process CA is zero as the volume remains constant. From (i),

$$3320 \text{ J} + W_{BC} = -1200 \text{ J}$$

or,

$$\begin{aligned}W_{BC} &\approx -4520 \text{ J} \\ &\approx -4500 \text{ J}.\end{aligned}$$

8. 2.00 mol of a monatomic ideal gas ($U = 1.5 nRT$) is enclosed in an adiabatic, fixed, vertical cylinder fitted with a smooth, light adiabatic piston. The piston is connected to a vertical spring of spring constant 200 N m^{-1} as shown in figure (26-W5). The area of cross section of the cylinder is 20.0 cm^2 . Initially, the spring is at its natural length and the temperature of the gas is 300 K. The atmospheric pressure is 100 kPa. The gas is heated slowly for some time by means of an electric heater so as to move the

piston up through 10 cm. Find (a) the work done by the gas (b) the final temperature of the gas and (c) the heat supplied by the heater.

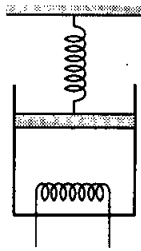


Figure 26-W5

Solution : (a) The force by the gas on the piston is

$$F = p_0 A + kx$$

where $p_0 = 100 \text{ kPa}$ is the atmospheric pressure, $A = 20 \text{ cm}^2$ is the area of cross section, $k = 200 \text{ N m}^{-1}$ is the spring constant and x is the compression of the spring. The work done by the gas as the piston moves through $l = 10 \text{ cm}$ is

$$\begin{aligned} W &= \int_0^l F dx \\ &= p_0 A l + \frac{1}{2} k l^2 \\ &= (100 \times 10^3 \text{ Pa}) \times (20 \times 10^{-4} \text{ m}^2) \times (10 \times 10^{-2} \text{ m}) \\ &\quad + \frac{1}{2} (200 \text{ N m}^{-1}) \times (10 \times 10^{-2} \text{ m})^2 \\ &= 20 \text{ J} + 1 \text{ J} = 21 \text{ J}. \end{aligned}$$

(b) The initial temperature is $T_1 = 300 \text{ K}$. Let the final temperature be T_2 . We have

$$nRT_1 = p_0 V_0$$

$$\begin{aligned} \text{and } nRT_2 &= pV_2 = \left(p_0 + \frac{kl}{A}\right)(V_0 + Al) \\ &= nRT_1 + p_0 Al + kl^2 + \frac{kl nRT_1}{Ap_0} \end{aligned}$$

$$\begin{aligned} \text{or, } T_2 &= T_1 + \frac{p_0 Al + kl^2}{nR} + \frac{klT_1}{Ap_0} \\ &= (300 \text{ K}) + \frac{20 \text{ J} + 1 \text{ J}}{(2.0 \text{ mol})(8.3 \text{ J K}^{-1} \text{ mol}^{-1})} \\ &\quad + \frac{(200 \text{ N m}^{-1}) \times (10 \times 10^{-2} \text{ m}) \times (300 \text{ K})}{(20 \times 10^{-4} \text{ m}^2) \times (100 \times 10^3 \text{ Pa})} \\ &= 300 \text{ K} + 1.325 \text{ K} + 30 \text{ K} \\ &\approx 331 \text{ K}. \end{aligned}$$

(c) The internal energy is $U = 1.5 nRT$.

The change in internal energy is

$$\begin{aligned} \Delta U &= 1.5 nR \Delta T \\ &= 1.5 \times (2.00 \text{ mol}) \times (8.3 \text{ J K}^{-1} \text{ mol}^{-1}) \times (31 \text{ K}) \\ &= 772 \text{ J}. \end{aligned}$$

From the first law,

$$\begin{aligned} \Delta Q &= \Delta U + \Delta W \\ &= 772 \text{ J} + 21 \text{ J} = 793 \text{ J}. \end{aligned}$$

9. A sample of an ideal gas has pressure p_0 , volume V_0 and temperature T_0 . It is isothermally expanded to twice its original volume. It is then compressed at constant pressure to have the original volume V_0 . Finally, the gas is heated at constant volume to get the original temperature. (a) Show the process in a V - T diagram (b) Calculate the heat absorbed in the process.

Solution :

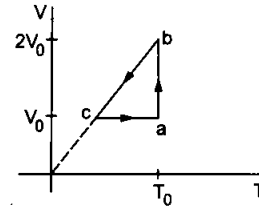


Figure 26-W6

(a) The V - T diagram for the process is shown in figure (26-W6). The initial state is represented by the point a . In the first step, it is isothermally expanded to a volume $2V_0$. This is shown by ab . Then the pressure is kept constant and the gas is compressed to the volume V_0 . From the ideal gas equation, V/T is constant at constant pressure. Hence, the process is shown by a line bc which passes through the origin. At point c , the volume is V_0 . In the final step, the gas is heated at constant volume to a temperature T_0 . This is shown by ca . The final state is the same as the initial state.

(b) The process is cyclic so that the change in internal energy is zero. The heat supplied is, therefore, equal to the work done by the gas. The work done during ab is

$$W_1 = nRT_0 \ln \frac{2V_0}{V_0} = nRT_0 \ln 2 = p_0 V_0 \ln 2.$$

Also from the ideal gas equation,

$$p_a V_a = p_b V_b$$

$$\text{or, } p_b = \frac{p_a V_a}{V_b} = \frac{p_0 V_0}{2V_0} = \frac{p_0}{2}.$$

In the step bc , the pressure remains constant. Hence, the work done is,

$$W_2 = \frac{p_0}{2} (V_0 - 2V_0) = -\frac{p_0 V_0}{2}.$$

In the step ca , the volume remains constant and so the work done is zero. The net work done by the gas in the cyclic process is

$$\begin{aligned} W &= W_1 + W_2 \\ &= p_0 V_0 [\ln 2 - 0.5] \\ &= 0.193 p_0 V_0. \end{aligned}$$

Hence, the heat supplied to the gas is $0.193 p_0 V_0$.

10. A sample of 100 g water is slowly heated from 27°C to 87°C. Calculate the change in the entropy of the water. Specific heat capacity of water = 4200 J kg⁻¹ K⁻¹.

Solution : The heat supplied to increase the temperature of the sample from T to $T + \Delta T$ is

$$\Delta Q = ms \Delta T,$$

where $m = 100 \text{ g} = 0.1 \text{ kg}$ and $C = 4200 \text{ J kg}^{-1} \text{ K}^{-1}$.

The change in entropy during this process is

$$\Delta S = \frac{\Delta Q}{T} = ms \frac{\Delta T}{T}.$$

The total change in entropy as the temperature rises from T_1 to T_2 is,

$$\begin{aligned} S_2 - S_1 &= \int_{T_1}^{T_2} ms \frac{dT}{T} \\ &= ms \ln \frac{T_2}{T_1}. \end{aligned}$$

Putting $T_1 = 27^\circ\text{C} = 300 \text{ K}$ and $T_2 = 87^\circ\text{C} = 360 \text{ K}$,

$$\begin{aligned} S_2 - S_1 &= (0.1 \text{ kg}) (4200 \text{ J kg}^{-1} \text{ K}^{-1}) \ln \frac{360}{300} \\ &= 76.6 \text{ J K}^{-1}. \end{aligned}$$

11. A heat engine operates between a cold reservoir at temperature $T_2 = 300 \text{ K}$ and a hot reservoir at temperature T_1 . It takes 200 J of heat from the hot reservoir and delivers 120 J of heat to the cold reservoir in a cycle. What could be the minimum temperature of the hot reservoir?

Solution : The work done by the engine in a cycle is

$$W = 200 \text{ J} - 120 \text{ J} = 80 \text{ J}.$$

The efficiency of the engine is

$$\eta = \frac{W}{Q} = \frac{80 \text{ J}}{200 \text{ J}} = 0.40.$$

From Carnot's theorem, no engine can have an efficiency greater than that of a Carnot engine.

$$\text{Thus, } 0.40 \leq 1 - \frac{T_2}{T_1} = 1 - \frac{300 \text{ K}}{T_1}$$

$$\text{or, } \frac{300 \text{ K}}{T_1} \leq 1 - 0.40 = 0.60$$

$$\text{or, } T_1 \geq \frac{300 \text{ K}}{0.60}$$

$$\text{or, } T_1 \geq 500 \text{ K}.$$

The minimum temperature of the hot reservoir has to be 500 K.

□

QUESTIONS FOR SHORT ANSWER

- Should the internal energy of a system necessarily increase if heat is added to it?
- Should the internal energy of a system necessarily increase if its temperature is increased?
- A cylinder containing a gas is lifted from the first floor to the second floor. What is the amount of work done on the gas? What is the amount of work done by the gas? Is the internal energy of the gas increased? Is the temperature of the gas increased?
- A force F is applied on a block of mass M . The block is displaced through a distance d in the direction of the force. What is the work done by the force on the block? Does the internal energy change because of this work?
- The outer surface of a cylinder containing a gas is rubbed vigorously by a polishing machine. The cylinder and its gas become warm. Is the energy transferred to the gas heat or work?
- When we rub our hands they become warm. Have we supplied heat to the hands?
- A closed bottle contains some liquid. The bottle is shaken vigorously for 5 minutes. It is found that the temperature of the liquid is increased. Is heat transferred to the liquid? Is work done on the liquid? Neglect expansion on heating.
- The final volume of a system is equal to the initial volume in a certain process. Is the work done by the system necessarily zero? Is it necessarily nonzero?
- Can work be done by a system without changing its volume?
- An ideal gas is pumped into a rigid container having diathermic walls so that the temperature remains constant. In a certain time interval, the pressure in the container is doubled. Is the internal energy of the contents of the container also doubled in the interval?
- When a tyre bursts, the air coming out is cooler than the surrounding air. Explain.
- When we heat an object, it expands. Is work done by the object in this process? Is heat given to the object equal to the increase in its internal energy?
- When we stir a liquid vigorously, it becomes warm. Is it a reversible process?
- What should be the condition for the efficiency of a Carnot engine to be equal to 1?
- When an object cools down, heat is withdrawn from it. Does the entropy of the object decrease in this process? If yes, is it a violation of the second law of thermodynamics stated in terms of increase in entropy?

OBJECTIVE I

- The first law of thermodynamics is a statement of
 - conservation of heat
 - conservation of work
 - conservation of momentum
 - conservation of energy.
- If heat is supplied to an ideal gas in an isothermal process,
 - the internal energy of the gas will increase
 - the gas will do positive work
 - the gas will do negative work
 - the said process is not possible.
- Figure (26-Q1) shows two processes *A* and *B* on a system. Let ΔQ_1 and ΔQ_2 be the heat given to the system in processes *A* and *B* respectively. Then
 - $\Delta Q_1 > \Delta Q_2$
 - $\Delta Q_1 = \Delta Q_2$
 - $\Delta Q_1 < \Delta Q_2$
 - $\Delta Q_1 \leq \Delta Q_2$.

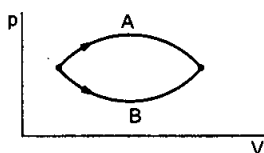


Figure 26-Q1

- Refer to figure (26-Q1). Let ΔU_1 and ΔU_2 be the changes in internal energy of the system in the processes *A* and *B*. Then
 - $\Delta U_1 > \Delta U_2$
 - $\Delta U_1 = \Delta U_2$
 - $\Delta U_1 < \Delta U_2$
 - $\Delta U_1 \neq \Delta U_2$.
- Consider the process on a system shown in figure (26-Q2). During the process, the work done by the system
 - continuously increases
 - continuously decreases
 - first increases then decreases
 - first decreases then increases.

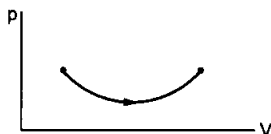


Figure 26-Q2

- Consider the following two statements.
 - If heat is added to a system, its temperature must increase.

- If positive work is done by a system in a thermodynamic process, its volume must increase.
 - Both A and B are correct.
 - A is correct but B is wrong.
 - B is correct but A is wrong.
 - Both A and B are wrong.

- An ideal gas goes from the state *i* to the state *f* as shown in figure (26-Q3). The work done by the gas during the process
 - is positive
 - is negative
 - is zero
 - cannot be obtained from this information.

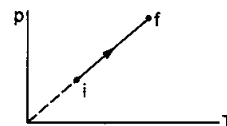


Figure 26-Q3

- Consider two processes on a system as shown in figure (26-Q4).

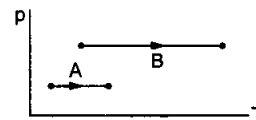


Figure 26-Q4

- The volumes in the initial states are the same in the two processes and the volumes in the final states are also the same. Let ΔW_1 and ΔW_2 be the work done by the system in the processes *A* and *B* respectively.
- $\Delta W_1 > \Delta W_2$.
 - $\Delta W_1 = \Delta W_2$.
 - $\Delta W_1 < \Delta W_2$.
 - Nothing can be said about the relation between ΔW_1 and ΔW_2 .

- A gas is contained in a metallic cylinder fitted with a piston. The piston is suddenly moved in to compress the gas and is maintained at this position. As time passes the pressure of the gas in the cylinder
 - increases
 - decreases
 - remains constant
 - increases or decreases depending on the nature of the gas.

OBJECTIVE II

- The pressure *p* and volume *V* of an ideal gas both increase in a process.
 - Such a process is not possible.
 - The work done by the system is positive.
 - The temperature of the system must increase.

- Heat supplied to the gas is equal to the change in internal energy.
- In a process on a system, the initial pressure and volume are equal to the final pressure and volume.
 - The initial temperature must be equal to the final temperature.

- (b) The initial internal energy must be equal to the final internal energy.
 (c) The net heat given to the system in the process must be zero.
 (d) The net work done by the system in the process must be zero.
3. A system can be taken from the initial state p_1, V_1 to the final state p_2, V_2 by two different methods. Let ΔQ and ΔW represent the heat given to the system and the work done by the system. Which of the following must be the same in both the methods?
 (a) ΔQ (b) ΔW (c) $\Delta Q + \Delta W$ (d) $\Delta Q - \Delta W$.
4. Refer to figure (26-Q5). Let ΔU_1 and ΔU_2 be the change in internal energy in processes A and B respectively, ΔQ be the net heat given to the system in process A+B and ΔW be the net work done by the system in the process A+B.

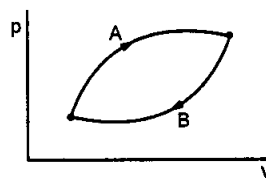


Figure 26-Q5

- (a) $\Delta U_1 + \Delta U_2 = 0$. (b) $\Delta U_1 - \Delta U_2 = 0$.
 (c) $\Delta Q - \Delta W = 0$. (d) $\Delta Q + \Delta W = 0$.
5. The internal energy of an ideal gas decreases by the same amount as the work done by the system.
 (a) The process must be adiabatic.
 (b) The process must be isothermal.
 (c) The process must be isobaric.
 (d) The temperature must decrease.

EXERCISES

1. A thermally insulated, closed copper vessel contains water at 15°C . When the vessel is shaken vigorously for 15 minutes, the temperature rises to 17°C . The mass of the vessel is 100 g and that of the water is 200 g. The specific heat capacities of copper and water are $420 \text{ J kg}^{-1} \text{ K}^{-1}$ and $4200 \text{ J kg}^{-1} \text{ K}^{-1}$ respectively. Neglect any thermal expansion. (a) How much heat is transferred to the liquid-vessel system? (b) How much work has been done on this system? (c) How much is the increase in internal energy of the system?
2. Figure (26-E1) shows a paddle wheel coupled to a mass of 12 kg through fixed frictionless pulleys. The paddle is immersed in a liquid of heat capacity 4200 J K^{-1} kept in an adiabatic container. Consider a time interval in which the 12 kg block falls slowly through 70 cm. (a) How much heat is given to the liquid? (b) How much work is done on the liquid? (c) Calculate the rise in the temperature of the liquid neglecting the heat capacity of the container and the paddle.

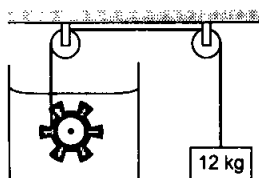


Figure 26-E1

3. A 100 kg block is started with a speed of 2.0 m s^{-1} on a long, rough belt kept fixed in a horizontal position. The coefficient of kinetic friction between the block and the belt is 0.20. (a) Calculate the change in the internal energy of the block-belt system as the block comes to a stop on the belt. (b) Consider the situation from a frame of reference moving at 2.0 m s^{-1} along the initial velocity of the block. As seen from this frame, the block is gently put on a moving belt and in due time the block starts

moving with the belt at 2.0 m s^{-1} . Calculate the increase in the kinetic energy of the block as it stops slipping past the belt. (c) Find the work done in this frame by the external force holding the belt.

4. Calculate the change in internal energy of a gas kept in a rigid container when 100 J of heat is supplied to it.
5. The pressure of a gas changes linearly with volume from 10 kPa, 200 cc to 50 kPa, 50 cc. (a) Calculate the work done by the gas. (b) If no heat is supplied or extracted from the gas, what is the change in the internal energy of the gas?
6. An ideal gas is taken from an initial state i to a final state f in such a way that the ratio of the pressure to the absolute temperature remains constant. What will be the work done by the gas?
7. Figure (26-E2) shows three paths through which a gas can be taken from the state A to the state B. Calculate the work done by the gas in each of the three paths.

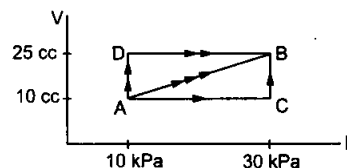


Figure 26-E2

8. When a system is taken through the process abc shown in figure (26-E3), 80 J of heat is absorbed by the system and 30 J of work is done by it. If the system does 10 J

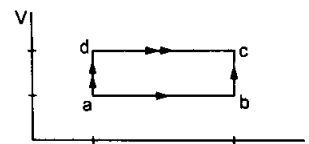


Figure 26-E3

of work during the process adc , how much heat flows into it during the process?

9. 50 cal of heat should be supplied to take a system from the state A to the state B through the path ACB as shown in figure (26-E4). Find the quantity of heat to be supplied to take it from A to B via ADB .

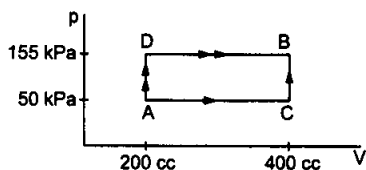


Figure 26-E4

10. Calculate the heat absorbed by a system in going through the cyclic process shown in figure (26-E5).

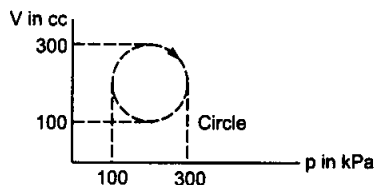


Figure 26-E5

11. A gas is taken through a cyclic process $ABCA$ as shown in figure (26-E6). If 2.4 cal of heat is given in the process, what is the value of J ?

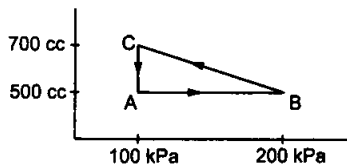


Figure 26-E6

12. A substance is taken through the process abc as shown in figure (26-E7). If the internal energy of the substance increases by 5000 J and a heat of 2625 cal is given to the system, calculate the value of J .

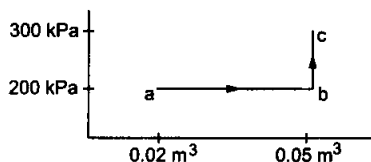


Figure 26-E7

13. A gas is taken along the path AB as shown in figure (26-E8). If 70 cal of heat is extracted from the gas in the

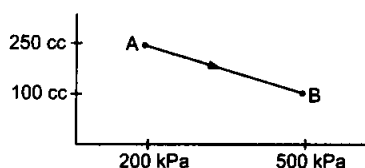


Figure 26-E8

process, calculate the change in the internal energy of the system.

14. The internal energy of a gas is given by $U = 1.5 pV$. It expands from 100 cm^3 to 200 cm^3 against a constant pressure of $1.0 \times 10^5 \text{ Pa}$. Calculate the heat absorbed by the gas in the process.
15. A gas is enclosed in a cylindrical vessel fitted with a frictionless piston. The gas is slowly heated for some time. During the process, 10 J of heat is supplied and the piston is found to move out 10 cm. Find the increase in the internal energy of the gas. The area of cross section of the cylinder = 4 cm^2 and the atmospheric pressure = 100 kPa.
16. A gas is initially at a pressure of 100 kPa and its volume is 2.0 m^3 . Its pressure is kept constant and the volume is changed from 2.0 m^3 to 2.5 m^3 . Its volume is now kept constant and the pressure is increased from 100 kPa to 200 kPa. The gas is brought back to its initial state, the pressure varying linearly with its volume. (a) Whether the heat is supplied to or extracted from the gas in the complete cycle? (b) How much heat was supplied or extracted?
17. Consider the cyclic process $ABCA$, shown in figure (26-E9), performed on a sample of 2.0 mol of an ideal gas. A total of 1200 J of heat is withdrawn from the sample in the process. Find the work done by the gas during the part BC .

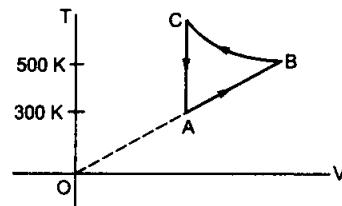


Figure 26-E9

18. Figure (26-E10) shows the variation in the internal energy U with the volume V of 2.0 mol of an ideal gas in a cyclic process $abcd$. The temperatures of the gas at b and c are 500 K and 300 K respectively. Calculate the heat absorbed by the gas during the process.

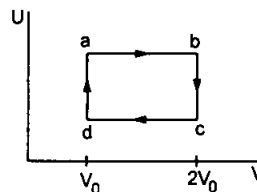


Figure 26-E10

19. Find the change in the internal energy of 2 kg of water as it is heated from 0°C to 4°C . The specific heat capacity of water is $4200 \text{ J kg}^{-1} \text{ K}^{-1}$ and its densities at 0°C and 4°C are 999.9 kg m^{-3} and 1000 kg m^{-3} respectively. Atmospheric pressure = 10^5 Pa .
20. Calculate the increase in the internal energy of 10 g of water when it is heated from 0°C to 100°C and converted

into steam at 100 kPa. The density of steam = 0.6 kg m^{-3} . Specific heat capacity of water = $4200 \text{ J kg}^{-1} \text{ } ^\circ\text{C}^{-1}$ and the latent heat of vaporization of water = $2.25 \times 10^6 \text{ J kg}^{-1}$.

21. Figure (26-E11) shows a cylindrical tube of volume V with adiabatic walls containing an ideal gas. The internal energy of this ideal gas is given by $1.5 nRT$. The tube is divided into two equal parts by a fixed diathermic wall. Initially, the pressure and the temperature are p_1, T_1 on the left and p_2, T_2 on the right. The system is left for sufficient time so that the temperature becomes equal on the two sides. (a) How

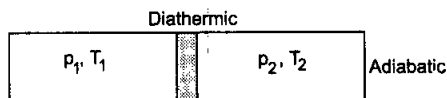


Figure 26-E11

much work has been done by the gas on the left part? (b) Find the final pressures on the two sides. (c) Find the final equilibrium temperature. (d) How much heat has flown from the gas on the right to the gas on the left?

22. An adiabatic vessel of total volume V is divided into two equal parts by a conducting separator. The separator is fixed in this position. The part on the left contains one mole of an ideal gas ($U = 1.5 nRT$) and the part on the right contains two moles of the same gas. Initially, the pressure on each side is p . The system is left for sufficient time so that a steady state is reached. Find (a) the work done by the gas in the left part during the process, (b) the temperature on the two sides in the beginning, (c) the final common temperature reached by the gases, (d) the heat given to the gas in the right part and (e) the increase in the internal energy of the gas in the left part.

□

ANSWERS

OBJECTIVE I

1. (d) 2. (b) 3. (a) 4. (b) 5. (a) 6. (c)
7. (c) 8. (c) 9. (b)

OBJECTIVE II

1. (b), (c) 2. (a), (b) 3. (d) 4. (a), (c)
5. (a), (d)

EXERCISES

1. (a) zero (b) 1764 J (c) 1764 J
2. (a) zero (b) 84 J (c) 0.02°C
3. (a) 200 J (b) 200 J (c) 400 J
4. 100 J
5. (a) -4.5 J (b) 4.5 J
6. zero
7. 0.30 J in AB , 0.450 J in ACB and 0.150 J in ADB
8. 60 J
9. 55 cal
10. 31.4 J

11. 4.17 J cal^{-1}
12. 4.19 J cal^{-1}
13. -241 J
14. 25 J
15. 6 J
16. (a) extracted (b) 25000 J
17. -4520 J
18. 2300 J
19. $(33600 - 0.02) \text{ J}$
20. $2.5 \times 10^4 \text{ J}$
21. (a) zero
(b) $\frac{p_1 T_2 (p_1 + p_2)}{\lambda}$ on the left and $\frac{p_2 T_1 (p_1 + p_2)}{\lambda}$ on the right
(c) $\frac{T_1 T_2 (p_1 + p_2)}{\lambda}$
(d) $\frac{3p_1 p_2 (T_2 - T_1) V}{4\lambda}$ where $\lambda = p_1 T_2 + p_2 T_1$
22. (a) zero (b) $\frac{pV}{(2 \text{ mol}) R}$, $\frac{pV}{(4 \text{ mol}) R}$
(c) $\frac{pV}{(3 \text{ mol}) R}$ (d) $\frac{pV}{4}$ (e) $\frac{-pV}{4}$

□