

Continuity Ex 9.1 Q23

We have given that the function is continuous at x = 2

$$LHL = RHL = f(2)....(1)$$

Now,

LHL =
$$\lim_{x \to 2^{-}} f(x) = \lim_{h \to 0} f(2-h) = \lim_{h \to 0} a(2-h) + 5 = 2a + 5$$

$$f(2) = 2a + 5$$

RHL =
$$\lim_{x \to 2^+} f(x) = \lim_{h \to 0} f(2+h) = \lim_{h \to 0} 2+h-1=1$$

:: Using (1),

$$2a+5=1 \Rightarrow a=-2$$

Continuity Ex 9.1 Q24

We have, at x = 0

$$\mathsf{LHL} = \lim_{x \to 0^+} f\left(x\right) = \lim_{h \to 0} f\left(0 - h\right) = \lim_{h \to 0} \frac{-h}{|-h| + 2\left(-h\right)^2} = \lim_{h \to 0} \frac{-h}{h + 2h^2} = \lim_{h \to 0} \frac{-1}{1 + 2h} = -1$$

$$f(0) = k$$

$$\mathsf{RHL} = \lim_{x \to 0^+} f\left(x\right) = \lim_{h \to 0} f\left(0 + h\right) = \lim_{h \to 0} \frac{h}{\left|h\right| + 2h^2} = \lim_{h \to 0} \frac{1}{1 + 2h} = 1$$

Since, LHL \neq RHL, function will remain discontinuous at x = 0, regardless the choice of k. Continuity Ex 9.1 Q25

Since f(x) is continuous at $x = \frac{\pi}{2}$, L.H.Limit = R.H.Limit.

$$\Rightarrow \lim_{x \to \frac{\pi}{2}^{-}} f(x) = \lim_{x \to \frac{\pi}{2}^{+}} f(x) = \lim_{x \to \frac{\pi}{2}^{+}} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \lim_{x \to \frac{\pi}{2}^{-}} \frac{k \cos x}{\pi - 2x} = 3$$

$$\Rightarrow k \lim_{x \to \frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right)}{2\left(\frac{\pi}{2} - x\right)} = 3$$

$$\Rightarrow \frac{k}{2} \lim_{x \to \frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\left(\frac{\pi}{2} - x\right)} = 3$$

$$\Rightarrow \frac{k}{2} = 3$$

$$\Rightarrow k = 6$$

Continuity Ex 9.1 Q26

We have given that the function is continuous at x = 0 LHL = RHL = f(0)....(1)

$$f(0) = c$$
LHL = $\lim_{x \to 0^{-}} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{\sin(a+1)(-h) + \sin(-h)}{-h} = \lim_{h \to 0} \frac{-\sin(ah+h) - \sinh(-h)}{-h}$

$$= \lim_{h \to 0} \frac{\sin(a+h)h}{h} + \lim_{h \to 0} \frac{\sinh h}{h}$$

$$= a+1+1=a+2$$

$$\begin{aligned} \text{RHL} &= \lim_{x \to 0^{+}} f\left(x\right) = \lim_{h \to 0} f\left(0 + h\right) = \lim_{h \to 0} \frac{\sqrt{h + bh^{2}} - \sqrt{h}}{bh^{\frac{3}{2}}} \\ &= \lim_{h \to 0} \frac{\sqrt{h + bh^{2}} - \sqrt{h}}{bh^{\frac{3}{2}}} \times \frac{\sqrt{h + bh^{2}} + \sqrt{h}}{\sqrt{h + bh^{2}} + \sqrt{h}} \\ &= \lim_{h \to 0} \frac{h + bh^{2} - h}{bh^{\frac{3}{2}} \left(\sqrt{h + bh^{2}} + \sqrt{h}\right)} = \lim_{h \to 0} \frac{bh^{2}}{bh^{2} \left(\sqrt{1 + bh} + 1\right)} = \frac{1}{2} \end{aligned}$$

: from (1),

$$a+2=\frac{1}{2} \Rightarrow a=\frac{-3}{2}$$

$$c = \frac{1}{2}$$
 and

$$b\in R-\left\{ 0\right\}$$

Hence,
$$a = \frac{-3}{2}$$
, $b \in R - \{0\}$, $c = \frac{1}{2}$