Chapter 9 Continuity Ex 9.2 Q3(i)

When $x \neq 1$

$f(x) = x^3 - x^2 + 2x - 2$ is a polynomial, so is continuous for $x < 1$ and $x > 1$

Now, consider the point $x = 1$

$$\text{LHL} = \lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} (1-h)^3 - (1-h)^2 + 2(1-h) - 2 = 1 - 1 + 2 - 2 = 0$$

$$\text{RHL} = \lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h) = \lim_{h \to 0} (1+h)^3 - (1+h)^2 + 2(1+h) - 2 = 1 - 1 + 2 - 2 = 0$$

$f(1) = 4$

$\text{LHL} = \text{RHL} \neq f(1)$

Thus, function is not discontinuous at $x = 1$

Chapter 9 Continuity Ex 9.2 Q3(ii)

When $x \neq 2$, we have,

$$f(x) = \frac{x^4 - 16}{x - 2} = \frac{(x^2+4)(x^2-4)}{x-2} = \frac{(x^2+4)(x+2)(x-2)}{x-2} = f(x) = (x^2+4)(x+2)$$

which is a polynomial, so the function is continuous when $x < 2$ or $x > 2$

Now, consider the point $x = 2$

$$\text{LHL} = \lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h) = \lim_{h \to 0} \frac{(2-h)^4 - 16}{(2-h) - 2}$$

$$= \lim_{h \to 0} \frac{2^4 - 4.8h + 6.4h^2 - 4.2h^3 + h^4 - 16}{-h}$$

$$= \lim_{h \to 0} \frac{16 - 32h + 24h^2 - 8h^3 + h^4 - 16}{-h}$$

$$= \lim_{h \to 0} 32 - 24h + 8h^2 - h^3 = 32$$

$$\text{RHL} = \lim_{x \to 2^+} f(x) = \lim_{h \to 0} f(2+h) = \lim_{h \to 0} \frac{(2+h)^4 - 16}{(2+h) - 2} = \lim_{h \to 0} \frac{16 + 32h + 24h^2 + 8h^3 + h^4 - 16}{h}$$

$$= \lim_{h \to 0} 32 + 24h + 8h^2 + h^3$$

$$= 32$$

Also, $f(2) = 16$

Thus, $\text{LHL} = \text{RHL} \neq f(2)$

Hence, the function is discontinuous at $x = 2$

Chapter 9 Continuity Ex 9.2 Q3(iii)

When $x < 0$, we have, $f(x) = \frac{\sin x}{x}$

We know that $\sin x$ and the identity function continuous for $x < 0$, so the quotient function $f(x) = \frac{\sin x}{x}$ is continuous for $x < 0$.

When $x > 0$ $f(x) = 2x + 3$, which is a polynomial of degree 1 so $f(x) = 2x + 3$ is continuous for $x > 0$.

Now, consider the point $x = 0$

LHL = $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{\sin(-h)}{-h} = \lim_{h \to 0} \frac{-\sin h}{-h} = 1$

RHL = $\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{\sin h}{h} = 1$

$f(0) = 2 \times 0 + 3 = 3$

Thus, L.H.L = R.H.L $\neq f(0)$

Hence, $f(x)$ is discontinuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(iv)

When $x \neq 0$ $f(x) = \frac{\sin 3x}{x}$

We know that $\sin 3x$ and the identity function x are continuous for $x < 0$ and $x > 0$.

So, the quotient function $f(x) = \frac{\sin 3x}{x}$ is continuous for $x < 0$ and $x > 0$.

Now, consider the point $x = 0$

LHL = $\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{\sin 3(-h)}{-h} = \lim_{h \to 0} \frac{-\sin 3h}{-h} = 3$

RHL = $\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{\sin 3h}{h} = 3$

$f(0) = 4$

Thus, LHL = RHL $\neq f(0)$

Hence, $f(x)$ is discontinuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(v)

When $x \neq 0$, we have, $f(x) = \frac{\sin x}{x} + \cos x$

We know that

$\sin x$ and $\cos x$ is continuous for $x < 0$ and $x > 0$.

The identity function x is also continuous for $x < 0$ and $x > 0$.

$\therefore$ The quotient function $f(x) = \frac{\sin x}{x}$ is continuous for $x < 0$ and $x > 0$.

And, the sum $\frac{\sin x}{x} + \cos x$ is also continuous for each $x < 0$ and $x > 0$.

Now, consider the point $x = 0$

LHL = $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{\sin(-h)}{-h} + \cos(-h) = \lim_{h \to 0} \frac{-\sin h}{-h} + \cos h = 1 + 1 = 2$

RHL = $\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{\sin h}{h} + \cos h = 1 + 1 = 2$

$f(0) = 5$

Thus, LHL = RHL $\neq f(0)$

Hence, $f(x)$ is discontinuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(vi)

When $x \neq 0$, we have, $f(x) = \dfrac{x^4 + x^3 + 2x^2}{tan^{-1} x}$

We know that a polynomial is continuous for $x < 0$ and $x > 0$, Also the inverse trignometric function is continuous in its domain.

Here, $x^4 + x^3 + 2x^2$ is polynomial, so is continuous for $x < 0$ and $x > 0$ and $tan^{-1} x$ is also continuous for $x < 0$ and $x > 0$

So, the quotient function $f(x) = \dfrac{x^4 + x^3 + 2x^2}{tan^{-1} x}$ is continuous for each $x < 0$ and $x > 0$.

Now, consider the point $x = 0$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{(-h)^4 + (-h)^3 + 2(-h)^2}{tan^{-1}(-h)} = \lim_{h \to 0} \frac{h^4 - h^3 + 2h^2}{tan^{-1} h} = 0$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{h^4 + h^3 + 2h^2}{tan^{-1} h} = 0$$

$f(0) = 10$

Thus, LHL = RHL $\neq f(0)$

Hence, the function is not continuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(vii)

When $x \neq 0$, we have,

$$f(x) = \frac{e^x - 1}{log_e(1 + 2x)}$$

We know that e^x and the constant function is continuous for $x < 0$ and $x > 0$

$\Rightarrow$ $e^x - 1$ is continuous for $x < 0$ and $x > 0$

Again, logarithmic function is continuous for $x < 0$ and $x > 0$

$\Rightarrow$ $log_e(1 + 2x)$ is continuous for $x > 0$ and $x < 0$

So, the quotient function $f(x) = \dfrac{e^x - 1}{log_e(1 + 2x)}$ is continuous for each $x < 0$ and $x > 0$.

Now, consider the point $x = 0$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{e^{-h} - 1}{log_e(1 - 2h)} = \lim_{h \to 0} \frac{\frac{e^{-h} - 1}{-h}}{\frac{log_e(1 - 2h)}{-2h} \times -2} = \frac{1}{2}$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{e^{h} - 1}{log_e(1 + 2h)} = \lim_{h \to 0} \frac{\frac{e^{h} - 1}{h}}{\frac{log_e(1 + 2h)}{2h} \times 2} = \frac{1}{2}$$

$f(0) = 7$

Thus, LHL = RHL $\neq f(0)$

Hence, $f(x)$ is not continuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(viii)

We know that

(i) The absolute value function $g(x) = |x|$ is continuous on IR

(ii) Polynomial function are every where continuous.

So, the only possible point of discontinuity of $f(x)$ can be $x = 1$

Now

$f(1) = |1-3| = |-2| = 2$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} |x-3| = 2$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \left(\frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}\right)$

$= \frac{1}{4} - \frac{3}{2} + \frac{13}{4} = \frac{8}{4} = 2$

Since

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1) = 2$

$\therefore$ $f(x)$ is continuous at x

Hence $f(x)$ has no point of discontinuity.

Chapter 9 Continuity Ex 9.2 Q3(ix)

When $x < -3$,

$f(x) = |x| + 3$

We know that $|x|$ is continuous for $x < -3$

$\therefore$ $|x| + 3$ is continuous for $x < -3$

When $x > 3$

$f(x) = 6x + 2$ which is a polynomial of degree 1, so $f(x) = 6x + 2$ is continuous for $x > 3$

When $-3 < x < 3$

$f(x) = -2x$ which is again a polynomial so, it is continuous for $-3 < x < 3$

Now, consider the point $x = -3$

LHL $= \lim_{x\to -3^-} f(x) = \lim_{h\to 0} f(-3-h) = \lim_{h\to 0} |-3-h| + 3 = \lim_{h\to 0} |3+h| + 3 = 6$

RHL $= \lim_{x\to -3^+} f(x) = \lim_{h\to 0} f(-3+h) = \lim_{h\to 0} -2(-3+h) = 6$

$f(-3) = |-3| + 3 = 6$

Thus, LHL = RHL = $f(-3) = 6$

So, the function is continuous at $x = -3$

Now, consider the point $x = 3$

LHL $= \lim_{x\to 3^-} f(x) = \lim_{h\to 0} f(3-h) = \lim_{h\to 0} -2(3-h) = -6$

Chapter 9 Continuity Ex 9.2 Q3(xi)

The given function is $f(x)=\begin{cases}2x, & \text{if } x<0\\ 0, & \text{if } 0\le x\le 1\\ 4x, & \text{if } x>1\end{cases}$

The given function is defined at all points of the real line.

Let c be a point on the real line.

Case I:

$$\text{If } c<0, \text{ then } f(c)=2c$$
$$\lim_{x\to c} f(x)=\lim_{x\to c}(2x)=2c$$
$$\therefore \lim_{x\to c} f(x)=f(c)$$

Therefore, f is continuous at all points x, such that $x<0$

Chapter 9 Continuity Ex 9.2 Q3(xii)

The given function f is $f(x)=\begin{cases}\sin x-\cos x, & \text{if } x\neq 0\\ -1 & \text{if } x=0\end{cases}$

It is evident that f is defined at all points of the real line.

Let c be a real number.

Case I:

$$\text{If } c\neq 0, \text{ then } f(c)=\sin c-\cos c$$
$$\lim_{x\to c} f(x)=\lim_{x\to c}(\sin x-\cos x)=\sin c-\cos c$$
$$\therefore \lim_{x\to c} f(x)=f(c)$$

Therefore, f is continuous at all points x, such that $x\neq 0$

Chapter 9 Continuity Ex 9.2 Q3(xiii)

The given function f is $f(x)=\begin{cases}-2, \text{ if } x\le -1\\ 2x, \text{ if } -1<x\le 1\\ 2, \text{ if } x>1\end{cases}$

The given function is defined at all points of the real line.

Let c be a point on the real line.

Case I:

If $c<-1$, then $f(c)=-2$ and $\lim\limits_{x\to c} f(x)=\lim\limits_{x\to c}(-2)=-2$

$\therefore \lim\limits_{x\to c} f(x)=f(c)$

Therefore, f is continuous at all points x, such that $x<-1$

Case II:

If $c=-1$, then $f(c)=f(-1)=-2$

The left hand limit of f at $x=-1$ is,

$\lim\limits_{x\to -1^-} f(x)=\lim\limits_{x\to -1^-}(-2)=-2$

The right hand limit of f at $x=-1$ is,

$\lim\limits_{x\to -1^+} f(x)=\lim\limits_{x\to -1^+}(2x)=2\times(-1)=-2$

$\therefore \lim\limits_{x\to -1} f(x)=f(-1)$

Therefore, f is continuous at $x=-1$

Case III:

If $-1<c<1$, then $f(c)=2c$

$\lim\limits_{x\to c} f(x)=\lim\limits_{x\to c}(2x)=2c$

$\therefore \lim\limits_{x\to c} f(x)=f(c)$

Therefore, f is continuous at all points of the interval $(-1, 1)$.

Case IV:

If $c=1$, then $f(c)=f(1)=2\times1=2$

The left hand limit of f at $x=1$ is,

$\lim\limits_{x\to1^-} f(x)=\lim\limits_{x\to1^-}(2x)=2\times1=2$

The right hand limit of f at $x=1$ is,

$\lim\limits_{x\to1^+} f(x)=\lim\limits_{x\to1^+}2=2$

$\therefore \lim\limits_{x\to1} f(x)=f(c)$

Therefore, f is continuous at $x=2$

Case V:

If $c>1$, then $f(c)=2$ and $\lim\limits_{x\to c} f(x)=\lim\limits_{x\to c}(2)=2$

$\lim\limits_{x\to c} f(x)=f(c)$

Therefore, f is continuous at all points x, such that $x>1$

Thus, from the above observations, it can be concluded that f is continuous at all points of the real line

*********** END **********