Binary Operations Ex 3.2 Q9

The binary operator $*$ defined as

$$a * b = \frac{a - b}{2} \text{ for all } a, b \in Q.$$

Now,

Associativity: Let $a, b, c \in Q$, then

$$\left(a * b\right) * c = \frac{a - b}{2} * c = \frac{\frac{a - b}{2} - c}{2}$$

$$= \frac{a - b - 2c}{4} \qquad --- (i)$$

and, $\quad a * \left(b * c\right) = a * \frac{b - c}{2} = \frac{a - \frac{b - c}{2}}{2}$

$$= \frac{2a - b + c}{4} = \qquad --- (ii)$$

From (i) & (ii)

$$\left(a * b\right) * c \neq a * \left(b * c\right)$$

Hence, '$*$' is not associative on Q.

Binary Operations Ex 3.2 Q10

The binary operator $*$ defined as

$$a * b = a + 3b - 4 \text{ for all } a, b \in Z$$

Now,

Commutativity: Let $a, b \in Z$, then

$$a * b = a + 3b - 4 \neq b + 3a - 4 = b * a$$

$\Rightarrow \qquad a * b \neq b * a$

$\Rightarrow \qquad$ '$*$' is not commutative on Z.

Associativity: Let $a, b, c \in Z$, then

$$\left(a * b\right) * c = \left(a + 3b - 4\right) * c = a + 3b - 4 + 3c - 4$$

$$= a + 3b + 3c - 8 \qquad --- (i)$$

and, $\quad a * \left(b * c\right) = a * \left(b + 3c - 4\right) = a + 3\left(b + 3c - 4\right) - 4$

$$= a + 3b + 9c - 16 \qquad --- (ii)$$

From (i) & (ii)

$$\left(a * b\right) * c \neq a * \left(b * c\right)$$

Hence, '$*$' is not associative on Z.

Binary Operations Ex 3.2 Q11

Q be the set of rational numbers and $*$ be a binary operation defined as

$$a * b = \frac{ab}{5} \text{ for all } a, b \in Q$$

Now,

Associativity: Let $a, b, c \in Q$, then

$$(a * b) * c = \frac{ab}{5} * c = \frac{abc}{25} \qquad\qquad ---(i)$$

and, $\quad a * (b * c) = a * \frac{bc}{5} = \frac{abc}{25} \qquad\qquad ---(ii)$

From (i) & (ii)

$\therefore \qquad (a * b) * c = a * (b * c)$

$\Rightarrow \qquad *$ is associative on Q.

Binary Operations Ex 3.2 Q12

The binary operator $*$ is defined as

$$a * b = \frac{ab}{7} \text{ for all } a, b \in Q$$

Now,

Associativity: Let $a, b, c \in Q$, then

$$(a * b) * c = \frac{ab}{7} * c = \frac{abc}{49} \qquad\qquad ---(i)$$

and, $\quad a * (b * c) = a * \frac{bc}{7} = \frac{abc}{49} \qquad\qquad ---(ii)$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

$\Rightarrow \qquad$ '$*$' is associative on Q.

Binary Operations Ex 3.2 Q13

The binary operator $*$ defined as

$$a * b = \frac{a+b}{2} \text{ for all } a, b \in Q.$$

Now,

Associativity: Let $a, b, c \in Q$, then

$$(a * b) * c = \frac{a+b}{2} * c = \frac{\frac{a+b}{2} + c}{2}$$

$$= \frac{a+b+2c}{4} \qquad\qquad ---(\text{i})$$

and, $\quad a * (b * c) = a * \frac{b+c}{2}$

$$= \frac{a + \frac{b+c}{2}}{2}$$

$$= \frac{2a+b+c}{4} = \qquad\qquad ---(\text{ii})$$

From (i) & (ii)

$$(a * b) * c \neq a * (b * c)$$

Hence, '$*$' is not associative on Q.

********** END *********