Polynomials Ex 2.1 Q17

Answer :

Given

$\alpha+\beta=24$ ......(i)

$\alpha-\beta=8$ ......(ii)

By subtracting equation (ii) from (i) we get

$$\begin{aligned}\alpha+\cancel{\beta}&=24\\ \alpha-\cancel{\beta}&=8\\ \hline 2\alpha&=32\end{aligned}$$

$$\alpha=\frac{32}{2}$$

$$\alpha=16$$

Substituting $\alpha=16$ in equation (i) we get,

$$\begin{aligned}\alpha+\beta&=24\\ 16+\beta&=24\\ \beta&=24-16\\ \beta&=8\end{aligned}$$

Let S and P denote respectively the sum and product of zeros of the required polynomial. then,

$$\begin{aligned}S&=\alpha+\beta\\ &=16+8\\ &=24\end{aligned}$$

$$\begin{aligned}P&=\alpha\beta\\ &=16\times8\\ &=128\end{aligned}$$

Hence, the required polynomial if $f(x)$ is given by

$$f(x)=k\left(x^2-Sx+P\right)$$

$$f(x)=k\left(x^2-24x+128\right)$$

Hence, required equation is $\boxed{f(x)=k\left(x^2-24x+128\right)}$ where k is any non-zeros real number.

Polynomials Ex 2.1 Q18

Answer :

Since α and β are the zeros of the quadratic polynomial $f(x)=x^2-p(x+1)-c$

Then

$$x^2-p(x+1)-c$$

$$x^2-px-p-c$$

$$\begin{aligned}\alpha+\beta&=\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}\\ &=\frac{-(-p)}{1}\\ &=p\end{aligned}$$

$$\begin{aligned}\alpha\beta&=\frac{\text{Constant term}}{\text{Coefficient of } x^2}\\ &=\frac{-p-c}{1}\\ &=-p-c\end{aligned}$$

We have to prove that $(\alpha+1)(\beta+1)=1-c$

$$\begin{aligned}(\alpha+1)(\beta+1)&=1-c\\ (\alpha+1)\beta+(\alpha+1)(1)&=1-c\\ \alpha\beta+\beta+\alpha+1&=1-c\\ \alpha\beta+(\alpha+\beta)+1&=1-c\end{aligned}$$

Substituting $\alpha+\beta=p$ and $\alpha\beta=-p-c$ we get,

$$-p-c+p+1=1-c$$

$$-\cancel{p}-c+\cancel{p}+1=1-c$$

$$1-c=1-c$$

Hence, it is shown that $\boxed{(\alpha+1)(\beta+1)=1-c}$.

*********** END **********