



Differentials Errors and Approximation Ex14.1 Q9(iv)

$$\text{Let } x = 400, x + \Delta x = 401$$

$$\Delta x = 401 - 400$$

$$\Delta x = 1$$

$$\text{Let } y = \sqrt{x}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\begin{aligned}\left(\frac{dy}{dx}\right)_{x=400} &= \frac{1}{2\sqrt{400}} \\ &= \frac{1}{40}\end{aligned}$$

$$\left(\frac{dy}{dx}\right)_{x=400} = 0.025$$

So,

$$\begin{aligned}\Delta y &= \left(\frac{dy}{dx}\right)_{x=400} \times \Delta x \\ &= (0.025)(1) \\ &= 0.025\end{aligned}$$

$$\begin{aligned}\sqrt{401} &= y + \Delta y \\ &= \sqrt{x} + 0.025 \\ &= \sqrt{400} + 0.025 \\ &= 20 + 0.025\end{aligned}$$

$$\sqrt{401} = 20.025$$

$$\text{Let } x = 16, x + \Delta x = 15$$

$$\Delta x = 15 - 16$$

$$\Delta x = -1$$

$$\text{Let } y = x^{\frac{1}{4}}$$

$$\frac{dy}{dx} = \frac{1}{4x^{\frac{3}{4}}}$$

$$\left(\frac{dy}{dx}\right)_{x=16} = \frac{1}{4(16)^{\frac{3}{4}}}$$

$$= \frac{1}{32}$$

$$= 0.03125$$

Now,

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=16} \times \Delta x$$

$$= (0.03125)(-1)$$

$$\Delta y = -0.03125$$

$$(15)^{\frac{1}{4}} = y + \Delta y$$

$$= (x)^{\frac{1}{4}} - 0.03125$$

$$= (16)^{\frac{1}{4}} - 0.03125$$

$$= 2 - 0.03125$$

$$(15)^{\frac{1}{4}} = 1.96875$$

Differentials Errors and Approximation Ex14.1 Q9(vi)

$$\text{Let } x = 256, \quad x + \Delta x = 255$$

$$\Delta x = 255 - 256$$

$$\Delta x = -1$$

$$\text{Let } y = x^{\frac{1}{4}}$$

$$\frac{dy}{dx} = \frac{1}{4x^{\frac{3}{4}}}$$

$$\begin{aligned} \left(\frac{dy}{dx} \right)_{x=256} &= \frac{1}{4(256)^{\frac{3}{4}}} \\ &= \frac{1}{256} \\ &= 0.00391 \end{aligned}$$

Now,

$$\begin{aligned} \Delta y &= \left(\frac{dy}{dx} \right)_{x=256} \times \Delta x \\ &= (0.00391)(-1) \\ \Delta y &= -0.00391 \end{aligned}$$

$$\begin{aligned} (255)^{\frac{1}{4}} &= y + \Delta y \\ &= (x)^{\frac{1}{4}} + (-0.00391) \end{aligned}$$

$$\begin{aligned}
 &= (256)^{\frac{1}{4}} - 0.00391 \\
 &= 4 - 0.00391
 \end{aligned}$$

$$(255)^{\frac{1}{4}} = 3.99609$$

Differentials Errors and Approximation Ex14.1 Q9(vii)

Let $x = 2, x + \Delta x = 2.002$

$$\Delta x = 2.002 - 2$$

$$\Delta x = 0.002$$

Let $y = \frac{1}{x^2}$

$$\frac{dy}{dx} = -\frac{2}{x^3}$$

$$\begin{aligned}
 \left(\frac{dy}{dx}\right)_{x=2} &= -\frac{2}{8} \\
 &= -0.25
 \end{aligned}$$

Now,

$$\begin{aligned}
 \Delta y &= \left(\frac{dy}{dx}\right)_{x=2} \times \Delta x \\
 &= (-0.25)(0.002) \\
 \Delta y &= -0.0005
 \end{aligned}$$

Now,

$$\begin{aligned}
 \frac{1}{(2.002)^3} &= y + \Delta y \\
 &= \frac{1}{8} + (-0.0005)
 \end{aligned}$$

$$\begin{aligned}
 & x^- \\
 &= \frac{1}{4} - 0.0005 \\
 &= 0.25 - 0.0005
 \end{aligned}$$

$$\frac{1}{(2.002)^3} = 0.2495$$

*****END*****