Arithmetic Progressions Ex 9.5 Q18

Answer :

In this problem, we need to find the sum of all the multiples of 9 lying between 100 and 550.
So, we know that the first multiple of 9 after 100 is 108 and the last multiple of 9 before 550 is 549.
Also, all these terms will form an A.P. with the common difference of 9.
So here,
First term (a) = 108
Last term (l) = 549
Common difference (d) = 9
So, here the first step is to find the total number of terms. Let us take the number of terms as n.
Now, as we know,

$a_n = a + (n-1)d$

So, for the last term,

$$549 = 108 + (n-1)9$$
$$549 = 108 + 9n - 9$$
$$549 = 99 + 9n$$
$$549 - 99 = 9n$$

Further simplifying,

$$450 = 9n$$
$$n = \frac{450}{9}$$
$$n = 50$$

Now, using the formula for the sum of n terms,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

We get,

$$S_n = \frac{50}{2}\left[2(108) + (50-1)9\right]$$
$$= 25\left[216 + (49)9\right]$$
$$= 25(216 + 441)$$
$$= 25(657)$$
$$= 16425$$

Therefore, the sum of all the multiples of 9 lying between 100 and 550 is $\boxed{S_n = 16425}$

Arithmetic Progressions Ex 9.5 Q19

Answer :

In the given problem, we need to find the number of terms of an A.P. Let us take the number of terms as n.
Here, we are given that,

$$a = 22$$
$$d = -4$$
$$S_n = 64$$

So, as we know the formula for the sum of n terms of an A.P. is given by,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Where; a = first term for the given A.P.
d = common difference of the given A.P.
n = number of terms
So, using the formula we get,

$$S_n = \frac{n}{2}\left[2(22) + (n-1)(-4)\right]$$
$$64 = \frac{n}{2}\left[44 - 4n + 4\right]$$
$$64(2) = n(48 - 4n)$$
$$128 = 48n - 4n^2$$

Further rearranging the terms, we get a quadratic equation,

$4n^2 - 48n + 128 = 0$

On taking 4 common, we get,

$n^2 - 12n + 32 = 0$

Further, on solving the equation for n by splitting the middle term, we get,

$$\begin{aligned} n^2 - 12n + 32 &= 0 \\ n^2 - 8n - 4n + 32 &= 0 \\ n(n-8) - 4(n-8) &= 0 \\ (n-8)(n-4) &= 0 \end{aligned}$$

So, we get,

$(n-8) = 0$

$n = 8$

Also,

$(n-4) = 0$

$n = 4$

Therefore, $\boxed{n = 4 \text{ or } 8}$

Arithmetic Progressions Ex 9.5 Q20

Answer :

In the given problem, let us take the first term as a and the common difference d

Here, we are given that,

$a_5 = 30 \quad \text{......}(1)$

$a_{12} = 65 \quad \text{......}(2)$

Also, we know,

$a_n = a + (n-1)d$

For the 5th term ($n = 5$),

$a_5 = a + (5-1)d$

$30 = a + 4d \quad (\text{Using } 1)$

$a = 30 - 4d \quad \text{......}(3)$

Similarly, for the 12th term ($n = 12$),

$a_{12} = a + (12-1)d$

$65 = a + 11d \quad (\text{Using } 2)$

$a = 65 - 11d \quad \text{......}(4)$

Subtracting (3) from (4), we get,

$$\begin{aligned} a - a &= (65 - 11d) - (30 - 4d) \\ 0 &= 65 - 11d - 30 + 4d \\ 0 &= 35 - 7d \\ 7d &= 35 \\ d &= 5 \end{aligned}$$

Now, to find a, we substitute the value of d in (4),

$a = 30 - 4(5)$

$a = 30 - 20$

$a = 10$

So, for the given A.P $d = 5$ and $a = 10$

So, to find the sum of first 20 terms of this A.P., we use the following formula for the sum of n terms of an A.P.,

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

Where; a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

So, using the formula for $n = 20$, we get,

$$\begin{aligned} S_{20} &= \frac{20}{2}\left[2(10) + (20-1)(5)\right] \\ &= (10)\left[20 + (19)(5)\right] \\ &= (10)\left[20 + 95\right] \\ &= (10)\left[115\right] \\ &= 1150 \end{aligned}$$

Therefore, the sum of first 20 terms for the given A.P. is $\boxed{S_{20} = 1150}$.

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