Indefinite Integrals Ex 19.8 Q16

$$\text{Let } I = \int \frac{1}{x\left(3+\log x\right)}dx \text{ - - - - - } (i)$$

$$\text{Let } 3+\log x = t \quad \text{then,}$$
$$d\left(3+\log x\right) = dt$$
$$\Rightarrow \quad \frac{1}{x}dx = dt$$
$$\Rightarrow \quad dx = x\,dt$$

Putting $3+\log x = t$ and $dx = x\,dt$ in equation (i), we get,

$$I = \int \frac{1}{x \times t} \times x dt$$
$$= \int \frac{dt}{t}$$
$$= \log|t| + c$$

$$= \log\left|\left(3+\log x\right)\right| + c$$
$$\therefore \quad I = \log\left|\left(3+\log x\right)\right| + c$$

Indefinite Integrals Ex 19.8 Q17

$$\text{Let } I = \int \frac{e^x+1}{e^x+x}dx \text{ - - - - - } (i)$$

$$\text{Let } e^x + x = t \quad \text{then,}$$
$$d\left(e^x+x\right) = dt$$
$$\Rightarrow \quad \left(e^x+x\right)dx = dt$$
$$\Rightarrow \quad dx = \frac{dt}{e^x+1}$$

Putting $e^x + x = t$ and $dx = \frac{dt}{e^x+1}$ in equation (i), we get,

$$I = \int \frac{e^x+1}{t} \times \frac{dt}{e^x+1}$$
$$= \int \frac{dt}{t}$$
$$= \log|t| + c$$

$$= \log\left|e^x+x\right| + c$$
$$\therefore \quad I = \log\left|e^x+x\right| + c$$

Indefinite Integrals Ex 19.8 Q18

$$\text{Let } I = \int \frac{1}{x \log x} dx \text{ ----- (i)}$$

$$\text{Let } \log x = t \quad \text{then,}$$

$$d(\log x) = dt$$

$$\Rightarrow \quad \frac{1}{x} dx = dt$$

$$\Rightarrow \quad dx = x\, dt$$

Putting $\log x = t$ and $dx = x\, dt$ in equation (i), we get,

$$I = \int \frac{1}{x \times t} \times x\, dt$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + c$$

$$= \log|(\log x)| + c$$

$$\therefore \quad I = \log|(\log x)| + c$$

Indefinite Integrals Ex 19.8 Q19

$$\text{Let } I = \int \frac{\sin 2x}{a\cos^2 x + b\sin^2 x} dx \text{ ----- (i)}$$

$$\text{Let } a\cos^2 x + b\sin^2 x = t \quad \text{then,}$$

$$d(a\cos^2 x + b\sin^2 x) = dt$$

$$\left[a(2\cos x(-\sin x)) + b(2\sin x \cos x)\right] dx = dt$$

$$\Rightarrow \quad \left[-a(2\sin x \cos x) + b(2\sin x \cos x)\right] dx = dt$$

$$\Rightarrow \quad \left[-a\sin 2x + b\sin 2x\right] dx = dt$$

$$\Rightarrow \quad \sin 2x (b - a)\, dx = dt$$

$$\Rightarrow \quad dx = \frac{dt}{(b-a)\sin 2x}$$

Putting $a\cos^2 x + b\sin^2 x = t$ and $dx = \frac{dt}{(b-a)\sin 2x}$ in equation (i), we get,

$$I = \int \frac{\sin 2x}{t} \times \frac{dt}{(b-a)\sin 2x}$$

$$= \frac{1}{b-a} \int \frac{dt}{t}$$

$$= \frac{1}{b-a} \log|t| + c$$

$$= \frac{1}{b-a} \log|a\cos^2 x + b\sin^2 x| + c$$

Indefinite Integrals Ex 19.8 Q20

Let $I = \int \frac{\cos x}{2+3\sin x} dx$ ----- (i)

Let $2+3\sin x = t$ then,

$d(2+3\sin x) = dt$

$\Rightarrow \quad 3\cos x dx = dt$

$\Rightarrow \quad dx = \frac{dt}{3\cos x}$

Putting $2+3\sin x = t$ and $dx = \frac{dt}{3\cos x}$ in equation (i), we get,

$$I = \int \frac{\cos x}{t} \times \frac{dt}{3\cos x}$$

$$= \frac{1}{3}\int \frac{dt}{t}$$

$$= \frac{1}{3}\log|t| + c$$

$$= \frac{1}{3}\log|2+3\sin x| + c$$

********** END **********