Combinations Ex 17.2 Q17

Total men = 6

Total women = 4

Total persons in committee = 5

(where at least are women has to be selected)

This can be done in

$$^4C_1 \times {}^6C_4 + {}^4C_2 \times {}^6C_3 + {}^4C_3 \times {}^6C_2 + {}^4C_4 \times {}^6C_1$$

$$\left({}^nC_r = \frac{n!}{r!(n-r)!}\right) \quad \left({}^nC_r = 1,\ {}^nC_1 = n\right)$$

$$= \left(\frac{4 \times 6!}{4! \times 2!}\right) + \left(\frac{4!}{2!\ 2!} \times \frac{6!}{3!\ 3!}\right) + \left(\frac{4!}{3!\ 1!} \times \frac{6!}{2!\ 4!}\right) + (1 \times 6)$$

$$= \left(\frac{4 \times 6 \times 5}{2}\right) + \left(\frac{4 \times 3}{2} \times \frac{6 \times 5 \times 4}{3 \times 2}\right) + \left(\frac{4 \times 6 \times 5}{2}\right) + (6)$$

$$= (60) + (120) + 60 + 6$$

$$= 246 \text{ ways}$$

Combinations Ex 17.2 Q18

52 families have at most 2 children, while 35 families have 2 children.
The selection of 20 families of which at least 18 families must have at most 2 children can be made as under

i) 18 families out of 52 and 2 families out of 35

ii) 19 families out of 52 and 1 family out of 35

iii) 20 families out of 52

Therefore the number of ways are $= {}^{52}C_{18} \times {}^{35}C_2 + {}^{52}C_{19} \times {}^{35}C_1 + {}^{52}C_{20} \times {}^{35}C_0$

Combinations Ex 17.2 Q19

i) Since, the team does not include any girl therefore, only boys are to be selected.

5 boys out of 7 boys can be selected in ${}^7C_5$ ways.

$$= {}^7C_5 = \frac{7!}{5!\,2!} = \frac{6 \times 7}{2} = 21$$

ii) Since, at least one boy and one girl are to be there in every team. The team consist of

a) 1 boy and 4 girls i.e. ${}^7C_1 \times {}^4C_4$

b) 2 boys and 3 girls i.e. ${}^7C_2 \times {}^4C_3$

c) 3 boys and 2 girls i.e. ${}^7C_3 \times {}^4C_2$

d) 4 boys and 1 girls i.e. ${}^7C_4 \times {}^4C_1$

$\therefore$ The required number of ways

$$= {}^7C_1 \times {}^4C_4 + {}^7C_2 \times {}^4C_3 + {}^7C_3 \times {}^4C_2 + {}^7C_4 \times {}^4C_1$$

$$= 7 + 84 + 210 + 140$$

$$= 441$$

iii) Since, the team has to consist of at least 3 girls, the team can consist of

a) 3 girls and 2 boys = ${}^7C_2 \times {}^4C_3$ ways

b) 4 girls and 1 boy = ${}^4C_4 \times {}^7C_1$, ways

$\therefore$ The required number of ways

$$= {}^4C_3 \times {}^7C_2 + {}^4C_4 \times {}^7C_1$$

$$= 84 + 7$$

$$= 91$$

Combinations Ex 17.2 Q20

The number of ways selecting of 3 people out of 5

$$= {}^5C_3 = \frac{5!}{3!\,2!} = \frac{5 \times 4}{2} = 10.$$

1 man can be selected from 2 men in ${}^2C_1$ ways and 2 women can be selected from 3 women in ${}^3C_2$ ways.

$\therefore$ The required number of committees

$$= {}^2C_1 \times {}^3C_2$$

$$= \frac{2!}{1!\,1!} \times \frac{3!}{2!\,1!}$$

$$= 6$$

********** END **********