



Definite Integrals Ex 20.1 Q39

$$\begin{aligned}
 \int_0^2 \frac{dx}{x+4-x^2} &= \int_0^2 \frac{dx}{-(x^2-x-4)} \\
 &= \int_0^2 \frac{dx}{-\left(x^2-x+\frac{1}{4}-\frac{1}{4}-4\right)} \\
 &= \int_0^2 \frac{dx}{-\left[\left(x-\frac{1}{2}\right)^2-\frac{17}{4}\right]} \\
 &= \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2}
 \end{aligned}$$

$$\text{Let } x-\frac{1}{2}=t \Rightarrow dx = dt$$

$$\text{When } x=0, t=-\frac{1}{2} \text{ and when } x=2, t=\frac{3}{2}$$

$$\therefore \int_0^2 \frac{dx}{\left(\frac{\sqrt{17}}{2}\right)^2-\left(x-\frac{1}{2}\right)^2} = \int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{dt}{\left(\frac{\sqrt{17}}{2}\right)^2-t^2}$$

$$\begin{aligned}
 &= \left[\frac{1}{2\left(\frac{\sqrt{17}}{2}\right)} \log \frac{\frac{\sqrt{17}}{2}+t}{\frac{\sqrt{17}}{2}-t} \right]_{-\frac{1}{2}}^{\frac{3}{2}} \\
 &= \frac{1}{\log} \left[\log \frac{\frac{\sqrt{17}}{2}+\frac{3}{2}}{\frac{\sqrt{17}}{2}-\frac{3}{2}} - \log \frac{\frac{\sqrt{17}}{2}-\frac{1}{2}}{\frac{\sqrt{17}}{2}+\frac{1}{2}} \right]
 \end{aligned}$$

$$= \frac{1}{\sqrt{17}} \left[\log \frac{\sqrt{17}-3}{2} - \log \frac{\sqrt{17}+1}{2} \right]$$

$$= \frac{1}{\sqrt{17}} \left[\log \frac{\sqrt{17}+3}{\sqrt{17}-3} - \log \frac{\sqrt{17}-1}{\sqrt{17}+1} \right]$$

$$= \frac{1}{\sqrt{17}} \log \frac{\sqrt{17}+3}{\sqrt{17}-3} \times \frac{\sqrt{17}+1}{\sqrt{17}-1}$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{17+3+4\sqrt{17}}{17+3-4\sqrt{17}} \right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{20+4\sqrt{17}}{20-4\sqrt{17}} \right]$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{5+\sqrt{17}}{5-\sqrt{17}} \right)$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{(5+\sqrt{17})(5+\sqrt{17})}{25-17} \right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{25+17+10\sqrt{17}}{8} \right]$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{42+10\sqrt{17}}{8} \right)$$

$$= \frac{1}{\sqrt{17}} \log \left(\frac{21+5\sqrt{17}}{4} \right)$$

Definite Integrals Ex 20.1 Q40

We have,

$$\begin{aligned}
 & \int_0^1 \frac{1}{2x^2 + x + 1} dx \\
 &= \frac{1}{2} \int_0^1 \frac{1 dx}{\left(x^2 + \frac{1}{2}x + \frac{1}{2}\right)} \\
 &= \frac{1}{2} \int_0^1 \frac{dx}{\left(x + \frac{1}{4}\right)^2 + \frac{1}{2} - \frac{1}{16}} \quad \left[\text{Adding } \frac{1}{16} \text{ \& subtracting } \frac{1}{16} \text{ in numerator} \right] \\
 &= \frac{1}{2} \int_0^1 \frac{dx}{\left(x + \frac{1}{4}\right)^2 + \frac{7}{16}} \\
 &= \frac{1}{2} \int_0^1 \frac{dx}{\left(x + \frac{1}{4}\right)^2 + \left(\frac{\sqrt{7}}{4}\right)^2} \\
 &= \frac{1}{2} \cdot \frac{4}{\sqrt{7}} \left[\tan^{-1} \left(\frac{x + \frac{1}{4}}{\frac{\sqrt{7}}{4}} \right) \right]_0^1 \\
 &= \frac{2}{\sqrt{7}} \left\{ \tan^{-1} \frac{5}{\sqrt{7}} - \tan^{-1} \left(\frac{1}{\sqrt{7}} \right) \right\} \\
 \therefore \int_0^1 \frac{1}{2x^2 + x + 1} dx &= \frac{2}{\sqrt{7}} \left\{ \tan^{-1} \frac{5}{\sqrt{7}} - \tan^{-1} \left(\frac{1}{\sqrt{7}} \right) \right\}
 \end{aligned}$$

Definite Integrals Ex 20.1 Q41

$$\text{Let } I = \int_0^1 \sqrt{x(1-x)} dx$$

$$\begin{aligned}
 \text{let } x &= \sin^2 \theta \\
 \Rightarrow dx &= 2 \sin \theta \cdot \cos \theta d\theta
 \end{aligned}$$

Now,

$$x = 0 \Rightarrow \theta = 0$$

$$x = 1 \Rightarrow \theta = \frac{\pi}{2}$$

$$\begin{aligned}
 \therefore I &= \int_0^{\frac{\pi}{2}} \sqrt{\sin^2 \theta (1 - \sin^2 \theta)} \cdot 2 \sin \theta \cdot \cos \theta d\theta \\
 &= \int_0^{\frac{\pi}{2}} 2 \sin^2 \theta \cdot \cos^2 \theta d\theta \\
 &= \frac{1}{2} \int_0^{\frac{\pi}{2}} 4 \sin^2 \theta \cdot \cos^2 \theta d\theta \\
 &= \frac{1}{2} \int_0^{\frac{\pi}{2}} (\sin^2 2\theta) d\theta
 \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2} \int_0^{\frac{\pi}{2}} \left(\frac{1 - \cos 4\theta}{2} \right) d\theta \\
&= \frac{1}{4} \int_0^{\frac{\pi}{2}} (1 - \cos 4\theta) d\theta \\
&= \frac{1}{4} \int_0^{\frac{\pi}{2}} d\theta - \frac{1}{4} \int_0^{\frac{\pi}{2}} \cos 4\theta d\theta \\
&= \frac{1}{4} [\theta]_0^{\frac{\pi}{2}} - \frac{1}{4} \left[\frac{\sin 4\theta}{4} \right]_0^{\frac{\pi}{2}} \\
&= \frac{1}{4} \left[\frac{\pi}{2} - 0 \right] - \frac{1}{16} [\sin \pi - \sin 0] \\
&= \frac{\pi}{8} - \frac{1}{16} [0 - 0] \\
&= \frac{\pi}{8} \\
I &= \frac{\pi}{8}
\end{aligned}$$

$$\therefore \int_0^1 \sqrt{x(1-x)} dx = \frac{\pi}{8}$$

Definite Integrals Ex 20.1 Q42

We have,

$$\int_0^2 \frac{dx}{\sqrt{3+2x-x^2}}$$

$$\int_0^2 \frac{dx}{\sqrt{3+1-(x^2-2x+1)}} \quad [\text{Add and subtract 1 in denominator}]$$

$$= \int_0^2 \frac{dx}{\sqrt{(2)^2(x-1)^2}} \quad \left[\because \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} \right]$$

$$= \left[\sin^{-1} \left(\frac{x-1}{2} \right) \right]_0^2$$

$$= \sin^{-1} \frac{1}{2} - \sin^{-1} \left(\frac{-1}{2} \right)$$

$$= \sin^{-1} \left(\sin \frac{\pi}{6} \right) - \sin^{-1} \left[\sin \left(\frac{-\pi}{6} \right) \right]$$

$$= \frac{\pi}{6} + \frac{\pi}{6}$$

$$= \frac{\pi}{3}$$

$$\therefore \int_0^2 \frac{dx}{\sqrt{3+2x-x^2}} = \frac{\pi}{3}$$

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