Chapter 6 Determinants Ex 6.2 Q25

We need to prove the following identity:

$$\begin{vmatrix} x+4 & x & x \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix} = 16(3x+4)$$

Let us consider the L.H.S of the above equation.

$$\Delta = \begin{vmatrix} x+4 & x & x \\ x & x+4 & x \\ x & x & x+4 \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, *we get,*

$$\Delta = \begin{vmatrix} 3x+4 & x & x \\ 3x+4 & x+4 & x \\ 3x+4 & x & x+4 \end{vmatrix}$$

Taking the common term $3x+4$, *we get,*

$$\Delta = (3x+4)\begin{vmatrix} 1 & x & x \\ 1 & x+4 & x \\ 1 & x & x+4 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ *and* $R_3 \to R_3 - R_1$, *we get,*

$$\Delta = (3x+4)\begin{vmatrix} 1 & x & x \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{vmatrix}$$

$$\Rightarrow \Delta = 16(3x+4)$$

Chapter 6 Determinants Ex 6.2 Q26

We need to prove the following identity:

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1$$

Let us consider the L.H.S of the above equation.

Applying $C_2 \to C_2 - pC_1$ *and* $C_3 \to C_3 - qC_1$, *we get*

$$\Delta = \begin{vmatrix} 1 & 1 & 1+p \\ 2 & 3 & 4+3p \\ 3 & 6 & 10+6p \end{vmatrix}$$

Applying $C_3 \to C_3 - pC_2$, *we get*

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 6 & 10 \end{vmatrix}$$

Applying $C_2 \to C_2 - pC_1$ *and* $C_3 \to C_3 - qC_1$, *we get*

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 2 \\ 3 & 3 & 7 \end{vmatrix}$$

$$\Rightarrow \Delta = 1[7-6] = 1$$

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$$\begin{vmatrix} a & b-c & c-b \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix}$$

$R_1 \to R_1 - R_2 - R_3$

$$= \begin{vmatrix} -a+c+b & -b-c+a & -c-b+a \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix}$$

$$= (b+c-a)\begin{vmatrix} 1 & -1 & -1 \\ a-c & b & c-a \\ a-b & b-a & c \end{vmatrix}$$

$$= (b+c-a)\begin{vmatrix} 1 & 0 & 0 \\ a-c & b+a-c & 0 \\ a-b & 0 & c+a-b \end{vmatrix}$$

$$= (a+b-c)(b+c-a)(c+a-b)$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q28

$$
\begin{aligned}
LHS &= \begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix} \\
&= \begin{vmatrix} a^2+b^2+2ab & 2ab & b^2 \\ a^2+b^2+2ab & a^2 & 2ab \\ a^2+b^2+2ab & b^2 & a^2 \end{vmatrix} \\
&= \left(a^2+b^2+2ab\right) \begin{vmatrix} 1 & 2ab & b^2 \\ 1 & a^2 & 2ab \\ 1 & b^2 & a^2 \end{vmatrix} \\
&= \left(a^2+b^2+2ab\right) \begin{vmatrix} 1 & 2ab & b^2 \\ 0 & a^2-2ab & 2ab-b^2 \\ 0 & b^2-2ab & a^2-b^2 \end{vmatrix} \\
&= \left(a^2+b^2+2ab\right) \begin{vmatrix} 1 & 2ab & b^2 \\ 0 & a^2-b^2 & 2ab-a^2 \\ 0 & b^2-2ab & a^2-b^2 \end{vmatrix} \\
&= (a+b)^2 \left[\left(a^2-b^2\right)\left(a^2-b^2\right)-\left(2ab-a^2\right)\left(b^2-2ab\right)\right] \\
&= (a+b)^2 \left(a^2+b^2-ab\right)^2 \\
&= \left(a^3+b^3\right)^2 \\
&= RHS
\end{aligned}
$$

********** END **********