Definite Integrals Ex 20.5 Q1

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----+f(a+(n-1)h)\right], \text{ where } h=\frac{b-a}{n}$$

Here, $a=0,\ b=3$ and $f(x)=(x+4)$

$h=\frac{3}{n}\Rightarrow nh=3$

Thus, we have,

$$\Rightarrow I=\int_0^3 (x+4)\,dx$$

$$\Rightarrow I=\lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f((n-1)h)\right]$$

$$\Rightarrow I=\lim_{h\to 0} h\left[4+(h+4)+(2h+4)+-----+((n-1)h+4)\right]$$

$$\Rightarrow I=\lim_{h\to 0} h\left[4n+h(1+2+3+-----+(n-1))\right]$$

$$\Rightarrow I=\lim_{h\to 0} h\left[4n+h\left(\frac{n(n-1)}{2}\right)\right] \qquad \left[\because h\to 0\ \&\ h=\frac{3}{n}\Rightarrow n\to\infty\right]$$

$$\Rightarrow I=\lim_{n\to\infty}\frac{3}{n}\left[4n+\frac{3}{n}\frac{(n^2-1)}{2}\right]$$

$$\Rightarrow I=\lim_{n\to\infty} 12+\frac{9}{2}\left(1-\frac{1}{n}\right)$$

$$=12+\frac{9}{2}=\frac{33}{2}$$

$$\therefore \int_0^3 (x+4)\,dx=\frac{33}{2}$$

Definite Integrals Ex 20.5 Q2

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----+f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 0,\ b = 2$

$$\Rightarrow h = \frac{2}{n}\ \&\ f(x) = x+3$$

Thus, we have,

$$I = \int_0^2 (x+3)\,dx$$

$$\Rightarrow I = \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f((n-1)h)\right]$$

$$\Rightarrow I = \lim_{h\to 0} h\left[3+(h+3)+(2h+3)+(3h+3)----+(n-1)h+3\right]$$

$$= \lim_{h\to 0} h\left[3n+h(1+2+3+----(n-1))\right]$$

$$= \lim_{h\to 0} h\left[3n+h\frac{n(n-1)}{2}\right]$$

$$\because h = \frac{2}{n}\ \&\text{if } h\to 0 \Rightarrow n\to\infty$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[3n+\frac{2}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} \left[6+\frac{2}{n}n^2\left(1-\frac{1}{n}\right)\right]$$

$$= 6+2 = 8$$

$$\therefore \int_0^2 (x+3)\,dx = 8$$

Definite Integrals Ex 20.5 Q3

We have,

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----+f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 1,\ b = 3$ and $f(x) = 3x-2$

$$h = \frac{2}{n} \Rightarrow nh = 2$$

Thus, we have,

$$I = \int_1^3 (3x-2)\,dx$$

$$\Rightarrow I = \lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(1+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[1+\{3(1+h)-2\}+\{3(1+2h)-2\}+-----+\ \{3(1+(n-1)h)-2\}\right]$$

$$= \lim_{h\to 0} h\left[n+3h(1+2+3+----(n-1))\right]$$

$$= \lim_{h\to 0} h\left[n+3h\frac{n(n-1)}{2}\right]$$

$$\because h = \frac{2}{n} \qquad \therefore \text{if } h\to 0 \Rightarrow n\to\infty$$

$$\therefore \lim_{n\to\infty} \frac{2}{n}\left[n+\frac{6}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} 2+\frac{6}{n^2}n^2\left(1-\frac{1}{n}\right)$$

$$= \lim_{n\to\infty} 2+6 = 8$$

$$\therefore \int_1^3 (3x-2)\,dx = 8$$

Definite Integrals Ex 20.5 Q4

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----+f(a+(n-1)h)\right]$$

where $h = \dfrac{b-a}{n}$

Here $a=-1$, $b=1$ and $f(x)=x+3$

$$\therefore h = \frac{2}{n} \Rightarrow nh = 2$$

Thus, we have,

$$I = \int_{-1}^{1}(x+3)dx$$

$$I = \lim_{h\to 0} h\left[f(-1)+f(-1+h)+f(-1+2h)+----+f(-1+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[2+(2+h)+(2+2h)+----+\{(n-1)h+2\}\right]$$

$$= \lim_{h\to 0} h\left[2n+h(1+2+3+----)\right]$$

$$= \lim_{h\to 0} h\left[2n+h\frac{n(n-1)}{2}\right] \qquad \left[\because h=\frac{2}{n} \text{ \& if } h\to 0 \Rightarrow n\to\infty\right]$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[2n+\frac{2}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} 4+\frac{2n^2}{n^2}\left(1-\frac{1}{n}\right)$$

$$= 4+2 = 6$$

$$\therefore \int_{-1}^{1}(x+3)dx = 6$$

*********** END **********