

Differentiation Ex 11.5 Q18(vii)

Let
$$y = (\cos x)^{x} + (\sin x)^{\frac{1}{x}}$$

 $y = e^{\log(\cos x)^{x}} + e^{\log(\sin^{\frac{1}{x}})}$ [Since, $\log a^{x} = b \log a$, $e^{\log a} = a$]
 $y = e^{x \log(\cos x)} + e^{\frac{1}{x} \log\sin x}$

Differentiating it with respect to \boldsymbol{x} using chain rule and product rule,

$$\begin{split} &\frac{dy}{dx} = \frac{d}{dx} e^{x\log\cos x} + \frac{d}{dx} e^{\frac{1}{k}\log\sin x} \\ &= e^{x\log\cos x} \times \frac{d}{dx} (x\log x) + e^{\frac{1}{k}\log\sin x} \frac{d}{dx} \Big(\frac{1}{x}\log\sin x\Big) \\ &= e^{\log(\cos x)} \times \left[x \frac{d}{dx} \log\cos x + \log\cos x \times \frac{d}{dx}(x) \right] + e^{\log(\sin x)^{\frac{1}{k}}} \times \left[\frac{1}{x} \frac{d}{dx} \log\sin x + \log\sin x \frac{d}{dx} \left(\frac{1}{x}\right) \right] \\ &= (\cos x)^{\frac{1}{k}} \left[x \times \left(\frac{1}{\cos x}\right) \frac{d}{dx} \cos x + \log\cos x (1) \right] + (\sin)^{\frac{1}{k}} \left[\frac{1}{x} \times \frac{1}{\sin x} \times \frac{d}{dx} (\sin x) + \log\sin x \left(-\frac{1}{x^2}\right) \right] \\ &= (\cos x)^{\frac{1}{k}} \left[x \times \left(\frac{1}{\cos x}\right) (-\sin x) + \log\cos x \right] + (\sin x)^{\frac{1}{k}} \left[\frac{1}{x} \times \frac{1}{\sin x} (\cos x) - \frac{1}{x^2} \log\sin x \right] \\ &\frac{dy}{dx} = (\cos x)^{\frac{1}{k}} \left[\log\cos x - x \tan x \right] + (\sin x)^{\frac{1}{k}} \left[\frac{\cot x}{x} - \frac{1}{x^2} \log\sin x \right] \end{split}$$

Differentiation Ex 11.5 Q18(vii)

Let
$$y = x^{x^2-3} + (x-3)^{x^2}$$

Also, let $u = x^{x^2-3}$ and $v = (x-3)^{x^2}$
 $\therefore y = u + v$

Differentiating both sides with respect to x, we obtain

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad \dots (1)$$

$$u = x^{x^2 - 3}$$

$$\therefore \log u = \log(x^{x^2 - 3})$$

$$\log u = (x^2 - 3) \log x$$

Differentiating with respect to x, we obtain

$$\frac{1}{u} \cdot \frac{du}{dx} = \log x \cdot \frac{d}{dx} (x^2 - 3) + (x^2 - 3) \cdot \frac{d}{dx} (\log x)$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = \log x \cdot 2x + (x^2 - 3) \cdot \frac{1}{x}$$

$$\Rightarrow \frac{du}{dx} = x^{x^2 - 3} \cdot \left[\frac{x^2 - 3}{x} + 2x \log x \right]$$

Also,

$$v = (x-3)^{x^2}$$

$$\therefore \log v = \log(x-3)^{x^2}$$

$$\Rightarrow \log v = x^2 \log(x-3)$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v} \cdot \frac{dv}{dx} = \log(x-3) \cdot \frac{d}{dx} (x^2) + x^2 \cdot \frac{d}{dx} \left[\log(x-3) \right]$$

$$\Rightarrow \frac{1}{v} \frac{dv}{dx} = \log(x-3) \cdot 2x + x^2 \cdot \frac{1}{x-3} \cdot \frac{d}{dx} (x-3)$$

$$\Rightarrow \frac{dv}{dx} = v \left[2x \log(x-3) + \frac{x^2}{x-3} \cdot 1 \right]$$

$$\Rightarrow \frac{dv}{dx} = (x-3)^{s^2} \left[\frac{x^2}{x-3} + 2x \log(x-3) \right]$$

Substituting the expressions of $\frac{du}{dx}$ and $\frac{dv}{dx}$ in equation (1), we obtain

$$\frac{dy}{dx} = x^{x^2 - 3} \left[\frac{x^2 - 3}{x} + 2x \log x \right] + (x - 3)^{x^2} \left[\frac{x^2}{x - 3} + 2x \log(x - 3) \right]$$

Here.

$$y = e^{x} + 10^{x} + x^{x}$$

$$= e^{x} + 10^{x} + e^{\log x^{x}}$$

$$y = e^{x} + 10^{x} + e^{x \log x}$$
[Since, $e^{\log_{a} x} = a, \log x^{b} = b \log x$]

Differentiating it with respect to x using product rule, chain rule,

$$\begin{split} &\frac{dy}{dx} = \frac{d}{dx} \left\{ e^x \right\} + \frac{d}{dx} \left\{ 10^x \right\} + \frac{d}{dx} \left\{ e^{x \log x} \right\} \\ &= e^x + 10^x \log 10 + e^{x \log x} \frac{d}{dx} \left(x \log x \right) \\ &= e^x + 10^x \log 10 + e^{x \log x} \left[x \times \frac{d}{dx} \left(\log x \right) + \log x \frac{d}{dx} \left(x \right) \right] \\ &= e^x + 10^x \log 10 + e^{\log x^x} \left[x \left(\frac{1}{x} \right) + \log x \left(1 \right) \right] \\ &= e^x + 10^x \log 10 + x^x \left[1 + \log x \right] \\ &= e^x + 10^x \log 10 + x^x \left[\log e + \log x \right] \end{split} \qquad \left[\text{Since, } \log_e e = 1 \right] \\ &\frac{dy}{dx} = e^x + 10^x \log 10 + x^x \left(\log e x \right) \end{aligned} \qquad \left[\text{Since } \log A + \log B = \log AB \right] \end{split}$$

Differentiation Ex 11.5 Q20

Here,

$$y = x^n + n^x + x^x + n^n$$

$$y = x^n + n^n + e^{\log x^x} + n^n$$

$$y = x^n + n^x + e^{\log x^x} + n^n$$
[Since, $e^{\log x^x} = a$ and $\log a^b = b \log a$]

Differentiating it with respect to \boldsymbol{x} using chain rule and product rule,

$$\begin{split} \frac{dy}{dx} &= \frac{d}{dx} \left\{ x^n \right\} + \frac{d}{dx} \left(n^x \right) + \frac{d}{dx} \left\{ e^{x \log x} \right\} + \frac{d}{dx} \left\{ n^n \right\} \\ &= nx^{n-1} + n^x \log n + e^{\log x^x} \left[d \frac{d}{dx} \log x + \log x \frac{d}{dx} (1) \right] \\ &= nx^{n-1} + n^x \log n + x^x \left[x \left(\frac{1}{x} \right) + \log x \right] \\ &= nx^{n-1} + n^x \log n + x^x \left[1 + \log x \right] \\ &= nx^{n-1} + n^x \log n + x^x \left[\log e + \log x \right] \end{split} \qquad \text{[Since, } \log_e e = 1 \text{ and } \log A + \log B = \log \left(AB \right) \text{]} \\ \frac{dy}{dx} &= nx^{n-1} + n^x \log n + x^x \log \left(ex \right) \end{split}$$

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