Tangents and Normals Ex 16.3 Q8(i)

We have,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \text{---(i)}$$

$$xy = c^2 \qquad \text{---(ii)}$$

Slope of (i)

$$\frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\therefore \qquad m_1 = \frac{dy}{dx} = \frac{x}{y} \times \frac{b^2}{a^2}$$

Slope of (ii)

$$y + x\frac{dy}{dx} = 0$$

$$\therefore \qquad m_2 = \frac{dy}{dx} = \frac{-y}{x}$$

$\therefore$ (i) and (ii) cuts orthogonally

$$\therefore \qquad m_1 \times m_2 = -1$$

$$\Rightarrow \qquad \frac{x}{y} \times \frac{-y}{x} \times \frac{a^2}{b^2} = -1$$

$$\Rightarrow \qquad a^2 = b^2$$

Tangents and Normals Ex 16.3 Q8(ii)

We have,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{---(i)}$$

$$\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1 \qquad \text{---(ii)}$$

Slope of (i)

$$\frac{2x}{a^2} + \frac{2y}{b^2} \times \frac{dy}{dx} = 0$$

$a^2 \quad b^2 \quad ax$

$\therefore \qquad m_1 = \frac{dy}{dx} = -\frac{x}{y}\frac{b^2}{a^2}$

Slope of (ii)

$$\frac{2x}{A^2} - \frac{2y}{B^2} \times \frac{dy}{dx} = 0$$

$\therefore \qquad m_2 = \frac{dy}{dx} = \frac{x}{y} \times \frac{B^2}{A^2}$

$\therefore$ (i) and (ii) cuts orthogonally

$\therefore \qquad m_1 \times m_2 = -1$

$\therefore \qquad \frac{-x}{y}\frac{b^2}{a^2} \times \frac{x}{y} \times \frac{B^2}{A^2} = -1$

$\Rightarrow \qquad \frac{x^2}{y^2} \times \frac{b^2 B^2}{a^2 A^2} = 1$

$\Rightarrow \qquad \frac{x^2}{y^2} = \frac{a^2 A^2}{b^2 B^2} \qquad$ ---(iii)

Now,

(i) − (ii) gives

$$x^2\left[\frac{1}{a^2} - \frac{1}{A^2}\right] + y^2\left[\frac{1}{b^2} + \frac{1}{B^2}\right] = 0$$

$\Rightarrow \qquad \frac{x^2}{y^2} = \frac{B^2 + b^2}{b^2 B^2} \times \frac{a^2 A^2}{a^2 - A^2}$

Put in (iii), we get

$$\frac{\left(B^2 + b^2\right)}{b^2 B^2} \times \frac{a^2 A^2}{\left(a^2 - A^2\right)} = \frac{a^2 A^2}{b^2 B^2}$$

$\Rightarrow \qquad B^2 + b^2 = a^2 - A^2$

$\Rightarrow \qquad a^2 - b^2 = A^2 + B^2$

Tangents and Normals Ex 16.3 Q9

We have,

$$\frac{x^2}{a^2+\lambda_1}+\frac{y^2}{b^2+\lambda_1}=1 \qquad \text{---(i)}$$

$$\frac{x^2}{a^2+\lambda_2}+\frac{y^2}{b^2+\lambda_2}=1 \qquad \text{---(ii)}$$

slope of (i)

$$\frac{2x}{a^2+\lambda_1}+\frac{2y}{b^2+\lambda_1}\times\frac{dy}{dx}=0$$

$$\Rightarrow \qquad m_1=\frac{dy}{dx}=\frac{-x}{y}\times\frac{b^2+\lambda_1}{a^2+\lambda_1}$$

Slope of (ii)

$$\frac{2x}{a^2+\lambda_2}+\frac{2y}{b^2+\lambda_2}\times\frac{dy}{dx}=0$$

$$\Rightarrow \qquad m_2=\frac{dy}{dx}=\frac{-x}{y}\times\frac{b^2+\lambda_2}{a^2+\lambda_2}$$

Now,

Subtracting (ii) from (i), we get

$$x^2\left[\frac{1}{a^2+\lambda_1}-\frac{1}{a^2+\lambda_2}\right]+y^2\left[\frac{1}{b^2+\lambda_1}-\frac{1}{b^2+\lambda_2}\right]=0$$

$$\Rightarrow \qquad \frac{x^2}{y^2}=\frac{\lambda_2-\lambda_1}{\left(b^2+\lambda_1\right)\left(b^2+\lambda_2\right)}\times\frac{1}{\frac{\lambda_1-\lambda_2}{\left(a^2+\lambda_1\right)\left(a^2+\lambda_2\right)}}$$

Now,

$$m_1\times m_2=\frac{x^2}{y^2}\times\frac{\left(b^2+\lambda_1\right)\left(b^2+\lambda_2\right)}{\left(a^2+\lambda_1\right)\left(a^2+\lambda_2\right)}$$

$$=\frac{\left(\lambda_2-\lambda_1\right)}{\left(b^2+\lambda_1\right)\left(b^2+\lambda_2\right)}\times-\frac{\left(a^2+\lambda_1\right)\left(a^2+\lambda_2\right)}{\lambda_2-\lambda_1}\times\frac{\left(b^2+\lambda_1\right)\left(b^2+\lambda_2\right)}{\left(a^2+\lambda_1\right)\left(a^2+\lambda_2\right)}$$

$$=-1$$

$\therefore$ (i) and (ii) cuts orthogonolly

Tangents and Normals Ex 16.3 Q10

Suppose the straight line $x\cos\alpha+y\sin\alpha=p$ touches the curve at $Q(x_1,y_1)$.

But equation of tangent to $\frac{x^2}{a^2}+\frac{y}{b^2}=1$ at $Q(x_1,y_1)$ is

$$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$$

Thus equation $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$ and $x\cos\alpha+y\sin\alpha=p$ represent the same line.

$$\therefore \frac{x_1/a^2}{\cos\alpha}+\frac{y_1/b^2}{\sin\alpha}=\frac{1}{p}$$

$$\Rightarrow x_1=\frac{a^2\cos\alpha}{p},\ y_1=\frac{b^2\sin\alpha}{p}\ldots\ldots\ldots\text{(i)}$$

The point $Q(x_1,y_1)$ lies on the curve $\frac{x^2}{a^2}+\frac{y}{b^2}=1$

$$\therefore \frac{a^4\cos^2\alpha}{p^2a^2}+\frac{b^4\sin^2\alpha}{p^2b^2}=1$$

$$\Rightarrow a^2\cos^2\alpha-b^2\sin^2\alpha=p^2$$

*********** END **********