



Adjoint and Inverse of Matrix Ex 7.1 Q2(i)

$$\text{Here } A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

Cofactors of A are:

$$\begin{array}{lll} C_{11} = -3 & C_{21} = +2 & C_{31} = 2 \\ C_{12} = +2 & C_{22} = -3 & C_{32} = 2 \\ C_{13} = 2 & C_{23} = 2 & C_{33} = -3 \end{array}$$

$$\begin{aligned} \therefore \text{adj } A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}^T \end{aligned}$$

Therefore,

$$\text{adj } A = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

Now,

$$(\text{adj } A) \cdot A = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\begin{aligned} |A| \cdot I &= \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= (-3 + 4 + 4) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \end{aligned}$$

$$A \cdot (\text{adj } A) = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\therefore (\text{adj } A) \cdot A = |A| \cdot I = A \cdot (\text{adj } A)$$

Adjoint and Inverse of Matrix Ex 7.1 Q2(ii)

Here, $A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 2 & C_{21} = 3 & C_{31} = -13 \\ C_{12} = -3 & C_{22} = 6 & C_{32} = 9 \\ C_{13} = 5 & C_{23} = -3 & C_{33} = -1 \end{array}$$

$$\begin{aligned} \therefore \text{adj } A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} 2 & -3 & 5 \\ 3 & 6 & -3 \\ -13 & 9 & -1 \end{bmatrix}^T \end{aligned}$$

Therefore,

$$\text{adj } A = \begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix}$$

Now,

$$\begin{aligned} (\text{adj } A) \cdot A &= \begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix} \end{aligned}$$

$$|A| \cdot I = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{vmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix}$$

$$A \cdot (\text{adj } A) = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix}$$

$$\therefore (\text{adj } A) \cdot A = |A| \cdot I = A \cdot (\text{adj } A)$$

Adjoint and Inverse of Matrix Ex 7.1 Q2(iii)

$$\text{Here, } A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{bmatrix}$$

Cofactors of A are:

$$\begin{array}{lll} C_{11} = -22 & C_{21} = 11 & C_{31} = -11 \\ C_{12} = 4 & C_{22} = -2 & C_{32} = 2 \\ C_{13} = 16 & C_{23} = -8 & C_{33} = 8 \end{array}$$

$$\begin{aligned} \therefore \text{adj} A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} -22 & 4 & 16 \\ 11 & -2 & -8 \\ -11 & 2 & 8 \end{bmatrix}^T \end{aligned}$$

Therefore,

$$\text{adj} A = \begin{bmatrix} -22 & 11 & -11 \\ 4 & -2 & 2 \\ 16 & -8 & 8 \end{bmatrix}$$

Now,

$$(\text{adj} A) \cdot A = \begin{bmatrix} -22 & 11 & -11 \\ 4 & -2 & 2 \\ 16 & -8 & 8 \end{bmatrix} \begin{bmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} |A| \cdot I &= \begin{vmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{vmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= (44 - 4 + 48) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0 \end{aligned}$$

$$A (\text{adj} A) = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{bmatrix} \begin{bmatrix} -22 & 11 & -11 \\ 4 & -2 & 2 \\ 16 & -8 & 8 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore (\text{adj} A) \cdot A = |A| I = A (\text{adj} A)$$

Adjoint and Inverse of Matrix Ex 7.1 Q2(iv)

$$\text{Here, } A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix}$$

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 3 & C_{21} = -1 & C_{31} = 1 \\ C_{12} = -15 & C_{22} = 7 & C_{32} = -5 \\ C_{13} = 4 & C_{23} = -2 & C_{33} = 2 \end{array}$$

$$\begin{aligned} \therefore \text{adj } A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} 3 & -15 & 4 \\ -1 & 7 & -2 \\ 1 & -5 & 2 \end{bmatrix}^T \end{aligned}$$

Therefore,

$$\text{adj } A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 7 & -5 \\ 4 & -2 & 2 \end{bmatrix}$$

Now,

$$(\text{adj } A)A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 7 & -5 \\ 4 & -2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{aligned} |A|.I &= \begin{vmatrix} 3 & -1 & 1 \\ -15 & 7 & -5 \\ 4 & -2 & 2 \end{vmatrix} I_3 \\ &= (6 - 4) I_3 = 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$A(\text{adj } A) = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ -15 & 7 & -5 \\ 4 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\therefore (\text{adj } A)A = |A|.I = A(\text{adj } A)$$

Here, $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 2 & 4 & 3 \end{bmatrix}$

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 15 & C_{21} = 6 & C_{31} = -15 \\ C_{12} = 0 & C_{22} = -3 & C_{32} = 0 \\ C_{13} = -10 & C_{23} = 0 & C_{33} = 5 \end{array}$$

$$\begin{aligned} \therefore \text{adj } A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} 15 & 0 & -10 \\ 6 & -3 & 0 \\ -15 & 0 & 5 \end{bmatrix}^T \end{aligned}$$

Therefore,

$$\text{adj } A = \begin{bmatrix} 15 & 6 & -15 \\ 0 & -3 & 0 \\ -10 & 0 & 5 \end{bmatrix}$$

Now,

$$(\text{adj } A)A = \begin{bmatrix} 15 & 6 & -15 \\ 0 & -3 & 0 \\ -10 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 2 & 4 & 3 \end{bmatrix} = \begin{bmatrix} -15 & 0 & 0 \\ 0 & -15 & 0 \\ 0 & 0 & -15 \end{bmatrix}$$

$$\begin{aligned} |A|.I &= \begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 2 & 4 & 3 \end{vmatrix} I_3 \\ &= (-15)I_3 = \begin{bmatrix} -15 & 0 & 0 \\ 0 & -15 & 0 \\ 0 & 0 & -15 \end{bmatrix} \end{aligned}$$

$$A.(\text{adj } A) = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 2 & 4 & 3 \end{bmatrix} \begin{bmatrix} 15 & 6 & -15 \\ 0 & -3 & 0 \\ -10 & 0 & 5 \end{bmatrix} = \begin{bmatrix} -15 & 0 & 0 \\ 0 & -15 & 0 \\ 0 & 0 & -15 \end{bmatrix}$$

$$\therefore (\text{adj } A)A = |A|.I = A.(\text{adj } A)$$

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