Geometric Progressions Ex 20.5 Q 1

Here,

$a,\ b,\ c$ are in G.P.

$b^2 = ac$ ---(i)

Now,

$$2\log b = \log b^2$$
$$= \log ac$$
$$2\log b = \log a + \log c$$
$$\log b - \log a = \log c - \log b$$

$\Rightarrow$ $\log a,\ \log b,\ \log c$ are in A.P.

Geometric Progressions Ex 20.5 Q 2

Here,

$a,\ b,\ c$ are in G.P., so

$b^2 = ac$

$$\frac{2}{\log_b m} = 2\log_m b$$
$$= \log_m b^2$$
$$= \log_m ac$$
$$= \log_m a + \log_m c$$
$$\frac{2}{\log_b m} = \frac{1}{\log_a m} + \frac{1}{\log_c m}$$
$$\Rightarrow \quad \frac{1}{\log_b m} - \frac{1}{\log_a m} = \frac{1}{\log_c m} - \frac{1}{\log_b m}$$

$\Rightarrow$ $\dfrac{1}{\log_a m}, \dfrac{1}{\log_b m}, \dfrac{1}{\log_c m}$ are in A.P.

Geometric Progressions Ex 20.5 Q 3

Here,

$a,\ b,\ c$ are in A.P.

$2b = a + c$ ---(i)

and $a,\ b,\ d$ are in G.P., so

$b^2 = ad$ ---(ii)

Now,

$$(a-b)^2 = a^2 + b^2 - 2ab$$
$$= a^2 + ad - a(a+c)$$

Using equation (i) and (ii)

$$= a^2 + ad - a^2 - ac$$
$$= ad - ac$$
$$(a-b)^2 = a(d-c)$$
$$\frac{(a-b)}{a} = \frac{(d-c)}{(a-b)}$$

$\Rightarrow$ $a,\ (a-b),\ (d-c)$ are in G.P.

Geometric Progressions Ex 20.5 Q 4

Here, Let R be common ratio,

a_p, a_q, a_r, a_s of AP are in GP

$$R = \frac{a_q}{a_p} = \frac{a_r}{a_q}$$
$$= \frac{a_q - a_r}{a_p - a_q} \quad \text{(Ratio property)}$$
$$= \frac{[a+(q-1)d]-[a+(r-1)d]}{[a+(p-1)d]-[a+(q-1)d]}$$
$$= \frac{(q-r)d}{(p-q)d}$$
$$R = \frac{q-r}{p-q} \text{------}(1)$$

Now,

$$R = \frac{a_r}{a_q} = \frac{a_s}{a_r}$$
$$= \frac{a_r - a_s}{a_q - a_r} \quad \text{(Ratio property)}$$
$$= \frac{[a+(r-1)d]-[a+(s-1)d]}{[a+(q-1)d]-[a+(r-1)d]}$$
$$= \frac{(r-s)d}{(q-r)d}$$
$$R = \frac{r-s}{q-r} \text{------}(2)$$

From equation as (1) and (2)

$$\frac{q-r}{p-q} = \frac{r-s}{p-r}$$

$\Rightarrow$ $(p-q), (q-r), (r-s)$ are in GP

Geometric Progressions Ex 20.5 Q 5

$$\frac{1}{a+b},\ \frac{1}{2b},\ \frac{1}{b+c} \text{ are in A.P.}$$

$$\frac{2}{2b} = \frac{1}{(a+b)} + \frac{1}{(b+c)}$$

$$\frac{1}{b} = \frac{b+c+a+b}{(a+b)(b+c)}$$

$$\frac{1}{b} = \frac{2b+c+a}{ab+ac+b^2+bc}$$

$$ab+ac+b^2+bc = 2b^2+bc+ba$$

$$b^2+ac = 2b^2$$

$$b^2 = ac$$

So,

a, b, c are in G.P.

*********** END **********