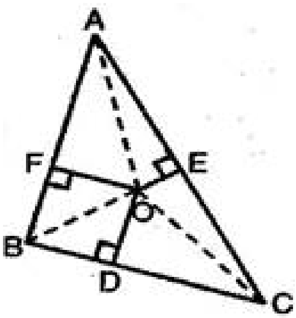
Exercise 6.5



Adding all these, we get

$OA^2 + OB^2 + OC^2 = AF^2 + BD^2 + CE^2 + OF^2 + OD^2 + OE^2$

$\Rightarrow \quad OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) In right Δ s ODB and ODC, we have

$OB^2 = BD^2 + OD^2$ and $OC^2 = OD^2 + CD^2$

$\Rightarrow \; OB^2 - OC^2 = BD^2 - CD^2$ ....(1)

Similarly, we have $OB^2 - OC^2 = BD^2 - CD^2$ ....(2)

and $OB^2 - OC^2 = BD^2 - CD^2$ ....(3)

Adding equations (1), (2) and (3), we get

$= \left(OB^2 - OC^2\right) + \left(OC^2 - OA^2\right) + \left(OA^2 - OB^2\right)$

$= \left(BD^2 - CD^2\right) + \left(CE^2 - AE^2\right) + \left(AF^2 - BF^2\right)$

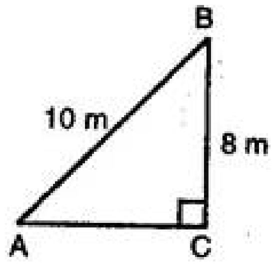
$\Rightarrow \; \left(BD^2 + CE^2 + AF^2\right) - \left(AE^2 + CD^2 + BF^2\right) = 0$

$\Rightarrow \; AF^2 + BD^2 + CE^2 = AE^2 + BF^2 + CD^2$

9. A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

Ans. Let AB be the ladder, B be the window and CB be the wall. Then, ABC is a

right triangle, right angled at C.



$\Rightarrow \quad AB^2 = AC^2 + BC^2$

$\Rightarrow \quad 10^2 = AC^2 + 8^2$

$\Rightarrow \quad AC^2 = 100 - 64$

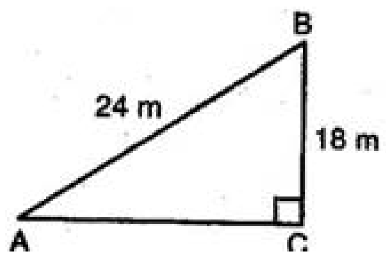
$\Rightarrow \quad AC^2 = 36$

$\Rightarrow \quad AC = 6$

Hence, the foot of the ladder is at a distance 6 m from the base of the wall.

10. A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other hand. How far from the base of the pole should the stake be driven so that the wire will be taut?

Ans. Let AB (= 24m) be a guy wire attached to a vertical pole. BC of height 18 m. To keep the wire taut, let it be fixed to a stake at A. Then, ABC is a right triangle, right angled at C.



$\therefore \quad AB^2 = AC^2 + BC^2$

$\Rightarrow \quad 24^2 = AC^2 + 18^2$

$\Rightarrow \quad AC^2 = 576 - 324$

$\Rightarrow \quad AC^2 = 252$

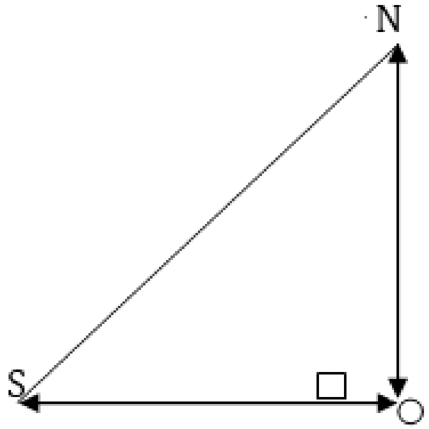
$\Rightarrow \quad AC = 6\sqrt{7}$

Hence, the stake may be placed at distance of 6 $\sqrt{7}$ m from the base of the pole.

11. An aeroplane leaves an airport and flies due north at a speed of 1000 km pwe hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Ans. Let the first aeroplane starts from O and goes up to A towards north where

OA = $\left(1000\times\frac{3}{2}\right)$ km = 1500 km



Let the second aeroplane starts from O at the same time and goes up to 1500 km

B towards west where

OB = $\left(1200\times\frac{3}{2}\right)$ km = 1800 km

According to the question the required distance = BA

In right angled triangle ABC, by Pythagoras theorem, we have,

$AB^2 = OA^2 + OB^2$

$= (1500)^2 + (1800)^2$

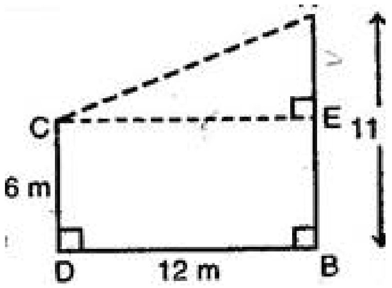
= 2250000 + 3240000

= 5490000 = $9\times61\times100\times100$

$\Rightarrow$ AB = $300\sqrt{61}$ km

12. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Ans. Let AB = 11 m and CD = 6 m be the two poles such that BD = 12 m A



Draw CE $\perp$ AB and join AC.

$\therefore$ CE = DB = 12 m

AE = AB – BE = AB – CD = (11 – 6)m = 5 m

In right angled triangle ACE, by Pythagoras theorem, we have

$AC^2 = CE^2 + AE^2 = 12^2 + 5^2$

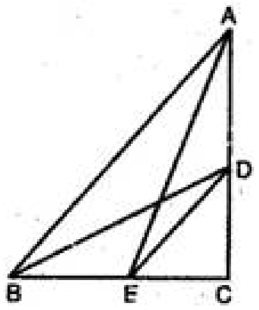
= 144 + 25 = 169

$\Rightarrow$ AC = 13 m

Hence, the distance between the tops of the two poles is 13 m.

13. D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

Ans. In right angled Δ s ACE and DCB, we have



$AE^2 = AC^2 + CE^2$ and $BD^2 = DC^2 + BC^2$

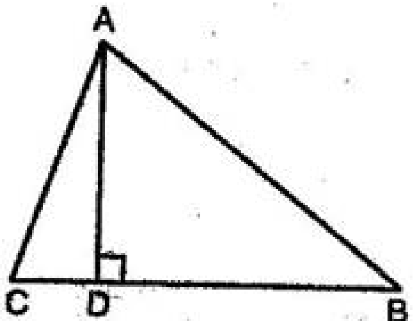
$\Rightarrow AE^2 + BD^2 = (AC^2 + CE^2) + (DC^2 + BC^2)$

$\Rightarrow AE^2 + BD^2 = (AC^2 + BC^2) + (DC^2 + CE^2)$

$\Rightarrow AE^2 + BD^2 = AB^2 + DE^2$

[By Pythagoras theorem, $AC^2 + BC^2 = AB^2$ and $DC^2 + CE^2 = DE^2$

14. The perpendicular from A on side BC of a Δ ABC intersects BC at D such that DB = 3CD (see figure). Prove that $2AB^2 = 2AC^2 + BC^2$.



Ans. We have, DB = 3CD

Now, BC = DB + CD

$\Rightarrow$ BC = 3CD + CD

$\Rightarrow$ BC = 4CD

$\therefore$ CD = $\frac{1}{4}$ BC and DB = 3CD = $\frac{3}{4}$ BC ....(1)

Since, Δ ABD is a right triangle, right angled at D. Therefore by Pythagoras theorem, we have,

$AB^2 = AD^2 + DB^2$ ....(2)

Similarly, from Δ ACD, we have, $AC^2 = AD^2 + CD^2$ ....(3)

From eq. (2) and (3) $AB^2 - AC^2 = DB^2 - CD^2$

$\Rightarrow$ $AB^2 - AC^2 = \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2$ [Using eq. (1)]

$\Rightarrow$ $AB^2 - AC^2 = \left(\frac{9}{16} - \frac{1}{16}\right)BC^2$

$\Rightarrow$ $AB^2 - AC^2 = \frac{1}{2}BC^2$

$\Rightarrow$ $2AB^2 - 2AC^2 = BC^2$

$\Rightarrow$ $2AB^2 = 2AC^2 + BC^2$

*********** END **********