Q 1 . If the zeroes of polynomial x^3-3x^2+x+1 are $a-b, a, a+b$, find *a* and *b*.

Answer :

$p(x)=x^3-3x^2+x+1$

Zeroes are *a* - *b*, *a* + *a* + *b*

Comparing the given polynomial with px^3+qx^2+rx+t, we obtain

p = 1, *q* = - 3, *r* = 1, *t* = 1

Sum of zeroes $= a-b+a+a+b$

$\frac{-q}{p}=3a$

$\frac{-(-3)}{1}=3a$

$3=3a$

$a=1$

The zeroes are $1-b, 1, 1+b$.

Multiplication of zeroes $=1(1-b)(1+b)$

$\frac{-t}{p}=1-b^2$

$\frac{-1}{1}=1-b^2$

$1-b^2=-1$

$1+1=b^2$

$b=\pm\sqrt{2}$

Hence, *a* = 1 and *b* = $\sqrt{2}$ or $-\sqrt{2}$.

Q 2 . It two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

Answer :

Given that $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are zeroes of the given polynomial.

Therefore, $\left(x - 2 - \sqrt{3}\right)\left(x - 2 + \sqrt{3}\right)$ = $x2$ + 4 - 4x - 3

= $x2$ - 4x + 1 is a factor of the given polynomial

For finding the remaining zeroes of the given polynomial, we will find the quotient by dividing $x^4 - 6x^3 - 26x^2 + 138x - 35$ by $x^2 - 4x + 1$.

$$
\begin{array}{r|l}
 & x^2 - 2x - 35 \\
\hline
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 - 4x^3 + x^2 \\
 & - \quad + \quad - \\
\hline
 & \quad -2x^3 - 27x^2 + 138x - 35 \\
 & \quad -2x^3 + 8x^2 - 2x \\
 & \quad + \quad - \quad + \\
\hline
 & \qquad -35x^2 + 140x - 35 \\
 & \qquad -35x^2 + 140x - 35 \\
 & \qquad + \quad - \quad + \\
\hline
 & \qquad\qquad 0
\end{array}
$$

Clearly, $x^4 - 6x^3 - 26x^2 + 138x - 35$ = $(x^2 - 4x + 1)(x^2 - 2x - 35)$

It can be observed that $(x^2 - 2x - 35)$ is also a factor of the given polynomial.

And $(x^2 - 2x - 35)$ = $(x - 7)(x + 5)$

Therefore, the value of the polynomial is also zero when $x - 7 = 0$ or $x + 5 = 0$

Or x = 7 or - 5

Hence, 7 and - 5 are also zeroes of this polynomial.

Q 3. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find k and a.

Answer :

By division algorithm,

Dividend = Divisor × Quotient + Remainder

Dividend - Remainder = Divisor × Quotient

$x^4 - 6x^3 + 16x^2 - 25x + 10 - x - a = x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ will be perfectly divisible by $x^2 - 2x + k$.

Let us divide $x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ by $x^2 - 2x + k$

$$
\begin{array}{rl}
 & \qquad\qquad\qquad\qquad x^2-4x+\left(8-k\right) \\
x^2-2x+k & \overline{\Big)\; x^4-6x^3+16x^2-26x+10-a} \\
 & x^4-2x^3+\;kx^2 \\
 & -\quad +\qquad - \\
\hline
 & \quad -4x^3+\left(16-k\right)x^2-26x \\
 & \quad -4x^3+\qquad 8x^2-\;4kx \\
 & \quad +\qquad -\qquad\quad + \\
\hline
 & \qquad\qquad \left(8-k\right)x^2-\left(26-4k\right)x+10-a \\
 & \qquad\qquad \left(8-k\right)x^2-\left(16-2k\right)x+\left(8k-k^2\right) \\
 & \qquad\qquad -\qquad\qquad +\qquad\qquad - \\
\hline
 & \qquad\qquad\qquad \left(-10+2k\right)x+\left(10-a-8k+k^2\right)
\end{array}
$$

It can be observed that $\left(-10+2k\right)x+\left(10-a-8k+k^2\right)$ will be 0.

Therefore, $\left(-10+2k\right) = 0$ and $\left(10-a-8k+k^2\right) = 0$

For $\left(-10+2k\right) = 0$,

$2k = 10$

And thus, $k = 5$

For $\left(10-a-8k+k^2\right) = 0$

$10 - a - 8 \times 5 + 25 = 0$

$10 - a - 40 + 25 = 0$

$-5 - a = 0$

Therefore, $a = -5$

Hence, $k = 5$ and $a = -5$

********** END *********