Definite Integrals Ex 20.1 Q5

Let $x^2 + 1 = t$

$\Rightarrow 2x\,dx = dt$

$\Rightarrow x\,dx = \dfrac{dt}{2}$

Now,

$x = 2 \Rightarrow t = 5$

$x = 3 \Rightarrow t = 10$

$$\therefore \int_2^3 \frac{x}{x^2+1}dx = \frac{1}{2}\int_5^{10}\frac{dt}{t} = \frac{1}{2}\left[\log t\right]_5^{10}$$

$$= \frac{1}{2}\left[\log 10 - \log 5\right]$$

$$= \frac{1}{2}\left[\log\frac{10}{5}\right]$$

$$= \frac{1}{2}\left[\log 2\right]$$

$$= \log\sqrt{2}$$

$$\therefore \int_2^3 \frac{x}{x^2+1} = \log\sqrt{2}$$

Definite Integrals Ex 20.1 Q6

We have,

$$\int_0^\infty \frac{1}{a^2+b^2x^2}dx = \frac{1}{b^2}\int_0^\infty \frac{1}{\left(\frac{a}{b}\right)^2+x^2}dx$$

We know that $\int \frac{1}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$

$$\therefore \frac{1}{b^2}\int_0^\infty \frac{1}{\left(\frac{a}{b}\right)^2+x^2}dx = \frac{1}{b^2}\left[\frac{b}{a}\tan^{-1}\left(\frac{bx}{a}\right)\right]_0^\infty$$

$$= \frac{1}{ab}\left[\tan^{-1}\left(\frac{bx}{a}\right)\right]_0^\infty$$

$$= \frac{1}{ab}\left[\tan^{-1}\infty - \tan^{-1}0\right]$$

$$= \frac{1}{ab}\left[\frac{\pi}{2} - 0\right]$$

$$= \frac{\pi}{2ab}$$

$$\Rightarrow \int_0^\infty \frac{1}{a^2+b^2x^2}dx = \frac{\pi}{2ab}$$

Definite Integrals Ex 20.1 Q7

We have,

$$\int_{-1}^{1} \frac{1}{1+x^2}\,dx$$

We know that $\int \frac{1}{1+x^2}\,dx = \tan^{-1} x$

Now,

$$\int_{-1}^{1} \frac{1}{1+x^2}\,dx$$

$$= \left[\tan^{-1} x\right]_{-1}^{1}$$

$$= \left[\tan^{-1} 1 - \tan^{-1}(-1)\right]$$

$$= \left[\frac{\pi}{4} - \left(\frac{-\pi}{4}\right)\right] \qquad \left[\because \tan^{-1}(-1) = \frac{-\pi}{4}\right]$$

$$= \left[\frac{\pi}{4} + \frac{\pi}{4}\right]$$

$$= \frac{2\pi}{4}$$

$$\therefore \int_{-1}^{1} \frac{1}{1+x^2}\,dx = \frac{\pi}{2}$$

Definite Integrals Ex 20.1 Q8

We have,

$$\int_0^{\infty} e^{-x} dx$$

We know that $\int e^{-x} dx = -e^{-x}$

Now,

$$\int_0^{\infty} e^{-x} dx$$

$$= \left[-e^{-x}\right]_0^{\infty}$$

$$= \left[-e^{-\infty} + e^{-0}\right] \qquad \left[\because e^{\infty} = 0, \quad e^{0} = 1\right]$$

$$= \left[-0 + 1\right]$$

$$\therefore \int_0^{\infty} e^{-x} dx = 1$$

*********** END **********