



Continuity Ex 9.1 Q37

It is given that the function is continuous at  $x = 3$  and at  $x = 5$

$$\therefore \text{LHL} = \text{RHL} = f(3) \dots (1) \text{ and}$$

$$\text{LHL} = \text{RHL} = f(5) \dots (2)$$

Now,

$$f(3) = 1$$

$$\text{RHL} = \lim_{x \rightarrow 3^+} f(x) = \lim_{h \rightarrow 0} f(3+h) = \lim_{h \rightarrow 0} a(3+h) + b = 3a + b$$

Thus, using (1), we get,

$$3a + b = 1 \dots (3)$$

$$f(5) = 7$$

$$\text{LHL} = \lim_{x \rightarrow 5^-} f(x) = \lim_{h \rightarrow 0} f(5-h) = \lim_{h \rightarrow 0} a(5-h) + b = 5a + b$$

Thus, using (2), we get

$$5a + b = 7 \dots (4)$$

Now, solving (3) and (4) we get,

$$a = 3 \text{ and } b = -8$$

Continuity Ex 9.1 Q38

We want to discuss the continuity of the function at  $x = 1$

We need to prove that

$$\text{LHL} = \text{RHL} = f(1)$$

$$f(1) = \frac{1^2}{2} = \frac{1}{2}$$

$$\text{LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1-h) = \lim_{h \rightarrow 0} \frac{(1-h)^2}{2} = \frac{1}{2}$$

$$\text{RHL} = \lim_{x \rightarrow 1^+} f(x) = \lim_{h \rightarrow 0} f(1+h) = \lim_{h \rightarrow 0} 2(1+h)^2 - 3(1+h) + \frac{3}{2} = 2 - 3 + \frac{3}{2} = \frac{1}{2}$$

$$\text{Thus, LHL} = \text{RHL} = f(1) = \frac{1}{2}$$

Hence, function is continuous at  $x = 1$

Continuity Ex 9.1 Q39

We want to discuss the continuity at  $x = 0$  and  $x = 1$

Now,

$$f(0) = 1$$

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} |-h| + |-h-1| = 1.$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} |h| + |h-1| = 1$$

$\therefore \text{LHL} = \text{RHL} = f(0) = 1$ , function is continuous at  $x = 0$ .

For  $x = 1$ .

$$f(1) = 1$$

$$\text{LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1-h) = \lim_{h \rightarrow 0} |1-h| + |1-h-1| = 1$$

$$\text{RHL} = \lim_{x \rightarrow 1^+} f(x) = \lim_{h \rightarrow 0} f(1+h) = \lim_{h \rightarrow 0} |1+h| + |1+h-1| = 1$$

$\therefore \text{LHL} = \text{RHL} = f(1) = 1$  function is continuous at  $x = 1$ .

For  $x = -1$

$$f(-1) = |-1-1| + |-1+1| = 2$$

$$\text{LHL} = \lim_{x \rightarrow -1^-} f(x) = \lim_{h \rightarrow 0} f(-1-h) = \lim_{h \rightarrow 0} |-1-h-1| + |-1-h+1| = 2$$

$$\text{RHL} = \lim_{x \rightarrow -1^+} f(x) = \lim_{h \rightarrow 0} f(-1+h) = \lim_{h \rightarrow 0} |-1+h-1| + |-1+h+1| = 2$$

Thus,  $\text{LHL} = \text{RHL} = f(-1) = 2$

Hence, function is continuous at  $x = -1$

For  $x = 1$

$$f(1) = |1-1| + |1+1| = 2$$

$$\text{LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1-h) = \lim_{h \rightarrow 0} |1-h-1| + |1-h+1| = 2$$

$$\text{RHL} = \lim_{x \rightarrow 1^+} f(x) = \lim_{h \rightarrow 0} f(1+h) = \lim_{h \rightarrow 0} |1+h-1| + |1+h+1| = 2$$

Thus,  $\text{LHL} = \text{RHL} = f(1) = 2$

Hence, function is continuous at  $x = 1$

Continuity Ex 9.1 Q40

Since  $f(x)$  is continuous at  $x = 0$ ,  $L.H.Limit = R.H.Limit$ .

Thus, we have

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)$$

$$\Rightarrow \lim_{x \rightarrow 0^-} a \sin \frac{\pi}{2}(x+1) = \lim_{x \rightarrow 0^+} \frac{\tan x - \sin x}{x^3}$$

$$\Rightarrow a \times 1 = \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$$

$$\Rightarrow a = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{\cos x} - \sin x}{x^3}$$

$$\Rightarrow a = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{x} \left( \frac{1}{\cos x} - 1 \right)}{x^2}$$

$$\Rightarrow a = \lim_{x \rightarrow 0} \frac{\frac{\sin x}{x} \left( \frac{1 - \cos x}{\cos x} \right)}{x^2}$$

$$\Rightarrow a = \lim_{x \rightarrow 0} \frac{\sin x}{x} \times \lim_{x \rightarrow 0} \frac{1}{\cos x} \times \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$$

$$\Rightarrow a = 1 \times 1 \times \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$$

$$\Rightarrow a = \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} \times \frac{1 + \cos x}{1 + \cos x}$$

$$\Rightarrow a = \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x^2(1 + \cos x)}$$

$$\Rightarrow a = \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2(1 + \cos x)}$$

$$\Rightarrow a = \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} \times \lim_{x \rightarrow 0} \frac{1}{1 + \cos x}$$

$$\Rightarrow a = 1 \times \lim_{x \rightarrow 0} \frac{1}{1 + \cos x}$$

$$\Rightarrow a = 1 \times \frac{1}{1 + 1}$$

$$\Rightarrow a = \frac{1}{2}$$

\*\*\*\*\* END \*\*\*\*\*