Transformation Formulae Ex 8.1 Q1

(i) $2\sin 3\theta\cos\theta$

$= \sin(3\theta+\theta)+\sin(3\theta-\theta)$ $\quad [\because 2\sin A\cos B = \sin(A+B)+\sin(A-B)]$

$= \sin 4\theta+\sin 2\theta$

(ii) $2\cos 3\theta\sin 2\theta$

$\because$ $2\cos A\sin B = \sin(A+B)-\sin(A-B)$

$\Rightarrow$ $2\cos 3\theta\sin 2\theta = \sin(3\theta+2\theta)-\sin(3\theta-2\theta)$

$=$ $\sin 5\theta-\sin\theta$

(iii) $2\sin 4\theta\sin 3\theta$

$\because$ $2\sin A\sin B = \cos(A-B)-\cos(A+B)$

$\Rightarrow$ $2\sin 4\theta\sin 3\theta = \cos(4\theta-3\theta)-\cos(4\theta+3\theta)$

$= \cos\theta-\cos 7\theta$

(iv) $2\cos 7\theta\cos 3\theta$

$\because$ $2\cos A\cos B = \cos(A+B)+\cos(A-B)$

$\Rightarrow$ $2\cos 7\theta\cos 3\theta = \cos(7\theta+3\theta)+\cos(7\theta-3\theta)$

$= \cos 10\theta+\cos 4\theta$

Transformation Formulae Ex 8.1 Q2

(i) $2\sin\frac{5\pi}{12}\sin\frac{\pi}{12}$

$\because$ $2\sin A\sin B = \cos(A-B)-\cos(A+B)$

$\Rightarrow$ $2\sin\frac{5\pi}{12}\sin\frac{\pi}{12} = \cos\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)-\cos\left(\frac{5\pi}{12}+\frac{\pi}{12}\right)$

$= \cos\left(\frac{4\pi}{12}\right)-\cos\left(\frac{6\pi}{12}\right)$

$= \cos\left(\frac{\pi}{3}\right)-\cos\left(\frac{\pi}{2}\right)$

$= \frac{1}{2}-0 = \frac{1}{2} = \text{RHS}$

(ii) $2\cos\frac{5\pi}{12}\cos\frac{\pi}{12} = \frac{1}{2}$

$\because$ $2\cos A\cos B = \cos(A+B)+\cos(A-B)$

$= \cos\left(\frac{5\pi}{12}+\frac{\pi}{12}\right)+\cos\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)$

$= \cos(\left(\frac{\pi}{2}\right)+\cos\left(\frac{\pi}{3}\right)$

$= 0+\frac{1}{2} = \frac{1}{2} = \text{RHS}$

(iii) $2\sin\frac{5\pi}{12}\cos\frac{\pi}{12}$

$\because$ $2\sin A\cos B = \sin(A+B)+\sin(A-B)$

$= \left(\frac{5\pi}{12}+\frac{\pi}{12}\right)+\sin\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)$

$= \sin\frac{\pi}{2}+\sin\frac{\pi}{3}$

$= 1+\frac{\sqrt{3}}{2} = \frac{2+\sqrt{3}}{2} = \text{RHS}$ (Taking LCM)

Transformation Formulae Ex 8.1 Q3(i)

$\sin 50° \cos 85° = \dfrac{1-\sqrt{2}\sin 35°}{2\sqrt{2}}$

LHS $= \sin 50° \cos 85° = \dfrac{2\sin 50° \cos 85°}{2}$

$\because \quad 2\sin A \cos B = \sin(A+B) + \sin(A-B)$

$$\Rightarrow \quad \frac{2\sin 50° \cos 85°}{2} = \frac{1}{2}\left[\sin(50° + 85°) + \sin(50° - 85°)\right]$$

$$= \frac{1}{2}\left[\sin 135° + \sin(-35°)\right]$$

$$= \frac{1}{2}\left[\sin(90° + 45°) - \sin 35°\right] \qquad [\because \sin(-\theta) = -\sin\theta]$$

$$= \frac{1}{2}\left[\cos 45° - \sin 35°\right] \qquad [\because \sin(90° + \theta) = \cos\theta]$$

Now,

$$\cos 45° = \frac{1}{\sqrt{2}}$$

$$= \frac{1}{2}\left[\frac{1}{\sqrt{2}} - \sin 35°\right]$$

$$= \frac{1-\sqrt{2}\sin 35°}{2\sqrt{2}}$$

Transformation Formulae Ex 8.1 Q3(ii)

$$\text{LHS} = \sin 25° \cos 115°$$

$$= \frac{2\sin 25° \cos 115°}{2}$$

We Know that

$$2\sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$= \frac{1}{2}\left[\sin(25° + 115°) + \sin(25° - 115°)\right]$$

$$= \frac{1}{2}\left[\sin 140° + \sin(-90°)\right]$$

$$\sin(-\theta) = -\sin\theta$$

And, $\sin(90° + \theta) = \cos\theta$

$$\Rightarrow \quad \frac{1}{2}\left[\sin(90° + 50°) - \sin 90°\right]$$

$$= \frac{1}{2}\left[\cos 50° - 1\right]$$

Also,

$$\cos\theta = \sin(90° - \theta)$$

$$\cos 50° = \sin(90° - 50°) = \sin 40°$$

$$\frac{1}{2}\left[\sin 40° - 1\right]$$

Transformation Formulae Ex 8.1 Q4.

We have,

$$\begin{aligned}
\text{LHS} &= 4\cos\theta\cos\left(\frac{\pi}{3}+\theta\right)\cos\left(\frac{\pi}{3}-\theta\right)\\
&= 2\cos\theta\left[2\cos\left(\frac{\pi}{3}+\theta\right)\cos\left(\frac{\pi}{3}-\theta\right)\right]\\
&= 2\cos\theta\left[2\cos\left(\frac{\pi}{3}+\theta+\frac{\pi}{3}-\theta\right)+\cos\left(\frac{\pi}{3}+\theta-\frac{\pi}{3}+\theta\right)\right]\\
&= 2\cos\theta\left[\cos\frac{2\pi}{3}+\cos 2\theta\right]\\
&= 2\cos\theta\left[\cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right)+\cos 2\theta\right]\\
&= 2\cos\theta\left[-\sin\frac{\pi}{6}+\cos 2\theta\right]\\
&= 2\cos\theta\left[-\frac{1}{2}+\cos 2\theta\right]\\
&= -2\cos\theta\times\frac{1}{2}+2\cos\theta\cos 2\theta\\
&= -\cos\theta+\left[\cos(\theta+2\theta)+\cos(2\theta-\theta)\right]\\
&= -\cos\theta+\cos 3\theta+\cos\theta\\
&= \cos 3\theta\\
&= \text{RHS}
\end{aligned}$$

$\therefore$ LHS = RHS Hence proved.

*********** END **********