



Differentiation Ex 11.3 Q45

Here,  $y = \tan^{-1} \left( \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right)$

Put  $x = \cos 2\theta$ , so

$$\begin{aligned} y &= \tan^{-1} \left( \frac{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}} \right) \\ &= \tan^{-1} \left( \frac{\sqrt{2\cos^2 \theta} - \sqrt{2\sin^2 \theta}}{\sqrt{2\cos^2 \theta} + \sqrt{2\sin^2 \theta}} \right) \\ &= \tan^{-1} \left( \frac{\sqrt{2}(\cos \theta - \sin \theta)}{\sqrt{2}(\cos \theta + \sin \theta)} \right) \\ &= \tan^{-1} \left( \frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} \right) \end{aligned}$$

[Dividing numerator and denominator by  $\cos \theta$ ]

$$\begin{aligned} &= \tan^{-1} \left( \frac{\frac{\cos \theta}{\cos \theta} - \frac{\sin \theta}{\cos \theta}}{\frac{\cos \theta}{\cos \theta} + \frac{\sin \theta}{\cos \theta}} \right) \\ &= \tan^{-1} \left( \frac{1 - \tan \theta}{1 + \tan \theta} \right) \\ &= \tan^{-1} \left( \frac{\frac{\tan \frac{\pi}{4}}{1} - \tan \theta}{1 + \frac{\tan \frac{\pi}{4}}{1} + \tan \theta} \right) \\ &= \tan^{-1} \left[ \tan \left( \frac{\pi}{4} - \theta \right) \right] \\ &= \frac{\pi}{4} - \theta \end{aligned}$$

$$y = \frac{\pi}{4} - \frac{1}{2} \cos^{-1} x$$

[Using  $x = \cos 2\theta$ ]

Differentiating it with respect to  $x$ ,

$$\frac{dy}{dx} = 0 - \frac{1}{2} \left( \frac{-1}{\sqrt{1-x^2}} \right)$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}$$

Differentiation Ex 11.3 Q46

$$\text{Here, } y = \cos^{-1} \left\{ \frac{2x - 3\sqrt{1-x^2}}{\sqrt{13}} \right\}$$

Let  $x = \cos \theta$ , so,

$$\begin{aligned} y &= \cos^{-1} \left\{ \frac{2 \cos \theta - 3\sqrt{1-\cos^2 \theta}}{\sqrt{13}} \right\} \\ &= \cos^{-1} \left\{ \frac{2}{\sqrt{13}} \cos \theta - \frac{3}{\sqrt{13}} \sin \theta \right\} \end{aligned}$$

$$\text{Let } \cos \phi = \frac{2}{\sqrt{13}}$$

$$\begin{aligned} \Rightarrow \sin \phi &= \sqrt{1 - \cos^2 \phi} \\ &= \sqrt{1 - \left( \frac{2}{\sqrt{13}} \right)^2} \\ &= \sqrt{\frac{13-4}{13}} \\ &= \sqrt{\frac{9}{13}} \\ \sin \phi &= \frac{3}{\sqrt{13}} \end{aligned}$$

So,

$$\begin{aligned} y &= \cos^{-1} \{ \cos \phi \cos \theta - \sin \phi \sin \theta \} \\ &= \cos^{-1} [ \cos (\theta + \phi) ] \\ y &= \phi + \theta \\ y &= \cos^{-1} \left( \frac{2}{\sqrt{13}} \right) + \cos^{-1} x \quad \left[ \text{Since, } x = \cos \theta, \cos \phi = \frac{2}{\sqrt{13}} \right] \end{aligned}$$

Differentiating it with respect to  $x$ ,

$$\frac{dy}{dx} = 0 + \left( -\frac{1}{\sqrt{1-x^2}} \right)$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q47

Consider the given expression:

$$\begin{aligned} y &= \sin^{-1} \left\{ \frac{2^{x+1} \times 3^x}{1 + (36)^x} \right\} \\ &= \sin^{-1} \left\{ \frac{2 \times 2^x \times 3^x}{1 + (6^2)^x} \right\} \\ y &= \sin^{-1} \left\{ \frac{2 \times 6^x}{1 + 6^{2x}} \right\} \dots (1) \end{aligned}$$

Substituting  $6^x = \tan \theta$  in the above equation, we get,

$$\begin{aligned} y &= \sin^{-1} \left\{ \frac{2 \times 6^x}{1 + 6^{2x}} \right\} \\ &= \sin^{-1} \left\{ \frac{2 \times \tan \theta}{1 + \tan^2 \theta} \right\} \\ &= \sin^{-1} (\sin 2\theta) \\ &= 2\theta \\ &= 2 \tan^{-1} (6^x) \end{aligned}$$

Differentiating the above function with respect to  $x$ , we have,

$$\begin{aligned} \frac{d}{dx} \left[ \sin^{-1} \left\{ \frac{2^{x+1} \times 3^x}{1 + (36)^x} \right\} \right] &= \frac{d}{dx} [2 \tan^{-1} (6^x)] \\ &= 2 \times \frac{1}{1 + (6^x)^2} \times 6^x \log 6 \\ &= \frac{2 \times 6^x \log 6}{1 + 6^{2x}} \end{aligned}$$

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