Areas of Parallelograms and Triangles Ex 15.3 Q24

Answer :

Given: In ΔABC

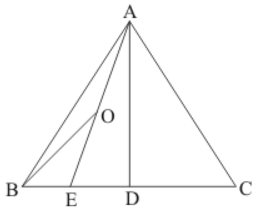
(1) D is the midpoint of the side BC

(2) E is the midpoint of the side BD

(3) O is the midpoint of the side AE

To prove: $\operatorname{ar}(\Delta BOE)=\frac{1}{8}\operatorname{ar}(\Delta ABC)$

Proof: We know that the median of a triangle divides the triangle into two triangles of equal area.



Since AD and AE are the medians of ΔABC and ΔABD respectively. And OB is the median of ΔABE

$$\operatorname{ar}(\Delta ABD)=\frac{1}{2}\operatorname{ar}(\Delta ABC) \quad \ldots\ldots (1)$$

$$\operatorname{ar}(\Delta ABE)=\frac{1}{2}\operatorname{ar}(\Delta ABD) \quad \ldots\ldots (2)$$

$$\operatorname{ar}(\Delta BOE)=\frac{1}{2}\operatorname{ar}(\Delta ABE) \quad \ldots\ldots (3)$$

Therefore

$$\operatorname{ar}(\Delta BOE)=\frac{1}{2}\left(\frac{1}{2}\operatorname{ar}(\Delta ABD)\right)(\text{from } 2)$$

$$\operatorname{ar}(\Delta BOE)=\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\operatorname{ar}(\Delta ABC)\right)\right)(\text{from } 1)$$

$$\operatorname{ar}(\Delta BOE)=\frac{1}{8}\operatorname{ar}(\Delta ABC)$$

Hence we have proved that

$$\boxed{\operatorname{ar}(\Delta BOE)=\frac{1}{8}\operatorname{ar}(\Delta ABC)}$$

Areas of Parallelograms and Triangles Ex 15.3 Q25

Answer :

Given:

(1) X and Y are the, midpoints of AC and AB respectively.

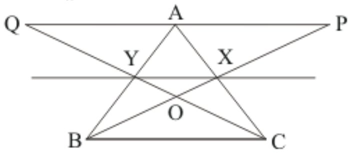
(2) QP|| BC

(3) CYQ and BXP are straight lines.

To prove: $ar(\Delta ABP) = ar(\Delta ACQ)$

Proof: Since X and Y are the, midpoints of AC and AB respectively.

So XY||BC



ΔBYC and ΔBXC are on the same base BC and between the same parallels XY and BC. Therefore

$$ar(\Delta BYC) = ar(\Delta BXC)$$

$$ar(\Delta BYC) - ar(\Delta BOC) = ar(\Delta BXC) - ar(\Delta BOC)$$

$$ar(\Delta BOY) = ar(\Delta COX)$$

$$ar(\Delta BOY) + ar(\Delta XOY) = ar(\Delta COX) + ar(\Delta XOY)$$

$$ar(\Delta BXY) = ar(\Delta CXY) \quad \ldots\ldots (1)$$

Similarly the quadrilaterals XYAP and XYQA are on the same base XY and between the same parallels XY and PQ. Therefore

$$ar(\text{quad XYAP}) = ar(\text{quad XYQA}) \quad \ldots\ldots (2)$$

Adding equation 1 and 2 we get

$$ar(\Delta BXY) + ar(\text{quad XYAP}) = ar(\Delta CXY) + ar(\text{quad XYQA})$$

$$\Rightarrow ar(\Delta ABP) = ar(\Delta ACQ)$$

Hence we had proved that

$$\boxed{ar(\Delta ABP) = ar(\Delta ACQ)}$$

*********** END **********