Increasing and Decreasing Functions Ex 17.2 Q14

We have,

$$f(x) = \tan x$$

$$\therefore \quad f'(x) = \sec^2 x$$

Now,

$$x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$$

$$\Rightarrow \quad \sec^2 x > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is increasing function on $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q15

We have,

$$f(x) = \tan^{-1}(\sin x + \cos x)$$

$$\therefore \quad f'(x) = \frac{1}{1+(\sin x + \cos x)^2} \times (\cos x - \sin x)$$

$$= \frac{\cos x - \sin x}{1 + \sin^2 x + \cos^2 x + 2\sin x \cos x}$$

$$= \frac{\cos x - \sin x}{2(1 + \sin x \cos x)}$$

Now,

$$x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

$$\Rightarrow \quad \cos x - \sin x < 0$$

$$\Rightarrow \quad \frac{\cos x - \sin x}{2(1 + \sin x \cos x)} < 0 \qquad \left[\because 2(1 + \sin x \cos x) > 0\right]$$

$$\Rightarrow \quad f'(x) < 0$$

Hence, $f(x)$ is decreasing function on $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q16

We have,

$$f(x) = \sin\left(2x + \frac{\pi}{4}\right)$$

$$\therefore \quad f'(x) = \cos\left(2x + \frac{\pi}{4}\right) \times 2$$

$$\therefore \quad f'(x) = 2\cos\left(2x + \frac{\pi}{4}\right)$$

Now,

$$x \in \left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$$

$$\Rightarrow \quad \frac{3\pi}{8} < x < \frac{5\pi}{8}$$

$$\Rightarrow \quad \frac{3\pi}{4} < 2x < \frac{5\pi}{4}$$

$$\Rightarrow \quad \pi < 2x < \frac{\pi}{4} < \frac{3\pi}{2}$$

$$\Rightarrow \quad 2x + \frac{\pi}{4} \text{ lies in IIIrd quadrant}$$

$$\Rightarrow \quad \cos\left(2x + \frac{\pi}{4}\right) < 0$$

$$\Rightarrow \quad 2\cos\left(2x + \frac{\pi}{4}\right) < 0$$

$$\Rightarrow \quad f'(x) < 0$$

Hence, $f(x)$ is decreasing on $\left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q17

We have,

$$f(x) = \tan x - 4x$$

$$\therefore \quad f'(x) = \sec^2 x - 4$$

$$= \frac{1 - 4\cos^2 x}{\cos^2 x}$$

$$= \frac{(1 + 2\cos x)(1 - 2\cos x)}{\cos^2 x}$$

$$= 4\sec^2 x \left(\frac{1}{2} + \cos x\right)\left(\frac{1}{2} - \cos x\right)$$

Now,

$$x \in \left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$$

$$\Rightarrow \quad -\frac{\pi}{3} < x < \frac{\pi}{3}$$

$$\Rightarrow \quad \cos x > \frac{1}{2}$$

$$\Rightarrow \quad \left(\frac{1}{2} - \cos x\right) < 0$$

$$\Rightarrow \quad 4\sec^2 x \left(\frac{1}{2} + \cos x\right)\left(\frac{1}{2} - \cos x\right) < 0$$

$$\Rightarrow \quad f'(x) < 0$$

Hence, $f(x)$ is decreasing function on $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$.

********** END **********