Indefinite Integrals Ex 19.9 Q15

Let $\quad I = \int \frac{1}{\sqrt{\tan^{-1} x}\,(1+x^2)} dx \; - - - - - (\text{i})$

Let $\quad \tan^{-1} x = t \quad$ then,

$d\left(\tan^{-1} x\right) = dt$

$\Rightarrow \quad \frac{1}{1+x^2} dx = dt$

Putting $\tan^{-1} x = t$ and $\frac{1}{1+x^2} dx = dt$ in equation (i), we get

$$I = \int \frac{1}{\sqrt{t}} dt$$

$$= \int t^{-\frac{1}{2}} dt$$

$$= 2t^{\frac{1}{2}} + c$$

$$= 2\sqrt{\tan^{-1} x} + c$$

$\therefore \quad I = 2\sqrt{\tan^{-1} x} + c$

Indefinite Integrals Ex 19.9 Q16

$$\text{Let } I = \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$$

$$= \int \frac{\sqrt{\tan x} \times \cos x}{\sin x \cos x \times \cos x} dx$$

$$= \int \frac{\sqrt{\tan x}}{\tan x \cos^2 x} dx$$

$$= \int \frac{\sec^2 x\, dx}{\sqrt{\tan x}}$$

Let $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$$\therefore I = \int \frac{dt}{\sqrt{t}}$$

$$= 2\sqrt{t} + C$$

$$= 2\sqrt{\tan x} + C$$

Indefinite Integrals Ex 19.9 Q17

Let $\quad I = \int \frac{1}{x} (\log x)^2 \, dx - - - - - (\text{i})$

Let $\quad \log x = t \quad$ then,

$d(\log x) = dt$

$\Rightarrow \quad \frac{1}{x} dx = dt$

Putting $\log x = t$ and $\frac{1}{x} dx = dt$ in equation (i), we get

$$I = \int t^2 dt$$
$$= \frac{t^3}{3} + c$$
$$= \frac{(\log x)^3}{3} + c$$

$\therefore \quad I = \frac{1}{3} (\log x)^3 + c$

Indefinite Integrals Ex 19.9 Q18

Let $\quad I = \int \sin^5 x \cos x \, dx - - - - - (\text{i})$

Let $\quad \sin x = t \quad$ then,

$d(\sin x) = dt$

$\Rightarrow \quad \cos x \, dx = dt$

Putting $\sin x = t$ and $\cos x \, dx = dt$ in equation (i), we get

$$I = \int t^5 dt$$
$$= \frac{t^6}{6} + c$$
$$= \frac{\sin^6 x}{6} + c$$

$\therefore \quad I = \frac{1}{6} \sin^6 x + c$

Indefinite Integrals Ex 19.9 Q19

Let $\quad I = \int \tan^{\frac{3}{2}} x \sec^2 x \, dx - - - - - (i)$

Let $\quad \tan x = t \quad$ then,

$d(\tan x) = dt$

$\Rightarrow \quad \sec^2 x \, dx = dt$

Putting $\tan x = t$ and $\sec^2 x \, dx = dt$ in equation (i), we get

$$I = \int t^{\frac{3}{2}} dt$$

$$= \frac{2}{5} t^{\frac{5}{2}} + c$$

$$= \frac{2}{5} (\tan x)^{\frac{5}{2}} + c$$

$$\therefore \quad I = \frac{2}{5} \tan^{\frac{5}{2}} x + c$$

*********** END **********