



Differentiation Ex 11.7 Q13

Here,

$$x = \frac{1-t^2}{1+t^2}$$

Differentiating it with respect to t using quotient rule,

$$\begin{aligned} \frac{dx}{dt} &= \left[\frac{(1+t^2) \frac{d}{dt}(1-t^2) - (1-t^2) \frac{d}{dt}(1+t^2)}{(1+t^2)^2} \right] \\ &= \left[\frac{(1+t^2)(-2t) - (1-t^2)(2t)}{(1+t^2)^2} \right] \\ &= \left[\frac{-2t - 2t^3 - 2t + 2t^3}{(1+t^2)^2} \right] \\ \frac{dx}{dt} &= \left(\frac{-4t}{(1+t^2)^2} \right) \quad \text{---(i)} \end{aligned}$$

And, $y = \frac{2t}{1+t^2}$

Differentiating it with respect to t using quotient rule,

$$\begin{aligned} \frac{dy}{dt} &= \left[\frac{(1+t^2) \frac{d}{dt}(2t) - (2t) \frac{d}{dt}(1+t^2)}{(1+t^2)^2} \right] \\ \frac{dy}{dt} &= \left[\frac{(1+t^2)(2) - (2t)(2t)}{(1+t^2)^2} \right] \\ &= \left[\frac{2+2t^2-4t^2}{(1+t^2)^2} \right] \\ \frac{dy}{dt} &= \frac{2(1-t^2)}{(1+t^2)^2} \quad \text{---(ii)} \end{aligned}$$

Differentiation Ex 11.7 Q14 Here, $x = 2\cos\theta - \cos 2\theta$

Differentiating it with respect to θ using chain rule,

$$\begin{aligned}\frac{dx}{d\theta} &= 2(-\sin\theta) - (-\sin 2\theta) \frac{d}{d\theta}(2\theta) \\ &= -2\sin\theta + 2\sin 2\theta \\ \frac{dx}{d\theta} &= 2(\sin 2\theta - \sin\theta) \quad \text{---(i)}\end{aligned}$$

And, $y = 2\sin\theta - \sin 2\theta$

Differentiating it with respect to θ using chain rule,

$$\begin{aligned}\frac{dy}{d\theta} &= 2\cos\theta - \cos 2\theta \frac{d}{d\theta}(2\theta) \\ &= 2\cos\theta - \cos 2\theta (2) \\ &= 2\cos\theta - 2\cos 2\theta \\ \frac{dy}{d\theta} &= 2(\cos\theta - \cos 2\theta) \quad \text{---(ii)}\end{aligned}$$

Dividing equation (ii) by equation (i),

$$\begin{aligned}\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} &= \frac{2(\cos\theta - \cos 2\theta)}{2(\sin 2\theta - \sin\theta)} \\ &= \frac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta} \\ \frac{dy}{dx} &= \frac{-2\sin\left(\frac{\theta+2\theta}{2}\right)\sin\left(\frac{\theta-2\theta}{2}\right)}{2\cos\left(\frac{2\theta+\theta}{2}\right)\sin\left(\frac{2\theta-\theta}{2}\right)} \quad \left[\begin{array}{l} \text{Since, } \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \\ \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \end{array} \right] \\ &= \frac{-\sin\left(\frac{3\theta}{2}\right)\sin\left(\frac{-\theta}{2}\right)}{\cos\left(\frac{3\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)} \\ &= \frac{-\sin\left(\frac{3\theta}{2}\right)\left(-\sin\frac{\theta}{2}\right)}{\cos\left(\frac{3\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)} \\ &= \frac{\sin\left(\frac{3\theta}{2}\right)}{\cos\left(\frac{3\theta}{2}\right)} \\ \frac{dy}{dx} &= \tan\left(\frac{3\theta}{2}\right)\end{aligned}$$

Differentiation Ex 11.7 Q15

Here,

$$x = e^{\cos 2t}$$

Differentiating it with respect to t using chain rule,

$$\begin{aligned}\frac{dx}{dt} &= \frac{d}{dt} (e^{\cos 2t}) \\ &= e^{\cos 2t} \frac{d}{dt} (\cos 2t) \\ &= e^{\cos 2t} (-\sin 2t) \frac{d}{dt} (2t) \\ &= -\sin 2t e^{\cos 2t} (2) \\ \frac{dx}{dt} &= -2 \sin 2t e^{\cos 2t} \quad \text{---(i)}\end{aligned}$$

And, $y = e^{\sin 2t}$

Differentiating it with respect to t using chain rule,

$$\begin{aligned}\frac{dy}{dt} &= \frac{d}{dt} (e^{\sin 2t}) \\ &= e^{\sin 2t} \frac{d}{dt} (\sin 2t) \\ &= e^{\sin 2t} (\cos 2t) \frac{d}{dt} (2t) \\ &= e^{\sin 2t} (\cos 2t) (2) \\ \frac{dy}{dt} &= 2 \cos 2t e^{\sin 2t} \quad \text{---(ii)}\end{aligned}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2 \cos 2t e^{\sin 2t}}{-2 \sin 2t e^{\cos 2t}}$$

$$\frac{dy}{dx} = -\frac{y \log x}{x \log y}$$

$$\left[\begin{array}{l} \text{Since, } x = e^{\cos 2t} \Rightarrow \log x = \cos 2t \\ y = e^{\sin 2t} \Rightarrow \log y = \sin 2t \end{array} \right]$$

Differentiation Ex 11.7 Q16

Here,

$$x = \cos t$$

Differentiating it with respect to t ,

$$\frac{dx}{dt} = \frac{d}{dt}(\cos t)$$

$$\frac{dx}{dt} = -\sin t \quad \text{---(i)}$$

and, $y = \sin t$

Differentiating it with respect to t ,

$$\frac{dy}{dt} = \frac{d}{dt}(\sin t)$$

$$\frac{dy}{dt} = \cos t \quad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t}{-\sin t}$$

$$\frac{dy}{dx} = -\cot t$$

$$\begin{aligned}\left(\frac{dy}{dx}\right) &= -\cot\left(\frac{2\pi}{3}\right) \\ &= -\cot\left(\pi - \frac{\pi}{3}\right) \\ &= -\left[-\cot\left(\frac{\pi}{3}\right)\right] \\ &= \cot\left(\frac{\pi}{3}\right)\end{aligned}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{3}}$$

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