Indefinite Integrals Ex 19.9 Q69

Let $\quad I = \int 4x^3\sqrt{5-x^2}\,dx ----(\text{i})$

Let $\quad 5 - x^2 = t^2 \qquad$ then,

$d\left(5 - x^2\right) = t^2$

$\Rightarrow \quad -2x\,dx = 2t\,dt$

$\Rightarrow \quad dx = \frac{-t}{x}dt$

Putting $5 - x^2 = t^2$ and $dx = \frac{-t}{x}dt$ in equation (i),

we get

$$I = \int 4x^3\sqrt{t^2} \times \frac{-t}{x}dt$$

$$= -4\int x^2 t \times t\,dt$$

$$= -4\int\left(5 - t^2\right)t^2 dt \qquad \left[\because \quad 5 - x^2 = t^2\right]$$

$$= -4\int\left(5t^2 - t^4\right)dt$$

$$= -20 \times \frac{t^3}{3} + 4\frac{t^5}{5} + c$$

$$= \frac{-20}{3} \times t^3 + \frac{4}{5} \times t^5 + c$$

$$= \frac{-20}{3} \times \left(5 - x^2\right)^{\frac{3}{2}} + \frac{4}{5} \times \left(5 - x^2\right)^{\frac{5}{2}} + c$$

$$\therefore \quad I = \frac{-20}{3} \times \left(5 - x^2\right)^{\frac{3}{2}} + \frac{4}{5} \times \left(5 - x^2\right)^{\frac{5}{2}} + c$$

Indefinite Integrals Ex 19.9 Q70

Let $\quad I = \int \frac{1}{\sqrt{x} + x} dx - - - - (i)$

Let $\quad \sqrt{x} = t \quad$ then,

$$d\left(\sqrt{x}\right) = dt$$

$$\Rightarrow \quad \frac{1}{2\sqrt{x}} dx = dt$$

$$\Rightarrow \quad dx = 2\sqrt{x}\, dt$$

Putting $\sqrt{x} = t$ and $2\sqrt{x}\, dt = dx$ in equation (i), we get

$$I = \int \frac{1}{t + t^2} 2t \times dt \qquad \begin{bmatrix} \because & \sqrt{x} = t \\ \Rightarrow & x = t^2 \end{bmatrix}$$

$$= \int \frac{2t}{t(1 + t)} dt$$

$$= 2\int \frac{t}{(1 + t)} dt$$

$$= 2 \log|1 + t| + c$$

$$= 2 \log\left|1 + \sqrt{x}\right| + c$$

$$\therefore \quad I = 2 \log\left|1 + \sqrt{x}\right| + c$$

Indefinite Integrals Ex 19.9 Q71

$$\frac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}$$

Multiplying and dividing by x^{-3}, we obtain

$$\frac{x^{-3}}{x^2\cdot x^{-3}\left(x^4+1\right)^{\frac{3}{4}}}=\frac{x^{-3}\left(x^4+1\right)^{\frac{-3}{4}}}{x^2\cdot x^{-3}}$$

$$=\frac{\left(x^4+1\right)^{\frac{-3}{4}}}{x^5\cdot\left(x^4\right)^{-\frac{3}{4}}}$$

$$=\frac{1}{x^5}\left(\frac{x^4+1}{x^4}\right)^{-\frac{3}{4}}$$

$$=\frac{1}{x^5}\left(1+\frac{1}{x^4}\right)^{-\frac{3}{4}}$$

Let $\frac{1}{x^4}=t \Rightarrow -\frac{4}{x^5}dx=dt \Rightarrow \frac{1}{x^5}dx=-\frac{dt}{4}$

$$\therefore \int\frac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}dx=\int\frac{1}{x^5}\left(1+\frac{1}{x^4}\right)^{-\frac{3}{4}}dx$$

$$=-\frac{1}{4}\int(1+t)^{-\frac{3}{4}}dt$$

$$=-\frac{1}{4}\left[\frac{(1+t)^{\frac{1}{4}}}{\frac{1}{4}}\right]+C$$

$$=-\frac{1}{4}\frac{\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}}{\frac{1}{4}}+C$$

$$=-\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}+C$$

Indefinite Integrals Ex 19.9 Q72

Let $\quad I = \int \frac{\sin^5 x}{\cos^4 x} dx ----(\text{i})$

Let $\quad \cos x = t \quad$ then,

$d(\cos x) = dt$

$\Rightarrow \quad -\sin x \, dx = dt$

$\Rightarrow \quad dx = -\frac{dt}{\sin x}$

Putting $\cos x = t$ and $dx = -\frac{dt}{\sin x}$ in equation (i),

we get

$$I = \int \frac{\sin^5 x}{t^4} \times -\frac{dt}{\sin x}$$

$$= -\int \frac{\sin^4 x}{t^4} dt$$

$$= -\int \frac{\left(1-\cos^2 x\right)^2}{t^4} dt$$

$$= -\int \frac{\left(1-t^2\right)^2}{t^4} dt$$

$$= -\int \frac{1+t^4-2t^2}{t^4} dt$$

$$= -\int \left(\frac{1}{t^4} + \frac{t^4}{t^4} - \frac{2t^2}{t^4}\right) dt$$

$$= -\int \left(t^{-4} + 1 - 2t^{-2}\right) dt$$

$$= -\left[\frac{t^{-3}}{-3} + t - 2\frac{t^{-1}}{-1}\right] + c$$

$$= -\left[-\frac{1}{3} \times \frac{1}{t^3} + t + \frac{2}{t}\right] + c$$

$$= \frac{1}{3} \times \frac{1}{t^3} - t - \frac{2}{t} + c$$

$$= \frac{1}{3} \times \frac{1}{\cos^3 x} - \cos x - \frac{2}{\cos x} + c$$

$\therefore \quad I = -\cos x - \frac{2}{\cos x} + \frac{1}{3\cos^3 x} + c$

*********** END **********