Adjoint and Inverse of Matrix Ex 7.1 Q18

$$A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$$

Now $A^2 + 4A - 42I = 0$

For this $A^2 = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix}$

Hence,

$$A^2 + 4A - 42I = \begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix} + \begin{bmatrix} -32 & 20 \\ 8 & 16 \end{bmatrix} - \begin{bmatrix} 42 & 0 \\ 0 & 42 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence proved.

Now, $A^2 + 4A - 42I = 0$

$\Rightarrow \quad A^{-1}.A.A + 4A^{-1}.A - 42A^{-1}.I = 0$

$\Rightarrow \quad IA + 4I - 42A^{-1} = 0$

$\Rightarrow \quad 42A^{-1} = A + 4I$

$\Rightarrow \quad A^{-1} = \frac{1}{42}\left[A + 4I\right] = \frac{1}{42}\left\{\begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}\right\} = \frac{1}{42}\begin{bmatrix} -4 & 5 \\ 2 & 8 \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.1 Q19

Here

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

Now,

$$A^2 - 5A + 7I = 0$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

So,

$$A^2 - 5A + 7I = 0$$

Px-multipling with A^{-1}

$$A^{-1}A^2 - 5A^{-1}A + 7IA^{-1} = 0$$

$$A - 5I + 7A^{-1} = 0$$

$$A^{-1} = \frac{1}{7}[5I - A]$$

$$= \frac{1}{7}\left\{\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\right\}$$

$$= \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q20

$$A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$$

$$\therefore\ A^2 = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix}$$

Now $A^2 - xA + yI = 0$

$$\Rightarrow \quad \begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix} - \begin{bmatrix} 4x & 3x \\ 2x & 5x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow \quad 22 - 4x + y = 0 \quad$ or $\quad 4x - y = 22$

$\Rightarrow \quad 18 - 2x = 0 \quad$ or $\quad x = 9$

$\therefore \quad y = 14$

Again,

$A^2 - 9A + 14I = 0$

$\Rightarrow \quad 9A = A^2 + 14I = 0$

$\Rightarrow \quad 9A^{-1}A = A^{-1}.A.A + 14A^{-1}$

$\Rightarrow \quad 9I = IA + 14A^{-1}$

$$\Rightarrow \quad A^{-1} = \frac{1}{14}\{9I - A\} = \frac{1}{14}\left\{\begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}\right\} = \frac{1}{14}\left\{\begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}\right\}$$

Adjoint and Inverse of Matrix Ex 7.1 Q21

$$A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

If $A^2 = \lambda A - 2I$

$$\lambda A = A^2 + 2I$$

$$= \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\lambda \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 3\lambda & -2\lambda \\ 4\lambda & -2\lambda \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$3\lambda = 3$$

$\lambda = 1$ Ans $\lambda = 1$

$$A^2 = A - 2I$$

Px multiplying by A^{-1}

$$A^{-1}.AA = A^{-1}.A - 2A^{-1}.I$$

$$A = I - 2A^{-1}$$

$$2A^{-1} = I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ -4 & 3 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{2}\begin{bmatrix} -2 & 2 \\ -4 & 3 \end{bmatrix}$$

*********** END **********