

Increasing and Decreasing Functions Ex 17.2 Q1(xxi)

$$f(x) = x^4 - 4x^3 + 4x^2 + 15$$

$$f'(x) = 4x^3 - 12x^2 + 8x$$

Critical points

$$f'(x) = 0$$

$$\Rightarrow 4x\left(x^2 - 3x + 2\right) = 0$$

$$\Rightarrow 4x(x-2)(x-1)=0$$

$$\Rightarrow$$
 $x = 0, 2, 1$

Clealry, f'(x) > 0 if 0 < x < 1 and x > 2

$$f'(x) < 0 \text{ if } x < 0 \text{ and } 1 < x < 2$$

Thus, f(x) increases in $(0,1) \cup (2,\infty)$, decreases in $(-\infty,0) \cup (1,2)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xxii)

We have,

$$f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}; x > 0$$

$$f'(x) = \frac{15}{2}x^{\frac{1}{2}} - \frac{15}{2}x^{\frac{3}{2}}$$

Critical points

$$f^+(x) = 0$$

$$\Rightarrow \frac{15}{2}x^{\frac{1}{2}} - \frac{15}{2}x^{\frac{3}{2}} = 0$$

$$\Rightarrow \frac{15}{2} x^{\frac{1}{2}} (1 - x) = 0$$

$$\Rightarrow$$
 $x = 0, 1$

Clearly,
$$f'(x) > 0$$
 if $0 < x < 1$

and
$$f'(x) < 0 \text{ if } x > 1$$

Thus, f(x) increases in (0,1), decreases in $(1,\infty)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xxiii)

We have,

$$f(x) = x^8 + 6x^2$$

$$f'(x) = 8x^7 + 12x$$

Critical points

$$f^+(x) = 0$$

$$\Rightarrow 8x^7 + 12x = 0$$

$$\Rightarrow 4x\left(2x^6+3\right)=0$$

$$\Rightarrow x = 0$$

Clearly,
$$f'(x) > 0$$
 if $x > 0$
 $f'(x) < 0$ if $x < 0$

Thus, f(x) increases in $(0,\infty)$, decreases in $(-\infty,0)$.

Increasing and Decreasing Functions Ex 17.2 Q1(xxiv) We have,

$$f(x) = x^3 - 6x^2 + 9x + 15$$

$$f'(x) = 3x^2 - 12x + 9$$

Critical points

$$f^+(x) = 0$$

$$\Rightarrow 3(x^2 - 4x + 3) = 0$$

$$\Rightarrow (x-3)(x-1)=0$$

$$\Rightarrow$$
 $x = 3, 1$

Clearly, f'(x) > 0 if x < 1 and x > 3

$$f'(x) < 0 \text{ if } 1 < x < 3$$

Thus, f(x) increases in $(-\infty,1) \cup (3,\infty)$, decreases in (1,3).

Increasing and Decreasing Functions Ex 17.2 Q1(xxv)

We have,

$$y = [x(x-2)]^2 = [x^2 - 2x]^2$$

$$\therefore \frac{dy}{dx} = y' = 2(x^2 - 2x)(2x - 2) = 4x(x - 2)(x - 1)$$

$$\therefore \frac{dy}{dx} = 0 \implies x = 0, x = 2, x = 1.$$

The points x = 0, x = 1, and x = 2 divide the real line into four disjoint intervals i.e., $(-\infty, 0)$, (0,1) (1,2), and $(2,\infty)$.

In intervals
$$\left(-\infty,0\right)$$
 and $\left(1,2\right)$, $\frac{dy}{dx}<0$.

 $\therefore y$ is strictly decreasing in intervals $(-\infty,0)$ and (1,2).

However, in intervals (0, 1) and (2, ∞), $\frac{dy}{dx} > 0$.

 $\therefore y$ is strictly increasing in intervals (0, 1) and (2, ∞).

 \therefore y is strictly increasing for $0 \le x \le 1$ and $x \ge 2$.

******* END *******