



Definite Integrals Ex 20.4B Q33

$$\text{Let } I = \int_0^2 x\sqrt{2-x} dx$$

$$I = \int_0^2 (2-x)\sqrt{x} dx \qquad \left(\int_0^a f(x) dx = \int_0^a f(a-x) dx \right)$$

$$= \int_0^2 \left\{ 2x^{\frac{1}{2}} - x^{\frac{3}{2}} \right\} dx$$

$$= \left[2 \left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right) - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} \right]_0^2$$

$$= \left[\frac{4}{3} x^{\frac{3}{2}} - \frac{2}{5} x^{\frac{5}{2}} \right]_0^2$$

$$= \frac{4}{3} (2)^{\frac{3}{2}} - \frac{2}{5} (2)^{\frac{5}{2}}$$

$$= \frac{4 \times 2\sqrt{2}}{3} - \frac{2}{5} \times 4\sqrt{2}$$

$$= \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5}$$

$$= \frac{40\sqrt{2} - 24\sqrt{2}}{15}$$

$$= \frac{16\sqrt{2}}{15}$$

Definite Integrals Ex 20.4B Q34

$$\text{Let } I = \int_0^1 \log\left(\frac{1}{x} - 1\right) dx$$

$$= \int_0^1 \log\left(\frac{1-x}{x}\right) dx$$

$$= \int_0^1 \log(1-x) dx - \int_0^1 \log(x) dx$$

$$\text{Applying the property, } \int_0^a f(x) dx = \int_0^a f(a-x) dx$$

$$\text{Thus, } I = \int_0^1 \log(1 - (1-x)) dx - \int_0^1 \log(x) dx$$

$$= \int_0^1 \log(1-1+x) dx - \int_0^1 \log(x) dx$$

$$= \int_0^1 \log(x) dx - \int_0^1 \log(x) dx$$

$$= 0$$

Definite Integrals Ex 20.4B Q35

$$I = \int_{-1}^1 |x \cos \pi x| dx$$

$$\text{Let } f(x) = |x \cos \pi x|$$

$$f(-x) = |-x \cos(-\pi x)| = |-x \cos(\pi x)| = |x \cos \pi x| = f(x)$$

$$\therefore I = \int_{-1}^1 |x \cos \pi x| dx = 2 \int_0^1 |x \cos \pi x| dx$$

Now,

$$f(x) = |x \cos \pi x| = \begin{cases} x \cos \pi x, & \text{if } 0 \leq x \leq \frac{1}{2} \\ -x \cos \pi x, & \text{if } \frac{1}{2} < x < 1 \end{cases}$$

$$\therefore I = 2 \int_0^1 |x \cos \pi x| dx$$

$$\Rightarrow I = 2 \left[\int_0^{\frac{1}{2}} x \cos \pi x dx + \int_{\frac{1}{2}}^1 -x \cos \pi x dx \right]$$

$$\Rightarrow I = 2 \left\{ \left[\frac{x}{\pi} \sin \pi x + \frac{1}{\pi^2} \cos \pi x \right]_0^{\frac{1}{2}} - \left[\frac{x}{\pi} \sin \pi x + \frac{1}{\pi^2} \cos \pi x \right]_{\frac{1}{2}}^1 \right\}$$

$$\Rightarrow I = 2 \left\{ \left[\frac{1}{2\pi} - \frac{1}{\pi^2} \right] - \left[-\frac{1}{\pi^2} - \frac{1}{2\pi} \right] \right\}$$

$$\Rightarrow I = \frac{2}{\pi}$$

Definite Integrals Ex 20.4B Q36

$$I = \int_0^{\pi} \left(\frac{x}{1 + \sin^2 x} + \cos^7 x \right) dx$$

$$I = \int_0^{\pi} \left(\frac{\pi - x}{1 + \sin^2(\pi - x)} + \cos^7(\pi - x) \right) dx$$

$$I = \int_0^{\pi} \left(\frac{\pi - x}{1 + \sin^2 x} - \cos^7 x \right) dx$$

$$2I = \int_0^{\pi} \left(\frac{\pi}{1 + \sin^2 x} \right) dx$$

$$2I = \pi \int_0^{\pi} \frac{1}{1 + \sin^2 x} dx$$

$$2I = \pi \int_0^{\pi} \frac{1}{1 + 2 \tan^2 x} \sec^2 x dx$$

$$I = \pi \int_0^{\frac{\pi}{2}} \frac{1}{1 + 2 \tan^2 x} \sec^2 x dx \dots \dots \dots \left[\because \int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx, \text{ if } f(2a - x) = f(x) \right]$$

$$\text{Let } \tan x = v$$

$$dv = \sec^2 x dx$$

$$\Rightarrow I = \pi \int_0^{\infty} \frac{1}{1 + 2v^2} dv$$

$$\Rightarrow I = \pi \left[\frac{\tan^{-1}(\sqrt{2}v)}{\sqrt{2}} \right]_0^{\infty}$$

$$\Rightarrow I = \pi \left[\frac{\pi}{2\sqrt{2}} \right]$$

$$\Rightarrow I = \frac{\pi^2}{2\sqrt{2}}$$

***** END *****

