Indefinite Integrals Ex 19.11 Q9

Let $I = \int \cot^n x \operatorname{cosec}^2 x dx,\ n \neq -1$ ---(i)

Let $\cot x = t$. Then

$$d(\cot x) = dt$$

$$\Rightarrow \quad -\operatorname{cosec}^2 x dx = dt$$

$$\Rightarrow \quad \operatorname{cosec}^2 x dx = -dt$$

Putting $\cot x = t$ and $\operatorname{cosec}^2 x dx = -dt$ in equation (i), we get

$$I = \int t^n \times (-dt)$$

$$= -\frac{t^{n+1}}{n+1} + c$$

$$\Rightarrow \quad = -\frac{(\cot x)^{n+1}}{n+1} + c$$

Indefinite Integrals Ex 19.11 Q10

Let $I = \int \cot^5 x \operatorname{cosec}^4 x dx$. Then,

$$I = \int \cot^5 x \operatorname{cosec}^2 x \operatorname{cosec}^2 x dx$$

$$= \int \cot^5 x \left(1 + \cot^2 x\right) \operatorname{cosec}^2 x dx$$

$$\Rightarrow \quad I = \int \left(\cot^5 x + \cot^7 x\right) \operatorname{cosec}^2 x dx$$

Substituting $\cot x = t$ and $-\operatorname{cosec}^2 x\, dx = dt$, we get

$$I = \int \left(t^5 + t^7\right)(-dt)$$

$$= -\frac{t^6}{6} - \frac{t^8}{8} + c$$

$$= -\frac{\cot^6 x}{6} - \frac{\cot^8 x}{8} + c$$

$$\therefore \quad I = -\frac{\cot^6 x}{6} - \frac{\cot^8 x}{8} + c$$

Indefinite Integrals Ex 19.11 Q11

Let $I = \int \cot^5 x dx$. Then,

$$I = \int \cot^3 x \times \cot^2 x dx$$

$$= \int \cot^3 x \times \left(\operatorname{cosec}^2 x - 1\right) dx$$

$$= \int \cot^3 x \operatorname{cosec}^2 x dx - \int \cot^3 x dx$$

$$= \int \cot^3 x \operatorname{cosec}^2 x dx - \int \left(\operatorname{cosec}^2 x - 1\right) \cot x dx$$

$$= \int \cot^3 x \operatorname{cosec}^2 x dx - \int \operatorname{cosec}^2 x \cot x dx + \int \cot x dx$$

$$\Rightarrow \quad I = \int \cot^3 x \operatorname{cosec}^2 x dx - \int \operatorname{cosec}^2 x \cot x dx + \int \cot x dx$$

Substituting $\cot x = t$ and $-\operatorname{cosec}^2 x\, dx = dt$ in first two integral, we get

$$I = \int t^3(-dt) - \int t \times (-dt) + \int \cot x dx$$

$$= -\frac{t^4}{4} + \frac{t^2}{2} + \log|\sin x| + c$$

$$= -\frac{1}{4}\cot^4 x + \frac{1}{2}\cot^2 x + \log|\sin x| + c$$

$$\therefore \quad I = -\frac{1}{4}\cot^4 x + \frac{1}{2}\cot^2 x + \log|\sin x| + c$$

Indefinite Integrals Ex 19.11 Q12

Let $I = \int \cot^6 x dx$. Then,

$$I = \int \cot^2 x \times \cot^4 x dx$$
$$= \int \left(cosec^2 x - 1\right) \times \cot^4 x dx$$
$$= \int \left(cosec^2 x \cot^4 x - \cot^4 x\right) dx$$
$$= \int cosec^2 x \times \cot^4 x dx - \int \cot^4 dx$$
$$= \int cosec^2 x \cot^4 x dx - \int \cot^2 x \left(cosec^2 - 1\right) dx$$
$$= \int cosec^2 x \cot^4 x dx - \int \cot^2 x\, cosec^2 x dx + \int \cot^2 x dx$$
$$\Rightarrow \quad I = \int cosec^2 x \cot^4 x dx - \int \cot^2 x\, cosec^2 x dx + \int \left(cosec^2 x - 1\right) dx$$

Substituting $\cot x = t$ and $-cosec^2 x dx = dt$ in first two integral, we get

$$I = \int t^4 (-dt) - \int t^2 (-dt) + \int cosec^2 x dx - \int dx$$
$$= -\frac{t^5}{5} + \frac{t^3}{3} - \cot x - x + c$$
$$= -\frac{\cot^5 x}{5} + \frac{\cot^3 x}{3} - \cot x - x + c$$
$$\therefore \quad I = -\frac{1}{5} \times \cot^5 x + \frac{1}{3} \times \cot^3 x - \cot x - x + c$$

*********** END **********