C. $1 - \alpha^2 - \beta\gamma = 0$

D. $1 + \alpha^2 - \beta\gamma = 0$

Answer

Answer: C

$$A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$$

$$\therefore A^2 = A \cdot A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \alpha\beta \\ \alpha\gamma - \alpha\gamma & \beta\gamma + \alpha^2 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix}$$

Now, $A^2 = I \Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

On comparing the corresponding elements, we have:

$$\alpha^2 + \beta\gamma = 1$$
$$\Rightarrow \alpha^2 + \beta\gamma - 1 = 0$$
$$\Rightarrow 1 - \alpha^2 - \beta\gamma = 0$$

Question 14:

If the matrix A is both symmetric and skew symmetric, then

A. A is a diagonal matrix

B. A is a zero matrix

C. A is a square matrix

D. None of these

Answer

Answer: B

If A is both symmetric and skew-symmetric matrix, then we should have

$$A' = A \text{ and } A' = -A$$
$$\Rightarrow A = -A$$
$$\Rightarrow A + A = O$$
$$\Rightarrow 2A = O$$
$$\Rightarrow A = O$$

Therefore, A is a zero matrix.

Question 15:

If A is square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

A. A **B.** $I - A$ **C.** I **D.** $3A$

Answer

Answer: C

$$(I + A)^3 - 7A = I^3 + A^3 + 3I^2 A + 3A^2 I - 7A$$
$$= I + A^3 + 3A + 3A^2 - 7A$$
$$= I + A^2 \cdot A + 3A + 3A - 7A \qquad [A^2 = A]$$
$$= I + A \cdot A - A$$
$$= I + A^2 - A$$
$$= I + A - A$$
$$= I$$

$$\therefore (I + A)^3 - 7A = I$$

********** END **********