

Maxima and Minima 18.4 Q2

Let
$$f(x) = 2x^3 - 24x + 107$$
.

$$f'(x) = 6x^2 - 24 = 6(x^2 - 4)$$

Now.

$$f'(x) = 0 \implies 6(x^2 - 4) = 0 \implies x^2 = 4 \implies x = \pm 2$$

We first consider the interval [1, 3].

Then, we evaluate the value of f at the critical point $x = 2 \in [1, 3]$ and at the end points of the interval [1, 3].

$$f(2) = 2(8) - 24(2) + 107 = 16 - 48 + 107 = 75$$

$$f(1) = 2(1) - 24(1) + 107 = 2 - 24 + 107 = 85$$

$$f(3) = 2(27) - 24(3) + 107 = 54 - 72 + 107 = 89$$

Hence, the absolute maximum value of f(x) in the interval [1, 3] is 89 occurring at x = 3.

Next, we consider the interval [-3, -1].

Evaluate the value of f at the critical point $x = -2 \in [-3, -1]$ and at the end points of the interval [1, 3].

$$f(-3) = 2(-27) - 24(-3) + 107 = -54 + 72 + 107 = 125$$

Maxima and Minima 18.4 O3

$$f(x) = \cos^2 x + \sin x$$

$$f'(x) = 2\cos x(-\sin x) + \cos x$$

$$=-2\sin x\cos x+\cos x$$

Now,
$$f'(x) = 0$$

$$\Rightarrow 2\sin x \cos x = \cos x \Rightarrow \cos x (2\sin x - 1) = 0$$

$$\Rightarrow \sin x = \frac{1}{2} \text{ or } \cos x = 0$$

$$\Rightarrow x = \frac{\pi}{6}$$
, or $\frac{\pi}{2}$ as $x \in [0,\pi]$

Now, evaluating the value of f at critical points $x = \frac{\pi}{2}$ and $x = \frac{\pi}{6}$ and at the end points of the interval $[0,\pi]$ (i.e., at x = 0 and $x = \pi$), we have:

$$f\left(\frac{\pi}{6}\right) = \cos^2\frac{\pi}{6} + \sin\frac{\pi}{6} = \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{1}{2} = \frac{5}{4}$$

$$f(0) = \cos^2 0 + \sin 0 = 1 + 0 = 1$$

$$f(\pi) = \cos^2 \pi + \sin \pi = (-1)^2 + 0 = 1$$

$$f\left(\frac{\pi}{2}\right) = \cos^2\frac{\pi}{2} + \sin\frac{\pi}{2} = 0 + 1 = 1$$

Hence, the absolute maximum value of f is $\frac{5}{4}$ occurring at $x = \frac{\pi}{6}$ and the absolute minimum value of f is 1 occurring at $x = 0, \frac{\pi}{2}$, and π .

Maxima and Minima 18.4 Q4

We have

$$f(x) = 12 x^{\frac{4}{3}} - 6x^{\frac{1}{3}}$$

$$f'(x) = 16 x^{\frac{1}{3}} - \frac{2}{\frac{2}{x^{\frac{2}{3}}}} = \frac{2(8x - 1)}{\frac{2}{x^{\frac{2}{3}}}}$$

Thus,
$$f'(x) = 0$$

Further note that f'(x) is not defined at x = 0.

So, the critical points are x = 0 and $x = \frac{1}{8}$.

Evaluating the value of f at critical points $x = 0, \frac{1}{8}$ and at end points of the

interval
$$x = -1$$
 and $x = 1$

$$f(-1) = 12(-1)^{4/3} - 6(-1)^{1/3} = 18$$

$$f(0) = 12(0) - 6(0) = 0$$

$$f\left(\frac{1}{8}\right) = 12\left(\frac{1}{8}\right)^{\frac{4}{3}} - 6\left(\frac{1}{8}\right)^{\frac{1}{3}} = \frac{-9}{4}$$

$$f(1) = 12(1)^{4/3} - 6(1)^{1/3} = 6$$

Hence we conclude that absolute maximum value of fis 18 at x=-1

and absolute minimum value of fix $\frac{-9}{4}$ at x = $\frac{1}{8}$.

Maxima and Minima 18.4 Q5 Given,

$$f(x) = 2x^3 - 15x^2 + 36x + 1$$

$$f'(x) = 6x^2 - 30x + 36 = 6(x^2 - 5x + 6) = 6(x - 2)(x - 3)$$

Note that f'(x) = 0 gives x = 2 and x = 3

We shall now evaluate the value of f at these points and at the end points of the interval [1,5],

i.e at x= 1, 2, 3 and 5

At
$$x = 1$$
, $f(1) = 2(1^3) - 15(1^2) + 36(1) + 1 = 24$

$$Atx = 2$$
, $f(2) = 2(2^3) - 15(2^2) + 36(2) + 1 = 29$

$$Atx = 3$$
, $f(3) = 2(3^3) - 15(3^2) + 36(3) + 1 = 28$

At x = 5,
$$f(5) = 2(5^3) - 15(5^2) + 36(5) + 1 = 56$$

Thus we conclude that the absolute maximum value of f on [1,5] is 56, occurring at x=5, and absolute minimum value of f on [1,5] is 24 which occurs at x=1.

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