Polynomials Ex 2.1 Q5

Answer :

Since α and β are the zeros of the quadratic polynomials $p(x)=4x^2-5x-1$

Sum of the zeros = $\dfrac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$$\alpha+\beta=-\left(-\frac{5}{4}\right)$$

$$\alpha+\beta=\frac{5}{4}$$

Product of zeros = $\dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

$$\alpha\beta=-\frac{1}{4}$$

We have, $\alpha^2\beta+\alpha\beta^2$

$$\alpha^2\beta+\alpha\beta^2=\alpha\beta(\alpha+\beta)$$

By substituting $\alpha+\beta=\frac{5}{4}$ and $\alpha\beta=-\frac{1}{4}$ in $\alpha^2\beta+\alpha\beta^2=\alpha\beta(\alpha+\beta)$, we get

$$\alpha^2\beta+\alpha\beta^2=-\frac{1}{4}\left(\frac{5}{4}\right)$$

$$\alpha^2\beta+\alpha\beta^2=-\frac{1}{4}\times\frac{5}{4}$$

$$\alpha^2\beta+\alpha\beta^2=-\frac{5}{16}$$

Hence, the value of $\alpha^2\beta+\alpha\beta^2$ is $\boxed{-\frac{5}{16}}$.

Polynomials Ex 2.1 Q6

Answer :

Since α and β are the zeros of the quadratic polynomials $f(x)=x^2+x-2$

Sum of the zeros = $\dfrac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$$\alpha+\beta=-\left[\frac{1}{1}\right]$$

$$\alpha+\beta=-\frac{1}{1}$$

$$\alpha+\beta=-1$$

Product if zeros = $\dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

$$\alpha\beta=\frac{-2}{1}$$

$$\alpha\beta=-2$$

We have, $\dfrac{1}{\alpha}-\dfrac{1}{\beta}$

$$\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^2=\left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2+\frac{2}{\alpha\beta}$$

$$\left(\frac{\alpha+\beta}{\alpha\beta}\right)^2=\left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2+\frac{2}{\alpha\beta}$$

$$\left(\frac{\alpha+\beta}{\alpha\beta}\right)^2-\frac{2}{\alpha\beta}=\left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2$$

By substituting $\alpha+\beta=-1$ and $\alpha\beta=-2$ we get ,

$$\left(\frac{-1}{-2}\right)^2-\frac{2}{-2}=\left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2$$

$$\frac{1}{4}+1=\left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2$$

$$\frac{1}{4}+\frac{1\times 4}{1\times 4}=\left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2$$

$$\frac{1+4}{4}=\left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2$$

$$\frac{5}{4}=\left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2$$

$$\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^2=\left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2-\frac{2}{\alpha\beta}$$

By substituting $\frac{5}{4}=\left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2$ in $\left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2-\frac{2}{\alpha\beta}$ we get ,

$$\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^2=\frac{5}{4}-\frac{2}{-2}$$

$$\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^2=\frac{5}{4}+1$$

$$\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^2=\frac{5}{4}+\frac{1\times 4}{1\times 4}$$

$$\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^2=\frac{5+4}{4}$$

$$\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^2=\frac{9}{4}$$

Taking square root on both sides we get

$$\sqrt{\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)^2}=\sqrt{\frac{3\times 3}{2\times 2}}$$

$$\frac{1}{\alpha}-\frac{1}{\beta}=\pm\frac{3}{2}$$

Hence, the value of $\frac{1}{\alpha}-\frac{1}{\beta}$ is $\boxed{\pm\frac{3}{2}}$.

*********** END **********