



EXERCISE.8.2

Question-1

Find the coefficient of x^5 in $(x + 3)^8$

Ans.

It is known that $(r + 1)^{\text{th}}$ term, (T_{r+1}) , in the binomial expansion of $(a + b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$.

Assuming that x^5 occurs in the $(r + 1)^{\text{th}}$ term of the expansion $(x + 3)^8$, we obtain

$$T_{r+1} = {}^8C_r (x)^{8-r} (3)^r$$

Comparing the indices of x in x^5 and in T_{r+1} , we obtain

$$r = 3$$

$$\text{Thus, the coefficient of } x^5 \text{ is } {}^8C_3 (3)^3 = \frac{8!}{3!5!} \times 3^3 = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{3 \cdot 2 \cdot 5!} \cdot 3^3 = 1512$$

Question-2

Find the coefficient of $a^5 b^7$ in $(a - 2b)^{12}$

Ans.

It is known that $(r + 1)^{\text{th}}$ term, (T_{r+1}) , in the binomial expansion of $(a + b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$.

Assuming that $a^5 b^7$ occurs in the $(r + 1)^{\text{th}}$ term of the expansion $(a - 2b)^{12}$, we obtain

$$T_{r+1} = {}^{12}C_r (a)^{12-r} (-2b)^r = {}^{12}C_r (-2)^r (a)^{12-r} (b)^r$$

Comparing the indices of a and b in $a^5 b^7$ and in T_{r+1} , we obtain

$$r = 7$$

Thus, the coefficient of $a^5 b^7$ is

$${}^{12}C_r (-2)^7 = -\frac{12!}{7!5!} \cdot 2^7 = -\frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 7!} \cdot 2^7 = -(792)(128) = -101376$$

Question-3

Write the general term in the expansion of $(x^2 - y)^6$

Ans.

It is known that the general term T_{r+1} {which is the $(r + 1)^{\text{th}}$ term} in the binomial expansion of $(a + b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$.

Thus, the general term in the expansion of $(x^2 - y^6)$ is

$$T_{r+1} = {}^nC_r (x^2)^{6-r} (-y)^r = (-1)^r {}^nC_r x^{12-2r} y^r$$

Question-4

Write the general term in the expansion of $(x^2 - yx)^{12}$, $x \neq 0$

Ans.

It is known that the general term T_{r+1} {which is the $(r + 1)^{\text{th}}$ term} in the binomial expansion of $(a + b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$.

Thus, the general term in the expansion of $(x^2 - yx)^{12}$ is

$$T_{r+1} = {}^{12}C_r (x^2)^{12-r} (-yx)^r = (-1)^r {}^{12}C_r x^{24-2r} y^r x^r = (-1)^r {}^{12}C_r x^{24-r} y^r$$

Question-5

Find the 4th term in the expansion of $(x - 2y)^{12}$.

Ans.

It is known that $(r + 1)^{\text{th}}$ term, (T_{r+1}) , in the binomial expansion of $(a + b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$.

Thus, the 4th term in the expansion of $(x - 2y)^{12}$ is

$$T_4 = T_{3+1} = {}^{12}C_3 (x)^{12-3} (-2y)^3 = (-1)^3 \cdot \frac{12!}{3!9!} \cdot x^9 \cdot (2)^3 \cdot y^3 = -\frac{12 \cdot 11 \cdot 10}{3 \cdot 2} \cdot (2)^3 x^9 y^3 = -1760 x^9 y^3$$

Question-6

Find the 13th term in the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$, $x \neq 0$.

Ans.

It is known that $(r + 1)^{\text{th}}$ term, (T_{r+1}) , in the binomial expansion of $(a + b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$.

Thus, 13th term in the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$ is

$$\begin{aligned} T_{13} = T_{12+1} &= {}^{18}C_{12} (9x)^{18-12} \left(-\frac{1}{3\sqrt{x}}\right)^{12} \\ &= (-1)^{12} \frac{18!}{12!6!} (9)^6 (x)^6 \left(\frac{1}{3}\right)^{12} \left(\frac{1}{\sqrt{x}}\right)^{12} \\ &= \frac{18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12!}{12! \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} \cdot x^6 \cdot \left(\frac{1}{x^6}\right) \cdot 3^{12} \left(\frac{1}{3^{12}}\right) \quad \left[9^6 = (3^2)^6 = 3^{12}\right] \\ &= 18564 \end{aligned}$$

Question-7

Find the middle terms in the expansions of $\left(3 - \frac{x^3}{6}\right)^7$

Ans.

It is known that in the expansion of $(a + b)^n$, if n is odd, then there are two middle terms, namely, $\left(\frac{n+1}{2}\right)^{\text{th}}$ term and $\left(\frac{n+1}{2}+1\right)^{\text{th}}$ term.

Therefore, the middle terms in the expansion of $\left(3 - \frac{x^3}{6}\right)^7$ are $\left(\frac{7+1}{2}\right)^{\text{th}} = 4^{\text{th}}$ term and $\left(\frac{7+1}{2}+1\right)^{\text{th}} = 5^{\text{th}}$ term

$$\begin{aligned} T_4 = T_{3+1} &= {}^7C_3 (3)^{7-3} \left(-\frac{x^3}{6}\right)^3 = (-1)^3 \frac{7!}{3!4!} \cdot 3^4 \cdot \frac{x^9}{6^3} \\ &= -\frac{7 \cdot 6 \cdot 5 \cdot 4!}{3 \cdot 2 \cdot 4!} \cdot 3^4 \cdot \frac{1}{2^3 \cdot 3^3} \cdot x^9 = -\frac{105}{8} x^9 \\ T_5 = T_{4+1} &= {}^7C_4 (3)^{7-4} \left(-\frac{x^3}{6}\right)^4 = (-1)^4 \frac{7!}{4!3!} (3)^3 \cdot \frac{x^{12}}{6^4} \\ &= \frac{7 \cdot 6 \cdot 5 \cdot 4!}{4! \cdot 3 \cdot 2} \cdot \frac{3^3}{2^4 \cdot 3^4} \cdot x^{12} = \frac{35}{48} x^{12} \end{aligned}$$

Thus, the middle terms in the expansion of $\left(3 - \frac{x^3}{6}\right)^7$ are $-\frac{105}{8} x^9$ and $\frac{35}{48} x^{12}$.

Question-8

Thus, the middle terms in the expansion of $\left(3 - \frac{x^3}{6}\right)^7$ are $-\frac{105}{8} x^9$ and $\frac{35}{48} x^{12}$.

Ans.

It is known that in the expansion $(a + b)^n$, if n is even, then the middle term is $\left(\frac{n}{2}+1\right)^{\text{th}}$ term.

Therefore, the middle term in the expansion of $\left(\frac{x}{3} + 9y\right)^{10}$ is $\left(\frac{10}{2}+1\right)^{\text{th}} = 6^{\text{th}}$ term

$$\begin{aligned} T_6 = T_{5+1} &= {}^{10}C_5 \left(\frac{x}{3}\right)^{10-5} (9y)^5 = \frac{10!}{5!5!} \cdot \frac{x^5}{3^5} \cdot 9^5 \cdot y^5 \\ &= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 5!} \cdot \frac{1}{3^5} \cdot 3^{10} \cdot x^5 y^5 \quad \left[9^5 = (3^2)^5 = 3^{10}\right] \\ &= 252 \times 3^5 \cdot x^5 \cdot y^5 = 61236 x^5 y^5 \end{aligned}$$

Thus, the middle term in the expansion of $\left(\frac{x}{3} + 9y\right)^{10}$ is $61236 x^5 y^5$.

Question-9

In the expansion of $(1 + a)^{m+n}$, prove that coefficients of a^m and a^n are equal.

Ans.

It is known that $(r + 1)^{\text{th}}$ term, (T_{r+1}) , in the binomial expansion of $(a + b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$.

Assuming that a^m occurs in the $(r + 1)^{\text{th}}$ term of the expansion $(1 + a)^{m+n}$, we obtain

$$T_{r+1} = {}^{m+n}C_r (1)^{m+n-r} (a)^r = {}^{m+n}C_r a^r$$

Comparing the indices of a in a^m and in T_{r+1} , we obtain

$$r = m$$

Therefore, the coefficient of a^m is

$${}^{m+n}C_m = \frac{(m+n)!}{m!(m+n-m)!} = \frac{(m+n)!}{m!n!} \quad \dots(1)$$

Assuming that a^n occurs in the $(k + 1)^{\text{th}}$ term of the expansion $(1 + a)^{m+n}$, we obtain

$$T_{k+1} = {}^{m+n}C_k (1)^{m+n-k} (a)^k = {}^{m+n}C_k (a)^k$$

Comparing the indices of a in a^n and in T_{k+1} , we obtain

$$k = n$$

Therefore, the coefficient of a^n is

$${}^{m+n}C_n = \frac{(m+n)!}{n!(m+n-n)!} = \frac{(m+n)!}{n!m!} \quad \dots(2)$$

Thus, from (1) and (2), it can be observed that the coefficients of a^m and a^n in the expansion of $(1 + a)^{m+n}$ are equal.

Question-10

The coefficients of the $(r - 1)^{\text{th}}$, r^{th} and $(r + 1)^{\text{th}}$ terms in the expansion of

$(x + 1)^n$ are in the ratio 1:3:5. Find n and r .

Ans.

It is known that $(k + 1)^{\text{th}}$ term, (T_{k+1}) , in the binomial expansion of $(a + b)^n$ is given by $T_{k+1} = {}^nC_k a^{n-k} b^k$.

Therefore, $(r - 1)^{\text{th}}$ term in the expansion of $(x + 1)^n$ is

$$T_{r-1} = {}^nC_{r-2} (x)^{n-(r-2)} (1)^{(r-2)} = {}^nC_{r-2} x^{n-r+2}$$

r^{th} term in the expansion of $(x + 1)^n$ is $T_r = {}^nC_{r-1} (x)^{n-(r-1)} (1)^{(r-1)} = {}^nC_{r-1} x^{n-r+1}$

$(r + 1)^{\text{th}}$ term in the expansion of $(x + 1)^n$ is $T_{r+1} = {}^nC_r (x)^{n-r} (1)^r = {}^nC_r x^{n-r}$

Therefore, the coefficients of the $(r - 1)^{\text{th}}$, r^{th} , and $(r + 1)^{\text{th}}$ terms in the expansion of $(x + 1)^n$ are ${}^nC_{r-2}$, ${}^nC_{r-1}$, and nC_r respectively. Since these coefficients are in the ratio 1:3:5, we obtain

$$\begin{aligned} \frac{{}^nC_{r-2}}{{}^nC_{r-1}} &= \frac{1}{3} \text{ and } \frac{{}^nC_{r-1}}{{}^nC_r} = \frac{3}{5} \\ \frac{{}^nC_{r-2}}{{}^nC_{r-1}} &= \frac{n!}{(r-2)!(n-r+2)!} \times \frac{(r-1)!(n-r+1)!}{n!} = \frac{(r-1)(r-2)!(n-r+1)!}{(r-2)!(n-r+2)(n-r+1)!} \\ &= \frac{r-1}{n-r+2} \end{aligned}$$

$$\begin{aligned}\therefore \frac{r-1}{n-r+2} &= \frac{1}{3} \\ \Rightarrow 3r-3 &= n-r+2 \\ \Rightarrow n-4r+5 &= 0 \quad \dots(1)\end{aligned}$$

$$\begin{aligned}\therefore \frac{r-1}{n-r+2} &= \frac{1}{3} \\ \Rightarrow 3r-3 &= n-r+2 \\ \Rightarrow n-4r+5 &= 0 \quad \dots(1)\end{aligned}$$

$$\begin{aligned}\frac{{}^nC_{r-1}}{{}^nC_r} &= \frac{n!}{(r-1)!(n-r+1)!} \times \frac{r!(n-r)!}{n!} = \frac{r(r-1)!(n-r)!}{(r-1)!(n-r+1)(n-r)!} \\ &= \frac{r}{n-r+1}\end{aligned}$$

$$\begin{aligned}\therefore \frac{r}{n-r+1} &= \frac{3}{5} \\ \Rightarrow 5r &= 3n-3r+3 \\ \Rightarrow 3n-8r+3 &= 0 \quad \dots(2)\end{aligned}$$

Multiplying (1) by 3 and subtracting it from (2), we obtain

$$4r - 12 = 0$$

$$r = 3$$

Putting the value of r in (1), we obtain

$$n - 12 + 5 = 0$$

$$n = 7$$

Thus, $n = 7$ and $r = 3$

Question-11

Prove that the coefficient of x^n in the expansion of $(1+x)^{2n}$ is twice the coefficient of x^n in the expansion of $(1+x)^{2n-1}$

Ans.

It is known that $(r + 1)^{\text{th}}$ term, (T_{r+1}) , in the binomial expansion of $(a + b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$.

Assuming that x^n occurs in the $(r + 1)^{\text{th}}$ term of the expansion of $(1 + x)^{2n}$, we obtain

$$T_{r+1} = {}^{2n}C_r (1)^{2n-r} (x)^r = {}^{2n}C_r (x)^r$$

Comparing the indices of x in x^n and in T_{r+1} , we obtain

$$r = n$$

Therefore, the coefficient of x^n in the expansion of $(1 + x)^{2n}$ is

$${}^{2n}C_n = \frac{(2n)!}{n!(2n-n)!} = \frac{(2n)!}{n!n!} = \frac{(2n)!}{(n!)^2} \quad \dots(1)$$

Assuming that x^n occurs in the $(k + 1)^{\text{th}}$ term of the expansion $(1 + x)^{2n-1}$, we obtain

$$T_{k+1} = {}^{2n-1}C_k (1)^{2n-1-k} (x)^k = {}^{2n-1}C_k (x)^k$$

Comparing the indices of x in x^n and T_{k+1} , we obtain

$$k = n$$

Therefore, the coefficient of x^n in the expansion of $(1 + x)^{2n-1}$ is

$$\begin{aligned} {}^{2n-1}C_n &= \frac{(2n-1)!}{n!(2n-1-n)!} = \frac{(2n-1)!}{n!(n-1)!} \\ &= \frac{2n \cdot (2n-1)!}{2n \cdot n!(n-1)!} = \frac{(2n)!}{2 \cdot n!n!} = \frac{1}{2} \left[\frac{(2n)!}{(n!)^2} \right] \quad \dots(2) \end{aligned}$$

From (1) and (2), it is observed that

$$\begin{aligned} \frac{1}{2} ({}^{2n}C_n) &= {}^{2n-1}C_n \\ \Rightarrow {}^{2n}C_n &= 2 ({}^{2n-1}C_n) \end{aligned}$$

Therefore, the coefficient of x^n in the expansion of $(1 + x)^{2n}$ is twice the coefficient of x^n in the expansion of $(1 + x)^{2n-1}$.

Hence, proved.

Question-12

Find a positive value of m for which the coefficient of x^2 in the expansion

$$(1 + x)^m \text{ is } 6.$$

Ans.

It is known that $(r + 1)^{\text{th}}$ term, (T_{r+1}) , in the binomial expansion of $(a + b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$.

Assuming that x^2 occurs in the $(r + 1)^{\text{th}}$ term of the expansion $(1 + x)^m$, we obtain

$$T_{r+1} = {}^m C_r (1)^{m-r} (x)^r = {}^m C_r (x)^r$$

Comparing the indices of x in x^2 and in T_{r+1} , we obtain

$$r = 2$$

Therefore, the coefficient of x^2 is ${}^m C_2$.

It is given that the coefficient of x^2 in the expansion $(1 + x)^m$ is 6.

$$\therefore {}^m C_2 = 6$$

$$\Rightarrow \frac{m!}{2!(m-2)!} = 6$$

$$\Rightarrow \frac{m(m-1)(m-2)!}{2 \times (m-2)!} = 6$$

$$\Rightarrow m(m-1) = 12$$

$$\Rightarrow m^2 - m - 12 = 0$$

$$\Rightarrow m^2 - 4m + 3m - 12 = 0$$

$$\Rightarrow m(m-4) + 3(m-4) = 0$$

$$\Rightarrow (m-4)(m+3) = 0$$

$$\Rightarrow (m-4) = 0 \text{ or } (m+3) = 0$$

$$\Rightarrow m = 4 \text{ or } m = -3$$

Thus, the positive value of m , for which the coefficient of x^2 in the expansion

$(1 + x)^m$ is 6, is 4.

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