Trigonometric Ratios Ex 5.2 Q12

Answer :

We have,

$$\cot^2 30^\circ - 2\cos^2 60^\circ - \frac{3}{4}\sec^2 45^\circ - 4\sec^2 30^\circ \text{ ...... (1)}$$

Now,

$$\cot 30^\circ = \sqrt{3},\ \cos 60^\circ = \frac{1}{2},\ \sec 45^\circ = \sqrt{2},\ \sec 30^\circ = \frac{2}{\sqrt{3}}$$

So by substituting above values in equation (1)

We get,

$$\cot^2 30^\circ - 2\cos^2 60^\circ - \frac{3}{4}\sec^2 45^\circ - 4\sec^2 30^\circ$$

$$= \left(\sqrt{3}\right)^2 - 2\left(\frac{1}{2}\right)^2 - \frac{3}{4}\left(\sqrt{2}\right)^2 - 4\left(\frac{2}{\sqrt{3}}\right)^2$$

$$= 3 - 2\times\frac{1^2}{2^2} - \frac{3}{4}\times 2 - 4\times\frac{2^2}{\left(\sqrt{3}\right)^2}$$

Now, in the third term 4 gets cancelled by 2 and 2 remains

Therefore,

$$\cot^2 30^\circ - 2\cos^2 60^\circ - \frac{3}{4}\sec^2 45^\circ - 4\sec^2 30^\circ$$

$$= 3 - 2\times\frac{1}{4} - \frac{3}{2} - 4\times\frac{4}{3}$$

Now in the second term, 4 gets cancelled by 2 and 2 remains

Therefore,

$$\cot^2 30^\circ - 2\cos^2 60^\circ - \frac{3}{4}\sec^2 45^\circ - 4\sec^2 30^\circ$$
$$= 3 - \frac{1}{2} - \frac{3}{2} - 4 \times \frac{4}{3}$$
$$= 3 - \frac{1}{2} - \frac{3}{2} - \frac{16}{3}$$

Now, LCM of denominator in the above expression is 6

Therefore by taking LCM

We get,

$$\cot^2 30^\circ - 2\cos^2 60^\circ - \frac{3}{4}\sec^2 45^\circ - 4\sec^2 30^\circ$$
$$= \frac{3\times 6}{1\times 6} - \frac{1\times 3}{2\times 3} - \frac{3\times 3}{2\times 3} - \frac{16\times 2}{3\times 2}$$
$$= \frac{18}{6} - \frac{3}{6} - \frac{9}{6} - \frac{32}{6}$$
$$= \frac{18-3-9-32}{6}$$
$$= \frac{18-12-32}{6}$$
$$= \frac{18-44}{6}$$
$$= \frac{-26}{6}$$

Now in the above expression, $\frac{-26}{6}$ gets reduced to $\frac{-13}{3}$

Therefore,

$$\cot^2 30^\circ - 2\cos^2 60^\circ - \frac{3}{4}\sec^2 45^\circ - 4\sec^2 30^\circ = \frac{-13}{3}$$

Trigonometric Ratios Ex 5.2 Q13

Answer :

We have,

$$\left(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ\right)\left(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ\right) \quad \ldots\ldots (1)$$

Now,

$$\sin 90^\circ = \cos 0^\circ = 1, \sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}, \sin 30^\circ = \cos 60^\circ = \frac{1}{2}$$

So by substituting above values in equation (1)

We get,

$$\left(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ\right)\left(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ\right)$$

$$= \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{2}\right)\left(1 - \frac{1}{\sqrt{2}} + \frac{1}{2}\right)$$

Now, LCM of both the product terms in the above expression is $2\sqrt{2}$

Therefore we get,

$$\left(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ\right)\left(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ\right)$$

$$= \left(\frac{1\times 2\sqrt{2}}{1\times 2\sqrt{2}} + \frac{1\times 2}{\sqrt{2}\times 2} + \frac{1\times\sqrt{2}}{2\times\sqrt{2}}\right)\times\left(\frac{1\times 2\sqrt{2}}{1\times 2\sqrt{2}} - \frac{1\times 2}{\sqrt{2}\times 2} + \frac{1\times\sqrt{2}}{2\times\sqrt{2}}\right)$$

$$= \left(\frac{2\sqrt{2}}{2\sqrt{2}} + \frac{2}{2\sqrt{2}} + \frac{\sqrt{2}}{2\sqrt{2}}\right)\times\left(\frac{2\sqrt{2}}{2\sqrt{2}} - \frac{2}{2\sqrt{2}} + \frac{\sqrt{2}}{2\sqrt{2}}\right)$$

$$= \left(\frac{2\sqrt{2} + 2 + \sqrt{2}}{2\sqrt{2}}\right)\times\left(\frac{2\sqrt{2} - 2 + \sqrt{2}}{2\sqrt{2}}\right)$$

Now by rearranging terms in the numerator of above expression

We get,

$$\left(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ\right)\left(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ\right)$$

$$= \left(\frac{2\sqrt{2} + \sqrt{2} + 2}{2\sqrt{2}}\right)\times\left(\frac{2\sqrt{2} + \sqrt{2} - 2}{2\sqrt{2}}\right)$$

$$= \frac{\left(2\sqrt{2} + \sqrt{2} + 2\right)\times\left(2\sqrt{2} + \sqrt{2} - 2\right)}{\left(2\sqrt{2}\right)\times\left(2\sqrt{2}\right)}$$

Now, by applying formula $\left[(a+b)(a-b) = a^2 - b^2\right]$ in the numerator of the above expression we get,

$$\left(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ\right)\left(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ\right)$$

$$= \frac{\left(2\sqrt{2} + \sqrt{2}\right)^2 - 2^2}{2\times 2\times\sqrt{2}\times\sqrt{2}}$$

$$= \frac{\left(2\sqrt{2} + \sqrt{2}\right)^2 - 2^2}{4\times 2} \quad \ldots\ldots (2)$$

Now, we know that $(a+b)^2 = a^2 + 2ab + b^2$

Therefore,

$$\left(2\sqrt{2} + \sqrt{2}\right)^2 = \left(2\sqrt{2}\right)^2 + 2\left(2\sqrt{2}\right)\left(\sqrt{2}\right) + \left(\sqrt{2}\right)^2$$

Now, by substituting the above value of $\left(2\sqrt{2} + \sqrt{2}\right)^2$ in equation (2)

We get,

$$\left(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ\right)\left(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ\right)$$

$$= \frac{\left[\left(2\sqrt{2}\right)^2 + 2\left(2\sqrt{2}\right)\left(\sqrt{2}\right) + \left(\sqrt{2}\right)^2\right] - 2^2}{4 \times 2}$$

$$= \frac{[8 + 8 + 2] - 4}{8}$$

$$= \frac{18 - 4}{8}$$

$$= \frac{14}{8}$$

Now $\frac{14}{8}$ gets reduced to $\frac{7}{4}$

Therefore,

$$\left(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ\right)\left(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ\right)$$

$$= \frac{7}{4}$$

Hence, $\left(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ\right)\left(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ\right) = \frac{7}{4}$

*********** END **********