



Functions Ex 3.3 Q3

We have,

$$f(x) = \frac{ax+b}{bx-a}$$

We observe that $f(x)$ is a rational function of x as $\frac{ax+b}{bx-a}$ is a rational expression.

Clearly, $f(x)$ assumes real values for all x except for the values of x for which $bx-a=0$ i.e., $bx=a$

$$\Rightarrow x = \frac{a}{b}$$

$$\therefore \text{Domain}(f) = R - \left\{ \frac{a}{b} \right\}$$

Range of f : Let $f(x) = y$

$$\Rightarrow \frac{ax+b}{bx-a} = y$$

$$\Rightarrow ax+b = y(bx-a)$$

$$\Rightarrow ax+b = bxy-ay$$

$$\Rightarrow b+ay = bxy-ax$$

$$\Rightarrow b+ay = x(by-a)$$

$$\Rightarrow \frac{b+ay}{b-ay} = x$$

$$\Rightarrow x = \frac{b+ay}{by-a}$$

Clearly, x will take real value for all $x \in R$ except for

$$by-a=0$$

$$\Rightarrow by=a$$

$$\Rightarrow y = \frac{a}{b}$$

$$\therefore \text{Range}(f) = R - \left\{ \frac{a}{b} \right\}.$$

We have,

$$f(x) = \frac{ax-b}{cx-d}$$

We observe that $f(x)$ is a rational function of x as $\frac{ax-b}{cx-d}$ is a rational expression.

Clearly, $f(x)$ assumes real values for all x except for all those values of x for which $cx-d=0$ i.e., $cx=d$

$$\Rightarrow x = \frac{d}{c}$$

$$\therefore \text{Domain}(f) = R - \left\{ \frac{d}{c} \right\}$$

Range: Let $f(x) = y$

$$\Rightarrow \frac{ax-b}{cx-d} = y$$

$$\Rightarrow ax-b = y(cx-d)$$

$$\Rightarrow ax-b = cxy-dy$$

$$\Rightarrow dy-b = cxy-ax$$

$$\Rightarrow dy-b = x(cy-a)$$

$$\Rightarrow \frac{dy-b}{cy-a} = x$$

Clearly, x assumes real values for all y except

$$cy-a=0 \text{ i.e., } y = \frac{a}{c}$$

$$\text{Hence, range}(f) = R - \left\{ \frac{a}{c} \right\}$$

We have,

$$f(x) = \sqrt{x-1}$$

Clearly, $f(x)$ assumes real values, if

$$x-1 \geq 0$$

$$\Rightarrow x \geq 1$$

$$\Rightarrow x \in [1, \infty)$$

Hence, $\text{domain}(f) = [1, \infty)$

Range: For $x \geq 1$, we have,

$$x-1 \geq 0$$

$$\Rightarrow \sqrt{x-1} \geq 0$$

$$\Rightarrow f(x) \geq 0$$

Thus, $f(x)$ takes all real values greater than zero.

Hence, $\text{range}(f) = [0, \infty)$

We have,

$$f(x) = \sqrt{x-3}$$

Clearly, $f(x)$ assumes real values, if

$$x-3 \geq 0$$

$$\Rightarrow x \geq 3$$

$$\Rightarrow x \in [3, \infty)$$

Hence, $\text{domain}(f) = [3, \infty)$

Range: For $x \geq 3$, we have,

$$x-3 \geq 0$$

$$\Rightarrow \sqrt{x-3} \geq 0$$

$$\Rightarrow f(x) \geq 0$$

Thus, $f(x)$ takes all real values greater than zero.

Hence, $\text{range}(f) = [0, \infty)$

We have,

$$f(x) = \frac{x-2}{2-x}$$

Domain of f : Clearly, $f(x)$ is defined for all $x \in R$ except for which

$$2-x \neq 0 \text{ i.e., } x \neq 2$$

Hence, $\text{domain}(f) = R - \{2\}$

Range of f : Let $f(x) = y$

$$\Rightarrow \frac{x-2}{2-x} = y$$

$$\Rightarrow \frac{-1(2-x)}{2-x} = y$$

$$\Rightarrow -1 = y$$

$$\Rightarrow y = -1$$

$\therefore \text{Range}(f) = \{-1\}$

We have,

$$f(x) = |x - 1|$$

Clearly, $f(x)$ is defined for all $x \in \mathbb{R}$

$$\Rightarrow \text{Domain}(f) = \mathbb{R}$$

Range: Let $f(x) = y$

$$\Rightarrow |x - 1| = y$$

$$\Rightarrow f(x) \geq 0 \quad \forall x \in \mathbb{R}$$

It follows from the above relation that y takes all real values greater or equal to zero.

$$\therefore \text{Range}(f) = [0, \infty)$$

As $|x|$ is defined for all real numbers, its domain is \mathbb{R} and range is only negative numbers because, $|x|$ is always positive real number for all real numbers and $-|x|$ is always negative real numbers.

In order to have $F(x)$ has defined value, term inside square root should always be greater than or equal to zero which gives domain as $-3 \leq x \leq 3$

Where as Range of above function is limited to $[0, 3]$

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