Differentiation Ex 11.4 Q6

Given,

$$x^5 + y^5 = 5xy$$

Differentiating with respect to x,

$$\frac{d}{dx}\left(x^5\right) + \frac{d}{dx}\left(y^5\right) = \frac{d}{dx}(5xy)$$

$$\Rightarrow \quad 5x^4 + 5y^4\frac{dy}{dx} = 5\left[x\frac{dy}{dx} + y\frac{dy}{dx}(x)\right] \qquad \text{[Using product rule]}$$

$$\Rightarrow \quad 5x^4 + 5y^4\frac{dy}{dx} = 5\left[x\frac{dy}{dx} + y\,(1)\right]$$

$$\Rightarrow \quad 5x^4 + 5y^4\frac{dy}{dx} = 5x\frac{dy}{dx} + 5y$$

$$\Rightarrow \quad 5y^4\frac{dy}{dx} - 5x\frac{dy}{dx} = 5y - 5x^4$$

$$\Rightarrow \quad 5\frac{dy}{dx}\left(y^4 - x\right) = 5\left(y - x^4\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{5\left(y - x^4\right)}{5\left(y^4 - x\right)}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y - x^4}{y^4 - x}$$

Differentiation Ex 11.4 Q7

Given,

$$(x + y)^2 = 2axy$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{d}{dx}(x + y)^2 = \frac{d}{dx}(2axy)$$

$$\Rightarrow \quad 2(x + y)\frac{d}{dx}(x + y) = 2a\left[x\frac{dy}{dx} + y\frac{d}{dx}(x)\right] \qquad \text{[Using chain rule and product rule]}$$

$$\Rightarrow \quad 2(x + y)\left[1 + \frac{dy}{dx}\right] = 2a\left[x\frac{dy}{dx} + y\,(1)\right]$$

$$\Rightarrow \quad 2(x + y) + 2(x + y)\frac{dy}{dx} = 2ax\frac{dy}{dx} + 2ay$$

$$\Rightarrow \quad \frac{dy}{dx}\left[2(x + y) - 2ax\right] = 2ay - 2(x + y)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2[ay - x - y]}{2[x + y - ax]}$$

$$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{ay - x - y}{x + y - ax}\right)$$

Differentiation Ex 11.4 Q8

Given,

$$\left(x^2 + y^2\right)^2 = xy$$

Differentiating with respect to x,

$$\frac{d}{dx}\left(\left(x^2 + y^2\right)^2\right) = \frac{d}{dx}(xy)$$

$$\Rightarrow \quad 2\left(x^2 + y^2\right)\frac{d}{dx}\left(x^2 + y^2\right) = x\frac{dy}{dx} + y\frac{d}{dx}(x) \qquad \text{[Using chain rule and product rule]}$$

$$\Rightarrow \quad 2\left(x^2 + y^2\right)\left(2x + 2y\frac{dy}{dx}\right) = x\frac{dy}{dx} + y\,(1)$$

$$\Rightarrow \quad 4x\left(x^2 + y^2\right) + 4y\left(x^2 + y^2\right)\frac{dy}{dx} = x\frac{dy}{dx} + y$$

$$\Rightarrow \quad 4y\left(x^2 + y^2\right)\frac{dy}{dx} - x\frac{dy}{dx} = y - 4x\left(x^2 + y^2\right)$$

$$\Rightarrow \quad \frac{dy}{dx}\left[4yx^2 + 4y^3 - x\right] = y - 4x^3 - 4xy^2$$

$$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{y - 4x^3 - 4xy^2}{4yx^2 + 4y^3 - x}\right)$$

Differentiation Ex 11.4 Q9

Here,

$$\tan^{-1}\left(x^2+y^2\right)=a$$

Differentiating with respect to x,

$$\frac{d}{dx}\left(\tan^{-1}\left(x^2+y^2\right)\right)=\frac{d}{dx}(a)$$

$$\Rightarrow \quad \frac{1}{1+\left(x^2+y^2\right)^2}\times\frac{d}{dx}\left(x^2+y^2\right)=0 \qquad \text{[Using chain rule]}$$

$$\Rightarrow \quad \left[\frac{1}{1+\left(x^2+y^2\right)^2}\right]\left(2x+2y\frac{dy}{dx}\right)=0$$

$$\Rightarrow \quad \left\{\frac{2x}{1+\left(x^2+y^2\right)^2}\right\}+\left\{\frac{2y}{1+\left(x^2+y^2\right)^2}\cdot\right\}\frac{dy}{dx}=0$$

$$\Rightarrow \quad \frac{2y}{1+\left(x^2+y^2\right)^2}\frac{dy}{dx}=-\frac{2x}{1+\left(x^2+y^2\right)^2}$$

$$\Rightarrow \quad \frac{dy}{dx}=-\left(\frac{2x}{1+\left(x^2+y^2\right)^2}\right)\left(\frac{1+\left(x^2+y^2\right)^2}{2y}\right)$$

$$\Rightarrow \quad \frac{dy}{dx}=-\left(\frac{x}{y}\right)$$

Differentiation Ex 11.4 Q10

Given,

$$e^{x-y}=\log\left(\frac{x}{y}\right)$$

Differentiating with respect to x,

$$\frac{d}{dx}\left(e^{x-y}\right)=\frac{d}{dx}\log\left(\frac{x}{y}\right)$$

$$\Rightarrow \quad e^{(x-y)}\frac{d}{dx}(x-y)=\frac{1}{\left(\frac{x}{y}\right)}\times\frac{d}{dx}\left(\frac{x}{y}\right) \qquad \text{[Using chain rule and quotient rule]}$$

$$\Rightarrow \quad e^{(x-y)}\left(1-\frac{dy}{dx}\right)=\frac{y}{x}\left[\frac{y\frac{d}{dx}(x)-x\frac{dy}{dx}}{y^2}\right]$$

$$\Rightarrow \quad e^{(x-y)}-e^{(x-y)}\frac{dy}{dx}=\frac{1}{xy}\left[y(1)-x\frac{dy}{dx}\right]$$

$$\Rightarrow \quad e^{(x-y)}-e^{(x-y)}\frac{dy}{dx}=\frac{y}{xy}-\frac{x}{xy}\frac{dy}{dx}$$

$$\Rightarrow \quad e^{(x-y)}-e\,(x-y)\frac{dy}{dx}=\frac{1}{x}-\frac{1}{y}\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx}-e^{(x-y)}\frac{dy}{dx}=\frac{1}{x}-e^{(x-y)}$$

$$\Rightarrow \quad \frac{dy}{dx}\left[\frac{1}{y}-\frac{e^{(x-y)}}{1}\right]=\frac{1}{x}-\frac{e^{(x-y)}}{1}$$

$$\Rightarrow \quad \frac{dy}{dx}\left[\frac{1-ye^{(x-y)}}{y}\right]=\frac{\left(1-xe^{(x-y)}\right)}{x}$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{y}{x}\left[\frac{1-xe^{(x-y)}}{1-ye^{(x-y)}}\right]$$

$$=\frac{-y}{-x}\left[\frac{xe^{(x-y)}-1}{ye^{(x-y)}-1}\right]$$

$$\frac{dy}{dx}=\frac{y}{x}\left[\frac{xe^{(x-y)}-1}{ye^{(x-y)}-1}\right]$$

*********** END **********