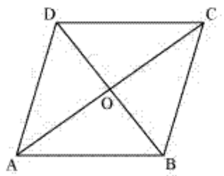
NCERT MISCELLANEOUS SOLUTIONS

Question 1:

Three vertices of a parallelogram ABCD are A (3, −1, 2), B (1, 2, −4) andC (−1, 1, 2). Find the coordinates of the fourth vertex.

Ans:

The three vertices of a parallelogram ABCD are given as A (3, −1, 2), B (1, 2, −4), and C (−1, 1, 2). Let the coordinates of the fourth vertex be D (x, y, z).



We know that the diagonals of a parallelogram bisect each other.

Therefore, in parallelogram ABCD, AC and BD bisect each other.

□Mid-point of AC = Mid-point of BD

$$\Rightarrow \left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2}\right) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2}\right)$$

$$\Rightarrow (1,0,2) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2}\right)$$

$$\Rightarrow \frac{x+1}{2} = 1,\ \frac{y+2}{2} = 0, \text{ and } \frac{z-4}{2} = 2$$

□ x = 1, y = −2, and z = 8

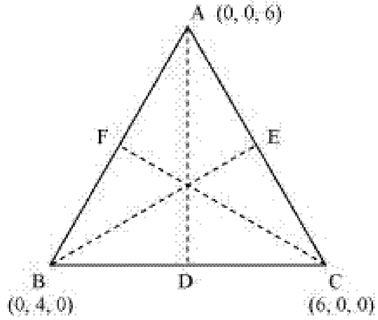
Thus, the coordinates of the fourth vertex are (1, −2, 8).

Question 2:

Find the lengths of the medians of the triangle with vertices A (0, 0, 6), B (0, 4, 0) and (6, 0, 0).

Ans:

Let AD, BE, and CF be the medians of the given triangle ABC.



Since AD is the median, D is the mid-point of BC.

∴Coordinates of point D = $\left(\frac{0+6}{2},\frac{4+0}{2},\frac{0+0}{2}\right)$ = (3, 2, 0)

$$AD=\sqrt{(0-3)^2+(0-2)^2+(6-0)^2}=\sqrt{9+4+36}=\sqrt{49}=7$$

Since BE is the median, E is the mid-point of AC.

$\therefore$ Coordinates of point E $=\left(\frac{0+6}{2},\frac{0+0}{2},\frac{6+0}{2}\right)=(3,0,3)$

$$BE=\sqrt{(3-0)^2+(0-4)^2+(3-0)^2}=\sqrt{9+16+9}=\sqrt{34}$$

Since CF is the median, F is the mid-point of AB.

$\therefore$ Coordinates of point F $=\left(\frac{0+0}{2},\frac{0+4}{2},\frac{6+0}{2}\right)=(0,2,3)$

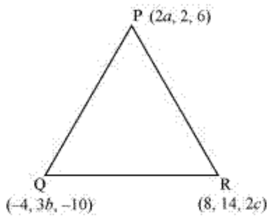
$$\text{Length of CF}=\sqrt{(6-0)^2+(0-2)^2+(0-3)^2}=\sqrt{36+4+9}=\sqrt{49}=7$$

Thus, the lengths of the medians of ΔABC are $7,\sqrt{34}$, and 7.

Question 3:

If the origin is the centroid of the triangle PQR with vertices P ($2a$, 2, 6), Q (−4, $3b$, −10) and R (8, 14, $2c$), then find the values of a, b and c.

Ans:



It is known that the coordinates of the centroid of the triangle, whose vertices are (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3), are $\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3},\frac{z_1+z_2+z_3}{3}\right)$.

Therefore, coordinates of the centroid of ΔPQR

$$=\left(\frac{2a-4+8}{3},\frac{2+3b+14}{3},\frac{6-10+2c}{3}\right)=\left(\frac{2a+4}{3},\frac{3b+16}{3},\frac{2c-4}{3}\right)$$

It is given that origin is the centroid of ΔPQR.

$$\therefore(0,0,0)=\left(\frac{2a+4}{3},\frac{3b+16}{3},\frac{2c-4}{3}\right)$$

$$\Rightarrow\frac{2a+4}{3}=0,\frac{3b+16}{3}=0\text{ and }\frac{2c-4}{3}=0$$

$$\Rightarrow a=-2,b=-\frac{16}{3}\text{ and }c=2$$

Thus, the respective values of a, b, and c are $-2,-\frac{16}{3}$, and 2.

Question 4:

Find the coordinates of a point on y-axis which are at a distance of $5\sqrt{2}$ from the point P (3, −2, 5).

Ans:

If a point is on the y-axis, then x-coordinate and the z-coordinate of the point are zero.

Let A (0, b, 0) be the point on the y-axis at a distance of $5\sqrt{2}$ from point P (3, −2, 5). Accordingly, $AP = 5\sqrt{2}$

$$\therefore AP^2 = 50$$
$$\Rightarrow (3-0)^2 + (-2-b)^2 + (5-0)^2 = 50$$
$$\Rightarrow 9 + 4 + b^2 + 4b + 25 = 50$$
$$\Rightarrow b^2 + 4b - 12 = 0$$
$$\Rightarrow b^2 + 6b - 2b - 12 = 0$$
$$\Rightarrow (b+6)(b-2) = 0$$
$$\Rightarrow b = -6 \text{ or } 2$$

Thus, the coordinates of the required points are (0, 2, 0) and (0, −6, 0).

Question 5:

A point R with x-coordinate 4 lies on the line segment joining the pointsP (2, −3, 4) and Q (8, 0, 10). Find the coordinates of the point R.

[**Hint** suppose R divides PQ in the ratio k: 1. The coordinates of the point R are given by $\left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1}\right)$]

Ans:

The coordinates of points P and Q are given as P (2, −3, 4) and Q (8, 0, 10).

Let R divide line segment PQ in the ratio k: 1.

Hence, by section formula, the coordinates of point R are given by $\left(\frac{k(8)+2}{k+1}, \frac{k(0)-3}{k+1}, \frac{k(10)+4}{k+1}\right) = \left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1}\right)$

It is given that the x-coordinate of point R is 4.

$$\therefore \frac{8k+2}{k+1} = 4$$
$$\Rightarrow 8k + 2 = 4k + 4$$
$$\Rightarrow 4k = 2$$
$$\Rightarrow k = \frac{1}{2}$$

Therefore, the coordinates of point R are $\left(4, \frac{-3}{\frac{1}{2}+1}, \frac{10\left(\frac{1}{2}\right)+4}{\frac{1}{2}+1}\right) = (4, -2, 6)$

Question 6:

If A and B be the points (3, 4, 5) and (−1, 3, −7), respectively, find the equation of the set of points P such that $PA^2 + PB^2 = k^2$, where k is a constant.

Ans:

The coordinates of points A and B are given as (3, 4, 5) and (−1, 3, −7) respectively.

Let the coordinates of point P be (x, y, z).

On using distance formula, we obtain

$$PA^2 = (x-3)^2 + (y-4)^2 + (z-5)^2$$
$$= x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 - 10z$$
$$= x^2 - 6x + y^2 - 8y + z^2 - 10z + 50$$
$$PB^2 = (x+1)^2 + (y-3)^2 + (z+7)^2$$
$$= x^2 + 2x + y^2 - 6y + z^2 + 14z + 59$$

Now, if $PA^2 + PB^2 = k^2$, then

$$\left(x^2 - 6x + y^2 - 8y + z^2 - 10z + 50\right) + \left(x^2 + 2x + y^2 - 6y + z^2 + 14z + 59\right) = k^2$$
$$\Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = k^2$$
$$\Rightarrow 2\left(x^2 + y^2 + z^2 - 2x - 7y + 2z\right) = k^2 - 109$$
$$\Rightarrow x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}$$

Thus, the required equation is $x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}$.

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