Differentiation Ex 11.7 Q10

Here,

$$x = e^{\theta}\left(\theta + \frac{1}{\theta}\right)$$

Differentiating it with respect to θ using product rule,

$$\frac{dx}{d\theta} = e^{\theta}\frac{d}{d\theta}\left(\theta + \frac{1}{\theta}\right) + \left(\theta + \frac{1}{\theta}\right)\frac{d}{d\theta}\left(e^{\theta}\right)$$

$$= e^{\theta}\left(1 - \frac{1}{\theta^2}\right) + \left(\frac{\theta^2 + 1}{\theta}\right)e^{\theta}$$

$$= e^{\theta}\left(1 - \frac{1}{\theta^2} + \frac{\theta^2 + 1}{\theta}\right)$$

$$= e^{\theta}\left(\frac{\theta^2 - 1 + \theta^3 + \theta}{\theta^2}\right)$$

$$\frac{dx}{d\theta} = \frac{e^{\theta}\left(\theta^3 + \theta^2 + \theta - 1\right)}{\theta^2} \qquad \text{---(i)}$$

And, $y = e^{\theta}\left(\theta - \frac{1}{\theta}\right)$

Differentiating it with respect to θ using product rule and chain rule,

$$\frac{dy}{d\theta} = e^{-\theta}\frac{d}{d\theta}\left(\theta - \frac{1}{\theta}\right) + \left(\theta - \frac{1}{\theta}\right)\frac{d}{d\theta}\left(e^{-\theta}\right)$$

$$= e^{-\theta}\left(1 + \frac{1}{\theta^2}\right) + \left(\theta - \frac{1}{\theta}\right)e^{-\theta}\frac{d}{d\theta}(-\theta)$$

$$= e^{-\theta}\left(1 + \frac{1}{\theta^2}\right) + \left(\theta - \frac{1}{\theta}\right)e^{-\theta}(-1)$$

$$\frac{dy}{d\theta} = e^{-\theta}\left[1 + \frac{1}{\theta^2} - \theta + \frac{1}{\theta}\right]$$

$$= e^{-\theta}\left[\frac{\theta^2 + 1 - \theta^3 + \theta}{\theta^2}\right]$$

$$\frac{dy}{d\theta} = e^{-\theta}\left[\frac{-\theta^3 + \theta^2 + \theta + 1}{\theta^2}\right] \qquad \text{---(ii)}$$

Differentiation Ex 11.7 Q11

Here,

$$x = \frac{2t}{1+t^2}$$

Differentiating it with respect to t using quotient rule,

$$\frac{dy}{dx} = \left[\frac{\left(1+t^2\right)\frac{d}{dt}(2t) - 2t\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{\left(1+t^2\right)(2) - 2t(2t)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{2+2t^2-4t^2}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{2-2t^2}{\left(1+t^2\right)^2}\right]$$

$$\frac{dx}{dt} = \frac{2\left(1-t^2\right)}{\left(1+t^2\right)^2} \qquad \text{---(i)}$$

And, $y = \dfrac{1-t^2}{1+t^2}$

Differentiating it with respect to t using quotient rule,

$$\frac{dy}{dt} = \left[\frac{\left(1+t^2\right)\frac{d}{dt}\left(1-t^2\right) - \left(1-t^2\right)\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{\left(1+t^2\right)(-2t) - \left(1-t^2\right)(2t)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{-2t-2t^3-2t+2t^3}{\left(1+t^2\right)^2}\right]$$

$$\frac{dy}{dt} = \left[\frac{-4t}{\left(1+t^2\right)^2}\right] \qquad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-4t}{\left(1+t^2\right)^2} \times \frac{\left(1+t^2\right)^2}{2\left(1-t^2\right)}$$

$$= \frac{-2t}{1-t^2}$$

$$\frac{dy}{dx} = -\frac{x}{y} \qquad \left[\text{Sicne, } \frac{x}{y} = \frac{2t}{1+t^2} \times \frac{1+t^2}{1-t^2} = \frac{2t}{1-t^2}\right]$$

Differentiation Ex 11.7 Q12

Here,

$$x = \cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$$

Differentiating it with respect to t using chain rule,

$$\frac{dx}{dt} = \frac{-1}{\sqrt{1-\left(\frac{1}{1+t^2}\right)^2}} \frac{d}{dt}\left(\frac{1}{\sqrt{1+t^2}}\right)$$

$$= \frac{-1}{\sqrt{1 - \frac{1}{\left(1+t^2\right)}}} \left\{ \frac{-\ 1}{2\left(1+t^2\right)^{\frac{3}{2}}} \right\} \frac{d}{dt}\left(1+t^2\right)$$

$$= \frac{\left(1+t^2\right)^{\frac{1}{2}}}{\sqrt{1+t^2-1}} \times \frac{-\ 1}{2\left(1+t^2\right)^{\frac{3}{2}}} (2t)$$

$$= \frac{-\ t}{\sqrt{t^2} \times \left(1+t^2\right)}$$

$$\frac{dx}{dt} = \frac{-1}{1+t^2} \qquad \text{---(i)}$$

Now, $\quad y = \sin^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$

Differentiating it with respect to t using chain rule,

$$\frac{dy}{dt} = \frac{1}{\sqrt{1 - \frac{1}{\left(\sqrt{1+t^2}\right)^2}}} \times \frac{d}{dt}\left(\frac{1}{\sqrt{1+t^2}}\right)$$

$$= \frac{\left(1+t^2\right)^{\frac{1}{2}}}{\sqrt{1+t^2-1}} \times \left(\frac{-1}{2\left(1+t^2\right)^{\frac{3}{2}}}\right) \frac{d}{dt}\left(1+t^2\right)$$

$$= \frac{-1}{2\sqrt{t^2}\left(1+t^2\right)} \times (2t)$$

$$\frac{dy}{dt} = \frac{-1}{\left(1+t^2\right)} \qquad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = -\frac{1}{\left(1+t^2\right)} \times \frac{\left(1+t^2\right)}{-1}$$

$$\frac{dy}{dx} = 1$$

*********** END **********