



### Maxima and Minima 18.5 Q13

Let the side of the square to be cut off be  $x$  cm. Then, the height of the box is  $x$ , the length is  $45 - 2x$ , and the breadth is  $24 - 2x$ .

Therefore, the volume  $V(x)$  of the box is given by,

$$\begin{aligned} V(x) &= x(45 - 2x)(24 - 2x) \\ &= x(1080 - 90x - 48x + 4x^2) \\ &= 4x^3 - 138x^2 + 1080x \\ \therefore V'(x) &= 12x^2 - 276x + 1080 \\ &= 12(x^2 - 23x + 90) \\ &= 12(x - 18)(x - 5) \\ V''(x) &= 24x - 276 = 12(2x - 23) \end{aligned}$$

Now,  $V'(x) = 0 \Rightarrow x = 18$  and  $x = 5$

It is not possible to cut off a square of side 18 cm from each corner of the rectangular sheet.

Thus,  $x$  cannot be equal to 18.

$$\therefore x = 5$$

$$\text{Now, } V''(5) = 12(10 - 23) = 12(-13) = -156 < 0$$

$\therefore$  By second derivative test,  $x = 5$  is the point of maxima.

Hence, the side of the square to be cut off to make the volume of the box maximum possible is 5 cm.

### Maxima and Minima 18.5 Q14

Let  $l$ ,  $b$ , and  $h$  represent the length, breadth, and height of the tank respectively.

Then, we have height ( $h$ ) = 2 m

$$\text{Volume of the tank} = 8\text{m}^3$$

$$\text{Volume of the tank} = l \times b \times h$$

$$\therefore 8 = l \times b \times 2$$

$$\Rightarrow lb = 4 \Rightarrow b = \frac{4}{l}$$

$$\text{Now, area of the base} = lb = 4$$

$$\text{Area of the 4 walls } (A) = 2h(l + b)$$

$$\therefore A = 4\left(l + \frac{4}{l}\right)$$

$$\Rightarrow \frac{dA}{dl} = 4\left(1 - \frac{4}{l^2}\right)$$

$$\text{Now, } \frac{dA}{dl} = 0$$

$$\Rightarrow 1 - \frac{4}{l^2} = 0$$

$$\Rightarrow l^2 = 4$$

$$\Rightarrow l = \pm 2$$

However, the length cannot be negative.

Therefore, we have  $l = 4$ .

$$\therefore b = \frac{4}{l} = \frac{4}{2} = 2$$

$$\text{Now, } \frac{d^2 A}{dl^2} = \frac{32}{l^3}$$

$$\text{When } l = 2, \frac{d^2 A}{dl^2} = \frac{32}{8} = 4 > 0.$$

Thus, by second derivative test, the area is the minimum when  $l = 2$ .

We have  $l = b = h = 2$ .

$$\therefore \text{Cost of building the base} = \text{Rs } 70 \times (lb) = \text{Rs } 70 (4) = \text{Rs } 280$$

$$\text{Cost of building the walls} = \text{Rs } 2h (l + b) \times 45 = \text{Rs } 90 (2) (2 + 2)$$

$$= \text{Rs } 8 (90) = \text{Rs } 720$$

$$\text{Required total cost} = \text{Rs } (280 + 720) = \text{Rs } 1000$$

Hence, the total cost of the tank will be Rs 1000.

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