Chapter 9 Continuity Ex 9.2 Q4(i)

We have given that the function is continuous at $x = 0$

$\therefore$ LHL = RHL = $f(0)$ ....(1)

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{\sin(-2h)}{5(-h)} = \lim_{h\to 0} \frac{-\sin 2h}{-5h} = \lim_{h\to 0} \frac{\sin 2h}{2h} \times \frac{2h}{5h} = \frac{2}{5}$$

$f(0) = 3k$

So, using (1) we get,

$\frac{2}{5} = 3k$

$k = \frac{2}{15}$

Chapter 9 Continuity Ex 9.2 Q4(ii)

It is given that the function is continuous

$\therefore$ LHL = RHL = $f(2)$ ....(1)

$$\text{LHL} = \lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h) = \lim_{h\to 0} k(2-h) + 5 \quad = 2k + 5$$

$$\text{RHL} = \lim_{x\to 2^+} f(x) = \lim_{h\to 0} f(2+h) = \lim_{h\to 0} (2+h) - 1 = 1$$

Thus, using (1), we get,

$2k + 5 = 1$

$k = -2$

Chapter 9 Continuity Ex 9.2 Q4(iii)

It is given that the function is continuous

LHL = RHL = $f(0)$ ....(1)

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} k\left((-h)^2 + 3(-h)\right) = \lim_{h\to 0} k\left(h^2 - 3h\right) = 0$$

$f(0) = \cos 2\times 0 = \cos 0^\circ = 1$

LHL $\neq f(0)$

Hence, no value of k can make f continuous

Chapter 9 Continuity Ex 9.2 Q4(iv)

First check the continuity of the function at $x = 3$

$f(3) = 2$ .....(A)

$$\text{RHL} = \lim_{x\to 3^+} f(x) = \lim_{h\to 0} f(3+h) = \lim_{h\to 0} a(3+h) + b \quad = 3a + b \text{.....(B)}$$

$\therefore$ $f(x)$ will be continuous at $x = 3$ if $3a + b = 2$.....(1)

Now, check the continuity at $x = 5$

$f(5) = 9$ ....(C)

$$\text{LHL} = \lim_{x\to 5^-} f(x) = \lim_{h\to 0} f(5-h) = \lim_{h\to 0} a(5-h) + b = 5a + b$$

$f(x)$ will be continuous at $x = 5$ if $5a + b = 9$....(2)

Solving (1) & (2), we get

$a = \frac{7}{2}$ and $b = \frac{-17}{2}$

Chapter 9 Continuity Ex 9.2 Q4(v)

It is given that the function is continuous

At $x = -1$

$f(-1) = 4$

RHL $= \lim_{x \to -1^+} f(x) = \lim_{h \to 0} f(-1+h) = \lim_{h \to 0} a(-1+h)^2 + b = a + b$

Since, $f(x)$ is continuous at $x = -1$

$\therefore\ a + b = 4 \qquad \ldots.(A)$

Now, at $x = 0$,

$f(0) = \cos 0^\circ = 1$

LHL $= \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} a(-h)^2 + b = b$

Since, $f(x)$ is continuous at $x = 0$

$\therefore\ f(0) = \text{LHL}$

$\Rightarrow b = 1$

$\therefore$ from (A)

$a = 3$

Thus, $a = 3,\ b = 1$

Chapter 9 Continuity Ex 9.2 Q4(vi)

It is given that the function is continuous.

At $x = 0$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{\sqrt{1-Ph} - \sqrt{1+Ph}}{-h} = \lim_{h \to 0} \frac{\left(\sqrt{1-Ph} - \sqrt{1+Ph}\right)}{-h} \times \frac{\left(\sqrt{1-Ph} + \sqrt{1+Ph}\right)}{\left(\sqrt{1-Ph} + \sqrt{1+Ph}\right)}$$

$$= \lim_{h \to 0} \frac{(1-Ph) - (1+Ph)}{-h\left(\sqrt{1-Ph} + \sqrt{1+Ph}\right)} = \frac{2P}{2} = P$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{2h+1}{h-2} = \frac{-1}{2}$$

Since, $f(x)$ is continuous so,

$P = \frac{-1}{2}$

Chapter 9 Continuity Ex 9.2 Q4(vii)

The given function f is $f(x)=\begin{cases}5, & \text{if } x\le 2\\ ax+b, & \text{if } 2<x<10\\ 21, & \text{if } x\ge 10\end{cases}$

It is evident that the given function f is defined at all points of the real line.

If f is a continuous function, then f is continuous at all real numbers.

In particular, f is continuous at $x=2$ and $x=10$

Since f is continuous at $x=2$, we obtain

$$\lim_{x\to 2^-} f(x)=\lim_{x\to 2^+} f(x)=f(2)$$
$$\Rightarrow \lim_{x\to 2^-}(5)=\lim_{x\to 2^+}(ax+b)=5$$
$$\Rightarrow 5=2a+b=5$$
$$\Rightarrow 2a+b=5 \qquad \ldots(1)$$

Since f is continuous at $x=10$, we obtain

$$\lim_{x\to 10^-} f(x)=\lim_{x\to 10^+} f(x)=f(10)$$
$$\Rightarrow \lim_{x\to 10^-}(ax+b)=\lim_{x\to 10^+}(21)=21$$
$$\Rightarrow 10a+b=21=21$$
$$\Rightarrow 10a+b=21 \qquad \ldots(2)$$

On subtracting equation (1) from equation (2), we obtain

$8a=16$

$a=2$

By putting $a=2$ in equation (1), we obtain

$2\times 2+b=5$

$4+b=5$

$b=1$

Therefore, the values of a and b for which f is a continuous function are 2 and 1 respectively.

Chapter 9 Continuity Ex 9.2 Q4(viii)

Since the function is continuous at $x = \frac{\pi}{2}$ therefore

LHL of $f(x)$ at $x = \frac{\pi}{2}$ is

$$= \lim_{x \to \frac{\pi}{2}^-} f(x)$$

$$= \lim_{h \to 0} f\left(h - \frac{\pi}{2}\right)$$

$$= \lim_{h \to 0} \frac{k \cos\left(h - \frac{\pi}{2}\right)}{\pi - 2\left(h - \frac{\pi}{2}\right)}$$

$$= \lim_{h \to 0} \frac{k \sinh}{2\pi - 2h}$$

$$= \frac{k}{2} \lim_{h \to 0} \frac{\sin(\pi - h)}{(\pi - h)}$$

$$= \frac{k}{2}$$

Again

$$f\left(\frac{\pi}{2}\right) = 3$$

Hence

$$LHL = f\left(\frac{\pi}{3}\right)$$

$$\frac{k}{2} = 3$$

$$k = 6$$

*********** END **********