Binary Operations Ex 3.5 Q6

$a \times_7 b$ = the remainder when the product of ab is divided by 7.

The composition table for $\times_7$ on $S = \{1, 2, 3, 4, 5, 6\}$

$\times_7$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

We know that 1 is the identity element with respect to multiplication

Also, b will be the inverse of a

if, $a \times_7 b = e = 1$

$\Rightarrow \quad 3 \times_7 b = 1$

From the above table $3 \times_7 5 = 1$

$\therefore \quad b = 3^{-1} = 5$

Now, $3^{-1} \times_7 4 = 5 \times_7 4 = 6$

Binary Operations Ex 3.5 Q7

$a \times_{11} b$ = the remainder when the product of ab is divided by 11.

The composition table for $\times_{11}$ on Z_{11}

$\times_{11}$	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	1	3	5	7	9
3	3	6	9	1	4	7	10	3	5	8
4	4	8	1	5	9	2	6	10	3	7
5	5	10	4	9	3	8	2	7	1	6
6	6	1	7	2	8	3	9	4	10	5
7	7	3	10	6	2	9	5	1	8	4
8	8	5	3	10	7	4	1	9	6	3
9	9	7	5	3	1	10	8	6	4	2
10	10	9	8	7	6	5	4	3	2	1

From the above table

$5 \times_{11} 9 = 1$ $\quad [\because 1 \text{ is the identity element}]$

$\therefore$ Inverse of 5 is 9.

Binary Operations Ex 3.5 Q8

$Z_5 = \{0, 1, 2, 3, 4\}$

$a \times_5 b$ = the remainder when the product of ab is divided by 5.

The composition table for $\times_5$ on $Z_5 = \{0, 1, 2, 3, 4\}$

$\times_5$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

Binary Operations Ex 3.5 Q9

(i)

From the above table we can say that

$$a*b=b*a=b$$
$$a*c=c*a=c$$
$$a*d=d*a=d$$
$$b*c=c*b=d$$
$$b*d=d*b=c$$
$$c*d=d*c=b$$

$\therefore$ '*' is commutative

Again, $a,b,c\in S$

$\Rightarrow$ $(a*b)*c=b*c=d$ and
$a*(b*c)=a*d=d$

$\therefore$ $(a*b)*c=a*(b*c)$

$\therefore$ $*$ is associative

We know that e will be identity element with respect to $*$ if
$a*e=e*a=a$ for all $a\in S$

$\Rightarrow$ $a*a=a,\ a*b=b,\ a*c=c,\ a*d=d$

$\therefore$ a will be the identity element

Again,
b will be the inverse of a if
$b*a=a*b=e$

From the above table
$a*a=a,$ $b*b=b, c*c=c$ and $d*d=d$

$\therefore$ Inverse of $a=a$
$b=b$
$c=c$
$d=d$

(ii)

From the above table, we can observe

$aob = boa,\quad boc = cob$

$aoc = coa,\quad bod = dob$

$aod = doa,\quad cod = doc$

$\therefore$ "o" is commutative on S

Again, for $a, b, c \in S$

$(aob)oc = aoc = a \qquad ---(i)$

$ao(boc) = aoc = a \qquad ---(ii)$

From (i) & (ii)

$(aob)oc = ao(boc)$

So, "o" is associative on S

Now, we have,

$aob = a$

$bob = b$

$cob = c$

$dob = d$

$\Rightarrow$ b is the identity element with respect to "o"

We know that x will be inverse of y

If $xoy = yox = e$

$\Rightarrow \quad xoy = yox = b \qquad [\because e = b]$

Now, from the above table we find that

$bob = b$

$cod = b$

$doc = b$

$\therefore \quad b^{-1} = b,\ c^{-1} = d$, and $d^{-1} = c$

Not: a^{-1} does ont exist.

Binary Operations Ex 3.5 Q10

Let $X = \{0, 1, 2, 3, 4, 5\}$.

The operation * on X is defined as:

$$a*b=\begin{cases}a+b & \text{if } a+b<6\\ a+b-6 & \text{if } a+b\geq 6\end{cases}$$

An element $e \in X$ is the identity element for the operation *, if

$a*e=a=e*a \ \forall a\in X.$

For $a\in X$, we observed that:

$$a*0=a+0=a \qquad [a\in X\Rightarrow a+0<6]$$

$$0*a=0+a=a \qquad [a\in X\Rightarrow 0+a<6]$$

$$\therefore a*0=a=0*a \ \forall a\in X$$

Thus, 0 is the identity element for the given operation *.

An element $a \in X$ is invertible if there exists $b \in X$ such that $a * b = 0 = b * a$.

$$\text{i.e.,}\begin{cases}a+b=0=b+a, & \text{if } a+b<6\\ a+b-6=0=b+a-6, & \text{if } a+b\geq 6\end{cases}$$

i.e.,

$a = -b$ or $b = 6 - a$

But, $X = \{0, 1, 2, 3, 4, 5\}$ and $a, b \in X$. Then, $a \neq -b$.

Therefore, $b = 6 - a$ is the inverse of a $a \in X$.

Hence, the inverse of an element $a \in X$, $a \neq 0$ is $6 - a$ i.e., $a^{-1} = 6 - a$.

*********** END **********