Differentiation Ex 11.8 Q18

Let $\quad u = \sin^{-1}\left(\sqrt{1-x^2}\right)$

Put $\quad x = \cos\theta \Rightarrow \theta = \cos^{-1}x$, so

$$u = \sin^{-1}(\sin\theta) \qquad \text{---(i)}$$

And,

Let $\quad v = \cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

$$= \cot^{-1}\left(\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}\right)$$

$$= \cot^{-1}\left(\frac{\cos\theta}{\sin\theta}\right)$$

$$v = \cot^{-1}(\cot\theta) \qquad \text{---(ii)}$$

Here, $\quad 0 < x < 1$

$\Rightarrow \quad 0 < \cos\theta < 1$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{2}$

So, from equation (i),

$$u = \theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta,\ \text{if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$u = \cos^{-1}x$$

Differentiation Ex 11.8 Q19

Let $\quad u = \sin^{-1}\left\{2ax\sqrt{1-a^2x^2}\right\}$

Put $\quad ax = \sin\theta \Rightarrow \theta = \sin^{-1}(ax)$

$$u = \sin^{-1}\left\{2\sin\theta\sqrt{1-\sin^2\theta}\right\}$$
$$= \sin^{-1}\{2\sin\theta\cos\theta\}$$
$$u = \sin^{-1}(\sin 2\theta) \qquad \text{---(i)}$$

And,

Let $\quad v = \sqrt{1-a^2x^2}$

Differentiating it with respect to x using chain rule,

$$\frac{dv}{dx} = \frac{1}{2\sqrt{1-a^2x^2}} \times \frac{d}{dx}\left(1-a^2x^2\right)$$
$$= \left(\frac{0-2a^2x}{2\sqrt{1-a^2x^2}}\right)$$
$$\frac{dv}{dx} = \frac{-a^2x}{\sqrt{1-a^2x^2}} \qquad \text{---(ii)}$$

Here, $\quad -\frac{1}{\sqrt{2}} < ax < \frac{1}{\sqrt{2}}$

$\Rightarrow \quad -\frac{1}{\sqrt{2}} < \sin\theta < \frac{1}{\sqrt{2}}$

$\Rightarrow \quad -\frac{\pi}{4} < \theta < \frac{\pi}{4}$

So, from equation (i),

$$u = 2\theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$
$$u = 2\sin^{-1} ax$$

Differentiation Ex 11.8 Q20

Let $u = \tan^{-1}\left(\frac{1-x}{1+x}\right)$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$, so

$$u = \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\frac{\tan\pi}{4} - \tan\theta}{1+\frac{\tan\pi}{4}\tan\theta}\right)$$

$$u = \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\theta\right)\right) \qquad \text{---(i)}$$

Here, $-1 < x < 1$

$\Rightarrow \quad -1 < \tan\theta < 1$

$\Rightarrow \quad -\frac{\pi}{4} < \theta < \frac{\pi}{4}$

So, from equation (i),

$$u = \left(\frac{\pi}{4}-\theta\right) \qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$$

$$u = \frac{\pi}{4} - \tan^{-1} x$$

Differentiating it with respect to x,

$$\frac{du}{dx} = 0 - \left(\frac{1}{1+x^2}\right)$$

$$\frac{du}{dx} = -\frac{1}{1+x^2} \qquad \text{---(ii)}$$

And,

Let $v = \sqrt{1-x^2}$

Differentiating it with respect to x using chain rule,

$$\frac{dv}{dx} = \frac{1}{2\sqrt{1-x^2}} \times \frac{d}{dx}\left(1-x^2\right)$$

$$= \frac{1}{2\sqrt{1-x^2}}(-2x)$$

$$\frac{dv}{dx} = \frac{-x}{\sqrt{1-x^2}} \qquad \text{---(iii)}$$

Dividing equation (ii) by (iii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = -\frac{1}{\left(1+x^2\right)} \times \frac{\sqrt{1-x^2}}{-x}$$

$$\frac{du}{dv} = \frac{\sqrt{1-x^2}}{x\left(1+x^2\right)}$$

*********** END **********