CHAPTER 7

CIRCULAR MOTION

7.1 ANGULAR VARIABLES

Suppose a particle P is moving in a circle of radius r (figure 7.1). Let O be the centre of the circle. Let O be the origin and OX the X-axis. The position of the particle P at a given instant may be described by the angle θ between OP and OX. We call θ the *angular position* of the particle. As the particle moves on the circle, its angular position θ changes. Suppose the particle goes to a nearby point P' in time Δt so that θ increases to $\theta + \Delta\theta$. The rate of change of angular position is called *angular velocity*. Thus, the angular velocity is

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \cdot$$

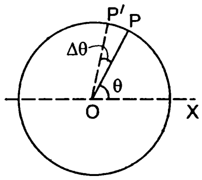

Figure 7.1

The rate of change of angular velocity is called *angular acceleration*. Thus, the angular acceleration is

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \cdot$$

If the angular acceleration α is constant, we have

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2 \qquad \dots (7.1)$$

$$\omega = \omega_0 + \alpha t \qquad \dots (7.2)$$

and $$\omega^2 = \omega_0^2 + 2\alpha\theta \qquad \dots (7.3)$$

where ω_0 and ω are the angular velocities at $t = 0$ and at time t and θ is the angular position at time t. The linear distance PP' travelled by the particle in time Δt is

$$\Delta s = r\,\Delta\theta$$

or, $$\frac{\Delta s}{\Delta t} = r \frac{\Delta\theta}{\Delta t}$$

or, $$v = r\,\omega \qquad \dots (7.4)$$

where v is the linear speed of the particle. Differentiating equation (7.4) with respect to time, the rate of change of speed is

$$a_t = \frac{dv}{dt} = r \frac{d\omega}{dt}$$

or, $$a_t = r\,\alpha. \qquad \dots (7.5)$$

Remember that $a_t = \frac{dv}{dt}$ is the rate of change of speed and is not the rate of the change of velocity. It is, therefore, not equal to the net acceleration.

We shall show that a_t is the component of acceleration along the tangent and hence we have used the suffix t. It is called the *tangential acceleration*.

Example 7.1

A particle moves in a circle of radius 20 cm *with a linear speed of* 10 m/s. *Find the angular velocity.*

Solution : The angular velocity is

$$\omega = \frac{v}{r} = \frac{10 \text{ m/s}}{20 \text{ cm}} = 50 \text{ rad/s}.$$

Example 7.2

A particle travels in a circle of radius 20 cm *at a speed that uniformly increases. If the speed changes from* 5·0 m/s *to* 6·0 m/s *in* 2·0 s, *find the angular acceleration.*

Solution : The tangential acceleration is given by

$$a_t = \frac{dv}{dt} = \frac{v_2 - v_1}{t_2 - t_1}$$

$$= \frac{6{\cdot}0 - 5{\cdot}0}{2{\cdot}0} \text{ m/s}^2 = 0{\cdot}5 \text{ m/s}^2.$$

The angular acceleration is $\alpha = a_t / r$

$$= \frac{0{\cdot}5 \text{ m/s}^2}{20 \text{ cm}} = 2{\cdot}5 \text{ rad/s}^2.$$

7.2 UNIT VECTORS ALONG THE RADIUS AND THE TANGENT

Consider a particle moving in a circle. Suppose the particle is at a point P in the circle at a given instant (figure 7.2). Take the centre of the circle to be the origin, a line OX as the X-axis and a perpendicular radius OY as the Y-axis. The angular position of the particle at this instant is θ.

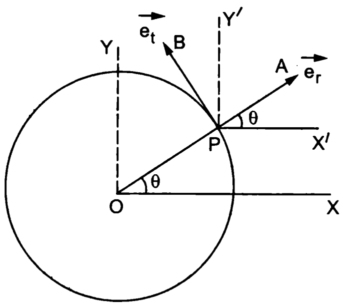

Figure 7.2

Draw a unit vector $\overrightarrow{PA} = \vec{e}_r$ along the outward radius and a unit vector $\overrightarrow{PB} = \vec{e}_t$ along the tangent in the direction of increasing θ. We call $\vec{e}_r$ the *radial unit vector* and $\vec{e}_t$ the *tangential unit vector*. Draw PX' parallel to the X-axis and PY' parallel to the Y-axis. From the figure,

$$\overrightarrow{PA} = \vec{i}\, PA \cos\theta + \vec{j}\, PA \sin\theta$$

or, $$\vec{e}_r = \vec{i} \cos\theta + \vec{j} \sin\theta, \quad \ldots (7.6)$$

where $\vec{i}$ and $\vec{j}$ are the unit vectors along the X and Y axes respectively. Similarly,

$$\overrightarrow{PB} = -\vec{i}\, PB \sin\theta + \vec{j}\, PB \cos\theta$$

or, $$\vec{e}_t = -\vec{i} \sin\theta + \vec{j} \cos\theta. \quad \ldots (7.7)$$

7.3 ACCELERATION IN CIRCULAR MOTION

Consider the situation shown in figure (7.2). It is clear from the figure that the position vector of the particle at time t is

$$\vec{r} = \overrightarrow{OP} = OP\, \vec{e}_r$$
$$= r\,(\vec{i} \cos\theta + \vec{j} \sin\theta). \quad \ldots \text{(i)}$$

Differentiating equation (i) with respect to time, the velocity of the particle at time t is

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}[r\,(\vec{i}\cos\theta + \vec{j}\sin\theta)]$$
$$= r\left[\vec{i}\left(-\sin\theta \frac{d\theta}{dt}\right) + \vec{j}\left(\cos\theta \frac{d\theta}{dt}\right)\right]$$
$$= r\,\omega[-\vec{i}\sin\theta + \vec{j}\cos\theta]. \quad \ldots \text{(ii)}$$

The term $r\,\omega$ is the speed of the particle at time t (equation 7.4) and the vector in the square bracket is the unit vector $\vec{e}_t$ along the tangent. Thus, the velocity of the particle at any instant is along the tangent to the circle and its magnitude is $v = r\,\omega$.

The acceleration of the particle at time t is $\vec{a} = \frac{d\vec{v}}{dt}$. From (ii),

$$\vec{a} = r\left[\omega \frac{d}{dt}[-\vec{i}\sin\theta + \vec{j}\cos\theta] + \frac{d\omega}{dt}[-\vec{i}\sin\theta + \vec{j}\cos\theta]\right]$$
$$= \omega r\left[-\vec{i}\cos\theta \frac{d\theta}{dt} - \vec{j}\sin\theta \frac{d\theta}{dt}\right] + r\frac{d\omega}{dt}\vec{e}_t$$
$$= -\omega^2 r\,[\vec{i}\cos\theta + \vec{j}\sin\theta] + r\frac{d\omega}{dt}\vec{e}_t$$
$$= -\omega^2 r\,\vec{e}_r + \frac{dv}{dt}\vec{e}_t, \quad \ldots (7.8)$$

where $\vec{e}_r$ and $\vec{e}_t$ are the unit vectors along the radial and tangential directions respectively and v is the speed of the particle at time t. We have used

$$r\frac{d\omega}{dt} = \frac{d}{dt}(r\,\omega) = \frac{dv}{dt}.$$

Uniform Circular Motion

If the particle moves in the circle with a uniform speed, we call it a *uniform circular motion*. In this case, $\frac{dv}{dt} = 0$ and equation (7.8) gives

$$\vec{a} = -\omega^2 r\,\vec{e}_r.$$

Thus, the acceleration of the particle is in the direction of $-\vec{e}_r$, that is, towards the centre. The magnitude of the acceleration is

$$a_r = \omega^2 r$$
$$= \frac{v^2}{r^2} r = \frac{v^2}{r}. \quad \ldots (7.9)$$

Thus, if a particle moves in a circle of radius r with a constant speed v, its acceleration is v^2/r directed towards the centre. This acceleration is called *centripetal acceleration*. Note that the speed remains constant, the direction continuously changes and hence the "velocity" changes and there is an acceleration during the motion.

Example 7.3

Find the magnitude of the linear acceleration of a particle moving in a circle of radius 10 cm *with uniform speed completing the circle in* 4 s.

Solution : The distance covered in completing the circle is $2\pi r = 2\pi \times 10$ cm. The linear speed is

$$v = 2\pi r / t$$

$$= \frac{2\pi \times 10 \text{ cm}}{4 \text{ s}} = 5\pi \text{ cm/s}.$$

The linear acceleration is

$$a = \frac{v^2}{r} = \frac{(5\pi \text{ cm/s})^2}{10 \text{ cm}} = 2{\cdot}5\,\pi^2 \text{ cm/s}^2.$$

This acceleration is directed towards the centre of the circle.

Nonuniform Circular Motion

If the speed of the particle moving in a circle is not constant, the acceleration has both the radial and the tangential components. According to equation (7.8), the radial and the tangential accelerations are

$$\left.\begin{aligned} a_r &= -\,\omega^2 r = -\,v^2/r \\ \text{and} \quad a_t &= \frac{dv}{dt} \end{aligned}\right| . \qquad \dots (7.10)$$

Thus, the component of the acceleration towards the centre is $\omega^2 r = v^2/r$ and the component along the tangent (along the direction of motion) is dv/dt. The magnitude of the acceleration is

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}.$$

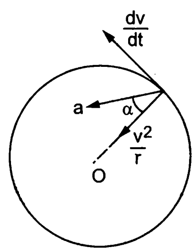

Figure 7.3

The direction of this resultant acceleration makes an angle α with the radius (figure 7.3) where

$$\tan\alpha = \left(\frac{dv}{dt}\right) \Big/ \left(\frac{v^2}{r}\right).$$

Example 7.4

A particle moves in a circle of radius 20 cm. *Its linear speed is given by* $v = 2t$, *where t is in second and v in metre/second. Find the radial and tangential acceleration at* $t = 3$ s.

Solution : The linear speed at $t = 3$ s is

$$v = 2t = 6 \text{ m/s}.$$

The radial acceleration at $t = 3$ s is

$$a_r = v^2/r = \frac{36 \text{ m}^2/\text{s}^2}{0{\cdot}20 \text{ m}} = 180 \text{ m/s}^2.$$

The tangential acceleration is

$$a_t = \frac{dv}{dt} = \frac{d\,(2t)}{dt} = 2 \text{ m/s}^2.$$

7.4 DYNAMICS OF CIRCULAR MOTION

If a particle moves in a circle as seen from an inertial frame, a resultant nonzero force must act on the particle. That is because a particle moving in a circle is accelerated and acceleration can be produced in an inertial frame only if a resultant force acts on it. If the speed of the particle remains constant, the acceleration of the particle is towards the centre and its magnitude is v^2/r. Here v is the speed of the particle and r is the radius of the circle. The resultant force must act towards the centre and its magnitude F must satisfy

$$a = \frac{F}{m}$$

$$\text{or,} \qquad \frac{v^2}{r} = \frac{F}{m}$$

$$\text{or,} \qquad F = \frac{mv^2}{r}. \qquad \dots (7.11)$$

Since this resultant force is directed towards the centre, it is called *centripetal force*. Thus, a centripetal force of magnitude mv^2/r is needed to keep the particle in uniform circular motion.

It should be clearly understood that "centripetal force" is another word for "force towards the centre". This force must originate from some external source such as gravitation, tension, friction, coulomb force, etc. Centripetal force is not a new kind of force, just as an "upward force" or a "downward force" is not a new kind of force.

Example 7.5

A small block of mass 100 g *moves with uniform speed in a horizontal circular groove, with vertical side walls, of radius* 25 cm. *If the block takes* 2·0 s *to complete one round, find the normal contact force by the side wall of the groove.*

Solution : The speed of the block is

$$v = \frac{2\pi \times (25 \text{ cm})}{2{\cdot}0 \text{ s}} = 0{\cdot}785 \text{ m/s}.$$

The acceleration of the block is

$$a = \frac{v^2}{r} = \frac{(0{\cdot}785 \text{ m/s})^2}{0{\cdot}25 \text{ m}} = 2{\cdot}5 \text{ m/s}^2$$

towards the centre. The only force in this direction is the normal contact force due to the side walls. Thus, from Newton's second law, this force is

$$\mathcal{N} = ma = (0{\cdot}100 \text{ kg})\,(2{\cdot}5 \text{ m/s}^2) = 0{\cdot}25 \text{ N}.$$

7.5 CIRCULAR TURNINGS AND BANKING OF ROADS

When vehicles go through turnings, they travel along a nearly circular arc. There must be some force which will produce the required acceleration. If the vehicle goes in a horizontal circular path, this resultant force is also horizontal. Consider the situation as shown in figure (7.4). A vehicle of mass M moving at a speed v is making a turn on the circular path of radius r. The external forces acting on the vehicle are

(i) weight Mg

(ii) Normal contact force $\mathcal{N}$ and

(iii) friction f_s.

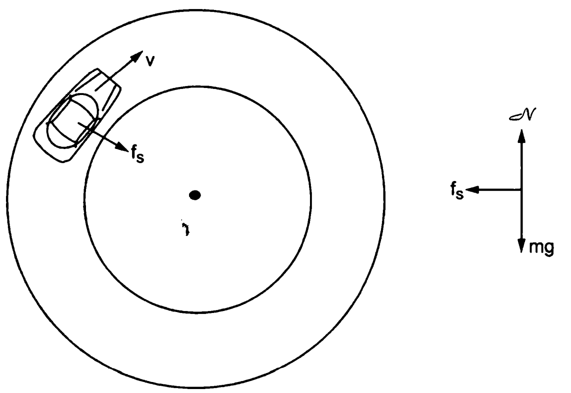

Figure 7.4

If the road is horizontal, the normal force $\mathcal{N}$ is vertically upward. The only horizontal force that can act towards the centre is the friction f_s. This is static friction and is self adjustable. The tyres get a tendency to skid outward and the frictional force which opposes this skidding acts towards the centre. Thus, for a safe turn we must have

$$\frac{v^2}{r} = \frac{f_s}{M}$$

or,
$$f_s = \frac{Mv^2}{r}.$$

However, there is a limit to the magnitude of the frictional force. If μ_s is the coefficient of static friction between the tyres and the road, the magnitude of friction f_s cannot exceed $\mu_s \mathcal{N}$. For vertical equilibrium $\mathcal{N} = Mg$, so that

$$f_s \le \mu_s Mg.$$

Thus, for a safe turn

$$\frac{Mv^2}{r} \le \mu_s Mg$$

or,
$$\mu_s \ge \frac{v^2}{rg}. \qquad \dots (7.12)$$

Friction is not always reliable at circular turns if high speeds and sharp turns are involved. To avoid dependence on friction, the roads are banked at the turn so that the outer part of the road is somewhat lifted up as compared to the inner part (figure 7.5).

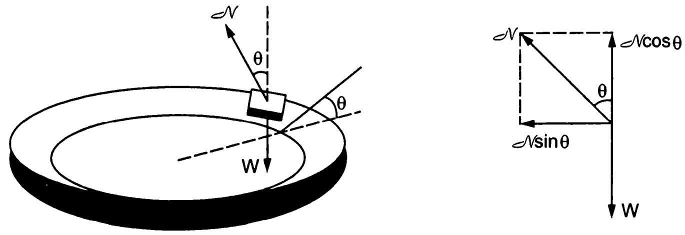

Figure 7.5

The surface of the road makes an angle θ with the horizontal throughout the turn. The normal force $\mathcal{N}$ makes an angle θ with the vertical. At the correct speed, the horizontal component of $\mathcal{N}$ is sufficient to produce the acceleration towards the centre and the self adjustable frictional force keeps its value zero. Applying Newton's second law along the radius and the first law in the vertical direction,

$$\mathcal{N} \sin\theta = \frac{Mv^2}{r}$$

and
$$\mathcal{N} \cos\theta = Mg.$$

These equations give

$$\tan\theta = \frac{v^2}{rg}. \qquad \dots (7.13)$$

The angle θ depends on the speed of the vehicle as well as on the radius of the turn. Roads are banked for the average expected speed of the vehicles. If the speed of a particular vehicle is a little less or a little more than the correct speed, the self adjustable static friction operates between the tyres and the road and the vehicle does not skid or slip. If the speed is too different from that given by equation (7.13), even the maximum friction cannot prevent a skid or a slip.

Example 7.6

The road at a circular turn of radius 10 m *is banked by an angle of* 10°. *With what speed should a vehicle move on the turn so that the normal contact force is able to provide the necessary centripetal force?*

Solution : If v is the correct speed,

$$\tan\theta = \frac{v^2}{rg}$$

or, $v = \sqrt{rg \tan \theta}$

$= \sqrt{(10 \text{ m})(9.8 \text{ m/s}^2)(\tan 10°)} = 4.2 \text{ m/s}.$

7.6 CENTRIFUGAL FORCE

We discussed in chapter 5 that Newton's laws of motion are not valid if one is working from a noninertial frame. If the frame translates with respect to an inertial frame with an acceleration $\vec{a}_0$, one must assume the existence of a pseudo force $-m\vec{a}_0$, acting on a particle of mass m. Once this pseudo force is included, one can use Newton's laws in their usual form. What pseudo force is needed if the frame of reference rotates at a constant angular velocity ω with respect to an inertial frame?

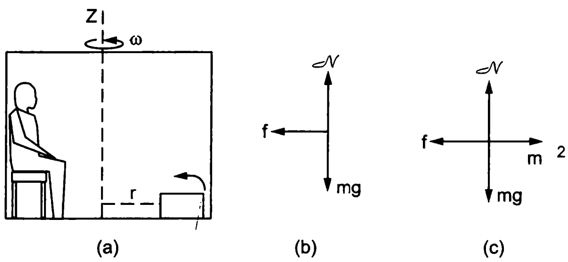

Figure 7.6

Suppose the observer is sitting in a closed cabin which is made to rotate about the vertical Z-axis at a uniform angular velocity ω (figure 7.6). The X and Y axes are fixed in the cabin. Consider a heavy box of mass m kept on the floor at a distance r from the Z-axis. Suppose the floor and the box are rough and the box does not slip on the floor as the cabin rotates. The box is at rest with respect to the cabin and hence is rotating with respect to the ground at an angular velocity ω. Let us first analyse the motion of the box from the ground frame. In this frame (which is inertial) the box is moving in a circle of radius r. It, therefore, has an acceleration $v^2/r = \omega^2 r$ towards the centre. The resultant force on the box must be towards the centre and its magnitude must be $m\omega^2 r$. The forces on the box are

(a) weight mg

(b) normal force $\mathcal{N}$ by the floor

(c) friction f by the floor.

Figure (7.6b) shows the free body diagram for the box. Since the resultant is towards the centre and its magnitude is $m\omega^2 r$, we should have

$$f = m\omega^2 r.$$

The floor exerts a force of static friction $f = m\omega^2 r$ towards the origin.

Now consider the same box when observed from the frame of the rotating cabin. The observer there finds that the box is at rest. If he or she applies Newton's laws, the resultant force on the box should be zero. The weight and the normal contact force balance each other but the frictional force $f = m\omega^2 r$ acts on the box towards the origin. To make the resultant zero, a pseudo force must be assumed which acts on the box away from the centre (radially outward) and has a magnitude $m\omega^2 r$. This pseudo force is called the *centrifugal force*. The analysis from the rotating frame is as follows :

The forces on the box are

(a) weight mg

(b) normal force $\mathcal{N}$

(c) friction f

(d) centrifugal force $m\omega^2 r$.

The free body diagram is shown in figure (7.6c). As the box is at rest, Newton's first law gives

$$f = m\omega^2 r.$$

Note that we get the same equation for friction as we got from the ground frame. But we had to apply Newton's second law from the ground frame and Newton's first law from the rotating frame. Let us now summarise our discussion.

Suppose we are working from a frame of reference that is rotating at a constant angular velocity ω with respect to an inertial frame. If we analyse the dynamics of a particle of mass m kept at a distance r from the axis of rotation, we have to assume that a force $m\omega^2 r$ acts radially outward on the particle. Only then we can apply Newton's laws of motion in the rotating frame. This radially outward pseudo force is called the centrifugal force.

In fact, centrifugal force is a sufficient pseudo force, only if we are analysing the particles at rest in a uniformly rotating frame. If we analyse the motion of a particle that moves in the rotating frame, we may have to assume other pseudo forces, together with the centrifugal force. Such forces are called the *coriolis forces*. The coriolis force is perpendicular to the velocity of the particle and also perpendicular to the axis of rotation of the frame. Once again, we emphasise that all these pseudo forces, centrifugal or coriolis, are needed only if the working frame is rotating. If we work from an inertial frame, there is no need to apply any pseudo force.

It is a common misconception among the beginners that centrifugal force acts on a particle because the particle goes on a circle. Centrifugal force acts (or is assumed to act) because we describe the particle from a rotating frame which is noninertial and still use Newton's laws.

7.7 EFFECT OF EARTH'S ROTATION ON APPARENT WEIGHT

The earth rotates about its axis at an angular speed of one revolution per 24 hours. The line joining the north and the south poles is the axis of rotation. Every point on the earth moves in a circle. A point at equator moves in a circle of radius equal to the radius of the earth and the centre of the circle is same as the centre of the earth. For any other point on the earth, the circle of rotation is smaller than this. Consider a place P on the earth (figure 7.7).

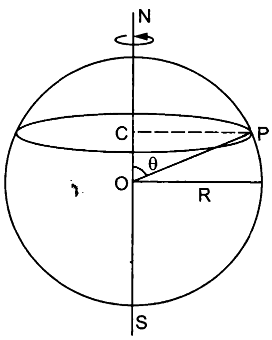

Figure 7.7

Drop a perpendicular PC from P to the axis SN. The place P rotates in a circle with the centre at C. The radius of this circle is CP. The angle between the axis SN and the radius OP through P is called the *colatitude* of the place P. We have

$$CP = OP \sin\theta$$

or, $$r = R \sin\theta$$

where R is the radius of the earth.

If we work from the frame of reference of the earth, we shall have to assume the existence of the pseudo forces. In particular, a centrifugal force $m\omega^2 r$ has to be assumed on any particle of mass m placed at P. Here ω is the angular speed of the earth. If we discuss the equilibrium of bodies at rest in the earth's frame, no other pseudo force is needed.

Consider a heavy particle of mass m suspended through a string from the ceiling of a laboratory at colatitude θ (figure 7.8). Looking from the earth's frame the particle is in equilibrium and the forces on it are

(a) gravitational attraction mg towards the centre of the earth, i.e., vertically downward,

(b) centrifugal force $m\omega^2 r$ towards CP and

(c) the tension in the string T along the string.

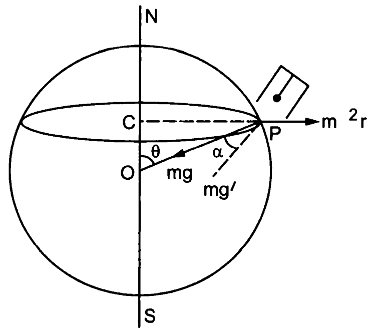

Figure 7.8

As the particle is in equilibrium (in the frame of earth), the three forces on the particle should add up to zero.

The resultant of mg and $m\omega^2 r$

$$= \sqrt{(mg)^2 + (m\omega^2 r)^2 + 2(mg)(m\omega^2 r)\cos(90° + \theta)}$$

$$= m\sqrt{g^2 + \omega^4 R^2 \sin^2\theta - 2g\omega^2 R \sin^2\theta}$$

$$= mg'$$

where $$g' = \sqrt{g^2 - \omega^2 R \sin^2\theta\,(2g - \omega^2 R)}. \quad \ldots (7.14)$$

Also, the direction of this resultant makes an angle α with the vertical OP, where

$$\tan\alpha = \frac{m\omega^2 r \sin(90° + \theta)}{mg + m\omega^2 r\cos(90° + \theta)}$$

$$= \frac{\omega^2 R \sin\theta\cos\theta}{g - \omega^2 R \sin^2\theta}. \quad \ldots (7.15)$$

As the three forces acting on the particle must add up to zero, the force of tension must be equal and opposite to the resultant of the rest two. Thus, the magnitude of the tension in the string must be mg' and the direction of the string should make an angle α with the true vertical.

The direction of g' is the apparent vertical direction, because a plumb line stays in this direction only. The walls of the buildings are constructed by making them parallel to g' and not to g. The water surface placed at rest is perpendicular to g'.

The magnitude of g' is also different from g. As $2g > \omega^2 R$, it is clear from equation (7.14) that $g' < g$. One way of measuring the weight of a body is to suspend it by a string and find the tension in the string. The tension itself is taken as a measure of the weight. As $T = mg'$, the weight so observed is less than the true weight mg. This is known as the *apparent weight*. Similarly, if a person stands on the platform of a weighing machine, the platform exerts a normal force $\mathcal{N}$ which is equal to mg'. The reading of the machine responds to the force exerted on it and hence the weight recorded is the apparent weight mg'.

At equator, $\theta = 90°$ and equation (7.14) gives

$$g' = \sqrt{g^2 - 2g\omega^2 R + \omega^4 R^2}$$

$$= g - \omega^2 R$$

or, $$mg' = mg - m\omega^2 R. \quad \ldots (7.16)$$

This can be obtained in a more straightforward way. At the equator, $m\omega^2 R$ is directly opposite to mg and the resultant is simply $mg - m\omega^2 R$. Also, this resultant is towards the centre of the earth so that at the equator the plumb line stands along the true vertical.

At poles, $\vartheta = 0$ and equation (7.14) gives $g' = g$ and equation (7.15) shows that $\alpha = 0$. Thus, there is no apparent change in g at the poles. This is because the poles themselves do not rotate and hence the effect of earth's rotation is not felt there.

Example 7.7

A body weighs 98 N *on a spring balance at the north pole. What will be its weight recorded on the same scale if it is shifted to the equator ? Use* $g = GM/R^2 = 9{\cdot}8\ \text{m/s}^2$ *and the radius of the earth* R = 6400 km.

Solution : At poles, the apparent weight is same as the true weight.

Thus,

$$98\ \text{N} = mg = m(9{\cdot}8\ \text{m/s}^2)$$

or, $$m = 10\ \text{kg}.$$

At the equator, the apparent weight is

$$mg' = mg - m\omega^2 R.$$

The radius of the earth is 6400 km and the angular speed is

$$\omega = \frac{2\pi\ \text{rad}}{24 \times 60 \times 60\ \text{s}} = 7{\cdot}27 \times 10^{-5}\ \text{rad/s}.$$

Thus,

$$mg' = 98\ \text{N} - (10\ \text{kg})\,(7{\cdot}27 \times 10^{-5}\ \text{s}^{-1})^2\,(6400\ \text{km})$$

$$= 97{\cdot}66\ \text{N}.$$

Worked Out Examples

1. *A car has to move on a level turn of radius* 45 m. *If the coefficient of static friction between the tyre and the road is* μ_s = 2.0, *find the maximum speed the car can take without skidding.*

Solution : Let the mass of the car be M. The forces on the car are

(a) weight Mg downward
(b) normal force $\mathcal{N}$ by the road upward
(c) friction f_s by the road towards the centre.

The car is going on a horizontal circle of radius R, so it is accelerating. The acceleration is towards the centre and its magnitude is v^2/R, where v is the speed. For vertical direction, acceleration = 0. Resolving the forces in vertical and horizontal directions and applying Newton's laws, we have

$$\mathcal{N} = mg$$

and $$f_s = Mv^2/R.$$

As we are looking for the maximum speed for no skidding, it is a case of limiting friction and hence $f_s = \mu_s \mathcal{N} = \mu_s Mg$.

So, we have

$$\mu_s Mg = Mv^2/R$$

or, $$v^2 = \mu_s gR.$$

Putting the values, $v = \sqrt{2 \times 10\ \text{m/s}^2 \times 45\ \text{m}}$
$= 30$ m/s $= 108$ km/hr.

2. *A circular track of radius* 600 m *is to be designed for cars at an average speed of* 180 km/hr. *What should be the angle of banking of the track ?*

Solution : Let the angle of banking be θ. The forces on the car are (figure 7-W1)

(a) weight of the car Mg downward and
(b) normal force $\mathcal{N}$.

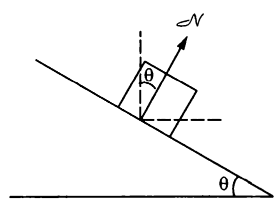

Figure 7-W1

For proper banking, static frictional force is not needed. For vertical direction the acceleration is zero. So,

$$\mathcal{N} \cos\theta = Mg. \quad \ldots \text{(i)}$$

For horizontal direction, the acceleration is v^2/r towards the centre, so that

$$\mathcal{N} \sin\theta = Mv^2/r. \quad \ldots \text{(ii)}$$

From (i) and (ii),

$$\tan\theta = v^2/rg.$$

Putting the values, $\tan\theta = \dfrac{(180\ \text{km/hr})^2}{(600\ \text{m})\,(10\ \text{m/s}^2)} = 0.4167$

or, $\theta = 22.6°$.

3. *A particle of mass m is suspended from a ceiling through a string of length L. The particle moves in a horizontal circle of radius r. Find (a) the speed of the particle and (b) the tension in the string. Such a system is called a conical pendulum.*

Solution : The situation is shown in figure (7-W2). The angle θ made by the string with the vertical is given by

$$\sin\theta = r/L. \qquad \ldots \text{(i)}$$

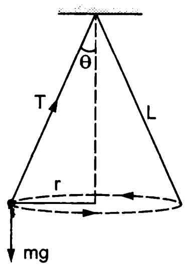

Figure 7-W2

The forces on the particle are

(a) the tension T along the string and

(b) the weight mg vertically downward.

The particle is moving in a circle with a constant speed v. Thus, the radial acceleration towards the centre has magnitude v^2/r. Resolving the forces along the radial direction and applying Newton's second law,

$$T\sin\theta = m\,(v^2/r). \qquad \ldots \text{(ii)}$$

As there is no acceleration in vertical direction, we have from Newton's first law,

$$T\cos\theta = mg. \qquad \ldots \text{(iii)}$$

Dividing (ii) by (iii),

$$\tan\theta = \frac{v^2}{rg}$$

or, $v = \sqrt{rg\tan\theta}$.

And from (iii),

$$T = \frac{mg}{\cos\theta}.$$

Using (i),

$$v = \frac{r\sqrt{g}}{(L^2 - r^2)^{1/4}} \quad \text{and} \quad T = \frac{mgL}{(L^2 - r^2)^{1/2}}.$$

4. *One end of a massless spring of spring constant* 100 N/m *and natural length* 0.5 m *is fixed and the other end is connected to a particle of mass* 0.5 kg *lying on a frictionless horizontal table. The spring remains horizontal. If the mass is made to rotate at an angular velocity of* 2 rad/s, *find the elongation of the spring.*

Solution : The particle is moving in a horizontal circle, so it is accelerated towards the centre with magnitude v^2/r. The horizontal force on the particle is due to the spring and equals kl, where l is the elongation and k is the spring constant. Thus,

$$kl = mv^2/r = m\omega^2 r = m\omega^2(l_0 + l).$$

Here ω is the angular velocity, l_0 is the natural length (0.5 m) and $l_0 + l$ is the total length of the spring which is also the radius of the circle along which the particle moves.

Thus, $(k - m\omega^2)l = m\omega^2 l_0$

or, $l = \dfrac{m\omega^2 l_0}{k - m\omega^2}$.

Putting the values,

$$l = \frac{0.5 \times 4 \times 0.5}{100 - 0.5 \times 4}\ \text{m} \approx \frac{1}{100}\ \text{m} = 1\ \text{cm}.$$

5. *A simple pendulum is constructed by attaching a bob of mass m to a string of length L fixed at its upper end. The bob oscillates in a vertical circle. It is found that the speed of the bob is v when the string makes an angle θ with the vertical. Find the tension in the string at this instant.*

Solution : The forces acting on the bob are (figure 7-W3)

(a) the tension T

(b) the weight mg.

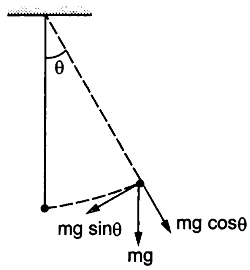

Figure 7-W3

As the bob moves in a vertical circle with centre at O, the radial acceleration is v^2/L towards O. Taking the components along this radius and applying Newton's second law, we get,

$$T - mg\cos\theta = mv^2/L$$

or, $T = m\,(g\cos\theta + v^2/L)$.

6. *A cylindrical bucket filled with water is whirled around in a vertical circle of radius r. What can be the minimum speed at the top of the path if water does not fall out from the bucket? If it continues with this speed, what normal contact force the bucket exerts on water at the lowest point of the path?*

Solution : Consider water as the system. At the top of the circle its acceleration towards the centre is vertically downward with magnitude v^2/r. The forces on water are (figure 7-W4)

(a) weight Mg downward and

(b) normal force by the bucket, also downward.

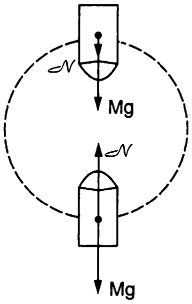

Figure 7-W4

So, from Newton's second law

$$Mg + \mathcal{N} = Mv^2/r.$$

For water not to fall out from the bucket, $\mathcal{N} \geq 0$.

Hence, $Mv^2/r \geq Mg$ or, $v^2 \geq rg$.

The minimum speed at the top must be $\sqrt{rg}$.

If the bucket continues on the circle with this minimum speed $\sqrt{rg}$, the forces at the bottom of the path are

(a) weight Mg downward and

(b) normal contact force $\mathcal{N}'$ by the bucket upward.

The acceleration is towards the centre which is vertically upward, so

$$\mathcal{N}' - Mg = Mv^2/r$$

or, $$\mathcal{N}' = M(g + v^2/r) = 2\,Mg.$$

7. *A fighter plane is pulling out for a dive at a speed of* 900 km/hr. *Assuming its path to be a vertical circle of radius* 2000 m *and its mass to be* 16000 kg, *find the force exerted by the air on it at the lowest point. Take* $g = 9{\cdot}8\ \text{m/s}^2$.

Solution : At the lowest point in the path the acceleration is vertically upward (towards the centre) and its magnitude is v^2/r.

The forces on the plane are

(a) weight Mg downward and

(b) force F by the air upward.

Hence, Newton's second law of motion gives

$$F - Mg = Mv^2/r$$

or, $$F = M(g + v^2/r).$$

Here $v = 900\ \text{km/hr} = \dfrac{9\times 10^5}{3600}\ \text{m/s} = 250\ \text{m/s}$

or, $F = 16000\left(9{\cdot}8 + \dfrac{62500}{2000}\right)\text{N} = 6{\cdot}56\times 10^5\ \text{N}$ (upward).

8. *Figure (7-W5) shows a rod of length* 20 cm *pivoted near an end and which is made to rotate in a horizontal plane with a constant angular speed. A ball of mass m is suspended by a string also of length* 20 cm *from the other end of the rod. If the angle* θ *made by the string with the vertical is* 30°, *find the angular speed of the rotation. Take* $g = 10\ \text{m/s}^2$.

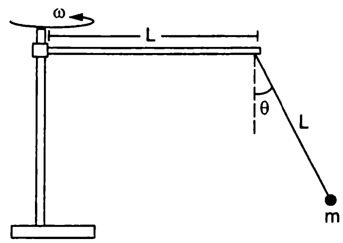

Figure 7-W5

Solution : Let the angular speed be ω. As is clear from the figure, the ball moves in a horizontal circle of radius $L + L\sin\theta$, where $L = 20$ cm. Its acceleration is, therefore, $\omega^2(L + L\sin\theta)$ towards the centre. The forces on the bob are (figure 7-W5)

(a) the tension T along the string and

(b) the weight mg.

Resolving the forces along the radius and applying Newton's second law,

$$T\sin\theta = m\omega^2 L\,(1 + \sin\theta). \qquad \text{... (i)}$$

Applying Newton's first law in the vertical direction,

$$T\cos\theta = mg. \qquad \text{... (ii)}$$

Dividing (i) by (ii),

$$\tan\theta = \frac{\omega^2 L(1 + \sin\theta)}{g}$$

or, $$\omega^2 = \frac{g\tan\theta}{L(1 + \sin\theta)} = \frac{(10\ \text{m/s}^2)\,(1/\sqrt{3})}{(0{\cdot}20\ \text{m})\,(1 + 1/2)}$$

or, $$\omega = 4{\cdot}4\ \text{rad/s}.$$

9. *Two blocks each of mass M are connected to the ends of a light frame as shown in figure (7-W6). The frame is rotated about the vertical line of symmetry. The rod breaks if the tension in it exceeds* T_0. *Find the maximum frequency with which the frame may be rotated without breaking the rod.*

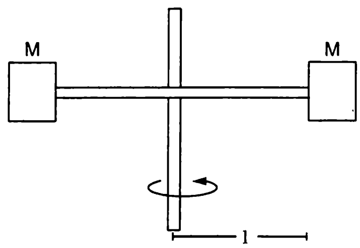

Figure 7-W6

Solution : Consider one of the blocks. If the frequency of revolution is f, the angular velocity is $\omega = 2\pi f$. The acceleration towards the centre is $v^2/l = \omega^2 l = 4\pi^2 f^2 l$, The only horizontal force on the block is the tension of the rod. At the point of breaking, this force is T_0. So from Newton's second law,

$$T_0 = M \cdot 4\pi^2 f^2 l$$

or, $$f = \frac{1}{2\pi}\left[\frac{T_0}{Ml}\right]^{1/2}$$

10. *In a rotor, a hollow vertical cylindrical structure rotates about its axis and a person rests against the inner wall. At a particular speed of the rotor, the floor below the person is removed and the person hangs resting against the wall without any floor. If the radius of the rotor is* 2 m *and the coefficient of static friction between the wall and the person is* 0·2, *find the minimum speed at which the floor may be removed. Take* $g = 10\ \text{m/s}^2$.

Solution : The situation is shown in figure (7-W7).

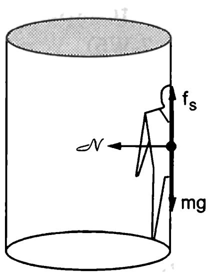

Figure 7-W7

When the floor is removed, the forces on the person are
(a) weight mg downward
(b) normal force $\mathcal{N}$ due to the wall, towards the centre
(c) frictional force f_s, parallel to the wall, upward.

The person is moving in a circle with a uniform speed, so its acceleration is v^2/r towards the centre.
Newton's law for the horizontal direction (2nd law) and for the vertical direction (1st law) give

$$\mathcal{N} = mv^2/r \qquad \text{... (i)}$$

and $$f_s = mg. \qquad \text{... (ii)}$$

For the minimum speed when the floor may be removed, the friction is limiting one and so equals $\mu_s \mathcal{N}$. This gives

$$\mu_s \mathcal{N} = mg$$

or, $$\frac{\mu_s m v^2}{r} = mg \quad \text{[using (i)]}$$

or, $$v = \sqrt{\frac{rg}{\mu_s}} = \sqrt{\frac{2\ \text{m} \times 10\ \text{m/s}^2}{0{\cdot}2}} = 10\ \text{m/s}.$$

11. *A hemispherical bowl of radius R is set rotating about its axis of symmetry which is kept vertical. A small block kept in the bowl rotates with the bowl without slipping on its surface. If the surface of the bowl is smooth, and the angle made by the radius through the block with the vertical is* θ, *find the angular speed at which the bowl is rotating.*

Solution : Suppose the angular speed of rotation of the bowl is ω. The block also moves with this angular speed. The forces on the block are (figure 7-W8)
(a) the normal force $\mathcal{N}$ and
(b) the weight mg.

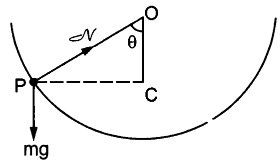

Figure 7-W8

The block moves in a horizontal circle with the centre at C, so that the radius is $PC = OP \sin\theta = R \sin\theta$. Its acceleration is, therefore, $\omega^2 R \sin\theta$. Resolving the forces along PC and applying Newton's second law,

$$\mathcal{N} \sin\theta = m\omega^2 R \sin\theta$$

or, $$\mathcal{N} = m\omega^2 R. \qquad \text{... (i)}$$

As there is no vertical acceleration,

$$\mathcal{N} \cos\theta = mg. \qquad \text{... (ii)}$$

Dividing (i) by (ii),

$$\frac{1}{\cos\theta} = \frac{\omega^2 R}{g}.$$

or, $$\omega = \sqrt{\frac{g}{R \cos\theta}}.$$

12. *A metal ring of mass m and radius R is placed on a smooth horizontal table and is set rotating about its own axis in such a way that each part of the ring moves with a speed v. Find the tension in the ring.*

Solution : Consider a small part ACB of the ring that subtends an angle $\Delta\theta$ at the centre as shown in figure (7-W9). Let the tension in the ring be T.

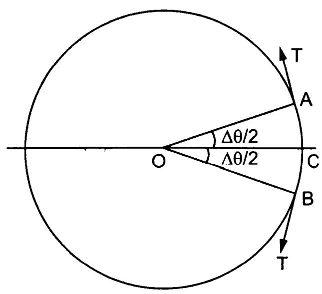

Figure 7-W9

The forces on this small part ACB are
(a) tension T by the part of the ring left to A,
(b) tension T by the part of the ring right to B,

(c) weight $(\Delta m)g$ and

(d) normal force $\mathcal{N}$ by the table.

The tension at A acts along the tangent at A and the tension at B acts along the tangent at B. As the small part ACB moves in a circle of radius R at a constant speed v, its acceleration is towards the centre (along CO) and has a magnitude $(\Delta m)v^2/R$.

Resolving the forces along the radius CO,

$$T\cos\left(90° - \frac{\Delta\theta}{2}\right) + T\cos\left(90° - \frac{\Delta\theta}{2}\right) = (\Delta m)\left(\frac{v^2}{R}\right)$$

or, $$2T\sin\frac{\Delta\theta}{2} = (\Delta m)\left(\frac{v^2}{R}\right) \cdot \qquad \text{... (i)}$$

The length of the part ACB is $R\Delta\theta$. As the total mass of the ring is m, the mass of the part ACB will be

$$\Delta m = \frac{m}{2\pi R}R\Delta\theta = \frac{m\Delta\theta}{2\pi} \cdot$$

Putting Δm in (i),

$$2T\sin\frac{\Delta\theta}{2} = \frac{m}{2\pi}\Delta\theta\left(\frac{v^2}{R}\right)$$

or, $$T = \frac{mv^2}{2\pi R}\frac{\Delta\theta/2}{\sin(\Delta\theta/2)}$$

As $\Delta\theta$ is very small, $\dfrac{\Delta\theta/2}{\sin(\Delta\theta/2)} \approx 1$ and $T = \dfrac{mv^2}{2\pi R} \cdot$

13. *A table with smooth horizontal surface is turning at an angular speed ω about its axis. A groove is made on the surface along a radius and a particle is gently placed inside the groove at a distance a from the centre. Find the speed of the particle as its distance from the centre becomes L.*

Solution : The situation is shown in figure (7-W10).

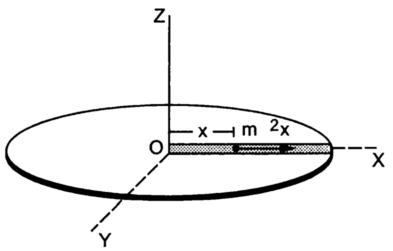

Figure 7-W10

Let us work from the frame of reference of the table. Let us take the origin at the centre of rotation O and the X-axis along the groove (figure 7-W10). The Y-axis is along the line perpendicular to OX coplanar with the surface of the table and the Z-axis is along the vertical. Suppose at time t the particle in the groove is at a distance x from the origin and is moving along the X-axis with a speed v. The forces acting on the particle (including the pseudo forces that we must assume because we have taken our frame on the table which is rotating and is nonintertial) are

(a) weight mg vertically downward,

(b) normal contact force $\mathcal{N}_1$ vertically upward by the bottom surface of the groove,

(c) normal contact force $\mathcal{N}_2$ parallel to the Y-axis by the side walls of the groove,

(d) centrifugal force $m\omega^2 x$ along the X-axis, and

(e) coriolis force along Y-axis (coriolis force is perpendicular to the velocity of the particle and the axis of rotation.)

As the particle can only move in the groove, its acceleration is along the X-axis. The only force along the X-axis is the centrifugal force $m\omega^2 x$. All the other forces are perpendicular to the X-axis and have no components along the X-axis.

Thus, the acceleration along the X-axis is

$$a = \frac{F}{m} = \frac{m\omega^2 x}{m} = \omega^2 x$$

or, $$\frac{dv}{dt} = \omega^2 x$$

or, $$\frac{dv}{dx}\cdot\frac{dx}{dt} = \omega^2 x$$

or, $$\frac{dv}{dx}\cdot v = \omega^2 x$$

or, $$v\,dv = \omega^2 x\,dx$$

or, $$\int_0^v v\,dv = \int_a^L \omega^2 x\,dx$$

or, $$\left[\frac{1}{2}v^2\right]_0^v = \left[\frac{1}{2}\omega^2 x^2\right]_a^L$$

or, $$\frac{v^2}{2} = \frac{1}{2}\omega^2(L^2 - a^2)$$

or, $$v = \omega\sqrt{L^2 - a^2}.$$

□

QUESTIONS FOR SHORT ANSWER

1. You are driving a motorcycle on a horizontal road. It is moving with a uniform velocity. Is it possible to accelerate the motorcyle without putting higher petrol input rate into the engine ?

2. Some washing machines have cloth driers. It contains a drum in which wet clothes are kept. As the drum rotates, the water particles get separated from the cloth. The general description of this action is that "the centrifugal force throws the water particles away from the drum". Comment on this statement from the viewpoint of an observer rotating with the drum and the observer who is washing the clothes.

3. A small coin is placed on a record rotating at $33\frac{1}{3}$ rev/minute. The coin does not slip on the record. Where does it get the required centripetal force from ?

4. A bird while flying takes a left turn, where does it get the centripetal force from ?

5. Is it necessary to express all angles in radian while using the equation $\omega = \omega_0 + \alpha t$?

6. After a good meal at a party you wash your hands and find that you have forgotten to bring your handkerchief. You shake your hands vigorously to remove the water as much as you can. Why is water removed in this process ?

7. A smooth block loosely fits in a circular tube placed on a horizontal surface. The block moves in a uniform circular motion along the tube (figure 7-Q1). Which wall (inner or outer) will exert a nonzero normal contact force on the block ?

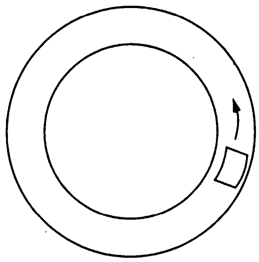

Figure 7-Q1

8. Consider the circular motion of the earth around the sun. Which of the following statements is more appropriate ?
(a) Gravitational attraction of the sun on the earth is equal to the centripetal force.
(b) Gravitational attraction of the sun on the earth is the centripetal force.

9. A car driver going at some speed v suddenly finds a wide wall at a distance r. Should he apply brakes or turn the car in a circle of radius r to avoid hitting the wall ?

10. A heavy mass m is hanging from a string in equilibrium without breaking it. When this same mass is set into oscillation, the string breaks. Explain.

OBJECTIVE I

1. When a particle moves in a circle with a uniform speed
(a) its velocity and acceleration are both constant
(b) its velocity is constant but the acceleration changes
(c) its acceleration is constant but the velocity changes
(d) its velocity and acceleration both change.

2. Two cars having masses m_1 and m_2 move in circles of radii r_1 and r_2 respectively. If they complete the circle in equal time, the ratio of their angular speeds ω_1/ω_2 is
(a) m_1/m_2 (b) r_1/r_2 (c) m_1r_1/m_2r_2 (d) 1.

3. A car moves at a constant speed on a road as shown in figure (7-Q2). The normal force by the road on the car is N_A and N_B when it is at the points A and B respectively.
(a) $N_A = N_B$ (b) $N_A > N_B$ (c) $N_A < N_B$ (d) insufficient information to decide the relation of N_A and N_B.

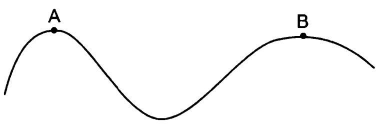

Figure 7-Q2

4. A particle of mass m is observed from an inertial frame of reference and is found to move in a circle of radius r with a uniform speed v. The centrifugal force on it is
(a) $\frac{mv^2}{r}$ towards the centre
(b) $\frac{mv^2}{r}$ away from the centre
(c) $\frac{mv^2}{r}$ along the tangent through the particle
(d) zero.

5. A particle of mass m rotates with a uniform angular speed ω. It is viewed from a frame rotating about the Z-axis with a uniform angular speed ω_0. The centrifugal force on the particle is
(a) $m\omega^2 a$ (b) $m\omega_0^2 a$ (c) $m\left(\frac{\omega+\omega_0}{2}\right)^2 a$ (d) $m\omega\omega_0 a$.

6. A particle is kept fixed on a turntable rotating uniformly. As seen from the ground the particle goes in a circle, its speed is 20 cm/s and acceleration is 20 cm/s^2. The particle is now shifted to a new position to make the radius half of the original value. The new values of the speed and acceleration will be
(a) 10 cm/s, 10 cm/s^2 (b) 10 cm/s, 80 cm/s^2
(c) 40 cm/s, 10 cm/s^2 (d) 40 cm/s, 40 cm/s^2.

7. Water in a bucket is whirled in a vertical circle with a string attached to it. The water does not fall down even when the bucket is inverted at the top of its path. We conclude that in this position

(a) $mg = \frac{mv^2}{r}$ (b) mg is greater than $\frac{mv^2}{r}$

(c) mg is not greater than $\frac{mv^2}{r}$

(d) mg is not less than $\frac{mv^2}{r}$.

8. A stone of mass m tied to a string of length l is rotated in a circle with the other end of the string as the centre. The speed of the stone is v. If the string breaks, the stone will move
(a) towards the centre (b) away from the centre
(c) along a tangent (d) will stop.

9. A coin placed on a rotating turntable just slips if it is placed at a distance of 4 cm from the centre. If the angular velocity of the turntable is doubled, it will just slip at a distance of
(a) 1 cm (b) 2 cm (c) 4 cm (d) 8 cm.

10. A motorcyle is going on an overbridge of radius R. The driver maintains a constant speed. As the motorcycle is ascending on the overbridge, the normal force on it
(a) increases (b) decreases
(c) remains the same (d) fluctuates.

11. Three identical cars A, B and C are moving at the same speed on three bridges. The car A goes on a plane bridge, B on a bridge convex upward and C goes on a bridge concave upward. Let F_A, F_B and F_C be the normal forces exerted by the cars on the bridges when they are at the middle of bridges.
(a) F_A is maximum of the three forces.
(b) F_B is maximum of the three forces.
(c) F_C is maximum of the three forces.
(d) $F_A = F_B = F_C$.

12. A train A runs from east to west and another train B of the same mass runs from west to east at the same speed along the equator. A presses the track with a force F_1 and B presses the track with a force F_2.
(a) $F_1 > F_2$. (b) $F_1 < F_2$. (c) $F_1 = F_2$.
(d) the information is insufficient to find the relation between F_1 and F_2.

13. If the earth stops rotating, the apparent value of g on its surface will
(a) increase everywhere
(b) decrease everywhere
(c) remain the same everywhere
(d) increase at some places and remain the same at some other places.

14. A rod of length L is pivoted at one end and is rotated with a uniform angular velocity in a horizontal plane. Let T_1 and T_2 be the tensions at the points $L/4$ and $3L/4$ away from the pivoted ends.
(a) $T_1 > T_2$. (b) $T_2 > T_1$. (c) $T_1 = T_2$. (d) The relation between T_1 and T_2 depends on whether the rod rotates clockwise or anticlockwise.

15. A simple pendulum having a bob of mass m is suspended from the ceiling of a car used in a stunt film shooting. The car moves up along an inclined cliff at a speed v and makes a jump to leave the cliff and lands at some distance. Let R be the maximum height of the car from the top of the cliff. The tension in the string when the car is in air is
(a) mg (b) $mg - \frac{mv^2}{R}$ (c) $mg + \frac{mv^2}{R}$ (d) zero.

16. Let θ denote the angular displacement of a simple pendulum oscillating in a vertical plane. If the mass of the bob is m, the tension in the string is $mg \cos\theta$
(a) always (b) never
(c) at the extreme positions
(d) at the mean position.

OBJECTIVE II

1. An object follows a curved path. The following quantities may remain constant during the motion
(a) speed (b) velocity
(c) acceleration (d) magnitude of acceleration.

2. Assume that the earth goes round the sun in a circular orbit with a constant speed of 30 km/s.
(a) The average velocity of the earth from 1st Jan, 90 to 30th June, 90 is zero.
(b) The average acceleration during the above period is 60 km/s^2.
(c) The average speed from 1st Jan, 90 to 31st Dec, 90 is zero.
(d) The instantaneous acceleration of the earth points towards the sun.

3. The position vector of a particle in a circular motion about the origin sweeps out equal area in equal time. Its
(a) velocity remains constant
(b) speed remains constant
(c) acceleration remains constant
(d) tangential acceleration remains constant.

4. A particle is going in a spiral path as shown in figure (7-Q3) with constant speed.

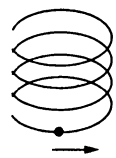

Figure 7-Q3

(a) The velocity of the particle is constant.
(b) The acceleration of the particle is constant.

(c) The magnitude of acceleration is constant.
(d) The magnitude of acceleration is decreasing continuously.

5. A car of mass M is moving on a horizontal circular path of radius r. At an instant its speed is v and is increasing at a rate a.
(a) The acceleration of the car is towards the centre of the path.
(b) The magnitude of the frictional force on the car is greater than $\frac{mv^2}{r}$.
(c) The friction coefficient between the ground and the car is not less than a/g.
(d) The friction coefficient between the ground and the car is $\mu = \tan^{-1}\frac{v^2}{rg}$.

6. A circular road of radius r is banked for a speed v = 40 km/hr. A car of mass m attempts to go on the circular road. The friction coefficient between the tyre and the road is negligible.
(a) The car cannot make a turn without skidding.
(b) If the car turns at a speed less than 40 km/hr, it will slip down.
(c) If the car turns at the correct speed of 40 km/hr, the force by the road on the car is equal to $\frac{mv^2}{r}$.
(d) If the car turns at the correct speed of 40 km/hr, the force by the road on the car is greater than mg as well as greater than $\frac{mv^2}{r}$.

7. A person applies a constant force $\vec{F}$ on a particle of mass m and finds that the particle moves in a circle of radius r with a uniform speed v as seen from an inertial frame of reference.
(a) This is not possible.
(b) There are other forces on the particle.
(c) The resultant of the other forces is $\frac{mv^2}{r}$ towards the centre.
(d) The resultant of the other forces varies in magnitude as well as in direction.

EXERCISES

1. Find the acceleration of the moon with respect to the earth from the following data : Distance between the earth and the moon = $3{\cdot}85 \times 10^5$ km and the time taken by the moon to complete one revolution around the earth = 27·3 days.
2. Find the acceleration of a particle placed on the surface of the earth at the equator due to earth's rotation. The diameter of earth = 12800 km and it takes 24 hours for the earth to complete one revolution about its axis.
3. A particle moves in a circle of radius 1·0 cm at a speed given by v = 2·0 t where v is in cm/s and t in seconds. (a) Find the radial acceleration of the particle at t = 1 s. (b) Find the tangential acceleration at t = 1 s. (c) Find the magnitude of the acceleration at t = 1 s.
4. A scooter weighing 150 kg together with its rider moving at 36 km/hr is to take a turn of radius 30 m. What horizontal force on the scooter is needed to make the turn possible ?
5. If the horizontal force needed for the turn in the previous problem is to be supplied by the normal force by the road, what should be the proper angle of banking ?
6. A park has a radius of 10 m. If a vehicle goes round it at an average speed of 18 km/hr, what should be the proper angle of banking ?
7. If the road of the previous problem is horizontal (no banking), what should be the minimum friction coefficient so that a scooter going at 18 km/hr does not skid ?
8. A circular road of radius 50 m has the angle of banking equal to 30°. At what speed should a vehicle go on this road so that the friction is not used ?
9. In the Bohr model of hydrogen atom, the electron is treated as a particle going in a circle with the centre at the proton. The proton itself is assumed to be fixed in an inertial frame. The centripetal force is provided by the Coloumb attraction. In the ground state, the electron goes round the proton in a circle of radius $5{\cdot}3 \times 10^{-11}$ m. Find the speed of the electron in the ground state. Mass of the electron = $9{\cdot}1 \times 10^{-31}$ kg and charge of the electron = $1{\cdot}6 \times 10^{-19}$ C.
10. A stone is fastened to one end of a string and is whirled in a vertical circle of radius R. Find the minimum speed the stone can have at the highest point of the circle.
11. A ceiling fan has a diameter (of the circle through the outer edges of the three blades) of 120 cm and rpm 1500 at full speed. Consider a particle of mass 1 g sticking at the outer end of a blade. How much force does it experience when the fan runs at full speed ? Who exerts this force on the particle ? How much force does the particle exert on the blade along its surface ?
12. A mosquito is sitting on an L.P. record disc rotating on a turn table at $33\frac{1}{3}$ revolutions per minute. The distance of the mosquito from the centre of the turn table is 10 cm. Show that the friction coefficient between the record and the mosquito is greater than $\pi^2/81$. Take $g = 10\ \text{m/s}^2$.
13. A simple pendulum is suspended from the ceiling of a car taking a turn of radius 10 m at a speed of 36 km/h. Find the angle made by the string of the pendulum with the vertical if this angle does not change during the turn. Take $g = 10\ \text{m/s}^2$.

14. The bob of a simple pendulum of length 1 m has mass 100 g and a speed of 1·4 m/s at the lowest point in its path. Find the tension in the string at this instant.

15. Suppose the bob of the previous problem has a speed of 1·4 m/s when the string makes an angle of 0·20 radian with the vertical. Find the tension at this instant. You can use $\cos\theta \approx 1 - \theta^2/2$ and $\sin\theta \approx \theta$ for small θ.

16. Suppose the amplitude of a simple pendulum having a bob of mass m is θ_0. Find the tension in the string when the bob is at its extreme position.

17. A person stands on a spring balance at the equator. (a) By what fraction is the balance reading less than his true weight? (b) If the speed of earth's rotation is increased by such an amount that the balance reading is half the true weight, what will be the length of the day in this case?

18. A turn of radius 20 m is banked for the vehicles going at a speed of 36 km/h. If the coefficient of static friction between the road and the tyre is 0·4, what are the possible speeds of a vehicle so that it neither slips down nor skids up?

19. A motorcycle has to move with a constant speed on an overbridge which is in the form of a circular arc of radius R and has a total length L. Suppose the motorcycle starts from the highest point. (a) What can its maximum velocity be for which the contact with the road is not broken at the highest point? (b) If the motorcycle goes at speed $1/\sqrt{2}$ times the maximum found in part (a), where will it lose the contact with the road? (c) What maximum uniform speed can it maintain on the bridge if it does not lose contact anywhere on the bridge?

20. A car goes on a horizontal circular road of radius R, the speed increasing at a constant rate $\frac{dv}{dt} = a$. The friction coefficient between the road and the tyre is μ. Find the speed at which the car will skid.

21. A block of mass m is kept on a horizontal ruler. The friction coefficient between the ruler and the block is μ. The ruler is fixed at one end and the block is at a distance L from the fixed end. The ruler is rotated about the fixed end in the horizontal plane through the fixed end. (a) What can the maximum angular speed be for which the block does not slip? (b) If the angular speed of the ruler is uniformly increased from zero at an angular acceleration α, at what angular speed will the block slip?

22. A track consists of two circular parts ABC and CDE of equal radius 100 m and joined smoothly as shown in figure (7-E1). Each part subtends a right angle at its centre. A cycle weighing 100 kg together with the rider travels at a constant speed of 18 km/h on the track. (a) Find the normal contact force by the road on the cycle when it is at B and at D. (b) Find the force of friction exerted by the track on the tyres when the cycle is at B, C and D. (c) Find the normal force between the road and the cycle just before and just after the cycle crosses C. (d) What should be the minimum friction coefficient between the road and the tyre, which will ensure that the cyclist can move with constant speed? Take $g = 10\ \text{m/s}^2$.

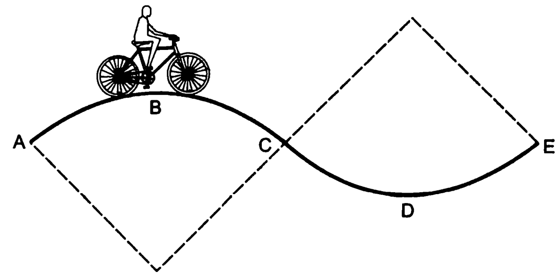

Figure 7-E1

23. In a children's park a heavy rod is pivoted at the centre and is made to rotate about the pivot so that the rod always remains horizontal. Two kids hold the rod near the ends and thus rotate with the rod (figure 7-E2). Let the mass of each kid be 15 kg, the distance between the points of the rod where the two kids hold it be 3·0 m and suppose that the rod rotates at the rate of 20 revolutions per minute. Find the force of friction exerted by the rod on one of the kids.

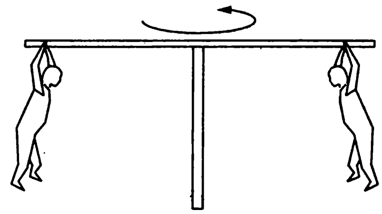

Figure 7-E2

24. A hemispherical bowl of radius R is rotated about its axis of symmetry which is kept vertical. A small block is kept in the bowl at a position where the radius makes an angle θ with the vertical. The block rotates with the bowl without any slipping. The friction coefficient between the block and the bowl surface is μ. Find the range of the angular speed for which the block will not slip.

25. A particle is projected with a speed u at an angle θ with the horizontal. Consider a small part of its path near the highest position and take it approximately to be a circular arc. What is the radius of this circle? This radius is called the radius of curvature of the curve at the point.

26. What is the radius of curvature of the parabola traced out by the projectile in the previous problem at a point where the particle velocity makes an angle $\theta/2$ with the horizontal?

27. A block of mass m moves on a horizontal circle against the wall of a cylindrical room of radius R. The floor of the room on which the block moves is smooth but the friction coefficient between the wall and the block is μ. The block is given an initial speed v_0. As a function of the speed v write (a) the normal force by the wall on the block, (b) the frictional force by the wall and (c) the tangential acceleration of the block. (d) Integrate the

tangential acceleration $\left(\frac{dv}{dt} = v\frac{dv}{ds}\right)$ to obtain the speed of the block after one revolution.

28. A table with smooth horizontal surface is fixed in a cabin that rotates with a uniform angular velocity ω in a circular path of radius R (figure 7-E3). A smooth groove AB of length $L(<<R)$ is made on the surface of the table. The groove makes an angle θ with the radius OA of the circle in which the cabin rotates. A small particle is kept at the point A in the groove and is released to move along AB. Find the time taken by the particle to reach the point B.

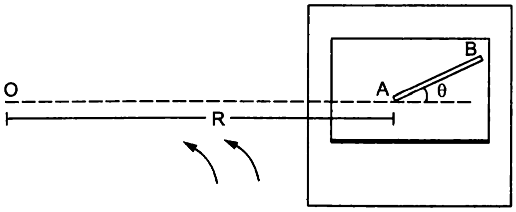

Figure 7-E3

29. A car moving at a speed of 36 km/hr is taking a turn on a circular road of radius 50 m. A small wooden plate is kept on the seat with its plane perpendicular to the radius of the circular road (figure 7-E4). A small block of mass 100 g is kept on the seat which rests against the plate. The friction coefficient between the block and the plate is $\mu = 0{\cdot}58$. (a) Find the normal contact force exerted by the plate on the block. (b) The plate is slowly turned so that the angle between the normal to the plate and the radius of the road slowly increases. Find the angle at which the block will just start sliding on the plate.

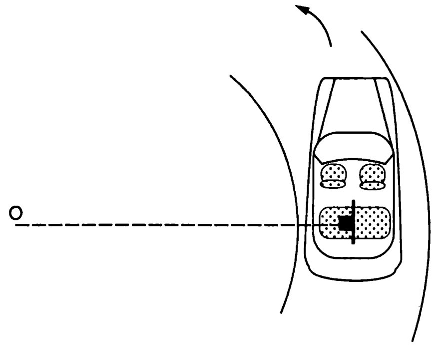

Figure 7-E4

30. A table with smooth horizontal surface is placed in a cabin which moves in a circle of a large radius R (figure 7-E5). A smooth pulley of small radius is fastened to the table. Two masses m and $2m$ placed on the table are connected through a string going over the pulley. Initially the masses are held by a person with the strings along the outward radius and then the system is released from rest (with respect to the cabin). Find the magnitude of the initial acceleration of the masses as seen from the cabin and the tension in the string.

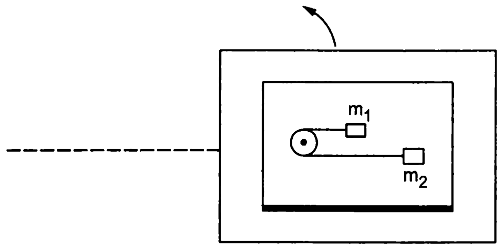

Figure 7-E5

□

ANSWERS

OBJECTIVE I

1. (d)	2. (d)	3. (c)	4. (d)	5. (b)	6. (a)
7. (c)	8. (c)	9. (a)	10. (a)	11. (c)	12. (a)
13. (d)	14. (a)	15. (d)	16. (c)		

OBJECTIVE II

1. (a), (d)	2. (d)	3. (b), (d)
4. (c)	5. (b), (c)	6. (b), (d)
7. (b), (d)		

EXERCISES

1. $2{\cdot}73 \times 10^{-3}$ m/s^2
2. $0{\cdot}0336$ m/s^2
3. (a) $4{\cdot}0$ cm/s^2 (b) $2{\cdot}0$ cm/s^2 (c) $\sqrt{20}$ cm/s^2
4. 500 N
5. $\tan^{-1}(1/3)$
6. $\tan^{-1}(1/4)$
7. 0·25
8. 17 m/s
9. $2{\cdot}2 \times 10^6$ m/s
10. $\sqrt{Rg}$
11. 14·8 N, 14·8 N
13. 45°
14. 1·2 N

15. 1·16 N

16. $mg\cos\theta_0$

17. (a) $3{\cdot}5 \times 10^{-3}$ (b) 2·0 hour

18. Between 14·7 km/h and 54 km/hr

19. (a) $\sqrt{Rg}$,
(b) a distance $\pi R/3$ along the bridge from the highest point,
(c) $\sqrt{gR\cos(L/2R)}$

20. $\left[(\mu^2 g^2 - a^2)R^2\right]^{1/4}$

21. (a) $\sqrt{\mu g/L}$ (b) $\left[\left(\frac{\mu g}{L}\right)^2 - \alpha^2\right]^{1/4}$

22. (a) 975 N, 1025 N (b) 0, 707 N, 0
(c) 682 N, 732 N (d) 1·037

23. $10\pi^2$

24. $\left[\frac{g(\sin\theta - \mu\cos\theta)}{R\sin\theta(\cos\theta + \mu\sin\theta)}\right]^{1/2}$ to $\left[\frac{g(\sin\theta + \mu\cos\theta)}{R\sin\theta(\cos\theta - \mu\sin\theta)}\right]^{1/2}$

25. $\frac{u^2\cos^2\theta}{g}$

26. $\frac{u^2\cos^2\theta}{g\cos^3(\theta/2)}$

27. (a) $\frac{mv^2}{R}$ (b) $\frac{\mu mv^2}{R}$ (c) $-\frac{\mu v^2}{R}$ (d) $v_0 e^{-2\pi\mu}$

28. $\sqrt{\frac{2L}{\omega^2 R\cos\theta}}$

29. (a) 0·2 N (b) 30°

30. $\frac{\omega^2 R}{3}, \frac{4}{3}m\omega^2 R$

□