Differentiation Ex 11.5 Q49

Here,

$$xy\log(x+y) = 1 \qquad ---(\text{i})$$

Differentiating with respect to x using chain rule, product rule,

$$\frac{dy}{dx}\left(xy\log(x+y)\right) = \frac{d}{dx}(1)$$

$$xy\frac{d}{dx}\log(x+y) + x\log(x+y)\frac{dy}{dx} + y\log(x+y)\frac{d}{dx}(x) = 0$$

$$\frac{xy}{(x+y)}\left(1+\frac{dy}{dx}\right) + x\log(x+y)\frac{dy}{dx} + y\log(x+y)(1) = 0$$

$$\left(\frac{xy}{x+y}\right)\left(1+\frac{dy}{dx}\right) + x\log(x+y)\frac{dy}{dx} + y\log(x+y) = 0$$

$$\left(\frac{xy}{x+y}\right)\frac{dy}{dx} + \frac{xy}{x+y} + x\left(\frac{1}{xy}\right)\frac{dy}{dx} + y\left(\frac{1}{xy}\right) = 0 \qquad [\text{Using equation (i)}]$$

$$\frac{dy}{dx}\left[\frac{xy}{x+y}+\frac{1}{y}\right] = -\left[\frac{1}{x}+\frac{xy}{x+y}\right]$$

$$\frac{dy}{dx}\left[\frac{xy^2+x+y}{(x+y)y}\right] = -\left[\frac{x+y+x^2y}{x(x+y)}\right]$$

$$\frac{dy}{dx} = -\frac{y}{x}\left(\frac{x+y+x^2y}{x+y+xy^2}\right)$$

Differentiation Ex 11.5 Q50

Here,

$$y = x\sin y \qquad ---(\text{i})$$

Differentiating it with respect to x using product rule,

$$\frac{dy}{dx} = \frac{d}{dx}(x\sin y)$$

$$= x\frac{d}{dx}(\sin y) + \sin y\frac{d}{dx}(x)$$

$$= x\cos y\frac{dy}{dx} + \sin y\,(1)$$

$$\frac{dy}{dx} - x\cos y\frac{dy}{dx} = \sin y$$

$$\frac{dy}{dx}(1 - x\cos y) = \sin y$$

$$\frac{dy}{dx} = \frac{\sin y}{(1 - x\cos y)}$$

Put the value of $\sin y = \frac{y}{x}$ form equation (i),

$$\frac{dy}{dx} = \frac{y}{x(1 - x\cos y)}$$

Differentiation Ex 11.5 Q51

Here,

$$f(x) = (1+x)\left(1+x^2\right)\left(1+x^4\right)\left(1+x^8\right)$$

Differentiating with respect to x using product rule and chain rule,

$$\Rightarrow \quad f'(x) = (1+x)\left(1+x^2\right)\frac{d}{dx}\left(1+x^8\right) + (1+x)\left(1+x^2\right)\left(1+x^8\right)\frac{d}{dx}\left(1+x^4\right) + (1+x)\left(1+x^4\right)\left(1+x^8\right)\frac{d}{dx}\left(1+x^2\right) + \left(1+x^2\right)\left(1+x^4\right)\left(1+x^8\right)\frac{d}{dx}(1+x)$$

$$\Rightarrow \quad f'(x) = (1+x)\left(1+x^2\right)\left(1+x^4\right)8x^7 + (1+x)\left(1+x^2\right)\left(1+x^8\right)\left(4x^3\right) + (1+x)\left(1+x^4\right)\left(1+x^8\right)(2x) + \left(1+x^2\right)\left(1+x^4\right)\left(1+x^8\right)(1)$$

$$f'(1) = (1+1)(1+1)(8) + (1+1)(1+1)(1+1)(4) + (1+1)(1+1)(1+1)(2) + (1+1)(1+1)(1+1)$$

$$\begin{aligned} f'(1) &= (2)(2)(2)(8) + (2)(2)(2)(4) + (2)(2)(2)(2) + (2)(2)(2) \\ &= 64 + 32 + 16 + 8 \\ &= 120 \end{aligned}$$

So,

$$f'(1) = 120$$

Differentiation Ex 11.5 Q52

Here,

$$y = \log\left(\frac{x^2+x+}{x^2-x+1}\right) + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}x}{1-x^2}\right)$$

Differentiating it with respect to x using chain rule and quotient rule,

$$\frac{dy}{dx} = \frac{d}{dx}\log\left(\frac{x^2+x+1}{x^2-x+1}\right) + \frac{2}{\sqrt{3}}\frac{d}{dx}\tan^{-1}\left(\frac{\sqrt{3}x}{1-x^2}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{\left(\frac{x^2+x+1}{x^2-x+1}\right)}\frac{d}{dx}\left(\frac{x^2+x+1}{x^2-x+1}\right) + \frac{2}{\sqrt{3}}\left\{\frac{1}{1+\left(\frac{\sqrt{3}x}{1-x^2}\right)}\right\}\frac{d}{dx}\left(\frac{\sqrt{3}x}{1-x^2}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{x^2-x+1}{x^2+x+1}\right)\left(\frac{\left(x^2-x+1\right)\frac{d}{dx}\left(x^2+x+1\right) - \left(x^2+x+1\right)\frac{d}{dx}\left(x^2-x+1\right)}{\left(x^2-x+1\right)^2}\right) + \frac{2}{\sqrt{3}}\left\{\frac{(1-x)^2}{1+x^4-2x^2+3x^2}\right\} \left\{\frac{\left(1-x^2\right)^2\frac{d}{dx}\left(\sqrt{3}x\right) - \sqrt{3}x\frac{d}{dx}(1-x)^2}{\left(1-x^2\right)^2}\right\}$$

$$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{1}{x^2+x+1}\right)\left(\frac{\left(x^2-x+1\right)(2x+1) - \left(x^2+x+1\right)(2x-1)}{\left(x^2-x+1\right)}\right) + \frac{2}{\sqrt{3}}\left(\frac{\left(1-x^2\right)^2}{1+x^2+x^4}\right)\left(\frac{\left(1-x^2\right)\left(\sqrt{3}\right) - \sqrt{3}x(-2x)}{\left(1-x^2\right)^2}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{2x^3-2x^2+2x+x^2-x+1-2x^3-2x^2-2x+x^2+x+1}{x^4+2x^2+1-x^2}\right) + \frac{2}{\sqrt{3}}\left(\frac{\sqrt{3}-\sqrt{3}x^2+2\sqrt{3}x^2}{1+x^2+x^4}\right)$$

$$\begin{aligned} &= \left(\frac{-2x^2+2}{x^4+x^2+1}\right) + \frac{2\sqrt{3}\left(x^2+1\right)}{\sqrt{3}\left(1+x^2+x^4\right)} \\ &= \frac{2\left(1-x^2\right)}{\left(x^4+x^2+1\right)} + \frac{2\left(x^2+1\right)}{1+x^2+x^4} \\ &= \frac{2\left(1-x^2+x^2+1\right)}{1+x^2+x^4} \end{aligned}$$

$$\frac{dy}{dx} = \frac{4}{1+x^2+x^4}$$

Differentiation Ex 11.5 Q53

Here,

$$y = (\sin x - \cos x)^{(\sin x - \cos x)}$$

Takig log on both the sides,

$$\Rightarrow \quad \log y = \log(\sin x - \cos x)^{(\sin x - \cos x)}$$
$$\Rightarrow \quad \log y = (\sin x - \cos x)\log(\sin x - \cos x)$$

Differentiating it with respect to x using product rule, chain rule,

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \log(\sin x - \cos x)\frac{d}{dx}(\sin x - \cos x) + (\sin x - \cos x)\frac{d}{dx}\log(\sin x - \cos x)$$
$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = \log(\sin x - \cos x) \times (\cos x + \sin x) + \frac{(\sin x - \cos x)}{(\sin x - \cos x)}\frac{d}{dx}(\sin x - \cos x)$$
$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = (\cos x + \sin x)\log(\sin x - \cos x) + (\cos x + \sin x)$$
$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = (\cos x + \sin x)\left(1 + \log(\sin x - \cos x)\right)$$
$$\Rightarrow \quad \frac{dy}{dx} = y\left[(\cos x + \sin x)\left(1 + \log(\sin x - \cos x)\right)\right]$$

Using equation (i),

$$\frac{dy}{dx} = (\sin x - \cos x)^{(\sin x - \cos x)}\left[(\cos x + \sin x)\left(1 + \log(\sin x - \cos x)\right)\right]$$

*********** END **********