Adjoint and Inverse of Matrix Ex 7.1 Q1(i)

Here, $A = \begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix}$

Cofactors of A are:

$$C_{11} = 4$$
$$C_{12} = -2$$
$$C_{21} = -5$$
$$C_{22} = -3$$

$$\therefore \quad \text{adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$$

$$\therefore \quad (\text{adj } A) = \begin{bmatrix} 4 & -2 \\ -5 & -3 \end{bmatrix}^T$$
$$= \begin{bmatrix} 4 & -5 \\ -2 & -3 \end{bmatrix}$$

Now, $(\text{adj } A)\, A = \begin{bmatrix} 4 & -5 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -22 & 0 \\ 0 & -22 \end{bmatrix}$

And, $|A|.I = \begin{vmatrix} -3 & 5 \\ 2 & 4 \end{vmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (-22)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -22 & 0 \\ 0 & -22 \end{bmatrix}$

Also,

$$A.(\text{adj } A) = \begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 4 & -5 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} -22 & 0 \\ 0 & -22 \end{bmatrix}$$

Therefore, $(\text{adj } A)\, A = |A|.I = A.(\text{adj } A)$

Adjoint and Inverse of Matrix Ex 7.1 Q1(ii)

Here, $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Cofactors of A are:

$$C_{11}=d$$
$$C_{12}=-c$$
$$C_{21}=-b$$
$$C_{22}=a$$

$$\therefore \quad \text{adj}A=\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$$

$$\therefore \quad \text{adj}A=\begin{bmatrix} d & -c \\ -b & a \end{bmatrix}^T = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Now, $(\text{adj}A)(A)=\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}=\begin{bmatrix} ad-bc & bd-bd \\ -ac+ac & ad-bc \end{bmatrix}=\begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$

And, $|A|.I=\begin{vmatrix} a & b \\ c & d \end{vmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=(ad-bc)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$

Also,

$$A(\text{adj}A)=\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}=\begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$$

$$\therefore \quad (\text{adj}A)(A)=|A|.I=A(\text{adj}A)$$

Adjoint and Inverse of Matrix Ex 7.1 Q1(iii)

Here, $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

Cofactors of A are:

$C_{11}=\cos\alpha$

$C_{12}=-\sin\alpha$

$C_{21}=-\sin\alpha$

$C_{22}=\cos\alpha$

$$\therefore \quad \operatorname{Adj} A=\begin{bmatrix}C_{11} & C_{12}\\ C_{21} & C_{22}\end{bmatrix}^T$$

$$\operatorname{Adj} A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}^T$$
$$=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$

Now, $$(\operatorname{adj}A).(A)=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$
$$=\begin{bmatrix}\cos^2\alpha-\sin^2\alpha & \cos\alpha\sin\alpha-\sin\alpha\cos\alpha\\ -\cos\alpha\sin\alpha+\sin\alpha\cos\alpha & -\sin^2\alpha+\cos^2\alpha\end{bmatrix}$$
$$=\begin{bmatrix}\cos 2\alpha & 0\\ 0 & \cos 2\alpha\end{bmatrix}$$

And, $$A(\operatorname{adj}A)=\begin{bmatrix}\cos\alpha & \sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & -\sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$
$$=\begin{bmatrix}\cos^2\alpha-\sin^2\alpha & -\cos\alpha\sin\alpha+\sin\alpha\cos\alpha\\ \sin\alpha\cos\alpha-\cos\alpha\sin\alpha & -\sin^2\alpha+\cos^2\alpha\end{bmatrix}$$
$$=\begin{bmatrix}\cos 2\alpha & 0\\ 0 & \cos 2\alpha\end{bmatrix}$$

Also,

$$|A|.I=\begin{vmatrix}\cos\alpha & \sin\alpha\\ \sin\alpha & \cos\alpha\end{vmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\left(\cos^2\alpha-\sin^2\alpha\right)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$=\begin{bmatrix}\cos^2\alpha-\sin^2\alpha & 0\\ 0 & \cos^2\alpha-\sin^2\alpha\end{bmatrix}$$

$$=\begin{bmatrix}\cos 2\alpha & 0\\ 0 & \cos 2\alpha\end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q1(iv)

We have,

$$A = \begin{bmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{bmatrix}$$

Cofactors of A are:

$c_{11} = 1, \quad c_{12} = -\left(-\tan \alpha/2\right) = \tan \alpha/2$

$c_{21} = -\tan \alpha/2, \quad c_{22} = 1$

$$\therefore \text{ adj } A = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -\tan \alpha/2 \\ \tan \alpha/2 & 1 \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{vmatrix}$$

$$= 1 + \tan^2 \alpha/2$$

$$= \sec^2 \alpha/2$$

We have,

$$A = \begin{bmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{bmatrix}$$

Cofactors of A are:

$c_{11} = 1, \quad c_{12} = -\left(-\tan \alpha/2\right) = \tan \alpha/2$

$c_{21} = -\tan \alpha/2, \quad c_{22} = 1$

$$\therefore \text{ adj } A = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -\tan \alpha/2 \\ \tan \alpha/2 & 1 \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{vmatrix}$$

$$= 1 + \tan^2 \alpha/2$$

$$= \sec^2 \alpha/2$$

*********** END **********