Differentiation Ex 11.1 Q5

Let $f(x) = e^{\sqrt{2x}}$

$\Rightarrow \quad f(x+h) = e^{\sqrt{2(x+h)}}$

$$\frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{e^{\sqrt{2(x+h)}} - e^{\sqrt{2x}}}{h}$$

$$= \lim_{h\to 0} e^{\sqrt{2x}} \frac{e^{\sqrt{2(x+h)}-\sqrt{2x}} - 1}{h}$$

$$= e^{\sqrt{2x}} \lim_{h\to 0} \left(\frac{\left(e^{\sqrt{2(x+h)}-\sqrt{2x}} - 1\right)}{\sqrt{2(x+h)} - \sqrt{2x}}\right)\left(\frac{\sqrt{2(x+h)} - \sqrt{2x}}{h}\right)$$

$$= e^{\sqrt{2x}} \lim_{h\to 0} \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h} \qquad \left[\text{Since, } \lim_{h\to 0} \frac{e^h - 1}{h} = 1\right]$$

$$= e^{\sqrt{2x}} \lim_{h\to 0} \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h} \times \frac{\sqrt{2(x+h)} + \sqrt{2x}}{\sqrt{2(x+h)} + \sqrt{2x}} \qquad [\text{Rationalizing the numerator}]$$

$$= de^{\sqrt{2x}} \lim_{h\to 0} \frac{2(x+h) - 2x}{h\left(\sqrt{2(x+h)} + \sqrt{2x}\right)}$$

$$= e^{\sqrt{2x}} \lim_{h\to 0} \frac{2x + 2h - 2x}{h\left(\sqrt{2(x+h)} + \sqrt{2x}\right)}$$

$$= e^{\sqrt{2x}} \lim_{h\to 0} \frac{2h}{h\left(\sqrt{2(x+h)} + \sqrt{2x}\right)}$$

$$= e^{\sqrt{2x}} \lim_{h\to 0} \frac{2}{\left(\sqrt{2(x+h)} + \sqrt{2x}\right)}$$

$$= \frac{e^{\sqrt{2x}}}{\sqrt{2x}}$$

So,

$$\frac{d}{dx}\left(e^{\sqrt{2x}}\right) = \frac{e^{\sqrt{2x}}}{\sqrt{2x}}$$

Differentiation Ex 11.1 Q6

Let $\quad f(x) = \log\cos x$

$\Rightarrow \quad f(x+h) = \log\cos(x+h)$

$$\therefore \quad \frac{d}{dx}\left(f(x)\right) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\log\cos(x+h) - \log\cos x}{h}$$

$$= \lim_{h\to 0} \frac{\log\frac{\cos(x+h)}{\cos x}}{h} \qquad \left[\text{Since, } \log A - \log B = \log\frac{A}{B}\right]$$

$$= \lim_{h\to 0} \frac{\log\left\{1 + \frac{\cos(x+h)}{\cos x} - 1\right\}}{h}$$

$$= \lim_{h\to 0} \frac{\log\left\{1 + \frac{\cos(x+h)}{\cos x}\right\}}{\left(\frac{\cos(x+h)}{\cos x}\right) h \times \left(\frac{\cos x}{\cos(x+h) - \cos x}\right)}$$

$$= \lim_{h\to 0} \frac{\cos(x+h) - \cos x}{\cos x \times h} \qquad \left[\text{Since, } \lim_{x\to 0} \frac{\log(1+x)}{x} = 1\right]$$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)}{\cos x \times h}$$

$$= -2\lim_{h\to 0} \frac{\sin\left(\frac{2x+h}{2}\right) \times \left(\sin\frac{h}{2}\right)}{2\cos x\left(\frac{h}{2}\right)}$$

$$= \frac{-2\sin x}{2\cos x} \qquad \left[\text{Since, } \lim_{x\to 0} \frac{\sin x}{x} = 1\right]$$

$$= -\tan x$$

So,

$$\frac{d}{dx}(\log\cos x) = -\tan x$$

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