Exercise 7.11 : Solutions of Questions on Page Number : **347**

Q1 : $\int_0^{\frac{\pi}{2}} \cos^2 x\,dx$

Answer :

$$I = \int_0^{\frac{\pi}{2}} \cos^2 x\,dx \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \cos^2\left(\frac{\pi}{2} - x\right)dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2 x dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \left(\sin^2 x + \cos^2 x\right)dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1\cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

Answer needs Correction? **Click Here**

Q2 : $\int_0^{\frac{\pi}{2}} \cos^2 x\,dx$

Answer :

$$I = \int_0^{\frac{\pi}{2}} \cos^2 x\,dx \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \cos^2\left(\frac{\pi}{2} - x\right)dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2 x dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \left(\sin^2 x + \cos^2 x\right)dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1\cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

Answer needs Correction? **Click Here**

Q3 : $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}dx$

Answer :

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}dx$$

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}dx \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}}dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos}}{\sqrt{\cos} + \sqrt{\sin x}}dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

Answer needs Correction? **Click Here**

Q4 : $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

Answer :

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}} dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos}}{\sqrt{\cos} + \sqrt{\sin x}} dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

Answer needs Correction? **Click Here**

Q5 : $\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x dx}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}$

Answer :

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right) + \cos^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)} dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

Answer needs Correction? **Click Here**

Q6 : $\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x dx}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}$

Answer :

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right) + \cos^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)} dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \qquad \ldots(2)$$

$\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

Answer needs Correction? **Click Here**

Q7 : $\int_0^{\frac{\pi}{2}} \frac{\cos^5 x dx}{\sin^5 x + \cos^5 x}$

Answer :

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x + \cos^5 x} dx \quad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^5\left(\frac{\pi}{2} - x\right)}{\sin^5\left(\frac{\pi}{2} - x\right) + \cos^5\left(\frac{\pi}{2} - x\right)} dx \quad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\sin^5 x + \cos^5 x} dx \quad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x + \cos^5 x}{\sin^5 x + \cos^5 x} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

Answer needs Correction? **Click Here**

Q8 : $\int_0^{\frac{\pi}{2}} \frac{\cos^5 x dx}{\sin^5 x + \cos^5 x}$

Answer :

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x + \cos^5 x} dx \quad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^5\left(\frac{\pi}{2} - x\right)}{\sin^5\left(\frac{\pi}{2} - x\right) + \cos^5\left(\frac{\pi}{2} - x\right)} dx \quad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\sin^5 x + \cos^5 x} dx \quad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x + \cos^5 x}{\sin^5 x + \cos^5 x} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

Answer needs Correction? **Click Here**

Q9 : $\int_{-5}^{5} |x+2| dx$

Answer :

$$\text{Let } I = \int_{-5}^{5} |x+2| dx$$

It can be seen that $(x + 2) \leq 0$ on [- 5, - 2] and $(x + 2) \geq 0$ on [- 2, 5].

$$\therefore I = \int_{-5}^{-2} -(x+2)dx + \int_{-2}^{5} (x+2)dx \quad \left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x)\right)$$

$$I = -\left[\frac{x^2}{2} + 2x\right]_{-5}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^{5}$$

$$= -\left[\frac{(-2)^2}{2} + 2(-2) - \frac{(-5)^2}{2} - 2(-5)\right] + \left[\frac{(5)^2}{2} + 2(5) - \frac{(-2)^2}{2} - 2(-2)\right]$$

$$= -\left[2-4-\frac{25}{2}+10\right]+\left[\frac{25}{2}+10-2+4\right]$$
$$= -2+4+\frac{25}{2}-10+\frac{25}{2}+10-2+4$$
$$= 29$$

Answer needs Correction? Click Here

Q10 : $\int_{-5}^{5}|x+2|\,dx$

Answer :

Let $I = \int_{-5}^{5}|x+2|\,dx$

It can be seen that $(x + 2) \le 0$ on [- 5, - 2] and $(x + 2) \ge 0$ on [- 2, 5].

$$\therefore I = \int_{-5}^{-2} -(x+2)\,dx + \int_{-2}^{5}(x+2)\,dx \qquad \left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x)\right)$$
$$I = -\left[\frac{x^2}{2}+2x\right]_{-5}^{-2} + \left[\frac{x^2}{2}+2x\right]_{-2}^{5}$$
$$= -\left[\frac{(-2)^2}{2}+2(-2)-\frac{(-5)^2}{2}-2(-5)\right]+\left[\frac{(5)^2}{2}+2(5)-\frac{(-2)^2}{2}-2(-2)\right]$$
$$= -\left[2-4-\frac{25}{2}+10\right]+\left[\frac{25}{2}+10-2+4\right]$$
$$= -2+4+\frac{25}{2}-10+\frac{25}{2}+10-2+4$$
$$= 29$$

Answer needs Correction? Click Here

Q11 : $\int_{2}^{8}|x-5|\,dx$

Answer :

Let $I = \int_{2}^{8}|x-5|\,dx$

It can be seen that $(x - 5) \le 0$ on [2, 5] and $(x - 5) \ge 0$ on [5, 8].

$$I = \int_{2}^{5} -(x-5)\,dx + \int_{5}^{8}(x-5)\,dx \qquad \left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x)\right)$$
$$= -\left[\frac{x^2}{2}-5x\right]_{2}^{5} + \left[\frac{x^2}{2}-5x\right]_{5}^{8}$$
$$= -\left[\frac{25}{2}-25-2+10\right]+\left[32-40-\frac{25}{2}+25\right]$$
$$= 9$$

Answer needs Correction? Click Here

Q12 : $\int_0^1 x(1-x)^n\,dx$

Answer :

Let $I = \int_0^1 x(1-x)^n\,dx$

$$\therefore I = \int_0^1 (1-x)(1-(1-x))^n\,dx$$
$$= \int_0^1 (1-x)(x)^n\,dx$$
$$= \int_0^1 \left(x^n - x^{n+1}\right)dx$$
$$= \left[\frac{x^{n+1}}{n+1}-\frac{x^{n+2}}{n+2}\right]_0^1 \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$
$$= \left[\frac{1}{n+1}-\frac{1}{n+2}\right]$$
$$= \frac{(n+2)-(n+1)}{(n+1)(n+2)}$$
$$= \frac{1}{(n+1)(n+2)}$$

Answer needs Correction? Click Here

Q13 : $\int_0^1 x(1-x)^n\,dx$

Answer :

Let $I = \int_0^1 x(1-x)^n\,dx$

$$\therefore I = \int_0^1 (1-x)(1-(1-x))^n\,dx$$
$$= \int_0^1 (1-x)(x)^n\,dx$$
$$= \int_0^1 \left(x^n - x^{n+1}\right)dx$$
$$= \left[\frac{x^{n+1}}{n+1}-\frac{x^{n+2}}{n+2}\right]_0^1 \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$=\left[\frac{1}{n+1}-\frac{1}{n+2}\right]$$
$$=\frac{(n+2)-(n+1)}{(n+1)(n+2)}$$
$$=\frac{1}{(n+1)(n+2)}$$

Answer needs Correction? Click Here

Q14 : $\int_0^{\frac{\pi}{4}} \log\,(1+\tan x)dx$

Answer :

Let $I=\int_0^{\frac{\pi}{4}} \log\,(1+\tan x)dx$ ...(1)

$\therefore I=\int_0^{\frac{\pi}{4}} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx$ $\left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$

$$\Rightarrow I=\int_0^{\frac{\pi}{4}} \log\left\{1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right\}dx$$
$$\Rightarrow I=\int_0^{\frac{\pi}{4}} \log\left\{1+\frac{1-\tan x}{1+\tan}\right\}dx$$
$$\Rightarrow I=\int_0^{\frac{\pi}{4}} \log\frac{2}{(1+\tan x)}dx$$
$$\Rightarrow I=\int_0^{\frac{\pi}{4}} \log 2\,dx-\int_0^{\frac{\pi}{4}} \log\,(1+\tan x)dx$$

$\Rightarrow I=\int_0^{\frac{\pi}{4}} \log 2dx-I$ $[\text{From (1)}]$

$$\Rightarrow 2I=[x\log 2]_0^{\frac{\pi}{4}}$$
$$\Rightarrow 2I=\frac{\pi}{4}\log 2$$
$$\Rightarrow I=\frac{\pi}{8}\log 2$$

Answer needs Correction? Click Here

Q15 : $\int_0^{\frac{\pi}{4}} \log\,(1+\tan x)dx$

Answer :

Let $I=\int_0^{\frac{\pi}{4}} \log\,(1+\tan x)dx$ ...(1)

$\therefore I=\int_0^{\frac{\pi}{4}} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx$ $\left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$

$$\Rightarrow I=\int_0^{\frac{\pi}{4}} \log\left\{1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right\}dx$$
$$\Rightarrow I=\int_0^{\frac{\pi}{4}} \log\left\{1+\frac{1-\tan x}{1+\tan}\right\}dx$$
$$\Rightarrow I=\int_0^{\frac{\pi}{4}} \log\frac{2}{(1+\tan x)}dx$$
$$\Rightarrow I=\int_0^{\frac{\pi}{4}} \log 2\,dx-\int_0^{\frac{\pi}{4}} \log\,(1+\tan x)dx$$

$\Rightarrow I=\int_0^{\frac{\pi}{4}} \log 2dx-I$ $[\text{From (1)}]$

$$\Rightarrow 2I=[x\log 2]_0^{\frac{\pi}{4}}$$
$$\Rightarrow 2I=\frac{\pi}{4}\log 2$$
$$\Rightarrow I=\frac{\pi}{8}\log 2$$

Answer needs Correction? Click Here

Q16 : $\int_0^2 x\sqrt{2-x}dx$

Answer :

Let $I=\int_0^2 x\sqrt{2-x}dx$

$I=\int_0^2 (2-x)\sqrt{x}dx$ $\left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$

$$=\int_0^2\left\{2x^{\frac{1}{2}}-x^{\frac{3}{2}}\right\}dx$$
$$=\left[2\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)-\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^2$$
$$=\left[\frac{4}{3}x^{\frac{3}{2}}-\frac{2}{5}x^{\frac{5}{2}}\right]_0^2$$
$$=\frac{4}{3}(2)^{\frac{3}{2}}-\frac{2}{5}(2)^{\frac{5}{2}}$$

$=\frac{4}{3}(2)^{\frac{3}{2}}-\frac{2}{5}(2)^{\frac{5}{2}}$

$=\frac{4\times 2\sqrt{2}}{3}-\frac{2}{5}\times 4\sqrt{2}$

$=\frac{8\sqrt{2}}{3}-\frac{8\sqrt{2}}{5}$

$=\frac{40\sqrt{2}-24\sqrt{2}}{15}$

$=\frac{16\sqrt{2}}{15}$

Answer needs Correction? Click Here

Q17 : $\int_0^2 x\sqrt{2-x}dx$

Answer :

Let $I=\int_0^2 x\sqrt{2-x}dx$

$I=\int_0^2 (2-x)\sqrt{x}dx \qquad \left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$

$=\int_0^2\left\{2x^{\frac{1}{2}}-x^{\frac{3}{2}}\right\}dx$

$=\left[2\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)-\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^2$

$=\left[\frac{4}{3}x^{\frac{3}{2}}-\frac{2}{5}x^{\frac{5}{2}}\right]_0^2$

$=\frac{4}{3}(2)^{\frac{3}{2}}-\frac{2}{5}(2)^{\frac{5}{2}}$

$=\frac{4\times 2\sqrt{2}}{3}-\frac{2}{5}\times 4\sqrt{2}$

$=\frac{8\sqrt{2}}{3}-\frac{8\sqrt{2}}{5}$

$=\frac{40\sqrt{2}-24\sqrt{2}}{15}$

$=\frac{16\sqrt{2}}{15}$

Answer needs Correction? Click Here

Q18 : $\int_0^{\frac{\pi}{2}}(2\log\sin x-\log\sin 2x)dx$

Answer :

Let $I=\int_0^{\frac{\pi}{2}}(2\log\sin x-\log\sin 2x)dx$

$\Rightarrow I=\int_0^{\frac{\pi}{2}}\{2\log\sin x-\log(2\sin x\cos x)\}dx$

$\Rightarrow I=\int_0^{\frac{\pi}{2}}\{2\log\sin x-\log\sin x-\log\cos x-\log 2\}dx$

$\Rightarrow I=\int_0^{\frac{\pi}{2}}\{\log\sin x-\log\cos x-\log 2\}dx \qquad ...(1)$

It is known that, $\left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$

$\Rightarrow I=\int_0^{\frac{\pi}{2}}\{\log\cos x-\log\sin x-\log 2\}dx \qquad ...(2)$

Adding (1) and (2), we obtain

$2I=\int_0^{\frac{\pi}{2}}(-\log 2-\log 2)dx$

$\Rightarrow 2I=-2\log 2\int_0^{\frac{\pi}{2}}1\,dx$

$\Rightarrow I=-\log 2\left[\frac{\pi}{2}\right]$

$\Rightarrow I=\frac{\pi}{2}(-\log 2)$

$\Rightarrow I=\frac{\pi}{2}\left[\log\frac{1}{2}\right]$

$\Rightarrow I=\frac{\pi}{2}\log\frac{1}{2}$

Answer needs Correction? Click Here

Q19 : $\int_0^{\frac{\pi}{2}}(2\log\sin x-\log\sin 2x)dx$

Answer :

Let $I=\int_0^{\frac{\pi}{2}}(2\log\sin x-\log\sin 2x)dx$

$\Rightarrow I=\int_0^{\frac{\pi}{2}}\{2\log\sin x-\log(2\sin x\cos x)\}dx$

$\Rightarrow I=\int_0^{\frac{\pi}{2}}\{2\log\sin x-\log\sin x-\log\cos x-\log 2\}dx$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \{\log \sin x - \log \cos x - \log 2\}\, dx \qquad ...(1)$$

It is known that, $\left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \{\log \cos x - \log \sin x - \log 2\}\, dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} (-\log 2 - \log 2)\, dx$$

$$\Rightarrow 2I = -2\log 2 \int_0^{\frac{\pi}{2}} 1\, dx$$

$$\Rightarrow I = -\log 2\left[\frac{\pi}{2}\right]$$

$$\Rightarrow I = \frac{\pi}{2}(-\log 2)$$

$$\Rightarrow I = \frac{\pi}{2}\left[\log \frac{1}{2}\right]$$

$$\Rightarrow I = \frac{\pi}{2}\log \frac{1}{2}$$

Answer needs Correction? **Click Here**

Q20 : $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x\, dx$

Answer :

Let $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x\, dx$

As $\sin^2(-x) = (\sin(-x))^2 = (-\sin x)^2 = \sin^2 x$, therefore, $\sin^2 x$ is an even function.

It is known that if $f(x)$ is an even function, then $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$

$$I = 2\int_0^{\frac{\pi}{2}} \sin^2 x\, dx$$

$$= 2\int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{2}\, dx$$

$$= \int_0^{\frac{\pi}{2}} (1-\cos 2x)\, dx$$

$$= \left[x - \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2}$$

*********** END **********