



Functions Ex 2.5 Q4

Given that

$$A = \{1, 2, 3, 4\}, \quad B = \{3, 5, 7, 9\}, \quad C = \{7, 23, 47, 79\}$$

$f: A \rightarrow B$ and $g: B \rightarrow C$ are two functions defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2$

Now,

$$g \circ f(x) = g(f(x)) = g(2x + 1) = (2x + 1)^2 - 2$$

$$\Rightarrow g \circ f(x) = 4x^2 + 4x - 1$$

Now,

$$f: A \rightarrow B \text{ given by } f(x) = 2x + 1$$

Clearly f is one-one and onto, $\therefore f$ is bijective

$$\Rightarrow f^{-1} \text{ exists}$$

$$\therefore f^{-1} = \{(3, 1), (5, 2), (7, 3), (9, 4)\}$$

Again, $g: B \rightarrow C$ given by $g(x) = x^2 - 2$

Clearly g is one-one and onto $\Rightarrow g^{-1}$ exists

$$g^{-1} = \{(7, 3), (23, 5), (47, 7), (79, 9)\}$$

$$f \circ g^{-1} = \{(7, 1), (23, 2), (47, 3), (79, 4)\} \dots\dots\dots (A)$$

$$\text{Now, } g \circ f(x) = 4x^2 + 4x - 1$$

Clearly $g \circ f$ is one-one and onto $\Rightarrow (g \circ f)^{-1}$ exists.

Hence,

$$(g \circ f)^{-1} = \{(7, 1), (23, 2), (47, 3), (79, 4)\} \dots\dots\dots (B)$$

$$\text{From (A) \& (B) we have } g \circ f^{-1} = f \circ g^{-1}$$

Functions Ex 2.5 Q5

Given that $f: Q \rightarrow Q$ defined by $f(x) = 3x + 5$.

To prove that f is invertible, we need to prove that f is one - one and onto.

Let $(x, y) \in Q$ be such that, $f(x) = f(y)$

$$\Rightarrow 3x + 5 = 3y + 5$$

$$\Rightarrow x = y$$

So, f is an injection.

Let y be an arbitrary element of Q such that $f(x) = y$.

$$\Rightarrow 3x + 5 = y$$

$$\Rightarrow 3x = y - 5$$

$$\Rightarrow x = \frac{y-5}{3}$$

Thus, for any $y \in Q$ there exists $x = \frac{y-5}{3} \in Q$ such that

$$f(x) = f\left(\frac{y-5}{3}\right) = 3 \frac{y-5}{3} + 5 = y$$

Thus, $f: Q \rightarrow Q$ is a bijection and hence invertible.

Let f^{-1} denotes the inverse of f .

Thus, $f \circ f^{-1}(x) = x$ for all $x \in Q$

$$\Rightarrow f[f^{-1}(x)] = x \text{ for all } x \in Q.$$

$$\Rightarrow 3f^{-1}(x) + 5 = x \text{ for all } x \in Q.$$

$$\Rightarrow f^{-1}(x) = \frac{x-5}{3} \text{ for all } x \in Q$$

Functions Ex 2.5 Q6

$f: \mathbf{R} \rightarrow \mathbf{R}$ is given by, $f(x) = 4x + 3$

One-one:

Let $f(x) = f(y)$.

$$\Rightarrow 4x + 3 = 4y + 3$$

$$\Rightarrow 4x = 4y$$

$$\Rightarrow x = y$$

Therefore f is a one-one function.

Onto:

For $y \in \mathbf{R}$, let $y = 4x + 3$.

$$\Rightarrow x = \frac{y-3}{4} \in \mathbf{R}$$

Therefore, for any $y \in \mathbf{R}$, there exists $x = \frac{y-3}{4} \in \mathbf{R}$ such that

$$f(x) = f\left(\frac{y-3}{4}\right) = 4\left(\frac{y-3}{4}\right) + 3 = y.$$

Therefore, f is onto.

Thus, f is one-one and onto and therefore, f^{-1} exists.

Let us define $g: \mathbf{R} \rightarrow \mathbf{R}$ by $g(x) = \frac{x-3}{4}$

$$\text{Now, } (g \circ f)(x) = g(f(x)) = g(4x+3) = \frac{(4x+3)-3}{4} = x$$

$$(f \circ g)(y) = f(g(y)) = f\left(\frac{y-3}{4}\right) = 4\left(\frac{y-3}{4}\right) + 3 = y - 3 + 3 = y$$

Therefore, $g \circ f = f \circ g = I_{\mathbf{R}}$

Hence, f is invertible and the inverse of f is given by

$$f^{-1}(y) = g(y) = \frac{y-3}{4}.$$

Functions Ex 2.5 Q7

$f: \mathbf{R}_+ \rightarrow [4, \infty)$ is given as $f(x) = x^2 + 4$.

One-one:

Let $f(x) = f(y)$.

$$\Rightarrow x^2 + 4 = y^2 + 4$$

$$\Rightarrow x^2 = y^2$$

$$\Rightarrow x = y \quad \left[\text{as } x = y \in \mathbf{R}_+ \right]$$

Therefore, f is a one-one function.

Onto:

For $y \in [4, \infty)$, let $y = x^2 + 4$.

$$\Rightarrow x^2 = y - 4 \geq 0 \quad \left[\text{as } y \geq 4 \right]$$

$$\Rightarrow x = \sqrt{y-4} \geq 0$$

Therefore, for any $y \in \mathbf{R}$, there exists $x = \sqrt{y-4} \in \mathbf{R}$ such that

$$f(x) = f(\sqrt{y-4}) = (\sqrt{y-4})^2 + 4 = y - 4 + 4 = y.$$

Therefore, f is onto.

Thus, f is one-one and onto and therefore, f^{-1} exists.

Let us define $g: [4, \infty) \rightarrow \mathbf{R}_+$ by,

$$g(y) = \sqrt{y-4}$$

$$\text{Now, } g \circ f(x) = g(f(x)) = g(x^2 + 4) = \sqrt{(x^2 + 4) - 4} = \sqrt{x^2} = x$$

$$\text{And, } f \circ g(y) = f(g(y)) = f(\sqrt{y-4}) = (\sqrt{y-4})^2 + 4 = (y-4) + 4 = y$$

Therefore, $g \circ f = \text{fog} = I_{\mathbf{R}}$

Hence, f is invertible and the inverse of f is given by

$$f^{-1}(y) = g(y) = \sqrt{y-4}.$$

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