Co-Ordinate Geometry Ex 14.2 Q21

Answer :

The distance *d* between two points (x_1, y_1) and (x_2, y_2) is given by the formula

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

The circumcentre of a triangle is the point which is equidistant from each of the three vertices of the triangle.

Here the three vertices of the triangle are given to be *A*(−2.−3), *B*(−1,0) and *C*(7,−6).

Let the circumcentre of the triangle be represented by the point *R*(*x*, *y*).

So we have $AR = BR = CR$

$AR = \sqrt{(-2-x)^2 + (-3-y)^2}$

$BR = \sqrt{(-1-x)^2 + (-y)^2}$

$CR = \sqrt{(7-x)^2 + (-6-y)^2}$

Equating the first pair of these equations we have,

$$AR = BR$$

$$\sqrt{(-2-x)^2 + (-3-y)^2} = \sqrt{(-1-x)^2 + (-y)^2}$$

Squaring on both sides of the equation we have,

$$\begin{aligned}
(-2-x)^2 + (-3-y)^2 &= (-1-x)^2 + (-y)^2 \\
4 + x^2 + 4x + 9 + y^2 + 6y &= 1 + x^2 + 2x + y^2 \\
2x + 6y &= -12 \\
x + 3y &= -6
\end{aligned}$$

Equating another pair of the equations we have,

$$AR = CR$$

$$\sqrt{(-2-x)^2 + (-3-y)^2} = \sqrt{(7-x)^2 + (-6-y)^2}$$

Squaring on both sides of the equation we have,

$$\begin{aligned}
(-2-x)^2 + (-3-y)^2 &= (7-x)^2 + (-6-y)^2 \\
4 + x^2 + 4x + 9 + y^2 + 6y &= 49 + x^2 - 14x + 36 + y^2 + 12y \\
18x - 6y &= 72 \\
3x - y &= 12
\end{aligned}$$

Now we have two equations for '*x*' and '*y*', which are

$x + 3y = -6$

$3x - y = 12$

From the second equation we have $y = 3x - 12$. Substituting this value of '*y*' in the first equation we have,

$$\begin{aligned}
x + 3(3x - 12) &= -6 \\
x + 9x - 36 &= -6 \\
10x &= 30 \\
x &= 3
\end{aligned}$$

Therefore the value of '*y*' is,

$$\begin{aligned}
y &= 3x - 12 \\
&= 9 - 12 \\
y &= -3
\end{aligned}$$

Hence the co-ordinates of the circumcentre of the triangle with the given vertices are $\boxed{(3,-3)}$.

Co-Ordinate Geometry Ex 14.2 Q22

Answer :

The distance *d* between two points (x_1, y_1) and (x_2, y_2) is given by the formula

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

In a right angled triangle the angle opposite the hypotenuse subtends an angle of 90°.

Here let the given points be *A*(0,100), *B*(10,0). Let the origin be denoted by *O*(0,0).

Let us find the distance between all the pairs of points

$$\begin{aligned}
AB &= \sqrt{(0-10)^2 + (100-0)^2} \\
&= \sqrt{(-10)^2 + (100)^2} \\
&= \sqrt{100 + 10000} \\
AB &= \sqrt{10100} \\
AO &= \sqrt{(0-0)^2 + (100-0)^2} \\
&= \sqrt{(0)^2 + (100)^2} \\
AO &= \sqrt{10000} \\
BO &= \sqrt{(10-0)^2 + (0-0)^2} \\
&= \sqrt{(10)^2 + (0)^2} \\
BO &= \sqrt{100}
\end{aligned}$$

Here we can see that $AB^2 = AO^2 + BO^2$.

So, ΔAOB is a right angled triangle with '*AB*' being the hypotenuse. So the angle opposite it has to be 90°. This angle is nothing but the angle subtended by the line segment '*AB*' at the origin.

Hence the angle subtended at the origin by the given line segment is $\boxed{90^\circ}$.

Co-Ordinate Geometry Ex 14.2 Q23

Answer :

The distance *d* between two points (x_1, y_1) and (x_2, y_2) is given by the formula

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

The centre of a circle is at equal distance from all the points on its circumference.

Here it is given that the circle passes through the points *A*(5,−8), *B*(2,−9) and *C*(2,1).

Let the centre of the circle be represented by the point *O*(*x*, *y*).

So we have $AO = BO = CO$

$$AO = \sqrt{(5-x)^2 + (-8-y)^2}$$
$$BO = \sqrt{(2-x)^2 + (-9-y)^2}$$
$$CO = \sqrt{(2-x)^2 + (1-y)^2}$$

Equating the first pair of these equations we have,

$$AO = BO$$
$$\sqrt{(5-x)^2 + (-8-y)^2} = \sqrt{(2-x)^2 + (-9-y)^2}$$

Squaring on both sides of the equation we have,

$$\begin{aligned}
(5-x)^2 + (-8-y)^2 &= (2-x)^2 + (-9-y)^2 \\
25 + x^2 - 10x + 64 + y^2 + 16y &= 4 + x^2 - 4x + 81 + y^2 + 18y \\
6x + 2y &= 4 \\
3x + y &= 2
\end{aligned}$$

Equating another pair of the equations we have,

$$AO = CO$$
$$\sqrt{(5-x)^2 + (-8-y)^2} = \sqrt{(2-x)^2 + (1-y)^2}$$

Squaring on both sides of the equation we have,

$$\begin{aligned}
(5-x)^2 + (-8-y)^2 &= (2-x)^2 + (1-y)^2 \\
25 + x^2 - 10x + 64 + y^2 + 16y &= 4 + x^2 - 4x + 1 + y^2 - 2y \\
6x - 18y &= 84 \\
x - 3y &= 14
\end{aligned}$$

Now we have two equations for '*x*' and '*y*', which are

$$3x + y = 2$$
$$x - 3y = 14$$

From the second equation we have $y = -3x + 2$. Substituting this value of '*y*' in the first equation we have,

$$\begin{aligned}
x - 3(-3x + 2) &= 14 \\
x + 9x - 6 &= 14 \\
10x &= 20 \\
x &= 2
\end{aligned}$$

Therefore the value of '*y*' is,

$$\begin{aligned}
y &= -3x + 2 \\
&= -3(2) + 2 \\
y &= -4
\end{aligned}$$

Hence the co-ordinates of the centre of the circle are $\boxed{(2, -4)}$.

*********** END **********