Complex Numbers Ex 13.4 Q1(vii)

The polar form of a complex number $z = x + iy$, is given by $z = |z|(\cos\theta + i\sin\theta)$ where,

$|z| = \sqrt{x^2 + y^2}$ and

$\arg(z) = \theta = \tan^{-1}\left(\frac{b}{a}\right)$

let $z = \sin 120^\circ - i\cos 120^\circ$

$$= \sin\left(\frac{\pi}{2} + \frac{\pi}{6}\right) - i\cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right) \qquad \left(\because 120^\circ = \frac{\pi}{2} + \frac{\pi}{6}\right)$$

$$\Rightarrow z = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} \qquad \left(\because \sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta \;\&\; \cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta\right)$$

Here z is already in polar form

with $|z| = 1$ & $\theta = \arg(z) = \frac{\pi}{6}$

Complex Numbers Ex 13.4 Q1(viii)

The polar form of a complex number $z = x + iy$, is given by $z = |z|(\cos\theta + i\sin\theta)$
where,
$|z| = \sqrt{x^2 + y^2}$ and
$\arg(z) = \theta = \tan^{-1}\left(\frac{b}{a}\right)$

let $z = \frac{-16}{1+i\sqrt{3}}$

$$= \frac{-16}{1+i\sqrt{3}} \times \frac{1-i\sqrt{3}}{1-i\sqrt{3}}$$
$$= \frac{-16\left(1-i\sqrt{3}\right)}{(1)^2 + \left(\sqrt{3}\right)^2}$$
$$= \frac{-16\left(1-i\sqrt{3}\right)}{1+3}$$
$$= \frac{-16}{4}\left(1-i\sqrt{3}\right)$$
$$= -4\left(1-i\sqrt{3}\right)$$
$$= -4 + 4\sqrt{3}i$$

$$\therefore |z| = \sqrt{(-4)^2 + \left(4\sqrt{3}\right)^2}$$
$$= \sqrt{16 + 48}$$
$$= \sqrt{64}$$
$$= 8$$

Here $x = -4 < 0$ & $y = 4R3 > 0, \therefore \theta$ lies in quadrant II

$$\theta = \arg(z) = \tan^{-1}\left(\frac{4\sqrt{3}}{-4}\right)$$
$$= \tan^{-1}\left(-\sqrt{3}\right)$$
$$= \tan^{-1}\left(-\tan\frac{\pi}{3}\right)$$
$$= \tan^{-1}\left(\tan\left(\pi - \frac{\pi}{3}\right)\right) \qquad \left(\because \tan(\pi-\theta) = -\tan\theta\right)$$
$$= \pi - \frac{\pi}{3}$$
$$= \frac{2\pi}{3}$$

The polar form is given by $z = 8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

Complex Numbers Ex 13.4 Q2

$$z = \left(i^{25}\right)^3 = (i)^3 = -i$$

$$|z| = 1,$$

$$\arg(z) = \tan^{-1}\left(\frac{-1}{0}\right) = -\frac{\pi}{2}$$

$$\text{Polar Form: } \cos\left(\frac{\pi}{2}\right) - i\sin\left(\frac{\pi}{2}\right) = \cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)$$

Complex Numbers Ex 13.4 Q3(i)

Let $z = 1 + i\tan\alpha$

$\tan\alpha$ is periodic function with period π.

So, let us take α lying in the interval $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$.

Case – I : When $\alpha \in \left[0, \frac{\pi}{2}\right)$

$|z| = \sqrt{1 + \tan^2\alpha} = \sqrt{\sec^2\alpha} = |\sec\alpha| = \sec\alpha$

Let β be acute angle given by $\tan\beta = \frac{|\mathrm{Im}(z)|}{|\mathrm{Re}(z)|}$.

$\tan\beta = |\tan\alpha| = \tan\alpha$

$\Rightarrow \beta = \alpha$

As z is represented by a point in first quadrant.

$\therefore \arg(z) = \beta = \alpha$.

So polar form of z is $\sec\alpha(\cos\alpha + i\sin\alpha)$

Case – II : When $\alpha \in \left(\frac{\pi}{2}, \pi\right]$

$|z| = \sqrt{1 + \tan^2\alpha} = \sqrt{\sec^2\alpha} = |\sec\alpha| = -\sec\alpha$

Let β be acute angle given by $\tan\beta = \frac{|\mathrm{Im}(z)|}{|\mathrm{Re}(z)|}$.

$\tan\beta = |\tan\alpha| = -\tan\alpha = \tan(\pi - \alpha)$

$\Rightarrow \beta = \pi - \alpha$

As z is represented by a point in fourth quadrant.

$\therefore \arg(z) = -\beta = \alpha - \pi$.

So polar form of z is $-\sec\alpha\left(\cos(\alpha - \pi) + i\sin(\alpha - \pi)\right)$.

*********** END **********