Algebra of Matrices Ex 5.3 Q66

Given,

A and B are square matices of same order

$$\begin{aligned}(A+B)^2 &= (A+B)(A+B)\\ &= A(A+B)+B(A+B) && \{\text{using distributive property}\}\\ &= A\times A+AB+BA+B^2\\ &= A^2+AB+BA+B^2\end{aligned}$$

But,

$(A+B)^2 = A^2+2AB+B^2$ is possible only when $AB = BA$

Here, we can not say that $AB = BA$

So,

$(A+B)^2 = A^2+2AB+B^2$ does not hold.

Algebra of Matrices Ex 5.3 Q67

Given, A and B are square matrices of same order.

(i) $$\begin{aligned}(A+B)^2 &= (A+B)(A+B)\\ &= A(A+B)+B(A+B) && \{\text{using distributive property}\}\\ &= A\times A+AB+BA+B\times B\\ &= A^2+AB+BA+B^2\\ &\neq A^2+2AB+B^2\end{aligned}$$

Since, in general matix multiplication is not commutative $(AB \neq BA)$

So, $(A+B)^2 \neq A^2+2AB+B^2$

(ii) $$\begin{aligned}(A-B)^2 &= (A-B)(A-B)\\ &= A(A-B)-B(A-B) && \{\text{using distributive property}\}.\\ &= A\times A-AB-BA+B\times B\\ &= A^2-AB-BA+B^2\\ &\neq A^2-2AB+B^2\end{aligned}$$

Since, in general matrix multiplication is not commutative $(AB \neq BA)$, so

So, $(A-B)^2 \neq A^2-2AB+B^2$

(iii) $$\begin{aligned}(A+B)(A-B) &= A(A-B)+B(A-B) && \{\text{using distributive property}\}\\ &= A\times A-AB+BA-B\times B\\ &= A^2-AB+BA-B^2\\ &= A^2-B^2\end{aligned}$$

Since, in general matix multiplication is not commutative $(AB \neq BA)$,

So, $(A+B)(A-B) \neq A^2-B^2$

Algebra of Matrices Ex 5.3 Q68

The given equality is true only when we choose A and B to be a square matrix in such a way that AB = BA else the result is not true in general.

Example: Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\text{Here } AB = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times 0+0\times 2+0\times 0 & 1\times 1+0\times 1+0\times 0 & 1\times 0+0\times 0+0\times 1 \\ 1\times 0+1\times 2+0\times 0 & 1\times 1+1\times 1+0\times 0 & 1\times 0+1\times 0+0\times 1 \\ 0\times 0+0\times 2+1\times 0 & 0\times 1+0\times 1+1\times 0 & 0\times 0+0\times 0+1\times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{and } BA = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0\times 1+1\times 1+0\times 0 & 0\times 0+1\times 1+0\times 0 & 0\times 0+1\times 0+0\times 1 \\ 2\times 1+1\times 1+0\times 0 & 2\times 0+1\times 1+0\times 0 & 2\times 0+1\times 0+0\times 1 \\ 0\times 1+0\times 1+1\times 0 & 0\times 0+0\times 1+1\times 0 & 0\times 0+0\times 0+1\times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$AB \neq BA$$

$$\text{Now, } (AB)^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0\times 0+1\times 1+0\times 0 & 0\times 1+1\times 2+0\times 0 & 0\times 0+1\times 0+0\times 1 \\ 1\times 0+2\times 1+0\times 0 & 1\times 1+1\times 2+0\times 0 & 1\times 0+2\times 0+0\times 1 \\ 0\times 0+0\times 1+1\times 0 & 0\times 1+0\times 2+1\times 0 & 0\times 0+0\times 0+1\times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times 1+0\times 1+0\times 0 & 1\times 0+0\times 1+0\times 0 & 1\times 0+0\times 0+0\times 1 \\ 1\times 1+1\times 1+0\times 0 & 1\times 0+1\times 1+0\times 0 & 1\times 0+1\times 0+0\times 1 \\ 0\times 1+0\times 1+1\times 0 & 0\times 0+0\times 1+1\times 0 & 0\times 0+0\times 0+1\times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0\times 0+1\times 2+0\times 0 & 0\times 1+1\times 1+0\times 0 & 0\times 0+1\times 0+0\times 1 \\ 2\times 0+1\times 2+0\times 0 & 2\times 1+1\times 1+0\times 0 & 2\times 0+1\times 0+0\times 1 \\ 0\times 0+0\times 2+1\times 0 & 0\times 1+0\times 1+1\times 0 & 0\times 0+0\times 0+1\times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^2B^2 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1+0\times1+0\times0 & 1\times1+0\times2+0\times0 & 1\times0+0\times0+0\times1 \\ 2\times1+1\times1+0\times0 & 2\times1+1\times2+0\times0 & 2\times0+1\times0+0\times1 \\ 0\times1+0\times1+1\times0 & 0\times1+0\times2+1\times0 & 0\times0+0\times0+1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We can see that if we have A and B two square matrices with $AB \neq BA$ then $(AB)^2 \neq A^2B^2$

*********** END **********