Exercise 7.1 : Solutions of Questions on Page Number : **299**

Q1 : sin $2x$

Answer :

The anti derivative of sin $2x$ is a function of x whose derivative is sin $2x$.

It is known that,

$$\frac{d}{dx}(\cos 2x) = -2\sin 2x$$
$$\Rightarrow \sin 2x = -\frac{1}{2}\frac{d}{dx}(\cos 2x)$$
$$\therefore \sin 2x = \frac{d}{dx}\left(-\frac{1}{2}\cos 2x\right)$$

Therefore, the anti derivative of $\sin 2x$ is $-\frac{1}{2}\cos 2x$

Q2 : Cos $3x$

Answer :

The anti derivative of cos $3x$ is a function of x whose derivative is cos $3x$.

It is known that,

$$\frac{d}{dx}(\sin 3x) = 3\cos 3x$$
$$\Rightarrow \cos 3x = \frac{1}{3}\frac{d}{dx}(\sin 3x)$$
$$\therefore \cos 3x = \frac{d}{dx}\left(\frac{1}{3}\sin 3x\right)$$

Therefore, the anti derivative of $\cos 3x$ is $\frac{1}{3}\sin 3x$.

Q3 : e^{2x}

Answer :

The anti derivative of e^{2x} is the function of x whose derivative is e^{2x}.

It is known that,

$$\frac{d}{dx}\left(e^{2x}\right) = 2e^{2x}$$
$$\Rightarrow e^{2x} = \frac{1}{2}\frac{d}{dx}\left(e^{2x}\right)$$
$$\therefore e^{2x} = \frac{d}{dx}\left(\frac{1}{2}e^{2x}\right)$$

Therefore, the anti derivative of e^{2x} is $\frac{1}{2}e^{2x}$.

Q4 : $(ax+b)^2$

Answer :

The anti derivative of $(ax+b)^2$ is the function of x whose derivative is $(ax+b)^2$.

It is known that,

$$\frac{d}{dx}(ax+b)^3 = 3a(ax+b)^2$$
$$\Rightarrow (ax+b)^2 = \frac{1}{3a}\frac{d}{dx}(ax+b)^3$$
$$\therefore (ax+b)^2 = \frac{d}{dx}\left(\frac{1}{3a}(ax+b)^3\right)$$

Therefore, the anti derivative of $(ax+b)^2$ is $\frac{1}{3a}(ax+b)^3$.

Q5 : $\sin 2x-4e^{3x}$

Answer :

The anti derivative of $\left(\sin 2x-4e^{3x}\right)$ is the function of x whose derivative is $\left(\sin 2x-4e^{3x}\right)$.

It is known that,

$$\frac{d}{dx}\left(-\frac{1}{2}\cos 2x-\frac{4}{3}e^{3x}\right)=\sin 2x-4e^{3x}$$

Therefore, the anti derivative of $\left(\sin 2x-4e^{3x}\right)$ is $\left(-\frac{1}{2}\cos 2x-\frac{4}{3}e^{3x}\right)$.

Answer needs Correction? Click Here

Q6 : $\int\left(4e^{3x}+1\right)dx$

Answer :

$$\int\left(4e^{3x}+1\right)dx$$
$$=4\int e^{3x}dx+\int 1dx$$
$$=4\left(\frac{e^{3x}}{3}\right)+x+C$$
$$=\frac{4}{3}e^{3x}+x+C$$

Answer needs Correction? Click Here

Q7 : $\int x^2\left(1-\frac{1}{x^2}\right)dx$

Answer :

$$\int x^2\left(1-\frac{1}{x^2}\right)dx$$
$$=\int\left(x^2-1\right)dx$$
$$=\int x^2dx-\int 1dx$$
$$=\frac{x^3}{3}-x+C$$

Answer needs Correction? Click Here

Q8 : $\int\left(ax^2+bx+c\right)dx$

Answer :

$$\int\left(ax^2+bx+c\right)dx$$
$$=a\int x^2dx+b\int xdx+c\int 1.dx$$
$$=a\left(\frac{x^3}{3}\right)+b\left(\frac{x^2}{2}\right)+cx+C$$
$$=\frac{ax^3}{3}+\frac{bx^2}{2}+cx+C$$

Answer needs Correction? Click Here

Q9 : $\int\left(2x^2+e^x\right)dx$

Answer :

$$\int\left(2x^2+e^x\right)dx$$
$$=2\int x^2dx+\int e^xdx$$
$$=2\left(\frac{x^3}{3}\right)+e^x+C$$
$$=\frac{2}{3}x^3+e^x+C$$

Answer needs Correction? Click Here

Q10 : $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2dx$

Answer :

$$\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2dx$$
$$=\int\left(x+\frac{1}{x}-2\right)dx$$
$$=\int xdx+\int\frac{1}{x}dx-2\int 1.dx$$
$$=\frac{x^2}{2}+\log|x|-2x+C$$

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Answer needs Correction? **Click Here**

Q11 : $\int \frac{x^3+5x^2-4}{x^2}dx$

Answer :

$\int \frac{x^3+5x^2-4}{x^2}dx$

$= \int\left(x+5-4x^{-2}\right)dx$

$= \int x dx + 5\int 1.dx - 4\int x^{-2}dx$

$= \frac{x^2}{2}+5x-4\left(\frac{x^{-1}}{-1}\right)+C$

$= \frac{x^2}{2}+5x+\frac{4}{x}+C$

Answer needs Correction? **Click Here**

Q12 : $\int \frac{x^3+3x+4}{\sqrt{x}}dx$

Answer :

$\int \frac{x^3+3x+4}{\sqrt{x}}dx$

$= \int\left(x^{\frac{5}{2}}+3x^{\frac{1}{2}}+4x^{-\frac{1}{2}}\right)dx$

$= \frac{x^{\frac{7}{2}}}{\frac{7}{2}}+\frac{3\left(x^{\frac{3}{2}}\right)}{\frac{3}{2}}+\frac{4\left(x^{\frac{1}{2}}\right)}{\frac{1}{2}}+C$

$= \frac{2}{7}x^{\frac{7}{2}}+2x^{\frac{3}{2}}+8x^{\frac{1}{2}}+C$

$= \frac{2}{7}x^{\frac{7}{2}}+2x^{\frac{3}{2}}+8\sqrt{x}+C$

Answer needs Correction? **Click Here**

Q13 : $\int \frac{x^3-x^2+x-1}{x-1}dx$

Answer :

$\int \frac{x^3-x^2+x-1}{x-1}dx$

On dividing, we obtain

$= \int\left(x^2+1\right)dx$

$= \int x^2 dx + \int 1 dx$

$= \frac{x^3}{3}+x+C$

Answer needs Correction? **Click Here**

Q14 : $\int(1-x)\sqrt{x}dx$

Answer :

$\int(1-x)\sqrt{x}dx$

$= \int\left(\sqrt{x}-x^{\frac{3}{2}}\right)dx$

$= \int x^{\frac{1}{2}}dx - \int x^{\frac{3}{2}}dx$

$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{x^{\frac{5}{2}}}{\frac{5}{2}}+C$

$= \frac{2}{3}x^{\frac{3}{2}}-\frac{2}{5}x^{\frac{5}{2}}+C$

Answer needs Correction? **Click Here**

Q15 : $\int\sqrt{x}\left(3x^2+2x+3\right)dx$

Answer :

$\int\sqrt{x}\left(3x^2+2x+3\right)dx$

$= \int\left(3x^{\frac{5}{2}}+2x^{\frac{3}{2}}+3x^{\frac{1}{2}}\right)dx$

$= 3\int x^{\frac{5}{2}}dx+2\int x^{\frac{3}{2}}dx+3\int x^{\frac{1}{2}}dx$

$$= 3\left(\frac{x^{\frac{7}{2}}}{\frac{7}{2}}\right) + 2\left(\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right) + 3\frac{\left(x^{\frac{3}{2}}\right)}{\frac{3}{2}} + C$$

$$= \frac{6}{7}x^{\frac{7}{2}} + \frac{4}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + C$$

Answer needs Correction? Click Here

Q16 : $\int\left(2x - 3\cos x + e^x\right)dx$

Answer :

$$\int\left(2x - 3\cos x + e^x\right)dx$$

$$= 2\int x dx - 3\int \cos x dx + \int e^x dx$$

$$= \frac{2x^2}{2} - 3\left(\sin x\right) + e^x + C$$

$$= x^2 - 3\sin x + e^x + C$$

Answer needs Correction? Click Here

Q17 : $\int\left(2x^2 - 3\sin x + 5\sqrt{x}\right)dx$

Answer :

$$\int\left(2x^2 - 3\sin x + 5\sqrt{x}\right)dx$$

$$= 2\int x^2 dx - 3\int \sin x dx + 5\int x^{\frac{1}{2}} dx$$

$$= \frac{2x^3}{3} - 3\left(-\cos x\right) + 5\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$

$$= \frac{2}{3}x^3 + 3\cos x + \frac{10}{3}x^{\frac{3}{2}} + C$$

Answer needs Correction? Click Here

Q18 : $\int \sec x\left(\sec x + \tan x\right)dx$

Answer :

$$\int \sec x\left(\sec x + \tan x\right)dx$$

$$= \int\left(\sec^2 x + \sec x \tan x\right)dx$$

$$= \int \sec^2 x dx + \int \sec x \tan x dx$$

$$= \tan x + \sec x + C$$

Answer needs Correction? Click Here

Q19 : $\int \frac{\sec^2 x}{\cos ec^2 x} dx$

Answer :

$$\int \frac{\sec^2 x}{\cos ec^2 x} dx$$

$$= \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}} dx$$

$$= \int \frac{\sin^2 x}{\cos^2 x} dx$$

$$= \int \tan^2 x dx$$

$$= \int\left(\sec^2 x - 1\right)dx$$

$$= \int \sec^2 x dx - \int 1 dx$$

$$= \tan x - x + C$$

Answer needs Correction? Click Here

Q20 : $\int \frac{2 - 3\sin x}{\cos^2 x} dx$

Answer :

$$\int \frac{2 - 3\sin x}{\cos^2 x} dx$$

$$= \int\left(\frac{2}{\cos^2 x} - \frac{3\sin x}{\cos^2 x}\right)dx$$

$$= \int 2\sec^2 x dx - 3\int \tan x \sec x dx$$

$$= 2\tan x - 3\sec x + C$$

Answer needs Correction? Click Here

Q21 : The anti derivative of $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$ equals

(A) $\frac{1}{3}x^{\frac{1}{3}}+2x^{\frac{1}{2}}+C$ (B) $\frac{2}{3}x^{\frac{2}{3}}+\frac{1}{2}x^{2}+C$

(C) $\frac{2}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+C$ (D) $\frac{3}{2}x^{\frac{3}{2}}+\frac{1}{2}x^{\frac{1}{2}}+C$

Answer :

$$\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx$$
$$=\int x^{\frac{1}{2}}dx+\int x^{-\frac{1}{2}}dx$$
$$=\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+C$$
$$=\frac{2}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+C$$

Hence, the correct answer is C.

Answer needs Correction? **Click Here**

Q22 : If $\frac{d}{dx}f(x)=4x^{3}-\frac{3}{x^{4}}$ such that $f(2)=0$, then $f(x)$ is

(A) $x^{4}+\frac{1}{x^{3}}-\frac{129}{8}$ (B) $x^{3}+\frac{1}{x^{4}}+\frac{129}{8}$

(C) $x^{4}+\frac{1}{x^{3}}+\frac{129}{8}$ (D) $x^{3}+\frac{1}{x^{4}}-\frac{129}{8}$

Answer :

It is given that,

$$\frac{d}{dx}f(x)=4x^{3}-\frac{3}{x^{4}}$$

$\therefore$Anti derivative of $4x^{3}-\frac{3}{x^{4}}=f(x)$

$$\therefore f(x)=\int 4x^{3}-\frac{3}{x^{4}}dx$$
$$f(x)=4\int x^{3}dx-3\int\left(x^{-4}\right)dx$$
$$\therefore f(x)=4\left(\frac{x^{4}}{4}\right)-3\left(\frac{x^{-3}}{-3}\right)+C$$
$$f(x)=x^{4}+\frac{1}{x^{3}}+C$$

Also,

$$f(2)=0$$
$$\therefore f(2)=(2)^{4}+\frac{1}{(2)^{3}}+C=0$$
$$\Rightarrow 16+\frac{1}{8}+C=0$$
$$\Rightarrow C=-\left(16+\frac{1}{8}\right)$$
$$\Rightarrow C=\frac{-129}{8}$$
$$\therefore f(x)=x^{4}+\frac{1}{x^{3}}-\frac{129}{8}$$

Hence, the correct answer is A.

Answer needs Correction? **Click Here**

*********** END **********