Algebraic Identities Ex 4.2 Q1

Answer :

In the given problem, we have to find expended form

(i) Given $(a+2b+c)^2$

We shall use the identity $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here $x=a, y=2b, z=c$

By applying in identity we get

$$\begin{aligned}(a+2b+c)^2 &= a^2+(2b)^2+c^2+2a\times 2b+2\times 2\times b\times c+2\times c\times a\\ &= a^2+4b^2+c^2+4ab+4bc+2ca\end{aligned}$$

Hence the expended form of $(a+2b+c)^2$ is $\boxed{a^2+4b^2+c^2+4ab+4bc+2ca}$

(ii) Given $(2a-3b-c)^2$

We shall use the identity $(x-y-z)^2 = x^2+y^2+z^2-2xy+2yz-2zx$

Here $x=2a, y=3b, z=c$

By applying in identity we get

$$\begin{aligned}(2a-3b-c)^2 &= (2a)^2+(3b)^2+(c)^2-2(2a)(3b)+2(3b)(c)+2(c)(2a)\\ &= 2a\times 2a+3b\times 3b+c\times c-2(2a)(3b)+2(3b)(c)+2(c)(2a)\\ &= 4a^2+9b^2+c^2-12ab+6bc-4ca\end{aligned}$$

Hence the expended form of $(2a-3b-c)^2$ is $\boxed{4a^2+9b^2+c^2-12ab+6bc-4ca}$

(iii) Given $(-3x+y+z)^2$

We shall use the identity $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

Here $a=3x, b=y, c=z$

By applying in identity we get

$$\begin{aligned}(-3x+y+z)^2 &= (-3x)^2+y^2+z^2-2\times 3x\times y+2yz-2\times(-3x)\times z\\ &= 3x\times 3x+y^2+3z\times 3z-2\times 3x\times y+2\times y\times z-2\times 3x\times z\\ &= 9x^2+y^2+z^2-6xy+2yz-6xz\end{aligned}$$

Hence the expended form of $(-3x+y+z)^2$ is $\boxed{9x^2+y^2+z^2-6xy+2yz-6xz}$.

(iv) Given $(m+2n-5p)^2$

We shall use the identity $(x+y-z)^2 = x^2+y^2+z^2+2xy-2yz-2zx$

Here $x=m, y=2n, z=5p$

By applying in identity we get

$$\begin{aligned}(m+2n-5p)^2 &= m^2+(2n)^2+(5p)^2+2\times m\times 2n-2\times 2n\times 5p-2\times 5p\times m\\ &= m\times m+2n\times 2n+5p\times 5p+2\times m\times 2n-2\times 2n\times 5p-2\times 5p\times m\\ &= m^2+4n^2+25p^2+4mn-20np-10mp\end{aligned}$$

Hence the expended form of $(m+2n-5p)^2$ is $\boxed{m^2+4n^2+25p^2+4mn-20np-10mp}$

(v) Given $(2+x-2y)^2$

We shall use the identity $(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

Here $a=2, b=x, c=-2y$

By applying in identity we get

$$\begin{aligned}(2+x-2y)^2 &= 2^2+x^2+(2y)^2+2\times2\times x-2\times x\times 2y-2\times 2y\times 2\\ &= 2\times2+x\times x+2y\times 2y+2\times 2\times x-2\times x\times 2y-2\times 2y\times 2\\ &= 4+x^2+4y^2+4x-4xy-8y\end{aligned}$$

Hence the expended form of $(2+x-2y)^2$ is $\boxed{4+x^2+4y^2+4x-4xy-8y}$.

(vi) Given $(a^2+b^2+c^2)^2$

We shall use the identity $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here $x=a^2, y=b^2, z=c^2$

By applying in identity we get

$$\begin{aligned}(a^2+b^2+c^2)^2 &= (a^2)^2+(b^2)^2+(c^2)^2+2\times a^2\times b^2+2\times b^2\times c^2+2\times c^2\times a^2\\ &= a^2\times a^2+b^2\times b^2+c^2\times c^2+2\times a^2\times b^2+2\times b^2\times c^2+2\times c^2\times a^2\\ &= a^4+b^4+c^4+2a^2b^2+2b^2c^2+2c^2a^2\end{aligned}$$

Hence the expended form of $(a^2+b^2+c^2)^2$ is $\boxed{a^4+b^4+c^4+2a^2b^2+2b^2c^2+2c^2a^2}$.

(vii) Given $(ab+bc+ca)^2$

We shall use the identity $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here $x=ab, y=bc, z=ca$

By applying in identity we get

$$\begin{aligned}(ab+bc+ca)^2 &= (ab)^2+(bc)^2+(ca)^2+2\times ab\times bc+2\times bc\times ca+2\times ca\times ab\\ &= ab\times ab+bc\times bc+ca\times ca+2\times ab\times bc+2\times bc\times ca+2\times ca\times ab\\ &= a^2b^2+b^2c^2+c^2a^2+2ab^2c+2bc^2a+2ca^2b\end{aligned}$$

Hence the expended form of $(ab+bc+ca)^2$ is $\boxed{a^2b^2+b^2c^2+c^2a^2+2ab^2c+2bc^2a+2ca^2b}$.

(viii) Given $\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)$

We shall use the identity $(a+b+c)^2 = a^2+b^2+c^2-2ab-2bc-2ca$

Here $a=\frac{x}{y}, b=\frac{y}{z}, c=\frac{z}{x}$

By applying in identity we get

$$\begin{aligned}\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)^2 &= \left(\frac{x}{y}\right)^2+\left(\frac{y}{z}\right)^2+\left(\frac{z}{x}\right)^2+2\times\frac{x}{y}\times\frac{y}{z}+2\times\frac{y}{z}\times\frac{z}{x}+2\times\frac{z}{x}\times\frac{x}{y}\\ &= \frac{x}{y}\times\frac{x}{y}+\frac{y}{z}\times\frac{y}{z}+\frac{z}{x}\times\frac{z}{x}+2\times\frac{x}{y}\times\frac{y}{z}+2\times\frac{y}{z}\times\frac{z}{x}+2\times\frac{z}{x}\times\frac{x}{y}\\ &= \frac{x^2}{y^2}+\frac{y^2}{z^2}+\frac{z^2}{x^2}++2\times\frac{x}{\cancel{y}}\times\frac{\cancel{y}}{z}+2\times\frac{y}{\cancel{z}}\times\frac{\cancel{z}}{x}+2\times\frac{z}{\cancel{x}}\times\frac{\cancel{x}}{y}\\ &= \frac{x^2}{y^2}+\frac{y^2}{z^2}+\frac{z^2}{x^2}+\frac{2x}{z}+\frac{2y}{x}+\frac{2z}{y}\end{aligned}$$

Hence the expended form of $\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)^2$ is $\boxed{\frac{x^2}{y^2}+\frac{y^2}{z^2}+\frac{z^2}{x^2}+\frac{2x}{z}+\frac{2y}{x}+\frac{2z}{y}}$

(ix) Given $\left(\frac{a}{bc}+\frac{b}{ca}+\frac{c}{ab}\right)^2$

We shall use the identity $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

Here $x=\frac{a}{bc}, y=\frac{b}{ca}, z=\frac{c}{ab}$

By applying in identity we get

$$\begin{aligned}\left(\frac{a}{bc}+\frac{b}{ca}+\frac{c}{ab}\right)^2 &=\left(\frac{a}{bc}\right)^2+\left(\frac{b}{ca}\right)^2+\left(\frac{c}{ab}\right)^2+2\times\frac{a}{bc}\times\frac{b}{ca}+2\times\frac{b}{ca}\times\frac{c}{ab}+2\times\frac{c}{ab}\times\frac{a}{bc}\\ &=\frac{a}{bc}\times\frac{a}{bc}+\frac{b}{ca}\times\frac{b}{ca}+\frac{c}{ab}\times\frac{c}{ab}+2\times\frac{a}{bc}\times\frac{b}{ca}+2\times\frac{b}{ca}\times\frac{c}{ab}+2\times\frac{c}{ab}\times\frac{a}{bc}\\ &=\frac{a^2}{b^2c^2}+\frac{b^2}{c^2a^2}+\frac{c^2}{a^2b^2}+2\times\frac{\cancel{a}}{\cancel{b}c}\times\frac{\cancel{b}}{c\cancel{a}}+2\times\frac{\cancel{b}}{\cancel{c}a}\times\frac{\cancel{c}}{a\cancel{b}}+2\times\frac{\cancel{c}}{\cancel{a}b}\times\frac{\cancel{a}}{b\cancel{c}}\\ &=\frac{a^2}{b^2c^2}+\frac{b^2}{c^2a^2}+\frac{c^2}{a^2b^2}+\frac{2}{c^2}+\frac{2}{a^2}+\frac{2}{b^2}\end{aligned}$$

Hence the expended form of $\left(\frac{a}{bc}+\frac{b}{ca}+\frac{c}{ab}\right)^2$ is $\boxed{\frac{a^2}{b^2c^2}+\frac{b^2}{c^2a^2}+\frac{c^2}{a^2b^2}+\frac{2}{c^2}+\frac{2}{a^2}+\frac{2}{b^2}}$

(x) Given $(x+2y+4z)^2$

We shall use the identity $(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$

Here $a=x,b=2y,c=4z$

By applying in identity we get

$$\begin{aligned}(x+2y+4z)^2 &=x^2+(2y)^2+(4z)^2+2\times x\times 2y+2\times 2y\times 4z+2\times 4z\times x\\ &=x\times x+2y\times 2y+4z\times 4z+2\times x\times 2y+2\times 2y\times 4z+2\times 4z\times x\\ &=x^2+4y^2+16z^2+4xy+16yz+8xy\end{aligned}$$

Hence the expended form of $(x+2y+4z)^2$ is $\boxed{x^2+4y^2+16z^2+4xy+16yz+8xy}$

(xi) Given $(2x-y+z)^2$

We shall use the identity $(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$

Here $a=2x,b=y,c=z$

By applying in identity we get

$$\begin{aligned}(2x-y+z)^2 &=(2x)^2+y^2+z^2-2\times 2x\times y-2\times y\times z+2\times z\times 2x\\ &=2x\times 2x+y\times y+z\times z-2\times 2x\times y-2\times y\times z+2\times z\times 2x\\ &=4x^2+y^2+z^2-4xy-2yz+4xy\end{aligned}$$

Hence the expended form of $(2x-y+z)^2$ is $\boxed{4x^2+y^2+z^2-4xy-2yz+4xy}$

(xii) Given $(-2x+3y+2z)^2$

We shall use the identity $(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$

Here $a=-2x,b=3y,c=2z$

By applying in identity we get

$$\begin{aligned}(-2x+3y+2z)^2 &=(-2x)^2+(3y)^2+(2z)^2-2\times 2x\times 3y+2\times 3y\times 2z-2\times 2z\times 2x\\ &=2x\times 2x+3y\times 3y+2z\times 2z-2\times 2x\times 3y+2\times 3y\times 2z-2\times 2z\times 2x\\ &=4x^2+9y^2+4z^2-12yx+12yz-8xz\end{aligned}$$

Hence the expended form of $(-2x+3y+2z)^2$ is $\boxed{4x^2+9y^2+4z^2-12yx+12yz-8xz}$.

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