Transformation Formulae Ex 8.2 Q 12.

We have,

$$\sin 2A = \lambda \sin 2B$$

$$\Rightarrow \qquad \lambda = \frac{\sin 2A}{\sin 2B}$$

Now,

$$\frac{\lambda + 1}{\lambda - 1} = \frac{\dfrac{\sin 2A}{\sin 2B} + 1}{\dfrac{\sin 2A}{\sin 2B} - 1}$$

$$= \frac{\dfrac{\sin 2A + \sin 2B}{\sin 2B}}{\dfrac{\sin 2A - \sin 2B}{\sin 2B}}$$

$$= \frac{\sin 2A + \sin 2B}{\sin 2A - \sin 2B}$$

$$= \frac{2 \sin\left(\dfrac{2A + 2B}{2}\right) \cos\left(\dfrac{2A - 2B}{2}\right)}{2 \sin\left(\dfrac{2A - 2B}{2}\right) \cos\left(\dfrac{2A + 2B}{2}\right)}$$

$$= \frac{\sin(A + B)\cos(A - B)}{\sin(A - B)\cos(A + B)}$$

$$= \frac{\sin(A + B)\cos(A - B)}{\cos(A + B)\sin(A - B)}$$

$$= \frac{\tan(A + B)}{\tan(A - B)}$$

$$\therefore \qquad \frac{\lambda + 1}{\lambda - 1} = \frac{\tan(A + B)}{\tan(A - B)}$$

$$\Rightarrow \qquad \frac{\tan(A + B)}{\tan(A - B)} = \frac{\lambda + 1}{\lambda - 1}$$

Hence proved.

Transformation Formulae Ex 8.2 Q13(i)

We have,

$$\text{LHS} = \frac{\cos(A+B+C)+\cos(-A+B+C)+\cos(A-B+C)+\cos(A+B-C)}{\sin(A+B+C)+\sin(-A+B+C)+\sin(A-B+C)-\sin(A+B-C)}$$

$$= \frac{2\cos\left\{\frac{A+B+C-A+B+C}{2}\right\}\cos\left\{\frac{A+B+C+A-B-C}{2}\right\}+2\cos\left\{\frac{A-B+C+A+B-C}{2}\right\}\cos\left\{\frac{A-B+C-A-B+C}{2}\right\}}{2\sin\left\{\frac{A+B+C-A+B+C}{2}\right\}\cos\left\{\frac{A+B+C+A-B-C}{2}\right\}+2\sin\left\{\frac{A-B+C-A-B+C}{2}\right\}\cos\left\{\frac{A-B+C+A+B-C}{2}\right\}}$$

$$= \frac{2\cos(B+C)\cos A+2\cos A\cos(C-B)}{2\sin(B+C)\cos A+2\sin(C-B)\cos A}$$

$$= \frac{2\cos A\left[\cos(B+C)+\cos(C-B)\right]}{2\cos A\left[\sin(B+C)+\sin(C-B)\right]}$$

$$= \frac{\cos(B+C)+\cos(C-B)}{\sin(B+C)+\sin(C-B)}$$

$$= \frac{2\cos\left\{\frac{B+C+C-B}{2}\right\}\cos\left\{\frac{B+C-C+B}{2}\right\}}{2\sin\left\{\frac{B+C+C-B}{2}\right\}\cos\left\{\frac{B+C-C+B}{2}\right\}}$$

$$= \frac{2\cos C\cos B}{2\sin C\cos B}$$

$$= \frac{\cos C}{\sin C}$$

$$= \cot C$$

$$= \text{RHS}$$

$$\therefore \quad \frac{\cos(A+B+C)+\cos(-A+B+C)+\cos(A-B+C)+\cos(A+B-C)}{\sin(A+B+C)+\sin(-A+B+C)+\sin(A-B+C)-\sin(A+B-C)} = \cot C.$$ Hence proved.

Transformation Formulae Ex 8.2 Q13(ii)

We have,

$$\text{LHS} = \sin(B-C)\cos(A-D)+\sin(C-A)\cos(B-D)+\sin(A-B)\cos(C-D)$$

$$= \frac{1}{2}\left[2\sin(B-C)\cos(A-D)+2\sin(C-A)\cos(B-D)+2\sin(A-B)\cos(C-D)\right]$$

$$= \frac{1}{2}\left[\begin{array}{r}\sin(B-C+A-D)+\sin(B-C-A+D)+\sin(C-A+B-D)+\sin(C-A-B+D)\\ +\sin(A-B+C-D)+\sin(A-B-C+D)\end{array}\right]$$

$$= \frac{1}{2}\left[\begin{array}{r}\sin(A+B-C-D)+\sin(B+D-C-A)+\sin(B+C-A-D)+\sin(C+D-A-B)\\ +\sin(A+C-B-D)+\sin(A+D-B-C)\end{array}\right]$$

$$= \frac{1}{2}\left[\begin{array}{r}\sin(A+B-C-D)-\sin(A+C-B-D)-\sin(A+D-B-C)-\sin(A+B-C-D)\\ +\sin(A+C-B-D)+\sin(A+D-B-C)\end{array}\right]$$

$$= \frac{1}{2}[0]$$

$$= 0$$

$$= \text{RHS}$$

$$\therefore \quad \sin(B-C)\cos(A-D)+\sin(C-A)\cos(B-D)+\sin(A-B)\cos(C-D) = 0$$ Hence proved.

Transformation Formulae Ex 8.2 Q 14.

We have,

$$\frac{\cos(A-B)}{\cos(A+B)}+\frac{\cos(C+D)}{\cos(C-D)}=0$$

$$\Rightarrow \quad \frac{\cos(A-B)}{\cos(A+B)}=-\frac{\cos(C+D)}{\cos(C-D)} \qquad \text{---(i)}$$

Now,

$$\frac{\cos(A-B)}{\cos(A+B)}=-\frac{\cos(C+D)}{\cos(C-D)}$$

$$\Rightarrow \quad \frac{\cos(A-B)}{\cos(A+B)}+1=\frac{-\cos(C+D)}{\cos(C-D)}+1$$

$$\Rightarrow \quad \frac{\cos(A-B)+\cos(A+B)}{\cos(A+B)}=\frac{-\cos(C+D)+\cos(C-D)}{\cos(C-D)}$$

$$\Rightarrow \quad \frac{\cos(A+B)+\cos(A-B)}{\cos(A+B)}=\frac{-\left[\cos(C+D)-\cos(C-D)\right]}{\cos(C-D)} \qquad \text{---(ii)}$$

Again,

$$\frac{\cos(A-B)}{\cos(A+B)}=\frac{-\cos(C+D)}{\cos(C-D)} \qquad \left[\text{By equation (i)}\right]$$

$$\Rightarrow \quad \frac{\cos(A-B)}{\cos(A+B)}-1=\frac{-\cos(C+D)}{\cos(C-D)}-1$$

$$\Rightarrow \quad \frac{\cos(A-B)-\cos(A+B)}{\cos(A+B)}=\frac{-\cos(C+D)-\cos(C-D)}{\cos(C-D)}$$

$$\Rightarrow \quad \frac{-\{\cos(A+B)-\cos(A-B)\}}{\cos(A+B)}=\frac{-\left[\cos(C+D)+\cos(C-D)\right]}{\cos(C-D)}$$

$$\Rightarrow \quad \frac{\cos(A+B)-\cos(A-B)}{\cos(A+B)}=\frac{\cos(C+D)+\cos(C-D)}{\cos(C-D)} \qquad \text{---(iii)}$$

Dividing equation (ii) by equation (iii), we get

$$\frac{\cos(A+B)+\cos(A-B)}{\cos(A+B)-\cos(A-B)}=\frac{-\left[\cos(C+D)-\cos(C-D)\right]}{\cos(C+D)+\cos(C-D)}$$

$$\Rightarrow \quad \frac{2\cos\left\{\frac{A+B+A-B}{2}\right\}\cos\left\{\frac{A+B-A+B}{2}\right\}}{-2\sin\left\{\frac{A+B+A-B}{2}\right\}\sin\left\{\frac{A+B-A+B}{2}\right\}}=\frac{-\left[2\sin\left\{\frac{C+D+C-D}{2}\right\}\sin\left\{\frac{C+D-C+D}{2}\right\}\right]}{2\cos\left\{\frac{C+D+C-D}{2}\right\}\cos\left\{\frac{C+D-C+D}{2}\right\}}$$

$$\Rightarrow \quad \frac{\cos A\cos B}{-\sin A\sin B}=\frac{\sin C\sin D}{\cos C\cos D}$$

$$\Rightarrow \quad \frac{1}{-\tan A\tan B}=\tan C\tan D$$

$$\Rightarrow \quad -1=\tan A\tan B\tan C\tan D$$

$\therefore \tan A\tan B\tan C\tan D=-1$ Hence proved.

Transformation Formulae Ex 8.2 Q 15.

We have,

$$\cos(\alpha+\beta)\sin(\gamma+\delta)=\cos(\alpha-\beta)\sin(\gamma-\delta)$$

$$\Rightarrow \quad \frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}=\frac{\sin(\gamma-\delta)}{\sin(\gamma+\delta)} \qquad \text{---(i)}$$

Now,

$$\frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}=\frac{\sin(\gamma-\delta)}{\sin(\gamma+\delta)}$$

$$\Rightarrow \quad \frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}+1=\frac{\sin(\gamma-\delta)}{\sin(\gamma+\delta)}+1$$

$$\Rightarrow \quad \frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{\cos(\alpha-\beta)}=\frac{\sin(\gamma-\delta)+\sin(\gamma+\delta)}{\sin(\gamma+\delta)} \qquad \text{---(ii)}$$

Again,

$$\frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}=\frac{\sin(\gamma-\delta)}{\sin(\gamma+\delta)} \qquad \left[\text{By equation (i)}\right]$$

$$\Rightarrow \quad \frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}-1=\frac{\sin(\gamma-\delta)}{\sin(\gamma+\delta)}-1$$

$$\Rightarrow \quad \frac{\cos(\alpha+\beta)-\cos(\alpha-\beta)}{\cos(\alpha-\beta)}=\frac{\sin(\gamma-\delta)-\sin(\gamma+\delta)}{\sin(\gamma+\delta)} \qquad \text{---(iii)}$$

Dividing equation (ii) by equation (iii), we get

$$\frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{\cos(\alpha+\beta)-\cos(\alpha-\beta)}=\frac{\sin(\gamma-\delta)+\sin(\gamma+\delta)}{\sin(\gamma-\delta)-\sin(\gamma+\delta)}$$

$$\Rightarrow \quad \frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{\cos(\alpha+\beta)-\cos(\alpha-\beta)}=-\left[\frac{\sin(\gamma+\delta)+\sin(\gamma-\delta)}{\sin(\gamma+\delta)-\sin(\gamma-\delta)}\right]$$

$$\Rightarrow \quad \frac{2\cos\left\{\frac{\alpha+\beta+\alpha-\beta}{2}\right\}\cos\left\{\frac{\alpha+\beta-\alpha+\beta}{2}\right\}}{-2\sin\left\{\frac{\alpha+\beta+\alpha-\beta}{2}\right\}\sin\left\{\frac{\alpha+\beta-\alpha+\beta}{2}\right\}}=-\left[\frac{2\sin\left\{\frac{\gamma+\delta+\gamma-\delta}{2}\right\}\cos\left\{\frac{\gamma+\delta-\gamma+\delta}{2}\right\}}{2\sin\left\{\frac{\gamma+\delta-\gamma+\delta}{2}\right\}\cos\left\{\frac{\gamma+\delta+\gamma-\delta}{2}\right\}}\right]$$

$$\Rightarrow \quad \frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta}=\frac{\sin\gamma\cos\delta}{\sin\delta\cos\gamma}$$

$$\Rightarrow \quad \cot\alpha\cot\beta=\frac{\sin\gamma\cos\delta}{\cos\gamma\sin\delta}$$

$$\Rightarrow \quad \cot\alpha\cot\beta=\frac{\cot\delta}{\cot\gamma}$$

$$\Rightarrow \quad \cot\alpha\cot\beta\cot\gamma=\cot\delta$$

$\therefore \cot\alpha\cot\beta\cot\gamma=\cot\delta$ Hence proved.

*********** END **********