Exercise 5.3

$\Rightarrow 320 = 2a \Rightarrow a = 160$

Therefore, value of first prize = Rs 160

Value of second prize = 160 – 20 = Rs 140

Value of third prize = 140 – 20 = Rs 120

Value of fourth prize = 120 – 20 = Rs 100

Value of fifth prize = 100 – 20 = Rs 80

Value of sixth prize = 80 – 20 = Rs 60

Value of seventh prize = 60 – 20 = Rs 40

17. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g, a section of Class I will plant 1 tree, a section of class II will plant two trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Ans. There are three sections of each class and it is given that the number of trees planted by any class is equal to class number.

The number of trees planted by class I = number of sections $\times 1 = 3 \times 1 = 3$

The number of trees planted by class II = number of sections $\times 2 = 3 \times 2 = 6$

The number of trees planted by class III = number of sections $\times 3 = 3 \times 3 = 9$

Therefore, we have sequence of the form 3, 6, 9 ... 12 terms

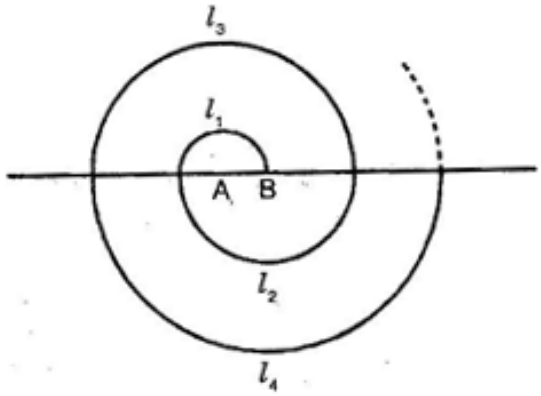
To find total number of trees planted by all the students, we need to find sum of the sequence 3, 6, 9, 12 ... 12 terms.

First term = a = 3, Common difference = d = 6 – 3 = 3 and n = 12

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP , we get

$$S_{12} = \frac{12}{2}[6 + (12-1)3] = 6(6+33) = 6 \times 39 = 234$$

18. A spiral is made up of successive semicircles, with centers alternatively at A and B, starting with center at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ... What is the total length of such a spiral made up of thirteen consecutive semicircles.



Ans. Length of semi–circle = $\frac{\text{Circumfere nce of circle}}{2} = \frac{2\pi r}{2} = \pi r$

Length of semi-circle of radii 0.5 cm = π (0.5) cm

Length of semi-circle of radii 1.0 cm = π (1.0) cm

Length of semi-circle of radii 1.5 cm = π (1.5) cm

Therefore, we have sequence of the form:

π (0.5), π (1.0), π (1.5) ... 13 terms {There are total of thirteen semi–circles}.

To find total length of the spiral, we need to find sum of the sequence π (0.5), π (1.0), π (1.5) ... 13 terms

Total length of spiral = π (0.5) + π (1.0) + π (1.5) ... 13 terms

⇒ Total length of spiral = π (0.5 + 1.0 + 1.5) ... 13 terms ... (1)

Sequence 0.5, 1.0, 1.5 ... 13 terms is an arithmetic progression.

Let us find the sum of this sequence.

First term = a = 0.5, Common difference = 1.0 – 0.5 = 0.5 and n = 13

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$S_{13} = \frac{13}{2}[1+(13-1)0.5] = 6.5(1+6) = 6.5\times 7 = 45.5$$

Therefore, 0.5 + 1.0 + 1.5 + 2.0 ... 13 terms = 45.5

Putting this in equation (1), we get

Total length of spiral= π (0.5 + 1.5 + 2.0 + ... 13 terms) = π (45.5) = 143 cm

19. 200 logs are stacked in the following manner:20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?

Ans. The number of logs in the bottom row = 20

The number of logs in the next row = 19

The number of logs in the next to next row = 18

Therefore, we have sequence of the form 20, 19, 18 ...

First term = a = 20, Common difference = d = 19 − 20 = −1

We need to find that how many rows make total of 200 logs.

Applying formula, $S_n = \frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP, we get

$$200 = \frac{n}{2}[40+(n-1)(-1)]$$

$\Rightarrow$ 400 = n (40 − n + 1)

$\Rightarrow$ 400 = $40n - n^2 + n$

$\Rightarrow$ $n^2 - 41n + 400 = 0$

It is a quadratic equation, we can factorize to solve the equation.

$\Rightarrow n^2 - 25n - 16n + 400 = 0$

$\Rightarrow n(n - 25) - 16(n - 25) = 0$

$\Rightarrow (n - 25)(n - 16)$

$\Rightarrow n = 25, 16$

We discard n = 25 because we cannot have more than 20 rows in the sequence. The sequence is of the form: 20, 19, 18 ...

At most, we can have 20 or less number of rows.

Therefore, n = 16 which means 16 rows make total number of logs equal to 200.

We also need to find number of logs in the 16th row.

Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP, we get

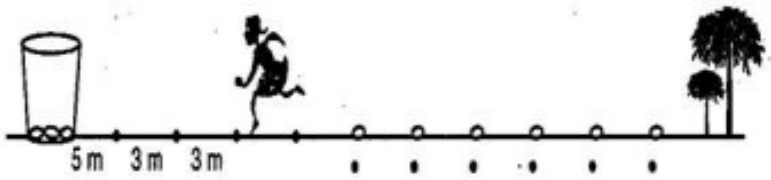
$200 = 8(20 + l)$

$\Rightarrow 200 = 160 + 8l$

$\Rightarrow 40 = 8l \Rightarrow l = 5$

Therefore, there are 5 logs in the top most row and there are total of 16 rows.

20. In a potato race, a bucket is placed at the starting point, which is 5 meters from the first potato, and the other potatoes are placed 3 meters apart in a straight line. There are ten potatoes in the line. A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?



Ans. The distance of first potato from the starting point = 5 meters

Therefore, the distance covered by competitor to pick up first potato and put it in bucket = 5×2 = 10 meters

The distance of Second potato from the starting point = 5 + 3 = 8 meters {All the potatoes are 3 meters apart from each other}

Therefore, the distance covered by competitor to pick up 2nd potato and put it in bucket = 8×2 = 16 meters

The distance of third potato from the starting point = 8 + 3 = 11 meters

Therefore, the distance covered by competitor to pick up 3rd potato and put it in bucket = 11×2 = 22 meters

Therefore, we have a sequence of the form 10, 16, 22 ... 10 terms

(There are ten terms because there are ten potatoes)

To calculate the total distance covered by the competitor, we need to find:

10 + 16 + 22 + ... 10 terms

First term = a = 10, Common difference = d = 16 − 10 = 6

n = 10{There are total of 10 terms in the sequence}

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$S_{n10} = \frac{10}{2}[20 + (10-1)6] = 5(20+54) = 5 \times 74 = 370$$

Therefore, total distance covered by competitor is equal to 370 meters.

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