AB will be parallel to CD, if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

$\frac{a_1}{a_2}=\frac{-2}{2}=-1$

$\frac{b_1}{b_2}=\frac{-4}{4}=-1$

$\frac{c_1}{c_2}=\frac{-4}{4}=-1$

$\therefore\ \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Thus, AB is parallel to CD.

Question 4:

Find the equation of the line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{i}+2\hat{j}-2\hat{k}$.

Answer

It is given that the line passes through the point A (1, 2, 3). Therefore, the position vector through A is $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$

$\vec{b}=3\hat{i}+2\hat{j}-2\hat{k}$

It is known that the line which passes through point A and parallel to $\vec{b}$ is given by

$\vec{r}=\vec{a}+\lambda\vec{b}$, where λ is a constant.

$\Rightarrow \vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda\left(3\hat{i}+2\hat{j}-2\hat{k}\right)$

This is the required equation of the line.

Question 5:

Find the equation of the line in vector and in Cartesian form that passes through the point with position vector $2\hat{i}-\hat{j}+4\hat{k}$ and is in the direction $\hat{i}+2\hat{j}-\hat{k}$.

Answer

It is given that the line passes through the point with position vector

$\vec{a}=2\hat{i}-\hat{j}+4\hat{k}$ ...(1)

$\vec{b}=\hat{i}+2\hat{j}-\hat{k}$ ...(2)

It is known that a line through a point with position vector $\vec{a}$ and parallel to $\vec{b}$ is given by the equation, $\vec{r}=\vec{a}+\lambda\vec{b}$

$\Rightarrow \vec{r}=2\hat{i}-\hat{j}+4\hat{k}+\lambda\left(\hat{i}+2\hat{j}-\hat{k}\right)$

This is the required equation of the line in vector form.

$\vec{r}=x\hat{i}-y\hat{j}+z\hat{k}$

$\Rightarrow x\hat{i}-y\hat{j}+z\hat{k}=(\lambda+2)\hat{i}+(2\lambda-1)\hat{j}+(-\lambda+4)\hat{k}$

Eliminating λ, we obtain the Cartesian form equation as

$\frac{x-2}{1}=\frac{y+1}{2}=\frac{z-4}{-1}$

This is the required equation of the given line in Cartesian form.

Question 6:

Find the Cartesian equation of the line which passes through the point (−2, 4, −5) and parallel to the line given by $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$

Answer

It is given that the line passes through the point (−2, 4, −5) and is parallel to

$\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$

$$\frac{x+3}{}=\frac{y-4}{}=\frac{z+8}{}$$

The direction ratios of the line, $\;3 \quad 5 \quad 6\;$, are 3, 5, and 6.

The required line is parallel to $\dfrac{x+3}{3}=\dfrac{y-4}{5}=\dfrac{z+8}{6}$

Therefore, its direction ratios are $3k$, $5k$, and $6k$, where $k \neq 0$

It is known that the equation of the line through the point (x_1, y_1, z_1) and with direction ratios, a, b, c, is given by $\dfrac{x-x_1}{a}=\dfrac{y-y_1}{b}=\dfrac{z-z_1}{c}$

Therefore the equation of the required line is

$$\frac{x+2}{3k}=\frac{y-4}{5k}=\frac{z+5}{6k}$$

$$\Rightarrow \frac{x+2}{3}=\frac{y-4}{5}=\frac{z+5}{6}=k$$

Question 7:

The Cartesian equation of a line is $\dfrac{x-5}{3}=\dfrac{y+4}{7}=\dfrac{z-6}{2}$. Write its vector form.

Answer

The Cartesian equation of the line is

$$\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2} \qquad \dots(1)$$

The given line passes through the point (5, −4, 6). The position vector of this point is

$$\vec{a}=5\hat{i}-4\hat{j}+6\hat{k}$$

Also, the direction ratios of the given line are 3, 7, and 2.

This means that the line is in the direction of vector, $\vec{b}=3\hat{i}+7\hat{j}+2\hat{k}$

It is known that the line through position vector $\vec{a}$ and in the direction of the vector $\vec{b}$ is given by the equation, $\vec{r}=\vec{a}+\lambda\vec{b}, \lambda \in R$

$$\Rightarrow \vec{r}=\left(5\hat{i}-4\hat{j}+6\hat{k}\right)+\lambda\left(3\hat{i}+7\hat{j}+2\hat{k}\right)$$

This is the required equation of the given line in vector form.

Question 8:

Find the vector and the Cartesian equations of the lines that pass through the origin and (5, −2, 3).

Answer

The required line passes through the origin. Therefore, its position vector is given by,

$$\vec{a}=\vec{0} \qquad \dots(1)$$

The direction ratios of the line through origin and (5, −2, 3) are

(5 − 0) = 5, (−2 − 0) = −2, (3 − 0) = 3

The line is parallel to the vector given by the equation, $\vec{b}=5\hat{i}-2\hat{j}+3\hat{k}$

The equation of the line in vector form through a point with position vector $\vec{a}$ and parallel to $\vec{b}$ is, $\vec{r}=\vec{a}+\lambda\vec{b}, \lambda \in R$

$$\Rightarrow \vec{r}=\vec{0}+\lambda\left(5\hat{i}-2\hat{j}+3\hat{k}\right)$$

$$\Rightarrow \vec{r}=\lambda\left(5\hat{i}-2\hat{j}+3\hat{k}\right)$$

The equation of the line through the point (x_1, y_1, z_1) and direction ratios a, b, c is given by, $\dfrac{x-x_1}{a}=\dfrac{y-y_1}{b}=\dfrac{z-z_1}{c}$

Therefore, the equation of the required line in the Cartesian form is

$$\frac{x-0}{5}=\frac{y-0}{-2}=\frac{z-0}{3}$$

$$\Rightarrow \frac{x}{5}=\frac{y}{-2}=\frac{z}{3}$$

Question 9:

Find the vector and the Cartesian equations of the line that passes through the points (3, −2, −5), (3, −2, 6).

Answer

Let the line passing through the points, P (3, −2, −5) and Q (3, −2, 6), be PQ.

Since PQ passes through P (3, −2, −5), its position vector is given by,

$$\vec{a}=3\hat{i}-2\hat{j}-5\hat{k}$$

The direction ratios of PQ are given by,

(3 − 3) = 0, (−2 + 2) = 0, (6 + 5) = 11

The equation of the vector in the direction of PQ is

$$\vec{b}=0.\hat{i}-0.\hat{j}+11\hat{k}=11\hat{k}$$

The equation of PQ in vector form is given by, $\vec{r}=\vec{a}+\lambda\vec{b},\ \lambda \in R$

$$\Rightarrow \vec{r} = \left(3\hat{i} - 2\hat{j} - 5\hat{k}\right) + 11\lambda\hat{k}$$

The equation of PQ in Cartesian form is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$ i.e.,

$$\frac{x - 3}{0} = \frac{y + 2}{0} = \frac{z + 5}{11}$$

Question 10:

Find the angle between the following pairs of lines:

(i) $\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda\left(3\hat{i} - 2\hat{j} + 6\hat{k}\right)$ and

$\vec{r} = 7\hat{i} - 6\hat{k} + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$

(ii) $\vec{r} = 3\hat{i} + \hat{j} - 2\hat{k} + \lambda\left(\hat{i} - \hat{j} - 2\hat{k}\right)$ and

$\vec{r} = 2\hat{i} - \hat{j} - 56\hat{k} + \mu\left(3\hat{i} - 5\hat{j} - 4\hat{k}\right)$

Answer

(i) Let Q be the angle between the given lines.

The angle between the given pairs of lines is given by, $\cos Q = \left|\frac{\vec{b}_1 \cdot \vec{b}_2}{\left|\vec{b}_1\right|\left|\vec{b}_2\right|}\right|$

The given lines are parallel to the vectors, $\vec{b}_1 = 3\hat{i} + 2\hat{j} + 6\hat{k}$ and $\vec{b}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$, respectively.

$$\therefore \left|\vec{b}_1\right| = \sqrt{3^2 + 2^2 + 6^2} = 7$$

$$\left|\vec{b}_2\right| = \sqrt{(1)^2 + (2)^2 + (2)^2} = 3$$

$$\begin{aligned}\vec{b}_1 \cdot \vec{b}_2 &= \left(3\hat{i} + 2\hat{j} + 6\hat{k}\right) \cdot \left(\hat{i} + 2\hat{j} + 2\hat{k}\right)\\ &= 3 \times 1 + 2 \times 2 + 6 \times 2\\ &= 3 + 4 + 12\\ &= 19\end{aligned}$$

*********** END **********