



(e) 9 mW; no power is expended when key K is open.

Speed of the rod, $v = 12 \text{ cm/s} = 0.12 \text{ m/s}$

Hence, power is given as:

$$\begin{aligned} P &= Fv \\ &= 75 \times 10^{-3} \times 0.12 \\ &= 9 \times 10^{-3} \text{ W} \\ &= 9 \text{ mW} \end{aligned}$$

When key K is open, no power is expended.

(f) 9 mW; power is provided by an external agent.

Power dissipated as heat $= I^2 R$

$$= (1)^2 \times 9 \times 10^{-3}$$

$$= 9 \text{ mW}$$

The source of this power is an external agent.

(g) Zero

In this case, no emf is induced in the coil because the motion of the rod does not cut across the field lines.

Question 6.15:

An air-cored solenoid with length 30 cm, area of cross-section 25 cm^2 and number of turns 500, carries a current of 2.5 A. The current is suddenly switched off in a brief time of 10^{-3} s . How much is the average back emf induced across the ends of the open switch in the circuit? Ignore the variation in magnetic field near the ends of the solenoid.

Answer

Length of the solenoid, $l = 30 \text{ cm} = 0.3 \text{ m}$

Area of cross-section, $A = 25 \text{ cm}^2 = 25 \times 10^{-4} \text{ m}^2$

Number of turns on the solenoid, $N = 500$

Current in the solenoid, $I = 2.5 \text{ A}$

Current flows for time, $t = 10^{-3} \text{ s}$

$$\text{Average back emf, } e = \frac{d\phi}{dt} \quad \dots (1)$$

Where,

$d\phi$ = Change in flux

$$= NAB \dots (2)$$

Where,

B = Magnetic field strength

$$= \mu_0 \frac{NI}{l} \quad \dots (3)$$

Where,

μ_0 = Permeability of free space $= 4\pi \times 10^{-7} \text{ T m A}^{-1}$

Using equations (2) and (3) in equation (1), we get

$$\begin{aligned} e &= \frac{\mu_0 N^2 I A}{lt} \\ &= \frac{4\pi \times 10^{-7} \times (500)^2 \times 2.5 \times 25 \times 10^{-4}}{0.3 \times 10^{-3}} = 6.5 \text{ V} \end{aligned}$$

Hence, the average back emf induced in the solenoid is 6.5 V.

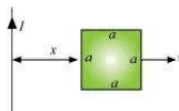
Question 6.16:

(a) Obtain an expression for the mutual inductance between a long straight wire and a square loop of side a as shown in Fig. 6.21.

(b) Now assume that the straight wire carries a current of 50 A and the loop is moved to the right with a constant velocity, $v = 10 \text{ m/s}$.

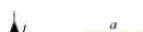
Calculate the induced emf in the loop at the instant when $x = 0.2 \text{ m}$.

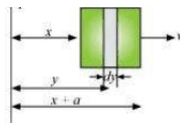
Take $a = 0.1 \text{ m}$ and assume that the loop has a large resistance.



Answer

(a) Take a small element dy in the loop at a distance y from the long straight wire (as shown in the given figure).





Magnetic flux associated with element dy , $d\phi = BdA$

Where,

dA = Area of element $dy = a \, dy$

B = Magnetic field at distance y

$$= \frac{\mu_0 I}{2\pi y}$$

I = Current in the wire

μ_0 = Permeability of free space = $4\pi \times 10^{-7} \text{ T m A}^{-1}$

$$\therefore d\phi = \frac{\mu_0 I a \, dy}{2\pi y}$$

$$\phi = \frac{\mu_0 I a}{2\pi} \int_y \frac{dy}{y}$$

y tends from x to $x+a$.

$$\therefore \phi = \frac{\mu_0 I a}{2\pi} \int_x^{x+a} \frac{dy}{y}$$

$$= \frac{\mu_0 I a}{2\pi} [\log_e y]_x^{x+a}$$

$$= \frac{\mu_0 I a}{2\pi} \log_e \left(\frac{x+a}{x} \right)$$

For mutual inductance M , the flux is given as:

$$\phi = MI$$

$$\therefore MI = \frac{\mu_0 I a}{2\pi} \log_e \left(\frac{x+a}{x} \right)$$

$$M = \frac{\mu_0 a}{2\pi} \log_e \left(\frac{x+a}{x} \right)$$

$$(b) \text{ Emf induced in the loop, } e = B'av = \left(\frac{\mu_0 I}{2\pi x} \right) av$$

Given,

$$I = 50 \text{ A}$$

$$x = 0.2 \text{ m}$$

$$a = 0.1 \text{ m}$$

$$v = 10 \text{ m/s}$$

$$e = \frac{4\pi \times 10^{-7} \times 50 \times 0.1 \times 10}{2\pi \times 0.2}$$

$$e = 5 \times 10^{-5} \text{ V}$$

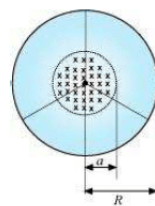
Question 6.17:

A line charge λ per unit length is lodged uniformly onto the rim of a wheel of mass M and radius R . The wheel has light non-conducting spokes and is free to rotate without friction about its axis (Fig. 6.22). A uniform magnetic field extends over a circular region within the rim. It is given by,

$$\mathbf{B} = -B_0 \mathbf{k} \quad (r \leq a; a < R)$$

$$= 0 \text{ (otherwise)}$$

What is the angular velocity of the wheel after the field is suddenly switched off?



Answer

$$= \lambda = \frac{\text{Total charge}}{\text{Length}} = \frac{Q}{2\pi r}$$

Line charge per unit length

Where,

r = Distance of the point within the wheel

Mass of the wheel = M

Radius of the wheel = R

Magnetic field, $\vec{B} = -B_0 \hat{k}$

At distance r , the magnetic force is balanced by the centripetal force i.e.,

$$BQv = \frac{Mv^2}{r}$$

Where,

v = linear velocity of the wheel

$$\therefore B2\pi r\lambda = \frac{Mv}{r}$$

$$v = \frac{B2\pi\lambda r^2}{M}$$

$$v = \frac{B}{M}$$

$$\therefore \text{Angular velocity, } \omega = \frac{v}{R} = \frac{B2\pi\lambda r^2}{MR}$$

For $r \leq a$ and $a < R$, we get:

$$\omega = -\frac{2B_0}{MR} \frac{a^2 \lambda}{k} \hat{k}$$

***** END *****