Factorisation of Polynomials Ex 6.4 Q19

Answer :

Let $f(x)=ax^3+x^2-2x+b$ be the given polynomial.

By factor theorem, if $(x+1)$ and $(x-1)$ both are factors of the polynomial f (x). if $f(-1)$ and $f(1)$ both are equal to zero.

Therefore,

$$f(-1)=a(-1)^3+(-1)^2-2(-1)+b=0$$
$$-a+1+2+b=0$$
$$-a+b=-3 \quad \text{...(i)}$$

And

$$f(1)=a(1)^3+(1)^2-2(1)+b=0$$
$$a+1-2+b=0$$
$$a+b=1 \quad \text{...(ii)}$$

Adding (i) and (ii), we get

$2b=-2$

$b=-1$

And putting this value in equation (ii), we get,

$a=2$

Hence, the value of a and b are 2 and -1 respectively.

Factorisation of Polynomials Ex 6.4 Q20

Answer :

Let $p(x)=x^3-3x^2-12x+19$ and $q(x)=x^2+x-6$ be the given polynomial.

When $p(x)$ is divided by $q(x)$, the reminder is a linear polynomial in x.

So, let $r(x)=ax+b$ is added to $p(x)$, so that $p(x)+r(x)$ is divisible by $q(x)$.

Let $f(x)=p(x)+r(x)$

Then,

$$f(x)=x^3-3x^2-12x+19+ax+b$$
$$=x^3-3x^2+(a-12)x+(19+b)$$

We have,

$$q(x)=x^2+x-6$$
$$=(x+3)(x-2)$$

Clearly, $q(x)$ is divisible by $(x+3)$ and $(x-2)$ i.e., $(x+3)$ and $(x-2)$ are the factors of $q(x)$.

Therefore, $f(x)$ is divisible by $q(x)$, if $(x+3)$ and $(x-2)$ are factors of $f(x)$, i.e.,

$f(-3)$ and $f(2)=0$

Now, $f(-3)=0$

$\Rightarrow f(-3)=(-3)^3-3(-3)^2+(a-12)(-3)+19+b=0$

$\Rightarrow -27-27-3a+36+19+b=0$

$\Rightarrow$ -27 - 27 - 3a + 36 + 19 + b = 0

$\Rightarrow$ -54 - 3a + b + 55 = 0

$\Rightarrow$ -3a + b + 1 = 0 ---- (i)

And

$$f(2) = (2)^3 - 3(2)^2 + (a-12) + 19 + b = 0$$
$$8 - 12 + 2a - 24 + 19 + b = 0$$
$$2a + b = 9 \qquad ...(ii)$$

Subtracting (i) from (ii), we get,

$$(2a+b) - (-3a+b) = 10$$
$$5a = 10$$
$$a = 2$$

Putting this value in equation (ii), we get,

$$\Rightarrow 2 \times 2 + b = 9$$
$$b = 5$$

Hence, $p(x)$ is divisible by $q(x)$ if $(2x+5)$ added to it.

Factorisation of Polynomials Ex 6.4 Q21

Answer :

By divisible algorithm, when $p(x) = x^3 - 6x^2 - 15x + 80$ is divided by $x^2 + x - 12,$ the reminder is a linear polynomial

Let $r(x) = a(x) + b$ be subtracted from $p(x)$ so that the result is divisible by $q(x)$.

Let

$$f(x) = p(x) - q(x)$$
$$= x^3 - 6x^2 - 15x + 80 - (ax + b)$$
$$= x^3 - 6x^2 - (a+15)x + 80 - b$$

We have,

$$q(x) = x^2 + x - 12$$
$$= x^2 + 4x - 3x - 12$$
$$= (x+4)(x-3)$$

Clearly, $(x+4)$ and $(x-3)$ are factors of $q(x)$, therefore, $f(x)$ will be divisible by $q(x)$ if $(x+4)$ and $(x-3)$ are factors of $f(x)$, i.e. $f(-4)$ and $f(3)$ are equal to zero.

Therefore,

$$f(-4) = (-4)^3 - 6(-4)^2 - (a+15)(-4) + 80 - b = 0$$
$$-64 - 96 + 4a + 60 + 80 - b = 0$$
$$-20 + 4a - b = 0$$
$$+4a - b = 20$$

and

$$f(3) = (3)^3 - 6(3)^2 - (a+15)(3) + 80 - b = 0$$
$$27 - 54 - 3a - 45 + 80 - b = 0$$
$$-3a - b = 8$$
$$3a + b = 8$$

Adding (i) and (ii), we get,

$\boxed{a = 4}$

Putting this value in equation (i), we get,

$b = -4$

Hence, $x^3 - 6x^2 - 15x + 80$ will be divisible by $x^2 + x - 12,$ if $4x - 4$ is subtracted from it

*********** END **********