CHAPTER 31

CAPACITORS

31.1 CAPACITOR AND CAPACITANCE

A combination of two conductors placed close to each other is called a *capacitor*. One of the conductors is given a positive charge and the other is given an equal negative charge. The conductor with the positive charge is called the *positive plate* and the other is called the *negative plate*. The charge on the positive plate is called *the charge on the capacitor* and the potential difference between the plates is called the *potential of the capacitor*. Figure (31.1a) shows two conductors. One of the conductors has a positive charge $+Q$ and the other has an equal, negative charge $-Q$. The first one is at a potential V_+ and the other is at a potential V_-. The charge on the capacitor is Q and the potential of the capacitor is $V = V_+ - V_-$. Note that the term *charge on a capacitor* does not mean the total charge given to the capacitor. This total charge is $+Q - Q = 0$. Figure (31.1b) shows the symbol used to represent a capacitor.

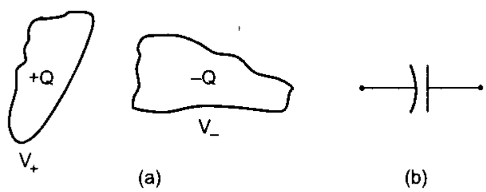

Figure 31.1

For a given capacitor, the charge Q on the capacitor is proportional to the potential difference V between the plates

Thus, $\quad Q \propto V$

or, $\quad Q = CV. \quad \ldots (31.1)$

The proportionality constant C is called the *capacitance* of the capacitor. It depends on the shape, size and geometrical placing of the conductors and the medium between them.

The SI unit of capacitance is coulomb per volt which is written as farad. The symbol F is used for it. This is a large unit on normal scales and microfarad (μF) is used more frequently.

To put equal and opposite charges on the two conductors, they may be connected to the terminals of a *battery*. We shall discuss in somewhat greater detail about the battery in the next chapter. Here we state the following properties of an ideal battery.

(a) A battery has two terminals.

(b) The potential difference V between the terminals is constant for a given battery. The terminal with higher potential is called the *positive terminal* and that with lower potential is called the *negative terminal*.

(c) The value of this fixed potential difference is equal to the *electromotive force* or *emf* of the battery. If a conductor is connected to a terminal of a battery, the potential of the conductor becomes equal to the potential of the terminal. When the two plates of a capacitor are connected to the terminals of a battery, the potential difference between the plates of the capacitor becomes equal to the emf of the battery.

(d) The total charge in a battery always remains zero. If its positive terminal supplies a charge Q, its negative terminal supplies an equal, negative charge $-Q$.

(e) When a charge Q passes through a battery of emf $\mathcal{E}$ from the negative terminal to the positive terminal, an amount $Q\mathcal{E}$ of work is done by the battery.

An ideal battery is represented by the symbol shown in figure (31.2). The potential difference between the facing parallel lines is equal to the emf $\mathcal{E}$ of the battery. The longer line is at the higher potential.

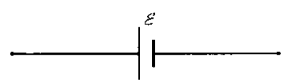

Figure 31.2

Example 31.1

A capacitor gets a charge of 60 μC *when it is connected to a battery of emf* 12 V. *Calculate the capacitance of the capacitor.*

Solution : The potential difference between the plates is the same as the emf of the battery which is 12 V. Thus, the capacitance is

$$C = \frac{Q}{V} = \frac{60\ \mu C}{12\ V} = 5\ \mu F.$$

31.2 CALCULATION OF CAPACITANCE

The procedure to calculate the capacitance of a given capacitor is simple. We assume that a charge $+Q$ is placed on the positive plate and a charge $-Q$ is placed on the negative plate of the capacitor. We calculate the electric field between the plates and from this the potential difference between the plates. The capacitance is then obtained using equation (31.1).

Parallel-plate Capacitor

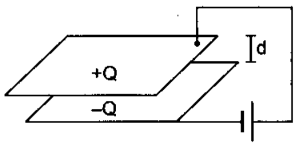

Figure 31.3

A parallel-plate capacitor consists of two large plane plates placed parallel to each other with a small separation between them (figure 31.3). Suppose, the area of each of the facing surfaces is A and the separation between the two plates is d. Also, assume that the space between the plates contains vacuum.

Let us put a charge Q on one plate and a charge $-Q$ on the other. The charges will appear on the facing surfaces. The charge density on each of these surfaces has a magnitude

$$\sigma = \frac{Q}{A}.$$

Suppose that the plates are large as compared to the separation between them. This means that any linear dimension of the plates is much larger than the separation d. For example, if the plates are square in shape, the length of a side should be much larger than d. If we use circular plates, the diameter should be much larger than d. The electric field between the plates is then uniform and perpendicular to the plates except for a small region near the edge. The magnitude of this uniform field E may be calculated using Gauss's law.

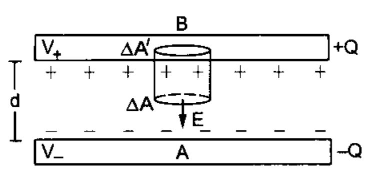

Figure 31.4

Let us draw a small area ΔA parallel to the plates and in between them (figure 31.4). Draw a cylinder with ΔA as a cross-section and terminate it by another symmetrically situated area $\Delta A'$ inside the positive plate. The cylinder and the two cross-sections ΔA and $\Delta A'$ form a Gaussian surface. The flux through $\Delta A'$ and through the curved part inside the plate is zero as the electric field is zero inside a conductor. The flux through the curved part outside the plates is also zero as the direction of the field E is parallel to this surface. The flux through ΔA is

$$\Phi = \vec{E} \cdot \Delta\vec{A} = E\,\Delta A.$$

The only charge inside the Gaussian surface is

$$\Delta Q = \sigma\,\Delta A = \frac{Q}{A}\,\Delta A.$$

From Gauss's law,

$$\oint \vec{E} \cdot d\vec{S} = Q_{in}/\varepsilon_0$$

or, $$E\,\Delta A = \frac{Q}{\varepsilon_0 A}\,\Delta A$$

or, $$E = \frac{Q}{\varepsilon_0 A}.$$

The potential difference between the plates is

$$V = V_+ - V_- = -\int_A^B \vec{E} \cdot d\vec{r}.$$

As one goes from A to B, the field $\vec{E}$ and the displacement $d\vec{r}$ are opposite in direction. Thus, $\vec{E} \cdot d\vec{r} = -E\,dr$ and

$$V = \int_A^B E\,dr$$

$$= Ed = \frac{Qd}{\varepsilon_0 A}.$$

The capacitance of the parallel-plate capacitor is

$$C = \frac{Q}{V} = \frac{Q\varepsilon_0 A}{Qd}$$

$$= \frac{\varepsilon_0 A}{d}. \qquad \dots (31.2)$$

Example 31.2

Show that the SI unit of ε_0 may be written as farad metre^{-1}.

Solution :

We have $C = \dfrac{\varepsilon_0 A}{d}$

or, $\varepsilon_0 = \dfrac{Cd}{A}$.

As the SI units of C, d and A are farad, metre and metre^2 respectively, the SI unit of ε_0 is farad metre^{-1}.

Example 31.3

Calculate the capacitance of a parallel-plate capacitor having $20\ \text{cm} \times 20\ \text{cm}$ square plates separated by a distance of 1·0 mm.

Solution : The capacitance is

$$C = \frac{\varepsilon_0 A}{d}$$

$$= \frac{8{\cdot}85 \times 10^{-12}\ \mathrm{F\ m^{-1}} \times 400 \times 10^{-4}\ \mathrm{m}^2}{1 \times 10^{-3}\ \mathrm{m}}$$

$$= 3{\cdot}54 \times 10^{-10}\ \mathrm{F} \approx 350\ \mathrm{pF}.$$

Spherical Capacitor

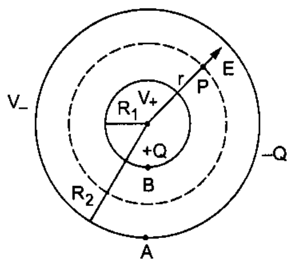

Figure 31.5

A spherical capacitor consists of a solid or a hollow spherical conductor surrounded by another concentric hollow spherical conductor. Suppose, the inner sphere has a radius R_1 and the outer sphere has a radius R_2. Suppose, the inner sphere is given a positive charge Q and the outer is given a negative charge $-Q$.

The field at any point P between the spheres is radially outward and its magnitude depends only on its distance r from the centre. Let us draw a sphere through P concentric with the given system. The flux of the electric field through this sphere is

$$\Phi = \oint \vec{E}\cdot d\vec{S} = \oint E\, dS$$

$$= E \oint dS = E\, 4\pi r^2.$$

The charge enclosed in this sphere is Q. Thus, from Gauss's law,

$$E\, 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

or,
$$E = \frac{Q}{4\pi\varepsilon_0 r^2}\cdot$$

The potential difference between the two conductors is

$$V = V_+ - V_- = -\int_A^B \vec{E}\cdot d\vec{r}$$

$$= -\int_{R_2}^{R_1} \frac{Q}{4\pi\varepsilon_0 r^2}\, dr$$

$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{Q(R_2 - R_1)}{4\pi\varepsilon_0 R_1 R_2}\cdot$$

The capacitance of the spherical capacitor is

$$C = \frac{Q}{V}$$

$$= \frac{4\pi\varepsilon_0 R_1 R_2}{R_2 - R_1}\cdot \qquad \ldots (31.3)$$

Isolated sphere

If we assume that the outer sphere is at infinity, we get an isolated single sphere of radius R_1. The capacitance of such a single sphere can be obtained from equation (31.3) by taking the limit as $R_2 \to \infty$. Then

$$C = \frac{4\pi\varepsilon_0 R_1 R_2}{R_2 - R_1}$$

$$\approx \frac{4\pi\varepsilon_0 R_1 R_2}{R_2} = 4\pi\varepsilon_0 R_1.$$

If a charge Q is placed on this sphere, its potential (with zero potential at infinity) becomes

$$V = \frac{Q}{C} = \frac{Q}{4\pi\varepsilon_0 R_1}\cdot$$

Parallel limit

If both R_1 and R_2 are made large but $R_2 - R_1 = d$ is kept fixed, we can write

$$4\pi R_1 R_2 \approx 4\pi R^2 = A$$

where R is approximately the radius of each sphere and A is the area. Equation (31.3) then becomes

$$C = \frac{\varepsilon_0 A}{d}$$

which is the same as the equation for the capacitance of a parallel-plate capacitor.

Cylindrical Capacitor

A cylindrical capacitor consists of a solid or a hollow cylindrical conductor surrounded by another coaxial hollow cylindrical conductor. Let the length of the cylinders be l and the radii of the inner and outer cylinders be R_1 and R_2 respectively. Suppose, a positive charge Q is placed on the inner cylinder and a negative charge $-Q$ is placed on the outer cylinder. If the cylinders are long as compared to the separation between them, the electric field at a point between the cylinders will be radial and its magnitude will depend only on the distance of the point from the axis. Let P be a point between the cylinders at a distance r from the axis (figure 31.6).

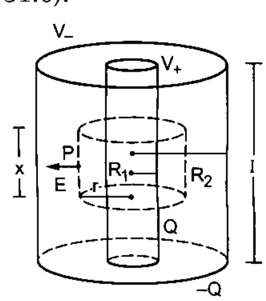

Figure 31.6

To calculate the electric field at the point P, let us draw a coaxial cylinder of length x through the point P. This cylinder together with its two cross sections forms a Gaussian surface. The flux through the cross sections is zero because the electric field is radial wherever it exists and hence is parallel to the cross sections. The flux through the curved part is

$$\Phi = \oint \vec{E}\cdot d\vec{S}$$
$$= \int E\, dS$$
$$= E\int dS = E\, 2\pi r x.$$

The charge enclosed by the Gaussian surface is

$$Q_{in} = \frac{Q}{l}x.$$

Thus, from Gauss's law,

$$E\, 2\pi r x = \left(\frac{Q}{l}x\right)/\varepsilon_0$$

or, $$E = \frac{Q}{2\pi\varepsilon_0 r l}.$$

The potential difference between the cylinders is

$$V = V_+ - V_-$$
$$= -\int_A^B \vec{E}\cdot d\vec{r} = -\int_{R_2}^{R_1} E\, dr$$
$$= -\int_{R_2}^{R_1} \frac{Q}{2\pi\varepsilon_0 r l}\, dr$$
$$= \frac{Q}{2\pi\varepsilon_0 l}\ln\frac{R_2}{R_1}.$$

The capacitance is

$$C = \frac{Q}{V} = \frac{2\pi\varepsilon_0 l}{\ln(R_2/R_1)}. \quad \dots (31.4)$$

31.3 COMBINATION OF CAPACITORS

Two or more capacitors may be connected in a number of ways. The combination should have two points which may be connected to a battery to apply a potential difference. The battery supplies positive and negative charges to the system. If V be the potential difference between the points and Q be the magnitude of the charge supplied by either terminal of the battery, we define *equivalent capacitance* of the combination *between the two points* to be

$$C = \frac{Q}{V}.$$

If the combination is replaced by a single capacitor of this capacitance, the single capacitor will store the same amount of charge for a given potential difference as the combination does.

Two special methods of combination are frequently used, one known as *series* combination and the other as *parallel* combination.

Series Combination

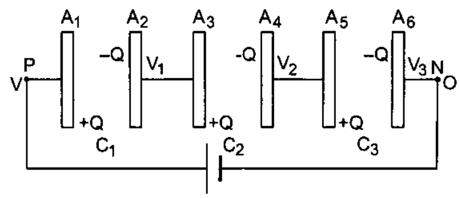

Figure 31.7

Figure (31.7) shows three capacitors connected in series. The capacitances are C_1, C_2 and C_3. The points P and N serve as the points through which a potential difference may be applied and a charge may be supplied to the combination. Let us connect the point P to the positive terminal and the point N to the negative terminal of a battery. The battery supplies a charge $+Q$ to the plate A_1 and a charge $-Q$ to the plate A_6. The charge $+Q$ given by the battery appears on the right surface of the plate A_1. The facing surface of A_2 must have a charge $-Q$ on it.

The plates A_2 and A_3 are connected and they together are isolated from everything else. The charge $-Q$ appearing on A_2 comes from the electrons drifted from the plate A_3 to A_2. This leaves a positive charge $+Q$ on the plate A_3. The facing surface of A_4 gets a charge $-Q$ from A_5 and a charge $+Q$ appears on the right surface of A_5. The facing surface of A_6 gets a charge $-Q$ from the battery. This completes the charge distribution. *In series combination, each capacitor has equal charge for any value of capacitances.*

Let us take the potential of the point N to be zero. The potential of the plate A_6 is also zero as it is connected to N by a conducting wire. The potential of the point P as well as that of the plate A_1 is V. The plates A_2 and A_3 are at the same potential, say, V_1. Similarly, A_4 and A_5 are at the same potential, say, V_2.

The charge on the first capacitor is Q and the potential difference is $V - V_1$. As the capacitance of this capacitor is C_1, we have

$$Q = C_1(V - V_1)$$

or, $$V - V_1 = \frac{Q}{C_1}. \quad \dots (i)$$

Similarly, considering the other capacitors,

$$V_1 - V_2 = \frac{Q}{C_2} \quad \dots (ii)$$

and $$V_2 - 0 = \frac{Q}{C_3}. \quad \dots (iii)$$

Adding (i), (ii) and (iii);

$$V = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right). \qquad \dots \text{(iv)}$$

If the equivalent capacitance of the combination between the points P and N is C, we have

$$C = \frac{Q}{V}$$

and equation (iv) becomes

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

The above analysis may be extended to any number of capacitors, the equivalent capacitance C is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots \qquad \dots \text{(31.5)}$$

Example 31.4

Calculate the charge on each capacitor shown in figure (31.8).

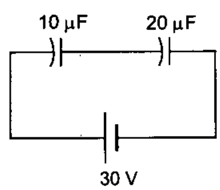

Figure 31.8

Solution : The two capacitors are joined in series. Their equivalent capacitance is given by $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

or, $$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{(10\ \mu\text{F})\,(20\ \mu\text{F})}{30\ \mu\text{F}} = \frac{20}{3}\ \mu\text{F}.$$

The charge supplied by the battery is

$$Q = CV$$

$$= \left(\frac{20}{3}\ \mu\text{F}\right)(30\ \text{V}) = 200\ \mu\text{C}.$$

In series combination, each capacitor has equal charge and this charge equals the charge supplied by the battery. Thus, each capacitor has a charge of 200 µC.

Parallel Combination

Figure (31.9) shows three capacitors connected in parallel. The capacitances are C_1, C_2 and C_3. The points

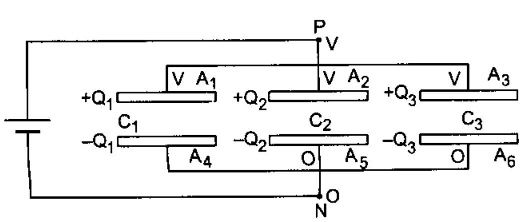

Figure 31.9

P and N are the two points through which a potential difference can be applied and charge can be supplied. Let us connect the point P to the positive terminal of a battery and the point N to its negative terminal. The battery supplies a charge $+Q$ which is distributed on the three positive plates A_1, A_2 and A_3 of the capacitors. Let the charges on the three plates A_1, A_2 and A_3 be Q_1, Q_2 and Q_3 respectively. The battery also supplies a charge $-Q$ which is distributed on the three plates A_4, A_5 and A_6. These plates must receive charges $-Q_1$, $-Q_2$ and $-Q_3$ respectively because the facing surfaces must have equal and opposite charges. We have

$$Q = Q_1 + Q_2 + Q_3. \qquad \dots \text{(i)}$$

Let us take the potential of the point N to be zero. The potentials of the plates A_4, A_5 and A_6 are also zero as they are all connected to N by conducting wires. Let the potential of the point P be V. This will also be the potential of the plates A_1, A_2 and A_3. Thus, *the potential differences of the capacitors connected in parallel are equal for any value of capacitances.* Using the equation $Q = CV$ for the three capacitors,

$$Q_1 = C_1 V \qquad \dots \text{(ii)}$$

$$Q_2 = C_2 V \qquad \dots \text{(iii)}$$

and $$Q_3 = C_3 V. \qquad \dots \text{(iv)}$$

Adding (ii), (iii) and (iv) and using (i),

$$Q = (C_1 + C_2 + C_3)V$$

or, $$\frac{Q}{V} = C_1 + C_2 + C_3.$$

But Q/V is the equivalent capacitance of the given combination. Thus,

$$C = C_1 + C_2 + C_3. \qquad \dots \text{(31.6)}$$

In parallel combination, all the positive plates are at the same potential and all the negative plates are at the same potential. The potential difference on each capacitor is the same in parallel combination but the charges on the capacitors may be different. In series combination, the charges on the capacitors are equal, the potential differences may be different.

Example 31.5

Find the equivalent capacitance of the combination shown in figure (31.10) between the points P and N.

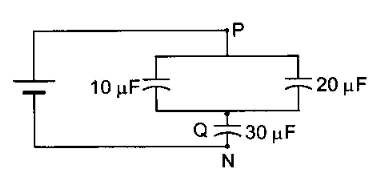

Figure 31.10

Solution : The 10 µF and 20 µF capacitors are connected in parallel. Their equivalent capacitance is

$10\ \mu F + 20\ \mu F = 30\ \mu F$. We can replace the $10\ \mu F$ and the $20\ \mu F$ capacitors by a single capacitor of capacitance $30\ \mu F$ between P and Q. This is connected in series with the given $30\ \mu F$ capacitor. The equivalent capacitance C of this combination is given by

$$\frac{1}{C} = \frac{1}{30\ \mu F} + \frac{1}{30\ \mu F} \quad \text{or,} \quad C = 15\ \mu F.$$

We have used series–parallel combination to solve the above example. Sometimes it may not be easy to find the equivalent capacitance of a combination using the equations for series–parallel combinations. We may then use the general method which was applied to derive the equivalent capacitance in series and parallel combinations. For any given combination, one may proceed as follows:

Step 1

Identify the two points between which the equivalent capacitance is to be calculated. Call any one of them as P and the other as N.

Step 2

Connect (mentally) a battery between P and N with the positive terminal connected to P and the negative terminal to N. Send a charge $+Q$ from the positive terminal of the battery and $-Q$ from the negative terminal of the battery.

Step 3

Write the charges appearing on each of the plates of the capacitors. The charge conservation principle may be used. The facing surfaces of a capacitor will always have equal and opposite charges. Assume variables Q_1, Q_2 ..., etc., for charges wherever needed.

Step 4

Take the potential of the negative terminal N to be zero and that of the positive terminal P to be V. Write the potential of each of the plates. If necessary, assume variables V_1, V_2 ... .

Step 5

Write the capacitor equation $Q = CV$ for each capacitor. Eliminate Q_1, Q_2, ... and V_1, V_2, ..., etc., to obtain the equivalent capacitance $C = Q/V$.

Example 31.6

Find the equivalent capacitance of the combination shown in figure (31.11a) between the points P and N.

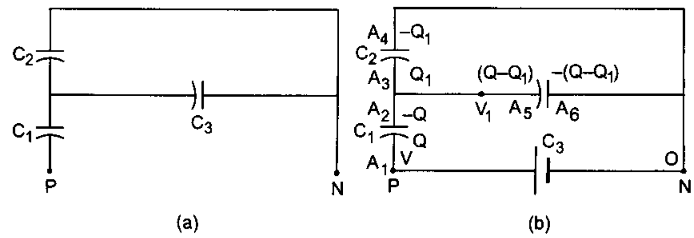

Figure 31.11

Solution : Let us connect a battery between the points P and N. The charges and the potentials are shown in figure (31.11b). The positive terminal of the battery supplies a charge $+Q$ which appears on the plate A_1. The facing plate A_2 gets a charge $-Q$. The plates A_2, A_3 and A_5 taken together form an isolated system. The total charge on these three plates should be zero. Let a charge Q_1 appear on A_3, then a charge $Q - Q_1$ will appear on A_5 to make the total charge zero on the three plates. The plate A_4 will get a charge $-Q_1$ (facing plate of A_3) and A_6 will get a charge $-(Q - Q_1)$ (facing plate of A_5). The total charge $-Q$ on A_4 and A_6 is supplied by the negative terminal of the battery. This completes the charge distribution.

Next, suppose the potential at the point N is zero and at P it is V. The potential of the plates A_4 and A_6 is also zero. The potential of the plate A_1 is V. The plates A_2, A_3 and A_5 are at the same potential. Let this common potential be V_1. This completes the potential distribution.

Applying the capacitor equation $Q = CV$ to the three capacitors,

$$Q = C_1(V - V_1) \qquad \text{... (i)}$$

$$Q_1 = C_2 V_1 \qquad \text{... (ii)}$$

and $$Q - Q_1 = C_3 V_1. \qquad \text{... (iii)}$$

Adding (ii) and (iii),

$$Q = (C_2 + C_3)V_1$$

or, $$\frac{Q}{C_2 + C_3} = V_1. \qquad \text{... (iv)}$$

From (i), $$\frac{Q}{C_1} = V - V_1. \qquad \text{... (v)}$$

Adding (iv) and (v),

$$\frac{Q}{C_2 + C_3} + \frac{Q}{C_1} = V$$

or, $$\frac{(C_1 + C_2 + C_3)Q}{C_1(C_2 + C_3)} = V$$

or, $$C = \frac{Q}{V} = \frac{C_1(C_2 + C_3)}{C_1 + C_2 + C_3}.$$

It may be noted that the above example could be solved by using the equations for series–parallel combinations. However, the general method was used to demonstrate its application.

Symmetry arguments play important role in simplifying the algebra involved in the problem. The use of symmetry arguments in writing the charges on different plates will be demonstrated later in the section of worked out examples.

31.4 FORCE BETWEEN THE PLATES OF A CAPACITOR

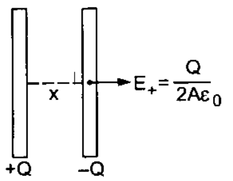

Figure 31.12

Consider a parallel-plate capacitor with plate area A. Suppose a positive charge $+Q$ is given to one plate and a negative charge $-Q$ to the other plate. The electric field due to only the positive plate is

$$E_+ = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$

at all points if the plate is large. The negative charge $-Q$ finds itself in the field of this positive charge. The force on $-Q$ is, therefore,

$$F = -QE_+$$

$$= (-Q)\frac{Q}{2A\varepsilon_0} = -\frac{Q^2}{2A\varepsilon_0}.$$

The magnitude of the force is

$$F = \frac{Q^2}{2A\varepsilon_0}.$$

This is the force with which the positive plate attracts the negative plate. This is also the force of attraction on the positive plate by the negative plate. Thus, the plates of a parallel-plate capacitor attract each other with a force

$$F = \frac{Q^2}{2A\varepsilon_0}. \qquad \ldots (31.7)$$

31.5 ENERGY STORED IN A CAPACITOR AND ENERGY DENSITY IN ELECTRIC FIELD

Let us consider a parallel-plate capacitor of plate area A (figure 31.13). Suppose the plates of the capacitor are almost touching each other and a charge Q is given to the capacitor. One of the plates, say a, is kept fixed and the other, say b, is slowly pulled away from a to increase the separation from zero to d. The attractive force on the plate b at any instant due to the first plate is, from equation (31.7),

$$F = \frac{Q^2}{2A\varepsilon_0}.$$

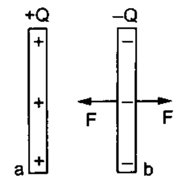

Figure 31.13

The person pulling the plate b must apply an equal force F in the opposite direction if the plate is only slowly moved.

The work done by the person during the displacement of the second plate is

$$W = Fd$$

$$= \frac{Q^2 d}{2A\varepsilon_0} = \frac{Q^2}{2C}$$

where C is the capacitance of the capacitor in the final position. The work done by the person must be equal to the increase in the energy of the system. Thus, a capacitor of capacitance C has a stored energy

$$U = \frac{Q^2}{2C} \qquad \ldots (31.8)$$

where Q is the charge given to it. Using $Q = CV$, the above equation may also be written as

$$U = \frac{1}{2}CV^2 \qquad \ldots (31.9)$$

or, $$U = \frac{1}{2}QV. \qquad \ldots (31.10)$$

Example 31.7

Find the energy stored in a capacitor of capacitance 100 μF *when it is charged to a potential difference of* 20 V.

Solution : The energy stored in the capacitor is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(100\,\mu\text{F})\,(20\,\text{V})^2 = 0{\cdot}02\ \text{J}.$$

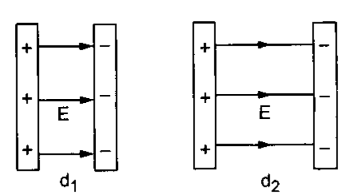

Figure 31.14

The energy stored in a capacitor is electrostatic potential energy. When we pull the plates of a capacitor apart, we have to do work against the electrostatic attraction between the plates. In which region of space is the energy stored ? When we increase the separation between the plates from d_1 to d_2, an amount $\frac{Q^2}{2A\varepsilon_0}(d_2 - d_1)$ of work is performed by us and this much energy goes into the capacitor. On the other hand, new electric field is created in a volume $A(d_2 - d_1)$ (figure 31.14). We conclude that the energy $\frac{Q^2}{2A\varepsilon_0}(d_2 - d_1)$ is stored in the volume $A(d_2 - d_1)$ which is now filled with the electric field. Thus, an electric field has energy associated with it. The energy stored per unit volume in the electric field is

$$u = \frac{\frac{Q^2(d_2 - d_1)}{2A\varepsilon_0}}{A(d_2 - d_1)} = \frac{Q^2}{2A^2\varepsilon_0}$$

$$= \frac{1}{2}\varepsilon_0\left(\frac{Q}{A\varepsilon_0}\right)^2 = \frac{1}{2}\varepsilon_0 E^2$$

where E is the intensity of the electric field.

Once it is established that a region containing electric field E has energy $\frac{1}{2}\varepsilon_0 E^2$ per unit volume, the result can be used for any electric field whether it is due to a capacitor or otherwise.

31.6 DIELECTRICS

In dielectric materials, effectively there are no free electrons. The monatomic materials are made of atoms. Each atom consists of a positively charged nucleus surrounded by electrons. In general, the centre of the negative charge coincides with the centre of the positive charge. Polyatomic materials, on the other hand, are made of molecules. The centre of the negative charge distribution in a molecule may or may not coincide with the centre of the positive charge distribution. If it does not coincide, each molecule has a permanent dipole moment $\vec{p}$. Such materials are known as *polar materials*. However, different molecules have different directions of the dipole moment because of the random thermal agitation in the material. In any volume containing a large number of molecules (say more than a thousand), the net dipole moment is zero. If such a material is placed in an electric field, the individual dipoles experience torque due to the field and they try to align along the field. On the other hand, thermal agitation tries to randomise the orientation and hence, there is a partial alignment. As a result, we get a net dipole moment in any volume of the material.

In nonpolar materials, the centre of the positive charge distribution in an atom or a molecule coincides with the centre of the negative charge distribution. The atoms or the molecules do not have any permanent dipole moment. If such a material is placed in an electric field, the electron charge distribution is slightly shifted opposite to the electric field. This induces dipole moment in each atom or molecule and thus, we get a dipole moment in any volume of the material.

Thus, when a dielectric material is placed in an electric field, dipole moment appears in any volume in it. This fact is known as *polarization* of the material. The *polarization vector* $\vec{P}$ is defined as the dipole moment per unit volume. Its magnitude P is often referred to as the polarization.

Consider a rectangular slab of a dielectric. The individual dipole moments are randomly oriented (figure 31.15a). In any volume containing a large number of molecules, the net charge is zero. When an electric field is applied, the dipoles get aligned along the field. Figure (31.15b) and (31.15c) show the effect of dipole alignment when a field is applied from left to right. We see that the interior is still charge free but the left surface of the slab gets negative charge and the right surface gets positive charge. The situation may be represented as in figure (31.15d). The charge appearing on the surface of a dielectric when placed in an electric field is called *induced charge*. As the induced charge appears due to a shift in the electrons bound to the nuclei, this charge is also called *bound charge*.

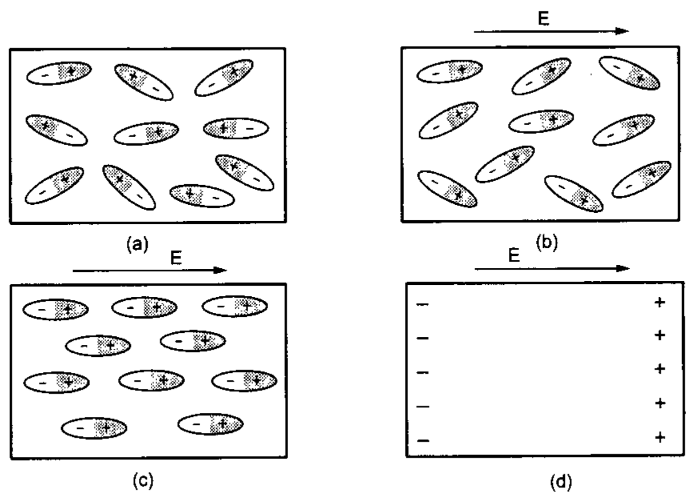

Figure 31.15

The surface charge density of the induced charge has a simple relationship with the polarization P. Suppose, the rectangular slab of figure (31.15) has a length l and area of cross-section A. Let σ_p be the magnitude of the induced charge per unit area on the faces. The dipole moment of the slab is then $(\sigma_p A)l = \sigma_p(Al)$. The polarization is dipole moment induced per unit volume. Thus,

$$P = \frac{\sigma_p(Al)}{Al} = \sigma_p. \qquad \ldots (31.11)$$

Although this result is deduced for a rectangular slab, it is true in general. The induced surface charge density is equal in magnitude to the polarization P.

Dielectric Constant

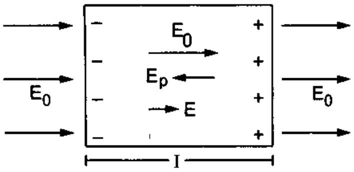

Figure 31.16

Because of the induced charges, an extra electric field is produced inside the material. Let $\vec{E}_0$ be the applied field due to external sources and $\vec{E}_P$ be the field due to polarization (figure 31.16). The resultant field is $\vec{E} = \vec{E}_0 + \vec{E}_P$. For homogeneous and isotropic dielectrics, the direction of $\vec{E}_P$ is opposite to the direction of $\vec{E}_0$. The resultant field $\vec{E}$ is in the same direction as the applied field $\vec{E}_0$ but its magnitude is reduced. We can write

$$\vec{E} = \frac{\vec{E}_0}{K}$$

where K is a constant for the given dielectric which has a value greater than one. This constant K is called the *dielectric constant* or *relative permittivity* of the dielectric. For vacuum, there is no polarization and hence $\vec{E} = \vec{E}_0$ and $K = 1$.

If a very high electric field is created in a dielectric, the outer electrons may get detached from their parent atoms. The dielectric then behaves like a conductor. This phenomenon is known as *dielectric breakdown*. The minimum field at which the breakdown occurs is called the *dielectric strength* of the material. Table (31.1) gives dielectric constants and dielectric strengths for some of the dielectrics.

31.7. PARALLEL-PLATE CAPACITOR WITH A DIELECTRIC

Consider a parallel-plate capacitor with plate area A and separation d between the plates (figure 31.17). A dielectric slab of dielectric constant K is inserted in the space between the plates. Suppose, the slab almost completely fills the space between the plates. A charge Q is given to the positive plate and $-Q$ to the negative plate of the capacitor. The electric field polarizes the dielectric so that induced charges $+Q_P$ and $-Q_P$ appear on the two faces of the slab.

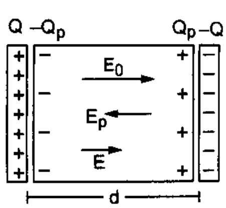

Figure 31.17

Table 31.1 : ***Dielectric constants and dielectric strengths***

Material	Dielectric constant	Dielectric strength ($kVmm^{-1}$)
Vacuum	1	∞
Pyrex Glass	5·6	≈ 14
Mica	3–6	12
Neoprene rubber	6·9	12
Bakelite	4·9	24
Plexiglas	3·40	40
Fused quartz	3·8	8
Paper	3·5	14
Polystyrene	2·6	25
Teflon	2·1	60
Strontium titanate	310	8
Titanium dioxide	100	6
Water	80	–
Glycerin	42·5	–
Benzene	2·3	–
Air (1 atm)	1·00059	3
Air (100 atm)	1·0548	–

The electric field at a point between the plates due to the charges $+Q$, $-Q$ on the capacitor plates is

$$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \qquad \text{... (i)}$$

in a direction left to right in the figure (31.17).

From the definition of dielectric constant, the resultant field is

$$E = \frac{E_0}{K} = \frac{Q}{\varepsilon_0 AK}. \qquad \text{... (ii)}$$

The potential difference between the plates is

$$V = Ed$$
$$= \frac{Qd}{\varepsilon_0 AK}.$$

The capacitance is

$$C = \frac{Q}{V} = \frac{K\varepsilon_0 A}{d} = KC_0 \qquad \text{... (31.12)}$$

where $C_0 = \frac{\varepsilon_0 A}{d}$ is the capacitance without the dielectric. Thus,

The capacitance of a capacitor is increased by a factor of K when the space between the plates is filled with a dielectric of dielectric constant K.

This result is often taken as the definition of the dielectric constant.

Magnitude of the Induced Charge

From (i), the electric field at a point between the plates due to the charges $+Q$, $-Q$ is

$$E_0 = \frac{Q}{A\varepsilon_0}.$$

The field due to the charges $Q_p, -Q_p$ is directed oppositely and has magnitude

$$E_p = \frac{\sigma_p}{\varepsilon_0} = \frac{Q_p}{A\varepsilon_0}.$$

The resultant field is

$$E = E_0 - E_p$$

$$= \frac{Q - Q_p}{A\varepsilon_0}. \qquad \text{... (iii)}$$

From equations (ii) and (iii),

$$\frac{Q - Q_p}{\varepsilon_0 A} = \frac{Q}{\varepsilon_0 AK}$$

or, $$Q - Q_p = \frac{Q}{K}$$

or, $$Q_p = Q\left(1 - \frac{1}{K}\right). \qquad \text{... (31.13)}$$

Example 31.8

Two parallel-plate capacitors, each of capacitance 40 μF, *are connected in series. The space between the plates of one capacitor is filled with a dielectric material of dielectric constant $K = 4$. Find the equivalent capacitance of the system.*

Solution : The capacitance of the capacitor with the dielectric is

$$C_1 = KC_0 = 4 \times 40\ \mu\text{F} = 160\ \mu\text{F}.$$

The other capacitor has capacitance $C_2 = 40\ \mu$F. As they are connected in series, the equivalent capacitance is

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{(160\ \mu\text{F})\,(40\ \mu\text{F})}{200\ \mu\text{F}} = 32\ \mu\text{F}.$$

Example 31.9

A parallel-plate capacitor has plate area A and plate separation d. The space between the plates is filled up to a thickness x (< d) with a dielectric of dielectric constant K. Calculate the capacitance of the system.

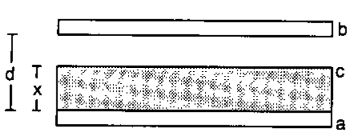

Figure 31.18

Solution :

The situation is shown in figure (31.18). The given system is equivalent to the series combination of two capacitors, one between a and c and the other between c and b. Here c represents the upper surface of the dielectric. This is because the potential at the upper surface of the dielectric is constant and we can imagine a thin metal plate being placed there.

The capacitance of the capacitor between a and c is

$$C_1 = \frac{K\varepsilon_0 A}{x}$$

and that between c and b is

$$C_2 = \frac{\varepsilon_0 A}{d - x}.$$

The equivalent capacitance is

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{K\varepsilon_0 A}{Kd - x(K - 1)}.$$

31.8 AN ALTERNATIVE FORM OF GAUSS'S LAW

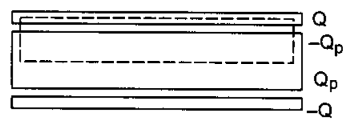

Figure 31.19

Let us again consider a parallel-plate capacitor with a charge Q. The space between the plates is filled with a dielectric slab of dielectric constant K. Let us consider a Gaussian surface as shown in figure (31.19). The charge enclosed by the surface is $Q - Q_p$. From Gauss's law,

$$\oint \vec{E}\cdot d\vec{S} = \frac{Q - Q_p}{\varepsilon_0} \qquad \text{... (i)}$$

$$= \frac{1}{\varepsilon_0}\left[Q - Q\left(1 - \frac{1}{K}\right)\right] = \frac{Q}{\varepsilon_0 K}$$

or, $$\oint K\vec{E}\cdot d\vec{S} = \frac{Q_{free}}{\varepsilon_0}. \qquad \text{... (31.14)}$$

Q_{free} is used in place of Q to emphasise that it is the free charge given to the plates and does not include the bound charge appearing due to polarization.

Equation (31.14) is taken as another form of Gauss's law. This form differs from the usual form of Gauss's law in two respects. Firstly, the charge Q_{free} appearing on the right-hand side is not the total charge inside the Gaussian surface. It is the free charge or external charge inside the Gaussian surface. The bound charge Q_p appearing due to polarization of the dielectric is left out. Secondly, an extra factor K appears on the left-hand side. The two differences compensate the effects of each other and the two forms of Gauss's law are identical. Either of the two may be used in any case.

Though we derived this result for a special case of parallel-plate capacitor, it is true in any situation where the dielectric used is homogeneous and isotropic. Let us now write Gauss's law in yet another form valid for any case.

Displacement Vector

The field due to the polarization is

$$E_p = \frac{\sigma_p}{\varepsilon_0} = \frac{P}{\varepsilon_0}$$

where P is the polarization (the dipole moment per unit volume). As the direction of E_p is opposite to the polarization vector $\vec{P}$, we write

$$\vec{E}_p = -\frac{\vec{P}}{\varepsilon_0}.$$

Now, $$\vec{E} = \vec{E}_0 + \vec{E}_p$$

or, $$\vec{E} = \vec{E}_0 - \frac{\vec{P}}{\varepsilon_0}$$

or, $$\varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \vec{E}_0 \quad \text{... (i)}$$

or, $$\oint (\varepsilon_0 \vec{E} + \vec{P}) \cdot d\vec{S} = \oint \varepsilon_0 \vec{E}_0 \cdot d\vec{S}$$

over any closed surface. As $\vec{E}_0$ is the field produced by the free charge Q_{free}, $\oint \varepsilon_0 \vec{E}_0 \cdot d\vec{S} = Q_{free}$ from Gauss's law. Thus,

$$\oint (\varepsilon_0 \vec{E} + \vec{P}) \cdot d\vec{S} = Q_{free}. \quad \text{... (ii)}$$

The quantity $\varepsilon_0 \vec{E} + \vec{P}$ is known as the *electric displacement vector* $\vec{D}$. Equation (ii) above may be written in terms of $\vec{D}$ as

$$\oint \vec{D} \cdot d\vec{S} = Q_{free} \quad \text{... (31.15)}$$

which is another form of Gauss's law.

If there is no polarization, $\vec{D} = \varepsilon_0 \vec{E}$ and Q_{free} is equal to the total charge inside the Gaussian surface. Equation (31.15) then reduces to the usual form of Gauss's law.

In case of homogeneous and isotropic dielectrics, $\vec{E}_0 = K\vec{E}$ so that equation (i) above gives $\vec{D} = \varepsilon_0 K\vec{E}$ and equation (31.15) reduces to (31.14).

31.9 ELECTRIC FIELD DUE TO A POINT CHARGE q PLACED IN AN INFINITE DIELECTRIC

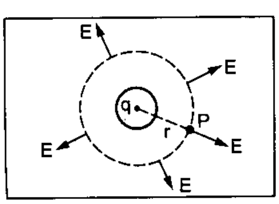

Figure 31.20

Suppose, a point charge q is placed inside an infinite dielectric and we wish to calculate the electric field at a point P at a distance r from the charge q (figure 31.20). We draw a spherical surface through P with the centre at q. From Gauss's law,

$$\oint K \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$$

or, $$KE\, 4\pi r^2 = \frac{q}{\varepsilon_0}$$

or, $$E = \frac{q}{4\pi\varepsilon_0 K r^2}. \quad \text{... (31.16)}$$

The field is radially away from the charge. Note that q is the total *free charge* inside the Gaussian surface.

It should be clear that the field $\frac{q}{4\pi\varepsilon_0 K r^2}$ is due to the free charge q and the polarization charges induced in the dielectric medium. Because of the radially outward field (assuming q to be positive), negative charges shift inward. This produces an induced charge $-q\left(1 - \frac{1}{K}\right)$ on the surface of the cavity in the dielectric in which the charge q is residing. The effective charge is, therefore, $q - q\left(1 - \frac{1}{K}\right) = q/K$ and hence the field is $\frac{q}{4\pi\varepsilon_0 K r^2}$.

31.10 ENERGY IN THE ELECTRIC FIELD IN A DIELECTRIC

Consider a parallel-plate capacitor filled with a dielectric of dielectric constant K. The energy stored in the capacitor is $U = \frac{1}{2} CV^2$. The energy density in the volume between the plates is

$$u = \frac{U}{Ad} = \frac{\frac{1}{2}\left(\frac{K\varepsilon_0 A}{d}\right)V^2}{Ad} = \frac{1}{2} K\varepsilon_0 \left(\frac{V}{d}\right)^2 = \frac{1}{2} K\varepsilon_0 E^2$$

where $E = V/d$ is the electric field between the plates.

We see that the energy density in dielectrics is greater than that in vacuum for the same electric field. The dipole moments interact with each other so as to give this additional energy.

31.11 CORONA DISCHARGE

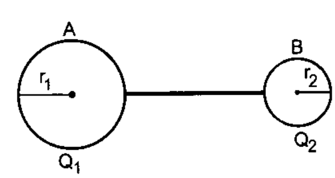

Figure 31.21

Let us consider two conducting spheres A and B connected to each other by a conducting wire. The radius of A is r_1 which is larger than the radius r_2 of B. A charge Q is given to this system. Suppose a part

Q_1 resides on the surface of A and the rest Q_2 on the surface of B. The potential of the sphere A is

$$V_1 = \frac{Q_1}{4\pi\varepsilon_0 r_1}$$

and that of the sphere B is

$$V_2 = \frac{Q_2}{4\pi\varepsilon_0 r_2}.$$

As the two spheres are connected by a conducting wire, their potentials must be the same. Thus,

$$\frac{Q_1}{4\pi\varepsilon_0 r_1} = \frac{Q_2}{4\pi\varepsilon_0 r_2}$$

or, $$\sigma_1 r_1 = \sigma_2 r_2$$

or, $$\frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1} \quad \ldots (31.17)$$

where σ_1 and σ_2 are charge densities on the two spheres. We see that the sphere with smaller radius has larger charge density to maintain the same potential.

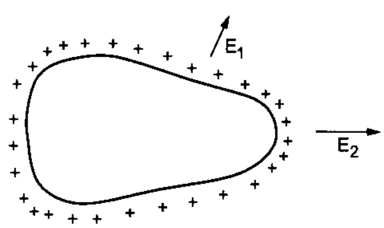

Figure 31.22

Now consider a single conductor with a nonspherical shape. If a charge is given to this conductor (figure 31.22), the charge density will not be uniform on the entire surface. A portion where the surface is more "flat" may be considered as part of a sphere of larger radius. The charge density at such a portion will be smaller from equation (31.17). At portions where the surface is more curved, the charge density will be larger. More precisely, the charge density will be larger where the radius of curvature is small.

The electric field just outside the surface of a conductor is σ/ε_0. Thus, the electric field near the portions of small radius of curvature (more curved part) is large as compared to the field near the portions of large radius of curvature (flatter part). If a conductor has a pointed shape like a needle and a charge is given to it, the charge density at the pointed end will be very high. Correspondingly, the electric field near these pointed ends will be very high which may cause dielectric breakdown in air. The charge may jump from the conductor to the air because of increased conductivity of the air. Often this discharge of air is accompanied by a visible glow surrounding the pointed end. This phenomenon is called *corona discharge.*

31.12 HIGH-VOLTAGE GENERATOR

In 1929, Robert J van de Graaff designed a machine which could produce large electrostatic potential difference, of the order of 10^7 volts. This machine, known as *van de Graaff generator,* is now described.

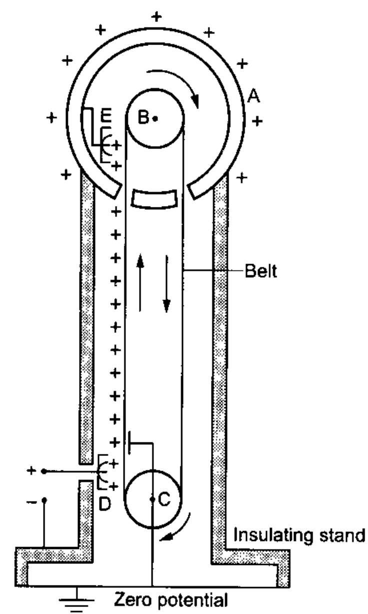

Figure 31.23

A hollow, metallic sphere A is mounted on an insulating stand. A pulley B is mounted at the centre of the sphere and another pulley C is mounted near the bottom. A belt of insulating material (such as silk) goes over the pulleys. The pulley C is continuously driven by an electric motor, or by hand for a smaller machine used for demonstration. The belt, therefore, continuously moves. Two comb-shaped conductors D and E, having a number of metallic needles, are mounted near the pulleys. The needles point towards the belt. The lower comb D is maintained at a positive potential of the order of 10^4 volts by a power supply system. The upper comb E is connected to the metallic sphere A.

Because of the high electric field near the needles of D, the air becomes conducting (corona discharge). The negative charges in the air move towards the needles and the positive charges towards the belt. This positive charge sticks to the belt. The negative charge neutralises some of the positive charge on the comb D. The power supply maintains the positive potential of the needles by supplying more positive charge to it. Effectively, positive charge is transferred from the power supply to the belt. As the belt moves, this positive charge is physically carried upwards. When it reaches near the upper comb E, corona discharge takes place and the air becomes conducting. The negative charges of the air move towards the belt and the positive charges towards the needles of the comb. The negative charges neutralise the positive charge on the belt. The positive charges of the air which have moved to the comb are transferred to the sphere. Effectively, the positive charge on the belt is transferred to the sphere. This positive charge quickly goes to the outer surface of the sphere.

The machine, thus, continuously transfers positive charge to the sphere. The potential of the sphere keeps on increasing. The main limiting factor on the value of this high potential is the radius of the sphere. If the electric field just outside the sphere is sufficient for dielectric breakdown of air, no more charge can be transferred to it. The dielectric strength of air is $3\times 10^{6}\ \mathrm{V\,m^{-1}}$. For a conducting sphere, the electric field just outside the sphere is $E=\frac{Q}{4\pi\varepsilon_0 R^2}$ and the potential of the sphere is $V=\frac{Q}{4\pi\varepsilon_0 R}\cdot$ Thus, $V=ER$. To have a field of $3\times 10^{6}\ \mathrm{V\,m^{-1}}$ with a sphere of radius 1 m, its potential should be 3×10^{6} V. Thus, the potential of a sphere of radius 1 m can be raised to 3×10^{6} V by this method. The potential can be increased by enclosing the sphere in a highly evacuated chamber.

Worked Out Examples

1. *A parallel-plate capacitor has plates of area* 200 cm^2 *and separation between the plates* 1·00 mm. *What potential difference will be developed if a charge of* 1·00 nC (*i.e.*, $1{\cdot}00\times 10^{-9}$ C) *is given to the capacitor? If the plate separation is now increased to* 2·00 mm, *what will be the new potential difference?*

Solution : The capacitance of the capacitor is $C=\frac{\varepsilon_0 A}{d}$

$$=8{\cdot}85\times 10^{-12}\ \mathrm{Fm^{-1}}\times\frac{200\times 10^{-4}\ \mathrm{m}^2}{1\times 10^{-3}\ \mathrm{m}}$$

$$=0{\cdot}177\times 10^{-9}\ \mathrm{F}=0{\cdot}177\ \mathrm{nF}.$$

The potential difference between the plates is

$$V=\frac{Q}{C}=\frac{1\ \mathrm{nC}}{0{\cdot}177\ \mathrm{nF}}=5{\cdot}65\ \text{volts}.$$

If the separation is increased from 1·00 mm to 2·00 mm, the capacitance is decreased by a factor of 2. If the charge remains the same, the potential difference will increase by a factor of 2. Thus, the new potential difference will be

$$5{\cdot}65\ \text{volts}\times 2=11{\cdot}3\ \text{volts}.$$

2. *An isolated sphere has a capacitance of* 50 pF. *(a) Calculate its radius. (b) How much charge should be placed on it to raise its potential to* 10^4 V?

Solution : (a) The capacitance of an isolated sphere is $C=4\pi\varepsilon_0 R$. Thus,

$$50\times 10^{-12}\ \mathrm{F}=\frac{R}{9\times 10^{9}\ \mathrm{mF^{-1}}}$$

or, $\quad R=50\times 10^{-12}\times 9\times 10^{9}\ \mathrm{m}=45\ \mathrm{cm}.$

(b) $\quad Q=CV$

$$=50\times 10^{-12}\ \mathrm{F}\times 10^{4}\ \mathrm{V}=0{\cdot}5\ \mu\mathrm{C}.$$

3. *Consider the connections shown in figure (31-W1). (a) Find the capacitance between the points A and B. (b) Find the charges on the three capacitors. (c) Taking the potential at the point B to be zero, find the potential at the point D.*

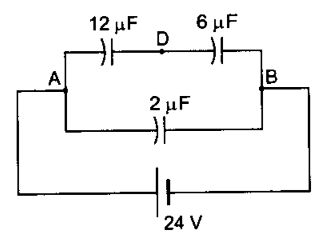

Figure 31-W1

Solution : (a) The 12 μF and 6 μF capacitors are joined in series. The equivalent of these two will have a capacitance given by

$$\frac{1}{C}=\frac{1}{12\ \mu\mathrm{F}}+\frac{1}{6\ \mu\mathrm{F}},$$

or, $\quad C=4\ \mu\mathrm{F}.$

The combination of these two capacitors is joined in parallel with the 2 μF capacitor. Thus, the equivalent capacitance between A and B is

$$4\ \mu\mathrm{F}+2\ \mu\mathrm{F}=6\ \mu\mathrm{F}.$$

(b) The charge supplied by the battery is

$$Q=CV=6\ \mu\mathrm{F}\times 24\ \mathrm{V}=144\ \mu\mathrm{C}.$$

The potential difference across the 2 μF capacitor is 24 V. The charge on this capacitor is, therefore,

$$2\ \mu\mathrm{F}\times 24\ \mathrm{V}=48\ \mu\mathrm{C}.$$

The charge on the 12 μF and 6 μF capacitor is, therefore,

$$144\ \mu\mathrm{C}-48\ \mu\mathrm{C}=96\ \mu\mathrm{C}.$$

(c) The potential difference across the 6 μF capacitor is

$$\frac{96\ \mu\mathrm{C}}{6\ \mu\mathrm{F}}=16\ \mathrm{V}.$$

As the potential at the point B is taken to be zero, the potential at the point D is 16 V.

4. *If* 100 volts *of potential difference is applied between a and b in the circuit of figure (31-W2a), find the potential difference between c and d.*

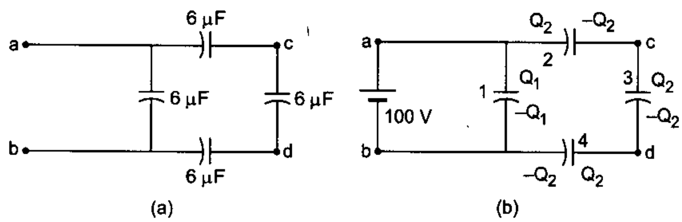

Figure 31-W2

Solution : The charge distribution on different plates is shown in figure (31-W2b). Suppose charge $Q_1 + Q_2$ is given by the positive terminal of the battery, out of which Q_1 resides on the positive plate of capacitor (1) and Q_2 on that of (2). The remaining plates will have charges as shown in the figure.

Take the potential at the point b to be zero. The potential at a will be 100 V. Let the potentials at points c and d be V_c and V_d respectively. Writing the equation $Q = CV$ for the four capacitors, we get,

$$Q_1 = 6\ \mu F \times 100\ V = 600\ \mu C \quad \text{... (i)}$$

$$Q_2 = 6\ \mu F \times (100\ V - V_c) \quad \text{... (ii)}$$

$$Q_2 = 6\ \mu F \times (V_c - V_d) \quad \text{... (iii)}$$

$$Q_2 = 6\ \mu F \times V_d. \quad \text{... (iv)}$$

From (ii) and (iii),

$$100\ V - V_c = V_c - V_d$$

or, $$2\ V_c - V_d = 100\ V \quad \text{... (v)}$$

and from (iii) and (iv),

$$V_c - V_d = V_d$$

or, $$V_c = 2\ V_d. \quad \text{... (vi)}$$

From (v) and (vi),

$$V_d = \frac{100}{3}\ V \text{ and } V_c = \frac{200}{3}\ V$$

so that $V_c - V_d = \dfrac{100}{3}$ V.

5. *Find the charges on the three capacitors shown in figure (31-W3a).*

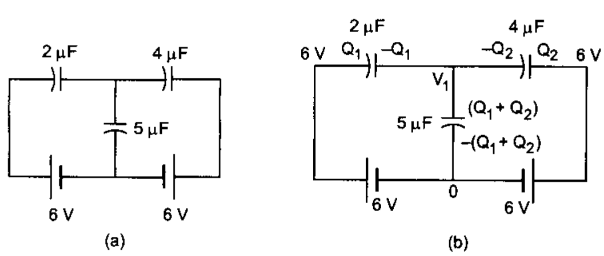

Figure 31-W3

Solution : Take the potential at the junction of the batteries to be zero. Let the left battery supply a charge Q_1 and the right battery a charge Q_2. The charge on the 5 μF capacitor will be $Q_1 + Q_2$. Let the potential at the junction of the capacitors be V_1. The charges at different plates and potentials at different points are shown in figure (31-W3b).

Note that the charges on the three plates which are in contact add to zero. It should be so, because, these plates taken together form an isolated system which cannot receive charges from the batteries. Applying the equation $Q = CV$ to the three capacitors, we get,

$$Q_1 = 2\ \mu F(6\ V - V_1) \quad \text{... (i)}$$

$$Q_2 = 4\ \mu F(6\ V - V_1) \quad \text{... (ii)}$$

and $$Q_1 + Q_2 = 5\ \mu F(V_1 - 0). \quad \text{... (iii)}$$

From (i) and (ii),

$$2\ Q_1 - Q_2 = 0 \text{ or, } Q_2 = 2\ Q_1.$$

From (ii) and (iii),

$$5\ Q_2 + 4(Q_1 + Q_2) = 20\ \mu F \times 6\ V$$

or, $$4\ Q_1 + 9\ Q_2 = 120\ \mu C$$

or, $$4\ Q_1 + 18\ Q_1 = 120\ \mu C$$

or, $$Q_1 = 5{\cdot}45\ \mu C \text{ and } Q_2 = 10{\cdot}9\ \mu C.$$

Thus, the charges on the 2 μF, 4 μF and 5 μF capacitors are 5·45 μC, 10·9 μC and 16·35 μC respectively.

6. *Find the equivalent capacitance of the system shown in figure (31-W4a) between the points a and b.*

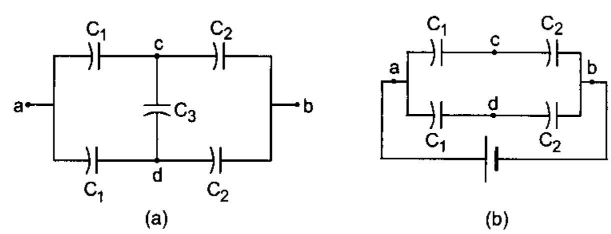

Figure 31-W4

Solution : Suppose, the capacitor C_3 is removed from the given system and a battery is connected between a and b. The remaining system is shown in figure (31-W4b). From the symmetry of the figure, the potential at c will be the same as the potential at d. Thus, if the capacitor C_3 is connected between c and d, it will have no charge. The charges of all the remaining four capacitors will remain unchanged. Thus, the system of capacitors in figure (31-W4a) is equivalent to that in the figure (31-W4b). The equivalent capacitance of the system in figure (31-W4b) can be calculated by applying the formulae for series and parallel combinations. C_1 and C_2 are connected in series. Their equivalent capacitance is

$$C = \frac{C_1 C_2}{C_1 + C_2}.$$

Two such capacitors are joined in parallel. So the equivalent capacitance of the given system is

$$2C = \frac{2\,C_1C_2}{C_1 + C_2}.$$

7. *Find the equivalent capacitance between the point A and B in figure (31-W5a).*

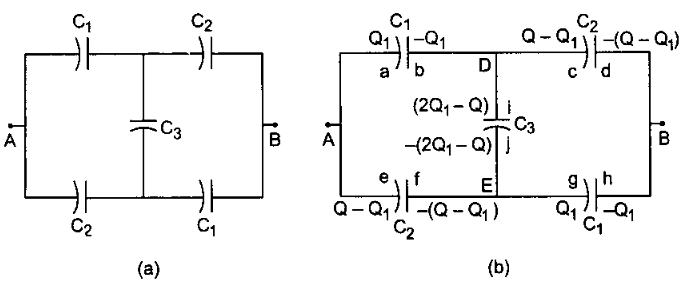

Figure 31-W5

Solution : Let us connect a battery between the points A and B. The charge distribution is shown in figure (31-W5b). Suppose the positive terminal of the battery supplies a charge $+Q$ and the negative terminal a charge $-Q$. The charge Q is divided between plates a and e. A charge Q_1 goes to the plate a and the rest $Q - Q_1$ goes to the plate e. The charge $-Q$ supplied by the negative terminal is divided between plates d and h. Using the symmetry of the figure, charge $-Q_1$ goes to the plate h and $-(Q - Q_1)$ to the plate d. This is because if you look into the circuit from A or from B, the circuit looks identical. The division of charge at A and at B should, therefore, be similar. The charges on the other plates may be written easily. The charge on the plate i is $2\,Q_1 - Q$ which ensures that the total charge on plates b, c and i remains zero as these three plates form an isolated system.

We have,

$$V_A - V_B = (V_A - V_D) + (V_D - V_B)$$

$$= \frac{Q_1}{C_1} + \frac{Q - Q_1}{C_2} \qquad \ldots \text{(i)}$$

Also, $$V_A - V_B = (V_A - V_D) + (V_D - V_E) + (V_E - V_B)$$

$$= \frac{Q_1}{C_1} + \frac{2\,Q_1 - Q}{C_3} + \frac{Q_1}{C_1}. \qquad \ldots \text{(ii)}$$

We have to eliminate Q_1 from these equations to get the equivalent capacitance $Q/(V_A - V_B)$.

The first equation may be written as

$$V_A - V_B = Q_1\left(\frac{1}{C_1} - \frac{1}{C_2}\right) + \frac{Q}{C_2}$$

or, $$\frac{C_1C_2}{C_2 - C_1}(V_A - V_B) = Q_1 + \frac{C_1}{C_2 - C_1}Q. \qquad \ldots \text{(iii)}$$

The second equation may be written as

$$V_A - V_B = 2\,Q_1\left(\frac{1}{C_1} + \frac{1}{C_3}\right) - \frac{Q}{C_3}$$

or, $$\frac{C_1C_3}{2(C_1 + C_3)}(V_A - V_B) = Q_1 - \frac{C_1}{2(C_1 + C_3)}Q. \qquad \ldots \text{(iv)}$$

Subtracting (iv) from (iii),

$$(V_A - V_B)\left[\frac{C_1C_2}{C_2 - C_1} - \frac{C_1C_3}{2(C_1 + C_3)}\right] = \left[\frac{C_1}{C_2 - C_1} + \frac{C_1}{2(C_1 + C_3)}\right]Q$$

or, $$(V_A - V_B)\,[2\,C_1C_2(C_1 + C_3) - C_1C_3(C_2 - C_1)] = C_1[2\,(C_1 + C_3) + (C_2 - C_1)]\,Q$$

or, $$C = \frac{Q}{V_A - V_B} = \frac{2\,C_1C_2 + C_2C_3 + C_3C_1}{C_1 + C_2 + 2\,C_3}.$$

8. *Twelve capacitors, each having a capacitance C, are connected to form a cube (figure 31-W6a). Find the equivalent capacitance between the diagonally opposite corners such as A and B.*

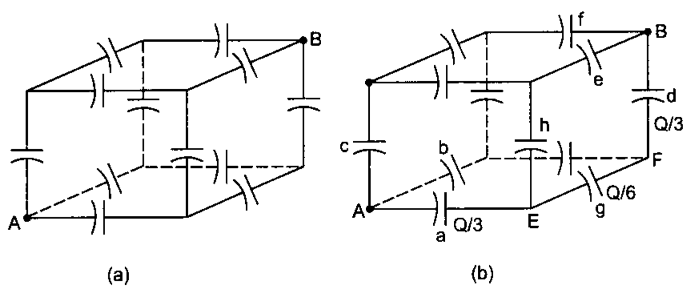

Figure 31-W6

Solution : Suppose the points A and B are connected to a battery. The charges appearing on some of the capacitors are shown in figure (31-W6b). Suppose the positive terminal of the battery supplies a charge $+Q$ through the point A. This charge is divided on the three plates connected to A. Looking from A, the three sides of the cube have identical properties and hence, the charge will be equally distributed on the three plates. Each of the capacitors a, b and c will receive a charge $Q/3$.

The negative terminal of the battery supplies a charge $-Q$ through the point B. This is again divided equally on the three plates connected to B. Each of the capacitors d, e and f gets equal charge $Q/3$.

Now consider the capacitors g and h. As the three plates connected to the point E form an isolated system, their total charge must be zero. The negative plate of the capacitor a has a charge $-Q/3$. The two plates of g and h connected to E should have a total charge $Q/3$. By symmetry, these two plates should have equal charges and hence each of these has a charge $Q/6$.

The capacitors a, g and d have charges $Q/3$, $Q/6$ and $Q/3$ respectively.

We have,

$$V_A - V_B = (V_A - V_E) + (V_E - V_F) + (V_F - V_B)$$

$$= \frac{Q/3}{C} + \frac{Q/6}{C} + \frac{Q/3}{C} = \frac{5\,Q}{6\,C}$$

or, $$C_{eq} = \frac{Q}{V_A - V_B} = \frac{6}{5}\,C.$$

9. *The negative plate of a parallel plate capacitor is given a charge of -20×10^{-8} C. Find the charges appearing on the four surfaces of the capacitor plates.*

Solution :

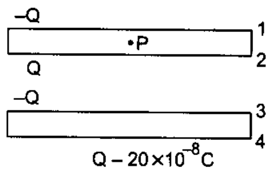

Figure 31-W7

Let the charge appearing on the inner surface of the negative plate be $-Q$. The charge on its outer surface will be $Q - 20 \times 10^{-8}$ C.

The charge on the inner surface of the positive plate will be $+Q$ from Gauss's law and that on the outer surface will be $-Q$ as the positive plate is electrically neutral. The distribution is shown in figure (31-W7).

To obtain the value of Q, consider the electric field at a point P inside the upper plate.

Field due to surface (1) $= \dfrac{Q}{2\varepsilon_0 A}$ upward,

due to surface (2) $= \dfrac{Q}{2\varepsilon_0 A}$ upward,

due to surface (3) $= \dfrac{Q}{2\varepsilon_0 A}$ downward

and due to surface (4) $= \dfrac{Q - 20 \times 10^{-8}\,\text{C}}{2\varepsilon_0 A}$ upward.

As P is a point inside the conductor, the field here must be zero. Thus,

$$Q = -Q + 20 \times 10^{-8}\ \text{C}$$

or, $$Q = 10 \times 10^{-8}\ \text{C}.$$

The charges on the four surfaces may be written immediately from figure (31-W7).

10. *Three capacitors of capacitances 2 μF, 3 μF and 6 μF are connected in series with a 12 V battery. All the connecting wires are disconnected, the three positive plates are connected together and the three negative plates are connected together. Find the charges on the three capacitors after the reconnection.*

Solution : The equivalent capacitance of the three capacitors joined in series is given by

$$\frac{1}{C} = \frac{1}{2\,\mu\text{F}} + \frac{1}{3\,\mu\text{F}} + \frac{1}{6\,\mu\text{F}}$$

or, $$C = 1\ \mu\text{F}.$$

The charge supplied by the battery $= 1\ \mu\text{F} \times 12\ \text{V}$
$= 12\ \mu\text{C}$.

As the capacitors are connected in series, 12 μC charge appears on each of the positive plates and -12 μC on each of the negative plates. The charged capacitors are now connected as shown in figure (31-W8).

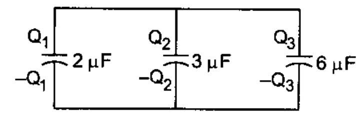

Figure 31-W8

The 36 μC charge on the three positive plates now redistribute as Q_1, Q_2 and Q_3 on the three connected positive plates. Similarly, -36 μC redistributes as $-Q_1$, $-Q_2$ and $-Q_3$. The three positive plates are now at a common potential and the three negative plates are also at a common potential. Let the potential difference across each capacitor be V. Then

$$Q_1 = (2\ \mu\text{F})\ V,$$
$$Q_2 = (3\ \mu\text{F})\ V,$$

and $$Q_3 = (6\ \mu\text{F})\ V.$$

Also, $$Q_1 + Q_2 + Q_3 = 36\ \mu\text{C}.$$

Solving these equations,

$$Q_1 = \frac{72}{11}\ \mu\text{C},\ Q_2 = \frac{108}{11}\ \mu\text{C} \text{ and } Q_3 = \frac{216}{11}\ \mu\text{C}.$$

11. *The connections shown in figure (31-W9a) are established with the switch S open. How much charge will flow through the switch if it is closed?*

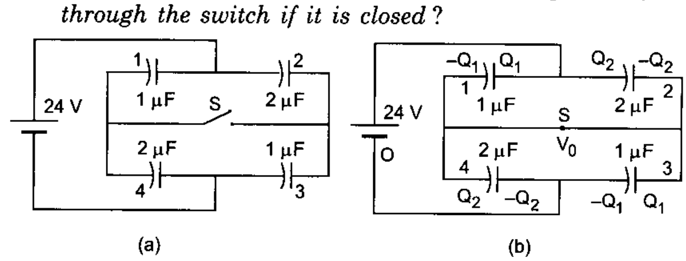

Figure 31-W9

Solution : When the switch is open, capacitors (2) and (3) are in series. Their equivalent capacitance is $\frac{2}{3}$ μF. The charge appearing on each of these capacitors is, therefore, $24\ \text{V} \times \frac{2}{3}\ \mu\text{F} = 16\ \mu\text{C}$.

The equivalent capacitance of (1) and (4), which are also connected in series, is also $\frac{2}{3}$ μF and the charge on each of these capacitors is also 16 μC. The total charge on the two plates of (1) and (4) connected to the switch is, therefore, zero.

The situation when the switch S is closed is shown in figure (31-W9b). Let the charges be distributed as shown in the figure. Q_1 and Q_2 are arbitrarily chosen for the positive plates of (1) and (2). The same magnitude of charges will appear at the negative plates of (3) and (4).

Take the potential at the negative terminal to be zero and at the switch to be V_0.

Writing equations for the capacitors (1), (2), (3) and (4),

$$Q_1 = (24\text{ V} - V_0) \times 1\ \mu\text{F} \qquad \ldots \text{(i)}$$

$$Q_2 = (24\text{ V} - V_0) \times 2\ \mu\text{F} \qquad \ldots \text{(ii)}$$

$$Q_1 = V_0 \times 1\ \mu\text{F} \qquad \ldots \text{(iii)}$$

$$Q_2 = V_0 \times 2\ \mu\text{F}. \qquad \ldots \text{(iv)}$$

From (i) and (iii), $V_0 = 12$ V.

Thus, from (iii) and (iv),

$$Q_1 = 12\ \mu\text{C} \text{ and } Q_2 = 24\ \mu\text{C}.$$

The charge on the two plates of (1) and (4) which are connected to the switch is, therefore, $Q_2 - Q_1 = 12\ \mu$C.

When the switch was open, this charge was zero. Thus, 12 μC of charge has passed through the switch after it was closed.

12. *Each of the three plates shown in figure (31-W10a) has an area of* 200 cm^2 *on one side and the gap between the adjacent plates is* 0·2 mm. *The emf of the battery is* 20 V. *Find the distribution of charge on various surfaces of the plates. What is the equivalent capacitance of the system between the terminal points?*

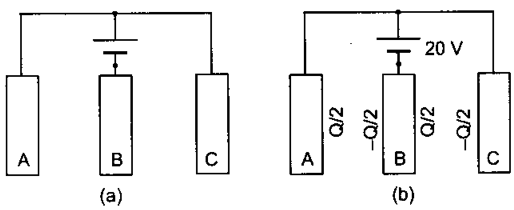

Figure 31-W10

Solution : Suppose the negative terminal of the battery gives a charge $-Q$ to the plate B. As the situation is symmetric on the two sides of B, the two faces of the plate B will share equal charges $-Q/2$ each. From Gauss's law, the facing surfaces will have charges $Q/2$ each. As the positive terminal of the battery has supplied just this much charge $(+Q)$ to A and C, the outer surfaces of A and C will have no charge. The distribution will be as shown in figure (31-W10b).

The capacitance between the plates A and B is

$$8{\cdot}85 \times 10^{-12}\text{ F m}^{-1} \times \frac{200 \times 10^{-4}\text{ m}^2}{2 \times 10^{-4}\text{ m}}$$

$$= 8{\cdot}85 \times 10^{-10}\text{ F} = 0{\cdot}885\text{ nF}.$$

Thus, $\quad \frac{Q}{2} = 0{\cdot}885\text{ nF} \times 20\text{ V} = 17{\cdot}7\text{ nC}.$

The distribution of charge on various surfaces may be written from figure (31-W10b).

The equivalent capacitance is

$$\frac{Q}{20\text{ V}} = 1{\cdot}77\text{ nF}.$$

13. *Find the capacitance of the infinite ladder shown in figure (31-W11).*

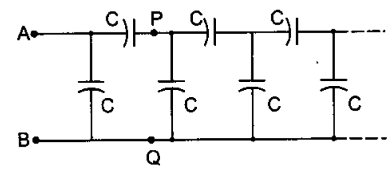

Figure 31-W11

Solution : As the ladder is infinitely long, the capacitance of the ladder to the right of the points P, Q is the same as that of the ladder to the right of the points A, B. If the equivalent capacitance of the ladder is C_1, the given ladder may be replaced by the connections shown in figure (31-W12).

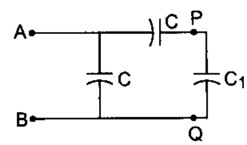

Figure 31-W12

The equivalent capacitance between A and B is easily found to be $C + \frac{CC_1}{C + C_1}$. But being equivalent to the original ladder, the equivalent capacitance is also C_1.

Thus, $$C_1 = C + \frac{CC_1}{C + C_1}$$

or, $$C_1 C + C_1^2 = C^2 + 2\,CC_1$$

or, $$C_1^2 - CC_1 - C^2 = 0$$

giving $$C_1 = \frac{C + \sqrt{C^2 + 4\,C^2}}{2} = \frac{1 + \sqrt{5}}{2}\,C.$$

Negative value of C_1 is rejected.

14. *Find the energy stored in the electric field produced by a metal sphere of radius R containing a charge Q.*

Solution : Consider a thin spherical shell of radius $x\ (> R)$ and thickness dx concentric with the given metal sphere. The energy density in the shell is

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 \left(\frac{Q}{4\pi\varepsilon_0 x^2}\right)^2.$$

The volume of the shell is $4\pi x^2 dx$. The energy contained in the shell is, therefore,

$$dU = \frac{1}{2}\varepsilon_0 \left(\frac{Q}{4\pi\varepsilon_0 x^2}\right)^2 \times 4\pi x^2 dx = \frac{Q^2 dx}{8\pi\varepsilon_0 x^2}.$$

The energy contained in the whole space outside the sphere is

$$U = \int_R^\infty \frac{Q^2 dx}{8\pi\varepsilon_0 x^2} = \frac{Q^2}{8\pi\varepsilon_0 R}.$$

As the field inside the sphere is zero, this is also the total energy stored in the field.

Alternative Method

Considering a concentric spherical shell at infinity, we have a spherical capacitor. The capacitance is $C = 4\pi\varepsilon_0 R$. The energy stored in this capacitor is the energy stored in the entire electric field. This energy is

$$U = \frac{Q^2}{2C} = \frac{Q^2}{8\pi\varepsilon_0 R}.$$

15. *A capacitor of capacitance C is charged by connecting it to a battery of emf $\mathcal{E}$. The capacitor is now disconnected and reconnected to the battery with the polarity reversed. Calculate the heat developed in the connecting wires.*

Solution : When the capacitor is connected to the battery, a charge $Q = C\mathcal{E}$ appears on one plate and $-Q$ on the other. When the polarity is reversed, a charge $-Q$ appears on the first plate and $+Q$ on the second. A charge $2Q$, therefore, passes through the battery from the negative to the positive terminal. The battery does a work

$$W = (2Q)\mathcal{E} = 2C\mathcal{E}^2$$

in the process. The energy stored in the capacitor is the same in the two cases. Thus, the work done by the battery appears as heat in the connecting wires. The heat produced is, therefore, $2C\mathcal{E}^2$.

16. *An uncharged capacitor is connected to a battery. Show that half the energy supplied by the battery is lost as heat while charging the capacitor.*

Solution : Suppose the capacitance of the capacitor is C and the emf of the battery is V. The charge given to the capacitor is $Q = CV$. The work done by the battery is

$$W = QV.$$

The battery supplies this energy. The energy stored in the capacitor is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV.$$

The remaining energy $QV - \frac{1}{2}QV = \frac{1}{2}QV$ is lost as heat. Thus, half the energy supplied by the battery is lost as heat.

17. *A parallel-plate capacitor having plate area* 100 cm ^{2} *and separation* 1·0 mm *holds a charge of* 0·12 μC *when connected to a* 120 V *battery. Find the dielectric constant of the material filling the gap.*

Solution :

The capacitance of the capacitor is

$$\frac{0.12\ \mu C}{120\ V} = 1.0 \times 10^{-9}\ F.$$

If K be the dielectric constant, the capacitance is also given by $\frac{K\varepsilon_0 A}{d}$. Thus,

$$\frac{K \times 8.85 \times 10^{-12}\ F\ m^{-1} \times 100 \times 10^{-4}\ m^2}{1.0 \times 10^{-3}\ m} = 1.0 \times 10^{-9}\ F$$

or, $$K = 11.3.$$

18. *A parallel-plate capacitor is formed by two plates, each of area* 100 cm ^{2}, *separated by a distance of* 1 mm. *A dielectric of dielectric constant* 5·0 *and dielectric strength* 1.9×10^7 V m^{-1} *is filled between the plates. Find the maximum charge that can be stored on the capacitor without causing any dielectric breakdown.*

Solution : If Q be the charge on the capacitor, the surface charge density is $\sigma = Q/A$ and the electric field is $\frac{Q}{KA\varepsilon_0}$. This should not exceed the dielectric strength 1.9×10^7 V m^{-1}. Thus, the maximum charge is given by

$$\frac{Q}{KA\varepsilon_0} = 1.9 \times 10^7\ V\ m^{-1}$$

or, $$Q = KA\varepsilon_0 \times 1.9 \times 10^7\ V\ m^{-1}$$

$$= (5.0)\,(10^{-2}\ m^2)\,(8.85 \times 10^{-12}\ F\ m^{-1}) \times (1.9 \times 10^7\ V\ m^{-1})$$

$$= 8.4 \times 10^{-6}\ C.$$

19. *The space between the plates of a parallel-plate capacitor of capacitance C is filled with three dielectric slabs of identical size as shown in figure (31-W13). If the dielectric constants of the three slabs are K_1, K_2 and K_3, find the new capacitance.*

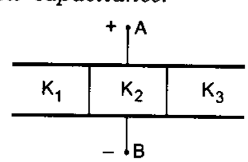

Figure 31-W13

Solution : Consider each one third of the assembly as a separate capacitor. The three positive plates are connected together at point A and the three negative plates are connected together at point B. Thus, the three capacitors are joined in parallel. As the plate area is one third of the original for each part, the capacitances of these parts will be $K_1C/3$, $K_2C/3$ and $K_3C/3$. The equivalent capacitance is, therefore,

$$C_{eq} = (K_1 + K_2 + K_3)\frac{C}{3}.$$

20. *Figure (31-W14a) shows a parallel-plate capacitor having square plates of edge a and plate-separation d. The gap between the plates is filled with a dielectric of dielectric constant K which varies parallel to an edge as*

$$K = K_0 + \alpha x$$

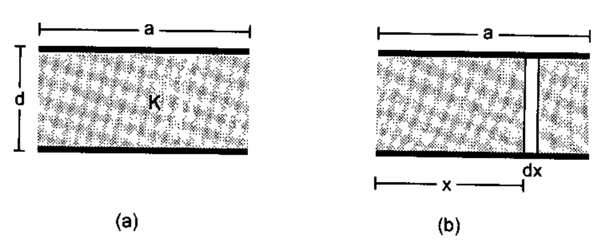

Figure 31-W14

where K and α are constants and x is the distance from the left end. Calculate the capacitance.

Solution :

Consider a small strip of width dx at a separation x from the left end (figure 31-W14b). This strip forms a small capacitor of plate area adx. Its capacitance is

$$dC = \frac{(K_0 + \alpha x)\varepsilon_0\, adx}{d}.$$

The given capacitor may be divided into such strips with x varying from 0 to a. All these strips are connected in parallel. The capacitance of the given capacitor is,

$$C = \int_0^a \frac{(K_0 + \alpha x)\varepsilon_0\, adx}{d}$$

$$= \frac{\varepsilon_0 a^2}{d}\left(K_0 + \frac{a\alpha}{2}\right).$$

21. *A parallel-plate capacitor of capacitance 100 μF is connected to a power supply of 200 V. A dielectric slab of dielectric constant 5 is now inserted into the gap between the plates. (a) Find the extra charge flown through the power supply and the work done by the supply. (b) Find the change in the electrostatic energy of the electric field in the capacitor.*

Solution : (a) The original capacitance was 100 μF. The charge on the capacitor before the insertion of the dielectric was, therefore,

$$Q_1 = 100\ \mu\text{F} \times 200\ \text{V} = 20\ \text{mC}.$$

After the dielectric slab is introduced, the capacitance is increased to 500 μF. The new charge on the capacitor is, therefore, 500 μF × 200 V = 100 mC. The charge flown through the power supply is, therefore, 100 mC − 20 mC = 80 mC. The work done by the power supply is 200 V × 80 mC = 16 J.

(b) The electrostatic field energy of the capacitor without the dielectric slab is

$$U_1 = \frac{1}{2}CV^2$$

$$= \frac{1}{2} \times (100\ \mu\text{F}) \times (200\ \text{V})^2 = 2\ \text{J}$$

and that after the slab is inserted is

$$U_2 = \frac{1}{2} \times (500\ \mu\text{F}) \times (200\ \text{V})^2 = 10\ \text{J}.$$

Thus, the energy is increased by 8 J.

22. *Figure (31-W15) shows a parallel-plate capacitor with plates of width b and length l. The separation between the plates is d. The plates are rigidly clamped and connected to a battery of emf V. A dielectric slab of thickness d and dielectric constant K is slowly inserted between the plates. (a) Calculate the energy of the system when a length x of the slab is introduced into the capacitor. (b) What force should be applied on the slab*

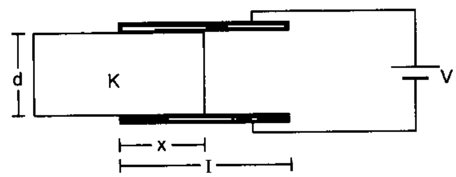

Figure 31-W15

to ensure that it goes slowly into the capacitor? Neglect any effect of friction or gravity.

Solution : (a) The plate area of the part with the dielectric is bx. Its capacitance is

$$C_1 = \frac{K\varepsilon_0\, bx}{d}.$$

Similarly, the capacitance of the part without the dielectric is

$$C_2 = \frac{\varepsilon_0\, b(l - x)}{d}.$$

These two parts are connected in parallel. The capacitance of the system is, therefore,

$$C = C_1 + C_2$$

$$= \frac{\varepsilon_0 b}{d}[l + x(K - 1)]. \quad \text{... (i)}$$

The energy of the capacitor is

$$U = \frac{1}{2}CV^2 = \frac{\varepsilon_0 bV^2}{2d}[l + x(K - 1)].$$

(b) Suppose, the electric field attracts the dielectric slab with a force F. An external force of equal magnitude F should be applied in opposite direction so that the plate moves slowly (no acceleration).

Consider the part of motion in which the dielectric moves a distance dx further inside the capacitor. The capacitance increases to $C + dC$. As the potential difference remains constant at V, the battery has to supply a further charge

$$dQ = (dC)V$$

to the capacitor. The work done by the battery is, therefore,

$$dW_b = V\,dQ = (dC)V^2.$$

The external force F does a work

$$dW_e = (-F\,dx)$$

during the displacement. The total work done on the capacitor is

$$dW_b + dW_e = (dC)V^2 - Fdx.$$

This should be equal to the increase dU in the stored energy. Thus,

$$\frac{1}{2}(dC)V^2 = (dC)V^2 - Fdx$$

or,

$$F = \frac{1}{2}V^2\frac{dC}{dx}.$$

Using equation (i),

$$F = \frac{\varepsilon_0 bV^2(K-1)}{2d}.$$

Thus, the electric field attracts the dielectric into the capacitor with a force $\frac{\varepsilon_0 bV^2(K-1)}{2d}$ and this much force should be applied in the opposite direction.

23. *A parallel-plate capacitor is placed in such a way that its plates are horizontal and the lower plate is dipped into a liquid of dielectric constant K and density ρ. Each plate has an area A. The plates are now connected to a battery which supplies a positive charge of magnitude Q to the upper plate. Find the rise in the level of the liquid in the space between the plates.*

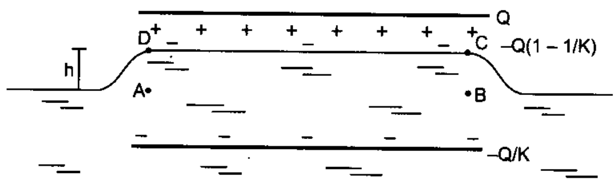

Figure 31-W16

Solution :

The situation is shown in figure (31-W16). A charge $-Q\left(1-\frac{1}{K}\right)$ is induced on the upper surface of the liquid and $Q\left(1-\frac{1}{K}\right)$ at the surface in contact with the lower plate. The net charge on the lower plate is $-Q+Q\left(1-\frac{1}{K}\right) = -\frac{Q}{K}$. Consider the equilibrium of the liquid in the volume *ABCD*. The forces on this liquid are

(a) the force due to the electric field at *CD*,
(b) the weight of the liquid,
(c) the force due to atmospheric pressure and
(d) the force due to the pressure of the liquid below *AB*.

As *AB* is in the same horizontal level as the outside surface, the pressure here is the same as the atmospheric pressure. The forces in (c) and (d), therefore, balance each other. Hence, for equilibrium, the forces in (a) and (b) should balance each other.

The electric field at *CD* due to the charge *Q* is

$$E_1 = \frac{Q}{2A\varepsilon_0}$$

in the downward direction. The field at *CD* due to the charge $-Q/K$ is

$$E_2 = \frac{Q}{2A\varepsilon_0 K}$$

also in the downward direction. The net field at *CD* is

$$E_1 + E_2 = \frac{(K+1)Q}{2A\varepsilon_0 K}.$$

The force on the charge $-Q\left(1-\frac{1}{K}\right)$ at *CD* is

$$F = Q\left(1-\frac{1}{K}\right)\frac{(K+1)Q}{2A\varepsilon_0 K}$$

$$= \frac{(K^2-1)Q^2}{2A\varepsilon_0 K^2}$$

in the upward direction. The weight of the liquid considered is $hA\rho g$. Thus,

$$hA\rho g = \frac{(K^2-1)Q^2}{2A\varepsilon_0 K^2}$$

or,
$$h = \frac{(K^2-1)Q^2}{2A^2K^2\varepsilon_0 \rho g}.$$

□

QUESTIONS FOR SHORT ANSWER

1. Suppose a charge $+Q_1$ is given to the positive plate and a charge $-Q_2$ to the negative plate of a capacitor. What is the "charge on the capacitor"?
2. As $C = \left(\frac{1}{V}\right)Q$, can you say that the capacitance C is proportional to the charge Q?
3. A hollow metal sphere and a solid metal sphere of equal radii are given equal charges. Which of the two will have higher potential?
4. The plates of a parallel-plate capacitor are given equal positive charges. What will be the potential difference between the plates? What will be the charges on the facing surfaces and on the outer surfaces?
5. A capacitor has capacitance C. Is this information sufficient to know what maximum charge the capacitor can contain? If yes, what is this charge? If no, what other information is needed?
6. The dielectric constant decreases if the temperature is increased. Explain this in terms of polarization of the material.
7. When a dielectric slab is gradually inserted between the plates of an isolated parallel-plate capacitor, the energy of the system decreases. What can you conclude about the force on the slab exerted by the electric field?

OBJECTIVE I

1. A capacitor of capacitance C is charged to a potential V. The flux of the electric field through a closed surface enclosing the capacitor is
(a) $\frac{CV}{\varepsilon_0}$ (b) $\frac{2CV}{\varepsilon_0}$ (c) $\frac{CV}{2\varepsilon_0}$ (d) zero.

2. Two capacitors each having capacitance C and breakdown voltage V are joined in series. The capacitance and the breakdown voltage of the combination will be
(a) $2C$ and $2V$ (b) $C/2$ and $V/2$
(c) $2C$ and $V/2$ (d) $C/2$ and $2V$.

3. If the capacitors in the previous question are joined in parallel, the capacitance and the breakdown voltage of the combination will be
(a) $2C$ and $2V$ (b) C and $2V$
(c) $2C$ and V (d) C and V.

4. The equivalent capacitance of the combination shown in figure (31-Q1) is
(a) C (b) $2C$ (c) $C/2$ (d) none of these.

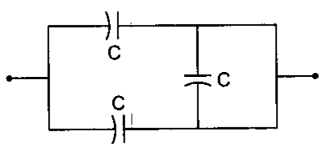

Figure 31-Q1

5. A dielectric slab is inserted between the plates of an isolated capacitor. The force between the plates will
(a) increase (b) decrease
(c) remain unchanged (d) become zero.

6. The energy density in the electric field created by a point charge falls off with the distance from the point charge as
(a) $\frac{1}{r}$ (b) $\frac{1}{r^2}$ (c) $\frac{1}{r^3}$ (d) $\frac{1}{r^4}$.

7. A parallel-plate capacitor has plates of unequal area. The larger plate is connected to the positive terminal of the battery and the smaller plate to its negative terminal. Let Q_+ and Q_- be the charges appearing on the positive and negative plates respectively.
(a) $Q_+ > Q_-$ (b) $Q_+ = Q_-$ (c) $Q_+ < Q_-$
(d) The information is not sufficient to decide the relation between Q_+ and Q_-.

8. A thin metal plate P is inserted between the plates of a parallel-plate capacitor of capacitance C in such a way that its edges touch the two plates (figure 31-Q2). The capacitance now becomes
(a) $C/2$ (b) $2C$ (c) 0 (d) indeterminate.

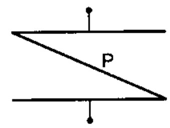

Figure 31-Q2

9. Figure (31-Q3) shows two capacitors connected in series and joined to a battery. The graph shows the variation in potential as one moves from left to right on the branch containing the capacitors.
(a) $C_1 > C_2$ (b) $C_1 = C_2$ (c) $C_1 < C_2$
(d) The information is not sufficient to decide the relation between C_1 and C_2.

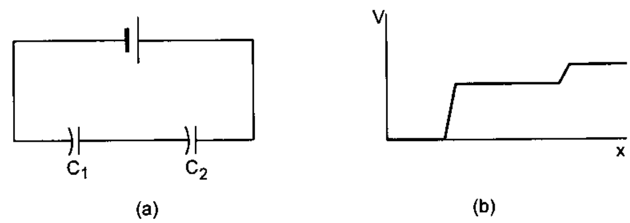

Figure 31-Q3

10. Two metal plates having charges Q, $-Q$ face each other at some separation and are dipped into an oil tank. If the oil is pumped out, the electric field between the plates will
(a) increase (b) decrease
(c) remain the same (d) become zero.

11. Two metal spheres of capacitances C_1 and C_2 carry some charges. They are put in contact and then separated. The final charges Q_1 and Q_2 on them will satisfy
(a) $\frac{Q_1}{Q_2} < \frac{C_1}{C_2}$ (b) $\frac{Q_1}{Q_2} = \frac{C_1}{C_2}$ (c) $\frac{Q_1}{Q_2} > \frac{C_1}{C_2}$ (d) $\frac{Q_1}{Q_2} = \frac{C_2}{C_1}$.

12. Three capacitors of capacitances 6 μF each are available. The minimum and maximum capacitances, which may be obtained are
(a) 6 μF, 18 μF (b) 3 μF, 12 μF
(c) 2 μF, 12 μF (d) 2 μF, 18 μF.

OBJECTIVE II

1. The capacitance of a capacitor does not depend on
(a) the shape of the plates
(b) the size of the plates
(c) the charges on the plates
(d) the separation between the plates.

2. A dielectric slab is inserted between the plates of an isolated charged capacitor. Which of the following quantities will remain the same?
(a) The electric field in the capacitor
(b) The charge on the capacitor

(c) The potential difference between the plates
(d) The stored energy in the capacitor

3. A dielectric slab is inserted between the plates of a capacitor. The charge on the capacitor is Q and the magnitude of the induced charge on each surface of the dielectric is Q'.
(a) Q' may be larger than Q.
(b) Q' must be larger than Q.
(c) Q' must be equal to Q.
(d) Q' must be smaller than Q.

4. Each plate of a parallel plate capacitor has a charge q on it. The capacitor is now connected to a battery. Now,
(a) the facing surfaces of the capacitor have equal and opposite charges
(b) the two plates of the capacitor have equal and opposite charges
(c) the battery supplies equal and opposite charges to the two plates
(d) the outer surfaces of the plates have equal charges

5. The separation between the plates of a charged parallel-plate capacitor is increased. Which of the following quantities will change ?
(a) Charge on the capacitor
(b) Potential difference across the capacitor
(c) Energy of the capacitor
(d) Energy density between the plates

6. A parallel-plate capacitor is connected to a battery. A metal sheet of negligible thickness is placed between the plates. The sheet remains parallel to the plates of the capacitor.
(a) The battery will supply more charge.
(b) The capacitance will increase.
(c) The potential difference between the plates will increase.
(d) Equal and opposite charges will appear on the two faces of the metal plate.

7. Following operations can be performed on a capacitor:
X – connect the capacitor to a battery of emf $\mathcal{E}$.
Y – disconnect the battery.
Z – reconnect the battery with polarity reversed.
W – insert a dielectric slab in the capacitor.
(a) In XYZ (perform X, then Y, then Z) the stored electric energy remains unchanged and no thermal energy is developed.
(b) The charge appearing on the capacitor is greater after the action XWY than after the action XYW.
(c) The electric energy stored in the capacitor is greater after the action WXY than after the action XYW.
(d) The electric field in the capacitor after the action XW is the same as that after WX.

EXERCISES

1. When $1{\cdot}0\times10^{12}$ electrons are transferred from one conductor to another, a potential difference of 10 V appears between the conductors. Calculate the capacitance of the two-conductor system.

2. The plates of a parallel-plate capacitor are made of circular discs of radii 5·0 cm each. If the separation between the plates is 1·0 mm, what is the capacitance ?

3. Suppose, one wishes to construct a 1·0 farad capacitor using circular discs. If the separation between the discs be kept at 1·0 mm, what would be the radius of the discs ?

4. A parallel-plate capacitor having plate area 25 cm^2 and separation 1·00 mm is connected to a battery of 6·0 V. Calculate the charge flown through the battery. How much work has been done by the battery during the process ?

5. A parallel-plate capacitor has plate area 25·0 cm^2 and a separation of 2·00 mm between the plates. The capacitor is connected to a battery of 12·0 V. (a) Find the charge on the capacitor. (b) The plate separation is decreased to 1·00 mm. Find the extra charge given by the battery to the positive plate.

6. Find the charges on the three capacitors connected to a battery as shown in figure (31-E1). Take $C_1 = 2{\cdot}0\ \mu F$, $C_2 = 4{\cdot}0\ \mu F$, $C_3 = 6{\cdot}0\ \mu F$ and $V = 12$ volts.

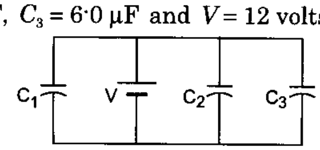

Figure 31-E1

7. Three capacitors having capacitances 20 μF, 30 μF and 40 μF are connected in series with a 12 V battery. Find the charge on each of the capacitors. How much work has been done by the battery in charging the capacitors ?

8. Find the charge appearing on each of the three capacitors shown in figure (31-E2).

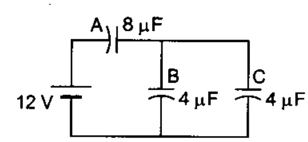

Figure 31-E2

9. Take $C_1 = 4{\cdot}0\ \mu F$ and $C_2 = 6{\cdot}0\ \mu F$ in figure (31-E3). Calculate the equivalent capacitance of the combination between the points indicated.

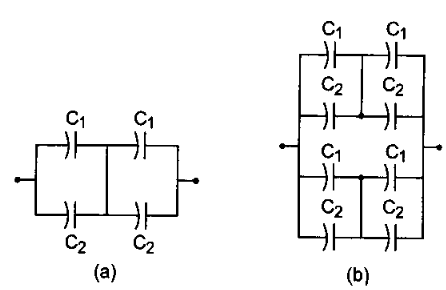

Figure 31-E3

10. Find the charge supplied by the battery in the arrangement shown in figure (31-E4).

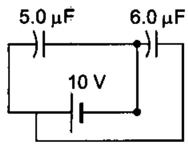

Figure 31-E4

11. The outer cylinders of two cylindrical capacitors of capacitance 2·2 μF each, are kept in contact and the inner cylinders are connected through a wire. A battery of emf 10 V is connected as shown in figure (31-E5). Find the total charge supplied by the battery to the inner cylinders.

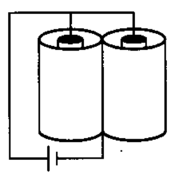

Figure 31-E5

12. Two conducting spheres of radii R_1 and R_2 are kept widely separated from each other. What are their individual capacitances ? If the spheres are connected by a metal wire, what will be the capacitance of the combination ? Think in terms of series–parallel connections.

13. Each of the capacitors shown in figure (31-E6) has a capacitance of 2 μF. Find the equivalent capacitance of the assembly between the points A and B. Suppose, a battery of emf 60 volts is connected between A and B. Find the potential difference appearing on the individual capacitors.

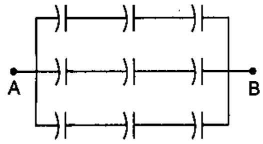

Figure 31-E6

14. It is required to construct a 10 μF capacitor which can be connected across a 200 V battery. Capacitors of capacitance 10 μF are available but they can withstand only 50 V. Design a combination which can yield the desired result.

15. Take the potential of the point B in figure (31-E7) to be zero. (a) Find the potentials at the points C and D. (b) If a capacitor is connected between C and D, what charge will appear on this capacitor ?

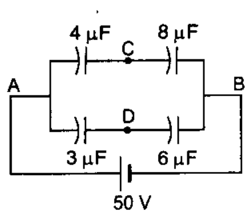

Figure 31-E7

16. Find the equivalent capacitance of the system shown in figure (31-E8) between the points a and b.

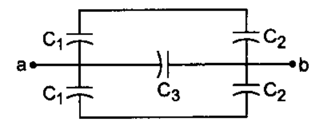

Figure 31-E8

17. A capacitor is made of a flat plate of area A and a second plate having a stair-like structure as shown in figure (31-E9). The width of each stair is a and the height is b. Find the capacitance of the assembly.

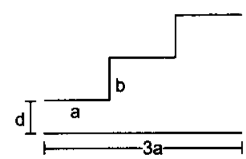

Figure 31-E9

18. A cylindrical capacitor is constructed using two coaxial cylinders of the same length 10 cm and of radii 2 mm and 4 mm. (a) Calculate the capacitance. (b) Another capacitor of the same length is constructed with cylinders of radii 4 mm and 8 mm. Calculate the capacitance.

19. A 100 pF capacitor is charged to a potential difference of 24 V. It is connected to an uncharged capacitor of capacitance 20 pF. What will be the new potential difference across the 100 pF capacitor ?

20. Each capacitor shown in figure (31-E10) has a capacitance of 5·0 μF. The emf of the battery is 50 V. How much charge will flow through AB if the switch S is closed ?

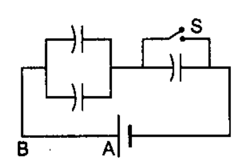

Figure 31-E10

21. The particle P shown in figure (31-E11) has a mass of 10 mg and a charge of $-0·01$ μC. Each plate has a surface area 100 cm^2 on one side. What potential difference V should be applied to the combination to hold the particle P in equilibrium ?

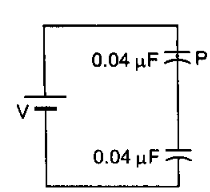

Figure 31-E11

22. Both the capacitors shown in figure (31-E12) are made of square plates of edge a. The separations between the plates of the capacitors are d_1 and d_2 as shown in the

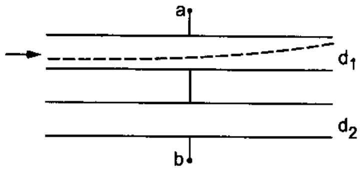

Figure 31-E12

figure. A potential difference V is applied between the points a and b. An electron is projected between the plates of the upper capacitor along the central line. With what minimum speed should the electron be projected so that it does not collide with any plate ? Consider only the electric forces.

23. The plates of a capacitor are 2·00 cm apart. An electron–proton pair is released somewhere in the gap between the plates and it is found that the proton reaches the negative plate at the same time as the electron reaches the positive plate. At what distance from the negative plate was the pair released ?

24. Convince yourself that parts (a), (b) and (c) of figure (31-E13) are identical. Find the capacitance between the points A and B of the assembly.

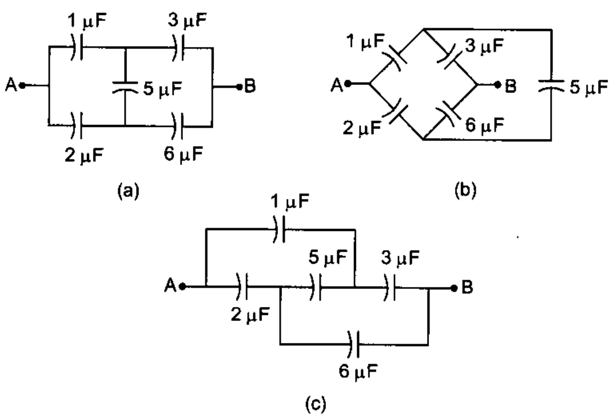

Figure 31-E13

25. Find the potential difference $V_a - V_b$ between the points a and b shown in each part of the figure (31-E14).

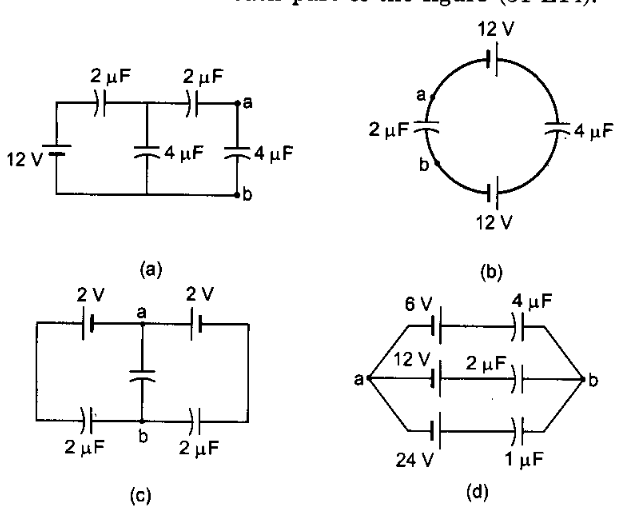

Figure 31-E14

26. Find the equivalent capacitances of the combinations shown in figure (31-E15) between the indicated points.

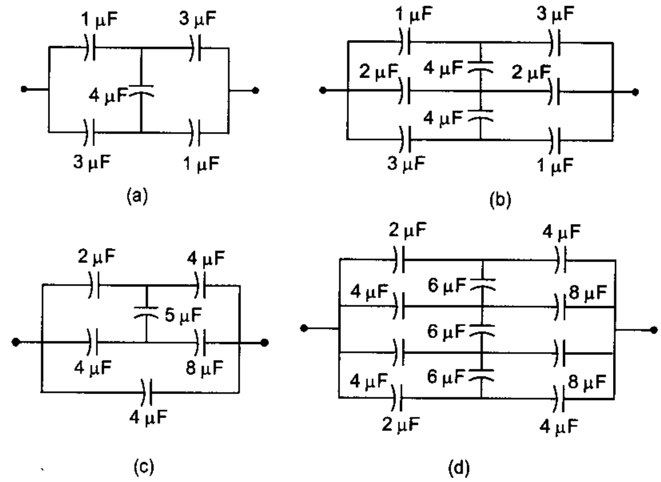

Figure 31-E15

27. Find the capacitance of the combination shown in figure (31-E16) between A and B.

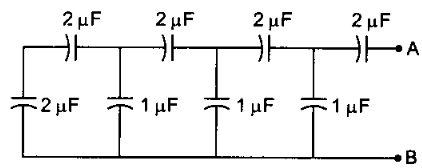

Figure 31-E16

28. Find the equivalent capacitance of the infinite ladder shown in figure (31-E17) between the points A and B.

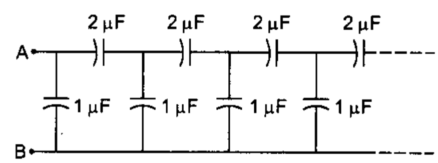

Figure 31-E17

29. A finite ladder is constructed by connecting several sections of 2 μF, 4 μF capacitor combinations as shown in figure (31-E18). It is terminated by a capacitor of capacitance C. What value should be chosen for C, such that the equivalent capacitance of the ladder between the points A and B becomes independent of the number of sections in between ?

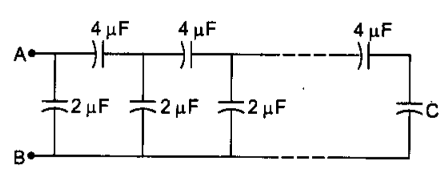

Figure 31-E18

30. A charge of $+ 2.0 \times 10^{-8}$ C is placed on the positive plate and a charge of $- 1.0 \times 10^{-8}$ C on the negative plate of a parallel-plate capacitor of capacitance 1.2×10^{-3} μF.

Calculate the potential difference developed between the plates.

31. A charge of 20 μC is placed on the positive plate of an isolated parallel-plate capacitor of capacitance 10 μF. Calculate the potential difference developed between the plates.

32. A charge of 1 μC is given to one plate of a parallel-plate capacitor of capacitance 0·1 μF and a charge of 2 μC is given to the other plate. Find the potential difference developed between the plates.

33. Each of the plates shown in figure (31-E19) has surface area $(96/\varepsilon_0) \times 10^{-12}$ Fm on one side and the separation between the consecutive plates is 4·0 mm. The emf of the battery connected is 10 volts. Find the magnitude of the charge supplied by the battery to each of the plates connected to it.

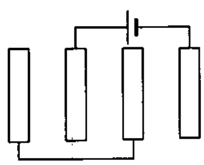

Figure 31-E19

34. The capacitance between the adjacent plates shown in figure (31-E20) is 50 nF. A charge of 1·0 μC is placed on the middle plate. (a) What will be the charge on the outer surface of the upper plate ? (b) Find the potential difference developed between the upper and the middle plates.

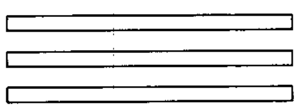

Figure 31-E20

35. Consider the situation of the previous problem. If 1·0 μC is placed on the upper plate instead of the middle, what will be the potential difference between (a) the upper and the middle plates and (b) the middle and the lower plates ?

36. Two capacitors of capacitances 20·0 pF and 50·0 pF are connected in series with a 6·00 V battery. Find (a) the potential difference across each capacitor and (b) the energy stored in each capacitor.

37. Two capacitors of capacitances 4·0 μF and 6·0 μF are connected in series with a battery of 20 V. Find the energy supplied by the battery.

38. Each capacitor in figure (31-E21) has a capacitance of 10 μF. The emf of the battery is 100 V. Find the energy stored in each of the four capacitors.

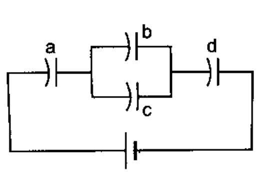

Figure 31-E21

39. A capacitor with stored energy 4·0 J is connected with an identical capacitor with no electric field in between. Find the total energy stored in the two capacitors.

40. A capacitor of capacitance 2·0 μF is charged to a potential difference of 12 V. It is then connected to an uncharged capacitor of capacitance 4·0 μF as shown in figure (31-E22). Find (a) the charge on each of the two capacitors after the connection, (b) the electrostatic energy stored in each of the two capacitors and (c) the heat produced during the charge transfer from one capacitor to the other.

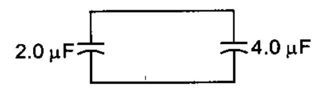

Figure 31-E22

41. A point charge Q is placed at the origin. Find the electrostatic energy stored outside the sphere of radius R centred at the origin.

42. A metal sphere of radius R is charged to a potential V. (a) Find the electrostatic energy stored in the electric field within a concentric sphere of radius 2 R. (b) Show that the electrostatic field energy stored outside the sphere of radius 2 R equals that stored within it.

43. A large conducting plane has a surface charge density $1{\cdot}0 \times 10^{-4}$ C m^{-2}. Find the electrostatic energy stored in a cubical volume of edge 1·0 cm in front of the plane.

44. A parallel-plate capacitor having plate area 20 cm^2 and separation between the plates 1·00 mm is connected to a battery of 12·0 V. The plates are pulled apart to increase the separation to 2·0 mm. (a) Calculate the charge flown through the circuit during the process. (b) How much energy is absorbed by the battery during the process ? (c) Calculate the stored energy in the electric field before and after the process. (d) Using the expression for the force between the plates, find the work done by the person pulling the plates apart. (e) Show and justify that no heat is produced during this transfer of charge as the separation is increased.

45. A capacitor having a capacitance of 100 μF is charged to a potential difference of 24 V. The charging battery is disconnected and the capacitor is connected to another battery of emf 12 V with the positive plate of the capacitor joined with the positive terminal of the battery. (a) Find the charges on the capacitor before and after the reconnection. (b) Find the charge flown through the 12 V battery. (c) Is work done by the battery or is it done on the battery ? Find its magnitude. (d) Find the decrease in electrostatic field energy. (e) Find the heat developed during the flow of charge after reconnection.

46. Consider the situation shown in figure (31-E23). The switch S is open for a long time and then closed. (a) Find the charge flown through the battery when the switch S is closed. (b) Find the work done by the battery.

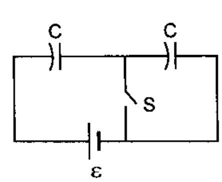

Figure 31-E23

(c) Find the change in energy stored in the capacitors.
(d) Find the heat developed in the system.

47. A capacitor of capacitance 5·00 μF is charged to 24·0 V and another capacitor of capacitance 6·0 μF is charged to 12·0 V. (a) Find the energy stored in each capacitor. (b) The positive plate of the first capacitor is now connected to the negative plate of the second and vice versa. Find the new charges on the capacitors. (c) Find the loss of electrostatic energy during the process. (d) Where does this energy go ?

48. A 5·0 μF capacitor is charged to 12 V. The positive plate of this capacitor is now connected to the negative terminal of a 12 V battery and vice versa. Calculate the heat developed in the connecting wires.

49. The two square faces of a rectangular dielectric slab (dielectric constant 4·0) of dimensions 20 cm × 20 cm × 1·0 mm are metal-coated. Find the capacitance between the coated surfaces.

50. If the above capacitor is connected across a 6·0 V battery, find (a) the charge supplied by the battery, (b) the induced charge on the dielectric and (c) the net charge appearing on one of the coated surfaces.

51. The separation between the plates of a parallel-plate capacitor is 0·500 cm and its plate area is 100 cm^2. A 0·400 cm thick metal plate is inserted into the gap with its faces parallel to the plates. Show that the capacitance of the assembly is independent of the position of the metal plate within the gap and find its value.

52. A capacitor stores 50 μC charge when connected across a battery. When the gap between the plates is filled with a dielectric, a charge of 100 μC flows through the battery. Find the dielectric constant of the material inserted.

53. A parallel-plate capacitor of capacitance 5 μF is connected to a battery of emf 6 V. The separation between the plates is 2 mm. (a) Find the charge on the positive plate. (b) Find the electric field between the plates. (c) A dielectric slab of thickness 1 mm and dielectric constant 5 is inserted into the gap to occupy the lower half of it. Find the capacitance of the new combination. (d) How much charge has flown through the battery after the slab is inserted ?

54. A parallel-plate capacitor has plate area 100 cm^2 and plate separation 1·0 cm. A glass plate (dielectric constant 6·0) of thickness 6·0 mm and an ebonite plate (dielectric constant 4·0) are inserted one over the other to fill the space between the plates of the capacitor. Find the new capacitance.

55. A parallel-plate capacitor having plate area 400 cm^2 and separation between the plates 1·0 mm is connected to a power supply of 100 V. A dielectric slab of thickness 0·5 mm and dielectric constant 5·0 is inserted into the gap. (a) Find the increase in electrostatic energy. (b) If the power supply is now disconnected and the dielectric slab is taken out, find the further increase in energy. (c) Why does the energy increase in inserting the slab as well as in taking it out ?

56. Find the capacitances of the capacitors shown in figure (31-E24). The plate area is A and the separation between the plates is d. Different dielectric slabs in a particular part of the figure are of the same thickness and the entire gap between the plates is filled with the dielectric slabs.

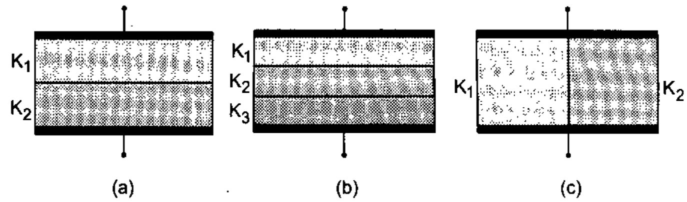

Figure 31-E24

57. A capacitor is formed by two square metal-plates of edge a, separated by a distance d. Dielectrics of dielectric constants K_1 and K_2 are filled in the gap as shown in figure (31-E25). Find the capacitance.

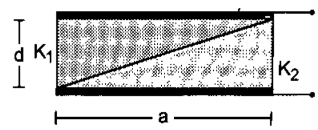

Figure 31-E25

58. Figure (31-E26) shows two identical parallel plate capacitors connected to a battery through a switch S. Initially, the switch is closed so that the capacitors are completely charged. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant 3. Find the ratio of the initial total energy stored in the capacitors to the final total energy stored.

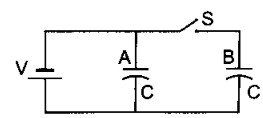

Figure 31-E26

59. A parallel-plate capacitor of plate area A and plate separation d is charged to a potential difference V and then the battery is disconnected. A slab of dielectric constant K is then inserted between the plates of the capacitor so as to fill the space between the plates. Find the work done on the system in the process of inserting the slab.

60. A capacitor having a capacitance of 100 μF is charged to a potential difference of 50 V. (a) What is the magnitude of the charge on each plate ? (b) The charging battery is disconnected and a dielectric of dielectric constant 2·5 is inserted. Calculate the new potential difference between the plates. (c) What charge would have produced this potential difference in absence of the dielectric slab. (d) Find the charge induced at a surface of the dielectric slab.

61. A sphercial capacitor is made of two conducting spherical shells of radii a and b. The space between the shells is filled with a dielectric of dielectric constant K

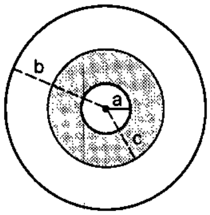

Figure 31-E27

up to a radius c as shown in figure (31-E27). Calculate the capacitance.

62. Consider an assembly of three conducting concentric spherical shells of radii a, b and c as shown in figure (31-E28). Find the capacitance of the assembly between the points A and B.

63. Suppose the space between the two inner shells of the previous problem is filled with a dielectric of dielectric constant K. Find the capacitance of the system between A and B.

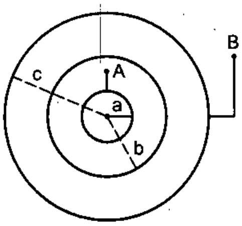

Figure 31-E28

64. An air-filled parallel-plate capacitor is to be constructed which can store 12 μC of charge when operated at 1200 V. What can be the minimum plate area of the capacitor? The dielectric strength of air is $3 \times 10^{6}\ \mathrm{V\ m^{-1}}$.

65. A parallel-plate capacitor with the plate area 100 cm^2 and the separation between the plates 1·0 cm is connected across a battery of emf 24 volts. Find the force of attraction between the plates.

66. Consider the situation shown in figure (31-E29). The width of each plate is b. The capacitor plates are rigidly clamped in the laboratory and connected to a battery of emf $\mathcal{E}$. All surfaces are frictionless. Calculate the value of M for which the dielectric slab will stay in equilibrium.

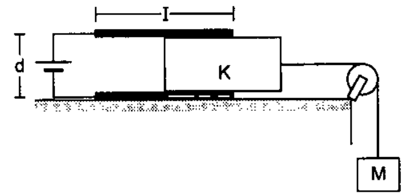

Figure 31-E29

67. Figure (31-E30) shows two parallel plate capacitors with fixed plates and connected to two batteries. The separation between the plates is the same for the two capacitors. The plates are rectangular in shape with width b and lengths l_1 and l_2. The left half of the dielectric slab has a dielectric constant K_1 and the right half K_2. Neglecting any friction, find the ratio of the emf of the left battery to that of the right battery for which the dielectric slab may remain in equilibrium.

68. Consider the situation shown in figure (31-E31). The plates of the capacitor have plate area A and are clamped in the laboratory. The dielectric slab is released from rest with a length a inside the capacitor. Neglecting any effect of friction or gravity, show that the slab will execute periodic motion and find its time period.

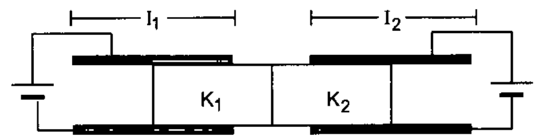

Figure 31-E30

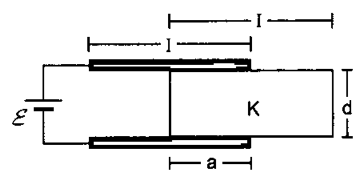

Figure 31-E31

□

ANSWERS

OBJECTIVE I

1. (d)	2. (d)	3. (c)	4. (b)	5. (c)	6. (d)
7. (b)	8. (d)	9. (c)	10. (a)	11. (b)	12. (d)

OBJECTIVE II

1. (c)	2. (b)	3. (d)	4. (a), (c), (d)	5. (b), (c)
6. (d)	7. (b), (c), (d)			

EXERCISES

1. $1{\cdot}6 \times 10^{-8}$ F
2. $6{\cdot}95 \times 10^{-5}$ μF
3. 6 km
4. $1{\cdot}33 \times 10^{-10}$ C, $8{\cdot}0 \times 10^{-10}$ J
5. (a) $1{\cdot}33 \times 10^{-10}$ C (b) $1{\cdot}33 \times 10^{-10}$ C

6. 24 μC, 48 μC, 72 μC
7. 110 μC on each, $1{\cdot}33 \times 10^{-3}$ J
8. 48 μC on the 8 μF capacitor and 24 μC on each of the 4 μF capacitors
9. (a) 5 μF (b) 10 μF
10. 110 μC
11. 44 μC
12. $4\pi\varepsilon_0 R_1$, $4\pi\varepsilon_0 R_2$; $4\pi\varepsilon_0 (R_1 + R_2)$
13. 2 μF, 20 V
15. (a) 50/3 μV at each point (b) zero
16. $C_3 + \dfrac{2\,C_1C_2}{C_1 + C_2}$
17. $\dfrac{\varepsilon_0 A(3\,d^2 + 6\,bd + 2\,b^2)}{3\,d(d+b)\,(d+2\,b)}$
18. (a) 8 pF (b) same as in (a)
19. 20 V
20. $3{\cdot}3 \times 10^{-4}$ C
21. 43 mV
22. $\left(\dfrac{Vea^2}{md_1(d_1 + d_2)}\right)^{1/2}$
23. $1{\cdot}08 \times 10^{-3}$ cm
24. 2·25 μF
25. (a) $\dfrac{12}{11}$ V (b) – 8 V (c) zero (d) – 10·3 V
26. (a) $\dfrac{11}{6}$ μF (b) $\dfrac{11}{4}$ μF (c) 8 μF (d) 8 μF
27. 1 μF
28. 2 μF
29. 4 μF
30. 12·5 V
31. 1 V
32. 5 V
33. 0·16 μC
34. (a) 0·50 μC (b) 10 V
35. (a) 10 V (b) 10 V
36. (a) 1·71 V, 4·29 V (b) 184 pJ, 73·5 pJ
37. 960 μJ
38. 8 mJ in (a) and (d), 2 mJ in (b) and (c)
39. 2·0 J
40. (a) 8 μC, 16 μC (b) 16 μJ, 32 μJ, (c) 96 μJ
41. $\dfrac{Q^2}{8\pi\varepsilon_0 R}$
42. (a) $\pi\varepsilon_0 RV^2$
43. $5{\cdot}6 \times 10^{-4}$ J
44. (a) $1{\cdot}06 \times 10^{-10}$ C (b) $12{\cdot}7 \times 10^{-10}$ J
    (c) $12{\cdot}7 \times 10^{-10}$ J, $6{\cdot}35 \times 10^{-10}$ J (d) $6{\cdot}35 \times 10^{-10}$ J
45. (a) 2400 μC, 1200 μC (b) 1200 μC (c) 14·4 mJ
    (d) 21·6 mJ (e) 7·2 mJ
46. (a) $C\mathcal{E}/2$, (b) $C\mathcal{E}^2/2$ (c) $C\mathcal{E}^2/4$ (d) $C\mathcal{E}^2/4$
47. (a) 1·44 mJ, 0·432 mJ (b) 21·8 μC, 26·2 μC, (c) 1·77 mJ
48. 1·44 mJ
49. 1·42 nF
50. (a) 8·5 nC (b) 6·4 nC (c) 2·1 nC
51. 88 pF
52. 3
53. (a) 30 μC (b) 3×10^3 V m^{-1} (c) 8·3 μF (d) 20 μC
54. 44 pF
55. (a) 1·18 μJ (b) 1·97 μJ
56. (a) $\dfrac{2\,K_1K_2\varepsilon_0 A}{d(K_1 + K_2)}$ (b) $\dfrac{3\varepsilon_0 A\,K_1K_2K_3}{d(K_1K_2 + K_2K_3 + K_3K_1)}$
    (c) $\dfrac{\varepsilon_0 A}{2\,d}(K_1 + K_2)$
57. $\dfrac{\varepsilon_0 K_1K_2 a^2 \ln \dfrac{K_1}{K_2}}{(K_1 - K_2)d}$
58. 3 : 5
59. $\dfrac{\varepsilon_0 AV^2}{2\,d}\left(\dfrac{1}{K} - 1\right)$
60. (a) 5 mC (b) 20 V (c) 2 mC (d) 3 mC
61. $\dfrac{4\pi\varepsilon_0 Kabc}{Ka(b-c) + b(c-a)}$
62. $\dfrac{4\pi\varepsilon_0\, ac}{c-a}$
63. $\dfrac{4\pi\varepsilon_0 Kabc}{Ka(c-b) + c(b-a)}$
64. 0·45 m^2
65. $2{\cdot}5 \times 10^{-7}$ N
66. $\dfrac{\varepsilon_0\, b\, \mathcal{E}^2(K-1)}{2\,dg}$
67. $\sqrt{\dfrac{K_2 - 1}{K_1 - 1}}$
68. $8\sqrt{\dfrac{(l-a)\,lmd}{\varepsilon_0 A\mathcal{E}^2(K-1)}}$

□