Definite Integrals Ex 20.4B Q44

$$I = \int_a^b x f(x)\,dx$$

$$\Rightarrow I = \int_a^b (a+b-x) f(a+b-x)\,dx$$

$$\Rightarrow I = \int_a^b (a+b-x) f(x)\,dx \ldots\ldots\ldots\ldots [\text{Given that } f(a+b-x) = f(x)]$$

$$\Rightarrow I = \int_a^b (a+b) f(x)\,dx - \int_a^b x f(x)\,dx$$

$$\Rightarrow I = \int_a^b (a+b) f(x)\,dx - I$$

$$\Rightarrow 2I = \int_a^b (a+b) f(x)\,dx$$

$$\Rightarrow I = \frac{a+b}{2} \int_a^b f(x)\,dx$$

Definite Integrals Ex 20.4B Q45

We have,

$$I = \int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_{0}^{a} f(x)\,dx$$

Let $x = -t$ then $dx = -dt$

$x = -a \Rightarrow t = a$

$x = 0 \Rightarrow t = 0$

$$\therefore \int_{-a}^{a} f(x)\,dx = \int_{a}^{0} f(-t)(-dt) = -\int_{a}^{0} f(-t)\,dt$$

$$\Rightarrow \int_{-a}^{a} f(x)\,dx = \int_{0}^{a} f(-t)\,dt$$

$$\Rightarrow \int_{-a}^{0} f(x)\,dx = \int_{0}^{a} f(-x)\,dx$$

$$\therefore \int_{-a}^{a} f(x)\,dx = \int_{0}^{a} f(-x)\,dx + \int_{0}^{a} f(x)\,dx$$

Hence,

$$\int_{-a}^{a} f(x)\,dx = \int_{0}^{a} \{f(-x) + f(x)\}\,dx$$

Proved

Definite Integrals Ex 20.4B Q46

$$I = \int_0^{\pi} x f(\sin x)dx$$

$$I = \int_0^{\pi} (\Pi - x) f(\sin(\Pi - x))dx$$

$$I = \int_0^{\pi} (\Pi - x) f(\sin x)dx$$

$$2I = \int_0^{\pi} \Pi f(\sin x)dx$$

$$I = \frac{\Pi}{2}\int_0^{\pi} f(\sin x)dx$$

*********** END **********