CHAPTER 40

ELECTROMAGNETIC WAVES

40.1 INTRODUCTION

We have seen that in certain situations light may be described as a wave. The wave equation for light propagating in x-direction in vacuum may be written as

$$E = E_0 \sin \omega(t - x/c)$$

where E is the sinusoidally varying electric field at the position x at time t. The constant c is the speed of light in vacuum. The electric field E is in the Y–Z plane, that is, perpendicular to the direction of propagation.

There is also a sinusoidally varying magnetic field associated with the electric field when light propagates. This magnetic field is perpendicular to the direction of propagation as well as to the electric field E. It is given by

$$B = B_0 \sin \omega(t - x/c).$$

Such a combination of mutually perpendicular electric and magnetic fields is referred to as an *electromagnetic wave* in vacuum. The theory of electromagnetic wave was mainly developed by Maxwell around 1864. We give a brief discussion of this theory.

40.2 MAXWELL'S DISPLACEMENT CURRENT

We have stated Ampere's law as

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i \qquad \ldots (40.1)$$

where i is the electric current crossing a surface bounded by a closed curve and the line integral of $\vec{B}$ (circulation) is calculated along that closed curve. This equation is valid only when the electric field at the surface does not change with time. This law tells us that an electric current produces magnetic field and gives a method to calculate the field.

Ampere's law in this form is not valid if the electric field at the surface varies with time. As an example, consider a parallel-plate capacitor with circular plates, being charged by a battery (figure 40.1). If we place a compass needle in the space between the plates, the needle, in general, deflects. This shows that there is a magnetic field in this region. Figure (40.1) also shows a closed curve γ which lies completely in the region between the plates. The plane surface S bounded by this curve is also parallel to the plates and lies completely inside the region between the plates.

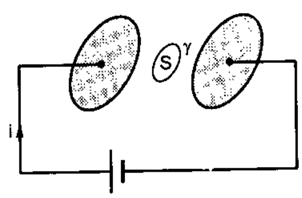

Figure 40.1

During the charging process, there is an electric current through the connecting wires. Charge is accumulated on the plates and the electric field at the points on the surface S changes. It is found that there is a magnetic field at the points on the curve γ and the circulation

$$\oint \vec{B} \cdot d\vec{l}$$

has a nonzero value. As no charge crosses the surface S, the electric current i through the surface is zero. Hence,

$$\oint \vec{B} \cdot d\vec{l} \neq \mu_0 i. \qquad \ldots \text{(i)}$$

Now, Ampere's law (40.1) can be deduced from Biot–Savart law. We can calculate the magnetic field due to each current element from Biot–Savart law and then its circulation along the closed curve γ. The circulation of the magnetic field due to these current elements must satisfy equation (40.1). If we denote this magnetic field by $\vec{B'}$,

$$\oint \vec{B'} \cdot d\vec{l} = 0. \qquad \ldots \text{(ii)}$$

This shows that the actual magnetic field $\vec{B}$ is different from the field $\vec{B'}$ produced by the electric currents only. So, there must be some other source of magnetic field. This other source is nothing but the changing electric field. As the capacitor gets charged, the electric field between the plates changes and this changing electric field produces magnetic field.

We know that a changing magnetic field produces an electric field. The relation between the two is given by Faraday's law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}.$$

Here, $\Phi_B = \int \vec{B} \cdot d\vec{S}$ is the flux of the magnetic field through the area bounded by the closed curve along which the circulation of $\vec{E}$ is calculated. Now we find that a changing electric field produces a magnetic field. The relation between the changing electric field and the magnetic field resulting from it is given by

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}. \quad \dots (40.2)$$

Here, Φ_E is the flux of the electric field through the area bounded by the closed curve along which the circulation of $\vec{B}$ is calculated. Equation (40.1) gives the magnetic field resulting from an electric current due to flow of charges and equation (40.2) gives the magnetic field due to the changing electric field. If there exists an electric current as well as a changing electric field, the resultant magnetic field is given by

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i + \mu_0 \varepsilon_0 \left(\frac{d\Phi_E}{dt} \right)$$

or, $$\oint \vec{B} \cdot d\vec{l} = \mu_0 (i + i_d) \quad \dots (40.3)$$

where $$i_d = \varepsilon_0 \frac{d\Phi_E}{dt}.$$

It was James Clerk Maxwell who generalised Ampere's law from equation (40.1) to equation (40.3). Maxwell named the term $i_d = \varepsilon_0 \frac{d\Phi_E}{dt}$ as *displacement current*. The current due to flow of charges is often called *conduction current* and is denoted by i_c.

Example 40.1

A parallel-plate capacitor is being charged. Show that the displacement current across an area in the region between the plates and parallel to it (figure 40.1) is equal to the conduction current in the connecting wires.

Solution :

The electric field between the plates is

$$E = \frac{Q}{\varepsilon_0 A}$$

where Q is the charge accumulated at the positive plate. The flux of this field through the given area is

$$\Phi_E = \frac{Q}{\varepsilon_0 A} \times A = \frac{Q}{\varepsilon_0}.$$

The displacement current is

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d}{dt} \left(\frac{Q}{\varepsilon_0} \right) = \frac{dQ}{dt}.$$

But $\frac{dQ}{dt}$ is the rate at which the charge is carried to the positive plate through the connecting wire. Thus, $i_d = i_c$.

40.3 CONTINUITY OF ELECTRIC CURRENT

Consider a closed surface enclosing a volume (figure 40.2). Suppose charges are entering into the volume and are also leaving it. If no charge is accumulated inside the volume, the total charge going into the volume in any time is equal to the total charge leaving it during the same time. The conduction current is then continuous.

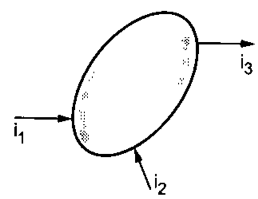

Figure 40.2

If charge is accumulated inside the volume, this continuity breaks. However, if we consider the conduction current plus the displacement current, the total current is still continuous. Any loss of conduction current i_c appears as displacement current i_d. This can be shown as follows.

Suppose a total conduction current i_1 goes into the volume and a total conduction current i_2 goes out of it. The charge going into the volume in a time dt is $i_1\,dt$ and that coming out is $i_2\,dt$. The charge accumulated inside the volume is

$$d(q_{inside}) = i_1\,dt - i_2\,dt$$

or, $$\frac{d}{dt}(q_{inside}) = i_1 - i_2. \quad \dots (i)$$

From Gauss's law,

$$\Phi_E = \oint \vec{E} \cdot d\vec{S} = \frac{q_{inside}}{\varepsilon_0}$$

or, $$\varepsilon_0 \frac{d\Phi_E}{dt} = \frac{d}{dt}(q_{inside})$$

or, $$i_d = \frac{d}{dt}(q_{inside}).$$

Comparing with (i),

$$i_1 - i_2 = i_d$$

or, $$i_1 = i_2 + i_d.$$

Thus, the total current (conduction + displacement) going into the volume is equal to the total current coming out of it.

40.4 MAXWELL'S EQUATIONS AND PLANE ELECTROMAGNETIC WAVES

The whole subject of electricity and magnetism may be described mathematically with the help of four fundamental equations:

Gauss's law for electricity $\oint \vec{E}\cdot d\vec{S} = \frac{q}{\varepsilon_0}$

Gauss's law for magnetism $\oint \vec{B}\cdot d\vec{S} = 0$

Faraday's law $\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$

Ampere's law $\oint \vec{B}\cdot d\vec{l} = \mu_0 i + \varepsilon_0\mu_0 \frac{d\Phi_E}{dt}$.

These equations are collectively known as Maxwell's equations.

In vacuum, there are no charges and hence no conduction currents. Faraday's law and Ampere's law take the form

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt} \quad \ldots \text{(i)}$$

and

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\,\varepsilon_0 \frac{d\Phi_E}{dt} \quad \ldots \text{(ii)}$$

respectively.

Let us check if these equations are satisfied by a plane electromagnetic wave given by

$$\left.\begin{aligned} E &= E_y = E_0 \sin \omega(t - x/c) \\ \text{and} \quad B &= B_z = B_0 \sin \omega(t - x/c). \end{aligned}\right] \quad \ldots (40.4)$$

The wave described above propagates along the positive x-direction, the electric field remains along the y-direction and the magnetic field along the z-direction. The magnitudes of the fields oscillate between $\pm E_0$ and $\pm B_0$ respectively. It is a linearly polarized light, polarized along the y-axis.

Faraday's Law

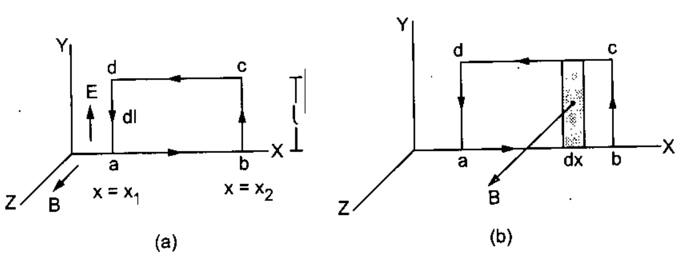

Figure 40.3

Let us consider the rectangular path $abcd$ in the x–y plane as shown in figure (40.3a). Let us evaluate the terms in the Faraday's law on this path. The electric field is parallel to the y-axis. The circulation of E is

$$\oint \vec{E}\cdot d\vec{l} = \int_a^b \vec{E}\cdot d\vec{l} + \int_b^c \vec{E}\cdot d\vec{l} + \int_c^d \vec{E}\cdot d\vec{l} + \int_d^a \vec{E}\cdot d\vec{l}$$

$$= 0 + E(x_2)\, l + 0 + E(x_1)\,(-l)$$

$$= E_0\, l\,[\sin \omega(t - x_2/c) - \sin \omega(t - x_1/c)]. \quad \ldots \text{(i)}$$

Next, let us calculate the flux of the magnetic field Φ_B, through the same rectangle $abcd$ (figure 40.3b). The flux through a strip of width dx at x is

$$B(x)\, l\, dx = B_0\,[\sin \omega(t - x/c)]\, l\, dx.$$

The flux through the rectangle $abcd$ is

$$\Phi_B = \int_{x_1}^{x_2} B_0\, l \sin \omega(t - x/c)\, dx$$

$$= -\frac{c}{\omega} B_0\, l[-\cos \omega(t - x_2/c) + \cos \omega(t - x_1/c)].$$

Thus,

$$\frac{d\Phi_B}{dt} = -cB_0\, l[\sin \omega(t - x_2/c) - \sin \omega(t - x_1/c)]. \quad \ldots \text{(ii)}$$

The Faraday's law for vacuum is

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}.$$

Putting from (i) and (ii) in this equation, we see that Faraday's law is satisfied by the wave given by equation (40.4a) if

$$E_0 = cB_0. \quad \ldots (40.5)$$

Ampere's Law

Let us consider the rectangular path $efgh$ in the x–z plane as shown in figure (40.4a).

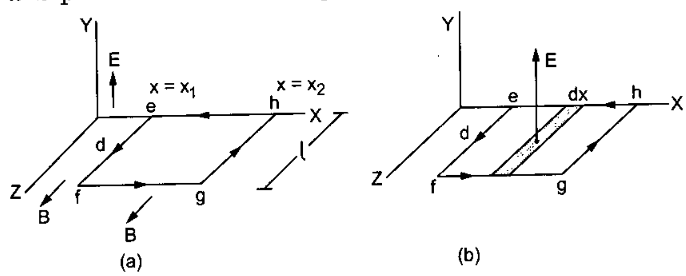

Figure 40.4

The circulation of $\vec{B}$ is

$$\oint \vec{B}\cdot d\vec{l} = \int_e^f \vec{B}\cdot d\vec{l} + \int_f^g \vec{B}\cdot d\vec{l} + \int_g^h \vec{B}\cdot d\vec{l} + \int_h^e \vec{B}\cdot d\vec{l}$$

$$= B(x_1)\, l + 0 - B(x_2)\, l + 0$$

$$= B_0\, l[\sin \omega(t - x_1/c) - \sin \omega(t - x_2/c)]. \quad \ldots \text{(i)}$$

The flux of the electric field through the same rectangle $efgh$ (figure 40.4b) is

$$\Phi_E = \int \vec{E}\cdot d\vec{S}$$

$$= \int_{x_1}^{x_2} E(x)\, l\, dx$$

$$= E_0\, l \int_{x_1}^{x_2} \sin \omega(t - x/c)\, dx$$

$$= -\frac{c}{\omega} E_0\, l[-\cos \omega(t - x_2/c) + \cos \omega(t - x_1/c)]$$

or, $$\frac{d\Phi_E}{dt} = -cE_0\, l[\sin \omega(t - x_2/c) - \sin \omega(t - x_1/c)]. \quad \dots \text{(ii)}$$

The Ampere's law for vacuum is

$$\oint \vec{B} \cdot d\vec{l} = \mu_0\, \varepsilon_0 \frac{d\Phi_E}{dt}.$$

Putting from (i) and (ii) in this equation, we see that Ampere's law is satisfied if

$$B_0 = \mu_0\, \varepsilon_0\, c\, E_0$$

or, $$\mu_0\, \varepsilon_0 = \frac{B_0}{E_0 c}.$$

Using equation (40.5),

$$\mu_0\, \varepsilon_0 = \frac{1}{c^2}$$

or, $$c = \frac{1}{\sqrt{\mu_0\, \varepsilon_0}}. \quad \dots (40.6)$$

Thus, Maxwell's equations have a solution giving a plane electromagnetic wave of the form (40.4) with $E_0 = cB_0$ and the speed of this wave is $\frac{1}{\sqrt{\mu_0\, \varepsilon_0}}$.

In older days, μ_0 and ε_0 were defined in terms of electric and magnetic measurements. Putting these values of μ_0 and ε_0, the speed of electromagnetic waves came out to be $c = 2{\cdot}99793 \times 10^8$ m/s which was the same as the measured speed of light in vacuum. This provided a confirmatory proof that light is an electromagnetic wave.

It may be recalled that the speed of electromagnetic waves, which is the same as the speed of light, is now an exactly defined constant. Similarly, the constant $\mu_0 = 4\pi \times 10^{-7}$ T–m/A is an exactly defined constant. The quantity ε_0 is defined by the equation (40.6).

Example 40.2

The maximum electric field in a plane electromagnetic wave is 600 N C^{-1}. The wave is going in the x-direction and the electric field is in the y-direction. Find the maximum magnetic field in the wave and its direction.

Solution :

We have $B_0 = \frac{E_0}{c} = \frac{600 \text{ N C}^{-1}}{3 \times 10^8 \text{ m s}^{-1}} = 2 \times 10^{-6}$ T.

As $\vec{E}, \vec{B}$ and the direction of propagation are mutually perpendicular, $\vec{B}$ should be along the z-direction.

40.5 ENERGY DENSITY AND INTENSITY

The electric and magnetic field in a plane electromagnetic wave are given by

$$E = E_0 \sin \omega(t - x/c)$$

and $$B = B_0 \sin \omega(t - x/c).$$

In any small volume dV, the energy of the electric field is

$$U_E = \frac{1}{2} \varepsilon_0\, E^2 dV$$

and the energy of the magnetic field is

$$U_B = \frac{1}{2\mu_0} B^2 dV.$$

Thus, the total energy is

$$U = \frac{1}{2} \varepsilon_0\, E^2\, dV + \frac{1}{2\mu_0} B^2 dV.$$

The energy density is $u = \frac{1}{2} \varepsilon_0\, E^2 + \frac{1}{2\mu_0} B^2$

$$= \frac{1}{2} \varepsilon_0\, E_0^2 \sin^2 \omega(t - x/c) + \frac{1}{2\,\mu_0} B_0^2 \sin^2 \omega(t - x/c).$$

If we take the average over a long time, the $\sin^2$ terms have an average value of 1/2. Thus,

$$u_{au} = \frac{1}{4} \varepsilon_0\, E_0^2 + \frac{1}{4\,\mu_0} B_0^2.$$

From equations (40.5) and (40.6),

$$E_0 = cB_0 \text{ and } \mu_0 \varepsilon_0 = \frac{1}{c^2} \text{ so that,}$$

$$\frac{1}{4\,\mu_0} B_0^2 = \frac{\varepsilon_0\, c^2}{4} \left(\frac{E_0}{c}\right)^2 = \frac{1}{4} \varepsilon_0\, E_0^2.$$

Thus, the electric energy density is equal to the magnetic energy density in average.

or, $$u_{av} = \frac{1}{4} \varepsilon_0\, E_0^2 + \frac{1}{4} \varepsilon_0\, E_0^2 = \frac{1}{2} \varepsilon_0\, E_0^2. \quad \dots (40.7)$$

Also, $$u_{av} = \frac{1}{4\,\mu_0} B_0^2 + \frac{1}{4\,\mu_0} B_0^2 = \frac{1}{2\,\mu_0} B_0^2. \quad \dots (40.8)$$

Example 40.3

The electric field in an electromagnetic wave is given by

$$E = (50 \text{ N C}^{-1}) \sin \omega(t - x/c).$$

Find the energy contained in a cylinder of cross-section 10 cm^2 and length 50 cm along the x-axis.

Solution :

The energy density is

$$u_{av} = \frac{1}{2} \varepsilon_0\, E_0^2 = \frac{1}{2} \times (8{\cdot}85 \times 10^{-12} \text{ C}^2\text{N}^{-1}\text{m}^{-2}) \times (50 \text{ N C}^{-1})^2$$

$$= 1{\cdot}1 \times 10^{-8} \text{ J m}^{-3}.$$

The volume of the cylinder is

$$V = 10 \text{ cm}^2 \times 50 \text{ cm} = 5 \times 10^{-4} \text{ m}^3.$$

The energy contained in this volume is

$$U = (1{\cdot}1 \times 10^{-8}\ \mathrm{J\ m^{-3}}) \times (5 \times 10^{-4}\ \mathrm{m^3})$$
$$= 5{\cdot}5 \times 10^{-12}\ \mathrm{J}.$$

Intensity

The energy crossing per unit area per unit time perpendicular to the direction of propagation is called the intensity of a wave.

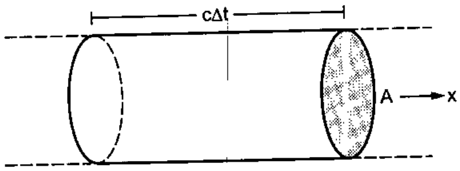

Figure 40.5

Consider a cylindrical volume with area of cross-section A and length $c\,\Delta t$ along the X-axis (figure 40.5). The energy contained in this cylinder crosses the area A in time Δt as the wave propagates at speed c. The energy contained is

$$U = u_{av}(c\,\Delta t)A.$$

The intensity is $I = \frac{U}{A\Delta t} = u_{av}\,c.$

In terms of maximum electric field,

$$I = \frac{1}{2}\varepsilon_0 E_0^2 c. \quad \dots (40.9)$$

Example 40.4

Find the intensity of the wave discussed in example (40.3).

Solution :

The intensity is

$$I = \frac{1}{2}\varepsilon_0 E_0^2 c = (1{\cdot}1 \times 10^{-8}\ \mathrm{J\ m^{-3}}) \times (3 \times 10^{8}\ \mathrm{m\ s^{-1}})$$
$$= 3{\cdot}3\ \mathrm{W\ m^{-2}}.$$

40.6 MOMENTUM

The electromagnetic wave also carries linear momentum with it. The linear momentum carried by the portion of wave having energy U is given by

$$p = \frac{U}{c}. \quad \dots (40.10)$$

Thus, if the wave incident on a material surface is completely absorbed, it delivers energy U and momentum $p = U/c$ to the surface. If the wave is totally reflected, the momentum delivered is $2U/c$ because the momentum of the wave changes from p to $-p$. It follows that electromagnetic waves incident on a surface exert a force on the surface.

40.7 ELECTROMAGNETIC SPECTRUM AND RADIATION IN ATMOSPHERE

Maxwell's equations are applicable for electromagnetic waves of all wavelengths. Visible light has wavelengths roughly in the range 380 nm to 780 nm. Today we are familiar with electromagnetic waves having wavelengths as small as 30 fm ($1\ \mathrm{fm} = 10^{-15}\ \mathrm{m}$) to as large as 30 km. Figure (40.6) shows the electromagnetic spectrum we are familiar with. The boundaries separating different regions of spectrum are not sharply defined. The gamma ray region and the X-ray region overlap considerably. We can only say that on the average, wavelengths of gamma rays are shorter than those of X-rays.

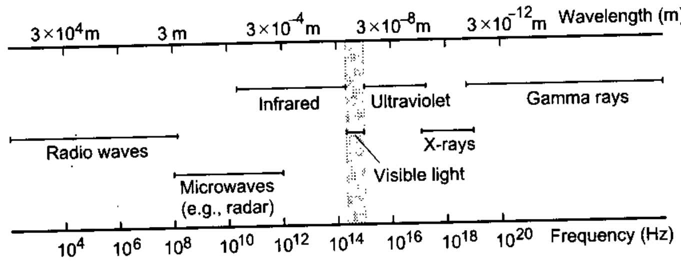

Figure 40.6

The basic source of electromagnetic waves is an accelerated charge. This produces changing electric field and changing magnetic field which constitute the wave. Radio waves may be produced by charges accelerating in AC circuits having an inductor and a capacitor. These waves are used in radio and TV communication. Microwaves are also produced by such electric circuits with oscillating current. They are used for radar systems among other applications. Microwave ovens are used for cooking. Infrared waves are emitted by the atoms and molecules of hot bodies. These waves are used in physical therapy. Among the electromagnetic waves, visible light is most familiar to us. This is emitted by atoms under suitable conditions. An atom contains electrons and the light emission is related to the acceleration of an electron inside the atom. The mechanism of emission of ultraviolet radiation is similar to that for visible light. The sun emits large amount of ultraviolet radiation. This radiation is harmful to us if absorbed in large amount. X-rays are produced most commonly when fast-moving electrons decelerate inside a metal target. X-rays are widely used in medical diagnosis. They are harmful to living tissues. Gamma rays are emitted by the nuclei and have the shortest wavelengths among the electromagnetic waves we generally deal with.

Radiation in Atmosphere

The earth is surrounded by atmosphere up to a height of about 300 km. The composition of atmosphere differs widely as one moves up. Most of the water droplets, vapour and ice particles forming clouds, are contained in a layer starting from the earth's surface up to height of about 12 km. This part is called *troposphere*. The density of air at the top of the troposphere is about one tenth of the density near the earth's surface. The atmosphere between the heights of 12 km and 50 km is called *stratosphere*. In the upper part of the stratosphere, we have a layer of ozone. The density of air at the top of the stratosphere is about 10^{-3} times the density at the surface of the earth. Then we have *mesosphere* between a height of 50 km and 80 km. The atmosphere above that is called *ionosphere*. There are no sharp boundaries between the above divisions and the numbers given are only a rough guide.

The main source of electromagnetic radiation in the atmosphere is the sun. The sun sends electromagnetic waves of different wavelengths towards the earth. A major part of it is absorbed by the atmosphere. Visible light is only weakly absorbed. Most of the infrared radiation is absorbed by the atmosphere and used to heat it. The radiation from the sun has a lot of ultraviolet radiation. The ozone layer absorbs most of this radiation and other radiations of lower wavelengths and thus protects us from their harmful effects. The ozone layer converts the ultraviolet radiation to infrared which is used to heat the atmosphere and the earth's surface. It is suspected that ozone layer is slowly being depleted and this is causing great concern to scientists and environmentalists.

Worked Out Examples

1. *A parallel-plate capacitor with plate area A and separation between the plates d, is charged by a constant current i. Consider a plane surface of area A/2 parallel to the plates and drawn symmetrically between the plates. Find the displacement current through this area.*

Solution :

Suppose the charge on the capacitor at time t is Q. The electric field between the plates of the capacitor is $E = \frac{Q}{\varepsilon_0 A}$. The flux through the area considered is

$$\Phi_E = \frac{Q}{\varepsilon_0 A} \cdot \frac{A}{2} = \frac{Q}{2\,\varepsilon_0}.$$

The displacement current is

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \left(\frac{1}{2\,\varepsilon_0}\right)\frac{dQ}{dt} = \frac{i}{2}.$$

2. *A plane electromagnetic wave propagating in the x-direction has a wavelength of* 5·0 mm. *The electric field is in the y-direction and its maximum magnitude is* 30 V m^{-1}. *Write suitable equations for the electric and magnetic fields as a function of x and t.*

Solution :

The equation for the electric and the magnetic fields in the wave may be written as

$$E = E_0 \sin \omega\left(t - \frac{x}{c}\right)$$

$$B = B_0 \sin \omega\left(t - \frac{x}{c}\right).$$

We have,

$$\omega = 2\pi\nu = \frac{2\pi}{\lambda}c.$$

Thus, $$E = E_0 \sin\left[\frac{2\pi}{\lambda}(ct - x)\right]$$

$$= (30\text{ V m}^{-1}) \sin\left[\frac{2\pi}{5\cdot0\text{ mm}}(ct - x)\right].$$

The maximum magnetic field is

$$B_0 = \frac{E_0}{c} = \frac{30\text{ V m}^{-1}}{3\times10^{8}\text{ m s}^{-1}} = 10^{-7}\text{ T}.$$

So, $$B = B_0 \sin\left[\frac{2\pi}{\lambda}(ct - x)\right]$$

$$= (10^{-7}\text{ T}) \sin\left[\frac{2\pi}{5\cdot0\text{ mm}}(ct - x)\right].$$

The magnetic field is along the z-axis.

3. *A light beam travelling in the x-direction is described by the electric field* $E_y = (300\text{ V m}^{-1}) \sin \omega(t - x/c)$. *An electron is constrained to move along the y-direction with a speed of* $2\cdot0\times10^{7}$ m s^{-1}. *Find the maximum electric force and the maximum magnetic force on the electron.*

Solution :

The maximum electric field is $E_0 = 300\text{ V m}^{-1}$. The maximum magnetic field is

$$B_0 = \frac{E_0}{c} = \frac{300\text{ V m}^{-1}}{3\times10^{8}\text{ m s}^{-1}} = 10^{-6}\text{ T}$$

along the z-direction.

The maximum electric force on the electron is

$$F_e = qE_0 = (1\cdot6\times10^{-19}\text{ C})\times(300\text{ V m}^{-1})$$

$$= 4\cdot8\times10^{-17}\text{ N}.$$

The maximum magnetic force on the electron is

$$F_b = |q\vec{v}\times\vec{B}|_{max} = qvB_0$$

$$= (1{\cdot}6 \times 10^{-19}\ \mathrm{C}) \times (2{\cdot}0 \times 10^{7}\ \mathrm{m\ s^{-1}}) \times (10^{-6}\ \mathrm{T})$$
$$= 3{\cdot}2 \times 10^{-18}\ \mathrm{N}.$$

4. *Find the energy stored in a* 60 cm *length of a laser beam operating at* 4 mW.

Solution :

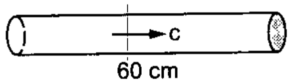

Figure 40-W1

The time taken by the electromagnetic wave to move through a distance of 60 cm is $t = \frac{60\ \mathrm{cm}}{c} = 2 \times 10^{-9}$ s. The energy contained in the 60 cm length passes through a cross-section of the beam in 2×10^{-9} s (figure 40-W1). But the energy passing through any cross section in 2×10^{-9} s is

$$U = (4\ \mathrm{mW}) \times (2 \times 10^{-9}\ \mathrm{s})$$
$$= (4 \times 10^{-3}\ \mathrm{Js^{-1}}) \times (2 \times 10^{-9}\ \mathrm{s})$$
$$= 8 \times 10^{-12}\ \mathrm{J}.$$

This is the energy contained in 60 cm length.

5. *Find the amplitude of the electric field in a parallel beam of light of intensity* $2{\cdot}0\ \mathrm{W\ m^{-2}}$.

Solution :

The intensity of a plane electromagnetic wave is

$$I = u_{av}\, c = \frac{1}{2} \varepsilon_0 E_0^2\, c$$

or, $$E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}}$$

$$= \sqrt{\frac{2 \times (2{\cdot}0\ \mathrm{W\ m^{-2}})}{(8{\cdot}85 \times 10^{-12}\ \mathrm{C^2N^{-1}m^{-2}}) \times (3 \times 10^{8}\ \mathrm{m\ s^{-1}})}}$$

$$= 38{\cdot}8\ \mathrm{N\ C^{-1}}.$$

□

QUESTIONS FOR SHORT ANSWER

1. In a microwave oven, the food is kept in a plastic container and the microwave is directed towards the food. The food is cooked without melting or igniting the plastic container. Explain.
2. A metal rod is placed along the axis of a solenoid carrying a high-frequency alternating current. It is found that the rod gets heated. Explain why the rod gets heated.
3. Can an electromagnetic wave be deflected by an electric field ? By a magnetic field ?
4. A wire carries an alternating current $i = i_0 \sin \omega t$. Is there an electric field in the vicinity of the wire ?
5. A capacitor is connected to an alternating-current source. Is there a magnetic field between the plates ?
6. Can an electromagnetic wave be polarized ?
7. A plane electromagnetic wave is passing through a region. Consider the quantities (a) electric field, (b) magnetic field, (c) electrical energy in a small volume and (d) magnetic energy in a small volume. Construct pairs of the quantities that oscillate with equal frequencies.

OBJECTIVE I

1. A magnetic field can be produced by
   (a) a moving charge (b) a changing electric field
   (c) none of them (d) both of them.
2. A compass needle is placed in the gap of a parallel plate capacitor. The capacitor is connected to a battery through a resistance. The compass needle
   (a) does not deflect
   (b) deflects for a very short time and then comes back to the original position
   (c) deflects and remains deflected as long as the battery is connected
   (d) deflects and gradually comes to the original position in a time which is large compared to the time constant.
3. Dimensions of $1/(\mu_0\varepsilon_0)$ is
   (a) L/T (b) T/L (c) L^2/T^2 (d) T^2/L^2.
4. Electromagnetic waves are produced by
   (a) a static charge (b) a moving charge
   (c) an accelerating charge (d) chargeless particles.
5. An electromagnetic wave going through vacuum is described by
   $$E = E_0 \sin(kx - \omega t);\ B = B_0 \sin(kx - \omega t).$$
   Then
   (a) $E_0\, k = B_0\, \omega$ (b) $E_0 B_0 = \omega k$
   (c) $E_0\, \omega = B_0\, k$ (d) none of these.

6. An electric field $\vec{E}$ and a magnetic field $\vec{B}$ exist in a region. The fields are not perpendicular to each other.
   (a) This is not possible.
   (b) No electromagnetic wave is passing through the region.
   (c) An electromagnetic wave may be passing through the region.
   (d) An electromagnetic wave is certainly passing through the region.
7. Consider the following two statements regarding a linearly polarized, plane electromagnetic wave:
   (A) The electric field and the magnetic field have equal average values.
   (B) The electric energy and the magnetic energy have equal average values.
   (a) Both A and B are true. (b) A is false but B is true.
   (c) B is false but A is true. (d) Both A and B are false.
8. A free electron is placed in the path of a plane electromagnetic wave. The electron will start moving
   (a) along the electric field
   (b) along the magnetic field
   (c) along the direction of propagation of the wave
   (d) in a plane containing the magnetic field and the direction of propagation.
9. A plane electromagnetic wave is incident on a material surface. The wave delivers momentum p and energy E.
   (a) $p = 0,\ E \neq 0$. (b) $p \neq 0,\ E = 0$.
   (c) $p \neq 0,\ E \neq 0$. (d) $p = 0,\ E = 0$.

OBJECTIVE II

1. An electromagnetic wave going through vacuum is described by
$$E = E_0 \sin(kx - \omega t).$$
   Which of the following is/are independent of the wavelength ?
   (a) k (b) ω (c) k/ω (d) $k\omega$.
2. Displacement current goes through the gap between the plates of a capacitor when the charge of the capacitor
   (a) increases (b) decreases
   (c) does not change (d) is zero.
3. Speed of electromagnetic waves is the same
   (a) for all wavelengths (b) in all media
   (c) for all intensities (d) for all frequencies.
4. Which of the following have zero average value in a plane electromagnetic wave ?
   (a) electric field (b) magnetic field
   (c) electric energy (d) magnetic energy.
5. The energy contained in a small volume through which an electromagnetic wave is passing oscillates with
   (a) zero frequency (b) the frequency of the wave
   (c) half the frequency of the wave
   (d) double the frequency of the wave.

EXERCISES

1. Show that the dimensions of the displacement current $\varepsilon_0 \frac{d\varphi_E}{dt}$ are that of an electric current.
2. A point charge is moving along a straight line with a constant velocity v. Consider a small area A perpendicular to the direction of motion of the charge (figure 40-E1). Calculate the displacement current through the area when its distance from the charge is x. The value of x is not large so that the electric field at any instant is essentially given by Coulomb's law.

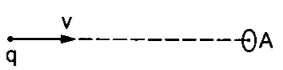

Figure 40-E1

3. A parallel-plate capacitor having plate-area A and plate separation d is joined to a battery of emf $\mathcal{E}$ and internal resistance R at $t = 0$. Consider a plane surface of area $A/2$, parallel to the plates and situated symmetrically between them. Find the displacement current through this surface as a function of time.
4. Consider the situation of the previous problem. Define displacement resistance $R_d = V/i_d$ of the space between the plates where V is the potential difference between the plates and i_d is the displacement current. Show that R_d varies with time as
$$R_d = R(e^{t/\tau} - 1).$$
5. Using $B = \mu_0 H$ find the ratio E_0/H_0 for a plane electromagnetic wave propagating through vacuum. Show that it has the dimensions of electric resistance. This ratio is a universal constant called the *impedance of free space*.
6. The sunlight reaching the earth has maximum electric field of 810 V m^{-1}. What is the maximum magnetic field in this light ?
7. The magnetic field in a plane electromagnetic wave is given by
$$B = (200\ \mu T) \sin[(4{\cdot}0 \times 10^{15}\ s^{-1})(t - x/c)].$$
   Find the maximum electric field and the average energy density corresponding to the electric field.
8. A laser beam has intensity $2{\cdot}5 \times 10^{14}$ W m^{-2}. Find the amplitudes of electric and magnetic fields in the beam.
9. The intensity of the sunlight reaching the earth is 1380 W m^{-2}. Assume this light to be a plane, monochromatic wave. Find the amplitudes of electric and magnetic fields in this wave.

□

ANSWERS

OBJECTIVE I

1. (d) 2. (d) 3. (c) 4. (c) 5. (a) 6. (c)
7. (a) 8. (a) 9. (c)

OBJECTIVE II

1. (c) 2. (a), (b) 3. (c), 4. (a), (b)
5. (d)

EXERCISES

2. $\frac{qAv}{2\pi x^3}$
3. $\frac{\mathcal{E}}{2R} e^{-\frac{td}{\varepsilon AR}}$
5. $377\ \Omega$
6. $2{\cdot}7\ \mu T$
7. $6 \times 10^4\ N\ C^{-1}$, $0{\cdot}016\ J\ m^{-3}$
8. $4{\cdot}3 \times 10^8\ N\ C^{-1}$, $1{\cdot}44\ T$
9. $1{\cdot}02 \times 10^3\ N\ C^{-1}$, $3{\cdot}40 \times 10^{-6}\ T$

□