Binomial Theorem Ex 18.2 Q23

We have,

$$(1+x)^n$$

Let the three consecutive terms are rth $(r+1)^{th}$ and $(r+2)^{th}$ i.e., T_r, T_{r+1} and T_{r+2}

$\therefore$ Coefficients of rth term $= {}^nC_{r-1} = 220$

Coefficients of $(r+1)^{th}$ term $= {}^nC_{r+1-1} = {}^nC_r = 495$

and, Coefficients of $(r+2)^{th}$ term $= {}^nC_{r+2-1} = {}^nC_{r+1} = 792$

Now,

$$\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{792}{495}$$

$$\Rightarrow \frac{n-(r+1)+1}{r+1} = \frac{792}{495} \qquad \left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}\right]$$

$$\Rightarrow \frac{n-r}{r+1} = \frac{792}{495} = \frac{72}{45} = \frac{8}{5}$$

$$\Rightarrow \frac{n-r}{r+1} = \frac{8}{5}$$

$$\Rightarrow 5n - 5r = 8r + 8$$

$$\Rightarrow 5n - 5r - 8r = 8$$

$$\Rightarrow 5n - 13r = 8 \qquad \text{---(i)}$$

and,

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{495}{220}$$

$$\Rightarrow \frac{n-r+1}{r} = \frac{495}{220} = \frac{45}{20} = \frac{9}{4}$$

$$\Rightarrow \frac{n-r+1}{r} = \frac{9}{4}$$

$$\Rightarrow 4n - 4r + 4 = 9r$$

$$\Rightarrow 4n - 4r - 9r = -4$$

$$\Rightarrow 4n - 13r = -4 \qquad \text{---(ii)}$$

Subtracting equation (ii) from equation (i),

$$n = 8 + 4$$

$$\Rightarrow n = 12$$

Binomial Theorem Ex 18.2 Q24

We have,

$$\left(1+x\right)^n$$

$\therefore$ Coefficients of 2nd term = ${}^nC_{2-1} = {}^nC_1$

Coefficients of 3rd term = ${}^nC_{3-1} = {}^nC_2$

and, Coefficients of 4th term = ${}^nC_{4-1} = {}^nC_3$

It is given that these coefficents are in A.P.

$\therefore \quad 2\,{}^nC_2 = {}^nC_1 + {}^nC_3$

$\Rightarrow \quad 2 = \dfrac{{}^nC_1}{{}^nC_2} + \dfrac{{}^nC_3}{{}^nC_2}$

$\Rightarrow \quad 2 = \dfrac{2}{n-2+1} + \dfrac{n-3+1}{3} \qquad \left[\because \dfrac{{}^nC_{r+1}}{{}^nC_r} = \dfrac{n-r}{r+1}\right]$

$\Rightarrow \quad 2 = \dfrac{2}{n-1} + \dfrac{n-2}{3}$

$\Rightarrow \quad 2 = \dfrac{6+\left(n-1\right)\left(n-2\right)}{3\left(n-1\right)}$

$\Rightarrow \quad 6\left(n-1\right) = 6+n^2-2n-n+2$

$\Rightarrow \quad 6n-6 = 8+n^2-3n$

$\Rightarrow \quad n^2-3n-6n+8+6 = 0$

$\Rightarrow \quad n^2-9n+14 = 0$

$\Rightarrow \quad n^2-7n-2n+14 = 0$

$\Rightarrow \quad n\left(n-7\right)-2\left(n-7\right) = 0$

$\Rightarrow \quad \left(n-2\right)\left(n-7\right) = 0 \qquad \left[\because n-2 \neq 0\right]$

$\Rightarrow \quad n = 7$

Binomial Theorem Ex 18.2 Q25

We have,

$$\left(1+x\right)^n$$

Coefficients of pth term = ${}^nC_{p-1}$

and, Coefficients of qth term = ${}^nC_{q-1}$

It is given that, these coefficients are equal.

$\therefore \quad {}^nC_{p-1} = {}^nC_{q-1}$

$\Rightarrow \quad p-1 = q-1$ or, $p-1+q-1 = n \qquad \left[\begin{array}{l}\because {}^nC_r = {}^nC_s \\ \Rightarrow r = s \text{ or, } r+s = n\end{array}\right]$

$\Rightarrow \quad p-q = 0$ or, $p+q = n+2$

$\therefore \quad p+q = n+2$ Hence proved.

Binomial Theorem Ex 18.2 Q26

We have,

$$\left(1+x\right)^n$$

Let the three consecutive terms are T_r, T_{r+1} and T_{r+2}

$\therefore$ Coefficients of T_r = ${}^nC_{r-1} = 56$

Coefficients of T_{r+1} = ${}^nC_{r+1-1}$ = ${}^nC_r = 70$

and, Coefficients of T_{r+2} = ${}^nC_{r+2-1}$ = ${}^nC_{r+1} = 56$

Now,

$$\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{56}{70}$$

$$\Rightarrow \quad \frac{n-\left(r+1\right)+1}{r+1} = \frac{4}{5} \qquad \left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}\right]$$

$$\Rightarrow \quad \frac{n-r}{r+1} = \frac{4}{5}$$

$$\Rightarrow \quad 5n - 5r = 4r + 4$$

$$\Rightarrow \quad 5n - 9r = 4 \qquad \text{---(i)}$$

and,

$$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{70}{56}$$

$$\Rightarrow \quad \frac{n-r+1}{r} = \frac{5}{4}$$

$$\Rightarrow \quad 4n - 4r + 4 = 5r$$

$$\Rightarrow \quad 4n - r = -4 \qquad \text{---(ii)}$$

Subtracting equation (ii) from (i), we get

$$n = 4 + 4 = 8$$

Put $n = 8$ in equation (i), we get

$$5 \times 8 - 9r = 4$$

$$\Rightarrow \quad -9r = 4 - 40$$

$$\Rightarrow \quad r = 4$$

$\therefore$ Three consecutive terms are 4th, 5th and 6th.

*********** END **********