EXERCISE 11.2

Question 1:

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $y^2 = 12x$

Ans:

The given equation is $y^2 = 12x$.

Here, the coefficient of x is positive. Hence, the parabola opens towards the right.

On comparing this equation with $y^2 = 4ax$, we obtain

$4a = 12$ □ $a = 3$

Coordinates of the focus = $(a, 0) = (3, 0)$

Since the given equation involves y^2, the axis of the parabola is the x-axis.

Equation of direcctrix, $x = -a$ i.e., $x = -3$ i.e., $x + 3 = 0$

Length of latus rectum = $4a = 4 \times 3 = 12$

Question 2:

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $x^2 = 6y$

Ans:

The given equation is $x^2 = 6y$.

Here, the coefficient of y is positive. Hence, the parabola opens upwards.

On comparing this equation with $x^2 = 4ay$, we obtain

$$4a = 6 \Rightarrow a = \frac{3}{2}$$

Coordinates of the focus = $(0, a)$ = $\left(0, \frac{3}{2}\right)$

Since the given equation involves x^2, the axis of the parabola is the y-axis.

Equation of directrix, $y = -a$ i.e., $y = -\frac{3}{2}$

Length of latus rectum = $4a = 6$

Question 3:

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $y^2 = -8x$

Ans:

The given equation is $y^2 = -8x$.

Here, the coefficient of x is negative. Hence, the parabola opens towards the left.

On comparing this equation with $y^2 = -4ax$, we obtain

$-4a = -8$ $a = 2$

Coordinates of the focus = $(-a, 0) = (-2, 0)$

Since the given equation involves y^2, the axis of the parabola is the x-axis.

Equation of directrix, $x = a$ i.e., $x = 2$

Length of latus rectum = $4a = 8$

Question 4:

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $x^2 = -16y$

Ans:

The given equation is $x^2 = -16y$.

Here, the coefficient of y is negative. Hence, the parabola opens downwards.

On comparing this equation with $x^2 = -4ay$, we obtain

$-4a = -16$ $a = 4$

Coordinates of the focus = $(0, -a) = (0, -4)$

Since the given equation involves x^2, the axis of the parabola is the y-axis.

Equation of directrix, $y = a$ i.e., $y = 4$

Length of latus rectum = $4a = 16$

Question 5:

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $y^2 = 10x$

Ans:

The given equation is $y^2 = 10x$.

Here, the coefficient of x is positive. Hence, the parabola opens towards the right.

On comparing this equation with $y^2 = 4ax$, we obtain

$$4a = 10 \Rightarrow a = \frac{5}{2}$$

Coordinates of the focus = $(a, 0) = \left(\frac{5}{2}, 0\right)$

Since the given equation involves y^2, the axis of the parabola is the x-axis.

Equation of directrix, $x = -a$, i.e., $x = -\frac{5}{2}$

Length of latus rectum = $4a = 10$

Question 6:

Find the coordinates of the focus, axis of the parabola, the equation of directrix and the length of the latus rectum for $x^2 = -9y$

Ans:

The given equation is $x^2 = -9y$.

Here, the coefficient of y is negative. Hence, the parabola opens downwards.

On comparing this equation with $x^2 = -4ay$, we obtain

$$-4a = -9 \Rightarrow b = \frac{9}{4}$$

Coordinates of the focus $= (0, -a) = \left(0, -\frac{9}{4}\right)$

Since the given equation involves x^2, the axis of the parabola is the y-axis.

Equation of directrix, $y = a$, i.e., $y = \frac{9}{4}$

Length of latus rectum = $4a$ = 9

Question 7:

Find the equation of the parabola that satisfies the following conditions: Focus (6, 0); directrix $x = -6$

Ans:

Focus (6, 0); directrix, $x = -6$

Since the focus lies on the x-axis, the x-axis is the axis of the parabola.

Therefore, the equation of the parabola is either of the form $y^2 = 4ax$ or

$y^2 = -4ax$.

It is also seen that the directrix, $x = -6$ is to the left of the y-axis, while the focus (6, 0) is to the right of the y-axis. Hence, the parabola is of the form $y^2 = 4ax$.

Here, $a = 6$

Thus, the equation of the parabola is $y^2 = 24x$.

Question 8:

Find the equation of the parabola that satisfies the following conditions: Focus (0, –3); directrix $y = 3$

Ans:

Focus = (0, –3); directrix $y = 3$

Since the focus lies on the y-axis, the y-axis is the axis of the parabola.

Therefore, the equation of the parabola is either of the form $x^2 = 4ay$ or

$x^2 = -4ay$.

It is also seen that the directrix, $y = 3$ is above the x-axis, while the focus

(0, –3) is below the x-axis. Hence, the parabola is of the form $x^2 = -4ay$.

Here, $a = 3$

Thus, the equation of the parabola is $x^2 = -12y$.

Question 9:

Find the equation of the parabola that satisfies the following conditions: Vertex (0, 0); focus (3, 0)

Ans:

Vertex (0, 0); focus (3, 0)

Since the vertex of the parabola is (0, 0) and the focus lies on the positive x-axis, x-axis is the axis of the parabola, while the equation of the parabola is of the form $y^2 = 4ax$.

Since the focus is (3, 0), $a = 3$.

Thus, the equation of the parabola is $y^2 = 4 \times 3 \times x$, i.e., $y^2 = 12x$

Question 10:

Find the equation of the parabola that satisfies the following conditions: Vertex (0, 0) focus (–2, 0)

Ans:

Vertex (0, 0) focus (–2, 0)

Since the vertex of the parabola is (0, 0) and the focus lies on the negative x-axis, x-axis is the axis of the parabola, while the equation of the parabola is of the form $y^2 = -4ax$.

Since the focus is (–2, 0), $a = 2$.

Thus, the equation of the parabola is $y^2 = -4(2)x$, i.e., $y^2 = -8x$

Question 11:

Find the equation of the parabola that satisfies the following conditions: Vertex (0, 0) passing through (2, 3) and axis is along x-axis

Ans:

Since the vertex is (0, 0) and the axis of the parabola is the x-axis, the equation of the parabola is either of the form $y^2 = 4ax$ or $y^2 = -4ax$.

The parabola passes through point (2, 3), which lies in the first quadrant.

Therefore, the equation of the parabola is of the form $y^2 = 4ax$, while point

(2, 3) must satisfy the equation $y^2 = 4ax$.

$$\therefore 3^2 = 4a(2) \Rightarrow a = \frac{9}{8}$$

Thus, the equation of the parabola is

$$y^2 = 4\left(\frac{9}{8}\right)x$$

$$y^2 = \frac{9}{2}x$$

$$2y^2 = 9x$$

Question 12:

Find the equation of the parabola that satisfies the following conditions: Vertex (0, 0), passing through (5, 2) and symmetric with respect to y-axis

Ans:

Since the vertex is (0, 0) and the parabola is symmetric about the y-axis, the equation of the parabola is either of the form $x^2 = 4ay$ or $x^2 = -4ay$.

The parabola passes through point (5, 2), which lies in the first quadrant.

Therefore, the equation of the parabola is of the form $x^2 = 4ay$, while point

(5, 2) must satisfy the equation $x^2 = 4ay$.

$$\therefore (5)^2 = 4 \times a \times 2 \Rightarrow 25 = 8a \Rightarrow a = \frac{25}{8}$$

Thus, the equation of the parabola is

$$x^2 = 4\left(\frac{25}{8}\right)y$$

$$2x^2 = 25y$$

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