Question 4. 11. A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cab man takes him along a circuitous path 23 km long and reaches the hotel in 28 min. What is (a) the average speed of the taxi, (b) the magnitude of average velocity? Are the two equal?

Answer:

Here, actual path length travelled, $s = 23$ km; Displacement = 10 km;

Time taken, $t = 28 \text{ min} = \frac{28}{60} \text{ h}$

(a) Average speed of taxi $= \frac{\text{actual path length}}{\text{time taken}} = \frac{23}{28/60} \text{ km/h} = 49.3 \text{ km/h}$

(b) Magnitude of average velocity $= \frac{\text{displacement}}{\text{time taken}} = \frac{10}{28/60} \text{ km/h} = 21.4 \text{ km/h}$

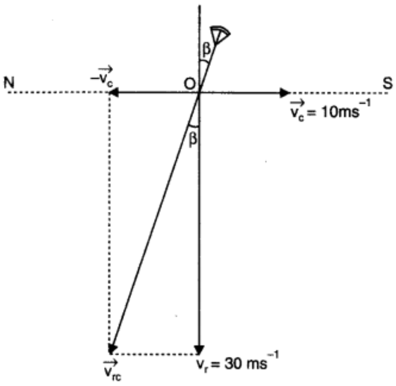
The average speed is not equal to the magnitude of average velocity. The two are equal for the motion of taxi along a straight path in one direction.

Question 4. 12. Rain is falling vertically with a speed of 30 ms^{1}. A woman rides a bicycle with a speed of 10 ms^{-1} in the north to south direction. What is the direction in which she should hold her umbrella ?

Answer:

The situation has been demonstrated in the figure below. Here $\vec{v}_r = 30 \text{ ms}^{-1}$ is the rain velocity in vertically downward direction and $\vec{v}_c = 10 \text{ ms}^{-1}$ is the velocity of cyclist woman in horizontal plane from north N to south S.

$\therefore$ Relative velocity of rain w.r.t. cyclist $\vec{v}_{rc}$ subtends an angle β with vertical such that



$$\tan \beta = \frac{|\vec{v}_c|}{|\vec{v}_r|} = \frac{10}{30} = \frac{1}{3}$$

$$\therefore \quad \beta = \tan^{-1}\left(\frac{1}{3}\right) = 18° 26'$$

Hence, the woman should hold her umbrella at 18° 26′ south of vertical.

Question 4. 13. A man can swim with a speed of 4.0 km h^{1} in still water. How long does he take to cross a river 1.0 km wide if the river flows steadily at 3.0 km h^{-1} and he makes his strokes normal to the river current? How far down the river does he go when he reaches the other bank?

Answer:

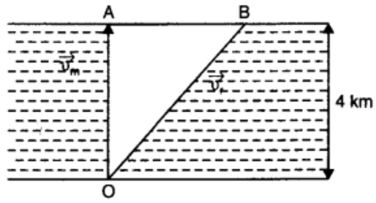
Here, $\overrightarrow{V_m} = 4 \text{ km h}^{-1}$; $\vec{v}_r = 3 \text{ km h}^{-1}$;

$OA = 1 \text{ km}$

Let t = time taken by man to reach the other bank.

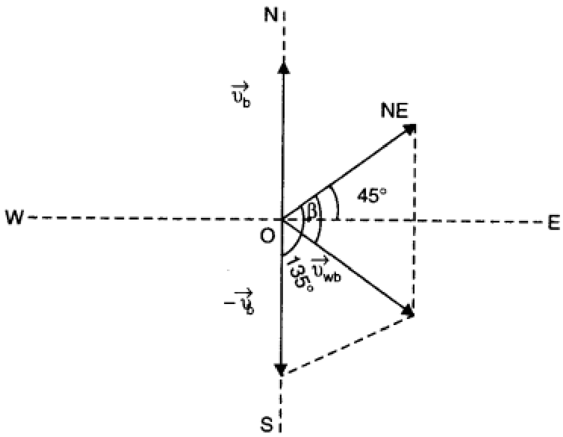
then $t = \frac{OA}{v_m} = \frac{1}{4} = 0.25 \text{ h}$

Distance, $AB = v_r \times t = 3 \times 0.25 = 0.75 \text{ km}$.



Question 4. 14. In a harbour, wind is blowing at the speed of 72 km/h and the flag on the mast of a boat anchored in the harbour flutters along the N-E direction. If the boat starts moving at a speed of 51 km/h to the north, what is the direction of the flag on the mast of the boat?

Answer: When the boat is anchored in the harbour, the flag flutters along the N-E direction. It shows that the velocity of wind is along the north-east direction. When the boat starts moving, the flag will flutter along the direction of relative velocity of wind w.r.t. boat. Let v_{wb} be the relative velocity of wind w.r.t. boat and P be the angle between v_{wb} and v_w (see fig. below).



Now, $\vec{v}_{wb} = \vec{v}_w + (-\vec{v}_b)$

Here, $|\vec{v}_w| = 72 \text{ km/h}$

$|-\vec{v}_b| = 51 \text{ km/h}$

Angle between $\vec{v}_w$ and $-\vec{v}_b$ is 135° *i.e.,* $\theta = 135°$. Then

$$\tan\beta = \frac{51 \sin 135°}{72 + 51 \cos 135°} = \frac{51 \sin 45°}{72 + 51(-\cos 45°)}$$

$$= \frac{51 \times (1/\sqrt{2})}{72 - 51(1/\sqrt{2})} = 1.0039$$

$\therefore$ $\beta = \tan^{-1}(1.0039) = 45.1°$

Angle w.r.t. east direction = 45.1° − 45° = 0.1°

It means the flag will flutter almost due east.

Question 4. 15. The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 ms^{-1} can go without hitting the ceiling of the hall?

Answer:

Maximum height h_{max} = 25 m; Horizontal range, R =?

Velocity of projection, v = 40 ms^{-1}

We know that $h_{max} = \frac{v^2 \sin^2\theta}{2g}$

or $\sin^2\theta = \frac{25\times 2\times 9.8}{40\times 40} = 0.30625$ or $\sin\theta = 0.5534$

$$\theta = \sin^{-1}(0.5534) = 33.6°$$

Again, $R = \frac{v^2 \sin 2\theta}{g} = \frac{40\times 40 \sin 67.2°}{9.8}$

or $R = \frac{1600}{9.8} \times 0.9219 \text{ m} = 150.5 \text{ m}.$

Question 4. 16. A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball?

Answer:

$$R_{max} = 100 \text{ m};$$

Since $R_{max} = \frac{v^2}{g} \Rightarrow 100 = \frac{v^2}{g}$

Using equation of motion

$$v^2 - u^2 = 2as$$

Here, $v = 0,\ a = -g,\ s = R_{max} = 100$ m

$\therefore$ $(0)^2 - u^2 = 2(-g) \times s$

$\Rightarrow$ $s = \frac{1}{2}\frac{u^2}{g}$

Since $u = v$

$\therefore$ $s = \frac{1}{2}\frac{v^2}{g} = \frac{1}{2}\times 100 = 50 \text{ m}.$

Question 4. 17. A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone?

Answer:

Here, $r = 80 \text{ cm} = 0.8 \text{ m};$

$$\nu = \frac{14}{25} \text{ rev/s}$$

$\therefore$ $\omega = 2\pi\nu = 2\times\frac{22}{7}\times\frac{14}{25} \text{ rad/s} = \frac{88}{25} \text{ rad}\cdot\text{s}^{-1}$

The centripetal acceleration,

$$a = \omega^2 r = \left(\frac{88}{25}\right)^2 \times 0.80 = 9.90 \text{ ms}^{-2}$$

The direction of centripetal acceleration is along the string directed towards the centre of circular path.

Question 4.18. An aircraft executes a horizontal loop of radius 1.00 km with a steady speed of 900 km/h. Compare its centripetal acceleration with the acceleration due to gravity.

Answer:

Here $r = 1 \text{ km} = 10^3 \text{ m},\ v = 900 \text{ km h}^{-1} = 900\times\frac{5}{18} = 250 \text{ ms}^{-1}$

Centripetal acceleration = $a_c = \frac{v^2}{r} = \frac{(250)^2}{10^3} = 62.5 \text{ ms}^{-2}$

Now, $\frac{a_c}{g} = \frac{62.5}{9.8} = 6.38.$

Question 4. 19. Read each statement below carefully and state, with reasons, if it is true or false:

(a) The net acceleration of a particle in circular motion is always along the radius of the circle towards the centre.

(b) The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point.
(c) The acceleration vector of a particle in uniform circular motion averaged over one cycle is a null vector.
Answer:
(a) False, the net acceleration of a particle in circular motion is along the radius of the circle towards the centre only in uniform circular motion.
(b) True, because while leaving the circular path, the particle moves tangentially to the circular path.
(c) True, the direction of acceleration vector in a uniform circular motion is directed towards the centre of circular path. It is constantly changing with time. The resultant of all these vectors will be a zero vector.

Question 4. 20. The position of a particle is given by

$$r = 3.0t\ \hat{i} - 2.0t^2\ \hat{j} + 4.0\ \hat{k}\ m$$

where t is in seconds and the coefficients have the proper units for **r** *to be in metres.*

(a) Find the $\vec{v}$ and $\vec{a}$ of the particle.
(b) What is the magnitude and direction of velocity of the particle at t = 2.0 s?

Answer:

Here $\vec{r}(t) = \left(3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\,\hat{k}\right)\text{m}$

(a) $\therefore$ $\vec{v}(t) = \dfrac{\vec{dr}}{dt} = \left(3.0\,\hat{i} - 4.0t\,\hat{j}\right)\text{m/s}$

and $\vec{a}(t) = \dfrac{\vec{dv}}{dt} = \left(-4.0\,\hat{j}\right)\text{m/s}^2$

(b) Magnitude of velocity at t = 2.0 s,

$$v_{(t=2s)} = \sqrt{(3.0)^2 + (-4.0\times 2)^2} = \sqrt{9+64} = \sqrt{73}$$
$$= 8.54\ \text{m s}^{-1}$$

This velocity will subtend an angle β from x-axis, where $\tan\beta = \dfrac{(-4.0\times 2)}{(3.0)} = -2.667.$

= -2.6667.

$\therefore$ $\beta = \tan^{-1}(-2.6667) = -69.44° = 69.44°$ from negative x-axis.

*********** END **********