



Differentiation Ex 11.5 Q34

Here,

$$x^{16}y^9 = (x^2 + y)^{17}$$

Taking \log on both the sides,

$$\begin{aligned} \log(x^{16}y^9) &= \log(x^2 + y)^{17} && [\text{Since, } \log(AB) = \log A + \log B, \log a^b = b \log a] \\ 16 \log x + 9 \log y &= 17 \log(x^2 + y) \end{aligned}$$

Differentiating it with respect to x using chain rule,

$$16 \frac{d}{dx}(\log x) + 9 \frac{d}{dx}(\log y) = 17 \frac{d}{dx} \log(x^2 + y)$$

$$\frac{16}{x} + \frac{9}{y} \frac{dy}{dx} = 17 \frac{1}{(x^2 + y)} \frac{d}{dx}(x^2 + y)$$

$$\frac{16}{x} + \frac{9}{y} \frac{dy}{dx} = \frac{17}{x^2 + y} \left[2x + \frac{dy}{dx} \right]$$

$$\frac{9}{y} \frac{dy}{dx} - \frac{17}{(x^2 + y)} \frac{dy}{dx} = \left(\frac{34x}{x^2 + y} \right) - \frac{16}{x}$$

$$\frac{dy}{dx} \left[\frac{9}{y} - \frac{17}{(x^2 + y)} \right] = \frac{34x^2 - 16x^2 - 16y}{x(x^2 + y)}$$

$$\frac{dy}{dx} \left[\frac{9x^2 + 9y - 17y}{y(x^2 + y)} \right] = \frac{18x^2 - 16y}{x(x^2 + y)}$$

$$\frac{dy}{dx} = \frac{y}{x} \left(\frac{2(9x^2 - 8y)}{9x^2 - 8y} \right)$$

$$\frac{dy}{dx} = \frac{2y}{x}$$

$$x \frac{dy}{dx} = 2y$$

Differentiation Ex 11.5 Q35

Here,

$$y = \sin(x^x) \quad \text{---(i)}$$

$$\text{Let } u = x^x \quad \text{---(ii)}$$

Taking log on both the sides,

$$\log u = \log x^x$$

$$\log u = x \log x$$

Differentiating it with respect to x ,

$$\frac{1}{u} \frac{du}{dx} = \frac{d}{dx} (x \log x)$$

$$= x \frac{d}{dx} (\log x) + \log x \frac{d}{dx} (x)$$

$$= x \left(\frac{1}{x} \right) + \log x (1)$$

$$\frac{1}{u} \frac{du}{dx} = 1 + \log x$$

$$\frac{du}{dx} = u (1 + \log x)$$

$$\frac{du}{dx} = x^x (1 + \log x) \quad \text{---(iii) [Using equation (ii)]}$$

Now, using equation (ii) in equation (i),

$$y = \sin u$$

Differentiating it with respect to x ,

$$\frac{dy}{dx} = \frac{d}{dx} (\sin u)$$

$$= \cos u \frac{du}{dx}$$

Using equation (ii) and (iii),

$$\frac{dy}{dx} = \cos(x^x) \times x^x (1 + \log x)$$

Differentiation Ex 11.5 Q36

Here,

$$x^x + y^y = 1$$

$$e^{\log x^x} + e^{\log y^y} = 1$$

$$e^{x \log x} + e^{y \log y} = 1 \quad \left[\text{Since, } e^{\log a^b} = a, \log a^b = b \log a \right]$$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{d}{dx} (e^{x \log x}) + \frac{d}{dx} (e^{y \log y}) = \frac{d}{dx} (1)$$

$$e^{x \log x} \frac{d}{dx} (x \log x) + e^{y \log y} \frac{d}{dx} (y \log y) = 0$$

$$e^{\log x^x} \left[x \frac{d}{dx} (\log x) + \log x \frac{d}{dx} (x) \right] + e^{\log y^y} \left[y \frac{d}{dx} (\log y) + \log y \frac{d}{dx} (y) \right] = 0$$

$$x^x \left[x \left(\frac{1}{x} \right) + \log x (1) \right] + y^y \left[x \left(\frac{1}{y} \right) \frac{dy}{dx} + \log y (1) \right] = 0$$

$$x^x [1 + \log x] + y^y \left(\frac{x}{y} \frac{dy}{dx} + \log y \right) = 0$$

$$y^y \times \frac{x}{y} \frac{dy}{dx} = -[x^x (1 + \log x) + y^y \log y]$$

$$(xy^{y-1}) \frac{dy}{dx} = -[x^x (1 + \log x) + y^y \log y]$$

$$\frac{dy}{dx} = - \left[\frac{x^x (1 + \log x) + y^y \log y}{xy^{y-1}} \right]$$

Differentiation Ex 11.5 Q37

Here,

$$x^y \times y^x = 1$$

Taking on both sides,

$$\log \{x^y \times y^x\} = \log(1)$$

$$y = \log x + x \log y = \log 1$$

$$[\text{Since, } \log(AB) = \log A + \log B, \log a^b = b \log a]$$

Differentiating it with respect to x using product rule,

$$\begin{aligned} \frac{d}{dx} (y \log x) + \frac{d}{dx} (x \log y) &= \frac{d}{dx} (\log 1) \\ \left[y \frac{d}{dx} (\log x) + \log x \frac{dy}{dx} \right] + \left[x \frac{d}{dx} (\log y) + \log y \frac{d}{dx} (x) \right] &= 0 \\ \left[y \left(\frac{1}{x} \right) + \log x \frac{dy}{dx} \right] + \left[x \left(\frac{1}{y} \frac{dy}{dx} \right) + \log y (1) \right] &= 0 \\ \frac{y}{x} + \log x \frac{dy}{dx} + \frac{x}{y} \frac{dy}{dx} + \log y &= 0 \\ \frac{dy}{dx} \left(\log x + \frac{x}{y} \right) &= - \left[\log y + \frac{y}{x} \right] \\ \frac{dy}{dx} \left[\frac{y \log x + x}{y} \right] &= - \left[\frac{x \log y + y}{x} \right] \\ \frac{dy}{dx} &= - \frac{y}{x} \left[\frac{x \log y + y}{y \log x + x} \right] \end{aligned}$$

Differentiation Ex 11.5 Q38

Here,

$$x^y + y^x = (x + y)^{x+y}$$

$$e^{\log x^y} + e^{\log y^x} = e^{\log(x+y)^{x+y}}$$

$$e^{y \log x} + e^{x \log y} = e^{(x+y) \log(x+y)}$$

$$[\text{Since, } e^{\log a} = a, \log a^b = b \log a]$$

Differentiating it with respect to x using chain rule, product rule,

$$\begin{aligned} \Rightarrow \frac{d}{dx} \{e^{y \log x}\} + \frac{d}{dx} \{e^{x \log y}\} &= \frac{d}{dx} e^{(x+y) \log(x+y)} \\ \Rightarrow e^{y \log x} \left[y \frac{d}{dx} (\log x) + \log x \frac{dy}{dx} \right] + e^{x \log y} \left[x \frac{d}{dx} \log y + \log y \frac{d}{dx} (x) \right] &= e^{(x+y) \log(x+y)} \frac{d}{dx} [(x+y) \log(x+y)] \\ \Rightarrow e^{\log x^y} \left[y \left(\frac{1}{x} \right) + \log x \frac{dy}{dx} \right] + e^{\log y^x} \left[x \frac{dy}{dx} + \log y (1) \right] &= e^{\log(x+y)^{x+y}} \left[(x+y) \frac{d}{dx} \log(x+y) + \log(x+y) \right] \\ \Rightarrow x^y \left[\frac{y}{x} + \log x \frac{dy}{dx} \right] + y^x \left[x \frac{dy}{dx} + \log y \right] &= (x+y)^{(x+y)} \left[(x+y) \frac{1}{(x+y)} \frac{d}{dx} (x+y) + \log(x+y) \left(1 + \frac{dy}{dx} \right) \right] \\ \Rightarrow x^y \times \frac{y}{x} + x^y \log x \frac{dy}{dx} + y^x \times x \frac{dy}{dx} + y^x \log y &= (x+y)^{(x+y)} \left[1 \times \left(1 + \frac{dy}{dx} \right) + \log(x+y) \left(1 + \frac{dy}{dx} \right) \right] \\ \Rightarrow x^{y-1} \times y + x^y \log x \frac{dy}{dx} + y^{x-1} \times x \frac{dy}{dx} + y^x \log y &= (x+y)^{(x+y)} + (x+y)^{(x+y)} \frac{dy}{dx} + (x+y)^{(x+y)} \log(x+y) \frac{dy}{dx} \\ \Rightarrow \frac{dy}{dx} \left[x^y \log x + x y^{x-1} - (x+y)^{(x+y)} \{1 + \log(x+y)\} \right] &= (x+y)^{(x+y)} \{1 + \log(x+y)\} - x^{y-1} \times y - y^x \log y \\ \Rightarrow \frac{dy}{dx} &= \left[\frac{(x+y)^{(x+y)} \{1 + \log(x+y)\} - x^{y-1} \times y - y^x \log y}{x^y \log x + x y^{x-1} - (x+y)^{(x+y)} \{1 + \log(x+y)\}} \right] \end{aligned}$$

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