Areas of Parallelograms and Triangles Ex 15.3 Q30

Answer :

Given:

(1) ABCD is a right angled triangle at A

(2) BCED, ACFG and ABMN are the squares on the sides of BC, CA and AB respectively.

(3) $AX \perp DE$, meets BC at Y.

To prove:

(i) $\Delta MBC \cong \Delta ABD$

(ii) $ar(BYXD) = 2ar(\Delta MBC)$

(iii) $ar(BYXD) = ar(ABMN)$

(iv) $\Delta FCB \cong \Delta ACE$

(v) $ar(CYXE) = 2ar(\Delta FCB)$

(vi) $ar(CYXE) = ar(ACFG)$

(vii) $ar(BCED) = ar(ABMN) + ar(ACFG)$

Proof:

(i) In ΔMBC and ΔABD

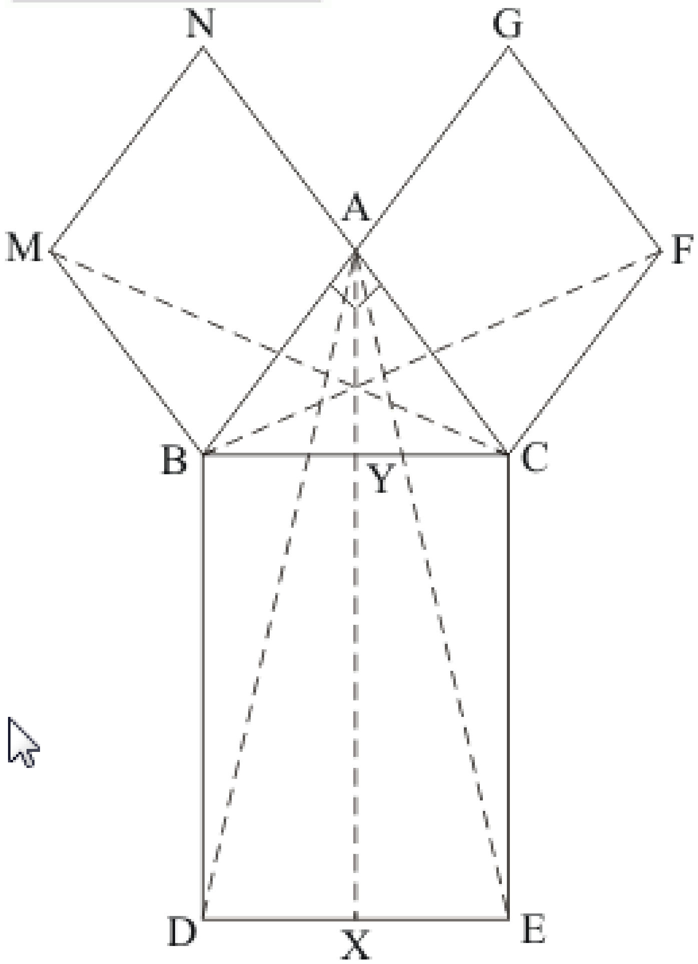
$MB = AB$

$BC = BD$

$\angle MBC = \angle ABD$ ($\angle MBC$ and $\angle ABD$are obtained by adding $\angle ABC$ to a right angle)

$$\Delta MBC \cong \Delta ABD \text{(SAS criteria)}$$

$$\boxed{\Delta MBC \cong \Delta ABD} \quad \ldots\ldots (1)$$



(ii) Triangle ABD and rectangle BYXD are on the same base BD and between the same parallels AX and BD.
Therefore

$$\mathrm{ar}(\Delta ABD) = \frac{1}{2}\mathrm{ar}(BYXD)$$
$$\Rightarrow 2\mathrm{ar}(\Delta ABD) = \mathrm{ar}(BYXD)$$
$$\Rightarrow \boxed{\mathrm{ar}(BYXD) = 2\mathrm{ar}(\Delta MBC)} \text{ (Using (1)) ...... (2)}$$

(iii) Since ΔMBC and square MBAN are on the same base MB and between the same parallels MB and NC.

$$\Rightarrow \mathrm{ar}(\Delta MBC) = \frac{1}{2}\mathrm{ar}(MBAN)$$
$$\Rightarrow 2\mathrm{ar}(\Delta MBC) = \mathrm{ar}(MBAN)$$
$$\Rightarrow \mathrm{ar}(MBAN) = 2\mathrm{ar}(\Delta MBC) \text{ ...... (3)}$$

From (2) and (3) we get

$$\boxed{\mathrm{ar}(MBAN) = \mathrm{ar}(BYXD)}$$

(iv) In triangle FCB and ACE

$BC = CE$

$AC = CF$

$\angle BCF = \angle ACE$ ($\angle BCF$ and $\angle ACE$ are obtained by adding $\angle ACB$ to a right angle)

$\Delta MBC \cong \Delta ABD$ (SAS criteria)

$\boxed{\Delta FCB = \Delta ACE}$ ...... (4)

(v) Since ΔACE and rectangle CYXE are on the same base CE and between the same parallels CE and AX.

$$\Rightarrow \mathrm{ar}(\Delta ACE) = \frac{1}{2}\mathrm{ar}(CYXE)$$
$$\Rightarrow 2\mathrm{ar}(\Delta ACE) = \mathrm{ar}(CYXE)$$
$$\Rightarrow \mathrm{ar}(CYXE) = 2\mathrm{ar}(\Delta ACE)$$
$$\Rightarrow \boxed{\mathrm{ar}(CYXE) = 2\mathrm{ar}(\Delta FCB)} \text{ ...... (5)}$$

(vi) Since ΔFCB and rectangle FCAG are on the same base FC and between the same parallels FC and BG

$$\Rightarrow \mathrm{ar}(\Delta FCB) = \frac{1}{2}\mathrm{ar}(FCAG)$$
$$\Rightarrow 2\mathrm{ar}(\Delta FCB) = \mathrm{ar}(FCAG)$$
$$\Rightarrow \mathrm{ar}(FCAG) = 2\mathrm{ar}(\Delta FCB) \text{ ...... (6)}$$

From (5) and (6) we get

$$\boxed{\mathrm{ar}(CYXE) = \mathrm{ar}(ACFG)}$$

(vii) Applying Pythagoras Theorem in ΔACB, WE get

$$BC^2 = AB^2 + AC^2$$
$$\Rightarrow BC \times BD = AB \times MB + AC \times FC$$
$$\boxed{\mathrm{ar}(BCED) = \mathrm{ar}(ABMN) + \mathrm{ar}(ACFG)}$$

*********** END **********