Functions Ex 2.3 Q7

We are given that f is a real function and g is a function given by $g(x) = 2x$

To prove; $g \circ f = f + f$.

L.H.S

$$g \circ f(x) = g(f(x)) = 2f(x)$$
$$= f(x) + f(x) = \text{R.H.S}$$
$$\Rightarrow \quad g \circ f = f + f$$

Functions Ex 2.3 Q8

$$f(x) = \sqrt{1 - x}\ , \qquad g(x) = \log_e^x$$

Domain of f and g are R.

$$\text{Range of } f = (-\infty, 1)$$
$$\text{Range of } g = (0, e)$$

Clearly Range $f \subset$ Domain $g \Rightarrow g \circ f$ exists

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$$\therefore\ g \circ f(x) = g(f(x)) = g\left(\sqrt{1 - x}\right)$$
$$g \circ f(x) = \log_e^{\sqrt{1-x}}$$

Again

$$f \circ g(x) = f(g(x)) = f\left(\log_e^x\right)$$
$$f \circ g(x) = \sqrt{1 - \log_e^x}$$

Functions Ex 2.3 Q9

$f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R$ and $g : [-1, 1] \to R$ defined as $f(x) = \tan x$ and $g(x) = \sqrt{1 - x^2}$

Range of f : let $y = f(x) \Rightarrow y = \tan x$

$\Rightarrow x = \tan^{-1} y$

Since $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), y \in (-\infty, \infty)$

$\therefore$ Range of f $\subset$ domain of $g = [-1, 1]$

$\therefore$ $g \circ f$ exists.

By similar argument $f \circ g$ exists.

Now,

$$f \circ g(x) = f(g(x)) = f\left(\sqrt{1 - x^2}\right)$$

$$f \circ g(x) = \tan \sqrt{1 - x^2}$$

Again

$$g \circ f(x) = g(f(x))$$

$$= g(\tan x)$$

$$g \circ f(x) = \sqrt{1 - \tan^2 x}$$

Functions Ex 2.3 Q10

$$f(x) = \sqrt{x + 3} \text{ and } g(x) = x^2 + 1$$

Now,

Range of $f = [-3, \infty]$ and

Range of $g = (1, \infty)$

Then, Range of $f \subset$ Domain g and

Range of $g \subset$ Domain f

$\therefore$ $f \circ g$ and $g \circ f$ exist.

Now,

$$f \circ g(x) = f(g(x)) = f(x^2 + 1)$$

$$f \circ g(x) = \sqrt{x^2 + 4}$$

Again,

$$g \circ f(x) = g(f(x)) = g\left(\sqrt{x + 3}\right)$$

$$= \left(\sqrt{x + 3}\right)^2 + 1$$

$$g \circ f(x) = x + 4$$

Functions Ex 2.3 Q11(i)

We have, $f(x) = \sqrt{x-2}$

Clearly, $\text{Domain}(f) = [2,\infty)$ and $\text{Range}(f) = [0,\infty)$.

We observe that range(f) is not a subset of domain of f.

$$\begin{aligned}\therefore \text{ Domain of } (fof) &= \{x : x \in \text{Domain } (f) \text{ and } f(x) \in \text{Domain } (f)\} \\ &= \{x : x \in [2,\infty) \text{ and } \sqrt{x-2} \in [2,\infty)\} \\ &= \{x : x \in [2,\infty) \text{ and } \sqrt{x-2} \geq 2\} \\ &= \{x : x \in [2,\infty) \text{ and } x-2 \geq 4\} \\ &= \{x : x \in [2,\infty) \text{ and } x \geq 6\} \\ &= [6,\infty)\end{aligned}$$

Now,

$$(fof)(x) = f(f(x)) = f\left(\sqrt{x-2}\right) = \sqrt{\sqrt{x-2}-2}$$

$\therefore$ $fof : [6,\infty) \rightarrow R$ defined as

$$(fof)(x) = \sqrt{\sqrt{x-2}-2}$$

*********** END **********