Trigonometric Identities Ex 6.1 Q33

Answer :

We need to prove $\frac{(1+\tan^2\theta)\cot\theta}{\operatorname{cosec}^2\theta} = \tan\theta$

Solving the L.H.S, we get

$$\frac{(1+\tan^2\theta)\cot\theta}{\operatorname{cosec}^2\theta} = \frac{\sec^2\theta(\cot\theta)}{\operatorname{cosec}^2\theta}$$

Using $\sec\theta = \frac{1}{\cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}$ and $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$, we get

$$\begin{aligned}
\frac{\sec^2\theta(\cot\theta)}{\operatorname{cosec}^2\theta} &= \frac{\frac{1}{\cos^2\theta}\left(\frac{\cos\theta}{\sin\theta}\right)}{\frac{1}{\sin^2\theta}} \\
&= \frac{\frac{1}{\cos\theta\sin\theta}}{\frac{1}{\sin^2\theta}} \\
&= \frac{\sin^2\theta}{\cos\theta\sin\theta} \\
&= \frac{\sin\theta}{\cos\theta} \\
&= \tan\theta
\end{aligned}$$

Hence proved.

Trigonometric Identities Ex 6.1 Q34

Answer :

We need to prove $\frac{1+\cos A}{\sin^2 A} = \frac{1}{1-\cos A}$

Using the property $\cos^2\theta + \sin^2\theta = 1$, we get

LHS = $\frac{1+\cos A}{\sin^2 A} = \frac{1+\cos A}{1-\cos^2 A}$

Further using the identity, $a^2 - b^2 = (a+b)(a-b)$, we get

$$\begin{aligned}
\frac{1+\cos A}{1-\cos^2 A} &= \frac{1+\cos A}{(1-\cos A)(1+\cos A)} \\
&= \frac{1}{1-\cos A}
\end{aligned}$$

= RHS

Hence proved.

Trigonometric Identities Ex 6.1 Q35

Answer :

We need to prove $\frac{\sec A - \tan A}{\sec A + \tan A} = \frac{\cos^2 A}{(1+\sin A)^2}$

Here, we will first solve the LHS.

Now, using $\sec\theta = \frac{1}{\cos\theta}$ and $\tan\theta = \frac{\sin\theta}{\cos\theta}$, we get

$$\frac{\sec A - \tan A}{\sec A + \tan A} = \frac{\frac{1}{\cos A} - \frac{\sin A}{\cos A}}{\frac{1}{\cos A} + \frac{\sin A}{\cos A}}$$
$$= \frac{\frac{1 - \sin A}{\cos A}}{\frac{1+\sin A}{\cos A}}$$
$$= \frac{1-\sin A}{1+\sin A}$$

Further, multiplying both numerator and denominator by $1+\sin A$, we get

$$\frac{1-\sin A}{1+\sin A} = \left(\frac{1-\sin A}{1+\sin A}\right)\left(\frac{1+\sin A}{1+\sin A}\right)$$
$$= \frac{(1-\sin A)(1+\sin A)}{(1+\sin A)^2}$$
$$= \frac{1-\sin^2 A}{(1+\sin A)^2}$$

Now, using the property $\cos^2\theta + \sin^2\theta = 1$, we get

So,

$\frac{1-\sin^2 A}{(1+\sin A)^2} = \frac{\cos^2 A}{(1+\sin A)^2}$ = RHS

Hence proved.

*********** END **********