Indefinite Integrals Ex 19.29 Q8

Let $I = \int (2x+3)\sqrt{x^2+4x+3}\,dx$

Let $(2x+3) = \lambda \frac{d}{dx}(x^2+4x+3) + \mu$

$= \lambda(2x+4) + \mu$

Equating similar terms, we get,

$\lambda = 1$ and $4\lambda + \mu = 3$

$\Rightarrow \quad \mu = -1$

So,

$I = \int ((2x+4)+(-1))\sqrt{x^2+4x+3}\,dx$

$= \int (2x+4)\sqrt{x^2+4x+3}\,dx - \int \sqrt{x^2+4x+3}\,dx$

Let $x^2+4x+3 = t$

$\Rightarrow \quad (2x+4)\,dx = dt$

$\therefore \quad I = \int \sqrt{t}\,dt - \int \sqrt{(x+2)^2-1}\,dx$

$= \frac{3}{2} t^{\frac{3}{2}} - \frac{(x+2)}{2}\sqrt{x^2+4x+3} + \frac{1}{2}\log\left|(x+2)+\sqrt{x^2+4x+3}\right| + c$

Hence,

$$I = \frac{2}{3}(x^2+4x+3)^{\frac{3}{2}} - \left(\frac{x+2}{2}\right)\sqrt{x^2+4x+3} + \frac{1}{2}\log\left|(x+2)+\sqrt{x^2+4x+3}\right| + c$$

Indefinite Integrals Ex 19.29 Q9

Let $I = \int (2x-5)\sqrt{x^2-4x+3}dx$

Let $(2x-5) = \lambda \frac{d}{dx}(x^2-4x+3) + \mu$

$= \lambda(2x-4) + \mu$

Equating similar terms, we get,

$\lambda = 1$ and $-4\lambda + \mu = -5$

$\Rightarrow \quad \mu = -1$

So,

$I = \int ((2x-4)-1)\sqrt{x^2-4x+3}dx$

$= \int (2x-4)\sqrt{x^2-4x+3}dx - \int \sqrt{x^2-4x+3}dx$

Let $x^2-4x+3 = t$

$\Rightarrow \quad 2x-4dx = dt$

$I = \int \sqrt{t}dt - \int \sqrt{(x-2)^2-1}dx$

$$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x-2)}{2}\sqrt{x^2-4x+3} + \frac{1}{2}\log\left|(x-2)+\sqrt{x^2-4x+3}\right| + c$$

Thus,

$$I = \frac{2}{3}(x^2-4x+3)^{\frac{3}{2}} - \frac{1}{2}(x-2)\sqrt{x^2-4x+3} + \frac{1}{2}\log\left|(x-2)+\sqrt{x^2-4x+3}\right| + c$$

Indefinite Integrals Ex 19.29 Q10

Let $I = \int x\sqrt{x^2+x}\,dx$

Let $x = \lambda \frac{d}{dx}\left(x^2+x\right)+\mu$
$= \lambda\left(2x+1\right)+\mu$

Equating similar terms, we get,

$2\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$\lambda+\mu = 0 \quad \Rightarrow \quad \mu = -\frac{1}{2}$

So,

$$I = \int\left(\frac{1}{2}\left(2x+1\right)-\frac{1}{2}\right)\sqrt{x^2+x}\,dx$$
$$= \frac{1}{2}\int\left(2x+1\right)\sqrt{x^2+x} - \frac{1}{2}\int\sqrt{x^2+x}\,dx$$

Let $x^2+x = t$
$\Rightarrow \quad \left(2x+1\right)dx = dt$

So,

$$I = \frac{1}{2}\int\sqrt{t}\,dt - \frac{1}{2}\int\sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}$$
$$I = \frac{1}{2}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{2}\left\{\frac{\left(x+\frac{1}{2}\right)}{2}\sqrt{x^2+x} - \frac{1}{8}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x}\right|+c\right.$$

Hence,

$$I = \frac{1}{3}\left(x^2+x\right)^{\frac{3}{2}} - \frac{1}{8}\left(x+\frac{1}{2}\right)\sqrt{x^2+x} + \frac{1}{16}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x}\right|+c$$

*********** END **********