Q17 : Find the relationship between *a* and *b* so that the function *f* defined by

$$f(x)=\begin{cases}ax+1, & \text{if } x\le 3\\ bx+3, & \text{if } x>3\end{cases}$$

is continuous at $x = 3$.

Answer :

The given function *f* is $f(x)=\begin{cases}ax+1, & \text{if } x\le 3\\ bx+3, & \text{if } x>3\end{cases}$

If *f* is continuous at *x*= 3, then

$$\lim_{x\to 3^-} f(x)=\lim_{x\to 3^+} f(x)=f(3) \qquad ...(1)$$

Also,

$$\lim_{x\to 3^-} f(x)=\lim_{x\to 3^-}(ax+1)=3a+1$$

$$\lim_{x\to 3^+} f(x)=\lim_{x\to 3^+}(bx+3)=3b+3$$

$$f(3)=3a+1$$

Therefore, from (1), we obtain

$$3a+1=3b+3=3a+1$$
$$\Rightarrow 3a+1=3b+3$$
$$\Rightarrow 3a=3b+2$$
$$\Rightarrow a=b+\frac{2}{3}$$

Therefore, the required relationship is given by, $a=b+\frac{2}{3}$

Answer needs Correction? **Click Here**

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Q18 : For what value of $\lambda$ is the function defined by

$$f(x)=\begin{cases}\lambda(x^2-2x), & \text{if } x\le 0\\ 4x+1, & \text{if } x>0\end{cases}$$

continuous at $x = 0$? What about continuity at *x*= 1?

Answer :

The given function *f* is $f(x)=\begin{cases}\lambda(x^2-2x), & \text{if } x\le 0\\ 4x+1, & \text{if } x>0\end{cases}$

If *f* is continuous at *x*= 0, then

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=f(0)$$

$$\Rightarrow \lim_{x\to 0^-}\lambda(x^2-2x)=\lim_{x\to 0^+}(4x+1)=\lambda(0^2-2\times 0)$$

$$\Rightarrow \lambda(0^2-2\times 0)=4\times 0+1=0$$

$$\Rightarrow 0=1=0, \text{ which is not possible}$$

Therefore, there is no value of ÃŽÂ»for which *f* is continuous at *x*= 0

At $x = 1$,

$f(1) = 4x + 1 = 4 \times 1 + 1 = 5$

$$\lim_{x\to 1}(4x+1)=4\times 1+1=5$$

$$\therefore \lim_{x\to 1} f(x)=f(1)$$

Therefore, for any values of ÃŽÂ», *f* is continuous at *x*= 1

Answer needs Correction? **Click Here**

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Q19 : Show that the function defined by $g(x)=x-[x]$ is discontinuous at all integral point. Here $[x]$ denotes the greatest integer less than or equal to *x*.

Answer :

The given function is $g(x)=x-[x]$

It is evident that *g* is defined at all integral points.

Let *n* be an integer.

Then,

$$g(n)=n-[n]=n-n=0$$

The left hand limit of *f* at $x = n$ is,

$\lim_{x \to n^-} g(x) = \lim_{x \to n^-} (x - [x]) = \lim_{x \to n^-} (x) - \lim_{x \to n^-} [x] = n - (n-1) = 1$

The right hand limit of *f* at *x* = *n* is,

$\lim_{x \to n^+} g(x) = \lim_{x \to n^+} (x - [x]) = \lim_{x \to n^+} (x) - \lim_{x \to n^+} [x] = n - n = 0$

It is observed that the left and right hand limits of *f* at *x* = *n* do not coincide.

Therefore, *f* is not continuous at *x* = *n*

Hence, *g* is discontinuous at all integral points.

Answer needs Correction? **Click Here**

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**Q20 : Is the function defined by $f(x) = x^2 - \sin x + 5$ continuous at *x* = ?**

**Answer :**

The given function is $f(x) = x^2 - \sin x + 5$

It is evident that *f* is defined at *x* = .

At $x = \pi$, $f(x) = f(\pi) = \pi^2 - \sin \pi + 5 = \pi^2 - 0 + 5 = \pi^2 + 5$

Consider $\lim_{x \to \pi} f(x) = \lim_{x \to \pi} (x^2 - \sin x + 5)$

Put $x = \pi + h$

If $x \to \pi$, *then it is evident that* $h \to 0$

$$\begin{aligned}
\therefore \lim_{x \to \pi} f(x) &= \lim_{x \to \pi} (x^2 - \sin x + 5) \\
&= \lim_{h \to 0} \left[ (\pi + h)^2 - \sin(\pi + h) + 5 \right] \\
&= \lim_{h \to 0} (\pi + h)^2 - \lim_{h \to 0} \sin(\pi + h) + \lim_{h \to 0} 5 \\
&= (\pi + 0)^2 - \lim_{h \to 0} [\sin \pi \cosh + \cos \pi \sinh] + 5 \\
&= \pi^2 - \lim_{h \to 0} \sin \pi \cosh - \lim_{h \to 0} \cos \pi \sinh + 5 \\
&= \pi^2 - \sin \pi \cos 0 - \cos \pi \sin 0 + 5 \\
&= \pi^2 - 0 \times 1 - (-1) \times 0 + 5 \\
&= \pi^2 + 5
\end{aligned}$$

$\therefore \lim_{x \to \pi} f(x) = f(\pi)$

Therefore, the given function *f* is continuous at $x = \pi$

Answer needs Correction? **Click Here**

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**Q21 : Discuss the continuity of the following functions.**

**(a) *f*(*x*) = sin *x* + cos *x***

**(b) *f*(*x*) = sin *x* - cos *x***

**(c) *f*(*x*) = sin *x* x cos x**

**Answer :**

It is known that if *g* and *h* are two continuous functions, then

$g + h$, $g - h$, and $g.h$ are also continuous.

It has to proved first that *g*(*x*) = sin *x* and *h*(*x*) = cos *x* are continuous functions.

Let *g* (*x*) = sin *x*

It is evident that *g*(*x*) = sin *x* is defined for every real number.

Let *c* be a real number. Put *x* = *c* + *h*

If *x* Ã¢â€ ' *c*, then *h* Ã¢â€ '0

$g(c) = \sin c$

$$\begin{aligned}
\lim_{x \to c} g(x) &= \lim_{x \to c} \sin x \\
&= \lim_{h \to 0} \sin(c + h) \\
&= \lim_{h \to 0} [\sin c \cos h + \cos c \sin h] \\
&= \lim_{h \to 0} (\sin c \cos h) + \lim_{h \to 0} (\cos c \sin h) \\
&= \sin c \cos 0 + \cos c \sin 0 \\
&= \sin c + 0 \\
&= \sin c
\end{aligned}$$

$\therefore \lim_{x \to c} g(x) = g(c)$

Therefore, *g* is a continuous function.

Let *h* (*x*) = cos *x*

It is evident that *h*(*x*) = cos *x* is defined for every real number.

Let *c* be a real number. Put *x* = *c* + *h*

If *x* Ã¢â€ ' *c*, then *h* Ã¢â€ '0

*h* (*c*) = cos *c*

$$\begin{aligned}
\lim_{x \to c} h(x) &= \lim_{x \to c} \cos x \\
&= \lim_{h \to 0} \cos(c + h) \\
&= \lim_{h \to 0} [\cos c \cos h - \sin c \sin h] \\
&= \lim_{h \to 0} \cos c \cos h - \lim_{h \to 0} \sin c \sin h \\
&= \cos c \cos 0 - \sin c \sin 0 \\
&= \cos c \times 1 - \sin c \times 0 \\
&= \cos c
\end{aligned}$$

$\therefore \lim_{x \to c} h(x) = h(c)$

Therefore, *h* is a continuous function.

Therefore, it can be concluded that

(a) $f(x) = g(x) + h(x) = \sin x + \cos x$ is a continuous function

(b) $f(x) = g(x) - h(x) = \sin x - \cos x$ is a continuous function

(c) $f(x) = g(x) \times h(x) = \sin x \times \cos x$ is a continuous function

Answer needs Correction? **Click Here**

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Q22 : Discuss the continuity of the cosine, cosecant, secant and cotangent functions,

**Answer :**

It is known that if *g* and *h* are two continuous functions, then

$$(i)\ \frac{h(x)}{g(x)},\ g(x) \neq 0 \text{ is continuous}$$

$$(ii)\ \frac{1}{g(x)},\ g(x) \neq 0 \text{ is continuous}$$

$$(iii)\ \frac{1}{h(x)},\ h(x) \neq 0 \text{ is continuous}$$

It has to be proved first that $g(x) = \sin x$ and $h(x) = \cos x$ are continuous functions.

Let $g(x) = \sin x$

It is evident that $g(x) = \sin x$ is defined for every real number.

Let *c* be a real number. Put $x = c + h$

If $x \to c$, then $h \to 0$

$$\begin{aligned}
g(c) &= \sin c \\
\lim_{x \to c} g(x) &= \lim_{x \to c} \sin x \\
&= \lim_{h \to 0} \sin(c+h) \\
&= \lim_{h \to 0} [\sin c \cos h + \cos c \sin h] \\
&= \lim_{h \to 0} (\sin c \cos h) + \lim_{h \to 0} (\cos c \sin h) \\
&= \sin c \cos 0 + \cos c \sin 0 \\
&= \sin c + 0 \\
&= \sin c \\
\therefore\ \lim_{x \to c} g(x) &= g(c)
\end{aligned}$$

Therefore, *g* is a continuous function.

Let $h(x) = \cos x$

It is evident that $h(x) = \cos x$ is defined for every real number.

Let *c* be a real number. Put $x = c + h$

If *x* Â® *c*, then *h* Â®0

$h(c) = \cos c$

$$\begin{aligned}
\lim_{x \to c} h(x) &= \lim_{x \to c} \cos x \\
&= \lim_{h \to 0} \cos(c+h) \\
&= \lim_{h \to 0} [\cos c \cos h - \sin c \sin h] \\
&= \lim_{h \to 0} \cos c \cos h - \lim_{h \to 0} \sin c \sin h \\
&= \cos c \cos 0 - \sin c \sin 0 \\
&= \cos c \times 1 - \sin c \times 0 \\
&= \cos c \\
\therefore \lim_{x \to c} h(x) &= h(c)
\end{aligned}$$

Therefore, $h(x) = \cos x$ is continuous function.

It can be concluded that,

$$\operatorname{cosec} x = \frac{1}{\sin x},\ \sin x \neq 0 \text{ is continuous}$$

$$\Rightarrow \operatorname{cosec} x,\ x \neq n\pi\ (n \in Z) \text{ is continuous}$$

Therefore, cosecant is continuous except at *x* = *n*p, *n* ÃƒÅ½ **Z**

$$\sec x = \frac{1}{\cos x},\ \cos x \neq 0 \text{ is continuous}$$

$$\Rightarrow \sec x,\ x \neq (2n+1)\frac{\pi}{2}\ (n \in \mathbf{Z}) \text{ is continuous}$$

Therefore, secant is continuous except at $x = (2n+1)\frac{\pi}{2}\ (n \in \mathbf{Z})$

$$\cot x = \frac{\cos x}{\sin x},\ \sin x \neq 0 \text{ is continuous}$$

$$\Rightarrow \cot x,\ x \neq n\pi\ (n \in Z) \text{ is continuous}$$

Therefore, cotangent is continuous except at *x* = *n*p, *n* ÃƒÅ½ **Z**

Answer needs Correction? **Click Here**

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Q23 : Find the points of discontinuity of *f*, where

$$f(x) = \begin{cases} \frac{\sin x}{x}, \text{ if } x < 0 \\ x+1,\ \text{if } x \geq 0 \end{cases}$$

**Answer :**

The given function $f$ is $f(x)=\begin{cases}\dfrac{\sin x}{x}, & \text{if } x<0\\ x+1, & \text{if } x\geq 0\end{cases}$

It is evident that $f$ is defined at all points of the real line.

Let $c$ be a real number.

Case I:

If $c<0$, then $f(c)=\dfrac{\sin c}{c}$ and $\lim\limits_{x\to c} f(x)=\lim\limits_{x\to c}\left(\dfrac{\sin x}{x}\right)=\dfrac{\sin c}{c}$

$\therefore \lim\limits_{x\to c} f(x)=f(c)$

Therefore, $f$ is continuous at all points $x$, such that $x<0$

Case II:

If $c>0$, then $f(c)=c+1$ and $\lim\limits_{x\to c} f(x)=\lim\limits_{x\to c}(x+1)=c+1$

$\therefore \lim\limits_{x\to c} f(x)=f(c)$

Therefore, $f$ is continuous at all points $x$, such that $x>0$

Case III:

If $c=0$, then $f(c)=f(0)=0+1=1$

The left hand limit of $f$ at $x=0$ is,

$\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1$

The right hand limit of $f$ at $x=0$ is,

$\lim\limits_{x\to 0^+} f(x)=\lim\limits_{x\to 0^+}(x+1)=1$

$\therefore \lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0^+} f(x)=f(0)$

Therefore, $f$ is continuous at $x=0$

From the above observations, it can be concluded that $f$ is continuous at all points of the real line.

Thus, $f$ has no point of discontinuity.

Answer needs Correction? **Click Here**

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**Q24 : Determine if $f$ defined by**

$f(x)=\begin{cases}x^2\sin\dfrac{1}{x}, & \text{if } x\neq 0\\ 0, & \text{if } x=0\end{cases}$

**is a continuous function?**

**Answer :**

The given function $f$ is $f(x)=\begin{cases}x^2\sin\dfrac{1}{x}, & \text{if } x\neq 0\\ 0, & \text{if } x=0\end{cases}$

It is evident that $f$ is defined at all points of the real line.

Let $c$ be a real number.

Case I:

If $c\neq 0$, then $f(c)=c^2\sin\dfrac{1}{c}$

$\lim\limits_{x\to c} f(x)=\lim\limits_{x\to c}\left(x^2\sin\dfrac{1}{x}\right)=\left(\lim\limits_{x\to c}x^2\right)\left(\lim\limits_{x\to c}\sin\dfrac{1}{x}\right)=c^2\sin\dfrac{1}{c}$

$\therefore \lim\limits_{x\to c} f(x)=f(c)$

Therefore, $f$ is continuous at all points $x\neq 0$

Case II:

If $c=0$, then $f(0)=0$

$\lim\limits_{x\to 0^-} f(x)=\lim\limits_{x\to 0^-}\left(x^2\sin\dfrac{1}{x}\right)=\lim\limits_{x\to 0}\left(x^2\sin\dfrac{1}{x}\right)$

It is known that, $-1\leq\sin\dfrac{1}{x}\leq 1,\ x\neq 0$

$\Rightarrow -x^2\leq\sin\dfrac{1}{x}\leq x^2$

$\Rightarrow \lim\limits_{x\to 0}\left(-x^2\right)\leq\lim\limits_{x\to 0}\left(x^2\sin\dfrac{1}{x}\right)\leq\lim\limits_{x\to 0}x^2$

$\Rightarrow 0\leq\lim\limits_{x\to 0}\left(x^2\sin\dfrac{1}{x}\right)\leq 0$

$\Rightarrow \lim\limits_{x\to 0}\left(x^2\sin\dfrac{1}{x}\right)=0$

$\therefore \lim\limits_{x\to 0^-} f(x)=0$

Similarly, $\lim\limits_{x\to 0^+} f(x)=\lim\limits_{x\to 0^+}\left(x^2\sin\dfrac{1}{x}\right)=\lim\limits_{x\to 0}\left(x^2\sin\dfrac{1}{x}\right)=0$

$\therefore \lim\limits_{x\to 0^-} f(x)=f(0)=\lim\limits_{x\to 0^+} f(x)$

Therefore, $f$ is continuous at $x=0$

From the above observations, it can be concluded that $f$ is continuous at every point of the real line.

Thus, $f$ is a continuous function.

Answer needs Correction? Click Here

Q25 : Examine the continuity of $f$, where $f$ is defined by

$$f(x)=\begin{cases}\sin x-\cos x, & \text{if } x\neq 0\\ -1 & \text{if } x=0\end{cases}$$

Answer :

The given function $f$ is $f(x)=\begin{cases}\sin x-\cos x, & \text{if } x\neq 0\\ -1 & \text{if } x=0\end{cases}$

It is evident that $f$ is defined at all points of the real line.

Let $c$ be a real number.

Case I:

$$\text{If } c\neq 0, \text{ then } f(c)=\sin c-\cos c$$
$$\lim_{x\to c} f(x)=\lim_{x\to c}(\sin x-\cos x)=\sin c-\cos c$$
$$\therefore \lim_{x\to c} f(x)=f(c)$$

Therefore, $f$ is continuous at all points $x$, such that $x\neq 0$

Case II:

$$\text{If } c=0, \text{ then } f(0)=-1$$

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0}(\sin x-\cos x)=\sin 0-\cos 0=0-1=-1$$
$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0}(\sin x-\cos x)=\sin 0-\cos 0=0-1=-1$$
$$\therefore \lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=f(0)$$

Therefore, $f$ is continuous at $x=0$

From the above observations, it can be concluded that $f$ is continuous at every point of the real line.

Thus, $f$ is a continuous function.

Answer needs Correction? Click Here

Q26 : Find the values of $k$ so that the function $f$ is continuous at the indicated point.

$$f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x}, & \text{if } x\neq\dfrac{\pi}{2}\\ 3, & \text{if } x=\dfrac{\pi}{2}\end{cases} \quad \text{at } x=\frac{\pi}{2}$$

Answer :

The given function $f$ is $f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x}, & \text{if } x\neq\dfrac{\pi}{2}\\ 3, & \text{if } x=\dfrac{\pi}{2}\end{cases}$

The given function $f$ is continuous at $x=\frac{\pi}{2}$, if $f$ is defined at $x=\frac{\pi}{2}$ and if the value of the $f$ at $x=\frac{\pi}{2}$ equals the limit of $f$ at $x=\frac{\pi}{2}$.

It is evident that $f$ is defined at $x=\frac{\pi}{2}$ and $f\left(\frac{\pi}{2}\right)=3$

$$\lim_{x\to\frac{\pi}{2}} f(x)=\lim_{x\to\frac{\pi}{2}}\frac{k\cos x}{\pi-2x}$$

$$\text{Put } x=\frac{\pi}{2}+h$$

$$\text{Then, } x\to\frac{\pi}{2}\Rightarrow h\to 0$$

$$\therefore \lim_{x\to\frac{\pi}{2}} f(x)=\lim_{x\to\frac{\pi}{2}}\frac{k\cos x}{\pi-2x}=\lim_{h\to 0}\frac{k\cos\left(\frac{\pi}{2}+h\right)}{\pi-2\left(\frac{\pi}{2}+h\right)}$$
$$=k\lim_{h\to 0}\frac{-\sin h}{-2h}=\frac{k}{2}\lim_{h\to 0}\frac{\sin h}{h}=\frac{k}{2}.1=\frac{k}{2}$$
$$\therefore \lim_{x\to\frac{\pi}{2}} f(x)=f\left(\frac{\pi}{2}\right)$$
$$\Rightarrow \frac{k}{2}=3$$
$$\Rightarrow k=6$$

Therefore, the required value of $k$ is 6.

Answer needs Correction? Click Here

Q27 : Find the values of $k$ so that the function $f$ is continuous at the indicated point.

$$f(x)=\begin{cases}kx^2, & \text{if } x\leq 2\\ 3, & \text{if } x>2\end{cases} \quad \text{at } x=2$$

Answer :

The given function is $f(x)=\begin{cases}kx^2, & \text{if } x\leq 2\\ 3, & \text{if } x>2\end{cases}$

The given function $f$ is continuous at $x=2$, if $f$ is defined at $x=2$ and if the value of $f$ at $x=2$ equals the limit of $f$ at $x=2$

limit of $f$ at $x = 2$

It is evident that $f$ is defined at $x$= 2 and $f(2)=k(2)^2=4k$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$$
$$\Rightarrow \lim_{x\to 2^-}(kx^2) = \lim_{x\to 2^+}(3) = 4k$$
$$\Rightarrow k\times 2^2 = 3 = 4k$$
$$\Rightarrow 4k = 3 = 4k$$
$$\Rightarrow 4k = 3$$
$$\Rightarrow k = \frac{3}{4}$$

Therefore, the required value of $k$ is $\frac{3}{4}$.

Answer needs Correction? Click Here

Q28 : Find the values of $k$ so that the function $f$ is continuous at the indicated point.

$$f(x)=\begin{cases} kx+1, \text{ if } x\le\pi \\ \cos x, \text{ if } x>\pi \end{cases} \quad \text{at } x=\pi$$

Answer :

The given function is $f(x)=\begin{cases} kx+1, \text{ if } x\le\pi \\ \cos x, \text{ if } x>\pi \end{cases}$

The given function $f$ is continuous at $x$= p, if $f$ is defined at $x$= pand if the value of $f$ at $x$= pequals the limit of $f$at$x$= p

It is evident that $f$ is defined at$x$= pand $f(\pi)=k\pi+1$

$$\lim_{x\to \pi^-} f(x) = \lim_{x\to \pi^+} f(x) = f(\pi)$$
$$\Rightarrow \lim_{x\to \pi^-}(kx+1) = \lim_{x\to \pi^+}\cos x = k\pi+1$$
$$\Rightarrow k\pi+1 = \cos\pi = k\pi+1$$
$$\Rightarrow k\pi+1 = -1 = k\pi+1$$
$$\Rightarrow k = -\frac{2}{\pi}$$

Therefore, the required value of $k$ is $-\frac{2}{\pi}$.

Answer needs Correction? Click Here

Q29 : Find the values of $k$ so that the function $f$ is continuous at the indicated point.

$$f(x)=\begin{cases} kx+1, \text{ if } x\le 5 \\ 3x-5, \text{ if } x>5 \end{cases} \quad \text{at } x=5$$

Answer :

The given function $f$ is $f(x)=\begin{cases} kx+1, \text{ if } x\le 5 \\ 3x-5, \text{ if } x>5 \end{cases}$

The given function $f$ is continuous at $x$= 5, if $f$ is defined at $x$= 5 and if the value of $f$ at $x$ = 5 equals the limit of $f$ at $x$ = 5

It is evident that $f$ is defined at $x$= 5 and $f(5)=kx+1=5k+1$

$$\lim_{x\to 5^-} f(x) = \lim_{x\to 5^+} f(x) = f(5)$$
$$\Rightarrow \lim_{x\to 5^-}(kx+1) = \lim_{x\to 5^+}(3x-5) = 5k+1$$
$$\Rightarrow 5k+1 = 15-5 = 5k+1$$
$$\Rightarrow 5k+1 = 10$$
$$\Rightarrow 5k = 9$$
$$\Rightarrow k = \frac{9}{5}$$

Therefore, the required value of $k$ is $\frac{9}{5}$.

Answer needs Correction? Click Here

Q30 : Find the values of $a$and $b$ such that the function defined by

$$f(x)=\begin{cases} 5, & \text{if } x\le 2 \\ ax+b, & \text{if } 2<x<10 \\ 21, & \text{if } x\ge 10 \end{cases}$$

is a continuous function.

Answer :

The given function $f$ is $f(x)=\begin{cases} 5, & \text{if } x\le 2 \\ ax+b, & \text{if } 2<x<10 \\ 21, & \text{if } x\ge 10 \end{cases}$

It is evident that the given function $f$ is defined at all points of the real line.

If $f$ is a continuous function, then $f$ is continuous at all real numbers.

In particular, $f$ is continuous at $x$ = 2 and $x$ = 10

Since $f$ is continuous at $x$ = 2. we obtain

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$$
$$\Rightarrow \lim_{x\to 2^-}(5) = \lim_{x\to 2^+}(ax+b) = 5$$
$$\Rightarrow 5 = 2a+b = 5$$
$$\Rightarrow 2a+b = 5 \qquad ...(1)$$

Since $f$ is continuous at $x = 10$, we obtain

$$\lim_{x\to 10^-} f(x) = \lim_{x\to 10^+} f(x) = f(10)$$
$$\Rightarrow \lim_{x\to 10^-}(ax+b) = \lim_{x\to 10^+}(21) = 21$$
$$\Rightarrow 10a+b = 21 = 21$$
$$\Rightarrow 10a+b = 21 \qquad ...(2)$$

On subtracting equation (1) from equation (2), we obtain

$8a = 16$

$\Rightarrow a = 2$

Byputting $a = 2$ in equation (1), we obtain

$2 \times 2 + b = 5$

$\Rightarrow 4 + b = 5$

$\Rightarrow b = 1$

Therefore, the values of $a$ and $b$ for which $f$ is a continuous function are 2 and 1 respectively.

Answer needs Correction? **Click Here**

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**Q31 : Show that the function defined by $f(x) = \cos(x^2)$ is a continuous function.**

**Answer :**

The given function is $f(x) = \cos(x^2)$

This function $f$ is defined for every real number and $f$ can be written as the composition of two functions as,

$f = g \text{ o } h$, where $g(x) = \cos x$ and $h(x) = x^2$

$$\left[\because (goh)(x) = g(h(x)) = g(x^2) = \cos(x^2) = f(x)\right]$$

It has to be first proved that $g(x) = \cos x$ and $h(x) = x^2$ are continuous functions.

It is evident that $g$ is defined for every real number.

Let $c$ be a real number.

Then, $g(c) = \cos c$

Put $x = c+h$

If $x \to c$, then $h \to 0$

$$\lim_{x\to c} g(x) = \lim_{x\to c} \cos x$$
$$= \lim_{h\to 0} \cos(c+h)$$
$$= \lim_{h\to 0}[\cos c \cos h - \sin c \sin h]$$
$$= \lim_{h\to 0} \cos c \cos h - \lim_{h\to 0} \sin c \sin h$$
$$= \cos c \cos 0 - \sin c \sin 0$$
$$= \cos c \times 1 - \sin c \times 0$$
$$= \cos c$$
$$\therefore \lim_{x\to c} g(x) = g(c)$$

Therefore, $g(x) = \cos x$ is continuous function.

$h(x) = x^2$

Clearly, $h$ is defined for every real number.

Let $k$ be a real number, then $h(k) = k^2$

$$\lim_{x\to k} h(x) = \lim_{x\to k} x^2 = k^2$$
$$\therefore \lim_{x\to k} h(x) = h(k)$$

Therefore, $h$ is a continuous function.

It is known that for real valued functions $g$ and $h$, such that ($g$ o $h$) is defined at $c$, if $g$ is continuous at $c$ and if $f$ is continuous at $g(c)$, then ($f$ o $g$) is continuous at $c$.

Therefore, $f(x) = (goh)(x) = \cos(x^2)$ is a continuous function.

Answer needs Correction? **Click Here**

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**Q32 : Show that the function defined by $f(x) = |\cos x|$ is a continuous function.**

**Answer :**

The given function is $f(x) = |\cos x|$

This function $f$ is defined for every real number and $f$ can be written as the composition of two functions as,

$f = g \text{ o } h$, where $g(x) = |x|$ and $h(x) = \cos x$

$$\left[\because (goh)(x) = g(h(x)) = g(\cos x) = |\cos x| = f(x)\right]$$

It has to be first proved that $g(x) = |x|$ and $h(x) = \cos x$ are continuous functions.

$g(x) = |x|$ can be written as

$$g(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

Clearly, *g* is defined for all real numbers.

Let *c* be a real number.

Case I:

If $c < 0$, then $g(c) = -c$ and $\lim_{x \to c} g(x) = \lim_{x \to c}(-x) = -c$

$\therefore \lim_{x \to c} g(x) = g(c)$

Therefore, *g* is continuous at all points *x*, such that $x < 0$

Case II:

If $c > 0$, then $g(c) = c$ and $\lim_{x \to c} g(x) = \lim_{x \to c} x = c$

$\therefore \lim_{x \to c} g(x) = g(c)$

Therefore, *g* is continuous at all points *x*, such that $x > 0$

Case III:

If $c = 0$, then $g(c) = g(0) = 0$

$$\lim_{x \to 0^-} g(x) = \lim_{x \to 0^-} (-x) = 0$$
$$\lim_{x \to 0^+} g(x) = \lim_{x \to 0^+} (x) = 0$$
$$\therefore \lim_{x \to 0^-} g(x) = \lim_{x \to 0^+} (x) = g(0)$$

Therefore, *g* is continuous at $x = 0$

From the above three observations, it can be concluded that *g* is continuous at all points.

*h* (*x*) = cos *x*

It is evident that *h*(*x*) = cos *x* is defined for every real number.

Let *c* be a real number. Put *x* = *c* + *h*

If *x* Ã¢â€ ' *c*, then *h* Ã¢â€ '0

*h* (*c*) = cos *c*

$$\begin{aligned} \lim_{x \to c} h(x) &= \lim_{x \to c} \cos x \\ &= \lim_{h \to 0} \cos(c+h) \\ &= \lim_{h \to 0} [\cos c \cos h - \sin c \sin h] \\ &= \lim_{h \to 0} \cos c \cos h - \lim_{h \to 0} \sin c \sin h \\ &= \cos c \cos 0 - \sin c \sin 0 \\ &= \cos c \times 1 - \sin c \times 0 \\ &= \cos c \end{aligned}$$
$$\therefore \lim_{x \to c} h(x) = h(c)$$

Therefore, *h* (*x*) = cos *x* is a continuous function.

It is known that for real valued functions *g* and *h*, such that (*g* o *h*) is defined at *c*, if *g* is continuous at *c* and if *f* is continuous at *g* (*c*), then (*f* o *g*) is continuous at *c*.

Therefore, $f(x) = (goh)(x) = g(h(x)) = g(\cos x) = |\cos x|$ is a continuous function.

Answer needs Correction? **Click Here**

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**Q33 : Examine that $\sin|x|$ is a continuous function.**

**Answer :**

Let $f(x) = \sin|x|$

This function *f* is defined for every real number and *f* can be written as the composition of two functions as,

*f* = *g o h*, where $g(x) = |x|$ and $h(x) = \sin x$

$$[\because (goh)(x) = g(h(x)) = g(\sin x) = |\sin x| = f(x)]$$

It has to be proved first that $g(x) = |x|$ and $h(x) = \sin x$ are continuous functions.

$g(x) = |x|$ can be written as

$$g(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

Clearly, *g* is defined for all real numbers.

Let *c* be a real number.

Case I:

If $c < 0$, then $g(c) = -c$ and $\lim_{x \to c} g(x) = \lim_{x \to c}(-x) = -c$

$\therefore \lim_{x \to c} g(x) = g(c)$

Therefore, *g* is continuous at all points *x*, such that $x < 0$

Case II:

If $c > 0$, then $g(c) = c$ and $\lim_{x \to c} g(x) = \lim_{x \to c} x = c$

$\therefore \lim_{x \to c} g(x) = g(c)$

Therefore, *g* is continuous at all points *x*, such that $x > 0$

Case III:

$\text{If } c=0, \text{ then } g(c)=g(0)=0$

$$\lim_{x\to 0^-} g(x)=\lim_{x\to 0^-}(-x)=0$$
$$\lim_{x\to 0^+} g(x)=\lim_{x\to 0^+}(x)=0$$
$$\therefore \lim_{x\to 0^-} g(x)=\lim_{x\to 0^+}(x)=g(0)$$

Therefore, $g$ is continuous at $x= 0$

From the above three observations, it can be concluded that $g$is continuous at all points.

$h(x) = \sin x$

It is evident that $h(x) = \sin x$ is defined for every real number.

Let $c$ be a real number. Put $x= c + k$

If $x$ Ã¢â€ ' $c$, then $k$ Ã¢â€ '0

$h(c) = \sin c$

$$h(c)=\sin c$$
$$\begin{aligned}\lim_{x\to c} h(x)&=\lim_{x\to c}\sin x\\ &=\lim_{k\to 0}\sin(c+k)\\ &=\lim_{k\to 0}[\sin c\cos k+\cos c\sin k]\\ &=\lim_{k\to 0}(\sin c\cos k)+\lim_{h\to 0}(\cos c\sin k)\\ &=\sin c\cos 0+\cos c\sin 0\\ &=\sin c+0\\ &=\sin c\end{aligned}$$
$$\therefore \lim_{x\to c} h(x)=g(c)$$

Therefore, $h$ is a continuous function.

It is known that for real valued functions $g$ and $h$,such that ($g$ o $h$) is defined at $c$, if $g$ is continuous at $c$ and if $f$ is continuous at $g$ ($c$), then ($f$ o $g$) is continuous at $c$.

Therefore, $f(x)=(goh)(x)=g(h(x))=g(\sin x)=|\sin x|$ is a continuous function.

Answer needs Correction? Click Here

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Q34 : Find all the points of discontinuity of $f$ defined by $f(x)=|x|-|x+1|$.

**Answer :**

The given function is $f(x)=|x|-|x+1|$

Thetwo functions, $g$and $h$, are defined as

$g(x)=|x| \text{ and } h(x)=|x+1|$

Then, $f = g - h$

Thecontinuity of $g$and $h$ is examined first.

$g(x)=|x| \text{ can be written as}$

$$g(x)=\begin{cases}-x, & \text{if } x<0\\ x, & \text{if } x\geq 0\end{cases}$$

Clearly, $g$ is defined for all real numbers.

Let $c$ be a real number.

Case I:

$\text{If } c<0, \text{ then } g(c)=-c \text{ and } \lim_{x\to c} g(x)=\lim_{x\to c}(-x)=-c$

$\therefore \lim_{x\to c} g(x)=g(c)$

Therefore, $g$ is continuous at all points $x$, such that $x < 0$

Case II:

$\text{If } c>0, \text{ then } g(c)=c \text{ and } \lim_{x\to c} g(x)=\lim_{x\to c} x=c$

$\therefore \lim_{x\to c} g(x)=g(c)$

Therefore, $g$ is continuous at all points $x$, such that $x> 0$

Case III:

$\text{If } c=0, \text{ then } g(c)=g(0)=0$

$$\lim_{x\to 0^-} g(x)=\lim_{x\to 0^-}(-x)=0$$
$$\lim_{x\to 0^+} g(x)=\lim_{x\to 0^+}(x)=0$$
$$\therefore \lim_{x\to 0^-} g(x)=\lim_{x\to 0^+}(x)=g(0)$$

Therefore, $g$ is continuous at $x= 0$

From the above three observations, it can be concluded that $g$is continuous at all points.

$h(x)=|x+1| \text{ can be written as}$

$$h(x)=\begin{cases}-(x+1), & \text{if, } x<-1\\ x+1, & \text{if } x\geq -1\end{cases}$$

Clearly, $h$ is defined for every real number.

Let $c$ be a real number.

Case I:

$\text{If } c<-1, \text{ then } h(c)=-(c+1) \text{ and } \lim_{x\to c} h(x)=\lim_{x\to c}[-(x+1)]=-(c+1)$

$\therefore \lim_{x\to c} h(x)=h(c)$

$\therefore \lim_{x \to c} h(x) = h(c)$

Therefore, *h* is continuous at all points *x*, such that *x* < - 1

Case II:

If $c > -1$, then $h(c) = c + 1$ and $\lim_{x \to c} h(x) = \lim_{x \to c} (x+1) = c + 1$

$\therefore \lim_{x \to c} h(x) = h(c)$

Therefore, *h* is continuous at all points *x*, such that *x*> - 1

Case III:

If $c = -1$, then $h(c) = h(-1) = -1 + 1 = 0$

$\lim_{x \to -1^-} h(x) = \lim_{x \to -1^-} \left[-(x+1)\right] = -(-1+1) = 0$

$\lim_{x \to -1^+} h(x) = \lim_{x \to -1^+} (x+1) = (-1+1) = 0$

$\therefore \lim_{x \to -1^-} h(x) = \lim_{h \to -1^+} h(x) = h(-1)$

Therefore, *h* is continuous at *x*= - 1

From the above three observations, it can be concluded that *h*is continuous at all points of the real line.



*********** END **********