Exercise 2.5

Q6. Write the following cubes in expanded form:

(i) $(2x+1)^3$

(ii) $(2a-3b)^3$

(iii) $\left(\frac{3}{2}x+1\right)^3$

(iv) $\left(x-\frac{2}{3}y\right)^3$

Ans: (i) $(2x+1)^3$

We know that $(x+y)^3 = x^3+y^3+3xy(x+y)$.

$$\begin{aligned} \therefore (2x+1)^3 &= (2x)^3+(1)^3+3\times 2x\times 1(2x+1) \\ &= 8x^3+1+6x(2x+1) \\ &= 8x^3+12x^2+6x+1. \end{aligned}$$

Therefore, the expansion of the expression $(2x+1)^3$ is $8x^3+12x^2+6x+1$.

(ii) $(2a-3b)^3$

We know that $(x-y)^3 = x^3-y^3-3xy(x-y)$.

$$\begin{aligned} \therefore (2a-3b)^3 &= (2a)^3-(3b)^3-3\times 2a\times 3b(2a-3b) \\ &= 8a^3-27b^3-18ab(2a-3b) \\ &= 8a^3-36a^2b+54ab^2-27b^3. \end{aligned}$$

Therefore, the expansion of the expression $(2a-3b)^3$ is $8a^3-36a^2b+54ab^2-27b^3$.

(iii) $\left(\frac{3}{2}x+1\right)^3$

$(x+y)^3 = x^3+y^3+3xy(x+y)$

We know that $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$.

$$\left(\frac{3}{2}x+1\right)^3 = \left(\frac{3}{2}x\right)^3 + (1)^3 + 3\times\frac{3}{2}x\times1\left(\frac{3}{2}x+1\right) \therefore$$

$$= \frac{27}{8}x^3 + 1 + \frac{9}{2}x\left(\frac{3}{2}x+1\right)$$

$$= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1.$$

Therefore, the expansion of the expression

$\left(\frac{3}{2}x+1\right)^3$ is $\frac{27}{8}x^3+\frac{27}{4}x^2+\frac{9}{2}x+1$.

(iv) $\left(x-\frac{2}{3}y\right)^3$

We know that $(x-y)^3 = x^3-y^3-3xy(x-y)$.

$$\therefore \left(x-\frac{2}{3}y\right)^3 = (x)^3-\left(\frac{2}{3}y\right)^3-3\times x\times\frac{2}{3}y\left(x-\frac{2}{3}y\right)$$

$$= x^3-\frac{8}{27}y^3-2xy\left(x-\frac{2}{3}y\right)$$

$$= x^3-2x^2y+\frac{4}{3}xy^2-\frac{8}{27}y^3.$$

Therefore, the expansion of the expression

$\left(x-\frac{2}{3}y\right)^3$ is $x^3-2x^2y+\frac{4}{3}xy^2-\frac{8}{27}y^3$.

Q7. Evaluate the following using suitable identities:

(i) $(99)^3$

(ii) $(102)^3$

(iii) $(998)^3$

Ans: (i) $(99)^3$

$(99)^3$ can also be written as $(100-1)^3$.

Using identity, $(x-y)^3 = x^3-y^3-3xy(x-y)$

$$(100-1)^3 = (100)^3-(1)^3-3\times100\times1(100-1)$$

=1000000-1-300(99)

=999999-29700

=970299

(ii) $(102)^3$

$(102)^3$ can also be written as $(100+2)^3$.

Using identity $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

$$(100+2)^3 = (100)^3 + (2)^3 + 3\times 100\times 2(100+2)$$

=1000000+8+600(102)

=1000008+61200

=1061208

(iii) $(998)^3$

$(998)^3$ can also be written as $(1000-2)^3$.

Using identity $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$$(1000-2)^3 = (1000)^3 - (2)^3 - 3\times 1000\times 2(1000-2)$$

=1000000000-8-6000(998)

=999999992-5988000

=994011992

Q8. Factorize each of the following:

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$ (ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

(iii) $27 - 125a^3 - 135a + 225a^2$ (iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Ans: (i) $8a^3 + b^3 + 12a^2b + 6ab^2$

The expression $8a^3 + b^3 + 12a^2b + 6ab^2$ can also be written as

$$= (2a)^3 + (b)^3 + 3\times 2a\times 2a\times b + 3\times 2a\times b\times b$$

$$= (2a)^3 + (b)^3 + 3\times 2a\times b(2a+b).$$

Using identity $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$ with respect to the expression $(2a)^3 + (b)^3 + 3\times 2a\times b(2a+b)$, we get $(2a+b)^3$.

Therefore, after factorizing the expression $8a^3 + b^3 + 12a^2b + 6ab^2$, we get $(2a+b)^3$.

(ii) $8a^3-b^3-12a^2b+6ab^2$

The expression $8a^3-b^3-12a^2b+6ab^2$ can also be written as

$=(2a)^3-(b)^3-3\times2a\times2a\times b+3\times2a\times b\times b$

$=(2a)^3-(b)^3-3\times2a\times b(2a-b).$

Using identity $(x-y)^3=x^3-y^3-3xy(x-y)$ with respect to the expression $(2a)^3-(b)^3-3\times2a\times b(2a-b)$, we get $(2a-b)^3$.

Therefore, after factorizing the expression $8a^3-b^3-12a^2b+6ab^2$, we get $(2a-b)^3$.

(iii) $27-125a^3-135a+225a^2$

The expression $27-125a^3-135a+225a^2$ can also be written as

$=(3)^3-(5a)^3-3\times3\times3\times5a+3\times3\times5a\times5a$

$=(3)^3-(5a)^3+3\times3\times5a(3-5a).$

Using identity $(x-y)^3=x^3-y^3-3xy(x-y)$ with respect to the expression $(3)^3-(5a)^3+3\times3\times5a(3-5a)$, we get $(3-5a)^3$.

Therefore, after factorizing the expression $27-125a^3-135a+225a^2$, we get $(3-5a)^3$.

(iv) $64a^3-27b^3-144a^2b+108ab^2$

The expression $64a^3-27b^3-144a^2b+108ab^2$ can also be written as

$=(4a)^3-(3b)^3-3\times4a\times4a\times3b+3\times4a\times3b\times3b$

$=(4a)^3-(3b)^3-3\times4a\times3b(4a-3b).$

Using identity $(x-y)^3=x^3-y^3-3xy(x-y)$ with respect to the expression $(4a)^3-(3b)^3-3\times4a\times3b(4a-3b)$, we get $(4a-3b)^3$.

Therefore, after factorizing the expression $64a^3-27b^3-144a^2b+108ab^2$, we get $(4a-3b)^3$.

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

The expression $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$ can also be written as

$$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3\times 3p\times 3p\times\frac{1}{6} + 3\times 3p\times\frac{1}{6}\times\frac{1}{6}$$

$$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3\times 3p\times\frac{1}{6}\left(3p - \frac{1}{6}\right).$$

Using identity $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$ with respect to the expression

$(3p)^3 - \left(\frac{1}{6}\right)^3 - 3\times 3p\times\frac{1}{6}\left(3p - \frac{1}{6}\right)$, to get $\left(3p - \frac{1}{6}\right)^3$.

Therefore, after factorizing the expression

$27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$, we get $\left(3p - \frac{1}{6}\right)^3$.

Q9. Verify:

(i) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

(ii) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

Ans: (i) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

We know that $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$.

$$\Rightarrow x^3 + y^3 = (x+y)^3 - 3xy(x+y)$$

$$= (x+y)\left[(x+y)^2 - 3xy\right]$$

$\because$ We know that $(x+y)^2 = x^2 + 2xy + y^2$

$$\therefore x^3 + y^3 = (x+y)(x^2 + 2xy + y^2 - 3xy)$$

$$= (x+y)(x^2 - xy + y^2)$$

Therefore, the desired result has been verified.

(ii) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

We know that $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$.

$$\Rightarrow x^3 - y^3 = (x-y)^3 + 3xy(x-y)$$

$$= (x-y)\left[(x-y)^2 + 3xy\right]$$

$\because$ We know that $(x-y)^2 = x^2 - 2xy + y^2$

$$\therefore\ x^3 - y^3 = (x-y)\left(x^2 - 2xy + y^2 + 3xy\right)$$

$$= (x-y)\left(x^2 + xy + y^2\right)$$

Therefore, the desired result has been verified.

Q10. Factorize:

(i) $27y^3 + 125z^3$

(ii) $64m^3 - 343n^3$

Ans: (i) $27y^3 + 125z^3$

The expression $27y^3 + 125z^3$ can also be written as $(3y)^3 + (5z)^3$.

We know that $x^3 + y^3 = (x+y)\left(x^2 - xy + y^2\right)$.

$$(3y)^3 + (5z)^3 = (3y+5z)\left[(3y)^2 - 3y \times 5z + (5z)^2\right]$$

$$= (3y+5z)\left(9y^2 - 15yz + 25z^2\right).$$

(ii) $64m^3 - 343n^3$

The expression $64m^3 - 343n^3$ can also be written as $(4m)^3 - (7n)^3$.

We know that $x^3 - y^3 = (x-y)\left(x^2 + xy + y^2\right)$.

$$(4m)^3 - (7n)^3 = (4m-7n)\left[(4m)^2 + 4m \times 7n + (7n)^2\right]$$

$$= (4m-7n)\left(16m^2 + 28mn + 49n^2\right)$$

.

Therefore, we conclude that after factorizing the expression $64m^3 - 343n^3$, we get $(4m-7n)\left(16m^2 + 28mn + 49n^2\right)$.

Q11. Factorize: $27x^3 + y^3 + z^3 - 9xyz$

Ans:

The expression $27x^3 + y^3 + z^3 - 9xyz$ can also be written as

$$(3x)^3 + (y)^3 + (z)^3 - 3 \times 3x \times y \times z.$$

We know that

$$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx).$$

$$\therefore (3x)^3+(y)^3+(z)^3-3\times 3x\times y\times z=(3x+y+z)\left[(3x)^2+(y)^2+(z)^2-3x\times y-y\times z-z\times 3x\right]$$

$$=(3x+y+z)(9x^2+y^2+z^2-3xy-yz-3xz).$$

Therefore, we conclude that after factorizing the expression $27x^3+y^3+z^3-9xyz$, we get $(3x+y+z)(9x^2+y^2+z^2-3xy-yz-3xz)$.

Q12. Verify that

$$x^3+y^3+z^3-3xyz=\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$$

Ans:

LHS is $x^3+y^3+z^3-3xyz$ and RHS is $\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$.

We know that

$$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$

.

And also, we know that $(x-y)^2=x^2-2xy+y^2$.

$$\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$$

$$\frac{1}{2}(x+y+z)\left[(x^2-2xy+y^2)+(y^2-2yz+z^2)+(z^2-2xz+x^2)\right]$$

$$\frac{1}{2}(x+y+z)(2x^2+2y^2+2z^2-2xy-2yz-2zx)$$

$$(x+y+z)(x^2+y^2+z^2-xy-yz-zx).$$

Therefore, we can conclude that the desired result is verified.

Q13. If $x+y+z=0$, show that $x^3+y^3+z^3=0$.

Ans: We know that

$$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx).$$

We need to substitute $x^3+y^3+z^3=0$ in

$$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx),$$

to get

$x^3+y^3+z^3-3xy = (0) = \left(x^2+y^2+z^2-xy-yz-zx\right)$,

$x^3+y^3+z^3-3xyz=0$

$\Rightarrow x^3+y^3+z^3=3xyz.$

Therefore, the desired result is verified.

Q14. Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3+(7)^3+(5)^3$

(ii) $(28)^3+(-15)^3+(-13)^3$

Ans: (i) $(-12)^3+(7)^3+(5)^3$

Let $a=-12, b=7$ and $c=5$

We know that, if $a+b+c=0$, then $a^3+b^3+c^3=3abc$

Here, $a+b+c=-12+7+5=0$

$\therefore (-12)^3+(7)^3+(5)^3=3(-12)(7)(5)$

$= -1260$

(ii) $(28)^3+(-15)^3+(-13)^3$

Let $a=28, b=-15$ and $c=-13$

We know that, if $a+b+c=0$, then $a^3+b^3+c^3=3abc$

Here, $a+b+c=28-15-13=0$

$\therefore (28)^3+(-15)^3+(-13)^3=3(28)(-15)(-13)$

$= 16380$

Q15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) $Are: 25a^2-35a+12$

(ii) $Are: 35y^2+13y-12$

Ans: (i) Area : $25a^2 - 35a + 12$

The expression $25a^2 - 35a + 12$ can also be written as $25a^2 - 15a - 20a + 12$.

$25a^2 - 15a - 20a + 12 = 5a(5a - 3) - 4(5a - 3)$

$= (5a - 4)(5a - 3)$.

Therefore, we can conclude that a possible expression for the length and breadth of a rectangle of area $25a^2 - 35a + 12$ is

Length $= (5a - 4)$ and Breadth $= (5a - 3)$.

(ii) Area : $35y^2 + 13y - 12$

The expression $35y^2 + 13y - 12$ can also be written as $35y^2 + 28y - 15y - 12$.

$35y^2 + 28y - 15y - 12 = 7y(5y + 4) - 3(5y + 4)$

$= (7y - 3)(5y + 4)$.

Therefore, we can conclude that a possible expression for the length and breadth of a rectangle of area $35y^2 + 13y - 12$ is

Length $= (7y - 3)$ and Breadth $= (5y + 4)$.

Q16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) $Volume: 3x^2 - 12x$

(ii) $Volume: 12ky^2 + 8ky - 20k$

Ans: (i) Volume : $3x^2 - 12x$

The expression $3x^2 - 12x$ can also be written as $3 \times x \times (x - 4)$.

Therefore, we can conclude that a possible expression for the dimension of a cuboid of volume $3x^2 - 12x$ is $3, x$ and $(x - 4)$.

(ii) Volume : $12ky^2 + 8ky - 20k$

The expression $12ky^2 + 8ky - 20k$ can also be written as $k(12y^2 + 8y - 20)$.

$= k\left[12y(y-1)+20(y-1)\right]$

$= k(12y+20)(y-1)$

$= 4k\times(3y+5)\times(y-1)$.

Therefore, we can conclude that a possible expression for the dimension of a cuboid of volume $12ky^2+8ky-20k$ is $4k$, $(3y+5)$ and $(y-1)$.

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