Definite Integrals Ex 20.2 Q9

Let $x^2 = t$

Differentiating w.r.t. x, we get

$$2x\,dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = 1 \Rightarrow t = 1$

$$\therefore \int_0^1 \frac{2x}{1+x^4}\,dx$$

$$= \int_0^1 \frac{dt}{1+t^2}$$

$$= \left[\tan^{-1} t\right]_0^1$$

$$= \left[\tan^{-1} 1 - \tan^{-1} 0\right] \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$$

$$= \frac{\pi}{4}$$

$$\therefore \int_0^1 \frac{2x}{1+x^4}\,dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q10

Let $x = a\sin\theta$

Differentiating w.r.t. x, we get

$$dx = a\cos\theta d\theta$$

Now,

$x = 0 \Rightarrow \theta = 0$

$x = a \Rightarrow \theta = \frac{\pi}{2}$

$$\therefore \int_0^a \sqrt{a^2 - x^2}\,dx$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{a^2\left(1 - \sin^2\theta\right)}\, a\cos\theta d\theta$$

$$= a^2 \int_0^{\frac{\pi}{2}} \cos^2\theta d\theta \qquad \left[\because \left(1 - \sin^2\theta\right) = \cos^2\theta \text{ and } \frac{1 + \cos 2\theta}{2} = \cos 2\theta\right]$$

$$= \frac{a^2}{2} \int_0^{\frac{\pi}{2}} \left(1 + \cos 2\theta\right) d\theta$$

$$= \frac{a^2}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{a^2}{2}\left[\frac{\pi}{2} + 0 - 0 - 0\right]$$

$$= \frac{\pi a^2}{4}$$

$$\therefore \int_0^a \sqrt{a^2 - x^2}\,dx = \frac{\pi a^2}{4}$$

Definite Integrals Ex 20.2 Q11

Let $I = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi} \cos^5\phi \, d\phi = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi} \cos^4\phi \cos\phi \, d\phi$

Also, let $\sin\phi = t \Rightarrow \cos\phi \, d\phi = dt$

When $\phi = 0$, $t = 0$ and when $\phi = \frac{\pi}{2}$, $t = 1$

$$\therefore I = \int_0^1 \sqrt{t}\left(1-t^2\right)^2 dt$$

$$= \int_0^1 t^{\frac{1}{2}}\left(1+t^4-2t^2\right)dt$$

$$= \int_0^1 \left[t^{\frac{1}{2}} + t^{\frac{9}{2}} - 2t^{\frac{5}{2}}\right]dt$$

$$= \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{11}{2}}}{\frac{11}{2}} - \frac{2t^{\frac{7}{2}}}{\frac{7}{2}}\right]_0^1$$

$$= \frac{2}{3} + \frac{2}{11} - \frac{4}{7}$$

$$= \frac{154+42-132}{231}$$

$$= \frac{64}{231}$$

Definite Integrals Ex 20.2 Q12

Let $\sin x = t$

Differentiating w.r.t. x, we get

$$\cos x\, dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\int_0^{\frac{\pi}{2}} \frac{\cos x}{1 + \sin^2 x} dx$$

$$= \int_0^1 \frac{dt}{1 + t^2}$$

$$= \left[\tan^{-1} t\right]_0^1$$

$$= \left[\tan^{-1} 1 - \tan^{-1} 0\right] \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\cos x}{1 + \sin^2 x} dx = \frac{\pi}{4}$$

Definite Integrals Ex 20.2 Q13

Let $1+\cos\theta = t^2$

Differentiating w.r.t. x, we get

$$-\sin\theta\, d\theta = 2t\, dt$$

$$\sin\theta\, d\theta = -2t\, dt$$

Now,

$x = 0 \Rightarrow t = \sqrt{2}$

$x = \frac{\pi}{2} \Rightarrow t = 1$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin\theta\, d\theta}{\sqrt{1+\cos\theta}}$$

$$= \int_{\sqrt{2}}^{1} \frac{-2t dt}{t}$$

$$= -2\int_{\sqrt{2}}^{1} dt$$

$$= -2\left[t\right]_{\sqrt{2}}^{1}$$

$$= -2\left[1-\sqrt{2}\right]$$

$$= 2\left[\sqrt{2}-1\right]$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\sin\theta\, d\theta}{\sqrt{1+\cos\theta}} = 2\left[\sqrt{2}-1\right]$$

*********** END **********