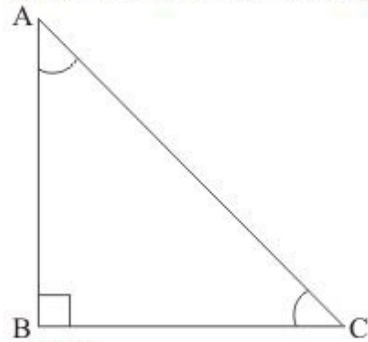




Trigonometric Ratios Ex 5.2 Q36

Answer :

(i) We have drawn the following figure related to given information



To find:

$$\sin A \cos C + \cos A \sin C \dots\dots (1)$$

Now we have,

$$\sin A = \frac{BC}{AC}, \sin A = \frac{AB}{AC}$$

$$\cos A = \frac{AB}{AC}, \cos C = \frac{BC}{AC}$$

Now by substituting the above values in equation (1)

We get,

$$\begin{aligned} \sin A \cos C + \cos A \sin C &= \frac{BC}{AC} \times \frac{BC}{AC} + \frac{AB}{AC} \times \frac{AB}{AC} \\ &= \frac{BC^2}{AC^2} + \frac{AB^2}{AC^2} \\ &= \frac{BC^2 + AB^2}{AC^2} \end{aligned}$$

Therefore,

$$\sin A \cos C + \cos A \sin C = \frac{BC^2 + AB^2}{AC^2} \dots\dots (2)$$

Now in right angled $\triangle ABC$

By applying Pythagoras theorem

We get,

$$AC^2 = AB^2 + BC^2$$

Now, by substituting above value of AC^2 in equation (2)

We get,

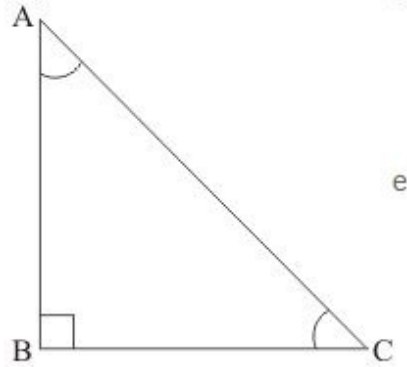
$$\begin{aligned} \sin A \cos C + \cos A \sin C &= \frac{BC^2 + AB^2}{AB^2 + BC^2} \\ \Rightarrow \sin A \cos C + \cos A \sin C &= \frac{AB^2 + BC^2}{AB^2 + BC^2} \end{aligned}$$

Now both numerator and denominator contains $AB^2 + BC^2$

Therefore it gets cancelled and 1 remains

Hence $\boxed{\sin A \cos C + \cos A \sin C = 1}$

(ii) We have drawn the following figure



To find:

$$\sin A \sin B + \cos A \cos B \dots\dots (1)$$

Now we know that sum of all the angles of any triangle is 180°

Therefore,

$$\angle A + \angle B + \angle C = 180^\circ$$

Since $\angle A = \angle C$ and $\angle B = 90^\circ$

Therefore,

$$\angle A + 90^\circ + \angle A = 180^\circ$$

$$\Rightarrow 2\angle A + 90^\circ = 180^\circ$$

$$\Rightarrow 2\angle A = 180^\circ - 90^\circ$$

$$\Rightarrow 2\angle A = 90^\circ$$

$$\Rightarrow \angle A = \frac{90^\circ}{2}$$

$$\Rightarrow \angle A = 45^\circ$$

It is given that $\angle A = \angle C$

Therefore,

$$\angle A = \angle C = 45^\circ \dots\dots (2)$$

Now we have,

$$\sin A = \frac{BC}{AC}, \sin B = \sin 90^\circ = 1$$

$$\cos A = \frac{AB}{AC}, \cos B = \cos 90^\circ = 0$$

Now by substituting the above values in equation (1)

We get,

$$\begin{aligned} \sin A \sin B + \cos A \cos B &= \sin 45^\circ \times 1 + \cos 45^\circ \times 0 \\ &= \sin 45^\circ \end{aligned}$$

$$\text{Since } \sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\text{Therefore } \boxed{\sin A \sin B + \cos A \cos B = \frac{1}{\sqrt{2}}}$$

***** END *****