Definite Integrals Ex 20.1 Q16

We have,

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1+\sin x}\,dx$$

We know,

$$\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

$$\therefore \frac{1}{1+\sin x} = \frac{1}{1+\left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)} = \frac{1+\tan^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2} = \frac{\sec^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2}$$

$$\Rightarrow \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1+\sin x}\,dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2}\,dx$$

If $f(x)$ is an even function $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$

So,

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2}\,dx = 2\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2}\,dx$$

let $1+\tan\frac{x}{2}=t$

$\Rightarrow \frac{1}{2}\sec^2\frac{x}{2}\,dx = dt$

Now,

$x=-\frac{\pi}{4} \Rightarrow t = 1-\tan\frac{\pi}{8}$

$x=\frac{\pi}{4} \Rightarrow t = 1+\tan\frac{\pi}{8}$

$$\therefore\ 2\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec^2\frac{x}{2}}{\left(1+\tan\frac{x}{2}\right)^2}dx = 2\int_{1-\tan\frac{\pi}{8}}^{1+\tan\frac{\pi}{8}} \frac{8dt}{t^2}$$

$$= 2\left[\frac{-1}{t}\right]_{1-\tan\frac{\pi}{8}}^{1+\tan\frac{\pi}{8}}$$

$$= 2\left[\frac{1}{1-\tan\frac{\pi}{8}}-\frac{1}{1+\tan\frac{\pi}{8}}\right]$$

$$= 2\left[\frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}\right]$$

$$= 2\tan\frac{\pi}{4} \qquad \left[\because \tan 2A = \frac{2\tan A}{1-\tan^2 A}\right]$$

$$= 2$$

$$\therefore \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1+\sin x}dx = 2$$

Definite Integrals Ex 20.1 Q17

Let $I=\int_0^{\frac{\pi}{2}}\cos^2 x\,dx$

$$\int\cos^2 x\,dx=\int\left(\frac{1+\cos 2x}{2}\right)dx=\frac{x}{2}+\frac{\sin 2x}{4}=\frac{1}{2}\left(x+\frac{\sin 2x}{2}\right)=F(x)$$

By second fundamental theorem of calculus, we obtain

$$I=\left[F\left(\frac{\pi}{2}\right)-F(0)\right]$$
$$=\frac{1}{2}\left[\left(\frac{\pi}{2}-\frac{\sin\pi}{2}\right)-\left(0+\frac{\sin 0}{2}\right)\right]$$
$$=\frac{1}{2}\left[\frac{\pi}{2}+0-0-0\right]$$
$$=\frac{\pi}{4}$$

Definite Integrals Ex 20.1 Q18

We have,

$$\int_0^{\frac{\pi}{2}}\cos^3 x\,dx$$

$$=\int_0^{\frac{\pi}{2}}\frac{\cos 3x+3\cos x}{4}dx \qquad \left[\because \cos 3x = 4\cos^3 x-3\cos x\right]$$

$$=\frac{1}{4}\int_0^{\frac{\pi}{2}}\left(\cos 3x+3\cos x\right)dx$$

$$=\frac{1}{4}\left[\frac{\sin 3x}{3}+3\sin x\right]_0^{\frac{\pi}{2}}$$

$$=\frac{1}{4}\left[\left(\frac{\sin 3\frac{\pi}{2}}{3}+3\sin\frac{\pi}{2}\right)-\left(\frac{\sin 0}{3}+3\sin 0\right)\right]$$

$$=\frac{1}{4}\left[\left(\frac{-1}{3}+3\right)-\left(0+0\right)\right]=\frac{2}{3}$$

$$=\frac{1}{4}\left[\frac{8}{3}\right]$$

$$=\frac{2}{3}$$

$$\therefore\int_0^{\frac{\pi}{2}}\cos^3 x\,dx=\frac{2}{3}$$

*********** END **********