Chapter 5 Trigonometric Functions Ex 5.3 Q 4

$$\text{LHS} = \sin^2\frac{\pi}{18} + \sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18} + \sin^2\frac{4\pi}{9}$$

$$= \sin^2\frac{\pi}{18} + \sin^2\frac{4\pi}{9} + \sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18}$$

$$= \sin^2\left(\frac{\pi}{2} - \frac{4\pi}{9}\right) + \sin^2\frac{4\pi}{9} + \sin^2\frac{\pi}{9} + \sin^2\left(\frac{\pi}{2} - \frac{\pi}{9}\right) \qquad \left(\because \frac{\pi}{18} = \frac{\pi}{2} - \frac{4\pi}{9} \text{ and } \frac{7\pi}{18} = \frac{\pi}{2} - \frac{\pi}{9}\right)$$

$$= \cos^2\frac{4\pi}{9} + \sin^2\frac{4\pi}{9} + \sin^2\frac{\pi}{9} + \cos^2\frac{\pi}{9} \qquad \left(\because \sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta\right)$$

$$= 1 + 1 \quad \left(\because \sin^2\theta + \cos^2\theta = 1\right)$$

$$= 2$$

$$= \text{RHS}$$

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 5

$$\text{LHS} = \sec\left(\frac{3\pi}{2} - \theta\right)\sec\left(\theta - \frac{5\pi}{2}\right) + \tan\left(\frac{5\pi}{2} + \theta\right)\tan\left(\theta - \frac{3\pi}{2}\right)$$

$$= \sec\left(\frac{3\pi}{2} - \theta\right)\sec\left(-\left(\frac{5\pi}{2} - \theta\right)\right) + \tan\left(\frac{5\pi}{2} + \theta\right)\tan\left(-\left(\frac{3\pi}{2} - \theta\right)\right)$$

$$= -\operatorname{cosec}\theta.\sec\left(\frac{5\pi}{2} - \theta\right) - \cot\theta \times (-)\tan\left(\frac{3\pi}{2} - \theta\right)$$

$$\left[\because \left(\sec\left(\frac{3\pi}{2} - \theta\right)\right) = -\operatorname{cosec}\theta,\ \sec(-\theta) = \sec\theta, \tan\left(\frac{5\pi}{2} + \theta\right) = -\cot\theta \ \&\ \tan(-\theta) = -\tan\theta\right]$$

$$= -\operatorname{cosec}\theta \times \operatorname{cosec}\theta - \cot\theta \times (-1) \times \cot\theta \qquad \left[\left(\because \sec\left(\frac{5\pi}{2} - \theta\right)\right) = \operatorname{cosec}\theta \ \&\ \tan\left(\frac{3\pi}{2} - \theta\right) = \cot\theta\right]$$

$$= -\operatorname{cosec}^2\theta + \cot^2\theta$$

$$= -\operatorname{cosec}^2\theta + \operatorname{cosec}^2\theta - 1 \qquad \left(\because \operatorname{cosec}^2\theta = 1 + \cot^2\theta\right)$$

$$= -1$$

$$= \text{RHS}$$

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 6

We have $A + B + C = \pi \qquad \left(\because \text{sum of 3 angles of a triangle is } \pi = 180^\circ\right)$

$\Rightarrow \quad A + B = \pi - C$

$\Rightarrow \quad \cos(A + B) = \cos(\pi - C)$

$\Rightarrow \quad = -\cos C \qquad \left(\because \cos(\pi - \theta) = -\cos\theta\right)$

$\Rightarrow \quad \cos(A + B) + \cos C = 0$

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 6 ii

We have $A + B + C = \pi \qquad \left(\because \text{sum of 3 angles of a triangle is } \pi = 180^\circ\right)$

$\Rightarrow \quad A + B = \pi - C$

$\Rightarrow \quad \frac{A + B}{2} = \frac{\pi - C}{2}$

$\Rightarrow \quad \frac{A + B}{2} = \frac{\pi}{2} - \frac{C}{2}$

$\Rightarrow \quad \cos = \left(\frac{A + B}{2}\right) = \cos\left(\frac{\pi}{2} - \frac{C}{2}\right)$

$\Rightarrow \quad = \sin\frac{C}{2} \qquad \left(\because \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta\right)$

Hence $\cos\left(\frac{A + B}{2}\right) = \sin\frac{C}{2}$

Proved

Chapter 5 Trigonometric Functions Ex 5.3 Q 6 iii

We have $A + B + C = \pi$ $\left(\because \text{ sum of 3 angles of a triangle is } \pi = 180^\circ\right)$

$\Rightarrow \quad A + B = \pi - C$

$\Rightarrow \quad \frac{A+B}{2} = \frac{\pi - C}{2}$

$\Rightarrow \quad \frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2}$

$\Rightarrow \quad \tan\left(\frac{A+B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right)$

$= \cot\frac{C}{2}$ $\left(\because \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta\right)$

Hence $\tan\left(\frac{A+B}{2}\right) = \cot\frac{C}{2}$

Proved

********** END **********