Maxima and Minima 18.3 Q4

The given function is $f(x) = \dfrac{\log x}{x}$.

$$f'(x) = \frac{x\left(\frac{1}{x}\right) - \log x}{x^2} = \frac{1 - \log x}{x^2}$$

Now, $f'(x) = 0$

$\Rightarrow 1 - \log x = 0$

$\Rightarrow \log x = 1$
$\Rightarrow \log x = \log e$
$\Rightarrow x = e$

$$\text{Now, } f''(x) = \frac{x^2\left(-\frac{1}{x}\right) - (1 - \log x)(2x)}{x^4}$$
$$= \frac{-x - 2x(1 - \log x)}{x^4}$$
$$= \frac{-3 + 2\log x}{x^3}$$

$$\text{Now, } f''(e) = \frac{-3 + 2\log e}{e^3} = \frac{-3+2}{e^3} = \frac{-1}{e^3} < 0$$

Therefore, by second derivative test, f is the maximum at $x = e$.

Maxima and Minima 18.3 Q5

$$f(x) = \frac{4}{x+2} + x$$

$\therefore \quad f'(x) = \frac{-4}{(x+2)^2} + 1$

$$f''(x) = \frac{8}{(x+2)^3}$$

For maximum and minimum value,

$$f'(x) = 0$$

$\Rightarrow \quad \frac{-4}{(x+2)^2} + 1 = 0$

$\Rightarrow \quad (x+2)^2 = 4$

$\Rightarrow \quad x^2 + 4x = 0$

$\Rightarrow \quad x(x+4) = 0$

$$x = 0, -4$$

Now,

$$f''(0) = 1 > 0$$

$\therefore \quad x = 0$ is point of minima

$$f''(-4) = -1 < 0$$

$\therefore \quad x = -4$ is point of maxima

$\therefore \quad$ local max value $= f(-4) = -6$

local min value $= f(0) = 2.$

Maxima and Minima 18.3 Q6

We have,

$$y = \tan x - 2x$$

$$\therefore \quad y' = \sec^2 x - 2$$

$$y'' = 2\sec^2 x \tan x$$

For maximum and minimum value,

$$y' = 0$$

$$\Rightarrow \quad \sec^2 x = 2$$

$$\Rightarrow \quad \sec x = \pm\sqrt{2}$$

$$\Rightarrow \quad x = \frac{\pi}{4}, \frac{3\pi}{4}$$

$$\therefore \quad y''\left(\frac{\pi}{4}\right) = 4 > 0$$

$$\therefore \quad x = \frac{\pi}{4} \text{ is point of minima}$$

$$y''\left(\frac{3\pi}{4}\right) = -4 < 0$$

$$\therefore \quad x = \frac{3\pi}{4} \text{ is point of maxima}$$

Hence,

$$\text{max value} = f\left(\frac{3\pi}{4}\right) = -1 - \frac{3\pi}{2}$$

$$\text{min value} = f\left(\frac{\pi}{4}\right) = 1 - \frac{\pi}{2}.$$

Maxima and Minima 18.3 Q7

Consider the function

$f(x) = x^3 + ax^2 + bx + c$

Then $f'(x) = 3x^2 + 2ax + b$

It is given that $f(x)$ is maximum at $x = -1$.

$\therefore f'(-1) = 3(-1)^2 + 2a(-1) + b = 0$

$\Rightarrow f'(-1) = 3 - 2a + b = 0 \ldots(1)$

It is given that $f(x)$ is minimum at $x = 3$.

$\therefore f'(3) = 3(3)^2 + 2a(3) + b = 0$

$\Rightarrow f'(3) = 27 + 6a + b = 0 \ldots(2)$

Solving equations (1) and (2), we have,

$a = -3$ and $b = -9$

Since $f'(x)$ is independent of constant c, it can be any real number.

*********** END **********