



Factorisation of Algebraic Expressions Ex 5.4 Q9

Answer :

The given expression to be factorized is

$$\left(\frac{x}{2} + y + \frac{z}{3}\right)^3 + \left(\frac{x}{3} - \frac{2y}{3} + z\right)^3 + \left(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right)^3$$

Let $a = \left(\frac{x}{2} + y + \frac{z}{3}\right)$, $b = \left(\frac{x}{3} - \frac{2y}{3} + z\right)$ and $c = \left(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right)$. Then the given expression becomes

$$\left(\frac{x}{2} + y + \frac{z}{3}\right)^3 + \left(\frac{x}{3} - \frac{2y}{3} + z\right)^3 + \left(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right)^3 = a^3 + b^3 + c^3$$

Note that

$$\begin{aligned} a + b + c &= \left(\frac{x}{2} + y + \frac{z}{3}\right) + \left(\frac{x}{3} - \frac{2y}{3} + z\right) + \left(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right) \\ &= \frac{x}{2} + y + \frac{z}{3} + \frac{x}{3} - \frac{2y}{3} + z - \frac{5x}{6} - \frac{y}{3} - \frac{4z}{3} \\ &= 0 \end{aligned}$$

Recall the formula

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

When $a + b + c = 0$, this becomes

$$\begin{aligned} a^3 + b^3 + c^3 - 3abc &= 0.(a^2 + b^2 + c^2 - ab - bc - ca) \\ &= 0 \end{aligned}$$

$$\Rightarrow a^3 + b^3 + c^3 = 3abc$$

So, we have the new formula

$$a^3 + b^3 + c^3 = 3abc, \text{ when } a + b + c = 0.$$

Using the above formula, the given expression can be written as $a^3 + b^3 + c^3 = 3abc$

$$\text{Put } a = \left(\frac{x}{2} + y + \frac{z}{3}\right), b = \left(\frac{x}{3} - \frac{2y}{3} + z\right) \text{ and } c = \left(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right).$$

Then we have

$$\left(\frac{x}{2} + y + \frac{z}{3}\right)^3 + \left(\frac{x}{3} - \frac{2y}{3} + z\right)^3 + \left(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right)^3 = 3\left(\frac{x}{2} + y + \frac{z}{3}\right)\left(\frac{x}{3} - \frac{2y}{3} + z\right)\left(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right)$$

We cannot further factorize the expression.

So, the required factorization is of $\left(\frac{x}{2} + y + \frac{z}{3}\right)^3 + \left(\frac{x}{3} - \frac{2y}{3} + z\right)^3 + \left(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right)^3$ is

$$3\left(\frac{x}{2} + y + \frac{z}{3}\right)\left(\frac{x}{3} - \frac{2y}{3} + z\right)\left(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right)$$

Factorisation of Algebraic Expressions Ex 5.4 Q10

Answer :

The given expression to be factorized is

$$(a - 3b)^3 + (3b - c)^3 + (c - a)^3$$

Let $x = (a - 3b)$, $y = (3b - c)$ and $z = (c - a)$. Then the given expression becomes

$$(a - 3b)^3 + (3b - c)^3 + (c - a)^3 = x^3 + y^3 + z^3$$

Note that

$$\begin{aligned} x + y + z &= (a - 3b) + (3b - c) + (c - a) \\ &= a - 3b + 3b - c + c - a \\ &= 0 \end{aligned}$$

Recall the formula $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

When $a + b + c = 0$, this becomes

$$\begin{aligned} a^3 + b^3 + c^3 - 3abc &= 0.(a^2 + b^2 + c^2 - ab - bc - ca) \\ &= 0 \end{aligned}$$

$$\Rightarrow a^3 + b^3 + c^3 = 3abc$$

So, we have the new formula

$a^3 + b^3 + c^3 = 3abc$, when $a + b + c = 0$.

Using the above formula, the given expression can be written as

$$x^3 + y^3 + z^3 = 3xyz$$

Put $x = (a - 3b)$, $y = (3b - c)$ and $z = (c - a)$. Then we have

$$(a - 3b)^3 + (3b - c)^3 + (c - a)^3 = 3(a - 3b)(3b - c)(c - a)$$

We cannot further factorize the expression.

So, the required factorization is of $(a - 3b)^3 + (3b - c)^3 + (c - a)^3$ is $\boxed{3(a - 3b)(3b - c)(c - a)}$.

Factorisation of Algebraic Expressions Ex 5.4 Q11

Answer :

The given expression to be factorized is

$$2\sqrt{2}a^3 + 3\sqrt{3}b^3 + c^3 - 3\sqrt{6}abc$$

This can be written in the form

$$2\sqrt{2}a^3 + 3\sqrt{3}b^3 + c^3 - 3\sqrt{6}abc = (\sqrt{2}a)^3 + (\sqrt{3}b)^3 + (c)^3 - 3.(\sqrt{2}a).(\sqrt{3}b).(c)$$

Recall the formula

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

Using the above formula, we have

$$2\sqrt{2}a^3 + 3\sqrt{3}b^3 + c^3 - 3\sqrt{6}abc$$

$$= \{(\sqrt{2}a) + (\sqrt{3}b) + c\} \{(\sqrt{2}a)^2 + (\sqrt{3}b)^2 + (c)^2 - (\sqrt{2}a).(\sqrt{3}b) - (\sqrt{3}b).(c) - (c).(\sqrt{2}a)\}$$

$$= (\sqrt{2}a + \sqrt{3}b + c)(2a^2 + 3b^2 + c^2 - \sqrt{6}ab - \sqrt{3}bc - \sqrt{2}ca)$$

We cannot further factorize the expression.

So, the required factorization is of $2\sqrt{2}a^3 + 3\sqrt{3}b^3 + c^3 - 3\sqrt{6}abc$ is

$$\boxed{(\sqrt{2}a + \sqrt{3}b + c)(2a^2 + 3b^2 + c^2 - \sqrt{6}ab - \sqrt{3}bc - \sqrt{2}ca)}.$$

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