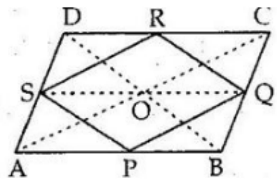
Exercise 10A

Question 21:

Given: ABCD is a parallelogram and P,Q,R and S are the midpoints of AB,BC,CD and DA respectively.



To Prove: PQRS is a parallelogram and ar(∥gmPQRS) $= \frac{1}{2}$ar (∥gm ABCD)

Construction: Join AC,BD and SQ.

Proof: As S and R are the midpoints of AD and CD. So, in ΔADC,

SR ∥ AC [By mid point theorem]

Also, as P and Q are the midpoints of AB and BC. So, in ΔABC,

PQ ∥ AC

$\therefore$ PQ ∥ AC ∥ SR

$\therefore$ PQ ∥ SR

Similarly, we can prove SP ∥ RQ.

Thus PQRS is a parallelogram as its opposite sides are parallel since diagonals of a parallelogram bisect each other.

So in ΔABD,

O is the midpoint of AC and S is the midpoint of AD.

$\therefore$ OS ∥ AB [By midpoint theorem]

Similarly in ΔABC, we can prove that,

OQ ∥AB

i.e. SQ ∥ AB

Thus, ABQS is a parallelogram.

Now, ar(ΔSPQ)$=\frac{1}{2}$ar (∥gm ABQS) ....(i)

[$\because$ ΔSPQ and ∥gm ABQS have the same base and lie between same parallel lines]

Similarly, we can prove that;

ar(ΔSRQ)$=\frac{1}{2}$ar (∥gm SQCD) ....(ii)

Adding (i) and (ii) we get

ar(ΔSPQ) + ar(ΔSRQ) $= \frac{1}{2}$[ar(∥gm ABQS) + ar(∥gm SQCD)]

$\therefore$ ar(∥gmPQRS) $= \frac{1}{2}$ar (∥gm ABCD)

*********** END **********