Trigonometric Ratios of Compound Angles Ex 7.2 Q1

Let $f(\theta) = 12\sin\theta - 5\cos\theta$

We know that

$$-\sqrt{(12)^2 + (-5)^2} \le f(\theta) \le \sqrt{(12)^2 + (-5)^2}$$

$$\Rightarrow \quad -\sqrt{144 + 25} \le f(\theta) \le \sqrt{144 + 25}$$

$$\Rightarrow \quad -\sqrt{169} \le f(\theta) \le \sqrt{169}$$

$$\Rightarrow \quad -13 \le f(\theta) \le 13$$

Hence, minimum and maximum values of $12\sin\theta - 5\cos\theta$ are -13 and 13 respectively.

Let $f(\theta) = 12\cos\theta + 5\sin\theta + 4$

We know that

$$-\sqrt{(12)^2 + (5)^2} \le 12\cos\theta + 5\sin\theta \le \sqrt{(12)^2 + (-5)^2}$$

$$\Rightarrow \quad -\sqrt{144 + 25} \le 12\cos\theta + 5\sin\theta \le \sqrt{144 + 25}$$

$$\Rightarrow \quad -\sqrt{169} \le 12\cos\theta + 5\sin\theta \le \sqrt{169}$$

$$\Rightarrow \quad -13 \le 12\cos\theta + 5\sin\theta \le 13$$

$$\Rightarrow \quad -13 + 4 \le 12\cos\theta + 5\sin\theta + 4 \le 13 + 4$$

$$\Rightarrow \quad -9 \le 12\cos\theta + 5\sin\theta + 4 \le 17$$

$$\Rightarrow \quad -9 \le f(\theta) \le 17$$

Hnece, minimum and maximum values of $12\cos\theta + 5\sin\theta + 4$ are -9 and 17 respectively.

Let $f(\theta) = 5\cos\theta + 3\sin\left(\frac{\pi}{6} - \theta\right) + 4$

Then, $f(\theta) = 5\cos\theta + 3\left[\sin\frac{\pi}{6}\cos\theta - \cos\frac{\pi}{6}\sin\theta\right] + 4$

$$= 5\cos\theta + 3\left[\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta\right] + 4$$

$$= 5\cos\theta + \frac{3}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 4$$

$$= \left(5 + \frac{3}{2}\right)\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 4$$

$$= \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 4$$

$$= \frac{13}{2}\cos\theta - \left(\frac{-3\sqrt{3}}{2}\right)\sin\theta + 4$$

We know that

$$-\sqrt{\left(\frac{13}{2}\right)^2 + \left(\frac{-3\sqrt{3}}{2}\right)^2} \le \frac{13}{2}\cos\theta - \left(\frac{-3\sqrt{3}}{2}\right)\sin\theta \le \sqrt{\left(\frac{13}{2}\right)^2 + \left(\frac{-3\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow \quad -\sqrt{\frac{169}{4} + \frac{27}{4}} \le \frac{13}{2}\cos\theta - \left(\frac{-3\sqrt{3}}{2}\right)\sin\theta \le \sqrt{\frac{169}{4} + \frac{27}{4}}$$

$$\Rightarrow \quad -\sqrt{\frac{196}{4}} \le \frac{13}{2}\cos\theta - \left(\frac{-3\sqrt{3}}{2}\right)\sin\theta \le \sqrt{\frac{196}{4}}$$

$$\Rightarrow \quad -\frac{14}{2} \le \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta \le \frac{14}{2}$$

$$\Rightarrow \quad -7 \le \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta \le 7$$

$$\Rightarrow \quad -7 + 4 \le \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 4 \le 7 + 4$$

$$\Rightarrow \quad -3 \le \frac{13}{2}\cos\theta - \left(\frac{-3\sqrt{3}}{2}\right)\sin\theta + 4 \le 11$$

$$\Rightarrow \quad -3 \le f(\theta) \le 11$$

Let $f(\theta) = \sin\theta - \cos\theta + 1$. Then,

$$f(\theta) = \sin\theta + (-1)\cos\theta + 1$$

$$= (-1)\cos\theta + \sin\theta + 1$$

We know that

$$-\sqrt{(-1)^2 + (1)^2} \le -\cos\theta + \sin\theta \le \sqrt{(-1)^2 + (1)^2}$$

$$\Rightarrow \quad -\sqrt{1+1} \le -\cos\theta + \sin\theta \le \sqrt{1+1}$$

$$\Rightarrow \quad -\sqrt{2} \le -\cos\theta + \sin\theta \le \sqrt{2}$$

$$\Rightarrow \quad -\sqrt{2} + 1 \le -\cos\theta + \sin\theta + 1 \le \sqrt{2} + 1$$

$$\Rightarrow \quad 1 - \sqrt{2} \le f(\theta) \le 1 + \sqrt{2}$$

Hence, minimum and maximum values of $\sin\theta - \cos\theta + 1$ are $1 - \sqrt{2}$ and $1 + \sqrt{2}$ respectively.

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