Exercise 6.3 : Solutions of Questions on Page Number : **211**

Q1 : Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at $x = 4$.

Answer :

The given curve is $y = 3x^4 - 4x$.

Then, the slope of the tangent to the given curve at $x = 4$ is given by,

$$\left.\frac{dy}{dx}\right]_{x=4} = 12x^3 - 4\Big]_{x=4} = 12(4)^3 - 4 = 12(64) - 4 = 764$$

Answer needs Correction? Click Here

Q2 : Find the slope of the tangent to the curve $y = \frac{x-1}{x-2}$, $x \neq 2$ at $x = 10$.

Answer :

The given curve is $y = \frac{x-1}{x-2}$.

$$\therefore \frac{dy}{dx} = \frac{(x-2)(1)-(x-1)(1)}{(x-2)^2}$$

$$= \frac{x-2-x+1}{(x-2)^2} = \frac{-1}{(x-2)^2}$$

Thus, the slope of the tangent at $x = 10$ is given by,

$$\left.\frac{dy}{dx}\right]_{x=10} = \left.\frac{-1}{(x-2)^2}\right]_{x=10} = \frac{-1}{(10-2)^2} = \frac{-1}{64}$$

Hence, the slope of the tangent at $x = 10$ is $\frac{-1}{64}$.

Answer needs Correction? Click Here

Q3 : Find the slope of the tangent to curve $y = x^3 - x + 1$ at the point whose x-coordinate is 2.

Answer :

The given curve is $y = x^3 - x + 1$.

$$\therefore \frac{dy}{dx} = 3x^2 - 1$$

The slope of the tangent to a curve at (x_0, y_0) is $\left.\frac{dy}{dx}\right]_{(x_0, y_0)}$.

It is given that $x_0 = 2$.

Hence, the slope of the tangent at the point where the x-coordinate is 2 is given by,

$$\left.\frac{dy}{dx}\right]_{x=2} = 3x^2 - 1\Big]_{x=2} = 3(2)^2 - 1 = 12 - 1 = 11$$

Answer needs Correction? Click Here

Q4 : Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose x-coordinate is 3.

Answer :

The given curve is $y = x^3 - 3x + 2$.

$$\therefore \frac{dy}{dx} = 3x^2 - 3$$

The slope of the tangent to a curve at (x_0, y_0) is $\left.\frac{dy}{dx}\right]_{(x_0, y_0)}$.

Hence, the slope of the tangent at the point where the x-coordinate is 3 is given by,

$$\left.\frac{dy}{dx}\right]_{x=3} = 3x^2 - 3\Big]_{x=3} = 3(3)^2 - 3 = 27 - 3 = 24$$

Answer needs Correction? Click Here

Q5 : Find the slope of the normal to the curve $x = a\cos^3\theta$, $y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$.

Answer :

It is given that $x = a\cos^3\theta$ and $y = a\sin^3\theta$.

$$\therefore \frac{dx}{d\theta} = 3a\cos^2\theta(-\sin\theta) = -3a\cos^2\theta\sin\theta$$
$$\frac{dy}{d\theta} = 3a\sin^2\theta(\cos\theta)$$
$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta} = -\frac{\sin\theta}{\cos\theta} = -\tan\theta$$

Therefore, the slope of the tangent at $\theta = \frac{\pi}{4}$ is given by,

$$\left.\frac{dy}{dx}\right]_{\theta=\frac{\pi}{4}} = -\tan\theta\Big]_{\theta=\frac{\pi}{4}} = -\tan\frac{\pi}{4} = -1$$

Hence, the slope of the normal at $\theta = \frac{\pi}{4}$ is given by,

$$\frac{1}{\text{slope of the tangent at } \theta = \frac{\pi}{4}} = \frac{-1}{-1} = 1$$

Answer needs Correction? Click Here

Q6 : Find the slope of the normal to the curve $x = 1 - a\sin\theta$, $y = b\cos^2\theta$ at $\theta = \frac{\pi}{2}$.

Answer :

It is given that $x = 1 - a\sin\theta$ and $y = b\cos^2\theta$.

$$\therefore \frac{dx}{d\theta} = -a\cos\theta \text{ and } \frac{dy}{d\theta} = 2b\cos\theta(-\sin\theta) = -2b\sin\theta\cos\theta$$
$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{-2b\sin\theta\cos\theta}{-a\cos\theta} = \frac{2b}{a}\sin\theta$$

Therefore, the slope of the tangent at $\theta = \frac{\pi}{2}$ is given by,

$$\left.\frac{dy}{dx}\right]_{\theta=\frac{\pi}{2}} = \frac{2b}{a}\sin\theta\Big]_{\theta=\frac{\pi}{2}} = \frac{2b}{a}\sin\frac{\pi}{2} = \frac{2b}{a}$$

Hence, the slope of the normal at $\theta = \frac{\pi}{2}$ is given by,

$$\frac{1}{\text{slope of the tangent at } \theta = \frac{\pi}{4}} = \frac{-1}{\left(\frac{2b}{a}\right)} = -\frac{a}{2b}$$

Answer needs Correction? Click Here

Q7 : Find points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$ is parallel to the x-axis.

Answer :

The equation of the given curve is $y = x^3 - 3x^2 - 9x + 7$.

$$\therefore \frac{dy}{dx} = 3x^2 - 6x - 9$$

Now, the tangent is parallel to the x-axis if the slope of the tangent is zero.

$$\therefore 3x^2 - 6x - 9 = 0 \Rightarrow x^2 - 2x - 3 = 0$$
$$\Rightarrow (x-3)(x+1) = 0$$
$$\Rightarrow x = 3 \text{ or } x = -1$$

When $x = 3$, $y = (3)^3 - 3(3)^2 - 9(3) + 7 = 27 - 27 - 27 + 7 = -20$.

When $x = -1$, $y = (-1)^3 - 3(-1)^2 - 9(-1) + 7 = -1 - 3 + 9 + 7 = 12$.

Hence, the points at which the tangent is parallel to the x-axis are (3, - 20) and (- 1, 12).

Answer needs Correction? Click Here

Q8 : Find a point on the curve $y = (x - 2)^2$ at which the tangent is parallel to the chord joining the points (2, 0) and (4, 4).

Answer :

If a tangent is parallel to the chord joining the points (2, 0) and (4, 4), then the slope of the tangent = the slope of the chord.

The slope of the chord is $\frac{4-0}{4-2} = \frac{4}{2} = 2$.

Now, the slope of the tangent to the given curve at a point (x, y) is given by,

$$\frac{dy}{dx} = 2(x-2)$$

Since the slope of the tangent = slope of the chord, we have:

$$2(x-2) = 2$$
$$\Rightarrow x - 2 = 1 \Rightarrow x = 3$$

When $x = 3$, $y = (3-2)^2 = 1$.

Hence, the required point is (3, 1).

Answer needs Correction? Click Here

Q9 : Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is $y = x - 11$.

Answer :

The equation of the given curve is $y = x^3 - 11x + 5$.

The equation of the tangent to the given curve is given as $y = x - 11$ (which is of the form $y = mx + c$).

$\therefore$Slope of the tangent = 1

Now, the slope of the tangent to the given curve at the point (x, y) is given by, $\frac{dy}{dx} = 3x^2 - 11$

Then, we have:

$$3x^2 - 11 = 1$$
$$\Rightarrow 3x^2 = 12$$
$$\Rightarrow x^2 = 4$$
$$\Rightarrow x = \pm 2$$

When $x = 2$, $y = (2)^3 - 11(2) + 5 = 8 - 22 + 5 = -9$.

When $x = -2$, $y = (-2)^3 - 11(-2) + 5 = -8 + 22 + 5 = 19$.

Hence, the required points are (2, - 9) and (- 2, 19).
But, both these points should satisfy the equation of the tangent as there would be point of contact between tangent and the curve.
$\therefore$ (2, - 9) is the required point as (- 2, 19) is not satisfying the given equation of tangent.

Answer needs Correction? Click Here

Q10 : Find the equation of all lines having slope - 1 that are tangents to the curve $y = \frac{1}{x-1}, x \neq 1$.

Answer :

The equation of the given curve is $y = \frac{1}{x-1}, x \neq 1$.

The slope of the tangents to the given curve at any point (x, y) is given by,

$$\frac{dy}{dx} = \frac{-1}{(x-1)^2}$$

If the slope of the tangent is - 1, then we have:

$$\frac{-1}{(x-1)^2} = -1$$
$$\Rightarrow (x-1)^2 = 1$$
$$\Rightarrow x - 1 = \pm 1$$
$$\Rightarrow x = 2,\ 0$$

When $x = 0$, $y = -1$ and when $x = 2$, $y = 1$.

Thus, there are two tangents to the given curve having slope - 1. These are passing through the points (0, - 1) and (2, 1).

$\therefore$The equation of the tangent through (0, - 1) is given by,

$$y - (-1) = -1(x - 0)$$
$$\Rightarrow y + 1 = -x$$
$$\Rightarrow y + x + 1 = 0$$

$\therefore$The equation of the tangent through (2, 1) is given by,

$y - 1 = -1(x - 2)$

$\Rightarrow y - 1 = -x + 2$

$\Rightarrow y + x - 3 = 0$

Hence, the equations of the required lines are $y + x + 1 = 0$ and $y + x - 3 = 0$.

Answer needs Correction? Click Here

Q11 : Find the equation of all lines having slope 2 which are tangents to the curve $y = \frac{1}{x-3}, x \neq 3$.

Answer :

The equation of the given curve is $y = \frac{1}{x-3}, x \neq 3$.

The slope of the tangent to the given curve at any point (x, y) is given by,

$$\frac{dy}{dx} = \frac{-1}{(x-3)^2}$$

If the slope of the tangent is 2, then we have:

$$\frac{-1}{(x-3)^2} = 2$$

$$\Rightarrow 2(x-3)^2 = -1$$
$$\Rightarrow (x-3)^2 = \frac{-1}{2}$$

This is not possible since the L.H.S. is positive while the R.H.S. is negative.

This is not possible since the L.H.S. is positive while the R.H.S. is negative.

Hence, there is no tangent to the given curve having slope 2.

Answer needs Correction? **Click Here**

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