Indefinite Integrals Ex 19.16 Q11

Let $I = \int \frac{1}{x\left(x^6+1\right)}dx$

$$= \int \frac{x^5}{x^6\left(x^6+1\right)}dx$$

Let $x^6 = t$

$$\Rightarrow \quad 6x^5\,dx = dt$$

$$\Rightarrow \quad x^5\,dx = \frac{dt}{6}$$

$$I = \frac{1}{6}\int \frac{dt}{t\left(t+1\right)}$$

$$= \frac{1}{6}\int \frac{dt}{t^2+t}$$

$$= \frac{1}{6}\int \frac{dt}{t^2+2t\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}$$

$$= \frac{1}{6}\int \frac{dt}{\left(t+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}$$

Let $t+\frac{1}{2} = u$

$$\Rightarrow \quad dt = du$$

$$I = \frac{1}{6}\int \frac{du}{u^2-\left(\frac{1}{2}\right)^2}$$

$$= \frac{1}{6}\times\frac{1}{2\left(\frac{1}{2}\right)}\log\left|\frac{u-\frac{1}{2}}{u+\frac{1}{2}}\right|+c \qquad \left[\text{Since } \int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c\right]$$

$$I = \frac{1}{6}\log\left|\frac{t+\frac{1}{2}-\frac{1}{2}}{t+\frac{1}{2}+\frac{1}{2}}\right|+c$$

$$I = \frac{1}{6}\log\left|\frac{x^6}{x^6+1}\right|+c$$

Indefinite Integrals Ex 19.16 Q12

Let $I = \int \frac{x}{x^4 - x^2 + 1} dx$

Let $x^2 = t$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad x\,dx = \frac{dt}{2}$

so, $\quad I = \frac{1}{2} \int \frac{dt}{t^2 - t + 1}$

$$= \frac{1}{2} \int \frac{dt}{t^2 - 2t \times \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1}$$

$$= \frac{1}{2} \int \frac{dt}{\left(t - \frac{1}{2}\right)^2 + \left(\frac{3}{4}\right)}$$

Let $\quad t - \frac{1}{2} = u$

$\Rightarrow \quad dt = du$

$$I = \frac{1}{2} \int \frac{du}{u^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1}{2} \times \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} \tan^{-1} \left(\frac{u}{\frac{\sqrt{3}}{2}} \right) + c \qquad \left[\text{Since} \quad \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a}\right) + c \right]$$

$$I = \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{t - \frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) + c$$

$$I = \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{2x^2 - 1}{\sqrt{3}} \right) + c$$

Indefinite Integrals Ex 19.16 Q13

Let $I = \int \frac{x}{3x^4 - 18x^2 + 11} dx$

$$= \frac{1}{3} \int \frac{x}{x^4 - 6x^2 + \frac{11}{3}} dx$$

Let $x^2 = t$

$\Rightarrow \quad 2x\, dx = dt$

$\Rightarrow \quad x\, dx = \frac{dt}{2}$

$$I = \frac{1}{3} \times \frac{1}{2} \int \frac{dt}{t^2 - 6t + \frac{11}{3}}$$

$$= \frac{1}{6} \int \frac{dt}{t^2 - 2t(3) + (3)^2 - (3)^2 + \frac{11}{3}}$$

$$= \frac{1}{6} \int \frac{dt}{(t-3)^2 - \left(\frac{16}{3}\right)}$$

Let $t-3 = u$

$\Rightarrow \quad dt = du$

$$I = \frac{1}{6} \int \frac{du}{u^2 - \left(\frac{4}{\sqrt{3}}\right)^2}$$

$$= \frac{1}{6} \times \frac{1}{2\left(\frac{4}{\sqrt{3}}\right)} \log \left| \frac{u - \frac{4}{\sqrt{3}}}{u + \frac{4}{\sqrt{3}}} \right| + c \quad \left[\text{Since} \quad \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c \right]$$

$$I = \frac{\sqrt{3}}{48} \log \left| \frac{t - 3 - \frac{4}{\sqrt{3}}}{t - 3 + \frac{4}{\sqrt{3}}} \right| + c$$

$$I = \frac{\sqrt{3}}{48} \log \left| \frac{x^2 - 3 - \frac{4}{\sqrt{3}}}{x^2 - 3 + \frac{4}{\sqrt{3}}} \right| + c$$

Indefinite Integrals Ex 19.16 Q14

To evaluate the following integral follow the steps:

Let $e^x = t$ therefore $e^x dx = dt$

Now

$$\int \frac{e^x}{(1+e^x)(2+e^x)} dx = \int \frac{dt}{(1+t)(2+t)}$$

$$= \int \frac{dt}{(1+t)} - \int \frac{dt}{(2+t)}$$

$$= \ln|1+t| - \ln|2+t| + c$$

$$= \ln\left|\frac{1+t}{2+t}\right| + c$$

$$= \ln\left|\frac{1+e^x}{2+e^x}\right| + c$$

*********** END **********