



Question 9:

Find x and y , if $2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

Answer

$$\begin{aligned} 2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} &= \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} &= \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} &= \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix} \end{aligned}$$

Comparing the corresponding elements of these two matrices, we have:

$$2 + y = 5$$

$$\Rightarrow y = 3$$

$$2x + 2 = 8$$

$$\Rightarrow x = 3$$

$$\therefore x = 3 \text{ and } y = 3$$

Question 10:

Solve the equation for x , y , z and t if

$$2 \begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3 \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

Answer

$$\begin{aligned} 2 \begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} &= 3 \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} &= \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} 2x+3 & 2z-3 \\ 2y & 2t+6 \end{bmatrix} &= \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix} \end{aligned}$$

Comparing the corresponding elements of these two matrices, we get:

$$2x + 3 = 9$$

$$\Rightarrow 2x = 6$$

$$\Rightarrow x = 3$$

$$2y = 12$$

$$\Rightarrow y = 6$$

$$2z - 3 = 15$$

$$\Rightarrow 2z = 18$$

$$\Rightarrow z = 9$$

$$2t + 6 = 18$$

$$\Rightarrow 2t = 12$$

$$\Rightarrow t = 6$$

$$\therefore x = 3, y = 6, z = 9, \text{ and } t = 6$$

Question 11:

If $x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, find values of x and y .

Answer

$$\begin{aligned} x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} &= \begin{bmatrix} 10 \\ 5 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} &= \begin{bmatrix} 10 \\ 5 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix} &= \begin{bmatrix} 10 \\ 5 \end{bmatrix} \end{aligned}$$

Comparing the corresponding elements of these two matrices, we get:

$$2x - y = 10 \text{ and } 3x + y = 5$$

Adding these two equations, we have:

$$5x = 15$$

$$\Rightarrow x = 3$$

$$\text{Now, } 3x + y = 5$$

$$\Rightarrow y = 5 - 3x$$

$$\Rightarrow y = 5 - 9 = -4$$

$$\therefore x = 3 \text{ and } y = -4$$

Question 12:

Given $3 \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$, find the values of x , y , z and w .

Answer

$$3 \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we get:

$$3x = x + 4$$

$$\Rightarrow 2x = 4$$

$$\Rightarrow x = 2$$

$$3y = 6 + x + y$$

$$\Rightarrow 2y = 6 + x = 6 + 2 = 8$$

$$\Rightarrow y = 4$$

$$3w = 2w + 3$$

$$\Rightarrow w = 3$$

$$3z = -1 + z + w$$

$$\Rightarrow 2z = -1 + w = -1 + 3 = 2$$

$$\Rightarrow z = 1$$

$$\therefore x = 2, y = 4, z = 1, \text{ and } w = 3$$

Question 13:

$F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$
If $F(x)F(y) = F(x+y)$, show that

Answer

$$F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}, F(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F(x+y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F(x)F(y)$$

$$= \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos x \cos y - \sin x \sin y + 0 & -\cos x \sin y - \sin x \cos y + 0 & 0 \\ \sin x \cos y + \cos x \sin y + 0 & -\sin x \sin y + \cos x \cos y + 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= F(x+y)$$

$$\therefore F(x)F(y) = F(x+y)$$

Question 14:

Show that

$$(i) \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

$$(ii) \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Answer

(i)

$$\begin{aligned} & \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 5(2)-1(3) & 5(1)-1(4) \\ 6(2)+7(3) & 6(1)+7(4) \end{bmatrix} \\ &= \begin{bmatrix} 10-3 & 5-4 \\ 12+21 & 6+28 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} & \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \\ &= \begin{bmatrix} 2(5)+1(6) & 2(-1)+1(7) \\ 3(5)+4(6) & 3(-1)+4(7) \end{bmatrix} \\ &= \begin{bmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{bmatrix} = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix} \end{aligned}$$

$$\therefore \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

(ii)

$$\begin{aligned} & \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 1(-1)+2(0)+3(2) & 1(1)+2(-1)+3(3) & 1(0)+2(1)+3(4) \\ 0(-1)+1(0)+0(2) & 0(1)+1(-1)+0(3) & 0(0)+1(1)+0(4) \\ 1(-1)+1(0)+0(2) & 1(1)+1(-1)+0(3) & 1(0)+1(1)+0(4) \end{bmatrix} \\ &= \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} & \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} -1(1)+1(0)+0(1) & -1(2)+1(1)+0(1) & -1(3)+1(0)+0(0) \\ 0(1)+(-1)(0)+1(1) & 0(2)+(-1)(1)+1(1) & 0(3)+(-1)(0)+1(0) \\ 2(1)+3(0)+4(1) & 2(2)+3(1)+4(1) & 2(3)+3(0)+4(0) \end{bmatrix} \\ &= \begin{bmatrix} -1 & -1 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix} \end{aligned}$$

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

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