Binomial Theorem Ex 18.2 Q16(vi)

$$\left(x-\frac{1}{x^2}\right)^{3n}$$

$$T_{r+1}=(-1)^r\,{}^{3n}C_r x^{3n-r}\left(\frac{1}{x^2}\right)^r$$

$$=(-1)^r\,{}^{3n}C_r x^{3n-r-2r}$$

Independent of $x \Rightarrow x^0$

$$x^{3n-3r}=x^0 \Rightarrow r=n$$

$$=(-1)^n\,{}^{3n}C_r$$

Binomial Theorem Ex 18.2 Q16(vii)

We have,

$$\left(\frac{1}{2}x^{\frac{1}{3}}+x^{\frac{-1}{5}}\right)^{8}$$

Let $(r+1)^{th}$ term be independent of x.

$$\therefore \quad T_{r+1} = {}^{8}C_{r}\left(\frac{1}{2}x^{\frac{1}{3}}\right)^{8-r}\left(x^{\frac{-1}{5}}\right)^{r}$$

$$= {}^{8}C_{r}\left(\frac{1}{2}\right)^{8-r}\times\left(x^{\frac{1}{3}}\right)^{8-r}\times\left(\frac{1}{x^{\frac{1}{5}}}\right)^{r}$$

$$= {}^{8}C_{r}\left(\frac{1}{2}\right)^{8-r}\times(x)^{\frac{8-r}{3}}\times\left(\frac{1}{x^{\frac{1}{5}}}\right)$$

$$= {}^{8}C_{r}\left(\frac{1}{2}\right)^{8-r}\times(x)^{\frac{8-r}{3}-\frac{r}{5}}$$

$$= {}^{8}C_{r}\left(\frac{1}{2}\right)^{8-r}\times(x)^{\frac{40-5r-3r}{15}}$$

$$= {}^{8}C_{r}\left(\frac{1}{2}\right)^{8-r}\times(x)^{\frac{40-8r}{15}}$$

If it is independent of x, we must have

$$\frac{40-8r}{15} = 0$$

$\Rightarrow \quad 8r = 40$

$\Rightarrow \quad r = 5$

$\therefore$ The term independet of $x = T_6$

Now,

$$T_6 = {}^{8}C_{r}\left(\frac{1}{2}x^{\frac{1}{3}}\right)^{8-5}\left(x^{\frac{-1}{5}}\right)^{5}$$

$$= 56\times\left(\frac{1}{2}\right)^{3}$$

$$= 56\times\frac{1}{8}$$

$$= 7$$

Hence, required term = 7

Binomial Theorem Ex 18.2 Q16(viii)

$$\left(1+x+2x^3\right)\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$$

$$= \left(1+x+2x^3\right)\left[\left(\frac{3}{2}x^2\right)^9 - {}^9C_1\left(\frac{3}{2}x^2\right)^8\frac{1}{3x}\ldots\ldots + {}^9C_6\left(\frac{3}{2}x^2\right)^3\left(\frac{1}{3x}\right)^6 - {}^9C_7\left(\frac{3}{2}x^2\right)^2\left(\frac{1}{3x}\right)^7\right]$$

In the second bracket, we have to search the term so x° and $\frac{1}{x^3}$ which when multiplying by 1 and $2x^3$ is first bracket will give the term in dependent of x. The term containing $\frac{1}{x}$ will not occur is second bracket.

The term independent of x

$$= 1\left[{}^9C_6\,\frac{3^3}{2^3}\times\frac{1}{3^6}\right] - 2x^3\left[{}^9C_7\,\frac{3^3}{2^3}\times\frac{1}{3^7}\times\frac{1}{x^3}\right]$$

$$= \left[\frac{9\times8\times7}{1\times2\times3}\times\frac{1}{8\times27}\right] - 2\left[\frac{9\times8}{1\times2} - \frac{1}{4\times243}\right]$$

$$= \frac{7}{18} - \frac{2}{27}$$

$$= \frac{17}{54}$$

Required term $= \frac{17}{54}$

*********** END **********