Indefinite Integrals Ex 19.9 Q50

Let $\quad I = \int \frac{e^{m\sin^{-1}x}}{\sqrt{1-x^2}}dx - - - - - (i)$

Let $\quad m\sin^{-1}x = t \quad$ then,

$$d\left(m\sin^{-1}x\right) = dt$$

$$\Rightarrow \quad m\frac{1}{\sqrt{1-x^2}}dx = dt$$

$$\Rightarrow \quad \frac{dx}{\sqrt{1-x^2}} = \frac{dt}{m}$$

Putting $m\sin^{-1}x = t$ and $\frac{dx}{\sqrt{1-x^2}} = \frac{dt}{m}$ in equation (i),

we get

$$I = \int e^t \frac{dt}{m}$$

$$= \frac{1}{m}e^t + c$$

$$= \frac{1}{m}e^{m\sin^{-1}x} + c$$

$$\therefore \quad I = \frac{1}{m}e^{m\sin^{-1}x} + c$$

Indefinite Integrals Ex 19.9 Q51

Let $\sqrt{x} = t$

$$\Rightarrow \frac{1}{2\sqrt{x}}dx = dt$$

$$\Rightarrow \int \frac{\cos\sqrt{x}}{\sqrt{x}}dx = 2\int \cos t\, dt$$

$$= 2\sin t + C$$

$$= 2\sin\sqrt{x} + C$$

Indefinite Integrals Ex 19.9 Q52

Let $\quad I = \int \frac{\sin\left(\tan^{-1} x\right)}{1+x^2} dx \text{ - - - - - } (i)$

Let $\quad \tan^{-1} x = t \quad$ then,

$d\left(\tan^{-1} x\right) = dt$

$\Rightarrow \quad \frac{1}{1+x^2} dx = dt$

Putting $\tan^{-1} x = t$ and $\frac{dx}{1+x^2} = dt$ in equation (i),

we get

$$I = \int \sin t \, dt$$
$$= -\cos t + c$$
$$= -\cos\left(\tan^{-1} x\right) + c$$

$\therefore \quad I = -\cos\left(\tan^{-1} x\right) + c$

Indefinite Integrals Ex 19.9 Q53

Let $\quad I = \int \frac{\sin(\log x)}{x} dx \text{ - - - - - } (i)$

Let $\quad \log x = t \quad$ then,

$d(\log x) = dt$

$\Rightarrow \quad \frac{1}{x} dx = dt$

Putting $\log x = t$ and $\frac{1}{x} dx = dt$ in equation (i),

we get

$$I = \int \sin t \, dt$$
$$= -\cos t + c$$
$$= -\cos(\log x) + c$$

$\therefore \quad I = -\cos(\log x) + c$

Indefinite Integrals Ex 19.9 Q54

Let $\tan^{-1}x = t$

Differentiating the above function with respect to, w, we have,

$$\frac{1}{1+x^2}dx = dt$$

$$\Rightarrow \int \frac{e^{m\tan^{-1}x}}{1+x^2} = \int e^{mt} \times dt$$

$$\Rightarrow \int \frac{e^{m\tan^{-1}x}}{1+x^2} = \frac{e^{mt}}{m}$$

Resubstituting the value of t in the above solution, we have,

$$\Rightarrow \int \frac{e^{m\tan^{-1}x}}{1+x^2} = \frac{e^{m\tan^{-1}x}}{m} + C$$

*********** END **********