

Higher Order Derivatives Ex 12.1 Q19

x = sint; y = sinpt

differentiating both w.r.t. t

$$\Rightarrow \frac{dy}{dt} = \cos t \dots (1); \quad \frac{dy}{dt} = P \cos pt \dots (2)$$

dividing (2) by (1)

$$\Rightarrow \frac{dy}{dx} = P \frac{\cos px}{\cos t}$$

differentiating w.r.t.x

$$\Rightarrow \frac{d\left(\frac{dy}{dx}\right)}{dt} = P\left\{\frac{P\cos t\left(-\sin pt\right) - \left(\cos pt\right)\left(-\sin t\right)}{\cos^2 t}\right\}$$
$$= P\left\{\frac{\sin t\cos pt - p\cos t\sin pt\left(-\sin t\right)}{\cos^2 t}\right\}......(3)$$

$$\Rightarrow$$
 dividing (3) by (1)

$$\Rightarrow \qquad \frac{d^2y}{dx^2} = P\left\{\frac{\sin t \cos pt - p \cos t \sin pt}{\cos^3 t}\right\} = \left\{\frac{\tan t \cos t - p \sin pt}{\cos^2 t}\right\}$$

$$\sin^2 t + \cos^2 t = 1$$

$$\Rightarrow 1 - \sin^2 t = \cos^2 t$$

$$\Rightarrow 1 - x^2 = \cos^2 t$$

$$\Rightarrow$$
 $1-x^2=\cos^2 t$

$$\Rightarrow \frac{d^2y}{dx^2} = P\left\{\frac{\tan t \cos pt - p \sin pt}{1 - x^2}\right\}$$

$$\Rightarrow \qquad \left(1 - X^2\right) \frac{d^2y}{dx^2} = p \frac{\sin t \cos pt}{\cos t} - p^2 \sin pt = \frac{dy}{dx} - p^2 y$$

$$\Rightarrow \qquad \Big(1-x^2\Big)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+p^2y=0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q20

$$y = e^{tan^{-1}} x$$

differentiating w.r.t. x

$$\Rightarrow \frac{dy}{dx} = e^{tan^{-1}} x \left(\frac{1}{1 + x^2} \right)$$

differentiating w.r.t.x

$$\Rightarrow \qquad \frac{d^2y}{dx^2} = \frac{\left(1+x^2\right)\left(e^{tan^{-1}}x\right)\times\frac{1}{1+x^2} - e^{tan^{-1}}x\left(2x\right)}{\left(1+x^2\right)^2}$$

$$\Rightarrow \left(1+x^2\right)\frac{d^2y}{dx^2} = \frac{e^{tan^{-1}}x - 2xe^{tan^{-1}}x}{1+x^2}$$

$$\Rightarrow \left(1 + x^{2}\right) \frac{d^{2}y}{dx^{2}} = \frac{e^{\tan^{-1}x}}{1 + x^{2}} \left(1 - 2x\right) = \frac{dy}{dx} \left(1 - 2x\right)$$

$$\Rightarrow \qquad \left(1 + x^2\right) \frac{d^2y}{dx^2} + \left(2x - 1\right) \frac{dy}{dx} = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q21

$$y = e^{tan^{-1}} x$$

differentiating w.r.t.x

$$\Rightarrow \qquad \frac{dy}{dx} = e^{tan^{-1}} x \left(\frac{1}{1+x^2} \right)$$

differentiating w.r.t.x

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{\left(1+x^2\right)\left(e^{tsn^4}x\right) \times \frac{1}{1+x^2} - e^{tsn^4}x\left(2x\right)}{\left(1+x^2\right)^2}$$

$$\Rightarrow \left(1 + x^{2}\right) \frac{d^{2}y}{dx^{2}} = \frac{e^{tan^{-1}}x - 2xe^{tan^{-1}}x}{1 + x^{2}}$$

$$\Rightarrow \qquad \left(1+x^2\right)\frac{d^2y}{dx^2} = \frac{e^{\tan^{-1}x}}{1+x^2}\left(1-2x\right) = \frac{dy}{dx}\left(1-2x\right)$$

$$\Rightarrow \left(1+x^2\right)\frac{d^2y}{dx^2} + \left(2x-1\right)\frac{dy}{dx} = 0$$

Hence proved!

It is given that, $y = 3\cos(\log x) + 4\sin(\log x)$

Then,

$$y_{1} = 3 \cdot \frac{d}{dx} \Big[\cos(\log x) \Big] + 4 \cdot \frac{d}{dx} \Big[\sin(\log x) \Big]$$

$$= 3 \cdot \Big[-\sin(\log x) \cdot \frac{d}{dx} (\log x) \Big] + 4 \cdot \Big[\cos(\log x) \cdot \frac{d}{dx} (\log x) \Big]$$

$$\therefore y_{1} = \frac{-3\sin(\log x)}{x} + \frac{4\cos(\log x)}{x} = \frac{4\cos(\log x) - 3\sin(\log x)}{x}$$

$$\therefore y_{2} = \frac{d}{dx} \Big(\frac{4\cos(\log x) - 3\sin(\log x)}{x} \Big)$$

$$= \frac{x \Big\{ 4\cos(\log x) - 3\sin(\log x) \Big\}' - \Big\{ 4\cos(\log x) - 3\sin(\log x) \Big\}(x)'}{x^{2}}$$

$$= \frac{x \Big[4 \Big\{ \cos(\log x) \Big\}' - 3 \Big\{ \sin(\log x) \Big\}' \Big] - \Big\{ 4\cos(\log x) - 3\sin(\log x) \Big\} \Big\} \Big]$$

$$= \frac{x \Big[-4\sin(\log x) \cdot (\log x)' - 3\cos(\log x) \cdot (\log x)' \Big] - 4\cos(\log x) + 3\sin(\log x)}{x^{2}}$$

$$= \frac{x \Big[-4\sin(\log x) \cdot \frac{1}{x} - 3\cos(\log x) \cdot \frac{1}{x} \Big] - 4\cos(\log x) + 3\sin(\log x)}{x^{2}}$$

$$= \frac{-4\sin(\log x) - 3\cos(\log x) - 4\cos(\log x) + 3\sin(\log x)}{x^{2}}$$

$$= \frac{-\sin(\log x) - 7\cos(\log x)}{x^{2}}$$

$$\therefore x^{2}y_{2} + yy_{1} + y$$

$$= x^{2} \Big(\frac{-\sin(\log x) - 7\cos(\log x)}{x^{2}} + x \Big(\frac{4\cos(\log x) - 3\sin(\log x)}{x} \Big) + 3\cos(\log x) + 4\sin(\log x)$$

$$= -\sin(\log x) - 7\cos(\log x) + 4\cos(\log x) - 3\sin(\log x) + 3\cos(\log x) + 4\sin(\log x)$$

$$= 0$$

Hence, proved.

********* FND *******