Question 14:

Using the method of integration find the area of the region bounded by lines:

$2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$

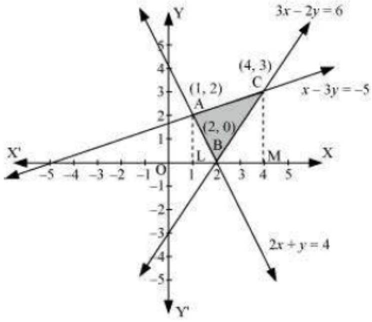
Answer

The given equations of lines are

$2x + y = 4$ ... (1)

$3x - 2y = 6$ ... (2)

And, $x - 3y + 5 = 0$ ... (3)



The area of the region bounded by the lines is the area of ΔABC. AL and CM are the perpendiculars on x-axis.

Area (ΔABC) = Area (ALMCA) − Area (ALB) − Area (CMB)

$$= \int_1^4 \left(\frac{x+5}{3}\right)dx - \int_1^2 (4-2x)dx - \int_2^4 \left(\frac{3x-6}{2}\right)dx$$

$$= \frac{1}{3}\left[\frac{x^2}{2}+5x\right]_1^4 - \left[4x-x^2\right]_1^2 - \frac{1}{2}\left[\frac{3x^2}{2}-6x\right]_2^4$$

$$= \frac{1}{3}\left[8+20-\frac{1}{2}-5\right] - [8-4-4+1] - \frac{1}{2}[24-24-6+12]$$

$$= \left(\frac{1}{3}\times\frac{45}{2}\right) - (1) - \frac{1}{2}(6)$$

$$= \frac{15}{2} - 1 - 3$$

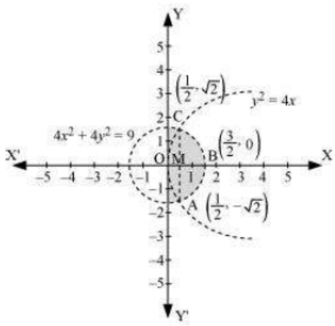
$$= \frac{15}{2} - 4 = \frac{15-8}{2} = \frac{7}{2} \text{ units}$$

Question 15:

Find the area of the region $\{(x, y): y^2 \le 4x, 4x^2 + 4y^2 \le 9\}$

Answer

The area bounded by the curves, $\{(x, y): y^2 \le 4x, 4x^2 + 4y^2 \le 9\}$, is represented as



The points of intersection of both the curves are $\left(\frac{1}{2}, \sqrt{2}\right)$ and $\left(\frac{1}{2}, -\sqrt{2}\right)$.

The required area is given by OABCO.

It can be observed that area OABCO is symmetrical about x-axis.

∴ Area OABCO = 2 × Area OBC

Area OBCO = Area OMC + Area MBC

$$= \int_0^{\frac{1}{2}} 2\sqrt{x}\ dx + \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{2}\sqrt{9-4x^2}\ dx$$

$$= \int_0^{\frac{1}{2}} 2\sqrt{x}\ dx + \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{2}\sqrt{(3)^2-(2x)^2}\ dx$$

Question 16:

Area bounded by the curve $y = x^3$, the x-axis and the ordinates $x = -2$ and $x = 1$ is

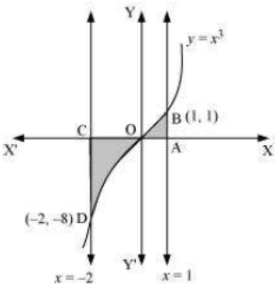
A. − 9

B. $-\frac{15}{4}$

C. $\frac{15}{4}$

D. $\frac{17}{4}$

Answer



Required area $= \int_{-2}^{1} y\,dx$

$$= \int_{-2}^{1} x^3\,dx$$

$$= \left[\frac{x^4}{4}\right]_{-2}^{1}$$

$$= \left[\frac{1}{4} - \frac{(-2)^4}{4}\right]$$

$$= \left(\frac{1}{4} - 4\right) = -\frac{15}{4} \text{ units}$$

Thus, the correct answer is B.

Question 17:

The area bounded by the curve $y = x|x|$, x-axis and the ordinates $x = -1$ and $x = 1$ is given by

[Hint: $y = x^2$ if $x > 0$ and $y = -x^2$ if $x < 0$]

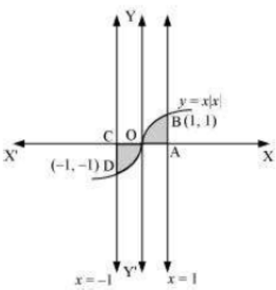
A. 0

B. $\frac{1}{3}$

C. $\frac{2}{3}$

D. $\frac{4}{3}$

Answer



Required area $= \int_{-1}^{1} y\,dx$

$$= \int_{-1}^{1} x|x|\,dx$$

$$= \int_{-1}^{0} x^2\,dx + \int_{0}^{1} x^2\,dx$$

$$= \left[\frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^3}{3}\right]_{0}^{1}$$
$$= -\left(-\frac{1}{3}\right) + \frac{1}{3}$$
$$= \frac{2}{3} \text{ units}$$

Thus, the correct answer is C.

Question 18:

The area of the circle $x^2 + y^2 = 16$ exterior to the parabola $y^2 = 6x$ is

A. $\frac{4}{3}\left(4\pi - \sqrt{3}\right)$

B. $\frac{4}{3}\left(4\pi + \sqrt{3}\right)$

C. $\frac{4}{3}\left(8\pi - \sqrt{3}\right)$

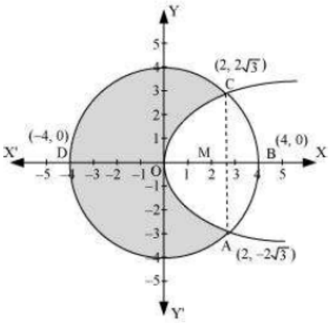
D. $\frac{4}{3}\left(4\pi + \sqrt{3}\right)$

Answer

The given equations are

$x^2 + y^2 = 16$ ... (1)

$y^2 = 6x$ ... (2)



Area bounded by the circle and parabola

$$= 2\left[\text{Area}(\text{OADO}) + \text{Area}(\text{ADBA})\right]$$
$$= 2\left[\int_0^2 \sqrt{16x}\,dx + \int_2^4 \sqrt{16 - x^2}\,dx\right]$$
$$= 2\left[\sqrt{6}\left\{\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right\}_0^2\right] + 2\left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_2^4$$
$$= 2\sqrt{6} \times \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^2 + 2\left[8 \cdot \frac{\pi}{2} - \sqrt{16 - 4} - 8\sin^{-1}\left(\frac{1}{2}\right)\right]$$
$$= \frac{4\sqrt{6}}{3}\left(2\sqrt{2}\right) + 2\left[4\pi - \sqrt{12} - 8\frac{\pi}{6}\right]$$
$$= \frac{16\sqrt{3}}{3} + 8\pi - 4\sqrt{3} - \frac{8}{3}\pi$$
$$= \frac{4}{3}\left[4\sqrt{3} + 6\pi - 3\sqrt{3} - 2\pi\right]$$
$$= \frac{4}{3}\left[\sqrt{3} + 4\pi\right]$$
$$= \frac{4}{3}\left[4\pi + \sqrt{3}\right] \text{ units}$$

Area of circle = $\pi (r)^2$

= $\pi (4)^2$

= 16π units

$$\therefore \text{Required area} = 16\pi - \frac{4}{3}\left[4\pi + \sqrt{3}\right]$$
$$= \frac{4}{3}\left[4 \times 3\pi - 4\pi - \sqrt{3}\right]$$
$$= \frac{4}{3}\left(8\pi - \sqrt{3}\right) \text{ units}$$

Thus, the correct answer is C.

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