



Definite Integrals Ex 20.4B Q37

$$I = \int_0^{\pi} \frac{x}{1 + \cos \alpha \sin x} dx$$

Then,

$$I = \int_0^{\pi} \frac{(\pi - x)}{1 + \cos \alpha \sin(\pi - x)} dx$$

$$I = \int_0^{\pi} \frac{(\pi - x)}{1 + \cos \alpha \sin x} dx$$

$$2I = \pi \int_0^{\pi} \frac{1}{1 + \cos \alpha \sin x} dx$$

$$2I = \pi \int_0^{\pi} \frac{1 + \tan^2\left(\frac{x}{2}\right)}{\left(1 + \tan^2\left(\frac{x}{2}\right)\right) + 2 \cos \alpha \tan\left(\frac{x}{2}\right)} dx$$

$$I = \frac{\pi}{2} \int_0^{\pi} \frac{\sec^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right) + 2 \cos \alpha \tan\left(\frac{x}{2}\right) + 1} dx$$

$$\text{Put } \tan\left(\frac{x}{2}\right) = t \text{ then } \sec^2\left(\frac{x}{2}\right) dx = 2dt$$

$$x = 0 \Rightarrow t = 0 \text{ and } x = \pi \Rightarrow t = \infty$$

$$I = \frac{\pi}{2} \int_0^{\infty} \frac{2}{t^2 + 2t \cos \alpha + 1} dt$$

$$I = \pi \int_0^{\infty} \frac{1}{(t + \cos \alpha)^2 + (1 - \cos^2 \alpha)} dt$$

$$I = \pi \int_0^{\infty} \frac{1}{(t + \cos \alpha)^2 + \sin^2 \alpha} dt$$

$$I = \frac{\pi}{\sin \alpha} \left[\tan^{-1} \left(\frac{t + \cos \alpha}{\sin \alpha} \right) \right]_0^{\infty}$$

$$I = \frac{\pi \alpha}{\sin \alpha}$$

Definite Integrals Ex 20.4B Q38

We know

$$\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a-x) dx$$

Also here

$$f(x) = f(2\pi - x)$$

So

$$I = \int_0^{2\pi} \sin^{100} x \cos^{101} x dx = 2 \int_0^{\pi} \sin^{100} x \cos^{101} x dx$$

$$I = 2 \int_0^{\pi} \sin^{100} (\pi - x) \cos^{101} (\pi - x) dx$$

$$I = -2 \int_0^{\pi} \sin^{100} x \cos^{101} x dx$$

Hence

$$2I = 0$$

$$I = 0$$

Definite Integrals Ex 20.4B Q39

$$I = \int_0^{\pi/2} \frac{a \sin x + b \cos x}{\sin x + \cos x} dx$$

Then,

$$I = \int_0^{\pi/2} \frac{a \sin\left(\frac{\pi}{2} - x\right) + b \cos\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$$I = \int_0^{\pi/2} \frac{a \cos x + b \sin x}{\cos x + \sin x} dx$$

$$2I = \int_0^{\pi/2} \frac{a \sin x + b \cos x}{\sin x + \cos x} dx + \int_0^{\pi/2} \frac{a \cos x + b \sin x}{\cos x + \sin x} dx$$

$$2I = (a+b) \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$I = \frac{(a+b)}{2} \int_0^{\pi/2} 1 dx$$

$$I = \frac{(a+b)\pi}{4}$$

Definite Integrals Ex 20.4B Q40

We have,

$$I = \int_0^{2a} f(x) dx$$

Then

$$I = \int_0^a f(x) dx + \int_a^{2a} f(x) dx$$

$$I = \int_0^a f(x) dx + I_1$$

$$\text{where, } I_1 = \int_a^{2a} f(x) dx$$

Let $2a - t = x$ then $dx = -dt$

If $t = a \Rightarrow x = a$

If $t = 2a \Rightarrow x = 0$

$$I_1 = \int_0^a f(x) dx = \int_a^0 f(2a - t) (-dt) = -\int_a^0 f(2a - t) dt$$

$$I_1 = \int_0^a f(2a - t) dt = \int_0^a f(2a - x) dx$$

$$\therefore I = \int_0^a f(x) dx + \int_0^a f(2a - x) dx$$

$$I = \int_0^a f(x) dx + \int_0^a f(x) dx = 2 \int_0^a f(x) dx \quad [f(2a - x) = f(x)]$$

Hence Proved.

***** END *****