Differentiation Ex 11.2 Q57

Consider

$$y = \log \sqrt{\frac{x-1}{x+1}}$$

Differentiating it with respect to $x$ and applying the chain and product rule, we get

$$y = \log\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$$

$$y = \frac{1}{2}\log\left(\frac{x-1}{x+1}\right)$$

$$y = \frac{1}{2}\left[\log(x-1) - \log(x+1)\right]$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{d}{dx}\log(x-1) - \frac{d}{dx}\log(x+1)\right]$$

$$= \frac{1}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)$$

$$= \frac{1}{2}\left(\frac{2}{x^2-1}\right)$$

$$\frac{dy}{dx} = \frac{1}{x^2-1}$$

Therefore,

$$\frac{dy}{dx} = \frac{1}{x^2-1}$$

Differentiation Ex 11.2 Q58

Here $y = \log\left\{\sqrt{x-1} - \sqrt{x+1}\right\}$

Differentiating it with respect to $x$ and applying the chain and product rule, we get

$$\frac{dy}{dx} = \frac{d}{dx}\log\left\{\sqrt{x-1} - \sqrt{x+1}\right\}$$

$$\frac{dy}{dx} = \frac{1}{\left\{\sqrt{x-1} - \sqrt{x+1}\right\}}\frac{d}{dx}\left(\sqrt{x-1} - \sqrt{x+1}\right)$$

$$= \frac{1}{\left\{\sqrt{x-1} - \sqrt{x+1}\right\}}\left[\frac{d}{dx}\sqrt{x-1} - \frac{d}{dx}\sqrt{x+1}\right]$$

$$= \frac{1}{\left\{\sqrt{x-1} - \sqrt{x+1}\right\}}\left[\frac{1}{2}(x-1)^{-\frac{1}{2}} - \frac{1}{2}(x+1)^{-\frac{1}{2}}\right]$$

$$= \frac{1}{2}\frac{1}{\left\{\sqrt{x-1} - \sqrt{x+1}\right\}}\left(\frac{1}{\sqrt{x-1}} - \frac{1}{\sqrt{x+1}}\right)$$

$$= \frac{1}{2}\frac{1}{\cancel{\left\{\sqrt{x-1} - \sqrt{x+1}\right\}}}\left(\frac{-\cancel{\left\{\sqrt{x-1} - \sqrt{x+1}\right\}}}{\left(\sqrt{x-1}\right)\left(\sqrt{x+1}\right)}\right)$$

$$= \frac{-1}{2}\left(\frac{1}{\left(\sqrt{x-1}\right)\left(\sqrt{x+1}\right)}\right)$$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{x^2-1}}$$

Therefore,

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{x^2-1}}$$

Differentiation Ex 11.2 Q59

Here $y=\sqrt{x+1}+\sqrt{x-1}$

Differentiating it with respect to $x$ and applying the chain and product rule, we get

$$\frac{dy}{dx}=\frac{d}{dx}\sqrt{x+1}+\frac{d}{dx}\sqrt{x-1}$$

$$=\frac{1}{2}(x+1)^{-\frac{1}{2}}+\frac{1}{2}(x-1)^{-\frac{1}{2}}$$

$$=\frac{1}{2}\left(\frac{1}{\sqrt{x+1}}+\frac{1}{\sqrt{x-1}}\right)$$

$$=\frac{1}{2}\left(\frac{\sqrt{x-1}+\sqrt{x+1}}{\left(\sqrt{x+1}\right)\left(\sqrt{x-1}\right)}\right)$$

$$\frac{dy}{dx}=\frac{1}{2}\left(\frac{y}{\left(\sqrt{x^2-1}\right)}\right)$$

$$\sqrt{x^2-1}\frac{dy}{dx}=\frac{1}{2}y$$

Differentiation Ex 11.2 Q60

Here $y=\frac{x}{x+2}$

Differentiating it with respect to $x$ and applying the chain and product rule, we get

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x}{x+2}\right)$$

$$=\frac{(x+2)\frac{dx}{dx}-x\frac{d}{dx}(x+2)}{(x+2)^2}$$

$$=\frac{x+2-x}{(x+2)^2}$$

$$=\frac{x+2}{(x+2)^2}-\frac{x}{(x+2)^2}$$

$$=\frac{1}{x+2}-\frac{xy^2}{x^2}\qquad\left[\text{Since } x+2=\frac{x}{y}\right]$$

$$=\frac{y}{x}-\frac{y^2}{x}$$

$$\frac{dy}{dx}=\frac{1}{x}y(1-y)$$

$$x\frac{dy}{dx}=(1-y)y$$

Differentiation Ex 11.2 Q61

Here $y=\log\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$

Differentiating it with respect to $x$ and applying the chain and product rule, we get

$$\frac{dy}{dx}=\frac{d}{dx}\log\left(x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)$$

$$=\frac{1}{x^{\frac{1}{2}}+x^{-\frac{1}{2}}}\frac{d}{dx}\left(x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)$$

$$=\frac{1}{\sqrt{x}+\frac{1}{\sqrt{x}}}\left(\frac{1}{2}x^{-\frac{1}{2}}-\frac{1}{2}x^{-\frac{1}{2}}x^{-1}\right)$$

$$=\frac{1}{2}\frac{\sqrt{x}}{x+1}\left(\frac{1}{\sqrt{x}}-\frac{1}{x\sqrt{x}}\right)$$

$$=\frac{1}{2}\frac{\cancel{\sqrt{x}}}{x+1}\left(\frac{x-1}{x\cancel{\sqrt{x}}}\right)$$

$$\frac{dy}{dx}=\frac{x-1}{2x(x+1)}$$

*********** END **********