Algebraic Expressions and Identities Ex 6.6 Q14

Answer :

Let us consider the following equation:

$$x+\frac{1}{x}=9$$

Squaring both sides, we get:

$$\left(x+\frac{1}{x}\right)^2=(9)^2=81$$
$$\Rightarrow\left(x+\frac{1}{x}\right)^2=81$$
$$\Rightarrow x^2+2\times x\times\frac{1}{x}+\left(\frac{1}{x}\right)^2=81$$
$$\Rightarrow x^2+2+\frac{1}{x^2}=81$$
$$\Rightarrow x^2+\frac{1}{x^2}=79 \qquad \text{(Subtracting 2 from both sides)}$$

Now, squaring both sides again, we get:

$$\left(x^2+\frac{1}{x^2}\right)^2=(79)^2=6241$$
$$\Rightarrow\left(x^2+\frac{1}{x^2}\right)^2=6241$$
$$\Rightarrow\left(x^2\right)^2+2\left(x^2\right)\left(\frac{1}{x^2}\right)+\left(\frac{1}{x^2}\right)^2=6241$$
$$\Rightarrow x^4+2+\frac{1}{x^4}=6241$$
$$\Rightarrow x^4+\frac{1}{x^4}=6239$$

Algebraic Expressions and Identities Ex 6.6 Q15

Answer :

Let us consider the following equation:

$x + \frac{1}{x} = 12$

Squaring both sides, we get:

$$\left(x + \frac{1}{x}\right)^2 = (12)^2 = 144$$
$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 144$$
$$\Rightarrow x^2 + 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 = 144 \qquad \left[\left(a + b\right)^2 = a^2 + b^2 + 2ab\right]$$
$$\Rightarrow x^2 + 2 + \frac{1}{x^2} = 144$$
$$\Rightarrow x^2 + \frac{1}{x^2} = 142 \qquad \text{(Subtracting 2 from both sides)}$$

Now

$$\left(x - \frac{1}{x}\right)^2 = x^2 - 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2 = x^2 - 2 + \frac{1}{x^2}$$
$$\left[\left(a - b\right)^2 = a^2 + b^2 - 2ab\right]$$
$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = x^2 - 2 + \frac{1}{x^2}$$
$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 142 - 2 \qquad \left(\because x^2 + \frac{1}{x^2} = 142\right)$$
$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 140$$
$$\Rightarrow x - \frac{1}{x} = \pm\sqrt{140} \qquad \text{(Taking square root)}$$

Algebraic Expressions and Identities Ex 6.6 Q16

Answer :

We will use the identity $(a + b)(a - b) = a^2 - b^2$ to obtain the value of *xy*.

Squaring $(2x + 3y)$ and $(2x - 3y)$ both and then subtracting them, we get :

$$(2x + 3y)^2 - (2x - 3y)^2 = \{(2x + 3y) + (2x - 3y)\}\{(2x + 3y) - (2x - 3y)\} = 4x \times 6y$$
$$= 24xy$$
$$\Rightarrow (2x + 3y)^2 - (2x - 3y)^2 = 24xy$$
$$\Rightarrow 24xy = (2x + 3y)^2 - (2x - 3y)^2$$
$$\Rightarrow 24xy = (14)^2 - (2)^2$$
$$\Rightarrow 24xy = (14 + 2)(14 - 2) \qquad \left(\because (a + b)(a - b) = a^2 - b^2\right)$$
$$\Rightarrow 24xy = 16 \times 12$$
$$\Rightarrow xy = \frac{16 \times 12}{24} \qquad \left(\text{Dividing both sides by } 24\right)$$
$$\Rightarrow xy = 8$$

*********** END **********