Mean Value Theorems Ex 15.1 Q7

Let $f(x) = 16 - x^2$, then $f'(x) = -2x$

$f(x)$ is continuous on $[-1, 1]$ because it is a polynomial function.

Also $f(-1) = 16 - (-1)^2 = 15$

$f(1) = 16 - (1)^2 = 15$

$f(-1) = f(1)$

There exists a $c \in [-1, 1]$ such that $f'(c) = 0$

$\Rightarrow -2c = 0$

$\Rightarrow \quad c = 0$

Thus, at $0 \in [-1, 1]$ the tangent is parallel to the x-axis.

Mean Value Theorems Ex 15.1 Q8(i)

Let $f(x) = x^2$, then $f'(x) = 2x$

$f(x)$ is continuous on $[-2, 2]$ because it is a polynomial function.

$f(x)$ is differentiable on $(-2, 2)$ as it is a polynomial function.

Also $f(-2) = (-2)^2 = 4$

$f(2) = 2^2 = 4$

$\Rightarrow \quad f(-2) = f(2)$

$\therefore \quad$ There exists $c \in (-2, 2)$ such that $f'(c) = 0$

$\Rightarrow \quad 2c = 0$

$\Rightarrow \quad c = 0$

Thus, at $0 \in [-2, 2]$ the tangent is parallel to the x-axis.

x = 0, then y = 0

Therefore, the point is (0, 0)

Mean Value Theorems Ex 15.1 Q8(ii)

Let $f(x) = e^{1-x^2}$ on $[-1,1]$

Since, $f(x)$ is a composition of two continuous functions, it is continuous on $[-1,1]$

Also $f(x) = -2xe^{1-x^2}$

$f(2) = 2^2 = 4$

$\therefore \quad f'(x)$ exists for every value of x in $(-1,1)$

$\Rightarrow \quad f(x)$ is differentiable on $(-1,1)$

By rolle's theorem, there exists $c \in (-1,1)$ such that $f'(c) = 0$

$\Rightarrow \quad -2ce^{1-c^2} = 0$

$\Rightarrow \quad c = 0$

Thus, at $c = 0 \in [-1,1]$ the tangent is parallel to the x-axis.

x = 0, then y = e

Therefore, the point is (0, e)

Mean Value Theorems Ex 15.1 Q8(iii)

Let $f(x) = 12(x+1)(x-2)$

Since, $f(x)$ is a polynomial function, it is continuous on $[-1,2]$ and differentiable on $(-1,2)$

Also $f'(x) = 12[(x-2)+(x+1)] = 12[2x-1]$

By rolle's theorem, there exists $c \in (-1,2)$ such that $f'(c) = 0$

$\Rightarrow \quad 12(2c-1) = 0$

$\Rightarrow \quad c = \frac{1}{2}$

Thus, at $c = \frac{1}{2} \in (-1,2)$ the tangent to $y = 12(x+1)(x-2)$ is parallel to x-axis

Mean Value Theorems Ex 15.1 Q9

It is given that $f:[-5,5] \to \mathbf{R}$ is a differentiable function.

Since every differentiable function is a continuous function, we obtain

(a) f is continuous on [−5, 5].

(b) f is differentiable on (−5, 5).

Therefore, by the Mean Value Theorem, there exists c □ (−5, 5) such that

$$f'(c) = \frac{f(5)-f(-5)}{5-(-5)}$$

$$\Rightarrow 10f'(c) = f(5) - f(-5)$$

It is also given that $f'(x)$ does not vanish anywhere.

$$\therefore f'(c) \neq 0$$
$$\Rightarrow 10f'(c) \neq 0$$
$$\Rightarrow f(5) - f(-5) \neq 0$$
$$\Rightarrow f(5) \neq f(-5)$$

Hence, proved.

*********** END **********