Polynomials Ex 2.1 Q19

Answer :

(i) Since α and β are the zeros of the quadratic polynomial $f(x)=x^2-2x+3$

$$\alpha+\beta=\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$
$$=\frac{-(-2)}{1}$$
$$=2$$

Product of the zeros = $\dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

$$=\frac{3}{1}$$
$$=3$$

Let S and P denote respectively the sums and product of the polynomial whose zeros $\alpha+2,\beta+2$

$$S=(\alpha+2)+(\beta+2)$$
$$S=\alpha+\beta+2+2$$
$$S=2+2+2$$
$$S=6$$
$$P=(\alpha+2)+(\beta+2)$$
$$P=\alpha\beta+2\beta+2\alpha+4$$
$$P=\alpha\beta+2(\alpha+\beta)+4$$
$$P=3+2(2)+4$$
$$P=3+4+4$$
$$P=11$$

Therefore the required polynomial *f (x)* is given by

$$f(x)=k(x^2-Sx+P)$$
$$=k(x^2-6x+11)$$

Hence, the required equation is $\boxed{f(x)=k(x^2-6x+11)}$.

(ii) Since α and β are the zeros of the quadratic polynomial $f(x)=x^2-2x+3$

$$\alpha+\beta=\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$
$$=\frac{-(-2)}{1}$$
$$=2$$

Product of the zeros = $\dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

$$=\frac{3}{1}$$
$$=3$$

Let S and P denote respectively the sums and product of the polynomial whose zeros $\dfrac{\alpha-1}{\alpha+1},\dfrac{\beta-1}{\beta+1}$

$$S=\frac{\alpha-1}{\alpha+1}+\frac{\beta-1}{\beta+1}$$
$$S=\frac{(\alpha-1)(\beta+1)+(\beta-1)(\alpha+1)}{(\alpha+1)(\beta+1)}$$

$$S=\frac{\alpha\beta-\beta+\alpha-1+\alpha\beta-\alpha+\beta-1}{\alpha\beta+\beta+\alpha+1}$$

$$S=\frac{\alpha\beta-\cancel{\beta}+\cancel{\alpha}+1-\alpha\beta-\cancel{\alpha}+\cancel{\beta}-1}{\alpha\beta+(\alpha+\beta)+1}$$

$$S=\frac{\alpha\beta+\alpha\beta-1-1}{\alpha\beta+(\alpha+\beta)+1}$$

By substituting $\alpha+\beta=2$ and $\alpha\beta=3$ we get ,

$$S=\frac{3+3-1-1}{3+2+1}$$

$$S=\frac{6-2}{6}$$

$$S=\frac{\cancel{4}^{2}}{\cancel{6}^{3}}$$

$$P=\left(\frac{\alpha-1}{\alpha+1}\right)\left(\frac{\beta-1}{\beta+1}\right)$$

$$P=\frac{\alpha\beta-\beta-\alpha+1}{\alpha\beta+\beta+\alpha+1}$$

$$P=\frac{\alpha\beta-(\beta+\alpha)+1}{\alpha\beta+(\alpha+\beta)+1}$$

$$P=\frac{3-2+1}{3+2+1}$$

$$P=\frac{2}{6}$$

$$P=\frac{\cancel{2}^{1}}{\cancel{6}_{3}}$$

$$P=\frac{1}{3}$$

The required polynomial *f (x)* is given by,

$$f(x)=k\left(x^2-Sx+P\right)$$

$$f(x)=k\left(x^2-\frac{2}{3}x+\frac{1}{3}\right)$$

Hence, the required equation is $f(x)=k\left(x^2-\frac{2}{3}x+\frac{1}{3}\right)$, where k is any non zero real number

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