Question 6. 21. The blades of a windmill sweep out a circle of area A.
(a) If the wind flows at a velocity v perpendicular to the circle, what is the mass of the air passing through it in time t?
(b) What is the kinetic energy of the air?
(c) Assume that the windmill converts 25% of the wind's energy into electrical energy, and that A = 30 m^2, v = 36 km/h and the density of air is 1.2 kg m^{-3} . What is the electrical power produced?
Answer:
(a) Volume of wind flowing per second = Av Mass of wind flowing per second = Avρ
Mass of air passing in second = Avρt

(b) Kinetic energy of air = $\frac{1}{2}mv^2 = \frac{1}{2}(Av\rho t)\,v^2 = \frac{1}{2}Av^3\rho t$

(c) Electrical energy produced = $\frac{25}{100} \times \frac{1}{2}Av^3\rho t = \frac{Av^3\rho t}{8}$

Electrical power = $\frac{Av^3\rho t}{8t} = \frac{Av^3\rho}{8}$

Now, $A = 30\text{ m}^2$, $v = 36\text{ kmh}^{-1} = 36 \times \frac{5}{18}\text{ms}^{-1}\ 10\text{ ms}^{-1}$
and $\rho = 1.2\text{ kg ms}^{-1}$

$\therefore$ Electrical power = $\frac{30 \times 10 \times 10 \times 10 \times 1.2}{8}\text{W} = 4500\text{ W} = 4.5\text{ KW}$.

Question 6. 22. A person trying to lose weight (dieter) lifts a 10 kg mass, one thousand times, to a height of 0.5 m each time. Assume that the potential energy lost each time she lowers the mass is dissipated,
(a) How much work does she do against the gravitational force?
(b) Fat supplies 3.8 x 10^7J of energy per kilogram which is converted to mechanical energy with a 20% efficiency rate. How much fat will the dieter use up?
Answer:
Here, m = 10 kg, h = 0.5 m, n = 1000
(a) work done against gravitational force.
W = n(mgh) = 1000 x (10 x 9.8 x 0.5) = 49000J.
(b) Mechanical energy supplied by 1 kg of fat = 3.8 x 10^7 x20/100
= 0.76 x10^7 J/kg
∴ Fat used up by the dieter = 1kg/(0.76 x 10^7) x 49000 = 6.45 x 10^{-3} kg

Question 6. 23. A family uses 8 kW of power, (a) Direct solar energy is incident on the horizontal surface at an average rate of 200 W per square meter. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8 kW? (b) Compare this area to that of the roof of a typical house.
Answer:
(a) Power used by family, p = 8 KW = 8000 W
As only 20% of solar energy can be converted to useful electrical energy, hence, power
8000 W to be supplied by solar energy = 8000 W/20 = 40000 W
As solar energy is incident at a rate of 200 Wm^{-2}, hence the area needed
A = 4000 W/200 Wm^{-2} =200 m^2
(b) The area needed is camparable to roof area of a large sized

house.

Question 6. 24. A bullet of mass 0.012 kg and horizontal speed 70 ms^{-1} strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by thin wire. Calculate the height to which the block rises. Also, estimate the amount of heat produced in the block.
Answer:
Here, m_1 = 0.012 kg, u_1 = 70 m/s
m_2 = 0.4 kg, u_2 = 0
As the bullet comes to rest with respect to the block, the two behave as one body. Let v be the velocity acquired by the combination. Applying principle of conservation of linear momentum, $(m_1 + m_2)$ v = $m_1H_1 + m_2u_2 = m_1u_1$

$$v = \frac{m_1u_1}{m_1+m_2} = \frac{0.012\times70}{0.012+0.4} = \frac{0.84}{0.412} = 2.04 \text{ ms}^{-1}$$

Let the block rise to a height h.

P.E. of the combination = K.E. of the combination

$$(m_1 + m_2)\, gh = \frac{1}{2}(m_1 + m_2)\, v^2$$

$$\therefore \quad h = \frac{v^2}{2g} = \frac{2.04\times2.04}{2\times9.8} = 0.212 \text{ m.}$$

For calculating heat produced, we calculate energy lost (W), where

W = intial K.E. of bullet – final K.E. of combination

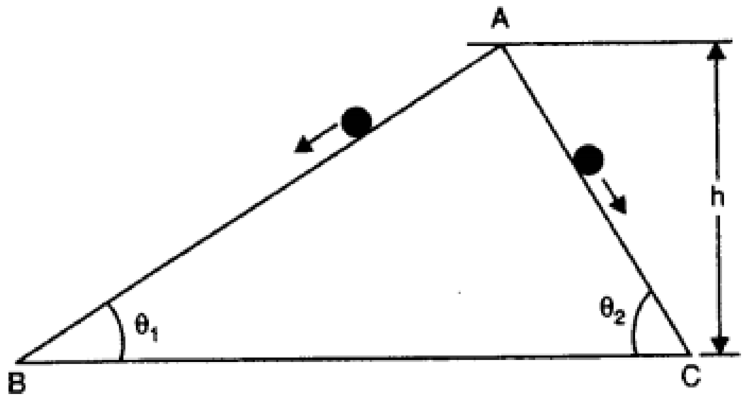
$$= \frac{1}{2}m_1u_1^2 - \frac{1}{2}(m_1+m_2)\, v^2$$

$$= \frac{1}{2}\times0.012\,(70)^2 - \frac{1}{2}(0.412)\,(2.04)^2$$

W = 29.4 – 0.86 = 28.54 joule

$$\therefore \text{ Heat produced,} \quad H = \frac{W}{J} = \frac{28.54}{4.2} = 6.8 \text{ cal.}$$

Question 6. 25. Two inclined friction less tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track (Fig). Will the stones reach the bottom at the same time? Will they reach there with the same speed? Explain. Given θ_1 = 30°, θ_2 = 60°, and h = 10 m, what are the speeds and times taken by the two stones?



Answer:

$$\frac{1}{2}mv^2 = mgh, \quad v = \sqrt{2gh}$$

$$= \sqrt{2\times10\times10}\ \text{ms}^{-1} = 14.14\ \text{ms}^{-1}$$

$$v_B = v_C = 14.14\ \text{ms}^{-1}, \quad l = \frac{1}{2}(g\sin\theta)\,t^2$$

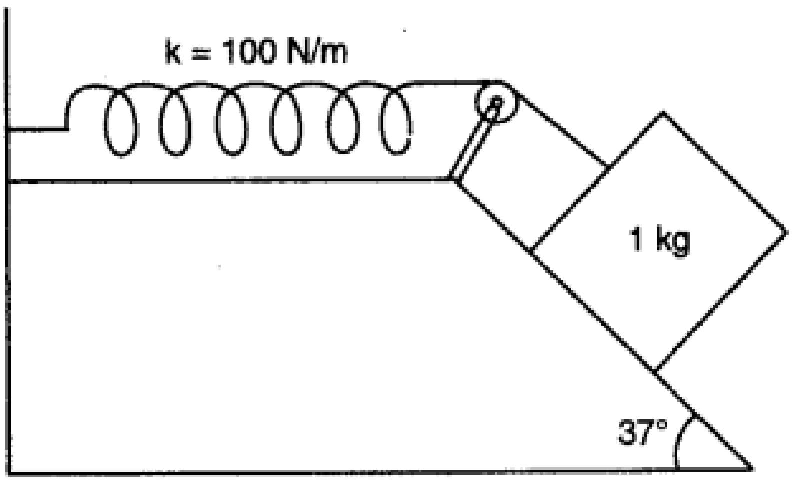
$$\sin\theta = \frac{h}{l}, \quad l = \frac{h}{\sin\theta}$$

$$\frac{h}{\sin\theta} = \frac{1}{2}g\sin\theta\,t^2 \quad \text{or} \quad t = \sqrt{\frac{2h}{g}}\cdot\frac{1}{\sin\theta}$$

$$t_B = \sqrt{\frac{2\times10}{10}}\cdot\frac{1}{\sin 30^\circ} = 2\sqrt{2}\text{s.}$$

$$t_C = \sqrt{\frac{2\times10}{10}}\cdot\frac{1}{\sin 60^\circ} = \frac{2\sqrt{2}}{\sqrt{3}}\text{s.}$$

Question 6. 26. A 1 kg block situated on a rough incline is connected to a spring with spring constant 100 Nm^{-1} as shown in Figure. The block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline. Assume that the spring has negligible mass and the pulley is friction less.



Answer:

From the above figure,

$$R = mg \cos \theta$$

$$F = \mu R = \mu\, mg \cos \theta$$

Net force on the block down the incline

$$= mg \sin \theta - F$$

$$= mg \sin \theta - \mu\, mg \cos \theta$$

$$= mg (\sin \theta - \mu \cos \theta)$$

Here distance moved $x = 10 \text{ cm} = 0.1 \text{ m}$

In equilibrium,

work done = Potential energy of stretched spring

$$mg (\sin \theta - \mu \cos \theta)\, x = \frac{1}{2} kx^2$$

or $$2mg (\sin \theta - \mu \cos \theta) = kx$$

or $$2 \times 1 \text{ kg} \times 10\text{ms}^{-2} (\sin 37° - \mu \cos 37°) = 100 \times 0.1 \text{ m}$$

or $$20(0.601 - \mu \times 0.798) = 10$$

or $$0.601 - 0.798\mu = \frac{10}{20} = 0.5$$

or $$-0.798\mu = 0.5 - 0.601 = -0.101$$

or $$\mu = \frac{-0.101}{-0.798} = \frac{101}{798} = 0.126$$

Hence $$\mu = \mathbf{0.126}$$

Question 6. 27. A bolt of mass 0.3 kg falls from the ceiling of an elevator moving down with a uniform speed of 7 ms^{-1}. It hits the floor of the elevator (length of elevator = 3 m) and does not rebound. What is the heat produced by the impact? Would your answer be different if the elevator were stationary ?

Answer:

P.E. of bolt = mgh = 0.3 x 9.8 x 3 = 8.82 J

The bolt does not rebound. So the whole of the energy is converted into heat. Since the value of acceleration due to gravity is the same in all inertial system, therefore the answer will not change even if the elevator is stationary.

Question 6. 28. A trolley of mass 200 kg moves with a uniform speed of 36 km h^{-1} on a friction less track. A child of mass 20 kg runs on the trolley from one end to the other (10 m away) with a speed of 4 ms^{-1} relative to the trolley in a direction opposite to the trolley's motion, and jumps out of the trolley. What is the final speed of the trolley? How much has the trolley moved from the time the child begins to run?

Answer:

Let there be an observer travelling parallel to the trolley with the same speed. He will observe the initial momentum of the trolley of mass M and child of mass m as zero. When the child jumps in opposite direction, he will observe the increase in the velocity of the trolley by Δv.

Let u be the velocity of the child. He will observe child landing at velocity (u - Δu) Therefore, initial momentum = 0

Final momentum = MΔ v - m (u - Δv)

Hence, MΔ v - m (u - Δv) = 0

Whence Δv = mu/ M + m

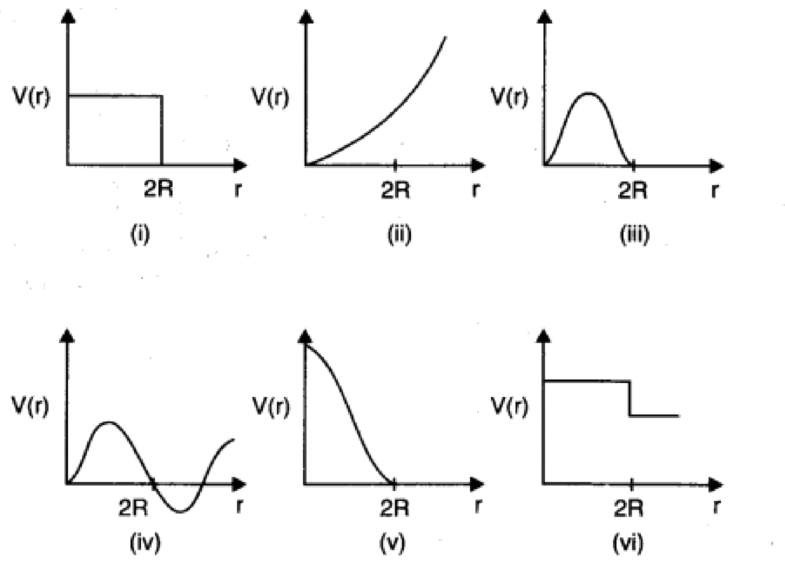
Putting values Δv =4 x 20/ 20 + 220 = ms^{-1}

∴ Final speed of trolley is 10.36 ms^{-1}.

The child take 2.5 s to run on the trolley.

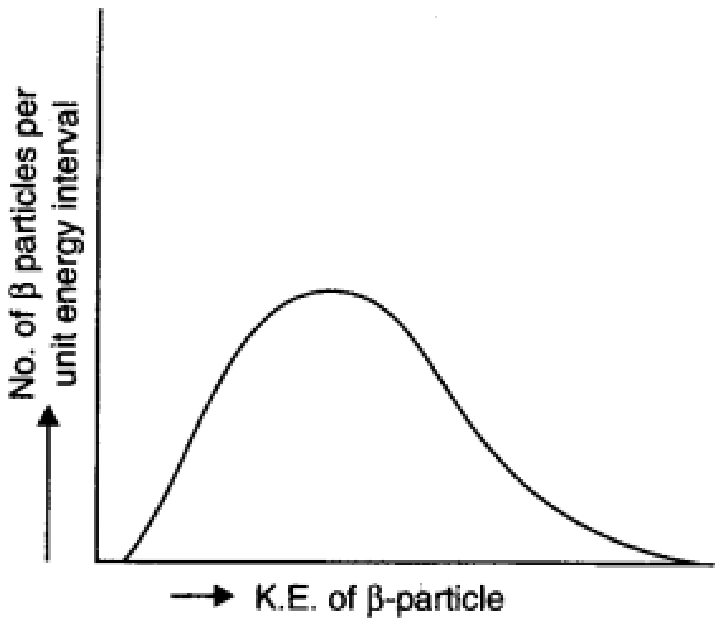
Therefore, the trolley moves a distance = 2.5 x 10.36 m = 25.9 m.

Question 6.29. Which of the following potential energy curves in Fig. cannot possibly describe the elastic collision of two billiard balls? Here r is distance between centres of the balls.



Answer: The potential energy of a system of two masses varies inversely as the distance (r) between 1 them i.e., V (r) α 1/r. When the two billiard balls touch each other, P.E. becomes zero i.e., at r = R + R = 2 R; V (r) = 0. Out of the given graphs, curve (v) only satisfies these two conditions. Therefore, all other curves cannot possibly describe the elastic collision of two billiard balls.

Question 6. 30. Consider the decay of a free neutron at rest: n > p + e^-. Show that the two body decay of this type must necessarily give an electron of fixed energy, and therefore, cannot account for the observed continuous energy distribution in the β -decay of a neutron or a nucleus, Fig.



Answer: Let the masses of the electron and proton be m and M respectively. Let v and V be the velocities of electron and proton respectively. Using law of conservation of momentum. Momentum of electron + momentum of proton = momentum of neutron

$\therefore \qquad mv + MV = 0 \Rightarrow V = -\frac{m}{M}v$

Clearly, the electron and the proton move in opposite directions. If mass Δm has been converted into energy in the reaction, then

$$\frac{1}{2}mv^2 + \frac{1}{2}MV^2 = \Delta m \times c^2$$

or $$\frac{1}{2}mv^2 + \frac{1}{2}M\left[-\frac{m}{M}\right]^2 v^2 = \Delta mc^2$$

or $$\frac{1}{2}mv^2\left[1+\frac{m}{M}\right] = \Delta mc^2$$

or $$v^2 = \frac{2M\Delta\, mc^2}{m\,(M+m)}$$

Thus, it is proved that the value of v^2 is fixed since all the quantities in right hand side are constant. It establishes that the emitted electron must have a fixed energy and thus we cannot account for the continuous energy distribution in the β-decay of a neutron.

*********** END **********