Complex Numbers Ex 13.2 Q7

$$\text{let } z = \frac{1+i}{1-i} - \frac{1-i}{1+i}$$

$$= \frac{(1+i)^2 - (1-i)^2}{(1-i)(1+i)}$$

$$= \frac{1^2 + i^2 + 2 \times 1 \times i - \left(1^2 + i^2 - 2 \times 1 \times i\right)}{1^2 + 1^2}$$

$$= \frac{1 - 1 + 2i - (1 - 1 - 2i)}{2}$$

$$= \frac{2i + 2i}{2}$$

$$= \frac{4i}{2}$$

$$\Rightarrow z = 2i$$

$$\therefore |z| = |2i|$$

$$= 2|i| \qquad \left(\because |z_1 z_2| = |z_1| \times |z_2|\right)$$

$$= 2 \times i \qquad \left(\because |i| = 1\right)$$

$$= 2$$

Complex Numbers Ex 13.2 Q8

$$x + iy = \frac{a+ib}{a-ib}$$

$$\Rightarrow \left(\overline{x+iy}\right) = \overline{\left(\frac{a+ib}{a-ib}\right)} \qquad \text{(on taking conjugate both sides)}$$

$$\Rightarrow \quad x - iy = \frac{\left(\overline{a+ib}\right)}{\left(\overline{a-ib}\right)} \qquad \left(\because \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}\right)$$

$$= \frac{a-ib}{a+ib}$$

$$\because (x+iy)(x-iy) = \frac{a+ib}{a-ib} \times \frac{a-ib}{a+ib}$$

$$\Rightarrow x^2 + y^2 = 1$$

proved

Complex Numbers Ex 13.2 Q9

For $n = 1$, we have,

$$\left(\frac{1+i}{1-i}\right)^1 = \frac{1+i}{1-i}$$

$$= \frac{1+i}{1-i} \times \frac{1+i}{1+i}$$

$$= \frac{(1+i)^2}{1^2+1^2}$$

$$= \frac{1^2+i^2+2\times 1\times i}{2}$$

$$= \frac{2i}{2} \qquad (\because i^2 = -1)$$

$= i$, which is not real

For $n = 2$, we have

$$\left(\frac{1+i}{1-i}\right)^2 = i^2 \qquad \left(\because \frac{1+i}{1-i} = 1 \text{ form above}\right)$$

$= -1$, which is real

Hence the least positive integral value of n is 2.

Complex Numbers Ex 13.2 Q10

$$\text{let } z = \frac{1+i\cos\theta}{1-2i\cos\theta}$$

$$= \frac{1+i\cos\theta}{1-2i\cos\theta} \times \frac{1+2i\cos\theta}{1+2i\cos\theta}$$

$$= \frac{1+2i\cos\theta + i\cos\theta(1+2i\cos\theta)}{1^2+(2\cos\theta)^2}$$

$$= \frac{1+2i\cos\theta + i\cos\theta - 2\cos^2\theta}{1+4\cos^2\theta}$$

$$= \frac{1-2\cos^2\theta + 3i\cos\theta}{1+4\cos^2\theta}$$

$$= \frac{1-2\cos^2\theta}{1+4\cos^2\theta} + \frac{3\cos\theta}{1+4\cos^2\theta}i$$

we know that z is purely real if and only if $\text{Im}\, z = 0$

$$\therefore \frac{3\cos\theta}{1+4\cos^2\theta} = 0 \qquad (\because z \text{ is given to be purely real})$$

$$\Rightarrow \quad 3\cos\theta = 0$$

$$\Rightarrow \quad \cos\theta = 0$$

$$\Rightarrow \quad \cos\theta = \cos\frac{\pi}{2}$$

$\therefore$ The general solution is given by

$$\theta = 2n\pi \pm \frac{\pi}{2}, n \in Z$$

*********** END **********