Indefinite Integrals Ex 19.13 Q14

Let $I = \int \frac{1}{\sqrt{\left(1-x^2\right)\left[9+\left(\sin^{-1}x\right)^2\right]}}dx$

Let $\sin^{-1}x = t$

$\Rightarrow \quad \frac{1}{\sqrt{1-x^2}}dx = dt$

$I = \int \frac{dt}{\sqrt{(3)^2+t^2}}$

$= \log\left|t+\sqrt{9+t^2}\right|+c \qquad \left[\text{Since} \quad \int \frac{1}{\sqrt{a^2+x^2}}dx = \log\left|x+\sqrt{a^2+x^2}\right|+c\right]$

$I = \log\left|\sin^{-1}x+\sqrt{9+\left(\sin^{-1}x\right)^2}\right|+c$

Indefinite Integrals Ex 19.13 Q15

Let $I = \int \frac{\cos x}{\sqrt{\sin^2 x - 2\sin x - 3}}dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x\, dx = dt$

$= \int \frac{dt}{\sqrt{t^2-2t-3}}$

$= \int \frac{dt}{\sqrt{t^2-2t+(1)^2-(1)^2-3}}$

$= \int \frac{dt}{\sqrt{(t-1)^2-(2)^2}}$

Let $t-1 = u$

$\Rightarrow \quad dt = du$

$I = \int \frac{du}{\sqrt{u^2-(2)^2}}$

$= \log\left|u+\sqrt{u^2-4}\right|+c \qquad \left[\text{Since} \quad \int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right|+c\right]$

$= \log\left|t-1+\sqrt{(t-1)^2+4}\right|+c$

$I = \log\left|\sin x - 1+\sqrt{\sin^2 x - 2\sin x - 3}\right|+c$

Indefinite Integrals Ex 19.13 Q16

Let $I = \int \sqrt{\operatorname{cosec} x - 1}\,dx$

$$= \int \sqrt{\frac{1-\sin x}{\sin x}}\,dx$$

$$= \int \sqrt{\frac{(1-\sin x)+(1+\sin x)}{\sin x\,(1+\sin x)}}\,dx$$

$$= \int \sqrt{\frac{\cos^2 x}{\sin^2 x+\sin x}}\,dx$$

$$= \int \frac{\cos x}{\sqrt{\sin^2 x+\sin x}}\,dx$$

Let $\sin x = t$

$\Rightarrow \quad \cos x\,dx = dt$

$$= \int \frac{dt}{\sqrt{t^2+t}}$$

$$= \int \frac{dt}{\sqrt{t^2+2t\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}}$$

$$== \int \frac{dt}{\sqrt{\left(t+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}}$$

Let, $t+\frac{1}{2} = u$

$\Rightarrow \quad dt = du$

$$= \int \frac{du}{\sqrt{u^2-\left(\frac{1}{2}\right)^2}}$$

$$== \log\left|u+\sqrt{u^2-\left(\frac{1}{2}\right)^2}\right|+c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2-a^2}}\,dx = \log\left|x+\sqrt{x^2-a^2}\right|+c\right]$$

$$= \log\left|\left(t+\frac{1}{2}\right)+\sqrt{\left(t+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}\right|+c$$

$$I = \log\left|\sin x+\frac{1}{2}+\sqrt{\sin^2 x+\sin x}\right|+c$$

Indefinite Integrals Ex 19.13 Q17

To evaluate the following integral follow the steps:

$$\int \frac{\sin x-\cos x}{\sqrt{\sin 2x}}\,dx = \int \frac{\sin x-\cos x}{\sqrt{(\sin x+\cos x)^2-1}}\,dx$$

Let $\sin x+\cos x = t$ therefore $(\cos x-\sin x)\,dx = dt$

Now

$$\int \frac{\sin x-\cos x}{\sqrt{(\sin x+\cos x)^2-1}}\,dx = -\int \frac{dt}{\sqrt{t^2-1}}$$

$$= -\ln\left|t+\sqrt{t^2-1}\right|+c$$

$$= -\ln\left|\sin x+\cos x+\sqrt{\sin 2x}\right|+c$$

*********** END **********