

### Exercise 7.3: Solutions of Questions on Page Number: 307

Q1:  $\sin^2(2x+5)$ 

# Answer:

$$\sin^{2}(2x+5) = \frac{1-\cos 2(2x+5)}{2} = \frac{1-\cos(4x+10)}{2}$$

$$\Rightarrow \int \sin^{2}(2x+5) dx = \int \frac{1-\cos(4x+10)}{2} dx$$

$$= \frac{1}{2} \int 1 dx - \frac{1}{2} \int \cos(4x+10) dx$$

$$= \frac{1}{2} x - \frac{1}{2} \left( \frac{\sin(4x+10)}{4} \right) + C$$

$$= \frac{1}{2} x - \frac{1}{8} \sin(4x+10) + C$$

Answer needs Correction? Click Here

Q2:  $\sin 3x \cos 4x$ 

### Answer:

It is known that,  $\sin A \cos B = \frac{1}{2} \left\{ \sin \left( A + B \right) + \sin \left( A - B \right) \right\}$ 

$$\int \sin 3x \cos 4x \, dx = \frac{1}{2} \int \{ \sin (3x + 4x) + \sin (3x - 4x) \} \, dx$$

$$= \frac{1}{2} \int \{ \sin 7x + \sin (-x) \} \, dx$$

$$= \frac{1}{2} \int \{ \sin 7x - \sin x \} \, dx$$

$$= \frac{1}{2} \int \sin 7x \, dx - \frac{1}{2} \int \sin x \, dx$$

$$= \frac{1}{2} \left( \frac{-\cos 7x}{7} \right) - \frac{1}{2} (-\cos x) + C$$

$$= \frac{-\cos 7x}{14} + \frac{\cos x}{2} + C$$

Answer needs Correction? Click Here

Q3: cos 2x cos 4x cos 6x

### Answer:

It is known that,  $\cos A \cos B = \frac{1}{2} \{\cos (A+B) + \cos (A-B)\}$ 

$$\int \cos 2x (\cos 4x \cos 6x) dx = \int \cos 2x \left[ \frac{1}{2} \left\{ \cos (4x + 6x) + \cos (4x - 6x) \right\} \right] dx$$

$$= \frac{1}{2} \int \left\{ \cos 2x \cos 10x + \cos 2x \cos (-2x) \right\} dx$$

$$= \frac{1}{2} \int \left\{ \cos 2x \cos 10x + \cos^2 2x \right\} dx$$

$$= \frac{1}{2} \int \left[ \left\{ \frac{1}{2} \cos (2x + 10x) + \cos (2x - 10x) \right\} + \left( \frac{1 + \cos 4x}{2} \right) \right] dx$$

$$= \frac{1}{4} \int (\cos 12x + \cos 8x + 1 + \cos 4x) dx$$

$$= \frac{1}{4} \left[ \frac{\sin 12x}{12} + \frac{\sin 8x}{8} + x + \frac{\sin 4x}{4} \right] + C$$

Answer needs Correction? Click Here

Q4: 
$$\sin^3(2x+1)$$

Let 
$$I = \int \sin^3(2x+1)$$

$$\Rightarrow \int \sin^3(2x+1) dx = \int \sin^2(2x+1) \cdot \sin(2x+1) dx$$
$$= \int (1 - \cos^2(2x+1)) \sin(2x+1) dx$$

$$Let \cos(2x+1) = t$$

$$\Rightarrow -2\sin(2x+1)dx = dt$$

$$\Rightarrow \sin(2x+1)dx = \frac{-dt}{2}$$

$$\Rightarrow I = \frac{-1}{2} \int (1 - t^2) dt$$

$$= \frac{-1}{2} \left\{ t - \frac{t^3}{3} \right\}$$

$$= \frac{-1}{2} \left\{ \cos(2x+1) - \frac{\cos^3(2x+1)}{3} \right\}$$

$$= \frac{-\cos(2x+1)}{2} + \frac{\cos^3(2x+1)}{6} + C$$

Q5:  $\sin^3 x \cos^3 x$ 

#### Answer:

Let 
$$I = \int \sin^3 x \cos^3 x \cdot dx$$
  

$$= \int \cos^3 x \cdot \sin^2 x \cdot \sin x \cdot dx$$
  

$$= \int \cos^3 x (1 - \cos^2 x) \sin x \cdot dx$$

Let 
$$\cos x = t$$
  

$$\Rightarrow -\sin x \cdot dx = dt$$

$$\Rightarrow I = -\int t^3 (1 - t^2) dt$$

$$= -\int (t^3 - t^5) dt$$

$$= -\left\{\frac{t^4}{4} - \frac{t^6}{6}\right\} + C$$

$$= -\left\{\frac{\cos^4 x}{4} - \frac{\cos^6 x}{6}\right\} + C$$

$$= \frac{\cos^6 x}{6} - \frac{\cos^4 x}{6} + C$$

Answer needs Correction? Click Here

Q6:  $\sin x \sin 2x \sin 3x$ 

#### Answer:

It is known that, 
$$\sin A \sin B = \frac{1}{2} \{\cos(A-B) - \cos(A+B)\}$$

$$\begin{split} & : \int \sin x \sin 2x \sin 3x \ dx = \int \left[ \sin x \cdot \frac{1}{2} \left\{ \cos \left( 2x - 3x \right) - \cos \left( 2x + 3x \right) \right\} \right] dx \\ & = \frac{1}{2} \int \left\{ \sin x \cos \left( -x \right) - \sin x \cos 5x \right) \ dx \\ & = \frac{1}{2} \int \left\{ \sin x \cos x - \sin x \cos 5x \right) \ dx \\ & = \frac{1}{2} \int \frac{\sin 2x}{2} \ dx - \frac{1}{2} \int \sin x \cos 5x \ dx \\ & = \frac{1}{4} \left[ \frac{-\cos 2x}{2} \right] - \frac{1}{2} \int \left\{ \frac{1}{2} \sin \left( x + 5x \right) + \sin \left( x - 5x \right) \right\} \ dx \\ & = \frac{-\cos 2x}{8} - \frac{1}{4} \int \left\{ \sin 6x + \sin \left( -4x \right) \right\} \ dx \\ & = \frac{-\cos 2x}{8} - \frac{1}{4} \left[ \frac{-\cos 6x}{3} + \frac{\cos 4x}{4} \right] + C \\ & = \frac{-\cos 2x}{8} - \frac{1}{8} \left[ \frac{-\cos 6x}{3} + \frac{\cos 4x}{2} \right] + C \\ & = \frac{1}{8} \left[ \frac{\cos 6x}{3} - \frac{\cos 4x}{2} - \cos 2x \right] + C \end{split}$$

Answer needs Correction? Click Here

Q7: sin 4x sin 8x

# Answer:

It is known that,  $\sin A \sin B = \frac{1}{2} \cos (A - B) - \cos (A + B)$ 

[Math Processing Error]

Answer needs Correction? Click Here

Q8: 
$$\frac{1-\cos x}{1+\cos x}$$

$$\frac{1-\cos x}{1+\cos x} = \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}} \qquad \left[ 2\sin^2\frac{x}{2} = 1-\cos x \text{ and } 2\cos^2\frac{x}{2} = 1+\cos x \right]$$
$$= \tan^2\frac{x}{2}$$
$$= \left(\sec^2\frac{x}{2} - 1\right)$$
$$\therefore \int \frac{1-\cos x}{1+\cos x} dx = \int \left(\sec^2\frac{x}{2} - 1\right) dx$$

$$= \left[\frac{\tan\frac{x}{2}}{\frac{1}{2}} - x\right] + C$$

$$= 2\tan\frac{x}{2} - x + C$$

Q9:  $\frac{\cos x}{1 + \cos x}$ 

Answer:

$$\frac{\cos x}{1 + \cos x} = \frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}} \qquad \left[\cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} \text{ and } \cos x = 2\cos^2 \frac{x}{2} - 1\right]$$

$$= \frac{1}{2} \left[1 - \tan^2 \frac{x}{2}\right]$$

$$\therefore \int \frac{\cos x}{1 + \cos x} dx = \frac{1}{2} \int \left(1 - \tan^2 \frac{x}{2}\right) dx$$

$$= \frac{1}{2} \int \left(1 - \sec^2 \frac{x}{2} + 1\right) dx$$

$$= \frac{1}{2} \int \left(2 - \sec^2 \frac{x}{2}\right) dx$$

$$= \frac{1}{2} \left[2x - \frac{\tan \frac{x}{2}}{\frac{1}{2}}\right] + C$$

$$= x - \tan \frac{x}{2} + C$$

Answer needs Correction? Click Here

Q10: sin<sup>4</sup> x

## Answer:

$$\sin^4 x = \sin^2 x \sin^2 x$$

$$= \left(\frac{1 - \cos 2x}{2}\right) \left(\frac{1 - \cos 2x}{2}\right)$$

$$= \frac{1}{4} \left(1 - \cos 2x\right)^2$$

$$= \frac{1}{4} \left[1 + \cos^2 2x - 2\cos 2x\right]$$

$$= \frac{1}{4} \left[1 + \left(\frac{1 + \cos 4x}{2}\right) - 2\cos 2x\right]$$

$$= \frac{1}{4} \left[1 + \frac{1}{2} + \frac{1}{2}\cos 4x - 2\cos 2x\right]$$

$$= \frac{1}{4} \left[\frac{3}{2} + \frac{1}{2}\cos 4x - 2\cos 2x\right]$$

$$\int \sin^4 x \, dx = \frac{1}{4} \int \left[ \frac{3}{2} + \frac{1}{2} \cos 4x - 2 \cos 2x \right] dx$$

$$= \frac{1}{4} \left[ \frac{3}{2} x + \frac{1}{2} \left( \frac{\sin 4x}{4} \right) - \frac{2 \sin 2x}{2} \right] + C$$

$$= \frac{1}{8} \left[ 3x + \frac{\sin 4x}{4} - 2 \sin 2x \right] + C$$

$$= \frac{3x}{8} - \frac{1}{4} \sin 2x + \frac{1}{32} \sin 4x + C$$

Answer needs Correction? Click Here

Q11: cos<sup>4</sup> 2*x* 

Answer:

$$\cos^{4} 2x = \left(\cos^{2} 2x\right)^{2}$$

$$= \left(\frac{1 + \cos 4x}{2}\right)^{2}$$

$$= \frac{1}{4} \left[1 + \cos^{2} 4x + 2\cos 4x\right]$$

$$= \frac{1}{4} \left[1 + \left(\frac{1 + \cos 8x}{2}\right) + 2\cos 4x\right]$$

$$= \frac{1}{4} \left[1 + \frac{1}{2} + \frac{\cos 8x}{2} + 2\cos 4x\right]$$

$$= \frac{1}{4} \left[\frac{3}{2} + \frac{\cos 8x}{2} + 2\cos 4x\right]$$

$$\therefore \int \cos^{4} 2x \, dx = \int \left(\frac{3}{8} + \frac{\cos 8x}{8} + \frac{\cos 4x}{2}\right) dx$$

$$= \frac{3}{8} x + \frac{\sin 8x}{64} + \frac{\sin 4x}{8} + C$$

Answer needs Correction? Click Here

Answer:

$$\frac{\sin^2 x}{1 + \cos x} = \frac{\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}} \quad \left[\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}; \cos x = 2\cos^2\frac{x}{2} - 1\right]$$

$$= \frac{4\sin^2\frac{x}{2}\cos^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$$

$$= 2\sin^2\frac{x}{2}$$

$$= 1 - \cos x$$

$$\therefore \int \frac{\sin^2 x}{1 + \cos x} dx = \int (1 - \cos x) dx$$

$$= x - \sin x + C$$

Answer needs Correction? Click Here

Q13:  $\frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}$ 

Answer:

$$\begin{split} \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} &= \frac{-2\sin \frac{2x + 2\alpha}{2} \sin \frac{2x - 2\alpha}{2}}{-2\sin \frac{x + \alpha}{2} \sin \frac{x - \alpha}{2}} \qquad \left[ \cos C - \cos D = -2\sin \frac{C + D}{2} \sin \frac{C - D}{2} \right] \\ &= \frac{\sin \left( x + \alpha \right) \sin \left( x - \alpha \right)}{\sin \left( \frac{x + \alpha}{2} \right) \sin \left( \frac{x - \alpha}{2} \right)} \\ &= \frac{\left[ 2\sin \left( \frac{x + \alpha}{2} \right) \cos \left( \frac{x + \alpha}{2} \right) \right] \left[ 2\sin \left( \frac{x - \alpha}{2} \right) \cos \left( \frac{x - \alpha}{2} \right) \right]}{\sin \left( \frac{x + \alpha}{2} \right) \sin \left( \frac{x - \alpha}{2} \right)} \\ &= 4\cos \left( \frac{x + \alpha}{2} \right) \cos \left( \frac{x - \alpha}{2} \right) \\ &= 2\left[ \cos \left( \frac{x + \alpha}{2} \right) + \cos \frac{x + \alpha}{2} - \frac{x - \alpha}{2} \right] \\ &= 2\left[ \cos \left( x \right) + \cos \alpha \right] \\ &= 2\cos x - \cos \alpha \\ &= 2\left[ \sin x + x \cos \alpha \right] + C \end{split}$$

Answer needs Correction? Click Here

Q14:  $\frac{\cos x - \sin x}{1 + \sin 2x}$ 

Answer:

$$\frac{\cos x - \sin x}{1 + \sin 2x} = \frac{\cos x - \sin x}{\left(\sin^2 x + \cos^2 x\right) + 2\sin x \cos x}$$

$$= \frac{\cos x - \sin x}{\left(\sin x + \cos x\right)^2}$$
Let  $\sin x + \cos x = t$ 

$$\therefore (\cos x - \sin x) dx = dt$$

$$\Rightarrow \int \frac{\cos x - \sin x}{1 + \sin 2x} dx = \int \frac{\cos x - \sin x}{\left(\sin x + \cos x\right)^2} dx$$

$$= \int \frac{dt}{t^2}$$

$$= \int t^2 dt$$

$$= -t^{-1} + C$$

$$= -\frac{1}{t} + C$$

$$= \frac{-1}{\sin x + \cos x} + C$$

Answer needs Correction? Click Here

Q15:  $\tan^{3} 2x \sec 2x$ 

$$\tan^3 2x \sec 2x = \tan^2 2x \tan 2x \sec 2x$$

$$= (\sec^2 2x - 1) \tan 2x \sec 2x$$

$$= \sec^2 2x \cdot \tan 2x \sec 2x - \tan 2x \sec 2x$$

$$\therefore \int \tan^3 2x \sec 2x \, dx = \int \sec^2 2x \tan 2x \sec 2x \, dx - \int \tan 2x \sec 2x \, dx$$

$$= \int \sec^2 2x \tan 2x \sec 2x \, dx - \frac{\sec 2x}{2} + C$$
Let  $\sec 2x = t$ 

$$\therefore 2 \sec 2x \tan 2x \, dx = dt$$

```
\therefore \int \tan^3 2x \sec 2x \, dx = \frac{1}{2} \int t^2 dt - \frac{\sec 2x}{2} + C
= \frac{t^3}{6} - \frac{\sec 2x}{2} + C
= \frac{(\sec 2x)^3}{6} - \frac{\sec 2x}{2} + C
```

Q16 : tan<sup>4</sup>x

#### Answer:

$$\tan^4 x$$

$$= \tan^2 x \cdot \tan^2 x$$

$$= (\sec^2 x - 1)\tan^2 x$$

$$= \sec^2 x \tan^2 x - \tan^2 x$$

$$= \sec^2 x \tan^2 x - (\sec^2 x - 1)$$

$$= \sec^2 x \tan^2 x - \sec^2 x + 1$$

$$\therefore \int \tan^4 x \, dx = \int \sec^2 x \tan^2 x \, dx - \int \sec^2 x \, dx + \int 1 \cdot dx$$

$$= \int \sec^2 x \tan^2 x \, dx - \tan x + x + C \qquad \dots (1)$$

Consider  $\int \sec^2 x \tan^2 x \, dx$ Let  $\tan x = t \Rightarrow \sec^2 x \, dx = dt$  $\Rightarrow \int \sec^2 x \tan^2 x dx = \int t^2 dt = \frac{t^3}{3} = \frac{\tan^3 x}{3}$ 

From equation (1), we obtain

$$\int \tan^4 x \ dx = \frac{1}{3} \tan^3 x - \tan x + x + C$$

# Answer needs Correction? Click Here

Q17: 
$$\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$$

#### Answer:

$$\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} = \frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x}$$

$$= \tan x \sec x + \cot x \csc x$$

$$\therefore \int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int (\tan x \sec x + \cot x \csc x) dx$$

$$= \sec x - \csc x + C$$

# Answer needs Correction? Click Here

Q18: 
$$\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$$

### Answer:

$$\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$$

$$= \frac{\cos 2x + (1 - \cos 2x)}{\cos^2 x} \qquad \left[\cos 2x = 1 - 2\sin^2 x\right]$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

$$\therefore \int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx = \int \sec^2 x dx = \tan x + C$$

Answer needs Correction? Click Here

Q19: 
$$\frac{1}{\sin x \cos^3 x}$$

Answer:

## Answer needs Correction? Click Here

Q20: 
$$\frac{\cos 2x}{\left(\cos x + \sin x\right)^2}$$

$$\frac{\cos 2x}{\left(\cos x + \sin x\right)^2} = \frac{\cos 2x}{\cos^2 x + \sin^2 x + 2\sin x \cos x} = \frac{\cos 2x}{1 + \sin 2x}$$

$$\therefore \int \frac{\cos 2x}{(\cos x + \sin x)^2} dx = \int \frac{\cos 2x}{(1 + \sin 2x)} dx$$
Let  $1 + \sin 2x = t$ 

$$\Rightarrow 2 \cos 2x dx = dt$$

$$\therefore \int \frac{\cos 2x}{(\cos x + \sin x)^2} dx = \frac{1}{2} \int \frac{1}{t} dt$$

$$= \frac{1}{2} \log|t| + C$$

$$= \frac{1}{2} \log|t| + \sin 2x| + C$$

$$= \frac{1}{2} \log|\sin x + \cos x|^2 + C$$

$$= \log|\sin x + \cos x| + C$$

Q21:  $\sin^{-1}(\cos x)$ 

#### Answer:

 $\sin^{-1}(\cos x)$ 

Let  $\cos x = t$ 

Then,  $\sin x = \sqrt{1-t^2}$ 

$$\Rightarrow (-\sin x) dx = dt$$

$$dx = \frac{-dt}{\sin x}$$

$$dx = \frac{-dt}{\sqrt{1-t^2}}$$

$$\therefore \int \sin^{-1}(\cos x)dx = \int \sin^{-1}t \left(\frac{-dt}{\sqrt{1-t^2}}\right)$$
$$= -\int \frac{\sin^{-1}t}{\sqrt{1-t^2}}dt$$

Let 
$$\sin^{-1} t = u$$

$$\Rightarrow \frac{1}{\sqrt{1-t^2}} dt = du$$

$$\int \sin^{-1}(\cos x)dx = \int 4du$$
$$= -\frac{u^2}{2} + C$$

$$= -\frac{u^{2}}{2} + C$$

$$= -\frac{(\sin^{1} t)^{2}}{2} + C$$

$$= -\frac{[\sin^{-1}(\cos x)]^{2}}{2} + C \qquad \dots (1)$$

It is known that,

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$
$$\therefore \sin^{-1} (\cos x) = \frac{\pi}{2} - \cos^{-1} (\cos x) = \left(\frac{\pi}{2} - x\right)$$

Substituting in equation (1), we obtain

$$\int \sin^{-1}(\cos x) dx = \frac{-\left[\frac{\pi}{2} - x\right]^2}{2} + C$$

$$= -\frac{1}{2} \left(\frac{\pi^2}{2} + x^2 - \pi x\right) + C$$

$$= -\frac{\pi^2}{8} - \frac{x^2}{2} + \frac{1}{2}\pi x + C$$

$$= \frac{\pi x}{2} - \frac{x^2}{2} + \left(C - \frac{\pi^2}{8}\right)$$

$$= \frac{\pi x}{2} - \frac{x^2}{2} + C_1$$

Answer needs Correction? Click Here

Q22: 
$$\frac{1}{\cos(x-a)\cos(x-b)}$$

$$\frac{1}{\cos(x-a)\cos(x-b)} = \frac{1}{\sin(a-b)} \left[ \frac{\sin(a-b)}{\cos(x-a)\cos(x-b)} \right]$$

$$= \frac{1}{\sin(a-b)} \left[ \frac{\sin[(x-b)-(x-a)]}{\cos(x-a)\cos(x-b)} \right]$$

$$= \frac{1}{\sin(a-b)} \left[ \frac{\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)} \right]$$

$$= \frac{1}{\sin(a-b)} \left[ \tan(x-b)-\tan(x-a) \right]$$

$$\Rightarrow \int \frac{1}{\cos(x-a)\cos(x-b)} dx = \frac{1}{\sin(a-b)} \int \left[ \tan(x-b)-\tan(x-a) \right] dx$$

$$= \frac{1}{\sin(a-b)} \left[ -\log|\cos(x-b)| + \log|\cos(x-a)| \right]$$

$$= \frac{1}{\sin(a-b)} \left[ \log \left| \frac{\cos(x-a)}{\cos(x-b)} \right| \right] + C$$

Q23:  $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$  is equal to

A.  $\tan x + \cot x + C$ 

B. tan x + cosec x + C

C. -  $\tan x + \cot x + C$ 

D. tan x + sec x + C

#### Answer:

$$\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx = \int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} - \frac{\cos^2 x}{\sin^2 x \cos^2 x}\right) dx$$
$$= \int \left(\sec^2 x - \csc^2 x\right) dx$$
$$= \tan x + \cot x + C$$

Hence, the correct answer is A.

# Answer needs Correction? Click Here

Q24: 
$$\int \frac{e^x(1+x)}{\cos^2(e^x x)} dx$$
 equals

A. -  $\cot(e^x x) + C$ 

B. tan (*xe*<sup>x</sup>) + C

C. tan (*e*<sup>x</sup>) + C

D.  $\cot(e^x) + C$ 

### Answer:

$$\int \frac{e^x (1+x)}{\cos^2(e^x x)} dx$$

Let  $e^{x}x = t$ 

$$\Rightarrow (e^x \cdot x + e^x \cdot 1) dx = dt$$

$$e^{x}(x+1)dx = dt$$

$$\therefore \int \frac{e^x (1+x)}{\cos^2(e^x x)} dx = \int \frac{dt}{\cos^2 t}$$

$$= \int \sec^2 t dt$$

$$= \tan t + C$$

$$= \tan(e^x \cdot x) + C$$

Hence, the correct answer is B.

Answer needs Correction? Click Here

\*\*\*\*\*\*\*\*\* END \*\*\*\*\*\*\*