Definite Integrals Ex 20.2 Q14

Let $3+4\sin x = t$

Differentiating w.r.t. x, we get

$$4\cos x dx = dt$$

$$\cos x dx = \frac{dt}{4}$$

Now,

$x = 0 \Rightarrow t = 3$

$x = \frac{\pi}{3} \Rightarrow t = 3+2\sqrt{3}$

$$\therefore \int_0^{\frac{\pi}{3}} \frac{\cos x}{3+4\sin x} dx$$

$$= \int_3^{3+2\sqrt{3}} \frac{dt}{4t}$$

$$= \frac{1}{4}\left[\log t\right]_3^{3+2\sqrt{3}}$$

$$= \frac{1}{4}\left[\log\left(3+2\sqrt{3}\right) - \log 3\right]$$

$$= \frac{1}{4}\log\left(\frac{3+2\sqrt{3}}{3}\right)$$

$$\therefore \int_0^{\frac{\pi}{3}} \frac{\cos x}{3+4\sin x} dx = \frac{1}{4}\log\left(\frac{3+2\sqrt{3}}{3}\right)$$

Definite Integrals Ex 20.2 Q15

Let $\tan^{-1}x = t$

Differentiating w.r.t. x, we get

$$\frac{1}{1+x^2}dx = dt$$

Now,

$x = 0 \Rightarrow t = 0$

$x = 1 \Rightarrow t = \frac{\pi}{4}$

$$\therefore \int_0^1 \frac{\sqrt{\tan^{-1}x}}{1+x^2}dx$$

$$= \int_0^{\frac{\pi}{4}} t^{1/2}\,dt$$

$$= \left[\frac{t^{3/2}}{3/2}\right]_0^{\frac{\pi}{4}}$$

$$= \frac{2}{3}\left[t^{3/2}\right]_0^{\frac{\pi}{4}}$$

$$= \frac{2}{3}\left[\left(\frac{\pi}{4}\right)^{3/2} - 0\right]$$

$$= \frac{1}{12}\pi^{3/2}$$

$$\therefore \int_0^1 \frac{\sqrt{\tan^{-1}x}}{1+x^2}dx = \frac{1}{12}\pi^{3/2}$$

Definite Integrals Ex 20.2 Q16

$$\int_0^2 x\sqrt{x+2}\,dx$$

Let $x + 2 = t^2 \Rightarrow dx = 2t\,dt$

When $x = 0$, $t = \sqrt{2}$ and when $x = 2$, $t = 2$

$$\therefore \int_0^2 x\sqrt{x+2}\,dx = \int_{\sqrt{2}}^2 \left(t^2-2\right)\sqrt{t^2}\,2t\,dt$$

$$= 2\int_{\sqrt{2}}^2 \left(t^2-2\right)t^2\,dt$$

$$= 2\int_{\sqrt{2}}^2 \left(t^4-2t^2\right)dt$$

$$= 2\left[\frac{t^5}{5}-\frac{2t^3}{3}\right]_{\sqrt{2}}^2$$

$$= 2\left[\frac{32}{5}-\frac{16}{3}-\frac{4\sqrt{2}}{5}+\frac{4\sqrt{2}}{3}\right]$$

$$= 2\left[\frac{96-80-12\sqrt{2}+20\sqrt{2}}{15}\right]$$

$$= 2\left[\frac{16+8\sqrt{2}}{15}\right]$$

$$= \frac{16\left(2+\sqrt{2}\right)}{15}$$

$$= \frac{16\sqrt{2}\left(\sqrt{2}+1\right)}{15}$$

Definite Integrals Ex 20.2 Q17

Let $x = \tan\theta$

Differentiating w.r.t. x, we get

$$dx = \sec^2\theta d\theta$$

Now,

$x = 0 \Rightarrow \theta = 0$

$x = 1 \Rightarrow \theta = \frac{\pi}{4}$

$$\int_0^1 \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$$

$$= \int_0^{\frac{\pi}{4}} \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)\sec^2\theta d\theta \qquad \left[\because \tan^2\theta = \frac{2\tan\theta}{1-\tan^2\theta}\right]$$

$$= \int_0^{\frac{\pi}{4}} \tan^{-1}(\tan 2\theta)\sec^2\theta d\theta$$

$$= \int_0^{\frac{\pi}{4}} 2\theta\sec^2\theta\, d\theta$$

Applying by parts, we get

$$= 2\left[\theta \int_0^{\frac{\pi}{4}} \sec^2\theta d\theta - \int_0^{\frac{\pi}{4}}\left(\sec^2\theta d\theta\right)\frac{d\theta}{d\theta}d\theta\right]$$

$$=2\left[\theta\tan\theta\right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \tan\theta d\theta\Big]$$

$$= 2\left[\theta\tan\theta + \log(\cos\theta)\right]_0^{\frac{\pi}{4}}$$

$$= 2\left[\frac{\pi}{4} + \log\left(\frac{1}{\sqrt{2}}\right) - 0 - 0\right]$$

$$= 2\left[\frac{\pi}{4} + \frac{1}{2}\log 2\right]$$

$$= \frac{\pi}{2} - \log 2$$

$$\therefore \int_0^1 \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx = \frac{\pi}{2} - \log 2$$

*********** END **********