$$\frac{\beta}{a\alpha+b}+\frac{\alpha}{a\beta+b}=\frac{a\left(\frac{b^2}{a^2}-2\times\frac{c}{a}\right)+\left(\frac{-b^2}{a}\right)}{a\times c-b^2+b^2}$$

$$\frac{\beta}{a\alpha+b}+\frac{\alpha}{a\beta+b}=\frac{a\left(\frac{b^2}{a^2}-\frac{2c\times a}{a\times a}\right)+\left(\frac{-b^2}{a}\right)}{a\times c-\cancel{b^2}+\cancel{b^2}}$$

$$\frac{\beta}{a\alpha+b}+\frac{\alpha}{a\beta+b}=\frac{a\left(\frac{b^2}{a^2}-\frac{2ca}{a^2}\right)+\left(\frac{-b^2}{a}\right)}{ac}$$

$$\frac{\beta}{a\alpha+b}+\frac{\alpha}{a\beta+b}=\frac{a\left(\frac{b^2-2ca}{a^2}\right)+\left(\frac{-b^2}{a}\right)}{ac}$$

$$\frac{\beta}{a\alpha+b}+\frac{\alpha}{a\beta+b}=\frac{\cancel{a}\left(\frac{b^2-2ca}{a^{\cancel{2}1}}\right)+\left(\frac{-b^2}{a}\right)}{ac}$$

$$\frac{\beta}{a\alpha+b}+\frac{\alpha}{a\beta+b}=\frac{\left(\frac{b^2-2ca}{a}\right)+\left(\frac{-b^2}{a}\right)}{ac}$$

$$\frac{\beta}{a\alpha+b}+\frac{\alpha}{a\beta+b}=\frac{\left(\frac{b^2-2ca-b^2}{a}\right)}{ac}$$

$$\frac{\beta}{a\alpha+b}+\frac{\alpha}{a\beta+b}=\frac{\left(\frac{\cancel{b^2}-2ca-\cancel{b^2}}{a}\right)}{ac}$$

$$\frac{\beta}{a\alpha+b}+\frac{\alpha}{a\beta+b}=\left(\frac{-2\cancel{ca}}{a}\right)\times\frac{1}{\cancel{ac}}$$

$$\frac{\beta}{a\alpha+b}+\frac{\alpha}{a\beta+b}=\frac{-2}{a}$$

Hence, the value of $\frac{\beta}{a\alpha+b}+\frac{\alpha}{a\beta+b}$ is $\boxed{\frac{-2}{a}}$.

(*viii*) Since α and β are the zeros of the quadratic polynomial $f(x)=ax^2+bx+c$

$$\alpha+\beta=\frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$=\frac{-b}{a}$$

$$\alpha\beta=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$=\frac{c}{a}$$

We have, $a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)$

$$a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)=a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)$$

$$a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)=a\left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right)+b\left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right)$$

$$a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)=a\left(\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}\right)+\left(\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}\right)$$

By substituting $\alpha+\beta=\frac{-b}{a}$ and $\alpha\beta=\frac{c}{a}$ we get ,

$$a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)=a\left(\frac{\left(\frac{-b}{a}\right)^3-3\times\frac{c}{a}\left(\frac{-b}{a}\right)}{\frac{c}{a}}\right)+b\left(\frac{\left(\frac{-b}{a}\right)^2-2\times\frac{c}{a}}{\frac{c}{a}}\right)$$

$$a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)=a\left(\frac{\frac{-b^3}{a^3}-\frac{3bc}{a^2}}{\frac{c}{a}}\right)+b\left(\frac{\frac{-b}{a}-\frac{2c}{a}}{\frac{c}{a}}\right)$$

$$a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)=a\left(\frac{-b^3+3bca}{a^{\cancel{3}^2}}\times\frac{\cancel{a}}{c}\right)+b\left(\frac{b^2-2ca}{a^{\cancel{2}^1}}\times\frac{\cancel{a}}{c}\right)$$

$$a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)=\cancel{a}\left(\frac{-b^3+3abc}{a^{\cancel{2}^1}c}\right)+b\left(\frac{b^2-2ca}{ac}\right)$$

$$a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)=\frac{-b^3+3abc}{ac}+\frac{b^3-2abc}{ac}$$

$$a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)=\frac{\cancel{-b^3}+3abc+\cancel{b^3}-2abc}{ac}$$

$$a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)=\frac{3abc-2abc}{ac}$$

$$a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)=\frac{\cancel{a}b\cancel{c}}{\cancel{a}\cancel{c}}$$

$$a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)=b$$

Hence, the value of $a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)$ is $\boxed{b}$.

*********** END **********