



Chapter 9 Continuity Ex 9.2 Q5

We have given that  $f(x)$  is continuous on  $[0, \infty]$

$\therefore f(x)$  is continuous at  $x = 1$  and  $x = \sqrt{2}$

$\therefore$  At  $x = 1$ , LHL = RHL =  $f(1)$  ..... (A)

$$f(1) = a \quad \dots (1)$$

$$\text{LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{h \rightarrow 0} f(1-h) = \lim_{h \rightarrow 0} \frac{(1-h)^2}{a} = \frac{1}{a}$$

Using (A) we get,

$$a = \frac{1}{a} \Rightarrow a^2 = 1 \Rightarrow a = \pm 1$$

$$\text{At } x = \sqrt{2} \text{ LHL} = \text{RHL} = f(\sqrt{2}) \quad \dots (B)$$

$$f(\sqrt{2}) = \frac{2b^2 - 4b}{(\sqrt{2})^2} = \frac{2b^2 - 4b}{2} = b^2 - 2b \quad \dots (2)$$

$$\text{LHL} = \lim_{x \rightarrow \sqrt{2}^-} f(x) = \lim_{h \rightarrow 0} f(\sqrt{2} - h) = \lim_{h \rightarrow 0} a = a.$$

So, using (B), we get,

$$b^2 - 2b = a$$

$$\text{For } a = 1, \quad b^2 - 2b - 1 = 0$$

$$\Rightarrow b = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$$

$$\text{For } a = -1 \quad b^2 - 2b + 1 = 0$$

$$\Rightarrow (b-1)^2 = 0 \Rightarrow b = 1$$

$$\text{Thus, } a = -1, \quad b = 1 \text{ or } a = 1, \quad b = 1 \pm \sqrt{2}$$

Chapter 9 Continuity Ex 9.2 Q6

Since,  $f(x)$  is continuous on  $[0, \pi]$

$f(x)$  is continuous at  $x = \frac{\pi}{4}$  and  $x = \frac{\pi}{2}$

At  $x = \frac{\pi}{4}$ ,

$$\text{LHL} = \text{RHL} = f\left(\frac{\pi}{4}\right) \dots (A)$$

$$\text{Now, } f\left(\frac{\pi}{4}\right) = 2 \cdot \frac{\pi}{4} \cdot \cot\left(\frac{\pi}{4}\right) + b = \frac{\pi}{2} \cdot 1 + b = \frac{\pi}{2} + b \dots (1)$$

$$\text{LHL} = \lim_{x \rightarrow \frac{\pi}{4}^-} f(x) = \lim_{h \rightarrow 0} f\left(\frac{\pi}{4} - h\right) = \lim_{h \rightarrow 0} \left(\frac{\pi}{4} - h\right) + a\sqrt{2} \sin\left(\frac{\pi}{4} - h\right) = \frac{\pi}{4} + a\sqrt{2} \cdot \frac{1}{\sqrt{2}} = \frac{\pi}{4} + a$$

Thus, using (A)

$$\frac{1}{2} + b = \frac{\pi}{4} + a$$

$$a - b = \frac{\pi}{4} \dots (B)$$

At  $x = \frac{\pi}{2}$

$$\text{LHL} = \text{RHL} = f\left(\frac{\pi}{2}\right) \dots (C)$$

$$\text{Now, } f\left(\frac{\pi}{2}\right) = a \cos 2 \cdot \frac{\pi}{2} - b \sin \frac{\pi}{2} = -a - b \dots (2)$$

$$\text{LHL} = \lim_{x \rightarrow \frac{\pi}{2}^-} f(x) = \lim_{h \rightarrow 0} f\left(\frac{\pi}{2} - h\right)$$

$$= \lim_{h \rightarrow 0} 2 \left(\frac{\pi}{2} - h\right) \cot\left(\frac{\pi}{2} - h\right) + b = \pi \times 0 + b = b$$

using (C), we get,

$$-a - b = b \Rightarrow 2b = -a \Rightarrow b = \frac{-a}{2}$$

$$\text{from (B), } a + \frac{a}{2} = \frac{\pi}{4}$$

$$\Rightarrow \frac{3}{2}a = \frac{\pi}{4}$$

$$\Rightarrow a = \frac{\pi}{6}$$

$$\text{and } b = \frac{-a}{2} = \frac{-\pi}{12}$$

$$\text{Thus, } a = \frac{\pi}{6}, \quad b = \frac{-\pi}{12}$$

It is given that the  $f(x)$  is continuous on  $[0, 8]$

$f(x)$  is continuous at  $x = 2$  and  $x = 4$ .

Now, At  $x = 2$

$$\text{LHL} = \text{RHL} = f(2) \dots (A)$$

$$f(2) = 3 \times 2 + 2 = 8 \dots (1)$$

$$\text{LHL} = \lim_{x \rightarrow 2^-} f(x) = \lim_{h \rightarrow 0} f(2-h) = \lim_{h \rightarrow 0} (2-h)^2 + a(2-h) + b = 4 + 2a + b$$

from (A)

$$4 + 2a + b = 8$$

$$2a + b = 4 \dots (B)$$

Now, At  $x = 4$

$$\text{LHL} = \text{RHL} = f(4) \dots (C)$$

$$f(4) = 3 \times 4 + 2 = 14 \dots (2)$$

$$\text{RHL} = \lim_{x \rightarrow 4^+} f(x) = \lim_{h \rightarrow 0} f(4+h) = \lim_{h \rightarrow 0} 2a(4+h) + 5b = 8a + 5b$$

From (C), we get,

$$8a + 5b = 14 \dots (D)$$

Solving (B) and (D), we get,

$$a = 3 \text{ and } b = -2$$

#### Chapter 9 Continuity Ex 9.2 Q8

The function will be continuous on  $\left[0, \frac{\pi}{2}\right]$  if it is continuous at every point in  $\left[0, \frac{\pi}{2}\right]$

Let us consider the point  $x = \frac{\pi}{4}$ ,

We must have,

$$\text{LHL} = \text{RHL} = f\left(\frac{\pi}{4}\right) \dots (A)$$

$$\text{LHL} = \lim_{x \rightarrow \frac{\pi}{4}^-} f(x) = \lim_{h \rightarrow 0} f\left(\frac{\pi}{4} - h\right) = \lim_{h \rightarrow 0} \frac{\tan\left(\frac{\pi}{4} - \frac{\pi}{4} + h\right)}{\cot + 2\left(\frac{\pi}{4} - h\right)} = \lim_{h \rightarrow 0} \frac{\tan h}{\tan 2h} \quad \left[ \because \cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta \right]$$

$$= \lim_{h \rightarrow 0} \frac{\frac{\tan h}{h}}{\frac{\tan 2h}{h}} = \frac{1}{2}$$

Thus, using (A) we get,

$$f\left(\frac{\pi}{4}\right) = \frac{1}{2}$$

Hence,  $f(x)$  will be continuous on  $\left[0, \frac{\pi}{2}\right]$  if  $f\left(\frac{\pi}{4}\right) = \frac{1}{2}$ .

#### Chapter 9 Continuity Ex 9.2 Q9

When  $x < 2$ , we have

$f(x) = 2x - 1$ , which is a polynomial of degree 1.

So,  $f(x)$  is continuous for  $x < 2$ .

When  $x > 2$ , we have

$f(x) = \frac{3x}{2}$ , which is again a polynomial of degree 1.

So,  $f(x)$  is continuous for  $x > 2$ .

Now, consider the point  $x = 2$

$$f(2) = \frac{3 \times 2}{2} = 3$$

$$\text{LHL} = \lim_{x \rightarrow 2^-} f(x) = \lim_{h \rightarrow 0} f(2 - h) = \lim_{h \rightarrow 0} 2(2 - h) - 1 = 3$$

$$\text{RHL} = \lim_{x \rightarrow 2^+} f(x) = \lim_{h \rightarrow 0} f(2 + h) = \lim_{h \rightarrow 0} \frac{3(2 + h)}{2} = 3$$

$$\text{LHL} = \text{RHL} = f(2) = 3$$

Thus,  $f(x)$  is continuous at  $x = 2$

Hence,  $f(x)$  is continuous every where.

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