Maxima and Minima 18.5 Q39

Let s be the sum of the surface areas of a sphere and a cube.

$$\therefore \quad s = 4\pi r^2 + 6l^2 \qquad \text{---(i)}$$

Let v = volume of sphere + volume of cube

$$\Rightarrow \quad v = \frac{4}{3}\pi r^3 + l^3 \qquad \text{---(ii)}$$

From (i)

$$l = \sqrt{\frac{s - 4\pi r^2}{6}}$$

$$\therefore \quad v = \frac{4}{3}\pi r^2 + \left(\frac{s - 4\pi r^2}{6}\right)^{\frac{3}{2}}$$

$$\therefore \quad \frac{dV}{dr} = 4\pi r^2 + \frac{3}{2}\left(\frac{s - 4\pi r^2}{6}\right)^{\frac{1}{2}} \times \left(\frac{-4\pi}{6}\right)^{2r}$$

For maxima and minima,

$$\frac{dv}{dr} = 0$$

$$\Rightarrow \quad 4\pi r^2 = \frac{\pi}{6}\left(s - 4\pi r^2\right)^{\frac{1}{2}} \times 2r = 0$$

$$\Rightarrow \quad 2r\pi\left[2r - l\right] = 0$$

$$\therefore \quad r = 0, \ \frac{l}{2}$$

Now,

$$\frac{d^2v}{dr^2} = 8\pi r - \frac{2\pi}{\sqrt{6}}\left[\left(s - 4\pi r^2\right)\right]^{\frac{1}{2}} - \frac{8\pi r^2}{2\left(s - 4\pi r^2\right)^{\frac{1}{2}}}$$

At $\quad r = \dfrac{l}{2}$

$$\frac{d^2v}{dr^2} = \pi\frac{l}{2} - \frac{2\pi}{\sqrt{6}}\left[\sqrt{6}l - \frac{8\pi\frac{l^2}{4}}{2\sqrt{6}l}\right] = 4\pi l - \frac{2\pi}{\sqrt{6}}\left[\frac{12l^2 - 2\pi l^2}{2\sqrt{6}l}\right]$$

Maxima and Minima 18.5 Q40

Let ABCDEF be a half cylinder with rectangular base and semi-circular ends.
Here AB = height of the cylinder
AB = h
Let r be the radius of the cylinder.

Volume of the half cylinder is $V = \frac{1}{2}\pi r^2 h$

$\Rightarrow \frac{2v}{\pi r^2} = h$

$\therefore$ TSA of the half cylinder is
S = LSA of the half cylinder + area of two semi-circular ends + area of the rectangle (base)

$S = \pi r h + \frac{\pi r^2}{2} + \frac{\pi r^2}{2} + h \times 2r$

$S = (\pi r + 2r)h + \pi r^2$

$S = (\pi r + 2r)\frac{2v}{\pi r^2} + \pi r^2$

$S = (\pi + 2)\frac{2v}{\pi r} + \pi r^2$

Differentiate S wrt r we get,

$\frac{ds}{dr} = \left[(\pi + 2) \times \frac{2v}{\pi}\left(\frac{-1}{r^2}\right) + 2\pi r\right]$

For maximum and minimum values of S, we have $\frac{ds}{dr} = 0$

$\Rightarrow (\pi + 2) \times \frac{2v}{\pi}\left(\frac{-1}{r^2}\right) + 2\pi r = 0$

$\Rightarrow (\pi + 2) \times \frac{2v}{\pi r^2} = 2\pi r$

But 2r = D

$\therefore h : D = \pi : \pi + 2$

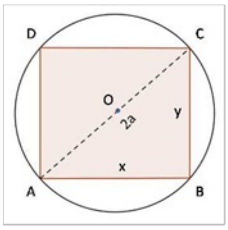
Differentiate $\frac{ds}{dr}$ wrt r we get,

$\frac{d^2s}{dr^2} = (\pi + 2)\frac{V}{\pi} \times \frac{2}{r^3} + 2\pi > 0$

Thus S will be minimum when h : 2r is π : π - 12.
Height of the cylinder : Diameter of the circular end
$\pi : \pi + 2$

Maxima and Minima 18.5 Q41

Let *ABCD* be the cross-sectional area of the beam which is cut from *a* circular log of radius *a*.



$\therefore \qquad AO = a \Rightarrow AC = 2a$

Let x be the width of log and y be the depth of log $ABCD$

Let S be the strength of the beam according to the question,

$$S = xy^2 \qquad \text{---(i)}$$

In ΔABC

$$x^2 + y^2 = (2a)^2$$

$$\Rightarrow \qquad y = (2a)^2 - x^2 \qquad \text{---(ii)}$$

From (i) and (ii), we get

$$S = x\left((2a)^2 - x^2\right)$$

$$\Rightarrow \qquad \frac{dS}{dx} = \left(4a^2 - x^2\right) - 2x^2$$

$$\Rightarrow \qquad \frac{dS}{dx} = 4a^2 - 3x^2$$

For maxima or minima

$$\frac{dS}{dx} = 0$$

$$\Rightarrow \qquad 4a^2 - 3x^2 = 0$$

$$\Rightarrow \qquad x^2 = \frac{4a^2}{3}$$

$$\therefore \qquad x = \frac{2a}{\sqrt{3}}$$

From (ii),

$$y^2 = 4a^2 - \frac{4a^2}{3} = \frac{8a^2}{3}$$

$$\therefore \qquad y = 2a \times \sqrt{\frac{2}{3}}$$

Now,

$$\frac{d^2S}{dx^2} = -6x$$

At $\qquad x = \frac{2a}{\sqrt{3}},\ y = \sqrt{\frac{2}{3}}2a,\ \ \frac{d^2S}{dx^2} = -\frac{12a}{\sqrt{3}} < 0$

$\therefore \qquad \left(x = \frac{2a}{\sqrt{3}}, y = \sqrt{\frac{2}{3}}2a\right)$ is the point of local maxima.

Hence,

The dimension of strongest beam is width = $x = \frac{2a}{\sqrt{3}}$ and depth = $y = \sqrt{\frac{2}{3}}2a$.

*********** END **********