Higher Order Derivatives Ex 12.1 Q48

$x = a\sin t - b\cos t; \quad y = a\cos t + b\sin t$

Differentiating both w.r.t. $t$

$$\Rightarrow \quad \frac{dx}{dt} = a\cos t + b\sin t; \quad \frac{dy}{dt} = -a\sin t + b\cos t$$

$$\Rightarrow \quad \frac{dx}{dt} = y \quad \ldots\ldots(1) \quad ; \quad \frac{dy}{dt} = -x \ldots\ldots(2)$$

Dividing (2) by (1)

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} == -\frac{x}{y}$$

Differentiating w.r.t. $t$

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{dt} = -\left\{\frac{y\frac{dx}{dt} - x\frac{dy}{dt}}{y^2}\right\}$$

Putting values from (1) and (2)

$$\Rightarrow \quad \frac{d\left(\frac{dy}{dx}\right)}{dt} = -\left\{\frac{y^2 + x^2}{y^2}\right\} \ldots\ldots\ldots(3)$$

Dividing (3) by (1)

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\left\{\frac{y^2 + x^2}{y^2 \times y}\right\} = -\left\{\frac{x^2 + y^2}{y^3}\right\}$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q49

$y = A\sin 3x + B\cos 3x$
differentiating w.r.t. $x$

$$\Rightarrow \quad \frac{dy}{dx} = 3A\cos 3x + 3B(-\sin 3x)$$

again differentiating w.r.t. $x$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 3A(-\sin 3x)\times 3 - 3B(\cos 3x)\times 3$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -9(A\sin 3x + B\cos 3x) = -9y$$

Now adding $\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 3y$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + \frac{4dy}{dx} + 3y = -9y + 4(3A\cos 3x - 3B\sin 3x) + 3y$$

$$= 12(A\cos 3x - B\sin 3x) - 6(A\sin 3x + B\cos 3x)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + \frac{4dy}{dx} + 3y = (12A - 6B)\cos 3x - (12B + 6A)\sin 3x$$

But given,

$$\frac{d^2y}{dx^2} + \frac{4dy}{dx} + 3y = 10\cos 3x$$

Thus, $12A - 6B = 10$ ...........$(1)$
and $-(12B + 6A) = 0$ ..........$(2)$

solving $(2)$
$12B + 6A = 0 \Rightarrow 6A = -12B \Rightarrow A = -2B$

Putting value of A in $(1)$
$\Rightarrow \quad 12(-2B) - 63 = 10$
$\Rightarrow \quad -24B - 6B = 10$
$\Rightarrow \quad -30B = 10$
$\Rightarrow \quad B = \frac{-1}{3}$
$\Rightarrow \quad \therefore \quad A = -2\times\frac{-1}{3} = \frac{2}{3}$

and $A = \frac{2}{3}; \quad B = \frac{-1}{3}$

*********** END **********