CHAPTER 36

PERMANENT MAGNETS

36.1 MAGNETIC POLES AND BAR MAGNETS

We have seen that a small current loop carrying a current i, produces a magnetic field

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{\mu}}{d^3} \quad \text{... (i)}$$

at an axial point. Here $\vec{\mu} = i\vec{A}$ is the magnetic dipole moment of the current loop. The vector $\vec{A}$ represents the area-vector of the current loop. Also, a current loop placed in a magnetic field $\vec{B}$ experiences a torque

$$\vec{\Gamma} = \vec{\mu} \times \vec{B}. \quad \text{... (ii)}$$

We also know that an electric dipole produces an electric field

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{d^3} \quad \text{... (iii)}$$

at an axial point and it experiences a torque

$$\vec{\Gamma} = \vec{p} \times \vec{E} \quad \text{... (iv)}$$

when placed in an electric field. Equations (i) and (ii) for a current loop are similar in structure to the equations (iii) and (iv) for an electric dipole with $\vec{\mu}$ taking the role of $\vec{p}$ and $\frac{\mu_0}{4\pi}$ taking the role of $\frac{1}{4\pi\varepsilon_0}\cdot$ The similarity suggests that the behaviour of a current loop can be described by the following hypothetical model:

(a) There are two types of magnetic charges, positive magnetic charge and negative magnetic charge. A magnetic charge m placed in a magnetic field $\vec{B}$ experiences a force

$$\vec{F} = m\vec{B}. \quad \text{... (36.1)}$$

The force on a positive magnetic charge is along the field and the force on a negative magnetic charge is opposite to the field.

(b) A magnetic charge m produces a magnetic field

$$B = \frac{\mu_0}{4\pi}\frac{m}{r^2} \quad \text{... (36.2)}$$

at a distance r from it. The field is radially outward if the magnetic charge is positive and is inward if it is negative.

(c) A magnetic dipole is formed when a negative magnetic charge $-m$ and a positive magnetic charge $+m$ are placed at a small separation d. The magnetic dipole moment is $\mu = md$ and its direction is from $-m$ to $+m$. The line joining $-m$ and $+m$ is called the *axis* of the dipole.

(d) A current loop of area A carrying a current i may be replaced by a magnetic dipole of dipole moment $\mu = md = iA$ placed along the axis of the loop. The area-vector $\vec{A}$ points in the direction $-m$ to $+m$.

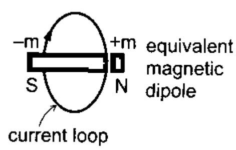

Figure 36.1

The model is very useful in studying magnetic effects and is widely used. It is customary to call a positive magnetic charge a *north pole* and a negative magnetic charge a *south pole*. They are represented by the letters N and S respectively. The quantity m is called *pole strength*. From the equation $md = iA$ or $F = mB$, we can easily see that the unit of pole strength is A–m. We can find the magnetic field due to a magnetic dipole at any point P using equation (36.2) for both the poles.

A solenoid very closely resembles a combination of circular loops placed side by side. If i be the current through it and A be the area of cross-section, the dipole moment of each turn is $\mu = iA$. In our model, each turn may be replaced by a small dipole placed at the centre of the loop along its axis. Suppose, each turn is

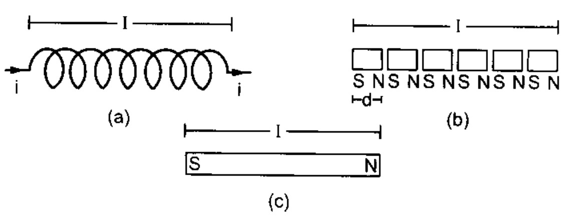

Figure 36.2

replaced by a magnetic dipole with pole strength m and separation d between the north and south poles. We have $md = iA$. Figure (36.2a) and (36.2b) show a current-carrying solenoid and its equivalent in terms of magnetic poles.

Suppose we take the value of d in such a way that the north pole of one dipole touches the south pole of the adjacent one. South poles and north poles, then, neutralise each other except at the ends. Thus, a current-carrying solenoid can be replaced by just a single south pole and a single north pole of pole strength m each, placed at a separation equal to the length of the solenoid (figure 36.2c).

We can obtain the field outside the solenoid using the above model. Each pole produces a field given by equation (36.2). The resultant field is the vector sum of the fields produced by the south pole and the north pole. Figure (36.3) shows the magnetic field lines due to a current-carrying solenoid.

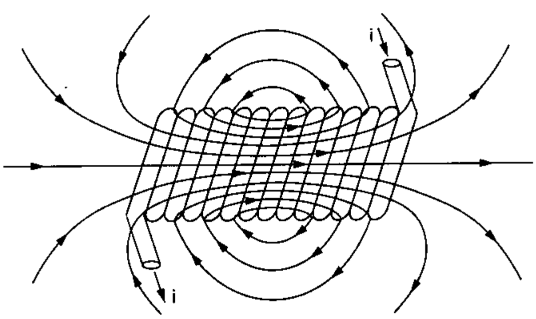

Figure 36.3

Note that the magnetic field inside the solenoid is opposite in direction from what one expects from the pole picture. The magnetic field lines are closed curves. They do not start or end at a point as is the case with electric field lines.

Example 36.1

A solenoid of length 10 cm *and radius* 1 cm *contains* 200 turns *and carries a current of* 10 A. *Find the magnetic field at a point on the axis at a distance of* 10 cm *from the centre.*

Solution : The dipole moment of each turn is

$$\mu = iA = (10\text{ A})(\pi\text{ cm}^2)$$

$$= \pi \times 10^{-3}\text{ A m}^2.$$

If each current loop is replaced by a dipole having pole strength m and separation between the poles d, we have

$$\mu = md.$$

As there are 200 turns,

$$200\,d = 10\text{ cm}$$

or, $$d = 5 \times 10^{-4}\text{ m}.$$

Thus,

$$m = \frac{\mu}{d} = \frac{\pi \times 10^{-3}\text{ A m}^2}{5 \times 10^{-4}\text{ m}} = 2\pi\text{ A m}.$$

We can replace the solenoid by a south pole and a north pole of equal pole strength 2π A m, separated by 10 cm. The equivalent picture is shown in figure (36.4).

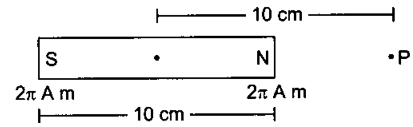

Figure 36.4

The magnetic field at P due to the north pole is

$$B_N = \frac{\mu_0}{4\pi}\frac{2\pi\text{ A m}}{(5\text{ cm})^2} = 2{\cdot}5 \times 10^{-4}\text{ T}.$$

The magnetic field at P due to the south pole is

$$B_S = \frac{\mu_0}{4\pi}\frac{2\pi\text{ A m}}{(15\text{ cm})^2} = 0{\cdot}3 \times 10^{-4}\text{ T}.$$

The field B_N is away from the poles and B_S is towards the poles. The resultant field at P is

$$B = B_N - B_S$$

$$= 2{\cdot}2 \times 10^{-4}\text{ T}$$

away from the solenoid.

In nature, we find certain objects whose magnetic behaviour may be described by assuming that there is a south pole placed at a certain point in the object and a north pole placed at a different point. Such an object is called a *magnet.* A magnet in the shape of a rod or a bar is called a *bar magnet.* The poles appear at points which are slightly inside the two ends. The line joining the positions of the assumed poles is called the *magnetic axis* of the bar magnet. The magnetic field lines due to a bar magnet are similar to those shown in figure (36.3). How can a rod produce magnetic field when no electric current is passed through it ? Let us now discuss this question.

A simple model tells us that matter is made of atoms and each atom contains electrons circulating around its nucleus. These moving electrons constitute electric currents at the atomic level. The actual description of these atomic currents is quite complicated but we can assume that these atomic currents are equivalent to small, circular current loops. In magnets, these loops are arranged nearly parallel to each other and have currents in the same sense.

Figure (36.5) shows the currents in a cross section of a cylindrical bar magnet. At any point inside the

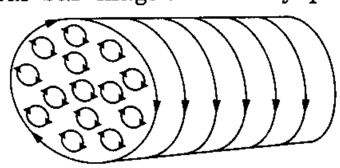

Figure 36.5

magnet, the net current is zero because the currents from the adjacent loops cancel each other. However, there is a net current along the surface as there is no cancellation of currents there. Due to such a surface current, the cylindrical magnet is equivalent to a closely-wound, current-carrying solenoid and hence produces a magnetic field similar to the solenoid. We can, therefore, treat the bar magnet as having a north pole and a south pole separated by a length l. Suppose the surface current is I per unit length of the magnet. The total current at the surface of the magnet of length l is Il. If the cross-sectional area is A, the magnetic dipole moment is

$$\mu = \text{current} \times \text{area} = IlA.$$

If the pole strength is m, the magnetic moment may also be written as $\mu = ml$.

Thus, $\quad ml = IlA$

or, $\quad m = IA. \quad \ldots (36.3)$

In the above discussion we have not considered the end effect. At the two ends of the magnet, the currents behave differently from those inside the magnet. Because of this effect, the magnetic poles appear slightly inside the bar. The distance between the locations of the assumed poles is called the *magnetic length* of the magnet. The distance between the ends is called the *geometrical length*. It is found that

$$\frac{\text{magnetic length}}{\text{geometrical length}} \approx 0{\cdot}84.$$

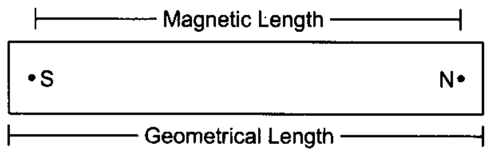

Figure 36.6

The magnetic moment of a bar magnet is conventionally denoted by M. Also, the magnetic length of a bar magnet is written as $2l$. If m be the pole strength and $2l$ the magnetic length of a bar magnet, its magnetic moment is

$$M = 2ml. \quad \ldots (36.4)$$

36.2 TORQUE ON A BAR MAGNET PLACED IN A MAGNETIC FIELD

Suppose a bar magnet of magnetic length $2l$ and pole strength m is placed in a uniform magnetic field $\vec{B}$ (figure 36.7). The angle between the magnet and the magnetic field is θ. The force on the north pole is mB along the field and that on the south pole is mB opposite to the field. The torque of these two forces is

$$\Gamma = mB\,l\sin\theta + mB\,l\sin\theta$$

$$= 2\,mB\,l\sin\theta = MB\sin\theta$$

where M is the magnetic moment of the magnet. This torque tries to rotate the magnet so as to align it with the field. We can write the torque as

$$\vec{\Gamma} = \vec{M} \times \vec{B}.$$

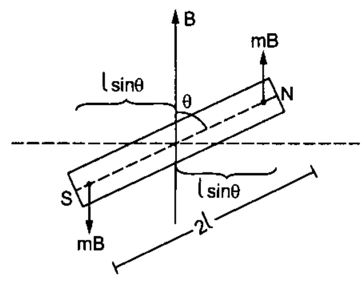

Figure 36.7

This equation is the same as that obtained earlier for a current loop. If an external agent rotates the magnet slowly, the agent has to exert a torque $MB\sin\theta$ opposite to that exerted by the field. The work done by the agent in changing the angle from θ to $\theta + d\theta$ is $dW = (MB\sin\theta)d\theta$. The work done in rotating the magnet from an angle θ_0 to θ is

$$W = \int_{\theta_0}^{\theta} MB\sin\theta\,d\theta = MB(\cos\theta_0 - \cos\theta).$$

This work is stored as the potential energy of the field–magnet system. Thus,

$$U(\theta) - U(\theta_0) = MB(\cos\theta_0 - \cos\theta).$$

If we take the potential energy at $\theta = 90°$ to be zero, the potential energy at an angle θ is

$$U(\theta) = U(\theta) - U(90°)$$

$$= -MB\cos\theta = -\vec{M}\cdot\vec{B}. \quad \ldots (36.5)$$

We see from this equation that the SI unit for the magnetic moment M may also be written as J T^{-1}.

Example 36.2

A bar magnet having a magnetic moment of $1{\cdot}0 \times 10^{4}$ J T^{-1} is free to rotate in a horizontal plane. A horizontal magnetic field $B = 4 \times 10^{-5}$ T exists in the space. Find the work done in rotating the magnet slowly from a direction parallel to the field to a direction 60° from the field.

Solution : The work done by the external agent = change in potential energy

$$= (-MB\cos\theta_2) - (-MB\cos\theta_1)$$

$$= -MB(\cos 60° - \cos 0°) = \frac{1}{2}MB$$

$$= \frac{1}{2} \times (1{\cdot}0 \times 10^{4}\ \text{J T}^{-1})\,(4 \times 10^{-5}\ \text{T}) = 0{\cdot}2\ \text{J}.$$

36.3 MAGNETIC FIELD DUE TO A BAR MAGNET

(a) End-on Position

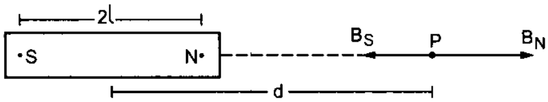

Figure 36.8

A position on the magnetic axis of a bar magnet is called an *end-on position.* Suppose *SN* is a bar magnet of magnetic length $2l$ and pole strength m. Let P be a point in end-on position at a distance d from the centre of the magnet. The magnetic field at P due to the north pole is

$$B_N = \frac{\mu_0}{4\pi} \frac{m}{(d-l)^2}$$

directed away from the magnet. The field due to the south pole is

$$B_S = \frac{\mu_0}{4\pi} \frac{m}{(d+l)^2}$$

directed towards the magnet. The resultant field is

$$B = B_N - B_S$$
$$= \frac{\mu_0 m}{4\pi} \left[\frac{1}{(d-l)^2} - \frac{1}{(d+l)^2} \right]$$
$$= \frac{\mu_0}{4\pi} \frac{2\,Md}{(d^2 - l^2)^2} \qquad \dots (36.6)$$

where $M = 2ml$ is the magnetic moment of the magnet. If $d >> l$, the magnet may be called a magnetic dipole and the field at an end-on position is

$$B = \frac{\mu_0}{4\pi} \frac{2M}{d^3}. \qquad \dots (36.7)$$

Example 36.3

A magnet is 10 cm *long and its pole strength is* 120 CGS units (1 CGS unit *of pole strength* = 0·1 A m). *Find the magnitude of the magnetic field B at a point on its axis at a distance* 20 cm *from it.*

Solution :

The pole strength is m = 120 CGS units = 12 A m.

Magnetic length is $2l$ = 10 cm or l = 0·05 m.

Distance from the magnet is d = 20 cm = 0·2 m. The field B at a point in end-on position is

$$B = \frac{\mu_0}{4\pi} \frac{2Md}{(d^2 - l^2)^2}$$
$$= \frac{\mu_0}{4\pi} \frac{4mld}{(d^2 - l^2)^2}$$
$$= \left(10^{-7} \frac{\text{T m}}{\text{A}}\right) \frac{4 \times (12\text{ A } m) \times (0{\cdot}05\text{ m}) \times (0{\cdot}2\text{ m})}{[(0{\cdot}2\text{ m})^2 - (0{\cdot}05\text{ m})^2]^2}$$
$$= 3{\cdot}4 \times 10^{-5}\text{ T}.$$

(b) Broadside-on Position

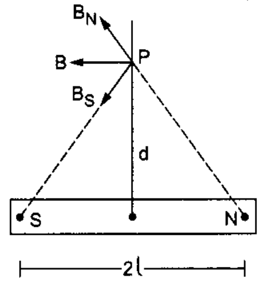

Figure 36.9

A position on a perpendicular bisector of the bar magnet is called *broadside-on position.* Let P be a point in the broadside-on position of the bar magnet at a distance d from its centre. The pole strength of the magnet is m and its magnetic length SN is $2l$. The field at P due to the north pole may be written as

$$\vec{B}_N = \frac{\mu_0}{4\pi} \frac{m\,\overrightarrow{NP}}{NP^3}.$$

This gives the magnitude as well as the direction of the field due to the north pole. The field due to the south pole is

$$\vec{B}_S = \frac{\mu_0}{4\pi} \frac{m\,\overrightarrow{PS}}{PS^3}.$$

Now, $NP = PS = (d^2 + l^2)^{1/2}$ so that the resultant field at P is

$$\vec{B} = \vec{B}_N + \vec{B}_S$$
$$= \frac{\mu_0}{4\pi} \frac{m}{(d^2 + l^2)^{3/2}} (\overrightarrow{NP} + \overrightarrow{PS})$$
$$= \frac{\mu_0}{4\pi} \frac{m\,\overrightarrow{NS}}{(d^2 + l^2)^{3/2}}.$$

The magnitude of the field is

$$B = \frac{\mu_0}{4\pi} \frac{m\,2l}{(d^2 + l^2)^{3/2}} = \frac{\mu_0}{4\pi} \frac{M}{(d^2 + l^2)^{3/2}} \qquad \dots (36.8)$$

where $M = 2ml$ is the magnetic moment of the magnet. The direction of the field is parallel to the axis, from the north pole to the south pole.

If $d >> l$, the magnet may be called a magnetic dipole and the magnetic field at a point in broadside-on position is

$$B = \frac{\mu_0}{4\pi} \frac{M}{d^3}. \qquad \dots (36.9)$$

36.4 MAGNETIC SCALAR POTENTIAL

Magnetic scalar potential is defined in the same way as gravitational or electrostatic potential. We define the change in potential $V(\vec{r}_2) - V(\vec{r}_1)$ by the equation

$$V(\vec{r_2}) - V(\vec{r_1}) = -\int_{\vec{r_1}}^{\vec{r_2}} \vec{B}\cdot d\vec{r}. \qquad \ldots (36.10)$$

Generally, the potential at infinity (a point far away from all sources of magnetic field) is taken to be zero. Taking $\vec{r_1}$ equal to ∞ and $\vec{r_2} = \vec{r}$, equation (36.10) gives

$$V(\vec{r}) = -\int_{\infty}^{\vec{r}} \vec{B}\cdot d\vec{r}.$$

The component of the magnetic field in any direction is given by

$$B_l = -\frac{dV}{dl} \qquad \ldots (36.11)$$

where dl is a small distance along the given direction.

For a pole of pole strength m, the field at a distance r is

$$B = \frac{\mu_0}{4\pi}\frac{m}{r^2}$$

radially away from the pole. So the potential at a distance r is

$$V(r) = -\int_{\infty}^{r} \frac{\mu_0}{4\pi} m \frac{dr}{r^2}$$

$$= \frac{\mu_0}{4\pi}\frac{m}{r}. \qquad \ldots (36.12)$$

Magnetic Scalar Potential due to a Magnetic Dipole

Suppose, SN is a magnetic dipole of length $2l$ and pole strength m (figure 36.10). The magnetic scalar potential is needed at a point P at a distance $OP = r (>> l)$ from the centre of the dipole. The angle $PON = \theta$.

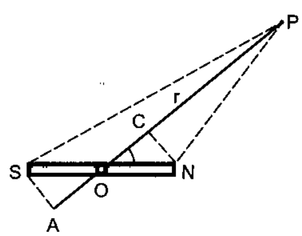

Figure 36.10

Let SA be the perpendicular from S to OP and NC be the perpendicular from N to OP. As $r >> l$,

$$PS \approx PA = PO + OA = r + l\cos\theta.$$

Similarly, $PN \approx PC = PO - OC = r - l\cos\theta.$

The magnetic scalar potential at P due to the north pole is

$$V_N = \frac{\mu_0}{4\pi}\frac{m}{NP}$$

$$= \frac{\mu_0}{4\pi}\frac{m}{r - l\cos\theta}$$

and that due to the south pole is

$$V_S = -\frac{\mu_0}{4\pi}\frac{m}{SP}$$

$$= -\frac{\mu_0}{4\pi}\frac{m}{r + l\cos\theta}.$$

The net potential at P due to the dipole is

$$V = V_N + V_S$$

$$= \frac{\mu_0 m}{4\pi}\left(\frac{1}{r - l\cos\theta} - \frac{1}{r + l\cos\theta}\right)$$

$$= \frac{\mu_0}{4\pi}\frac{m(2l\cos\theta)}{(r^2 - l^2\cos^2\theta)}$$

$$\approx \frac{\mu_0}{4\pi}\frac{M\cos\theta}{r^2}. \qquad \ldots (36.13)$$

Magnetic Field due to a Dipole

Let SN be a magnetic dipole and P be a point far away from the dipole (figure 36.11). The distance $OP = r$ and the angle $PON = \theta$. If we move a small distance PQ in the direction of OP, the value of r is changed to $r + dr$ while θ remains unchanged. Similarly, if we move a small distance PR in the direction perpendicular to OP, θ is changed from θ to $\theta + d\theta$ while r remains very nearly constant. The distance moved is $rd\theta$.

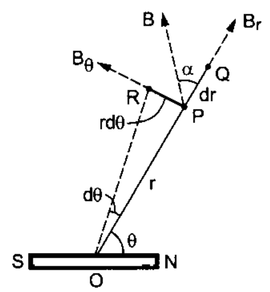

Figure 36.11

The component of magnetic field along OP is

$$B_r = -\frac{dV}{PQ} = -\left[\frac{dV}{dr}\right]_{\theta = \text{constant}}$$

$$= -\frac{\partial V}{\partial r}$$

$$= -\frac{\partial}{\partial r}\left(\frac{\mu_0}{4\pi}\frac{M\cos\theta}{r^2}\right)$$

$$= \frac{\mu_0}{4\pi}\frac{2M\cos\theta}{r^3}. \qquad \ldots (i)$$

The component perpendicular to OP is

$$B_\theta = -\frac{dV}{PR} = -\left[\frac{dV}{rd\theta}\right]_{r = \text{constant}}$$

$$= -\frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{\mu_0}{4\pi}\frac{M\cos\theta}{r^2}\right)$$

$$= \frac{\mu_0}{4\pi} \frac{M \sin\theta}{r^3} . \qquad \ldots \text{(ii)}$$

The resultant magnetic field at P is

$$B = \sqrt{B_r^2 + B_\theta^2}$$

$$= \frac{\mu_0}{4\pi} \frac{M}{r^3} \sqrt{(2\cos\theta)^2 + (\sin\theta)^2}$$

$$= \frac{\mu_0}{4\pi} \frac{M}{r^3} \sqrt{1 + 3\cos^2\theta} . \qquad \ldots (36.14)$$

If it makes an angle α with OP,

$$\tan\alpha = \frac{B_\theta}{B_r} .$$

From (i) and (ii),

$$\tan\alpha = \frac{\sin\theta}{2\cos\theta} = \frac{\tan\theta}{2} . \qquad \ldots (36.15)$$

Example 36.4

Find the magnetic field due to a dipole of magnetic moment 1·2 A m² at a point 1 m away from it in a direction making an angle of 60° with the dipole-axis.

Solution : The magnitude of the field is

$$B = \frac{\mu_0}{4\pi} \frac{M}{r^3} \sqrt{1 + 3\cos^2\theta}$$

$$= \left(10^{-7} \frac{\text{T m}}{\text{A}}\right) \frac{1{\cdot}2\ \text{A m}^2}{1\ \text{m}^3} \sqrt{1 + 3\cos^2 60^\circ}$$

$$= 1{\cdot}6 \times 10^{-7}\ \text{T}.$$

The direction of the field makes an angle α with the radial line where

$$\tan\alpha = \frac{\tan\theta}{2} = \frac{\sqrt{3}}{2} .$$

36.5 TERRESTRIAL MAGNETISM

Earth is a natural source of magnetic field. We have magnetic field present everywhere near the earth's surface. The magnitude and direction of this field can be obtained approximately by assuming that the earth has a magnetic dipole of dipole moment about $8{\cdot}0 \times 10^{22}\ \text{J T}^{-1}$ located at its centre (figure 36.12). The axis of this dipole makes an angle of about 11·5° with the earth's axis of rotation. The dipole-axis cuts the earth's surface at two points, one near the geographical north pole and the other near the geographical south pole. The first of these points is called *geomagnetic north pole* and the other is called *geomagnetic south pole.*

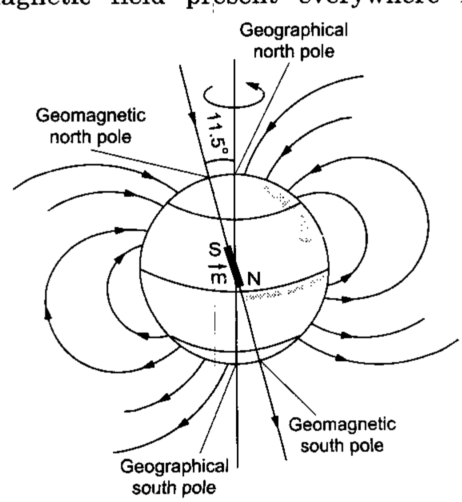

Figure 36.12

If we suspend a bar magnet freely at a point near the earth's surface, it will stay along the magnetic field there. The north pole will point towards the direction of the magnetic field. At the geomagnetic poles, the magnetic field is vertical. If we suspend the bar magnet near the geomagnetic north pole, it will become vertical with its north pole towards the earth's surface. Similarly, if we suspend a bar magnet near the geomagnetic south pole, it will become vertical with its south pole pointing towards the earth's surface. Geomagnetic poles may, therefore, be defined as "the points where a freely suspended bar magnet becomes vertical".

If we treat the assumed magnetic dipole inside the earth as a pair of north and south poles (figure 36.12), the south pole will be towards the geomagnetic north pole and the north pole will be towards the geomagnetic south pole. This may be easily remembered by using the fact that the north pole of the suspended magnet should be attracted by the south pole of the assumed dipole.

Earth's magnetic field changes both in magnitude and direction as time passes. It is fairly constant over a span of a few days, but noticeable changes occur in say, ten years. Studies of magnetic rocks have revealed that the magnetic field may even reverse its direction. It appears that in the past $7{\cdot}6 \times 10^7$ years, already 171 such reversals have taken place. The latest reversal in earth's magnetic field is believed to have occurred around 10,000 years ago.

The theory of earth's magnetic field is not yet well-understood. At present, it seems that the field results mainly due to circulating electric currents induced in the molten liquid and other conducting material inside the earth.

Elements of the Earth's Magnetic Field

The earth's magnetic field at a point on its surface is usually characterised by three quantities: (a) declination (b) inclination or dip and (c) horizontal component of the field. These are known as the *elements of the earth's magnetic field.*

Declination

A plane passing through the geographical poles (that is, through the axis of rotation of the earth) and a given point P on the earth's surface is called the *geographical meridian* at the point P. Similarly, the plane passing through the geomagnetic poles (that is, through the dipole-axis of the earth) and the point P is called the *magnetic meridian* at the point P. In other words, the magnetic meridian is a vertical plane through the point P that contains the geomagnetic poles. The magnetic field due to the earth at P must be in this plane (magnetic meridian).

The angle made by the magnetic meridian at a point with the geographical meridian is called the *declination* at that point. The knowledge of declination fixes the vertical plane in which the earth's magnetic field lies.

Navigators often use a magnetic compass needle to locate direction. A compass needle is a short and light magnetic needle, free to rotate about a vertical axis. The needle is enclosed in a small case with a glass-top. The needle stays in equilibrium when it is in magnetic meridian. Hence the north direction shown by the needle makes an angle equal to the declination with the true north and navigators have to take care of it.

Inclination or dip

The angle made by the earth's magnetic field with the horizontal direction in the magnetic meridian, is called the *inclination* or *dip* at that point.

In the magnetic nothern hemisphere, the vertical component of the earth's magnetic field points downwards. The north pole of a freely suspended magnet, therefore, *dips* (goes down).

The knowledge of declination and inclination completely specifies the direction of the earth's magnetic field.

Horizontal component of the earth's magnetic field

As the name indicates, the *horizontal component* is component of the earth's magnetic field in the horizontal direction in the magnetic meridian. This direction is towards the magnetic north.

Figure (36.13) shows the three elements. Starting from the geographical meridian we draw the magnetic meridian at an angle θ (declination). In the magnetic meridian we draw the horizontal direction specifying magnetic north. The magnetic field is at an angle δ (dip) from this direction. The horizontal component B_H and the total field B are related as

$$B_H = B\cos\delta$$

or,
$$B = B_H/\cos\delta.$$

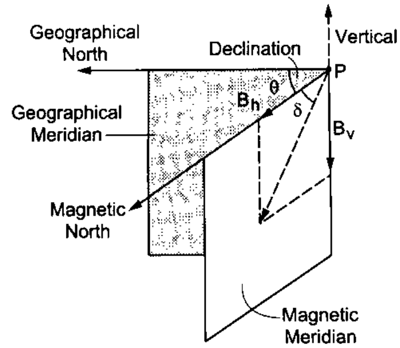

Figure 36.13

Thus, from the knowledge of the three elements, both the magnitude and direction of the earth's magnetic field can be obtained.

Example 36.5

The horizontal component of the earth's magnetic field is $3{\cdot}6 \times 10^{-5}$ T *where the dip is* 60°. *Find the magnitude of the earth's magnetic field.*

Solution : We have $B_H = B\cos\delta$

or, $$B = \frac{B_H}{\cos\delta} = \frac{3{\cdot}6\times10^{-5}\,\text{T}}{\cos 60^\circ} = 7{\cdot}2\times10^{-5}\,\text{T}.$$

36.6 DETERMINATION OF DIP AT A PLACE

Dip Circle

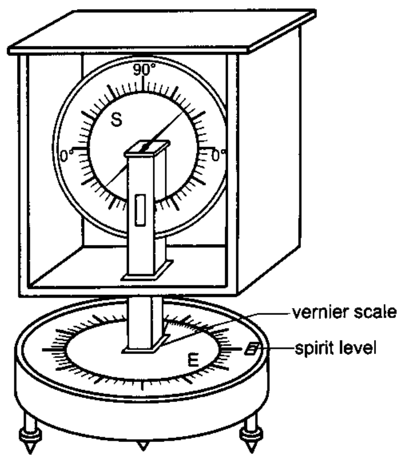

Figure 36.14

The dip at a place can be determined by an apparatus known as *dip circle*. It consists of a vertical circular scale S and a magnetic needle (a small pointed permanent magnet) pivoted at the centre of the scale. The needle can rotate freely in the vertical plane of the scale. The pointed ends move over the graduations on the scale which are marked 0°–0° in the horizontal and 90°–90° in the vertical direction. The scale S together with the needle is enclosed in a glass cover which can be rotated about a vertical axis. The angle rotated can be read from a horizontal angular scale E, fixed with the base, and a vernier scale fixed with the stand supporting the glass cover. The base can be made horizontal by levelling screws fixed with it. A spirit-level fixed to the apparatus helps in levelling.

Determination of Dip

Determination of magnetic meridian

At the beginning of the experiment, the base of the dip circle is made horizontal with the help of the levelling screws and the magnetic needle is pivoted in its place. The glass cover containing the vertical scale S and the needle is rotated about the vertical axis till the needle becomes vertical and reads 90°–90° on the vertical scale. In this condition, the plane of the circular scale S is perpendicular to the magnetic meridian. The horizontal component B_H is perpendicular to this plane and hence does not take part in rotating the needle. The needle is aligned with the vertical component B_V and hence reads 90°–90°. The reading of the vernier is noted and the glass cover is rotated exactly through 90° from this position. The plane of the circular scale S is now the same as the magnetic meridian.

Measurement of dip

When the plane of the vertical scale S is the same as the magnetic meridian, the earth's magnetic field $\vec{B}$ is in this same plane. In this case, the needle rests in the direction of $\vec{B}$. The readings of the ends of the needle on the vertical scale now directly give the value of the dip.

Possible errors and their remedies

Errors may occur because of several imperfections in the instrument. Some of the possible errors and their remedies are given below.

(a) *The centre of the needle is not at the centre of the vertical scale.*

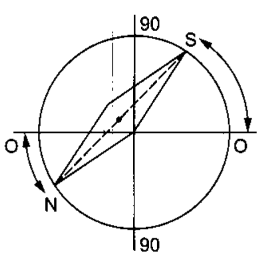

Figure 36.15

If the centre of the needle does not coincide with the centre of the scale, the readings do not represent the true dip. The reading of one end of the needle is less than the true value of the dip and the reading of the other end is greater by the same amount. Thus, both ends are read and the average is taken.

(b) 0°–0° *line is not horizontal.*

If the 0°–0° line on the scale is not horizontal, the value of dip will have some error. This error may be removed as described below.

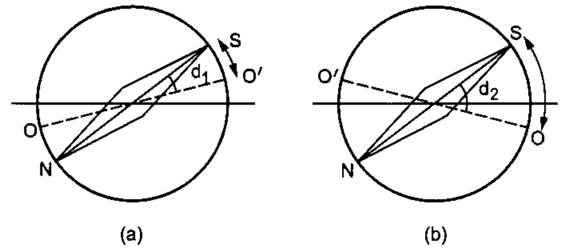

Figure 36.16

Bring the vertical scale in the magnetic meridian and note the readings of the ends of the needle (figure 36.16a). Now, rotate the circle through 180° about the vertical axis and again note the readings (figure 36.16b). The average of these readings will not have the error due to 0°–0° line.

(c) *The magnetic and the geometrical axes of the needle are different.*

In the experiment, we read the angles corresponding to the ends of the needle. If the magnetic axis is inclined at an angle with the line joining the ends, the dip obtained is in error. This error can be removed by inverting the needle on its bearing and repeating the previous readings. The average of these readings is free of this error.

(d) *Centre of mass of the needle does not coincide with the pivot.*

If the centre of mass of the needle is not at the pivot, its weight mg will have a torque and will affect the equilibrium position. To remove this error, one has to read the dip and then take out the needle. The needle should be demagnetized and then remagnetized in opposite direction. Thus, the position of the north and south poles are interchanged. The centre of mass now appears on the other side of the pivot and hence the effect of mg is also reversed. The dip is again determined with this needle and the average is taken.

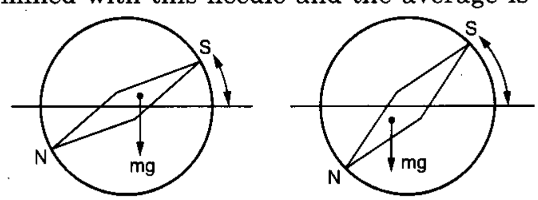

Figure 36.17

Thus, one should take 16 readings of dip and an average of all these gives the true dip.

Apparent Dip

If the dip circle is not kept in the magnetic meridian, the needle will not show the correct direction of earth's magnetic field. The angle made by the needle with the horizontal is called the *apparent dip* for this plane. If the dip circle is at an angle α to the meridian, the effective horizontal component in this plane is $B'_H = B_H \cos\alpha$. The vertical component is still B_v. If δ' is the apparent dip and δ is the true dip, we have

$$\tan\delta' = \frac{B_v}{B'_H}$$

or, $$\cot\delta' = \frac{B'_H}{B_v} = \frac{B_H \cos\alpha}{B_v}$$

or, $$\cot\delta' = \cot\delta \cos\alpha. \quad \ldots \text{(i)}$$

Now suppose, the dip circle is rotated through an angle of 90° from this position. It will now make an angle $(90° - \alpha)$ with the meridian. The effective horizontal component in this plane is $B''_H = B_H \sin\alpha$. If δ'' be the apparent dip, we shall have

$$\cot\delta'' = \cot\delta \sin\alpha. \quad \ldots \text{(ii)}$$

Squaring and adding (i) and (ii),

$$\cot^2\delta' + \cot^2\delta'' = \cot^2\delta. \quad \ldots (36.16)$$

Thus, one can get the true dip δ without locating the magnetic meridian.

Example 36.6

At 45° to the magnetic meridian, the apparent dip is 30°. Find the true dip.

Solution : At 45° to the magnetic meridian, the effective horizontal component of the earth's magnetic field is $B'_H = B_H \cos 45° = \frac{1}{\sqrt{2}} B_H$. The apparent dip δ' is given by

$$\tan\delta' = \frac{B_v}{B'_H} = \frac{\sqrt{2}\, B_v}{B_H} = \sqrt{2} \tan\delta$$

where δ is the true dip. Thus,

$$\tan 30° = \sqrt{2} \tan\delta$$

or, $$\delta = \tan^{-1}\sqrt{1/6}.$$

36.7 NEUTRAL POINT

Suppose at a point, the horizontal component of the magnetic field due to a magnet is equal and opposite to the earth's horizontal magnetic field. The net horizontal field is zero at such a point. If a compass needle is placed at such a point, it can stay in any position. Such a point is called a *neutral point.*

36.8 TANGENT GALVANOMETER

Tangent galvanometer is an instrument to measure an electric current. The essential parts are a vertical circular coil C of conducting wire and a small compass needle A pivoted at the centre of the coil (figure 36.18). The coil C together with its frame is fixed to a horizontal base B provided with levelling screws. Terminals T_1 and T_2 connected to the coil are

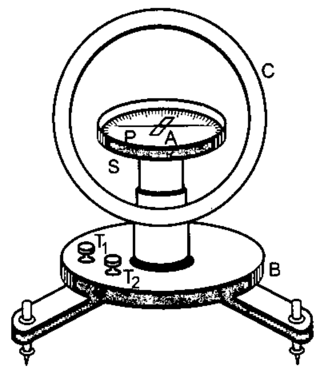

Figure 36.18

provided on this base for connecting the galvanometer to an external circuit. An aluminium pointer P is rigidly attached with the compass needle and perpendicular to it. The compass needle together with the pointer can rotate freely about the vertical axis. The ends of the pointer move over a graduated, horizontal circular scale. The graduations are marked from 0° to 90° in each quadrant. The scale, the pointer and the compass needle are enclosed in a closed cylindrical box which is placed with its centre coinciding with the centre of the coil. The box can also be rotated about the vertical axis. The upper surface of the box is made of glass so that the things inside it are visible. To avoid the errors due to parallax, a plane mirror is fixed at the lower surface of the box. While noting the reading of the pointer, the eye should be properly positioned so that the image of the pointer is just below the pointer.

When there is no current through the galvanometer, the compass needle is in magnetic north–south direction. To measure a current with the tangent galvanometer, the base is rotated in such a way that the plane of the coil is parallel to the compass needle. The plane then coincides with the magnetic meridian. The box containing the needle is rotated so that the aluminium pointer reads 0°–0° on the scale.

The current to be measured is passed through the coil. The current through the coil produces a magnetic field at the centre and the compass needle deflects under its action. The pointer deflects through the same angle and the deflection of both the ends are read from the horizontal scale. The average of these two is calculated.

Suppose the current through the coil is i. The radius of the coil is r and the number of turns in it is n. The magnetic field produced at the centre is

$$B = \frac{\mu_0 in}{2r}. \quad \ldots \text{(i)}$$

This field is perpendicular to the plane of the coil. This direction is horizontal and perpendicular to the magnetic meridian and hence to the horizontal component B_H of the earth's magnetic field. The resultant horizontal magnetic field is

$$B_r = \sqrt{B^2 + B_H^2}$$

in a direction making an angle θ with B_H (figure 36.19) where

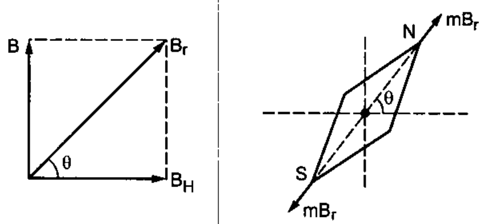

Figure 36.19

$$\tan\theta = B/B_H. \qquad \text{... (ii)}$$

If m be the pole strength of the needle, the force on the north pole of the needle is mB_r along B_r and on the south pole is mB_r, opposite to B_r. The needle will stay in equilibrium when its length is parallel to B_r, because then no torque is produced by the two forces. Thus, the deflection of the needle from its original position is θ as given by (ii). Using (i) and (ii),

$$B_H \tan\theta = \frac{\mu_0 in}{2r}$$

or, $$i = \frac{2r B_H}{\mu_0 n} \tan\theta$$

or, $$i = K \tan\theta, \qquad \text{... (36.17)}$$

where $K = \frac{2r B_H}{\mu_0 n}$ is a constant for the given galvanometer at a given place. This constant is called the *reduction factor* of the galvanometer. The reduction factor may be obtained by passing a known current i through the galvanometer, measuring θ and then using (36.17).

Equation (i) is strictly valid only at the centre of the coil. The poles of the needle are slightly away from the centre. Thus, the length of the needle should be small as compared to the radius of the coil.

Sensitivity

Good sensitivity means that the change in deflection is large for a given fractional change in current.

We have

$$i = K \tan\theta$$

or, $$di = K \sec^2\theta \, d\theta$$

or, $$\frac{di}{i} = \frac{K \sec^2\theta \, d\theta}{K \tan\theta} = \frac{2 \, d\theta}{\sin 2\theta}$$

or, $$d\theta = \frac{1}{2} \sin 2\theta \left(\frac{di}{i}\right)$$

Thus, for good sensitivity, sin 2θ should be as large as possible. This is the case when θ = 45°. So, the tangent galvanometer is most sensitive when the deflection is around 45°.

Example 36.7

A tangent galvanometer has 66 *turns and the diameter of its coil is* 22 cm. *It gives a deflection of* 45° *for* 0·10 A *current. What is the value of the horizontal component of the earth's magnetic field ?*

Solution : For a tangent galvanometer

$$i = K \tan\theta$$

$$= \frac{2r B_H}{\mu_0 n} \tan\theta$$

or, $$B_H = \frac{\mu_0 ni}{2r \tan\theta}$$

$$= \frac{\left(4\pi \times 10^{-7} \frac{\text{T m}}{\text{A}}\right) \times 66 \times (0{\cdot}1 \text{ A})}{(22 \times 10^{-2} \text{ m}) (\tan 45°)}$$

$$= 3{\cdot}8 \times 10^{-5} \text{ T}.$$

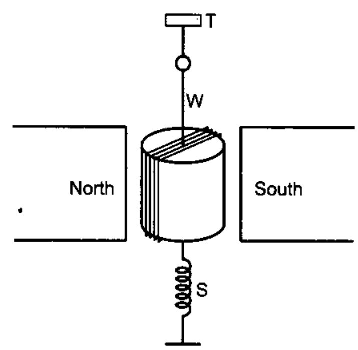

Figure 36.20

36.9 MOVING-COIL GALVANOMETER

The main parts of a moving-coil galvanometer are shown in figure (36.20). A rectangular coil of several turns is wound over a soft-iron core. The wire of the coil is coated with an insulating material so that each turn is insulated from the other and from the iron core. The coil is suspended between the two pole pieces of a strong permanent magnet. A fine strip W of phosphor bronze is used to suspend the coil. The upper end of this strip is attached to a torsion head T. The lower end of the coil is attached to a spring S also made of phosphor bronze. A small mirror is fixed on the suspension strip and is used to measure the deflection of the coil with the help of a lamp–scale arrangement. Terminals are connected to the suspension strip W and the spring S. These terminals are used to pass current through the galvanometer.

The current to be measured is passed through the galvanometer. As the coil is in the magnetic field $\vec{B}$ of the permanent magnet, a torque $\vec{\Gamma} = ni\vec{A} \times \vec{B}$ acts on the coil. Here n = number of turns, i = current in the coil, $\vec{A}$ = area-vector of the coil and $\vec{B}$ = magnetic field at the site of the coil. This torque deflects the coil from

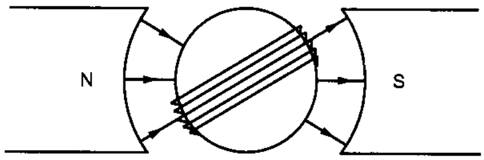

Figure 36.21

its equilibrium position.

The pole pieces are made cylindrical. As a result, the magnetic field at the arms of the coil remains parallel to the plane of the coil everywhere even as the coil rotates. The deflecting torque is then $\Gamma = niAB$. As the upper end of the suspension strip W is fixed, the strip gets twisted when the coil rotates. This produces a restoring torque acting on the coil. If the deflection of the coil is θ and the torsional constant of the suspension strip is k, the restoring torque is $k\theta$. The coil will stay at a deflection θ where

$$niAB = k\theta$$

or, $$i = \frac{k}{nAB}\theta. \qquad \dots (36.18)$$

Hence, the current is proportional to the deflection. The constant $\frac{k}{nAB}$ is called the *galvanometer constant* and may be found by passing a known current, measuring the deflection θ and putting these values in equation (36.18).

Sensitivity

The sensitivity of a moving-coil galvanometer is defined as θ/i. From equation (36.18), the sensitivity is $\frac{nAB}{k}\cdot$ For large sensitivity, the field B should be large. The presence of soft-iron core increases the magnetic field. We shall discuss in a later chapter how soft iron increases magnetic field.

36.10 SHUNT

A galvanometer is usually a delicate and sensitive instrument and only a small current is sufficient to deflect the coil to its maximum allowed value. If a current larger than this permissible value is passed through the galvanometer, it may get damaged. A small resistance R_s called *shunt*, is connected in parallel with the galvanometer to save it from such accidents. The main current i is divided in two parts,

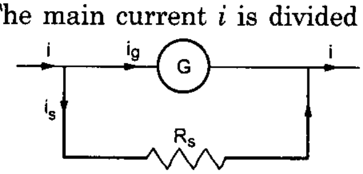

Figure 36.22

i_g through the galvanometer and remaining $i_s = i - i_g$ through the shunt. If the resistance of the galvanometer coil is R_g, we have

$$i_g = \frac{R_s}{R_s + R_g}\, i. \qquad \dots (36.19)$$

As R_s is much smaller than R_g, only a small fraction goes through the galvanometer.

Example 36.8

A galvanometer having a coil of resistance 20 Ω needs 20 mA current for full-scale deflection. In order to pass a maximum current of 2 A through the galvanometer, what resistance should be added as a shunt?

Solution : Out of the main current of 2 A, only 20 mA should go through the coil. The current through the coil is

$$i_g = \frac{R_s}{R_s + R_g}\, i$$

or, $$20\text{ mA} = \frac{R_s}{R_s + (20\ \Omega)}\, 2\text{ A}$$

or, $$\frac{20\text{ mA}}{2\text{ A}} = \frac{R_s}{R_s + (20\ \Omega)}$$

or, $$(10^{-2})(R_s + 20\ \Omega) = R_s$$

or, $$R_s = \frac{20}{99}\ \Omega \approx 0{\cdot}2\ \Omega.$$

36.11 TANGENT LAW OF PERPENDICULAR FIELDS

When a compass needle is placed in the earth's magnetic field, it stays along the horizontal component B_H of the field. The magnetic forces mB_H and $-mB_H$ on the poles do not produce any torque in this case. If an external horizontal magnetic field B is produced which is perpendicular to B_H, the needle deflects from its position. The situation is the same as that shown in figure (36.19). The resultant of B and B_H is

$$B_r = \sqrt{B^2 + B_H^2}$$

making an angle θ with B_H so that

$$\tan\theta = \frac{B}{B_H}\cdot \qquad \dots (\text{i})$$

The forces on the poles are mB_r, $-mB_r$ which may produce a torque and deflect the needle. The needle can stay in a position parallel to the resultant horizontal field B_r. Thus, the deflection of the needle in equilibrium is θ. Using (i), the external magnetic field B may be written in terms of B_H and θ as

$$B = B_H \tan\theta. \qquad \dots (36.20)$$

This is known as the *tangent law of perpendicular fields*.

36.12 DEFLECTION MAGNETOMETER

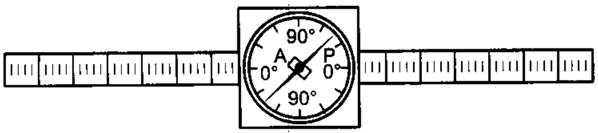

Figure 36.23

A deflection magnetometer (figure 36.23) consists of a small compass needle A pivoted at the centre of a graduated circular scale. The graduations are marked from 0° to 90° in each quadrant. An aluminium pointer P is rigidly fixed with the needle and perpendicular to it. The ends of the pointer move on the circular scale. These are enclosed in a cylindrical box known as the magnetometer box. The upper cover of the box is made of glass so that the things inside are visible. A plane mirror is fixed on the lower surface so that the pointer may be read without parallax. This arrangement is the same as that used in a tangent galvanometer.

The magnetometer box is kept in a wooden frame having two long arms. Metre scales are fitted on the two arms. The reading of a scale at any point directly gives the distance of that point from the centre of the compass needle.

The basic use of a deflection magnetometer is to determine M/B_H for a permanent bar magnet. Here M is the magnetic moment of the magnet and B_H is the horizontal component of the earth's magnetic field. This quantity M/B_H can be measured in two standard positions of the magnetometer. One is called Tan-A position of Gauss and the other is called Tan-B position of Gauss.

Tan-*A* Position

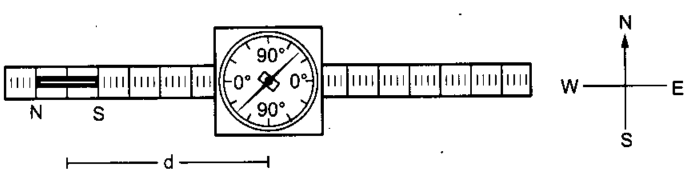

Figure 36.24

In this, the arms of the magnetometer are kept along the magnetic east–west direction. The aluminium pointer shows this direction when no extra magnets or magnetic materials are present nearby. The magnetometer box is rotated in its plane till the pointer reads 0°–0°. The magnet is now kept on one of the arms, parallel to its length (figure 36.24). The needle deflects and the deflection θ in its equilibrium position is read from the circular scale. The distance of the centre of the magnet from the centre of the compass is calculated from the linear scale on the arm.

Let d be this distance and $2l$ be the magnetic length of the magnet. In Tan-A position of the magnetometer, the compass needle is in end-on position of the bar magnet. The magnetic field due to the bar magnet at the site of the needle is, therefore,

$$B = \frac{\mu_0}{4\pi} \frac{2Md}{(d^2 - l^2)^2}.$$

This field is along the length of the magnet, that is, towards east or towards west. It is, therefore, perpendicular to the earth's field B_H. From the tangent law, we have,

$$B = B_H \tan\theta$$

or, $$\frac{\mu_0}{4\pi} \frac{2Md}{(d^2 - l^2)^2} = B_H \tan\theta$$

or, $$\frac{M}{B_H} = \frac{4\pi}{\mu_0} \frac{(d^2 - l^2)^2}{2d} \tan\theta. \qquad \dots (36.21)$$

Knowing all the quantities on the right-hand side, one gets M/B_H.

Possible errors and their remedies

Errors may occur due to various reasons.

(a) The pivot of the needle may not be at the centre of the circular scale (figure 36.25). To remove this error both ends of the pointer are read and the mean is taken.

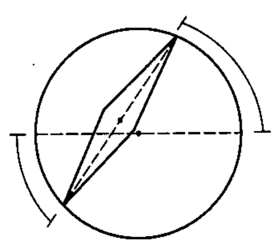

Figure 36.25

(b) The magnetic centre of the bar magnet may not coincide with its geometrical centre (figure 36.26). The measured distance d is more or less than the actual distance d_1 of the centre of the needle from the magnetic centre of the magnet. To remove the error due to this, the magnet is rotated through 180° about the vertical so that the positions of the north pole and the south pole are interchanged. The deflections are again noted with both ends of the pointer. These readings correspond to the distance d_2. The average of the sets of deflections corresponding to the distances d_1 and d_2 give the correct value approximately.

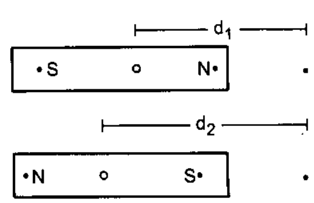

Figure 36.26

(c) The geometrical axis of the bar magnet may not coincide with the magnetic axis (figure 36.27). To

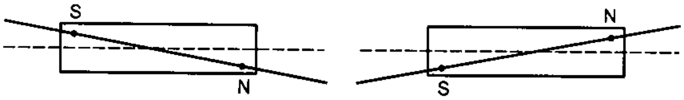

Figure 36.27

avoid error due to this, the magnet is put upside down at the same position and the readings are taken.

(d) The zero of the linear scale may not coincide with the centre of the circular scale. To remove the error due to this, the magnet is kept on the other arm of the magnetometer at the same distance from the needle and all the readings are repeated.

Thus, one gets sixteen values of θ for the same distance d. The mean of these sixteen values gives the correct value of θ.

Tan-B Position

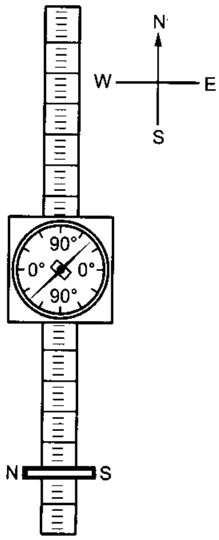

Figure 36.28

In this position, the arms of the magnetometer are kept in the magnetic north to south direction. The box is rotated so that the pointer reads 0°–0°. The bar magnet is placed on one of the arms symmetrically and at right angles to it (figure 36.28). The distance d of the centre of the magnet from the centre of the compass needle is calculated from the linear scale. The deflection θ of the needle is noted from the circular scale. To remove the errors due to the reasons described above, the deflections are read in various situations mentioned below. Both ends of the pointer are read. The magnet is put upside down and readings are taken. The bar magnet is rotated through 180° to interchange the positions of north and south poles and again both ends of the pointer are read. The magnet is again put upside down and readings are taken for both ends of the pointer. The magnet is kept on the other arm at the same distance and the corresponding readings are taken. The mean of these sixteen values gives correct θ for this d.

In tan-B position, the compass is in broadside-on position of the magnet. The magnetic field at the compass due to the magnet is, therefore,

$$B = \frac{\mu_0}{4\pi} \frac{M}{(d^2 + l^2)^{3/2}}.$$

The field is parallel to the axis of the magnet and hence it is towards east or towards west. The earth's magnetic field is from south to north. Using tangent law,

$$B = B_H \tan\theta$$

or, $$\frac{\mu_0}{4\pi} \frac{M}{(d^2 + l^2)^{3/2}} = B_H \tan\theta$$

or, $$\frac{M}{B_H} = \frac{4\pi}{\mu_0} (d^2 + l^2)^{3/2} \tan\theta. \quad \ldots (36.22)$$

Applications of a Deflection Magnetometer

A variety of quantities may be obtained from the basic measurement of M/B_H using a deflection magnetometer. Here are some of the examples.

(a) Comparison of the magnetic moments M_1 and M_2 of two magnets

One can find M_1/B_H and M_2/B_H separately for the two magnets and then get the ratio M_1/M_2. There is another simple method known as *null method* to get M_1/M_2. The experiment can be done either in Tan-A position or in Tan-B position. The two magnets are placed on the two arms of the magnetometer. The distances of the magnets from the centre of the magnetometer are so adjusted that the deflection of the needle is zero. In this case, the magnetic field at the needle due to the first magnet is equal in magnitude to the field due to the other magnet. If the magnetometer is used in Tan-A position,

$$\frac{\mu_0}{4\pi} \frac{2 M_1 d_1}{(d_1^2 - l_1^2)^2} = \frac{\mu_0}{4\pi} \frac{2 M_2 d_2}{(d_2^2 - l_2^2)^2}$$

or, $$\frac{M_1}{M_2} = \frac{d_2 (d_1^2 - l_1^2)^2}{d_1 (d_2^2 - l_2^2)^2}.$$

If the magnetometer is used in Tan-B position,

$$\frac{\mu_0}{4\pi} \frac{M_1}{(d_1^2 + l_1^2)^{3/2}} = \frac{\mu_0}{4\pi} \frac{M_2}{(d_2^2 + l_2^2)^{3/2}}$$

or, $$\frac{M_1}{M_2} = \frac{(d_1^2 + l_1^2)^{3/2}}{(d_2^2 + l_2^2)^{3/2}}.$$

Null method is easier and better than finding M_1/B_H and M_2/B_H separately and then calculating M_1/M_2. This is because, here the deflection remains zero and the possible errors in the measurement of θ do not occur. Also, it is more sensitive, because even a small deflection from 0°–0° gives the indication that the adjustment is not perfect.

(b) Verification of inverse square law for magnetic field due to a magnetic pole

Equations (36.21) and (36.22) for M/B_H are deduced from the basic equation (36.2) giving the magnetic field due to a magnetic pole. Equation (36.2) shows that the magnetic field due to a magnetic pole is inversely proportional to the square of the distance. Thus, if we verify equation (36.21) or (36.22), inverse square law is verified.

For Tan-*A* position, from equation (36.21),

$$\cot\theta = \frac{4\pi}{\mu_0}\frac{B_H}{M}\frac{(d^2 - l^2)^2}{2d}. \qquad \ldots \text{(i)}$$

A magnet is placed at a distance d in Tan-*A* position of the magnetometer and the corresponding value of deflection θ is noted. The experiment is repeated for different values of d and a graph between $\cot\theta$ and $\frac{(d^2 - l^2)^2}{2d}$ is plotted. The graph turns out to be a straight line passing through the origin (figure 36.29a). This is consistent with equation (i) above and hence the inverse square law is verified.

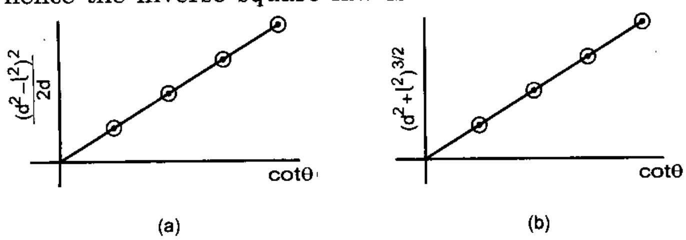

Figure 36.29

One can also do the experiment in Tan-*B* position. The deflection θ and the distance d are related through equation (36.22). We have,

$$\cot\theta = \frac{4\pi}{\mu_0}\frac{B_H}{M}(d^2 + l^2)^{3/2}. \qquad \ldots \text{(ii)}$$

The values of deflection θ are noted for different values of the distance d. A graph is drawn between $\cot\theta$ and $(d^2 + l^2)^{3/2}$. The graph turns out to be a straight line passing through the origin (figure 36.29b). This verifies the inverse square law.

(c) Comparison of the horizontal components of the earth's magnetic field at two places

Suppose the horizontal component of the earth's magnetic field is B_{H1} at the first place and B_{H2} at the second. A deflection magnetometer is taken and a bar magnet is kept at a distance d in Tan-*A* position. The deflection θ_1 of the needle is noted. The magnetometer is now taken to the second place and the same magnet is kept at the same distance d in Tan-*A* position. The deflection θ_2 is noted. We have from equation (36.21),

$$\frac{M}{B_{H1}} = \frac{4\pi}{\mu_0}\frac{(d^2 - l^2)^2}{2d}\tan\theta_1$$

and $$\frac{M}{B_{H2}} = \frac{4\pi}{\mu_0}\frac{(d^2 - l^2)^2}{2d}\tan\theta_2.$$

Thus, $$\frac{B_{H1}}{B_{H2}} = \frac{\tan\theta_2}{\tan\theta_1}.$$

The experiment can also be done in Tan-*B* position. Using equation (36.22), we again get the same relation for $\frac{B_{H1}}{B_{H2}}$.

36.13 OSCILLATION MAGNETOMETER

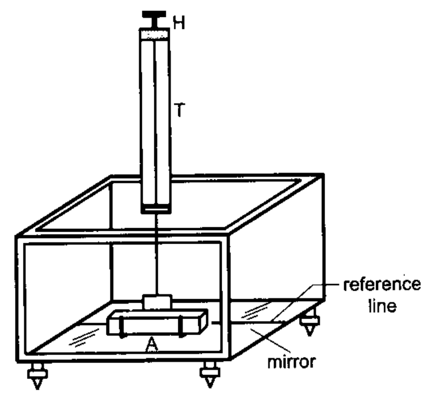

Figure 36.30

The design of an oscillation magnetometer is shown in figure (36.30). It consists of a rectangular wooden box having glass walls at the sides and at the top. Three of the walls are fixed and the fourth may be slid in and out. This is used to close or open the box. A plane mirror is fixed on the inner surface of the bottom of the box. A line parallel to the length of the box is drawn on the mirror in the middle. This is known as the *reference line*. There are levelling screws at the bottom on which the box rests. A vertical cylindrical glass tube T, having a torsion head H at its top, is fitted to the top plate of the box. A hanger A is suspended in the box through an unspun silk thread. The upper end of the thread is attached to the torsion head.

Measuring MB_H for a Bar Magnet

The basic use of an oscillation magnetometer is to measure MB_H for a bar magnet. To start with, the instrument is made horizontal with the help of a spirit level and the levelling screws. A compass needle is placed on the reference line and the box is rotated to make the reference line parallel to the needle. The reference line is now in the magnetic meridian. The needle is removed. A nonmagnetic heavy piece (generally made of brass) is put on the hanger. Due to the twist in the thread, the bar rotates and finally stays in equilibrium when the twist is removed. With the help of the torsion head, the bar is made parallel to the reference line in equilibrium.

The magnet is now gently placed in the hanger after removing the heavy piece. The north pole should be towards the north. The magnet is set into angular oscillation about the thread by giving it a slight angular deflection. This may be done by bringing another magnet close to the box and then removing it. The time period of oscillation is obtained by measuring the time required for, say 20 oscillations. To measure the time period accurately, one should look at the magnet through the top glass cover of the box. The eye should be positioned in such a way that the image of the magnet in the bottom mirror should be just below the magnet. Oscillations are counted as the magnet crosses the reference line. The box is kept closed at the time of oscillations, so that air currents do not disturb the oscillations.

Expression for time period

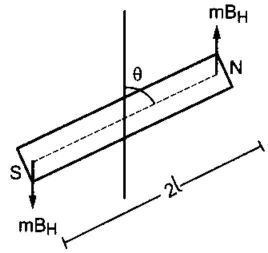

Figure 36.31

Figure (36.31) shows the position of the magnet when it is deflected through an angle θ from the mean position during its oscillation. The magnetic field in the horizontal direction is B_H from south to north. If m be the pole strength, the force on the north pole is mB_H towards north and on the south pole it is mB_H towards south. The length of the magnet is $2l$.

The torque of each of the two forces about the vertical axis is $mB_H l \sin\theta$ and it tries to bring the magnet towards the equilibrium position. The net torque about the vertical axis is

$$\Gamma = -2\, mB_H\, l \sin\theta$$
$$= -MB_H \sin\theta. \qquad \dots \text{(i)}$$

We neglect the torque due to the small twist produced in the thread as the magnet rotates. If the angular amplitude of oscillations is small, $\sin\theta \approx \theta$ and equation (i) becomes

$$\Gamma = -MB_H\theta.$$

Also, $\Gamma = I\alpha$

where I is the moment of inertia of the magnet about the vertical axis and α is the angular acceleration. Thus,

$$\alpha = \frac{\Gamma}{I} = -\frac{MB_H}{I}\theta$$

or, $\alpha = -\omega^2\theta$

where $\omega = \sqrt{\frac{MB_H}{I}}$. This is an equation of angular simple harmonic motion. The time period is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{MB_H}} \cdot \qquad \dots (36.23)$$

Calculation of MB_H

From equation (36.23), $MB_H = \frac{4\pi^2 I}{T^2} \cdot \qquad \dots (36.24)$

For a magnet of rectangular cross section, the moment of inertia about the axis of rotation is

$$I = \frac{W(a^2 + b^2)}{12}$$

where a is the geometrical length, b is the breadth and W is the mass of the magnet. Measuring these quantities and the time period, MB_H can be obtained from equation (36.24).

Example 36.9

A compass needle oscillates 20 times per minute *at a place where the dip is* 45° *and* 30 times per minute *where the dip is* 30°. *Compare the total magnetic field due to the earth at the two places.*

Solution : The time period of oscillation is given by

$$T = 2\pi\sqrt{\frac{I}{MB_H}}.$$

The time period at the first place is T_1 = 1/20 minute = 3·0 s and at the second place it is T_2 = 1/30 minute = 2·0 s.

If the total magnetic field at the first place is B_1, the horizontal component of the field is

$$B_{H1} = B_1 \cos 45° = B_1/\sqrt{2}.$$

Similarly, if the total magnetic field at the second place is B_2, the horizontal component is

$$B_{H2} = B_2 \cos 30° = B_2\sqrt{3}/2.$$

We have,

$$T_1 = 2\pi\sqrt{\frac{I}{MB_{H1}}} \text{ and } T_2 = 2\pi\sqrt{\frac{I}{MB_{H2}}}.$$

Thus,

$$\frac{T_1}{T_2} = \sqrt{\frac{B_{H2}}{B_{H1}}} \quad \text{or,} \quad \frac{B_{H2}}{B_{H1}} = \frac{T_1^2}{T_2^2}$$

or, $$\frac{B_2\sqrt{3}/2}{B_1/\sqrt{2}} = \frac{T_1^2}{T_2^2}$$

or, $$\frac{B_2}{B_1} = \sqrt{\frac{2}{3}}\frac{T_1^2}{T_2^2} = \sqrt{\frac{2}{3}} \times \frac{9}{4} = 1{\cdot}83.$$

Once we know how to measure MB_H, we can easily compare magnetic moments M_1 and M_2 by measuring

M_1B_H and M_2B_H. Similarly, we can compare the horizontal components of the earth's magnetic field at two places.

36.14 DETERMINATION OF M AND B_H

The magnetic moment of a bar magnet can be obtained by measuring M/B_H using a deflection magnetometer and MB_H using an oscillation magnetometer. If

$$M/B_H = X \text{ and } MB_H = Y,$$

we have

$$M = \sqrt{\left(\frac{M}{B_H}\right)(MB_H)} = \sqrt{XY}.$$

One can also determine the horizontal component B_H of earth's magnetic field at any place. Using any magnet one can find M/B_H and MB_H as above.

We have,

$$B_H = \sqrt{\frac{MB_H}{M/B_H}} = \sqrt{\frac{Y}{X}}.$$

If M/B_H is measured in Tan-A position,

$$X = \frac{M}{B_H} = \frac{4\pi}{\mu_0} \frac{(d^2 - l^2)^2}{2d} \tan\theta$$

and $$Y = MB_H = \frac{4\pi^2 I}{T^2}.$$

Then $$M = \sqrt{XY} = \frac{2\pi(d^2 - l^2)}{T} \sqrt{\frac{4\pi}{\mu_0} \frac{I \tan\theta}{2d}}$$

and $$B_H = \sqrt{\frac{Y}{X}} = \frac{2\pi}{T(d^2 - l^2)} \sqrt{\frac{\mu_0}{4\pi} \frac{2Id}{\tan\theta}}.$$

We can also measure M/B_H and MB_H in Tan-B position and get M and B_H as above.

36.15 GAUSS'S LAW FOR MAGNETISM

From Coulomb's law

$$E = \frac{q}{4\pi\varepsilon_0 r^2},$$

we can derive the Gauss's law for electric field, i.e.,

$$\oint \vec{E}\cdot d\vec{S} = \frac{q_{\text{inside}}}{\varepsilon_0}$$

where $\oint \vec{E}\cdot d\vec{S}$ is the electric flux and q_{inside} is the "net charge" enclosed by the closed surface.

Similarly, from the equation

$$B = \frac{\mu_0}{4\pi} \frac{m}{r^2}$$

we can derive Gauss's law for magnetism as

$$\oint \vec{B}\cdot d\vec{S} = \mu_0\, m_{\text{inside}},$$

where $\oint \vec{B}\cdot d\vec{S}$ is the magnetic flux and m_{inside} is the "net pole strength" inside the closed surface. However, there is an additional feature for the magnetic case. We do not have an isolated magnetic pole in nature. At least none has been found to exist till date. The smallest unit of the source of magnetic field is a magnetic dipole where the "net magnetic pole" is zero. Hence, the net magnetic pole enclosed by any closed surface is always zero. Correspondingly, the flux of the magnetic field through any closed surface is zero. *Gauss's law for magnetism,* therefore, states that

$$\oint \vec{B}\cdot d\vec{S} = 0. \qquad \ldots (36.25)$$

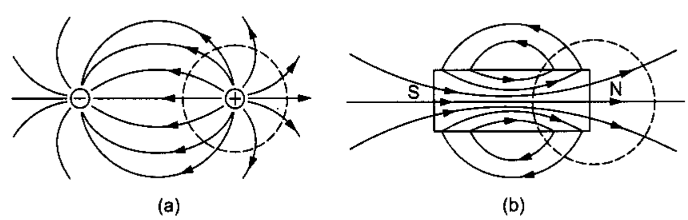

Figure 36.32

Figure (36.32a) shows the electric field lines through a closed surface which encloses one charge of an electric dipole. Figure (36.32b) shows the magnetic field lines through a closed surface which encloses one end of a bar magnet. One can see that the electric field lines only go out of the surface, giving a nonzero flux. On the other hand, the number of magnetic field lines going out of the surface is equal to the number going into it. The magnetic flux is positive at some places on the surface and is negative at others giving the total flux equal to zero.

Worked Out Examples

1. *A bar magnet has a pole strength of* 3·6 A m *and magnetic length* 8 cm. *Find the magnetic field at (a) a point on the axis at a distance of* 6 cm *from the centre towards the north pole and (b) a point on the perpendicular bisector at the same distance.*

Solution :

(a) The point in question is in end-on position, so the magnetic field is,

$$B = \frac{\mu_0}{4\pi} \frac{2Md}{(d^2 - l^2)^2}$$

$$= 10^{-7} \frac{\text{T m}}{\text{A}} \times \frac{2 \times 3{\cdot}6 \text{ A m} \times 0{\cdot}08 \text{ m} \times 0{\cdot}06 \text{ m}}{[(0{\cdot}06 \text{ m})^2 - (0{\cdot}04 \text{ m})^2]^2}$$

$$= 8{\cdot}6 \times 10^{-4} \text{ T}.$$

The field will be away from the magnet.

(b) In this case the point is in broadside-on position so that the field is

$$B = \frac{\mu_0}{4\pi} \frac{M}{(d^2 + l^2)^{3/2}}$$

$$= 10^{-7} \frac{\text{T m}}{\text{A}} \times \frac{3{\cdot}6 \text{ A m} \times 0{\cdot}08 \text{ m}}{[(0{\cdot}06 \text{ m})^2 + (0{\cdot}04 \text{ m})^2]^{3/2}}$$

$$= 7{\cdot}7 \times 10^{-5} \text{ T}.$$

The field will be parallel to the magnet.

2. *A magnet is suspended by a vertical string attached to its middle point. Find the position in which the magnet can stay in equilibrium. The horizontal component of the earth's magnetic field* = 25 μT *and its vertical component* = 40 μT. *Assume that the string makes contact with the magnet only at a single point.*

Solution : The magnetic field of earth is in the north–south plane (magnetic meridian) making an angle θ with the horizontal such that

$$\tan\theta = \frac{B_V}{B_H} = \frac{40}{25}$$

or, $$\theta = 58°.$$

As the tension and the force of gravity act through the centre, their torque about the centre is zero. To make the net torque acting on the magnet zero, it must stay in the direction of the resultant magnetic field. Hence, it stays in the magnetic meridian making an angle of 58° with the horizontal.

3. *A magnetic needle having magnetic moment* 10 A m^2 *and length* 2·0 cm *is clamped at its centre in such a way that it can rotate in the vertical east–west plane. A horizontal force towards east is applied at the north pole to keep the needle fixed at an angle of* 30° *with the vertical. Find the magnitude of the applied force. The vertical component of the earth's magnetic field is* 40 μT.

Solution :

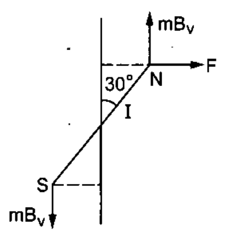

Figure 36-W1

The situation is shown in figure (36-W1). As the needle is in equilibrium, the torque of all the forces about the centre should be zero. As the needle can rotate in the vertical east–west plane, the horizontal component of earth's magnetic field is ineffective. This gives,

$$mB_V l \sin 30° + mB_V l \sin 30° = Fl \cos 30°$$

or, $$F = 2\, mB_V \tan 30°$$

$$= 2 \frac{M}{2l} B_V \tan 30°$$

$$= \frac{(10 \text{ A–m}^2)(40 \times 10^{-6} \text{ T})}{(1{\cdot}0 \times 10^{-2} \text{ m})\sqrt{3}}$$

$$= 2{\cdot}3 \times 10^{-2} \text{ N}.$$

4. *The magnetic scalar potential due to a magnetic dipole at a point on its axis situated at a distance of* 20 cm *from its centre is found to be* 1·2 × 10^{-5} T m. *Find the magnetic moment of the dipole.*

Solution : The magnetic potential due to a dipole is

$$V = \frac{\mu_0}{4\pi} \frac{M \cos\theta}{r^2}$$

or, $$1{\cdot}2 \times 10^{-5} \text{ T m} = \left(10^{-7} \frac{\text{T m}}{\text{A}}\right) \frac{M}{(0{\cdot}2 \text{ m})^2}$$

or, $$M = 4{\cdot}8 \text{ A m}^2.$$

5. *A bar magnet of magnetic moment* 2·0 A m^2 *is free to rotate about a vertical axis through its centre. The magnet is released from rest from the east–west position. Find the kinetic energy of the magnet as it takes the north–south position. The horizontal component of the earth's magnetic field is* B = 25 μT.

Solution : The magnetic potential energy of the dipole in a uniform magnetic field is given by $U = -MB\cos\theta$. As the earth's magnetic field is from south to north, the initial value of θ is π/2 and final value of θ is 0. Thus, the decrease in magnetic potential energy during the rotation is

$$U_i - U_f = -MB\cos(\pi/2) + MB\cos 0$$

$$= 2{\cdot}0 \text{ A m}^2 \times 25\ \mu\text{T} = 50\ \mu\text{J}.$$

Thus, the kinetic energy in the north–south position is 50 μJ.

6. *Figure (36-W2a) shows two identical magnetic dipoles a and b of magnetic moments M each, placed at a separation d, with their axes perpendicular to each other. Find the magnetic field at the point P midway between the dipoles.*

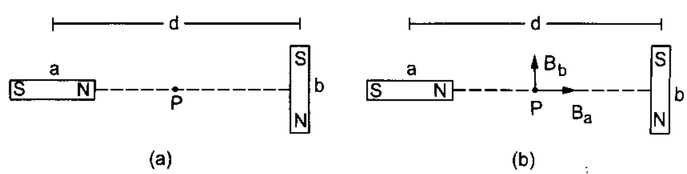

Figure 36-W2

Solution : The point P is in end-on position for the dipole a and in broadside-on position for the dipole b. The magnetic field at P due to a is $B_a = \frac{\mu_0}{4\pi} \frac{2M}{(d/2)^3}$ along the axis of a, and that due to b is $B_b = \frac{\mu_0}{4\pi} \frac{M}{(d/2)^3}$ parallel to the axis of b as shown in figure (36-W2b). The resultant field at P is, therefore,

$$B = \sqrt{B_a^2 + B_b^2}$$
$$= \frac{\mu_0 M}{4\pi (d/2)^3} \sqrt{1^2 + 2^2}$$
$$= \frac{2\sqrt{5}\, \mu_0 M}{\pi d^3}.$$

The direction of this field makes an angle α with B_a such that $\tan\alpha = B_b/B_a = 1/2$.

7. *A bar magnet of length* 8 cm *and having a pole strength of* 1·0 A m *is placed vertically on a horizontal table with its south pole on the table. A neutral point is found on the table at a distance of* 6·0 cm *north of the magnet. Calculate the earth's horizontal magnetic field.*

Solution :

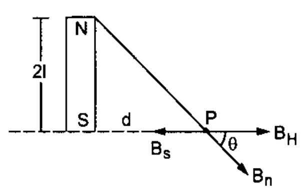

Figure 36-W3

The situation is shown in figure (36-W3). The magnetic field at P due to the south pole is

$$B_s = \frac{\mu_0}{4\pi} \frac{m}{d^2}$$

towards south and that due to the north pole is

$$B_n = \frac{\mu_0}{4\pi} \frac{m}{(d^2 + 4l^2)}$$

along NP. The horizontal component of this field will be towards north and will have the magnitude

$$= \frac{\mu_0}{4\pi} \frac{m}{(d^2 + 4l^2)} \cdot \frac{d}{(d^2 + 4l^2)^{1/2}}.$$

The resultant horizontal field due to the magnet is, therefore,

$$\frac{\mu_0}{4\pi} \frac{m}{d^2} - \frac{\mu_0}{4\pi} \frac{md}{(d^2 + 4l^2)^{3/2}}$$

towards south. As P is a neutral point, this field should be equal in magnitude to the earth's magnetic field B_H which is towards north. Thus,

$$B_H = \frac{\mu_0 m}{4\pi} \left[\frac{1}{d^2} - \frac{d}{(d^2 + 4l^2)^{3/2}} \right]$$
$$= \frac{\mu_0 m}{4\pi} \left[\frac{1}{36\ \text{cm}^2} - \frac{6\ \text{cm}}{(36\ \text{cm}^2 + 64\ \text{cm}^2)^{3/2}} \right]$$
$$= 10^{-7}\ \text{T m A}^{-1} \times (1{\cdot}0\ \text{A m}) \times \left[\frac{1}{36\ \text{cm}^2} - \frac{6}{1000\ \text{cm}^2} \right]$$
$$= (10^{-7}\ \text{T m}^2)\ [0{\cdot}028 - 0{\cdot}006] \times 10^4\ \text{m}^{-2}$$
$$= 22 \times 10^{-6}\ \text{T} = 22\ \mu\text{T}.$$

8. *The magnetic field at a point on the magnetic equator is found to be* $3{\cdot}1 \times 10^{-5}$ T. *Taking the earth's radius to be* 6400 km, *calculate the magnetic moment of the assumed dipole at the earth's centre.*

Solution :

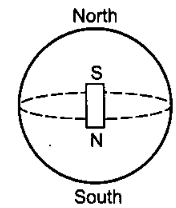

Figure 36-W4

A point on the magnetic equator is in broadside-on position of the earth's assumed dipole (figure 36-W4). The field is, therefore,

$$B = \frac{\mu_0}{4\pi} \frac{M}{R^3}$$

or, $$M = \frac{4\pi}{\mu_0} BR^3$$
$$= 10^7\ \text{A m}^{-1}\ \text{T}^{-1} \times 3{\cdot}1 \times 10^{-5}\ \text{T} \times (6400 \times 10^3)^3\ \text{m}^3$$
$$= 8{\cdot}1 \times 10^{22}\ \text{A m}^2.$$

9. *The earth's magnetic field at geomagnetic poles has a magnitude* $6{\cdot}2 \times 10^{-5}$ T. *Find the magnitude and the direction of the field at a point on the earth's surface where the radius makes an angle of* 135° *with the axis of the earth's assumed magnetic dipole. What is the inclination (dip) at this point?*

Solution :

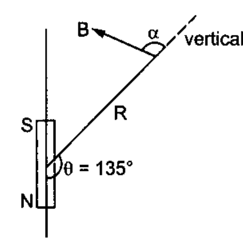

Figure 36-W5

Assuming the earth's field to be due to a dipole at the centre, geomagnetic poles are in end-on position (figure 36-W5).

The magnetic field B at geomagnetic poles is

$$B_p = \frac{\mu_0}{4\pi} \frac{2M}{R^3}.$$

The magnetic field due to a dipole at a distance R away from its centre has a magnitude

$$B = \frac{\mu_0}{4\pi} \frac{M}{R^3} (1 + 3\cos^2\theta)^{1/2} = \frac{1}{2} B_p (1 + 3\cos^2\theta)^{1/2}.$$

This field is in a direction making an angle α with the radial direction such that $\tan\alpha = (\tan\theta)/2$, as shown in the figure. At the point given, $\theta = 135°$ and thus the field B is

$$B = \frac{B_p}{2}(1 + 3\cos^2 135°)^{1/2}$$
$$= \frac{1}{2} \times 6{\cdot}2 \times 10^{-5}\ \text{T} \times 1{\cdot}58$$
$$= 4{\cdot}9 \times 10^{-5}\ \text{T}.$$

The angle α of this field with the vertical is given by

$$\tan\alpha = \frac{\tan\theta}{2} = \frac{\tan 135°}{2} = -0{\cdot}5$$

giving $\alpha = 153°$.

The inclination (dip) is the angle made by the earth's magnetic field with the horizontal plane. Here it is $153° - 90° = 63°$ below the horizontal.

10. *A magnetic needle free to rotate in a fixed vertical plane stays in a direction making an angle of 60° with the horizontal. If the dip at that place is 37°, find the angle of the fixed vertical plane with the meridian.*

Solution : If the vertical plane makes an angle θ with the meridian, the horizontal component of the earth's field in that plane will be $B_H \cos\theta$. Thus the apparent dip δ_1, i.e., the angle between the needle in equilibrium and the horizontal will be given by

$$\tan\delta_1 = \frac{B_V}{B_H \cos\theta} = \frac{\tan\delta}{\cos\theta}$$

or, $$\cos\theta = \frac{\tan\delta}{\tan\delta_1}$$
$$= \frac{\tan 37°}{\tan 60°} = \frac{3}{4\sqrt{3}} = \frac{\sqrt{3}}{4}$$

or, $$\theta = 64°.$$

11. *A dip circle shows an apparent dip of 60° at a place where the true dip is 45°. If the dip circle is rotated through 90°, what apparent dip will it show ?*

Solution : If δ_1 and δ_2 be the apparent dips shown by the dip circle in the two perpendicular positions, the true dip δ is given by

$$\cot^2\delta = \cot^2\delta_1 + \cot^2\delta_2$$

or, $$\cot^2 45° = \cot^2 60° + \cot^2\delta_2$$

or, $$\cot^2\delta_2 = 2/3$$

or, $$\cot\delta_2 = 0{\cdot}816 \text{ giving } \delta_2 = 51°.$$

12. *A magnetic needle of length* 10 cm, *suspended at its middle point through a thread, stays at an angle of* 45° *with the horizontal. The horizontal component of the earth's magnetic field is* 18 μT. *(a) Find the vertical component of this field. (b) If the pole strength of the needle is* 1·6 A–m, *what vertical force should be applied to an end so as to keep it in horizontal position?*

Solution :

(a)

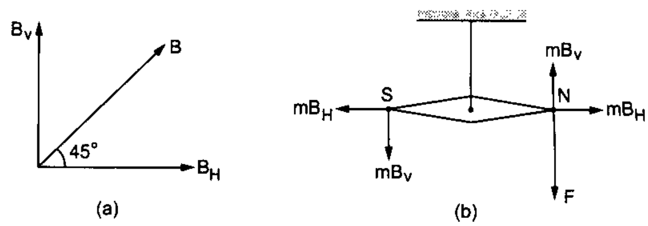

Figure 36-W6

Without the applied force, the needle will stay in the direction of the resultant magnetic field of the earth. Thus, the dip δ at the place is 45°. From figure (36-W6a),

$$\tan 45° = B_V / B_H$$

or, $$B_V = B_H = 18\ \mu\text{T}.$$

(b) When the force F is applied (figure 36-W6b), the needle stays in horizontal position. Taking torque about the centre of the magnet,

$$2mB_V \times l = F \times l$$

or, $$F = 2mB_V$$
$$= 2 \times (1{\cdot}6\ \text{A m}) \times (18 \times 10^{-6}\ \text{T})$$
$$= 5{\cdot}8 \times 10^{-5}\ \text{N}.$$

13. *A tangent galvanometer has a coil of* 50 *turns and a radius of* 20 cm. *The horizontal component of the earth's magnetic field is* $B_H = 3 \times 10^{-5}$ T. *Find the current which gives a deflection of* 45°.

Solution : We have

$$i = K\tan\theta = \frac{2rB_H}{\mu_0 n}\tan\theta$$
$$= \frac{2 \times (0{\cdot}20\ \text{m}) \times (3 \times 10^{-5}\ \text{T})}{4\pi \times 10^{-7}\ \text{T m A}^{-1} \times 50}\tan 45° = 0{\cdot}19\ \text{A}.$$

14. *A moving-coil galvanometer has* 100 *turns and each turn has an area* 2·0 cm^2. *The magnetic field produced by the magnet is* 0·01 T. *The deflection in the coil is* 0·05 *radian when a current of* 10 mA *is passed through it. Find the torsional constant of the suspension wire.*

Solution : We have

$$i = \frac{K}{nAB}\theta$$

or, $$K = \frac{inAB}{\theta}$$
$$= \frac{(10 \times 10^{-3}\ \text{A}) \times 100 \times (2{\cdot}0 \times 10^{-4}\ \text{m}^2) \times 0{\cdot}01\ \text{T}}{0{\cdot}05\ \text{rad}}$$
$$= 4{\cdot}0 \times 10^{-5}\ \text{N m rad}^{-1}.$$

15. *A galvanometer coil has a resistance of* 100 Ω. *When a current passes through the galvanometer,* 1% *of the current goes through the coil and the rest through the shunt. Find the resistance of the shunt.*

Solution :

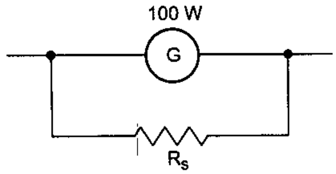

Figure 36-W7

The situation is shown in figure (36-W7). As the potential differences across the 100 Ω coil and across the shunt R_s are the same,

$$0{\cdot}01\, i \times 100\, \Omega = 0{\cdot}99\, i \times R_s$$

or, $$R_s = \frac{0{\cdot}01 \times 100}{0{\cdot}99}\, \Omega = \frac{100}{99}\, \Omega \cdot$$

16. *The needle of a deflection magnetometer deflects through* 45° *from north to south when the instrument is used in Tan-A position with a magnet of length* 10 cm *placed at a distance of* 25 cm. *(a) Find the magnetic moment of the magnet if the earth's horizontal magnetic field is* 20 μT. *(b) If the magnetometer is used in Tan-B position with the same magnet at the same separation from the needle, what will be the deflection ?*

Solution : (a) In Tan-*A* position, the needle is in end-on position of the magnet so that the field at the needle due to the magnet is

$$B = \frac{\mu_0}{4\pi} \frac{2Md}{(d^2 - l^2)^2} \cdot$$

Thus, $$\frac{\mu_0}{4\pi} \frac{2Md}{(d^2 - l^2)^2} = B_H \tan\theta$$

or, $$M = \frac{4\pi}{\mu_0} \frac{B_H \tan\theta (d^2 - l^2)^2}{2d}$$

$$= 10^7\ \mathrm{A\ m^{-1}\ T^{-1}} \times \frac{20 \times 10^{-6}\ \mathrm{T} \times 1 \times (625 - 25)^2 \times 10^{-8}\ \mathrm{m^4}}{2 \times 25 \times 10^{-2}\ \mathrm{m}}$$

$$= 1{\cdot}44\ \mathrm{A\ m^2}.$$

(b) In Tan-*B* position, the needle is in broadside-on position of the manget, so that

$$B = \frac{\mu_0}{4\pi} \frac{M}{(d^2 + l^2)^{3/2}} = B_H \tan\theta$$

or, $$\tan\theta = \frac{10^{-7}\ \mathrm{T\ m\ A^{-1}} \times 1{\cdot}44\ \mathrm{A\ m^2}}{(625 + 25)^{3/2} \times 10^{-6}\ \mathrm{m^3} \times 20 \times 10^{-6}\ \mathrm{T}}$$

$$= 0{\cdot}43$$

or, $$\theta = 23{\cdot}5°.$$

17. *Figure (36-W8) shows a short magnet executing small oscillations in an oscillation magnetometer in earth's magnetic field having horizontal component* 24 μT. *The time period of oscillation is* 0·10 s. *An upward electric current of* 18 A *is established in the vertical wire placed* 20 cm *east of the magnet by closing the switch S. Find the new time period.*

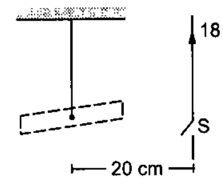

Figure 36-W8

Solution : The magnetic field at the site of the short magnet due to the vertical current is

$$B = \frac{\mu_0\, i}{2\pi d} = (2 \times 10^{-7}\ \mathrm{T\ m\ A^{-1}}) \frac{18\ \mathrm{A}}{0{\cdot}20\ \mathrm{m}}$$

$$= 18\ \mu\mathrm{T}.$$

As the wire is east of the magnet, this magnetic field will be from north to south according to the right-hand thumb rule. The earth's magnetic field has horizontal component 24 μT from south to north. Thus, the resultant field will be 6·0 μT from south to north. If T_1 and T_2 be the time periods without and with the current,

$$T_1 = \sqrt{\frac{I}{MB_H}} \quad \text{and} \quad T_2 = \sqrt{\frac{I}{M(B_H - B)}}$$

or, $$\frac{T_2}{T_1} = \sqrt{\frac{B_H}{B_H - B}} = \sqrt{\frac{24\ \mu\mathrm{T}}{6\ \mu\mathrm{T}}} = 2$$

or, $$T_2 = 2T_1 = 0{\cdot}20\ \mathrm{s}.$$

18. *The frequency of oscillation of the magnet in an oscillation magnetometer in the earth's magnetic field is* 40 *oscillations per minute. A short bar magnet is placed to the north of the magnetometer, at a separation of* 20 cm *from the oscillating magnet, with its north pole pointing towards north (figure 36-W9). The frequency of oscillation is found to increase to* 60 *oscillations per minute. Calculate the magnetic moment of this short bar magnet. Horizontal component of the earth's magnetic field is* 24 μT.

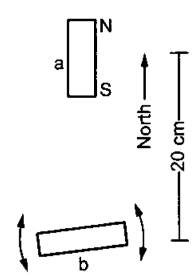

Figure 36-W9

Solution : Let the magnetic field due to the short magnet have magnitude *B* at the site of the oscillating magnet. From the figure, this magnetic field will be towards north and hence the resultant horizontal field will be $B_H + B$. Let *M* and *M′* denote the magnetic moments of the oscillating magnet and the other magnet respectively. If ν and ν′ be the frequencies without and with the other magnet, we have

$$\nu = \frac{1}{2\pi} \sqrt{\frac{MB_H}{I}} \quad \text{and}$$

$$v' = \frac{1}{2\pi}\sqrt{\frac{M(B_H + B)}{I}}$$

or, $$\frac{v'^2}{v^2} = \frac{B_H + B}{B_H}$$

or, $$\left(\frac{60}{40}\right)^2 = 1 + \frac{B}{B_H}$$

or, $$\frac{B}{B_H} = 1{\cdot}25$$

or, $$B = 1{\cdot}25 \times 24\ \mu T = 30 \times 10^{-6}\ T.$$

The oscillating magnet is in end-on position of the short magnet. Thus, the field B can be written as

$$B = \frac{\mu_0}{4\pi}\frac{2M'}{d^3}$$

or, $$M' = \frac{2\pi}{\mu_0} Bd^3$$

$$= 0{\cdot}5 \times 10^7\ A\ m^{-1}\ T^{-1} \times (30 \times 10^{-6}\ T) \times (20 \times 10^{-2}\ m)^3$$

$$= 1{\cdot}2\ A\ m^2.$$

19. *A bar magnet of mass* 100 g, *length* 7·0 cm, *width* 1·0 cm *and height* 0·50 cm *takes* $\pi/2$ seconds *to complete an oscillation in an oscillation magnetometer placed in a horizontal magnetic field of* 25 μT. *(a) Find the magnetic moment of the magnet. (b) If the magnet is put in the magnetometer with its* 0·50 cm *edge horizontal, what would be the time period?*

Solution : (a) The moment of inertia of the magnet about the axis of rotation is

$$I = \frac{m'}{12}(L^2 + b^2)$$

$$= \frac{100 \times 10^{-3}}{12}[(7 \times 10^{-2})^2 + (1 \times 10^{-2})^2]\ kg\ m^2$$

$$= \frac{25}{6} \times 10^{-5}\ kg\ m^2.$$

We have,

$$T = 2\pi\sqrt{\frac{I}{MB}} \qquad \text{... (i)}$$

or, $$M = \frac{4\pi^2 I}{BT^2} = \frac{4\pi^2 \times 25 \times 10^{-5}\ kg\ m^2}{6 \times (25 \times 10^{-6}\ T) \times \frac{\pi^2}{4}\ s^2}$$

$$\approx 27\ A\ m^2.$$

(b) In this case the moment of inertia becomes

$$I' = \frac{m'}{12}(L^2 + b'^2) \text{ where } b' = 0{\cdot}5\ cm.$$

The time period would be

$$T' = \sqrt{\frac{I'}{MB}}. \qquad \text{... (ii)}$$

Dividing by equation (i),

$$\frac{T'}{T} = \sqrt{\frac{I'}{I}}$$

$$= \frac{\sqrt{\frac{m'}{12}(L^2 + b'^2)}}{\sqrt{\frac{m'}{12}(L^2 + b^2)}} = \frac{\sqrt{(7\ cm)^2 + (0{\cdot}5\ cm)^2}}{\sqrt{(7\ cm)^2 + (1{\cdot}0\ cm)^2}}$$

$$= 0{\cdot}992$$

or, $$T' = \frac{0{\cdot}992 \times \pi}{2}\ s = 0{\cdot}496\pi\ s.$$

□

QUESTIONS FOR SHORT ANSWER

1. Can we have a single north pole, or a single south pole?
2. Do two distinct poles actually exist at two nearby points in a magnetic dipole?
3. An iron needle is attracted to the ends of a bar magnet but not to the middle region of the magnet. Is the material making up the ends of a bar magnet different from that of the middle region?
4. Compare the direction of the magnetic field inside a solenoid with that of the field there if the solenoid is replaced by its equivalent combination of north pole and south pole.
5. Sketch the magnetic field lines for a current-carrying circular loop near its centre. Replace the loop by an equivalent magnetic dipole and sketch the magnetic field lines near the centre of the dipole. Identify the difference.
6. The force on a north pole, $\vec{F} = m\vec{B}$, is parallel to the field $\vec{B}$. Does it contradict our earlier knowledge that a magnetic field can exert forces only perpendicular to itself?
7. Two bar magnets are placed close to each other with their opposite poles facing each other. In absence of other forces, the magnets are pulled towards each other and their kinetic energy increases. Does it contradict our earlier knowledge that magnetic forces cannot do any work and hence cannot increase kinetic energy of a system?
8. Magnetic scalar potential is defined as

$$U(\vec{r}_2) - U(\vec{r}_1) = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{B}\cdot d\vec{l}.$$

Apply this equation to a closed curve enclosing a long straight wire. The RHS of the above equation is then $-\mu_0 i$ by Ampere's law. We see that $U(\vec{r_2}) \neq U(\vec{r_1})$ even when $\vec{r_2} = \vec{r_1}$. Can we have a magnetic scalar potential in this case ?

9. Can the earth's magnetic field be vertical at a place ? What will happen to a freely suspended magnet at such a place ? What is the value of dip here ?

10. Can the dip at a place be (a) zero (b) 90° ?

11. The reduction factor K of a tangent galvanometer is written on the instrument. The manual says that the current is obtained by multiplying this factor to $\tan\theta$. The procedure works well at Bhuwaneshwar. Will the procedure work if the instrument is taken to Nepal ? If there is some error, can it be corrected by correcting the manual or the instrument will have to be taken back to the factory ?

OBJECTIVE I

1. A circular loop carrying a current is replaced by an equivalent magnetic dipole. A point on the axis of the loop is in
   (a) end-on position (b) broadside-on position
   (c) both (d) none.

2. A circular loop carrying a current is replaced by an equivalent magnetic dipole. A point on the loop is in
   (a) end-on position (b) broadside-on position
   (c) both (d) none.

3. When a current in a circular loop is equivalently replaced by a magnetic dipole,
   (a) the pole strength m of each pole is fixed
   (b) the distance d between the poles is fixed
   (c) the product md is fixed
   (d) none of the above.

4. Let r be the distance of a point on the axis of a bar magnet from its centre. The magnetic field at such a point is proportional to
   (a) $\frac{1}{r}$ (b) $\frac{1}{r^2}$ (c) $\frac{1}{r^3}$ (d) none of these.

5. Let r be the distance of a point on the axis of a magnetic dipole from its centre. The magnetic field at such a point is proportional to
   (a) $\frac{1}{r}$ (b) $\frac{1}{r^2}$ (c) $\frac{1}{r^3}$ (d) none of these.

6. Two short magnets of equal dipole moments M are fastened perpendicuarly at their centres (figure 36-Q1). The magnitude of the magnetic field at a distance d from the centre on the bisector of the right angle is
   (a) $\frac{\mu_0}{4\pi}\frac{M}{d^3}$ (b) $\frac{\mu_0}{4\pi}\frac{\sqrt{2}M}{d^3}$ (c) $\frac{\mu_0}{4\pi}\frac{2\sqrt{2}M}{d^3}$ (d) $\frac{\mu_0}{4\pi}\frac{2M}{d^3}$.

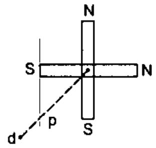

Figure 36-Q1

7. Magnetic meridian is
   (a) a point (b) a line along north–south
   (c) a horizontal plane (d) a vertical plane.

8. A compass needle which is allowed to move in a horizontal plane is taken to a geomagnetic pole. It
   (a) will stay in north–south direction only
   (b) will stay in east–west direction only
   (c) will become rigid showing no movement
   (d) will stay in any position.

9. A dip circle is taken to geomagnetic equator. The needle is allowed to move in a vertical plane perpendicular to the magnetic meridian. The needle will stay
   (a) in horizontal direction only
   (b) in vertical direction only
   (c) in any direction except vertical and horizontal
   (d) in any direction it is released.

10. Which of the following four graphs may best represent the current–deflection relation in a tangent galvanometer ?

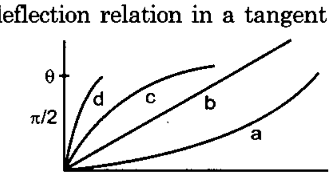

Figure 36-Q2

11. A tangent galvanometer is connected directly to an ideal battery. If the number of turns in the coil is doubled, the deflection will
    (a) increase (b) decrease
    (c) remain unchanged (d) either increase or decrease.

12. If the current is doubled, the deflection is also doubled in
    (a) a tangent galvanometer
    (b) a moving-coil galvanometer
    (c) both (d) none.

13. A very long bar magnet is placed with its north pole coinciding with the centre of a circular loop carrying an electric current i. The magnetic field due to the magnet at a point on the periphery of the wire is B. The radius of the loop is a. The force on the wire is
    (a) very nearly $2\pi aiB$ perpendicular to the plane of the wire
    (b) $2\pi aiB$ in the plane of the wire
    (c) πaiB along the magnet (d) zero.

OBJECTIVE II

1. Pick the correct options.
   (a) Magnetic field is produced by electric charges only.
   (b) Magnetic poles are only mathematical assumptions having no real existence.
   (c) A north pole is equivalent to a clockwise current and a south pole is equivalent to an anticlockwise current.
   (d) A bar magnet is equivalent to a long, straight current.
2. A horizontal circular loop carries a current that looks clockwise when viewed from above. It is replaced by an equivalent magnetic dipole consisting of a south pole S and a north pole N.
   (a) The line SN should be along a diameter of the loop.
   (b) The line SN should be perpendicular to the plane of the loop.
   (c) The south pole should be below the loop.
   (d) The north pole should be below the loop.
3. Consider a magnetic dipole kept in the north to south direction. Let P_1, P_2, Q_1, Q_2 be four points at the same distance from the dipole towards north, south, east and west of the dipole respectively. The directions of the magnetic field due to the dipole are the same at
   (a) P_1 and P_2 (b) Q_1 and Q_2
   (c) P_1 and Q_1 (d) P_2 and Q_2.
4. Consider the situation of the previous problem. The directions of the magnetic field due to the dipole are opposite at
   (a) P_1 and P_2 (b) Q_1 and Q_2
   (c) P_1 and Q_1 (d) P_2 and Q_2.
5. To measure the magnetic moment of a bar magnet, one may use
   (a) a tangent galvanometer
   (b) a deflection galvanometer if the earth's horizontal field is known
   (c) an oscillation magnetometer if the earth's horizontal field is known
   (d) both deflection and oscillation magnetometer if the earth's horizontal field is not known.

EXERCISES

1. A long bar magnet has a pole strength of 10 Am. Find the magnetic field at a point on the axis of the magnet at a distance of 5 cm from the north pole of the magnet.
2. Two long bar magnets are placed with their axes coinciding in such a way that the north pole of the first magnet is 2·0 cm from the south pole of the second. If both the magnets have a pole strength of 10 Am, find the force exerted by one magnet on the other.
3. A uniform magnetic field of $0{\cdot}20 \times 10^{-3}$ T exists in the space. Find the change in the magnetic scalar potential as one moves through 50 cm along the field.
4. Figure (36-E1) shows some of the equipotential surfaces of the magnetic scalar potential. Find the magnetic field B at a point in the region.

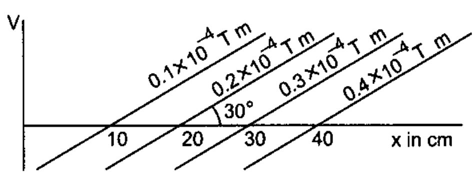

Figure 36-E1

5. The magnetic field at a point, 10 cm away from a magnetic dipole, is found to be $2{\cdot}0 \times 10^{-4}$ T. Find the magnetic moment of the dipole if the point is (a) in end-on position of the dipole and (b) in broadside-on position of the dipole.
6. Show that the magnetic field at a point due to a magnetic dipole is perpendicular to the magnetic axis if the line joining the point with the centre of the dipole makes an angle of $\tan^{-1}(\sqrt{2})$ with the magnetic axis.
7. A bar magnet has a length of 8 cm. The magnetic field at a point at a distance 3 cm from the centre in the broadside-on position is found to be 4×10^{-6} T. Find the pole strength of the magnet.
8. A magnetic dipole of magnetic moment $1{\cdot}44\ \text{A m}^2$ is placed horizontally with the north pole pointing towards north. Find the position of the neutral point if the horizontal component of the earth's magnetic field is 18 μT.
9. A magnetic dipole of magnetic moment $0{\cdot}72\ \text{A m}^2$ is placed horizontally with the north pole pointing towards south. Find the position of the neutral point if the horizontal component of the earth's magnetic field is 18 μT.
10. A magnetic dipole of magnetic moment $0{\cdot}72\sqrt{2}\ \text{A m}^2$ is placed horizontally with the north pole pointing towards east. Find the position of the neutral point if the horizontal component of the earth's magnetic field is 18 μT.
11. The magnetic moment of the assumed dipole at the earth's centre is $8{\cdot}0 \times 10^{22}\ \text{A m}^2$. Calculate the magnetic field B at the geomagnetic poles of the earth. Radius of the earth is 6400 km.
12. If the earth's magnetic field has a magnitude $3{\cdot}4 \times 10^{-5}$ T at the magnetic equator of the earth, what would be its value at the earth's geomagnetic poles ?
13. The magnetic field due to the earth has a horizontal component of 26 μT at a place where the dip is 60°. Find the vertical component and the magnitude of the field.
14. A magnetic needle is free to rotate in a vertical plane which makes an angle of 60° with the magnetic meridian. If the needle stays in a direction making an angle of $\tan^{-1}(2/\sqrt{3})$ with the horizontal, what would be the dip at that place ?

15. The needle of a dip circle shows an apparent dip of 45° in a particular position and 53° when the circle is rotated through 90°. Find the true dip.

16. A tangent galvanometer shows a deflection of 45° when 10 mA of current is passed through it. If the horizontal component of the earth's magnetic field is $B_H = 3{\cdot}6 \times 10^{-5}$ T and radius of the coil is 10 cm, find the number of turns in the coil.

17. A moving-coil galvanometer has a 50-turn coil of size 2 cm × 2 cm. It is suspended between the magnetic poles producing a magnetic field of 0·5 T. Find the torque on the coil due to the magnetic field when a current of 20 mA passes through it.

18. A short magnet produces a deflection of 37° in a deflection magnetometer in Tan-*A* position when placed at a separation of 10 cm from the needle. Find the ratio of the magnetic moment of the magnet to the earth's horizontal magnetic field.

19. The magnetometer of the previous problem is used with the same magnet in Tan-*B* position. Where should the magnet be placed to produce a 37° deflection of the needle ?

20. A deflection magnetometer is placed with its arms in north–south direction. How and where should a short magnet having $M/B_H = 40$ A m^2 T^{-1} be placed so that the needle can stay in any position ?

21. A bar magnet takes $\pi/10$ second to complete one oscillation in an oscillation magnetometer. The moment of inertia of the magnet about the axis of rotation is $1{\cdot}2 \times 10^{-4}$ kg m^2 and the earth's horizontal magnetic field is 30 μT. Find the magnetic moment of the magnet.

22. The combination of two bar magnets makes 10 oscillations per second in an oscillation magnetometer when like poles are tied together and 2 oscillations per second when unlike poles are tied together. Find the ratio of the magnetic moments of the magnets. Neglect any induced magnetism.

23. A short magnet oscillates in an oscillation magnetometer with a time period of 0·10 s where the earth's horizontal magnetic field is 24 μT. A downward current of 18 A is established in a vertical wire placed 20 cm east of the magnet. Find the new time period.

24. A bar magnet makes 40 oscillations per minute in an oscillation magnetometer. An identical magnet is demagnetized completely and is placed over the magnet in the magnetometer. Find the time taken for 40 oscillations by this combination. Neglect any induced magnetism.

25. A short magnet makes 40 oscillations per minute when used in an oscillation magnetometer at a place where the earth's horizontal magnetic field is 25 μT. Another short magnet of magnetic moment 1·6 A m^2 is placed 20 cm east of the oscillating magnet. Find the new frequency of oscillation if the magnet has its north pole (a) towards north and (b) towards south.

□

ANSWERS

OBJECTIVE I

1. (a)	2. (b)	3. (c)	4. (d)	5. (c)	6. (c)
7. (d)	8. (d)	9. (d)	10. (c)	11. (c)	12. (b)
13. (a)					

OBJECTIVE II

1. (a), (b)	2. (b), (d)	3. (a), (b)
4. (c), (d)	5. (b), (c), (d)	

EXERCISES

1. 4×10^{-4} T
2. $2{\cdot}5 \times 10^{-2}$ N
3. decreases by $0{\cdot}10 \times 10^{-3}$ T m
4. $2{\cdot}0 \times 10^{-4}$ T
5. (a) 1·0 A m^2 and (b) 2·0 A m^2
7. 6×10^{-5} A m
8. at a distance of 20 cm in the plane bisecting the dipole
9. 20 cm south of the dipole
10. 20 cm from the dipole, $\tan^{-1}\sqrt{2}$ south of east
11. 60 μT
12. $6{\cdot}8 \times 10^{-5}$ T
13. 45 μT, 52 μT
14. 30°
15. 39°
16. 570
17. 2×10^{-4} N m
18. $3{\cdot}75 \times 10^{3} \dfrac{\text{A m}^2}{\text{T}}$
19. 7·9 cm from the centre
20. 2·0 cm from the needle, north pole pointing towards south
21. 1600 A m^2
22. 13 : 12
23. 0·076 s
24. $\sqrt{2}$ minutes
25. (a) 18 oscillations/min (b) 54 oscillations/min

□