Indefinite Integrals Ex 19.8 Q36

Let $\quad I = \int \frac{1-\sin 2x}{x+\cos^2 x} dx - - - - - (i)$

Let $x + \cos^2 x = t \quad$ then,

$d\left(x+\cos^2 x\right) = dt$

$\Rightarrow \quad (1 - 2\cos x \sin x) dx = dt$

$\Rightarrow \quad dx = \frac{dt}{1 - 2\cos x \sin x}$

Putting $x + \cos^2 x = t$ and $dx = \frac{dt}{1 - 2\cos x \sin x}$ in equation (i), we get

$$I = \int \frac{1-\sin 2x}{t} \times \frac{dt}{1 - 2\cos x \sin x}$$
$$= \int \frac{1-\sin 2x}{t} \times \frac{dt}{1-\sin 2x}$$
$$= \int \frac{dt}{t}$$
$$= \log|t| + c$$
$$= \log\left|x + \cos^2 x\right| + c$$

$\therefore \quad I = \log\left|x + \cos^2 x\right| + c$

Indefinite Integrals Ex 19.8 Q37

Let $\quad I = \int \frac{1+\tan x}{x + \log\sec x} dx - - - - - - - (i)$

Let $x + \log\sec x = t \quad$ then,

$d(x + \log\sec x) = dt$

$\Rightarrow \quad (1+\tan x) dx = dt \quad \left[\because \quad \frac{d}{dx}(\log\sec x) = \tan x\right]$

$\Rightarrow \quad dx = \frac{dt}{1+\tan x}$

Putting $x + \log\sec x = t$ and $dx = \frac{dt}{1+\tan x}$ in equation (i), we get,

$$I = \int \frac{1+\tan x}{t} \times \frac{dt}{1+\tan x}$$
$$= \int \frac{dt}{t}$$
$$= \log|t| + c$$

$\Rightarrow \quad I = \log|x + \log\sec x| + c$

Indefinite Integrals Ex 19.8 Q38

Let $I = \int \frac{\sin 2x}{a^2 + b^2 \sin^2 x} dx$ - - - - - - - (i)

Let $a^2 + b^2 \sin^2 x = t$ then,

$d\left(a^2 + b^2 \sin^2 x\right) = dt$

$\Rightarrow$ $b^2 \left(2 \sin x \cos x\right) dx = dt$

$\Rightarrow$ $dx = \frac{dt}{b^2 \left(2 \sin x \cos x\right)}$

$= \frac{dt}{b^2 \sin 2x}$

Putting $a^2 + b^2 \sin^2 x = t$ and $dx = \frac{dt}{b^2 \sin 2x}$ in equation (i),we get,

$I = \int \frac{\sin 2x}{t} \times \frac{dt}{b^2 \sin 2x}$

$= \frac{1}{b^2} \int \frac{dt}{t}$

$= \frac{1}{b^2} \log|t| + c$

$= \frac{1}{b^2} \log\left|a^2 + b^2 \sin^2 x\right| + c$

$\Rightarrow$ $I = \frac{1}{b^2} \log\left|a^2 + b^2 \sin^2 x\right| + c$

Indefinite Integrals Ex 19.8 Q39

Let $I = \int \frac{x+1}{x\left(x + \log x\right)} dx$ - - - - - - - (i)

Let $\left(x + \log x\right) = t$ then,

$d\left(x + \log x\right) = dt$

$\Rightarrow$ $\left(1 + \frac{1}{x}\right) dx = dt$

$\Rightarrow$ $\left(\frac{x+1}{x}\right) dx = dt$

$\Rightarrow$ $dx = \frac{x}{x+1} dt$

Putting $\left(x + \log x\right) = t$ and $dx = \frac{x}{x+1}$ in equation (i),we get,

$I = \int \frac{x+1}{x \times t} \times \frac{x}{x+1} dt$

$= \int \frac{dt}{t}$

$= \log|t| + c$

$= \log\left|x + \log x\right| + c$

$\Rightarrow$ $I = \log\left|x + \log x\right| + c$

Indefinite Integrals Ex 19.8 Q40

Let $\quad I = \int \frac{1}{\sqrt{1-x^2}\left(2+3\sin^{-1}x\right)}dx$ ------- (i)

Let $2+3\sin^{-1}x = t$ then,

$d\left(2+3\sin^{-1}x\right) = dt$

$$\Rightarrow \quad 3 \times \frac{1}{\sqrt{1-x^2}}dx = dt$$

$$\Rightarrow \quad dx = \frac{\sqrt{1-x^2}}{3}dt$$

Putting $2+3\sin^{-1}x = t$ and $dx = \frac{\sqrt{1-x^2}}{3}$ in equation (i), we get,

$$I = \int \frac{\sqrt{1-x^2}}{3} \times \frac{1}{\sqrt{1-x^2}\,t}dt$$

$$= \frac{1}{3}\int \frac{dt}{t}$$

$$= \frac{1}{3}\log|t| + c$$

$$= \frac{1}{3}\log\left|2+3\sin^{-1}x\right| + c$$

$$\Rightarrow \quad I = \frac{1}{3}\log\left|2+3\sin^{-1}x\right| + c$$

*********** END **********