



EXERCISE 3.3

Question 1:

Prove

$$\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4} = -\frac{1}{2}$$

Ans:

$$\text{L.H.S.} = \sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4}$$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - (1)^2$$

$$= \frac{1}{4} + \frac{1}{4} - 1 = -\frac{1}{2}$$

$$= \text{R.H.S.}$$

Question 2:

$$\text{Prove that } 2\sin^2 \frac{\pi}{6} + \operatorname{cosec}^2 \frac{7\pi}{6} \cos^2 \frac{\pi}{3} = \frac{3}{2}$$

Ans:

$$\begin{aligned}
\text{L.H.S.} &= 2 \sin^2 \frac{\pi}{6} + \operatorname{cosec}^2 \frac{7\pi}{6} \cos^2 \frac{\pi}{3} \\
&= 2 \left(\frac{1}{2} \right)^2 + \operatorname{cosec}^2 \left(\pi + \frac{\pi}{6} \right) \left(\frac{1}{2} \right)^2 \\
&= 2 \times \frac{1}{4} + \left(-\operatorname{cosec} \frac{\pi}{6} \right)^2 \left(\frac{1}{4} \right) \\
&= \frac{1}{2} + (-2)^2 \left(\frac{1}{4} \right) \\
&= \frac{1}{2} + \frac{4}{4} = \frac{1}{2} + 1 = \frac{3}{2} \\
&= \text{R.H.S.}
\end{aligned}$$

Question 3:

Prove that $\cot^2 \frac{\pi}{6} + \operatorname{cosec} \frac{5\pi}{6} + 3 \tan^2 \frac{\pi}{6} = 6$

Ans:

$$\begin{aligned}
\text{L.H.S.} &= \cot^2 \frac{\pi}{6} + \operatorname{cosec} \frac{5\pi}{6} + 3 \tan^2 \frac{\pi}{6} \\
&= (\sqrt{3})^2 + \operatorname{cosec} \left(\pi - \frac{\pi}{6} \right) + 3 \left(\frac{1}{\sqrt{3}} \right)^2 \\
&= 3 + \operatorname{cosec} \frac{\pi}{6} + 3 \times \frac{1}{3} \\
&= 3 + 2 + 1 = 6 \\
&= \text{R.H.S.}
\end{aligned}$$

Question 4:

Prove that $2 \sin^2 \frac{3\pi}{4} + 2 \cos^2 \frac{\pi}{4} + 2 \sec^2 \frac{\pi}{3} = 10$

Ans:

$$\begin{aligned}
\text{L.H.S} &= 2 \sin^2 \frac{3\pi}{4} + 2 \cos^2 \frac{\pi}{4} + 2 \sec^2 \frac{\pi}{3} \\
&= 2 \left\{ \sin \left(\pi - \frac{\pi}{4} \right) \right\}^2 + 2 \left(\frac{1}{\sqrt{2}} \right)^2 + 2(2)^2 \\
&= 2 \left\{ \sin \frac{\pi}{4} \right\}^2 + 2 \times \frac{1}{2} + 8 \\
&= 2 \left(\frac{1}{\sqrt{2}} \right)^2 + 1 + 8 \\
&= 1 + 1 + 8 \\
&= 10 \\
&= \text{R.H.S}
\end{aligned}$$

Question 5:

Find the value of:

(i) $\sin 75^\circ$

(ii) $\tan 15^\circ$

Ans:

$$(i) \sin 75^\circ = \sin (45^\circ + 30^\circ)$$

$$= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$$

$$[\sin (x+y) = \sin x \cos y + \cos x \sin y]$$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$(ii) \tan 15^\circ = \tan (45^\circ - 30^\circ)$$

$$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ} \quad \left[\tan (x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y} \right]$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1\left(\frac{1}{\sqrt{3}}\right)} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{3+1-2\sqrt{3}}{(\sqrt{3})^2 - (1)^2}$$

$$= \frac{4-2\sqrt{3}}{3-1} = 2-\sqrt{3}$$

Question 6:

$$\text{Prove that: } \cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right) - \sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right) = \sin(x+y)$$

Ans:

$$\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right) - \sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$$

$$= \frac{1}{2} \left[2 \cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right) \right] + \frac{1}{2} \left[-2 \sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right) \right]$$

$$= \frac{1}{2} \left[\cos\left\{\left(\frac{\pi}{4}-x\right) + \left(\frac{\pi}{4}-y\right)\right\} + \cos\left\{\left(\frac{\pi}{4}-x\right) - \left(\frac{\pi}{4}-y\right)\right\} \right]$$

$$+ \frac{1}{2} \left[\cos\left\{\left(\frac{\pi}{4}-x\right) + \left(\frac{\pi}{4}-y\right)\right\} - \cos\left\{\left(\frac{\pi}{4}-x\right) - \left(\frac{\pi}{4}-y\right)\right\} \right]$$

$$\left[\begin{array}{l} \because 2 \cos A \cos B = \cos(A+B) + \cos(A-B) \\ -2 \sin A \sin B = \cos(A+B) - \cos(A-B) \end{array} \right]$$

$$= 2 \times \frac{1}{2} \left[\cos\left\{\left(\frac{\pi}{4}-x\right) + \left(\frac{\pi}{4}-y\right)\right\} \right]$$

$$= \cos\left[\frac{\pi}{2} - (x+y)\right]$$

$$= \sin(x+y)$$

$$= \text{R.H.S}$$

Question 7:

Prove that:
$$\frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \left(\frac{1 + \tan x}{1 - \tan x}\right)^2$$

Ans:

It is known that $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ and $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

\therefore L.H.S. =

$$\frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \frac{\left(\frac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4} \tan x}\right)}{\left(\frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4} \tan x}\right)} = \frac{\left(\frac{1 + \tan x}{1 - \tan x}\right)}{\left(\frac{1 - \tan x}{1 + \tan x}\right)} = \left(\frac{1 + \tan x}{1 - \tan x}\right)^2 = \text{R.H.S.}$$

Question 8:

Prove that
$$\frac{\cos(\pi + x) \cos(-x)}{\sin(\pi - x) \cos\left(\frac{\pi}{2} + x\right)} = \cot^2 x$$

Ans:

$$\begin{aligned} \text{L.H.S.} &= \frac{\cos(\pi + x) \cos(-x)}{\sin(\pi - x) \cos\left(\frac{\pi}{2} + x\right)} \\ &= \frac{[-\cos x][\cos x]}{(\sin x)(-\sin x)} \\ &= \frac{-\cos^2 x}{-\sin^2 x} \\ &= \cot^2 x \\ &= \text{R.H.S.} \end{aligned}$$

Question 9:

$$\cos\left(\frac{3\pi}{2} + x\right) \cos(2\pi + x) \left[\cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x) \right] = 1$$

Ans:

$$\begin{aligned}
\text{L.H.S.} &= \cos\left(\frac{3\pi}{2} + x\right) \cos(2\pi + x) \left[\cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x) \right] \\
&= \sin x \cos x [\tan x + \cot x] \\
&= \sin x \cos x \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right) \\
&= (\sin x \cos x) \left[\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \right] \\
&= 1 = \text{R.H.S.}
\end{aligned}$$

Question 10:

Prove that $\sin(n+1)x \sin(n+2)x + \cos(n+1)x \cos(n+2)x = \cos x$

Ans:

$$\begin{aligned}
\text{L.H.S.} &= \sin(n+1)x \sin(n+2)x + \cos(n+1)x \cos(n+2)x \\
&= \frac{1}{2} [2 \sin(n+1)x \sin(n+2)x + 2 \cos(n+1)x \cos(n+2)x] \\
&= \frac{1}{2} \left[\cos\{(n+1)x - (n+2)x\} - \cos\{(n+1)x + (n+2)x\} \right. \\
&\quad \left. + \cos\{(n+1)x + (n+2)x\} + \cos\{(n+1)x - (n+2)x\} \right] \\
&\quad \left[\begin{aligned} \because -2 \sin A \sin B &= \cos(A+B) - \cos(A-B) \\ 2 \cos A \cos B &= \cos(A+B) + \cos(A-B) \end{aligned} \right] \\
&= \frac{1}{2} \times 2 \cos\{(n+1)x - (n+2)x\} \\
&= \cos(-x) = \cos x = \text{R.H.S.}
\end{aligned}$$

Question 11:

Prove that $\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{2} \sin x$

Ans:

It is known that $\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \cdot \sin\left(\frac{A-B}{2}\right)$.

$$\begin{aligned}
 \therefore \text{L.H.S.} &= \cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) \\
 &= -2 \sin\left\{\frac{\left(\frac{3\pi}{4} + x\right) + \left(\frac{3\pi}{4} - x\right)}{2}\right\} \cdot \sin\left\{\frac{\left(\frac{3\pi}{4} + x\right) - \left(\frac{3\pi}{4} - x\right)}{2}\right\} \\
 &= -2 \sin\left(\frac{3\pi}{4}\right) \sin x \\
 &= -2 \sin\left(\pi - \frac{\pi}{4}\right) \sin x \\
 &= -2 \sin \frac{\pi}{4} \sin x \\
 &= -2 \times \frac{1}{\sqrt{2}} \times \sin x \\
 &= -\sqrt{2} \sin x \\
 &= \text{R.H.S.}
 \end{aligned}$$

Question 12:

Prove that $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

Ans:

It is known that

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right), \quad \sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$\begin{aligned}
 \therefore \text{L.H.S.} &= \sin^2 6x - \sin^2 4x \\
 &= (\sin 6x + \sin 4x) (\sin 6x - \sin 4x) \\
 &= \left[2 \sin\left(\frac{6x+4x}{2}\right) \cos\left(\frac{6x-4x}{2}\right)\right] \left[2 \cos\left(\frac{6x+4x}{2}\right) \sin\left(\frac{6x-4x}{2}\right)\right] \\
 &= (2 \sin 5x \cos x) (2 \cos 5x \sin x) \\
 &= (2 \sin 5x \cos 5x) (2 \sin x \cos x) \\
 &= \sin 10x \sin 2x \\
 &= \text{R.H.S.}
 \end{aligned}$$

Question 13:

Prove that $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

Ans:

It is known that

$$\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right), \quad \cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S.} = \cos^2 2x - \cos^2 6x$$

$$= (\cos 2x + \cos 6x) (\cos 2x - \cos 6x)$$

$$= \left[2 \cos\left(\frac{2x+6x}{2}\right) \cos\left(\frac{2x-6x}{2}\right) \right] \left[-2 \sin\left(\frac{2x+6x}{2}\right) \sin\left(\frac{2x-6x}{2}\right) \right]$$

$$= [2 \cos 4x \cos(-2x)] [-2 \sin 4x \sin(-2x)]$$

$$= [2 \cos 4x \cos 2x] [-2 \sin 4x (-\sin 2x)]$$

$$= (2 \sin 4x \cos 4x) (2 \sin 2x \cos 2x)$$

$$= \sin 8x \sin 4x$$

$$= \text{R.H.S.}$$

Question 14:

$$\text{Prove that } \sin 2x + 2 \sin 4x + \sin 6x = 4 \cos^2 x \sin 4x$$

Ans:

$$\text{L.H.S.} = \sin 2x + 2 \sin 4x + \sin 6x$$

$$= [\sin 2x + \sin 6x] + 2 \sin 4x$$

$$= \left[2 \sin\left(\frac{2x+6x}{2}\right) \cos\left(\frac{2x-6x}{2}\right) \right] + 2 \sin 4x$$

$$\left[\because \sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) \right]$$

$$= 2 \sin 4x \cos(-2x) + 2 \sin 4x$$

$$= 2 \sin 4x \cos 2x + 2 \sin 4x$$

$$= 2 \sin 4x (\cos 2x + 1)$$

$$= 2 \sin 4x (2 \cos^2 x - 1 + 1)$$

$$= 2 \sin 4x (2 \cos^2 x)$$

$$= 4 \cos^2 x \sin 4x$$

Question 15:

$$\text{Prove that } \cot 4x (\sin 5x + \sin 3x) = \cot x (\sin 5x - \sin 3x)$$

Ans:

$$\text{L.H.S} = \cot 4x (\sin 5x + \sin 3x)$$

$$= \frac{\cos 4x}{\sin 4x} \left[2 \sin \left(\frac{5x+3x}{2} \right) \cos \left(\frac{5x-3x}{2} \right) \right]$$

$$\left[\because \sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right) \right]$$

$$= \left(\frac{\cos 4x}{\sin 4x} \right) [2 \sin 4x \cos x]$$

$$= 2 \cos 4x \cos x$$

$$\text{R.H.S.} = \cot x (\sin 5x - \sin 3x)$$

$$= \frac{\cos x}{\sin x} \left[2 \cos \left(\frac{5x+3x}{2} \right) \sin \left(\frac{5x-3x}{2} \right) \right]$$

$$\left[\because \sin A - \sin B = 2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right) \right]$$

$$= \frac{\cos x}{\sin x} [2 \cos 4x \sin x]$$

$$= 2 \cos 4x \cos x$$

$$\text{L.H.S.} = \text{R.H.S.}$$

Question 16:

$$\text{Prove that } \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\frac{\sin 2x}{\cos 10x}$$

Ans:

It is known that

$$\cos A - \cos B = -2 \sin \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right), \quad \sin A - \sin B = 2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$$

$$\therefore \text{L.H.S} = \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$$

$$= \frac{-2 \sin \left(\frac{9x+5x}{2} \right) \cdot \sin \left(\frac{9x-5x}{2} \right)}{2 \cos \left(\frac{17x+3x}{2} \right) \cdot \sin \left(\frac{17x-3x}{2} \right)}$$

$$= \frac{-2 \sin 7x \cdot \sin 2x}{2 \cos 10x \cdot \sin 7x}$$

$$= -\frac{\sin 2x}{\cos 10x}$$

$$= \text{R.H.S.}$$

Question 17:

Prove that $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$

Ans:

It is known that

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right), \quad \cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S.} = \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$$

$$\begin{aligned} &= \frac{2 \sin\left(\frac{5x+3x}{2}\right) \cdot \cos\left(\frac{5x-3x}{2}\right)}{2 \cos\left(\frac{5x+3x}{2}\right) \cdot \cos\left(\frac{5x-3x}{2}\right)} \\ &= \frac{2 \sin 4x \cdot \cos x}{2 \cos 4x \cdot \cos x} \\ &= \frac{\sin 4x}{\cos 4x} \\ &= \tan 4x = \text{R.H.S.} \end{aligned}$$

Question 18:

Prove that $\frac{\sin x - \sin y}{\cos x + \cos y} = \tan \frac{x-y}{2}$

Ans:

It is known that

$$\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right), \quad \cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S.} = \frac{\sin x - \sin y}{\cos x + \cos y}$$

$$\begin{aligned} &= \frac{2 \cos\left(\frac{x+y}{2}\right) \cdot \sin\left(\frac{x-y}{2}\right)}{2 \cos\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)} \\ &= \frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)} \\ &= \tan\left(\frac{x-y}{2}\right) = \text{R.H.S.} \end{aligned}$$

Question 19:

Prove that $\frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x$

Ans:

It is known that

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right), \quad \cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

$$\therefore \text{L.H.S.} = \frac{\sin x + \sin 3x}{\cos x + \cos 3x}$$

$$\begin{aligned} &= \frac{2 \sin\left(\frac{x+3x}{2}\right) \cos\left(\frac{x-3x}{2}\right)}{2 \cos\left(\frac{x+3x}{2}\right) \cos\left(\frac{x-3x}{2}\right)} \\ &= \frac{\sin 2x}{\cos 2x} \\ &= \tan 2x \\ &= \text{R.H.S.} \end{aligned}$$

Question 20:

Prove that
$$\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2 \sin x$$

Ans:

It is known that

$$\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right), \quad \cos^2 A - \sin^2 A = \cos 2A$$

$$\therefore \text{L.H.S.} = \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$$

$$\begin{aligned} &= \frac{2 \cos\left(\frac{x+3x}{2}\right) \sin\left(\frac{x-3x}{2}\right)}{-\cos 2x} \\ &= \frac{2 \cos 2x \sin(-x)}{-\cos 2x} \\ &= -2 \times (-\sin x) \\ &= 2 \sin x = \text{R.H.S.} \end{aligned}$$

Question 21:

Prove that
$$\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$$

Ans:

$$\text{L.H.S.} = \frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$$

$$\begin{aligned} &= \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x} \\ &= \frac{2 \cos\left(\frac{4x+2x}{2}\right) \cos\left(\frac{4x-2x}{2}\right) + \cos 3x}{2 \sin\left(\frac{4x+2x}{2}\right) \cos\left(\frac{4x-2x}{2}\right) + \sin 3x} \end{aligned}$$

$$\begin{aligned} &\left[\because \cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right), \quad \sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) \right] \\ &= \frac{2 \cos 3x \cos x + \cos 3x}{2 \sin 3x \cos x + \sin 3x} \\ &= \frac{\cos 3x (2 \cos x + 1)}{\sin 3x (2 \cos x + 1)} \\ &= \cot 3x = \text{R.H.S.} \end{aligned}$$

Question 22:

Prove that $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

Ans:

$$\text{L.H.S.} = \cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x$$

$$= \cot x \cot 2x - \cot 3x (\cot 2x + \cot x)$$

$$= \cot x \cot 2x - \cot (2x + x) (\cot 2x + \cot x)$$

$$= \cot x \cot 2x - \left[\frac{\cot 2x \cot x - 1}{\cot x + \cot 2x} \right] (\cot 2x + \cot x)$$

$$\left[\because \cot(A + B) = \frac{\cot A \cot B - 1}{\cot A + \cot B} \right]$$

$$= \cot x \cot 2x - (\cot 2x \cot x - 1)$$

$$= 1 = \text{R.H.S.}$$

Question 23:

Prove that $\tan 4x = \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$

Ans:

It is known that $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$.

$$\therefore \text{L.H.S.} = \tan 4x = \tan 2(2x)$$

$$= \frac{2 \tan 2x}{1 - \tan^2 (2x)}$$

$$= \frac{2 \left(\frac{2 \tan x}{1 - \tan^2 x} \right)}{1 - \left(\frac{2 \tan x}{1 - \tan^2 x} \right)^2}$$

$$= \frac{\left(\frac{4 \tan x}{1 - \tan^2 x} \right)}{\left[1 - \frac{4 \tan^2 x}{(1 - \tan^2 x)^2} \right]}$$

$$= \frac{\left(\frac{4 \tan x}{1 - \tan^2 x} \right)}{\left[\frac{(1 - \tan^2 x)^2 - 4 \tan^2 x}{(1 - \tan^2 x)^2} \right]}$$

$$\begin{aligned}
&= \frac{\left(\frac{4 \tan x}{1 - \tan^2 x} \right)}{\left[\frac{(1 - \tan^2 x)^2 - 4 \tan^2 x}{(1 - \tan^2 x)^2} \right]} \\
&= \frac{4 \tan x (1 - \tan^2 x)}{(1 - \tan^2 x)^2 - 4 \tan^2 x} \\
&= \frac{4 \tan x (1 - \tan^2 x)}{1 + \tan^4 x - 2 \tan^2 x - 4 \tan^2 x} \\
&= \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x} = \text{R.H.S.}
\end{aligned}$$

Question 24:

Prove that $\cos 4x = 1 - 8 \sin^2 x \cos^2 x$

Ans:

$$\text{L.H.S.} = \cos 4x$$

$$= \cos 2(2x)$$

$$= 1 - 2 \sin^2 2x \quad [\cos 2A = 1 - 2 \sin^2 A]$$

$$= 1 - 2(2 \sin x \cos x)^2 \quad [\sin 2A = 2 \sin A \cos A]$$

$$= 1 - 8 \sin^2 x \cos^2 x$$

$$= \text{R.H.S.}$$

Question 25:

Prove that: $\cos 6x = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$

Ans:

$$\text{L.H.S.} = \cos 6x$$

$$= \cos 3(2x)$$

$$= 4 \cos^3 2x - 3 \cos 2x [\cos 3A = 4 \cos^3 A - 3 \cos A]$$

$$= 4 [(2 \cos^2 x - 1)^3 - 3 (2 \cos^2 x - 1) [\cos 2x = 2 \cos^2 x - 1]]$$

$$= 4 [(2 \cos^2 x)^3 - (1)^3 - 3 (2 \cos^2 x)^2 + 3 (2 \cos^2 x)] - 6 \cos^2 x + 3$$

$$= 4 [8 \cos^6 x - 1 - 12 \cos^4 x + 6 \cos^2 x] - 6 \cos^2 x + 3$$

$$= 32 \cos^6 x - 4 - 48 \cos^4 x + 24 \cos^2 x - 6 \cos^2 x + 3$$

$$= 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$$

$$= \text{R.H.S.}$$

***** END *****