

Exercise 7A

Question 7

$$\begin{aligned} & LHS = \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} \\ & = \frac{\sin\theta \left(1-\cos\theta\right)}{\left(1+\cos\theta\right)\left(1-\cos\theta\right)} + \frac{1+\cos\theta}{\sin\theta} \\ & = \frac{\sin\theta \left(1-\cos\theta\right)}{1-\cos^2\theta} + \frac{1+\cos\theta}{\sin\theta} \\ & = \frac{\sin\theta \left(1-\cos\theta\right)}{\sin^2\theta} + \frac{1+\cos\theta}{\sin\theta} = \frac{\left(1-\cos\theta\right)}{\sin\theta} + \frac{1+\cos\theta}{\sin\theta} \\ & = \frac{1-\cos\theta+1+\cos\theta}{\sin\theta} = \frac{2}{\sin\theta} = 2\cos\theta = RHS \end{aligned}$$

Question 8

$$\begin{aligned} \mathsf{LHS} &= \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} \\ &= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}} \\ & \left[ \because \tan\theta = \frac{\sin\theta}{\cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta} \right] \end{aligned}$$

$$&= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$&= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$&= \frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$$

$$&= \frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$$

$$&= \frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{(\sin\theta-\cos\theta)\sin\theta\cos\theta}$$

$$&= \frac{(\sin\theta-\cos\theta)\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

 $= \frac{1}{\sin\theta\cos\theta} + 1 = 1 + \sec\theta\cos\theta\cos\theta = RHS$