Increasing and Decreasing Functions Ex 17.2 Q22

A function $f(x)$ is said to be increasing on $[a,b]$ if $f'(x) > 0$

Now, we have,

$$f(x) = x^2 - 6x + 3$$

$$\therefore \quad f'(x) = 2x - 6$$

$$= 2(x - 3)$$

Again,

$$x \in [4,6]$$

$$\Rightarrow \quad 4 \le x \le 6$$

$$\Rightarrow \quad 1 \le x - 3 \le 3$$

$$\Rightarrow \quad (x - 3) > 0$$

$$\Rightarrow \quad 2(x - 3) > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is an increasing function for $x \in [4,6]$.

Increasing and Decreasing Functions Ex 17.2 Q23

We have,

$$f(x) = \sin x - \cos x$$

$$\therefore \quad f'(x) = \cos x + \sin x$$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right)$$

$$= \sqrt{2}\left(\frac{\sin \pi}{4}\cos x + \frac{\cos \pi}{4}\sin x\right)$$

$$= \sqrt{2}\sin\left(\frac{\pi}{4} + x\right)$$

Now,

$$x \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$$

$$\Rightarrow \quad -\frac{\pi}{4} < x < \frac{\pi}{4}$$

$$\Rightarrow \quad 0 < \frac{\pi}{4} + x < \frac{\pi}{2}$$

$$\Rightarrow \quad \sin 0° < \sin\left(\frac{\pi}{4} + x\right) < \sin\frac{\pi}{2}$$

$$\Rightarrow \quad 0 < \sin\left(\frac{\pi}{4} + x\right) < 1$$

$$\Rightarrow \quad \sqrt{2}\sin\left(\frac{\pi}{4} + x\right) > 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is an increasing function on $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$.

Increasing and Decreasing Functions Ex 17.2 Q24

We have,

$$f(x) = \tan^{-1} x - x$$

$$\therefore \quad f'(x) = \frac{1}{1+x^2} - 1$$

$$= \frac{-x^2}{1+x^2}$$

Now,

$$x \in R$$

$$\Rightarrow \quad x^2 > 0 \text{ and } 1 + x^2 > 0$$

$$\Rightarrow \quad \frac{x^2}{1+x^2} > 0$$

$$\Rightarrow \quad \frac{-x^2}{1+x^2} < 0$$

$$\Rightarrow \quad f'(x) < 0$$

Hence, $f(x)$ is a decreasing function for $x \in R$.

Increasing and Decreasing Functions Ex 17.2 Q25

We have,

$$f(x) = -\frac{x}{2} + \sin x$$

$$\therefore \quad f'(x) = -\frac{1}{2} + \cos x$$

Now,

$$x \in \left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$$

$$\Rightarrow \quad -\frac{\pi}{3} < x < \frac{\pi}{3}$$

$$\Rightarrow \quad \cos\left(-\frac{\pi}{3}\right) < \cos x < \cos\frac{\pi}{3}$$

$$\Rightarrow \quad \cos\frac{\pi}{3} < \cos x < \cos\frac{\pi}{3}$$

$$\Rightarrow \quad \frac{1}{2} < \cos x < \frac{1}{2}$$

$$\Rightarrow \quad -\frac{1}{2} + \cos x + 0$$

$$\Rightarrow \quad f'(x) > 0$$

Hence, $f(x)$ is an increasing function on $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$.

********** END **********