Definite Integrals Ex 20.1 Q19

We have,

$$\int_0^{\frac{\pi}{6}} \cos x \cos 2x dx \qquad \left[\because 2\cos C \cos D = \cos(C+D) - \cos(C-D)\right]$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{6}} 2\cos x \cos 2x dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{6}} (\cos 3x + \cos x) dx$$

$$= \frac{1}{2}\int \left[\frac{\sin 3x}{3} + \sin x\right]_0^{\frac{\pi}{6}}$$

$$= \frac{1}{2}\left[\left(\frac{\sin 3\frac{\pi}{6}}{3} + \sin\frac{\pi}{6}\right) - (\sin 0 - \sin 0)\right]$$

$$= \frac{1}{2}\left[\frac{\sin\frac{\pi}{2}}{3} + \sin\frac{\pi}{6}\right]$$

$$= \frac{1}{2}\left(\frac{1}{3} + \frac{1}{2}\right)$$

$$= \frac{1}{2}\left(\frac{5}{6}\right)$$

$$= \frac{5}{12}$$

$$\therefore \int_0^{\frac{\pi}{6}} \cos x \cos 2x dx = \frac{5}{12}$$

Definite Integrals Ex 20.1 Q20

We have,

$$\int_0^{\frac{\pi}{2}} \sin x \sin 2x dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} 2\sin x \sin 2x dx \qquad [\because 2\sin C \times \sin D = \cos(D-C) - \cos(D+C)]$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} (\cos x - \cos 3x) dx$$

$$= \frac{1}{2}\left[\sin x - \frac{\sin 3x}{3}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left[\left(\sin\frac{\pi}{2} - \sin 0\right) - \left(\frac{\sin 3\frac{\pi}{2}}{3} - \frac{\sin 0}{3}\right)\right]$$

$$= \frac{1}{2}\left[(1-0) - \left(\frac{-1}{3} - 0\right)\right] \qquad \left[\because \sin 3\frac{\pi}{2} = -1\right]$$

$$= \frac{1}{2} \times \frac{4}{3}$$

$$= \frac{2}{3}$$

$$\therefore \int_0^{\frac{\pi}{2}} \sin x \sin 2x dx = \frac{2}{3}$$

Definite Integrals Ex 20.1 Q21

We have,

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\tan x + \cot x\right)^2 dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{\sin^2 x + \cot^2 x}{\sin x \cos x}\right)^2 dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{1}{\sin x \cos x}\right)^2 dx$$

Multiplying numerator and denominator by 2

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{2}{2 \sin x \cos x}\right)^2 dx$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\frac{2}{\sin 2x}\right)^2 dx \qquad \left[\because 2 \sin x \cos x = \sin 2x\right]$$

$$= 4\int_{\frac{\pi}{3}}^{\frac{\pi}{4}} cosec^2 x dx$$

$$= 4\left[-\frac{\cot 2x}{2}\right]_{\frac{\pi}{3}}^{\frac{\pi}{4}}$$

$$= 2\left[-\cot\frac{\pi}{2} + \cot 2\frac{\pi}{3}\right]$$

$$= 2\left[\frac{-1}{\sqrt{3}} - 0\right]$$

$$= \frac{-2}{\sqrt{3}}$$

$$\therefore \int_{\frac{\pi}{3}}^{\frac{\pi}{4}} \left(\tan x + \cot x\right)^2 dx = \frac{-2}{\sqrt{3}}$$

Definite Integrals Ex 20.1 Q22

We have,

$$\int_0^{\frac{\pi}{2}} \cos^4 x dx$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \left(1+\cos 2x\right)^2 dx \qquad \left[\because 2\cos^2 x = 1+\cos 2x\right]$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \left(1+\cos^2 2x + 2\cos 2x\right) dx$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \left(1+\frac{1+\cos 4x}{2} + 2\cos 2x\right) dx$$

$$= \frac{1}{4}\left[x+\frac{1}{2}x+\frac{\sin 4x}{8}+\sin 2x\right]_0^{\frac{\pi}{2}} \qquad \left[\because \int \cos 4x dx = \frac{\sin 4x}{4}\right]$$

$$= \frac{1}{4}\left[\frac{\pi}{2}+\frac{\pi}{4}+0+0-0-0-0-0\right]$$

$$= \frac{1}{4}\times\frac{3\pi}{4}$$

$$= \frac{3\pi}{16}$$

$$\therefore \int_0^{\frac{\pi}{2}} \cos^4 x dx = \frac{3\pi}{16}$$

*********** END **********