Definite Integrals Ex 20.2 Q28

We have,

$$\int_0^{\frac{\pi}{2}} \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx$$

Dividing numerator and denominator by $\cos^2 x$

$$\int_0^{\frac{\pi}{2}} \left(\frac{\frac{1}{\cos^2 x}}{a^2 \frac{\sin^2 x}{\cos^2 x} + b^2 \frac{\cos^2 x}{\cos^2 x}} \right) dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{\sec^2 x}{a^2 \tan^2 x + b^2} \right) dx$$

$$= \frac{1}{a^2} \int_0^{\frac{\pi}{2}} \left(\frac{\sec^2 x}{\tan^2 x + \left(\frac{b}{a}\right)^2} \right) dx$$

Let $\tan x = t$

Differentiating w.r.t. x, we get

$$\sec^2 x \, dx = dt$$

When $x = 0 \Rightarrow t = 0$

$$x = \frac{\pi}{2} \Rightarrow t = \infty$$

$$\therefore \frac{1}{a^2} \int_0^{\frac{\pi}{2}} \left(\frac{\sec^2 x}{\tan^2 x + \left(\frac{b}{a}\right)^2} \right) dx$$

$$= \frac{1}{a^2} \int_0^{\infty} \frac{dt}{\left(\frac{b}{a}\right)^2 + t^2}$$

$$= \frac{1}{a^2} \left[\frac{a}{b} \tan^{-1} \frac{at}{b} \right]_0^{\infty}$$

$$= \frac{1}{a^2} \frac{a}{b} \left[\tan^{-1} \infty - \tan^{-1} 0 \right]$$

$$= \frac{1}{ab} \left[\tan^{-1} \tan \frac{\pi}{2} \right] = \frac{\pi}{2ab}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx = \frac{\pi}{2ab}$$

Definite Integrals Ex 20.2 Q29

$$I = \int_0^{\frac{\pi}{2}} \frac{x+\sin x}{1+\cos x} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{x+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}} dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{x\sec^2\frac{x}{2}}{2} + \tan\frac{x}{2} \right) dx$$

$$= \left[x\tan\left(\frac{x}{2}\right) - \int_0^{\frac{\pi}{2}} \tan\frac{x}{2} dx + \int_0^{\frac{\pi}{2}} \tan\frac{x}{2} dx \right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2}$$

$$\therefore I = \int_0^{\frac{\pi}{2}} \frac{x+\sin x}{1+\cos x} dx = \frac{\pi}{2}$$

Definite Integrals Ex 20.2 Q30

$$I = \int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$$

Let $t = \tan^{-1} x$

$$dt = \frac{1}{1+x^2} dx$$

$$x = 0, t = 0$$

$$x = 1, t = \frac{\pi}{4}$$

$$I = \int_0^{\frac{\pi}{4}} t dt$$

$$= \left[\frac{t^2}{2} \right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{2}\frac{\pi^2}{16}$$

$$= \frac{\pi^2}{32}$$

Definite Integrals Ex 20.2 Q31

$$I = \int_0^{\pi/4} \frac{\sin x + \cos x}{3 + \sin 2x}\, dx$$

$$I = \int_0^{\pi/4} \left(\frac{\sin x + \cos x}{3 + 1 - (\cos x - \sin x)^2} \right) dx$$

$$I = \int_0^{\pi/4} \left(\frac{\sin x + \cos x}{4 - (\cos x - \sin x)^2} \right) dx$$

$$I = \frac{1}{4} \left[\log \left| \frac{2 + \sin x - \cos x}{2 - \sin x + \cos x} \right| \right]_0^{\pi/4}$$

$$I = -\frac{1}{4} \log \left(\frac{1}{3} \right)$$

$$I = \frac{1}{4} \log_e 3$$

********** END **********