Arithmetic Progressions Ex 9.5 Q22

Answer :

In the given problem, we need to find the sum of terms for different arithmetic progressions. So, here we use the following formula for the sum of n terms of an A.P.,

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

Where; a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

(i) First 15 multiples of 8.

So, we know that the first multiple of 8 is 8 and the last multiple of 8 is 120.

Also, all these terms will form an A.P. with the common difference of 8.

So here,

First term (a) = 8

Number of terms (n) = 15

Common difference (d) = 8

Now, using the formula for the sum of n terms, we get

$$\begin{aligned} S_n &= \frac{15}{2}\left[2(8)+(15-1)8\right] \\ &= \frac{15}{2}\left[16+(14)8\right] \\ &= \frac{15}{2}(16+112) \\ &= \frac{15}{2}(128) \\ &= 960 \end{aligned}$$

Therefore, the sum of the first 15 multiples of 8 is $\boxed{960}$

(a) First 40 positive integers divisible by 3

So, we know that the first multiple of 3 is 3 and the last multiple of 3 is 120.

Also, all these terms will form an A.P. with the common difference of 3.

So here,

First term (a) = 3

Number of terms (n) = 40

Common difference (d) = 3

Now, using the formula for the sum of n terms, we get

$$\begin{aligned} S_n &= \frac{40}{2}\left[2(3)+(40-1)3\right] \\ &= 20\left[6+(39)3\right] \\ &= 20(6+117) \\ &= 20(123) \\ &= 2460 \end{aligned}$$

Therefore, the sum of first 40 multiples of 3 is $\boxed{2460}$

(b) First 40 positive integers divisible by 5

So, we know that the first multiple of 5 is 5 and the last multiple of 5 is 200.

Also, all these terms will form an A.P. with the common difference of 5.

So here,

First term (a) = 5

Number of terms (n) = 40

Common difference (d) = 5

Now, using the formula for the sum of n terms, we get

$$S_n = \frac{40}{2}\left[2(5)+(40-1)5\right]$$
$$= 20\left[10+(39)5\right]$$
$$= 20(10+195)$$
$$= 20(205)$$
$$= 4100$$

Therefore, the sum of first 40 multiples of 3 is $\boxed{4100}$

(c) First 40 positive integers divisible by 6

So, we know that the first multiple of 6 is 6 and the last multiple of 6 is 240.

Also, all these terms will form an A.P. with the common difference of 6.

So here,

First term (a) = 6

Number of terms (n) = 40

Common difference (d) = 6

Now, using the formula for the sum of n terms, we get

$$S_n = \frac{40}{2}\left[2(6)+(40-1)6\right]$$
$$= 20\left[12+(39)6\right]$$
$$= 20(12+234)$$
$$= 20(246)$$
$$= 4920$$

Therefore, the sum of first 40 multiples of 3 is $\boxed{4920}$

(ii) All 3 digit natural number which are divisible by 13

So, we know that the first 3 digit multiple of 13 is 104 and the last 3 digit multiple of 13 is 988.

Also, all these terms will form an A.P. with the common difference of 13.

So here,

First term (a) = 104

Last term (l) = 988

Common difference (d) = 13

So, here the first step is to find the total number of terms. Let us take the number of terms as n.

Now, as we know,

$$a_n = a+(n-1)d$$

So, for the last term,

$$988 = 104+(n-1)13$$
$$988 = 104+13n-13$$
$$988 = 91+13n$$

Further simplifying,

$$n = \frac{988-91}{13}$$
$$n = \frac{897}{13}$$

$n = 69$

Now, using the formula for the sum of n terms, we get

$$S_n = \frac{69}{2}\left[2(104) + (69-1)13\right]$$
$$= \frac{69}{2}\left[208 + (68)13\right]$$
$$= \frac{69}{2}(208 + 884)$$

On further simplification, we get,

$$S_n = \frac{69}{2}(1092)$$
$$= 69(546)$$
$$= 37674$$

Therefore, the sum of all the 3 digit multiples of 13 is $\boxed{S_n = 37674}$.

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