Exercise 2.1

Question 1:

Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$

Answer

Let $\sin^{-1}\left(-\frac{1}{2}\right)=y.$ Then sin y = $-\frac{1}{2}=-\sin\left(\frac{\pi}{6}\right)=\sin\left(-\frac{\pi}{6}\right).$

We know that the range of the principal value branch of $\sin^{-1}$ is

$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ and $\sin\left(-\frac{\pi}{6}\right)=-\frac{1}{2}.$

Therefore, the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is $-\frac{\pi}{6}.$

Question 2:

Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Answer

$\text{Let } \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=y. \text{ Then, } \cos y=\frac{\sqrt{3}}{2}=\cos\left(\frac{\pi}{6}\right).$

We know that the range of the principal value branch of $\cos^{-1}$ is

$[0,\pi]$ and $\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$.

Therefore, the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{6}$.

Question 3:

Find the principal value of cosec^{-1} (2)

Answer

Let cosec^{-1} (2) = y. Then, $\text{cosec}\, y=2=\text{cosec}\left(\frac{\pi}{6}\right).$

We know that the range of the principal value branch of cosec^{-1} is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}.$

Therefore, the principal value of $\text{cosec}^{-1}(2)$ is $\frac{\pi}{6}.$

Question 4:

Find the principal value of $\tan^{-1}\left(-\sqrt{3}\right)$

Answer

$\text{Let } \tan^{-1}\left(-\sqrt{3}\right)=y. \text{ Then, } \tan y=-\sqrt{3}=-\tan\frac{\pi}{3}=\tan\left(-\frac{\pi}{3}\right).$

We know that the range of the principal value branch of $\tan^{-1}$ is

$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{3}\right)$ is $-\sqrt{3}.$

Therefore, the principal value of $\tan^{-1}\left(\sqrt{3}\right)$ is $-\frac{\pi}{3}.$

Question 5:

Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$

Answer

$\text{Let } \cos^{-1}\left(-\frac{1}{2}\right)=y. \text{ Then, } \cos y=-\frac{1}{2}=-\cos\left(\frac{\pi}{3}\right)=\cos\left(\pi-\frac{\pi}{3}\right)=\cos\left(\frac{2\pi}{3}\right).$

We know that the range of the principal value branch of $\cos^{-1}$ is

$[0,\pi]$ and $\cos\left(\frac{2\pi}{3}\right)=-\frac{1}{2}$.

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is $\frac{2\pi}{3}$.

Question 6:

Find the principal value of $\tan^{-1}(-1)$

Answer

Let $\tan^{-1}(-1) = y$. Then,

$$\tan y=-1=-\tan\left(\frac{\pi}{4}\right)=\tan\left(-\frac{\pi}{4}\right).$$

We know that the range of the principal value branch of $\tan^{-1}$ is

$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{4}\right)=-1$.

Therefore, the principal value of $\tan^{-1}(-1)$ is $-\frac{\pi}{4}$.

Question 7:

Find the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Answer

$$\text{Let } \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)=y. \text{ Then, } \sec y=\frac{2}{\sqrt{3}}=\sec\left(\frac{\pi}{6}\right).$$

We know that the range of the principal value branch of $\sec^{-1}$ is

$[0,\pi]-\left\{\frac{\pi}{2}\right\}$ and $\sec\left(\frac{\pi}{6}\right)=\frac{2}{\sqrt{3}}$.

Therefore, the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{6}$.

Question 8:

Find the principal value of $\cot^{-1}\left(\sqrt{3}\right)$

Answer

$$\text{Let } \cot^{-1}\left(\sqrt{3}\right)=y. \text{ Then, } \cot y=\sqrt{3}=\cot\left(\frac{\pi}{6}\right).$$

We know that the range of the principal value branch of $\cot^{-1}$ is (0,π) and

$\cot\left(\frac{\pi}{6}\right)=\sqrt{3}$.

Therefore, the principal value of $\cot^{-1}\left(\sqrt{3}\right)$ is $\frac{\pi}{6}$.

Question 9:

Find the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

Answer

$$\text{Let } \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)=y. \text{ Then, } \cos y=-\frac{1}{\sqrt{2}}=-\cos\left(\frac{\pi}{4}\right)=\cos\left(\pi-\frac{\pi}{4}\right)=\cos\left(\frac{3\pi}{4}\right).$$

We know that the range of the principal value branch of $\cos^{-1}$ is [0,π] and

$\cos\left(\frac{3\pi}{4}\right)=-\frac{1}{\sqrt{2}}$.

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$.

Question 10:

Find the principal value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$

Answer

$$\text{Let } \operatorname{cosec}^{-1}\left(-\sqrt{2}\right)=y. \text{ Then, } \operatorname{cosec} y=-\sqrt{2}=-\operatorname{cosec}\left(\frac{\pi}{4}\right)=\operatorname{cosec}\left(-\frac{\pi}{4}\right).$$

We know that the range of the principal value branch of $\operatorname{cosec}^{-1}$ is

$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$ and $\operatorname{cosec}\left(-\frac{\pi}{4}\right)=-\sqrt{2}$.

Therefore, the principal value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$ is $-\frac{\pi}{4}$.

Question 11:

Find the value of $\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$

Answer

Let $\tan^{-1}(1) = x$. Then, $\tan x = 1 = \tan\frac{\pi}{4}$.

$\therefore \tan^{-1}(1) = \frac{\pi}{4}$

Let $\cos^{-1}\left(-\frac{1}{2}\right) = y$. Then, $\cos y = -\frac{1}{2} = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$.

$\therefore \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

Let $\sin^{-1}\left(-\frac{1}{2}\right) = z$. Then, $\sin z = -\frac{1}{2} = -\sin\left(\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right)$.

$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

$\therefore \tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$

$= \frac{3\pi + 8\pi - 2\pi}{12} = \frac{9\pi}{12} = \frac{3\pi}{4}$

Question 12:

Find the value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$

Answer

Let $\cos^{-1}\left(\frac{1}{2}\right) = x$. Then, $\cos x = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$.

$\therefore \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

Let $\sin^{-1}\left(\frac{1}{2}\right) = y$. Then, $\sin y = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$.

$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\therefore \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + \frac{2\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$

Question 13:

Find the value of if $\sin^{-1} x = y$, then

(A) $0 \le y \le \pi$ **(B)** $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

(C) $0 < y < \pi$ **(D)** $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Answer

It is given that $\sin^{-1} x = y$.

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Therefore, $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

Question 14:

Find the value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to

(A) π **(B)** $-\frac{\pi}{3}$ **(C)** $\frac{\pi}{3}$ **(D)** $\frac{2\pi}{3}$

Answer

********** END **********