Trigonometric Identities Ex 6.1 Q28

Answer :

We have to prove $\frac{1+\tan^2\theta}{1+\cot^2\theta}=\left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2=\tan^2\theta$

Consider the expression

$$\begin{aligned}\frac{1+\tan^2\theta}{1+\cot^2\theta}&=\frac{1+\tan^2\theta}{1+\frac{1}{\tan^2\theta}}\\&=\frac{1+\tan^2\theta}{\frac{\tan^2\theta+1}{\tan^2\theta}}\\&=\tan^2\theta\frac{1+\tan^2\theta}{1+\tan^2\theta}\\&=\tan^2\theta\end{aligned}$$

Again, we have

$$\begin{aligned}\left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2&=\left(\frac{1-\tan\theta}{1-\frac{1}{\tan\theta}}\right)^2\\&=\left(\frac{1-\tan\theta}{\frac{\tan\theta-1}{\tan\theta}}\right)^2\end{aligned}$$

$$= \tan^2\theta\left(\frac{1-\tan\theta}{\tan\theta-1}\right)^2$$

$$= \tan^2\theta\left(-\frac{1-\tan\theta}{1-\tan\theta}\right)^2$$

$$= \tan^2\theta(-1)^2$$

$$= \tan^2\theta$$

Trigonometric Identities Ex 6.1 Q29

Answer :

We have to prove $\frac{1+\sec\theta}{\sec\theta} = \frac{\sin^2\theta}{1-\cos\theta}$

We know that, $\sin^2\theta + \cos^2\theta = 1$

$$\frac{1+\sec\theta}{\sec\theta} = \frac{1+\frac{1}{\cos\theta}}{\frac{1}{\cos\theta}}$$

$$= \frac{\frac{\cos\theta+1}{\cos\theta}}{\frac{1}{\cos\theta}}$$

$$= \frac{1+\cos\theta}{1}$$

Multiplying the numerator and denominator by $(1-\cos\theta)$, we have

$$\frac{1+\sec\theta}{\sec\theta} = \frac{(1+\cos\theta)(1-\cos\theta)}{(1-\cos\theta)}$$

$$= \frac{1-\cos^2\theta}{1-\cos\theta}$$

$$= \frac{\sin^2\theta}{1-\cos\theta}$$

*********** END **********