Trigonometric Ratios of Compound Angles Ex 7.1 Q8

We have,

$$\cos A = \frac{-12}{13} \text{ and } \cot B = \frac{24}{7}$$

$$\therefore \quad \sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\left(\frac{-12}{13}\right)^2} = \sqrt{1-\frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

and,

$$\operatorname{cosec} B = -\sqrt{1+\cot^2 B} \qquad [\because \text{cosec is negative in third quadrant}]$$

$$= -\sqrt{1+\left(\frac{24}{7}\right)^2} = -\sqrt{1+\frac{576}{49}} = -\sqrt{\frac{49+576}{49}} = -\sqrt{\frac{625}{49}} = -\frac{25}{7}$$

$$\Rightarrow \quad \sin B = \frac{-7}{25} \qquad \left[\because \operatorname{cosec} B = \frac{1}{\sin B}\right]$$

Now,

$$\cos B = -\sqrt{1-\sin^2 B} \qquad [\because \cos\theta \text{ is negative in third quadrant}]$$

$$= -\sqrt{1-\left(\frac{-7}{25}\right)^2} = -\sqrt{1-\frac{49}{625}} = -\sqrt{\frac{625-49}{625}} = -\sqrt{\frac{576}{625}} = \frac{-24}{25}$$

Now,

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$= \frac{5}{13} \times \left(\frac{-24}{25}\right) + \left(\frac{-12}{13}\right) \times \left(\frac{-7}{25}\right)$$

$$= \frac{-120}{325} + \frac{84}{325}$$

$$= \frac{-120+84}{325}$$

$$= \frac{-36}{325}$$

We have,

$$\cos A = \frac{-12}{13} \text{ and } \cot B = \frac{24}{7}$$

$$\therefore \quad \sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\left(\frac{-12}{13}\right)^2} = \sqrt{1-\frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

and,

$$\operatorname{cosec} B = -\sqrt{1+\cot^2 B} \qquad [\because \text{cosec is negative in third quadrant}]$$

$$= -\sqrt{1+\left(\frac{24}{7}\right)^2} = -\sqrt{1+\frac{576}{49}} = -\sqrt{\frac{49+576}{49}} = -\sqrt{\frac{625}{49}} = -\frac{25}{7}$$

$$\Rightarrow \quad \sin B = \frac{-7}{25} \qquad \left[\because \operatorname{cosec} B = \frac{1}{\sin B}\right]$$

Now,

$$\cos B = -\sqrt{1-\sin^2 B} \qquad [\because \cos\theta \text{ is negative in third quadrant}]$$

$$= -\sqrt{1-\left(\frac{-7}{25}\right)^2} = -\sqrt{1-\frac{49}{625}} = -\sqrt{\frac{625-49}{625}} = -\sqrt{\frac{576}{625}} = \frac{-24}{25}$$

Now,

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$= \left(\frac{-12}{13}\right)\times\left(\frac{-24}{25}\right) - \left(\frac{5}{13}\right)\times\left(\frac{-7}{25}\right)$$

$$= \frac{288}{325} + \frac{35}{325}$$

$$= \frac{323}{325}$$

We have,

$$\cos A = \frac{-12}{13} \text{ and } \cot B = \frac{24}{7}$$

$$\therefore \quad \sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\left(\frac{-12}{13}\right)^2} = \sqrt{1-\frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

and,

$$\operatorname{cosec} B = -\sqrt{1+\cot^2 B} \qquad [\because \text{cosec is negative in third quadrant}]$$

$$= -\sqrt{1+\left(\frac{24}{7}\right)^2} = -\sqrt{1+\frac{576}{49}} = -\sqrt{\frac{49+576}{49}} = -\sqrt{\frac{625}{49}} = -\frac{25}{7}$$

$$\Rightarrow \quad \sin B = \frac{-7}{25} \qquad \left[\because \operatorname{cosec} B = \frac{1}{\sin B}\right]$$

Now,

$$\cos B = -\sqrt{1-\sin^2 B} \qquad [\because \cos\theta \text{ is negative in third quadrant}]$$

$$= -\sqrt{1-\left(\frac{-7}{25}\right)^2} = -\sqrt{1-\frac{49}{625}} = -\sqrt{\frac{625-49}{625}} = -\sqrt{\frac{576}{625}} = \frac{-24}{25}$$

Now,

$$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{5}{13}}{\frac{-12}{13}} = \frac{-5}{12} \qquad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$\text{and,} \quad \tan B = \frac{\sin B}{\cos B} = \frac{\frac{-7}{25}}{\frac{-24}{25}} = \frac{7}{24} \qquad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$\therefore \quad \tan(A+B) = \frac{\tan A + \tan B}{1-\tan A \tan B}$$

$$= \frac{\frac{-5}{12}+\frac{7}{24}}{1-\left(\frac{-5}{12}\right)\times\frac{7}{24}}$$

$$= \frac{\frac{-10+7}{24}}{1+\frac{35}{288}}$$

$$= \frac{\frac{-3}{24}}{\frac{288+35}{288}}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q9

LHS: $\cos 105° + \cos 15°$

$= \cos(90° + 15°) + \cos(90° - 75°)$

$= -\sin 15° + \sin 75°$ $\quad \left[\begin{array}{l} \because \cos(90+\theta) = -\sin\theta \\ \text{and } \cos(90-\theta) = \sin\theta \end{array}\right]$

$= \sin 75° - \sin 15°$

$\therefore \quad \cos 105° + \cos 15° = \sin 75° - \sin 15°$

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q10

LHS: $\dfrac{\tan A + \tan B}{\tan A - \tan B}$

$= \dfrac{\dfrac{\sin A}{\cos A} + \dfrac{\sin B}{\cos B}}{\dfrac{\sin A}{\cos A} - \dfrac{\sin B}{\cos B}}$ $\quad \left[\because \tan\theta = \dfrac{\sin\theta}{\cos\theta}\right]$

$= \dfrac{\dfrac{\sin A\cos B + \cos A\sin B}{\cos A\cos B}}{\dfrac{\sin A\cos B - \cos A\sin B}{\cos A\cos B}}$

$= \dfrac{\sin A\cos B + \cos A\sin B}{\sin A\cos B - \cos A\sin B}$

$= \dfrac{\sin(A+B)}{\sin(A-B)}$ $\quad \left[\begin{array}{l} \because \quad \sin(A+B) = \sin A\cos B + \cos A\sin B \\ \text{and, } \sin(A-B) = \sin A\cos B - \cos A\sin B \end{array}\right]$

$\therefore \quad \dfrac{\tan A + \tan B}{\tan A - \tan B} = \dfrac{\sin(A+B)}{\sin(A-B)}$

Hence proved.

*********** END **********