Trigonometric Ratios of Compound Angles Ex 7.1 Q20

We have,

$$\tan A = x \tan B$$

$$\frac{\sin A}{\cos A} = x \frac{\sin B}{\cos B} \qquad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$\Rightarrow \quad \sin A \cos B = x \cos A \sin B \qquad \text{---(i)}$$

Now, $$\frac{\sin(A-B)}{\sin(A+B)} = \frac{\sin A \cos B - \sin B \cos A}{\sin A \cos B + \cos A \sin B}$$

$$= \frac{x \cos A \sin B - \cos A \sin B}{x \cos A \sin B + \cos A \sin B} \qquad [\text{Using equation (i)}]$$

$$= \frac{\cos A \sin B (x-1)}{\cos A \sin B (x+1)}$$

$$= \frac{x-1}{x+1}$$

$$\therefore \quad \frac{\sin(A-B)}{\sin(A+B)} = \frac{x-1}{x+1}$$

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q21

We have,

$$\tan(A+B) = x \text{ and } \tan(A-B) = y$$

Now, $$\tan 2A = \tan[(A+B)+(A-B)]$$

$$= \frac{\tan(A+B) + \tan(A-B)}{1 - \tan(A+B) \times \tan(A-B)}$$

$$= \frac{x+y}{1-xy}$$

$$\therefore \quad \tan 2A = \frac{x+y}{1-xy}$$

Now, $$\tan 2B = \tan[(A+B)-(A-B)]$$

$$= \frac{\tan(A+B) - \tan(A-B)}{1 + \tan(A+B) \times \tan(A-B)}$$

$$= \frac{x-y}{1+xy}$$

$$\therefore \quad \tan 2B = \frac{x-y}{1+xy}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q22

We have,

$$\cos A + \sin B = m \text{ and } \sin A + \cos B = n$$

Now, $m^2 + n^2 - 2$

$= (\cos A + \sin B)^2 + (\sin A + \cos B)^2 - 2$

$= \cos^2 A + \sin^2 B + 2\cos A \sin B + \sin^2 A + \cos^2 B + 2\sin A \cos B - 2$

$= (\sin^2 A + \cos^2 A) + (\sin^2 B + \cos^2 B) + 2\cos A \sin B + 2\sin A \cos B - 2$

$= 1 + 1 + 2\cos A \sin B + 2\sin A \cos B - 2$

$= 2 + 2(\sin A \cos B + \cos A \sin B) - 2$

$= 2(\sin A \cos B + \cos A \sin B)$

$= 2\sin(A+B)$ $\quad [\because \sin(A+B) = \sin A \cos B + \cos A \sin B]$

$\therefore \quad 2\sin(A+B) = m^2 + n^2 - 2$

Hence proved

Trigonometric Ratios of Compound Angles Ex 7.1 Q23

We have,

$$\tan A + \tan B = a \text{ and } \cot A + \cot B = b$$

Now, $\cot A + \cot B = b$

$\Rightarrow \quad \dfrac{1}{\tan A} + \dfrac{1}{\tan B} = b$ $\quad \left[\because \cot\theta = \dfrac{1}{\tan\theta}\right]$

$\Rightarrow \quad \dfrac{\tan B + \tan A}{\tan A \tan B} = b$

$\Rightarrow \quad \dfrac{a}{\tan A \tan B} = b$ $\quad [\because \tan A + \tan B = a]$

$\Rightarrow \quad \dfrac{a}{b} = \tan A \tan B$

$\because \quad \cot(A+B) = \dfrac{1}{\tan(A+B)}$

$= \dfrac{1}{\dfrac{\tan A + \tan B}{1 - \tan A \tan B}}$

$= \dfrac{1 - \tan A \tan B}{\tan A + \tan B}$

$= \dfrac{1 - \dfrac{a}{b}}{a}$ $\quad \left[\because \tan A \tan B = \dfrac{a}{b}\right]$

$= \dfrac{b-a}{ab}$

$= \dfrac{b}{ab} - \dfrac{a}{ab}$

$= \dfrac{1}{a} - \dfrac{1}{b}$

$\therefore \quad \cot(A+B) = \dfrac{1}{a} - \dfrac{1}{b}$

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q24

We have,

$$\cos\theta = \frac{8}{17}$$

$$\therefore \quad \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{64}{289}}$$
$$= \sqrt{\frac{225}{289}}$$
$$= \frac{15}{17}$$

Now, $\cos\left(\frac{\pi}{6}+\theta\right)+\cos\left(\frac{\pi}{4}-\theta\right)+\cos\left(\frac{2\pi}{3}-\theta\right)$

$$= \left[\cos\frac{\pi}{6}\cos\theta - \sin\frac{\pi}{6}\sin\theta\right] + \left[\cos\frac{\pi}{4}\cos\theta + \sin\frac{\pi}{4}\sin\theta\right]$$
$$+\left[\cos\frac{2\pi}{3}\cos\theta + \sin\frac{2\pi}{3}\sin\theta\right]$$
$$= \left[\cos\frac{\pi}{6}+\cos\frac{\pi}{4}+\cos\frac{2\pi}{3}\right]\cos\theta + \sin\theta\left[-\sin\frac{\pi}{6}+\sin\frac{\pi}{4}+\sin\frac{2\pi}{3}\right]$$
$$= \left[\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}+\cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right)\right]\times\frac{8}{17}+\frac{15}{17}\times\left[-\frac{1}{2}+\frac{1}{\sqrt{2}}+\sin\left(\frac{\pi}{2}+\frac{\pi}{6}\right)\right]$$
$$= \left[\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}-\sin\frac{\pi}{6}\right]\times\frac{8}{17}+\frac{15}{17}\times\left[-\frac{1}{2}+\frac{1}{\sqrt{2}}+\cos\frac{\pi}{6}\right]$$

$$= \left[\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}-\frac{1}{2}\right]\times\frac{8}{17}+\frac{15}{17}\times\left[-\frac{1}{2}+\frac{1}{\sqrt{2}}+\frac{\sqrt{3}}{2}\right]$$
$$\left[\because \cos A \text{ is negative in second quadrant}\right]$$
$$= \left[\frac{\sqrt{3}-1}{2}+\frac{1}{\sqrt{2}}\right]\times\frac{8}{17}+\frac{15}{17}\times\left[\frac{\sqrt{3}-1}{2}+\frac{1}{\sqrt{2}}\right]$$
$$= \left(\frac{\sqrt{3}-1}{2}+\frac{1}{\sqrt{2}}\right)\left(\frac{8}{17}+\frac{15}{17}\right)$$
$$= \left(\frac{\sqrt{3}-1}{2}+\frac{1}{\sqrt{2}}\right)\left(\frac{8+15}{17}\right)$$
$$= \left(\frac{\sqrt{3}-1}{2}+\frac{1}{\sqrt{2}}\right)\times\frac{23}{17}$$

$$\therefore \quad \cos\left(\frac{\pi}{6}+\theta\right)+\cos\left(\frac{\pi}{4}-\theta\right)+\cos\left(\frac{2\pi}{3}-\theta\right) = \left(\frac{\sqrt{3}-1}{2}+\frac{1}{\sqrt{2}}\right)\times\frac{23}{17}$$

Hence proved.

*********** END **********