CHAPTER 34

MAGNETIC FIELD

34.1 INTRODUCTION

If a charge q is placed at rest at a point P near a metallic wire carrying a current i, it experiences almost no force. We conclude that there is no appreciable electric field at the point P. This is expected because in any volume of wire (which contains several thousand atoms) there are equal amounts of positive and negative charges. The wire is electrically neutral and does not produce an electric field.*

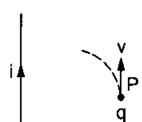

Figure 34.1

However, if the charge q is projected from the point P in the direction of the current (figure 34.1), it is deflected towards the wire (q is assumed positive). There must be a field at P which exerts a force on the charge when it is projected, but not when it is kept at rest. This field is different from the electric field which always exerts a force on a charged particle whether it is at rest or in motion. This new field is called *magnetic field* and is denoted by the symbol $\vec{B}$. The force exerted by a magnetic field is called *magnetic force.*

34.2 DEFINITION OF MAGNETIC FIELD $\vec{B}$

If a charged particle is projected in a magnetic field, in general, it experiences a magnetic force. By projecting the particle in different directions from the same point P with different speeds, we can observe the following facts about the magnetic force:

(a) There is one line through the point P, such that, if the velocity of the particle is along this line, there is no magnetic force. We define the direction of the magnetic field to be along this line (the direction is not uniquely defined yet, because there are two opposite directions along any line).

(b) If the speed of the particle is v and it makes an angle θ with the line identified in (a), i.e., with the direction of the magnetic field, the magnitude of the magnetic force is proportional to $|v \sin\theta|$.

(c) The direction of the magnetic force is perpendicular to the direction of the magnetic field as well as to the direction of the velocity.

(d) The force is proportional to the magnitude of the charge q and its direction is opposite for positive and negative charges.

All the above facts may be explained if we define the magnetic field by the equation

$$\vec{F} = q\vec{v} \times \vec{B}. \qquad \dots (34.1)$$

By measuring the magnetic force $\vec{F}$ acting on a charge q moving at velocity $\vec{v}$, we can obtain $\vec{B}$. If $\vec{v} \parallel \vec{B}$, the force is zero. By taking magnitudes in equation (34.1), we see that the force is proportional to $|v \sin\theta|$. By the rules of vector product, the force is perpendicular to both $\vec{B}$ and $\vec{v}$. Also, the observation (d) follows from equation (34.1).

Equation (34.1) uniquely determines the direction of $\vec{B}$ from the rules of vector product. The SI unit of magnetic field is newton metre/ampere. This is written as tesla and abbreviated as T. Another unit in common use is gauss (G). The relation between gauss and tesla is $1\ \mathrm{T} = 10^4\ \mathrm{G}$.

The unit weber/meter^2 is also used for magnetic field and is the same as tesla. Tesla is quite a large unit for many practical applications. We have a magnetic field of the order of 10^{-5} T near the earth's surface. Large superconducting magnets are needed to produce a field of the order of 10 T in laboratories.

* In fact, there is a small charge density on the surface of the wire which does produce an electric field near the wire. This field is very small and we shall neglect it.

In the older days, the term *magnetic induction* was used for this field $\vec{B}$.

Example 34.1

A proton is projected with a speed of $3\times10^{6}\ \mathrm{m\ s^{-1}}$ *horizontally from east to west. A uniform magnetic field* $\vec{B}$ *of strength* $2\cdot0\times10^{-3}$ T *exists in the vertically upward direction. (a) Find the force on the proton just after it is projected. (b) What is the acceleration produced?*

Solution :

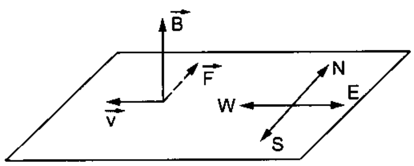

Figure 34.2

(a) The situation is shown in figure (34.2). The force is perpendicular to $\vec{B}$ hence it is in the horizontal plane through the proton. In this plane, it is perpendicular to the velocity $\vec{v}$. Thus, it is along the north-south line. The rule for vector product shows that $\vec{v}\times\vec{B}$ is towards north. As the charge on the proton is positive, the force $\vec{F}=q\vec{v}\times\vec{B}$ is also towards north. The magnitude of the force is

$$F = qvB\sin\theta$$
$$= (1\cdot6\times10^{-19}\ \mathrm{C})\,(3\cdot0\times10^{6}\ \mathrm{m\ s^{-1}})\,(2\cdot0\times10^{-3}\ \mathrm{T})$$
$$= 9\cdot6\times10^{-16}\ \mathrm{N}.$$

(b) The acceleration of the proton is

$$a=\frac{F}{m}=\frac{9\cdot6\times10^{-16}\ \mathrm{N}}{1\cdot67\times10^{-27}\ \mathrm{kg}}$$
$$=5\cdot8\times10^{11}\ \mathrm{m\ s^{-2}}.$$

34.3 RELATION BETWEEN ELECTRIC AND MAGNETIC FIELDS

Figure (34.3) shows a long wire carrying a current i and a charge q having a velocity v parallel to the current as seen by an observer S. There is no electric field, but there is a magnetic field which exerts a force on the charge and the charge is attracted towards the wire.

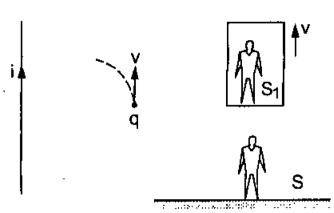

Figure 34.3

Now consider another observer S_1 who is moving at a uniform velocity v parallel to the wire. In this frame the charge is at rest and hence, the magnetic field (if any) cannot exert any force on the charge. However, the observer S_1 also sees that the charge is attracted by the wire. In fact, the acceleration of the charge is the same for both S and S_1 as they are unaccelerated with respect to each other. Hence, there must be an electric field in the frame of S_1. What was a pure magnetic field in the frame of S turns out to be a combination of electric field and magnetic field in the frame of S_1. We conclude that the electric field and the magnetic field are not basically independent. They are two aspects of the same entity which we call *electromagnetic field.* Whether the electromagnetic field will show up as an electric field or a magnetic field or a combination, depends on the frame from which we are looking at the field. If we are confined to a particular frame, we can treat the electric field and the magnetic field as separate entities.

34.4 MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

As the magnetic force on a particle is perpendicular to the velocity, it does not do any work on the particle. Hence, the kinetic energy or the speed of the particle does not change due to the magnetic force.

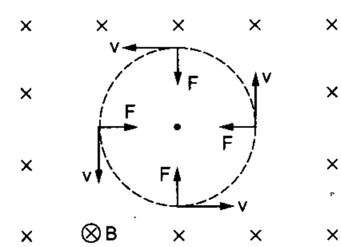

Figure 34.4

In figure (34.4), the magnetic field B is perpendicular to the paper and going into it. (This direction is, by convention, shown as ⊗. The direction coming out of the paper is, by convention, shown as ⊙. The circle around the cross or around the dot is quite often omitted.) A charge q is projected with a speed v in the plane of the paper. The velocity is perpendicular to the magnetic field. The force is $F = qvB$ in the direction perpendicular to both v and B. This force will deflect the particle without changing the speed and the particle will move along a circle perpendicular to the field. The magnetic force provides the centripetal force. If r be the radius of the circle,

$$qvB = m\frac{v^2}{r}$$

or, $$r=\frac{mv}{qB}\cdot \qquad \ldots (34.2)$$

The time taken to complete the circle (time period) is

$$T=\frac{2\pi r}{v}=\frac{2\pi m}{qB}\cdot \qquad \ldots (34.3)$$

We see that the time period is independent of the speed v. If the particle moves faster, the radius is larger, it has to move a longer distance to complete the circle so that the time taken is the same.

The frequency of revolution is

$$\nu = \frac{1}{T} = \frac{qB}{2\pi m}. \qquad \dots (34.4)$$

This frequency is called the *cyclotron frequency.*

Example 34.2

A particle having a charge of 100 μC *and a mass of* 10 mg *is projected in a uniform magnetic field of* 25 mT *with a speed of* 10 m s^{-1}. *If the velocity is perpendicular to the magnetic field, how long will it take for the particle to come back to its original position for the first time after being projected.*

Solution : The particle moves along a circle and returns to its original position after completing the circle, that is after one time period. The time period is

$$T = \frac{2\pi m}{qB}$$

$$= \frac{2\pi \times (10 \times 10^{-6}\ \text{kg})}{(100 \times 10^{-6}\ \text{C}) \times (25 \times 10^{-3}\ \text{T})} = 25\ \text{s}.$$

If the velocity of the charge is not perpendicular to the magnetic field, we can break the velocity in two components— $v_{\parallel}$, parallel to the field and $v_{\perp}$, perpendicular to the field. The component $v_{\parallel}$ remains unchanged as the force $q\vec{v} \times \vec{B}$ is perpendicular to it. In the plane perpendicular to the field, the particle traces a circle of radius $r = \frac{mv_{\perp}}{qB}$ as given by equation (34.2). The resultant path is a helix (figure 34.5).

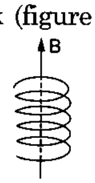

Figure 34.5

34.5 MAGNETIC FORCE ON A CURRENT-CARRYING WIRE

Suppose a conducting wire, carrying a current i, is placed in a magnetic field $\vec{B}$. Consider a small element dl of the wire (figure 34.6). The free electrons drift with a speed v_d opposite to the direction of the current. The relation between the current i and the drift speed v_d is

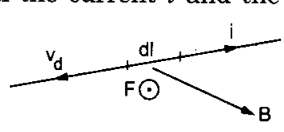

Figure 34.6

$$i = jA = nev_d A. \qquad \dots (i)$$

Here A is the area of cross-section of the wire and n is the number of free electrons per unit volume. Each electron experiences an average magnetic force

$$\vec{f} = -e\vec{v}_d \times \vec{B}.$$

The number of free electrons in the small element considered is $nAdl$. Thus, the magnetic force on the wire of length dl is

$$d\vec{F} = (nAdl)(-e\vec{v}_d \times \vec{B}).$$

If we denote the length dl along the direction of the current by $d\vec{l}$, the above equation becomes

$$d\vec{F} = nAev_d\, d\vec{l} \times \vec{B}.$$

Using (i),

$$d\vec{F} = i\, d\vec{l} \times \vec{B}. \qquad \dots (34.5)$$

The quantity $i\,d\vec{l}$ is called a *current element.*

If a straight wire of length l carrying a current i is placed in a uniform magnetic field $\vec{B}$, the force on it is

$$\vec{F} = i\,\vec{l} \times \vec{B}. \qquad \dots (34.6)$$

Example 34.3

Figure (34.7) shows a triangular loop PQR carrying a current i. The triangle is equilateral with edge-length l. A uniform magnetic field B exists in a direction parallel to PQ. Find the forces acting on the three wires PQ, QR and RP separately.

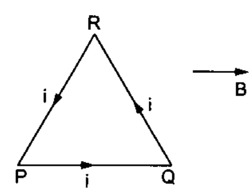

Figure 34.7

Solution : The force on the wire PQ is

$$\vec{F}_1 = i\,\vec{PQ} \times \vec{B} = 0$$

as the field $\vec{B}$ is parallel to $\vec{PQ}$.

The force on QR is

$$\vec{F}_2 = i\,\vec{QR} \times \vec{B}$$

or, $$F_2 = ilB \sin 120°$$

$$= \frac{\sqrt{3}}{2} ilB.$$

From the rule of vector product, this force is perpendicular to the plane of the diagram and is going into it.

The force on RP is

$$\vec{F}_3 = i\,\vec{RP} \times \vec{B}$$

or, $$F_3 = i\,lB \sin 120° = \frac{\sqrt{3}}{2} ilB.$$

From the rule of vector product, this force is perpendicular to the plane of the diagram and is coming out of it.

34.6 TORQUE ON A CURRENT LOOP

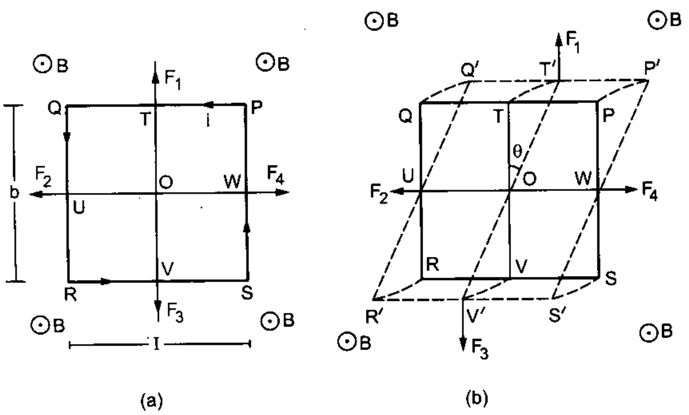

Figure 34.8

Figure (34.8a) shows a rectangular loop $PQRS$ carrying a current i and placed in a uniform magnetic field B. The magnetic forces F_1, F_2, F_3 and F_4 on the wires PQ, QR, RS and SP are obtained by using the equation $\vec{F} = i\,\vec{l} \times \vec{B}$. These forces act from the middle points T, U, V, W of the respective sides. Clearly, $F_1 = F_3 = ilB$ and $F_2 = F_4 = ibB$ in figure (34.8a). The resultant force is, therefore, zero. Also, F_1 and F_3 have the same line of action so they together produce no torque. Similarly, F_2 and F_4 together produce no torque. Hence, the resultant torque on the loop is zero.

Now suppose the loop is rotated through an angle θ about the line WU (figure 34.8b). The wire PQ shifts parallel to itself so that the force $\vec{F_1} = i\,\vec{l} \times \vec{B}$ on it remains unchanged in magnitude and direction. Its point of application T shifts to T'. Similarly, the force on RS remains $\vec{F_3}$ but the point of application shifts to V'. The line TV gets rotated by an angle θ to take the position $T'V'$. This line makes an angle θ with the force F_1 and F_3. The torque of F_1 about O has magnitude

$$\left| \vec{OT'} \times \vec{F_1} \right| = \left(\frac{b}{2} \right) \times F_1 \times \sin\theta = \frac{b}{2}\,(ilB) \sin\theta.$$

This torque acts along the line UW. The torque of F_3 about O is also $\frac{b}{2}\,(ilB)\sin\theta$ along the same direction.

As the wire QR rotates about WU, the plane containing the wire and the magnetic field does not change. The force on the wire is perpendicular to this plane and hence its direction remains unchanged. Also, the point of application U remains the same. Similar is the case for the wire SP. The forces on QR and SP are, therefore, equal and opposite and act along the same line. They together produce no torque.

The net torque acting on the loop is, therefore,

$$\Gamma = \frac{b}{2}\,(ilB) \sin\theta + \frac{b}{2}\,(ilB) \sin\theta = b(ilB) \sin\theta$$
$$= i\,AB \sin\theta.$$

Let us define the *area-vector* $\vec{A}$ of the loop in the following way. The magnitude of $\vec{A}$ is equal to the area enclosed by the loop and the direction of $\vec{A}$ is perpendicular to the plane of the loop and is towards the side from which the current looks anticlockwise. Thus, in figure (34.8a), the area-vector $\vec{A}$ points towards the viewer. It is drawn from the centre O of the loop. Another way to get the direction of $\vec{A}$ is to use the right-hand thumb rule. If you curl your fingers of the right hand along the current, the stretched thumb gives the direction of $\vec{A}$.

The definition of area-vector is valid for a closed loop of any shape, not necessarily rectangular.

In figure (34.8a), the angle between the area-vector $\vec{A}$ and the magnetic field $\vec{B}$ is zero. As the loop rotates, the area-vector also rotates by an angle θ and hence the angle between $\vec{A}$ and $\vec{B}$ becomes θ. Taking the direction of the torque (along UW) into consideration,

$$\vec{\Gamma} = i\vec{A} \times \vec{B} = \vec{\mu} \times \vec{B} \qquad \dots (34.7)$$

where $\vec{\mu} = i\vec{A}$ is called the *magnetic dipole moment* or simply *magnetic moment* of the current loop.

We have already discussed an electric dipole. A pair of charges $-q$, $+q$ separated by a distance l forms an electric dipole of dipole moment $p = ql$. The direction is from $-q$ to $+q$. If such a dipole is placed in a uniform electric field, a torque

$$\vec{\Gamma} = \vec{p} \times \vec{E}$$

acts on the dipole. Equation (34.7) is similar to this equation in structure and hence $\vec{\mu}$ is called magnetic dipole moment.

If there are n turns in the loop, each turn experiences a torque. The net torque is

$$\vec{\Gamma} = ni\vec{A} \times \vec{B}.$$

We still write it as $\vec{\Gamma} = \vec{\mu} \times \vec{B}$ with the magnetic dipole moment defined as

$$\vec{\mu} = ni\vec{A}. \qquad \dots (34.8)$$

Equations (34.7) and (34.8) are obtained by considering a rectangular loop. However, these equations are valid for plane loops of any shape.

Example 34.4

A current of 10·0 nA *is established in a circular loop of radius* 5·0 cm. *Find the magnetic dipole moment of the current loop.*

Solution : The magnetic dipole moment is

$$\vec{\mu} = i\vec{A}.$$

Thus, $\mu = i\pi r^2 = (10 \times 10^{-9}\ \text{A})\,(3{\cdot}14) \times (5 \times 10^{-2}\ \text{m})^2$

$= 7{\cdot}85 \times 10^{-11}\ \text{A m}^2.$

Worked Out Examples

1. *A charge of* 2·0 μC *moves with a speed of* $2{\cdot}0 \times 10^6$ m s^{-1} *along the positive x-axis. A magnetic field* $\vec{B}$ *of strength* $(0{\cdot}20\,\vec{j} + 0{\cdot}40\,\vec{k})$T *exists in space. Find the magnetic force acting on the charge.*

Solution : The force on the charge

$$= q\vec{v} \times \vec{B}$$

$$= (2{\cdot}0 \times 10^{-6}\ \text{C})\,(2{\cdot}0 \times 10^{6}\ \text{m s}^{-1}\,\vec{i}) \times (0{\cdot}20\,\vec{j} + 0{\cdot}40\,\vec{k})\ \text{T}$$

$$= 4{\cdot}0(0{\cdot}20\,\vec{i} \times \vec{j} + 0{\cdot}40\,\vec{i} \times \vec{k})\ \text{N}$$

$$= (0{\cdot}8\,\vec{k} - 1{\cdot}6\,\vec{j})\ \text{N}.$$

2. *A wire is bent in the form of an equilateral triangle PQR of side* 10 cm *and carries a current of* 5·0 A. *It is placed in a magnetic field B of magnitude* 2·0 T *directed perpendicularly to the plane of the loop. Find the forces on the three sides of the triangle.*

Solution :

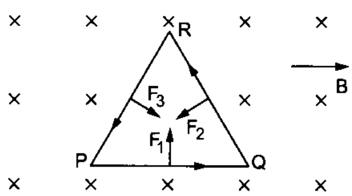

Figure 34-W1

Suppose the field and the current have directions as shown in figure (34-W1). The force on PQ is

$$\vec{F_1} = i\,\vec{l} \times \vec{B}$$

or, $\qquad F_1 = 5{\cdot}0\ \text{A} \times 10\ \text{cm} \times 2{\cdot}0\ \text{T} = 1{\cdot}0\ \text{N}.$

The rule of vector product shows that the force F_1 is perpendicular to PQ and is directed towards the inside of the triangle.

The forces $\vec{F_2}$ and $\vec{F_3}$ on QR and RP can also be obtained similarly. Both the forces are 1·0 N directed perpendicularly to the respective sides and towards the inside of the triangle.

The three forces $\vec{F_1}$, $\vec{F_2}$, and $\vec{F_3}$ will have zero resultant, so that there is no net magnetic force on the triangle. This result can be generalised. Any closed current loop, placed in a homogeneous magnetic field, does not experience a net magnetic force.

3. *Figure (34-W2) shows two long metal rails placed horizontally and parallel to each other at a separation l. A uniform magnetic field B exists in the vertically downward direction. A wire of mass m can slide on the rails. The rails are connected to a constant current source which drives a current i in the circuit. The friction coefficient between the rails and the wire is* μ. *(a) What should be the minimum value of* μ *which can prevent the wire from sliding on the rails? (b) Describe the motion of the wire if the value of* μ *is half the value found in the previous part.*

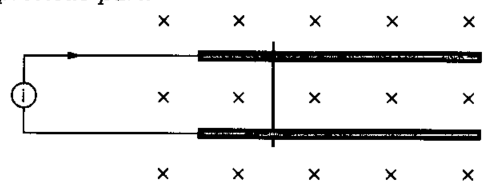

Figure 34-W2

Solution :

(a) The force on the wire due to the magnetic field is

$$\vec{F} = i\,\vec{l} \times \vec{B}$$

or, $\qquad F = ilB.$

It acts towards right in the given figure. If the wire does not slide on the rails, the force of friction by the rails should be equal to F. If μ_0 be the minimum coefficient of friction which can prevent sliding, this force is also equal to $\mu_0\, mg$. Thus,

$$\mu_0\, mg = ilB$$

or, $\qquad \mu_0 = \dfrac{ilB}{mg}.$

(b) If the friction coefficient is $\mu = \dfrac{\mu_0}{2} = \dfrac{ilB}{2\,mg}$, the wire will slide towards right. The frictional force by the rails is

$$f = \mu mg = \frac{ilB}{2}\ \text{towards left.}$$

The resultant force is $ilB - \dfrac{ilB}{2} = \dfrac{ilB}{2}$ towards right. The acceleration will be $a = \dfrac{ilB}{2\,m}$. The wire will slide towards right with this acceleration.

4. *A proton, a deuteron and an alpha particle moving with equal kinetic energies enter perpendicularly into a magnetic field. If* r_p, r_d *and* r_a *are the respective radii of the circular paths, find the ratios* r_p/r_d *and* r_p/r_a.

Solution : We have $r = \dfrac{mv}{qB} = \dfrac{\sqrt{2\,mK}}{qB}$

where $K = \dfrac{1}{2}mv^2 =$ kinetic energy.

Thus, $r_p = \frac{\sqrt{2\, m_p K}}{q_p B}$, $r_d = \frac{\sqrt{2\, m_d K}}{q_d B}$

and $r_\alpha = \frac{\sqrt{2\, m_\alpha K}}{q_\alpha B}$.

We get $\frac{r_p}{r_d} = \frac{q_d}{q_p}\sqrt{\frac{m_p}{m_d}} = \frac{q_p}{q_p}\sqrt{\frac{m_p}{2\, m_p}} = \frac{1}{\sqrt{2}}$

and $\frac{r_p}{r_\alpha} = \frac{q_\alpha}{q_p}\sqrt{\frac{m_p}{m_\alpha}} = \frac{2\, q_p}{q_p}\sqrt{\frac{m_p}{4\, m_p}} = 1.$

5. *Singly charged magnesium ($A = 24$) ions are accelerated to kinetic energy 2 keV and are projected perpendicularly into a magnetic field B of magnitude 0·6 T. (a) Find the radius of the circle formed by the ions. (b) If there are also singly charged ions of the isotope magnesium-26, what would be the radius for these particles?*

Solution : The radius is given by

$$r = \frac{mv}{qB} = \frac{\sqrt{2\, mK}}{qB}$$

For ^{24}Mg ions, $m = 24 \times m_p$ approximately and $q = 1{\cdot}6 \times 10^{-19}$ C.

Putting the values,

$$r = \frac{\sqrt{2 \times 24 \times 1{\cdot}67 \times 10^{-27}\ \text{kg} \times 2000 \times 1{\cdot}6 \times 10^{-19}\ \text{J}}}{1{\cdot}6 \times 10^{-19}\ \text{C} \times 0{\cdot}6\ \text{T}}$$

$= 0{\cdot}053$ m $= 5{\cdot}3$ cm.

For ^{26}Mg, the radius r' will be given by

$$r' = \frac{\sqrt{2\, m'K}}{qB}$$

or, $$r' = r\sqrt{\frac{m'}{m}} = 5{\cdot}3\ \text{cm}\sqrt{\frac{26}{24}} = 5{\cdot}5\ \text{cm}.$$

6. *A particle having a charge 20 μC and mass 20 μg moves along a circle of radius 5·0 cm under the action of a magnetic field $B = 1{\cdot}0$ T. When the particle is at a point P, a uniform electric field is switched on and it is found that the particle continues on the tangent through P with a uniform velocity. Find the electric field.*

Solution :

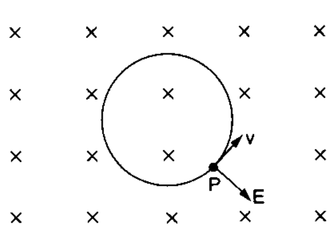

Figure 34-W3

When the particle moves along a circle in the magnetic field B, the magnetic force is radially inward. If an electric field of proper magnitude is switched on which is directed radially outwards, the particle may experience no force. It will then move along a straight line with uniform velocity. This will be the case when

$$qE = qvB \text{ or, } E = vB.$$

The radius of the circle in a magnetic field is given by

$$r = \frac{mv}{qB}$$

or, $$v = \frac{rqB}{m}$$

$$= \frac{(5{\cdot}0 \times 10^{-2}\ \text{m})\,(20 \times 10^{-6}\ \text{C})\,(1{\cdot}0\ \text{T})}{20 \times 10^{-9}\ \text{kg}} = 50\ \text{m s}^{-1}.$$

The required electric field is

$$E = vB = (50\ \text{m s}^{-1})\,(1.0\ \text{T})$$
$$= 50\ \text{V m}^{-1}.$$

This field will be in a direction which is radially outward at P.

7. *A particle of mass $m = 1{\cdot}6 \times 10^{-27}$ kg and charge $q = 1{\cdot}6 \times 10^{-19}$ C moves at a speed of $1{\cdot}0 \times 10^{7}$ m s^{-1}. It enters a region of uniform magnetic field at a point E, as shown in figure (34-W4). The field has a strength of 1·0 T. (a) The magnetic field is directed into the plane of the paper. The particle leaves the region of the field at the point F. Find the distance EF and the angle θ. (b) If the field is coming out of the paper, find the time spent by the particle in the region of the magnetic field after entering it at E.*

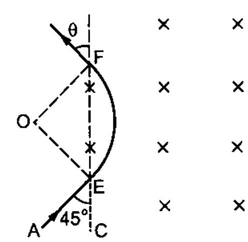

Figure 34-W4

Solution : (a) As the particle enters the magnetic field, it will travel in a circular path. The centre will be on the line perpendicular to its velocity and the radius r will be $\frac{mv}{qB}$. The direction of the force $q\vec{v} \times \vec{B}$ shows that the centre will be outside the field as shown in figure (34-W4). As $\angle AEO = 90°$ (as AE is tangent and OE is radius) and $\angle AEC = 45°$, we have $\angle OEF = 45°$. As $OE = OF$ (they are radii of the circular arc), $\angle OFE = \angle OEF = 45°$. Also, OF is perpendicular to the velocity of the particle at F, so that $\theta = 45°$. From triangle OEF,

$$EF = 2.OE \cos \angle OEF$$
$$= 2 \cdot \frac{mv}{qB} \cdot \frac{1}{\sqrt{2}}$$
$$= \frac{\sqrt{2} \times (1{\cdot}6 \times 10^{-27}\ \text{kg}) \times (10^{7}\ \text{m s}^{-1})}{(1{\cdot}6 \times 10^{-19}\ \text{C}) \times 1{\cdot}0\ \text{T}}$$
$$= \sqrt{2} \times 10^{-1}\ \text{m} = 14\ \text{cm}.$$

(b)

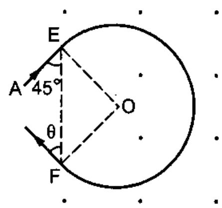

Figure 34-W5

If the magnetic field is coming out of the paper, the direction of the force $q\vec{v} \times \vec{B}$ shows that the centre O will be inside the field region as shown in figure (34-W5). Again $\angle AEO = 90°$, giving

$$\angle OEF = \angle OFE = 45°.$$

Thus, the angle $EOF = 90°$. The particle describes three fourths of the complete circle inside the field. As the speed v is uniform, the time spent in the magnetic field will be

$$\frac{3}{4} \times \frac{2\pi r}{v} = \frac{3\pi mv}{2vqB} = \frac{3\pi m}{2qB}$$

$$= \frac{3 \times 3{\cdot}14 \times 1{\cdot}6 \times 10^{-27}\ \text{kg}}{2 \times 1{\cdot}6 \times 10^{-19}\ \text{C} \times 1{\cdot}0\ \text{T}} = 4{\cdot}7 \times 10^{-8}\ \text{s}.$$

8. *A beam of protons with a velocity of* 4×10^{5} m s^{-1} *enters a uniform magnetic field of* 0·3 T. *The velocity makes an angle of* 60° *with the magnetic field. Find the radius of the helical path taken by the proton beam and the pitch of the helix.*

Solution : The components of the proton's velocity along and perpendicular to the magnetic field are

$$v_{\parallel} = (4 \times 10^{5}\ \text{m s}^{-1}) \cos 60° = 2 \times 10^{5}\ \text{m s}^{-1}.$$

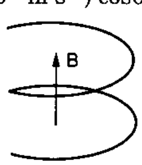

Figure 34-W6

and $\quad v_{\perp} = (4 \times 10^{5}\ \text{m s}^{-1}) \sin 60° = 2\sqrt{3} \times 10^{5}\ \text{m s}^{-1}.$

As the force $q\vec{v} \times \vec{B}$ is perpendicular to the magnetic field, the component $v_{\parallel}$ will remain constant. In the plane perpendicular to the field, the proton will describe a circle whose radius is obtained from the equation

$$qv_{\perp}B = \frac{mv_{\perp}^{2}}{r}$$

or, $$r = \frac{mv_{\perp}}{qB} = \frac{(1{\cdot}67 \times 10^{-27}\ \text{kg}) \times (2\sqrt{3} \times 10^{5}\ \text{m s}^{-1})}{(1{\cdot}6 \times 10^{-19}\ \text{C}) \times (0{\cdot}3\ \text{T})}$$

$$\approx 0{\cdot}012\ \text{m} = 1{\cdot}2\ \text{cm}.$$

The time taken in one complete revolution in the plane perpendicular to B is

$$T = \frac{2\pi r}{v_{\perp}} = \frac{2 \times 3{\cdot}14 \times 0{\cdot}012\ \text{m}}{2\sqrt{3} \times 10^{5}\ \text{m s}^{-1}}.$$

The distance moved along the field during this period, i.e., the pitch

$$= \frac{(2 \times 10^{5}\ \text{m s}^{-1}) \times 2 \times 3{\cdot}14 \times 0{\cdot}012\ \text{m}}{2\sqrt{3} \times 10^{5}\ \text{m s}^{-1}}$$

$$= 0{\cdot}044\ \text{m} = 4{\cdot}4\ \text{cm}.$$

The qualitative nature of the path of the protons is shown in figure (34-W6).

9. *A rectangular coil of size* 3·0 cm × 4·0 cm *and having* 100 turns, *is pivoted about the z-axis as shown in figure (34-W7). The coil carries an electric current of* 2·0 A *and a magnetic field of* 1·0 T *is present along the y-axis. Find the torque acting on the coil if the side in the x–y plane makes an angle* θ = 37° *with the x-axis.*

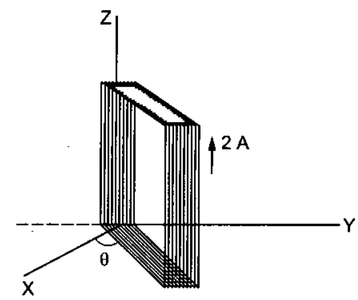

Figure 34-W7

Solution : The magnetic moment of the loop is $\vec{\mu} = ni\vec{A}$ where n is the number of turns, i is the current and $\vec{A}$ is the area-vector. The direction of $\vec{A}$ is determined by the sense of the current and in this case it lies in the fourth quadrant making an angle $\theta = 37°$ with the negative y-axis.

Torque $\vec{\Gamma} = \vec{\mu} \times \vec{B} = ni\vec{A} \times \vec{B}$.

Thus, $\quad \Gamma = 100 \times (2\ \text{A}) \times (12 \times 10^{-4}\ \text{m}^{2}) \times 1\ \text{T} \times \sin 37°$

$$= 0{\cdot}14\ \text{Nm}.$$

The torque is along the positive z-axis.

10. *An electron moves with a constant speed v along a circle of radius r. (a) Find the equivalent current through a point on its path. (b) Find the magnetic moment of the circulating electron. (c) Find the ratio of the magnetic moment to the angular momentum of the electron.*

Solution : (a) Consider a point P on the path of the electron. In one revolution of the electron, a charge e crosses the point P. As the frequency of revolution is $v/(2\pi r)$, the charge crossing P in unit time, i.e., the electric current is

$$i = \frac{ev}{2\pi r}.$$

(b) The area A enclosed by this circular current is πr^{2} so that the magnetic moment of the current is

$$\mu = iA = \left(\frac{ev}{2\pi r}\right)(\pi r^{2}) = \frac{evr}{2}$$

in a direction perpendicular to the loop.

(c) The angular momentum of the electron is $l = mvr$. Its direction is opposite to that of the magnetic moment. Thus,

$$\frac{\mu}{l} = \frac{-evr}{2\,mvr} = \frac{-e}{2\,m}.$$

11. *An electron is released from the origin at a place where a uniform electric field E and a uniform magnetic field B exist along the negative y-axis and the negative z-axis respectively. Find the displacement of the electron along the y-axis when its velocity becomes perpendicular to the electric field for the first time.*

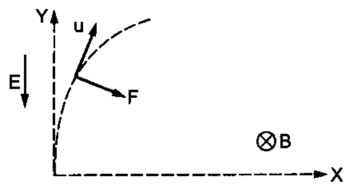

Figure 34-W8

Solution : Let us take axes as shown in figure (34-W8). According to the right-handed system, the z-axis is upward in the figure and hence the magnetic field is shown downwards. At any time, the velocity of the electron may be written as

$$\vec{u} = u_x \vec{i} + u_y \vec{j}.$$

The electric and magnetic fields may be written as

$$\vec{E} = -E\vec{j}$$

and $$\vec{B} = -B\vec{k}$$

respectively. The force on the electron is

$$\vec{F} = -e(\vec{E} + \vec{u} \times \vec{B})$$
$$= eE\vec{j} + eB(u_y \vec{i} - u_x \vec{j}).$$

Thus, $$F_x = eu_y B$$

and $$F_y = e(E - u_x B).$$

The components of the acceleration are

$$a_x = \frac{du_x}{dt} = \frac{eB}{m} u_y \qquad \text{... (i)}$$

and $$a_y = \frac{du_y}{dt} = \frac{e}{m}(E - u_x B). \qquad \text{... (ii)}$$

We have,

$$\frac{d^2 u_y}{dt^2} = -\frac{eB}{m}\frac{du_x}{dt}$$
$$= -\frac{eB}{m}\cdot\frac{eB}{m}u_y$$
$$= -\omega^2 u_y$$

where $$\omega = \frac{eB}{m}. \qquad \text{... (iii)}$$

This equation is similar to that for a simple harmonic motion. Thus,

$$u_y = A\sin(\omega t + \delta) \qquad \text{... (iv)}$$

and hence,

$$\frac{du_y}{dt} = A\,\omega\cos(\omega t + \delta). \qquad \text{... (v)}$$

At $t = 0$, $u_y = 0$ and $\dfrac{du_y}{dt} = \dfrac{F_y}{m} = \dfrac{eE}{m}$.

Putting in (iv) and (v),

$$\delta = 0 \text{ and } A = \frac{eE}{m\omega} = \frac{E}{B}.$$

Thus, $$u_y = \frac{E}{B}\sin\omega t.$$

The path of the electron will be perpendicular to the y-axis when $u_y = 0$. This will be the case for the first time at t where

$$\sin\omega t = 0$$

or, $$\omega t = \pi$$

or, $$t = \frac{\pi}{\omega} = \frac{\pi m}{eB}.$$

Also, $$u_y = \frac{dy}{dt} = \frac{E}{B}\sin\omega t$$

or, $$\int_0^y dy = \frac{E}{B}\int_0^t \sin\omega t\, dt$$

or, $$y = \frac{E}{B\omega}(1 - \cos\omega t).$$

At $$t = \frac{\pi}{\omega},$$

$$y = \frac{E}{B\omega}(1 - \cos\pi) = \frac{2E}{B\omega}.$$

Thus, the displacement along the y-axis is

$$\frac{2E}{B\omega} = \frac{2Em}{BeB} = \frac{2Em}{eB^2}.$$

□

QUESTIONS FOR SHORT ANSWER

1. Suppose a charged particle moves with a velocity v near a wire carrying an electric current. A magnetic force, therefore, acts on it. If the same particle is seen from a frame moving with velocity v in the same direction, the charge will be found at rest. Will the magnetic force become zero in this frame? Will the magnetic field become zero in this frame?

2. Can a charged particle be accelerated by a magnetic field ? Can its speed be increased ?
3. Will a current loop placed in a magnetic field always experience a zero force ?
4. The free electrons in a conducting wire are in constant thermal motion. If such a wire, carrying no current, is placed in a magnetic field, is there a magnetic force on each free electron? Is there a magnetic force on the wire?
5. Assume that the magnetic field is uniform in a cubical region and is zero outside. Can you project a charged particle from outside into the field so that the particle describes a complete circle in the field ?
6. An electron beam projected along the positive x-axis deflects along the positive y-axis. If this deflection is caused by a magnetic field, what is the direction of the field ? Can we conclude that the field is parallel to the z-axis ?
7. Is it possible for a current loop to stay without rotating in a uniform magnetic field ? If yes, what should be the orientation of the loop ?
8. The net charge in a current-carrying wire is zero. Then, why does a magnetic field exert a force on it ?
9. The torque on a current loop is zero if the angle between the positive normal and the magnetic field is either $\theta = 0$ or $\theta = 180°$. In which of the two orientations, the equilibrium is stable ?
10. Verify that the units weber and volt second are the same.

OBJECTIVE I

1. A positively charged particle projected towards east is deflected towards north by a magnetic field. The field may be
(a) towards west (b) towards south
(c) upward (d) downward.
2. A charged particle is whirled in a horizontal circle on a frictionless table by attaching it to a string fixed at one point. If a magnetic field is switched on in the vertical direction, the tension in the string
(a) will increase (b) will decrease
(c) will remain the same (d) may increase or decrease.
3. Which of the following particles will experience maximum magnetic force (magnitude) when projected with the same velocity perpendicular to a magnetic field ?
(a) Electron (b) Proton (c) He^{+} (d) Li^{++}
4. Which of the following particles will describe the smallest circle when projected with the same velocity perpendicular to a magnetic field ?
(a) Electron (b) Proton (c) He^{+} (d) Li^{+}
5. Which of the following particles will have minimum frequency of revolution when projected with the same velocity perpendicular to a magnetic field ?
(a) Electron (b) Proton (c) He^{+} (d) Li^{+}
6. A circular loop of area 1 cm^2, carrying a current of 10 A, is placed in a magnetic field of 0·1 T perpendicular to the plane of the loop. The torque on the loop due to the magnetic field is
(a) zero (b) 10^{-4} N m (c) 10^{-2} N m (d) 1 N m
7. A beam consisting of protons and electrons moving at the same speed goes through a thin region in which there is a magnetic field perpendicular to the beam. The protons and the electrons
(a) will go undeviated
(b) will be deviated by the same angle and will not separate
(c) will be deviated by different angles and hence separate
(d) will be deviated by the same angle but will separate.
8. A charged particle moves in a uniform magnetic field. The velocity of the particle at some instant makes an acute angle with the magnetic field. The path of the particle will be
(a) a straight line (b) a circle
(c) a helix with uniform pitch
(d) a helix with nonuniform pitch.
9. A particle moves in a region having a uniform magnetic field and a parallel, uniform electric field. At some instant, the velocity of the particle is perpendicular to the field direction. The path of the particle will be
(a) a straight line (b) a circle
(c) a helix with uniform pitch
(d) a helix with nonuniform pitch.
10. An electric current i enters and leaves a uniform circular wire of radius a through diametrically opposite points. A charged particle q moving along the axis of the circular wire passes through its centre at speed v. The magnetic force acting on the particle when it passes through the centre has a magnitude
(a) $qv\dfrac{\mu_0 i}{2a}$ (b) $qv\dfrac{\mu_0 i}{2\pi a}$ (c) $qv\dfrac{\mu_0 i}{a}$ (d) zero.

OBJECTIVE II

1. If a charged particle at rest experiences no electromagnetic force,
(a) the electric field must be zero
(b) the magnetic field must be zero
(c) the electric field may or may not be zero
(d) the magnetic field may or may not be zero.

2. If a charged particle kept at rest experiences an electromagnetic force,
   (a) the electric field must not be zero
   (b) the magnetic field must not be zero
   (c) the electric field may or may not be zero
   (d) the magnetic field may or may not be zero.
3. If a charged particle projected in a gravity-free room deflects,
   (a) there must be an electric field
   (b) there must be a magnetic field
   (c) both fields cannot be zero
   (d) both fields can be nonzero.
4. A charged particle moves in a gravity-free space without change in velocity. Which of the following is/are possible ?
   (a) $E = 0, B = 0$ (b) $E = 0, B \neq 0$
   (c) $E \neq 0, B = 0$ (d) $E \neq 0, B \neq 0$
5. A charged particle moves along a circle under the action of possible constant electric and magnetic fields. Which of the following are possible ?
   (a) $E = 0, B = 0$ (b) $E = 0, B \neq 0$
   (c) $E \neq 0, B = 0$ (d) $E \neq 0, B \neq 0$
6. A charged particle goes undeflected in a region containing electric and magnetic field. It is possible that
   (a) $\vec{E} \parallel \vec{B}$, $\vec{v} \parallel \vec{E}$ (b) $\vec{E}$ is not parallel to $\vec{B}$
   (c) $\vec{v} \parallel \vec{B}$ but $\vec{E}$ is not parallel to $\vec{B}$
   (d) $\vec{E} \parallel \vec{B}$ but $\vec{v}$ is not parallel to $\vec{E}$.
7. If a charged particle goes unaccelerated in a region containing electric and magnetic fields,
   (a) $\vec{E}$ must be perpendicular to $\vec{B}$
   (b) $\vec{v}$ must be perpendicular to $\vec{E}$
   (c) $\vec{v}$ must be perpendicular to $\vec{B}$
   (d) E must be equal to vB.
8. Two ions have equal masses but one is singly-ionized and the other is doubly-ionized. They are projected from the same place in a uniform magnetic field with the same velocity perpendicular to the field.
   (a) Both ions will go along circles of equal radii.
   (b) The circle described by the singly-ionized charge will have a radius double that of the other circle.
   (c) The two circles do not touch each other.
   (d) The two circles touch each other.
9. An electron is moving along the positive x-axis. You want to apply a magnetic field for a short time so that the electron may reverse its direction and move parallel to the negative x-axis, This can be done by applying the magnetic field along
   (a) y-axis (b) z-axis (c) y-axis only (d) z-axis only.
10. Let $\vec{E}$ and $\vec{B}$ denote electric and magnetic fields in a frame S and $\vec{E'}$ and $\vec{B'}$ in another frame S' moving with respect to S at a velocity $\vec{v}$. Two of the following equations are wrong. Identify them.
    (a) $B_y' = B_y + \frac{vE_z}{c^2}$ (b) $E_y' = E_y - \frac{vB_z}{c^2}$
    (c) $B_y' = B_y + vE_z$ (d) $E_y' = E_y + vB_z$

EXERCISES

1. An alpha particle is projected vertically upward with a speed of $3{\cdot}0 \times 10^4$ km s^{-1} in a region where a magnetic field of magnitude 1·0 T exists in the direction south to north. Find the magnetic force that acts on the α-particle.
2. An electron is projected horizontally with a kinetic energy of 10 keV. A magnetic field of strength $1{\cdot}0 \times 10^{-7}$ T exists in the vertically upward direction. (a) Will the electron deflect towards right or towards left of its motion ? (b) Calculate the sideways deflection of the electron in travelling through 1 m. Make appropriate approximations.
3. A magnetic field of $(4{\cdot}0 \times 10^{-3} \vec{k})$ T exerts a force of $(4{\cdot}0 \vec{i} + 3{\cdot}0 \vec{j}) \times 10^{-10}$ N on a particle having a charge of $1{\cdot}0 \times 10^{-9}$ C and going in the x–y plane. Find the velocity of the particle.
4. An experimenter's diary reads as follows: "a charged particle is projected in a magnetic field of $(7{\cdot}0 \vec{i} - 3{\cdot}0 \vec{j}) \times 10^{-3}$ T. The acceleration of the particle is found to be $(\square \vec{i} + 7{\cdot}0 \vec{j}) \times 10^{-6}$ m s^{-2}". The number to the left of $\vec{i}$ in the last expression was not readable. What can this number be ?
5. A 10 g bullet having a charge of 4·00 μC is fired at a speed of 270 m s^{-1} in a horizontal direction. A vertical magnetic field of 500 μT exists in the space. Find the deflection of the bullet due to the magnetic field as it travels through 100 m. Make appropriate approximations.
6. When a proton is released from rest in a room, it starts with an initial acceleration a_0 towards west. When it is projected towards north with a speed v_0, it moves with an initial acceleration $3a_0$ towards west. Find the electric field and the maximum possible magnetic field in the room.
7. Consider a 10-cm long portion of a straight wire carrying a current of 10 A placed in a magnetic field of 0·1 T making an angle of 53° with the wire. What magnetic force does the wire experience ?
8. A current of 2 A enters at the corner d of a square frame $abcd$ of side 20 cm and leaves at the opposite corner b. A magnetic field $B = 0{\cdot}1$ T exists in the space in a direction perpendicular to the plane of the frame as shown in figure

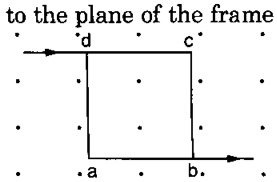

Figure 34-E1

(34-E1). Find the magnitude and direction of the magnetic forces on the four sides of the frame.

9. A magnetic field of strength 1·0 T is produced by a strong electromagnet in a cylindrical region of radius 4·0 cm as shown in figure (34-E2). A wire, carrying a current of 2·0 A, is placed perpendicular to and intersecting the axis of the cylindrical region. Find the magnitude of the force acting on the wire.

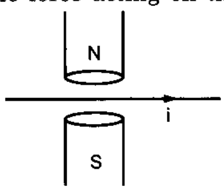

Figure 34-E2

10. A wire of length l carries a current i along the x-axis. A magnetic field exists which is given as $\vec{B} = B_0(\vec{i} + \vec{j} + \vec{k})$ T. Find the magnitude of the magnetic force acting on the wire.

11. A current of 5·0 A exists in the circuit shown in figure (34-E3). The wire PQ has a length of 50 cm and the magnetic field in which it is immersed has a magnitude of 0·20 T. Find the magnetic force acting on the wire PQ.

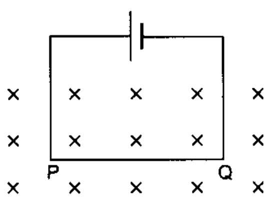

Figure 34-E3

12. A circular loop of radius a, carrying a current i, is placed in a two-dimensional magnetic field. The centre of the loop coincides with the centre of the field (figure 34-E4). The strength of the magnetic field at the periphery of the loop is B. Find the magnetic force on the wire.

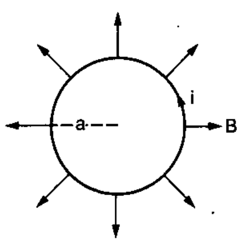

Figure 34-E4

13. A hypothetical magnetic field existing in a region is given by $\vec{B} = B_0\,\vec{e}_r$, where $\vec{e}_r$ denotes the unit vector along the radial direction. A circular loop of radius a, carrying a current i, is placed with its plane parallel to the x–y plane and the centre at (0, 0, d). Find the magnitude of the magnetic force acting on the loop.

14. A rectangular wire-loop of width a is suspended from the insulated pan of a spring balance as shown in figure (34-E5). A current i exists in the anticlockwise direction in the loop. A magnetic field B exists in the lower region. Find the change in the tension of the spring if the current in the loop is reversed.

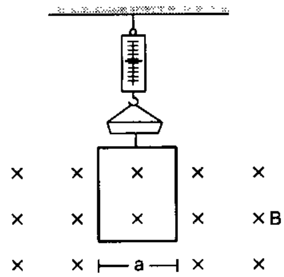

figure 34-E5

15. A current loop of arbitrary shape lies in a uniform magnetic field B. Show that the net magnetic force acting on the loop is zero.

16. Prove that the force acting on a current-carrying wire, joining two fixed points a and b in a uniform magnetic field, is independent of the shape of the wire.

17. A semicircular wire of radius 5·0 cm carries a current of 5·0 A. A magnetic field B of magnitude 0·50 T exists along the perpendicular to the plane of the wire. Find the magnitude of the magnetic force acting on the wire.

18. A wire, carrying a current i, is kept in the x–y plane along the curve $y = A\sin\left(\frac{2\pi}{\lambda}x\right)$. A magnetic field B exists in the z-direction. Find the magnitude of the magnetic force on the portion of the wire between $x = 0$ and $x = \lambda$.

19. A rigid wire consists of a semicircular portion of radius R and two straight sections (figure 34-E6). The wire is partially immersed in a perpendicular magnetic field B as shown in the figure. Find the magnetic force on the wire if it carries a current i.

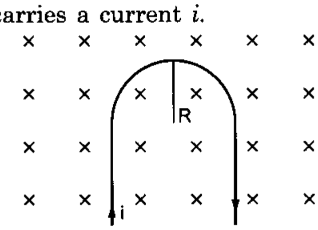

Figure 34-E6

20. A straight horizontal wire of mass 10 mg and length 1·0 m carries a current of 2·0 A. What minimum magnetic field B should be applied in the region so that the magnetic force on the wire may balance its weight ?

21. Figure (34-E7) shows a rod PQ of length 20·0 cm and mass 200 g suspended through a fixed point O by two threads of lengths 20·0 cm each. A magnetic field of strength 0·500 T exists in the vicinity of the wire PQ as shown in the figure. The wires connecting PQ with the battery are loose and exert no force on PQ. (a) Find the tension in the threads when the switch S is open. (b) A current of 2·0 A is established when the switch S is closed. Find the tension in the threads now.

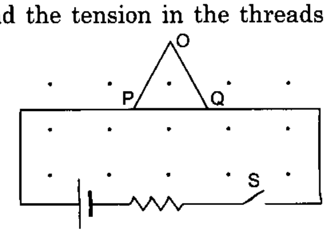

Figure 34-E7

22. Two metal strips, each of length l, are clamped parallel to each other on a horizontal floor with a separation b between them. A wire of mass m lies on them perpendicularly as shown in figure (34-E8). A vertically upward magnetic field of strength B exists in the space. The metal strips are smooth but the coefficient of friction between the wire and the floor is μ. A current i is established when the switch S is closed at the instant $t = 0$. Discuss the motion of the wire after the switch is closed. How far away from the strips will the wire reach ?

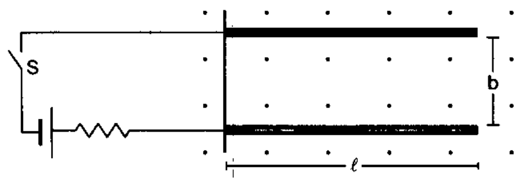

Figure 34-E8

23. A metal wire PQ of mass 10 g lies at rest on two horizontal metal rails separated by 4·90 cm (figure 34-E9). A vertically downward magnetic field of magnitude 0·800 T exists in the space. The resistance of the circuit is slowly decreased and it is found that when the resistance goes below 20·0 Ω, the wire PQ starts sliding on the rails. Find the coefficient of friction.

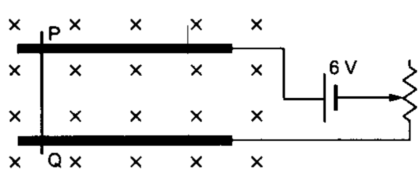

Figure 34-E9

24. A straight wire of length l can slide on two parallel plastic rails kept in a horizontal plane with a separation d. The coefficient of friction between the wire and the rails is μ. If the wire carries a current i, what minimum magnetic field should exist in the space in order to slide the wire on the rails.

25. Figure (34-E10) shows a circular wire-loop of radius a, carrying a current i, placed in a perpendicular magnetic field B. (a) Consider a small part dl of the wire. Find the force on this part of the wire exerted by the magnetic field. (b) Find the force of compression in the wire.

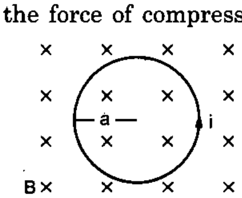

Figure 34-E10

26. Suppose that the radius of cross section of the wire used in the previous problem is r. Find the increase in the radius of the loop if the magnetic field is switched off. The Young modulus of the material of the wire is Y.

27. The magnetic field existing in a region is given by

$$\vec{B} = B_0\left(1 + \frac{x}{l}\right)\vec{k}.$$

A square loop of edge l and carrying a current i, is placed with its edges parallel to the x–y axes. Find the magnitude of the net magnetic force experienced by the loop.

28. A conducting wire of length l, lying normal to a magnetic field B, moves with a velocity v as shown in figure (34-E11). (a) Find the average magnetic force on a free electron of the wire. (b) Due to this magnetic force, electrons concentrate at one end resulting in an electric field inside the wire. The redistribution stops when the electric force on the free electrons balances the magnetic force. Find the electric field developed inside the wire when the redistribution stops. (c) What potential difference is developed between the ends of the wire ?

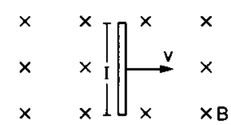

Figure 34-E11

29. A current i is passed through a silver strip of width d and area of cross section A. The number of free electrons per unit volume is n. (a) Find the drift velocity v of the electrons. (b) If a magnetic field B exists in the region as shown in figure (34-E12), what is the average magnetic force on the free electrons ? (c) Due to the magnetic force, the free electrons get accumulated on one side of the conductor along its length. This produces a transverse electric field in the conductor which opposes the magnetic force on the electrons. Find the magnitude of the electric field which will stop further accumulation of electrons. (d) What will be the potential difference developed across the width of the conductor due to the electron-accumulation ? The appearance of a transverse emf, when a current-carrying wire is placed in a magnetic field, is called *Hall effect*.

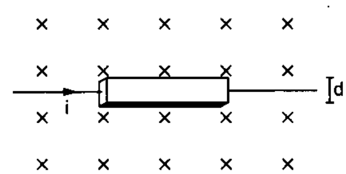

Figure 34-E12

30. A particle having a charge of $2{\cdot}0 \times 10^{-8}$ C and a mass of $2{\cdot}0 \times 10^{-10}$ g is projected with a speed of $2{\cdot}0 \times 10^{3}\ \mathrm{m\ s^{-1}}$ in a region having a uniform magnetic field of 0·10 T. The velocity is perpendicular to the field. Find the radius of the circle formed by the particle and also the time period.

31. A proton describes a circle of radius 1 cm in a magnetic field of strength 0·10 T. What would be the radius of the circle described by an α-particle moving with the same speed in the same magnetic field ?

32. An electron having a kinetic energy of 100 eV circulates in a path of radius 10 cm in a magnetic field. Find the magnetic field and the number of revolutions per second made by the electron.

33. Protons having kinetic energy K emerge from an accelerator as a *narrow beam*. The beam is bent by a perpendicular magnetic field so that it just misses a

plane target kept at a distance l in front of the accelerator. Find the magnetic field.

34. A charged particle is accelerated through a potential difference of 12 kV and acquires a speed of $1 \cdot 0 \times 10^{6}$ m s^{-1}. It is then injected perpendicularly into a magnetic field of strength 0·2 T. Find the radius of the circle described by it.

35. Doubly ionized helium ions are projected with a speed of 10 km s^{-1} in a direction perpendicular to a uniform magnetic field of magnitude 1·0 T. Find (a) the force acting on an ion, (b) the radius of the circle in which it circulates and (c) the time taken by an ion to complete the circle.

36. A proton is projected with a velocity of 3×10^{6} m s^{-1} perpendicular to a uniform magnetic field of 0·6 T. Find the acceleration of the proton.

37. (a) An electron moves along a circle of radius 1 m in a perpendicular magnetic field of strength 0·50 T. What would be its speed? Is it reasonable? (b) If a proton moves along a circle of the same radius in the same magnetic field, what would be its speed?

38. A particle of mass m and positive charge q, moving with a uniform velocity v, enters a magnetic field B as shown in figure (34-E13). (a) Find the radius of the circular arc it describes in the magnetic field. (b) Find the angle subtended by the arc at the centre. (c) How long does the particle stay inside the magnetic field? (d) Solve the three parts of the above problem if the charge q on the particle is negative.

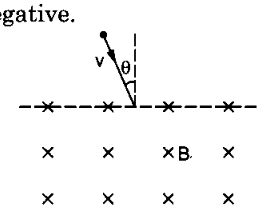

Figure 34-E13

39. A particle of mass m and charge q is projected into a region having a perpendicular magnetic field B. Find the angle of deviation (figure 34-E14) of the particle as it comes out of the magnetic field if the width d of the region is very slightly smaller than

(a) $\frac{mv}{qB}$ (b) $\frac{mv}{2qB}$ (c) $\frac{2mv}{qB}$.

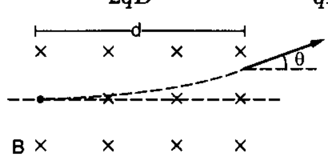

Figure 34-E14

40. A narrow beam of singly-charged carbon ions, moving at a constant velocity of $6 \cdot 0 \times 10^{4}$ m s^{-1}, is sent perpendicularly in a rectangular region having uniform magnetic field $B = 0 \cdot 5$ T (figure 34-E15). It is found that two beams emerge from the field in the backward direction, the separations from the incident beam being 3·0 cm and 3·5 cm. Identify the isotopes present in the ion beam. Take the mass of an ion $= A(1 \cdot 6 \times 10^{-27})$ kg, where A is the mass number.

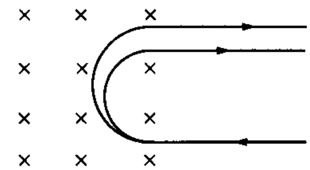

Figure 34-E15

41. Fe^{+} ions are accelerated through a potential difference of 500 V and are injected normally into a homogeneous magnetic field B of strength 20·0 mT. Find the radius of the circular paths followed by the isotopes with mass numbers 57 and 58. Take the mass of an ion $= A$ $(1 \cdot 6 \times 10^{-27})$ kg where A is the mass number.

42. A narrow beam of singly charged potassium ions of kinetic energy 32 keV is injected into a region of width 1·00 cm having a magnetic field of strength 0·500 T as shown in figure (34-E16). The ions are collected at a screen 95·5 cm away from the field region. If the beam contains isotopes of atomic weights 39 and 41, find the separation between the points where these isotopes strike the screen. Take the mass of a potassium ion $= A$ $(1 \cdot 6 \times 10^{-27})$ kg where A is the mass number.

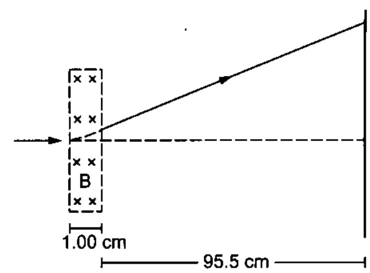

Figure 34-E16

43. Figure (34-E17) shows a convex lens of focal length 12 cm lying in a uniform magnetic field B of magnitude 1·2 T parallel to its principal axis. A particle having a charge $2 \cdot 0 \times 10^{-3}$ C and mass $2 \cdot 0 \times 10^{-5}$ kg is projected perpendicular to the plane of the diagram with a speed of 4·8 m s^{-1}. The particle moves along a circle with its centre on the principal axis at a distance of 18 cm from the lens. Show that the image of the particle goes along a circle and find the radius of that circle.

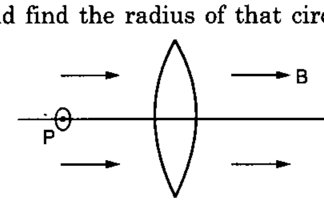

Figure 34-E17

44. Electrons emitted with negligible speed from an electron gun are accelerated through a potential difference V

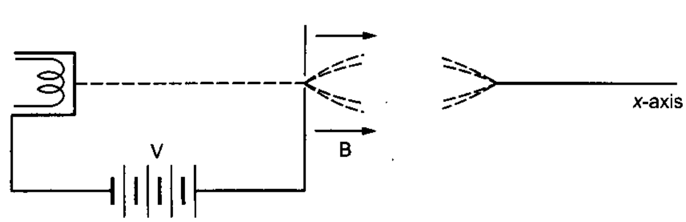

Figure 34-E18

along the x-axis. These electrons emerge from a narrow hole into a uniform magnetic field B directed along this axis. However, some of the electrons emerging from the hole make slightly divergent angles as shown in figure (34-E18). Show that these paraxial electrons are refocussed on the x-axis at a distance

$$\sqrt{\frac{8\pi^2 mV}{eB^2}}.$$

45. Two particles, each having a mass m are placed at a separation d in a uniform magnetic field B as shown in figure (34-E19). They have opposite charges of equal magnitude q. At time $t = 0$, the particles are projected towards each other, each with a speed v. Suppose the Coulomb force between the charges is switched off. (a) Find the maximum value v_m of the projection speed so that the two particles do not collide. (b) What would be the minimum and maximum separation between the particles if $v = v_m/2$? (c) At what instant will a collision occur between the particles if $v = 2v_m$? (d) Suppose $v = 2v_m$ and the collision between the particles is completely inelastic. Describe the motion after the collision.

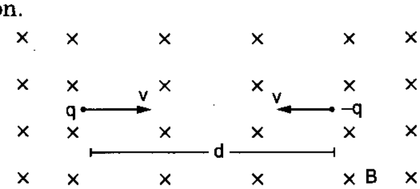

Figure 34-E19

46. A uniform magnetic field of magnitude 0·20 T exists in space from east to west. With what speed should a particle of mass 0·010 g and having a charge $1{\cdot}0 \times 10^{-5}$ C be projected from south to north so that it moves with a uniform velocity ?

47. A particle moves in a circle of diameter 1·0 cm under the action of a magnetic field of 0·40 T. An electric field of 200 V m^{-1} makes the path straight. Find the charge/mass ratio of the particle.

48. A proton goes undeflected in a crossed electric and magnetic field (the fields are perpendicular to each other) at a speed of $2{\cdot}0 \times 10^5$ m s^{-1}. The velocity is perpendicular to both the fields. When the electric field is switched off, the proton moves along a circle of radius 4·0 cm. Find the magnitudes of the electric and the magnetic fields. Take the mass of the proton $= 1{\cdot}6 \times 10^{-27}$ kg.

49. A particle having a charge of 5·0 μC and a mass of $5{\cdot}0 \times 10^{-12}$ kg is projected with a speed of 1·0 km s^{-1} in a magnetic field of magnitude 5·0 mT. The angle between the magnetic field and the velocity is $\sin^{-1}(0{\cdot}90)$. Show that the path of the particle will be a helix. Find the diameter of the helix and its pitch.

50. A proton projected in a magnetic field of 0·020 T travels along a helical path of radius 5·0 cm and pitch 20 cm. Find the components of the velocity of the proton along and perpendicular to the magnetic field. Take the mass of the proton $= 1{\cdot}6 \times 10^{-27}$ kg.

51. A particle having mass m and charge q is released from the origin in a region in which electric field and magnetic field are given by

$$\vec{B} = -B_0\vec{j} \text{ and } \vec{E} = E_0\vec{k}.$$

Find the speed of the particle as a function of its z-coordinate.

52. An electron is emitted with negligible speed from the negative plate of a parallel plate capacitor charged to a potential difference V. The separation between the plates is d and a magnetic field B exists in the space as shown in figure (34-E20). Show that the electron will fail to strike the upper plate if

$$d > \left(\frac{2m_e V}{eB_0^2}\right)^{\frac{1}{2}}.$$

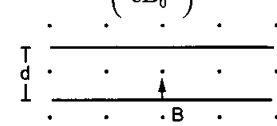

Figure 34-E20

53. A rectangular coil of 100 turns has length 5 cm and width 4 cm. It is placed with its plane parallel to a uniform magnetic field and a current of 2 A is sent through the coil. Find the magnitude of the magnetic field B, if the torque acting on the coil is 0·2 N m^{-1}.

54. A 50-turn circular coil of radius 2·0 cm carrying a current of 5·0 A is rotated in a magnetic field of strength 0·20 T. (a) What is the maximum torque that acts on the coil ? (b) In a particular position of the coil, the torque acting on it is half of this maximum. What is the angle between the magnetic field and the plane of the coil ?

55. A rectangular loop of sides 20 cm and 10 cm carries a current of 5·0 A. A uniform magnetic field of magnitude 0·20 T exists parallel to the longer side of the loop. (a) What is the force acting on the loop ? (b) What is the torque acting on the loop ?

56. A circular coil of radius 2·0 cm has 500 turns in it and carries a current of 1·0 A. Its axis makes an angle of 30° with the uniform magnetic field of magnitude 0·40 T that exists in the space. Find the torque acting on the coil.

57. A circular loop carrying a current i has wire of total length L. A uniform magnetic field B exists parallel to the plane of the loop. (a) Find the torque on the loop. (b) If the same length of the wire is used to form a square loop, what would be the torque ? Which is larger?

58. A square coil of edge l having n turns carries a current i. It is kept on a smooth horizontal plate. A uniform magnetic field B exists in a direction parallel to an edge. The total mass of the coil is M. What should be the minimum value of B for which the coil will start tipping over ?

59. Consider a nonconducting ring of radius r and mass m which has a total charge q distributed uniformly on it. The ring is rotated about its axis with an angular speed ω. (a) Find the equivalent electric current in the ring. (b) Find the magnetic moment μ of the ring. (c) Show that $\mu = \frac{q}{2m} l$ where l is the angular momentum of the ring about its axis of rotation.

60. Consider a nonconducting plate of radius r and mass m which has a charge q distributed uniformly over it. The plate is rotated about its axis with an angular speed ω. Show that the magnetic moment μ and the angular momentum l of the plate are related as $\mu = \frac{q}{2m} l$.

61. Consider a solid sphere of radius r and mass m which has a charge q distributed uniformly over its volume. The sphere is rotated about a diameter with an angular speed ω. Show that the magnetic moment μ and the angular momentum l of the sphere are related as $\mu = \frac{q}{2m} l$.

□

ANSWERS

OBJECTIVE I

1. (d)	2. (d)	3. (d)	4. (a)	5. (d)	6. (a)
7. (c)	8. (c)	9. (d)	10. (d)		

OBJECTIVE II

1. (a), (d)	2. (a), (d)	3. (c), (d)
4. (a), (b), (d)	5. (b)	6. (a), (b)
7. (a), (b)	8. (b), (d)	9. (a), (b)
10. (b), (c)		

EXERCISES

1. 9.6×10^{-12} N towards west
2. (a) left (b) ≈ 1.5 cm
3. $(-75\vec{i} + 100\vec{j})$ m s^{-1}
4. 3·0
5. 3.7×10^{-6} m
6. $\frac{ma_0}{e}$ towards west, $\frac{2ma_0}{ev_0}$ downward
7. 0·08 N perpendicular to both the wire and the field
8. 0·02 N on each wire, on da and cb towards left and on dc and ab downward
9. 0·16 N
10. $\sqrt{2}\, B_0 il$
11. 0·50 N towards the inside of the circuit
12. $2\pi aiB$, perpendicular to the plane of the figure going into it
13. $\frac{2\pi a^2 iB_0}{\sqrt{a^2 + d^2}}$
14. $2iBa$
17. 0·25 N
18. $i\lambda B$
19. $2iRB$, upward in the figure
20. 4.9×10^{-5} T
21. (a) 1·13 N (b) 1·25 N
22. $\frac{ilbB}{\mu mg}$
23. 0·12
24. $\frac{\mu mg}{il}$
25. (a) $idlB$ towards the centre (b) iaB
26. $\frac{ia^2 B}{\pi r^2 Y}$
27. $iB_0 l$
28. (a) evB (b) vB (c) lBv
29. (a) $\frac{i}{Ane}$ (b) $\frac{iB}{An}$ upwards in the figure (c) $\frac{iB}{Ane}$ (d) $\frac{iBd}{Ane}$
30. 20 cm, 6.3×10^{-4} s
31. 2 cm
32. 3.4×10^{-4} T, 9.4×10^{6}
33. $\frac{\sqrt{2m_p K}}{el}$ where m_p = mass of a proton
34. 12 cm
35. (a) 3.2×10^{-15} N (b) 2.1×10^{-4} m (c) 1.31×10^{-7} s
36. 1.72×10^{14} m s^{-2}
37. (a) 8.8×10^{10} m s^{-1} (b) 4.8×10^{7} m s^{-1}
38. (a) $\frac{mv}{qB}$ (b) $\pi - 2\theta$ (c) $\frac{m}{qB}(\pi - 2\theta)$ (d) $\frac{mv}{qB}$, $\pi + 2\theta$, $\frac{m}{qB}(\pi + 2\theta)$
39. (a) $\pi/2$ (b) $\pi/6$ (c) π
40. ^{12}C and ^{14}C
41. 119 cm and 120 cm
42. 0·75 mm
43. 8 cm
45. (a) $\frac{qBd}{2\text{ m}}$ (b) $\frac{d}{2}$, $\frac{3d}{2}$ (c) $\frac{\pi m}{6qB}$ (d) the particles stick together and the combined mass moves with constant speed v_m along the straight line drawn upward in the plane of figure through the point of collision

46. $49\ \mathrm{m\ s^{-1}}$

47. $2{\cdot}5 \times 10^{5}\ \mathrm{C\ kg^{-1}}$

48. $1{\cdot}0 \times 10^{4}\ \mathrm{N\ C^{-1}}$, $0{\cdot}05\ \mathrm{T}$

49. 36 cm, 55 cm

50. $6{\cdot}4 \times 10^{4}\ \mathrm{m\ s^{-1}}$, $1{\cdot}0 \times 10^{5}$

51. $\sqrt{\dfrac{2qE_0 z}{m}}$

53. $0{\cdot}5\ \mathrm{T}$

54. (a) $6{\cdot}3 \times 10^{-2}\ \mathrm{N\ m}$ (b) $60°$

55. (a) zero (b) $0{\cdot}02$ N m parallel to the shorter side.

56. $0{\cdot}13$ N m

57. (a) $\dfrac{iL^2B}{4\pi}$ (b) $\dfrac{iL^2B}{16}$

58. $\dfrac{Mg}{2nil}$

59. (a) $\dfrac{q\omega}{2\pi}$ (b) $\dfrac{q\omega r^2}{2}$

□