Definite Integrals Ex 20.1 Q47

We have,

$$\int_1^2 \left(\frac{x-1}{x^2}\right)e^x dx = \int_1^2 \frac{xe^x}{x^2} - \int_1^2 \frac{e^x}{x^2}dx = \int_1^2 \frac{e^x dx}{x} - \int_1^2 \frac{e^x}{x^2}dx$$

Expanding 1^{st} integral by by parts we get

$$= \frac{1}{x}\int_1^2 e^x dx - \int_1^2 \left(\int e^x . \frac{d\left(1/x\right)}{dx} dx\right) - \int_1^2 \frac{e^x}{x^2}$$

$$= \left[\frac{e^x}{x}\right]_1^2 + \int_1^2 \frac{e^x}{x^2}dx - \int_1^2 \frac{e^x}{x^2}$$

$$= \left[\frac{e^x}{x}\right]_1^2$$

$$= \frac{e^2}{2} - e$$

$$\therefore \int_1^2 \left(\frac{x-1}{x^2}\right)e^x dx = \frac{e^2}{2} - e$$

Definite Integrals Ex 20.1 Q48

We have,

$$\int_0^1 \left(xe^{2x} + \sin\frac{\pi x}{2}\right)dx = \int_0^1 \underset{\text{I}}{x}\,\underset{\text{II}}{e^{2x}}\, dx + \int_0^1 \sin\frac{\pi x}{2}dx$$

Applying by parts in first integral

$$= x\int_0^1 e^{2x}dx - \int_0^1 \left(\int e^{2x}dx.\right)\frac{dx}{dx}dx + \left[\frac{-\cos\frac{\pi x}{2}}{\frac{\pi}{2}}\right]_0^1$$

$$= \frac{xe^{2x}}{2} - \frac{1}{2}\int_0^1 e^{2x}dx + \frac{2}{\pi}[1-0]$$

$$= \frac{xe^{2x}}{2} - \frac{1}{2}\int_0^1 e^{2x}dx + \frac{2}{\pi}[1-0]$$

$$= \left[\frac{xe^{2x}}{2} - \frac{1}{4}e^{2x}\right]_0^1 + \frac{2}{\pi}[1-0]$$

$$= \frac{e^2}{2} - \frac{1}{4}e^2 + \frac{1}{4} + \frac{2}{\pi}[1-0]$$

$$= \frac{e^2}{4} + \frac{2}{\pi} + \frac{1}{4}$$

$$= \frac{e^2}{4} + \frac{1}{4} + \frac{2}{\pi}$$

$$\therefore \int_0^1 \left(xe^{2x} + \sin\frac{\pi x}{2}\right)dx = \frac{e^2}{4} + \frac{1}{4} + \frac{2}{\pi}$$

Definite Integrals Ex 20.1 Q49

We have,

$$\int_0^1\left(xe^x+\cos\frac{\pi x}{4}\right)dx$$

$$=\int_0^1 \underset{\mathrm{I}}{x}\,\underset{\mathrm{II}}{e^x}\,dx+\int_0^1\cos\frac{\pi x}{4}dx$$

Applying by by parts in 1st integral we get,

$$=x\int_0^1 e^x dx-\int_0^1\left(\int e^x dx\right)\frac{dx}{dx}.dx+\int_0^1\cos\frac{\pi x}{4}dx$$

$$=\left[xe^x\right]_0^1-\int_0^1 e^x dx+\left[\frac{\sin\frac{\pi x}{4}}{\frac{\pi}{4}}\right]_0^1$$

$$=\left[xe^x-e^x\right]_0^1+\frac{4}{\pi}\left[\frac{1}{\sqrt{2}}-0\right]$$

$$=\left[e^x(x-1)\right]_0^1+\frac{4}{\pi}\left[\frac{1}{\sqrt{2}}\right]$$

$$=0+1+\frac{4}{\pi\sqrt{2}}$$

$$=1+\frac{2\sqrt{2}}{\pi}$$

$$\therefore\int_0^1\left(xe^x+\cos\frac{\pi x}{4}\right)dx=1+\frac{2\sqrt{2}}{\pi}$$

Definite Integrals Ex 20.1 Q50

$$\int_{\frac{\pi}{2}}^{\pi}e^x\frac{1-\sin x}{1-\cos x}dx=\int_{\frac{\pi}{2}}^{\pi}e^x\frac{1-2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}dx\qquad\left[1-\cos x=2\sin^2\frac{x}{2}\right]$$

$$=-\int_{\frac{\pi}{2}}^{\pi}e^x\left(-\frac{1}{2}\csc^2\frac{x}{2}+\cot\frac{x}{2}\right)dx$$

$$=-e^x\cot\frac{x}{2}\Big|_{\frac{\pi}{2}}^{\pi}\qquad\left[\int e^x\left(F(x)+F'(x)\right)dx=e^xF(x)\right]$$

$$=e^{\frac{\pi}{2}}$$

*********** END **********