Functions Ex 2.1 Q17

Let $f_1 : R \to R$ and $f_2 : R \to R$ be two functions given by:

$$f_1(x) = x$$
$$f_2(x) = -x$$

We can earily verify that $f_1$ and $f_2$ are one-one functions.

Now,

$$(f_1 + f_2)(x) = f_1(x) + f_2(x) = x - x = 0$$

$\therefore$ $f_1 + f_2 : R \to R$ is a function given by

$$(f_1 + f_2)(x) = 0$$

Since $f_1 + f_2$ is a constant function, it is not one-one.

Functions Ex 2.1 Q18

Let $f_1 : Z \to Z$ defined by $f_1(x) = x$ and

$f_2 : Z \to Z$ defined by $f_2(x) = -x$

Then $f_1$ *and* $f_2$ are surjective functions.

Now,

$f_1 + f_2 : Z \to Z$ is given by

$$(f_1 + f_2)(x) = f_1(x) + f_2(x) = x - x = 0$$

Since $f_1 + f_2$ is a constant function, it is not surjective.

Functions Ex 2.1 Q19

Let $f_1 : R \to R$ be defined by $f_1(x) = x$

and $f_2 : R \to R$ be defined by $f_2(x) = x$

clearly $f_1$ *and* $f_2$ are one-one functions.

Now,

$F = f_1 \times f_2 : R \to R$ *is* defined by

$$F(X) = (f_1 \times f_2)(x) = f_1(x) \times f_2(x) = x^2 \quad \ldots\ldots\ldots (i)$$

Clearly, $F(-1) = 1 = F(1)$

$\therefore$ $F$ is not one-one

Hence, $f_1 \times f_2 : R \to R$ is not one-one.

Functions Ex 2.1 Q20

Let $f_1 : R \to R$ and $f_2 : R \to R$ are two functions defined by $f_1(x) = x^3$ and

$$f_2(x) = x$$

clearly $f_1$ & $f_2$ are one-one functions.

Now,

$$\frac{f_1}{f_2} : R \to R \text{ given by}$$

$$\left(\frac{f_1}{f_2}\right)(x) = \frac{f_1(x)}{f_2(x)} = x^2 \text{ for all } x \in R.$$

let $\quad \frac{f_1}{f_2} = f$

$\therefore \quad F = R \to R$ defined by $f(x) = x^2$

now, $\quad F(1) = 1 = F(-1)$

$\therefore \qquad F$ is not one-one

$\therefore \quad \frac{f_1}{f_2} = R \to R$ is not one-one.

Functions Ex 2.1 Q22

We have $f : R \to R$ given by $f(x) = x - [x]$

Now,

check for injectivity:

$\because \ f(x) = x - [x] \ \Rightarrow \ f(x) = 0 \ \text{ for } x \in Z$

$\therefore$ Range of $f = [0, 1] \neq R$

$\therefore$ $f$ is not one-one, where as many-one

Again, Range of $f = [0, 1] \neq R$

$\therefore$ $f$ is an into function

Functions Ex 2.1 23

Suppose $f(n_1) = f(n_2)$

If $n_1$ is odd and $n_2$ is even, then we have

$n_1 + 1 = n_2 - 1 \Rightarrow n_2 - n_1 = 2$, not possible

If $n_1$ is even and $n_2$ is odd, then we have

$n_1 - 1 = n_2 + 1 \Rightarrow n_1 - n_2 = 2$, not possible

Therefore, both $n_1$ and $n_2$ must be either odd or even.

Suppose both $n_1$ and $n_2$ are odd.

Then, $f(n_1) = f(n_2) \Rightarrow n_1 + 1 = n_2 + 1 \Rightarrow n_1 = n_2$

Suppose both $n_1$ and $n_2$ are even.

Then, $f(n_1) = f(n_2) \Rightarrow n_1 - 1 = n_2 - 1 \Rightarrow n_1 = n_2$

Thus, f is one – one.

Also, any odd number $2r + 1$ in the co – domain $\mathbb{N}$ will have an even number as image in domain $\mathbb{N}$ which is

$f(n) = 2r + 1 \Rightarrow n - 1 = 2r + 1 \Rightarrow n = 2r + 2$

any even number $2r$ in the co – domain $\mathbb{N}$ will have an odd number as image in domain $\mathbb{N}$ which is

$f(n) = 2r \Rightarrow n + 1 = 2r \Rightarrow n = 2r - 1$

Thus, f is onto.

*********** END **********