



Permutations Ex 16.3 Q12

We have,

$$\text{LHS} = 1 \cdot P(1, 1) + 2 \cdot P(2, 2) + 3 \cdot P(3, 3) + \dots + n \cdot P(n, n)$$

$$= 1 \cdot 1 + 2 \cdot 2! + 3 \cdot 3! + \dots + n \cdot n! \quad \left[ \because P(n, n) = n! \right]$$

$$= \sum_{r=1}^n r \cdot r!$$

$$= \sum_{r=1}^n [(r+1)r! - r!]$$

$$= \sum_{r=1}^n [(r+1)! - r!] \quad \left[ \because (r+1)r! = (r+1)! \right]$$

$$= [(2! - 1!) + (3! - 2!) + (4! - 3!) + \dots + (n+1)! - n!]$$

$$= (n+1)! - 1!$$

$$= {}^{n+1}P_{n+1} - 1! \quad \left[ \because {}^nP_n = n! \right]$$

$$= P(n+1, n+1) - 1$$

$\Rightarrow$  LHS = RHS

Hence proved.

Permutations Ex 16.3 Q13

We have,

$$P(15, r-1) = P(16, r-2) = 3 : 4$$

$$\Rightarrow \frac{P(15, r-1)}{P(16, r-2)} = \frac{3}{4}$$

$$\Rightarrow \frac{\frac{15!}{[15-(r-1)]!}}{\frac{16!}{[16-(r-2)]!}} = \frac{3}{4}$$

$$\Rightarrow \frac{\frac{15!}{[16-r]!}}{\frac{16!}{[18-r]!}} = \frac{3}{4}$$

$$\Rightarrow \frac{15!}{(16-r)!} \times \frac{(18-r)!}{16!} = \frac{3}{4}$$

$$\Rightarrow \frac{15! \times (18-r)(17-r)(16-r)!}{(16-r)! \times 16 \times 15!} = \frac{3}{4}$$

$$\Rightarrow \frac{(18-r)(17-r)}{16} = \frac{3}{4}$$

$$\Rightarrow 306 - 18r - 17r + r^2 = \frac{3}{4} \times 16$$

$$\Rightarrow r^2 - 35r + 306 = 12$$

$$\Rightarrow r^2 - 35r + 306 - 12 = 0$$

$$\Rightarrow r^2 - 35r + 294 = 0$$

$$\Rightarrow r^2 - 21r - 14r + 294 = 0$$

$$\Rightarrow r(r-21) - 14(r-21) = 0$$

$$\Rightarrow (r-21)(r-14) = 0$$

$$\Rightarrow r-14 = 0$$

$$\Rightarrow r = 14$$

$$[\because r = 21 \neq 0]$$

Hence,  $r = 14$

Permutations Ex 16.3 Q14

We have,

$${}^{n+5}P_{n+1} = \frac{11(n-1)}{2} {}^n P_n$$

$$\Rightarrow \frac{(n+5)!}{[n+5-(n+1)]!} = \frac{11(n-1)}{2} \times \frac{(n+3)!}{[n+3-n]!}$$

$$\Rightarrow \frac{(n+5)!}{[n+5-n-1]!} = \frac{11(n-1)}{2} \times \frac{(n+3)!}{3!}$$

$$\Rightarrow \frac{(n+5)!}{4!} = \frac{11(n-1)}{2} \times \frac{(n+3)!}{3!}$$

$$\Rightarrow \frac{(n+5)(n+4)(n+3)!}{4!} = \frac{11(n-1)}{2} \times \frac{(n+3)!}{3!}$$

$$\Rightarrow \frac{(n+5)(n+4)}{4 \times 3!} = \frac{11(n-1)}{2 \times 3!}$$

$$\Rightarrow (n+5)(n+4) = \frac{11(n-1) \times 4}{2}$$

$$\Rightarrow (n+5)(n+4) = 22(n-1)$$

$$\Rightarrow n^2 + 4n + 5n + 20 = 22n - 22$$

$$\Rightarrow n^2 + 9n - 22n + 20 + 22 = 0$$

$$\Rightarrow n^2 - 13n + 42 = 0$$

$$\Rightarrow n^2 - 6n - 7n + 42 = 0$$

$$\Rightarrow n(n-6) - 7(n-6) = 0$$

$$\Rightarrow n = 6 \quad \text{or,} \quad n = 7$$

Hence,  $n = 6$  or,  $7$

\*\*\*\*\* END \*\*\*\*\*