Co-Ordinate Geometry Ex 14.2 Q37

Answer :

The distance *d* between two points (x_1, y_1) and (x_2, y_2) is given by the formula

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

In a square all the sides are of equal length. The diagonals are also equal to each other. Also in a square the diagonal is equal to $\sqrt{2}$ times the side of the square.

Here let the two points which are said to be the opposite vertices of a diagonal of a square be $A(-1,2)$ and $C(3,2)$.

Let us find the distance between them which is the length of the diagonal of the square.

$$AC = \sqrt{(-1-3)^2 + (2-2)^2}$$
$$= \sqrt{(-4)^2 + (0)^2}$$
$$= \sqrt{16}$$
$$AC = 4$$

Now we know that in a square,

$$\text{Side of the square} = \frac{\text{Diagonal of the square}}{\sqrt{2}}$$

Substituting the value of the diagonal we found out earlier in this equation we have,

$$\text{Side of the square} = \frac{4}{\sqrt{2}}$$
$$\text{Side of the square} = 2\sqrt{2}$$

Now, a vertex of a square has to be at equal distances from each of its adjacent vertices.

Let $P(x, y)$ represent another vertex of the same square adjacent to both '*A*' and '*C*'.

$$AP = \sqrt{(-1-x)^2 + (2-y)^2}$$
$$CP = \sqrt{(3-x)^2 + (2-y)^2}$$

But these two are nothing but the sides of the square and need to be equal to each other.

$$AP = CP$$
$$\sqrt{(-1-x)^2 + (2-y)^2} = \sqrt{(3-x)^2 + (2-y)^2}$$

Squaring on both sides we have,

$$(-1-x)^2 + (2-y)^2 = (3-x)^2 + (2-y)^2$$
$$1 + x^2 + 2x + 4 + y^2 - 4y = 9 + x^2 - 6x + 4 + y^2 - 4y$$
$$8x = 8$$
$$x = 1$$

Substituting this value of '*x*' and the length of the side in the equation for '*AP*' we have,

$$AP = \sqrt{(-1-x)^2 + (2-y)^2}$$
$$2\sqrt{2} = \sqrt{(-1-1)^2 + (2-y)^2}$$
$$2\sqrt{2} = \sqrt{(-2)^2 + (2-y)^2}$$

Squaring on both sides,

$$8 = (-2)^2 + (2-y)^2$$
$$8 = 4 + 4 + y^2 - 4y$$
$$0 = y^2 - 4y$$

We have a quadratic equation. Solving for the roots of the equation we have,

$$y^2 - 4y = 0$$
$$y(y-4) = 0$$

The roots of this equation are 0 and 4.

Therefore the other two vertices of the square are $\boxed{(1,0) \text{ and } (1,4)}$.

*********** END **********