



Trigonometric Ratios of Compound Angles Ex 7.1 Q31

We have,

$$\tan \alpha = x + 1 \text{ and } \tan \beta = x - 1$$

$$\begin{aligned} \text{Now, } 2 \cot(\alpha - \beta) &= \frac{2}{\tan(\alpha - \beta)} \\ &= \frac{2}{\frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}} \\ &= \frac{2(1 + \tan \alpha \tan \beta)}{\tan \alpha - \tan \beta} \\ &= \frac{2[1 + (x + 1)(x - 1)]}{x + 1 - (x - 1)} \\ &= \frac{2[1 + x^2 - 1]}{x + 1 - x + 1} \\ &= \frac{2 \times x^2}{2} = x^2 \end{aligned}$$

$$\therefore 2 \cot(\alpha - \beta) = x^2$$

Hence proved

Trigonometric Ratios of Compound Angles Ex 7.1 Q32

Let the two parts of the angle be θ and $\theta - \phi$.

$$\begin{aligned} \tan(\theta - \phi) &= \lambda \tan \phi \quad [\text{According to question}] \\ \Rightarrow \frac{\tan(\theta - \phi)}{\tan \phi} &= \frac{\lambda}{1} \\ \Rightarrow \frac{\tan(\theta - \phi)}{\tan \phi} &= \frac{\lambda}{1} \\ \Rightarrow \frac{\tan(\theta - \phi) + \tan \phi}{\tan(\theta - \phi) - \tan \phi} &= \frac{\lambda + 1}{\lambda - 1} \quad [\text{Using Componendo and Dividendo}] \\ \Rightarrow \frac{\frac{\tan \theta - \tan \phi}{1 + \tan \theta \tan \phi} + \tan \phi}{\frac{\tan \theta - \tan \phi}{1 + \tan \theta \tan \phi} - \tan \phi} &= \frac{\lambda + 1}{\lambda - 1} \\ \Rightarrow \frac{\frac{\tan \theta - \tan \phi + \tan \phi(1 + \tan \theta \tan \phi)}{1 + \tan \theta \tan \phi}}{\frac{\tan \theta - \tan \phi - \tan \phi(1 + \tan \theta \tan \phi)}{1 + \tan \theta \tan \phi}} &= \frac{\lambda + 1}{\lambda - 1} \\ \Rightarrow \frac{\tan \theta - \tan \phi + \tan \phi + \tan \theta \tan^2 \phi}{\tan \theta - \tan \phi - \tan \phi - \tan \theta \tan^2 \phi} &= \frac{\lambda + 1}{\lambda - 1} \\ \Rightarrow \frac{\tan \theta + \tan \theta \tan^2 \phi}{\tan \theta - 2 \tan \phi - \tan \theta \tan^2 \phi} &= \frac{\lambda + 1}{\lambda - 1} \\ \Rightarrow \tan & \end{aligned}$$

Trigonometric Ratios of Compound Angles Ex 7.1 33

$$\begin{aligned}\tan \theta &= \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha} \\ \Rightarrow \tan \theta &= \frac{\tan \alpha - 1}{\tan \alpha + 1} \text{ [Dividing both Numerator and Denominator by } \cos \alpha \text{]} \\ \Rightarrow \tan \theta &= \frac{\tan \alpha - \tan \frac{\pi}{4}}{1 + \tan \frac{\pi}{4} \cdot \tan \alpha} \\ \Rightarrow \tan \theta &= \tan \left(\alpha - \frac{\pi}{4} \right) \\ \Rightarrow \theta &= \alpha - \frac{\pi}{4} \text{ [Removing } \tan \text{ from both sides]} \\ \Rightarrow \cos \theta &= \cos \left(\alpha - \frac{\pi}{4} \right) \text{ [Taking } \cos \text{ on both sides]} \\ \Rightarrow \cos \theta &= \cos \alpha \cdot \cos \frac{\pi}{4} + \sin \alpha \cdot \sin \frac{\pi}{4} \\ \Rightarrow \cos \theta &= \cos \alpha \cdot \frac{1}{\sqrt{2}} + \sin \alpha \cdot \frac{1}{\sqrt{2}} \\ \Rightarrow \cos \theta &= \frac{\cos \alpha + \sin \alpha}{\sqrt{2}} \\ \Rightarrow \sqrt{2} \cos \theta &= \sin \alpha + \cos \alpha \\ \text{Hence Proved}\end{aligned}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q34

RHS,

$$\begin{aligned}& \frac{p+q}{1-pq} \\ &= \frac{\tan(A+B) + \tan(A-B)}{1 - \tan(A+B) \cdot \tan(A-B)} \\ &= \frac{\frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} + \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}}{1 - \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} \cdot \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}} \\ &= \frac{(\tan A + \tan B)(1 + \tan A \cdot \tan B) + (\tan A - \tan B)(1 - \tan A \cdot \tan B)}{(1 - \tan A \cdot \tan B)(1 + \tan A \cdot \tan B) - (\tan A + \tan B)(\tan A - \tan B)} \\ &= \frac{\tan A + \tan B + \tan^2 A \cdot \tan B + \tan A \cdot \tan^2 B + \tan A - \tan B - \tan^2 A \cdot \tan B + \tan A \cdot \tan^2 B}{1 - \tan^2 A \cdot \tan^2 B - \tan^2 A + \tan^2 B} \\ &= \frac{2 \tan A + 2 \tan A \cdot \tan^2 B}{(1 - \tan^2 A)(1 + \tan^2 B)} = \frac{2 \tan A(1 + \tan^2 B)}{(1 - \tan^2 A)(1 + \tan^2 B)} = \frac{2 \tan A}{1 - \tan^2 A} = \tan 2A = LHS\end{aligned}$$

Hence Proved

***** END *****