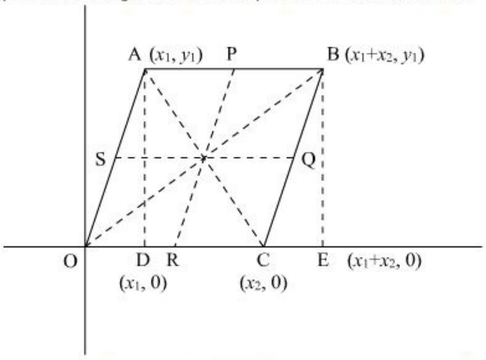
Co-Ordinate Geometry Ex 14.4 Q4

Answer :

Let us consider a Cartesian plane having a parallelogram OABC in which O is the origin.
We have to prove that middle point of the opposite sides of a quadrilateral and the join of the mid-points of its diagonals meet in a point and bisect each other.



Let the co-ordinate of A be (x_1, y_1). So the coordinates of other vertices of the quadrilateral are- O (0, 0); B$(x_1 + x_2, y_1)$; C$(x_2, 0)$

Let P, Q, R and S be the mid-points of the sides AB, BC, CD, DA respectively.

In general to find the mid-point $P(x, y)$ of two points $A(x_1, y_1)$ and $B(x_2, y_2)$ we use section formula as,

$$P(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

So co-ordinate of point P,

$$=\left(\frac{x_1+x_2+x_1}{2},\frac{y_1+y_1}{2}\right)$$

$$=\left(\frac{2x_1+x_2}{2},y_1\right)$$

Similarly co-ordinate of point Q,

$$=\left(\frac{x_1+x_2+x_2}{2},\frac{y_1}{2}\right)$$

$$=\left(\frac{2x_2+x_1}{2},\frac{y_1}{2}\right)$$

Similarly co-ordinate of point R,

$$=\left(\frac{x_2}{2},0\right)$$

Similarly co-ordinate of point S,

$$=\left(\frac{x_1}{2},\frac{y_1}{2}\right)$$

Let us find the co-ordinates of mid-point of PR as,

$$=\left(\frac{\frac{2x_1+x_2}{2}+\frac{x_2}{2}}{2},\frac{y_1}{2}\right)$$

$$=\left(\frac{x_1+x_2}{2},\frac{y_1}{2}\right)$$

Similarly co-ordinates of mid-point of QS as,

$$=\left(\frac{x_1+x_2}{2},\frac{y_1}{2}\right)$$

Now the mid-point of diagonal AC,

$$=\left(\frac{x_1+x_2}{2},\frac{y_1}{2}\right)$$

Similarly the mid–point of diagonal OA,

$$=\left(\frac{x_1+x_2}{2},\frac{y_1}{2}\right)$$

Hence the mid-points of PR, QS, AC and OA coincide.

Thus, middle point of the opposite sides of a quadrilateral and the join of the mid-points of its diagonals meet in a point and bisect each other.

Co-Ordinate Geometry Ex 14.4 Q5

Answer :

Let ΔABC be any triangle whose coordinates are $A(x_1, y_1); B(x_2, y_2); C(x_3, y_3)$. Let P be the origin and G be the centroid of the triangle.

We have to prove that,

$PA^2 + PB^2 + PC^2 = GA^2 + GB^2 + GC^2 + 3GP^2$ ...... (1)

We know that the co-ordinates of the centroid G of a triangle whose vertices are $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is-

$$G\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

In general, the distance between $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by,

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

So,

$$\begin{aligned} PA^2 &= (x_1 - 0)^2 + (y_1 - 0)^2 \\ &= x_1^2 + y_1^2 \\ PB^2 &= (x_2 - 0)^2 + (y_2 - 0)^2 \\ &= x_2^2 + y_2^2 \\ PC^2 &= (x_3 - 0)^2 + (y_3 - 0)^2 \\ &= x_3^2 + y_3^2 \end{aligned}$$

Now,

$$\begin{aligned} GP^2 &= \left(\frac{x_1 + x_2 + x_3}{3} - 0\right)^2 + \left(\frac{y_1 + y_2 + y_3}{3} - 0\right)^2 \\ &= \frac{(x_1 + x_2 + x_3)^2}{9} + \frac{(y_1 + y_2 + y_3)^2}{9} \\ GA^2 &= \left(x_1 - \frac{x_1 + x_2 + x_3}{3}\right)^2 + \left(y_1 - \frac{y_1 + y_2 + y_3}{3}\right)^2 \\ &= \frac{(2x_1 - x_2 - x_3)^2}{9} + \frac{(2y_1 - y_2 - y_3)^2}{9} \\ GB^2 &= \left(x_2 - \frac{x_1 + x_2 + x_3}{3}\right)^2 + \left(y_2 - \frac{y_1 + y_2 + y_3}{3}\right)^2 \\ &= \frac{(2x_2 - x_1 - x_3)^2}{9} + \frac{(2y_2 - y_1 - y_3)^2}{9} \\ GC^2 &= \left(x_3 - \frac{x_1 + x_2 + x_3}{3}\right)^2 + \left(y_3 - \frac{y_1 + y_2 + y_3}{3}\right)^2 \\ &= \frac{(2x_3 - x_1 - x_2)^2}{9} + \frac{(2y_3 - y_1 - y_2)^2}{9} \end{aligned}$$

So we get the value of left hand side of equation (1) as,

$$PA^2 + PB^2 + PC^2 = x_1^2 + x_2^2 + x_3^2 + y_1^2 + y_2^2 + y_3^2$$

Similarly we get the value of right hand side of equation (1) as,

$$\begin{aligned} GA^2 + GB^2 + GC^2 + 3GP^2 &= \left[\frac{(2x_1 - x_2 - x_3)^2}{9} + \frac{(2y_1 - y_2 - y_3)^2}{9}\right] + \left[\frac{(2x_2 - x_1 - x_3)^2}{9} + \frac{(2y_2 - y_1 - y_3)^2}{9}\right] \\ &\quad + \left[\frac{(2x_3 - x_1 - x_2)^2}{9} + \frac{(2y_3 - y_1 - y_2)^2}{9}\right] + 3\left[\frac{(x_1 + x_2 + x_3)^2}{9} + \frac{(y_1 + y_2 + y_3)^2}{9}\right] \\ &= \left[\frac{2}{3}(x_1^2 + x_2^2 + x_3^2) + \frac{1}{3}(x_1^2 + x_2^2 + x_3^2)\right] + \left[\frac{2}{3}(y_1^2 + y_2^2 + y_3^2) + \frac{1}{3}(y_1^2 + y_2^2 + y_3^2)\right] \\ &= x_1^2 + x_2^2 + x_3^2 + y_1^2 + y_2^2 + y_3^2 \end{aligned}$$

Hence,

$$PA^2 + PB^2 + PC^2 = GA^2 + GB^2 + GC^2 + 3GP^2$$

*********** END **********