Chapter 6 Determinants Ex 6.2 Q33

$$\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$$

$$LHS = \begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix}$$

Multiply R_1, R_2 and R_3 by a, b and c respectively.

$$= \frac{1}{abc}\begin{vmatrix} ab^2+ac^2 & a^2b & a^2c \\ b^2a & bc^2+ba^2 & b^2c \\ c^2a & c^2b & ca^2+cb^2 \end{vmatrix}$$

Take a, b and c common from C_1, C_2 and C_3 respectively.

$$= \frac{abc}{abc}\begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

Now apply $R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} 2\left(b^2+c^2\right) & 2\left(c^2+a^2\right) & 2\left(a^2+b^2\right) \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

$$= 2\begin{vmatrix} \left(b^2+c^2\right) & \left(c^2+a^2\right) & \left(a^2+b^2\right) \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

$$= 2\begin{vmatrix} c^2 & 0 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

$$= 2\left[c^2\left\{\left(c^2+a^2\right)\left(a^2+b^2\right) - b^2c^2\right\} + a^2\left\{b^2c^2 - \left(c^2+a^2\right)c^2\right\}\right]$$

$$= 4a^2b^2c^2$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q34

$$\begin{vmatrix} 0 & b^2a & c^2a \\ a^2b & 0 & c^2b \\ a^2c & b^2c & 0 \end{vmatrix} = 2a^3b^3c^3$$

$$\text{LHS} = \begin{vmatrix} 0 & b^2a & c^2a \\ a^2b & 0 & c^2b \\ a^2c & b^2c & 0 \end{vmatrix}$$

$$= a^2b^2c^2 \begin{vmatrix} 0 & a & a \\ b & 0 & b \\ c & c & 0 \end{vmatrix}$$

$$= a^3b^3c^3 \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= a^3b^3c^3 \begin{vmatrix} 0 & 0 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= 2a^3b^3c^3$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q35

$$\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{c^2+a^2}{b} \end{vmatrix}$$

$$= \frac{1}{abc} \begin{vmatrix} a^2+b^2 & c^2 & c^2 \\ a^2 & c^2+b^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$$

$$= \frac{1}{abc} \begin{vmatrix} 0 & -2b^2 & -2a^2 \\ a^2 & c^2+b^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$$

$$= \frac{-2}{abc} \begin{vmatrix} 0 & b^2 & a^2 \\ a^2 & c^2+b^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix}$$

$$= \frac{-2}{abc} \begin{vmatrix} 0 & b^2 & a^2 \\ a^2 & c^2+b^2 & a^2 \\ b^2 & 0 & c^2 \end{vmatrix}$$

$$= \frac{-2}{abc} \begin{vmatrix} 0 & b^2 & a^2 \\ a^2 & c^2 & 0 \\ b^2 & 0 & c^2 \end{vmatrix}$$

$$= \frac{-2}{abc}\left[\left(-a^2\right)\left(b^2c^2\right) + \left(b^2\right)\left(-a^2c^2\right)\right]$$

$$= \frac{-2}{abc}\left(-2a^2b^2c^2\right)$$

$$= 4abc$$

$$= RHS$$

Chapter 6 Determinants Ex 6.2 Q36

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}$$

Multiply R_1, R_2 and R_3 by a, b and c respectively

$$= \frac{1}{abc}\begin{vmatrix} -abc & ab^2+abc & ac^2+abc \\ a^2b+abc & -abc & bc^2+abc \\ a^2c+abc & b^2c+abc & -abc \end{vmatrix}$$

Take a, b and c common from C_1, C_2 and C_3 respectively.

$$= \frac{abc}{abc}\begin{vmatrix} -bc & ab+ac & ac+ab \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

Apply: $R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} ab+bc+ca & ab+bc+ca & ab+bc+ca \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

$$= (ab+bc+ca)\begin{vmatrix} 1 & 1 & 1 \\ ab+bc & -ac & bc+ab \\ ac+bc & bc+ac & -ab \end{vmatrix}$$

$$= (ab+bc+ca)\begin{vmatrix} 0 & 1 & 0 \\ ab+bc+ac & -ac & bc+ab+ac \\ 0 & bc+ac & -ab-bc-ac \end{vmatrix}$$

$$= (ab+bc+ca)^3\begin{vmatrix} 0 & 1 & 0 \\ 1 & -ac & 1 \\ 0 & bc+ac & -1 \end{vmatrix}$$

$$= (ab+bc+ca)^3$$

$$= RHS$$

*********** END **********