Indefinite Integrals Ex 19.9 Q60

Let $\quad I = \int \frac{e^{2x}}{1+e^x} dx \text{ - - - -}(i)$

Let $\quad 1+e^x = t \qquad$ then,

$$d\left(1+e^x\right) = dt$$

$$\Rightarrow \quad e^x \, dx = dt$$

$$\Rightarrow \quad dx = \frac{dt}{e^x}$$

Putting $1+e^x = t$ and $dx = \frac{dt}{e^x}$ in equation $(i)$,

we get

$$I = \int \frac{e^{2x}}{t} \times \frac{dt}{e^x}$$

$$= \int \frac{e^x}{t} dt$$

$$= \int \frac{t-1}{t} dt$$

$$= \int \left(\frac{t}{t} - \frac{1}{t}\right) dt$$

$$= t - \log|t| + c$$

$$= \left(1+e^x\right) - \log\left|1+e^x\right| + c$$

$$\therefore \quad I = 1 + e^x - \log\left|1+e^x\right| + c$$

Indefinite Integrals Ex 19.9 Q61

Let $\quad I = \int \frac{\sec^2 \sqrt{x}}{\sqrt{x}} dx$ ---- (i)

Let $\quad \sqrt{x} = t$ then,

$d\left(\sqrt{x}\right) = dt$

$\Rightarrow \quad \frac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow \quad dx = 2\sqrt{x}\, dt$

$\Rightarrow \quad dx = 2t\,dt \quad \left[\because \quad \sqrt{x} = t\right]$

Putting $\sqrt{x} = t$ and $dx = 2t\,dt$ in equation (i), we get

$$I = \int \frac{\sec^2 t}{t} \times 2t\, dt$$
$$= 2\int \sec^2 t\, dt$$
$$= 2 \tan t + c$$
$$= 2 \tan \sqrt{x} + c$$

$\therefore \quad I = 2 \tan \sqrt{x} + c$

Indefinite Integrals Ex 19.9 Q62

$$\tan^3 2x \sec 2x = \tan^2 2x \tan 2x \sec 2x$$
$$= \left(\sec^2 2x - 1\right) \tan 2x \sec 2x$$
$$= \sec^2 2x \cdot \tan 2x \sec 2x - \tan 2x \sec 2x$$
$$\therefore \int \tan^3 2x \sec 2x\, dx = \int \sec^2 2x \tan 2x \sec 2x\, dx - \int \tan 2x \sec 2x\, dx$$
$$= \int \sec^2 2x \tan 2x \sec 2x\, dx - \frac{\sec 2x}{2} + C$$

Let $\sec 2x = t$

$\therefore 2 \sec 2x \tan 2x\, dx = dt$

$$\therefore \int \tan^3 2x \sec 2x\, dx = \frac{1}{2} \int t^2 dt - \frac{\sec 2x}{2} + C$$
$$= \frac{t^3}{6} - \frac{\sec 2x}{2} + C$$
$$= \frac{(\sec 2x)^3}{6} - \frac{\sec 2x}{2} + C$$

Indefinite Integrals Ex 19.9 Q63

Let $\quad I = \int \frac{x+\sqrt{x+1}}{x+2}\,dx \text{-----}(i)$

Let $\quad x+1 = t^2 \qquad$ then,

$d(x+1) = d(t^2)$

$\Rightarrow \qquad dx = 2t\,dt$

Putting $x+1=t^2$ and $dx = 2t\,dt$ in equation $(i)$, we get

$$
\begin{aligned}
I &= \int \frac{x+\sqrt{t^2}}{x+2} 2t\,dt \\
&= 2\int \frac{(t^2-1)+t}{(t^2-1)+2} t\,dt \qquad [\because \quad x+1=t^2] \\
&= 2\int \frac{t^2+t-1}{t^2+1} t\,dt \\
&= 2\int \frac{t^3+t^2-t}{t^2+1} dt \\
&= 2\left[\int \frac{t^3}{t^2+1}dt + \int \frac{t^2}{t^2+1}dt - \int \frac{t}{t^2+1}dt\right]
\end{aligned}
$$

$\therefore \qquad I = 2\left[\int \frac{t^3}{t^2+1}dt + \int \frac{t^2}{t^2+1}dt - \int \frac{t}{t^2+1}dt\right] \text{----}(ii)$

Let $I_1 = \int \frac{t^3}{t^2+1}dt$

$I_2 = \int \frac{t^2}{t^2+1}dt$

and $\quad I_3 = \int \frac{t}{t^2+1}dt$

Now, $\quad I_1 = \int \frac{t^3}{t^2+1}dt$

$= \int \left(t - \frac{t}{t^2+1}\right)dt$

$= \frac{t^2}{2} - \frac{1}{2}\log(t^2+1)$

$\therefore \qquad I = \frac{t^2}{2} - \frac{1}{2}\log(t^2+1) + c_1 \text{----}(iii)$

Since, $\quad I_2 = \int \frac{t^2}{t^2+1}dt$

$= \int \frac{t^2+1-1}{t^2+1}dt$

$= \int \frac{t^2+1}{t^2+1}dt - \int \frac{1}{t^2+1}dt$

$= \int dt - \int \frac{1}{t^2+1}dt$

$\Rightarrow \qquad I_2 = t - \tan^{-1}(t^2) + c_2 \text{----}(iv)$

and, $\quad I_3 = \int \frac{t}{t^2+1} dt$

$= \frac{1}{2}\log\left(1+t^2\right) + c_3 \text{ - - - - - - } (v)$

Using equations $(ii), (iii), (iv)$ and $(v)$, we get

$$I = 2\left[\frac{t^2}{2} - \frac{1}{2}\log\left(t^2+1\right) + c_1 + t - \tan^{-1}\left(t^2\right) + c_2 - \frac{1}{2}\log\left(1+t^2\right) + c_3\right]$$

$$= 2\left[\frac{t^2}{2} + t - \tan^{-1}\left(t^2\right) - \log\left(1+t^2\right) + c_1 + c_2 + c_3\right]$$

$$= 2\left[\frac{t^2}{2} + t - \tan^{-1}\left(t^2\right) - \log\left(1+t^2\right) + c_4\right] \qquad \left[\text{Putting } c_1 + c_2 + c_3 = c_4\right]$$

$$= t^2 + 2t - 2\tan^{-1}\left(t^2\right) - 2\log\left(1+t^2\right) + 2c_4$$

$$= \left(x+1\right) + 2\sqrt{x+1} - 2\tan^{-1}\left(\sqrt{x+1}\right) - 2\log\left(1+x+1\right) + 2c_4$$

$$= \left(x+1\right) + 2\sqrt{x+1} - 2\tan^{-1}\left(\sqrt{x+1}\right) - 2\log\left(x+2\right) + c \qquad \left[\text{Putting } 2c_4 = c\right]$$

$$\therefore \quad I = \left(x+1\right) + 2\sqrt{x+1} - 2\tan^{-1}\left(\sqrt{x+1}\right) - 2\log\left(x+2\right) + c$$

*********** END **********