Differentiation Ex 11.4 Q16

Given,

$$x\sqrt{1+y} + y\sqrt{1+x} = 0$$

$$\Rightarrow \quad x\sqrt{1+y} = -y\sqrt{1+x}$$

Squaring both the sides,

$$\Rightarrow \quad \left(x\sqrt{1+y}\right)^2 = \left(-y\sqrt{1+x}\right)^2$$

$$\Rightarrow \quad x^2(1+y) = y^2(1+x)$$

$$\Rightarrow \quad x^2 + x^2y = y^2 + y^2x$$

$$\Rightarrow \quad x^2 - y^2 = y^2x - x^2y$$

$$\Rightarrow \quad (x-y)(x+y) = xy(y-x)$$

$$\Rightarrow \quad (x+y) = -xy$$

$$\Rightarrow \quad y + xy = -x$$

$$\Rightarrow \quad y(1+x) = -x$$

$$\Rightarrow \quad y = \frac{-x}{(1+x)}$$

Differentiating with respect to x using quotient rule,

$$\Rightarrow \quad \frac{dy}{dx} = \left[\frac{-(1+x)\frac{d}{dx}(x) + (-x)\frac{d}{dx}(x+1)}{(1+x)^2}\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = \left[\frac{-(1+x)(1) + x(1)}{(1+x)^2}\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = \left[\frac{-1-x+x}{(1+x)^2}\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-1}{(1+x)^2}$$

$$\Rightarrow \quad (1+x)^2\frac{dy}{dx} = -1$$

$$\Rightarrow \quad (1+x)^2\frac{dy}{dx} + 1 = 0$$

Differentiation Ex 11.4 Q17

Here,

$$\log\sqrt{x^2+y^2} = \tan^{-1}\left(\frac{x}{y}\right)$$

$$\Rightarrow \quad \log\left(x^2+y^2\right)^{\frac{1}{2}} = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\Rightarrow \quad \frac{1}{2}\log\left(x^2+y^2\right) = \tan^{-1}\left(\frac{y}{x}\right)$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{1}{2}\frac{d}{dx}\log\left(x^2+y^2\right) = \frac{d}{dx}\tan^{-1}\left(\frac{y}{x}\right)$$

$$\Rightarrow \quad \frac{1}{2}\times\left(\frac{1}{x^2+y^2}\right)\frac{d}{dx}\left(x^2+y^2\right) = \frac{1}{1+\left(\frac{y}{x}\right)^2}\frac{d}{dx}\left(\frac{y}{x}\right) \qquad \text{[Using chain rule, quotient rule]}$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{1}{x^2+y^2}\right)\left[2x+2y\frac{dy}{dx}\right] = \frac{x^2}{\left(x^2+y^2\right)}\left[\frac{x\frac{dy}{dx}-y\frac{d}{dx}(x)}{x^2}\right]$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{1}{x^2+y^2}\right)\times 2\left(x+y\frac{dy}{dx}\right) = \frac{x^2}{\left(x^2+y^2\right)}\left[\frac{x\frac{dy}{dx}-y(1)}{x^2}\right]$$

$$\Rightarrow \quad x+y\frac{dy}{dx} = x\frac{dy}{dx}-y$$

$$\Rightarrow \quad y\frac{dy}{dx}-x\frac{dy}{dx} = -y-x$$

$$\Rightarrow \quad \frac{dy}{dx}(y-x) = -(y+x)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-(y+x)}{y-x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x+y}{x-y}$$

Differentiation Ex 11.4 Q18

Here,

$$\sec\left(\frac{x+y}{x-y}\right) = a$$

$$\Rightarrow \quad \frac{x+y}{x-y} = \sec^{-1}(a)$$

Differentiating with respect to x,

$$\Rightarrow \quad \left[\frac{(x-y)\frac{d}{dx}(x+y)-(x+y)\frac{d}{dx}(x-y)}{(x-y)^2}\right] = 0 \qquad \text{[Using quotient rule]}$$

$$\Rightarrow \quad (x-y)\left(1+\frac{dy}{dx}\right)-(x+y)\left(1-\frac{dy}{dx}\right) = 0$$

$$\Rightarrow \quad (x-y)+(x-y)\frac{dy}{dx}-(x+y)+(x+y)\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx}[x-y+x+y] = x+y-x+y$$

$$\Rightarrow \quad \frac{dy}{dx}(2x) = 2y$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x}$$

Differentiation Ex 11.4 Q19

Here,

$$\tan^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = a$$

$$\Rightarrow \quad \frac{x^2-y^2}{x^2+y^2} = \tan a$$

$$\Rightarrow \quad x^2-y^2 = \tan a\left(x^2+y^2\right)$$

Differentiating with respect to x,

$$\Rightarrow \quad \frac{d}{dx}\left(x^2-y^2\right) = \tan a\frac{d}{dx}\left(x^2+y^2\right)$$

$$\Rightarrow \quad \left(2x-2y\frac{dy}{dx}\right) = \tan a\left(2x+2y\frac{dy}{dx}\right)$$

$$\Rightarrow \quad 2x-2y\frac{dy}{dx} = 2x\tan a+2t\tan a\frac{dy}{dx}$$

$$\Rightarrow \quad 2y\tan a\frac{dy}{dx}+2y\frac{dy}{dx} = 2x-2x\tan a$$

$$\Rightarrow \quad 2y\frac{dy}{dx}(1+\tan a) = 2x(1-\tan a)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x}{y}\left(\frac{1-\tan a}{1+\tan a}\right)$$

Differentiation Ex 11.4 Q20

Here,

$$xy\log(x+y)=1$$

Differentiating it with respect to x,

$$\Rightarrow \quad \frac{d}{dx}\left[xy\log(x+y)\right]=\frac{d}{dx}(1)$$

$$\Rightarrow \quad xy\frac{d}{dx}\log(x+y)+x\log(x+y)\frac{dy}{dx}+y\log(x+y)\frac{d}{dx}(x)=0$$

[Using chain rule and product rule]

$$\Rightarrow \quad xy\times\left(\frac{1}{x+y}\right)\frac{d}{dx}(x+y)+x\log(x+y)\frac{dy}{dx}+y\log(x+y)(1)=0$$

$$\Rightarrow \quad \left(\frac{xy}{x+y}\right)\left(1+\frac{dy}{dx}\right)+x\log(x+y)\frac{dy}{dx}+y\log(x+y)=0$$

$$\Rightarrow \quad \left(\frac{xy}{x+y}\right)\frac{dy}{dx}+\left(\frac{xy}{x+y}\right)+x\left(\frac{1}{xy}\right)\frac{dy}{dx}+y\left(\frac{1}{xy}\right)=0$$

$$\left[\text{Sicne from equation (i) } \log(x+y)=\frac{1}{xy}\right]$$

$$\Rightarrow \quad \frac{dy}{dx}\left[\frac{xy}{x+y}+\frac{1}{y}\right]=-\left[\frac{1}{x}+\frac{xy}{x+y}\right]$$

$$\frac{dy}{dx}\left[\frac{xy^2+x+y}{(x+y)y}\right]=-\left[\frac{x+y+x^2y}{x(x+y)}\right]$$

$$\frac{dy}{dx}=-\left(\frac{x+y+x^2y}{x(x+y)}\right)\left(\frac{y(x+y)}{xy^2+x+y}\right)$$

$$=-\frac{y}{x}\left(\frac{x+y+x^2y}{x+y+xy^2}\right)$$

So,

$$\frac{dy}{dx}=-\frac{y}{x}\left(\frac{x^2y+x+y}{xy^2+x+y}\right)$$

*********** END **********