



Definite Integrals Ex 20.5 Q28

$$\text{Let } I = \int_0^5 (x+1) dx$$

It is known that,

$$\int_a^b f(x) dx = (b-a) \lim_{n \rightarrow \infty} \frac{1}{n} \left[f(a) + f(a+h) + \dots + f\left(a + (n-1)h\right) \right], \text{ where } h = \frac{b-a}{n}$$

Here, $a = 0$, $b = 5$, and $f(x) = (x+1)$

$$\Rightarrow h = \frac{5-0}{n} = \frac{5}{n}$$

$$\begin{aligned} \therefore \int_0^5 (x+1) dx &= (5-0) \lim_{n \rightarrow \infty} \frac{1}{n} \left[f(0) + f\left(\frac{5}{n}\right) + \dots + f\left((n-1)\frac{5}{n}\right) \right] \\ &= 5 \lim_{n \rightarrow \infty} \frac{1}{n} \left[1 + \left(\frac{5}{n} + 1\right) + \dots + \left\{ 1 + \left(\frac{5(n-1)}{n}\right) \right\} \right] \\ &= 5 \lim_{n \rightarrow \infty} \frac{1}{n} \left[\left(1 + 1 + 1 \dots 1\right) + \left[\frac{5}{n} + 2 \cdot \frac{5}{n} + 3 \cdot \frac{5}{n} + \dots + (n-1) \frac{5}{n} \right] \right] \\ &= 5 \lim_{n \rightarrow \infty} \frac{1}{n} \left[n + \frac{5}{n} \{1 + 2 + 3 \dots (n-1)\} \right] \\ &= 5 \lim_{n \rightarrow \infty} \frac{1}{n} \left[n + \frac{5}{n} \cdot \frac{(n-1)n}{2} \right] \\ &= 5 \lim_{n \rightarrow \infty} \frac{1}{n} \left[n + \frac{5(n-1)}{2} \right] \\ &= 5 \lim_{n \rightarrow \infty} \left[1 + \frac{5}{2} \left(1 - \frac{1}{n}\right) \right] \\ &= 5 \left[1 + \frac{5}{2} \right] \\ &= 5 \left[\frac{7}{2} \right] \\ &= \frac{35}{2} \end{aligned}$$

Definite Integrals Ex 20.5 Q29

It is known that,

$$\int_a^b f(x) dx = (b-a) \lim_{n \rightarrow \infty} \frac{1}{n} [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)], \text{ where } h = \frac{b-a}{n}$$

Here, $a = 2$, $b = 3$, and $f(x) = x^2$

$$\Rightarrow h = \frac{3-2}{n} = \frac{1}{n}$$

$$\begin{aligned} \therefore \int_2^3 x^2 dx &= (3-2) \lim_{n \rightarrow \infty} \frac{1}{n} \left[f(2) + f\left(2 + \frac{1}{n}\right) + f\left(2 + \frac{2}{n}\right) + \dots + f\left(2 + (n-1)\frac{1}{n}\right) \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \left[(2)^2 + \left(2 + \frac{1}{n}\right)^2 + \left(2 + \frac{2}{n}\right)^2 + \dots + \left(2 + \frac{(n-1)}{n}\right)^2 \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \left[2^2 + \left\{ 2^2 + \left(\frac{1}{n}\right)^2 + 2 \cdot 2 \cdot \frac{1}{n} \right\} + \dots + \left\{ (2)^2 + \frac{(n-1)^2}{n^2} + 2 \cdot 2 \cdot \frac{(n-1)}{n} \right\} \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \left[\left(2^2 + \dots + 2^2 \right) + \left\{ \left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \dots + \left(\frac{n-1}{n}\right)^2 \right\} + 2 \cdot 2 \cdot \left\{ \frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \dots + \frac{(n-1)}{n} \right\} \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \left[4n + \frac{1}{n^2} \{ 1^2 + 2^2 + 3^2 + \dots + (n-1)^2 \} + \frac{4}{n} \{ 1 + 2 + \dots + (n-1) \} \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \left[4n + \frac{1}{n^2} \left\{ \frac{n(n-1)(2n-1)}{6} \right\} + \frac{4}{n} \left\{ \frac{n(n-1)}{2} \right\} \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \left[4n + \frac{n \left(1 - \frac{1}{n}\right) \left(2 - \frac{1}{n}\right)}{6} + \frac{4n-4}{2} \right] \\ &= \lim_{n \rightarrow \infty} \left[4 + \frac{1}{6} \left(1 - \frac{1}{n}\right) \left(2 - \frac{1}{n}\right) + 2 - \frac{2}{n} \right] \\ &= 4 + \frac{2}{6} + 2 \\ &= \frac{19}{3} \end{aligned}$$

Definite Integrals Ex 20.5 Q30

We have

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)]$$

$$\text{Where } h = \frac{b-a}{n}$$

Here

$$a = 1, b = 3 \text{ and } f(x) = x^2 + x$$

Now

$$h = \frac{2}{n}$$

$$nh = 2$$

Thus, we have

$$\begin{aligned} I &= \int_1^3 (x^2 + x) dx \\ &= \lim_{h \rightarrow 0} h [f(1) + f(1+h) + f(1+2h) + \dots + f(1+(n-1)h)] \\ &= \lim_{h \rightarrow 0} h \left[(1^2 + 1) + \{ (1+h)^2 + (1+h) \} + \{ (1+2h)^2 + (1+2h) \} + \dots \right] \\ &= \lim_{h \rightarrow 0} h \left[(1^2 + (1+h)^2 + (1+2h)^2 + \dots) + \{ 1 + (1+h) + (1+2h) + \dots \} \right] \\ &= \lim_{h \rightarrow 0} h \left[(n + 2h(1+2+3+\dots) + h^2(1+2^2+3^2+\dots)) + (n + h(1+2+3+\dots)) \right] \\ &= \lim_{h \rightarrow 0} h \left[(2n + 3h(1+2+3+\dots + (n-1))) + h^2(1+2^2+3^2+\dots + (n-1)^2) \right] \\ \therefore h &= \frac{2}{n} \text{ \& if } h \rightarrow 0 \Rightarrow n \rightarrow \infty \\ &= \lim_{n \rightarrow \infty} \frac{2}{n} \left[2n + \frac{9}{n} \frac{n(n-1)}{2} + \frac{9}{n^2} \frac{n(n-1)(2n-1)}{6} \right] \\ &= \frac{38}{3} \end{aligned}$$

***** END *****

