Functions Ex 2.5 Q19

Given: $f : R \to R$ is a function defined by

$$f(x) = \cos(x+2)$$

Injectivity: let $x, y \in R$ such that

$$f(x) = f(y)$$

$\Rightarrow \quad \cos(x+2) = \cos(y+2)$

$\Rightarrow \quad x + 2 = 2n\pi \pm y + 2$

$\Rightarrow \quad x = 2n\pi \pm y$

$\Rightarrow \quad x \neq y$

$\Rightarrow \quad f$ is not one-one

Hence, f is not bijective

$\Rightarrow \quad f$ is not invertible

Functions Ex 2.5 Q20

We have, $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c, d\}$

We know that a function from A to B is said to be bijection if it is one-one and onto. This means different elements of A has different image in B. Also each element of B has preimage in A.

Let f_1, f_2, f_3 and f_4 are the functions from A to B.

$$f_1 = \{(1, a), (2, b), (3, c), (4, d)\}$$
$$f_2 = \{(1, b), (2, c), (3, d), (4, a)\}$$
$$f_3 = \{(1, c), (2, d), (3, a), (4, b)\}$$
$$f_4 = \{(1, d), (2, a), (3, b), (4, c)\}$$

we can verify that f_1, f_2, f_3 and f_4 are bijective from A to B.

Now,

$$f_1^{-1} = \{(a, 1), (b, 2), (c, 3), (d, 4)\}$$
$$f_2^{-1} = \{(b, 1), (c, 2), (d, 3), (a, 4)\}$$
$$f_3^{-1} = \{(c, 1), (d, 2), (a, 3), (b, 4)\}$$
$$f_4^{-1} = \{(d, 1), (a, 2), (b, 3), (c, 4)\}$$

Functions Ex 2.5 Q21

Given: A and B are two sets with finite elements.

$f : A \to B$ and $g : B \to A$ are injective map.

To prove: f in bijective

Proof: *Since*, $f : A \to B$ in injective we need to show f in surjective only.
Now,

$g : B \to A$ in injective

$\Rightarrow$ each element of B has image in A.

Functions Ex 2.5 Q22

We have,

$f : Q \to Q$ and $g : Q \to Q$ are two function defined by

$f(x) = 2x$ and $g(x) = x + 2$

Now, $f : Q \to Q$ defined by $f(x) = 2x$

Injectivity: let $x, y \in Q$ such that

$f(x) = f(y) \Rightarrow 2x = 2y \quad \Rightarrow x = y$

$\Rightarrow$ f in one-one

Surjectivity: let $y \in Q$ such that

$f(x) = y \quad \Rightarrow \quad 2x = y \quad \Rightarrow x = y/2 \in Q$

$\therefore$ For each $y \in Q$ (co-domain) there exist $x = y/2 \in Q$ (domain) such that $f(x) = y$

$\Rightarrow$ f is onto

$\therefore$ f in bijective

Again for $g : Q \to Q$ defined by

$g(x) = x + 2$

Injectivity: let $x, y \in Q$ such that

$g(y) = g(x) \Rightarrow \quad y + 2 = x + 2 \Rightarrow \quad y = x$

$\Rightarrow$ g is one-one

Surjectivity: let $y \in Q$ be arbitrary such that

$g(x) = y \Rightarrow \quad x + 2 = y \Rightarrow \quad x = y - 2 \in Q$

Thus, for each $y \in Q$ (co-domain), there exist $x = y - 2 \in Q$ such that $g(x) = y$

$\therefore$ g in onto

Hence, g is bijective.

$$g \circ f(x) = g(f(x)) = g(2x) = 2x + 2$$

$$\Rightarrow \; gof(x) = 2x + 2$$

f and g are bijective $\Rightarrow g \circ f$ is bijective

$$\Rightarrow \quad (g \circ f)^{-1} \text{ exist}$$

$$Now, \quad (g \circ f)(x) = 2x + 2$$

$$\Rightarrow \; (g \circ f)^{-1}(2x + 2) = x$$

$$\Rightarrow (g \circ f)^{-1}(2x) = x - 2$$

$$(g \circ f)^{-1}(x) = \frac{1}{2}(x - 2) \qquad \ldots A$$

Again,

f is bijective $\Rightarrow f^{-1}$ exist

$\therefore \quad f^{-1} : Q \to Q$ defined by

$$f^{-1}(x) = x/2$$

Also, g is bijective $\Rightarrow g^{-1}$ exist.

$\therefore \quad g^{-1} : Q \to Q$ defined by

$$g^{-1}(x) = x - 2$$

$$\therefore \quad f^{-1} \circ g^{-1}(x) = f^{-1}(g^{-1}(x))$$

$$= f^{-1}(x - 2)$$

$$(f^{-1} \circ g^{-1})(x) = \frac{1}{2}(x - 2) \; \ldots\ldots\ldots\ldots\ldots (B)$$

From (A) & (B)

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$$

*********** END **********