Definite Integrals Ex 20.2 Q57

$$I = \int_0^{\pi/2} \frac{\tan x}{1 + m^2 \tan^2 x} dx$$

$$I = \int_0^{\pi/2} \frac{\sin x \cos x}{\cos^2 x + m^2 \sin^2 x} dx$$

Put $\sin^2 x = t$ then $2\sin x \cos x \, dx = dt$

$x = 0 \Rightarrow t = 0$ and $x = \frac{\pi}{2} \Rightarrow t = 1$

$$I = \frac{1}{2}\int_0^1 \frac{1}{(1 - t) + m^2 t} dt$$

$$I = \frac{1}{2}\int_0^1 \frac{1}{(m^2 - 1)t + 1} dt$$

$$I = \frac{1}{2}\left[\frac{1}{m^2 - 1}\log\left|(m^2 - 1)t + 1\right|\right]_0^1$$

$$I = \frac{1}{2}\left[\frac{1}{m^2 - 1}\log\left|m^2\right| - \frac{1}{m^2 - 1}\ln|1|\right]$$

$$I = \frac{1}{2}\left[\frac{\log\left|m^2\right|}{m^2 - 1}\right]$$

$$I = \frac{1}{2}\left[\frac{2\log|m|}{m^2 - 1}\right]$$

$$I = \frac{\log|m|}{m^2 - 1}$$

Definite Integrals Ex 20.2 Q58

$$I = \int_0^{\frac{1}{2}} \frac{1}{(1+x^2)\sqrt{1-x^2}}\,dx$$

Let $x = \sin u$

$dx = \cos u\, du$

$$I = \int_0^{\frac{\pi}{6}} \frac{1}{(1+\sin^2 u)}\,du$$

$$I = \int_0^{\frac{\pi}{6}} \frac{\sec^2 u}{(1+2\tan^2 u)}\,du$$

Let $\tan u = v$

$dv = \sec^2 u\, du$

$$I = \int_0^{\frac{1}{\sqrt{3}}} \frac{1}{(1+2v^2)}\,dv$$

$$I = \frac{1}{\sqrt{2}}\left[\tan^{-1}(\sqrt{2}v)\right]_0^{\frac{1}{\sqrt{3}}}$$

$$I = \frac{1}{\sqrt{2}}\left[\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)\right]$$

Definite Integrals Ex 20.2 Q59

$$I = \int_{\frac{1}{3}}^{1} \frac{(x-x^3)^{\frac{1}{3}}}{x^4}\,dx$$

$$I = \int_{\frac{1}{3}}^{1} \frac{\left(\frac{1}{x^2}-1\right)^{\frac{1}{3}}}{x^3}\,dx$$

Let $\frac{1}{x^2} - 1 = t$

$\frac{-2}{x^3}dx = dt$

$x = \frac{1}{3} \Rightarrow t = 8$ and $x = 1 \Rightarrow t = 0$

$$I = -\frac{1}{2}\int_8^0 (t)^{\frac{1}{3}}\,dt$$

$$I = -\frac{1}{2}\left[\frac{t^{\frac{4}{3}}}{\frac{4}{3}}\right]_8^0$$

$$I = -\frac{1}{2}[0-12]$$

$I = 6$

*********** END **********