

NCERT MISCELLANEOUS EXERCISE

Question 1:

Prove that:
$$2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} = 0$$

Ans:

L.H.S.

$$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$$

$$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\left(\frac{3\pi}{13} + \frac{5\pi}{13}\right)\cos\left(\frac{3\pi}{13} - \frac{5\pi}{13}\right)$$

$$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\frac{4\pi}{13}\cos\left(\frac{-\pi}{13}\right)$$

$$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\frac{4\pi}{13}\cos\frac{\pi}{13}$$

$$= 2\cos\frac{\pi}{13}\left[\cos\frac{9\pi}{13} + \cos\frac{4\pi}{13}\right]$$

$$= 2\cos\frac{\pi}{13}\left[\cos\frac{9\pi}{13} + \cos\frac{4\pi}{13}\right]$$

$$= 2\cos\frac{\pi}{13}\left[2\cos\left(\frac{9\pi}{13} + \frac{4\pi}{13}\right)\cos\left(\frac{9\pi}{13} - \frac{4\pi}{13}\right)\right]$$

$$= 2\cos\frac{\pi}{13}\left[2\cos\frac{9\pi}{13} + \cos\frac{5\pi}{26}\right]$$

$$= 2\cos\frac{\pi}{13}\left[2\cos\frac{\pi}{2}\cos\frac{5\pi}{26}\right]$$

$$= 2\cos\frac{\pi}{13}\times2\times0\times\cos\frac{5\pi}{26}$$

=0=R.H.S

Question 2:

Prove that: $(\sin 3x + \sin x) \sin x + (\cos 3x - \cos x) \cos x = 0$

Ans:

L.H.S.

$$= (\sin 3x + \sin x) \sin x + (\cos 3x - \cos x) \cos x$$

$$= \sin 3x \sin x + \sin^2 x + \cos 3x \cos x - \cos^2 x$$

$$= \cos 3x \cos x + \sin 3x \sin x - (\cos^2 x - \sin^2 x)$$

$$= \cos(3x - x) - \cos 2x$$
 $\left[\cos(A - B) = \cos A \cos B + \sin A \sin B\right]$

$$=\cos 2x - \cos 2x$$

= 0

=RH.S.

Question 3:

Prove that:
$$(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4\cos^2\frac{x+y}{2}$$

Ans:

L.H.S. =
$$(\cos x + \cos y)^2 + (\sin x - \sin y)^2$$

= $\cos^2 x + \cos^2 y + 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y$
= $(\cos^2 x + \sin^2 x) + (\cos^2 y + \sin^2 y) + 2(\cos x \cos y - \sin x \sin y)$
= $1 + 1 + 2\cos(x + y)$ [$\cos(A + B) = (\cos A \cos B - \sin A \sin B)$]
= $2 + 2\cos(x + y)$
= $2[1 + \cos(x + y)]$
= $2[1 + 2\cos^2(\frac{x + y}{2}) - 1]$ [$\cos 2A = 2\cos^2 A - 1$]
= $4\cos^2(\frac{x + y}{2}) = R.H.S.$

Question 4:

Prove that:
$$(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2 \frac{x - y}{2}$$

Ans:

L.H.S. =
$$(\cos x - \cos y)^2 + (\sin x - \sin y)^2$$

$$\begin{aligned} &= \cos^2 x + \cos^2 y - 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y \\ &= \left(\cos^2 x + \sin^2 x\right) + \left(\cos^2 y + \sin^2 y\right) - 2\left[\cos x \cos y + \sin x \sin y\right] \\ &= 1 + 1 - 2\left[\cos(x - y)\right] & \left[\cos(A - B) = \cos A \cos B + \sin A \sin B\right] \\ &= 2\left[1 - \cos(x - y)\right] \\ &= 2\left[1 - \left\{1 - 2\sin^2\left(\frac{x - y}{2}\right)\right\}\right] & \left[\cos 2A = 1 - 2\sin^2 A\right] \\ &= 4\sin^2\left(\frac{x - y}{2}\right) = \text{R.H.S.} \end{aligned}$$

Question 5:

Prove that: $\sin x + \sin 3x + \sin 5x + \sin 7x = 4\cos x \cos 2x \sin 4x$ Ans:

It is known that
$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)$$
.

$$\therefore$$
 L.H.S. = $\sin x + \sin 3x + \sin 5x + \sin 7x$

$$= (\sin x + \sin 5x) + (\sin 3x + \sin 7x)$$

$$= 2\sin\left(\frac{x+5x}{2}\right) \cdot \cos\left(\frac{x-5x}{2}\right) + 2\sin\left(\frac{3x+7x}{2}\right) \cos\left(\frac{3x-7x}{2}\right)$$

$$= 2\sin 3x \cos(-2x) + 2\sin 5x \cos(-2x)$$

$$= 2\sin 3x \cos 2x + 2\sin 5x \cos 2x$$

$$= 2\cos 2x \left[\sin 3x + \sin 5x\right]$$

$$= 2\cos 2x \left[2\sin\left(\frac{3x+5x}{2}\right) \cdot \cos\left(\frac{3x-5x}{2}\right) \right]$$

$$= 2\cos 2x \left[2\sin 4x \cdot \cos(-x) \right]$$

$$= 4\cos 2x \sin 4x \cos x = R.H.S.$$

Question 6:

Prove that:
$$\frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x$$

Ans:

It is known that

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right), \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)$$

L.H.S. =
$$\frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)}$$

$$= \frac{\left[2\sin\left(\frac{7x+5x}{2}\right)\cdot\cos\left(\frac{7x-5x}{2}\right)\right] + \left[2\sin\left(\frac{9x+3x}{2}\right)\cdot\cos\left(\frac{9x-3x}{2}\right)\right]}{\left[2\cos\left(\frac{7x+5x}{2}\right)\cdot\cos\left(\frac{7x-5x}{2}\right)\right] + \left[2\cos\left(\frac{9x+3x}{2}\right)\cdot\cos\left(\frac{9x-3x}{2}\right)\right]}$$

$$= \frac{\left[2\sin 6x \cdot \cos x\right] + \left[2\sin 6x \cdot \cos 3x\right]}{\left[2\cos 6x \cdot \cos x\right] + \left[2\cos 6x \cdot \cos 3x\right]}$$
$$= \frac{2\sin 6x\left[\cos x + \cos 3x\right]}{2\cos 6x\left[\cos x + \cos 3x\right]}$$

$$= \frac{2\sin 6x \left[\cos x + \cos 3x\right]}{2\cos 6x \left[\cos x + \cos 3x\right]}$$

 $= \tan 6x$

=R.H.S

Question 7:

Prove that:
$$\sin 3x + \sin 2x - \sin x = 4\sin x \cos \frac{x}{2} \cos \frac{3x}{2}$$

Ans:

$$L.H.S. = \sin 3x + \sin 2x - \sin x$$

$$= \sin 3x + \left(\sin 2x - \sin x\right)$$

$$= \sin 3x + \left[2\cos\left(\frac{2x + x}{2}\right)\sin\left(\frac{2x - x}{2}\right)\right] \qquad \left[\sin A - \sin B = 2\cos\left(\frac{A + B}{2}\right)\sin\left(\frac{A - B}{2}\right)\right]$$

$$= \sin 3x + \left[2\cos\left(\frac{3x}{2}\right)\sin\left(\frac{x}{2}\right)\right]$$

$$= \sin 3x + 2\cos\frac{3x}{2}\sin\frac{x}{2}$$

$$= 2\sin\frac{3x}{2}\cdot\cos\frac{3x}{2} + 2\cos\frac{3x}{2}\sin\frac{x}{2}$$

$$= 2\sin\left(\frac{3x}{2}\right)\left[\sin\left(\frac{3x}{2}\right) + \sin\left(\frac{x}{2}\right)\right]$$

$$= 2\cos\left(\frac{3x}{2}\right)\left[\sin\left(\frac{3x}{2}\right) + \sin\left(\frac{x}{2}\right)\right]$$

$$= 2\cos\left(\frac{3x}{2}\right)\left[2\sin\left(\frac{3x}{2}\right) + \left(\frac{x}{2}\right)\right]$$

$$= 2\cos\left(\frac{3x}{2}\right) \cdot 2\sin x \cos\left(\frac{x}{2}\right)$$

$$= 4\sin x \cos\left(\frac{x}{2}\right)\cos\left(\frac{3x}{2}\right) = R.H.S.$$

Question 8:

Find
$$\sin \frac{x}{2}$$
, $\cos \frac{x}{2}$ and $\tan \frac{x}{2}$, if

$$\tan x = -\frac{4}{3}$$
, x in quadrant II

Ans:

Here, x is in quadrant II.

i.e.,
$$\frac{\pi}{2} < x < \pi$$

$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

Therefore, $\sin \frac{x}{2}$, $\cos \frac{x}{2}$ and $\tan \frac{x}{2}$ are all positive.

It is given that $\tan x = -\frac{4}{3}$.

$$\sec^2 x = 1 + \tan^2 x = 1 + \left(\frac{-4}{3}\right)^2 = 1 + \frac{16}{9} = \frac{25}{9}$$

$$\therefore \cos^2 x = \frac{9}{25}$$

$$\Rightarrow \cos x = \pm \frac{3}{5}$$

As x is in quadrant II, cosx is negative.

$$\therefore \cos x = \frac{-3}{5}$$

Now,
$$\cos x = 2\cos^2 \frac{x}{2} - 1$$

$$\Rightarrow \frac{-3}{5} = 2\cos^2\frac{x}{2} - 1$$

$$\Rightarrow 2\cos^2\frac{x}{2} = 1 - \frac{3}{5}$$

$$\Rightarrow 2\cos^2\frac{x}{2} = \frac{2}{5}$$

$$\Rightarrow \cos^2 \frac{x}{2} = \frac{1}{5}$$

$$\Rightarrow \cos \frac{x}{2} = \frac{1}{\sqrt{5}}$$

$$\therefore \cos \frac{x}{2} = \frac{\sqrt{5}}{5}$$

$$\left[\because \cos\frac{x}{2} \text{ is positive}\right]$$

$$\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} = 1$$

$$\Rightarrow \sin^2 \frac{x}{2} + \left(\frac{1}{\sqrt{5}}\right)^2 = 1$$

$$\Rightarrow \sin^2 \frac{x}{2} = 1 - \frac{1}{5} = \frac{4}{5}$$

$$\Rightarrow \sin \frac{x}{2} = \frac{2}{\sqrt{5}}$$

$$\therefore \sin \frac{x}{2} = \frac{2\sqrt{5}}{5}$$

$$\therefore \sin \frac{x}{2} = \frac{2\sqrt{5}}{5}$$

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\left(\frac{2}{\sqrt{5}}\right)}{\left(\frac{1}{\sqrt{5}}\right)} = 2$$

Thus, the respective values of $\sin \frac{x}{2}$, $\cos \frac{x}{2}$ and $\tan \frac{x}{2}$ are $\frac{2\sqrt{5}}{5}$, $\frac{\sqrt{5}}{5}$, and 2. Ouestion 9:

Find
$$\sin \frac{x}{2}$$
, $\cos \frac{x}{2}$ and $\tan \frac{x}{2}$ for $\cos x = -\frac{1}{3}$, x in quadrant III

Ans:

Here, x is in quadrant III.

i.e.,
$$\pi < x < \frac{3\pi}{2}$$

$$\Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$$

Therefore, $\cos \frac{x}{2}$ and $\tan \frac{x}{2}$ are negative, whereas $\sin \frac{x}{2}$ is positive.

It is given that $\cos x = -\frac{1}{3}$.

$$\cos x = 1 - 2\sin^2\frac{x}{2}$$

$$\Rightarrow \sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}$$

$$\Rightarrow \sin^2 \frac{x}{2} = \frac{1 - \left(-\frac{1}{3}\right)}{2} = \frac{\left(1 + \frac{1}{3}\right)}{2} = \frac{\frac{4}{3}}{2} = \frac{2}{3}$$

$$\Rightarrow \sin \frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}} \qquad \left[\because \sin \frac{x}{2} \text{ is positive} \right]$$
$$\therefore \sin \frac{x}{2} = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$$

Now,
$$\cos x = 2\cos^2\frac{x}{2} - 1$$

$$\Rightarrow \cos^2 \frac{x}{2} = \frac{1 + \cos x}{2} = \frac{1 + \left(-\frac{1}{3}\right)}{2} = \frac{\left(\frac{3 - 1}{3}\right)}{2} = \frac{\left(\frac{2}{3}\right)}{2} = \frac{1}{3}$$

$$\Rightarrow \cos \frac{x}{2} = -\frac{1}{\sqrt{3}}$$

$$\left[\because \cos\frac{x}{2} \text{ is negative}\right]$$

$$\therefore \cos\frac{x}{2} = -\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$$

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\left(\frac{\sqrt{2}}{\sqrt{3}}\right)}{\left(\frac{-1}{\sqrt{3}}\right)} = -\sqrt{2}$$

Thus, the respective values of $\sin \frac{x}{2}$, $\cos \frac{x}{2}$ and $\tan \frac{x}{2} \operatorname{are} \frac{\sqrt{6}}{3}$, $\frac{-\sqrt{3}}{3}$, and $-\sqrt{2}$ Question 10:

Find
$$\sin \frac{x}{2}$$
, $\cos \frac{x}{2}$ and $\tan \frac{x}{2}$ for $\sin x = \frac{1}{4}$, x in quadrant II

Ans:

Here, x is in quadrant II.

i.e.,
$$\frac{\pi}{2} < x < \pi$$

$$\Rightarrow \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

Therefore, $\sin \frac{x}{2}$, $\cos \frac{x}{2}$, and $\tan \frac{x}{2}$ are all positive.

It is given that $\sin x = \frac{1}{4}$.

$$\cos^2 x = 1 - \sin^2 x = 1 - \left(\frac{1}{4}\right)^2 = 1 - \frac{1}{16} = \frac{15}{16}$$

$$\Rightarrow$$
 cos x = $-\frac{\sqrt{15}}{4}$ [cosx is negative in quadrant II]

$$\sin^2 \frac{x}{2} = \frac{1 - \cos x}{2} = \frac{1 - \left(-\frac{\sqrt{15}}{4}\right)}{2} = \frac{4 + \sqrt{15}}{8}$$

$$\Rightarrow \sin \frac{x}{2} = \sqrt{\frac{4 + \sqrt{15}}{8}} \qquad \left[\because \sin \frac{x}{2} \text{ is positive}\right]$$

$$= \sqrt{\frac{4 + \sqrt{15}}{8} \times \frac{2}{2}}$$

$$= \sqrt{\frac{8 + 2\sqrt{15}}{16}}$$

$$= \frac{\sqrt{8 + 2\sqrt{15}}}{4}$$

$$\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2} = \frac{1 + \left(-\frac{\sqrt{15}}{4}\right)}{2} = \frac{4 - \sqrt{15}}{8}$$

$$\Rightarrow \cos \frac{x}{2} = \sqrt{\frac{4 - \sqrt{15}}{8}}$$

$$= \sqrt{\frac{4 - \sqrt{15}}{8} \times \frac{2}{2}}$$

$$= \sqrt{\frac{8 - 2\sqrt{15}}{16}}$$

$$= \frac{\sqrt{8 - 2\sqrt{15}}}{4}$$

$$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\left(\frac{\sqrt{8 + 2\sqrt{15}}}{4}\right)}{\left(\frac{\sqrt{8 - 2\sqrt{15}}}{4}\right)} = \frac{\sqrt{8 + 2\sqrt{15}}}{\sqrt{8 - 2\sqrt{15}}}$$
$$= \sqrt{\frac{8 + 2\sqrt{15}}{8 - 2\sqrt{15}}} \times \frac{8 + 2\sqrt{15}}{8 + 2\sqrt{15}}$$
$$= \sqrt{\frac{\left(8 + 2\sqrt{15}\right)^{2}}{64 - 60}} = \frac{8 + 2\sqrt{15}}{2} = 4 + \sqrt{15}$$

Thus, the respective values of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are $\frac{\sqrt{8+2\sqrt{15}}}{4}$, $\frac{\sqrt{8-2\sqrt{15}}}{4}$, and $4+\sqrt{15}$

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