Indefinite Integrals Ex 19.2 Q36

$$\int \cos^{-1}(\sin x)dx$$

$$= \int \cos^{-1}\left[\cos\left(\frac{\pi}{2} - x\right)\right]dx$$

$$= \int\left(\frac{\pi}{2} - x\right)dx$$

$$= \frac{\pi}{2}\int dx - \int x dx$$

$$= \frac{\pi}{2} \times x - \frac{x^2}{2} + c$$

$$\therefore \quad \int \cos^{-1}(\sin x)dx = \frac{\pi}{2} \times x - \frac{x^2}{2} + c.$$

Indefinite Integrals Ex 19.2 Q37

$$\int \cos^{-1}(\sin x)dx$$

$$= \int \cot^{-1}\left[\frac{\sin 2x}{1 - \cos 2x}\right]dx$$

$$= \int \cot^{-1}\left(\frac{\cos x}{\sin x}\right)dx$$

$$= \int \cot^{-1}(\cot x)dx$$

$$= \int x dx$$

$$= \frac{x^2}{2} + c$$

$$\therefore \quad \int \cot^{-1}\left[\frac{\sin 2x}{1 - \cos 2x}\right]dx = \frac{x^2}{2} + c.$$

Indefinite Integrals Ex 19.2 Q38

$$\int \sin^{-1}\left(\frac{2\tan x}{1 + \tan^2 x}\right)dx$$

$$= \int \sin^{-1}(\sin 2x)dx \qquad \left[\because \sin 2x = \frac{2\tan x}{1 + \tan^2 x}\right]$$

$$= \int 2x dx$$

$$= 2\int x\, dx$$

$$= \frac{2x^2}{2} + c$$

$$= x^2 + c$$

$$\therefore \quad \int \sin^{-1}\left(\frac{2\tan x}{1 + \tan^2 x}\right) = x^2 + c.$$

Indefinite Integrals Ex 19.2 Q39

$$\int \frac{\left(x^3+8\right)(x-1)}{x^2-2x+4}dx$$

$$= \int \frac{(x+2)\left(x^2-2x+4\right)(x-1)}{x^2-2x+4}dx$$
$$= \int (x+2)(x-1)\,dx$$
$$= \int \left(x^2-x+2x-2\right)dx$$
$$= \int \left(x^2+x-2\right)dx$$
$$= \frac{x^3}{3}+\frac{x^2}{2}-2x+c$$

$$\therefore \quad \int \frac{\left(x^3+8\right)(x-1)}{x^2-2x+4}dx = \frac{x^3}{3}+\frac{x^2}{2}-2x+c.$$

Indefinite Integrals Ex 19.2 Q40

$$\int (a\tan x + b\cot x)^2 dx$$

$$= \int \left(a^2\tan^2 x + b^2\cot^2 x + 2ab\tan x\cot x\right)dx$$
$$= \int \left[a^2\left(\sec^2 x-1\right)+b^2\left(\operatorname{cosec}^2 x-1\right)+2ab\right]dx$$
$$= \int \left[a^2\sec^2 x - a^2 + b^2\operatorname{cosec}^2 x - b^2 + 2ab\right]dx$$
$$= a^2\tan x - a^2x - b^2\cot x - b^2x + 2abx + c$$
$$= a^2\tan x + -b^2\cot x - \left(a^2+b^2-2ab\right)x + c$$

$$\therefore \quad \int (a\tan x + b\cot x)^2 dx = a^2\tan x - b^2\cot x - \left(a^2+b^2-2ab\right)x + c.$$

Indefinite Integrals Ex 19.2 Q41

$$\int \frac{x^3-3x^2+5x-7+x^2a^x}{2x^2}dx$$

$$= \frac{1}{2}\int \frac{x^3}{x^2}dx - \frac{3}{2}\int \frac{x^2}{x^2}dx + \frac{5}{2}\int x\,\frac{x}{x^2}dx - \frac{7}{2}\int x^{-2}dx + \frac{1}{2}\int \frac{x^2a^x}{x^2}dx$$
$$= \frac{1}{2}\times\frac{x^2}{2} - \frac{3}{2}x + \frac{5}{2}\log x - \frac{7}{2}x^{-1} + \frac{1}{2}\frac{a^x}{\log a} + c$$
$$= \frac{1}{2}\left[\frac{x^2}{2} - 3x + 5\log x + \frac{7}{x} + \frac{a^x}{\log a}\right] + c$$

$$\therefore \quad \int \frac{x^3-3x^2+5x-7+x^2a^x}{2x^2}dx = \frac{1}{2}\left[\frac{x^2}{2} - 3x + 5\log x + \frac{7}{x} + \frac{a^x}{\log a}\right] + c$$

Indefinite Integrals Ex 19.2 Q42

$$\frac{\cos x}{1+\cos x}=\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}} \qquad \left[\cos x=\cos^2\frac{x}{2}-\sin^2\frac{x}{2} \text{ and } \cos x=2\cos^2\frac{x}{2}-1\right]$$

$$=\frac{1}{2}\left[1-\tan^2\frac{x}{2}\right]$$

$$\therefore \int\frac{\cos x}{1+\cos x}dx=\frac{1}{2}\int\left(1-\tan^2\frac{x}{2}\right)dx$$

$$=\frac{1}{2}\int\left(1-\sec^2\frac{x}{2}+1\right)dx$$

$$=\frac{1}{2}\int\left(2-\sec^2\frac{x}{2}\right)dx$$

$$=\frac{1}{2}\left[2x-\frac{\tan\frac{x}{2}}{\frac{1}{2}}\right]+C$$

$$=x-\tan\frac{x}{2}+C$$

********** END **********