Co-Ordinate Geometry Ex 14.2 Q13

Answer :

The distance d between two points (x_1, y_1) and (x_2, y_2) is given by the formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

In an equilateral triangle all the sides are of equal length.

Here we are given that *A* (3, 4) and *B* (−2, 3) are two vertices of an equilateral triangle. Let *C*(*x*, *y*) be the third vertex of the equilateral triangle.

First let us find out the length of the side of the equilateral triangle.

$$\begin{aligned} AB &= \sqrt{(3+2)^2 + (4-3)^2} \\ &= \sqrt{(5)^2 + (1)^2} \\ &= \sqrt{25+1} \\ AB &= \sqrt{26} \end{aligned}$$

Hence the side of the equilateral triangle measures $\sqrt{26}$ units.

Now, since it is an equilateral triangle, all the sides need to measure the same length.

Hence we have $BC = AC$

$$BC = \sqrt{(-2-x)^2 + (3-y)^2}$$

$$AC = \sqrt{(3-x)^2 + (4-y)^2}$$

Equating both these equations we have,

$$\sqrt{(-2-x)^2 + (3-y)^2} = \sqrt{(3-x)^2 + (4-y)^2}$$

Squaring on both sides we have,

$$\begin{aligned} (-2-x)^2 + (3-y)^2 &= (3-x)^2 + (4-y)^2 \\ 4 + x^2 + 4x + 9 + y^2 - 6y &= 9 + x^2 - 6x + 16 + y^2 - 8y \\ 10x + 2y &= 12 \\ 5x + y &= 6 \end{aligned}$$

From the above equation we have, $y = 6 - 5x$

Substituting this and the value of the side of the triangle in the equation for one of the sides we have,

$$BC = \sqrt{(-2-x)^2 + (3-y)^2}$$

$$\sqrt{26} = \sqrt{(-2-x)^2 + (3-6+5x)^2}$$

Squaring on both sides,

$$\begin{aligned} 26 &= (-2-x)^2 + (-3+5x)^2 \\ 26 &= 4 + x^2 + 4x + 9 + 25x^2 - 30x \\ 13 &= 26x^2 - 26x \\ 1 &= 2x^2 - 2x \end{aligned}$$

Now we have a quadratic equation for '*x*'. Solving for the roots of this equation,

$$2x^2 - 2x - 1 = 0$$

$$\begin{aligned} x &= \frac{2 \pm \sqrt{4 - 4(2)(-1)}}{4} \\ &= \frac{2 \pm \sqrt{12}}{4} \\ x &= \frac{1 \pm \sqrt{3}}{2} \end{aligned}$$

We know that $y = 6 - 5x$. Substituting the value of '*x*' we have,

$$\begin{aligned} y &= 6 - 5\left(\frac{1 \pm \sqrt{3}}{2}\right) \\ &= \frac{12 - 5 \mp 5\sqrt{3}}{2} \\ y &= \frac{7 \mp 5\sqrt{3}}{2} \end{aligned}$$

Hence the two possible values of the third vertex are $\boxed{\frac{1+\sqrt{3}}{2}, \frac{7-5\sqrt{3}}{2} \text{ and } \frac{1-\sqrt{3}}{2}, \frac{7+5\sqrt{3}}{2}}$.

Co-Ordinate Geometry Ex 14.2 Q14

Answer :

The distance *d* between two points (x_1, y_1) and (x_2, y_2) is given by the formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

In a rhombus all the sides are equal in length.

Here the four points are *A* (2,−1), *B* (3, 4), *C* (−2, 3) and *D* (−3, −2).

First let us check if all the four sides are equal.

$$AB = \sqrt{(2-3)^2 + (-1-4)^2}$$
$$= \sqrt{(-1)^2 + (-5)^2}$$
$$= \sqrt{1+25}$$
$$AB = \sqrt{26}$$
$$BC = \sqrt{(3+2)^2 + (4-3)^2}$$
$$= \sqrt{(5)^2 + (1)^2}$$
$$= \sqrt{25+1}$$
$$BC = \sqrt{26}$$
$$CD = \sqrt{(-2-3)^2 + (-3+2)^2}$$
$$= \sqrt{(-5)^2 + (-1)^2}$$
$$= \sqrt{25+1}$$
$$CD = \sqrt{26}$$
$$AD = \sqrt{(2+3)^2 + (-1+2)^2}$$
$$= \sqrt{(5)^2 + (1)^2}$$
$$= \sqrt{25+1}$$
$$AD = \sqrt{26}$$

Here, we see that all the sides are equal, so it has to be a rhombus.

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