NCERT MISCELLANEOUS SOLUTIONS

Question-1

Find the derivative of the following functions from first principle:

(i) $-x$ (ii) $(-x)^{-1}$ (iii) $\sin(x + 1)$

(iv) $\cos\left(x-\frac{\pi}{8}\right)$

Ans.

(i) Let $f(x) = -x$. Accordingly, $f(x+h) = -(x+h)$

By first principle,

$$\begin{aligned} f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0}\frac{-(x+h)-(-x)}{h} \\ &= \lim_{h\to 0}\frac{-x-h+x}{h} \\ &= \lim_{h\to 0}\frac{-h}{h} \\ &= \lim_{h\to 0}(-1) = -1 \end{aligned}$$

(ii) Let $f(x)=(-x)^{-1}=\frac{1}{-x}=\frac{-1}{x}$. Accordingly, $f(x+h)=\frac{-1}{(x+h)}$

By first principle,

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{1}{h}\left[\frac{-1}{x+h}-\left(\frac{-1}{x}\right)\right]$$

$$=\lim_{h\to 0}\frac{1}{h}\left[\frac{-1}{x+h}+\frac{1}{x}\right]$$

$$=\lim_{h\to 0}\frac{1}{h}\left[\frac{-x+(x+h)}{x(x+h)}\right]$$

$$=\lim_{h\to 0}\frac{1}{h}\left[\frac{-x+x+h}{x(x+h)}\right]$$

$$=\lim_{h\to 0}\frac{1}{h}\left[\frac{h}{x(x+h)}\right]$$

$$=\lim_{h\to 0}\frac{1}{x(x+h)}$$

$$=\frac{1}{x\cdot x}=\frac{1}{x^2}$$

(iii) Let $f(x)$ = sin $(x + 1)$. Accordingly, $f(x+h)=\sin(x+h+1)$

By first principle,

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{1}{h}\left[\sin(x+h+1)-\sin(x+1)\right]$$

$$=\lim_{h\to 0}\frac{1}{h}\left[2\cos\left(\frac{x+h+1+x+1}{2}\right)\sin\left(\frac{x+h+1-x-1}{2}\right)\right]$$

$$=\lim_{h\to 0}\frac{1}{h}\left[2\cos\left(\frac{2x+h+2}{2}\right)\sin\left(\frac{h}{2}\right)\right]$$

$$=\lim_{h\to 0}\left[\cos\left(\frac{2x+h+2}{2}\right)\cdot\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}\right]$$

$$=\lim_{h\to 0}\cos\left(\frac{2x+h+2}{2}\right)\cdot\lim_{\frac{h}{2}\to 0}\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}\qquad\left[\text{As } h\to 0\Rightarrow\frac{h}{2}\to 0\right]$$

$$=\cos\left(\frac{2x+0+2}{2}\right)\cdot 1\qquad\left[\lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

$$=\cos(x+1)$$

(iv) Let $f(x) = \cos\left(x - \frac{\pi}{8}\right)$. Accordingly, $f(x+h) = \cos\left(x + h - \frac{\pi}{8}\right)$

By first principle,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{1}{h}\left[\cos\left(x + h - \frac{\pi}{8}\right) - \cos\left(x - \frac{\pi}{8}\right)\right]$$
$$= \lim_{h\to 0} \frac{1}{h}\left[-2\sin\left(\frac{x + h - \frac{\pi}{8} + x - \frac{\pi}{8}}{2}\right)\sin\left(\frac{x + h - \frac{\pi}{8} - x + \frac{\pi}{8}}{2}\right)\right]$$
$$= \lim_{h\to 0} \frac{1}{h}\left[-2\sin\left(\frac{2x + h - \frac{\pi}{4}}{2}\right)\sin\frac{h}{2}\right]$$
$$= \lim_{h\to 0}\left[-\sin\left(\frac{2x + h - \frac{\pi}{4}}{2}\right)\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}\right]$$
$$= \lim_{h\to 0}\left[-\sin\left(\frac{2x + h - \frac{\pi}{4}}{2}\right)\right] \cdot \lim_{\frac{h}{2}\to 0} \frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)} \qquad \left[\text{As } h \to 0 \Rightarrow \frac{h}{2} \to 0\right]$$
$$= -\sin\left(\frac{2x + 0 - \frac{\pi}{4}}{2}\right).1$$
$$= -\sin\left(x - \frac{\pi}{8}\right)$$

Question-2

Find the derivative of the following functions (it is to be understood that *a*, *b*, *c*, *d*, *p*, q, *r* and *s* are fixed non-zero constants and *m* and *n* are integers): $(x + a)$

Ans.

Let $f(x) = x + a$. Accordingly, $f(x+h) = x + h + a$

By first principle,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{x + h + a - x - a}{h}$$
$$= \lim_{h\to 0}\left(\frac{h}{h}\right)$$
$$= \lim_{h\to 0}(1)$$
$$= 1$$

Question-3

Find the derivative of the following functions (it is to be understood that *a*, *b*, *c*, *d*, *p*, q, *r* and *s* are fixed non-zero constants and *m* and *n* are integers): $(px+q)\left(\frac{r}{x}+s\right)$

Ans.

$$\text{Let } f(x) = (px+q)\left(\frac{r}{x}+s\right)$$

By Leibnitz product rule,

$$f'(x) = (px+q)\left(\frac{r}{x}+s\right)' + \left(\frac{r}{x}+s\right)(px+q)'$$

$$= (px+q)\left(rx^{-1}+s\right)' + \left(\frac{r}{x}+s\right)(p)$$

$$= (px+q)\left(-rx^{-2}\right) + \left(\frac{r}{x}+s\right)p$$

$$= (px+q)\left(\frac{-r}{x^2}\right) + \left(\frac{r}{x}+s\right)p$$

$$= \frac{-pr}{x} - \frac{qr}{x^2} + \frac{pr}{x} + ps$$

$$= ps - \frac{qr}{x^2}$$

Question-4

Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(ax + b)(cx + d)^2$

Ans.

Let $f(x) = (ax+b)(cx+d)^2$

By Leibnitz product rule,

$$f'(x) = (ax+b)\frac{d}{dx}(cx+d)^2 + (cx+d)^2\frac{d}{dx}(ax+b)$$

$$= (ax+b)\frac{d}{dx}\left(c^2x^2+2cdx+d^2\right) + (cx+d)^2\frac{d}{dx}(ax+b)$$

$$= (ax+b)\left[\frac{d}{dx}\left(c^2x^2\right)+\frac{d}{dx}(2cdx)+\frac{d}{dx}d^2\right] + (cx+d)^2\left[\frac{d}{dx}ax+\frac{d}{dx}b\right]$$

$$= (ax+b)\left(2c^2x+2cd\right) + \left(cx+d^2\right)a$$

$$= 2c(ax+b)(cx+d) + a(cx+d)^2$$

Question-5

Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{ax+b}{cx+d}$

Ans.

Let $f(x)=\frac{ax+b}{cx+d}$

By quotient rule,

$$f'(x)=\frac{(cx+d)\frac{d}{dx}(ax+b)-(ax+b)\frac{d}{dx}(cx+d)}{(cx+d)^2}$$
$$=\frac{(cx+d)(a)-(ax+b)(c)}{(cx+d)^2}$$
$$=\frac{acx+ad-acx-bc}{(cx+d)^2}$$
$$=\frac{ad-bc}{(cx+d)^2}$$

Question-6

Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{1+\frac{1}{x}}{1-\frac{1}{x}}$$

Ans.

$$\text{Let } f(x)=\frac{1+\frac{1}{x}}{1-\frac{1}{x}}=\frac{\frac{x+1}{x}}{\frac{x-1}{x}}=\frac{x+1}{x-1},\ \text{where } x\neq 0$$

By quotient rule,

$$f'(x)=\frac{(x-1)\frac{d}{dx}(x+1)-(x+1)\frac{d}{dx}(x-1)}{(x-1)^2},\ x\neq 0,\ 1$$
$$=\frac{(x-1)(1)-(x+1)(1)}{(x-1)^2},\ x\neq 0,\ 1$$
$$=\frac{x-1-x-1}{(x-1)^2},\ x\neq 0,\ 1$$
$$=\frac{-2}{(x-1)^2},\ x\neq 0,\ 1$$

Question-7

Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{1}{ax^2+bx+c}$

Ans.

Let $f(x)=\dfrac{1}{ax^2+bx+c}$

By quotient rule,

$$f'(x)=\frac{\left(ax^2+bx+c\right)\frac{d}{dx}(1)-\frac{d}{dx}\left(ax^2+bx+c\right)}{\left(ax^2+bx+c\right)^2}$$
$$=\frac{\left(ax^2+bx+c\right)(0)-(2ax+b)}{\left(ax^2+bx+c\right)^2}$$
$$=\frac{-(2ax+b)}{\left(ax^2+bx+c\right)^2}$$

Question-8

Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\dfrac{ax+b}{px^2+qx+r}$

Ans.

Let $f(x)=\dfrac{ax+b}{px^2+qx+r}$

By quotient rule,

$$f'(x)=\frac{\left(px^2+qx+r\right)\frac{d}{dx}(ax+b)-(ax+b)\frac{d}{dx}\left(px^2+qx+r\right)}{\left(px^2+qx+r\right)^2}$$
$$=\frac{\left(px^2+qx+r\right)(a)-(ax+b)(2px+q)}{\left(px^2+qx+r\right)^2}$$
$$=\frac{apx^2+aqx+ar-2apx^2-aqx-2bpx-bq}{\left(px^2+qx+r\right)^2}$$
$$=\frac{-apx^2-2bpx+ar-bq}{\left(px^2+qx+r\right)^2}$$

Question-9

Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\dfrac{px^2+qx+r}{ax+b}$

Ans.

$$\text{Let } f(x)=\frac{px^2+qx+r}{ax+b}$$

By quotient rule,

$$f'(x)=\frac{(ax+b)\frac{d}{dx}(px^2+qx+r)-(px^2+qx+r)\frac{d}{dx}(ax+b)}{(ax+b)^2}$$
$$=\frac{(ax+b)(2px+q)-(px^2+qx+r)(a)}{(ax+b)^2}$$
$$=\frac{2apx^2+aqx+2bpx+bq-apx^2-aqx-ar}{(ax+b)^2}$$
$$=\frac{apx^2+2bpx+bq-ar}{(ax+b)^2}$$

Question-10

Find the derivative of the following functions (it is to be understood that *a*, *b*, *c*, *d*, *p*, *q*, *r* and *s* are fixed non-zero constants and *m* and *n* are integers): $\frac{a}{x^4}-\frac{b}{x^2}+\cos x$

Ans.

$$\text{Let } f(x)=\frac{a}{x^4}-\frac{b}{x^2}+\cos x$$
$$f'(x)=\frac{d}{dx}\left(\frac{a}{x^4}\right)-\frac{d}{dx}\left(\frac{b}{x^2}\right)+\frac{d}{dx}(\cos x)$$
$$=a\frac{d}{dx}\left(x^{-4}\right)-b\frac{d}{dx}\left(x^{-2}\right)+\frac{d}{dx}(\cos x)$$
$$=a\left(-4x^{-5}\right)-b\left(-2x^{-3}\right)+(-\sin x)\qquad\left[\frac{d}{dx}\left(x^n\right)=nx^{n-1}\text{and}\frac{d}{dx}(\cos x)=-\sin x\right]$$
$$=\frac{-4a}{x^5}+\frac{2b}{x^3}-\sin x$$

Question-11

Find the derivative of the following functions (it is to be understood that *a*, *b*, *c*, *d*, *p*, *q*, *r* and *s* are fixed non-zero constants and *m* and *n* are integers): $4\sqrt{x}-2$

Ans.

$$\text{Let } f(x)=4\sqrt{x}-2$$
$$f'(x)=\frac{d}{dx}\left(4\sqrt{x}-2\right)=\frac{d}{dx}\left(4\sqrt{x}\right)-\frac{d}{dx}(2)$$
$$=4\frac{d}{dx}\left(x^{\frac{1}{2}}\right)-0=4\left(\frac{1}{2}x^{\frac{1}{2}-1}\right)$$
$$=\left(2x^{-\frac{1}{2}}\right)=\frac{2}{\sqrt{x}}$$

Question-12

Find the derivative of the following functions (it is to be understood that *a*, *b*, *c*, *d*, *p*, *q*, *r* and *s* are fixed non-zero constants and *m* and *n* are integers): $(ax+b)^n$

Ans.

Let $f(x)=(ax+b)^n$. Accordingly, $f(x+h)=\{a(x+h)+b\}^n=(ax+ah+b)^n$

By first principle,

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to 0}\frac{(ax+ah+b)^n-(ax+b)^n}{h}$$
$$=\lim_{h\to 0}\frac{(ax+b)^n\left(1+\frac{ah}{ax+b}\right)^n-(ax+b)^n}{h}$$
$$=(ax+b)^n\lim_{h\to 0}\frac{\left(1+\frac{ah}{ax+b}\right)^n-1}{h}$$
$$=(ax+b)^n\lim_{h\to 0}\frac{1}{n}\left[\left\{1+n\left(\frac{ah}{ax+b}\right)+\frac{n(n-1)}{\underline{|2}}\left(\frac{ah}{ax+b}\right)^2+\ldots\right\}-1\right]$$
(Using binomial theorem)
$$=(ax+b)^n\lim_{h\to 0}\frac{1}{h}\left[n\left(\frac{ah}{ax+b}\right)+\frac{n(n-1)a^2h^2}{\underline{|2}(ax+b)^2}+\ldots(\text{Terms containing higher degrees of } h)\right]$$
$$=(ax+b)^n\lim_{h\to 0}\left[\frac{na}{(ax+b)}+\frac{n(n-1)a^2h}{\underline{|2}(ax+b)^2}+\ldots\right]$$
$$=(ax+b)^n\left[\frac{na}{(ax+b)}+0\right]$$
$$=na\frac{(ax+b)^n}{(ax+b)}$$
$$=na(ax+b)^{n-1}$$

Question-13

Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(ax + b)^n (cx + d)^m$

Ans.

Let $f(x)=(ax+b)^n(cx+d)^m$

By Leibnitz product rule,

$$f'(x)=(ax+b)^n\frac{d}{dx}(cx+d)^m+(cx+d)^m\frac{d}{dx}(ax+b)^n \qquad \ldots(1)$$

Now, let $f_1(x)=(cx+d)^m$

$f_1(x+h)=(cx+ch+d)^m$

$$f_1'(x)=\lim_{h\to 0}\frac{f_1(x+h)-f_1(x)}{h}$$
$$=\lim_{h\to 0}\frac{(cx+ch+d)^m-(cx+d)^m}{h}$$
$$=(cx+d)^m\lim_{h\to 0}\frac{1}{h}\left[\left(1+\frac{ch}{cx+d}\right)^m-1\right]$$
$$=(cx+d)^m\lim_{h\to 0}\frac{1}{h}\left[\left(1+\frac{mch}{(cx+d)}+\frac{m(m-1)}{2}\frac{(c^2h^2)}{(cx+d)^2}+\ldots\right)-1\right]$$
$$=(cx+d)^m\lim_{h\to 0}\frac{1}{h}\left[\frac{mch}{(cx+d)}+\frac{m(m-1)c^2h^2}{2(cx+d)^2}+\ldots(\text{Terms containing higher degrees of } h)\right]$$
$$=(cx+d)^m\lim_{h\to 0}\left[\frac{mc}{(cx+d)}+\frac{m(m-1)c^2h}{2(cx+d)^2}+\ldots\right]$$
$$=(cx+d)^m\left[\frac{mc}{cx+d}+0\right]$$

$$= \frac{mc(cx+d)^m}{(cx+d)}$$

$$= mc(cx+d)^{m-1}$$

$$\frac{d}{dx}(cx+d)^m = mc(cx+d)^{m-1} \qquad ...(2)$$

$$\text{Similarly, } \frac{d}{dx}(ax+b)^n = na(ax+b)^{n-1} \qquad ...(3)$$

Therefore, from (1), (2), and (3), we obtain

$$f'(x) = (ax+b)^n \left\{mc(cx+d)^{m-1}\right\} + (cx+d)^m \left\{na(ax+b)^{n-1}\right\}$$

$$= (ax+b)^{n-1}(cx+d)^{m-1}\left[mc(ax+b)+na(cx+d)\right]$$

Question-14

Find the derivative of the following functions (it is to be understood that *a*, *b*, *c*, *d*, *p*, q, *r* and *s* are fixed non-zero constants and *m* and *n* are integers): sin (*x* + *a*)

Ans.

Let $f(x) = \sin(x+a)$

$$f(x+h) = \sin(x+h+a)$$

By first principle,

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\sin(x+h+a)-\sin(x+a)}{h}$$

$$= \lim_{h\to 0}\frac{1}{h}\left[2\cos\left(\frac{x+h+a+x+a}{2}\right)\sin\left(\frac{x+h+a-x-a}{2}\right)\right]$$

$$= \lim_{h\to 0}\frac{1}{h}\left[2\cos\left(\frac{2x+2a+h}{2}\right)\sin\left(\frac{h}{2}\right)\right]$$

$$= \lim_{h\to 0}\left[\cos\left(\frac{2x+2a+h}{2}\right)\left\{\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}\right\}\right]$$

$$= \lim_{h\to 0}\cos\left(\frac{2x+2a+h}{2}\right)\lim_{\frac{h}{2}\to 0}\left\{\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}\right\} \qquad \left[\text{As } h\to 0 \Rightarrow \frac{h}{2}\to 0\right]$$

$$= \cos\left(\frac{2x+2a}{2}\right)\times 1 \qquad \left[\lim_{x\to 0}\frac{\sin x}{x} = 1\right]$$

$$= \cos(x+a)$$

*********** END **********