Trigonometric Ratios Ex 5.3 Q6

Answer :

(i) We have to prove: $\tan\left(\frac{C+A}{2}\right)=\cot\frac{B}{2}$

Since we know that in triangle ABC

$$A+B+C=180$$

$$\Rightarrow C+A=180^{\circ}-B$$

$$\Rightarrow \frac{C+A}{2}=90^{\circ}-\frac{B}{2}$$

$$\Rightarrow \tan\frac{C+A}{2}=\tan\left(90^{\circ}-\frac{B}{2}\right)$$

$$\Rightarrow \boxed{\tan\frac{C+A}{2}=\cot\frac{B}{2}}$$

Proved

(ii) We have to prove: $\sin\left(\frac{B+C}{2}\right)=\cos\frac{A}{2}$

Since we know that in triangle ABC

$$A+B+C=180$$

$$\Rightarrow B+C=180^{\circ}-A$$

$$\Rightarrow \frac{B+C}{2}=90^{\circ}-\frac{A}{2}$$

$$\Rightarrow \sin\frac{B+C}{2}=\sin\left(90^{\circ}-\frac{A}{2}\right)$$

$$\Rightarrow \boxed{\sin\frac{B+C}{2}=\cos\frac{A}{2}}$$

Proved

Trigonometric Ratios Ex 5.3 Q7

Answer :

We are asked to find the value of $\tan 20^\circ \tan 35^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ$

(i) Therefore

$$\begin{aligned}&\tan 20^\circ \tan 35^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ\\ &= \tan\left(90^\circ - 70^\circ\right)\tan\left(90^\circ - 55^\circ\right)\tan 45^\circ \tan 55^\circ \tan 70^\circ\\ &= \cot 70^\circ \cot 55^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ\\ &= \left(\tan 70^\circ \cot 70^\circ\right)\left(\tan 55^\circ \cot 55^\circ\right)\tan 45^\circ\\ &= 1\times 1\times 1\\ &= \boxed{1}\end{aligned}$$

Proved

(ii) We will simplify the left hand side

$$\begin{aligned}\sin 48^\circ.\sec 48^\circ + \cos 48^\circ.\text{cosec}42^\circ &= \sin 48^\circ.\sec\left(90^\circ - 48^\circ\right) + \cos 48^\circ.\text{cosec}\left(90^\circ - 48^\circ\right)\\ &= \sin 48.\cos 48^\circ + \cos 48^\circ.\sin 48^\circ\\ &= 1+1\\ &= \boxed{2}\end{aligned}$$

Proved

(iii) We have, $\frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\text{cosec}20^\circ}{\sec 70^\circ} - 2\cos 70^\circ.\text{cosec}20^\circ = 0$

So we will calculate left hand side

$$\begin{aligned}\frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\text{cosec}20^\circ}{\sec 70^\circ} - 2\cos 70^\circ.\text{cosec}20^\circ &= \frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\cos 70^\circ}{\sin 20^\circ} - 2\cos 70^\circ.\text{cosec}\left(90^\circ - 70^\circ\right)\\ &= \frac{\sin\left(90^\circ - 20^\circ\right)}{\cos 20^\circ} + \frac{\cos\left(90^\circ - 20^\circ\right)}{\sin 20^\circ} - 2\cos 70^\circ.\sec 70^\circ\\ &= \frac{\cos 20^\circ}{\cos 20^\circ} + \frac{\sin 20^\circ}{\sin 20^\circ} - 2\times 1\\ &= 1+1-2\\ &= 2-2\\ &= 0\end{aligned}$$

Proved

(iv) We have $\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ.\cos ec31^\circ = 2$

We will simplify the left hand side

$$\begin{aligned}\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ.\text{cosec}31^\circ &= \frac{\cos\left(90^\circ - 10^\circ\right)}{\sin 10} + \cos 59^\circ.\text{cosec}\left(90^\circ - 59^\circ\right)\\ &= \frac{\sin 10^\circ}{\sin 10^\circ} + \cos 59^\circ.\sec 59^\circ\\ &= 1+1\\ &= \boxed{2}\end{aligned}$$

Proved

*********** END **********