



Indefinite Integrals Ex 19.9 Q20

$$\text{Let } I = \int \frac{x^3}{(x^2 + 1)^3} x \, dx \text{ ----- (i)}$$

$$\text{Let } 1 + x^2 = t \quad \text{then,}$$

$$d(1 + x^2) = dt$$

$$\Rightarrow 2x \, dx = dt$$

$$\Rightarrow x \, dx = \frac{dt}{2}$$

Putting  $1 + x^2 = t$  and  $x \, dx = \frac{dt}{2}$  in equation (i), we get

$$\begin{aligned} I &= \int \frac{x^2}{t^3} \times \frac{dt}{2} \\ &= \frac{1}{2} \int \frac{(t-1)}{t^3} dt \quad \left[ \because 1 + x^2 = t \right] \\ &= \frac{1}{2} \int \left[ \frac{t}{t^3} - \frac{1}{t^3} \right] dt \\ &= \frac{1}{2} \int (t^{-2} - t^{-3}) dt \\ &= \frac{1}{2} \left[ -1t^{-1} - \frac{t^{-2}}{-2} \right] + C \\ &= \frac{1}{2} \left[ -\frac{1}{t} + \frac{1}{2t^2} \right] + C \\ &= -\frac{1}{2t} + \frac{1}{4t^2} + C \end{aligned}$$

$$\begin{aligned} &= -\frac{1}{2(1+x^2)} + \frac{1}{4(1+x^2)^2} + C \\ &= \frac{-2(1+x^2) + 1}{4(1+x^2)^2} + C \\ &= \frac{-2 - 2x^2 + 1}{4(1+x^2)^2} + C \\ &= \frac{-2x^2 - 1}{4(1+x^2)^2} + C \\ &= -\frac{(1 + 2x^2)}{4(x^2 + 1)^2} + C \end{aligned}$$

$$\therefore I = \frac{-(1 + 2x^2)}{4(x^2 + 1)^2} + C$$

$$\text{Let } x^2 + x + 1 = t$$

$$(2x + 1)dx = dt$$

$$\int (4x + 2) \sqrt{x^2 + x + 1} \, dx$$

$$= \int 2\sqrt{t} \, dt$$

$$= 2 \int \sqrt{t} \, dt$$

$$= 2 \left( \frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + C$$

$$= \frac{4}{3} (x^2 + x + 1)^{\frac{3}{2}} + C$$

Indefinite Integrals Ex 19.9 Q22

$$\text{Let } I = \int \frac{4x + 3}{\sqrt{2x^3 + 3x + 1}} \, dx \text{ ----- (i)}$$

$$\text{Let } 2x^3 + 3x + 1 = t \quad \text{then,}$$

$$d(2x^3 + 3x + 1) = dt$$

$$\Rightarrow (4x + 3) \, dx = dt$$

Putting  $2x^3 + 3x + 1 = t$  and  $(4x + 3) \, dx = dt$  in equation (i), we get

$$I = \int \frac{dt}{\sqrt{t}}$$

$$= \int t^{-\frac{1}{2}} \, dt$$

$$= 2t^{\frac{1}{2}} + c$$

$$= 2\sqrt{t} + c$$

$$\therefore I = 2\sqrt{2x^3 + 3x + 1} + c$$

Indefinite Integrals Ex 19.9 Q23

$$\text{Let } I = \int \frac{1}{1+\sqrt{x}} dx \text{ ----- (i)}$$

$$\text{Let } x = t^2 \text{ then,}$$

$$dx = d(t^2)$$

$$\Rightarrow dx = 2t dt$$

Putting  $x = t^2$  and  $dx = 2t dt$  in equation (i), we get

$$\begin{aligned} I &= \int \frac{2t}{1+\sqrt{t^2}} dt \\ &= \int \frac{2t}{1+t} dt \\ &= 2 \int \frac{t}{1+t} dt \\ &= 2 \int \frac{1+t-1}{1+t} dt \\ &= 2 \int \left[ \frac{1+t}{1+t} - \frac{1}{1+t} \right] dt \\ &= 2 \int dt - 2 \int \frac{1}{1+t} dt \\ &= 2t - 2 \log(1+t) + c \\ &= 2\sqrt{x} - 2 \log(1+\sqrt{x}) + c \end{aligned}$$

$$\therefore I = 2\sqrt{x} - 2 \log(1+\sqrt{x}) + c$$

Indefinite Integrals Ex 19.9 Q24

$$\text{Let } I = \int e^{\cos^2 x} \sin 2x dx \text{ ----- (i)}$$

$$\text{Let } \cos^2 x = t \text{ then,}$$

$$d(\cos^2 x) = dt$$

$$\Rightarrow -2 \cos x \sin x dx = dt$$

$$\Rightarrow -\sin 2x dx = dt$$

$$\Rightarrow \sin 2x dx = -dt$$

Putting  $\cos^2 x = t$  and  $\sin 2x dx = -dt$  in equation (i), we get

$$\begin{aligned} I &= \int e^t (-dt) \\ &= -e^t + c \\ &= -e^{\cos^2 x} + c \end{aligned}$$

$$\therefore I = -e^{\cos^2 x} + c$$

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