Differentials Errors and Approximation Ex14.1 Q13

Let r be the radius of the sphere and Δr be the error in measuring the radius.

Then,

$r = 9$ m and $\Delta r = 0.03$ m

Now, the surface area of the sphere (S) is given by,

$S = 4\pi r^2$

$$\therefore \frac{dS}{dr} = 8\pi r$$

$$\therefore dS = \left(\frac{dS}{dr}\right)\Delta r$$

$$= (8\pi r)\Delta r$$

$$= 8\pi(9)(0.03)\ \text{m}^2$$

$$= 2.16\pi\ \text{m}^2$$

Hence, the approximate error in calculating the surface area is 2.16π m^2.

Differentials Errors and Approximation Ex14.1 Q14

The surface area of a cube (S) of side x is given by $S = 6x^2$.

$$\therefore \frac{dS}{dx} = \left(\frac{dS}{dx}\right)\Delta x$$

$$= (12x)\Delta x$$

$$= (12x)(0.01x) \qquad [\text{as } 1\% \text{ of } x \text{ is } 0.01x]$$

$$= 0.12x^2$$

Hence, the approximate change in the surface area of the cube is $0.12x^2$ m^2.

Differentials Errors and Approximation Ex14.1 Q15

Let r be the radius of the sphere and Δr be the error in measuring the radius.

Then,

$r = 7$ m and $\Delta r = 0.02$ m

Now, the volume V of the sphere is given by,

$$V = \frac{4}{3}\pi r^3$$
$$\therefore \frac{dV}{dr} = 4\pi r^2$$
$$\therefore dV = \left(\frac{dV}{dr}\right)\Delta r$$
$$= \left(4\pi r^2\right)\Delta r$$
$$= 4\pi(7)^2(0.02)\ \text{m}^3 = 3.92\pi\ \text{m}^3$$

Hence, the approximate error in calculating the volume is $3.92\ \pi\ \text{m}^3$.

Differentials Errors and Approximation Ex14.1 Q16

The volume of a cube (V) of side x is given by $V = x^3$.

$$\therefore dV = \left(\frac{dV}{dx}\right)\Delta x$$
$$= \left(3x^2\right)\Delta x$$
$$= \left(3x^2\right)(0.01x) \qquad [\text{as } 1\% \text{ of } x \text{ is } 0.01x]$$
$$= 0.03x^3$$

Hence, the approximate change in the volume of the cube is $0.03x^3\ \text{m}^3$.

********** END **********