

Trigonometric Identities Ex 6.1 Q64 **Answer:**

We have to prove
$$\frac{\sin A}{\sec A + \tan A - 1} + \frac{\cos A}{\csc A + \cot A - 1} = 1$$

We know that, $\sin^2 A + \cos^2 A = 1$
So,

$$\frac{\sin A}{\sec A + \tan A - 1} + \frac{\cos A}{\csc A + \cot A - 1}$$

$$= \frac{\sin A}{\frac{1}{\cos A} + \frac{\sin A}{\cos A} - 1} + \frac{\cos A}{\frac{1}{\sin A} + \frac{\cos A}{\sin A} - 1}$$

$$= \frac{\sin A}{\frac{1 + \sin A - \cos A}{\cos A}} + \frac{\cos A}{\frac{1 + \cos A - \sin A}{\sin A}}$$

$$= \frac{\sin A \cos A}{1 + \sin A - \cos A} + \frac{\sin A \cos A}{1 + \cos A - \sin A}$$

$$= \frac{\sin A \cos A(1 + \cos A - \sin A) + \sin A \cos A(1 + \sin A - \cos A)}{(1 + \sin A - \cos A)(1 + \cos A - \sin A)}$$

$$= \frac{\sin A \cos A(1 + \cos A - \sin A) + \sin A \cos A(1 + \sin A - \cos A)}{\{1 + (\sin A - \cos A)\}\{1 - (\sin A - \cos A)\}}$$

$$= \frac{2 \sin A \cos A}{1 - (\sin A - \cos A)^2}$$

$$= \frac{2\sin A \cos A}{1 - (\sin^2 A - 2\sin A \cos A + \cos^2 A)}$$

$$= \frac{2\sin A \cos A}{1 - (\sin^2 A + \cos^2 A - 2\sin A \cos A)}$$

$$= \frac{2\sin A \cos A}{1 - (1 - 2\sin A \cos A)}$$

$$= \frac{2\sin A \cos A}{1 - 1 + 2\sin A \cos A}$$

$$= \frac{2\sin A \cos A}{2\sin A \cos A}$$

$$= 1$$

Hence proved.

Answer:

We have to prove
$$\frac{\tan A}{(1+\tan^2 A)^2} + \frac{\cot A}{(1+\cot^2 A)^2} = \sin A \cos A$$
 We know that, $\sin^2 A + \cos^2 A = 1$. So,

$$\frac{\tan A}{(1+\tan^2 A)^2} + \frac{\cot A}{(1+\cot^2 A)^2}$$

$$= \frac{\tan A}{(\sec^2 A)^2} + \frac{\cot A}{(\csc^2 A)^2}$$

$$= \frac{\tan A}{\sec^4 A} + \frac{\cot A}{\csc^4 A}$$

$$= \frac{\sin A}{\cos^4 A} + \frac{\cos A}{\sin^4 A}$$

$$= \frac{\sin A \cos^4 A}{\cos A} + \frac{\cos A \sin^4 A}{\sin A}$$

$$= \sin A \cos^3 A + \cos A \sin^3 A$$

$$= \sin A \cos A(\cos^2 A + \sin^2 A)$$

$$= \sin A \cos A$$

Hence proved.

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