Continuity Ex 9.1 Q36

The given function will be continuous at $x = 0$ if

LHL = RHL = $f(0) \ldots . (1)$

$f(0) = 8 \ldots .. (A)$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{1 - \cos 2k(-h)}{(-h)^2} = \lim_{h \to 0} \frac{1 - \cos 2kh}{h^2} = \lim_{h \to 0} \frac{2 \sin^2 kh}{h^2}$$

$$= \lim_{h \to 0} 2 \left(\frac{\sin kh}{kh} \right)^2 . k^2$$

$$= 2k^2$$

Thus, using (1) we get,

$2k^2 = 8 \quad \Rightarrow k^2 = 4 \quad \Rightarrow k = \pm 2$

Hence, $k = \pm 2$

Let $x - 1 = y$

$\Rightarrow x = y + 1$

Thus,

$$\lim_{x \to 1} (x - 1)\tan\frac{\pi x}{2} = \lim_{y \to 0} y\tan\frac{\pi(y + 1)}{2}$$

$$= \lim_{y \to 0} y\tan\left(\frac{\pi y}{2} + \frac{\pi}{2}\right)$$

$$= -\lim_{y \to 0} y\cot\frac{\pi y}{2}$$

$$= -\lim_{y \to 0} y\frac{\cos\frac{\pi y}{2}}{\sin\frac{\pi y}{2}}$$

$$= -\lim_{y \to 0} y\frac{\cos\frac{\pi y}{2}}{\frac{\left(\sin\frac{\pi y}{2}\right)\frac{\pi}{2}}{\frac{\pi}{2}}}$$

$$= -\lim_{y \to 0} \frac{\cos\frac{\pi y}{2}}{\frac{\left(\sin\frac{\pi y}{2}\right)\frac{\pi}{2}}{\frac{\pi y}{2}}}$$

$$= -\lim_{y \to 0} \frac{2}{\pi}\frac{\cos\frac{\pi y}{2}}{\frac{\left(\sin\frac{\pi y}{2}\right)}{\frac{\pi y}{2}}}$$

$$= -\frac{2}{\pi}\lim_{y \to 0} \cos\frac{\pi y}{2}$$

$$= -\frac{2}{\pi}$$

Since the function is continuous, $L.H.Limit = R.H.Limit$

Thus, $k = -\dfrac{2}{\pi}$

Since the function is continuous at every point, therefore

$$LHL = RHL = f(0)$$

Now

$$f(0) = \cos 0$$
$$= 1$$

Again

$$LHL = \lim_{x \to 0} k\left(x^2 - 2x\right)$$
$$= \lim_{h \to 0^+} k\left(h^2 - 2h\right)$$
$$= 0$$

Therefore there is no value of k

Since the function is continuous at every point, therefore

$$LHL = RHL = f(\pi)$$

Now

$$f(\pi) = k\pi + 1$$

Again

$$RHL = \lim_{x \to \pi^+} \cos x$$
$$= \lim_{h \to 0^+} \cos(\pi - h)$$
$$= -\lim_{h \to 0^+} \cosh$$
$$= -1$$

Therefore we can write

$$k\pi + 1 = -1$$
$$k = -\frac{2}{\pi}$$

We are given that function is continuous at $x = 5$.

$\therefore$ LHL = RHL = $f(5)$.... (1)

$f(5) = 5k + 1$

LHL = $\lim_{x \to 5^+} f(x) = \lim_{h \to 0} f(5+h) = \lim_{h \to 0} 3(5+h) - 5 = 10$

Thus, using (1), we get,

$5k + 1 = 10$

$5k = 9$

$k = \frac{9}{5}$

We know that the function will be continuous at $x = 5$. if

LHL = RHL = $f(5)$..... (1)

$f(5) = k$

LHL = $\lim_{x \to 5^-} f(x) = \lim_{h \to 0} f(5-h) = \lim_{h \to 0} \frac{(5-h)^2 - 25}{(5-h) - 5} = \lim_{h \to 0} \frac{h^2 - 10h}{-h} = \lim_{h \to 0} -h + 10 = 10$

Thus, using (1), we get,

$k = 10$

We know that a function will be continuous at $x = 1$. if

LHL = RHL = $f(1)$ .... (1)

$f(1) = k.1^2 = k$

LHL = $\lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} 4 = 4$

Thus, using (1), we get,

$k = 4$

We know that a function will be continuous at $x = 0$. if

LHL = RHL = $f(0) \ldots\ldots (1)$

$$f(0) = k(0+2) = 2k$$

$$\text{LHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} 3(h) + 1 = 1$$

Thus, using (1), we get,

$2k = 1$

$$k = \frac{1}{2}$$

*********** END **********