Definite Integrals Ex 20.1 Q13

$$\text{Let } I=\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \, dx$$

$$\int \operatorname{cosec} x \, dx=\log |\operatorname{cosec} x-\cot x|=\mathrm{F}(x)$$

By second fundamental theorem of calculus, we obtain

$$\begin{aligned}
I &= \mathrm{F}\left(\frac{\pi}{4}\right)-\mathrm{F}\left(\frac{\pi}{6}\right) \\
&= \log \left|\operatorname{cosec} \frac{\pi}{4}-\cot \frac{\pi}{4}\right|-\log \left|\operatorname{cosec} \frac{\pi}{6}-\cot \frac{\pi}{6}\right| \\
&= \log |\sqrt{2}-1|-\log |2-\sqrt{3}| \\
&= \log \left(\frac{\sqrt{2}-1}{2-\sqrt{3}}\right)
\end{aligned}$$

Definite Integrals Ex 20.1 Q14

We have,

$$\int_0^1 \frac{1-x}{1+x}dx$$

Let $x = \cos 2\theta \Rightarrow dx = -2\sin 2\theta d\theta$

Now,

$$x = 0 \Rightarrow \theta = \frac{\pi}{4}$$
$$x = 1 \Rightarrow \theta = 0$$

Now,

$$\int_0^1 \frac{1-x}{1+x}dx$$
$$= \int_{\frac{\pi}{4}}^{0} \frac{1-\cos 2\theta}{1+\cos 2\theta} \times (-2\sin 2\theta)d\theta$$
$$= \int_0^{\frac{\pi}{4}} \frac{2\sin^2\theta}{2\cos^2\theta} \times 2\sin 2\theta d\theta \qquad \left[\because -\int_a^b f(x)dx = \int_b^a f(x)dx\right]$$
$$= \int_0^{\frac{\pi}{4}} \frac{4\sin^3\theta}{\cos\theta}d\theta$$

Let $\cos\theta = t$

$\Rightarrow -\sin\theta d\theta = dt$

Now,

$\theta = 0 \Rightarrow t = 1$

$\theta = \frac{\pi}{4} \Rightarrow t = \frac{1}{\sqrt{2}}$

$$\therefore \int_0^{\frac{\pi}{4}} \frac{4\sin^3\theta}{\cos\theta}d\theta$$
$$= -4\int_1^{\frac{1}{\sqrt{2}}} \frac{(1-t^2)}{t}dt$$
$$= -4\left[\log t - \frac{t^2}{2}\right]_1^{\frac{1}{\sqrt{2}}}$$
$$= -4\left[\log\left(\frac{1}{\sqrt{2}}\right) - \frac{1}{4} - 0 + \frac{1}{2}\right]$$
$$= -4\left[-\log\sqrt{2} + \frac{1}{4}\right]$$

$$\therefore \int_0^1 \frac{1-x}{1+x}dx = 2\log 2 - 1$$

Definite Integrals Ex 20.1 Q15

$$I = \int_0^{\pi} \frac{1}{1+\sin x} dx$$

Multiplying Numerator and Denominator by $(1-\sin x)$

$$\begin{aligned}
I &= \int_0^{\pi} \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x} dx \\
&= \int_0^{\pi} \frac{\left(1-\sin x\right)}{\left(1^2-\sin^2 x\right)} dx \\
&= \int_0^{\pi} \frac{1-\sin x}{\left(\cos^2 x\right)} dx \\
&= \int_0^{\pi} \frac{1}{\cos^2 x} dx - \int_0^{\pi} \frac{\sin x}{\cos^2 x} dx \\
&= \int_0^{\pi} \sec^2 x dx - \int_0^{\pi} \tan x . \sec x dx \\
&= \left[\tan x\right]_0^{\pi} - \left[\sec x\right]_0^{\pi} \\
&= \left[\tan \pi - \tan 0\right] - \left[\sec \pi - \sec 0\right] \\
&= \left[0-0\right] - \left[-1-1\right] \\
&= 2 \\
I &= 2
\end{aligned}$$

$$\therefore \int_0^{\pi} \frac{1}{1+\sin x} dx = 2$$

*********** END **********