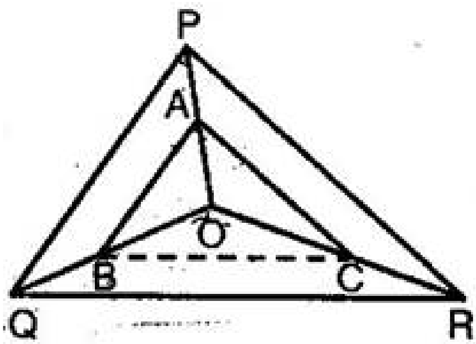
Exercise 6.2

6. In figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.



Ans. Given: O is any point in ΔPQR, in which AB || PQ and AC || PR.

To prove: BC || QR

Construction: Join BC.

Proof: In ΔOPQ, AB || PQ

$\therefore \frac{OA}{AP} = \frac{OB}{PQ}$ [Basic Proportionality theorem] ...(i)

And in ΔOPR, AC || PR

$\therefore \frac{OA}{AP} = \frac{OC}{CR}$ [Basic Proportionality theorem] ....(ii)

From eq. (i) and (ii), we have

$\frac{OB}{BQ} = \frac{OC}{CR}$

$\because$ In ΔOQR, B and C are points dividing the sides OQ and OR in the same ratio.

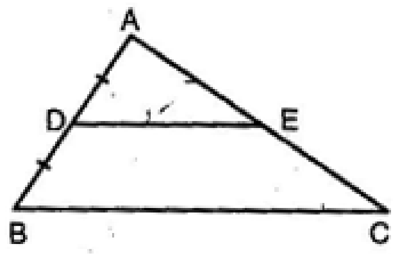
$\therefore$ By the converse of Basic Proportionality theorem,

$\Rightarrow$ BC || QR

7. Using Theorem 6.1, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Ans. Given: A triangle ABC, in which D is the mid-point of side AB

and the line DE is drawn parallel to BC, meeting AC at E.



To prove: AE = EC

Proof: Since DE || BC

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ [Basic Proportionality theorem] .... (i)

But AD = DB [Given]

$\Rightarrow \frac{AD}{DB} = 1$

$\Rightarrow \frac{AE}{EC} = 1$ [From eq. (i)]

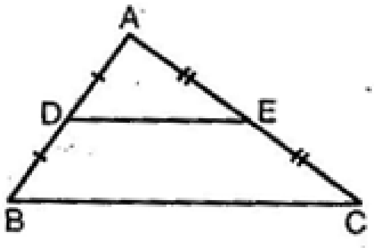
$\Rightarrow$ AE = EC

Hence, E is the mid-point of the third side AC.

8. Using Theorem 6.2, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Ans. Given: A triangle ABC, in which D and E are the mid-points of

sides AB and AC respectively.



To Prove: DE $\parallel$ BC

Proof: Since D and E are the mid-points of AB and AC

respectively.

$\therefore$ AD = DB and AE = EC

Now, AD = DB

$\Rightarrow \dfrac{AD}{DB} = 1$ and AE = EC

$\Rightarrow \dfrac{AE}{EC} = 1$

$\therefore \dfrac{AD}{DB} = \dfrac{AE}{EC} = 1$

$\Rightarrow \dfrac{AD}{DB} = \dfrac{AE}{EC}$

Thus, in triangle ABC, D and E are points dividing the sides AB and AC in the same ratio.

Therefore, by the converse of Basic Proportionality theorem, we have

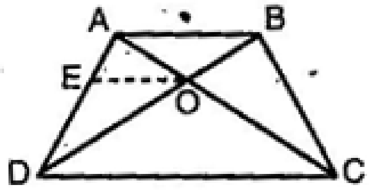
DE $\parallel$ BC

9. ABCD is a trapezium in which AB $\parallel$ DC and its diagonals intersect each other at the point O. Show that $\dfrac{AO}{BO} = \dfrac{CO}{DO}$.

Ans. Given: A trapezium ABCD, in which AB $\parallel$ DC and its diagonals

AC and BD intersect each other at O.

AC and BD intersect each other at O.



To Prove: $\frac{AO}{BO}=\frac{CO}{DO}$

Construction: Through O, draw OE || AB, i.e. OE || DC.

Proof: In ΔADC, we have OE || DC

$\therefore \frac{AE}{ED}=\frac{AO}{CO}$ [By Basic Proportionality theorem] ……….(i)

Again, in ΔABD, we have OE || AB[Construction]

$\therefore \frac{ED}{AE}=\frac{DO}{BO}$ [By Basic Proportionality theorem]

$\Rightarrow \frac{AE}{ED}=\frac{BO}{DO}$ ……….(ii)

From eq. (i) and (ii), we get

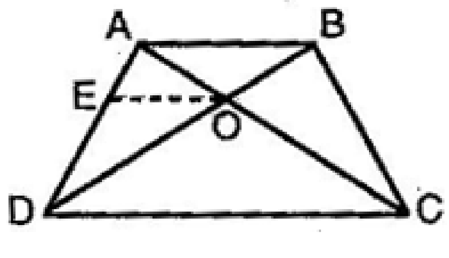
$\frac{AO}{CO}=\frac{BO}{DO}$

$\Rightarrow \frac{AO}{BO}=\frac{CO}{DO}$

10. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO}=\frac{CO}{DO}$. Such that ABCD is a trapezium.

Ans. Given: A quadrilateral ABCD, in which its diagonals AC and

BD intersect each other at O such that $\frac{AO}{BO}=\frac{CO}{DO}$, i.e.



$\frac{AO}{CO} = \frac{BO}{DO}$.

To Prove: Quadrilateral ABCD is a trapezium.

Construction: Through O, draw OE $\parallel$ AB meeting AD at E.

Proof: In ΔADB, we have OE $\parallel$ AB [By construction]

$\therefore \frac{DE}{EA} = \frac{OD}{BO}$ [By Basic Proportionality theorem]

$\Rightarrow \frac{EA}{DE} = \frac{BO}{DO}$

$\Rightarrow \frac{EA}{DE} = \frac{BO}{DO} = \frac{AO}{CO}$

$\left[\because \frac{AO}{CO} = \frac{BO}{DO}\right]$

$\Rightarrow \frac{EA}{DE} = \frac{AO}{CO}$

Thus in ΔADC, E and O are points dividing the sides AD and AC in the same ratio. Therefore by the converse of Basic Proportionality theorem, we have

EO $\parallel$ DC

But EO $\parallel$ AB[By construction]

$\therefore$ AB $\parallel$ DC

$\therefore$ Quadrilateral ABCD is a trapezium.

*********** END *********