



Higher Order Derivatives Ex 12.1 Q1(i)

We have $f(x) = x^3 + \tan x$

$$\Rightarrow f'(x) = 3x^2 + \sec^2 x$$

$$\Rightarrow f''(x) = 6x + 2 \sec x \times \sec x \tan x$$

$$\Rightarrow f''(x) = 6x + 2 \sec^2 x \tan x.$$

Higher Order Derivatives Ex 12.1 Q1(ii)

Let $y = \sin(\log x)$

Then,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} [\sin(\log x)] = \cos(\log x) \cdot \frac{d}{dx} (\log x) = \frac{\cos(\log x)}{x} \\ \therefore \frac{d^2 y}{dx^2} &= \frac{d}{dx} \left[\frac{\cos(\log x)}{x} \right] \\ &= \frac{x \cdot \frac{d}{dx} [\cos(\log x)] - \cos(\log x) \cdot \frac{d}{dx} (x)}{x^2} \\ &= \frac{x \cdot \left[-\sin(\log x) \cdot \frac{d}{dx} (\log x) \right] - \cos(\log x) \cdot 1}{x^2} \\ &= \frac{-x \sin(\log x) \cdot \frac{1}{x} - \cos(\log x)}{x^2} \\ &= \frac{-[\sin(\log x) + \cos(\log x)]}{x^2} \end{aligned}$$

Higher Order Derivatives Ex 12.1 Q1(iii)

Let $y = \log(\sin x)$

Differentiating with respect to x , we get,

$$\frac{dy}{dx} = \frac{\cos x}{\sin x}$$

Again differentiating with respect to x , we get,

$$\frac{d^2y}{dx^2} = \frac{-\sin x \times \sin x - \cos x \times \cos x}{\sin^2 x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-1}{\sin^2 x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\operatorname{cosec}^2 x$$

Higher Order Derivatives Ex 12.1 Q1(iv)

Let $y = e^x \sin 5x$

Then,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(e^x \sin 5x) = \sin 5x \cdot \frac{d}{dx}(e^x) + e^x \frac{d}{dx}(\sin 5x) \\ &= \sin 5x \cdot e^x + e^x \cdot \cos 5x \cdot \frac{d}{dx}(5x) = e^x \sin 5x + e^x \cos 5x \cdot 5 \\ &= e^x (\sin 5x + 5 \cos 5x) \\ \therefore \frac{d^2y}{dx^2} &= \frac{d}{dx}[e^x (\sin 5x + 5 \cos 5x)] \\ &= (\sin 5x + 5 \cos 5x) \cdot \frac{d}{dx}(e^x) + e^x \cdot \frac{d}{dx}(\sin 5x + 5 \cos 5x) \\ &= (\sin 5x + 5 \cos 5x) e^x + e^x \left[\cos 5x \cdot \frac{d}{dx}(5x) + 5(-\sin 5x) \cdot \frac{d}{dx}(5x) \right] \\ &= e^x (\sin 5x + 5 \cos 5x) + e^x (5 \cos 5x - 25 \sin 5x) \\ &= e^x (10 \cos 5x - 24 \sin 5x) = 2e^x (5 \cos 5x - 12 \sin 5x)\end{aligned}$$

Higher Order Derivatives Ex 12.1 Q1(v)

Let $y = e^{6x} \cos 3x$

Then,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(e^{6x} \cdot \cos 3x) = \cos 3x \cdot \frac{d}{dx}(e^{6x}) + e^{6x} \cdot \frac{d}{dx}(\cos 3x) \\ &= \cos 3x \cdot e^{6x} \cdot \frac{d}{dx}(6x) + e^{6x} \cdot (-\sin 3x) \cdot \frac{d}{dx}(3x) \\ &= 6e^{6x} \cos 3x - 3e^{6x} \sin 3x \quad \dots (1) \\ \therefore \frac{d^2y}{dx^2} &= \frac{d}{dx}(6e^{6x} \cos 3x - 3e^{6x} \sin 3x) = 6 \cdot \frac{d}{dx}(e^{6x} \cos 3x) - 3 \cdot \frac{d}{dx}(e^{6x} \sin 3x) \\ &= 6 \cdot [6e^{6x} \cos 3x - 3e^{6x} \sin 3x] - 3 \cdot \left[\sin 3x \cdot \frac{d}{dx}(e^{6x}) + e^{6x} \cdot \frac{d}{dx}(\sin 3x) \right] \quad [\text{Using (1)}] \\ &= 36e^{6x} \cos 3x - 18e^{6x} \sin 3x - 3 \left[\sin 3x \cdot e^{6x} \cdot 6 + e^{6x} \cdot \cos 3x \cdot 3 \right] \\ &= 36e^{6x} \cos 3x - 18e^{6x} \sin 3x - 18e^{6x} \sin 3x - 9e^{6x} \cos 3x \\ &= 27e^{6x} \cos 3x - 36e^{6x} \sin 3x \\ &= 9e^{6x} (3 \cos 3x - 4 \sin 3x)\end{aligned}$$

***** END *****

