Adjoint and Inverse of Matrix Ex 7.1 Q15

Here

$$(AB)^{-1} = B^{-1}A^{-1}$$

Now we need to find A^{-1}.

We have

$$A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

So,

$$|A| = -5 + 4 = -1$$

Co-factors of A are

$$C_{11} = -1 \qquad C_{21} = 8 \qquad C_{31} = -12$$
$$C_{12} = 0 \qquad C_{22} = 1 \qquad C_{32} = -2$$
$$C_{13} = 1 \qquad C_{33} = -10 \qquad C_{33} = 15$$

Therefore,

$$adjA = \begin{bmatrix} -1 & 8 & -12 \\ 0 & 1 & -2 \\ 1 & -10 & 15 \end{bmatrix}$$

So,

$$A^{-1} = \frac{1}{|A|} adjA = \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$$

Hence,

$$(AB)^{-1} = B^{-1}A^{-1}$$
$$= \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$$
$$= \begin{bmatrix} -2 & 19 & -27 \\ -2 & 18 & -25 \\ -3 & 29 & -42 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q16(i)

$$F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow |F(\alpha)| = \cos^2\alpha + \sin^2\alpha = 1$$

$C_{11} = \cos\alpha \quad C_{21} = +\sin\alpha \quad C_{31} = 0$

$C_{12} = -\sin\alpha \quad C_{22} = \cos\alpha \quad C_{32} = 0$

$C_{13} = 0 \quad C_{23} = 0 \quad C_{33} = 1$

$$[F(\alpha)]^{-1} = \frac{adj(F(\alpha))}{|F(\alpha)|} = \frac{1}{1}\begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad ---(1)$$

Now

$$F(-\alpha) = \begin{bmatrix} \cos(-\alpha) & -\sin(-\alpha) & 0 \\ \sin(-\alpha) & \cos(-\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad ---(2)$$

From (1) & (2) $F(-\alpha) = [F(\alpha)]^{-1}$

Hence, proved

Adjoint and Inverse of Matrix Ex 7.1 Q16(ii)

$$G(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \Rightarrow |G(\beta)| = \cos^2\beta + \sin^2$$

$C_{11} = \cos\beta \quad C_{21} = 0 \quad C_{31} = \sin\beta$

$C_{12} = +0 \quad C_{22} = 1 \quad C_{32} = 0$

$C_{13} = \sin\beta \quad C_{23} = 0 \quad C_{33} = \cos\beta$

$$[G(\beta)]^{1} = \frac{adj(G(\beta))}{|G(\beta)|} = \frac{1}{1}\begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \qquad ---(1)$$

Now

$$G(-\beta) = \begin{bmatrix} \cos(-\beta) & 0 & \sin(-\beta) \\ 0 & 1 & 0 \\ -\sin(-\beta) & 0 & \cos(-\beta) \end{bmatrix}$$

$$= \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \qquad ---(2)$$

From (1) & (2)

$[G(\beta)]^{1} = G(-\beta)$

Adjoint and Inverse of Matrix Ex 7.1 Q16(iii)

We have to show that

$$\left[F(\alpha)G(\beta)\right]^{-1} = G(-\beta)F(-\alpha)$$

We have already shown that

$$G(-\beta) = \left[G(\beta)\right]^{-1}$$

and $F(-\beta) = \left[F(\beta)\right]^{-1}$

$\therefore$ LHS $= \left[F(\alpha)G(\beta)\right]^{-1}$

$= \left[G(\beta)\right]^{-1}\left[F(\alpha)\right]^{-1}$ $\quad\left[\because (AB)^{-1} = B^{-1}A^{-1}\right]$

$= G(-\beta) \times F(-\alpha)$

$=$ RHS

Adjoint and Inverse of Matrix Ex 7.1 Q17

We have $A^2 = A.A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$

Hence $A^2 - 4A + I = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} - 4\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 7-8+1 & 12-12+0 \\ 4-4+0 & 7-8+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

Now, $A^2 - 4A + I = O$

$\Rightarrow A.A - 4A = -I$

Post multiplying both sides by A^{-1}, since $|A| \neq 0$

$AA\left(A^{-1}\right) - 4AA^{-1} = -IA^{-1}$

$\Rightarrow A\left(AA^{-1}\right) - 4I = -A^{-1}$

$\Rightarrow AI - 4I = -A^{-1}$

or $A^{-1} = 4I - A = 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4-2 & 0-3 \\ 0-1 & 4-2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

*********** END **********