∴ Area BCAB = Area (OBCAO) − Area (OBAO)

$$= \int_0^3 2\sqrt{1-\frac{x^2}{9}}\,dx - \int_0^3 2\left(1-\frac{x}{3}\right)dx$$

$$= \frac{2}{3}\left[\int_0^3 \sqrt{9-x^2}\,dx\right] - \frac{2}{3}\int_0^3 (3-x)\,dx$$

$$= \frac{2}{3}\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3 - \frac{2}{3}\left[3x-\frac{x^2}{2}\right]_0^3$$

$$= \frac{2}{3}\left[\frac{9}{2}\left(\frac{\pi}{2}\right)\right] - \frac{2}{3}\left[9-\frac{9}{2}\right]$$

$$= \frac{2}{3}\left[\frac{9\pi}{4}-\frac{9}{2}\right]$$

$$= \frac{2}{3}\times\frac{9}{4}(\pi-2)$$

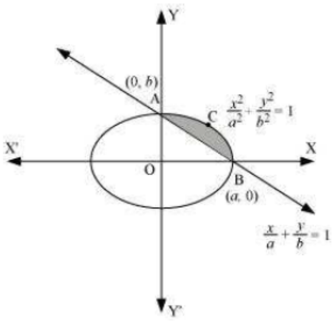
$$= \frac{3}{2}(\pi-2) \text{ units}$$

Question 9:

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1$

Answer

The area of the smaller region bounded by the ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, and the line, $\frac{x}{a}+\frac{y}{b}=1$, is represented by the shaded region BCAB as



∴ Area BCAB = Area (OBCAO) − Area (OBAO)

$$= \int_0^a b\sqrt{1-\frac{x^2}{a^2}}\,dx - \int_0^a b\left(1-\frac{x}{a}\right)dx$$

$$= \frac{b}{a}\int_0^a \sqrt{a^2-x^2}\,dx - \frac{b}{a}\int_0^a (a-x)\,dx$$

$$= \frac{b}{a}\left[\left\{\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right\}_0^a - \left\{ax-\frac{x^2}{2}\right\}_0^a\right]$$

$$= \frac{b}{a}\left[\left\{\frac{a^2}{2}\left(\frac{\pi}{2}\right)\right\} - \left\{a^2-\frac{a^2}{2}\right\}\right]$$

$$= \frac{b}{a}\left[\frac{a^2\pi}{4}-\frac{a^2}{2}\right]$$

$$= \frac{ba^2}{2a}\left[\frac{\pi}{2}-1\right]$$

$$= \frac{ab}{2}\left[\frac{\pi}{2}-1\right]$$

$$= \frac{ab}{4}(\pi-2)$$

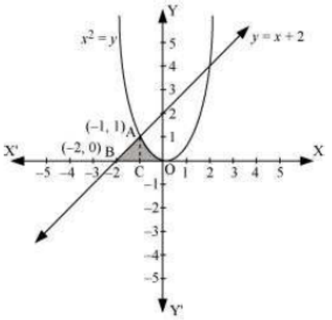
Question 10:

Find the area of the region enclosed by the parabola $x^2 = y$, the line $y = x + 2$ and x-axis

axis

Answer

The area of the region enclosed by the parabola, $x^2 = y$, the line, $y = x + 2$, and x-axis is represented by the shaded region OABCO as



The point of intersection of the parabola, $x^2 = y$, and the line, $y = x + 2$, is A (−1, 1).

∴ Area OABCO = Area (BCA) + Area COAC

$$= \int_{-2}^{-1}(x+2)dx + \int_{-1}^{0} x^2 dx$$
$$= \left[\frac{x^2}{2} + 2x\right]_{-2}^{-1} + \left[\frac{x^3}{3}\right]_{-1}^{0}$$
$$= \left[\frac{(-1)^2}{2} + 2(-1) - \frac{(-2)^2}{2} - 2(-2)\right] + \left[-\frac{(-1)^3}{3}\right]$$
$$= \left[\frac{1}{2} - 2 - 2 + 4 + \frac{1}{3}\right]$$
$$= \frac{5}{6} \text{ units}$$

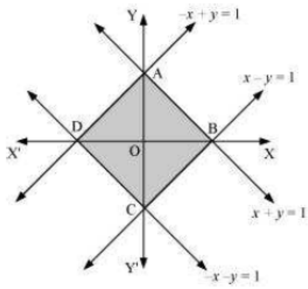
Question 11:

Using the method of integration find the area bounded by the curve $|x| + |y| = 1$

[**Hint:** the required region is bounded by lines $x + y = 1$, $x - y = 1$, $-x + y = 1$ and $-x - y = 11$]

Answer

The area bounded by the curve, $|x| + |y| = 1$, is represented by the shaded region ADCB as



The curve intersects the axes at points A (0, 1), B (1, 0), C (0, −1), and D (−1, 0).

It can be observed that the given curve is symmetrical about x-axis and y-axis.

∴ Area ADCB = 4 × Area OBAO

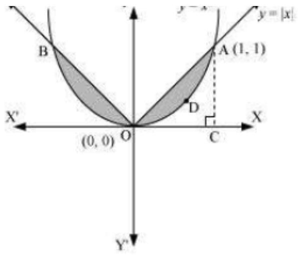
$$= 4\int_0^1 (1-x)dx$$
$$= 4\left(x - \frac{x^2}{2}\right)_0^1$$
$$= 4\left[1 - \frac{1}{2}\right]$$
$$= 4\left(\frac{1}{2}\right)$$
$$= 2 \text{ units}$$

Question 12:

Find the area bounded by curves $\{(x, y) : y \geq x^2 \text{ and } y = |x|\}$

Answer

The area bounded by the curves, $\{(x, y) : y \geq x^2 \text{ and } y = |x|\}$, is represented by the shaded region as



It can be observed that the required area is symmetrical about y-axis.

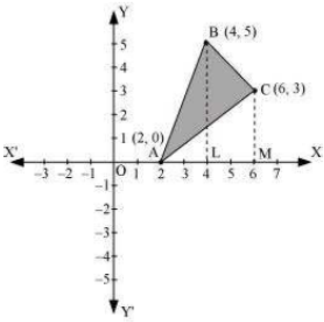
$$\text{Required area} = 2\left[\text{Area}(\text{OCAO}) - \text{Area}(\text{OCADO})\right]$$
$$= 2\left[\int_0^1 x\,dx - \int_0^1 x^2\,dx\right]$$
$$= 2\left[\left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1\right]$$
$$= 2\left[\frac{1}{2} - \frac{1}{3}\right]$$
$$= 2\left[\frac{1}{6}\right] = \frac{1}{3}\ units$$

Question 13:

Using the method of integration find the area of the triangle ABC, coordinates of whose vertices are A (2, 0), B (4, 5) and C (6, 3)

Answer

The vertices of ΔABC are A (2, 0), B (4, 5), and C (6, 3).



Equation of line segment AB is

$$y - 0 = \frac{5-0}{4-2}(x-2)$$
$$2y = 5x - 10$$
$$y = \frac{5}{2}(x-2) \qquad ...(1)$$

Equation of line segment BC is

$$y - 5 = \frac{3-5}{6-4}(x-4)$$
$$2y - 10 = -2x + 8$$
$$2y = -2x + 18$$
$$y = -x + 9 \qquad ...(2)$$

Equation of line segment CA is

$$y - 3 = \frac{0-3}{2-6}(x-6)$$
$$-4y + 12 = -3x + 18$$
$$4y = 3x - 6$$
$$y = \frac{3}{4}(x-2) \qquad ...(3)$$

Area (ΔABC) = Area (ABLA) + Area (BLMCB) – Area (ACMA)

$$= \int_2^4 \frac{5}{2}(x-2)\,dx + \int_4^6 (-x+9)\,dx - \int_2^6 \frac{3}{4}(x-2)\,dx$$
$$= \frac{5}{2}\left[\frac{x^2}{2} - 2x\right]_2^4 + \left[\frac{-x^2}{2} + 9x\right]_4^6 - \frac{3}{4}\left[\frac{x^2}{2} - 2x\right]_2^6$$
$$= \frac{5}{2}[8 - 8 - 2 + 4] + [-18 + 54 + 8 - 36] - \frac{3}{4}[18 - 12 - 2 + 4]$$
$$= 5 + 8 - \frac{3}{4}(8)$$
$$= 13 - 6$$
$$= 7\text{ units}$$

*********** END **********