Algebraic Identities Ex 4.5 Q4

Answer :

In the given problem, we have to find value of $a^3+b^3+c^3-3abc$

Given $a+b+c=9, a^2+b^2+c^2=35$

We shall use the identity

$$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$$

$$(a+b+c)^2=35+2(ab+bc+ca)$$

$$(9)^2=35+2(ab+bc+ca)$$

$$81-35=2(ab+bc+ca)$$

$$\frac{46}{2}=ab+bc+ca$$

$$23=ab+bc+ca$$

We know that

$$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

$$a^3+b^3+c^3-3abc=(a+b+c)\left[(a^2+b^2+c^2)-(ab+bc+ca)\right]$$

Here substituting $a+b+c=9, a^2+b^2+c^2=35, ab+bc+ca=23$ we get

$$a^3+b^3+c^3-3abc=9\left[(35-23)\right]$$

$$=9\times 12$$

$$=108$$

Hence the value of $a^3+b^3+c^3-3abc$ is $\boxed{108}$.

Algebraic Identities Ex 4.5 Q5

Answer :

In the given problem we have to evaluate the following

(i) Given $25^3-75^3+50^3$

We shall use the identity $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

Let Take $a=25, b=75, c=50$

$$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

$$a^3+b^3+c^3=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)+3abc$$

$$a^3+b^3+c^3=(-75+25+50)(a^2+b^2+c^2-ab-bc-ca)+3abc$$

$$a^3+b^3+c^3=0\times(a^2+b^2+c^2-ab-bc-ca)+3abc$$

$$a^3+b^3+c^3=+3abc$$

$$25^3-75^3+50^3=3\times 25\times 50\times -75$$

$$=-281250$$

Hence the value of $25^3-75^3+50^3$ is $\boxed{-281250}$

(ii) Given $48^3 - 30^3 - 18^3$

We shall use the identity $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Let Take $a = 48, b = -30, c = -18$

$$a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$a^3 + b^3 + c^3 = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$$

$$a^3 + b^3 + c^3 = (48 - 30 - 18)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$$

$$a^3 + b^3 + c^3 = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$$

$$a^3 - b^3 - c^3 = +3abc$$

$$48^3 - 30^3 - 18^3 = 3 \times 48 \times -30 \times -18$$

$$= 77760$$

Hence the value of $25^3 - 75^3 + 50^3$ is $\boxed{77760}$

Applying least common multiple we get,

$$a^3 + b^3 + c^3 = \left(\frac{1}{2} + \frac{1}{3} - \frac{5}{6}\right)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$$

$$a^3 + b^3 + c^3 = \left(\frac{1 \times 6}{2 \times 6} + \frac{1 \times 4}{3 \times 4} - \frac{5}{6}\right)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$$

$$a^3 + b^3 + c^3 = \frac{6}{12} + \frac{4}{12} - \frac{10}{12}(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$$

$$a^3 + b^3 + c^3 = 0(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$$

$$a^3 + b^3 + c^3 = +3abc$$

$$\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3 = 3 \times \frac{1}{2} \times \frac{1}{3} \times -\frac{5}{6}$$

$$= \cancel{3} \times \frac{1}{2} \times \frac{1}{\cancel{3}} \times -\frac{5}{6}$$

$$= -\frac{5}{12}$$

Hence the value of $\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3$ is $\boxed{-\frac{5}{12}}$

(iv) Given $(0.2)^3 - (0.3)^3 + (0.1)^3$

We shall use the identity $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Let Take $a = 0.2, b = 0.3, c = 0.1$

$$a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$a^3 + b^3 + c^3 = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$$

$$a^3 + b^3 + c^3 = (0.2 - 0.3 + 0.1)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$$

$$a^3 + b^3 + c^3 = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$$

$$a^3 + b^3 + c^3 = +3abc$$

$$(0.2)^3 - (0.3)^3 + (0.1)^3 = 3 \times 0.2 \times 0.3 \times 0.1$$

$$= -0.018$$

Hence the value of $(0.2)^3 - (0.3)^3 + (0.1)^3$ is $\boxed{-0.018}$

********** END **********