Differentiation Ex 11.2 Q47

Let $y = \left(\sin^{-1} x^4\right)^4$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sin^{-1} x^4\right)^4$$

$$= 4\left(\sin^{-1} x^4\right)\frac{d}{dx}\left(\sin^{-1} x^4\right) \qquad \text{[Using chain rule]}$$

$$= 4\left(\sin^{-1} x^4\right)^3 \frac{1}{\sqrt{1-\left(x^4\right)^2}}\frac{d}{dx}\left(x^4\right)$$

$$= 4\left(\sin^{-1} x^4\right)^3 \frac{4x^3}{\sqrt{1-x^8}}$$

$$= \frac{16x^3\left(\sin^{-1} x^4\right)^3}{\sqrt{1-x^8}}$$

So,

$$\frac{d}{dx}\left(\sin^{-1} x^4\right) = \frac{16x^3\left(\sin^{-1} x^4\right)^3}{\sqrt{1-x^8}}.$$

Differentiation Ex 11.2 Q48

Let $y = \sin^{-1}\left(\frac{x}{\sqrt{x^2+a^2}}\right)$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\sin^{-1}\left(\frac{x}{\sqrt{x^2+a^2}}\right)$$

$$= \frac{1}{\sqrt{1-\left(\frac{x}{\sqrt{x^2+a^2}}\right)^2}} \times \frac{d}{dx}\left(\frac{x}{\sqrt{x^2+a^2}}\right) \qquad \text{[Using chain rule and quotient rule]}$$

$$= \frac{1}{\sqrt{1-\left(\frac{x}{\sqrt{x^2+a^2}}\right)^2}} \times \left[\frac{\left(x^2+a^2\right)^{\frac{1}{2}}\frac{d}{dx}(x) - \frac{d}{dx}\left(x^2+a^2\right)^{\frac{1}{2}}}{\left[\left(x^2+a^2\right)^{\frac{1}{2}}\right]^2}\right]$$

$$= \frac{\sqrt{x^2+a^2}}{\sqrt{x^2+a^2-x^2}}\left[\frac{\sqrt{x^2+a^2} - x \times \frac{1}{2\sqrt{x^2+a^2}}\frac{d}{dx}\left(x^2+a^2\right)}{\left(x^2+a^2\right)}\right]$$

$$= \frac{\sqrt{x^2+a^2}}{a\left(x^2+a^2\right)}\left[\sqrt{x^2+a^2} - \frac{x}{2\sqrt{x^2+a^2}} \times 2x\right]$$

$$= \frac{\sqrt{x^2+a^2}}{a\left(x^2+a^2\right)}\left[\frac{x^2+a^2-x^2}{\sqrt{x^2+a^2}}\right]$$

$$= \frac{a^2}{a\left(x^2+a^2\right)}$$

$$= \frac{a}{\left(a^2+x^2\right)}$$

So,

$$\frac{d}{dx}\left(\sin^{-1}\frac{x}{\sqrt{x^2+a^2}}\right) = \frac{a}{a^2+x^2}$$

Differentiation Ex 11.2 Q49

Consider

$$y=\frac{e^x\sin x}{\left(x^2+2\right)^3}$$

Differentiating it with respect toxand applying the chain and product rule, we get

$$\frac{dy}{dx}=\frac{\left(x^2+2\right)^3\frac{d}{dx}\left(e^x\sin x\right)-e^x\sin x\frac{d}{dx}\left(x^2+2\right)^3}{\left[\left(x^2+2\right)^3\right]^2}$$

$$=\frac{\left(x^2+2\right)^3\left[e^x\cos x+\sin xe^x\right]-e^x\sin x3\left(x^2+2\right)^2(2x)}{\left(x^2+2\right)^6}$$

$$=\frac{\left(x^2+2\right)^3\left[e^x\cos x+\sin xe^x\right]-6xe^x\sin x\left(x^2+2\right)^2}{\left(x^2+2\right)^6}$$

$$=\frac{\left(x^2+2\right)^2\left[\left(x^2+2\right)\left(e^x\cos x+\sin xe^x\right)-6xe^x\sin x\right]}{\left(x^2+2\right)^6}$$

$$=\frac{x^2e^x\cos x+x^2\sin xe^x+2e^x\cos x+2\sin xe^x-6xe^x\sin x}{\left(x^2+2\right)^4}$$

$$=\frac{e^x\sin x}{\left(x^2+2\right)^3}+\frac{e^x\cos x}{\left(x^2+2\right)^3}-\frac{6xe^x\sin x}{\left(x^2+2\right)^4}$$

Therefore,

$$\frac{dy}{dx}=\frac{e^x\sin x}{\left(x^2+2\right)^3}+\frac{e^x\cos x}{\left(x^2+2\right)^3}-\frac{6xe^x\sin x}{\left(x^2+2\right)^4}$$

Differentiation Ex 11.2 Q50

Consider

$$y=3e^{-3x}\log\left(1+x\right)$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx}=3\frac{d}{dx}\left[e^{-3x}\log\left(1+x\right)\right]$$

$$\frac{dy}{dx}=3\left(e^{-3x}\frac{1}{1+x}+\log\left(1+x\right)\left(-3e^{-3x}\right)\right)$$

$$=3\left(\frac{e^{-3x}}{1+x}-3e^{-3x}\log\left(1+x\right)\right)$$

$$=3e^{-3x}\left(\frac{1}{1+x}-3\log\left(1+x\right)\right)$$

Differentiation Ex 11.2 Q51

Consider

$$y=\frac{x^2+2}{\sqrt{\cos x}}$$

Differentiating it with respect to x and applying the chain and product rule, we get

$$\frac{dy}{dx}=\frac{\sqrt{\cos x}\frac{d}{dx}\left(x^2+2\right)-\left(x^2+2\right)\frac{d}{dx}\sqrt{\cos x}}{\left(\sqrt{\cos x}\right)^2}$$

$$=\frac{2x\sqrt{\cos x}-\left(x^2+2\right)\left(-\frac{1}{2}\frac{\sin x}{\sqrt{\cos x}}\right)}{\cos x}$$

$$=\frac{2x\sqrt{\cos x}+\frac{\left(x^2+2\right)\sin x}{2\sqrt{\cos x}}}{\cos x}$$

$$=\frac{4x\cos x+\left(x^2+2\right)\sin x}{2\left(\cos x\right)^{\frac{3}{2}}}$$

$$=\frac{2x}{\sqrt{\cos x}}+\frac{1}{2}\frac{\left(x^2+2\right)\sin x}{\left(\cos x\right)^{\frac{3}{2}}}$$

Therefore,

$$\frac{dy}{dx}=\frac{2x}{\sqrt{\cos x}}+\frac{1}{2}\frac{\left(x^2+2\right)\sin x}{\left(\cos x\right)^{\frac{3}{2}}}$$

********** END **********