Indefinite Integrals Ex 19.30 Q32

$$\text{Let } \frac{x^2+x-1}{(x+1)^2(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+2)}$$

$$\Rightarrow \quad x^2+x-1 = A(x+1)(x+2) + B(x+2) + C(x+1)^2$$
$$= (A+C)x^2 + (3A+B+2C)x + (2A+2B+C)$$

Equating similar terms

$$A+C=1,\ 3A+B+2C=1,\ 2A+2B+C=-1$$

Solving, we get, $A=0,\ B=-1,\ C=1$

Thus,

$$I = 0.\int \frac{dx}{x+1} + (-1)\int \frac{dx}{(x+1)^2} + 1.\int \frac{dx}{(x+2)}$$
$$= +\frac{1}{x+1} + \log|x+2| + c$$

$$I = \frac{1}{x+1} + \log|x+2| + c$$

Indefinite Integrals Ex 19.30 Q33

$$\text{Let } \frac{2x^2+7x-3}{x^2(2x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(2x+1)}$$

$$\Rightarrow \quad 2x^2+7x-3 = Ax(2x+1) + B(2x+1) + Cx^2$$

Equating similar terms, we get,

$$2A+C=2,\ A+2B=7,\ B=-3$$

Solving, we get, $A=13,\ C=-24$

Thus,

$$I = \int \frac{13dx}{x} - \int \frac{3dx}{x^2} - 24\int \frac{dx}{2x+1}$$

$$I = 13\log|x| + \frac{3}{x} - 12\log|2x+1| + c$$

Indefinite Integrals Ex 19.30 Q34

Let $I = \int \frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x} dx$

$= \int \frac{5x^2 + 20x + 6}{x(x+1)^2} dx$

Now,

Let $\frac{5x^2 + 20x + 6}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$\Rightarrow \quad 5x^2 + 20x + 6 = A(x+1)^2 + Bx(x+1) + Cx$

Equating similar terms, we get,

$A + B = 5,\ 2A + B + C = 20,\ A = 6$

Solving, we get, $B = -1,\ C = 9$

Thus,

$I = \int \frac{6dx}{x} - 1\int \frac{dx}{x+1} + 9\int \frac{dx}{(x+1)^2}$

$\therefore \quad I = 6\log|x| - \log|x+1| - \frac{9}{x+1} + c$

Indefinite Integrals Ex 19.30 Q35

Let $\frac{18}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+4}$

$\Rightarrow \quad 18 = A(x^2+4) + (Bx+C)(x+2)$

$18 = (A+B)x^2 + (2B+C)x + (4A+2C)$

Equating similar terms, we get,

$A + B = 0,\ 2B + C = 0,\ 4A + 2C = 18$

Solving, we get, $A = \frac{9}{4},\ B = -\frac{9}{4},\ C = \frac{9}{2}$

Thus,

$I = \frac{9}{4}\int \frac{dx}{x+2} + \left(-\frac{9}{4}\right)\int \frac{x}{x^2+4} dx + \frac{9}{2}\int \frac{dx}{x^2+4}$

$I = \frac{9}{4}\log|x+2| - \frac{9}{8}\log|x^2+4| + \frac{9}{4}\tan^{-1}\left(\frac{x}{2}\right) + c \qquad \left[\because \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{9}\right]$

Indefinite Integrals Ex 19.30 Q36

Let $\frac{5}{(x^2+1)(x+2)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+2}$

$\Rightarrow \qquad 5 = (Ax+B)(x+2) + C(x^2+1)$

Equating similar terms, we get,

$A + C = 0,\ 2A + B = 0,\ 2B + C = 5$

Solving, we get, $A = -1,\ B = 2,\ C = 1$

Thus,

$$I = \int \frac{-x+2}{x^2+1}dx + \int \frac{dx}{x+2}$$
$$= \int \frac{-xdx}{x^2+1} + 2\int \frac{dx}{x^2+1} + \int \frac{dx}{x+2}$$

$$I = -\frac{1}{2}\log|x^2+1| + 2\tan^{-1}x + \log|x+2| + c$$

********** END **********