

Exercise 7B

$$\begin{split} (\sin\theta + \cos\theta) &= m \quad \text{and} \quad (\sec\theta + \cos ec\theta) = n \\ \text{LHS} &= n \Big(m^2 - 1 \Big) = (\sec\theta + \cos ec\theta) \Big[(\sin\theta + \cos\theta)^2 - 1 \Big] \\ &= (\sec\theta + \csc\theta) \times \Big[\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1 \Big] \\ &= \Big(\frac{1}{\cos\theta} + \frac{1}{\sin\theta} \Big) (2\sin\theta\cos\theta) \\ &= \Big(\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta} \Big) \times 2\sin\theta\cos\theta \\ &= 2 \left(\sin\theta + \cos\theta \right) - - - - (1) \\ \text{RHS} &= 2m = 2 \left(\sin\theta + \cos\theta \right) - - - - (2) \end{split}$$

 $(\cot \theta + \tan \theta) = m$ and $(\sec \theta - \cos \theta) = n$

:. LHS = RHS

Question 9:

$$\Rightarrow \left(\frac{1}{\tan\theta} + \tan\theta\right) = m \text{ and } \left(\frac{1}{\cos\theta} - \cos\theta\right) = n$$

$$\Rightarrow \left(\frac{1 + \tan^2\theta}{\tan\theta}\right) = m \text{ and } \frac{\left(1 - \cos^2\theta\right)}{\cos\theta} = n$$

$$\Rightarrow \left(\frac{\sec^2\theta}{\tan\theta}\right) = m \text{ and } \frac{\left(1 - \cos^2\theta\right)}{\cos\theta} = n$$

$$\Rightarrow \frac{1}{\cos^2\theta \times \frac{\sin\theta}{\cos\theta}} = m \text{ and } \frac{\sin^2\theta}{\cos\theta} = n$$

$$\Rightarrow \frac{1}{\cos\theta\sin\theta} = m \text{ and } \frac{\sin^2\theta}{\cos\theta} = n$$

$$\therefore \left(m^2n\right)^{\frac{2}{3}} - \left(mn^2\right)^{\frac{2}{3}} = \left[\frac{1}{\cos^2\theta\sin^2\theta} \times \frac{\sin^2\theta}{\cos^2\theta}\right]^{\frac{2}{3}}$$

$$= \left(\frac{1}{\cos^3\theta}\right)^{\frac{2}{3}} - \left(\frac{\sin^3\theta}{\cos^3\theta}\right)^{\frac{2}{3}} = \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \sec^2\theta - \tan^2\theta = 1 \quad \left[\because \sec^2\theta = 1 + \tan^2\theta\right]$$
Hence, $\left(m^2n\right)^{\frac{2}{3}} - \left(mn^2\right)^{\frac{2}{3}} = 1$

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