Differentiation Ex 11.8 Q1

Let $u = x^2, \; v = x^3$

Differentiating u with respect to x,

$$\frac{du}{dx} = 2x \qquad \text{---(i)}$$

Differentiating v with respect to x,

$$\frac{dv}{dx} = 3x^2 \qquad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2x}{3x^2}$$

$$\frac{du}{dv} = \frac{2}{3x}$$

Differentiation Ex 11.8 Q2

Let $\quad u = \log\left(1 + x^2\right)$

Differentiating it with respect to x using chain rule,

$$\frac{du}{dx} = \frac{1}{\left(1 + x^2\right)} \frac{d}{dx}\left(1 + x^2\right)$$

$$= \frac{1}{\left(1 + x^2\right)}(2x)$$

$$\frac{du}{dx} = \frac{2x}{\left(1 + x^2\right)} \qquad \text{---(i)}$$

Let $\quad v = \tan^{-1} x$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{1}{1 + x^2} \qquad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2x}{\left(1 + x^2\right)} \times \frac{\left(1 + x^2\right)}{1}$$

$$\frac{du}{dv} = 2x$$

Differentiation Ex 11.8 Q3

Let $u = (\log x)^x$

Taking log on both the sides,

$$\log u = \log(\log x)^x$$
$$\log u = x\log(\log x) \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating it with respect to x using chain rule, product rule,

$$\frac{1}{u}\frac{du}{dx} = x\frac{d}{dx}\log(\log x) + \log(\log x)\frac{d}{dx}(x)$$
$$\frac{1}{u}\frac{du}{dx} = x\left(\frac{1}{\log x}\right)\frac{d}{dx}(\log x) + \log\log x\,(1)$$
$$\frac{du}{dx} = u\left[\frac{x}{\log x}\left(\frac{1}{x}\right) + \log\log x\right]$$
$$\frac{du}{dx} = (\log x)^x\left[\frac{1}{\log x} + \log\log x\right] \qquad \text{---(i)}$$

Again, let $v = \log x$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{1}{x} \qquad \text{---(ii)}$$

Dividing equation (i) by (ii),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{(\log x)^x\left[\frac{1}{\log x} + \log\log x\right]}{\frac{1}{x}}$$
$$\frac{du}{dv} = \frac{(\log x)^x\left[\frac{1 + \log x \times \log\log x}{\log x}\right]}{\frac{1}{x}}$$

$$\frac{du}{dv} = (\log x)^{x-1}\ (1 + \log x \times \log\log x) \times x$$

Differentiation Ex 11.8 Q4(i)

Let $u = \sin^{-1}\sqrt{1-x^2}$

Put $x = \cos\theta$, so,

$$u = \sin^{-1}\sqrt{1-\cos^2\theta}$$

$$u = \sin^{-1}(\sin\theta) \qquad \text{---(i)}$$

And, $v = \cos^{-1}x \qquad$ ---(ii)

Now, $x \in (0,1)$

$\Rightarrow \quad \cos\theta \in (0,1)$

$\Rightarrow \quad \theta \in \left(0, \frac{\pi}{2}\right)$

So, from equation (i),

$$u = \theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$$

$$u = \cos^{-1}x \qquad [\text{Since, } \cos\theta = x]$$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{-1}{\sqrt{1-x^2}} \qquad \text{---(ii)}$$

From equation (ii),

$$v = \cos^{-1}x$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{-1}{\sqrt{1-x^2}} \qquad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{-1}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{-1}$$

$$\frac{du}{dv} = 1$$

*********** END **********