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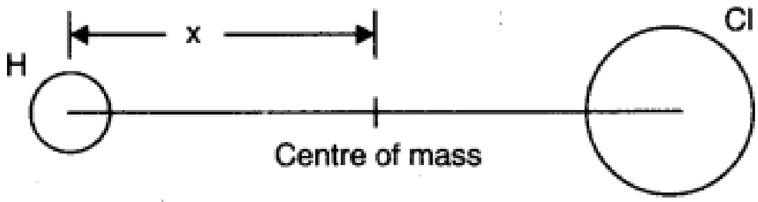
Question 7. 1. Give the location of the centre of mass of a (i) sphere, (ii) cylinder, (iii) ring, and (iv) cube, each of uniform mass density. Does the centre of mass of a body necessarily lie inside the body?
Answer: In all the four cases, as the mass density is uniform, centre of mass is located at their respective geometrical centres.
No, it is not necessary that the centre of mass of a body should lie on the body. For example, in case of a circular ring, centre of mass is at the centre of the ring, where there is no mass.

Question 7. 2. In the HCl molecule, the separation between the nuclei of the two atoms is about 1.27 A (1 A = 10^{-10}m). Find the approximate location of the CM of the molecule, given that a chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus.
Answer: Let us choose the nucleus of the hydrogen atom as the origin for measuring distance. Mass of hydrogen atom,m_1= 1 unit (say) Since cholorine atom is 35.5 times as massive as hydrogen atom,
∴ mass of cholorine atom, m_2 = 35.5 units



Now, $x_1 = 0$ and $x_2 = 1.27\ \text{Å} = 1.27 \times 10^{-10}$ m
Distance of centre of mass of HCl molecule from the origin is given by

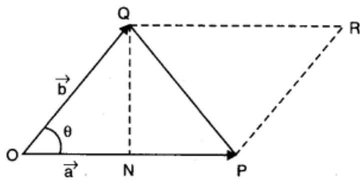
$$X = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{1 \times 0 + 35.5 \times 1.27 \times 10^{-10}}{1 + 35.5}\ \text{m}$$

$$= \frac{35.5 \times 1.27}{36.5} \times 10^{-10}\ \text{m} = 1.235 \times 10^{-10}\text{m} = 1.235\ \text{Å}$$

Question 7. 3. A child sits stationary at one end of a long trolley moving uniformly with a speed V on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, what is the speed of the CM of the (trolley + child) system?
Answer: When the child gets up and runs about on the trolley, the speed of the centre of mass of the trolley and child remains unchanged irrespective of the manner of motion of child. It is because here child and trolley constitute one single system and forces involved are purely internal forces. As there is no external force, there is no change in momentum of the system and velocity remains unchanged.

Question 7. 4.
Show that the area of the triangle contained between the vectors $\vec{a}$ and $\vec{b}$ is one half of the magnitude of $\vec{a} \times \vec{b}$.
Answer:

Let $\vec{a}$ be represented $\overrightarrow{OP}$ and $\vec{b}$ be represented by $\overrightarrow{OQ}$. Let $\angle POQ = \theta$, Fig.
Complete the $\parallel$ gm $OPRQ$. Join PQ.
Draw $QN \perp OP$



In ΔOQN, $\quad \sin\theta = \dfrac{QN}{OQ} = \dfrac{QN}{b}$

$$QN = b \sin\theta$$

Now, by definition, $|\vec{a} \times \vec{b}| = ab \sin\theta = (OP)(QN)$

$$= \frac{2(OP)(QN)}{2} = 2 \times \text{area of } \Delta OPQ$$

$\therefore \quad$ area of $\Delta OPQ = \dfrac{1}{2}|\vec{a} \times \vec{b}|$, which was to be proved.

Question 7. 5.

Show that $\vec{a}\,.\,(\vec{b} \times \vec{c})$ is equal in magnitude to the volume of the parallelopiped formed on the three vectors, $\vec{a}$, $\vec{b}$ and $\vec{c}$.

Answer:

Let *a* parallelopiped be formed on the three vectors.

$$\overrightarrow{OA} = \vec{a}, \quad \overrightarrow{OB} = \vec{b}$$

and $\quad \overrightarrow{OC} = \vec{c}$

Now, $\quad \vec{b} \times \vec{c} = bc \sin 90^\circ\ \hat{n} = bc\ \hat{n}$

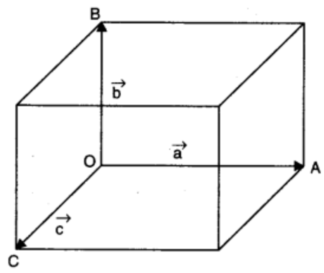
where $\hat{n}$ is unit vector along $\overrightarrow{OA}$ perpendicular to the plane containing $\vec{b}$ and $\vec{c}$.

Now $\quad \vec{a}\,.(\vec{b} \times \vec{c}) = \vec{a}\,.\,bc\ \hat{n}$

$$= (a)(bc)\cos 0^\circ$$

$$= a\ b\ c$$

which is equal in magnitude to the volume of the parallelopiped.



Question 7. 6. Find the components along the x, y, z-axes of the angular momentum l of a particle, whose position vector is r with components x, y, z and momentum is p with components p_x, p_y and p_z. Show that if the particle moves only in the x-y plane the angular momentum has only a z- component.

Answer:

We know that angular momentum $\vec{l}$ of a particle having position vector $\vec{r}$ and momentum $\vec{p}$ is given by

$$\vec{l} = \vec{r} \times \vec{p}$$

But $\quad \vec{r} = [x\hat{i} + y\hat{j} + z\hat{k}]$, where x, y, z are the components of $\vec{r}$ and $\vec{p} = [p_x\hat{i} + p_y\hat{j} + p_z\hat{k}]$

$\therefore \quad \vec{l} = \vec{r} \times \vec{p} = [x\hat{i} + y\hat{j} + z\hat{k}] \times [p_x\hat{i} + p_y\hat{j} + p_z\hat{k}]$

or $\quad (l_x\hat{i} + l_y\hat{j} + l_z\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$

$$= (yp_z - zp_y)\,\hat{i} + (zp_x - xp_z)\,\hat{j} + (xp_y - yp_x)\,\hat{k}$$

From this relation, we conclude that

$$l_x = yp_z - zp_y, \quad l_y = zp_x - xp_z \quad \text{and} \quad l_z = xp_y - yp_x$$

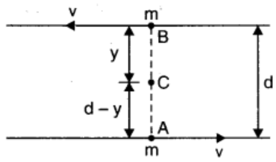
If the given particle moves only in the $x - y$ plane, then $z = 0$ and $p_z = 0$ and hence, $\vec{l} = (xp_y - yp_x)\,\hat{k}$, which is only the *z*-comonent of $\vec{l}$.

It means that for a particle moving only in the $x - y$ plane, the angular momentum has only the *z*-component.

Question 7. 7. Two particles, each of mass m and speed v, travel in opposite directions along parallel lines separated by a distance d. Show that the vector angular momentum of the two particle system the same whatever be the point about which the angular momentum is taken.

Answer:

Angular momentum about A,



$$L_A = mv \times 0 + mv \times d$$
$$= mvd$$

Angular momentum about B,

$$L_B = mv \times d + mv \times 0$$
$$= mvd$$

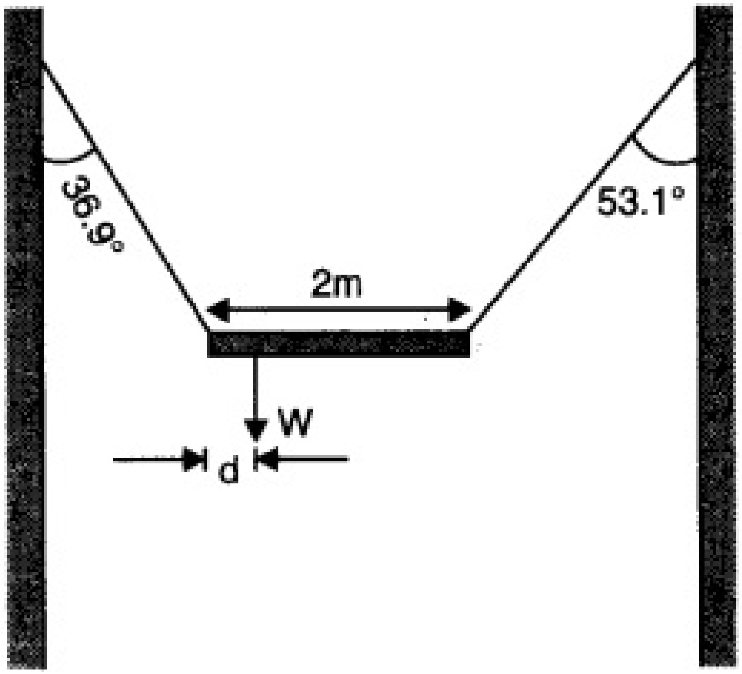
Angular momentum about C,

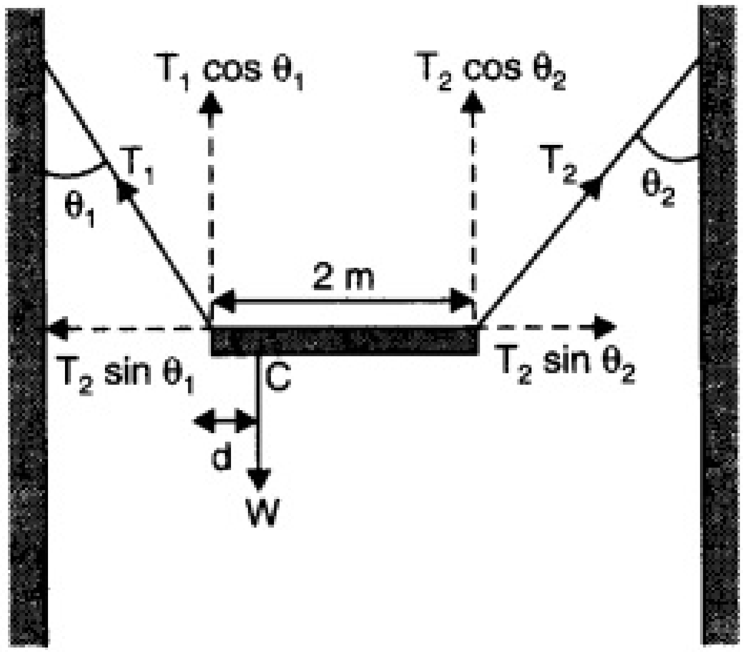
$$L_c = mv \times y + mv \times (d - y) = mvd$$

In all the three cases, the direction of angular momentum is the same.

$$\therefore \quad \vec{L}_A = \vec{L}_B = \vec{L}_C$$

Question 7. 8. A non-uniform bar of weight W is suspended at rest by two strings of negligible weight as shown in Fig. The angles made by the strings with the vertical are 36.9° and 53.2° respectively. The bar is 2 m long. Calculate the distance d of the centre of gravity of the bar from its left end.





Answer:

As it is clear from Fig.,

$$\theta_1 = 36.9°, \quad \theta_2 = 53.1°.$$

If T_1, T_2 are the tensions in the two strings, then for equilibrium along the horizontal,

$$T_1 \sin\theta_1 = T_2 \sin\theta_2$$

or

$$\frac{T_1}{T_2} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{\sin 53.1°}{\sin 36.9°}$$

$$= \frac{0.7404}{0.5477} = 1.3523$$

Let d be the distance of centre of gravity C of the bar from the left end.

For rotational equilibrium about C,

$$T_1 \cos\theta_1 \times d = T_2 \cos\theta_2 (2-d)$$

$$T_1 \cos 36.9° \times d = T_2 \cos 53.1° (2-d)$$

$$T_1 \times 0.8366\, d = T_2 \times 0.6718 (2-d)$$

Put

$$T_1 = 13523\ \text{T}_2 \text{ and solve to get}$$

$$d = 0.745 \text{ m}$$

$$\tau = I_1\alpha_1 = I_2\alpha_2$$

or

$$\frac{\alpha_1}{\alpha_2} = \frac{I_2}{I_1} = \frac{(2/5)MR^2}{MR^2} = \frac{2}{5}$$

or

$$\alpha_2 = \frac{5}{2}\alpha_1.$$

Question 7. 9. A car weighs 1800 kg. The distance between its front and back axles is 1.8 m. Its centre of gravity is 1.05 m behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel.

Answer: Let F_1 and F_2 be the forces exerted by the level ground on front wheels and back wheels respectively.

Considering rotational equilibrium about the front wheels,F_2 x 1.8 = mg x 1.05 or F_2 = 1.05/1.8 x 1800 x 9.8 N =10290 N

Force on each back wheel is = 10290/2 N or 5145 N.

Considering rotational equilibrium about the back wheels.

F_1 x 1.8 = mg (1.8 - 1.05) = 0.75 x 1800 x 9.8

or F_1=0.75 x 1800 x 9.8/1.8 = 7350 N

Force on each front wheel is 7350/2 N or 3675 N.

Question 7. 10. (a) Find the moment of inertia of a sphere about a tangent to the sphere, given the moment of inertia of the sphere about any of its diameters to be 2 MR^2/5, where M is the mass of the sphere and R is the radius of the sphere.

(b) Given the moment of inertia of a disc of mass M and radius R about any of its diameters to 1 be 1/4 MR^2, find the moment of inertia about an axis normal to the disc passing through a point on its edge.

Answer:

(a) Moment of inertia of sphere about any diameter = 2/5 MR^2

Applying theorem of parallel axes,Moment of inertia of sphere about a tangent to the sphere = 2/5 MR^2 +M$(R)^2$ =7/5 MR^2

(b) We are given, moment of inertia of the disc about any of its

diameters = 1/4 MR^2
(i) Using theorem of perpendicular axes, moment of inertia of the disc about an axis passing through its centre and normal to the disc = 2 x 1/4 MR^2 = 1/2 MR^2.
(ii) Using theorem axes, moment of inertia of the disc passing through a point on its edge and normal to the dies = 1/2 MR^2+ MR^2 = 3/2 MR^2.

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