Exercise 6.5 : Solutions of Questions on Page Number : 231

Q1 : Find the maximum and minimum values, if any, of the following functions given by

(i) $f(x) = (2x - 1)^2 + 3$ (ii) $f(x) = 9x^2 + 12x + 2$

(iii) $f(x) = -(x - 1)^2 + 10$ (iv) $g(x) = x^3 + 1$

Answer :

(i) The given function is $f(x) = (2x - 1)^2 + 3$.

It can be observed that $(2x - 1)^2 \geq 0$ for every $x \in \mathbf{R}$.

Therefore, $f(x) = (2x - 1)^2 + 3 \geq 3$ for every $x \in \mathbf{R}$.

The minimum value of f is attained when $2x - 1 = 0$.

$2x - 1 = 0 \Rightarrow x = \frac{1}{2}$

$\therefore$Minimum value of $f = f\left(\frac{1}{2}\right) = \left(2 \cdot \frac{1}{2} - 1\right)^2 + 3 = 3$

Hence, function f does not have a maximum value.

(ii) The given function is $f(x) = 9x^2 + 12x + 2 = (3x + 2)^2 - 2$.

It can be observed that $(3x + 2)^2 \geq 0$ for every $x \in \mathbf{R}$.

Therefore, $f(x) = (3x + 2)^2 - 2 \geq -2$ for every $x \in \mathbf{R}$.

The minimum value of f is attained when $3x + 2 = 0$.

$3x + 2 = 0 \Rightarrow x = \frac{-2}{3}$

$\therefore$Minimum value of $f = f\left(-\frac{2}{3}\right) = \left(3\left(\frac{-2}{3}\right) + 2\right)^2 - 2 = -2$

Hence, function f does not have a maximum value.

(iii) The given function is $f(x) = -(x - 1)^2 + 10$.

It can be observed that $(x - 1)^2 \geq 0$ for every $x \in \mathbf{R}$.

Therefore, $f(x) = -(x - 1)^2 + 10 \leq 10$ for every $x \in \mathbf{R}$.

The maximum value of f is attained when $(x - 1) = 0$.

$(x - 1) = 0 \Rightarrow x = 1$

$\therefore$Maximum value of $f = f(1) = -(1 - 1)^2 + 10 = 10$

Hence, function f does not have a minimum value.

(iv) The given function is $g(x) = x^3 + 1$.

Hence, function g neither has a maximum value nor a minimum value.

Q2 : Find the maximum and minimum values, if any, of the following functions given by

(i) $f(x) = |x + 2| - 1$ (ii) $g(x) = -|x + 1| + 3$

(iii) $h(x) = \sin(2x) + 5$ (iv) $f(x) = |\sin 4x + 3|$

(v) $h(x) = x + 4, x \in (-1, 1)$

Answer :

(i) $f(x) = |x+2|-1$

We know that $|x+2| \geq 0$ for every $x \in \mathbf{R}$.

Therefore, $f(x) = |x+2|-1 \geq -1$ for every $x \in \mathbf{R}$.

The minimum value of f is attained when $|x+2| = 0$.

$|x+2| = 0$
$\Rightarrow x = -2$

$\therefore$Minimum value of $f = f(-2) = = |-2+2|-1 = -1$

Hence, function f does not have a maximum value.

(ii) $g(x) = -|x+1|+3$

We know that $-|x+1| \leq 0$ for every $x \in \mathbf{R}$.

Therefore, $g(x) = -|x+1|+3 \leq 3$ for every $x \in \mathbf{R}$.

The maximum value of g is attained when $|x+1| = 0$.

$|x+1| = 0$
$\Rightarrow x = -1$
$\therefore$ Maximum value of $g = g(-1) = -|-1+1|+3 = 3$

Hence, function g does not have a minimum value.

(iii) $h(x) = \sin 2x + 5$

We know that $-1 \le \sin 2x \le 1$.

$\Rightarrow -1 + 5 \le \sin 2x + 5 \le 1 + 5$

$\Rightarrow 4 \le \sin 2x + 5 \le 6$

Hence, the maximum and minimum values of h are 6 and 4 respectively.

(iv) $f(x) = |\sin 4x + 3|$

We know that $-1 \le \sin 4x \le 1$.

$\Rightarrow 2 \le \sin 4x + 3 \le 4$

$\Rightarrow 2 \le |\sin 4x + 3| \le 4$

Hence, the maximum and minimum values of f are 4 and 2 respectively.

(v) $h(x) = x + 1$, $x \in (-1, 1)$

Here, if a point x_0 is closest to -1, then we find $\frac{x_0}{2}+1 < x_0 + 1$ for all $x_0 \in (-1, 1)$.

Also, if x_1 is closest to 1, then $x_1 + 1 < \frac{x_1+1}{2}+1$ for all $x_1 \in (-1, 1)$.

Hence, function $h(x)$ has neither maximum nor minimum value in $(-1, 1)$.

Answer needs Correction? Click Here

Q3 : Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be:

(i). $f(x) = x^2$
(ii). $g(x) = x^3 - 3x$
(iii). $h(x) = \sin x + \cos x$, $0 < x < \frac{\pi}{2}$
(iv). $f(x) = \sin x - \cos x$, $0 < x < 2\pi$
(v). $f(x) = x^3 - 6x^2 + 9x + 15$
(vi). $g(x) = \frac{x}{2} + \frac{2}{x}, x > 0$
(vii). $g(x) = \frac{1}{x^2+2}$
(viii). $f(x) = x\sqrt{1-x}, x > 0$

Answer :

(i) $f(x) = x^2$

$\therefore f'(x) = 2x$
Now,
$f'(x) = 0 \Rightarrow x = 0$

Thus, $x = 0$ is the only critical point which could possibly be the point of local maxima or local minima of f.

We have $f''(0) = 2$, which is positive.

Therefore, by second derivative test, $x = 0$ is a point of local minima and local minimum value of f at $x = 0$ is $f(0) = 0$.

(ii) $g(x) = x^3 - 3x$

$\therefore g'(x) = 3x^2 - 3$
Now,
$g'(x) = 0 \Rightarrow 3x^2 = 3 \Rightarrow x = \pm 1$
$g'(x) = 6x$
$g'(1) = 6 > 0$
$g'(-1) = -6 < 0$

By second derivative test, $x = 1$ is a point of local minima and local minimum value of g at $x = 1$ is $g(1) = 1^3 - 3 = 1 - 3 = -2$. However,

$x = -1$ is a point of local maxima and local maximum value of g at

$x = -1$ is $g(1) = (-1)^3 - 3(-1) = -1 + 3 = 2$.

(iii) $h(x) = \sin x + \cos x$, $0 < x < \frac{\pi}{2}$

$\therefore h'(x) = \cos x - \sin x$
$h'(x) = 0 \Rightarrow \sin x = \cos x \Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4} \in \left(0, \frac{\pi}{2}\right)$
$h''(x) = -\sin x - \cos x = -(\sin x + \cos x)$
$h''\left(\frac{\pi}{4}\right) = -\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = -\frac{2}{\sqrt{2}} = -\sqrt{2} < 0$

Therefore, by second derivative test, $x = \frac{\pi}{4}$ is a point of local maxima and the local maximum value of h at $x = \frac{\pi}{4}$ is $h\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4} + \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$.

(iv) $f(x) = \sin x - \cos x$, $0 < x < 2\pi$

$\therefore$

Q4 : Prove that the following functions do not have maxima or minima:

(i) $f(x) = e^x$ (ii) $g(x) = \log x$

(iii) $h(x) = x^3 + x^2 + x + 1$

Answer :

i. We have,

$f(x) = e^x$

$\therefore f'(x) = e^x$

Now, if $f'(x) = 0$, then $e^x = 0$. But, the exponential function can never assume 0 for any value of x.

Therefore, there does not exist $c \in \mathbf{R}$ such that $f'(c) = 0$.

Hence, function f does not have maxima or minima.

ii. We have,

$g(x) = \log x$

$\therefore g'(x) = \frac{1}{x}$

Since $\log x$ is defined for a positive number x, $g'(x) > 0$ for any x.

Therefore, there does not exist $c \in \mathbf{R}$ such that $g'(c) = 0$.

Hence, function g does not have maxima or minima.

iii. We have,

$h(x) = x^3 + x^2 + x + 1$

$\therefore h'(x) = 3x^2 + 2x + 1$

Now,

$h(x) = 0 \Rightarrow 3x^2 + 2x + 1 = 0 \Rightarrow x = \frac{-2 \pm 2\sqrt{2}i}{6} = \frac{-1 \pm \sqrt{2}i}{3} \notin \mathbf{R}$

Therefore, there does not exist $c \in \mathbf{R}$ such that $h'(c) = 0$.

Hence, function h does not have maxima or minima.

Q5 : Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals:

(i) $f(x) = x^3, x \in [-2, 2]$ (ii) $f(x) = \sin x + \cos x, x \in [0, \pi]$

(iii) $f(x) = 4x - \frac{1}{2}x^2, x \in \left[-2, \frac{9}{2}\right]$

(iv) $f(x) = (x-1)^2 + 3, x \in [-3, 1]$

Answer :

(i) The given function is $f(x) = x^3$.

$\therefore f'(x) = 3x^2$

Now,

$f'(x) = 0 \Rightarrow x = 0$

Then, we evaluate the value of f at critical point $x = 0$ and at end points of the interval [- 2, 2].

$f(0) = 0$

$f(-2) = (-2)^3 = -8$

$f(2) = (2)^3 = 8$

Hence, we can conclude that the absolute maximum value of f on [- 2, 2] is 8 occurring at $x = 2$. Also, the absolute minimum value of f on [- 2, 2] is - 8 occurring at $x = -2$.

(ii) The given function is $f(x) = \sin x + \cos x$.

$\therefore f'(x) = \cos x - \sin x$

Now,

$f'(x) = 0 \Rightarrow \sin x = \cos x \Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}$

Then, we evaluate the value of f at critical point $x = \frac{\pi}{4}$ and at the end points of the interval [0, π].

$$f\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4} + \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$
$$f(0) = \sin 0 + \cos 0 = 0 + 1 = 1$$
$$f(\pi) = \sin\pi + \cos\pi = 0 - 1 = -1$$

Hence, we can conclude that the absolute maximum value of f on [0, π] is $\sqrt{2}$ occurring at $x = \frac{\pi}{4}$ and the absolute minimum value of f on [0, π] is - 1 occurring at $x = \pi$.

(iii) The given function is $f(x) = 4x - \frac{1}{2}x^2$.

$\therefore f'(x) = 4 - \frac{1}{2}(2x) = 4 - x$

Now,

$f'(x)=0 \Rightarrow x=4$

Then, we evaluate the value of f at critical point $x = 4$ and at the end points of the interval $\left[-2, \frac{9}{2}\right]$.

$f(4)=16-\frac{1}{2}(16)=16-8=8$

$f(-2)=-8-\frac{1}{2}(4)=-8-2=-10$

$f\left(\frac{9}{2}\right)=4\left(\frac{9}{2}\right)-\frac{1}{2}\left(\frac{9}{2}\right)^2=18-\frac{81}{8}=18-10.125=7.875$

Hence, we can conclude that the absolute maximum value of f on $\left[-2, \frac{9}{2}\right]$ is 8 occurring at $x = 4$ and the absolute minimum value of f on $\left[-2, \frac{9}{2}\right]$ is - 10 occurring at $x = - 2$.

(iv) The given function is $f(x)=(x-1)^2+3$.

$\therefore f'(x)=2(x-1)$

Now,

$f'(x)=0 \Rightarrow 2(x - 1) = 0 \Rightarrow x = 1$

Then, we evaluate the value of f at critical point $x = 1$ and at the end points of the interval [- 3, 1].

$f(1)=(1-1)^2+3=0+3=3$

$f(-3)=(-3-1)^2+3=16+3=19$

Hence, we can conclude that the absolute maximum value of f on [- 3, 1] is 19 occurring at $x = - 3$ and the minimum value of f on [- 3, 1] is 3 occurring at $x = 1$.

Answer needs Correction? **Click Here**

Q6 : Find the maximum profit that a company can make, if the profit function is given by

$p(x) = 41 - 72x - 18x^2$

Answer :

The profit function is given as $p(x) = 41 - 72x - 18x^2$.

$\therefore$

Also,

Answer needs Correction? **Click Here**

Q7 : Find both the maximum value and the minimum value of

$3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval [0, 3]

Answer :

Let $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$.

$$\begin{aligned}\therefore f'(x)&=12x^3-24x^2+24x-48\\&=12\left(x^3-2x^2+2x-4\right)\\&=12\left[x^2(x-2)+2(x-2)\right]\\&=12(x-2)\left(x^2+2\right)\end{aligned}$$

Now, $f'(x)=0$ gives $x = 2$ or $x^2 + 2 = 0$ for which there are no real roots.

Therefore, we consider only $x = 2 \in [0, 3]$.

Now, we evaluate the value of f at critical point $x = 2$ and at the end points of the interval [0, 3].

$$\begin{aligned}f(2)&=3(16)-8(8)+12(4)-48(2)+25\\&=48-64+48-96+25\\&=-39\\f(0)&=3(0)-8(0)+12(0)-48(0)+25\\&=25\\f(3)&=3(81)-8(27)+12(9)-48(3)+25\\&=243-216+108-144+25=16\end{aligned}$$

Hence, we can conclude that the absolute maximum value of f on [0, 3] is 25 occurring at $x = 0$ and the absolute minimum value of f at [0, 3] is - 39 occurring at $x = 2$.

Answer needs Correction? **Click Here**

Q8 : At what points in the interval $[0, 2\pi]$, does the function $\sin 2x$ attain its maximum value?

Answer :

Let $f(x) = \sin 2x$.

$\therefore f'(x)=2\cos 2x$

Now,

$f'(x)=0 \Rightarrow \cos 2x=0$

$\Rightarrow 2x=\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$\Rightarrow x=\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Then, we evaluate the values of f at critical points $x=\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ and at the end points of the interval [0, 2π].

$$f\left(\frac{\pi}{4}\right)=\sin\frac{\pi}{2}=1, f\left(\frac{3\pi}{4}\right)=\sin\frac{3\pi}{2}=-1$$
$$f\left(\frac{5\pi}{4}\right)=\sin\frac{5\pi}{2}=1, f\left(\frac{7\pi}{4}\right)=\sin\frac{7\pi}{2}=-1$$
$$f(0)=\sin 0=0, f(2\pi)=\sin 2\pi=0$$

Hence, we can conclude that the absolute maximum value of f on [0, 2π] is occurring at $x=\frac{\pi}{4}$ and $x=\frac{5\pi}{4}$.

Answer needs Correction? Click Here

Q9 : What is the maximum value of the function sin x + cos x?

Answer :

Let $f(x)$ = sin x + cos x.

$$\therefore f'(x)=\cos x-\sin x$$
$$f'(x)=0 \Rightarrow \sin x=\cos x \Rightarrow \tan x=1 \Rightarrow x=\frac{\pi}{4}, \frac{5\pi}{4}...,$$
$$f''(x)=-\sin x-\cos x=-(\sin x+\cos x)$$

Now, $f''(x)$ will be negative when (sin x + cos x) is positive i.e., when sin x and cos x are both positive. Also, we know that sin x and cos x both are positive in the first quadrant. Then, $f''(x)$ will be negative when $x\in\left(0,\frac{\pi}{2}\right)$.

Thus, we consider $x=\frac{\pi}{4}$.

$$f''\left(\frac{\pi}{4}\right)=-\left(\sin\frac{\pi}{4}+\cos\frac{\pi}{4}\right)=-\left(\frac{2}{\sqrt{2}}\right)=-\sqrt{2}<0$$

∴By second derivative test, f will be the maximum at $x=\frac{\pi}{4}$ and the maximum value of f is

$$f\left(\frac{\pi}{4}\right)=\sin\frac{\pi}{4}+\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}=\frac{2}{\sqrt{2}}=\sqrt{2}.$$

Answer needs Correction? Click Here

Q10 : Find the maximum value of $2x^3 - 24x + 107$ in the interval [1, 3]. Find the maximum value of the same function in [-3, -1].

Answer :

Let $f(x) = 2x^3 - 24x + 107$.

$$\therefore f'(x)=6x^2-24=6(x^2-4)$$
Now,
$$f'(x)=0 \Rightarrow 6(x^2-4)=0 \Rightarrow x^2=4 \Rightarrow x=\pm 2$$

We first consider the interval [1, 3].

Then, we evaluate the value of f at the critical point $x = 2 \in [1, 3]$ and at the end points of the interval [1, 3].

$f(2) = 2(8) - 24(2) + 107 = 16 - 48 + 107 = 75$

$f(1) = 2(1) - 24(1) + 107 = 2 - 24 + 107 = 85$

$f(3) = 2(27) - 24(3) + 107 = 54 - 72 + 107 = 89$

Hence, the absolute maximum value of $f(x)$ in the interval [1, 3] is 89 occurring at $x = 3$.

Next, we consider the interval [- 3, - 1].

Evaluate the value of f at the critical point $x = - 2 \in [- 3, - 1]$ and at the end points of the interval [1, 3].

$f(- 3) = 2 (- 27) - 24(- 3) + 107 = - 54 + 72 + 107 = 125$

$f(- 1) = 2(- 1) - 24 (- 1) + 107 = - 2 + 24 + 107 = 129$

$f(- 2) = 2(- 8) - 24 (- 2) + 107 = - 16 + 48 + 107 = 139$

Hence, the absolute maximum value of $f(x)$ in the interval [- 3, - 1] is 139 occurring at $x = - 2$.

Answer needs Correction? Click Here

Q11 : It is given that at $x = 1$, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value, on the interval [0, 2]. Find the value of a.

Answer :

Let $f(x) = x^4 - 62x^2 + ax + 9$.

$$\therefore f'(x)=4x^3-124x+a$$

It is given that function f attains its maximum value on the interval [0, 2] at $x = 1$.

$$\therefore f'(1)=0$$
$$\Rightarrow 4-124+a=0$$
$$\Rightarrow a=120$$

Hence, the value of a is 120.

Q12 : Find the maximum and minimum values of $x + \sin 2x$ on $[0, 2\pi]$.

Answer :

Let $f(x) = x + \sin 2x$.

$$\therefore f'(x) = 1 + 2\cos 2x$$

$$\text{Now, } f'(x) = 0 \Rightarrow \cos 2x = -\frac{1}{2} = -\cos\frac{\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\frac{2\pi}{3}$$

$$2x = 2n\pi \pm \frac{2\pi}{3},\ n \in \mathbb{Z}$$

$$\Rightarrow x = n\pi \pm \frac{\pi}{3},\ n \in \mathbb{Z}$$

$$\Rightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \in [0, 2\pi]$$

Then, we evaluate the value of f at critical points $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ and at the end points of the interval $[0, 2\pi]$.

$$f\left(\frac{\pi}{3}\right) = \frac{\pi}{3} + \sin\frac{2\pi}{3} = \frac{\pi}{3} + \frac{\sqrt{3}}{2}$$

$$f\left(\frac{2\pi}{3}\right) = \frac{2\pi}{3} + \sin\frac{4\pi}{3} = \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$$

$$f\left(\frac{4\pi}{3}\right) = \frac{4\pi}{3} + \sin\frac{8\pi}{3} = \frac{4\pi}{3} + \frac{\sqrt{3}}{2}$$

$$f\left(\frac{5\pi}{3}\right) = \frac{5\pi}{3} + \sin\frac{10\pi}{3} = \frac{5\pi}{3} - \frac{\sqrt{3}}{2}$$

$$f(0) = 0 + \sin 0 = 0$$

$$f(2\pi) = 2\pi + \sin 4\pi = 2\pi + 0 = 2\pi$$

Hence, we can conclude that the absolute maximum value of $f(x)$ in the interval $[0, 2\pi]$ is 2π occurring at $x = 2\pi$ and the absolute minimum value of $f(x)$ in the interval $[0, 2\pi]$ is 0 occurring at $x = 0$.

Q13 : Find two numbers whose sum is 24 and whose product is as large as possible.

Answer :

Let one number be x. Then, the other number is $(24 - x)$.

Let $P(x)$ denote the product of the two numbers. Thus, we have:

$$P(x) = x(24 - x) = 24x - x^2$$

$$\therefore P'(x) = 24 - 2x$$

$$P''(x) = -2$$

Now,

$$P'(x) = 0 \Rightarrow x = 12$$

Also,

$$P''(12) = -2 < 0$$

$\therefore$By second derivative test, $x = 12$ is the point of local maxima of P. Hence, the product of the numbers is the maximum when the numbers are 12 and 24 - 12 = 12.

Q14 : Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

Answer :

The two numbers are x and y such that $x + y = 60$.

$\Rightarrow y = 60 - x$

Let $f(x) = xy^3$.

$$\Rightarrow f(x) = x(60 - x)^3$$

$$\therefore f'(x) = (60 - x)^3 - 3x(60 - x)^2$$

$$= (60 - x)^2 [60 - x - 3x]$$

$$= (60 - x)^2 (60 - 4x)$$

$$\text{And, } f''(x) = -2(60 - x)(60 - 4x) - 4(60 - x)^2$$

$$= -2(60 - x)[60 - 4x + 2(60 - x)]$$

$$= -2(60 - x)(180 - 6x)$$

$$= -12(60 - x)(30 - x)$$

$$\text{Now, } f'(x) = 0 \Rightarrow x = 60 \text{ or } x = 15$$

$$\text{When } x = 60, f''(x) = 0.$$

$$\text{When } x = 15, f''(x) = -12(60 - 15)(30 - 15) = -12 \times 45 \times 15 < 0.$$

$\therefore$By second derivative test, $x = 15$ is a point of local maxima of f. Thus, function xy^3 is maximum when $x = 15$ and $y = 60 - 15 = 45$.

Hence, the required numbers are 15 and 45.

Q15 : Find two positive numbers x and y such that their sum is 35 and the product x^2y^5 is a maximum

Answer :

Let one number be x. Then, the other number is $y = (35 - x)$.

Let $P(x) = x^2y^5$. Then, we have:

$$P(x) = x^2(35-x)^5$$
$$\therefore P'(x) = 2x(35-x)^5 - 5x^2(35-x)^4$$
$$= x(35-x)^4\left[2(35-x)-5x\right]$$
$$= x(35-x)^4(70-7x)$$
$$= 7x(35-x)^4(10-x)$$

$$\text{And, } P''(x) = 7(35-x)^4(10-x) + 7x\left[-(35-x)^4 - 4(35-x)^3(10-x)\right]$$
$$= 7(35-x)^4(10-x) - 7x(35-x)^4 - 28x(35-x)^3(10-x)$$
$$= 7(35-x)^3\left[(35-x)(10-x) - x(35-x) - 4x(10-x)\right]$$
$$= 7(35-x)^3\left[350 - 45x + x^2 - 35x + x^2 - 40x + 4x^2\right]$$
$$= 7(35-x)^3\left(6x^2 - 120x + 350\right)$$

Now, $P'(x) = 0 \Rightarrow x = 0$, $x = 35$, $x = 10$

When $x = 35$, $f'(x) = f(x) = 0$ and $y = 35 - 35 = 0$. This will make the product $x^2\,y^5$ equal to 0.

When $x = 0$, $y = 35 - 0 = 35$ and the product x^2y^2 will be 0.

$\therefore$ $x = 0$ and $x = 35$ cannot be the possible values of x.

When $x = 10$, we have:

$$P''(x) = 7(35-10)^3(6\times100 - 120\times10 + 350)$$
$$= 7(25)^3(-250) < 0$$

$\therefore$ By second derivative test, $P(x)$ will be the maximum when $x = 10$ and $y = 35 - 10 = 25$.

Hence, the required numbers are 10 and 25.

Q16 : Find two positive numbers whose sum is 16 and the sum of whose cubes is minimum.

Answer :

Let one number be x. Then, the other number is $(16 - x)$.

Let the sum of the cubes of these numbers be denoted by $S(x)$. Then,

$$S(x) = x^3 + (16-x)^3$$
$$\therefore S'(x) = 3x^2 - 3(16-x)^2,\ S''(x) = 6x + 6(16-x)$$
$$\text{Now, } S'(x) = 0 \Rightarrow 3x^2 - 3(16-x)^2 = 0$$
$$\Rightarrow x^2 - (16-x)^2 = 0$$
$$\Rightarrow x^2 - 256 - x^2 + 32x = 0$$
$$\Rightarrow x = \frac{256}{32} = 8$$

Now, $S''(8) = 6(8) + 6(16-8) = 48 + 48 = 96 > 0$

$\therefore$ By second derivative test, $x = 8$ is the point of local minima of S.

Hence, the sum of the cubes of the numbers is the minimum when the numbers are 8 and 16 - 8 = 8.

*********** END **********