

Indefinite Integrals Ex 19.25 Q51

Let
$$I = \int (\tan^{-1} x^2) x dx$$

Let $x^2 = t$
 $2x dx = dt$
 $I = \frac{1}{2} \int \tan^{-1} t dt$
 $= \frac{1}{2} \int 1 \tan^{-1} t dt$
 $= \frac{1}{2} \left[\tan^{-1} t \int dt - \left(\int \frac{1}{1+t^2} \int dt \right) dt \right]$
 $= \frac{1}{2} \left[t \tan^{-1} t - \int \frac{t}{1+t^2} dt \right]$
 $= \frac{1}{2} t \tan^{-1} t - \frac{1}{4} \int \frac{2t}{1+t^2} dt$
 $= \frac{1}{2} t \tan^{-1} t - \frac{1}{4} \log |1+t^2| + c$
 $I = \frac{1}{2} x^2 \tan^{-1} x^2 - \frac{1}{4} \log |1+x^4| + c$

Indefinite Integrals Ex 19.25 Q52

Let first function be $\sin^{-1} x$ and second dunction be $\frac{x}{\sqrt{1-x^2}}$. First we find the intergral of the second function, i.e., $\int \frac{x dx}{\sqrt{1-x^2}}$.

Put
$$t = 1 - x^2$$
. Then $dt = -2x \ dx$ Therefore,
$$\int \frac{x dx}{\sqrt{1 - x^2}} = -\frac{1}{2} \int \frac{dt}{\sqrt{t}} = -\sqrt{t} = -\sqrt{1 - x^2}$$
 Hence,
$$\int \frac{x \sin^{-1} x}{\sqrt{1 - x^2}} \ dx = \left(\sin^{-1} x\right) \left(-\sqrt{1 - x^2}\right) - \int \frac{1}{\sqrt{1 - x^2}} \left(-\sqrt{1 - x^2}\right) \ dx = -\sqrt{1 - x^2} \sin^{-1} x + x + C = x - \sqrt{1 - x^2} \sin^{-1} x + C$$

Indefinite Integrals Ex 19.25 Q53

Let
$$I = \int \sin^3 \sqrt{x} \, dx$$
$$\sqrt{x} = t$$
$$x = t^2$$
$$dx = 2t \, dt$$
$$I = 2\int t \sin^3 t \, dt$$
$$= 2\int t \left(\frac{3 \sin t - \sin 3t}{4}\right) dt$$
$$= \frac{1}{2}\int t \left(3 \sin t - \sin 3t\right) dt$$

Using integration by parts,

$$\begin{split} I &= \frac{1}{2} \left[t \left(-3\cos t + \frac{1}{3}\cos 3t \right) - \int \left(-3\cos t + \frac{\cos 3t}{3} \right) dt \right] \\ &= \frac{1}{2} \left[\frac{-9t\cos t + t\cos 3t}{3} - \left\{ -3\sin t + \frac{\sin 3t}{9} \right\} \right] + c \\ &= \frac{1}{2} \left[\frac{-9t\cos t + t\cos 3t}{3} + \frac{27\sin t - 3\sin 3t}{9} \right] + c \\ &= \frac{1}{18} \left[-27t\cos t + 3t\cos 3t + 27\sin t - 3\sin 3t \right] + c \end{split}$$

$$I = \frac{1}{18} \left[3\sqrt{x} \cos 3\sqrt{x} + 27\sin \sqrt{x} - 27\sqrt{x} \cos \sqrt{x} - 3\sin 3\sqrt{x} \right] + c$$

Indefinite Integrals Ex 19.25 Q54

Let
$$I = \int x \sin^3 x \, dx$$
$$= \int x \left(\frac{3 \sin x - \sin 3x}{4} \right) dx$$
$$= \frac{1}{4} \int x \left(3 \sin x - \sin 3x \right) dx$$

Using integration by parts,

$$I = \frac{1}{4} \left[x \int (3\sin x - \sin 3x) dx - \int (1) \left(3\sin x - \sin 3x\right) dx \right]$$

$$= \frac{1}{4} \left[x \left(-3\cos x + \frac{\cos 3x}{3} \right) - \int \left(-3\cos x + \frac{\cos 3x}{3} \right) dx \right]$$

$$= \frac{1}{4} \left[-3x\cos x + \frac{x\cos 3x}{3} + 3\sin x - \frac{\sin 3x}{9} \right] + c$$

$$I = \frac{1}{36} [3x \cos 3x - 27x \cos x + 27 \sin x - \sin 3x] + c$$

Indefinite Integrals Ex 19.25 Q55

Let
$$I = \int \cos^3 \sqrt{x} \, dx$$
Let
$$x = t^2$$

$$dx = 2t dt$$

$$= 2 \int t \cos^3 t \, dt$$

$$= 2 \int t \left(\frac{3 \cos t + \cos 3t}{4} \right) dt$$

$$= \frac{1}{2} \int t \left(3 \cos t + \cos 3t \right) dt$$

Using integration by parts,

$$\begin{split} I &= \frac{1}{2} \bigg[t \bigg(3 \sin t + \frac{1}{3} \sin 3t \bigg) + \int \bigg(1 \times 3 \sin t + \frac{\sin 3t}{3} \bigg) dt \bigg] \\ &= \frac{1}{2} \bigg[t \bigg(\frac{9 \sin t + \sin 3t}{3} \bigg) + 3 \cos t + \frac{\cos 3t}{9} \bigg] + c \\ &= \frac{1}{18} \bigg[27t \sin t + 3t \sin 3t + 9 \cos t + \cos 3t \bigg] + c \end{split}$$

$$I = \frac{1}{18} \left[27\sqrt{x} \sin \sqrt{x} + 3\sqrt{x} \sin 3\sqrt{x} + 9\cos \sqrt{x} + \cos 3\sqrt{x} \right] + c$$

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