Exercise 7A

Question 17

$$\text{LHS} = \frac{1+\tan^2\theta}{1+\cot^2\theta} = \frac{\left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)}{\left(1+\frac{\cos^2\theta}{\sin^2\theta}\right)} = \frac{\frac{\left(\cos^2\theta+\sin^2\theta\right)}{\cos^2\theta}}{\frac{\left(\sin^2\theta+\cos^2\theta\right)}{\sin^2\theta}}$$

$$= \frac{\left(\frac{1}{\cos^2\theta}\right)}{\left(\frac{1}{\sin^2\theta}\right)} = \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$$

$$\text{RHS} = \frac{(1-\tan\theta)^2}{(1-\cot\theta)^2}$$

$$= \frac{\left(1-\frac{\sin\theta}{\cos\theta}\right)^2}{\left(1-\frac{\cos\theta}{\sin\theta}\right)^2}$$

$$= \frac{\frac{(\cos\theta-\sin\theta)^2}{\cos^2\theta}}{\frac{(\sin\theta-\cos\theta)^2}{\sin^2\theta}} = \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \tan^2\theta$$

$\therefore$ LHS = RHS

Question 18

(i) $$\text{LHS} = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$= \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\left(\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}\right)}{\left(\frac{\cos^2\theta+\sin^2\theta}{\cos^2}\right)} = \frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta}$$

$$= \frac{\left(\cos^2\theta-\sin^2\theta\right)}{1} = \left(\cos^2\theta-\sin^2\theta\right) = \text{RHS}$$

$\therefore$ LHS = RHS

(ii) $$\text{LHS} = \frac{1-\tan^2\theta}{\cot^2\theta-1}$$

$$= \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta}-1} = \frac{\left(\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}\right)}{\left(\frac{\cos^2\theta-\sin^2\theta}{\sin^2\theta}\right)}$$

$$= \left(\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}\right) \times \frac{\sin^2\theta}{\left(\cos^2\theta-\sin^2\theta\right)}$$

$$= \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta = \text{RHS}$$

$\therefore$ LHS = RHS

*********** END **********