



Trigonometric Ratios of Compound Angles Ex 7.1 Q29

$$\begin{aligned}
 \text{LHS} &= \frac{1}{\sin(x-a)\sin(x-b)} \\
 &= \frac{1}{\sin(a-b)} \left[\frac{\sin(a-b)}{\sin(x-a)\sin(x-b)} \right] \\
 &= \frac{1}{\sin(a-b)} \left[\frac{\sin\{(x-b)-(x-a)\}}{\sin(x-a)\sin(x-b)} \right] \\
 &= \frac{1}{\sin(a-b)} \left[\frac{\sin(x-b)\cos(x-a) - \cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)} \right] \\
 &= \frac{1}{\sin(a-b)} \left[\frac{\sin(x-b)\cos(x-a)}{\sin(x-a)\sin(x-b)} - \frac{\cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)} \right] \\
 &= \frac{1}{\sin(a-b)} [\cot(x-a) - \cot(x-b)] \\
 &= \frac{\cot(x-a) - \cot(x-b)}{\sin(a-b)} \\
 &= \text{RHS}
 \end{aligned}$$

∴ LHS=RHS

Hence proved

$$\begin{aligned}
 \text{LHS} &= \frac{1}{\sin(x-a)\cos(x-b)} \\
 &= \frac{1}{\cos(a-b)} \left[\frac{\cos(a-b)}{\sin(x-a)\cos(x-b)} \right] \\
 &= \frac{1}{\cos(a-b)} \left[\frac{\cos\{(x-b)-(x-a)\}}{\sin(x-a)\cos(x-b)} \right] \\
 &= \frac{1}{\cos(a-b)} \left[\frac{\cos(x-b)\cos(x-a) + \sin(x-b)\sin(x-a)}{\sin(x-a)\cos(x-b)} \right] \\
 &= \frac{1}{\cos(a-b)} \left[\frac{\cos(x-b)\cos(x-a)}{\sin(x-a)\cos(x-b)} + \frac{\sin(x-b)\sin(x-a)}{\sin(x-a)\cos(x-b)} \right] \\
 &= \frac{1}{\cos(a-b)} \left[\frac{\cos(x-a)}{\sin(x-a)} + \frac{\sin(x-b)}{\cos(x-b)} \right] \\
 &= \frac{1}{\cos(a-b)} [\cot(x-a) + \tan(x-b)] \\
 &= \frac{\cot(x-a) + \tan(x-b)}{\cos(a-b)} \\
 &= \text{RHS}
 \end{aligned}$$

∴ LHS=RHS

Hence proved

$$\begin{aligned}
\text{LHS} &= \frac{1}{\cos(x-a)\cos(x-b)} \\
&= \frac{1}{\sin(a-b)} \left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)} \right] \\
&= \frac{1}{\sin(a-b)} \left[\frac{\sin\{(x-b)-(x-a)\}}{\cos(x-a)\cos(x-b)} \right] \\
&= \frac{1}{\sin(a-b)} \left[\frac{\sin(x-b)\cos(x-a) - \cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)} \right] \\
&= \frac{1}{\sin(a-b)} \left[\frac{\sin(x-b)\cos(x-a)}{\cos(x-a)\cos(x-b)} - \frac{\cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)} \right] \\
&= \frac{1}{\sin(a-b)} \left[\frac{\sin(x-b)}{\cos(x-b)} - \frac{\sin(x-a)}{\cos(x-a)} \right] \\
&= \frac{1}{\sin(a-b)} [\tan(x-b) - \tan(x-a)] \\
&= \frac{\tan(x-b) - \tan(x-a)}{\sin(a-b)} \\
&= \text{RHS}
\end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved

Trigonometric Ratios of Compound Angles Ex 7.1 Q30

We have,

$$\sin \alpha \sin \beta - \cos \alpha \cos \beta + 1 = 0$$

$$\Rightarrow -(\cos \alpha \cos \beta - \sin \alpha \sin \beta) = -1$$

$$\Rightarrow \cos(\alpha + \beta) = 1 \quad \text{--- (i)}$$

$$\begin{aligned}
\therefore \sin(\alpha + \beta) &= \sqrt{1 - \cos^2(\alpha + \beta)} \\
&= \sqrt{1 - 1^2} \\
&= 0
\end{aligned}$$

$$\Rightarrow \sin(\alpha + \beta) = 0 \quad \text{--- (ii)}$$

Now,

$$\begin{aligned}
1 + \cot \alpha \tan \beta &= 1 + \frac{\cos \alpha}{\sin \alpha} \times \frac{\sin \beta}{\cos \beta} \\
&= \frac{\sin \alpha \times \cos \beta + \cos \alpha \times \sin \beta}{\sin \alpha \times \cos \beta} \\
&= \frac{\sin(\alpha + \beta)}{\sin \alpha \times \cos \beta} \\
&= \frac{0}{\sin \alpha \times \cos \beta} \quad \text{[Using equation (ii)]} \\
&= 0
\end{aligned}$$

$$\therefore 1 + \cot \alpha \tan \beta = 0$$

Hence proved

***** END *****