CHAPTER 44

X-RAYS

44.1 PRODUCTION OF X-RAYS

When highly energetic electrons are made to strike a metal target, electromagnetic radiation comes out. A large part of this radiation has wavelength of the order of 0·1 nm (≈ 1 Å) and is known as *X-ray*.

X-ray was discovered by the German physicist W C Roentgen in 1895. He found that photographic film wrapped light-tight in black paper became exposed when placed near a cathode-ray tube. He concluded that some invisible radiation was coming from the cathode-ray tube which penetrated the black paper to affect the photographic plate. He named this radiation as X-ray because its nature and properties could not be known at that time. In mathematics, we generally use the symbol x for unknown quantities. However, after some calculation we finally get the value of this unknown x. Similarly, we now know about the nature and properties of X-rays.

A device used to produce X-rays is generally called an *X-ray tube*. Figure (41.1) shows a schematic diagram of such a device. This was originally designed by Coolidge and is known as *Coolidge tube* to produce X-rays.

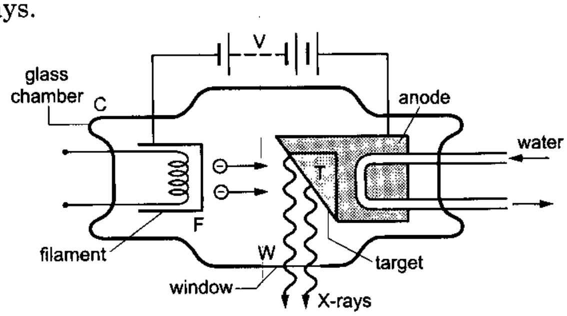

Figure 44.1

A filament F and a metallic target T are fixed in an evacuated glass chamber C. The filament is heated electrically and emits electrons by thermionic emission. A constant potential difference of several kilovolts is maintained between the filament and the target using a DC power supply so that the target is at a higher potential than the filament. The electrons emitted by the filament are, therefore, accelerated by the electric field set up between the filament and the target and hit the target with a very high speed. These electrons are stopped by the target and in the process X-rays are emitted. These X-rays are brought out of the tube through a window W made of thin mica or mylar or some such material which does not absorb X-rays appreciably.

In the process, large amount of heat is developed, and thus an arrangement is provided to cool down the tube continuously by running water.

The exact design of the X-ray tube depends on the type of use for which these X-rays are required.

44.2 CONTINUOUS AND CHARACTERISTIC X-RAYS

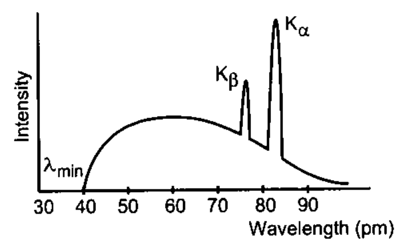

Figure 44.2

If the X-rays coming from a Coolidge tube are examined for the wavelengths present, and the intensity of different wavelength components are measured, we obtain a plot of the nature shown in figure (44.2). We see that there is a minimum wavelength below which no X-ray is emitted. This is called the *cutoff wavelength* or the *threshold wavelength*. The X-rays emitted can be clearly divided in two categories. At certain sharply defined wavelengths, the intensity of X-rays is very large as marked K_α, K_β in figure (44.2). These X-rays are known as *characteristic X-rays*. At other wavelengths the intensity varies gradually and these X-rays are called *continuous X-rays*. Let us examine the origin of these two types of X-rays.

Suppose, the potential difference applied between the target and the filament is V and electrons are emitted by the filament with negligible speed. The electrons are accelerated in their journey from the filament to the target. The kinetic energy of an electron when it hits the target is

$$K = eV. \qquad \dots \ (44.1)$$

As the electron enters into the target material, it readily loses its kinetic energy and is brought to rest inside the metal. The electron before finally being stopped, makes several collisions with the atoms in the target. At each collision, one of the following two processes may occur:

(a) The kinetic energy of the electron is reduced. A part of this lost kinetic energy is converted into a photon of electromagnetic radiation and the remaining part increases the kinetic energy of the colliding particle of the target. The energy received by the colliding particle goes into heating the target. The electron makes another collision with its reduced energy.

(b) The electron knocks out an inner electron of the atom with which it collides.

The fraction of kinetic energy appearing as the energy of a photon varies from collision to collision. In a certain collision, the electron may lose its entire kinetic energy to bring out a photon or it may not create a photon at all. Thus, the energy of the photon created can be anything between 0 and eV depending on how much energy has already been lost to the target and what fraction of the available energy is converted into the photon. The maximum energy of such a photon can be $E = eV$ when the electron converts all its kinetic energy into a photon in the first encounter itself.

The wavelength of the X-ray and the energy of the corresponding photon are related through the equation

$$\lambda = \frac{hc}{E}. \qquad \dots \ (44.2)$$

As E can take any value between zero and eV, the wavelength λ can take any value between infinity and hc/eV. This explains the origin of continuous X-rays and the cutoff wavelength. We have,

$$\lambda_{min} = \frac{hc}{eV}. \qquad \dots \ (44.3)$$

We see that the cutoff wavelength λ_{min} depends only on the accelerating voltage V applied between the target and the filament. It does not depend on the material of the target.

We shall now discuss what happens if the electron knocks out an inner electron from the atom with which it collides. The electrons in an atom occupy different quantum states characterized by the quantum numbers n, l, m_l, m_s. The energy primarily depends on the principal quantum number n. The two electrons corresponding to $n = 1$ are said to be in K shell, those corresponding to $n = 2$ are in L shell, etc. Suppose, the incident electron knocks out an electron from the K shell. This will create a vacancy in the K shell in the sense that now there is only one electron with $n = 1$, whereas two could be accommodated by Pauli exclusion principle. An electron from a higher energy state may make a transition to this vacant state. When such a transition takes place, the difference of energy ΔE is converted into an X-ray photon of wavelength $\lambda = hc/\Delta E$. *X-rays emitted due to electronic transition from a higher energy state to a vacancy created in the K shell are called K X-rays.*

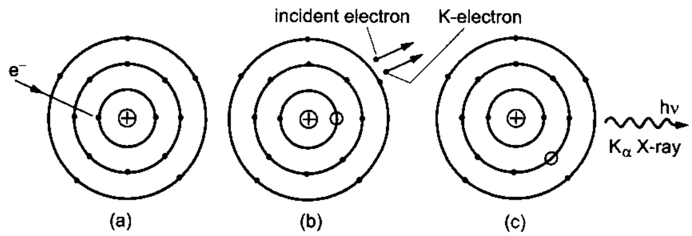

Figure 44.3

Figure (44.3) shows the process schematically. If an electron from the L shell (i.e., with $n = 2$) makes transition to the vacant state in the K shell, the X-ray emitted is called *K_α X-ray.* If an electron from the M shell makes transition to the K shell, a K_β X-ray is emitted. Similarly one defines K_γ X-ray. If a photon of K_α X-ray is emitted, the vacancy in the K shell is filled up but a vacancy is created in the L shell. This vacancy can be filled up by a transition of electron from higher shells giving L X-ray. If an electron jumps from the M shell to the vacant state in the L shell, we obtain L_α X-ray. If the vacancy in L shell is filled up by an electron of N shell ($n = 4$), L_β X-ray is emitted, and so on.

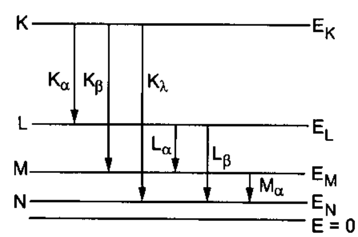

Figure 44.4

Figure (44.4) shows the energy levels of the atom when one electron is knocked out. The lowest line corresponds to the atom with all its electrons intact. This has been taken as zero energy. The energy level with label E_K is the energy of the atom when an electron from the K shell is knocked out. Similar is the interpretation for E_L, E_M, E_N, etc. Note the convention of choosing $E = 0$ in the ground state. In hydrogen atom we had chosen $E = 0$ when the electron was knocked out. The ground state of hydrogen atom then had an energy of $-13{\cdot}6$ eV. Here the convention is opposite and the energy in the ground state is chosen to be zero. The energy in the ionized state is then positive. As the electrons in K shell are most tightly bound, maximum energy is to be given to the atom to knock out an electron from the K shell. That is why, in figure (44.4), the energy level of the atom with a vacancy in the K shell is shown highest.

The energies E_K, E_L, ..., etc., are characteristic properties of the material. For different materials, the values of these energies will be different. The values of $E_K - E_L$, $E_L - E_M$, etc., also have definite values for a given material. The wavelengths of the X-rays emitted corresponding to these transitions are

$$\lambda = \frac{hc}{E_K - E_L} \text{ for } K_\alpha,$$

$$\lambda = \frac{hc}{E_K - E_M} \text{ for } K_\beta,$$

$$\lambda = \frac{hc}{E_L - E_M} \text{ for } L_\alpha,$$

etc. These wavelengths, therefore, have definite values for a particular material. The X-rays emitted in this way are the characteristic X-rays shown in figure (44.2). They are so named because their wavelengths may be used to identify the element from which they originate.

44.3 SOFT AND HARD X-RAYS

If the accelerating voltage applied between the filament and the target is increased, the cutoff wavelength λ_{min} decreases further (equation 44.3). The X-rays of low wavelengths are called *hard X-rays* and those of large wavelength are called *soft X-rays*. Hard and soft are simply relative terms. In terms of energy, harder X-rays means more energy in each photon. So, if the voltage between the filament and the target is increased, we get harder X-rays. If the filament current in a Coolidge tube is increased by increasing the voltage in the filament circuit, more electrons are emitted per unit time. This results in an increase in the number of X-ray photons emitted per unit time and hence the intensity of X-rays is increased. The cutoff wavelength λ_{min} remains unchanged as the maximum kinetic energy of the electrons reaching the target is not affected by the filament current.

44.4 MOSELEY'S LAW

Moseley's experiments (1913–1914) on characteristic X-rays played a very important role in developing the concept of atomic number. In those days, the elements were arranged in periodic table in the increasing order of atomic weight. The periodicity in chemical properties of elements was brought out from such arrangement, though some anomalies were present. Bohr had proposed his model in the same year and there was no concept of distribution of electrons in different energy levels. During those days, Moseley measured the frequencies of characteristic X-rays from a large number of elements and plotted the square root of the frequency against its position number in the periodic table. He discovered that the plot is very close to a straight line. A portion of Moseley's plot is shown in figure (44.5) where $\sqrt{\nu}$ of K_α X-rays is plotted against the position number. From this linear relation, Moseley concluded that there must be a fundamental property of the atom which increases by regular steps as one moves from one element to the other. This quantity was later identified to be the number of protons in the nucleus and was referred to as the *atomic number*.

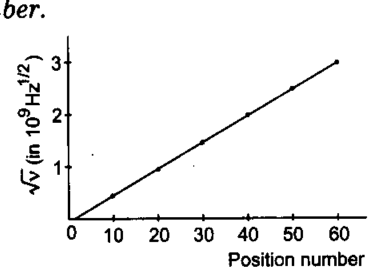

Figure 44.5

Thus, elements should be arranged in the ascending order of atomic number and not of atomic weight. This removed several discrepancies existing in the periodic table. For example, nickel has atomic weight $58{\cdot}7$ whereas the atomic weight of cobalt is $58{\cdot}9$. However, the frequency of K_α X-ray from cobalt is less than the frequency of K_α X-ray from nickel. Thus, Moseley rearranged the sequence as Co, Ni instead of Ni, Co. Similarly, several other rearrangements were made.

Moseley's observations can be mathematically expressed as

$$\sqrt{\nu} = a(Z - b) \qquad \text{... (44.4)}$$

where a and b are constants. This relation is known as *Moseley's law*. We can understand Moseley's law qualitatively from Bohr's atomic model.

Consider an atom from which an electron from the K shell has been knocked out. Consider an electron from the L shell which is about to make a transition to the vacant site. It finds the nucleus of charge Ze screened by the spherical cloud of the remaining one electron in the K shell (figure 44.6). If we neglect the effect of the outer electrons and the other L electrons, the electron making the transition finds a charge $(Z-1)e$ at the centre. One, therefore, may expect Bohr's model to give reasonable results if Z is replaced by $Z - b$ with $b \approx 1$.

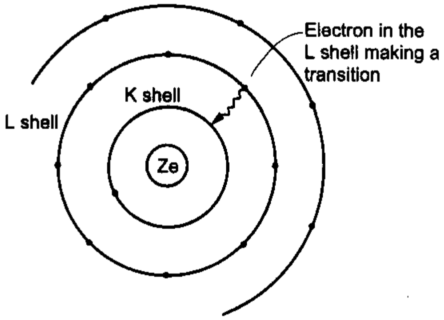

Figure 44.6

According to Bohr's model, the energy released during the transition from $n = 2$ to $n = 1$ is given by

$$\Delta E = h\nu = Rhc(Z-b)^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$$

so that $\sqrt{\nu} = \sqrt{\frac{3Rc}{4}}\,(Z-b)$

which is the same as equation (44.4) with $a = \sqrt{3Rc/4}$.

Moseley was killed in the First World War at an early age of 27 years.

44.5 BRAGG'S LAW

X-rays are electromagnetic waves of short wavelengths and may be diffracted by suitable diffracting centres. However, the diffraction effects are appreciable only when the diffracting apertures are of the order of the wavelength, i.e., of the order of 0·1 nm. This is almost the size of an atom and it is difficult to construct slits with such small gaps so that X-rays can be appreciably diffracted.

In solid crystals, atoms are arranged in fairly regular pattern with interatomic gaps of the order of 0·1 nm. Common salt is an example of such a crystalline solid. Almost all the metals at ordinary temperature are crystalline. These metals may act as natural three-dimensional gratings for the diffraction of X-rays.

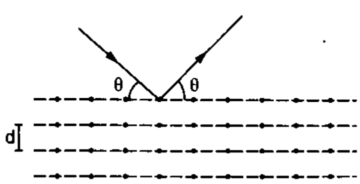

Figure 44.7

The structure of a solid can be viewed as a series of parallel planes of atoms separated by a distance d (figure 44.7). Suppose, an X-ray beam is incident on a solid, making an angle θ with the planes of the atoms. These X-rays are diffracted by different atoms and the diffracted rays interfere. In certain directions, the interference is constructive and we obtain strong reflected X-rays. The analysis shows that there will be a strong reflected X-ray beam only if

$$2d\sin\theta = n\lambda \qquad \ldots\ (44.5)$$

where n is an integer. For monochromatic X-rays, λ is fixed and there are some specific angles θ_1, θ_2, θ_3, ..., etc., corresponding to $n = 1, 2, 3, \ldots$, etc., in equation (44.5). Thus, if the X-rays are incident at one of these angles, they are reflected; otherwise they are absorbed. When they are reflected, the laws of reflection are obeyed, i.e., (a) the angle of incidence is equal to the angle of reflection and (b) the incident ray, the reflected ray and the normal to the reflecting plane are coplanar.

Equation (44.5) is known as *Bragg's law.*

By using a monochromatic X-ray beam and noting the angles of strong reflection, the interplanar spacing d and several informations about the structure of the solid can be obtained.

44.6 PROPERTIES AND USES OF X-RAYS

As discussed earlier, X-rays are electromagnetic waves of short wavelengths. Accordingly, it has many properties common with light. Here are some of the properties of X-rays.

(a) X-rays travel in straight lines in vacuum at a speed equal to that of light (3×10^8 m/s).

(b) X-rays are diffracted by crystals in accordance with Bragg's law.

(c) X-rays are not deflected by electric or magnetic field as it contains no charged particles.

(d) X-rays affect a photographic plate more strongly than visible light.

(e) When incident on certain materials such as barium platinocyanide, X-rays cause fluorescence (light is emitted from the material).

(f) When passed through a gas, X-rays ionize the molecules of the gas.

(g) X-rays can penetrate into several metals and other materials. Thus, they can pass through small thicknesses of aluminium, woods, plastics, human flesh etc. They are stopped by materials of high density and high atomic number.

The penetrating power of X-ray has made it popular and familiar to the general public. It is used extensively to detect diseases inside the body. It passes quite freely through the flesh but is stopped by the bones. So it can photograph the bones inside the body on a photographic film. Such a photograph is called a *radiograph.* This is used to detect and study bone fractures due to an accident. Chest radiographs are used to study diseases in lungs. Dentists also use X-ray to study teeth-decay. X-ray is also used in cancer therapy.

X-ray is used in industry and material science research. It can detect structural defects, fault of joints, welding etc. X-ray machines are used to inspect suitcases, wooden boxes etc. without opening them and can be typically found at the custom, security counters at airports etc.

X-ray can be used to detect diseases and to cure them. At the same time, random and excess exposure to X-ray may induce diseases. X-ray has a damaging effect on the living cells of a body which may lead to cell-death. High exposure for a long period (say for years) may lead to cancer or genetic defects.

TV, computer terminals, oscilloscopes etc. use a cathode-ray tube in which a highly energetic electron beam strikes the screen. A fraction of kinetic energy is converted into X-rays which may come out. Screens of these equipments are generally designed to absorb these X-rays.

Worked Out Examples

1. *Find the maximum frequency of the X-rays emitted by an X-ray tube operating at* 30 kV.

Solution : For maximum frequency, the total kinetic energy (eV) should be converted into an X-ray photon. Thus,

$$h\nu = eV$$

or,
$$\nu = \frac{e}{h}V$$
$$= \frac{e \times 30 \times 10^{3}\ \text{V}}{4{\cdot}14 \times 10^{-15}\ \text{eV–s}}$$
$$= \frac{30}{4{\cdot}14} \times 10^{18}\ \text{Hz} \approx 7{\cdot}2 \times 10^{18}\ \text{Hz}.$$

2. *An X-ray tube operates at* 20 kV. *A particular electron loses* 5% *of its kinetic energy to emit an X-ray photon at the first collision. Find the wavelength corresponding to this photon.*

Solution : Kinetic energy acquired by the electron is

$$K = eV = 20 \times 10^{3}\ \text{eV}.$$

The energy of the photon

$$= 0{\cdot}05 \times 20 \times 10^{3}\ \text{eV} = 10^{3}\ \text{eV}.$$

Thus,
$$\frac{hc}{\lambda} = 10^{3}\ \text{eV}$$

or,
$$\lambda = \frac{hc}{10^{3}\ \text{eV}}$$
$$= \frac{(4{\cdot}14 \times 10^{-15}\ \text{eVs}) \times (3 \times 10^{8}\ \text{m s}^{-1})}{10^{3}\ \text{eV}}$$
$$= \frac{1242\ \text{eV nm}}{10^{3}\ \text{eV}} = 1{\cdot}24\ \text{nm}.$$

3. *An X-ray tube is operated at* 20 kV *and the current through the tube is* 0·5 mA. *Find (a) the number of electrons hitting the target per second, (b) the energy falling on the target per second as the kinetic energy of the electrons and (c) the cutoff wavelength of the X-rays emitted.*

Solution : (a)
$$i = ne = 0{\cdot}5 \times 10^{-3}\ \text{A}$$

or,
$$n = \frac{0{\cdot}5 \times 10^{-3}\ \text{A}}{1{\cdot}6 \times 10^{-19}\ \text{C}} = 3{\cdot}1 \times 10^{15}.$$

(b) The kinetic energy of an electron reaching the target is $K = eV$. The energy falling on the target per second

$$= n\,eV = iV = (0{\cdot}5 \times 10^{-3}\ \text{A}) \times (20 \times 10^{3}\ \text{V})$$
$$= 10\ \text{J s}^{-1}.$$

(c)
$$\frac{hc}{\lambda_{min}} = eV$$

or,
$$\lambda_{min} = \frac{hc}{eV}$$
$$= \frac{1242\ \text{eV nm}}{e(20 \times 10^{3}\ \text{V})} = 0{\cdot}062\ \text{nm}.$$

4. *Find the constants a and b in Moseley's equation* $\sqrt{\nu} = a(Z - b)$ *from the following data.*

Element	Z	Wavelength of K_α X-ray
Mo	42	71 pm
Co	27	178·5 pm

Solution : Moseley's equation is

$$\sqrt{\nu} = a(Z - b).$$

Thus,
$$\sqrt{\frac{c}{\lambda_1}} = a(Z_1 - b) \qquad \dots \text{(i)}$$

and
$$\sqrt{\frac{c}{\lambda_2}} = a(Z_2 - b). \qquad \dots \text{(ii)}$$

From (i) and (ii),

$$\sqrt{c}\left(\frac{1}{\sqrt{\lambda_1}} - \frac{1}{\sqrt{\lambda_2}}\right) = a(Z_1 - Z_2)$$

or,
$$a = \frac{\sqrt{c}}{(Z_1 - Z_2)}\left(\frac{1}{\sqrt{\lambda_1}} - \frac{1}{\sqrt{\lambda_2}}\right)$$
$$= \frac{(3 \times 10^{8}\ \text{m s}^{-1})^{1/2}}{42 - 27}\left[\frac{1}{(71 \times 10^{-12}\ \text{m})^{1/2}} - \frac{1}{(178{\cdot}5 \times 10^{-12}\ \text{m})^{1/2}}\right]$$
$$= 5{\cdot}0 \times 10^{7}\ (\text{Hz})^{1/2}.$$

Dividing (i) by (ii),

$$\sqrt{\frac{\lambda_2}{\lambda_1}} = \frac{Z_1 - b}{Z_2 - b}$$

or,
$$\sqrt{\frac{178{\cdot}5}{71}} = \frac{42 - b}{27 - b}$$

or,
$$b = 1{\cdot}37.$$

5. *The K_α X-ray of molybdenum has wavelength 71 pm. If the energy of a molybdenum atom with a K electron knocked out is 23·32 keV, what will be the energy of this atom when an L electron is knocked out?*

Solution : K_α X-ray results from the transition of an electron from L shell to K shell. If the energy of the atom with a vacancy in the K shell is E_K and the energy with a vacancy in the L shell is E_L, the energy of the photon emitted is $E_K - E_L$. The energy of the 71 pm photon is

$$E = \frac{hc}{\lambda}$$

$$= \frac{1242 \text{ eV nm}}{71 \times 10^{-3} \text{ nm}} = 17{\cdot}5 \text{ keV}.$$

Thus, $E_K - E_L = 17{\cdot}5$ keV

or, $E_L = E_K - 17{\cdot}5$ keV

$= 23{\cdot}32 \text{ keV} - 17{\cdot}5 \text{ keV} = 5{\cdot}82 \text{ keV}.$

6. *Show that the frequency of K_β X-ray of a material equals the sum of the frequencies of K_α and L_α X-rays of the same material.*

Solution :

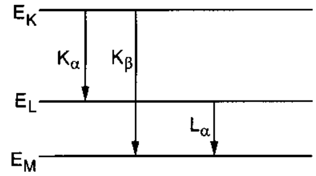

Figure 44-W1

The energy level diagram of an atom with one electron knocked out is shown in figure (44-W1).

Energy of K_α X-ray is $E_{K_\alpha} = E_K - E_L$

of K_β X-ray is $E_{K_\beta} = E_K - E_M$,

and of L_α X-ray is $E_{L_\alpha} = E_L - E_M$.

Thus, $E_{K_\beta} = E_{K_\alpha} + E_{L_\alpha}$

or, $h\nu_{K_\beta} = h\nu_{K_\alpha} + h\nu_{L_\alpha}$

or, $\nu_{K_\beta} = \nu_{K_\alpha} + \nu_{L_\alpha}$.

□

QUESTIONS FOR SHORT ANSWER

1. When a Coolidge tube is operated for some time it becomes hot. Where does the heat come from?
2. In a Coolidge tube, electrons strike the target and stop inside it. Does the target get more and more negatively charged as time passes?
3. Can X-rays be used for photoelectric effect?
4. Can X-rays be polarized?
5. X-ray and visible light travel at the same speed in vacuum. Do they travel at the same speed in glass?
6. Characteristic X-rays may be used to identify the element from which they are coming. Can continuous X-rays be used for this purpose?
7. Is it possible that in a Coolidge tube characterstic L_α X-rays are emitted but not K_α X-rays?
8. Can L_α X-ray of one material have shorter wavelength than K_α X-ray of another?
9. Can a hydrogen atom emit characteristic X-ray?
10. Why is exposure to X-ray injurious to health but exposure to visible light is not, when both are electromagnetic waves?

OBJECTIVE I

1. X-ray beam can be deflected
(a) by an electric field (b) by a magnetic field
(c) by an electric field as well as by a magnetic field
(d) neither by an electric field nor by a magnetic field.

2. Consider a photon of continuous X-ray coming from a Coolidge tube. Its energy comes from
(a) the kinetic energy of the striking electron
(b) the kinetic energy of the free electrons of the target
(c) the kinetic energy of the ions of the target
(d) an atomic transition in the target.

3. The energy of a photon of characteristic X-ray from a Coolidge tube comes from
(a) the kinetic energy of the striking electron
(b) the kinetic energy of the free electrons of the target
(c) the kinetic energy of the ions of the target
(d) an atomic transition in the target.

4. If the potential difference applied to the tube is doubled and the separation between the filament and the target is also doubled, the cutoff wavelength
(a) will remain unchanged (b) will be doubled

(c) will be halved
(d) will become four times the original.

5. If the current in the circuit for heating the filament is increased, the cutoff wavelength
(a) will increase (b) will decrease
(c) will remain unchanged (d) will change.

6. Moseley's law for characteristic X-rays is $\sqrt{\nu} = a(Z - b)$. In this,
(a) both a and b are independent of the material
(b) a is independent but b depends on the material
(c) b is independent but a depends on the material
(d) both a and b depend on the material.

7. Frequencies of K_α X-rays of different materials are measured. Which one of the graphs in figure (44-Q1) may represent the relation between the frequency ν and the atomic number Z.

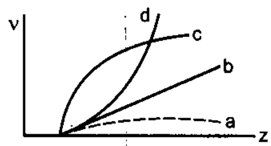

Figure 44-Q1

8. The X-ray beam coming from an X-ray tube
(a) is monochromatic
(b) has all wavelengths smaller than a certain maximum wavelength
(c) has all wavelengths greater than a certain minimum wavelength
(d) has all wavelengths lying between a minimum and a maximum wavelength.

9. One of the following wavelengths is absent and the rest are present in the X-rays coming from a Coolidge tube. Which one is the absent wavelength ?
(a) 25 pm (b) 50 pm (c) 75 pm (d) 100 pm.

10. Figure (44-Q2) shows the intensity–wavelength relations of X-rays coming from two different Coolidge tubes. The solid curve represents the relation for the tube A in which the potential difference between the target and the filament is V_A and the atomic number of the target material is Z_A. These quantities are V_B and Z_B for the other tube. Then,
(a) $V_A > V_B, Z_A > Z_B$ (b) $V_A > V_B, Z_A < Z_B$
(c) $V_A < V_B, Z_A > Z_B$ (d) $V_A < V_B, Z_A < Z_B$.

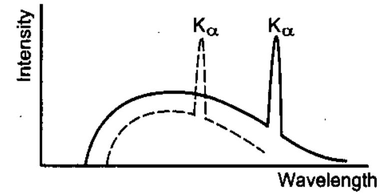

Figure 44-Q2

11. 50% of the X-ray coming from a Coolidge tube is able to pass through a 0·1 mm thick aluminium foil. If the potential difference between the target and the filament is increased, the fraction of the X-ray passing through the same foil will be
(a) 0% (b) < 50% (c) 50% (d) > 50%.

12. 50% of the X-ray coming from a Coolidge tube is able to pass through a 0·1 mm thick aluminium foil. The potential difference between the target and the filament is increased. The thickness of aluminium foil, which will allow 50% of the X-ray to pass through, will be
(a) zero (b) < 0·1 mm (c) 0·1 mm (d) > 0·1 mm.

13. X-ray from a Coolidge tube is incident on a thin aluminium foil. The intensity of the X-ray transmitted by the foil is found to be I_0. The heating current is increased so as to increase the temperature of the filament. The intensity of the X-ray transmitted by the foil will be
(a) zero (b) $< I_0$ (c) I_0 (d) $> I_0$.

14. Visible light passing through a circular hole forms a diffraction disc of radius 0·1 mm on a screen. If X-ray is passed through the same set-up, the radius of the diffraction disc will be
(a) zero (b) < 0·1 mm (c) 0·1 mm (d) > 0·1 mm.

OBJECTIVE II

1. For harder X-rays,
(a) the wavelength is higher
(b) the intensity is higher
(c) the frequency is higher
(d) the photon energy is higher.

2. Cutoff wavelength of X-rays coming from a Coolidge tube depends on the
(a) target material (b) accelerating voltage
(c) separation between the target and the filament
(d) temperature of the filament.

3. Mark the correct options.
(a) An atom with a vacancy has smaller energy than a neutral atom.
(b) K X-ray is emitted when a hole makes a jump from the K shell to some other shell.
(c) The wavelength of K X-ray is smaller than the wavelength of L X-ray of the same material.
(d) The wavelength of K_α X-ray is smaller than the wavelength of K_β X-ray of the same material.

4. For a given material, the energy and wavelength of characterstic X-rays satisfy
(a) $E(K_\alpha) > E(K_\beta) > E(K_\gamma)$ (b) $E(M_\alpha) > E(L_\alpha) > E(K_\alpha)$
(c) $\lambda(K_\alpha) > \lambda(K_\beta) > \lambda(K_\gamma)$ (d) $\lambda(M_\alpha) > \lambda(L_\alpha) > \lambda(K_\alpha)$.

5. The potential difference applied to an X-ray tube is increased. As a result, in the emitted radiation,
(a) the intensity increases
(b) the minimum wavelength increases
(c) the intensity remains unchanged
(d) the minimum wavelength decreases.

6. When an electron strikes the target in a Coolidge tube, its entire kinetic energy

(a) is converted into a photon
(b) may be converted into a photon
(c) is converted into heat
(d) may be converted into heat.

7. X-ray incident on a material
(a) exerts a force on it (b) transfers energy to it
(c) transfers momentum to it
(d) transfers impulse to it.

8. Consider a photon of continuous X-ray and a photon of characteristic X-ray of the same wavelength. Which of the following is/are different for the two photons ?
(a) Frequency (b) Energy
(c) Penetrating power (d) Method of creation

EXERCISES

Planck constant $h = 4{\cdot}14 \times 10^{-15}$ eV s^{-1}, speed of light $c = 3 \times 10^{8}$ m s^{-1}.

---

1. Find the energy, the frequency and the momentum of an X-ray photon of wavelength 0·10 nm.
2. Iron emits K_α X-ray of energy 6·4 keV and calcium emits K_α X-ray of energy 3·69 keV. Calculate the times taken by an iron K_α photon and a calcium K_α photon to cross through a distance of 3 km.
3. Find the cutoff wavelength for the continuous X-rays coming from an X-ray tube operating at 30 kV.
4. What potential difference should be applied across an X-ray tube to get X-ray of wavelength not less than 0·10 nm ? What is the maximum energy of a photon of this X-ray in joule ?
5. The X-ray coming from a Coolidge tube has a cutoff wavelength of 80 pm. Find the kinetic energy of the electrons hitting the target.
6. If the operating potential in an X-ray tube is increased by 1%, by what percentage does the cutoff wavelength decrease ?
7. The distance between the cathode (filament) and the target in an X-ray tube is 1·5 m. If the cutoff wavelength is 30 pm, find the electric field between the cathode and the target.
8. The short-wavelength limit shifts by 26 pm when the operating voltage in an X-ray tube is increased to 1·5 times the original value. What was the original value of the operating voltage ?
9. The electron beam in a colour TV is accelerated through 32 kV and then strikes the screen. What is the wavelength of the most energetic X-ray photon ?
10. When 40 kV is applied across an X-ray tube, X-ray is obtained with a maximum frequency of $9{\cdot}7 \times 10^{18}$ Hz. Calculate the value of Planck constant from these data.
11. An X-ray tube operates at 40 kV. Suppose the electron converts 70% of its energy into a photon at each collision. Find the lowest three wavelengths emitted from the tube. Neglect the energy imparted to the atom with which the electron collides.
12. The wavelength of K_α X-ray of tungsten is 21·3 pm. It takes 11·3 keV to knock out an electron from the L shell of a tungsten atom. What should be the minimum accelerating voltage across an X-ray tube having tungsten target which allows production of K_α X-ray ?
13. The K_β X-ray of argon has a wavelength of 0·36 nm. The minimum energy needed to ionize an argon atom is 16 eV. Find the energy needed to knock out an electron from the K shell of an argon atom.
14. The K_α X-rays of aluminium ($Z = 13$) and zinc ($Z = 30$) have wavelengths 887 pm and 146 pm respectively. Use Moseley's law $\sqrt{\nu} = a(Z - b)$ to find the wavelength of the K_α X-ray of iron ($Z = 26$).
15. A certain element emits K_α X-ray of energy 3·69 keV. Use the data from the previous problem to identify the element.
16. The K_β X-rays from certain elements are given below. Draw a Moseley-type plot of $\sqrt{\nu}$ versus Z for K_β radiation.

Element	Ne	P	Ca	Mn	Zn	Br
Energy (keV)	0·858	2·14	4·02	6·51	9·57	13·3.

17. Use Moseley's law with $b = 1$ to find the frequency of the K_α X-ray of La($Z = 57$) if the frequency of the K_α X-ray of Cu($Z = 29$) is known to be $1{\cdot}88 \times 10^{18}$ Hz.
18. The K_α and K_β X-rays of molybdenum have wavelengths 0·71 Å and 0·63 Å respectively. Find the wavelength of L_α X-ray of molybdenum.
19. The wavelengths of K_α and L_α X-rays of a material are 21·3 pm and 141 pm respectively. Find the wavelength of K_β X-ray of the material.
20. The energy of a silver atom with a vacancy in K shell is 25·31 keV, in L shell is 3·56 keV and in M shell is 0·530 keV higher than the energy of the atom with no vacancy. Find the frequency of K_α, K_β and L_α X-rays of silver.
21. Find the maximum potential difference which may be applied across an X-ray tube with tungsten target without emitting any characteristic K or L X-ray. The energy levels of the tungsten atom with an electron knocked out are as follows.

Cell containing vacancy	K	L	M
Energy in keV	69.5	11.3	2.3

22. The electric current in an X-ray tube (from the target to the filament) operating at 40 kV is 10 mA. Assume that on an average, 1% of the total kinetic energy of the electrons hitting the target are converted into X-rays. (a) What is the total power emitted as X-rays and (b) how much heat is produced in the target every second ?
23. Heat at the rate of 200 W is produced in an X-ray tube operating at 20 kV. Find the current in the circuit. Assume that only a small fraction of the kinetic energy of electrons is converted into X-rays.

24. Continuous X-rays are made to strike a tissue paper soaked with polluted water. The incoming X-rays excite the atoms of the sample by knocking out the electrons from the inner shells. Characteristic X-rays are subsequently emitted. The emitted X-rays are analysed and the intensity is plotted against the wavelength (figure 44-E1). Assuming that only K_α intensities are detected, list the elements present in the sample from the plot. Use Moseley's equation

$$\nu = (25 \times 10^{14}\ \text{Hz})(Z-1)^2.$$

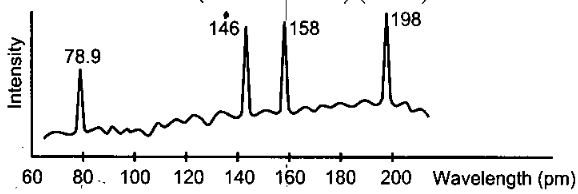

Figure 44-E1

25. A free atom of iron emits K_α X-rays of energy 6·4 keV. Calculate the recoil kinetic energy of the atom. Mass of an iron atom $= 9{\cdot}3 \times 10^{-26}$ kg.

26. The stopping potential in a photoelectric experiment is linearly related to the inverse of the wavelength ($1/\lambda$) of the light falling on the cathode. The potential difference applied across an X-ray tube is linearly related to the inverse of the cutoff wavelength ($1/\lambda$) of the X-ray emitted. Show that the slopes of the lines in the two cases are equal and find its value.

27. Suppose a monochromatic X-ray beam of wavelength 100 pm is sent through a Young's double slit and the interference pattern is observed on a photographic plate placed 40 cm away from the slit. What should be the separation between the slits so that the successive maxima on the screen are separated by a distance of 0·1 mm?

□

ANSWERS

OBJECTIVE I

1. (d)	2. (a)	3. (d)	4. (c)	5. (c)	6. (a)
7. (d)	8. (c)	9. (a)	10. (b)	11. (d)	12. (d)
13. (d)	14. (b)				

OBJECTIVE II

1. (c), (d)	2. (b)	3. (b), (c)
4. (c), (d)	5. (c), (d)	6. (b), (d)
7. all	8. (d)	

EXERCISES

1. 12·4 keV, 3×10^{18} Hz, $6{\cdot}62 \times 10^{-24}$ kg m s^{-1}
2. 10 μs by both
3. 41·4 pm
4. 12·4 kV, $2{\cdot}0 \times 10^{-15}$ J
5. 15·5 keV
6. approximately 1%
7. 27·7 kV m^{-1}
8. 15·9 kV
9. 38·8 pm
10. $4{\cdot}12 \times 10^{-15}$ eVs
11. 44·3 pm, 148 pm, 493 pm
12. 69·5 kV
13. 3·47 keV
14. 198 pm
15. calcium
17. $7{\cdot}52 \times 10^{18}$ Hz
18. 5·64 Å
19. 18·5 pm
20. $5{\cdot}25 \times 10^{18}$ Hz, $5{\cdot}98 \times 10^{18}$ Hz, $7{\cdot}32 \times 10^{17}$ Hz
21. less than 11·3 kV
22. (a) 4 W (b) 396 J
23. 10 mA
24. Zr, Zn, Cu, Fe
25. $3{\cdot}9 \times 10^{-4}$ eV
26. $\frac{hc}{e} = 1{\cdot}242 \times 10^{-6}$ Vm
27. 4×10^{-7} m

□