Functions Ex 2.3 Q 1(i)

$f(x) = e^x$ and $g(x) = \log_e x$

Now, $f \circ g(x) = f(g(x)) = f(\log_e x) = e^{\log_e x} = x$

$$f \circ g(x) = x$$

$g \circ f(x) = g(f(x)) = g(e^x) = \log_e e^x = x$

$\Rightarrow \quad g \circ f(x) = x$

Functions Ex 2.3 Q 1(ii)

$f(x) = x^2, \quad g(x) = \cos x$

Domain of f and Domain of $g = R$

Range of $f = (0, \infty)$

Range of $g = (-1, 1)$

$\therefore$ Range of $f \subset$ domain of $g \Rightarrow g \circ f$ exist

Range of $g \subset$ domain of $f \Rightarrow f \circ g$ exist

Now,

$$g \circ f(x) = g(f(x)) = g(x^2) = \cos x^2$$

And

$$f \circ g(x) = f(f(x)) = f(\cos x) = \cos^2 x$$

Functions Ex 2.3 Q1(iii)

$f(x) = |x|$ and $g(x) = \sin x$

Range of $f = (0, \infty) \subset$ Domain $g(R) \Rightarrow g \circ f$ exist

Range of $g = [-1, 1] \subset$ Domain of $(R) \Rightarrow f \circ g$ exist

Now,

$$f \circ g(x) = f(g(x)) = f(\sin x) = |\sin x|$$

And

$$g \circ f(x) = g(f(x)) = g(|x|) = \sin |x|$$

Functions Ex 2.3 Q1(iv)

$f(x) = x + 1$ and $g(x) = e^x$

$$\text{Range of } f = R \subset \text{ Domain of } g = R \Rightarrow g \circ f \text{ exist}$$
$$\text{Range of } g = (0, \infty) \subset \text{ Domain of } f = R \Rightarrow f \circ g \text{ exist}$$

Now,

$$g \circ f(x) = g(f(x)) = g(x+1) = e^{x+1}$$

And

$$f \circ g(x) = f(g(x)) = f(e^x) = e^x + 1$$

Functions Ex 2.3 Q1(v)

$f(x) = sin^{-1} x$ and $g(x) = x^2$

$$\text{Range of } f = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \subset \text{ Domain of } g = R \Rightarrow g \circ f \text{ exist}$$
$$\text{Range of } g = (0, \infty) \subseteq \text{ Domain of } f = R \Rightarrow f \circ g \text{ exist}$$

Now,

$$f \circ g(x) = f(g(x)) = f(x^2) = sin^{-1} x^2$$

And

$$g \circ f(x) = g(f(x)) = g(sin^{-1} x) = (sin^{-1} x)^2$$

Functions Ex 2.3 Q 1(vi)

$f(x) = x + 1$ and $g(x) = sin x$

$$\text{Range of } f = R \subset \text{Domain of } g = R \Rightarrow g \circ f \text{ exists}$$
$$\text{Range of } g = [-1, 1] \subset \text{ Domain of } f = R \Rightarrow f \circ g \text{ exists}$$

Now,

$$f \circ g(x) = f(g(x)) = f(sin x) = sin x + 1$$

And

$$g \circ f(x) = g(f(x)) = g(x+1) = sin(x+1)$$

Functions Ex 2.3 Q1(vii)

$f(x) = x + 1$ and $g(x) = 2x + 3$

$$\text{Range of } f = R \subseteq \text{Domain of } g = R \Rightarrow g \circ f \text{ exist}$$
$$\text{Range of } g = R \subseteq \text{Domain of } R = R \Rightarrow f \circ g \text{ exist}$$

Now,

$$f \circ g(x) = f(g(x)) = f(2x+3) = (2x+3) + 1 = 2x + 4$$

And

$$g \circ f(x) = g(f(x)) = g(x+1) = 2(x+1) + 3$$

$$\Rightarrow \quad g \circ f(x) = 2x + 5$$

Functions Ex 2.3 Q1(viii)

$f(x) = c, \qquad c \in R$ and

$g(x) = \sin x^2$

Range of $f = R \subset$ Domain of $g = R \Rightarrow g \circ f$ exist

Range of $g = [-1, 1] \subset$ Domain of $f = R \Rightarrow f \circ g$ exist

Now,

$$g \circ f(x) = g(f(x)) = g(c) = \sin c^2$$

And

$$f \circ g(x) = f(g(x)) = f(\sin x^2) = c$$

Functions Ex 2.3 Q1(ix)

$f(x) = x^2 + 2$ and $g(x) = 1 - \frac{1}{1-x}$

Range of $f = (2, \infty) \subset$ Domain of $g = R \Rightarrow g \circ f$ exist

Range of $g = R - [1] \subset$ Domain of $f = R \Rightarrow f \circ g$ exist

Now,

$$f \circ g(x) = f(g(x)) = f\left(\frac{-x}{1-x}\right) = \frac{x^2}{(1-x)^2} + 2$$

And

$$g \circ f(x) = g(f(x)) = g(x^2 + 2) = \frac{-(x^2 + 2)}{1 - (x^2 + 2)}$$

$$\Rightarrow \qquad g \circ f(x) = \frac{x^2 + 2}{x^2 + 1}$$

*********** END **********