CHAPTER 28

HEAT TRANSFER

Heat can be transferred from one place to another by three different methods, namely, conduction, convection and radiation. Conduction usually takes place in solids, convection in liquids and gases, and no medium is required for radiation.

28.1 THERMAL CONDUCTION

If one end of a metal rod is placed in a stove, the temperature of the other end gradually increases. Heat is transferred from one end of the rod to the other end. This transfer takes place due to molecular collisions and the process is called *heat conduction.* The molecules at one end of the rod gain heat from the stove and their average kinetic energy increases. As these molecules collide with the neighbouring molecules having less kinetic energy, the energy is shared between these two groups. The kinetic energy of these neighbouring molecules increases. As they collide with their neighbours on the colder side, they transfer energy to them. This way, heat is passed along the rod from molecule to molecule. The average position of a molecule does not change and hence, there is no mass movement of matter.

Thermal Conductivity

The ability of a material to conduct heat is measured by *thermal conductivity* (defined below) of the material.

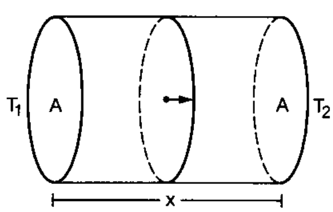

Figure 28.1

Consider a slab of uniform cross section A and length x. Let one face of the slab be maintained at temperature T_1 and the other at T_2. Also, suppose the remaining surface is covered with a nonconducting material so that no heat is transferred through the sides. After sufficient time, steady state is reached and the temperature at any point remains unchanged as time passes. In such a case, the amount of heat crossing per unit time through any cross section of the slab is equal. If ΔQ amount of heat crosses through any cross section in time Δt, $\Delta Q/\Delta t$ is called the *heat current.* It is found that in steady state the heat current is proportional to the area of cross section A, proportional to the temperature difference $(T_1 - T_2)$ between the ends and inversely proportional to the length x. Thus,

$$\frac{\Delta Q}{\Delta t} = K\frac{A(T_1 - T_2)}{x} \quad \dots (28.1)$$

where K is a constant for the material of the slab and is called the *thermal conductivity* of the material.

If the area of cross section is not uniform or if the steady-state conditions are not reached, the equation can only be applied to a thin layer of material perpendicular to the heat flow. If A be the area of cross section at a place, dx be a small thickness along the direction of heat flow, and dT be the temperature difference across the layer of thickness dx, the heat current through this cross section is

$$\frac{\Delta Q}{\Delta t} = -KA\frac{dT}{dx} \cdot \quad \dots (28.2)$$

The quantity dT/dx is called the *temperature gradient.* The minus sign indicates that dT/dx is negative along the direction of the heat flow.

The unit of thermal conductivity can be easily worked out using equation (28.1) or (28.2). The SI unit is $Js^{-1}m^{-1}K^{-1}$ or $Wm^{-1}K^{-1}$. As a change of 1 K and a change of 1°C are the same, the unit may also be written as $Wm^{-1}{}^{\circ}C^{-1}$.

Example 28.1

One face of a copper cube of edge 10 cm *is maintained at* 100°C *and the opposite face is maintained at* 0°C. *All other surfaces are covered with an insulating material.*

Find the amount of heat flowing per second through the cube. Thermal conductivity of copper is 385 $Wm^{-1}{}^{\circ}C^{-1}$.

Solution : The heat flows from the hotter face towards the colder face. The area of cross section perpendicular to the heat flow is

$$A = (10 \text{ cm})^2.$$

The amount of heat flowing per second is

$$\frac{\Delta Q}{\Delta t} = KA \frac{T_1 - T_2}{x}$$

$$= (385 \text{ Wm}^{-1}{}^{\circ}\text{C}^{-1}) \times (0{\cdot}1 \text{ m})^2 \times \frac{(100^{\circ}\text{C} - 0^{\circ}\text{C})}{0{\cdot}1 \text{ m}}$$

$$= 3850 \text{ W}.$$

In general, solids are better conductors than liquids and liquids are better conductors than gases. Metals are much better conductors than nonmetals. This is because, in metals we have a large number of "free electrons" which can move freely anywhere in the body of the metal. These free electrons help in carrying the thermal energy from one place to another in a metal.

Table (28.1) gives thermal conductivities of some materials.

Table 28.1 : *Thermal conductivities*

Material	$K(\text{Wm}^{-1}\text{K}^{-1})$
Aluminium	209
Brass	109
Copper	385
Silver	414
Steel	46·0
Water	0·585
Transformer oil	0·176
Air	0·0238
Hydrogen	0·167
Oxygen	0·0242
Brick	0·711
Celotex (Sugarcane fibre)	0·50
Concrete	1·30
Glass	0·669
Masonite	0·046
Rock & Galls wool	0·042
Wood (oak)	0·146
Wood (pine)	0·117

We can now understand why cooking utensils are made of metals whereas their handles are made of plastic or wood. When a rug is placed in bright sun on a tiled floor, both the rug and the floor acquire the same temperature. But it is much more difficult to stay bare foot on the tiles than to stay on the rug. This is because the conductivity of the rug is lesser than the tiles, and hence, the heat current going into the foot is smaller.

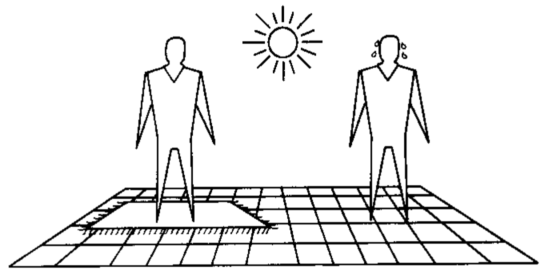

Figure 28.2

Thermal Resistance

The quantity $\frac{x}{KA}$ in equation (28.1) is called the *thermal resistance* R. Writing the heat current $\Delta Q/\Delta t$ as i, we have

$$i = \frac{T_1 - T_2}{R}. \qquad \dots (28.3)$$

This is mathematically equivalent to Ohm's law to be introduced in a later chapter. Many results derived from Ohm's law are also valid for thermal conduction.

Example 28.2

Find the thermal resistance of an aluminium rod of length 20 cm *and area of cross section* 1 cm^2. *The heat current is along the length of the rod. Thermal conductivity of aluminium* = 200 $Wm^{-1}K^{-1}$.

Solution : The thermal resistance is

$$R = \frac{x}{KA} = \frac{20 \times 10^{-2} \text{ m}}{(200 \text{ Wm}^{-1}\text{K}^{-1})(1 \times 10^{-4} \text{ m}^2)} = 10 \text{ KW}^{-1}.$$

28.2 SERIES AND PARALLEL CONNECTION OF RODS

(a) Series Connection

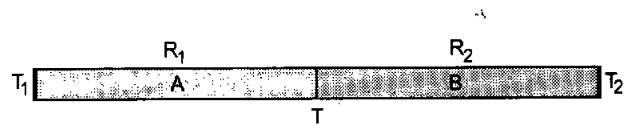

Figure 28.3

Consider two rods of thermal resistances R_1 and R_2 joined one after the other as shown in figure (28.3). The free ends are kept at temperatures T_1 and T_2 with $T_1 > T_2$. In steady state, any heat that goes through the first rod also goes through the second rod. Thus, the same heat current passes through the two rods. Such a connection of rods is called a *series connection*. Suppose, the temperature of the junction is T. From equation (28.3), the heat current through the first rod is

$$i = \frac{\Delta Q}{\Delta t} = \frac{T_1 - T}{R_1}$$

or, $$T_1 - T = R_1 i \qquad \dots \text{(i)}$$

and that through the second rod is

$$i = \frac{\Delta Q}{\Delta t} = \frac{T - T_2}{R_2}$$

or, $$T - T_2 = R_2 i. \qquad \dots \text{(ii)}$$

Adding (i) and (ii),

$$T_1 - T_2 = (R_1 + R_2)\, i$$

or, $$i = \frac{T_1 - T_2}{R_1 + R_2}.$$

Thus, the two rods together is equivalent to a single rod of thermal resistance $R_1 + R_2$.

If more than two rods are joined in series, the equivalent thermal resistance is given by

$$R = R_1 + R_2 + R_3 + \dots$$

(b) Parallel Connection

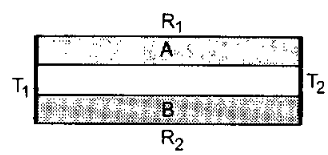

Figure 28.4

Now, suppose the two rods are joined at their ends as shown in figure (28.4). The left ends of both the rods are kept at temperature T_1 and the right ends are kept at temperature T_2. So the same temperature difference is maintained between the ends of each rod. Such a connection of rods is called a *parallel connection*. The heat current going through the first rod is

$$i_1 = \frac{\Delta Q_1}{\Delta t} = \frac{T_1 - T_2}{R_1}$$

and that through the second rod is

$$i_2 = \frac{\Delta Q_2}{\Delta t} = \frac{T_1 - T_2}{R_2}.$$

The total heat current going through the left end is

$$i = i_1 + i_2$$

$$= (T_1 - T_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

or, $$i = \frac{T_1 - T_2}{R}$$

where $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}. \qquad \dots \text{(i)}$$

Thus, the system of the two rods is equivalent to a single rod of thermal resistance R given by (i).

If more than two rods are joined in parallel, the equivalent thermal resistance R is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

28.3 MEASUREMENT OF THERMAL CONDUCTIVITY OF A SOLID

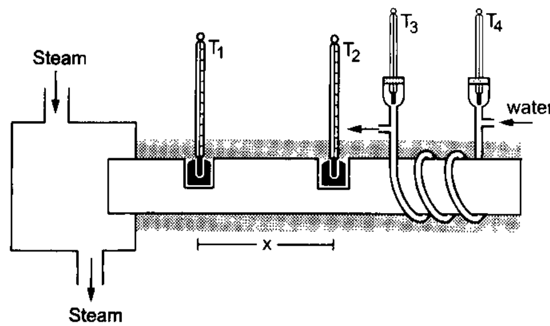

Figure 28.5

Figure (28.5) shows Searle's apparatus to measure the thermal conductivity of a solid. The solid is taken in the form of a cylindrical rod. One end of the rod goes into a steam chamber. A copper tube is coiled around the rod near the other end of the rod. A steady flow of water is maintained in the copper tube. Water enters the tube at the end away from the steam chamber and it leaves at the end nearer to it. Thermometers T_3 and T_4 are provided to measure the temperatures of the outgoing and incoming water. Two holes are drilled in the rod and mercury is filled in these holes to measure the temperature of the rod at these places with the help of thermometers T_1 and T_2. The whole apparatus is covered properly with layers of an insulating material like wool or felt so as to prevent any loss of heat from the sides.

Steam is passed in the steam chamber and a steady flow of water is maintained. The temperatures of all the four thermometers rise initially and ultimately become constant in time as the steady state is reached. The readings θ_1, θ_2, θ_3 and θ_4 are noted in steady state.

A beaker is weighed and the water coming out of the copper tube is collected in it for a fixed time t measured by a stop clock. The beaker together with the water is weighed. The mass m of the water collected is then calculated. The area of cross section of the rod is calculated by measuring its radius with a slide calipers. The distance between the holes in the rod is measured with the help of a divider and a metre scale.

Let the length of the rod between the holes be x and the area of cross section of the rod be A. If the thermal conductivity of the material of the rod is K, the rate of heat flow (heat current) from the steam chamber to the rod is

$$\frac{\Delta Q}{\Delta t} = K\frac{A(\theta_1 - \theta_2)}{x}.$$

In a time t, the chamber supplies a heat

$$Q = K\frac{A(\theta_1 - \theta_2)}{x}\, t. \quad \text{... (i)}$$

As the mass of the water collected in time t is m, the heat taken by the water is

$$Q = ms(\theta_3 - \theta_4) \quad \text{... (ii)}$$

where s is the specific heat capacity of water.

As the entire rod is covered with an insulating material and the temperature of the rod does not change with time at any point, any heat given by the steam chamber must go into the flowing water. Hence, the same Q is used in (i) and (ii). Thus,

$$K\frac{A(\theta_1 - \theta_2)}{x}\, t = ms(\theta_3 - \theta_4)$$

or,
$$K = \frac{x\, ms(\theta_3 - \theta_4)}{A(\theta_1 - \theta_2)t}. \quad \text{... (28.4)}$$

28.4 CONVECTION

In convection, heat is transferred from one place to the other by the actual motion of heated material. For example, in a hot air blower, air is heated by a heating element and is blown by a fan. The air carries the heat wherever it goes. When water is kept in a vessel and heated on a stove, the water at the bottom gets heat due to conduction through the vessel's bottom. Its density decreases and consequently it rises. Thus, the heat is carried from the bottom to the top by the actual movement of the parts of the water. If the heated material is forced to move, say by a blower or a pump, the process of heat transfer is called *forced convection*. If the material moves due to difference in density, it is called *natural* or *free convection*.

Natural convection and the anomalous expansion of water play important roles in saving the lives of aquatic animals like fishes when the atmospheric temperature goes below 0°C. As the water at the surface is cooled, it becomes denser and goes down. The less cold water from the bottom rises up to the surface and gets cooled. This way the entire water is cooled to 4°C. As the water at the surface is further cooled, it expands and the density decreases. Thus, it remains at the surface and gets further cooled. Finally, it starts freezing. Heat is now lost to the atmosphere by the water only due to conduction through the ice. As ice is a poor conductor of heat, the further freezing is very slow. The temperature of the water at the bottom remains constant at 4°C for a large period of time. The atmospheric temperature ultimately improves and the animals are saved.

The main mechanism for heat transfer inside a human body is forced convection. Heart serves as the pump and blood as the circulating fluid. Heat is lost to the atmosphere through all the three processes conduction, convection and radiation. The rate of loss depends on the clothing, the tiredness and perspiration, atmospheric temperature, air current, humidity and several other factors. The system, however, transports the just required amount of heat and hence maintains a remarkably constant body temperature.

28.5 RADIATION

The process of radiation does not need any material medium for heat transfer. Energy is emitted by a body and this energy travels in the space just like light. When it falls on a material body, a part is absorbed and the thermal energy of the receiving body is increased. The energy emitted by a body in this way is called *radiant energy, thermal radiation* or simply *radiation*. Thus, the word "radiation" is used in two meanings. It refers to the process by which the energy is emitted by a body, is transmitted in space and falls on another body. It also refers to the energy itself which is being transmitted in space. The heat from the sun reaches the earth by this process, travelling millions of kilometres of empty space.

28.6 PRÉVOST THEORY OF EXCHANGE

Way back in 1792, Pierre Prévost put forward the theory of radiation in a systematic way now known as the *theory of exchange*. According to this theory, all bodies radiate thermal radiation at all temperatures. The amount of thermal radiation radiated per unit time depends on the nature of the emitting surface, its area and its temperature. The rate is faster at higher temperatures. Besides, a body also absorbs part of the thermal radiation emitted by the surrounding bodies when this radiation falls on it. If a body radiates more than what it absorbs, its temperature falls. If a body radiates less than what it absorbs, its temperature rises.

Now, consider a body kept in a room for a long time. One finds that the temperature of the body remains constant and is equal to the room temperature. The body is still radiating thermal radiation. But it is also absorbing part of the radiation emitted by the surrounding objects, walls, etc. We thus conclude that when the temperature of a body is equal to the temperature of its surroundings, it radiates at the same rate as it absorbs. If we place a hotter body in the room, it radiates at a faster rate than the rate at which it absorbs. Thus, the body suffers a net loss of thermal energy in any given time and its temperature decreases. Similarly, if a colder body is kept in a warm surrounding, it radiates less to the surrounding than what it absorbs from the surrounding. Consequently, there is a net increase in the thermal energy of the body and the temperature rises.

28.7 BLACKBODY RADIATION

Consider two bodies of equal surface areas suspended in a room. One of the bodies has polished surface and the other is painted black. After sufficient time, the temperature of both the bodies will be equal to the room temperature. As the surface areas of the bodies are the same, equal amount of radiation falls on the two surfaces. The polished surface reflects a large part of it and absorbs a little, while the black-painted surface reflects a little and absorbs a large part of it. As the temperature of each body remains constant, we conclude that the polished surface radiates at a slower rate and the black-painted surface radiates at a faster rate. So, good absorbers of radiation are also good emitters.

A body that absorbs all the radiation falling on it is called a *blackbody*. Such a body will emit radiation at the fastest rate. The radiation emitted by a blackbody is called *blackbody radiation*. The radiation inside an enclosure with its inner walls maintained at a constant temperature has the same properties as the blackbody radiation and is also called blackbody radiation. A blackbody is also called an *ideal radiator*. A perfect blackbody, absorbing 100% of the radiation falling on it, is only an ideal concept. Among the materials, lampblack is close to a blackbody. It reflects only about 1% of the radiation falling on it. If an enclosure is painted black from inside and a small hole is made in the wall (figure 28.6) the hole acts as a very good blackbody.

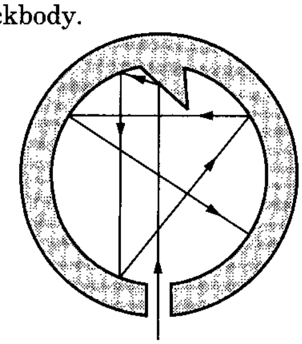

Figure 28.6

Any radiation that falls on the hole goes inside. This radiation has little chance to come out of the hole again and it gets absorbed after multiple reflections. The cone directly opposite to the hole (figure 28.6) ensures that the incoming radiation is not directly reflected back to the hole.

28.8 KIRCHHOFF'S LAW

We have learnt that good absorbers of radiation are also good radiators. This aspect is described quantitatively by Kirchhoff's law of radiation. Before stating the law let us define certain terms.

Emissive Power

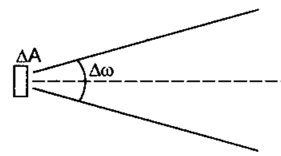

Figure 28.7

Consider a small area ΔA of a body emitting thermal radiation. Consider a small solid angle $\Delta\omega$ (see the chapter "Gauss's Law") about the normal to the radiating surface. Let the energy radiated by the area ΔA of the surface in the solid angle $\Delta\omega$ in time Δt be ΔU. We define *emissive power* of the body as

$$E = \frac{\Delta U}{(\Delta A)\,(\Delta\omega)\,(\Delta t)}.$$

Thus, emissive power denotes the energy radiated per unit area per unit time per unit solid angle along the normal to the area.

Absorptive Power

Absorptive power of a body is defined as the fraction of the incident radiation that is absorbed by the body. If we denote the absorptive power by a,

$$a = \frac{\text{energy absorbed}}{\text{energy incident}}.$$

As all the radiation incident on a blackbody is absorbed, the absorptive power of a blackbody is unity.

Note that the absorptive power is a dimensionless quantity but the emissive power is not.

Kirchhoff's Law

The ratio of emissive power to absorptive power is the same for all bodies at a given temperature and is equal to the emissive power of a blackbody at that temperature. Thus,

$$\frac{E(\text{body})}{a(\text{body})} = E(\text{blackbody}).$$

Kirchhoff's law tells that if a body has high emissive power, it should also have high absorptive power to have the ratio E/a same. Similarly, a body having low emissive power should have low absorptive power. Kirchhoff's law may be easily proved by a simple argument as described below.

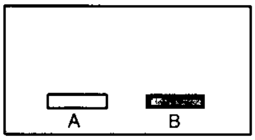

Figure 28.8

Consider two bodies A and B of identical geometrical shapes placed in an enclosure. Suppose A is an arbitrary body and B is a blackbody. In thermal equilibrium, both the bodies will have the same temperature as the temperature of the enclosure. Suppose an amount ΔU of radiation falls on the body A in a given time Δt. As A and B have the same geometrical shapes, the radiation falling on the blackbody B is also ΔU. The blackbody absorbs all of this ΔU. As the temperature of the blackbody remains constant, it also emits an amount ΔU of radiation in that time. If the emissive power of the blackbody is E_0, we have

$$\Delta U \propto E_0 \text{ or } \Delta U = kE_0 \qquad \text{... (i)}$$

where k is a constant.

Let the absorptive power of A be a. Thus, it absorbs an amount $a\Delta U$ of the radiation falling on it in time Δt. As its temperature remains constant, it must also emit the same amount $a\Delta U$ in that time. If the emissive power of the body A is E, we have

$$a\Delta U = kE. \qquad \text{... (ii)}$$

The same proportionality constant k is used in (i) and (ii) because the two bodies have identical geometrical shapes and radiation emitted in the same time Δt is considered.

From (i) and (ii),

$$a = \frac{E}{E_0}$$

or, $$\frac{E}{a} = E_0$$

or, $$\frac{E(\text{body})}{a(\text{body})} = E(\text{blackbody}).$$

This proves Kirchhoff's law.

28.9 NATURE OF THERMAL RADIATION

Thermal radiation, once emitted, is an electromagnetic wave like light. It, therefore, obeys all the laws of wave theory. The wavelengths are still small compared to the dimensions of usual obstacles encountered, so the rules of geometrical optics are valid, i.e., it travels in a straight line, casts shadow, is reflected and refracted at the change of medium, etc. The radiation emitted by a body is a mixture of waves of different wavelengths. However, only a small range of wavelength has significant contribution in the total radiation. The radiation emitted by a body at a temperature of 300 K (room temperature) has significant contribution from wavelengths around 9550 nm which is in long infrared region (visible light has a range of about 380–780 nm). As the temperature of the emitter increases, this dominant wavelength decreases. At around 1100 K, the radiation has a good contribution from red region of wavelengths and the object appears red. At temperatures around 3000 K, the radiation contains enough shorter wavelengths and the object appears white. Even at such a high temperature most significant contributions come from wavelengths around 950 nm.

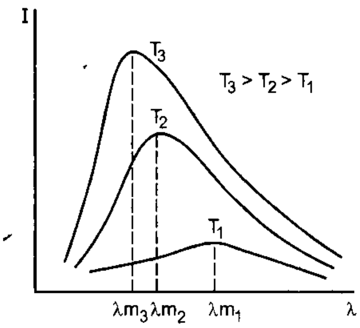

Figure 28.9

The relative importance of different wavelengths in a thermal radiation can be studied qualitatively from figure (28.9). Here the intensity of radiation near a given wavelength is plotted against the wavelength for different temperatures. We see that as the temperature is increased, the wavelength corresponding to the highest intensity decreases. In fact, this wavelength λ_m is inversely proportional to the absolute temperature of the emitter. So,

$$\lambda_m T = b \qquad \text{... (28.5)}$$

where b is a constant.

This equation is known as the *Wien's displacement law*. For a blackbody, the constant b appearing in equation (28.5) is measured to be 0·288 cmK and is known as the *Wien constant*.

Example 28.3

The light from the sun is found to have a maximum intensity near the wavelength of 470 nm. *Assuming that the surface of the sun emits as a blackbody, calculate the temperature of the surface of the sun.*

Solution : For a blackbody, $\lambda_m T = 0{\cdot}288$ cmK.

Thus, $$T = \frac{0{\cdot}288 \text{ cmK}}{470 \text{ nm}} = 6130 \text{ K}.$$

Distribution of radiant energy among different wavelengths has played a very significant role in the development of quantum mechanics and in our understanding of nature in a new way. Classical physics had predicted a very different and unrealistic wavelength distribution. Planck put forward a bold hypothesis that radiation can be emitted or absorbed only in discrete steps, each step involving an amount of energy given by $E = nh\nu$ where ν is the frequency of the radiation and n is an integer. A new fundamental constant h named as *Planck constant* was introduced in physics. This opened the gateway of modern physics through which we look into the atomic and subatomic world.

28.10 STEFAN-BOLTZMANN LAW

The energy of thermal radiation emitted per unit time by a blackbody of surface area A is given by

$$u = \sigma A T^4 \qquad \dots (28.6)$$

where σ is a universal constant known as Stefan–Boltzmann constant and T is its temperature on absolute scale. The measured value of σ is $5{\cdot}67 \times 10^{-8}$ $\mathrm{W m^{-2} K^{-4}}$. Equation (28.6) itself is called the *Stefan–Boltzmann law.* Stefan had suggested this law from experimental data available on radiation and Boltzmann derived it from thermodynamical considerations. The law is also quoted as *Stefan's law* and the constant σ as *Stefan constant.*

A body which is not a blackbody, emits less radiation than given by equation (28.6). It is, however, proportional to T^4. The energy emitted by such a body per unit time is written as

$$u = e\sigma A T^4 \qquad \dots (28.7)$$

where e is a constant for the given surface having a value between 0 and 1. This constant is called the *emissivity* of the surface. It is zero for completely reflecting surface and is unity for a blackbody.

Using Kirchhoff's law,

$$\frac{E(\text{body})}{E(\text{blackbody})} = a \qquad \dots (\text{i})$$

where a is the absorptive power of the body. The emissive power E is proportional to the energy radiated per unit time, that is, proportional to u. Using equations (28.6) and (28.7) in (i),

$$\frac{e\sigma A T^4}{\sigma A T^4} = a \ \text{ or } \ e = a.$$

Thus, emissivity and absorptive power have the same value.

Consider a body of emissivity e kept in thermal equilibrium in a room at temperature T_0. The energy of radiation absorbed by it per unit time should be equal to the energy emitted by it per unit time. This is because the temperature remains constant. Thus, the energy of the radiation absorbed per unit time is

$$u = e\sigma A T_0^4.$$

Now, suppose the temperature of the body is changed to T but the room temperature remains T_0. The energy of the thermal radiation emitted by the body per unit time is

$$u = e\sigma A T^4.$$

The energy absorbed per unit time by the body is

$$u_0 = e\sigma A T_0^4.$$

Thus, the net loss of thermal energy per unit time is

$$\Delta u = u - u_0$$
$$= e\sigma A (T^4 - T_0^4). \qquad \dots (28.8)$$

Example 28.4

A blackbody of surface area $10\ \mathrm{cm}^2$ *is heated to* $127°\mathrm{C}$ *and is suspended in a room at temperature* $27°\mathrm{C}$. *Calculate the initial rate of loss of heat from the body to the room.*

Solution : For a blackbody at temperature T, the rate of emission is $u = \sigma A T^4$. When it is kept in a room at temperature T_0, the rate of absorption is $u_0 = \sigma A T_0^4$.

The net rate of loss of heat is $u - u_0 = \sigma A (T^4 - T_0^4)$.

Here $A = 10 \times 10^{-4}\ \mathrm{m}^2$, $T = 400$ K and $T_0 = 300$ K . Thus,

$u - u_0$

$= (5{\cdot}67 \times 10^{-8}\ \mathrm{W m^{-2} K^{-4}})\,(10 \times 10^{-4}\ \mathrm{m}^2)\,(400^4 - 300^4)\ \mathrm{K}^4$

$= 0{\cdot}99$ W.

28.11 NEWTON'S LAW OF COOLING

Suppose, a body of surface area A at an absolute temperature T is kept in a surrounding having a lower temperature T_0. The net rate of loss of thermal energy from the body due to radiation is

$$\Delta u_1 = e\sigma A (T^4 - T_0^4).$$

If the temperature difference is small, we can write

$$T = T_0 + \Delta T$$

or,
$$T^4 - T_0^4 = (T_0 + \Delta T)^4 - T_0^4$$
$$= T_0^4 \left(1 + \frac{\Delta T}{T_0}\right)^4 - T_0^4$$
$$= T_0^4 \left[1 + 4\frac{\Delta T}{T_0} + \text{higher powers of } \frac{\Delta T}{T_0}\right] - T_0^4$$
$$\approx 4\,T_0^3 \Delta T = 4\,T_0^3 (T - T_0).$$

Thus,
$$\Delta u_1 = 4\,e\sigma A T_0^3 (T - T_0)$$

$$= b_1 A(T - T_0).$$

The body may also lose thermal energy due to convection in the surrounding air. For small temperature difference, the rate of loss of heat due to convection is also proportional to the temperature difference and the area of the surface. This rate may, therefore, be written as

$$\Delta u_2 = b_2 A(T - T_0).$$

The net rate of loss of thermal energy due to convection and radiation is

$$\Delta u = \Delta u_1 + \Delta u_2 = (b_1 + b_2)A(T - T_0).$$

If s be the specific heat capacity of the body and m its mass, the rate of fall of temperature is

$$\frac{-dT}{dt} = \frac{\Delta u}{ms} = \frac{b_1 + b_2}{ms} A(T - T_0)$$
$$= bA(T - T_0).$$

Thus, for small temperature difference between a body and its surrounding, the rate of cooling of the body is directly proportional to the temperature difference and the surface area exposed. We can write

$$\frac{dT}{dt} = -bA(T - T_0). \quad \ldots (28.9)$$

This is known as *Newton's law of cooling*. The constant b depends on the nature of the surface involved and the surrounding conditions. The minus sign indicates that if $T > T_0$, dT/dt is negative, that is, the temperature decreases with time. As the difference in temperature is the same for absolute and Celsius scale, equation (28.9) may also be written as

$$\frac{d\theta}{dt} = -bA(\theta - \theta_0) = -k(\theta - \theta_0)$$

where θ refers to temperature in Celsius scale.

Example 28.5

A liquid cools from 70°C *to* 60°C *in* 5 minutes. *Calculate the time taken by the liquid to cool from* 60°C *to* 50°C, *if the temperature of the surrounding is constant at* 30°C.

Solution : The average temperature of the liquid in the first case is

$$\theta_1 = \frac{70°C + 60°C}{2} = 65°C.$$

The average temperature difference from the surrounding is

$$\theta_1 - \theta_0 = 65°C - 30°C = 35°C.$$

The rate of fall of temperature is

$$-\frac{d\theta_1}{dt} = \frac{70°C - 60°C}{5\ \text{min}} = 2°C\ \text{min}^{-1}.$$

From Newton's law of cooling,

$$2°C\ \text{min}^{-1} = bA(35°C)$$

or, $$bA = \frac{2}{35\ \text{min}}. \quad \ldots (i)$$

In the second case, the average temperature of the liquid is

$$\theta_2 = \frac{60°C + 50°C}{2} = 55°C$$

so that, $\theta_2 - \theta_0 = 55°C - 30°C = 25°C.$

If it takes a time t to cool down from 60°C to 50°C, the rate of fall of temperature is

$$-\frac{d\theta_2}{dt} = \frac{60°C - 50°C}{t} = \frac{10°C}{t}.$$

From Newton's law of cooling and (i),

$$\frac{10°C}{t} = \frac{2}{35\ \text{min}} \times 25°C$$

or, $t = 7$ min.

28.12 DETECTION AND MEASUREMENT OF RADIATION

Several instruments are used to detect and measure the amount of thermal radiation. We describe two of them here, a bolometer and a thermopile.

Bolometer

The bolometer is based on the theory of Wheatstone bridge which was introduced while discussing resistance thermometer. A thin (a small fraction of a millimetre) foil of platinum is taken and strips are cut from it to leave a grid-type structure as shown in figure (28.10a). Four such grids, identical in all respect, are prepared and joined with a battery and a galvanometer to form a Wheatstone bridge (figure 28.10b). Grid 1 faces the radiation to be detected or measured. The particular arrangement of the four grids ensures that the radiation passing through the empty spaces in grid 1 falls on the strips of grid 4. Grids 2 and 3 are protected from the radiation.

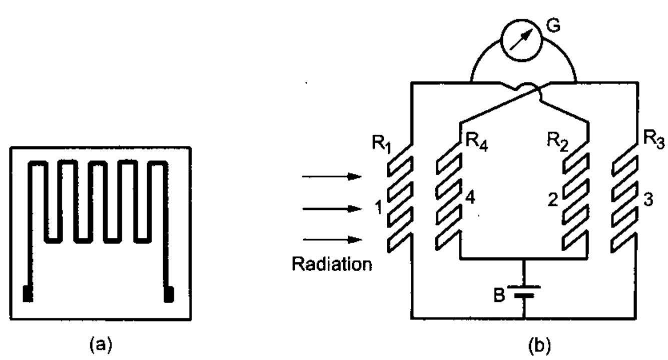

Figure 28.10

When no radiation falls on the bolometer, all the grids have the same resistance so that

$R_1 = R_2 = R_3 = R_4$. Thus, $\frac{R_1}{R_2} = \frac{R_3}{R_4}$ and the bridge is balanced. There is no deflection in the galvanometer. When radiation falls on the bolometer, grids 1 and 4 get heated. As the temperature increases, the resistances R_1 and R_4 increase and hence the product $R_1 R_4$ increases. On the other hand, $R_2 R_3$ remains unchanged. Thus,

$$R_1 R_4 > R_2 R_3$$

or, $$\frac{R_1}{R_2} > \frac{R_3}{R_4}.$$

The bridge becomes unbalanced and there is a deflection in the galvanometer which indicates the presence of radiation. The magnitude of deflection is related to the amount of radiation falling on the bolometer.

The bolometer is usually enclosed in a glass bulb evacuated to low pressures. This increases the sensitivity.

Thermopile

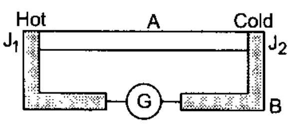

Figure 28.11

A thermopile is based on the principle of *Seebeck effect.* Figure (28.11) illustrates the principle. Two dissimilar metals A and B are joined to form two junctions J_1 and J_2. A galvanometer is connected between the junctions through the metal B. If the junctions are at the same temperature, there is no deflection in the galvanometer. But if the temperatures of the junctions are different, the galvanometer deflects. Such an instrument is called a *thermocouple.*

In a thermopile, a number of thermocouples are joined in series. The thermocouples are made from antimony and bismuth metals. The free ends are joined to a galvanometer (figure 28.12). The series connection of thermocouples increases the sensitivity of the system.

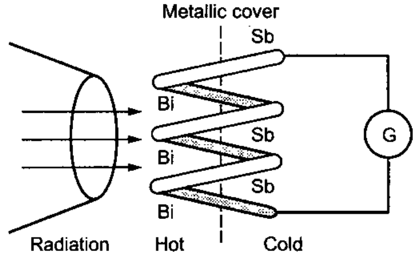

Figure 28.12

The junctions are arranged in such a way that all the hot junctions lie on a plane face and all the cold junctions lie on the opposite plane face. The face of the hot junctions is blackened so that it may absorb large fraction of radiation falling on it. This face is exposed to the radiation and the other face is protected from it by a metallic cover. A metallic cone generally concentrates the radiation on the hot face.

The radiation is detected and measured by observing the deflection in the galvanometer.

Worked Out Examples

1. *The lower surface of a slab of stone of face-area* 3600 cm ^{2} *and thickness* 10 cm *is exposed to steam at* 100°C. *A block of ice at* 0°C *rests on the upper surface of the slab.* 4·8 g *of ice melts in one hour. Calculate the thermal conductivity of the stone. Latent heat of fusion of ice* $= 3{\cdot}36 \times 10^5$ J kg^{-1}.

Solution : The amount of heat transferred through the slab to the ice in one hour is

$$Q = (4{\cdot}8 \times 10^{-3}\ \text{kg}) \times (3{\cdot}36 \times 10^5\ \text{J kg}^{-1})$$
$$= 4{\cdot}8 \times 336\ \text{J}.$$

Using the equation

$$Q = \frac{KA(\theta_1 - \theta_2)t}{x},$$

$$4{\cdot}8 \times 336\ \text{J} = \frac{K(3600\ \text{cm}^2)(100°\text{C}) \times (3600\ \text{s})}{10\ \text{cm}}$$

or, $$K = 1{\cdot}24 \times 10^{-3}\ \text{W m}^{-1}\,°\text{C}^{-1}.$$

2. *An icebox made of* 1·5 cm *thick styrofoam has dimensions* 60 cm × 60 cm × 30 cm. *It contains ice at* 0°C *and is kept in a room at* 40°C. *Find the rate at which the ice is melting. Latent heat of fusion of ice* $= 3{\cdot}36 \times 10^5$ J kg^{-1}. *and thermal conductivity of styrofoam* $= 0{\cdot}04$ W m^{-1}°C^{-1}.

Solution : The total surface area of the walls
$= 2(60\ \text{cm} \times 60\ \text{cm} + 60\ \text{cm} \times 30\ \text{cm} + 60\ \text{cm} \times 30\ \text{cm})$
$= 1{\cdot}44\ \text{m}^2$.

The thickness of the walls = 1·5 cm = 0·015 m.

The rate of heat flow into the box is

$$\frac{\Delta Q}{\Delta t} = \frac{KA(\theta_1 - \theta_2)}{x}$$

$$= \frac{(0{\cdot}04\ \text{W m}^{-1}\,°\text{C}^{-1})\ (1{\cdot}44\ \text{m}^2)\ (40°\text{C})}{0{\cdot}015\ \text{m}} = 154\ \text{W}.$$

The rate at which the ice melts is

$$= \frac{154\ \text{W}}{3{\cdot}36 \times 10^5\ \text{J kg}^{-1}} = 0{\cdot}46\ \text{g s}^{-1}.$$

3. *A closed cubical box is made of perfectly insulating material and the only way for heat to enter or leave the box is through two solid cylindrical metal plugs, each of cross sectional area* $12\ \text{cm}^2$ *and length* $8\ \text{cm}$ *fixed in the opposite walls of the box. The outer surface of one plug is kept at a temperature of* $100°\text{C}$ *while the outer surface of the other plug is maintained at a temperature of* $4°\text{C}$*. The thermal conductivity of the material of the plug is* $2{\cdot}0\ \text{W m}^{-1}{}°\text{C}^{-1}$*. A source of energy generating* $13\ \text{W}$ *is enclosed inside the box. Find the equilibrium temperature of the inner surface of the box assuming that it is the same at all points on the inner surface.*

Solution :

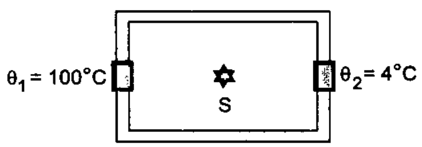

Figure 28-W1

The situation is shown in figure (28-W1). Let the temperature inside the box be θ. The rate at which heat enters the box through the left plug is

$$\frac{\Delta Q_1}{\Delta t} = \frac{KA(\theta_1 - \theta)}{x}.$$

The rate of heat generation in the box = 13 W. The rate at which heat flows out of the box through the right plug is

$$\frac{\Delta Q_2}{\Delta t} = \frac{KA(\theta - \theta_2)}{x}.$$

In the steady state

$$\frac{\Delta Q_1}{\Delta t} + 13\ \text{W} = \frac{\Delta Q_2}{\Delta t}$$

or, $$\frac{KA}{x}(\theta_1 - \theta) + 13\ \text{W} = \frac{KA}{x}(\theta - \theta_2)$$

or, $$2\frac{KA}{x}\theta = \frac{KA}{x}(\theta_1 + \theta_2) + 13\ \text{W}$$

or, $$\theta = \frac{\theta_1 + \theta_2}{2} + \frac{(13\ \text{W})\,x}{2KA}$$

$$= \frac{100°\text{C} + 4°\text{C}}{2} + \frac{(13\ \text{W}) \times 0{\cdot}08\ \text{m}}{2 \times (2{\cdot}0\ \text{W m}^{-1}{}°\text{C}^{-1})\,(12 \times 10^{-4}\ \text{m}^2)}$$

$$= 52°\text{C} + 216{\cdot}67°\text{C} \approx 269°\text{C}.$$

4. *A bar of copper of length* $75\ \text{cm}$ *and a bar of steel of length* $125\ \text{cm}$ *are joined together end to end. Both are of circular cross section with diameter* $2\ \text{cm}$*. The free ends of the copper and the steel bars are maintained at* $100°\text{C}$ *and* $0°\text{C}$ *respectively. The curved surfaces of the bars are thermally insulated. What is the temperature of the copper–steel junction? What is the amount of heat transmitted per unit time across the junction? Thermal conductivity of copper is* $386\ \text{J s}^{-1}\ \text{m}^{-1}{}°\text{C}^{-1}$ *and that of steel is* $46\ \text{J s}^{-1}\ \text{m}^{-1}{}°\text{C}^{-1}$.

Solution :

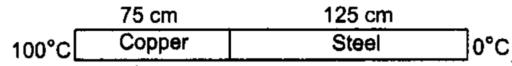

Figure 28-W2

The situation is shown in figure (28-W2). Let the temperature at the junction be θ (on Celsius scale). The same heat current passes through the copper and the steel rods. Thus,

$$\frac{\Delta Q}{\Delta t} = \frac{K_{cu}\,A(100°\text{C} - \theta)}{75\ \text{cm}} = \frac{K_{steel}\,A\theta}{125\ \text{cm}}$$

or, $$\frac{K_{cu}\,(100°\text{C} - \theta)}{75} = \frac{K_{steel}\,\theta}{125}$$

or, $$\frac{100°\text{C} - \theta}{\theta} = \frac{75\,K_{steel}}{125\,K_{cu}} = \frac{3}{5} \times \frac{46}{386}$$

or, $$\theta = 93°\text{C}.$$

The rate of heat flow is

$$\frac{\Delta Q}{\Delta t} = \frac{K_{steel}A\theta}{125\ \text{cm}}$$

$$= \frac{(46\ \text{J s}^{-1}\ \text{m}^{-1}{}°\text{C}^{-1})\,(\pi \times 1\ \text{cm}^2) \times 93°\text{C}}{125\ \text{cm}}$$

$$= 1{\cdot}07\ \text{J s}^{-1}.$$

5. *Two parallel plates A and B are joined together to form a compound plate (figure 28-W3). The thicknesses of the plates are* $4{\cdot}0\ \text{cm}$ *and* $2{\cdot}5\ \text{cm}$ *respectively and the area of cross section is* $100\ \text{cm}^2$ *for each plate. The thermal conductivities are* $K_A = 200\ \text{W m}^{-1}{}°\text{C}^{-1}$ *for the plate A and* $K_B = 400\ \text{W m}^{-1}{}°\text{C}^{-1}$ *for the plate B. The outer surface of the plate A is maintained at* $100°\text{C}$ *and the outer surface of the plate B is maintained at* $0°\text{C}$*. Find (a) the rate of heat flow through any cross section, (b) the temperature at the interface and (c) the equivalent thermal conductivity of the compound plate.*

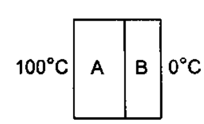

Figure 28-W3

Solution : (a) Let the temperature of the interface be **θ**. The area of cross section of each plate is $A = 100\ \text{cm}^2 = 0{\cdot}01\ \text{m}^2$. The thicknesses are $x_A = 0{\cdot}04\ \text{m}$ and $x_B = 0{\cdot}025\ \text{m}$.

The thermal resistance of the plate A is

$$R_1 = \frac{1}{K_A}\frac{x_A}{A}$$

and that of the plate B is

$$R_2 = \frac{1}{K_B}\frac{x_B}{A}.$$

The equivalent thermal resistance is

$$R = R_1 + R_2 = \frac{1}{A}\left(\frac{x_A}{K_A} + \frac{x_B}{K_B}\right). \quad \ldots \text{(i)}$$

Thus, $\frac{\Delta Q}{\Delta t} = \frac{\theta_1 - \theta_2}{R}$

$$= \frac{A(\theta_1 - \theta_2)}{x_A/K_A + x_B/K_B}$$

$$= \frac{(0{\cdot}01\ \text{m}^2)\ (100°\text{C})}{(0{\cdot}04\ \text{m})/(200\ \text{W m}^{-1}°\text{C}^{-1}) + (0{\cdot}025\ \text{m})/(400\ \text{W m}^{-1}°\text{C}^{-1})}$$

$= 3810$ W.

(b) We have $\frac{\Delta Q}{\Delta t} = \frac{A(\theta - \theta_2)}{x_B/K_B}$

or, $$3810\ \text{W} = \frac{(0{\cdot}01\ \text{m}^2)\ (\theta - 0°\text{C})}{(0{\cdot}025\ \text{m})/(400\ \text{W m}^{-1}°\text{C}^{-1})}$$

or, $\theta = 24°$C.

(c) If K is the equivalent thermal conductivity of the compound plate, its thermal resistance is

$$R = \frac{1}{A}\frac{x_A + x_B}{K}.$$

Comparing with (i),

$$\frac{x_A + x_B}{K} = \frac{x_A}{K_A} + \frac{x_B}{K_B}$$

or, $$K = \frac{x_A + x_B}{x_A/K_A + x_B/K_B}$$

$$= 248\ \text{W m}^{-1}°\text{C}^{-1}.$$

6. *A room has a* $4\ \text{m} \times 4\ \text{m} \times 10\ \text{cm}$ *concrete roof* ($K = 1{\cdot}26$ $\text{Wm}^{-1}°\text{C}^{-1}$). *At some instant, the temperature outside is* 46°C *and that inside is* 32°C. *(a) Neglecting convection, calculate the amount of heat flowing per second into the room through the roof. (b) Bricks* ($K = 0{\cdot}65\ \text{Wm}^{-1}°\text{C}^{-1}$) *of thickness* 7·5 cm *are laid down on the roof. Calculate the new rate of heat flow under the same temperature conditions.*

Solution : The area of the roof

$$= 4\ \text{m} \times 4\ \text{m} = 16\ \text{m}^2.$$

The thickness $x = 10\ \text{cm} = 0{\cdot}10$ m.

(a) The thermal resistance of the roof is

$$R_1 = \frac{1}{K}\frac{x}{A} = \frac{1}{1{\cdot}26\ \text{W m}^{-1}°\text{C}^{-1}}\ \frac{0{\cdot}10\ \text{m}}{16\ \text{m}^2}$$

$$= 4{\cdot}96 \times 10^{-3}\ °\text{C W}^{-1}.$$

The heat current is

$$\frac{\Delta Q}{\Delta t} = \frac{\theta_1 - \theta_2}{R_1} = \frac{46°\text{C} - 32°\text{C}}{4{\cdot}96 \times 10^{-3}\ °\text{C W}^{-1}}$$

$$= 2822\ \text{W}.$$

(b) The thermal resistance of the brick layer is

$$R_2 = \frac{1}{K}\frac{x}{A} = \frac{1}{0{\cdot}65\ \text{W m}^{-1}°\text{C}^{-1}}\ \frac{7{\cdot}5 \times 10^{-2}\ \text{m}}{16\ \text{m}^2}$$

$$= 7{\cdot}2 \times 10^{-3}\ °\text{C W}^{-1}.$$

The equivalent thermal resistance is

$$R = R_1 + R_2 = (4{\cdot}96 + 7{\cdot}2) \times 10^{-3}\ °\text{C W}^{-1}$$

$$= 1{\cdot}216 \times 10^{-2}\ °\text{C W}^{-1}.$$

The heat current is

$$\frac{\Delta Q}{\Delta t} = \frac{\theta_1 - \theta_2}{R} = \frac{46°\text{C} - 32°\text{C}}{1{\cdot}216 \times 10^{-2}\ °\text{C W}^{-1}}.$$

$$= 1152\ \text{W}.$$

7. *An electric heater is used in a room of total wall area* $137\ \text{m}^2$ *to maintain a temperature of* 20°C *inside it, when the outside temperature is* – 10°C. *The walls have three different layers of materials. The innermost layer is of wood of thickness* 2·5 cm, *the middle layer is of cement of thickness* 1·0 cm *and the outermost layer is of brick of thickness* 25·0 cm. *Find the power of the electric heater. Assume that there is no heat loss through the floor and the ceiling. The thermal conductivities of wood, cement and brick are* $0{\cdot}125\ \text{Wm}^{-1}°\text{C}^{-1}$, $1{\cdot}5\ \text{Wm}^{-1}°\text{C}^{-1}$ *and* $1{\cdot}0\ \text{Wm}^{-1}°\text{C}^{-1}$ *respectively.*

Solution :

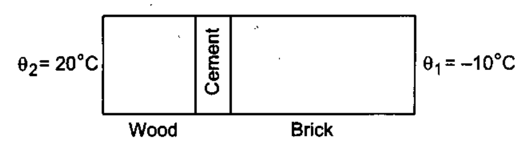

Figure 28-W4

The situation is shown in figure (28-W4).

The thermal resistances of the wood, the cement and the brick layers are

$$R_W = \frac{1}{K}\frac{x}{A}$$

$$= \frac{1}{0{\cdot}125\ \text{W m}^{-1}°\text{C}^{-1}}\ \frac{2{\cdot}5 \times 10^{-2}\ \text{m}}{137\ \text{m}^2}$$

$$= \frac{0{\cdot}20}{137}\ °\text{C W}^{-1},$$

$$R_C = \frac{1}{1{\cdot}5\ \text{W m}^{-1}°\text{C}^{-1}}\ \frac{1{\cdot}0 \times 10^{-2}\ \text{m}}{137\ \text{m}^2}$$

$$= \frac{0{\cdot}0067}{137}\ °\text{C W}^{-1}$$

and $$R_B = \frac{1}{1{\cdot}0\ \text{W m}^{-1}°\text{C}^{-1}}\ \frac{25{\cdot}0 \times 10^{-2}\ \text{m}}{137\ \text{m}^2}$$

$$= \frac{0{\cdot}25}{137}\ °\text{C W}^{-1}.$$

As the layers are connected in series, the equivalent thermal resistance is

$$R = R_W + R_C + R_B$$

$$= \frac{0{\cdot}20 + 0{\cdot}0067 + 0{\cdot}25}{137}\ {}^\circ\text{C W}^{-1}$$

$$= 3{\cdot}33 \times 10^{-3}\ {}^\circ\text{C W}^{-1}.$$

The heat current is

$$i = \frac{\theta_1 - \theta_2}{R}$$

$$= \frac{20^\circ\text{C} - (-10^\circ\text{C})}{3{\cdot}33 \times 10^{-3}\ {}^\circ\text{C W}^{-1}} \approx 9000\ \text{W}.$$

The heater must supply 9000 W to compensate the outflow of heat.

8. *Three rods of material x and three of material y are connected as shown in figure (28-W5). All the rods are identical in length and cross sectional area. If the end A is maintained at 60°C and the junction E at 10°C, calculate the temperature of the junction B. The thermal conductivity of x is* $800\ \text{W m}^{-1}{}^\circ\text{C}^{-1}$ *and that of y is* $400\ \text{W m}^{-1}{}^\circ\text{C}^{-1}$.

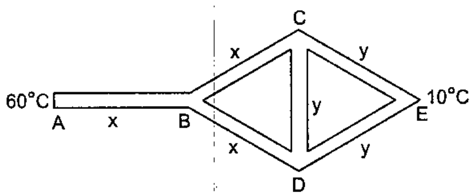

Figure 28-W5

Solution : It is clear from the symmetry of the figure that the points C and D are equivalent in all respect and hence, they are at the same temperature, say θ. No heat will flow through the rod CD. We can, therefore, neglect this rod in further analysis.

Let l and A be the length and the area of cross section of each rod. The thermal resistances of AB, BC and BD are equal. Each has a value

$$R_1 = \frac{1}{K_x}\frac{l}{A}. \qquad \text{... (i)}$$

Similarly, thermal resistances of CE and DE are equal, each having a value

$$R_2 = \frac{1}{K_y}\frac{l}{A}. \qquad \text{... (ii)}$$

As the rod CD has no effect, we can say that the rods BC and CE are joined in series. Their equivalent thermal resistance is

$$R_3 = R_{BC} + R_{CE} = R_1 + R_2.$$

Also, the rods BD and DE together have an equivalent thermal resistance $R_4 = R_{BD} + R_{DE} = R_1 + R_2$.

The resistances R_3 and R_4 are joined in parallel and hence their equivalent thermal resistance is given by

$$\frac{1}{R_5} = \frac{1}{R_3} + \frac{1}{R_4} = \frac{2}{R_3}$$

or, $$R_5 = \frac{R_3}{2} = \frac{R_1 + R_2}{2}.$$

This resistance R_5 is connected in series with AB. Thus, the total arrangement is equivalent to a thermal resistance

$$R = R_{AB} + R_5 = R_1 + \frac{R_1 + R_2}{2} = \frac{3R_1 + R_2}{2}.$$

Figure (28-W6) shows the successive steps in this reduction.

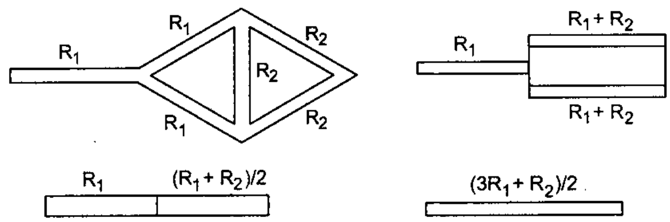

Figure 28-W6

The heat current through A is

$$i = \frac{\theta_A - \theta_E}{R} = \frac{2(\theta_A - \theta_E)}{3R_1 + R_2}.$$

This current passes through the rod AB. We have

$$i = \frac{\theta_A - \theta_B}{R_{AB}}$$

or, $$\theta_A - \theta_B = (R_{AB})i$$

$$= R_1 \frac{2(\theta_A - \theta_E)}{3R_1 + R_2}.$$

Putting from (i) and (ii),

$$\theta_A - \theta_B = \frac{2K_y(\theta_A - \theta_E)}{K_x + 3K_y}$$

$$= \frac{2 \times 400}{800 + 3 \times 400} \times 50^\circ\text{C} = 20^\circ\text{C}$$

or, $$\theta_B = \theta_A - 20^\circ\text{C} = 40^\circ\text{C}.$$

9. *A rod CD of thermal resistance* $5{\cdot}0\ \text{K W}^{-1}$ *is joined at the middle of an identical rod AB as shown in figure (28-W7). The ends A, B and D are maintained at 100°C, 0°C and 25°C respectively. Find the heat current in CD.*

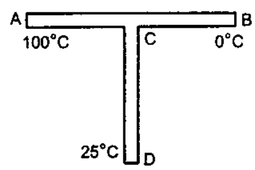

Figure 28-W7

Solution : The thermal resistance of AC is equal to that of CB and is equal to $2{\cdot}5\ \text{K W}^{-1}$. Suppose, the temperature at C is θ. The heat currents through AC, CB and CD are

$$\frac{\Delta Q_1}{\Delta t} = \frac{100^\circ\text{C} - \theta}{2{\cdot}5\ \text{K W}^{-1}},$$

$$\frac{\Delta Q_2}{\Delta t} = \frac{\theta - 0°C}{2{\cdot}5 \text{ K W}^{-1}}$$

and $$\frac{\Delta Q_3}{\Delta t} = \frac{\theta - 25°C}{5{\cdot}0 \text{ K W}^{-1}}.$$

We also have

$$\frac{\Delta Q_1}{\Delta t} = \frac{\Delta Q_2}{\Delta t} + \frac{\Delta Q_3}{\Delta t}$$

or, $$\frac{100°C - \theta}{2{\cdot}5} = \frac{\theta - 0°C}{2{\cdot}5} + \frac{\theta - 25°C}{5}$$

or, $$225°C = 5\theta$$

or, $$\theta = 45°C.$$

Thus, $$\frac{\Delta Q_3}{\Delta t} = \frac{45°C - 25°C}{5{\cdot}0 \text{ K W}^{-1}} = \frac{20 \text{ K}}{5{\cdot}0 \text{ K W}^{-1}}$$

$$= 4{\cdot}0 \text{ W}.$$

10. *Two thin metallic spherical shells of radii r_1 and r_2 $(r_1 < r_2)$ are placed with their centres coinciding. A material of thermal conductivity K is filled in the space between the shells. The inner shell is maintained at temperature θ_1 and the outer shell at temperature θ_2 $(\theta_1 < \theta_2)$. Calculate the rate at which heat flows radially through the material.*

Solution :

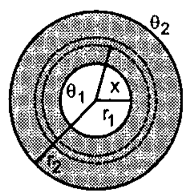

Figure 28-W8

Let us draw two spherical shells of radii x and $x + dx$ concentric with the given system. Let the temperatures at these shells be θ and $\theta + d\theta$ respectively. The amount of heat flowing radially inward through the material between x and $x + dx$ is

$$\frac{\Delta Q}{\Delta t} = \frac{K\, 4\pi x^2\, d\theta}{dx}.$$

Thus,

$$K\, 4\pi \int_{\theta_1}^{\theta_2} d\theta = \frac{\Delta Q}{\Delta t} \int_{r_1}^{r_2} \frac{dx}{x^2}$$

or, $$K\, 4\pi(\theta_2 - \theta_1) = \frac{\Delta Q}{\Delta t}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

or, $$\frac{\Delta Q}{\Delta t} = \frac{4\pi K r_1 r_2 (\theta_2 - \theta_1)}{r_2 - r_1}.$$

11. *On a cold winter day, the atmospheric temperature is $-\theta$ (on Celsius scale) which is below 0°C. A cylindrical drum of height h made of a bad conductor is completely filled with water at 0°C and is kept outside without any lid. Calculate the time taken for the whole mass of water to freeze. Thermal conductivity of ice is K and its latent heat of fusion is L. Neglect expansion of water on freezing.*

Solution :

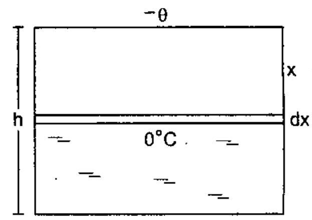

Figure 28-W9

Suppose, the ice starts forming at time $t = 0$ and a thickness x is formed at time t. The amount of heat flown from the water to the surrounding in the time interval t to $t + dt$ is

$$\Delta Q = \frac{KA\theta}{x}\, dt.$$

The mass of the ice formed due to the loss of this amount of heat is

$$dm = \frac{\Delta Q}{L} = \frac{KA\theta}{xL}\, dt.$$

The thickness dx of ice formed in time dt is

$$dx = \frac{dm}{A\rho} = \frac{K\theta}{\rho x L}\, dt$$

or, $$dt = \frac{\rho L}{K\theta}\, x\, dx.$$

Thus, the time T taken for the whole mass of water to freeze is given by

$$\int_0^T dt = \frac{\rho L}{K\theta} \int_0^h x\, dx$$

or, $$T = \frac{\rho L h^2}{2K\theta}.$$

12. *Figure (28-W10) shows a large tank of water at a constant temperature θ_0 and a small vessel containing a mass m of water at an initial temperature $\theta_1 (< \theta_0)$. A metal rod of length L, area of cross section A and thermal conductivity K connects the two vessels. Find the time taken for the temperature of the water in the smaller vessel to become $\theta_2 (\theta_1 < \theta_2 < \theta_0)$. Specific heat capacity of water is s and all other heat capacities are negligible.*

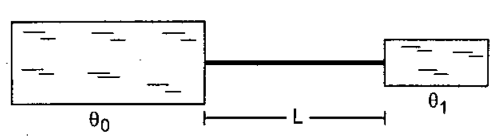

Figure 28-W10

Solution : Suppose, the temperature of the water in the smaller vessel is θ at time t. In the next time interval dt, a heat ΔQ is transferred to it where

$$\Delta Q = \frac{KA}{L}(\theta_0 - \theta)\, dt. \qquad \text{... (i)}$$

This heat increases the temperature of the water of mass m to $\theta + d\theta$ where

$$\Delta Q = ms\, d\theta. \qquad \text{... (ii)}$$

From (i) and (ii),

$$\frac{KA}{L}(\theta_0 - \theta)\, dt = ms\, d\theta$$

or, $$dt = \frac{Lms}{KA}\frac{d\theta}{\theta_0 - \theta}$$

or, $$\int_0^T dt = \frac{Lms}{KA}\int_{\theta_1}^{\theta_2}\frac{d\theta}{\theta_0 - \theta}$$

where T is the time required for the temperature of the water to become θ_2.

Thus, $$T = \frac{Lms}{KA}\ln\frac{\theta_0 - \theta_1}{\theta_0 - \theta_2}.$$

13. *One mole of an ideal monatomic gas is kept in a rigid vessel. The vessel is kept inside a steam chamber whose tempreature is 97°C. Initially, the temperature of the gas is 5·0°C. The walls of the vessel have an inner surface of area 800 cm² and thickness 1·0 cm. If the temperature of the gas increases to 9·0°C in 5·0 seconds, find the thermal conductivity of the material of the walls.*

Solution : The initial temperature difference is $97°C - 5°C = 92°C$ and at 5·0 s the temperature difference becomes $97°C - 9°C = 88°C$. As the change in the temperature difference is small, we work with the average temperature difference

$$\frac{92°C + 88°C}{2} = 90°C = 90\text{ K}.$$

The rise in the temperature of the gas is

$$9{\cdot}0°C - 5{\cdot}0°C = 4°C = 4\text{ K}.$$

The heat supplied to the gas in 5·0 s is

$$\Delta Q = nC_v\,\Delta T$$
$$= (1\text{ mol}) \times \left(\frac{3}{2} \times 8{\cdot}3\text{ JK}^{-1}\text{ mol}^{-1}\right) \times (4\text{ K})$$
$$= 49{\cdot}8\text{ J}.$$

If the thermal conductivity is K,

$$49{\cdot}8\text{ J} = \frac{K(800 \times 10^{-4}\text{ m}^2) \times (90\text{ K})}{1{\cdot}0 \times 10^{-2}\text{ m}} \times 5{\cdot}0\text{ s}$$

or, $$K = \frac{49{\cdot}8\text{ J}}{3600\text{ msK}} = 0{\cdot}014\text{ J s}^{-1}\text{ m}^{-1}\text{ K}^{-1}.$$

14. *A monatomic ideal gas is contained in a rigid container of volume V with walls of total inner surface area A, thickness x and thermal conductivity K. The gas is at an initial temperature T_0 and pressure p_0. Find the pressure of the gas as a function of time if the temperature of the surrounding air is T_s. All temperatures are in absolute scale.*

Solution : As the volume of the gas is constant, a heat ΔQ given to the gas increases its temperature by $\Delta T = \Delta Q / C_v$. Also, for a monatomic gas, $C_v = \frac{3}{2}R$. If the temperature of the gas at time t is T, the heat current into the gas is

$$\frac{\Delta Q}{\Delta t} = \frac{KA(T_s - T)}{x}$$

or, $$\frac{\Delta T}{\Delta t} = \frac{2\,KA}{3\,xR}(T_s - T)$$

or, $$\int_{T_0}^{T}\frac{dT}{T_s - T} = \int_0^t \frac{2\,KA}{3\,xR}\,dt$$

or, $$\ln\frac{T_s - T_0}{T_s - T} = \frac{2\,KA}{3\,xR}\,t$$

or, $$T_s - T = (T_s - T_0)\,e^{-\frac{2KA}{3xR}t}$$

or, $$T = T_s - (T_s - T_0)\,e^{-\frac{2KA}{3xR}t}.$$

As the volume remains constant,

$$\frac{p}{T} = \frac{p_0}{T_0}$$

or, $$p = \frac{p_0}{T_0}T$$
$$= \frac{p_0}{T_0}\left[T_s - (T_s - T_0)\,e^{-\frac{2KA}{3xR}t}\right].$$

15. *Consider a cubical vessel of edge a having a small hole in one of its walls. The total thermal resistance of the walls is r. At time $t = 0$, it contains air at atmospheric pressure p_a and temperature T_0. The temperature of the surrounding air is $T_a (> T_0)$. Find the amount of the gas (in moles) in the vessel at time t. Take C_v of air to be $5\,R/2$.*

Solution : As the gas can leak out of the hole, the pressure inside the vessel will be equal to the atmospheric pressure p_a. Let n be the amount of the gas (moles) in the vessel at time t. Suppose an amount ΔQ of heat is given to the gas in time dt. Its temperature increases by dT where

$$\Delta Q = nC_p dT.$$

If the temperature of the gas is T at time t, we have

$$\frac{\Delta Q}{dt} = \frac{T_a - T}{r}$$

or, $$(C_p r)n\, dT = (T_a - T)dt. \qquad \text{... (i)}$$

We have, $$p_a a^3 = nRT$$

or, $$n\,dT + T\,dn = 0$$

or, $$n\,dT = -T\,dn. \qquad \text{... (ii)}$$

Also, $$T = \frac{p_a a^3}{nR}. \qquad \dots \text{(iii)}$$

Using (ii) and (iii) in (i),

$$\frac{-C_p\, r\, p_a a^3}{nR}\, dn = \left(T_a - \frac{p_a a^3}{nR}\right) dt$$

or, $$\frac{dn}{nR\left(T_a - \frac{p_a a^3}{nR}\right)} = -\frac{dt}{C_p\, r\, p_a a^3}$$

or, $$\int_{n_0}^{n} \frac{dn}{nRT_a - p_a a^3} = -\int_0^t \frac{dt}{C_p\, r\, p_a a^3}$$

where $n_0 = \frac{p_a a^3}{RT_0}$ is the initial amount of the gas in the vessel. Thus,

$$\frac{1}{RT_a} \ln \frac{nRT_a - p_a a^3}{n_0RT_a - p_a a^3} = -\frac{t}{C_p\, r\, p_a a^3}$$

or, $$nRT_a - p_a a^3 = (n_0RT_a - p_a a^3)\, e^{-\frac{RT_a}{C_p r p_a a^3}t}.$$

Writing $$n_0 = \frac{p_a a^3}{RT_0} \text{ and } C_p = C_v + R = \frac{7R}{2},$$

$$n = \frac{p_a a^3}{RT_a}\left[1 + \left(\frac{T_a}{T_0} - 1\right) e^{-\frac{2T_a}{7 r p_a a^3}t}\right].$$

16 *A blackbody of surface area* 1 cm ^{2} *is placed inside an enclosure. The enclosure has a constant temperature* 27°C *and the blackbody is maintained at* 327°C *by heating it electrically. What electric power is needed to maintain the temperature?* $\sigma = 6{\cdot}0 \times 10^{-8}$ W m $^{-2}$ K $^{-4}$.

Solution : The area of the blackbody is $A = 10^{-4}$ m ^{2}, its temperature is $T_1 = 327°\text{C} = 600$ K and the temperature of the enclosure is $T_2 = 27°\text{C} = 300$ K. The blackbody emits radiation at the rate of $A\sigma T_1^4$. The radiation falls on it (and gets absorbed) at the rate of $A\sigma T_2^4$. The net rate of loss of energy is $A\sigma(T_1^4 - T_2^4)$. The heater must supply this much of power. Thus, the power needed is

$A\sigma(T_1^4 - T_2^4)$

$= (10^{-4}\ \text{m}^2)(6{\cdot}0 \times 10^{-8}\ \text{W m}^{-2}\ \text{K}^{-4})\,[(600\ \text{K})^4 - (300\ \text{K})^4]$

$= 0{\cdot}73$ W.

17. *An electric heater emits* 1000 W *of thermal radiation. The coil has a surface area of* $0{\cdot}020$ m ^{2}. *Assuming that the coil radiates like a blackbody, find its temperature.* $\sigma = 6{\cdot}00 \times 10^{-8}$ W m $^{-2}$ K $^{-4}$.

Solution : Let the temperature of the coil be T. The coil will emit radiation at a rate $A\sigma T^4$. Thus,

$$1000\ \text{W} = (0{\cdot}020\ \text{m}^2) \times (6{\cdot}0 \times 10^{-8}\ \text{W m}^{-2}\ \text{K}^{-4}) \times T^4$$

or, $$T^4 = \frac{1000}{0{\cdot}020 \times 6{\cdot}00 \times 10^{-8}}\ \text{K}^4$$

$$= 8{\cdot}33 \times 10^{11}\ \text{K}^4$$

or, $$T = 955\ \text{K}.$$

18. *The earth receives solar radiation at a rate of* $8{\cdot}2$ J cm $^{-2}$ min $^{-1}$. *Assuming that the sun radiates like a blackbody, calculate the surface temperature of the sun. The angle subtended by the sun on the earth is* $0{\cdot}53°$ *and the Stefan constant* $\sigma = 5{\cdot}67 \times 10^{-8}$ W m $^{-2}$ K $^{-4}$.

Solution :

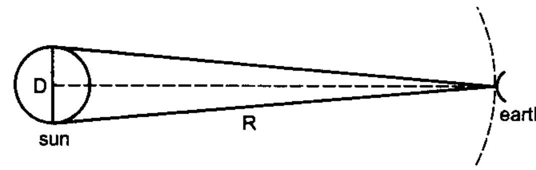

Figure 28-W11

Let the diameter of the sun be D and its distance from the earth be R. From the question,

$$\frac{D}{R} \approx 0{\cdot}53 \times \frac{\pi}{180}$$

$$= 9{\cdot}25 \times 10^{-3}. \qquad \dots \text{(i)}$$

The radiation emitted by the surface of the sun per unit time is

$$4\pi\left(\frac{D}{2}\right)^2 \sigma T^4 = \pi D^2 \sigma T^4.$$

At distance R, this radiation falls on an area $4\pi R^2$ in unit time. The radiation received at the earth's surface per unit time per unit area is, therefore,

$$\frac{\pi D^2 \sigma T^4}{4\pi R^2} = \frac{\sigma T^4}{4}\left(\frac{D}{R}\right)^2.$$

Thus, $$\frac{\sigma T^4}{4}\left(\frac{D}{R}\right)^2 = 8{\cdot}2\ \text{J cm}^{-2}\ \text{min}^{-1}$$

or, $$\frac{1}{4} \times (5{\cdot}67 \times 10^{-8}\ \text{W m}^{-2}\ \text{K}^{-4})\, T^4 \times (9{\cdot}25 \times 10^{-3})^2$$

$$= \frac{8{\cdot}2}{10^{-4} \times 60}\ \text{W m}^{-2}$$

or, $$T = 5794\ \text{K} \approx 5800\ \text{K}.$$

19. *The temperature of a body falls from* 40°C *to* 36°C *in* 5 *minutes when placed in a surrounding of constant temperature* 16°C. *Find the time taken for the temperature of the body to become* 32°C.

Solution : As the temperature differences are small, we can use Newton's law of cooling.

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

or, $$\frac{d\theta}{\theta - \theta_0} = -k\,dt \qquad \dots \text{(i)}$$

where k is a constant, θ is the temperature of the body at time t and $\theta_0 = 16°\text{C}$ is the temperature of the surrounding. We have,

$$\int_{40°C}^{36°C} \frac{d\theta}{\theta - \theta_0} = -k(5 \text{ min})$$

or, $$\ln \frac{36°C - 16°C}{40°C - 16°C} = -k(5 \text{ min})$$

or, $$k = -\frac{\ln(5/6)}{5 \text{ min}}.$$

If t be the time required for the temperature to fall from 36°C to 32°C then by (i),

$$\int_{36°C}^{32°C} \frac{d\theta}{\theta - \theta_0} = -kt$$

or, $$\ln \frac{32°C - 16°C}{36°C - 16°C} = \frac{\ln(5/6)t}{5 \text{ min}}$$

or, $$t = \frac{\ln(4/5)}{\ln(5/6)} \times 5 \text{ min}$$

$$= 6{\cdot}1 \text{ min}.$$

Alternative method

The mean temperature of the body as it cools from 40°C to 36°C is $\frac{40°C + 36°C}{2} = 38°C$. The rate of decrease of temperature is $\frac{40°C - 36°C}{5 \text{ min}} = 0{\cdot}80°C \text{ min}^{-1}$.

Newton's law of cooling is

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

or, $$-0{\cdot}8°C \text{ min}^{-1} = -k(38°C - 16°C) = -k(22°C)$$

or, $$k = \frac{0{\cdot}8}{22} \text{ min}^{-1}.$$

Let the time taken for the temperature to become 32°C be t.

During this period,

$$\frac{d\theta}{dt} = -\frac{36°C - 32°C}{t} = -\frac{4°C}{t}.$$

The mean temperature is $\frac{36°C + 32°C}{2} = 34°C$.

Now,

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

or, $$\frac{-4°C}{t} = -\frac{0{\cdot}8}{22} \times (34°C - 16°C) \text{ min}^{-1}$$

or, $$t = \frac{22 \times 4}{0{\cdot}8 \times 18} \text{ min} = 6{\cdot}1 \text{ min}.$$

□

20. *A hot body placed in air is cooled down according to Newton's law of cooling, the rate of decrease of temperature being k times the temperature difference from the surrounding. Starting from t = 0, find the time in which the body will lose half the maximum heat it can lose.*

Solution : We have,

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

where θ_0 is the temperature of the surrounding and θ is the temperature of the body at time t. Suppose $\theta = \theta_1$ at $t = 0$.

Then,

$$\int_{\theta_1}^{\theta} \frac{d\theta}{\theta - \theta_0} = -k \int_0^t dt$$

or, $$\ln \frac{\theta - \theta_0}{\theta_1 - \theta_0} = -kt$$

or, $$\theta - \theta_0 = (\theta_1 - \theta_0)\, e^{-kt}. \quad \text{... (i)}$$

The body continues to lose heat till its temperature becomes equal to that of the surrounding. The loss of heat in this entire period is

$$\Delta Q_m = ms(\theta_1 - \theta_0).$$

This is the maximum heat the body can lose. If the body loses half this heat, the decrease in its temperature will be,

$$\frac{\Delta Q_m}{2\, ms} = \frac{\theta_1 - \theta_0}{2}.$$

If the body loses this heat in time t_1, the temperature at t_1 will be

$$\theta_1 - \frac{\theta_1 - \theta_0}{2} = \frac{\theta_1 + \theta_0}{2}.$$

Putting these values of time and temperature in (i),

$$\frac{\theta_1 + \theta_0}{2} - \theta_0 = (\theta_1 - \theta_0)\, e^{-kt_1}$$

or, $$e^{-kt_1} = \frac{1}{2}$$

or, $$t_1 = \frac{\ln 2}{k}.$$

QUESTIONS FOR SHORT ANSWER

1. The heat current is written as $\frac{\Delta Q}{\Delta t}$. Why don't we write $\frac{dQ}{dt}$?

2. Does a body at 20°C radiate in a room, where the room temperature is 30°C? If yes, why does its temperature not fall further?

3. Why does blowing over a spoonful of hot tea cools it ? Does evaporation play a role ? Does radiation play a role ?
4. On a hot summer day we want to cool our room by opening the refrigerator door and closing all the windows and doors. Will the process work ?
5. On a cold winter night you are asked to sit on a chair. Would you like to choose a metal chair or a wooden chair ? Both are kept in the same lawn and are at the same temperature.
6. Two identical metal balls one at $T_1 = 300$ K and the other at $T_2 = 600$ K are kept at a distance of 1 m in vacuum. Will the temperatures equalise by radiation ? Will the rate of heat gained by the colder sphere be proportional to $T_2^4 - T_1^4$ as may be expected from the Stefan's law ?
7. An ordinary electric fan does not cool the air, still it gives comfort in summer. Explain.
8. The temperature of the atmosphere at a high altitude is around 500°C. Yet an animal there would freeze to death and not boil. Explain.
9. Standing in the sun is more pleasant on a cold winter day than standing in shade. Is the temperature of air in the sun considerably higher than that of the air in shade ?
10. Cloudy nights are warmer than the nights with clean sky. Explain.
11. Why is a white dress more comfortable than a dark dress in summer ?

OBJECTIVE I

1. The thermal conductivity of a rod depends on
   (a) length (b) mass
   (c) area of cross section (d) material of the rod.
2. In a room containing air, heat can go from one place to another
   (a) by conduction only (b) by convection only
   (c) by radiation only (d) by all the three modes.
3. A solid at temperature T_1 is kept in an evacuated chamber at temperature $T_2 > T_1$. The rate of increase of temperature of the body is proportional to
   (a) $T_2 - T_1$ (b) $T_2^2 - T_1^2$
   (c) $T_2^3 - T_1^3$ (d) $T_2^4 - T_1^4$.
4. The thermal radiation emitted by a body is proportional to T^n where T is its absolute temperature. The value of n is exactly 4 for
   (a) a blackbody (b) all bodies
   (c) bodies painted black only (d) polished bodies only.
5. Two bodies A and B having equal surface areas are maintained at temperatures 10°C and 20°C. The thermal radiation emitted in a given time by A and B are in the ratio
   (a) 1 : 1·15 (b) 1 : 2
   (c) 1 : 4 (d) 1 : 16.
6. One end of a metal rod is kept in a furnace. In steady state, the temperature of the rod
   (a) increases (b) decreases
   (c) remains constant (d) is nonuniform.
7. Newton's law of cooling is a special case of
   (a) Wien's displacement law (b) Kirchhoff's law
   (c) Stefan's law (d) Planck's law.
8. A hot liquid is kept in a big room. Its temperature is plotted as a function of time. Which of the following curves may represent the plot ?

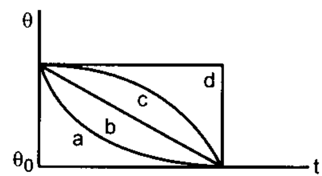

Figure 28-Q1

9. A hot liquid is kept in a big room. The logarithm of the numerical value of the temperature difference between the liquid and the room is plotted against time. The plot will be very nearly
   (a) a straight line (b) a circular arc
   (c) a parabola (d) an ellipse.
10. A body cools down from 65°C to 60°C in 5 minutes. It will cool down from 60°C to 55°C in
   (a) 5 minutes (b) less than 5 minutes
   (c) more than 5 minutes
   (d) less than or more than 5 minutes depending on whether its mass is more than or less than 1 kg.

OBJECTIVE II

1. One end of a metal rod is dipped in boiling water and the other is dipped in melting ice.
   (a) All parts of the rod are in thermal equilibrium with each other.
   (b) We can assign a temperature to the rod.
   (c) We can assign a temperature to the rod after steady state is reached.
   (d) The state of the rod does not change after steady state is reached.
2. A blackbody does not
   (a) emit radiation (b) absorb radiation
   (c) reflect radiation (d) refract radiation.

3. In summer, a mild wind is often found on the shore of a calm river. This is caused due to
(a) difference in thermal conductivity of water and soil
(b) convection currents
(c) conduction between air and the soil
(d) radiation from the soil.

4. A piece of charcoal and a piece of shining steel of the same surface area are kept for a long time in an open lawn in bright sun.
(a) The steel will absorb more heat than the charcoal.
(b) The temperature of the steel will be higher than that of the charcoal.
(c) If both are picked up by bare hands, the steel will be felt hotter than the charcoal.
(d) If the two are picked up from the lawn and kept in a cold chamber, the charcoal will lose heat at a faster rate than the steel.

5. A heated body emits radiation which has maximum intensity near the frequency ν_0. The emissivity of the material is 0·5. If the absolute temperature of the body is doubled,
(a) the maximum intensity of radiation will be near the frequency $2\nu_0$
(b) the maximum intensity of radiation will be near the frequency $\nu_0/2$
(c) the total energy emitted will increase by a factor of 16
(d) the total energy emitted will increase by a factor of 8.

6. A solid sphere and a hollow sphere of the same material and of equal radii are heated to the same temperature.
(a) Both will emit equal amount of radiation per unit time in the biginning.
(b) Both will absorb equal amount of radiation from the surrounding in the biginning.
(c) The initial rate of cooling (dT/dt) will be the same for the two spheres.
(d) The two spheres will have equal temperatures at any instant.

EXERCISES

1. A uniform slab of dimension 10 cm × 10 cm × 1 cm is kept between two heat reservoirs at temperatures 10°C and 90°C. The larger surface areas touch the reservoirs. The thermal conductivity of the material is 0·80 W $m^{-1}°C^{-1}$. Find the amount of heat flowing through the slab per minute.

2. A liquid-nitrogen container is made of a 1-cm thick styrofoam sheet having thermal conductivity 0·025 J s^{-1} $m^{-1}°C^{-1}$. Liquid nitrogen at 80 K is kept in it. A total area of 0·80 m^2 is in contact with the liquid nitrogen. The atmospheric temperature is 300 K. Calculate the rate of heat flow from the atmosphere to the liquid nitrogen.

3. The normal body-temperature of a person is 97°F. Calculate the rate at which heat is flowing out of his body through the clothes assuming the following values. Room temperature = 47°F, surface of the body under clothes = 1·6 m^2, conductivity of the cloth = 0·04 J s^{-1} $m^{-1}°C^{-1}$, thickness of the cloth = 0·5 cm.

4. Water is boiled in a container having a bottom of surface area 25 cm^2, thickness 1·0 mm and thermal conductivity 50 W $m^{-1}°C^{-1}$. 100 g of water is converted into steam per minute in the steady state after the boiling starts. Assuming that no heat is lost to the atmosphere, calculate the temperature of the lower surface of the bottom. Latent heat of vaporization of water = 2·26 × 10^6 J kg^{-1}.

5. One end of a steel rod (K = 46 J s^{-1} $m^{-1}°C^{-1}$) of length 1·0 m is kept in ice at 0°C and the other end is kept in boiling water at 100°C. The area of cross section of the rod is 0·04 cm^2. Assuming no heat loss to the atmosphere, find the mass of the ice melting per second. Latent heat of fusion of ice = 3·36 × 10^5 J kg^{-1}.

6. An icebox almost completely filled with ice at 0°C is dipped into a large volume of water at 20°C. The box has walls of surface area 2400 cm^2, thickness 2·0 mm and thermal conductivity 0·06 W $m^{-1}°C^{-1}$. Calculate the rate at which the ice melts in the box. Latent heat of fusion of ice = 3·4 × 10^5 J kg^{-1}.

7. A pitcher with 1-mm thick porous walls contains 10 kg of water. Water comes to its outer surface and evaporates at the rate of 0·1 g s^{-1}. The surface area of the pitcher (one side) = 200 cm^2. The room temperature = 42°C, latent heat of vaporization = 2·27 × 10^6 J kg^{-1}, and the thermal conductivity of the porous walls = 0·80 J s^{-1} $m^{-1}°C^{-1}$. Calculate the temperature of water in the pitcher when it attains a constant value.

8. A steel frame (K = 45 W $m^{-1}°C^{-1}$) of total length 60 cm and cross sectional area 0·20 cm^2, forms three sides of a square. The free ends are maintained at 20°C and 40°C. Find the rate of heat flow through a cross section of the frame.

9. Water at 50°C is filled in a closed cylindrical vessel of height 10 cm and cross sectional area 10 cm^2. The walls of the vessel are adiabatic but the flat parts are made of 1-mm thick aluminium (K = 200 J s^{-1} $m^{-1}°C^{-1}$). Assume that the outside temperature is 20°C. The density of water is 1000 kg m^{-3}, and the specific heat capacity of water = 4200 J $kg^{-1}°C^{-1}$. Estimate the time taken for the temperature to fall by 1·0°C. Make any simplifying assumptions you need but specify them.

10. The left end of a copper rod (length = 20 cm, area of cross section = 0·20 cm^2) is maintained at 20°C and the right end is maintained at 80°C. Neglecting any loss of heat through radiation, find (a) the temperature at a point 11 cm from the left end and (b) the heat current through the rod. Thermal conductivity of copper = 385 W $m^{-1}°C^{-1}$.

11. The ends of a metre stick are maintained at 100°C and 0°C. One end of a rod is maintained at 25°C. Where should its other end be touched on the metre stick so that there is no heat current in the rod in steady state ?

12. A cubical box of volume 216 cm^3 is made up of 0·1 cm thick wood. The inside is heated electrically by a 100 W heater. It is found that the temperature difference between the inside and the outside surface is 5°C in steady state. Assuming that the entire electrical energy spent appears as heat, find the thermal conductivity of the material of the box.

13. Figure (28-E1) shows water in a container having 2·0-mm thick walls made of a material of thermal conductivity 0·50 W m^{-1}°C^{-1}. The container is kept in a melting-ice bath at 0°C. The total surface area in contact with water is 0·05 m^2. A wheel is clamped inside the water and is coupled to a block of mass *M* as shown in the figure. As the block goes down, the wheel rotates. It is found that after some time a steady state is reached in which the block goes down with a constant speed of 10 cm s^{-1} and the temperature of the water remains constant at 1·0°C. Find the mass *M* of the block. Assume that the heat flows out of the water only through the walls in contact. Take $g = 10$ m s^{-2}.

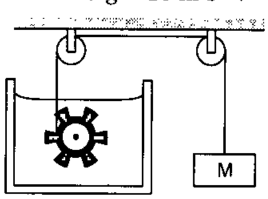

Figure 28-E1

14. On a winter day when the atmospheric temperature drops to −10°C, ice forms on the surface of a lake. (a) Calculate the rate of increase of thickness of the ice when 10 cm of ice is already formed. (b) Calculate the total time taken in forming 10 cm of ice. Assume that the temperature of the entire water reaches 0°C before the ice starts forming. Density of water = 1000 kg m^{-3}, latent heat of fusion of ice = $3·36 \times 10^5$ J kg^{-1} and thermal conductivity of ice = 1·7 W m^{-1}°C^{-1}. Neglect the expansion of water on freezing.

15. Consider the situation of the previous problem. Assume that the temperature of the water at the bottom of the lake remains constant at 4°C as the ice forms on the surface (the heat required to maintain the temperature of the bottom layer may come from the bed of the lake). The depth of the lake is 1·0 m. Show that the thickness of the ice formed attains a steady state maximum value. Find this value. The thermal conductivity of water = 0·50 W m^{-1}°C^{-1}. Take other relevant data from the previous problem.

16. Three rods of lengths 20 cm each and area of cross section 1 cm^2 are joined to form a triangle *ABC*. The conductivities of the rods are $K_{AB} = 50$ J s^{-1} m^{-1}°C^{-1}, $K_{BC} = 200$ J s^{-1} m^{-1}°C^{-1} and $K_{AC} = 400$ J s^{-1} m^{-1}°C^{-1}. The junctions *A*, *B* and *C* are maintained at 40°C, 80°C and 80°C respectively. Find the rate of heat flowing through the rods *AB*, *AC* and *BC*.

17. A semicircular rod is joined at its end to a straight rod of the same material and the same cross-sectional area. The straight rod forms a diameter of the other rod. The junctions are maintained at different temperatures. Find the ratio of the heat transferred through a cross section of the semicircular rod to the heat transferred through a cross section of the straight rod in a given time.

18. A metal rod of cross sectional area 1·0 cm^2 is being heated at one end. At one time, the temperature gradient is 5·0°C cm^{-1} at cross section *A* and is 2·5°Ccm^{-1} at cross section *B*. Calculate the rate at which the temperature is increasing in the part *AB* of the rod. The heat capacity of the part *AB* = 0·40 J°C^{-1}, thermal conductivity of the material of the rod = 200 W m^{-1}°C^{-1}. Neglect any loss of heat to the atmosphere.

19. Steam at 120°C is continuously passed through a 50-cm long rubber tube of inner and outer radii 1·0 cm and 1·2 cm. The room temperature is 30°C. Calculate the rate of heat flow through the walls of the tube. Thermal conductivity of rubber = 0·15 J s^{-1} m^{-1}°C^{-1}.

20. A hole of radius r_1 is made centrally in a uniform circular disc of thickness *d* and radius r_2. The inner surface (a cylinder of length *d* and radius r_1) is maintained at a temperature θ_1 and the outer surface (a cylinder of length *d* and radius r_2) is maintained at a temperature $\theta_2 (\theta_1 > \theta_2)$. The thermal conductivity of the material of the disc is *K*. Calculate the heat flowing per unit time through the disc.

21. A hollow tube has a length *l*, inner radius R_1 and outer radius R_2. The material has a thermal conductivity *K*. Find the heat flowing through the walls of the tube if (a) the flat ends are maintained at temperatures T_1 and $T_2 (T_2 > T_1)$ (b) the inside of the tube is maintained at temperature T_1 and the outside is maintained at T_2.

22. A composite slab is prepared by pasting two plates of thicknesses L_1 and L_2 and thermal conductivities K_1 and K_2. The slabs have equal cross-sectional area. Find the equivalent conductivity of the composite slab.

23. Figure (28-E2) shows a copper rod joined to a steel rod. The rods have equal length and equal cross sectional area. The free end of the copper rod is kept at 0°C and that of the steel rod is kept at 100°C. Find the temperature at the junction of the rods. Conductivity of copper = 390 W m^{-1}°C^{-1} and that of steel = 46 W m^{-1}°C^{-1}.

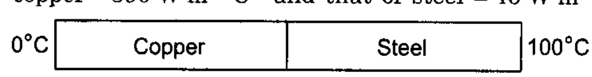

Figure 28-E2

24. An aluminium rod and a copper rod of equal length 1·0 m and cross-sectional area 1 cm^2 are welded together as shown in figure (28-E3). One end is kept at a temperature of 20°C and the other at 60°C. Calculate the amount of heat taken out per second from the hot end. Thermal conductivity of aluminium = 200 Wm^{-1}°C^{-1} and of copper = 390 W m^{-1}°C^{-1}.

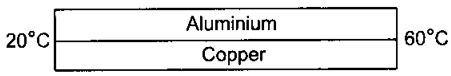

Figure 28-E3

25. Figure (28-E4) shows an aluminium rod joined to a copper rod. Each of the rods has a length of 20 cm and area of cross section $0{\cdot}20\ \mathrm{cm}^2$. The junction is maintained at a constant temperature 40°C and the two ends are maintained at 80°C. Calculate the amount of heat taken out from the cold junction in one minute after the steady state is reached. The conductivities are $K_{Al} = 200\ \mathrm{W\ m^{-1}{}^\circ C^{-1}}$ and $K_{Cu} = 400\ \mathrm{W\ m^{-1}{}^\circ C^{-1}}$.

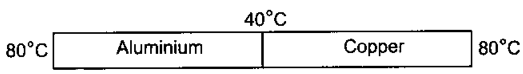

Figure 28-E4

26. Consider the situation shown in figure (28-E5). The frame is made of the same material and has a uniform cross-sectional area everywhere. Calculate the amount of heat flowing per second through a cross section of the bent part if the total heat taken out per second from the end at 100°C is 130 J.

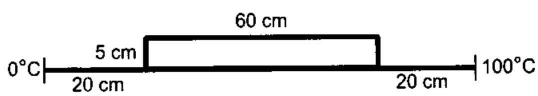

Figure 28-E5

27. Suppose the bent part of the frame of the previous problem has a thermal conductivity of $780\ \mathrm{J\ s^{-1}\ m^{-1}{}^\circ C^{-1}}$ whereas it is $390\ \mathrm{J\ s^{-1}\ m^{-1}{}^\circ C^{-1}}$ for the straight part. Calculate the ratio of the rate of heat flow through the bent part to the rate of heat flow through the straight part.

28. A room has a window fitted with a single $1{\cdot}0\ \mathrm{m} \times 2{\cdot}0\ \mathrm{m}$ glass of thickness 2 mm. (a) Calculate the rate of heat flow through the closed window when the temperature inside the room is 32°C and that outside is 40°C. (b) The glass is now replaced by two glasspanes, each having a thickness of 1 mm and separated by a distance of 1 mm. Calculate the rate of heat flow under the same conditions of temperature. Thermal conductivity of window glass $= 1{\cdot}0\ \mathrm{J\ s^{-1}\ m^{-1}{}^\circ C^{-1}}$ and that of air $= 0{\cdot}025\ \mathrm{J\ s^{-1}\ m^{-1}{}^\circ C^{-1}}$.

29. The two rods shown in figure (28-E6) have identical geometrical dimensions. They are in contact with two heat baths at temperatures 100°C and 0°C. The temperature of the junction is 70°C. Find the temperature of the junction if the rods are interchanged.

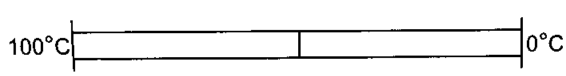

Figure 28-E6

30. The three rods shown in figure (28-E7) have identical geometrical dimensions. Heat flows from the hot end at a rate of 40 W in the arrangement (a). Find the rates of heat flow when the rods are joined as in arrangement (b) and in (c). Thermal conductivities of aluminium and copper are $200\ \mathrm{W\ m^{-1}{}^\circ C^{-1}}$ and $400\ \mathrm{W\ m^{-1}{}^\circ C^{-1}}$ respectively.

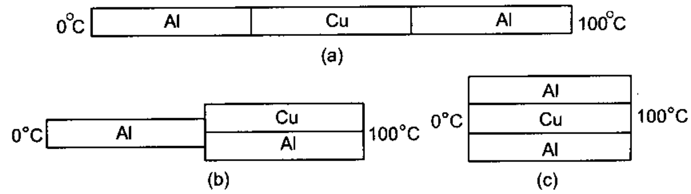

Figure 28-E7

31. Four identical rods AB, CD, CF and DE are joined as shown in figure (28-E8). The length, cross-sectional area and thermal conductivity of each rod are l, A and K respectively. The ends A, E and F are maintained at temperatures T_1, T_2 and T_3 respectively. Assuming no loss of heat to the atmosphere, find the temperature at B.

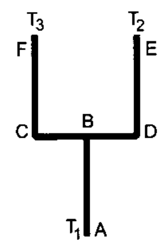

Figure 28-E8

32. Seven rods A, B, C, D, E, F and G are joined as shown in figure (28-E9). All the rods have equal cross-sectional area A and length l. The thermal conductivities of the rods are $K_A = K_C = K_0$, $K_B = K_D = 2K_0$, $K_E = 3K_0$, $K_F = 4K_0$ and $K_G = 5K_0$. The rod E is kept at a constant temperature T_1 and the rod G is kept at a constant temperature $T_2 (T_2 > T_1)$. (a) Show that the rod F has a uniform temperature $T = (T_1 + 2T_2)/3$. (b) Find the rate of heat flowing from the source which maintains the temperature T_2.

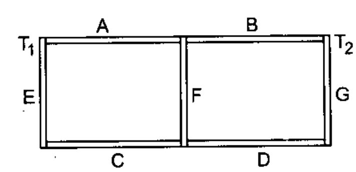

Figure 28-E9

33. Find the rate of heat flow through a cross section of the rod shown in figure (28-E10) $(\theta_2 > \theta_1)$. Thermal conductivity of the material of the rod is K.

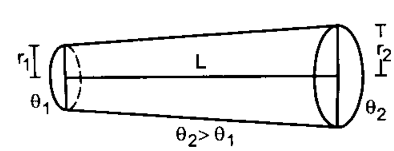

Figure 28-E10

34. A rod of negligible heat capacity has length 20 cm, area of cross section $1 \cdot 0\ \mathrm{cm}^2$ and thermal conductivity $200\ \mathrm{W\ m^{-1} {}^\circ C^{-1}}$. The temperature of one end is maintained at 0°C and that of the other end is slowly and linearly varied from 0°C to 60°C in 10 minutes. Assuming no loss of heat through the sides, find the total heat transmitted through the rod in these 10 minutes.

35. A hollow metallic sphere of radius 20 cm surrounds a concentric metallic sphere of radius 5 cm. The space between the two spheres is filled with a nonmetallic material. The inner and outer spheres are maintained at 50°C and 10°C respectively and it is found that 100 J of heat passes from the inner sphere to the outer sphere per second. Find the thermal conductivity of the material between the spheres.

36. Figure (28-E11) shows two adiabatic vessels, each containing a mass m of water at different temperatures. The ends of a metal rod of length L, area of cross section A and thermal conductivity K, are inserted in the water as shown in the figure. Find the time taken for the difference between the temperatures in the vessels to become half of the original value. The specific heat capacity of water is s. Neglect the heat capacity of the rod and the container and any loss of heat to the atmosphere.

Figure 28-E11

37. Two bodies of masses m_1 and m_2 and specific heat capacities s_1 and s_2 are connected by a rod of length l, cross-sectional area A, thermal conductivity K and negligible heat capacity. The whole system is thermally insulated. At time $t = 0$, the temperature of the first body is T_1 and the temperature of the second body is T_2 ($T_2 > T_1$). Find the temperature difference between the two bodies at time t.

38. An amount n (in moles) of a monatomic gas at an initial temperature T_0 is enclosed in a cylindrical vessel fitted with a light piston. The surrounding air has a temperature $T_s (> T_0)$ and the atmospheric pressure is p_a. Heat may be conducted between the surrounding and the gas through the bottom of the cylinder. The bottom has a surface area A, thickness x and thermal conductivity K. Assuming all changes to be slow, find the distance moved by the piston in time t.

39. Assume that the total surface area of a human body is $1 \cdot 6\ \mathrm{m}^2$ and that it radiates like an ideal radiator. Calculate the amount of energy radiated per second by the body if the body temperature is 37°C. Stefan constant σ is $6 \cdot 0 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$.

40. Calculate the amount of heat radiated per second by a body of surface area $12\ \mathrm{cm}^2$ kept in thermal equilibrium in a room at temperature 20°C. The emissivity of the surface $= 0 \cdot 80$ and $\sigma = 6 \cdot 0 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$.

41. A solid aluminium sphere and a solid copper sphere of twice the radius are heated to the same temperature and are allowed to cool under identical surrounding temperatures. Assume that the emissivity of both the spheres is the same. Find the ratio of (a) the rate of heat loss from the aluminium sphere to the rate of heat loss from the copper sphere and (b) the rate of fall of temperature of the aluminium sphere to the rate of fall of temperature of the copper sphere. The specific heat capacity of aluminium $= 900\ \mathrm{J\ kg^{-1} {}^\circ C^{-1}}$ and that of copper $= 390\ \mathrm{J\ kg^{-1} {}^\circ C^{-1}}$. The density of copper $= 3 \cdot 4$ times the density of aluminium.

42. A 100 W bulb has tungsten filament of total length 1·0 m and radius 4×10^{-5} m. The emissivity of the filament is 0·8 and $\sigma = 6 \cdot 0 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{4}}$. Calculate the temperature of the filament when the bulb is operating at correct wattage.

43. A spherical ball of surface area $20\ \mathrm{cm}^2$ absorbs any radiation that falls on it. It is suspended in a closed box maintained at 57°C. (a) Find the amount of radiation falling on the ball per second. (b) Find the net rate of heat flow to or from the ball at an instant when its temperature is 200°C. Stefan constant $= 6 \cdot 0 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$.

44. A spherical tungsten piece of radius 1·0 cm is suspended in an evacuated chamber maintained at 300 K. The piece is maintained at 1000 K by heating it electrically. Find the rate at which the electrical energy must be supplied. The emissivity of tungsten is 0·30 and the Stefan constant σ is $6 \cdot 0 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$.

45. A cubical block of mass 1·0 kg and edge 5·0 cm is heated to 227°C. It is kept in an evacuated chamber maintained at 27°C. Assuming that the block emits radiation like a blackbody, find the rate at which the temperature of the block will decrease. Specific heat capacity of the material of the block is $400\ \mathrm{J\ kg^{-1}\ K^{-1}}$.

46. A copper sphere is suspended in an evacuated chamber maintained at 300 K. The sphere is maintained at a constant temperature of 500 K by heating it electrically. A total of 210 W of electric power is needed to do it. When the surface of the copper sphere is completely blackened, 700 W is needed to maintain the same temperature of the sphere. Calculate the emissivity of copper.

47. A spherical ball A of surface area $20\ \mathrm{cm}^2$ is kept at the centre of a hollow spherical shell B of area $80\ \mathrm{cm}^2$. The surface of A and the inner surface of B emit as blackbodies. Both A and B are at 300 K. (a) How much is the radiation energy emitted per second by the ball A? (b) How much is the radiation energy emitted per second by the inner surface of B? (c) How much of the energy emitted by the inner surface of B falls back on this surface itself ?

48. A cylindrical rod of length 50 cm and cross sectional area $1\ \mathrm{cm}^2$ is fitted between a large ice chamber at 0°C and an evacuated chamber maintained at 27°C as shown in figure (28-E12). Only small portions of the rod are inside the chambers and the rest is thermally insulated from the surrounding. The cross section going into the evacuated chamber is blackened so that it completely absorbs any radiation falling on it. The temperature of the blackened end is 17°C when steady state is reached. Stefan constant $\sigma = 6 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$. Find the thermal conductivity of the material of the rod.

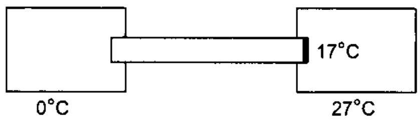

Figure 28-E12

49. One end of a rod of length 20 cm is inserted in a furnace at 800 K. The sides of the rod are covered with an insulating material and the other end emits radiation like a blackbody. The temperature of this end is 750 K in the steady state. The temperature of the surrounding air is 300 K. Assuming radiation to be the only important mode of energy transfer between the surrounding and the open end of the rod, find the thermal conductivity of the rod. Stefan constant $\sigma = 6{\cdot}0 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$.

50. A calorimeter of negligible heat capacity contains 100 cc of water at 40°C. The water cools to 35°C in 5 minutes. The water is now replaced by K-oil of equal volume at 40°C. Find the time taken for the temperature to become 35°C under similar conditions. Specific heat capacities of water and K-oil are $4200\ \mathrm{J\ kg^{-1}\ K^{-1}}$ and $2100\ \mathrm{J\ kg^{-1}\ K^{-1}}$ respectively. Density of K-oil $= 800\ \mathrm{kg\ m^{-3}}$.

51. A body cools down from 50°C to 45°C in 5 minutes and to 40°C in another 8 minutes. Find the temperature of the surrounding.

52. A calorimeter contains 50 g of water at 50°C. The temperature falls to 45°C in 10 minutes. When the calorimeter contains 100 g of water at 50°C, it takes 18 minutes for the temperature to become 45°C. Find the water equivalent of the calorimeter.

53. A metal ball of mass 1 kg is heated by means of a 20 W heater in a room at 20°C. The temperature of the ball becomes steady at 50°C. (a) Find the rate of loss of heat to the surrounding when the ball is at 50°C. (b) Assuming Newton's law of cooling, calculate the rate of loss of heat to the surrounding when the ball is at 30°C. (c) Assume that the temperature of the ball rises uniformly from 20°C to 30°C in 5 minutes. Find the total loss of heat to the surrounding during this period. (d) Calculate the specific heat capacity of the metal.

54. A metal block of heat capacity $80\ \mathrm{J°C^{-1}}$ placed in a room at 20°C is heated electrically. The heater is switched off when the temperature reaches 30°C. The temperature of the block rises at the rate of $2\ \mathrm{°C\ s^{-1}}$ just after the heater is switched on and falls at the rate of $0{\cdot}2\ \mathrm{°C\ s^{-1}}$ just after the heater is switched off. Assume Newton's law of cooling to hold. (a) Find the power of the heater. (b) Find the power radiated by the block just after the heater is switched off. (c) Find the power radiated by the block when the temperature of the block is 25°C. (d) Assuming that the power radiated at 25°C represents the average value in the heating process, find the time for which the heater was kept on.

55. A hot body placed in a surrounding of temperature θ_0 obeys Newton's law of cooling $\frac{d\theta}{dt} = -k(\theta - \theta_0)$. Its temperature at $t = 0$ is θ_1. The specific heat capacity of the body is s and its mass is m. Find (a) the maximum heat that the body can lose and (b) the time starting from $t = 0$ in which it will lose 90% of this maximum heat.

□

ANSWERS

OBJECTIVE I

1. (d)	2. (d)	3. (d)	4. (b)	5. (a)	6. (d)
7. (c)	8. (a)	9. (a)	10. (c)		

OBJECTIVE II

1. (d)	2. (c), (d)	3. (b)
4. (c), (d)	5. (a), (c)	6. (a), (b)

EXERCISES

1. 3840 J
2. 440 W
3. $356\ \mathrm{J\ s^{-1}}$
4. 130°C
5. $5{\cdot}5 \times 10^{-5}$ g
6. $1{\cdot}5\ \mathrm{kg\ h^{-1}}$
7. 28°C
8. 0·03 W
9. 0·035 s
10. (a) 53°C (b) $2{\cdot}31\ \mathrm{J\ s^{-1}}$
11. 25 cm from the cold end
12. $0{\cdot}92\ \mathrm{W\ m^{-1}°C^{-1}}$
13. 12·5 kg

14. (a) $5{\cdot}0 \times 10^{-7}$ m s^{-1} (b) 27·5 hours
15. 89 cm
16. 1 W, 8 W, zero
17. $2 : \pi$
18. 12·5°C s^{-1}
19. 233 J s^{-1}
20. $\frac{2\pi K d(\theta_1 - \theta_2)}{\ln(r_2/r_1)}$
21. (a) $\frac{K\pi(R_2^2 - R_1^2)(T_2 - T_1)}{l}$ (b) $\frac{2\pi K l(T_2 - T_1)}{\ln(R_2/R_1)}$
22. $\frac{K_1 K_2(L_1 + L_2)}{L_1 K_2 + L_2 K_1}$
23. 10·6°C
24. 2·36 J
25. 144 J
26. 60 J
27. 12 : 7
28. (a) 8000 J s^{-1} (b) 381 J s^{-1}
29. 30°C
30. 75 W, 400 W
31. $\frac{3\,T_1 + 2(T_2 + T_3)}{7}$
32. (b) $\frac{4\,K_0\,A(T_2 - T_1)}{3l}$
33. $\frac{K\pi r_1 r_2(\theta_2 - \theta_1)}{L}$
34. 1800 J
35. 3·0 W m^{-1}°C^{-1}
36. $\frac{Lms}{2KA}\ln 2$
37. $(T_2 - T_1)\,e^{-\lambda t}$ where $\lambda = \frac{KA(m_1 s_1 + m_2 s_2)}{l m_1 m_2 s_1 s_2}$
38. $\frac{nR}{P_a A}(T_s - T_0)(1 - e^{-2KAt/5Rnx})$
39. 887 J
40. 0·42 J
41. (a) 1 : 4 (b) 2·9 : 1
42. 1700 K
43. (a) 1·4 J (b) 4·58 W from the ball
44. 22 W
45. 0·12°C s^{-1}
46. 0·3
47. (a) 0.94 J (b) 3.8 J (c) 2.8 J
48. 1·8 W m^{-1}°C^{-1}
49. 74 W m^{-1} K^{-1}
50. 2 min
51. 34°C
52. 12·5 g
53. (a) 20 W (b) $\frac{20}{3}$ W (c) 1000 J (d) 500 J kg^{-1} K^{-1}
54. (a) 160 W (b) 16 W (c) 8 W (d) 5·2 s
55. (a) $ms(\theta_1 - \theta_0)$ (b) $\frac{\ln 10}{k}$

□