



$$\Delta = \begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$, we have:

$$\Delta = \begin{vmatrix} a+b+c & -a+b & -a+c \\ a+b+c & 3b & -b+c \\ a+b+c & -c+b & 3c \end{vmatrix}$$

$$= (a+b+c) \begin{vmatrix} 1 & -a+b & -a+c \\ 1 & 3b & -b+c \\ 1 & -c+b & 3c \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we have:

$$\Delta = (a+b+c) \begin{vmatrix} 1 & -a+b & -a+c \\ 0 & 2b+a & a-b \\ 0 & a-c & 2c+a \end{vmatrix}$$

Expanding along C_1 , we have:

$$\begin{aligned} \Delta &= (a+b+c) [(2b+a)(2c+a) - (a-b)(a-c)] \\ &= (a+b+c) [4bc + 2ab + 2ac + a^2 - a^2 + ac + ba - bc] \\ &= (a+b+c) (3ab + 3bc + 3ac) \\ &= 3(a+b+c)(ab + bc + ca) \end{aligned}$$

Hence, the given result is proved.

Question 14:

Using properties of determinants, prove that:

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1$$

Answer

$$\Delta = \begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - 2R_1$ and $R_3 \rightarrow R_3 - 3R_1$, we have:

$$\Delta = \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & 2+p \\ 0 & 3 & 7+3p \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 - 3R_2$, we have:

$$\Delta = \begin{vmatrix} 1 & 1+p & 1+p+q \\ 0 & 1 & 2+p \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding along C_1 , we have:

$$\Delta = 1 \begin{vmatrix} 2+p & 1 \\ 0 & 1 \end{vmatrix} = 1(1-0) = 1$$

Hence, the given result is proved.

Question 15:

Using properties of determinants, prove that:

$$\begin{vmatrix} \sin \alpha & \cos \alpha & \cos(\alpha + \delta) \\ \sin \beta & \cos \beta & \cos(\beta + \delta) \\ \sin \gamma & \cos \gamma & \cos(\gamma + \delta) \end{vmatrix} = 0$$

Answer

$$\Delta = \begin{vmatrix} \sin \alpha & \cos \alpha & \cos(\alpha + \delta) \\ \sin \beta & \cos \beta & \cos(\beta + \delta) \\ \sin \gamma & \cos \gamma & \cos(\gamma + \delta) \end{vmatrix}$$

$$\begin{vmatrix} \sin \gamma & \cos \gamma & \cos(\gamma + \delta) \\ \sin \alpha \sin \delta & \cos \alpha \cos \delta & \cos \alpha \cos \delta - \sin \alpha \sin \delta \\ \sin \beta \sin \delta & \cos \beta \cos \delta & \cos \beta \cos \delta - \sin \beta \sin \delta \\ \sin \gamma \sin \delta & \cos \gamma \cos \delta & \cos \gamma \cos \delta - \sin \gamma \sin \delta \end{vmatrix}$$

$$= \frac{1}{\sin \delta \cos \delta} \begin{vmatrix} \sin \alpha \sin \delta & \cos \alpha \cos \delta & \cos \alpha \cos \delta - \sin \alpha \sin \delta \\ \sin \beta \sin \delta & \cos \beta \cos \delta & \cos \beta \cos \delta - \sin \beta \sin \delta \\ \sin \gamma \sin \delta & \cos \gamma \cos \delta & \cos \gamma \cos \delta - \sin \gamma \sin \delta \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_3$, we have:

$$\Delta = \frac{1}{\sin \delta \cos \delta} \begin{vmatrix} \cos \alpha \cos \delta & \cos \alpha \cos \delta & \cos \alpha \cos \delta - \sin \alpha \sin \delta \\ \cos \beta \cos \delta & \cos \beta \cos \delta & \cos \beta \cos \delta - \sin \beta \sin \delta \\ \cos \gamma \cos \delta & \cos \gamma \cos \delta & \cos \gamma \cos \delta - \sin \gamma \sin \delta \end{vmatrix}$$

Here, two columns C_1 and C_2 are identical.

$\therefore \Delta = 0$.

Hence, the given result is proved.

Question 16:

Solve the system of the following equations

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

Answer

$$\frac{1}{x} = p, \frac{1}{y} = q, \frac{1}{z} = r.$$

Let

Then the given system of equations is as follows:

$$2p + 3q + 10r = 4$$

$$4p - 6q + 5r = 1$$

$$6p + 9q - 20r = 2$$

This system can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}.$$

Now,

$$\begin{aligned} |A| &= 2(120 - 45) - 3(-80 - 30) + 10(36 + 36) \\ &= 150 + 330 + 720 \\ &= 1200
\end{aligned}$$

A

Thus, A is non-singular. Therefore, its inverse exists.

Now,

$$A_{11} = 75, A_{12} = 110, A_{13} = 72$$

$$A_{21} = 150, A_{22} = -100, A_{23} = 0$$

$$A_{31} = 75, A_{32} = 30, A_{33} = -24$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj}A$$

$$= \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

Now,

$$X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{1200} \begin{bmatrix} 300 + 150 + 150 \\ 440 - 100 + 60 \\ 288 + 0 - 48 \end{bmatrix}$$

$$= \frac{1}{1200} \begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$$\therefore p = \frac{1}{2}, q = \frac{1}{3}, \text{ and } r = \frac{1}{5}$$

Hence, $x = 2$, $y = 3$, and $z = 5$.

Question 17:

Choose the correct answer.

If a, b, c , are in A.P., then the determinant

$$\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \end{vmatrix}$$

$$\begin{vmatrix} x+4 & x+5 & x+2c \end{vmatrix}$$

A. 0 B. 1 C. x D. $2x$

Answer

Answer: A

$$\begin{aligned} \Delta &= \begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} \\ &= \begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+(a+c) \\ x+4 & x+5 & x+2c \end{vmatrix} \quad (2b = a + c \text{ as } a, b, \text{ and } c \text{ are in A.P.}) \end{aligned}$$

Applying $R_1 \rightarrow R_1 - R_2$ and $R_3 \rightarrow R_3 - R_2$, we have:

$$\Delta = \begin{vmatrix} -1 & -1 & a-c \\ x+3 & x+4 & x+(a+c) \\ 1 & 1 & c-a \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_3$, we have:

$$\Delta = \begin{vmatrix} 0 & 0 & 0 \\ x+3 & x+4 & x+a+c \\ 1 & 1 & c-a \end{vmatrix}$$

Question 18:

Choose the correct answer.

$$A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix} \text{ is}$$

If x, y, z are nonzero real numbers, then the inverse of matrix

$$\text{A. } \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix} \quad \text{B. } \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$$

$$\text{C. } \frac{1}{xyz} \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix} \quad \text{D. } \frac{1}{xyz} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Answer

Answer: A

$$A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$$

$$\therefore |A| = x(yz - 0) = xyz \neq 0$$

$$\text{Now, } A_{11} = yz, A_{12} = 0, A_{13} = 0$$

$$A_{21} = 0, A_{22} = xz, A_{23} = 0$$

$$A_{31} = 0, A_{32} = 0, A_{33} = xy$$

$$\therefore \text{adj}A = \begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj}A$$

$$= \frac{1}{xyz} \begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}$$

$$= \begin{bmatrix} \frac{yz}{xyz} & 0 & 0 \\ 0 & \frac{xz}{xyz} & 0 \\ 0 & 0 & \frac{xy}{xyz} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{x} & 0 & 0 \\ 0 & \frac{1}{y} & 0 \\ 0 & 0 & \frac{1}{z} \end{bmatrix} = \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$$

The correct answer is A.

Question 19:

Choose the correct answer.

$$A = \begin{bmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{bmatrix}, \text{ where } 0 \leq \theta \leq 2\pi, \text{ then}$$

A. $\text{Det}(A) = 0$

B. $\text{Det}(A) \in (2, \infty)$

C. $\text{Det}(A) \in (2, 4)$

D. $\text{Det}(A) \in [2, 4]$

Answer

Answer: D

$$A = \begin{bmatrix} 1 & \sin \theta & 1 \\ -\sin \theta & 1 & \sin \theta \\ -1 & -\sin \theta & 1 \end{bmatrix}$$

$$\begin{aligned} \therefore |A| &= 1(1 + \sin^2 \theta) - \sin \theta(-\sin \theta + \sin \theta) + 1(\sin^2 \theta + 1) \\ &= 1 + \sin^2 \theta + \sin^2 \theta + 1 \\ &= 2 + 2\sin^2 \theta \\ &= 2(1 + \sin^2 \theta) \end{aligned}$$

Now, $0 \leq \theta \leq 2\pi$

$$\Rightarrow 0 \leq \sin \theta \leq 1$$

$$\Rightarrow 0 \leq \sin^2 \theta \leq 1$$

$$\Rightarrow 1 \leq 1 + \sin^2 \theta \leq 2$$

$$\Rightarrow 2 \leq 2(1 + \sin^2 \theta) \leq 4$$

$$\therefore \text{Det}(A) \in [2, 4]$$

The correct answer is D.

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