Solution of Simultaneous Linear Equations Ex 8.1 Q5

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 2+4+0 & 2-2+0 & -4+4+0 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ 0-4+4 & 0+2-2 & 0-4+10 \end{bmatrix}$$

$$AB = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$AB = 6I$, where I is a 3×3 unit matrix

or $A^{-1} = \frac{1}{6}B$ [By def. of inverse]

$$= \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

Now, the ginven system of equations can be written as

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

or $AX = B$

or $X = A^{-1}B$

$$= \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 6+34-28 \\ -12+34-28 \\ 6-17+35 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

Hence, $x = 2, y = -1, z = 4$

Solution of Simultaneous Linear Equations Ex 8.1 Q6

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$$|A| = 2(0) + 3(-2) + 5(1) = -1 \neq 0$$

Also, $C_{11} = 0$ $\quad C_{21} = -1$ $\quad C_{31} = 2$

$C_{12} = 2$ $\quad C_{22} = -9$ $\quad C_{32} = 23$

$C_{13} = 1$ $\quad C_{23} = -5$ $\quad C_{33} = 13$

$$(\text{adj } A) = \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

The given system of equations can be written as

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ -1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

or $\quad AX = B$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} -5+6 \\ -22+45+69 \\ -11-25+39 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, $x = 1, y = 2, z = 3$

Solution of Simultaneous Linear Equations Ex 8.1 Q7

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$$

$$|A| = 1(1+3) - 2(-1+2) + 5(5) = 4 - 2 + 25 = 27 \neq 0$$

$$\begin{array}{lll} C_{11} = 4 & C_{21} = 17 & C_{31} = 3 \\ C_{12} = -1 & C_{22} = -11 & C_{32} = 6 \\ C_{13} = 5 & C_{23} = 1 & C_{33} = -3 \end{array}$$

$$A^{-1} = \frac{1}{|A|} \times \operatorname{adj} A = \frac{1}{27} \begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$$

Now, the given set of equations can be represented as

$x + 2y + 5z = 10$

$x - y - z = -2$

$2x + 3y - z = -11$

or $$\begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ -2 \\ -11 \end{bmatrix}$$

or $$\begin{aligned} X &= A^{-1} \times B \\ &= \frac{1}{27} \begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix} \begin{bmatrix} 10 \\ -2 \\ -11 \end{bmatrix} \\ &= \frac{1}{27} \begin{bmatrix} 40 - 34 - 33 \\ -10 + 22 - 66 \\ 50 - 2 + 33 \end{bmatrix} = \frac{1}{27} \begin{bmatrix} -27 \\ -54 \\ 81 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix} \end{aligned}$$

Hence, $x = -1, y = -2, z = 3$

Solution of Simultaneous Linear Equations Ex 8.1 Q8

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$$

$$|A| = 1(7) + 2(2) = 11$$

$$\begin{array}{lll} C_{11} = 7 & C_{21} = 2 & C_{31} = -6 \\ C_{12} = -2 & C_{22} = 1 & C_{32} = -3 \\ C_{13} = -4 & C_{23} = 2 & C_{33} = 5 \end{array}$$

$$A^{-1} = \frac{1}{|A|} \operatorname{adj} A = \frac{1}{11} \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$$

Now, $x - 2y = 10$

$2x + y + 3z = 8$

$-2y + z = 7$

or $$\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

or $$X = A^{-1} \times B$$

$$= \frac{1}{11} \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$= \frac{1}{11} \begin{bmatrix} 70 + 16 - 42 \\ -20 + 8 - 21 \\ -40 + 16 + 35 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 44 \\ -33 \\ 11 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$$

Hence, $x = 4, y = -3, z = 1$

Solution of Simultaneous Linear Equations Ex 8.1 Q8(ii)

$$A=\begin{bmatrix}3 & -4 & 2\\ 2 & 3 & 5\\ 1 & 0 & 1\end{bmatrix}$$

$$|A|=3(3)+4(-3)+2(-3)=-9$$

$$\begin{array}{lll} C_{11}=3 & C_{21}=4 & C_{31}=-26\\ C_{12}=3 & C_{22}=1 & C_{32}=-11\\ C_{13}=-3 & C_{23}=-4 & C_{33}=17\end{array}$$

$$A^{-1}=\frac{1}{|A|}\text{adj}A=\frac{1}{-9}\begin{bmatrix}3 & 4 & -26\\ 3 & 1 & -11\\ -3 & -4 & 17\end{bmatrix}$$

Now,

$$\begin{aligned}3x-4y+2z&=-1\\ 2x+3y+5z&=7\\ x+z&=2\end{aligned}$$

Or $$\begin{bmatrix}3 & -4 & 2\\ 2 & 3 & 5\\ 1 & 0 & 1\end{bmatrix}=\begin{bmatrix}-1\\ 7\\ 2\end{bmatrix}$$

$$X=A^{-1}\times B$$

Or $$=\frac{1}{-9}\begin{bmatrix}3 & 4 & -26\\ 3 & 1 & -11\\ -3 & -4 & 17\end{bmatrix}\begin{bmatrix}-1\\ 7\\ 2\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{-9}\begin{bmatrix}-27\\ -18\\ 9\end{bmatrix}$$

Hence $x=3, y=2, z=-1$

Solution of Simultaneous Linear Equations Ex 8.1 Q8(iii)

$$A=\begin{bmatrix}1 & -2 & 0\\ 2 & 1 & 3\\ 0 & -2 & 1\end{bmatrix} \text{ and } B=\begin{bmatrix}7 & 2 & -6\\ -2 & 1 & -3\\ -4 & 2 & 5\end{bmatrix}$$

$$A\times B=\begin{bmatrix}11 & 0 & 0\\ 0 & 11 & 0\\ 0 & 0 & 11\end{bmatrix}$$

$AB=11I$, where I is a 3×3 unit matrix

$$A^{-1}=\frac{1}{11}B \qquad \text{[By def. of inverse]}$$

Or
$$=\frac{1}{11}\begin{bmatrix}7 & 2 & -6\\ -2 & 1 & -3\\ -4 & 2 & 5\end{bmatrix}$$

Now, the given system of equations can be written as

$$\begin{bmatrix}1 & -2 & 0\\ 2 & 1 & 3\\ 0 & -2 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}10\\ 8\\ 7\end{bmatrix}$$

Or $AX=B$

$$X=A^{-1}B$$

Or
$$=\frac{1}{11}\begin{bmatrix}7 & 2 & -6\\ -2 & 1 & -3\\ -4 & 2 & 5\end{bmatrix}\begin{bmatrix}10\\ 8\\ 7\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\frac{1}{11}\begin{bmatrix}44\\ -33\\ 11\end{bmatrix}=\begin{bmatrix}4\\ -3\\ 1\end{bmatrix}$$

Hence, $x=4, y=-3, z=1$

*********** END **********