



Trigonometric Ratios Ex 5.1 Q28

**Answer :**

Given:

$$\sin \theta = \frac{a}{b} \dots\dots (1)$$

To find:  $\sec \theta + \tan \theta$

Now we know,  $\sin \theta$  is defined as follows

$$\sin \theta = \frac{\text{Perpendicular side opposite to } \angle \theta}{\text{Hypotenuse}} \dots\dots (2)$$

Now by comparing (1) and (2)

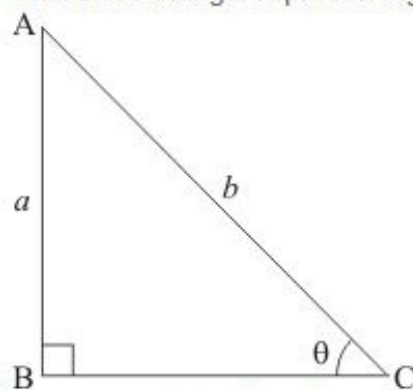
We get,

Perpendicular side opposite to  $\angle \theta = a$

and

Hypotenuse =  $b$

Therefore triangle representing angle  $\theta$  is as shown below



Here side  $BC$  is unknown

Now we find side  $BC$  by applying Pythagoras theorem to right angled  $\triangle ABC$

Therefore,

$$AC^2 = AB^2 + BC^2$$

Now by substituting the value of sides  $AB$  and  $AC$  from figure (a)

We get,

$$b^2 = a^2 + BC^2$$

Therefore,

$$BC^2 = b^2 - a^2$$

Now by taking square root on both sides

We get,

$$BC = \sqrt{b^2 - a^2}$$

Therefore,

$$\text{Base side } BC = \sqrt{b^2 - a^2} \dots\dots (3)$$

Now we know,  $\cos \theta$  is defined as follows

$$\cos \theta = \frac{\text{Base side adjacent to } \angle \theta}{\text{Hypotenuse}}$$

Therefore from figure (a) and equation (3)

We get,

$$\begin{aligned} \cos \theta &= \frac{BC}{AC} \\ &= \frac{\sqrt{b^2 - a^2}}{b} \end{aligned}$$

$$\cos \theta = \frac{\sqrt{b^2 - a^2}}{b} \dots\dots (4)$$

$$\text{Now we know, } \sec \theta = \frac{1}{\cos \theta}$$

Therefore,

$$\sec \theta = \frac{1}{\frac{\sqrt{b^2 - a^2}}{b}}$$

Therefore,

$$\sec \theta = \frac{b}{\sqrt{b^2 - a^2}} \dots\dots (5)$$

$$\text{Now we know, } \tan \theta = \frac{\sin \theta}{\cos \theta}$$

Now by substituting the values from equation (1) and (3)

We get,

$$\begin{aligned} \tan \theta &= \frac{\frac{a}{b}}{\frac{\sqrt{b^2 - a^2}}{b}} \\ &= \frac{a}{b} \times \frac{b}{\sqrt{b^2 - a^2}} \\ &= \frac{a}{\sqrt{b^2 - a^2}} \end{aligned}$$

Therefore,

$$\tan \theta = \frac{a}{\sqrt{b^2 - a^2}} \dots\dots (6)$$

Now we need to find  $\sec \theta + \tan \theta$

Now by substituting the value of  $\sec \theta$  and  $\tan \theta$  from equation (5) and (6) respectively

We get,

$$\sec \theta + \tan \theta = \frac{b}{\sqrt{b^2 - a^2}} + \frac{a}{\sqrt{b^2 - a^2}}$$

$$\sec \theta + \tan \theta = \frac{b + a}{\sqrt{b^2 - a^2}} \dots\dots (7)$$

Now we have the following formula which says

$$x^2 - y^2 = (x + y) \times (x - y)$$

Therefore by applying above formula in equation (7)

We get,

$$\begin{aligned} \sec \theta + \tan \theta &= \frac{b + a}{\sqrt{(b + a) \times (b - a)}} \\ &= \frac{b + a}{\sqrt{(b + a)} \times \sqrt{(b - a)}} \end{aligned}$$

Now by substituting  $(b + a) = \sqrt{(b + a)} \times \sqrt{(b + a)}$  in above expression

We get,

$$\sec \theta + \tan \theta = \frac{\sqrt{(b + a)} \times \sqrt{(b + a)}}{\sqrt{(b + a)} \times \sqrt{(b - a)}}$$

Now  $\sqrt{(b + a)}$  present in the numerator as well as denominator of above expression gets cancels

and we get,

$$\sec \theta + \tan \theta = \frac{\sqrt{(b + a)}}{\sqrt{(b - a)}}$$

Square root is present in the numerator as well as denominator of above expression

Therefore we can place both numerator as well as denominator under a common square root sign

$$\text{Therefore, } \sec \theta + \tan \theta = \sqrt{\frac{b + a}{b - a}}$$

\*\*\*\*\* END \*\*\*\*\*