Exponents of Real Numbers Ex 2.1 Q3

Answer :

(i) We have to prove that $\frac{\sqrt{3\times5^{-3}}}{\sqrt[3]{3^{-1}}\sqrt{5}}\times\sqrt[6]{3\times5^{6}}=\frac{3}{5}$

By using rational exponent $a^{-n}=\frac{1}{a^{n}}$ we get,

$$\frac{\sqrt{3\times5^{-3}}}{\sqrt[3]{3^{-1}}\sqrt{5}}\times\sqrt[6]{3\times5^{6}}=\frac{\sqrt{3\times\frac{1}{5^{3}}}}{\sqrt[3]{\frac{1}{3}}\sqrt{5}}\times\sqrt[6]{3\times5^{6}}$$

$$\Rightarrow\frac{\sqrt{3\times5^{-3}}}{\sqrt[3]{3^{-1}}\sqrt{5}}\times\sqrt[6]{3\times5^{6}}=\frac{3^{\frac{1}{2}}\times\frac{1}{5^{3\times\frac{1}{2}}}}{\frac{1}{3^{\frac{1}{3}}}\times5^{\frac{1}{2}}}\times3^{\frac{1}{6}}\times5^{6\times\frac{1}{6}}$$

$$\frac{\sqrt{3\times5^{-3}}}{\sqrt[3]{3^{-1}}\sqrt{5}}\times\sqrt[6]{3\times5^{6}}=\frac{3^{\frac{1}{2}}\times\frac{1}{5^{\frac{3}{2}}}}{\frac{1}{3^{\frac{1}{3}}}\times5^{\frac{1}{2}}}\times3^{\frac{1}{6}}\times5^{\cancel{6}\times\frac{1}{\cancel{6}}}$$

$$= \frac{\dfrac{3^{\frac{1}{2}}}{5^{\frac{3}{2}}}}{\dfrac{5^{\frac{1}{2}}}{3^{\frac{1}{3}}}} \times 3^{\frac{1}{6}} \times 5^{1}$$

$$= \frac{3^{\frac{1}{2}}}{5^{\frac{3}{2}}} \times \frac{3^{\frac{1}{3}}}{5^{\frac{1}{2}}} \times 3^{\frac{1}{6}} \times 5^{1}$$

$$\frac{\sqrt{3 \times 5^{-3}}}{\sqrt[3]{3^{-1}\sqrt{5}}} \times \sqrt[6]{3 \times 5^{6}} = 3^{\frac{1}{2}} \times 3^{\frac{1}{3}} \times 5^{-\frac{3}{2}} \times 5^{-\frac{1}{2}} \times 3^{\frac{1}{6}} \times 5^{1}$$

$$= 3^{\frac{1}{2}+\frac{1}{3}+\frac{1}{6}} \times 5^{-\frac{3}{2}-\frac{1}{2}+1}$$

$$\frac{\sqrt{3 \times 5^{-3}}}{\sqrt[3]{3^{-1}\sqrt{5}}} \times \sqrt[6]{3 \times 5^{6}} = 3^{\frac{1\times 3}{2\times 3}+\frac{1\times 2}{3\times 2}+\frac{1}{6}} \times 5^{-\frac{3}{2}-\frac{1}{2}+\frac{1\times 2}{1\times 2}}$$

$$= 3^{\frac{3+2+1}{6}} \times 5^{\frac{-3-1+2}{2}}$$

$$\Rightarrow \frac{\sqrt{3 \times 5^{-3}}}{\sqrt[3]{3^{-1}\sqrt{5}}} \times \sqrt[6]{3 \times 5^{6}}\, 3^{\frac{\cancel{6}}{\cancel{6}}} \times 5^{-\frac{\cancel{2}}{\cancel{2}}} = 3^{1} \times 5^{-1} = \frac{3}{5}$$

Hence, $\boxed{\dfrac{\sqrt{3 \times 5^{-3}}}{\sqrt[3]{3^{-1}\sqrt{5}}} \times \sqrt[6]{3 \times 5^{6}} = \dfrac{3}{5}}$

(ii) We have to prove that $9^{\frac{3}{2}} - 3\times 5^0 - \left(\frac{1}{81}\right)^{-\frac{1}{2}} = 15$. So,

$$9^{\frac{3}{2}} - 3\times 5^0 - \left(\frac{1}{81}\right)^{-\frac{1}{2}} = 3^{2\times\frac{3}{2}} - 3\times 5^0 - \frac{1}{81^{-\frac{1}{2}}}$$

$$= 3^{\cancel{2}\times\frac{3}{\cancel{2}}} - 3\times 1 - \frac{1}{\frac{1}{\sqrt{81}}}$$

$$9^{\frac{3}{2}} - 3\times 5^0 - \left(\frac{1}{81}\right)^{-\frac{1}{2}} = 3^3 - 3 - \frac{1}{\frac{1}{\sqrt[2]{9\times 9}}}$$

$$= 27 - 3 - \frac{1}{\frac{1}{9}}$$

$$= 27 - 3 - 1\times\frac{9}{1}$$

$$= 27 - 12 = 15$$

Hence, $\boxed{9^{\frac{3}{2}} - 3\times 5^0 - \left(\frac{1}{81}\right)^{-\frac{1}{2}} = 15}$

*********** END **********