Higher Order Derivatives Ex 12.1 Q31

$y = ae^{2x} + be^{-x}$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = 2ae^{2x} + be^{-x}(-1) = 2ae^{2x} - be^{-x}$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 2ae^{2x}(2) - be^{-x}(-1) = 4ae^{2x} + be^{-x}$$

Adding and subtracting be^{-x} on RHS

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 4ae^{2x} + 2be^{-x} - be^{-x} = 2\left(ae^{2x} + be^{-x}\right) + 2ae^{2x} - be^{-x} = 2y + \frac{dy}{dx}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$$

Higher Order Derivatives Ex 12.1 Q32

$$y = e^x\left(\sin x + \cos x\right)$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = e^x\left(\cos x - \sin x\right) + \left(\sin x + \cos x\right)e^x$$

$$\Rightarrow \quad \frac{dy}{dx} = y + e^x\left(\cos x - \sin x\right)$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{dy}{dx} + e^x\left(-\sin x - \cos x\right) + \left(\cos x - \sin x\right)e^x$$

$$= \frac{dy}{dx} - y + \left(\cos x - \sin x\right)e^x$$

Adding and subtracting y on RHS

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{dy}{dx} - y + \left(\cos x - \sin x\right)e^x + y - y = 2\frac{dy}{dx} - 2y$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q33

It is given that, $y = \cos^{-1} x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\cos^{-1} x\right) = \frac{-1}{\sqrt{1-x^2}} = -\left(1-x^2\right)^{\frac{-1}{2}}$$

$$\frac{d^2 y}{dx^2} = \frac{d}{dx}\left[-\left(1-x^2\right)^{\frac{-1}{2}}\right]$$

$$= -\left(-\frac{1}{2}\right)\cdot\left(1-x^2\right)^{\frac{-3}{2}}\cdot\frac{d}{dx}\left(1-x^2\right)$$

$$= \frac{1}{2\sqrt{\left(1-x^2\right)^3}}\times(-2x)$$

$$\Rightarrow \frac{d^2 y}{dx^2} = \frac{-x}{\sqrt{\left(1-x^2\right)^3}} \qquad \ldots(\text{i})$$

$y = \cos^{-1} x \Rightarrow x = \cos y$

Putting $x = \cos y$ in equation (i), we obtain

$$\frac{d^2 y}{dx^2} = \frac{-\cos y}{\sqrt{\left(1-\cos^2 y\right)^3}}$$

$$\Rightarrow \frac{d^2 y}{dx^2} = \frac{-\cos y}{\sqrt{\left(\sin^2 y\right)^3}}$$

$$= \frac{-\cos y}{\sin^3 y}$$

$$= \frac{-\cos y}{\sin y}\times\frac{1}{\sin^2 y}$$

$$\Rightarrow \frac{d^2 y}{dx^2} = -\cot y\cdot\operatorname{cosec}^2 y$$

Higher Order Derivatives Ex 12.1 Q34

*********** END **********