Indefinite Integrals Ex 19.21 Q15

Let $I = \int \frac{2x+1}{\sqrt{x^2+4x+3}}dx$

Let $2x+1 = \lambda \frac{d}{dx}\left(x^2+4x+3\right) + \mu$

$= \lambda\left(2x+4\right) + \mu$

$2x+1 = \left(2\lambda\right)x + 4\lambda + \mu$

Comparing the coefficients of like powers of x,

$2\lambda = 2 \quad \Rightarrow \quad \lambda = 1$

$4\lambda + \mu = 1 \quad \Rightarrow \quad 4\left(1\right) + \mu = 1$

$\Rightarrow \quad \mu = -3$

so, $I = \int \frac{\left(2x+4\right)-3}{\sqrt{x^2+4x+3}}dx$

$= \int \frac{\left(2x+4\right)}{\sqrt{x^2+4x+3}}dx - 3\int \frac{1}{\sqrt{x^2+2x\left(2\right)+\left(2\right)^2-\left(2\right)^2+3}}dx$

$= \int \frac{\left(2x+4\right)}{\sqrt{x^2+4x+3}}dx - 3\int \frac{1}{\sqrt{\left(x+2\right)^2-1}}dx$

$I = 2\sqrt{x^2+4x+3} - 3\log\left|x+2+\sqrt{\left(x+2\right)^2-1}\right| + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right| + c\right]$

$I = 2\sqrt{x^2+4x+3} - 3\log\left|x+2+\sqrt{x^2+4x+3}\right| + c$

Indefinite Integrals Ex 19.21 Q16

Let $I = \int \frac{2x+3}{\sqrt{x^2+4x+5}}dx$

Let $2x+3 = \lambda \frac{d}{dx}\left(x^2+4x+5\right) + \mu$

$= \lambda\left(2x+4\right) + \mu$

$2x+3 = \left(2\lambda\right)x + 4\lambda + \mu$

Comparing the coefficients of like powers of x,

$2\lambda = 2 \quad \Rightarrow \quad \lambda = 1$

$4\lambda + \mu = 3 \quad \Rightarrow \quad 4\left(1\right) + \mu = 3$

$\Rightarrow \quad \mu = -1$

so, $I = \int \frac{\left(2x+4\right)-1}{\sqrt{x^2+4x+5}}dx$

$= \int \frac{\left(2x+4\right)}{\sqrt{x^2+4x+5}}dx - \int \frac{1}{\sqrt{x^2+2x\left(2\right)+\left(2\right)^2-\left(2\right)^2+5}}dx$

$= \int \frac{\left(2x+4\right)}{\sqrt{x^2+4x+5}}dx - \int \frac{1}{\sqrt{\left(x+2\right)^2+\left(1\right)^2}}dx$

$I = 2\sqrt{x^2+4x+5} - \log\left|x+2+\sqrt{\left(x+2\right)^2+1}\right| + c \qquad \left[\text{since, } \int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}+c,\ \int \frac{1}{\sqrt{x^2+a^2}}dx = \log\left|x+\sqrt{x^2+a^2}\right| + c\right]$

$I = 2\sqrt{x^2+4x+5} - \log\left|x+2+\sqrt{x^2+4x+5}\right| + c$

Indefinite Integrals Ex 19.21 Q17

$$\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx$$

$\rightarrow$ *let* $5x+3 = \lambda(2x+4) + \mu$

$\lambda = \frac{5}{2}, \mu = -7$

$$\int \frac{\lambda(2x+4)+\mu}{\sqrt{x^2+4x+10}}dx = \int \frac{\frac{5}{2}(2x+4)-7}{\sqrt{x^2+4x+10}}dx$$

$$= \int \frac{\frac{5}{2}(2x+4)}{\sqrt{x^2+4x+10}}dx - \int \frac{7}{\sqrt{x^2+4x+10}}dx$$

$$= \int \frac{\frac{5}{2}dt}{\sqrt{t}} - \int \frac{7}{\sqrt{(x+2)^2+6}}dx$$

$$= 5\sqrt{x^2+4x+10} - 7\log\left|(x+2)+\sqrt{x^2+4x+10}\right| + C$$

Indefinite Integrals Ex 19.21 Q18

Let I$=\int \frac{x+2}{\sqrt{x^2+2x+3}}$

$x+2 = A\frac{d}{dx}\left[x^2+2x+3\right]+B$

$\Rightarrow x+2 = 2Ax+2A+B$

Comparing the coefficients, we have,

2A=1 and $2A+B=2$

$\Rightarrow A=\frac{1}{2}$

Substituting the value of A in $2A+B=2$, we have,

$2\times\frac{1}{2}+B=2$

$\Rightarrow 1+B=2$

$\Rightarrow B=2-1$

$\Rightarrow B=1$

Thus we have,

$x+2=\frac{1}{2}\left[2x+2\right]+1$

Hence,

$$I=\int \frac{x+2}{\sqrt{x^2+2x+3}}dx$$

$$=\int \frac{\left[\frac{1}{2}\left[2x+2\right]+1\right]}{\sqrt{x^2+2x+3}}dx$$

$$=\int \frac{\left[\frac{1}{2}\left[2x+2\right]\right]}{\sqrt{x^2+2x+3}}dx+\int \frac{dx}{\sqrt{x^2+2x+3}}$$

$$=\frac{1}{2}\int \frac{\left[2x+2\right]}{\sqrt{x^2+2x+3}}dx+\int \frac{dx}{\sqrt{x^2+2x+3}}$$

Substituting t=x^2+2x+3 and dt=2x+2

in the first integrand, we have,

$$I=\frac{1}{2}\int \frac{dt}{\sqrt{t}}+\int \frac{dx}{\sqrt{x^2+2x+3}}$$

$$=\frac{1}{2}\times 2\sqrt{t}+\int \frac{dx}{\sqrt{x^2+2x+1+2}}+C$$

$$=\sqrt{t}+\int \frac{dx}{\sqrt{\left(x+1\right)^2+\left(\sqrt{2}\right)^2}}+C$$

$$I=\sqrt{x^2+2x+3}+\log\left[\left|x+1\right|+\sqrt{\left(x+1\right)^2+\left(\sqrt{2}\right)^2}\right]+C$$

$$\Rightarrow I=\sqrt{x^2+2x+3}+\log\left[\left|x+1\right|+\sqrt{x^2+2x+3}\right]+C$$

*********** END **********