Q1 : $\dfrac{2x}{1+x^2}$

Answer :

Let $1+x^2 = t$

$\therefore 2x \, dx = dt$

$\Rightarrow \displaystyle\int \dfrac{2x}{1+x^2}\,dx = \int \dfrac{1}{t}\,dt$

$= \log|t| + C$

$= \log\left|1+x^2\right| + C$

$= \log\left(1+x^2\right) + C$

Q2 : $\dfrac{\left(\log x\right)^2}{x}$

Answer :

Let $\log|x| = t$

$\therefore \dfrac{1}{x}\,dx = dt$

$\Rightarrow \displaystyle\int \dfrac{\left(\log|x|\right)^2}{x}\,dx = \int t^2 \, dt$

$\qquad\qquad = \dfrac{t^3}{3} + C$

$\qquad\qquad = \dfrac{\left(\log|x|\right)^3}{3} + C$

Q3 : $\dfrac{1}{x + x\log x}$

Answer :

$\dfrac{1}{x + x\log x} = \dfrac{1}{x\left(1 + \log x\right)}$

Let $1 + \log x = t$

$\therefore \dfrac{1}{x}\,dx = dt$

$\Rightarrow \displaystyle\int \dfrac{1}{x\left(1+\log x\right)}\,dx = \int \dfrac{1}{t}\,dt$

$= \log|t| + C$

$= \log|1 + \log x| + C$

Q4 : $\sin x \cdots \sin(\cos x)$

Answer :

$\sin x \cdots \sin(\cos x)$

Let $\cos x = t$

$\therefore -\sin x \, dx = dt$

$\Rightarrow \displaystyle\int \sin x \cdot \sin(\cos x)\,dx = -\int \sin t \, dt$

$\qquad\qquad = -\left[-\cos t\right] + C$

$\qquad\qquad = \cos t + C$

$\qquad\qquad = \cos(\cos x) + C$

Q5 : $\sin\left(ax + b\right)\cos\left(ax + b\right)$

Answer :

$$\sin(ax+b)\cos(ax+b) = \frac{2\sin(ax+b)\cos(ax+b)}{2} = \frac{\sin 2(ax+b)}{2}$$

Let $2(ax+b) = t$

$\therefore 2adx = dt$

$$\Rightarrow \int \frac{\sin 2(ax+b)}{2}dx = \frac{1}{2}\int \frac{\sin t \; dt}{2a}$$

$$= \frac{1}{4a}[-\cos t] + C$$

$$= \frac{-1}{4a}\cos 2(ax+b) + C$$

Q6 : $\sqrt{ax+b}$

Answer :

Let $ax + b = t$

$\Rightarrow adx = dt$

$\therefore dx = \dfrac{1}{a}dt$

$$\Rightarrow \int (ax+b)^{\frac{1}{2}} dx = \frac{1}{a}\int t^{\frac{1}{2}} dt$$

$$= \frac{1}{a}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$

$$= \frac{2}{3a}(ax+b)^{\frac{3}{2}} + C$$

Q7 : $x\sqrt{x+2}$

Answer :

Let $(x+2) = t$

$\therefore dx = dt$

$$\Rightarrow \int x\sqrt{x+2}\,dx = \int (t-2)\sqrt{t}\,dt$$

$$= \int \left(t^{\frac{3}{2}} - 2t^{\frac{1}{2}}\right) dt$$

$$= \int t^{\frac{3}{2}} dt - 2\int t^{\frac{1}{2}} dt$$

$$= \frac{t^{\frac{5}{2}}}{\frac{5}{2}} - 2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$

$$= \frac{2}{5}t^{\frac{5}{2}} - \frac{4}{3}t^{\frac{3}{2}} + C$$

$$= \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C$$

Q8 : $x\sqrt{1+2x^2}$

Answer :

Let $1 + 2x^2 = t$

$\therefore 4xdx = dt$

$$\Rightarrow \int x\sqrt{1+2x^2}\,dx = \int \frac{\sqrt{t}\,dt}{4}$$

$$= \frac{1}{4}\int t^{\frac{1}{2}} dt$$

$$= \frac{1}{4}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$

$$= \frac{1}{6}(1+2x^2)^{\frac{3}{2}} + C$$

Q9 : $(4x+2)\sqrt{x^2+x+1}$

Answer :

Let $x^2 + x + 1 = t$

$\therefore (2x+1)dx = dt$

$$\int (4x+2)\sqrt{x^2+x+1}\ dx$$

$$= \int 2\sqrt{t}\ dt$$

$$= 2\int \sqrt{t}\ dt$$

$$= 2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$$

$$= \frac{4}{3}(x^2+x+1)^{\frac{3}{2}}+C$$

Q10 : $\dfrac{1}{x-\sqrt{x}}$

Answer :

$$\frac{1}{x-\sqrt{x}} = \frac{1}{\sqrt{x}\left(\sqrt{x}-1\right)}$$

Let $\left(\sqrt{x}-1\right)=t$

$$\therefore \frac{1}{2\sqrt{x}}dx = dt$$

$$\Rightarrow \int \frac{1}{\sqrt{x}\left(\sqrt{x}-1\right)}dx = \int \frac{2}{t}dt$$

$$= 2\log|t|+C$$

$$= 2\log\left|\sqrt{x}-1\right|+C$$

Q11 : $\dfrac{x}{\sqrt{x+4}}, x>0$

Answer :

Let $x+4=t$

$\therefore dx = dt$

********** END **********