Algebraic Identities Ex 4.3 Q16

Answer :

In the given problem, we have to find the value of $x^3+\frac{1}{x^3}, x^2+\frac{1}{x^2}, x+\frac{1}{x}$

Given $x^4+\frac{1}{x^4}=194$

By adding and subtracting $2\times x^2\times\frac{1}{x^2}$ in left hand side of $x^4+\frac{1}{x^4}=194$ we get,

$$x^4+\frac{1}{x^4}+2\times x^2\times\frac{1}{x^2}-2\times x^2\times\frac{1}{x^2}=194$$

$$x^4+\frac{1}{x^4}+2\times x^2\times\frac{1}{x^2}-2\times\left(\cancel{x^2}\times\frac{1}{\cancel{x^2}}\right)=194$$

$$\left(x^2+\frac{1}{x^2}\right)^2-2=194$$

$$\left(x^2+\frac{1}{x^2}\right)^2=194+2$$

$$\left(x^2+\frac{1}{x^2}\right)^2=196$$

$$\left(x^2+\frac{1}{x^2}\right)^2=(14)^2$$

$$\left(x^2+\frac{1}{x^2}\right)=14$$

Again by adding and subtracting $2\times x\ \times\frac{1}{x}$ in left hand side of $\left(x^2+\frac{1}{x^2}\right)=14$ we get,

$$x^2+\frac{1}{x^2}+2\times x\times\frac{1}{x}-2\times x\times\frac{1}{x}=14$$

$$\left(x+\frac{1}{x}\right)^2-2\times\cancel{x}\times\frac{1}{\cancel{x}}=14$$

$$\left(x+\frac{1}{x}\right)^2-2=14$$

$$\left(x+\frac{1}{x}\right)^2=14+2$$

$$\left(x+\frac{1}{x}\right)^2=16$$

$$\left(x+\frac{1}{x}\right)^2=4\times4$$

$$\left(x+\frac{1}{x}\right)=4$$

Now cubing on both sides of $\left(x+\frac{1}{x}\right)=4$ we get

$$\left(x+\frac{1}{x}\right)^3=4^3$$

we shall use identity $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$$x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 4 \times 4 \times 4$$

$$x^3 + \frac{1}{x^3} + 3 \times \cancel{x} \times \frac{1}{\cancel{x}} \times 4 = 64$$

$$x^3 + \frac{1}{x^3} + 12 = 64$$

$$x^3 + \frac{1}{x^3} = 64 - 12$$

$$x^3 + \frac{1}{x^3} = 52$$

Hence the value of $x^3 + \frac{1}{x^3}, x^2 + \frac{1}{x^2}, x + \frac{1}{x}$ is $\boxed{52, 14, 4}$ respectively.

Algebraic Identities Ex 4.3 Q17

Answer :

In the given problem, we have to find the value of $27x^3 + 8y^3$

(i) Given $3x + 2y = 14, xy = 8$

On cubing both sides we get,

$$(3x + 2y)^3 = (14)^3$$

We shall use identity $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$$27x^3 + 8y^3 + 3(3x)(2y)(3x + 2y) = 14 \times 14 \times 14$$

$$27x^3 + 8y^3 + 18(xy)(3x + 2y) = 14 \times 14 \times 14$$

$$27x^3 + 8y^3 + 18(8)(14) = 2744$$

$$27x^3 + 8y^3 + 2016 = 2744$$

$$27x^3 + 8y^3 = 2744 - 2016$$

$$27x^3 + 8y^3 = 728$$

Hence the value of $27x^3 + 8y^3$ is $\boxed{728}$

(ii) Given $3x + 2y = 20, xy = \frac{14}{9}$

On cubing both sides we get,

$$(3x + 2y)^3 = (20)^3$$

We shall use identity $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$$27x^3 + 8y^3 + 3(3x)(2y)(3x + 2y) = 20 \times 20 \times 20$$

$$27x^3 + 8y^3 + 18(xy)(3x + 2y) = 8000$$

$$27x^3 + 8y^3 + 18\left(\frac{14}{9}\right)(20) = 8000$$

$$27x^3 + 8y^3 + 560 = 8000$$

$$27x^3 + 8y^3 = 8000 - 560$$

$$27x^3 + 8y^3 = 7440$$

Hence the value of $27x^3 + 8y^3$ is $\boxed{7440}$.

*********** END **********