Geometric Progressions Ex 20.3 Q 3

$$\sum_{n=1}^{11}\left(2+3^{n}\right)$$
$$=\left(2+3^{1}\right)+\left(2+3^{2}\right)+\left(2+3^{3}\right)+\ldots+\left(2+3^{11}\right)$$
$$=2\times 11+3^{1}+3^{2}+3^{3}+\ldots+3^{11}$$
$$=22+\frac{3\left(3^{11}-1\right)}{(3-1)}$$
$$=22+\frac{3\left(3^{11}-1\right)}{2}$$
$$=\frac{44+3(177147-1)}{2}$$
$$=\frac{44+3(177146)}{2}$$
$$=265741$$

So,
$$\sum_{n=1}^{11}\left(2+3^{n}\right)=265741$$

$$\sum_{k=1}^{n}\left(2^{k}+3^{k-1}\right)$$

$$=\left(2+3^{0}\right)+\left(2^{2}+3\right)+\left(2^{3}+3^{2}\right)+\ldots+\left(2^{n}+3^{n-1}\right)$$
$$=\left(2+2^{2}+2^{3}+\ldots+2^{n}\right)+\left(3^{0}+3^{1}+3^{2}+\ldots+3^{n-1}\right)$$
$$=S_n+S_m$$

$$S_n \Rightarrow a=2,\ n=n,\ r=\frac{2^2}{2}=2$$
$$s_n=\frac{a\left(r^{n}-1\right)}{r-1}=\frac{2\left(2^{n}-1\right)}{2-1}=2\left(2^{n}-1\right)$$

Also, $S_m=S_{n-1}$

$a=1,\ r=3,\ n=n-1$
$$S_{n-1}=\frac{1\left(3^{n-1}-1\right)}{3-1}=\frac{1}{2}\left(3^{n}-1\right)$$

$$\therefore \quad \sum_{k=1}^{n}\left(2^{k}+3^{k-1}\right)=2\left(2^{n}-1\right)+\frac{1}{2}\left(3^{n}-1\right)$$
$$=\frac{1}{2}\left[2^{n+2}+3^{n}-4-1\right]$$
$$=\frac{1}{2}\left[2^{n+2}+3^{n}-5\right]$$

$$\sum_{n=2}^{10} 4^n$$

$$= 4^2 + 4^3 + 4^4 + \ldots + 4^{10}$$

$$a = 4^2,\ r = \frac{4^3}{4} = 4,\ n = 9$$

$$S_{10} = \frac{a\left(r^9 - 1\right)}{1 - r}$$

$$= \frac{4^2\left(4^9 - 1\right)}{4 - 1}$$

$$= \frac{1}{3}\left[4^{11} - 16\right]$$

$$= \frac{16}{3}\left[4^9 - 1\right]$$

*********** END **********