Trigonometric Functions Ex 5.1 Q16

$$\text{LHS} = \cos\theta\,(\tan\theta + 2)(2\tan\theta + 1)$$

$$= \cos\theta\left(\frac{\sin\theta}{\cos\theta} + 2\right)\left(\frac{2\sin\theta}{\cos\theta} + 1\right)\left(\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right)$$

$$= \cos\frac{(\sin\theta + 2\cos\theta)(2\sin\theta + \cos\theta)}{\cos\theta.\cos\theta}$$

$$= \frac{\left(2\sin^2\theta + \sin\theta\cos\theta + 4\sin\theta\cos\theta + 2\cos^2\theta\right)}{\cos\theta}$$

$$= \frac{2\left(\sin^2\theta + \cos^2\theta\right) + 5\sin\theta\cos\theta}{\cos\theta}$$

$$= \frac{2 + 5\sin\theta\cos\theta}{\cos\theta}\ \left(\because \sin^2\theta + \cos^2\theta\right) = 1$$

$$= \frac{2}{\cos\theta} + \frac{5\sin\theta\cos\theta}{\cos\theta}$$

$$= 2\sec\theta + 5\sin\theta$$

$= \text{RHS}$

Proved

Trigonometric Functions Ex 5.1 Q17

$$\frac{2\sin\theta}{1+\cos\theta+\sin\theta} = x$$

$$\Rightarrow \frac{2\sin\theta(1-\cos\theta+\sin\theta)}{(1+\cos\theta+\sin\theta)(1-\cos\theta+\sin\theta)} = x \quad [\textit{Rationalizing the denominator}]$$

$$\Rightarrow \frac{2\sin\theta(1-\cos\theta+\sin\theta)}{(1+\sin\theta)^2-\cos^2\theta} = x$$

$$\Rightarrow \frac{2\sin\theta - 2\sin\theta\cos\theta + 2\sin^2\theta}{1+\sin^2\theta + 2\sin\theta - \cos^2\theta} = x$$

$$\Rightarrow \frac{2\sin\theta(1+\cos\theta-\sin\theta)}{2\sin^2\theta + 2\sin\theta} = x$$

$$\Rightarrow \frac{2\sin\theta(1+\cos\theta-\sin\theta)}{2\sin\theta(1+\sin\theta)} = x$$

$$\Rightarrow \frac{1+\cos\theta-\sin\theta}{1+\sin\theta} = x\ [\textit{Cancelling the } 2\sin\theta \textit{ in both Numerator and Denominator}]$$

Hence Proved

Trigonometric Functions Ex 5.1 Q18

Now, $\cos\theta = \sqrt{1-\sin^2\theta}$

$$= \sqrt{\frac{1-\left(a^2-b^2\right)^2}{\left(a^2+b^2\right)^2}} \qquad \left[\because \sin\theta = \frac{a^2-b^2}{a^2+b^2}\right]$$

$$= \sqrt{\frac{\left(a^2+b^2\right)^2-\left(a^2-b^2\right)^2}{\left(a^2+b^2\right)^2}}$$

$$= \sqrt{\frac{\left(a^2+b^2+a^2-b^2\right)\left(a^2+b^2-a^2+b^2\right)}{a^2+b^2}} \quad \left(\text{Using } x^2-y^2 = (x-y)(x+y)\right)$$

$$= \sqrt{\frac{2a^2\times 2b^2}{a^2+b^2}}$$

$$= \frac{2ab}{a^2+b^2} \ \ldots\ldots\ldots\ldots (ii)$$

Now $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$$= \frac{\frac{a^2-b^2}{a^2+b^2}}{\frac{2ab}{a^2+b^2}}$$

$$= \frac{a^2-b^2}{2ab}$$

$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{a^2+b^2}{2ab}$ (from (ii))

and $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta} = \dfrac{a^2+b^2}{a^2-b^2}$ (from (i))

Trigonometric Functions Ex 5.1 Q19

$$\sqrt{\frac{a+b}{a-b}}+\sqrt{\frac{a-b}{a+b}}$$

$$= \sqrt{\frac{\frac{a}{b}+1}{\frac{a}{b}-1}}+\sqrt{\frac{\frac{a}{b}-1}{\frac{a}{b}+1}} \quad [\textit{Dividing both Numerator and denominator by } b]$$

$$= \sqrt{\frac{\tan\theta+1}{\tan\theta-1}}+\sqrt{\frac{\tan\theta-1}{\tan\theta+1}}$$

$$= \sqrt{\frac{\frac{\sin\theta}{\cos\theta}+1}{\frac{\sin\theta}{\cos\theta}-1}}+\sqrt{\frac{\frac{\sin\theta}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}+1}}$$

$$= \sqrt{\frac{\frac{\sin\theta+\cos\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\cos\theta}}}+\sqrt{\frac{\frac{\sin\theta-\cos\theta}{\cos\theta}}{\frac{\sin\theta+\cos\theta}{\cos\theta}}}$$

$$= \sqrt{\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}}+\sqrt{\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}}$$

$$= \frac{\sin\theta+\cos\theta+\sin\theta-\cos\theta}{\sqrt{\sin^2\theta-\cos^2\theta}}$$

$$= \frac{2\sin\theta}{\sqrt{\sin^2\theta-\cos^2\theta}}$$

Trigonometric Functions Ex 5.1 Q20

Given = $\tan\theta = \dfrac{a}{b}$

To show: $\dfrac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \dfrac{a^2 - b^2}{a^2 + b^2}$

Since, $\tan\theta = \dfrac{a}{b}$

$\Rightarrow \quad \dfrac{\sin\theta}{\cos\theta} = \dfrac{a}{b}$

$\Rightarrow \quad b\sin\theta = a\cos\theta = \lambda$ (say)

$\Rightarrow \quad \sin\theta = \dfrac{\lambda}{b}$ and $\cos\theta = \dfrac{\lambda}{a}$

how $\quad \dfrac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \dfrac{\dfrac{a.\lambda}{b} - \dfrac{b.\lambda}{a}}{\dfrac{a.\lambda}{b} + \dfrac{b.\lambda}{a}}$

$$= \frac{\lambda\left(\dfrac{a}{b} - \dfrac{b}{a}\right)}{\lambda\left(\dfrac{a}{b} + \dfrac{b}{a}\right)}$$

$$= \frac{\dfrac{a}{b} - \dfrac{b}{a}}{\dfrac{a}{b} + \dfrac{b}{a}}$$

$$= \frac{\dfrac{a^2 - b^2}{ab}}{\dfrac{a^2 + b^2}{ab}}$$

$$= \frac{a^2 - b^2}{a^2 + b^2}$$

Proved

*********** END **********