Mean Value Theorems Ex 15.2 Q8

Here,

$$y = x^3 - 3x$$

y is a polynomial function, so it is continuous differentiable, so
Lagrange's mean value theorem is applicable thus there exists a point c such that,

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow \quad 3c^2 - 3 = \frac{f(2) - f(1)}{2 - 1}$$

$$\Rightarrow \quad 3c^2 - 3 = \frac{2 + 2}{1}$$

$$\Rightarrow \quad 3c^2 = 7$$

$$\Rightarrow \quad c = \pm\sqrt{\frac{7}{3}}$$

$$\Rightarrow \quad y = \mp\frac{2}{3}\sqrt{\frac{7}{3}}$$

So, $(c, y) = \left(\pm\sqrt{\frac{7}{3}}, \mp\frac{2}{3}\sqrt{\frac{7}{3}}\right)$ is the required point.

Mean Value Theorems Ex 15.2 Q9

Here,

$$y = x^3 + 1$$

It is a polynomial function, so it is continuous differentiable.

$\Rightarrow$ Lagrange's mean value theorem is applicable, so there exists a point c such that,

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow \quad 3c^2 = \frac{f(3) - f(1)}{3 - 1}$$

$$\Rightarrow \quad 3c^2 = \frac{28 - 2}{2}$$

$$\Rightarrow \quad c^2 = \frac{13}{3}$$

$$\Rightarrow \quad c = \sqrt{\frac{13}{3}}$$

$$\Rightarrow \quad y = \left(\frac{13}{3}\right)^{\frac{3}{2}} + 1$$

So, $(c, y) = \left(\sqrt{\frac{13}{3}}, \left(\frac{13}{3}\right)^{\frac{3}{2}} + 1\right)$ is the required point.

Mean Value Theorems Ex 15.2 Q10

Trigonometric functions are continuous and differentiable.

Thus, the curve C is continuous between the points $(a,0)$ and $(0,a)$ and is differentiable on $[a,a]$

Therefore, by Lagrange's Mean Value Theorem, there exists a real number $c \in (a,a)$ such that

$$f'(c) = \frac{a-0}{0-a} = -1$$

Now consider the parametric functions of the given function

$x = a\cos^3\theta$

and

$y = a\sin^3\theta$

$$\Rightarrow \frac{dx}{d\theta} = 3a\cos^2\theta(-\sin\theta)$$

and

$$\Rightarrow \frac{dy}{d\theta} = 3a\sin^2\theta(\cos\theta)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3a\sin^2\theta(\cos\theta)}{3a\cos^2\theta(-\sin\theta)}$$

$$\Rightarrow \frac{dy}{dx} = -\tan\theta$$

Slope of the chord joining the points $(a,0)$ and $(0,a)$

=Slope of the tangent at $(c, f(c))$, where c lies on the curve

$$\Rightarrow \frac{a-0}{0-a} = -\tan\theta$$

$$\Rightarrow -1 = -\tan\theta$$

$$\Rightarrow \tan\theta = 1$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

Now substituting $\theta = \frac{\pi}{4}$, in the parametric representations, we have,

$x = a\cos^3\theta, y = a\sin^3\theta$

$$\Rightarrow x = a\cos^3\left(\frac{\pi}{4}\right), y = a\sin^3\left(\frac{\pi}{4}\right)$$

$$\Rightarrow x = \frac{a}{2\sqrt{2}}, y = \frac{a}{2\sqrt{2}}$$

Thus, $P\left(\frac{a}{2\sqrt{2}}, \frac{a}{2\sqrt{2}}\right)$ is a point on C, where the tangent is parallel to the chord joining the points $(a,0)$ and $(0,a)$.

Mean Value Theorems Ex 15.2 Q11

Consider the function as

$$f(x) = \tan x, \qquad \left\{x \in [a,b] \text{ such that } 0 < a < b < \frac{\pi}{2}\right\}$$

We know that $\tan x$ is continuous and differentiable in $\left(0, \frac{\pi}{2}\right)$, so, Lagrange's mean value theorem is applicable on (a,b), so there exists a point c such that,

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow \quad \sec^2 c = \frac{\tan b - \tan a}{b - a} \qquad \text{---(i)}$$

Now,

$$c \in (a,b)$$

$$\Rightarrow \quad a < c < b$$

$$\Rightarrow \quad \sec^2 a < \sec^2 c < \sec^2 b$$

$$\Rightarrow \quad \sec^2 a < \left(\frac{\tan b - \tan a}{b - a}\right) < \sec^2 b$$

Using equation (i),

$$\Rightarrow \quad (b - a)\sec^2 a < (\tan b - \tan a) < (b - a)\sec^2 b$$

*********** END **********