Binomial Theorem Ex 18.2 Q30

It is given that,

$$T_6 = 112, T_7 = 7, T_8 = \frac{1}{4}$$

$\therefore \quad T_6 = {}^nC_{n-5}x^{n-5} \times a^5 = 112$

$T_7 = {}^nC_{n-6}x^{n-6} \times a^6 = 7$

and, $\quad T_8 = {}^nC_{n-7}x^{n-7} \times a^7 = \frac{1}{4}$

Now,

$$\frac{T_7}{T_6} = \frac{{}^nC_{n-6}x^{n-6} \times a^6}{{}^nC_{n-5}x^{n-5} \times a^5} = \frac{7}{112}$$

$$\Rightarrow \quad \frac{{}^nC_{n-6}}{{}^nC_{n-5}} \times \frac{a}{x} = \frac{1}{16}$$

$$\Rightarrow \quad \frac{n-6+1}{n-(n-5)+1} \times \frac{a}{x} = \frac{1}{16} \qquad \left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}\right]$$

$$\Rightarrow \quad \frac{n-5}{6} \times \frac{a}{x} = \frac{1}{16}$$

$$\Rightarrow \quad \frac{a}{x} = \frac{6}{16} \times \frac{1}{n-5}$$

$$\Rightarrow \quad \frac{a}{x} = \frac{3}{8} \times \frac{1}{(n-5)} \qquad \text{---(i)}$$

and,

$$\frac{T_8}{T_7} = \frac{{}^nC_{n-7}x^{n-7} \times a^7}{{}^nC_{n-6}x^{n-6} \times a^6} = \frac{\frac{1}{4}}{7}$$

$$\Rightarrow \quad \frac{T_8}{T_7} = \frac{{}^nC_{n-7}}{{}^nC_{n-6}} \times \frac{a}{x} = \frac{1}{28}$$

$$\Rightarrow \quad \frac{{}^nC_{n-7}}{{}^nC_{n-6}} \times \frac{a}{x} = \frac{1}{28}$$

$$\Rightarrow \quad \frac{n-7+1}{n-(n-6)+1} \times \frac{a}{x} = \frac{1}{28}$$

$$\Rightarrow \quad \frac{n-6}{7} \times \frac{a}{x} = \frac{1}{28}$$

$$\Rightarrow \quad \frac{a}{x} = \frac{1}{4(n-6)} \qquad \text{---(ii)}$$

Comparing equation (i) and (ii), we get

$$\frac{3}{8}\times\frac{1}{(n-5)}=\frac{1}{4(n-6)}$$

$$\Rightarrow \quad \frac{3}{2}\times\frac{1}{(n-5)}=\frac{1}{(n-6)}$$

$$\Rightarrow \quad 3(n-6)=2(n-5)$$

$$\Rightarrow \quad 3n-18=2n-10$$

$$\Rightarrow \quad 3n-2n=18-10$$

$$\Rightarrow \quad n=8$$

Putting $n=8$ in equation (ii), we get

$$\frac{a}{x}=\frac{1}{4(8-6)}$$

$$\Rightarrow \quad \frac{a}{x}=\frac{1}{8}$$

$$\Rightarrow \quad x=8a$$

Now,

$$T6=112$$

$$\Rightarrow \quad {}^{n}C_{n-5}\times x^{n-5}\times a^{5}=112$$

$$\Rightarrow \quad {}^{8}C_{3}\times x^{3}\times a^{5}=112 \qquad [\because n=8]$$

$$\Rightarrow \quad {}^{8}C_{3}\times(8a)^{3}a^{5}=112 \qquad [\because x=8a]$$

$$\Rightarrow \quad \frac{8!}{(8-3)!3!}\times 8^{3}\times a^{8}=112$$

$$\Rightarrow \quad \frac{8!}{5!3!}\times 512\times a^{8}=112$$

$$\Rightarrow \quad \frac{8\times7\times6\times5!}{5!3!}\times 512\times a^{8}=112$$

$$\Rightarrow \quad a^{8}=\frac{112}{56\times512}$$

$$\Rightarrow \quad a^{8}=\frac{2}{512}$$

$$\Rightarrow \quad a^{8}=\frac{1}{256}$$

$$\Rightarrow \quad a^{8}=\left(\frac{1}{2}\right)^{8}$$

$$\Rightarrow \quad a=\frac{1}{2}$$

Putting $a=\frac{1}{2}$ in $x=8a$, we get

$$x=8\times\frac{1}{2}=4$$

Hence, $x=4$, $a=\frac{1}{2}$ and $n=8$.

Binomial Theorem Ex 18.2 Q31

It is given that

$$T_2 = 240$$
$$T_3 = 720$$
$$T_4 = 1080$$

$\therefore \quad T_2 = {}^nC_1 \times x^{n-1} \times a = 240$

$T_3 = {}^nC_2 \times x^{n-2} \times a^2 = 720$

and, $\quad T_4 = {}^nC_3 \times x^{n-3} \times a^3 = 1080$

Now,

$$\frac{T_4}{T_3} = \frac{{}^nC_3 \times x^{n-3} \times a^3}{{}^nC_2 \times x^{n-2} \times a^2} = \frac{1080}{720}$$

$\Rightarrow \quad \dfrac{{}^nC_3 a}{{}^nC_2 x} = \dfrac{3}{2}$

$\Rightarrow \quad \dfrac{n-3+1}{2+1} \times \dfrac{a}{x} = \dfrac{3}{2}$

$\Rightarrow \quad \dfrac{n-2}{3} \times \dfrac{a}{x} = \dfrac{3}{2}$

$\Rightarrow \quad \dfrac{a}{x} = \dfrac{9}{2(n-2)} \qquad$ ---(i)

and,

$$\frac{T_3}{T_2} = \frac{{}^nC_2 \times x^{n-2} \times a^2}{{}^nC_1 \times x^{n-1} \times a} = \frac{720}{240}$$

$\Rightarrow \quad \dfrac{{}^nC_2}{{}^nC_1} \times \dfrac{a}{x} = 3$

$\Rightarrow \quad \dfrac{n-2+1}{2} \times \dfrac{a}{x} = 3$

$\Rightarrow \quad \dfrac{n-1}{2} \times \dfrac{a}{x} = 3$

$\Rightarrow \quad \dfrac{a}{x} = \dfrac{6}{n-1} \qquad$ ---(ii)

Comparing equation (i) and equation (ii), we get

$$\frac{6}{n-1} = \frac{9}{2(n-2)}$$

$\Rightarrow \quad 12(n-2) = 9(n-1)$

$\Rightarrow \quad 12n - 24 = 9n - 9$

$\Rightarrow \quad 3n = 24 - 9$

$\Rightarrow \quad 3n = 15$

$\Rightarrow \quad n = 5$

Putting $n = 5$ in equation (ii), we get

$$\frac{a}{x} = \frac{6}{5-1}$$

$\Rightarrow \quad \dfrac{a}{x} = \dfrac{6}{4}$

$\Rightarrow \quad \dfrac{a}{x} = \dfrac{3}{2}$

$\Rightarrow \quad a = \dfrac{3}{2}x$

Now,

$$T_2 = {}^nC_1 \times x^{n-1} \times a = 240$$

$\Rightarrow \quad {}^5C_1 \times x^4 \times \left(\dfrac{3}{2}x\right) = 240 \qquad \left[\because n = 5 \text{ and } a = \dfrac{3}{2}x\right]$

$\Rightarrow \quad x^5 = \dfrac{240 \times 2}{5 \times 3}$

$\Rightarrow \quad x^5 = 32$

$\Rightarrow \quad x^5 = 2^5$

$\Rightarrow \quad x = 2$

Putting $x = 2$ in $a = \dfrac{3}{2}x$, we get

$$a = \frac{3}{2} \times 2 = 3$$

Hence, $x = 2$, $a = 3$ and $n = 5$.

*********** END **********