



Differentiation Ex 11.5 Q18(iv)

$$\text{Let } y = (x \cos x)^x + (x \sin x)^{\frac{1}{x}}$$

$$\text{Also, let } u = (x \cos x)^x \text{ and } v = (x \sin x)^{\frac{1}{x}}$$

$$\therefore y = u + v$$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \dots(1)$$

$$u = (x \cos x)^x$$

$$\Rightarrow \log u = \log (x \cos x)^x$$

$$\Rightarrow \log u = x \log (x \cos x)$$

$$\Rightarrow \log u = x [\log x + \log \cos x]$$

$$\Rightarrow \log u = x \log x + x \log \cos x$$

Differentiating both sides with respect to x , we obtain

$$\frac{1}{u} \frac{du}{dx} = \frac{d}{dx} (x \log x) + \frac{d}{dx} (x \log \cos x)$$

$$\Rightarrow \frac{du}{dx} = u \left[\left\{ \log x \cdot \frac{d}{dx} (x) + x \cdot \frac{d}{dx} (\log x) \right\} + \left\{ \log \cos x \cdot \frac{d}{dx} (x) + x \cdot \frac{d}{dx} (\log \cos x) \right\} \right]$$

$$\Rightarrow \frac{du}{dx} = (x \cos x)^x \left[\left(\log x \cdot 1 + x \cdot \frac{1}{x} \right) + \left(\log \cos x \cdot 1 + x \cdot \frac{1}{\cos x} \cdot \frac{d}{dx} (\cos x) \right) \right]$$

$$\Rightarrow \frac{du}{dx} = (x \cos x)^x \left[(\log x + 1) + \left\{ \log \cos x + \frac{x}{\cos x} \cdot (-\sin x) \right\} \right]$$

$$\Rightarrow \frac{du}{dx} = (x \cos x)^x \left[(1 + \log x) + (\log \cos x - x \tan x) \right]$$

$$\Rightarrow \frac{du}{dx} = (x \cos x)^x \left[1 - x \tan x + (\log x + \log \cos x) \right]$$

$$\Rightarrow \frac{du}{dx} = (x \cos x)^x \left[1 - x \tan x + \log (x \cos x) \right] \quad \dots(2)$$

$$\begin{aligned}
v &= (x \sin x)^{\frac{1}{x}} \\
\Rightarrow \log v &= \log (x \sin x)^{\frac{1}{x}} \\
\Rightarrow \log v &= \frac{1}{x} \log (x \sin x) \\
\Rightarrow \log v &= \frac{1}{x} (\log x + \log \sin x) \\
\Rightarrow \log v &= \frac{1}{x} \log x + \frac{1}{x} \log \sin x
\end{aligned}$$

Differentiating both sides with respect to x , we obtain

$$\begin{aligned}
\frac{1}{v} \frac{dv}{dx} &= \frac{d}{dx} \left(\frac{1}{x} \log x \right) + \frac{d}{dx} \left[\frac{1}{x} \log (\sin x) \right] \\
\Rightarrow \frac{1}{v} \frac{dv}{dx} &= \left[\log x \cdot \frac{d}{dx} \left(\frac{1}{x} \right) + \frac{1}{x} \cdot \frac{d}{dx} (\log x) \right] + \left[\log (\sin x) \cdot \frac{d}{dx} \left(\frac{1}{x} \right) + \frac{1}{x} \cdot \frac{d}{dx} \{ \log (\sin x) \} \right] \\
\Rightarrow \frac{1}{v} \frac{dv}{dx} &= \left[\log x \cdot \left(-\frac{1}{x^2} \right) + \frac{1}{x} \cdot \frac{1}{x} \right] + \left[\log (\sin x) \cdot \left(-\frac{1}{x^2} \right) + \frac{1}{x} \cdot \frac{1}{\sin x} \cdot \frac{d}{dx} (\sin x) \right] \\
\Rightarrow \frac{1}{v} \frac{dv}{dx} &= \frac{1}{x^2} (1 - \log x) + \left[-\frac{\log (\sin x)}{x^2} + \frac{1}{x \sin x} \cdot \cos x \right] \\
\Rightarrow \frac{dv}{dx} &= (x \sin x)^{\frac{1}{x}} \left[\frac{1 - \log x}{x^2} + \frac{-\log (\sin x) + x \cot x}{x^2} \right] \\
\Rightarrow \frac{dv}{dx} &= (x \sin x)^{\frac{1}{x}} \left[\frac{1 - \log x - \log (\sin x) + x \cot x}{x^2} \right] \\
\Rightarrow \frac{dv}{dx} &= (x \sin x)^{\frac{1}{x}} \left[\frac{1 - \log (x \sin x) + x \cot x}{x^2} \right] \quad \dots (3)
\end{aligned}$$

From (1), (2), and (3), we obtain

$$\frac{dy}{dx} = (x \cos x)^x [1 - x \tan x + \log (x \cos x)] + (x \sin x)^{\frac{1}{x}} \left[\frac{x \cot x + 1 - \log (x \sin x)}{x^2} \right]$$

Differentiation Ex 11.5 Q18(v)

$$\text{Let } y = \left(x + \frac{1}{x}\right)^x + x^{\left(1 + \frac{1}{x}\right)}$$

$$\text{Also, let } u = \left(x + \frac{1}{x}\right)^x \text{ and } v = x^{\left(1 + \frac{1}{x}\right)}$$

$$\therefore y = u + v$$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad \dots(1)$$

$$\text{Then, } u = \left(x + \frac{1}{x}\right)^x$$

$$\Rightarrow \log u = \log \left(x + \frac{1}{x}\right)^x$$

$$\Rightarrow \log u = x \log \left(x + \frac{1}{x}\right)$$

Differentiating both sides with respect to x , we obtain

$$\frac{1}{u} \cdot \frac{du}{dx} = \frac{d}{dx}(x) \times \log \left(x + \frac{1}{x}\right) + x \times \frac{d}{dx} \left[\log \left(x + \frac{1}{x}\right) \right]$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = 1 \times \log \left(x + \frac{1}{x}\right) + x \times \frac{1}{\left(x + \frac{1}{x}\right)} \cdot \frac{d}{dx} \left(x + \frac{1}{x}\right)$$

$$\Rightarrow \frac{du}{dx} = u \left[\log \left(x + \frac{1}{x}\right) + \frac{x}{\left(x + \frac{1}{x}\right)} \times \left(1 - \frac{1}{x^2}\right) \right]$$

$$\Rightarrow \frac{du}{dx} = \left(x + \frac{1}{x}\right)^x \left[\log \left(x + \frac{1}{x}\right) + \frac{\left(x - \frac{1}{x}\right)}{\left(x + \frac{1}{x}\right)} \right]$$

$$\Rightarrow \frac{du}{dx} = \left(x + \frac{1}{x}\right)^x \left[\log \left(x + \frac{1}{x}\right) + \frac{x^2 - 1}{x^2 + 1} \right]$$

$$\Rightarrow \frac{du}{dx} = \left(x + \frac{1}{x}\right)^x \left[\frac{x^2 - 1}{x^2 + 1} + \log \left(x + \frac{1}{x}\right) \right]$$

$$v = x^{\left(1 + \frac{1}{x}\right)}$$

$$\Rightarrow \log v = \log \left[x^{\left(1 + \frac{1}{x}\right)} \right]$$

$$\Rightarrow \log v = \left(1 + \frac{1}{x}\right) \log x$$

Differentiating both sides with respect to x , we obtain

$$\frac{1}{v} \cdot \frac{dv}{dx} = \left[\frac{d}{dx} \left(1 + \frac{1}{x}\right) \right] \times \log x + \left(1 + \frac{1}{x}\right) \cdot \frac{d}{dx} \log x$$

$$\Rightarrow \frac{1}{v} \frac{dv}{dx} = \left(-\frac{1}{x^2}\right) \log x + \left(1 + \frac{1}{x}\right) \cdot \frac{1}{x}$$

$$\Rightarrow \frac{1}{v} \frac{dv}{dx} = -\frac{\log x}{x^2} + \frac{1}{x} + \frac{1}{x^2}$$

$$\Rightarrow \frac{dv}{dx} = v \left[\frac{-\log x + x + 1}{x^2} \right]$$

$$\Rightarrow \frac{dv}{dx} = x^{\left(1 + \frac{1}{x}\right)} \left(\frac{x + 1 - \log x}{x^2} \right) \quad \dots(3)$$

Therefore, from (1), (2), and (3), we obtain

$$\frac{dy}{dx} = \left(x + \frac{1}{x}\right)^x \left[\frac{x^2 - 1}{x^2 + 1} + \log \left(x + \frac{1}{x}\right) \right] + x^{\left(1 + \frac{1}{x}\right)} \left(\frac{x + 1 - \log x}{x^2} \right)$$

Differentiation Ex 11.5 Q18(vi)

$$\text{Let } y = e^{\sin x} + (\tan x)^x$$

$$y = e^{\sin x} + e^{\log(\tan x)^x}$$

$$y = e^{\sin x} + e^{x \log(\tan x)} \quad \left[\text{Since, } \log a^b = b \log a, e^{\log a} = a \right]$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx} (e^{\sin x}) + \frac{d}{dx} (e^{x \log(\tan x)})$$

$$= e^{\sin x} \frac{d}{dx} (\sin x) + e^{x \log(\tan x)} \times \frac{d}{dx} (x \log \tan x)$$

$$= e^{\sin x} (\cos x) + e^{\log(\tan x)^x} \left[x \frac{d}{dx} \log \tan x + \log \tan x \frac{d}{dx} (x) \right]$$

$$= e^{\sin x} (\cos x) + (\tan x)^x \left[\frac{x}{\tan x} \frac{d}{dx} (\tan x) + \log \tan x (1) \right]$$

$$\frac{dy}{dx} = \cos x e^{\sin x} + (\tan x)^x \left[\frac{x}{\tan x} (\sec^2 x) + \log \tan x \right]$$

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