1 u = 931.5 MeV/c^2

Energy of the highest level is given as:

$$E = \left[m\left({}^{198}_{78}\mathrm{Au} \right) - m\left({}^{190}_{80}\mathrm{Hg} \right) \right]$$

$$= 197.968233 - 197.966760 = 0.001473 \text{ u}$$
$$= 0.001473 \times 931.5 = 1.3720995 \text{ MeV}$$

β_1 decays from the 1.3720995 MeV level to the 1.088 MeV level

∴Maximum kinetic energy of the β_1 particle = 1.3720995 − 1.088

= 0.2840995 MeV

β_2 decays from the 1.3720995 MeV level to the 0.412 MeV level

∴Maximum kinetic energy of the β_2 particle = 1.3720995 − 0.412

= 0.9600995 MeV

Question 13.30:

Calculate and compare the energy released by a) fusion of 1.0 kg of hydrogen deep within Sun and b) the fission of 1.0 kg of ^{235}U in a fission reactor.

Answer

(a) Amount of hydrogen, m = 1 kg = 1000 g

1 mole, i.e., 1 g of hydrogen (${}^{1}_{1}\mathrm{H}$) contains 6.023 × 10^{23} atoms.

∴1000 g of ${}^{1}_{1}\mathrm{H}$ contains 6.023 × 10^{23} × 1000 atoms.

Within the sun, four ${}^{1}_{1}\mathrm{H}$ nuclei combine and form one ${}^{4}_{2}\mathrm{He}$ nucleus. In this process 26 MeV of energy is released.

Hence, the energy released from the fusion of 1 kg ${}^{1}_{1}\mathrm{H}$ is:

$$E_1 = \frac{6.023 \times 10^{23} \times 26 \times 10^3}{4}$$
$$= 39.1495 \times 10^{26} \text{ MeV}$$

(b) Amount of ${}^{235}_{92}\mathrm{U}$ = 1 kg = 1000 g

1 mole, i.e., 235 g of ${}^{235}_{92}\mathrm{U}$ contains 6.023 × 10^{23} atoms.

∴1000 g of ${}^{235}_{92}\mathrm{U}$ contains $\frac{6.023 \times 10^{23} \times 1000}{235}$ atoms

It is known that the amount of energy released in the fission of one atom of ${}^{235}_{92}\mathrm{U}$ is 200 MeV.

Hence, energy released from the fission of 1 kg of ${}^{235}_{92}\mathrm{U}$ is:

$$E_2 = \frac{6 \times 10^{23} \times 1000 \times 200}{235}$$
$$= 5.106 \times 10^{26} \text{ MeV}$$

$$\therefore \frac{E_1}{E_1} = \frac{39.1495 \times 10^{26}}{5.106 \times 10^{26}} = 7.67 \approx 8$$

Therefore, the energy released in the fusion of 1 kg of hydrogen is nearly 8 times the energy released in the fission of 1 kg of uranium.

Question 13.31:

Suppose India had a target of producing by 2020 AD, 200,000 MW of electric power, ten percent of which was to be obtained from nuclear power plants. Suppose we are given that, on an average, the efficiency of utilization (i.e. conversion to electric energy) of thermal energy produced in a reactor was 25%. How much amount of fissionable uranium would our country need per year by 2020? Take the heat energy per fission of ^{235}U to be about 200MeV.

Answer

Amount of electric power to be generated, $P = 2 \times 10^5$ MW

10% of this amount has to be obtained from nuclear power plants.

∴Amount of nuclear power, $P_1 = \frac{10}{100} \times 2 \times 10^5$

$= 2 \times 10^4$ MW

$= 2 \times 10^4 \times 10^6$ J/s

$= 2 \times 10^{10} \times 60 \times 60 \times 24 \times 365$ J/y

Heat energy released per fission of a ^{235}U nucleus, E = 200 MeV

Efficiency of a reactor = 25%

Hence, the amount of energy converted into the electrical energy per fission is calculated as:

$$\frac{25}{100} \times 200 = 50 \text{ MeV}$$

$$= 50 \times 1.6 \times 10^{-19} \times 10^6 = 8 \times 10^{-12} \text{ J}$$

Number of atoms required for fission per year:

$$\frac{2 \times 10^{10} \times 60 \times 60 \times 24 \times 365}{8 \times 10^{-12}} = 78840 \times 10^{24} \text{ atoms}$$

1 mole, i.e., 235 g of U^{235} contains 6.023×10^{23} atoms.

∴Mass of 6.023×10^{23} atoms of U^{235} = 235 g = 235×10^{-3} kg

∴Mass of 78840×10^{24} atoms of U^{235}

$$= \frac{235 \times 10^{-3}}{6.023 \times 10^{23}} \times 78840 \times 10^{24}$$

$$= 3.076 \times 10^4 \text{ kg}$$

Hence, the mass of uranium needed per year is 3.076×10^4 kg.

*********** END **********