Trigonometric Ratios of Compound Angles Ex 7.1 Q5

We have,

$$\sin A = \frac{1}{2} \text{ and } \cos B = \frac{12}{13}$$

$$\therefore \quad \cos A = -\sqrt{1-\sin^2 A} \text{ and } \sin B = -\sqrt{1-\cos^2 B}$$

$$\left[\because \text{ cosine is negative in second quadrant and sine is negative in fourth quadrant}\right]$$

$$\Rightarrow \quad \cos A = -\sqrt{1-\left(\frac{1}{2}\right)^2} \text{ and } \sin B = -\sqrt{1-\left(\frac{12}{13}\right)^2}$$

$$\Rightarrow \quad \cos A = -\sqrt{1-\frac{1}{4}} \text{ and } \sin B = -\sqrt{1-\frac{144}{169}}$$

$$\Rightarrow \quad \cos A = -\sqrt{\frac{3}{4}} \text{ and } \sin B = -\sqrt{\frac{25}{169}}$$

$$\Rightarrow \quad \cos A = -\frac{\sqrt{3}}{2} \text{ and } \sin B = -\frac{5}{13}$$

$$\therefore \quad \tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{2}}{\frac{-\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}}$$

$$\text{and,} \quad \tan B = \frac{\sin B}{\cos B} = \frac{-\frac{5}{13}}{\frac{12}{13}} = \frac{-5}{12}$$

$$\text{Now,} \quad \tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$$

$$= \frac{\frac{-1}{\sqrt{3}} - \left(\frac{-5}{12}\right)}{1+\left(\frac{-1}{\sqrt{3}}\right)\times\left(\frac{-5}{12}\right)}$$

$$= \frac{\frac{-1}{\sqrt{3}} + \frac{5}{12}}{1+\frac{5}{12\sqrt{3}}}$$

$$= \frac{\frac{-12+5\sqrt{3}}{12\sqrt{3}}}{\frac{12\sqrt{3}+5}{12\sqrt{3}}}$$

$$= \frac{5\sqrt{3}-12}{5+12\sqrt{3}}$$

$$\Rightarrow \quad \tan(A-B) = \frac{5\sqrt{3}-12}{5+12\sqrt{3}}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q6

We have,

$$\sin A = \frac{1}{2} \text{ and } \cos B = \frac{\sqrt{3}}{2}$$

$$\therefore \quad \cos A = -\sqrt{1-\sin^2 A} \text{ and } \sin B = \sqrt{1-\cos^2 B}$$

[$\because$ cosine is negative in second quadrant]

$$\Rightarrow \quad \cos A = -\sqrt{1-\left(\frac{1}{2}\right)^2} \text{ and } \sin B = \sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow \quad \cos A = -\sqrt{1-\frac{1}{4}} \text{ and } \sin B = \sqrt{1-\frac{3}{4}}$$

$$\Rightarrow \quad \cos A = -\sqrt{\frac{3}{4}} \text{ and } \sin B = \sqrt{\frac{1}{4}}$$

$$\Rightarrow \quad \cos A = -\frac{\sqrt{3}}{2} \text{ and } \sin B = \frac{1}{2}$$

$$\therefore \quad \tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{2}}{\frac{-\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}}$$

$$\text{and,} \quad \tan B = \frac{\sin B}{\cos B} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

Now,

$$\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A \tan B}$$

$$= \frac{\frac{-1}{\sqrt{3}} + \frac{1}{\sqrt{3}}}{1-\left(\frac{-1}{\sqrt{3}}\right)\times\left(\frac{1}{\sqrt{3}}\right)}$$

$$= 0$$

$$\therefore \quad \tan(A+B) = 0$$

We have,

$$\sin A = \frac{1}{2} \text{ and } \cos B = \frac{\sqrt{3}}{2}$$

$$\therefore \quad \cos A = -\sqrt{1-\sin^2 A} \text{ and } \sin B = \sqrt{1-\cos^2 B}$$ [$\because$ cosine is negative in second quadrant]

$$\Rightarrow \quad \cos A = -\sqrt{1-\left(\frac{1}{2}\right)^2} \text{ and } \sin B = \sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow \quad \cos A = -\sqrt{1-\frac{1}{4}} \text{ and } \sin B = \sqrt{1-\frac{3}{4}}$$

$$\Rightarrow \quad \cos A = -\sqrt{\frac{3}{4}} \text{ and } \sin B = \sqrt{\frac{1}{4}}$$

$$\Rightarrow \quad \cos A = -\frac{\sqrt{3}}{2} \text{ and } \sin B = \frac{1}{2}$$

$$\therefore \quad \tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{2}}{\frac{-\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}}$$

$$\text{and,} \quad \tan B = \frac{\sin B}{\cos B} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

$$\text{Now,} \quad \tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$$

$$= \frac{\frac{-1}{\sqrt{3}} - \left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{-1}{\sqrt{3}}\right)\times\frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{-2}{\sqrt{3}}}{1-\frac{1}{3}}$$

$$= \frac{\frac{-2}{\sqrt{3}}}{\frac{3-1}{3}}$$

$$= \frac{\frac{-2}{\sqrt{3}}}{\frac{2}{3}}$$

$$= \frac{-3}{\sqrt{3}}$$

$$= \frac{-\sqrt{3}\times\sqrt{3}}{\sqrt{3}} = -\sqrt{3}$$

$$\therefore \quad \tan(A-B) = -\sqrt{3}$$

Trigonometric Ratios of Compound Angles Ex 7.1 Q7

(i)

$\sin 78° \cos 18° - \cos 78° \sin 18°$ $\quad [\sin(A - B) = \sin A \cos B - \cos A \sin B]$

$= \sin(78° - 18°)$

$= \sin 60°$

$= \frac{\sqrt{3}}{2}$

(ii)

$\cos 47° \cos 13° - \sin 47° \sin 13°$ $\quad [\cos(A + B) = \cos A \cos B - \sin A \sin B]$

$= \cos(47° + 13°)$

$= \cos 60°$

$= \frac{1}{2}$

(iii)

$\sin 36° \cos 9° + \cos 36° \sin 9°$ $\quad [\sin(A + B) = \sin A \cos B + \cos A \sin B]$

$= \sin(36° + 9°)$

$= \sin 45°$

$= \frac{1}{\sqrt{2}}$

(iv)

$\cos 80° \cos 20° + \sin 80° \sin 20°$ $\quad [\cos(A - B) = \cos A \cos B + \sin A \sin B]$

$= \cos(80° - 20°)$

$= \cos 60°$

$= \frac{1}{2}$

*********** END **********