Binary Operations Ex 3.2 Q5

The binary operator o defined on $Q - \{-1\}$ is given by

$$a \circ b = a + b - ab \text{ for all } a, b \in Q - \{-1\}$$

Commutativity:

Let $a, b \in Q - \{-1\}$, then

$$a \circ b = a + b - ab = b + a - ba = b \circ a$$

$$\Rightarrow \qquad a \circ b = b \circ a$$

$$\Rightarrow \qquad \text{'o' is commutative on } Q - \{-1\}.$$

Binary Operations Ex 3.2 Q6

The binary operator $*$ defined on Z and is given by

$$a * b = 3a + 7b$$

Commutativity: Let $a, b \in Z$, then

$$a * b = 1a + 7b \text{ and}$$
$$b * a = 3b + 7a$$

$$\therefore \qquad a * b \neq b * a$$

Hence, '*' is not commutative on Z.

Binary Operations Ex 3.2 Q7

We have, $*$ is a binary operator defined on Z is given by

$$a * b = ab + 1 \text{ for all } a, b \in Z$$

Associativity: Let $a, b, c \in Z$, then

$$(a * b) * c = (ab + 1) * c$$
$$= abc + c + 1 \qquad \qquad - - - (i)$$

and, $a * (b * c) = a * (bc + 1)$
$$= abc + a + 1 \qquad \qquad - - - (ii)$$

From (i) & (ii)

$$\therefore \qquad (a * b) * c \neq a * (b * c)$$

Hence, '*' is not associative on Z.

Binary Operations Ex 3.2 Q8

We have, set of real numbers except -1 and $*$ is an operator given by

$$a * b = a + b + ab \text{ for all } a, b \in S = R - \{-1\}$$

Now, $\forall\, a, b \in S$

$$a * b = a + b + ab \in S$$

$\because$ if $\quad a + b + ab = -1$

$\Rightarrow \quad a + b(1 + a) + 1 = 0$

$\Rightarrow \quad (a + 1)(b + 1) = 0$

$\Rightarrow \quad a = -1 \text{ or } b = -1$

but $a \neq -1$ and $b \neq -1$ (given)

$\therefore \quad a + b + ab \neq -1$

$\Rightarrow \quad a * b \in S \text{ for } ab \in S$

$\Rightarrow \quad$ '$*$' is a binary operator on S

Commutativity: Let $a, b \in S$

$\Rightarrow \quad a * b = a + b + ab = b + a + ba = b * a$

$\Rightarrow \quad a * b = b * a$

and, $\quad a * (b * c) = a * (b + c + bc)$

$$= a + b + c + bc + ab + ac + abc \qquad\qquad ---(ii)$$

From (i) and (ii)

$$(a * b) * c = a * (b * c)$$

$\therefore \quad$ '$*$' is associative on S.

Now, $(2 * x) * 3 = 7$

$\Rightarrow \quad (2 + x + 2x) * 3 = 7$

$\Rightarrow \quad 2 + x + 2x + 3 + 6 + 3x + 6x = 7$

$\Rightarrow \quad 11 + 12x = 7$

$\Rightarrow \quad 12x = -4$

$\Rightarrow \quad x = \dfrac{-4}{12} \qquad \Rightarrow x = \dfrac{-1}{3}$

********** END *********