CHAPTER 25

CALORIMETRY

25.1 HEAT AS A FORM OF ENERGY

When two bodies at different temperatures are placed in contact, the hotter body cools down and the colder body warms up. Energy is thus transferred from a body at higher temperature to a body at lower temperature when they are brought in contact.

The energy being transferred between two bodies or between adjacent parts of a body as a result of temperature difference is called *heat*.

Thus, heat is a form of energy. It is energy in transit whenever temperature differences exist. Once it is transferred it becomes the internal energy of the receiving body. It should be clearly understood that the word "heat" is meaningful only as long as the energy is being transferred. Thus, expressions like "heat in a body" or "heat of a body" are meaningless.

25.2 UNITS OF HEAT

As heat is just energy in transit, its unit in SI is joule. However, another unit of heat "calorie" is in wide use. This unit was formulated much before it was recognised that heat is a form of energy. The old day definition of calorie is as follows:

The amount of heat needed to increase the temperature of 1 g *of water from* 14·5°C to 15·5°C *at a pressure of* 1 atm *is called* 1 calorie.

The amount of heat needed to raise the temperature of 1 g of water by 1°C depends slightly on the actual temperature of water and the pressure. That is why, the range 14·5°C to 15·5°C and the pressure 1 atm was specified in the definition. We shall ignore this small variation and use one calorie to mean the amount of heat needed to increase the temperature of 1 g of water by 1°C at any region of temperature and pressure.

The calorie is now defined in terms of joule as 1 cal = 4·186 joule. We also use the unit kilocalorie which is equal to 1000 calorie as the name indicates.

Example 25.1

What is the kinetic energy of a 10 kg *mass moving at a speed of* 36 km h^{-1} *in calorie*?

Solution :

The kinetic energy is

$$\frac{1}{2}mv^2 = \frac{1}{2} \times 10 \text{ kg} \times \left(\frac{36 \times 10^3 \text{ m}}{3600 \text{ s}}\right)^2$$

$$= 500 \text{ J} = \frac{500}{4{\cdot}186} \text{ cal} \approx 120 \text{ cal.}$$

25.3 PRINCIPLE OF CALORIMETRY

A simple calorimeter is a vessel generally made of copper with a stirrer of the same material. The vessel is kept in a wooden box to isolate it thermally from the surrounding. A thermometer is used to measure the temperature of the contents of the calorimeter.

Objects at different temperatures are made to come in contact with each other in the calorimeter. As a result, heat is exchanged between the objects as well as with the calorimeter. Neglecting any heat exchange with the surrounding, the principle of calorimetry states that *the total heat given by the hot objects equals the total heat received by the cold objects.*

25.4 SPECIFIC HEAT CAPACITY AND MOLAR HEAT CAPACITY

When we supply heat to a body, its temperature increases. The amount of heat absorbed depends on the mass of the body, the change in temperature, the material of the body as well as the surrounding conditions, such as pressure. We write the equation

$$Q = ms\Delta\theta \qquad \ldots (25.1)$$

where $\Delta\theta$ is the change in temperature, m is the mass of the body, Q is the heat supplied, and s is a constant for the given material under the given surrounding conditions. The constant s is called *specific heat capacity* of the substance. When a solid or a liquid is kept open in the atmosphere and heated, the pressure remains constant. Table (25.1) gives the specific heat capacities of some of the solids and liquids under constant pressure condition. As can be seen from equation (25.1), the SI unit for specific heat capacity is $J\,kg^{-1}\,K^{-1}$ which is the same as $J\,kg^{-1}\,°C^{-1}$. The specific heat capacity may also be expressed in $cal\,g^{-1}\,K^{-1}$ (same as $cal\,g^{-1}\,°C^{-1}$). Specific heat capacity is also called *specific heat* in short.

Table 25.1 : Specific heat capacities of some materials

Material	$cal\,g^{-1}\,°C^{-1}$	$J\,kg^{-1}\,K^{-1}$	Material	$Cal\,g^{-1}\,°C^{-1}$	$J\,kg^{-1}\,K^{-1}$
Water	1·00	4186	Glass	0·1–0·2	419–837
Ethanol	0·55	2302	Iron	0·112	470
Paraffin	0·51	2135	Copper	0·093	389
Ice	0·50	2093	Mercury	0·033	138
Steam	0·46	1926	Lead	0·031	130
Aluminium	0·215	900			

The amount of substance in the given body may also be measured in terms of the number of moles. Equation (25.1) may be rewritten as

$$Q = nC\Delta\theta$$

where n is the number of moles in the sample. The constant C is called *molar heat capacity.*

Example 25.2

A copper block of mass 60 g *is heated till its temperature is increased by* 20°C. *Find the heat supplied to the block. Specific heat capacity of copper* = $0{\cdot}09\ cal\,g^{-1}\,°C^{-1}$.

Solution :

The heat supplied is $Q = ms\Delta\theta$

$$= (60\ g)\,(0{\cdot}09\ cal\,g^{-1}\,°C^{-1})\,(20°C) = 108\ cal.$$

The quantity ms is called the *heat capacity of the body*. Its unit is $J\,K^{-1}$. The mass of water having the same heat capacity as a given body is called the *water equivalent* of the body.

25.5 DETERMINATION OF SPECIFIC HEAT CAPACITY IN LABORATORY

Figure (25.1) shows Regnault's apparatus to determine the specific heat capacity of a solid heavier than water, and insoluble in it. A wooden partition P separates a steam chamber O and a calorimeter C. The steam chamber O is a double-walled cylindrical vessel. Steam can be passed in the space between the two walls through an inlet A and it can escape through an outlet B. The upper part of the vessel is closed by a cork. The given solid may be suspended in the vessel by a thread passing through the cork. A thermometer T_1 is also inserted into the vessel to record the temperature of the solid. The steam chamber is kept on a wooden platform with a removable wooden disc D closing the bottom hole of the chamber.

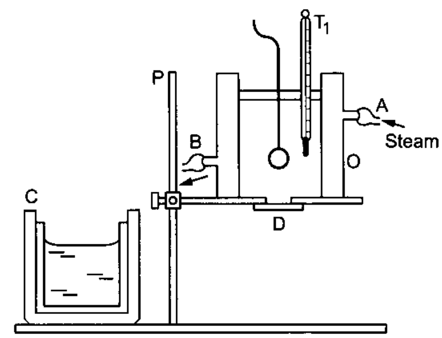

Figure 25.1

To start with, the experimental solid (in the form of a ball or a block) is weighed and then suspended in the steam chamber. Steam is prepared by boiling water in a separate boiler and is passed through the steam chamber. A calorimeter with a stirrer is weighed and sufficient amount of water is kept in it so that the solid may be completely immersed in it. The calorimeter is again weighed with water to get the mass of the water. The initial temperature of the water is noted.

When the temperature of the solid becomes constant (say for 15 minutes), the partition P is removed, the calorimeter is taken below the steam chamber, the wooden disc D is removed and the thread is cut to drop the solid in the calorimeter. The calorimeter is taken to its original place and is stirred. The maximum temperature of the mixture is noted.

Calculation:

Let the mass of the solid	$= m_1$
mass of the calorimeter and the stirrer	$= m_2$
mass of the water	$= m_3$
specific heat capacity of the solid	$= s_1$
specific heat capacity of the material of the calorimeter (and stirrer)	$= s_2$
specific heat capacity of water	$= s_3$
initial temperature of the solid	$= \theta_1$
initial temperature of the calorimeter, stirrer and water	$= \theta_2$
final temperature of the mixture	$= \theta.$

We have,

heat lost by the solid $= m_1 s_1(\theta_1 - \theta)$

heat gained by the calorimeter (and the stirrer) $= m_2 s_2(\theta - \theta_2)$

and heat gained by the water $= m_3s_3(\theta - \theta_2)$.

Assuming no loss of heat to the surrounding, the heat lost by the solid goes into the calorimeter, stirrer and water. Thus,

$$m_1s_1(\theta_1 - \theta) = m_2s_2(\theta - \theta_2) + m_3s_3(\theta - \theta_2) \quad \ldots \text{(i)}$$

or, $$s_1 = \frac{(m_2s_2 + m_3s_3)(\theta - \theta_2)}{m_1(\theta_1 - \theta)}.$$

Knowing the specific heat capacity of water ($s_3 = 4186 \text{ J kg}^{-1} \text{K}^{-1}$) and that of the material of the calorimeter and the stirrer ($s_2 = 389 \text{ J kg}^{-1} \text{K}^{-1}$ if the material be copper), one can calculate s_1.

Specific heat capacity of a liquid can also be measured with the Regnault's apparatus. Here a solid of known specific heat capacity s_1 is used and the experimental liquid is taken in the calorimeter in place of water. The solid should be denser than the liquid. Using the same procedure and with the same symbols we get an equation identical to equation (i) above, that is,

$$m_1s_1(\theta_1 - \theta) = m_2s_2(\theta - \theta_2) + m_3s_3(\theta - \theta_2)$$

in which s_3 is the specific heat capacity of the liquid. We get,

$$s_3 = \frac{m_1s_1(\theta_1 - \theta)}{m_3(\theta - \theta_2)} - \frac{m_2s_2}{m_3}.$$

25.6 SPECIFIC LATENT HEAT OF FUSION AND VAPORIZATION

Apart from raising the temperature, heat supplied to a body may cause a phase change such as solid to liquid or liquid to vapour.

During this process of melting or vaporization, the temperature remains constant. The amount of heat needed to melt a solid of mass m may be written as

$$Q = mL \quad \ldots (25.2)$$

where L is a constant for the given material (and surrounding conditions). This constant L is called *specific latent heat of fusion*. The term *latent heat of fusion* is also used to mean the same thing. Equation (25.2) is also valid when a liquid changes its phase to vapour. The constant L in this case is called the *specific latent heat of vaporization* or simply *latent heat of vaporization*. When a vapour condenses or a liquid solidifies, heat is released to the surrounding.

In solids, the forces between the molecules are large and the molecules are almost fixed in their positions inside the solid. In a liquid, the forces between the molecules are weaker and the molecules may move freely inside the volume of the liquid. However, they are not able to come out of the surface. In vapours or gases, the intermolecular forces are almost negligible and the molecules may move freely anywhere in the container. When a solid melts, its molecules move apart against the strong molecular attraction. This needs energy which must be supplied from outside. Thus, the internal energy of a given body is larger in liquid phase than in solid phase. Similarly, the internal energy of a given body in vapour phase is larger than that in liquid phase.

Example 25.3

A piece of ice of mass 100 g *and at temperature* 0°C *is put in* 200 g *of water at* 25°C. *How much ice will melt as the temperature of the water reaches* 0°C ? *The specific heat capacity of water* = $4200 \text{ J kg}^{-1} \text{K}^{-1}$ *and the specific latent heat of fusion of ice* = $3{\cdot}4 \times 10^5 \text{ J kg}^{-1}$.

Solution :

The heat released as the water cools down from 25°C to 0°C is

$$Q = ms\Delta\theta = (0{\cdot}2 \text{ kg})(4200 \text{ J kg}^{-1} \text{K}^{-1})(25 \text{ K}) = 21000 \text{ J}.$$

The amount of ice melted by this much heat is given by

$$m = \frac{Q}{L} = \frac{21000 \text{ J}}{3{\cdot}4 \times 10^5 \text{ J kg}^{-1}} = 62 \text{ g}.$$

25.7 MEASUREMENT OF SPECIFIC LATENT HEAT OF FUSION OF ICE

An empty calorimeter (together with a stirrer) is weighed. About two third of the calorimeter is filled with water and is weighed again. Thus, one gets the mass of the water. The initial temperature of the water is measured with the help of a thermometer. A piece of ice is taken and as it starts melting it is dried with a blotting paper and put into the calorimeter. The water is stirred keeping the ice always fully immersed in it. The minimum temperature reached is recorded. This represents the temperature when all the ice has melted. The calorimeter with its contents is weighed again. Thus, one can get the mass of the ice that has melted in the calorimeter.

Calculation:

Let the mass of the calorimeter (with stirrer)	$= m_1$
mass of water	$= m_2$
mass of ice	$= m_3$
initial temperature of the calorimeter and the water (in Celsius)	$= \theta_1$
final temperature of the calorimeter and the water (in Celsius)	$= \theta_2$
temperature of the melting ice	$= 0°\text{C}$
specific latent heat of fusion of ice	$= L$

specific heat capacity of the material of the calorimeter (and stirrer) $= s_1$

specific heat capacity of water $= s_2$.

We have,

heat lost by the calorimeter (and the stirrer)

$$= m_1 s_1(\theta_1 - \theta_2)$$

heat lost by the water kept initially in the calorimeter $= m_2 s_2(\theta_1 - \theta_2)$

heat gained by the ice during its fusion to water $= m_3 L$

heat gained by this water in coming from 0°C to θ_2

$$= m_3 s_2 \theta_2.$$

Assuming no loss of heat to the surrounding,

$$m_1 s_1(\theta_1 - \theta_2) + m_2 s_2(\theta_1 - \theta_2) = m_3 L + m_3 s_2 \theta_2$$

or, $$L = \frac{(m_1 s_1 + m_2 s_2)(\theta_1 - \theta_2)}{m_3} - s_2 \theta_2.$$

Knowing the specific heat capacity of water and that of the material of the calorimeter and the stirrer, one can calculate the specific latent heat of fusion of ice L.

25.8 MEASUREMENT OF SPECIFIC LATENT HEAT OF VAPORIZATION OF WATER

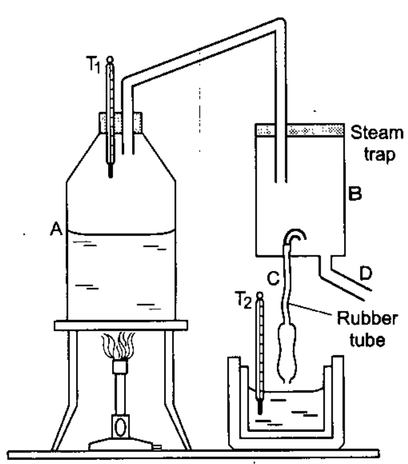

Figure 25.2

Figure (25.2) shows the arrangement used to measure the specific latent heat of vaporization of water. Steam is prepared by boiling water in a boiler A. The cork of the boiler has two holes. A thermometer T_1 is inserted into one to measure the temperature of the steam and the other contains a bent glass tube to carry the steam to a steam trap B. A tube C with one end bent and the other end terminated in a jet is fitted in the steam trap. Another tube D is fitted in the trap which is used to drain out the extra steam and water condensed at the bottom.

To start the experiment, an empty calorimeter (together with the stirrer) is weighed. About half of it is filled with water and is weighed again. Thus, one gets the mass of the water. The initial temperature of the water and the calorimeter is measured by a thermometer T_2 . Water is kept in the boiler A and is heated. As it boils, the steam passes to the steam trap and then comes out through the tubes C and D. After some steam has gone out (say for 5 minutes), the temperature of the steam is noted. The calorimeter with the water is kept below the tube C so that steam goes into the calorimeter. The water in the calorimeter is continuously stirred and the calorimeter is removed after the temperature in it increases by about 5°C. The final temperature of the water in the calorimeter is noted. The calorimeter together with the water (including the water condensed) is weighed. From this, one gets the mass of the steam that condensed in the calorimeter.

Let the mass of the calorimeter (with the stirrer)	$= m_1$
mass of the water	$= m_2$
mass of the steam condensed	$= m_3$
temperature of the steam	$= \theta_1$
initial temperature of the water in the calorimeter	$= \theta_2$
final temperature of the water in the calorimeter	$= \theta_3$
specific latent heat of vaporization of water	$= L$
specific heat capacity of the material of the calorimeter (and the stirrer)	$= s_1$
specific heat capacity of water	$= s_2$.

We have,

heat gained by the calorimeter (and the stirrer) $= m_1 s_1(\theta_3 - \theta_2)$

heat gained by the water kept initially in the calorimeter $= m_2 s_2(\theta_3 - \theta_2)$

heat lost by the steam in condensing $= m_3 L$

heat lost by the condensed water in cooling from temperature θ_1 to $\theta_3 = m_3 s_2(\theta_1 - \theta_3)$.

Assuming no loss of heat to the surrounding,

$$m_1 s_1(\theta_3 - \theta_2) + m_2 s_2(\theta_3 - \theta_2) = m_3 L + m_3 s_2(\theta_1 - \theta_3)$$

or, $$L = \frac{(m_1 s_1 + m_2 s_2)(\theta_3 - \theta_2)}{m_3} - s_2(\theta_1 - \theta_3).$$

Knowing the specific heat capacity of water and that of the material of the calorimeter, one can calculate the specific latent heat of vaporization of water L.

Example 25.4

A calorimeter of water equivalent 15 g *contains* 165 g *of water at* 25°C. *Steam at* 100°C *is passed through the water for some time. The temperature is increased to* 30°C *and the mass of the calorimeter and its contents is increased by* 1·5 g. *Calculate the specific latent heat of vaporization of water. Specific heat capacity of water is* $1\ \text{cal g}^{-1}\ {}^\circ\text{C}^{-1}$.

Solution : Let L be the specific latent heat of vaporization of water. The mass of the steam condensed is 1·5 g. Heat lost in condensation of steam is

$$Q_1 = (1{\cdot}5\ \text{g})\ L.$$

The condensed water cools from 100°C to 30°C. Heat lost in this process is

$$Q_2 = (1{\cdot}5\ \text{g})\ (1\ \text{cal g}^{-1}\ {}^\circ\text{C}^{-1})\ (70^\circ\text{C}) = 105\ \text{cal}.$$

Heat supplied to the calorimeter and to the cold water during the rise in temperature from 25°C to 30°C is

$$Q_3 = (15\ \text{g} + 165\ \text{g})\ (1\ \text{cal g}^{-1}\ {}^\circ\text{C}^{-1})\ (5^\circ\text{C}) = 900\ \text{cal}.$$

If no heat is lost to the surrounding,

$$(1{\cdot}5\ \text{g})\ L + 105\ \text{cal} = 900\ \text{cal}$$

or, $$L = 530\ \text{cal g}^{-1}.$$

25.9 MECHANICAL EQUIVALENT OF HEAT

In early days heat was not recognised as a form of energy. Heat was supposed to be something needed to raise the temperature of a body or to change its phase. Calorie was defined as the unit of heat. A number of experiments were performed to show that the temperature may also be increased by doing mechanical work on the system. These experiments established that heat is equivalent to mechanical energy and measured how much mechanical energy is equivalent to a calorie. If mechanical work W produces the same temperature change as heat H, we write,

$$W = JH \qquad \dots (25.3)$$

where J is called *mechanical equivalent of heat*. It is clear that if W and H are both measured in the same unit then $J = 1$. If W is measured in joule (work done by a force of 1 N in displacing an object by 1 m in its direction) and H in calorie (heat required to raise the temperature of 1 g of water by 1°C) then J is expressed in joule per calorie. The value of J gives how many joules of mechanical work is needed to raise the temperature of 1 g of water by 1°C. We describe below a laboratory method to measure the mechanical equivalent of heat.

Searle's Cone Method

Figure (25.3) shows the apparatus. A conical vessel B just fits in another conical vessel A of the same material. The outer vessel A is connected to a spindle C which may be rotated at high speed by an electric motor or by hand. The number of rotations made in a given time can be recorded.

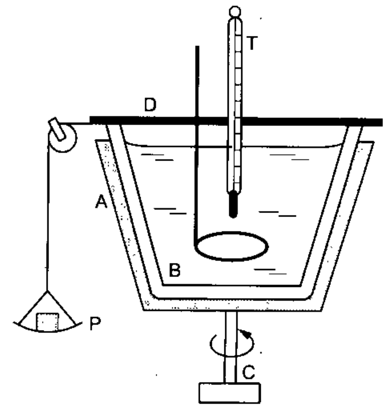

Figure 25.3

The inner vessel B is fitted with a grooved wooden disc D. A cord is wound around the groove and is connected to a hanging pan P after passing through a fixed pulley. Weights can be put on this pan.

The wooden disc D contains two holes through which a thermometer and a stirrer can pass into the inner vessel. If the outer vessel is rotated, it tries to drag the inner vessel with it due to the friction between the surfaces of the cones. The friction produces a net torque Γ about the central axis. The hanging weights also produce a torque about the central axis which is equal to Mgr where M is the total mass of the pan and the weights on it. If the direction of rotation is properly chosen, these torques may be opposite to each other. Also, the value of M may be adjusted for a given speed of the outer vessel so that $Mgr = \Gamma$. In such a case the inner vessel does not move.

To start the experiment, a measured mass of water is taken in the inner vessel and the thermometer and the stirrer are placed in their positions. The masses of the vessels A and B are also known. The outer vessel A is rotated by rotating the spindle either by a motor or by hand. The direction of rotation is chosen to make sure that the frictional torque and the torque due to the weight Mg oppose each other. The value of M is so adjusted that the inner vessel does not move. The temperature of the water is noted at an initial instant after the adjustments are made. The water is continuously stirred with the help of the stirrer and the temperature is noted at the final instant when it is increased roughly by 5°C. The number of revolutions made by the spindle during this period is noted.

Suppose,

the mass of the water taken	$= m_1$
mass of the two vessels taken together	$= m_2$
mass of the pan and the weights on it	$= M$
initial temperature of water	$= \theta_1$
final temperature of water	$= \theta_2$
number of revolutions made by the outer vessel	$= n$
radius of the disc D	$= r$
specific heat capacity of water	$= s_1$
specific heat capacity of the material of the vessels	$= s_2$.

The torque due to friction
= The torque due to the weights $= Mgr$.

Work done by this torque as the outer vessel slides on the inner one $= \Gamma\theta$

$$= Mgr.2\pi n.$$

The heat needed to raise the temperature of water

$$= m_1 s_1(\theta_2 - \theta_1).$$

Heat needed to raise the temperature of the vessels

$$= m_2 s_2(\theta_2 - \theta_1).$$

The total amount of heat needed to raise the temperature is $(m_1 s_1 + m_2 s_2)(\theta_2 - \theta_1)$. Thus, the mechanical work $2\pi n\, Mgr$ produces the same effect as the heat $(m_1 s_1 + m_2 s_2)(\theta_2 - \theta_1)$.

Thus, $\quad 2\pi n\, Mgr = J\,(m_1 s_1 + m_2 s_2)(\theta_2 - \theta_1)$

or, $$J = \frac{2\pi n\, Mgr}{(m_1 s_1 + m_2 s_2)(\theta_2 - \theta_1)}.$$

Putting s_1, s_2 in cal gm^{-1} °C^{-1} (for water $s = 1$ cal g^{-1} °C^{-1}), one gets J in joule per calorie. Experiments give a value $J = 4{\cdot}186$ J cal^{-1}.

Although heat is the energy in transit due to temperature difference, the word "heat" is also used for the mechanical work that raises the temperature of a body or that which causes a phase change. Thus, if a block slides on a rough surface, its kinetic energy may be used to increase the temperature of the block and the surface. We say that "heat is developed" when the block slides on the surface. Such a use of the word "heat" is made only due to tradition, though it is not strictly correct. It is better to say that thermal energy is produced.

Worked Out Examples

1. *Calculate the amount of heat required to convert* 1·00 kg *of ice at* –10°C *into steam at* 100°C *at normal pressure. Specific heat capacity of ice* = 2100 J kg^{-1} K^{-1}, *latent heat of fusion of ice* = 3·36 × 10^{5} J kg^{-1}, *specific heat capacity of water* = 4200 J kg^{-1} K^{-1} *and latent heat of vaporization of water* = 2·25 × 10^{6} J kg^{-1}.

Solution : Heat required to take the ice from –10°C to 0°C

$$= (1\text{ kg})\,(2100\text{ J kg}^{-1}\text{ K}^{-1})\,(10\text{ K}) = 21000\text{ J}.$$

Heat required to melt the ice at 0°C to water

$$= (1\text{ kg})\,(3{\cdot}36 \times 10^{5}\text{ J kg}^{-1}) = 336000\text{ J}.$$

Heat required to take 1 kg of water from 0°C to 100°C

$$= (1\text{ kg})\,(4200\text{ J kg}^{-1}\text{ K}^{-1})\,(100\text{ K}) = 420000\text{ J}.$$

Heat required to convert 1 kg of water at 100°C into steam

$$= (1\text{ kg})\,(2{\cdot}25 \times 10^{6}\text{ J kg}^{-1}) = 2{\cdot}25 \times 10^{6}\text{ J}.$$

Total heat required = 3·03 × 10^{6} J.

2. *A* 5 g *piece of ice at* –20°C *is put into* 10 g *of water at* 30°C. *Assuming that heat is exchanged only between the ice and the water, find the final temperature of the mixture. Specific heat capacity of ice* = 2100 J kg^{-1} °C^{-1}, *specific heat capacity of water* = 4200 J kg^{-1} °C^{-1} *and latent heat of fusion of ice* = 3·36 × 10^{5} J kg^{-1}.

Solution : The heat given by the water when it cools down from 30°C to 0°C is

$$(0{\cdot}01\text{ kg})\,(4200\text{ J kg}^{-1}\,°\text{C}^{-1})\,(30°\text{C}) = 1260\text{ J}.$$

The heat required to bring the ice to 0°C is

$$(0{\cdot}005\text{ kg})\,(2100\text{ J kg}^{-1}\,°\text{C}^{-1})\,(20°\text{C}) = 210\text{ J}.$$

The heat required to melt 5 g of ice is

$$(0{\cdot}005\text{ kg})\,(3{\cdot}36 \times 10^{5}\text{ J kg}^{-1}) = 1680\text{ J}.$$

We see that whole of the ice cannot be melted as the required amount of heat is not provided by the water. Also, the heat is enough to bring the ice to 0°C. Thus the final temperature of the mixture is 0°C with some of the ice melted.

3. *An aluminium container of mass* 100 g *contains* 200 g *of ice at* –20°C. *Heat is added to the system at a rate of* 100 cal s^{-1}. *What is the temperature of the system after* 4 *minutes ? Draw a rough sketch showing the variation in the temperature of the system as a function of time. Specific heat capacity of ice* = 0.5 cal g^{-1} °C^{-1}, *specific heat capacity of aluminium* = 0·2 cal g^{-1} °C^{-1}, *specific heat capacity of water* = 1 cal g^{-1} °C^{-1} *and latent heat of fusion of ice* = 80 cal g^{-1}.

Solution : Total heat supplied to the system in 4 minutes is $Q = 100 \text{ cal s}^{-1} \times 240 \text{ s} = 2{\cdot}4 \times 10^4 \text{ cal}$.

The heat required to take the system from $-20°C$ to $0°C$

$$= (100 \text{ g}) \times (0{\cdot}2 \text{ cal g}^{-1} °C^{-1}) \times (20°C) + (200 \text{ g}) \times (0{\cdot}5 \text{ cal g}^{-1} °C^{-1}) \times (20°C)$$
$$= 400 \text{ cal} + 2000 \text{ cal} = 2400 \text{ cal}.$$

The time taken in this process $= \frac{2400}{100} \text{ s} = 24 \text{ s}$.

The heat required to melt the ice at 0°C

$$= (200 \text{ g}) (80 \text{ cal g}^{-1}) = 16000 \text{ cal}.$$

The time taken in this process $= \frac{16000}{100} \text{ s} = 160 \text{ s}$.

If the final temperature is θ, the heat required to take the system to the final temperature is

$$= (100 \text{ g}) (0{\cdot}2 \text{ cal g}^{-1} °C^{-1}) \theta + (200 \text{ g}) (1 \text{ cal g}^{-1} °C^{-1}) \theta.$$

Thus,

$$2{\cdot}4 \times 10^4 \text{ cal} = 2400 \text{ cal} + 16000 \text{ cal} + (220 \text{ cal } °C^{-1}) \theta$$

or, $$\theta = \frac{5600 \text{ cal}}{220 \text{ cal } °C^{-1}} = 25{\cdot}5°C.$$

The variation in the temperature as a function of time is sketched in figure (25-W1).

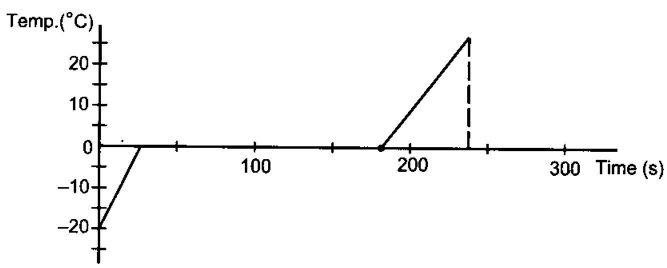

Figure 25-W1

4. *A thermally isolated vessel contains* 100 g *of water at* 0°C. *When air above the water is pumped out, some of the water freezes and some evaporates at* 0°C *itself. Calculate the mass of the ice formed if no water is left in the vessel. Latent heat of vaporization of water at* $0°C = 2{\cdot}10 \times 10^6 \text{ J kg}^{-1}$ *and latent heat of fusion of ice* $= 3{\cdot}36 \times 10^5 \text{ J kg}^{-1}$.

Solution : Total mass of the water $= M = 100$ g.

Latent heat of vaporization of water at 0°C

$$= L_1 = 21{\cdot}0 \times 10^5 \text{ J kg}^{-1}.$$

Latent heat of fusion of ice $= L_2 = 3{\cdot}36 \times 10^5 \text{ J kg}^{-1}$.

Suppose, the mass of the ice formed $= m$.

Then the mass of water evaporated $= M - m$.

Heat taken by the water to evaporate $= (M - m) L_1$

and heat given by the water in freezing $= mL_2$.

Thus, $mL_2 = (M - m)L_1$

or, $$m = \frac{ML_1}{L_1 + L_2}$$
$$= \frac{(100 \text{ g}) (21{\cdot}0 \times 10^5 \text{ J kg}^{-1})}{(21{\cdot}0 + 3{\cdot}36) \times 10^5 \text{ J kg}^{-1}} = 86 \text{ g}.$$

5. *A lead bullet penetrates into a solid object and melts. Assuming that 50% of its kinetic energy was used to heat it, calculate the initial speed of the bullet. The initial temperature of the bullet is 27°C and its melting point is 327°C. Latent heat of fusion of lead* $= 2{\cdot}5 \times 10^4 \text{ J kg}^{-1}$ *and specific heat capacity of lead* $= 125 \text{ J kg}^1 \text{ K}^1$.

Solution : Let the mass of the bullet $= m$.

Heat required to take the bullet from 27°C to 327°C

$$= m \times (125 \text{ J kg}^{-1} \text{ K}^{-1}) (300 \text{ K})$$
$$= m \times (3{\cdot}75 \times 10^4 \text{ J kg}^{-1}).$$

Heat required to melt the bullet

$$= m \times (2{\cdot}5 \times 10^4 \text{ J kg}^{-1}).$$

If the initial speed be v, the kinetic energy is $\frac{1}{2} mv^2$ and hence the heat developed is $\frac{1}{2}\left(\frac{1}{2} mv^2\right) = \frac{1}{4} mv^2$. Thus,

$$\frac{1}{4} mv^2 = m(3{\cdot}75 + 2{\cdot}5) \times 10^4 \text{ J kg}^{-1}$$

or, $$v = 500 \text{ m s}^{-1}.$$

6. *A lead ball at 30°C is dropped from a height of* 6·2 km. *The ball is heated due to the air resistance and it completely melts just before reaching the ground. The molten substance falls slowly on the ground. Calculate the latent heat of fusion of lead. Specific heat capacity of lead* $= 126 \text{ J kg}^{-1} °C^{-1}$ *and melting point of lead* $= 330°C$. *Assume that any mechanical energy lost is used to heat the ball. Use* $g = 10 \text{ m s}^{-2}$.

Solution : The initial gravitational potential energy of the ball

$$= mgh$$
$$= m \times (10 \text{ m s}^{-2}) \times (6{\cdot}2 \times 10^3 \text{ m})$$
$$= m \times (6{\cdot}2 \times 10^4 \text{ m}^2 \text{ s}^{-2}) = m \times (6{\cdot}2 \times 10^4 \text{ J kg}^{-1}).$$

All this energy is used to heat the ball as it reaches the ground with a small velocity. Energy required to take the ball from 30°C to 330°C is

$$m \times (126 \text{ J kg}^{-1} °C^{-1}) \times (300°C)$$
$$= m \times 37800 \text{ J kg}^{-1}$$

and energy required to melt the ball at 330°C

$$= mL$$

where $L =$ latent heat of fusion of lead.

Thus,

$$m \times (6{\cdot}2 \times 10^4 \text{ J kg}^{-1}) = m \times 37800 \text{ J kg}^{-1} + mL$$

or, $$L = 2{\cdot}4 \times 10^4 \text{ J kg}^{-1}.$$

QUESTIONS FOR SHORT ANSWER

1. Is heat a conserved quantity ?
2. The calorie is defined as 1 cal = 4·186 joule. Why not as 1 cal = 4 J to make the conversions easy ?
3. A calorimeter is kept in a wooden box to insulate it thermally from the surroundings. Why is it necessary ?
4. In a calorimeter, the heat given by the hot object is assumed to be equal to the heat taken by the cold object. Does it mean that heat of the two objects taken together remains constant ?
5. In Regnault's apparatus for measuring specific heat capacity of a solid, there is an inlet and an outlet in the steam chamber. The inlet is near the top and the outlet is near the bottom. Why is it better than the opposite choice where the inlet is near the bottom and the outlet is near the top ?
6. When a solid melts or a liquid boils, the temperature does not increase even when heat is supplied. Where does the energy go ?
7. What is the specific heat capacity of (a) melting ice (b) boiling water ?
8. A person's skin is more severely burnt when put in contact with 1 g of steam at 100°C than when put in contact with 1 g of water at 100°C. Explain.
9. The atmospheric temperature in the cities on sea-coast change very little. Explain.
10. Should a thermometer bulb have large heat capacity or small heat capacity ?

OBJECTIVE I

1. The specific heat capacity of a body depends on
   (a) the heat given (b) the temperature raised
   (c) the mass of the body (d) the material of the body.
2. Water equivalent of a body is measured in
   (a) kg (b) calorie (c) kelvin (d) m^3.
3. When a hot liquid is mixed with a cold liquid, the temperature of the mixture
   (a) first decreases then becomes constant
   (b) first increases then becomes constant
   (c) continuously increases
   (d) is undefined for some time and then becomes nearly constant.
4. Which of the following pairs represent units of the same physical quantity ?
   (a) Kelvin and joule (b) Kelvin and calorie
   (c) Newton and calorie (d) Joule and calorie
5. Which of the following pairs of physical quantities may be represented in the same unit ?
   (a) Heat and temperature (b) Temperature and mole
   (c) Heat and work (d) Specific heat and heat
6. Two bodies at different temperatures are mixed in a calorimeter. Which of the following quantities remains conserved ?
   (a) Sum of the temperatures of the two bodies
   (b) Total heat of the two bodies
   (c) Total internal energy of the two bodies
   (d) Internal energy of each body
7. The mechanical equivalent of heat
   (a) has the same dimension as heat
   (b) has the same dimension as work
   (c) has the same dimension as energy
   (d) is dimensionless.

OBJECTIVE II

1. The heat capacity of a body depends on
   (a) the heat given (b) the temperature raised
   (c) the mass of the body (d) the material of the body.
2. The ratio of specific heat capacity to molar heat capacity of a body
   (a) is a universal constant
   (b) depends on the mass of the body
   (c) depends on the molecular weight of the body
   (d) is dimensionless.
3. If heat is supplied to a solid, its temperature
   (a) must increase (b) may increase
   (c) may remain constant (d) may decrease.
4. The temperature of a solid object is observed to be constant during a period. In this period
   (a) heat may have been supplied to the body
   (b) heat may have been extracted from the body
   (c) no heat is supplied to the body
   (d) no heat is extracted from the body.
5. The temperature of an object is observed to rise in a period. In this period
   (a) heat is certainly supplied to it
   (b) heat is certainly not supplied to it
   (c) heat may have been supplied to it
   (d) work may have been done on it.
6. Heat and work are equivalent. This means,
   (a) when we supply heat to a body we do work on it
   (b) when we do work on a body we supply heat to it
   (c) the temperature of a body can be increased by doing work on it
   (d) a body kept at rest may be set into motion along a line by supplying *heat to it*.

EXERCISES

1. An aluminium vessel of mass 0·5 kg contains 0·2 kg of water at 20°C. A block of iron of mass 0·2 kg at 100°C is gently put into the water. Find the equilibrium temperature of the mixture. Specific heat capacities of aluminium, iron and water are $910\ \mathrm{J\ kg^{-1}\ K^{-1}}$, $470\ \mathrm{J\ kg^{-1}\ K^{-1}}$ and $4200\ \mathrm{J\ kg^{-1}\ K^{-1}}$ respectively.
2. A piece of iron of mass 100 g is kept inside a furnace for a long time and then put in a calorimeter of water equivalent 10 g containing 240 g of water at 20°C. The mixture attains an equilibrium temperature of 60°C. Find the temperature of the furnace. Specific heat capacity of iron = $470\ \mathrm{J\ kg^{-1}\ °C^{-1}}$.
3. The temperatures of equal masses of three different liquids A, B and C are 12°C, 19°C and 28°C respectively. The temperature when A and B are mixed is 16°C, and when B and C are mixed, it is 23°C. What will be the temperature when A and C are mixed?
4. Four 2 cm × 2 cm × 2 cm cubes of ice are taken out from a refrigerator and are put in 200 ml of a drink at 10°C. (a) Find the temperature of the drink when thermal equilibrium is attained in it. (b) If the ice cubes do not melt completely, find the amount melted. Assume that no heat is lost to the outside of the drink and that the container has negligible heat capacity. Density of ice = $900\ \mathrm{kg\ m^{-3}}$, density of the drink = $1000\ \mathrm{kg\ m^{-3}}$, specific heat capacity of the drink = $4200\ \mathrm{J\ kg^{-1}\ K^{-1}}$, latent heat of fusion of ice = $3{\cdot}4 \times 10^{5}\ \mathrm{J\ kg^{-1}}$.
5. Indian style of cooling drinking water is to keep it in a pitcher having porous walls. Water comes to the outer surface very slowly and evaporates. Most of the energy needed for evaporation is taken from the water itself and the water is cooled down. Assume that a pitcher contains 10 kg of water and 0·2 g of water comes out per second. Assuming no backward heat transfer from the atmosphere to the water, calculate the time in which the temperature decreases by 5°C. Specific heat capacity of water = $4200\ \mathrm{J\ kg^{-1}\ °C^{-1}}$ and latent heat of vaporization of water = $2{\cdot}27 \times 10^{6}\ \mathrm{J\ kg^{-1}}$.
6. A cube of iron (density = $8000\ \mathrm{kg\ m^{-3}}$, specific heat capacity = $470\ \mathrm{J\ kg^{-1}\ K^{-1}}$) is heated to a high temperature and is placed on a large block of ice at 0°C. The cube melts the ice below it, displaces the water and sinks. In the final equilibrium position, its upper surface just goes inside the ice. Calculate the initial temperature of the cube. Neglect any loss of heat outside the ice and the cube. The density of ice = $900\ \mathrm{kg\ m^{-3}}$ and the latent heat of fusion of ice = $3{\cdot}36 \times 10^{5}\ \mathrm{J\ kg^{-1}}$.
7. 1 kg of ice at 0°C is mixed with 1 kg of steam at 100°C. What will be the composition of the system when thermal equilibrium is reached? Latent heat of fusion of ice = $3{\cdot}36 \times 10^{5}\ \mathrm{J\ kg^{-1}}$ and latent heat of vaporization of water = $2{\cdot}26 \times 10^{6}\ \mathrm{J\ kg^{-1}}$.
8. Calculate the time required to heat 20 kg of water from 10°C to 35°C using an immersion heater rated 1000 W. Assume that 80% of the power input is used to heat the water. Specific heat capacity of water = $4200\ \mathrm{J\ kg^{-1}\ K^{-1}}$.
9. On a winter day the temperature of the tap water is 20°C whereas the room temperature is 5°C. Water is stored in a tank of capacity $0{\cdot}5\ \mathrm{m^{3}}$ for household use. If it were possible to use the heat liberated by the water to lift a 10 kg mass vertically, how high can it be lifted as the water comes to the room temperature? Take $g = 10\ \mathrm{m\ s^{-2}}$.
10. A bullet of mass 20 g enters into a fixed wooden block with a speed of $40\ \mathrm{m\ s^{-1}}$ and stops in it. Find the change in internal energy during the process.
11. A 50 kg man is running at a speed of $18\ \mathrm{km\ h^{-1}}$. If all the kinetic energy of the man can be used to increase the temperature of water from 20°C to 30°C, how much water can be heated with this energy?
12. A brick weighing 4·0 kg is dropped into a 1·0 m deep river from a height of 2·0 m. Assuming that 80% of the gravitational potential energy is finally converted into thermal energy, find this thermal energy in calorie.
13. A van of mass 1500 kg travelling at a speed of $54\ \mathrm{km\ h^{-1}}$ is stopped in 10 s. Assuming that all the mechanical energy lost appears as thermal energy in the brake mechanism, find the average rate of production of thermal energy in $\mathrm{cal\ s^{-1}}$.
14. A block of mass 100 g slides on a rough horizontal surface. If the speed of the block decreases from $10\ \mathrm{m\ s^{-1}}$ to $5\ \mathrm{m\ s^{-1}}$, find the thermal energy developed in the process.
15. Two blocks of masses 10 kg and 20 kg moving at speeds of $10\ \mathrm{m\ s^{-1}}$ and $20\ \mathrm{m\ s^{-1}}$ respectively in opposite directions, approach each other and collide. If the collision is completely inelastic, find the thermal energy developed in the process.
16. A ball is dropped on a floor from a height of 2·0 m. After the collision it rises up to a height of 1·5 m. Assume that 40% of the mechanical energy lost goes as thermal energy into the ball. Calculate the rise in the temperature of the ball in the collision. Heat capacity of the ball is $800\ \mathrm{J\ K^{-1}}$.
17. A copper cube of mass 200 g slides down on a rough inclined plane of inclination 37° at a constant speed. Assume that any loss in mechanical energy goes into the copper block as thermal energy. Find the increase in the temperature of the block as it slides down through 60 cm. Specific heat capacity of copper = $420\ \mathrm{J\ kg^{-1}\ K^{-1}}$.
18. A metal block of density $6000\ \mathrm{kg\ m^{-3}}$ and mass 1·2 kg is suspended through a spring of spring constant $200\ \mathrm{N\ m^{-1}}$. The spring–block system is dipped in water kept in a vessel. The water has a mass of 260 g and the block is at a height 40 cm above the bottom of the vessel. If the support to the spring is broken, what will be the rise in the temperature of the water. Specific heat capacity of the block is $250\ \mathrm{J\ kg^{-1}\ K^{-1}}$ and that of water is $4200\ \mathrm{J\ kg^{-1}\ K^{-1}}$. Heat capacities of the vessel and the spring are negligible.

ANSWERS

OBJECTIVE I

1. (d) 2. (a) 3. (d) 4. (d) 5. (c) 6. (c)
7. (d)

OBJECTIVE II

1. (c), (d) 2. (c) 3. (b), (c)
4. (a), (b) 5. (c), (d) 6. (c)

EXERCISES

1. 25°C
2. 950°C
3. 20·3°C
4. (a) 0°C (b) 25 g
5. 7·7 min
6. 80°C
7. 665 g steam and 1·335 kg water
8. 44 min
9. 315 km
10. 16 J
11. 15 g
12. 23 cal
13. 4000 cal s^{-1}
14. 3·75 J
15. 3000 J
16. $2·5 \times 10^{-3}$°C
17. $8·6 \times 10^{-3}$°C
18. 0·003°C

□