Maxima and Minima 18.3 Q1(i)

$f(x) = x^4 - 62x^2 + 120x + 9$

$\therefore \quad f'(x) = 4x^3 - 124x + 120 = 4(x^3 - 31x + 30)$

$f''(x) = 12x^2 - 124 = 4(3x^2 - 31)$

For maxima and minima,

$f'(x) = 0$

$\Rightarrow \quad 4(x^3 - 31x + 30) = 0$

$\Rightarrow \quad 4(x^3 - 31x + 30) = 0$

$\Rightarrow \quad x = 5, 1, -6$

Now,

$f''(5) = 176 > 0$

$\Rightarrow \quad x = 5$ is point of local minima

$f''(1) = -112 < 0$

$\Rightarrow \quad x = 1$ is point of local maxima

$f''(-6) = 308 > 0$

$\Rightarrow \quad x = -6$ is point of local minima

$\therefore \quad$ local max value $= f(1) = 68$

local min value $= f(5) = -316$

and $= f(-6) = -1647.$

Maxima and Minima 18.3 Q1(ii)

We have,

$$f(x) = x^3 - 6x^2 + 9x + 15$$

$$\therefore \quad f'(x) = 3x^2 - 12x + 9$$

$$= 3(x^2 - 4x + 3)$$

$$f''(x) = 6x - 12$$

$$= 6(x - 2)$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad 3(x^2 - 4x + 3) = 0$$

$$\Rightarrow \quad 3(x - 3)(x - 1) = 0$$

$$\Rightarrow \quad x = 3, 1$$

Now,

$$f''(3) = 6 > 0$$

$\therefore$ $x = 3$ is point of local minima

$$f''(1) = -6 < 0$$

$\therefore$ $x = 1$ is point of local maxima

$\therefore$ local max value $= f(1) = 19$

local min value $= f(3) = 15$.

Maxima and Minima 18.3 Q1(iii)

We have,

$$f(x) = (x-1)(x+2)^2$$

$$\therefore \quad f'(x) = (x+2)^2 + 2(x-1)(x+2)$$

$$= (x+2)(x+2+2x-2)$$

$$= (x+2)(3x)$$

and, $$f''(x) = 3(x+2) + 3x$$

$$= 6x + 6$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad 3x(x+2) = 0$$

$$\Rightarrow \quad x = 0, -2$$

Now,

$$f''(0) = 6 > 0$$

$\therefore$ $x = 0$ is point of local minima

$$f''(-2) = -6 < 0$$

$\therefore$ $x = -2$ is point of local maxima

$\therefore$ local max value $= f(-2) = 0$

local min value $= f(0) = -4.$

Maxima and Minima 18.3 Q1(iv)

We have,

$$f(x) = \frac{2}{x} - \frac{2}{x^2}, \ x > 0$$

$\therefore \qquad f'(x) = \frac{-2}{x^2} + \frac{4}{x^3}$

and, $\quad f''(x) = \frac{+4}{x^3} - \frac{12}{x^4}$

For maxima and minima,

$$f'(x) = 0$$

$\Rightarrow \qquad \frac{-2}{x^2} + \frac{4}{x^3} = 0$

$\Rightarrow \qquad \frac{-2(x-2)}{x^3} = 0$

$\Rightarrow \qquad x = 2$

Now,

$$f''(2) = \frac{4}{8} - \frac{12}{6} = \frac{1}{2} - \frac{3}{4} = \frac{-1}{4} < 0$$

$\therefore \qquad x = 2$ is point of local maxima

local max value $= f(2) = \frac{1}{2}$.

*********** END **********