Indefinite Integrals Ex 19.25 Q56

Let $I = \int x \cos^3 x \, dx$

$$= \int x \left(\frac{3\cos x + \cos 3x}{4} \right) dx$$

$$= \frac{1}{4} \int x \left(3\cos x + \cos 3x \right) dx$$

Using integration by parts,

$$I = \frac{1}{4} \left[x \int \left(3\cos x + \cos 3x \right) dx - \int \left(1 \int \left(3\cos x + \cos 3x \right) dx \right) dx \right]$$

$$= \frac{1}{4} \left[x \left(3\sin x + \frac{\sin 3x}{3} \right) - \int \left(3\sin x + \frac{\sin 3x}{3} \right) dx \right]$$

$$= \frac{1}{4} \left[3x \sin x + \frac{x \sin 3x}{3} + 3\cos x + \frac{\cos 3x}{9} \right] + c$$

$$I = \frac{3x \sin x}{4} + \frac{x \sin 3x}{12} + \frac{3\cos x}{4} + \frac{\cos 3x}{36} + c$$

Indefinite Integrals Ex 19.25 Q57

Let $I = \int \tan^{-1} \sqrt{\frac{1-x}{1+x}}$ and $x = \cos\theta \Rightarrow dx = -\sin\theta d\theta$

$$\therefore I = \int \tan^{-1} \left(\tan \frac{\theta}{2} \right) (-\sin\theta) d\theta$$

$$= -\frac{1}{2} \int \theta \sin\theta d\theta$$

$Let\, \theta = u \text{ and } \sin\theta d\theta = v \text{ so that } \sin\theta = \int v \, d\theta$

$$Then, \int uv \, dx = u \int (v \, dx) - \left(\int \frac{du}{dx} \int v \, dx \right) dx$$

$$Hence, I = -\frac{1}{2} \left(-\theta\cos\theta - \int -\cos\theta d\theta \right)$$

$$= -\frac{1}{2} \left(-\theta\cos\theta + \sin\theta \right) + c$$

$$= -\frac{1}{2} \left(-\theta\cos\theta + \sqrt{1-\cos^2\theta} \right) + c$$

$$= -\frac{1}{2} \left(-x\cos^{-1} x + \sqrt{1-x^2} \right) + c$$

Indefinite Integrals Ex 19.25 Q58

Let $I = \int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx$

Let $x = a\tan^2\theta$

$dx = 2a\tan\theta\sec^2\theta\,d\theta$

$$I = \int\left(\sin^{-1}\sqrt{\frac{a\tan^2\theta}{a+a\tan^2\theta}}\right)\left(2a\tan\theta\sec^2\theta\right)d\theta$$

$$= \int\left(\sin^{-1}\sqrt{\frac{\tan^2\theta}{\sec^2\theta}}\right)\left(2a\tan\theta\sec^2\theta\right)d\theta$$

$$= \int \sin^{-1}\left(\sin\theta\right)\left(2a\tan\theta\sec^2\theta\right)d\theta$$

$$= \int 2\theta a\tan\theta\sec^2\theta\,d\theta$$

$$= 2a\int\theta\left(\tan\theta\sec^2\theta\right)d\theta$$

$$= 2a\left[\theta\int\tan\theta\sec^2\theta\,d\theta - \int\left(\int\tan\theta\sec^2\theta\,d\theta\right)d\theta\right]$$

$$= 2a\left[\theta\frac{\tan^2\theta}{2} - \int\frac{\tan^2\theta}{2}\,d\theta\right]$$

$$= a\theta\tan^2\theta - \frac{2a}{2}\int\left(\sec^2\theta - 1\right)d\theta$$

$$= a\theta\tan^2\theta - a\tan\theta + a\theta + c$$

$$= a\left(\tan^{-1}\sqrt{\frac{x}{a}}\right)\frac{x}{a} - a\sqrt{\frac{x}{a}} + a\tan^{-1}\sqrt{\frac{x}{a}} + c$$

$$I = x\tan^{-1}\sqrt{\frac{x}{a}} - \sqrt{ax} + a\tan^{-1}\sqrt{\frac{x}{a}} + c$$

Indefinite Integrals Ex 19.25 Q59

Let $I = \int\frac{x^3\sin^{-1}x^2}{\sqrt{1-x^4}}\,dx$

Let $\sin^{-1}x^2 = t$

$$\frac{1}{\sqrt{1-x^4}}\left(2x\right)dx = dt$$

$$I = \int\frac{x^2\sin^{-1}x^2}{\sqrt{1-x^2}}\,x\,dx$$

$$= \int\left(\sin t\right)t\frac{dt}{2}$$

$$= \frac{1}{2}\int t\sin t\,dt$$

$$= \frac{1}{2}\left[t\int\sin t\,dt - \int\left(1\int\sin t\,dt\right)dt\right]$$

$$= \frac{1}{2}\left[t\left(-\cos t\right) - \int\left(-\cos t\right)dt\right]$$

$$= \frac{1}{2}\left[-t\cos t + \sin t\right] + c$$

$$I = \frac{1}{2}\left[x^2 - \sqrt{1-x^4}\,\sin^{-1}x^2\right] + c$$

Indefinite Integrals Ex 19.25 Q60

Let $I = \int \frac{x^2 \sin^{-1} x}{\left(1 - x^2\right)^{\frac{3}{2}}} dx$

Let $\sin^{-1} x = t$

$\frac{1}{\sqrt{1 - x^2}} dx = dt$

$$I = \int \frac{\sin^2 t \times t}{\left(1 - \sin^2 t\right)} dt$$

$$= \int \frac{t \sin^2 t}{\cos^2 t} dt$$

$$= \int t \tan^2 t dt$$

$$= \int t \left(\sec^2 t - 1\right) dt$$

$$= \int t \sec^2 t \, dt - \frac{t^2}{2}$$

$$= t \int \sec^2 t \, dt - \int \left(1 \int \sec^2 t \, dt\right) dt - \frac{t^2}{2}$$

$$= t \tan t - \int \tan t \, dt - \frac{t^2}{2}$$

$$= t \tan t - \log \sec t - \frac{t^2}{2} + c$$

$$I = \frac{x}{\sqrt{1 - x^2}} \sin^{-1} x + \log\left|1 - x^2\right| - \frac{1}{2}\left(\sin^{-1} x\right)^2 + c$$

*********** END **********