Higher Order Derivatives Ex 12.1 Q27

$$y = \left[\log\left(x + \sqrt{1+x^2}\right)\right]^2$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = 2\log\left(x + \sqrt{1+x^2}\right) \times \frac{1}{x + \sqrt{1+x^2}} \times \left(1 + \frac{1 \times 2x}{2\sqrt{1+x^2}}\right)$$

$$\Rightarrow \quad y_1 = \frac{2\log\left(x + \sqrt{1+x^2}\right)}{x + \sqrt{1+x^2}} \times \frac{\sqrt{1+x^2} + x}{\sqrt{1+x^2}} = \frac{2\log\left(x + \sqrt{1+x^2}\right)}{\sqrt{1+x^2}}$$

squaring both sides

$$\Rightarrow \quad (y_1)^2 = \frac{4}{1+x^2}\left[\log\left(x + \sqrt{1+x^2}\right)\right]^2 = \frac{4y}{1+x^2}$$

$$\Rightarrow \quad \left(1+x^2\right)(y_1)^2 = 4y$$

differentiating w.r.t. x

$$\Rightarrow \quad \left(1+x^2\right)2y_1y_2 + 2x\,(y_1)^2 = 4y_1$$

$$\Rightarrow \quad \left(1+x^2\right)y_2 + xy_1 = 2$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q28

The given relationship is $y = \left(\tan^{-1} x\right)^2$

Then,

$$y_1 = 2\tan^{-1} x \frac{d}{dx}\left(\tan^{-1} x\right)$$

$$\Rightarrow y_1 = 2\tan^{-1} x.\frac{1}{1+x^2}$$

$$\Rightarrow \left(1+x^2\right)y_1 = 2\tan^{-1} x$$

Again differentiating with respect to x on both the sides, we obtain

$$\left(1+x^2\right)y_2 + 2xy_1 = 2\left(\frac{1}{1+x^2}\right)$$

$$\Rightarrow \left(1+x^2\right)^2 y_2 + 2x\left(1+x^2\right)y_1 = 2$$

Hence, proved.

Higher Order Derivatives Ex 12.1 Q29

$y = \cot x$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = -\operatorname{cosec}^2 x$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\left[2\operatorname{cosec}x\left(-\operatorname{cosec}x \cot x\right)\right] = 2\operatorname{cosec}^2 x \cot x = -2\frac{dy}{dx}.y$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + 2y\frac{dy}{dx} = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q30

$$y = \log\left(\frac{x^2}{e^2}\right)$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{x^2/e^2} \times \frac{1}{e^2} \times 2x = \frac{2}{x}$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 2\left(\frac{-1}{x^2}\right) = \frac{-2}{x^2}$$

*********** END **********