Trigonometric Identities Ex 6.1 Q57

Answer :

In the given question, we need to prove

$$\left(\frac{1}{\sec^2\theta-\cos^2\theta}+\frac{1}{\operatorname{cosec}^2\theta-\sin^2\theta}\right)\sin^2\theta\cos^2\theta=\left(\frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}\right)$$

Now, using $\sec\theta=\frac{1}{\cos\theta}$ and $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$ in L.H.S, we get

$$L.H.S.=\left(\frac{1}{\left(\frac{1}{\cos^2\theta}\right)-\cos^2\theta}+\frac{1}{\left(\frac{1}{\sin^2\theta}\right)-\sin^2\theta}\right)\sin^2\theta\cos^2\theta$$

$$=\left(\frac{1}{\left(\frac{1-\cos^4\theta}{\cos^2\theta}\right)}+\frac{1}{\left(\frac{1-\sin^4\theta}{\sin^2\theta}\right)}\right)\sin^2\theta\cos^2\theta$$

$$=\left(\frac{\cos^2\theta}{1-\cos^4\theta}+\frac{\sin^2\theta}{1-\sin^4\theta}\right)\sin^2\theta\cos^2\theta$$

Further using the identity $a^2-b^2=(a+b)(a-b)$, we get

$$L.H.S.=\left(\frac{\cos^2\theta}{(1-\cos^2\theta)(1+\cos^2\theta)}+\frac{\sin^2\theta}{(1-\sin^2\theta)(1+\sin^2\theta)}\right)\sin^2\theta\cos^2\theta$$

$$=\left(\frac{\cos^2\theta}{\sin^2\theta(1+\cos^2\theta)}+\frac{\sin^2\theta}{\cos^2\theta(1+\sin^2\theta)}\right)\sin^2\theta\cos^2\theta$$

$$=\left(\frac{\cos^2\theta\left(\cos^2\theta(1+\sin^2\theta)\right)+\sin^2\theta\left(\sin^2\theta(1+\cos^2\theta)\right)}{\sin^2\theta\cos^2\theta(1+\cos^2\theta)(1+\sin^2\theta)}\right)\sin^2\theta\cos^2\theta$$

$$=\left(\frac{\cos^4\theta(1+\sin^2\theta)+\sin^4\theta(1+\cos^2\theta)}{(1+\cos^2\theta)(1+\sin^2\theta)}\right)$$

Further using the identity $\sin^2\theta+\cos^2\theta=1$, we get

$$L.H.S.=\left(\frac{\cos^4\theta+\cos^4\theta\sin^2\theta+\sin^4\theta+\sin^4\theta\cos^2\theta}{1+\cos^2\theta+\sin^2\theta+\sin^2\theta\cos^2\theta}\right)$$

$$=\left(\frac{\cos^4\theta+\sin^4\theta+\cos^2\theta\sin^2\theta(\cos^2\theta+\sin^2\theta)}{2+\sin^2\theta\cos^2\theta}\right)$$

$$=\left(\frac{\cos^4\theta+\sin^4\theta+\cos^2\theta\sin^2\theta(1)}{2+\sin^2\theta\cos^2\theta}\right)$$

Now, from the identity $a^2+b^2=(a+b)^2-2ab$, we get

So,

$$L.H.S. = \left(\frac{\left(\cos^2\theta + \sin^2\theta\right)^2 - 2\cos^2\theta\sin^2\theta + \cos^2\theta\sin^2\theta}{2 + \sin^2\theta\cos^2\theta} \right)$$

$$= \left(\frac{(1)^2 - \cos^2\theta\sin^2\theta}{2 + \sin^2\theta\cos^2\theta} \right)$$

$$= \left(\frac{1 - \sin^2\theta\cos^2\theta}{2 + \sin^2\theta\cos^2\theta} \right)$$

Hence proved.

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