Q12 : $\left(x^3-1\right)^{\frac{1}{3}}x^5$

Answer :

Let $x^3-1=t$

$\therefore\ 3x^2dx=dt$

$$\Rightarrow \int\left(x^3-1\right)^{\frac{1}{3}}x^5dx=\int\left(x^3-1\right)^{\frac{1}{3}}x^3\cdot x^2dx$$
$$=\int t^{\frac{1}{3}}(t+1)\frac{dt}{3}$$
$$=\frac{1}{3}\int\left(t^{\frac{4}{3}}+t^{\frac{1}{3}}\right)dt$$
$$=\frac{1}{3}\left[\frac{t^{\frac{7}{3}}}{\frac{7}{3}}+\frac{t^{\frac{4}{3}}}{\frac{4}{3}}\right]+C$$
$$=\frac{1}{3}\left[\frac{3}{7}t^{\frac{7}{3}}+\frac{3}{4}t^{\frac{4}{3}}\right]+C$$
$$=\frac{1}{7}\left(x^3-1\right)^{\frac{7}{3}}+\frac{1}{4}\left(x^3-1\right)^{\frac{4}{3}}+C$$

Answer needs Correction? **Click Here**

Q13 : $\dfrac{x^2}{\left(2+3x^3\right)^3}$

Answer :

Let $2+3x^3=t$

$\therefore\ 9x^2\,dx=dt$

$$\Rightarrow \int\frac{x^2}{\left(2+3x^3\right)^3}dx=\frac{1}{9}\int\frac{dt}{(t)^3}$$
$$=\frac{1}{9}\left[\frac{t^{-2}}{-2}\right]+C$$
$$=\frac{-1}{18}\left(\frac{1}{t^2}\right)+C$$
$$=\frac{-1}{18\left(2+3x^3\right)^2}+C$$

Answer needs Correction? **Click Here**

Q14 : $\dfrac{1}{x(\log x)^m}$, $x>0$

Answer :

Let $\log x=t$

$\therefore\ \frac{1}{x}dx=dt$

$$\Rightarrow \int\frac{1}{x(\log x)^m}dx=\int\frac{dt}{(t)^m}$$
$$=\left(\frac{t^{-m+1}}{1-m}\right)+C$$
$$=\frac{(\log x)^{1-m}}{(1-m)}+C$$

Answer needs Correction? **Click Here**

Q15 : $\dfrac{x}{9-4x^2}$

Answer :

Let $9-4x^2=t$

$\therefore\ -8x\,dx=dt$

$$\Rightarrow \int\frac{x}{9-4x^2}dx=\frac{-1}{8}\int\frac{1}{t}dt$$

$$=\frac{-1}{8}\log|t|+C$$
$$=\frac{-1}{8}\log\left|9-4x^2\right|+C$$

Answer needs Correction? **Click Here**

Q16 : e^{2x+3}

Answer :

Let $2x+3=t$

$\therefore 2dx = dt$

$$\Rightarrow \int e^{2x+3}dx=\frac{1}{2}\int e^t dt$$
$$=\frac{1}{2}\left(e^t\right)+C$$
$$=\frac{1}{2}e^{(2x+3)}+C$$

Answer needs Correction? **Click Here**

Q17 : $\frac{x}{e^{x^2}}$

Answer :

Let $x^2=t$

$\therefore 2xdx = dt$

$$\Rightarrow \int \frac{x}{e^{x^2}}dx=\frac{1}{2}\int\frac{1}{e^t}dt$$

$$=\frac{1}{2}\int e^{-t}dt$$
$$=\frac{1}{2}\left(\frac{e^{-t}}{-1}\right)+C$$
$$=-\frac{1}{2}e^{-x^2}+C$$
$$=\frac{-1}{2e^{x^2}}+C$$

Answer needs Correction? **Click Here**

Q18 : $\frac{e^{\tan^{-1}x}}{1+x^2}$

Answer :

Let $\tan^{-1}x=t$

$\therefore \frac{1}{1+x^2}dx=dt$

$$\Rightarrow \int\frac{e^{\tan^{-1}x}}{1+x^2}dx=\int e^t dt$$

$$=e^t+C$$
$$=e^{\tan^{-1}x}+C$$

Answer needs Correction? **Click Here**

Q19 : $\frac{e^{2x}-1}{e^{2x}+1}$

Answer :

$\frac{e^{2x}-1}{e^{2x}+1}$

Dividing numerator and denominator by e^x, we obtain

$$\frac{\frac{\left(e^{2x}-1\right)}{e^x}}{\frac{\left(e^{2x}+1\right)}{e^x}}=\frac{e^x-e^{-x}}{e^x+e^{-x}}$$

Let $e^x+e^{-x}=t$

$\therefore \left(e^x-e^{-x}\right)dx=dt$

$$\Rightarrow \int\frac{e^{2x}-1}{e^{2x}+1}dx=\int\frac{e^x-e^{-x}}{e^x+e^{-x}}dx$$

$$=\int\frac{dt}{t}$$
$$=\log|t|+C$$
$$=\log\left|e^x+e^{-x}\right|+C$$

Answer needs Correction? **Click Here**

Q20 : $e^{2x}-e^{-2x}$

$\overline{e^{2x}+e^{-2x}}$

Answer :

Let $e^{2x}+e^{-2x}=t$

$\therefore \left(2e^{2x}-2e^{-2x}\right)dx=dt$

$\Rightarrow 2\left(e^{2x}-e^{-2x}\right)dx=dt$

$$\Rightarrow \int\left(\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\right)dx=\int\frac{dt}{2t}$$
$$=\frac{1}{2}\int\frac{1}{t}dt$$
$$=\frac{1}{2}\log|t|+C$$
$$=\frac{1}{2}\log\left|e^{2x}+e^{-2x}\right|+C$$

Answer needs Correction? **Click Here**

Q21 : $\tan^2(2x-3)$

Answer :

$\tan^2(2x-3)=\sec^2(2x-3)-1$

Let $2x-3=t$

$\therefore 2dx=dt$

$$\Rightarrow \int\tan^2(2x-3)dx=\int\left[\left(\sec^2(2x-3)\right)-1\right]dx$$
$$=\frac{1}{2}\int\left(\sec^2 t\right)dt-\int 1dx$$
$$=\frac{1}{2}\int\sec^2 t\,dt-\int 1dx$$
$$=\frac{1}{2}\tan t-x+C$$
$$=\frac{1}{2}\tan(2x-3)-x+C$$

Answer needs Correction? **Click Here**

Q22 : $\sec^2(7-4x)$

Answer :

Let $7-4x=t$

$\therefore -4dx=dt$

$$\therefore \int\sec^2(7-4x)dx=\frac{-1}{4}\int\sec^2 t\,dt$$
$$=\frac{-1}{4}(\tan t)+C$$
$$=\frac{-1}{4}\tan(7-4x)+C$$

Answer needs Correction? **Click Here**

Q23 : $\dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$

Answer :

Let $\sin^{-1}x=t$

$\therefore \dfrac{1}{\sqrt{1-x^2}}dx=dt$

$$\Rightarrow \int\frac{\sin^{-1}x}{\sqrt{1-x^2}}dx=\int t\,dt$$

$$=\frac{t^2}{2}+C$$
$$=\frac{\left(\sin^{-1}x\right)^2}{2}+C$$

Answer needs Correction? **Click Here**

Q24 : $\dfrac{2\cos x-3\sin x}{6\cos x+4\sin x}$

Answer :

$$\frac{2\cos x-3\sin x}{6\cos x+4\sin x}=\frac{2\cos x-3\sin x}{2(3\cos x+2\sin x)}$$

Let $3\cos x+2\sin x=t$

$\therefore (-3\sin x+2\cos x)dx=dt$

$$\int\frac{2\cos x-3\sin x}{6\cos x+4\sin x}dx=\int\frac{dt}{2t}$$

$$= \frac{1}{2}\int \frac{1}{t}dt$$
$$= \frac{1}{2}\log|t| + C$$
$$= \frac{1}{2}\log|2\sin x + 3\cos x| + C$$

Answer needs Correction? **Click Here**

Q25 : $\frac{1}{\cos^2 x(1-\tan x)^2}$

Answer :

$$\frac{1}{\cos^2 x(1-\tan x)^2} = \frac{\sec^2 x}{(1-\tan x)^2}$$

Let $(1-\tan x) = t$

$\therefore\ -\sec^2 x dx = dt$

$$\Rightarrow \int \frac{\sec^2 x}{(1-\tan x)^2}dx = \int \frac{-dt}{t^2}$$
$$= -\int t^{-2} dt$$
$$= +\frac{1}{t} + C$$
$$= \frac{1}{(1-\tan x)} + C$$

Answer needs Correction? **Click Here**

Q26 : $\frac{\cos\sqrt{x}}{\sqrt{x}}$

Answer :

Let $\sqrt{x} = t$

$\therefore\ \frac{1}{2\sqrt{x}}dx = dt$

$$\Rightarrow \int \frac{\cos\sqrt{x}}{\sqrt{x}}dx = 2\int \cos t\, dt$$
$$= 2\sin t + C$$
$$= 2\sin\sqrt{x} + C$$

Answer needs Correction? **Click Here**

Q27 : $\sqrt{\sin 2x}\cos 2x$

Answer :

Let $\sin 2x = t$

$\therefore\ 2\cos 2x\, dx = dt$

$$\Rightarrow \int \sqrt{\sin 2x}\ \cos 2x\, dx = \frac{1}{2}\int \sqrt{t}\, dt$$
$$= \frac{1}{2}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$
$$= \frac{1}{3}t^{\frac{3}{2}} + C$$
$$= \frac{1}{3}(\sin 2x)^{\frac{3}{2}} + C$$

Answer needs Correction? **Click Here**

Q28 : $\frac{\cos x}{\sqrt{1+\sin x}}$

Answer :

Let $1+\sin x = t$

$\therefore \cos x\, dx = dt$

$$\Rightarrow \int \frac{\cos x}{\sqrt{1+\sin x}}dx = \int \frac{dt}{\sqrt{t}}$$
$$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$$
$$= 2\sqrt{t} + C$$
$$= 2\sqrt{1+\sin x} + C$$

Answer needs Correction? **Click Here**

Q29 : $\cot x \log \sin x$

Answer :

Let log sin $x = t$

$$\Rightarrow \frac{1}{\sin x} \cdot \cos x \; dx = dt$$
$$\therefore \cot x \; dx = dt$$

$$\begin{aligned}\Rightarrow \int \cot x \log \sin x \; dx &= \int t \, dt \\ &= \frac{t^2}{2} + C \\ &= \frac{1}{2}(\log \sin x)^2 + C\end{aligned}$$

Answer needs Correction? Click Here

Q30 : $\dfrac{\sin x}{1+\cos x}$

Answer :

Let 1 + cos $x = t$

$\therefore$ - sin $x \, dx = dt$

$$\begin{aligned}\Rightarrow \int \frac{\sin x}{1+\cos x} dx &= \int -\frac{dt}{t} \\ &= -\log|t| + C \\ &= -\log|1+\cos x| + C\end{aligned}$$

Answer needs Correction? Click Here

Q31 : $\dfrac{\sin x}{(1+\cos x)^2}$

Answer :

Let 1 + cos $x = t$

$\therefore$ - sin $x \, dx = dt$

$$\begin{aligned}\Rightarrow \int \frac{\sin x}{(1+\cos x)^2} dx &= \int -\frac{dt}{t^2} \\ &= -\int t^{-2} dt \\ &= \frac{1}{t} + C \\ &= \frac{1}{1+\cos x} + C\end{aligned}$$

Answer needs Correction? Click Here

Q32 : $\dfrac{1}{1+\cot x}$

Answer :

$$\begin{aligned}\text{Let } I &= \int \frac{1}{1+\cot x} dx \\ &= \int \frac{1}{1+\frac{\cos x}{\sin x}} dx \\ &= \int \frac{\sin x}{\sin x + \cos x} dx \\ &= \frac{1}{2} \int \frac{2 \sin x}{\sin x + \cos x} dx \\ &= \frac{1}{2} \int \frac{(\sin x + \cos x) + (\sin x - \cos x)}{(\sin x + \cos x)} dx \\ &= \frac{1}{2} \int 1 \, dx + \frac{1}{2} \int \frac{\sin x - \cos x}{\sin x + \cos x} dx \\ &= \frac{1}{2}(x) + \frac{1}{2} \int \frac{\sin x - \cos x}{\sin x + \cos x} dx\end{aligned}$$

Let sin x + cos $x = t \Rightarrow$ (cos x - sin x) $dx = dt$

$$\begin{aligned}\therefore I &= \frac{x}{2} + \frac{1}{2} \int \frac{-(dt)}{t} \\ &= \frac{x}{2} - \frac{1}{2} \log|t| + C \\ &= \frac{x}{2} - \frac{1}{2} \log|\sin x + \cos x| + C\end{aligned}$$

Answer needs Correction? Click Here

Q33 : $\dfrac{1}{1-\tan x}$

Answer :

$$\begin{aligned}\text{Let } I &= \int \frac{1}{1-\tan x} dx \\ &= \int \frac{1}{1-\frac{\sin x}{\cos x}} dx\end{aligned}$$

$$= \int \frac{\cos x}{\cos x - \sin x} dx$$
$$= \frac{1}{2}\int \frac{2\cos x}{\cos x - \sin x} dx$$
$$= \frac{1}{2}\int \frac{(\cos x - \sin x) + (\cos x + \sin x)}{(\cos x - \sin x)} dx$$
$$= \frac{1}{2}\int 1\, dx + \frac{1}{2}\int \frac{\cos x + \sin x}{\cos x - \sin x} dx$$
$$= \frac{x}{2} + \frac{1}{2}\int \frac{\cos x + \sin x}{\cos x - \sin x} dx$$

Put cos x - sin x = t ⇒ (- sin x - cos x) dx = dt

$$\therefore I = \frac{x}{2} + \frac{1}{2}\int \frac{-(dt)}{t}$$
$$= \frac{x}{2} - \frac{1}{2}\log|t| + C$$
$$= \frac{x}{2} - \frac{1}{2}\log|\cos x - \sin x| + C$$

Answer needs Correction? **Click Here**

Q34 : $\frac{\sqrt{\tan x}}{\sin x \cos x}$

Answer :

$$\text{Let } I = \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$$
$$= \int \frac{\sqrt{\tan x} \times \cos x}{\sin x \cos x \times \cos x} dx$$
$$= \int \frac{\sqrt{\tan x}}{\tan x \cos^2 x} dx$$
$$= \int \frac{\sec^2 x\, dx}{\sqrt{\tan x}}$$

Let $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$$\therefore I = \int \frac{dt}{\sqrt{t}}$$
$$= 2\sqrt{t} + C$$
$$= 2\sqrt{\tan x} + C$$

Answer needs Correction? **Click Here**

Q35 : $\frac{(1+\log x)^2}{x}$

Answer :

Let 1 + log x = t

$$\therefore \frac{1}{x} dx = dt$$

$$\Rightarrow \int \frac{(1+\log x)^2}{x} dx = \int t^2 dt$$
$$= \frac{t^3}{3} + C$$
$$= \frac{(1+\log x)^3}{3} + C$$

Answer needs Correction? **Click Here**

Q36 : $\frac{(x+1)(x+\log x)^2}{x}$

Answer :

$$\frac{(x+1)(x+\log x)^2}{x} = \left(\frac{x+1}{x}\right)(x+\log x)^2 = \left(1+\frac{1}{x}\right)(x+\log x)^2$$

Let $(x+\log x) = t$

$$\therefore \left(1+\frac{1}{x}\right) dx = dt$$

$$\Rightarrow \int \left(1+\frac{1}{x}\right)(x+\log x)^2 dx = \int t^2 dt$$
$$= \frac{t^3}{3} + C$$
$$= \frac{1}{3}(x+\log x)^3 + C$$

Answer needs Correction? **Click Here**

Q37 : $\frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8}$

Answer :

Let $x^4 = t$

$\therefore 4x^3 dx = dt$

$$\Rightarrow \int \frac{x^3 \sin\left(\tan^{-1} x^4\right)}{1+x^8} dx = \frac{1}{4}\int \frac{\sin\left(\tan^{-1} t\right)}{1+t^2} dt \qquad ...(1)$$

Let $\tan^{-1} t = u$

$\therefore \frac{1}{1+t^2} dt = du$

From (1), we obtain

$$\int \frac{x^3 \sin\left(\tan^{-1} x^4\right) dx}{1+x^8} = \frac{1}{4}\int \sin u \, du$$
$$= \frac{1}{4}(-\cos u) + C$$

$$= \frac{-1}{4}\cos\left(\tan^{-1} t\right) + C$$
$$= \frac{-1}{4}\cos\left(\tan^{-1} x^4\right) + C$$

Answer needs Correction? **Click Here**

Q38 : $\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx$ equals

(A) $10^x - x^{10} + C$ (B) $10^x + x^{10} + C$

(C) $\left(10^x - x^{10}\right)^{-1} + C$ (D) $\log\left(10^x + x^{10}\right) + C$

Answer :

Let $x^{10} + 10^x = t$

$\therefore \left(10x^9 + 10^x \log_e 10\right) dx = dt$

$$\Rightarrow \int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx = \int \frac{dt}{t}$$

$= \log t + C$

$= \log\left(10^x + x^{10}\right) + C$

Hence, the correct answer is D.

Answer needs Correction? **Click Here**

Q39 : $\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx$ equals

(A) $10^x - x^{10} + C$ (B) $10^x + x^{10} + C$

(C) $\left(10^x - x^{10}\right)^{-1} + C$ (D) $\log\left(10^x + x^{10}\right) + C$

Answer :

Let $x^{10} + 10^x = t$

$\therefore \left(10x^9 + 10^x \log_e 10\right) dx = dt$

$$\Rightarrow \int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx = \int \frac{dt}{t}$$

$= \log t + C$

$= \log\left(10^x + x^{10}\right) + C$

Hence, the correct answer is D.

Answer needs Correction? **Click Here**

*********** END **********