Tangents and Normals Ex 16.2 Q12

The given equations are,

$$y = x \log_e x \quad \text{---(i)}$$

$$2x - 2y + 3 = 0 \quad \text{---(ii)}$$

Slope m_1 of (i)

$$m_1 = \frac{dy}{dx} = \log_e x + 1$$

slope m_2 of (ii)

$$m_2 = 1$$

Tangents and Normals Ex 16.2 Q13

The equation of the given curve is $y = x^2 - 2x + 7$.

On differentiating with respect to x, we get:

$$\frac{dy}{dx} = 2x - 2$$

(a) The equation of the line is $2x - y + 9 = 0$.

$2x - y + 9 = 0 \Rightarrow y = 2x + 9$

This is of the form $y = mx + c$.

$\therefore$Slope of the line $= 2$

If a tangent is parallel to the line $2x - y + 9 = 0$, then the slope of the tangent is equal to the slope of the line.

Therefore, we have:

$2 = 2x - 2$

$\Rightarrow 2x = 4$

$\Rightarrow x = 2$

Now, $x = 2$

$\Rightarrow y = 4 - 4 + 7 = 7$

Thus, the equation of the tangent passing through (2, 7) is given by,

$y - 7 = 2(x - 2)$
$\Rightarrow y - 2x - 3 = 0$

Hence, the equation of the tangent line to the given curve (which is parallel to line $2x - y + 9 = 0$) is $y - 2x - 3 = 0$.

(b) The equation of the line is $5y - 15x = 13$.

$5y - 15x = 13 \Rightarrow y = 3x + \frac{13}{5}$

This is of the form $y = mx + c$.

$\therefore$Slope of the line = 3

If a tangent is perpendicular to the line $5y - 15x = 13$, then the slope of the tangent is $\frac{-1}{\text{slope of the line}} = \frac{-1}{3}$.

$\Rightarrow 2x - 2 = \frac{-1}{3}$

$\Rightarrow 2x = \frac{-1}{3} + 2$

$\Rightarrow 2x = \frac{5}{3}$

$\Rightarrow x = \frac{5}{6}$

Now, $x = \frac{5}{6}$

$\Rightarrow y = \frac{25}{36} - \frac{10}{6} + 7 = \frac{25 - 60 + 252}{36} = \frac{217}{36}$

Thus, the equation of the tangent passing through $\left(\frac{5}{6}, \frac{217}{36}\right)$ is given by,

$y - \frac{217}{36} = -\frac{1}{3}\left(x - \frac{5}{6}\right)$
$\Rightarrow \frac{36y - 217}{36} = \frac{-1}{18}(6x - 5)$
$\Rightarrow 36y - 217 = -2(6x - 5)$
$\Rightarrow 36y - 217 = -12x + 10$
$\Rightarrow 36y + 12x - 227 = 0$

Hence, the equation of the tangent line to the given curve (which is perpendicular to line $5y - 15x = 13$) is $36y + 12x - 227 = 0$.

********** END **********