Exercise 7A

Question 30

$$\text{LHS} = \frac{\cos\theta\,\text{cosec}\theta - \sin\theta\sec\theta}{\cos\theta + \sin\theta}$$

$$= \frac{\cos\theta \times \frac{1}{\sin\theta} - \sin\theta \times \frac{1}{\cos\theta}}{\cos\theta + \sin\theta} = \frac{\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}}{(\cos\theta + \sin\theta)}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta(\cos\theta + \sin\theta)}$$

$$= \frac{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}{\sin\theta\cos\theta(\cos\theta + \sin\theta)}$$

$$= \frac{\cos\theta}{\sin\theta\cos\theta} - \frac{\sin\theta}{\sin\theta\cos\theta}$$

$$= \text{cosec}\theta - \sec\theta = \text{RHS}$$

$\therefore$ LHS = RHS

Question 31

$$\text{LHS} = (1 + \tan\theta + \cot\theta)(\sin\theta - \cos\theta)$$

$$= \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)(\sin\theta - \cos\theta)$$

$$= \left(\frac{\cos\theta\sin\theta + \sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)(\sin\theta - \cos\theta)$$

$$= \frac{(\cos\theta\sin\theta + 1)}{\cos\theta\sin\theta}(\sin\theta - \cos\theta)$$

$$\text{RHS} = \left(\frac{\sec\theta}{\text{cosec}^2\theta} - \frac{\text{cosec}\theta}{\sec^2\theta}\right) = \left(\frac{\frac{1}{\cos\theta}}{\frac{1}{\sin^2\theta}} - \frac{\frac{1}{\sin\theta}}{\frac{1}{\cos^2\theta}}\right)$$

$$= \left(\frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}\right) = \frac{\sin^3\theta - \cos^3\theta}{\cos\theta\sin\theta}$$

$$= \frac{(\sin\theta - \cos\theta)\left(\sin^2\theta + \cos^2\theta + \cos\theta\sin\theta\right)}{\cos\theta\sin\theta}$$

$$\left[\because a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)\right]$$

$$= \frac{(\sin\theta - \cos\theta)(1 + \cos\theta\sin\theta)}{\cos\theta\sin\theta}$$

$\therefore$ LHS = RHS

********** END **********