



Binary Operations Ex 3.2 Q1

We have,

$$a * b = \text{l.c.m.}(a, b) \text{ for all } a, b \in N$$

(1)

Now,

$$2 * 4 = \text{l.c.m.}(2, 4) = 4$$

$$3 * 5 = \text{l.c.m.}(3, 5) = 15$$

$$1 * 6 = \text{l.c.m.}(1, 6) = 6$$

(ii)

Commutativity:

Let  $a, b \in N$  then,

$$\begin{aligned} a * b &= \text{l.c.m.}(a, b) \\ &= \text{l.c.m.}(b, a) \\ &= b * a \end{aligned}$$

$$\Rightarrow a * b = b * a$$

$\therefore$   $*$  is commutative on  $N$ .

Associativity:

Let  $a, b, c \in N$  then,

$$\begin{aligned} (a * b) * c &= \text{l.c.m.}(a, b) * c \\ &= \text{l.c.m.}(a, b, c) \end{aligned} \quad \text{--- (i)}$$

$$\begin{aligned} \text{and, } a * (b * c) &= a * \text{l.c.m.}(b, c) \\ &= \text{l.c.m.}(a, b, c) \end{aligned} \quad \text{--- (ii)}$$

From (i) and (ii)

$$(a * b) * c = a * (b * c)$$

$\therefore$   $*$  is associative on  $N$ .

Binary Operations Ex 3.2 Q2

(i) Clearly, by definition  $a * b = 1 = b * a$ ,  $\forall a, b \in \mathbb{N}$

Also,  $(a * b) * c = (1 * c) = 1$

and  $a * (b * c) = (a * 1) = 1 \quad \forall a, b, c \in \mathbb{N}$

Hence,  $\mathbb{N}$  is both associative and commutative.

(ii)  $a * b = \frac{a+b}{2} = \frac{b+a}{2} = b * a$ ,

which shows  $*$  is commutative.

$$\text{Further, } (a * b) * c = \left( \frac{a+b}{2} \right) * c = \frac{\left( \frac{a+b}{2} \right) + c}{2} = \frac{a+b+2c}{4}$$

$$a * (b * c) = a * \left( \frac{b+c}{2} \right) = \frac{a + \left( \frac{b+c}{2} \right)}{2} = \frac{2a+b+c}{2} \neq \frac{a+b+2c}{4}$$

Hence,  $*$  is not associative.

Binary Operations Ex 3.2 Q3

We have, binary operator  $*$  defined on  $A$  and is given by

$$a * b = b \text{ for all } a, b \in A$$

Commutativity: Let  $a, b \in A$ , then

$$a * b = b \neq a = b * a$$

$$\Rightarrow a * b \neq b * a$$

$\therefore$   $'*$  is not commutative on  $A$ .

Associativity: Let  $a, b, c \in A$ , then

$$(a * b) * c = b * c = c \quad \text{--- (i)}$$

$$\text{and, } a * (b * c) = a * c = c \quad \text{--- (ii)}$$

From (i) and (ii)

$$(a * b) * c = a * (b * c)$$

$$\Rightarrow ' * ' \text{ is associative on } A.$$

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