Trigonometric Ratios Ex 5.1 Q15

Answer :

Given: $\cot\theta = \frac{1}{\sqrt{3}}$ ......(1)

To show that $\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{3}{5}$

Now, we know that $\cot\theta = \frac{1}{\tan\theta}$

Since $\tan\theta = \frac{\text{Perpendicular side opposite to}\angle\theta}{\text{Base side adjacent to}\angle\theta}$

Therefore,

$$\cot\theta = \frac{1}{\frac{\text{Perpendicular side opposite to}\angle\theta}{\text{Base side adjacent to}\angle\theta}}$$

Therefore,

$$\cot\theta = \frac{\text{Base side adjacent to}\angle\theta}{\text{Perpendicular side opposite to}\angle\theta} \quad \text{...... (2)}$$

Comparing Equation (1) and (2)

We get,

Base side adjacent to $\angle\theta$ = 1

Perpendicular side opposite to $\angle\theta = \sqrt{3}$

Therefore, Triangle representing angle θ is as shown below

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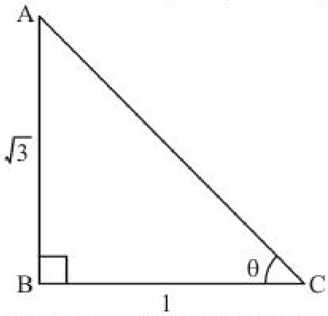

Hypotenuse *AC* is unknown and it can be found by using Pythagoras theorem

Therefore by applying Pythagoras theorem

We get,

$$AC^2 = AB^2 + BC^2$$

Therefore by substituting the values of known sides

We get,

$$AC^2 = \left(\sqrt{3}\right)^2 + 1^2$$

Therefore,

$$AC^2 = 3 + 1$$
$$AC^2 = 4$$
$$AC = \sqrt{4}$$

Therefore,

$$AC = 2 \;\ldots\ldots\; (3)$$

Now, we know that

$$\sin\theta = \frac{\text{Perpendicular side opposite to} \angle\theta}{\text{Hypotenuse}}$$

Now from figure (*a*)

We get,

$$\sin\theta = \frac{AB}{AC}$$

Therefore from figure (a) and equation (3) ,

$$\sin\theta = \frac{\sqrt{3}}{2} \;\ldots\ldots\; (4)$$

Now, we know that

$$\cos\theta = \frac{\text{Base side adjacent to} \angle\theta}{\text{Hypotenuse}}$$

Now from figure (*a*)

We get,

$$\cos\theta = \frac{BC}{AC}$$

Therefore from figure (*a*) and equation (3) ,

$$\cos\theta = \frac{1}{2} \;\ldots\ldots\; (5)$$

Now, L.H.S of the equation to be proved is as follows

$$L.H.S = \frac{1 - \cos^2\theta}{2 - \sin^2\theta}$$

Substituting the value of $\sin\theta$ and $\cos\theta$ from equation (4) and (5)

We get,

$$L.H.S = \frac{1-\left(\frac{1}{2}\right)^2}{2-\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$L.H.S = \frac{1-\frac{(1)^2}{(2)^2}}{2-\frac{\left(\sqrt{3}\right)^2}{(2)^2}}$$

$$L.H.S = \frac{1-\frac{1}{4}}{2-\frac{3}{4}}$$

Now by taking L.C.M. in numerator as well as denominator

We get,

$$L.H.S = \frac{\frac{(4\times 1)-1}{4}}{\frac{(4\times 2)-3}{4}}$$

Therefore,

$$L.H.S = \frac{\frac{4-1}{4}}{\frac{8-3}{4}}$$

$$L.H.S = \frac{\frac{3}{4}}{\frac{5}{4}}$$

Therefore,

$$L.H.S = \frac{3}{4} \times \frac{4}{5}$$

$$L.H.S = \frac{3}{5} = R.H.S$$

Therefore,

$$\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{3}{5}$$

Hence proved that $\boxed{\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{3}{5}}$

*********** END **********