Indefinite Integrals Ex 19.32 Q6

Let $\quad I = \int \dfrac{x}{\left(x^2 + 1\right)\sqrt{x}}\,dx$

Let $x = t^2$

$\Rightarrow \qquad dx = 2t\,dt$

$\therefore \qquad 2\int \dfrac{t\,dt}{\left(t^2 + 1\right)t}$

$= 2\left|\dfrac{dt}{t^4 + 1}\right|$

Dividing numerator and denominator by t^2

$I = 2\int \dfrac{\dfrac{t}{t^2}}{\left(t^2 + \dfrac{1}{t^2}\right)}\,dt$

$= \int \dfrac{\left(1 + \dfrac{1}{t^2}\right) - \left(1 - \dfrac{1}{t^2}\right)}{\left(t^2 + \dfrac{1}{t^2}\right)}\,dt$

$= \int \dfrac{\left(1 + \dfrac{1}{t^2}\right)}{\left(t - \dfrac{1}{t}\right)^2 + 2}\,dt - \int \dfrac{\left(1 - \dfrac{1}{t^2}\right)}{\left(1 + \dfrac{1}{t}\right)^2 - 2}\,dt$

Let $t - \dfrac{1}{t} = z$

$\Rightarrow \qquad \left(1 + \dfrac{1}{t^2}\right)dt = dz \qquad\qquad$ [For Ist part]

and,

$\qquad t + \dfrac{1}{t} = y$

$\Rightarrow \qquad \left(1 - \dfrac{1}{t^2}\right)dt = dy \qquad$ [For IInd part]

$\therefore \qquad I = \int \dfrac{dz}{z^2 + 2} - \int \dfrac{dy}{y^2 - 2}$

$= \dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{z}{\sqrt{2}}\right) - \dfrac{1}{2\sqrt{2}}\log\left|\dfrac{y - \sqrt{2}}{y + \sqrt{2}}\right| + c$

$= \dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{t^2 - 1}{\sqrt{2}t}\right) - \dfrac{1}{2\sqrt{2}}\log\left|\dfrac{x + 1 - \sqrt{2x}}{x + 1 + \sqrt{2x}}\right| + c$

Indefinite Integrals Ex 19.32 Q7

Let $\quad I = \int \dfrac{x}{\left(x^2 + 2X + 2\right)\sqrt{x+1}}\, dx$

Let $x + 1 = t^2$

$\Rightarrow \qquad dx = 2t\, dt$

$$= 2\int \dfrac{\left(t^2 - 1\right) t\, dt}{\left(t^4 + 1\right) t}$$

$$= 2\int \dfrac{\left(t^2 - 1\right) dt}{\left(t^4 + 1\right)}$$

$$= 2\int \dfrac{\left(1 - \dfrac{1}{t^2}\right) dt}{t^2 + \dfrac{1}{t^2}}$$

$$= 2\int \dfrac{\left(1 - \dfrac{1}{t^2}\right) dt}{\left(t + \dfrac{1}{t}\right)^2 - 2}$$

Let $t + \dfrac{1}{t} = y$

$\Rightarrow \qquad \left(1 - \dfrac{1}{t^2}\right) dt = dy$

$\therefore \qquad I = 2\int \dfrac{dy}{y^2 - 2}$

$$= \dfrac{2}{2\sqrt{2}} \log \left|\dfrac{y - \sqrt{2}}{y + \sqrt{2}}\right| + c$$

Thus,

$$I = \dfrac{1}{\sqrt{2}} \log \left|\dfrac{t^2 + 1 - \sqrt{2}t}{t^2 + 1 + \sqrt{2}t}\right| + c$$

Hence,

$$I = \dfrac{1}{\sqrt{2}} \log \left|\dfrac{x + 2 - \sqrt{2(x+1)}}{x + 2 + \sqrt{2(x+1)}}\right| + c$$

Indefinite Integrals Ex 19.32 Q8

Let $I = \int \dfrac{1}{(x-1)\sqrt{x^2+1}}\,dx$

Let $x - 1 = \dfrac{1}{t}$

$\Rightarrow \quad dx = -\dfrac{1}{t^2}\,dt$

$\therefore \quad I = -\int \dfrac{\dfrac{1}{t^2}\,dt}{\dfrac{1}{t}\sqrt{\left(\dfrac{1}{t}+1\right)^2 + 1}}$

$\quad = -\int \dfrac{dt}{\sqrt{2t^2+2t+1}}$

$\quad = -\dfrac{1}{\sqrt{2}}\int \dfrac{dt}{\sqrt{t^2+t+\dfrac{1}{2}}}$

$\quad = -\dfrac{1}{\sqrt{2}}\int \dfrac{dt}{\sqrt{\left(t+\dfrac{1}{2}\right)^2 + \dfrac{1}{4}}}$

$\therefore \quad I = -\dfrac{1}{\sqrt{2}}\log\left|\left(t+\dfrac{1}{2}\right)+\sqrt{\left(t+\dfrac{1}{2}\right)^2 + \dfrac{1}{4}}\right| + c \quad \left[\text{When } t = \dfrac{1}{x-1}\right]$

Indefinite Integrals Ex 19.32 Q9

Let $I = \int \dfrac{1}{(x+1)\sqrt{x^2+x+1}}\,dx$

Let $x + 1 = \dfrac{1}{t}$

$\Rightarrow \quad dx = -\dfrac{1}{t^2}\,dt$

$\therefore \quad I = -\int \dfrac{\dfrac{1}{t^2}\,dt}{\dfrac{1}{t}\sqrt{\left(\dfrac{1}{t^2}+\dfrac{1}{t}-1\right)}}$

$\quad = -\int \dfrac{dt}{\sqrt{1+t-t^2}}$

$\quad = -\int \dfrac{dt}{\sqrt{\dfrac{5}{4}-\left(\dfrac{1}{4}-t+t^2\right)}}$

$\quad = -\int \dfrac{dt}{\sqrt{\dfrac{5}{4}-\left(t-\dfrac{1}{2}\right)^2}}$

$\quad = -\sin^{-1}\left(\dfrac{t-\dfrac{1}{2}}{\dfrac{\sqrt{5}}{2}}\right) + c$

$\therefore \quad I = -\sin^{-1}\left(\dfrac{2t-1}{\sqrt{5}}\right) + c \qquad\qquad \left[\text{When } t = \dfrac{1}{x+1}\right]$

Indefinite Integrals Ex 19.32 Q10

Let $I = \int \dfrac{1}{(x^2 - 1)\sqrt{x^2 + 1}} \, dx$

Let $x = \dfrac{1}{t}$

$\Rightarrow \quad dx = -\dfrac{1}{t^2} \, dt$

$\therefore \quad I = -\int \dfrac{\dfrac{1}{t^2} \, dt}{\left(\dfrac{1}{t^2} - 1\right)\sqrt{\left(\dfrac{1}{t^2} + 1\right)}}$

$\qquad = -\int \dfrac{t \, dt}{\left(1 - t^2\right)\sqrt{1 + t^2}}$

Let $1 + t^2 = u^2$

$\Rightarrow \quad 2t \, dt = 2u \, du$

$\qquad I = \int \dfrac{u \, du}{\left(u^2 - 2\right) u}$

$\qquad = \int \dfrac{du}{u^2 - 2}$

$\therefore \quad I = \dfrac{1}{2\sqrt{2}} \log \left|\dfrac{u - \sqrt{2}}{u + \sqrt{2}}\right| + c$

$\qquad = \dfrac{1}{2\sqrt{2}} \log \left|\dfrac{\sqrt{1 + t^2} - \sqrt{2}}{\sqrt{1 + t^2} + \sqrt{2}}\right| + c$

Hence,

$\qquad I = -\dfrac{1}{2\sqrt{2}} \log \left|\dfrac{\sqrt{2}x + \sqrt{x^2 + 1}}{\sqrt{2}x - \sqrt{x^2 + 1}}\right| + c$

********** END **********