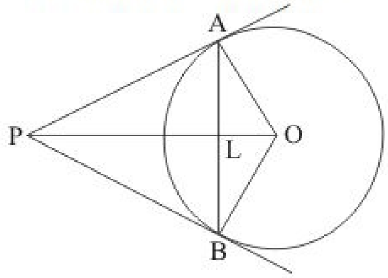
Circles Ex 10.2 Q12

Answer :

Let us first put the given data in the form of a diagram.



Consider ΔPOA and ΔPOB. We have,

PO is the common side for both the triangles.

$PA = PB$(Tangents drawn from an external point will be equal in length)

$OB = OA$(Radii of the same circle)

Therefore, by SSS postulate of congruency, we have

$\Delta POA \cong \Delta POB$

Hence,

$\angle OPA = \angle OPB$ ...... (1)

Now let us consider ΔPLA and ΔPLB. We have,

PL is the common side for both the triangles.

$\angle OPA = \angle OPB$ (From equation (1))

$PA = PB$ (Tangents drawn from an external point will be equal in length)

From SAS postulate of congruent triangles,

$\Delta PLA \cong \Delta PLB$

Therefore,

$PL = LB$ ...... (2)

$\angle PLA = \angle PLB$

Since AB is a straight line,

$\angle ALB = 180^\circ$

$\angle PLA + \angle PLB = 180^\circ$

$2\angle PLA = 180^\circ$

$\angle PLA = 90^\circ$

$\angle PLB = 90^\circ$

Let us now take up ΔOPB. We know that the radius of a circle will always be perpendicular to the tangent at the point of contact. Therefore,

$\angle OBP = 90^\circ$

By Pythagoras theorem we have,

$PB^2 = OP^2 - OB^2$

It is given that

OP = diameter of the circle

Therefore,

$OP = 2OB$

Hence,

$PB^2 = (2OB)^2 - OB^2$

$PB^2 = 4OB^2 - OB^2$

$PB^2 = 3OB^2$

$PB = \sqrt{3}OB$

Consider ΔPLB. We have,

$LB^2 = PB^2 - PL^2$

But we have found that,

$PB = \sqrt{3}OB$

Also from the figure, we can say

$PL = PO - OL$

Therefore,

$LB^2 = 3OB^2 - [PO - OL]^2$ …… (3)

Also, from ΔOLB, we have

$LB^2 = OB^2 - OL^2$ …… (4)

Since Left Hand Sides of equation (3) and equation (4) are same, we can equate the Right Hand Sides of the two equations. Thus we have,

$OB^2 - OL^2 = 3OB^2 - [PO - OL]^2$

$OB^2 - OL^2 = 3OB^2 - [PO^2 + OL^2 - 2.PO.OL]$

$OB^2 - OL^2 = 3OB^2 - PO^2 - OL^2 + 2.PO.OL$

We know from the given data, that OP = 2.OB. Let us substitute 2OB in place of PO in the above equation. We get,

$OB^2 - OL^2 = 3OB^2 - (2OB)^2 - OL^2 + 2.2.OB.OL$

$OB^2 - OL^2 = 3OB^2 - 4OB^2 - OL^2 + 4.OB.OL$

$2OB^2 = 4.OB.OL$

$$OL = \frac{OB}{2}$$

Substituting the value of OL and also PO in equation (3), we get,

$$LB^2 = 3OB^2 - [2OB - \left(\frac{OB}{2}\right)]^2$$

$$LB^2 = 3OB^2 - [4OB^2 + \left(\frac{OB^2}{4}\right) - 2.OB^2]$$

$$LB^2 = 3OB^2 - 4OB^2 - \left(\frac{OB^2}{4}\right) + 2.OB^2$$

$$LB^2 = \frac{3OB^2}{4}$$

$$LB = \frac{\sqrt{3}OB}{2}$$

Also from the figure we get,

$AB = PL + LB$

From equation (2), we know that $PL = LB$. Therefore,

$AB = 2.LB$

$$AB = 2 \times \frac{\sqrt{3}OB}{2}$$

$$AB = \sqrt{3}OB$$

We have also found that $PB = \sqrt{3}OB$. We know that tangents drawn from an external point will be equal in length. Therefore, we have

$PA = PB$

Hence,

$PA = \sqrt{3}OB$

Now, consider ΔPAB. We have,

$PA = \sqrt{3}OB$

$PB = \sqrt{3}OB$

$AB = \sqrt{3}OB$

Since all the sides of the triangle are of equal length, ΔPAB is an equilateral triangle. Thus we have proved.

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