Q 1. Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following:

(i) $p(x) = x^3 - 3x^2 + 5x - 3, \quad g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5, \quad g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6, \quad g(x) = 2 - x^2$

Answer :

(i) $p(x) = x^3 - 3x^2 + 5x - 3$

$q(x) = x^2 - 2$

$$
\begin{array}{r|l}
 & x - 3 \\
\cline{2-2}
x^2 - 2 & x^3 - 3x^2 + 5x - 3 \\
 & x^3 \qquad\quad - 2x \\
 & - \qquad\quad\;\; + \\
\cline{2-2}
 & \qquad -3x^2 + 7x - 3 \\
 & \qquad -3x^2 \qquad + 6 \\
 & \qquad + \qquad\qquad - \\
\cline{2-2}
 & \qquad\qquad\quad 7x - 9 \\
\cline{2-2}
\end{array}
$$

Quotient = x - 3

Remainder = $7x$ - 9

(ii) $p(x) = x^4 - 3x^2 + 4x + 5 = x^4 + 0.x^3 - 3x^2 + 4x + 5$

$q(x) = x^2 + 1 - x = x^2 - x + 1$

$$
\begin{array}{r|l}
 & x^2 + x - 3 \\
\hline
x^2 - x + 1 & x^4 + 0.x^3 - 3x^2 + 4x + 5 \\
 & x^4 - x^3 + x^2 \\
 & - \quad + \quad - \\
\hline
 & \quad x^3 - 4x^2 + 4x + 5 \\
 & \quad x^3 - x^2 + x \\
 & \quad - \quad + \quad - \\
\hline
 & \quad -3x^2 + 3x + 5 \\
 & \quad -3x^2 + 3x - 3 \\
 & \quad + \quad - \quad + \\
\hline
 & \qquad 8 \\
\hline
\end{array}
$$

Quotient = $x^2 + x - 3$

Remainder = 8

(iii) $p(x) = x^4 - 5x + 6 = x^4 + 0.x^2 - 5x + 6$

$q(x) = 2 - x^2 = -x^2 + 2$

$$
\begin{array}{r|l}
 & -x^2 - 2 \\
\hline
-x^2 + 2 & x^4 + 0.x^2 - 5x + 6 \\
 & x^4 - 2x^2 \\
 & - \quad + \\
\hline
 & \quad 2x^2 - 5x + 6 \\
 & \quad 2x^2 \quad - 4 \\
 & \quad - \quad + \\
\hline
 & \qquad -5x + 10 \\
\hline
\end{array}
$$

Quotient = $-x^2 - 2$

Remainder = $-5x + 10$

Q 2 . Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3 + x^2 - 5x + 2; \quad \frac{1}{2}, 1, -2$

(ii) $x^3 - 4x^2 + 5x - 2; \quad 2, 1, 1$

Answer :

(i) $p(x) = 2x^3 + x^2 - 5x + 2.$

Zeroes for this polynomial are $\frac{1}{2}, 1, -2$

$$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$$
$$= \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2$$
$$= 0$$

$$p(1) = 2 \times 1^3 + 1^2 - 5 \times 1 + 2$$
$$= 0$$

$$p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$$
$$= -16 + 4 + 10 + 2 = 0$$

Therefore, $\frac{1}{2}$, 1, and - 2 are the zeroes of the given polynomial.

Comparing the given polynomial with
ax^3+bx^2+cx+d, we obtain a = 2, b = 1, c = - 5, d = 2

We can take $\alpha=\frac{1}{2}, \beta=1, \gamma=-2$

$$\alpha+\beta+\gamma=\frac{1}{2}+1+(-2)=-\frac{1}{2}=\frac{-b}{a}$$

$$\alpha\beta+\beta\gamma+\alpha\gamma=\frac{1}{2}\times1+1(-2)+\frac{1}{2}(-2)=\frac{-5}{2}=\frac{c}{a}$$

$$\alpha\beta\gamma=\frac{1}{2}\times1\times(-2)=\frac{-1}{1}=\frac{-(2)}{2}=\frac{-d}{a}$$

Therefore, the relationship between the zeroes and the coefficients is verified.

(ii) $p(x)=x^3-4x^2+5x-2$

Zeroes for this polynomial are 2, 1, 1.

$$p(2)=2^3-4(2^2)+5(2)-2$$
$$=8-16+10-2=0$$
$$p(1)=1^3-4(1)^2+5(1)-2$$
$$=1-4+5-2=0$$

Therefore, 2, 1, 1 are the zeroes of the given polynomial.

Comparing the given polynomial with
ax^3+bx^2+cx+d, we obtain a = 1, b = - 4, c = 5, d = - 2.

Verification of the relationship between zeroes and coefficient of the given polynomial

$$\text{Sum of zeroes}=2+1+1=4=\frac{-(-4)}{1}=\frac{-b}{a}$$

Multiplication of zeroes taking two at a time = (2)(1) + (1)(1) + (2)(1) =2 + 1 + 2 = 5 $= \frac{(5)}{1} = \frac{c}{a}$

Multiplication of zeroes = 2 × 1 × 1 = 2

$= \frac{-(-2)}{1} = \frac{-d}{a}$

Hence, the relationship between the zeroes and the coefficients is verified.

*********** END **********