Mean Value Theorems Ex 15.1 Q2(iv)

Here,

$$f(x) = x(x-1)^2 \text{ on } [0,1]$$

$f(x)$ is continuous on $[0,1]$ and differentiable on $(0,1)$ as it is a polynomial function.

Now,

$$f(0) = 0(0-1)^2 = 0$$

$$f(1) = 1(1-1)^2 = 0$$

$$\Rightarrow \quad f(0) = f(1)$$

So, Rolle's theorem is applicable on $f(x)$ in $[0,1]$ therefore, we should show that there exist a $c \in (0,1)$ such that $f'(c) = 0$

Now,

$$f(x) = x(x-1)^2$$

$$f'(x) = (x-1)^2 + x \times 2(x-1)$$

$$= (x-1)(x-1+2x)$$

$$f'(x) = (x-1)(3x-1)$$

So, $\quad f'(c) = 0$

$$(c-1)(3c-1) = 0$$

$$\Rightarrow \quad c = 1 \text{ or } c = \frac{1}{3} \in (0,1)$$

Thus, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q2(v)

Here,

$$f(x) = (x^2 - 1)(x - 2) \text{ on } [-1, 2]$$

$f(x)$ is continuous is $[-1,2]$ and differentiable in $(-1,2)$ as it is a polynomial functions.

Now,

$$f(-1) = (1 - 1)(-1 - 2) = 0$$
$$f(2) = (4 - 1)(2 - 2) = 0$$
$$\Rightarrow \quad f(-1) = f(2)$$

So, Rolle's theorem is applicable on $f(x)$ is $[-1,2]$ therefore, we have to show that there exist a $c \in (-1,2)$ such that $f'(c) = 0$

Now,

$$f(x) = (x^2 - 1)(x - 2)$$
$$f'(x) = 2x(x - 2) + (x^2 - 1)$$
$$= 2x^2 - 4 + x^2 - 1$$
$$f'(x) = 3x^2 - 5$$

Now,

$$f'(c) = 0$$
$$\Rightarrow \quad 3x^2 - 5 = 0$$
$$\Rightarrow \quad x = -\sqrt{\frac{5}{3}} \text{ or } x = \sqrt{\frac{5}{3}} \in (-1,2)$$

Thus, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q2(vi)

Here, $f(x) = x(x - 4)^2$ on $[0, 4]$

$f(x)$ is continuous is $[0, 4]$ and differentiable is $(0, 4)$ since $f(x)$ is a polynomial function.

Now,

$$f(x) = x(x - 4)^2$$
$$f(0) = 0(0 - 4)^2$$
$$f(0) = 0 \qquad \text{---(i)}$$
$$f(4) = 4(4 - 4)^2$$
$$f(4) = 0 \qquad \text{---(ii)}$$

From equation (i) and (ii),

$$f(0) = f(4)$$

So, Rolle's theorem is applicable, therefore, we have to show that $f'(c) = 0$ for $c \in (0, 4)$

$$f'(x) = x \times 2(x - 4) + (x - 4)^2$$
$$= 2x^2 - 8x + x^2 + 16 - 8x$$

So, $f'(c) = 3c^2 - 16c + 16$

$$0 = 3c^2 - 12c - 4c + 16$$
$$0 = 3c(c - 4) - 4(c - 4)$$
$$0 = (c - 4)(3c - 4)$$
$$\Rightarrow \quad c = 4 \text{ or } c = \frac{4}{3} \in (0, 4)$$

So, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q2(vii)

Here, $f(x) = x(x-2)^2$ on $[0,2]$

$f(x)$ is continuous is $[0,2]$ and differentiable is $(0,2)$ as it is a polynomial function.

And $f(0) = 0(0-2)^2 = 0$

$f(2) = 2(2-2)^2 = 0$

$\Rightarrow \quad f(0) = f(2)$

So, Rolle's theorem is applicable on $f(x)$ is $[0,2]$, therefore, we have to show that $f'(c) = 0$ as $c \in (0,2)$

$$f(x) = x(x-2)^2$$
$$f'(x) = x \times 2(x-2) + (x-2)$$
$$f'(x) = 2x(x-2) + (x-2)$$
$$\Rightarrow \quad f'(c) = 0$$
$$2c(c-2) + (c-2) = 0$$
$$(c-2)(2c+1) = 0$$
$$c = 2 \text{ or } c = -\frac{1}{2}$$
$$\Rightarrow \quad c = 2 \in (0,2)$$

So, Rolle's theorem is verified.

*********** END **********