Solution of Simultaneous Linear Equations Ex 8.1 Q2(i)

The given system can be written in matrix form as:

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$$

or $\quad AX = B$

Where,

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$$

Now, $\quad |A| = 1\begin{bmatrix} 3 & 1 \\ -1 & -7 \end{bmatrix} - 1\begin{bmatrix} 2 & 1 \\ 3 & -7 \end{bmatrix} - 1\begin{bmatrix} 2 & 3 \\ 3 & -1 \end{bmatrix}$

$$= (-20) - 1(-17) - 1(-11)$$

$$= -20 + 17 + 11 = 8 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$C_{11} = -20 \qquad C_{21} = 8 \qquad C_{31} = 4$

$C_{12} = -(-17) = 17 \qquad C_{22} = -4 \qquad C_{32} = -3$

$C_{13} = -11 \qquad C_{23} = -(-4) = 4 \qquad C_{33} = 1$

$$\text{adj } A = \begin{bmatrix} -20 & 17 & -11 \\ 8 & -4 & 4 \\ 4 & -3 & 1 \end{bmatrix}^T = \begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix}$$

Now, $\quad X = A^{-1}B = \dfrac{1}{8}\begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ 8 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

Hence, $x = 3$

$y = 1$

$z = 1$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(ii)

The above system can be written in matrix form as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ -9 \end{bmatrix}$$

or $\quad AX = B$

Where,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & -3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ -1 \\ -9 \end{bmatrix}$$

Since, $|A| = 14 \neq 0$, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll} C_{11} = 2 & C_{21} = 4 & C_{31} = 2 \\ C_{12} = 8 & C_{22} = -5 & C_{32} = 1 \\ C_{13} = 4 & C_{23} = 1 & C_{33} = -3 \end{array}$$

$$\text{Adj}\, A = \begin{bmatrix} 2 & 8 & 4 \\ 4 & -5 & 1 \\ 2 & 1 & -3 \end{bmatrix}^T = \begin{bmatrix} 2 & 4 & 2 \\ 8 & -5 & 1 \\ 4 & 1 & -3 \end{bmatrix}$$

Now, $\quad X = A^{-1}B = \dfrac{1}{|A|} \times \text{Adj}\, A \times B$

$$= \frac{1}{14}\begin{bmatrix} 2 & 4 & 2 \\ 8 & -5 & 1 \\ 4 & 1 & -3 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \\ -9 \end{bmatrix}$$

$$= \frac{1}{14}\begin{bmatrix} -16 \\ 20 \\ 38 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{-8}{7} \\ \frac{10}{7} \\ \frac{19}{7} \end{bmatrix}$$

Hence, $x = \dfrac{-8}{7}$, $y = \dfrac{10}{7}$, $z = \dfrac{19}{7}$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(iii)

The above system can be written in matrix form as:

$$\begin{bmatrix} 6 & -12 & 25 \\ 4 & 15 & -20 \\ 2 & 18 & 15 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 10 \end{bmatrix}$$

or $\quad AX = B$

Where,

$$A = \begin{bmatrix} 6 & -12 & 25 \\ 4 & 15 & -20 \\ 2 & 18 & 15 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 4 \\ 3 \\ 10 \end{bmatrix}$$

Now,

$$\begin{aligned} |A| &= 6(225+360)+12(60+40)+25(72-30) \\ &= 6(585)+1200+25(42) \\ &= 3510+1200+1050 \\ &= 5760 \neq 0 \end{aligned}$$

So, the above system will have a unique solution, given by

$$X = A^{-1}B$$

$C_{11} = 585 \qquad C_{21} = -(-180-450) = 630 \qquad C_{31} = -135$

$C_{12} = -100 \qquad C_{22} = 40 \qquad C_{32} = 220$

$C_{13} = 42 \qquad C_{23} = -132 \qquad C_{33} = 138$

$$X = A^{-1}B = \frac{1}{|A|}(\mathrm{Adj}\,A)\times B = \frac{1}{5760}\begin{bmatrix} 585 & 630 & -135 \\ -100 & 40 & 220 \\ 42 & -132 & 138 \end{bmatrix}\begin{bmatrix} 2880 \\ 1920 \\ 1152 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5760}\begin{bmatrix} 2880 \\ 1920 \\ 1152 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

Hence, $x = \frac{1}{2}$

$y = \frac{1}{3}$

$z = \frac{1}{5}$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(iv)

The above system can be written as

$$\begin{bmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 1 & 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$$

or $\quad AX = B$

$$\begin{aligned} |A| &= 3(-3) - 4(-9) + 7(5) \\ &= -9 + 36 + 35 \\ &= 62 \neq 0 \end{aligned}$$

So, the above system will have a unique solution, given by

$$X = A^{-1}B$$

Now, $\quad C_{11} = -3 \qquad C_{21} = 26 \qquad C_{31} = 19$

$C_{12} = 9 \qquad C_{22} = -16 \qquad C_{32} = 5$

$C_{13} = 5 \qquad C_{23} = -2 \qquad C_{33} = -11$

$$\text{adj } A = \begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}$$

$$\begin{aligned} X = A^{-1}B &= \frac{1}{|A|}(\text{Adj } A)B \\ &= \frac{1}{62}\begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}\begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix} \\ &= \frac{1}{62}\begin{bmatrix} -42 + 104 + 0 \\ 126 - 64 + 0 \\ 70 - 8 + 0 \end{bmatrix} = \frac{1}{62}\begin{bmatrix} 62 \\ 62 \\ 62 \end{bmatrix} \end{aligned}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Hence, $x = 1, y = 1, z = 1$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(v)

The above system can be written as

$$\begin{bmatrix} 2 & 6 & 0 \\ 3 & 0 & -1 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \\ -3 \end{bmatrix}$$

Or $\quad AX = B$

$$|A| = 2(-1) - 6(5) + 0(-3) = -32 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$C_{11} = -1 \quad C_{21} = -6 \quad C_{31} = -6$$
$$C_{12} = -5 \quad C_{22} = 2 \quad C_{32} = 2$$
$$C_{13} = -3 \quad C_{23} = 14 \quad C_{33} = -18$$

$$\text{adj}A = \begin{bmatrix} -1 & -5 & -3 \\ -6 & 2 & 14 \\ -6 & 2 & -18 \end{bmatrix}^T = \begin{bmatrix} -1 & -6 & -6 \\ -5 & 2 & 2 \\ -3 & 14 & -18 \end{bmatrix}$$

Now,

$$X = A^{-1}B = \frac{1}{|A|}(\text{Adj}A) \times B$$

$$= \frac{1}{-32}\begin{bmatrix} -1 & -6 & -6 \\ -5 & 2 & 2 \\ -3 & 14 & -18 \end{bmatrix}\begin{bmatrix} 2 \\ -8 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-32}\begin{bmatrix} 64 \\ -32 \\ -64 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

Hence, $x = -2, y = 1, z = 2$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(v)

Let $\frac{1}{x} = u, \frac{1}{y} = v, \frac{1}{z} = w$

$$2u - 3v + 3w = 10$$
$$u + v + w = 10$$
$$3u - v + 2w = 13$$

Which can be written as

$$\begin{bmatrix} 2 & -3 & 3 \\ 1 & 1 & 1 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 13 \end{bmatrix}$$

$$|A| = 2(3) + 3(-1) + 3(-4)$$
$$= 6 - 3 - 12 = -9 \neq 0$$

Hence, the system has a unique solution, given by

$$X = A^{-1} \times B$$

$$\begin{array}{lll} C_{11} = 3 & C_{21} = 3 & C_{31} = -6 \\ C_{12} = 1 & C_{22} = -5 & C_{32} = 1 \\ C_{13} = -4 & C_{23} = -7 & C_{33} = 5 \end{array}$$

$$X = \frac{1}{|A|}(\mathrm{Adj}\,A) \times (B)$$
$$= \frac{1}{-9} \begin{bmatrix} 3 & 3 & -6 \\ 1 & -5 & 1 \\ -4 & -7 & 5 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \\ 13 \end{bmatrix}$$
$$= \frac{-1}{9} \begin{bmatrix} 30 + 30 - 78 \\ 10 - 50 + 13 \\ -40 - 70 + 65 \end{bmatrix}$$
$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{-1}{9} \begin{bmatrix} -18 \\ -27 \\ -45 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

Hence, $x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{5}$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(vi)

$$\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix}$$

or $\qquad AX = B$

$$|A| = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$$

$$= 5(-2) - 3(5) + 1(3)$$

$$= -10 - 15 + 3 = -22 \neq 0$$

Hence, it has a unique solution, given by

$$X = A^{-1} \times B$$

$$\begin{array}{lll} C_{11} = -2 & C_{21} = -10 & C_{31} = 8 \\ C_{12} = -5 & C_{22} = 19 & C_{32} = -13 \\ C_{13} = 3 & C_{23} = -7 & C_{33} = -1 \end{array}$$

$$X = A^{-1} \times B = \frac{1}{|A|}(\text{Adj}\, A) \times B$$

$$= \frac{1}{-22} \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix} \begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix}$$

$$= \frac{-1}{22} \begin{bmatrix} -32 - 190 + 200 \\ -80 + 361 - 325 \\ 48 - 133 - 25 \end{bmatrix}$$

$$= \frac{-1}{22} \begin{bmatrix} -22 \\ -44 \\ -110 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

Hence, $x = 1, y = 2, z = 5$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(vii)

$$\begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix}$$

or $\quad A X = B$

$$\begin{aligned} |A| &= 3(6) - 4(3) + 2(-2) \\ &= 18 - 12 - 4 \\ &= 2 \neq 0 \end{aligned}$$

Hence, the system has a unique solution, given by

$$X = A^{-1}B$$

$$\begin{array}{lll} C_{11} = 6 & C_{21} = -28 & C_{31} = -16 \\ C_{12} = -3 & C_{22} = 16 & C_{32} = 9 \\ C_{13} = -2 & C_{23} = 10 & C_{33} = 6 \end{array}$$

Next, $X = A^{-1}B = \dfrac{1}{|A|}(\text{Adj } A) \times B$

$$= \frac{1}{2}\begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & 9 \\ 2 & 10 & 6 \end{bmatrix}\begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 48 - 84 + 32 \\ -24 + 48 - 18 \\ -16 + 30 - 12 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} -4 \\ 6 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$$

Hence, $x = -2, y = 3, z = 1$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(viii)

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & -1 \\ 3 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$$

$$\begin{aligned} |A| &= 2(-5) - 1(1) + 1(-8) \\ &= -10 - 1 - 8 = -19 \neq 0 \end{aligned}$$

Hence, the unique solution, given by

$$X = A^{-1} \times B$$

$$\begin{array}{lll} C_{11} = -5 & C_{21} = 3 & C_{31} = -4 \\ C_{12} = -1 & C_{22} = -7 & C_{32} = 3 \\ C_{13} = -8 & C_{23} = 1 & C_{33} = 5 \end{array}$$

Next, $X = A^{-1} \times B$

$$\begin{aligned} &= \frac{1}{|A|}\begin{bmatrix} -5 & 3 & -4 \\ -1 & -7 & 3 \\ -8 & 1 & 5 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} \\ &= \frac{1}{-19}\begin{bmatrix} -10+15-24 \\ -2-35+18 \\ -16+5+30 \end{bmatrix} \\ &= \frac{-1}{19}\begin{bmatrix} -19 \\ -19 \\ 19 \end{bmatrix} \end{aligned}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

Hence, $x = 1, y = 1, z = -1$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(x)

The above system of equations can be written as

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

or $\quad AX = B$

$$|A| = 1(1) + 1(-2) + 1(4) = 1 - 2 + 4 = 3 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll} C_{11} = 1 & C_{21} = 1 & C_{31} = +1 \\ C_{12} = 2 & C_{22} = -1 & C_{32} = 2 \\ C_{13} = 4 & C_{23} = -2 & C_{33} = 1 \end{array}$$

$$\text{adj } A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & -1 & -2 \\ +1 & 2 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 1 & +1 \\ 2 & -1 & 2 \\ 4 & -2 & 1 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{|A|}(\text{Adj } A) \times B$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & +1 \\ 2 & -1 & 2 \\ 4 & -2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, $x = 1, y = 2, z = 3$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(xi)

The above system can be written as

$$\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 18 \\ 5 \\ 5 \end{bmatrix}$$

or $\quad AX = B$

$$|A| = 8(-1) - 4(1) + 3(3) = -8 - 4 + 9 = -3 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$C_{11} = -1 \qquad C_{21} = 2 \qquad C_{31} = 1$

$C_{12} = -1 \qquad C_{22} = 5 \qquad C_{32} = -2$

$C_{13} = 3 \qquad C_{23} = -12 \qquad C_{33} = 0$

$$\text{adj } A = \begin{bmatrix} -1 & -1 & 3 \\ 2 & 5 & -12 \\ 1 & -2 & 0 \end{bmatrix}^T = \begin{bmatrix} -1 & 2 & 1 \\ -1 & 5 & -2 \\ 3 & -12 & 0 \end{bmatrix}$$

Now, $\quad X = A^{-1}B = \frac{1}{|A|}(\text{Adj } A) \times B$

$$= \frac{-1}{3}\begin{bmatrix} -1 & 2 & 1 \\ -1 & 5 & -2 \\ 3 & -12 & 0 \end{bmatrix} \begin{bmatrix} 18 \\ 5 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{3}\begin{bmatrix} -3 \\ -3 \\ -6 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

Hence, $x = 1, y = 1, z = 2$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(xii)

This system can be written as

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

or $\quad AX = B$

$$|A| = 1(-2) - 1(-5) + 1(1) = -2 + 5 + 1 = 4 \neq 0$$

So, $AX = B$ has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll} C_{11} = -2 & C_{21} = 0 & C_{31} = 2 \\ C_{12} = +5 & C_{22} = -2 & C_{32} = -1 \\ C_{13} = 1 & C_{23} = 2 & C_{33} = -1 \end{array}$$

$$\text{adj } A = \begin{bmatrix} -2 & 5 & 1 \\ 0 & -2 & 2 \\ 2 & -1 & -1 \end{bmatrix}^T = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

$$X = A^{-1} \times B = \frac{1}{|A|}(\text{Adj } A) \times B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -12 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

Hence, $x = -3, y = 1, z = 2$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(xiii)

Let

$$\frac{1}{x}=u,\frac{1}{y}=v,\frac{1}{z}=w$$

The above system can be written as

$$\begin{bmatrix}2 & 3 & 10\\ 4 & -6 & 5\\ 6 & 9 & -20\end{bmatrix}\begin{bmatrix}u\\ v\\ w\end{bmatrix}=\begin{bmatrix}4\\ 1\\ 2\end{bmatrix}$$

Or $\quad AX = B$

$$|A|=2(75)-3(-110)+10(72)=1200\neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{matrix}C_{11}=75 & C_{21}=150 & C_{31}=75\\ C_{12}=110 & C_{22}=-100 & C_{32}=30\\ C_{13}=72 & C_{23}=0 & C_{33}=-24\end{matrix}$$

$$\text{adj}A=\begin{bmatrix}75 & 110 & 72\\ 150 & -100 & 0\\ 75 & 30 & -24\end{bmatrix}^T=\begin{bmatrix}75 & 150 & 75\\ 110 & -100 & 30\\ 72 & 0 & -24\end{bmatrix}$$

Now,

$$X = A^{-1}B=\frac{1}{|A|}(\text{Adj}A)\times B$$

$$=\frac{1}{1200}\begin{bmatrix}75 & 150 & 75\\ 110 & -100 & 30\\ 72 & 0 & -24\end{bmatrix}\begin{bmatrix}4\\ 1\\ 2\end{bmatrix}$$

$$\begin{bmatrix}u\\ v\\ w\end{bmatrix}=\frac{1}{1200}\begin{bmatrix}600\\ 400\\ 240\end{bmatrix}=\begin{bmatrix}\frac{1}{2}\\ \frac{1}{3}\\ \frac{1}{5}\end{bmatrix}$$

Hence, $x=2, y=3, z=5$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(xiv)

The above system can be written as

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

Or $\quad AX = B$

$$|A| = 1(7) + 1(19) + 2(-11) = 4 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$C_{11} = 7 \qquad C_{21} = 1 \qquad C_{31} = -3$$
$$C_{12} = -19 \qquad C_{22} = -1 \qquad C_{32} = 11$$
$$C_{13} = -11 \qquad C_{23} = -1 \qquad C_{33} = 7$$

$$\operatorname{adj}A = \begin{bmatrix} 7 & -19 & -11 \\ 1 & -1 & -1 \\ -3 & 11 & 7 \end{bmatrix}^T = \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

Now,

$$X = A^{-1}B = \frac{1}{|A|}(\operatorname{Adj}A) \times B$$

$$= \frac{1}{4} \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix} \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Hence, $x = 2, y = 1, z = 3$

*********** END **********