



Differentiation Ex 11.8 Q16

$$\text{Let } u = \cos^{-1}(4x^3 - 3x)$$

$$\text{Put } x = \cos \theta \Rightarrow \theta = \cos^{-1} x, \text{ so}$$

$$\begin{aligned} u &= \cos^{-1}(4\cos^3 \theta - 3\cos \theta) \\ u &= \cos^{-1}(\cos 3\theta) \end{aligned} \quad \text{---(i)}$$

$$\begin{aligned} \text{Let } v &= \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) \\ &= \tan^{-1}\left(\frac{\sqrt{1-\cos^2 \theta}}{\cos \theta}\right) \\ &= \tan^{-1}\left(\frac{\sin \theta}{\cos \theta}\right) \\ v &= \tan^{-1}(\tan \theta) \end{aligned} \quad \text{---(ii)}$$

Here,

$$\begin{aligned} \frac{1}{2} &< x < 1 \\ \Rightarrow \frac{1}{2} &< \cos \theta < 1 \\ \Rightarrow 0 &< \theta < \frac{\pi}{3} \end{aligned}$$

So, from equation (i),

$$\begin{aligned} u &= 3\theta \\ u &= 3\cos^{-1} x \end{aligned} \quad \left[\text{Since, } \cos^{-1}(\cos \theta) = \theta, \text{ if } \theta \in [0, \pi] \right]$$

Differentiating it with respect to x ,

$$\frac{du}{dx} = \frac{-3}{\sqrt{1-x^2}} \quad \text{---(iii)}$$

From equation (ii),

$$\begin{aligned} v &= \theta \\ v &= \cos^{-1} x \end{aligned} \quad \left[\text{Since, } \tan^{-1}(\tan \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \right]$$

Differentiating it with respect to x ,

$$\frac{dv}{dx} = \frac{-1}{\sqrt{1-x^2}} \quad \text{---(iv)}$$

Dividing equation (iii) by (iv),

$$\begin{aligned} \frac{\frac{du}{dx}}{\frac{dv}{dx}} &= \left(\frac{-3}{\sqrt{1-x^2}} \right) \left(-\frac{\sqrt{1-x^2}}{1} \right) \\ \frac{du}{dv} &= 3 \end{aligned}$$

Differentiation Ex 11.8 Q17

$$\text{Let } u = \tan^{-1} \left(\frac{x}{\sqrt{1-x^2}} \right)$$

$$\text{Put } x = \cos \theta \Rightarrow \theta = \sin^{-1} x, \text{ so}$$

$$\begin{aligned} u &= \tan^{-1} \left(\frac{\sin \theta}{\sqrt{1-\sin^2 \theta}} \right) \\ &= \tan^{-1} \left(\frac{\sin \theta}{\cos \theta} \right) \\ u &= \tan^{-1} (\tan \theta) \end{aligned} \quad \text{---(i)}$$

And,

$$\begin{aligned} \text{Let } v &= \sin^{-1} (2x\sqrt{1-x^2}) \\ v &= \sin^{-1} (2\sin \theta \sqrt{1-\sin^2 \theta}) \\ &= \sin^{-1} (2\sin \theta \cos \theta) \\ v &= \sin^{-1} (\sin 2\theta) \end{aligned} \quad \text{---(ii)}$$

$$\begin{aligned} \text{Here, } & -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}} \\ \Rightarrow & -\frac{1}{\sqrt{2}} < \sin \theta < \frac{1}{\sqrt{2}} \\ \Rightarrow & -\frac{\pi}{4} < \theta < \frac{\pi}{4} \end{aligned}$$

So, from equation (i),

$$\begin{aligned} u &= \theta & \left[\text{Since, } \tan^{-1} (\tan \theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right) \right] \\ u &= \sin^{-1} x \end{aligned}$$

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