Arithematic Progressions Ex 19.5 Q1(i)

$\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ will be in A.P if $\frac{c+a}{b} - \frac{b+c}{a} = \frac{a+b}{c} - \frac{c+a}{b}$

if $\quad \frac{ca+a^2-b^2-cb}{ab} = \frac{ab+b^2-c^2-ac}{bc}$

$$\text{LHS} \Rightarrow \frac{ca+a^2-b^2-cb}{ab}$$
$$\Rightarrow \frac{c^2a+a^2c-b^2c-c^2b}{abc}$$
$$\Rightarrow \frac{c(a-b)[a+b+c]}{abc} \qquad \text{---(i)}$$

$$\text{RHS} \Rightarrow \frac{ab+b^2-c^2-ac}{bc}$$
$$\Rightarrow \frac{a^2b+ab^2-ac^2-a^2c}{abc}$$
$$\Rightarrow \frac{a(b-c)[a+b+c]}{abc} \qquad \text{---(ii)}$$

and since $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$
$$c(b-a) = a(b-c) \qquad \text{---(iii)}$$

$\therefore$ LHS = RHS and the given terms are in A.P.

Arithematic Progressions Ex 19.5 Q1(ii)

$a(b+c), b(c+a), c(a+b)$ are in A.P if $b(c+a) - a(b+c) = c(a+b) - b(c+a)$

$$\text{LHS} = b(c+a) - a(b+c)$$
$$= bc + ab - ab - ac$$
$$= c(b-a) \qquad \text{---(i)}$$

$$\text{RHS} = c(a+b) - b(c+a)$$
$$= ca + cb - bc - ba$$
$$= a(c-b) \qquad \text{---(ii)}$$

and $\quad \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$\therefore \quad \frac{1}{a} - \frac{1}{b} = \frac{1}{b} - \frac{1}{c}$

or $\quad c(b-a) = a(c-b) \qquad \text{---(iii)}$

From (i), (ii) and (iii)

$a(b+c), b(c+a), c(a+b)$ are in A.P

Arithematic Progressions Ex 19.5 Q2

$\frac{a}{b+c}, \frac{b}{a+c}, \frac{c}{a+b}$ are in A.P if $\frac{b}{a+c} - \frac{a}{b+c} = \frac{c}{a+b} - \frac{b}{a+c}$

LHS $= \frac{b}{a+c} - \frac{a}{b+c}$

$\Rightarrow \quad \frac{b^2 + bc - a^2 - ac}{(a+c)(b+c)}$

$\Rightarrow \quad \frac{(b-a)(a+b+c)}{(a+c)(b+c)}$ ---(i)

RHS $= \frac{a}{a+b} - \frac{b}{a+c}$

$\Rightarrow \quad \frac{ca + c^2 - b^2 - ab}{(a+b)(b+c)}$

$\Rightarrow \quad \frac{(c-b)(a+b+c)}{(a+b)(b+c)}$ ---(ii)

and a^2, b^2, c^2 are in A.P

$\therefore \quad b^2 - a^2 = c^2 - b^2$ ---(iii)

Substituting $b^2 - a^2$ with $c^2 - b^2$

(i) = (ii)

$\therefore \quad \frac{a}{b+c}, \frac{b}{a+c}, \frac{c}{a+b}$ are in A.P

Arithematic Progressions Ex 19.5 Q3(i)

$a^2(b+c), b^2(c+a), c^2(a+b)$ are in A.P.

If $b^2(c+a) - a^2(b+c) = c^2(a+b) - b^2(a+c)$

$\Rightarrow \quad b^2c + b^2a - a^2b - a^2c = c^2a + c^2b - b^2a - b^2c$

Given, $b - a = c - b$ $\quad [a, b, c$ are in A.P$]$

$c\left(b^2 - a^2\right) + ab(b-a) = a\left(c^2 - b^2\right) + bc(c-b)$

$(b-a)(ab + bc + ca) = (c-b)(ab + bc + ca)$

Cancelling $ab + bc + ca$ from both sides

$b - a = c - b$

$2b = c + a$ which is true

Hence, $a^2(b+c), (c+a)b^2$ and $c^2(a+b)$ are also in A.P.

*********** END **********