



Chapter 10 Differentiability Ex 10.1 Q7(i)

$$f(x) = \begin{cases} x^m \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$\begin{aligned} (\text{LHD at } x = 0) &= \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0} \\ &= \lim_{h \rightarrow 0} \frac{f(0 - h) - f(0)}{(0 - h) - 0} \\ &= \lim_{h \rightarrow 0} \frac{(0 - h)^m \sin\left(\frac{1}{-h}\right) - 0}{-h} \\ &= \lim_{h \rightarrow 0} \frac{(-h)^m \sin\left(-\frac{1}{h}\right)}{-h} \\ &= \lim_{h \rightarrow 0} (-h)^{m-1} \sin\left(-\frac{1}{h}\right) \\ &= \lim_{h \rightarrow 0} (-h)^{m-1} \sin\left(\frac{1}{h}\right) \\ &= 0 \times k \quad [\text{When } -1 \leq k \leq 1] \\ &= 0 \end{aligned}$$

$$\begin{aligned} (\text{RHD at } x = 0) &= \lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0} \\ &= \lim_{h \rightarrow 0} \frac{f(0 + h) - f(0)}{(0 + h) - 0} \\ &= \lim_{h \rightarrow 0} \frac{h^m \sin\left(\frac{1}{h}\right) - 0}{h} \\ &= \lim_{h \rightarrow 0} \left(h^{m-1}\right) \sin\left(\frac{1}{h}\right) \\ &= 0 \times k' \quad [\text{Since } -1 \leq k' \leq 1] \\ &= 0 \end{aligned}$$

$$(\text{LHD at } x = 0) = (\text{RHD at } x = 0)$$

$\therefore f(x)$ is differentiable at $x = 0$

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$$\begin{aligned}
\text{LHL} &= \lim_{x \rightarrow 0^-} f(x) \\
&= \lim_{h \rightarrow 0} f(0-h) \\
&= \lim_{h \rightarrow 0} (-h)^m \sin\left(-\frac{1}{h}\right) \\
&= -\lim_{h \rightarrow 0} (-h)^m \sin\left(\frac{1}{h}\right) \\
&= 0 \times k \quad \quad \quad [\text{When } -1 \leq k \leq 1] \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\text{RHL} &= \lim_{x \rightarrow 0^+} f(x) \\
&= \lim_{h \rightarrow 0} f(0+h) \\
&= \lim_{h \rightarrow 0} (0+h)^m \sin \frac{1}{(0+h)} \\
&= \lim_{h \rightarrow 0} h^m \sin\left(\frac{1}{h}\right) \\
&= 0 \times k' \quad \quad \quad [\text{Where } -1 \leq k' \leq 1] \\
&= 0
\end{aligned}$$

$$\text{LHL} = f(0) = \text{RHL}$$

$\therefore f(x)$ is continuous at $x = 0$
For differentiability at $x = 0$

$$\begin{aligned}
(\text{LHD at } x = 0) &= \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0} \\
&= \lim_{h \rightarrow 0} \frac{(0-h) - f(0)}{(0-h) - 0} \\
&= \lim_{h \rightarrow 0} \frac{(-h)^m \sin\left(-\frac{1}{h}\right)}{-h} \\
&= \lim_{h \rightarrow 0} -(-h)^{m-1} \sin\left(\frac{1}{h}\right) \\
&= \text{Not defined} \quad \quad \quad [\text{Since } 0 < m < 1]
\end{aligned}$$

$$\begin{aligned}
(\text{RHD at } x = 0) &= \lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0} \\
&= \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{0+h-0} \\
&= \lim_{h \rightarrow 0} \frac{h^m \sin\left(\frac{1}{h}\right)}{h}
\end{aligned}$$

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$$\begin{aligned}
\text{LHL} &= \lim_{x \rightarrow 0^-} f(x) \\
&= \lim_{h \rightarrow 0} f(0 - h) \\
&= \lim_{h \rightarrow 0} (-h)^m \sin\left(-\frac{1}{h}\right) \\
&= \text{Not defined as } m \leq 0 \\
\text{RHL} &= \lim_{x \rightarrow 0^+} f(x) \\
&= \lim_{h \rightarrow 0} f(0 + h) \\
&= \lim_{h \rightarrow 0} h^m \sin\left(\frac{1}{h}\right) \\
&= \text{Not defined, as } m \leq 0
\end{aligned}$$

Since RHL and LHL are not defined, so $f(x)$ is not continuous

Let $x = 0$ for $m \leq 0$.

Now,

$$\begin{aligned}
(\text{LHD at } x = 0) &= \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0} \\
&= \lim_{h \rightarrow 0} \frac{f(0 - h) - 0}{0 - h - 0} \\
&= \lim_{h \rightarrow 0} \frac{(-h)^m \sin\left(-\frac{1}{h}\right)}{-h} \\
&= \lim_{h \rightarrow 0} -(-h)^{m-1} \sin\left(\frac{1}{h}\right) \\
&= \text{Not defined, as } m \leq 0 \\
\text{RHD} &= \lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0} \\
&= \lim_{h \rightarrow 0} \frac{f(0 + h) - f(0)}{0 + h - 0} \\
&= \lim_{h \rightarrow 0} \frac{h^m \sin\left(\frac{1}{h}\right)}{h} \\
&= \lim_{h \rightarrow 0} \left(h^{m-1}\right) \sin\left(\frac{1}{h}\right) \\
&= \text{Not defined, as } m \leq 0
\end{aligned}$$

Thus,

$f(x)$ is neither continuous nor differentiable at $x = 0$ for $m \leq 0$.

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