



Indefinite Integrals Ex 19.21 Q1

$$\text{Let } I = \int \frac{x}{\sqrt{x^2 + 6x + 10}} dx$$

$$\begin{aligned} \text{Let } x &= \lambda \frac{d}{dx} \{x^2 + 6x + 10\} + \mu \\ &= \lambda (2x + 6) + \mu \\ x &= (2\lambda)x + 6\lambda + \mu \end{aligned}$$

Comparing the coefficients of like powers of x ,

$$2\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$$

$$6\lambda + \mu = 0 \quad \Rightarrow \quad 6\left(\frac{1}{2}\right) + \mu = 0$$

$$\mu = -3$$

$$\begin{aligned} \text{so, } I_1 &= \int \frac{\frac{1}{2}(2x+6) - 3}{\sqrt{x^2 + 6x + 10}} dx \\ &= \frac{1}{2} \int \frac{(2x+6)}{\sqrt{x^2 + 6x + 10}} dx - 3 \int \frac{1}{\sqrt{x^2 + 2x(3) + (3)^2 - (3)^2 + 10}} dx \\ I_1 &= \frac{1}{2} \int \frac{2x+6}{\sqrt{x^2 + 6x + 10}} dx - 3 \int \frac{1}{\sqrt{(x+3)^2 + (1)^2}} dx \end{aligned}$$

$$I_1 = \frac{1}{2} [2\sqrt{x^2 + 6x + 10}] - 3 \log |x + 3 + \sqrt{(x+3)^2 + 1}| + c$$

$$\left[\text{since, } \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + c, \int \frac{1}{\sqrt{x^2 + a^2}} dx = \log |x + \sqrt{x^2 + a^2}| + c \right]$$

$$I = \sqrt{x^2 + 6x + 10} - 3 \log |x + 3 + \sqrt{x^2 + 6x + 10}| + c$$

Indefinite Integrals Ex 19.21 Q2

$$\text{Let } I = \int \frac{2x+1}{\sqrt{x^2 + 2x - 1}} dx$$

$$\begin{aligned} \text{Let } 2x+1 &= \lambda \frac{d}{dx} \{x^2 + 2x - 1\} + \mu \\ &= \lambda (2x + 2) + \mu \\ 2x+1 &= (2\lambda)x + 2\lambda + \mu \end{aligned}$$

Comparing the coefficients of like powers of x ,

$$2\lambda = 2 \quad \Rightarrow \quad \lambda = 1$$

$$2\lambda + \mu = 1 \quad \Rightarrow \quad 2(1) + \mu = 1$$

$$\mu = -1$$

$$\begin{aligned} \text{so, } I &= \int \frac{(2x+2) - 1}{\sqrt{x^2 + 2x - 1}} dx \\ &= \int \frac{(2x+2)}{\sqrt{x^2 + 2x - 1}} dx - \int \frac{1}{\sqrt{x^2 + 2x + (1)^2 - (1)^2 - 1}} dx \\ I &= \int \frac{2x+2}{\sqrt{x^2 + 2x - 1}} dx - \int \frac{1}{\sqrt{(x+1)^2 - (\sqrt{2})^2}} dx \end{aligned}$$

$$I = [2\sqrt{x^2 + 2x - 1}] - \log |x + 1 + \sqrt{(x+1)^2 - (\sqrt{2})^2}| + c$$

$$\left[\text{since, } \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + c, \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log |x + \sqrt{x^2 - a^2}| + c \right]$$

$$I = 2\sqrt{x^2 + 2x - 1} - \log |x + 1 + \sqrt{x^2 + 2x - 1}| + c$$

Indefinite Integrals Ex 19.21 Q3

$$\text{Let } I = \int \frac{x+1}{\sqrt{4+5x-x^2}} dx$$

$$\text{Let } x+1 = \lambda \frac{d}{dx} (4+5x-x^2) + \mu$$

$$= \lambda (5-2x) + \mu$$

$$x = (-2\lambda)x + 5\lambda + \mu$$

Comparing the coefficients of like powers of x,

$$-2\lambda = 1 \quad \Rightarrow \quad \lambda = -\frac{1}{2}$$

$$5\lambda + \mu = 1 \quad \Rightarrow \quad 5\left(-\frac{1}{2}\right) + \mu = 1$$

$$\mu = \frac{7}{2}$$

$$\text{so, } I = \int \frac{-\frac{1}{2}(5-2x) + \frac{7}{2}}{\sqrt{4+5x-x^2}} dx$$

$$= -\frac{1}{2} \int \frac{(5-2x)}{\sqrt{4+5x-x^2}} dx + \frac{7}{2} \int \frac{1}{\sqrt{[x^2-5x-4]}} dx$$

$$I = -\frac{1}{2} \int \frac{5-2x}{\sqrt{4+5x-x^2}} dx + \frac{7}{2} \int \frac{1}{\sqrt{-\left[x^2-2x\left(\frac{5}{2}\right) + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 - 4]\right]} dx$$

$$I = -\frac{1}{2} \int \frac{5-2x}{\sqrt{4+5x-x^2}} dx + \frac{7}{2} \int \frac{1}{\sqrt{-\left[\left(x-\frac{5}{2}\right)^2 - \left(\frac{\sqrt{41}}{2}\right)^2\right]}} dx$$

$$I = -\frac{1}{2} \int \frac{5-2x}{\sqrt{4+5x-x^2}} dx + \frac{7}{2} \int \frac{1}{\sqrt{\left[\left(\frac{\sqrt{41}}{2}\right)^2 - \left(x-\frac{5}{2}\right)^2\right]}} dx$$

$$I = -\frac{1}{2} \left(2\sqrt{4+5x-x^2} \right) + \frac{7}{2} \sin^{-1} \left(\frac{x-\frac{5}{2}}{\frac{\sqrt{41}}{2}} \right) + c \quad \left[\text{since, } \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + c, \int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + c \right]$$

$$I = -\sqrt{4+5x-x^2} + \frac{7}{2} \sin^{-1} \left(\frac{2x-5}{\sqrt{41}} \right) + c$$

Indefinite Integrals Ex 19.21 Q4

$$\text{Let } I = \int \frac{6x-5}{\sqrt{3x^2-5x+1}} dx$$

$$\text{Let } 3x^2-5x+1 = t$$

$$(6x-5)dx = dt$$

$$I = \int \frac{dt}{\sqrt{t}}$$

$$= 2\sqrt{t} + c$$

$$I = 2\sqrt{3x^2-5x+1} + c$$

Indefinite Integrals Ex 19.21 Q5

$$\text{Let } I = \int \frac{3x+1}{\sqrt{5-2x-x^2}} dx$$

$$\text{Let } 3x+1 = \lambda \frac{d}{dx} (5-2x-x^2) + \mu$$

$$= \lambda (-2-2x) + \mu$$

$$3x+1 = (-2\lambda)x - 2\lambda + \mu$$

Comparing the coefficients of like powers of x,

$$-2\lambda = 3 \quad \Rightarrow \quad \lambda = -\frac{3}{2}$$

$$-2\lambda + \mu = 1 \quad \Rightarrow \quad -2\left(-\frac{3}{2}\right) + \mu = 1$$

$$\mu = -2$$

$$\text{so, } I = \int \frac{-\frac{3}{2}(-2-2x) - 2}{\sqrt{5-2x-x^2}} dx$$

$$= -\frac{3}{2} \int \frac{(-2-2x)}{\sqrt{5-2x-x^2}} dx - 2 \int \frac{1}{\sqrt{[x^2+2x-5]}} dx$$

$$I = -\frac{3}{2} \int \frac{(-2-2x)}{\sqrt{5-2x-x^2}} dx - 2 \int \frac{1}{\sqrt{[x^2+2x+(1)^2-(1)^2-5]}} dx$$

$$I = -\frac{3}{2} \int \frac{(-2-2x)}{\sqrt{5-2x-x^2}} dx - 2 \int \frac{1}{\sqrt{[(x+1)^2-(\sqrt{6})^2]}} dx$$

$$I = -\frac{3}{2} \int \frac{(-2-2x)}{\sqrt{5-2x-x^2}} dx - 2 \int \frac{1}{\sqrt{[(\sqrt{6})^2-(x+1)^2]}} dx$$

$$I = -\frac{3}{2} \times 2\sqrt{5-2x-x^2} - 2 \sin^{-1} \left(\frac{x+1}{\sqrt{6}} \right) + c \quad \left[\text{since, } \int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + c, \int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + c \right]$$

$$I = -3\sqrt{5-2x-x^2} - 2 \sin^{-1} \left(\frac{x+1}{\sqrt{6}} \right) + c$$

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