Inverse Trigonometric Functions Ex 4.1 Q2.

$$\text{Let } \cos^{-1}\left(\frac{1}{2}\right) = x. \text{ Then, } \cos x = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right).$$

$$\therefore \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

$$\text{Let } \sin^{-1}\left(\frac{1}{2}\right) = y. \text{ Then, } \sin y = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right).$$

$$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

$$\therefore \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + \frac{2\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\therefore \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$$

$$\therefore \tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] = \tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right]$$

$$= \tan^{-1}\left[2\cos\frac{\pi}{3}\right] = \tan^{-1}\left[2\times\frac{1}{2}\right]$$

$$= \tan^{-1}1 = \frac{\pi}{4}$$

Concept Insight:

Solve the innermost bracket first, so first find the principal value of $\sin^{-1}(1/2)$

$$\text{Let } \tan^{-1}(1) = x. \text{ Then, } \tan x = 1 = \tan\frac{\pi}{4}.$$

$$\therefore \tan^{-1}(1) = \frac{\pi}{4}$$

$$\text{Let } \cos^{-1}\left(-\frac{1}{2}\right) = y. \text{ Then, } \cos y = -\frac{1}{2} = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right).$$

$$\therefore \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$$

$$\text{Let } \sin^{-1}\left(-\frac{1}{2}\right) = z. \text{ Then, } \sin z = -\frac{1}{2} = -\sin\left(\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right).$$

$$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

$$\therefore \tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$$

$$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$$

$$= \frac{3\pi + 8\pi - 2\pi}{12} = \frac{9\pi}{12} = \frac{3\pi}{4}$$

$\tan^{-1}\left(\sqrt{3}\right)$ = Angle in $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$ whose tangent is $\sqrt{3}$

$= \frac{\pi}{3}$

$\sec^{-1}(-2)$ = An angle in $[0,\pi]-\left\{\frac{\pi}{2}\right\}$ whose secant is (-2)

$= \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

$\operatorname{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$ = An angle in $\left[\frac{-\pi}{2},\frac{\pi}{2}\right]-\{0\}$ whose cosecant is $\left(\frac{2}{\sqrt{3}}\right)$

$= \frac{\pi}{3}$

Hence,

$$\tan^{-1}\sqrt{3} - \sec^{-1}(-2) + \operatorname{cosec}^{-1}\frac{2}{\sqrt{3}}$$

$$= \frac{\pi}{3} - \frac{2\pi}{3} + \frac{\pi}{3}$$

$$= 0$$

$$\therefore\ \tan^{-1}\sqrt{3} - \sec^{-1}\left(-\sqrt{2}\right) + \operatorname{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right) = 0$$

*********** END **********