Polynomials Ex 2.2 Q2

**Answer :**

If $\alpha, \beta$ and $\gamma$ are the zeros of a cubic polynomial $f(x)$, then

$f(x) = k\{x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\beta\gamma+\gamma\alpha)x - \alpha\beta\gamma\}$ where $k$ is any non-zero real number.

Here,

$$\alpha+\beta+\gamma = 3,$$
$$\alpha\beta+\beta\gamma+\gamma\alpha = -1$$
$$\alpha\beta\gamma = -3$$

Therefore

$$f(x) = k\{x^3 - (3)x^2 + (-1)x - (-3)\}$$
$$f(x) = k\{x^3 - 3x^2 - 1x + 3\}$$

Hence, cubic polynomial is $\boxed{f(x) = k\{x^3 - 3x^2 - 1x + 3\}}$, where $k$ is any non-zero real number.

Polynomials Ex 2.2 Q3

**Answer :**

Let $\alpha = a-d, \beta = a$ and $\gamma = a+d$ be the zeros of the polynomial

$f(x) = 2x^3 - 15x^2 + 37x - 30$

Therefore

$$\alpha+\beta+\gamma = \frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$$
$$= -\left(\frac{-15}{2}\right)$$
$$= \frac{15}{2}$$

$$\alpha\beta\gamma = \frac{-\text{Constant term}}{\text{Coefficient of } x^2}$$
$$= -\left(\frac{-30}{2}\right)$$
$$= 15$$

Sum of the zeros = $\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

$$(a-d)+a+(a+d) = \frac{15}{2}$$
$$a+a+a-\cancel{d}+\cancel{d} = \frac{15}{2}$$
$$3a = \frac{15}{2}$$

$$a=\frac{\cancel{15}}{2}\times\frac{1}{\cancel{3}}$$

$$a=\frac{5}{2}$$

Product of the zeros = $\dfrac{-\text{Constant term}}{\text{Coefficient of } x^2}$

$$\alpha\beta\gamma=15$$

$$(a-d)+a+(a+d)=15$$

$$a\left(a^2-d^2\right)=15$$

Substituting $a=\dfrac{5}{2}$ we get

$$\frac{5}{2}\left(\left(\frac{5}{2}\right)^2-d^2\right)=15$$

$$\frac{5}{2}\left(\frac{25}{4}-d^2\right)=15$$

$$\frac{25}{4}-d^2=\cancel{15}^{3}\times\frac{2}{\cancel{5}}$$

$$\frac{25}{4}-d^2=3\times2$$

$$\frac{25}{4}-d^2=6$$

$$-d^2=6-\frac{25}{4}$$

$$-d^2 = \frac{24-25}{4}$$

$$\cancel{-}d^2 = \frac{\cancel{-}1}{4}$$

$$d \times d = \frac{1}{2} \times \frac{1}{2}$$

$$d = \frac{1}{2}$$

Therefore, substituting $a = \frac{5}{2}$ and $d = \frac{1}{2}$ in $\alpha = a - d$, $\beta = a$ and $\gamma = a + d$

$$\alpha = a - d$$

$$\alpha = \frac{5}{2} - \frac{1}{2}$$

$$\alpha = \frac{5-1}{2}$$

$$\alpha = \frac{4}{2}$$

$$\alpha = 2$$

$$\beta = a$$

$$\beta = \frac{5}{2}$$

$$\gamma = a + d$$

$$\gamma = \frac{5}{2} + \frac{1}{2}$$

$$\gamma = \frac{5+1}{2}$$

$$\gamma = \frac{6}{2}$$

$$\gamma = 3$$

Hence, the zeros of the polynomial are $\boxed{2, \frac{5}{2}, 3}$.

*********** END **********