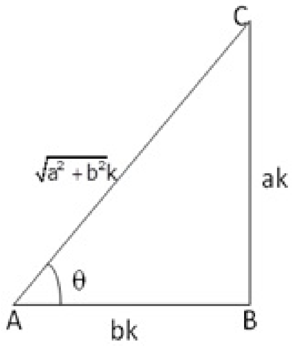
Question 13

Given: $tan\theta = \frac{a}{b} = \frac{ak}{bk} = \frac{BC}{AB}$

Let us draw a ΔABC in which ∠B = 90^0 and ∠A = θ



By Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2 = b^2k^2 + a^2k^2$$

$$\therefore \quad AC = \sqrt{a^2+b^2}\ k$$

$$\sin\theta = \frac{BC}{AC} = \frac{ak}{\sqrt{a^2+b^2}k} = \frac{a}{\sqrt{a^2+b^2}}$$

$$\cos\theta = \frac{AB}{AC} = \frac{bk}{\sqrt{a^2+b^2}k} = \frac{b}{\sqrt{a^2+b^2}}$$

$$\text{L.H.S.} = \frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$$

$$= \frac{a.\frac{a}{\sqrt{a^2+b^2}} - b.\frac{b}{\sqrt{a^2+b^2}}}{a.\frac{a}{\sqrt{a^2+b^2}} + b.\frac{b}{\sqrt{a^2+b^2}}} = \frac{\frac{a^2-b^2}{\sqrt{a^2+b^2}}}{\frac{a^2+b^2}{\sqrt{a^2+b^2}}} = \frac{a^2-b^2}{a^2+b^2}$$

$$= \text{R.H.S.}$$

*********** END **********