Functions Ex 2.1 Q5(xvi)

$f : R \to R$ defined by $f(x) = 1 + x^2$

Injective: let $x, y \in R$ such that

$$f(x) = f(y)$$

$\Rightarrow \quad 1 + x^2 = 1 + y^2$

$\Rightarrow \quad x^2 - y^2 = 0$

$\Rightarrow \quad (x - y)(x + y) = 0$

either $x = y$ or $x = -y$ or $x \neq y$

$\therefore \quad f$ is not one-one.

Surjective: let $y \in R$ be arbitrary, then

$$f(x) = y$$

$\Rightarrow \quad 1 + x^2 = y$

$\Rightarrow \quad x^2 + 1 - y = 0$

$\therefore \quad x = \pm\sqrt{y - 1} \notin R$ for $y < 1$

$\therefore \quad f$ is not onto.

Functions Ex 2.1 Q6

Given, $f : A \to B$ is injective such that range$(f) = \{a\}$

We know that in injective map different elements have different images.

$\therefore$ A has only one element.

Functions Ex 2.1 Q7

A = **R** − {3}, B = **R** − {1}

f: A → B is defined as $f(x)=\left(\dfrac{x-2}{x-3}\right)$.

Let $x,\ y \in \mathrm{A}$ such that $f(x)=f(y)$.

$$\Rightarrow \frac{x-2}{x-3}=\frac{y-2}{y-3}$$
$$\Rightarrow (x-2)(y-3)=(y-2)(x-3)$$
$$\Rightarrow xy-3x-2y+6=xy-3y-2x+6$$
$$\Rightarrow -3x-2y=-3y-2x$$
$$\Rightarrow 3x-2x=3y-2y$$
$$\Rightarrow x=y$$

Therefore, f is one-one.

Let y ∈B = **R** − {1}.

Then, $y \neq 1$.

The function f is onto if there exists x ∈A such that $f(x) = y$.

Now,

$$f(x)=y$$
$$\Rightarrow \frac{x-2}{x-3}=y$$
$$\Rightarrow x-2=xy-3y$$
$$\Rightarrow x(1-y)=-3y+2$$
$$\Rightarrow x=\frac{2-3y}{1-y}\in \mathrm{A} \qquad [y\neq 1]$$

Thus, for any $y \in$ B, there exists $\dfrac{2-3y}{1-y}\in \mathrm{A}$ such that

$$f\left(\frac{2-3y}{1-y}\right)=\frac{\left(\frac{2-3y}{1-y}\right)-2}{\left(\frac{2-3y}{1-y}\right)-3}=\frac{2-3y-2+2y}{2-3y-3+3y}=\frac{-y}{-1}=y.$$

$\therefore f$ is onto.

Hence, function f is one-one and onto.

Functions Ex 2.1 Q8

We have $f : R \to R$ given by $f(x) = x - [x]$

Now,

check for injectivity:

$\because \ f(x) = x - [x] \ \Rightarrow \ f(x) = 0$ for $x \in Z$

$\therefore$ Range of $f = [0, 1] \neq R$

$\therefore$ f is not one-one, where as many-one

Again, Range of $f = [0, 1] \neq R$

$\therefore$ f is an into function

Functions Ex 2.1 Q9

Suppose $f(n_1) = f(n_2)$

If n_1 is odd and n_2 is even, then we have

$n_1 + 1 = n_2 - 1 \Rightarrow n_2 - n_1 = 2$, not possible

If n_1 is even and n_2 is odd, then we have

$n_1 - 1 = n_2 + 1 \Rightarrow n_1 - n_2 = 2$, not possible

Therefore, both n_1 and n_2 must be either odd or even.

Suppose both n_1 and n_2 are odd.

Then, $f(n_1) = f(n_2) \Rightarrow n_1 + 1 = n_2 + 1 \Rightarrow n_1 = n_2$

Suppose both n_1 and n_2 are even.

Then, $f(n_1) = f(n_2) \Rightarrow n_1 - 1 = n_2 - 1 \Rightarrow n_1 = n_2$

Thus, f is one – one.

Also, any odd number $2r + 1$ in the co – domain $\mathbb{N}$ will have an even number as image in domain $\mathbb{N}$ which is

$f(n) = 2r + 1 \Rightarrow n - 1 = 2r + 1 \Rightarrow n = 2r + 2$

any even number $2r$ in the co – domain $\mathbb{N}$ will have an odd number as image in domain $\mathbb{N}$ which is

$f(n) = 2r \Rightarrow n + 1 = 2r \Rightarrow n = 2r - 1$

Thus, f is onto.

Functions Ex 2.1 Q10

We have $A = \{1, 2, 3\}$

All one-one functions from $A = \{1, 2, 3\}$ to itself are obtained by re-arranging elements of A.

Thus all possible one-one functions are:

i) $f(1) = 1, \quad f(2) = 2, \quad f(3) = 3$

ii) $f(1) = 2, \quad f(2) = 3, \quad f(3) = 1$

iii) $f(1) = 3, \quad f(2) = 1, \quad f(3) = 2$

iv) $f(1) = 1, \quad f(2) = 3, \quad f(3) = 2$

v) $f(1) = 3, \quad f(2) = 2, \quad f(3) = 1$

vi) $f(1) = 2, \quad f(2) = 1, \quad f(3) = 3$

*********** END **********