



Definite Integrals Ex 20.2 Q40

$$I = \int_0^{\pi/2} \frac{\cos^2 x}{1 + 3 \sin^2 x} dx$$

$$I = \int_0^{\pi/2} \frac{\sec^2 x}{\sec^2 x (\sec^2 x + 3 \tan^2 x)} dx$$

Put $\tan x = t$

$$\sec^2 x dx = dt$$

$$x = 0 \Rightarrow t = 0 \text{ and } x = \frac{\pi}{2} \Rightarrow t = \infty$$

$$I = \int_0^{\infty} \frac{1}{(1+t^2)(1+4t^2)} dt$$

$$I = -\frac{1}{3} \int_0^{\infty} \left[\frac{1}{(1+t^2)} - \frac{1}{(1+4t^2)} \right] dt$$

$$I = -\frac{1}{3} \left[\tan^{-1} t - 2 \tan^{-1} 2t \right]_0^{\infty}$$

$$I = \frac{\pi}{6}$$

Definite Integrals Ex 20.2 Q41

$$\text{Let } I = \int_0^{\pi/4} \sin^3 2t \cos 2t dt. \text{ consider } \int \sin^3 2t \cos 2t dt$$

$$\text{Put } \sin 2t = u \text{ so that } 2 \cos 2t dt = du \text{ or } \cos 2t dt = \frac{1}{2} du$$

$$\begin{aligned} \text{So } \int \sin^3 2t \cos 2t dt &= \frac{1}{2} \int u^3 du \\ &= \frac{1}{8} [u^4] = \frac{1}{8} \sin^4 2t = F(t) \text{ say} \end{aligned}$$

Therefore, by the second fundamental theorem of integrals calculus

$$I = F\left(\frac{\pi}{4}\right) - F(0) = \frac{1}{8} \left[\sin^4 \frac{\pi}{2} - \sin^4 0 \right] = \frac{1}{8}$$

Definite Integrals Ex 20.2 Q42

$$\text{Let } 5 - 4 \cos \theta = t$$

Differentiating w.r.t. x , we get

$$4 \sin \theta d\theta = dt$$

$$\text{Now, } \theta = 0 \Rightarrow t = 1$$

$$\theta = \pi \Rightarrow t = 9$$

$$\therefore \int_0^{\pi} 5(5 - 4 \cos \theta)^{\frac{1}{4}} \sin \theta d\theta$$

$$= \frac{5}{4} \int_1^9 t^{\frac{1}{4}} dt$$

$$= \frac{5}{4} \left[\frac{4}{5} t^{\frac{5}{4}} \right]_1^9$$

$$= 3^{\frac{5}{2}} - 1$$

$$= 9\sqrt{3} - 1$$

$$\therefore \int_0^{\pi} 5(5 - 4 \cos \theta)^{\frac{1}{4}} \sin \theta d\theta = 9\sqrt{3} - 1$$

Definite Integrals Ex 20.2 Q43

We have,

$$\int_0^{\frac{\pi}{6}} \cos^{-3} 2\theta \sin 2\theta d\theta$$

$$= \int_0^{\frac{\pi}{6}} \frac{\sin 2\theta}{\cos^3 2\theta} d\theta$$

$$= \int_0^{\frac{\pi}{6}} \tan 2\theta \cdot \sec^2 2\theta d\theta$$

Let $\tan 2\theta = t$

Differentiating w.r.t. x , we get

$$2 \sec^2 2\theta d\theta = dt$$

Now, $\theta = 0 \Rightarrow t = 0$

$$\theta = \frac{\pi}{6} \Rightarrow t = \sqrt{3}$$

$$\begin{aligned} \therefore \int_0^{\frac{\pi}{6}} \tan 2\theta \cdot \sec^2 2\theta d\theta &= \frac{1}{2} \int_0^{\sqrt{3}} t dt = \frac{1}{2} \left[\frac{t^2}{2} \right]_0^{\sqrt{3}} \\ &= \frac{3}{4} \end{aligned}$$

$$\therefore \int_0^{\frac{\pi}{6}} \cos^{-3} 2\theta \sin 2\theta d\theta = \frac{3}{4}$$

***** END *****