Indefinite Integrals Ex 19.16 Q1

Let $I = \int \frac{\sec^2 x}{1 - \tan^2 x} dx$

Let $\tan x = t$

$\Rightarrow \quad \sec^2 x \, dx = dt$

$I = \int \frac{dt}{(1)^2 - t^2}$

$= \frac{1}{2(1)} \log\left|\frac{1+t}{1-t}\right| + c \quad \left[\text{Since, } \int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + c\right]$

$I = \frac{1}{2} \log\left|\frac{1 + \tan x}{1 - \tan x}\right| + c$

Indefinite Integrals Ex 19.16 Q2

Let $I = \int \frac{e^x}{1 + e^{2x}} dx$

Let $\tan e^x = t$

$\Rightarrow \quad e^x \, dx = dt$

so, $\quad I = \int \frac{dt}{1 + t^2}$

$= \tan^{-1}(t) + c \quad \left[\text{Since, } \int \frac{1}{1 + x^2} dx = \tan^{-1} x + c\right]$

$I = \tan^{-1}(e^x) + c$

Indefinite Integrals Ex 19.16 Q3

$$\text{Let } I = \int \frac{\cos x}{\sin^2 x + 4\sin x + 5} dx$$

$$\text{Let } \sin x = t$$

$$\Rightarrow \quad \cos x\, dx = dt$$

$$\text{so,} \quad I = \int \frac{dx}{t^2 + 4t + 5}$$

$$= \int \frac{dt}{t^2 + 2t(2) + (2)^2 - (2)^2 + 5}$$

$$= \int \frac{dt}{(t+2)^2 + 1}$$

$$\text{Again, Let } (t+2) = u$$

$$dt = du$$

$$I = \int \frac{dt}{u^2 + 1}$$

$$= \tan^{-1}(u) + c \qquad \left[\text{Since, } \int \frac{1}{x^2+1} dx = \tan^{-1} x + c\right]$$

$$I = \tan^{-1}(t+2) + c$$

$$I = \tan^{-1}(\sin x + 2) + c$$

Indefinite Integrals Ex 19.16 Q4

$$\text{Let } I = \int \frac{e^x}{e^{2x} + 5e^x + 6} dx$$

$$\text{Let } e^x = t$$

$$\Rightarrow \quad e^x\, dx = dt$$

$$\text{so,} \quad I = \int \frac{dt}{t^2 + 5t + 6}$$

$$= \int \frac{dt}{t^2 + 2t\left(\frac{5}{2}\right) + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 6}$$

$$= \int \frac{dt}{\left(t + \frac{5}{2}\right)^2 - \frac{1}{4}}$$

$$\text{Put } \left(t + \frac{5}{2}\right) = u$$

$$\Rightarrow \quad dt = du$$

$$I = \int \frac{dt}{u^2 - \left(\frac{1}{2}\right)^2}$$

$$= \frac{1}{2\left(\frac{1}{2}\right)} \log\left|\frac{u - \frac{1}{2}}{u + \frac{1}{2}}\right| + c \qquad \left[\text{Since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a}\left|\frac{x-a}{x+a}\right| + c\right]$$

$$I = \log\left|\frac{2u-1}{2u+1}\right| + c$$

$$I = \log\left|\frac{2\left(t + \frac{5}{2}\right) - 1}{2\left(t + \frac{5}{2}\right) + 1}\right| + c$$

$$I = \log\left|\frac{e^x + 2}{e^x + 3}\right| + c$$

Indefinite Integrals Ex 19.16 Q5

Let $I = \int \frac{e^{3x}}{4e^{6x} - 9} dx$

Let $e^{3x} = t$

$\Rightarrow \quad 3e^{3x}\, dx = dt$

$\Rightarrow \quad e^{3x} dx = \frac{dt}{3}$

$$I = \frac{1}{3} \int \frac{dt}{4t^2 - 9}$$

$$= \frac{1}{12} \int \frac{dt}{t^2 - \frac{9}{4}}$$

$$= \frac{1}{12} \int \frac{dt}{t^2 - \left(\frac{3}{2}\right)^2}$$

$$= \frac{1}{12} \times \frac{1}{2\left(\frac{3}{2}\right)} \log \left| \frac{t - \frac{3}{2}}{t + \frac{3}{2}} \right| + c \qquad \left[\text{Since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \left| \frac{x - a}{x + a} \right| + c \right]$$

$$I = \frac{1}{36} \log \left| \frac{2t - 3}{2t + 3} \right| + c$$

$$I = \frac{1}{36} \log \left| \frac{2e^{3x} - 3}{2e^{3x} + 3} \right| + c$$

*********** END **********