Mean Value Theorems Ex 15.1 Q1(v)

Here, $f(x) = x^{\frac{2}{3}}$ on $[-1,1]$

$$f'(x) = \frac{2}{3x^{\frac{1}{3}}}$$

$$f'(0) = \frac{2}{3(0)^{\frac{1}{3}}}$$

$$f'(0) = \infty$$

So, $f'(x)$ does not exist at $x = 0 \in (-1,1)$

$\Rightarrow$ $f(x)$ is not differentiatable in $x \in (-1,1)$

So, rolle's theorem is not applicable on $f(x)$ in $[-1,1]$.

Mean Value Theorems Ex 15.1 Q1(vi)

Here, $f(x) = \begin{cases} -4x + 5, & 0 \le x \le 1 \\ 2x - 3, & 1 < x \le 2 \end{cases}$

For $n = 1$

$$\begin{aligned} \text{LHS} &= \lim_{x \to (1-h)} (-4x + 5) \\ &= \lim_{h \to 0} [-4(1-h) + 5] \\ &= -4 + 5 \\ \text{LHS} &= 1 \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \lim_{x \to (1+h)} (2x - 3) \\ &= \lim_{h \to 0} [2(1+h) - 3] \\ &= 2 - 3 \\ \text{RHS} &= -1 \end{aligned}$$

So, LHS $\ne$ RHS

$\Rightarrow$ $f(x)$ is not contiuuous at $x = 1 \in [0,2]$

$\Rightarrow$ Rolle's theorem is not applicable on $f(x)$ in $[0,2]$.

Mean Value Theorems Ex 15.1 Q2(i)

Here,

$$f(x) = x^2 - 8x + 12 \text{ on } [2,6]$$

$f(x)$ is continuous is $[2,6]$ and differentiable is $(2,6)$ as it is a polynomial function

And $f(2) = (2)^2 - 8(2) + 12 = 0$

$f(6) = (6)^2 - 8(6) + 12 = 0$

$\Rightarrow f(2) = f(6)$

So, Rolle's theorem is applicable, therefore we show have $f'(c) = 0$ such that $c \in (2,6)$

So, $f(x) = x^2 - 8x + 12$

$\Rightarrow f'(x) = 2x - 8$

So, $f'(c) = 0$

$2c - 8 = 0$

$c = 4 \in (2,6)$

Therefore, Rolle's theorem is verified.

Mean Value Theorems Ex 15.1 Q2(ii)

The given function is $f(x) = x^2 - 4x + 3$

f, being a polynomial function, is continuous in $[1, 4]$ and is differentiable in $(1, 4)$ whose derivative is $2x - 4$.

$f(1) = 1^2 - 4\times1 + 3 = 0$, $f(4) = 4^2 - 4\times4 + 3 = 3$

$\therefore \frac{f(b)-f(a)}{b-a} = \frac{f(4)-f(1)}{4-1} = \frac{3-(0)}{3} = \frac{3}{3} = 1$

Mean Value Theorem states that there is a point $c \in (1, 4)$ such that $f'(c) = 1$

$f'(c) = 1$

$\Rightarrow 2c - 4 = 1$

$\Rightarrow c = \frac{5}{2}$, where $c = \frac{5}{2} \in (1, 4)$

Hence, Mean Value Theorem is verified for the given function

Mean Value Theorems Ex 15.1 Q2(iii)

Here,

$$f(x) = (x-1)(x-2)^2 \text{ on } (1,2)$$

$f(x)$ is cantinuous is $[1,2]$ and differentiable is $(1,2)$ since it is a polynomial function.

And $f(1) = (1-1)(1-2)^2 = 0$

$f(2) = (2-1)(2-2)^2 = 0$

$\Rightarrow$ $f(1) = f(2)$

So, Rolle's theorem is applicable on $f(x)$ in $[1,2]$, therefore, there exist a $c \in (1,2)$ such that $f'(c) = 0$

Now,

$$f(x) = (x-1)(x-2)^2$$

$$f'(x) = (x-1)\times 2(x-2) + (x-2)^2$$

$$f'(x) = (x-2)(3x-4)$$

So, $f'(c) = 0$

$(c-2)(3c-4) = 0$

$\Rightarrow$ $c = 2$ or $c = \frac{4}{3} \in (1,2)$

Thus, Rolle's theorem is verified.

*********** END **********