Exercise 7.8 : Solutions of Questions on Page Number : **334**

Q1 : $\int_a^b x\,dx$

Answer :

It is known that,

$$\int_a^b f(x)dx=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+\ldots+f(a+(n-1)h)\right],\text{ where }h=\frac{b-a}{n}$$

Here, $a=a,\ b=b,$ and $f(x)=x$

$$\begin{aligned}
\therefore \int_a^b x\,dx &=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[a+(a+h)\ldots(a+2h)\ldots a+(n-1)h\right]\\
&=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{a+a+a+\ldots+a}_{n\text{ times}}\right)+\left(h+2h+3h+\ldots+(n-1)h\right)\right]\\
&=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[na+h\left(1+2+3+\ldots+(n-1)\right)\right]\\
&=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[na+h\left\{\frac{(n-1)(n)}{2}\right\}\right]\\
&=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[na+\frac{n(n-1)h}{2}\right]\\
&=(b-a)\lim_{n\to\infty}\frac{n}{n}\left[a+\frac{(n-1)h}{2}\right]\\
&=(b-a)\lim_{n\to\infty}\left[a+\frac{(n-1)h}{2}\right]\\
&=(b-a)\lim_{n\to\infty}\left[a+\frac{(n-1)(b-a)}{2n}\right]\\
&=(b-a)\lim_{n\to\infty}\left[a+\frac{\left(1-\frac{1}{n}\right)(b-a)}{2}\right]\\
&=(b-a)\left[a+\frac{(b-a)}{2}\right]\\
&=(b-a)\left[\frac{2a+b-a}{2}\right]\\
&=\frac{(b-a)(b+a)}{2}\\
&=\frac{1}{2}\left(b^2-a^2\right)
\end{aligned}$$

Answer needs Correction? **Click Here**

Q2 : $\int_0^5 (x+1)dx$

Answer :

Let $I=\int_0^5 (x+1)dx$

It is known that,

$$\int_a^b f(x)dx=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)\ldots f(a+(n-1)h)\right],\text{ where }h=\frac{b-a}{n}$$

Here, $a=0,\ b=5,$ and $f(x)=(x+1)$

$$\Rightarrow h=\frac{5-0}{n}=\frac{5}{n}$$

$$\begin{aligned}
\therefore \int_0^5 (x+1)dx &=(5-0)\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f\left(\frac{5}{n}\right)+\ldots+f\left((n-1)\frac{5}{n}\right)\right]\\
&=5\lim_{n\to\infty}\frac{1}{n}\left[1+\left(\frac{5}{n}+1\right)+\ldots\left\{1+\left(\frac{5(n-1)}{n}\right)\right\}\right]\\
&=5\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{1+1+1\ldots1}_{n\text{ times}}\right)+\left[\frac{5}{n}+2\cdot\frac{5}{n}+3\cdot\frac{5}{n}+\ldots(n-1)\frac{5}{n}\right]\right]\\
&=5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5}{n}\left\{1+2+3\ldots(n-1)\right\}\right]\\
&=5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5}{n}\cdot\frac{(n-1)n}{2}\right]\\
&=5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5(n-1)}{2}\right]\\
&=5\lim_{n\to\infty}\left[1+\frac{5}{2}\left(1-\frac{1}{n}\right)\right]\\
&=5\left[1+\frac{5}{\ }\right.
\end{aligned}$$

$$= 5\left[\frac{7}{2}\right]$$
$$= \frac{35}{2}$$

Answer needs Correction? Click Here

Q3 : $\int_2^3 x^2 dx$

Answer :

It is known that,

$$\int_a^b f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+f(a+2h)\ldots f\{a+(n-1)h\}\right], \text{ where } h=\frac{b-a}{n}$$

Here, $a = 2$, $b = 3$, and $f(x) = x^2$

$$\Rightarrow h = \frac{3-2}{n} = \frac{1}{n}$$

$$\therefore \int_2^3 x^2 dx = (3-2)\lim_{n\to\infty}\frac{1}{n}\left[f(2)+f\left(2+\frac{1}{n}\right)+f\left(2+\frac{2}{n}\right)\ldots f\left\{2+(n-1)\frac{1}{n}\right\}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[(2)^2+\left(2+\frac{1}{n}\right)^2+\left(2+\frac{2}{n}\right)^2+\ldots\left(2+\frac{(n-1)}{n}\right)^2\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[2^2+\left\{2^2+\left(\frac{1}{n}\right)^2+2\cdot2\cdot\frac{1}{n}\right\}+\ldots+\left\{(2)^2+\frac{(n-1)^2}{n^2}+2\cdot2\cdot\frac{(n-1)}{n}\right\}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{2^2+\ldots+2^2}_{n\text{ times}}\right)+\left\{\left(\frac{1}{n}\right)^2+\left(\frac{2}{n}\right)^2+\ldots+\left(\frac{n-1}{n}\right)^2\right\}+2\cdot2\cdot\left\{\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\ldots+\frac{(n-1)}{n}\right\}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[4n+\frac{1}{n^2}\left\{1^2+2^2+3^2\ldots+(n-1)^2\right\}+\frac{4}{n}\left\{1+2+\ldots+(n-1)\right\}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[4n+\frac{1}{n^2}\left\{\frac{n(n-1)(2n-1)}{6}\right\}+\frac{4}{n}\left\{\frac{n(n-1)}{2}\right\}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[4n+\frac{n\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}{6}+\frac{4n-4}{2}\right]$$

$$= \lim_{n\to\infty}\left[4+\frac{1}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+2-\frac{2}{n}\right]$$

$$= 4+\frac{2}{6}+2$$

$$= \frac{19}{3}$$

Answer needs Correction? Click Here

Q4 : $\int_1^4 (x^2-x)dx$

Answer :

Let $I = \int_1^4 (x^2-x)dx$

$$= \int_1^4 x^2\,dx - \int_1^4 x\,dx$$

Let $I = I_1 - I_2$, where $I_1 = \int_1^4 x^2 dx$ and $I_2 = \int_1^4 x\,dx$ ...(1)

It is known that,

$$\int_a^b f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+f(a+(n-1)h)\right], \text{ where } h=\frac{b-a}{n}$$

For $I_1 = \int_1^4 x^2 dx$,

$a = 1, b = 4$, and $f(x) = x^2$

$$\therefore h = \frac{4-1}{n} = \frac{3}{n}$$

$$I_1 = \int_1^4 x^2 dx = (4-1)\lim_{n\to\infty}\frac{1}{n}\left[f(1)+f(1+h)+\ldots+f(1+(n-1)h)\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[1^2+\left(1+\frac{3}{n}\right)^2+\left(1+2\cdot\frac{3}{n}\right)^2+\ldots\left(1+\frac{(n-1)3}{n}\right)^2\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[1^2+\left\{1^2+\left(\frac{3}{n}\right)^2+2\cdot\frac{3}{n}\right\}+\ldots+\left\{1^2+\left(\frac{(n-1)3}{n}\right)^2+\frac{2\cdot(n-1)\cdot3}{n}\right\}\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{1^2+\ldots+1^2}_{n\text{ times}}\right)+\left(\frac{3}{n}\right)^2\left\{1^2+2^2+\ldots+(n-1)^2\right\}+2\cdot\frac{3}{n}\left\{1+2+\ldots+(n-1)\right\}\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{9}{n^2}\left\{\frac{(n-1)(n)(2n-1)}{6}\right\}+\frac{6}{n}\left\{\frac{(n-1)(n)}{2}\right\}\right]$$

$$= 3\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{9n}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+\frac{6n-6}{2}\right]$$

$$= 3\lim_{n\to\infty}\left[1+\frac{9}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+3-\frac{3}{n}\right]$$

$$= 3[1+3+3]$$

$$= 3[7]$$

$I_1 = 21$ ...(2)

For $I_2 = \int_1^4 x dx$,

$a = 1, b = 4$, and $f(x) = x$

$$\Rightarrow h=\frac{4-1}{n}=\frac{3}{n}$$

$$\therefore I_2=(4-1)\lim_{n\to\infty}\frac{1}{n}\left[f(1)+f(1+h)+\dots f\left(a+(n-1)h\right)\right]$$

$$=3\lim_{n\to\infty}\frac{1}{n}\left[1+(1+h)+\dots+\left(1+(n-1)h\right)\right]$$

$$=3\lim_{n\to\infty}\frac{1}{n}\left[1+\left(1+\frac{3}{n}\right)+\dots+\left\{1+(n-1)\frac{3}{n}\right\}\right]$$

$$=3\lim_{n\to\infty}\frac{1}{n}\left[\left(\underbrace{1+1+\dots+1}_{n\text{ times}}\right)+\frac{3}{n}\left(1+2+\dots+(n-1)\right)\right]$$

$$=3\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{3}{n}\left\{\frac{(n-1)n}{2}\right\}\right]$$

$$=3\lim_{n\to\infty}\frac{1}{n}\left[1+\frac{3}{2}\left(1-\frac{1}{n}\right)\right]$$

$$=3\left[1+\frac{3}{2}\right]$$

$$=3\left[\frac{5}{2}\right]$$

$$I_2=\frac{15}{2} \qquad \dots(3)$$

From equations (2) and (3), we obtain

$$I=I_1+I_2=21-\frac{15}{2}=\frac{27}{2}$$

Answer needs Correction? **Click Here**

Q5 : $\int_{-1}^{1}e^x dx$

Answer :

$$\text{Let } I=\int_{-1}^{1}e^x dx \qquad \dots(1)$$

It is known that,

$$\int_a^b f(x)\,dx=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)\dots f\left(a+(n-1)h\right)\right],\text{ where } h=\frac{b-a}{n}$$

Here, $a=-1$, $b=1$, and $f(x)=e^x$

$$\therefore h=\frac{1+1}{n}=\frac{2}{n}$$

$$\therefore I=(1+1)\lim_{n\to\infty}\frac{1}{n}\left[f(-1)+f\left(-1+\frac{2}{n}\right)+f\left(-1+2\cdot\frac{2}{n}\right)+\dots+f\left(-1+\frac{(n-1)2}{n}\right)\right]$$

$$=2\lim_{n\to\infty}\frac{1}{n}\left[e^{-1}+e^{\left(-1+\frac{2}{n}\right)}+e^{\left(-1+2\cdot\frac{2}{n}\right)}+\dots e^{\left(-1+(n-1)\frac{2}{n}\right)}\right]$$

$$=2\lim_{n\to\infty}\frac{1}{n}\left[e^{-1}\left\{1+e^{\frac{2}{n}}+e^{\frac{4}{n}}+e^{\frac{6}{n}}+e^{(n-1)\frac{2}{n}}\right\}\right]$$

$$=2\lim_{n\to\infty}\frac{e^{-1}}{n}\left[\frac{e^{\frac{2n}{n}-1}}{e^{\frac{2}{n}-1}}\right]$$

$$=e^{-1}\times2\lim_{n\to\infty}\frac{1}{n}\left[\frac{e^2-1}{e^{\frac{2}{n}-1}}\right]$$

$$=\frac{e^{-1}\times2\left(e^2-1\right)}{\lim\limits_{\frac{2}{n}\to0}\left(\frac{e^{\frac{2}{n}}-1}{\frac{2}{n}}\right)\times2}$$

$$=e^{-1}\left[\frac{2\left(e^2-1\right)}{2}\right] \qquad \left[\lim_{h\to0}\left(\frac{e^h-1}{h}\right)=1\right]$$

$$=\frac{e^2-1}{e}$$

$$=\left(e-\frac{1}{e}\right)$$

Answer needs Correction? **Click Here**

Q6 : $\int_0^4\left(x+e^{2x}\right)dx$

Answer :

It is known that,

$$\int_a^b f(x)\,dx=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+\dots+f\left(a+(n-1)h\right)\right],\text{ where } h=\frac{b-a}{n}$$

Here, $a=0$, $b=4$, and $f(x)=x+e^{2x}$

$$\therefore h=\frac{4-0}{n}=\frac{4}{n}$$

$$\Rightarrow\int_0^4\left(x+e^{2x}\right)dx=(4-0)\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f(h)+f(2h)+\dots+f\left((n-1)h\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\left(0+e^0\right)+\left(h+e^{2h}\right)+\left(2h+e^{2\cdot2h}\right)+\dots+\left\{(n-1)h+e^{2(n-1)h}\right\}\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[1+\left(h+e^{2h}\right)+\left(2h+e^{4h}\right)+\dots+\left\{(n-1)h+e^{2(n-1)h}\right\}\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[\left\{h+2h+3h+\dots+(n-1)h\right\}+\left(1+e^{2h}+e^{4h}+\dots+e^{2(n-1)h}\right)\right]$$

$$=4\lim_{n\to\infty}\frac{1}{n}\left[h\left\{1+2+\dots(n-1)\right\}+\left(\frac{e^{2hn}-1}{e^{2h}-1}\right)\right]$$

$$= 4\lim_{n\to\infty}\frac{1}{n}\left[\frac{(h(n-1)n)}{2}+\left(\frac{e^{2nh}-1}{e^{2h}-1}\right)\right]$$

$$= 4\lim_{n\to\infty}\frac{1}{n}\left[\frac{4}{n}\cdot\frac{(n-1)n}{2}+\left(\frac{e^{8}-1}{e^{\frac{8}{n}}-1}\right)\right]$$

$$= 4(2)+4\lim_{n\to\infty}\frac{\left(e^{8}-1\right)}{\left(\frac{e^{\frac{8}{n}}-1}{\frac{8}{n}}\right)8}$$

$$= 8+\frac{4\cdot\left(e^{8}-1\right)}{8} \qquad \left(\lim_{x\to 0}\frac{e^{x}-1}{x}=1\right)$$

$$= 8+\frac{e^{8}-1}{2}$$

$$= \frac{15+e^{8}}{2}$$

*********** END **********