$$\Rightarrow \vec{l} = \left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}-5\hat{k}\right)$$

Question 8:

Find the equation of the plane passing through (a, b, c) and parallel to the plane $\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=2$

Answer

Any plane parallel to the plane, $\vec{r}_1\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=2$, is of the form

$$\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=\lambda \qquad \text{...(1)}$$

The plane passes through the point (a, b, c). Therefore, the position vector $\vec{r}$ of this point is $\vec{r}=a\hat{i}+b\hat{j}+c\hat{k}$

Therefore, equation (1) becomes

$$\left(a\hat{i}+b\hat{j}+c\hat{k}\right)\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=\lambda$$

$$\Rightarrow a+b+c=\lambda$$

Substituting $\lambda=a+b+c$ in equation (1), we obtain

$$\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=a+b+c \qquad \text{...(2)}$$

This is the vector equation of the required plane.

Substituting $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ in equation (2), we obtain

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=a+b+c$$

$$\Rightarrow x+y+z=a+b+c$$

Question 9:

Find the shortest distance between lines $\vec{r}=6\hat{i}+2\hat{j}+2\hat{k}+\lambda\left(\hat{i}-2\hat{j}+2\hat{k}\right)$ and $\vec{r}=-4\hat{i}-\hat{k}+\mu\left(3\hat{i}-2\hat{j}-2\hat{k}\right)$.

Answer

The given lines are

$$\vec{r}=6\hat{i}+2\hat{j}+2\hat{k}+\lambda\left(\hat{i}-2\hat{j}+2\hat{k}\right) \qquad \text{...(1)}$$

$$\vec{r}=-4\hat{i}-\hat{k}+\mu\left(3\hat{i}-2\hat{j}-2\hat{k}\right) \qquad \text{...(2)}$$

It is known that the shortest distance between two lines, $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\lambda\vec{b}_2$, is given by

$$d=\left|\frac{\left(\vec{b}_1\times\vec{b}_2\right)\cdot\left(\vec{a}_2-\vec{a}_1\right)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right| \qquad \text{...(3)}$$

Comparing $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\lambda\vec{b}_2$ to equations (1) and (2), we obtain

$$\vec{a}_1=6\hat{i}+2\hat{j}+2\hat{k}$$

$$\vec{b}_1=\hat{i}-2\hat{j}+2\hat{k}$$

$$\vec{a}_2=-4\hat{i}-\hat{k}$$

$$\vec{b}_2=3\hat{i}-2\hat{j}-2\hat{k}$$

$$\Rightarrow \vec{a}_2-\vec{a}_1=\left(-4\hat{i}-\hat{k}\right)-\left(6\hat{i}+2\hat{j}+2\hat{k}\right)=-10\hat{i}-2\hat{j}-3\hat{k}$$

$$\Rightarrow \vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 2\\ 3 & -2 & -2\end{vmatrix}=(4+4)\hat{i}-(-2-6)\hat{j}+(-2+6)\hat{k}=8\hat{i}+8\hat{j}+4\hat{k}$$

$$\therefore \left|\vec{b}_1\times\vec{b}_2\right|=\sqrt{(8)^2+(8)^2+(4)^2}=12$$

$$\left(\vec{b}_1\times\vec{b}_2\right)\cdot\left(\vec{a}_2-\vec{a}_1\right)=\left(8\hat{i}+8\hat{j}+4\hat{k}\right)\cdot\left(-10\hat{i}-2\hat{j}-3\hat{k}\right)=-80-16-12=-108$$

Substituting all the values in equation (1), we obtain

$$d=\left|\frac{-108}{12}\right|=9$$

Therefore, the shortest distance between the two given lines is 9 units.

Therefore, the shortest distance between the two given lines is 9 units.

Question 10:

Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the YZ-plane

Answer

It is known that the equation of the line passing through the points, (x_1, y_1, z_1) and (x_2, y_2, z_2), is $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$

The line passing through the points, (5, 1, 6) and (3, 4, 1), is given by,

$$\frac{x-5}{3-5}=\frac{y-1}{4-1}=\frac{z-6}{1-6}$$

$$\Rightarrow \frac{x-5}{-2}=\frac{y-1}{3}=\frac{z-6}{-5}=k\left(\text{say}\right)$$

$$\Rightarrow x=5-2k,\ y=3k+1,\ z=6-5k$$

Any point on the line is of the form $(5-2k, 3k+1, 6-5k)$.

The equation of YZ-plane is $x = 0$

Since the line passes through YZ-plane,

$5 - 2k = 0$

$$\Rightarrow k=\frac{5}{2}$$

$$\Rightarrow 3k+1=3\times\frac{5}{2}+1=\frac{17}{2}$$

$$6-5k=6-5\times\frac{5}{2}=\frac{-13}{2}$$

Therefore, the required point is $\left(0,\frac{17}{2},\frac{-13}{2}\right)$.

Question 11:

Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the ZX − plane.

Answer

It is known that the equation of the line passing through the points, (x_1, y_1, z_1) and (x_2, y_2, z_2), is $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$

The line passing through the points, (5, 1, 6) and (3, 4, 1), is given by,

$$\frac{x-5}{3-5}=\frac{y-1}{4-1}=\frac{z-6}{1-6}$$

$$\Rightarrow \frac{x-5}{-2}=\frac{y-1}{3}=\frac{z-6}{-5}=k\left(\text{say}\right)$$

$$\Rightarrow x=5-2k,\ y=3k+1,\ z=6-5k$$

Any point on the line is of the form $(5-2k, 3k+1, 6-5k)$.

Since the line passes through ZX-plane,

$$3k+1=0$$

$$\Rightarrow k=-\frac{1}{3}$$

$$\Rightarrow 5-2k=5-2\left(-\frac{1}{3}\right)=\frac{17}{3}$$

$$6-5\text{k}=6-5\left(-\frac{1}{3}\right)=\frac{23}{3}$$

Therefore, the required point is $\left(\frac{17}{3},0,\frac{23}{3}\right)$.

Question 12:

Find the coordinates of the point where the line through (3, −4, −5) and (2, − 3, 1) crosses the plane $2x + y + z = 7$).

Answer

It is known that the equation of the line through the points, (x_1, y_1, z_1) and (x_2, y_2, z_2), is

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

Since the line passes through the points, (3, −4, −5) and (2, −3, 1), its equation is given by,

$$\frac{x-3}{2-3}=\frac{y+4}{-3+4}=\frac{z+5}{1+5}$$

$$\Rightarrow \frac{x-3}{-1}=\frac{y+4}{1}=\frac{z+5}{6}=k\left(\text{say}\right)$$

$$\Rightarrow x=3-k,\ y=k-4,\ z=6k-5$$

Therefore, any point on the line is of the form $(3-k, k-4, 6k-5)$.

This point lies on the plane, $2x + y + z = 7$

$\therefore 2(3-k) + (k-4) + (6k-5) = 7$

$$\Rightarrow 5k-3=7$$

$$\Rightarrow k=2$$

$\Rightarrow k = 2$

Hence, the coordinates of the required point are (3 − 2, 2 − 4, 6 × 2 − 5) i.e., (1, −2, 7).

Question 13:

Find the equation of the plane passing through the point (−1, 3, 2) and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

Answer

The equation of the plane passing through the point (−1, 3, 2) is

$a(x + 1) + b(y - 3) + c(z - 2) = 0$ ... (1)

where, a, b, c are the direction ratios of normal to the plane.

It is known that two planes, $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$, are perpendicular, if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Plane (1) is perpendicular to the plane, $x + 2y + 3z = 5$

$$\therefore a \cdot 1 + b \cdot 2 + c \cdot 3 = 0$$

$$\Rightarrow a + 2b + 3c = 0 \qquad \ldots(2)$$

Also, plane (1) is perpendicular to the plane, $3x + 3y + z = 0$

$$\therefore a \cdot 3 + b \cdot 3 + c \cdot 1 = 0$$

$$\Rightarrow 3a + 3b + c = 0 \qquad \ldots(3)$$

From equations (2) and (3), we obtain

$$\frac{a}{2\times1-3\times3} = \frac{b}{3\times3-1\times1} = \frac{c}{1\times3-2\times3}$$

$$\Rightarrow \frac{a}{-7} = \frac{b}{8} = \frac{c}{-3} = k\,(\text{say})$$

*********** END **********