



Indefinite Integrals Ex 19.29 Q5

$$\text{Let } I = \int (4x + 1) \sqrt{x^2 - x - 2} dx$$

$$\begin{aligned} \text{Let } 4x + 1 &= \lambda \frac{d}{dx} (x^2 - x - 2) + \mu \\ &= \lambda (2x - 1) + \mu \end{aligned}$$

Equating similar terms, we get,

$$\begin{aligned} 2\lambda &= 4 & \Rightarrow & \lambda = 2 \\ -\lambda + \mu &= 1 & \Rightarrow & \mu = 3 \end{aligned}$$

So,

$$\begin{aligned} I &= \int [2(2x - 1) + 3] \sqrt{x^2 - x - 2} dx \\ &= 2 \int (2x - 1) \sqrt{x^2 - x - 2} dx + 3 \int \sqrt{x^2 - x - 2} dx \end{aligned}$$

$$\begin{aligned} \text{Let } x^2 - x - 2 &= t \\ (2x - 1) dx &= dt \end{aligned}$$

$$\therefore I = 2 \int \sqrt{t} dt + 3 \int \sqrt{\left(x - \frac{1}{2}\right)^2 - \left(\frac{3}{2}\right)^2} dx$$

$$\Rightarrow I = 2 \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + 3 \left\{ \frac{\left(x - \frac{1}{2}\right)}{2} \sqrt{x^2 - x - 2} - \frac{9}{8} \log \left| \left(x - \frac{1}{2}\right) + \sqrt{x^2 - x - 2} \right| \right\} + c$$

Hence,

$$I = \frac{4}{3} (x^2 - x - 2)^{\frac{3}{2}} + \frac{3}{4} (2x - 1) \sqrt{x^2 - x - 2} - \frac{27}{8} \log \left| \left(x - \frac{1}{2}\right) + \sqrt{x^2 - x - 2} \right| + c$$

Indefinite Integrals Ex 19.29 Q6

$$\text{Let } I = \int (x-2) \sqrt{2x^2 - 6x + 5} dx$$

$$\begin{aligned} \text{Let } x-2 &= \lambda \frac{d}{dx} (2x^2 - 6x + 5) + \mu \\ &= \lambda (4x - 6) + \mu \end{aligned}$$

Equating similar terms, we get,

$$4\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{4}$$

$$-6\lambda + \mu = -2 \quad \Rightarrow \quad \mu = -2 + 6\lambda = -2 + \frac{3}{2} = -\frac{1}{2}$$

$$\therefore \mu = -\frac{1}{2}$$

So,

$$\begin{aligned} I &= \int \left(\frac{1}{4} (4x - 6) + \left(-\frac{1}{2} \right) \right) \sqrt{2x^2 - 6x + 5} dx \\ &= \frac{1}{4} \int (4x - 6) \sqrt{2x^2 - 6x + 5} dx - \frac{1}{2} \int \sqrt{2x^2 - 6x + 5} dx \end{aligned}$$

$$\text{Let } 2x^2 - 6x + 5 = t$$

$$(4x - 6) dx = dt$$

$$\therefore I = \frac{1}{4} \int \sqrt{t} dt - \frac{\sqrt{2}}{2} \int \sqrt{x^2 - 3x + \frac{5}{2}} dx$$

$$\Rightarrow I = \frac{1}{4} \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{\sqrt{2}} \int \sqrt{\left(x - \frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2} dx$$

$$= \frac{1}{6} (2x^2 - 6x + 5)^{\frac{3}{2}} - \frac{1}{\sqrt{2}} \left\{ \frac{\left(x - \frac{3}{2}\right)}{2} \sqrt{x^2 - 3x + \frac{5}{2}} + \frac{1}{8} \log \left| \left(x - \frac{3}{2}\right) + \sqrt{x^2 - 3x + \frac{5}{2}} \right| \right\} + c$$

Hence,

$$I = \frac{1}{6} (2x^2 - 6x + 5)^{\frac{3}{2}} - \frac{1}{8} (2x - 3) \sqrt{2x^2 - 6x + 5} - \frac{1}{8\sqrt{2}} \log \left| \left(x - \frac{3}{2}\right) + \sqrt{x^2 - 3x + \frac{5}{2}} \right| + c$$

Indefinite Integrals Ex 19.29 Q7

$$\text{Let } I = \int (x+1) \sqrt{x^2+x+1} dx$$

$$\begin{aligned} \text{Let } x+1 &= \lambda \frac{d}{dx} (x^2+x+1) + \mu \\ &= \lambda (2x+1) + \mu \end{aligned}$$

Equating similar terms, we get,

$$2\lambda = 1 \Rightarrow \lambda = \frac{1}{2}$$

$$\lambda + \mu = 1 \Rightarrow \mu = \frac{1}{2}$$

So,

$$\begin{aligned} I &= \int \left(\frac{1}{2} (2x+1) + \frac{1}{2} \right) \sqrt{x^2+x+1} dx \\ &= \frac{1}{2} \int (2x+1) \sqrt{x^2+x+1} dx + \frac{1}{2} \int \sqrt{x^2+x+1} dx \end{aligned}$$

$$\text{Let } x^2+x+1 = t$$

$$\Rightarrow (2x+1) dx = dt$$

$$= \frac{1}{2} \int \sqrt{t} dt + \frac{1}{2} \int \sqrt{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx$$

$$= \frac{1}{2} \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{1}{2} \left\{ \frac{\left(x + \frac{1}{2}\right)}{2} \sqrt{x^2+x+1} + \frac{3}{8} \log \left| \left(x + \frac{1}{2}\right) + \sqrt{x^2+x+1} \right| \right\} + c$$

$$\Rightarrow I = \frac{1}{3} (x^2+x+1)^{\frac{3}{2}} + \frac{1}{8} (2x+1) \sqrt{x^2+x+1} + \frac{3}{16} \log \left| \left(x + \frac{1}{2}\right) + \sqrt{x^2+x+1} \right| + c$$

Hence,

$$I = \frac{1}{3} (x^2+x+1)^{\frac{3}{2}} + \frac{1}{8} (2x+1) \sqrt{x^2+x+1} + \frac{3}{16} \log \left| \left(x + \frac{1}{2}\right) + \sqrt{x^2+x+1} \right| + c$$

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