Definite Integrals Ex 20.5 Q25

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a = 0,\ b = 2$ and $f(x) = x^2 + 2x + 1$

$$\therefore \quad h = \frac{2}{n} \quad \Rightarrow nh = 2$$

Thus, we have,

$$I = \int_0^2 \left(x^2 + 2x + 1\right)dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f(0+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[1+\left(h^2+2h+1\right)+\left\{(2h)^2+2\times 2h+1\right\}+----\right]$$

$$= \lim_{h\to 0} h\left[n+h^2\left(1+2^2+3^2+----(n-1)^2+2h\left(1+2+3----(n-1)\right)\right]\right.$$

$$\because h = \frac{2}{n} \quad \text{if } h\to 0 \Rightarrow n\to\infty$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[n+\frac{4}{n^2}\frac{n(n-1)(2n-1)}{6}+\frac{4}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} 2+\frac{4}{3n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+\frac{4}{n^2}n^2\left(1-\frac{1}{n}\right)$$

$$= 2+\frac{8}{3}+4 = \frac{26}{3}$$

$$\therefore \int_0^2 \left(x^2+2x+1\right)dx = \frac{26}{3}$$

Definite Integrals Ex 20.5 Q26

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here, $a=0,\ b=3$ and $f(x) = 2x^2+3x+5$

$$\therefore \quad h = \frac{3}{n} \quad \Rightarrow nh = 3$$

Thus, we have,

$$I = \int_0^3 \left(2x^2+3x+5\right)dx$$

$$= \lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f((n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[5+\left(2h^2+3h+5\right)+\left\{2(2h)^2+3\times 2h+5\right\}+----\right]$$

$$= \lim_{h\to 0} h\left[5n+2h^2\left(1+2^2+3^2+----(n-1)^2+3h\left(1+2+3----(n-1)\right)\right)\right]$$

$$\because h = \frac{3}{n} \quad \& \text{ if } h\to 0 \Rightarrow n\to\infty$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[5n+\frac{18}{n^2}\frac{n(n-1)(2n-1)}{6}+\frac{9}{n}\frac{n(n-1)}{2}\right]$$

$$= \lim_{n\to\infty} 15+\frac{9}{n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+\frac{27}{2n^2}n^2\left(1-\frac{1}{n}\right)$$

$$= 15+18+\frac{27}{2} \quad = \frac{93}{2}$$

$$\therefore \quad \int_0^3 \left(2x^2+3x+5\right)dx = \frac{93}{2}$$

Definite Integrals Ex 20.5 Q27

It is known that,

$$\int_a^b f(x)\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\Big[f(a)+f(a+h)+\ldots+f\big(a+(n-1)h\big)\Big],\ \text{where } h=\frac{b-a}{n}$$

Here, $a=a,\ b=b,$ and $f(x)=x$

$$\therefore \int_a^b x\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\big[a+(a+h)\ldots(a+2h)\ldots a+(n-1)h\big]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\Big[\big(\underbrace{a+a+a+\ldots+a}_{n\text{ times}}\big)+\big(h+2h+3h+\ldots+(n-1)h\big)\Big]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\Big[na+h\big(1+2+3+\ldots+(n-1)\big)\Big]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[na+h\left\{\frac{(n-1)(n)}{2}\right\}\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[na+\frac{n(n-1)h}{2}\right]$$

$$=(b-a)\lim_{n\to\infty}\frac{n}{n}\left[a+\frac{(n-1)h}{2}\right]$$

$$=(b-a)\lim_{n\to\infty}\left[a+\frac{(n-1)h}{2}\right]$$

$$=(b-a)\lim_{n\to\infty}\left[a+\frac{(n-1)(b-a)}{2n}\right]$$

$$=(b-a)\lim_{n\to\infty}\left[a+\frac{\left(1-\frac{1}{n}\right)(b-a)}{2}\right]$$

$$=(b-a)\left[a+\frac{(b-a)}{2}\right]$$

$$=(b-a)\left[\frac{2a+b-a}{2}\right]$$

$$=\frac{(b-a)(b+a)}{2}$$

$$=\frac{1}{2}\left(b^2-a^2\right)$$

*********** END **********