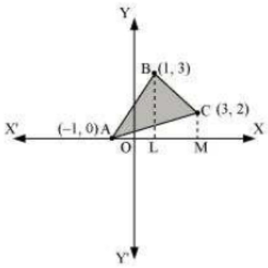


Equation of line segment AB is

$$y-0=\frac{3-0}{1+1}(x+1)$$

$$y=\frac{3}{2}(x+1)$$

$$\therefore \text{Area}(\text{ALBA})=\int_{-1}^{1}\frac{3}{2}(x+1)dx=\frac{3}{2}\left[\frac{x^2}{2}+x\right]_{-1}^{1}=\frac{3}{2}\left[\frac{1}{2}+1-\frac{1}{2}+1\right]=3\text{ units}$$

Equation of line segment BC is

$$y-3=\frac{2-3}{3-1}(x-1)$$

$$y=\frac{1}{2}(-x+7)$$

$$\therefore \text{ Area}(\text{BLMCB})=\int_{1}^{3}\frac{1}{2}(-x+7)dx=\frac{1}{2}\left[-\frac{x^2}{2}+7x\right]_{1}^{3}=\frac{1}{2}\left[-\frac{9}{2}+21+\frac{1}{2}-7\right]=5\text{ units}$$

Equation of line segment AC is

$$y-0=\frac{2-0}{3+1}(x+1)$$

$$y=\frac{1}{2}(x+1)$$

$$\therefore \text{Area}(\text{AMCA})=\frac{1}{2}\int_{-1}^{3}(x+1)dx=\frac{1}{2}\left[\frac{x^2}{2}+x\right]_{-1}^{3}=\frac{1}{2}\left[\frac{9}{2}+3-\frac{1}{2}+1\right]=4\text{ units}$$

Therefore, from equation (1), we obtain

Area (ΔABC) = (3 + 5 – 4) = 4 units

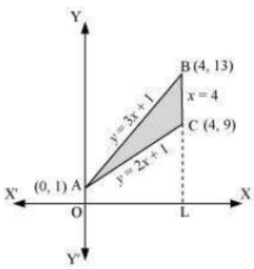
Question 5:

Using integration find the area of the triangular region whose sides have the equations $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

Answer

The equations of sides of the triangle are $y = 2x + 1$, $y = 3x + 1$, and $x = 4$.

On solving these equations, we obtain the vertices of triangle as A(0, 1), B(4, 13), and C (4, 9).



It can be observed that,

Area (ΔACB) = Area (OLBAO) –Area (OLCAO)

$$=\int_0^4(3x+1)dx-\int_0^4(2x+1)dx$$

$$=\left[\frac{3x^2}{2}+x\right]_0^4-\left[\frac{2x^2}{2}+x\right]_0^4$$

$$=(24+4)-(16+4)$$

$$=28-20$$

$= 8$ units

Question 6:

Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ is

A. 2 (π – 2)

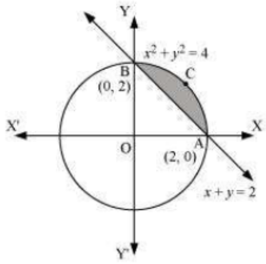
B. π – 2

C. 2π – 1

D. 2 (π + 2)

Answer

The smaller area enclosed by the circle, $x^2 + y^2 = 4$, and the line, $x + y = 2$, is represented by the shaded area ACBA as



It can be observed that,

Area ACBA = Area OACBO – Area (ΔOAB)

$$= \int_0^2 \sqrt{4-x^2}\,dx - \int_0^2 (2-x)\,dx$$

$$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - \left[2x - \frac{x^2}{2}\right]_0^2$$

$$= \left[2\cdot\frac{\pi}{2}\right] - [4-2]$$

$$= (\pi - 2) \text{ units}$$

Thus, the correct answer is B.

Question 7:

Area lying between the curve $y^2 = 4x$ and $y = 2x$ is

A. $\frac{2}{3}$

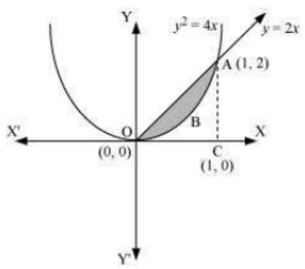
B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. $\frac{3}{4}$

Answer

The area lying between the curve, $y^2 = 4x$ and $y = 2x$, is represented by the shaded area OBAO as



The points of intersection of these curves are O (0, 0) and A (1, 2).

We draw AC perpendicular to x-axis such that the coordinates of C are (1, 0).

∴ Area OBAO = Area (ΔOCA) – Area (OCABO)

$$= \int_0^1 2x\,dx - \int_0^1 2\sqrt{x}\,dx$$

$$= 2\left[\frac{x^2}{2}\right]_0^1 - 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1$$

$$= \left|1 - \frac{4}{3}\right|$$

$$= \left|-\frac{1}{3}\right|$$

$= \frac{1}{3}$ units

Thus, the correct answer is B.

Miscellaneous Solutions

Question 1:

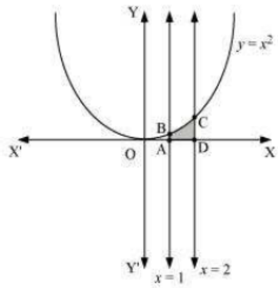
Find the area under the given curves and given lines:

(i) $y = x^2$, $x = 1$, $x = 2$ and x-axis

(ii) $y = x^4$, $x = 1$, $x = 5$ and x –axis

Answer

i. The required area is represented by the shaded area ADCBA as



$$\text{Area ADCBA} = \int_1^2 y\,dx$$

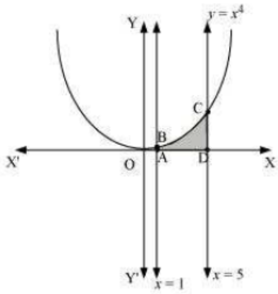
$$= \int_1^2 x^2\,dx$$

$$= \left[\frac{x^3}{3}\right]_1^2$$

$$= \frac{8}{3} - \frac{1}{3}$$

$$= \frac{7}{3} \text{ units}$$

ii. The required area is represented by the shaded area ADCBA as



$$\text{Area ADCBA} = \int_1^5 x^4\,dx$$

$$= \left[\frac{x^5}{5}\right]_1^5$$

$$= \frac{(5)^5}{5} - \frac{1}{5}$$

$$= (5)^4 - \frac{1}{5}$$

$$= 625 - \frac{1}{5}$$

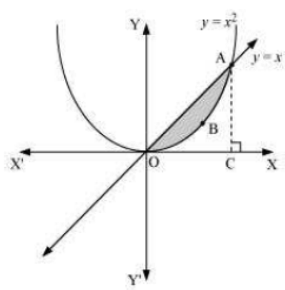
$$= 624.8 \text{ units}$$

Question 2:

Find the area between the curves $y = x$ and $y = x^2$

Answer

The required area is represented by the shaded area OBAO as



The points of intersection of the curves, $y = x$ and $y = x^2$, is A (1, 1).

*********** END **********