Indefinite Integrals Ex 19.15 Q1

Let $I = \int \frac{1}{4x^2+12x+5}dx$

$$= \frac{1}{4}\int \frac{1}{x^2+3x+\frac{5}{4}}dx$$

$$= \frac{1}{4}\int \frac{1}{x^2+2\times x\times\left(\frac{3}{2}\right)+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+\frac{5}{4}}dx$$

$$I = \frac{1}{4}\int \frac{1}{\left(x+\frac{3}{2}\right)^2-1}dx$$

Let $\left(x+\frac{3}{2}\right) = t$ - - - - - - - (i)

$\Rightarrow$ $dx = dt$

so,

$$I = \frac{1}{4}\int \frac{1}{t^2-(1)^2}dt$$

$$I = \frac{1}{4}\times\frac{1}{2\times(1)}\log\left|\frac{t-1}{t+1}\right|+c \qquad \left[\text{Since, } \int\frac{1}{x^2-a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c\right]$$

$$I = \frac{1}{8}\log\left|\frac{x+\frac{3}{2}-1}{x+\frac{3}{2}+1}\right|+c \qquad [\text{using (i)}]$$

$$I = \frac{1}{8}\log\left|\frac{2x+1}{2x+5}\right|+c$$

Indefinite Integrals Ex 19.15 Q2

Let $I = \int \frac{1}{x^2-10x+34}dx$

$$= \int \frac{1}{x^2-2x\times 5+(5)^2-(5)^2+34}dx$$

$$= \int \frac{1}{(x-5)^2+9}dx$$

Let $(x-1) = t$ - - - - - - - (i)

$\Rightarrow$ $dx = dt$

so,

$$I = \int \frac{1}{t^2+(3)^2}dt$$

$$I = \frac{1}{3}\tan^{-1}\left(\frac{t}{3}\right)+c \qquad \left[\text{Since, } \int\frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{2}\right)+c\right]$$

$$I = \frac{1}{3}\tan^{-1}\left(\frac{x-5}{3}\right)+c \qquad [\text{using (i)}]$$

Indefinite Integrals Ex 19.15 Q3

$$\int \frac{1}{1+x-x^2}dx = \int \frac{1}{-\left(x^2-x-1\right)}dx$$

adding and subtracting $\frac{1}{4}$ in the denominator to make it a perfect square

$$= \int \frac{1}{-\left(x^2-x+\frac{1}{4}-1-\frac{1}{4}\right)}dx$$

$$= \int \frac{1}{-\left(\left[x^2-x+\frac{1}{4}\right]-1-\frac{1}{4}\right)}dx$$

$$= \int \frac{1}{-\left(\left[x-\frac{1}{2}\right]^2-1-\frac{1}{4}\right)}dx = \int \frac{1}{-\left(\left[x-\frac{1}{2}\right]^2-\frac{5}{4}\right)}dx$$

$$= \int \frac{1}{\left(\left(\frac{\sqrt{5}}{2}\right)^2-\left[x-\frac{1}{2}\right]^2\right)}d$$

$$= \frac{1}{2\left(\frac{\sqrt{5}}{2}\right)}\log\left|\frac{\frac{\sqrt{5}}{2}+\left(x-\frac{1}{2}\right)}{\frac{\sqrt{5}}{2}-\left(x-\frac{1}{2}\right)}\right|$$

$$= \frac{1}{\sqrt{5}}\log\left|\frac{\sqrt{5}+2x-1}{\sqrt{5}-2x+1}\right|$$

$$= \frac{1}{\sqrt{5}}\log\left|\frac{\sqrt{5}-1+2x}{\sqrt{5}+1-2x}\right|$$

Indefinite Integrals Ex 19.15 Q4

$$\text{Let } I = \int \frac{1}{2x^2 - x - 1}dx$$

$$= \frac{1}{2}\int \frac{1}{x^2 - \frac{x}{2} - \frac{1}{2}}dx$$

$$= \frac{1}{2}\int \frac{1}{x^2 - 2x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - \frac{1}{2}}dx$$

$$= \frac{1}{2}\int \frac{1}{\left(x - \frac{1}{4}\right)^2 - \frac{9}{16}}dx$$

$$\text{Let } x - \frac{1}{4} = t$$

$$\Rightarrow \quad dx = dt$$

$$I = \frac{1}{2}\int \frac{1}{t^2 - \left(\frac{3}{4}\right)^2}dt$$

$$I = \frac{1}{2} \times \frac{1}{2 \times \left(\frac{3}{4}\right)}\log\left|\frac{t - \frac{3}{4}}{t + \frac{3}{4}}\right| + c \quad \left[\text{Since, } \int \frac{1}{x^2 - a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c\right]$$

$$I = \frac{1}{3}\log\left|\frac{x - \frac{1}{4} - \frac{3}{4}}{x - \frac{1}{4} + \frac{3}{4}}\right| + c$$

$$I = \frac{1}{3}\log\left|\frac{x-1}{2x+1}\right| + c$$

Indefinite Integrals Ex 19.15 Q5

We have $x^2 + 6x + 13 = x^2 + 6x + 3^2 - 3^2 + 13 = (x + 3)^2 + 4$

Sol, $\int \frac{dx}{x^2 + 6x + 13} = \int \frac{1}{(x+3)^2 + 2^2}\,dx$

Let $x + 3 = t$. Then $dx = dt$

Therefore, $\int \frac{dx}{x^2 + 6x + 13} = \int \frac{dt}{t^2 + 2^2} = \frac{1}{2}\tan^{-1}\frac{t}{2} + C$ [by 7.4 (3)]

$$= \frac{1}{2}\tan^{-1}\frac{x+3}{2} + C$$

*********** END **********