Differentiation Ex 11.5 Q39

Here,

$$x^m y^n = 1$$

Taking log on both the side,

$$\log\left(x^m y^n\right) = \log(1)$$
$$m\log x + n\log y = \log(1)$$

Differentiating it with respect to x,

$$\frac{dy}{dx}(m\log x) + \frac{d}{dx}(n\log y) = \frac{d}{dx}\left(\log(1)\right)$$
$$\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = -\frac{m}{x} \times \frac{y}{n}$$

$$\frac{dy}{dx} = -\frac{my}{nx}$$

Differentiation Ex 11.5 Q40

Here,

$$y^x = e^{y-x}$$

Taking log on both the sides,

$$\log y^x = \log e^{(y-x)} \qquad \left[\text{Since, } \log a^b = b\log a \text{ and } \log_e e = 1\right]$$
$$x\log y = (y-x)\log e$$
$$x\log y = y - x \qquad \text{---(i)}$$

Differentiating it with respect to x using product rule,

$$\frac{d}{dx}(x\log y) = \frac{d}{dx}(y-x)$$
$$\left[x\frac{d}{dx}(\log y) + \log y\frac{d}{dx}(x)\right] = \frac{dy}{dx} - 1$$
$$x\left(\frac{1}{y}\right)\frac{dy}{dx} + \log y\,(1) = \frac{dy}{dx} - 1$$
$$\frac{dy}{dx}\left(\frac{x}{y} - 1\right) = -1 - \log y$$
$$\frac{dy}{dx}\left(\frac{y}{(1+\log y)y}\right) = -(1+\log y) \qquad \left[\text{Since, from equation (i), } x = \frac{y}{(1+\log y)}\right]$$

$$\frac{dy}{dx}\left[\frac{1-1-\log y}{(1+\log y)}\right] = -(1+\log y)$$
$$\frac{dy}{dx} = -\frac{(1+\log y)^2}{-\log y}$$

$$\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$$

Differentiation Ex 11.5 Q41

Here,

$$(\sin x)^y = (\cos y)^x$$

Taking log on both the sides,

$$\log(\sin x)^y = \log(\cos y)^x \qquad \left[\text{Using } \log a^b = b\log a\right]$$
$$y\log(\sin x) = x\log(\cos y)$$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{d}{dx}\left[y\log\sin x\right] = \frac{d}{dx}\left[x\log\cos y\right]$$
$$y\frac{d}{dx}(\log\sin x) + \log\sin x\frac{dy}{dx} = x\frac{dy}{dx}\log\cos y + \log\cos y\frac{d}{dx}(x)$$
$$y\left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x) + \log\sin x\frac{dy}{dx} = \frac{x}{\cos y}\frac{d}{dx}(\cos y) + \log\cos y\,(1)$$
$$\frac{y}{\sin x}(\cos x) + \log\sin x\frac{dy}{dx} = \frac{x}{\cos y}(-\sin y)\frac{dy}{dx} + \log\cos y$$
$$y\cot x + \log\sin x\frac{dy}{dx} = -x\tan y\frac{dy}{dx} + \log\cos y$$
$$\frac{dy}{dx}(\log\sin x + x\tan y) = \log\cos y - y\cot x$$

$$\frac{dy}{dx} = \frac{\log\cos y - y\cot x}{\log\sin x + x\tan y}$$

Differentiation Ex 11.5 Q42

Here,

$$(\cos x)^y = (\tan y)^x$$

Taking log on both the sides,

$$\log(\cos x)^y = \log(\tan y)^x$$
$$y\log\cos x = x\log\tan y \qquad \left[\text{Since, } \log a^b = b\log a\right]$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{d}{dx}(y\log\cos x) = \frac{d}{dx}(x\log\tan y)$$
$$\left(y\frac{d}{dx}\log\cos x + \log\cos x\frac{dy}{dx}\right) = \left(x\frac{d}{dx}\log\tan y + \log\tan y\frac{d}{dx}(x)\right)$$
$$\left(y\left(\frac{1}{\cos x}\right)\frac{d}{dx}(\cos x) + \log\cos x\frac{dy}{dx}\right) = \left(x\frac{1}{\tan y}\frac{d}{dx}(\tan y) + \log\tan y\,(1)\right)$$
$$\left(\frac{y}{\cos x}(-\sin x) + \log\cos x\frac{dy}{dx}\right) = \left(\frac{x}{\tan y}\left(\sec^2 y\right)\right)\frac{dy}{dx} + \log\tan y - y\tan x + \log\cos x\frac{dy}{dx}$$
$$= \left(\sec y\,\mathrm{cosec}\,y \times x\frac{dy}{dx} + \log\tan y\right)$$

$$\frac{dy}{dx}\left[\log\cos x - x\sec y\,\mathrm{cosec}\,y\right] = \log\tan y + y\tan x$$

$$\frac{dy}{dx} = \left[\frac{\log\tan y + y\tan x}{\log\cos x - x\sec y\,\mathrm{cosec}\,y}\right]$$

Differentiation Ex 11.5 Q43

Here,

$$e^x + e^y = e^{x+y} \qquad \text{---(i)}$$

Differentiating both the sides using chain rule,

$$\frac{d}{dx}\left(e^x\right) + \frac{d}{dx}\left(e^y\right) = \frac{d}{dx}\left(e^{x+y}\right)$$

$$e^x + e^y \frac{dy}{dx} = e^{x+y} \frac{d}{dx}(x + y)$$

$$e^x + e^y \frac{dy}{dx} = e^{x+y}\left[1 + \frac{dy}{dx}\right]$$

$$e^y \frac{dy}{dx} - e^{x+y} \frac{dy}{dx} = e^{x+y} - e^x$$

$$\frac{dy}{dx} = \frac{e^{x+y} - e^x}{e^y - e^{x+y}}$$

$$= \left(\frac{e^x + e^y - e^x}{e^y - e^x - e^y}\right) \qquad \left[\text{Using equation (i)}\right]$$

$$\frac{dy}{dx} = -e^{y-x}$$

$$\frac{dy}{dx} + e^{y-x} = 0$$

*********** END **********