Exercise 5A

(v) $\left\{\left(\frac{-3}{4}\right)^3-\left(\frac{-5}{2}\right)^3\right\}\times 4^2=\left\{\left(\frac{-3^3}{4^3}\right)-\left(\frac{-5^3}{2^3}\right)\right\}\times 4^2$

$=\left\{\left(\frac{-27}{64}\right)-\left(\frac{-125}{8}\right)\right\}\times 16$

$=\left\{\frac{-27}{64}+\frac{125}{8}\right\}\times 16$

$=\left(\frac{-27+1000}{64}\right)\times 16$

$=\left(\frac{973}{64}\times 16\right)=\frac{973}{4}$

Q8

Answer :

(i) $\left(\frac{4}{9}\right)^6\times\left(\frac{4}{9}\right)^{-4}=\left(\frac{4}{9}\right)^{6+(-4)}$ $\quad\left[\text{since } a^n\times a^m=a^{n+m}\right]$

$=\left(\frac{4}{9}\right)^2=\frac{(4)^2}{(9)^2}=\frac{4\times4}{9\times9}=\frac{16}{81}$

(ii) $\left(\frac{-7}{8}\right)^{-3}\times\left(\frac{-7}{8}\right)^2=\left(\frac{-7}{8}\right)^{(-3)+2}$ $\quad\left[\text{since } a^n\times a^m=a^{n+m}\right]$

$=\left(\frac{-7}{8}\right)^{-1}$

$=\left(\frac{8}{-7}\right)^1$ $\quad\left[\text{since } \left(\frac{a}{b}\right)^{-1}=\left(\frac{b}{a}\right)^1\right]$

$=\left(\frac{8\times-1}{-7\times-1}\right)=\frac{-8}{7}$

(iii) $\left(\frac{4}{3}\right)^{-3}\times\left(\frac{4}{3}\right)^{-2}=\left(\frac{4}{3}\right)^{(-3)+(-2)}$ $\quad\left[\text{since } a^n\times a^m=a^{n+m}\right]$

$=\left(\frac{4}{3}\right)^{-5}$

$=\left(\frac{3}{4}\right)^5$ $\quad\left[\text{since } \left(\frac{a}{b}\right)^{-1}=\left(\frac{b}{a}\right)^1\right]$

$=\frac{(3)^5}{(4)^5}=\frac{3\times3\times3\times3\times3}{4\times4\times4\times4\times4}=\frac{243}{1024}$

Q9

(i) $5^{-3}=\left(\frac{5}{1}\right)^{-3}=\left(\frac{1}{5}\right)^3=\frac{(1)^3}{(5)^3}=\frac{1}{125}$

(ii) $(-2)^{-5}=\left(\frac{-2}{1}\right)^{-5}=\left(\frac{1}{-2}\right)^5=\frac{(1)^5}{(-2)^5}=\frac{1\times-1}{-32\times-1}=\frac{-1}{32}$

(iii) $\left(\frac{1}{4}\right)^{-4}=\left(\frac{4}{1}\right)^4=\frac{(4)^4}{(1)^4}=\frac{256}{1}=256$

(iv) $\left(\frac{-3}{4}\right)^{-3}=\left(\frac{4}{-3}\right)^3=\frac{(4)^3}{(-3)^3}=\frac{64}{-27}=\frac{64\times-1}{-27\times-1}=\frac{-64}{27}$

(v) $(-3)^{-1}\times\left(\frac{1}{3}\right)^{-1}=\left(\frac{1}{-3}\right)^1\times\left(\frac{3}{1}\right)^1=\left(\frac{1\times3}{-3\times1}\right)^1=\left(\frac{3}{-3}\right)^1=\frac{1}{-1}=\frac{1\times-1}{-1\times-1}=\frac{-1}{1}=-1$

(vi) $\left(\frac{5}{7}\right)^{-1}\times\left(\frac{7}{4}\right)^{-1}=\left(\frac{7}{5}\right)^1\times\left(\frac{4}{7}\right)^1=\left(\frac{7\times4}{5\times7}\right)^1=\frac{4}{5}$

(vii) $\left(5^{-1}-7^{-1}\right)^{-1}=\left(\frac{1}{5}-\frac{1}{7}\right)^{-1}=\left(\frac{7-5}{35}\right)^{-1}$

$=\left(\frac{2}{35}\right)^{-1}=\left(\frac{35}{2}\right)^1=\frac{35}{2}$

(viii) $\left\{\left(\frac{4}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}^{-1}=\left\{\left(\frac{3}{4}\right)^{1}-\left(\frac{4}{1}\right)^{1}\right\}^{-1}=\left(\frac{3}{4}-\frac{4}{1}\right)^{-1}$

$$=\left(\frac{3-16}{4}\right)^{-1}=\left(\frac{-13}{4}\right)^{-1}$$

$$=\left(\frac{4}{-13}\right)^{1}=\left(\frac{4\times-1}{-13\times-1}\right)$$

$$=\frac{-4}{13}$$

(ix) $\left\{\left(\frac{3}{2}\right)^{-1}\div\left(\frac{-2}{5}\right)^{-1}\right\}=\left\{\left(\frac{2}{3}\right)^{1}\div\left(\frac{5}{-2}\right)^{1}\right\}$

$$=\left(\frac{2}{3}\times\frac{-2}{5}\right)$$

$$=\frac{-4}{15}$$

(x) $\left(\frac{23}{25}\right)^{0}=1$ [since $a^0 = 1$ for every integer a]

Q10

Answer :

(i)

$$\left[\left\{\left(-\frac{1}{4}\right)^{2}\right\}^{-2}\right]^{-1}=\left[\left(-\frac{1}{4}\right)^{2\times-2}\right]^{-1} \qquad \left[\text{since } \left\{\left(\frac{a}{b}\right)^{m}\right\}^{n}=\left(\frac{a}{b}\right)^{mn}\right]$$

$$=\left[\left(-\frac{1}{4}\right)^{-4}\right]^{-1}$$

$$=\left(-\frac{1}{4}\right)^{(-4)\times(-1)}$$

$$=\left(-\frac{1}{4}\right)^{4}=\frac{(-1)^4}{(4)^4}$$

(ii)

$$\left\{\left(\frac{-2}{3}\right)^{2}\right\}^{3}=\left(\frac{-2}{3}\right)^{2\times3} \qquad \left[\text{since } \left\{\left(\frac{a}{b}\right)^{m}\right\}^{n}=\left(\frac{a}{b}\right)^{mn}\right]$$

$$=\left(\frac{-2}{3}\right)^{6}$$

$$=\frac{(-2)^6}{(3)^6}=\frac{64}{729}$$

[since $(-2)^6 = 64$ and $(3)^6 = 729$]

(iii)

$$\left(\frac{-3}{2}\right)^{3}\div\left(\frac{-3}{2}\right)^{6}=\left(\frac{-3}{2}\right)^{3-6} \qquad [\text{since } a^m\div a^n=a^{m-n}]$$

$$=\left(\frac{-3}{2}\right)^{-3}$$

$$=\left(\frac{2}{-3}\right)^{3} \qquad \left[\text{since } \left(\frac{a}{b}\right)^{-1}=\left(\frac{b}{a}\right)^{1}\right]$$

$$=\left(\frac{2\times-1}{-3\times-1}\right)^{3}=\left(\frac{-2}{3}\right)^{3}$$

$$=\frac{(-2)^3}{(3)^3}=\frac{-8}{27}$$

(iv)

$$\left(\frac{-2}{3}\right)^{7}\div\left(\frac{-2}{3}\right)^{4}=\left(\frac{-2}{3}\right)^{7-4} \qquad [\text{since } a^m\div a^n=a^{m-n}]$$

$$=\left(\frac{-2}{3}\right)^{3}$$

$$=\frac{(-2)^3}{(3)^3}=\frac{-8}{27}$$

Q11

Answer :

Let the required number be x.

$(-5)^{-1} \times x = (8)^{-1}$

$\Rightarrow \frac{1}{-5} \times x = \frac{1}{8}$

$\therefore x = \frac{1}{8} \times (-5) = \frac{-5}{8}$

Hence, the required number is $\frac{-5}{8}$.

Q12

Answer :

Let the required number be x.

$(3)^{-3} \times x = 4$

$\Rightarrow \frac{1}{3^3} \times x = 4$

$\Rightarrow \frac{1}{27} \times x = 4$

$\therefore x = 4 \times 27 = 108$

Hence, the required number is 108.

Q13

Answer :

Let the required number be x.

$(-30)^{-1} \div x = 6^{-1}$

$\Rightarrow \frac{1}{(-30)} \times \frac{1}{x} = \frac{1}{6}$

$\Rightarrow \frac{1}{(-30x)} = \frac{1}{6}$

$\therefore x = \frac{6}{(-30)} = \frac{1}{-5}$

$= \frac{-1}{5}$

Hence, the required number is $\frac{-1}{5}$.

Q14

Answer :

$$\left(\frac{3}{5}\right)^{3}\times\left(\frac{3}{5}\right)^{-6}=\left(\frac{3}{5}\right)^{2x-1}$$
$$\Rightarrow\left(\frac{3}{5}\right)^{3+(-6)}=\left(\frac{3}{5}\right)^{2x-1}\qquad\left[since\ \mathrm{a^m\times a^n=a^{m+n}}\right]$$
$$\Rightarrow\left(\frac{3}{5}\right)^{-3}=\left(\frac{3}{5}\right)^{2x-1}$$

On equating the exponents:

$-3 = 2x - 1$

$\Rightarrow 2x = -3 + 1$

$\Rightarrow 2x = -2$

$\therefore x = \left(\frac{-2}{2}\right) = -1$

Q15

Answer :

$$\frac{3^5\times10^5\times25}{5^7\times6^5}=\frac{3^5\times(2\times5)^5\times5^2}{5^7\times(2\times3)^5}$$
$$=\frac{3^5\times2^5\times5^5\times5^2}{5^7\times2^5\times3^5}$$
$$=\frac{3^5\times2^5\times5^7}{3^5\times2^5\times5^7}$$
$$=3^{5-5}\times2^{5-5}\times5^{7-7}$$
$$=3^0\times2^0\times5^0$$
$$=1\times1\times1=1$$

Q16

Answer :

$$\frac{16\times2^{n+1}-4\times2^{n}}{16\times2^{n+2}-2\times2^{n+2}}$$
$$\Rightarrow \frac{2^{4}\times2^{n+1}-2^{2}\times2^{n}}{2^{4}\times2^{n+2}-2^{n+1}\times2^{2}}$$
$$\Rightarrow \frac{2^{2}\times\left(2^{n+3}-2^{n}\right)}{2^{2}\times\left(2^{n+4}-2^{n+1}\right)}$$
$$\Rightarrow \frac{2^{n}\times2^{3}-2^{n}}{2^{n}\times2^{4}-2^{n}\times2}$$
$$\Rightarrow \frac{2^{n}\left(2^{3}-1\right)}{2^{n}\left(2^{4}-2\right)}=\frac{8-1}{16-2}=\frac{7}{14}=\frac{1}{2}$$

Q17

Answer :

(i) $5^{2n} \times 5^{3} = 5^{9}$

$5^{2n+3} = 5^{9}$ [since $a^{n} \times a^{m} = a^{m+n}$]

On equating the coefficients:

$2n + 3 = 9$

$\Rightarrow 2n = 9 - 3$

$\Rightarrow 2n = 6$

$\therefore n = \frac{6}{2} = 3$

(ii) $8 \times 2^{n+2} = 32$

$\Rightarrow (2)^{3} \times 2^{n+2} = (2)^{5}$ [since $2^{3} = 8$ and $2^{5} = 32$]

$\Rightarrow (2)^{3+(n+2)} = (2)^{5}$

On equating the coefficients:

$3 + n + 2 = 5$

$\Rightarrow n + 5 = 5$

$\Rightarrow n = 5 - 5$

$\therefore n = 0$

(iii) $6^{2n+1} \div 36 = 6^3$

$\Rightarrow 6^{2n+1} \div 6^2 = 6^3$ [since $36 = 6^2$]

$\Rightarrow \frac{6^{2n+1}}{6^2} = 6^3$

$\Rightarrow 6^{2n+1-2} = 6^3$ [since $\frac{a^m}{a^n} = a^{m-n}$]

$\Rightarrow 6^{2n-1} = 6^3$

On equating the coefficients:

$2n - 1 = 3$

$\Rightarrow 2n = 3 + 1$

$\Rightarrow 2n = 4$

$\therefore n = \frac{4}{2} = 2$

Q18

Answer :

$2^{n-7} \times 5^{n-4} = 1250$

$\Rightarrow \frac{2^n}{2^7} \times \frac{5^n}{5^4} = 2 \times 5^4$ [since $1250 = 2 \times 5^4$]

$\Rightarrow \frac{2^n \times 5^n}{2^7 \times 5^4} = 2 \times 5^4$

$\Rightarrow 2^n \times 5^n = 2 \times 5^4 \times 2^7 \times 5^4$ [using cross multiplication]

$\Rightarrow 2^n \times 5^n = 2^{1+7} \times 5^{4+4}$ [since $a^m \times a^n = a^{m+n}$]

$\Rightarrow 2^n \times 5^n = 2^8 \times 5^8$

$\Rightarrow (2 \times 5)^n = (2 \times 5)^8$ [since $a^n \times b^n = (a \times b)^n$]

$\Rightarrow 10^n = 10^8$

$\Rightarrow n = 8$

*********** END **********