Indefinite Integrals Ex 19.26 Q14

Let $I = \int \frac{\sqrt{1-\sin x}}{1+\cos x} e^{-\frac{x}{2}} dx$

Put $= \frac{x}{2} = t$

$\Rightarrow \qquad x = 2t$

$dx = 2dt$

$\therefore \qquad \int \frac{\sqrt{1-\sin x}}{1+\cos x} e^{-\frac{x}{2}} dx$

$= 2\int \frac{\sqrt{1-\sin 2t}}{1+\cos 2t} e^{-t} dt \qquad \left[\because \sin^2 t + \cos^2 t = 1\right]$

$= 2\int \frac{\sqrt{\sin^2 t + \cos^2 t - 2\sin t \cos t}}{1+\cos 2t} e^{-t} dt$

$= 2\int \frac{\sqrt{(\cos t - \sin t)^2}}{2\cos^2 t} e^{-t} dt$

$= 2\int \frac{(\cos t - \sin t)}{2\cos^2 t} e^{-t} dt$

$= \int (\sec t - \tan t \sec t) e^{-t} dt$

$= \int \sec t\, e^{-t} dt - \int \tan t \sec t\, e^{-t} dt$

Integrating by parts

$= e^{-t} \sec t + \int e^{-t} \frac{d}{dt}(\sec t) dt - \int \tan t \sec t\, e^{-t} dt$

$= -e^{-t} \sec t + \int e^{-t} \sec t \tan t dt - \int \sec t \tan t\, e^{-t} dt$

$= -e^{-t} \sec t + c$

Putting the value of t

$= -e^{-\frac{x}{2}} \sec \frac{x}{2} + c$

Indefinite Integrals Ex 19.26 Q15

We have,

$$I = \int e^x \left(\log x + \frac{1}{x}\right) dx$$

We know that

$$\int e^{ax}\left(af(x) + f'(x)\right) dx = e^{ax} f(x) + c$$

Here $f(x) = \log x$ and $f'(x) = \frac{1}{x}$

$$\therefore \qquad \int e^x \left(\log x + \frac{1}{x}\right) dx = e^x \log x + c$$

Indefinite Integrals Ex 19.26 Q16

We have,

$$I = \int e^x \left(\log x + \frac{1}{x^2}\right) dx$$

$$= \int e^x \left(\log x + \frac{1}{x} - \frac{1}{x} + \frac{1}{x^2}\right) dx$$

$$= \int e^x \left(\log x - \frac{1}{x}\right) dx + \int e^x \left(\frac{1}{x} + \frac{1}{x^2}\right) dx$$

Integrating by parts

$$= e^x \left(\log x - \frac{1}{x}\right) - \int e^x \frac{d}{dx}\left(\log x - \frac{1}{x}\right) dx + \int e^x \left(\frac{1}{x} + \frac{1}{x^2}\right) dx$$

$$= e^x \left(\log x - \frac{1}{x}\right) - \int e^x \left(\frac{1}{x} + \frac{1}{x^2}\right) dx + \int e^x \left(\frac{1}{x} + \frac{1}{x^2}\right) dx$$

$$= e^x \left(\log x - \frac{1}{x}\right) + c$$

Indefinite Integrals Ex 19.26 Q17

We have,

$$I = \int \frac{e^x}{x}\left\{x(\log x)^2 + 2\log x\right\}dx$$

$$= \int e^x\left\{(\log x)^2 + \frac{2}{x}\log x\right\}dx$$

$$= \int e^x(\log x)^2 + 2\int \frac{e^x}{x}\log x dx$$

Integrating by parts

$$= e^x(\log x)^2 - \int e^x \frac{d}{dx}(\log x)^2 dx + 2\int e^x \frac{1}{x}\log x dx$$

$$= e^x(\log x)^2 - \int e^x \frac{2\log x}{x}dx + 2\int e^x \frac{\log x}{x}dx$$

$$= e^x(\log x)^2 + c$$

*********** END **********