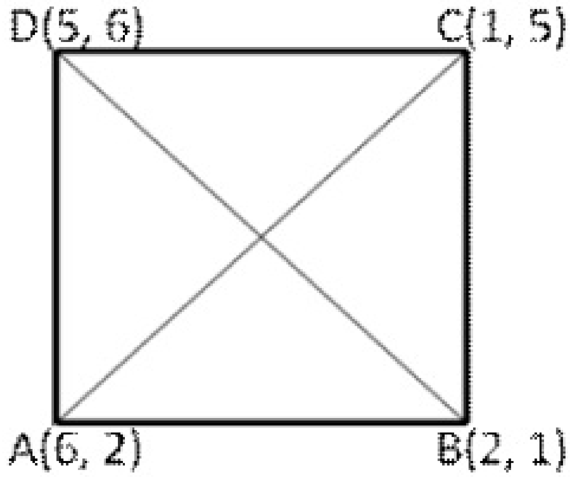
Exercise 16A

Question 21:

(i) Let A(6,2), B(2,1), C(1,5) and D(5,6) be the angular points of quad. ABCD. Join AC and BD



Now,

$$AB = \sqrt{(2-6)^2 + (1-2)^2} = \sqrt{(-4)^2 + (-1)^2} = \sqrt{16+1} = \sqrt{17} \text{ units}$$

$$BC = \sqrt{(1-2)^2 + (5-1)^2} = \sqrt{(-1)^2 + (4)^2} = \sqrt{1+16} = \sqrt{17} \text{ units}$$

$$CD = \sqrt{(5-1)^2 + (6-5)^2} = \sqrt{(4)^2 + (1)^2} = \sqrt{16+1} = \sqrt{17} \text{ unit}$$

$$DA = \sqrt{(6-5)^2 + (2-6)^2} = \sqrt{(1)^2 + (-4)^2} = \sqrt{1+16} = \sqrt{17} \text{ unit}$$

thus, $AB = BC = CD = DA$

$$\text{Diag. } AC = \sqrt{(1-6)^2 + (5-2)^2} = \sqrt{(-5)^2 + (3)^2} = \sqrt{25+9} = \sqrt{34} \text{ units}$$

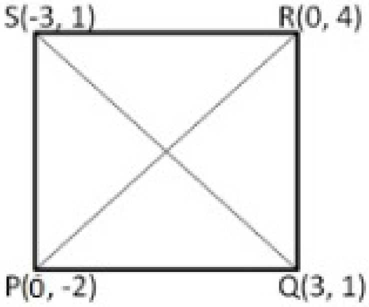
$$\text{Diag.} BD = \sqrt{(5-2)^2 + (6-1)^2} = \sqrt{(3)^2 + (5)^2} = \sqrt{9+25} = \sqrt{34} \text{ units}$$

$\therefore$ Diag. AC = Diag. BD

Thus, ABCD is a quadrilateral in which all sides are equal and the diagonals are equal.

Hence, quad ABCD is a square.

(ii) Let P(0, -2), Q(3,1), R(0,4) and S(-3,1) be the angular points of quad. ABCD



Join PR and QSD

Now,

$PQ = \sqrt{(3-0)^2 + (1+2)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units

$QR = \sqrt{(0-3)^2 + (4-1)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ unit

$RS = \sqrt{(-3-0)^2 + (1-4)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ unit

$SP = \sqrt{(0+3)^2 + (-2-1)^2} = \sqrt{(3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ unit

Thus, $PQ = QR = RS = SP$

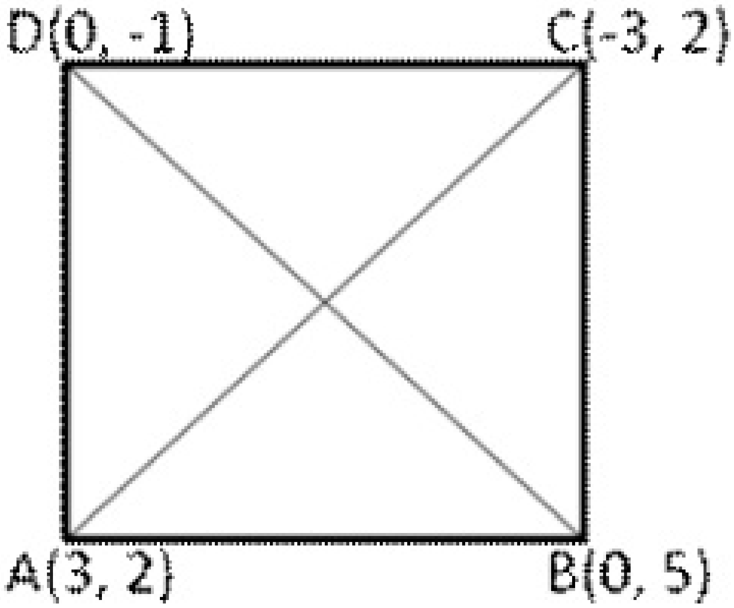
Diag. $PR = \sqrt{(0-0)^2 + (4+2)^2} = \sqrt{(6)^2} = 6$ units

Diag. $QS = \sqrt{(-3-3)^2 + (1-1)^2} = \sqrt{(-6)^2} = 6$ units

$\therefore$ Diag. PR = Diag. QS

Thus, PQRS is a quadrilateral in which all sides are equal and the diagonals are equal.
Hence, quad. PQRS is a square.

(iii) The angular points of quadrilateral ABCD are A(3,2), B(0,5), C(-3,2) and D(0,-1)



$$AB = \sqrt{(0-3)^2 + (5-2)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$BC = \sqrt{(-3-0)^2 + (2-5)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$CD = \sqrt{(0+3)^2 + (-1-2)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$DA = \sqrt{(3-0)^2 + (2+1)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$\therefore AB = BC = CD + DA = 3\sqrt{2}$$

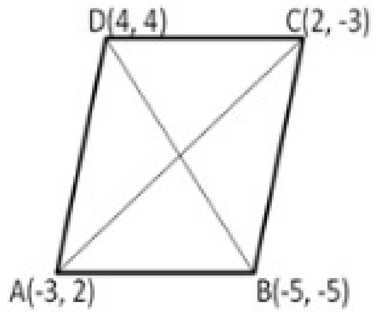
$$\text{Diag. } AC = \sqrt{(-3-3)^2 + (2-2)^2} = 6$$

$$\text{Diag. } BD = \sqrt{(0-0)^2 + (-1-5)^2} = 6$$

$$\therefore \text{Diag. } AC = \text{Diag. } BD$$

Thus, all sides of quad. ABCD are equal and diagonals are also equal.
Quad. ABCD is a square.

Question 22:
Let A(-3,2), B(-5, -5), C(2, -3) and D(4,4) be the angular point of quad ABCD. Join AC and BD.



$$\text{Now,}\quad AB = \sqrt{(-5+3)^2 + (-5-2)^2}$$
$$= \sqrt{(-2)^2 + (-7)^2} = \sqrt{4+49} = \sqrt{53} \text{ units}$$
$$BC = \sqrt{(2+5)^2 + (-3+5)^2}$$
$$= \sqrt{(7) + (2)^2} = \sqrt{49+4} = \sqrt{53} \text{ units}$$
$$CD = \sqrt{(4-2)^2 + (4+3)^2}$$
$$= \sqrt{(2)^2 + (7)^2} = \sqrt{4+49} = \sqrt{53} \text{ units}$$
$$DA = \sqrt{(-3-4)^2 + (2-4)^2}$$
$$= \sqrt{(-7)^2 + (2)^2} = \sqrt{4+49} = \sqrt{53} \text{ units}$$

$\therefore$ $AB = BC = CD = DA = \sqrt{53}$ units

$$\text{Diag. } AC = \sqrt{(2+3)^2 + (-3-2)^2} = \sqrt{(5)^2 + (-5)^2}$$
$$= \sqrt{25+25} = \sqrt{50} = 5\sqrt{2} \text{ unit}$$
$$\text{Diag. } BD = \sqrt{(4+5)^2 + (4+5)^2} = \sqrt{(9)^2 + (9)^2}$$
$$= \sqrt{81+81} = \sqrt{162} = 9\sqrt{2} \text{ unit}$$

$\therefore$ Diag. AC $\neq$ Diag. BD

Thus, ABCD is a quadrilateral having all sides equal but diagonals are unequal.
Hence, ABCD is a rhombus.

$$\text{Area of rhombus ABCD} = \frac{1}{2} \times \text{Product of diagonals}$$
$$= \left(\frac{1}{2} \times AC \times BD\right) = \left(\frac{1}{2} \times 5\sqrt{2} \times 9\sqrt{2}\right)$$
$$= 45 \text{ sq. unit}$$

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