$M_{23} = \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = 10 - 3 = 7$

$\therefore A_{23}$ = cofactor of $a_{23} = (-1)^{2+3} M_{23} = -7$

We know that Δ is equal to the sum of the product of the elements of the second row with their corresponding cofactors.

$\therefore \Delta = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23} = 2(7) + 0(7) + 1(-7) = 14 - 7 = 7$

Question 4:

Using Cofactors of elements of third column, evaluate $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$

Answer

The given determinant is $\begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$.

We have:

$M_{13} = \begin{vmatrix} 1 & y \\ 1 & z \end{vmatrix} = z - y$

$M_{23} = \begin{vmatrix} 1 & x \\ 1 & z \end{vmatrix} = z - x$

$M_{33} = \begin{vmatrix} 1 & x \\ 1 & y \end{vmatrix} = y - x$

$\therefore A_{13}$ = cofactor of $a_{13} = (-1)^{1+3} M_{13} = (z - y)$

A_{23} = cofactor of $a_{23} = (-1)^{2+3} M_{23} = -(z - x) = (x - z)$

A_{33} = cofactor of $a_{33} = (-1)^{3+3} M_{33} = (y - x)$

We know that Δ is equal to the sum of the product of the elements of the second row with their corresponding cofactors.

$$\begin{aligned}
\therefore \Delta &= a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33} \\
&= yz(z-y) + zx(x-z) + xy(y-x) \\
&= yz^2 - y^2z + x^2z - xz^2 + xy^2 - x^2y \\
&= (x^2z - y^2z) + (yz^2 - xz^2) + (xy^2 - x^2y) \\
&= z(x^2 - y^2) + z^2(y-x) + xy(y-x) \\
&= z(x-y)(x+y) + z^2(y-x) + xy(y-x) \\
&= (x-y)\left[zx + zy - z^2 - xy\right] \\
&= (x-y)\left[z(x-z) + y(z-x)\right] \\
&= (x-y)(z-x)\left[-z+y\right] \\
&= (x-y)(y-z)(z-x)
\end{aligned}$$

Hence, $\Delta = (x-y)(y-z)(z-x)$.

Question 5:

For the matrices A and B, verify that $(AB)' = B'A'$ where

(i) $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$

Answer

(i)

$AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} \begin{bmatrix} -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \end{bmatrix}$

$$\begin{matrix} 3 \end{matrix} \qquad \begin{matrix} -3 & 6 & 3 \end{matrix}$$

$$\therefore (AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

Now, $A' = \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}, B' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$

$$\therefore B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

Hence, we have verified that $(AB)' = B'A'$.

(ii)

$$AB = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}$$

$$\therefore (AB)' = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

Now, $A' = \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}, B' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$

$$\therefore B'A' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

Hence, we have verified that $(AB)' = B'A'$.

Exercise 4.5

Question 1:

Find adjoint of each of the matrices.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Answer

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

We have,

$A_{11} = 4,\ A_{12} = -3,\ A_{21} = -2,\ A_{22} = 1$

$$\therefore adjA = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

Question 2:

Find adjoint of each of the matrices.

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$$

Answer

Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$.

We have,

$$A_{11} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3 - 0 = 3$$

$$A_{12} = -\begin{vmatrix} 2 & 5 \\ -2 & 1 \end{vmatrix} = -(2+10) = -12$$

$$A_{13} = \begin{vmatrix} 2 & 3 \\ -2 & 0 \end{vmatrix} = 0 + 6 = 6$$

$$A_{21} = -\begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = -(-1-0) = 1$$

$$A_{22} = \begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} = 1 + 4 = 5$$

$$A_{23} = -\begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = -(0-2) = 2$$

$$A_{31} = \begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} = -5 - 6 = -11$$

$$A_{32} = -\begin{vmatrix} 1 & 2 \end{vmatrix} = -(5-4) = -1$$

$A_{32} = -\begin{vmatrix} 2 & 5 \end{vmatrix} = -(5-4) = -1$

$$A_{33} = \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 3+2 = 5$$

$$\text{Hence, } adjA = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}.$$

Question 3:

Verify *A* (*adj A*) = (*adj A*) *A* = $|A|I$.

$$\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$$

Answer

$$A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$$

we have,

$$|A| = -12 - (-12) = -12 + 12 = 0$$

$$\therefore |A|I = 0\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Now,

$$A_{11} = -6, A_{12} = 4, A_{21} = -3, A_{22} = 2$$

$$\therefore adjA = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$$

Now,

*********** END **********