Adjoint and Inverse of Matrix Ex 7.1 Q7 (i)

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Now, $|A| = 1 \neq 0$

Hence A^{-1} exists.

Cofactors of A are:

$$C_{11} = \cos\theta \qquad C_{21} = -\sin\theta$$
$$C_{12} = \sin\theta \qquad C_{22} = \cos\theta$$

$$\therefore \quad \operatorname{adj} A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$$
$$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\therefore \quad \operatorname{adj} A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Now, $A^{-1} = \dfrac{1}{|A|} . \operatorname{adj} A$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q7 (ii)

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Now, $|A| = -1 \neq 0$

Hence A^{-1} exists.

Cofactors of A are:

$$C_{11} = 0 \qquad C_{12} = -1$$
$$C_{21} = -1 \qquad C_{22} = 0$$

$$\therefore \quad \operatorname{adj} A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$$
$$= \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

Also, $A^{-1} = \frac{1}{|A|}(\operatorname{adj} A)$

$$A^{-1} = \frac{1}{(-1)}\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q7 (iii)

$$A=\begin{bmatrix} a & b \\ c & \dfrac{1+bc}{a} \end{bmatrix}$$

Now, $|A| = \dfrac{a+abc}{a} - bc = \dfrac{a+abc-abc}{a} = 1 \neq 0$

Hence A^{-1} exists.

Now, cofactors of A are:

$$C_{11} = \frac{1+bc}{a} \qquad C_{12} = -c$$

$$C_{21} = -b \qquad C_{22} = a$$

$$\therefore \quad \text{adj}\,A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$$

$$= \begin{bmatrix} \dfrac{1+bc}{a} & -c \\ -b & a \end{bmatrix}^T$$

$$\therefore \quad \text{adj}\,A = \begin{bmatrix} \dfrac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$$

Also, $A^{-1} = \dfrac{1}{|A|}.\text{adj}\,A$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} \dfrac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$$

or $$A^{-1} = \begin{bmatrix} \dfrac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q7 (iv)

$$A=\begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$$

Now, $|A| = 2+15 = 17 \neq 0$

Hence A^{-1} exists.

Now, cofactors of A are:

$C_{11} = 1 \qquad C_{12} = 3$

$C_{21} = -5 \qquad C_{22} = 2$

$$\therefore \quad \operatorname{adj} A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & 3 \\ -5 & 2 \end{bmatrix}$$

$$\therefore \quad \operatorname{adj} A = \begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot (\operatorname{adj} A)$$

$$A^{-1} = \frac{1}{17} \begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix}$$

Hence, $A^{-1} = \frac{1}{17} \begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix}$

*********** END **********