



Indefinite Integrals Ex 19.11 Q5

Let $I = \int \tan^5 x dx$. Then

$$\begin{aligned} I &= \int \tan^2 x \tan^3 x dx \\ &= \int (\sec^2 x - 1) \tan^3 x dx \\ &= \int \sec^2 x \tan^3 x dx - \int \tan^3 x dx \\ &= \int \sec^2 x \tan^3 x dx - \int (\sec^2 x - 1) \tan x dx \\ &= \int \sec^2 x \tan^3 x dx - \int \sec^2 x \tan x dx + \int \tan x dx \end{aligned}$$

Substituting $\tan x = t$ and $\sec^2 x dx = dt$ in first two integral, we get

$$\begin{aligned} I &= \int t^3 dt - \int t dt + \int \tan x dx \\ &= \frac{t^4}{4} - \frac{t^2}{2} + \log |\sec x| + c \\ &= \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \log |\sec x| + c \end{aligned}$$

$$\therefore I = \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \log |\sec x| + c$$

Indefinite Integrals Ex 19.11 Q6

Let $I = \int \sqrt{\tan x} \sec^4 x dx$. Then

$$\begin{aligned} I &= \int \sqrt{\tan x} \sec^2 x \sec^2 x dx \\ &= \int \sqrt{\tan x} (1 + \tan^2 x) \sec^2 x dx \\ &= \int \tan x^{\frac{1}{2}} (1 + \tan^2 x) \sec^2 x dx \\ \Rightarrow I &= \int \left(\tan x^{\frac{1}{2}} + \tan x^{\frac{5}{2}} \right) \sec^2 x dx \end{aligned}$$

Substituting $\tan x = t$ and $\sec^2 x dx = dt$, we get

$$\begin{aligned} I &= \int \left(t^{\frac{1}{2}} + t^{\frac{5}{2}} \right) dt \\ &= \frac{2}{3} t^{\frac{3}{2}} + \frac{2}{7} t^{\frac{7}{2}} + c \\ &= \frac{2}{3} (\tan x)^{\frac{3}{2}} + \frac{2}{7} (\tan x)^{\frac{7}{2}} + c \end{aligned}$$

$$\therefore I = \frac{2}{3} \tan^{\frac{3}{2}} x + \frac{2}{7} \tan^{\frac{7}{2}} x + c$$

Indefinite Integrals Ex 19.11 Q7

Let $I = \int \sec^4 2x dx$. Then

$$\begin{aligned} I &= \int \sec^2 2x \sec^2 2x dx \\ &= \int (1 + \tan^2 2x) \sec^2 2x dx \\ &= \int (\sec^2 2x + \sec^2 2x \tan^2 2x) dx \end{aligned}$$

$$\Rightarrow I = \int \sec^2 2x dx + \int \sec^2 2x \tan^2 2x dx$$

$$\Rightarrow I = \int \sec^2 2x \tan^2 2x dx + \int \sec^2 2x dx$$

Substituting $\tan 2x = t$ and $\sec^2 2x dx = \frac{dt}{2}$ in first integral, we get

$$\begin{aligned} I &= \int t^2 \frac{dt}{2} + \int \sec^2 2x dx \\ &= \frac{1}{2} \times \frac{t^3}{3} + \frac{1}{2} \tan 2x + c \end{aligned}$$

$$\Rightarrow I = \frac{1}{6} \tan^3 2x + \frac{1}{2} \tan 2x + c$$

$$\therefore I = \frac{1}{2} \tan 2x + \frac{1}{6} \tan^3 2x + c$$

Indefinite Integrals Ex 19.11 Q8

Let $I = \int \operatorname{cosec}^4 3x dx$. Then

$$\begin{aligned} I &= \int \operatorname{cosec}^2 3x \operatorname{cosec}^2 3x dx \\ &= \int (1 + \cot^2 3x) \operatorname{cosec}^2 3x dx \\ &= \int (\operatorname{cosec}^2 3x + \cot^2 3x \operatorname{cosec}^2 3x) dx \end{aligned}$$

$$\Rightarrow I = \int \operatorname{cosec}^2 3x dx + \int \cot^2 3x \operatorname{cosec}^2 3x dx$$

Substituting $\cot 3x = t$ and $\operatorname{cosec}^2 3x dx = -dt$ in 2nd integral, we get

$$\begin{aligned} I &= \int \operatorname{cosec}^2 3x dx - \int t^2 \frac{dt}{3} \\ &= -\frac{1}{3} \cot 3x - \frac{t^3}{9} + c \\ &= -\frac{1}{3} \cot 3x - \frac{\cot^3 3x}{9} + c \end{aligned}$$

$$\therefore I = -\frac{1}{3} \cot 3x - \frac{1}{9} \cot^3 3x + c$$

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