



$$= \frac{1}{40} \begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Hence, $x = 1$, $y = 2$, and $z = -1$.

Question 14:

Solve system of linear equations, using matrix method.

$$x - y + 2z = 7$$

$$3x + 4y - 5z = -5$$

$$2x - y + 3z = 12$$

Answer

The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}.$$

Now,

$$|A| = 1(12 - 5) + 1(9 + 10) + 2(-3 - 8) = 7 + 19 - 22 = 4 \neq 0$$

Thus, A is non-singular. Therefore, its inverse exists.

Now, $A_{11} = 7, A_{12} = -19, A_{13} = -11$

$$A_{21} = 1, A_{22} = -1, A_{23} = -1$$

$$A_{31} = -3, A_{32} = 11, A_{33} = 7$$

$$\therefore A^{-1} = \frac{1}{|A|} (\text{adj} A) = \frac{1}{4} \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{4} \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix} \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 49 - 5 - 36 \\ -133 + 5 + 132 \\ -77 + 5 + 84 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Hence, $x = 2, y = 1$, and $z = 3$.

Question 15:

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

If A^{-1} exists, find A^{-1} . Using A^{-1} solve the system of equations

$$2x - 3y + 5z = 11$$

$$3x + 2y - 4z = -5$$

$$x + y - 2z = -3$$

Answer

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$$\therefore |A| = 2(-4 + 4) + 3(-6 + 4) + 5(3 - 2) = 0 - 6 + 5 = -1 \neq 0$$

Now, $A_{11} = 0, A_{12} = 2, A_{13} = 1$

$$A_{21} = -1, A_{22} = -9, A_{23} = -5$$

$$A_{31} = 2, A_{32} = 23, A_{33} = 13$$

$$\therefore A^{-1} = \frac{1}{|A|} \begin{bmatrix} 0 & -1 & 2 \\ 1 & 9 & 5 \\ 2 & -23 & -13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} (\text{adj}A) = - \begin{bmatrix} 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \quad \dots(1)$$

Now, the given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}.$$

The solution of the system of equations is given by $X = A^{-1}B$.

$$\begin{aligned} X &= A^{-1}B \\ \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} &= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} \quad [\text{Using (1)}] \\ &= \begin{bmatrix} 0-5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \end{aligned}$$

Hence, $x = 1$, $y = 2$, and $z = 3$.

Question 16:

The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs 60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is Rs 90. The cost of 6 kg onion 2 kg wheat and 3 kg rice is Rs 70. Find cost of each item per kg by matrix method.

Answer

Let the cost of onions, wheat, and rice per kg be Rs x , Rs y , and Rs z respectively.

Then, the given situation can be represented by a system of equations as:

$$4x + 3y + 2z = 60$$

$$2x + 4y + 6z = 90$$

$$6x + 2y + 3z = 70$$

This system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}.$$

$$|A| = 4(12-12) - 3(6-36) + 2(4-24) = 0 + 90 - 40 = 50 \neq 0$$

$$\begin{aligned} \text{Now, } A_{11} &= 0, A_{12} = 30, A_{13} = -20 \\ A_{21} &= -5, A_{22} = 0, A_{23} = 10 \\ A_{31} &= 10, A_{32} = -20, A_{33} = 10 \end{aligned}$$

$$\begin{aligned} \therefore \text{adj}A &= \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix} \\ \therefore A^{-1} &= \frac{1}{|A|} \text{adj}A = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix} \end{aligned}$$

Now,

$$X = A^{-1}B$$

$$\begin{aligned} \Rightarrow X &= \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix} \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} &= \frac{1}{50} \begin{bmatrix} 0-450+700 \\ 1800+0-1400 \\ -1200+900+700 \end{bmatrix} \\ &= \frac{1}{50} \begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix} \\ &= \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix} \end{aligned}$$

$\therefore x = 5, y = 8$, and $z = 8$.

Hence, the cost of onions is Rs 5 per kg, the cost of wheat is Rs 8 per kg, and the cost of rice is Rs 8 per kg.

Miscellaneous Solutions

Question 1:

$$| \begin{matrix} x & \sin \theta & \cos \theta \end{matrix} |$$

Prove that the determinant $\begin{vmatrix} -\sin \theta & -x & 1 \\ \cos \theta & 1 & x \end{vmatrix}$ is independent of θ .

Answer

$$\begin{aligned} \Delta &= \begin{vmatrix} x & \sin \theta & \cos \theta \\ -\sin \theta & -x & 1 \\ \cos \theta & 1 & x \end{vmatrix} \\ &= x(x^2 - 1) - \sin \theta(-x \sin \theta - \cos \theta) + \cos \theta(-\sin \theta + x \cos \theta) \\ &= x^3 - x + x \sin^2 \theta + \sin \theta \cos \theta - \sin \theta \cos \theta + x \cos^2 \theta \\ &= x^3 - x + x(\sin^2 \theta + \cos^2 \theta) \\ &= x^3 - x + x \\ &= x^3 \text{ (Independent of } \theta) \end{aligned}$$

Hence, Δ is independent of θ .

Question 2:

Without expanding the determinant, prove that

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

Answer

$$\begin{aligned} \text{L.H.S.} &= \begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} \\ &= \frac{1}{abc} \begin{vmatrix} a^2 & a^3 & abc \\ b^2 & b^3 & abc \\ c^2 & c^3 & abc \end{vmatrix} && [R_1 \rightarrow aR_1, R_2 \rightarrow bR_2, \text{ and } R_3 \rightarrow cR_3] \\ &= \frac{1}{abc} \cdot abc \begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix} && [\text{Taking out factor } abc \text{ from } C_3] \\ &= \begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix} \\ &= \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} && [\text{Applying } C_1 \leftrightarrow C_3 \text{ and } C_2 \leftrightarrow C_3] \end{aligned}$$

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