Indefinite Integrals Ex 19.30 Q22

$$\text{Let } I = \int \frac{dx}{x\left[6(\log x)^2 + 7\log x + 2\right]}$$
$$= \int \frac{1}{x(2\log x + 1)(3\log x + 2)} dx$$

Now,

$$\text{Let } \frac{1}{x(2\log x + 1)(3\log x + 2)} = \frac{A}{x(2\log x + 1)} + \frac{B}{x(3\log x + 2)}$$

$$\Rightarrow \quad 1 = A(3\log x + 2) + B(2\log x + 1)$$

Put $x = 10^{-\frac{1}{2}}$

$$\Rightarrow \quad 1 = \frac{1}{2}A \Rightarrow \quad A = 2$$

Put $x = 10^{-\frac{2}{3}}$

$$\Rightarrow \quad 1 = -\frac{1}{3}B \quad \Rightarrow \quad B = -3$$

$$\therefore \quad I = \int \frac{2dx}{x(2\log x + 1)} - \int \frac{3dx}{x(3\log x + 2)}$$
$$= \log|2\log x + 1| - \log|3\log x + 2| + c$$

$$\therefore \quad I = \log\left|\frac{2\log x + 1}{3\log x + 2}\right| + c$$

Indefinite Integrals Ex 19.30 Q23

$$\frac{1}{x\left(x^n+1\right)}$$

Multiplying numerator and denominator by x^{n-1}, we obtain

$$\frac{1}{x\left(x^n+1\right)}=\frac{x^{n-1}}{x^{n-1}x\left(x^n+1\right)}=\frac{x^{n-1}}{x^n\left(x^n+1\right)}$$

Let $x^n=t \Rightarrow x^{n-1}dx=dt$

$$\therefore \int\frac{1}{x\left(x^n+1\right)}dx=\int\frac{x^{n-1}}{x^n\left(x^n+1\right)}dx=\frac{1}{n}\int\frac{1}{t\left(t+1\right)}dt$$

Let $\dfrac{1}{t\left(t+1\right)}=\dfrac{A}{t}+\dfrac{B}{\left(t+1\right)}$

$$1=A\left(1+t\right)+Bt \qquad \ldots(1)$$

Substituting $t = 0, -1$ in equation (1), we obtain

$A = 1$ and $B = -1$

$$\therefore \frac{1}{t\left(t+1\right)}=\frac{1}{t}-\frac{1}{\left(1+t\right)}$$

$$\Rightarrow \int\frac{1}{x\left(x^n+1\right)}dx=\frac{1}{n}\int\left\{\frac{1}{t}-\frac{1}{\left(t+1\right)}\right\}dx$$

$$=\frac{1}{n}\left[\log|t|-\log|t+1|\right]+C$$

$$=-\frac{1}{n}\left[\log\left|x^n\right|-\log\left|x^n+1\right|\right]+C$$

$$=\frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right|+C$$

Indefinite Integrals Ex 19.30 Q24

Let $\displaystyle\int\frac{x}{\left(x^2-a^2\right)\left(x^2-b^2\right)}=\frac{Ax+B}{\left(x^2-a^2\right)}+\frac{Cx+D}{\left(x^2-b^2\right)}$

$$\Rightarrow \qquad x=\left(Ax+B\right)\left(x^2-b^2\right)+\left(Cx+D\right)\left(x^2-a^2\right)$$

$$x=\left(A+C\right)x^3+\left(B+D\right)x^2+\left(-Ab^2-Ca^2\right)x+\left(-Bb^2-Da^2\right)$$

$$\Rightarrow \qquad A+C=0,\ B+D=0,\ -Ab^2-Ca^2=1,\ -Bb^2-Da^2=0$$

We get $B=0$, $D=0$, $C=\dfrac{1}{b^2-a^2}$, $A=-\dfrac{1}{b^2-a^2}$

Thus,

$$I=-\frac{1}{b^2-a^2}\int\frac{xdx}{x^2-a^2}+\frac{1}{b^2-a^2}\int\frac{xdx}{x^2-b^2}$$

$$I=-\frac{1}{2\left(b^2-a^2\right)}\log\left|x^2-a^2\right|+\frac{1}{2\left(b^2-a^2\right)}\log\left|x^2-b^2\right|+c$$

Indefinite Integrals Ex 19.30 Q25

Consider the integral

$$I=\int \frac{x^2+1}{(x^2+4)(x^2+25)}dx$$

Let $y=x^2$

Thus,

$$\frac{x^2+1}{(x^2+4)(x^2+25)}=\frac{y+1}{(y+4)(y+25)}$$

$$\Rightarrow \frac{y+1}{(y+4)(y+25)}=\frac{A}{y+4}+\frac{B}{y+25}$$

$$\Rightarrow \frac{y+1}{(y+4)(y+25)}=\frac{A(y+25)+B(y+4)}{(y+4)(y+25)}$$

$\Rightarrow y+1=Ay+25A+By+4B$

Comparing the coefficients, we have,

$A+B=1$ and $25A+4B=1$

Solving the above equations, we have,

$A=\frac{-1}{7}$ and $B=\frac{8}{7}$

Thus, $\int \frac{x^2+1}{(x^2+4)(x^2+25)}dx$

$$=\int \frac{\frac{-1}{7}}{x^2+4}dx+\int \frac{\frac{8}{7}}{x^2+25}dx$$

$$=\frac{-1}{7}\int \frac{1}{x^2+4}dx+\frac{8}{7}\int \frac{1}{x^2+25}dx$$

$$=\frac{-1}{7}\times\frac{1}{2}\tan^{-1}\frac{x}{2}+\frac{8}{7}\times\frac{1}{5}\tan^{-1}\frac{x}{5}+C$$

$$=\frac{-1}{14}\tan^{-1}\frac{x}{2}+\frac{8}{35}\tan^{-1}\frac{x}{5}+C$$

Indefinite Integrals Ex 19.30 Q26

Let $I = \int \frac{x^3 + x + 1}{x^2 - 1} dx$

$= \int \left(x + \frac{2x + 1}{x^2 - 1} \right) dx$

Now,

Let $\frac{2x + 1}{x^2 - 1} = \frac{A}{x + 1} + \frac{B}{x - 1}$

$\Rightarrow \quad 2x + 1 = A(x - 1) + B(x + 1)$

Put $x = 1$

$\Rightarrow \quad 3 = 2B \Rightarrow \quad B = \frac{3}{2}$

Put $x = -1$

$\Rightarrow \quad -1 = -2A \Rightarrow \quad A = \frac{1}{2}$

$\therefore \quad I = \int x dx + \frac{1}{2} \int \frac{dx}{x + 1} + \frac{3}{2} \int \frac{dx}{x - 1}$

$I = \frac{x^2}{2} + \frac{1}{2} \log |x + 1| + \frac{3}{2} \log |x - 1| + c$

*********** END **********