Exercise 7A

Question 27

(i)

$$LHS = \frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}$$

On dividing the numerator and denominator of LHS by $\cos^{\theta}$,We get

$$LHS = \frac{\sec\theta+1+\tan\theta}{\sec\theta+1-\tan\theta}$$

$$= \frac{(\sec\theta+\tan\theta)+\left(\sec^2\theta-\tan^2\theta\right)}{1+\sec\theta-\tan\theta}$$

$$\text{writing } 1 = \left(\sec^2\theta-\tan^2\theta\right)$$

$$= \frac{(\sec\theta+\tan\theta)+(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)}{(1+\sec\theta-\tan\theta)}$$

$$= \frac{(\sec\theta+\tan\theta)(1+\sec\theta-\tan\theta)}{(1+\sec\theta-\tan\theta)}$$

$$= (\sec\theta+\tan\theta) = \left(\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}\right)$$

$$= \left(\frac{1+\sin\theta}{\cos\theta}\right) = RHS$$

$\therefore$ LHS = RHS

(ii)

$$LHS = \frac{\sin\theta+1-\cos\theta}{\cos\theta-1+\sin\theta}$$

On dividing the numerator and denominator of LHS by $\cos^{\theta}$,We get

$$LHS = \frac{\tan\theta+\sec\theta-1}{1-\sec\theta+\tan\theta}$$

$$= \frac{(\tan\theta+\sec\theta)-\left(\sec^2\theta-\tan^2\theta\right)}{(1-\sec\theta+\tan\theta)}$$

$$\left(\text{writing } 1 = \sec^2\theta-\tan^2\theta\right)$$

$$= \frac{(\tan\theta+\sec\theta)-(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)}{(1-\sec\theta+\tan\theta)}$$

$$= \frac{(\tan\theta+\sec\theta)(1-\sec\theta+\tan\theta)}{(1-\sec\theta+\tan\theta)}$$

$$= \tan\theta+\sec\theta = \frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta} = \frac{\sin\theta+1}{\cos\theta} = RHS$$

$\therefore$ LHS = RHS

********** END **********