Trigonometric Ratios Ex 5.2 Q16

Answer :

We have,

$$4\left(\sin^4 30^\circ + \cos^2 60^\circ\right) - 3\left(\cos^2 45^\circ - \sin^2 90^\circ\right) - \sin^2 60^\circ \quad \ldots\ldots (1)$$

Now,

$$\sin 30^\circ = \cos 60^\circ = \frac{1}{2}, \sin 90^\circ = 1, \cos 45^\circ = \frac{1}{\sqrt{2}}, \sin 60^\circ = \frac{\sqrt{3}}{2}$$

So by substituting above values in equation (1)

We get,

$$4\left(\sin^4 30^\circ + \cos^2 60^\circ\right) - 3\left(\cos^2 45^\circ - \sin^2 90^\circ\right) - \sin^2 60^\circ$$

$$= 4\left(\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^2\right) - 3\left(\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right) - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 4\left(\frac{1^4}{2^4} + \frac{1^2}{2^2}\right) - 3\left(\frac{1^2}{(\sqrt{2})^2} - 1\right) - \frac{(\sqrt{3})^2}{2^2}$$

$$= 4\left(\frac{1}{16} + \frac{1}{4}\right) - 3\left(\frac{1}{2} - 1\right) - \frac{3}{4}$$

LCM of 16 and 4 in the first term of above expression is 16 and

Similarly LCM of 2 and 1 in the second term of above expression is 2

Therefore,

$$4\left(\sin^4 30^\circ + \cos^2 60^\circ\right) - 3\left(\cos^2 45^\circ - \sin^2 90^\circ\right) - \sin^2 60^\circ$$

$$= 4\left(\frac{1}{16} + \frac{1 \times 4}{4 \times 4}\right) - 3\left(\frac{1}{2} - \frac{1 \times 2}{1 \times 2}\right) - \frac{3}{4}$$

$$= 4\left(\frac{1}{16} + \frac{4}{16}\right) - 3\left(\frac{1}{2} - \frac{2}{2}\right) - \frac{3}{4}$$

$$= 4\left(\frac{1+4}{16}\right) - 3\left(\frac{1-2}{2}\right) - \frac{3}{4}$$

$$= 4\left(\frac{5}{16}\right) - 3\left(\frac{-1}{2}\right) - \frac{3}{4}$$

Now in the second term of the above expression $-3 \times -1 = +3$

Therefore,

$$4\left(\sin^4 30^\circ + \cos^2 60^\circ\right) - 3\left(\cos^2 45^\circ - \sin^2 90^\circ\right) - \sin^2 60^\circ$$

$$= 4\left(\frac{5}{16}\right) + \frac{3}{2} - \frac{3}{4}$$

Now, in the above expression 4 cancels 16 and 4 remains in the denominator of first term

Therefore,

$$4\left(\sin^4 30^\circ + \cos^2 60^\circ\right) - 3\left(\cos^2 45^\circ - \sin^2 90^\circ\right) - \sin^2 60^\circ$$

$$= \frac{5}{4} + \frac{3}{2} - \frac{3}{4}$$

Now by taking LCM =4 in the above expression

We get,

$$4\left(\sin^4 30^\circ + \cos^2 60^\circ\right) - 3\left(\cos^2 45^\circ - \sin^2 90^\circ\right) - \sin^2 60^\circ$$

$$= \frac{5}{4} + \frac{3 \times 2}{2 \times 2} - \frac{3}{4}$$

$$= \frac{5}{4} + \frac{6}{4} - \frac{3}{4}$$

$$= \frac{5+6-3}{4}$$

$$= \frac{11-3}{4}$$

$$= \frac{8}{4}$$

Now, in the above expression $\frac{8}{4}$ gets reduced to 2

Therefore,

$$4\left(\sin^4 30^\circ + \cos^2 60^\circ\right) - 3\left(\cos^2 45^\circ - \sin^2 90^\circ\right) - \sin^2 60^\circ = 2$$

Trigonometric Ratios Ex 5.2 Q17

Answer :

We have,

$$\frac{\tan^2 60^\circ + 4\cos^2 45^\circ + 3\sec^2 30^\circ + 5\cos^2 90^\circ}{\cosec 30^\circ + \sec 60^\circ - \cot^2 30^\circ} \quad \ldots\ldots (1)$$

Now,

$$\tan 60^\circ = \cot 30^\circ = \sqrt{3}, \ \cos 45^\circ = \frac{1}{\sqrt{2}}, \ \sec 30^\circ = \frac{2}{\sqrt{3}}, \ \sec 60^\circ = 2, \ \cosec 30^\circ = 2, \ \cos 90^\circ = 0$$

So by substituting above values in equation (1)

We get,

$$\frac{\tan^2 60^\circ + 4\cos^2 45^\circ + 3\sec^2 30^\circ + 5\cos^2 90^\circ}{\cosec 30^\circ + \sec 60^\circ - \cot^2 30^\circ}$$

$$= \frac{\left(\sqrt{3}\right)^2 + 4\left(\frac{1}{\sqrt{2}}\right)^2 + 3\left(\frac{2}{\sqrt{3}}\right)^2 + 5(0)^2}{2 + 2 - \left(\sqrt{3}\right)^2}$$

$$= \frac{3 + \frac{4}{2} + 3\left(\frac{4}{3}\right) + 0}{4 - 3}$$

Now,

3 gets cancel in numerator and we get,

$$\frac{\tan^2 60^\circ + 4\cos^2 45^\circ + 3\sec^2 30^\circ + 5\cos^2 90^\circ}{\cosec 30^\circ + \sec 60^\circ - \cot^2 30^\circ}$$

$$= \dfrac{3 + \dfrac{4}{2} + 4}{1}$$

$$= \dfrac{7 + \dfrac{4}{2}}{1}$$

Now, $\dfrac{4}{2}$ in the numerator get reduced to 2and we get,

$$\dfrac{\tan^2 60° + 4\cos^2 45° + 3\sec^2 30° + 5\cos^2 90°}{\operatorname{cosec} 30° + \sec 60° - \cot^2 30°}$$

$$= \dfrac{7 + 2}{1}$$

$$= \dfrac{9}{1}$$

$$= 9$$

Therefore,

$$\boxed{\dfrac{\tan^2 60° + 4\cos^2 45° + 3\sec^2 30° + 5\cos^2 90°}{\operatorname{cosec} 30° + \sec 60° - \cot^2 30°} = 9}$$

********** END *********