



### EXERCISE 3.4

Question 1:

Find the principal and general solutions of the equation  $\tan x = \sqrt{3}$

Ans:

$$\tan x = \sqrt{3}$$

It is known that  $\tan \frac{\pi}{3} = \sqrt{3}$  and  $\tan \left( \frac{4\pi}{3} \right) = \tan \left( \pi + \frac{\pi}{3} \right) = \tan \frac{\pi}{3} = \sqrt{3}$

Therefore, the principal solutions are  $x = \frac{\pi}{3}$  and  $\frac{4\pi}{3}$ .

$$\text{Now, } \tan x = \tan \frac{\pi}{3}$$

$$\Rightarrow x = n\pi + \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}$$

Therefore, the general solution is  $x = n\pi + \frac{\pi}{3}$ , where  $n \in \mathbb{Z}$

Question 2:

Find the principal and general solutions of the equation  $\sec x = 2$

Ans:

$$\sec x = 2$$

It is known that  $\sec \frac{\pi}{3} = 2$  and  $\sec \frac{5\pi}{3} = \sec \left( 2\pi - \frac{\pi}{3} \right) = \sec \frac{\pi}{3} = 2$

Therefore, the principal solutions are  $x = \frac{\pi}{3}$  and  $\frac{5\pi}{3}$ .

$$\text{Now, } \sec x = \sec \frac{\pi}{3}$$

$$\Rightarrow \cos x = \cos \frac{\pi}{3} \quad \left[ \sec x = \frac{1}{\cos x} \right]$$

$$\Rightarrow x = 2n\pi \pm \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}$$

Therefore, the general solution is  $x = 2n\pi \pm \frac{\pi}{3}$ , where  $n \in \mathbb{Z}$

Question 3:

Find the principal and general solutions of the equation  $\cot x = -\sqrt{3}$

Ans:

$$\cot x = -\sqrt{3}$$

It is known that  $\cot \frac{\pi}{6} = \sqrt{3}$

$$\therefore \cot\left(\pi - \frac{\pi}{6}\right) = -\cot \frac{\pi}{6} = -\sqrt{3} \text{ and } \cot\left(2\pi - \frac{\pi}{6}\right) = -\cot \frac{\pi}{6} = -\sqrt{3}$$

$$\text{i.e., } \cot \frac{5\pi}{6} = -\sqrt{3} \text{ and } \cot \frac{11\pi}{6} = -\sqrt{3}$$

Therefore, the principal solutions are  $x = \frac{5\pi}{6}$  and  $\frac{11\pi}{6}$ .

$$\text{Now, } \cot x = \cot \frac{5\pi}{6}$$

$$\Rightarrow \tan x = \tan \frac{5\pi}{6} \quad \left[ \cot x = \frac{1}{\tan x} \right]$$

$$\Rightarrow x = n\pi + \frac{5\pi}{6}, \text{ where } n \in \mathbb{Z}$$

Therefore, the general solution is  $x = n\pi + \frac{5\pi}{6}$ , where  $n \in \mathbb{Z}$

Question 4:

Find the general solution of  $\operatorname{cosec} x = -2$

Ans:

It is known that

$$\operatorname{cosec} \frac{\pi}{6} = 2$$

$$\therefore \operatorname{cosec}\left(\pi + \frac{\pi}{6}\right) = -\operatorname{cosec} \frac{\pi}{6} = -2 \text{ and } \operatorname{cosec}\left(2\pi - \frac{\pi}{6}\right) = -\operatorname{cosec} \frac{\pi}{6} = -2$$

$$\text{i.e., } \operatorname{cosec} \frac{7\pi}{6} = -2 \text{ and } \operatorname{cosec} \frac{11\pi}{6} = -2$$

Therefore, the principal solutions are  $x = \frac{7\pi}{6}$  and  $\frac{11\pi}{6}$ .

$$\text{Now, } \operatorname{cosec} x = \operatorname{cosec} \frac{7\pi}{6}$$

$$\Rightarrow \sin x = \sin \frac{7\pi}{6} \quad \left[ \operatorname{cosec} x = \frac{1}{\sin x} \right]$$

$$\Rightarrow x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in \mathbb{Z}$$

Therefore, the general solution is  $x = n\pi + (-1)^n \frac{7\pi}{6}$ , where  $n \in \mathbb{Z}$

Question 5:

Find the general solution of the equation  $\cos 4x = \cos 2x$

Ans:

$$\cos 4x = \cos 2x$$

$$\Rightarrow \cos 4x - \cos 2x = 0$$

$$\Rightarrow -2 \sin\left(\frac{4x+2x}{2}\right) \sin\left(\frac{4x-2x}{2}\right) = 0$$

$$\left[ \because \cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right) \right]$$

$$\Rightarrow \sin 3x \sin x = 0$$

$$\Rightarrow \sin 3x = 0 \quad \text{or} \quad \sin x = 0$$

$$\therefore 3x = n\pi \quad \text{or} \quad x = n\pi, \text{ where } n \in \mathbb{Z}$$

$$\Rightarrow x = \frac{n\pi}{3} \quad \text{or} \quad x = n\pi, \text{ where } n \in \mathbb{Z}$$

Question 6:

Find the general solution of the equation  $\cos 3x + \cos x - \cos 2x = 0$

Ans:

$$\cos 3x + \cos x - \cos 2x = 0$$

$$\Rightarrow 2 \cos\left(\frac{3x+x}{2}\right) \cos\left(\frac{3x-x}{2}\right) - \cos 2x = 0 \quad \left[ \cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) \right]$$

$$\Rightarrow 2 \cos 2x \cos x - \cos 2x = 0$$

$$\Rightarrow \cos 2x (2 \cos x - 1) = 0$$

$$\Rightarrow \cos 2x = 0 \quad \text{or} \quad 2 \cos x - 1 = 0$$

$$\Rightarrow \cos 2x = 0 \quad \text{or} \quad \cos x = \frac{1}{2}$$

$$\therefore 2x = (2n+1)\frac{\pi}{2} \quad \text{or} \quad \cos x = \cos \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}$$

$$\Rightarrow x = (2n+1)\frac{\pi}{4} \quad \text{or} \quad x = 2n\pi \pm \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}$$

Question 7:

Find the general solution of the equation  $\sin 2x + \cos x = 0$

Ans:

$$\sin 2x + \cos x = 0$$

$$\Rightarrow 2 \sin x \cos x + \cos x = 0$$

$$\Rightarrow \cos x (2 \sin x + 1) = 0$$

$$\Rightarrow \cos x = 0 \quad \text{or} \quad 2 \sin x + 1 = 0$$

$$\text{Now, } \cos x = 0 \Rightarrow \cos x = (2n+1)\frac{\pi}{2}, \text{ where } n \in \mathbb{Z}$$

$$2 \sin x + 1 = 0$$

$$\Rightarrow \sin x = \frac{-1}{2} = -\sin \frac{\pi}{6} = \sin\left(\pi + \frac{\pi}{6}\right) = \sin\left(\pi + \frac{\pi}{6}\right) = \sin \frac{7\pi}{6}$$

$$\Rightarrow x = n\pi + (-1)^n \frac{7\pi}{6}, \text{ where } n \in \mathbb{Z}$$

Therefore, the general solution is  $(2n+1)\frac{\pi}{2}$  or  $n\pi + (-1)^n \frac{7\pi}{6}$ ,  $n \in \mathbb{Z}$ .

Question 8:

Find the general solution of the equation  $\sec^2 2x = 1 - \tan 2x$

Ans:

$$\sec^2 2x = 1 - \tan 2x$$

$$\Rightarrow 1 + \tan^2 2x = 1 - \tan 2x$$

$$\Rightarrow \tan^2 2x + \tan 2x = 0$$

$$\Rightarrow \tan 2x (\tan 2x + 1) = 0$$

$$\Rightarrow \tan 2x = 0 \quad \text{or} \quad \tan 2x + 1 = 0$$

$$\text{Now, } \tan 2x = 0$$

$$\Rightarrow \tan 2x = \tan 0$$

$$\Rightarrow 2x = n\pi + 0, \text{ where } n \in \mathbb{Z}$$

$$\Rightarrow x = \frac{n\pi}{2}, \text{ where } n \in \mathbb{Z}$$

$$\tan 2x + 1 = 0$$

$$\Rightarrow \tan 2x = -1 = -\tan \frac{\pi}{4} = \tan \left( \pi - \frac{\pi}{4} \right) = \tan \frac{3\pi}{4}$$

$$\Rightarrow 2x = n\pi + \frac{3\pi}{4}, \text{ where } n \in \mathbb{Z}$$

$$\Rightarrow x = \frac{n\pi}{2} + \frac{3\pi}{8}, \text{ where } n \in \mathbb{Z}$$

Therefore, the general solution is  $\frac{n\pi}{2}$  or  $\frac{n\pi}{2} + \frac{3\pi}{8}$ ,  $n \in \mathbb{Z}$ .

Question 9:

Find the general solution of the equation  $\sin x + \sin 3x + \sin 5x = 0$

Ans:

$$\sin x + \sin 3x + \sin 5x = 0$$

$$\Rightarrow \sin 3x (2 \cos 2x + 1) = 0$$

$$\Rightarrow \sin 3x = 0 \quad \text{or} \quad 2 \cos 2x + 1 = 0$$

$$\text{Now, } \sin 3x = 0 \Rightarrow 3x = n\pi, \text{ where } n \in \mathbb{Z}$$

$$\text{i.e., } x = \frac{n\pi}{3}, \text{ where } n \in \mathbb{Z}$$

$$2 \cos 2x + 1 = 0$$

$$\Rightarrow \cos 2x = \frac{-1}{2} = -\cos \frac{\pi}{3} = \cos \left( \pi - \frac{\pi}{3} \right)$$

$$\Rightarrow \cos 2x = \cos \frac{2\pi}{3}$$

$$\Rightarrow 2x = 2n\pi \pm \frac{2\pi}{3}, \text{ where } n \in \mathbb{Z}$$

$$\Rightarrow x = n\pi \pm \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}$$

$$(\sin x + \sin 5x) + \sin 3x = 0$$

$$\Rightarrow \left[ 2 \sin \left( \frac{x+5x}{2} \right) \cos \left( \frac{x-5x}{2} \right) \right] + \sin 3x = 0 \quad \left[ \sin A + \sin B = 2 \sin \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right) \right]$$

$$\Rightarrow 2 \sin 3x \cos (-2x) + \sin 3x = 0$$

$$\Rightarrow 2 \sin 3x \cos 2x + \sin 3x = 0$$

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