



Question 5:

Find the vector and Cartesian equation of the planes

(a) that passes through the point $(1, 0, -2)$ and the normal to the plane is $\hat{i} + \hat{j} - \hat{k}$.

(b) that passes through the point $(1, 4, 6)$ and the normal vector to the plane is $\hat{i} - 2\hat{j} + \hat{k}$.

Answer

(a) The position vector of point $(1, 0, -2)$ is $\vec{a} = \hat{i} - 2\hat{k}$

The normal vector \vec{N} perpendicular to the plane is $\vec{N} = \hat{i} + \hat{j} - \hat{k}$

The vector equation of the plane is given by, $(\vec{r} - \vec{a}) \cdot \vec{N} = 0$

$$\Rightarrow [\vec{r} - (\hat{i} - 2\hat{k})] \cdot (\hat{i} + \hat{j} - \hat{k}) = 0 \quad \dots(1)$$

\vec{r} is the position vector of any point P (x, y, z) in the plane.

$$\therefore \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

Therefore, equation (1) becomes

$$[(x\hat{i} + y\hat{j} + z\hat{k}) - (\hat{i} - 2\hat{k})] \cdot (\hat{i} + \hat{j} - \hat{k}) = 0$$

$$\Rightarrow [(x-1)\hat{i} + y\hat{j} + (z+2)\hat{k}] \cdot (\hat{i} + \hat{j} - \hat{k}) = 0$$

$$\Rightarrow (x-1) + y - (z+2) = 0$$

$$\Rightarrow x + y - z - 3 = 0$$

$$\Rightarrow x + y - z = 3$$

This is the Cartesian equation of the required plane.

(b) The position vector of the point $(1, 4, 6)$ is $\vec{a} = \hat{i} + 4\hat{j} + 6\hat{k}$

The normal vector \vec{N} perpendicular to the plane is $\vec{N} = \hat{i} - 2\hat{j} + \hat{k}$

The vector equation of the plane is given by, $(\vec{r} - \vec{a}) \cdot \vec{N} = 0$

$$\Rightarrow [\vec{r} - (\hat{i} + 4\hat{j} + 6\hat{k})] \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0 \quad \dots(1)$$

\vec{r} is the position vector of any point P (x, y, z) in the plane.

$$\therefore \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

Therefore, equation (1) becomes

$$[(x\hat{i} + y\hat{j} + z\hat{k}) - (\hat{i} + 4\hat{j} + 6\hat{k})] \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0$$

$$\Rightarrow [(x-1)\hat{i} + (y-4)\hat{j} + (z-6)\hat{k}] \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0$$

$$\Rightarrow (x-1) - 2(y-4) + (z-6) = 0$$

$$\Rightarrow x - 2y + z + 1 = 0$$

This is the Cartesian equation of the required plane.

Question 6:

Find the equations of the planes that passes through three points.

(a) $(1, 1, -1)$, $(6, 4, -5)$, $(-4, -2, 3)$

(b) $(1, 1, 0)$, $(1, 2, 1)$, $(-2, 2, -1)$

Answer

(a) The given points are A $(1, 1, -1)$, B $(6, 4, -5)$, and C $(-4, -2, 3)$.

$$\begin{vmatrix} 1 & 1 & -1 \\ 6 & 4 & -5 \\ -4 & -2 & 3 \end{vmatrix} = (12-10) - (18-20) - (-12+16) \\ = 2 + 2 - 4 \\ = 0$$

Since A, B, C are collinear points, there will be infinite number of planes passing through the given points.

(b) The given points are A $(1, 1, 0)$, B $(1, 2, 1)$, and C $(-2, 2, -1)$.

$$\begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ -2 & 2 & -1 \end{vmatrix} = (-2-2) - (2+2) = -8 \neq 0$$

Therefore, a plane will pass through the points A, B, and C.

It is known that the equation of the plane through the points, (x_1, y_1, z_1) , (x_2, y_2, z_2) , and (x_3, y_3, z_3) , is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & y-1 & z \\ 0 & 1 & 1 \\ -3 & 1 & -1 \end{vmatrix} = 0$$

$$\Rightarrow (-2)(x-1) - 3(y-1) + 3z = 0$$

$$\Rightarrow -2x - 3y + 3z + 2 + 3 = 0$$

$$\Rightarrow -2x - 3y + 3z = -5$$

$$\Rightarrow 2x + 3y - 3z = 5$$

This is the Cartesian equation of the required plane.

Question 7:

Find the intercepts cut off by the plane $2x + y - z = 5$

Answer

$$2x + y - z = 5 \quad \dots(1)$$

Dividing both sides of equation (1) by 5, we obtain

$$\frac{2}{5}x + \frac{y}{5} - \frac{z}{5} = 1$$

$$\Rightarrow \frac{x}{\frac{5}{2}} + \frac{y}{5} - \frac{z}{-5} = 1 \quad \dots(2)$$

It is known that the equation of a plane in intercept form is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, where a, b, c are the intercepts cut off by the plane at x, y , and z axes respectively.

Therefore, for the given equation,

$$a = \frac{5}{2}, b = 5, \text{ and } c = -5$$

Thus, the intercepts cut off by the plane are $\frac{5}{2}, 5$, and -5 .

Question 8:

Find the equation of the plane with intercept 3 on the y -axis and parallel to ZOX plane.

Answer

The equation of the plane ZOX is

$$y = 0$$

Any plane parallel to it is of the form, $y = a$

Since the y -intercept of the plane is 3,

$$\therefore a = 3$$

Thus, the equation of the required plane is $y = 3$

Question 9:

Find the equation of the plane through the intersection of the planes

$$3x - y + 2z - 4 = 0 \text{ and } x + y + z - 2 = 0 \text{ and the point } (2, 2, 1)$$

Answer

The equation of any plane through the intersection of the planes,

$$3x - y + 2z - 4 = 0 \text{ and } x + y + z - 2 = 0, \text{ is}$$

$$(3x - y + 2z - 4) + \alpha(x + y + z - 2) = 0, \text{ where } \alpha \in \mathbb{R} \quad \dots(1)$$

The plane passes through the point $(2, 2, 1)$. Therefore, this point will satisfy equation (1).

$$\therefore (3 \times 2 - 2 + 2 \times 1 - 4) + \alpha(2 + 2 + 1 - 2) = 0$$

$$\Rightarrow 2 + 3\alpha = 0$$

$$\Rightarrow \alpha = -\frac{2}{3}$$

Substituting $\alpha = -\frac{2}{3}$ in equation (1), we obtain

$$(3x - y + 2z - 4) - \frac{2}{3}(x + y + z - 2) = 0$$

$$\Rightarrow 3(3x - y + 2z - 4) - 2(x + y + z - 2) = 0$$

$$\Rightarrow (9x - 3y + 6z - 12) - 2(x + y + z - 2) = 0$$

$$\Rightarrow 7x - 5y + 4z - 8 = 0$$

This is the required equation of the plane.

Question 10:

Find the vector equation of the plane passing through the intersection of the planes

$$\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 7, \quad \vec{r} \cdot (2\hat{i} + 5\hat{j} + 3\hat{k}) = 9$$

and through the point (2, 1, 3)

Answer

The equations of the planes are $\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 7$ and $\vec{r} \cdot (2\hat{i} + 5\hat{j} + 3\hat{k}) = 9$

$$\Rightarrow \vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) - 7 = 0 \quad \dots(1)$$

$$\vec{r} \cdot (2\hat{i} + 5\hat{j} + 3\hat{k}) - 9 = 0 \quad \dots(2)$$

The equation of any plane through the intersection of the planes given in equations (1) and (2) is given by,

$$[\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) - 7] + \lambda [\vec{r} \cdot (2\hat{i} + 5\hat{j} + 3\hat{k}) - 9] = 0, \text{ where } \lambda \in R$$

$$\vec{r} \cdot [(2\hat{i} + 2\hat{j} - 3\hat{k}) + \lambda(2\hat{i} + 5\hat{j} + 3\hat{k})] = 9\lambda + 7$$

$$\vec{r} \cdot [(2 + 2\lambda)\hat{i} + (2 + 5\lambda)\hat{j} + (3\lambda - 3)\hat{k}] = 9\lambda + 7 \quad \dots(3)$$

The plane passes through the point (2, 1, 3). Therefore, its position vector is given by,

$$\vec{r} = 2\hat{i} + \hat{j} + 3\hat{k}$$

Substituting in equation (3), we obtain

$$(2\hat{i} + \hat{j} + 3\hat{k}) \cdot [(2 + 2\lambda)\hat{i} + (2 + 5\lambda)\hat{j} + (3\lambda - 3)\hat{k}] = 9\lambda + 7$$

$$\Rightarrow (2 + 2\lambda) + (2 + 5\lambda) + (3\lambda - 3) = 9\lambda + 7$$

$$\Rightarrow 18\lambda - 3 = 9\lambda + 7$$

$$\Rightarrow 9\lambda = 10$$

***** END *****