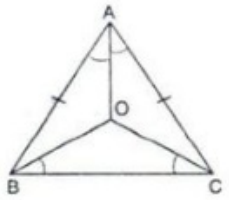
Exercise 5A

Question 21:

Given : A ΔABC in which AB = AC, BO and CO are bisectors of $\angle B$ and $\angle C$



To Prove : In ΔBOC, we have,

$$\angle OBC = \frac{1}{2}\angle B$$

and, $\angle OCB = \frac{1}{2}\angle C$

But, $\angle B = \angle C$ [$\because$ AB = AC (given)]

So, $\angle OBC = \angle OCB$

Since base angles are equal, sides are equal

$\Rightarrow$ OB = OC ....(1)

Since OB and OC are the bisectors of angles, $\angle B$ and $\angle C$ respectively, we have

$$\angle ABO = \frac{1}{2}\angle B$$

$$\angle ACO = \frac{1}{2}\angle C$$

$\Rightarrow \angle ABO = \angle ACO$ .....(2)

Now, in ΔABO and ΔACO

AB = AC [Given]

$\angle ABO = \angle ACO$ [from (2)]

BO = OC [from (1)]

Thus, by Side-Angle-Side criterion of congruence, we have

$\Delta ABO \cong \Delta ACO$ [By SAS]

The corresponding parts of the congruent triangles are equal

$\therefore$ $\angle BAO = \angle CAO$ [By cpct]

i.e. AO bisects $\angle A$.

********** END **********