Indefinite Integrals Ex 19.12 Q1

Let $I = \int \sin^4 x \cos^3 x dx$

Here, power of $\cos x$ is odd, so we substitute

$$\sin x = t$$

$$\Rightarrow \quad \cos x dx = dt$$

$$\Rightarrow \quad dx = \frac{dt}{\cos x}$$

$$\therefore \quad I = \int t^4 \cos^3 x \frac{dt}{\cos x}$$

$$= \int t^4 \cos^2 x dt$$

$$= \int t^4 \left(1 - \sin^2 x\right) dt$$

$$= \int t^4 \left(1 - t^2\right) dt$$

$$= \int \left(t^4 - t^6\right) dt$$

$$= \frac{t^5}{5} - \frac{t^7}{7} + c$$

$$\therefore \quad I = \frac{1}{5} \times \sin^5 x - \frac{1}{7} \times \sin^7 x + c$$

Indefinite Integrals Ex 19.12 Q2

Let $I = \int \sin^5 x dx$. Then

$$I = \int \sin^3 x \sin^2 x dx$$

$$= \int \sin^3 x \left(1 - \cos^2 x\right) dx$$

$$= \int \left(\sin^3 x - \sin^3 x \cos^2 x\right) dx$$

$$= \int \left[\sin x \left(1 - \cos^2 x\right) - \sin^3 x \cos^2 x\right] dx$$

$$= \int \left(\sin x - \sin x \cos^2 x - \sin^3 x \cos^2 x\right) dx$$

$$\Rightarrow \quad I = \int \sin x dx - \int \sin x \cos^2 x dx - \int \sin^3 x \cos^2 x dx$$

Putting $\cos x = t$ and $-\sin x dx = dt$ in 2nd and 3rd integrals, we get

$$I = \int \sin x dx - \int t^2 (-dt) + \int \sin^2 x t^2 dt$$

$$= \int \sin x dx + \int t^2 dt + \int \left(1 - \cos^2 x\right) t^2 dt$$

$$= \int sixdx + \int t^2 dt + \int \left(1 - t^2\right) t^2 dt$$

$$= -\cos x + \frac{t^3}{3} + \frac{t^3}{3} - \frac{t^5}{5} + c$$

$$= -\cos x + \frac{2}{3} t^3 - \frac{1}{5} t^5 + c$$

$$= -\cos x + \frac{2}{3}\left(\cos^3 x\right) - \frac{1}{5}\left(\cos^5 x\right) + c$$

$$\therefore \quad I = -\left[\cos x - \frac{2}{3}\cos^3 x + \frac{1}{5}\cos^5 x\right] + c$$

Indefinite Integrals Ex 19.12 Q3

Let $I = \int \cos^5 x dx$. Then

$$I = \int \cos^2 x \cos^3 x dx$$
$$= \int \left(1 - \sin^2 x\right) \cos^3 x dx$$
$$= \int \cos^3 x dx - \int \sin^2 x \cos^3 x dx$$
$$= \int \cos^2 x \cos x dx - \int \sin^2 x \left(1 - \sin^2 x\right) \cos x dx$$
$$= \int \left(\cos x - \sin^2 x \cos x\right) dx - \int \left(\sin^2 x \cos x - \sin^4 x \cos x\right) dx$$
$$\Rightarrow \quad I = \int \cos x dx - 2\int \sin^2 x \cos x dx + \int \sin^4 x \cos x dx$$

Putting $\sin x = t$ and $\cos x dx = dt$ in 2nd and 3rd integrals, we get

$$I = \int \cos x dx - 2\int t^2 dt + \int t^4 dt$$
$$= \sin x - \frac{2}{3} t^3 + \frac{t^5}{5} + c$$
$$= \sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x + c$$

$$\therefore \quad I = \sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x + c$$

Indefinite Integrals Ex 19.12 Q4

Let $I = \int \sin^5 x \cos x \; dx$ ---(i)

Let $\sin x = t$. Then,

$$d\left(\sin x\right) = dt$$
$$\Rightarrow \quad \cos x dx = dt$$

Putting $\sin x = t$ and $\cos x dx = dt$ in equation (i), we get

$$I = \int t^5 dt$$
$$= \frac{t^6}{6} + c$$
$$= \frac{\sin^6 x}{6} + c$$
$$\therefore \quad I = \frac{1}{6} \sin^6 x + c$$

Indefinite Integrals Ex 19.12 Q5

Let $I = \int \sin^3 x \cos^6 x dx$

Here, power of $\sin x$ is odd, so we substitute

$$\cos x = t$$
$$\Rightarrow \quad -\sin x dx = dt$$
$$\therefore \quad I = \int \sin^2 x t^6 \left(-dt\right)$$
$$= -\int \left(1 - \cos^2 x\right) t^6 dt$$
$$= -\int \left(t^6 - t^8\right) dt$$
$$= -\frac{t^7}{7} + \frac{t^9}{9} + c$$
$$= -\frac{\cos^7 x}{7} + \frac{\cos^9 x}{9} + c$$

$$\therefore \quad I = -\frac{1}{7} \cos^7 x + \frac{1}{9} \cos^9 x + c$$

*********** END **********