Binomial Theorem Ex 18.2 Q15(i)

$$\left(x-\frac{1}{x}\right)^{10}$$

Here, $n=10$, which is even, $\therefore$ it has 11 terms

$\therefore$ middle term is $\left(\frac{n}{2}+1\right)=6^{\text{th}}$ term

$$T_n = T_{r+1} = (-1)^r \; {}^nC_r x^{n-r} y^r$$

$$T_6 = T_{5+1} = (-1)^5 \; {}^{10}C_5 (x)^{10-5}\left(\frac{1}{x}\right)^5$$

$$= \frac{-10\times9\times8\times7\times6}{5\times4\times3\times2}\times\frac{x^5}{x^5}$$

$$= -3\times2\times7\times6$$

$$= -252$$

Binomial Theorem Ex 18.2 Q15(ii)

$$\left(1-2x+x^2\right)^n$$

Here, n is odd, $\therefore$ $\left(1-2x+x^2\right)$ has $n+1=$ even term

$\therefore$ middle term is $\left(\frac{n+1}{2}\right)^{\text{th}}$ term

$$T_n = T_{r+1} = {}^nC_r x^{n-r} y^r$$

$$T_{\frac{n+1}{2}} = T_{\frac{n}{2}} = {}^nC_{\frac{n}{2}} (1-2x)^{n-\frac{n}{2}}\left(x^2\right)^{\frac{n}{2}}$$

$$= \frac{n!}{\frac{n}{2}!\frac{n}{2}!}(1-2x)^{\frac{n}{2}} x^{\frac{2n}{2}}$$

$$= \frac{(2n)!}{(n!)^2}(-1)^n x^n \qquad \left[\because (1-x)^n = 1-nx\right]$$

Binomial Theorem Ex 18.2 Q15(iii)

$$\left(1+3x+3x^2+x^3\right)^{2n}$$

This expansion is $\left((1+x)^3\right)^{2n} = (1+x)^{6n}$

Since $6n$ is even $\therefore$ it has $6n+1=$ odd terms has middle term is

$$\left(\frac{6n}{2}+1\right)^{\text{th}} = (4n)^{\text{th}} \text{ term}$$

$$T_n = T_{r+1} = {}^nC_r x^{n-r} y^r$$

$$T_{4n} = T_{3n+1} = {}^{6n}C_{3n} (1)^{6n-3n} (x)^{3n}$$

$$= \frac{(6n)!}{(3n)!(3n)!} x^{3n} \qquad \left[\because 1^{6n-3n} = 1\right]$$

Binomial Theorem Ex 18.2 Q15(iv)

$\left(2x-\frac{x^2}{4}\right)^9$

4^{th} and 5^{th} terms are middle terms

$\binom{9}{4}(2x)^5\left(-\frac{x^2}{4}\right)^4+\binom{9}{5}(2x)^4\left(-\frac{x^2}{4}\right)^5$

$\frac{63}{4}x^{13},-\frac{63}{32}x^{14}$

Binomial Theorem Ex 18.2 Q15(v)

$\left(x-\frac{1}{x}\right)^{2n+1}$

$2n+1$ is odd hence this expansion will have $2n+2$ = even terms.

Hene, middle terms is $\frac{2n+1}{2}=n+1,n+2$

Term formula is

$$T_n=T_{r+1}=(-1)^r\ {}^nC_r x^{n-r}y^r$$

$$T_{n+1}=T_{n+1}=(-1)^n\ {}^{2n+1}C_n\,(x)^{2n+1-n}\left(\frac{1}{x}\right)^n$$

$$=(-1)^n\ {}^{2n+1}C_n x^{n+1-n}$$

$$=(-1)^n\ {}^{2n+1}C_n x$$

$$T_{n+2}=T_{n+1+1}=(-1)^{n+1}\ {}^{2n+1}C_{n+1}\,(x)^{2n+1-n-1}\left(\frac{1}{x}\right)^{n+1}$$

$$=(-1)^{n+1}\ {}^{2n+1}C_{n+1}x^{-1}$$

$$=(-1)^{n+1}\ {}^{2n+1}C_{n+1}\frac{1}{x}$$

$$=(-1)^{n+1}\ {}^{2n+1}C_n\frac{1}{x}\qquad\left[\because {}^nC_r={}^nC_{r-1}\right]$$

*********** END **********