Adjoint and Inverse of Matrix Ex 7.1 Q37

$$\text{Let } B = A^T = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$$

$$|B| = \begin{vmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{vmatrix} = (-1-8) - 0 - 2(-8+3) = -9 + 10 = 1 \neq 0$$

So, B is invertible matrix.

$$B_{11} = (-1)^{1+1}(-9) = -9;\ B_{12} = (-1)^{1+2}(-8) = 8;\ B_{13} = (-1)^{1+3}(-5) = -5$$
$$B_{21} = (-1)^{2+1}(8) = -8;\ B_{22} = (-1)^{2+2}(7) = 7;\ B_{23} = (-1)^{2+3}(4) = -4$$
$$B_{31} = (-1)^{3+1}(-2) = -2;\ B_{32} = (-1)^{3+2}(-2) = 2;\ B_{33} = (-1)^{3+3}(-1) = -1$$

$$\operatorname{adj} B = \begin{bmatrix} -9 & 8 & -5 \\ -8 & 7 & -4 \\ -2 & 2 & -1 \end{bmatrix}^T = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \operatorname{adj} B$$

$$\Rightarrow B^{-1} = \frac{1}{1} \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

$$\Rightarrow \left(A^T\right)^{-1} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q38

$$|A| = \begin{vmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{vmatrix} = -1(1-4) + 2(2+4) - 2(-4-2) = 3 + 12 + 12 = 27$$

$$A_{11} = (-1)^{1+1}(-3) = -3;\ A_{12} = (-1)^{1+2}(6) = -6;\ A_{13} = (-1)^{1+3}(-6) = -6$$
$$A_{21} = (-1)^{2+1}(-6) = 6;\ A_{22} = (-1)^{2+2}(3) = 3;\ A_{23} = (-1)^{2+3}(6) = -6$$
$$A_{31} = (-1)^{3+1}(6) = 6;\ A_{32} = (-1)^{3+2}(6) = -6;\ A_{33} = (-1)^{3+3}(3) = 3$$

$$\operatorname{adj} A = \begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix}^T = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$A(\operatorname{adj} A) = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$\Rightarrow A(\operatorname{adj} A) = \begin{bmatrix} 27 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 27 \end{bmatrix}$$

$$\Rightarrow A(\operatorname{adj} A) = 27 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A(\operatorname{adj} A) = |A| I_3$$

Adjoint and Inverse of Matrix Ex 7.1 Q39

$$|A| = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 0 - 1(0-1) + 1(1-0) = 0 + 1 + 1 = 2 \neq 0$$

So, A is invertible matrix.

$$A_{11} = (-1)^{1+1}(-1) = -1;\ A_{12} = (-1)^{1+2}(-1) = 1;\ A_{13} = (-1)^{1+3}(1) = 1$$
$$A_{21} = (-1)^{2+1}(-1) = 1;\ A_{22} = (-1)^{2+2}(-1) = -1;\ A_{23} = (-1)^{2+3}(-1) = 1$$
$$A_{31} = (-1)^{3+1}(1) = 1;\ A_{32} = (-1)^{3+2}(-1) = 1;\ A_{33} = (-1)^{3+3}(-1) = -1$$

$$\text{adj } A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \ldots\ldots\ldots\ldots(i)$$

$$A^2 - 3I = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$A^2 - 3I = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$
$$A^2 - 3I = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

From (i) and (ii) we can see that,

$$A^{-1} = \frac{1}{2}\left(A^2 - 3I\right)$$

*********** END **********