

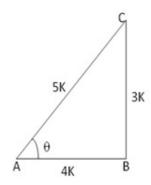
Question 10

Given:
$$sec\theta = \frac{AC}{AB} = \frac{5}{4}$$

Let AC = 5k and AB = 4k,

Where k is positive

Let us draw a $\triangle ABC$ in which $\angle B = 90^{\circ}$ and $\angle BAC = \theta$



By pythagoras theorem, we have

$$\Rightarrow AC^{2} = (AB)^{2} + (BC)^{2} \Rightarrow BC^{2} = (AC^{2} - AB^{2})$$

$$BC^{2} = [(5k)^{2} - (4k)^{2}]$$

$$= (25k^{2} - 16k^{2})$$

$$\Rightarrow$$
 BC² = 9k²

$$\sin\theta = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}, \sec\theta = \frac{5}{4}$$

and
$$\cos \theta = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \left(\frac{3}{5} \times \frac{5}{4}\right) = \frac{3}{4}$$

L.H.S. =
$$\frac{\tan \theta}{1 + \tan^2 \theta} = \frac{\left(\frac{3}{4}\right)}{\left(1 + \frac{9}{16}\right)} = \frac{\left(\frac{3}{4}\right)}{\left(\frac{25}{16}\right)} = \frac{3}{4} \times \frac{16}{25} = \frac{12}{25}$$

R.H.S. =
$$\frac{\sin \theta}{\sec \theta} = \frac{\left(\frac{3}{5}\right)}{\left(\frac{5}{4}\right)} = \left(\frac{3}{5} \times \frac{4}{5}\right) = \frac{12}{25}$$

Hence,
$$\frac{\tan \theta}{1 + \tan^2 \theta} = \frac{\sin \theta}{\cos \theta}$$

******* END *******