NCERT Solutions For Class 10 Chapter 6 Triangles Exercise 6.5

1. Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.

(i)7 cm, 24 cm, 25 cm

(ii) 3 cm, 8 cm, 6 cm

(iii)50 cm, 80 cm, 100 cm

(iv)13 cm, 12 cm, 5 cm

Ans. (i) Let a = 7 cm, b = 24 cm and c = 25 cm

Here the larger side is c = 25 cm.

We have, $a^2+b^2=7^2+24^2=49+576=625=c^2$

So, the triangle with the given sides is a right triangle. Its hypotenuse = 25 cm

(ii) Let a = 3 cm, b = 8 cm and c = 6 cm

Here the larger side is b = 8 cm.

We have, $a^2+c^2=3^2+6^2=9+36=45\neq b^2$

So, the triangle with the given sides is not a right triangle.

(iii) Let a = 50 cm, b = 80 cm and c = 100 cm

Here the larger side is c = 100 cm.

We have, $a^2+b^2=50^2+80^2=2500+6400=8900\neq c^2$

So, the triangle with the given sides is not a right triangle.

(iv) Let a = 13 cm, b = 12 cm and c = 5 cm

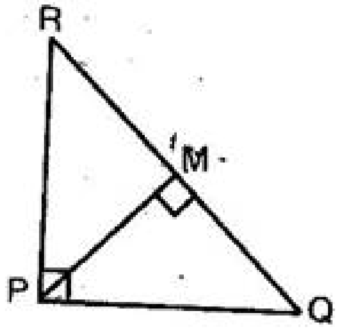
Here the larger side is a = 13 cm.

We have, $b^2+c^2=12^2+5^2=144+25=169=a^2$

So, the triangle with the given sides is a right triangle. Its hypotenuse = 13 cm

2. PQR is a triangle right angled at P and M is a point on QR such that PM$\perp$ QR. Show that PM^2 = QM.MR.

Ans. Given: PQR is a triangle right angles at P and PM$\perp$ QR



To Prove: $PM^2 = QM.MR$

Proof: Since $PM \perp QR$

$\therefore \Delta PQM \sim \Delta PRM$

$$\Rightarrow \frac{PM}{QM} = \frac{MR}{PM}$$

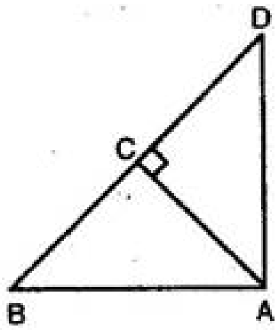
$$\Rightarrow PM^2 = QM.MR$$

3. In figure, ABD is a triangle right angled at A and $AC \perp BD$. Show that:

(i) $AB^2 = BC.BD$

(ii) $AC^2 = BC.DC$

(iii) $AD^2 = BD.CD$



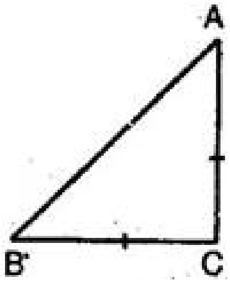
Ans. Given: ABD is a triangle right angled at A and $AC \perp BD$.

To Prove: **(i)** $AB^2 = BC.BD$, **(ii)** $AC^2 = BC.DC$, **(iii)** $AD^2 = BD.CD$

Proof:(i) Since $AC \perp BD$

$\therefore \Delta ABC \sim \Delta ADC$ and each triangle is similar to ΔABD

$\therefore \Delta ABC \sim \Delta ABD$



$$\Rightarrow \frac{AB}{BD} = \frac{BC}{AB}$$

$$\Rightarrow AB^2 = BC.BD$$

(ii) Since Δ ABC~ Δ ADC

$$\Rightarrow \frac{AC}{BC} = \frac{DC}{AC}$$

$$\Rightarrow AC^2 = BC.DC$$

(iii) Since Δ ACD~ Δ ABD

$$\Rightarrow \frac{AD}{CD} = \frac{BD}{AD}$$

$$\Rightarrow AD^2 = BD.CD$$

4. ABC is an isosceles triangle right angled at C. Prove that $\mathbf{AB^2 = 2AC^2}$.

Ans. Since ABC is an isosceles right triangle, right angled at C.

$$\Rightarrow AB^2 = AC^2 + BC^2$$

$$\Rightarrow AB^2 = AC^2 + AC^2 \text{ [BC = AC, given]}$$

$$\Rightarrow AB^2 = 2AC^2$$

5. ABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, prove that ABC is a right triangle.

Ans. Since ABC is an isosceles right triangle with AC = BC and $AB^2 = 2AC^2$

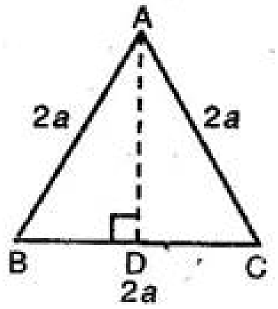
$$\Rightarrow AB^2 = AC^2 + AC^2$$

$$\Rightarrow AB^2 = AC^2 + BC^2 \text{ [BC = AC, given]}$$

$\therefore$ Δ ABC is right angled at C.

6. ABC is an equilateral triangle of side $2a$. Find each of its altitudes.

Ans. Let ABC be an equilateral triangle of side $2a$ units.



Draw AD $\perp$ BC. Then, D is the mid-point of BC.

$\Rightarrow$ BD = $\frac{1}{2}$ BC = $\frac{1}{2} \times 2a = a$

Since, ABD is a right triangle, right triangle at D.

$\therefore$ $AB^2 = AD^2 + BD^2$

$\Rightarrow$ $(2a)^2 = AD^2 + (a)^2$

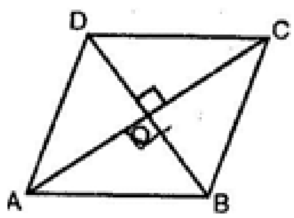
$\Rightarrow$ $AD^2 = 4a^2 - a^2 = 3a^2$

$\therefore$ Each of its altitude = $\sqrt{3}a$

7. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of squares of its diagonals.

Ans. Let the diagonals AC and BD of rhombus ABCD intersect each other at O. Since the diagonals of a rhombus bisect each other at right angles.

$\therefore$ $\angle AOB = \angle BOC = \angle COD = \angle DOA = 90^\circ$ and AO = CO, BO = OD



Since AOB is a right triangle, right angled at O.

$\therefore \quad AB^2 = OA^2 + OB^2$

$\Rightarrow \quad AB^2 = \left(\frac{1}{2}AC\right)^2 + \left(\frac{1}{2}BD\right)^2$

[$\because$ OA = OC and OB = OD]

$\Rightarrow \quad 4AB^2 = AC^2 + BD^2$ ....(1)

Similarly, we have $4BC^2 = AC^2 + BD^2$ ....(2)

$4CD^2 = AC^2 + BD^2$ ....(3)

$4AD^2 = AC^2 + BD^2$ ....(4)

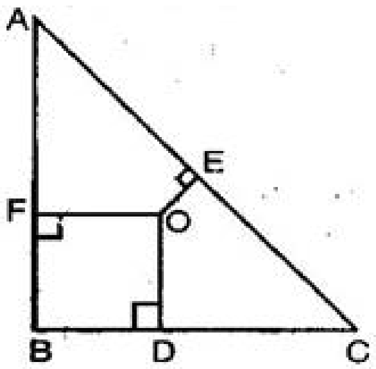
Adding all these results, we get

$4\left(AB^2 + BC^2 + CD^2 + DA^2\right) = 4\left(AC^2 + BD^2\right)$

$\Rightarrow \quad AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

8. In figure, O is a point in the interior of a triangle ABC, OD $\perp$ BC,

OE $\perp$ AC and OF $\perp$ AB. Show that:



(i) $\mathbf{OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2}$

(ii) $\mathbf{AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2}$

Ans. Join AO, BO and CO.

(i) In right Δ s OFA, ODB and OEC, we have

$OA^2 = AF^2 + OF^2$, $OB^2 = BD^2 + OD^2$ and $OC^2 = CE^2 + OE^2$

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