

Answer

It is given that $\vec{a}\cdot\vec{a}=0$ and $\vec{a}\cdot\vec{b}=0$

Now.

$$\vec{a} \cdot \vec{a} = 0 \Rightarrow |\vec{a}|^2 = 0 \Rightarrow |\vec{a}| = 0$$

 \vec{a} is a zero vector.

Hence, vector \vec{b} satisfying $\vec{a} \cdot \vec{b} = 0$ can be any vector.

Question 14:

If either vector $\vec{a}=\vec{0}$ or $\vec{b}=\vec{0}$, then $\vec{a}\cdot\vec{b}=0$. But the converse need not be true. Justify your answer with an example.

Answer

Consider $\vec{a} = 2\hat{i} + 4\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 3\hat{j} - 6\hat{k}$.

Then,

$$\vec{a} \cdot \vec{b} = 2.3 + 4.3 + 3(-6) = 6 + 12 - 18 = 0$$

We now observe that:

$$|\vec{a}| = \sqrt{2^2 + 4^2 + 3^2} = \sqrt{29}$$

$$\vec{a} \neq \vec{0}$$

$$|\vec{b}| = \sqrt{3^2 + 3^2 + (-6)^2} = \sqrt{54}$$

$$\vec{b} \neq \vec{0}$$

Hence, the converse of the given statement need not be true.

Question 15:

If the vertices A, B, C of a triangle ABC are (1, 2, 3), (-1, 0, 0), (0, 1, 2), respectively,

then find $\Box {\sf ABC}.$ [$\Box {\sf ABC}$ is the angle between the vectors \overline{BA} and \overline{BC}]

Answer

The vertices of ΔABC are given as A (1, 2, 3), B (-1, 0, 0), and C (0, 1, 2).

Also, it is given that $\square ABC$ is the angle between the vectors \overrightarrow{BA} and \overrightarrow{BC} .

$$\overrightarrow{BA} = \{1 - (-1)\}\hat{i} + (2 - 0)\hat{j} + (3 - 0)\hat{k} = 2\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\overrightarrow{BC} = \{0 - (-1)\}\hat{i} + (1 - 0)\hat{j} + (2 - 0)\hat{k} = \hat{i} + \hat{j} + 2\hat{k}$$

$$\therefore \overrightarrow{BA} \cdot \overrightarrow{BC} = (2\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 2 \times 1 + 2 \times 1 + 3 \times 2 = 2 + 2 + 6 = 10$$

$$|\overline{BA}| = \sqrt{2^2 + 2^2 + 3^2} = \sqrt{4 + 4 + 9} = \sqrt{17}$$

$$|\overrightarrow{BC}| = \sqrt{1+1+2^2} = \sqrt{6}$$

Now, it is known that:

$$\overrightarrow{BA} \cdot \overrightarrow{BC} = |\overrightarrow{BA}| |\overrightarrow{BC}| \cos(\angle ABC)$$

$$\therefore 10 = \sqrt{17} \times \sqrt{6} \cos(\angle ABC)$$

$$\Rightarrow$$
 cos $(\angle ABC) = \frac{10}{\sqrt{17} \times \sqrt{6}}$

$$\Rightarrow \angle ABC = \cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$$

Question 16:

Show that the points A (1, 2, 7), B (2, 6, 3) and C (3, 10, -1) are collinear.

Δnswer

The given points are A (1, 2, 7), B (2, 6, 3), and C (3, 10, -1).

$$\therefore \overrightarrow{AB} = (2-1)\hat{i} + (6-2)\hat{j} + (3-7)\hat{k} = \hat{i} + 4\hat{j} - 4\hat{k}$$

$$\overrightarrow{BC} = (3-2)\hat{i} + (10-6)\hat{j} + (-1-3)\hat{k} = \hat{i} + 4\hat{j} - 4\hat{k}$$

$$\overrightarrow{AC} = (3-1)\hat{i} + (10-2)\hat{j} + (-1-7)\hat{k} = 2\hat{i} + 8\hat{j} - 8\hat{k}$$

$$|\overrightarrow{AB}| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{1 + 16 + 16} = \sqrt{33}$$

$$|\overline{BC}| = \sqrt{1^2 + 4^2 + (-4)^2} = \sqrt{1 + 16 + 16} = \sqrt{33}$$

$$|\overrightarrow{AC}| = \sqrt{2^2 + 8^2 + 8^2} = \sqrt{4 + 64 + 64} = \sqrt{132} = 2\sqrt{33}$$

 $|\overrightarrow{AC}| = |\overrightarrow{AB}| + |\overrightarrow{BC}|$

Hence, the given points A, B, and C are collinear.

Show that the vectors $2\hat{i}-\hat{j}+\hat{k},~\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$ form the vertices of a right

Answer

Let vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ be position vectors of points A. B. and C.

i.e.,
$$\overrightarrow{OA} = 2\hat{i} - \hat{j} + \hat{k}$$
, $\overrightarrow{OB} = \hat{i} - 3\hat{j} - 5\hat{k}$ and $\overrightarrow{OC} = 3\hat{i} - 4\hat{j} - 4\hat{k}$

Now, vectors \overrightarrow{AB} , \overrightarrow{BC} , and \overrightarrow{AC} represent the sides of $\triangle ABC$.

i.e.,
$$\overrightarrow{OA} = 2\hat{i} - \hat{j} + \hat{k}$$
, $\overrightarrow{OB} = \hat{i} - 3\hat{j} - 5\hat{k}$, and $\overrightarrow{OC} = 3\hat{i} - 4\hat{j} - 4\hat{k}$

$$\therefore \overrightarrow{AB} = (1-2)\hat{i} + (-3+1)\hat{j} + (-5-1)\hat{k} = -\hat{i} - 2\hat{j} - 6\hat{k}$$

$$\overrightarrow{BC} = (3-1)\hat{i} + (-4+3)\hat{j} + (-4+5)\hat{k} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\overrightarrow{AC} = (2-3)\hat{i} + (-1+4)\hat{j} + (1+4)\hat{k} = -\hat{i} + 3\hat{j} + 5\hat{k}$$

$$|\overline{AB}| = \sqrt{(-1)^2 + (-2)^2 + (-6)^2} = \sqrt{1 + 4 + 36} = \sqrt{41}$$

$$|\overrightarrow{BC}| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{4 + 1 + 1} = \sqrt{6}$$

$$|\overrightarrow{AC}| = \sqrt{(-1)^2 + 3^2 + 5^2} = \sqrt{1 + 9 + 25} = \sqrt{35}$$

$$|\overrightarrow{BC}|^2 + |\overrightarrow{AC}|^2 = 6 + 35 = 41 = |\overrightarrow{AB}|^2$$

Hence, $\triangle ABC$ is a right-angled triangle.

If \vec{a} is a nonzero vector of magnitude 'a' and λ a nonzero scalar, then λ \vec{a} is unit vector if

(A)
$$\lambda = 1$$
 (B) $\lambda = -1$ (C) $a = |\lambda|$

$$a = \frac{1}{|\lambda|}$$

Vector $\lambda \vec{a}$ is a unit vector if $|\lambda \vec{a}| = 1$

Now.

$$|\lambda \vec{a}| = 1$$

$$\Rightarrow |\lambda||\vec{a}| = 1$$

$$\Rightarrow |\vec{a}| = \frac{1}{|\lambda|}$$

$$\Rightarrow a = \frac{1}{|\lambda|}$$
 $\left[|\vec{a}| = a \right]$

 $a = \frac{1}{|\lambda|}$ Hence, vector $\lambda \vec{a}$ is a unit vector if

The correct answer is D.

Exercise 10.4

Ouestion 1:

$$\operatorname{Find}_{\vec{a}} |\vec{a} \times \vec{b}|_{\text{if } \vec{a} = \hat{i} - 7\hat{j} + 7\hat{k} \text{ and } \vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}}$$

Answer

We have.

$$\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$$
 and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$$

$$=\hat{i}\left(-14+14\right)-\hat{j}\left(2-21\right)+\hat{k}\left(-2+21\right)=19\hat{j}+19\hat{k}$$
$$\therefore \left|\vec{a}\times\vec{b}\right|=\sqrt{(19)^2+(19)^2}=\sqrt{2\times(19)^2}=19\sqrt{2}$$

Question 2:

Find a unit vector perpendicular to each of the vector $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, where $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

Answer

$$\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}_{and} \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$$

$$\vec{a} + \vec{b} = 4\hat{i} + 4\hat{j}, \ \vec{a} - \vec{b} = 2\hat{i} + 4\hat{k}$$

$$(a+b) \times (a-b) = \begin{vmatrix} 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix} = i(16) - j(16) + k(-8) = 16i - 16j - 8k$$

$$\therefore |(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})| = \sqrt{16^2 + (-16)^2 + (-8)^2}$$

$$= \sqrt{2^2 \times 8^2 + 2^2 \times 8^2 + 8^2}$$

$$= 8\sqrt{2^2 + 2^2 + 1} = 8\sqrt{9} = 8 \times 3 = 24$$

Hence, the unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ is given by the relation

$$= \pm \frac{\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} - \vec{b}\right)}{\left|\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} - \vec{b}\right)\right|} = \pm \frac{16\hat{i} - 16\hat{j} - 8\hat{k}}{24}$$
$$= \pm \frac{2\hat{i} - 2\hat{j} - \hat{k}}{3} = \pm \frac{2}{3}\hat{i} \mp \frac{2}{3}\hat{j} \mp \frac{1}{3}\hat{k}$$

Question 3:

If a unit vector \vec{a} makes an angles $\frac{\pi}{3}$ with $\hat{i}, \frac{\pi}{4}$ with \hat{j} and an acute angle θ with \hat{k} , then find θ and hence, the compounds of \vec{a} .

Answe

Let unit vector \vec{a} have (a_1, a_2, a_3) components.

Since \vec{a} is a unit vector, $|\vec{a}| = 1$

Also, it is given that \vec{a} makes angles $\frac{\pi}{3}$ with $\hat{i}, \frac{\pi}{4}$ with \hat{j} , and an acute angle θ with \hat{k} . Then, we have:

$$\cos \frac{\pi}{3} = \frac{a_1}{|\vec{a}|}$$

$$\Rightarrow \frac{1}{2} = a_1 \qquad \qquad \left[|\vec{a}| = 1 \right]$$

$$\cos \frac{\pi}{4} = \frac{a_2}{|\vec{a}|}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = a_2 \qquad \qquad \left[|\vec{a}| = 1 \right]$$
Also, and $a = \frac{a_3}{3}$

Also,
$$\cos \theta = \frac{a_3}{|\vec{a}|}$$
.
 $\Rightarrow a_3 = \cos \theta$

Now,

$$|a| = 1$$

$$\Rightarrow \sqrt{a_1^2 + a_2^2 + a_3^2} = 1$$

$$\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2 \theta = 1$$

$$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2 \theta = 1$$

$$\Rightarrow \frac{3}{4} + \cos^2 \theta = 1$$

$$\Rightarrow \cos^2 \theta = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$$\therefore a_3 = \cos \frac{\pi}{3} = \frac{1}{2}$$

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