Differentiation Ex 11.5 Q30

Here,

$$y = (\tan x)^{\log x} + \cos^2\left(\frac{\pi}{4}\right)$$

$$y = e^{\log(\tan x)^{\cos x}} + \cos^2\left(\frac{\pi}{4}\right)$$

$$y = e^{\log x \log \tan x} + \cos x^2\left(\frac{\pi}{4}\right) \qquad \left[\text{Since, } e^{\log a} = a \text{ and } \log a^b = b \log a\right]$$

Differentiating it using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\log x \log \tan x}\right) + \frac{d}{dx}\cos^2\left(\frac{\pi}{4}\right)$$

$$= e^{\log x \log \tan x}\frac{d}{dx}(\log x \log \tan x) + 0$$

$$= e^{\log(\tan x)^{\log x}}\left[\log x \frac{d}{dx}(\log \tan x) + \log \tan x \frac{d}{dx}(\log x)\right]$$

$$= (\tan x)^{\log x}\left[\log x\left(\frac{1}{\tan x}\right)\frac{d}{dx}(\tan x) + \log \tan x\left(\frac{1}{x}\right)\right]$$

$$= (\tan x)^{\log x}\left[\log x\left(\frac{1}{\tan x}\right)(\sec^2 x) + \frac{\log \tan x}{x}\right]$$

$$\frac{dy}{dx} = (\tan x)^{\log x}\left[\log x\left(\frac{\sec^2 x}{\tan x}\right) + \frac{\log \tan x}{x}\right]$$

Differentiation Ex 11.5 Q31

Here,

$$y = x^x + x^{\frac{1}{x}}$$

$$= e^{\log x^x} + e^{\log x^{\frac{1}{x}}}$$

$$y = e^{x \log x} + e^{\left(\frac{1}{x}\log x\right)} \qquad \left[\text{Since, } e^{\log a} = a, \log a^b = b \log a\right]$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{x \log x}\right) + \frac{d}{dx}\left(e^{\frac{1}{x}\log x}\right)$$

$$= e^{x \log x} + \frac{d}{dx}(x \log x) + e^{\frac{1}{x}\log x}\frac{d}{dx}\left(\frac{1}{x}\log x\right)$$

$$= e^{\log x^x}\left[x\frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x)\right] + e^{\log x^{\frac{1}{x}}}\left[\frac{1}{x}\frac{d}{dx}(\log x) + \log x \frac{d}{dx}\left(\frac{1}{x}\right)\right]$$

$$= x^x\left[x\left(\frac{1}{x}\right) + \log x\,(1)\right] + x^{\frac{1}{x}}\left[\left(\frac{1}{x}\right)\left(\frac{1}{x}\right) + \log x\left(-\frac{1}{x^2}\right)\right]$$

$$= x^x\left[1 + \log x\right] + x^{\frac{1}{x}}\left(\frac{1}{x^2} - \frac{1}{x^2}\log x\right)$$

$$\frac{dy}{dx} = x^x\left[1 + \log x\right] + x^{\frac{1}{x}}\frac{(1 - \log x)}{x^2}$$

Differentiation Ex 11.5 Q32

Let $y=(\log x)^{x}+x^{\log x}$

Also, let $u=(\log x)^{x}$ and $v=x^{\log x}$

$\therefore y=u+v$

$$\Rightarrow \frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx} \qquad ...(1)$$

$u=(\log x)^{x}$

$$\Rightarrow \log u=\log\left[(\log x)^{x}\right]$$
$$\Rightarrow \log u=x\log(\log x)$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{u}\frac{du}{dx}=\frac{d}{dx}(x)\times\log(\log x)+x\cdot\frac{d}{dx}\left[\log(\log x)\right]$$
$$\Rightarrow \frac{du}{dx}=u\left[1\times\log(\log x)+x\cdot\frac{1}{\log x}\cdot\frac{d}{dx}(\log x)\right]$$
$$\Rightarrow \frac{du}{dx}=(\log x)^{x}\left[\log(\log x)+\frac{x}{\log x}\cdot\frac{1}{x}\right]$$
$$\Rightarrow \frac{du}{dx}=(\log x)^{x}\left[\log(\log x)+\frac{1}{\log x}\right]$$
$$\Rightarrow \frac{du}{dx}=(\log x)^{x}\left[\frac{\log(\log x)\cdot\log x+1}{\log x}\right]$$

$$\frac{1}{v}\cdot\frac{dv}{dx}=\frac{d}{dx}\left[(\log x)^{2}\right]$$
$$\Rightarrow \frac{1}{v}.\frac{dv}{dx}=2(\log x)\cdot\frac{d}{dx}(\log x)$$
$$\Rightarrow \frac{dv}{dx}=2v(\log x)\cdot\frac{1}{x}$$
$$\Rightarrow \frac{dv}{dx}=2x^{\log x}\frac{\log x}{x}$$
$$\Rightarrow \frac{dv}{dx}=2x^{\log x-1}\cdot\log x \qquad ...(3)$$

Therefore, from (1), (2), and (3), we obtain

$$\frac{dy}{dx}=(\log x)^{x-1}\left[1+\log x.\log(\log x)\right]+2x^{\log x-1}\cdot\log x$$

Differentiation Ex 11.5 Q33

Here,

$$x^{13}y^{7} = (x+y)^{20}$$

Taking log on both the sides,

$$\log\left(x^{13}y^{7}\right) = \log(x+y)^{20}$$

$$13\log x + 7\log y = 20\log(x+y) \qquad \left[\text{Since, } \log(AB) = \log A + \log B, \log a^{b} = b\log a\right]$$

Differentiating it with respect to x using chain rule,

$$13\frac{d}{dx}(\log x) + 7\frac{d}{dx}(\log y) = 20\frac{d}{dx}\log(x+y)$$

$$\frac{13}{x} + \frac{7}{y}\frac{dy}{dx} = \frac{20}{x+y}\frac{d}{dx}(x+y)$$

$$\frac{13}{x} + \frac{7}{y}\frac{dy}{dx} = \frac{20}{(x+y)}\left[1+\frac{dy}{dx}\right]$$

$$\frac{7}{y}\frac{dy}{dx} - \frac{20}{(x+y)} = \frac{20}{(x+y)} - \frac{13}{x}$$

$$\frac{dy}{dx}\left[\frac{7}{y} - \frac{20}{(x+y)}\right] = \frac{20}{(x+y)} - \frac{13}{x}$$

$$\frac{dy}{dx}\left[\frac{7(x+y) - 20y}{y(x+y)}\right] = \left[\frac{20x - 13(x+y)}{x(x+y)}\right]$$

$$\frac{dy}{dx} = \left[\frac{20x - 13x - 13y}{x(x+y)}\right]\left(\frac{y(x+y)}{7x + 7y - 20y}\right)$$

$$= \frac{y}{x}\left(\frac{7x - 13y}{7x - 13y}\right)$$

$$\frac{dy}{dx} = \frac{y}{x}$$

*********** END **********