Differentiation Ex 11.3 Q16

Let $\quad y = \tan^{-1}\left\{\frac{4x}{1-4x^2}\right\}$

Put $\quad 2x = \tan\theta$, so

$$y = \tan^{-1}\left\{\frac{2\tan\theta}{1-\tan^2\theta}\right\}$$

$$y = \tan^{-1}\{\tan 2\theta\} \qquad \text{---(i)}$$

Here, $\quad -\frac{1}{2} < x < \frac{1}{2}$

$\Rightarrow \quad -1 < 2x < 1$

$\Rightarrow \quad -1 < \tan\theta < 1$

$\Rightarrow \quad -\frac{\pi}{4} < \theta < \frac{\pi}{4}$

$\Rightarrow \quad -\frac{\pi}{2} < (2\theta) < \frac{\pi}{2}$

So, from equation (i),

$$y = 2\theta \qquad \left[\text{Sicne, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$y = 2\tan^{-1}(2x) \qquad [\text{Since, } 2x = \tan\theta]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = 2\left(\frac{1}{1+(2x)^2}\right)\frac{d}{dx}(2x)$$

$$\frac{dy}{dx} = \frac{4}{1+4x^2}.$$

Differentiation Ex 11.3 Q17

Let $\quad y = \tan^{-1}\left\{\frac{2^{x+1}}{1-4^x}\right\}$

Put $\quad 2^x = \tan\theta$, so,

$$= \tan^{-1}\left\{\frac{2^x \times 2}{1-\left(2^x\right)^2}\right\}$$

$$= \tan^{-1}\left\{\frac{2\tan\theta}{1-\tan^2\theta}\right\}$$

$$y = \tan^{-1}\left\{\tan\left(2\theta\right)\right\} \qquad ---(i)$$

Here, $\quad -\infty < x < 0$

$\Rightarrow \quad 2^{-\infty} < 2^x < 2^\circ$

$\Rightarrow \quad 0 < 2^x < 1$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{4}$

$\Rightarrow \quad 0 < \left(2\theta\right) < \frac{\pi}{2}$

From equatoin (i),

$$y = 2\theta \qquad \left[\text{Since, } \tan^{-1}\left(\tan\theta\right) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$$

$$y = 2\tan^{-1}\left(2^x\right)$$

Differentiate it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{2}{1+\left(2^x\right)^2}\frac{d}{dx}\left(2^x\right)$$

$$= \frac{2 \times 2^x \log 2}{1+4^x}$$

$$\frac{dy}{dx} = \frac{2^{x+1}\log 2}{1+4^x}.$$

Differentiation Ex 11.3 Q18

Let $\quad y = \tan^{-1}\left\{\frac{2a^x}{1-a^{2x}}\right\}$

Put $\quad a^x = \tan\theta$,

$$y = \tan^{-1}\left\{\frac{2\tan\theta}{1-\tan^2\theta}\right\}$$

$$y = \tan^{-1}\left\{\tan\left(2\theta\right)\right\} \qquad ---(i)$$

Here, $\quad -\infty < x < 0$

$\Rightarrow \quad a^{-\infty} < a^x < 2^\circ$

$\Rightarrow \quad 0 < \tan\theta < 1$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{4}$

$\Rightarrow \quad 0 < \left(2\theta\right) < \frac{\pi}{2}$

From equatoin (i),

$$y = 2\theta \qquad \left[\text{Since, } \tan^{-1}\left(\tan\theta\right) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$$

$$y = 2\tan^{-1}\left(a^x\right)$$

Differentiate it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{2}{1+\left(a^x\right)^2}\frac{d}{dx}\left(a^x\right)$$

$$\frac{dy}{dx} = \frac{2a^x \log a}{1+a^{2x}}.$$

Differentiation Ex 11.3 Q19

Let $y = \sin^{-1}\left\{\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right\}$

Put $x = \cos 2\theta$, so,

$$= \sin^{-1}\left\{\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{2}\right\}$$

$$= \sin^{-1}\left\{\frac{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}{2}\right\}$$

$$= \sin^{-1}\left\{\frac{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}{2}\right\}$$

$$= \sin^{-1}\left\{\cos\theta\left(\frac{1}{\sqrt{2}}\right)+\left(\frac{1}{\sqrt{2}}\right)\sin\theta\right\}$$

$$= \sin^{-1}\left\{\cos\theta\sin\left(\frac{\pi}{4}\right)+\cos\frac{\pi}{4}\sin\theta\right\}$$

$$y = \sin^{-1}\left\{\sin\left(\theta+\frac{\pi}{4}\right)\right\} \qquad \text{---(i)}$$

Here, $0 < x < 1$

$\Rightarrow \quad 0 < \cos 2\theta < 1$

$\Rightarrow \quad 0 < 2\theta < \frac{\pi}{2}$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{4}$

$\Rightarrow \quad \frac{\pi}{4} < \left(\theta+\frac{\pi}{4}\right) < \frac{\pi}{2}$

So, from equatoin (i),

$$y = \theta+\frac{\pi}{4} \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)\right]$$

$$y = \frac{1}{2}\cos^{-1}x+\frac{\pi}{4}$$

Differentiate it with respect to x,

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{-1}{\sqrt{1-x^2}}\right)+0$$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q20

Let $y = \tan^{-1}\left(\frac{\sqrt{1+a^2x^2}-1}{ax}\right)$

Put $ax = \tan\theta$

$$y = \tan^{-1}\left(\frac{\sqrt{1+a^2x^2}-1}{ax}\right)$$

$$= \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$y = \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$= \frac{\theta}{2}$$

$$y = \frac{1}{2}\tan^{-1}(ax)$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{2}\times\left(\frac{1}{1+(ax)^2}\right)\frac{d}{dx}(ax)$$

$$\frac{dy}{dx} = \frac{1}{2\left(1+a^2x^2\right)}(a)$$

$$\frac{dy}{dx} = \frac{a}{2\left(1+a^2x^2\right)}.$$

*********** END **********