Indefinite Integrals Ex 19.30 Q37

Let $\frac{x}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$

$$\Rightarrow \quad x = A(x^2+1) + (Bx+C)(x+1)$$

Equating similar terms, we get,

$A+B = 0,\ B+C = 1,\ \ A+C = 0$

Solving, we get, $A = -\frac{1}{2},\ \ B = \frac{1}{2},\ C = \frac{1}{2}$

Thus,

$$I = -\frac{1}{2}\int\frac{dx}{x+1} + \frac{1}{2}\int\frac{xdx}{x^2+1} + \frac{1}{2}\int\frac{dx}{x^2+1}$$

$$I = -\frac{1}{2}\log|x+1| + \frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1}x + c$$

Indefinite Integrals Ex 19.30 Q38

Let $I = \int \frac{dx}{1 + x + x^2 + x^3}$

$$\Rightarrow \qquad I = \int \frac{dx}{\left(x^2 + 1\right)\left(x + 1\right)}$$

Now,

Let $\frac{1}{\left(x^2 + 1\right)\left(x + 1\right)} = \frac{Ax + B}{x^2 + 1} + \frac{C}{x + 1}$

$$\Rightarrow \qquad 1 = \left(Ax + B\right)\left(x + 1\right) + C\left(x^2 + 1\right)$$

Equating similar terms, we get,

$A + C = 0,\ A + B = 0,\ B + C = 1$

Solving, we get, $A = -\frac{1}{2},\ B = \frac{1}{2},\ C = \frac{1}{2}$

Thus,

$$I = -\frac{1}{2}\int \frac{xdx}{x^2 + 1} + \frac{1}{2}\int \frac{dx}{x^2 + 1} + \frac{1}{2}\int \frac{dx}{x + 1}$$

$$I = -\frac{1}{4}\log\left|x^2 + 1\right| + \frac{1}{2}\tan^{-1} x + \frac{1}{2}\log\left|x + 1\right| + c$$

Indefinite Integrals Ex 19.30 Q39

Let $\frac{1}{(x + 1)^2\left(x^2 + 1\right)} = \frac{A}{x + 1} + \frac{B}{(x + 1)^2} + \frac{Cx + D}{x^2 + 1}$

$$\Rightarrow \qquad 1 = A(x + 1)\left(x^2 + 1\right) + B\left(x^2 + 1\right) + (Cx + D)(x + 1)^2$$
$$= (A + C)x^3 + (A + B + 2C + D)x^2 + (A + C + 2D)x + (A + B + D)$$

Equating similar terms, we get,

$A + C = 0,\ A + B + 2C + D = 0,\ A + C + 2D = 0,\ A + B + D = 1$

Solving, we get, $A = \frac{1}{2},\ B = \frac{1}{2},\ C = -\frac{1}{2},\ D = 0$

Thus,

$$I = \frac{1}{2}\int \frac{dx}{x + 1} + \frac{1}{2}\int \frac{dx}{(x + 1)^2} - \frac{1}{2}\int \frac{xdx}{x^2 + 1}$$

$$I = \frac{1}{2}\log|x + 1| - \frac{1}{2(x + 1)} - \frac{1}{4}\log\left|x^2 + 1\right| + c$$

Indefinite Integrals Ex 19.30 Q40

Let $I = \int \frac{2x}{x^3 - 1} dx = \int \frac{2x}{(x-1)(x^2+x+1)} dx$

Now,

Let $\frac{2x}{(x-1)(x^2+x+1)} = \frac{A}{(x-1)} + \frac{Bx+C}{x^2+x+1}$

$$\Rightarrow \quad 2x = A(x^2+x+1) + (Bx+C)(x-1)$$
$$= (A+B)x^2 + (A-B+C)x + (A-C)$$

Equating similar terms,

$A + B = 0,\ A - B + C = 2,\ \ A - C = 0,$

Solving, we get, $A = \frac{2}{3},\ \ B = -\frac{2}{3},\ C = \frac{2}{3}$

Thus,

$$I = \frac{2}{3}\int \frac{dx}{x-1} - \frac{2}{3}\int \frac{(x-1)dx}{x^2+x+1}$$

$$= \frac{2}{3}\int \frac{dx}{x-1} - \frac{2}{3}\cdot\frac{1}{2}\int \frac{(2x-2)dx}{x^2+x+1}$$

$$\Rightarrow \quad I = \frac{2}{3}\int \frac{dx}{x-1} - \frac{1}{3}\int \frac{2x+1}{x^2+x+1}dx + \int \frac{dx}{x^2+x+1}$$

$$= \frac{2}{3}\int \frac{dx}{x-1} - \frac{1}{3}\int \frac{2x+1}{x^2+x+1}dx + \int \frac{dx}{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{2}{3}\log|x-1| - \frac{1}{3}\log|x^2+x+1| + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c$$

Hence,

$$I = \frac{2}{3}\log|x-1| - \frac{1}{3}\log|x^2+x+1| + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + c$$

Indefinite Integrals Ex 19.30 Q41

Let $\dfrac{1}{(x^2+1)(x^2+4)} = \dfrac{Ax+B}{(x^2+1)} + \dfrac{Cx+D}{x^2+4}$

$$\Rightarrow \quad 1 = (Ax+B)(x^2+4) + (Cx+D)(x^2+1)$$
$$= (A+C)x^3 + (B+D)x^2 + (4A+C)x + 4B + D$$

Equating similar terms, we get,

$A + C = 0,\ B + D = 0,\ 4A + C = 0,\ \ 4B + D = 1$

Solving, we get, $A = 0,\ \ B = \dfrac{1}{3},\ C = 0,\ \ D = -\dfrac{1}{3}$

Thus,

$$I = \int \frac{\frac{1}{3}dx}{(x^2+1)} - \int \frac{\frac{1}{3}dx}{(x^2+4)}$$

$$= \frac{1}{3}\tan^{-1}x - \frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right) + c \qquad \left[\because \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}\right]$$

$$\therefore \qquad I = \frac{1}{3}\tan^{-1}x - \frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right) + c$$

********** END **********