Indefinite Integrals Ex 19.24 Q1

Let $\quad I = \int \frac{1}{1 - \cot x} dx$

$= \int \frac{1}{1 - \frac{\cos x}{\sin x}} dx$

$= \int \frac{\sin x}{\sin x - \cos x} dx$

Let $\quad \sin x = \lambda \frac{d}{dx}\left(\sin x - \cos x\right) + \mu\left(\sin x - \cos x\right) + \nu$

$\sin x = \lambda \frac{d}{dx}\left(\cos x + \sin x\right) + \mu\left(\sin x - \cos x\right) + \nu$

$\sin x = \cos\left(\lambda - \mu\right) + \sin x\left(\lambda + \mu\right) + \nu$

Comparing the coefficients of $\sin x$ & $\cos x$ on the both the sides,

$\lambda + \mu = 1 \qquad \ldots(1)$

$\lambda - \mu = 1 \qquad \ldots(2)$

$\nu = 0 \qquad \ldots(3)$

Equation $(1),(2),(3)$ gives

$\lambda = \frac{1}{2}, \mu = \frac{1}{2}, \nu = 0$

$I = \int \frac{\frac{1}{2}\left(\cos x + \sin x\right) + \frac{1}{2}\left(\sin x - \cos x\right)}{\left(\sin x - \cos x\right)} dx$

$= \frac{1}{2}\int \frac{\left(\cos x + \sin x\right)}{\left(\sin x - \cos x\right)} dx + \frac{1}{2}\int dx$

$I = \frac{1}{2}\log\left|\sin x - \cos x\right| + \frac{1}{2}x + c$

Indefinite Integrals Ex 19.24 Q2

Let $\quad I = \int \frac{1}{1-\tan x} dx$

$= \int \frac{1}{1-\frac{\sin x}{\cos x}} dx$

$= \int \frac{\cos x}{\cos x - \sin x} dx$

Let $\quad \cos x = \lambda \frac{d}{dx}(\cos x - \sin x) + \mu(\cos x - \sin x) + \nu$

$= \lambda \frac{d}{dx}(-\sin x - \cos x) + \mu(\cos x - \sin x) + \nu$

$\cos x = \sin x(-\lambda - \mu) + \cos x(-\lambda + \mu) + \nu$

Camparing the coefficients of $\cos x$ & $\sin x$ on the both the sides,

$-\lambda - \mu = 0 ------(1)$

$-\lambda + \mu = 1 ------(2)$

$\nu = 0 ----------(3)$

Equation $(1),(2),(3)$ gives

$\lambda = -\frac{1}{2}, \mu = \frac{1}{2}, \nu = 0$

$I = \int \frac{-\frac{1}{2}(-\sin x - \cos x) + \frac{1}{2}(\cos x - \sin x)}{(\cos x - \sin x)} dx$

$= \frac{1}{2}\int \frac{(\sin x + \cos x)}{(\cos x - \sin x)} dx + \frac{1}{2}\int dx$

$= -\frac{1}{2}\log|\cos x - \sin x| + \frac{1}{2}x + c$

$I = \frac{1}{2}x - \frac{1}{2}\log|\cos x - \sin x| + c$

Indefinite Integrals Ex 19.24 Q3

Let $I = \int \frac{3+2\cos x+4\sin x}{2\sin x+\cos x+3}dx$

Let $3+2\cos x+4\sin x = \lambda \frac{d}{dx}(2\sin x+\cos x+3)+\mu(2\sin x+\cos x+3)+\nu$

$3+2\cos x+4\sin x = \lambda(2\cos x-\sin x)+\mu(2\sin x+\cos x+3)+\nu$

$3+2\cos x+4\sin x = (-\lambda+2\mu)\sin x+(2\lambda+\mu)\cos x+3\mu+\nu$

Camparing the coefficients of $\sin x$ & $\cos x$ on the both the sides,

$-\lambda+2\mu = 4 ------(1)$

$2\lambda+\mu = 2 ------(2)$

$2\mu+\nu = 3 ---------(3)$

Solving equation $(1),(2)$ and $(3)$, we get

$\lambda = 0, \mu = 2, \nu = -3$

$I = \int \frac{2(2\sin x+\cos x+3)-3}{(2\sin x+\cos x+3)}dx$

$= 2\int dx - 3\int \frac{1}{2\sin x+\cos x+3}dx$

$I = 2x - 3I_1 + C_1 -------(4)$

Let $I_1 = \int \frac{1}{2\sin x+\cos x+3}dx$

Put $\sin x = \frac{2\tan\frac{x}{2}}{\left(1+\tan^2\frac{x}{2}\right)}, \quad \cos x = \frac{1-\tan^2\frac{x}{2}}{\left(1+\tan^2\frac{x}{2}\right)}$

$I_1 = \int \frac{1}{2\left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)+\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)+3}dx$

$= \int \frac{\left(1+\tan^2\frac{x}{2}\right)}{4\tan\frac{x}{2}+1-\tan^2\frac{x}{2}+3\left(1+\tan^2\frac{x}{2}\right)}dx$

$= \int \frac{\sec^2\frac{x}{2}}{2\tan^2\frac{x}{2}+4\tan\frac{x}{2}+4}dx$

Let $\tan\frac{x}{2} = t$

$\frac{1}{2}\sec^2\frac{x}{2} = dt$

$I_1 = \int \frac{2dt}{2t^2+4t+4}$

$= \frac{2}{2}\int \frac{dt}{t^2+2t+2}$

$= \int \frac{dt}{t^2+2t+1-1+2}$

$= \int \frac{dt}{(t+1)^2+1}$

$= \tan^{-1}(t+1)+C_2$

$= \tan^{-1}\left(\tan\frac{x}{2}+1\right)+C_2$

Now, using equation $(1)$,

$I = 2x - 3\tan^{-1}\left(\tan\frac{x}{2}+1\right)+C$

*********** END **********