

Polynomials Ex 2.1 Q11

Answer:

Since α and β are the zeros of the quadratic polynomial $f(x) = x^2 - px + q$

$$\alpha + \beta = \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$$
$$= \frac{-(-p)}{1}$$
$$= p$$

$$\alpha \beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$= \frac{q}{1}$$
$$= q$$

We have,

$$\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{\alpha^2 \times \alpha^2}{\beta^2 \times \alpha^2} + \frac{\beta^2 \times \beta^2}{\alpha^2 \times \beta^2}$$
$$\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{\alpha^4}{\beta^2 \alpha^2} + \frac{\beta^4}{\alpha^2 \beta^2}$$
$$\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{\alpha^4 + \beta^4}{\alpha^2 \beta^2}$$

$$\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2} = \frac{\left(\alpha^2 + \beta^2\right)^2 - 2\alpha^2\beta^2}{\alpha^2\beta^2}$$

$$\frac{\alpha^{2}}{\beta^{2}} + \frac{\beta^{2}}{\alpha^{2}} = \frac{\left[(\alpha + \beta)^{2} - 2\alpha\beta \right]^{2} - 2(\alpha\beta)^{2}}{(\alpha\beta)^{2}}$$

$$\frac{\alpha^{2}}{\beta^{2}} + \frac{\beta^{2}}{\alpha^{2}} = \frac{\left[(p)^{2} - 2q \right]^{2} - 2(q)^{2}}{q^{2}}$$

$$\frac{\alpha^{2}}{\beta^{2}} + \frac{\beta^{2}}{\alpha^{2}} = \frac{\left[p^{2} - 2q \right]^{2} - 2q^{2}}{q^{2}}$$

$$\frac{\alpha^{2}}{\beta^{2}} + \frac{\beta^{2}}{\alpha^{2}} = \frac{\left[p^{2} - 2q \right]^{2} - 2q^{2}}{q^{2}}$$

$$\frac{\alpha^{2}}{\beta^{2}} + \frac{\beta^{2}}{\alpha^{2}} = \frac{\left[p^{4} - 4p^{2}q + 4q^{2} \right] - 2q^{2}}{q^{2}}$$

$$\frac{\alpha^{2}}{\beta^{2}} + \frac{\beta^{2}}{\alpha^{2}} = \frac{p^{4} - 4p^{2}q + 4q^{2} - 2q^{2}}{q^{2}}$$

$$\frac{\alpha^{2}}{\beta^{2}} + \frac{\beta^{2}}{\alpha^{2}} = \frac{p^{4} - 4p^{2}q + 2q^{2}}{q^{2}}$$

$$\frac{\alpha^{2}}{\beta^{2}} + \frac{\beta^{2}}{\alpha^{2}} = \frac{p^{4}}{q^{2}} - \frac{4p^{2}q}{q^{2}} + \frac{2q^{2}}{q^{2}}$$

$$\frac{\alpha^{2}}{\beta^{2}} + \frac{\beta^{2}}{\alpha^{2}} = \frac{p^{4}}{q^{2}} - \frac{4p^{2}p}{q^{2}} + \frac{2p^{2}p}{q^{2}}$$

$$\frac{\alpha^{2}}{\beta^{2}} + \frac{\beta^{2}}{\alpha^{2}} = \frac{p^{4}}{q^{2}} - \frac{4p^{2}p}{q^{2}} + \frac{2p^{2}p}{q^{2}}$$

Hence, it is proved that $\frac{\alpha^2}{\beta^2} + \frac{\beta^2}{\alpha^2}$ is equal to $\left[\frac{p^4}{q^2} - \frac{4p^2}{q} + 2 \right]$.

******* END *******