Geometric Progressions Ex 20.5 Q 6

$$x^a = x^{\frac{b}{2}} z^{\frac{b}{2}} = z^c = \lambda \text{ (say)}$$

$$x = \lambda^{\frac{1}{a}},\ z = \lambda^{\frac{1}{c}}$$

$$x^{\frac{b}{2}} \times z^{\frac{b}{2}} = \lambda$$

$$\lambda^{\frac{1}{a}\left(\frac{b}{2}\right)} \times \lambda^{\frac{b}{2}\times\frac{1}{c}} = \lambda$$

$$\lambda^{\frac{b}{2a}+\frac{b}{2c}} = \lambda^1$$

$$\frac{b}{2a} + \frac{b}{2c} = 1$$

$$\frac{1}{a} + \frac{1}{c} = \frac{2}{b}$$

$$\Rightarrow \quad \frac{1}{a},\ \frac{1}{b},\ \frac{1}{c} \text{ are in A.P.}$$

Geometric Progressions Ex 20.5 Q 7

$k + 9,\ k - 6,\ 4$ are in G.P.

$(k-6)^2 = (k+9)4$

$k^2 + 36 - 12k = 4k + 36$

$k^2 - 16k = 0$

$k(k-16) = 0$

$k = 0,\ k = 16$

Geometric Progressions Ex 20.5 Q 8

Let $a-d,\ a,\ a+d$ be numbers in A.P.

Here,

$$a-d+a+a+d=15$$
$$3a=15$$
$$a=5$$

Find

$$\left[(5-d)+1\right],\ (5+3),\ \left[(5+d)+9\right] \text{ are in G.P.}$$

$\Rightarrow$ $(6-d),\ 8,\ (14+d)$ are in G.P.

$$(8)^2=(6-d)(14+d)$$
$$64=84+6d-14d-d^2$$
$$d^2+8d-20=0$$
$$(d+10)(d-2)=0$$
$$d=2,\ -10$$

So,

Numbers are $3,5,7$ or $15,5,-5$

Geometric Progressions Ex 20.5 Q 9

Let three numbers in A.P. be $a-d,\ a+d$

Here,

$$a-d+a+a+d=21$$
$$3a=21$$
$$a=7$$

And,

$(7-d),\ (7-1),\ (7+d)+1$ are in G.P.

$(7-d),\ 6,\ (8+d)$ are in G.P.

$$(6)^2=(7-d)(8+d)$$
$$36=56+7d-8d-d^2$$
$$d^2+d-20=0$$
$$(d+5)(d-4)=0$$
$$d=4,\ -5$$

So,

Numbers are $3,7,11$ or $12,7,2$.

Geometric Progressions Ex 20.5 Q 10

Here,

a,b,c are in A.P.

Let $a=A-d,\ b=A,\ c=A+d$

Here,

$$a+b+c=18$$
$$A-d+A+A+d=18$$
$$3A=18$$
$$A=6$$

And,

$(a+4),\ (b+4),\ (c+36)$ are in G.P.

$(6-d+4),\ (6+4),\ (6+d+36)$ are in G.P.

$(10-d),\ (10),\ (42+d)$ are in G.P.

$$(10)^2=(10-d)(42+d)$$
$$100=420+10d-42d-d^2$$
$$d^2+32d-320=0$$
$$(d+40)(d-8)=0$$
$$d=-40,\ 8$$

So,

Numbers of $-2,6,14$ or $46,6,-34$.

Geometric Progressions Ex 20.5 Q 11

Let numbers are a, ar, ar^2

$$a + ar + ar^2 = 56 \text{ - - - - - } (1)$$

$(a-1), (ar-7), (ar^2-21)$ are in AP

$$\Rightarrow \quad 2(ar-7) = a - 1 + ar^2 - 21$$

$$= (ar^2 + a) - 22$$

$$2ar - 14 = (56 - ar) - 22 \qquad \text{[using equation (1)]}$$

$$2ar - 14 = 34 - ar$$

$$3ar = 48$$

$$ar = 16 \text{ - - - - - - - } (2)$$

$$a = \frac{16}{r}$$

Put a in equation (1),

$$\frac{16 + 16r + 16r^2}{r} = 56$$

$$16 + 16r + 16r^2 = 56r$$

$$16r^2 - 40r + 16 = 0$$

$$2r^2 - 5r + 2 = 0$$

$$2r^2 - 4r - r + 2 = 0$$

$$2r(r-2) - 1(r-2) = 0$$

$$(r-2)(2r-1) = 0$$

$$r = 2, \ \frac{1}{2}$$

Put r in equation (2),

$$ar = 16$$

for $r = \frac{2}{a} = 8$

for $r = \frac{1}{2}$, $a = 32$

thus, there numbers are

8,16,32

in both cases.

********** END **********