



Areas of Parallelograms and Triangles Ex 15.3 Q17

Answer :

Given:

- (1) ABCD is a parallelogram.
- (2) E is a point on BA such that $BE = 2EA$
- (3) F is a point on DC such that $DF = 2FC$.

To find:

$$\text{Area of parallelogram AECF} = \frac{1}{3} \text{ Area of parallelogram ABCD}$$

Proof: We have,

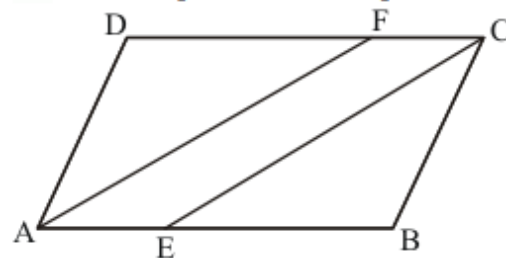
$$BE = 2EA \text{ and } DF = 2FC$$

$$\Rightarrow AB - AE = 2AE \text{ and } DC - FC = 2FC$$

$$\Rightarrow AB = 3AE \text{ and } DC = 3FC$$

$$\Rightarrow AE = \frac{1}{3} AB \text{ and } FC = \frac{1}{3} DC$$

$$\Rightarrow AE = FC \text{ [since } AB = DC]$$



Thus, $AE \parallel FC$ such that $AE = FC$

Therefore AECF is a parallelogram.

Clearly, parallelograms ABCD and AECF have the same altitude and

$$AE = \frac{1}{3} AB.$$

Therefore

$$\text{Area of a parallelogram AECF} = \frac{1}{3} \text{ Area of parallelogram ABCD}$$

Hence proved that $\text{Area of parallelogram AECF} = \frac{1}{3} \text{ Area of parallelogram ABCD}$

Areas of Parallelograms and Triangles Ex 15.3 Q18

Answer :

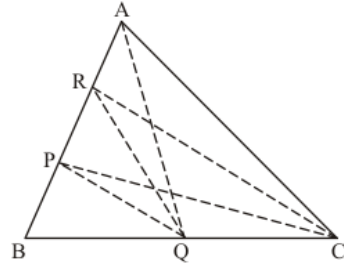
Given:

- (1) In a triangle ABC, P is the mid-point of AB.
- (2) Q is mid-point of BC.
- (3) R is mid-point of AP.

To prove:

- (a) Area of $\triangle PBQ$ = Area of $\triangle ARC$
- (b) Area of $\triangle PRQ = \frac{1}{2}$ Area of $\triangle ARC$
- (c) Area of $\triangle RQC = \frac{3}{8}$ Area of $\triangle ABC$

Proof: We know that each median of a triangle divides it into two triangles of equal area.



- (a) Since CR is a median of $\triangle CAP$

$$\text{Therefore } \text{ar}(\triangle CRA) = \frac{1}{2} \text{ar}(\triangle CAP) \dots\dots (1)$$

Also, CP is a median of $\triangle CAB$.

$$\text{Therefore } \text{ar}(\triangle CRA) = \frac{1}{2} \text{ar}(\triangle CAP) \dots\dots (2)$$

From equation (1) and (2), we get

$$\text{Therefore } \text{ar}(\triangle ARC) = \frac{1}{4} \text{ar}(\triangle CAB) \dots\dots (3)$$

PQ is a median of $\triangle ABQ$

$$\text{Therefore } \text{ar}(\triangle CPB) = 2\text{ar}(\triangle PBQ)$$

$$\text{Since } \text{ar}(\triangle CPB) = \frac{1}{2} \text{ar}(\triangle ABC)$$

Put this value in the above equation we get

$$\frac{1}{2} \text{ar}(\triangle ABC) = 2\text{ar}(\triangle PBQ)$$

$$\Rightarrow \text{ar}(\triangle PBQ) = \frac{1}{4} \text{ar}(\triangle ABC) \dots\dots (4)$$

From equation (3) and (4), we get

$$\text{Therefore } \text{ar}(\triangle ARC) = \text{ar}(\triangle PBQ) \dots\dots (5)$$

- (b) Since QP is a median of $\triangle QAB$ and QR medians of $\triangle QAP$

$$\text{Therefore } \text{ar}(\triangle QAP) = \text{ar}(\triangle QBP) \dots\dots (6)$$

$$\text{and, ar}(\Delta QAP) = 2 \text{ ar}(\Delta QRP) \dots\dots (7)$$

From equation (6) and (7)

$$\text{ar}(\Delta PRQ) = \frac{1}{2} \text{ar}(\Delta PBQ) \dots\dots (8)$$

From equation (7) and (8)

$$\boxed{\text{ar}(\Delta PRQ) = \frac{1}{2} \text{ar}(\Delta ARC)}$$

(c) Since CR is a median of ΔCAP .

$$\begin{aligned} \text{ar}(\Delta ARC) &= \frac{1}{2} \text{ar}(\Delta CAP) \\ \Rightarrow &= \frac{1}{2} \left\{ \frac{1}{2} \text{ar}(\Delta ABC) \right\} \quad [\text{Since CP is a median of DABC}] \\ &= \frac{1}{4} \text{ar}(\Delta ABC) \dots\dots (9) \end{aligned}$$

Since RQ is a median of ΔRBC .

$$\begin{aligned} \Rightarrow \text{ar}(\Delta RQC) &= \frac{1}{2} \text{ar}(\Delta RBC) \\ &= \frac{1}{2} \{ \text{ar}(\Delta ABC) - \text{ar}(\Delta ARC) \} \\ &= \frac{1}{2} \left\{ \text{ar}(\Delta ABC) - \frac{1}{4} \text{ar}(\Delta ABC) \right\} \quad [\text{from equation (9)}] \\ &\boxed{= \frac{3}{8} \text{ar}(\Delta ABC)} \end{aligned}$$

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