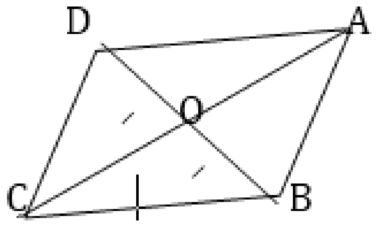
Exercise 9.3

Q6. In figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD. If AB = CD, then show that:

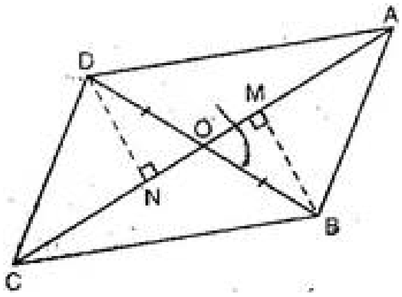


(i) ar (DOC) = ar (AOB)

(ii) ar (DCB) = ar (ACB)

(iii) DA $\parallel$ CB or ABCD is a parallelogram.

Ans. (i) Draw BM $\perp$ AC and DN $\perp$ AC.



In ΔDON and ΔBOM,

OD = OB [Given]

$\angle$DNO = $\angle$BMO = 90° [By construction]

$\angle$DON = $\angle$BOM [Vertically opposite]

$\therefore$ ΔDON $\cong$ ΔBOM [By RHS congruency]

$\Rightarrow$ DN = BM [By CPCT]

Also ar (ΔDON) = ar (ΔBOM) ....(i)

Again, In ΔDCN and ΔABM,

CD = AB [Given]

$\angle DNC = \angle BMA = 90^\circ$ [By construction]

DN = BM [Prove above]

$\therefore \Delta DCN \cong \Delta BAM$ [By RHS congruency]

$\therefore$ ar (ΔDCN) = ar (ΔBAM) ……….(ii)

Adding eq. (i) and (ii),

ar (ΔDON) + ar (ΔDCN) = ar (ΔBOM) + ar (ΔBAM)

$\Rightarrow$ ar (ΔDOC) = ar (ΔAOB)

(ii) Since ar (ΔDOC) = ar (ΔAOB)

Adding ar ΔBOC both sides,

ar (ΔDOC) + ar ΔBOC = ar (ΔAOB) + ar ΔBOC

$\Rightarrow$ ar (ΔDCB) = ar (ΔACB)

(iii) Since ar (ΔDCB) = ar (ΔACB)

Therefore, these two triangles in addition to be on the same base CB lie between two same parallels CB and DA.

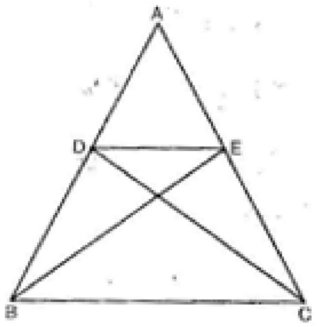
$\therefore$ DA $\parallel$ CB

Now AB = CD and DA $\parallel$ CB

Therefore, ABCD is a parallelogram.

Q7. D and E are points on sides AB and AC respectively of ΔABC such that ar (DBC) = ar (EBC). Prove that DE $\parallel$ BC.

Ans. Given: ar (ΔDBC) = ar (ΔEBC)

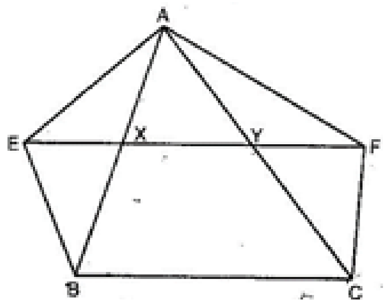


Since two triangles of equal area have common base BC.

Therefore, DE $\parallel$ BC [$\because$ Two triangles having same base (or equal bases) and equal areas lie between the same parallel]

Q8. XY is a line parallel to side BC of triangle ABC. If BE $\parallel$ AC and CF $\parallel$ AB meet XY at E and F respectively, show that ar (ABE) = ar (ACF).

Ans. ΔABE and parallelogram BCYE lie on the same base BE and between the same parallels BE and AC.



$\therefore$ ar (ΔABE) = $\frac{1}{2}$ ar ($\parallel$gm BCYE) ....(i)

Also ΔACF and $\parallel$gm BCFX lie on the same base CF and between same parallel BX and CF.

$\therefore$ ar (ΔACF) = $\frac{1}{2}$ ar (|| gm BCFX) ....(ii)

But || gm BCYE and || gm BCFX lie on the same base BC and between the same parallels BC and EF.

$\therefore$ ar (|| gm BCYE) = ar (|| gm BCFX) ....(iii)

From eq. (i), (ii) and (iii), we get,

ar (ΔABE) = ar (ΔACF)

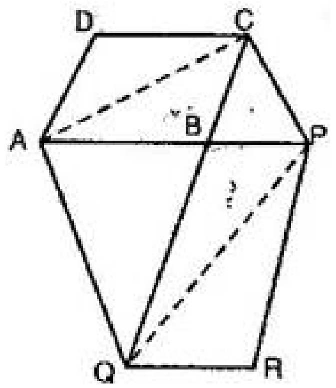
Q9. The side AB of parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed. Show that ar (ABCD) = ar (PBQR).

Ans. Given: ABCD is a parallelogram, CP || AQ and PBQR is a parallelogram.

To prove: ar (ABCD) = ar (PBQR)

Construction: Join AC and QP.

Proof: Since AQ || CP



$\therefore$ ar (ΔAQC) = ar (ΔAQP)

[Triangles on the same base and between the same parallels are equal in area]

Subtracting ar (ΔABQ) from both sides, we get

ar (ΔAQC – ar (ΔABQ) = ar (ΔAQP) – ar (Δ ABQ)

$\Rightarrow$ ar (ΔABC) = ar (ΔQBP) ....(i)

Now ar (ΔABC) = $\frac{1}{2}$ ar (|| gm ABCD)

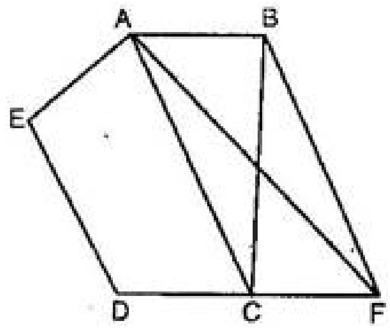
[Diagonal divides a parallelogram in two parts of equal area]

And ar (ΔPQB) = $\frac{1}{2}$ ar (|| gm PBQR)

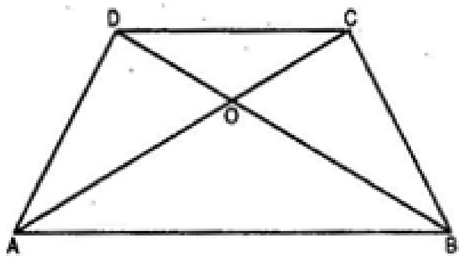
From eq. (i), (ii) and (iii), we get

ar (|| gm ABCD) = ar (|| gm PBQR)

Q10. Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O. Prove that ar(AOD) = ar (BOC).



Ans. ΔABD and ΔABC lie on the same base AB and between the same parallels AB and DC.



$\therefore$ ar (ΔABD) = ar (Δ ABC)

Subtracting ar (ΔAOB) from both sides,

ar (ΔABD) – ar (Δ AOB)

= ar (ΔABC) – ar (ΔAOB)

$\Rightarrow$ ar (ΔAOD) = ar (ΔBOC)

Q11. In figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that:

(i) ar (ACB = ar (ACF)

(ii) ar (AEDF) = ar (ABCDE)

Ans. (i) Given that BF|| AC

Δ ACB and Δ ACF lie on the same base AC and between the same parallels AC and BF.

$\therefore$ ar (ΔACB) = ar (ΔACF) ....(i)

(ii) Now ar (ABCDE) = ar (trap. AEDC) + ar (Δ ABC) ....(ii)

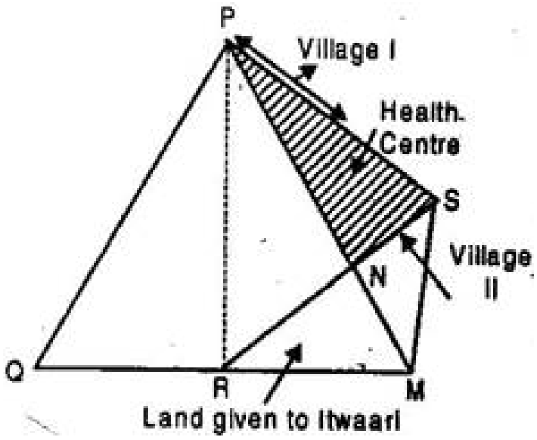
$\Rightarrow$ ar (ABCDE) = ar (trap. AEDC) + ar (Δ ACF) = ar (quad. AEDF) [Using (i)]

$\Rightarrow$ ar (AEDF) = ar (ABCDE)

Q12. A villager Itwaari has a plot of land of the shape of quadrilateral. The Gram Panchyat of two villages decided to take over some portion of his plot from one of the corners to construct a health centre. Itwaari agrees to the above personal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

Ans. Let Itwari has land in shape of quadrilateral PQRS.

Draw a line through 5 parallel to PR, which meets QR produced at M.



Let diagonals PM and RS of new formed quadrilateral intersect each other at point N.

We have PR|| SM [By construction]

$\therefore$ ar (Δ PRS) = ar (Δ PMR)

[Triangles on the same base and same parallel are equal in area]

Subtracting ar (Δ PNR) from both sides,

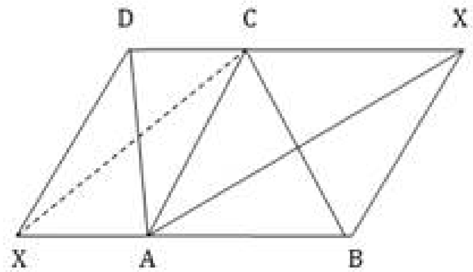
ar (Δ PRS) – ar (Δ PNR) = ar (Δ PMR) – ar (Δ PNR)

$\Rightarrow$ ar (Δ PSN) = ar (Δ MNR)

It implies that Itwari will give corner triangular shaped plot PSN to the Grampanchayat for health centre and will take equal amount of land (denoted by Δ MNR) adjoining his plot so as to form a triangular plot PQM.

Q13. ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y. Prove that ar (ADX) = ar (ACY).

Ans. Join CX, ΔADX and ACX lie on the same base XA and between the same parallels XA and DC.



$\therefore$ ar (Δ ADX) = ar (Δ ACX) ....(i)

Also Δ ACX and Δ ACY lie on the same base

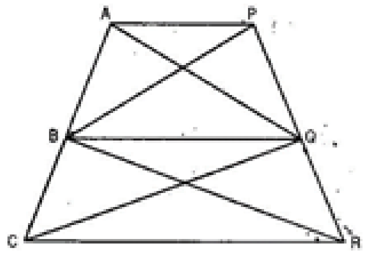
AC and between same parallels CY and XA.

$\therefore$ ar (Δ ACX) = ar (Δ ACY) ....(ii)

From (i) and (ii),

ar (Δ ADX) = ar (Δ ACY)

Q14. In figure, AP || BQ || CR. Prove that ar (AQC) = ar (PBR).



Ans. ΔABQ and BPQ lie on the same base BQ and between same parallels AP and BQ.

$\therefore$ ar (ΔABQ) = ar (ΔBPQ) ....(i)

ΔBQC and ΔBQR lie on the same base BQ and between same parallels BQ and CR.

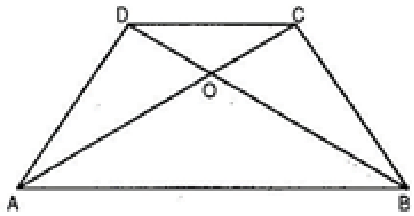
$\therefore$ ar (ΔBQC) = ar (ΔBQR) ....(ii)

Adding eq (i) and (ii), ar (ΔABQ) + ar (ΔBQC) = ar (ΔBPQ) + ar (ΔBQR)

$\Rightarrow$ ar (ΔAQC) = ar (ΔPBR)

15. Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that ar (AOD) = ar(BOC). Prove that ABCD is a trapezium.

Ans. Given that ar (ΔAOD) = ar (ΔBOC)



Adding ΔAOB both sides,

ar (ΔAOD) + ar (ΔAOB) = ar (ΔBOC) + ar (ΔAOB)

$\Rightarrow$ ar (ΔABD) = ar (ΔABC)

Since if two triangles equal in area, lie on the same base then, they lie between same parallels. We have ΔABD and ΔABC lie on common base AB and are equal in area.

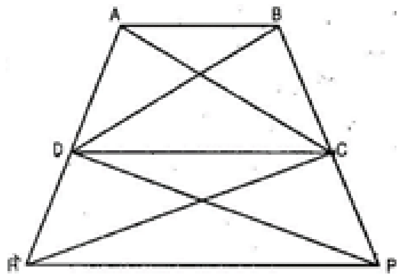
$\therefore$ They lie in same parallels AB and DC.

$\Rightarrow$ AB $\parallel$ DC

Now in quadrilateral ABCD, we have AB $\parallel$ DC

Therefore, ABCD is trapezium. [$\because$ In trapezium one pair of opposite sides is parallel]

Q16. In figure, ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC). Show that both the quadrilaterals ABCD and DCPR are trapeziums.



Ans. Given that ΔDRC and ΔDPC lie on the same base DC and ar (ΔDPC) = ar (ΔDRC) .....(i)

$\therefore$ DC $\parallel$ RP

[If two triangles equal in area, lie on the same base then, they lie between same parallels]

Therefore, DCPR is trapezium. [$\because$ In trapezium one pair of opposite sides is parallel]

Also ar (ΔBDP) = ar (ΔARC) ....(ii)

Subtracting eq. (i) from (ii),

ar (ΔBDP) – ar (ΔDPC) = ar (ΔARC) – ar (ΔDRC)

$\Rightarrow$ ar (ΔBDC) = ar (ΔADC)

Therefore, AB $\parallel$ DC [If two triangles equal in area, lie on the same base then, they lie between same parallels]

Therefore, ABCD is trapezium.

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