



= R.H.S.

Hence, the given result is proved.

Question 3:

Evaluate
$$\begin{vmatrix} \cos \alpha \cos \beta & \cos \alpha \sin \beta & -\sin \alpha \\ -\sin \beta & \cos \beta & 0 \\ \sin \alpha \cos \beta & \sin \alpha \sin \beta & \cos \alpha \end{vmatrix}$$

Answer

$$\Delta = \begin{vmatrix} \cos \alpha \cos \beta & \cos \alpha \sin \beta & -\sin \alpha \\ -\sin \beta & \cos \beta & 0 \\ \sin \alpha \cos \beta & \sin \alpha \sin \beta & \cos \alpha \end{vmatrix}$$

Expanding along C_3 , we have:

$$\begin{aligned} \Delta &= -\sin \alpha (-\sin \alpha \sin^2 \beta - \cos^2 \beta \sin \alpha) + \cos \alpha (\cos \alpha \cos^2 \beta + \cos \alpha \sin^2 \beta) \\ &= \sin^2 \alpha (\sin^2 \beta + \cos^2 \beta) + \cos^2 \alpha (\cos^2 \beta + \sin^2 \beta) \\ &= \sin^2 \alpha (1) + \cos^2 \alpha (1) \\ &= 1 \end{aligned}$$

Question 4:

$$\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$$

If a , b and c are real numbers, and

Show that either $a + b + c = 0$ or $a = b = c$.

Answer

$$\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$, we have:

$$\Delta = \begin{vmatrix} 2(a+b+c) & 2(a+b+c) & 2(a+b+c) \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

$$= 2(a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, we have:

$$\Delta = 2(a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ c+a & b-c & b-a \\ a+b & c-a & c-b \end{vmatrix}$$

Expanding along R_1 , we have:

$$\begin{aligned} \Delta &= 2(a+b+c)(1)[(b-c)(c-b) - (b-a)(c-a)] \\ &= 2(a+b+c)[-b^2 - c^2 + 2bc - bc + ba + ac - a^2] \\ &= 2(a+b+c)[ab + bc + ca - a^2 - b^2 - c^2] \end{aligned}$$

It is given that $\Delta = 0$.

$$(a+b+c)[ab + bc + ca - a^2 - b^2 - c^2] = 0$$

$$\Rightarrow \text{Either } a+b+c=0, \text{ or } ab+bc+ca-a^2-b^2-c^2=0.$$

Now,

$$ab + bc + ca - a^2 - b^2 - c^2 = 0$$

$$\Rightarrow -2ab - 2bc - 2ca + 2a^2 + 2b^2 + 2c^2 = 0$$

$$\Rightarrow (a-b)^2 + (b-c)^2 + (c-a)^2 = 0$$

$$\Rightarrow (a-b)^2 = (b-c)^2 = (c-a)^2 = 0$$

$$[(a-b)^2, (b-c)^2, (c-a)^2 \text{ are non-negative}]$$

$$\Rightarrow (a-b) = (b-c) = (c-a) = 0$$

$$\Rightarrow (a-b) = (b-c) = (c-a) = 0$$

$$\Rightarrow a = b = c$$

Hence, if $\Delta = 0$, then either $a + b + c = 0$ or $a = b = c$.

Question 5:

$$\text{Solve the equations } \begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0, a \neq 0$$

Answer

$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$, we get:

$$\begin{vmatrix} 3x+a & 3x+a & 3x+a \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

$$\Rightarrow (3x+a) \begin{vmatrix} 1 & 1 & 1 \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, we have:

$$(3x+a) \begin{vmatrix} 1 & 0 & 0 \\ x & a & 0 \\ x & 0 & a \end{vmatrix} = 0$$

Expanding along R_1 , we have:

$$(3x+a)[1 \times a^2] = 0$$

$$\Rightarrow a^2(3x+a) = 0$$

But $a \neq 0$.

Therefore, we have:

$$3x+a=0$$

$$\Rightarrow x = -\frac{a}{3}$$

Question 6:

$$\text{Prove that } \begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

Answer

$$\Delta = \begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$$

Taking out common factors a , b , and c from C_1 , C_2 , and C_3 , we have:

$$\Delta = abc \begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we have:

$$\Delta = abc \begin{vmatrix} a & c & a+c \\ b & b-c & -c \\ b-a & b & -a \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 + R_1$, we have:

$$\Delta = abc \begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b-a & b & -a \end{vmatrix}$$

Applying $R_3 \rightarrow R_3 + R_2$, we have:

$$\Delta = abc \begin{vmatrix} a & c & a+c \\ a+b & b & a \\ 2b & 2b & 0 \end{vmatrix}$$

$$= 2ab^2c \begin{vmatrix} a & c & a+c \\ a+b & b & a \\ 1 & 1 & 0 \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$, we have:

$$\Delta = 2ab^2c \begin{vmatrix} a & c-a & a+c \\ a+b & -a & a \\ 1 & 0 & 0 \end{vmatrix}$$

Expanding along R_3 , we have:

$$\begin{aligned}\Delta &= 2ab^2c[a(c-a) + a(a+c)] \\ &= 2ab^2c[ac - a^2 + a^2 + ac] \\ &= 2ab^2c(2ac) \\ &= 4a^2b^2c^2\end{aligned}$$

Hence, the given result is proved.

Question 8:

$$A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

Let \quad verify that

$$(i) [adj A]^{-1} = adj(A^{-1})$$

$$(ii) (A^{-1})^{-1} = A$$

Answer

$$A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

$$\therefore |A| = 1(15-1) + 2(-10-1) + 1(-2-3) = 14 - 22 - 5 = -13$$

$$\text{Now, } A_{11} = 14, A_{12} = 11, A_{13} = -5$$

$$A_{21} = 11, A_{22} = 4, A_{23} = -3$$

$$A_{31} = -5, A_{32} = -3, A_{33} = -1$$

$$\therefore adj A = \begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} (adj A)$$

$$= -\frac{1}{13} \begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix} = \frac{1}{13} \begin{bmatrix} -14 & -11 & 5 \\ -11 & -4 & 3 \\ 5 & 3 & 1 \end{bmatrix}$$

(i)

$$\begin{aligned}|adj A| &= 14(-4-9) - 11(-11-15) - 5(-33+20) \\ &= 14(-13) - 11(-26) - 5(-13) \\ &= -182 + 286 + 65 = 169\end{aligned}$$

We have,

***** END *****