Indefinite Integrals Ex 19.8 Q6

$$\frac{\cos 2x}{(\cos x+\sin x)^2}=\frac{\cos 2x}{\cos^2 x+\sin^2 x+2\sin x\cos x}=\frac{\cos 2x}{1+\sin 2x}$$

$$\therefore \int\frac{\cos 2x}{(\cos x+\sin x)^2}\,dx=\int\frac{\cos 2x}{(1+\sin 2x)}\,dx$$

Let $1+\sin 2x=t$

$\Rightarrow 2\cos 2x\,dx=dt$

$$\therefore \int\frac{\cos 2x}{(\cos x+\sin x)^2}dx=\frac{1}{2}\int\frac{1}{t}dt$$

$$=\frac{1}{2}\log|t|+C$$

$$=\frac{1}{2}\log|1+\sin 2x|+C$$

$$=\frac{1}{2}\log\left|(\sin x+\cos x)^2\right|+C$$

$$=\log|\sin x+\cos x|+C$$

Indefinite Integrals Ex 19.8 Q7

Let $I=\int\frac{\sin(x-a)}{\sin(x-b)}dx$ then

$$I=\int\frac{\sin(x-a+b-b)}{\sin(x-b)}dx$$

$$=\int\frac{\sin(x-b+b-a)}{\sin(x-b)}dx$$

$$=\int\frac{\sin(x-b)\cos(b-a)+\cos(x-b)\sin(b-a)}{\sin(x-b)}dx$$

$$=\int(\cos(b-a)+\cot(x-b)\sin(b-a))\,dx$$

$$=\cos(b-a)\int dx+\sin(b-a)\int\cot(x-b)\,dx$$

$$=x\cos(b-a)+\sin(b-a)\log|\sin(x-b)|+c$$

$$\therefore \quad I=x\cos(b-a)+\sin(b-a)\log|\sin(x-b)|+c$$

Indefinite Integrals Ex 19.8 Q8

Let $I = \int \frac{\sin(x-\alpha)}{\sin(x+\alpha)}dx$ then,

$$I = \int \frac{\sin(x-\alpha+\alpha-\alpha)}{\sin(x+\alpha)}dx$$
$$= \int \frac{\sin(x+\alpha-2\alpha)}{\sin(x+\alpha)}dx$$
$$= \int \frac{\sin(x+\alpha)\cos 2\alpha - \cos(x+\alpha)\sin 2\alpha}{\sin(x+\alpha)}dx$$
$$= \int \left[\frac{\sin(x+\alpha)\cos 2\alpha}{\sin(x+\alpha)} - \frac{\cos(x+\alpha)\sin 2\alpha}{\sin(x+\alpha)}\right]dx$$
$$= \int \left(\cos 2\alpha - \cot(x+\alpha)\sin 2\alpha\right)dx$$
$$= \cos 2\alpha \int dx - \sin 2\alpha \int \cot(x+\alpha)dx$$
$$= x\cos 2\alpha - \sin 2\alpha \log\left|\sin(x+\alpha)\right| + c$$

$\therefore \quad I = x\cos 2\alpha - \sin 2\alpha \log\left|\sin(x+\alpha)\right| + c$

Indefinite Integrals Ex 19.8 Q9

Let $I = \int \frac{1+\tan x}{1-\tan x}dx$

$$I = \int \frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}}dx$$
$$= \int \frac{\frac{\cos x+\sin x}{\cos x}}{\frac{\cos x-\sin x}{\cos x}}dx$$

$\Rightarrow \quad I = \int \frac{\cos x+\sin x}{\cos x-\sin x}dx$ ------ (i)

Let $\cos x - \sin x = t$ then,
$d(\cos x - \sin x) = dt$

$\Rightarrow \quad (-\sin x - \cos x)dx = dt$
$\Rightarrow \quad -(\sin x + \cos x)dx = dt$
$\Rightarrow \quad dx = -\frac{dt}{\sin x + \cos x}$

Putting $\cos x - \sin x = t$ and $dx = \frac{-dt}{\sin x+\cos x}$ in equation (i), we get

$$I = \int \frac{\cos x+\sin x}{t} \times \frac{-dt}{\sin x+\cos x}$$
$$= -\int \frac{dt}{t}$$
$$= -\log|t| + c$$
$$= -\log|\cos x - \sin x| + c$$

$\therefore \quad I = -\log|\cos x - \sin x| + c$

Indefinite Integrals Ex 19.8 Q10

Let $I = \int \frac{\cos x}{\cos(x-a)}\,dx$ then,

$$I = \int \frac{\cos(x+a-a)}{\cos(x-a)}\,dx$$

$$= \int \frac{\cos(x-a+a)}{\cos(x-a)}\,dx$$

$$= \int \frac{\cos(x-a)\cos a - \sin(x-a)\sin a}{\cos(x-a)}\,dx$$

$$= \int \frac{\cos(x-a)\cos a}{\cos(x-a)}\,dx - \int \frac{\sin(x-a)\sin a}{\cos(x-a)}\,dx$$

$$= \cos a \int dx - \sin a \int \tan(x-a)\,dx$$

$$= x\cos a - \sin a \log\left|\sec(x-a)\right| + c$$

********** END **********