Tangents and Normals Ex 16.2 Q3(xiv)

Differentiating $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2$ with respect to x, we get

$$\frac{2}{3}x^{\frac{-1}{3}} + \frac{2}{3}y^{\frac{-1}{3}}\frac{dy}{dx} = 0$$

$$\Rightarrow \qquad \frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$$

Therefore, the slope of the tangent at $(1,1)$ is $\left.\frac{dy}{dx}\right]_{(1,1)} = -1$

So, the equation of the tangent at $(1,1)$ is

$$y - 1 = -1(x - 1)$$

$$\Rightarrow \qquad y + x - 2 = 0$$

Also, the slope of the normal at $(1,1)$ is given by $\frac{-1}{\text{slope of tangent at } (1,1)} = 1$

$\therefore$ the equation of the normal at $(1,1)$ is

$$y - 1 = 1(x - 1)$$

$$\Rightarrow \qquad y - x = 0$$

Tangents and Normals Ex 16.2 Q3(xv)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad (A) \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad (B) \quad \text{Normal}$$

Where m is the slope

We have,

$$x^2 = 4y \qquad P = (2, 1)$$

$$\therefore \qquad 2x = \frac{4dy}{dx}$$

$$\Rightarrow \qquad \frac{dy}{dx} = \frac{x}{2}$$

$$\therefore \qquad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = 1$$

From (A)

Equation of tangent is

$$y - 1 = 1(x - 2)$$

$$\Rightarrow \qquad x - y = 1$$

From (B)

Equation of normal is

$$(y - 1) = -1(x - 2)$$

$$\Rightarrow \qquad x + y = 3$$

Tangents and Normals Ex 16.2 Q3(xvi)

The equation of the given curve is $y^2 = 4x$.

Differentiating with respect to x, we have:

$$2y\frac{dy}{dx} = 4$$
$$\Rightarrow \frac{dy}{dx} = \frac{4}{2y} = \frac{2}{y}$$
$$\therefore \left.\frac{dy}{dx}\right]_{(1,2)} = \frac{2}{2} = 1$$

Now, the slope at point (1, 2) is $\frac{-1}{\left.\frac{dy}{dx}\right]_{(1,2)}} = \frac{-1}{1} = -1$.

$\therefore$Equation of the tangent at (1, 2) is $y - 2 = -1(x - 1)$.

$\Rightarrow y - 2 = -x + 1$

$\Rightarrow x + y - 3 = 0$

Equation of the normal is ,

$$y - 2 = -(-1)(x - 1)$$
$$y - 2 = x - 1$$
$$x - y + 1 = 0$$

Tangents and Normals Ex 16.2 Q3(xix)

Let $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be the equation of the curve.

Rewriting the above equation as,

$$\frac{y^2}{b^2} = \frac{x^2}{a^2} - 1$$
$$\Rightarrow y^2 = \frac{b^2}{a^2}x^2 - b^2$$

Differentiating the above function w.r.t. x, we get,

$$2y\frac{dy}{dx} = \frac{b^2}{a^2}2x$$
$$\Rightarrow \frac{dy}{dx} = \frac{b^2}{a^2}\frac{x}{y}$$

$$\Rightarrow \left[\frac{dy}{dx}\right]_{(\sqrt{2}a,\, b)} = \frac{b^2}{a^2}\frac{\sqrt{2}a}{b} = \frac{\sqrt{2}b}{a}$$

Slope of the tangent $m = \frac{\sqrt{2}b}{a}$

Equation of the tangent is

$$(y - y_1) = m(x - x_1)$$
$$\Rightarrow (y - b) = \frac{\sqrt{2}b}{a}(x - \sqrt{2}a)$$
$$\Rightarrow a(y - b) = \sqrt{2}b(x - \sqrt{2}a)$$
$$\Rightarrow \sqrt{2}bx - ay + ab - 2ab = 0$$
$$\Rightarrow \sqrt{2}bx - ay - ab = 0$$

Slope of the normal is $-\frac{1}{\frac{\sqrt{2}b}{a}} = -\frac{a}{b\sqrt{2}}$

Equation of the normal is

$$(y - y_1) = m(x - x_1)$$
$$\Rightarrow (y - b) = \frac{-a}{\sqrt{2}b}(x - \sqrt{2}a)$$
$$\Rightarrow \sqrt{2}b(y - b) = -a(x - \sqrt{2}a)$$
$$\Rightarrow ax + \sqrt{2}by - \sqrt{2}b^2 - \sqrt{2}a^2 = 0$$
$$\Rightarrow ax + \sqrt{2}by - \sqrt{2}(a^2 + b^2) = 0$$

Tangents and Normals Ex 16.2 Q4

The given equations are,

$$x=\theta+\sin\theta \quad , \quad y=1+\cos\theta$$

$$\frac{dx}{d\theta}=1+\cos\theta \quad , \quad \frac{dy}{d\theta}=-\sin\theta$$

$$\therefore \quad \frac{dx}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{-\sin\theta}{1+\cos\theta}$$

Slope,

$$m=\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{4}}=\frac{-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}$$

$$=-1+\frac{1}{\sqrt{2}}$$

Thus, equation of tangent is,

$$y-y_1=m(x-x_1)$$

$$y-\left(1+\frac{1}{\sqrt{2}}\right)=\left(-1+\frac{1}{\sqrt{2}}\right)\left(x-\left(\frac{\pi}{4}+\frac{1}{\sqrt{2}}\right)\right)$$

$$y-1-\frac{1}{\sqrt{2}}=\left(-1+\frac{1}{\sqrt{2}}\right)\left(x-\frac{\pi}{4}-\frac{1}{\sqrt{2}}\right)$$

*********** END **********