Q2 : Differentiate the function with respect to *x*.

$$\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$$

Answer :

Let $y = \sqrt{\dfrac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$

Taking logarithm on both the sides, we obtain

$$\log y = \log \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$$

$$\Rightarrow \log y = \frac{1}{2}\log\left[\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}\right]$$

$$\Rightarrow \log y = \frac{1}{2}\left[\log\{(x-1)(x-2)\} - \log\{(x-3)(x-4)(x-5)\}\right]$$

$$\Rightarrow \log y = \frac{1}{2}\left[\log(x-1) + \log(x-2) - \log(x-3) - \log(x-4) - \log(x-5)\right]$$

Differentiating both sides with respect to *x*, we obtain

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\begin{array}{l}\dfrac{1}{x-1}\cdot\dfrac{d}{dx}(x-1) + \dfrac{1}{x-2}\cdot\dfrac{d}{dx}(x-2) - \dfrac{1}{x-3}\cdot\dfrac{d}{dx}(x-3) \\ -\dfrac{1}{x-4}\cdot\dfrac{d}{dx}(x-4) - \dfrac{1}{x-5}\cdot\dfrac{d}{dx}(x-5)\end{array}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{2}\left(\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right)$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}\left[\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right]$$

Answer needs Correction? **Click Here**

Q3 : Differentiate the function with respect to *x*.

$$(\log x)^{\cos x}$$

Answer :

Let $y = (\log x)^{\cos x}$

Taking logarithm on both the sides, we obtain

$$\log y = \cos x \cdot \log(\log x)$$

Differentiating both sides with respect to *x*, we obtain

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{d}{dx}(\cos x)\times\log(\log x) + \cos x\times\frac{d}{dx}\left[\log(\log x)\right]$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -\sin x\log(\log x) + \cos x\times\frac{1}{\log x}\cdot\frac{d}{dx}(\log x)$$

$$\Rightarrow \frac{dy}{dx} = y\left[-\sin x\log(\log x) + \frac{\cos x}{\log x}\times\frac{1}{x}\right]$$

$$\therefore \frac{dy}{dx} = (\log x)^{\cos x}\left[\frac{\cos x}{x\log x} - \sin x\log(\log x)\right]$$

Answer needs Correction? **Click Here**

Q4 : Differentiate the function with respect to *x*.

$$x^x - 2^{\sin x}$$

Answer :

Let $y = x^x - 2^{\sin x}$

Also, let $x^x = u$ and $2^{\sin x} = v$

$\therefore y = u - v$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$$

$u = x^x$

Taking logarithm on both the sides, we obtain

$$\log u = x\log x$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{u}\frac{du}{dx} = \left[\frac{d}{dx}(x) \times \log x + x \times \frac{d}{dx}(\log x)\right]$$

$$\Rightarrow \frac{du}{dx} = u\left[1 \times \log x + x \times \frac{1}{x}\right]$$

$$\Rightarrow \frac{du}{dx} = x^x(\log x + 1)$$

$$\Rightarrow \frac{du}{dx} = x^x(1 + \log x)$$

$v = 2^{\sin x}$

Taking logarithm on both the sides with respect to x, we obtain

$$\log v = \sin x \cdot \log 2$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v} \cdot \frac{dv}{dx} = \log 2 \cdot \frac{d}{dx}(\sin x)$$

$$\Rightarrow \frac{dv}{dx} = v \log 2 \cos x$$

$$\Rightarrow \frac{dv}{dx} = 2^{\sin x} \cos x \log 2$$

$$\therefore \frac{dy}{dx} = x^x(1 + \log x) - 2^{\sin x} \cos x \log 2$$

Answer needs Correction? **Click Here**

Q5 : Differentiate the function with respect to *x*.

$$(x+3)^2 \cdot (x+4)^3 \cdot (x+5)^4$$

Answer :

Let $y = (x+3)^2 \cdot (x+4)^3 \cdot (x+5)^4$

Taking logarithm on both the sides, we obtain

$$\log y = \log(x+3)^2 + \log(x+4)^3 + \log(x+5)^4$$

$$\Rightarrow \log y = 2\log(x+3) + 3\log(x+4) + 4\log(x+5)$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{y} \cdot \frac{dy}{dx} = 2 \cdot \frac{1}{x+3} \cdot \frac{d}{dx}(x+3) + 3 \cdot \frac{1}{x+4} \cdot \frac{d}{dx}(x+4) + 4 \cdot \frac{1}{x+5} \cdot \frac{d}{dx}(x+5)$$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5}\right]$$

$$\Rightarrow \frac{dy}{dx} = (x+3)^2(x+4)^3(x+5)^4 \cdot \left[\frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5}\right]$$

$$\Rightarrow \frac{dy}{dx} = (x+3)^2(x+4)^3(x+5)^4 \cdot \left[\frac{2(x+4)(x+5) + 3(x+3)(x+5) + 4(x+3)(x+4)}{(x+3)(x+4)(x+5)}\right]$$

$$\Rightarrow \frac{dy}{dx} = (x+3)(x+4)^2(x+5)^3 \cdot \left[2(x^2+9x+20) + 3(x^2+8x+15) + 4(x^2+7x+12)\right]$$

$$\therefore \frac{dy}{dx} = (x+3)(x+4)^2(x+5)^3(9x^2 + 70x + 133)$$

Answer needs Correction? **Click Here**

Q6 : Differentiate the function with respect to *x*.

$$(x+3)^2 \cdot (x+4)^3 \cdot (x+5)^4$$

Answer :

Let $y = (x+3)^2 \cdot (x+4)^3 \cdot (x+5)^4$

Taking logarithm on both the sides, we obtain

$$\log y = \log(x+3)^2 + \log(x+4)^3 + \log(x+5)^4$$

$$\Rightarrow \log y = 2\log(x+3) + 3\log(x+4) + 4\log(x+5)$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{y} \cdot \frac{dy}{dx} = 2 \cdot \frac{1}{x+3} \cdot \frac{d}{dx}(x+3) + 3 \cdot \frac{1}{x+4} \cdot \frac{d}{dx}(x+4) + 4 \cdot \frac{1}{x+5} \cdot \frac{d}{dx}(x+5)$$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5}\right]$$

$$\Rightarrow \frac{dy}{dx} = (x+3)^2(x+4)^3(x+5)^4 \cdot \left[\frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5}\right]$$

$$\Rightarrow \frac{dy}{dx} = (x+3)^2(x+4)^3(x+5)^4 \cdot \left[\frac{2(x+4)(x+5) + 3(x+3)(x+5) + 4(x+3)(x+4)}{(x+3)(x+4)(x+5)}\right]$$

$$\Rightarrow \frac{dy}{dx} = (x+3)(x+4)^2(x+5)^3 \cdot \left[2(x^2+9x+20) + 3(x^2+8x+15) + 4(x^2+7x+12)\right]$$

$$\therefore \frac{dy}{dx} = (x+3)(x+4)^2(x+5)^3(9x^2 + 70x + 133)$$

Answer needs Correction? **Click Here**

Q7 : Differentiate the function with respect to *x*.

$$(\log x)^x + x^{\log x}$$

Answer :

Let $y = (\log x)^x + x^{\log x}$

Also, let $u = (\log x)^x$ and $v = x^{\log x}$

$\therefore y = u + v$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad ...(1)$$

$u = (\log x)^x$

$$\Rightarrow \log u = \log\left[(\log x)^x\right]$$

$$\Rightarrow \log u = x \log(\log x)$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(x) \times \log(\log x) + x \cdot \frac{d}{dx}\left[\log(\log x)\right]$$

$$\Rightarrow \frac{du}{dx} = u\left[1 \times \log(\log x) + x \cdot \frac{1}{\log x} \cdot \frac{d}{dx}(\log x)\right]$$

$$\Rightarrow \frac{du}{dx} = (\log x)^x\left[\log(\log x) + \frac{x}{\log x} \cdot \frac{1}{x}\right]$$

$$\Rightarrow \frac{du}{dx} = (\log x)^x\left[\log(\log x) + \frac{1}{\log x}\right]$$

$$\Rightarrow \frac{du}{dx} = (\log x)^x\left[\frac{\log(\log x) \cdot \log x + 1}{\log x}\right]$$

$$\Rightarrow \frac{du}{dx} = (\log x)^{x-1}\left[1 + \log x. \log(\log x)\right] \qquad ...(2)$$

$v = x^{\log x}$

$$\Rightarrow \log v = \log\left(x^{\log x}\right)$$

$$\Rightarrow \log v = \log x \log x = (\log x)^2$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v} \cdot \frac{dv}{dx} = \frac{d}{dx}\left[(\log x)^2\right]$$

$$\Rightarrow \frac{1}{v} \cdot \frac{dv}{dx} = 2(\log x) \cdot \frac{d}{dx}(\log x)$$

$$\Rightarrow \frac{dv}{dx} = 2v(\log x) \cdot \frac{1}{x}$$

$$\Rightarrow \frac{dv}{dx} = 2x^{\log x}\frac{\log x}{x}$$

$$\Rightarrow \frac{dv}{dx} = 2x^{\log x-1} \cdot \log x \qquad ...(3)$$

Therefore, from (1), (2), and (3), we obtain

$$\frac{dy}{dx} = (\log x)^{x-1}\left[1 + \log x. \log(\log x)\right] + 2x^{\log x-1} \cdot \log x$$

Answer needs Correction? **Click Here**

Q8 : Differentiate the function with respect to *x*.

$$(\sin x)^x + \sin^{-1}\sqrt{x}$$

Answer :

Let $y = (\sin x)^x + \sin^{-1}\sqrt{x}$

Also, let $u = (\sin x)^x$ and $v = \sin^{-1}\sqrt{x}$

$\therefore y = u + v$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad ...(1)$$

$u = (\sin x)^x$

$$\Rightarrow \log u = \log(\sin x)^x$$

$$\Rightarrow \log u = x \log(\sin x)$$

Differentiating both sides with respect to x, we obtain

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(x) \times \log(\sin x) + x \times \frac{d}{dx}\left[\log(\sin x)\right]$$

$$\Rightarrow \frac{du}{dx} = u\left[1 \cdot \log(\sin x) + x \cdot \frac{1}{\sin x} \cdot \frac{d}{dx}(\sin x)\right]$$

$$\Rightarrow \frac{du}{dx} = (\sin x)^x\left[\log(\sin x) + \frac{x}{\sin x} \cdot \cos x\right]$$

$$\Rightarrow \frac{du}{dx} = (\sin x)^x(x \cot x + \log \sin x) \qquad ...(2)$$

$v = \sin^{-1}\sqrt{x}$

Differentiating both sides with respect to x, we obtain

$$\frac{dv}{dx} = \frac{1}{\sqrt{1 - (\sqrt{x})^2}} \cdot \frac{d}{dx}(\sqrt{x})$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}}$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{2\sqrt{x-x^2}} \qquad ...(3)$$

$$\frac{d}{dx}\; 2\sqrt{x-x^2}$$

Therefore, from (1), (2), and (3), we obtain

$$\frac{dy}{dx} = (\sin x)^x (x\cot x + \log \sin x) + \frac{1}{2\sqrt{x-x^2}}$$

Q9 : Differentiate the function with respect to x.

$$x^{\sin x} + (\sin x)^{\cos x}$$

Answer :

Let $y = x^{\sin x} + (\sin x)^{\cos x}$

Also, let $u = x^{\sin x}$ and $v = (\sin x)^{\cos x}$

$\therefore y = u + v$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad\qquad ...(1)$$

$u = x^{\sin x}$

$\Rightarrow \log u = \log\left(x^{\sin x}\right)$

$\Rightarrow \log u = \sin x \log x$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(\sin x)\cdot \log x + \sin x \cdot \frac{d}{dx}(\log x)$$

$$\Rightarrow \frac{du}{dx} = u\left[\cos x \log x + \sin x \cdot \frac{1}{x}\right]$$

$$\Rightarrow \frac{du}{dx} = x^{\sin x}\left[\cos x \log x + \frac{\sin x}{x}\right] \qquad\qquad ...(2)$$

$v = (\sin x)^{\cos x}$

$\Rightarrow \log v = \log(\sin x)^{\cos x}$

$\Rightarrow \log v = \cos x \log(\sin x)$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v}\frac{dv}{dx} = \frac{d}{dx}(\cos x)\times \log(\sin x) + \cos x \times \frac{d}{dx}\left[\log(\sin x)\right]$$

$$\Rightarrow \frac{dv}{dx} = v\left[-\sin x.\log(\sin x) + \cos x \cdot \frac{1}{\sin x}\cdot \frac{d}{dx}(\sin x)\right]$$

$$\Rightarrow \frac{dv}{dx} = (\sin x)^{\cos x}\left[-\sin x \log \sin x + \frac{\cos x}{\sin x}\cos x\right]$$

$$\Rightarrow \frac{dv}{dx} = (\sin x)^{\cos x}\left[-\sin x \log \sin x + \cot x \cos x\right]$$

$$\Rightarrow \frac{dv}{dx} = (\sin x)^{\cos x}\left[\cot x \cos x - \sin x \log \sin x\right] \qquad\qquad ...(3)$$

From (1), (2), and (3), we obtain

$$\frac{dy}{dx} = x^{\sin x}\left(\cos x \log x + \frac{\sin x}{x}\right) + (\sin x)^{\cos x}\left[\cos x \cot x - \sin x \log \sin x\right]$$

Q10 : Differentiate the function with respect to x.

$$x^{x\cos x} + \frac{x^2+1}{x^2-1}$$

Answer :

Let $y = x^{x\cos x} + \frac{x^2+1}{x^2-1}$

Also, let $u = x^{x\cos x}$ and $v = \frac{x^2+1}{x^2-1}$

$\therefore y = u + v$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad\qquad ...(1)$$

$u = x^{x\cos x}$

$\Rightarrow \log u = \log\left(x^{x\cos x}\right)$

$\Rightarrow \log u = x\cos x \log x$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(x)\cdot \cos x \cdot \log x + x \cdot \frac{d}{dx}(\cos x)\cdot \log x + x\cos x \cdot \frac{d}{dx}(\log x)$$

$$\Rightarrow \frac{du}{dx} = u\left[1\cdot \cos x \cdot \log x + x \cdot (-\sin x)\log x + x\cos x \cdot \frac{1}{x}\right]$$

$$\Rightarrow \frac{du}{dx} = x^{x\cos x}(\cos x \log x - x\sin x \log x + \cos x)$$

$$\Rightarrow \frac{du}{dx} = x^{x\cos x}\left[\cos x(1 + \log x) - x\sin x \log x\right] \qquad\qquad ...(2)$$

$v = \frac{x^2+1}{x^2-1}$

$\Rightarrow \log v = \log(x^2+1) - \log(x^2-1)$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v}\frac{dv}{dx}=\frac{2x}{x^2+1}-\frac{2x}{x^2-1}$$

$$\Rightarrow\frac{dv}{dx}=v\left[\frac{2x(x^2-1)-2x(x^2+1)}{(x^2+1)(x^2-1)}\right]$$

$$\Rightarrow\frac{dv}{dx}=\frac{x^2+1}{x^2-1}\times\left[\frac{-4x}{(x^2+1)(x^2-1)}\right]$$

$$\Rightarrow\frac{dv}{dx}=\frac{-4x}{(x^2-1)^2}\qquad\qquad...(3)$$

From (1), (2), and (3), we obtain

$$\frac{dy}{dx}=x^{x\cos x}\left[\cos x(1+\log x)-x\sin x\log x\right]-\frac{4x}{(x^2-1)^2}$$

Answer needs Correction? **Click Here**

Q11 : Differentiate the function with respect to *x*.

$$(x\cos x)^x+(x\sin x)^{\frac{1}{x}}$$

Answer :

Let $y=(x\cos x)^x+(x\sin x)^{\frac{1}{x}}$

Also, let $u=(x\cos x)^x$ and $v=(x\sin x)^{\frac{1}{x}}$

$\therefore y=u+v$

$$\Rightarrow\frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx}\qquad\qquad...(1)$$

$u=(x\cos x)^x$

$\Rightarrow\log u=\log(x\cos x)^x$

$\Rightarrow\log u=x\log(x\cos x)$

$\Rightarrow\log u=x\left[\log x+\log\cos x\right]$

$\Rightarrow\log u=x\log x+x\log\cos x$

Differentiating both sides with respect to *x*, we obtain

$$\frac{1}{u}\frac{du}{dx}=\frac{d}{dx}(x\log x)+\frac{d}{dx}(x\log\cos x)$$

$$\Rightarrow\frac{du}{dx}=u\left[\left\{\log x\cdot\frac{d}{dx}(x)+x\cdot\frac{d}{dx}(\log x)\right\}+\left\{\log\cos x\cdot\frac{d}{dx}(x)+x\cdot\frac{d}{dx}(\log\cos x)\right\}\right]$$

$$\Rightarrow\frac{du}{dx}=(x\cos x)^x\left[\left(\log x\cdot1+x\cdot\frac{1}{x}\right)+\left\{\log\cos x\cdot1+x\cdot\frac{1}{\cos x}\cdot\frac{d}{dx}(\cos x)\right\}\right]$$

$$\Rightarrow\frac{du}{dx}=(x\cos x)^x\left[(\log x+1)+\left\{\log\cos x+\frac{x}{\cos x}\cdot(-\sin x)\right\}\right]$$

$$\Rightarrow\frac{du}{dx}=(x\cos x)^x\left[(1+\log x)+(\log\cos x-x\tan x)\right]$$

$$\Rightarrow\frac{du}{dx}=(x\cos x)^x\left[1-x\tan x+(\log x+\log\cos x)\right]$$

$$\Rightarrow\frac{du}{dx}=(x\cos x)^x\left[1-x\tan x+\log(x\cos x)\right]\qquad\qquad...(2)$$

$v=(x\sin x)^{\frac{1}{x}}$

$\Rightarrow\log v=\log(x\sin x)^{\frac{1}{x}}$

$\Rightarrow\log v=\frac{1}{x}\log(x\sin x)$

$\Rightarrow\log v=\frac{1}{x}(\log x+\log\sin x)$

$\Rightarrow\log v=\frac{1}{x}\log x+\frac{1}{x}\log\sin x$

Differentiating both sides with respect to *x*, we obtain

$$\frac{1}{v}\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{x}\log x\right)+\frac{d}{dx}\left[\frac{1}{x}\log(\sin x)\right]$$

$$\Rightarrow\frac{1}{v}\frac{dv}{dx}=\left[\log x\cdot\frac{d}{dx}\left(\frac{1}{x}\right)+\frac{1}{x}\cdot\frac{d}{dx}(\log x)\right]+\left[\log(\sin x)\cdot\frac{d}{dx}\left(\frac{1}{x}\right)+\frac{1}{x}\cdot\frac{d}{dx}\{\log(\sin x)\}\right]$$

$$\Rightarrow\frac{1}{v}\frac{dv}{dx}=\left[\log x\cdot\left(-\frac{1}{x^2}\right)+\frac{1}{x}\cdot\frac{1}{x}\right]+\left[\log(\sin x)\cdot\left(-\frac{1}{x^2}\right)+\frac{1}{x}\cdot\frac{1}{\sin x}\cdot\frac{d}{dx}(\sin x)\right]$$

$$\Rightarrow\frac{1}{v}\frac{dv}{dx}=\frac{1}{x^2}(1-\log x)+\left[-\frac{\log(\sin x)}{x^2}+\frac{1}{x\sin x}\cdot\cos x\right]$$

$$\Rightarrow\frac{dv}{dx}=(x\sin x)^{\frac{1}{x}}\left[\frac{1-\log x}{x^2}+\frac{-\log(\sin x)+x\cot x}{x^2}\right]$$

$$\Rightarrow\frac{dv}{dx}=(x\sin x)^{\frac{1}{x}}\left[\frac{1-\log x-\log(\sin x)+x\cot x}{x^2}\right]$$

$$\Rightarrow\frac{dv}{dx}=(x\sin x)^{\frac{1}{x}}\left[\frac{1-\log(x\sin x)+x\cot x}{x^2}\right]\qquad\qquad...(3)$$

From (1), (2), and (3), we obtain

$$\frac{dy}{dx}=(x\cos x)^x\left[1-x\tan x+\log(x\cos x)\right]+(x\sin x)^{\frac{1}{x}}\left[\frac{x\cot x+1-\log(x\sin x)}{x^2}\right]$$

Answer needs Correction? **Click Here**

Q12 : Find $\dfrac{dy}{dx}$ of function.

$x^y + y^x = 1$

Answer :

The given function is $x^y + y^x = 1$

Let $x^y = u$ and $y^x = v$

Then, the function becomes $u + v = 1$

$\therefore \dfrac{du}{dx} + \dfrac{dv}{dx} = 0$ ...(1)

$u = x^y$

$\Rightarrow \log u = \log\left(x^y\right)$

$\Rightarrow \log u = y \log x$

Differentiating both sides with respect to x, we obtain

$\dfrac{1}{u}\dfrac{du}{dx} = \log x \dfrac{dy}{dx} + y \cdot \dfrac{d}{dx}(\log x)$

$\Rightarrow \dfrac{du}{dx} = u\left[\log x \dfrac{dy}{dx} + y \cdot \dfrac{1}{x}\right]$

$\Rightarrow \dfrac{du}{dx} = x^y\left(\log x \dfrac{dy}{dx} + \dfrac{y}{x}\right)$ ...(2)

$v = y^x$

$\Rightarrow \log v = \log\left(y^x\right)$

$\Rightarrow \log v = x \log y$

Differentiating both sides with respect to x, we obtain

$\dfrac{1}{v} \cdot \dfrac{dv}{dx} = \log y \cdot \dfrac{d}{dx}(x) + x \cdot \dfrac{d}{dx}(\log y)$

$\Rightarrow \dfrac{dv}{dx} = v\left(\log y \cdot 1 + x \cdot \dfrac{1}{y} \cdot \dfrac{dy}{dx}\right)$

$\Rightarrow \dfrac{dv}{dx} = y^x\left(\log y + \dfrac{x}{y}\dfrac{dy}{dx}\right)$ ...(3)

From (1), (2), and (3), we obtain

$x^y\left(\log x \dfrac{dy}{dx} + \dfrac{y}{x}\right) + y^x\left(\log y + \dfrac{x}{y}\dfrac{dy}{dx}\right) = 0$

$\Rightarrow \left(x^y \log x + xy^{x-1}\right)\dfrac{dy}{dx} = -\left(yx^{y-1} + y^x \log y\right)$

$\therefore \dfrac{dy}{dx} = -\dfrac{yx^{y-1} + y^x \log y}{x^y \log x + xy^{x-1}}$

Answer needs Correction? **Click Here**

Q13 : Find $\dfrac{dy}{dx}$ of function.

$y^x = x^y$

Answer :

The given function is $y^x = x^y$

Taking logarithm on both the sides, we obtain

$x \log y = y \log x$

Differentiating both sides with respect to x, we obtain

$\log y \cdot \dfrac{d}{dx}(x) + x \cdot \dfrac{d}{dx}(\log y) = \log x \cdot \dfrac{d}{dx}(y) + y \cdot \dfrac{d}{dx}(\log x)$

$\Rightarrow \log y \cdot 1 + x \cdot \dfrac{1}{y} \cdot \dfrac{dy}{dx} = \log x \cdot \dfrac{dy}{dx} + y \cdot \dfrac{1}{x}$

$\Rightarrow \log y + \dfrac{x}{y}\dfrac{dy}{dx} = \log x \dfrac{dy}{dx} + \dfrac{y}{x}$

$\Rightarrow \left(\dfrac{x}{y} - \log x\right)\dfrac{dy}{dx} = \dfrac{y}{x} - \log y$

$\Rightarrow \left(\dfrac{x - y\log x}{y}\right)\dfrac{dy}{dx} = \dfrac{y - x\log y}{x}$

$\therefore \dfrac{dy}{dx} = \dfrac{y}{x}\left(\dfrac{y - x\log y}{x - y\log x}\right)$

Answer needs Correction? **Click Here**

Q14 : Find $\dfrac{dy}{dx}$ of function.

$(\cos x)^y = (\cos y)^x$

Answer :

The given function is $(\cos x)^y = (\cos y)^x$

Taking logarithm on both the sides, we obtain

$y \log \cos x = x \log \cos y$

Differentiating both sides, we obtain

$\log \cos x \cdot \dfrac{dy}{dx} + y \cdot \dfrac{d}{dx}(\log \cos x) = \log \cos y \cdot \dfrac{d}{dx}(x) + x \cdot \dfrac{d}{dx}(\log \cos y)$

$\Rightarrow \log \cos x \dfrac{dy}{dx} + y \cdot \dfrac{1}{\cos x} \cdot \dfrac{d}{dx}(\cos x) = \log \cos y \cdot 1 + x \cdot \dfrac{1}{\cos y} \cdot \dfrac{d}{dx}(\cos y)$

$\Rightarrow \log \cos x \dfrac{dy}{dx} + \dfrac{y}{\cos x} \cdot (-\sin x) = \log \cos y + \dfrac{x}{\cos y}(-\sin y) \cdot \dfrac{dy}{dx}$

$\Rightarrow \log \cos x \dfrac{dy}{dx} - y \tan x = \log \cos y - x \tan y \dfrac{dy}{dx}$

$\Rightarrow (\log \cos x + x \tan y)\dfrac{dy}{dx} = y \tan x + \log \cos y$

$\therefore \dfrac{dy}{dx} = \dfrac{y \tan x + \log \cos y}{x \tan y + \log \cos x}$

Answer needs Correction? **Click Here**

Q15 : Find $\dfrac{dy}{dx}$ of function.

$xy = e^{(x-y)}$

Answer :

The given function is $xy = e^{(x-y)}$

Taking logarithm on both the sides, we obtain

$\log (xy) = \log\left(e^{x-y}\right)$

$\Rightarrow \log x + \log y = (x-y)\log e$

$\Rightarrow \log x + \log y = (x-y)\times 1$

$\Rightarrow \log x + \log y = x - y$

Differentiating both sides with respect to x, we obtain

$\dfrac{d}{dx}(\log x) + \dfrac{d}{dx}(\log y) = \dfrac{d}{dx}(x) - \dfrac{dy}{dx}$

$\Rightarrow \dfrac{1}{x} + \dfrac{1}{y}\dfrac{dy}{dx} = 1 - \dfrac{dy}{dx}$

$\Rightarrow \left(1 + \dfrac{1}{y}\right)\dfrac{dy}{dx} = 1 - \dfrac{1}{x}$

$\Rightarrow \left(\dfrac{y+1}{y}\right)\dfrac{dy}{dx} = \dfrac{x-1}{x}$

$\therefore \dfrac{dy}{dx} = \dfrac{y(x-1)}{x(y+1)}$

Answer needs Correction? **Click Here**

Q16 : Find the derivative of the function given by $f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$ and hence find $f'(1)$.

Answer :

The given relationship is $f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$

Taking logarithm on both the sides, we obtain

$\log f(x) = \log(1+x) + \log(1+x^2) + \log(1+x^4) + \log(1+x^8)$

Differentiating both sides with respect to x, we obtain

$\dfrac{1}{f(x)} \cdot \dfrac{d}{dx}[f(x)] = \dfrac{d}{dx}\log(1+x) + \dfrac{d}{dx}\log(1+x^2) + \dfrac{d}{dx}\log(1+x^4) + \dfrac{d}{dx}\log(1+x^8)$

$\Rightarrow \dfrac{1}{f(x)} \cdot f'(x) = \dfrac{1}{1+x} \cdot \dfrac{d}{dx}(1+x) + \dfrac{1}{1+x^2} \cdot \dfrac{d}{dx}(1+x^2) + \dfrac{1}{1+x^4} \cdot \dfrac{d}{dx}(1+x^4) + \dfrac{1}{1+x^8} \cdot \dfrac{d}{dx}(1+x^8)$

$\Rightarrow f'(x) = f(x)\left[\dfrac{1}{1+x} + \dfrac{1}{1+x^2} \cdot 2x + \dfrac{1}{1+x^4} \cdot 4x^3 + \dfrac{1}{1+x^8} \cdot 8x^7\right]$

$\therefore f'(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)\left[\dfrac{1}{1+x} + \dfrac{2x}{1+x^2} + \dfrac{4x^3}{1+x^4} + \dfrac{8x^7}{1+x^8}\right]$

Hence, $f'(1) = (1+1)(1+1^2)(1+1^4)(1+1^8)\left[\dfrac{1}{1+1} + \dfrac{2\times 1}{1+1^2} + \dfrac{4\times 1^3}{1+1^4} + \dfrac{8\times 1^7}{1+1^8}\right]$

$= 2\times 2\times 2\times 2\left[\dfrac{1}{2} + \dfrac{2}{2} + \dfrac{4}{2} + \dfrac{8}{2}\right]$

$= 16\times\left(\dfrac{1+2+4+8}{2}\right)$

$= 16\times\dfrac{15}{2} = 120$

Answer needs Correction? **Click Here**

Q17 : Differentiate $(x^5 - 5x + 8)(x^3 + 7x + 9)$ in three ways mentioned below

(i) By using product rule.

(ii) By expanding the product to obtain a single polynomial.

(iii By logarithmic differentiation.

Answer :

Let $y = (x^5 - 5x + 8)(x^3 + 7x + 9)$

(i)

Let $x^2 - 5x + 8 = u$ and $x^3 + 7x + 9 = v$

$\therefore y = uv$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{du}{dx} \cdot v + u \cdot \dfrac{dv}{dx}$ (By using product rule)

$\Rightarrow \dfrac{dy}{dx} = \dfrac{d}{dx}(x^2 - 5x + 8) \cdot (x^3 + 7x + 9) + (x^2 - 5x + 8) \cdot \dfrac{d}{dx}(x^3 + 7x + 9)$

$\Rightarrow \dfrac{dy}{dx} = (2x - 5)(x^3 + 7x + 9) + (x^2 - 5x + 8)(3x^2 + 7)$

$\Rightarrow \dfrac{dy}{dx} = 2x(x^3 + 7x + 9) - 5(x^3 + 7x + 9) + x^2(3x^2 + 7) - 5x(3x^2 + 7) + 8(3x^2 + 7)$

$\Rightarrow \dfrac{dy}{dx} = (2x^4 + 14x^2 + 18x) - 5x^3 - 35x - 45 + (3x^4 + 7x^2) - 15x^3 - 35x + 24x^2 + 56$

$\therefore \dfrac{dy}{dx} = 5x^4 - 20x^3 + 45x^2 - 52x + 11$

(ii)

$y = (x^2 - 5x + 8)(x^3 + 7x + 9)$

$= x^2(x^3 + 7x + 9) - 5x(x^3 + 7x + 9) + 8(x^3 + 7x + 9)$

$= x^5 + 7x^3 + 9x^2 - 5x^4 - 35x^2 - 45x + 8x^3 + 56x + 72$

$= x^5 - 5x^4 + 15x^3 - 26x^2 + 11x + 72$

$\therefore \dfrac{dy}{dx} = \dfrac{d}{dx}(x^5 - 5x^4 + 15x^3 - 26x^2 + 11x + 72)$

$\quad = \dfrac{d}{dx}(x^5) - 5\dfrac{d}{dx}(x^4) + 15\dfrac{d}{dx}(x^3) - 26\dfrac{d}{dx}(x^2) + 11\dfrac{d}{dx}(x) + \dfrac{d}{dx}(72)$

$\quad = 5x^4 - 5 \times 4x^3 + 15 \times 3x^2 - 26 \times 2x + 11 \times 1 + 0$

$\quad = 5x^4 - 20x^3 + 45x^2 - 52x + 11$

(iii) $y = (x^2 - 5x + 8)(x^3 + 7x + 9)$

Taking logarithm on both the sides, we obtain

$\log y = \log(x^2 - 5x + 8) + \log(x^3 + 7x + 9)$

Differentiating both sides with respect to x, we obtain

$\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{d}{dx}\log(x^2 - 5x + 8) + \dfrac{d}{dx}\log(x^3 + 7x + 9)$

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{1}{x^2 - 5x + 8} \cdot \dfrac{d}{dx}(x^2 - 5x + 8) + \dfrac{1}{x^3 + 7x + 9} \cdot \dfrac{d}{dx}(x^3 + 7x + 9)$

$\Rightarrow \dfrac{dy}{dx} = y\left[\dfrac{1}{x^2 - 5x + 8} \times (2x - 5) + \dfrac{1}{x^3 + 7x + 9} \times (3x^2 + 7)\right]$

$\Rightarrow \dfrac{dy}{dx} = (x^2 - 5x + 8)(x^3 + 7x + 9)\left[\dfrac{2x - 5}{x^2 - 5x + 8} + \dfrac{3x^2 + 7}{x^3 + 7x + 9}\right]$

$\Rightarrow \dfrac{dy}{dx} = (x^2 - 5x + 8)(x^3 + 7x + 9)\left[\dfrac{(2x - 5)(x^3 + 7x + 9) + (3x^2 + 7)(x^2 - 5x + 8)}{(x^2 - 5x + 8)(x^3 + 7x + 9)}\right]$

$\Rightarrow \dfrac{dy}{dx} = 2x(x^3 + 7x + 9) - 5(x^3 + 7x + 9) + 3x^2(x^2 - 5x + 8) + 7(x^2 - 5x + 8)$

$\Rightarrow \dfrac{dy}{dx} = (2x^4 + 14x^2 + 18x) - 5x^3 - 35x - 45 + (3x^4 - 15x^3 + 24x^2) + (7x^2 - 35x + 56)$

$\Rightarrow \dfrac{dy}{dx} = 5x^4 - 20x^3 + 45x^2 - 52x + 11$

From the above three observations, it can be concluded that all the results of $\dfrac{dy}{dx}$ are same.

Answer needs Correction? **Click Here**

Q18 : If u, v and w are functions of x, then show that

$$\dfrac{d}{dx}(u.v.w) = \dfrac{du}{dx}v.w + u.\dfrac{dv}{dx}.w + u.v.\dfrac{dw}{dx}$$

in two ways-first by repeated application of product rule, second by logarithmic differentiation.

Answer :

Let $y = u.v.w = u.(v.w)$

By applying product rule, we obtain

$\dfrac{dy}{dx} = \dfrac{du}{dx} \cdot (v.w) + u.\dfrac{d}{dx}(v \cdot w)$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{du}{dx}v \cdot w + u\left[\dfrac{dv}{dx} \cdot w + v \cdot \dfrac{dw}{dx}\right]$ (Again applying product rule)

$\Rightarrow \dfrac{dy}{dx} = \dfrac{du}{dx} \cdot v \cdot w + u \cdot \dfrac{dv}{dx} \cdot w + u \cdot v \cdot \dfrac{dw}{dx}$

By taking logarithm on both sides of the equation $y = u.v.w$, we obtain

$\log y = \log u + \log v + \log w$

Differentiating both sides with respect to x, we obtain

$\dfrac{1}{y} \cdot \dfrac{dy}{dx} = \dfrac{d}{dx}(\log u) + \dfrac{d}{dx}(\log v) + \dfrac{d}{dx}(\log w)$

$\Rightarrow \dfrac{1}{y} \cdot \dfrac{dy}{dx} = \dfrac{1}{u}\dfrac{du}{dx} + \dfrac{1}{v}\dfrac{dv}{dx} + \dfrac{1}{w}\dfrac{dw}{dx}$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{u}\cdot\frac{du}{dx} + \frac{1}{v}\cdot\frac{dv}{dx} + \frac{1}{w}\cdot\frac{dw}{dx}$$

$$\Rightarrow \frac{dy}{dx} = y\left(\frac{1}{u}\frac{du}{dx} + \frac{1}{v}\frac{dv}{dx} + \frac{1}{w}\frac{dw}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx} = u.v.w.\left(\frac{1}{u}\frac{du}{dx} + \frac{1}{v}\frac{dv}{dx} + \frac{1}{w}\frac{dw}{dx}\right)$$

$$\therefore \frac{dy}{dx} = \frac{du}{dx}\cdot v\cdot w + u\cdot\frac{dv}{dx}\cdot w + u\cdot v\cdot\frac{dw}{dx}$$

********** END **********