Definite Integrals Ex 20.3 Q3

We have,

$$\int_{-3}^{3} |x+1| dx$$

$$= \int_{-3}^{-1} -(x+1) dx + \int_{-1}^{3} (x+1) dx$$

$$= -\left[\frac{x^2}{2}+x\right]_{-3}^{-1} + \left[\frac{x^2}{2}+x\right]_{-1}^{3}$$

$$= -\left[\left(\frac{1}{2}-1\right)-\left(\frac{9}{2}-3\right)\right]+\left[\left(\frac{9}{2}+3\right)-\left(\frac{1}{2}-1\right)\right]$$

$$= -\left[\left(-\frac{1}{2}\right)-\left(1\frac{1}{2}\right)\right]+\left[\left(7\frac{1}{2}\right)-\left(-\frac{1}{2}\right)\right]$$

$$= -\left[-\frac{1}{2}-1\frac{1}{2}\right]+\left[7\frac{1}{2}+\frac{1}{2}\right]$$

$$= [-2]+[8]$$

$$= 2+8$$

$$= 10$$

$$\therefore \int_{-3}^{3} |x+1| dx = 10$$

Definite Integrals Ex 20.3 Q4

We have,

$$\int_{-1}^{1} |2x+1|\,dx$$

$$= \int_{-1}^{-\frac{1}{2}} -(2x+1)\,dx + \int_{-\frac{1}{2}}^{1} (2x+1)\,dx$$

$$= -\left[\frac{2x^2}{2}+x\right]_{-1}^{-\frac{1}{2}} + \left[\frac{2x^2}{2}+x\right]_{-\frac{1}{2}}^{1}$$

$$= -\left[\left(\frac{2}{8}-\frac{1}{2}\right)-\left(\frac{2}{2}-1\right)\right]+\left[\left(\frac{2}{2}+1\right)-\left(\frac{2}{8}-\frac{1}{2}\right)\right]$$

$$= -\left[\left(\frac{1}{4}-\frac{1}{2}\right)-(1-1)\right]+\left[(1+1)-\left(\frac{1}{4}-\frac{1}{2}\right)\right]$$

$$= -\left[-\frac{1}{4}\right]+\left[2+\frac{1}{4}\right]$$

$$= \frac{1}{4}+2+\frac{1}{4}$$

$$= 2\frac{1}{2}$$

$$\therefore \int_{-1}^{1} |2x+1|\,dx = \frac{5}{2}$$

Definite Integrals Ex 20.3 Q5

(i)

$$\int_{-2}^{2} |2x + 3| dx$$

$$= \int_{-2}^{-\frac{3}{2}} -(2x+3)\,dx + \int_{-\frac{3}{2}}^{2} (2x+3)\,dx$$

$$= -\left[\frac{2x^2}{2}+3x\right]_{-2}^{-\frac{3}{2}} + \left[\frac{2x^2}{2}+3x\right]_{-\frac{3}{2}}^{2}$$

$$= -\left[\left(\frac{2\times 9}{2\times 4}-\frac{9}{2}\right)-\left(\frac{2\times 4}{2}-6\right)\right]+\left[\left(\frac{2\times 4}{2}+6\right)-\left(\frac{2\times 9}{2\times 4}-\frac{9}{2}\right)\right]$$

$$= -\left[\left(\frac{18}{8}-\frac{9}{2}\right)-\left(\frac{8}{2}-6\right)\right]+\left[\left(\frac{8}{2}+6\right)-\left(\frac{18}{8}-\frac{9}{2}\right)\right]$$

$$= -\left[\left(\frac{9}{4}-\frac{9}{2}\right)-(-2)\right]+\left[(10)-\left(\frac{9}{4}-\frac{9}{2}\right)\right]$$

$$= \left[-\frac{9}{4}+2\right]+\left[10+\frac{9}{4}\right]$$

$$= \frac{9}{4}-2+10+\frac{9}{4}$$

$$\Rightarrow 8\,\frac{9}{2}$$

$$= 12\,\frac{1}{2}$$

$$\therefore \int_{-2}^{2} |2x+3|\,dx = \frac{25}{2}$$

Definite Integrals Ex 20.3 Q6

(ii)

We have,

$$f(x) = \left|x^2 - 3x + 2\right|$$

$$= \left|(x-1)(x-2)\right|$$

$$= \begin{cases} x^2 - 3x + 2 & 0 \le x \le 1 \\ -\left(x^2 - 3x + 2\right) & 1 \le x \le 2 \end{cases}$$

Hence,

$$\int_0^2 \left|x^2 - 3x + 2\right| dx$$

$$= \int_0^1 \left(x^2 - 3x + 2\right) dx + \int_1^2 -\left(x^2 - 3x + 2\right) dx$$

$$= \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_0^1 - \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_1^2$$

$$= \left[\frac{1}{3} - \frac{3}{2} + 2 - 0\right] - \left[\frac{8}{3} - \frac{12}{2} + 4 - \frac{1}{3} + \frac{3}{2} + 2\right]$$

$$= \left[\frac{1}{6}\right] - \left[-\frac{5}{6}\right]$$

$$= \frac{1}{6} + \frac{5}{6}$$

$$= 1$$

$$\therefore \int_0^2 \left|x^2 - 3x + 2\right| dx = 1$$

*********** END **********