Binomial Theorem Ex 18.1 Q1(ix)

Let $y = x+1$, then

$$\left(x+1-\frac{1}{x}\right)^3 = \left(y-\frac{1}{x}\right)^3$$

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $\left(y-\frac{1}{x}\right)^3$ has 4 terms.

Using binomial theorem to expand, we get

$$\left(y-\frac{1}{x}\right)^3 = {}^3C_0 y^3\left(\frac{1}{x}\right)^0 - {}^3C_1 y^2\left(\frac{1}{x}\right) + {}^3C_2 y\left(\frac{1}{x}\right)^2 - {}^3C_3 y^0\left(\frac{1}{x}\right)^3$$

$$= y^3 - 3y^2 \times \frac{1}{x} + 3y \times \frac{1}{x^2} - \frac{1}{x^3}$$

Putting $y = x+1$, we get

$$\left(x+1-\frac{1}{x}\right)^3 = (x+1)^3 - 3(x+1)^2 \times \frac{1}{x} + 3(x+1) \times \frac{1}{x^2} - \frac{1}{x^3}$$

$$= x^3 + 1 + 3x^2 + 3x - 3x - \frac{3}{x} - 6 + \frac{3}{x} + \frac{3}{x^2} - \frac{1}{x^3}$$

$$= x^3 + 3x^2 - 5 + \frac{3}{x^2} - \frac{1}{x^3}$$

Binomial Theorem Ex 18.1 Q1(x)

Let $y = 1-2x$, then

$$\left(1-2x+3x^2\right)^3 = \left(y+3x^2\right)^3$$

The expansion of $(x+y)^n$ has $n+1$ terms so the expansion of $\left(y+3x^2\right)^3$ has 4 terms.

Using binomial theorem to expand, we get

$$\left(y+3x^2\right)^3 = {}^3C_0 y^3\left(3x^2\right)^0 + {}^3C_1 y^2\left(3x^2\right)^1 + {}^3C_2 y\left(3x^2\right)^2 + {}^3C_3 y^0\left(3x^2\right)^3$$

$$= y^3 + 3y^2\left(3x^2\right) + 3y\left(9x^2\right) + \left(27x^6\right)$$

Substituting $y = 1-2x$, we get,

$$\left(1-2x+3x^2\right)^3 = (1-2x)^3 + 3\left(1+4x^2-4x\right)\left(3x^2\right) + 3(1-2x)\left(9x^2\right) + \left(27x^6\right)$$

$$= 1 - 8x^3 - 6x + 12x^2 + 9x^2 + 36x^4 - 36x^3 + 27x^2 - 54x^3 + 27x^6$$

$$= 1 - 6x + 21x^2 - 44x^3 + 63x^4 - 54x^5 + 27x^6$$

Binomial Theorem Ex 18.1 Q2(i)

$$\left(\sqrt{x+1}+\sqrt{x-1}\right)^6 + \left(\sqrt{x+1}-\sqrt{x-1}\right)^6$$

$$= {}^6C_0\left(\sqrt{x+1}\right)^6 + {}^6C_1\left(\sqrt{x+1}\right)^5\left(\sqrt{x-1}\right) + {}^6C_2\left(\sqrt{x+1}\right)^4\left(\sqrt{x-1}\right)^2 - {}^6C_3\left(\sqrt{x+1}\right)^3\left(\sqrt{x-1}\right)^3$$
$$+ {}^6C_4\left(\sqrt{x+1}\right)^2\left(\sqrt{x-1}\right)^4 + {}^6C_5\left(\sqrt{x+1}\right)\left(\sqrt{x-1}\right)^5 + {}^6C_6\left(\sqrt{x-1}\right)^6 + {}^6C_0\left(\sqrt{x+1}\right)^6 -$$
$${}^6C_1\left(\sqrt{x+1}\right)^5\left(\sqrt{x-1}\right) + {}^6C_2\left(\sqrt{x+1}\right)^4 \times \left(\sqrt{x-1}\right)^2 - {}^6C_3\left(\sqrt{x+1}\right)^3\left(\sqrt{x-1}\right)^3 +$$
$${}^6C_4\left(\sqrt{x+1}\right)^2\left(\sqrt{x-1}\right)^4 - {}^6C_5\left(\sqrt{x+1}\right)\left(\sqrt{x-1}\right)^5 + {}^6C_6\left(\sqrt{x-1}\right)^6$$

$$= 2\left[(x+1)^3 + 15(x+1)^2(x-1) + 15(x+1)(x-1)^2 + (x-1)^3\right]$$

$$= 2\begin{bmatrix} x^3+1+3x+3x^2+15x^3-15x^2+15x-15+30x^2-30x \\ +15x^3+15x^2+15x+15-30x^2-30x+x^3-1-3x^2+3x \end{bmatrix}$$

$$= 64x^3 - 48x$$
$$= 16x\left(4x^2-3\right)$$

Binomial Theorem Ex 18.1 Q2(ii)

$$\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6$$

$$=2\left[{}^6C_0x^6+{}^6C_2x^4\left(\sqrt{x^2-1}\right)^2+{}^6C_4x^2\left(\sqrt{x^2-1}\right)^4+{}^6C_6\left(\sqrt{x^2-1}\right)^6\right]$$

$$=2\left[x^6+15x^4\left(x^2-1\right)+15x^2\left(x^2-1\right)^2+\left(x^2-1\right)^3\right]$$

$$=2\left[x^6+15x^6-15x^4+15x^6+15x^2-30x^4+x^6-1-3x^4+3x^2\right]$$

$$=64x^6-96x^4+36x^2-2$$

*********** END **********