We draw AC perpendicular to x-axis.

∴ Area (OBAO) = Area (ΔOCA) – Area (OCABO) ... (1)

$$= \int_0^1 x\,dx - \int_0^1 x^2\,dx$$

$$= \left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1$$

$$= \frac{1}{2} - \frac{1}{3}$$

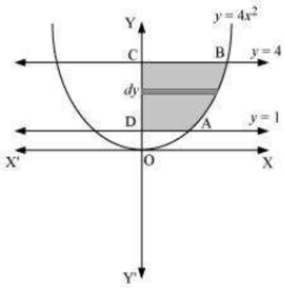
$$= \frac{1}{6} \text{ units}$$

Question 3:

Find the area of the region lying in the first quadrant and bounded by $y = 4x^2$, $x = 0$, $y = 1$ and $y = 4$

Answer

The area in the first quadrant bounded by $y = 4x^2$, $x = 0$, $y = 1$, and $y = 4$ is represented by the shaded area ABCDA as



$$\therefore \text{Area ABCD} = \int_1^4 x\,dx$$

$$= \int_1^4 \frac{\sqrt{y}}{2}\,dx$$

$$= \frac{1}{2}\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4$$

$$= \frac{1}{3}\left[(4)^{\frac{3}{2}} - 1\right]$$

$$= \frac{1}{3}[8-1]$$

$$= \frac{7}{3} \text{ units}$$

Question 4:

Sketch the graph of $y = |x+3|$ and evaluate $\int_{-6}^{0} |x+3|\,dx$

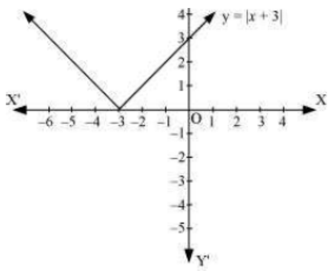
Answer

The given equation is $y = |x+3|$

The corresponding values of x and y are given in the following table.

x	– 6	– 5	– 4	– 3	– 2	– 1	0
y	3	2	1	0	1	2	3

On plotting these points, we obtain the graph of $y = |x+3|$ as follows.



It is known that, $(x+3)\le 0$ for $-6\le x\le -3$ and $(x+3)\ge 0$ for $-3\le x\le 0$

$$\therefore \int_{-6}^{0}|(x+3)|dx = -\int_{-6}^{-3}(x+3)dx + \int_{-3}^{0}(x+3)dx$$

$$= -\left[\frac{x^2}{2}+3x\right]_{-6}^{-3} + \left[\frac{x^2}{2}+3x\right]_{-3}^{0}$$

$$= -\left[\left(\frac{(-3)^2}{2}+3(-3)\right)-\left(\frac{(-6)^2}{2}+3(-6)\right)\right]+\left[0-\left(\frac{(-3)^2}{2}+3(-3)\right)\right]$$

$$= -\left[-\frac{9}{2}\right]-\left[-\frac{9}{2}\right]$$

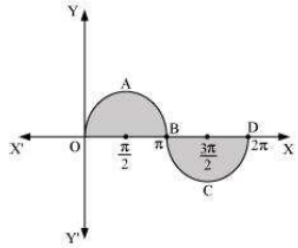
$$= 9$$

Question 5:

Find the area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$

Answer

The graph of $y = \sin x$ can be drawn as



∴ Required area = Area OABO + Area BCDB

$$= \int_0^{\pi} \sin x\, dx + \left|\int_{\pi}^{2\pi} \sin x\, dx\right|$$

$$= [-\cos x]_0^{\pi} + \left|[-\cos x]_{\pi}^{2\pi}\right|$$

$$= [-\cos\pi + \cos 0] + |-\cos 2\pi + \cos\pi|$$

$$= 1 + 1 + |(-1-1)|$$

$$= 2 + |-2|$$

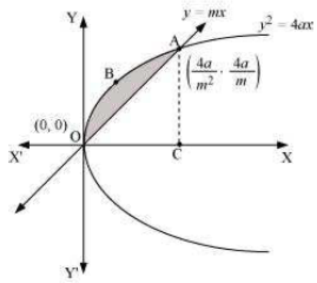
$$= 2 + 2 = 4 \text{ units}$$

Question 6:

Find the area enclosed between the parabola $y^2 = 4ax$ and the line $y = mx$

Answer

The area enclosed between the parabola, $y^2 = 4ax$, and the line, $y = mx$, is represented by the shaded area OABO as



The points of intersection of both the curves are (0, 0) and $\left(\frac{4a}{m^2}, \frac{4a}{m}\right)$.

We draw AC perpendicular to x-axis.

∴ Area OABO = Area OCABO − Area (ΔOCA)

$$= \int_0^{\frac{4a}{m^2}} 2\sqrt{ax}\,dx - \int_0^{\frac{4a}{m^2}} mx\,dx$$

$$= 2\sqrt{a}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{\frac{4a}{m^2}} - m\left[\frac{x^2}{2}\right]_0^{\frac{4a}{m^2}}$$

$$= \frac{4}{3}\sqrt{a}\left(\frac{4a}{m^2}\right)^{\frac{3}{2}} - \frac{m}{2}\left[\left(\frac{4a}{m^2}\right)^2\right]$$

$$= \frac{32a^2}{3m^3} - \frac{m}{2}\left(\frac{16a^2}{m^4}\right)$$

$$= \frac{32a^2}{3m^3} - \frac{8a^2}{m^3}$$

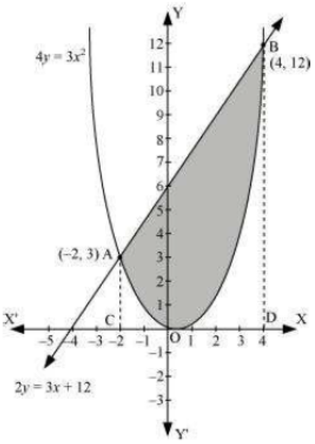
$$= \frac{8a^2}{3m^3} \text{ units}$$

Question 7:

Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$

Answer

The area enclosed between the parabola, $4y = 3x^2$, and the line, $2y = 3x + 12$, is represented by the shaded area OBAO as



The points of intersection of the given curves are A (−2, 3) and (4, 12).

We draw AC and BD perpendicular to x-axis.

∴ Area OBAO = Area CDBA − (Area ODBO + Area OACO)

$$= \int_{-2}^{4} \frac{1}{2}(3x+12)\,dx - \int_{-2}^{4} \frac{3x^2}{4}\,dx$$

$$= \frac{1}{2}\left[\frac{3x^2}{2}+12x\right]_{-2}^{4} - \frac{3}{4}\left[\frac{x^3}{3}\right]_{-2}^{4}$$

$$= \frac{1}{2}[24+48-6+24] - \frac{1}{4}[64+8]$$

$$= \frac{1}{2}[90] - \frac{1}{4}[72]$$

$$= 45 - 18$$

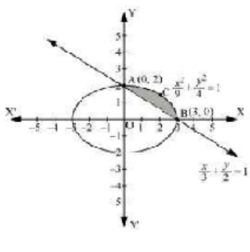
$$= 27 \text{ units}$$

Question 8:

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$

Answer

The area of the smaller region bounded by the ellipse, $\frac{x^2}{9}+\frac{y^2}{4}=1$, and the line, $\frac{x}{3}+\frac{y}{2}=1$, is represented by the shaded region BCAB as



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