Definite Integrals Ex 20.4A Q7

We know

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

Hence

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{1+e^{\tan x}}dx = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{1+e^{-\tan x}}dx$$

If

$$I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{1+e^{\tan x}}dx$$

Then

$$I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{1+e^{-\tan x}}dx$$

So

$$2I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{1+e^{\tan x}} + \frac{1}{1+e^{-\tan x}}dx$$

$$2I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{1+e^{\tan x}} + \frac{e^{\tan x}}{1+e^{\tan x}}dx$$

$$2I = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 1dx$$

$$2I = \frac{2\Pi}{3}$$

$$I = \frac{\Pi}{3}$$

Definite Integrals Ex 20.4A Q8

We know

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

Hence

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^x}dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2(-x)}{1+e^{-x}}dx$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^x}dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^{-x}}dx$$

If

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^x}dx$$

Then

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^{-x}}dx$$

So

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^x} + \frac{\cos^2 x}{1+e^{-x}}dx$$

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^x} + \frac{e^x\cos^2 x}{1+e^x}dx$$

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{(1+e^x)\cos^2 x}{1+e^x}dx$$

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 x dx$$

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1+\cos 2x}{2}dx$$

$$I = \frac{1}{4}\left\{x + \frac{\sin 2x}{2}\right\}_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$I = \frac{1}{4}\left\{\left(\frac{\Pi}{2}\right) - \left(-\frac{\Pi}{2}\right)\right\}$$

$$I = \frac{\Pi}{4}$$

Note: Answer given in the book is incorrect.

Definite Integrals Ex 20.4A Q9

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x^{11}-3x^9+5x^7-x^5+1}{\cos^2 x}dx$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x^{11}-3x^9+5x^7-x^5}{\cos^2 x}+\frac{1}{\cos^2 x}dx$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x^{11}-3x^9+5x^7-x^5}{\cos^2 x}dx+\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x dx$$

If f(x) is even

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$$

If f(x) is odd

$$\int_{-a}^{a} f(x)dx = 0$$

Here

$\dfrac{x^{11}-3x^9+5x^7-x^5}{\cos^2 x}$ is odd and

$\sec^2 x$ is even. Hence

$$0+2\int_{0}^{\frac{\pi}{4}} \sec^2 x dx$$

$$2\{\tan x\}_0^{\frac{\pi}{4}}$$

$$2$$

*********** END **********