



Indefinite Integrals Ex 19.25 Q29

$$\begin{aligned}\text{Let } I &= \int \operatorname{cosec}^3 x \, dx \\ &= \int \operatorname{cosec} x - \operatorname{cosec}^2 x \, dx\end{aligned}$$

Using integration by parts,

$$\begin{aligned}&= \operatorname{cosec} x \times \int \operatorname{cosec}^2 x \, dx + \int (\operatorname{cosec} x \cot x) \operatorname{cosec}^2 x \, dx \\ &= \operatorname{cosec} x \times (-\cot x) + \int \operatorname{cosec} x \cot x (-\cot x) \, dx \\ &= -\operatorname{cosec} x \cot x - \int \operatorname{cosec} x \cot^2 x \, dx \\ &= -\operatorname{cosec} x \cot x - \int \operatorname{cosec} x (\operatorname{cosec}^2 x - 1) \, dx \\ &= -\operatorname{cosec} x \cot x - \int \operatorname{cosec}^3 x \, dx + \int \operatorname{cosec} x \, dx \\ I &= -\operatorname{cosec} x \cot x - I + \log \left| \tan \frac{x}{2} \right| + c_1 \\ 2I &= -\operatorname{cosec} x \cot x + \log \left| \tan \frac{x}{2} \right| + c_1 \\ I &= -\frac{1}{2} \operatorname{cosec} x \cot x + \frac{1}{2} \log \left| \tan \frac{x}{2} \right| + c\end{aligned}$$

Indefinite Integrals Ex 19.25 Q30

$$\begin{aligned}\text{Let } I &= \int \sec^{-1} \sqrt{x} \, dx \\ \text{Let } \sqrt{x} &= t \\ x &= t^2 \\ dx &= 2t \, dt \\ I &= \int 2t \sec^{-1} t \, dt \\ &= 2 \left[\sec^{-1} t \int t \, dt - \int \left(\frac{1}{t\sqrt{t^2-1}} \int t \, dt \right) dt \right] \\ &= 2 \left[\frac{t^2}{2} \sec^{-1} t - \int \left(\frac{t}{2t\sqrt{t^2-1}} \right) dt \right] \\ &= t^2 \sec^{-1} t - \int \frac{t}{\sqrt{t^2-1}} \, dt \\ &= t^2 \sec^{-1} t - \frac{1}{2} \int \frac{2t}{\sqrt{t^2-1}} \, dt \\ &= t^2 \sec^{-1} t - \frac{1}{2} \times 2\sqrt{t^2-1} + c \\ I &= x \sec^{-1} \sqrt{x} - \sqrt{x-1} + c\end{aligned}$$

Indefinite Integrals Ex 19.25 Q31

$$\int \sin^{-1} \sqrt{x} \, dx =$$

$$\text{let } x = t^2 \rightarrow dx = 2t \, dt$$

$$\int \sin^{-1} \sqrt{x} \, dx = \int \sin^{-1} \sqrt{t^2} \, 2t \, dt = \int \sin^{-1} t \, 2t \, dt$$

$$= \sin^{-1} t \int 2t \, dt - \left(\int \frac{d \sin^{-1} t}{dt} \left(\int 2t \, dt \right) dt \right)$$

$$= \sin^{-1} t (t^2) - \int \frac{1}{\sqrt{1-t^2}} (t^2) \, dt$$

$$\text{Let's solve } \int \frac{1}{\sqrt{1-t^2}} (t^2) \, dt$$

$$\int \frac{1}{\sqrt{1-t^2}} (t^2) \, dt = \int \frac{t^2 - 1 + 1}{\sqrt{1-t^2}} \, dt = \int \frac{t^2 - 1}{\sqrt{1-t^2}} \, dt + \int \frac{1}{\sqrt{1-t^2}} \, dt$$

$$\text{we know that, value of } \int \frac{1}{\sqrt{1-t^2}} \, dt = \sin^{-1} t$$

$$\text{Remaining integral to evaluate is } \int \frac{t^2 - 1}{\sqrt{1-t^2}} \, dt = \int -\sqrt{1-t^2} \, dt$$

$$\text{sub } t = \sin u, \, dt = \cos u \, du$$

$$\int -\sqrt{1-t^2} \, dt = \int -\cos^2 u \, du = -\int \left[\frac{1 + \cos 2u}{2} \right] du$$

$$= -\frac{u}{2} - \frac{\sin 2u}{4}$$

$$\text{Substitute back } u = \sin^{-1} t \text{ and } t = \sqrt{x}$$

$$= -\frac{\sin^{-1} \sqrt{x}}{2} - \frac{\sin(2 \sin^{-1} \sqrt{x})}{4}$$

$$\int \sin^{-1} \sqrt{x} \, dx = x \sin^{-1} \sqrt{x} - \frac{\sin^{-1} \sqrt{x}}{2} - \frac{\sin(2 \sin^{-1} \sqrt{x})}{4}$$

$$\sin(2 \sin^{-1} \sqrt{x}) = 2\sqrt{x} \sqrt{1-x}$$

$$\int \sin^{-1} \sqrt{x} \, dx = x \sin^{-1} \sqrt{x} - \frac{\sin^{-1} \sqrt{x}}{2} - \frac{\sqrt{x(1-x)}}{2}$$

Indefinite Integrals Ex 19.25 Q32

$$\text{Let } I = \int x \tan^2 x \, dx$$

$$= \int x (\sec^2 x - 1) \, dx$$

$$= \int x \sec^2 x \, dx - \int x \, dx$$

$$= \left[x \int \sec^2 x \, dx - \int (1) \sec^2 x \, dx \right] - \frac{x^2}{2}$$

$$= x \tan x - \int \tan x \, dx - \frac{x^2}{2}$$

$$I = x \tan x - \log \sec x - \frac{x^2}{2} + c$$

Indefinite Integrals Ex 19.25 Q33

$$\begin{aligned}
\text{Let } I &= \int x \left(\frac{\sec 2x - 1}{\sec 2x + 1} \right) dx \\
&= \int x \left(\frac{1 - \cos 2x}{1 + \cos 2x} \right) dx \\
&= \int x \left(\frac{\sec^2 x}{\cos^2 x} \right) dx \\
&= \int x \tan^2 x dx \\
&= \int x (\sec^2 x - 1) dx \\
&= \int x \sec^2 x dx - \int dx \\
&= \left[x \int \sec^2 x dx - \int (1 \int \sec^2 x dx) dx \right] - \frac{x^2}{2} \\
&= x \tan x - \int \tan x dx - \frac{x^2}{2} \\
\\
I &= x \tan x - \log \sec x - \frac{x^2}{2} + c
\end{aligned}$$

Indefinite Integrals Ex 19.25 Q34

$$\text{Let } I = \int (x + 1) e^x \log(xe^x) dx$$

$$\text{Let } xe^x = t$$

$$(1 \times e^x + xe^x) dx = dt$$

$$(x + 1) e^x dx = dt$$

$$I = \int \log t dt$$

$$= \int 1 \times \log t dt$$

$$= \log t \int dt - \int \left(\frac{1}{t} \int dt \right) dt$$

$$= t \log t - \int \left(\frac{1}{t} t \right) dt$$

$$= t \log t - \int dt$$

$$= t \log t - t + c$$

$$= t (\log t - 1) + c$$

$$I = xe^x (\log xe^x - 1) + c$$

***** END *****