Co-Ordinate Geometry Ex 14.3 Q14

Answer :

The ratio in which the x–axis divides two points (x_1, y_1) and (x_2, y_2) is $-y_1 : y_2$

The ratio in which the y-axis divides two points (x_1, y_1) and (x_2, y_2) is $-x_1 : x_2$

The co-ordinates of the point dividing two points (x_1, y_1) and (x_2, y_2) in the ratio $m : n$ is given as,

$$(x, y) = \left(\left(\frac{\lambda x_2 + x_1}{\lambda + 1} \right), \left(\frac{\lambda y_2 + y_1}{\lambda + 1} \right) \right) \text{ Where } \lambda = \frac{m}{n}$$

Here the two given points are $A(-2,-3)$ and $B(5,6)$.

The ratio in which the x-axis divides these points is

$$-y_1 : y_2$$
$$-(-3) : 6$$
$$3 : 6$$
$$1 : 2$$

Let point $P(x, y)$ divide the line joining 'AB' in the ratio $1 : 2$

Substituting these values in the earlier mentioned formula we have,

$$(x, y) = \left(\left(\frac{\frac{1}{2}(5) + (-2)}{\frac{1}{2} + 1} \right), \left(\frac{\frac{1}{2}(6) + (-3)}{\frac{1}{2} + 1} \right) \right)$$

$$(x, y) = \left(\left(\frac{\frac{5 + 2(-2)}{2}}{\frac{1 + 2}{2}} \right), \left(\frac{\frac{6 + 2(-3)}{2}}{\frac{1 + 2}{2}} \right) \right)$$

$$(x, y) = \left(\left(\frac{1}{3} \right), \left(\frac{0}{3} \right) \right)$$

$$(x, y) = \left(\frac{1}{3}, 0 \right)$$

Thus the ratio in which the x–axis divides the two given points and the co-ordinates of the point is

$$\boxed{\begin{array}{l} 1:2 \\ \left(\frac{1}{3}, 0 \right) \end{array}}.$$

The ratio in which the y-axis divides these points is

$$-x_1 : x_2$$
$$-(-2) : 5$$
$$2 : 5$$

Let point $P(x, y)$ divide the line joining 'AB' in the ratio $2 : 5$

Substituting these values in the earlier mentioned formula we have,

$$(x, y) = \left(\left(\frac{\frac{2}{5}(5) + (-2)}{\frac{2}{5} + 1} \right), \left(\frac{\frac{2}{5}(6) + (-3)}{\frac{2}{5} + 1} \right) \right)$$

$$(x, y) = \left(\left(\frac{\frac{10 + 5(-2)}{5}}{\frac{2 + 5}{5}} \right), \left(\frac{\frac{12 + 5(-3)}{5}}{\frac{2 + 5}{5}} \right) \right)$$

$$(x, y) = \left(\left(\frac{0}{7} \right), \left(-\frac{3}{7} \right) \right)$$

$$(x, y) = \left(0, -\frac{3}{7} \right)$$

Thus the ratio in which the x-axis divides the two given points and the co-ordinates of the point is

$$\boxed{2:5 \text{ and } \left(0, -\frac{3}{7} \right)}.$$

Co-Ordinate Geometry Ex 14.3 Q15

Answer :

Let A (4, 5); B (7, 6); C (6, 3) and D (3, 2) be the vertices of a quadrilateral. We have to prove that the quadrilateral ABCD is a parallelogram.

We should proceed with the fact that if the diagonals of a quadrilateral bisect each other than the quadrilateral is a parallelogram.

Now to find the mid-point $\mathrm{P}(x, y)$ of two points $\mathrm{A}(x_1, y_1)$ and $\mathrm{B}(x_2, y_2)$ we use section formula as,

$$\mathrm{P}(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

So the mid-point of the diagonal AC is,

$$\mathrm{Q}(x, y) = \left(\frac{4+6}{2}, \frac{5+3}{2}\right)$$
$$= (5, 4)$$

Similarly mid-point of diagonal BD is,

$$R(x, y) = \left(\frac{7+3}{2}, \frac{6+2}{2}\right)$$
$$= (5, 4)$$

Therefore the mid-points of the diagonals are coinciding and thus diagonal bisects each other.

Hence ABCD is a parallelogram.

Now to check if ABCD is a rectangle, we should check the diagonal length.

$$\mathrm{AC} = \sqrt{(6-4)^2 + (3-5)^2}$$
$$= \sqrt{4+4}$$
$$= 2\sqrt{2}$$

Similarly,

$$\mathrm{BD} = \sqrt{(7-3)^2 + (6-2)^2}$$
$$= \sqrt{16+16}$$
$$= 4\sqrt{2}$$

Diagonals are of different lengths.

Hence ABCD is not a rectangle.

*********** END **********