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Question 4. 1. State, for each of the following physical quantities, if it is a scalar or a vector: volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.
Answer:
Scalars: Volume, mass, speed, density, number of moles, angular frequency.
Vectors: Acceleration, velocity, displacement, angular velocity.

Question 4. 2. Pick out the two scalar quantities in the following list: force, angular momentum, work, current, linear momentum, electric field, average velocity, magnetic moment, relative velocity.
Answer: Work and current are the scalar quantities in the, given list.

Question 4. 3. Pick out the only vector quantity in the following list: Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.
Answer: Impulse.

Question 4. 4. State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful:
(a) adding any two scalars,
(b) adding a scalar to a vector of the same dimensions,
(c) multiplying any vector by any scalar,
(d) multiplying any two scalars,
(e) adding any two vectors,
(f) adding a component of a vector to the same vector.
Answer:
(a) No, because only the scalars of same dimensions can be added.
(b) No, because a scalar cannot be added to a vector.
(c) Yes, multiplying a vector with a scalar gives the scalar (number) times the vector quantity which makes sense and one gets a bigger vector. For example, when acceleration A is multiplied by mass m, we get a force F = ml
(d) Yes, two scalars multiplied yield a meaningful result, for example multiplication of rise in temperature of water and its mass gives the amount of heat absorbed by that mass of water.
(e) No, because the two vectors of same dimensions can be added.
(f) Yes, because both are vectors of the same dimensions.

Question 4.5. Read each statement below carefully and state with reasons, if it is true or false:
(a) The magnitude of a vector is always a scalar.
(b) Each component of a vector is always a scalar.
(c) The total path length is always equal to the magnitude of the displacement vector of a particle.
(d) The average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time.
(e) Three vectors not lying in a plane can never add up to give a null vector.
Answer:

(a) True, magnitude of the velocity of a body moving in a straight line may be equal to the speed of the body.
(b) False, each component of a vector is always a vector, not scalar.
(c) False, total path length can also be more than the magnitude of displacement vector of a particle.
(d) True, because the total path length is either greater than or equal to the magnitude of the displacement vector.
(e) True, this is because the resultant of two vectors will not lie in the plane of third vector and hence cannot cancel its effect to give null vector.

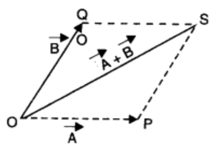
Question 4. 6. Establish the following inequalities geometrically or otherwise:

(a) $\left|\vec{A}+\vec{B}\right| \le \left|\vec{A}\right| + \left|\vec{B}\right|$ (b) $\left|\vec{A}+\vec{B}\right| \ge \left|\left|\vec{A}\right| - \left|\vec{B}\right|\right|$

(c) $\left|\vec{A}-\vec{B}\right| \le \left|\vec{A}\right| + \left|\vec{B}\right|$ (d) $\left|\vec{A}-\vec{B}\right| \ge \left|\left|\vec{A}\right| - \left|\vec{B}\right|\right|$

When does the equality sign above apply?
Answer:

Consider two vectors $\vec{A}$ and $\vec{B}$ be represented by the sides $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ of a parallelogram OPSQ. According to parallelogram law of vector addition; $(\vec{A}+\vec{B})$ will be represented by $\overrightarrow{OS}$ as shown in Fig. Thus



$$OP = \left|\vec{A}\right|, \quad OQ = PS = \left|\vec{B}\right|$$

and $$OS = \left|\vec{A}+\vec{B}\right|$$

(a) **To prove** $\left|\vec{A}+\vec{B}\right| \le \left|\vec{A}\right| + \left|\vec{B}\right|$

We know that the length of one side of a triangle is always less than the sum of the lengths of the other two sides. Hence from Δ OPS, we have

$$OS < OP + PS \quad \text{or} \quad OS < OP + OQ \quad \text{or} \quad \left|\vec{A}+\vec{B}\right| < \left|\vec{A}\right| + \left|\vec{B}\right| \qquad ...(i)$$

If the two vectors $\vec{A}$ and $\vec{B}$ are acting along the same straight line and in the same direction

then $$\left|\vec{A}+\vec{B}\right| = \left|\vec{A}\right| + \left|\vec{B}\right| \qquad ...(ii)$$

Combining the conditions mentioned in (i) and (ii) we have

$$\left|\vec{A}+\vec{B}\right| \le \left|\vec{A}\right| + \left|\vec{B}\right|$$

(b) **To prove** $\left|\vec{A}+\vec{B}\right| \ge \left|\left|\vec{A}\right| - \left|\vec{B}\right|\right|$

From Δ OPS, we have $OS + PS > OP$ or $OS > |OP - PS|$ or $OS > |OP - OQ|$

...(iii) ($\because PS = OQ$)

The modulus of $(OP - PS)$ has been taken because the L.H.S. is always positive but the R.H.S. may be negative if $OP < PS$. Thus from (iii) we have.

$$\left|\vec{A}+\vec{B}\right| > \left|\left|\vec{A}\right| - \left|\vec{B}\right|\right| \qquad ...(iv)$$

If the two vectors $\vec{A}$ and $\vec{B}$ are acting along a straight line in opposite directions, then

$$\left|\vec{A}+\vec{B}\right| = \left|\left|\vec{A}\right| - \left|\vec{B}\right|\right| \qquad ...(v)$$

Combining the conditions mentioned in (iv) and (v) we get.

$$\left|\vec{A}+\vec{B}\right| \ge \left|\left|\vec{A}\right| - \left|\vec{B}\right|\right|$$

(c) **To prove** $\left|\vec{A}-\vec{B}\right| \le \left|\vec{A}\right| + \left|\vec{B}\right|$

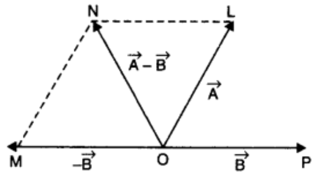
In fig. $\overrightarrow{OL}$ and $\overrightarrow{OM}$ represents vectors $\vec{A}$ and $\vec{B}$ respectively. Here $\overrightarrow{ON}$ represents $\vec{A}-\vec{B}$.

Consider the Δ OMN,

$$ON < MN + OM$$

or $$\left|\vec{A}-\vec{B}\right| < \left|\vec{A}\right| + \left|-\vec{B}\right|$$

or $\left|\vec{A}-\vec{B}\right| < \left|\vec{A}\right|+\left|\vec{B}\right|$ ...(*vi*)



When $\vec{A}$ and $\vec{B}$ are along the same straight line, but point in the opposite direction, then

$$\left|\vec{A}-\vec{B}\right| = \left|\vec{A}\right|+\left|\vec{B}\right| \quad ...(vii)$$

Combining equation (*vi*) and (*vii*), we get

$$\left|\vec{A}-\vec{B}\right| \le \left|\vec{A}\right|+\left|\vec{B}\right|$$

(*d*) **To prove** $\left|\vec{A}-\vec{B}\right| \ge \left|\left|\vec{A}\right|-\left|\vec{B}\right|\right|$

Let us consider the Δ OMN,

$$ON + OM > MN \quad \text{or} \quad ON > |MN - OM|$$

Since $MN = OL \quad \therefore \quad ON > |OL - OM|$

or $\left|\vec{A}-\vec{B}\right| > \left|\left|\vec{A}\right|-\left|\vec{B}\right|\right|$ ...(*viii*)

When $\vec{A}$ and $\vec{B}$ are along the same straight line and point in the same direction, then

$$\left|\vec{A}-\vec{B}\right| = \left|\vec{A}\right|-\left|\vec{B}\right| \quad ...(ix)$$

Combining equations (*viii*) and (*ix*), we get

$$\left|\vec{A}-\vec{B}\right| \ge \left|\left|\vec{A}\right|-\left|\vec{B}\right|\right|$$

Question 4. 7.

Given $\vec{a}+\vec{b}+\vec{c}+\vec{d} = \vec{0}$, which of the following statements are correct:

(a) $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ must each be a null vector,

(b) The magnitude of $(\vec{a}+\vec{c})$ equals the magnitude of $(\vec{b}+\vec{d})$.

(c) The magnitude of $\vec{a}$ can never be greater than the sum of the magnitudes of $\vec{b}, \vec{c}$ and $\vec{d}$.

(d) $\vec{b}+\vec{c}$ must lie in the plane of $\vec{a}$ and $\vec{d}$ if $\vec{a}$ and $\vec{d}$ are not collinear, and in the line of $\vec{a}$ and $\vec{d}$, if they are collinear?

Answer:

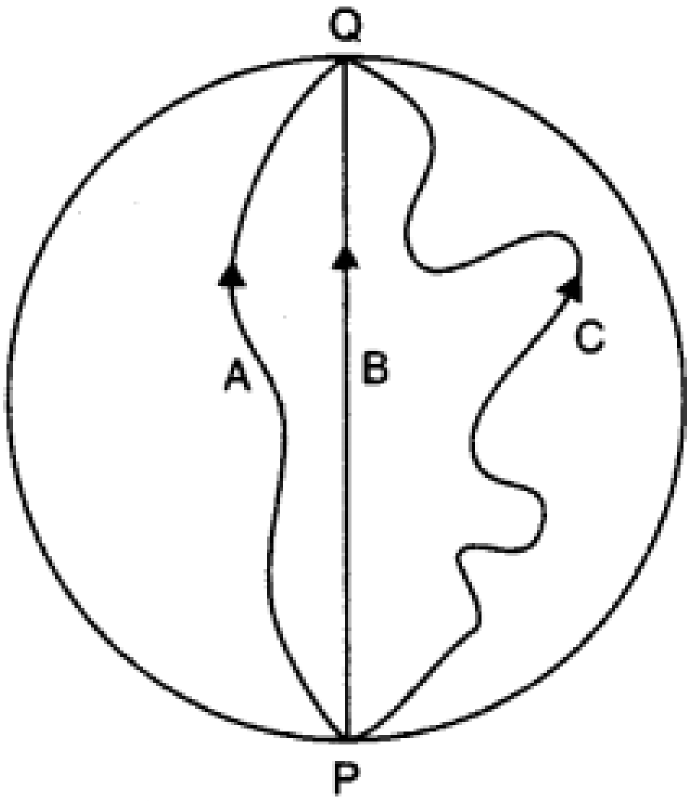
(*a*) This statement is not correct. Each need not be a null vector. Even when $\vec{a} = -\vec{b}$ and $\vec{c} = -\vec{d}$, they can form a null vector.

(*b*) This statement is correct. When $\left|\vec{a}+\vec{c}\right| = \left|\vec{b}+\vec{d}\right|$, the addition may be a null vector, if $\vec{a}+\vec{c} = -(\vec{b}+\vec{d})$ and are collinear.

(*c*) This statement is correct. Let $\left|\vec{a}\right| > \left|\vec{b}+\vec{c}+\vec{d}\right|$. If it is true the vector sum cannot be zero. Even if $\vec{b}, \vec{c}, \vec{d}$ form a triangle, the vector sum $\vec{b}+\vec{c}+\vec{d} = 0$ but then $\vec{a}+\vec{b}+\vec{c}+\vec{d}$ is not zero.

(*d*) This statement is correct. If $\vec{b}+\vec{c}$ do not lie in the plane of $\vec{a}+\vec{d}$, the vector sum $(\vec{a}+\vec{b})+(\vec{c}+\vec{d})$ is not zero because the addends will have different magnitude and different direction.

Question 4. 8. Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in Fig. What is the magnitude of the displacement vector for each? For which girl is this equal to the actual length of path skate?



Answer:

Displacement for each girl = $\overrightarrow{PQ}$.

$\therefore$ Magnitude of the displacement for each girl

$$= PQ = \text{diameter of circular ice ground}$$

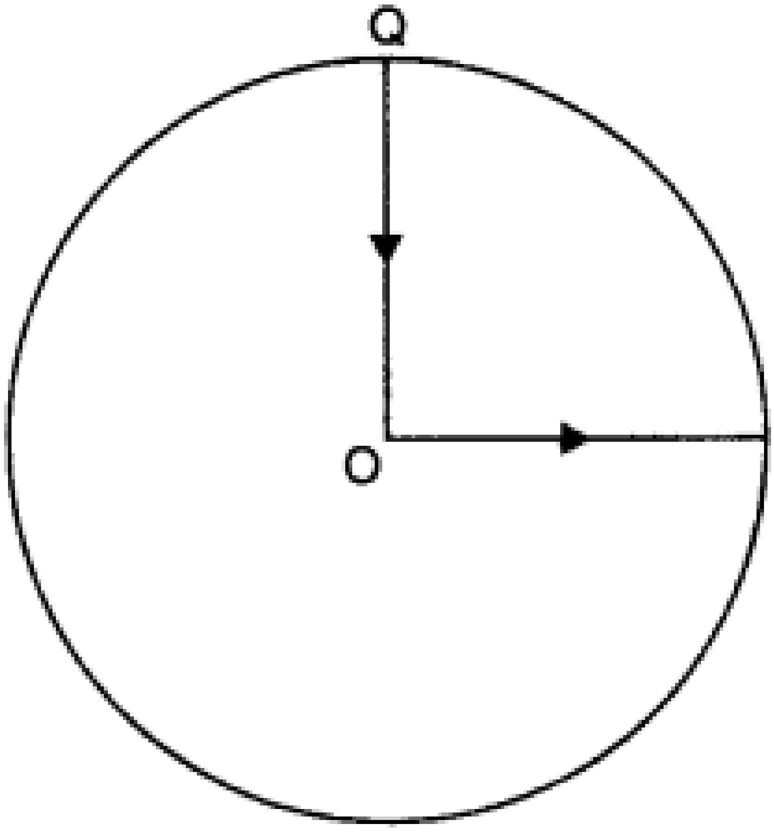
$$= 2 \times 200 = 400 \text{ m}.$$

Question 4. 9. A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference, and returns to the centre along QO as shown in Fig. If the round trip takes 10 min, what is the

(a) net displacement,

(b) average velocity, and

(c) average speed of the cyclist?



Answer: (a) Since both the initial and final positions are the same therefore the net displacement is zero.
(b) Average velocity is the ratio of net displacement and total time taken. Since the net displacement is zero therefore the average velocity is also zero.

(c) $$\text{Average speed} = \frac{\text{distance covered}}{\text{time taken}}$$

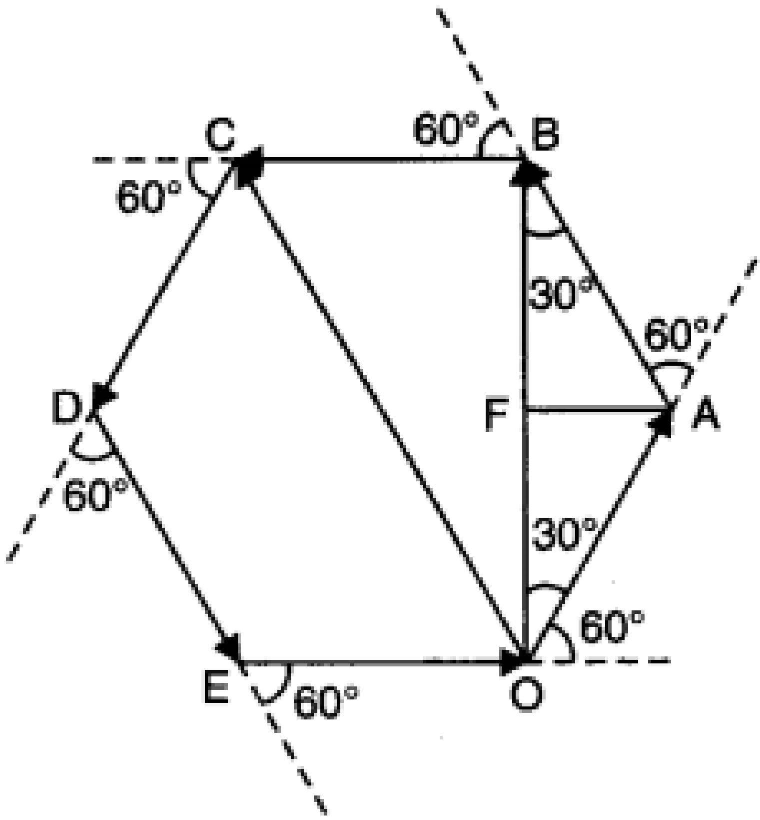
$$= \frac{OP + \text{Actual distance } PQ + QO}{10 \text{ minute}}$$

$$= \frac{1\text{ km} + \frac{1}{4} \times 2\pi \times 1\text{ km} + 1\text{ km}}{10/60\ h}$$

$$= 6\left(2 + \frac{22}{14}\right)\text{km h}^{-1} = 6 \times \frac{50}{14}\text{ km h}^{-1}$$

$$= 21.43 \text{ km h}^{-1}.$$

Question 4. 10. On an open ground, a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.



Answer: (i) The path followed by the motorist will be a closed hexagonal path.
Suppose the motorist starts his journey from the , point O. He takes the turn at the point C.

$$\text{Displacement} = \overrightarrow{OC}$$

Here $$OC = \sqrt{(OB)^2 + (BC)^2} = \sqrt{(OF + FB)^2 + (BC)^2}$$

$$= \sqrt{(500 \cos 30° + 500 \cos 30°)^2 + (500)^2}$$

$$= \sqrt{\left(2 \times 500 \times \frac{\sqrt{3}}{2}\right)^2 + (500)^2}$$

$$= 500\sqrt{4} = 1000 \text{ m} = 1 \text{ km}$$

Total path length = 500 m + 500 m + 500 m = 1500 m = 1.5 km

$$\frac{\text{Magnitude of displacement}}{\text{Total path length}} = \frac{1}{1.5} = \frac{2}{3} = 0.67$$

(*ii*) The motorist will take the sixth turn at O.
Displacement is zero. So, displacement vector is a null vector.
Path length is 3000 m, *i.e.,* 3 km.
Ratio of magnitude of displacement and path length is zero.

(*iii*) The motorist will take the 8th turn at *B*.

Magnitude of displacement = $2 \times 500 \cos 30° = 500\sqrt{3}\text{m} = \frac{\sqrt{3}}{2}$ km = 0.866 km

Path length = 8 × 500 m = 4 km

Ratio of magnitude of displacement and path length is $\frac{\sqrt{3}/2}{4}$ *i.e.,* $\frac{\sqrt{3}}{8} = 0.22$

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