NCERT Solutions For Class 10 Chapter 8 Introduction to Trigonometry Exercise 8.4

Q1. Express the trigonometric ratios $\sin A$, $\sec A$ and $\tan A$ in terms of $\cot A$

Ans: For $\sin A$,

By using identity $\operatorname{cosec}^2 A - \cot^2 A = 1$

$$\Rightarrow \operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\Rightarrow \frac{1}{\sin^2 A} = 1 + \cot^2 A$$

$$\Rightarrow \sin^2 A = \frac{1}{1 + \cot^2 A}$$

$$\Rightarrow \sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$$

For $\sec A$,

By using identity $\sec^2 A - \tan^2 A = 1$

$$\Rightarrow \sec^2 A = 1 + \tan^2 A$$

$$\Rightarrow \sec^2 A = 1 + \frac{1}{\cot^2 A} = \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\Rightarrow \sec^2 A = \frac{1 + \cot^2 A}{\cot^2 A}$$

$$\Rightarrow \sec A = \frac{\sqrt{1 + \cot^2 A}}{\cot A}$$

For $\tan A$,

$$\tan A = \frac{1}{\cot A}$$

Q2. Write the other trigonometric ratios of A in terms of $\sec A$

Ans: For $\sin A$,

By using identity, $\sin^2 A + \cos^2 A = 1$

$\Rightarrow \sin^2 A = 1 - \cos^2 A$

$\Rightarrow \sin^2 A = 1 - \dfrac{1}{\sec^2 A} = \dfrac{\sec^2 A - 1}{\sec^2 A}$

$\Rightarrow \sin A = \dfrac{\sqrt{\sec^2 A - 1}}{\sec A}$

For $\cos A$,

$= \sin 25°.\sin 25° + \cos 25°.\cos 25°$

$\left[\because \sin\left(90^\circ - \theta\right) = \cos\theta, \cos\left(90^\circ - \theta\right) = \sin\theta\right]$

$= \sin^2 25° + \cos^2 25° = 1$

$\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

Q4. Choose the correct option. Justify your choice:

(i) $9\sec^2 A - 9\tan^2 A =$

(A) 1

(B) 9

(C) 8

(D) 0

(ii) $\left(1 + \tan\theta + \sec\theta\right)\left(1 + \cot\theta - \operatorname{cosec}\theta\right) =$

(A) 0

(B) 1

(C) 2

(D) none of these

(iii) $\left(\sec A + \tan A\right)\left(1 - \sin A\right) =$

(A) $\sec A$

(B) $\sin A$

(C) $\operatorname{cosec} A$

(D) $\cos A$

(iv) $\dfrac{1 + \tan^2 A}{1 + \cot^2 A} =$

(A) $\sec^2 A$

(B) -1

(C) $\cot^2 A$

(D) none of these

Ans: (i) (B) $9\sec^2 A - 9\tan^2 A$

$= 9\left(\sec^2 A - \tan^2 A\right)$

$= 9 \times 1 = 9$

(ii) (C) $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)$

$$= \left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$$

$$= \left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$$

$$= \frac{(\cos\theta+\sin\theta)^2-(1)^2}{\cos\theta.\sin\theta}$$

$$= \frac{\cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta-1}{\cos\theta.\sin\theta}$$

$$= \frac{1+2\cos\theta\sin\theta-1}{\cos\theta.\sin\theta}$$

$$\left[\because \sin^2\theta+\cos^2\theta=1\right]$$

$$= \frac{2\cos\theta\sin\theta}{\cos\theta.\sin\theta} = 2$$

(iii)(D) $(\sec A+\tan A)(1-\sin A)$

$$= \left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A)$$

$$= \left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)$$

$$= \frac{1-\sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A}$$

$$= \cos A \quad \left[\because 1-\sin^2 A=\cos^2 A\right]$$

(iv)(D) $\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sec^2 A-\tan^2 A+\tan^2 A}{\cos ec^2 A-\cot^2 A+\cot^2 A}$

$$= \frac{\sec^2 A}{\cos ec^2 A} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

Q5. Prove the following identities, where the angles involved are acute angles for which the expressions are defined:

(i) $(\cos ec\theta-\cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

(ii) $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$

(iii) $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\operatorname{cosec}\theta$

(iv) $\frac{1+\sec A}{\sec A}=\frac{\sin^2 A}{1-\cos A}$

(v) $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\operatorname{cosec}A+\cot A$, using the identity $\operatorname{cosec}^2 A=1+\cot^2 A$

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}}=\sec A+\tan A$

(vii) $\frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}=\tan\theta$

(viii) $(\sin A+\operatorname{cosec}A)^2+(\cos A+\sec A)^2$

$=7+\tan^2 A+\cot^2 A$

(ix) $(\operatorname{cosec}A-\sin A)(\sec A-\cos A)$

$=\frac{1}{\tan A+\cot A}$

(x) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\frac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A$

Ans: (i) L.H.S. $(\operatorname{cosec}\theta-\cot\theta)^2$

$=\operatorname{cosec}^2\theta+\cot^2\theta-2\operatorname{cosec}\theta\cot\theta$

$=\frac{1}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}-2\times\frac{1}{\sin\theta}\cdot\frac{\cos\theta}{\sin\theta}$

$=\frac{1+\cos^2\theta}{\sin^2\theta}-\frac{2\cos\theta}{\sin^2\theta}$

$=\frac{1+\cos^2\theta-2\cos\theta}{\sin^2\theta}$

$=\frac{(1-\cos\theta)^2}{\sin^2\theta}$ $\left[\because a^2+b^2-2ab=(a-b)^2\right]$

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