Solution of Simultaneous Linear Equations Ex 8.1 Q3(i)

The above system can be written as

$$\begin{bmatrix} 6 & 4 \\ 9 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

or $\quad A\,X = B$

$$|A| = 36 - 36 = 0$$

So, A is singular. Now, X will be consistent if $(\text{adj}\,A) \times B = 0$

$$C_{11} = 6$$
$$C_{12} = -9$$
$$C_{21} = -4$$
$$C_{22} = 6$$

$$\text{adj}\,A = \begin{bmatrix} 6 & -9 \\ -4 & 6 \end{bmatrix}^T = \begin{bmatrix} 6 & -4 \\ -9 & 6 \end{bmatrix}$$

$$(\text{Adj}\,A) \times B = \begin{bmatrix} 6 & -4 \\ -9 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix}$$
$$= \begin{bmatrix} 12 - 12 \\ -18 + 18 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Thus, $AX = B$ will have infinite solutions.

Let $y = k$

Hence, $6x = 2 - 4k \quad$ or $\quad 9x = 3 - 6k$

$$x = \frac{1 - 2k}{3} \quad \text{or} \quad x = \frac{1 - 2k}{3}$$

Hence, $x = \dfrac{1 - 2k}{3}, y = k$

Solution of Simultaneous Linear Equations Ex 8.1 Q3(ii)

The system can be written as

$$\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 15 \end{bmatrix}$$

or $\quad AX = B$

$$|A| = 18 - 18 = 0$$

So, A is singular. Now the system will be inconsistent if $(\text{adj } A) \times B \neq 0$

$$C_{11} = 9 \qquad C_{21} = -3$$
$$C_{12} = -6 \qquad C_{22} = 2$$

$$\text{adj } A = \begin{bmatrix} 9 & -6 \\ -3 & 2 \end{bmatrix}^T = \begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}$$

$$(\text{Adj } A) \times B = \begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 15 \end{bmatrix}$$
$$= \begin{bmatrix} 45 - 45 \\ -30 + 30 \end{bmatrix} = [0]$$

Since, $(\text{Adj } A \times B) = 0$, the system will have infinite solutions.

Now,

Let $y = k$

$$2x = 5 - 3k \quad \text{or} \quad x = \frac{5 - 3k}{2}$$
$$x = 15 - 9k \quad \text{or} \quad x = \frac{5 - 3k}{2}$$

Hence, $x = \dfrac{5 - 3k}{2}, y = k$

Solution of Simultaneous Linear Equations Ex 8.1 Q3(iii)

This can be written as

$$\begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}$$

or $\quad AX = B$

$$\begin{aligned} |A| &= 5(256) - 3(16) + 7(6 - 182) \\ &= 0 \end{aligned}$$

So, A is singular. Thus, the given system is either inconsistent or it is consistent with infinitely many solutions according as

$$(\text{Adj}\, A) \times B \neq 0 \quad \text{or} \quad (\text{Adj}\, A) \times B = 0$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll} C_{11} = 256 & C_{21} = -16 & C_{31} = -176 \\ C_{12} = -16 & C_{22} = 1 & C_{32} = 11 \\ C_{13} = -176 & C_{23} = 11 & C_{33} = 121 \end{array}$$

$$\text{adj}\, A = \begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix}^T = \begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix}$$

$$\text{adj}\, A \times B = \begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Thus, $AX = B$ has infinite many solutions.

Now, $\quad$ let $z = k$

then, $\quad 5x + 3y = 4 - 7k$

$\qquad\quad 3x + 26y = 9 - 2k$

Which can be written as

$$\begin{bmatrix} 5 & 3 \\ 3 & 26 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 - 7k \\ 9 - 2k \end{bmatrix}$$

or $\quad AX = B$

$$|A| = 2$$

$$\text{adj } A = \begin{bmatrix} 26 & -3 \\ -3 & 5 \end{bmatrix}$$

Now, $\quad X = A^{-1}B = \dfrac{1}{|A|} \times \text{adj } A \times B$

$$= \frac{1}{121}\begin{bmatrix} 26 & -3 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 4-7k \\ 9-2k \end{bmatrix}$$

$$= \frac{1}{121}\begin{bmatrix} 77-176k \\ 11k+33 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \dfrac{7-16k}{11} \\ \dfrac{k+3}{11} \end{bmatrix}$$

There values of x, y, z satisfies the third eq.

Hence, $x = \dfrac{7-16k}{11}, y = \dfrac{k+3}{11}, z = k$

Solution of Simultaneous Linear Equations Ex 8.1 Q3(iv)

This above system can be written as

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ -1 & -2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

or $\quad A\,X = B$

$$\begin{aligned} |A| &= 1(2-2)+1(4-1)+1(-3) \\ &= 0+3-3 \\ &= 0 \end{aligned}$$

So, A is singular. Thus, the given system is either inconsistent or consistent with infinitely many solutions according as

$$(\text{Adj}\,A)\times(B) \neq 0 \quad \text{or} \quad (\text{Adj}\,A)\times B = 0$$

Let C_{ij} be the co-factor of a_{ij} in A

$C_{11} = 0 \qquad C_{21} = 0 \qquad C_{31} = 0$

$C_{12} = -3 \qquad C_{22} = 3 \qquad C_{32} = 3$

$C_{13} = -3 \qquad C_{23} = -3 \qquad C_{33} = 3$

$$\text{adj}\,A = \begin{bmatrix} 0 & -3 & -3 \\ 0 & 3 & 3 \\ 0 & 3 & 3 \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & 0 \\ -3 & 3 & 3 \\ -3 & 3 & 3 \end{bmatrix}$$

$$(\text{adj}\,A)\times B = \begin{bmatrix} 0 & 0 & 0 \\ -3 & 3 & 3 \\ -3 & 3 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Thus, $AX = B$ has infinite many solutions.

Now, $\quad$ let $z = k$

So, $\quad x - y = 3 - k$

$\qquad 2x + y = 2 + k$

Which can be written as

$$\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3-k \\ 2+k \end{bmatrix}$$

or $\quad A X = B$

$$|A| = 1 + 2 = 3 \neq 0$$

$$\text{adj } A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$$

and, $\quad X = A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{|A|}\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 3-5 \\ 2+k \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 3-k+2+k \\ -6+2k+2+k \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{5}{3} \\ \frac{3k-4}{3} \end{bmatrix}$$

Hence, $x = \frac{5}{3}, y = k - \frac{4}{3}, z = k$

Solution of Simultaneous Linear Equations Ex 8.1 Q3(v)

This system can be written as

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix}$$

or $\quad A\,X = B$

$$\begin{aligned} |A| &= 1(2) - 1(4) + 1(2) \\ &= 2 - 4 + 2 \\ &= 0 \end{aligned}$$

So, A is singular. Thus, the given system has either no solution or infinite solutions depending on as

$$(\text{Adj } A) \times (B) \neq 0 \quad \text{or} \quad (\text{Adj } A) \times (B) = 0$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll} C_{11} = 2 & C_{21} = -3 & C_{31} = 1 \\ c_{12} = -4 & C_{22} = 6 & C_{32} = -2 \\ c_{13} = 2 & C_{23} = -3 & C_{33} = 1 \end{array}$$

$$\text{adj } A = \begin{bmatrix} 2 & -4 & 2 \\ -3 & 6 & -3 \\ 1 & -2 & 1 \end{bmatrix}^T = \begin{bmatrix} 2 & -3 & 1 \\ -4 & 6 & -2 \\ 2 & -3 & 1 \end{bmatrix}$$

$$(\text{adj } A) \times B = \begin{bmatrix} 2 & -3 & 1 \\ -4 & 6 & -2 \\ 2 & -3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix} = \begin{bmatrix} 12 - 42 + 30 \\ -24 + 84 - 60 \\ 12 - 42 + 30 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

So, $AX = B$ has infinite solutions.

Now, $\quad$ let $z = k$

So, $\quad x + y = 6 - k$

$\quad\quad x + 2y = 14 - 3k$

Which can be written as

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 - k \\ 14 - 3k \end{bmatrix}$$

or $\quad A\,X = B$

$$|A| = 1 \neq 0$$

$$\text{adj } A = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}^T = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{|A|}\,\text{adj}\,A \times B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{1}\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 6-k \\ 14-3k \end{bmatrix}$$

$$= \begin{bmatrix} 12-2k-14+3k \\ -6+k+14-3k \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2+k \\ 8-2k \end{bmatrix}$$

Hence, $x = k - 2$

$y = 8 - 2k$

$z = k$

Solution of Simultaneous Linear Equations Ex 8.1 Q3(vi)

This system can be written as

$$\begin{bmatrix} 2 & 2 & -2 \\ 4 & 4 & -1 \\ 6 & 6 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

or $\quad A\,X = B$

$$|A| = 2(14) - 2(14) - 2(0) = 0$$

So, A is singular and the system has either no solution or infinite solutions according as

$$(\text{Adj } A) \times (B) \neq 0 \quad \text{or} \quad (\text{Adj } A) \times (B) = 0$$

Let C_{ij} be the co-factor of a_{ij} in A

$C_{11} = 14 \qquad C_{21} = -16 \qquad C_{31} = 6$

$C_{12} = -14 \qquad C_{22} = 16 \qquad C_{32} = -6$

$C_{13} = 0 \qquad C_{23} = 0 \qquad C_{33} = 0$

$$\text{adj } A = \begin{bmatrix} 14 & -14 & 0 \\ -16 & 16 & 0 \\ 6 & -6 & 0 \end{bmatrix}^T = \begin{bmatrix} 14 & -16 & 6 \\ -14 & 16 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(\text{adj } A) \times B = \begin{bmatrix} 14 & -16 & 6 \\ -14 & 16 & -6 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 14 - 32 + 18 \\ -14 + 32 - 18 \\ 0 + 0 + 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

So, $AX = B$ has infinite solutions.

Now, $\quad$ let $z = k$

So, $\quad 2x + 2y = 1 + 2k$

$\quad\quad 4x + 4y = 2 + k$

Which can be written as

$$\begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 + 2k \\ 2 + k \end{bmatrix}$$

or $\quad A\,X = B$

$$|A| = 0, \; z = 0$$

Again,

$2x + 2y = 1$

$4x + 4y = 2$

*********** END **********