

Indefinite Integrals Ex 19.9 Q40

$$\frac{(x+1)(x+\log x)^2}{x} = \left(\frac{x+1}{x}\right)(x+\log x)^2 = \left(1+\frac{1}{x}\right)(x+\log x)^2$$

Let 
$$(x + \log x) = t$$

$$\Rightarrow \left(1 + \frac{1}{x}\right) dx = dt$$

$$\Rightarrow \int \left(1 + \frac{1}{x}\right) (x + \log x)^2 dx = \int t^2 dt$$

$$= \frac{t^3}{3} + C$$

$$= \frac{1}{3} (x + \log x)^3 + C$$

Indefinite Integrals Ex 19.9 Q41

Let 
$$I = \int \tan x \sec^2 x \sqrt{1 - \tan^2 x} dx - - - - - (i)$$

Let 
$$1 - \tan^2 x = t$$
 then,  
 $d(1 - \tan^2 x) = dt$ 

$$\Rightarrow$$
 -2 tan x sec<sup>2</sup> x dx = dt

$$\Rightarrow -2 \tan x \sec^2 x \, dx = dt$$

$$\Rightarrow \tan x \sec^2 x \, dx = \frac{-dt}{2}$$

Putting 1 –  $\tan^2 x = t$  and  $\tan x \sec^2 x \, dx = -\frac{dt}{2}$  in equation (i),

$$I = \int \sqrt{t} \times \frac{-dt}{2}$$
$$= -\frac{1}{2} \int t^{\frac{1}{2}} dt$$
$$= -\frac{1}{2} \times \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$$
$$= -\frac{1}{3} t^{\frac{3}{2}} + c$$

$$I = -\frac{1}{3} \left[ 1 - \tan^2 x \right]^{\frac{3}{2}} + c$$

Indefinite Integrals Ex 19.9 Q42

Let 
$$I = \int \log x \frac{\sin(1 + (\log x)^2)}{x} dx - \cdots - (i)$$

Let 
$$1 + (\log x)^2 = t$$
 then,  

$$d(1 + (\log x)^2) = dt$$

$$\Rightarrow 2\log x \, \frac{1}{x} dx = dt$$

$$\Rightarrow \frac{\log x}{x} dx = \frac{dt}{2}$$

Putting  $1 + (\log x)^2 = t$  and  $\frac{\log x}{x} dx = \frac{dt}{2}$  in equation (i), we get

$$I = \int \sin t \times \frac{dt}{2}$$

$$= \frac{1}{2} \int \sin t \, dt$$

$$\therefore I = -\frac{1}{2} \cos t + c$$

$$= -\frac{1}{2} \cos \left[ 1 + (\log x)^2 \right] + c$$

$$I = -\frac{1}{2}\cos\left[1+\left(\log x\right)^2\right] + c$$

Indefinite Integrals Ex 19.9 Q43

Let 
$$I = \int \frac{1}{x^2} \cos^2 \left(\frac{1}{x}\right) dx - - - - - (i)$$

Let 
$$\frac{1}{x} = t$$
 then, 
$$d\left(\frac{1}{x}\right) = dt$$

$$\Rightarrow \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \frac{1}{x^2}dx = -dt$$

Putting  $\frac{1}{x} = t$  and  $\frac{1}{x^2} dx = -dt$  in equation (i), we get

$$I = \int \cos^2 t \, \left(-dt\right)$$

$$= -\int \cos^2 t \, dt$$

$$= -\int \frac{\cos^2 2t + 1}{2} \, dt$$

$$= -\frac{1}{2} \int \cos 2t \, dt - \frac{1}{2} \int \, dt$$

$$= -\frac{1}{2} \times \frac{\sin 2t}{2} - \frac{1}{2}t + c$$

$$\therefore I = -\frac{1}{4} \sin 2t - \frac{1}{2}t + c$$

$$= -\frac{1}{4} \sin 2 \times \frac{1}{x} - \frac{1}{2} \times \frac{1}{x} + c$$

$$I = -\frac{1}{4}\sin\left(\frac{2}{x}\right) - \frac{1}{2}\left(\frac{1}{x}\right) + C$$

Indefinite Integrals Ex 19.9 Q44

Let 
$$I = \int \sec^4 x \tan x \, dx - - - - (i)$$

Let 
$$\tan x = t$$
 then,  
 $d(\tan x) = dt$ 

$$\Rightarrow$$
  $\sec^2 x \, dx = dt$ 

$$\Rightarrow dx = \frac{dt}{\sec^2 x}$$

Putting  $\tan x = t$  and  $dx = \frac{dt}{\sec^2 x}$  in equation (i), we get

$$I = \int \sec^4 x \tan x \frac{dt}{\sec^2 x}$$

$$= \int \sec^2 x t dt$$

$$= \int \left(1 + \tan^2 x\right) t dt$$

$$= \int \left(1 + t^2\right) t dt$$

$$= \int \left(t + t^3\right) dt$$

$$= \frac{t^2}{2} + \frac{t^4}{4} + c$$

$$= \frac{\tan^2 x}{2} + \frac{\tan^4 x}{4} + c$$

$$I = \frac{1}{2} \tan^2 x + \frac{1}{4} \tan^4 x + c$$

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