

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 1 L.H.S,

$$sin 5\theta = sin (3\theta + 2\theta)$$

$$= sin 3\theta \cos 2\theta + \cos 3\theta. sin 2\theta$$

$$= (3 sin \theta - 4 sin^3 \theta) (1 - 2 sin^2 \theta) + (4 \cos^3 \theta - 3 \cos \theta) 2 sin \theta \cos \theta.$$

$$= 3 sin \theta - 4 sin^3 \theta - 6 sin^3 \theta + 8 sin^5 \theta + (8 \cos^4 \theta - 6 \cos^2 \theta) sin \theta$$

$$= 3 sin \theta - 10 sin^3 \theta + 8 sin^5 \theta + 8 sin \theta ((1 - sin^2 \theta)^2 - 6 sin \theta (1 - sin^2 \theta))$$

$$= 3 sin \theta - 10 sin^3 \theta + 8 sin^5 \theta + 8 sin \theta - 16 sin^3 \theta + 8 sin^5 \theta - 6 sin \theta + 6 sin^3 \theta$$

$$= 5 sin \theta - 20 sin^3 \theta + 16 sin^5 \theta = RHS$$

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 2

Consider the L.H.S of the given equation

$$4 \left(\cos^{3} 10^{\circ} + \sin^{3} 20^{\circ}\right) = 3 \left(\cos 10^{\circ} + \sin 20^{\circ}\right)$$
Since  $\sin 30^{\circ} = \cos 60^{\circ} = \frac{1}{2}$ 
and  $\sin 60^{\circ} = \cos 30^{\circ} = \frac{\sqrt{3}}{2}$ 

$$\Rightarrow \sin 3.20^{\circ} = \cos 3.10^{\circ}$$

$$\Rightarrow 3\sin 20^{\circ} - 4\sin^{3} 20^{\circ} = 4\cos^{3} 10^{\circ} - 3\cos 10^{\circ}$$

$$\Rightarrow 4 \left(\cos^{3} 10^{\circ} + \sin^{3} 20^{\circ}\right) = 3 \left(\cos 10^{\circ} + \sin 20^{\circ}\right)$$

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 3  $\cos^3\theta \sin 3\theta + \sin^3\theta \cos 3\theta = \frac{3}{4}\sin 4\theta$ 

LHS = 
$$\cos^3\theta \sin 3\theta + \sin^3\theta \cos 3\theta$$
  
=  $\left(\frac{\cos 3\theta + 3\cos \theta}{4}\right) \sin 3\theta + \left(\frac{3\sin \theta - \sin 3\theta}{4}\right) \cos 3\theta$   $\left\{\because \sin 3\theta = 3\sin \theta - 4\sin^3\theta\right\}$   
=  $\frac{1}{4} \left[3\left(\sin 3\theta \cos \theta + \sin \theta \cos 3\theta\right) + \cos 3\theta \sin 3\theta - \sin 3\theta \cos 3\theta\right]$   
=  $\frac{1}{4} \left[3\sin \left(3\theta + \theta\right) + 0\right]$   
=  $\frac{3}{4} \sin 4\theta$   
So,  
 $\cos^3\theta \sin 3\theta + \sin^3\theta \cos 3\theta = \frac{3}{4} \sin 4\theta$ 

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 4

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\sin 5A = 5\cos^4 A \sin A - 10\cos^2 A \sin^3 A + \sin^5 A
L.H.S= sin 5A = sin (3A + 2A)
                        = sin 3A cos 2A + cos 3A. sin 2A
                        = (3 \sin A - 4 \sin^3 A)(2 \cos^2 A - 1) + (4 \cos^3 A - 3 \cos A) 2 \sin A \cos A.
                        = -3 \sin A + 4 \sin^3 A + 6 \sin A \cos^2 A - 8 \sin^3 A \cos^2 A + 8 \cos^4 A \sin A - 6 \cos^2 A \sin A
                        = 8 \cos^4 A \sin A - 8 \sin^3 A \cos^2 A - 3 \sin A + 4 \sin^3 A
                       =5\cos^{4}A\sin A-10\sin^{3}A\cos^{2}A-3\sin A+3\cos^{4}A\sin A+4\sin^{3}A+2\sin^{3}A\cos^{2}A
                        =5\cos^4 A \sin A - 10\sin^3 A \cos^2 A - 3\sin A(1-\cos^4 A) + 2\sin^3 A(2+\cos^2 A)
                        =5\cos^4 A \sin A - 10\sin^3 A \cos^2 A - 3\sin A(1-\cos^2 A)(1+\cos^2 A) + 2\sin^3 A\left(2+\cos^2 A\right)
                        = 5\cos^4 A \sin A - 10\sin^3 A \cos^2 A - 3\sin^3 A(1 + \cos^2 A) + 2\sin^3 A(2 + \cos^2 A)
                        = 5\cos^4 A \sin A - 10\sin^3 A \cos^2 A - \sin^3 A \left[3(1 + \cos^2 A) - 2(2 + \cos^2 A)\right]
                        = 5\cos^4 A \sin A - 10\sin^3 A \cos^2 A - \sin^3 A \left[3 + 3\cos^2 A - 4 - 2\cos^2 A\right]
                        = 5\cos^4 A \sin A - 10\sin^3 A \cos^2 A - \sin^3 A \left[\cos^2 A - 1\right]
                       =5\cos^4 A\sin A - 10\sin^3 A\cos^2 A + \sin^5 A
                        = RHS
Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 5
 tan A \times tan(A+60^{\circ}) + tan A \times tan(A-60^{\circ}) + tan(A+60^{\circ})tan(A-60^{\circ})
\begin{split} &= tan(A) \frac{\left[tan(A) - tan(60^\circ)\right]}{\left[1 + tan(A)tan(60^\circ)\right]} \\ &+ tan(A) \frac{\left[tan(A) + tan(60^\circ)\right]}{\left[1 - tan(A)tan(60^\circ)\right]} \end{split}
 +\left.\left\{\frac{[\tan(A) - \tan(60^\circ)]}{[1 + \tan(A)\tan(60^\circ)]}\right.\right\}\left\{\frac{[\tan(A) + \tan(60^\circ)]}{[1 - \tan(A)\tan(60^\circ)]}\right.\right\}
\begin{split} &= tan(A) \frac{[tan(A) - tan(60^\circ)][1 - tan(A)tan(60^\circ)]}{[1 - tan^2(A)tan^2(60^\circ)]} \\ &+ tan(A) \frac{[tan(A) + tan(60^\circ)][1 + tan(A)tan(60^\circ)]}{[1 - tan^2(A)tan^2(60^\circ)]} \\ &+ \frac{[tan(A) - tan(60^\circ)][tan(A) + tan(60^\circ)]}{[1 - tan^2(A)tan^2(60^\circ)]} \end{split}
 = \tan(A) \frac{[\tan(A) - \sqrt{3}][1 - \sqrt{3}\tan(A)]}{[1 - 3\tan^2(A)]}
 +\tan(A)\frac{[\tan(A)+\sqrt{3}\,][1+\sqrt{3}\tan(A)]}{[1\,-\,3\tan^2(A)]}
 + \frac{[\tan(A) - \sqrt{3}][\tan(A) + \sqrt{3}]}{[1 - 3\tan^2(A)]}
 = tan(A) \frac{[4tan(A) - \sqrt{3} - \sqrt{3}tan^{2}(A)]}{[1 - 3tan^{2}(A)]}
 +\tan(A)\frac{[4tan(A)+\sqrt{3}+\sqrt{3}tan^2(A)]}{[1\ -\ 3tan^2(A)]}
 +\frac{[\tan^2(A) - 3]}{[1 - 3\tan^2(A)]}
= \frac{[9\tan^2(A) - 3]}{[1 - 3\tan^2(A)]}
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We have to prove that

Trigonometric Ratios of multiple and Submultiple Angles Ex 9.2 Q 6

$$tan A + tan (60^{\circ} + A) - tan (60^{\circ} - A) = 3 tan 3A$$

$$LHS = tan A + tan (60^{\circ} + A) - tan (60^{\circ} - A)$$

$$= tan A + \frac{tan 60^{\circ} + tan A}{1 - tan 60^{\circ} tan A} - \frac{tan 60^{\circ} - tan A}{1 + tan 60^{\circ} tan A}$$

$$= tan A + \frac{\sqrt{3} + tan A}{1 - \sqrt{3} tan A} - \frac{\sqrt{3} - tan A}{1 + \sqrt{3} tan A}$$

$$= tan A + \left[ \frac{\sqrt{3} + 3tan A + tan A + \sqrt{3} tan^{2} A + \sqrt{3} + 3tan A + tan A - \sqrt{3} tan^{2} A}{(1 - \sqrt{3} tan A)(1 + \sqrt{3} tan A)} \right]$$

$$= tan A + \frac{8 tan A}{1 - 3 tan^{2} A}$$

$$= \frac{tan A - 3tan^{3} A + 8 tan A}{1 - 3tan^{2} A}$$

$$= \frac{9 tan A - 3tan^{3} A}{1 - 3tan^{2} A}$$

$$= 3\left(\frac{3tan A - tan^{3} A}{1 - 3tan^{2} A}\right)$$

$$= 3tan 3A$$

= 3 tan 3

 $tanA + tan(60^{\circ} + A) - tan(60^{\circ} - A) = 3tan 3A$ 

\*\*\*\*\*\*\*\*\* END \*\*\*\*\*\*\*