$$E = \frac{1}{2} m_e v^2$$

$$m_e v = \sqrt{2Em_e}$$

Where,

v = Velocity of the electron

$m_e v$ = Momentum (p) of the electron

According to the de Broglie principle, the de Broglie wavelength is given as:

$$\lambda = \frac{h}{p} = \frac{h}{m_e v} = \frac{h}{\sqrt{2Em_e}}$$

$$\therefore E = \frac{h^2}{2\lambda^2 m_e}$$

$$= \frac{\left(6.6 \times 10^{-34}\right)^2}{2 \times \left(10^{-10}\right)^2 \times 9.11 \times 10^{-31}} = 2.39 \times 10^{-17} \text{ J}$$

$$= \frac{2.39 \times 10^{-17}}{1.6 \times 10^{-19}} = 149.375 \text{ eV}$$

Energy of a photon, $E' = \frac{hc}{\lambda e}$ eV

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{10^{-10} \times 1.6 \times 10^{-19}}$$

$$= 12.375 \times 10^3 \text{ eV} = 12.375 \text{ keV}$$

Hence, a photon has a greater energy than an electron for the same wavelength.

Question 11.32:

(a) Obtain the de Broglie wavelength of a neutron of kinetic energy 150 eV. As you have seen in Exercise 11.31, an electron beam of this energy is suitable for crystal diffraction experiments. Would a neutron beam of the same energy be equally suitable? Explain. (m_n= 1.675 × 10^{-27} kg)

(b) Obtain the de Broglie wavelength associated with thermal neutrons at room temperature (27 ºC). Hence explain why a fast neutron beam needs to be thermalised with the environment before it can be used for neutron diffraction experiments.

Answer

(a) De Broglie wavelength $= 2.327 \times 10^{-12}$ m; neutron is not suitable for the diffraction experiment

Kinetic energy of the neutron, K = 150 eV

= 150 × 1.6 × 10^{-19}

= 2.4 × 10^{-17} J

Mass of a neutron, m_n = 1.675 × 10^{-27} kg

The kinetic energy of the neutron is given by the relation:

$$K = \frac{1}{2} m_n v^2$$
$$m_n v = \sqrt{2\,K m_n}$$

Where,

v = Velocity of the neutron

$m_n v$ = Momentum of the neutron

De-Broglie wavelength of the neutron is given as:

$$\lambda = \frac{h}{m_n v} = \frac{h}{\sqrt{2\,K m_n}}$$

It is clear that wavelength is inversely proportional to the square root of mass.

Hence, wavelength decreases with increase in mass and vice versa.

$$\therefore \lambda = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 2.4 \times 10^{-17} \times 1.675 \times 10^{-27}}}$$
$$= 2.327 \times 10^{-12} \text{ m}$$

It is given in the previous problem that the inter-atomic spacing of a crystal is about 1 Å, i.e., 10^{-10} m. Hence, the inter-atomic spacing is about a hundred times greater. Hence, a neutron beam of energy

150 eV is not suitable for diffraction experiments.

(b) De Broglie wavelength = 1.447×10^{-10} m

Room temperature, T = 27°C = 27 + 273 = 300 K

The average kinetic energy of the neutron is given as:

$$E = \frac{3}{2} kT$$

Where,

k = Boltzmann constant = 1.38 × 10^{-23} J mol^{-1} K^{-1}

The wavelength of the neutron is given as:

$$\lambda = \frac{h}{\sqrt{2 m_n E}} = \frac{h}{\sqrt{3\, m_n kT}}$$
$$= \frac{6.6 \times 10^{-34}}{\sqrt{3 \times 1.675 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}}$$
$$= 1.447 \times 10^{-10} \text{ m}$$

This wavelength is comparable to the inter-atomic spacing of a crystal. Hence, the high-energy neutron beam should first be thermalised, before using it for diffraction.

Question 11.33:

An electron microscope uses electrons accelerated by a voltage of 50 kV. Determine the de Broglie wavelength associated with the electrons. If other factors (such as numerical aperture, etc.) are taken to be roughly the same, how does the resolving power of an electron microscope compare with that of an optical microscope which uses yellow light?

Answer

Electrons are accelerated by a voltage, V = 50 kV = 50 × 10^3 V

Charge on an electron, e = 1.6 × 10^{-19} C

Mass of an electron, m_e = 9.11 × 10^{-31} kg

Wavelength of yellow light = 5.9 × 10^{-7} m

The kinetic energy of the electron is given as:

$E = eV$

= 1.6 × 10^{-19} × 50 × 10^3

= 8 × 10^{-15} J

De Broglie wavelength is given by the relation:

$$\lambda = \frac{h}{\sqrt{2 m_e E}}$$
$$= \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 8 \times 10^{-15}}}$$
$$= 5.467 \times 10^{-12} \text{ m}$$

This wavelength is nearly 10^5 times less than the wavelength of yellow light.

The resolving power of a microscope is inversely proportional to the wavelength of light used. Thus, the resolving power of an electron microscope is nearly 10^5 times that of an optical microscope.

Question 11.34:

The wavelength of a probe is roughly a measure of the size of a structure that it can probe in some detail. The quark structure of protons and neutrons appears at the minute length-scale of 10^{-15} m or less. This structure was first probed in early 1970's using high energy electron beams produced by a linear accelerator at Stanford, USA. Guess what might have been the order of energy of these electron beams. (Rest mass energy of electron = 0.511 MeV.)

Answer

Wavelength of a proton or a neutron, $\lambda \approx 10^{-15}$ m

Rest mass energy of an electron:

$m_0c^2 = 0.511$ MeV

$= 0.511 \times 10^6 \times 1.6 \times 10^{-19}$

$= 0.8176 \times 10^{-13}$ J

Planck's constant, $h = 6.6 \times 10^{-34}$ Js

Speed of light, $c = 3 \times 10^8$ m/s

The momentum of a proton or a neutron is given as:

$$p = \frac{h}{\lambda}$$

$$= \frac{6.6 \times 10^{-34}}{10^{-15}} = 6.6 \times 10^{-19} \text{ kg m/s}$$

The relativistic relation for energy (E) is given as:

$$E^2 = p^2c^2 + m_0^2c^4$$

$$= \left(6.6 \times 10^{-19} \times 3 \times 10^8\right)^2 + \left(0.8176 \times 10^{-13}\right)^2$$

$$= 392.04 \times 10^{-22} + 0.6685 \times 10^{-26}$$

$$\approx 392.04 \times 10^{-22}$$

$$\therefore E = 1.98 \times 10^{-10} \text{ J}$$

$$= \frac{1.98 \times 10^{-10}}{1.6 \times 10^{-19}}$$

$$= 1.24 \times 10^9 \text{ eV} = 1.24 \text{ BeV}$$

Thus, the electron energy emitted from the accelerator at Stanford, USA might be of the order of 1.24 BeV.

Question 11.35:

Find the typical de Broglie wavelength associated with a He atom in helium gas at room temperature (27 ºC) and 1 atm pressure; and compare it with the mean separation between two atoms under these conditions.

Answer

De Broglie wavelength associated with He atom $= 0.7268 \times 10^{-10}$ m

Room temperature, $T = 27°C = 27 + 273 = 300$ K

Atmospheric pressure, $P = 1$ atm $= 1.01 \times 10^5$ Pa

Atomic weight of a He atom = 4

Avogadro's number, $N_A = 6.023 \times 10^{23}$

Boltzmann constant, $k = 1.38 \times 10^{-23}$ J mol^{-1} K^{-1}

Average energy of a gas at temperature T,is given as:

$$E = \frac{3}{2}kT$$

De Broglie wavelength is given by the relation:

$$\lambda = \frac{h}{\sqrt{2mE}}$$

Where,

m = Mass of a He atom

$$= \frac{\text{Atomic weight}}{\text{N}_A}$$

$$= \frac{4}{6.023 \times 10^{23}}$$

$$= 6.64 \times 10^{-24} \text{ g} = 6.64 \times 10^{-27} \text{ kg}$$

$$\therefore \lambda = \frac{h}{\sqrt{3mkT}}$$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{3 \times 6.64 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}}$$

$$= 0.7268 \times 10^{-10} \text{ m}$$

We have the ideal gas formula:

$PV = RT$

$PV = kNT$

$$\frac{V}{N} = \frac{kT}{P}$$

Where,

V = Volume of the gas

N = Number of moles of the gas

Mean separation between two atoms of the gas is given by the relation:

$$r = \left(\frac{V}{N}\right)^{\frac{1}{3}} = \left(\frac{kT}{P}\right)^{\frac{1}{3}}$$

$$= \left[\frac{1.38 \times 10^{-23} \times 300}{1.01 \times 10^5}\right]^{\frac{1}{3}}$$

$$= 3.35 \times 10^{-9} \text{ m}$$

Hence, the mean separation between the atoms is much greater than the de Broglie wavelength.

Question 11.36:

Compute the typical de Broglie wavelength of an electron in a metal at 27 ºC and compare it with the mean separation between two electrons in a metal which is given to be about 2×10^{-10} m.

[*Note:* Exercises 11.35 and 11.36 reveal that while the wave-packets associated with gaseous molecules under ordinary conditions are non-overlapping, the electron wave-packets in a metal strongly overlap with one another. This suggests that whereas molecules in an ordinary gas can be distinguished apart, electrons in a metal cannot be distinguished apart from one another. This indistinguishibility has many fundamental implications which you will explore in more advanced Physics courses.]

Answer

Temperature, T = 27°C = 27 + 273 = 300 K

Mean separation between two electrons, $r = 2 \times 10^{-10}$ m

De Broglie wavelength of an electron is given as:

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

Where,

h = Planck's constant = 6.6×10^{-34} Js

m = Mass of an electron = 9.11×10^{-31} kg

k = Boltzmann constant = 1.38×10^{-23} J mol^{-1} K^{-1}

$$\therefore \lambda = \frac{6.6 \times 10^{-34}}{\sqrt{3 \times 9.11 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}}$$

$$\approx 6.2 \times 10^{-9} \text{ m}$$

Hence, the de Broglie wavelength is much greater than the given inter-electron separation.

Question 11.37:

Answer the following questions:

(a) Quarks inside protons and neutrons are thought to carry fractional charges [(+2/3)*e* ; (−1/3)*e*]. Why do they not show up in Millikan's oil-drop experiment?

(b) What is so special about the combination *e*/*m*? Why do we not simply talk of *e* and *m* separately?

(c) Why should gases be insulators at ordinary pressures and start conducting at very low pressures?

(d) Every metal has a definite work function. Why do all photoelectrons not come out with the same energy if incident radiation is monochromatic? Why is there an energy distribution of photoelectrons?

(e) The energy and momentum of an electron are related to the frequency and wavelength of the associated matter wave by the relations:

$E = h\nu$, $p = \frac{h}{\lambda}$

But while the value of λ is physically significant, the value of ν (and therefore, the value of the phase speed $\nu\lambda$) has no physical significance. Why?

Answer

(a) Quarks inside protons and neutrons carry fractional charges. This is because nuclear force increases extremely if they are pulled apart. Therefore, fractional charges may exist in nature; observable charges are still the integral multiple of an electrical charge.

(b) The basic relations for electric field and magnetic field are

$$\left(eV = \frac{1}{2}mv^2\right) \text{ and } \left(eBv = \frac{mv^2}{r}\right) \text{ respectively}$$

These relations include *e* (electric charge), *v* (velocity), *m* (mass), *V* (potential), *r* (radius), and *B* (magnetic field). These relations give the value of velocity of an electron as $\left(v = \sqrt{2V\left(\frac{e}{m}\right)}\right)$ and

$$\left(v = Br\left(\frac{e}{m}\right)\right) \text{ respectively.}$$

It can be observed from these relations that the dynamics of an electron is determined not by *e* and *m* separately, but by the ratio e/*m*.

(c) At atmospheric pressure, the ions of gases have no chance of reaching their respective electrons because of collision and recombination with other gas molecules. Hence, gases are insulators at atmospheric pressure. At low pressures, ions have a chance of reaching their respective electrodes and constitute a current. Hence, they conduct electricity at these pressures.

(d) The work function of a metal is the minimum energy required for a conduction electron to get out of the metal surface. All the electrons in an atom do not have the same energy level. When a ray having some photon energy is incident on a metal surface, the electrons come out from different levels with different energies. Hence, these emitted electrons show different energy distributions.

(e) The absolute value of energy of a particle is arbitrary within the additive constant. Hence, wavelength (λ) is significant, but the frequency (ν) associated with an electron has no direct physical significance.

Therefore, the product $\nu\lambda$(phase speed)has no physical significance.

Group speed is given as:

$$v_G = \frac{d\nu}{dk}$$

$$= \frac{d\nu}{d\left(\frac{1}{\lambda}\right)} = \frac{dE}{dp} = \frac{d\left(\frac{p^2}{2m}\right)}{dp} = \frac{p}{m}$$

This quantity has a physical meaning.

*********** END **********