Trigonometric Ratios of Compound Angles Ex 7.1 Q14

We have,

$$\tan A = \frac{5}{6} \text{ and } \tan B = \frac{1}{11}$$

Now,

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\frac{5}{6} + \frac{1}{11}}{1 - \frac{5}{6} \times \frac{1}{11}}$$

$$= \frac{\frac{55+6}{66}}{1 - \frac{5}{66}}$$

$$= \frac{\frac{61}{66}}{\frac{66-5}{66}}$$

$$= \frac{\frac{61}{66}}{\frac{61}{66}}$$

$$= \frac{61}{66} \times \frac{66}{61}$$

$$= 1$$

$$= \tan\frac{\pi}{4} \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$$

$$\Rightarrow \quad \tan(A+B) = \tan\frac{\pi}{4}$$

$$\Rightarrow \quad A + B = \frac{\pi}{4}$$

Hence proved.

We have,

$$\tan A = \frac{m}{m-1} \text{ and } \tan B = \frac{1}{2m-1}$$

Now, $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$

$$= \frac{\frac{m}{m-1} - \frac{1}{2m-1}}{1+\frac{m}{m-1}\times\frac{1}{2m-1}}$$

$$= \frac{\frac{m(2m-1)-(m-1)}{(m-1)(2m-1)}}{1+\frac{m}{(m-1)(2m-1)}}$$

$$= \frac{\frac{m(2m-1)-(m-1)}{(m-1)(2m-1)}}{\frac{(m-1)(2m-1)+(m)}{(m-1)(2m-1)}}$$

$$= \frac{m(2m-1)-(m-1)}{(m-1)(2m-1)+(m)}$$

$$= \frac{2m^2-m-m+1}{2m^2-m-2m+1+m}$$

$$= \frac{2m^2-m-m+1}{2m^2-2m+1}$$

$$= \frac{2m^2-2m+1}{2m^2-2m+1}$$

$$= 1$$

$\therefore \quad \tan(A-B) = 1 = \tan\left(\frac{\pi}{4}\right) \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$

$\Rightarrow \quad \tan(A-B) = \tan\left(\frac{\pi}{4}\right)$

$\Rightarrow \quad A - B = \left(\frac{\pi}{4}\right)$

Trigonometric Ratios of Compound Angles Ex 7.1 Q15

LHS: $\cos^2 45° - \sin^2 15°$

$= \left(\frac{1}{\sqrt{2}}\right)^2 - \sin^2 15° \qquad \left[\because \cos 45 = \frac{1}{\sqrt{2}}\right]$

$= \frac{1}{2} - \left(\frac{1-\cos 2\times 15°}{2}\right) \qquad \left[\because \cos 2\theta = 1-\sin^2\theta\right]$

$= \frac{1}{2} - \left(\frac{1-\cos 30°}{2}\right)$

$= \frac{1-1+\cos 30°}{2}$

$= \frac{\cos 30°}{2}$

$= \frac{\sqrt{3}}{2}\times\frac{1}{2}$

$= \frac{\sqrt{3}}{4}$

$=$ RHS

$\therefore \quad$ LHS = RHS

Hence proved.

We have,

LHS $\sin^2(n+1)A - \sin^2 nA$

$= \sin\left[(n+1)A + nA\right]\sin\left[(n+1)A - nA\right]$

$\left[\because \sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)\right]$

$= \sin\left[nA + A + nA\right]\sin\left[nA + A - nA\right]$

$= \sin(2nA + A)\sin(A)$

$= \sin(2n+1)A\sin A$

$=$ RHS

$\because$ LHS = RHS

Hence proved.

*********** END **********