Indefinite Integrals Ex 19.19 Q10

Let $I = \int \frac{x+2}{2x^2+6x+5}dx$

Let $x+2 = \lambda \frac{d}{dx}\left(2x^2+6x+5\right)+\mu$

$= \lambda\left(4x+6\right)+\mu$

$x+2 = \left(4\lambda\right)x+\left(6\lambda+\mu\right)$

Comparing the coefficients of like powers of x,

$4\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{4}$

$6\lambda+\mu = 2 \quad \Rightarrow \quad 6\left(\frac{1}{4}\right)+\mu = 2$

$\mu = \frac{1}{2}$

so, $I = \int \frac{\frac{1}{4}\left(4x+6\right)+\frac{1}{2}}{2x^2+6x+5}dx$

$$I = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5}dx + \frac{1}{2}\int \frac{1}{2x^2+6x+5}dx$$

$$I = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5}dx + \frac{1}{4}\int \frac{1}{x^2+3x+\frac{5}{2}}dx$$

$$I = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5}dx + \frac{1}{4}\int \frac{1}{x^2+2x\left(\frac{3}{2}\right)+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^3+\frac{5}{2}}dx$$

$$= \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5}dx + \frac{1}{4}\int \frac{1}{\left(x+\frac{3}{2}\right)^2+\frac{1}{4}}dx$$

$$I = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5}dx + \frac{1}{4}\int \frac{1}{\left(x+\frac{3}{2}\right)^2+\left(\frac{1}{2}\right)^2}dx$$

$$I = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5}dx + \frac{1}{4}\times\frac{1}{\frac{1}{2}}\tan^{-1}\left(\frac{x+\frac{3}{2}}{\frac{1}{2}}\right)+c\,dx \qquad \left[\text{since, } \int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c\right]$$

$$I = \frac{1}{4}\log\left|2x^2+6x+5\right|+\frac{1}{2}\tan^{-1}\left(2x+3\right)+c$$

Indefinite Integrals Ex 19.19 Q11

Let $I=\int \frac{(3\sin x-2)\cos x}{5-\cos^2 x-4\sin x}dx$

$\therefore I=\int \frac{(3\sin x-2)\cos x}{5-(1-\sin^2 x)-4\sin x}dx$

$\Rightarrow I=\int \frac{(3\sin x-2)\cos x}{5-1+\sin^2 x-4\sin x}dx$

Substitute sinx=t

$\Rightarrow$ cosx dx = dt

Thus,

$I=\int \frac{(3t-2)}{4+t^2-4t}dt$

$\Rightarrow I=\int \frac{(3t-2)}{t^2-4t+4}dt$

$\Rightarrow I=\int \frac{(3t-2)}{(t-2)^2}dt$

Now let us separate the integrand into the simplest form using partial fractions.

$\frac{(3t-2)}{(t-2)^2}=\frac{A}{(t-2)}+\frac{B}{(t-2)^2}$

$=\frac{A(t-2)+B}{(t-2)^2}$

$=\frac{At-2A+B}{(t-2)^2}$

$\Rightarrow 3t-2=At-2A+B$

Comparing the coefficients, we have,

A=3

and

$-2A+B=-2$

Susbtituting the value of A=3 in the above equation, we have,

$\Rightarrow -2\times 3+B=-2$

$\Rightarrow -6+B=-2$

$\Rightarrow B=6-2$

$\Rightarrow B=4$

Thus, $I=\int \frac{(3t-2)}{(t-2)^2}dt$ becomes,

$I=\int \frac{3}{(t-2)}dt+\int \frac{4}{(t-2)^2}dt$

$=3\log|t-2|-4\left(\frac{1}{t-2}\right)+C$

$=3\log|2-t|+4\left(\frac{1}{2-t}\right)+C$

Now substituting t=sinx, we have,

$I=3\log|2-\text{sinx}|+4\left(\frac{1}{2-\text{sinx}}\right)+C$

Indefinite Integrals Ex 19.19 Q12

Let $I=\int \frac{5x-2}{1+2x+3x^2}dx$

Rewriting the numerator we have,

$$5x-2=A\frac{d}{dx}\left(1+2x+3x^2\right)+B$$

$$\Rightarrow 5x-2=A\left(2+6x\right)+B$$

$$\Rightarrow 5x-2=6xA+2A+B$$

Comparing the coefficients, we have,

$6A=5$ and $2A+B=-2$

$$\Rightarrow A=\frac{5}{6}$$

Substituting the value of A in $2A+B=-2, we have,$

$$2\times\frac{5}{6}+B=-2$$

$$\Rightarrow \frac{10}{6}+B=-2$$

$$\Rightarrow B=-2-\frac{10}{6}$$

$$\Rightarrow B=\frac{-12-10}{6}$$

$$\Rightarrow B=\frac{-22}{6}$$

$$\Rightarrow B=\frac{-11}{3}$$

$$5x-2=\frac{5}{6}\left(2+6x\right)-\frac{11}{3}$$

Thus, $I=\int \frac{5x-2}{1+2x+3x^2}dx$ becomes,

$$I=\int \frac{\left[\frac{5}{6}\left(2+6x\right)-\frac{11}{3}\right]}{3x^2+2x+1}dx$$

$$=\frac{5}{6}\int \frac{\left(2+6x\right)}{3x^2+2x+1}dx-\frac{11}{3}\int \frac{dx}{3x^2+2x+1}$$

$$=\frac{5}{6}\log\left(3x^2+2x+1\right)-\frac{11}{3\times 3}\int \frac{dx}{x^2+\frac{2}{3}x+\frac{1}{3}}+C$$

$$=\frac{5}{6}\log\left(3x^2+2x+1\right)-\frac{11}{9}\int \frac{dx}{x^2+\frac{2}{3}x+\left(\frac{4}{3}\right)^2+\frac{1}{3}-\left(\frac{4}{3}\right)^2}+C$$

$$=\frac{5}{6}\log\left(3x^2+2x+1\right)-\frac{11}{9}\int \frac{dx}{\left(x+\frac{1}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2}+C$$

$$=\frac{5}{6}\log\left(3x^2+2x+1\right)-\frac{11}{9}\times\frac{1}{\frac{\sqrt{2}}{3}}\tan^{-1}\left[\frac{\left(x+\frac{1}{3}\right)}{\frac{\sqrt{2}}{3}}\right]+C$$

$$=\frac{5}{6}\log\left(3x^2+2x+1\right)-\frac{11}{9}\times\frac{3}{\sqrt{2}}\tan^{-1}\left[\frac{\left(\frac{3x+1}{3}\right)}{\frac{\sqrt{2}}{3}}\right]+C$$

$$=\frac{5}{6}\log\left(3x^2+2x+1\right)-\frac{11}{3\sqrt{2}}\tan^{-1}\left[\frac{3x+1}{\sqrt{2}}\right]+C$$

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