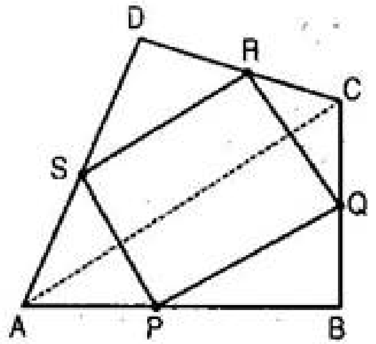
NCERT solutions for class 9 maths chapter 8 quadrilaterals Ex 8.2

1Q. ABCD is a quadrilateral in which P, Q, R and S are the mid-points of sides AB, BC, CD and DA respectively (See figure). AC is a diagonal. Show that:



(i) SR $\parallel$ AC and SR = $\frac{1}{2}$ AC

(ii) PQ = SR

(iii) PQRS is a parallelogram.

Ans. In Δ ABC, P is the mid-point of AB and Q is the mid-point of BC.

Then PQ $\parallel$ AC and PQ = $\frac{1}{2}$ AC

(i) In Δ ACD, R is the mid-point of CD and S is the mid-point of AD.

Then SR $\parallel$ AC and SR = $\frac{1}{2}$ AC

(ii) Since PQ = $\frac{1}{2}$ AC and SR = $\frac{1}{2}$ AC

Therefore, PQ = SR

(iii) Since PQ || AC and SR || AC

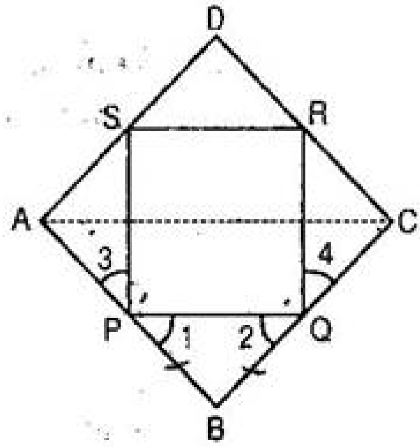
Therefore, PQ || SR [two lines parallel to given line are parallel to each other]

Now PQ = SR and PQ || SR

Therefore, PQRS is a parallelogram.

Q2. ABCD is a rhombus and P, Q, R, S are mid-points of AB, BC, CD and DA respectively. Prove that quadrilateral PQRS is a rectangle.

Ans. Given: P, Q, R and S are the mid-points of respective sides AB, BC, CD and DA of rhombus. PQ, QR, RS and SP are joined.



To prove: PQRS is a rectangle.

Construction: Join A and C.

Proof: In $\triangle$ ABC, P is the mid-point of AB and Q is the mid-point of BC.

$\therefore$ PQ || AC and PQ = $\frac{1}{2}$ AC ....(i)

In $\triangle$ ADC, R is the mid-point of CD and S is the mid-point of AD.

$\therefore$ SR $\parallel$ AC and SR = $\frac{1}{2}$ AC ....(ii)

From eq. (i) and (ii), PQ $\parallel$ SR and PQ = SR

$\therefore$ PQRS is a parallelogram.

Now ABCD is a rhombus. [Given]

$\therefore$ AB = BC

$\Rightarrow \frac{1}{2}$ AB = $\frac{1}{2}$ BC $\Rightarrow$ PB = BQ

$\therefore \angle 1 = \angle 2$ [Angles opposite to equal sides are equal]

Now in triangles APS and CQR, we have,

AP = CQ [P and Q are the mid-points of AB and BC and AB = BC]

Similarly, AS = CR and PS = QR [Opposite sides of a parallelogram]

$\therefore \Delta$ APS $\cong \Delta$ CQR [By SSS congreuancy]

$\Rightarrow \angle 3 = \angle 4$ [By C.P.C.T.]

Now we have $\angle 1 + \angle SPQ + \angle 3 = 180^\circ$

And $\angle 2 + \angle PQR + \angle 4 = 180^\circ$ [Linear pairs]

$\therefore \angle 1 + \angle SPQ + \angle 3 = \angle 2 + \angle PQR + \angle 4$

Since $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ [Proved above]

$\therefore \angle SPQ = \angle PQR$ ....(iii)

Now PQRS is a parallelogram [Proved above]

$\therefore \angle SPQ + \angle PQR = 180^\circ$ ....(iv) [Interior angles]

Using eq. (iii) and (iv),

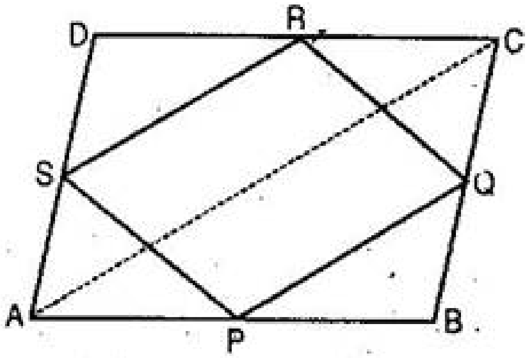
$\angle SPQ + \angle SPQ = 180^\circ \Rightarrow 2\angle SPQ = 180^\circ$

$\Rightarrow \angle SPQ = 90^\circ$

Hence PQRS is a rectangle.

Q3. ABCD is a rectangle and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Ans. Given: A rectangle ABCD in which P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. PQ, QR, RS and SP are joined.



To prove: PQRS is a rhombus.

Construction: Join AC.

Proof: In Δ ABC, P and Q are the mid-points of sides AB, BC respectively.

$\therefore$ PQ $\parallel$ AC and PQ = $\frac{1}{2}$ AC ....(i)

In Δ ADC, R and S are the mid-points of sides CD, AD respectively.

$\therefore$ SR $\parallel$ AC and SR = $\frac{1}{2}$ AC ....(ii)

From eq. (i) and (ii), PQ $\parallel$ SR and PQ = SR ....(iii)

$\therefore$ PQRS is a parallelogram.

Now ABCD is a rectangle. [Given]

$\therefore$ AD = BC

$\Rightarrow \frac{1}{2}$ AD = $\frac{1}{2}$ BC $\Rightarrow$ AS = BQ ....(iv)

In triangles APS and BPQ,

AP = BP [P is the mid-point of AB]

$\angle$ PAS = $\angle$ PBQ [Each 90°]

And AS = BQ [From eq. (iv)]

$\therefore$ Δ APS $\cong$ Δ BPQ [By SAS congruency]

$\Rightarrow$ PS = PQ [By C.P.C.T.] ....(v)

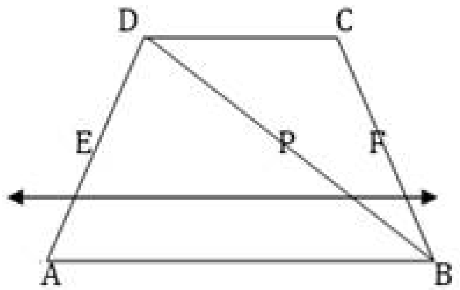
From eq. (iii) and (v), we get that PQRS is a parallelogram.

$\Rightarrow$ PS = PQ

$\Rightarrow$ Two adjacent sides are equal.

Hence, PQRS is a rhombus.

Q4. ABCD is a trapezium, in which AB $\parallel$ DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E, parallel to AB intersecting BC at F (See figure). Show that F is the mid-point of BC.



Ans. Let diagonal BD intersect line EF at point P.

In Δ DAB,

E is the mid-point of AD and EP $\parallel$ AB [$\because$ EF $\parallel$ AB (given) P is the part of EF]

$\therefore$ P is the mid-point of other side, BD of Δ DAB.

[A line drawn through the mid-point of one side of a triangle, parallel to another side intersects the third side at the mid-point]

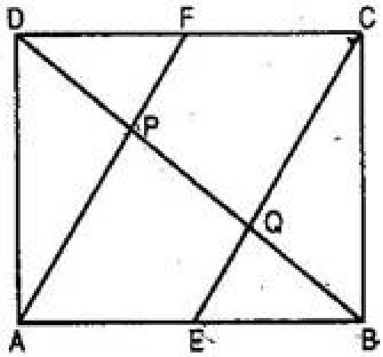
Now in Δ BCD,

P is the mid-point of BD and PF $\parallel$ DC [$\because$ EF $\parallel$ AB (given) and AB $\parallel$ DC (given)]

$\therefore$ EF $\parallel$ DC and PF is a part of EF.

$\therefore$ F is the mid-point of other side, BC of Δ BCD. [Converse of mid-point of theorem]

Q5. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (See figure). Show that the line segments AF and EC trisect the diagonal BD.



Ans. Since E and F are the mid-points of AB and CD respectively.

$\therefore$ AE = $\frac{1}{2}$ AB and CF = $\frac{1}{2}$ CD..........(i)

But ABCD is a parallelogram.

$\therefore$ AB = CD and AB $\parallel$ DC

$\Rightarrow \frac{1}{2}$ AB = $\frac{1}{2}$ CD and AB $\parallel$ DC

$\Rightarrow$ AE = FC and AE $\parallel$ FC [From eq. (i)]

$\therefore$ AECF is a parallelogram.

$\Rightarrow$ FA $\parallel$ CE $\Rightarrow$ FP $\parallel$ CQ [FP is a part of FA and CQ is a part of CE] ....(ii)

Since the segment drawn through the mid-point of one side of a triangle and parallel to the other side bisects the third side.

In Δ DCQ, F is the mid-point of CD and $\Rightarrow$ FP $\parallel$ CQ

$\therefore$ P is the mid-point of DQ.

$\Rightarrow$ DP = PQ ....(iii)

Similarly, In Δ ABP, E is the mid-point of AB and $\Rightarrow$ EQ $\parallel$ AP

$\therefore$ Q is the mid-point of BP.

$\Rightarrow$ BQ = PQ ....(iv)

From eq. (iii) and (iv),

DP = PQ = BQ ....(v)

Now BD = BQ + PQ + DP = BQ + BQ + BQ = 3BQ

$\Rightarrow$ BQ = $\frac{1}{3}$ BD ....(vi)

From eq. (v) and (vi),

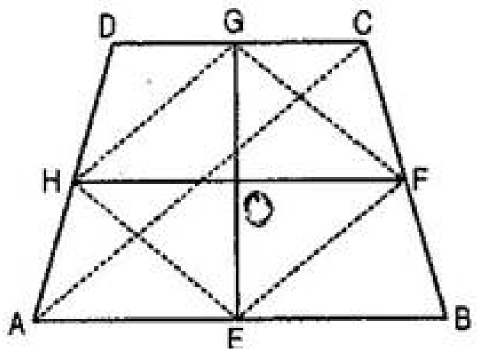
DP = PQ = BQ = $\frac{1}{3}$ BD

$\Rightarrow$ Points P and Q trisects BD.

So AF and CE trisects BD.

Q6. Show that the line segments joining the mid-points of opposite sides of a quadrilateral bisect each other.

Ans. Given: A quadrilateral ABCD in which EG and FH are the line-segments joining the mid-points of opposite sides of a quadrilateral.



To prove: EG and FH bisect each other.

Construction: Join AC, EF, FG, GH and HE.

Proof: In Δ ABC, E and F are the mid-points of respective sides AB and BC.

$\therefore$ EF $\parallel$ AC and EF $\frac{1}{2}$ AC ....(i)

Similarly, in Δ ADC,

G and H are the mid-points of respective sides CD and AD.

$\therefore$ HG $\parallel$ AC and HG $\frac{1}{2}$ AC ....(ii)

From eq. (i) and (ii),

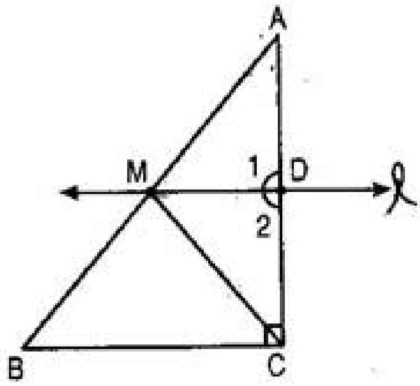
EF $\parallel$ HG and EF = HG

$\therefore$ EFGH is a parallelogram.

Since the diagonals of a parallelogram bisect each other, therefore line segments (i.e. diagonals) EG and FH (of parallelogram EFGH) bisect each other.

Q7. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D.

Ans. (i) In Δ ABC, M is the mid-point of AB [Given]



$MD \parallel BC$

$\therefore$ AD = DC [Converse of mid-point theorem]

Thus D is the mid-point of AC.

(ii) $l \parallel BC$ (given) consider AC as a transversal.

$\therefore \angle 1 = \angle C$ [Corresponding angles]

$\Rightarrow \angle 1 = 90^\circ$ [$\angle C = 90^\circ$]

Thus $MD \perp AC$.

(iii) In Δ AMD and Δ CMD,

AD = DC [proved above]

$\angle 1 = \angle 2 = 90^\circ$ [proved above]

MD = MD [common]

$\therefore \Delta AMD \cong \Delta CMD$ [By SAS congruency]

$\Rightarrow$ AM = CM [By C.P.C.T.] ....(i)

Given that M is the mid-point of AB.

$\therefore AM = \frac{1}{2} AB$ ....(ii)

From eq. (i) and (ii),

$CM = AM = \frac{1}{2} AB$

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