

Functions Ex 2.5 Q8

It is given that
$$f(x) = \frac{4x+3}{6x-4}$$
, $x \neq \frac{2}{3}$.

$$(fof)(x) = f(f(x)) = f\left(\frac{4x+3}{6x-4}\right)$$

$$= \frac{4\left(\frac{4x+3}{6x-4}\right) + 3}{6\left(\frac{4x+3}{6x-4}\right) - 4} = \frac{16x+12+18x-12}{24x+18-24x+16} = \frac{34x}{34} = x$$

Therefore, fof (x) = x, for all $x \neq \frac{2}{3}$.

$$\Rightarrow$$
 fof = I

Hence, the given function f is invertible and the inverse of f is f itself. Functions Ex 2.5 Q9

 $f: \mathbf{R}_+ \to [-5, \infty)$ is given as $f(x) = 9x^2 + 6x - 5$. Let y be an arbitrary element of $[-5, \infty)$.

Let
$$y = 9x^2 + 6x - 5$$
.

$$\Rightarrow y = (3x+1)^2 - 1 - 5 = (3x+1)^2 - 6$$

$$\Rightarrow (3x+1)^2 = y+6$$

$$\Rightarrow 3x+1 = \sqrt{y+6}$$
 [as $y \ge -5 \Rightarrow y+6 > 0$]

$$\Rightarrow x = \frac{\sqrt{y+6}-1}{3}$$

Therefore, f is onto, thereby range $f = [-5, \infty)$.

Let us define
$$g: [-5, \infty) \to \mathbb{R}_+ \text{ as } g(y) = \frac{\sqrt{y+6}-1}{3}$$
.

We now have:

$$(gof)(x) = g(f(x)) = g(9x^{2} + 6x - 5)$$

$$= g((3x+1)^{2} - 6)$$

$$= \frac{\sqrt{(3x+1)^{2} - 6 + 6} - 1}{3}$$

$$= \frac{3x+1-1}{3} = x$$

And,
$$(f \circ g)(y) = f(g(y)) = f\left(\frac{\sqrt{y+6}-1}{3}\right)$$

$$= \left[3\left(\frac{\sqrt{y+6}-1}{3}\right)+1\right]^2 - 6$$

$$= \left(\sqrt{y+6}\right)^2 - 6 = y+6-6 = y$$

Therefore, $gof = I_R$ and $fog = I_{(-5, \infty)}$ Hence, f is invertible and the inverse of f is given by

$$f^{-1}(y) = g(y) = \frac{\sqrt{y+6}-1}{3}$$
.

Functions Ex 2.5 Q10

 $f:R\to R$ be a function defined by

$$f(x) = x^3 - 3$$

Injectivity:

$$\operatorname{let} f(x_1) = f(x_2)$$

$$\Rightarrow$$
 $x_1^3 - 3 = x_2^3 - 3$

$$\Rightarrow$$
 $x_1^3 = x_2^3$

$$\Rightarrow$$
 $x_1 = x_2$

Surjectivity: let $y \in R$ be arbitrary such that

$$f(x) = y$$

$$\Rightarrow x^3 - 3 - y = 0$$

We know that an equation of odd degree must have atleast one real solution.

let $x = \alpha$ be that solution

$$\alpha^3 - 3 = y$$

$$\Rightarrow$$
 $f(\alpha) = y$

so, for each $y \in R$ in co-domain there exist $\alpha \in R$ in domain

⇒ f in onto

Thus, f in one-one and onto, so

 f^{-1} exists

Now,

$$f(x) = x^3 - 3 = y$$

$$\Rightarrow x^3 = 3 + y$$

$$\Rightarrow x = 3\sqrt{3+y}$$

$$\Rightarrow f^{-1}(x) = 3\sqrt{3} + x$$

Thus, $f^{-1}: R \to R$ be the inverse function defined by $f^{-1}(x) = (x+3)^{1/3}$

finally,

$$f^{-1}(24) = (24 + 3)^{\frac{1}{3}} = 3$$

$$f^{-1}(5) = (5+3)^{\frac{1}{3}} = 2$$

Functions Ex 2.5 Q11

We have,

 $f: R \to R$ in a function defined by

$$f(x) = x^3 + 4$$

Injectivity: let $f(x_1) = f(x_2)$ for $x_1x_2 \in R$

$$\Rightarrow \qquad x_1^3 + 4 = x_2^3 + 4$$

$$\Rightarrow x_1^3 = x_2^3$$

$$\Rightarrow$$
 $x_1 = x_2$

$$\Rightarrow$$
 finone-one

Surjectivity: let $y \in R$ be artritrary such that

$$f(x) = y$$

$$\Rightarrow x^3 + 4 = y$$

$$\Rightarrow x^3 + 4 - y = 0$$

We know that an odd degree equation must have a real root.

$$\Rightarrow$$
 $\alpha^3 + 4 = y \Rightarrow f(\alpha) = y$

$$\Rightarrow$$
 f in onto

Since f in one-one and onto

finally.

$$f(x) = y$$

$$\Rightarrow x^3 + 4 = y$$

$$\Rightarrow \qquad x^3 = y - 4$$

$$\Rightarrow \qquad x = \left(y - 4\right)^{\frac{1}{3}}$$

$$f^{-1}(x) = (x-4)^{\frac{1}{3}}$$

$$f^{-1}(3) = (3-4)^{\frac{1}{3}} = -1$$

******* END *******