Mathematical Induction Ex 12.2 Q27

Let $P(n) : 11^{n+2} + 12^{2n+1}$ is divisible by 133

For $n = 1$

$11^3 + 12^3$

$= 1331 + 1728$

$= 3059$

it is divisible of 133

$\Rightarrow \quad P(n)$ is true for $n = 1$

Let $P(n)$ is true for $n = k$, so

$11^{k+2} + 12^{2k+1}$ is divisible by 133

$11^{k+2} + 12^{2k+1} = 133\lambda \qquad ---(1)$

We have to show that,

$11^{k+3} + 12^{2k+3}$ is divisible by 133

Now,

$11^{k+2}.11 + 12^{2k+1}.12^2$

$= \left(133\lambda - 12^{2k+1}\right)11 + 12^{2k+1}.144$

$= 11.133\lambda - 11.12^{2k+1} + 144.12^{2k+1}$

$= 11.133\lambda + 133.12^{2k+1}$

$= 133\left(11\lambda + 12^{2k+1}\right)$

$= 133\mu$

$\Rightarrow \quad P(n)$ is true for $n = k + 1$

$\Rightarrow \quad P(n)$ is true for all $n \in N$ by PMI

Mathematical Induction Ex 12.2 Q28

Consider equation

$1\times 1! + 2\times 2! + 3\times 3! + \ldots\ldots n\times n!$

Lets take $(n+1)!-n! = n!\,(n+1-1)=n\times n!$

Now substitue $n=1,2,3,4,\ldots n$ in above equation we get

$2!-1! = 1\times 1!$

$3!-2! = 2\times 2!$

$4!-3! = 3\times 3!$

....

$(n+1)!-n! = n\times n!$

Adding all the above terms gives

$1\times 1! + 2\times 2! + 3\times 3! + \ldots\ldots n\times n! = 2!-1!+3!-2!+4!-3!\ \ldots\ldots+(n+1)!-n!$

$1\times 1! + 2\times 2! + 3\times 3! + \ldots\ldots n\times n! = (n+1)!-1$

Mathematical Induction Ex 12.2 Q29

Let $P(n)$ be the statement given by

$P(n): n^3-7n+3$ is divisible by 3.

Step I:

$P(1): 1^3-7(1)+3$ is divisible by 3

$\because 1-7+3=-3$ is divisible by 3

$\therefore P(1)$ is true.

Step II:

Let $P(m)$ is true. Then,

m^3-7m+3 is divisible by 3

$\Rightarrow m^3-7m+3=3\lambda$ for some $\lambda\in N$ ....(i)

We have to prove that $P(m+1)$ is true.

$$\begin{aligned}(m+1)^3-7(m+1)+3 &= m^3+3m^2+3m+1-7m-7+3\\ &= m^3-7m+3+3m^2+3m+1-7\\ &= (m^3-7m+3)+3(m^2+m-2)\\ &= 3\lambda+3(m^2+m-2)\ldots\ldots\ldots\ldots\ldots\ldots[\text{Using (i)}]\\ &= 3\left[\lambda+(m^2+m-2)\right] \text{ which is divisible by 3}\end{aligned}$$

$\Rightarrow P(m+1)$ is true.

Hence by the principle of mathematical induction, the given result is true for all $n\in N$.

*********** END **********