Question 1:
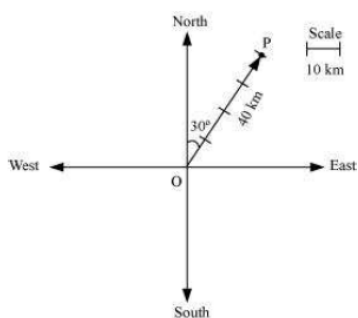
Represent graphically a displacement of 40 km, 30° east of north.

Answer

Here, vector $\overline{OP}$ represents the displacement of 40 km, 30° East of North.

Question 2:

Classify the following measures as scalars and vectors.

(i) 10 kg (ii) 2 metres north-west (iii) 40°

(iv) 40 watt (v) 10^{-19} coulomb (vi) 20 m/s²

Answer

(i) 10 kg is a scalar quantity because it involves only magnitude.

(ii) 2 meters north-west is a vector quantity as it involves both magnitude and direction.

(iii) 40° is a scalar quantity as it involves only magnitude.

(iv) 40 watts is a scalar quantity as it involves only magnitude.

(v) 10^{-19} coulomb is a scalar quantity as it involves only magnitude.

(vi) 20 m/s² is a vector quantity as it involves magnitude as well as direction.

Question 3:

Classify the following as scalar and vector quantities.

(i) time period (ii) distance (iii) force

(iv) velocity (v) work done

Answer

(i) Time period is a scalar quantity as it involves only magnitude.

(ii) Distance is a scalar quantity as it involves only magnitude.

(iii) Force is a vector quantity as it involves both magnitude and direction.

(iv) Velocity is a vector quantity as it involves both magnitude as well as direction.

(v) Work done is a scalar quantity as it involves only magnitude.

Question 4:

In Figure, identify the following vectors.
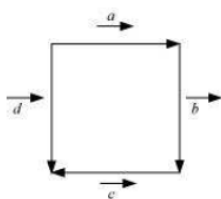
(i) Coinitial (ii) Equal (iii) Collinear but not equal

Answer

(i) Vectors $\vec{a}$ and $\vec{d}$ are coinitial because they have the same initial point.

(ii) Vectors $\vec{b}$ and $\vec{d}$ are equal because they have the same magnitude and direction.

(iii) Vectors $\vec{a}$ and $\vec{c}$ are collinear but not equal. This is because although they are parallel, their directions are not the same.

Question 5:

Answer the following as true or false.

(i) $\vec{a}$ and $-\vec{a}$ are collinear.

(ii) Two collinear vectors are always equal in magnitude.

(iii) Two vectors having same magnitude are collinear.

(iv) Two collinear vectors having the same magnitude are equal.

Answer

(i) True.

Vectors $\vec{a}$ and $-\vec{a}$ are parallel to the same line.

(ii) False.

Collinear vectors are those vectors that are parallel to the same line.

(iii) False.

Exercise 10.2

Question 1:

Compute the magnitude of the following vectors:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}; \quad \vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}; \quad \vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$$

Answer

The given vectors are:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}; \quad \vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}; \quad \vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$$

$$|\vec{a}| = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3}$$

$$|\vec{b}| = \sqrt{(2)^2 + (-7)^2 + (-3)^2}$$
$$= \sqrt{4 + 49 + 9}$$
$$= \sqrt{62}$$

$$|\vec{c}| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(-\frac{1}{\sqrt{3}}\right)^2}$$
$$= \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = 1$$

Question 2:

Write two different vectors having same magnitude.

Answer

Consider $\vec{a} = \left(\hat{i} - 2\hat{j} + 3\hat{k}\right)$ and $\vec{b} = \left(2\hat{i} + \hat{j} - 3\hat{k}\right)$.

It can be observed that $|\vec{a}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$ and

$|\vec{b}| = \sqrt{2^2 + 1^2 + (-3)^2} = \sqrt{4 + 1 + 9} = \sqrt{14}$.

Hence, $\vec{a}$ and $\vec{b}$ are two different vectors having the same magnitude. The vectors are different because they have different directions.

Question 3:

Write two different vectors having same direction.

Answer

Consider $\vec{p} = \left(\hat{i} + \hat{j} + \hat{k}\right)$ and $\vec{q} = \left(2\hat{i} + 2\hat{j} + 2\hat{k}\right)$.

The direction cosines of $\vec{p}$ are given by,

$$l = \frac{1}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}, \quad m = \frac{1}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}, \quad \text{and } n = \frac{1}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}.$$

The direction cosines of $\vec{q}$ are given by

$$l = \frac{2}{\sqrt{2^2 + 2^2 + 2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}, \quad m = \frac{2}{\sqrt{2^2 + 2^2 + 2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}},$$

$$\text{and } n = \frac{2}{\sqrt{2^2 + 2^2 + 2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}.$$

The direction cosines of $\vec{p}$ and $\vec{q}$ are the same. Hence, the two vectors have the same direction.

Question 4:

Find the values of x and y so that the vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ are equal

Answer

The two vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ will be equal if their corresponding components are equal.

Hence, the required values of x and y are 2 and 3 respectively.

Question 5:

Find the scalar and vector components of the vector with initial point (2, 1) and terminal point (−5, 7).

Answer

The vector with the initial point P (2, 1) and terminal point Q (−5, 7) can be given by,

$$\overrightarrow{PQ} = (-5-2)\hat{i} + (7-1)\hat{j}$$
$$\Rightarrow \overrightarrow{PQ} = -7\hat{i} + 6\hat{j}$$

Hence, the required scalar components are −7 and 6 while the vector components are $-7\hat{i}$ and $6\hat{j}$.

Question 6:

Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.

Answer

The given vectors are $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.

$$\therefore \vec{a} + \vec{b} + \vec{c} = (1-2+1)\hat{i} + (-2+4-6)\hat{j} + (1+5-7)\hat{k}$$
$$= 0 \cdot \hat{i} - 4\hat{j} - 1 \cdot \hat{k}$$
$$= -4\hat{j} - \hat{k}$$

Question 7:

Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$.

Answer

The unit vector $\hat{a}$ in the direction of vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ is given by $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$.

$$|\vec{a}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{1+1+4} = \sqrt{6}$$

$$\therefore \hat{a} = \dfrac{\vec{a}}{|\vec{a}|} = \dfrac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}} = \dfrac{1}{\sqrt{6}}\hat{i} + \dfrac{1}{\sqrt{6}}\hat{j} + \dfrac{2}{\sqrt{6}}\hat{k}$$

Question 8:

Find the unit vector in the direction of vector $\overrightarrow{PQ}$, where P and Q are the points (1, 2, 3) and (4, 5, 6), respectively.

Answer

The given points are P (1, 2, 3) and Q (4, 5, 6).

$$\therefore \overrightarrow{PQ} = (4-1)\hat{i} + (5-2)\hat{j} + (6-3)\hat{k} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$
$$|\overrightarrow{PQ}| = \sqrt{3^2 + 3^2 + 3^2} = \sqrt{9+9+9} = \sqrt{27} = 3\sqrt{3}$$

Hence, the unit vector in the direction of $\overrightarrow{PQ}$ is

$$\dfrac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|} = \dfrac{3\hat{i} + 3\hat{j} + 3\hat{k}}{3\sqrt{3}} = \dfrac{1}{\sqrt{3}}\hat{i} + \dfrac{1}{\sqrt{3}}\hat{j} + \dfrac{1}{\sqrt{3}}\hat{k}$$

Question 9:

For given vectors, $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$, find the unit vector in the direction of the vector $\vec{a} + \vec{b}$

Answer

The given vectors are $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$.

$$\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$$
$$\vec{b} = -\hat{i} + \hat{j} - \hat{k}$$
$$\therefore \vec{a} + \vec{b} = (2-1)\hat{i} + (-1+1)\hat{j} + (2-1)\hat{k} = 1\hat{i} + 0\hat{j} + 1\hat{k} = \hat{i} + \hat{k}$$
$$|\vec{a} + \vec{b}| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

Hence, the unit vector in the direction of $(\vec{a} + \vec{b})$ is

$$\dfrac{(\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|} = \dfrac{\hat{i} + \hat{k}}{\sqrt{2}} = \dfrac{1}{2}\hat{i} + \dfrac{1}{\sqrt{2}}\hat{k}$$

Question 10:

Find a vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units.

Answer

Let $\vec{a} = 5\hat{i} - \hat{j} + 2\hat{k}$.

$$\therefore |\vec{a}| = \sqrt{5^2 + (-1)^2 + 2^2} = \sqrt{25+1+4} = \sqrt{30}$$

$$\therefore \hat{a} = \dfrac{\vec{a}}{|\vec{a}|} = \dfrac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}}$$

Hence, the vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units is given by,

$$8\hat{a} = 8\left(\frac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}}\right) = \frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$$

$$= 8\left(\frac{5\vec{i} - \vec{j} + 2\vec{k}}{\sqrt{30}}\right)$$

$$= \frac{40}{\sqrt{30}}\vec{i} - \frac{8}{\sqrt{30}}\vec{j} + \frac{16}{\sqrt{30}}\vec{k}$$

Question 11:

Show that the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.

Answer

********** END *********