



Indefinite Integrals Ex 19.30 Q27

$$\text{Let } \frac{3x-2}{(x+1)^2(x+3)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+3)}$$

$$\begin{aligned}\Rightarrow 3x-2 &= A(x+1)(x+3) + B(x+3) + C(x+1)^2 \\ &= (A+C)x^2 + (4A+B+2C)x + (3A+3B+C)\end{aligned}$$

Equating similar terms, we get,

$$\begin{aligned}A+C &= 0 \quad \Rightarrow \quad A = -C \\ 4A+B+2C &= 3 \quad \Rightarrow \quad B = -2C = 3 \\ 3A+3B+C &= -2 \Rightarrow 3B-2C = -2\end{aligned}$$

$$\text{Solving, we get, } B = -\frac{5}{2}, C = -\frac{11}{4} \text{ \& } A = \frac{11}{4}$$

Thus,

$$I = \frac{11}{4} \int \frac{dx}{x+1} - \frac{5}{2} \int \frac{dx}{(x+1)^2} - \frac{11}{4} \int \frac{dx}{x+3}$$

$$I = \frac{11}{4} \log|x+1| + \frac{5}{2(x+1)} - \frac{11}{4} \log|x+3| + c$$

Indefinite Integrals Ex 19.30 Q28

$$\text{Let } \frac{2x+1}{(x+2)(x-3)^2} = \frac{A}{x+2} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$$

$$\begin{aligned}\Rightarrow 2x+1 &= A(x-3)^2 + B(x+2)(x-3) + C(x+2) \\ &= (A+B)x^2 + (-6A-B+C)x + (9A-6B+2C)\end{aligned}$$

Equating similar terms, we get,

$$\begin{aligned}A+B &= 0 & \Rightarrow & A = -B \\ -6A-B+C &= 2 & \Rightarrow & 5B+C=2 \\ 9A-6B+2C &= 1 & \Rightarrow & -15B+2C=1\end{aligned}$$

$$\text{Solving, we get, } B = \frac{3}{25}, C = \frac{7}{5}, A = -\frac{3}{25}$$

Thus,

$$I = -\frac{3}{25} \int \frac{dx}{x+2} + \frac{3}{25} \int \frac{dx}{x-3} + \frac{7}{5} \int \frac{dx}{(x-3)^2}$$

$$I = -\frac{3}{25} \log|x+2| + \frac{3}{25} \log|x-3| - \frac{7}{5(x-3)} + c$$

Indefinite Integrals Ex 19.30 Q29

$$\text{Let } \frac{x^2+1}{(x-2)^2(x+3)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+3}$$

$$\begin{aligned}\Rightarrow x^2+1 &= A(x-2)(x+3) + B(x+3) + C(x-2)^2 \\ &= (A+C)x^2 + (A+B-4C)x + (-6A+3B+4C)\end{aligned}$$

Equating similar terms, we get,

$$A+C=1, A+B-4C=0, -6A+3B+4C=1$$

$$\text{Solving, we get, } A = \frac{3}{5}, B = 1, C = \frac{2}{5}$$

Thus,

$$I = \frac{3}{5} \int \frac{dx}{x-2} + \int \frac{dx}{(x-2)^2} + \frac{2}{5} \int \frac{dx}{x+3}$$

$$I = \frac{3}{5} \log|x-2| - \frac{1}{(x-2)} + \frac{2}{5} \log|x+3| + c$$

Indefinite Integrals Ex 19.30 Q30

$$\text{Let } \frac{x}{(x-1)^2(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$$

$$x = A(x-1)(x+2) + B(x+2) + C(x-1)^2$$

Substituting $x = 1$, we obtain

$$B = \frac{1}{3}$$

Equating the coefficients of x^2 and constant term, we obtain

$$A + C = 0$$

$$-2A + 2B + C = 0$$

On solving, we obtain

$$A = \frac{2}{9} \text{ and } C = -\frac{2}{9}$$

$$\begin{aligned} \therefore \frac{x}{(x-1)^2(x+2)} &= \frac{2}{9(x-1)} + \frac{1}{3(x-1)^2} - \frac{2}{9(x+2)} \\ \Rightarrow \int \frac{x}{(x-1)^2(x+2)} dx &= \frac{2}{9} \int \frac{1}{(x-1)} dx + \frac{1}{3} \int \frac{1}{(x-1)^2} dx - \frac{2}{9} \int \frac{1}{(x+2)} dx \\ &= \frac{2}{9} \log|x-1| + \frac{1}{3} \left(\frac{-1}{x-1} \right) - \frac{2}{9} \log|x+2| + C \\ &= \frac{2}{9} \log \left| \frac{x-1}{x+2} \right| - \frac{1}{3(x-1)} + C \end{aligned}$$

Indefinite Integrals Ex 19.30 Q31

$$\text{Let } \frac{x^2}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$$

$$\Rightarrow x^2 = A(x+1)^2 + B(x-1)(x+1) + C(x-1) \\ = (A+B)x^2 + (2A+C)x + (A-B-C)$$

Equating similar terms,

$$A+B=1, 2A+C=0, A-B-C=0$$

$$\text{Solving, we get, } A = \frac{1}{4}, B = \frac{3}{4}, C = -\frac{1}{2}$$

Thus,

$$I = \frac{1}{4} \int \frac{dx}{x-1} + \frac{3}{4} \int \frac{dx}{x+1} - \frac{1}{2} \int \frac{dx}{(x+1)^2} \\ = \frac{1}{4} \log|x-1| + \frac{3}{4} \log|x+1| + \frac{1}{2(x+1)} + C$$

$$I = \frac{1}{4} \log|x-1| + \frac{3}{4} \log|x+1| + \frac{1}{2(x+1)} + C$$

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