Trigonometric Ratios of Compound Angles Ex 7.1 Q17

We have,

$$8\theta = 6\theta + 2\theta$$

$\Rightarrow \quad \tan 8\theta = \tan(6\theta + 2\theta)$

$\Rightarrow \quad \tan 8\theta = \dfrac{\tan 6\theta + \tan 2\theta}{1 - \tan 6\theta \tan 2\theta} \qquad \left[\because \tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}\right]$

$\Rightarrow \quad \tan 8\theta(1 - \tan 6\theta \tan 2\theta) = \tan 6\theta + \tan 2\theta$

$\Rightarrow \quad \tan 8\theta - \tan 8\theta \tan 6\theta \tan 2\theta = \tan 6\theta + \tan 2\theta$

$\Rightarrow \quad \tan 8\theta - \tan 6\theta - \tan 2\theta = \tan 8\theta \tan 6\theta \tan 2\theta$

Hence proved.

We have,

$$45° = 30° + 15°$$

$\Rightarrow \quad \tan 45° = \tan(30° + 15°)$

$\Rightarrow \quad 1 = \dfrac{\tan 30° + \tan 15°}{1 - \tan 30° \tan 15°} \qquad \left[\because \tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}\right]$

$\Rightarrow \quad 1 - \tan 30° \tan 15° = \tan 15° + \tan 30°$

$\Rightarrow \quad 1 = \tan 15° + \tan 30° + \tan 30° \tan 15°$

$\Rightarrow \quad \tan 15° + \tan 30° + \tan 15° \tan 30° = 1$

Hence proved.

We have,

$$45° = 9° + 36°$$

$\Rightarrow \quad \tan 45° = \tan(9° + 36°)$

$\Rightarrow \quad 1 = \dfrac{\tan 9° + \tan 36°}{1 - \tan 9° \tan 36°} \qquad \left[\because \tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}\right]$

$\Rightarrow \quad 1 - \tan 9° \tan 36° = \tan 9° + \tan 36°$

$\Rightarrow \quad 1 = \tan 9° + \tan 36° + \tan 9° \tan 36°$

$\Rightarrow \quad \tan 9° + \tan 36° + \tan 9° \tan 36° = 1$

Hence proved.

We have,

$$13\theta = 9\theta + 4\theta$$

$\Rightarrow \quad \tan 13\theta = \tan(9\theta + 4\theta)$

$\Rightarrow \quad \tan 13\theta = \dfrac{\tan 9\theta + \tan 4\theta}{1 - \tan 9\theta \tan 4\theta} \qquad \left[\because \tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}\right]$

$\Rightarrow \quad \tan 13\theta(1 - \tan 9\theta \tan 4\theta) = \tan 9\theta + \tan 4\theta$

$\Rightarrow \quad \tan 13\theta - \tan 13\theta \tan 9\theta \tan 4\theta = \tan 9\theta + \tan 4\theta$

$\Rightarrow \quad \tan 13\theta - \tan 9\theta - \tan 4\theta = \tan 13\theta \tan 9\theta \tan 4\theta$

Hence proved.

Trigonometric Ratios of Compound Angles Ex 7.1 Q18

We have,

RHS $= \tan 3\theta \tan\theta$

$= \tan(2\theta+\theta) \times \tan(2\theta-\theta)$

$= \left[\dfrac{\tan 2\theta + \tan\theta}{1 - \tan 2\theta \tan\theta}\right] \times \left[\dfrac{\tan 2\theta - \tan\theta}{1 + \tan 2\theta \tan\theta}\right]$

$= \dfrac{(\tan 2\theta + \tan\theta)(\tan 2\theta - \tan\theta)}{(1 - \tan 2\theta \tan\theta)(1 + \tan 2\theta \tan\theta)}$

$= \dfrac{\tan^2 2\theta - \tan^2\theta}{1 - \tan^2 2\theta \tan^2\theta}$ $\left[\because (a-b)(a+b) = a^2 - b^2\right]$

= LHS

$\therefore$ LHS = RHS

Hence proved

Trigonometric Ratios of Compound Angles Ex 7.1 Q19

$\dfrac{\sin x.\cos y + \sin y.\cos x}{\sin x.\cos y - \sin y.\cos x} = \dfrac{a+b}{a-b}$

$\Rightarrow \dfrac{\sin x.\cos y + \sin y.\cos x + \sin x.\cos y - \sin y.\cos x}{\sin x.\cos y + \sin y.\cos x - \sin x.\cos y + \sin y.\cos x} = \dfrac{a+b+a-b}{a+b-a+b}$ [*Using Componendo and Dividendo*]

$\Rightarrow \dfrac{2\sin x.\cos y}{2\sin y.\cos x} = \dfrac{2a}{2b}$

$\Rightarrow \dfrac{\tan x}{\tan y} = \dfrac{a}{b}$

Hence Proved

*********** END **********