



Indefinite Integrals Ex 19.3 Q16

Let $I = \int \frac{1 + \cos 4x}{\cot x - \tan x} \times dx$. Then,

$$\begin{aligned}
 I &= \int \frac{2 \cos^2 2x}{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}} dx \\
 &= \int \frac{2 \cos^2 2x}{\frac{\cos^2 x - \sin^2 x}{\sin x \cos x}} dx \\
 &= \int \frac{2 \cos^2 2x \times \sin x \cos x}{\cos^2 x - \sin^2 x} dx \\
 &= \int \frac{\cos^2 2x \times \sin 2x}{\cos^2 2x} dx \\
 &= \int \cos 2x \times \sin 2x \times dx \\
 &= \frac{1}{2} \int 2 \sin 2x \cos 2x dx \\
 &= \frac{1}{2} \int [\sin(2x + 2x) + \sin(2x - 2x)] dx \\
 &= \frac{1}{2} \int (\sin 4x + \sin 0) dx \\
 &= \frac{1}{2} \int (\sin 4x + 0) dx \\
 &= \frac{1}{2} \int \sin 4x \\
 &= -\frac{1}{2} \times \frac{\cos 4x}{4} + C \\
 &= -\frac{1}{8} \times \cos 4x + C
 \end{aligned}$$

Indefinite Integrals Ex 19.3 Q17

Let $I = \int \frac{1}{\sqrt{x+3} - \sqrt{x+2}} dx$. Then,

$$\begin{aligned} I &= \int \frac{1}{\sqrt{x+3} - \sqrt{x+2}} \times \frac{\sqrt{x+3} + \sqrt{x+2}}{\sqrt{x+3} + \sqrt{x+2}} dx \\ &= \int \frac{\sqrt{x+3} + \sqrt{x+2}}{x+3-x-2} dx \\ &= \int \left[(x+3)^{\frac{1}{2}} + (x+2)^{\frac{1}{2}} \right] dx \\ &= \frac{(x+3)^{\frac{3}{2}}}{\frac{3}{2}} + \frac{(x+2)^{\frac{3}{2}}}{\frac{3}{2}} + C \\ &= \frac{2}{3} \times (x+3)^{\frac{3}{2}} + \frac{2}{3} (x+2)^{\frac{3}{2}} + C \\ &= \frac{2}{3} \left\{ (x+3)^{\frac{3}{2}} + (x+2)^{\frac{3}{2}} \right\} + C \end{aligned}$$

$$\therefore I = \frac{2}{3} \left\{ (x+3)^{\frac{3}{2}} + (x+2)^{\frac{3}{2}} \right\} + C$$

Indefinite Integrals Ex 19.3 Q18

$$\tan^2(2x-3) = \sec^2(2x-3) - 1$$

$$\text{Let } 2x - 3 = t$$

$$\Rightarrow 2dx = dt$$

$$\begin{aligned} \Rightarrow \int \tan^2(2x-3) dx &= \int \left[\sec^2(2x-3) - 1 \right] dx \\ &= \frac{1}{2} \int (\sec^2 t) dt - \int 1 dx \\ &= \frac{1}{2} \int \sec^2 t dt - \int 1 dx \\ &= \frac{1}{2} \tan t - x + C \\ &= \frac{1}{2} \tan(2x-3) - x + C \end{aligned}$$

Indefinite Integrals Ex 19.3 Q19

$$\text{Consider } I = \int \frac{1}{\cos^2 x (1 - \tan x)^2} dx$$

$$= \int \frac{1}{\cos^2 x \left(1 - \frac{\sin x}{\cos x}\right)^2} dx$$

$$= \int \frac{1}{(\cos x - \sin x)^2} dx$$

$$= \int \frac{1}{1 - \sin 2x} dx$$

$$= \int \frac{1}{1 + \cos\left(\frac{\pi}{2} + 2x\right)} dx$$

$$= \int \frac{1}{2 \cos^2\left(\frac{\pi}{4} + x\right)} dx$$

$$= \frac{1}{2} \int \sec^2\left(\frac{\pi}{4} + x\right) dx$$

***** END *****