Algebra of Matrices Ex 5.3 Q55

We have,

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

Then ,

$$A^2 = A.A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 2\times2+0\times2+1\times1 & 2\times0+0\times1+1\times-1 & 2\times1+0\times3+1\times0 \\ 2\times2+1\times2+3\times1 & 2\times0+0\times1+1\times-1 & 2\times1+1\times3+3\times0 \\ 1\times2+-1\times2+0\times1 & 1\times0+-1\times1+0\times-1 & 1\times1+-1\times3+0\times0 \end{bmatrix}.$$

$$-5A = \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix},\quad 4I = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Hence, $A^2-5A+4I = \begin{bmatrix} 5 & -1 & 5 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} + \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$$A^2-5A+4I = \begin{bmatrix} 5-10+4 & -1+0+0 & 5-5+0 \\ 9-10+0 & -2-5+4 & 5-15+0 \\ 0-5-0 & -1+5+0 & -2+0+4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$$

Now, given is $A^2-5A+4I+X=O$

$\Rightarrow X = -(A^2-5A+4I)$

$$X = -\begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & 4 & 2 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q56

Given,

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

To prove $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ we will use the principle of mathematical induction.

Step 1: Put $n = 1$

$$A^1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

So,

A^n is true for $n = 1$

Step 2: Let, A^n be true for $n = k$, then

$$A^k = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \qquad \text{---(i)}$$

Step 3: We have to show that $A^{k+1} = \begin{bmatrix} 1 & k+1 \\ 0 & 1 \end{bmatrix}$

So,

$$A^{k+1} = A^k \times A$$

$$= \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \qquad \{\text{using equation (i) and given}\}$$

$$= \begin{bmatrix} 1+0 & 1+k \\ 0+0 & 0+1 \end{bmatrix}$$

$$A^{k+1} = \begin{bmatrix} 1 & 1+k \\ 0 & 1 \end{bmatrix}$$

This shows that A^n is true for $n = k+1$ whenever it is true for $n = k$

Hence, by the principle of mathematical induction A^n is true for all positive integer.

*********** END **********