Tangents and Normals Ex 16.3 Q2(i)

We know that two curves intersects orthogonally if

$$m_1 \times m_2 = -1 \qquad \text{---(A)}$$

Where $m_1$ and $m_2$ are the slopes of two curves

$$y = x^3 \qquad \text{---(i)}$$

$$6y = 7 - x^2 \qquad \text{---(ii)}$$

Slope of (i)

$$\frac{dy}{dx} = 3x^2 = m_1$$

Slope of (ii)

$$\frac{dy}{dx} = -\frac{2}{6}x = m_2$$

Point of intersection of (i) and (ii) is

$$6x^3 = 7 - x^2$$

$$\Rightarrow \quad 6x^3 + x^2 - 7 = 0$$

$$\Rightarrow \quad x = 1$$

$$\therefore \quad y = 1$$

$$\therefore \quad P = (1, 1)$$

$$\therefore \quad m_1 = 3 \text{ and } m_2 = -\frac{1}{3}$$

Now,

$$m_1 \times m_2 = 3 \times -\frac{1}{3} = -1$$

$\therefore$ (i) and (ii) cuts orthogonally.

Tangents and Normals Ex 16.3 Q2(ii)

We know that two curves intersects orthogonally if

$$m_1 \times m_2 = -1 \qquad \text{---(A)}$$

Where $m_1$ and $m_2$ are the slopes of two curves

$$x^3 - 3xy^2 = -2 \qquad \text{---(i)}$$
$$3x^2y - y^3 = 2 \qquad \text{---(ii)}$$

Point of intersection of (i) and (ii)

$$(i) + (ii)$$
$$\Rightarrow \quad x^3 - 3xy^2 + 3x^2y - y^3 = 0$$
$$\Rightarrow \quad (x - y)^3 = 0$$
$$\Rightarrow \quad x = y$$

$\therefore$ from (i)

$$x^3 - 3x^2 = -2$$
$$\Rightarrow \quad -2x^3 = -2$$
$$\Rightarrow \quad x = 1$$

$\therefore$ $P = (1,1)$ is the point of intersection

Now,

Slope of (i)

$$3x^2 - 3y^2 - 6xy\frac{dy}{dx} = 0$$
$$\therefore \quad m_1 = \frac{dy}{dx} = \frac{3\left(x^2 - y^2\right)}{6xy}$$

Slope of (ii)

$$6xy + 3x^2\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 0$$
$$\Rightarrow \quad m_2 = \frac{dy}{dx} = \frac{-6xy}{3\left(x^2 - y^2\right)}$$

$$\therefore \quad m_1 \times m_2 = \frac{\left(x^2 - y^2\right)}{2xy} \times \frac{-2xy}{\left(x^2 - y^2\right)} = -1$$

Tangents and Normals Ex 16.3 Q2(iii)

We know that two curves intersects orthogonally if

$$m_1 \times m_2 = -1 \qquad \text{---(A)}$$

Where $m_1$ and $m_2$ are the slopes of two curves

$$x^2 + 4y^2 = 8 \qquad \text{---(i)}$$

$$x^2 - 2y^2 = 4 \qquad \text{---(ii)}$$

Point of intersection of (i) and (ii) is (i) − (ii), we get

$$6y^2 = 4$$

$$\Rightarrow \quad y = \sqrt{\frac{2}{3}}$$

$$\therefore \quad x^2 = 4 + \frac{8}{6}$$

$$x^2 = \frac{32}{6}$$

$$\Rightarrow \quad x = \frac{4}{\sqrt{3}}$$

Now,

Slope of (i)

$$2x + 8y\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{x}{4y}$$

$$\Rightarrow \quad m_1 = -\frac{1}{4} \times \frac{4}{\sqrt{2}} = -\frac{1}{\sqrt{2}} \qquad \left[\because \frac{x}{y} = \frac{4}{\sqrt{2}}\right]$$

Slope of (ii)

$$2x - 4y\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x}{2y}$$

$$\Rightarrow \quad m_2 = \frac{1}{2} \times \frac{4}{\sqrt{2}} = \sqrt{2}$$

$$\therefore \quad m_1 \times m_2 = -\frac{1}{\sqrt{2}} \times \sqrt{2} = -1$$

$\therefore$ (i) and (ii) cuts orthogonally.

********** END *********