Exercise 13.3

Unless stated otherwise, take $\pi = \dfrac{22}{7}$.

1. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Ans. For sphere, Radius $(r) = 4.2$ cm

Volume $= \dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi(4.2)^3$ cm^3

For cylinder, Radius (R) = 6 cm

Let the height of the cylinder be H cm.

Then, Volume $= \pi R^2 H = \pi(6)^2 H$ cm^3

According to question, Volume of sphere = Volume of cylinder

$\Rightarrow \dfrac{4}{3}\pi(4.2)^3 = \pi(6)^2 H$

$\Rightarrow H = \dfrac{4(4.2)^3}{3(6)^2}$

$\Rightarrow H = 2.74$ cm

2. Metallic spheres of radii 6 cm, 8 cm and 10 cm respectively are melted to form a single solid sphere. Find the radius of the resulting sphere.

Ans. Let the volume of resulting sphere be r cm.

According to question,

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3 + \frac{4}{3}\pi(8)^3 + \frac{4}{3}\pi(10)^3$$

$$\Rightarrow r^3 = (6)^3 + (8)^3 + (10)^3$$

$$\Rightarrow r^3 = 216 + 512 + 1000$$

$$\Rightarrow r^3 = 1728$$

$$\Rightarrow r = 12 \text{ cm}$$

3. A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

Ans. Diameter of well = 7 m

$\therefore$ Radius of well $(r) = \dfrac{7}{2}$ m

And Depth of earth dug (h) = 20 m

Length of platform (l) = 22 m, Breadth of platform (b) = 14 m

Let height of the platform be h' m

According to question,

Volume of earth dug = Volume of platform

$$\Rightarrow \pi r^2 h = l \times b \times h'$$

$$\Rightarrow \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20 = 22 \times 14 \times h'$$

$$\Rightarrow h' = \frac{22 \times 7 \times 7}{7 \times 2 \times 2 \times 22 \times 14}$$

$$\Rightarrow h' = 2.5 \text{ m}$$

4. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

Ans. Diameter of well = 3 m

∴ Radius of well $(r) = \dfrac{3}{2}$ m and Depth of earth dug

$(h) = 14$ m

Width of the embankment = 4 m

∴ Radius of the well with embankment $r' = \dfrac{3}{2} + 4 = \dfrac{11}{2}$ m

Let the height of the embankment be h' m

According to the question,

Volume of embankment = Volume of the earth dug

$\Rightarrow \pi\left[(r')^2 - r^2\right]h' = \pi r^2 h$

$\Rightarrow \left[\left(\dfrac{11}{2}\right)^2 - \left(\dfrac{3}{2}\right)^2\right]h' = \left(\dfrac{3}{2}\right)^2 \times 14$

$\Rightarrow \left[\dfrac{121}{4} - \dfrac{9}{4}\right]h' = \dfrac{9}{4} \times 14$

$\Rightarrow \dfrac{112}{4} \times h' = \dfrac{9}{4} \times 14$

$\Rightarrow h' = \dfrac{9 \times 14 \times 4}{4 \times 112}$

$\Rightarrow h' = 1.125$ m

5. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

Ans. For right circular cylinder, Diameter = 12 cm

∴ Radius (r) = $\dfrac{12}{2}$ = 6 cm and height (h) = 15 cm

For cone, Diameter = 6 cm

∴ Radius (r_1) = $\dfrac{6}{2}$ = 3 cm and height (h_1) = 12 cm

Let n cones be filled with ice cream.

Then, According to question,

Volume of n cones = Volume of right circular cylinder

$\Rightarrow n.\dfrac{1}{3}\pi r_1^2 h_1 = \pi r^2 h$

$\Rightarrow n \times \dfrac{1}{3} \times \dfrac{22}{7} \times (3)^2 \times 12 = \dfrac{22}{7}(6)^2 \times 15$

$\Rightarrow n = \dfrac{22 \times 6 \times 6 \times 15 \times 3 \times 7}{7 \times 22 \times 3 \times 3 \times 12}$

$\Rightarrow n = 15$

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