



Indefinite Integrals Ex 19.2 Q43

$$\begin{aligned}\frac{1 - \cos x}{1 + \cos x} &= \frac{2 \sin^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}} & \left[2 \sin^2 \frac{x}{2} = 1 - \cos x \text{ and } 2 \cos^2 \frac{x}{2} = 1 + \cos x \right] \\ &= \tan^2 \frac{x}{2} \\ &= \left(\sec^2 \frac{x}{2} - 1 \right) \\ \therefore \int \frac{1 - \cos x}{1 + \cos x} dx &= \int \left(\sec^2 \frac{x}{2} - 1 \right) dx \\ &= \left[\frac{\tan \frac{x}{2}}{1/2} - x \right] + C \\ &= 2 \tan \frac{x}{2} - x + C\end{aligned}$$

Indefinite Integrals Ex 19.2 Q44

$$\begin{aligned}&\int \left\{ 3 \sin x - 4 \cos x + \frac{5}{\cos^2 x} - \frac{6}{\sin^2 x} + \tan^2 x - \cot^2 x \right\} dx \\ &= 3 \int \sin x dx - 4 \int \cos x dx + 5 \int \sec^2 x dx - 6 \int \operatorname{cosec}^2 x dx + \int \tan^2 x dx - \int \cot^2 x dx \\ &= 3 \int \sin x dx - 4 \int \cos x dx + 5 \int \sec^2 x dx - 6 \int \operatorname{cosec}^2 x dx + \int (\sec^2 x - 1) dx - \int (\operatorname{cosec}^2 x - 1) dx \\ &= 3 \int \sin x dx - 4 \int \cos x dx + 5 \int \sec^2 x dx - 7 \int \operatorname{cosec}^2 x dx \\ &= -3 \cos x - 4 \sin x + 5 \tan x + 7 \cot x + c\end{aligned}$$

Indefinite Integrals Ex 19.2 Q45

It is given that $f'(x) = x - \frac{1}{x^2}$

$$\therefore \int f'(x) = \int \left(x - \frac{1}{x^2} \right) dx$$

$$\Rightarrow f(x) = \int x dx - \int \frac{1}{x^2} dx$$

$$= \frac{x^2}{2} + x^{-1} + c$$

$$= \frac{x^2}{2} + \frac{1}{x} + c$$

$$\Rightarrow f(x) = \frac{x^2}{2} + \frac{1}{x} + c \quad \text{---(i)}$$

Now,

$$f(1) = \frac{1}{2} \quad \text{[given]}$$

$$\Rightarrow \frac{1^2}{2} + \frac{1}{1} + c = \frac{1}{2}$$

$$\Rightarrow c = -1$$

Putting $c = -1$ in (i), we get

$$f(x) = \frac{x^2}{2} + \frac{1}{x} - 1.$$

Indefinite Integrals Ex 19.2 Q46

It is given that $f'(x) = x + b$

$$\therefore \int f'(x) = \int (x + b) dx$$

$$\Rightarrow f(x) = \frac{x^2}{2} + bx + c \quad \text{---(i)}$$

Since,

$$f(1) = 5$$

$$\therefore \frac{1^2}{2} + b \times 1 + c = 5$$

$$\Rightarrow \frac{1}{2} + b + c = 5$$

$$\Rightarrow b + c = \frac{9}{2} \quad \text{---(ii)}$$

$$\text{and, } f(2) = 13$$

$$\Rightarrow \frac{(2)^2}{2} + b \times 2 + c = 13$$

$$\Rightarrow 2 + 2b + c = 13$$

$$\Rightarrow 2b + c = 11 \quad \text{---(iii)}$$

Subtracting equation (ii) from equation (iii), we get

$$b = 11 - \frac{9}{2}$$

$$\Rightarrow b = \frac{13}{2}$$

Putting $b = \frac{13}{2}$ in equation (ii), we get

$$\frac{13}{2} + c = \frac{9}{2}$$

$$\Rightarrow c = \frac{9}{2} - \frac{13}{2}$$

$$\Rightarrow c = \frac{9 - 13}{2} = \frac{-4}{2} = -2$$

Putting $b = \frac{13}{2}$ and $c = -2$ in equation (i), we get

$$f(x) = \frac{x^2}{2} + \frac{13}{2}x - 2$$

$$\therefore f(x) = \frac{x^2}{2} + \frac{13}{2}x - 2$$

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