The values of z at these corner points are as follows.

Corner point	$z = 16x + 20y$	
A (10, 0)	160	
B (2, 4)	112	→ Minimum
C (1, 5)	116	
D (0, 8)	160	

As the feasible region is unbounded, therefore, 112 may or may not be the minimum value of z.

For this, we draw a graph of the inequality, $16x + 20y < 112$ or $4x + 5y < 28$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $4x + 5y < 28$

Therefore, the minimum value of z is 112 at (2, 4).

Thus, the mixture should contain 2 kg of food X and 4 kg of food Y. The minimum cost of the mixture is Rs 112.

Question 4:

A manufacturer makes two types of toys A and B. Three machines are needed for this purpose and the time (in minutes) required for each toy on the machines is given below:

Type of toys	Machines		
	I	II	III
A	12	18	6
B	6	0	9

Each machine is available for a maximum of 6 hours per day. If the profit on each toy of type A is Rs 7.50 and that on each toy of type B is Rs 5, show that 15 toys of type A and 30 of type B should be manufactured in a day to get maximum profit.

Answer

Let x and y toys of type A and type B respectively be manufactured in a day.

The given problem can be formulated as follows.

Maximize $z = 7.5x + 5y$ ... (1)

subject to the constraints,

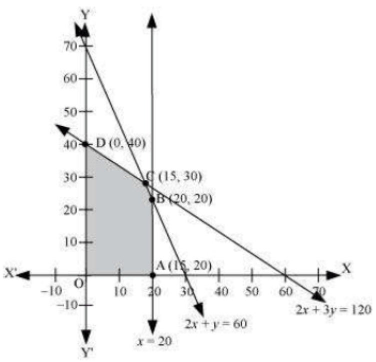
$$2x + y \le 60 \quad \text{...(2)}$$

$$x \le 20 \quad \text{...(3)}$$

$$2x + 3y \le 120 \quad \text{...(4)}$$

$$x, y \ge 0 \quad \text{...(5)}$$

The feasible region determined by the constraints is as follows.



The corner points of the feasible region are A (20, 0), B (20, 20), C (15, 30), and D (0, 40).

The values of z at these corner points are as follows.

Corner point	Z = $7.5x + 5y$	
A(20, 0)	150	
B(20, 20)	250	

C(15, 30)	262.5	→ Maximum
O(0, 40)	200	

The maximum value of z is 262.5 at (15, 30).

Thus, the manufacturer should manufacture 15 toys of type A and 30 toys of type B to maximize the profit.

Question 5:

An aeroplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit?

Answer

Let the airline sell x tickets of executive class and y tickets of economy class.

The mathematical formulation of the given problem is as follows.

Maximize $z = 1000x + 600y$ ... (1)

subject to the constraints,

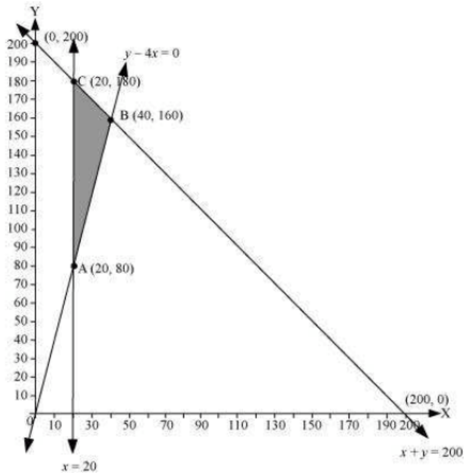
$$x + y \le 200 \quad \text{...(2)}$$

$$x \ge 20 \quad \text{...(3)}$$

$$y - 4x \ge 0 \quad \text{...(4)}$$

$$x, y \ge 0 \quad \text{...(5)}$$

The feasible region determined by the constraints is as follows.



The corner points of the feasible region are A (20, 80), B (40, 160), and C (20, 180).

The values of z at these corner points are as follows.

Corner point	$z = 1000x + 600y$	
A (20, 80)	68000	
B (40, 160)	136000	→ Maximum
C (20, 180)	128000	

The maximum value of z is 136000 at (40, 160).

Thus, 40 tickets of executive class and 160 tickets of economy class should be sold to maximize the profit and the maximum profit is Rs 136000.

Question 6:

Two godowns A and B have grain capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops, D, E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops are given in the following table:

Transportation cost per quintal (in Rs)		
From/To	**A**	**B**
D	6	4
E	3	2
F	2.50	3

How should the supplies be transported in order that the transportation cost is minimum? What is the minimum cost?

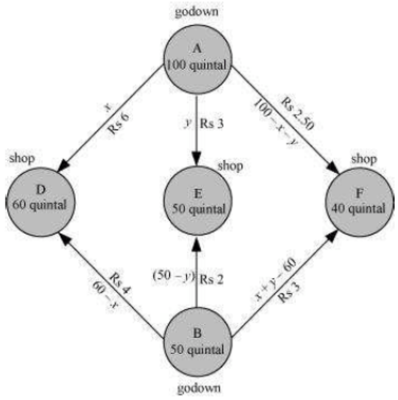
Answer

Let godown A supply x and y quintals of grain to the shops D and E respectively. Then, $(100 - x - y)$ will be supplied to shop F.

The requirement at shop D is 60 quintals since x quintals are transported from godown A. Therefore, the remaining $(60 - x)$ quintals will be transported from godown B.

Similarly, $(50 - y)$ quintals and $40 - (100 - x - y) = (x + y - 60)$ quintals will be transported from godown B to shop E and F respectively.

The given problem can be represented diagrammatically as follows.



$x \geq 0,\ y \geq 0,$ and $100 - x - y \geq 0$

$\Rightarrow x \geq 0,\ y \geq 0,$ and $x + y \leq 100$

*********** END **********