EXERCISE.10.3

Question-1

Reduce the following equations into slope-intercept form and find their slopes and the y-intercepts.

(i) $x + 7y = 0$ (ii) $6x + 3y - 5 = 0$ (iii) $y = 0$

Ans.

(i) The given equation is $x + 7y = 0$.

It can be written as

$$y = -\frac{1}{7}x + 0 \qquad \ldots(1)$$

This equation is of the form $y = mx + c$, where $m = -\frac{1}{7}$ and $c = 0$.

Therefore, equation (1) is in the slope-intercept form, where the slope and the y-intercept are $-\frac{1}{7}$ and 0 respectively.

(ii) The given equation is $6x + 3y - 5 = 0$.

It can be written as

$$y = \frac{1}{3}\left(-6x + 5\right)$$

$$y = -2x + \frac{5}{3} \qquad \ldots(2)$$

This equation is of the form $y = mx + c$, where $m = -2$ and $c = \frac{5}{3}$.

Therefore, equation (2) is in the slope-intercept form, where the slope and the y-intercept are -2 and $\frac{5}{3}$ respectively.

(iii) The given equation is $y = 0$.

It can be written as

$y = 0.x + 0$ ... (3)

This equation is of the form $y = mx + c$, where $m = 0$ and $c = 0$.

Therefore, equation (3) is in the slope-intercept form, where the slope and the y-intercept are 0 and 0 respectively.

Question-2

Reduce the following equations into intercept form and find their intercepts on the axes.

(i) $3x + 2y - 12 = 0$ (ii) $4x - 3y = 6$ (iii) $3y + 2 = 0$.

Ans.

(i) The given equation is 3x + 2y − 12 = 0.

It can be written as

$$3x + 2y = 12$$
$$\frac{3x}{12} + \frac{2y}{12} = 1$$

i.e., $\dfrac{x}{4} + \dfrac{y}{6} = 1$ ...(1)

This equation is of the form $\dfrac{x}{a} + \dfrac{y}{b} = 1$, where $a = 4$ and $b = 6$.

Therefore, equation (1) is in the intercept form, where the intercepts on the x and y axes are 4 and 6 respectively.

(ii) The given equation is 4x − 3y = 6.

It can be written as

$$\frac{4x}{6} - \frac{3y}{6} = 1$$
$$\frac{2x}{3} - \frac{y}{2} = 1$$

i.e., $\dfrac{x}{\left(\dfrac{3}{2}\right)} + \dfrac{y}{(-2)} = 1$ ...(2)

This equation is of the form $\dfrac{x}{a} + \dfrac{y}{b} = 1$, where $a = \dfrac{3}{2}$ and $b = -2$.

Therefore, equation (2) is in the intercept form, where the intercepts on the x and y axes are $\dfrac{3}{2}$ and –2 respectively.

(iii) The given equation is 3y + 2 = 0.

It can be written as

$$3y = -2$$

i.e., $\dfrac{y}{\left(-\dfrac{2}{3}\right)} = 1$ ...(3)

This equation is of the form $\dfrac{x}{a} + \dfrac{y}{b} = 1$, where $a = 0$ and $b = -\dfrac{2}{3}$.

Therefore, equation (3) is in the intercept form, where the intercept on the y-axis is $-\dfrac{2}{3}$ and it has no intercept on the x-axis.

Question-3

Reduce the following equations into normal form. Find their perpendicular distances from the origin and angle between perpendicular and the positive x-axis.

(i) $x - \sqrt{3}y + 8 = 0$ (ii) y − 2 = 0 (iii) x − y = 4

Ans.

(i) The given equation is $x - \sqrt{3}y + 8 = 0$.

It can be reduced as:

$$x - \sqrt{3}y = -8$$
$$\Rightarrow -x + \sqrt{3}y = 8$$

On dividing both sides by $\sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{4} = 2$, we obtain

$$-\frac{x}{2} + \frac{\sqrt{3}}{2}y = \frac{8}{2}$$
$$\Rightarrow \left(-\frac{1}{2}\right)x + \left(\frac{\sqrt{3}}{2}\right)y = 4$$
$$\Rightarrow x\cos 120° + y\sin 120° = 4 \qquad \dots(1)$$

Equation (1) is in the normal form.

On comparing equation (1) with the normal form of equation of line

$x \cos \omega + y \sin \omega = p$, we obtain $\omega = 120°$ and $p = 4$.

Thus, the perpendicular distance of the line from the origin is 4, while the angle between the perpendicular and the positive x-axis is 120°.

(ii) The given equation is $y - 2 = 0$.

It can be reduced as $0.x + 1.y = 2$

On dividing both sides by $\sqrt{0^2 + 1^2} = 1$, we obtain $0.x + 1.y = 2$

$\Rightarrow x \cos 90° + y \sin 90° = 2 \dots$ (1)

Equation (1) is in the normal form.

On comparing equation (1) with the normal form of equation of line

$x \cos \omega + y \sin \omega = p$, we obtain $\omega = 90°$ and $p = 2$.

Thus, the perpendicular distance of the line from the origin is 2, while the angle between the perpendicular and the positive x-axis is 90°.

(iii) The given equation is $x - y = 4$.

It can be reduced as $1.x + (-1) y = 4$

On dividing both sides by $\sqrt{1^2 + (-1)^2} = \sqrt{2}$, we obtain

$$\frac{1}{\sqrt{2}}x + \left(-\frac{1}{\sqrt{2}}\right)y = \frac{4}{\sqrt{2}}$$
$$\Rightarrow x\cos\left(2\pi - \frac{\pi}{4}\right) + y\sin\left(2\pi - \frac{\pi}{4}\right) = 2\sqrt{2}$$
$$\Rightarrow x\cos 315° + y\sin 315° = 2\sqrt{2} \qquad \dots(1)$$

Equation (1) is in the normal form.

On comparing equation (1) with the normal form of equation of line

$x \cos \omega + y \sin \omega = p$, we obtain $\omega = 315°$ and $p = 2\sqrt{2}$.

Thus, the perpendicular distance of the line from the origin is $2\sqrt{2}$, while the angle between the perpendicular and the positive x-axis is 315°.

Question-4
Find the distance of the point (−1, 1) from the line 12(x + 6) = 5(y − 2).

Ans.

The given equation of the line is $12(x + 6) = 5(y - 2)$.

$\Rightarrow 12x + 72 = 5y - 10$

$\Rightarrow 12x - 5y + 82 = 0 \dots (1)$

On comparing equation (1) with general equation of line $Ax + By + C = 0$, we obtain $A = 12$, $B = -5$, and $C = 82$.

It is known that the perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by $d = \dfrac{\left|Ax_1 + By_1 + C\right|}{\sqrt{A^2 + B^2}}$.

The given point is $(x_1, y_1) = (-1, 1)$.

Therefore, the distance of point $(-1, 1)$ from the given line

$= \dfrac{\left|12(-1) + (-5)(1) + 82\right|}{\sqrt{(12)^2 + (-5)^2}}$ units $= \dfrac{\left|-12 - 5 + 82\right|}{\sqrt{169}}$ units $= \dfrac{\left|65\right|}{13}$ units $= 5$ units

Question-5

Find the points on the x-axis, whose distances from the line $\dfrac{x}{3} + \dfrac{y}{4} = 1$ are 4 units.

Ans.

The given equation of line is

$\dfrac{x}{3} + \dfrac{y}{4} = 1$

or, $4x + 3y - 12 = 0 \qquad \dots (1)$

On comparing equation (1) with general equation of line $Ax + By + C = 0$, we obtain $A = 4$, $B = 3$, and $C = -12$.

Let $(a, 0)$ be the point on the x-axis whose distance from the given line is 4 units.

It is known that the perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by $d = \dfrac{\left|Ax_1 + By_1 + C\right|}{\sqrt{A^2 + B^2}}$.

Therefore,

$4 = \dfrac{\left|4a + 3 \times 0 - 12\right|}{\sqrt{4^2 + 3^2}}$

$\Rightarrow 4 = \dfrac{\left|4a - 12\right|}{5}$

$\Rightarrow \left|4a - 12\right| = 20$

$\Rightarrow \pm(4a - 12) = 20$

$\Rightarrow (4a - 12) = 20$ or $-(4a - 12) = 20$

$\Rightarrow 4a = 20 + 12$ or $4a = -20 + 12$

$\Rightarrow a = 8$ or -2

Thus, the required points on the x-axis are $(-2, 0)$ and $(8, 0)$.

Question-6

Find the distance between parallel lines

(i) $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$

(ii) $l(x + y) + p = 0$ and $l(x + y) - r = 0$

Ans.

It is known that the distance (d) between parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by $d = \dfrac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$.

(i) The given parallel lines are $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$.

Here, $A = 15, B = 8, C_1 = -34$, and $C_2 = 31$.

Therefore, the distance between the parallel lines is

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}} = \frac{|-34 - 31|}{\sqrt{(15)^2 + (8)^2}} \text{ units} = \frac{|-65|}{17} \text{ units} = \frac{65}{17} \text{ units}$$

(ii) The given parallel lines are $l(x + y) + p = 0$ and $l(x + y) - r = 0$.

$lx + ly + p = 0$ and $lx + ly - r = 0$

Here, $A = l, B = l, C_1 = p$, and $C_2 = -r$.

Therefore, the distance between the parallel lines is

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}} = \frac{|p + r|}{\sqrt{l^2 + l^2}} \text{ units} = \frac{|p + r|}{\sqrt{2l^2}} \text{ units} = \frac{|p + r|}{l\sqrt{2}} \text{ units} = \frac{1}{\sqrt{2}}\left|\frac{p + r}{l}\right| \text{ units}$$

Question-7

Find equation of the line parallel to the line $3x - 4y + 2 = 0$ and passing through the point ($-2, 3$).

Ans.

The equation of the given line is

$$3x - 4y + 2 = 0$$
$$\text{or } y = \frac{3x}{4} + \frac{2}{4}$$

$$\text{or } y = \frac{3}{4}x + \frac{1}{2}, \text{ which is of the form } y = mx + c$$

$\therefore$ Slope of the given line $= \dfrac{3}{4}$

It is known that parallel lines have the same slope.

$\therefore$ Slope of the other line = $m = \dfrac{3}{4}$

Now, the equation of the line that has a slope of $\dfrac{3}{4}$ and passes through the point ($-2, 3$) is

$$(y - 3) = \frac{3}{4}\{x - (-2)\}$$
$$4y - 12 = 3x + 6$$

i.e., $3x - 4y + 18 = 0$

Question-8

Find equation of the line perpendicular to the line $x - 7y + 5 = 0$ and having x intercept 3.

Ans.

The given equation of line is $x - 7y + 5 = 0$.

Or, $y = \dfrac{1}{7}x + \dfrac{5}{7}$, which is of the form $y = mx + c$

$\therefore$ Slope of the given line $= \dfrac{1}{7}$

The slope of the line perpendicular to the line having a slope of $\dfrac{1}{7}$ is

$$m = -\dfrac{1}{\left(\dfrac{1}{7}\right)} = -7$$

The equation of the line with slope –7 and x-intercept 3 is given by

$y = m\,(x - d)$

$\Rightarrow y = -7\,(x - 3)$

$\Rightarrow y = -7x + 21$

$\Rightarrow 7x + y = 21$

Question-9
Find angles between the lines $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$

Ans.

The given lines are $\sqrt{3}x + y = 1$ and $x + \sqrt{3}y = 1$.

$$y = -\sqrt{3}x + 1 \qquad \ldots(1) \qquad \text{and } y = -\dfrac{1}{\sqrt{3}}x + \dfrac{1}{\sqrt{3}} \qquad \ldots(2)$$

The slope of line (1) is $m_1 = -\sqrt{3}$, while the slope of line (2) is $m_2 = -\dfrac{1}{\sqrt{3}}$.

The acute angle i.e., θ between the two lines is given by

$$\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan\theta = \left|\dfrac{-\sqrt{3} + \dfrac{1}{\sqrt{3}}}{1 + \left(-\sqrt{3}\right)\left(-\dfrac{1}{\sqrt{3}}\right)}\right|$$

$$\tan\theta = \left|\dfrac{\dfrac{-3+1}{\sqrt{3}}}{1+1}\right| = \left|\dfrac{-2}{2 \times \sqrt{3}}\right|$$

$$\tan\theta = \dfrac{1}{\sqrt{3}}$$

$$\theta = 30°$$

Thus, the angle between the given lines is either 30° or 180° – 30° = 150°.

Question-10
The line through the points (h, 3) and (4, 1) intersects the line 7x – 9y – 19 = 0. at right angle. Find the value of h.

Ans.

The slope of the line passing through points $(h, 3)$ and $(4, 1)$ is

$$m_1 = \frac{1-3}{4-h} = \frac{-2}{4-h}$$

The slope of line $7x - 9y - 19 = 0$ or $y = \frac{7}{9}x - \frac{19}{9}$ is $m_2 = \frac{7}{9}$.

It is given that the two lines are perpendicular.

$$\therefore m_1 \times m_2 = -1$$

$$\Rightarrow \left(\frac{-2}{4-h}\right) \times \left(\frac{7}{9}\right) = -1$$

$$\Rightarrow \frac{-14}{36-9h} = -1$$

$$\Rightarrow 14 = 36 - 9h$$

$$\Rightarrow 9h = 36 - 14$$

$$\Rightarrow h = \frac{22}{9}$$

Thus, the value of h is $\frac{22}{9}$.

Question-11

Prove that the line through the point (x_1, y_1) and parallel to the line $Ax + By + C = 0$ is $A(x - x_1) + B(y - y_1) = 0$.

Ans.

The slope of line $Ax + By + C = 0$ or $y = \left(\frac{-A}{B}\right)x + \left(\frac{-C}{B}\right)$ is $m = -\frac{A}{B}$

It is known that parallel lines have the same slope.

$\therefore$ Slope of the other line $= m = -\frac{A}{B}$

The equation of the line passing through point (x_1, y_1) and having a slope $m = -\frac{A}{B}$ is

$$y - y_1 = m(x - x_1)$$

$$y - y_1 = -\frac{A}{B}(x - x_1)$$

$$B(y - y_1) = -A(x - x_1)$$

$$A(x - x_1) + B(y - y_1) = 0$$

Hence, the line through point (x_1, y_1) and parallel to line $Ax + By + C = 0$ is

$A(x - x_1) + B(y - y_1) = 0$

Question-12

Two lines passing through the point $(2, 3)$ intersects each other at an angle of $60°$. If slope of one line is 2, find equation of the other line.

Ans.

It is given that the slope of the first line, $m_1 = 2$.

Let the slope of the other line be m_2.

The angle between the two lines is $60°$.

$$\therefore \tan 60° = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\Rightarrow \sqrt{3} = \left|\frac{2 - m_2}{1 + 2m_2}\right|$$

$$\Rightarrow \sqrt{3} = \pm\left(\frac{2 - m_2}{1 + 2m_2}\right)$$

$$\Rightarrow \sqrt{3} = \frac{2 - m_2}{1 + 2m_2} \text{ or } \sqrt{3} = -\left(\frac{2 - m_2}{1 + 2m_2}\right)$$

$$\Rightarrow \sqrt{3}(1 + 2m_2) = 2 - m_2 \text{ or } \sqrt{3}(1 + 2m_2) = -(2 - m_2)$$

$$\Rightarrow \sqrt{3} + 2\sqrt{3}m_2 + m_2 = 2 \text{ or } \sqrt{3} + 2\sqrt{3}m_2 - m_2 = -2$$

$$\Rightarrow \sqrt{3} + (2\sqrt{3} + 1)m_2 = 2 \text{ or } \sqrt{3} + (2\sqrt{3} - 1)m_2 = -2$$

$$\Rightarrow m_2 = \frac{2 - \sqrt{3}}{(2\sqrt{3} + 1)} \text{ or } m_2 = \frac{-(2 + \sqrt{3})}{(2\sqrt{3} - 1)}$$

Case I : $\qquad m_2 = \left(\frac{2 - \sqrt{3}}{2\sqrt{3} + 1}\right)$

The equation of the line passing through point (2, 3) and having a slope of $\frac{(2 - \sqrt{3})}{(2\sqrt{3} + 1)}$ is

$$(y - 3) = \frac{2 - \sqrt{3}}{2\sqrt{3} + 1}(x - 2)$$

$$(2\sqrt{3} + 1)y - 3(2\sqrt{3} + 1) = (2 - \sqrt{3})x - 2(2 - \sqrt{3})$$

$$(\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = -4 + 2\sqrt{3} + 6\sqrt{3} + 3$$

$$(\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = -1 + 8\sqrt{3}$$

In this case, the equation of the other line is $(\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = -1 + 8\sqrt{3}$.

Case II : $\qquad m_2 = \frac{-(2 + \sqrt{3})}{(2\sqrt{3} - 1)}$

The equation of the line passing through point (2, 3) and having a slope of $\frac{-(2 + \sqrt{3})}{(2\sqrt{3} - 1)}$ is

$$(y - 3) = \frac{-(2 + \sqrt{3})}{(2\sqrt{3} - 1)}(x - 2)$$

$$(2\sqrt{3} - 1)y - 3(2\sqrt{3} - 1) = -(2 + \sqrt{3})x + 2(2 + \sqrt{3})$$

$$(2\sqrt{3} - 1)y + (2 + \sqrt{3})x = 4 + 2\sqrt{3} + 6\sqrt{3} - 3$$

$$(2 + \sqrt{3})x + (2\sqrt{3} - 1)y = 1 + 8\sqrt{3}$$

In this case, the equation of the other line is $(2 + \sqrt{3})x + (2\sqrt{3} - 1)y = 1 + 8\sqrt{3}$.

Thus, the required equation of the other line is $(\sqrt{3} - 2)x + (2\sqrt{3} + 1)y = -1 + 8\sqrt{3}$ or $(2 + \sqrt{3})x + (2\sqrt{3} - 1)y = 1 + 8\sqrt{3}$.

Question-13
Find the equation of the right bisector of the line segment joining the points (3, 4) and (−1, 2).

Ans.

The right bisector of a line segment bisects the line segment at 90°.

The end-points of the line segment are given as A (3, 4) and B (–1, 2).

Accordingly, mid-point of AB $= \left(\frac{3-1}{2}, \frac{4+2}{2} \right) = (1, 3)$

Slope of AB $= \frac{2-4}{-1-3} = \frac{-2}{-4} = \frac{1}{2}$

∴ Slope of the line perpendicular to AB $= -\frac{1}{\left(\frac{1}{2} \right)} = -2$

The equation of the line passing through (1, 3) and having a slope of –2 is
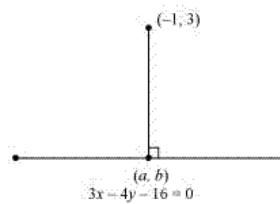
(y – 3) = –2 (x – 1)

y – 3 = –2x + 2

2x + y = 5

Thus, the required equation of the line is 2x + y = 5.

Question-14

Find the coordinates of the foot of perpendicular from the point (–1, 3) to the line 3x – 4y – 16 = 0.

Ans.

Let (a, b) be the coordinates of the foot of the perpendicular from the point (–1, 3) to the line 3x – 4y – 16 = 0.

(–1, 3)

(a, b)
3x – 4y – 16 = 0

Slope of the line joining (–1, 3) and (a, b), $m_1 = \frac{b-3}{a+1}$

Slope of the line 3x – 4y – 16 = 0 or $y = \frac{3}{4}x - 4$, $m_2 = \frac{3}{4}$

Since these two lines are perpendicular, $m_1 m_2 = -1$

∴ $\left(\frac{b-3}{a+1} \right) \times \left(\frac{3}{4} \right) = -1$

$\Rightarrow \frac{3b-9}{4a+4} = -1$

$\Rightarrow 3b - 9 = -4a - 4$

$\Rightarrow 4a + 3b = 5$ ...(1)

Point (a, b) lies on line 3x – 4y = 16.

∴ 3a – 4b = 16 ... (2)

On solving equations (1) and (2), we obtain

$a = \frac{68}{25}$ and $b = -\frac{49}{25}$

Thus, the required coordinates of the foot of the perpendicular are $\left(\frac{68}{25}, -\frac{49}{25} \right)$.

Question-15

The perpendicular from the origin to the line $y = mx + c$ meets it at the point

(−1, 2). Find the values of m and c.

Ans.

The given equation of line is $y = mx + c$.

It is given that the perpendicular from the origin meets the given line at (−1, 2).

Therefore, the line joining the points (0, 0) and (−1, 2) is perpendicular to the given line.

∴ Slope of the line joining (0, 0) and (−1, 2) $= \dfrac{2}{-1} = -2$

The slope of the given line is m.

$$\therefore m \times -2 = -1 \qquad \left[\text{The two lines are perpendicular}\right]$$
$$\Rightarrow m = \dfrac{1}{2}$$

Since point (−1, 2) lies on the given line, it satisfies the equation $y = mx + c$.

$$\therefore 2 = m(-1) + c$$
$$\Rightarrow 2 = \dfrac{1}{2}(-1) + c$$
$$\Rightarrow c = 2 + \dfrac{1}{2} = \dfrac{5}{2}$$

Thus, the respective values of m and c are $\dfrac{1}{2}$ and $\dfrac{5}{2}$.

Question-16

If p and q are the lengths of perpendiculars from the origin to the lines $x \cos\theta - y \sin\theta = k \cos 2\theta$ and $x \sec\theta + y \csc\theta = k$, respectively, prove that $p^2 + 4q^2 = k^2$

Ans.

The equations of given lines are

$x \cos\theta - y \sin\theta = k \cos 2\theta$ ... (1)

$x \sec\theta + y \csc\theta = k$ ... (2)

The perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is

given by $d = \dfrac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$.

On comparing equation (1) to the general equation of line i.e., $Ax + By + C = 0$, we obtain $A = \cos\theta$, $B = -\sin\theta$, and $C = -k \cos 2\theta$.

It is given that p is the length of the perpendicular from (0, 0) to line (1).

$$\therefore p = \dfrac{|A(0) + B(0) + C|}{\sqrt{A^2 + B^2}} = \dfrac{|C|}{\sqrt{A^2 + B^2}} = \dfrac{|-k \cos 2\theta|}{\sqrt{\cos^2\theta + \sin^2\theta}} = |-k \cos 2\theta| \qquad ...(3)$$

On comparing equation (2) to the general equation of line i.e., $Ax + By + C = 0$, we obtain $A = \sec\theta$, $B = \csc\theta$, and $C = -k$.

It is given that q is the length of the perpendicular from (0, 0) to line (2).

$$\therefore q = \frac{|A(0) + B(0) + C|}{\sqrt{A^2 + B^2}} = \frac{|C|}{\sqrt{A^2 + B^2}} = \frac{|-k|}{\sqrt{\sec^2\theta + \csc^2\theta}} \qquad \ldots(4)$$

From (3) and (4), we have

$$p^2 + 4q^2 = \left(|-k\cos 2\theta|\right)^2 + 4\left(\frac{|-k|}{\sqrt{\sec^2\theta + \csc^2\theta}}\right)^2$$

$$= k^2\cos^2 2\theta + \frac{4k^2}{\left(\sec^2\theta + \csc^2\theta\right)}$$

$$= k^2\cos^2 2\theta + \frac{4k^2}{\left(\dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin^2\theta}\right)}$$

$$= k^2\cos^2 2\theta + \frac{4k^2}{\left(\dfrac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta}\right)}$$

$$= k^2\cos^2 2\theta + \frac{4k^2}{\left(\dfrac{1}{\sin^2\theta\cos^2\theta}\right)}$$

$$= k^2\cos^2 2\theta + 4k^2\sin^2\theta\cos^2\theta$$

$$= k^2\cos^2 2\theta + k^2\left(2\sin\theta\cos\theta\right)^2$$

$$= k^2\cos^2 2\theta + k^2\sin^2 2\theta$$

$$= k^2\left(\cos^2 2\theta + \sin^2 2\theta\right)$$
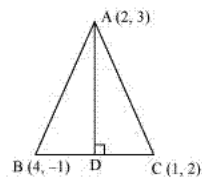
$$= k^2$$

Hence, we proved that p² + 4q² = k².

Question-17

In the triangle ABC with vertices A (2, 3), B (4, −1) and C (1, 2), find the equation and length of altitude from the vertex A.

Ans.

Let AD be the altitude of triangle ABC from vertex A.

Accordingly, AD⊥BC

The equation of the line passing through point (2, 3) and having a slope of 1 is

(y − 3) = 1 (x − 2)

⇒ x − y + 1 = 0

⇒ y − x = 1

Therefore, equation of the altitude from vertex A = y − x = 1.

Length of AD = Length of the perpendicular from A (2, 3) to BC

The equation of BC is

$$(y+1) = \frac{2+1}{1-4}(x-4)$$
$$\Rightarrow (y+1) = -1(x-4)$$
$$\Rightarrow y+1 = -x+4$$
$$\Rightarrow x+y-3 = 0 \qquad\qquad\qquad ...(1)$$

The perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by $d = \dfrac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$.

On comparing equation (1) to the general equation of line $Ax + By + C = 0$, we obtain $A = 1$, $B = 1$, and $C = -3$.

$\therefore$ Length of AD $= \dfrac{|1 \times 2 + 1 \times 3 - 3|}{\sqrt{1^2 + 1^2}}$ units $= \dfrac{|2|}{\sqrt{2}}$ units $= \dfrac{2}{\sqrt{2}}$ units $= \sqrt{2}$ units

Thus, the equation and the length of the altitude from vertex A are $y - x = 1$ and $\sqrt{2}$ units respectively.

Question-18

If p is the length of perpendicular from the origin to the line whose intercepts on the axes are a and b, then show that $\dfrac{1}{p^2} = \dfrac{1}{a^2} + \dfrac{1}{b^2}$.

Ans.

It is known that the equation of a line whose intercepts on the axes are a and b is

$$\frac{x}{a} + \frac{y}{b} = 1$$
or $bx + ay = ab$
or $bx + ay - ab = 0 \qquad\qquad ...(1)$

The perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by $d = \dfrac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$.

On comparing equation (1) to the general equation of line $Ax + By + C = 0$, we obtain $A = b$, $B = a$, and $C = -ab$.

Therefore, if p is the length of the perpendicular from point $(x_1, y_1) = (0, 0)$ to line (1), we obtain

$$p = \frac{|A(0) + B(0) - ab|}{\sqrt{b^2 + a^2}}$$
$$\Rightarrow p = \frac{|-ab|}{\sqrt{a^2 + b^2}}$$

On squaring both sides, we obtain

$$p^2 = \frac{(-ab)^2}{a^2 + b^2}$$
$$\Rightarrow p^2(a^2 + b^2) = a^2 b^2$$
$$\Rightarrow \frac{a^2 + b^2}{a^2 b^2} = \frac{1}{p^2}$$
$$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

Hence, we showed that $\dfrac{1}{p^2} = \dfrac{1}{a^2} + \dfrac{1}{b^2}$.

*********** END **********

*********** END **********