Complex Numbers Ex 13.2 Q11

$$\text{let } z = \frac{(1+i)^n}{(1-i)^{n-2}}$$

$$= \frac{(1+i)^n}{(1-i)^n}(1-i)^2$$

$$= \left(\frac{1+i}{1-i}\right)^n \times (1-i)^2$$

$$= i^n\left(1+i^2-2\times 1\times i\right) \qquad \left(\because \frac{1+i}{1-i} = i, \text{using problem 10}\right)$$

$$= i^n(1-1-2i)$$

$$= -2i \times i^n$$

$$= -2i^{n+1}$$

$\therefore$ For $n = 1$

$z = -2i^{1+1}$

$= -2i^2$

$= 2$, which is a real number

$\therefore$ The smallest positive integer value of n is 1.

Complex Numbers Ex 13.2 Q12

$$\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = x+iy$$

$$\Rightarrow \left(\frac{(1+i)(1+i)}{(1-i)(1+i)}\right)^3 - \left(\frac{(1-i)(1-i)}{(1+i)(1-i)}\right)^3 = x+iy \text{[Rationalizing the denomiantor]}$$

$$\Rightarrow \left(\frac{1+2i-1}{1+1}\right)^3 - \left(\frac{1-2i-1}{1+1}\right)^3 = x+iy$$

$$\Rightarrow \left(\frac{2i}{2}\right)^3 - \left(\frac{-2i}{2}\right)^3 = x+iy$$

$$\Rightarrow i^3 - (-i)^3 = x+iy$$

$$\Rightarrow -i-i = x+iy$$

$$\Rightarrow -2i = x+iy$$

Comparing the real and imaginary parts,

$(x, y) = (0, 2)$

Complex Numbers Ex 13.2 Q13

$$\frac{(1+i)^2}{2-i} = x + iy$$

$$\Rightarrow \frac{(1+2i-1)}{2-i} = x + iy$$

$$\Rightarrow \frac{2i}{2-i} = x + iy$$

$$\Rightarrow \frac{2i(2+i)}{(2-i)(2+i)} = x + iy \text{[Rationalizing the denominator]}$$

$$\Rightarrow \frac{2(2i-1)}{4+1} = x + iy$$

$$\Rightarrow \frac{4i-2}{5} = x + iy$$

$$\Rightarrow -\frac{2}{5} + i\frac{4}{5} = x + iy$$

Comparing the real and imaginary parts, we get

$$x = -\frac{2}{5}, y = \frac{4}{5}$$

$$x + y = \frac{2}{5}$$

Complex Numbers Ex 13.2 Q14

$$\left(\frac{1-i}{1+i}\right)^{100} = a + ib$$

$$\Rightarrow \left(\frac{(1-i)(1-i)}{(1+i)(1-i)}\right)^{100} = a+ib \text{[Rationalizing the denominator]}$$

$$\Rightarrow \left(\frac{(1-2i-1)}{(1+1)}\right)^{100} = a+ib$$

$$\Rightarrow \left(\frac{-2i}{2}\right)^{100} = a+ib$$

$$\Rightarrow (-i)^{100} = a+ib$$

$$\Rightarrow 1 = a+ib$$

Comparing, we get (a,b)=(1,0)

*********** END **********