



Quadratic Equations Ex 8.6 Q13

Answer :

The quadric equation is $(x-a)(x-b) + (x+b)(x-c) + (x-c)(x-a) = 0$

Here,

After simplifying the equation

$$x^2 - (a+b)x + ab + x^2 - (b+c)x + bc + x^2 - (c+a)x + ca = 0$$

$$3x^2 - 2(a+b+c)x + (ab+bc+ca) = 0$$

$$a=3, b=2(a+b+c) \text{ and } c=(ab+bc+ca)$$

As we know that $D = b^2 - 4ac$

Putting the value of $a=3, b=2(a+b+c)$ and $c=(ab+bc+ca)$

$$\begin{aligned} D &= \{2(a+b+c)\}^2 - 4 \times 3 \times (ab+bc+ca) \\ &= 4(a^2 + b^2 + c^2 + 2ab + 2bc + 2ca) - 12(ab+bc+ca) \\ &= 4(a^2 + b^2 + c^2 + 2ab + 2bc + 2ca - 3ab - 3bc - 3ca) \\ &= 4(a^2 + b^2 + c^2 - ab - bc - ca) \end{aligned}$$

$$\begin{aligned} D &= 4(a^2 + b^2 + c^2 - ab - bc - ca) \\ &= 2[2a^2 + 2b^2 + 2c^2 - 2ab - 2ac - 2bc] \\ &= 2[(a-b)^2 + (b-c)^2 + (c-a)^2] \end{aligned}$$

Since, $D > 0$. So the solutions are real

Let $a=b=c$

Then

$$\begin{aligned} D &= 4(a^2 + b^2 + c^2 - ab - bc - ca) \\ &= 4(a^2 + b^2 + c^2 - aa - bb - cc) \\ &= 4(\cancel{a^2} + \cancel{b^2} + \cancel{c^2} - \cancel{a^2} - \cancel{b^2} - \cancel{c^2}) \\ &= 4 \times 0 \end{aligned}$$

Thus, the value of $D = 0$

Therefore, the roots of the given equation are real and but they are equal only when, $a=b=c$

Hence proved

Quadratic Equations Ex 8.6 Q14

Answer :

The given equations are

$$ax^2 + bx + c = 0 \quad \dots\dots (1)$$

$$-ax^2 + bx + c = 0 \quad \dots\dots (2)$$

Roots are simultaneously real

Let D_1 and D_2 be the discriminants of equation (1) and (2) respectively,

Then,

$$\begin{aligned} D_1 &= (b)^2 - 4ac \\ &= b^2 - 4ac \end{aligned}$$

And

$$\begin{aligned} D_2 &= (b)^2 - 4 \times (-a) \times c \\ &= b^2 + 4ac \end{aligned}$$

Both the given equation will have real roots, if $D_1 \geq 0$ and $D_2 \geq 0$.

Thus,

$$b^2 - 4ac \geq 0$$

$$b^2 \geq 4ac \quad \dots\dots (3)$$

And,

$$b^2 + 4ac \geq 0 \quad \dots\dots (4)$$

Now given that a, b, c are real number and $ac \neq 0$ as well as from equations (3) and (4) we get

At least one of the given equation has real roots

Hence, proved

Quadratic Equations Ex 8.6 Q15

Answer :

The given equation $(1+m^2)x^2 + 2mcx + (c^2 - a^2) = 0$, has equal roots

Then prove that $c^2 = (1+m^2)a^2$.

Here,

$$a = (1+m^2), b = 2mc \text{ and } c = (c^2 - a^2)$$

As we know that $D = b^2 - 4ac$

Putting the value of $a = (1+m^2), b = 2mc$ and $c = (c^2 - a^2)$

$$\begin{aligned} D &= b^2 - 4ac \\ &= \{2mc\}^2 - 4 \times (1+m^2) \times (c^2 - a^2) \\ &= 4(m^2c^2) - 4(c^2 - a^2 + m^2c^2 - m^2a^2) \\ &= \cancel{4m^2c^2} - 4c^2 + 4a^2 - \cancel{4m^2c^2} + 4m^2a^2 \\ &= 4a^2 + 4m^2a^2 - 4c^2 \end{aligned}$$

The given equation will have real roots, if $D = 0$

$$4a^2 + 4m^2a^2 - 4c^2 = 0$$

$$4a^2 + 4m^2a^2 = 4c^2$$

$$4a^2(1+m^2) = 4c^2$$

$$a^2(1+m^2) = c^2$$

Hence, $\boxed{c^2 = a^2(1+m^2)}$

***** END *****