QUESTIONS FROM TEXTBOOK

Question 13. 1. Estimate the fraction of molecular volume to the actual volume occupied by oxygen gas at STP.' Take the diameter of an oxygen molecule to be 3 A.

Answer: Diameter of an oxygen molecule, d = 3A = 3 x 10^{-10} m.

Consider one mole of oxygen gas at STP, which contain total NA = 6.023 x 10^{23} molecules.

Actual molecular volume of 6.023×10^{23} oxygen molecules

$$V_{actual} = \frac{4}{3}\pi r^3 \cdot N_A$$
$$= \frac{4}{3} \times 3.14 \times (1.5)^3 \times 10^{-3} \times 6.02 \times 10^{23}\ m^3$$
$$= 8.51 \times 10^{-6}\ m^3$$
$$= 8.51 \times 10^{-3}\ \text{litre} \qquad [\because 1\ m^3 = 10^3\ \text{litre}]$$

$\therefore$ Molecular volume of one mole of oxygen

$$V_{actual} = 8.51 \times 10^{-3}\ \text{litre}$$

At STP, the volume of one mole of oxygen

$$V_{molar} = 22.4\ \text{litre}$$
$$\frac{V_{actual}}{V_{molar}} = \frac{8.51 \times 10^{-3}}{22.4} = 3.8 \times 10^{-4} \simeq 4 \times 10^{-4}.$$

Question 13. 2. Molar volume is the volume occupied by 1 mol of any (ideal) gas at standard temperature and pressure (STP : 1 atmospheric pressure, 0 °C). Show that it is 22.4 litres.

Answer:

For one mole of an ideal gas, we have

$$PV = RT \Rightarrow V = \frac{RT}{P}$$

Putting $R = 8.31$ J mol^{-1} K^{-1}, $T = 273K$ and $P = 1$ atmosphere $= 1.013 \times 10^5\ Nm^{-2}$

$$\therefore \quad V = \frac{8.31 \times 273}{1.013 \times 10^5} = 0.0224\ m^3$$
$$= 0.0224 \times 10^6\ cm^3 = 22400\ ml \qquad [1\ cm^3 = 1ml]$$

Question 13. 3. Following figure shows plot of PV/T versus P for 1.00 x 10^{-3} kg of oxygen gas at two different temperatures.

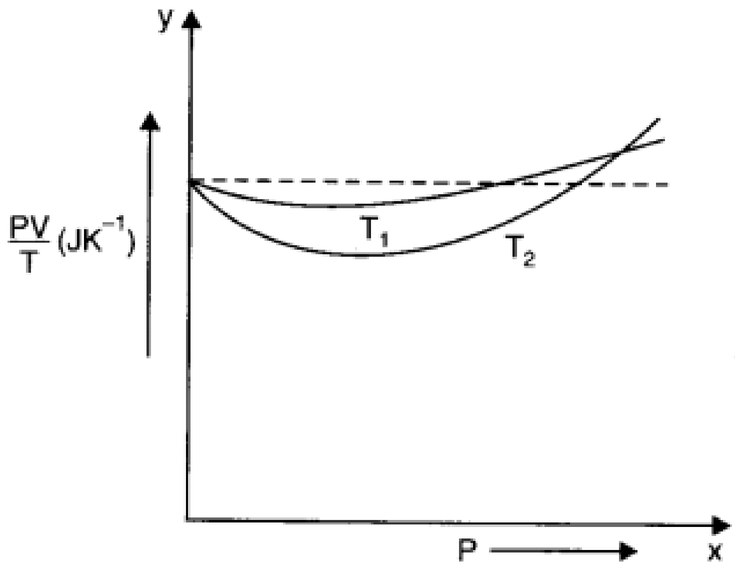
(a) What does the dotted plot signify?

(b) Which is true : $T_1 > T_2$ or $T_1 < T_2$?

(c) What is the value of PV/T where the curves meet on the y-axis?

(d) If we obtained similar plots for 1.00 x 10^{-3} kg of hydrogen, would we get the same value of PV/T at the point where the curves meet on the y-axis? If not, what mass of hydrogen yields the same value of PV/T (for low pressure high temperature region of the plot)

? (Molecular mass of H_2 = 2.02 u, of O_2 = 32.0 u, R = 8.31 J mol^{-1} K^{-1}.)



Answer:
(a) The dotted plot corresponds to 'ideal' gas behaviour as it is parallel to P-axis and it tells that value of PV/T remains same even when P is changed.
(b) The upper position of PV/T shows that its value is lesser for T_1 thus $T_1 > T_2$. This is because the curve at T_1 is more close to dotted plot than the curve at T_2 Since the behaviour of a real gas approaches the perfect gas behaviour, as the temperature is increased.
(c) Where the two curves meet, the value of PV/T on y-axis is equal to μR. Since ideal gas equation for μ moles is PV = μRT

where, $\mu = \frac{1.00 \times 10^{-3} kg}{32 \times 10^{-3} kg} = \frac{1}{32}$

$\therefore$ Value of $\frac{PV}{T} = \mu R = \frac{1}{32} \times 8.31\ JK^{-1} = 0.26\ JK^{-1}$

(d) If we obtained similar plots for 1.00×10^{-3} kg of hydrogen, we will not get the same value of $\frac{PV}{T}$ at the point, where the curves meet on the *y-axis*. This is because molecular mass of hydrogen is different from that of oxygen.

For the same value of $\frac{PV}{T}$, mass of hydrogen required is obtained from

$$\frac{PV}{T} = nR = \frac{m}{2.02} \times 8.31 = 0.26$$

$$m = \frac{2.02 \times 0.26}{8.31} \text{ gram} = 6.32 \times 10^{-2} \text{ gram.}$$

Question 13. 4. An oxygen cylinder of volume 30 Hire has an initial gauge pressure of 15 atmosphere and a temperature of 27 °C. After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atmosphere and its temperature drops to 17 °C. Estimate the mass of oxygen taken out of the cylinder. ($R = 8.31\ J\ mol^{-1}\ K^{-1}$, molecular mass of O_2 = 32 u.)
Answer:

Initial volume, $V_1 = 30$ litre $= 30 \times 10^3$ cm^3

$= 30 \times 10^3 \times 10^{-6}$ m^3 $= 30 \times 10^{-3}$ m^3

Initial pressure, $P_1 = 15$ atm

$= 15 \times 1.013 \times 10^5$ N m^{-2}

Initial temperature, $T_1 = (27 + 273)$ K $= 300$ K

Initial number of moles,

$$\mu_1 = \frac{P_1V_1}{RT_1} = \frac{15 \times 1.013 \times 10^5 \times 30 \times 10^{-3}}{8.31 \times 300} = 18.3$$

Final pressure, $P_2 = 11$ atm

$= 11 \times 1.013 \times 10^5$ N m^{-2}

Final volume, $V_2 = 30$ litre $= 30 \times 10^{-3}$ cm^3

Final temperature, $T_2 = 17 + 273 = 290$ K

Final number of moles,

$$\mu_2 = \frac{P_2V_2}{RT_2} = \frac{11 \times 1.013 \times 10^5 \times 30 \times 10^{-3}}{8.31 \times 290} = 13.9$$

Number of moles taken out of cylinder

$= 18.3 - 13.9 = 4.4$

Mass of gas taken out of cylinder

$= 4.4 \times 32$ g $= 140.8$ g $= 0.141$ kg.

Question 13. 5. An air bubble of volume 1.0 cm^3 rises from the bottom of a lake 40 m deep at a temperature of 12°C. To what volume does it grow when it reaches the surface, which is at a temperature of 35 °C.

Answer:

Volume of the bubble inside, $V_1 = 1.0$ cm^3 $= 1 \times 10^{-6}$ m^3

Pressure on the bubble, P_1 = Pressure of water + Atmospheric pressure

$= pgh + 1.01 \times 10^5 = 1000 \times 9.8 \times 40 + 1.01 \times 10^5$

$= 3.92 \times 10^5 + 1.01 \times 10^5 = 4.93 \times 10^5$ Pa

Temperature, $T_1 = 12$ °C $= 273 + 12 = 285$ K

Also, pressure outside the lake, $P_2 = 1.01 \times 10^5$ N m^{-2}

Temperature, $T_2 = 35$ °C $= 273 + 35 = 308$ K, volume V_2 =?

Now $$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\therefore \quad V_2 = \frac{P_1V_1}{T_1} \cdot \frac{T_2}{P_2} = \frac{4.93 \times 10^5 \times 1 \times 10^{-6} \times 308}{285 \times 1.01 \times 10^5} = 5.3 \times 10^{-6} \text{ m}^{-3}$$

Question 13. 6. Estimate the total number of air molecules (inclusive of oxygen, nitrogen, water vapour and other constituents) in a room of capacity 25.0 m^3 at a temperature of 27 °C and 1 atm pressure.

Answer:

Here, volume of room, $V = 25.0$ m^3, temperature, $T = 27$ °C $= 300$ K and

Pressure, $P = 1$ atm $= 1.01 \times 10^5$ Pa

According to gas equation,

$$PV = \mu RT = \mu N_A \cdot k_B T$$

Hence, total number of air molecules in the volume of given gas,

$$N = \mu \cdot N_A = \frac{PV}{k_BT}$$

$$\therefore \quad N = \frac{1.01 \times 10^5 \times 25.0}{\left(1.38 \times 10^{-23}\right) \times 300} = 6.1 \times 10^{26}.$$

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