Factorisation of Polynomials Ex 6.4 Q22

Answer :

By division algorithm, when $p(x)=3x^3+x^2-22x+9$ is divided by $3x^2+7x-6$, the reminder is a linear polynomial. So, let $r(x) = ax + b$ be added to $p(x)$ so that the result is divisible by $q(x)$

Let

$$f(x)=p(x)+r(x)$$
$$=3x^3+x^2-22x+9+ax+b$$
$$=3x^3+x^2+(a-22)x+9+b$$

We have,

$$q(x) = 3x^2 + 7x - 6$$
$$= 3x^2 + 9x - 2x - 6$$
$$= 3x(x + 3) - 2(x + 3)$$
$$= (3x - 2)(x + 3)$$

Clearly, $(3x - 2)$ and $(x + 3)$ are factors of $q(x)$.

Therefore, $f(x)$ will be divisible by $q(x)$ if $(3x-2)$ and $(x+3)$ are factors of $f(x)$, i.e.,

$f\left(\frac{2}{3}\right)$ and $f(-3)$ are equal to zero.

Now,

$$f\left(\tfrac{2}{3}\right) = 0$$
$$\Rightarrow 3\left(\tfrac{2}{3}\right)^3 + \left(\tfrac{2}{3}\right)^2 + (a - 22)\left(\tfrac{2}{3}\right) + 9 + b = 0$$
$$\Rightarrow 3 \times \tfrac{8}{27} + \tfrac{4}{9} + \tfrac{2a}{3} - \tfrac{44}{3} + 9 + b = 0$$
$$\Rightarrow \frac{8}{9} + \frac{4}{9} - \frac{44}{3} + 9 + \frac{2a}{3} + b = 0$$
$$\Rightarrow \frac{8+4-132+81}{9} + \frac{2a}{3} + b = 0$$
$$\Rightarrow -\frac{39}{9} + \frac{2a}{3} + b = 0$$
$$\Rightarrow \frac{2a}{3} + b = \frac{13}{3}$$
$$\Rightarrow 2a + 3b = 13 \qquad \ldots\ldots\ldots(\text{i})$$

And

$$f(-3) = 0$$
$$\Rightarrow 3(-3)^3 + (-3)^2 + (a - 22)(-3) + 9 + b = 0$$
$$\Rightarrow -81 + 9 - 3a + 66 + 9 + b = 0$$
$$\Rightarrow -3a + b = -3$$
$$\Rightarrow b = -3 + 3a \qquad \ldots\ldots\ldots(\text{ii})$$

Substituting the value of b from (ii) in (i), we get,

$$2a + 3(3a - 3) = 13$$
$$\Rightarrow 2a + 9a - 9 = 13$$
$$\Rightarrow 11a = 13 + 9$$
$$\Rightarrow 11a = 22$$
$$\Rightarrow a = 2$$

Now, from (ii), we get

$$b = -3 + 3(2) = -3 + 6 = 3$$

So, we have $a = 2$ and $b = 3$

Hence, $p(x)$ is divisible by $q(x)$, if $2x+3$ is added to it.

Factorisation of Polynomials Ex 6.4 Q23

Answer :

(i) Let $f(x)=x^3-2ax^2+ax-1$ be the given polynomial.

By factor theorem, if $(x-2)$ is a factor of $f(x)$, then $f(2)=0$

Therefore,

$$\begin{aligned} f(2)=(2)^3-2a(2)^2+a(2)-1&=0\\ 8-8a+2a-1&=0\\ -6a+7&=0\\ a&=7/6 \end{aligned}$$

Thus the value of a is 7/6.

(ii) Let $f(x)=x^5-3x^4-ax^3+3ax^2+2ax+4$ be the given polynomial.

By the factor theorem, $(x-2)$ is a factor of $f(x)$, if $f(2)=0$

Therefore,

$$\begin{aligned} f(2)=(2)^5-3(2)^4-a(2)^3+3a(2)^2+2a(2)+4&=0\\ 32-48-8a+12a+4a+4&=0\\ -12+8a&=0\\ a&=3/2 \end{aligned}$$

Thus, the value of a is $\frac{3}{2}$.

Factorisation of Polynomials Ex 6.4 Q24

Answer :

(i) Let $f(x)=x^6-ax^5+x^4-ax^3+3x-a+2$ be the given polynomial.

By factor theorem, $(x-a)$ is a factor of the polynomial if $f(a)=0$

Therefore,

$$\begin{aligned} f(a)=a^6-a(a)^5+(a)^4+(a)^4-a(a)^3+3(a)-a+2&=0\\ \cancel{a^6}-\cancel{a^6}+\cancel{a^4}-\cancel{a^4}+2a+2&=0\\ 2a+2&=0\\ a&=-1 \end{aligned}$$

Thus, the value of a is -1.

(ii) Let $f(x)=x^5-a^2x^3+2x+a+1$ be the given polynomial.

By factor theorem, $(x-a)$ is a factor of $f(x)$, if $f(a)=0$.

Therefore,

$$\begin{aligned} \Rightarrow f(a)=(a)^5-a^2(a)^3+2(a)+a+1&=0\\ \cancel{a^5}-\cancel{a^5}+2a+a+1&=0\\ 3a+1&=0\\ a&=-1/3 \end{aligned}$$

Thus, the value of a is $-1/3$.

Factorisation of Polynomials Ex 6.4 Q25

Answer :

(i) Let $f(x)=x^3+ax^2-2x+a+4$ be the given polynomial.

By the factor theorem, (+ a) is the factor of $f(x)$, if $f(-a)=0$, i.e.,

$$f(-a)=(-a)^3+a(-a)^2-2(-a)+a+4=0$$
$$-a^3+a^3+2a+a+4=0$$
$$3a+4=0$$
$$a=-4/3$$

Thus, the value of a is $-4/3$.

(ii) Let $f(x)=x^4-a^2x^2+3x-a$ be the polynomial. By factor theorem, $(x+a)$ is a factor of the $f(x)$, if $f(-a)=0$, i.e.,

$$f(-a)=(-a)^4-a^2(-a)^2+3(-a)-a=0$$
$$a^4-a^4-3a-a=0$$
$$-4a=0$$
$$a=0$$

Thus, the value of a is 0.

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