Binary Operations Ex 3.1 Q7

We have,

$$M = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : \ a, b \in R - \{0\} \right\} \text{ and}$$

$$A * B = AB \text{ for all } A, B \in M$$

Let $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \in M$ and $B = \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} \in M$

Now, $\quad AB = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} = \begin{bmatrix} ac & 0 \\ 0 & bd \end{bmatrix}$

$\therefore \qquad a \in R, \ b \in R, \ c \in R, \ \& \ d \in R$

$\Rightarrow \qquad ac \in R \text{ and } bd \in R$

$\Rightarrow \qquad \begin{bmatrix} ac & 0 \\ 0 & bd \end{bmatrix} \in M$

$\Rightarrow \qquad A * B \in M$

Thus, the operater $*$ difines a binary operation on M

Binary Operations Ex 3.1 Q8

S = set of rational numbers of the form $\frac{m}{n}$ where $m \in Z$ and $n = 1, 2, 3$

Also, $\quad a * b = ab$

Let $a \in S$ and $b \in S$

$\Rightarrow \qquad ab \notin S$

For example $\quad a = \frac{7}{3}$ and $b = \frac{5}{2}$

$\Rightarrow \qquad ab = \frac{35}{6} \notin S$

$\therefore \qquad a * b \notin S$

Hence, the operater $*$ does not define a binary operation on S

Binary Operations Ex 3.1 Q9

It is given that, $a*b = 2a + b$

Now

$$(2*3) = 2 \times 2 + 3$$
$$= 4 + 3$$
$$= 7$$
$$(2*3)*4 = 7*4 = 2 \times 7 + 4$$
$$= 14 + 4$$
$$= 18$$

Binary Operations Ex 3.1 Q10

It is given that, $a*b = LCM\ (a, b)$

Now

$$5*7 = LCM\ (5, 7)$$
$$= 35$$

********** END **********