



Co-Ordinate Geometry Ex 14.5 Q6

Answer :

The formula for the area 'A' encompassed by three points (x_1, y_1) , (x_2, y_2) and (x_3, y_3) is given by the formula,

$$A = \frac{1}{2} \begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_2 - x_3 & y_2 - y_3 \end{vmatrix}$$

$$A = \frac{1}{2} \{ (x_1 - x_2)(y_2 - y_3) - (x_2 - x_3)(y_1 - y_2) \}$$

If three points are collinear the area encompassed by them is equal to 0.

The three given points are $A(a, 0)$, $B(0, b)$ and $C(1, 1)$.

$$A = \frac{1}{2} \begin{vmatrix} a - 0 & 0 - b \\ 0 - 1 & b - 1 \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} a & -b \\ -1 & b - 1 \end{vmatrix}$$

$$= \frac{1}{2} \{ (a)(b - 1) - (-1)(-b) \}$$

$$= \frac{1}{2} \{ ab - a - b \}$$

$$A = ab - (a + b)$$

It is given that $\frac{1}{a} + \frac{1}{b} = 1$

So we have,

$$\frac{1}{a} + \frac{1}{b} = 1$$

$$\frac{a + b}{ab} = 1$$

$$a + b = ab$$

Using this in the previously arrived equation for area we have,

$$A = ab - (a + b)$$

$$A = ab - ab$$

$$A = 0$$

Since the area enclosed by the three points is equal to 0, the three points need to be **collinear**.

Co-Ordinate Geometry Ex 14.5 Q7

Answer :

GIVEN: point A divides the line segment joining P $(-5, 1)$ and Q $(3, -5)$ in the ratio $k: 1$

Coordinates of point B $(1, 5)$ and C $(7, -2)$

TO FIND: The value of k

PROOF: point A divides the line segment joining P $(-5, 1)$ and Q $(3, -5)$ in the ratio $k: 1$

So the coordinates of A are $\left[\frac{3k+1}{k+1}, \frac{5k+1}{k+1} \right]$

We know area of triangle formed by three points (x_1, y_1) , (x_2, y_2) and (x_3, y_3) is given by

$$\Delta = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Now Area of $\Delta ABC = 2$ sq units.

Taking three points A $\left[\frac{3k+1}{k+1}, \frac{5k+1}{k+1} \right]$, B $(1, 5)$ and C $(7, -2)$

$$2 = \frac{1}{2} \left| \left(\frac{3k-5}{k+1} \right) 5 - 2 + 7 \left(\frac{5k+1}{k+1} \right) - \left(\frac{5k+1}{k+1} \right) + 35 - 2 \left(\frac{3k-5}{k+1} \right) \right|$$

$$2 = \frac{1}{2} \left| \left(\frac{15k-25}{k+1} \right) - 2 + \left(\frac{35k+7}{k+1} \right) - \left(\frac{5k+1}{k+1} \right) + 35 - \left(\frac{6k-10}{k+1} \right) \right|$$

$$2 = \frac{1}{2} \left| \frac{15k-25-2k+35k+7}{k+1} - \left(\frac{5k+1+35k+35-6k+10}{k+1} \right) \right|$$

$$2 = \frac{1}{2} \left| \frac{(48k-20)-(34k+46)}{k+1} \right|$$

$$2 = \frac{1}{2} \left(\frac{14k - 66}{k + 1} \right)$$

$$2 = \left(\frac{7k - 33}{k + 1} \right)$$

$$\pm 2 = \left(\frac{7k - 33}{k + 1} \right)$$

$$\Rightarrow 7k - 33 = \pm 2(k + 1)$$

$$\Rightarrow 7k - 33 = 2(k + 1), 7k - 33 = -2(k + 1)$$

$$\Rightarrow 7k - 33 = 2k + 2, 7k - 33 = -2k - 2$$

$$\Rightarrow 7k - 2k = 33 + 2, 7k + 2k = +33 - 2$$

$$\Rightarrow 5k = 35, 9k = 31$$

$$\Rightarrow k = \frac{35}{5}, k = \frac{31}{9}$$

$$\Rightarrow k = 7, k = \frac{31}{9}$$

Hence $\boxed{k = 7 \text{ or } \frac{31}{9}}$

***** END *****