Definite Integrals Ex 20.1 Q9

We have,

$$\int_0^1 \frac{x}{x+1}\,dx \qquad \left[\text{Add and subtract 1 in numerator}\right]$$

$$= \int_0^1 \frac{(x+1)-1}{x+1}\,dx$$

$$= \int_0^1 1\,dx - \int_0^1 \frac{1}{x+1}\,dx$$

$$= \left[x\right]_0^1 - \left[\log(x+1)\right]_0^1$$

$$= 1 - \left[\log 2 - \log 1\right]$$

$$= 1 - \log\frac{2}{1}$$

$$= 1 - \log 2$$

$$= \log e - \log 2 \qquad \left[\because \log e = 1\right]$$

$$= \log\frac{e}{2}$$

$$\therefore \int_0^1 \frac{x}{x+1}\,dx = \log\frac{e}{2}$$

Definite Integrals Ex 20.1 Q10

We have,

$$\int_0^{\frac{\pi}{2}} (\sin x + \cos x)dx$$

$$= \int_0^{\frac{\pi}{2}} \sin x dx + \int_0^{\frac{\pi}{2}} \cos x dx$$

$$= \left[-\cos x\right]_0^{\frac{\pi}{2}} + \left[\sin x\right]_0^{\frac{\pi}{2}}$$

$$= \left[\cos\frac{\pi}{2} + \cos 0\right] + \left[\sin\frac{\pi}{2} - \sin 0\right]$$

$$= \left[-0 + 1\right] + 1$$

$$= 1 + 1$$

$$= 2$$

$$\therefore \int_0^{\frac{\pi}{2}} (\sin x + \cos x)dx = 2$$

Definite Integrals Ex 20.1 Q11

We have,

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x dx$$

We know that $\int \cot x dx = \log(\sin x)$

Now,

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x dx$$

$$= \left[\log(\sin x)\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \left[\log\left(\sin \frac{\pi}{2}\right) - \log\left(\sin \frac{\pi}{4}\right)\right]$$

$$= \left[\log 1 - \log \frac{1}{\sqrt{2}}\right]$$

$$= \left[0 - (\log 1 - \log\sqrt{2})\right]$$

$$= \log\sqrt{2} \qquad \left[\because \log a^n = n \log a\right]$$

$$= \frac{1}{2}\log 2$$

$$\therefore \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x dx = \frac{1}{2}\log 2$$

Definite Integrals Ex 20.1 Q12

We have,

$$\int_0^{\frac{\pi}{4}} \sec x dx$$

We know that $\int \sec x dx = \log(\sec x + \tan x)$

$$\therefore \int_0^{\frac{\pi}{4}} \sec x dx$$

$$= \left[\log(\sec x + \tan x)\right]_0^{\frac{\pi}{4}}$$

$$= \left[\log(\sqrt{2} + 1) - \log(1 + 0)\right]$$

$$= \log(\sqrt{2} + 1) \qquad \left[\because \log 1 = 0\right]$$

$$\therefore \int_0^{\frac{\pi}{4}} \sec x dx = \log(\sqrt{2} + 1)$$

*********** END **********