



Differentials Errors and Approximation Ex14.1 Q9(xii)

$$\text{Let } x = 25, x + \Delta x = 25.1$$

$$\Delta x = 25.1 - 25$$

$$\Delta x = 0.1$$

$$\text{Let } y = \frac{1}{\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{2}{2x^{\frac{3}{2}}}$$

$$\begin{aligned}\left(\frac{dy}{dx}\right)_{x=25} &= -\frac{1}{2(25)^{\frac{3}{2}}} \\ &= -\frac{1}{250} \\ &= -0.004\end{aligned}$$

Now,

$$\begin{aligned}\Delta y &= \left(\frac{dy}{dx}\right)_{x=25} \times (\Delta x) \\ &= (-0.004)(0.1) \\ &= -0.0004\end{aligned}$$

$$\frac{1}{\sqrt{x}} = v + \Delta v$$

$$\begin{aligned}
 \sqrt{25.1} &= \sqrt{25 + 0.1} \\
 &= \frac{1}{\sqrt{x}} + (-0.0004) \\
 &= \frac{1}{\sqrt{25}} - 0.0004 \\
 &= \frac{1}{5} - 0.0004 \\
 &= 0.2 - 0.0004
 \end{aligned}$$

$$\frac{1}{\sqrt{25.1}} = 0.1996$$

Differentials Errors and Approximation Ex14.1 Q9(xiii)

$$\text{Let } x = \frac{\pi}{2}, \quad x + \Delta x = \frac{22}{14}$$

$$\Delta x = \left(\frac{22}{14} - \frac{\pi}{2} \right)$$

$$\Delta x = \sin x$$

$$\text{Let } y = \sin x$$

$$\frac{dy}{dx} = \cos x$$

$$\left(\frac{dy}{dx} \right)_{x=\frac{\pi}{2}} = \cos \frac{\pi}{2}$$

$$\left(\frac{dy}{dx} \right)_{\frac{\pi}{2}} = 0$$

$$\begin{aligned}
 \Delta y &= \left(\frac{dy}{dx} \right)_{x=\frac{\pi}{2}} \times (\Delta x) \\
 &= 0 \times \left(\frac{22}{14} - \frac{\pi}{2} \right) \\
 &= 0
 \end{aligned}$$

So,

$$\begin{aligned}
 \sin\left(\frac{22}{14}\right) &= y + \Delta y \\
 &= \sin x + 0 \\
 &= \sin\left(\frac{\pi}{2}\right)
 \end{aligned}$$

$$\sin\left(\frac{22}{14}\right) = 1$$

Differentials Errors and Approximation Ex14.1 Q9(xiv)

$$\text{Let } x = \frac{\pi}{3}, \quad x + \Delta x = \frac{11\pi}{36}$$

$$\begin{aligned}
 \Delta x &= \frac{11\pi}{36} - \frac{\pi}{3} \\
 &= -\frac{\pi}{36} \\
 &= -\frac{22}{7 \times 36} \\
 &= -0.0873
 \end{aligned}$$

$$\text{Let } y = \cos x$$

$$\frac{dy}{dx} = -\sin x$$

dx

$$\begin{aligned}\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{3}} &= -\sin\frac{\pi}{3} \\ &= -\frac{\sqrt{3}}{2} \\ &= -0.866\end{aligned}$$

$$\begin{aligned}\Delta y &= \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{3}} \times (\Delta x) \\ &= (-0.866)(-0.0873) \\ &= 0.0756\end{aligned}$$

$$\begin{aligned}\cos\left(\frac{11\pi}{36}\right) &= y + \Delta y \\ &= \cos x + (0.0756) \\ &= \cos\frac{\pi}{3} + 0.0756 \\ &= \frac{1}{2} + 0.0756 \\ &= 0.5 + 0.0756\end{aligned}$$

$$\cos\frac{11\pi}{36} = 0.7546$$

$$\begin{aligned}\text{Let } x &= 36, \quad x + \Delta x = 37 \\ \Delta x &= 37 - 36 \\ &= 1\end{aligned}$$

$$\begin{aligned}\text{Let } y &= \sqrt{x} \\ \frac{dy}{dx} &= \frac{1}{2\sqrt{x}} \\ \left(\frac{dy}{dx}\right)_{x=36} &= \frac{1}{2\sqrt{36}} \\ &= \frac{1}{12} \\ &= 0.0833\end{aligned}$$

$$\begin{aligned}\Delta y &= \left(\frac{dy}{dx}\right)_{x=36} \times (\Delta x) \\ &= (0.0833)(1) \\ &= 0.0833\end{aligned}$$

$$\begin{aligned}\sqrt{37} &= y + \Delta y \\ &= \sqrt{x} + 0.0833 \\ &= \sqrt{36} + 0.0833\end{aligned}$$

$$\sqrt{37} = 6.0833$$

***** END *****

