Trigonometric Ratios Ex 5.1 Q8

Answer :

Given: $3\cot A = 4$

To check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not

$3\cot A = 4$

Dividing by 3 on both sides,

We get,

$\cot A = \frac{4}{3}$ ...... (1)

By definition,

$\cot A = \frac{1}{\tan A}$

Therefore,

$$\cot A = \frac{1}{\frac{\text{Perpendicular side opposite to} \angle \text{A}}{\text{Base side adjacent to} \angle \text{A}}}$$

$$\cot A = \frac{\text{Base side adjacent to} \angle \text{A}}{\text{Perpendicular side opposite to} \angle \text{A}} \quad \text{...... (2)}$$

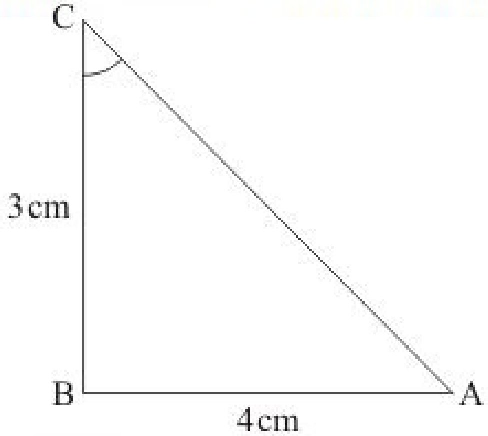
Comparing Equation (1) and (2)

We get,

Base side adjacent to $\angle A$ = 4

Perpendicular side opposite to $\angle A$ = 3

Hence, ΔABC is as shown in figure below



In ΔABC, Hypotenuse is unknown

Hence, It can be found by using Pythagoras theorem

Therefore by applying Pythagoras theorem in ΔABC

We get

$$AC^2 = AB^2 + BC^2$$

Substituting values of sides from the above figure

$$AC^2 = 4^2 + 3^2$$

$$AC^2 = 16 + 9$$

$$AC^2 = 25$$

$$AC = \sqrt{25}$$
$$AC = 5$$

Hence, Hypotenuse = 5

To check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not

We get the values of $\tan A, \cos A, \sin A$

By definition,

$$\tan A = \frac{1}{\cot A}$$

Substituting the value $\cot A$ from Equation (1)

We get,

$$\tan A = \frac{1}{\frac{4}{3}}$$

$$\tan A = \frac{3}{4} \quad \ldots\ldots. \ (3)$$

Now by definition,

$$\sin A = \frac{\text{Perpendicular side opposite to } \angle A}{\text{Hypotenuse}}$$

$$\sin A = \frac{BC}{AC}$$

Substituting values of sides from the above figure

$$\sin A = \frac{3}{5} \quad \ldots\ldots \ (4)$$

Now by definition,

$$\cos A = \frac{\text{Base side adjacent to} \angle A}{\text{Hypotenuse}}$$

$$\cos A = \frac{AB}{AC}$$

Substituting values of sides from the above figure

$$\cos A = \frac{4}{5} \quad \ldots\ldots \ (5)$$

Now we first take L.H.S of Equation $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

$$L.H.S = \frac{1-\tan^2 A}{1+\tan^2 A}$$

Substituting value of $\tan A$ from equation (3)

We get,

$$L.H.S = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2}$$

$$\frac{1-\tan^2 A}{1+\tan^2 A}=\frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2}$$

$$\frac{1-\tan^2 A}{1+\tan^2 A}=\frac{1-\left(\frac{3^2}{4^2}\right)}{1+\left(\frac{3^2}{4^2}\right)}$$

$$\frac{1-\tan^2 A}{1+\tan^2 A}=\frac{1-\frac{9}{16}}{1+\frac{9}{16}}$$

Taking L.C.M on both numerator and denominator

We get,

$$\frac{1-\tan^2 A}{1+\tan^2 A}=\frac{\frac{16-9}{16}}{\frac{16+9}{16}}$$

$$\frac{1-\tan^2 A}{1+\tan^2 A}=\frac{7}{25} \quad \ldots\ldots (6)$$

Now we take R.H.S of Equation $\frac{1-\tan^2 A}{1+\tan^2 A}=\cos^2 A-\sin^2 A$

$$R.H.S=\cos^2 A-\sin^2 A$$

Substituting value of $\sin A$ and $\cos A$ from equation (4) and (5)

We get,

$$R.H.S=\left(\frac{4}{5}\right)^2-\left(\frac{3}{5}\right)^2$$

$$\cos^2 A-\sin^2 A=\frac{4^2}{5^2}-\frac{3^2}{5^2}$$

$$\cos^2 A-\sin^2 A=\frac{16}{25}-\frac{9}{25}$$

$$\cos^2 A-\sin^2 A=\frac{16-9}{25}$$

$$\cos^2 A-\sin^2 A=\frac{7}{25} \quad \ldots\ldots (7)$$

Comparing Equation (6) and (7)

We get,

$$\frac{1-\tan^2 A}{1+\tan^2 A}=\cos^2 A-\sin^2 A$$

Answer: Yes $\frac{1-\tan^2 A}{1+\tan^2 A}=\cos^2 A-\sin^2 A$

*********** END **********