Indefinite Integrals Ex 19.25 Q35

Let $\quad I = \int \sin^{-1}\left(3x - 4x^3\right) dx$

Let $\quad x = \sin\theta$

$dx = \cos\theta\, d\theta$

$= \int \sin^{-1}\left(3\sin\theta - 4\sin^3\theta\right)\cos\theta\, d\theta$

$= \int \sin^{-1}\left(\sin 3\theta\right)\cos\theta\, d\theta$

$= \int 3\theta \cos\theta\, d\theta$

$= 3\left[\theta \int \cos\theta\, d\theta - \int \left(1 \int \cos\theta\, d\theta\right) d\theta\right]$

$= 3\left[\theta\sin\theta - \int \sin\theta\, d\theta\right]$

$= 3\left[\theta\sin\theta + \cos\theta\right] + c$

$I = 3\left[x\sin^{-1}x + \sqrt{1 - x^2}\right] + c$

Indefinite Integrals Ex 19.25 Q36

Let $x = \tan\theta \;\Rightarrow\; dx = \sec^2\theta\, d\theta$

$\therefore \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) = \sin^{-1}\left(\dfrac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta$

$\Rightarrow \int \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) dx = \int 2\theta \cdot \sec^2\theta\, d\theta = 2\int \theta \cdot \sec^2\theta\, d\theta$

Integrating by parts, we obtain

$2\left[\theta \cdot \int \sec^2\theta d\theta - \int\left\{\left(\dfrac{d}{d\theta}\theta\right)\int \sec^2\theta d\theta\right\} d\theta\right]$

$= 2\left[\theta \cdot \tan\theta - \int \tan\theta d\theta\right]$

$= 2\left[\theta\tan\theta + \log|\cos\theta|\right] + C$

$= 2\left[x\tan^{-1}x + \log\left|\dfrac{1}{\sqrt{1+x^2}}\right|\right] + C$

$= 2x\tan^{-1}x + 2\log\left(1+x^2\right)^{-\frac{1}{2}} + C$

$= 2x\tan^{-1}x + 2\left[-\dfrac{1}{2}\log\left(1+x^2\right)\right] + C$

$= 2x\tan^{-1}x - \log\left(1+x^2\right) + C$

Indefinite Integrals Ex 19.25 Q37

Let $\quad I = \int \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)dx$

Let $\quad x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$I = \int \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2 x}\right)\sec^2\theta\, d\theta$

$= \int \tan^{-1}\left(\tan 3\theta\right)\sec^2\theta\, d\theta$

$= \int 3\theta \sec^2\theta\, d\theta$

$= 3\left[\theta\int \sec^2\theta\, d\theta - \int\left(1\int \sec^2\theta\, d\theta\right)d\theta\right]$

$= 3\left[\theta\tan\theta - \int \tan\theta\, d\theta\right]$

$= 3\left[\theta\tan\theta + \log\sec\theta\right] + c$

$= 3\left[x\tan^{-1}x - \log\sqrt{1 + x^2}\right] + c$

$I = 3x\tan^{-1}x - \frac{3}{2}\log\left|1 + x^2\right| + c$

Indefinite Integrals Ex 19.25 Q38

Let $\quad I = \int x^2 \sin^{-1}x\, dx$

$I = \sin^{-1}x\int x^2\, dx - \int\left(\frac{1}{\sqrt{1 - x^2}}\int x^2\, dx\right)dx$

$= \frac{x^3}{3}\sin^{-1}x - \int\frac{x^3}{3\sqrt{1 - x^2}}dx$

$I = \frac{x^3}{3}\sin^{-1}x - \frac{1}{3}I_1 + c_1 \text{ ------ } (1)$

$I_1 = \int\frac{x^3}{\sqrt{1 - x^2}}dx$

Let $\quad 1 - x^2 = t^2$

$-2x\, dx = 2t\, dt$

$-x\, dx = t\, dt$

$I_1 = -\int\frac{\left(1 - t^2\right)t\, dt}{t}$

$= \int\left(t^2 - 1\right)dt$

$= \frac{t^3}{3} - t + c_2$

$= \frac{\left(1 - x^2\right)^{\frac{3}{2}}}{3} - \left(1 - x^2\right)^{\frac{1}{2}} + c_2$

Now,

$I = \frac{x^3}{3}\sin^{-1}x - \frac{1}{9}\left(1 - x^2\right)^{\frac{3}{2}} + \frac{1}{3}\left(1 - x^2\right)^{\frac{1}{2}} + c$

Indefinite Integrals Ex 19.25 Q39

Let $\quad I = \int \frac{\sin^{-1} x}{x^2} dx$

$$= \int \left(\frac{1}{x^2}\right)\left(\sin^{-1} x\right) dx$$

$$I = \left[\sin^{-1} x \int \frac{1}{x^2} dx - \int \left(\frac{1}{\sqrt{1-x^2}} \int \frac{1}{x^2} dx\right) dx\right]$$

$$= \sin^{-1} x \left(-\frac{1}{x}\right) - \int \frac{1}{\sqrt{1-x^2}} \left(-\frac{1}{x}\right) dx$$

$$I = -\frac{1}{x} \sin^{-1} x + \int \frac{1}{x\sqrt{1-x^2}} dx$$

$$I = -\frac{1}{x} \sin^{-1} x + I_1 ------(1)$$

Where,

$$I_1 = \int \frac{1}{x\sqrt{1-x^2}} dx$$

Let $\quad 1 - x^2 = t^2$

$$-2x\, dx = 2t\, dt$$

$$I_1 = \int \frac{x}{x^2\sqrt{1-x^2}} dx$$

$$= -\int \frac{t\,dt}{\left(1-t^2\right)\sqrt{t}}$$

$$= -\int \frac{dt}{\left(1-t^2\right)}$$

$$= \int \frac{1}{t^2-1} dt$$

$$= \frac{1}{2} \log\left|\frac{t-1}{t+1}\right|$$

$$= \frac{1}{2} \log\left|\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1}\right| + c_1$$

Now,

$$I = -\frac{\sin^{-1} x}{x} + \frac{1}{2}\log\left|\left(\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1}\right)\left(\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}-1}\right)\right| + c$$

$$= -\frac{\sin^{-1} x}{x} + \frac{1}{2}\log\left|\frac{\left(\sqrt{1-x^2}-1\right)^2}{1-x^2-1}\right| + c$$

$$= -\frac{\sin^{-1} x}{x} + \frac{1}{2}\log\left|\frac{\left(\sqrt{1-x^2}-1\right)^2}{-x^2}\right| + c$$

$$= -\frac{\sin^{-1} x}{x} + \log\left|\frac{\sqrt{1-x^2}-1}{-x}\right| + c$$

$$I = -\frac{\sin^{-1} x}{x} + \log\left|\frac{1-\sqrt{1-x^2}}{x}\right| + c$$

*********** END **********