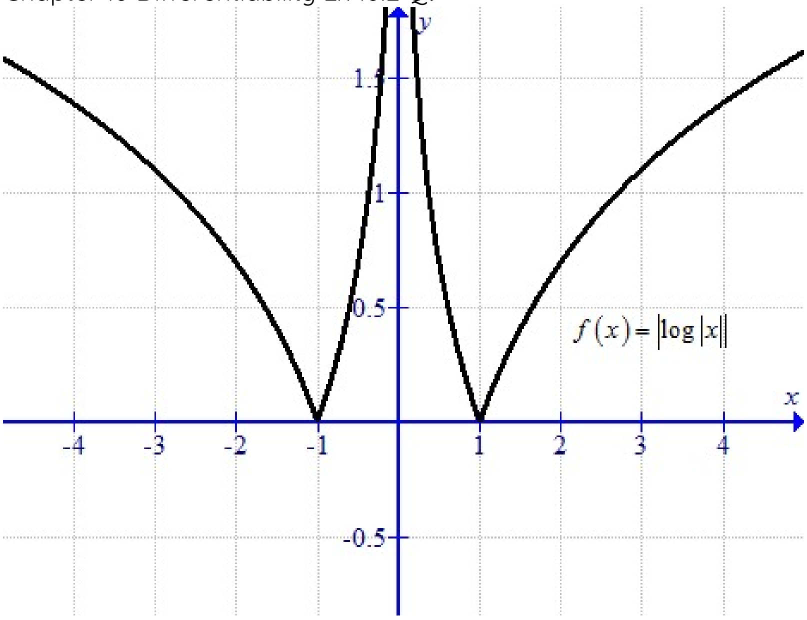
Chapter 10 Differentiability Ex 10.2 Q9



$f(x) = |\log|x||$

Since, it is an absolute function. So, it is continuous function.
The graph of the function is as below:-

Chapter 10 Differentiability Ex 10.2 Q10

$$f(x) = e^{|x|}$$

$$f(x) = \begin{cases} e^{-x} & , x < 0 \\ e^{x} & , x \geq 0 \end{cases}$$

For continuity at $x = 0$

$$\begin{aligned}
\text{RHL} &= \lim_{x \to 0^+} f(x) \\
&= \lim_{h \to 0} f(0+h) \\
&= \lim_{h \to 0} e^{(0+h)} \\
&= \lim_{h \to 0} e^{h} \\
&= e^{0} \\
\text{RHL} &= 1 \\
\text{LHL} &= \lim_{x \to 0^-} f(x) \\
&= \lim_{h \to 0} f(0-h) \\
&= \lim_{h \to 0} e^{-(0-h)} \\
&= \lim_{h \to 0} e^{h} \\
\text{LHL} &= 1
\end{aligned}$$

$$\begin{aligned} f(0) &= e^{0} \\ &= 1 \end{aligned}$$

Now,

LHL $= f(0) =$ RHL

So, $f(x)$ is continuous at $x = 0$

For differentiablility at $x = 0$

$$\begin{aligned}
\text{LHD} &= \lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} \\
&= \lim_{h \to 0} \frac{f(0-h) - e^{0}}{(0-h) - 0} \\
&= \lim_{h \to 0} \frac{e^{-(0-h)} - 1}{-h} \\
&= \lim_{h \to 0} \frac{e^{h} - 1}{-h} \\
&= 1 \qquad \left[\text{Since } \lim_{x \to 0} \frac{e^{x} - 1}{x} = 1\right]
\end{aligned}$$

$$\begin{aligned}
\text{RHD} &= \lim_{hx \to 0^+} \frac{f(x) - f(0)}{x - 0} \\
&= \lim_{h \to 0} \frac{f(0+h) - f(0)}{(0+h) - 0} \\
&= \lim_{h \to 0} \frac{e^{x} - e^{0}}{h} \\
&= \lim_{h \to 0} \frac{e^{h} - 1}{h} \\
&= 1 \qquad \left[\text{Since } \lim_{x \to 0} \frac{e^{x} - 1}{x} = 1\right]
\end{aligned}$$

Clearly,

LHD $\neq$ RHD

So,

$f(x)$ is not differentiable at $x = 0$.

Differentiability Ex 10.2 Q11

$$f(x) = \begin{cases} (x-c)\cos\frac{1}{(x-c)} & , x \neq c \\ 0 & , x = c \end{cases}$$

$$(\text{LHL at } x = c) = \lim f(x)$$

$$(\text{LHL at } x = c) = \lim_{x \to c^-} f(x)$$

$$= \lim_{h \to 0} f(-h)$$

$$= \lim_{h \to 0} (c - h - c)\cos\left(\frac{1}{c - h - c}\right)$$

$$= \lim_{h \to 0} - h\cos\left(-\frac{1}{h}\right)$$

$$= \lim_{h \to 0} - h\cos\left(\frac{1}{h}\right)$$

$$= 0$$

$$(\text{RHL at } x = c) = \lim_{x \to c^+} f(x)$$

$$= \lim_{h \to 0} f(c + h)$$

$$= \lim_{h \to 0} (c + h - c)\cos\left(\frac{1}{c + h - c}\right)$$

$$= \lim_{h \to 0} h\cos\left(\frac{1}{h}\right)$$

$$= 0$$

$$f(e) = 0$$

Since, LHL = $f(x)$ = RHL at $x = c$

$\Rightarrow$ $f(x)$ is continuous at $x = c$

$$(\text{LHD at } x = c) = \lim_{h \to 0} \frac{f(c - h) - f(c)}{-h}$$

$$= \lim_{h \to 0} \frac{(c - h - c)\cos\left(\frac{1}{c - h - c}\right) - 0}{-h}$$

$$= \lim_{h \to 0} \cos\left(-\frac{1}{h}\right)$$

$$= \lim_{h \to 0} \cos\left(\frac{1}{h}\right)$$

$$= k$$

$$(\text{RHD at } x = c) = \lim_{h \to 0} \frac{f(c + h) - f(c)}{h}$$

$$= \lim_{h \to 0} \frac{(c + h - c)\cos\left(\frac{1}{c + h - c}\right) - 0}{h}$$

$$= \lim_{h \to 0} \frac{h\cos\left(\frac{1}{h}\right)}{h}$$

$$= \lim_{h \to 0} \cos\left(\frac{1}{h}\right)$$

$$= k$$

$$(\text{LHD at } x = c) = (\text{RHD at } x = c)$$

So,

$f(x)$ is differentiable and continuous at $x = c$.

Differentiability Ex 10.2 Q12

$$f(x) = |\sin x| = \begin{cases} -\sin x & , \ x < n\pi \\ \sin x & , \ x \geq n\pi \end{cases}$$

For $x = n\pi$ (n even)

$$(\text{LHD at } x = n\pi) = \lim_{x \to n\pi^{-1}} \frac{f(x) - f(n\pi)}{x - n\pi}$$

$$= \lim_{h \to 0} \frac{-\sin(n\pi - h) - \sin n\pi}{n\pi - h - n\pi}$$

$$= \lim_{h \to 0} \frac{\sin h - 0}{-h}$$

$$= -1$$

$$(\text{RHD at } x = n\pi) = \lim_{h \to 0} \frac{\sin(n\pi + h) - \sin n\pi}{h}$$

$$= \lim_{h \to 0} \frac{\sin h}{h}$$

$$= 1$$

$$(\text{LHD at } x = n\pi) \neq (\text{RHD at } x = n\pi)$$

For $x = n\pi$ (n is odd)

$$(\text{LHD at } x = n\pi) = \lim_{h \to 0} \frac{-\sin(n\pi - h) - \sin n\pi}{-h}$$

$$= \lim_{h \to 0} \frac{-\sin h}{-h}$$

$$= 1$$

$$(\text{RHD at } x = n\pi) = \lim_{h \to 0} \frac{\sin(n\pi + h) - \sin n\pi}{h}$$

$$= \lim_{h \to 0} \frac{-\sin h - 0}{h}$$

$$= -1$$

$$(\text{LHD at } x = n\pi) \neq (\text{RHD at } x = n\pi)$$

Thus,

$f(x) = |\sin x|$ is not differentiable at $x = n\pi$

$f(x) = \cos|x|$

Since, $\cos(-x) = \cos x$

$\Rightarrow \quad f(x) = \cos x$

$\Rightarrow \quad f(x) = \cos|x|$ is differnetiable everywhere.

*********** END **********