CHAPTER 38

ELECTROMAGNETIC INDUCTION

38.1 FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

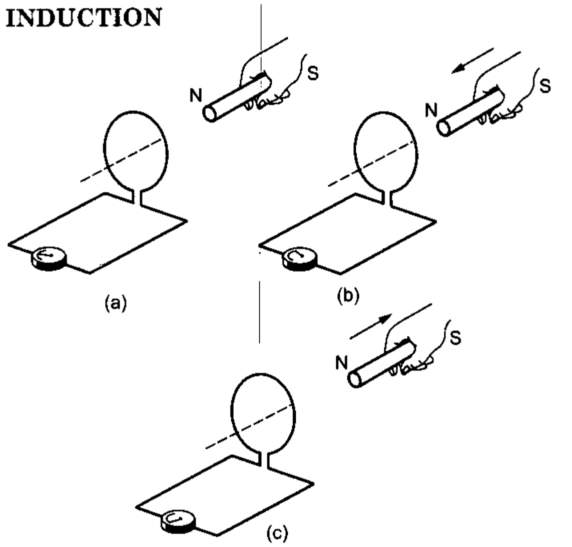


Figure 38.1

Figure (38.1a) shows a bar magnet placed along the axis of a conducting loop containing a galvanometer. There is no current in the loop and correspondingly no deflection in the galvanometer. If we move the magnet towards the loop (figure 38.1b), there is a deflection in the galvanometer showing that there is an electric current in the loop. If the magnet is moved away from the loop (figure 38.1c), again there is a current but the current is in the opposite direction. The current exists as long as the magnet is moving. Faraday studied this behaviour in detail by performing a number of experiments and discovered the following law of nature:

Whenever the flux of magnetic field through the area bounded by a closed conducting loop changes, an emf is produced in the loop. The emf is given by

$$\mathcal{E} = -\frac{d\Phi}{dt} \qquad \dots (38.1)$$

where $\Phi = \int \vec{B} \cdot d\vec{S}$ *is the flux of the magnetic field through the area.*

We shall call the quantity Φ *magnetic flux.* The SI unit of magnetic flux is called weber which is equivalent to tesla metre^2.

The law described by equation (38.1) is called *Faraday's law of electromagnetic induction.* The flux may be changed in a number of ways. One can change the magnitude of the magnetic field $\vec{B}$ at the site of the loop, the area of the loop or the angle between the area-vector $d\vec{S}$ and the magnetic field $\vec{B}$. In any case, as long as the flux keeps changing, the emf is present. The emf so produced drives an electric current through the loop. If the resistance of the loop is R, the current is

$$i = \frac{\mathcal{E}}{R} = -\frac{1}{R}\frac{d\Phi}{dt}. \qquad \dots (38.2)$$

The emf developed by a changing flux is called *induced emf* and the current produced by this emf is called *induced current.*

Direction of Induced Current

The direction of the induced current in a loop may be obtained using equation (38.1) or (38.2). The procedure to decide the direction is as follows:

Put an arrow on the loop to choose the positive sense of current. This choice is arbitrary. Using right-hand thumb rule find the positive direction of the normal to the area bounded by the loop. If the fingers curl along the loop in the positive sense, the thumb represents the positive direction of the normal. Calculate the flux $\Phi = \int \vec{B} \cdot d\vec{S}$ through the area bounded by the loop. If the flux increases with time, $\frac{d\Phi}{dt}$ is positive and $\mathcal{E}$ is negative from equation (38.1). Correspondingly, the current is negative. It is, therefore, in the direction opposite to the arrow put on the loop. If Φ decreases with time, $\frac{d\Phi}{dt}$ is negative, $\mathcal{E}$ is positive and the current is along the arrow.

Example 38.1

Figure (38.2) shows a conducting loop placed near a long, straight wire carrying a current i as shown. If the current increases continuously, find the direction of the induced current in the loop.

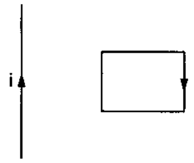


Figure 38.2

Solution : Let us put an arrow on the loop as shown in the figure. The right-hand thumb rule shows that the positive normal to the loop is going into the plane of the diagram. Also, the same rule shows that the magnetic field at the site of the loop due to the current is also going into the plane of the diagram. Thus, $\vec{B}$ and $d\vec{S}$ are along the same direction everywhere so that the flux $\Phi = \int \vec{B} \cdot d\vec{S}$ is positive. If i increases, the magnitude of Φ increases. Since Φ is positive and its magnitude increases, $\frac{d\Phi}{dt}$ is positive. Thus, $\mathscr{E}$ is negative and hence, the current is negative. The current is, therefore, induced in the direction opposite to the arrow.

38.2 LENZ'S LAW

Another way to find the direction of the induced current in a conducting loop is to use Lenz's law. The current is induced by the changing magnetic flux. The induced current itself produces a magnetic field and hence a magnetic flux. This magnetic flux may have the same sign as the original flux or it may have the opposite sign. It strengthens the original flux if it has the same sign and weakens it otherwise. Lenz's law states:

The direction of the induced current is such that it opposes the change that has induced it.

If a current is induced by an increasing flux, it will weaken the original flux. If a current is induced by a decreasing flux, it will strengthen the original flux.

Figure (38.3) shows some situations. In figure (38.3a), a magnet is brought towards a circular loop. The north pole faces the loop. As the magnet gets closer to the loop, the magnetic field increases and hence, the flux of the magnetic field through the area of the loop increases. The induced current should weaken the flux. The original field is away from the magnet, so the induced field should be towards the magnet. Using the right-hand thumb rule, we can find the direction of the current that produces a field towards the magnet.

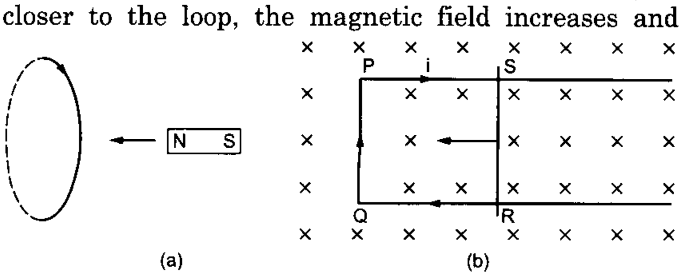


Figure 38.3

In figure (38.3b), the wire RS slides towards left so that the area of $PQRS$ decreases. As a result, the magnetic flux through the area $PQRS$ decreases. The induced current should strengthen the original flux. The induced current should produce a magnetic field along the original field which is going into the plane of the diagram. Using the right-hand thumb rule, we find that the induced current should be clockwise as shown in the figure.

38.3 THE ORIGIN OF INDUCED EMF

An electric current is established in a conducting wire when an electric field exists in it. The flow of charges tend to destroy the field and some external mechanism is needed to maintain the electric field in the wire. It is the work done per unit charge by this external mechanism that we call emf. When the magnetic flux through a closed loop changes, an electric current results. What is the external mechanism that maintains the electric field in the loop to drive the current ? In other words, what is the mechanism to produce an emf ? Let us now investigate this question.

The flux $\int \vec{B} \cdot d\vec{S}$ can be changed by

(a) keeping the magnetic field constant as time passes and moving whole or part of the loop

(b) keeping the loop at rest and changing the magnetic field

(c) combination of (a) and (b), that is, by moving the loop (partly or wholely) as well as by changing the field.

The mechanism by which emf is produced is different in the two basic processes (a) and (b). We now study them under the headings *motional emf* and *induced electric field.*

Motional Emf

Figure (38.4) shows a rod PQ of length l moving in a magnetic field $\vec{B}$ with a constant velocity $\vec{v}$. The length of the rod is perpendicular to the magnetic field and the velocity is perpendicular to both the magnetic

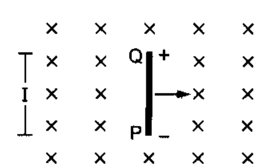


Figure 38.4

field and the rod. The free electrons of the wire also move with this velocity $\vec{v}$ together with the random velocity they have in the rod. The magnetic force due to the random velocity is zero on the average. Thus, the magnetic field exerts an average force $\vec{F}_b = q\vec{v} \times \vec{B}$ on each free electron where $q = -1.6 \times 10^{-19}$ C is the charge on the electron. This force is towards QP and hence the free electrons will move towards P. Negative charge is accumulated at P and positive charge appears at Q. An electrostatic field E is developed within the wire from Q to P. This field exerts a force $\vec{F}_e = q\vec{E}$ on each free electron. The charge keeps on accumulating until a situation comes when $F_b = F_e$

or, $\quad |q\vec{v} \times \vec{B}| = |q\vec{E}| \quad$ or, $\quad vB = E.$

After this, there is no resultant force on the free electrons of the wire PQ. The potential difference between the ends Q and P is

$$V = El = vBl.$$

Thus, it is the magnetic force on the moving free electrons that maintains the potential difference $V = vBl$ and hence produces an emf

$$\mathcal{E} = vBl. \quad \dots (38.3)$$

As this emf is produced due to the motion of a conductor, it is called *motional emf*.

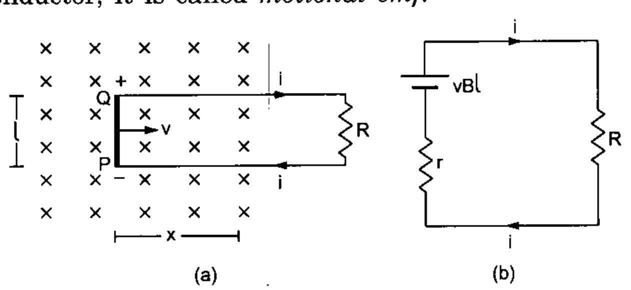


Figure 38.5

If the ends P and Q are connected by an external resistor (figure 38.5a), an electric field is produced in this resistor due to the potential difference. A current is established in the circuit. The electrons flow from P to Q via the external circuit and this tries to neutralise the charges accumulated at P and Q. The magnetic force qvB on the free electrons in the wire QP, however, drives the electrons back from Q to P to maintain the potential difference and hence the current.

Thus, we can replace the moving rod QP by a battery of emf vBl with the positive terminal at Q and the negative terminal at P. The resistance r of the rod QP may be treated as the internal resistance of the battery. Figure (38.5b) shows the equivalent circuit. The current is $i = \frac{vBl}{R + r}$ in the clockwise direction (induced current).

We can also find the induced emf and the induced current in the loop in figure (38.5a) from Faraday's law of electromagnetic induction. If x be the length of the circuit in the magnetic field at time t, the magnetic flux through the area bounded by the loop is

$$\Phi = Blx.$$

The magnitude of the induced emf is

$$\mathcal{E} = \left|\frac{d\Phi}{dt}\right| = \left|Bl\frac{dx}{dt}\right|$$

$$= vBl.$$

The current is $i = \frac{vBl}{R + r}$. The direction of the current can be worked out from Lenz's law.

Example 38.2

Figure (38.6a) shows a rectangular loop MNOP being pulled out of a magnetic field with a uniform velocity v by applying an external force F. The length MN is equal to l and the total resistance of the loop is R. Find (a) the current in the loop, (b) the magnetic force on the loop, (c) the external force F needed to maintain the velocity, (d) the power delivered by the external force and (e) the thermal power developed in the loop.

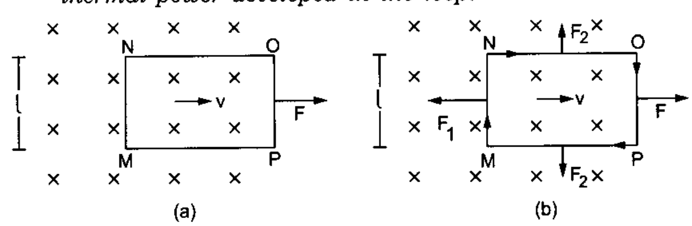


Figure 38.6

Solution : (a) The emf induced in the loop is due to the motion of the wire MN. The emf is $\mathcal{E} = vBl$ with the positive end at N and the negative end at M. The current is

$$i = \frac{\mathcal{E}}{R} = \frac{vBl}{R}$$

in the clockwise direction (figure 38.6b).

(b) The magnetic force on the wire MN is $\vec{F}_1 = i\vec{l} \times \vec{B}$. The magnitude is $F_1 = ilB = \frac{vB^2l^2}{R}$ and is opposite to the velocity. The forces on the parts of the wire NO and PM, lying in the field, cancel each other. The resultant magnetic force on the loop is, therefore, $F_1 = \frac{B^2l^2v}{R}$ opposite to the velocity.

(c) To move the loop at a constant velocity, the resultant force on it should be zero. Thus, one should pull the loop with a force

$$F = F_1 = \frac{vB^2l^2}{R}.$$

(d) The power delivered by the external force is

$$P = F\,v = \frac{v^2 B^2 l^2}{R}.$$

(e) The thermal power developed is

$$P = i^2 R = \left(\frac{vBl}{R}\right)^2 \cdot R = \frac{v^2 B^2 l^2}{R}.$$

We see that the power delivered by the external force is equal to the thermal power developed in the loop. This is consistent with the principle of conservation of energy.

Induced Electric Field

Consider a conducting loop placed at rest in a magnetic field $\vec{B}$. Suppose, the field is constant till $t = 0$ and then changes with time. An induced current starts in the loop at $t = 0$.

The free electrons were at rest till $t = 0$ (we are not interested in the random motion of the electrons). The magnetic field cannot exert force on electrons at rest. Thus, the magnetic force cannot start the induced current. The electrons may be forced to move only by an electric field and hence we conclude that an electric field appears at $t = 0$. This electric field is produced by the changing magnetic field and not by charged particles according to the Coulomb's law or the Gauss's law. The electric field produced by the changing magnetic field is nonelectrostatic and nonconservative in nature. We cannot define a potential corresponding to this field. We call it *induced electric field*. The lines of induced electric field are closed curves. There are no starting and terminating points of the field lines.

If $\vec{E}$ be the induced electric field, the force on a charge q placed in the field is $q\vec{E}$. The work done per unit charge as the charge moves through $\vec{dl}$ is $\vec{E}\cdot\vec{dl}$. The emf developed in the loop is, therefore,

$$\mathcal{E} = \oint \vec{E}\cdot\vec{dl}.$$

Using Faraday's law of induction,

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

or, $$\oint \vec{E}\cdot\vec{dl} = -\frac{d\Phi}{dt}. \qquad \text{... (38.4)}$$

The presence of a conducting loop is not necessary to have an induced electric field. As long as $\vec{B}$ keeps changing, the induced electric field is present. If a loop is there, the free electrons start drifting and consequently an induced current results.

38.4 EDDY CURRENT

Consider a solid plate of metal which enters a region having a magnetic field (figure 38.7a). Consider a loop drawn on the plate, a part of which is in the

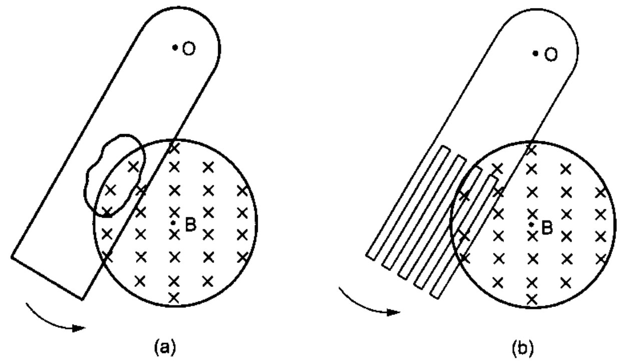


Figure 38.7

field. As the plate moves, the magnetic flux through the area bounded by the loop changes and hence a current is induced. There may be a number of such loops on the plate and hence currents are induced on the surface along a variety of paths. Such currents are called *eddy currents*. The basic idea is that we do not have a definite conducting loop to guide the induced current. The system itself looks for the loops on the surface along which eddy currents are induced. Because of the eddy currents in the metal plate, thermal energy is produced in it. This energy comes at the cost of the kinetic energy of the plate and the plate slows down. This is known as *electromagnetic damping*. To reduce electromagnetic damping, one can cut slots in the plate (figure 38.7b). This reduces the possible paths of the eddy current considerably.

38.5 SELF-INDUCTION

When a current is established in a closed conducting loop, it produces a magnetic field. This magnetic field has its flux through the area bounded by the loop. If the current changes with time, the flux through the loop changes and hence an emf is induced in the loop. This process is called *self-induction*. The name is so chosen because the emf is induced in the loop by changing the current in the same loop.

The magnetic field at any point due to a current is proportional to the current. The magnetic flux through the area bounded by a current-carrying loop is, therefore, proportional to the current. We can write

$$\Phi = Li \qquad \text{... (38.5)}$$

where L is a constant depending on the geometrical construction of the loop. This constant is called *self-inductance* of the loop. The induced emf $\mathcal{E}$, when the current in the coil changes, is given by

$$\mathcal{E} = -\frac{d\Phi}{dt}.$$

Using equation (38.5),

$$\mathcal{E} = -L\frac{di}{dt}. \qquad \text{... (38.6)}$$

The SI unit of self-inductance L is weber ampere^{-1} from equation (38.5) or volt second ampere^{-1} from (38.6). It is given a special name henry and is abbreviated as H.

If we have a coil or a solenoid of N turns, the flux through each turn is $\int \vec{B} \cdot d\vec{S}$. If this flux changes, an emf is induced in each turn. The net emf induced between the ends of the coil is the sum of all these. Thus,

$$\mathcal{E} = -N \frac{d}{dt} \int \vec{B} \cdot d\vec{S}.$$

One can compare this with equation (38.6) to get the inductance.

Example 38.3

An average induced emf of 0·20 V appears in a coil when the current in it is changed from 5·0 A in one direction to 5·0 A in the opposite direction in 0·20 s. Find the self-inductance of the coil.

Solution :

Average $\frac{di}{dt} = \frac{(-5{\cdot}0\text{ A}) - (5{\cdot}0\text{ A})}{0{\cdot}20\text{ s}} = -50\text{ A/s}.$

Using $\mathcal{E} = -L\frac{di}{dt}$,

$0{\cdot}2\text{ V} = L(50\text{ A/s})$

or, $L = \frac{0{\cdot}2\text{ V}}{50\text{ A/s}} = 4{\cdot}0\text{ mH}.$

Self-inductance of a Long Solenoid

Consider a long solenoid of radius r having n turns per unit length. Suppose a current i is passed through the solenoid. The magnetic field produced inside the solenoid is $B = \mu_0 ni$. The flux through each turn of the solenoid is

$$\Phi = \int \vec{B} \cdot d\vec{S} = (\mu_0 ni)\pi r^2.$$

The emf induced in each turn is

$$-\frac{d\Phi}{dt} = -\mu_0 n \pi r^2 \frac{di}{dt}.$$

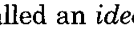
As there are nl turns in a length l of the solenoid, the net emf across a length l is

$$\mathcal{E} = -(nl)(\mu_0 n \pi r^2)\frac{di}{dt}.$$

Comparing with $\mathcal{E} = -L\frac{di}{dt}$, the self-inductance is

$$L = \mu_0 n^2 \pi r^2 l. \quad \ldots (38.7)$$

We see that the self-inductance depends only on geometrical factors.

A coil or a solenoid made from thick wire has negligible resistance but a considerable self-inductance. Such an element is called an *ideal inductor* and is indicated by the symbol  .

The self-induced emf in a coil opposes the change in the current that has induced it. This is in accordance with the Lenz's law. If the current is increasing, the induced current will be opposite to the original current. If the current is decreasing, the induced current will be along the original current.

Example 38.4

Consider the circuit shown in figure (38.8). The sliding contact is being pulled towards right so that the resistance in the circuit is increasing. Its value at the instant shown is 12 Ω. Will the current be more than 0·50 A *or less than it at this instant?*

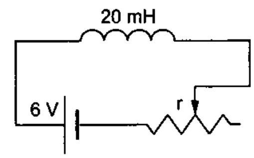


Figure 38.8

Solution : As the sliding contact is being pulled, the current in the circuit changes. An induced emf $\mathcal{E} = -L\frac{di}{dt}$ is produced across the inductor. The net emf in the circuit is $6\text{ V} - L\frac{di}{dt}$ and hence the current is

$$i = \frac{6\text{ V} - L\frac{di}{dt}}{12\ \Omega} \quad \ldots \text{(i)}$$

at the instant shown. Now the resistance in the circuit is increasing, the current is decreasing and so $\frac{di}{dt}$ is negative. Thus, the numerator of (i) is more than 6 V and hence i is greater than $\frac{6\text{ V}}{12\ \Omega} = 0{\cdot}50\text{ A}.$

38.6 GROWTH AND DECAY OF CURRENT IN AN LR CIRCUIT

Growth of Current

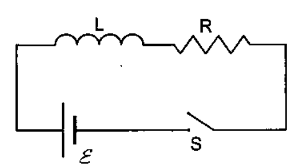


Figure 38.9

Figure (38.9) shows an inductance L, a resistance R and a source of emf $\mathcal{E}$ connected in series through a switch S. Initially, the switch is open and there is no current in the circuit. At $t = 0$, the switch is closed and the circuit is completed. As the current increases in the inductor, a self-induced emf $\left(-L\frac{di}{dt}\right)$ is produced. Using Kirchhoff's loop law,

$$\mathcal{E} - L\frac{di}{dt} = Ri$$

or, $$L\frac{di}{dt} = \mathcal{E} - Ri$$

or, $$\frac{di}{\mathcal{E} - Ri} = \frac{dt}{L}.$$

At $t = 0$, $i = 0$ and at time t the current is i. Thus,

$$\int_0^i \frac{di}{\mathcal{E} - Ri} = \int_0^t \frac{dt}{L}$$

or, $$-\frac{1}{R}\ln\frac{\mathcal{E} - Ri}{\mathcal{E}} = \frac{t}{L}$$

or, $$\frac{\mathcal{E} - Ri}{\mathcal{E}} = e^{-tR/L}$$

or, $$\mathcal{E} - Ri = \mathcal{E}e^{-tR/L}$$

or, $$i = \frac{\mathcal{E}}{R}(1 - e^{-tR/L}). \quad \text{... (38.8)}$$

The constant L/R has dimensions of time and is called the *time constant* of the *LR* circuit. Writing $L/R = \tau$ and $\mathcal{E}/R = i_0$, equation (38.8) becomes

$$i = i_0(1 - e^{-t/\tau}). \quad \text{... (38.9)}$$

Figure (38.10) shows the plot of the current versus time. The current gradually rises from $t = 0$ and attains the maximum value i_0 after a long time. At $t = \tau$, the current is

$$i = i_0\left(1 - \frac{1}{e}\right) = 0{\cdot}63\, i_0.$$

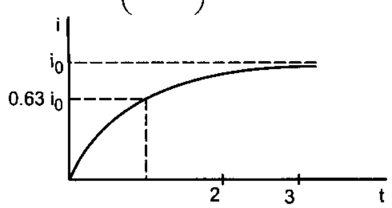


Figure 38.10

Thus, in one time constant, the current reaches 63% of the maximum value. The time constant tells us how fast will the current grow. If the time constant is small, the growth is steep. Equation (38.9) shows that $i = i_0$ at $t = \infty$. In principle, it takes infinite time for the current to attain its maximum value. In practice, however, a small number of time constants may be sufficient for the current to reach almost the maximum value.

Example 38.5

An inductor ($L = 20$ mH), a resistor ($R = 100\ \Omega$) and a battery ($\mathcal{E} = 10$ V) are connected in series. Find (a) the time constant, (b) the maximum current and (c) the time elapsed before the current reaches 99% of the maximum value.

Solution : (a) The time constant is

$$\tau = \frac{L}{R} = \frac{20\ \text{mH}}{100\ \Omega} = 0{\cdot}20\ \text{ms}.$$

(b) The maximum current is

$$i = \mathcal{E}/R = \frac{10\ \text{V}}{100\ \Omega} = 0{\cdot}10\ \text{A}.$$

(c) Using $\quad i = i_0(1 - e^{-t/\tau}),$

$$0{\cdot}99\, i_0 = i_0(1 - e^{-t/\tau})$$

or, $$e^{-t/\tau} = 0{\cdot}01$$

or, $$\frac{t}{\tau} = -\ln(0{\cdot}01)$$

or, $$t = (0{\cdot}20\ \text{ms})\ln(100) = 0{\cdot}92\ \text{ms}.$$

Decay of Current

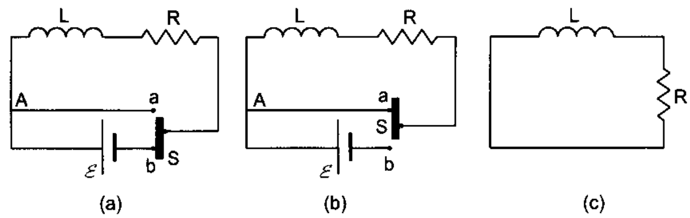


Figure 38.11

Consider the arrangement shown in figure (38.11a). The sliding switch S can be slid up and down. The circuit is complete and a steady current $i = i_0$ is maintained through the circuit. Suddenly at $t = 0$, the switch S is moved to connect the point a. This completes the circuit through the wire Aa and disconnects the battery from the circuit (figure 38.11b). The special arrangement of the switch ensures that the circuit through the wire Aa is completed before the battery is disconnected. The equivalent circuit is redrawn in figure (38.11c).

As the battery is disconnected, the current decreases in the circuit. This induces an emf $\left(-L\frac{di}{dt}\right)$ in the inductor. As this is the only emf in the circuit,

$$-L\frac{di}{dt} = Ri$$

or, $$\frac{di}{i} = -\frac{R}{L}dt.$$

At $t = 0$, $i = i_0$. If the current at time t be i,

$$\int_{i_0}^{i} \frac{di}{i} = \int_0^t -\frac{R}{L}dt$$

or, $$\ln\frac{i}{i_0} = -\frac{R}{L}t$$

or, $$i = i_0 e^{-tR/L} \quad \text{... (38.10)}$$

or, $$i = i_0 e^{-t/\tau} \quad \text{... (38.11)}$$

where $\tau = L/R$ is the time constant of the circuit.

We see that the current does not suddenly fall to zero. It gradually decreases as time passes. At $t = \tau$,

$$i = i_0 / e = 0{\cdot}37\, i_0.$$

The current reduces to 37% of the initial value in one time constant, i.e., 63% of the decay is complete. If the time constant is small, the decay is steep. Figure (38.12) shows the plot of the current against time.

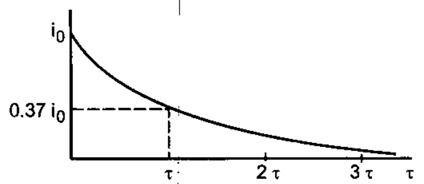


Figure 38.12

Example 38.6

An inductor ($L = 20$ mH), a resistor ($R = 100\ \Omega$) and a battery ($\mathcal{E} = 10$ V) are connected in series. After a long time the circuit is short-circuited and then the battery is disconnected. Find the current in the circuit 1 ms after short-circuiting.

Solution :

The initial current is $i = i_0 = \mathcal{E}/R = \dfrac{10\ \text{V}}{100\ \Omega} = 0{\cdot}10$ A.

The time constant is $\tau = L/R = \dfrac{20\ \text{mH}}{100\ \Omega} = 0{\cdot}20$ ms.

The current at $t = 1$ ms is

$$i = i_0\, e^{-t/\tau}$$

$$= (0{\cdot}10\ \text{A})\, e^{-(1\ \text{ms}/0{\cdot}20\ \text{ms})}$$

$$= (0{\cdot}10\ \text{A})\, e^{-5} = 6{\cdot}7 \times 10^{-4}\ \text{A}.$$

38.7 ENERGY STORED IN AN INDUCTOR

When a capacitor is charged, electric field builds up between its plates and energy is stored in it. Similarly, when an inductor carries a current, a magnetic field builds up in it and magnetic energy is stored in it.

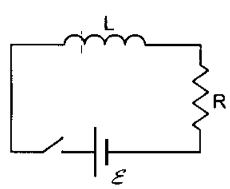


Figure 38.13

Consider the circuit shown in figure (38.13). As the connections are made, the current grows in the circuit and the magnetic field increases in the inductor. Part of the work done by the battery during the process is stored in the inductor as magnetic field energy and the rest appears as thermal energy in the resistor. After sufficient time, the current and hence the magnetic field becomes constant and further work done by the battery appears completely as thermal energy. If i be the current in the circuit at time t, we have

$$\mathcal{E} - L\frac{di}{dt} = iR$$

or, $$\mathcal{E}\, i\, dt = i^2 R\, dt + L\, i\, di$$

or, $$\int_0^t \mathcal{E}\, i\, dt = \int_0^t i^2 R\, dt + \int_0^i L\, i\, di$$

or, $$\int_0^t \mathcal{E}\, i\, dt = \int_0^t i^2 R\, dt + \frac{1}{2} L\, i^2. \qquad \ldots \text{(i)}$$

Now ($i\, dt$) is the charge flowing through the circuit during the time t to $t + dt$. Thus, ($\mathcal{E}\, i\, dt$) is the work done by the battery in this period. The quantity on the left-hand side of equation (i) is, therefore, the total work done by the battery in time 0 to t. Similarly, the first term on the right-hand side of equation (i) is the total thermal energy (Joule heat) developed in the resistor in time t. Thus $\frac{1}{2} Li^2$ is the energy stored in the inductor as the current in it increases from 0 to i. As the energy is zero when the current is zero, the energy stored in an inductor, carrying a current i, is

$$U = \frac{1}{2} L\, i^2. \qquad \ldots (38.12)$$

Energy Density in Magnetic Field

Consider a long solenoid of radius r, length l and having n turns per unit length. If it carries a current i, the magnetic field within it is

$$B = \mu_0\, ni.$$

Neglecting the end effects, the field outside is zero. The self-inductance of this solenoid is, from equation (38.7),

$$L = \mu_0\, n^2 \pi r^2 l.$$

The magnetic energy is, therefore,

$$U = \frac{1}{2} Li^2 = \frac{1}{2} \mu_0\, n^2 \pi r^2 l\, i^2$$

$$= \frac{1}{2\mu_0} (\mu_0\, ni)^2\, V = \frac{B^2}{2\mu_0} V$$

where $V = \pi r^2 l$ is the volume enclosed by the solenoid. As the field is assumed uniform throughout the volume of the solenoid and zero outside, the energy per unit volume in the magnetic field, i.e., the energy density, is

$$u = \frac{U}{V} = \frac{B^2}{2\mu_0}. \qquad \ldots (38.13)$$

In deriving this equation, we have assumed that there is no magnetic material at the site of the field.

Example 38.7

Calculate the energy stored in an inductor of inductance 50 mH when a current of 2·0 A is passed through it.

Solution : The energy stored is

$$U = \frac{1}{2} Li^2 = \frac{1}{2}(50 \times 10^{-3}\ \text{H})\ (2{\cdot}0\ \text{A})^2 = 0{\cdot}10\ \text{J}.$$

38.8 MUTUAL INDUCTION

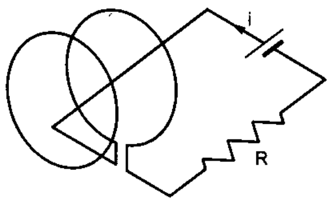


Figure 38.14

Suppose two closed circuits are placed close to each other and a current i is passed in one. It produces a magnetic field and this field has a flux Φ through the area bounded by the other circuit. As the magnetic field at a point is proportional to the current producing it, we can write

$$\Phi = Mi \qquad \dots (38.14)$$

where M is a constant depending on the geometrical shapes of the two circuits and their placing. This constant is called *mutual inductance* of the given pair of circuits. If the same current i is passed in the second circuit and the flux is calculated through the area bounded by the first circuit, the same proportionality constant M appears. If there are more than one turns in a circuit, one has to add the flux through each turn before applying equation (38.14).

If the current i in one circuit changes with time, the flux through the area bounded by the second circuit also changes. Thus, an emf is induced in the second circuit. This phenomenon is called *mutual induction.* From equation (38.14), the induced emf is

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

$$= -M\frac{di}{dt} \cdot \qquad \dots (38.15)$$

Example 38.8

A solenoid S_1 is placed inside another solenoid S_2 as shown in figure (38.15). The radii of the inner and the outer solenoids are r_1 and r_2 respectively and the numbers of turns per unit length are n_1 and n_2 respectively. Consider a length l of each solenoid. Calculate the mutual inductance between them.

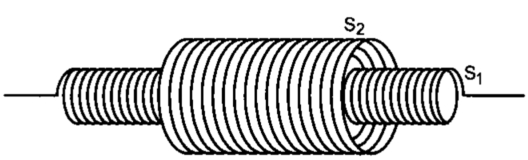


Figure 38.15

Solution : Suppose a current i is passed through the inner solenoid S_1. A magnetic field $B = \mu_0 n_1 i$ is produced inside S_1 whereas the field outside it is zero. The flux through each turn of S_2 is

$$B\pi r_1^2 = \mu_0 n_1 i \pi r_1^2.$$

The total flux through all the turns in a length l of S_2 is

$$\Phi = (\mu_0 n_1 i \pi r_1^{\,2}) n_2 l = (\mu_0 n_1 n_2 \pi r_1^{\,2} l)\, i.$$

Thus, $\qquad M = \mu_0 n_1 n_2 \pi r_1^{\,2} l. \qquad \dots$ (i)

38.9 INDUCTION COIL

An induction coil is used to produce a large emf from a source of low emf. The schematic design of an induction coil known as Ruhmkorff's induction coil is shown in figure (38.16). It consists of a primary coil T wound over a laminated soft-iron core and a secondary coil S wound coaxially over the primary coil. The secondary coil is connected to two rods G_1 and G_2. The separation between the rods may be adjusted.

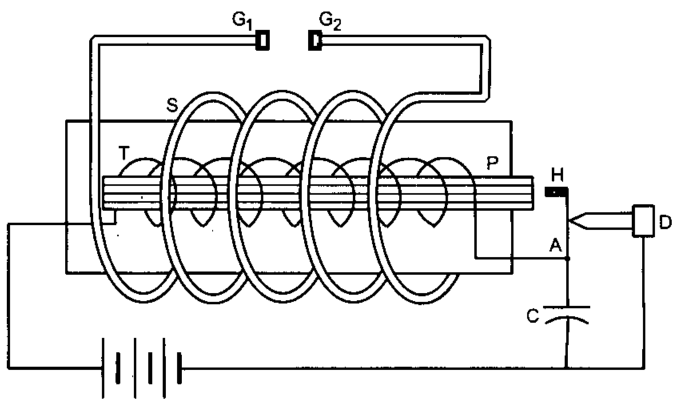


Figure 38.16

The primary circuit contains a battery and a circuit interrupter. The circuit interrupter may be formed as follows. One end of the primary coil is connected to a thin metallic strip A with a soft-iron hammer H at one end. The hammer is close to the soft-iron core of the primary. The other end of the primary is connected to a screw D through the battery. The screw just touches the metallic strip. This arrangement forms the circuit interrupter. A capacitor C is connected in parallel to the circuit interrupter as shown in the figure.

Working

(a) *Make and break*: When the screw D touches the strip A, the primary circuit is completed and a current is established in the primary circuit. Because of the current in the primary coil, the soft-iron core becomes magnetized. It attracts the iron hammer H and the contact between the screw D and the strip A is broken. The current stops in the primary circuit, the iron core is demagnetized and the strip acquires its natural position. The screw D again touches the strip and the current is established. This process of successive 'make and break' continues.

(b) *Growth and decay of current in the primary*: When the contact between the screw and the strip is made, the current grows in the primary circuit. Because of the self-inductance of the primary coil, the growth will be slow. When the contact is broken, the current suddenly decreases to zero. The rate of decay of the current when the circuit is broken is quite high as compared to the rate of growth of the current when the circuit is closed.

(c) *Induced emf in the secondary*: As the current i in the primary changes, the magnetic flux linked with the secondary also changes. This induces an emf across the ends of the secondary coil. This emf appears between the rods G_1 and G_2. When the circuit breaks, $\left|\frac{di}{dt}\right|$ is very large and hence the emf induced in the secondary is also very large. With suitable separation between the rods, one can see sparks jumping from one rod to the other. One can easily produce emf of the order of 50000 V starting from a 12 V battery using the above arrangement. The secondary emf induced at the time of break is in the opposite direction to that induced at the time of make. The high emf produced between the rods can be used to operate equipments like a discharge tube.

(d) *Role of the capacitor*: When the circuit is broken, a large emf is produced due to the self-induction in the primary which tends to drive a current in the direction of the original current. A large potential difference appears between the screw D and the strip A. This may cause sparks to jump and hence the current to continue in the same direction. This reduces the rate of decay of the current thereby reducing the emf across the secondary. Secondly, repeated sparks between the screw and the strip damage the surfaces. The capacitor provides an alternative path to the current when the circuit is broken. The current charges the capacitor. Thus, sparks do not occur across the screw–strip gap and the current drops more rapidly. This increases $\left|\frac{di}{dt}\right|$ and hence the induced emf across the secondary. Not only this, the charged capacitor soon gets discharged by sending a current through the primary in the opposite direction. Thus, the currrent changes not only from i to 0 but from i to almost $-i$. The change in flux is almost doubled and correspondingly, the induced emf across the secondary is also increased.

Worked Out Examples

1. *A conducting circular loop is placed in a uniform magnetic field $B = 0{\cdot}020$ T with its plane perpendicular to the field. Somehow, the radius of the loop starts shrinking at a constant rate of $1{\cdot}0$ mm s^{-1}. Find the induced emf in the loop at an instant when the radius is 2 cm.*

Solution : Let the radius be r at time t. The flux of the magnetic field at this instant is $\Phi = \pi r^2 B$.

Thus, $$\frac{d\Phi}{dt} = 2\pi r B \frac{dr}{dt}.$$

The induced emf when $r = 2{\cdot}0$ cm is, therefore,

$$\mathcal{E} = 2\pi(2\text{ cm})\,(0{\cdot}02\text{ T})\,(1{\cdot}0\text{ mm s}^{-1}) = 2{\cdot}5\ \mu\text{V}.$$

2. *A uniform magnetic field B exists in a direction perpendicular to the plane of a square frame made of copper wire. The wire has a diameter of 2 mm and a total length of 40 cm. The magntic field changes with time at a steady rate $dB/dt = 0{\cdot}02$ T s^{-1}. Find the current induced in the frame. Resistivity of copper $= 1{\cdot}7 \times 10^{-8}$ Wm.*

Solution :

The area A of the loop $= \left(\frac{40\text{ cm}}{4}\right)\left(\frac{40\text{ cm}}{4}\right) = 0{\cdot}01\text{ m}^2$.

If the magnetic field at an instant is B, the flux through the frame at that instant will be $\Phi = BA$. As the area remains constant, the magnitude of the emf induced will be

$$\mathcal{E} = \frac{d\Phi}{dt} = A\frac{dB}{dt}$$

$$= (0{\cdot}01\text{ m}^2)\,(0{\cdot}02\text{ T/s}) = 2 \times 10^{-4}\text{ V}.$$

The resistance of the loop is $R = \rho\frac{l}{A}$

$$= \frac{(1{\cdot}7 \times 10^{-8}\ \Omega\text{m})\,(40\text{ cm})}{\pi \times 1\text{ mm}^2}$$

$$= \frac{(1{\cdot}7 \times 10^{-8}\ \Omega\text{m})\,(40 \times 10^{-2}\text{ m})}{3{\cdot}14 \times 1 \times 10^{-6}\text{ m}^2} = 2{\cdot}16 \times 10^{-3}\ \Omega.$$

Hence, the current induced in the loop will be

$$i = \frac{2 \times 10^{-4}\text{ V}}{2{\cdot}16 \times 10^{-3}\ \Omega} = 9{\cdot}3 \times 10^{-2}\text{ A}.$$

3. *A conducting circular loop of face area $2{\cdot}5 \times 10^{-3}\text{ m}^2$ is placed perpendicular to a magnetic field which varies as $B = (0{\cdot}20\text{ T})\sin[(50\pi\text{ s}^{-1})t]$. (a) Find the charge flowing through any cross-section during the time $t = 0$ to $t = 40$ ms. (b) If the resistance of the loop is 10 Ω, find the thermal energy developed in the loop in this period.*

Solution : The face area of the loop is $A = 2{\cdot}5 \times 10^{-3}\ \text{m}^2$ and the magnetic field changes as $B = B_0 \sin \omega t$ where $B_0 = 0{\cdot}20$ T and $\omega = 50\pi\ \text{s}^{-1}$. The resistance of the loop is $R = 10\ \Omega$.

The flux through the loop at time t is

$$\Phi = B_0 A \sin \omega t.$$

The emf induced is

$$\mathcal{E} = -\frac{d\Phi}{dt} = -B_0 A\omega \cos \omega t.$$

The current is $i = \dfrac{\mathcal{E}}{R} = -\dfrac{B_0 A\omega}{R} \cos \omega t = -i_0 \cos \omega t.$

The current changes sinusoidally with the time period

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{50\pi\ \text{s}^{-1}} = 40\ \text{ms}.$$

(a) The charge flowing through any cross-section in 40 ms is

$$Q = \int_0^T i dt = -i_0 \int_0^T \cos \omega t\, dt$$

$$= -\frac{i_0}{\omega}[\sin \omega t]_0^T = 0.$$

(b) The thermal energy produced in 40 ms is

$$H = \int_0^T i^2 R dt = i_0^2 R \int_0^T \cos^2 \omega t\, dt$$

$$= \frac{i_0^2 R}{2} \int_0^T (1 + \cos 2\omega t)\, dt$$

$$= \frac{i_0^2 R}{2}\left[t + \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{i_0^2 RT}{2} = \frac{B_0^2 A^2 \omega^2}{2R^2} R\left(\frac{2\pi}{\omega}\right) = \frac{\pi B_0^2 A^2 \omega}{R}$$

$$= \frac{\pi \times (0{\cdot}20\ \text{T})^2 \times (2{\cdot}5 \times 10^{-3}\ \text{m}^2)^2 \times (50\pi\ \text{s}^{-1})}{10\ \Omega}$$

$$= 1{\cdot}25 \times 10^{-5}\ \text{J}.$$

4. *A long solenoid of radius* 2 cm *has* 100 turns/cm *and is surrounded by a* 100-turn *coil of radius* 4 cm *having a total resistance of* 20 Ω. *The coil is connected to a galvanometer as shown in figure (38-W1). If the current in the solenoid is changed from* 5 A *in one direction to* 5 A *in the opposite direction, find the charge which flows through the galvanometer.*

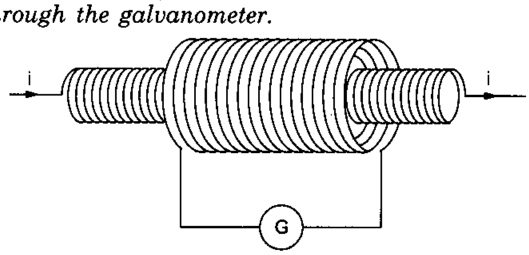


Figure 38-W1

Solution : If the current in the solenoid is i, the magnetic field inside the solenoid is $B = \mu_0 ni$ parallel to its axis. Outside the solenoid, the field will be zero. The flux of the magnetic field through the coil will be $\Phi = B\pi r^2 N$ where r is the radius of the solenoid and N is the number of turns in the coil. The induced emf will have magnitude

$$\frac{d\Phi}{dt} = N\pi r^2 \frac{dB}{dt} = \pi r^2 N \mu_0 n \frac{di}{dt}.$$

If R denotes the resistance of the coil, the current through the galvanometer is

$$I = \frac{\pi r^2 N}{R} \mu_0 n \frac{di}{dt}$$

or, $$I\, dt = \frac{\pi r^2 N}{R} \mu_0 n\, di.$$

The total charge passing through the galvanometer is

$$\Delta Q = \int I\, dt = \frac{\pi r^2 N}{R} \mu_0 n \int di$$

$$= \frac{\pi r^2 N \mu_0 n}{R} \Delta i$$

$$= \frac{\pi (2\ \text{cm})^2 \times 100 \times 4\pi \times 10^{-7}\ \text{TmA}^{-1} \times 100\ \text{cm}^{-1} \times 10\ \text{A}}{20\ \Omega}$$

$$\approx 8 \times 10^{-4}\ \text{C} = 800\ \mu\text{C}.$$

5. *The magnetic field B shown in figure (38-W2) is directed into the plane of the paper. ACDA is a semicircular conducting loop of radius r with the centre at O. The loop is now made to rotate clockwise with a constant angular velocity ω about an axis passing through O and perpendicular to the plane of the paper. The resistance of the loop is R. Obtain an expression for the magnitude of the induced current in the loop. Plot a graph between the induced current i and ωt, for two periods of rotation.*

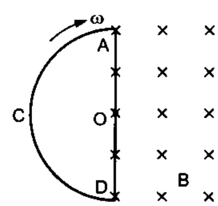


Figure 38-W2

Solution : When the loop rotates through an angle θ, which is less than π (figure 38-W3a), the area inside the field region is

$$A = \frac{\theta}{\pi}\frac{\pi r^2}{2} = \frac{\theta r^2}{2} = \frac{\omega t r^2}{2}.$$

The flux of the magnetic field at time t is

$$\Phi = BA = B\frac{\omega t r^2}{2}.$$

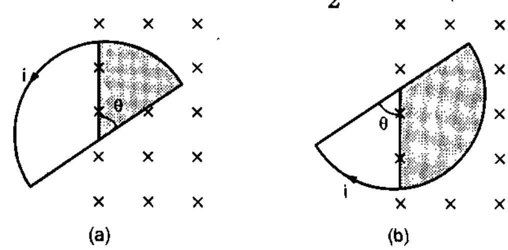


Figure 38-W3

The induced emf $= -\dfrac{d\Phi}{dt} = -\dfrac{B\omega r^2}{2}$.

The magnitude of the induced current will be

$$i = \frac{B\omega r^2}{2R}.$$

As the flux is increasing, the direction of the induced current will be anticlockwise so that the field due to the induced current is opposite to the original field.

After half a rotation, the area in the field region will start decreasing (figure 38-W3b) and will be given by

$$A(t) = \frac{\pi r^2}{2} - \frac{\omega t r^2}{2}.$$

Hence, the induced current will have the same magnitude but opposite sense. The plot for two time periods is shown in figure (38-W4).

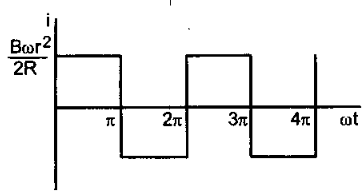


Figure 38-W4

6. *Figure (38-W5) shows a square loop having* 100 *turns, an area of* $2{\cdot}5 \times 10^{-3}\ m^2$ *and a resistance of* 100 Ω. *The magnetic field has a magnitude* $B = 0{\cdot}40$ T. *Find the work done in pulling the loop out of the field, slowly and uniformly in* 1·0 s.

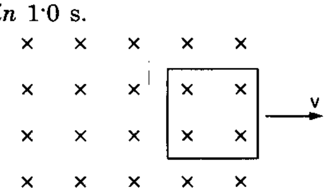


Figure 38-W5

Solution :

The side of the square is

$$l = \sqrt{2{\cdot}5 \times 10^{-3}\ m^2} = 0{\cdot}05\ m.$$

As it is uniformly pulled out in 1·0 s, the speed of the loop is

$$v = 0{\cdot}05\ m\ s^{-1}.$$

The emf induced in the left arm of the loop is

$$\mathcal{E} = NvBl$$
$$= 100 \times (0{\cdot}05\ m\ s^{-1}) \times (0{\cdot}40\ T) \times (0{\cdot}05\ m)$$
$$= 0{\cdot}1\ V.$$

The current in the loop is

$$i = \frac{0{\cdot}1\ V}{100\ \Omega} = 1{\cdot}0 \times 10^{-3}\ A.$$

The force on the left arm due to the magnetic field is

$$F = NilB = 100 \times (1{\cdot}0 \times 10^{-3}\ A) \times (0{\cdot}05\ m) \times (0{\cdot}40\ T)$$
$$= 2{\cdot}0 \times 10^{-3}\ N.$$

This force is towards left in the figure. To pull the loop uniformly, an external force of $2{\cdot}0 \times 10^{-3}$ N towards right must be applied. The work done by this force is

$$W = (2{\cdot}0 \times 10^{-3}\ N) \times (0{\cdot}05\ m) = 1{\cdot}0 \times 10^{-4}\ J.$$

7. *Magadh Express takes* 16 hours *to cover the distance of* 960 km *between Patna and Gaziabad. The rails are separated by* 130 cm *and the vertical component of the earth's magnetic field is* $4{\cdot}0 \times 10^{-5}$ T. *(a) Find the average emf induced across the width of the train. (b) If the leakage resistance between the rails is* 100 Ω, *find the retarding force on the train due to the magnetic field.*

Solution : As the train moves in a magnetic field, a motional emf $\mathcal{E} = vBl$ is produced across its width. Here B is the component of the magnetic field in a direction perpendicular to the plane of the motion, i.e., the vertical component.

The speed of the train is $v = \dfrac{960\ km}{16\ h} = 16{\cdot}67\ m\ s^{-1}$.

Thus, $\mathcal{E} = (16{\cdot}67\ m\ s^{-1})\,(4{\cdot}0 \times 10^{-5}\ T)\,(1{\cdot}30\ m)$

$$= 8{\cdot}6 \times 10^{-4}\ V.$$

The leakage current is $i = \mathcal{E}/R$ and the retarding force is

$$F = ilB = \frac{8{\cdot}6 \times 10^{-4}\ V}{100\ \Omega} \times 1{\cdot}3\ m \times 4{\cdot}0 \times 10^{-5}\ T$$
$$= 4{\cdot}47 \times 10^{-10}\ N.$$

8. *A square loop of edge a having n turns is rotated with a uniform angular velocity ω about one of its diagonals which is kept fixed in a horizontal position (figure 38-W6). A uniform magnetic field B exists in the vertical direction. Find the emf induced in the coil.*

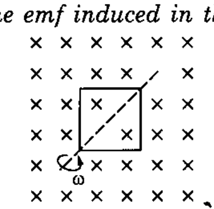


Figure 38-W6

Solution : The area of the square frame is $A = a^2$. If the normal to the frame makes an angle $\theta = 0$ with the magnetic field at $t = 0$, this angle will become $\theta = \omega t$ at time t. The flux of the magnetic field at this time is

$$\Phi = nBA \cos\theta = nBa^2 \cos\omega t.$$

The induced emf is

$$\mathcal{E} = -\frac{d\Phi}{dt} = nBa^2\omega \sin\omega t.$$

Thus, an alternating emf is induced in the coil.

9. *A conducting circular loop of radius r is rotated about its diameter at a constant angular velocity ω in a*

magnetic field B perpendicular to the axis of rotation. In what position of the loop is the induced emf zero?

Solution : Suppose, the normal to the loop is parallel to the magnetic field at $t = 0$. At time t, the normal will make an angle $\theta = \omega t$ with this position. The flux of the magnetic field through the loop is

$$\Phi = B\pi r^2 \cos \omega t$$

so that the induced emf at time t is

$$\mathcal{E} = \omega B\pi r^2 \sin \omega t.$$

This is zero when $\omega t = n\pi$, i.e., when $\theta = 0, \pi, 2\pi, \ldots$, etc. These are the positions when the plane of the loop is normal to the magnetic field. It may be noted that at these positions, the flux has the maximum magnitude.

10. *Figure (38-W7) shows a horizontal magnetic field which is uniform above the dotted line and is zero below it. A long, rectangular, conducting loop of width L, mass m and resistance R is placed partly above and partly below the dotted line with the lower edge parallel to it. With what velocity should it be pushed downwards so that it may continue to fall without any acceleration?*

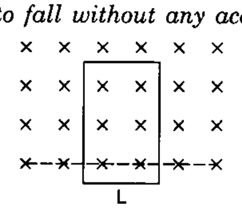


Figure 38-W7

Solution : Let the uniform velocity of fall be v. The emf is induced across the upper wire and its magnitude is $\mathcal{E} = vBl$. The current induced in the frame is

$$i = \frac{vBl}{R}$$

so that, the magnetic force on the upper arm is $F = ilB = \frac{vB^2l^2}{R}\cdot$ This force is in the upward direction. As the frame falls uniformly, this force should balance its weight. Thus,

$$mg = \frac{vB^2l^2}{R}$$

or,
$$v = \frac{mgR}{B^2l^2}.$$

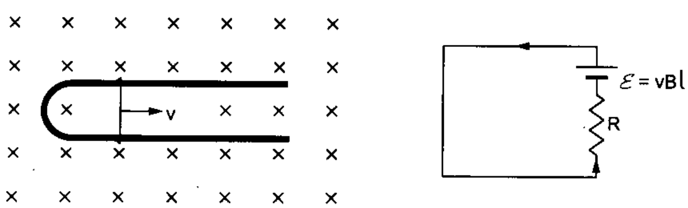
11. *Figure (38-W8a) shows a wire of length l which can slide on a U-shaped rail of negligible resistance. The resistance of the wire is R. The wire is pulled to the right with a constant speed v. Draw an equivalent circuit diagram representing the induced emf by a battery. Find the current in the wire using this diagram.*

Solution : The emf is induced due to the moving wire. The magnitude of this emf is $\mathcal{E} = vBl$. As the wire moves towards right, the force $q\vec{v} \times \vec{B}$ on a positive charge acts in the upward direction in the figure. The positive terminal of the equivalent battery appears upwards. The resistance of the wire acts as the internal resistance of the equivalent battery. The equivalent circuit is drawn in figure (38-W8b).

The current in the circuit is, from Ohm's law,

$$i = \frac{\mathcal{E}}{R} = \frac{vBl}{R}.$$

It is in the upward direction along the wire.

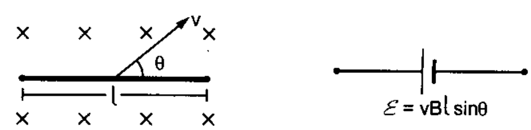


(a) (b)

Figure 38-W8

12. *A rod of length l is translating at a velocity v making an angle θ with its length. A uniform magnetic field B exists in a direction perpendicular to the plane of motion. Calculate the emf induced in the rod. Draw a figure representing the induced emf by an equivalent battery.*

Solution :



(a) (b)

Figure 38-W9

The situation is shown in figure (38-W9a). The component of the velocity perpendicular to the length of the rod is $v_\perp = v \sin\theta$. Only this component is effective in producing the emf in the rod. As the magnetic field is perpendicular to the plane of motion, the emf induced across the ends is

$$\mathcal{E} = v_\perp Bl = vBl \sin\theta.$$

In the figure shown, the positive charges of the rod shift towards left due to the force $q\vec{v} \times \vec{B}$. Thus, the left side of the rod is electrically positive. Figure (38-W9b) shows the equivalent battery.

13. *The horizontal component of the earth's magnetic field at a place is* $3{\cdot}0 \times 10^{-4}$ T *and the dip is* 53°. *A metal rod of length* 25 cm *is placed in the north–south direction and is moved at a constant speed of* 10 cm s^{-1} *towards east. Calculate the emf induced in the rod.*

Solution : The induced emf is $\mathcal{E} = vBl$ where l is the length of the rod, v is its speed in the perpendicular direction and B is the component of the magnetic field perpendicular to both l and v. In the present case, B is the vertical component of the earth's magnetic field.

The dip at a place is given by

$$\tan \delta = \frac{B_V}{B_H}$$

or, $$B_V = B_H \tan \delta$$

$$= (3{\cdot}0 \times 10^{-4}\ \text{T}) \tan 53° = 4{\cdot}0 \times 10^{-4}\ \text{T}.$$

The emf induced is $\mathcal{E} = vBl$

$$= (0{\cdot}10\ \text{m s}^{-1}) \times (4{\cdot}0 \times 10^{-4}\ \text{T}) \times (0{\cdot}25\ \text{m})$$

$$= 1{\cdot}0 \times 10^{-5}\ \text{V} = 10\ \mu\text{V}.$$

14. *An angle aob made of a conducting wire moves along its bisector through a magnetic field B as suggested by figure (38-W10a). Find the emf induced between the two free ends if the magnetic field is perpendicular to the plane of the angle.*

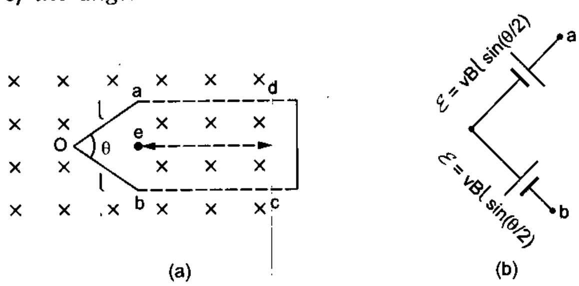


Figure 38-W10

Solution : Consider the circuit as being closed externally, as shown in the figure. If $ad = x$, the area of the rectangular part *abcd* is $A = 2xl \sin(\theta/2)$. As the angle moves towards right, the flux of the magnetic field through this rectangular area decreases at the rate

$$\frac{d\Phi}{dt} = B\frac{dA}{dt} = 2Blv \sin(\theta/2).$$

This is also the rate of decrease of the flux through the closed circuit shown in the figure. So the induced emf is

$$\mathcal{E} = 2Blv \sin(\theta/2).$$

As the emf is induced solely because of the movement of the angle, this is the emf induced between its ends.

Alternative method

The rod *oa* is equivalent to a battery of emf $vBl \sin(\theta/2)$. The positive charges of *oa* shift towards *a* due to the force $q\vec{v} \times \vec{B}$. The positive terminal of the battery appears towards *a*. Similarly, the rod *ob* is equivalent to a battery of emf $vBl \sin(\theta/2)$ with the positive terminal towards *o*. The equivalent circuit is shown in figure (38-W10b). Clearly, the emf between the points *a* and *b* is $2\,Blv \sin(\theta/2)$.

15. *Figure (38-W11a) shows a wire ab of length l and resistance R which can slide on a smooth pair of rails.*

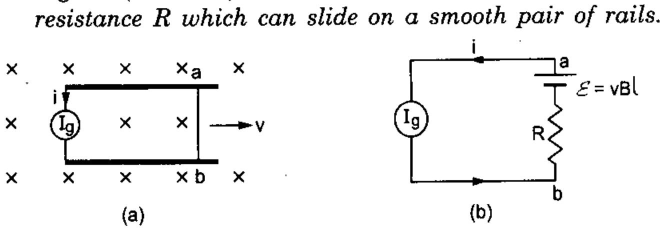


Figure 38-W11

I_g is a current generator which supplies a constant current i in the circuit. If the wire ab slides at a speed v towards right, find the potential difference between a and b.

Solution : The moving wire *ab* is equivalent to a battery of emf vBl having a resistance R. If it moves towards right and the magnetic field is going into the plane of the figure, the force $q\vec{v} \times \vec{B}$ will push the positive charges towards *a*. Thus the positive terminal of the equivalent battery is towards *a*. An equivalent circuit is shown in figure (38-W13b). The potential difference between *a* and *b* is

$$V_a - V_b = vBl - iR.$$

16. *A square loop of side* 10 cm *and resistance* 1 Ω *is moved towards right with a constant velocity v_0 as shown in figure (38-W12). The left arm of the loop is in a uniform magnetic field of* 2 T. *The field is perpendicular to the plane of the drawing and is going into it. The loop is connected to a network of resistors each of value* 3 Ω. *With what speed should the loop be moved so that a steady current of* 1 mA *flows in the loop.*

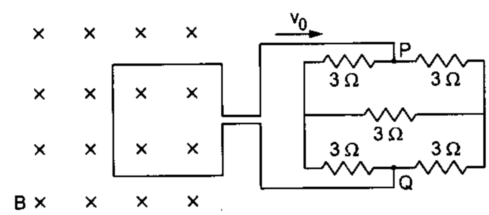


Figure 38-W12

Solution : The equivalent resistance of the network of the resistors, between P and Q will be 3 Ω. The total resistance of the circuit is 1 Ω + 3 Ω = 4 Ω.

The emf induced in the loop is

$$\mathcal{E} = vBl = v_0(2\ \text{T})\ (10\ \text{cm}).$$

The current in the loop will be $i = \frac{\mathcal{E}}{R}$

or, $$1 \times 10^{-3}\ \text{A} = \frac{v_0(2\ \text{T})\ (0{\cdot}1\ \text{m})}{4\ \Omega}$$

giving $$v_0 = \frac{(4\ \Omega)\ (1 \times 10^{-3}\ \text{A})}{0{\cdot}2\ \text{Tm}} = 2\ \text{cm s}^{-1}.$$

17. *A metal rod of length l rotates about an end with a uniform angular velocity ω. A uniform magnetic field $\vec{B}$ exists in the direction of the axis of rotation. Calculate the emf induced between the ends of the rod. Neglect the centripetal force acting on the free electrons as they move in circular paths.*

Solution :

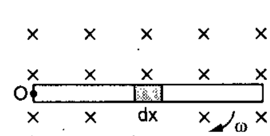


Figure 38-W13

Consider an element dx of the rod at a distance x from the axis of rotation. The linear speed of this element is ωx. The element moves in a direction perpendicular to its length as well as perpendicular to the magnetic field. The emf induced between the ends of this element is

$$d\mathcal{E} = B\,\omega x\,dx.$$

The emfs of all such elements will add to give the net emf between the ends of the rod. This emf is, therefore,

$$\mathcal{E} = \int d\mathcal{E} = \int_0^l B\,\omega x\,dx = \frac{1}{2} B\omega l^2.$$

18. *Figure (38-W14a) shows a conducting circular loop of radius a placed in a uniform, perpendicular magnetic field B. A metal rod OA is pivoted at the centre O of the loop. The other end A of the rod touches the loop. The rod OA and the loop are resistanceless but a resistor having a resistance R is connected between O and a fixed point C on the loop. The rod OA is made to rotate anticlockwise at a small but uniform angular speed ω by an external force. Find (a) the current in the resistance R and (b) the torque of the external force needed to keep the rod rotating with the constant angular velocity ω.*

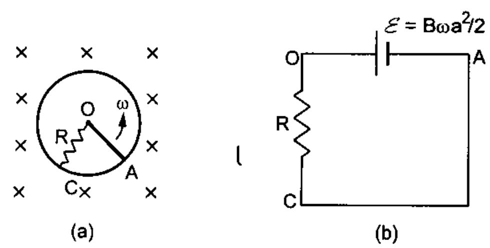


Figure 38-W14

Solution : The emf between the ends of the rotating rod is

$$\mathcal{E} = \int d\mathcal{E} = \int_0^a B\omega x\,dx = \frac{1}{2} B\omega a^2.$$

The positive charges of the rod will be pushed towards O by the magnetic field. Thus, the rod may be replaced by a battery of emf $= \frac{1}{2} B\omega a^2$ with the positive terminal towards O. The equivalent circuit diagram is shown in figure (38-W14b). The circular loop joins A to C by a resistanceless path.

(a) The current in the resistance R is

$$i = \frac{\mathcal{E}}{R} = \frac{B\omega a^2}{2R}.$$

(b) The force on the rod due to the magnetic field is $F = iaB$. As the force is uniformly distributed over OA, it may be assumed to act at the middle point of OA. The torque is, therefore,

$$\Gamma = (iaB)\frac{a}{2} = \frac{B^2\omega a^4}{4R}$$

in clockwise direction. To keep the rod rotating at uniform angular velocity, an external torque $\frac{B^2\omega a^4}{4R}$ in anticlockwise direction is needed.

19. *Figure (38-W15) shows a conducting loop abcdefa made of six segments ab, bc, cd, de, ef and fa, each of length l. Each segment makes a right angle with the next so that abc is in the x–z plane, cde in the x–y plane and efa is in the y–z plane. A uniform magnetic field B exists along the x-axis. If the magnetic field changes at a rate $\frac{dB}{dt}$, find the emf induced in the loop.*

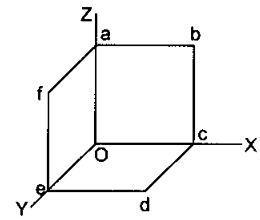


Figure 38-W15

Solution : As the magnetic field is along the x-axis, the flux through the loop is equal to the magnetic field multiplied by the area of projection of the loop on the y–z plane. This projection on the y–z plane will be *aoef* which has an area l^2. Thus, the flux is $\Phi = Bl^2$. The induced emf is $\mathcal{E} = \frac{dB}{dt} l^2$.

20. *A wire of mass m and length l can freely slide on a pair of parallel, smooth, horizontal rails placed in a vertical magnetic field B (figure 38-W16). The rails are connected by a capacitor of capacitance C. The electric resistance of the rails and the wire is zero. If a constant force F acts on the wire as shown in the figure, find the acceleration of the wire.*

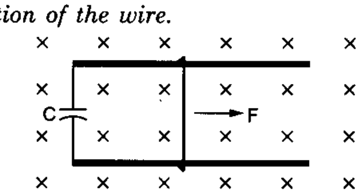


Figure 38-W16

Solution : Suppose the velocity of the wire is v at time t. The induced emf is $\mathcal{E} = vBl$. As there is no resistance anywhere, the charge on the capacitor will be

$$q = C\mathcal{E} = CvBl$$

at time t. The current in the circuit will be

$$i = \frac{dq}{dt} = CBl\frac{dv}{dt} = CBla.$$

Because of this current through the wire, there will be a magnetic force

$$F' = ilB = CB^2l^2a$$

towards left. The net force on the wire is $F - F'$.

From Newton's law,

$$F - F' = ma$$

or, $$F - CB^2 l^2 a = ma$$

or, $$a = \frac{F}{m + CB^2 l^2}.$$

21. *An inductor coil stores* 32 J *of magnetic field energy and dissipates energy as heat at the rate of* 320 W *when a current of* 4 A *is passed through it. Find the time constant of the circuit when this coil is joined across an ideal battery.*

Solution : The magnetic field energy stored in an inductor is $U = \frac{1}{2} Li^2.$

Thus, $$32 \text{ J} = \frac{1}{2} L(4 \text{ A})^2$$

or, $$L = 4 \text{ H}.$$

The power dissipated as heat is given by

$$P = i^2 R$$

or, $$320 \text{ W} = (4 \text{ A})^2 R, \text{ giving } R = 20\ \Omega.$$

The time constant of the circuit is

$$\tau = \frac{L}{R} = \frac{4 \text{ H}}{20\ \Omega} = 0{\cdot}2 \text{ s}.$$

22. *A* 12 V *battery connected to a* 6 Ω, 10 H *coil through a switch drives a constant current in the circuit. The switch is suddenly opened. Assuming that it took* 1 ms *to open the switch, calculate the average emf induced across the coil.*

Solution : The steady-state current is $\frac{12 \text{ V}}{6\ \Omega} = 2$ A. The final current is zero. Thus,

$$\frac{di}{dt} = -\frac{2 \text{ A}}{1 \text{ ms}} = -2 \times 10^3 \text{ A s}^{-1}.$$

The induced emf is $\mathcal{E} = -\frac{d\Phi}{dt} = -L\frac{di}{dt}$

$$= -(10 \text{ H}) \times (-2 \times 10^3 \text{ A s}^{-1}) = 20000 \text{ V}.$$

Such a high emf may cause sparks across the open switch.

23. *A solenoid of inductance* 50 mH *and resistance* 10 Ω *is connected to a battery of* 6 V. *Find the time elapsed before the current acquires half of its steady-state value.*

Solution : The time constant of the circuit is

$$\tau = L/R = 50 \text{ mH}/10\ \Omega = 5 \text{ ms}.$$

The current at time t is given by

$$i = i_0(1 - e^{-t/\tau}).$$

For $$i = i_0/2,$$

$$i_0/2 = i_0(1 - e^{-t/\tau})$$

or, $$e^{-t/\tau} = \frac{1}{2}$$

or, $$\frac{t}{\tau} = \ln 2$$

giving $t = \tau \ln 2 = (5 \text{ ms})(0{\cdot}693) = 3{\cdot}5$ ms.

24. *An LR circuit having* $L = 4{\cdot}0$ H, $R = 1{\cdot}0\ \Omega$ *and* $\mathcal{E} = 6{\cdot}0$ V *is switched on at* $t = 0$. *Find the power dissipated in Joule heating at* $t = 4{\cdot}0$ s.

Solution : The time constant of the circuit is

$$\tau = L/R = 4{\cdot}0 \text{ H}/1{\cdot}0\ \Omega = 4{\cdot}0 \text{ s}.$$

The current at $t = 4{\cdot}0$ s is, therefore,

$$i = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau}) = (6 \text{ A})\left(1 - \frac{1}{e}\right)$$

$$= (6 \text{ A}) \times (0{\cdot}63) = 3{\cdot}8 \text{ A}.$$

The power dissipated in Joule heating $= i^2 R$

$$= (3{\cdot}8 \text{ A})^2 \times 10\ \Omega \approx 140\ \Omega.$$

25. *An LR combination is connected to an ideal battery. If* $L = 10$ mH, $R = 2{\cdot}0\ \Omega$ *and* $\mathcal{E} = 2{\cdot}0$ V, *how much time will it take for the current to reach* 0·63 A?

Solution : The steady-state current in the LR circuit is

$$i_0 = \mathcal{E}/R = 2{\cdot}0 \text{ V}/2{\cdot}0\ \Omega = 1 \text{ A}.$$

Thus, 0·63 A is 63% of the steady-state current. As we know, it takes one time constant for the current to reach 63% of its steady-state value. Hence the required time

$$= L/R = 10 \text{ mH}/2{\cdot}0\ \Omega = 5{\cdot}0 \text{ ms}.$$

26. *An inductor–resistance–battery circuit is switched on at* $t = 0$. *If the emf of the battery is* $\mathcal{E}$, *find the charge which passes through the battery in one time constant* τ.

Solution : The current at time t is given by

$$i = i_0(1 - e^{-t/\tau}) \text{ where } i_0 = \mathcal{E}/R.$$

The charge passed through the battery during the period t to $t + dt$ is $i\,dt$. Thus, the total charge passed during 0 to τ is

$$Q = \int_0^\tau i\,dt = i_0 \int_0^\tau (1 - e^{-t/\tau})\,dt = i_0\left[t - \frac{e^{-t/\tau}}{-1/\tau}\right]_0^\tau$$

$$= i_0[\tau + \tau(e^{-1} - 1)] = i_0\,\tau/e.$$

27. *A coil of inductance* 1·0 H *and resistance* 100 Ω *is connected to a battery of emf* 12 V. *Find the energy stored in the magnetic field associated with the coil at an instant* 10 ms *after the circuit is switched on.*

Solution : The energy in the magnetic field associated with the coil is

$$U = \frac{1}{2} Li^2 = \frac{1}{2} L\left[\frac{\mathcal{E}}{R}(1 - e^{-t/\tau})\right]^2. \quad \text{... (i)}$$

The time constant of the circuit is

$$\tau = \frac{L}{R} = \frac{1{\cdot}0 \text{ H}}{100\ \Omega} = 10 \text{ ms}.$$

Putting the numerical values in (i), the energy at $t = 10$ ms is

$$\frac{1}{2} \times (1{\cdot}0 \text{ H}) \times [0{\cdot}12 \text{ A}(1 - 1/e)]^2$$

$= 2{\cdot}8$ mJ.

28. *An inductance L and a resistance R are connected in series with a battery of emf $\mathcal{E}$. Find the maximum rate at which the energy is stored in the magnetic field.*

Solution :

The energy stored in the magnetic field at time t is

$$U = \frac{1}{2} Li^2 = \frac{1}{2} Li_0^2 (1 - e^{-t/\tau})^2.$$

The rate at which the energy is stored is

$$P = \frac{dU}{dt} = Li_0^2(1 - e^{-t/\tau})(-e^{-t/\tau})\left(-\frac{1}{\tau}\right)$$

$$= \frac{Li_0^2}{\tau}(e^{-t/\tau} - e^{-2t/\tau}). \quad \dots \text{(i)}$$

This rate will be maximum when $\frac{dP}{dt} = 0$

or, $$\frac{Li_0^2}{\tau}\left(-\frac{1}{\tau}e^{-t/\tau} + \frac{2}{\tau}e^{-2t/\tau}\right) = 0$$

or, $$e^{-t/\tau} = \frac{1}{2}.$$

Putting in (i),

$$P_{max} = \frac{Li_0^2}{\tau}\left(\frac{1}{2} - \frac{1}{4}\right)$$

$$= \frac{L\mathcal{E}^2}{4R^2(L/R)} = \frac{\mathcal{E}^2}{4R}.$$

29. *Two conducting circular loops of radii R_1 and R_2 are placed in the same plane with their centres coinciding. Find the mutual inductance between them assuming $R_2 << R_1$.*

Solution : Suppose a current i is established in the outer loop. The magnetic field at the centre will be

$$B = \frac{\mu_0 i}{2R_1}.$$

As the radius R_2 of the inner coil is small compared to R_1, the flux of magnetic field through it will be approximately

$$\Phi = \frac{\mu_0 i}{2R_1} \pi R_2^2$$

so that the mutual inductance is

$$M = \frac{\Phi}{i} = \frac{\mu_0 \pi R_2^2}{2R_1}.$$

QUESTIONS FOR SHORT ANSWER

1. A metallic loop is placed in a nonuniform magnetic field. Will an emf be induced in the loop ?
2. An inductor is connected to a battery through a switch. Explain why the emf induced in the inductor is much larger when the switch is opened as compared to the emf induced when the switch is closed.
3. The coil of a moving-coil galvanometer keeps on oscillating for a long time if it is deflected and released. If the ends of the coil are connected together, the oscillation stops at once. Explain.
4. A short magnet is moved along the axis of a conducting loop. Show that the loop repels the magnet if the magnet is approaching the loop and attracts the magnet if it is going away from the loop.
5. Two circular loops are placed coaxially but separated by a distance. A battery is suddenly connected to one of the loops establishing a current in it. Will there be a current induced in the other loop ? If yes, when does the current start and when does it end ? Do the loops attract each other or do they repel ?
6. The battery discussed in the previous question is suddenly disconnected. Is a current induced in the other loop ? If yes, when does it start and when does it end ? Do the loops attract each other or repel ?
7. If the magnetic field outside a copper box is suddenly changed, what happens to the magnetic field inside the box ? Such low-resistivity metals are used to form enclosures which shield objects inside them against varying magnetic fields.
8. Metallic (nonferromagnetic) and nonmetallic particles in a solid waste may be separated as follows. The waste is allowed to slide down an incline over permanent magnets. The metallic particles slow down as compared to the nonmetallic ones and hence are separated. Discuss the role of eddy currents in the process.
9. A pivoted aluminium bar falls much more slowly through a small region containing a magnetic field than a similar bar of an insulating material. Explain.
10. A metallic bob A oscillates through the space between the poles of an electromagnet (figure 38-Q1). The oscillations are more quickly damped when the circuit

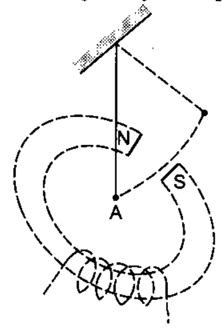


Figure 38-Q1

is on, as compared to the case when the circuit is off. Explain.

11. Two circular loops are placed with their centres separated by a fixed distance. How would you orient the loops to have (a) the largest mutual inductance (b) the smallest mutual inductance?

12. Consider the self-inductance per unit length of a solenoid at its centre and that near its ends. Which of the two is greater?

13. Consider the energy density in a solenoid at its centre and that near its ends. Which of the two is greater?

OBJECTIVE I

1. A rod of length l rotates with a small but uniform angular velocity ω about its perpendicular bisector. A uniform magnetic field B exists parallel to the axis of rotation. The potential difference between the centre of the rod and an end is

(a) zero (b) $\frac{1}{8}\omega Bl^2$ (c) $\frac{1}{2}\omega Bl^2$ (d) $B\omega l^2$.

2. A rod of length l rotates with a uniform angular velocity ω about its perpendicular bisector. A uniform magnetic field B exists parallel to the axis of rotation. The potential difference between the two ends of the rod is

(a) zero (b) $\frac{1}{2}Bl\omega^2$ (c) $Bl\omega^2$ (d) $2Bl\omega^2$.

3. Consider the situation shown in figure (38-Q2). If the switch is closed and after some time it is opened again, the closed loop will show
(a) an anticlockwise current-pulse
(b) a clockwise current-pulse
(c) an anticlockwise current-pulse and then a clockwise current-pulse
(d) a clockwise current-pulse and then an anticlockwise current-pulse.

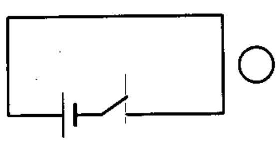


Figure 38-Q2

4. Solve the previous question if the closed loop is completely enclosed in the circuit containing the switch.

5. A bar magnet is released from rest along the axis of a very long, vertical copper tube. After some time the magnet
(a) will stop in the tube
(b) will move with almost contant speed
(c) will move with an acceleration g
(d) will oscillate.

6. Figure (38-Q3) shows a horizontal solenoid connected to a battery and a switch. A copper ring is placed on a frictionless track, the axis of the ring being along the axis of the solenoid. As the switch is closed, the ring will
(a) remain stationary
(b) move towards the solenoid
(c) move away from the solenoid
(d) move towards the solenoid or away from it depending on which terminal (positive or negative) of the battery is connected to the left end of the solenoid.

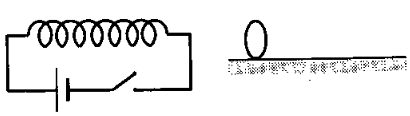


Figure 38-Q3

7. Consider the following statements:
(A) An emf can be induced by moving a conductor in a magnetic field.
(B) An emf can be induced by changing the magnetic field.
(a) Both A and B are true. (b) A is true but B is false.
(c) B is true but A is false. (d) Both A and B are false.

8. Consider the situation shown in figure (38-Q4). The wire AB is slid on the fixed rails with a constant velocity. If the wire AB is replaced by a semicircular wire, the magnitude of the induced current will
(a) increase (b) remain the same (c) decrease
(d) increase or decrease depending on whether the semicircle bulges towards the resistance or away from it.

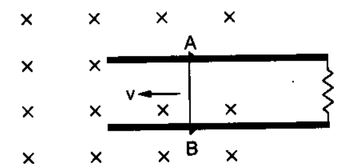


Figure 38-Q4

9. Figure (38-Q5a) shows a conducting loop being pulled out of a magnetic field with a speed v. Which of the four plots shown in figure (38-Q5b) may represent the power delivered by the pulling agent as a function of the speed v?

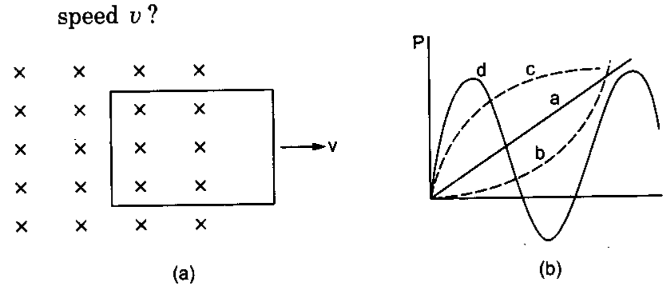


Figure 38-Q5

10. Two circular loops of equal radii are placed coaxially at some separation. The first is cut and a battery is inserted in between to drive a current in it. The current changes slightly because of the variation in resistance with temperature. During this period, the two loops
(a) attract each other (b) repel each other
(c) do not exert any force on each other
(d) attract or repel each other depending on the sense of the current.

11. A small, conducting circular loop is placed inside a long solenoid carrying a current. The plane of the loop contains the axis of the solenoid. If the current in the solenoid is varied, the current induced in the loop is
(a) clockwise (b) anticlockwise (c) zero
(d) clockwise or anticlockwise depending on whether the resistance is increased or decreased.

12. A conducting square loop of side l and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A uniform and constant magnetic field B exists along the perpendicular to the plane of the loop as shown in figure (38-Q6). The current induced in the loop is
(a) Blv/R clockwise (b) Blv/R anticlockwise
(c) $2Blv/R$ anticlockwise (d) zero.

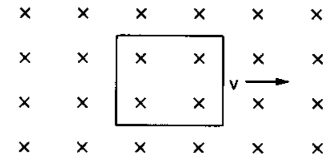


Figure 38-Q6

OBJECTIVE II

1. A bar magnet is moved along the axis of a copper ring placed far away from the magnet. Looking from the side of the magnet, an anticlockwise current is found to be induced in the ring. Which of the following may be true ?
(a) The south pole faces the ring and the magnet moves towards it.
(b) The north pole faces the ring and the magnet moves towards it.
(c) The south pole faces the ring and the magnet moves away from it.
(d) The north pole faces the ring and the magnet moves away from it.

2. A conducting rod is moved with a constant velocity v in a magnetic field. A potential difference appears across the two ends
(a) if $\vec{v} \parallel \vec{l}$ (b) if $\vec{v} \parallel \vec{B}$ (c) if $\vec{l} \parallel \vec{B}$
(d) none of these.

3. A conducting loop is placed in a uniform magnetic field with its plane perpendicular to the field. An emf is induced in the loop if
(a) it is translated
(b) it is rotated about its axis
(c) it is rotated about a diameter
(d) it is deformed.

4. A metal sheet is placed in front of a strong magnetic pole. A force is needed to
(a) hold the sheet there if the metal is magnetic
(b) hold the sheet there if the metal is nonmagnetic
(c) move the sheet away from the pole with uniform velocity if the metal is magnetic
(d) move the sheet away from the pole with uniform velocity if the metal is nonmagnetic.
Neglect any effect of paramagnetism, diamagnetism and gravity.

5. A constant current i is maintained in a solenoid. Which of the following quantities will increase if an iron rod is inserted in the solenoid along its axis ?
(a) magnetic field at the centre
(b) magnetic flux linked with the solenoid
(c) self-inductance of the solenoid
(d) rate of Joule heating.

6. Two solenoids have identical geometrical construction but one is made of thick wire and the other of thin wire. Which of the following quantities are different for the two solenoids ?
(a) self-inductance
(b) rate of Joule heating if the same current goes through them
(c) magnetic field energy if the same current goes through them
(d) time constant if one solenoid is connected to one battery and the other is connected to another battery.

7. An LR circuit with a battery is connected at $t = 0$. Which of the following quantities is not zero just after the connection ?
(a) Current in the circuit
(b) Magnetic field energy in the inductor
(c) Power delivered by the battery
(d) Emf induced in the inductor

8. A rod AB moves with a uniform velocity v in a uniform magnetic field as shown in figure (38-Q7).
(a) The rod becomes electrically charged.
(b) The end A becomes positively charged.
(c) The end B becomes positively charged.
(d) The rod becomes hot because of Joule heating.

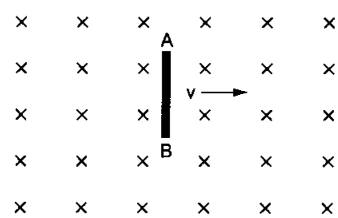


Figure 38-Q7

9. L, C and R represent the physical quantities inductance, capacitance and resistance respectively. Which of the following combinations have dimensions of frequency ?
(a) $\frac{1}{RC}$ (b) $\frac{R}{L}$ (c) $\frac{1}{\sqrt{LC}}$ (d) C/L.

10. The switches in figure (38-Q8a) and (38-8b) are closed at $t = 0$ and reopened after a long time at $t = t_0$.

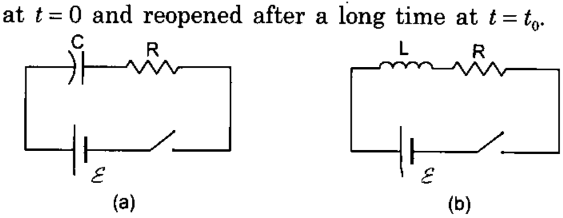


Figure 38-Q8

(a) The charge on C just after $t=0$ is $\mathcal{E}C$.

(b) The charge on C long after $t=0$ is $\mathcal{E}C$.

(c) The current in L just before $t=t_0$ is $\mathcal{E}/R$.

(d) The current in L long after $t=t_0$ is $\mathcal{E}/R$.

EXERCISES

1. Calculate the dimensions of (a) $\int \vec{E}\cdot d\vec{l}$, (b) vBl and (c) $\frac{d\Phi_B}{dt}$. The symbols have their usual meanings.
2. The flux of magnetic field through a closed conducting loop changes with time according to the equation, $\Phi = at^2 + bt + c$. (a) Write the SI units of a, b and c. (b) If the magnitudes of a, b and c are 0·20, 0·40 and 0·60 respectively, find the induced emf at $t = 2$ s.
3. (a) The magnetic field in a region varies as shown in figure (38-E1). Calculate the average induced emf in a conducting loop of area $2{\cdot}0\times10^{-3}\ \mathrm{m}^2$ placed perpendicular to the field in each of the 10 ms intervals shown. (b) In which intervals is the emf not constant? Neglect the behaviour near the ends of 10 ms intervals.

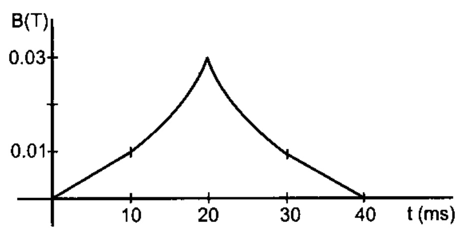


Figure 38-E1

4. A conducting circular loop having a radius of 5·0 cm, is placed perpendicular to a magnetic field of 0·50 T. It is removed from the field in 0·50 s. Find the average emf produced in the loop during this time.
5. A conducting circular loop of area 1 mm^2 is placed coplanarly with a long, straight wire at a distance of 20 cm from it. The straight wire carries an electric current which changes from 10 A to zero in 0·1 s. Find the average emf induced in the loop in 0·1 s.
6. A square-shaped copper coil has edges of length 50 cm and contains 50 turns. It is placed perpendicular to a 1·0 T magnetic field. It is removed from the magnetic field in 0·25 s and restored in its original place in the next 0·25 s. Find the magnitude of the average emf induced in the loop during (a) its removal, (b) its restoration and (c) its motion.
7. Suppose the resistance of the coil in the preivous problem is 25 Ω. Assume that the coil moves with uniform velocity during its removal and restoration. Find the thermal energy developed in the coil during (a) its removal, (b) its restoration and (c) its motion.
8. A conducting loop of area 5·0 cm^2 is placed in a magnetic field which varies sinusoidally with time as $B = B_0 \sin \omega t$ where $B_0 = 0{\cdot}20$ T and $\omega = 300\ \mathrm{s}^{-1}$. The normal to the coil makes an angle of 60° with the field. Find (a) the maximum emf induced in the coil, (b) the emf induced at $\tau = (\pi/900)$s and (c) the emf induced at $t = (\pi/600)$ s.
9. Figure (38-E2) shows a conducting square loop placed parallel to the pole-faces of a ring magnet. The pole-faces have an area of 1 cm^2 each and the field between the poles is 0·10 T. The wires making the loop are all outside the magnetic field. If the magnet is removed in 1·0 s, what is the average emf induced in the loop?

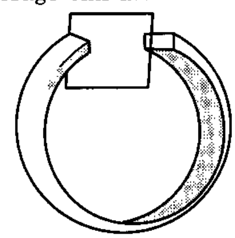


Figure 38-E2

10. A conducting square loop having edges of length 2·0 cm is rotated through 180° about a diagonal in 0·20 s. A magnetic field B exists in the region which is perpendicular to the loop in its initial position. If the average induced emf during the rotation is 20 mV, find the magnitude of the magnetic field.
11. A conducting loop of face-area A and resistance R is placed perpendicular to a magnetic field B. The loop is withdrawn completely from the field. Find the charge which flows through any cross-section of the wire in the process. Note that it is independent of the shape of the loop as well as the way it is withdrawn.
12. A long solenoid of radius 2 cm has 100 turns/cm and carries a current of 5 A. A coil of radius 1 cm having 100 turns and a total resistance of 20 Ω is placed inside the solenoid coaxially. The coil is connected to a galvanometer. If the current in the solenoid is reversed in direction, find the charge flown through the galvanometer.
13. Figure (38-E3) shows a metallic square frame of edge a in a vertical plane. A uniform magnetic field B exists in the space in a direction perpendicular to the plane of the figure. Two boys pull the opposite corners of the square to deform it into a rhombus. They start pulling the corners at $t=0$ and displace the corners at a uniform speed u. (a) Find the induced emf in the frame at the instant when the angles at these corners reduce to 60°. (b) Find the induced current in the frame at this instant if the total resistance of the frame is R. (c) Find the total charge which flows through a side of the frame by the time the square is deformed into a straight line.

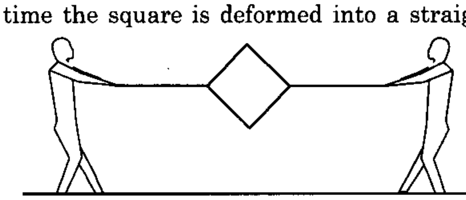


Figure 38-E2

14. The north pole of a magnet is brought down along the axis of a horizontal circular coil (figure 38-E4). As a result, the flux through the coil changes from 0·35 weber to 0·85 weber in an interval of half a second. Find the average emf induced during this period. Is the induced current clockwise or anticlockwise as you look into the coil from the side of the magnet ?

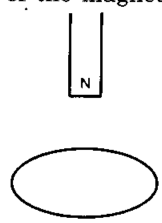


Figure 38-E4

15. A wire-loop confined in a plane is rotated in its own plane with some angular velocity. A uniform magnetic field exists in the region. Find the emf induced in the loop.

16. Figure (38-E5) shows a square loop of side 5 cm being moved towards right at a constant speed of 1 cm/s. The front edge enters the 20 cm wide magnetic field at $t = 0$. Find the emf induced in the loop at (a) $t = 2$ s, (b) $t = 10$ s, (c) $t = 22$ s and (d) $t = 30$ s.

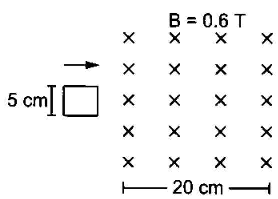


Figure 38-E5

17. Find the total heat produced in the loop of the previous problem during the interval 0 to 30 s if the resistance of the loop is 4·5 mΩ.

18. A uniform magnetic field B exists in a cylindrical region of radius 10 cm as shown in figure (38-E6). A uniform wire of length 80 cm and resistance 4·0 Ω is bent into a square frame and is placed with one side along a diameter of the cylindrical region. If the magnetic field increases at a constant rate of 0·010 T/s, find the current induced in the frame.

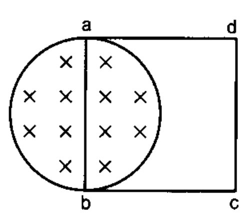


Figure 38-E6

19. The magnetic field in the cylindrical region shown in figure (38-E7) increases at a constant rate of 20·0 mT/s. Each side of the square loop *abcd* and *defa* has a length of 1·00 cm and a resistance of 4·00 Ω. Find the current (magnitude and sense) in the wire *ad* if (a) the switch S_1 is closed but S_2 is open, (b) S_1 is open but S_2 is closed, (c) both S_1 and S_2 are open and (d) both S_1 and S_2 are closed.

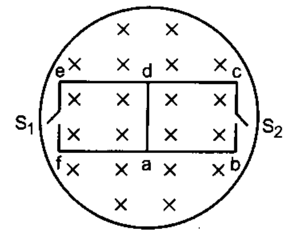


Figure 38-E7

20. Figure (38-E8) shows a circular coil of N turns and radius a, connected to a battery of emf $\mathcal{E}$ through a rheostat. The rheostat has a total length L and resistance R. The resistance of the coil is r. A small circular loop of radius a' and resistance r' is placed coaxially with the coil. The centre of the loop is at a distance x from the centre of the coil. In the beginning, the sliding contact of the rheostat is at the left end and then onwards it is moved towards right at a constant speed v. Find the emf induced in the small circular loop at the instant (a) the contact begins to slide and (b) it has slid through half the length of the rheostat.

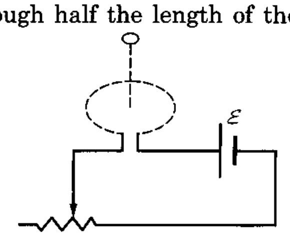


Figure 38-E8

21. A circular coil of radius 2·00 cm has 50 turns. A uniform magnetic field $B = 0{\cdot}200$ T exists in the space in a direction parallel to the axis of the loop. The coil is now rotated about a diameter through an angle of 60·0°. The operation takes 0·100 s. (a) Find the average emf induced in the coil. (b) If the coil is a closed one (with the two ends joined together) and has a resistance of 4·00 Ω, calculate the net charge crossing a cross-section of the wire of the coil.

22. A closed coil having 100 turns is rotated in a uniform magnetic field $B = 4{\cdot}0 \times 10^{-4}$ T about a diameter which is perpendicular to the field. The angular velocity of rotation is 300 revolutions per minute. The area of the coil is 25 cm^2 and its resistance is 4·0 Ω. Find (a) the average emf developed in half a turn from a position where the coil is perpendicular to the magnetic field, (b) the average emf in a full turn and (c) the net charge displaced in part (a).

23. A coil of radius 10 cm and resistance 40 Ω has 1000 turns. It is placed with its plane vertical and its axis parallel to the magnetic meridian. The coil is connected to a galvanometer and is rotated about the vertical diameter through an angle of 180°. Find the charge which flows through the galvanometer if the horizontal component of the earth's magnetic field is $B_H = 3{\cdot}0 \times 10^{-5}$ T.

24. A circular coil of one turn of radius 5·0 cm is rotated about a diameter with a constant angular speed of 80 revolutions per minute. A uniform magnetic field $B = 0{\cdot}010$ T exists in a direction perpendicular to the axis of rotation. Find (a) the maximum emf induced, (b)

the average emf induced in the coil over a long period and (c) the average of the squares of emf induced over a long period.

25. Suppose the ends of the coil in the previous problem are connected to a resistance of 100 Ω. Neglecting the resistance of the coil, find the heat produced in the circuit in one minute.

26. Figure (38-E9) shows a circular wheel of radius 10·0 cm whose upper half, shown dark in the figure, is made of iron and the lower half of wood. The two junctions are joined by an iron rod. A uniform magnetic field B of magnitude $2{\cdot}00 \times 10^{-4}$ T exists in the space above the central line as suggested by the figure. The wheel is set into pure rolling on the horizontal surface. If it takes 2·00 seconds for the iron part to come down and the wooden part to go up, find the average emf induced during this period.

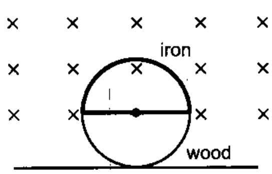


Figure 38-E9

27. A 20 cm long conducting rod is set into pure translation with a uniform velocity of 10 cm s^{-1} perpendicular to its length. A uniform magnetic field of magnitude 0·10 T exists in a direction perpendicular to the plane of motion. (a) Find the average magnetic force on the free electrons of the rod. (b) For what electric field inside the rod, the electric force on a free elctron will balance the magnetic force? How is this electric field created? (c) Find the motional emf between the ends of the rod.

28. A metallic metre stick moves with a velocity of 2 m s^{-1} in a direction perpendicular to its length and perpendicular to a uniform magnetic field of magnitude 0·2 T. Find the emf induced between the ends of the stick.

29. A 10 m wide spacecraft moves through the interstellar space at a speed 3×10^{7} m s^{-1}. A magnetic field $B = 3 \times 10^{-10}$ T exists in the space in a direction perpendicular to the plane of motion. Treating the spacecraft as a conductor, calculate the emf induced across its width.

30. The two rails of a railway track, insulated from each other and from the ground, are connected to a millivoltmeter. What will be the reading of the millivoltmeter when a train travels on the track at a speed of 180 km h^{-1}? The vertical component of earth's magnetic field is $0{\cdot}2 \times 10^{-4}$ T and the rails are separated by 1 m.

31. A right-angled triangle abc, made from a metallic wire, moves at a uniform speed v in its plane as shown in figure (38-E10). A uniform magnetic field B exists in the perpendicular direction. Find the emf induced (a) in the loop abc, (b) in the segment bc, (c) in the segment ac and (d) in the segment ab.

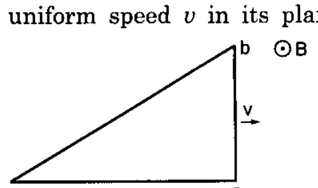


Figure 38-E10

32. A copper wire bent in the shape of a semicircle of radius r translates in its plane with a constant velocity v. A uniform magnetic field B exists in the direction perpendicular to the plane of the wire. Find the emf induced between the ends of the wire if (a) the velocity is perpendicular to the diameter joining free ends, (b) the velocity is parallel to this diameter.

33. A wire of length 10 cm translates in a direction making an angle of 60° with its length. The plane of motion is perpendicular to a uniform magnetic field of 1·0 T that exists in the space. Find the emf induced between the ends of the rod if the speed of translation is 20 cm s^{-1}.

34. A circular copper-ring of radius r translates in its plane with a constant velocity v. A uniform magnetic field B exists in the space in a direction perpendicular to the plane of the ring. Consider different pairs of diametrically opposite points on the ring. (a) Between which pair of points is the emf maximum? What is the value of this maximum emf? (b) Between which pair of points is the emf minimum? What is the value of this minimum emf?

35. Figure (38-E11) shows a wire sliding on two parallel, conducting rails placed at a separation l. A magnetic field B exists in a direction perpendicular to the plane of the rails. What force is necessary to keep the wire moving at a constant velocity v?

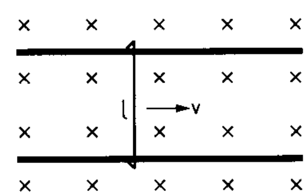


Figure 38-E11

36. Figure (38-E12) shows a long U-shaped wire of width l placed in a perpendicular magnetic field B. A wire of length l is slid on the U-shaped wire with a constant velocity v towards right. The resistance of all the wires is r per unit length. At $t = 0$, the sliding wire is close to the left edge of the U-shaped wire. Draw an equivalent circuit diagram, showing the induced emf as a battery. Calculate the current in the circuit.

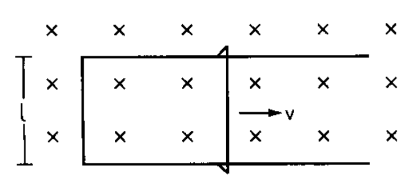


Figure 38-E12

37. Consider the situation of the previous problem. (a) Calculate the force needed to keep the sliding wire moving with a constant velocity v. (b) If the force needed just after $t = 0$ is F_0, find the time at which the force needed will be $F_0/2$.

38. Consider the situation shown in figure (38-E13). The wire PQ has mass m, resistance r and can slide on the smooth, horizontal parallel rails separated by a distance l. The resistance of the rails is negligible. A uniform magnetic field B exists in the rectangular region and a resistance R connects the rails outside the field region. At $t = 0$, the wire PQ is pushed towards right with a speed v_0. Find (a) the current in the loop at an instant when the speed of the wire PQ is v, (b) the acceleration of the wire at this instant, (c) the velocity v as a function of x and (d) the maximum distance the wire will move.

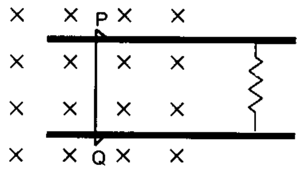


Figure 38-E13

39. A rectangular frame of wire $abcd$ has dimensions 32 cm × 8·0 cm and a total resistance of 2·0 Ω. It is pulled out of a magnetic field $B = 0{\cdot}020$ T by applying a force of $3{\cdot}2 \times 10^{-5}$ N (figure 38-E14). It is found that the frame moves with constant speed. Find (a) this constant speed, (b) the emf induced in the loop, (c) the potential difference between the points a and b and (d) the potential difference between the points c and d.

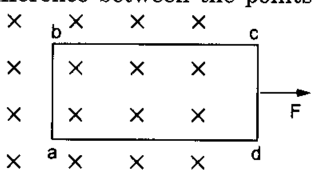


Figure 38-E14

40. Figure (38-E15) shows a metallic wire of resistance 0·20 Ω sliding on a horizontal, U-shaped metallic rail. The separation between the parallel arms is 20 cm. An electric current of 2·0 μA passes through the wire when it is slid at a rate of 20 cm s^{-1}. If the horizontal component of the earth's magnetic field is $3{\cdot}0 \times 10^{-5}$ T, calculate the dip at the place.

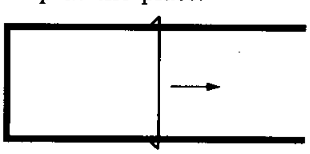


Figure 38-E15

41. A wire ab of length l, mass m and resistance R slides on a smooth, thick pair of metallic rails joined at the bottom as shown in figure (38-E16). The plane of the rails makes an angle θ with the horizontal. A vertical magnetic field B exists in the region. If the wire slides on the rails at a constant speed v, show that

$$B = \sqrt{\frac{mg\,R\sin\theta}{vl^2\cos^2\theta}}.$$

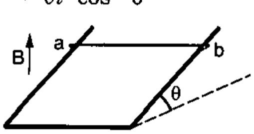


Figure 38-E16

42. Consider the situation shown in figure (38-E17). The wires P_1Q_1 and P_2Q_2 are made to slide on the rails with the same speed 5 cm s^{-1}. Find the electric current in the 19 Ω resistor if (a) both the wires move towards right and (b) if P_1Q_1 moves towards left but P_2Q_2 moves towards right.

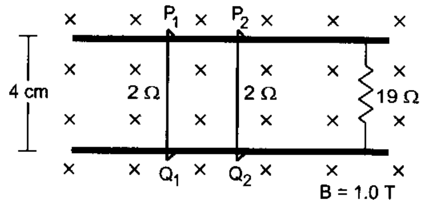


Figure 38-E17

43. Suppose the 19 Ω resistor of the previous problem is disconnected. Find the current through P_2Q_2 in the two situations (a) and (b) of that problem.

44. Consider the situation shown in figure (38-E18). The wire PQ has a negligible resistance and is made to slide on the three rails with a constant speed of 5 cm s^{-1}. Find the current in the 10 Ω resistor when the switch S is thrown to (a) the middle rail (b) the bottom rail.

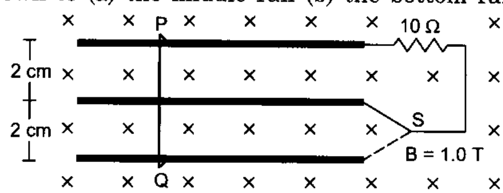


Figure 38-E18

45. The current generator I_g, shown in figure (38-E19), sends a constant current i through the circuit. The wire cd is fixed and ab is made to slide on the smooth, thick rails with a constant velocity v towards right. Each of these wires has resistance r. Find the current through the wire cd.

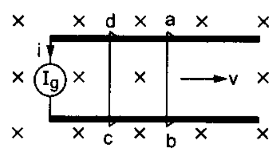


Figure 38-E19

46. The current generator I_g, shown in figure (38-E20), sends a constant current i through the circuit. The wire ab has a length l and mass m and can slide on the smooth, horizontal rails connected to I_g. The entire system lies in a vertical magnetic field B. Find the velocity of the wire as a function of time.

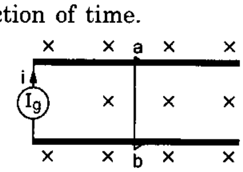


Figure 38-E20

47. The system containing the rails and the wire of the previous problem is kept vertically in a uniform horizontal magnetic field B that is perpendicular to the plane of the rails (figure 38-E21). It is found that the wire stays in equilibrium. If the wire ab is replaced by

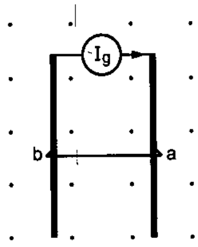


Figure 38-E21

another wire of double its mass, how long will it take in falling through a distance equal to its length ?

48. The rectangular wire-frame, shown in figure (38-E22), has a width d, mass m, resistance R and a large length. A uniform magnetic field B exists to the left of the frame. A constant force F starts pushing the frame into the magnetic field at $t = 0$. (a) Find the acceleration of the frame when its speed has increased to v. (b) Show that after some time the frame will move with a constant velocity till the whole frame enters into the magnetic field. Find this velocity v_0. (c) Show that the velocity at time t is given by

$$v = v_0(1 - e^{-Ft/mv_0}).$$

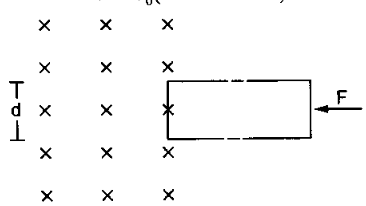


Figure 38-E22

49. Figure (38-E23) shows a smooth pair of thick metallic rails connected across a battery of emf $\mathcal{E}$ having a negligible internal resistance. A wire ab of length l and resistance r can slide smoothly on the rails. The entire system lies in a horizontal plane and is immersed in a uniform vertical magnetic field B. At an instant t, the wire is given a small velocity v towards right. (a) Find the current in it at this instant. What is the direction of the current ? (b) What is the force acting on the wire at this instant ? (c) Show that after some time the wire ab will slide with a constant velocity. Find this velocity.

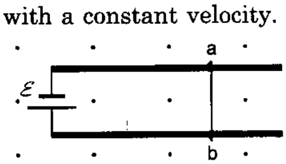


Figure 38-E23

50. A conducting wire ab of length l, resistance r and mass m starts sliding at $t = 0$ down a smooth, vertical, thick pair of connected rails as shown in figure (38-E24). A uniform magnetic field B exists in the space in a direction perpendicular to the plane of the rails. (a) Write the induced emf in the loop at an instant t when the speed of the wire is v. (b) What would be the magnitude and direction of the induced current in the wire ? (c) Find the downward acceleration of the wire at this instant. (d) After sufficient time, the wire starts moving with a constant velocity. Find this velocity v_m. (e) Find the velocity of the wire as a function of time. (f) Find the displacement of the wire as a function of time. (g) Show that the rate of heat developed in the wire is equal to the rate at which the gravitational potential energy is decreased after steady state is reached.

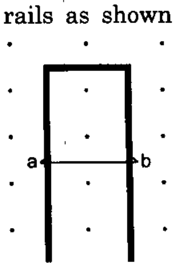


Figure 38-E24

51. A bicycle is resting on its stand in the east–west direction and the rear wheel is rotated at an angular speed of 100 revolutions per minute. If the length of each spoke is 30·0 cm and the horizontal component of the earth's magnetic field is $2{\cdot}0 \times 10^{-5}$ T, find the emf induced between the axis and the outer end of a spoke. Neglect centripetal force acting on the free electrons of the spoke.

52. A conducting disc of radius r rotates with a small but constant angular velocity ω about its axis. A uniform magnetic field B exists parallel to the axis of rotation. Find the motional emf between the centre and the periphery of the disc.

53. Figure (38-E25) shows a conducting disc rotating about its axis in a perpendicular magnetic field B. A resistor of resistance R is connected between the centre and the rim. Calculate the current in the resistor. Does it enter the disc or leave it at the centre ? The radius of the disc is 5·0 cm, angular speed $\omega = 10$ rad/s, $B = 0{\cdot}40$ T and $R = 10\ \Omega$.

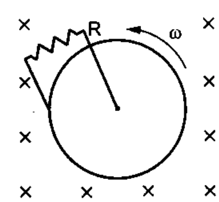


Figure 38-E25

54. The magnetic field in a region is given by $\vec{B} = \vec{k}\frac{B_0}{L}y$ where L is a fixed length. A conducting rod of length L lies along the Y-axis between the origin and the point $(0, L, 0)$. If the rod moves with a velocity $v = v_0\vec{i}$, find the emf induced between the ends of the rod.

55. Figure (38-E26) shows a straight, long wire carrying a current i and a rod of length l coplanar with the wire and perpendicular to it. The rod moves with a constant velocity v in a direction parallel to the wire. The distance of the wire from the centre of the rod is x. Find the motional emf induced in the rod.

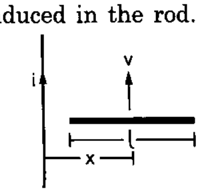


Figure 38-E26

56. Consider a situation similar to that of the previous problem except that the ends of the rod slide on a pair of thick metallic rails laid parallel to the wire. At one end the rails are connected by resistor of resistance R. (a) What force is needed to keep the rod sliding at a constant speed v? (b) In this situation what is the current in the resistance R? (c) Find the rate of heat developed in the resistor. (d) Find the power delivered by the external agent exerting the force on the rod.

57. Figure (38-E27) shows a square frame of wire having a total resistance r placed coplanarly with a long, straight wire. The wire carries a current i given by $i = i_0 \sin \omega t$. Find (a) the flux of the magnetic field through the square frame, (b) the emf induced in the frame and (c) the heat developed in the frame in the time interval 0 to $\frac{20\pi}{\omega}$.

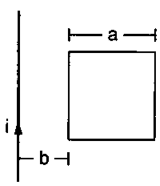


Figure 38-E27

58. A rectangular metallic loop of length l and width b is placed coplanarly with a long wire carrying a current i (figure 38-E28). The loop is moved perpendicular to the wire with a speed v in the plane containing the wire and the loop. Calculate the emf induced in the loop when the rear end of the loop is at a distance a from the wire. Solve by using Faraday's law for the flux through the loop and also by replacing different segments with equivalent batteries.

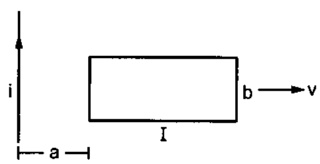


Figure 38-E28

59. Figure (38-E29) shows a conducting circular loop of radius a placed in a uniform, perpendicular magnetic field B. A thick metal rod OA is pivoted at the centre O. The other end of the rod touches the loop at A. The centre O and a fixed point C on the loop are connected by a wire OC of resistance R. A force is applied at the middle point of the rod OA perpendicularly, so that the rod rotates clockwise at a uniform angular velocity ω. Find the force.

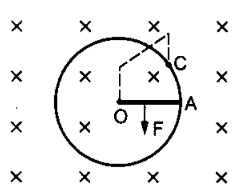


Figure 38-E29

60. Consider the situation shown in the figure of the previous problem. Suppose the wire connecting O and C has zero resistance but the circular loop has a resistance R uniformly distributed along its length. The rod OA is made to rotate with a uniform angular speed ω as shown in the figure. Find the current in the rod when $\angle AOC = 90°$.

61. Consider a variation of the previous problem (figure 38-E29). Suppose the circular loop lies in a vertical plane. The rod has a mass m. The rod and the loop have negligible resistances but the wire connecting O and C has a resistance R. The rod is made to rotate with a uniform angular velocity ω in the clockwise direction by applying a force at the midpoint of OA in a direction perpendicular to it. Find the magnitude of this force when the rod makes an angle θ with the vertical.

62. Figure (38-E30) shows a situation similar to the previous problem. All parameters are the same except that a battery of emf $\mathcal{E}$ and a variable resistance R are connected between O and C. Neglect the resistance of the connecting wires. Let θ be the angle made by the rod from the horizontal position (shown in the figure), measured in the clockwise direction. During the part of the motion $0 < \theta < \pi/4$ the only forces acting on the rod are gravity and the forces exerted by the magnetic field and the pivot. However, during the part of the motion, the resistance R is varied in such a way that the rod continues to rotate with a constant angular velocity ω. Find the value of R in terms of the given quantities.

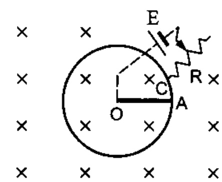


Figure 38-E30

63. A wire of mass m and length l can slide freely on a pair of smooth, vertical rails (figure 38-E31). A magnetic field B exists in the region in the direction perpendicular to the plane of the rails. The rails are connected at the top end by a capacitor of capacitance C. Find the acceleration of the wire neglecting any electric resistance.

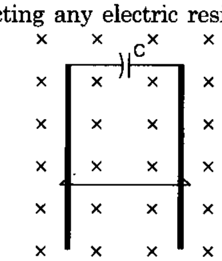


Figure 38-E31

64. A uniform magnetic field B exists in a cylindrical region, shown dotted in figure (38-E32). The magnetic field increases at a constant rate $\frac{dB}{dt}$. Consider a circle of

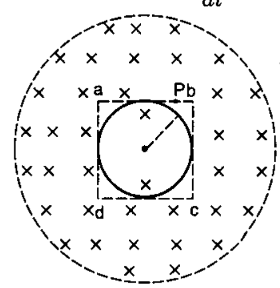


Figure 38-E32

radius r coaxial with the cylindrical region. (a) Find the magnitude of the electric field E at a point on the circumference of the circle. (b) Consider a point P on the side of the square circumscribing the circle. Show that the component of the induced electric field at P along ba is the same as the magnitude found in part (a).

65. The current in an ideal, long solenoid is varied at a uniform rate of 0·01 As^{-1}. The solenoid has 2000 turns/m and its radius is 6·0 cm. (a) Consider a circle of radius 1·0 cm inside the solenoid with its axis coinciding with the axis of the solenoid. Write the change in the magnetic flux through this circle in 2·0 seconds. (b) Find the electric field induced at a point on the circumference of the circle. (c) Find the electric field induced at a point outside the solenoid at a distance 8·0 cm from its axis.

66. An average emf of 20 V is induced in an inductor when the current in it is changed from 2·5 A in one direction to the same value in the opposite direction in 0·1 s. Find the self-inductance of the inductor.

67. A magnetic flux of 8×10^{-4} weber is linked with each turn of a 200-turn coil when there is an electric current of 4 A in it. Calculate the self-inductance of the coil.

68. The current in a solenoid of 240 turns, having a length of 12 cm and a radius of 2 cm, changes at a rate of 0·8 A s^{-1}. Find the emf induced in it.

69. Find the value of t/τ for which the current in an LR circuit builds up to (a) 90%, (b) 99% and (c) 99·9% of the steady-state value.

70. An inductor-coil carries a steady-state current of 2·0 A when connected across an ideal battery of emf 4·0 V. If its inductance is 1·0 H, find the time constant of the circuit.

71. A coil having inductance 2·0 H and resistance 20 Ω is connected to a battery of emf 4·0 V. Find (a) the current at the instant 0·20 s after the connection is made and (b) the magnetic field energy at this instant.

72. A coil of resistance 40 Ω is connected across a 4·0 V battery. 0·10 s after the battery is connected, the current in the coil is 63 mA. Find the inductance of the coil.

73. An inductor of inductance 5·0 H, having a negligible resistance, is connected in series with a 100 Ω resistor and a battery of emf 2·0 V. Find the potential difference across the resistor 20 ms after the circuit is switched on.

74. The time constant of an LR circuit is 40 ms. The circuit is connected at $t = 0$ and the steady-state current is found to be 2·0 A. Find the current at (a) $t = 10$ ms (b) $t = 20$ ms, (c) $t = 100$ ms and (d) $t = 1$ s.

75. An LR circuit has $L = 1·0$ H and $R = 20$ Ω. It is connected across an emf of 2·0 V at $t = 0$. Find di/dt at (a) $t = 100$ ms, (b) $t = 200$ ms and (c) $t = 1·0$ s.

76. What are the values of the self-induced emf in the circuit of the previous problem at the times indicated therein?

77. An inductor-coil of inductance 20 mH having resistance 10 Ω is joined to an ideal battery of emf 5·0 V. Find the rate of change of the induced emf at $t = 0$, (b) $t = 10$ ms and (c) $t = 1·0$ s.

78. An LR circuit contains an inductor of 500 mH, a resistor of 25·0 Ω and an emf of 5·00 V in series. Find the potential difference across the resistor at t = (a) 20·0 ms, (b) 100 ms and (c) 1·00 s.

79. An inductor-coil of resistance 10 Ω and inductance 120 mH is connected across a battery of emf 6 V and internal resistance 2 Ω. Find the charge which flows through the inductor in (a) 10 ms, (b) 20 ms and (c) 100 ms after the connections are made.

80. An inductor-coil of inductance 17 mH is constructed from a copper wire of length 100 m and cross-sectional area 1 mm^2. Calculate the time constant of the circuit if this inductor is joined across an ideal battery. The resistivity of copper $= 1·7 \times 10^{-8}$ Ωm.

81. An LR circuit having a time constant of 50 ms is connected with an ideal battery of emf $\mathcal{E}$. Find the time elapsed before (a) the current reaches half its maximum value, (b) the power dissipated in heat reaches half its maximum value and (c) the magnetic field energy stored in the circuit reaches half its maximum value.

82. A coil having an inductance L and a resistance R is connected to a battery of emf $\mathcal{E}$. Find the time taken for the magnetic energy stored in the circuit to change from one fourth of the steady-state value to half of the steady-state value.

83. A solenoid having inductance 4·0 H and resistance 10 Ω is connected to a 4·0 V battery at $t = 0$. Find (a) the time constant, (b) the time elapsed before the current reaches 0·63 of its steady-state value, (c) the power delivered by the battery at this instant and (d) the power dissipated in Joule heating at this instant.

84. The magnetic field at a point inside a 2·0 mH inductor-coil becomes 0·80 of its maximum value in 20 μs when the inductor is joined to a battery. Find the resistance of the circuit.

85. An LR circuit with emf $\mathcal{E}$ is connected at $t = 0$. (a) Find the charge Q which flows through the battery during 0 to t. (b) Calculate the work done by the battery during this period. (c) Find the heat developed during this period. (d) Find the magnetic field energy stored in the circuit at time t. (e) Verify that the results in the three parts above are consistent with energy conservation.

86. An inductor of inductance 2·00 H is joined in series with a resistor of resistance 200 Ω and a battery of emf 2·00 V. At $t = 10$ ms, find (a) the current in the circuit, (b) the power delivered by the battery, (c) the power dissipated in heating the resistor and (d) the rate at which energy is being stored in magnetic field.

87. Two coils A and B have inductances 1·0 H and 2·0 H respectively. The resistance of each coil is 10 Ω. Each coil is connected to an ideal battery of emf 2·0 V at $t = 0$. Let i_A and i_B be the currents in the two circuit at time t. Find the ratio i_A / i_B at (a) $t = 100$ ms, (b) $t = 200$ ms and (c) $t = 1$ s.

88. The current in a discharging LR circuit without the battery drops from 2·0 A to 1·0 A in 0·10 s. (a) Find the time constant of the circuit. (b) If the inductance of the circuit is 4·0 H, what is its resistance?

89. A constant current exists in an inductor-coil connected to a battery. The coil is short-circuited and the battery is removed. Show that the charge flown through the coil after the short-circuiting is the same as that which flows in one time constant before the short-circuiting.

90. Consider the circuit shown in figure (38-E33). (a) Find the current through the battery a long time after the switch S is closed. (b) Suppose the switch is again opened at $t = 0$. What is the time constant of the discharging circuit ? (c) Find the current through the inductor after one time constant.

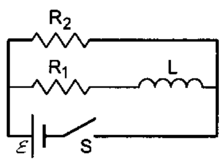


Figure 38-E33

91. A current of 1·0 A is established in a tightly wound solenoid of radius 2 cm having 1000 turns/metre. Find the magnetic energy stored in each metre of the solenoid.

92. Consider a small cube of volume 1 mm^3 at the centre of a circular loop of radius 10 cm carrying a current of 4 A. Find the magnetic energy stored inside the cube.

93. A long wire carries a current of 4·00 A. Find the energy stored in the magnetic field inside a volume of 1·00 mm^3 at a distance of 10·0 cm from the wire.

94. The mutual inductance between two coils is 2·5 H. If the current in one coil is changed at the rate of 1 As^{-1}, what will be the emf induced in the other coil ?

95. Find the mutual inductance between the straight wire and the square loop of figure (38-E27).

96. Find the mutual inductance between the circular coil and the loop shown in figure (38-E8).

97. A solenoid of length 20 cm, area of cross-section 4·0 cm^2 and having 4000 turns is placed inside another solenoid of 2000 turns having a cross-sectional area 8·0 cm^2 and length 10 cm. Find the mutual inductance between the solenoids.

98. The current in a long solenoid of radius R and having n turns per unit length is given by $i = i_0 \sin \omega t$. A coil having N turns is wound around it near the centre. Find (a) the induced emf in the coil and (b) the mutual inductance between the solenoid and the coil.

□

ANSWERS

OBJECTIVE I

1. (b)	2. (a)	3. (d)	4. (c)	5. (b)	6. (c)
7. (a)	8. (b)	9. (b)	10. (a)	11. (c)	12. (d)

OBJECTIVE II

1. (b), (c)	2. (d)	3. (c), (d)
4. (a), (c), (d)	5. (a), (b), (c)	6. (b), (d)
7. (d)	8. (b)	9. (a), (b), (c)
10. (b), (c)		

EXERCISE

1. $ML^2I^{-1}T^{-3}$ in each case
2. (a) volt/sec, volt, volt-sec (or weber) (b) 1·2 volt
3. (a) – 2·0 mV, – 4·0 mV, 4·0 mV, 2·0 mV (b)10 ms to 20 ms and 20 ms to 30 ms
4. $7{\cdot}8 \times 10^{-3}$ V
5. 1×10^{-10} V
6. (a) 50 V (b) 50 V (c) zero
7. (a) 25 J (b) 25 J (c) 50 J
8. (a) 0·015 V (b) $7{\cdot}5 \times 10^{-3}$ V (c) zero
9. 10 μV
10. 5·0 T
11. BA/R
12. 2×10^{-4} C
13. (a) $2Bav$ (b) $2Bav/R$ (c) $a^2 B/R$
14. $\mathcal{E} = 1{\cdot}0$ V, anticlockwise
15. zero
16. (a) 3×10^{-4} V, (b) zero, (c) 3×10^{-4} V and (d) zero
17. 2×10^{-4} J
18. $3{\cdot}9 \times 10^{-5}$ A
19. (a) $1{\cdot}25 \times 10^{-7}$ A, a to d (b) $1{\cdot}25 \times 10^{-7}$ A, d to a, (c) zero (d) zero
20. $\dfrac{\pi\mu_0 N a^2 a'^2 \mathcal{E} R v}{2L(a^2 + x^2)^{3/2}(R' + r)^2}$ where $R' = R$ for part (a) and $R/2$ for part (b)
21. (a) $6{\cdot}28 \times 10^{-2}$ V (b) $1{\cdot}57 \times 10^{-3}$ C
22. (a) $2{\cdot}0 \times 10^{-3}$ V (b) zero (c) $5{\cdot}0 \times 10^{-5}$ C
23. $4{\cdot}7 \times 10^{-5}$ C
24. (a) $6{\cdot}6 \times 10^{-4}$ V (b) zero (c) $2{\cdot}2 \times 10^{-7}$ V^2
25. $1{\cdot}3 \times 10^{-7}$ J

26. 1.57×10^{-6} V
27. (a) 1.6×10^{-21} N (b) 1.0×10^{-2} Vm^{-1} (c) 2.0×10^{-3} V
28. 0·4 V
29. 0·09 V
30. 1 mV
31. (a) zero (b) $vB(bc)$, positive at c (c) zero
(d) $vB(bc)$, positive at a
32. (a) $2rvB$ (b) zero
33. 17×10^{-3} V
34. (a) at the ends of the diameter perpendicular to the velocity, $2\,rvB$ (b) at the ends of the diameter parallel to the velocity, zero
35. zero
36. $\dfrac{Blv}{2r(l+vt)}$
37. (a) $\dfrac{B^2l^2v}{2r(l+vt)}$ (b) l/v
38. (a) $\dfrac{Blv}{R+r}$ (b) $\dfrac{B^2l^2v}{m(R+r)}$ towards left (c) $v = v_0 - \dfrac{B^2l^2x}{m(R+r)}$
(d) $\dfrac{mv_0(R+r)}{B^2l^2}$
39. (a) 25 m s^{-1} (b) 4.0×10^{-2} V (c) 3.6×10^{-2} V
(d) 4.0×10^{-3} V
40. $\tan^{-1}(1/3)$
42. (a) 0·1 mA (b) zero
43. (a) zero (b) 1 mA
44. (a) 0·1 mA (b) 0·2 mA
45. $\dfrac{ir - Blv}{2r}$
46. $ilBt/m$, away from the generator
47. $2\sqrt{l/g}$
48. (a) $\dfrac{RF - vl^2B^2}{mR}$ (b) $\dfrac{RF}{l^2B^2}$
49. (a) $\dfrac{1}{r}(E - vBl)$ from b to a (b) $\dfrac{lB}{r}(E - vBl)$ towards right (c) $\dfrac{E}{Bl}$
50. (a) vBl (b) $\dfrac{vBl}{r}$, b to a (c) $g - \dfrac{B^2l^2}{mr}v$ (d) $\dfrac{mgr}{B^2l^2}$
(e) $v_m(1 - e^{-gt/v_m})$ (f) $v_m t - \dfrac{v_m^2}{g}(1 - e^{-gt/v_m})$
51. 9.4×10^{-6} V
52. $\dfrac{1}{2}\omega r^2 B$
53. 0·5 mA, leaves
54. $\dfrac{B_0 v_0 l}{2}$
55. $\dfrac{\mu_0 iv}{2\pi}\ln\left(\dfrac{2x+l}{2x-l}\right)$
56. (a) $\dfrac{v}{R}\left\{\dfrac{\mu_0 i}{2\pi}\ln\dfrac{2x+l}{2x-l}\right\}^2$ (b) $\dfrac{\mu_0 iv}{2\pi R}\ln\dfrac{2x+l}{2x-l}$
(c) $\dfrac{1}{R}\left\{\dfrac{\mu_0 iv}{2\pi}\ln\dfrac{2x+l}{2x-l}\right\}^2$ (d) same as (c)
57. (a) $\dfrac{\mu_0 ia}{2\pi}\ln\left(1+\dfrac{a}{b}\right)$ (b) $\dfrac{\mu_0 ai_0\omega\cos\omega t}{2\pi}\ln\left(1+\dfrac{a}{b}\right)$
(c) $\dfrac{5\mu_0^2 a^2 i_0^2\omega}{2\pi r}\ln^2\left(1+\dfrac{a}{b}\right)$
58. $\dfrac{\mu\, ilvb}{2\pi a(a+l)}$
59. $\dfrac{\omega a^3B^2}{2R}$ to the right of OA in the figure
60. $\dfrac{8}{3}\dfrac{\omega a^2B}{R}$
61. $\dfrac{\omega a^3B^2}{2R} - mg\sin\theta$
62. $\dfrac{aB(2\mathcal{E} + \omega a^2B)}{2\,mg\cos\theta}$
63. $\dfrac{mg}{m + CB^2l^2}$
64. (a) $\dfrac{r}{2}\dfrac{dB}{dt}$
65. (a) 1.6×10^{-8} weber (b) 1.2×10^{-7} V m^{-1}
(c) 5.6×10^{-7} V m^{-1}
66. 0·4 H
67. 4×10^{-2} H
68. 6×10^{-4} V
69. 2·3, 4·6, 6·9
70. 0·50 s
71. (a) 0·17 A (b) 0·03 J
72. 4·0 H
73. 0·66 V
74. (a) 0·44 A (b) 0·79 A (c) 1·8 A and (d) 2·0 A
75. (a) 0·27 A s^{-1} (b) 0·036 A s^{-1} and (c) 4.1×10^{-9} A s^{-1}
76. (a) 0·27 V (b) 0·036 V (c) 4.1×10^{-9} V
77. (a) 2.5×10^3 V s^{-1} (b) 17 V s^{-1} and (c) 0·00 V s^{-1}
78. (a) 3·16 V (b) 4·97 V and (c) 5·00 V
79. (a) 1·8 mC (b) 5·7 mC and (c) 45 mC
80. 10 ms
81. (a) 35 ms (b) 61 ms (c) 61 ms
82. $\tau\ln\dfrac{1}{2-\sqrt{2}}$
83. (a) 0·40 s (b) 0·40 s (c) 1·0 W and (d) 0·64 W
84. 160 Ω

85. (a) $\frac{\mathcal{E}}{R}\{t-\frac{L}{R}(1-x)\}$

(b) $\frac{\mathcal{E}^2}{R}\{t-\frac{L}{R}(1-x)\}$

(c) $\frac{\mathcal{E}^2}{R}\{t-\frac{L}{2R}(3-4x+x^2)\}$

(d) $\frac{L\mathcal{E}^2}{2R^2}(1-x)^2$, where $x=e^{-Rt/L}$

86. (a) 6·3 mA (b) 12·6 mW (c) 8·0 mW and (d) 4·6 mW

87. (a) 1·6 (b) 1·4 (c) 1·0

88. (a) 0·14 s (b) 28 Ω

90. (a) $\frac{\mathcal{E}(R_1+R_2)}{R_1R_2}$ (b) $\frac{L}{R_1+R_2}$ (c) $\frac{\mathcal{E}}{R_1e}$

91. $7{\cdot}9\times10^{-4}$ J

92. $8\pi\times10^{-14}$ J

93. $2{\cdot}55\times10^{-14}$ J

94. 2·5 V

95. $\frac{\mu_0 a}{2\pi}\ln\left(1+\frac{a}{b}\right)$

96. $N\frac{\mu_0\pi a^2 a'^{1/2}}{2(a^2+x^2)^{3/2}}$

97. $2{\cdot}0\times10^{-2}$ H

98. (a) $\pi\mu_0 i_0 nN\omega R^2\cos\omega t$ (b) $\pi\mu_0 nNR^2$

□