Tangents and Normals Ex 16.3 Q1(viii)

We know that angle of intersection of two curves is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \quad \text{---(A)}$$

Where m_1 and m_2 are slopes of curves.

$$x^2 + y^2 = 2x \quad \text{---(i)}$$
$$y^2 = x \quad \text{---(ii)}$$

Solving (i) and (ii)

$$x^2 + x = 2x$$
$$\Rightarrow \quad x^2 - x = 0$$
$$\Rightarrow \quad x(x-1) = 0$$
$$\Rightarrow \quad x = 0,1$$

$$\therefore \quad y = 0 \text{ or } 1$$

$\therefore$ The points of intersection is $P = (0,0)$, $Q = (1,1)$

$\therefore$ Slope of (i)

$$2y\frac{dy}{dx} = 2 - 2x$$
$$\therefore \quad \frac{dy}{dx} = \frac{2-2x}{2y} = \frac{1-x}{y}$$
$$\therefore \quad m_1 = 0$$

Slope of (ii)

$$m_2 = \frac{1}{2y} = \frac{1}{2}$$

From (A)

$$\tan\theta = \left|\frac{\frac{1}{2} - 0}{1 + \frac{1}{2}\times 0}\right| = \frac{1}{2}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{1}{2}\right)$$

Tangents and Normals Ex 16.3 Q1(ix)

$y = 4 - x^2$......(i)
$y = x^2$.......(ii)

Substituting eq (ii) in (i) we get,
$x^2 = 4 - x^2$
$\Rightarrow x^2 = 2$
$\Rightarrow x = \pm\sqrt{2}$

From(i) when $x = \sqrt{2}$,we get $y = 2$ and when $x = -\sqrt{2}$,we get $y = 2$
Thus the two curves intersect at $\left(\sqrt{2}, 2\right)$ and $\left(-\sqrt{2}, 2\right)$.

Differnentiating (i) wrt x, we get

$$\frac{dy}{dx} = 0 - 2x = -2x$$

Differnentiating (ii) wrt x, we get

$$\frac{dy}{dx} = 2x$$

Angle of intersection at $\left(\sqrt{2}, 2\right)$

$$m_1 = \left(\frac{dy}{dx}\right)_{\left(\sqrt{2},\ 2\right)} = -2\sqrt{2}$$

Angle of intersection at $\left(-\sqrt{2}, 2\right)$

$$m_2 = \left(\frac{dy}{dx}\right)_{\left(-\sqrt{2},\ 2\right)} = 2\sqrt{2}$$

Let θ be the angle of intersection of the two curves.

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{2\sqrt{2} + 2\sqrt{2}}{1 + \left(2\sqrt{2}\right)\left(-2\sqrt{2}\right)}\right| = \left|\frac{4\sqrt{2}}{-7}\right| = \frac{4\sqrt{2}}{7}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{4\sqrt{2}}{7}\right)$$

*********** END **********