Chapter 10 Differentiability Ex 10.1 Q10

$$\text{Her,e}\quad f(x)=\begin{cases} ax^2-b & ,\ \text{if } |x|<1 \\ \dfrac{1}{|x|} & ,\ \text{if } |x|\geq 1 \end{cases}$$

$$=\begin{cases} -\dfrac{1}{x} & ,\ \text{if } x\leq -1 \\ ax^2-b & ,\ \text{if } -1<x< 1 \\ \dfrac{1}{x} & ,\ \text{if } x\geq 1 \end{cases}$$

$$\text{LHL} = \lim_{x\to 1^-} f(x)$$

$$= \lim_{h\to 0} f(1-h)$$

$$= \lim_{h\to 0} a(1-h)^2-b$$

$$= a-b$$

$$\text{RHL} = \lim_{x\to 1^+} f(x)$$

$$= \lim_{h\to 0} f(1+h)$$

$$= \lim_{h\to 0} \frac{1}{1+h}$$

$$= 1$$

Since, $f(x)$ is continuous, so

LHS = RHS

$$a-b=1 \qquad \text{---(i)}$$

$$(\text{LHD at } x=1) = \lim_{x\to 1^-}\frac{f(x)-f(1)}{x-1}$$

$$= \lim_{h\to 0}\frac{f(1-h)-1}{1-h-1}$$

$$= \lim_{h\to 0}\frac{a(1-h)^2-b-1}{-h}$$

$$= \lim_{h\to 0}\frac{a(1-h)^2-(a-1)-1}{-h}$$

Using equation (i),

$$= \lim_{h\to 0}\frac{a+ah^2-2ah-a+1-1}{-h}$$

$$= \lim_{h\to 0}\frac{ah^2-2ah}{-h}$$

$$= \lim_{h\to 0}(2a-ah)$$

$$= 2a$$

$$(\text{RHD at } x=1) = \lim_{x\to 1^+}\frac{f(x)-f(1)}{x-1}$$

$$= \lim_{h\to 0}\frac{f(1+h)-f(1)}{1+h-1}$$

$$= \lim_{h\to 0}\frac{\frac{1}{1+h}-1}{h}$$

$$= \lim_{h\to 0}\frac{1-1-h}{(1+h)h}$$

$$= \lim_{h\to 0}\frac{-1}{1+h}$$

$$= -1$$

Since $f(x)$ is differentiable at $x=1$,

$$(\text{LHD at } x=1) = (\text{RHD at } x=1)$$

$$2a = -1$$

$$a = \frac{-1}{2}$$

Put $a = \frac{-1}{2}$ in equation (i),

$$a - b = 1$$

$$\left(\frac{-1}{2}\right) - b = 1$$

$$b = \frac{-1}{2} - 1$$

$$b = \frac{-3}{2}$$

$$a = \frac{-1}{2}$$

********** END **********