Trigonometric Identities Ex 6.1 Q60

Answer :

We have to prove $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$

We know that, $\sin^2 A + \cos^2 A = 1$.

So,

$$
\begin{aligned}
(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) &= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right) \\
&= \left(\frac{\sin A + \cos A - 1}{\sin A}\right)\left(\frac{\cos A + \sin A + 1}{\cos A}\right) \\
&= \frac{(\sin A + \cos A - 1)(\sin A + \cos A + 1)}{\sin A \cos A} \\
&= \frac{\{(\sin A + \cos A) - 1\}\{(\sin A + \cos A) + 1\}}{\sin A \cos A} \\
&= \frac{(\sin A + \cos A)^2 - 1}{\sin A \cos A} \\
&= \frac{\sin^2 A + 2\sin A \cos A + \cos^2 A - 1}{\sin A \cos A} \\
&= \frac{(\sin^2 A + \cos^2 A) + 2\sin A \cos A - 1}{\sin A \cos A} \\
&= \frac{1 + 2\sin A \cos A - 1}{\sin A \cos A} \\
&= \frac{2\sin A \cos A}{\sin A \cos A} \\
&= 2
\end{aligned}
$$

Hence proved.

Trigonometric Identities Ex 6.1 Q61

Answer :

We have to prove

$(\operatorname{cosec}\theta - \sec\theta)(\cot\theta - \tan\theta) = (\operatorname{cosec}\theta + \sec\theta)(\sec\theta\operatorname{cosec}\theta - 2)$

We know that, $\sin^2\theta + \cos^2\theta = 1$

Consider the LHS.

$$
\begin{aligned}
(\operatorname{cosec}\theta - \sec\theta)(\cot\theta - \tan\theta) &= \left(\frac{1}{\sin\theta} - \frac{1}{\cos\theta}\right)\left(\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}\right) \\
&= \left(\frac{\cos\theta - \sin\theta}{\sin\theta\cos\theta}\right)\left(\frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta}\right) \\
&= \frac{(\cos\theta - \sin\theta)}{\sin\theta\cos\theta}\frac{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}{\sin\theta\cos\theta} \\
&= \frac{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)^2}{\sin^2\theta\cos^2\theta}
\end{aligned}
$$

Now, consider the RHS.

$$(\cosec\theta + \sec\theta)(\sec\theta\cosec\theta - 2) = \left(\frac{1}{\sin\theta} + \frac{1}{\cos\theta}\right)\left(\frac{1}{\cos\theta}\frac{1}{\sin\theta} - 2\right)$$

$$= \left(\frac{\cos\theta + \sin\theta}{\sin\theta\cos\theta}\right)\left(\frac{1 - 2\sin\theta\cos\theta}{\sin\theta\cos\theta}\right)$$

$$= \frac{(\cos\theta + \sin\theta)}{\sin\theta\cos\theta}\frac{(\cos^2\theta + \sin^2\theta - 2\cos\theta\sin\theta)}{\sin\theta\cos\theta}$$

$$= \frac{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)^2}{\sin^2\theta\cos^2\theta}$$

∴ LHS = RHS

Hence proved.

********** END *********