

Exercise 3D

Question 17:

$$2x + 3y - 7 = 0$$

 $(k - 1)x + (k + 2)y - 3k = 0$

These are of the form

$$a_1 \times + b_1 y + c_1 = 0$$
, $a_2 \times + b_2 y + c_2 = 0$
where, $a_1 = 2$, $b_1 = 3$, $c_1 = -7$
 $a_2 = (k-1)$, $b_2 = (k+2)$, $c_2 = -3k$

For infinitely many solutions, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

This hold only when

$$\frac{2}{k-1} = \frac{3}{k+2} = \frac{-7}{-3k}$$
$$\frac{2}{k-1} = \frac{3}{k+2} = \frac{7}{3k}$$

Now the following cases arises

Case: I

$$\frac{2}{k-1} = \frac{3}{k+2}$$

$$\Rightarrow 2(k+2) = 3(k-1) \Rightarrow 2k+4=3k-3$$

$$\Rightarrow k=7$$

Case: II

$$\frac{3}{k+2} = \frac{7}{3k}$$

$$\Rightarrow 7(k+2) = 9k \Rightarrow 7k + 14 = 9k$$

$$\Rightarrow k = 7$$

Case III

$$\frac{2}{k-1} = \frac{7}{3k}$$

$$\Rightarrow 7k - 7 = 6k$$

$$\Rightarrow k = 7$$

For k = 7, there are infinitely many solutions of the given system of equations.

Question 18:

$$2x + (k - 2)y - k = 0$$

 $6x + (2k - 1)y - (2k + 5) = 0$

These are of the form

$$a_1 \times + b_1 y + c_1 = 0$$
, $a_2 \times + b_2 y + c_2 = 0$
where $a_1 = 2$, $b_1 = (k - 2)$, $c_1 = -k$
 $a_2 = 6$, $b_2 = (2k - 1)$, $c_2 = -(2k + 5)$

For infinite number of solutions, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

This hold only when

$$\frac{2}{6} = \frac{k-2}{2k-1} = \frac{-k}{-(2k+5)}$$

$$\frac{1}{3} = \frac{k-2}{2k-1} = \frac{k}{2k+5}$$

$$\frac{1}{3} = \frac{k-2}{2k-1} [takingI and II]$$

$$\Rightarrow$$
 2k - 1 = 3k - 6 \Rightarrow k = 5

$$\frac{k-2}{2k-1} = \frac{k}{2k+5} \quad \text{[Taking II and III]}$$

$$(k-2)(2k+5) = k(2k-1)$$

$$\Rightarrow 2k^2 + 5k - 4k - 10 = 2k^2 - k$$

$$\Rightarrow k+k = 10 \Rightarrow 2k = 10$$

$$k = \frac{10}{2} = 5 \quad \text{[taking I and III]}$$

$$\frac{1}{3} = \frac{k}{2k+5}$$

$$2k+5 = 3k \Rightarrow 3k-2k=5$$

$$k=5$$

Thus, for k = 5 there are infinitely many solutions.

********** END *******