Algebra of Matrices Ex 5.3 Q62

Given,

$$A = \text{diag}(a, b, c)$$

Show that,

$$A^n = \text{diag}\left(a^n, b^n, c^n\right)$$

Step 1: Put $n = 1$

$$A^1 = \text{diag}\left(a^1, b^1, c^1\right)$$

$$A = \text{diag}(a, b, c)$$

So,

A^n is true for $n = 1$

Step 2: Let, A^n be true for $n = k$, so,

$$A^k = \text{diag}\left(a^k, b^k, c^k\right) \qquad \text{---(i)}$$

Step 3: Now, we have to show that,

$$A^{k+1} = \text{diag}\left(a^{k+1}, b^{k+1}, c^{k+1}\right)$$

Now,

$$A^{k+1} = A^k \times k3$$

$$= \text{diag}\left(a^k, b^k, c^k\right) \times \text{diag}(a, b, c) \qquad \{\text{using equation (i) and given}\}$$

$$A^{k+1} = \begin{bmatrix} a^k & 0 & 0 \\ 0 & b^k & 0 \\ 0 & 0 & c^k \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$= \begin{bmatrix} a^k \times a + 0 + 0 & 0 + 0 + 0 & 0 + 0 + 0 \\ 0 + 0 + 0 & 0 + b^k \times b + 0 & 0 + 0 + 0 \\ 0 + 0 + 0 & 0 + 0 + 0 & 0 + 0 + c^k \times c \end{bmatrix}$$

$$= \begin{bmatrix} a^{k+1} & 0 & 0 \\ 0 & b^{k+1} & 0 \\ 0 & 0 & c^{k+1} \end{bmatrix}$$

$$A^{k+1} = \text{diag}\left(a^{k+1}, b^{k+1}, c^{k+1}\right)$$

So, $P(n)$ is true for $n = k + 1$ whenever $P(n)$ is true for $n = k$.

Hence, by principle of mathematical induction A^n is true for all positive integer.

Algebra of Matrices Ex 5.3 Q64

Given,

order of matrix $X = (a+b) \times (a+2)$

order of matrix $Y = (b+1) \times (a+3)$

Given, $X_{(a+b)\times(a+2)} \cdot Y_{(b+1)\times(a+3)}$ exist.

$\Rightarrow \quad a+2 = b+1$

$\Rightarrow \quad a-b = -1 \qquad \text{---(i)}$

And

$Y_{(b+1)\times(a+3)} \cdot X_{(a+b)\times(a+2)}$ exists.

$\Rightarrow \quad a+3 = a+b$

$\Rightarrow \quad b = 3$

Put $b = 3$ in equation (i),

$a - b = -1$

$a - 3 = -1$

$a = 3 - 1$

$a = 2$

So, $a = 2, b = 3$

So,

$Order\ of\ X = (a+b) \times (a+2)$

$= (2+3) \times (2+2)$

$= 5 \times 4$

$Order\ of\ Y = (b+1) \times (a+3)$

$= (3+1) \times (2+3)$

$= 4 \times 5$

$Order\ of\ X_{5\times 4} \cdot Y_{4\times 5} = 5 \times 5$

$Order\ of\ X_{4\times 5} \cdot Y_{5\times 4} = 4 \times 4$

So, *order* of XY and YX are not same and they are not equal but both are square matrices.

*********** END **********