



# Complex Numbers Ex 13.2 Q15

$$\begin{aligned}
 a &= \cos \theta + i \sin \theta \\
 \frac{1+a}{1-a} &= \frac{1+\cos \theta + i \sin \theta}{1-\cos \theta - i \sin \theta} \\
 &= \frac{(1+\cos \theta + i \sin \theta)(1-\cos \theta + i \sin \theta)}{(1-\cos \theta - i \sin \theta)(1-\cos \theta + i \sin \theta)} \quad [\text{Rationalizing the denominator}] \\
 &= \frac{(1+\cos \theta + i \sin \theta)(1-\cos \theta + i \sin \theta)}{(1-\cos \theta)^2 - (i \sin \theta)^2} \\
 &= \frac{(1+i \sin \theta)^2 - \cos^2 \theta}{1-2 \cos \theta + \cos^2 \theta + \sin^2 \theta} \\
 &= \frac{1+2i \sin \theta - \sin^2 \theta - \cos^2 \theta}{1-2 \cos \theta + \cos^2 \theta + \sin^2 \theta} \\
 &= \frac{1+2i \sin \theta - 1}{1-2 \cos \theta + \cos^2 \theta + \sin^2 \theta} \quad [\because \cos^2 \theta + \sin^2 \theta = 1] \\
 &= \frac{2i \sin \theta}{1-2 \cos \theta + 1} \\
 &= \frac{2i \sin \theta}{2-2 \cos \theta} \\
 &= \frac{i \sin \theta}{1-\cos \theta} \\
 &= \frac{i 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \sin^2 \frac{\theta}{2}} \\
 &= \frac{i \cos \frac{\theta}{2}}{\sin \frac{\theta}{2}} = i \cot \frac{\theta}{2}
 \end{aligned}$$

# Complex Numbers Ex 13.2 Q16(i)

We have,

$$x = \frac{3-5i}{2}$$

$$\begin{aligned}
 \Rightarrow 2x &= 3-5i \\
 \Rightarrow 2x-3 &= -5i \\
 \Rightarrow (2x-3)^2 &= (-5i)^2 \\
 \Rightarrow 4x^2+9-12x &= -25 \\
 \Rightarrow 4x^2-12x+34 &= 0 \\
 \Rightarrow 2(2x^2-6x+17) &= 0 \\
 \Rightarrow 2x^2-6x+17 &= 0 \quad \dots\dots\dots(i)
 \end{aligned}$$

$$\begin{aligned}
 \therefore 2x^3+2x^2-7x+72 &= x(2x^2-6x+17)+6x^2-17x+2x^2-7x+72 \quad \{\text{adding and subtracting } 6x^2 \text{ and } 17x\} \\
 &= x \times 0 + 8x^2-24x+72 \quad \{\text{using (i)}\} \\
 &= 4(2x^2-6x+17)+4 \\
 &= 4 \times 0 + 4 \quad \{\text{using (i)}\} \\
 &= 4
 \end{aligned}$$

# Complex Numbers Ex 13.2 Q16(ii)

We have,

$$x = 3 + 2i$$

$$\Rightarrow x - 3 = 2i$$

$$\Rightarrow (x - 3)^2 = (2i)^2$$

$$\Rightarrow x^2 + 3^2 - 2 \times 3 \times x = -4$$

$$\Rightarrow x^2 + 9 - 6x + 4 = 0$$

$$\Rightarrow x^2 - 6x + 13 = 0 \quad \dots\dots\dots(i)$$

Now,

$$x^4 - 4x^3 + 4x^2 + 8x + 44$$

$$= x^2(x^2 - 6x + 13) + 6x^2 - 13x^2 - 4x^3 + 4x^2 + 8x + 44 \quad \text{(adding and subtracting } 6x^3 \text{ and } 13x^2 \text{)}$$

$$= x^2 \times 0 + 2x^3 - 9x^2 + 8x + 44 \quad \text{(using (i))}$$

$$= 2x(x^2 - 6x + 13) + 12x^2 - 26x - 9x^2 + 8x + 44 \quad \text{(adding and subtracting } 12x^2 \text{ and } 26x \text{)}$$

$$= 2x \times 0 + 3x^2 - 18x + 44 \quad \text{(using (i))}$$

$$= 3(x^2 - 6x + 13) + 5$$

$$= 3 \times 0 + 5 \quad \text{(using (i))}$$

$$= 5$$

### Complex Numbers Ex 13.2 Q16(iii)

We have,

$$x = -1 + i\sqrt{2}$$

$$\Rightarrow x + 1 = i\sqrt{2}$$

$$\Rightarrow (x + 1)^2 = (i\sqrt{2})^2 \quad \text{(squaring both sides)}$$

$$\Rightarrow x^2 + 1 + 2x = -2$$

$$\Rightarrow x^2 + 2x + 3 = 0 \dots\dots\dots(ii)$$

Now,

$$x^4 + 4x^3 + 6x^2 + 4x + 9$$

$$= x^2(x^2 + 2x + 3) + 2x^3 + 3x^2 + 4x + 9$$

$$= x^2 \times 0 + 2x(x^2 + 2x + 3) - x^2 - 2x + 9 \quad \text{(using (ii))}$$

$$= 2x \times 0 - (x^2 + 2x + 3) + 3 + 9 \quad \text{(using (ii) and adding and subtracting 3)}$$

$$= -0 + 3 + 9 \quad \text{(using (ii))}$$

$$= 12$$

\*\*\*\*\* END \*\*\*\*\*