



Question 4:

If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A+2B)'$

Answer

We know that $A = (A')'$

$$\therefore A = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$$

$$\therefore A+2B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + 2 \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$$

$$\therefore (A+2B)' = \begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$$

Question 5:

For the matrices A and B , verify that $(AB)' = B'A'$ where

$$(i) \quad A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$$

$$(ii) \quad A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$$

Answer

(i)

$$AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} \begin{bmatrix} -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$$

$$\therefore (AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

$$\text{Now, } A' = \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}, B' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

$$\therefore B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

Hence, we have verified that $(AB)' = B'A'$.

(ii)

$$AB = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}$$

$$\therefore (AB)' = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

$$\text{Now, } A' = \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}, B' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$$

$$\therefore B'A' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

Hence, we have verified that $(AB)' = B'A'$.

Question 6:

If (i) $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$, then verify that $A'A = I$

(ii) $A = \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix}$, then verify that $A'A = I$

Answer

(i)

$$A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$$

$$\therefore A' = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

$$\begin{aligned} A'A &= \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \\ &= \begin{bmatrix} (\cos \alpha)(\cos \alpha) + (-\sin \alpha)(-\sin \alpha) & (\cos \alpha)(\sin \alpha) + (-\sin \alpha)(\cos \alpha) \\ (\sin \alpha)(\cos \alpha) + (\cos \alpha)(-\sin \alpha) & (\sin \alpha)(\sin \alpha) + (\cos \alpha)(\cos \alpha) \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & \sin \alpha \cos \alpha - \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha - \sin \alpha \cos \alpha & \sin^2 \alpha + \cos^2 \alpha \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \end{aligned}$$

Hence, we have verified that $A'A = I$.

(ii)

$$A = \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix}$$

$$\therefore A' = \begin{bmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix}$$

$$\begin{aligned} A'A &= \begin{bmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix} \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix} \\ &= \begin{bmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix} \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix} \\ &= \begin{bmatrix} (\sin \alpha)(\sin \alpha) + (-\cos \alpha)(-\cos \alpha) & (\sin \alpha)(\cos \alpha) + (-\cos \alpha)(\sin \alpha) \\ (\cos \alpha)(\sin \alpha) + (\sin \alpha)(-\cos \alpha) & (\cos \alpha)(\cos \alpha) + (\sin \alpha)(\sin \alpha) \end{bmatrix} \\ &= \begin{bmatrix} \sin^2 \alpha + \cos^2 \alpha & \sin \alpha \cos \alpha - \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha - \sin \alpha \cos \alpha & \cos^2 \alpha + \sin^2 \alpha \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \end{aligned}$$

Hence, we have verified that $A'A = I$.

Question 7:

(i) Show that the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a symmetric matrix

(ii) Show that the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is a skew symmetric matrix

Answer

(i) We have:

$$A' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} = A$$

$\therefore A' = A$

Hence, A is a symmetric matrix.

(ii) We have:

$$A' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = -A$$

$$\therefore A' = -A$$

Hence, A is a skew-symmetric matrix.

Question 8:

For the matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify that

(i) $(A + A')$ is a symmetric matrix

(ii) $(A - A')$ is a skew symmetric matrix

Answer

$$A' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$$

$$(i) \quad A + A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$$

$$\therefore (A + A')' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = A + A'$$

Hence, $(A + A')$ is a symmetric matrix.

$$(ii) \quad A - A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$(A - A')' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = -(A - A')$$

Hence, $(A - A')$ is a skew-symmetric matrix.

Question 9:

Find $\frac{1}{2}(A + A')$ and $\frac{1}{2}(A - A')$, when $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

Answer

$$A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

The given matrix is

$$\therefore A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

$$A + A' = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} + \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \frac{1}{2}(A + A') = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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