Complex Numbers Ex 13.4 Q1(iii)

Modulus, $|1-i| = \sqrt{1^2 + 1^2} = \sqrt{2}$

Argument, $\arg(1-i) = \tan^{-1}\left(\dfrac{-1}{1}\right) = \tan^{-1}(-1) = -\dfrac{\pi}{4}$

Polar form, $\sqrt{2}\left(\cos\dfrac{\pi}{4} - i\sin\dfrac{\pi}{4}\right)$

Complex Numbers Ex 13.4 Q1(iv)

$\dfrac{1-i}{1+i} = \dfrac{(1-i)(1-i)}{(1+i)(1-i)} = \dfrac{(1-i)^2}{1^2 - i^2} = \dfrac{1-2i-1}{1+1} = \dfrac{-2i}{2} = -i$

Modulus, $\left|\dfrac{1-i}{1+i}\right| = |-i| = 1$

Argument, $\tan^{-1}\left(\dfrac{-1}{0}\right) = -\dfrac{\pi}{2}$

Polar Form, $z = r(\cos\theta + i\sin\theta)$

$z = \left(\cos\dfrac{\pi}{2} - i\sin\dfrac{\pi}{2}\right)$

Complex Numbers Ex 13.4 Q1(v)

Modulus, $\left|\dfrac{1}{1+i}\right|$

$= \left|\dfrac{1(1-i)}{(1+i)(1-i)}\right|$ [Rationalizing the denominator]

$= \left|\dfrac{1-i}{1^2 - i^2}\right| = \left|\dfrac{1-i}{2}\right| = \sqrt{\left(\dfrac{1}{2}\right)^2 + \left(-\dfrac{1}{2}\right)^2} = \sqrt{\dfrac{1}{2}} = \dfrac{1}{\sqrt{2}}$

Argument, $\tan^{-1}(-1) = -\dfrac{\pi}{4}$

Polar Form $= \cos\left(\dfrac{\pi}{4}\right) - i\sin\left(\dfrac{\pi}{4}\right)$

Complex Numbers Ex 13.4 Q1(vi)

The polar form of a complex number $z = x + iy$, is given by $z = |z|(\cos\theta + i\sin\theta)$ where,

$|z| = \sqrt{x^2 + y^2}$ and

$\arg(z) = \theta = \tan^{-1}\left(\dfrac{b}{a}\right)$

let $z = \dfrac{1+2i}{1-3i}$

$\quad = \dfrac{1+2i}{1-3i} \times \dfrac{1+3i}{1+3i}$

$\quad = \dfrac{1(1+3i) + 2i(1+3i)}{1^2 + 3^2}$

$\quad = \dfrac{1+3i+2i-6}{1+9}$

$\quad = \dfrac{-5+5i}{10}$

$\quad = \dfrac{-5}{10} + \dfrac{5}{10}i$

$\quad = \dfrac{-1}{2} + \dfrac{1}{2}i$

$\therefore |z| = \sqrt{\left(\dfrac{-1}{2}\right)^2 + \left(\dfrac{1}{2}\right)^2}$

$\quad = \sqrt{\dfrac{1}{4} + \dfrac{1}{4}}$

$\quad = \sqrt{\dfrac{2}{4}}$

$\quad = \dfrac{1}{\sqrt{2}}$

Here $x = \dfrac{-1}{2} < 0$ & $y = \dfrac{1}{2} > 0, \therefore \theta$ lies in quadrant II

$\theta = \arg(z) = \tan^{-1}\dfrac{\frac{1}{2}}{-\frac{1}{2}}$

$\quad = \tan^{-1}(-1)$

$\quad = \tan^{-1}\left(-\tan\dfrac{\pi}{4}\right)$

$\quad = \tan^{-1}\left(\tan\left(\pi - \dfrac{\pi}{4}\right)\right) \qquad (\because \tan(\pi - \theta) = -\tan\theta)$

$\quad = \pi - \dfrac{\pi}{4}$

$\quad = \dfrac{3\pi}{4}$

The polar form is given by $z = \dfrac{1}{\sqrt{2}}\left(\cos\dfrac{3\pi}{4} + i\sin\dfrac{3\pi}{4}\right)$

*********** END **********