Indefinite Integrals Ex 19.30 Q11

$$\text{Let } \int \frac{\sin 2x}{(1+\sin x)(2+\sin x)}dx = \frac{A}{1+\sin x} + \frac{B}{2+\sin x}$$

$$\Rightarrow \qquad \sin 2x = A(2+\sin x) + B(1+\sin x)$$

$$\Rightarrow \qquad 2\sin x\cos x = (2A+B) + (A+B)\sin x$$

Equating similar terms, we get,

$$2A + B = 0 \qquad \Rightarrow \qquad B = -2A \text{ and}$$

$$A + B = 2\cos x \Rightarrow \qquad -A = 2\cos x$$

$$\Rightarrow \qquad A = -2\cos x$$

$$\text{and } B = +4\cos x$$

Thus,

$$I = \int -\frac{2\cos x}{1+\sin x}dx + \int \frac{4\cos x}{2+\sin x}dx$$

$$= -2\log|1+\sin x| + 4\log|2+\sin x| + c$$

$$I = \log\left|\frac{(2+\sin x)^4}{(1+\sin x)^2}\right| + c$$

Indefinite Integrals Ex 19.30 Q12

Let $\int \frac{2x}{\left(x^2+1\right)\left(x^2+3\right)}dx = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+3}$

$$\Rightarrow \quad 2x = (Ax+B)\left(x^2+3\right) + (Cx+D)\left(x^2+1\right)$$
$$= (A+C)x^3 + (B+D)x^2 + (3A+C)x + (3B+D)$$

Equating similar terms, we get,

$A+C=0,\ B+D=0,\ 3A+C=2$ and $3B+D=0$

$\Rightarrow \quad A=-C,\ B=D=0\ 2A=2 \quad \Rightarrow \quad A=1\ \&\ C=-1$

Thus,

$$I = \int \frac{xdx}{x^2+1} - \int \frac{xdx}{x^2+3}$$
$$= \frac{1}{2}\log\left|x^2+1\right| - \frac{1}{2}\log\left|x^2+3\right| + c$$

$$I = \frac{1}{2}\log\left|\frac{x^2+1}{x^2+3}\right| + c$$

Indefinite Integrals Ex 19.30 Q13

Let $\int \frac{1}{x\log x\left(2+\log x\right)} = \frac{A}{x\log x} + \frac{B}{x\left(2+\log x\right)}$

$\Rightarrow \quad 1 = A\left(2+\log x\right) + B\log x$

Put $x=1$

$\Rightarrow \quad 1 = 2A \Rightarrow \quad A = \frac{1}{2}$

Put $x = 10^{-2}$

$\Rightarrow \quad 1 = -2B \quad \Rightarrow \quad B = -\frac{1}{2}$

Thus,

$$I = \frac{1}{2}\int \frac{dx}{x\log x} + \left(-\frac{1}{2}\right)\int \frac{dx}{x\left(2+\log x\right)}$$
$$= \frac{1}{2}\log\left|\log x\right| - \frac{1}{2}\log\left|2+\log x\right| + c$$

$$I = \frac{1}{2}\log\left|\frac{\log x}{2+\log x}\right| + c$$

Indefinite Integrals Ex 19.30 Q15

Let $\frac{ax^2+bx+c}{(x-a)(x-b)(x-c)} = \frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{x-c}$

$\Rightarrow \quad ax^2+bx+c = A(x-b)(x-c)+B(x-a)(x-c)+C(x-a)(x-b)$

Put $x = a$

$\Rightarrow \quad a^3+ba+c = (a-b)(a-c)A \Rightarrow \qquad A = \frac{a^3+ba+c}{(a-b)(a-c)}$

Put $x = b$

$\Rightarrow \quad ab^2+b^2+c = (b-a)(b-c)B \Rightarrow \qquad B = \frac{ab^2+b^2+c}{(b-a)(b-c)}$

Put $x = c$

$\Rightarrow \quad ac^2+bc+c = (c-a)(c-b)C \Rightarrow \qquad C = \frac{ac^2+bc+c}{(c-a)(c-b)}$

Thus,

$$I = \frac{a^3+ba+c}{(a-b)(a-c)}\int\frac{dx}{x-a}+\frac{ab^2+b^2+c}{(b-a)(b-c)}\int\frac{dx}{x-b}+\frac{ac^2+bc+c}{(c-a)(c-b)}\int\frac{dx}{x-c}$$

Hence,

$$I = \frac{a^3+ba+c}{(a-b)(a-c)}\log|x-a|+\frac{ab^2+b^2+c}{(b-a)(b-c)}\log|x-b|+\frac{ac^2+bc+c}{(c-a)(c-b)}\log|x-c|+c$$

Indefinite Integrals Ex 19.30 Q16

Consider the integral

$$I=\int \frac{x}{\left(x^2+1\right)\left(x-1\right)}dx$$

Now let us separate the fraction $\frac{x}{\left(x^2+1\right)\left(x-1\right)}$

through partial fractions.

$$\frac{x}{\left(x^2+1\right)\left(x-1\right)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+1}$$

$$\Rightarrow \frac{x}{\left(x^2+1\right)\left(x-1\right)}=\frac{A\left(x^2+1\right)+\left(Bx+C\right)\left(x-1\right)}{\left(x^2+1\right)\left(x-1\right)}$$

$$\Rightarrow x = A\left(x^2+1\right)+\left(Bx+C\right)\left(x-1\right)$$

$$\Rightarrow x = Ax^2+A+Bx^2-Bx+Cx-C$$

Comparing the coefficients, we have,

A+B=0, $-B+C=1$ and A $-C$=0

Solving the equations, we get,

$$\Rightarrow A=\frac{1}{2}, B=-\frac{1}{2} \text{ and } C=\frac{1}{2}$$

$$\Rightarrow \frac{x}{\left(x^2+1\right)\left(x-1\right)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+1}$$

$$\Rightarrow \frac{x}{\left(x^2+1\right)\left(x-1\right)}=\frac{1}{2}\times\frac{1}{x-1}-\frac{1}{2}\times\frac{x-1}{x^2+1}$$

$$\Rightarrow \frac{x}{\left(x^2+1\right)\left(x-1\right)}=\frac{1}{2\left(x-1\right)}-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2\left(x^2+1\right)}$$

Thus, we have,

$$I=\int \frac{x}{\left(x^2+1\right)\left(x-1\right)}dx$$

$$=\int\left[\frac{1}{2\left(x-1\right)}-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2\left(x^2+1\right)}\right]dx$$

$$=\int\frac{dx}{2\left(x-1\right)}-\int\frac{xdx}{2\left(x^2+1\right)}+\int\frac{dx}{2\left(x^2+1\right)}$$

$$=\frac{1}{2}\int\frac{dx}{\left(x-1\right)}-\frac{1}{2}\int\frac{xdx}{\left(x^2+1\right)}+\frac{1}{2}\int\frac{dx}{\left(x^2+1\right)}$$

$$=\frac{1}{2}\int\frac{dx}{\left(x-1\right)}-\frac{1}{2}\times\frac{1}{2}\int\frac{2xdx}{\left(x^2+1\right)}+\frac{1}{2}\int\frac{dx}{\left(x^2+1\right)}$$

$$=\frac{1}{2}\log\left|x-1\right|-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\tan^{-1}x+C$$

*********** END **********