Indefinite Integrals Ex 19.30 Q42

Let $\dfrac{x^2}{\left(x^2+1\right)\left(3x^2+4\right)} = \dfrac{Ax+B}{\left(x^2+1\right)} + \dfrac{Cx+D}{\left(3x^2+4\right)}$

$$\Rightarrow \quad x^2 = \left(Ax+B\right)\left(3x^2+4\right) + \left(Cx+D\right)\left(x^2+1\right)$$
$$= \left(3A+C\right)x^3 + \left(3B+D\right)x^2 + \left(4A+C\right)x + 4B+D$$

Equating similar terms, we get,

$3A+C=0,\ 3B+D=1,\ 4A+C=0,\ 4B+D=0$

Solving, we get, $A=0,\ B=-1,\ C=0,\ D=4$

Thus,

$$I = \int \frac{-dx}{\left(x^2+1\right)} + \int \frac{4dx}{\left(3x^2+4\right)}$$

$$= -\tan^{-1}x + \frac{4}{3}\int \frac{dx}{x^2+\left(\frac{2}{\sqrt{3}}\right)^2}$$

$$= -\tan^{-1}x + \frac{4}{3}\cdot\frac{\sqrt{3}}{2}\tan^{-1}\left(\frac{\sqrt{3}x}{2}\right) + c$$

$$I = \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}x}{2}\right) - \tan^{-1}x + c$$

Indefinite Integrals Ex 19.30 Q43

To evaluate the integral follow the steps:

$$\int \frac{3x+5}{x^3-x^2-x+1}dx$$

Let $\dfrac{3x+5}{(x-1)^2(x+1)} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x+1}$

$$3x+5 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$$

For $x=1 \quad B=4$

For $x=-1\ C=\dfrac{1}{2}$

For $x=0 \quad A=-\dfrac{1}{2}$,

Therefore

$$\int \frac{3x+5}{(x-1)^2(x+1)}dx = -\frac{1}{2}\int \frac{dx}{x-1} + 4\int \frac{dx}{(x-1)^2} + \frac{1}{2}\int \frac{dx}{x+1}$$

$$= -\frac{1}{2}\ln|(x-1)| - \frac{4}{(x-1)} + \frac{1}{2}\ln|(x+1)| + c$$

$$= \frac{1}{2}\ln\left|\frac{x+1}{x-1}\right| - \frac{4}{(x-1)} + c$$

Indefinite Integrals Ex 19.30 Q44

Let $I = \int \frac{x^3 - 1}{x^3 + x} dx$

$$= \int 1 - \frac{(x+1)}{x^3 + x} dx$$

$$= \int dx - \int \frac{x+1}{x^3 + x} dx$$

Let $\frac{x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 1}$

$$\Rightarrow \quad x + 1 = A(x^2 + 1) + (Bx + C)(x)$$

$$= (A + B)x^2 + (B + C)x + A$$

Equating similar terms, we get,

$A + B = 0,\ C = 1,\ A = 1$

Solving, we get, $A = 1,\ B = -1,\ C = 1$

Thus,

$$I = -\int \frac{dx}{x} - \int \frac{-x+1}{x^2+1} dx + \int dx$$

$$= -\log|x| + \int \frac{x dx}{x^2 + 1} - \int \frac{dx}{x^2 + 1} + \int dx$$

$$\therefore \quad I = x - \log|x| + \frac{1}{2}\log|x^2 + 1| - \tan^{-1} x + c$$

$$\therefore \quad I = x - \log|x| + \frac{1}{2}\log|x^2 + 1| - \tan^{-1} x + c$$

Indefinite Integrals Ex 19.30 Q45

To evaluate the integral follow the steps:

$$\int \frac{x^2 + x + 1}{(x+1)^2 (x+2)} dx$$

Let $\frac{x^2 + x + 1}{(x+1)^2 (x+2)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+2}$

$x^2 + x + 1 = A(x+1)(x+2) + B(x+2) + C(x+1)^2$

For $x = -1$ $B = 1$

For $x = -2$ $C = 3$

For $x = 0$ $A = -2$,

Therefore

$$\int \frac{x^2 + x + 1}{(x+1)^2 (x+2)} dx = -2\int \frac{dx}{x+1} + \int \frac{dx}{(x+1)^2} + 3\int \frac{dx}{x+2}$$

$$= -2\ln|x+1| - \frac{1}{x+1} + 3\ln|x+2| + c$$

Indefinite Integrals Ex 19.30 Q46

Let $\dfrac{1}{x\left(x^4+1\right)} = \dfrac{A}{x} + \dfrac{Bx^3 + Cx^2 + Dx + E}{x^4+1}$

$$\Rightarrow \quad 1 = A\left(x^4+1\right) + \left(Bx^3 + Cx^2 + Dx + E\right)x$$
$$= \left(A+B\right)x^4 + Cx^3 + Dx^2 + Ex + A$$

Equating similar terms, we get,

$A + B = 0,\ C = 0,\ D = 0,\ E = 0,\ A = 1$

$\therefore \quad B = -1$

Thus,

$$I = \int \frac{dx}{x} + \int -\frac{x^3 dx}{x^4+1}$$

$$= \log\left|x\right| - \frac{1}{4}\log\left|x^4+1\right| + c$$

$$I = \frac{1}{4}\log\left|\frac{x^4}{x^4+1}\right| + c$$

*********** END **********