Indefinite Integrals Ex 19.14 Q6

Let $I = \int \frac{1}{\sqrt{a^2 + b^2x^2}} dx$

Let $bx = t$

$$\Rightarrow \quad bdx = dt$$

$$dx = \frac{dt}{b}$$

$$I = \frac{1}{b} \int \frac{1}{\sqrt{a^2 + t^2}} dt$$

$$I = \frac{1}{b} \log\left|t + \sqrt{a^2 + t^2}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 + x^2}} dx = \log\left|x + \sqrt{a^2 + x^2}\right| + c\right]$$

$$I = \frac{1}{b} \log\left|bx + \sqrt{a^2 + b^2x^2}\right| + c \qquad [\text{since } t = bx]$$

Indefinite Integrals Ex 19.14 Q7

Let $I = \int \frac{1}{\sqrt{a^2 - b^2x^2}} dx$

Let $bx = t$

$$\Rightarrow \quad bdx = dt$$

$$dx = \frac{dt}{b}$$

so, $$I = \frac{1}{b} \int \frac{1}{\sqrt{a^2 - t^2}} dt$$

$$I = \frac{1}{b} \sin^{-1}\left(\frac{t}{a}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \frac{1}{b} \sin^{-1}\left(\frac{bx}{a}\right) + c \qquad [\text{since } bx = t]$$

Indefinite Integrals Ex 19.14 Q8

Let $I = \int \frac{1}{\sqrt{(2 - x)^2 + 1}} dx$

Let $2 - x = t$

$$\Rightarrow \quad -dx = dt$$

$$dx = -dt$$

so, $$I = -\int \frac{1}{\sqrt{t^2 + (1)^2}} dt$$

$$I = -\log\left|t + \sqrt{t^2 + 1}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2 + a^2}} dx = \log\left|x + \sqrt{x^2 + a^2}\right| + c\right]$$

$$I = -\log\left|(2 - x) + \sqrt{(2 - x)^2 + 1}\right| + c \qquad [\text{since } t = (2 - x)]$$

Indefinite Integrals Ex 19.14 Q9

Let $I = \int \frac{1}{\sqrt{(2-x)^2 - 1}} dx$

Let $2 - x = t$

$\Rightarrow \quad -dx = dt$

$dx = -dt$

so, $I = -\int \frac{1}{\sqrt{t^2 - (1)^2}} dt$

$I = -\log\left|t + \sqrt{t^2 - (1)^2}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c\right]$

$I = -\log\left|(2-x) + \sqrt{(2-x)^2 - 1}\right| + c \qquad [\text{since } t = (2-x)]$

Indefinite Integrals Ex 19.14 Q10

Let $I = \int \frac{x^4 + 1}{x^2 + 1} dx$

$I = \int \frac{(x^2+1)^2 - 2x^2}{x^2 + 1} dx \qquad \left[a^2 + b^2 = (a+b)^2 - 2ab\right]$

$I = \int \frac{(x^2+1)^2}{x^2+1} dx - \int \frac{2x^2}{(x^2+1)} dx$

$I = \int (x^2+1) dx - \int \frac{2x^2 + 2 - 2}{(x^2+1)} dx$

$I = \int (x^2+1) dx - \int \frac{2(x^2+1)}{(x^2+1)} dx + 2\int \frac{1}{x^2+1} dx$

$I = \int (x^2+1) dx - \int 2dx + 2\int \frac{1}{x^2+1} dx$

$I = \frac{x^3}{3} + x - 2x + 2 \times \tan^{-1}(x) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2+1}} dx = \tan^{-1}(x) + c\right]$

$I = \frac{x^3}{3} - x + 2\tan^{-1}(x) + c$

*********** END **********