

CHAPTER 22

PHOTOMETRY

We see an object when light coming from the object enters our eyes and excites the sensation of vision. The brightness sensed by the eye depends on the amount of light energy entering into it and the wavelength distribution of this energy. In this chapter, we shall study the factors responsible for the sensation of brightness.

22.1 TOTAL RADIANT FLUX

The total energy of radiation emitted by a source per unit time is called its *total radiant flux*. This radiation contains components of various wavelengths extending even beyond the visible range. However, not all wavelengths have equal contribution in making up the total radiation. In calculating total radiant flux of a source, the total energy emitted per unit time in the whole range of wavelengths must be calculated.

The SI unit of total radiant flux of a source is *watt*.

22.2 LUMINOSITY OF RADIANT FLUX

The brightness produced by radiation depends on the wavelength of the radiation besides depending on the total radiant flux. For example, consider two 10 W sources of light, one emitting yellow light and the other red light. Though both emit equal energy per unit time, yellow will look brighter than the red. The *luminosity of radiant flux* measures the capacity to produce brightness sensation in eye. A relative comparison of luminosity of radiant flux of different wavelengths can be made by the curve in figure (22.1). The figure represents relative luminosity under normal light conditions for an average person. The scale on the vertical axis is chosen arbitrarily. We see that for normal light conditions, the luminosity is maximum for wavelength around 555 nm and falls off on both sides. Radiation is “visible” if its luminosity is not zero. As the luminosity falls off gradually, there are no sharp cut-offs of visible region.

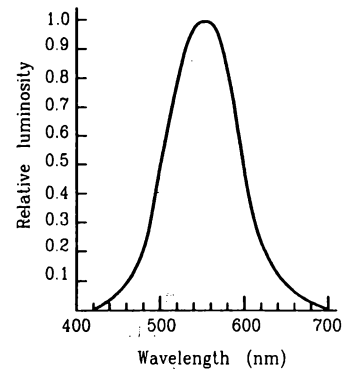


Figure 22.1

22.3 LUMINOUS FLUX : RELATIVE LUMINOSITY

In general, the radiation emitted by a source has components corresponding to a wide range of wavelengths. Different component wavelengths have different energies (in a given time) and different brightness producing capacities. The radiant flux is a quantity directly representing the total energy emitted per unit time. The *luminous flux* is a quantity directly representing the total brightness producing capacity of the source. Its unit is called *lumen*. The luminous flux of a source of 1/685 W emitting monochromatic light of wavelength 555 nm is called one lumen. In other words, a 1 W source emitting monochromatic light of wavelength 555 nm emits 685 lumen.

Relative luminosity of a wavelength refers to the fraction

$$\frac{\text{luminous flux of a source of given wavelength}}{\text{luminous flux of a 555 nm source of same power}}$$

It is often represented as a percentage. Thus, figure (22.1) represents the relative luminosity as a function of wavelength.

It should be clear that the luminous flux depends on the radiant flux as well as on the wavelength distribution.

Example 22.1

Find the luminous flux of a 10 W source of 600 nm. The relative luminosity at 600 nm is 0.6.

Solution : The luminous flux of a 1 W source of 555 nm = 685 lumen. Thus, the luminous flux of a 10 W source of 555 nm = 6850 lumen. The luminous flux of a 10 W source of 600 nm is, therefore, 0.6×6850 lumen = 4110 lumen.

For radiation having a range of wavelengths, the luminous flux gets contribution from each wavelength.

22.4 LUMINOUS EFFICIENCY

Total luminous flux per unit radiant flux is called *luminous efficiency*. Thus,

$$\text{Luminous efficiency} = \frac{\text{Total luminous flux}}{\text{Total radiant flux}} \quad \dots (22.1)$$

The luminous efficiency of a monochromatic source of 555 nm is 685 lumen/watt by definition. The luminous efficiency of a monochromatic source of any other wavelength is the relative luminosity of that wavelength multiplied by 685 lumen/watt.

An electric lamp glows when electric energy is given to it. However, not all the electric power given to it is converted into radiant flux. The term luminous efficiency is used in a slightly wider sense for such a light source. It is defined as the luminous flux divided by the power input to the source. Thus, it is the efficiency with which the power input to the source is used to produce brightness. We may call it *overall luminous efficiency*. Overall luminous efficiency

$$= \frac{\text{Luminous flux emitted}}{\text{Power input to the source}} \quad \dots (22.2)$$

A good fraction of power given to a filament lamp is used to heat the filament to a certain temperature at which it glows. Also, a good fraction of the emitted radiation has a wavelength where the relative luminosity is small or zero. The overall luminous efficiency of a filament lamp is rarely more than 50 lumen/watt.

22.5 LUMINOUS INTENSITY OR ILLUMINATING POWER

In the chapter on Gauss's law, we shall describe in detail what is a solid angle. In brief, the solid angle measures the angular divergence of a cone and is defined as

$$\omega = \frac{A}{R^2},$$

where A is the area intercepted by the cone on a sphere of radius R centred at the apex of the cone (figure 22.2).

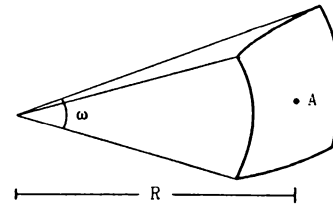


Figure 22.2

It is clear that the solid angle does not depend on the radius of the sphere. The SI unit of solid angle is called a *steradian* written in short as *sr*.

The *luminous intensity* of a source in a given direction is defined as

$$I = \frac{dF}{d\omega}, \quad \dots (22.3)$$

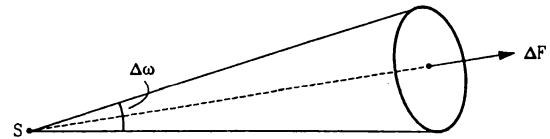


Figure 22.3

where dF is the luminous flux of the radiation emitted by the source in a small cone of solid angle $d\omega$ constructed around the given direction. The luminous intensity is also called just *intensity* in short. An ideal point source emits radiation uniformly in all directions. If the total luminous flux of the source is F , its intensity in any direction is

$$I = \frac{F}{(4\pi \text{ sr})}$$

as the total solid angle at a point is 4π sr. For an extended source, the intensity is different in different directions.

The SI unit of luminous intensity is lumen/steradian. This is called a *candela* written in short as "cd". Luminous intensity is also called *illuminating power*.

Candela is one of the seven base units of SI. It is defined precisely as the luminous intensity of a blackbody of surface area $\frac{1}{60} \text{ cm}^2$ placed at the freezing temperature of platinum at a pressure of 101,325 N/m² in the direction perpendicular to the surface.

22.6 ILLUMINANCE

When radiation strikes a surface, the surface gets illuminated. We define the *illuminance* of a small area as follows. If dF be the luminous flux of the radiation

striking a surface area dA , the illuminance of the area is defined as

$$E = \frac{dF}{dA} \quad \dots (22.4)$$

The illuminance is, therefore, the luminous flux incident per unit area.

It is the illuminance which is directly related to the brightness of an illuminated area. The SI unit of illuminance is lumen/m² and is called *lux*.

22.7 INVERSE SQUARE LAW

Consider a point source S and a small area ΔA around the point P at a distance r from the source (figure 22.4). Suppose, the angle between SP and the normal PN to the area is θ . Also suppose, the luminous intensity of the source in the direction SP is I .

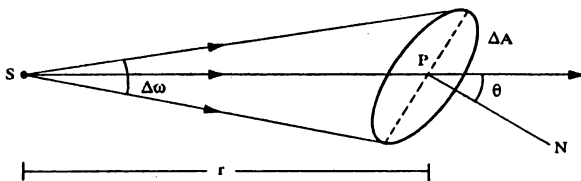


Figure 22.4

The solid angle subtended by the area ΔA at the source is

$$\Delta\omega = \frac{\Delta A \cos\theta}{r^2}$$

The luminous flux going through this solid angle is

$$\begin{aligned} \Delta F &= I \Delta\omega \\ &= I \frac{\Delta A \cos\theta}{r^2} \end{aligned}$$

The illuminance at ΔA is

$$E = \frac{\Delta F}{\Delta A} \quad \dots (22.5)$$

or,

$$E = \frac{I \cos\theta}{r^2}$$

We note that

(a) the illuminance of a small area is inversely proportional to the square of the distance of the area from the source and

(b) the illuminance of a small area is proportional to $\cos\theta$ where θ is the angle made by the normal to the area with the direction of incident radiation.

The first observation is known as the *inverse square law*.

22.8 LAMBERT'S COSINE LAW

An ideal point source emits radiation uniformly in all directions. In general, sources are extended and such a source has different luminous intensity in

different directions. If the source is in the form of a small plane surface, the radiation is emitted only in the forward half that is in a solid angle 2π around the forward normal. Even in this half, the intensity is different in different directions. The intensity is maximum along the normal to the surface and decreases as we consider directions away from this normal. For many surfaces, if the luminous intensity along the normal is I_0 , it is

$$I = I_0 \cos\theta \quad \dots (22.6)$$

in a direction making an angle θ with the normal. Equation (22.6) is called *Lambert's cosine law*. The surfaces which radiate according to the Lambert's cosine law are called *perfectly diffused*.

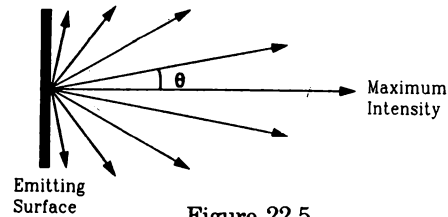


Figure 22.5

22.9 PHOTOMETERS

A photometer is used to compare the intensities of two point sources. The basic principle is as follows. Two screens are placed side by side. One screen is illuminated by the source S_1 only and the other screen by the source S_2 only. Light falls on the two screens at equal angles. The distances d_1 and d_2 of the sources from the screens are so adjusted that the two screens look equally bright. If I_1 and I_2 be the intensities of the sources, we must have for equal illuminance

$$\frac{I_1}{d_1^2} = \frac{I_2}{d_2^2}$$

or,

$$\frac{I_1}{I_2} = \frac{d_1^2}{d_2^2} \quad \dots (22.7)$$

A simple design proposed by Bunsen is now described (figure 22.6). It consists of an optical bench fitted with three vertical stands. The stands can slide along a straight rail on the bench.

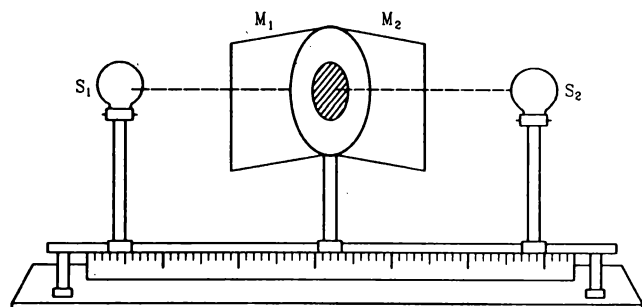


Figure 22.6

The distance between any two points on the rail may be read from a meter scale attached to the bench. The central stand contains a white paper with a grease spot. The other two stands carry the sources S_1 and S_2 to be compared. Two plane mirrors M_1 and M_2 are placed behind the central stand at proper inclination so that one side of the spot is imaged in one mirror and the other side of the spot is imaged in the other mirror. The two images can be seen simultaneously.

One of the sources is kept fixed at a distance from the spot and the position of the other is adjusted till the two spots seen in the mirrors appear equally bright. The distances d_1 and d_2 of the sources from the spot are measured in this condition. In this condition, the light falling on the spot from the two sources has equal intensity. If I_1 and I_2 be the intensity of the two sources, we have for equal illuminance,

$$\frac{I_1}{d_1^2} = \frac{I_2}{d_2^2} \quad \text{or,} \quad \frac{I_1}{I_2} = \frac{d_1^2}{d_2^2}$$

Worked Out Examples

1. A source emits 12.0 J of light of wavelength 620 nm and 8.0 J of light of wavelength 580 nm per second. The relative luminosity at 620 nm is 35% and that at 580 nm is 80%. Find (a) the total radiant flux, (b) the total luminous flux and (c) the luminous efficiency.

Solution :

(a) The total radiant flux = Total energy radiated per unit time = 12 J/s + 8 J/s = 20 J/s = 20 W.

(b) The luminous flux corresponding to the 12 W of 620 nm radiation is

$$0.35 \times (12 \text{ W}) \times 685 \text{ lumen/W} = 2877 \text{ lumen.}$$

Similarly, the luminous flux corresponding to the 8 W of 580 nm radiation is

$$0.80 \times (8 \text{ W}) \times 685 \text{ lumen/W} = 4384 \text{ lumen.}$$

The luminous flux of the source is 2877 lumen + 4384 lumen

$$= 7261 \text{ lumen} \approx 7260 \text{ lumen.}$$

(c) The luminous efficiency = $\frac{\text{Total luminous flux}}{\text{Total radiant flux}}$

$$= \frac{7260 \text{ lumen}}{20 \text{ W}} = 363 \text{ lumen/W.}$$

2. A circular area of radius 1.0 cm is placed at a distance of 2.0 m from a point source. The source emits light uniformly in all directions. The line joining the source to the centre of the area is normal to the area. It is found that 2.0×10^{-3} lumen of luminous flux is incident on the area. Calculate the total luminous flux emitted by the source and the luminous intensity of the source along the axis of the area.

Solution : The solid angle subtended by the area on the point source is

$$\Delta\omega = \frac{\pi(1.0 \text{ cm})^2}{(2.0 \text{ m})^2} = \frac{\pi}{4} \times 10^{-4} \text{ sr.}$$

Thus, 2.0×10^{-3} lumen of flux is emitted in $\frac{\pi}{4} \times 10^{-4}$ sr.

The total solid angle at the source is 4π . As the source

radiates uniformly in all directions, the total luminous flux is

$$F = \frac{4\pi}{\frac{\pi}{4} \times 10^{-4}} \times 2.0 \times 10^{-3} \text{ lumen} \\ = 320 \text{ lumen.}$$

The luminous intensity = $\Delta F / \Delta\omega$

$$= \frac{2.0 \times 10^{-3} \text{ lumen}}{\frac{\pi}{4} \times 10^{-4} \text{ sr}} = 25 \text{ cd.}$$

3. The overall luminous efficiency of a 100 W electric lamp is 25 lumen/W. Assume that light is emitted by the lamp only in the forward half, and is uniformly distributed in all directions in this half. Calculate the luminous flux falling on a plane object of area 1 cm² placed at a distance of 50 cm from the lamp and perpendicular to the line joining the lamp and the object.

Solution : The power input to the bulb = 100 W.

The luminous flux emitted by the bulb
= (25 lumen/W) \times 100 W
= 2500 lumen.

Since light is emitted only in the forward half and is distributed uniformly in this half, the luminous intensity is

$$I = \Delta F / \Delta\omega \\ = \frac{2500 \text{ lumen}}{2\pi \text{ sr}}$$

The solid angle subtended by the object on the lamp is

$$\Delta\omega = \frac{1 \text{ cm}^2}{(50 \text{ cm})^2} = \frac{1}{2500} \text{ sr.}$$

The luminous flux emitted in this solid angle is

$$\Delta F = I \Delta\omega \\ = \left(\frac{2500 \text{ lumen}}{2\pi \text{ sr}} \right) \left(\frac{1}{2500} \text{ sr} \right) \\ = \frac{1}{2\pi} \text{ lumen} = 0.16 \text{ lumen.}$$

4. A point source emitting uniformly in all directions is placed above a table-top at a distance of 0.50 m from it. The luminous flux of the source is 1570 lumen. Find the illuminance at a small surface area of the table-top (a) directly below the source and (b) at a distance of 0.80 m from the source.

Solution : Consider the situation shown in figure (22-W1). Let A be the point directly below the source S and B be the point at 0.80 m from the source.

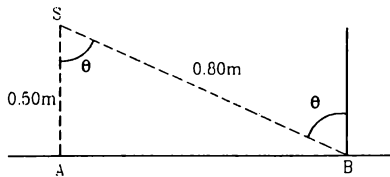


Figure 22-W1

The luminous flux of 1570 lumen is emitted uniformly in the solid angle 4π . The luminous intensity of the source in any direction is

$$I = \frac{1570 \text{ lumen}}{4\pi \text{ sr}} = 125 \text{ cd.}$$

The illuminance is

$$E = \frac{I \cos \theta}{r^2}.$$

At the point A, $r = 0.50 \text{ m}$ and $\theta = 0$. Thus,

$$E_A = \frac{125 \text{ cd}}{0.25 \text{ m}^2} = 500 \text{ lux.}$$

At the point B, $r = 0.80 \text{ m}$ and $\cos \theta = \frac{SA}{SB} = \frac{0.50}{0.80} = \frac{5}{8}$.

□

QUESTIONS FOR SHORT ANSWER

1. What is the luminous flux of a source emitting radio waves?
2. The luminous flux of a 1 W sodium vapour lamp is more than that of a 10 kW source of ultraviolet radiation. Comment.
3. Light is incident normally on a small plane surface. If the surface is rotated by an angle of 30° about the incident light, does the illuminance of the surface increase, decrease or remain same? Does your answer change if the light did not fall normally on the surface?
4. A bulb is hanging over a table. At which portion of the table is the illuminance maximum? If a plane mirror is

Thus,

$$E_B = \frac{(125 \text{ cd}) \times \frac{5}{8}}{0.64 \text{ m}^2} = 122 \text{ lux.}$$

5. The luminous intensity of a small plane source of light along the forward normal is 160 candela. Assuming the source to be perfectly diffused, find the luminous flux emitted into a cone of solid angle 0.02 sr around a line making an angle of 60° with the forward normal.

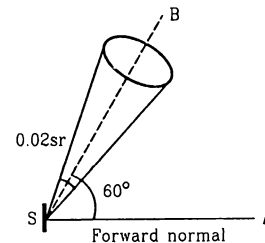


Figure 22-W2

Solution : The situation is shown in figure (22-W2). By Lambert's cosine law, the intensity in the direction SB is

$$I = I_0 \cos 60^\circ,$$

where $I_0 = 160 \text{ candela}$ is the intensity along the forward normal.

Thus,

$$I = (160 \text{ candela}) \left(\frac{1}{2} \right) = 80 \text{ candela.}$$

The luminous flux emitted in the cone shown in the figure is

$$\begin{aligned} \Delta F &= I \Delta \omega \\ &= (80 \text{ candela}) (0.02 \text{ sr}) \\ &= 1.6 \text{ lumen.} \end{aligned}$$

placed above the bulb facing the table, will the illuminance on the table increase?

5. The sun is less bright at morning and evening as compared to at noon although its distance from the observer is almost the same. Why?
6. Why is the luminous efficiency small for a filament bulb as compared to a mercury vapour lamp?
7. The yellow colour has a greater luminous efficiency as compared to the other colours. Can we increase the illuminating power of a white light source by putting a yellow plastic paper around this source?

OBJECTIVE I

- The one parameter that determines the brightness of a light source sensed by an eye is
 - energy of light entering the eye per second
 - wavelength of the light
 - total radiant flux entering the eye
 - total luminous flux entering the eye.
- Three light sources A , B and C emit equal amount of radiant energy per unit time. The wavelengths emitted by the three sources are 450 nm, 555 nm and 700 nm respectively. The brightness sensed by an eye for the sources are X_A , X_B and X_C respectively. Then,
 - $X_A > X_B$, $X_C > X_B$
 - $X_A > X_B$, $X_B > X_C$
 - $X_B > X_A$, $X_B > X_C$
 - $X_B > X_A$, $X_C > X_B$.
- As the wavelength is increased from violet to red, the luminosity
 - continuously increases
 - continuously decreases
 - increases then decreases
 - decreases then increases.
- An electric bulb is hanging over a table at a height of 1 m above it. The illuminance on the table directly below the bulb is 40 lux. The illuminance at a point on the table 1 m away from the first point will be about
 - 10 lux
 - 14 lux
 - 20 lux
 - 28 lux.
- Light from a point source falls on a screen. If the separation between the source and the screen is increased by 1%, the illuminance will decrease (nearly) by
 - 0.5%
 - 1%
 - 2%
 - 4%.
- A battery-operated torch is adjusted to send an almost parallel beam of light. It produces an illuminance of 40 lux when the light falls on a wall 2 m away. The illuminance produced when it falls on a wall 4 m away is close to
 - 40 lux
 - 20 lux
 - 10 lux
 - 5 lux.
- The intensity produced by a long cylindrical light source at a small distance r from the source is proportional to
 - $\frac{1}{r^2}$
 - $\frac{1}{r^3}$
 - $\frac{1}{r}$
 - none of these.
- A photographic plate placed at a distance of 5 cm from a weak point source is exposed for 3 s. If the plate is kept at a distance of 10 cm from the source, the time needed for the same exposure is
 - 3 s
 - 12 s
 - 24 s
 - 48 s.
- A photographic plate is placed directly in front of a small diffused source in the shape of a circular disc. It takes 12 s to get a good exposure. If the source is rotated by 60° about one of its diameters, the time needed to get the same exposure will be
 - 6 s
 - 12 s
 - 24 s
 - 48 s.
- A point source of light moves in a straight line parallel to a plane table. Consider a small portion of the table directly below the line of movement of the source. The illuminance at this portion varies with its distance r from the source as
 - $I \propto \frac{1}{r}$
 - $I \propto \frac{1}{r^2}$
 - $I \propto \frac{1}{r^3}$
 - $I \propto \frac{1}{r^4}$.
- Figure (22-Q1) shows a glowing mercury tube. The intensities at point A , B and C are related as
 - $B > C > A$
 - $A > C > B$
 - $B = C > A$
 - $B = C < A$.

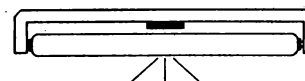


Figure 22-Q1

OBJECTIVE II

- The brightness producing capacity of a source
 - does not depend on its power
 - does not depend on the wavelength emitted
 - depends on its power
 - depends on the wavelength emitted.
- A room is illuminated by an extended source. The illuminance at a particular portion of a wall can be increased by
 - moving the source
 - rotating the source
 - bringing some mirrors in proper positions
 - changing the colour of the source.
- Mark the correct options.
 - The luminous efficiency of a monochromatic source is always greater than that of a white light source of same power.
 - The luminous efficiency of a monochromatic source of wavelength 555 nm is always greater than that of a white light source of same power.
 - The illuminating power of a monochromatic source of wavelength 555 nm is always greater than that of a white light source of same power.
 - The illuminating power of a monochromatic source is always greater than that of a white light source of same power.
- Mark the correct options.
 - Luminous flux and radiant flux have same dimensions.
 - Luminous flux and luminous intensity have same dimensions.
 - Radiant flux and power have same dimensions.
 - Relative luminosity is a dimensionless quantity.

EXERCISES

1. A source emits 45 joules of energy in 15 s. What is the radiant flux of the source?
2. A photographic plate records sufficiently intense lines when it is exposed for 12 s to a source of 10 W. How long should it be exposed to a 12 W source radiating the light of same colour to get equally intense lines?
3. Using figure (22.1), find the relative luminosity of wavelength (a) 480 nm, (b) 520 nm (c) 580 nm and (d) 600 nm.
4. The relative luminosity of wavelength 600 nm is 0.6. Find the radiant flux of 600 nm needed to produce the same brightness sensation as produced by 120 W of radiant flux at 555 nm.
5. The luminous flux of a monochromatic source of 1 W is 450 lumen/watt. Find the relative luminosity at the wavelength emitted.
6. A source emits light of wavelengths 555 nm and 600 nm. The radiant flux of the 555 nm part is 40 W and of the 600 nm part is 30 W. The relative luminosity at 600 nm is 0.6. Find (a) the total radiant flux, (b) the total luminous flux, (c) the luminous efficiency.
7. A light source emits monochromatic light of wavelength 555 nm. The source consumes 100 W of electric power and emits 35 W of radiant flux. Calculate the overall luminous efficiency.
8. A source emits 31.4 W of radiant flux distributed uniformly in all directions. The luminous efficiency is 60 lumen/watt. What is the luminous intensity of the source?
9. A point source emitting 628 lumen of luminous flux uniformly in all directions is placed at the origin. Calculate the illuminance on a small area placed at (1.0 m, 0, 0) in such a way that the normal to the area makes an angle of 37° with the X-axis.
10. The illuminance of a small area changes from 900 lumen/m^2 to 400 lumen/m^2 when it is shifted along its normal by 10 cm. Assuming that it is illuminated by a point source placed on the normal, find the distance between the source and the area in the original position.
11. A point source emitting light uniformly in all directions is placed 60 cm above a table-top. The illuminance at a point on the table-top, directly below the source, is 15 lux. Find the illuminance at a point on the table-top 80 cm away from the first point.
12. Light from a point source falls on a small area placed perpendicular to the incident light. If the area is rotated about the incident light by an angle of 60° , by what fraction will the illuminance change?
13. A student is studying a book placed near the edge of a circular table of radius R . A point source of light is suspended directly above the centre of the table. What should be the height of the source above the table so as to produce maximum illuminance at the position of the book?
14. Figure (22-E1) shows a small diffused plane source S placed over a horizontal table-top at a distance of 2.4 m with its plane parallel to the table-top. The illuminance at the point A directly below the source is 25 lux. Find the illuminance at a point B of the table at a distance of 1.8 m from A.

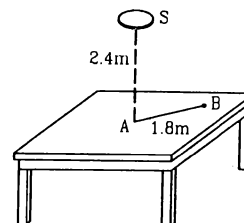


Figure 22-E1

15. An electric lamp and a candle produce equal illuminance at a photometer screen when they are placed at 80 cm and 20 cm from the screen respectively. The lamp is now covered with a thin paper which transmits 49% of the luminous flux. By what distance should the lamp be moved to balance the intensities at the screen again?
16. Two light sources of intensities 8 cd and 12 cd are placed on the same side of a photometer screen at a distance of 40 cm from it. Where should a 80 cd source be placed to balance the illuminance?

□

ANSWERS

OBJECTIVE I

1. (d) 2. (c) 3. (c) 4. (b) 5. (c) 6. (a)
7. (c) 8. (b) 9. (c) 10. (c) 11. (d)

OBJECTIVE II

1. (c), (d) 2. (a), (b), (c), (d) 3. (b), (c)
4. (b), (c), (d)

EXERCISES

1. 3 W
2. 10 s
3. 0.14 (b) 0.68 (c) 0.92 (d) 0.66
4. 200 W
5. 66%
6. (a) 70 W (b) 39730 lumen (c) 568 lumen/W

- 7. 240 lumen/W
- 8. 150 cd
- 9. 40 lux
- 10. 20 cm
- 11. 3.24 lux

- 12. it will not change
- 13. $R/\sqrt{2}$
- 14. 6.1 lux
- 15. 24 cm
- 16. 80 cm

□