



Indefinite Integrals Ex 19.24 Q4

$$\begin{aligned}\text{Let } I &= \int \frac{1}{p+q \tan x} dx \\ &= \int \frac{1}{p+q \left(\frac{\sin x}{\cos x} \right)} dx \\ &= \int \frac{\cos x}{p \cos x + q \sin x} dx\end{aligned}$$

$$\begin{aligned}\text{Let } \cos x &= \lambda \frac{d}{dx} (p \cos x + q \sin x) + \mu (p \cos x + q \sin x) + v \\ \cos x &= \lambda (-p \sin x + q \cos x) + \mu (p \cos x + q \sin x) + v \\ \cos x &= (-p\lambda + q\mu) \sin x + (q\lambda + p\mu) \cos x + v\end{aligned}$$

Comparing the coefficients of $\sin x, \cos x$ on the both the sides,

$$-p\lambda + q\mu = 0 \text{ ----- (1)}$$

$$q\lambda + p\mu = 1 \text{ ----- (2)}$$

$$v = 0 \text{ ----- (3)}$$

Solving equation (1), (2) and (3),

$$\lambda = \frac{q}{(p^2 + q^2)}$$

$$\mu = \frac{p}{(p^2 + q^2)}$$

$$v = 0$$

Now,

$$I = \int \frac{q}{(p^2 + q^2)} \frac{(-p \sin x + q \cos x)}{(p \cos x + q \sin x)} dx + \int \frac{p}{(p^2 + q^2)} \frac{(p \cos x + q \sin x)}{(p \cos x + q \sin x)} dx$$

$$I = \frac{q}{(p^2 + q^2)} (\log |p \cos x + q \sin x|) + \frac{p}{(p^2 + q^2)} x + c$$

Indefinite Integrals Ex 19.24 Q5

$$\text{Let } I = \int \frac{5 \cos x + 6}{2 \cos x + \sin x + 3} dx$$

$$\begin{aligned} \text{Let } (5 \cos x + 6) &= \lambda \frac{d}{dx} (2 \cos x + \sin x + 3) + \mu (2 \cos x + \sin x + 3) + v \\ (5 \cos x + 6) &= \lambda (-2 \sin x + \cos x) + \mu (2 \cos x + \sin x + 3) + v \\ (5 \cos x + 6) &= (-2\lambda + \mu) \sin x + (\lambda + 2\mu) \cos x + (3\mu + v) \end{aligned}$$

Comparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$$-2\lambda + \mu = 0 \text{ --- (1)}$$

$$\lambda + 2\mu = 5 \text{ --- (2)}$$

$$3\mu + v = 6 \text{ --- (3)}$$

Solving equation (1), (2) and (3),

$$\lambda = 1$$

$$\mu = 2$$

$$v = 0$$

Now,

$$I = \int \frac{(-2 \sin x + \cos x)}{(2 \cos x + \sin x + 3)} dx + 2 \int dx$$

$$I = \log|2 \cos x + \sin x + 3| + 2x + c$$

Indefinite Integrals Ex 19.24 Q6

$$\text{Let } I = \int \frac{2 \sin x + 3 \cos x}{3 \sin x + 4 \cos x} dx$$

$$\begin{aligned} \text{Let } (2 \sin x + 3 \cos x) &= \lambda \frac{d}{dx} (3 \sin x + 4 \cos x) + \mu (3 \sin x + 4 \cos x) + v \\ (2 \sin x + 3 \cos x) &= \lambda (3 \cos x - 4 \sin x) + \mu (3 \sin x + 4 \cos x) + v \\ (2 \sin x + 3 \cos x) &= (3\lambda + 4\mu) \cos x + (-4\lambda + 3\mu) \sin x + v \end{aligned}$$

Comparing the coefficients of $\sin x, \cos x$ on the both the sides,

$$3\lambda + 4\mu = 3 \text{ --- (1)}$$

$$-4\lambda + 3\mu = 2 \text{ --- (2)}$$

$$v = 0 \text{ --- (3)}$$

Solving the equation (1), (2) and (3),

$$\lambda = \frac{1}{25}$$

$$\mu = \frac{18}{25}$$

$$v = 0$$

$$I = \frac{1}{25} \int \frac{(3 \cos x - 4 \sin x)}{(3 \sin x + 4 \cos x)} dx + \frac{18}{25} \int dx$$

$$I = \frac{1}{25} \log|3 \sin x + 4 \cos x| + \frac{18}{25} x + c$$

Indefinite Integrals Ex 19.24 Q7

$$\begin{aligned}\text{Let } I &= \int \frac{1}{3+4\cot x} dx \\ &= \int \frac{\sin x}{3\sin x + 4\cos x} dx\end{aligned}$$

$$\begin{aligned}\text{Let } \sin x &= \lambda \frac{d}{dx}(3\sin x + 4\cos x) + \mu(3\sin x + 4\cos x) + v \\ \sin x &= \lambda(3\cos x - 4\sin x) + \mu(3\sin x + 4\cos x) + v \\ \sin x &= (3\lambda + 4\mu)\cos x + (-4\lambda + 3\mu)\sin x + v\end{aligned}$$

Comparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$$3\lambda + 4\mu = 0 \text{ ----- (1)}$$

$$-4\lambda + 3\mu = 1 \text{ ----- (2)}$$

$$v = 0 \text{ ----- (3)}$$

Solving the equation (1), (2) and (3), we get

$$\lambda = -\frac{4}{25}$$

$$\mu = \frac{3}{25}$$

$$v = 0$$

$$I = -\frac{4}{25} \int \frac{(3\cos x - 4\sin x)}{(3\sin x + 4\cos x)} dx + \frac{3}{25} \int dx$$

$$I = -\frac{4}{25} \log|3\sin x + 4\cos x| + \frac{3}{25}x + c$$

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