Q11 : Prove that the function *f* given by $f(x) = x^2 - x + 1$ is neither strictly increasing nor strictly decreasing on (-1, 1).

Answer :

The given function is $f(x) = x^2 - x + 1$.

$\therefore f'(x) = 2x - 1$

$\text{Now, } f'(x) = 0 \Rightarrow x = \frac{1}{2}.$

The point $\frac{1}{2}$ divides the interval (- 1, 1) into two disjoint intervals i.e., $\left(-1,\ \frac{1}{2}\right)$ and $\left(\frac{1}{2},\ 1\right)$.

Now, in interval $\left(-1,\ \frac{1}{2}\right), f'(x) = 2x - 1 < 0.$

Therefore, *f* is strictly decreasing in interval $\left(-1,\ \frac{1}{2}\right)$.

However, in interval $\left(\frac{1}{2},\ 1\right), f'(x) = 2x - 1 > 0.$

Therefore, *f* is strictly increasing in interval $\left(\frac{1}{2},\ 1\right)$.

Hence, *f* is neither strictly increasing nor decreasing in interval (- 1, 1).

Answer needs Correction? **Click Here**

Q12 : Which of the following functions are strictly decreasing on $\left(0, \frac{\pi}{2}\right)$?

(A) cos *x* (B) cos 2*x* (C) cos 3*x* (D) tan *x*

Answer :

(A) Let $f_1(x) = \cos x.$

$\therefore f_1'(x) = -\sin x$

In interval $\left(0, \frac{\pi}{2}\right), f_1'(x) = -\sin x < 0.$

$\therefore f_1(x) = \cos x$ is strictly decreasing in interval $\left(0,\ \frac{\pi}{2}\right)$.

(B) Let $f_2(x) = \cos 2x.$

$\therefore f_2'(x) = -2\sin 2x$

$\text{Now, } 0 < x < \frac{\pi}{2} \Rightarrow 0 < 2x < \pi \Rightarrow \sin 2x > 0 \Rightarrow -2\sin 2x < 0$

$\therefore f_2'(x) = -2\sin 2x < 0 \text{ on } \left(0, \frac{\pi}{2}\right)$

$\therefore f_2(x) = \cos 2x$ is strictly decreasing in interval $\left(0,\ \frac{\pi}{2}\right)$.

(C) Let $f_3(x) = \cos 3x.$

$\therefore f_3'(x) = -3\sin 3x$

$\text{Now, } f_3'(x) = 0.$

$\Rightarrow \sin 3x = 0 \Rightarrow 3x = \pi, \text{ as } x \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow x = \frac{\pi}{3}$

The point $x = \frac{\pi}{3}$ divides the interval $\left(0,\ \frac{\pi}{2}\right)$ into two disjoint intervals

i.e., 0 $\left(0,\ \frac{\pi}{3}\right)$ and $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$.

$\text{Now, in interval } \left(0, \frac{\pi}{3}\right), f_3(x) = -3\sin 3x < 0 \left[\text{as } 0 < x < \frac{\pi}{3} \Rightarrow 0 < 3x < \pi\right]. \therefore$

$f_3 \text{ is strictly decreasing in interval } \left(0,\ \frac{\pi}{3}\right).$

$\text{However, in interval } \left(\frac{\pi}{3},\ \frac{\pi}{2}\right), f_3(x) = -3\sin 3x > 0 \left[\text{as } \frac{\pi}{3} < x < \frac{\pi}{2} \Rightarrow \pi < 3x < \frac{3\pi}{2}\right].$

$\therefore f_3$ is strictly increasing in interval $\left(\frac{\pi}{3},\ \frac{\pi}{2}\right)$.

Hence, f_3 is neither increasing nor decreasing in interval $\left(0, \frac{\pi}{2}\right)$.

(D) Let $f_4(x) = \tan x$.

$\therefore f_4'(x) = \sec^2 x$

In interval $\left(0, \frac{\pi}{2}\right)$, $f_4'(x) = \sec^2 x > 0$.

$\therefore$ f_4 is strictly increasing in interval

Answer needs Correction? **Click Here**

Q13 : On which of the following intervals is the function *f* given by $f(x) = x^{100} + \sin x - 1$ strictly decreasing?

(A) $(0, 1)$ (B) $\left(\frac{\pi}{2}, \pi\right)$

(C) $\left(0, \frac{\pi}{2}\right)$ (D) None of these

Answer :

We have,

$f(x) = x^{100} + \sin x - 1$

$\therefore f'(x) = 100x^{99} + \cos x$

In interval $(0, 1)$, $\cos x > 0$ and $100x^{99} > 0$.

$\therefore f'(x) > 0$.

Thus, function *f* is strictly increasing in interval (0, 1).

In interval $\left(\frac{\pi}{2}, \pi\right)$, $\cos x < 0$ and $100x^{99} > 0$. Also, $100x^{99} > \cos x$

$\therefore f'(x) > 0$ in $\left(\frac{\pi}{2}, \pi\right)$.

Thus, function *f* is strictly increasing in interval $\left(\frac{\pi}{2}, \pi\right)$.

In interval $\left(0, \frac{\pi}{2}\right)$, $\cos x > 0$ and $100x^{99} > 0$.

$\therefore 100x^{99} + \cos x > 0$

$\Rightarrow f'(x) > 0$ on $\left(0, \frac{\pi}{2}\right)$

$\therefore$ *f* is strictly increasing in interval $\left(0, \frac{\pi}{2}\right)$.

Hence, function *f* is strictly decreasing in none of the intervals.

The correct answer is D.

Answer needs Correction? **Click Here**

Q14 : Find the least value of *a* such that the function *f* given $f(x) = x^2 + ax + 1$ is strictly increasing on (1, 2).

Answer :

We have,

$f(x) = x^2 + ax + 1$

$\therefore f'(x) = 2x + a$

Now, function *f* will be increasing in (1, 2), if $f'(x) > 0$ in (1, 2).

$f'(x) > 0$

$\Rightarrow 2x + a > 0$

$\Rightarrow 2x > -a$

$\Rightarrow x > \frac{-a}{2}$

Therefore, we have to find the least value of *a* such that

$x > \frac{-a}{2}$, when $x \in (1, 2)$.

$\Rightarrow x > \frac{-a}{2}$ (when $1 < x < 2$)

Thus, the least value of *a* for *f* to be increasing on (1, 2) is given by,

$\frac{-a}{2} = 1$

$\frac{-a}{2} = 1 \Rightarrow a = -2$

Hence, the required value of *a* is - 2.

Answer needs Correction? **Click Here**

Q15 : Let I be any interval disjoint from (- 1, 1). Prove that the function *f* given by

$f(x)=x+\frac{1}{x}$ is strictly increasing on I.

Answer :

We have,

$f(x)=x+\frac{1}{x}$

$\therefore f'(x)=1-\frac{1}{x^2}$

Now,

$f'(x)=0 \Rightarrow \frac{1}{x^2}=1 \Rightarrow x=\pm 1$

The points x = 1 and x = - 1 divide the real line in three disjoint intervals i.e., $(-\infty,-1),(-1,\ 1)$, and $(1,\ \infty)$.

In interval (- 1, 1), it is observed that:

$-1<x<1$

$\Rightarrow x^2<1$

$\Rightarrow 1<\frac{1}{x^2},\ x\neq 0$

$\Rightarrow 1-\frac{1}{x^2}<0,\ x\neq 0$

$\therefore f'(x)=1-\frac{1}{x^2}<0$ on $(-1,\ 1)\sim\{0\}$.

$\therefore$ *f* is strictly decreasing on $(-1,\ 1)\sim\{0\}$.

In intervals $(-\infty,-1)$ and $(1,\ \infty)$, it is observed that:

$x<-1$ or $1<x$

$\Rightarrow x^2>1$

$\Rightarrow 1>\frac{1}{x^2}$

$\Rightarrow 1-\frac{1}{x^2}>0$

$\therefore f'(x)=1-\frac{1}{x^2}>0$ on $(-\infty,-1)$ and $(1,\ \infty)$.

$\therefore$ *f* is strictly increasing on $(-\infty,\ 1)$ and $(1,\ \infty)$.

Hence, function *f* is strictly increasing in interval I disjoint from (- 1, 1).

Hence, the given result is proved.

Q16 : Prove that the function *f* given by *f*(*x*) = log sin *x* is strictly increasing on $\left(0,\frac{\pi}{2}\right)$ and strictly decreasing on $\left(\frac{\pi}{2},\pi\right)$.

Answer :

We have,

$f(x)=\log\sin x$

$\therefore f'(x)=\frac{1}{\sin x}\cos x=\cot x$

In interval $\left(0,\ \frac{\pi}{2}\right)$, $f'(x)=\cot x>0$.

$\therefore$ *f* is strictly increasing in $\left(0,\ \frac{\pi}{2}\right)$.

In interval $\left(\frac{\pi}{2},\ \pi\right)$, $f'(x)=\cot x<0$.

$\therefore$ *f* is strictly decreasing in $\left(\frac{\pi}{2},\pi\right)$.

Q17 : Prove that the function *f* given by *f*(*x*) = log cos *x* is strictly decreasing on $\left(0,\ \frac{\pi}{2}\right)$ and strictly increasing on $\left(\frac{\pi}{2},\pi\right)$.

Answer :

We have,

$f(x)=\log\cos x$

$\therefore f'(x)=\frac{1}{\cos x}(-\sin x)=-\tan x$

In interval $\left(0,\ \frac{\pi}{2}\right)$, $\tan x>0 \Rightarrow -\tan x<0$.

$\therefore f'(x)<0$ on $\left(0,\ \frac{\pi}{2}\right)$

$\therefore f$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$.

In interval $\left(\frac{\pi}{2}, \pi\right)$, $\tan x < 0 \Rightarrow -\tan x > 0$.

$\therefore f'(x) > 0$ on $\left(\frac{\pi}{2}, \pi\right)$

$\therefore f$ is strictly increasing on $\left(\frac{\pi}{2}, \pi\right)$.

Answer needs Correction? **Click Here**

Q18 : Prove that the function given by $f(x) = x^3 - 3x^2 + 3x - 100$ is increasing in R.

Answer :

We have,

$f(x) = x^3 - 3x^2 + 3x - 100$

$$\begin{aligned} f'(x) &= 3x^2 - 6x + 3 \\ &= 3(x^2 - 2x + 1) \\ &= 3(x-1)^2 \end{aligned}$$

For any $x \in$ **R**, $(x - 1)^2 > 0$.

Thus, $f'(x)$ is always positive in **R**.

Hence, the given function (*f*) is increasing in **R**.

Answer needs Correction? **Click Here**

Q19 : The interval in which $y = x^2 e^{-x}$ is increasing is

(A) $(-\infty, \infty)$ (B) (- 2, 0) (C) $(2, \infty)$ (D) (0, 2)

Answer :

We have,

$y = x^2 e^{-x}$

$\therefore \frac{dy}{dx} = 2xe^{-x} - x^2 e^{-x} = xe^{-x}(2 - x)$

$\text{Now}, \frac{dy}{dx} = 0.$

$\Rightarrow x = 0 \text{ and } x = 2$

The points $x = 0$ and $x = 2$ divide the real line into three disjoint intervals i.e., $(-\infty, 0)$, $(0, 2)$, and $(2, \infty)$.

In intervals $(-\infty, 0)$ and $(2, \infty)$, $f'(x) < 0$ as e^{-x} is always positive.

$\therefore f$ is decreasing on $(-\infty, 0)$ and $(2, \infty)$.

In interval (0, 2), $f'(x) > 0$.

$\therefore$ *f* is strictly increasing on (0, 2).

Hence, *f* is strictly increasing in interval (0, 2).

The correct answer is D.

Answer needs Correction? **Click Here**

*********** END **********