CHAPTER 39

ALTERNATING CURRENT

39.1 ALTERNATING CURRENT

When a resistor is connected across the terminals of a battery, a current is established in the circuit. The current has a unique direction, it goes from the positive terminal to the negative terminal via the external resistor. The magnitude of the current also remains almost constant. If the direction of the current in a resistor or in any other element changes alternately, the current is called an *alternating current* (AC). In this chapter, we shall study the alternating current that varies sinusoidally with time. Such a current is given by

$$i = i_0 \sin(\omega t + \varphi). \qquad \dots (39.1)$$

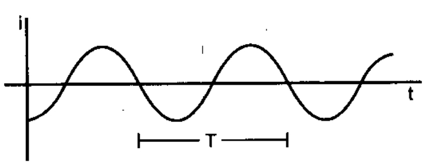

Figure 39.1

The current repeats its value after each time interval $T = 2\pi/\omega$. This time interval is called the *time period*. The current is positive for half the time period and is negative for the remaining half period. This means, its direction reverses after each half time period. The maximum value of the current is i_0 which is called the *peak current* or the *current amplitude*. To get sinusoidally varying alternating current, we need a source which can generate sinusoidally varying emf. An *AC generator,* also called an *AC dynamo*, can be used as such a source. It converts mechanical energy into electrical energy, producing an alternating emf.

39.2 AC GENERATOR, OR AC DYNAMO

Construction

A schematic design of an AC dynamo is shown in figure (39.2a). A simplified diagram of the same is shown in figure (39.2b). It consists of three main parts: a magnet, an armature with slip rings and brushes.

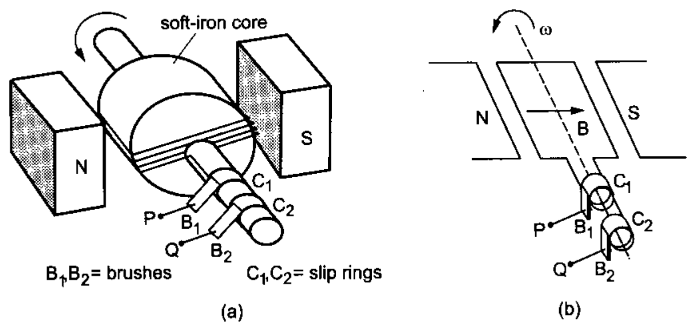

Figure 39.2

Magnet: It may be a permanent magnet or an electromagnet. The poles of the magnet face each other so that a strong uniform magnetic field $\vec{B}$ is produced between the poles.

Armature: It is a coil generally wound over a soft-iron core. The core increases the magnetic field due to its magnetization. The two ends of the coil are connected to two slip rings C_1 and C_2. The coil together with the rings can rotate in the magnetic field. The axis of rotation is in the plane of the coil but perpendicular to the magnetic field.

Brushes: Two graphite brushes B_1 and B_2 permanently touch the slip rings. As the armature rotates, the slip rings C_1 and C_2 slip against the brushes so that the contact is maintained all the time. These brushes are connected to two terminals P and Q. The external circuit is connected to these terminals.

emf Induced as the Coil Rotates

Suppose the area of the coil is A, it contains N turns and it is rotated at a constant angular velocity ω. Suppose, the plane of the coil is perpendicular to the magnetic field at $t = 0$. The total magnetic flux through each turn of the coil is BA in this position. In time t, the coil rotates through an angle $\theta = \omega t$. The flux through each turn of the coil at this time t is

$$\Phi = BA \cos \omega t.$$

Using Faraday's law, the emf induced in each turn of the coil is

$$-\frac{d\Phi}{dt} = BA\omega \sin \omega t.$$

The total emf induced in the coil is,

$$\mathcal{E} = NBA\, \omega \sin \omega t$$
$$= \mathcal{E}_0 \sin \omega t. \quad \dots (39.2)$$

We see that the emf varies sinusoidally with time with an angular frequency ω and hence with a time period $T = 2\pi/\omega$. The maximum magnitude of the emf, known as *peak emf*, is $\mathcal{E}_0$.

If the terminals P and Q are connected to an external circuit, this emf drives a current in the circuit which also varies sinusoidally with time as shown in figure (39.1).

Household Power Generation

The electricity that we use in our houses is generally AC electricity and is produced in power plants using the same principle as described above. The armature is connected to a *turbine*. The turbine has a rotor with blades. Steam at high pressure, water from a height or air at high speed strikes the blades. This rotates the rotor of the turbine. As the armature is connected to the turbine, the armature also rotates and alternating emf is produced. Gensets, which are used in houses at the time of power failure, at marriage functions, at public meetings, in fields where regular electric power is not available, etc., also work on the same principle. Here a diesel or a petrol engine drives the armature.

39.3 INSTANTANEOUS AND RMS CURRENT

An alternating current is given by

$$i = i_0 \sin(\omega t + \varphi). \quad \dots (i)$$

This equation gives the instantaneous current at any instant t. The current changes with time, sometimes it is positive and sometimes negative. We define the *average current* or *mean current* over a time interval 0 to t as

$$\bar{i} = \frac{\int_0^t i\, dt}{\int_0^t dt} = \frac{1}{T}\int_0^t i dt \cdot$$

Using (i),

$$\bar{i} = \frac{i_0}{t}\int_0^t \sin(\omega t + \varphi)\, dt = -\frac{i_0}{t}\left[\frac{\cos(\omega t + \varphi)}{\omega}\right]_0^t.$$

or, $$\bar{i} = -\frac{i_0}{t}\left[\frac{\cos(\omega t + \varphi) - \cos\varphi}{\omega}\right]. \quad \dots (ii)$$

For a time period, $t = T$ and $\omega T = 2\pi$ so that,

$$\bar{i} = -\frac{i_0}{T\omega}[\cos(2\pi + \varphi) - \cos \varphi] = 0.$$

If we take the average over a long time, the value of i will be the same as for one time period. This can be easily seen from equation (ii). As cosine of an angle must remain between ± 1, the numerator has a finite value. If t is large, the denominator is large and the average current $\bar{i}$ tends to zero.

The instantaneous current i could be positive or negative at a given instant but the quantity i^2 always remains positive and hence its average is also positive. The average of i^2 over a time period is

$$\overline{i^2} = \frac{\int_0^T i^2\, dt}{\int_0^T dt}$$
$$= \frac{1}{T}\int_0^T i_0^2 \sin^2(\omega t + \varphi)\, dt$$
$$= \frac{i_0^2}{2T}\int_0^T [1 - \cos 2(\omega t + \varphi)]dt$$
$$= \frac{i_0^2}{2T}\left[t - \frac{\sin 2(\omega t + \varphi)}{2\omega}\right]_0^T$$
$$= \frac{i_0^2}{2T}\left[T - \frac{\sin(4\pi + 2\varphi) - \sin 2\varphi}{2\omega}\right] = \frac{i_0^2}{2}.$$

This is known as the *mean square current*. The square root of mean square current is called *root-mean-square current* or rms current. This is also known as the *virtual current*. Thus, the rms current or the virtual current corresponding to the current $i = i_0 \sin(\omega t + \varphi)$ is

$$i_{rms} = \sqrt{\overline{i^2}} = \frac{i_0}{\sqrt{2}}. \quad \dots (39.3)$$

The equations for mean square current and root-mean-square current are derived for one time period. They are also valid if the average is calculated over a long period of time.

An alternating voltage (potential difference) may be written as

$$V = V_0 \sin(\omega t + \varphi).$$

This gives the instantaneous voltage. The mean voltage $\overline{V}$ over a complete cycle is zero, the mean square voltage over a cycle is $V_0^2/2$ and the root-mean-square voltage (rms voltage or virtual voltage) is $V_0/\sqrt{2}$. The significance of rms current and rms voltage may be shown by considering a resistor of resistance R carrying a current

$$i = i_0 \sin(\omega t + \varphi). \quad \dots (i)$$

The voltage across the resistor is

$$V = Ri = (i_0 R)\sin(\omega t + \varphi). \qquad \dots \text{(ii)}$$

The thermal energy developed in the resistor during the time t to $t + dt$ is

$$i^2 R\, dt = i_0^2 R \sin^2(\omega t + \varphi) dt.$$

The thermal energy developed in one time period is

$$U = \int_0^T i^2 R dt$$

$$= R \int_0^T i_0^2 \sin^2(\omega t + \varphi) dt$$

$$= RT\left[\frac{1}{T}\int_0^T i_0^2 \sin^2(\omega t + \varphi) dt\right]$$

$$= i_{rms}^2 RT.$$

Thus, if we pass a constant current i_{rms} through the resistor, it will produce the same thermal energy in a time period as that produced when the alternating current i passes through it. Similarly, a constant voltage V_{rms} applied across a resistor produces the same thermal energy as that produced by the voltage $V = V_0 \sin(\omega t + \varphi)$. These statements are also valid if we consider a long period of time. The alternating voltage and the alternating current are generally measured and mentioned in terms of their rms values. When we say that the household supply is 220 V AC we mean that the rms value is 220 V. The peak value would be (220 V) $\sqrt{2} = 311$ V.

Example 39.1

The peak value of an alternating current is 5 A *and its frequency is* 60 Hz. *Find its rms value. How long will the current take to reach the peak value starting from zero ?*

Solution :

The rms current is

$$i_{rms} = \frac{i_0}{\sqrt{2}} = \frac{5\text{ A}}{\sqrt{2}} = 3{\cdot}5 \text{ A}.$$

The time period is

$$T = \frac{1}{\nu} = \frac{1}{60} \text{ s}.$$

The current takes one fourth of the time period to reach the peak value starting from zero. Thus, the time required is

$$t = \frac{T}{4} = \frac{1}{240} \text{ s}.$$

39.4 SIMPLE AC CIRCUITS

AC Circuit Containing only a Resistor

Figure (39.3) shows a circuit containing an AC source $\mathcal{E} = \mathcal{E}_0 \sin \omega t$ and a resistor of resistance R.

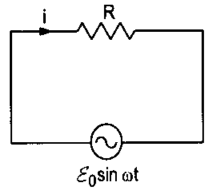

Figure 39.3

Such a circuit is also known as a *purely resistive circuit.* Notice the symbol for an AC source.

If the current at time t is i, Kirchhoff's loop law gives

$$\mathcal{E}_0 \sin \omega t = Ri$$

or, $$i = \frac{\mathcal{E}_0}{R} \sin \omega t$$

$$= i_0 \sin \omega t \qquad \dots (39.4)$$

where $$i_0 = \frac{\mathcal{E}_0}{R}. \qquad \dots (39.5)$$

AC Circuit Containing only a Capacitor

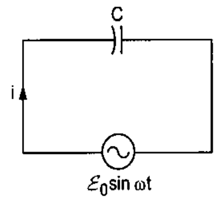

Figure 39.4

Figure (39.4) shows an AC source connected across a capacitor. The resistance of the circuit is assumed to be zero. Such a circuit is also known as a *purely capacitive circuit.* Suppose the charge on the capacitor is q and the current is i at time t. Any charge that goes through a wire accumulates on the capacitor, so that

$$idt = dq$$

or, $$i = \frac{dq}{dt}.$$

Using Kirchhoff's loop law,

$$\mathcal{E}_0 \sin \omega t = \frac{q}{C}$$

or, $$q = C\mathcal{E}_0 \sin \omega t$$

or, $$i = \frac{dq}{dt} = C\mathcal{E}_0 \omega \cos \omega t$$

or, $$i = i_0 \cos \omega t \qquad \dots (39.6)$$

where $$i_0 = C\mathcal{E}_0 \omega = \frac{\mathcal{E}_0}{1/\omega C}. \qquad \dots (39.7)$$

There are several points to be discussed. If a battery is connected across a capacitor, there is a current only for a short time in which the capacitor gets charged. After this the current becomes negligible. In case of an AC source, the current exists as along as the source is connected. We say that *a capacitor* *stops direct current but allows alternating current.* The physical reason behind it is obvious. The charge on a capacitor is determined by the emf of the source. In case of an AC source, the emf keeps on changing. Accordingly, the charge q keeps on changing and we get continuous current through the connecting wires and the source.

Another important point to note is the relation between the peak emf and the peak current. We have

$$i_0 = \frac{\mathcal{E}_0}{1/\omega C}$$

or, $$i_0 = \frac{\mathcal{E}_0}{X_c} \text{ where } X_c = \frac{1}{\omega C}.$$

We see that $X_c = 1/\omega C$ plays the role of effective resistance. It is called the *reactance* of the capacitor and its unit is ohm. It depends on the capacitance of the capacitor as well as on the frequency of the AC source. For a source of high frequency, the reactance $X_c = 1/\omega C$ is small and the peak current i_0 is large. For a small frequency, the reactance $1/\omega C$ is large and consequently the peak current is small.

If the frequency is zero, we get a direct-current (DC) source producing a constant emf. In this case, the reactance $1/\omega C$ is infinity and $i_0 = 0$. So the response of a capacitor to an alternating-current source depends on the frequency of the source.

The third important point concerns the phase difference between the emf and the current. We have,

$$\mathcal{E} = \mathcal{E}_0 \sin \omega t$$

and $$i = i_0 \cos \omega t = i_0 \sin(\omega t + \pi/2).$$

Thus, the current leads the emf by $\pi/2$. When the emf $\mathcal{E}$ is zero, the current has maximum magnitude. When the emf has maximum magnitude, the current is zero. Figure (39.5) shows variations in the current through the capacitor and in the emf as time passes.

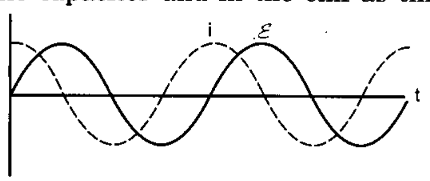

Figure 39.5

Example 39.1

Find the reactance of a capacitor (C = 200 μF) when it is connected to (a) a 10 Hz AC source, (b) a 50 Hz AC source and (c) a 500 Hz AC source.

Solution :

The reactance is $X_c = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$.

(a) $$X_c = \frac{1}{2\pi(10\text{ Hz})(200 \times 10^{-6}\text{ F})}$$

$$= 80\ \Omega.$$

Similarly, the reactance is 16 Ω for 50 Hz and 1·6 Ω for 500 Hz.

AC Circuit Containing only an Inductor

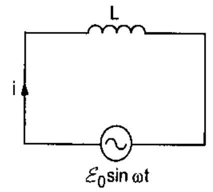

Figure 39.6

Figure (39.6) shows an inductor connected to an AC source. Such a circuit is also known as a *purely inductive circuit.*

The induced emf across the inductor is $-L\frac{di}{dt}$ so that from Kirchhoff's loop law,

$$\mathcal{E}_0 \sin \omega t - L\frac{di}{dt} = 0$$

or, $$\frac{di}{dt} = \frac{\mathcal{E}_0}{L} \sin \omega t$$

or, $$i = -\frac{\mathcal{E}_0}{\omega L} \cos \omega t + c \quad \text{... (i)}$$

where c is a constant. Now, average of $\cos \omega t$ over one time period is zero. Also, in the circuit we are discussing, the emf is sinusoidal and we expect the current to be sinusoidal too. Thus, average of i must be zero over one time period. Hence, from (i), $c = 0$ and

$$i = -\frac{\mathcal{E}_0}{\omega L} \cos \omega t$$

or, $$i = \frac{\mathcal{E}_0}{\omega L} \sin(\omega t - \pi/2) \quad \text{... (39.8)}$$

or, $$i = i_0 \sin(\omega t - \pi/2)$$

where $$i_0 = \frac{\mathcal{E}_0}{\omega L}. \quad \text{... (39.9)}$$

The constant $X_L = \omega L$ plays the role of effective resistance in this circuit. It is called the *reactance* of the inductor. It is zero for direct current ($\omega = 0$) and increases as the frequency is increased. We see from equation (39.8) that the phase of the current is $\pi/2$ less than that of the emf. The current lags behind the emf. Figure (39.7) shows plots of the current through an inductor and of the emf as time passes.

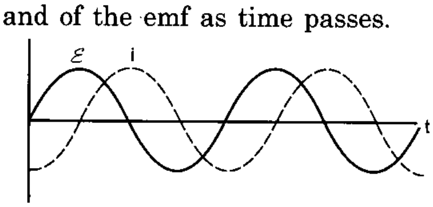

Figure 39.7

Example 39.2

An inductor (L = 200 mH) *is connected to an AC source of peak emf* 210 V *and frequency* 50 Hz. *Calculate the peak current. What is the instantaneous voltage of the source when the current is at its peak value?*

Solution : The reactance of the inductor is

$$X_L = \omega L = (2\pi \times 50 \text{ s}^{-1}) \times (200 \times 10^{-3} \text{ H})$$

$$= 62{\cdot}8 \ \Omega.$$

The peak current is

$$i_0 = \frac{\mathcal{E}_0}{X_L} = \frac{210 \text{ V}}{62{\cdot}8 \ \Omega} = 3{\cdot}3 \text{ A}.$$

As the current lags behind the voltage by $\pi/2$, the voltage is zero when the current has its peak value.

Impedance

The peak current and the peak emf in all the three circuits discussed above may be written as

$$i_0 = \frac{\mathcal{E}_0}{Z} \quad \dots \text{(39.10)}$$

where $Z = R$ for a purely resistive circuit

$Z = \dfrac{1}{\omega C}$ for a purely capacitive circuit

and $Z = \omega L$ for a purely inductive circuit.

The peak current and the peak emf are related by equation (39.10) for any series circuit (one-loop circuit) having an AC source. The general name for Z is *impedance*. Thus, the impedance of a purely resistive circuit is R, that of a purely capacitive circuit is $1/\omega C$ and that of a purely inductive circuit is ωL.

Phase factor

We have seen that the current and the emf are, in general, not in phase in an AC circuit. If the emf is

$$\mathcal{E} = \mathcal{E}_0 \sin \omega t,$$

the current may be written as

$$i = i_0 \sin(\omega t + \varphi).$$

For a purely resistive circuit, $\varphi = 0$; for a purely capacitive circuit, $\varphi = \pi/2$ and for a purely inductive circuit, $\varphi = -\pi/2$. We shall call the constant φ the phase factor.

39.5 VECTOR METHOD TO FIND THE CURRENT IN AN AC CIRCUIT

Let us now describe a simple method by which we can calculate the current in an AC circuit. We shall confine the discussion to series circuits only. Suppose an emf

$$\mathcal{E} = \mathcal{E}_0 \sin \omega t$$

is applied in a series AC circuit which may contain a resistance, a capacitor, an inductor or any combination of these. Let us represent the resistance of a resistor by a vector of magnitude R, the reactance of a capacitance by a vector of magnitude $X_c = 1/\omega C$ and the reactance of an inductor by a vector of magnitude $X_L = \omega L$. The vector corresponding to the resistance is drawn along the X-axis, the vector for the capacitive reactance is drawn $\pi/2$ ahead of the resistance, that is, along the positive Y-axis and the vector for the inductive reactance is drawn $\pi/2$ behind the resistance, that is, along the negative Y-axis.

The impedance of the circuit, Z, and the phase factor φ are obtained by the vector sum of these three vectors. The magnitude of the vector sum gives the impedance Z, and its angle with the X-axis gives the phase factor.

Thus, if the resistance of the circuit is R and the net reactance is X, the impedance is $Z = \sqrt{R^2 + X^2}$ and $\tan\varphi = \frac{X}{R}$.

Once Z and φ are obtained, the current in the circuit can be easily written as

$$i = \frac{\mathcal{E}_0}{Z} \sin(\omega t + \varphi). \quad \dots \text{(39.11)}$$

It should be clearly understood that the resistance, capacitance, inductance, etc., are not vector quantities. The above description is only a method to derive easily the equations for the current in an AC circuit. Figure (39.8) shows the construction of vector diagrams for the three circuits discussed above.

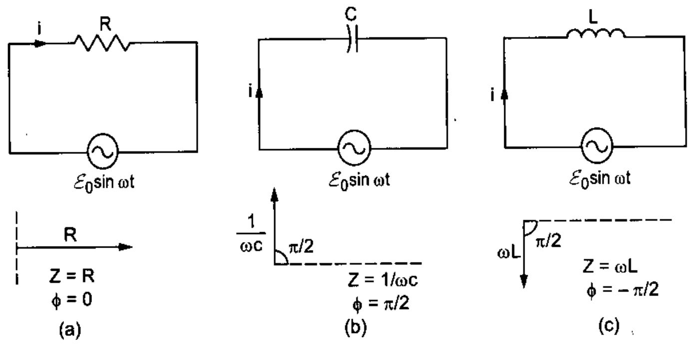

Figure 39.8

39.6 MORE AC CIRCUITS

When an AC source is connected in a circuit with a resistance and a reactance, the current varies initially in a complex way. After sufficient time, a sinusoidally varying current persists in the circuit. This steady-state current has a frequency equal to that of the source and may have a phase difference with the source. This steady-state current may be obtained by the vector method described above.

***CR* Circuit**

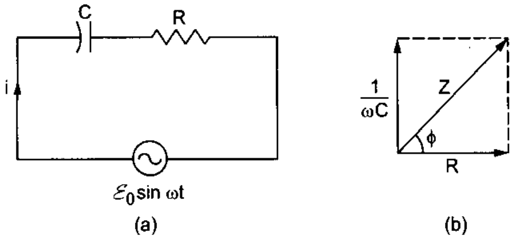

Figure 39.9

Let us find the current in a *CR* circuit using the vector method. The circuit and the corresponding vector diagram are drawn in figure (39.9). The resistance is represented by a vector of magnitude R along the x-axis and the capacitive reactance by a vector of magnitude $1/\omega C$ along the positive y-axis. The impedance of the circuit is given by the magnitude of the resultant of these two. It is

$$Z = \sqrt{R^2 + (1/\omega C)^2} \quad \dots \text{(i)}$$

and hence the peak current is

$$i_0 = \frac{\mathcal{E}_0}{Z} = \frac{\mathcal{E}_0}{\sqrt{R^2 + (1/\omega C)^2}} \cdot$$

Also, the direction of the resultant makes an angle φ with the x-axis where

$$\tan\varphi = \frac{1}{\omega CR} \cdot \quad \dots \text{(ii)}$$

The steady-state current in the circuit is

$$i = \frac{\mathcal{E}_0}{Z} \sin(\omega t + \varphi)$$

where Z and φ are given by equations (i) and (ii).

The reactance of the circuit is $1/\omega C$. We see that the current *leads* the emf.

***LR* Circuit**

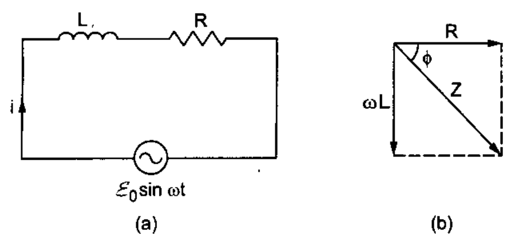

Figure 39.10

Figure (39.10) shows an inductor, a resistor and an AC source connected in series together with its vector diagram. The resistance is represented by a vector of magnitude R along the x-axis and the inductive reactance by a vector of magnitude ωL along the negative y-axis. The impedance of the circuit is equal to the magnitude of the resultant of these two. Its value is

$$Z = \sqrt{R^2 + \omega^2 L^2}. \quad \dots \text{(i)}$$

The resultant is at an angle φ below the x-axis where

$$\tan\varphi = \frac{\omega L}{R} \cdot \quad \dots \text{(ii)}$$

The current in steady state is, therefore, given by

$$i = \frac{\mathcal{E}_0}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t - \varphi)$$

where φ is given by equation (ii). The reactance of the circuit is ωL. We see that the current *lags* behind the emf.

***LCR* Circuit**

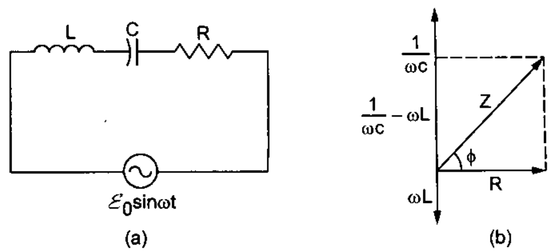

Figure 39.11

Figure (39.11) shows an inductor, a capacitor and a resistor connected in series with an AC source and the vector diagram to find the steady-state current.

The resultant of $1/\omega C$ and ωL is

$$X = X_c - X_L = \left(\frac{1}{\omega C} - \omega L\right)$$

in the direction of the positive y-axis. This is the net reactance of the circuit. The resultant of the vector for R and that for the reactance $\left(\frac{1}{C\omega} - L\omega\right)$ has a magnitude

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2} \quad \dots (39.12)$$

which is the impedance of the circuit. This resultant makes an angle φ with the x-axis where

$$\tan\varphi = \frac{\frac{1}{\omega C} - \omega L}{R} \cdot \quad \dots (39.13)$$

The steady-state current in the circuit is given by

$$i = \frac{\mathcal{E}_0}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}} \sin(\omega t + \varphi)$$

where φ is given by equation (39.13).

If $X_c = 1/\omega C$ is greater than $X_L = \omega L$, the vector for the net reactance $X_c - X_L$ is along the positive Y-axis. From equation (39.13), the phase factor φ is positive. Thus, the current *leads* the emf. If $X_c < X_L$, the vector for the net reactance is along the negative Y-axis and φ is negative. In this case, the current *lags* behind the emf. If $X_L = X_C$, the net reactance is zero. It behaves as purely resistive circuit and the vector for Z is along the X-axis. The current is in phase with the emf in this case.

If we vary the angular frequency ω of the AC source, the peak current

$$i_0 = \frac{\mathcal{E}_0}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}}$$

also varies. It is maximum when

$$\frac{1}{\omega C} - \omega L = 0$$

or,
$$\omega = \sqrt{\frac{1}{LC}}.$$

The corresponding frequency is

$$\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}. \qquad \dots (39.14)$$

This frequency is known as the *resonant frequency* of the given circuit. The peak current in this case is $i_0 = \mathcal{E}_0/R$ and the reactance is zero.

Figure (39.12) shows the variation in the peak current i_0 with the applied frequency ν of the AC source in two different circuits. The values of L as well as the values of C are the same for the two circuits. We see that if R is small, the resonance is sharp. This means, if the applied frequency is close to the resonant frequency ν_0, the current is high, otherwise it is small. An LCR circuit used at a frequency close to the resonance frequency is called *resonant circuit.*

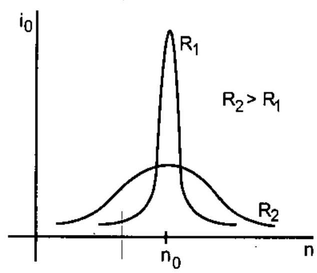

Figure 39.12

The tuning circuit of a radio or a television is an example of LCR resonant circuit. Signals are transmitted by different stations at different frequencies. The antenna receives these signals and drives a current in the tuning circuit. Only the signal corresponding to the resonant frequency is able to drive appreciable current and is further processed. When we 'tune' a radio, we change the capacitance of the tuning circuit and hence the resonant frequency changes. When this frequency matches with the frequency of the signal from the desired station, the tuning is complete.

***LC* oscillations**

If the resistance R in an LCR circuit is zero, the peak current at resonance is

$$i = \frac{\mathcal{E}_0}{zero}.$$

This means, there can be a finite current in the pure LC circuit even without any applied emf. This is the case when a charged capacitor is connected to a pure inductor. There is a current in the circuit at frequency $\nu = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}\cdot$ The capacitor gets discharged sending a current in the inductor and induced emf in the inductor charges the capacitor again. Thus, the energy oscillates between electric field energy in the capacitor and magnetic field energy in the inductor. This phenomenon is called *LC oscillation.*

Example 39.3

An LCR series circuit with $L = 100$ mH, $C = 100\,\mu$F, $R = 120\ \Omega$ is connected to an AC source of emf $\mathcal{E} = (30\text{ V})\sin(100\text{ s}^{-1})t$. Find the impedance, the peak current and the resonant frequency of the circuit.

Solution :

The reactance of the circuit is

$$X = \frac{1}{\omega C} - \omega L$$

$$= \frac{1}{(100\text{ s}^{-1})(100\times10^{-6}\text{ F})} - (100\text{ s}^{-1})\times(100\times10^{-3}\text{ H})$$

$$= 100\ \Omega - 10\ \Omega = 90\ \Omega.$$

The resistance is $R = 120\ \Omega$.

The impedance is

$$Z = \sqrt{R^2 + X^2}$$

$$= \sqrt{(120\ \Omega)^2 + (90\ \Omega)^2} = 150\ \Omega.$$

The peak current is

$$i_0 = \frac{\mathcal{E}_0}{Z} = \frac{30\text{ V}}{150\ \Omega} = 0{\cdot}2\text{ A}.$$

The resonant frequency of the circuit is

$$\nu = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{(100\times10^{-3}\text{ H})(100\times10^{-6}\text{ F})}}$$

$$\approx 50\text{ Hz}.$$

39.7 POWER IN AC CIRCUITS

Suppose an emf $\mathcal{E} = \mathcal{E}_0 \sin\omega t$ is applied in a circuit and a current $i = i_0\sin(\omega t + \varphi)$ results. The work done by the source during the time interval t to $t + dt$ is

$$dW = \mathcal{E}\, i\, dt$$

$= \mathcal{E}_0 i_0 \sin \omega t \sin(\omega t + \varphi)\, dt$

$= \mathcal{E}_0 i_0 (\sin^2 \omega t \cos \varphi + \sin \omega t \cos \omega t \sin \varphi) dt.$

The total work done in a complete cycle is

$$W = \mathcal{E}_0 i_0 \cos\varphi \int_0^T \sin^2 \omega t\, dt + \mathcal{E}_0 i_0 \sin \varphi \int_0^T \sin \omega t \cos \omega t\, dt$$

$$= \frac{1}{2} \mathcal{E}_0 i_0 \cos \varphi \int_0^T (1 - \cos 2\omega t) dt + \frac{1}{2} \mathcal{E}_0 i_0 \sin \varphi \int_0^T \sin 2\omega t\, dt$$

$$= \frac{1}{2} \mathcal{E}_0 i_0 T \cos \varphi.$$

The average power delivered by the source is, therefore,

$$P = \frac{W}{T} = \frac{1}{2} \mathcal{E}_0 i_0 \cos\varphi = \left(\frac{\mathcal{E}_0}{\sqrt{2}}\right)\left(\frac{i_0}{\sqrt{2}}\right)(\cos\varphi)$$

$$= \mathcal{E}_{rms} i_{rms} \cos\varphi. \qquad \ldots (39.15)$$

This equation is derived for the average power in a complete cycle. It also represents the average power delivered in a long time.

The term $\cos\varphi$ is called the *power factor* of the circuit. For a purely resistive circuit, $\varphi = 0$ so that $\cos \varphi = 1$ and $P = \mathcal{E}_{rms} i_{rms}$. For purely reactive circuits (no resistance, only capacitance and/or inductance), $\varphi = \pi/2$ or $-\pi/2$. In these cases, $\cos\varphi = 0$ and hence no power is absorbed in such circuits.

39.8 CHOKE COIL

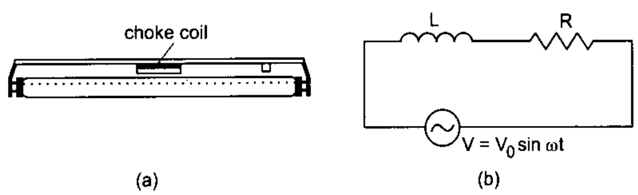

Figure 39.13

Choke coil is simply a coil having a large inductance but a small resistance. Choke coils are used with fluorescent mercury-tube fittings in houses (figure 39.13a).

At most places, the household electric power is supplied at 220 V, 50 Hz. If such a source is directly connected to a mercury tube, the tube will be damaged. To reduce the current, a choke coil is connected in series with the tube. Representing the tube by a resistor and the choke coil by an ideal inductor, the equivalent circuit is drawn in figure (39.13b). This is a simple LR circuit with impedance $Z = \sqrt{R^2 + \omega^2 L^2}$.

If the voltage applied is $V = V_0 \sin \omega t$, the peak current through the circuit is

$$i_0 = \frac{V_0}{\sqrt{R^2 + \omega^2 L^2}}.$$

The rms current is

$$i_{rms} = \frac{i_0}{\sqrt{2}} = \frac{V_0/\sqrt{2}}{\sqrt{R^2 + \omega^2 L^2}} = \frac{V_{rms}}{\sqrt{R^2 + \omega^2 L^2}}.$$

The rms voltage appearing across the resistor is

$$V_{R,\,rms} = R\, i_{rms} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} V_{rms}.$$

If the choke coil were not used, the voltage across the resistor would be the same as the applied voltage. Thus, by using the choke coil, the voltage across the resistor is reduced by a factor

$$\frac{R}{\sqrt{R^2 + \omega^2 L^2}}.$$

The advantage of using a choke coil to reduce the voltage is that an inductor does not consume power. Hence, we do not lose electric energy in the form of heat. If we connect an additional resistor in series with the tube to reduce the voltage, power will be lost in heating this additional resistor.

39.9 HOT-WIRE INSTRUMENTS

In an ordinary ammeter or voltmeter, a coil is free to rotate in the magnetic field of a fixed magnet. To measure a current or a voltage, current is passed through the coil and the coil deflects due to the torque acting on it. If an alternating current is passed through such a coil, the torque will reverse its direction each time the current changes direction and the average value of the torque will be zero. Because of friction, etc., the coil does not quickly respond to the changing torque and remains undeflected. To measure alternating currents or voltages, one would have to use a property so that the deflection of the moving part depends on i^2 and not on i. This ensures that the deflection remains independent of the direction of the current. The average of i^2 is not zero and hence a steady deflection may be obtained. Hot-wire instruments are designed to work on this principle.

Hot-wire Ammeter

The construction of a hot-wire ammeter is shown in figure (39.14). A platinum–iridium wire AB is fixed tightly between two fixed ends A and B. A spring is fixed at one end C and is permanently connected to a thin wire at the other end. The thin wire is wound several times over a cylinder D and the end is connected to the middle point of AB. The cylinder can rotate about its axis. A pointer connected to the

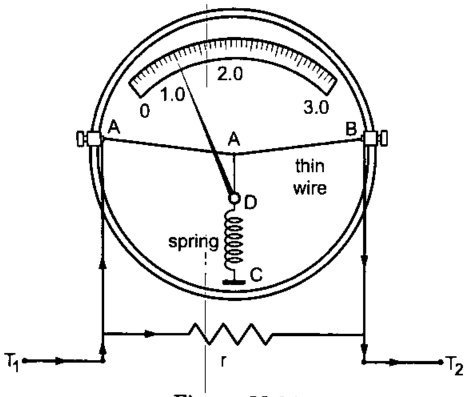

Figure 39.14

cylinder moves along a graduated scale when the cylinder rotates. A small resistance r is connected in parallel to the wire AB as a shunt. This makes the total resistance of the ammeter small so that it does not appreciably alter the current in the circuit. The points A and B are connected to the outer terminals T_1 and T_2.

The current to be measured is passed through the instrument via T_1, T_2. The wire AB gets heated due to the current, the rise in temperature being proportional to i_{rms}^2. The length of the wire increases and consequently its tension decreases. Because of the tension in the spring on the other side, the cylinder rotates a little and the pointer deflects along the scale. The deflection is proportional to i_{rms}^2 but the scale is graduated in such a way that the reading gives directly the rms current.

Hot-wire Voltmeter

The construction of a hot-wire voltmeter is almost identical to a hot-wire ammeter except that a high resistance R is connected in series with the wire AB in place of the shunt r (figure 39.15). The alternating voltage to be measured is applied across T_1 and T_2. A current passes through AB and the pointer attached to the cylinder deflects. The deflection is proportional to V_{rms}^2. The scale is graduated in such a way that it reads directly the rms voltage.

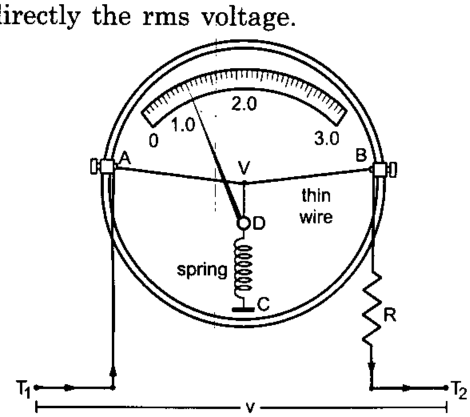

Figure 39.15

38.10 DC DYNAMO

An AC dynamo converts mechanical energy into electrical energy and it supplies alternating current in the circuit connected to it. A DC dynamo also converts mechanical energy into electrical energy but it supplies current in one direction only in the circuit connected to it.

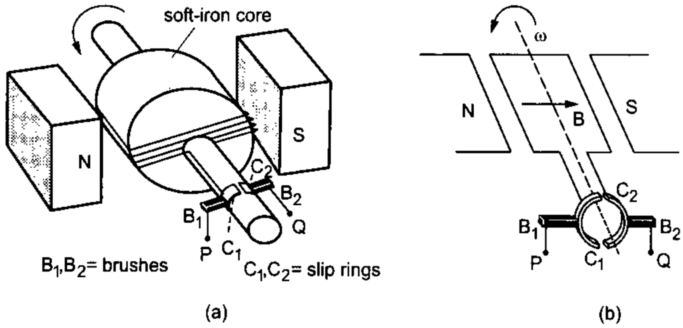

Figure 39.16

The basic design of a DC dynamo (figure 39.16a) is the same as that of an AC dynamo except for the slip rings. Figure (39.16b) shows a simplified diagram of the same. The slip rings are in the form of a split cylinder (figure 39.16). The ends of the armature (coil) are connected separately to the two halves C_1 and C_2 of the cylinders. The armature is rotated by some external agency. The split cylinder rotates with the armature. Two carbon brushes B_1 and B_2 press against the rotating halves C_1 and C_2. As the gaps pass under the brushes, the contacts to the external circuit are reversed. For half of a period of rotation, the terminal P is connected to C_1 and the terminal Q to C_2. For the other half of the period, P is connected to C_2 and Q to C_1. It is arranged in such a way that the gaps pass under the brushes at the time the emf becomes zero. Thus, although emf becomes negative, the current in the external circuit continues in the same direction (figure 39.17a). The system consisting of the split cylinders with brushes is also called a *slip-ring commutator.*

Although the current is unidirectional, its magnitude oscillates in time. To reduce the variation in the current, another coil perpendicular to the first

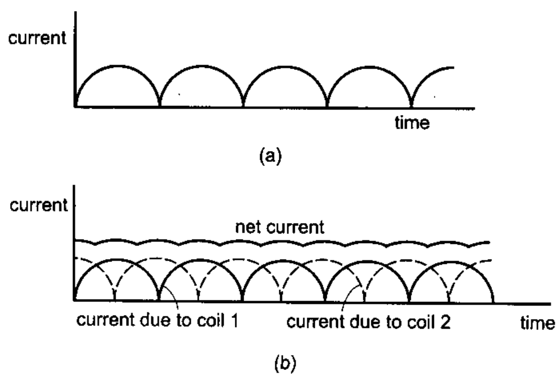

Figure 39.17

one is added in the system. The emf from this coil, again arranged properly with slip-ring commutator, is fed to the external circuit. The emf from this coil is maximum when the emf from the first coil is zero and vice versa. The sum of the two contains less oscillations and the current is more nearly constant (figure 39.17b). One can increase the number of coils to reduce the variation further.

39.11 DC MOTOR

A motor is used to convert electrical energy into mechanical energy and rotate a mechanical load. The principle of a DC motor is the same as that of a moving-coil galvanometer. The arrangement is basically the same as that of a DC dynamo. We can refer to figure (39.16) for its description. It has the field magnets, the armature, the slip rings and the brushes. A battery or the output of a DC generator is connected to the brushes through the outer terminals P and Q. The battery drives a current in the coil and because of the magnetic field, a torque acts on it. This torque rotates the coil which is on a shaft to which the mechanical load is attached. This way the load is rotated. The torque depends on the orientation of the coil besides the strength of the current in it. It is zero when the coil is perpendicular to the field and is maximum in magnitude when it is parallel to the field. As the coil rotates, an induced emf e is produced opposite to the applied emf $\mathcal{E}$. If the resistance of the circuit is R, the current at any instant is given by $i = (\mathcal{E} - e)/R$.

39.12 TRANSFORMER

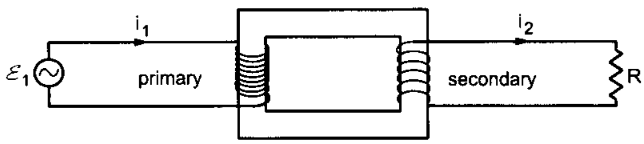

Figure 39.18

A transformer is used either to obtain a high AC voltage from a low-voltage AC source or to obtain a low AC voltage from a high-voltage AC source. The design of a simple transformer is shown in figure (39.18). Two coils are wound separately on a laminated soft-iron core. One of the coils is called the *primary* and the other is called the *secondary*. The original source of alternating voltage is connected across the primary. An induced emf appears across the ends of the secondary which is used to drive current in the desired circuit.

Suppose there are N_1 turns in the primary and N_2 turns in the secondary. An alternating emf $\mathcal{E}_1$ is applied across the primary which produces a current i_1 in the primary circuit and a current i_2 in the secondary circuit. The currents in the coils produce a magnetization in the soft-iron core and there is a corresponding magnetic field B inside the core. The field due to magnetization of the core is large as compared to the field due to the currents in the coils. We assume that the field is constant in magnitude everywhere in the core and hence its flux (BA) through each turn is the same for the primary as well as for the secondary coil. Let the flux through each turn be Φ. The emf induced in the primary is $-N_1 \frac{d\Phi}{dt}$ and that induced in the secondary is $-N_2 \frac{d\Phi}{dt} = \mathcal{E}_2$. If we neglect the resistance in the primary circuit, Kirchhoff's loop law applied to the primary circuit gives

$$\mathcal{E}_1 - N_1 \frac{d\Phi}{dt} = 0$$

or, $$\mathcal{E}_1 = N_1 \frac{d\Phi}{dt} \cdot \qquad \text{... (i)}$$

Also, $$\mathcal{E}_2 = -N_2 \frac{d\Phi}{dt} \cdot \qquad \text{... (ii)}$$

From (i) and (ii),

$$\mathcal{E}_2 = -\frac{N_2}{N_1} \mathcal{E}_1. \qquad \text{... (39.16)}$$

The minus sign shows that $\mathcal{E}_2$ is 180° out of phase with $\mathcal{E}_1$. Equations (i), (ii) and (39.16) are valid for all values of currents in the primary and the secondary circuits.

Power Transfer

Let us first consider the case when the terminals of the secondary are not connected to any external circuit. The secondary circuit is incomplete and the current through it is zero. Suppose, the current in the primary is i_s in this case (the subscript s stands for the source and not for the secondary). As we have neglected the resistance in the primary circuit, it is a purely inductive circuit. The current has a phase difference of 90° with the applied emf $\mathcal{E}_1$ and hence the power delivered by the AC source is zero. The power in the secondary circuit is anyway zero as there is no current in this circuit.

Now suppose, the terminals of the secondary are joined to a resistance R. There will be an alternating current i_2 through R. There will be additional emf's induced in the primary as well as in the secondary due to i_2. But the net induced emf in the primary should remain equal and opposite to the source-emf $\mathcal{E}_1$ by (i). So, there will be an additional current i_1 in the primary circuit which will cancel the emf induced due to i_2. Thus the current in the primary will be $i_s + i_1$ and in the secondary i_2. The emf in the secondary will remain $\mathcal{E}_2$ as given by (ii).

As the induced emf's due to i_1 and i_2 always cancel each other, the two alternating currents should be 180° out of phase. Also, i_2 is in phase with $\mathcal{E}_2$ (purely resistive circuit), and $\mathcal{E}_1$ is 180° out of phase with $\mathcal{E}_2$ (equation 39.16). This shows that i_1 is in phase with $\mathcal{E}_1$ (figure 39.19).

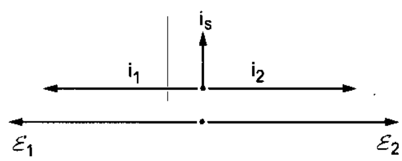

Figure 39.19

The primary current (current in the primary circuit) $i_s + i_1$, therefore, has a component i_s which is at a phase difference of 90° from the applied emf $\mathcal{E}_1$ and a component i_1 which is in phase with this emf. The power delivered by the AC source is, therefore, $\mathcal{E}_1 i_1$. The power consumed by the resistance in the secondary circuit is $\mathcal{E}_2 i_2$. Neglecting any loss of energy elsewhere,

$$\mathcal{E}_1 i_1 = \mathcal{E}_2 i_2. \quad \text{... (i)}$$

Using equation (39.16),

$$i_2 = -\frac{N_1}{N_2} i_1. \quad \text{... (39.17)}$$

The minus sign shows that i_2 is 180° out of phase with i_1.

Quite often, the additional current i_1 in the primary is much larger than the original current i_s. This can be easily shown by connecting an electric bulb in series with the primary. The bulb glows much brighter when the secondary circuit is completed than when it is open. If i_s is negligible as compared to i_1, equation (39.17) gives the relation between the net currents.

Step-up and Step-down Transformers

If $N_2 > N_1$, the secondary emf $\mathcal{E}_2$ is larger in magnitude than the primary emf $\mathcal{E}_1$. This type of transformer is called a *step-up* transformer. The secondary current is less than the primary current. The primary coil is made from a thick wire so that it can sustain the high current.

If $N_2 < N_1$, the emf in the secondary circuit is smaller in magnitude than the primary emf. This type of transformer is called a *step-down* transformer. The secondary current is more than the primary current and the wire used to make the secondary coil should be sufficiently thick to carry the high current.

Efficiency of a Transformer

In an ordinary transformer, there is some loss of energy due to primary resistance, hysteresis in the core, eddy currents in the core, etc. The efficiency of a transformer is defined as

$$\eta = \frac{\text{output power}}{\text{input power}}.$$

Efficiencies of the order of 99% can be easily achieved.

Example 39.5

A radio set operates at 6 V DC. A transformer with 18 turns in the secondary coil is used to step down the input 220 V AC emf to 6 V AC emf. This AC emf is then rectified by another circuit to give 6 V DC which is fed to the radio. Find the number of turns in the primary.

Solution :

We have,

$$\left|\frac{\mathcal{E}_2}{\mathcal{E}_1}\right| = \frac{N_2}{N_1}$$

or,

$$N_1 = \left|\frac{\mathcal{E}_1}{\mathcal{E}_2}\right| N_2 = \frac{220}{6} \times 18 = 660.$$

Transmission of Power

The fact that an AC voltage can be stepped up or stepped down, has application in transmission of power from the electricity generation plants to the users. Generally, these plants are quite far away from the actual areas where the power is used. Power is transmitted through several hundred kilometres of wires before it is used. Because of the resistance of these wires, some energy is lost in Joule heating in the form of i^2Rt. The plant can supply a fixed power depending on its capacity. If this power is supplied at a high voltage, the current is small. Correspondingly, the loss of power in transmission is small. So, the voltage at the electricity generation plant is stepped up to, say, 66 kV and fed to the transmission lines. In a town or city, the voltage is stepped down to the required value such as 220 V.

Worked Out Examples

1. *A resistance of* $20\,\Omega$ *is connected to a source of alternating current rated* 110 V, 50 Hz. *Find (a) the rms current, (b) the maximum instantaneous current in the resistor and (c) the time taken by the current to change from its maximum value to the rms value.*

Solution :

(a) The rms potential difference = 110 V and so

the rms current $= \dfrac{110\text{ V}}{20\,\Omega} = 5{\cdot}5$ A.

(b) The maximum instantaneous current

$$= \sqrt{2}\text{ (rms current)}$$

$$= \sqrt{2} \times 5{\cdot}5\text{ A} = 7{\cdot}8\text{ A}.$$

(c) Let the current be $i = i_0 \sin \omega t$.

If t_1 and t_2 be the time instants for consecutive appearances of the maximum value and the rms value of the current,

$$i_0 = i_0 \sin \omega t_1$$

and
$$\frac{i_0}{\sqrt{2}} = i_0 \sin \omega t_2.$$

If $\omega t_1 = \dfrac{\pi}{2}$, $\omega t_2 = \dfrac{\pi}{2} + \dfrac{\pi}{4}$.

Hence, $t_2 - t_1 = \dfrac{\pi}{4\omega}$

$$= \frac{\pi}{4 \times 2\pi\nu} = \frac{1}{8 \times 50}\text{ s} = 2{\cdot}5\text{ ms}.$$

2. *The electric current in a circuit is given by* $i = i_0(t/\tau)$ *for some time. Calculate the rms current for the period* $t = 0$ *to* $t = \tau$.

Solution :

The mean square current is

$$\overline{i^2} = \frac{1}{\tau}\int_0^{\tau} i_0^2 (t/\tau)^2\, dt = \frac{i_0^2}{\tau^3}\int_0^{\tau} t^2\, dt = \frac{i_0^2}{3}.$$

Thus, the rms current is

$$i_{rms} = \sqrt{\overline{i^2}} = \frac{i_0}{\sqrt{3}}.$$

3. *A coil having a resistance of* $50{\cdot}0\ \Omega$ *and an inductance of* 0·500 *henry is connected to an AC source of* 110 *volts,* 50·0 cycle/s. *Find the rms value of the current in the circuit.*

Solution :

The angular frequency $\omega = 2\pi\nu = 100\pi\text{ s}^{-1}$.

The impedance of the coil

$$= \sqrt{R^2 + L^2\omega^2}$$

$$= \sqrt{(50\,\Omega)^2 + (0{\cdot}50\text{ H} \times 100\pi\text{ s}^{-1})^2}$$

$$= \sqrt{2500\,\Omega^2 + 2500\,\pi^2\Omega^2} = 164{\cdot}8\,\Omega.$$

The rms current is $\dfrac{\mathcal{E}_{rms}}{Z} = \dfrac{110\text{ V}}{164{\cdot}8\,\Omega} \approx 0{\cdot}667$ A.

The peak current $= \sqrt{2}$ (rms current) $\approx 0{\cdot}943$ A.

4. *A capacitor of capacitance* $100\ \mu\text{F}$ *and a coil of resistance* $50\ \Omega$ *and inductance* 0·5 H *are connected in series with a* 110 V, 50 Hz *AC source. Find the rms value of the current.*

Solution :

The resistance of the circuit is $R = 50\ \Omega$.

The reactance of the capacitor $= \dfrac{1}{\omega C}$

$$= \frac{1}{(2\pi \times 50\text{ s}^{-1})\,(100 \times 10^{-6}\text{ F})} = 31{\cdot}8\ \Omega.$$

The reactance of the inductor $= \omega L$

$$= (2\pi \times 50\text{ s}^{-1})\,(0{\cdot}5\text{ henry}) = 157\ \Omega.$$

The reactance of the circuit $= X = \dfrac{1}{\omega C} - L\omega$

$$= 31{\cdot}8\ \Omega - 157\ \Omega = -125{\cdot}2\ \Omega.$$

Hence, the impedance $Z = \sqrt{R^2 + X^2}$

$$= \sqrt{(50\ \Omega)^2 + (125{\cdot}2\ \Omega)^2} \approx 134{\cdot}6\ \Omega.$$

The rms current $= \dfrac{E_{rms}}{Z} = \dfrac{110\text{ V}}{134{\cdot}6\ \Omega} = 0{\cdot}82$ A.

5. *A capacitor of capacitance* $12{\cdot}0\ \mu\text{F}$ *is joined to an AC source of frequency* 200 Hz. *The rms current in the circuit is* 2·00 A. *(a) Find the rms voltage across the capacitor. (b) Find the average energy stored in the electric field between the plates of the capacitor.*

Solution :

(a) The impedance of the capacitor $= \dfrac{1}{\omega C}$

$$= \frac{1}{(2\pi \times 200\text{ s}^{-1})\,(12\ \mu\text{F})} = 66{\cdot}3\ \Omega.$$

The rms voltage across the capacitor

$$= i_{rms} Z = 2{\cdot}0\text{ A} \times 66{\cdot}3\ \Omega \approx 133\text{ V}.$$

(b) The energy stored in the electric field $= \dfrac{1}{2}CV^2$.

Hence the average energy stored $= \dfrac{1}{2}C\overline{V^2}$.

But $\quad \overline{V^2} = (V_{rms})^2$.

Thus, the average energy stored

$$= \frac{1}{2} \times (12\ \mu\text{F}) \times (133\text{ V})^2 \approx 0{\cdot}106\text{ J}.$$

6. *A series AC circuit contains an inductor* (20 mH), *a capacitor* (100 μF), *a resistor* (50 Ω) *and an AC source of* 12 V, 50 Hz. *Find the energy dissipated in the circuit in* 1000 s.

Solution :

The time period of the source is

$$T = 1/\nu = 20 \text{ ms}.$$

The given time 1000 s is much larger than the time period. Hence we can write the average power dissipated as

$$P_{av} = V_{rms}\, i_{rms} \cos\varphi$$

where $\cos\varphi = R/Z$ is the power factor. Thus,

$$P_{av} = V_{rms} \frac{V_{rms}}{Z}\frac{R}{Z} = \frac{R\, V_{rms}^2}{Z^2}$$

$$= \frac{(50\ \Omega)\,(12\ \text{V})^2}{Z^2}$$

$$= \frac{7200}{Z^2}\ \Omega \text{V}^2. \quad \ldots \text{(i)}$$

The capacitive reactance $\dfrac{1}{\omega C} = \dfrac{1}{2\pi \times 50 \times 100 \times 10^{-6}}\ \Omega$

$$= \frac{100}{\pi}\ \Omega.$$

The inductive reactance $= \omega L$

$$= 2\pi \times 50 \times 20 \times 10^{-3}\ \Omega = 2\pi\ \Omega.$$

The net reactance is $X = \dfrac{1}{\omega C} - \omega L$

$$= \frac{100}{\pi}\ \Omega - 2\pi\ \Omega \approx 25{\cdot}5\ \Omega.$$

Thus,

$$Z^2 = (50\ \Omega)^2 + (25{\cdot}5\ \Omega)^2 = 3150\ \Omega^2.$$

From (i), average power $P_{av} = \dfrac{7200\ \Omega\text{–V}^2}{3150\ \Omega^2} = 2{\cdot}286$ W.

The energy dissipated in 1000 s $= P_{av} \times 1000$ s

$$\approx 2{\cdot}3 \times 10^3 \text{ J}.$$

7. *An inductor of inductance* 100 mH *is connected in series with a resistance, a variable capacitance and an AC source of frequency* 2·0 kHz. *What should be the value of the capacitance so that maximum current may be drawn into the circuit ?*

Solution :

This is an *LCR* series circuit. The current will be maximum when the net reactance is zero. For this,

$$\frac{1}{\omega C} = \omega L$$

or, $$C = \frac{1}{\omega^2 L} = \frac{1}{4\pi^2 \times (2{\cdot}0 \times 10^3\ \text{s}^{-1})^2\,(0{\cdot}1\ \text{H})}$$

$$= 63 \text{ nF}.$$

8. *An inductor coil joined to a* 6 V *battery draws a steady current of* 12 A. *This coil is connected to a capacitor and an AC source of rms voltage* 6 V *in series. If the current in the circuit is in phase with the emf, find the rms current.*

Solution :

The resistance of the coil is $R = \dfrac{6\ \text{V}}{12\ \text{A}} = 0{\cdot}5\ \Omega.$

In the AC circuit, the current is in phase with the emf. This means that the net reactance of the circuit is zero. The impedance is equal to the resistance, i.e., $Z = 0{\cdot}5\ \Omega$.

The rms current $= \dfrac{\text{rms voltage}}{Z} = \dfrac{6\ \text{V}}{0{\cdot}5\ \Omega} = 12$ A.

□

QUESTIONS FOR SHORT ANSWER

1. What is the reactance of a capacitor connected to a constant DC source ?
2. The voltage and current in a series AC circuit are given by
   $$V = V_0 \cos\omega t \text{ and } i = i_0 \sin\omega t.$$
   What is the power dissipated in the circuit ?
3. Two alternating currents are given by
   $$i_1 = i_0 \sin\omega t \text{ and } i_2 = i_0 \sin\left(\omega t + \frac{\pi}{3}\right).$$
   Will the rms values of the currents be equal or different ?
4. Can the peak voltage across the inductor be greater than the peak voltage of the source in an *LCR* circuit ?
5. In a circuit containing a capacitor and an AC source, the current is zero at the instant the source voltage is maximum. Is it consistent with Ohm's law ?
6. An AC source is connected to a capacitor. Will the rms current increase, decrease or remain constant if a dielectric slab is inserted into the capacitor ?
7. When the frequency of the AC source in an *LCR* circuit equals the resonant frequency, the reactance of the circuit is zero. Does it mean that there is no current through the inductor or the capacitor ?
8. When an AC source is connected to a capacitor there is a steady-state current in the circuit. Does it mean that

the charges jump from one plate to the other to complete the circuit ?

9. A current $i_1 = i_0 \sin \omega t$ passes through a resistor of resistance R. How much thermal energy is produced in one time period ? A current $i_2 = -i_0 \sin \omega t$ passes through the resistor. How much thermal energy is produced in one time period ? If i_1 and i_2 both pass through the resistor simultaneously, how much thermal energy is produced ? Is the principle of superposition obeyed in this case ?

10. Is energy produced when a transformer steps up the voltage ?

11. A transformer is designed to convert an AC voltage of 220 V to an AC voltage of 12 V. If the input terminals are connected to a DC voltage of 220 V, the transformer usually burns. Explain.

12. Can you have an AC series circuit in which there is a phase difference of (a) 180° (b) 120° between the emf and the current ?

13. A resistance is connected to an AC source. If a capacitor is included in the series circuit, will the average power absorbed by the resistance increase or decrease ? If an inductor of small inductance is also included in the series circuit, will the average power absorbed increase or decrease further ?

14. Can a hot-wire ammeter be used to measure a direct current having a constant value ? Do we have to change the graduations ?

OBJECTIVE I

1. A capacitor acts as an infinite resistance for
(a) DC (b) AC
(c) DC as well as AC (d) neither AC nor DC.

2. An AC source producing emf
$$\mathcal{E} = \mathcal{E}_0\left[\cos(100\,\pi\,\text{s}^{-1})t + \cos(500\,\pi\,\text{s}^{-1})t\right]$$
is connected in series with a capacitor and a resistor. The steady-state current in the circuit is found to be
$$i = i_1 \cos\left[(100\,\pi\,\text{s}^{-1})t + \varphi_1\right] + i_2 \cos\left[(500\,\pi\,\text{s}^{-1})t + \phi_2\right].$$
(a) $i_1 > i_2$ (b) $i_1 = i_2$ (c) $i_1 < i_2$
(d) The information is insufficient to find the relation between i_1 and i_2.

3. The peak voltage in a 220 V AC source is
(a) 220 V (b) about 160 V
(c) about 310 V (d) 440 V.

4. An AC source is rated 220 V, 50 Hz. The average voltage is calculated in a time interval of 0·01 s. It
(a) must be zero (b) may be zero
(c) is never zero (d) is $(220/\sqrt{2})$V.

5. The magnetic field energy in an inductor changes from maximum value to minimum value in 5·0 ms when connected to an AC source. The frequency of the source is
(a) 20 Hz (b) 50 Hz (c) 200 Hz (d) 500 Hz.

6. Which of the following plots may represent the reactance of a series LC combination ?

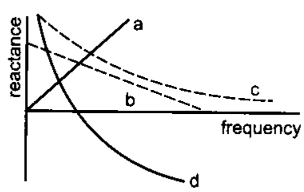

Figure 39-Q1

7. A series AC circuit has a resistance of 4 Ω and a reactance of 3 Ω. The impedance of the circuit is
(a) 5 Ω (b) 7 Ω (c) 12/7 Ω (d) 7/12 Ω.

8. Transformers are used
(a) in DC circuits only (b) in AC circuits only
(c) in both DC and AC circuits
(d) neither in DC nor in AC circuits.

9. An alternating current is given by
$$i = i_1 \cos \omega t + i_2 \sin \omega t.$$
The rms current is given by
(a) $\dfrac{i_1 + i_2}{\sqrt{2}}$ (b) $\dfrac{|i_1 + i_2|}{\sqrt{2}}$ (c) $\sqrt{\dfrac{i_1^2 + i_2^2}{2}}$ (d) $\sqrt{\dfrac{i_1^2 + i_2^2}{\sqrt{2}}}$.

10. An alternating current having peak value 14 A is used to heat a metal wire. To produce the same heating effect, a constant current i can be used where i is
(a) 14 A (b) about 20 A (c) 7 A (d) about 10 A.

11. A constant current of 2·8 A exists in a resistor. The rms current is
(a) 2·8 A (b) about 2 A
(c) 1·4 A (d) undefined for a direct current.

OBJECTIVE II

1. An inductor, a resistor and a capacitor are joined in series with an AC source. As the frequency of the source is slightly increased from a very low value, the reactance
(a) of the inductor increases
(b) of the resistor increases
(c) of the capacitor increases
(d) of the circuit increases.

2. The reactance of a circuit is zero. It is possible that the circuit contains
(a) an inductor and a capacitor
(b) an inductor but no capacitor

(c) a capacitor but no inductor
(d) neither an inductor nor a capacitor.

3. In an AC series circuit, the instantaneous current is zero when the instantaneous voltage is maximum. Connected to the source may be a
(a) pure inductor (b) pure capacitor
(c) pure resistor
(d) combination of an inductor and a capacitor.

4. An inductor-coil having some resistance is connected to an AC source. Which of the following quantities have zero average value over a cycle ?
(a) Current (b) Induced emf in the inductor
(c) Joule heat
(d) Magnetic energy stored in the inductor

5. The AC voltage across a resistance can be measured using
(a) a potentiometer (b) a hot-wire voltmeter
(c) a moving-coil galvanometer
(d) a moving-magnet galvanometer.

6. To convert mechanical energy into electrical energy, one can use
(a) DC dynamo (b) AC dynamo
(c) motor (d) transformer.

7. An AC source rated 100 V (rms) supplies a current of 10 A (rms) to a circuit. The average power delivered by the source
(a) must be 1000 W (b) may be 1000 W
(c) may be greater than 1000 W
(d) may be less than 1000 W.

EXERCISES

1. Find the time required for a 50 Hz alternating current to change its value from zero to the rms value.

2. The household supply of electricity is at 220 V (rms value) and 50 Hz. Find the peak voltage and the least possible time in which the voltage can change from the rms value to zero.

3. A bulb rated 60 W at 220 V is connected across a household supply of alternating voltage of 220 V. Calculate the maximum instantaneous current through the filament.

4. An electric bulb is designed to operate at 12 volts DC. If this bulb is connected to an AC source and gives normal brightness, what would be the peak voltage of the source ?

5. The peak power consumed by a resistive coil when connected to an AC source is 80 W. Find the energy consumed by the coil in 100 seconds which is many times larger than the time period of the source.

6. The dielectric strength of air is $3{\cdot}0 \times 10^{6}$ V/m. A parallel-plate air-capacitor has area 20 cm^2 and plate separation 0·10 mm. Find the maximum rms voltage of an AC source which can be safely connected to this capacitor.

7. The current in a discharging LR circuit is given by $i = i_0 e^{-t/\tau}$ where τ is the time constant of the circuit. Calculate the rms current for the period $t = 0$ to $t = \tau$.

8. A capacitor of capacitance 10 μF is connected to an oscillator giving an output voltage $\mathcal{E} = (10\ \mathrm{V})\sin \omega t$. Find the peak currents in the circuit for $\omega = 10\ s^{-1}$, $100\ s^{-1}$, $500\ s^{-1}$, $1000\ s^{-1}$.

9. A coil of inductance 5·0 mH and negligible resistance is connected to the oscillator of the previous problem. Find the peak currents in the circuit for $\omega = 100\ s^{-1}$, $500\ s^{-1}$, $1000\ s^{-1}$.

10. A coil has a resistance of 10 Ω and an inductance of 0·4 henry. It is connected to an AC source of 6·5 V, $\frac{30}{\pi}$ Hz. Find the average power consumed in the circuit.

11. A resistor of resistance 100 Ω is connected to an AC source $\mathcal{E} = (12\ \mathrm{V}) \sin (250\,\pi\ s^{-1})t$. Find the energy dissipated as heat during $t = 0$ to $t = 1{\cdot}0$ ms.

12. In a series RC circuit with an AC source, $R = 300\ \Omega$, $C = 25\ \mu F$, $\mathcal{E}_0 = 50$ V and $\nu = 50/\pi$ Hz. Find the peak current and the average power dissipated in the circuit.

13. An electric bulb is designed to consume 55 W when operated at 110 volts. It is connected to a 220 V, 50 Hz line through a choke coil in series. What should be the inductance of the coil for which the bulb gets correct voltage ?

14. In a series LCR circuit with an AC source, $R = 300\ \Omega$, $C = 20\ \mu F$, $L = 1{\cdot}0$ henry, $\mathcal{E}_{rms} = 50$ V and $\nu = 50/\pi$ Hz. Find (a) the rms current in the circuit and (b) the rms potential differences across the capacitor, the resistor and the inductor. Note that the sum of the rms potential differences across the three elements is greater than the rms voltage of the source.

15. Consider the situation of the previous problem. Find the average electric field energy stored in the capacitor and the average magnetic field energy stored in the coil.

16. An inductance of 2·0 H, a capacitance of 18 μF and a resistance of 10 kΩ are connected to an AC source of 20 V with adjustable frequency. (a) What frequency should be chosen to maximise the current in the circuit ? (b) What is the value of this maximum current ?

17. An inductor-coil, a capacitor and an AC source of rms voltage 24 V are connected in series. When the frequency of the source is varied, a maximum rms current of 6·0 A is observed. If this inductor coil is connected to a battery of emf 12 V and internal resistance 4·0 Ω, what will be the current ?

18. Figure (39-E1) shows a typical circuit for low-pass filter. An AC input $V_i = 10$ mV is applied at the left end and

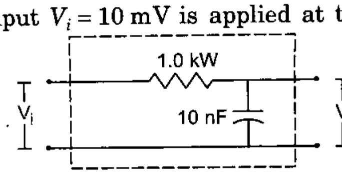

Figure 39-E1

the output V_0 is received at the right end. Find the output voltages for $\nu = 10$ kHz, 100kHz, 1·0 MHz and 10·0 MHz. Note that as the frequency is increased the output decreases and hence the name low-pass filter.

19. A transformer has 50 turns in the primary and 100 in the secondary. If the primary is connected to a 220 V DC supply, what will be the voltage across the secondary ?

□

ANSWERS

OBJECTIVE I

1. (a)	2. (c)	3. (c)	4. (b)	5. (b)	6. (d)
7. (a)	8. (b)	9. (c)	10. (d)	11. (a)	

OBJECTIVE II

1. (a)	2. (a), (d)	3. (a), (b), (d)
4. (a), (b)	5. (b)	6. (a), (b)
7. (b), (d)		

EXERCISES

1. 2·5 ms
2. 311 V, 2·5 ms
3. 0·39 A
4. 17 volts
5. 4·0 kJ
6. 210 V
7. $\frac{i_0}{e}\sqrt{(e^2-1)/2}$
8. $1{\cdot}0 \times 10^{-3}$ A, 0·01 A, 0·05 A, 0·1 A
9. 20 A, 4·0 A, 0·20 A
10. 5/8 W
11. $2{\cdot}61 \times 10^{-4}$ J
12. 0·10 A, 1·5 W
13. 1·2 H
14. (a) 0·10 A (b) 50 V, 30 V, 10 V
15. 25 mJ, 5 mJ
16. (a) 27 Hz (b) 2 mA
17. 1·5 A
18. 8·5 mV, 1·6 mV, 0·16 mV, 16 μV
19. zero.

□