Chapter 10 Differentiability Ex 10.1 Q5

f(x) = |x|+|x-1| in theinterval (-1, 2).

$$f(x)=\begin{cases} x+x+1 & -1<x<0 \\ 1 & 0\le x\le 1 \\ -x-x+1 & 1<x<2 \end{cases}$$

$$f(x)=\begin{cases} 2x+1 & -1<x<0 \\ 1 & 0\le x\le 1 \\ -2x+1 & 1<x<2 \end{cases}$$

We know that a polynomial and a constant function is continuous and differentiable everywhere.
So, f(x) is continuous and differentiable for $x\in(-1,\ 0)$, $x\in(0,\ 1)$ and (1, 2).

We need to check continuity and differentiability at x = 0 and x = 1.

Continuity at x = 0

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 2x+1 = 1$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 1 = 1$$

$$f(0) = 1$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$$

$\therefore$ f(x) is continuous at x = 0.

Continuity at x = 1

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 1 = 1$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 1 = 1$$

$$f(1) = 1$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(1)$$

$\therefore$ f(x) is continuous at x = 1.

Differentiability at x = 0

$$(\text{LHD at } x=0) = \lim_{x\to 0^-}\frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^-}\frac{2x+1-1}{x-0} = \lim_{x\to 0^-}\frac{2x}{x} = 2$$

$$(\text{RHD at } x=0) = \lim_{x\to 0^+}\frac{f(x)-f(0)}{x-0} = \lim_{x\to 0^+}\frac{1-1}{x} = \lim_{x\to 0^+}\frac{0}{x} = 0$$

$\therefore$ (LHD at x = 0) $\neq$ (RHD at x = 0)

So, f(x) is differentiable at x = 0.

Differentiability at $x = 1$

$$(\text{LHD at } x = 1) = \lim_{x\to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^-} \frac{1-1}{x-1} = 0$$

$$(\text{RHD at } x = 1) = \lim_{x\to 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^+} \frac{-2x+1-1}{x-1} \to \infty$$

$\therefore$ (LHD at $x = 1$) $\neq$ (RHD at $x = 1$)

So, $f(x)$ is not differentiable at $x = 1$.

So, $f(x)$ is continuous on $(-1, 2)$ but not differentiable at $x = 0, 1$.

Chapter 10 Differentiability Ex 10.1 Q6

$$f(x) = \begin{cases} x, & x \le 1 \\ 2 - x, & 1 \le x \le 2 \\ -2 + 3x - x^2, & x > 2 \end{cases}$$

Differentiability at $x = 1$

$$(\text{LHD at } x = 1) = \lim_{x\to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^-} \frac{x-1}{x-1} = 1$$

$$(\text{RHD at } x = 1) = \lim_{x\to 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^+} \frac{2-x-1}{x-1} = \lim_{x\to 1^+} \frac{1-x}{x-1} = -1$$

$\therefore$ (LHD at $x = 1$) $\neq$ (RHD at $x = 1$)

So, $f(x)$ is not differentiable at $x = 1$.

Differentiability at $x = 2$

$$(\text{LHD at } x = 2) = \lim_{x\to 2^-} \frac{f(x) - f(2)}{x - 2} = \lim_{x\to 2^-} \frac{2-x-0}{x-2} = \lim_{x\to 2^-} \frac{2-x}{x-2} = -1$$

$$(\text{RHD at } x = 2) = \lim_{x\to 2^+} \frac{f(x) - f(2)}{x - 2} = \lim_{x\to 2^+} \frac{-2+3x-x^2-0}{x-2} = \lim_{x\to 2^+} \frac{(1-x)(x-2)}{x-2} = -1$$

$\therefore$ (LHD at $x = 2$) = (RHD at $x = 2$)

So, $f(x)$ is differentiable at $x = 2$.

*********** END **********