



Mathematical Induction Ex 12.2 Q7

$$\text{Let } P(n) : \frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$

Put $n = 1$

$$P(1) : \frac{1}{1.4} = \frac{1}{4}$$

$$\frac{1}{4} = \frac{1}{4}$$

$\Rightarrow P(n)$ is true for $n = 1$

Let $P(n)$ is true for $n = k$, so

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots + \frac{1}{(3k-2)(3k+1)} = \frac{k}{3k+1} \quad \text{--- (1)}$$

We have to show that,

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots + \frac{1}{(3k-2)(3k+1)} + \frac{1}{(3k+1)(3k+4)} = \frac{(k+1)}{(3k+4)}$$

Now,

$$\left\{ \frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots + \frac{1}{(3k-2)(3k+1)} \right\} + \frac{1}{(3k+1)(3k+4)}$$

Now,

$$\begin{aligned}
 & \left\{ \frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \dots + \frac{1}{(3k-2)(3k+1)} \right\} + \frac{1}{(3k+1)(3k+4)} \\
 &= \frac{k}{(3k+1)} + \frac{1}{(3k+1)(3k+4)} \\
 &= \frac{1}{(3k+1)} \left[\frac{k}{1} + \frac{1}{(3k+4)} \right] \\
 &= \frac{1}{(3k+1)} \left[\frac{k(3k+4)+1}{(3k+4)} \right] \\
 &= \frac{1}{(3k+1)} \left[\frac{3k^2+4k+1}{(3k+4)} \right] \\
 &= \frac{1}{(3k+1)} \frac{(3k^2+3k+k+1)}{(3k+4)} \\
 &= \frac{3k(k+1)+(k+1)}{(3k+1)(3k+4)} \\
 &= \frac{(k+1)(3k+1)}{(3k+1)(3k+4)} \\
 &= \frac{(k+1)}{(3k+4)}
 \end{aligned}$$

$\Rightarrow P(n)$ is true for $n = k + 1$

$\Rightarrow P(n)$ is true for all $n \in N$ by *PMI*

Mathematical Induction Ex 12.2 Q8

$$\text{Let } P(n): \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$$

Put $n = 1$

$$\begin{aligned}
 \frac{1}{3.5} &= \frac{1}{3(5)} \\
 \frac{1}{15} &= \frac{1}{15}
 \end{aligned}$$

$\Rightarrow P(n)$ is true for $n = 1$

Let $P(n)$ is true for $n = k$, so

$$\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots + \frac{1}{(2k+1)(2k+3)} = \frac{k}{3(2k+3)} \quad \dots (1)$$

We have to show that,

$$\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots + \frac{1}{(2k+1)(2k+3)} + \frac{1}{(2k+3)(2k+5)} = \frac{(k+1)}{3(2k+5)}$$

Now,

$$\begin{aligned}
 & \left\{ \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots + \frac{1}{(2k+1)(2k+3)} \right\} + \frac{1}{(2k+3)(2k+5)} \\
 &= \frac{k}{3(2k+3)} + \frac{1}{(2k+3)(2k+5)} \quad \text{[Using equation (1)]}
 \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{(2k+3)} \left[\frac{k}{3} + \frac{1}{(2k+5)} \right] \\
&= \frac{1}{(2k+3)} \left[\frac{k(2k+5)+3}{(2k+5)} \right] \\
&= \frac{1}{(2k+3)} \left[\frac{2k^2+5k+3}{(2k+5)} \right] \\
&= \frac{1}{(2k+3)} \left[\frac{2k^2+2k+3k+3}{(2k+5)} \right] \\
&= \frac{1}{(2k+3)} \left[\frac{2k(k+1)+3(k+1)}{(2k+5)} \right] \\
&= \frac{1}{(2k+3)} \left[\frac{(k+1)(2k+3)}{(2k+5)} \right] \\
&= \frac{(k+1)}{2k+5}
\end{aligned}$$

$\Rightarrow P(n)$ is true for $n = k + 1$

$\Rightarrow P(n)$ is true for all $n \in \mathbb{N}$ by *PMI*

***** END *****