



Trigonometric Ratios Ex 5.1 Q19

Answer :

Given: $\cos \theta = \frac{5}{13}$ (1)

To Find:

The value of expression $\frac{\sin^2 \theta - \cos^2 \theta}{2 \sin \theta \cos \theta} \times \frac{1}{\tan^2 \theta}$

Now, we know that

$$\cos \theta = \frac{\text{Base side adjacent to } \angle \theta}{\text{Hypotenuse}} \dots\dots (2)$$

Now when we compare equation (1) and (2)

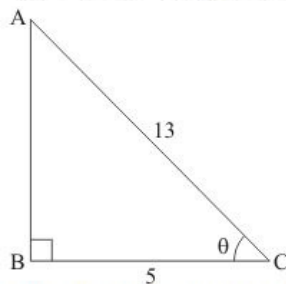
We get,

$$\text{Base side adjacent to } \angle \theta = 5$$

And

$$\text{Hypotenuse} = 13$$

Therefore, Triangle representing angle θ is as shown below



Perpendicular side AB is unknown and it can be found by using Pythagoras theorem

Therefore by applying Pythagoras theorem

We get,

$$AC^2 = AB^2 + BC^2$$

Therefore by substituting the values of known sides

We get,

$$13^2 = AB^2 + 5^2$$

Therefore,

$$AB^2 = 13^2 - 5^2$$

$$AB^2 = 169 - 25$$

$$AB^2 = 144$$

$$AB = \sqrt{144}$$

Therefore,

$$AB = 12 \dots\dots (3)$$

Now, we know that

$$\sin \theta = \frac{\text{Perpendicular side opposite to } \angle \theta}{\text{Hypotenuse}}$$

Now from figure (a)

We get,

$$\sin \theta = \frac{AB}{AC}$$

Therefore from figure (a) and equation (3) ,

$$\sin \theta = \frac{12}{13} \dots\dots (4)$$

Now we know that,

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

Therefore, substituting the value of $\sin \theta$ and $\cos \theta$ from equation (1) and (4)

We get,

$$\tan \theta = \frac{\frac{12}{13}}{\frac{5}{13}}$$

$$\tan \theta = \frac{12}{13} \times \frac{13}{5}$$

Therefore 13 gets cancelled and we get

$$\tan \theta = \frac{12}{5} \dots\dots (5)$$

Now we substitute the value of $\cos \theta$, $\sin \theta$ and $\tan \theta$ from equation (1) , (4) and (5) respectively in the expression below

$$\frac{\sin^2 \theta - \cos^2 \theta}{2 \sin \theta \cos \theta} \times \frac{1}{\tan^2 \theta}$$

Therefore,

We get,

$$\frac{\sin^2 \theta - \cos^2 \theta}{2 \sin \theta \cos \theta} \times \frac{1}{\tan^2 \theta} = \frac{\left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2}{2 \times \left(\frac{12}{13}\right) \times \left(\frac{5}{13}\right)} \times \frac{1}{\left(\frac{12}{5}\right)^2}$$

Therefore by further simplifying we get,

$$\frac{\sin^2 \theta - \cos^2 \theta}{2 \sin \theta \cos \theta} \times \frac{1}{\tan^2 \theta} = \frac{\frac{(12)^2}{(13)^2} - \frac{(5)^2}{(13)^2}}{2 \times \left(\frac{12}{13}\right) \times \left(\frac{5}{13}\right)} \times \frac{1}{\left(\frac{12}{5}\right)^2}$$

$$\begin{aligned} &= \frac{\frac{144}{169} - \frac{25}{169}}{2 \times \frac{12}{13} \times \frac{5}{13}} \times \frac{25}{144} \\ &= \frac{\frac{144 - 25}{169}}{\frac{120}{169}} \times \frac{25}{144} \end{aligned}$$

Now we substitute the value of $\cos \theta$, $\sin \theta$ and $\tan \theta$ from equation (1) , (4) and (5) respectively in the expression below

$$\frac{\sin^2 \theta - \cos^2 \theta}{2 \sin \theta \cos \theta} \times \frac{1}{\tan^2 \theta}$$

Therefore,

We get,

$$\frac{\sin^2 \theta - \cos^2 \theta}{2 \sin \theta \cos \theta} \times \frac{1}{\tan^2 \theta} = \frac{\left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2}{2 \times \left(\frac{12}{13}\right) \times \left(\frac{5}{13}\right)} \times \frac{1}{\left(\frac{12}{5}\right)^2}$$

Therefore by further simplifying we get,

$$\frac{\sin^2 \theta - \cos^2 \theta}{2 \sin \theta \cos \theta} \times \frac{1}{\tan^2 \theta} = \frac{\frac{(12)^2}{(13)^2} - \frac{(5)^2}{(13)^2}}{2 \times \left(\frac{12}{13}\right) \times \left(\frac{5}{13}\right)} \times \frac{1}{\left(\frac{12}{5}\right)^2}$$

$$\begin{aligned} &= \frac{\frac{144}{169} - \frac{25}{169}}{2 \times \frac{12}{13} \times \frac{5}{13}} \times \frac{25}{144} \\ &= \frac{\frac{144 - 25}{169}}{\frac{120}{169}} \times \frac{25}{144} \\ &= \frac{144 - 25}{169} \times \frac{25}{144} \end{aligned}$$

$$= \frac{119}{169} \times \frac{169}{120} \times \frac{25}{144}$$

Now 169 gets cancelled and $\frac{25}{120}$ gets reduced to $\frac{5}{24}$

Therefore

$$\begin{aligned} \frac{\sin^2 \theta - \cos^2 \theta}{2 \sin \theta \cos \theta} \times \frac{1}{\tan^2 \theta} &= \frac{119}{1} \times \frac{1}{24} \times \frac{5}{144} \\ &= \frac{119 \times 5}{24 \times 144} \\ &= \frac{595}{3456} \end{aligned}$$

Therefore the value of $\frac{\sin^2 \theta - \cos^2 \theta}{2 \sin \theta \cos \theta} \times \frac{1}{\tan^2 \theta}$ is $\frac{595}{3456}$

That is $\frac{\sin^2 \theta - \cos^2 \theta}{2 \sin \theta \cos \theta} \times \frac{1}{\tan^2 \theta} = \frac{595}{3456}$

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