Differentials Errors and Approximation Ex14.1 Q9(viii)

Let $x = 4,\ x + \Delta x = 4.04$

$\Delta x = 4.04 - 4$

$\Delta x = 0.04$

Let $y = \log x$

$$\frac{dy}{dx} = \frac{1}{x}$$

$$\left(\frac{dy}{dx}\right)_{x=4} = \frac{1}{4}$$

$$= 0.25$$

Now,

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=4} \times \Delta x$$

$$= (0.25)(0.04)$$

$$\Delta y = 0.01$$

$$\log_e 4.04 = y + \Delta y$$

$$= \log x + (0.01)$$

$$= \log_e 4 + 0.01$$

$$= \frac{\log_e 4}{\log_{10} e} + 0.01$$

$$= \frac{0.6021}{0.4343} + 0.01 \qquad \left[\text{Since, } \log_a b = \frac{\log_c b}{\log_c a}\right]$$

$$= 1.38637 + 0.01$$

$$\log_e 4.04 = 1.39637$$

Differentials Errors and Approximation Ex14.1 Q9(ix)

Let $x = 10,\ x + \Delta x = 10.02$

$\Delta x = 10.02 - 10$

$\Delta x = 0.02$

Let $y = \log_e x$

$$\frac{dy}{dx} = \frac{1}{x}$$

$$\left(\frac{dy}{dx}\right)_{x=10} = \frac{1}{10}$$

$$\left(\frac{dy}{dx}\right)_{x=10} = 0.1$$

Now,

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=10} \times \Delta x$$

$$= (0.1)(0.02)$$

$$\Delta y = 0.002$$

$$\log_e (10.02) = y + \Delta y$$

$$= \log_e x + 0.002$$

$$= \log_e 10 + 0.002$$

$$= 2.3026 + 0.002$$

$$\log_e (10.02) = 2.3046$$

Differentials Errors and Approximation Ex14.1 Q9(x)

Let $x = 10,\ x + \Delta x = 10.1$

$\Delta x = 10.1 - 10$

$\Delta x = 0.1$

Let $y = \log_{10} x$

$$= \frac{\log_e x}{\log_e 10} \qquad \left[\text{Since, } \log_a b = \frac{\log_c a}{\log_c b}\right]$$

$$\left(\frac{dy}{dx}\right) = \frac{1}{x \log_e 10}$$

$$\left(\frac{dy}{dx}\right)_{x=10} = \frac{1}{10 \log_e 10}$$

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=10} \times \Delta x$$

$$= \frac{1}{10(\log_e 10)} \times 0.1$$

$$\Delta y = \frac{0.01}{(\log_e 10)}$$

$$\log_{10} (10.1) = y + \Delta y$$

$$= \log_{10} x + \frac{0.01}{\log_e 10}$$

$$= \log_{10} 10 + 0.01 \log_{10} e \qquad \left[\text{Since, } \log_a b = \frac{1}{\log_b a}\right]$$

$$= 1 + (0.01)(0.4343)$$

$$\log_{10} (10.1) = 1.004343$$

Differentials Errors and Approximation Ex14.1 Q9(xi)

Let $x = 60°,\ x + \Delta x = 61°$

$$\Delta x = 61° - 60°$$

$$\Delta x = 1° = \frac{\pi}{18°} = 0.01745$$

Let $y = \cos x$

$$\frac{dy}{dx} = -\sin x$$

$$\left(\frac{dy}{dx}\right)_{x=60°} = -\sin\left(60^{\circ}\right)$$

$$= -\frac{\sqrt{3}}{2}$$

$$= -0.866$$

$$\Delta y = \left(\frac{dy}{dx}\right)_{x=60°} \times (\Delta x)$$

$$= (-0.866)(0.01745)$$

$$= -0.01511$$

So,

$$\cos 61° = y + \Delta y$$

$$= \cos 60° - 0.01511$$

$$= \frac{1}{2} - 0.01511$$

$$= 0.5 - 0.01511$$

$$\cos 61° = 0.48489$$

********** END **********