Tangents and Normals Ex 16.2 Q5(i)

We know that the equation of tangent and normal to any curve at the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \qquad \text{---(A) Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{---(B) Normal}$$

Where m is slope.

$$x = \theta + \sin\theta, \quad y = 1 + \cos\theta, \quad \theta = \frac{\pi}{2}$$

$$\therefore \qquad P = \left[\left(\frac{\pi}{2} + 1\right), 1\right]$$

and $\qquad \dfrac{dx}{d\theta} = 1 + \cos\theta, \dfrac{dy}{d\theta} = -\sin\theta$

$$\therefore \qquad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \left(\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}\right) = \frac{-1}{+1} = -1$$

Equation of tangent from (A)

$$(y - 1) = -1\left(x - \left(\frac{\pi}{2} + 1\right)\right)$$

$$\Rightarrow \qquad x + y = \frac{\pi}{2} + 1 + 1$$

$$\Rightarrow \qquad 2(x + y) = \pi + 4$$

From (B)

Equation of normal is

$$(y - 1) = 1\left(x - \left(\frac{\pi}{2} + 1\right)\right)$$

$$\Rightarrow \qquad 2(x - y) = \pi$$

Tangents and Normals Ex 16.2 Q5(ii)

We know that the equation of tangent and normal to any curve at the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \qquad \text{---(A) Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{---(B) Normal}$$

Where m is slope.

$$x = \frac{2at^2}{1+t^2}, \qquad y = \frac{2at^3}{1+t^2}, \qquad t = \frac{1}{2}$$

$$\therefore \qquad P = \left(x = \frac{a}{2+\frac{1}{2}} = \frac{2a}{5}, y = \frac{a}{4+1} = \frac{a}{5}\right)$$

Now,

$$\frac{dx}{dt} = \frac{4a + \left(1+t^2\right) - 2at^2\,(2t)}{\left(1+t^2\right)^2}$$

$$= \frac{4at}{\left(1+t^2\right)^2}$$

$$\frac{dy}{dt} = \frac{6at^2\left(1+t^2\right) - \left(2at^3\right)(2t)}{\left(1+t^2\right)^2}$$

$$= \frac{6at^2 - 2at^4}{\left(1+t^2\right)^2}$$

$$\therefore \qquad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6at^2 + 2at^4}{4at}$$

$$\therefore \qquad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{\frac{3a}{2} + \frac{a}{8}}{2a} = \frac{13}{16}$$

From (A)

Equation of tangent is,

$$\left(y - \frac{a}{5}\right) = \frac{13}{16}\left(x - \frac{2a}{5}\right)$$

$$16y - \frac{16a}{5} = 13x - \frac{26a}{5}$$

$$13x - 16y - 2a = 0$$

Equation of normal is,

$$\left(y - \frac{a}{5}\right) = -\frac{16}{13}\left(x - \frac{2a}{5}\right)$$

$$13y - \frac{13a}{5} = -16x + \frac{32a}{5}$$

$$16x + 13y - 9a = 0$$

Tangents and Normals Ex 16.2 Q5(iii)

We know that the equation of tangent and normal to any curve at the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \qquad \text{---(A) Tangent}$$
$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{---(B) Normal}$$

Where m is slope.

$x = at^2, \qquad y = 2at, \qquad t = 1$

$\therefore \qquad P = (a, 2a)$

and

$$\frac{dx}{dt} = 2at, \qquad \frac{dy}{dt} = 2a$$

$$\therefore \qquad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2a} = 1$$

From (A)

Equation of tangent is

$$(y - 2a) = 1(x - a)$$

$\Rightarrow \qquad x - y + a = 0$

From (B)

Equation of normaol is

$$(y - 2a) = -1(x - a)$$

$\Rightarrow \qquad x + y = 3a$

*********** END **********