Question-1

If $^nC_8 = {^nC_2}$, find $^nC_2$.

Ans.

It is known that, $^nC_a = {^nC_b} \Rightarrow a = b$ or $n = a + b$

Therefore,

$^nC_8 = {^nC_2} \Rightarrow n = 8 + 2 = 10$

$\therefore {^nC_2} = {^{10}C_2} = \dfrac{10!}{2!(10-2)!} = \dfrac{10!}{2!8!} = \dfrac{10 \times 9 \times 8!}{2 \times 1 \times 8!} = 45$

Question-2

Determine n if

(i) $^{2n}C_3 : {^nC_3} = 12 : 1$ (ii) $^{2n}C_3 : {^nC_3} = 11 : 1$

Ans.

$$\frac{^{2n}C_3}{^nC_3} = \frac{12}{1}$$

$$\Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n!} = \frac{12}{1}$$

$$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!} \times \frac{(n-3)!}{n(n-1)(n-2)(n-3)!} = 12$$

$$\Rightarrow \frac{2(2n-1)(2n-2)}{(n-1)(n-2)} = 12$$

$$\Rightarrow \frac{4(2n-1)(n-1)}{(n-1)(n-2)} = 12$$

$$\Rightarrow \frac{(2n-1)}{(n-2)} = 3$$

$$\Rightarrow 2n-1 = 3(n-2)$$

$$\Rightarrow 2n-1 = 3n-6$$

$$\Rightarrow 3n-2n = -1+6$$

$$\Rightarrow n = 5$$

(ii)

$$\frac{^{2n}C_3}{^nC_3} = \frac{11}{1}$$

$$\Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n!} = 11$$

$$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!} \times \frac{(n-3)!}{n(n-1)(n-2)(n-3)!} = 11$$

$$\Rightarrow \frac{2(2n-1)(2n-2)}{(n-1)(n-2)} = 11$$

$$\Rightarrow \frac{4(2n-1)(n-1)}{(n-1)(n-2)} = 11$$

$$\Rightarrow \frac{4(2n-1)}{n-2} = 11$$

$$\Rightarrow 4(2n-1) = 11(n-2)$$

$$\Rightarrow 8n-4 = 11n-22$$

$$\Rightarrow 11n-8n = -4+22$$

$$\Rightarrow 3n = 18$$

$$\Rightarrow n = 6$$

Question-3

How many chords can be drawn through 21 points on a circle?

Ans.

For drawing one chord on a circle, only 2 points are required.

To know the number of chords that can be drawn through the given 21 points on a circle, the number of combinations have to be counted.

Therefore, there will be as many chords as there are combinations of 21 points taken 2 at a time.

Thus, required number of chords $= {}^{21}C_2 = \frac{21!}{2!(21-2)!} = \frac{21!}{2!19!} = \frac{21 \times 20}{2} = 210$

Question-4

In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4 girls?

Ans.

A team of 3 boys and 3 girls is to be selected from 5 boys and 4 girls.

3 boys can be selected from 5 boys in $^5C_3$ ways.

3 girls can be selected from 4 girls in $^4C_3$ ways.

Therefore, by multiplication principle, number of ways in which a team of 3 boys and 3 girls can be selected $= {}^5C_3 \times {}^4C_3 = \dfrac{5!}{3!2!} \times \dfrac{4!}{3!1!}$

$$= \dfrac{5 \times 4 \times 3!}{3! \times 2} \times \dfrac{4 \times 3!}{3!}$$
$$= 10 \times 4 = 40$$

Question-5

Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour.

Ans.

There are a total of 6 red balls, 5 white balls, and 5 blue balls.

9 balls have to be selected in such a way that each selection consists of 3 balls of each colour.

Here,

3 balls can be selected from 6 red balls in $^6C_3$ ways.

3 balls can be selected from 5 white balls in $^5C_3$ ways.

3 balls can be selected from 5 blue balls in $^5C_3$ ways.

Thus, by multiplication principle, required number of ways of selecting 9 balls

$$= {}^6C_3 \times {}^5C_3 \times {}^5C_3 = \dfrac{6!}{3!3!} \times \dfrac{5!}{3!2!} \times \dfrac{5!}{3!2!}$$
$$= \dfrac{6 \times 5 \times 4 \times 3!}{3! \times 3 \times 2} \times \dfrac{5 \times 4 \times 3!}{3! \times 2 \times 1} \times \dfrac{5 \times 4 \times 3!}{3! \times 2 \times 1}$$
$$= 20 \times 10 \times 10 = 2000$$

Question-6

Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination.

Ans.

In a deck of 52 cards, there are 4 aces. A combination of 5 cards have to be made in which there is exactly one ace.

Then, one ace can be selected in $^4C_1$ ways and the remaining 4 cards can be selected out of the 48 cards in $^{48}C_4$ ways.

Thus, by multiplication principle, required number of 5 card combinations

$$= {}^{48}C_4 \times {}^4C_1 = \dfrac{48!}{4!44!} \times \dfrac{4!}{1!3!}$$
$$= \dfrac{48 \times 47 \times 46 \times 45}{4 \times 3 \times 2 \times 1} \times 4$$
$$= 778320$$

Question-7

In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

Ans.

Out of 17 players, 5 players are bowlers.

A cricket team of 11 players is to be selected in such a way that there are exactly 4 bowlers.

4 bowlers can be selected in $^5C_4$ ways and the remaining 7 players can be selected out of the 12 players in $^{12}C_7$ ways.

Thus, by multiplication principle, required number of ways of selecting cricket team
$$= {}^5C_4 \times {}^{12}C_7 = \frac{5!}{4!1!} \times \frac{12!}{7!5!} = 5 \times \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1} = 3960$$

Question-8

A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.

Ans.

There are 5 black and 6 red balls in the bag.

2 black balls can be selected out of 5 black balls in $^5C_2$ ways and 3 red balls can be selected out of 6 red balls in $^6C_3$ ways.

Thus, by multiplication principle, required number of ways of selecting 2 black and 3 red balls $= {}^5C_2 \times {}^6C_3 = \frac{5!}{2!3!} \times \frac{6!}{3!3!} = \frac{5 \times 4}{2} \times \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 10 \times 20 = 200$

Question-9

In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 specific courses are compulsory for every student?

Ans.

There are 9 courses available out of which, 2 specific courses are compulsory for every student.

Therefore, every student has to choose 3 courses out of the remaining 7 courses. This can be chosen in $^7C_3$ ways.

Thus, required number of ways of choosing the programme

$$= {}^7C_3 = \frac{7!}{3!4!} = \frac{7 \times 6 \times 5 \times 4!}{3 \times 2 \times 1 \times 4!} = 35$$

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