Trigonometric Identities Ex 6.1 Q45

Answer :

In the given question, we need to prove $\frac{\tan^2 A}{1+\tan^2 A}+\frac{\cot^2 A}{1+\cot^2 A}=1$.

Here, we will first solve the LHS.

Now, using $\tan\theta=\frac{\sin\theta}{\cos\theta}$ and $\cot\theta=\frac{\cos\theta}{\sin\theta}$, we get

$$\begin{aligned}\frac{\tan^2 A}{1+\tan^2 A}+\frac{\cot^2 A}{1+\cot^2 A}&=\frac{\left(\frac{\sin^2 A}{\cos^2 A}\right)}{\left(1+\frac{\sin^2 A}{\cos^2 A}\right)}+\frac{\left(\frac{\cos^2 A}{\sin^2 A}\right)}{\left(1+\frac{\cos^2 A}{\sin^2 A}\right)}\\&=\frac{\left(\frac{\sin^2 A}{\cos^2 A}\right)}{\left(\frac{\cos^2 A+\sin^2 A}{\cos^2 A}\right)}+\frac{\left(\frac{\cos^2 A}{\sin^2 A}\right)}{\left(\frac{\sin^2 A+\cos^2 A}{\sin^2 A}\right)}\\&=\frac{\left(\frac{\sin^2 A}{\cos^2 A}\right)}{\left(\frac{1}{\cos^2 A}\right)}+\frac{\left(\frac{\cos^2 A}{\sin^2 A}\right)}{\left(\frac{1}{\sin^2 A}\right)}\qquad\left(\text{Using } \sin^2\theta+\cos^2\theta=1\right)\end{aligned}$$

On further solving by taking the reciprocal of the denominator, we get,

$$\begin{aligned}\frac{\left(\frac{\sin^2 A}{\cos^2 A}\right)}{\left(\frac{1}{\cos^2 A}\right)}+\frac{\left(\frac{\cos^2 A}{\sin^2 A}\right)}{\left(\frac{1}{\sin^2 A}\right)}&=\left(\frac{\sin^2 A}{\cos^2 A}\right)\left(\frac{\cos^2 A}{1}\right)+\left(\frac{\cos^2 A}{\sin^2 A}\right)\left(\frac{\sin^2 A}{1}\right)\\&=\sin^2 A+\cos^2 A\qquad\left(\text{Using } \sin^2\theta+\cos^2\theta=1\right)\\&=1\end{aligned}$$

Hence proved.

Trigonometric Identities Ex 6.1 Q46

Answer :

In the given question, we need to prove $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$

Here, we will first solve the LHS.

Now, using $\cot\theta = \frac{\cos\theta}{\sin\theta}$, we get

$$\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\left(\frac{\cos A}{\sin A} - \cos A\right)}{\left(\frac{\cos A}{\sin A} + \cos A\right)}$$

$$= \frac{\left(\frac{\cos A - \cos A \sin A}{\sin A}\right)}{\left(\frac{\cos A + \cos A \sin A}{\sin A}\right)}$$

On further solving by taking the reciprocal of the denominator, we get,

$$\frac{\left(\frac{\cos A - \cos A \sin A}{\sin A}\right)}{\left(\frac{\cos A + \cos A \sin A}{\sin A}\right)} = \left(\frac{\cos A - \cos A \sin A}{\sin A}\right)\left(\frac{\sin A}{\cos A + \cos A \sin A}\right)$$

$$= \left(\frac{\cos A - \cos A \sin A}{\cos A + \cos A \sin A}\right)$$

Now, taking $\cos A \sin A$ common from both the numerator and the denominator, we get

$$\left(\frac{\cos A - \cos A \sin A}{\cos A + \cos A \sin A}\right) = \frac{\cos A \sin A\left(\frac{1}{\sin A} - 1\right)}{\cos A \sin A\left(\frac{1}{\sin A} + 1\right)}$$

$$= \frac{\left(\frac{1}{\sin A} - 1\right)}{\left(\frac{1}{\sin A} + 1\right)}$$

$$= \frac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1} \qquad \left(\text{Using } \frac{1}{\sin\theta} = \operatorname{cosec}\theta\right)$$

Hence proved.

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