Indefinite Integrals Ex 19.8 Q46

Let $I = \int \tan 2x \tan 3x \tan 5x \, dx$ ---------(i)

Now,

$$\tan(5x) = \tan(2x + 3x)$$

$$= \frac{\tan 2x + \tan 3x}{1 - \tan 2x \tan 3x}$$

$\Rightarrow \quad \tan 5x = \frac{\tan 2x + \tan 3x}{1 - \tan 2x \tan 3x}$

$\Rightarrow \quad \tan 5x - \tan 2x \tan 3x \tan 5x = \tan 2x + \tan 3x$

$\Rightarrow \quad \tan 5x - \tan 2x - \tan 3x = \tan 2x \tan 3x \tan 5x$ -----(ii)

Using equation (i) and equation (ii), we get

$I = \int \left[\tan 5x - \tan 2x - \tan 3x\right] dx$

$= \frac{1}{5}\log|\sec 5x| - \frac{1}{2}\log|\sec 2x| - \frac{1}{3}\log|\sec 3x| + c$

$\therefore \quad I = \frac{1}{5}\log|\sec 5x| - \frac{1}{2}\log|\sec 2x| - \frac{1}{3}\log|\sec 3x| + c$

Indefinite Integrals Ex 19.8 Q47

Since,

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$\therefore \quad \tan(x + \theta - x) = \frac{\tan(x + \theta) - \tan x}{1 + \tan(x + \theta)\tan x}$

$\Rightarrow \quad 1 + \tan(x + \theta)\tan x = \frac{\tan(x + \theta) - \tan x}{\tan \theta}$

$\Rightarrow \quad \int 1 + \tan(x + \theta)\tan x \, dx$

$= \frac{1}{\tan \theta}\left[\int \tan(x + \theta)\,dx - \int \tan x \, dx\right]$

$= \frac{1}{\tan \theta}\left[-\log|\cos(x + \theta)| + \log|\cos x|\right] + C$

$= \frac{1}{\tan \theta}\left[\log|\cos x| - \log|\cos(x + \theta)|\right] + C$

$= \frac{1}{\tan \theta}\log\left|\frac{\cos x}{\cos(x + \theta)}\right| + C$

Indefinite Integrals Ex 19.8 Q48

Consider $I = \int \left(\dfrac{\sin 2x}{\sin\left(x - \dfrac{\pi}{6}\right)\sin\left(x + \dfrac{\pi}{6}\right)} \right) dx$

$= \int \left(\dfrac{\sin 2x}{\left(\dfrac{3}{4}\sin^2 x - \dfrac{1}{4}\cos^2 x\right)} \right) dx$

$= \int \left(\dfrac{\sin 2x}{\left(\dfrac{3}{4}(1 - \cos^2 x) - \dfrac{1}{4}\cos^2 x\right)} \right) dx$

$= \int \left(\dfrac{\sin 2x}{\left(\dfrac{3}{4} - \cos^2 x\right)} \right) dx$

let $\cos^2 x = t \rightarrow \sin 2x\, dx = -dt$

$I = \int \left(\dfrac{-dt}{\left(\dfrac{3}{4} - t\right)} \right)$

$I = \log\left|\sin^2 x - \dfrac{1}{4}\right| + C$

*********** END *********