Indefinite Integrals Ex 19.9 Q64

Let $\quad I = \int 5^{5^{5^x}} 5^{5^x} 5^x \, dx - - - - - (i)$

Let $\quad 5^{5^{5^x}} = t$ then,

$$d\left(5^{5^{5^x}}\right) = dt$$

$$\Rightarrow \quad 5^{5^{5^x}} 5^{5^x} 5^x (\log 5)^3 \, dx = dt$$

$$\Rightarrow \quad 5^{5^{5^x}} 5^{5^{5^x}} 5^x \, dx = \frac{dt}{(\log 5)^3}$$

Putting $5^{5^{5^x}} = t$ and $5^{5^{5^x}} 5^{5^{5^x}} 5^x \, dx = \dfrac{dt}{(\log 5)^3}$ in equation (i), we get

$$I = \int \frac{dt}{(\log 5)^3}$$

$$= \frac{1}{(\log 5)^3} \int dt$$

$$= \frac{t}{(\log 5)^3} + c$$

$$\therefore \quad I = \frac{5^{5^{5^x}}}{(\log 5)^3} + c$$

Indefinite Integrals Ex 19.9 Q65

Let $\quad I = \int \frac{1}{x\sqrt{x^4 - 1}} dx \text{ - - - - - (i)}$

Let $\quad x^2 = t$ then,

$d\left(x^2\right) = dt$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad dx = \frac{dt}{2x}$

Putting $x^2 = t$ and $dx = \frac{dt}{2x}$ in equation (i),

we get

$$I = \int \frac{1}{x\sqrt{t^2 - 1}} \times \frac{dt}{2x}$$
$$= \frac{1}{2} \int \frac{1}{x^2\sqrt{t^2 - 1}} dt$$
$$= \frac{1}{2} \int \frac{1}{t\sqrt{t^2 - 1}} dt$$
$$= \frac{1}{2} \sec^{-1} t + c$$
$$= \frac{1}{2} \sec^{-1} x^2 + c$$

$\therefore \quad I = \frac{1}{2} \sec^{-1}\left(x^2\right) + c$

Indefinite Integrals Ex 19.9 Q66

Let $\quad I = \int \sqrt{e^x - 1}\, dx$ - - - - - (i)

Let $\quad e^x - 1 = t^2 \quad$ then,

$d\left(e^x - 1\right) = dt\left(t^2\right)$

$\Rightarrow \quad e^x\, dx = 2t\, dt$

$\Rightarrow \quad dx = \dfrac{2t}{e^x} dt$

$\Rightarrow \quad dx = \dfrac{2t}{t^2 + 1} dt \qquad \left[\because \quad e^x - 1 = t^2\right]$

Putting $e^x - 1 = t^2$ and $dx = \dfrac{2t\, dt}{t^2 + 1}$ in equation (i),

we get

$$
\begin{aligned}
I &= \int \sqrt{t^2} \times \frac{2t\, dt}{t^2 + 1} \\
&= 2\int \frac{t \times t}{t^2 + 1} dt \\
&= 2\int \frac{t^2}{t^2 + 1} dt \\
&= 2\int \frac{t^2 + 1 - 1}{t^2 + 1} dt \\
&= 2\int \left[\frac{t^2 + 1}{t^2 + 1} - \frac{1}{t^2 + 1}\right] dt \\
&= 2\int dt - 2\int \frac{1}{t^2 + 1} dt \\
&= 2t - 2\tan^{-1}\left(t\right) + c \\
&= 2\sqrt{\left(e^x - 1\right)} - 2\tan^{-1}\left(\sqrt{e^x - 1}\right) + c
\end{aligned}
$$

$\therefore \quad I = 2\sqrt{e^x - 1} - 2\tan^{-1}\sqrt{e^x - 1} + c$

Indefinite Integrals Ex 19.9 Q67

$$I = \int \frac{1}{(x+1)\left(x^2+2x+2\right)} dx$$

$$= \int \frac{1}{(x+1)\left((x+1)^2+1\right)} dx$$

Let $x + 1 = \tan u$

$\Rightarrow \quad dx = \sec^2 u \; du$

$$\therefore \quad I = \int \frac{\sec^2 u}{\tan u \left(\tan^2 u + 1\right)} du$$

$$= \int \frac{\cos u}{\sin u} du$$

$$= \log|\sin u| + C$$

$$= \log\left|\frac{\tan u}{\sec^2 u}\right| + C$$

$$= \log\left|\frac{x+1}{\sqrt{x^2+2x+2}}\right| + C$$

Indefinite Integrals Ex 19.9 Q68

Let $\quad I = \int \frac{x^5}{\sqrt{1+x^3}} dx ----(i)$

Let $\quad 1 + x^3 = t^2$ then,

$d\left(1 + x^3\right) = d\left(t^2\right)$

$\Rightarrow \quad 3x^2 dx = dt\, 2t$

$\Rightarrow \quad dx = \frac{dt}{3x^2} 2t$

Putting $1 + x^3 = t^2$ and $dx = \frac{2t}{3x^2} dt$ in equation (i),

we get

$$I = \int \frac{x^5}{\sqrt{t^2}} \times \frac{2t}{3x^2} dt$$

$$= \int \frac{x^5}{t} \times \frac{2t}{3x^2} dt$$

$$= \frac{2}{3} \int x^3 dt$$

$$= \frac{2}{3} \int \left(t^2 - 1\right) dt$$

$$= \frac{2}{3} \times \frac{t^3}{3} - \frac{2}{3} t + c$$

$$\therefore \quad I = \frac{2}{9}\left(1 + x^3\right)^{\frac{3}{2}} - \frac{2}{3}\sqrt{1 + x^3} + c$$

*********** END **********