CHAPTER 46

THE NUCLEUS

At the centre of an atom exists the *nucleus* which contains protons and neutrons. The electrons surround this nucleus to form the atom. As discussed earlier, this structure of atom was revealed by the experiments of Rutherford in which a beam of alpha particles was made to strike a thin gold foil. Most of the alpha particles crossed the foil without being appreciably deviated, but there were some alpha particles which suffered large deviation from their original lines of motion. The data suggested that positive charges in an atom are concentrated in a small volume which we call nucleus and this nucleus is responsible for the large deviation of alpha particles. Later on, the existence of protons and neutrons in the nucleus was established. In this chapter, we shall discuss the physics of the nucleus.

46.1 PROPERTIES OF A NUCLEUS

Nuclear Constituents

A nucleus is made of protons and neutrons. A proton has a positive charge of magnitude equal to that of an electron and has a mass of about 1840 times the mass of an electron. A neutron has a mass slightly greater than that of a proton. The masses of a proton and a neutron are

$$m_p = 1{\cdot}6726231 \times 10^{-27}\ \mathrm{kg}$$

and $$m_n = 1{\cdot}6749286 \times 10^{-27}\ \mathrm{kg}.$$

It is customary in nuclear physics and high energy physics to represent mass in energy units according to the conversion formula $E = mc^2$. (Matter can be viewed as a condensed form of energy. Theory of relativity reveals that a mass m is equivalent to an energy E where $E = mc^2$.) For example, the mass of an electron is $m_e = 9{\cdot}1093897 \times 10^{-31}$ kg and the equivalent energy is

$$m_e c^2 = 510{\cdot}99\ \mathrm{keV}.$$

Thus, the mass of an electron is $510{\cdot}99\ \mathrm{keV\ c^{-2}}$. Similarly, the mass of a proton is $938{\cdot}27231\ \mathrm{MeV\ c^{-2}}$ and the mass of a neutron is $939{\cdot}56563\ \mathrm{MeV\ c^{-2}}$. The energy corresponding to the mass of a particle when it is at rest is called its *rest mass energy*.

Another unit which is widely used in describing mass in nuclear physics as well as in atomic physics is *unified atomic mass unit* denoted by the symbol u. It is 1/12 of the mass of a neutral carbon atom in its lowest energy state which contains six protons, six neutrons and six electrons. We have

$$1\ \mathrm{u} = 1{\cdot}6605402 \times 10^{-27}\ \mathrm{kg} = 931{\cdot}478\ \mathrm{MeV\ c^{-2}}.$$

Protons and neutrons are fermions and obey the Pauli exclusion principle like electrons. No two protons or two neutrons can have the same quantum state. But one proton and one neutron can exist in the same quantum state. Protons and neutrons are collectively called *nucleons.*

The number of protons in a nucleus is denoted by Z, the number of neutrons by N and the total number of nucleons by A. Thus, $A = Z + N$. The total number of nucleons A is also called the *mass number* of the nucleus. The number of protons Z is called the *atomic number*. A nucleus is symbolically expressed as ${}^{A}_{Z}\mathrm{X}$ in which X is the chemical symbol of the element. Thus, ${}^{4}_{2}\mathrm{He}$ represents helium nucleus which contains 2 protons and a total of 4 nucleons. So it contains 2 neutrons. Similarly, ${}^{238}_{92}\mathrm{U}$ represents a uranium nucleus which contains 92 protons and 146 neutrons.

The distribution of electrons around the nucleus is determined by the number of protons Z and hence the chemical properties of an element are also determined by Z. The nuclei having the same number of protons but different number of neutrons are called *isotopes*. Nuclei with the same neutron number N but different atomic number Z are called *isotones* and the nuclei with the same mass number A are called *isobars*. All nuclei with a given Z and N are collectively called a *nuclide*. Thus, all the ${}^{56}_{26}\mathrm{Fe}$ nuclei taken together is one nuclide and all the ${}^{32}_{16}\mathrm{S}$ nuclei taken together is another nuclide.

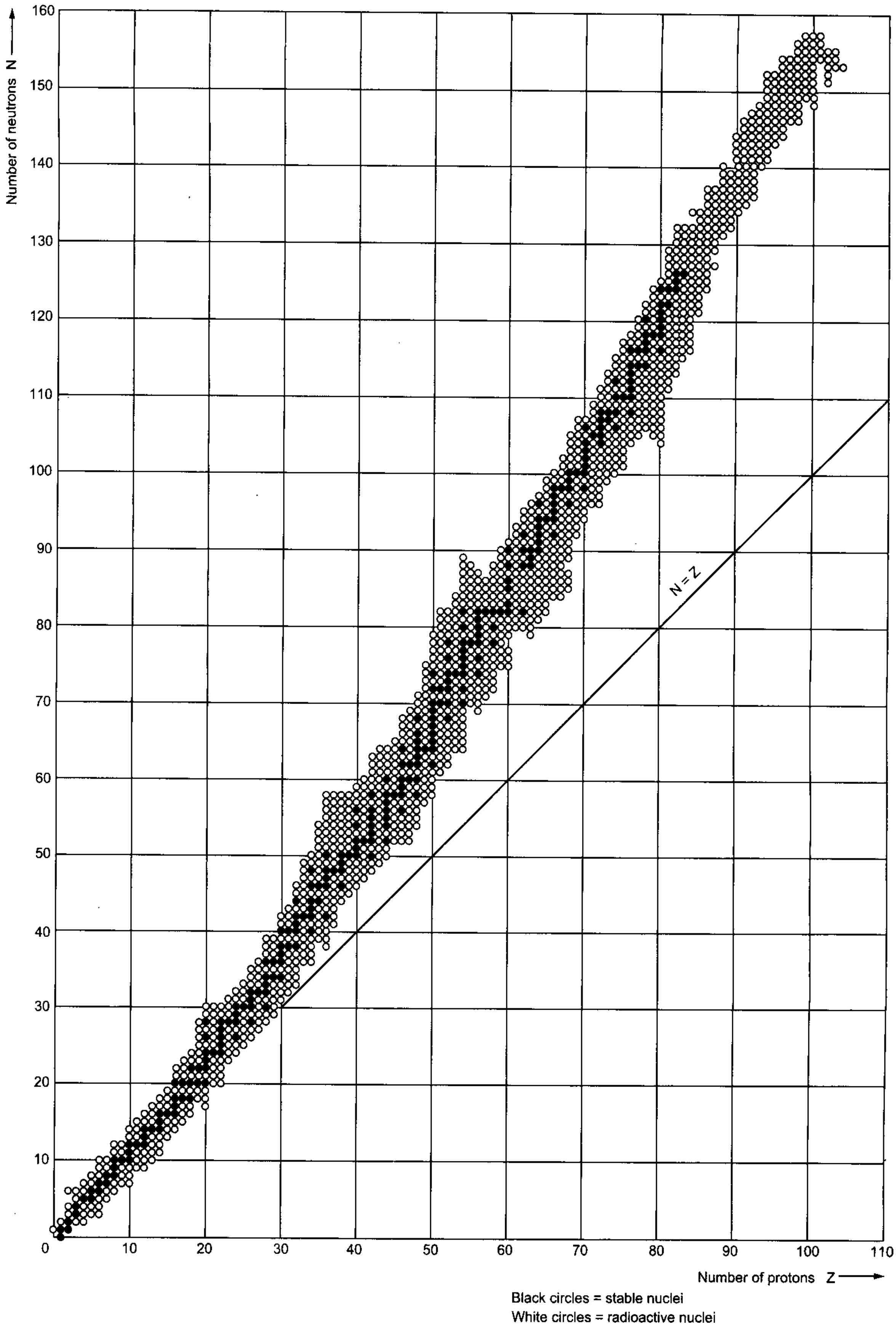
Number of neutrons N
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
10
20
30
40
50
60
70
80
90
100
110
N = Z
Number of protons Z
Black circles = stable nuclei
White circles = radioactive nuclei

Figure 46.1

Nuclear Stability

More than 1000 nuclides have been identified but not all are stable. An unstable nucleus emits some kind of particle and changes its constitution. A stable nucleus maintains its constitution all the time. Figure (46.1) shows a plot of neutron number N versus proton number Z for the nuclides observed. The black circles represent the stable nuclides. For light stable nuclides, the neutron number is equal to the proton number so that the ratio N/Z is equal to 1. The ratio N/Z increases for the heavier nuclides and becomes about 1·6 for heaviest stable nuclides.

The points (Z, N) for stable nuclides fall in a rather well-defined narrow region. There are nuclides to the left of the stability region as well as to the right of it. They are represented in figure (46.1) by white circles. The nuclides to the left of the stability region have excess neutrons, whereas, those to the right of the stability region have excess protons. These nuclides are unstable and decay with time according to the laws of radioactive disintegration discussed later in this chapter. They are called *radioactive nuclide.*

Nuclear Radius

The nucleus is so small a particle that we cannot define a sharp boundary for it. For such small particles, the description must be given in terms of the wave functions only. The magnitude of the wave function becomes very small as one moves some distance away from the centre of the nucleus. A rough estimate of nuclear size may be made by finding the region where the wave function has appreciable magnitude. Experiments show that the average radius R of a nucleus may be written as

$$R = R_0 A^{1/3} \quad \dots (46.1)$$

where $R_0 = 1{\cdot}1 \times 10^{-15}$ m = 1·1 fm and A is the mass number.

The volume of a nucleus is

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A. \quad \dots \text{(i)}$$

As the masses of a proton and a neutron are roughly equal, say m, the mass of a nucleus M is also roughly proportional to the mass number A.

We have, $$M = mA. \quad \dots \text{(ii)}$$

From (i) and (ii), the density within a nucleus (mass per unit volume) $\rho = M/V$ is independent of A.

Example 46.1

Calculate the radius of ^{70}Ge.

Solution : We have,

$$R = R_0 A^{1/3} = (1{\cdot}1 \text{ fm})\,(70)^{1/3}$$

$$= (1{\cdot}1 \text{ fm})\,(4{\cdot}12) = 4{\cdot}53 \text{ fm}.$$

Nuclear Spin

The protons and neutrons inside a nucleus move in well-defined quantum states and thus have orbital angular momenta. Apart from this, protons and neutrons also have internal spin angular momentum. The spin quantum number m_s is $+\frac{1}{2}$ or $-\frac{1}{2}\cdot$ The total angular momentum of the nucleus is the resultant of all the spin and orbital angular momenta of the individual nucleons. This total angular momentum of a nucleus is called the *nuclear spin* of that nucleus.

46.2 NUCLEAR FORCES

When nucleons are kept at a separation of the order of a femtometre (10^{-15} m), a new kind of force, called *nuclear force* starts acting. Nuclear force is much stronger than gravitational and electromagnetic forces if the separation between the interacting nucleons is of the order of 1 fm. Nuclear forces are basically attractive and are responsible for keeping the nucleons bound in a nucleus. The protons exert repulsive Coulomb forces on each other. The neutrons take no part in electric interaction as they are chargeless particles. The nuclear forces operate between proton and proton, neutron and neutron as well as between proton and neutron. The overall effect of this attractive nuclear force is much stronger than that of the repulsive Coulomb forces between the protons and thus the nucleus stays bound.

Unlike gravitational or electromagnetic force, nuclear force is not represented by a simple formula. In fact, the nuclear force is not yet completely understood and physicists are still working out the details. Some of the qualitative properties of nuclear forces are as follows:

(a) Nuclear forces are short-ranged. They are most effective only up to a distance of the order of a femtometre or less. The nuclear force between two nucleons decreases rapidly as the separation between them increases and becomes negligible at separations more than, say, 10 fm. The range up to which the nuclear force is effective is called *nuclear range.*

(b) Nuclear forces are, on an average, much stronger than electromagnetic forces ($\approx$ 50–60 times stronger) in the nuclear range.

(c) Nuclear forces are independent of charge. The nuclear force between two protons is the same as that between two neutrons or between a proton and a neutron. Remember, the Coulomb force between two protons acts according to the well-defined Coulomb's law, over and above the nuclear force. The nuclear force itself is independent of charge.

(d) An important property of nuclear force is that it is not a central force. The force between a pair of nucleons is not solely determined by the distance between the necleons. For example, the nuclear force depends on the directions of the spins of the nucleons. The force is stronger if the spins of the nucleons are parallel (i.e., both nucleons have $m_s = +1/2$ or $-1/2$) and is weaker if the spins are antiparallel (i.e., one nucleon has $m_s = +1/2$ and the other has $m_s = -1/2$).

Many of the nuclear properties may be understood on the basis of these qualitative properties of nuclear forces.

Because of the short-range nature of nuclear force, each nucleon in a nucleus interacts only with a small number of neighbouring nucleons through the nuclear force. This explains why the density of the nucleons is roughly the same in all the nuclei.

Because of the Pauli exclusion principle, each quantum state can contain at the most two protons (with opposite spins) and two neutrons (again with opposite spins). Thus, nuclear forces favour pairing of two protons and two neutrons together. In light nuclei, the nuclear forces between the nucleons are much dominant over the Coulomb repulsion and hence the neutron number N and the proton number Z tend to be equal in light nuclei. In a heavier nucleus, the radius is large and for many nucleon-pairs, the interaction through nuclear force is not effective. On the other hand, Coulomb force is a long-range force and even the diametrically opposite protons repel each other. Thus, Coulomb repulsion becomes more effective for nuclei of larger mass number A. Stability is achieved by having more neutrons than protons because neutrons do not take part in Coulomb interaction. That is why N/Z increases with A for stable nuclides. However, one should not expect greater stability with too many neutrons because then many of these neutrons will not have pairing with protons. This will increase the energy and hence, decrease the stability. A very large nucleus cannot be stable for any value of N/Z. The heaviest stable nuclide is $^{209}_{83}\mathrm{Bi}$.

46.3 BINDING ENERGY

We already know about the concept of binding energy of a hydrogen atom. If the constituents of a hydrogen atom (a proton and an electron) are brought from infinity to form the atom, 13·6 eV of energy is released. Thus, the binding energy of a hydrogen atom in ground state is 13·6 eV. Also, 13·6 eV energy must be supplied to the hydrogen atom in ground state to separate the constituents to large distances. Similarly, the nucleons are bound together in a nucleus and energy must be supplied to the nucleus to separate the constituent nucleons to large distances (figure 46.2a). The amount of energy needed to do this is called the *binding energy of the nucleus*. If the nucleons are initially well-separated and are brought to form the nucleus, this much energy is released (figure 46.2b).

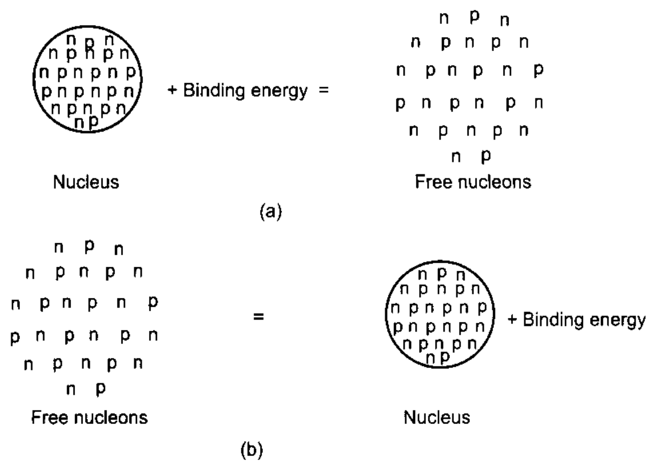

Figure 46.2

Thus, the rest mass energy of a nucleus is smaller than the rest mass energy of its constituent nucleons in free state. The difference of the two energies is the binding energy. The rest mass energy of a free proton is $m_p c^2$ and that of a free neutron is $m_n c^2$. If the nucleus has a mass M, its rest mass energy is Mc^2. If it contains Z protons and N neutrons, the rest mass energy of its nucleons in free state is $Zm_p c^2 + Nm_n c^2$. If the binding energy of the nucleus is B, we have,

$$B = (Zm_p + Nm_n - M)c^2. \qquad \ldots \text{(i)}$$

We can also use the atomic masses in place of nuclear masses. The above equation then becomes

$$B = \left[Zm\left({}^1_1\mathrm{H}\right) + N\,m_n - m\left({}^{Z+N}_{\;\;Z}\mathrm{X}\right)\right]c^2. \qquad \ldots (46.2)$$

Here, $m\left({}^1_1\mathrm{H}\right)$ is the mass of a hydrogen atom and $m\left({}^{Z+N}_{\;\;Z}\mathrm{X}\right)$ is the mass of an atom with Z protons and N neutrons. Verify that the masses of electrons cancel out in this equation. There are Z electrons in Z hydrogen atoms as well as in the atom ${}^{Z+N}_{\;\;Z}\mathrm{X}$. Hence, equation (46.2) is equivalent to (i) above. Such cancellations often occur and atomic masses are rather frequently used in place of nuclear masses. The small difference due to binding energy of electrons with the nucleus is neglected. As we shall see later, such cancellations do not work in the case of β-decays. Unless stated otherwise, mass of ${}^A_Z\mathrm{X}$ in this chapter will refer to the atomic mass which includes the mass of Z electrons.

Example 46.2

Calculate the binding energy of an alpha particle from the following data:

mass of $^{1}_{1}H$ *atom* $= 1{\cdot}007825$ u

mass of neutron $= 1{\cdot}008665$ u

mass of $^{4}_{2}He$ *atom* $= 4{\cdot}00260$ u.

Take $1\ u = 931\ MeV\ c^{-2}$.

Solution : The alpha particle contains 2 protons and 2 neutrons. The binding energy is

$$B = (2 \times 1{\cdot}007825\ u + 2 \times 1{\cdot}008665\ u - 4{\cdot}00260\ u)c^2$$
$$= (0{\cdot}03038\ u)c^2$$
$$= 0{\cdot}03038 \times 931\ MeV = 28{\cdot}3\ MeV.$$

If two protons and two neutrons combine to form an alpha particle, 28·3 MeV of energy will be released.

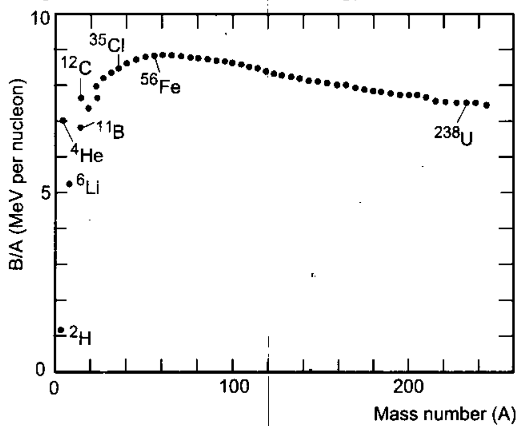

Figure 46.3

A very useful quantity in nuclear physics is binding energy per nucleon, i.e., binding energy divided by the mass number. Figure (46.3) shows a plot of binding energy per nucleon against the mass number. The binding energy of deuteron (the nucleus of heavy hydrogen containing a proton and a neutron) is 2·22 MeV so that the binding energy per nucleon is $\frac{2{\cdot}22\ MeV}{2} = 1{\cdot}11$ MeV. As we consider nuclei with increasing mass number, the binding energy per nucleon increases on an average and reaches a maximum of about 8·7 MeV for $A \approx 50$–80. For still heavier nuclei, the binding energy per nucleon slowly decreases as A increases. For uranium, one of the heaviest natural elements ($A \approx 238$), the value drops to about 7·5 MeV. It follows that the nuclei in the intermediate region, $A \approx 50$–80, are most stable as maximum energy is needed to bring out any individual nucleon from such a nucleus.

The behaviour of binding energy per nucleon versus A can be roughly understood in terms of the short-range nature of nuclear force. Because of the short range, a nucleon inside the nucleus can interact with a fixed number of nucleons surrounding it. If each nucleon is visualised as a sphere and these spheres are assumed to be closely packed, each nucelon has 12 neighbouring nucleons touching it. Thus, 12 pairs are formed with each nucleon for nuclear interaction. If all the A nucleons could be in the interior of the nucleus, there would have been $6A$ pairs. The binding energy resulting from these pairs will, therefore, be proportional to A. This energy is called *volume energy* and is written as $b_v = a_1 A$. However, all the nucleons are not in the interior. A nucleon near the surface does not interact with 12 nucleons. Thus, there is a decrease in binding energy and this decrease will be proportional to the surface area or to R^2. As $R = R_0 A^{1/3}$, this surface energy is $b_s = -a_2 A^{2/3}$. The negative sign is used because the surface effect decreases the binding energy whereas the volume effect increases it. Another factor contributing to the binding energy is the Coulomb repulsion between the protons. As Coulomb force is a long-range force, all proton-pairs participate in it. The Coulomb potential energy is, therefore, proportional to $Z(Z-1)/2$ and is inversely proportional to the nuclear radius R as the average separation between the protons will be proportional to the nuclear radius. The Coulomb contribution to the binding energy is, therefore, written as $b_c = -a_3 Z(Z-1)/A^{1/3}$. The binding energy per nucleon is, therefore,

$$\frac{B}{A} = a_1 - \frac{a_2}{A^{1/3}} - a_3 \frac{Z(Z-1)}{A^{4/3}}. \qquad \dots (46.3)$$

With suitable choices of a_1, a_2 and a_3, this equation agrees well with the general nature of the plot shown in figure (46.3). There are other effects such as that due to Pauli exclusion principle which should be included in (46.3) to make it more accurate.

Mass Excess

Consider a nucleus of mass number A. Let the mass of the neutral atom containing this nucleus be m atomic mass units. Also, let A' represent the mass A atomic mass units. Thus,

mass of the atom $= m$ u

and $A' = A$ u.

We define *mass excess* as

$$(\text{mass of the atom} - A')\,c^2 \qquad \dots (46.4)$$
$$= (m\ u - A\ u)c^2$$
$$= (m - A)\left(\frac{931\ MeV}{c^2}\right)c^2$$
$$= 931(m - A) MeV. \qquad \dots (46.5)$$

Example 46.3

The atomic mass of ^1_1H *is* 1·00783 u. *Calculate the mass excess of hydrogen.*

Solution : The mass excess of hydrogen is $931(m-A)$MeV

$$= 931(1{\cdot}00783 - 1)\text{MeV} = 7{\cdot}29 \text{ MeV}.$$

46.4 RADIOACTIVE DECAY

Stable nuclides have definite Z, N combinations as shown by the black circles in figure (46.1). Nuclides with other Z, N combinations are also found in nature or may be prepared in laboratory. However, these nuclides are unstable and they decay into other nuclides by various processes. Two main processes by which an unstable nucleus decays are *alpha decay* and *beta decay.*

Alpha Decay

In alpha decay, the unstable nucleus emits an alpha particle reducing its proton number Z as well as its neutron number N by 2. The alpha decay process may be represented as

$$^A_ZX \rightarrow {}^{A-4}_{Z-2}Y + {}^4_2He. \qquad \dots (46.6)$$

As the proton number Z is changed, the element itself is changed and hence the chemical symbol of the residual nucleus is different from that of the original nucleus. The nucleus before the decay is called the *parent nucleus* and that resulting after the decay is called the *daughter nucleus.* An example of alpha decay is

$$^{212}_{83}Bi \rightarrow {}^{208}_{81}Tl + {}^4_2He.$$

The parent nucleus bismuth had 83 protons and 129 neutrons, the daughter nucleus is thallium with 81 protons and 127 neutrons.

Alpha decay occurs in all nuclei with mass number $A > 210$. We have seen that too heavy a nucleus will be unstable because of the Coulomb repulsive force. By emitting alpha particle the nucleus decreases its mass number to move towards stability.

The rest mass energy of $^{212}_{83}Bi$ is larger than the sum of the rest mass energies of the products $^{208}_{81}Tl$ and ^4_2He. The difference between the rest mass energy of the initial constituents and that of the final products is called the *Q-value* of the process. Thus, if U_i is the rest mass energy of the initial constituents and U_f is that of the final products,

$$Q = U_i - U_f.$$

This definition is valid not only for alpha decay but for any nuclear process. This much energy is made available as the kinetic energy of the products. In an α-decay given by equation (46.6), the Q-value is

$$Q = \left[m\left(^A_ZX\right) - m\left(^{A-4}_{Z-2}Y\right) - m\left(^4_2He\right)\right]c^2. \qquad \dots (46.7)$$

A stream of alpha particles coming from a bulk material is called *alpha ray.*

Beta Decay

Beta decay is a process in which either a neutron is converted into a proton or a proton is converted into a neutron. Thus, the ratio N/Z is altered in beta decay. If a nucleus is formed with more number of neutrons than needed for stability, a neutron will convert itself into a proton to move towards stability. Similarly, if a nucleus is formed with more number of protons than needed for stability, a proton will convert itself into a neutron. Such transformations take place because of *weak forces* operating within a neutron or a proton. When a neutron is converted into a proton, an electron and a new particle named *antineutrino* are created and emitted from the nucleus,

$$n \rightarrow p + e + \bar{\nu}. \qquad \dots (46.8)$$

The antineutrino is denoted by the symbol $\bar{\nu}$. It is supposed to have zero rest mass like photon, is chargeless and has spin quantum number ± 1/2. The electron emitted from the nucleus is called a *beta particle* and is denoted by the symbol β^-. A stream of such beta particles coming from a bulk of unstable nuclei is called *beta ray.* The beta decay process may be represented by

$$^A_ZX \rightarrow {}_{Z+1}^{\;\;A}Y + e + \bar{\nu}$$

$$\text{or,} \quad ^A_ZX \rightarrow {}_{Z+1}^{\;\;A}Y + \beta^- + \bar{\nu}. \qquad \dots (46.9)$$

It is also called *beta minus decay* as negatively charged beta particles are emitted. The rest mass energy of the initial constituents is

$$U_i = \left[m\left(^A_ZX\right) - Zm_e\right]c^2$$

and that of the final constituents is

$$U_f = \left[m\left({}_{Z+1}^{\;\;A}Y\right) - (Z+1)m_e + m_e\right]c^2.$$

The kinetic energy available to the product particles is,

$$Q = U_i - U_f = \left[m\left(^A_ZX\right) - m\left({}_{Z+1}^{\;\;A}Y\right)\right]c^2 \qquad \dots (46.10)$$

where atomic masses are used. It may be noted that the rest mass energy of the electron created is not explicitly subtracted in this equation. Because of the large mass, the residual nucleus ${}_{Z+1}^{\;\;A}Y$ does not share appreciable kinetic energy. Thus, the energy Q is shared by the antineutrino and the beta particle. Depending on the fraction taken away by the antineutrino, the kinetic energy of the beta particle can be anything between zero and a maximum value Q.

If the unstable nucleus has excess protons than needed for stability, a proton converts itself into a neutron. In the process, a *positron* and a *neutrino* are created and emitted from the nucleus,

$$p \rightarrow n + e^{+} + \nu. \qquad \ldots (46.11)$$

The positron e^{+} has a positive electric charge equal in magnitude to the charge on an electron and has a mass equal to the mass of an electron. Positron is called the *antiparticle* of electron. When an electron and a positron collide, both the particles are destroyed and energy is made available. Similarly, neutrino and antineutrino are antiparticles of each other. When a proton in a nucleus converts itself into a neutron, the decay process is represented as

$$^{A}_{Z}X \rightarrow {}^{\;\;A}_{Z-1}Y + e^{+} + \nu$$

or,

$$^{A}_{Z}X \rightarrow {}^{\;\;A}_{Z-1}Y + \beta^{+} + \nu. \qquad \ldots (46.12)$$

This process is called *beta plus decay*. The positron so emitted is called a *beta plus* particle.

Verify that the Q-value of this decay is given by

$$Q = \left[m\left({}^{A}_{Z}X\right) - m\left({}^{\;\;A}_{Z-1}Y\right) - 2m_e\right]c^2. \qquad \ldots (46.13)$$

Can an isolated proton decay to a neutron emitting a positron and a neutrino as suggested by equation (46.11) ? The mass of a neutron is larger than the mass of a proton and hence the Q-value of such a process would be negative. So, an isolated proton does not beta decay to a neutron. On the other hand, an isolated neutron decays to a proton as suggested by equation (46.8).

A similar process, known as *electron capture*, takes place in certain nuclides. In this process, the nucleus captures one of the atomic electrons (most likely an electron from the K shell). A proton in the nucleus combines with this electron and converts itself into a neutron. A neutrino is created in the process and emitted from the nucleus,

$$p + e \rightarrow n + \nu.$$

The process may be represented as

$$^{A}_{Z}X + e \rightarrow {}^{\;\;A}_{Z-1}Y + \nu. \qquad \ldots (46.14)$$

The Q-value of the process is

$$Q = \left[m\left({}^{A}_{Z}X\right) - m\left({}^{\;\;A}_{Z-1}Y\right)\right]c^2. \qquad \ldots (46.15)$$

When an atomic electron is captured, a vacancy is created in the atomic shell and X-rays are emitted following the capture.

The daughter nucleus formed as a result of an alpha decay or a beta decay may not be stable and undergo another alpha or beta decay. Thus, a series of decays proceed till a stable nucleus is formed. An example of such a series of decays starting from ^{238}U and ending at ^{206}Pb is shown in figure (46.4).

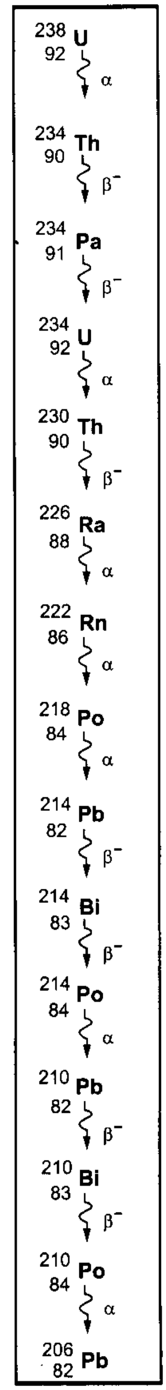

Figure 46.4

Gamma Decay

The protons and neutrons inside a nucleus move in discrete quantum states with definite energies. In the ground state, the nucleons occupy such quantum states which minimise the total energy of the nucleus. However, higher energy states are also available to the nucleons and if appropriate energy is supplied, the nucleus may be excited to higher energies. The energy differences in the allowed energy levels of a nucleus are generally large, of the order of MeVs. It is, therefore, difficult to excite the nucleus to higher energy levels by usual methods of supplying energy as heating. However, when an alpha or a beta decay takes place, the daughter nucleus is generally formed in one of its excited states. Such a nucleus in an excited state eventually comes to its ground state by emitting a photon or photons of electromagnetic radiation. The process is similar to that in a hydrogen atom when an electron jumps from a higher energy orbit to a lower energy orbit emitting a photon. A typical situation is schematically shown in figure (46.5). The parent nucleus ^{57}Co in its ground state decays to the daughter nucleus ^{57}Fe by β^{+}-decay. The nucleus ^{57}Fe is formed in its second excited state with energy 136 keV above the ground state. This nucleus in excited state may emit a photon of 136 keV and reach its ground state. Or, it may emit a photon of 122 keV and drop to its first excited state and then drop to the ground state by emitting another photon of energy 14 keV. If bulk ^{57}Co is taken, many ^{57}Fe nuclei will be formed; some will drop directly to their ground states and the rest will go via the first excited states. Thus, one will observe a stream of β^{+}-particles, 136 keV photons, 122 keV photons and 14 keV photons coming from the ^{57}Co source.

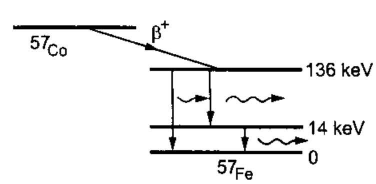

Figure 46.5

The electromagnetic radiation emitted in nuclear transitions is called *gamma ray*. The wavelength of this radiation is given by the usual relation

$$\lambda = \frac{hc}{E}$$

where E is the energy of the photon. The process of a nucleus coming down to a lower energy level by emitting a photon is called *gamma decay.*

Note that in gamma decay neither the proton number nor the neutron number changes. Only the quantum states of the nucleons change.

Alpha, beta and gamma decays are collectively called *radioactive decay* and the materials capable of undergoing radioactive decay are called *radioactive materials.* The α-, β- and γ-rays are collectively called *nuclear radiation.*

46.5 LAW OF RADIOACTIVE DECAY

A radioactive nucleus decays by emitting some nuclear radiation (α, β or γ). Suppose an alpha-active nucleus is prepared at $t = 0$. When will this nucleus emit alpha particle ? The answer to this question is that there is no fixed time at which it must decay. No rule in physics predicts the time at which it will decay. Only when it decays we know that it has done so. Putting in another way, suppose several identical active nuclei are prepared at the same instant and are kept in identical environment. They will, in general, not decay simultaneously. Some will decay quite early and some will live longer, some might live for very long periods. In classical physics of Newton, if all the conditions at present are known, the equations of motion completely determine the future course of all particles. This is called a *deterministic world* in which everything is predetermined. Radioactive decay cannot be described by classical physics. One has to use quantum mechanics to understand it. In quantum mechanics, the equations of physics do not predict the exact future of a system in terms of the present conditions. The equations only give the probability of a particular particle behaving in a particular fashion. In fact, Einstein never wholeheartedly accepted this indeterministic nature of the world.

Suppose there are N active nuclei at an instant t. How many of these nuclei will decay in the next small time interval dt ? The number will be proportional to N and to dt. Each active nucleus has a chance to decay in time interval dt. So, more the number of active nuclei at time t, more will decay in the next dt. Similarly, if you take dt slightly longer, more nuclei will decay in dt because each nucleus will have an increased chance of decaying. Thus,

$$dN = -\lambda\, N dt. \qquad \dots (46.16)$$

The minus sign is used because the number of active nuclei is decreasing. The constant of proportionality λ is called *decay constant* and is a constant for a given decay scheme. From equation (46.16),

$$\frac{dN}{N} = -\lambda\, dt$$

or, $$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$$

or, $$\ln \frac{N}{N_0} = -\lambda\, t$$

or, $$N = N_0\, e^{-\lambda t} \qquad \dots (46.17)$$

where N_0 is the number of active nuclei at $t = 0$. Also from (46.16), the rate of decay is

$$-\frac{dN}{dt} = \lambda N. \qquad \dots (46.18)$$

The quantity $\left(-\frac{dN}{dt}\right)$ gives the number of decays per unit time and is called the *activity* of the sample. Thus, the activity of a radioactive sample is $A = \lambda N$.

From equation (46.17), we have

$$A = A_0\, e^{-\lambda t}. \qquad \dots (46.19)$$

Unit of Activity

The activity of a radioactive material is measured in terms of the disintegrations per unit time. Its SI unit is becquerel which is the same as 1 disintegration per second. It is denoted by the symbol Bq. However, the popular unit of activity is curie defined as

$$1 \text{ curie} = 3{\cdot}7 \times 10^{10} \text{ disintegrations s}^{-1}.$$

The unit 'curie' is represented by the symbol Ci.

The activity per unit mass is called *specific activity.*

Example 46.4

The decay constant for the radioactive nuclide ^{64}Cu *is* $1{\cdot}516 \times 10^{-5}\ \text{s}^{-1}$. *Find the activity of a sample containing* 1 μg *of* ^{64}Cu. *Atomic weight of copper* = 63·5 g mole^{-1}. *Neglect the mass difference between the given radioisotope and normal copper.*

Solution : 63·5 g of copper has 6×10^{23} atoms. Thus, the number of atoms in 1 μg of Cu is

$$N = \frac{6 \times 10^{23} \times 1\ \mu\text{g}}{63{\cdot}5\ \text{g}} = 9{\cdot}45 \times 10^{15}.$$

The activity $= \lambda N$

$$= (1{\cdot}516 \times 10^{-5}\ \text{s}^{-1}) \times (9{\cdot}45 \times 10^{15})$$

$$= 1{\cdot}43 \times 10^{11} \text{ disintegrations s}^{-1}$$

$$= \frac{1{\cdot}43 \times 10^{11}}{3{\cdot}7 \times 10^{10}}\ \text{Ci} = 3{\cdot}86\ \text{Ci}.$$

Half-life

The time elapsed before half the active nuclei decay is called *half-life* and is denoted by $t_{1/2}$. Suppose there are N_0 active nuclei at $t = 0$. The half-life $t_{1/2}$ is the time elapsed before $N_0/2$ nuclei have decayed and $N_0/2$ remain active. From equation (46.17),

$$\frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}}$$

or, $$e^{\lambda t_{1/2}} = 2$$

or, $$\lambda t_{1/2} = \ln 2$$

or, $$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0{\cdot}693}{\lambda} \cdot \qquad \dots (46.20)$$

Equation (46.20) relates the decay constant λ and the half-life $t_{1/2}$. As the activity $\left(-\frac{dN}{dt}\right)$ is proportional to N, the activity also reduces to half its value in one half-life.

Using equation (46.20), equation (46.17) may be rewritten as

$$N = N_0 e^{-(\ln 2)\frac{t}{t_{1/2}}} = \frac{N_0}{\left(e^{\ln 2}\right)^{t/t_{1/2}}}$$

$$= \frac{N_0}{2^{t/t_{1/2}}} \cdot \qquad \dots (46.21)$$

Similarly, the activity at time t is

$$A = \frac{A_0}{2^{t/t_{1/2}}} \cdot \qquad \dots (46.22)$$

Example 46.5

The half-life of a radioactive nuclide is 20 hours. What fraction of original activity will remain after 40 hours?

Solution : We have

$$\frac{t}{t_{1/2}} = \frac{40 \text{ hours}}{20 \text{ hours}} = 2.$$

Thus,

$$A = \frac{A_0}{2^{t/t_{1/2}}} = \frac{A_0}{2^2} = \frac{A_0}{4}$$

or, $$\frac{A}{A_0} = \frac{1}{4} \cdot$$

So one fourth of the original activity will remain after 40 hours.

Average-life

Consider a sample containing N_0 radioactive nuclei at time $t = 0$. The number of nuclei which decay between the time t and $t + dt$ is $\lambda N\, dt$. The life of these nuclei is approximately t each. The sum of the lives of these dN nuclei is $t \lambda N\, dt$. The sum of all the lives of all the N nuclei that were active at $t = 0$ will be

$$S = \int_0^\infty t \lambda N\, dt$$

$$= \lambda N_0 \int_0^\infty t e^{-\lambda t}\, dt$$

$$= \lambda N_0 \left[\left(t \frac{e^{-\lambda t}}{-\lambda} \right)_0^\infty - \int_0^\infty \frac{e^{-\lambda t}}{-\lambda}\, dt \right]$$

$$= -\lambda N_0 \left(\frac{e^{-\lambda t}}{\lambda^2} \right)_0^\infty = \frac{N_0}{\lambda} \cdot$$

Thus, the average-life of the nuclei is

$$t_{av} = \frac{S}{N_0} = \frac{1}{\lambda} \cdot \qquad \dots (46.23)$$

Using equation (46.20),

$$t_{av} = \frac{t_{1/2}}{0{\cdot}693} \cdot \qquad \dots (46.24)$$

All the equations derived above are statistical in nature. They do not predict the exact behaviour of each radioactive nucleus, they only predict the total numbers. In one half-life, half of the active particles will decay. But which of these particles will decay in one half-life is never predicted. Suppose a traffic controller is stationed at the junction of three roads A, B and C. He counts the total number of vehicles coming towards the junction from the side A and the number turning towards B and towards C. From his observations over a long period, he can formulate a statistical law that out of the vehicles coming from the side A, 60% turn towards B and 40% turn towards C. This rule is statistical and will work well on a normal day (it will not work, say, on the Independence Day when a procession of hundreds of vehicles on road A may turn towards B). But the traffic controller will not be able to predict whether a particular vehicle coming from the side A will turn towards B or towards C. Also, the rule works only when a large number of vehicles are considered. If he considers just 5 vehicles, 4 may turn towards B, and only 1 towards C and the rule may be a total failure. Similarly, the equations developed for radioactive decay work well only when N is large.

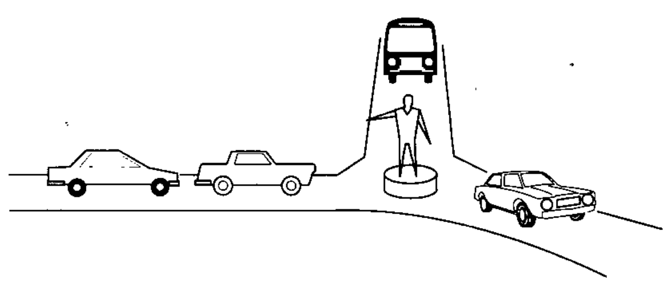

Figure 46.6

No description of radioactivity can be complete without mentioning the Curie couple. Marie Curie and her teacher-turned-husband Pierre Curie worked hard to separate the radioactive material radium chloride ($RaCl_2$) from uranium ore. They succeeded in 1902 when about 0·19 g of $RaCl_2$ was separated and its radioactivity was studied. They shared the Nobel Prize in physics for 1903 with Henri Becquerel for this work. The unit Ci is in their honour.

46.6 PROPERTIES AND USES OF NUCLEAR RADIATION

Alpha ray

(a) It is a stream of alpha particles, each particle containing two protons and two neutrons. An alpha particle is nothing but a helium nucleus.

(b) Being made of positively charged particles, alpha ray can be deflected by an electric field as well as by a magnetic field.

(c) Its penetrating power is low. Even in air, its intensity falls down to very small values within a few centimetres.

(d) Alpha rays coming from radioactive materials travel at large speeds of the order of 10^6 m s^{-1}.

(e) All the alpha particles coming from a particular decay scheme have the same energy.

(f) Alpha ray produces scintillation (flashes of light) when it strikes certain fluorescent materials, such as barium platinocyanide.

(g) It causes ionization in gases.

Beta ray

(a) It is a stream of electrons coming from the nuclei. Thus, the properties of beta ray, cathode ray, thermions, photoelectrons, etc., are all identical except for their origin. Beta particles are created at the time of nuclear transformation, whereas, in cathode ray, thermions, etc., the electrons are already present and get ejected.

(b) Being made of negatively charged particles, beta ray can be deflected by an electric field as well as by a magnetic field.

(c) Its penetrating power is greater than that of alpha ray. Typically, it can travel several metres in air before its intensity drops to small values.

(d) The ionizing power is less than that of alpha rays.

(e) Beta ray also produces scintillation in fluorescent materials, but the scintillation is weak.

(f) The energy of the beta particles coming from the same decay scheme are not equal. This is because the available energy is shared by antineutrinos. The energy of beta particles thus varies between zero and a maximum.

Beta-plus ray

Beta-plus ray has all the properties of beta ray, except that it is made of positively charged particles.

Gamma ray

(a) Gamma ray is an electromagnetic radiation of short wavelength. Its wavelength is, in general, smaller than X-rays. Many of its properties are the same as those of X-rays.

(b) Being chargeless, it is not deflected by electric or magnetic field.

(c) It has the least ionizing power and the largest penetrating power among different types of nuclear radiation.

(d) All the photons coming from a particular gamma decay scheme have the same energy.

(e) Being an electromagnetic wave, gamma ray travels in vacuum with the velocity c.

Nuclear radiation, specially gamma ray, is used in medicine for cancer therapy and other treatments. The ionizing power of nuclear radiation is used in factories to avoid accumulation of charge on moving parts due to friction. Presence of radioactive material ionizes the air and any charge accumulated leaks away. Carbon dating, which is based on radioactive decay of ^{14}C, is a reliable technique to estimate the 'age' of archeological samples which have carbon contents. Excess exposure to nuclear radiation is harmful for human body.

46.7 ENERGY FROM THE NUCLEUS

Energy in various forms is available around us. Matter itself is a concentrate of energy. All atoms, molecules, nuclei, etc., are in continuous motion and have large amount of kinetic energy. But the energy that we need for our daily tasks is required in specific forms. We require energy in the form of heat to cook our food. We require energy in the form of electric current for our fans and electric lamps. Cooking gas and oxygen in air contain energy but this internal energy is not in the form required to cook our food. When the two are engaged in a chemical reaction, heat is produced which is in usable form. Sources of usable energy is something not in plenty and man is now concerned about energy conservation. Traditional sources of usable energy are wood, coal, petroleum, etc., and they are only in limited amounts and might be exhausted in a few hundred years. We are getting a large amount of energy from the sun but to use an appreciable fraction of it has been a challenging task. Several centres throughout the world are working hard to develop efficient solar cells which can convert the energy from the sun to usable forms. Satellites and spaceships receive energy from the sun through such solar cells.

A solution to the energy crisis has been presented by nuclear energy. Nuclear energy may be obtained either by breaking a heavy nucleus into two nuclei of middle weight or by combining two light nuclei to form a middle-weight nucleus. The former process is called *nuclear fission* and the latter *nuclear fusion*.

The physics of fission or fusion lies in the relation between the binding energy per nucleon versus A (figure 46.3). The middle-weight nuclei are more tightly bound than heavy-weight nuclei. When the nucleons of a heavy nucleus regroup in two middle-weight nuclei, called fragments, the total binding energy increases and hence the rest mass energy decreases. The difference in energy appears as the kinetic energy of the fragments or in some other form. This is the basic principle of fission.

Similar arguments hold for fusion. Again, the light-weight nuclei are less-tightly bound than the middle-weight nuclei. Thus, if two light nuclei combine to form a middle-weight nucleus, the binding energy increases and the rest mass decreases. Energy is released in the form of kinetic energy or in some other external form.

Example 46.6

The binding energy per nucleon is 8·5 MeV *for* $A \approx 120$ *and is* 7·6 MeV *for* $A = 240$ *(see figure 46.3). Suppose a nucleus with* $A = 240$ *breaks into two nuclei of nearly equal mass numbers. Calculate the energy released in the process.*

Solution :

Suppose the heavy nucleus had Z protons and N neutrons. The rest mass energy of this nucleus would be

$$E = Mc^2 = (Zm_p + Nm_n)c^2 - B_1$$
$$= (Zm_p + Nm_n)c^2 - 7{\cdot}6 \times 240 \text{ MeV}.$$

If there are Z_1 protons and N_1 neutrons in the first fragment, its rest mass energy will be

$$E_1 = M_1c^2 = (Z_1m_p + N_1m_n)c^2 - B_2$$
$$= (Z_1m_p + N_1m_n)c^2 - (8{\cdot}5 \text{ MeV})(Z_1 + N_1).$$

Similarly, if there are Z_2 protons and N_2 neutrons in the second fragment, its rest mass energy will be

$$E_2 = (Z_2m_p + N_2m_n)c^2 - (8{\cdot}5 \text{ MeV})(Z_2 + N_2).$$

The energy released due to the breaking is

$$E - (E_1 + E_2)$$
$$= \left[(Z - Z_1 - Z_2)m_p c^2 + (N - N_1 - N_2)m_n c^2\right]$$
$$+ [(Z_1 + Z_2 + N_1 + N_2) \times 8{\cdot}5 - 240 \times 7{\cdot}6] \text{ MeV}$$
$$= 240 \times (8{\cdot}5 - 7{\cdot}6) \text{ MeV} = 216 \text{ MeV}.$$

We have used the fact that $Z_1 + Z_2 = Z$, $N_1 + N_2 = N$ and $Z_1 + Z_2 + N_1 + N_2 = Z + N = 240$. Thus, 216 MeV of energy will be released when this nucleus breaks.

46.8 NUCLEAR FISSION

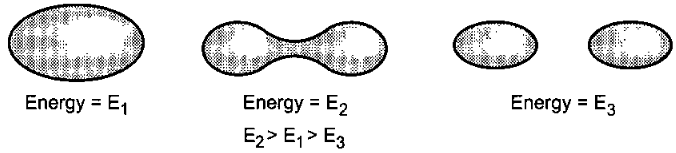

Figure 46.7

We have seen that a heavy nucleus has larger rest mass energy than that of its two middle-weight fragments. It is thus energetically favourable for the heavy nucleus to break into two middle-weight nuclei. However, before finally breaking into two parts, the heavy nucleus has to undergo a distortion which gradually increases to break the nucleus. The situation is shown in figure (46.7). The rest mass energy E_1 of the heavy nucleus is larger than the combined rest mass energy E_3 of its fragments but the energy E_2 in the intermediate state happens to be larger than E_1. Thus, it is not simple for the heavy nucleus to break spontaneously. In fact, according to classical physics, the process is impossible unless energy is supplied to the heavy nucleus to reach the intermediate state. Once it reaches the intermediate state, it can break into two parts and release energy. But the amount $E_2 - E_1$ has to be supplied to the heavy nucleus so that it may reach the intermediate state. Left to itself, the heavy nucleus will not break according to classical physics.

The world of subatomic particles is much different from that of our common day experience. According to quantum mechanics, if the final state has lesser energy than the energy in the initial state, there is a chance that the process will take place even if the intermediate state has energy greater than the initial one and no energy is supplied externally (figure 46.8).

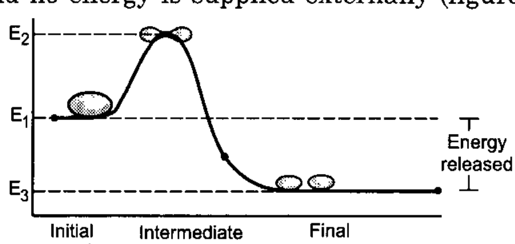

Figure 46.8

Such processes are called *barrier penetration*. The energy seems to be created out of nothing, a violation of energy conservation ! But this is a fact of the physics of small particles. The energy conservation in the usual sense may be violated for 'short times'. The amount of energy seems to be created and the time for which it is created are related through Heisenberg uncertainty relation,

$$\Delta E.\Delta t \approx h/2\pi$$

where h is the Planck constant.

Barrier penetration, though possible, is not easy. Greater the energy difference $\Delta E = E_2 - E_1$, smaller is the probability of a successful barrier penetration. This extra energy ΔE is called height of the barrier. Similarly, larger the duration of intermediate state, smaller is the probability of barrier penetration. These parameters are different for different nuclides and hence the probability that a heavy nucleus will break in a given time is different for different nuclides. Generally, this probability is extremely small except in a few nuclides. For example, the half-life of $^{238}_{92}U$ for fission reaction is about 10^{16} years. If you start with N nuclei of $^{238}_{92}U$ today, only $N/2$ will disintegrate in the next 10^{16} years. Table (46.1) shows some of the better cases where the probability of fission is appreciable.

Table 46.1 : *Fission probability*

Nuclide	Fission probability relative to ^{236}U
^{236}U	1 (arbitrarily assumed)
^{239}U	$< 1 \times 10^{-3}$
^{240}Pu	1·5
^{244}Am	$< 2 \times 10^{-4}$

46.9 URANIUM FISSION REACTOR

The most attractive bid, from a practical point of view, to achieve energy from nuclear fission is to use $^{236}_{92}U$ as the fission material. This nuclide is highly fissionable and hence is not found in nature. Natural uranium contains about 99·3% of $^{238}_{92}U$ and 0·7% of $^{235}_{92}U$. The technique is to hit a uranium sample by slow-moving neutrons (kinetic energy ≈ 0·04 eV, also called thermal neutrons). A $^{235}_{92}U$ nucleus has large probability of absorbing a slow neutron and forming $^{236}_{92}U$ nucleus. This nucleus then fissions into two parts. A variety of combinations of the middle-weight nuclei may be formed due to the fission. For example, one may have

$$^{236}_{92}U \rightarrow {}^{137}_{53}I + {}^{97}_{39}Y + 2n,$$

$$^{236}_{92}U \rightarrow {}^{140}_{56}Ba + {}^{94}_{36}Kr + 2n$$

and a number of other combinations.

During a fission event, in general, two or three neutrons are emitted. If the total number of neutrons emitted in a large number of events is divided by the number of events, the average comes out to be around 2·47. We say that on an average 2·47 neutrons (or 2·5 as a round figure) are emitted in each fission event. The two fragments generally have unequal mass numbers as is the case in the above examples. If the relative yield of different nuclei are plotted against their mass number, a plot of the type shown in figure (46.9) results. The most probable mass numbers of the fragments are around $A = 95$ and 140. The probability of having nearly equal fragments is small.

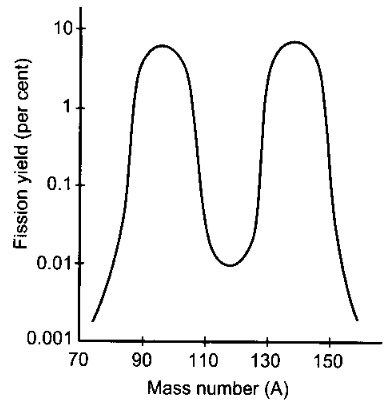

Figure 46.9

The ratio N/Z is larger in heavier nuclei than in the middle-weight nuclei (figure 46.1). Thus, the fragments will have N/Z ratio larger than that needed for stability. As a result, 2 or 3 neutrons are emitted together with the fission fragments. The fragments reduce their N/Z ratio further via beta decay in which a neutron is converted into a proton. These daughter nuclei are generally formed in excited states and consequently emit gamma rays. At some stage, a daughter nucleus can also emit another neutron. Thus, neutrons, beta particles (electrons), antineutrinos and gamma photons accompany nuclear fission. For example, in one of the reactions, ^{236}U breaks into $^{97}_{39}Y$ and $^{137}_{53}I$. These nuclei undergo the following changes:

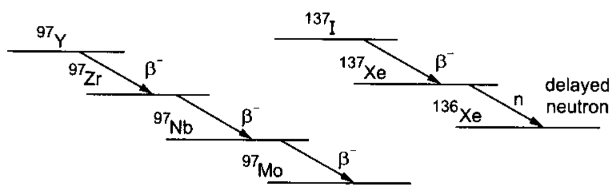

Figure 46.10

The neutron emitted by ^{137}Xe comes out an appreciable time interval after the fission as against the two neutrons emitted almost simultaneously with the fission. This is because ^{137}I decays to ^{137}Xe with its own half-life and only then ^{137}Xe emits the neutron. Such neutrons are called *delayed neutrons* and play an important role in controlling the fission rate.

In each fission event, about 200 MeV of energy is released a large part of which appears in the form of kinetic energies of the two fragments. Neutrons take away about 5 MeV. As the fragments decay, an additional energy of about 15–20 MeV is released in the form of kinetic energy of beta particles, antineutrino and photons. The fragments, formed with so much kinetic energy, are immediately stopped in the bulk uranium solid in which they are formed. This produces large amount of heat which is taken away by passing a cold liquid in pipes through the reaction area.

Chain Reaction

A very important and interesting feature of neutron-induced fission is the chain reaction. Once a neutron starts the fission by being absorbed in ^{235}U, the fission itself produces 2 or 3 new neutrons which may be absorbed by another nearby ^{235}U causing another fission. Such a process is known as a chain reaction. The number of neutrons may thus go on increasing in each generation of fission and the rate of fission may likewise increase in geometrical progression. If that happens, the whole of the material will fission out in a small time. The large amount of heat produced in such a short time will be uncontrollable and will only lead to disaster. There are ways to control the rate of chain reaction. If the fission event takes place near the surface of the bulk uranium material taken, there is a good chance that the neutrons produced will escape from the material without coming in contact with another ^{235}U. The fraction of the neutrons lost in this way will be larger if smaller pieces of uranium are taken. Controlling the size will thus control the rate of fission. Another important point is that the neutrons produced in the fission have kinetic energy $\approx$ 2 MeV. They are called *fast neutrons*. The ^{235}U nucleus has a good probability of absorbing slow neutrons ($\approx$ 0·04 eV), but has a poor chance of absorbing fast neutrons. The neutron may not get absorbed in ^{235}U even if the two meet and collide. If the material is large enough, the neutron will suffer a number of collisions with other nuclei. In each collision, it will lose its kinetic energy and after some time may slow down to thermal energies $\approx$ 0·04 eV. It may then be absorbed by ^{235}U. But ^{238}U nuclei, which are present in large numbers (99·3%), are extremely good neutron absorbers if they get neutrons of energy 1–100 eV. These neutrons get absorbed in ^{238}U to form ^{239}U which decays generally by means other than fission. A fast neutron has to go through this 1–100 eV range before slowing down to thermal energies $\approx$ 0·04 eV and has a fair chance of being absorbed by ^{238}U. This neutron is, therefore, lost as far as fission is concerned.

Nuclear Reactor

Let us now consider the design and working of a typical uranium nuclear reactor (figure 46.11). Uranium is taken in the form of cylindrical rods arranged in a regular pattern in the active reactor core. The volume in the core is filled with a low-Z material such as heavy water (D_2O), graphite, beryllium, etc. This material is called *moderator*.

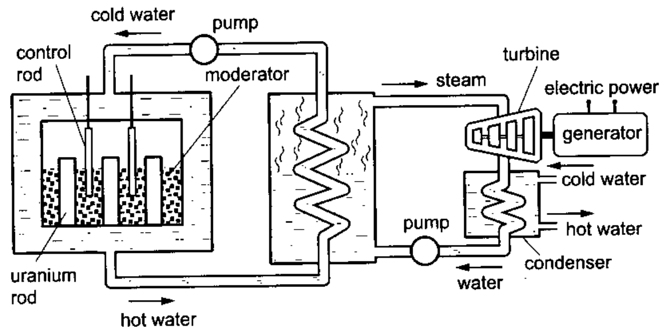

Figure 46.11

When fission takes place in a uranium rod, most of the fast neutrons produced escape from the rod and enter into the moderator. These neutrons make collisions with the particles of the moderator and thus slow down. About 25 collisions with deuteron (present in heavy water) or 100 collisions with carbon or beryllium are sufficient to slow down a neutron from 2 MeV to thermal energies. The distances between the rods are adjusted in such a way that a neutron coming from one rod is generally slowed down to thermal energies before entering the other rod. This eliminates the possibility of a neutron being absorbed by ^{238}U in 1–100 eV region. The geometry of the core is such that out of the average 2·5 neutrons produced per fission, 1 neutron is used to trigger the next fission and the remaining are lost without triggering any fission. The reaction is then sustained at a constant rate. If the rate of the loss of neutrons is decreased further, the fission rate will keep on increasing which may lead to explosion. If the rate of loss of neutrons is increased, the rate of fission will keep on decreasing and ultimately the chain reaction will stop. The finer control of fission rate is made by the *control rods* which are made of cadmium and are inserted up to a certain depth in the moderator. Cadmium is a very good neutron absorber. If the stage is set for stable chain reaction and the cadmium rods are pushed into the moderator, the reactor will be shut off. Pulling the cadmium rods out will start the reactor.

Some coolant liquid such as water at high pressure or molten sodium is passed through the reactor-core area which withdraws the heat produced in the core. The heat is used to prepare steam from water. The steam so prepared is used to run steam turbines and produce electric power. The amount of ^{235}U goes on decreasing as the uranium rod is used for fission. When ^{235}U is finished and only ^{238}U is left, the rods have to be changed.

Breeder Reactors

Although fission generates large amount of energy and the world is heavily depending on fission for its energy requirement, uranium resources are also limited. Table (46.1) shows that fission can easily take place with ^{240}Pu besides ^{236}U. But ^{239}Pu is not a naturally occurring isotope. However, ^{238}U can capture a neutron to produce ^{239}Pu which can be used as a nuclear fuel.

$$^{238}_{92}U + n \rightarrow {}^{239}_{92}U \xrightarrow{\beta^-} {}^{239}_{93}Np \xrightarrow{\beta^-} {}^{239}_{94}Pu.$$

Suppose, used uraninum rods, which contain only ^{238}U, are kept in or around a uranium-reactor core. Also suppose, the geometry is such that out of the average 2·5 neutrons produced in a fission, one neutron is absorbed by a ^{238}U nucleus in these rods resulting in ^{239}Pu. Then we produce as much nuclear fuel in the form of ^{239}Pu as we consume in the form of ^{235}U. If more than one neutron can be absorbed by these ^{238}U rods per fission then we produce more fuel than what we consume. Thus, apart from nuclear energy, these reactors give us fresh nuclear fuel which often exceeds the nuclear fuel used. Such a reactor is called a *breeder reactor*.

46.10 NUCLEAR FUSION

When two light nuclei come close to one another, within the range of attractive nuclear force (≈ 1 fm), they may combine to form a bigger nucleus. The process is possible from an energy point of view because the binding energy per nucleon is small for light nuclei and increases with A until A is about 50. To bring the light nuclei within the separation of about a femtometre is, however, a difficult task. Any bulk material is composed of atoms and an atom typically has a radius of a few angstroms (1 angstrom $= 10^{-10}$ m $= 10^5$ fm). When atoms are pushed closer, their electrons cause them to repel each other. Even if all the electrons are stripped off, the nuclei themselves are positively charged and strongly repel each other. The technique thus is to heat a gas to an extremely high temperature so that the electrons are completely detached and the nuclei move within the gas with large random speeds. Two nuclei moving towards each other may come close enough to fuse into one nucleus. What must be the order of temperature which will ensure enough fusion ?

Example 46.7

Consider two deuterons moving towards each other with equal speeds in a deuteron gas. What should be their kinetic energies (when they are widely separated) so that the closest separation between them becomes 2 fm ? Assume that the nuclear force is not effective for separations greater than 2 fm. At what temperature will the deuterons have this kinetic energy on an average ?

Solution :

As the deuterons move, the Coulomb repulsion will slow them down. The loss in kinetic energy will be equal to the gain in Coulomb potential energy. At the closest separation, the kinetic energy is zero and the potential energy is $\frac{e^2}{4\pi\varepsilon_0 r}$. If the initial kinetic energy of each deuteron is K and the closest separation is 2 fm, we shall have

$$2K = \frac{e^2}{4\pi\varepsilon_0 (2\text{ fm})}$$

$$= \frac{(1{\cdot}6\times10^{-19}\text{ C})^2\times(9\times10^9\text{ N m}^2\text{C}^{-2})}{2\times10^{-15}\text{ m}}$$

or, $$K = 5{\cdot}7\times10^{-14}\text{ J}.$$

If the temperature of the gas is T, the average kinetic energy of random motion of each nucleus will be $1{\cdot}5\,kT$. The temperature needed for the deuterons to have the average kinetic energy of $5{\cdot}7\times10^{-14}$ J will be given by

$$1{\cdot}5\,kT = 5{\cdot}7\times10^{-14}\text{ J}$$

or, $$T = \frac{5{\cdot}7\times10^{-14}\text{ J}}{1{\cdot}5\times1{\cdot}38\times10^{-23}\text{ J K}}$$

$$= 2{\cdot}8\times10^9\text{ K}.$$

One needs a temperature of the order of 10^9 K if deuterons are to be fused. The temperature inside the sun is estimated to be around $1{\cdot}5\times10^7$ K. Yet fusion is the main source of energy in the sun which it ultimately radiates to the universe including the earth. There are two main reasons why fusion can take place at a temperature even hundred times smaller than that calculated in example (46.7). One is that the energy of all the particles is not equal to the average energy. Although the average kinetic energy is $1{\cdot}5\,kT$, there are particles which have kinetic energy much larger than $1{\cdot}5\,kT$. Secondly, even if the kinetic energy of the two interacting nuclei is less than that needed to bring them within the nuclear range, there is a small chance of fusion through the process of *barrier penetration*.

As these reactions take place at high temperatures, they are also called *thermonuclear fusion* or *thermonuclear reactions*.

Fusion in Sun

Among the celestial bodies in which energy is produced, the sun is relatively cooler. There are stars with temperatures around 10^8 K inside. In sun and other stars, where the temperature is less than or around 10^7 K, fusion takes place dominantly by *proton–proton cycle* as follows:

$$\begin{aligned}
{}^{1}H + {}^{1}H &\rightarrow {}^{2}H + e^{+} + \nu \\
{}^{2}H + {}^{1}H &\rightarrow {}^{3}He + \gamma \\
{}^{3}He + {}^{3}He &\rightarrow {}^{4}He + 2\,{}^{1}H \\
\hline
4\,{}^{1}H &\rightarrow {}^{4}He + 2e^{+} + 2\nu + 2\gamma
\end{aligned}$$

Note that the first two reactions should occur twice to produce two ${}^{3}He$ nuclei and initiate the third reaction. As a result of this cycle, effectively, four hydrogen nuclei combine to form a helium nucleus. About 26·7 MeV energy is released in the cycle. Thus, hydrogen is the fuel which 'burns' into helium to release energy. The sun is estimated to have been radiating energy for the last $4{\cdot}5 \times 10^9$ years and will continue to do so till all the hydrogen in it is used up. It is estimated that the present store of hydrogen in the sun is sufficient for the next 5×10^9 years.

In hotter stars where the temperature is $\approx 10^8$ K, another cycle known as proton–carbon cycle takes place.

$$\begin{aligned}
{}^{1}H + {}^{12}C &\rightarrow {}^{13}N + \gamma \\
{}^{13}N &\rightarrow {}^{13}C + e^{+} + \nu \\
{}^{1}H + {}^{13}C &\rightarrow {}^{14}N + \gamma \\
{}^{1}H + {}^{14}N &\rightarrow {}^{15}O + \gamma \\
{}^{15}O &\rightarrow {}^{15}N + e^{+} + \nu \\
{}^{1}H + {}^{15}N &\rightarrow {}^{12}C + {}^{4}He
\end{aligned}$$

The end result of this cycle is again the fusion of four hydrogen nuclei into a helium nucleus. Carbon nucleus acts only as a catalyst.

It is clear that Coulomb repulsion becomes more and more obstructive to fusion as Z increases. Thus, it needs still higher temperatures for heavier elements to fuse. When the temperature inside a star rises, such fusions do take place to produce heavier nuclei such as

$${}^{4}He + {}^{4}He + {}^{4}He \rightarrow {}^{12}C + \gamma.$$

The process can continue, finally producing elements in iron region ($A = 56$) where the binding energy per nucleon is maximum. Elements heavier than iron can be produced by neutron absorption and subsequent beta decay.

46.11 FUSION IN LABORATORY

In stellar objects, the material remains confined at high temperature due to gravitational pull. If we wish to make fusion as energy-producing device in laboratory, the major problem is to confine the hot plasma (when all the electrons are detached from the atoms, we get a plasma) in a small volume for extended time intervals. Producing a high temperature is obviously a major task but confinement at such high temperatures is more challenging. Solid walls can't be used as containers because no solid can sustain the high temperatures needed for fusion. The easiest thermonuclear reaction that can be handled on earth is the fusion of two deuterons (*D–D* reaction) or fusion of a deuteron with a triton (*D–T* reaction).

$$\begin{aligned}
{}^{2}_{1}H + {}^{2}_{1}H &\rightarrow {}^{3}_{2}He + n + 3{\cdot}3 \text{ MeV } (D\text{–}D) \\
{}^{2}_{1}H + {}^{2}_{1}H &\rightarrow {}^{3}_{1}H + {}^{1}_{1}H + 4{\cdot}0 \text{ MeV } (D\text{–}D) \\
{}^{2}_{1}H + {}^{3}_{1}H &\rightarrow {}^{4}_{2}He + n + 17{\cdot}6 \text{ MeV } (D\text{–}T)
\end{aligned}$$

One starts with deuterium gas (heavy hydrogen) or a mixture of deuterium with tritium, heat the gas to high temperatures, ensuring its confinement for reasonable period, and looks for the fusion.

Lawson criterion

J.D. Lawson showed that in order to get an energy output greater than the energy input, a fusion reactor should achieve

$$n\tau > 10^{14} \text{ s cm}^{-3}$$

where n is the density of the interacting particles and τ is the confinement time. The quantity $n\tau$ in s cm^{-3} is called *Lawson number.*

The ratio of the energy output to the energy input is known as Q of the fusion machine. For a viable fusion machine, Q should be greater than 1.

Tokamak Design

In one of the methods receiving serious attention, one uses the so-called *Tokamak* design. The deuterium plasma is contained in a toroidal region by specially designed magnetic field. The directions and magnitudes of the magnetic field are so managed in the toroidal space that whenever a charged plasma particle attempts to go out, the $q\vec{v} \times \vec{B}$ force tends to push it back into the toroidal volume. It is a difficult task to design a magnetic field which will push the particles moving in random directions with random speeds into a specified volume, but it is possible and has been done. The plasma is, therfore, confined by the magnetic field. Such confinement has been achieved for short durations ($\approx$ few microseconds) in which some fusion occurs. Fusion thus proceeds in bursts or pulses. The heating is accomplished by passing high frequency oscillating current through the plasma gas. A schematic design is shown in figure (46.12).

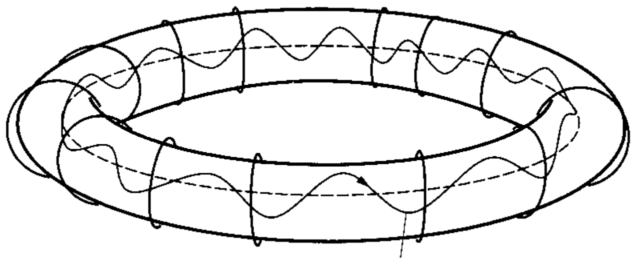

Figure 46.12

A large fusion machine known as *Joint European Torus* (JET) is designed to achieve fusion energy on this principle. A value of $Q \approx 1$ is already achieved with JET. Scientists working on this machine expect to get $Q \approx 40$ in the next 40 to 50 years.

At the Institute for Plasma Research (IPR), Ahmedabad, a small machine named *Aditya* is functioning on the Tokamak design. This machine is being used to study the properties of a plasma and a value of $n\tau \approx 10^{11}$ s cm^{-3} has been achieved for Lawson number.

Inertial Confinement

In another method known as inertial confinement, laser beams are used to confine the plasma. A small solid pellet is made which contains deuterium and tritium. Intense laser beams are directed on the pellet from many directions distributed over all sides. The lasers first vaporize the pellet converting it into plasma and then compress it from all directions because of the large pressure exerted. The density increases by 10^3 to 10^4 times the initial density and the temperature rises to high values. The fusion occurs in this period. The α-particles (He nuclei) generated by the fusion are also forced to remain inside the plasma. Their kinetic energy is lost into the plasma itself contributing further rise in temperature. Again the lasers are operated in pulses of short duration.

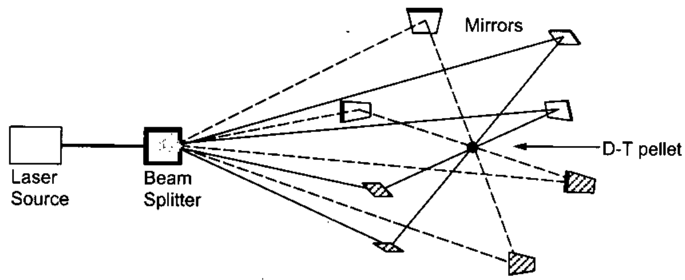

Figure 46.13

The research in fusion energy is going on. Fusion is the definite and ultimate answer to our energy problems. The 'fuel' used for fusion on earth is deuterium which is available in natural water (0·03%). And with oceans as the almost unlimited source of water, we can be sure of fuel supply for thousands of years. Secondly, fusion reactions are neat and clean. Radioactive radiation accompanying fission reactors will not be there with fusion reactors.

Worked Out Examples

1. *Calculate the electric potential energy due to the electric repulsion between two nuclei of* ^{12}C *when they 'touch' each other at the surface.*

Solution : The radius of a ^{12}C nucleus is

$$R = R_0 A^{1/3}$$
$$= (1{\cdot}1 \text{ fm})(12)^{1/3} = 2{\cdot}52 \text{ fm}.$$

The separation between the centres of the nuclei is $2R = 5{\cdot}04$ fm. The potential energy of the pair is

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$
$$= (9\times10^{9}\ \text{N m}^2\text{C}^{-2})\frac{(6\times1{\cdot}6\times10^{-19}\ \text{C})^2}{5{\cdot}04\times10^{-15}\ \text{m}}$$
$$= 1{\cdot}64\times10^{-12}\ \text{J} = 10{\cdot}2\ \text{MeV}.$$

2. *Find the binding energy of* $^{56}_{26}$Fe. *Atomic mass of* ^{56}Fe *is* 55·9349 u *and that of* ^{1}H *is* 1·00783 u. *Mass of neutron* = 1·00867 u.

Solution : The number of protons in $^{56}_{26}$Fe = 26 and the number of neutrons = 56 − 26 = 30. The binding energy of $^{56}_{26}$Fe is

$$= [26\times1{\cdot}00783\ \text{u} + 30\times1{\cdot}00867\ \text{u} - 55{\cdot}9349\ \text{u}]c^2$$
$$= (0{\cdot}52878\ \text{u})c^2$$
$$= (0{\cdot}52878\ \text{u})(931\ \text{MeV u}^{-1}) = 492\ \text{MeV}.$$

3. *Find the kinetic energy of the α-particle emitted in the decay* ^{238}Pu → ^{234}U + α. *The atomic masses needed are as follows:*

^{238}Pu	^{234}U	^{4}He
238·04955 u	234·04095 u	4·002603 u

Neglect any recoil of the residual nucleus.

Solution : Using energy conservation,

$$m(^{238}\text{Pu})c^2 = m(^{234}\text{U})c^2 + m(^4\text{He})c^2 + K$$

or, $$K = [m(^{238}\text{Pu}) - m(^{234}\text{U}) - m(^4\text{He})]c^2$$

$$= [238{\cdot}04955\ \text{u} - 234{\cdot}04095\ \text{u} - 4{\cdot}002603\ \text{u}](931\ \text{MeV u}^{-1})$$
$$= 5{\cdot}58\ \text{MeV}.$$

4. *Calculate the Q-value in the following decays:*
(a) $^{19}O \rightarrow {}^{19}F + e + \bar{\nu}$
(b) $^{25}Al \rightarrow {}^{25}Mg + e^{+} + \nu$.
The atomic masses needed are as follows:

^{19}O	^{19}F	^{25}Al	^{25}Mg
19·003576 u	18·998403 u	24·990432 u	24·985839 u

Solution :

(a) The Q-value of β^{-}-decay is

$$Q = [m(^{19}O) - m(^{19}F)]c^2$$
$$= [19{\cdot}003576\ u - 18{\cdot}998403\ u]\ (931\ MeV\ u^{-1})$$
$$= 4{\cdot}816\ MeV.$$

(b) The Q-value of β^{+}-decay is

$$Q = [m(^{25}Al) - m(^{25}Mg) - 2m_e]c^2$$
$$= [24{\cdot}990432\ u - 24{\cdot}985839\ u - 2 \times 0{\cdot}511\ MeV{\cdot}c^{-2}]c^2$$
$$= (0{\cdot}004593\ u)\ (931\ MeV\ u^{-1}) - 1{\cdot}022\ MeV$$
$$= 4{\cdot}276\ MeV - 1{\cdot}022\ MeV = 3{\cdot}254\ MeV.$$

5. *Find the maximum energy that a beta particle can have in the following decay*

$$^{176}Lu \rightarrow {}^{176}Hf + e + \bar{\nu}.$$

Atomic mass of ^{176}Lu *is* 175·942694 u *and that of* ^{176}Hf is 175·941420 u.

Solution : The kinetic energy available for the beta particle and the antineutrino is

$$Q = [m(^{176}Lu) - m(^{176}Hf)]c^2$$
$$= (175{\cdot}942694\ u - 175{\cdot}941420\ u)\ (931\ MeVu^{-1})$$
$$= 1{\cdot}182\ MeV.$$

This energy is shared by the beta particle and the antineutrino. The maximum kinetic energy of a beta particle in this decay is, therefore, 1·182 MeV when the antineutrino practically does not get any share.

6. *Consider the beta decay*

$$^{198}Au \rightarrow {}^{198}Hg^{*} + \beta^{-} + \bar{\nu}$$

where $^{198}Hg^{*}$ *represents a mercury nucleus in an excited state at energy* 1·088 MeV *above the ground state. What can be the maximum kinetic energy of the electron emitted ? The atomic mass of* ^{198}Au *is* 197·968233 u *and that of* ^{198}Hg *is* 197·966760 u.

Solution : If the product nucleus ^{198}Hg is formed in its ground state, the kinetic energy available to the electron and the antineutrino is

$$Q = [m(^{198}Au) - m(^{198}Hg)]c^2.$$

As $^{198}Hg^{*}$ has energy 1·088 MeV more than ^{198}Hg in ground state, the kinetic energy actually available is

$$Q = [m(^{198}Au) - m(^{198}Hg)]c^2 - 1{\cdot}088\ MeV$$
$$= (197{\cdot}968233\ u - 197{\cdot}966760\ u)\ (931\ MeV\ u^{-1}) - 1{\cdot}088\ MeV$$
$$= 1{\cdot}3686\ MeV - 1{\cdot}088\ MeV = 0{\cdot}2806\ MeV$$

This is also the maximum possible kinetic energy of the electron emitted.

7. *The half-life of* ^{198}Au *is* 2·7 days. *Calculate (a) the decay constant, (b) the average-life and (c) the activity of* 1·00 mg *of* ^{198}Au. *Take atomic weight of* ^{198}Au *to be* 198 g mol^{-1}.

Solution : (a) The half-life and the decay constant are related as

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0{\cdot}693}{\lambda}$$

or,
$$\lambda = \frac{0{\cdot}693}{t_{1/2}} = \frac{0{\cdot}693}{2{\cdot}7\ days}$$
$$= \frac{0{\cdot}693}{2{\cdot}7 \times 24 \times 3600\ s} = 2{\cdot}9 \times 10^{-6}\ s^{-1}.$$

(b) The average-life is $t_{av} = \frac{1}{\lambda} = 3{\cdot}9$ days.

(c) The activity is $A = \lambda N$. Now, 198 g of ^{198}Au has 6×10^{23} atoms. The number of atoms in 1·00 mg of ^{198}Au is

$$N = 6 \times 10^{23} \times \frac{1{\cdot}0\ mg}{198\ g} = 3{\cdot}03 \times 10^{18}.$$

Thus,
$$A = \lambda N$$
$$= (2{\cdot}9 \times 10^{-6}\ s^{-1})\ (3{\cdot}03 \times 10^{18})$$
$$= 8{\cdot}8 \times 10^{12}\ \text{disintegrations}\ s^{-1}$$
$$= \frac{8{\cdot}8 \times 10^{12}}{3{\cdot}7 \times 10^{10}}\ Ci = 240\ Ci.$$

8. *A radioactive sample has* $6{\cdot}0 \times 10^{18}$ *active nuclei at a certain instant. How many of these nuclei will still be in the same active state after two half-lives ?*

Solution : In one half-life the number of active nuclei reduces to half the original number. Thus, in two half-lives the number is reduced to $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ of the original number. The number of remaining active nuclei is, therefore,

$$6{\cdot}0 \times 10^{18} \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)$$
$$= 1{\cdot}5 \times 10^{18}.$$

9. *The activity of a radioactive sample falls from* 600 s^{-1} *to* 500 s^{-1} *in* 40 minutes. *Calculate its half-life.*

Solution : We have,

$$A = A_0\, e^{-\lambda t}$$

or, $500\ \mathrm{s}^{-1} = (600\ \mathrm{s}^{-1})e^{-\lambda t}$

or, $e^{-\lambda t} = \frac{5}{6}$

or, $\lambda t = \ln(6/5)$

or, $\lambda = \frac{\ln(6/5)}{t} = \frac{\ln(6/5)}{40\ \mathrm{min}}$.

The half-life is $t_{1/2} = \frac{\ln 2}{\lambda}$

$$= \frac{\ln 2}{\ln(6/5)} \times 40\ \mathrm{min}$$

$$= 152\ \mathrm{min}.$$

10. *The number of* ^{238}U *atoms in an ancient rock equals the number of* ^{206}Pb *atoms. The half-life of decay of* ^{238}U *is* $4{\cdot}5 \times 10^{9}$ y. *Estimate the age of the rock assuming that all the* ^{206}Pb *atoms are formed from the decay of* ^{238}U.

Solution : Since the number of ^{206}Pb atoms equals the number of ^{238}U atoms, half of the original ^{238}U atoms have decayed. It takes one half-life to decay half of the active nuclei. Thus, the sample is $4{\cdot}5 \times 10^{9}$ y old.

11. *Equal masses of two samples of charcoal A and B are burnt separately and the resulting carbon dioxide are collected in two vessels. The radioactivity of* ^{14}C *is measured for both the gas samples. The gas from the charcoal A gives* 2100 *counts per week and the gas from the charcoal B gives* 1400 *counts per week. Find the age difference between the two samples. Half-life of* ^{14}C = 5730 y.

Solution : The activity of sample A is 2100 counts per week. After a certain time t, its activity will be reduced to 1400 counts per week. This is because a fraction of the active ^{14}C nuclei will decay in time t. The sample B must be a time t older than the sample A.

We have,

$$A = A_0 e^{-\lambda t}$$

or, $1400\ \mathrm{s}^{-1} = 2100\ \mathrm{s}^{-1} e^{-\lambda t}$

or, $e^{-\lambda t} = \frac{2}{3}$

$$t = \frac{\ln(3/2)}{\lambda}$$

$$= \frac{\ln(3/2)}{0{\cdot}693} t_{1/2}$$

$$= \frac{0{\cdot}4055}{0{\cdot}693} \times 5730\ \mathrm{y} = 3352\ \mathrm{y}.$$

12. *Suppose, the daughter nucleus in a nuclear decay is itself radioactive. Let* λ_p *and* λ_d *be the decay constants of the parent and the daughter nuclei. Also, let* N_p *and* N_d *be the number of parent and daughter nuclei at time t. Find the condition for which the number of daughter nuclei becomes constant.*

Solution : The number of parent nuclei decaying in a short time interval t to $t + dt$ is $\lambda_p N_p dt$. This is also the number of daughter nuclei produced in this interval. The number of daughter nuclei decaying during the same time interval is $\lambda_d N_d dt$. The number of the daughter nuclei will be constant if

$$\lambda_p N_p dt = \lambda_d N_d dt$$

or, $\lambda_p N_p = \lambda_d N_d$.

13. *A radioactive sample decays with an average-life of* 20 ms. *A capacitor of capacitance* 100 μF *is charged to some potential and then the plates are connected through a resistance R. What should be the value of R so that the ratio of the charge on the capacitor to the activity of the radioactive sample remains constant in time ?*

Solution : The activity of the sample at time t is given by

$$A = A_0 e^{-\lambda t}$$

where λ is the decay constant and A_0 is the activity at time $t = 0$ when the capacitor plates are connected. The charge on the capacitor at time t is given by

$$Q = Q_0 e^{-t/CR}$$

where Q_0 is the charge at $t = 0$ and $C = 100\ \mu\mathrm{F}$ is the capacitance. Thus,

$$\frac{Q}{A} = \frac{Q_0}{A_0} \frac{e^{-t/CR}}{e^{-\lambda t}}.$$

It is independent of t if $\lambda = \frac{1}{CR}$

or, $R = \frac{1}{\lambda C} = \frac{t_{av}}{C} = \frac{20 \times 10^{-3}\ \mathrm{s}}{100 \times 10^{-6}\ \mathrm{F}} = 200\ \Omega.$

14. *A radioactive nucleus can decay by two different processes. The half-life for the first process is* t_1 *and that for the second process is* t_2. *Show that the effective half-life t of the nucleus is given by*

$$\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}.$$

Solution : The decay constant for the first process is $\lambda_1 = \frac{\ln 2}{t_1}$ and for the second process it is $\lambda_2 = \frac{\ln 2}{t_1}$. The probability that an active nucleus decays by the first process in a time interval dt is $\lambda_1 dt$. Similarly, the probability that it decays by the second process is $\lambda_2 dt$. The probability that it either decays by the first process or by the second process is $\lambda_1 dt + \lambda_2 dt$. If the effective decay constant is λ, this probability is also equal to λdt. Thus,

$$\lambda dt = \lambda_1 dt + \lambda_2 dt$$

or, $\lambda = \lambda_1 + \lambda_2$

or, $$\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}.$$

15. *Calculate the energy released when three alpha particles combine to form a ^{12}C nucleus. The atomic mass of ^4_2He is 4·002603 u.*

Solution : The mass of a ^{12}C atom is exactly 12 u. The energy released in the reaction $3\left(^4_2He\right) \rightarrow {}^{12}_6C$ is

$$[3\, m(^4_2He) - m(^{12}_6C)]c^2$$

$$= [3 \times 4{\cdot}002603 \text{ u} - 12 \text{ u}]\ (931 \text{ MeV u}^{-1}) = 7{\cdot}27 \text{ MeV}.$$

□

QUESTIONS FOR SHORT ANSWER

1. If neutrons exert only attractive force, why don't we have a nucleus containing neutrons alone ?
2. Consider two pairs of neutrons. In each pair, the separation between the neutrons is the same. Can the force between the neutrons have different magnitudes for the two pairs ?
3. A molecule of hydrogen contains two protons and two electrons. The nuclear force between these two protons is always neglected while discussing the behaviour of a hydrogen molecule. Why ?
4. Is it easier to take out a nucleon (a) from carbon or from iron (b) from iron or from lead ?
5. Suppose we have 12 protons and 12 neutrons. We can assemble them to form either a ^{24}Mg nucleus or two ^{12}C nuclei. In which of the two cases more energy will be liberated ?
6. What is the difference between cathode rays and beta rays ? When the two are travelling in space, can you make out which is the cathode ray and which is the beta ray ?
7. If the nucleons of a nucleus are separated from each other, the total mass is increased. Where does this mass come from ?
8. In beta decay, an electron (or a positron) is emitted by a nucleus. Does the remaining atom get oppositely charged ?
9. When a boron nucleus ($^{10}_5B$) is bombarded by a neutron, an α-particle is emitted. Which nucleus will be formed as a result ?
10. Does a nucleus lose mass when it suffers gamma decay ?
11. In a typical fission reaction, the nucleus is split into two middle-weight nuclei of unequal masses. Which of the two (heavier or lighter) has greater kinetic energy ? Which one has greater linear momentum ?
12. If three helium nuclei combine to form a carbon nucleus, energy is liberated. Why can't helium nuclei combine on their own and minimise the energy ?

OBJECTIVE I

1. The mass of a neutral carbon atom in ground state is
   (a) exact 12 u (b) less than 12 u
   (c) more than 12 u
   (d) depends on the form of carbon such as graphite or charcoal.
2. The mass number of a nucleus is equal to
   (a) the number of neutrons in the nucleus
   (b) the number of protons in the nucleus
   (c) the number of nucleons in the nucleus
   (d) none of them.
3. As compared to ^{12}C atom, ^{14}C atom has
   (a) two extra protons and two extra electrons
   (b) two extra protons but no extra electron
   (c) two extra neutrons and no extra electron
   (d) two extra neutrons and two extra electrons.
4. The mass number of a nucleus is
   (a) always less than its atomic number
   (b) always more than its atomic number
   (c) equal to its atomic number
   (d) sometimes more than and sometimes equal to its atomic number.
5. The graph of $\ln(R/R_0)$ versus $\ln A$ (R = radius of a nucleus and A = its mass number) is
   (a) a straight line (b) a parabola
   (c) an ellipse (d) none of them.
6. Let F_{pp}, F_{pn} and F_{nn} denote the magnitudes of the nuclear force by a proton on a proton, by a proton on a neutron and by a neutron on a neutron respectively. When the separation is 1 fm,
   (a) $F_{pp} > F_{pn} = F_{nn}$ (b) $F_{pp} = F_{pn} = F_{nn}$
   (c) $F_{pp} > F_{pn} > F_{nn}$ (d) $F_{pp} < F_{pn} = F_{nn}$.
7. Let F_{pp}, F_{pn} and F_{nn} denote the magnitudes of the net force by a proton on a proton, by a proton on a neutron and by a neutron on a neutron respectively. Neglect gravitational force. When the separation is 1 fm,
   (a) $F_{pp} > F_{pn} = F_{nn}$ (b) $F_{pp} = F_{pn} = F_{nn}$
   (c) $F_{pp} > F_{pn} > F_{nn}$ (d) $F_{pp} < F_{pn} = F_{nn}$.
8. Two protons are kept at a separation of 10 nm. Let F_n and F_e be the nuclear force and the electromagnetic force between them.
   (a) $F_e = F_n$ (b) $F_e >> F_n$ (c) $F_e << F_n$
   (d) F_e and F_n differ only slightly.

9. As the mass number A increases, the binding energy per nucleon in a nucleus
(a) increases (b) decreases (c) remains the same
(d) varies in a way that depends on the actual value of A.

10. Which of the following is a wrong description of binding energy of a nucleus ?
(a) It is the energy required to break a nucleus into its constituent nucleons.
(b) It is the energy made available when free nucleons combine to form a nucleus.
(c) It is the sum of the rest mass energies of its nucleons minus the rest mass energy of the nucleus.
(d) It is the sum of the kinetic energy of all the nucleons in the nucleus.

11. In one average-life,
(a) half the active nuclei decay
(b) less than half the active nuclei decay
(c) more than half the active nuclei decay
(d) all the nuclei decay.

12. In a radioactive decay, neither the atomic number nor the mass number changes. Which of the following particles is emitted in the decay ?
(a) Proton (b) Neutron (c) Electron (d) Photon

13. During a negative beta decay,
(a) an atomic electron is ejected
(b) an electron which is already present within the nucleus is ejected
(c) a neutron in the nucleus decays emitting an electron
(d) a proton in the nucleus decays emitting an electron.

14. A freshly prepared radioactive source of half-life 2 h emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with this source is
(a) 6 h (b) 12 h (c) 24 h (d) 128 h.

15. The decay constant of a radioactive sample is λ. The half-life and the average-life of the sample are respectively
(a) $1/\lambda$ and $(\ln 2/\lambda)$ (b) $(\ln 2/\lambda)$ and $1/\lambda$
(c) $\lambda(\ln 2)$ and $1/\lambda$ (d) $\lambda/(\ln 2)$ and $1/\lambda$.

16. An α-particle is bombarded on ^{14}N. As a result, a ^{17}O nucleus is formed and a particle is emitted. This particle is a
(a) neutron (b) proton (c) electron (d) positron.

17. Ten grams of ^{57}Co kept in an open container beta-decays with a half-life of 270 days. The weight of the material inside the container after 540 days will be very nearly
(a) 10 g (b) 5 g (c) 2·5 g (d) 1·25 g.

18. Free ^{238}U nuclei kept in a train emit alpha particles. When the train is stationary and a uranium nucleus decays, a passenger measures that the separation between the alpha particle and the recoiling nucleus becomes x in time t after the decay. If a decay takes place when the train is moving at a uniform speed v, the distance between the alpha particle and the recoiling nucleus at a time t after the decay, as measured by the passenger will be
(a) $x + vt$ (b) $x - vt$ (c) x
(d) depends on the direction of the train.

19. During a nuclear fission reaction,
(a) a heavy nucleus breaks into two fragments by itself
(b) a light nucleus bombarded by thermal neutrons breaks up
(c) a heavy nucleus bombarded by thermal neutrons breaks up
(d) two light nuclei combine to give a heavier nucleus and possibly other products.

OBJECTIVE II

1. As the mass number A increases, which of the following quantities related to a nucleus do not change ?
(a) Mass (b) Volume (c) Density (d) Binding energy

2. The heavier nuclei tend to have larger N/Z ratio because
(a) a neutron is heavier than a proton
(b) a neutron is an unstable particle
(c) a neutron does not exert electric repulsion
(d) Coulomb forces have longer range compared to the nuclear forces.

3. A free neutron decays to a proton but a free proton does not decay to a neutron. This is because
(a) neutron is a composite particle made of a proton and an electron whereas proton is a fundamental particle
(b) neutron is an uncharged particle whereas proton is a charged particle
(c) neutron has larger rest mass than the proton
(d) weak forces can operate in a neutron but not in a proton.

4. Consider a sample of a pure beta-active material.
(a) All the beta particles emitted have the same energy.
(b) The beta particles originally exist inside the nucleus and are ejected at the time of beta decay.
(c) The antineutrino emitted in a beta decay has zero mass and hence zero momentum.
(d) The active nucleus changes to one of its isobars after the beta decay.

5. In which of the following decays the element does not change ?
(a) α-decay (b) β^+-decay (c) β^--decay (d) γ-decay

6. In which of the following decays the atomic number decreases ?
(a) α-decay (b) β^+-decay (c) β^--decay (d) γ-decay

7. Magnetic field does not cause deflection in
(a) α-rays (b) beta-plus rays
(c) beta-minus rays (d) gamma rays.

8. Which of the following are electromagnetic waves ?
(a) α-rays (b) Beta-plus rays
(c) Beta-minus rays (d) Gamma rays

9. Two lithium nuclei in a lithium vapour at room temperature do not combine to form a carbon nucleus because
(a) a lithium nucleus is more tightly bound than a

carbon nucleus
(b) carbon nucleus is an unstable particle
(c) it is not energetically favourable
(d) Coulomb repulsion does not allow the nuclei to come very close.

10. For nuclei with $A > 100$,
(a) the binding energy of the nucleus decreases on an average as A increases
(b) the binding energy per nucleon decreases on an average as A increases
(c) if the nucleus breaks into two roughly equal parts, energy is released
(d) if two nuclei fuse to form a bigger nucleus, energy is released.

EXERCISES

Mass of proton $m_p = 1{\cdot}007276$ u, Mass of ${}^1_1$H atom $= 1{\cdot}007825$ u, Mass of neutron $m_n = 1{\cdot}008665$ u, Mass of electron $= 0{\cdot}0005486$ u ≈ 511 keV c^{-2}, 1 u = 931 MeV c^{-2}.

1. Assume that the mass of a nucleus is approximately given by $M = Am_p$ where A is the mass number. Estimate the density of matter in kgm^{-3} inside a nucleus. What is the specific gravity of nuclear matter?
2. A neutron star has a density equal to that of the nuclear matter. Assuming the star to be spherical, find the radius of a neutron star whose mass is $4{\cdot}0 \times 10^{30}$ kg (twice the mass of the sun).
3. Calculate the mass of an α-particle. Its binding energy is 28·2 MeV.
4. How much energy is released in the following reaction:
$${}^7\text{Li} + p \rightarrow \alpha + \alpha.$$
Atomic mass of ^{7}Li = 7·0160 u and that of ^{4}He = 4·0026 u.
5. Find the binding energy per nucleon of ${}^{197}_{79}$Au if its atomic mass is 196·96 u.
6. (a) Calculate the energy released if ^{238}U emits an α-particle. (b) Calculate the energy to be supplied to ^{238}U if two protons and two neutrons are to be emitted one by one. The atomic masses of ^{238}U, ^{234}Th and ^{4}He are 238·0508 u, 234·04363 u and 4·00260 u respectively.
7. Find the energy liberated in the reaction
$${}^{223}\text{Ra} \rightarrow {}^{209}\text{Pb} + {}^{14}\text{C}.$$
The atomic masses needed are as follows.

^{223}Ra	^{209}Pb	^{14}C
223·018 u	208·981 u	14·003 u

8. Show that the minimum energy needed to separate a proton from a nucleus with Z protons and N neutrons is
$$\Delta E = (M_{Z-1,N} + M_H - M_{Z,N})c^2$$
where $M_{Z,N}$ = mass of an atom with Z protons and N neutrons in the nucleus and M_H = mass of a hydrogen atom. This energy is known as *proton-separation energy*.
9. Calculate the minimum energy needed to separate a neutron from a nucleus with Z protons and N neutrons in terms of the masses $M_{Z,N}$, $M_{Z,N-1}$ and the mass of the neutron.
10. ^{32}P beta-decays to ^{32}S. Find the sum of the energy of the antineutrino and the kinetic energy of the β-particle. Neglect the recoil of the daughter nucleus. Atomic mass of ^{32}P = 31·974 u and that of ^{32}S = 31·972 u.
11. A free neutron beta-decays to a proton with a half-life of 14 minutes. (a) What is the decay constant? (b) Find the energy liberated in the process.
12. Complete the following decay schemes.
(a) ${}^{226}_{88}\text{Ra} \rightarrow \alpha +$
(b) ${}^{19}_{8}\text{O} \rightarrow {}^{19}_{9}\text{F} +$
(c) ${}^{25}_{13}\text{Al} \rightarrow {}^{25}_{12}\text{Mg} +$
13. In the decay ${}^{64}\text{Cu} \rightarrow {}^{64}\text{Ni} + e^+ + \nu$, the maximum kinetic energy carried by the positron is found to be 0·650 MeV. (a) What is the energy of the neutrino which was emitted together with a positron of kinetic energy 0·150 MeV? (b) What is the momentum of this neutrino in kg m s^{-1}? Use the formula applicable to a photon.
14. Potassium-40 can decay in three modes. It can decay by β^--emission, β^+-emission or electron capture. (a) Write the equations showing the end products. (b) Find the Q-values in each of the three cases. Atomic masses of ${}^{40}_{18}$Ar, ${}^{40}_{19}$K and ${}^{40}_{20}$Ca are 39·9624 u, 39·9640 u and 39·9626 u respectively.
15. Lithium ($Z = 3$) has two stable isotopes ^{6}Li and ^{7}Li. When neutrons are bombarded on lithium sample, electrons and α-particles are ejected. Write down the nuclear processes taking place.
16. The masses of ^{11}C and ^{11}B are respectively 11·0114 u and 11·0093 u. Find the maximum energy a positron can have in the β^+-decay of ^{11}C to ^{11}B.
17. ^{228}Th emits an alpha particle to reduce to ^{224}Ra. Calculate the kinetic energy of the alpha particle emitted in the following decay:
$${}^{228}\text{Th} \rightarrow {}^{224}\text{Ra}^* + \alpha$$
$${}^{224}\text{Ra}^* \rightarrow {}^{224}\text{Ra} + \gamma\,(217\text{ keV}).$$
Atomic mass of ^{228}Th is 228·028726 u, that of ^{224}Ra is 224·020196 u and that of ${}^4_2$He is 4·00260 u.
18. Calculate the maximum kinetic energy of the beta particle emitted in the following decay scheme:
$${}^{12}\text{N} \rightarrow {}^{12}\text{C}^* + e^+ + \nu$$
$${}^{12}\text{C}^* \rightarrow {}^{12}\text{C} + \gamma\,(4{\cdot}43\text{ MeV}).$$
The atomic mass of ^{12}N is 12·018613 u.
19. The decay constant of ${}^{197}_{80}$Hg (electron capture to ${}^{197}_{79}$Au) is $1{\cdot}8 \times 10^{-4}$ s^{-1}. (a) What is the half-life? (b) What is the average-life? (c) How much time will it take to convert 25% of this isotope of mercury into gold?

20. The half-life of ^{198}Au is 2·7 days. (a) Find the activity of a sample containing 1·00 µg of ^{198}Au. (b) What will be the activity after 7 days? Take the atomic weight of ^{198}Au to be 198 g mol^{-1}.

21. Radioactive ^{131}I has a half-life of 8·0 days. A sample containing ^{131}I has activity 20 µCi at $t = 0$. (a) What is its activity at $t = 4·0$ days? (b) What is its decay constant at $t = 4·0$ days?

22. The decay constant of ^{238}U is $4·9 \times 10^{-18}\ s^{-1}$. (a) What is the average-life of ^{238}U? (b) What is the half-life of ^{238}U? (c) By what factor does the activity of a ^{238}U sample decrease in 9×10^{9} years?

23. A certain sample of a radioactive material decays at the rate of 500 per second at a certain time. The count rate falls to 200 per second after 50 minutes. (a) What is the decay constant of the sample? (b) What is its half-life?

24. The count rate from a radioactive sample falls from $4·0 \times 10^{6}$ per second to $1·0 \times 10^{6}$ per second in 20 hours. What will be the count rate 100 hours after the beginning?

25. The half-life of ^{226}Ra is 1602 y. Calculate the activity of 0·1 g of $RaCl_2$ in which all the radium is in the form of ^{226}Ra. Taken atomic weight of Ra to be 226 g mol^{-1} and that of Cl to be 35·5 g mol^{-1}.

26. The half-life of a radioisotope is 10 h. Find the total number of disintegrations in the tenth hour measured from a time when the activity was 1 Ci.

27. The selling rate of a radioactive isotope is decided by its activity. What will be the second-hand rate of a one month old $^{32}P(t_{1/2} = 14·3$ days) source if it was originally purchased for 800 rupees?

28. ^{57}Co decays to ^{57}Fe by β^+-emission. The resulting ^{57}Fe is in its excited state and comes to the ground state by emitting γ-rays. The half-life of β^+-decay is 270 days and that of the γ-emission is 10^{-8} s. A sample of ^{57}Co gives $5·0 \times 10^{9}$ gamma rays per second. How much time will elapse before the emission rate of gamma rays drops to $2·5 \times 10^{9}$ per second?

29. Carbon ($Z = 6$) with mass number 11 decays to boron ($Z = 5$). (a) Is it a β^+-decay or a β^--decay? (b) The half-life of the decay scheme is 20·3 minutes. How much time will elapse before a mixture of 90% carbon-11 and 10% boron-11 (by the number of atoms) converts itself into a mixture of 10% carbon-11 and 90% boron-11?

30. 4×10^{23} tritium atoms are contained in a vessel. The half-life of decay of tritium nuclei is 12·3 y. Find (a) the activity of the sample, (b) the number of decays in the next 10 hours (c) the number of decays in the next 6·15 y.

31. A point source emitting alpha particles is placed at a distance of 1 m from a counter which records any alpha particle falling on its 1 cm^2 window. If the source contains $6·0 \times 10^{16}$ active nuclei and the counter records a rate of 50000 counts/second, find the decay constant. Assume that the source emits alpha particles uniformly in all directions and the alpha particles fall nearly normally on the window.

32. ^{238}U decays to ^{206}Pb with a half-life of $4·47 \times 10^{9}$ y. This happens in a number of steps. Can you justify a single half-life for this chain of processes? A sample of rock is found to contain 2·00 mg of ^{238}U and 0·600 mg of ^{206}Pb. Assuming that all the lead has come from uranium, find the life of the rock.

33. When charcoal is prepared from a living tree, it shows a disintegration rate of 15·3 disintegrations of ^{14}C per gram per minute. A sample from an ancient piece of charcoal shows ^{14}C activity to be 12·3 disintegrations per gram per minute. How old is this sample? Half-life of ^{14}C is 5730 y.

34. Natural water contains a small amount of tritium ($^{3}_{1}H$). This isotope beta-decays with a half-life of 12·5 years. A mountaineer while climbing towards a difficult peak finds debris of some earlier unsuccessful attempt. Among other things he finds a sealed bottle of whisky. On return he analyses the whisky and finds that it contains only 1·5 per cent of the $^{3}_{1}H$ radioactivity as compared to a recently purchased bottle marked '8 years old'. Estimate the time of that unsuccessful attempt.

35. The count rate of nuclear radiation coming from a radioactive sample containing ^{128}I varies with time as follows.

Time t (minute):	0	25	50	75	100
Count rate $R(10^{9}\ s^{-1})$:	30	16	8·0	3·8	2·0

(a) Plot $\ln(R_0/R)$ against t. (b) From the slope of the best straight line through the points, find the decay constant λ. (c) Calculate the half-life $t_{1/2}$.

36. The half-life of ^{40}K is $1·30 \times 10^{9}$ y. A sample of 1·00 g of pure KCl gives 160 counts s^{-1}. Calculate the relative abundance of ^{40}K (fraction of ^{40}K present) in natural potassium.

37. $^{197}_{80}Hg$ decays to $^{197}_{79}Au$ through electron capture with a decay constant of 0·257 per day. (a) What other particle or particles are emitted in the decay? (b) Assume that the electron is captured from the K shell. Use Moseley's law $\sqrt{\nu} = a(Z - b)$ with $a = 4·95 \times 10^{7}\ s^{-1/2}$ and $b = 1$ to find the wavelength of the K_α X-ray emitted following the electron capture.

38. A radioactive isotope is being produced at a constant rate $dN/dt = R$ in an experiment. The isotope has a half-life $t_{1/2}$. Show that after a time $t >> t_{1/2}$, the number of active nuclei will become constant. Find the value of this constant.

39. Consider the situation of the previous problem. Suppose the production of the radioactive isotope starts at $t = 0$. Find the number of active nuclei at time t.

40. In an agricultural experiment, a solution containing 1 mole of a radioactive material ($t_{1/2} = 14·3$ days) was injected into the roots of a plant. The plant was allowed 70 hours to settle down and then activity was measured in its fruit. If the activity measured was 1 µCi, what per cent of activity is transmitted from the root to the fruit in steady state?

41. A vessel of volume 125 cm^3 contains tritium (^{3}H, $t_{1/2} = 12·3$ y) at 500 kPa and 300 K. Calculate the activity of the gas.

42. ${}^{212}_{83}\text{Bi}$ can disintegrate either by emitting an α-particle or by emitting a β^--particle. (a) Write the two equations showing the products of the decays. (b) The probabilities of disintegration by α- and β-decays are in the ratio 7/13. The overall half-life of ${}^{212}\text{Bi}$ is one hour. If 1 g of pure ${}^{212}\text{Bi}$ is taken at 12·00 noon, what will be the composition of this sample at 1 p.m. the same day ?

43. A sample contains a mixture of ${}^{108}\text{Ag}$ and ${}^{110}\text{Ag}$ isotopes each having an activity of $8\cdot0 \times 10^8$ disintegrations per second. ${}^{110}\text{Ag}$ is known to have larger half-life than ${}^{108}\text{Ag}$. The activity A is measured as a function of time and the following data are obtained.

Time (s)	Activity (A) (10^8 disintegrations s^{-1})	Time (s)	Activity (A) (10^8 disintegrations s^{-1})
20	11·799	200	3·0828
40	9·1680	300	1·8899
60	7·4492	400	1·1671
80	6·2684	500	0·7212
100	5·4115		

(a) Plot $\ln(A/A_0)$ versus time. (b) See that for large values of time, the plot is nearly linear. Deduce the half-life of ${}^{110}\text{Ag}$ from this portion of the plot. (c) Use the half-life of ${}^{110}\text{Ag}$ to calculate the activity corresponding to ${}^{108}\text{Ag}$ in the first 50 s. (d) Plot $\ln(A/A_0)$ versus time for ${}^{108}\text{Ag}$ for the first 50 s. (e) Find the half-life of ${}^{108}\text{Ag}$.

44. A human body excretes (removes by waste discharge, sweating, etc.) certain materials by a law similar to radioactivity. If technitium is injected in some form in a human body, the body excretes half the amount in 24 hours. A patient is given an injection containing ${}^{99}\text{Tc}$. This isotope is radioactive with a half-life of 6 hours. The activity from the body just after the injection is 6 μCi. How much time will elapse before the activity falls to 3 μCi ?

45. A charged capacitor of capacitance C is discharged through a resistance R. A radioactive sample decays with an average-life τ. Find the value of R for which the ratio of the electrostatic field energy stored in the capacitor to the activity of the radioactive sample remains constant in time.

46. Radioactive isotopes are produced in a nuclear physics experiment at a constant rate $dN/dt = R$. An inductor of inductance 100 mH, a resistor of resistance 100 Ω and a battery are connected to form a series circuit. The circuit is switched on at the instant the production of radioactive isotope starts. It is found that i/N remains constant in time where i is the current in the circuit at time t and N is the number of active nuclei at time t. Find the half-life of the isotope.

47. Calculate the energy released by 1 g of natural uranium assuming 200 MeV is released in each fission event and that the fissionable isotope ${}^{235}\text{U}$ has an abundance of 0·7% by weight in natural uranium.

48. A uranium reactor develops thermal energy at a rate of 300 MW. Calculate the amount of ${}^{235}\text{U}$ being consumed every second. Average energy released per fission is 200 MeV.

49. A town has a population of 1 million. The average electric power needed per person is 300 W. A reactor is to be designed to supply power to this town. The efficiency with which thermal power is converted into electric power is aimed at 25%. (a) Assuming 200 MeV of thermal energy to come from each fission event on an average, find the number of events that should take place every day. (b) Assuming the fission to take place largely through ${}^{235}\text{U}$, at what rate will the amount of ${}^{235}\text{U}$ decrease ? Express your answer in kg per day. (c) Assuming that uranium enriched to 3% in ${}^{235}\text{U}$ will be used, how much uranium is needed per month (30 days) ?

50. Calculate the Q-values of the following fusion reactions:

(a) ${}^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \rightarrow {}^{3}_{1}\text{H} + {}^{1}_{1}\text{H}$

(b) ${}^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \rightarrow {}^{3}_{2}\text{He} + n$

(c) ${}^{2}_{1}\text{H} + {}^{3}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + n$.

Atomic masses are $m({}^{2}_{1}\text{H}) = 2\cdot014102$ u, $m({}^{3}_{1}\text{H}) = 3\cdot016049$ u, $m({}^{3}_{2}\text{He}) = 3\cdot016029$ u, $m({}^{4}_{2}\text{He}) = 4\cdot002603$ u.

51. Consider the fusion in helium plasma. Find the temperature at which the average thermal energy $1\cdot5\ kT$ equals the Coulomb potential energy at 2 fm.

52. Calculate the Q-value of the fusion reaction

$${}^{4}\text{He} + {}^{4}\text{He} = {}^{8}\text{Be}.$$

Is such a fusion energetically favourable ? Atomic mass of ${}^{8}\text{Be}$ is 8·0053 u and that of ${}^{4}\text{He}$ is 4·0026 u.

53. Calculate the energy that can be obtained from 1 kg of water through the fusion reaction

$${}^{2}\text{H} + {}^{2}\text{H} \rightarrow {}^{3}\text{H} + p.$$

Assume that $1\cdot5 \times 10^{-2}$ % of natural water is heavy water D_2O (by number of molecules) and all the deuterium is used for fusion.

□

ANSWERS

OBJECTIVE I

1. (a)	2. (c)	3. (c)	4. (d)	5. (a)	6. (b)
7. (d)	8. (b)	9. (d)	10. (d)	11. (c)	12. (d)
13. (c)	14. (b)	15. (b)	16. (b)	17. (a)	18. (c)
19. (c)					

OBJECTIVE II

1. (c)	2. (c), (d)	3. (c)
4. (d)	5. (d)	6. (a), (b)
7. (d)	8. (d)	9. (d)
10. (b), (c)		

EXERCISES

1. 3×10^{17} kg m^{-3}, 3×10^{14}
2. 15 km
3. 4·0016 u
4. 17·34 MeV
5. 7·94 MeV
6. (a) 4·255 MeV (b) 24·03 MeV
7. 31·65 MeV
9. $(M_{Z,N-1} + m_n - M_{Z,N})c^2$
10. 1·86 MeV
11. (a) $8{\cdot}25 \times 10^{-4}$ s^{-1} (b) 782 keV
12. (a) ${}^{222}_{86}\mathrm{Rn}$ (b) $\bar{e} + \bar{\nu}$ (c) $e^+ + \nu$
13. (a) 500 keV (b) $2{\cdot}67 \times 10^{-22}$ kg m s^{-1}
14. (a) ${}^{40}_{19}\mathrm{K} \rightarrow {}^{40}_{20}\mathrm{Ca} + e^- + \bar{\nu}$, ${}^{40}_{19}\mathrm{K} \rightarrow {}^{40}_{18}\mathrm{Ar} + e^+ + \nu$,
    ${}^{40}_{19}\mathrm{K} + e^- \rightarrow {}^{40}_{18}\mathrm{Ar} + \nu$
    (b) 1·3034 MeV, 0·4676 MeV, 1·490 MeV
15. ${}^{6}_{3}\mathrm{Li} + n \rightarrow {}^{7}_{3}\mathrm{Li}$, ${}^{7}_{3}\mathrm{Li} + n \rightarrow {}^{8}_{3}\mathrm{Li} \rightarrow {}^{8}_{4}\mathrm{Be} + e^- + \bar{\nu}$,
    ${}^{8}_{4}\mathrm{Be} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He}$
16. 933·6 keV
17. 5·304 MeV
18. 11·88 MeV
19. (a) 64 min (b) 92 min (c) 1600 s
20. (a) 0·244 Ci (b) 0·040 Ci
21. (a) 14 μCi (b) $1{\cdot}4 \times 10^{-6}$ s^{-1}
22. $6{\cdot}49 \times 10^{9}$ y (b) $4{\cdot}5 \times 10^{9}$ y (c) 4
23. $3{\cdot}05 \times 10^{-4}$ s (b) 38 min
24. $3{\cdot}9 \times 10^{3}$ per second
25. $2{\cdot}8 \times 10^{9}$ disintegrations s^{-1}
26. $6{\cdot}91 \times 10^{13}$
27. 187 rupees
28. 270 days
29. (a) β^+ (b) 64 min
30. (a) $7{\cdot}146 \times 10^{14}$ disintegrations s^{-1}
    (b) $2{\cdot}57 \times 10^{19}$ (c) $1{\cdot}17 \times 10^{23}$
31. $1{\cdot}05 \times 10^{-7}$ s^{-1}
32. $1{\cdot}92 \times 10^{9}$ y
33. 1800 y
34. about 83 years ago
35. (b) 0·028 min^{-1} approx. (c) 25 min approx.
36. 0·12%
37. (a) neutrino (b) 20 pm
38. $\dfrac{Rt_{1/2}}{0{\cdot}693}$
39. $\dfrac{R}{\lambda}(1 - e^{-\lambda t})$
40. $1{\cdot}26 \times 10^{-11}$ %
41. 724 Ci
42. (a) ${}^{212}_{83}\mathrm{Bi} \rightarrow {}^{208}_{81}\mathrm{Tl} + \alpha$, ${}^{212}_{83}\mathrm{Bi} \rightarrow {}^{212}_{84}\mathrm{Bi} \rightarrow {}^{212}_{84}\mathrm{Po} + e^- + \bar{\nu}$
    (b) 0·50 g Bi, 0·175 g Tl, 0·325 g Po
43. the half-life of ${}^{110}\mathrm{Ag} = 24{\cdot}4$ s and of ${}^{108}\mathrm{Ag} = 144$ s
44. 4·8 hours
45. $2\,\tau/C$
46. $6{\cdot}93 \times 10^{-4}$ s
47. $5{\cdot}7 \times 10^{8}$ J
48. 3·7 mg
49. (a) $3{\cdot}24 \times 10^{24}$ (b) 1·264 kg per day (c) 1263 kg
50. (a) 4·05 MeV (b) 3·25 MeV (c) 17·57 MeV
51. $2{\cdot}23 \times 10^{10}$ K
52. –93·1 keV, no
53. 3200 MJ

□