Differentiation Ex 11.5 Q21

Here,

$$y=\frac{\left(x^2-1\right)^3(2x-1)}{\sqrt{(x-3)(4x-1)}} \qquad \text{---(i)}$$

$$y=\frac{\left(x^2-1\right)^3(2x-1)}{(x-3)^{\frac{1}{2}}(4x-)^{\frac{1}{2}}}$$

Taking log on both the sides,

$$\log y=\log\left[\frac{\left(x^2-1\right)^3(2x-1)}{(x-3)^{\frac{1}{2}}(4x-1)^{\frac{1}{2}}}\right]$$

$$=\log\left(x^2-1\right)^3+\log(2x-1)-\log(x-3)^{\frac{1}{2}}-\log(4x-1)^{\frac{1}{2}}$$

$$\left[\text{Since, }\log(AB)=\log A+\log B,\log\left(\frac{A}{B}\right)=\log A-\log B\right]$$

$$=3\log\left(x^2-1\right)+\log(2x-1)-\frac{1}{2}\log(x-3)-\frac{1}{2}\log(4x-1)$$

Differentiating it with respect to x using chain rule,

$$\frac{1}{y}\frac{dy}{dx}=3\frac{d}{dx}\log\left(x^2-1\right)+\frac{d}{dx}\log(2x-1)-\frac{1}{2}\frac{d}{dx}\log(x-3)-\frac{1}{2}\log(4x-1)$$

$$=3\left(\frac{1}{x^2-1}\right)\frac{d}{dx}\left(x^2-1\right)+\frac{1}{(2x-1)}\frac{d}{dx}(2x-1)-\frac{1}{2}\left(\frac{1}{x-3}\right)\frac{d}{dx}(x-3)-\frac{1}{2}\frac{1}{(4x-1)}\frac{d}{dx}(4x-1)$$

$$=3\left(\frac{1}{x^2-1}\right)(2x)+\frac{1}{(2x-1)}(2)-\frac{1}{2}\left(\frac{1}{x-3}\right)(1)-\frac{1}{2}\left(\frac{1}{4x-1}\right)(4)$$

$$\frac{1}{y}\frac{dy}{dx}=\left[\frac{6x}{x^2-1}+\frac{2}{2x-1}-\frac{1}{2(x-3)}-\frac{2}{4x-1}\right]$$

$$\frac{dy}{dx}=y\left[\frac{6x}{x^2-1}+\frac{2}{2x-1}-\frac{1}{2(x-3)}-\frac{2}{4x-1}\right]$$

$$\frac{dy}{dx}=\frac{\left(x^2-1\right)^3(2x-1)}{\sqrt{(x-3)(4x-1)}}\left[\frac{6x}{x^2-1}+\frac{2}{2x-1}-\frac{1}{2(x-3)}-\frac{2}{4x-1}\right] \qquad \text{[Using equaiton (i)]}$$

Differentiation Ex 11.5 Q22

Here,

$$y=\frac{e^{ax}\sec^x x\log x}{\sqrt{1-2x}} \qquad \text{---(i)}$$

$$\Rightarrow \quad y=\frac{e^{ax}\times\sec^x x\log x}{(1-2x)^{\frac{1}{2}}}$$

Taking log on both the sides,

$$\log y=\log e^{ax}+\log^{\sec x}+\log\log x-\frac{1}{2}\log(1-2x) \qquad \left[\begin{array}{l}\text{Since, }\log\left(\frac{A}{B}\right)=\log A-\log B,\\ \log(AB)=\log A+\log B\end{array}\right]$$

$$\log y=ax+\log^{\sec x}+\log\log x-\frac{1}{2}\log(1-2x) \qquad \left[\text{Since, }\log a^b=b\log a\text{ and }\log_e e=1\right]$$

Differentiating it with respect to x using chain rule,

$$\frac{1}{y}\frac{dy}{dx}=\frac{d}{dx}(ax)+\frac{d}{dx}(\log\sec x)+\frac{d}{dx}(\log\log x)-\frac{1}{2}\log(1-2x)$$

$$\frac{1}{y}\frac{dy}{dx}=a+\frac{1}{\sec x}\frac{d}{dx}(\sec x)+\frac{1}{\log x}\frac{d}{dx}(\log x)-\frac{1}{2}\left(\frac{1}{1-2x}\right)\frac{d}{dx}(1-2x)$$

$$\frac{1}{y}\frac{dy}{dx}=a+\frac{\sec x\tan x}{\sec x}+\frac{1}{(\log x)}\left(\frac{1}{x}\right)-\frac{1}{2}\left(\frac{1}{1-2x}\right)(-2)$$

$$\frac{dy}{dx}=y\left[a+\tan x+\frac{1}{x\log x}+\frac{1}{1-2x}\right]$$

$$\frac{dy}{dx}=\frac{e^{ax}\sec x\log x}{\sqrt{1-2x}}\left[a+\tan x+\frac{1}{x\log x}+\frac{1}{1-2x}\right] \qquad \text{[Using equaton (i)]}$$

Differentiation Ex 11.5 Q23

Here,

$$y = e^{3x} \times \sin 4x \times 2^x \qquad \text{---(i)}$$

Taking log on both the sides,

$$\log y = \log e^{3x} + \log \sin 4x + \log 2^x \qquad \left[\text{Since, } \log(AB) = \log A + \log B\right]$$
$$\log y = 3x \log e + \log \sin 4x + x \log 2 \qquad \left[\text{Since, } \log_e e = 1, \log a^b = b \log a\right]$$
$$\log y = 3x + \log \sin 4x + x \log 2$$

Differentiating it with respect to x,

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(3x) + \frac{d}{dx}(\log \sin 4x) + \frac{d}{dx}(x \log 2)$$
$$= 3 + \frac{1}{\sin 4x}\frac{d}{dx}(\sin 4x) + \log 2(1)$$
$$= 3 + \frac{1}{\sin 4x}(\cos 4x)\frac{d}{dx}(4x) + \log 2$$
$$= 3 + \cot x(4) + \log 2$$
$$\frac{1}{y}\frac{dy}{dx} = 3 + 4\cot 4x + \log 2$$
$$\frac{dy}{dx} = y\left[3 + 4\cot 4x + \log 2\right]$$

$$\frac{dy}{dx} = e^{3x} \times \sin 4x \times 2^x\left[3 + 4\cot 4x + \log 2\right]$$

Differentiation Ex 11.5 Q24

Here,

$$y = \sin x \sin 2x \sin 3x \sin 4x \qquad \text{---(i)}$$

Taking log on both the sides,

$$\log y = \log(\sin x \sin 2x \sin 3x \sin 4x)$$
$$\log y = \log \sin x + \log \sin 2x + \log \sin 3x + \log \sin 4x$$

Differentiating it with respect to x using chain rule,

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}\log \sin x + \frac{d}{dx}\log \sin 2x + \frac{d}{dx}\log \sin 3x + \frac{d}{dx}\log \sin 4x$$
$$= \frac{1}{\sin x}\frac{d}{dx}(\sin x) + \frac{1}{\sin 2x}\frac{d}{dx}(\sin 2x) + \frac{1}{\sin 3x}\frac{d}{dx}(\sin 3x) + \frac{1}{\sin 4x}\frac{d}{dx}(\sin 4x)$$
$$= \frac{1}{\sin x}(\cos x) + \frac{1}{\sin 2x}(\cos 2x)\frac{d}{dx}(2x) + \frac{1}{\sin 3x}(\cos 3x)\frac{d}{dx}(3x) + \frac{1}{\sin 4x}(\cos 4x)\frac{d}{dx}(4x)$$
$$\frac{1}{y}\frac{dy}{dx} = \left[\cot x + \cot 2x(2) + \cot 3x(3) + \cot 4x(4)\right]$$
$$\frac{dy}{dx} = y\left[\cot x + 2\cot 2x + 3\cot x 3x + 4\cot 4x\right]$$

$$\frac{dy}{d} = (\sin x \sin 2x \sin 3x \sin 4x)\left[\cot x + 2\cot 2x + 3\cot x 3x + 4\cot 4x\right] \qquad \left[\text{Using equation (i)}\right]$$

Differentiation Ex 11.5 Q25

Let $y = x^{\sin x} + (\sin x)^x$

Also, let $u = x^{\sin x}$ and $v = (\sin x)^x$

$\therefore y = u + v$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \qquad ...(1)$$

$u = x^{\sin x}$

$\Rightarrow \log u = \log\left(x^{\sin x}\right)$

$\Rightarrow \log u = \sin x \log x$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(\sin x)\cdot\log x + \sin x\cdot\frac{d}{dx}(\log x)$$

$$\Rightarrow \frac{du}{dx} = u\left[\cos x \log x + \sin x\cdot\frac{1}{x}\right]$$

$$\Rightarrow \frac{du}{dx} = x^{\sin x}\left[\cos x \log x + \frac{\sin x}{x}\right] \qquad ...(2)$$

$v = (\sin x)^x$

$\Rightarrow \log v = \log(\sin x)^x$

$\Rightarrow \log v = x\log(\sin x)$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{v}\frac{dv}{dx} = \frac{d}{dx}(x)\times\log(\sin x) + x\times\frac{d}{dx}\left[\log(\sin x)\right]$$

$$\Rightarrow \frac{dv}{dx} = v\left[\log(\sin x) + x\cdot\frac{1}{\sin x}\cdot\frac{d}{dx}(\sin x)\right]$$

$$\Rightarrow \frac{dv}{dx} = (\sin x)^x\left[\log\sin x + \frac{x}{\sin x}\cos x\right]$$

$$\Rightarrow \frac{dv}{dx} = (\sin x)^x\left[\log\sin x + x\cot x\right]$$

$$\Rightarrow \frac{dv}{dx} = (\sin x)^x\left[\log\sin x + x\cot x\right] \qquad ...(3)$$

From (1), (2), and (3), we obtain

$$\frac{dy}{dx} = x^{\sin x}\left(\cos x\log x + \frac{\sin x}{x}\right) + (\sin x)^x\left[\log\sin x + x\cot x\right]$$

*********** END **********