Differentiation Ex 11.5 Q11

Let $y=(\log x)^{\log x}$

Taking logarithm on both the sides, we obtain

$$\log y=\log x\cdot\log(\log x)$$

Differentiating both sides with respect to x, we obtain

$$\frac{1}{y}\frac{dy}{dx}=\frac{d}{dx}\left[\log x\cdot\log(\log x)\right]$$

$$\Rightarrow\frac{1}{y}\frac{dy}{dx}=\log(\log x).\frac{d}{dx}(\log x)+\log x\cdot\frac{d}{dx}\left[\log(\log x)\right]$$

$$\Rightarrow\frac{dy}{dx}=y\left[\log(\log x).\frac{1}{x}+\log x\cdot\frac{1}{\log x}\cdot\frac{d}{dx}(\log x)\right]$$

$$\Rightarrow\frac{dy}{dx}=y\left[\frac{1}{x}\log(\log x)+\frac{1}{x}\right]$$

$$\therefore\frac{dy}{dx}=(\log x)^{\log x}\left[\frac{1}{x}+\frac{\log(\log x)}{x}\right]$$

Differentiation Ex 11.5 Q12

Let $y=10^{(10x)}$ ---(i)

Taking log on both the siedes,

$$\log y=\log 10^{(10x)}$$
$$\log y=10^{x}\log 10$$

Differentiating it with respect to x,

$$\frac{1}{y}\frac{dy}{dx}=\log 10\times 10^{x}\log 10$$
$$\frac{1}{y}\frac{dy}{dx}=10^{x}\times(\log 10)^{2}$$
$$\frac{dy}{dx}=10^{(10^{x})}\times 10^{x}(\log 10)^{2}$$ [Using equation (i)]

Differentiation Ex 11.5 Q13

Let $\quad y = \sin x^x$

$\Rightarrow \quad \sin^{-1} y = x^x$

Taking log on both the siedes,

$$\log\left(\sin^{-1} y\right) = \log x^x$$
$$\log\left(\sin^{-1} y\right) = x \log x \qquad \left[\text{Since, } \log a^b = b \log a\right]$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{1}{\sin^{-1} y} \frac{dy}{dx} = \left(\sin^{-1} y\right) = x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x)$$
$$\frac{1}{\sin^{1} y} \times \left(\frac{1}{\sqrt{1 - y^2}}\right) \frac{dy}{dx} = x\left(\frac{1}{x}\right) + \log x\,(1)$$
$$\frac{dy}{dx} = \sin^{-1} y \sqrt{1 - y^2}\,(1 + \log x)$$
$$= \sin^{-1} \quad \left(\sin x^x\right) \sqrt{1 - \left(\sin x^x\right)^2}\,(1 + \log x)$$
$$= x^x \sqrt{\cos^2 x^x}\,(1 + \log x) \qquad \left[\text{Using equation (i)}\right]$$

$$\frac{dy}{dx} = x^x \cos x^x\,(1 + \log x)$$

Differentiation Ex 11.5 Q14

Let $\quad y = \left(\sin^{-1} x\right)^x$

Taking log on both the sides,

$$\log y = \log\left(\sin^{-1} x\right)^x$$
$$\log y = x \log\left(\sin^{-1} x\right) \qquad \left[\text{Since, } \log a^b = b \log a\right]$$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{1}{y} \frac{dy}{dx} = x \frac{d}{dx}\left(\log \sin^{-1} x\right) + \log \sin^{-1} x \frac{d}{dx}(x)$$
$$= x \times \frac{1}{\sin^{-1} x} \frac{d}{dx}\left(\sin^{-1} x\right) + \log \sin^{-1} x\,(1)$$
$$\frac{1}{y} \frac{dy}{dx} = \frac{x}{\sin^{-1} x}\left(\frac{1}{\sqrt{1 - x^2}}\right) + \log \sin^{-1} x$$
$$\frac{dy}{dx} = y\left[\log \sin^{-1} x + \frac{x}{\sin^{-1} x\left(\sqrt{1 - x^2}\right)}\right]$$
$$\frac{dy}{dx} = \left(\sin^{-1} x\right)^2\left[\log \sin^{-1} x + \frac{x}{\sin^{-1} x \sqrt{1 - x^2}}\right] \qquad \left[\text{Using equation (i)}\right]$$

Differentiation Ex 11.5 Q15

Let $\quad y = x^{\sin^{-1} x} \qquad$ ---(i)

Taking log on both the sides,

$$\log y = \log x^{\sin^{-1} x}$$
$$\log y = \sin^{-1} x \log x \qquad \left[\text{Since, } \log a^b = b \log a\right]$$

Differentiating it with respect to x using product rule,

$$\frac{1}{y} \frac{dy}{dx} = \sin^{-1} x \frac{d}{dx}(\log x) + (\log x) \frac{d}{dx}\left(\sin^{-1} x\right)$$
$$\frac{1}{y} \frac{dy}{dx} = \sin^{-1} x\left(\frac{1}{x}\right) + (\log x)\left(\frac{1}{\sqrt{1 - x^2}}\right)$$
$$\frac{dy}{dx} = y\left[\frac{\sin^{-1} x}{x} + \frac{\log x}{\sqrt{1 - x^2}}\right]$$

$$\frac{dy}{dx} = x^{\sin^{-1} x}\left[\frac{\sin^{-1} x}{x} + \frac{\log x}{\sqrt{1 - x^2}}\right] \qquad \left[\text{Using equation (i)}\right]$$

*********** END **********