Trigonometric Identities Ex 6.1 Q47

Answer :

(i) We have to prove the following identity-

$$\frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}=\frac{1+\sin\theta}{\cos\theta}$$

Consider the LHS.

$$\frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}$$

$$=\left(\frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}\right)\left(\frac{1+\cos\theta+\sin\theta}{1+\cos\theta+\sin\theta}\right)$$

$$=\frac{(1+\cos\theta+\sin\theta)^2}{(1+\cos\theta)^2-\sin^2\theta}$$

$$=\frac{2+2(\cos\theta+\sin\theta+\sin\theta\cos\theta)}{2\cos^2\theta+2\cos\theta}$$

$$=\frac{2(1+\cos\theta)(1+\sin\theta)}{2\cos\theta(1+\cos\theta)}$$

$$=\frac{1+\sin\theta}{\cos\theta}$$

= RHS

Hence proved.

(ii) We have to prove the following identity-

$$\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=\frac{1}{\sec\theta-\tan\theta}$$

Consider the LHS.

$$\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}$$

$$=\left(\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}\right)\left(\frac{\sin\theta+\cos\theta+1}{\sin\theta+\cos\theta+1}\right)$$

$$=\frac{(\sin\theta+1)^2-\cos^2\theta}{(\sin\theta+\cos\theta)^2-1}$$

$$=\frac{2\sin^2\theta+2\sin\theta}{2\sin\theta\cos\theta}$$

$$=\frac{2\sin\theta(1+\sin\theta)}{2\sin\theta\cos\theta}$$

$$=\frac{1+\sin\theta}{\cos\theta}$$

$$=\left(\frac{1+\sin\theta}{\cos\theta}\right)\left(\frac{1-\sin\theta}{1-\sin\theta}\right)$$

$$=\frac{\cos\theta}{1-\sin\theta}$$

$$=\frac{1}{\sec\theta-\tan\theta}$$ (Divide numerator and denominator by $\cos\theta$)

RHS

Hence proved.

(iii) We have to prove the following identity-

$$\frac{\cos\ \theta - \ \sin\ \theta + 1}{\cos\ \theta + \ \sin\ \theta - 1} = \operatorname{cosec}\theta + \cot\theta$$

Consider the LHS.

$$\frac{\cos\ \theta - \ \sin\ \theta + 1}{\cos\ \theta + \ \sin\ \theta - 1}$$

$$= \frac{\cos\ \theta - \ \sin\ \theta + 1}{\cos\ \theta + \ \sin\ \theta - 1} \times \frac{\cos\ \theta + \ \sin\ \theta + 1}{\cos\ \theta + \ \sin\ \theta + 1}$$

$$= \frac{(\cos\ \theta + 1)^2 - (\sin\theta)^2}{(\cos\ \theta + \ \sin\ \theta)^2 - (1)^2}$$

$$= \frac{\cos^2\theta + 1 + 2\ \cos\ \theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta + 2\ \cos\ \theta\ \sin\ \theta - 1}$$

$$= \frac{\cos^2\theta + 1 + 2\ \cos\ \theta - (1 - \cos^2\theta)}{1 + 2\ \cos\ \theta\ \sin\ \theta - 1}$$

$$= \frac{2\ \cos^2\theta + 2\ \cos\ \theta}{2\ \cos\ \theta\ \sin\ \theta}$$

$$= \frac{2\ \cos\ \theta(\cos\ \theta + 1)}{2\ \cos\ \theta\ \sin\ \theta}$$

$$= \frac{\cos\ \theta + 1}{\sin\ \theta}$$

$$= \frac{\cos\ \theta}{\sin\ \theta} + \frac{1}{\sin\ \theta}$$

$$= \cot\theta + \operatorname{cosec}\theta$$

= RHS

Hence proved.

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