∴Mass defect of this nucleus, $\Delta m' = 29 \times m_H + 34 \times m_n - m$

Where,
Mass of a proton, $m_H$ = 1.007825 u
Mass of a neutron, $m_n$ = 1.008665 u

∴$\Delta m'$ = 29 × 1.007825 + 34 × 1.008665 − 62.9296

= 0.591935 u
Mass defect of all the atoms present in the coin, $\Delta m = 0.591935 \times 2.868 \times 10^{22}$
= $1.69766958 \times 10^{22}$ u
But 1 u = 931.5 MeV/$c^2$

∴$\Delta m = 1.69766958 \times 10^{22} \times 931.5$ MeV/$c^2$

Hence, the binding energy of the nuclei of the coin is given as:
$E_b = \Delta mc^2$
$= 1.69766958 \times 10^{22} \times 931.5 \left(\frac{\text{MeV}}{c^2}\right) \times c^2$
$= 1.581 \times 10^{25}$ MeV
But 1 MeV = $1.6 \times 10^{-13}$ J
$E_b = 1.581 \times 10^{25} \times 1.6 \times 10^{-13}$
$= 2.5296 \times 10^{12}$ J
This much energy is required to separate all the neutrons and protons from the given coin.

**Question 13.6:**

Write nuclear reaction equations for

**(i)** $\alpha$-decay of ${}^{226}_{88}\text{Ra}$ **(ii)** $\alpha$-decay of ${}^{242}_{94}\text{Pu}$

**(iii)** $\beta^-$-decay of ${}^{32}_{15}\text{P}$ **(iv)** $\beta^-$-decay of ${}^{210}_{83}\text{Bi}$

**(v)** $\beta^+$-decay of ${}^{11}_{6}\text{C}$ **(vi)** $\beta^+$-decay of ${}^{97}_{43}\text{Tc}$

**(vii)** Electron capture of ${}^{120}_{54}\text{Xe}$

Answer

α is a nucleus of helium $\left({}_2\text{He}^4\right)$ and $\beta$ is an electron ($e^-$ for $\beta^-$ and $e^+$ for $\beta^+$). In every $\alpha$-decay, there is a loss of 2 protons and 4 neutrons. In every $\beta^+$-decay, there is a loss of 1 proton and a neutrino is emitted from the nucleus. In every $\beta^-$-decay, there is a gain of 1 proton and an antineutrino is emitted from the nucleus.

For the given cases, the various nuclear reactions can be written as:

(i) ${}_{88}\text{Ra}^{226} \longrightarrow {}_{86}\text{Rn}^{222} + {}_2\text{He}^4$

(ii) ${}^{242}_{94}\text{Pu} \longrightarrow {}^{238}_{92}\text{U} + {}^4_2\text{He}$

(iii) ${}^{32}_{15}\text{P} \longrightarrow {}^{32}_{16}\text{S} + e^- + \overline{\nu}$

(iv) ${}^{210}_{83}\text{B} \longrightarrow {}^{210}_{84}\text{PO} + e^- + \overline{\nu}$

(v) ${}^{11}_{6}\text{C} \longrightarrow {}^{11}_{5}\text{B} + e^+ + \nu$

(vi) ${}^{97}_{43}\text{Tc} \longrightarrow {}^{97}_{42}\text{MO} + e^+ + \nu$

(vii) ${}^{120}_{54}\text{Xe} + e^+ \longrightarrow {}^{120}_{53}\text{I} + \nu$

**Question 13.7:**

A radioactive isotope has a half-life of $T$ years. How long will it take the activity to reduce to a) 3.125%, b) 1% of its original value?

Answer

Half-life of the radioactive isotope = $T$ years

Original amount of the radioactive isotope = $N_0$

**(a)** After decay, the amount of the radioactive isotope = $N$

It is given that only 3.125% of $N_0$ remains after decay. Hence, we can write:

$$\frac{N}{N_0} = 3.125\% = \frac{3.125}{100} = \frac{1}{32}$$

$$\text{But } \frac{N}{N_0} = e^{-\lambda t}$$

Where,

$\lambda$ = Decay constant

$t$ = Time

$$\therefore -\lambda t = \frac{1}{32}$$

$$-\lambda t = \ln 1 - \ln 32$$
$$-\lambda t = 0 - 3.4657$$

$$t = \frac{3.4657}{\lambda}$$

$$\text{Since } \lambda = \frac{0.693}{T}$$

$$\therefore t = \frac{3.466}{\frac{0.693}{T}} \approx 5T \text{ years}$$

Hence, the isotope will take about $5T$ years to reduce to 3.125% of its original value.

**(b)** After decay, the amount of the radioactive isotope = $N$

It is given that only 1% of $N_0$ remains after decay. Hence, we can write:

$$\frac{N}{N_0} = 1\% = \frac{1}{100}$$

$$\text{But } \frac{N}{N_0} = e^{-\lambda t}$$

$$\therefore e^{-\lambda t} = \frac{1}{100}$$
$$-\lambda t = \ln 1 - \ln 100$$
$$-\lambda t = 0 - 4.6052$$
$$t = \frac{4.6052}{\lambda}$$

Since, $\lambda = 0.693/T$

$$\therefore t = \frac{4.6052}{\frac{0.693}{T}} = 6.645T \text{ years}$$

Hence, the isotope will take about $6.645T$ years to reduce to 1% of its original value.

**Question 13.8:**

The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive ${}^{14}_{6}C$ present with the stable carbon isotope ${}^{12}_{6}C$. When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life (5730 years) of ${}^{14}_{6}C$, and the measured activity, the age of the specimen can be approximately estimated. This is the principle of ${}^{14}_{6}C$ dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilisation.

Answer

Decay rate of living carbon-containing matter, $R$ = 15 decay/min

Let N be the number of radioactive atoms present in a normal carbon- containing matter.

Half life of ${}^{14}_{6}C$, $T_{1/2}$ = 5730 years

The decay rate of the specimen obtained from the Mohenjodaro site:

$R'$ = 9 decays/min

Let N' be the number of radioactive atoms present in the specimen during the Mohenjodaro period.

Therefore, we can relate the decay constant, $\lambda$and time, $t$ as:

$$\frac{N}{N'}=\frac{R}{R'}=e^{-\lambda t}$$

$$e^{-\lambda t}=\frac{9}{15}=\frac{3}{5}$$

$$-\lambda t=\log_e\frac{3}{5}=-0.5108$$

$$\therefore t=\frac{0.5108}{\lambda}$$

$$\text{But } \lambda=\frac{0.693}{T_{1/2}}=\frac{0.693}{5730}$$

$$\therefore t=\frac{0.5108}{\frac{0.693}{5730}}=4223.5 \text{ years}$$

Hence, the approximate age of the Indus-Valley civilisation is 4223.5 years.

### Question 13.9:

Obtain the amount of ${}^{60}_{27}\text{Co}$ necessary to provide a radioactive source of 8.0 mCi strength. The half-life of ${}^{60}_{27}\text{Co}$ is 5.3 years.

Answer

The strength of the radioactive source is given as:

$$\frac{dN}{dt}=8.0 \text{ mCi}$$
$$=8\times10^{-3}\times3.7\times10^{10}$$
$$=29.6\times10^{7} \text{ decay/s}$$

Where,

$N$ = Required number of atoms

Half-life of ${}^{60}_{27}\text{Co}$, $T_{1/2}$ = 5.3 years

= 5.3 × 365 × 24 × 60 × 60

= 1.67 × $10^8$ s

For decay constant $\lambda$, we have the rate of decay as:

$$\frac{dN}{dt}=\lambda N$$

Where, $\lambda$ $=\frac{0.693}{T_{1/2}}=\frac{0.693}{1.67\times10^{8}} \text{ s}^{-1}$

$$\therefore N=\frac{1}{\lambda}\frac{dN}{dt}$$

$$=\frac{29.6\times10^{7}}{\frac{0.693}{1.67\times10^{8}}}=7.133\times10^{16} \text{ atoms}$$

For ${}_{27}\text{Co}^{60}$:

Mass of 6.023 × $10^{23}$ (Avogadro's number) atoms = 60 g

$\therefore$Mass of $7.133\times10^{16}$ atoms $=\frac{60\times7.133\times10^{16}}{6.023\times10^{23}}=7.106\times10^{-6}$ g

Hence, the amount of ${}_{27}\text{Co}^{60}$ necessary for the purpose is 7.106 × $10^{-6}$ g.

### Question 13.10:

The half-life of ${}^{90}_{38}\text{Sr}$ is 28 years. What is the disintegration rate of 15 mg of this isotope?

Answer

*********** END **********