Definite Integrals Ex 20.1 Q63

$$I = \int_0^{\pi/4} (\tan x + \cot x)^{-2}\, dx$$

$$I = \int_0^{\pi/4} \frac{1}{(\tan x + \cot x)^2}\, dx$$

$$I = \int_0^{\pi/4} \frac{1}{\left(\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right)^2}\, dx$$

$$I = \int_0^{\pi/4} (\sin x \cos x)^2\, dx$$

$$I = \int_0^{\pi/4} \sin^2 x (1 - \sin^2 x)\, dx$$

$$I = \int_0^{\pi/4} \sin^2 x\, dx - \int_0^{\pi/4} \sin^4 x\, dx$$

We know that by reduction formula,

$$\int \sin^n x\, dx = \frac{n-1}{n} \int \sin^{n-2} x\, dx - \frac{\cos x \sin^{n-1} x}{n}$$

For $n = 2$

$$\int \sin^2 x\, dx = \frac{2-1}{2} \int 1\, dx - \frac{\cos x \sin x}{2}$$

$$\int \sin^2 x\, dx = \frac{1}{2} x - \frac{\cos x \sin x}{2}$$

For $n = 4$

$$\int \sin^4 x\, dx = \frac{4-1}{4} \int \sin^2 x\, dx - \frac{\cos x \sin^3 x}{4}$$

$$\int \sin^4 x\, dx = \frac{3}{4}\left\{\frac{1}{2} x - \frac{\cos x \sin x}{2}\right\} - \frac{\cos x \sin^3 x}{4}$$

Hence,

$$I = \left\{\frac{1}{2} x - \frac{\cos x \sin x}{2}\right\}_0^{\pi/4} - \left\{\frac{3}{4}\left\{\frac{1}{2} x - \frac{\cos x \sin x}{2}\right\} - \frac{\cos x \sin^3 x}{4}\right\}_0^{\pi/4}$$

$$= \left\{\frac{\pi}{8} - \frac{1}{4}\right\} - \left\{\frac{3}{4}\left(\frac{\pi}{8} - \frac{1}{4}\right) - \frac{1}{16}\right\}$$

$$= \frac{\pi}{32}$$

$$\int_0^{\frac{\pi}{2}} (\sin x \cos x)^2 dx$$

$$\int_0^{\frac{\pi}{2}} \sin^2 x(1-\sin^2 x)dx$$

$$\int_0^{\frac{\pi}{2}} \sin^2 x - \sin^4 xdx$$

$$\int_0^{\frac{\pi}{2}} \sin^2 xdx - \int_0^{\frac{\pi}{2}} \sin^4 xdx$$

We know , By reduction formula

$$\int \sin^n xdx = \frac{n-1}{n}\int \sin^{n-2} xdx - \frac{\cos x \sin^{n-1} x}{n}$$

For n=2

$$\int \sin^2 xdx = \frac{2-1}{2}\int 1dx - \frac{\cos x \sin x}{2}$$

$$\int \sin^2 xdx = \frac{1}{2}x - \frac{\cos x \sin x}{2}$$

For n=4

$$\int \sin^4 xdx = \frac{4-1}{4}\int \sin^2 xdx - \frac{\cos x \sin^3 x}{4}$$

$$\int \sin^4 xdx = \frac{3}{4}\left\{\frac{1}{2}x - \frac{\cos x \sin x}{2}\right\} - \frac{\cos x \sin^3 x}{4}$$

Hence

$$\left\{\frac{1}{2}x - \frac{\cos x \sin x}{2}\right\}_0^{\frac{\pi}{2}} - \left\{\frac{3}{4}\left\{\frac{1}{2}x - \frac{\cos x \sin x}{2}\right\} - \frac{\cos x \sin^3 x}{4}\right\}_0^{\frac{\pi}{2}}$$

$$\frac{\Pi}{4} - \frac{3}{4}\left\{\frac{\Pi}{4}\right\}$$

$$\frac{\Pi}{16}$$

Definite Integrals Ex 20.1 Q64

Using Integration By parts

$$\int f'g = fg - \int fg'$$

$$f' = x, g = \log(2x+1)$$

$$f = \frac{x^2}{2}, g' = \frac{2}{2x+1}$$

$$\int_0^1 x\log(1+2x)\,dx$$

$$= \left[\frac{x^2\log(1+2x)}{2}\right]_0^1 - \int_0^1 \frac{2x^2}{2(2x+1)}\,dx$$

$$= \frac{\log(3)}{2} - \int_0^1 \frac{x}{2} - \frac{1}{4} + \frac{1}{4(2x+1)}\,dx$$

$$= \frac{\log(3)}{2} - \left[\frac{x^2}{4} - \frac{x}{4} + \frac{1}{8}\log|2x+1|\right]_0^1$$

$$= \frac{\log(3)}{2} - \frac{1}{8}\log(3)$$

$$= \frac{3}{8}\log_e(3)$$

*********** END **********