



Geometric Progressions Ex 20.4 Q 1

$$S_{\infty} = 1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \dots$$

$$\Rightarrow a = 1, r = -\frac{1}{3}$$

$$\begin{aligned} S_{\infty} &= \frac{a}{1-r} \\ &= \frac{1}{1 + \frac{1}{3}} \end{aligned}$$

$$S_{\infty} = \frac{3}{4}$$

$$S_{\infty} = 8 + 4\sqrt{2} + 4 + \dots$$

$$\Rightarrow a = 8, r = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\begin{aligned} S_{\infty} &= \frac{a}{1-r} \\ &= \frac{8}{1 - \frac{1}{\sqrt{2}}} \\ &= \frac{8\sqrt{2}}{\sqrt{2}-1} \times \frac{(\sqrt{2}+1)}{(\sqrt{2}+1)} \\ &= \frac{8(2+\sqrt{2})}{2-1} \\ S_{\infty} &= 8(2+\sqrt{2}) \end{aligned}$$

$$\begin{aligned}
 S_{\infty} &= \frac{2}{5} + \frac{3}{5^2} + \frac{2}{5^3} + \frac{3}{5^4} + \dots \\
 &= \left( \frac{2}{5} + \frac{2}{5^3} + \dots \right) + \left( \frac{3}{5^2} + \frac{3}{5^4} + \dots \right) \\
 S_{\infty} &= S'_{\infty} + S''_{\infty}
 \end{aligned}$$

For

$$\begin{aligned}
 S'_{\infty} &= \frac{a}{1-r} \\
 &= \frac{\frac{2}{5}}{1 - \frac{1}{25}} \\
 &= \frac{2}{5} \times \frac{25}{24} \\
 S'_{\infty} &= \frac{5}{12} \\
 S''_{\infty} &= \frac{\frac{3}{25}}{1 - \frac{1}{25}} \\
 &= \frac{3}{25} \times \frac{25}{24} \\
 &= \frac{3}{24} \\
 S_{\infty} &= S'_{\infty} + S''_{\infty} \\
 &= \frac{5}{12} + \frac{3}{24} \\
 &= \frac{13}{24} \\
 S_{\infty} &= \frac{13}{24}
 \end{aligned}$$

This infinite G.P has first term  $a = 10$  and common ratio  $r = -\frac{9}{10} = -0.9$

Thus the sum of the infinite G.P will be:

$$\begin{aligned}
 10 - 9 + 8.9 - 7.29 + \dots \infty &= \frac{a}{1-r} \quad [\text{Since } |r| < 1] \\
 &= \frac{10}{1 - (-0.9)} \\
 &= \frac{10}{1.9} \\
 &= \frac{100}{19}
 \end{aligned}$$

The G.P can be written as follows:

$$\begin{aligned}
 \frac{1}{3} + \frac{1}{5^2} + \frac{1}{3^3} + \frac{1}{5^4} + \frac{1}{3^5} + \frac{1}{5^6} + \dots \infty &= \left( \frac{1}{3} + \frac{1}{3^3} + \frac{1}{3^5} + \dots \infty \right) + \left( \frac{1}{5^2} + \frac{1}{5^4} + \frac{1}{5^6} + \dots \infty \right) \\
 &= \frac{\frac{1}{3}}{1 - \frac{1}{3^2}} + \frac{\frac{1}{5^2}}{1 - \frac{1}{5^2}} \\
 &= \frac{3}{8} + \frac{1}{24} \\
 &= \frac{10}{24} \\
 &= \frac{5}{12}
 \end{aligned}$$

\*\*\*\*\* END \*\*\*\*\*