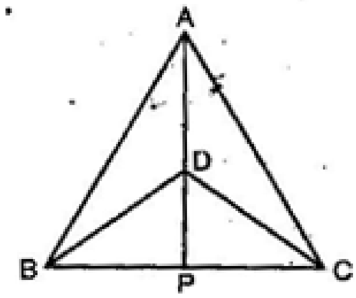
NCERT solutions for class 9 Maths Triangles Ex 7.3

Q1. ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (See figure). If AD is extended to intersect BC at P, show that:



(i) $\Delta ABD \cong \Delta ACD$

(ii) $\Delta ABP \cong \Delta ACP$

(iii) AP bisects $\angle A$ as well as $\angle D$.

(iv) AP is the perpendicular bisector of BC.

Ans. (i) ΔABC is an isosceles triangle.

$\therefore AB = AC$

ΔDBC is an isosceles triangle.

$\therefore BD = CD$

Now in ΔABD and ΔACD,

AB = AC [Given]

BD = CD [Given]

AD = AD [Common]

$\therefore \Delta ABD \cong \Delta ACD$ [By SSS congruency]

$\Rightarrow \angle BAD = \angle CAD$ [By C.P.C.T.] ....(i)

(ii) Now in ΔABP and ΔACP,

AB = AC [Given]

$\angle BAD = \angle CAD$ [From eq. (i)]

AP = AP

$\therefore$ Δ ABP $\cong$ Δ ACP [By SAS congruency]

(iii) Since Δ ABP $\cong$ Δ ACP [From part (ii)]

$\Rightarrow \angle$ BAP = $\angle$ CAP [By C.P.C.T.]

$\Rightarrow$ AP bisects $\angle$ A.

Since Δ ABD $\cong$ Δ ACD [From part (i)]

$\Rightarrow$ $\angle$ ADB = $\angle$ ADC [By C.P.C.T.] ....(ii)

Now $\angle$ ADB + $\angle$ BDP = 180° [Linear pair] .... (iii)

And $\angle$ ADC + $\angle$ CDP = 180° [Linear pair] .... (iv)

From eq. (iii) and (iv),

$\angle$ ADB + $\angle$ BDP = $\angle$ ADC + $\angle$ CDP

$\Rightarrow \angle$ ADB + $\angle$ BDP = $\angle$ ADB + $\angle$ CDP [Using (ii)]

$\Rightarrow \angle$ BDP = $\angle$ CDP

$\Rightarrow$ DP bisects $\angle$ D or AP bisects $\angle$ D.

(iv) Since Δ ABP $\cong$ Δ ACP [From part (ii)]

$\therefore$ BP = PC [By C.P.C.T.] ....(v)

And $\angle$ APB = $\angle$ APC [By C.P.C.T.] ....(vi)

Now $\angle$ APB + $\angle$ APC = 180° [Linear pair]

$\Rightarrow$ $\angle$ APB + $\angle$ APC = 180° [Using eq. (vi)]

$\Rightarrow$ $2\angle$ APB = 180°

$\Rightarrow$ $\angle$ APB = 90°

$\Rightarrow$ AP $\perp$ BC ....(vii)

From eq. (v), we have BP PC and from (vii), we have proved AP $\perp$ B. So, collectively AP is perpendicular bisector of BC.

Q2. AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that:

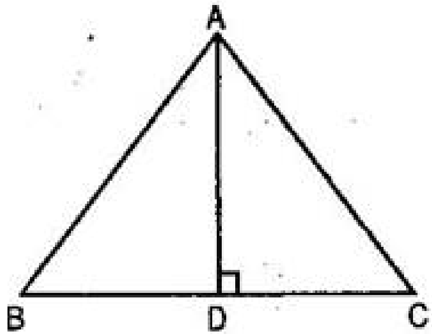
(i) AD bisects BC.

(ii) AD bisects $\angle$A.

Ans. In ΔABD and ΔACD,

AB = AC [Given]

$\angle$ ADB = $\angle$ ADC = 90° [AD $\perp$ BC]



AD = AD [Common]

$\therefore$ ΔABD $\cong$ Δ ACD [RHS rule of congruency]

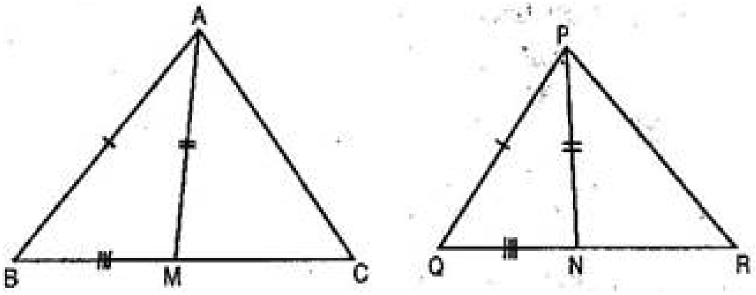
$\Rightarrow$ BD = DC [By C.P.C.T.]

$\Rightarrow$ AD bisects BC

Also $\angle$ BAD = $\angle$ CAD [By C.P.C.T.]

$\Rightarrow$ AD bisects $\angle$A.

Q3. Two sides AB and BC and median AM of the triangle ABC are respectively equal to side PQ and QR and median PN of Δ PQR (See figure). Show that:



(i) $\Delta ABM \cong \Delta PQN$

(ii) $\Delta ABC \cong \Delta PQR$

Ans. AM is the median of Δ ABC.

$\therefore$ BM = MC = $\frac{1}{2}$ BC ....(i)

PN is the median of Δ PQR.

$\therefore$ QN = NR = $\frac{1}{2}$ QR ....(ii)

Now BC = QR [Given] $\Rightarrow \frac{1}{2}$ BC = $\frac{1}{2}$ QR

$\therefore$ BM = QN ....(iii)

(i) Now in Δ ABM and Δ PQN,

AB = PQ [Given]

AM = PN [Given]

BM = QN [From eq. (iii)]

$\therefore$ $\Delta ABM \cong \Delta PQN$ [By SSS congruency]

$\Rightarrow \angle B = \angle Q$ [By C.P.C.T.] ....(iv)

(ii) In ΔABC and ΔPQR,

AB = PQ [Given]

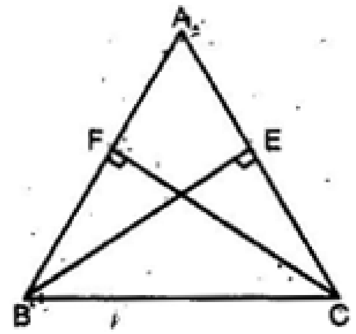
$\angle$B = $\angle$Q [Prove above]

BC = QR [Given]

$\therefore$ ΔABC $\cong$ ΔPQR [By SAS congruency]

Q4. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Ans. In ΔBEC and ΔCFB,



$\angle$BEC = $\angle$CFB [Each 90°]

BC = BC [Common]

BE = CF [Given]

$\therefore$ ΔBEC $\cong$ ΔCFB [RHS congruency]

$\Rightarrow$ EC = FB [By C.P.C.T.] .....(i)

Now In ΔAEB and ΔAFC

$\angle$AEB = $\angle$AFC [Each 90°]

$\angle$A = $\angle$A [Common]

BE = CF [Given]

$\therefore \Delta AEB \cong \Delta AFC$ [ASA congruency]

$\Rightarrow$ AE = AF [By C.P.C.T.] ....(ii)

Adding eq. (i) and (ii), we get,

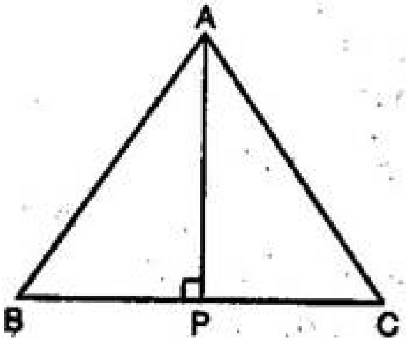
EC + AE = FB + AF

$\Rightarrow$ AB = AC

$\Rightarrow$ ABC is an isosceles triangle.

Q5. ABC is an isosceles triangles with AB = AC. Draw AP $\perp$ BC and show that $\angle$B = $\angle$C.

Ans. Given: ABC is an isosceles triangle in which AB = AC



To prove: $\angle$B = $\angle$C

Construction: Draw AP $\perp$ BC

Proof: In ΔABP and ΔACP

$\angle$APB = $\angle$APC = 90° [By construction]

AB = AC [Given]

AP = AP [Common]

$\therefore \Delta ABP \cong \Delta ACP$ [RHS congruency]

$\Rightarrow \angle$B = $\angle$C [By C.P.C.T.]

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