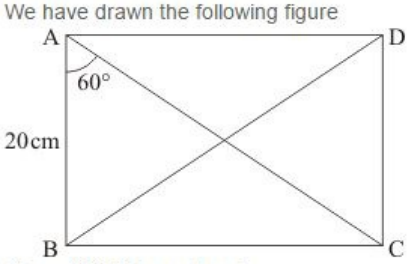
Trigonometric Ratios Ex 5.2 Q32

Answer :

We have drawn the following figure



Since *ABCD* is a rectangle

Therefore,

$\angle ABC = \angle BCD = 90°$

Now, consider ΔABC

We know that sum of all the angles of any triangle is $180°$

Therefore,

$\angle BAC + \angle ABC + \angle ACB = 180°$ ..... (1)

Now by substituting the values of known angles $\angle BAC$ and $\angle ABC$ in equation (1)

We get,

$$60° + 90° + \angle ACB = 180°$$
$$\Rightarrow \quad 150° + \angle ACB = 180°$$
$$\Rightarrow \quad \angle ACB = 180° - 150°$$
$$\Rightarrow \quad \angle ACB = 30°$$

Now in ΔABC

We know that,

$$\cos A = \cos 60°$$
$$\Rightarrow \frac{AB}{AC} = \cos 60° \qquad \ldots\ldots (2)$$

Now we have,

AB = 20cm and $\cos 60° = \frac{1}{2}$

Therefore by substituting above values in equation (2)

We get,

$$\cos A = \cos 60°$$
$$\Rightarrow \frac{20}{AC} = \frac{1}{2}$$

Now by cross multiplying we get,

$$20 \times 2 = 1 \times AC$$
$$\Rightarrow \quad 40 = AC$$
$$\Rightarrow AC = 40$$

Therefore,

$\boxed{AC = 40 \text{ cm}}$ ..... (3)

Now in ΔABC

We know that,

$$\sin A = \sin 60°$$
$$\Rightarrow \frac{BC}{AC} = \sin 60° \qquad \ldots\ldots (4)$$

Now we have from equation (3),

AC=40cm and $\sin 60° = \frac{\sqrt{3}}{2}$

Therefore by substituting above values in equation (4)

We get,

$$\sin A = \sin 60°$$

$$\Rightarrow \frac{BC}{40} = \frac{\sqrt{3}}{2}$$

Now by cross multiplying we get,

$$BC \times 2 = \sqrt{3} \times 40$$

$$\Rightarrow BC = \frac{\sqrt{3} \times 40}{2}$$

$$\Rightarrow BC = 20\sqrt{3}$$

Therefore,

$$\boxed{BC = 20\sqrt{3} \text{ m}} \quad \ldots\ldots (5)$$

Since $ABCD$ is a rectangle

Therefore,

$$AB = CD = 20 \text{ cm} \quad \ldots\ldots (6)$$

And

$$BC = AD = 20\sqrt{3} \text{ cm} \quad \ldots\ldots (7)$$

Now in ΔBCD

We know that,

$$\tan B = \frac{CD}{BC}$$

Now by substituting the values of sides from equation (6) and (7)

We get,

$$\Rightarrow \tan B = \frac{20}{20\sqrt{3}}$$

$$\Rightarrow \tan B = \frac{1}{\sqrt{3}}$$

Since

$$\tan 30° = \frac{1}{\sqrt{3}}$$

Therefore, $\angle B = 30°$

That is in ΔBCD

$\angle DBC = 30°$ ..... (8)

Now in ΔBCD

We know that,

$$\cos B = \frac{BC}{BD}$$

From equation (7)and (8)

$$\Rightarrow \cos 30° = \frac{20\sqrt{3}}{BD}$$

Since

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

Therefore,

$$\frac{\sqrt{3}}{2} = \frac{20\sqrt{3}}{BD}$$

Now by cross multiplying we get,

$$\sqrt{3} \times BD = 20\sqrt{3} \times 2$$

$$\Rightarrow \sqrt{3} \times BD = 40\sqrt{3}$$

$$\Rightarrow \quad BD = \frac{40\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow \quad BD = 40$$

Therefore,

$$\boxed{BD = 40 \text{ cm}} \quad \ldots\ldots (9)$$

Hence from equation (3), (5) and (9)

$$\boxed{AC = 40 \text{ cm}}, \boxed{BC = 20\sqrt{3} \text{ cm}}, \boxed{BD = 40 \text{ cm}}$$

********** END **********