Differentiation Ex 11.3 Q38

$$\text{Let } y = \cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right] \qquad ...(1)$$

$$\text{Then, } \frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}$$

$$= \frac{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)^2}{\left(\sqrt{1+\sin x}-\sqrt{1-\sin x}\right)\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}$$

$$= \frac{(1+\sin x)+(1-\sin x)+2\sqrt{(1-\sin x)(1+\sin x)}}{(1+\sin x)-(1-\sin x)}$$

$$= \frac{2+2\sqrt{1-\sin^2 x}}{2\sin x}$$

$$= \frac{1+\cos x}{\sin x}$$

$$= \frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$= \cot\frac{x}{2}$$

Therefore, equation (1) becomes

$$y = \cot^{-1}\left(\cot\frac{x}{2}\right)$$

$$\Rightarrow y = \frac{x}{2}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}(x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}$$

Differentiation Ex 11.3 Q39

Here, $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right) + \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

$y = \tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Put $x = \tan\theta$,

$y = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) + \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$y = \tan^{-1}(\tan 2\theta) + \cos^{-1}(\cos 2\theta)$ ---(i)

Here, $< x < \infty$

$\Rightarrow$ $0 < \tan\theta < \infty$

$\Rightarrow$ $0 < \theta < \frac{\pi}{2}$

$\Rightarrow$ $0 < 2\theta < \pi$

So, from equation (i),

$y = 2\theta + 2\theta$ $\left[\begin{array}{l}\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ \text{and } \cos^{-1}(\cos\theta) = \theta, \text{ if } \theta \in [0, \pi]\end{array}\right]$

$y = 4\theta$

$y = 4\tan^{-1} x$ [Using $x = \tan\theta$]

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{4}{1+x^2}$$

Differentiation Ex 11.3 Q40

Here, $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$

$y = \cos^{-1}\left(\frac{x-1}{x+1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$ $\left[\text{Since, } \sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)\right]$

$y = \frac{\pi}{2}$ $\left[\text{Since, } \cos^{-1} x + \sin^{-1} x = \frac{\pi}{2}\right]$

Differentiating with respect to x,

$$\frac{dy}{dx} = 0$$

Differentiation Ex 11.3 Q41

Here, $y = \sin\left[2\tan^{-1}\left[\sqrt{\frac{1-x}{1+x}}\right]\right]$

Put $x = \cos 2\theta$, so,

$y = \sin\left[2\tan^{-1}\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}\right]$

$= \sin\left[2\tan^{-1}\sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}}\right]$

$= \sin\left[2\tan^{-1}\sqrt{\tan^2\theta}\right]$

$= \sin\left[2\tan^{-1}(\tan\theta)\right]$

$= \sin(2\theta)$

$= \sin\left[2 \times \frac{1}{2}\cos^{-1} x\right]$ [Since, $x = \cos 2\theta$]

$= \sin\left(\sin^{-1}\sqrt{1-x^2}\right)$

$y = \sqrt{1-x^2}$

Differentiating with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}\frac{d}{dx}\left(1-x^2\right).$$

*********** END **********