Arithmetic Progressions Ex 9.5 Q5

Answer :

In the given problem, we have the sum of the certain number of terms of an A.P. and we need to find the number of terms.

(i) A.P. is $18, 16, 14, \dots$

So here, let us find the number of terms whose sum is 0. For that, we will use the formula,

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

Where; a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

The first term (a) = 18

The sum of n terms (S_n) = 0

Common difference of the A.P. (d) = $a_2 - a_1$

$= 16 - 18$

$= -2$

So, on substituting the values in the formula for the sum of n terms of an A.P., we get,

$0 = \frac{n}{2}\left[2(18) + (n-1)(-2)\right]$

$0 = \left(\frac{n}{2}\right)\left[36 + (-2n + 2)\right]$

$0 = \left(\frac{n}{2}\right)\left[38 - 2n\right]$

Further,

$\frac{n}{2} = 0$

$n = 0$

Or,

$38 - 2n = 0$

$n = \frac{-38}{-2}$

$n = 19$

Since, the number of terms cannot be zero, the number of terms (n) is $\boxed{n = 19}$.

(ii) Here, let us take the common difference as d.

So, we are given,

First term (a_1) = −14

Fifth term (a_5) = 2

Sum of terms (s_n) = 40

Now,

$a_5 = a_1 + 4d$

$2 = -14 + 4d$

$2 + 14 = 4d$

$d = \frac{16}{4}$

$d = 4$

Further, let us find the number of terms whose sum is 40. For that, we will use the formula,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Where; a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

The first term (a_1) = −14

The sum of n terms (S_n) = 40

Common difference of the A.P. (d) = 4

So, on substituting the values in the formula for the sum of n terms of an A.P., we get,

$$40 = \frac{n}{2}\left[2(-14) + (n-1)(4)\right]$$

$$40 = \left(\frac{n}{2}\right)\left[-28 + (4n-4)\right]$$

$$40 = \left(\frac{n}{2}\right)\left[-32 + 4n\right]$$

$$40(2) = -32n + 4n^2$$

So, we get the following quadratic equation,

$$4n^2 - 32n - 80 = 0$$

$$n^2 - 8n + 20 = 0$$

On solving by splitting the middle term, we get,

$$n^2 - 10n + 2n + 20 = 0$$

$$n(n-10) + 2(n-10) = 0$$

$$(n+2)(n-10) = 0$$

Further,

$$n + 2 = 0$$

$$n = -2$$

Or,

$$n - 10 = 0$$

$$n = 10$$

Since the number of terms cannot be negative. Therefore, the number of terms (n) is $\boxed{n = 10}$

(iii) A.P. is $9, 17, 25, \ldots$

So here, let us find the number of terms whose sum is 636. For that, we will use the formula,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Where; a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

The first term (a) = 9

The sum of n terms (S_n) = 636

Common difference of the A.P. (d) = $a_2 - a_1$

$= 17 - 9$

$= 8$

So, on substituting the values in the formula for the sum of n terms of an A.P., we get,

$$636 = \frac{n}{2}\left[2(9) + (n-1)(8)\right]$$

$$636=\left(\frac{n}{2}\right)\left[18+(8n-8)\right]$$

$$636=\left(\frac{n}{2}\right)\left[10+8n\right]$$

$$636(2)=10n+8n^2$$

So, we get the following quadratic equation,

$$8n^2+10n-1272=0$$

$$4n^2+5n-636=0$$

On solving by splitting the middle term, we get,

$$4n^2-48n+53n-636=0$$

$$4n(n-12)-53(n-12)=0$$

$$(4n-53)(n-12)=0$$

Further,

$$4n-53=0$$

$$n=\frac{53}{4}$$

Or,

$$n-12=0$$

$$n=12$$

Since, the number of terms cannot be a fraction, the number of terms (n) is $\boxed{12}$.

(iv) A.P. is $63, 60, 57, \ldots$

So here, let us find the number of terms whose sum is 693. For that, we will use the formula,

$$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$$

Where; a = first term for the given A.P.

d = common difference of the given A.P.

n = number of terms

The first term (a) = 63

The sum of n terms (S_n) = 693

Common difference of the A.P. (d) = a_2-a_1

$=60-63$

$=-3$

So, on substituting the values in the formula for the sum of n terms of an A.P., we get,

$$693=\frac{n}{2}\left[2(63)+(n-1)(-3)\right]$$

$$693=\left(\frac{n}{2}\right)\left[126+(-3n+3)\right]$$

$$693=\left(\frac{n}{2}\right)\left[129-3n\right]$$

$$693(2)=129n-3n^2$$

So, we get the following quadratic equation,

$$3n^2-129n+1386=0$$

$$n^2-43n+462=0$$

On solving by splitting the middle term, we get,

$$n^2-22n-21n+462=0$$

$$n(n-22)-21(n-22)=0$$

$$(n-22)(n-21)=0$$

Further,

$$n-22=0$$

$$n=22$$

Or,

$$n-21=0$$

$$n=21$$

Here, 22nd term will be

$$a_{22}=a_1+21d$$

$$=63+21(-3)$$

$$=63-63$$

$$=0$$

So, the sum of 22 as well as 21 terms is 693. Therefore, the number of terms (n) is $\boxed{21 \text{ or } 22}$.

*********** END **********