Algebra of Matrices Ex 5.3 Q46

Given,

$$A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$A^2 - 7A + 10I_3$

$$= \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} - 7\begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} + 10\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9+2+0 & 6+8+0 & 0+0+0 \\ 3+4+0 & 2+16+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+25 \end{bmatrix} - \begin{bmatrix} 21 & 14 & 0 \\ 7 & 28 & 0 \\ 0 & 0 & 35 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{bmatrix} - \begin{bmatrix} 21 & 14 & 0 \\ 7 & 28 & 0 \\ 0 & 0 & 35 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 11-21+10 & 14-14+0 & 0-0+0 \\ 7-7+0 & 18-28+10 & 0-0+0 \\ 0-0+0 & 0-0+0 & 25-35+10 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$= 0$

Hence,

$$A^2 - 7A + 10I_3 = 0$$

Algebra of Matrices Ex 5.3 Q47

Given,

$$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} x & y \\ z & u \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 5x - 7z & 5y - 7u \\ -2x + 3z & -2y + 3u \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$$5x - 7z = -16 \qquad \text{---(i)}$$

$$-2x + 3z = 7 \qquad \text{---(ii)}$$

$$5y - 7u = -6 \qquad \text{---(iii)}$$

$$-2y + 3u = 2 \qquad \text{---(iv)}$$

Solving equation (i) and (ii)

$$10x - 14z = -32$$

$$\underline{-10x + 15z = 35}$$

$$z = 3$$

Put the value of z in equation (i)

$$5x - 7(3) = -16$$

$$\Rightarrow \quad 5x = 16 + 21$$

$$\Rightarrow \quad 5x = 5$$

$$\Rightarrow \quad x = 1$$

Solving equation (iii) and (iv)

$$10y - 14u = -12$$

$$\underline{-10y + 15u = 10}$$

$$u = -2$$

Put the value of u in equation (iii)

$$5y - 7u = -6$$

$$\Rightarrow \quad 5y - 7(-2) = -6$$

$$\Rightarrow \quad 5y + 14 = -6$$

$$\Rightarrow \quad 5y = -20$$

$$\Rightarrow \quad y = -4$$

Algebra of Matrices Ex 5.3 Q48

Given,

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

Since, $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}_{2\times 2}$ $\quad A = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}_{2\times 3}$

$\Rightarrow \quad A$ is a matrix of order 2×3

So,

$$\text{Let } A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} a+d & b+e & c+c \\ 0+d & 0+e & 0+f \end{bmatrix} = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$d = 1,\ e = 0,\ f = 1$

And $\quad a + d = 3$

$a + 1 = 3$

$a = 3 - 1$

$a = 2$

$b + e = 3$

$b + 0 = 3$

$b = 3$

And $\quad c + f = 5$

$c + 1 = 5$

$c = 4$

Hence,

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 1 \end{bmatrix}$$

Algebra of Matrices Ex 5.3 Q48(ii)

It is given that:

$$A\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$$

The matrix given on the R.H.S. of the equation is a 2×3 matrix and the one given on the L.H.S. of the equation is a 2×3 matrix. Therefore, X has to be a 2×2 matrix.

Now, let $X=\begin{bmatrix}a & c\\ b & d\end{bmatrix}$

Therefore, we have:

$$\begin{bmatrix}a & c\\ b & d\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}a+4c & 2a+5c & 3a+6c\\ b+4d & 2b+5d & 3b+6d\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$$

Equating the corresponding elements of the two matrices, we have:

$a+4c=-7,\quad 2a+5c=-8,\quad 3a+6c=-9$

$b+4d=2,\quad 2b+5d=4,\quad 3b+6d=6$

Now, $a+4c=-7 \Rightarrow a=-7-4c$

$\therefore 2a+5c=-8 \Rightarrow -14-8c+5c=-8$

$\Rightarrow -3c=6$

$\Rightarrow c=-2$

$\therefore a=-7-4(-2)=-7+8=1$

Now, $b+4d=2 \Rightarrow b=2-4d$

$\therefore 2b+5d=4 \Rightarrow 4-8d+5d=4$

$\Rightarrow -3d=0$

$\Rightarrow d=0$

$\therefore b=2-4(0)=2$

Thus, $a=1$, $b=2$, $c=-2$, $d=0$

Hence, the required matrix X is $\begin{bmatrix}1 & -2\\ 2 & 0\end{bmatrix}$.

Algebra of Matrices Ex 5.3 Q48(iii)

We know that two matrices B and C are eligible for the product BC only when number of columns of B is equal to number of rows in C. So, from the given definition we can conclude that the order of matrix A is 1×3 i.e. we can assume $A=\begin{bmatrix}x_1 & x_2 & x_3\end{bmatrix}$.

Therefore,

$$\begin{bmatrix}4\\1\\3\end{bmatrix}_{3\times1}\begin{bmatrix}x_1 & x_2 & x_3\end{bmatrix}_{1\times3}=\begin{bmatrix}-4 & 8 & 4\\ -1 & 2 & 1\\ -3 & 6 & 3\end{bmatrix}_{3\times3},$$

$$\Rightarrow \begin{bmatrix}4\times(x_1) & 4\times(x_2) & 4\times(x_3)\\ 1\times(x_1) & 1\times(x_2) & 1\times(x_3)\\ 3\times(x_1) & 3\times(x_2) & 3\times(x_3)\end{bmatrix}_{3\times3}=\begin{bmatrix}-4 & 8 & 4\\ -1 & 2 & 1\\ -3 & 6 & 3\end{bmatrix}_{3\times3}$$

$$\Rightarrow \begin{bmatrix}4x_1 & 4x_2 & 4x_3\\ x_1 & x_2 & x_3\\ 3x_1 & 3x_2 & 3x_3\end{bmatrix}_{3\times3}=\begin{bmatrix}-4 & 8 & 4\\ -1 & 2 & 1\\ -3 & 6 & 3\end{bmatrix}_{3\times3}$$

$\Rightarrow \quad 4x_1=-4,\quad 4x_2=8,\ 4x_3=4$

Solving $\quad x_1=-1,\ x_2=2,\ x_3=1$

So, matrix $A=\begin{bmatrix}-1 & 2 & 1\end{bmatrix}$.

Algebra of Matrices Ex 5.3 Q48(iv)

Using matrix multiplication,

Let, $A_1 = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}$, $A_2 = \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and $A_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

Now, $A_1.A_2 = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} =$

$= \begin{bmatrix} (2\times-1)+(1\times-1)+(3\times0) & (2\times0)+(1\times1)+(3\times1) & (2\times-1)+(1\times0)+(3\times1) \end{bmatrix}$

$= \begin{bmatrix} -3 & 4 & 1 \end{bmatrix}$

and $(A_1.A_2)A_3 = \begin{bmatrix} -3 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$= \begin{bmatrix} (-3\times1)+(4\times0)+(1\times-1) \end{bmatrix}$

$(A_1.A_2)A_3 = \begin{bmatrix} -4 \end{bmatrix} = A$

Therefore matrix $A = \begin{bmatrix} -4 \end{bmatrix}$

Note : The problem can also be solved by calculating (A_2A_3) first then pre multiplying it with A_1 as matrix multiplication is associative but one must not change the order of multiplication.

*********** END **********