Differentiation Ex 11.7 Q6

Here $x = a(1-\cos\theta)$ and $y = a(\theta + \sin\theta)$

Then,

$$\frac{dx}{d\theta} = \frac{d}{d\theta}\left[a(1-\cos\theta)\right] = a(\sin\theta)$$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}\left[a(\theta+\sin\theta)\right] = a(1+\cos\theta)$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \left.\frac{a(1+\cos\theta)}{a(\sin\theta)}\right|_{\theta=\frac{\pi}{2}} = \frac{a(1+0)}{a} = 1$$

Differentiation Ex 11.7 Q7

Here,

$$x = \frac{e^t + e^{-t}}{2}$$

Differentiating it with respect to t,

$$\frac{dx}{dt} = \frac{1}{2}\left[\frac{d}{dt}\left(e^t\right) + \frac{d}{dt}\left(e^{-t}\right)\right]$$

$$= \frac{1}{2}\left[e^t + e^{-t}\frac{d}{dt}(-t)\right]$$

$$\frac{dx}{dt} = \frac{1}{2}\left(e^t - e^{-t}\right) = y \qquad \text{---(i)}$$

And, $$y = \frac{e^t - e^{-t}}{2}$$

Differentiating it with respect to t,

$$\frac{dy}{dt} = \frac{1}{2}\left[\frac{d}{dt}\left(e^t\right) - \frac{d}{dt}e^{-t}\right]$$

$$= \frac{1}{2}\left[e^t - e^{-t}\frac{d}{dt}\left(e^{-t}\right)\right]$$

$$= \frac{1}{2}\left(e^t - e^{-t}(-1)\right)$$

$$\frac{dy}{dt} = \frac{1}{2}\left(e^t + e^{-t}\right) = x \qquad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{x}{y}$$

$$\frac{dy}{dt} = \frac{x}{y}$$

Differentiation Ex 11.7 Q8

Here,

$$x = \frac{3at}{1+t^2}$$

Differentiating it with respect to t using quotiont rule,

$$\frac{dx}{dt} = \left[\frac{\left(1+t^2\right)\frac{d}{dt}(3at) - 3at\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{\left(1+t^2\right)(3a) - 3at(2t)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{3a+3at^2-6at^2}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{3a-3at^2}{\left(1-t^2\right)^2}\right]$$

$$\frac{dx}{dt} = \frac{3a\left(1-t^2\right)}{\left(1+t^2\right)^2} \qquad \text{---(i)}$$

And, $y = \dfrac{3at^2}{1+t^2}$

Differentiating it with respect to t using quotient rule,

$$\frac{dy}{dt} = \left[\frac{\left(1+t^2\right)\frac{d}{dt}\left(3at^2\right) - 3at^2\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right]$$

$$\frac{dy}{dt} = \left[\frac{\left(1+t^2\right)(6at) - \left(3at^2\right)(2t)}{\left(1+t^2\right)^2}\right]$$

$$= \left[\frac{6at+6at^3-6at^3}{\left(1+t^2\right)^2}\right]$$

$$\frac{dy}{dt} = \frac{6at}{\left(1+t^2\right)^2} \qquad \text{---(ii)}$$

Dividing equation (ii) by (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6at}{\left(1+t^2\right)^2} \times \frac{\left(1+t^2\right)^2}{3a\left(1-t^2\right)}$$

$$\frac{dy}{dx} = \frac{2t}{1-t^2}$$

Differentiation Ex 11.7 Q9

The given equations are $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$

$$\text{Then, } \frac{dx}{d\theta} = a\left[\frac{d}{d\theta}\cos\theta + \frac{d}{d\theta}(\theta\sin\theta)\right] = a\left[-\sin\theta + \theta\frac{d}{d\theta}(\sin\theta) + \sin\theta\frac{d}{d\theta}(\theta)\right]$$

$$= a[-\sin\theta + \theta\cos\theta + \sin\theta] = a\theta\cos\theta$$

$$\frac{dy}{d\theta} = a\left[\frac{d}{d\theta}(\sin\theta) - \frac{d}{d\theta}(\theta\cos\theta)\right] = a\left[\cos\theta - \left\{\theta\frac{d}{d\theta}(\cos\theta) + \cos\theta\cdot\frac{d}{d\theta}(\theta)\right\}\right]$$

$$= a[\cos\theta + \theta\sin\theta - \cos\theta]$$

$$= a\theta\sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta$$

*********** END **********