



Arithmetic Progressions Ex 19.5 Q1(i)

$\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ will be in A.P if $\frac{c+a}{b} - \frac{b+c}{a} = \frac{a+b}{c} - \frac{c+a}{b}$

$$\text{if } \frac{ca+a^2-b^2-cb}{ab} = \frac{ab+b^2-c^2-ac}{bc}$$

$$\begin{aligned} \text{LHS} &\Rightarrow \frac{ca+a^2-b^2-cb}{ab} \\ &\Rightarrow \frac{c^2a+a^2c-b^2c-c^2b}{abc} \\ &\Rightarrow \frac{c(a-b)[a+b+c]}{abc} \end{aligned} \quad \text{---(i)}$$

$$\begin{aligned} \text{RHS} &\Rightarrow \frac{ab+b^2-c^2-ac}{bc} \\ &\Rightarrow \frac{a^2b+ab^2-ac^2-a^2c}{abc} \\ &\Rightarrow \frac{a(b-c)[a+b+c]}{abc} \end{aligned} \quad \text{---(ii)}$$

and since $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$$\begin{aligned} \frac{1}{b} - \frac{1}{a} &= \frac{1}{c} - \frac{1}{b} \\ c(b-a) &= a(b-c) \end{aligned} \quad \text{---(iii)}$$

\therefore LHS = RHS and the given terms are in A.P.

Arithmetic Progressions Ex 19.5 Q1(ii)

$a(b+c), b(c+a), c(a+b)$ are in A.P if $b(c+a) - a(b+c) = c(a+b) - b(c+a)$

$$\begin{aligned} \text{LHS} &= b(c+a) - a(b+c) \\ &= bc+ab-ab-ac \\ &= c(b-a) \end{aligned} \quad \text{---(i)}$$

$$\begin{aligned} \text{RHS} &= c(a+b) - b(c+a) \\ &= ca+cb-bc-ba \\ &= a(c-b) \end{aligned} \quad \text{---(ii)}$$

and $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$$\begin{aligned} \therefore \frac{1}{a} - \frac{1}{b} &= \frac{1}{b} - \frac{1}{c} \\ \text{or } c(b-a) &= a(c-b) \end{aligned} \quad \text{---(iii)}$$

From (i), (ii) and (iii)

$a(b+c), b(c+a), c(a+b)$ are in A.P

Arithmetic Progressions Ex 19.5 Q2

$$\frac{a}{b+c}, \frac{b}{a+c}, \frac{c}{a+b} \text{ are in A.P if } \frac{b}{a+c} - \frac{a}{b+c} = \frac{c}{a+b} - \frac{b}{a+c}$$

$$\begin{aligned} \text{LHS} &= \frac{b}{a+c} - \frac{a}{b+c} \\ \Rightarrow & \frac{b^2 + bc - a^2 - ac}{(a+c)(b+c)} \\ \Rightarrow & \frac{(b-a)(a+b+c)}{(a+c)(b+c)} \quad \text{---(i)} \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \frac{a}{a+b} - \frac{b}{a+c} \\ \Rightarrow & \frac{ca + c^2 - b^2 - ab}{(a+b)(b+c)} \\ \Rightarrow & \frac{(c-b)(a+b+c)}{(a+b)(b+c)} \quad \text{---(ii)} \end{aligned}$$

$$\begin{aligned} \text{and } a^2, b^2, c^2 \text{ are in A.P} \\ \therefore b^2 - a^2 = c^2 - b^2 \quad \text{---(iii)} \end{aligned}$$

$$\begin{aligned} \text{Substituting } b^2 - a^2 \text{ with } c^2 - b^2 \\ \text{(i)} = \text{(ii)} \end{aligned}$$

$$\therefore \frac{a}{b+c}, \frac{b}{a+c}, \frac{c}{a+b} \text{ are in A.P}$$

Arithmetic Progressions Ex 19.5 Q3(i)

$a^2(b+c), b^2(c+a), c^2(a+b)$ are in A.P.

$$\begin{aligned} \text{If } b^2(c+a) - a^2(b+c) &= c^2(a+b) - b^2(a+c) \\ \Rightarrow b^2c + b^2a - a^2b - a^2c &= c^2a + c^2b - b^2a - b^2c \end{aligned}$$

$$\begin{aligned} \text{Given, } b-a &= c-b \quad [a, b, c \text{ are in A.P}] \\ c(b^2 - a^2) + ab(b-a) &= a(c^2 - b^2) + bc(c-b) \\ (b-a)(ab+bc+ca) &= (c-b)(ab+bc+ca) \end{aligned}$$

Cancelling $ab+bc+ca$ from both sides

$$\begin{aligned} b-a &= c-b \\ 2b &= c+a \text{ which is true} \end{aligned}$$

Hence, $a^2(b+c), (c+a)b^2$ and $c^2(a+b)$ are also in A.P.

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