Inverse Trigonometric Functions Ex 4.1 Q3.

Let $\sin^{-1}\left(\frac{1}{2}\right)=x$. Then, $\sin x=\frac{1}{2}=\sin\left(\frac{\pi}{6}\right)$

$\therefore \sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$

Let $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=y$. Then, $\sin y=\frac{1}{\sqrt{2}}=\sin\left(\frac{\pi}{4}\right)$

$\therefore \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4}$

$\therefore \sin^{-1}\left(\frac{1}{2}\right)-2\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{6}-\frac{2\pi}{4}=\frac{\pi}{6}-\frac{\pi}{2}=\frac{\pi-3\pi}{6}=-\frac{\pi}{3}$

Let $\sin^{-1}\left(-\frac{1}{2}\right)=x$. Then, $\sin x=-\frac{1}{2}=\sin\left(-\frac{\pi}{6}\right)$

$\therefore \sin^{-1}\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$

Let $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)=y$. Then, $\cos y=\frac{-\sqrt{3}}{2}=\cos\left(\pi-\frac{\pi}{6}\right)$

$\therefore \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)=\frac{5\pi}{6}$

$\therefore \sin^{-1}\left(-\frac{1}{2}\right)+2\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)=-\frac{\pi}{6}+\frac{10\pi}{6}=\frac{-\pi+10\pi}{6}=\frac{9\pi}{6}=\frac{3\pi}{2}$

Let $\tan^{-1}(-1)=x$. Then, $\tan x=-1=-\tan\left(\frac{\pi}{4}\right)=\tan\left(\pi-\frac{\pi}{4}\right)$

$\therefore \tan^{-1}(-1)=\frac{3\pi}{4}$

Let $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)=y$. Then, $\cos y=\frac{-1}{\sqrt{2}}=-\cos\left(\frac{\pi}{4}\right)=\cos\left(\pi-\frac{\pi}{4}\right)$

$\therefore \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)=\frac{3\pi}{4}$

$\therefore \tan^{-1}(-1)+\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)=\frac{3\pi}{4}+\frac{3\pi}{4}=\frac{6\pi}{4}=\frac{3\pi}{2}$

Let $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)=x$. Then, $\sin x=-\frac{\sqrt{3}}{2}=-\sin\left(\frac{\pi}{3}\right)=\sin\left(\pi-\frac{\pi}{3}\right)$

$\therefore \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)=\frac{2\pi}{3}$

Let $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=y$. Then, $\cos y=\frac{\sqrt{3}}{2}=\cos\left(\frac{\pi}{6}\right)$

$\therefore \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{6}$

$\therefore \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)+\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=\frac{2\pi}{3}+\frac{\pi}{6}=\frac{4\pi+\pi}{6}=\frac{5\pi}{6}$

Let $\tan^{-1}\left(\sqrt{3}\right)=x$. Then, $\tan x=\sqrt{3}=\tan\left(\frac{\pi}{3}\right)$

$\therefore \tan^{-1}\left(\sqrt{3}\right)=\frac{\pi}{3}$

Let $\sec^{-1}(-2)=y$. Then, $\sec y=-2=\sec\left(\pi-\frac{\pi}{3}\right)$

$\therefore \sec^{-1}(-2)=\frac{2\pi}{3}$

$\therefore \tan^{-1}\left(\sqrt{3}\right)-\sec^{-1}(-2)=\frac{\pi}{3}-\frac{2\pi}{3}=\frac{\pi-2\pi}{3}=-\frac{\pi}{3}$

********** END **********