Indefinite Integrals Ex 19.9 Q55

Let $\quad I = \int \frac{x}{\sqrt{x^2+a^2}+\sqrt{x^2-a^2}}\,dx$

$$\therefore \quad I = \int \frac{x}{\sqrt{x^2+a^2}+\sqrt{x^2-a^2}} \times \frac{\sqrt{x^2+a^2}-\sqrt{x^2-a^2}}{\sqrt{x^2+a^2}-\sqrt{x^2-a^2}}\,dx$$

$$= \int \frac{x\left(\sqrt{x^2+a^2}-\sqrt{x^2-a^2}\right)}{x^2+a^2-x^2+a^2}\,dx$$

$$= \int \frac{x}{2a^2}\left(\sqrt{x^2+a^2}-\sqrt{x^2-a^2}\right)dx$$

$$\therefore \quad I = \frac{1}{2a^2}\int x\left(\sqrt{x^2+a^2}-\sqrt{x^2-a^2}\right)dx \text{ ------- (i)}$$

Let $\quad x^2 = t \quad$ then,

$d\left(x^2\right) = dt$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad x\,dx = \frac{dt}{2}$

Putting $x^2 = t$ and $x\,dx = \frac{dt}{2}$ in equation (i),

we get

$$I = \frac{1}{2a^2}\int\left(\sqrt{t+a^2}-\sqrt{t-a^2}\right)\frac{dx}{2}$$

$$= \frac{1}{4a^2}\left[\frac{2}{3}\left(t+a^2\right)^{\frac{3}{2}}-\frac{2}{3}\left(t-a^2\right)^{\frac{3}{2}}\right]+c$$

$$\therefore \quad I = \frac{1}{4a^2}\left[\frac{2}{3}\left(x^2+a^2\right)^{\frac{3}{2}}-\frac{2}{3}\left(x^2-a^2\right)^{\frac{3}{2}}\right]+c$$

$$= \frac{1}{6a^2}\left[\left(x^2+a^2\right)^{\frac{3}{2}}-\left(x^2-a^2\right)^{\frac{3}{2}}\right]+c$$

Indefinite Integrals Ex 19.9 Q56

Let $\quad I = \int x \dfrac{\tan^{-1} x^2}{1 + x^4} dx - - - - - (i)$

Let $\quad \tan^{-1} x^2 = t \quad$ then,

$$d\left(\tan^{-1} x^2\right) = dt$$

$$\Rightarrow \quad \frac{1 \times 2x}{1 + \left(x^2\right)^2} dx = dt$$

$$\Rightarrow \quad \frac{1 \times x}{1 + x^4} dx = \frac{dt}{2}$$

Putting $\tan^{-1} x^2 = t$ and $\dfrac{x}{1 + x^4} dx = \dfrac{dt}{2}$ in equation (i),

we get

$$I = \int t \frac{dx}{2}$$

$$= \frac{1}{2} \int t \, dt$$

$$= \frac{1}{2} \times \frac{t^2}{2} + c$$

$$\therefore \quad I = \frac{t^2}{4} + c$$

$$= \frac{\left(\tan^{-1} x^2\right)^2}{4} + c$$

$$\therefore \quad I = \frac{1}{4}\left(\tan^{-1} x^2\right)^2 + c$$

Indefinite Integrals Ex 19.9 Q57

Let $\quad I = \int \frac{\left(\sin^{-1} x\right)^3}{\sqrt{1-x^2}} dx \text{ ----(i)}$

Let $\quad \sin^{-1} x = t \qquad$ then,

$d\left(\sin^{-1} x\right) = dt$

$\Rightarrow \quad \frac{1}{\sqrt{1-x^2}} dx = dt$

Putting $\sin^{-1} x = t$ and $\frac{1}{\sqrt{1-x^2}} dx = dt$ in equation (i),

we get

$$I = \int t^3 dt$$
$$= \frac{t^4}{4} + c$$

$\therefore \quad I = \frac{1}{4}\left(\sin^{-1} x\right)^4 + c$

Indefinite Integrals Ex 19.9 Q58

Let $\quad I = \int \frac{\sin\left(2 + 3\log x\right)}{x} dx \text{ ----(i)}$

Let $\quad 2 + 3\log x = t \qquad$ then,

$d\left(2 + 3\log x\right) = dt$

$\Rightarrow \quad 3\frac{1}{x} dx = dt$

$\Rightarrow \quad \frac{dx}{x} = \frac{dt}{3}$

Putting $2 + 3\log x = t$ and $\frac{dx}{x} = \frac{dt}{3}$ in equation (i),

we get

$$I = \int \sin t \frac{dt}{3}$$
$$= \frac{1}{3}\left(-\cos t\right) + c$$
$$= -\frac{1}{3}\cos\left(2 + 3\log x\right) + c$$

$\therefore \quad I = -\frac{1}{3}\cos\left(2 + 3\log x\right) + c$

Indefinite Integrals Ex 19.9 Q59

Let $I = \int x e^{x^2} dx ----(i)$

Let $x^2 = t$ then,

$d\left(x^2\right) = dt$

$\Rightarrow \quad 2x\, dx = dt$

$\Rightarrow \quad x\, dx = \frac{dt}{2}$

Putting $x^2 = t$ and $x\, dx = \frac{dt}{2}$ in equation (i), we get

$$I = \int e^t \frac{dt}{2}$$
$$= \frac{1}{2} e^t + c$$
$$= \frac{1}{2} e^{x^2} + c$$

$\therefore \quad I = \frac{1}{2} e^{x^2} + c$

*********** END **********