Complex Numbers Ex 13.4 Q4

$|z_1| = |z_2|$

Let $\arg(z_1) = \theta$

$\therefore \arg(z_2) = \pi - \theta$

In polar form, $z_1 = |z_1|(\cos\theta + i\sin\theta)\ldots\ldots(i)$

$z_2 = |z_2|\{\cos(\pi - \theta) + i\sin(\pi - \theta)\}$

$= |z_2|(-\cos\theta + i\sin\theta)$

$= -|z_2|(\cos\theta - i\sin\theta)$

Finding conjugate of

$\overline{z_2} = -|z_2|(\cos\theta + i\sin\theta)\ldots\ldots(ii)$

(i)/(ii) is equal to

$\dfrac{z_1}{\overline{z_2}} = -\dfrac{|z_1|(\cos\theta + i\sin\theta)}{|z_2|(\cos\theta + i\sin\theta)}$

$\dfrac{z_1}{\overline{z_2}} = -\dfrac{|z_1|}{|z_1|} \qquad [\because |z_1| = |z_2|]$

$\dfrac{z_1}{\overline{z_2}} = -1$

$z_1 = -\overline{z_2}$

Hence Proved.

Complex Numbers Ex 13.4 Q5

$z_1, z_2$ are conjugates implies $z_2 = \overline{z_1}$

$z_3, z_4$ are conjugates implies $z_4 = \overline{z_3}$

Also we know that $\arg(z_1) + \arg(\overline{z_1}) = 0$

$\arg\left(\dfrac{z_1}{z_4}\right) + \arg\left(\dfrac{z_2}{z_3}\right)$

$= \arg(z_1) - \arg(z_4) + \arg(z_2) - \arg(z_3) \qquad [\because \arg\left(\dfrac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)]$

$= \arg(z_1) - \arg(\overline{z_3}) + \arg(\overline{z_1}) - \arg(z_3)$

$= \arg(z_1) + \arg(\overline{z_1}) - \arg(\overline{z_3}) - \arg(z_3)$

$= \arg(z_1) + \arg(\overline{z_1}) - \left[\arg(\overline{z_3}) + \arg(z_3)\right][\because \arg(z_1) + \arg(\overline{z_1}) = 0]$

$= 0 + 0 = 0$

Complex Numbers Ex 13.4 Q6

$$\sin\frac{\pi}{5} + i\left(1 - \cos\frac{\pi}{5}\right)$$

$$= 2\sin\frac{\pi}{10}\cos\frac{\pi}{10} + i2\sin^2\frac{\pi}{10}\left[\text{Using }\sin 2\theta = 2\sin\theta\cos\theta \ \& \ 1 - \cos 2\theta = 2\sin^2\theta\right]$$

$$= 2\sin\frac{\pi}{10}\left(\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right)$$

********** END *********