Maxima and Minima 18.3 Q1(ix)

$$Local\ Maximum\ value = f(4)$$
$$= 4\sqrt{32-4^2}$$
$$= 4\sqrt{32-16}$$
$$= 4\sqrt{16}$$
$$= 16$$

$$Local\ minimum\ at\ x = -4;$$
$$Local\ Minimum\ value = f(-4)$$
$$= -4\sqrt{32-(-4)^2}$$
$$= -4\sqrt{32-16}$$
$$= -4\sqrt{16}$$
$$= -16$$

Maxima and Minima 18.3 Q1(x)

$$f(x) = x + \frac{a^2}{x}$$

$$\therefore \quad f'(x) = 1 - \frac{a^2}{x^2}$$

$$f''(x) = \frac{2a^2}{x^3}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad 1 - \frac{a^2}{x^2} = 0$$

$$\Rightarrow \quad x^2 - a^2 = 0$$

$$\Rightarrow \quad x = \pm a$$

Now,

$$f''(a) = \frac{2}{a} > 0 \text{ as } a > 0$$

$$\therefore \quad x = a \text{ is point of minima}$$

$$f''(-a) = \frac{-2}{a} < 0 \text{ as } a > 0$$

$$\therefore \quad x = -a \text{ is point of maxima}$$

Hence,

local max value $= f(-a) = -2a$

local min value $= f(a) = 2a$.

Maxima and Minima 18.3 Q1(xi)

$$f(x) = x\sqrt{2-x^2}$$

$$\therefore \quad f'(x) = \sqrt{2-x^2} - \frac{2x^2}{2\sqrt{2-x^2}}$$

$$= \frac{2\left(2-x^2\right) - 2x^2}{2\sqrt{2-x^2}}$$

$$= \frac{2-2x^2}{\sqrt{2-x^2}}$$

$$f''(x) = \frac{\sqrt{2-x^2}\,(-4x) + \frac{\left(2-2x^2\right)2x}{\sqrt{2-x^2}}}{\left(\sqrt{2-x^2}\right)^2}$$

$$= \frac{-\left(2-x^2\right)4x + 4x - 4x^3}{\left(2-x^2\right)^{\frac{3}{2}}}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad \frac{2\left(1-x^2\right)}{\sqrt{2-x^2}} = 0$$

$$\Rightarrow \quad x = \pm 1$$

Now,

$$f''(1) < 0$$

$\Rightarrow$ $x = 1$ is point of local maxima

$$f''(-1) > 0$$

$\Rightarrow$ $x = -1$ is point of local minima

Hence,

local max value $= f(1) = 1$

local min value $= f(-1) = -1.$

Maxima and Minima 18.3 Q1(xii)

$$f(x) = x + \sqrt{1-x}$$

$$\therefore \qquad f'(x) = 1 - \frac{1}{2\sqrt{1-x}} = \frac{2\sqrt{1-x} - 1}{2\sqrt{1-x}}$$

$$\therefore \qquad f'(x) = \frac{2\sqrt{1-x}\left(\frac{-1}{\sqrt{1-x}}\right) + \frac{\left(2\sqrt{1-x} - 1\right)}{\sqrt{1-x}}}{4(1-x)}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \qquad \frac{2\sqrt{1-x} - 1}{2\sqrt{1-x}} = 0$$

$$\Rightarrow \qquad \sqrt{1-x} = \frac{1}{2}$$

$$\Rightarrow \qquad x = 1 - \frac{1}{4} = \frac{3}{4}$$

Now,

$$f''\left(\frac{3}{4}\right) < 0$$

$\Rightarrow \qquad x = \frac{3}{4}$ is point of local maxima

Hence,

local max value $= f\left(\frac{3}{4}\right) = \frac{5}{4}$.

*********** END **********