Differentiation Ex 11.5 Q26

Here,

$$y = (\sin x)^{\cos x} + (\cos x)^{\sin x}$$
$$y = e^{\log(\sin x)^{\cos x}} + e^{\log(\cos x)^{\sin x}}$$
$$y = e^{\cos x \log \sin x} + e^{\sin x \log \cos x} \qquad \left[\text{Since, } \log_e e = 1 \text{ and } \log a^b = b \log a\right]$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\cos x \log \sin x}\right) + \frac{d}{dx}\left(e^{\sin x \log \cos x}\right)$$
$$= e^{\cos x \log \sin x} \frac{d}{dx}(\cos x \log \sin x) + e^{\sin x \log \cos x} \frac{d}{dx}(\sin x \log \cos x)$$
$$= e^{\log(\sin x)^{\cos x}}\left[\cos x \frac{d}{dx} \log \sin x + \log \sin x \frac{d}{dx}(\cos x)\right] + e^{\log(\cos x)^{\sin x}}\left[\sin x \frac{d}{dx} \log \cos x + \log \cos x \frac{d}{dx}(\sin x)\right]$$
$$= (\sin)^{\cos x}\left[\cos x\left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x) + \log \sin x \times (-\sin x)\right] + (\cos x)^{\sin x}\left[\sin x\left(\frac{1}{\cos x}\right)\frac{d}{dx}(\cos x) + \log \cos x (\cos x)\right]$$
$$= (\sin x)^{\cos x}\left[\cot x \times \cos x - \sin x \log \sin x\right] + (\cos x)^{\sin x}\left[\tan x(-\sin x) + \cos x \log \cos x\right]$$

$$\frac{dy}{dx} = (\sin x)^{\cos x}\left[\cot x \times \cos x - \sin x \log \sin x\right] + (\cos x)^{\sin x}\left[\cos x \log \cos x - \sin x \tan x\right]$$

Differentiation Ex 11.5 Q27

Here,

$$y = (\tan x)^{\cot x} + (\cot x)^{\tan x}$$
$$y = e^{\log(\tan x)^{\cot x}} + e^{\log(\cot x)^{\tan x}} \qquad \left[\text{Since, } \log_e e = 1, \log a^b = b \log a\right]$$
$$y = e^{\cot x \log \tan x} + e^{\tan x \log(\cot x)}$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\cot x \log \tan x}\right) + \frac{d}{dx}\left(e^{\tan x \log \cot x}\right)$$
$$= e^{\cot x \log \tan x} \frac{d}{dx}(\cot x \log \tan x) + e^{\tan x \log \cot x} \frac{d}{dx}(\tan x \log \cot x)$$
$$= e^{\log(\tan x)^{\cot x}}\left[\cot x \frac{d}{dx} \log \tan x + \log \tan x \frac{d}{dx} \cot x\right] + e^{\log(\cot x)^{\tan x}}\left[\begin{array}{l}\tan x \frac{d}{dx} \log \cot x + \\ \log \cot x \frac{d}{dx}(\tan x)\end{array}\right]$$
$$= (\tan x)^{\cot x}\left[\cot x \times \left(\frac{1}{\tan x}\right)\frac{d}{dx}(\tan x) + \log \tan x \left(-\operatorname{cosec}^2 x\right)\right] + (\cot x)^{\tan x}\left[\begin{array}{l}\tan x \left(\frac{1}{\cot x}\right)\frac{d}{dx}(\cot x) \\ + \log \cot x \left(\sec^2 x\right)\end{array}\right]$$
$$= \tan x^{\cot x}\left[(1)\left(\sec^2 x\right) - \operatorname{cosec}^2 x \log \tan x\right] + (\cot)^{\tan x}\left[(1)\left(-\operatorname{cosec}^2 x\right) + \sec^2 x \log \cot x\right]$$

$$\frac{dy}{dx} = (\tan)^{\cot x}\left[\sec^{2x} - \operatorname{cosec}^2 x \log \tan x\right] + (\cot x)^{\tan x}\left[\sec^2 x \log \cot x - \operatorname{cosec}^2 x\right]$$

Differentiation Ex 11.5 Q28

Here,

$$y = (\sin x)^x + \sin^{-1}\sqrt{x}$$
$$= e^{\log(\sin x)^x} + \sin^{-1}\sqrt{x}$$
$$y = e^{x \operatorname{lgo} \sin x} + \sin^{-1}\sqrt{x} \qquad \left[\text{Since, } \log_e e = 1, \log a^b = b \log a\right]$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{x \log \sin x}\right) + \frac{d}{dx} \sin^{-1}\left(\sqrt{x}\right)$$
$$= e^{x \log \sin x} \frac{d}{dx}(x \log \sin x) + \frac{1}{\sqrt{1 - \left(\sqrt{x}\right)^2}} \frac{d}{dx}\left(\sqrt{x}\right)$$
$$= e^{\log(\sin x)^x}\left[x \frac{d}{dx} \log \sin x + \log \sin x \frac{d}{dx}(x) + \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}}\right]$$
$$= (\sin x)^x\left[x \times \frac{1}{\sin x} \frac{d}{dx}(\sin x) + \log \sin x (1)\right] + \frac{1}{2\sqrt{x - x^2}}$$
$$= (\sin x)^x\left[\frac{x}{\sin x}(\cos x) + \log \sin x\right] + \frac{1}{2\sqrt{x - x^2}}$$

$$\frac{dy}{dx} = (\sin x)^x\left[x \cot x + \log \sin x\right] + \frac{1}{2\sqrt{x - x^2}}$$

Differentiation Ex 11.5 Q29

Here,

$$y = x^{\cos x} + (\sin x)^{\tan x}$$
$$y = e^{\log x^{\cos x}} + e^{\log(\sin x)^{\tan x}} \qquad \left[\text{Since, } e^{\log_e a} = a \text{ and } \log a^b = b\log a\right]$$
$$y = e^{\cos x \log x} + e^{\tan x \log \sin x}$$

Differentiating it with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{\cos x \log x}\right) + \frac{d}{dx}\left(e^{\tan x \log \sin x}\right)$$
$$= e^{\cos x \log x}\frac{d}{dx}(\cos x \log x) + e^{\tan x \log \sin x} \times \frac{d}{dx}(\tan x \log \sin x)$$
$$= e^{\log x^{\cos x}}\left[\cos x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(\cos x)\right] + e^{\log(\sin x)^{\tan x}}\left[\tan x \frac{d}{dx}\log \sin x + \log \sin x \frac{d}{dx}(\tan x)\right]$$
$$= x^{\cos x}\left[\cos x\left(\frac{1}{x}\right) + \log x(-\sin x)\right] + (\sin x)^{\tan x}\left[\tan x\left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x) + \log \sin x\left(\sec^2 x\right)\right]$$
$$= x^{\cos x}\left[\frac{\cos x}{x} - \sin x \log x\right] + (\sin x)^{\tan x}\left[\tan x\left(\frac{1}{\sin x}\right)(\cos x) + \sec^2 x \log \sin x\right]$$

$$\frac{dy}{dx} = x^{\cos x}\left[\frac{\cos x}{x} - \sin x \log x\right] + (\sin x)^{\tan x}\left[1 + \sec^2 x \log \sin x\right]$$

Here,

$$y = x^x + (\sin x)^x$$
$$= e^{\log x^x} + e^{\log(\sin x)^x}$$
$$y = e^{x\log x} + e^{x \log \sin x} \qquad \left[\text{Using } e^{\log a} = a \text{ and } \log a^b = b\log a\right]$$

Differentiating with respect to x using chain rule and product rule,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^{x\log x}\right) + \frac{d}{dx}\left(e^{x \log \sin x}\right)$$
$$= e^{x\log x}\frac{d}{dx}(x\log x) + e^{x\log \sin x}\frac{d}{dx}(x \log \sin x)$$
$$= e^{\log x^x}\left[x\frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x)\right] + e^{\log(\sin x)^x}\left[x\frac{d}{dx}(\log \sin x) + \log \sin x \frac{d}{dx}(x)\right]$$
$$= x^x\left[x\left(\frac{1}{x}\right) + \log x(1)\right] + (\sin x)^x\left[x \times \left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x) + \log \sin x(1)\right]$$
$$= x^x[1 + \log x] + (\sin x)^x\left[x\left(\frac{1}{\sin x}\right)(\cos x) + \log \sin x\right]$$

$$\frac{dy}{dx} = x^x(1 + \log x) + (\sin x)^x[x \cot x + \log \sin x]$$

*********** END **********