



Algebraic Identities Ex 4.3 Q12

Answer :

In the given problem, we have to find the value of $x^3 - \frac{1}{x^3}$

$$\text{Given } x^4 + \frac{1}{x^4} = 119$$

We shall use the identity $(x + y)^2 = x^2 + y^2 + 2xy$

Here putting $x^4 + \frac{1}{x^4} = 119$,

$$\left(x^2 + \frac{1}{x^2}\right)^2 = x^4 + \frac{1}{x^4} + 2 \times x^2 \times \frac{1}{x^2}$$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = x^4 + \frac{1}{x^4} + 2 \times \cancel{x^2} \times \frac{1}{\cancel{x^2}}$$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = x^4 + \frac{1}{x^4} + 2$$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 119 + 2$$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 121$$

$$x^2 + \frac{1}{x^2} = \sqrt{11 \times 11}$$

$$x^2 + \frac{1}{x^2} = \pm 11$$

In order to find $\left(x - \frac{1}{x}\right)$ we are using identity $(x - y)^2 = x^2 + y^2 - 2xy$

$$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x}$$

$$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$$

$$\left(x - \frac{1}{x}\right)^2 = 11 - 2$$

$$\left(x - \frac{1}{x}\right)^2 = 9$$

$$\left(x - \frac{1}{x}\right) = \sqrt{9}$$

$$\left(x - \frac{1}{x}\right) = \sqrt{3 \times 3}$$

$$\left(x - \frac{1}{x}\right) = \pm 3$$

In order to find $x^3 - \frac{1}{x^3}$ we are using identity $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$$x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right) \left(x^2 + \frac{1}{x^2} + x \times \frac{1}{x}\right) \text{ Here } x^2 + \frac{1}{x^2} = 11 \text{ and } \left(x - \frac{1}{x}\right) = 3$$

$$x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right) \left(x^2 + \frac{1}{x^2} + \cancel{x} \times \frac{\cancel{1}}{\cancel{x}}\right)$$

$$= 3(11 + 1)$$

$$= 3 \times 12$$

$$= 36$$

Hence the value of $x^3 - \frac{1}{x^3}$ is 36.

***** END *****