CHAPTER 29

ELECTRIC FIELD AND POTENTIAL

29.1 WHAT IS ELECTRIC CHARGE ?

Matter is made of certain elementary particles. With the advancement in technology, we have discovered hundreds of elementary particles. Many of them are rare and of no concern to us in the present course. The three most common elementary particles are electrons, protons and neutrons having masses $m_e = 9{\cdot}10940 \times 10^{-31}$ kg, $m_p = 1{\cdot}67262 \times 10^{-27}$ kg and $m_n = 1{\cdot}67493 \times 10^{-27}$ kg. Because of their mass these particles attract each other by gravitational forces. Thus, an electron attracts another electron, placed 1 cm away, with a gravitational force

$$F = \frac{Gm_1m_2}{r^2}$$

$$= \frac{(6{\cdot}67 \times 10^{-11}\ \mathrm{N\ m^2\ kg^{-2}}) \times (9{\cdot}1 \times 10^{-31}\ \mathrm{kg})^2}{(10^{-2}\ \mathrm{m})^2}$$

$$= 5{\cdot}5 \times 10^{-67}\ \mathrm{N}.$$

However, an electron is found to repel another electron at 1 cm with a force of $2{\cdot}3 \times 10^{-24}$ N. This extra force is called the *electric force*. The electric force is very large as compared to the gravitational force. The electrons must have some additional property, apart from their mass, which is responsible for the electric force. We call this property *charge*. Just as masses are responsible for the gravitational force, charges are responsible for the electric force. Two protons placed at a distance of 1 cm also repel each other with a force of $2{\cdot}3 \times 10^{-24}$ N. Thus, protons also have charge. Two neutrons placed at a distance of 1 cm attract each other with a force of $1{\cdot}9 \times 10^{-60}$ N which is equal to $\frac{Gm_1m_2}{r^2}$. Thus, neutrons exert only gravitational force on each other and experience no electric force. The neutrons have mass but no charge.

Two Kinds of Charges

As mentioned above, the electric force between two electrons is the same as the electric force between two protons placed at the same separation. We may guess that the amount of charge on an electron is the same as that on a proton. However, if a proton and an electron are placed 1 cm apart, they attract each other with a force of $2{\cdot}3 \times 10^{-24}$ N. Certainly this force is electric, but it is attractive and not repulsive. The charge on an electron repels the charge on another electron but attracts the charge on a proton. Thus, although the charge on an electron and that on a proton have the same strength, they are of two different nature. Also, if we pack a proton and an electron together in a small volume, the combination does not attract or repel another electron or proton placed at a distance. The net charge on the proton–electron system seems to be zero. It is, therefore, convenient to define one charge as positive and the other as negative. We arbitrarily call the charge on a proton as positive and that on an electron as negative. This assignment of positive and negative signs to the proton charge and the electron charge is purely a convention. It does not mean that the charge on an electron is "less" than the charge on a proton.

Unit of Charge

The above discussion suggests that charge is a basic property associated with the elementary particles and its definition is as difficult as the definition of mass or time or length. We can measure the charge on a system by comparing it with the charge on a standard body but we do not know what exactly it is that we intend to measure. The SI unit of charge is coulomb abbreviated as C. 1 coulomb is the charge flowing through a wire in 1 *s* if the electric current in it is 1 A. The charge on a proton is

$$e = 1{\cdot}60218 \times 10^{-19}\ \mathrm{C}.$$

The charge on an electron is the negative of this value.

Charge is Quantized

If protons and electrons are the only charge carriers in the universe, all observable charges must be integral multiples of e. If an object contains n_1 protons and n_2 electrons, the net charge on the object is

$$n_1(e) + n_2(-e) = (n_1 - n_2)e.$$

Indeed, there are elementary particles other than protons and electrons, which carry charge. However, they all carry charges which are integral multiples of e. Thus, the charge on any object is always an integral multiple of e and can be changed only in steps of e, i.e., charge is quantized.

The step size e is usually so small that we can easily neglect the quantization. If we rub a glass rod with a silk cloth, typically charges of the order of a microcoulomb appear on the rubbed objects. Now, 1 μC contains n units of basic charge e where

$$n = \frac{1\ \mu\text{C}}{1{\cdot}6 \times 10^{-19}\ \text{C}} \approx 6 \times 10^{12}.$$

The step size is thus very small as compared to the charges usually found and in many cases we can assume a continuous charge variation.

Charge is Conserved

The charge of an isolated system is conserved. It is possible to create or destroy charged particles but it is not possible to create or destroy *net charge*. In a beta decay process, a neutron converts itself into a proton and a fresh electron is created. The charge however, remains zero before and after the event.

Frictional Electricity : Induction

The simplest way to experience electric charges is to rub certain solid bodies against each other. Long ago, around 600 BC, the Greeks knew that when amber is rubbed with wool, it acquires the property of attracting light objects such as small pieces of paper. This is because amber becomes electrically charged. If we pass a comb through dry hair, the comb becomes electrically charged and can attract small pieces of paper. An automobile becomes charged when it travels through the air. A paper sheet becomes charged when it passes through a printing machine. A gramophone record becomes charged when cleaned with a dry cloth.

The explanation of appearance of electric charge on rubbing is simple. All material bodies contain large number of electrons and equal number of protons in their normal state. When rubbed against each other, some electrons from one body may pass on to the other body. The body that receives the extra electrons, becomes negatively charged. The body that donates the electrons, becomes positively charged because it has more protons than electrons. Thus, when a glass rod is rubbed with a silk cloth, electrons are transferred from the glass rod to the silk cloth. The glass rod becomes positively charged and the silk cloth becomes negatively charged.

If we take a positively charged glass rod near small pieces of paper, the rod attracts the pieces. Why does the rod attract paper pieces which are uncharged ? This is because the positively charged rod attracts the electrons of a paper piece towards itself. Some of the electrons accumulate at that edge of the paper piece which is closer to the rod. At the farther end of the piece there is a deficiency of electrons and hence positive charge appears there. Such a redistribution of charge in a material, due to the presence of a nearby charged body, is called *induction*. The rod exerts larger attraction on the negative charges of the paper piece as compared to the repulsion on the positive charges. This is because the negative charges are closer to the rod. Hence, there is a net attraction between the rod and the paper piece.

29.2 COULOMB'S LAW

The experiments of Coulomb and others established that the force exerted by a charged particle on the other is given by

$$F = \frac{kq_1q_2}{r^2}, \qquad \ldots (29.1)$$

where q_1 and q_2 are the charges on the particles, r is the separation between them and k is a constant. The force is attractive if the charges are of opposite signs and is repulsive if they are of the same sign. We can write Coulomb's law as

$$\vec{F} = \frac{kq_1q_2\,\vec{r}}{r^3},$$

where $\vec{r}$ is the position vector of the force-experiencing particle with respect to the force-exerting particle. In this form, the equation includes the direction of the force.

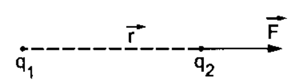

Figure 29.1

As F, q_1, q_2 and r are all independently defined quantities, the constant k can be measured experimentally. In SI units, the constant k is measured to be $8{\cdot}98755 \times 10^{9}\ \text{N m}^2\ \text{C}^{-2}$.

The constant k is often written as $\frac{1}{4\pi\varepsilon_0}$ so that equation (29.1) becomes

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}. \qquad \ldots (29.2)$$

The constant ε_0 is called the *permittivity of free space* and its value is

$$\varepsilon_0 = \frac{1}{4\pi k} = 8{\cdot}85419 \times 10^{-12}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}.$$

29.3 ELECTRIC FIELD

We have already discussed in the chapter on gravitation that a particle cannot directly interact with another particle kept at a distance. A particle creates a gravitational field around it and this field exerts force on another particle placed in it. The electric force between two charged particles is also seen as a two-step process. A charge produces something called an *electric field* in the space around it and this electric field exerts a force on any charge (except the source charge itself) placed in it. The electric field has its own existence and is present even if there is no additional charge to experience the force. The field takes finite time to propagate. Thus, if a charge is displaced from its position, the field at a distance r will change after a time $t = r/c$, where c is the speed of light. We define the *intensity of electric field* at a point as follows:

Bring a charge q at the given point without disturbing any other charge that has produced the field. If the charge q experiences an electric force $\vec{F}$, we define the intensity of electric field at the given point as

$$\vec{E} = \frac{\vec{F}}{q}. \qquad \ldots (29.3)$$

The charge q used to define $\vec{E}$ is called a *test charge.*

One way to ensure that the test charge q does not disturb other charges is to keep its magnitude very small. If this magnitude is not small, the positions of the other charges may change. Equation (29.3) then gives the electric field due to the charges in the changed positions. The intensity of electric field is often abbreviated as *electric field.*

The electric field at a point is a vector quantity. Suppose, $\vec{E}_1$ is the field at a point due to a charge Q_1 and $\vec{E}_2$ is the field at the same point due to a charge Q_2. The resultant field when both the charges are present, is $\vec{E} = \vec{E}_1 + \vec{E}_2$.

Electric Field due to a Point Charge

Consider a point charge Q placed at a point A (figure 29.2). We are interested in the electric field $\vec{E}$ at a point P at a distance r from Q. Let us imagine a test charge q placed at P. The charge Q creates a field $\vec{E}$ at P and this field exerts a force $\vec{F} = q\vec{E}$ on the charge q. But, from Coulomb's law the force on the charge q in the given situation is

$$F = \frac{Qq}{4\pi\varepsilon_0 r^2}$$

along AP. The electric field at P is, therefore,

$$E = \frac{F}{q} = \frac{Q}{4\pi\varepsilon_0 r^2} \qquad \ldots (29.4)$$

along AP.

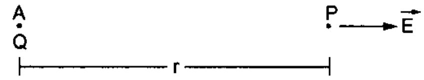

Figure 29.2

The electric field due to a set of charges may be obtained by finding the fields due to each individual charge and then adding these fields according to the rules of vector addition.

Example 29.1

Two charges 10 μC *and* −10 μC *are placed at points A and B separated by a distance of* 10 cm. *Find the electric field at a point P on the perpendicular bisector of AB at a distance of* 12 cm *from its middle point.*

Solution :

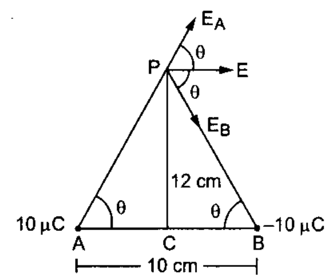

Figure 29.3

The situation is shown in figure (29.3). The distance $AP = BP = \sqrt{(5\text{ cm})^2 + (12\text{ cm})^2} = 13$ cm.

The field at the point P due to the charge 10 μC is

$$E_A = \frac{10\,\mu\text{C}}{4\pi\varepsilon_0\,(13\text{ cm})^2} = \frac{(10 \times 10^{-6}\text{ C}) \times (9 \times 10^9\text{ N m}^2\text{ C}^{-2})}{169 \times 10^{-4}\text{ m}^2}$$

$$= 5{\cdot}3 \times 10^6\text{ N C}^{-1}.$$

This field is along AP. The field due to −10 μC at P is $E_B = 5{\cdot}3 \times 10^6\text{ N C}^{-1}$ along PB. As E_A and E_B are equal in magnitude, the resultant will bisect the angle between the two. The geometry of the figure shows that this resultant is parallel to the base AB. The magnitude of the resultant field is

$$E = E_A \cos\theta + E_B \cos\theta$$

$$= 2 \times (5{\cdot}3 \times 10^6\text{ N C}^{-1}) \times \frac{5}{13}$$

$$= 4{\cdot}1 \times 10^6\text{ N C}^{-1}.$$

If a given charge distribution is continuous, we can use the technique of integration to find the resultant electric field at a point. A small element dQ is chosen in the distribution and the field $d\vec{E}$ due to dQ is calculated. The resultant field is then calculated by integrating the components of $d\vec{E}$ under proper limits.

Example 29.2

A ring of radius a contains a charge q distributed uniformly over its length. Find the electric field at a point on the axis of the ring at a distance x from the centre.

Solution :

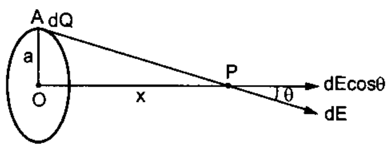

Figure 29.4

Figure (29.4) shows the situation. Let us consider a small element of the ring at the point A having a charge dQ. The field at P due to this element is

$$dE = \frac{dQ}{4\pi\varepsilon_0 (AP)^2}.$$

By symmetry, the field at P will be along the axis OP. The component of dE along this direction is

$$dE\cos\theta = \frac{dQ}{4\pi\varepsilon_0 (AP)^2}\left(\frac{OP}{AP}\right)$$

$$= \frac{x\,dQ}{4\pi\varepsilon_0 (a^2 + x^2)^{3/2}}.$$

The net field at P is

$$E = \int dE\cos\theta = \int \frac{x\,dQ}{4\pi\varepsilon_0 (a^2 + x^2)^{3/2}}$$

$$= \frac{x}{4\pi\varepsilon_0 (a^2 + x^2)^{3/2}} \int dQ = \frac{xQ}{4\pi\varepsilon_0 (a^2 + x^2)^{3/2}}.$$

29.4 LINES OF ELECTRIC FORCE

The electric field in a region can be graphically represented by drawing certain curves known as *lines of electric force* or *electric field lines*. Lines of force are drawn in such a way that the tangent to a line of force gives the direction of the resultant electric field there. Thus, the electric field due to a positive point charge is represented by straight lines originating from the charge (figure 29.5a). The electric field due to a negative point charge is represented by straight lines terminating at the charge (figure 29.5b). If we draw the lines isotropically (the lines are drawn uniformly in all directions, originating from the point charge), we can compare the intensities of the field at two points by just looking at the distribution of the lines of force.

Consider two points P_1 and P_2 in figure (29.5). Draw equal small areas through P_1 and P_2 perpendicular to the lines. More number of lines pass through the area at P_1 and less number of lines pass through the area at P_2. Also, the intensity of electric field is more at P_1 than at P_2. In fact, the electric field is proportional to the lines per unit area if the lines originate isotropically from the charge.

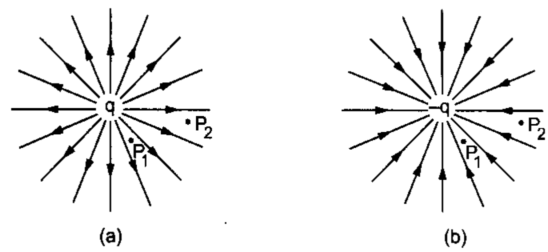

Figure 29.5

We can draw the lines of force for a charge distribution containing more than one charge. From each charge we can draw the lines isotropically. The lines may not be straight as one moves away from a charge. Figure (29.6) shows the shapes of these lines for some charge distributions.

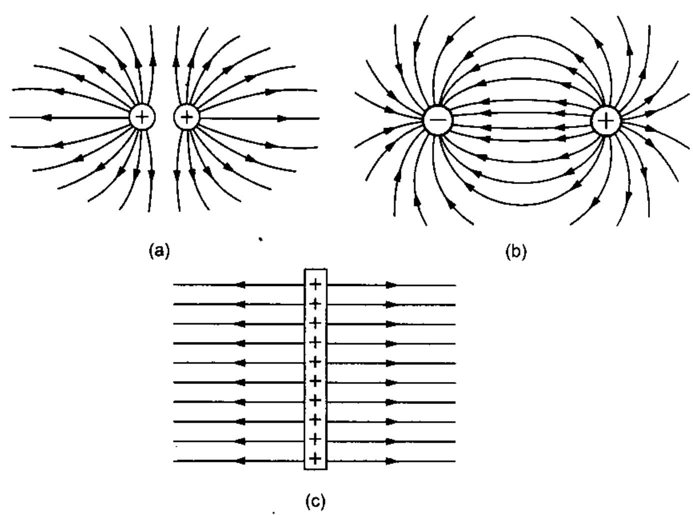

Figure 29.6

The lines of force are purely a geometrical construction which help us to visualise the nature of electric field in a region. They have no physical existence.

29.5 ELECTRIC POTENTIAL ENERGY

Consider a system of charges. The charges of the system exert electric forces on each other. If the position of one or more charges is changed, work may be done by these electric forces. We define *change in electric potential energy* of the system as negative of the work done by the electric forces as the configuration of the system changes.

Consider a system of two charges q_1 and q_2. Suppose, the charge q_1 is fixed at a point A and the charge q_2 is taken from a point B to a point C along the line ABC (figure 29.7).

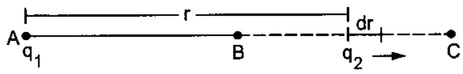

Figure 29.7

Let the distance $AB = r_1$ and the distance $AC = r_2$.

Consider a small displacement of the charge q_2 in which its distance from q_1 changes from r to $r + dr$. The electric force on the charge q_2 is

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} \text{ towards } \overrightarrow{AB}.$$

The work done by this force in the small displacement dr is

$$dW = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} dr.$$

The total work done as the charge q_2 moves from B to C is

$$W = \int_{r_1}^{r_2} \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} dr = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

No work is done by the electric force on the charge q_1 as it is kept fixed. The change in potential energy $U(r_2) - U(r_1)$ is, therefore,

$$U(r_2) - U(r_1) = -W = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right). \quad \ldots (29.5)$$

We choose the potential energy of the two-charge system to be zero when they have infinite separation (that means when they are widely separated). This means $U(\infty) = 0$. The potential energy when the separation is r is

$$U(r) = U(r) - U(\infty)$$

$$= \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{\infty}\right) = \frac{q_1 q_2}{4\pi\varepsilon_0 r}. \quad \ldots (29.6)$$

The above equation is derived by assuming that one of the charges is fixed and the other is displaced. However, the potential energy depends essentially on the separation between the charges and is independent of the spatial location of the charged particles. Equations (29.5) and (29.6) are, therefore, general.

Equation (29.6) gives the electric potential energy of a pair of charges. If there are three charges q_1, q_2 and q_3, there are three pairs. Similarly for an N-particle system, the potential energy of the system is equal to the sum of the potential energies of the N pairs of charged particles.

Example 29.3

Three particles, each having a charge of 10 μC, *are placed at the vertices of an equilateral triangle of side* 10 cm. *Find the work done by a person in pulling them apart to infinite separations.*

Solution : The potential energy of the system in the initial condition is

$$U = \frac{3 \times (10\,\mu\text{C}) \times (10\,\mu\text{C})}{4\pi\varepsilon_0\,(10\text{ cm})}$$

$$= \frac{(3 \times 10^{-10}\text{ C}^2) \times (9 \times 10^9\text{ N m}^2\text{ C}^{-2})}{0{\cdot}1\text{ m}} = 27\text{ J}.$$

When the charges are infinitely separated, the potential energy is reduced to zero. If we assume that the charges do not get kinetic energy in the process, the total mechanical energy of the system decreases by 27 J. Thus, the work done by the person on the system is –27 J.

29.6 ELECTRIC POTENTIAL

The electric field in a region of space is described by assigning a vector quantity $\vec{E}$ at each point. The same field can also be described by assigning a scalar quantity V at each point. We now define this scalar quantity known as *electric potential.*

Suppose, a test charge q is moved in an electric field from a point A to a point B while all the other charges in question remain fixed. If the electric potential energy changes by $U_B - U_A$ due to this displacement, we define the *potential difference* between the point A and the point B as

$$V_B - V_A = \frac{U_B - U_A}{q}. \quad \ldots (29.7)$$

Conversely, if a charge q is taken through a potential difference $V_B - V_A$, the electric potential energy is increased by $U_B - U_A = q(V_B - V_A)$. This equation defines potential difference between any two points in an electric field. We can define absolute electric potential at any point by choosing a reference point P and saying that the potential at this point is zero. The electric potential at a point A is then given by (equation 29.7)

$$V_A = V_A - V_P = \frac{U_A - U_P}{q}. \quad \ldots (29.8)$$

So, *the potential at a point A is equal to the change in electric potential energy per unit test charge when it is moved from the reference point to the point A.*

Suppose, the test charge is moved in an electric field without changing its kinetic energy. The total work done on the charge should be zero from the work–energy theorem. If W_{ext} and W_{el} be the work done by the external agent and by the electric field as the charge moves, we have,

$$W_{ext} + W_{el} = 0$$

or, $$W_{ext} = -W_{el} = \Delta U,$$

where ΔU is the change in electric potential energy. Using this equation and equation (29.8), the potential at a point A may also be defined as follows:

The potential at a point A is equal to the work done per unit test charge by an external agent in moving the test charge from the reference point to the point A (without changing its kinetic energy).

The choice of reference point is purely ours. Generally, a point widely separated from all charges in question is taken as the reference point. Such a point is assumed to be at infinity.

As potential energy is a scalar quantity, potential is also a scalar quantity. Thus, if V_1 is the potential at a given point due to a charge q_1 and V_2 is the potential at the same point due to a charge q_2, the potential due to both the charges is $V_1 + V_2$.

29.7 ELECTRIC POTENTIAL DUE TO A POINT CHARGE

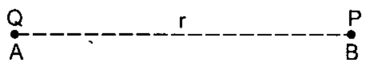

Figure 29.8

Consider a point charge Q placed at a point A (figure 29.8). We have to find the electric potential at a point P where $AP = r$. Let us take the reference point at $r = \infty$. Suppose, a test charge q is moved from $r = \infty$ to the point P. The change in electric potential energy of the system is, from equation (29.6),

$$U_P - U_\infty = \frac{Qq}{4\pi\varepsilon_0 r}.$$

The potential at P is, from equation (29.8),

$$V_P = \frac{U_P - U_\infty}{q} = \frac{Q}{4\pi\varepsilon_0 r}. \qquad \dots (29.9)$$

The electric potential due to a system of charges may be obtained by finding potentials due to the individual charges using equation (29.9) and then adding them. Thus,

$$V = \frac{1}{4\pi\varepsilon_0}\sum \frac{Q_i}{r_i}.$$

Example 29.4

Two charges $+10\,\mu C$ *and* $+20\,\mu C$ *are placed at a separation of* 2 cm. *Find the electric potential due to the pair at the middle point of the line joining the two charges.*

Solution : Using the equation $V = \frac{Q}{4\pi\varepsilon_0 r}$, the potential due to $+10\,\mu C$ is

$$V_1 = \frac{(10\times 10^{-6}\,C)\times(9\times 10^{9}\,N\,m^2C^{-2})}{1\times 10^{-2}\,m} = 9\,MV.$$

The potential due to $+20\,\mu C$ is

$$V_2 = \frac{(20\times 10^{-6}\,C)\times(9\times 10^{9}\,N\,m^2C^{-2})}{1\times 10^{-2}\,m} = 18\,MV.$$

The net potential at the given point is

$$9\,MV + 18\,MV = 27\,MV.$$

If the charge distribution is continuous, we may use the technique of integration to find the electric potential.

29.8 RELATION BETWEEN ELECTRIC FIELD AND POTENTIAL

Suppose, the electric field at a point $\vec{r}$ due to a charge distribution is $\vec{E}$ and the electric potential at the same point is V. Suppose, a point charge q is displaced slightly from the point $\vec{r}$ to $\vec{r} + d\vec{r}$. The force on the charge is

$$\vec{F} = q\vec{E}$$

and the work done by the electric field during the displacement is

$$dW = \vec{F}\cdot d\vec{r} = q\vec{E}\cdot d\vec{r}.$$

The change in potential energy is

$$dU = -\,dW = -\,q\vec{E}\cdot d\vec{r}.$$

The change in potential is

$$dV = \frac{dU}{q}$$

or, $$dV = -\,\vec{E}\cdot d\vec{r}. \qquad \dots (29.10)$$

Integrating between the points $\vec{r}_1$ and $\vec{r}_2$, we get

$$V_2 - V_1 = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{E}\cdot d\vec{r} \qquad \dots (29.11)$$

where V_1 and V_2 are the potentials at $\vec{r}_1$ and $\vec{r}_2$ respectively. If we choose $\vec{r}_1$, at the reference point (say at infinity) and $\vec{r}_2$ at $\vec{r}$, equation (29.11) becomes

$$V(\vec{r}) = -\int_{\infty}^{\vec{r}} \vec{E}\cdot d\vec{r}. \qquad \dots (29.12)$$

Example 29.5

Figure (29.9) shows two metallic plates A and B placed parallel to each other at a separation d. A uniform electric field E exists between the plates in the direction from plate B to plate A. Find the potential difference between the plates.

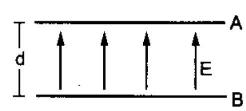

Figure 29.9

Solution : Let us take the origin at plate A and x-axis along the direction from plate A to plate B. We have

$$V_B - V_A = -\int_{\vec{r}_A}^{\vec{r}_B} \vec{E} \cdot d\vec{r} = -\int_0^d -E\,dx = Ed.$$

If we work in Cartesian coordinate system

$$E = E_x \vec{i} + E_y \vec{j} + E_z \vec{k}$$

and $\quad d\vec{r} = dx\,\vec{i} + dy\,\vec{j} + dz\,\vec{k}.$

Thus, from (29.10)

$$dV = -E_x\,dx - E_y\,dy - E_z\,dz. \quad \text{... (i)}$$

If we change x to $x + dx$ keeping y and z constant, $dy = dz = 0$ and from (i),

$$E_x = -\frac{\partial V}{\partial x}.$$

Similarly, $$E_y = -\frac{\partial V}{\partial y} \quad \text{... (29.13)}$$

and $$E_z = -\frac{\partial V}{\partial z}.$$

The symbols $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}$, etc., are used to indicate that while differentiating with respect to one coordinate, the others are kept constant.

If we know the electric field in a region, we can find the electric potential using equation (29.12) and if we know the electric potential in a region, we can find the electric field using (29.13).

Equation (29.10) may also be written as

$$dV = -E\,dr\cos\theta$$

where θ is the angle between the field $\vec{E}$ and the small displacement $d\vec{r}$. Thus,

$$-\frac{dV}{dr} = E\cos\theta. \quad \text{... (29.14)}$$

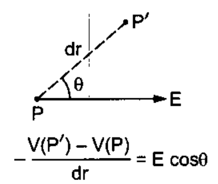

Figure 29.10

We see that, $-\frac{dV}{dr}$ gives the component of the electric field in the direction of displacement $d\vec{r}$. In figure (29.10), we show a small displacement $PP' = dr$. The electric field is E making an angle θ with PP'. We have

$$dV = V(P') - V(P)$$

so that $$\frac{V(P) - V(P')}{dr} = E\cos\theta.$$

This gives us a method to get the component of the electric field in any given direction if we know the potential. Move a small distance dr in the given direction and see the change dV in the potential. The component of electric field along that direction is $-\frac{dV}{dr}$.

If we move a distance dr in the direction of the field, θ is zero and $-\frac{dV}{dr} = E$ is maximum. Thus, *the electric field is along the direction in which the potential decreases at the maximum rate.*

If a small displacement $d\vec{r}$ perpendicular to the electric field is considered, $\theta = 90°$ and $dV = -\vec{E} \cdot d\vec{r} = 0$. The potential does not vary in a direction perpendicular to the electric field.

Equipotential Surfaces

If we draw a surface in such a way that the electric potential is the same at all the points of the surface, it is called an *equipotential surface.* The component of electric field parallel to an equipotential surface is zero, as the potential does not change in this direction. Thus, the electric field is perpendicular to the equipotential surface at each point of the surface. For a point charge, the electric field is radial and the equipotential surfaces are concentric spheres with centres at the charge (figure 29.11).

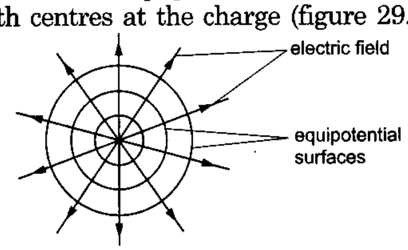

Figure 29.11

29.9 ELECTRIC DIPOLE

A combination of two charges $+q$ and $-q$ separated by a small distance d constitutes an *electric dipole.* The *electric dipole moment* of this combination is defined as a vector

$$\vec{p} = q\vec{d}, \quad \text{... (29.15)}$$

where $\vec{d}$ is the vector joining the negative charge to the positive charge. The line along the direction of the dipole moment is called the *axis of the dipole.*

Electric Potential due to a Dipole

Suppose, the negative charge $-q$ is placed at a point A and the positive charge q is placed at a point

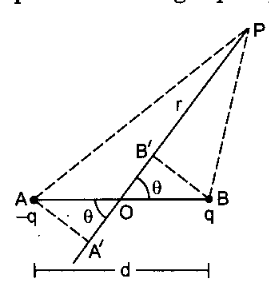

Figure 29.12

B (figure 29.12), the separation $AB = d$. The middle point of AB is O. The potential is to be evaluated at a point P where $OP = r$ and $\angle POB = \theta$. Also, let $r >> d$.

Let AA' be the perpendicular from A to PO and BB' be the perpendicular from B to PO. As d is very small compared to r,

$$AP \approx A'P = OP + OA'$$
$$= OP + AO\cos\theta = r + \frac{d}{2}\cos\theta.$$

Similarly, $BP \approx B'P = OP - OB'$

$$= r - \frac{d}{2}\cos\theta.$$

The potential at P due to the charge $-q$ is

$$V_1 = -\frac{1}{4\pi\varepsilon_0}\frac{q}{AP} \approx -\frac{1}{4\pi\varepsilon_0}\frac{q}{r + \frac{d}{2}\cos\theta}$$

and that due to the charge $+q$ is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP} \approx \frac{1}{4\pi\varepsilon_0}\frac{q}{r - \frac{d}{2}\cos\theta}.$$

The net potential at P due to the dipole is

$$V = V_1 + V_2$$
$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r - \frac{d}{2}\cos\theta} - \frac{q}{r + \frac{d}{2}\cos\theta}\right]$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{q\,d\cos\theta}{r^2 - \frac{d^2}{4}\cos^2\theta}$$
$$\approx \frac{1}{4\pi\varepsilon_0}\frac{q\,d\cos\theta}{r^2}$$

or, $$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}. \quad \ldots (29.16)$$

Generalised Definition of Electric Dipole

The potential at a distance r from a point charge q is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}.$$

It is inversely proportional to r and is independent of direction. The potential due to a dipole is inversely proportional to r^2 and depends on direction as shown by the term $\cos\theta$ in equation (29.16). In general, any charge distribution that produces electric potential given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

is called an electric dipole. The constant p is called its dipole moment and the direction from which the angle θ is measured to get the above equation is called the direction of the dipole moment.

Electric Field due to a Dipole

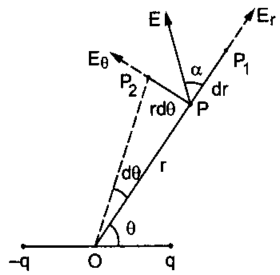

Figure 29.13

We can find the electric field due to an electric dipole using the expression (29.16) for the electric potential. In figure (29.13), PP_1 is a small displacement in the direction of OP and PP_2 is a small displacement perpendicular to OP. Thus, PP_1 is in radial direction and PP_2 is in transverse direction. In going from P to P_1, the angle θ does not change and the distance OP changes from r to $r + dr$. Thus, $PP_1 = dr$. In going from P to P_2, the angle θ changes from θ to $\theta + d\theta$ while the distance r remains almost constant. Thus, $PP_2 = r\,d\theta$. From equation (29.14), the component of the electric field at P in the radial direction PP_1 is

$$E_r = -\frac{dV}{PP_1} = -\frac{dV}{dr} = -\frac{\partial V}{\partial r}. \quad \ldots \text{(i)}$$

The symbol ∂ specifies that θ should be treated as constant while differentiating with respect to r.

Similarly, the component of the electric field at P in the transverse direction PP_2 is

$$E_\theta = -\frac{dV}{PP_2} = -\frac{dV}{r\,d\theta} = -\frac{1}{r}\frac{\partial V}{\partial\theta}. \quad \ldots \text{(ii)}$$

As $$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2},$$

$$E_r = -\frac{\partial V}{\partial r} = -\frac{1}{4\pi\varepsilon_0}\frac{\partial}{\partial r}\left(\frac{p\cos\theta}{r^2}\right)$$
$$= -\frac{1}{4\pi\varepsilon_0}(p\cos\theta)\frac{d}{dr}\left(\frac{1}{r^2}\right)$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3} \quad \ldots \text{(iii)}$$

and $$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial\theta} = -\frac{1}{r}\frac{1}{4\pi\varepsilon_0}\frac{\partial}{\partial\theta}\left(\frac{p\cos\theta}{r^2}\right)$$
$$= -\frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\frac{d}{d\theta}(\cos\theta)$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}. \quad \ldots \text{(iv)}$$

The resultant electric field at P (figure 29.13) is

$$E = \sqrt{E_r^2 + E_\theta^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\sqrt{\left(\frac{2p\cos\theta}{r^3}\right)^2 + \left(\frac{p\sin\theta}{r^3}\right)^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{3\cos^2\theta + 1}. \quad \text{... (29.17)}$$

If the resultant field makes an angle α with the radial direction OP, we have

$$\tan\alpha = \frac{E_\theta}{E_r} = \frac{p\sin\theta/r^3}{2p\cos\theta/r^3} = \frac{1}{2}\tan\theta$$

or,
$$\alpha = \tan^{-1}\left(\frac{1}{2}\tan\theta\right). \quad \text{... (29.18)}$$

Special Cases

(a) $\theta = 0$

In this case, the point P is on the axis of the dipole. From equation (29.16), the electric potential is $V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$.

The field at such a point is, from equation (29.17), $E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$ along the axis. Such a position of the point P is called an *end-on position.*

(b) $\theta = 90°$

In this case the point P is on the perpendicular bisector of the dipole. The potential here is zero while the field is, from equation (29.17), $E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$.

The angle α is given by

$$\tan\alpha = \frac{\tan\theta}{2} = \infty$$

or,
$$\alpha = 90°.$$

The field is antiparallel to the dipole axis. Such a position of the point P is called a *broadside-on position.*

29.10 TORQUE ON AN ELECTRIC DIPOLE PLACED IN AN ELECTRIC FIELD

Consider an electric dipole placed in a uniform electric field $\vec{E}$. The dipole consists of charges $-q$ placed at A and $+q$ placed at B (figure 29.14). The mid-point of AB is O and the length $AB = d$. Suppose the axis of the dipole AB makes an angle θ with the electric field at a certain instant.

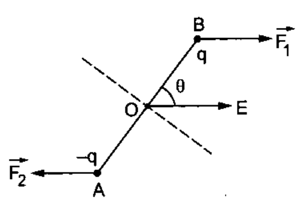

Figure 29.14

The force on the charge $+q$ is $\vec{F}_1 = q\vec{E}$ and the force on the charge $-q$ is $\vec{F}_2 = -q\vec{E}$. Let us calculate the torques $(\vec{r} \times \vec{F})$ of these forces about O.

The torque of $\vec{F}_1$ about O is

$$\vec{\Gamma}_1 = \vec{OB} \times \vec{F}_1 = q(\vec{OB} \times \vec{E})$$

and the torque of $\vec{F}_2$ about O is

$$\vec{\Gamma}_2 = \vec{OA} \times \vec{F}_2 = -q(\vec{OA} \times \vec{E}) = q(\vec{AO} \times \vec{E}).$$

The net torque acting on the dipole is

$$\vec{\Gamma} = \vec{\Gamma}_1 + \vec{\Gamma}_2$$
$$= q(\vec{OB} \times \vec{E}) + q(\vec{AO} \times \vec{E})$$
$$= q(\vec{OB} + \vec{AO}) \times \vec{E}$$
$$= q\,\vec{AB} \times \vec{E} = \vec{p} \times \vec{E}. \quad \text{... (29.19)}$$

The direction of the torque is perpendicular to the plane containing the dipole axis and the electric field. In figure (29.14), this is perpendicular to the plane of paper and is going into the page. The magnitude is $\Gamma = |\vec{\Gamma}| = pE\sin\theta$.

29.11 POTENTIAL ENERGY OF A DIPOLE PLACED IN A UNIFORM ELECTRIC FIELD

When an electric dipole is placed in an electric field $\vec{E}$, a torque $\vec{\Gamma} = \vec{p} \times \vec{E}$ acts on it (figure 29.14). If we rotate the dipole through a small angle $d\theta$, the work done by the torque is

$$dW = \Gamma d\theta$$
$$= -pE\sin\theta\, d\theta.$$

The work is negative as the rotation $d\theta$ is opposite to the torque.

The change in electric potential energy of the dipole is, therefore,

$$dU = -\,dW = pE\sin\theta\, d\theta.$$

If the angle θ is changed from 90° to θ, the change in potential energy is

$$U(\theta) - U(90°) = \int_{90°}^{\theta} pE\sin\theta\, d\theta$$
$$= pE\,[-\cos\theta]_{90°}^{\theta}$$
$$= -pE\cos\theta = -\vec{p}\cdot\vec{E}.$$

If we choose the potential energy of the dipole to be zero when $\theta = 90°$ (dipole axis is perpendicular to the field), $U(90°) = 0$ and the above equation becomes

$$U(\theta) = -\vec{p}\cdot\vec{E}. \quad \text{... (29.20)}$$

29.12 CONDUCTORS, INSULATORS AND SEMICONDUCTORS

Any piece of matter of moderate size contains millions and millions of atoms or molecules. Each atom contains a positively charged nucleus and several electrons going round it.

In gases, the atoms or molecules almost do not interact with each other. In solids and liquids, the interaction is comparatively stronger. It turns out that the materials may be broadly divided into three categories according to their behaviour when they are placed in an electric field.

In some materials, the outer electrons of each atom or molecule are only weakly bound to it. These electrons are almost free to move throughout the body of the material and are called *free electrons*. They are also known as *conduction electrons*. When such a material is placed in an electric field, the free electrons move in a direction opposite to the field. Such materials are called *conductors*.

Another class of materials is called *insulators* in which all the electrons are tightly bound to their respective atoms or molecules. Effectively, there are no *free electrons*. When such a material is placed in an electric field, the electrons may slightly shift opposite to the field but they can't leave their parent atoms or molecules and hence can't move through long distances. Such materials are also called *dielectrics*.

In *semiconductors*, the behaviour is like an insulator at the temperature 0 K. But at higher temperatures, a small number of electrons are able to free themselves and they respond to the applied electric field. As the number of free electrons in a semiconductor is much smaller than that in a conductor, its behaviour is in between a conductor and an insulator and hence, the name semiconductor. A freed electron in a semiconductor leaves a vacancy in its normal bound position. These vacancies also help in conduction.

We shall learn more about conductivity in later chapters. At the moment we accept the simple approximate model described above. The conductors have large number of free electrons everywhere in the material whereas the insulators have none. The discussion of semiconductors is deferred to a separate chapter.

Roughly speaking, the metals are conductors and the nonmetals are insulators. The above discussion may be extended to liquids and gases. Some of the liquids, such as mercury, and ionized gases are conductors.

29.13 THE ELECTRIC FIELD INSIDE A CONDUCTOR

Consider a conducting plate placed in a region. Initially, there is no electric field and the conduction electrons are almost uniformly distributed within the plate (shown by dots in figure 29.15a). In any small volume (which contains several thousand molecules) the number of electrons is equal to the number of protons in the nuclei. The net charge in the volume is zero.

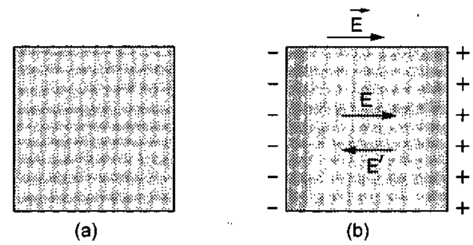

Figure 29.15

Now, suppose an electric field $\vec{E}$ is created in the direction left to right (figure 29.15b). This field exerts force on the free electrons from right to left. The electrons move towards left, the number of electrons on the left face increases and the number on the right face decreases. The left face becomes negatively charged and the right face becomes positively charged. These extra charges produce an extra electric field E' inside the plate from right to left. The electrons continue to drift and the internal field $\vec{E'}$ becomes stronger and stronger. A situation comes when the field $\vec{E'}$ due to the redistribution of free electrons becomes equal in magnitude to $\vec{E}$. The net electric field inside the plate is then zero. The free electrons there do not experience any net force and the process of further drifting stops. Thus, a steady state is reached in which some positive and negative charges appear at the surface of the plate and there is no electric field inside the plate.

Whenever a conductor is placed in an electric field some of the free electrons redistribute themselves on the surface of the conductor. The redistribution takes place in such a way that the electric field is zero at all the points inside the conductor. The redistribution takes a time which is, in general, less than a millisecond. Thus, *there can be no electric field inside a conductor in electrostatics.*

Worked Out Examples

1. *Charges* $5 \cdot 0 \times 10^{-7}$ C, $-2 \cdot 5 \times 10^{-7}$ C *and* $1 \cdot 0 \times 10^{-7}$ C *are held fixed at the three corners A, B, C of an equilateral triangle of side* $5 \cdot 0$ cm. *Find the electric force on the charge at C due to the rest two.*

Solution :

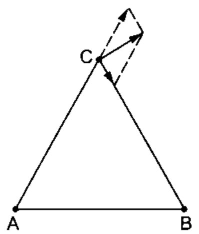

Figure 29-W1

The force on C due to A

$$= \frac{1}{4\pi\varepsilon_0} \frac{(5 \times 10^{-7}\ \text{C})\,(1 \times 10^{-7}\ \text{C})}{(0 \cdot 05\ \text{m})^2}$$

$$= 9 \times 10^9\ \text{Nm}^2\text{C}^{-2} \times \frac{5 \times 10^{-14}\ \text{C}^2}{25 \times 10^{-4}\ \text{m}^2} = 0 \cdot 18\ \text{N}.$$

This force acts along AC. The force on C due to B

$$= \frac{1}{4\pi\varepsilon_0} \frac{(2 \cdot 5 \times 10^{-7}\ \text{C})\,(1 \times 10^{-7}\ \text{C})}{(0 \cdot 05\ \text{m})^2} = 0 \cdot 09\ \text{N}.$$

This attractive force acts along CB. As the triangle is equilateral, the angle between these two forces is 120°. The resultant electric force on C is

$$[(0 \cdot 18\ \text{N})^2 + (0 \cdot 09\ \text{N})^2 + 2(0 \cdot 18\ \text{N})\,(0 \cdot 09\ \text{N})\,(\cos 120°)]^{1/2}$$

$= 0 \cdot 16$ N.

The angle made by this resultant with CB is

$$\tan^{-1} \frac{0 \cdot 18 \sin 120°}{0 \cdot 09 + 0 \cdot 18 \cos 120°} = 90°.$$

2. *Two particles A and B having charges* $8 \cdot 0 \times 10^{-6}$ C *and* $-2 \cdot 0 \times 10^{-6}$ C *respectively are held fixed with a separation of* 20 cm. *Where should a third charged particle be placed so that it does not experience a net electric force ?*

Solution : As the net electric force on C should be equal to zero, the force due to A and B must be opposite in direction. Hence, the particle should be placed on the line AB. As A and B have charges of opposite signs, C cannot be between A and B. Also, A has larger magnitude of charge than B. Hence, C should be placed closer to B than A. The situation is shown in figure (29-W2).

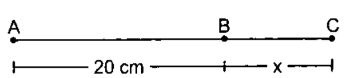

Figure 29-W2

Suppose $BC = x$ and the charge on C is Q.

The force due to $A = \dfrac{(8 \cdot 0 \times 10^{-6}\ \text{C})Q}{4\pi\varepsilon_0(20\ \text{cm} + x)^2}$.

The force due to $B = \dfrac{(2 \cdot 0 \times 10^{-6}\ \text{C})Q}{4\pi\varepsilon_0 x^2}$.

They are oppositely directed and to have a zero resultant, they should be equal in magnitude. Thus,

$$\frac{8}{(20\ \text{cm} + x)^2} = \frac{2}{x^2}$$

or, $$\frac{20\ \text{cm} + x}{x} = 2, \text{ giving } x = 20\ \text{cm}.$$

3. *Three equal charges, each having a magnitude of* $2 \cdot 0 \times 10^{-6}$ C, *are placed at the three corners of a right-angled triangle of sides* 3 cm, 4 cm *and* 5 cm. *Find the force on the charge at the right-angle corner.*

Solution :

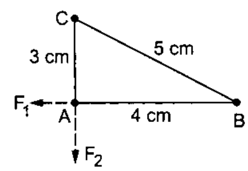

Figure 29-W3

The situation is shown in figure (29-W3). The force on A due to B is

$$F_1 = \frac{(2 \cdot 0 \times 10^{-6}\ \text{C})\,(2 \cdot 0 \times 10^{-6}\ \text{C})}{4\pi\varepsilon_0\,(4\ \text{cm})^2}$$

$$= 9 \times 10^9\ \text{N m}^2\ \text{C}^{-2} \times 4 \times 10^{-12}\ \text{C}^2 \times \frac{1}{16 \times 10^{-4}\ \text{m}^2}$$

$= 22 \cdot 5$ N.

This force acts along BA. Similarly, the force on A due to C is $F_2 = 40$ N in the direction of CA. Thus, the net electric force on A is

$$F = \sqrt{F_1^2 + F_2^2}$$

$$= \sqrt{(22 \cdot 5\ \text{N})^2 + (40\ \text{N})^2} = 45 \cdot 9\ \text{N}.$$

This resultant makes an angle θ with BA where

$$\tan\theta = \frac{40}{22 \cdot 5} = \frac{16}{9}.$$

4. *Two small iron particles, each of mass* 280 mg, *are placed at a distance* 10 cm *apart. If* $0 \cdot 01\%$ *of the electrons of one particle are transferred to the other, find the electric force between them. Atomic weight of iron is* 56 g mol^{-1} *and there are* 26 *electrons in each atom of iron.*

Solution : The atomic weight of iron is 56 g mol^{-1}. Thus, 56 g of iron contains 6×10^{23} atoms and each atom contains 26 electrons. Hence, 280 mg of iron contains

$$\frac{280 \text{ mg} \times 6 \times 10^{23} \times 26}{56 \text{ g}} = 7{\cdot}8 \times 10^{22} \text{ electrons.}$$

The number of electrons transferred from one particle to another

$$= \frac{0{\cdot}01}{100} \times 7{\cdot}8 \times 10^{22} = 7{\cdot}8 \times 10^{18}.$$

The charge transferred is, therefore,

$$1{\cdot}6 \times 10^{-19} \text{ C} \times 7{\cdot}8 \times 10^{18} = 1{\cdot}2 \text{ C}.$$

The electric force between the particles is

$$(9 \times 10^{9} \text{ N m}^2 \text{ C}^{-2}) \frac{(1{\cdot}2 \text{ C})^2}{(10 \times 10^{-2} \text{ m})^2}$$

$$= 1{\cdot}3 \times 10^{12} \text{ N}.$$

This equals the load of approximately 2000 million grown-up persons !

5. *A charge Q is to be divided on two objects. What should be the values of the charges on the objects so that the force between the objects can be maximum ?*

Solution : Suppose one object receives a charge q and the other $Q - q$. The force between the objects is

$$F = \frac{q(Q-q)}{4\pi\varepsilon_0 d^2},$$

where d is the separation between them. For F to be maximum, the quantity

$$y = q(Q-q) = Qq - q^2$$

should be maximum. This is the case when,

$$\frac{dy}{dq} = 0 \text{ or, } Q - 2q = 0 \text{ or, } q = Q/2.$$

Thus, the charge should be divided equally on the two objects.

6. *Two particles, each having a mass of* 5 g *and charge* $1{\cdot}0 \times 10^{-7}$ C, *stay in limiting equilibrium on a horizontal table with a separation of* 10 cm *between them. The coefficient of friction between each particle and the table is the same. Find the value of this coefficient.*

Solution : The electric force on one of the particles due to the other is

$$F = (9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}) \times (1{\cdot}0 \times 10^{-7} \text{C})^2 \times \frac{1}{(0{\cdot}10 \text{ m})^2}$$

$$= 0{\cdot}009 \text{ N}.$$

The frictional force in limiting equilibrium

$$f = \mu \times (5 \times 10^{-3} \text{ kg}) \times 9{\cdot}8 \text{ m s}^{-2}$$

$$= (0{\cdot}049\,\mu) \text{ N}.$$

As these two forces balance each other,

$$0{\cdot}049\,\mu = 0{\cdot}009$$

or, $$\mu = 0{\cdot}18.$$

7. *A vertical electric field of magnitude* $4{\cdot}00 \times 10^5$ N C^{-1} *just prevents a water droplet of mass* $1{\cdot}00 \times 10^{-4}$ kg *from falling. Find the charge on the droplet.*

Solution : The forces acting on the droplet are

(i) the electric force $q\vec{E}$ and

(ii) the force of gravity $m\vec{g}$.

To just prevent from falling, these two forces should be equal and opposite. Thus,

$$q(4{\cdot}00 \times 10^5 \text{ N C}^{-1}) = (1{\cdot}00 \times 10^{-4} \text{ kg}) \times (9{\cdot}8 \text{ m s}^{-2})$$

or, $$q = 2{\cdot}45 \times 10^{-9} \text{ C}.$$

8. *Three charges, each equal to q, are placed at the three corners of a square of side a. Find the electric field at the fourth corner.*

Solution :

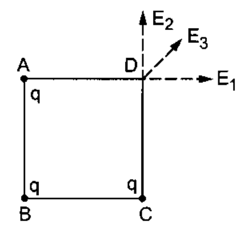

Figure 29-W4

Let the charges be placed at the corners A, B and C (figure 29-W4). We shall calculate the electric field at the fourth corner D. The field E_1 due to the charge at A will have the magnitude $\frac{q}{4\pi\varepsilon_0 a^2}$ and will be along AD. The field E_2 due to the charge at C will have the same magnitude and will be along CD.

The field E_3 due to the charge at B will have the magnitude $\frac{q}{4\pi\varepsilon_0 (\sqrt{2}a)^2}$ and will be along BD. As E_1 and E_2 are equal in magnitude, their resultant will be along the bisector of the angle between E_1, E_2 and hence along E_3. The magnitude of this resultant is $\sqrt{E_1^2 + E_2^2}$ as the angle between E_1 and E_2 is $\pi/2$. The resultant electric field at D is, therefore, along E_3 and has magnitude

$$\sqrt{E_1^2 + E_2^2} + E_3$$

$$= \sqrt{\left(\frac{q}{4\pi\varepsilon_0 a^2}\right)^2 + \left(\frac{q}{4\pi\varepsilon_0 a^2}\right)^2} + \frac{q}{4\pi\varepsilon_0 (\sqrt{2}a)^2}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{\sqrt{2}}{a^2} + \frac{1}{2a^2}\right] = (2\sqrt{2} + 1)\frac{q}{8\pi\varepsilon_0 a^2}.$$

9. *A charged particle of mass* $1{\cdot}0$ g *is suspended through a silk thread of length* 40 cm *in a horizontal electric field of* $4{\cdot}0 \times 10^4$ N C^{-1}. *If the particle stays at a distance of* 24 cm *from the wall in equilibrium, find the charge on the particle.*

Solution :

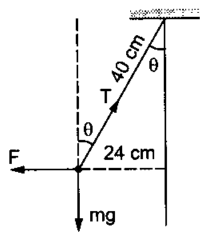

Figure 29-W5

The situation is shown in figure (29-W5).
The forces acting on the particle are
(i) the electric force $F = qE$ horizontally,
(ii) the force of gravity mg downward and
(iii) the tension T along the thread.
As the particle is at rest, these forces should add to zero. Taking components along horizontal and vertical,

$$T\cos\theta = mg \quad \text{and} \quad T\sin\theta = F$$

or, $$F = mg\tan\theta \qquad \dots \text{(i)}$$

From the figure,

$$\sin\theta = \frac{24}{40} = \frac{3}{5}\cdot$$

Thus, $\tan\theta = \frac{3}{4}\cdot$ From (i),

$$q(4{\cdot}0 \times 10^{4}\ \mathrm{N\ C^{-1}}) = (1{\cdot}0 \times 10^{-3}\ \mathrm{kg})\,(9{\cdot}8\ \mathrm{m\ s^{-2}})\,\frac{3}{4},$$

giving $$q = 1{\cdot}8 \times 10^{-7}\ \mathrm{C}.$$

10. *A particle A having a charge of* $5{\cdot}0 \times 10^{-7}$ C *is fixed in a vertical wall. A second particle B of mass* 100 g *and having equal charge is suspended by a silk thread of length* 30 cm *from the wall. The point of suspension is* 30 cm *above the particle A. Find the angle of the thread with the vertical when it stays in equilibrium.*

Solution :

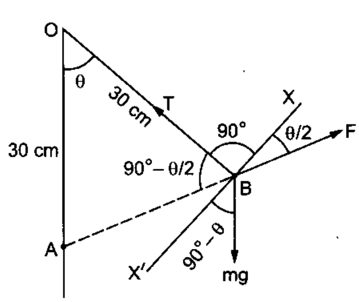

Figure 29-W6

The situation is shown in figure (29-W6). Suppose the point of suspension is O and let θ be the angle between the thread and the vertical. Forces on the particle B are
(i) weight mg downward
(ii) tension T along the thread and
(iii) electric force of repulsion F along AB.

For equilibrium, these forces should add to zero. Let $X'BX$ be the line perpendicular to OB. We shall take the components of the forces along BX. This will give a relation between F, mg and θ.
The various angles are shown in the figure. As

$$OA = OB,\ \angle OBA = \angle OAB = 90° - \frac{\theta}{2}\cdot$$

The other angles can be written down directly.
Taking components along BX, we get

$$F\cos\frac{\theta}{2} = mg\cos(90° - \theta)$$
$$= 2\,mg\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

or, $$\sin\frac{\theta}{2} = \frac{F}{2\,mg}\cdot \qquad \dots \text{(i)}$$

Now, $$F = (9 \times 10^{9}\ \mathrm{N\ m^{2}\ C^{-2}}) \times (5{\cdot}0 \times 10^{-7}\ \mathrm{C})^{2} \times \frac{1}{AB^{2}}$$

and $$AB = 2(OA)\sin\frac{\theta}{2}\cdot$$

Thus, $$F = \frac{9 \times 10^{9} \times 25 \times 10^{-14}}{4 \times (30 \times 10^{-2})^{2} \times \sin^{2}\frac{\theta}{2}}\ \mathrm{N}. \qquad \dots \text{(ii)}$$

From (i) and (ii),

$$\sin\frac{\theta}{2} = \frac{F}{2\,mg} = \frac{9 \times 10^{9} \times 25 \times 10^{-14}\ \mathrm{N}}{4 \times (30 \times 10^{-2})^{2} \times \sin^{2}\frac{\theta}{2}}\cdot\frac{1}{2\,mg}$$

or,

$$\sin^{3}\frac{\theta}{2} = \frac{9 \times 10^{9} \times 25 \times 10^{-14}\ \mathrm{N}}{4 \times 9 \times 10^{-2} \times 2 \times (100 \times 10^{-3}\ \mathrm{kg}) \times 9{\cdot}8\ \mathrm{m\ s^{-2}}}$$
$$= 0{\cdot}0032$$

or, $$\sin\frac{\theta}{2} = 0{\cdot}15,\ \text{giving}\ \theta = 17°.$$

11. *Four particles, each having a charge q, are placed on the four vertices of a regular pentagon. The distance of each corner from the centre is a. Find the electric field at the centre of the pentagon.*

Solution :

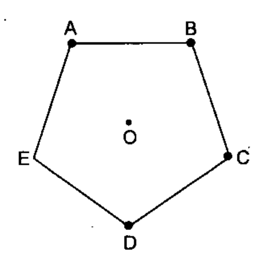

Figure 29-W7

Let the charges be placed at the vertices A, B, C and D of the pentagon $ABCDE$. If we put a charge q at the corner E also, the field at O will be zero by symmetry. Thus, the field at the *centre due to the charges at* A, B,

C and D is equal and opposite to the field due to the charge q at E alone.

The field at O due to the charge q at E is

$$\frac{q}{4\pi\varepsilon_0 a^2} \text{ along } EO.$$

Thus, the field at O due to the given system of charges is $\frac{q}{4\pi\varepsilon_0 a^2}$ along OE.

12. *Find the electric field at a point P on the perpendicular bisector of a uniformly charged rod. The length of the rod is L, the charge on it is Q and the distance of P from the centre of the rod is a.*

Solution :

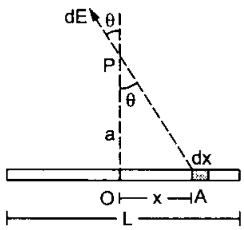

Figure 29-W8

Let us take an element of length dx at a distance x from the centre of the rod (figure 29-W8). The charge on this element is

$$dQ = \frac{Q}{L}dx.$$

The electric field at P due to this element is

$$dE = \frac{dQ}{4\pi\varepsilon_0 (AP)^2}.$$

By symmetry, the resultant field at P will be along OP (if the charge is positive). The component of dE along OP is

$$dE\cos\theta = \frac{dQ}{4\pi\varepsilon_0 (AP)^2}\cdot\frac{OP}{AP} = \frac{a\,Q\,dx}{4\pi\varepsilon_0 L(a^2+x^2)^{3/2}}$$

Thus, the resultant field at P is

$$E = \int dE\cos\theta$$

$$= \frac{aQ}{4\pi\varepsilon_0 L}\int_{-L/2}^{L/2}\frac{dx}{(a^2+x^2)^{3/2}}. \quad \dots \text{(i)}$$

We have $x = a\tan\theta$ or $dx = a\sec^2\theta\, d\theta$.

$$\text{Thus, } \int\frac{dx}{(a^2+x^2)^{3/2}} = \int\frac{a\sec^2\theta\,d\theta}{a^3\sec^3\theta}$$

$$= \frac{1}{a^2}\int\cos\theta\,d\theta = \frac{1}{a^2}\sin\theta = \frac{1}{a^2}\frac{x}{(x^2+a^2)^{1/2}}.$$

From (i),

$$E = \frac{aQ}{4\pi\varepsilon_0 La^2}\left[\frac{x}{(x^2+a^2)^{1/2}}\right]_{-L/2}^{L/2}$$

$$= \frac{aQ}{4\pi\varepsilon_0 La^2}\left[\frac{2L}{(L^2+4a^2)^{1/2}}\right]$$

$$= \frac{Q}{2\pi\varepsilon_0 a\sqrt{L^2+4a^2}}.$$

13. *A uniform electric field E is created between two parallel, charged plates as shown in figure (29-W9). An electron enters the field symmetrically between the plates with a speed v_0. The length of each plate is l. Find the angle of deviation of the path of the electron as it comes out of the field.*

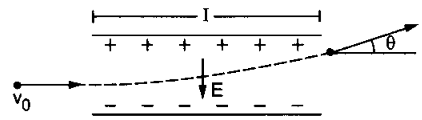

Figure 29-W9

Solution : The acceleration of the electron is $a = \frac{eE}{m}$ in the upward direction. The horizontal velocity remains v_0 as there is no acceleration in this direction. Thus, the time taken in crossing the field is

$$t = \frac{l}{v_0}. \quad \dots \text{(i)}$$

The upward component of the velocity of the electron as it emerges from the field region is

$$v_y = at = \frac{eEl}{mv_0}.$$

The horizontal component of the velocity remains

$$v_x = v_0.$$

The angle θ made by the resultant velocity with the original direction is given by

$$\tan\theta = \frac{v_y}{v_x} = \frac{eEl}{mv_0^2}.$$

Thus, the electron deviates by an angle

$$\theta = \tan^{-1}\frac{eEl}{mv_0^2}.$$

14. *In a circuit,* 10 C *of charge is passed through a battery in a given time. The plates of the battery are maintained at a potential difference of* 12 V. *How much work is done by the battery?*

Solution : By definition, the work done to transport a charge q through a potential difference V is qV. Thus, work done by the battery

$$= 10\text{ C}\times 12\text{ V} = 120\text{ J}.$$

15. *Charges* $2{\cdot}0\times10^{-6}$ C *and* $1{\cdot}0\times10^{-6}$ C *are placed at corners A and B of a square of side* 5·0 cm *as shown in figure (29-W10). How much work will be done against the electric field in moving a charge of* $1{\cdot}0\times10^{-6}$ C *from C to D?*

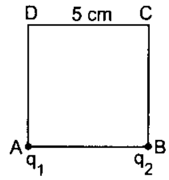

Figure 29-W10

Solution : The electric potential at C

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{AC} + \frac{q_2}{BC}\right)$$

$$= 9 \times 10^9\ \text{Nm}^{-2}\text{C}^{-2}\left(\frac{2{\cdot}0 \times 10^{-6}\ \text{C}}{\sqrt{2} \times 0{\cdot}05\ \text{m}} + \frac{1{\cdot}0 \times 10^{-6}\ \text{C}}{0{\cdot}05\ \text{m}}\right)$$

$$= (9000\ \text{V})\left(\frac{2 + \sqrt{2}}{\sqrt{2} \times 0{\cdot}05}\right).$$

The electric potential at D

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{AD} + \frac{q_2}{BD}\right)$$

$$= 9 \times 10^9\ \text{Nm}^{-2}\text{C}^{-2}\left(\frac{2{\cdot}0 \times 10^{-6}\ \text{C}}{0{\cdot}05\ \text{m}} + \frac{1{\cdot}0 \times 10^{-6}\ \text{C}}{\sqrt{2} \times 0{\cdot}05\ \text{m}}\right)$$

$$= (9000\ \text{V})\left(\frac{2\sqrt{2} + 1}{\sqrt{2} \times 0{\cdot}05}\right).$$

The work done against the electric field in moving the charge $1{\cdot}0 \times 10^{-6}$ C from C to D is $q(V_D - V_C)$

$$= (1{\cdot}0 \times 10^{-6}\ \text{C})\ (9000\ \text{V})\left(\frac{2\sqrt{2} + 1 - 2 - \sqrt{2}}{\sqrt{2} \times 0{\cdot}05}\right)$$

$$= 0{\cdot}053\ \text{J}.$$

16. *The electric field in a region is given by* $\vec{E} = (A/x^3)\,\vec{i}$. *Write a suitable SI unit for A. Write an expression for the potential in the region assuming the potential at infinity to be zero.*

Solution : The SI unit of electric field is N C^{-1} or V m^{-1}. Thus, the unit of A is $\text{N m}^3\ \text{C}^{-1}$ or V m^{-2}.

$$V(x,\ y,\ z) = -\int_{\infty}^{(x,\ y,\ z)} \vec{E}\cdot\vec{dr}$$

$$= -\int_{\infty}^{(x,\ y,\ z)} \frac{A\,dx}{x^3} = \frac{A}{2x^2}.$$

17. *Three point charges q, 2q and 8q are to be placed on a 9 cm long straight line. Find the positions where the charges should be placed such that the potential energy of this system is minimum. In this situation, what is the electric field at the charge q due to the other two charges?*

Solution : The maximum contribution may come from the charge $8q$ forming pairs with others. To reduce its effect, it should be placed at a corner and the smallest charge q in the middle. This arrangement shown in figure (29-W11) ensures that the charges in the strongest pair $2q$, $8q$ are at the largest separation.

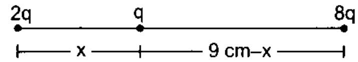

Figure 29-W11

The potential energy is

$$U = \frac{q^2}{4\pi\varepsilon_0}\left[\frac{2}{x} + \frac{16}{9\ \text{cm}} + \frac{8}{9\ \text{cm} - x}\right].$$

This will be minimum if

$$A = \frac{2}{x} + \frac{8}{9\ \text{cm} - x} \text{ is minimum.}$$

For this, $$\frac{dA}{dx} = -\frac{2}{x^2} + \frac{8}{(9\ \text{cm} - x)^2} = 0 \qquad \dots \text{(i)}$$

or, $9\ \text{cm} - x = 2x$ or, $x = 3$ cm.

The electric field at the position of charge q is

$$\frac{q}{4\pi\varepsilon_0}\left(\frac{2}{x^2} - \frac{8}{(9\ \text{cm} - x)^2}\right)$$

$$= 0 \qquad \text{from (i).}$$

18. *An* HCl *molecule has a dipole moment of* $3{\cdot}4 \times 10^{-30}$ Cm. *Assuming that equal and opposite charges lie on the two atoms to form a dipole, what is the magnitude of this charge? The separation between the two atoms of* HCl *is* $1{\cdot}0 \times 10^{-10}$ m.

Solution : If the charges on the two atoms are q, $-q$,

$$q(1{\cdot}0 \times 10^{-10}\ \text{m}) = 3{\cdot}4 \times 10^{-30}\ \text{Cm}$$

or, $$q = 3{\cdot}4 \times 10^{-20}\ \text{C}.$$

Note that this is less than the charge of a proton. Can you explain, how such a charge can appear on an atom?

19. *Figure (29-W12) shows an electric dipole formed by two particles fixed at the ends of a light rod of length l. The mass of each particle is m and the charges are −q and +q. The system is placed in such a way that the dipole axis is parallel to a uniform electric field E that exists in the region. The dipole is slightly rotated about its centre and released. Show that for small angular displacement, the motion is angular simple harmonic and find its time period.*

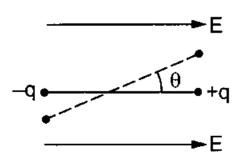

Figure 29-W12

Solution : Suppose, the dipole axis makes an angle θ with the electric field at an instant. The magnitude of the torque on it is

$$|\tau| = |\vec{p} \times \vec{E}|$$
$$= qlE \sin\theta.$$

This torque will tend to rotate the dipole back towards the electric field. Also, for small angular displacement $\sin\theta \approx \theta$ so that

$$\tau = -qlE\theta.$$

The moment of inertia of the system about the axis of rotation is

$$I = 2 \times m\left(\frac{l}{2}\right)^2 = \frac{ml^2}{2}.$$

Thus, the angular acceleration is

$$\alpha = \frac{\tau}{I} = -\frac{2qE}{ml}\theta = -\omega^2\theta$$

where $\omega^2 = \frac{2qE}{ml}$.

Thus, the motion is angular simple harmonic and the time period is $T = 2\pi\sqrt{\frac{ml}{2qE}}$.

□

QUESTIONS FOR SHORT ANSWER

1. The charge on a proton is $+1.6 \times 10^{-19}$ C and that on an electron is -1.6×10^{-19} C. Does it mean that the electron has a charge 3.2×10^{-19} C less than the charge of a proton ?
2. Is there any lower limit to the electric force between two particles placed at a separation of 1 cm ?
3. Consider two particles A and B having equal charges and placed at some distance. The particle A is slightly displaced towards B. Does the force on B increase as soon as the particle A is displaced ? Does the force on the particle A increase as soon as it is displaced ?
4. Can a gravitational field be added vectorially to an electric field to get a total field ?
5. Why does a phonograph-record attract dust particles just after it is cleaned ?
6. Does the force on a charge due to another charge depend on the charges present nearby ?
7. In some old texts it is mentioned that 4π lines of force originate from each unit positive charge. Comment on the statement in view of the fact that 4π is not an integer.
8. Can two equipotential surfaces cut each other ?
9. If a charge is placed at rest in an electric field, will its path be along a line of force ? Discuss the situation when the lines of force are straight and when they are curved.
10. Consider the situation shown in figure (29-Q1). What are the signs of q_1 and q_2 ? If the lines are drawn in proportion to the charge, what is the ratio q_1/q_2 ?

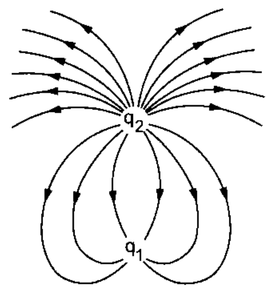

Figure 29-Q1

11. A point charge is taken from a point A to a point B in an electric field. Does the work done by the electric field depend on the path of the charge ?
12. It is said that the separation between the two charges forming an electric dipole should be small. Small compared to what ?
13. The number of electrons in an insulator is of the same order as the number of electrons in a conductor. What is then the basic difference between a conductor and an insulator ?
14. When a charged comb is brought near a small piece of paper, it attracts the piece. Does the paper become charged when the comb is brought near it ?

OBJECTIVE I

1. Figure (29-Q2) shows some of the electric field lines corresponding to an electric field. The figure suggests that

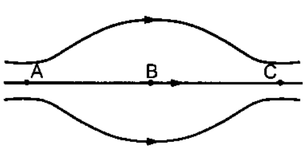

Figure 29-Q2

(a) $E_A > E_B > E_C$ (b) $E_A = E_B = E_C$
(c) $E_A = E_C > E_B$ (d) $E_A = E_C < E_B$.

2. When the separation between two charges is increased, the electric potential energy of the charges
(a) increases (b) decreases
(c) remains the same (d) may increase or decrease.

3. If a positive charge is shifted from a low-potential region to a high-potential region, the electric potential energy

(a) increases (b) decreases
(c) remains the same (d) may increase or decrease.

4. Two equal positive charges are kept at points A and B. The electric potential at the points between A and B (excluding these points) is studied while moving from A to B. The potential
(a) continuously increases
(b) continuously decreases
(c) increases then decreases
(d) decreases then increases.

5. The electric field at the origin is along the positive x-axis. A small circle is drawn with the centre at the origin cutting the axes at points A, B, C and D having coordinates $(a, 0)$, $(0, a)$, $(-a, 0)$, $(0, -a)$ respectively. Out of the points on the periphery of the circle, the potential is minimum at
(a) A (b) B (c) C (d) D.

6. If a body is charged by rubbing it, its weight
(a) remains precisely constant
(b) increases slightly
(c) decreases slightly
(d) may increase slightly or may decrease slightly.

7. An electric dipole is placed in a uniform electric field. The net electric force on the dipole
(a) is always zero
(b) depends on the orientation of the dipole
(c) can never be zero
(d) depends on the strength of the dipole.

8. Consider the situation of figure (29-Q3). The work done in taking a point charge from P to A is W_A, from P to B is W_B and from P to C is W_C.
(a) $W_A < W_B < W_C$ (b) $W_A > W_B > W_C$
(c) $W_A = W_B = W_C$ (d) None of these

Figure 29-Q3

9. A point charge q is rotated along a circle in the electric field generated by another point charge Q. The work done by the electric field on the rotating charge in one complete revolution is
(a) zero (b) positive (c) negative
(d) zero if the charge Q is at the centre and nonzero otherwise.

OBJECTIVE II

1. Mark out the correct options.
(a) The total charge of the universe is constant.
(b) The total positive charge of the universe is constant.
(c) The total negative charge of the universe is constant.
(d) The total number of charged particles in the universe is constant.

2. A point charge is brought in an electric field. The electric field at a nearby point
(a) will increase if the charge is positive
(b) will decrease if the charge is negative
(c) may increase if the charge is positive
(d) may decrease if the charge is negative.

3. The electric field and the electric potential at a point are E and V respectively.
(a) If $E = 0$, V must be zero.
(b) If $V = 0$, E must be zero.
(c) If $E \neq 0$, V cannot be zero.
(d) If $V \neq 0$, E cannot be zero.

4. The electric potential decreases uniformly from 120 V to 80 V as one moves on the x-axis from $x = -1$ cm to $x = +1$ cm. The electric field at the origin
(a) must be equal to 20 Vcm^{-1}
(b) may be equal to 20 Vcm^{-1}
(c) may be greater than 20 Vcm^{-1}
(d) may be less than 20 Vcm^{-1}.

5. Which of the following quantities do not depend on the choice of zero potential or zero potential energy ?
(a) Potential at a point
(b) Potential difference between two points
(c) Potential energy of a two-charge system
(d) Change in potential energy of a two-charge system.

6. An electric dipole is placed in an electric field generated by a point charge.
(a) The net electric force on the dipole must be zero.
(b) The net electric force on the dipole may be zero.
(c) The torque on the dipole due to the field must be zero.
(d) The torque on the dipole due to the field may be zero.

7. A proton and an electron are placed in a uniform electric field.
(a) The electric forces acting on them will be equal.
(b) The magnitudes of the forces will be equal.
(c) Their accelerations will be equal.
(d) The magnitudes of their accelerations will be equal.

8. The electric field in a region is directed outward and is proportional to the distance r from the origin. Taking the electric potential at the origin to be zero,
(a) it is uniform in the region
(b) it is proportional to r
(c) it is proportional to r^2
(d) it increases as one goes away from the origin.

EXERCISES

1. Find the dimensional formula of ε_0.
2. A charge of 1·0 C is placed at the top of your college building and another equal charge at the top of your house. Take the separation between the two charges to be 2·0 km. Find the force exerted by the charges on each other. How many times of your weight is this force ?
3. At what separation should two equal charges, 1·0 C each, be placed so that the force between them equals the weight of a 50 kg person ?
4. Two equal charges are placed at a separation of 1·0 m. What should be the magnitude of the charges so that the force between them equals the weight of a 50 kg person ?
5. Find the electric force between two protons separated by a distance of 1 fermi (1 fermi = 10^{-15} m). The protons in a nucleus remain at a separation of this order.
6. Two charges $2{\cdot}0 \times 10^{-6}$ C and $1{\cdot}0 \times 10^{-6}$ C are placed at a separation of 10 cm. Where should a third charge be placed such that it experiences no net force due to these charges ?
7. Suppose the second charge in the previous problem is $-1{\cdot}0 \times 10^{-6}$ C. Locate the position where a third charge will not experience a net force.
8. Two charged particles are placed at a distance 1·0 cm apart. What is the minimum possible magnitude of the electric force acting on each charge ?
9. Estimate the number of electrons in 100 g of water. How much is the total negative charge on these electrons ?
10. Suppose all the electrons of 100 g water are lumped together to form a negatively charged particle and all the nuclei are lumped together to form a positively charged particle. If these two particles are placed 10·0 cm away from each other, find the force of attraction between them. Compare it with your weight.
11. Consider a gold nucleus to be a sphere of radius 6·9 fermi in which protons and neutrons are distributed. Find the force of repulsion between two protons situated at largest separation. Why do these protons not fly apart under this repulsion ?
12. Two insulating small spheres are rubbed against each other and placed 1 cm apart. If they attract each other with a force of 0·1 N, how many electrons were transferred from one sphere to the other during rubbing ?
13. NaCl molecule is bound due to the electric force between the sodium and the chlorine ions when one electron of sodium is transferred to chlorine. Taking the separation between the ions to be $2{\cdot}75 \times 10^{-8}$ cm, find the force of attraction between them. State the assumptions (if any) that you have made.
14. Find the ratio of the electric and gravitational forces between two protons.
15. Suppose an attractive nuclear force acts between two protons which may be written as $F = Ce^{-kr}/r^2$. (a) Write down the dimensional formulae and appropriate SI units of C and k. (b) Suppose that $k = 1$ fermi^{-1} and that the repulsive electric force between the protons is just balanced by the attractive nuclear force when the separation is 5 fermi. Find the value of C.
16. Three equal charges, $2{\cdot}0 \times 10^{-6}$ C each, are held fixed at the three corners of an equilateral triangle of side 5 cm. Find the Coulomb force experienced by one of the charges due to the rest two.
17. Four equal charges $2{\cdot}0 \times 10^{-6}$ C each are fixed at the four corners of a square of side 5 cm. Find the Coulomb force experienced by one of the charges due to the rest three.
18. A hydrogen atom contains one proton and one electron. It may be assumed that the electron revolves in a circle of radius 0·53 angstrom (1 angstrom = 10^{-10} m and is abbreviated as Å) with the proton at the centre. The hydrogen atom is said to be in the ground state in this case. Find the magnitude of the electric force between the proton and the electron of a hydrogen atom in its ground state.
19. Find the speed of the electron in the ground state of a hydrogen atom. The description of ground state is given in the previous problem.
20. Ten positively charged particles are kept fixed on the x-axis at points $x = 10$ cm, 20 cm, 30 cm, ..., 100 cm. The first particle has a charge $1{\cdot}0 \times 10^{-8}$ C, the second 8×10^{-8} C, the third 27×10^{-8} C and so on. The tenth particle has a charge 1000×10^{-8} C. Find the magnitude of the electric force acting on a 1 C charge placed at the origin.
21. Two charged particles having charge $2{\cdot}0 \times 10^{-8}$ C each are joined by an insulating string of length 1 m and the system is kept on a smooth horizontal table. Find the tension in the string.
22. Two identical balls, each having a charge of $2{\cdot}00 \times 10^{-7}$ C and a mass of 100 g, are suspended from a common point by two insulating strings each 50 cm long. The balls are held at a separation 5·0 cm apart and then released. Find (a) the electric force on one of the charged balls (b) the components of the resultant force on it along and perpendicular to the string (c) the tension in the string (d) the acceleration of one of the balls. Answers are to be obtained only for the instant just after the release.
23. Two identical pith balls are charged by rubbing against each other. They are suspended from a horizontal rod through two strings of length 20 cm each, the separation between the suspension points being 5 cm. In equilibrium, the separation between the balls is 3 cm. Find the mass of each ball and the tension in the strings. The charge on each ball has a magnitude $2{\cdot}0 \times 10^{-8}$ C.
24. Two small spheres, each having a mass of 20 g, are suspended from a common point by two insulating strings of length 40 cm each. The spheres are identically charged and the separation between the balls at

equilibrium is found to be 4 cm. Find the charge on each sphere.

25. Two identical pith balls, each carrying a charge q, are suspended from a common point by two strings of equal length l. Find the mass of each ball if the angle between the strings is 2θ in equilibrium.

26. A particle having a charge of $2{\cdot}0 \times 10^{-4}$ C is placed directly below and at a separation of 10 cm from the bob of a simple pendulum at rest. The mass of the bob is 100 g. What charge should the bob be given so that the string becomes loose?

27. Two particles A and B having charges q and $2q$ respectively are placed on a smooth table with a separation d. A third particle C is to be clamped on the table in such a way that the particles A and B remain at rest on the table under electrical forces. What should be the charge on C and where should it be clamped ?

28. Two identically charged particles are fastened to the two ends of a spring of spring constant $100\ \mathrm{N\ m^{-1}}$ and natural length 10 cm. The system rests on a smooth horizontal table. If the charge on each particle is $2{\cdot}0 \times 10^{-8}$ C, find the extension in the length of the spring. Assume that the extension is small as compared to the natural length. Justify this assumption after you solve the problem.

29. A particle A having a charge of $2{\cdot}0 \times 10^{-6}$ C is held fixed on a horizontal table. A second charged particle of mass 80 g stays in equilibrium on the table at a distance of 10 cm from the first charge. The coefficient of friction between the table and this second particle is $\mu = 0{\cdot}2$. Find the range within which the charge of this second particle may lie.

30. A particle A having a charge of $2{\cdot}0 \times 10^{-6}$ C and a mass of 100 g is placed at the bottom of a smooth inclined plane of inclination 30°. Where should another particle B, having same charge and mass, be placed on the incline so that it may remain in equilibrium ?

31. Two particles A and B, each having a charge Q, are placed a distance d apart. Where should a particle of charge q be placed on the perpendicular bisector of AB so that it experiences maximum force ? What is the magnitude of this maximum force ?

32. Two particles A and B, each carrying a charge Q, are held fixed with a separation d between them. A particle C having mass m and charge q is kept at the middle point of the line AB. (a) If it is displaced through a distance x perpendicular to AB, what would be the electric force experienced by it. (b) Assuming $x << d$, show that this force is proportional to x. (c) Under what conditions will the particle C execute simple harmonic motion if it is released after such a small displacement ? Find the time period of the oscillations if these conditions are satisfied.

33. Repeat the previous problem if the particle C is displaced through a distance x along the line AB.

34. The electric force experienced by a charge of $1{\cdot}0 \times 10^{-6}$ C is $1{\cdot}5 \times 10^{-3}$ N. Find the magnitude of the electric field at the position of the charge.

35. Two particles A and B having charges of $+2{\cdot}00 \times 10^{-6}$ C and of $-4{\cdot}00 \times 10^{-6}$ C respectively are held fixed at a separation of 20·0 cm. Locate the point(s) on the line AB where (a) the electric field is zero (b) the electric potential is zero.

36. A point charge produces an electric field of magnitude $5{\cdot}0\ \mathrm{N\ C^{-1}}$ at a distance of 40 cm from it. What is the magnitude of the charge ?

37. A water particle of mass 10·0 mg and having a charge of $1{\cdot}50 \times 10^{-6}$ C stays suspended in a room. What is the magnitude of electric field in the room ? What is its direction ?

38. Three identical charges, each having a value $1{\cdot}0 \times 10^{-8}$ C, are placed at the corners of an equilateral triangle of side 20 cm. Find the electric field and potential at the centre of the triangle.

39. Positive charge Q is distributed uniformly over a circular ring of radius R. A particle having a mass m and a negative charge q, is placed on its axis at a distance x from the centre. Find the force on the particle. Assuming $x << R$, find the time period of oscillation of the particle if it is released from there.

40. A rod of length L has a total charge Q distributed uniformly along its length. It is bent in the shape of a semicircle. Find the magnitude of the electric field at the centre of curvature of the semicircle.

41. A 10-cm long rod carries a charge of +50 μC distributed uniformly along its length. Find the magnitude of the electric field at a point 10 cm from both the ends of the rod.

42. Consider a uniformly charged ring of radius R. Find the point on the axis where the electric field is maximum.

43. A wire is bent in the form of a regular hexagon and a total charge q is distributed uniformly on it. What is the electric field at the centre ? You may answer this part without making any numerical calculations.

44. A circular wire-loop of radius a carries a total charge Q distributed uniformly over its length. A small length dL of the wire is cut off. Find the electric field at the centre due to the remaining wire.

45. A positive charge q is placed in front of a conducting solid cube at a distance d from its centre. Find the electric field at the centre of the cube due to the charges appearing on its surface.

46. A pendulum bob of mass 80 mg and carrying a charge of 2×10^{-8} C is at rest in a uniform, horizontal electric field of $20\ \mathrm{kVm^{-1}}$. Find the tension in the thread.

47. A particle of mass m and charge q is thrown at a speed u against a uniform electric field E. How much distance will it travel before coming to momentary rest ?

48. A particle of mass 1 g and charge $2{\cdot}5 \times 10^{-4}$ C is released from rest in an electric field of $1{\cdot}2 \times 10^{4}\ \mathrm{N\ C^{-1}}$. (a) Find the electric force and the force of gravity acting on this particle. Can one of these forces be neglected in comparison with the other for approximate analysis ? (b) How long will it take for the particle to travel a distance of 40 cm ? (c) What will be the speed of the particle after travelling this distance ? (d) How much is the work done by the electric force on the particle *during this period* ?

49. A ball of mass 100 g and having a charge of $4{\cdot}9 \times 10^{-5}$ C is released from rest in a region where a horizontal electric field of $2{\cdot}0 \times 10^{4}$ N C^{-1} exists. (a) Find the resultant force acting on the ball. (b) What will be the path of the ball? (c) Where will the ball be at the end of 2 s?

50. The bob of a simple pendulum has a mass of 40 g and a positive charge of $4{\cdot}0 \times 10^{-6}$ C. It makes 20 oscillations in 45 s. A vertical electric field pointing upward and of magnitude $2{\cdot}5 \times 10^{4}$ N C^{-1} is switched on. How much time will it now take to complete 20 oscillations?

51. A block of mass m having a charge q is placed on a smooth horizontal table and is connected to a wall through an unstressed spring of spring constant k as shown in figure (29-E1). A horizontal electric field E parallel to the spring is switched on. Find the amplitude of the resulting SHM of the block.

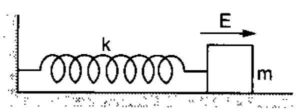

Figure 29-E1

52. A block of mass m containing a net positive charge q is placed on a smooth horizontal table which terminates in a vertical wall as shown in figure (29-E2). The distance of the block from the wall is d. A horizontal electric field E towards right is switched on. Assuming elastic collisions (if any) find the time period of the resulting oscillatory motion. Is it a simple harmonic motion?

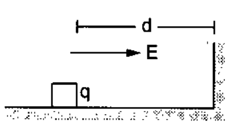

Figure 29-E2

53. A uniform electric field of 10 N C^{-1} exists in the vertically downward direction. Find the increase in the electric potential as one goes up through a height of 50 cm.

54. 12 J of work has to be done against an existing electric field to take a charge of 0·01 C from A to B. How much is the potential difference $V_B - V_A$?

55. Two equal charges, $2{\cdot}0 \times 10^{-7}$ C each, are held fixed at a separation of 20 cm. A third charge of equal magnitude is placed midway between the two charges. It is now moved to a point 20 cm from both the charges. How much work is done by the electric field during the process?

56. An electric field of 20 N C^{-1} exists along the x-axis in space. Calculate the potential difference $V_B - V_A$ where the points A and B are given by,
(a) $A = (0,\ 0)$; $B = (4\text{ m},\ 2\text{ m})$
(b) $A = (4\text{ m},\ 2\text{ m})$; $B = (6\text{ m},\ 5\text{ m})$
(c) $A = (0,\ 0)$; $B = (6\text{ m},\ 5\text{ m})$.
Do you find any relation between the answers of parts (a), (b) and (c)?

57. Consider the situation of the previous problem. A charge of $-2{\cdot}0 \times 10^{-4}$ C is moved from the point A to the point B. Find the change in electrical potential energy $U_B - U_A$ for the cases (a), (b) and (c).

58. An electric field $\vec{E} = (\vec{i}\,20 + \vec{j}\,30)$ N C^{-1} exists in the space. If the potential at the origin is taken to be zero, find the potential at (2 m, 2 m).

59. An electric field $\vec{E} = \vec{i}\,Ax$ exists in the space, where $A = 10$ V m^{-2}. Take the potential at (10 m, 20 m) to be zero. Find the potential at the origin.

60. The electric potential existing in space is $V(x,\ y,\ z) = A(xy + yz + zx)$. (a) Write the dimensional formula of A. (b) Find the expression for the electric field. (c) If A is 10 SI units, find the magnitude of the electric field at (1 m, 1 m, 1 m).

61. Two charged particles, having equal charges of $2{\cdot}0 \times 10^{-5}$ C each, are brought from infinity to within a separation of 10 cm. Find the increase in the electric potential energy during the process.

62. Some equipotential surfaces are shown in figure (29-E3). What can you say about the magnitude and the direction of the electric field?

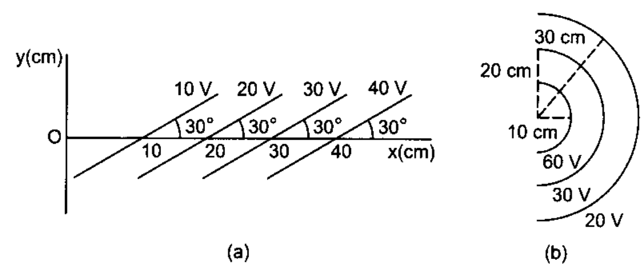

Figure 29-E3

63. Consider a circular ring of radius r, uniformly charged with linear charge density λ. Find the electric potential at a point on the axis at a distance x from the centre of the ring. Using this expression for the potential, find the electric field at this point.

64. An electric field of magnitude 1000 N C^{-1} is produced between two parallel plates having a separation of 2·0 cm as shown in figure (29-E4). (a) What is the potential difference between the plates? (b) With what minimum speed should an electron be projected from the lower plate in the direction of the field so that it may reach the upper plate? (c) Suppose the electron is projected from the lower plate with the speed calculated in part (b). The direction of projection makes an angle of 60° with the field. Find the maximum height reached by the electron.

Figure 29-E4

65. A uniform field of $2{\cdot}0\ N\ C^{-1}$ exists in space in x-direction. (a) Taking the potential at the origin to be zero, write an expression for the potential at a general point (x, y, z). (b) At which points, the potential is 25 V ? (c) If the potential at the origin is taken to be 100 V, what will be the expression for the potential at a general point ? (d) What will be the potential at the origin if the potential at infinity is taken to be zero ? Is it practical to choose the potential at infinity to be zero ?

66. How much work has to be done in assembling three charged particles at the vertices of an equilateral triangle as shown in figure (29-E5) ?

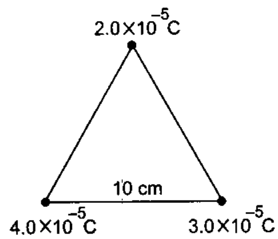

Figure 29-E5

67. The kinetic energy of a charged particle decreases by 10 J as it moves from a point at potential 100 V to a point at potential 200 V. Find the charge on the particle.

68. Two identical particles, each having a charge of $2{\cdot}0\times10^{-4}$ C and mass of 10 g, are kept at a separation of 10 cm and then released. What would be the speeds of the particles when the separation becomes large ?

69. Two particles have equal masses of 5·0 g each and opposite charges of $+4{\cdot}0\times10^{-5}$ C and $-4{\cdot}0\times10^{-5}$ C. They are released from rest with a separation of 1·0 m between them. Find the speeds of the particles when the separation is reduced to 50 cm.

70. A sample of HCl gas is placed in an electric field of $2{\cdot}5\times10^{4}\ N\ C^{-1}$. The dipole moment of each HCl molecule is $3{\cdot}4\times10^{-30}$ Cm. Find the maximum torque that can act on a molecule.

71. Two particles A and B, having opposite charges $2{\cdot}0\times10^{-6}$ C and $-2{\cdot}0\times10^{-6}$ C, are placed at a separation of 1·0 cm. (a) Write down the electric dipole moment of this pair. (b) Calculate the electric field at a point on the axis of the dipole 1·0 cm away from the centre. (c) Calculate the electric field at a point on the perpendicular bisector of the dipole and 1·0 m away from the centre.

72. Three charges are arranged on the vertices of an equilateral triangle as shown in figure (29-E6). Find the dipole moment of the combination.

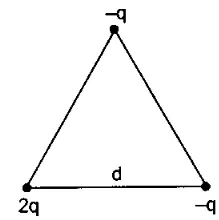

Figure 29-E6

73. Find the magnitude of the electric field at the point P in the configuration shown in figure (29-E7) for $d >> a$. Take $2qa = p$.

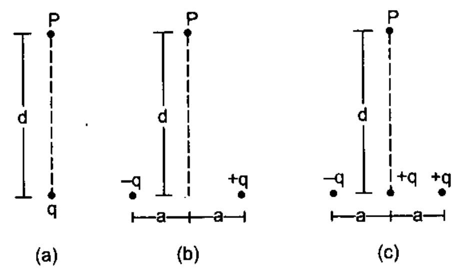

Figure 29-E7

74. Two particles, carrying charges $-q$ and $+q$ and having equal masses m each, are fixed at the ends of a light rod of length a to form a dipole. The rod is clamped at an end and is placed in a uniform electric field E with the axis of the dipole along the electric field. The rod is slightly tilted and then released. Neglecting gravity find the time period of small oscillations.

75. Assume that each atom in a copper wire contributes one free electron. Estimate the number of free electrons in a copper wire having a mass of 6·4 g (take the atomic weight of copper to be 64 g mol^{-1}).

□

ANSWERS

OBJECTIVE I

1. (c)	2. (d)	3. (a)	4. (d)	5. (a)	6. (d)
7. (a)	8. (c)	9. (a)			

OBJECTIVE II

1. (a)	2. (c), (d)	3. none
4. (b), (c)	5. (b), (d)	6. (d)
7. (b)	8. (c)	

EXERCISES

1. $I^2M^{-1}L^{-3}T^4$
2. $2{\cdot}25 \times 10^3$ N
3. $4{\cdot}3 \times 10^3$ m
4. $2{\cdot}3 \times 10^{-4}$ C
5. 230 N
6. 5·9 cm from the larger charge in between the two charges
7. 34·1 cm from the larger charge on the line joining the charge in the side of the smaller charge
8. $2{\cdot}3 \times 10^{-24}$ N
9. $3{\cdot}35 \times 10^{25}$, $5{\cdot}35 \times 10^6$ C
10. $2{\cdot}56 \times 10^{25}$ N
11. 1·2 N
12. 2×10^{11}
13. $3{\cdot}05 \times 10^{-9}$ N
14. $1{\cdot}23 \times 10^{36}$
15. (a) ML^3T^{-2}, L^{-1}, $N\,m^2$, m^{-1} (b) $3{\cdot}4 \times 10^{-26}$ N m^2
16. 24·9 N at 30° with the extended sides from the charge under consideration
17. 27·5 N at 45° with the extended sides of the square from the charge under consideration
18. $8{\cdot}2 \times 10^{-8}$ N
19. $2{\cdot}18 \times 10^6$ m s^{-1}
20. $4{\cdot}95 \times 10^5$ N
21. $3{\cdot}6 \times 10^{-6}$ N
22. (a) 0·144 N
    (b) zero, 0·095 N away from the other charge
    (c) 0·986 N and (d) 0·95 m s^{-2} perpendicular to the string and going away from the other charge
23. 8·2 g, $8{\cdot}2 \times 10^{-2}$ N
24. $4{\cdot}17 \times 10^{-8}$ C
25. $\dfrac{q^2 \cot\theta}{16\pi\varepsilon_0 g l^2 \sin^2\theta}$
26. $5{\cdot}4 \times 10^{-9}$ C
27. $-(6 - 4\sqrt{2})\,q$, between q and $2q$ at a distance of $(\sqrt{2} - 1)\,d$ from q
28. $3{\cdot}6 \times 10^{-6}$ m
29. between $\pm 8{\cdot}71 \times 10^{-8}$ C
30. 27 cm from the bottom
31. $d/2\sqrt{2}$, $3{\cdot}08 \dfrac{Qq}{4\pi\varepsilon_0 d^2}$
32. (a) $\dfrac{Qqx}{2\pi\varepsilon_0\left(x^2 + \dfrac{d^2}{4}\right)^{3/2}}$ (c) $\left[\dfrac{m\pi^3\varepsilon_0 d^3}{Qq}\right]^{\frac{1}{2}}$
33. time period $= \left[\dfrac{\pi^3\varepsilon_0 m d^3}{2Qq}\right]^{\frac{1}{2}}$
34. $1{\cdot}5 \times 10^3$ N C^{-1}
35. (a) 48·3 cm from A along BA
    (b) 20 cm from A along BA and $\dfrac{20}{3}$ cm from A along AB
36. $8{\cdot}9 \times 10^{-11}$ C
37. 65·3 N C^{-1}, upward
38. zero, $2{\cdot}3 \times 10^3$ V
39. $\left[\dfrac{16\pi^3\varepsilon_0 m R^3}{Qq}\right]^{1/2}$
40. $\dfrac{Q}{2\varepsilon_0 L^2}$
41. $5{\cdot}2 \times 10^7$ N C^{-1}
42. $R/\sqrt{2}$
43. zero
44. $\dfrac{QdL}{8\pi^2\varepsilon_0 a^3}$
45. $\dfrac{q}{4\pi\varepsilon_0 d^2}$ towards the charge q
46. $8{\cdot}8 \times 10^{-4}$ N
47. $\dfrac{mu^2}{2qE}$
48. (a) 3·0 N, $9{\cdot}8 \times 10^{-3}$ N (b) $1{\cdot}63 \times 10^{-2}$ s
    (c) 49·0 m s^{-1} (d) 1·20 J
49. (a) 1·4 N making an angle of 45° with $\vec{g}$ and $\vec{E}$
    (b) straight line along the resultant force
    (c) 28 m from the starting point on the line of motion
50. 52 s
51. qE/k
52. $\sqrt{\dfrac{8\,md}{qE}}$
53. 5 V
54. 1200 volts
55. $3{\cdot}6 \times 10^{-3}$ J
56. (a) −80 V (b) −40 V (c) −120 V
57. 0·016 J, 0·008 J, 0·024 J
58. −100 V
59. 500 V
60. (a) $MT^{-3}I^{-1}$ (b) $-A\,\{\vec{i}(y+z) + \vec{j}(z+x) + \vec{k}(x+y)\}$
    (c) 35 N C^{-1}
61. 36 J
62. (a) 200 V m^{-1} making an angle 120° with the x-axis
    (b) radially outward, decreasing with distance as $E = \dfrac{6\text{ Vm}}{r^2}$.
63. $\dfrac{r\lambda}{2\varepsilon_0(r^2 + x^2)^{1/2}}$, $\dfrac{r\lambda x}{2\varepsilon_0(r^2 + x^2)^{3/2}}$

64. (a) 20 V (b) $2{\cdot}65 \times 10^{6}$ m s^{-1} (c) 0·50 cm
65. (a) $-(2{\cdot}0 \text{ V m}^{-1})\, x$
    (b) points on the plane $x = -12{\cdot}5$ m
    (c) $100 \text{ V} - (2{\cdot}0 \text{ V m}^{-1})\, x$
    (d) infinity
66. 234 J
67. 0·1 C
68. 600 m s^{-1}
69. 54 m s^{-1} for each particle
70. $8{\cdot}5 \times 10^{-26}$ Nm
71. (a) $2{\cdot}0 \times 10^{-8}$ Cm (b) 360 N C^{-1} (c) 180 N C^{-1}
72. $qd\sqrt{3}$, along the bisector of the angle at $2q$, away from the triangle
73. (a) $\dfrac{q}{4\pi\varepsilon_0 d^2}$ (b) $\dfrac{p}{4\pi\varepsilon_0 d^3}$ (c) $\dfrac{1}{4\pi\varepsilon_0 d^3}\sqrt{q^2 d^2 + p^2}$
74. $2\pi\sqrt{\dfrac{ma}{qE}}$
75. 6×10^{22}

□