Question 13.1:

(a) Two stable isotopes of lithium ${}^{6}_{3}\mathrm{Li}$ and ${}^{7}_{3}\mathrm{Li}$ have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.01512 u and 7.01600 u, respectively. Find the atomic mass of lithium.

(b) Boron has two stable isotopes, ${}^{10}_{5}\mathrm{B}$ and ${}^{11}_{5}\mathrm{B}$. Their respective masses are 10.01294 u and 11.00931 u, and the atomic mass of boron is 10.811 u. Find the abundances of ${}^{10}_{5}\mathrm{B}$ and ${}^{11}_{5}\mathrm{B}$.

Answer

(a) Mass of lithium isotope ${}^{6}_{3}\mathrm{Li}$, m_1 = 6.01512 u

Mass of lithium isotope ${}^{7}_{3}\mathrm{Li}$, m_2 = 7.01600 u

Abundance of ${}^{6}_{3}\mathrm{Li}$, η_1 = 7.5%

Abundance of ${}^{7}_{3}\mathrm{Li}$, η_2 = 92.5%

The atomic mass of lithium atom is given as:

$$m = \frac{m_1\eta_1 + m_2\eta_2}{\eta_1 + \eta_2}$$

$$= \frac{6.01512 \times 7.5 + 7.01600 \times 92.5}{92.5 + 7.5}$$

$$= 6.940934 \text{ u}$$

(b) Mass of boron isotope ${}^{10}_{5}\mathrm{B}$, m_1 = 10.01294 u

Mass of boron isotope ${}^{11}_{5}\mathrm{B}$, m_2 = 11.00931 u

Abundance of ${}^{10}_{5}\mathrm{B}$, η_1 = x%

Abundance of ${}^{11}_{5}\mathrm{B}$, η_2 = (100 − x)%

Atomic mass of boron, m = 10.811 u

The atomic mass of boron atom is given as:

$$m = \frac{m_1\eta_1 + m_2\eta_2}{\eta_1 + \eta_2}$$

$$10.811 = \frac{10.01294 \times x + 11.00931 \times (100 - x)}{x + 100 - x}$$

$$1081.11 = 10.01294x + 1100.931 - 11.00931x$$

$$\therefore x = \frac{19.821}{0.99637} = 19.89\%$$

And 100 − x = 80.11%

Hence, the abundance of ${}^{10}_{5}\mathrm{B}$ is 19.89% and that of ${}^{11}_{5}\mathrm{B}$ is 80.11%.

Question 13.2:

The three stable isotopes of neon: ${}^{20}_{10}\mathrm{Ne}$, ${}^{21}_{10}\mathrm{Ne}$ and ${}^{22}_{10}\mathrm{Ne}$ have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u, respectively. Obtain the average atomic mass of neon.

Answer

Atomic mass of ${}^{20}_{10}\mathrm{Ne}$, m_1 = 19.99 u

Abundance of ${}^{20}_{10}\mathrm{Ne}$, η_1 = 90.51%

Atomic mass of ${}^{21}_{10}\mathrm{Ne}$, m_2 = 20.99 u

Abundance of ${}_{10}\text{Ne}$, η_2 = 0.27%

Atomic mass of ${}^{22}_{10}\text{Ne}$, m_3 = 21.99 u

Abundance of ${}^{22}_{10}\text{Ne}$, η_3 = 9.22%

The average atomic mass of neon is given as:

$$m = \frac{m_1\eta_1 + m_2\eta_2 + m_3\eta_3}{\eta_1 + \eta_2 + \eta_3}$$

$$= \frac{19.99 \times 90.51 + 20.99 \times 0.27 + 21.99 \times 9.22}{90.51 + 0.27 + 9.22}$$

$$= 20.1771 \text{ u}$$

Question 13.3:

Obtain the binding energy (in MeV) of a nitrogen nucleus $\left({}^{14}_{7}\text{N}\right)$, given $m\left({}^{14}_{7}\text{N}\right)$=14.00307 u

Answer

Atomic mass of nitrogen $\left({}_{7}\text{N}^{14}\right)$, m = 14.00307 u

A nucleus of nitrogen ${}_{7}\text{N}^{14}$ contains 7 protons and 7 neutrons.

Hence, the mass defect of this nucleus, $\Delta m = 7m_H + 7m_n - m$

Where,

Mass of a proton, m_H = 1.007825 u

Mass of a neutron, m_n= 1.008665 u

$\therefore \Delta m$ = 7 × 1.007825 + 7 × 1.008665 − 14.00307

= 7.054775 + 7.06055 − 14.00307

= 0.11236 u

But 1 u = 931.5 MeV/c^2

$\therefore \Delta m$ = 0.11236 × 931.5 MeV/c^2

Hence, the binding energy of the nucleus is given as:

$E_b = \Delta mc^2$

Where,

c = Speed of light

$\therefore E_b$ = 0.11236 × 931.5 $\left(\frac{\text{MeV}}{c^2}\right) \times c^2$

= 104.66334 MeV

Hence, the binding energy of a nitrogen nucleus is 104.66334 MeV.

Question 13.4:

Obtain the binding energy of the nuclei ${}^{56}_{26}\text{Fe}$ and ${}^{209}_{83}\text{Bi}$ in units of MeV from the following data:

$m\left({}^{56}_{26}\text{Fe}\right)$ = 55.934939 u $m\left({}^{209}_{83}\text{Bi}\right)$ = 208.980388 u

Answer

Atomic mass of ${}^{56}_{26}\text{Fe}$, m_1 = 55.934939 u

${}^{56}_{26}\text{Fe}$ nucleus has 26 protons and (56 − 26) = 30 neutrons

Hence, the mass defect of the nucleus, $\Delta m = 26 \times m_H + 30 \times m_n - m_1$

Where,

Mass of a proton, m_H = 1.007825 u

Mass of a neutron, m_n = 1.008665 u

$\therefore \Delta m$ = 26 × 1.007825 + 30 × 1.008665 − 55.934939

= 26.20345 + 30.25995 − 55.934939

= 0.528461 u

But 1 u = 931.5 MeV/c^2

But 1 u = 931.5 MeV/c^2

$\therefore \Delta m = 0.528461 \times 931.5\ \text{MeV}/c^2$

The binding energy of this nucleus is given as:

$E_{b1} = \Delta mc^2$

Where,

c = Speed of light

$\therefore E_{b1} = 0.528461 \times 931.5 \left(\frac{\text{MeV}}{c^2}\right) \times c^2$

$= 492.26\ \text{MeV}$

Average binding energy per nucleon $= \frac{492.26}{56} = 8.79\ \text{MeV}$

Atomic mass of ${}^{209}_{83}\text{Bi}$, $m_2 = 208.980388$ u

${}^{209}_{83}\text{Bi}$ nucleus has 83 protons and (209 − 83) 126 neutrons.

Hence, the mass defect of this nucleus is given as:

$\Delta m' = 83 \times m_H + 126 \times m_n - m_2$

Where,

Mass of a proton, $m_H = 1.007825$ u

Mass of a neutron, $m_n = 1.008665$ u

$\therefore \Delta m' = 83 \times 1.007825 + 126 \times 1.008665 - 208.980388$

$= 83.649475 + 127.091790 - 208.980388$

$= 1.760877$ u

But 1 u = 931.5 MeV/c^2

$\therefore \Delta m' = 1.760877 \times 931.5\ \text{MeV}/c^2$

Hence, the binding energy of this nucleus is given as:

$E_{b2} = \Delta m'c^2$

$= 1.760877 \times 931.5 \left(\frac{\text{MeV}}{c^2}\right) \times c^2$

$= 1640.26\ \text{MeV}$

Average bindingenergy per nucleon = $\frac{1640.26}{209} = 7.848\ \text{MeV}$

Question 13.5:

A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of ${}^{63}_{29}\text{Cu}$ atoms (of mass 62.92960 u).

Answer

Mass of a copper coin, $m' = 3$ g

Atomic mass of ${}_{29}\text{Cu}^{63}$ atom, $m = 62.92960$ u

The total number of ${}_{29}\text{Cu}^{63}$ atoms in the coin, $N = \frac{N_A \times m'}{\text{Mass number}}$

Where,

N_A = Avogadro's number = 6.023×10^{23} atoms /g

Mass number = 63 g

$\therefore N = \frac{6.023 \times 10^{23} \times 3}{63} = 2.868 \times 10^{22}$ atoms

${}_{29}\text{Cu}^{63}$ nucleus has 29 protons and (63 − 29) 34 neutrons

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