



Exercise 5.1

**Q4.** If a point  $C$  lies between two points  $A$  and  $B$  such that  $AC = BC$ , then prove that  $AC = \frac{1}{2} AB$ .

Explain by drawing the figure.

**Ans:** We are given that a point  $C$  lies between two points  $A$  and  $B$ , such that  $AC = BC$ .

We need to prove that  $AC = \frac{1}{2} AB$ .

Let us consider the given below figure.



We are given that  $AC = BC$ ....(i)

An axiom of the Euclid says that "If equals are added to equals, the wholes are equal."

Let us add  $AC$  to both sides of equation (i).

$$AC + AC = BC + AC.$$

An axiom of the Euclid says that "Things which coincide with one another are equal to one another."

We can conclude that  $BC + AC$  coincide with  $AB$ ,  
or

$$AB = BC + AC$$
....(ii)

An axiom of the Euclid says that "Things which are equal to the same thing are equal to one another."

From equations (i) and (ii), we can conclude that  $AC + AC = AB$ , or  $2AC = AB$ .

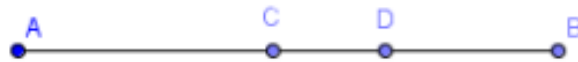
An axiom of the Euclid says that “Things which are halves of the same things are equal to one another.”

Therefore, we can conclude that  $AC = \frac{1}{2} AB$ .

**Q5.** In the above question, point C is called a mid-point of line segment AB, prove that every line segment has one and only one mid-point.

**Ans:** We need to prove that every line segment has one and only one mid-point.

Let us consider the given below line segment AB and assume that C and D are the mid-points of the line segment AB.



If C is the mid-point of line segment AB, then  $AC = CB$ .

An axiom of the Euclid says that “If equals are added to equals, the wholes are equal.”

$$AC + AC = CB + AC \dots (i)$$

From the figure, we can conclude that  $CB + AC$  will coincide with AB.

An axiom of the Euclid says that “Things which coincide with one another are equal to one another.”

$$AC + AC = AB \dots (ii)$$

An axiom of the Euclid says that “Things which are equal to the same thing are equal to one another.”

Let us compare equations (i) and (ii), to get  
 $AC + AC = AB$ , or  $2AC = AB$  .... (iii)

If  $D$  is the mid-point of line segment  $AB$ , then  
 $AD = DB$ .

An axiom of the Euclid says that “If equals are added to equals, the wholes are equal.”

$AD + AD = DB + AD$  .....(iv)

From the figure, we can conclude that  $DB + AD$  will coincide with  $AB$ .

An axiom of the Euclid says that “Things which coincide with one another are equal to one another.”

$AD + AD = AB$  .....(v)

An axiom of the Euclid says that “Things which are equal to the same thing are equal to one another.”

Let us compare equations (iv) and (v), to get

$$AD + AD = AB, \text{ or}$$

$$2AD = AB. \text{(vi)}$$

An axiom of the Euclid says that “Things which are equal to the same thing are equal to one another.”

Let us compare equations (iii) and (vi), to get

$$2AC = 2AD.$$

An axiom of the Euclid says that “Things which are halves of the same things are equal to one another.”

$$AC = AD.$$

Therefore, we can conclude that the assumption that we made previously is false and a line segment has one and only one mid-point.

**Q6.** In the following figure, if  $AC = BD$ , then prove that  $AB = CD$ .



**Ans:** We are given that  $AC = BD$ .

We need to prove that  $AB = CD$  in the figure given below.



From the figure, we can conclude that

$AC = AB + BC$ , and

$BD = CD + BC$ .

An axiom of the Euclid says that “Things which are equal to the same thing are equal to one another.”

$AB + BC = CD + BC$ . (i)

An axiom of the Euclid says that “when equals are subtracted from equals, the remainders are also equal.”

We need to subtract  $BC$  from equation (i), to get

$AB + BC - BC = CD + BC - BC$

$AB = CD$ .

Therefore, we can conclude that the desired result is proved.

**Q7.** Why is axiom 5, in the list of Euclid’s axioms, considered as a ‘universal truth’? (Note that the question is not about fifth postulate)

**Ans:** We need to prove that Euclid's fifth axiom is considered as a universal truth.

Euclid's fifth axiom states that "the whole is greater than the part."

The above given axiom is a universal truth. We can apply the fifth axiom not only mathematically but also universally in daily life.

Mathematical proof:

Let us consider a quantity  $z$ , which has different parts as  $a$ ,  $b$ ,  $x$  and  $y$ .

$$z = a + b + x + y .$$

Therefore, we can conclude that  $z$  will always be greater than its corresponding parts  $a$ ,  $b$ ,  $x$  and  $y$ .

Universal proof:

We know that Mumbai is located in Maharashtra and Maharashtra is located in India.

In other words, we can conclude that Mumbai is a part of Maharashtra and Maharashtra is a part of India.

Therefore, we can conclude that whole India will be greater than Mumbai or Maharashtra or both.

Therefore, we can conclude that Euclid's fifth axiom is considered as a 'Universal truth'.

\*\*\*\*\* END \*\*\*\*\*