Mathematical Induction Ex 12.2 Q38

Let $P(n): x^{2n-1}+y^{2n-1}$ is divisible by $(x+y)$

For $n=1$

$x^{2(1)-1}+y^{2(1)-1}$

$=x+y$

$\Rightarrow \quad P(n)$ is true for $n=1$

Let $P(n)$ is true for $n=k$,

$x^{2k-1}+y^{2k-1}$ is divisible by $(x+y)$

$$x^{2k-1}+y^{2k-1}=(x+y)\lambda \qquad ---(1)$$

We have to show that,

$$x^{2k+1}+y^{2k+1}=(x+y)\mu$$

Now,

$x^{2k+1}+y^{2k+1}$

$$=x^{2k-1}.x^2+y^{2k-1}.y^2$$
$$=\left[(x+y)\lambda-y^{2k-1}\right]x^2+y^{2k-1}.y^2$$
$$=(x+y)\lambda x^2-y^{2k-1}.x^2+y^{2k-1}.y^2$$
$$=(x+y)\lambda x^2-y^{2k-1}\left(x^2-y^2\right)$$
$$=(x+y)\lambda x^2-y^{2k-1}(x+y)(x-y)$$
$$=(x+y)\left[\lambda x^2-y^{2k-1}(x-y)\right]$$

$$=(x+y)\mu$$

$\Rightarrow \quad P(n)$ is true for $n=k+1$

$\Rightarrow \quad P(n)$ is true for all $n\in N$ by PMI

Mathematical Induction Ex 12.2 Q39

Let $P(n): \sin x + \sin 3x + \ldots + \sin(2n-1)x = \frac{\sin^2 nx}{\sin x}$

For $n = 1$

$$\sin x = \frac{\sin^2 x}{\sin x}$$

$$\sin x = \sin x$$

$\Rightarrow \quad P(n)$ is true for $n = 1$

Let $P(n)$ is true for $n = k$, so

$$\sin x + \sin 3x + \ldots + \sin(2k-1)x = \frac{\sin^2 kx}{\sin x} \qquad \text{---(i)}$$

We have to show that

$$\sin x + \sin 3x + \ldots + \sin(2k-1)x + \sin(2k+1)x = \frac{\sin^2 (k+1)x}{\sin x}$$

Now,

$$\left\{\sin x + \sin 3x + \ldots + \sin(2k-1)x\right\} + \sin(2k+1)x$$

$$= \frac{\sin^2 kx}{\sin x} + \frac{\sin(2k+1)x}{1}$$

Using equation (i),

$$= \frac{\sin^2 kx + \sin(2k+1)x \sin x}{\sin x}$$

$$= \frac{2\sin^2 kx + \cos\left[(2k+1)x - x\right] - \cos\left[2kx + x + x\right]}{2\sin x}$$

$$= \frac{2\sin^2 kx + \cos 2kx - \cos(2kx + 2x)}{2\sin x}$$

$$= \frac{1 - \cos 2kx + \cos 2kx - \cos 2x(k+1)}{2\sin x}$$

$$= \frac{1 - \cos 2x(k+1)}{2\sin x}$$

$$= \frac{2\sin^2 x(k+1)}{2\sin x}$$

$$= \frac{\sin^2 x(k+1)}{\sin x}$$

$\Rightarrow \quad P(n)$ is true for $n = k+1$

$\Rightarrow \quad P(n)$ is true for all $n \in N$ by PMI.

Mathematical Induction Ex 12.2 Q40

Let P(n) be the statement given by

$$P(n): \cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \cdots\cdots + \cos(\alpha+(n-1)\beta)$$

$$= \frac{\cos\left\{\alpha+\left(\frac{n-1}{2}\right)\beta\right\}\sin\left(\frac{n\beta}{2}\right)}{\sin\frac{\beta}{2}} \text{ for all } n \in N.$$

Step I:

$$P(1): \cos\alpha = \frac{\cos\left\{\alpha+\left(\frac{1-1}{2}\right)\beta\right\}\sin\left(\frac{1\beta}{2}\right)}{\sin\frac{\beta}{2}}$$

$$\Rightarrow \cos\alpha = \frac{\cos\{\alpha+0\}\sin\left(\frac{\beta}{2}\right)}{\sin\frac{\beta}{2}}$$

$$\Rightarrow \cos\alpha = \cos\alpha$$

$\therefore$ P(1) is true.

Step II:

Let P(m) is true. Then,

$$\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \cdots\cdots + \cos(\alpha+(m-1)\beta)$$

$$= \frac{\cos\left\{\alpha+\left(\frac{m-1}{2}\right)\beta\right\}\sin\left(\frac{m\beta}{2}\right)}{\sin\frac{\beta}{2}}$$

We have to prove that P(m + 1) is true.

$$\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \cdots\cdots + \cos(\alpha+(m)\beta)$$

$$= \cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \cdots\cdots + \cos(\alpha+(m-1)\beta) + \cos(\alpha+(m)\beta)$$

$$= \frac{\cos\left\{\alpha+\left(\frac{m-1}{2}\right)\beta\right\}\sin\left(\frac{m\beta}{2}\right)}{\sin\frac{\beta}{2}} + \cos(\alpha+(m)\beta) \ldots\ldots\ldots\ldots\ldots\ldots\ldots [\text{Using (i)}]$$

$$= \frac{\sin\frac{\beta}{2}\cos\ \left(\alpha+(m)\beta\right)+\cos\left\{\alpha+\left(\frac{m-1}{2}\right)\beta\right\}\sin\left(\frac{m\beta}{2}\right)}{\sin\frac{\beta}{2}}$$

$$= \frac{\frac{1}{2}\left[\sin\left(\alpha+\left(\frac{2m+1}{2}\right)\beta\right)-\sin\left(\alpha+\left(\frac{2m-1}{2}\right)\beta\right)\right]+\cos\left\{\alpha+\left(\frac{m-1}{2}\right)\beta\right\}\sin\left(\frac{m\beta}{2}\right)}{\sin\frac{\beta}{2}}$$

$$= \frac{\frac{1}{2}\left[\sin\left(\alpha+\left(\frac{2m+1}{2}\right)\beta\right)-\sin\left(\alpha+\left(\frac{2m-1}{2}\right)\beta\right)\right]+\frac{1}{2}\left[\sin\left(\alpha+\left(\frac{2m-1}{2}\right)\beta\right)+\sin\left(-\alpha+\frac{\beta}{2}\right)\right]}{\sin\frac{\beta}{2}}$$

$$= \frac{\frac{1}{2}\left[\sin\left(\alpha+\left(\frac{2m+1}{2}\right)\beta\right)\right]+\frac{1}{2}\left[\sin\left(-\alpha+\frac{\beta}{2}\right)\right]}{\sin\frac{\beta}{2}}$$

$$= \frac{\frac{1}{2}\left[\sin\alpha\cos\left(\frac{2m+1}{2}\right)\beta+\cos\alpha\sin\left(\frac{2m+1}{2}\right)\beta+\sin\frac{\beta}{2}\cos\alpha-\cos\frac{\beta}{2}\sin\alpha\right]}{\sin\frac{\beta}{2}}$$

$$= \frac{\frac{1}{2}\left[\sin\alpha\left(\cos\left(\frac{2m+1}{2}\right)\beta-\cos\frac{\beta}{2}\right)+\cos\alpha\left(\sin\left(\frac{2m+1}{2}\right)\beta+\sin\frac{\beta}{2}\right)\right]}{\sin\frac{\beta}{2}}$$

$$= \frac{\frac{1}{2}\left[-2\sin\alpha\left(\left(\sin\left(\frac{m+1}{2}\right)\beta\right)\sin\frac{m\beta}{2}\right)+2\cos\alpha\left(\left(\sin\left(\frac{m+1}{2}\right)\beta\right)\cos\frac{m\beta}{2}\right)\right]}{\sin\frac{\beta}{2}}$$

$$= \frac{\sin\left(\frac{(m+1)\beta}{2}\right)\left[\cos\alpha\cos\frac{m\beta}{2}-\sin\alpha\sin\left(\frac{(m+1)\beta}{2}\right)\right]}{\sin\frac{\beta}{2}}$$

$$= \frac{\sin\left(\frac{(m+1)\beta}{2}\right)\cos\left(\alpha+\frac{m\beta}{2}\right)}{\sin\frac{\beta}{2}}$$

$\Rightarrow P(m+1)$ is true.

Hence by the principle of mathematical induction, the given result is true for all $n \in N$.

*********** END **********