



Trigonometric Ratios Ex 5.2 Q29

Answer :

Given:

$$\sin(A - B) = \sin A \cos B - \cos A \sin B \dots\dots (1)$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B \dots\dots (2)$$

To find:

The values of $\sin 15^\circ$ and $\cos 15^\circ$

In this problem we need to find $\sin 15^\circ$ and $\cos 15^\circ$

Hence to get 15° angle we need to choose the value of A and B such that $(A - B) = 15^\circ$

So If we choose $A = 45^\circ$ and $B = 30^\circ$

Then we get, $(A - B) = 15^\circ$

Therefore by substituting $A = 45^\circ$ and $B = 30^\circ$ in equation (1)

We get,

$$\sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$$

Therefore,

$$\sin(15^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ \dots\dots (3)$$

Now we know that,

$$\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}, \sin 30^\circ = \frac{1}{2}, \cos 30^\circ = \frac{\sqrt{3}}{2}$$

Now by substituting above values in equation (3)

We get,

$$\begin{aligned}\sin(15^\circ) &= \left(\frac{1}{\sqrt{2}}\right) \times \left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right) \times \left(\frac{1}{2}\right) \\ &= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} \\ &= \frac{\sqrt{3}-1}{2\sqrt{2}}\end{aligned}$$

Therefore,

$$\sin(15^\circ) = \frac{\sqrt{3}-1}{2\sqrt{2}} \dots\dots (4)$$

Now by substituting $A = 45^\circ$ and $B = 30^\circ$ in equation (2)

We get,

$$\cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$

Therefore,

$$\cos(15^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ \dots\dots (5)$$

Now we know that,

$$\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}, \sin 30^\circ = \frac{1}{2}, \cos 30^\circ = \frac{\sqrt{3}}{2}$$

Now by substituting above values in equation (5)

We get,

$$\begin{aligned}\cos(15^\circ) &= \left(\frac{1}{\sqrt{2}}\right) \times \left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right) \times \left(\frac{1}{2}\right) \\ &= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} \\ &= \frac{\sqrt{3}+1}{2\sqrt{2}}\end{aligned}$$

Therefore,

$$\cos(15^\circ) = \frac{\sqrt{3}+1}{2\sqrt{2}} \dots\dots (6)$$

Therefore from equation (4) and (6)

$$\sin(15^\circ) = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\cos(15^\circ) = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

***** END *****