Sine and Cosine Formulae and their Applications Ex-10.2 Q5

$$b(c\cos A - a\cos C) = c^2 - a^2$$

$$RHS$$

$$= c^2 - a^2$$

$$= k^2\sin^2 C - k^2\sin^2 A$$

$$= k^2(\sin^2 C - \sin^2 A)$$

$$= k^2\sin(C+A).\sin(C-A)$$

$$= k^2\sin(\pi - B).\sin(C-A)$$

$$= k^2\sin B.\sin(C-A)$$

$$= k\sin B.k\sin(C-A)$$

$$= bk\sin(C-A)$$

$$= bk(\sin C.\cos A - \sin A.\cos C)$$

$$= b(k\sin C.\cos A - k\sin A.\cos C)$$

$$= b(c\cos A - a\cos C) = LHS$$

Sine and Cosine Formulae and their Applications Ex-10.2 Q6

$$c(a\cos B - b\cos A)$$

$$= ac.\cos B - bc\cos A$$

$$= ac.\frac{a^2+c^2-b^2}{2ac} - bc.\frac{b^2+c^2-a^2}{2bc}$$

$$= \frac{a^2+c^2-b^2}{2} - \frac{b^2+c^2-a^2}{2}$$

$$= \frac{a^2+c^2-b^2-b^2-c^2+a^2}{2}$$

$$= \frac{2a^2-2b^2}{2} = \left(a^2-b^2\right) = RHS$$

Sine and Cosine Formulae and their Applications Ex-10.2 Q7

$$2(bc\cos A + ca\cos B + ab\cos C) = a^2 + b^2 + c^2$$

$$LHS$$

$$= 2bc\cos A + 2ca\cos B + 2ab\cos C$$

$$= 2bc\frac{b^2+c^2-a^2}{2bc} + 2ca\frac{a^2+c^2-b^2}{2ca} + 2ab\frac{a^2+b^2-c^2}{2ab}$$

$$= b^2+c^2-a^2+a^2+c^2-b^2+a^2+b^2-c^2$$

$$= a^2+b^2+c^2 = RHS$$

Sine and Cosine Formulae and their Applications Ex-10.2 Q8

For any $\triangle ABC$, we have

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

therefore,

$$\begin{aligned}\left(c^2+b^2-a^2\right)\tan A &= \left(c^2+b^2-a^2\right)\frac{\sin A}{\cos A}\\ &= \left(c^2+b^2-a^2\right)\frac{ka}{\frac{b^2+c^2-a^2}{2bc}}\\ &= 2kabc\end{aligned}$$

Also,

$$\begin{aligned}\left(a^2+c^2-b^2\right)\tan B &= \left(a^2+c^2-b^2\right)\frac{\sin B}{\cos B}\\ &= \left(a^2+c^2-b^2\right)\frac{kb}{\frac{a^2+c^2-b^2}{2ac}}\\ &= 2kabc\end{aligned}$$

Now,

$$\begin{aligned}\left(a^2+b^2-c^2\right)\tan C &= \left(a^2+b^2-c^2\right)\frac{\sin C}{\cos C}\\ &= \left(a^2+b^2-c^2\right)\frac{kc}{\frac{a^2+b^2-c^2}{2ab}}\\ &= 2kabc\end{aligned}$$

Hence proved.

Sine and Cosine Formulae and their Applications Ex-10.2 Q9

$$\frac{c-b\cos A}{b-c\cos A} = \frac{\cos B}{\cos C}$$

LHS

$$\begin{aligned}&= \frac{c-b\cos A}{b-c\cos A}\\ &= \frac{k\sin C - k\sin B\cos A}{k\sin B - k\sin C\cos A}\\ &= \frac{\sin(\pi-(A+B)) - \sin B\cos A}{\sin(\pi-(A+C)) - \sin C\cos A}\\ &= \frac{\sin(A+B) - \sin B\cos A}{\sin(A+C) - \sin C\cos A}\\ &= \frac{\sin A\cos B + \cos A\sin B - \sin B\cos A}{\sin A\cos C + \cos A\sin C - \sin C\cos A}\\ &= \frac{\sin A\cos B}{\sin A\cos C}\\ &= \frac{\cos B}{\cos C} = RHS\end{aligned}$$

********** END **********