Exercise 1E

Question 15:

For rationalising the denominator of a number, we multiply its numerator and denominator by its rationalising factor.

If a and b are integers, then $(a+\sqrt{b})$ and $(a-\sqrt{b})$ are rationalising factor of each other, as $(a+\sqrt{b})(a-\sqrt{b}) = (a^2 - b)$, which is rational.

Let us rationalise the denominator of the first term on the Left hand side.

We have,

$$\frac{4+\sqrt{5}}{4-\sqrt{5}} = \frac{4+\sqrt{5}}{4-\sqrt{5}} \times \frac{4+\sqrt{5}}{4+\sqrt{5}}$$

$$= \frac{(4+\sqrt{5})^2}{(4)^2 - (\sqrt{5})^2}$$

$$= \frac{(4)^2 + 2(4)(\sqrt{5}) + (\sqrt{5})^2}{16-5}$$

$$\frac{4+\sqrt{5}}{4-\sqrt{5}} = \frac{16+8\sqrt{5}+5}{11} = \frac{21+8\sqrt{5}}{11} \ldots\ldots(1)$$

Now consider the denominator of the second term on the left hand side:

$$\frac{4-\sqrt{5}}{4+\sqrt{5}} = \frac{4-\sqrt{5}}{4+\sqrt{5}} \times \frac{4-\sqrt{5}}{4-\sqrt{5}}$$

$$= \frac{(4-\sqrt{5})^2}{(4)^2 - (\sqrt{5})^2}$$

$$= \frac{(4)^2 - 2(4)(\sqrt{5}) + (\sqrt{5})^2}{16-5}$$

$$\frac{4-\sqrt{5}}{4+\sqrt{5}} = \frac{16-8\sqrt{5}+5}{11} = \frac{21-8\sqrt{5}}{11} \ldots\ldots(2)$$

Adding equations (1) and (2), we have,

$$\therefore \frac{4+\sqrt{5}}{4-\sqrt{5}} + \frac{4-\sqrt{5}}{4+\sqrt{5}} = \frac{21+8\sqrt{5}}{11} + \frac{21-8\sqrt{5}}{11}$$

$$= \frac{21+8\sqrt{5}+21-8\sqrt{5}}{11} = \frac{42}{11}.$$

********** END **********