

Mean Value Theorems Ex 15.2 Q1(v) Here,

$$f(x) = 2x^2 - 3x + 1$$
 on [1,3]

We know that, polynomial function is continuous and differentiable. So, f(x) is continuous in [1,3] and f(x) is differentiable in (1,3). So, Lagrange's mean value theorem is applicable, so there exist a point $c \in (1,3)$ such that

$$f'(c) = \frac{f(3) - f(1)}{3 - 1}$$

$$\Rightarrow 4c - 3 = \frac{\left(2(3)^2 - 3(3) + 1\right) - (2 - 3 + 1)}{3 - 1}$$

$$\Rightarrow 4c - 3 = \frac{10}{2}$$

$$\Rightarrow 4c = 5 + 3$$

$$\Rightarrow 4c = 8$$

$$\Rightarrow c = 2 \in (1, 3)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(vi)

$$f(x) = x^2 - 2x + 4$$
 on [1,5]

We know that, polynomial is always continuous and differentiable. So, f(x) is continuous in [1,5] and it is differentiable in (1,5). So, Lagrange's mean value theorem is applicable. Thus, there exists a point $c \in (1,5)$ such that

$$f'(c) = \frac{f(5) - f(1)}{5 - 1}$$

$$\Rightarrow 2c - 2 = \frac{\left((5)^2 - 2(5) + 4\right) - (1 - 2 + 4)}{4}$$

$$\Rightarrow 2c - 2 = \frac{19 - 3}{4}$$

$$\Rightarrow 2c - 2 = 4$$

$$\Rightarrow 2c = 6$$

$$\Rightarrow c = 3 \in (1, 5)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(vii) Here,

$$f(x) = 2x - x^2$$
 on $[0,1]$

We know that, polynomial is continuous and differentiable. So, f(x) is continuous in [0,1] and differentiable in (0,1). So, Lagrange's mean value theorem is applicable. Thus, there exists a point $c \in (0,1)$ such that

$$f'(c) = \frac{f(1) - f(0)}{1 - 0}$$

$$\Rightarrow 2 - 2c = \frac{\left(2(1) - (1)^2\right) - (0)}{1}$$

$$\Rightarrow 2 - 2c = 1$$

$$\Rightarrow 1 = 2c$$

$$\Rightarrow c = \frac{1}{2} \in (0, 1)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(viii)

$$f(x) = (x-1)(x-2)(x-3)$$
 on $[0,4]$

We know that, polynomial is continuous and differentiable every where. So, f(x) is continuous in [0,4] and differentiable in (0,4). So, Lagrange's mean value theorem is applicable. Thus, there exists a point $c \in (0,4)$ such that

$$f'(c) = \frac{f(4) - f(0)}{4 - 0}$$

$$\Rightarrow (c - 1)(c - 2) + (c - 2)(c - 3) + (c - 1)(c - 3) = \frac{(3)(2)(1) - (-1)(-2)(-3)}{4 - 0}$$

$$\Rightarrow c^2 - 3c + 2 + c^2 + 5c + 6 + c^2 - 4c + 3 = \frac{6 + 6}{4}$$

$$\Rightarrow 3c^2 - 12c + 11 = 3$$

$$\Rightarrow 3c^2 = 12c + 8 = 0$$

$$\Rightarrow c = \frac{-(-12) \pm \sqrt{144 - 4 \times 3 \times 8}}{6}$$

$$\Rightarrow c = \frac{12 \pm \sqrt{48}}{6}$$

$$\Rightarrow c = 2 \pm \frac{2\sqrt{3}}{3} \in (0, 4)$$

$$\Rightarrow c = 2 \pm \frac{2}{\sqrt{3}} \in (0, 4)$$

Hence, Lagrange's mean value theorem is verified.

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