Exercise 12.1

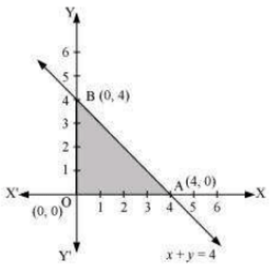
Question 1:

Maximise Z = 3*x* + 4*y*

Subject to the constraints: $x + y \le 4, x \ge 0, y \ge 0.$

Answer

The feasible region determined by the constraints, $x + y \le 4$, $x \ge 0$, $y \ge 0$, is as follows.



The corner points of the feasible region are O (0, 0), A (4, 0), and B (0, 4). The values of Z at these points are as follows.

Corner point	Z = 3x + 4y	
O(0, 0)	0	
A(4, 0)	12	
B(0, 4)	16	→ Maximum

Therefore, the maximum value of Z is 16 at the point B (0, 4).

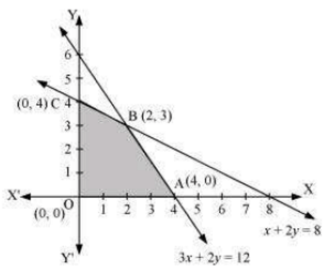
Question 2:

Minimise Z = −3*x* + 4*y*

subject to $x + 2y \le 8,\ 3x + 2y \le 12,\ x \ge 0,\ y \ge 0$.

Answer

The feasible region determined by the system of constraints, $x + 2y \le 8,\ 3x + 2y \le 12,$ $x \ge 0$, and $y \ge 0$, is as follows.



The corner points of the feasible region are O (0, 0), A (4, 0), B (2, 3), and C (0, 4). The values of Z at these corner points are as follows.

Corner point	Z = −3x + 4y	
0(0, 0)	0	
A(4, 0)	−12	→ Minimum
B(2, 3)	6	
C(0, 4)	16	

Therefore, the minimum value of Z is −12 at the point (4, 0).

Question 3:

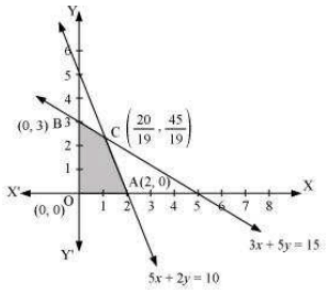
Maximise Z = 5*x* + 3*y*

subject to $3x + 5y \le 15,\ 5x + 2y \le 10,\ x \ge 0,\ y \ge 0$.

Answer

The feasible region determined by the system of constraints, $3x + 5y \le 15$,

$5x + 2y \le 10$, $x \ge 0$, and $y \ge 0$, are as follows.



The corner points of the feasible region are O (0, 0), A (2, 0), B (0, 3), and $C\left(\frac{20}{19}, \frac{45}{19}\right)$.

The values of Z at these corner points are as follows.

Corner point	Z = 5*x* + 3*y*	
O(0, 0)	0	
A(2, 0)	10	
B(0, 3)	9	
$C\left(\frac{20}{19}, \frac{45}{19}\right)$	$\frac{235}{19}$	→ Maximum

Therefore, the maximum value of Z is $\frac{235}{19}$ at the point $\left(\frac{20}{19}, \frac{45}{19}\right)$.

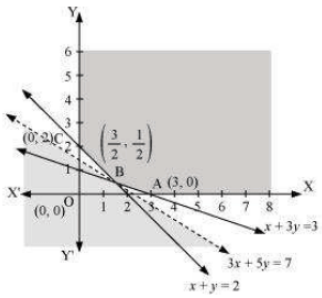
Question 4:

Minimise Z = 3*x* + 5*y*

such that $x + 3y \ge 3,\ x + y \ge 2,\ x, y \ge 0$.

Answer

The feasible region determined by the system of constraints, $x + 3y \ge 3, x + y \ge 2$, and *x*, $y \ge 0$, is as follows.



It can be seen that the feasible region is unbounded.

The corner points of the feasible region are A (3, 0), $B\left(\frac{3}{2}, \frac{1}{2}\right)$, and C (0, 2).

The values of Z at these corner points are as follows.

Corner point	Z = 3*x* + 5*y*	
A(3, 0)	9	
$B\left(\frac{3}{2}, \frac{1}{2}\right)$	7	→ Smallest
C(0, 2)	10	

As the feasible region is unbounded, therefore, 7 may or may not be the minimum value of Z.

For this, we draw the graph of the inequality, $3x + 5y < 7$, and check whether the resulting half plane has points in common with the feasible region or not.

It can be seen that the feasible region has no common point with $3x + 5y < 7$ Therefore, the minimum value of Z is 7 at $\left(\frac{3}{2}, \frac{1}{2}\right)$.

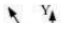
Question 5:

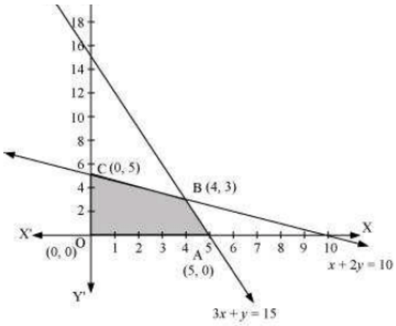
Maximise Z = 3*x* + 2*y*

subject to $x + 2y \le 10, 3x + y \le 15, x, y \ge 0$.

Answer

The feasible region determined by the constraints, $x + 2y \le 10$, $3x + y \le 15$, $x \ge 0$, and $y \ge 0$, is as follows.





The corner points of the feasible region are A (5, 0), B (4, 3), and C (0, 5).

The values of Z at these corner points are as follows.

Corner point	**Z = 3*x* + 2*y***	
A(5, 0)	15	
B(4, 3)	18	→ Maximum
C(0, 5)	10	

Therefore, the maximum value of Z is 18 at the point (4, 3).

Question 6:

Minimise Z = $x + 2y$

subject to $2x + y \geq 3,\ x + 2y \geq 6,\ x, y \geq 0$.

Answer

*********** END **********