Differentiation Ex 11.3 Q11

Let $\quad y=\cos^{-1}\left\{\frac{\cos x+\sin x}{\sqrt{2}}\right\}$

$$y=\cos^{-1}\left\{\cos x\left(\frac{1}{\sqrt{2}}\right)+\sin x\left(\frac{1}{\sqrt{2}}\right)\right\}$$

$$=\cos^{-1}\left\{\cos x\cos\left(\frac{\pi}{4}\right)+\sin x\sin x\left(\frac{\pi}{4}\right)\right\}$$

$$y=\cos^{-1}\left[\cos\left(x-\frac{\pi}{4}\right)\right] \qquad ---(i)$$

Here, $\quad -\frac{\pi}{4}<x<\frac{\pi}{4}$

$\Rightarrow \quad \left(-\frac{\pi}{4}-\frac{\pi}{4}\right)<\left(x-\frac{\pi}{4}\right)<\left(\frac{\pi}{4}-\frac{\pi}{4}\right)$

$\Rightarrow \quad -\frac{\pi}{2}<\left(x-\frac{\pi}{4}\right)<0$

So, from equation (i),

$$y=-\left(x-\frac{\pi}{4}\right) \qquad \left[\text{Since, } \cos^{-1}(\cos\theta)=-\theta, \text{ if } \theta\in[-\pi,0]\right]$$

$$y=-x+\frac{\pi}{4}$$

Differentiating it with respect to x,

$$\frac{dy}{dx}=-1.$$

Differentiation Ex 11.3 Q12

Let $\quad y=\tan^{-1}\left\{\frac{x}{1+\sqrt{1-x^2}}\right\}$

Put $\quad x=\sin\theta$, so

$$y=\tan^{-1}\left\{\frac{\sin\theta}{1+\sqrt{1-\sin^2\theta}}\right\}$$

$$=\tan^{-1}\left\{\frac{\sin\theta}{1+\cos\theta}\right\}$$

$$=\tan^{-1}\left\{\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}\right\}$$

$$y=\tan^{-1}\left\{\tan\frac{\theta}{2}\right\} \qquad ---(i)$$

Here, $\quad -1<x<1$

$\Rightarrow \quad -1<\sin\theta<1$

$\Rightarrow \quad -\frac{\pi}{2}<\theta<\frac{\pi}{2}$

$\Rightarrow \quad -\frac{\pi}{4}<\frac{\theta}{2}<\frac{\pi}{4}$

So, from equation (i),

$$y=\frac{\theta}{2} \qquad \left[\text{Since, } \tan^{-1}(\tan\theta)=\theta, \text{ if } \theta\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\right]$$

$$y=\frac{1}{2}\sin^{-1}x \qquad [\text{Since, } x=\sin\theta]$$

Differentiating it with respect to x,

$$\frac{dy}{dx}=\frac{1}{2\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q13

Let $y = \tan^{-1}\left\{\frac{x}{a+\sqrt{a^2-x^2}}\right\}$

Put $x = a\sin\theta$, so

$$y = \tan^{-1}\left\{\frac{a\sin\theta}{a+\sqrt{a^2-a^2\sin^2\theta}}\right\}$$

$$= \tan^{-1}\left\{\frac{a\sin\theta}{a+\sqrt{a^2\left(1-\sin^2\theta\right)}}\right\}$$

$$= \tan^{-1}\left\{\frac{a\sin\theta}{a+a\cos\theta}\right\}$$

$$= \tan^{-1}\left\{\frac{a\sin\theta}{a\left(1+\cos\theta\right)}\right\}$$

$$= \tan^{-1}\left(\frac{\sin\theta}{1+\cos\theta}\right)$$

$$= \tan^{-1}\left(\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}\right)$$

$$y = \tan^{-1}\left(\tan\frac{\theta}{2}\right) \qquad \text{---(i)}$$

Here, $-a < x < a$

$\Rightarrow \quad -1 < \frac{x}{a} < 1$

$\Rightarrow \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\Rightarrow \quad -\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{4}$

So, from equation (i),

$$y = \frac{\theta}{2} \qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$y = \frac{1}{2} + \sin^{-1}\left(\frac{x}{a}\right) \qquad \left[\text{Since, } x = a\sin\theta\right]$$

Differentiating it with respect to x using chain rule,

$$\frac{dy}{dx} = \frac{1}{2}\times\frac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}\frac{d}{dx}\left(\frac{x}{a}\right)$$

$$= \frac{a}{2\sqrt{a^2-x^2}}\times\left(\frac{1}{a}\right)$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{a^2-x^2}}.$$

Differentiation Ex 11.3 Q14

Let $\quad y = \sin^{-1}\left\{\frac{x+\sqrt{1-x^2}}{\sqrt{2}}\right\}$

Put $\quad x = \sin\theta$, so

$$= \sin^{-1}\left\{\frac{\sin\theta+\sqrt{1-\sin^2\theta}}{\sqrt{2}}\right\}$$

$$= \sin^{-1}\left\{\frac{\sin\theta+\cos\theta}{\sqrt{2}}\right\}$$

$$= \sin^{-1}\left\{\sin\theta\left(\frac{1}{\sqrt{2}}\right)+\cos\theta\left(\frac{1}{\sqrt{2}}\right)\right\}$$

$$= \sin^{-1}\left\{\sin\theta\cos\frac{\pi}{4}+\cos\theta\sin\frac{\pi}{4}\right\}$$

$$y = \sin^{-1}\left\{\sin\left(\theta+\frac{\pi}{4}\right)\right\} \qquad \text{---(i)}$$

Here, $\quad -1 < x < 1$

$\Rightarrow \quad -1 < \sin\theta < 1$

$\Rightarrow \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\Rightarrow \quad \left(-\frac{\pi}{2}+\frac{\pi}{4}\right) < \left(\frac{\pi}{4}+\theta\right) < \frac{3\pi}{4}$

So, from equation (i),

$$y = \theta+\frac{\pi}{4} \qquad \left[\text{Since, } \sin^{-1}(\sin\theta)=\theta, \text{ as } \theta\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\right]$$

$$y = \sin^{-1}x+\frac{\pi}{4} \qquad [\text{Since, } \sin\theta = x]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}+0$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}.$$

Differentiation Ex 11.3 Q15

Let $\quad y = \cos^{-1}\left\{\frac{x+\sqrt{1-x^2}}{\sqrt{2}}\right\}$

Put $\quad x = \sin\theta$, so

$$y = \cos^{-1}\left\{\frac{\sin\theta+\sqrt{1-\sin^2\theta}}{\sqrt{2}}\right\}$$

$$= \cos^{-1}\left\{\frac{\sin\theta+\cos\theta}{\sqrt{2}}\right\}$$

$$= \cos^{-1}\left\{\sin\theta\left(\frac{1}{\sqrt{2}}\right)+\cos\theta\left(\frac{1}{\sqrt{2}}\right)\right\}$$

$$= \cos^{-1}\left\{\sin\theta\times\sin\frac{\pi}{4}+\cos\theta\times\cos\frac{\pi}{4}\right\}$$

$$y = \cos^{-1}\left\{\cos\left(\theta-\frac{\pi}{4}\right)\right\} \qquad \text{---(i)}$$

Here, $\quad -1 < x < 1$

$\Rightarrow \quad -1 < \sin\theta < 1$

$\Rightarrow \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\Rightarrow \quad -\frac{\pi}{2}+\frac{\pi}{4} < \left(\theta-\frac{\pi}{4}\right) < \frac{\pi}{2}-\frac{\pi}{4}$

$\Rightarrow \quad \left(-\frac{3\pi}{4}\right) < \left(\theta-\frac{\pi}{4}\right) < \left(\frac{\pi}{4}\right)$

So, from equation (i),

$$y = -\left(\theta-\frac{\pi}{4}\right) \qquad [\text{Since, } \cos^{-1}(\cos\theta) = -\theta, \text{ if } \theta\in[-\pi,0]]$$

$$y = -\theta+\frac{\pi}{4}$$

$$y = -\sin^{-1}x+\frac{\pi}{4} \qquad [\text{Since, } x = \sin\theta]$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}+0$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}.$$

*********** END **********