Indefinite Integrals Ex 19.25 Q22

Let $I = \int e^{\sqrt{x}}\, dx$

Let $\sqrt{x} = t$

$x = t^2$

$dx = 2t\,dt$

$I = 2\int e^t t\,dt$

$I = 2\left[t\int e^t dt - \int\left(1\int e^t dt\right)dt\right]$

$I = 2\left[te^t - \int e^t dt\right]$

$= 2\left[te^t - e^t\right] + c$

$= 2e^t\left(t - 1\right) + c$

$I = 2e^{\sqrt{x}}\left(\sqrt{x} - 1\right) + c$

Indefinite Integrals Ex 19.25 Q23

Let $I = \int \frac{\log\left(x+2\right)}{\left(x+2\right)^2}dx$

Let $\frac{1}{x+2} = t$

$-\frac{1}{\left(x+2\right)^2}dx = dt$

$I = -\int \log\left(\frac{1}{t}\right)dt$

$= -\int \log t^{-1}\, dt$

$= -\int 1 \times \log t\, dt$

Using integration by parts,

$I = \log t \int dt - \int\left(\frac{1}{t}\int dt\right)dt$

$= t\log t - \int\left(\frac{1}{t} \times t\right)dt$

$= t\log t - \int dt$

$= t\log t - t + c$

$= \frac{1}{x+2}\left(\log\left(x+2\right)^{-1} - 1\right) + c$

$I = \frac{-1}{x+2} - \frac{\log\left(x+2\right)}{x+2} + c$

Indefinite Integrals Ex 19.25 Q24

Let $\quad I = \int \frac{x + \sin x}{1 + \cos x} dx$

$= \int \frac{x}{2\cos^2 \frac{x}{2}} dx + \int \frac{2\sin \frac{x}{2} \cos \frac{x}{2}}{2\cos^2 \frac{x}{2}} dx$

$= \frac{1}{2} \int x \sec^2 \frac{x}{2} dx + \int \tan \frac{x}{2} dx$

Using integration by parts,

$= \frac{1}{2}\left[x \int \sec^2 \frac{x}{2} dx - \int \left(1 \int \sec^2 \frac{x}{2} dx\right) dx\right] + \int \tan \frac{x}{2} dx$

$= \frac{1}{2}\left[2x \tan \frac{x}{2} - 2\int \tan \frac{x}{2} dx\right] + \int \tan \frac{x}{2} dx + c$

$= x \tan \frac{x}{2} - \int \tan \frac{x}{2} dx + \int \tan \frac{x}{2} dx + c$

$I = x \tan \frac{x}{2} + c$

Indefinite Integrals Ex 19.25 Q25

Let $\quad I = \int \log_{10} x \, dx$

$= \int \frac{\log x}{\log 10} dx$

$= \frac{1}{\log 10} \int 1 \times \log x \, dx$

Using integration by parts,

$= \frac{1}{\log 10}\left[\log x \int dx - \int \left(\frac{1}{x} \int dx\right) dx\right]$

$= \frac{1}{\log 10}\left[x \log x - \int \left(\frac{x}{x}\right) dx\right]$

$= \frac{1}{\log 10}\left[x \log x - x\right]$

$I = \frac{x}{\log 10}(\log x - 1)$

Indefinite Integrals Ex 19.25 Q26

Let $\quad I = \int \cos \sqrt{x} \, dx$

$\sqrt{x} = t$

$x = t^2$

$dx = 2t \, dt$

$= \int 2t \cos t \, dt$

$I = 2\int t \cos t \, dt$

$I = 2\left[t \int \cos t \, dt - \int \left(1 \int \cos t \, dt\right) dt\right]$

$= 2\left[t \sin t - \int \sin t \, dt\right]$

$= 2\left[t \sin t + \cos t\right] + c$

$I = 2\left[\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x}\right] + c$

Indefinite Integrals Ex 19.25 Q27

Let $I=\int \frac{x\cos^{-1}x}{\sqrt{1-x^2}}dx$

Let us substitute, $t=\cos^{-1}x$

$\Rightarrow dt=\frac{-1}{\sqrt{1-x^2}}dx$

Also, $\cos t=x$

Thus,

$I=-\int t\cos t\,dt$

Now let us solve this by the 'by parts' method.

Let $u=t$; $du=dt$

$\int \cos t\,dt=\int dv$

$\Rightarrow \sin t=v$

Thus, $I=-\left[t\sin t-\int \sin t\,dt\right]$

$\Rightarrow I=-\left[t\sin t+\cos t\right]+C$

Substituting the value $t=\cos^{-1}x$, we have,

$I=-\left[\cos^{-1}x\sin t+x\right]+C$

$\Rightarrow I=-\left[\cos^{-1}x\sqrt{1-x^2}+x\right]+C$

*********** END **********