Trigonometric Identities Ex 6.1 Q40

Answer :

We need to prove $\frac{1-\cos A}{1+\cos A}=(\cot A-\operatorname{cosec} A)^2$

Now, rationalising the L.H.S, we get

$$\frac{1-\cos A}{1+\cos A}=\left(\frac{1-\cos A}{1+\cos A}\right)\left(\frac{1-\cos A}{1-\cos A}\right)$$

$$=\frac{(1-\cos A)^2}{1-\cos^2 A} \qquad \left(\text{Using } a^2-b^2=(a+b)(a-b)\right)$$

$$=\frac{1+\cos^2 A-2\cos A}{\sin^2 A} \qquad \left(\text{Using } \sin^2\theta=1-\cos^2\theta\right)$$

$$=\frac{1}{\sin^2 A}+\frac{\cos^2 A}{\sin^2 A}-\frac{2\cos A}{\sin^2 A}$$

Using $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$ and $\cot\theta=\frac{\cos\theta}{\sin\theta}$, we get

$$\frac{1}{\sin^2 A}+\frac{\cos^2 A}{\sin^2 A}-\frac{2\cos A}{\sin^2 A}=\operatorname{cosec}^2 A+\cot^2 A-2\cot A\operatorname{cosec} A$$

$$=(\cot A-\operatorname{cosec} A)^2 \qquad \left(\text{Using } (a+b)^2=a^2+b^2+2ab\right)$$

Hence proved.

Trigonometric Identities Ex 6.1 Q41

Answer :

We need to prove $\frac{1}{\sec A-1}+\frac{1}{\sec A+1}=2\operatorname{cosec} A\cot A$

Solving the L.H.S, we get

$$\frac{1}{\sec A-1}+\frac{1}{\sec A+1}=\frac{\sec A+1+\sec A-1}{(\sec A-1)(\sec A+1)}$$

$$=\frac{2\sec A}{\sec^2 A-1}$$

Further using the property $1+\tan^2\theta=\sec^2\theta$, we get

So,

$$\frac{2\sec A}{\sec^2 A-1}=\frac{2\sec A}{\tan^2 A}$$

$$=\frac{2\left(\frac{1}{\cos A}\right)}{\frac{\sin^2 A}{\cos^2 A}}$$

$$=2\frac{1}{\cos A}\times\frac{\cos^2 A}{\sin^2 A}$$

$$=2\left(\frac{\cos A}{\sin A}\right)\times\frac{1}{\sin A}$$

$$=2\operatorname{cosec} A\cot A$$

Hence proved.

Trigonometric Identities Ex 6.1 Q42

Answer :

We need to prove $\frac{\cos A}{1-\tan A}+\frac{\sin A}{1-\cot A}=\sin A+\cos A$

Solving the L.H.S, we get

$$\begin{aligned}
\frac{\cos A}{1-\tan A}+\frac{\sin A}{1-\cot A} &= \frac{\cos A}{1-\frac{\sin A}{\cos A}}+\frac{\sin A}{1-\frac{\cos A}{\sin A}} \\
&= \frac{\cos A}{\frac{\cos A-\sin A}{\cos A}}+\frac{\sin A}{\frac{\sin A-\cos A}{\sin A}} \\
&= \frac{\cos^2 A}{\cos A-\sin A}+\frac{\sin^2 A}{\sin A-\cos A} \\
&= \frac{\cos^2 A-\sin^2 A}{\cos A-\sin A} \\
&= \frac{(\cos A+\sin A)(\cos A-\sin A)}{\cos A-\sin A}\left[\text{using } a^2-b^2=(a+b)(a-b)\right] \\
&= \cos A+\sin A
\end{aligned}$$

= RHS

Hence proved.

*********** END **********