Exercise 4D

Question 22:
Given : In ΔABC, bisectors of $\angle B$ and $\angle C$ meet at O and $\angle A = 70^o$
In ΔBOC, we have,

$$\Rightarrow \angle BOC + \frac{1}{2}\angle B + \frac{1}{2}\angle C = 180^o$$

$$\Rightarrow \angle BOC = 180^o - \frac{1}{2}\angle B - \frac{1}{2}\angle C$$

$$= 180^o - \frac{1}{2}(\angle B + \angle C)$$

$$= 180^o - \frac{1}{2}\left[180^o - \angle A\right]$$

$$\left[\because \angle A + \angle B + \angle C = 180^o\right]$$

$$= 180^o - \frac{1}{2}\left[180^o - 70^o\right]$$
$$= 180^o - \frac{1}{2} \times 110^o$$

$\angle BOC + \angle OBC + \angle OCB = 180^o$

$= 180^o - 55^o = 125^o$

$\therefore \angle BOC = 125^o$.

Question 23:
We have a ΔABC whose sides AB and AC have been procued to D and E. A = 40^o and bisectors of $\angle CBD$ and $\angle BCE$ meet at O.

In ΔABC, we have,

Exterior $\angle CBD = C + 40^\circ$

$$\Rightarrow \quad \angle CBO = \frac{1}{2}\text{ Ext. }\angle CBD = \frac{1}{2}\left(\angle C + 40^\circ\right) = \frac{1}{2}\angle C + 20^\circ$$

And exterior $\angle BCE = B + 40^\circ$

$$\Rightarrow \quad \angle BCO = \frac{1}{2}\text{ Ext. }\angle BCE = \frac{1}{2}\left(\angle B + 40^\circ\right) = \frac{1}{2}\angle B + 20^\circ.$$

Now, in ΔBCO, we have,

$$\angle BOC = 180^\circ - \angle CBO - \angle BCO$$
$$= 180^\circ - \frac{1}{2}\angle C - 20^\circ - \frac{1}{2}\angle B - 20^\circ$$
$$= 180^\circ - \frac{1}{2}\angle C - \frac{1}{2}\angle B - 20^\circ - 20^\circ$$
$$= 180^\circ - \frac{1}{2}(\angle B + \angle C) - 40^\circ$$
$$= 140^\circ - \frac{1}{2}(\angle B + \angle C)$$
$$= 140^\circ - \frac{1}{2}\left[180^\circ - \angle A\right]$$

$$= 140^\circ - 90^\circ + \frac{1}{2}\angle A$$
$$= 50^\circ + \frac{1}{2}\angle A$$
$$= 50^\circ + \frac{1}{2}\times 40^\circ$$

$= 50^\circ + 20^\circ$

$= 70^\circ$

Thus, $\angle BOC = 70^\circ$

*********** END **********