Trigonometric Identities Ex 6.1 Q30

Answer :

We need to prove $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\tan\theta+\cot\theta$

Now, using $\cot\theta=\frac{1}{\tan\theta}$ in the L.H.S, we get

$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=\frac{\tan\theta}{\left(1-\frac{1}{\tan\theta}\right)}+\frac{\left(\frac{1}{\tan\theta}\right)}{1-\tan\theta}$$

$$=\frac{\tan\theta}{\left(\frac{\tan\theta-1}{\tan\theta}\right)}+\frac{1}{\tan\theta(1-\tan\theta)}$$

$$=\left(\frac{\tan\theta}{\tan\theta-1}\right)(\tan\theta)+\frac{1}{\tan\theta(1-\tan\theta)}$$

$$=\frac{\tan^2\theta}{\tan\theta-1}-\frac{1}{\tan\theta(\tan\theta-1)}$$

$$=\frac{\tan^3\theta-1}{\tan\theta(\tan\theta-1)}$$

Further using the identity $a^3-b^3=(a-b)(a^2+ab+b^2)$, we get

$$\frac{\tan^3\theta-1}{\tan\theta(\tan\theta-1)}=\frac{(\tan\theta-1)(\tan^2\theta+\tan\theta+1)}{\tan\theta(\tan\theta-1)}$$

$$=\frac{\tan^2\theta+\tan\theta+1}{\tan\theta}$$

$$=\frac{\tan^2\theta}{\tan\theta}+\frac{\tan\theta}{\tan\theta}+\frac{1}{\tan\theta}$$

$$=\tan\theta+1+\cot\theta$$

Hence $\boxed{\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\tan\theta+\cot\theta}$

Trigonometric Identities Ex 6.1 Q31

Answer :

We need to prove $\sec^6\theta = \tan^6\theta + 3\tan^2\theta\sec^2\theta + 1$

Solving the L.H.S, we get

$$\sec^6\theta = \left(\sec^2\theta\right)^3$$
$$= \left(1+\tan^2\theta\right)^3$$

Further using the identity $(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$, we get

$$\left(1+\tan^2\theta\right)^3 = 1 + \tan^6\theta + 3(1)^2\left(\tan^2\theta\right) + 3(1)\left(\tan^2\theta\right)^2$$
$$= 1 + \tan^6\theta + 3\tan^2\theta + 3\tan^4\theta$$
$$= 1 + \tan^6\theta + 3\tan^2\theta(1+\tan^2\theta)$$
$$= 1 + \tan^6\theta + 3\tan^2\theta\sec^2\theta \qquad (\text{using } 1+\tan^2\theta = \sec^2\theta)$$

Hence proved.

Trigonometric Identities Ex 6.1 Q32

Answer :

We need to prove $\text{cosec}^6\theta = \cot^6\theta + 3\cot^2\theta\,\text{cosec}^2\theta + 1$

Solving the L.H.S, we get

$$\text{cosec}^6\theta = \left(\text{cosec}^2\theta\right)^3$$
$$= \left(1+\cot^2\theta\right)^3 \qquad \left(1+\cot^2\theta = \text{cosec}^2\theta\right)$$

Further using the identity $(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$, we get

$$\left(1+\cot^2\theta\right)^3 = 1 + \cot^6\theta + 3(1)^2\left(\cot^2\theta\right) + 3(1)\left(\cot^2\theta\right)^2$$
$$= 1 + \cot^6\theta + 3\cot^2\theta + 3\cot^4\theta$$
$$= 1 + \cot^6\theta + 3\cot^2\theta(1+\cot^2\theta)$$
$$= 1 + \cot^6\theta + 3\cot^2\theta\,\text{cosec}^2\theta \qquad (\text{using } 1+\cot^2\theta = \text{cosec}^2\theta)$$

Hence proved.

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