



Definite Integrals Ex 20.1 Q35

We have,

$$\int_1^e \frac{\log x}{x} dx$$

Let  $\log x = t$

$$\Rightarrow \frac{1}{x} dx = dt$$

Now,

$$x = 1 \Rightarrow t = 0$$

$$x = e \Rightarrow t = 1$$

$$\therefore \int_1^e \frac{\log x}{x} dx = \int_0^1 t dt$$

$$= \left[ \frac{t^2}{2} \right]_0^1$$

$$= \frac{1}{2}$$

$$\therefore \int_1^e \frac{\log x}{x} dx = \frac{1}{2}$$

Definite Integrals Ex 20.1 Q36

We have,

$$\int_e^{e^2} \left\{ \frac{1}{\log x} - \frac{1}{(\log x)^2} \right\} dx$$

$$I = \int \frac{1}{\log x} \cdot 1 dx = \frac{1}{\log x} \int dx - \int \left( \int dx \right) \cdot \frac{d}{dx} \left( \frac{1}{\log x} \right) dx = \frac{x}{\log x} + \int \frac{1}{(\log x)^2} \cdot x \cdot \frac{1}{x} dx$$

$$= \frac{x}{\log x} + \int \frac{dx}{(\log x)^2}$$

$$\int_e^{e^2} \left\{ \frac{1}{\log x} - \frac{1}{(\log x)^2} \right\} dx = \left[ \frac{x}{\log x} \right]_e^{e^2} + \int_e^{e^2} \frac{dx}{(\log x)^2} - \int_e^{e^2} \frac{dx}{(\log x)^2}$$

$$= \left[ \frac{x}{\log x} \right]_e^{e^2}$$

$$= \frac{e^2}{2} - e$$

Definite Integrals Ex 20.1 Q37

We have,

$$\int_1^2 \frac{x+3}{x(x+2)} dx$$

$$= \int_1^2 \frac{x}{x(x+2)} dx + \int_1^2 \frac{3}{x(x+2)} dx$$

$$= \int_1^2 \frac{dx}{(x+2)} + \int_1^2 \frac{3}{x(x+2)} dx$$

$$= [\log(x+2)]_1^2 + \frac{3}{2} \int_1^2 \frac{1}{x} - \frac{1}{x+2} dx \quad [\text{using partial fraction}]$$

$$= [\log(x+2)]_1^2 + \left[ \frac{3}{2} \log x - \frac{3}{2} \log(x+2) \right]_1^2$$

$$= \left[ \frac{3}{2} \log x - \frac{1}{2} \log(x+2) \right]_1^2$$

$$= \frac{1}{2} [3 \log 2 - \log 4 + \log 3]$$

$$= \frac{1}{2} [3 \log 2 - 2 \log 2 + \log 3] \quad [\because \log 4 = 2 \log 2]$$

$$= \frac{1}{2} [\log 2 + \log 3]$$

$$= \frac{1}{2} [\log 6]$$

$$= \frac{1}{2} \log 6$$

$$\therefore \int_1^2 \frac{x+3}{x(x+2)} dx = \frac{1}{2} \log 6$$

Definite Integrals Ex 20.1 Q38

$$\text{Let } I = \int \frac{2x+3}{5x^2+1} dx$$

$$\int \frac{2x+3}{5x^2+1} dx = \frac{1}{5} \int \frac{5(2x+3)}{5x^2+1} dx$$

$$= \frac{1}{5} \int \frac{10x+15}{5x^2+1} dx$$

$$= \frac{1}{5} \int \frac{10x}{5x^2+1} dx + 3 \int \frac{1}{5x^2+1} dx$$

$$= \frac{1}{5} \int \frac{10x}{5x^2+1} dx + 3 \int \frac{1}{5\left(x^2+\frac{1}{5}\right)} dx$$

$$= \frac{1}{5} \log(5x^2+1) + \frac{3}{5} \cdot \frac{1}{\frac{1}{\sqrt{5}}} \tan^{-1} \frac{x}{\frac{1}{\sqrt{5}}}$$

$$= \frac{1}{5} \log(5x^2+1) + \frac{3}{\sqrt{5}} \tan^{-1}(\sqrt{5}x)$$

$$= F(x)$$

\*\*\*\*\* END \*\*\*\*\*