Maxima and Minima Ex 18.2 Q5

$f(x) = (x-1)^3 (x+1)^2$

$\therefore \quad f'(x) = 3(x-1)^2 (x+1)^2 + 2(x-1)^3 (x+1)$

$= (x-1)^2 (x+1) \left(3(x+1) + 2(x-1)\right)$

$= (x-1)^2 (x+1)(5x+1)$

For the point of local maxima and minima,

$f'(x) = 0$

$\Rightarrow \quad (x-1)^2 (x+1)(5x+1) = 0$

$\Rightarrow \quad x = 1, -1, -\frac{1}{5}$

Here,

At $x = -1$ $f'(x)$ changes from $+$ve to $-$ve so $x = -1$ is point of maxima.

At $x = -\frac{1}{5}$, $f'(x)$ changes from $-$ve to $+$ve so $x = -\frac{1}{5}$ is point of minima

Hence, local max value $= 0$

local min value $= -\frac{3456}{3125}$.

Maxima and Minima Ex 18.2 Q6

$f(x) = x^3 - 6x^2 + 9x + 15$

$\therefore \quad f'(x) = 3x^2 - 12x + 9$

$= 3\left(x^2 - 4x + 3\right)$

$= 3(x-3)(x-1)$

For, the point of local maxima and minima,

$f'(x) = 0$

$\Rightarrow \quad 3(x-3)(x-1) = 0$

$\Rightarrow \quad x = 3, 1$

At $\quad x = -1,\ f'(x)$ changes from $+$ve to $-$ve

$\therefore \quad x = 1$ is point of local maxima

At $\quad x = 3,\ f'(x)$ changes from $-$ve to $+$ve

$\therefore \quad x = 3$ is point of local manima

Hence, local max value $= f(1) = 19$

local min value $= f(3) = 15$.

Maxima and Minima Ex 18.2 Q7

$f(x) = \sin 2x,\ 0 < x, \pi$

$\therefore \quad f'(x) = 2\cos 2x$

For, the point of local maxima and minima,

$f'(x) = 0$

$\Rightarrow \quad 2x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\Rightarrow \quad x = \frac{\pi}{4}, \frac{3\pi}{4}$

At $\quad x = \frac{\pi}{4}$, $f'(x)$ changes from $+$ve to $-$ve

$\therefore \quad x = \frac{\pi}{4}$ is point of local maxima

At $\quad x = \frac{3\pi}{4}$, $f'(x)$ changes from $-$ve to $+$ve

$\therefore \quad x = \frac{3\pi}{4}$ is point of local minima,

Hence, local max value $= f\left(\frac{\pi}{4}\right) = 1$

local min value $= f\left(\frac{3\pi}{4}\right) = -1.$

Maxima and Minima Ex 18.2 Q8

$f(x) = \sin x - \cos x,\ 0 < x < 2\pi$

$\therefore f'(x) = \cos x + \sin x$

$f'(x) = 0 \Rightarrow \cos x = -\sin x \Rightarrow \tan x = -1 \Rightarrow x = \frac{3\pi}{4}, \frac{7\pi}{4} \in (0, 2\pi)$

$f''(x) = -\sin x + \cos x$

$f''\left(\frac{3\pi}{4}\right) = -\sin\frac{3\pi}{4} + \cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\sqrt{2} < 0$

$f''\left(\frac{7\pi}{4}\right) = -\sin\frac{7\pi}{4} + \cos\frac{7\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2} > 0$

Therefore, by second derivative test, $x = \frac{3\pi}{4}$ is a point of local maxima and the local maximum value of f at $x = \frac{3\pi}{4}$ is

$f\left(\frac{3\pi}{4}\right) = \sin\frac{3\pi}{4} - \cos\frac{3\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$. However, $x = \frac{7\pi}{4}$ is a point of local minima and the local minimum value of f at $x = \frac{7\pi}{4}$ is $f\left(\frac{7\pi}{4}\right) = \sin\frac{7\pi}{4} - \cos\frac{7\pi}{4} = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\sqrt{2}$.

*********** END **********