EXERCISE 9.1

Question 1:

Write the first five terms of the sequences whose n$^{\text{th}}$ term is $a_n = n(n+2)$

Ans:

$a_n = n(n+2)$

Substituting n = 1, 2, 3, 4, and 5, we obtain

$$a_1 = 1(1+2) = 3$$
$$a_2 = 2(2+2) = 8$$
$$a_3 = 3(3+2) = 15$$
$$a_4 = 4(4+2) = 24$$
$$a_5 = 5(5+2) = 35$$

Therefore, the required terms are 3, 8, 15, 24, and 35.

Question 2:

Write the first five terms of the sequences whose n$^{\text{th}}$ term is $a_n = \frac{n}{n+1}$

Ans:

$a_n = \frac{n}{n+1}$

Substituting n = 1, 2, 3, 4, 5, we obtain

$$a_1 = \frac{1}{1+1} = \frac{1}{2},\ a_2 = \frac{2}{2+1} = \frac{2}{3},\ a_3 = \frac{3}{3+1} = \frac{3}{4},\ a_4 = \frac{4}{4+1} = \frac{4}{5},\ a_5 = \frac{5}{5+1} = \frac{5}{6}$$

Therefore, the required terms are $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$, and $\frac{5}{6}$.

Question 3:

Write the first five terms of the sequences whose n$^{\text{th}}$ term is $a_n = 2^n$

Ans:

$a_n = 2^n$

Substituting n = 1, 2, 3, 4, 5, we obtain

$$a_1 = 2^1 = 2$$
$$a_2 = 2^2 = 4$$
$$a_3 = 2^3 = 8$$
$$a_4 = 2^4 = 16$$
$$a_5 = 2^5 = 32$$

Therefore, the required terms are 2, 4, 8, 16, and 32.

Question 4:

Write the first five terms of the sequences whose n^{th} term is $a_n = \frac{2n-3}{6}$

Ans:

Substituting n = 1, 2, 3, 4, 5, we obtain

$$a_1 = \frac{2\times1-3}{6} = \frac{-1}{6}$$
$$a_2 = \frac{2\times2-3}{6} = \frac{1}{6}$$
$$a_3 = \frac{2\times3-3}{6} = \frac{3}{6} = \frac{1}{2}$$
$$a_4 = \frac{2\times4-3}{6} = \frac{5}{6}$$
$$a_5 = \frac{2\times5-3}{6} = \frac{7}{6}$$

Therefore, the required terms are $\frac{-1}{6}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6}$, and $\frac{7}{6}$.

Question 5:

Write the first five terms of the sequences whose n^{th} term is $a_n = (-1)^{n-1} 5^{n+1}$

Ans:

Substituting n = 1, 2, 3, 4, 5, we obtain

$$a_1 = (-1)^{1-1} 5^{1+1} = 5^2 = 25$$
$$a_2 = (-1)^{2-1} 5^{2+1} = -5^3 = -125$$
$$a_3 = (-1)^{3-1} 5^{3+1} = 5^4 = 625$$
$$a_4 = (-1)^{4-1} 5^{4+1} = -5^5 = -3125$$
$$a^5 = (-1)^{5-1} 5^{5+1} = 5^6 = 15625$$

Therefore, the required terms are 25, –125, 625, –3125, and 15625.

Question 6:

Write the first five terms of the sequences whose n^{th} term is $a_n = n\frac{n^2+5}{4}$

Ans:

Substituting n = 1, 2, 3, 4, 5, we obtain

$$a_1 = 1 \cdot \frac{1^2+5}{4} = \frac{6}{4} = \frac{3}{2}$$
$$a_2 = 2 \cdot \frac{2^2+5}{4} = 2 \cdot \frac{9}{4} = \frac{9}{2}$$
$$a_3 = 3 \cdot \frac{3^2+5}{4} = 3 \cdot \frac{14}{4} = \frac{21}{2}$$
$$a_4 = 4 \cdot \frac{4^2+5}{4} = 21$$
$$a_5 = 5 \cdot \frac{5^2+5}{4} = 5 \cdot \frac{30}{4} = \frac{75}{2}$$

Therefore, the required terms are $\frac{3}{2}$, $\frac{9}{2}$, $\frac{21}{2}$, 21, and $\frac{75}{2}$.

Question 7:

Find the 17th term in the following sequence whose n^{th} term is $a_n = 4n - 3; a_{17}, a_{24}$

Ans:

Substituting n = 17, we obtain

$$a_{17} = 4(17) - 3 = 68 - 3 = 65$$

Substituting n = 24, we obtain

$$a_{24} = 4(24) - 3 = 96 - 3 = 93$$

Question 8:

Find the 7th term in the following sequence whose n^{th} term is $a_n = \frac{n^2}{2n}; a_7$

Ans:

Substituting n = 7, we obtain

$$a_7 = \frac{7^2}{2^7} = \frac{49}{128}$$

Question 9:

Find the 9th term in the following sequence whose n^{th} term is $a_n = (-1)^{n-1} n^3; a_9$

Ans:

Substituting $n = 9$, we obtain

$$a_9 = (-1)^{9-1}(9)^3 = (9)^3 = 729$$

Question 10:

Find the 20th term in the following sequence whose n^{th} term is $a_n = \frac{n(n-2)}{n+3}; a_{20}$

Ans:

Substituting $n = 20$, we obtain

$$a_{20} = \frac{20(20-2)}{20+3} = \frac{20(18)}{23} = \frac{360}{23}$$

Question 11:

Write the first five terms of the following sequence and obtain the corresponding series:

$a_1 = 3, a_n = 3a_{n-1} + 2$ for all $n > 1$

Ans:

$a_1 = 3, a_n = 3a_{n-1} + 2$ for all $n > 1$

$$\Rightarrow a_2 = 3a_1 + 2 = 3(3) + 2 = 11$$
$$a_3 = 3a_2 + 2 = 3(11) + 2 = 35$$
$$a_4 = 3a_3 + 2 = 3(35) + 2 = 107$$
$$a_5 = 3a_4 + 2 = 3(107) + 2 = 323$$

Hence, the first five terms of the sequence are 3, 11, 35, 107, and 323.

The corresponding series is 3 + 11 + 35 + 107 + 323 + ...

Question 12:

Write the first five terms of the following sequence and obtain the corresponding series:

$a_1 = -1, a_n = \frac{a_{n-1}}{n}, n \geq 2$

Ans:

$a_1 = -1, a_n = \frac{a_{n-1}}{n}, n \geq 2$

$$\Rightarrow a_2 = \frac{a_1}{2} = \frac{-1}{2}$$
$$a_3 = \frac{a_2}{3} = \frac{-1}{6}$$
$$a_4 = \frac{a_3}{4} = \frac{-1}{24}$$
$$a_5 = \frac{a_4}{4} = \frac{-1}{120}$$

Hence, the first five terms of the sequence are $-1, \frac{-1}{2}, \frac{-1}{6}, \frac{-1}{24}$, and $\frac{-1}{120}$.

The corresponding series is $(-1) + \left(\frac{-1}{2}\right) + \left(\frac{-1}{6}\right) + \left(\frac{-1}{24}\right) + \left(\frac{-1}{120}\right) + \ldots$

Question 13:

Write the first five terms of the following sequence and obtain the corresponding series:

$a_1 = a_2 = 2, a_n = a_{n-1} - 1, n > 2$

Ans:

$a_1 = a_2 = 2, a_n = a_{n-1} - 1, n > 2$

$\Rightarrow a_3 = a_2 - 1 = 2 - 1 = 1$

$a_4 = a_3 - 1 = 1 - 1 = 0$

$a_5 = a_4 - 1 = 0 - 1 = -1$

Hence, the first five terms of the sequence are 2, 2, 1, 0, and –1.

The corresponding series is 2 + 2 + 1 + 0 + (–1) + ...

Question 14:

The Fibonacci sequence is defined by

$1 = a_1 = a_2$ and $a_n = a_{n-1} + a_{n-2}, n > 2$

Find $\frac{a_{n+1}}{a_n}$, for n = 1, 2, 3, 4, 5

Ans:

$$1 = a_1 = a_2$$

$$a_n = a_{n-1} + a_{n-2}, n > 2$$

$$\therefore a_3 = a_2 + a_1 = 1 + 1 = 2$$

$$a_4 = a_3 + a_2 = 2 + 1 = 3$$

$$a_5 = a_4 + a_3 = 3 + 2 = 5$$

$$a_6 = a_5 + a_4 = 5 + 3 = 8$$

$$\therefore \text{For } n = 1, \quad \frac{a_n + 1}{a_n} = \frac{a_2}{a_1} = \frac{1}{1} = 1$$

$$\text{For } n = 2, \quad \frac{a_n + 1}{a_n} = \frac{a_3}{a_2} = \frac{2}{1} = 2$$

$$\text{For } n = 3, \frac{a_n + 1}{a_n} = \frac{a_4}{a_3} = \frac{3}{2}$$

$$\text{For } n = 4, \quad \frac{a_n + 1}{a_n} = \frac{a_5}{a_4} = \frac{5}{3}$$

$$\text{For } n = 5, \quad \frac{a_n + 1}{a_n} = \frac{a_6}{a_5} = \frac{8}{5}$$

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