Definite Integrals Ex 20.3 Q19

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x<0 \end{cases}$$

$$|x-2| = \begin{cases} x-2, & x \geq 2 \\ 2-x, & x<2 \end{cases}$$

$$|x-4| = \begin{cases} x-4, & x \geq 4 \\ 4-x, & x < 4 \end{cases}$$

Splitting the limits of the integral, we get

$$\int_0^4 (|x| + |x-2| + |x-4|)\,dx$$

$$= \int_0^2 (|x| + |x-2| + |x-4|)\,dx + \int_2^4 (|x| + |x-2| + |x-4|)\,dx$$

$$= \int_0^2 (x + 2 - x + 4 - x)\,dx + \int_2^4 (x + x - 2 + 4 - x)\,dx$$

$$= \int_0^2 (6-x)\,dx + \int_2^4 (2+x)\,dx$$

$$= \left[6x - \frac{x^2}{2}\right]_0^2 + \left[2x + \frac{x^2}{2}\right]_2^4$$

$$= [12-2]+[16-6]$$

$$= 10+10$$

$$= 20$$

Definite Integrals Ex 20.3 Q20

$$\int_{-1}^{2} |x+1|\,dx + \int_{-1}^{2} |x|\,dx + \int_{-1}^{2} |x-1|\,dx$$

$$\int_{-1}^{2} (x+1)dx - \int_{-1}^{0} xdx + \int_{0}^{2} xdx - \int_{-1}^{1} (x-1)dx + \int_{1}^{2} (x-1)dx$$

$$\left\{\frac{x^2}{2}+x\right\}_{-1}^{2} - \left\{\frac{x^2}{2}\right\}_{-1}^{0} + \left\{\frac{x^2}{2}\right\}_{0}^{2} - \left\{\frac{x^2}{2}-x\right\}_{-1}^{1} + \left\{\frac{x^2}{2}-x\right\}_{1}^{2}$$

$$\left\{(4)-(-\frac{1}{2})\right\} - \left\{-\frac{1}{2}\right\} + \{2\} - \left\{(-\frac{1}{2})-(\frac{3}{2})\right\} + \left\{(0)-(-\frac{1}{2})\right\}$$

$$\left\{4+\frac{1}{2}\right\} + \left\{\frac{1}{2}\right\} + \{2\} + \{2\} + \left\{\frac{1}{2}\right\}$$

$$\frac{19}{2}$$

Definite Integrals Ex 20.3 Q21

$$\int_{-2}^{0} xe^{-x}dx + \int_{0}^{2} xe^{x}dx$$

For

$$\int_{-2}^{0} xe^{-x}dx$$

Using Integration By parts

$$\int f'g = fg - \int fg'$$

$$f' = e^{-x}, g = x$$

$$f = -e^{-x}, g' = 1$$

$$\int_{-2}^{0} xe^{-x}dx = \left\{-xe^{-x}\right\}_{-2}^{0} + \int_{-2}^{0} e^{-x}dx$$

$$\int_{-2}^{0} xe^{-x}dx = \left\{-xe^{-x} - e^{-x}\right\}_{-2}^{0}$$

$$\int_{-2}^{0} xe^{-x}dx = \left\{(-1) - (2e^{2} - e^{2})\right\}$$

$$\int_{-2}^{0} xe^{-x}dx = \left\{-1 - e^{2}\right\}$$

For

$$\int_{0}^{2} xe^{x}dx$$

Using Integration By parts

$$\int f'g = fg - \int fg'$$

$$f' = e^{x}, g = x$$

$$f = e^{x}, g' = 1$$

$$\int_{0}^{2} xe^{x}dx = \left\{xe^{x}\right\}_{0}^{2} - \int_{0}^{2} e^{x}dx$$

$$\int_{0}^{2} xe^{x}dx = \left\{xe^{x} - e^{x}\right\}_{0}^{2}$$

$$\int_{0}^{2} xe^{x}dx = 2e^{2} - e^{2} + 1$$

$$\int_{0}^{2} xe^{x}dx = e^{2} + 1$$

Hence answer is,

$$\int_{-2}^{2} xe^{|x|}dx = -1 - e^{2} + e^{2} + 1 = 0$$

Definite Integrals Ex 20.3 Q22

$$-\int_{-\frac{\pi}{4}}^{0} \sin^2 x dx + \int_{0}^{\frac{\pi}{2}} \sin^2 x dx$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$-\int_{-\frac{\pi}{4}}^{0} \frac{1-\cos 2x}{2} dx + \int_{0}^{\frac{\pi}{2}} \frac{1-\cos 2x}{2} dx$$

$$-\frac{1}{2}\left\{x - \frac{\sin 2x}{2}\right\}_{-\frac{\pi}{4}}^{0} + \frac{1}{2}\left\{x - \frac{\sin 2x}{2}\right\}_{0}^{\frac{\pi}{2}}$$

$$-\frac{1}{2}\left\{-(-\frac{\Pi}{4}+\frac{1}{2})\right\} + \frac{1}{2}\left\{\frac{\Pi}{2}\right\}$$

$$\left\{-\frac{\Pi}{8}+\frac{1}{4}\right\} + \left\{\frac{\Pi}{4}\right\}$$

$$\frac{\Pi}{8}+\frac{1}{4}$$

$$\frac{\Pi+2}{8}$$

*********** END **********