



Indefinite Integrals Ex 19.14 Q1

$$\begin{aligned}
 \text{Let } I &= \int \frac{1}{a^2 - b^2 x^2} dx \\
 &= \frac{1}{b^2} \int \frac{1}{\frac{a^2}{b^2} - x^2} dx \\
 &= \frac{1}{b^2} \int \frac{1}{\left(\frac{a}{b}\right)^2 - x^2} dx \\
 I &= \frac{1}{b^2} \times \frac{1}{2 \times \left(\frac{a}{b}\right)} \log \left| \frac{\frac{a}{b} + x}{\frac{a}{b} - x} \right| + c \quad \left[\text{Since } \int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{x+a}{x-a} \right| + c \right]
 \end{aligned}$$

$$I = \frac{1}{2ab} \log \left| \frac{a+bx}{a-bx} \right| + c$$

Indefinite Integrals Ex 19.14 Q2

$$\begin{aligned}
 \text{Let } I &= \int \frac{1}{a^2 x^2 - b^2} dx \\
 &= \frac{1}{a^2} \int \frac{1}{x^2 - \frac{b^2}{a^2}} dx \\
 &= \frac{1}{a^2} \int \frac{1}{x^2 - \left(\frac{b}{a}\right)^2} dx \\
 I &= \frac{1}{a^2} \times \frac{1}{2 \times \left(\frac{b}{a}\right)} \times \log \left| \frac{x - \frac{b}{a}}{x + \frac{b}{a}} \right| + c \quad \left[\text{Since } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c \right]
 \end{aligned}$$

$$I = \frac{1}{2ab} \log \left| \frac{ax-b}{ax+b} \right| + c$$

Indefinite Integrals Ex 19.14 Q3

$$\begin{aligned}
 \text{Let } I &= \int \frac{1}{a^2 x^2 + b^2} dx \\
 &= \frac{1}{a^2} \int \frac{1}{x^2 + \frac{b^2}{a^2}} dx \\
 &= \frac{1}{a^2} \int \frac{1}{x^2 + \left(\frac{b}{a}\right)^2} dx \\
 I &= \frac{1}{a^2} \times \frac{1}{\left(\frac{b}{a}\right)} \tan^{-1} \left(\frac{x}{\frac{b}{a}} \right) + c \quad \left[\text{Since } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c \right]
 \end{aligned}$$

$$I = \frac{1}{ab} \tan^{-1} \left(\frac{ax}{b} \right) + c$$

Indefinite Integrals Ex 19.14 Q4

$$\text{Let } I = \int \frac{x^2 - 1}{x^2 - 4} dx$$

$$\begin{aligned} \text{Let } I &= \int \frac{x^2 - 1}{x^2 + 4} dx \\ &= \int \frac{(x^2 + 4) - 4 - 1}{x^2 + 4} dx \\ &= \int \frac{x^2 + 4}{x^2 + 4} dx - \int \frac{5}{x^2 + 4} dx \\ &= \int dx - 5 \int \frac{1}{x^2 + (2)^2} dx \end{aligned}$$

$$I = x - 5 \times \frac{1}{2} \tan^{-1} \left(\frac{x}{2} \right) + C \quad \left[\text{Since } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) \right]$$

$$I = x - \frac{5}{2} \tan^{-1} \left(\frac{x}{2} \right) + C$$

Indefinite Integrals Ex 19.14 Q5

$$\text{Let } 2x = t$$

$$\Rightarrow 2dx = dt$$

$$\begin{aligned} \Rightarrow \int \frac{1}{\sqrt{1+4x^2}} dx &= \frac{1}{2} \int \frac{dt}{\sqrt{1+t^2}} \\ &= \frac{1}{2} \left[\log |t + \sqrt{t^2 + 1}| \right] + C \quad \left[\int \frac{1}{\sqrt{x^2 + a^2}} dt = \log |x + \sqrt{x^2 + a^2}| \right] \\ &= \frac{1}{2} \log |2x + \sqrt{4x^2 + 1}| + C \end{aligned}$$

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