Geometric Progressions Ex 20.3 Q16.

Let the G.P. be $2n, 2, 2n+4, \ldots$

Then, $S_n = \dfrac{a(r^n - 1)}{r - 1}$, $a = 2n$, $r = 2$

$\therefore \quad S_n = \dfrac{2n(2^n - 1)}{2 - 1} = 2n^{n+1} - 2n$

Then the G.P. of odd term

$a_1 + a_3 + a_5 + \ldots a_{2n-1}$

According to the question

Sum of all terms = 5 (sum of terms occupying the odd places)

$$a_1 + a_2 + a_3 + \ldots + a_{2n} = 5(a_1 + a_3 + a_5 + \ldots + a_{2n-1})$$

$$a + ar + ar^2 + \ldots + ar^{2n-1} = 5\left(a + ar^2 + ar^4 + \ldots + ar^{2n-2}\right)$$

$$\frac{a(1 - r^{2n})}{1 - r} = 5\left(\frac{a\left(1 - \left((r)^2\right)^n\right)}{1 - r^2}\right)$$

$\dfrac{a}{1 - r}$ is cancelled on both side

$$1 - r^{2n} = \frac{5(1 - r^{2n})}{1 + r}$$

$$1 + r - r^{2n} - r^{2n+1} = 5 - 5r^{2n}$$

$$r^{2n+1} - 4r^{2n} - r + 4 = 0$$

$$r^{2n}(r - 4) - 1(r - 4) = 0$$

$$r^{2n} = 1, \ r = 4$$

$\Rightarrow \quad r = 4$

Geometric Progressions Ex 20.3 Q17

Given $\sum_{n=1}^{100} a_{2n} = \alpha$

$\Rightarrow \quad a_2 + a_4 + a_6 + \ldots + a_{200} = \alpha \qquad$ ---(i)

also, $\sum_{n=1}^{100} a_{2n-1} = \beta$

$\Rightarrow \quad a_1 + a_3 + a_5 + \ldots + a_{199} = \beta \qquad$ ---(ii)

Sum of G.P,

$$S_n = \frac{a(1 - r^n)}{1 - n}$$

$$= a = a_2, r = r^2, n = 100$$

$$ar + ar^3 + ar^5 + \ldots + ar^{199} = \alpha$$

$$ar\frac{\left(1 - (r^2)^{100}\right)}{1 - r^2} = \alpha \qquad \text{---(iii)}$$

$$a + ar^2 + ar^4 + \ldots + ar^{198} = \beta$$

$$\frac{a\left(1 - (r^2)^{100}\right)}{1 - r^2} = \beta \qquad \text{---(iv)}$$

$$r(\beta) = \alpha$$

$$r = \frac{\alpha}{\beta} \qquad [\text{From (iv) and (v)}]$$

Geometric Progressions Ex 20.3 18

Let the seried be $a_1 + a_2 + a_3 + \ldots + a_{2n}$

It is given that $a_1 = 1, a_2 = a, a_3 = ac, a_4 = a^2c, a_5 = a^2c^2, \ldots$

$\therefore$ Sum of $2n$ term

$$a_1 + a_2 + a_3 + \ldots + a_{2n}$$
$$= 1 + a + ac + a^2c + a^2c^2 + \ldots + 2n \text{ term}$$
$$= (1+a) + ac(1+a) + a^2c^2(1+a) + \ldots + n \text{ term}$$
$$= (1+a)\frac{\left(1-(ac)^n\right)}{1-ac}$$
$$= (a+1)\frac{\left((ac)^n - 1\right)}{ac-1}.$$

Geometric Progressions Ex 20.3 Q19.

Sum of first n term of G.P.

$$= a + a_2 + a_3 + \ldots + a_n$$
$$= a + ar + ar^2 + \ldots + ar^{n-1} \qquad \left[\because t_n = ar^{n-1}\right] \text{ ---(i)}$$

Also sum of term from

$(n+1)^{th}$ to $(2n)^{th}$ term is

$$= a_{n+1} + a_{n+2} + \ldots + a_{2n}$$
$$= ar^n + ar^{n-1} + \ldots + ar^{2n-1} \qquad \text{---(ii)}$$

Ratio of (i) and (ii) is

$$= \frac{a + ar + ar^2 + \ldots ar^{n-1}}{ar^n + ar^{n-1} + \ldots + ar^{2n-1}} \qquad \left[\because S_n = \frac{a\left(1-r^n\right)}{1-r}\right]$$
$$= \frac{\dfrac{a\left(1-r^n\right)}{1-r}}{\dfrac{ar^n\left(1-r^n\right)}{1-r}}$$
$$= \frac{1}{r^n}$$

Geometric Progressions Ex 20.3 Q20

Given,

a, b are roots of the equation $x^2 - 3x + p = 0$

$\Rightarrow \quad a + b = 3, \; ab = p$

and $\quad c, d$ are roots of the equation $x^2 - 12x + q = 0$

$\Rightarrow \quad c + d = 12, \; cd = q$

Let $b = ar, \; c = ar^2$ and $d = ar^3$, then $a + b = 3$ and $c + d = 12$

$$a(1+r) = 3 \text{ and } ar^2(1+r) = 12$$
$$\Rightarrow \quad \frac{ar^2(1+r)}{a(1+r)} = \frac{12}{3}$$
$$\Rightarrow \quad r = 2$$

and $a(r+1) = 3$

$$\Rightarrow \quad a = 1$$
$$p = ab$$
$$= a \times ar$$
$$p = 2$$
$$q = cd$$
$$= ar^2 \times ar^3$$
$$= 2^5$$
$$a = 32$$
$$\frac{q+p}{q-p} = \frac{32+2}{32-2}$$
$$= \frac{34}{30}$$
$$(q+p):(q-p) = 17:15$$

Geometric Progressions Ex 20.3 Q21.

$$\text{Sum} = \frac{3069}{512} = \frac{3(1-\frac{1}{2^n})}{\frac{1}{2}}$$

$$1-\frac{1}{2^n} = \frac{3069}{512\times 6} = \frac{1023}{512\times 2}$$

$$1-\frac{1023}{1024} = \frac{1}{2^n}$$

$$\frac{1}{2^n} = \frac{1}{1024}$$

$$n = 10$$

Geometric Progressions Ex 20.3 Q22.

To find number of ancestors, we will find the sum of $2, 2^2, 2^3, \ldots$

$$\text{Number of ancestors} = \frac{2(2^{10}-1)}{2-1}$$

$$= 2(1024-1)$$

$$= 2\times 1023$$

$$= 2046$$

*********** END **********