



Q 1. Prove that $\sqrt{5}$ is irrational.

Answer :

Let $\sqrt{5}$ is a rational number.

Therefore, we can find two integers a, b ($b \neq 0$)

such that $\sqrt{5} = \frac{a}{b}$

Let a and b have a common factor other than 1.
Then we can divide them by the common factor,
and assume that a and b are co-prime.

$$a = \sqrt{5}b$$

$$a^2 = 5b^2$$

Therefore, a^2 is divisible by 5 and it can be said
that a is divisible by 5.

Let $a = 5k$, where k is an integer

$(5k)^2 = 5b^2$ This means that b^2 is divisible by 5
and hence, b is divisible by 5.

$$b^2 = 5k^2$$

This implies that a and b have 5 as
a common factor.

And this is a contradiction to the fact that a and
 b are co-prime.

Hence, $\sqrt{5}$ cannot be expressed as $\frac{p}{q}$ or it can

be said that $\sqrt{5}$ is irrational.

Q 2 . Prove that $3+2\sqrt{5}$ is irrational.

Answer :

Let $3+2\sqrt{5}$ is rational.

Therefore, we can find two integers a, b ($b \neq 0$) such that

$$3+2\sqrt{5} = \frac{a}{b}$$

$$2\sqrt{5} = \frac{a}{b} - 3$$

$$\sqrt{5} = \frac{1}{2} \left(\frac{a}{b} - 3 \right)$$

Since a and b are integers, $\frac{1}{2} \left(\frac{a}{b} - 3 \right)$ will also be rational and therefore, $\sqrt{5}$ is rational.

This contradicts the fact that $\sqrt{5}$ is irrational.

Hence, our assumption that $3+2\sqrt{5}$ is rational is false. Therefore, $3+2\sqrt{5}$ is irrational.

Q 3. Prove that the following are irrationals:

(i) $\frac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$ (iii) $6+\sqrt{2}$

Answer :

(i) $\frac{1}{\sqrt{2}}$

Let $\frac{1}{\sqrt{2}}$ is rational.

Therefore, we can find two integers a, b ($b \neq 0$) such that

$$\frac{1}{\sqrt{2}} = \frac{a}{b}$$

$$\sqrt{2} = \frac{b}{a}$$

$\frac{b}{a}$ is rational as a and b are integers.

Therefore, $\sqrt{2}$ is rational which contradicts to the fact that $\sqrt{2}$ is irrational.

Hence, our assumption is false and $\frac{1}{\sqrt{2}}$ is irrational.

(ii) $7\sqrt{5}$

Let $7\sqrt{5}$ is rational.

Therefore, we can find two integers a, b ($b \neq 0$) such that

$$7\sqrt{5} = \frac{a}{b} \text{ for some integers } a \text{ and } b$$

$$\therefore \sqrt{5} = \frac{a}{7b}$$

$\frac{a}{7b}$ is rational as a and b are integers.

Therefore, $\sqrt{5}$ should be rational.

This contradicts the fact that $\sqrt{5}$ is irrational.

Therefore, our assumption that $7\sqrt{5}$ is rational is false. Hence, $7\sqrt{5}$ is irrational.

(iii) $6 + \sqrt{2}$

Let $6 + \sqrt{2}$ be rational.

Therefore, we can find two integers a, b ($b \neq 0$) such that

$$6 + \sqrt{2} = \frac{a}{b}$$

$$\sqrt{2} = \frac{a}{b} - 6$$

Since a and b are integers, $\frac{a}{b} - 6$ is also rational

and hence, $\sqrt{2}$ should be rational. This contradicts the fact that $\sqrt{2}$ is irrational.

Therefore, our assumption is false and hence

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