Question 2 .11. 'The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures.
Answer:
As Area = (4.234 x 1.005) x 2
= 8.51034 = 8.5 m^2
Volume = (4.234 x 1.005) x (2.01 x 10^{-2})
= 8.55289 x 10^{-2} = 0.0855 m^3.

Question 2. 12. The mass of a box measured by a grocer's balance is 2.3 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) the total mass of the box (b) the difference in the masses of the pieces to correct significant figures?
Answer:
(a) Total mass of the box = (2.3 + 0.0217 + 0.0215) kg = 2.3442 kg
Since the least number of decimal places is 1, therefore, the total mass of the box = 2.3 kg.
(b) Difference of mass = 2.17 - 2.15 = 0.02 g
Since the least number of decimal places is 2 so the difference in masses to the correct significant figures is 0.02 g.

Question 2. 13. A physical quantity P is related to four observables a, b, c and d as follows:

$$P = a^3 b^2 / \left(\sqrt{c d}\right)$$

The percentage errors of measurement in a, b, c and d are 1%, 3%, 4% and 2%, respectively. What is the percentage error in the quantity P? If the value of P calculated using the above relation turns out to be 3.763, to what value should you round off the result?
Answer:

As $P = \dfrac{a^3 b^2}{\left(\sqrt{c}\, d\right)} = a^3\, b^2\, c^{-1/2}\, d^{-1}$

∴ Maximum fractional error in the mearurement

$$\frac{\Delta P}{P} = 3\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + \frac{\Delta d}{d}$$

As $\dfrac{\Delta a}{a} = 1\%$, $\dfrac{\Delta b}{b} = 3\%$, $\dfrac{\Delta c}{c} = 4\%$ and $\dfrac{\Delta d}{d} = 2\%$

∴ Maximum fractional error in the measurement

$$\frac{\Delta P}{P} = 3 \times 1\% + 2 \times 3\% + \frac{1}{2} \times 4\% + 2\%$$
$$= 3\% + 6\% + 2\% + 2\% = 13\%$$

If $P = 3.763$, then $\Delta P = 13\%$ of P

$$= \frac{13P}{100} = \frac{13 \times 3.763}{100} = 0.489$$

As the error lies in first decimal place, the answer should be rounded off to first decimal place. Hence, we shall express the value of P after rounding it off as P = 3.8.

Question 2. 14. A book with many printing errors contains four different formulas for the displacement y of a particle undergoing a certain periodic motion:

(a) $y = a \sin \frac{2\pi t}{T}$ (b) $y = a \sin vt$

(c) $y = \left(\frac{a}{T}\right) \sin \frac{t}{a}$ (d) $y = \left(\frac{a}{\sqrt{2}}\right)\left(\sin \frac{2\pi t}{T} + \cos \frac{2\pi t}{T}\right)$

(a = maximum displacement of the particle, v = speed of the particle, T = time-period of motion) Rule out the wrong formulas on dimensional grounds.

Answer: According to dimensional analysis an equation must be dimensionally homogeneous.

(a) $y = a \sin \frac{2\pi t}{T}$

Here, $[\text{L.H.S.}] = [y] = [L]$

and $[\text{R.H.S.}] = \left[a \sin \frac{2\pi t}{T}\right] = \left[L \sin \frac{T}{T}\right] = [L]$

So, it is correct.

(b) $y = a \sin vt$

Here, $[y] = [L]$ and $[a \sin vt] = [L \sin (LT^{-1}. T)] = [L \sin L]$

So, the equation is wrong.

(c) $y = \left(\frac{a}{T}\right) \sin \frac{t}{a}$

Here, $[y] = [L]$ and $\left[\left(\frac{a}{T}\right) \sin \frac{t}{a}\right] = \left[\frac{L}{T} \sin \frac{T}{L}\right] = [LT^{-1} \sin TL^{-1}]$

So, the equation is wrong.

(d) $y = (a\sqrt{2})\left(\sin \frac{2\pi t}{T} + \cos \frac{2\pi t}{T}\right)$

Here, $[y] = [L], [a\sqrt{2}] = [L]$

and $\left[\sin \frac{2\pi t}{T} + \cos \frac{2\pi t}{T}\right] = \left[\sin \frac{T}{T} + \cos \frac{T}{T}\right]$ = dimensionless

So, the equation is correct.

Question 2. 15. A famous relation in physics relates 'moving mass' m to the 'rest mass' m0 of a particle in terms of its speed v and the speed of light c. (This relation first arose as a consequence of special relativity due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant c. He writes:

$$m = \frac{m_0}{(1 - v^2)^{1/2}}$$

Guess where to put the missing c.

Answer:

From the given equation, $\frac{m_0}{m} = \sqrt{1 - v^2}$

Left hand side is dimensionless.

Therefore, right hand side should also be dimensionless.

It is possible only when $\sqrt{1 - v^2}$ should be $\sqrt{1 - \frac{v^2}{c^2}}$.

Thus, the correct formula is $m = m_0 \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$

Question 2. 16. The unit of length convenient on the atomic scale is known as an angstrom and is denoted by A: 1 A = 10^{-10} m. The size of a hydrogen atom is about 0.5 A. What is the total atomic volume in m^3 of a mole of hydrogen atoms?

Answer:

Volume of one hydrogen atom = 4/3 πr^3 (volume of sphere)

= 4/3 x 3.14 x (0.5 x 10^{-10}) m^3 = 5.23 x 10^{-31} m^3

According to Avagadro's hypothesis, one mole of hydrogen contains 6.023 x 10^{23} atoms.

Atomic volume of 1 mole of hydrogen atoms

= 6.023 x 1023 x 5.23 x 10^{-31} = 3.15 x $10^{-7}$$m^3$.

Question 2. 17. One mole of an ideal gas at standard temperature and pressure occupies 22.4 L (molar volume). What is the ratio of molar volume to the atomic volume of a mole of hydrogen? (Take the size of hydrogen molecule to be about 1 A.) Why is this ratio so large?

Answer:

Volume of one mole of ideal gas, V_g

= 22.4 litre = 22.4 x 10^{-3} m^3

Radius of hydrogen molecule = 1A/2

= 0.5 A = 0.5 x 10^{-10} m

Volume of hydrogen molecule = 4/3 πr^3

= 4/3 x 22/7 (0.5 x 10^{-10})^3 m^3

= 0.5238 x 10^{-30} m^3

One mole contains 6.023 x 10^{23} molecules.

Volume of one mole of hydrogen, VH

= 0.5238 x 10^{-30} x 6.023 x 10^{23} m^3 = 3.1548 x 10^{-7} m^3

Now V_g/V_H=22.4 x 10^{-3}/3.1548 x 10^{-7} =7.1 x 10^4

The ratio is very large. This is because the interatomic separation in the gas is very large compared to the size of a hydrogen molecule.

Question 2. 18. Explain this common observation clearly: If you look out of the window of a fast moving train, the nearby trees, houses etc., seem to move rapidly in a direction opposite to the train's motion, but the distant objects (hill tops, the Moon, the stars etc.) seem to be stationary. (In fact, since you are aware that you are moving, these distant objects seem to move with you).

Answer: The line joining a given object to our eye is known as the line of sight. When a train moves rapidly, the line of sight of a passenger sitting in the train for nearby trees changes its direction rapidly. As a result, the nearby trees and other objects appear to run in a direction opposite to the train's motion. However, the line of sight of distant and large size objects e.g., hill tops, the Moon, the stars etc., almost remains unchanged (or changes by an extremely small angle). As a result, the distant object seems to be stationary.

Question 2. 19. The principle of 'parallax' is used in the determination of distances of very distant stars. The baseline AB is the line joining the Earth's two locations six months apart in its orbit around the Sun. That is, the baseline is about the diameter of the Earth's orbit

=3 x 10 nm. However, even the nearest stars are so distant that with such a long baseline, they show parallel only of the order of 1" (second) of arc or so. A parsec is a convenient unit of length on the astronomical scale. It is the distance of an object that will show a parallax of 1" (second) of arc from opposite ends of a baseline equal to the distance from the Earth to the Sun. How much is a parsec in terms of metres?
Answer:
From parallax method we can say
θ = b/D,where b=baseline ,D = distance of distant object or star
Since, θ =1" (s) and b=3 x 10^{11} m
D = b/20 =3 x 10^{11}/2 x 4.85 x 10^{-6} m
or D = 3 x 10^{11}/9.7 x 10^{-6} m = 30 x 10^{16}/9.7 m
= 3.09 x 10^{16} m = 3 x 10^{16} m.

Question 2. 20. The nearest star to our solar system is 4.29 light years away. How much is this distance in terms of parsecs? How much parallax would this star (named Alpha Centauri) show when viewed from two locations of the Earth six months apart in its orbit around the Sun?
Answer:
As we know, 1 light year = 9.46 x 10^{15} m
∴ 4.29 light years = 4.29 x 9.46 x 10^{15} = 4.058 x 10^{16} m
Also, 1 parsec = 3.08 x 10^{16} m
∴ 4.29 light years =4.508 x 10^{16}/3.80 x 10^{16} = 1.318 parsec = 1.32 parsec.
As a parsec distance subtends a parallax angle of 1" for a basis of radius of Earth's orbit around the Sun (r).In present problem base is the distance between two locations of the Earth six months apart in its orbit around the Sun = diameter of Earth's orbit (b = 2r).
∴ Parallax angle subtended by 1 parsec distance at this basis = 2 second (by definition of parsec).
∴ Parallax angle subtended by the star Alpha Centauri at the given basis θ = 1.32 x 2 = 2.64".

*********** END **********