



Indefinite Integrals Ex 19.32 Q11

$$I = \int \frac{x}{(x^2 + 4)\sqrt{x^2 + 1}} dx$$

$$\text{Let } x^2 + 1 = u^2$$

$$\Rightarrow 2x dx = 2u du$$

$$\begin{aligned} \therefore I &= \int \frac{u}{(u^2 + 3)u} du \\ &= \int \frac{1}{u^2 + 3} du \\ &= \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{u}{\sqrt{3}} \right) + C \end{aligned}$$

$$= \frac{1}{\sqrt{3}} \tan^{-1} \left(\sqrt{\frac{x^2 + 1}{3}} \right) + C$$

Indefinite Integrals Ex 19.32 Q12

$$\text{Let } I = \int \frac{1}{(1+x^2)\sqrt{1-x^2}} dx$$

$$\text{Let } x = \frac{1}{t}$$

$$\Rightarrow dx = -\frac{1}{t^2} dt$$

$$\begin{aligned} \therefore I &= \int \frac{-\frac{1}{t^2} dt}{\left(\frac{1}{t^2} + 1\right) \sqrt{\left(1 - \frac{1}{t^2}\right)}} \\ &= -\int \frac{tdt}{(t^2 + 1)\sqrt{t^2 - 1}} \end{aligned}$$

$$\text{Let } t^2 - 1 = u^2$$

$$\Rightarrow 2tdt = 2udu$$

$$\begin{aligned} I &= -\int \frac{udu}{(u^2 + 2)u} \\ &= -\int \frac{du}{u^2 + 2} \\ &= -\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + c \\ &= -\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{\sqrt{t^2 - 1}}{\sqrt{2}}\right) + c \end{aligned}$$

Thus,

$$I = -\frac{1}{\sqrt{2}} \tan^{-1}\left|\sqrt{\frac{1-x^2}{2x^2}}\right| + c$$

Indefinite Integrals Ex 19.32 Q13

$$\text{Let } I = \int \frac{1}{(2x^2 + 3)\sqrt{x^2 - 4}} dx$$

$$\text{Let } x = \frac{1}{t}$$

$$\Rightarrow dx = -\frac{1}{t^2} dt$$

$$\begin{aligned} \therefore I &= \int \frac{-\frac{1}{t^2} dt}{\left(\frac{2}{t^2} + 3\right)\sqrt{\left(\frac{1}{t^2} - 4\right)}} \\ &= -\int \frac{tdt}{(2 + 3t^2)\sqrt{1 - 4t^2}} \end{aligned}$$

$$\text{Let } 1 - 4t^2 = u^2$$

$$\Rightarrow -8tdt = 2u du$$

$$\begin{aligned} \therefore I &= \frac{1}{4} \int \frac{u du}{\left(\frac{11 - 3u^2}{4}\right)u} \\ &= \frac{1}{3} \int \frac{du}{\frac{11}{3} - u^2} \\ &= \frac{1}{2\sqrt{33}} \log \left| \frac{u - \sqrt{\frac{11}{3}}}{u + \sqrt{\frac{11}{3}}} \right| + c \\ &= \frac{1}{2\sqrt{33}} \log \left| \frac{\sqrt{1 - 4t^2} - \sqrt{\frac{11}{3}}}{\sqrt{1 - 4t^2} + \sqrt{\frac{11}{3}}} \right| + c \end{aligned}$$

Hence,

$$I = \frac{1}{2\sqrt{33}} \log \left| \frac{\sqrt{11}x + \sqrt{3x^2 - 12}}{\sqrt{11}x - \sqrt{3x^2 - 12}} \right| + c$$

Indefinite Integrals Ex 19.32 Q14

$$I = \int \frac{x}{(x^2 + 4)\sqrt{x^2 + 9}} dx$$

$$\text{Let } x^2 + 9 = u^2$$

$$\Rightarrow \quad 2x dx = 2u du$$

$$\begin{aligned} \therefore \quad I &= \int \frac{u}{(u^2 - 5)u} du \\ &= \int \frac{du}{u^2 - 5} \\ &= \frac{1}{2\sqrt{5}} \log \left(\frac{u - \sqrt{5}}{u + \sqrt{5}} \right) + C \\ &= \frac{1}{2\sqrt{5}} \log \left(\frac{\sqrt{x^2 + 9} - \sqrt{5}}{\sqrt{x^2 + 9} + \sqrt{5}} \right) + C \end{aligned}$$

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