Mean Value Theorems Ex 15.2 Q1(i)

Here,

$$f(x) = x^2 - 1 \text{ on } [2,3]$$

It is a polynomial function so it is continuous in $[2,3]$ and differentiable in $(2,3)$. So, both conditions of Lagrange's mean value theorem are satisfied.

Therefore, there exist a point $c \in (2,3)$ such that

$$f'(c) = \frac{f(3) - f(2)}{3 - 2}$$

$$2c = \frac{\left((3)^2 - 1\right) - \left((2)^2 - 1\right)}{1}$$

$$2c = (8 - 3)$$

$$c = \frac{5}{2} \in (2,3)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(ii)

Here,

$$f(x) = x^3 - 2x^2 - x + 3 \text{ on } [0,1]$$

Since, $f(x)$ is a polynomial function. So, $f(x)$ is continuous in $[0,1]$ and differentiable in $(0,1)$. So, Lagrange's mean value theorem is applicable. Thus, there exists a point $c \in (0,1)$ such that

$$f'(c) = \frac{f(1) - f(0)}{1 - 0}$$

$$\Rightarrow \quad 3c^2 - 4c - 1 = \frac{\left[(1)^3 - 2(1)^2 - (1) + 3\right] - 3}{1}$$

$$\Rightarrow \quad 3c^2 - 4c - 1 = 1 - 3$$

$$\Rightarrow \quad 3c^2 - 4c + 1 = 0$$

$$\Rightarrow \quad 3c^2 - 3c - c + 1 = 0$$

$$\Rightarrow \quad 3c(c - 1) - 1(c - 1) = 0$$

$$\Rightarrow \quad (3c - 1)(c - 1) = 0$$

$$\Rightarrow \quad c = \frac{1}{3} \in (0,1)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(iii)

Here,

$$f(x) = x(x - 1)$$

$$f(x) = x^2 - x \quad \text{on } [1,2]$$

We know that, polynomial function is continuous and differentiable. So, $f(x)$ is continuous in $[1,2]$ and $f(x)$ is differentiable in $(1,2)$. So, Lagrange's mean value theorem is applicable. Thus, there exists a point $c \in (1,2)$ such that

$$f'(c) = \frac{f(2) - f(1)}{2 - 1}$$

$$\Rightarrow \quad 2c - 1 = \frac{(4 - 2) - (1 - 1)}{1}$$

$$\Rightarrow \quad 2c - 1 = \frac{2 - 0}{1}$$

$$\Rightarrow \quad 2c = 3$$

$$\Rightarrow \quad c = \frac{3}{2} \in (1,2)$$

Hence, Lagrange's mean value theorem is verified.

Mean Value Theorems Ex 15.2 Q1(iv)

Here,

$$f(x) = x^2 - 3x + 2 \text{ on } [-1, 2]$$

We know that, polynomial function is continuous and differentiable. So, $f(x)$ is continuous in $[-1, 2]$ and differentiable in $(-1, 2)$. So, Lagrange's mean value theorem is applicable, so there exist a point $c \in (-1, 2)$ such that

$$f'(c) = \frac{f(2) - f(-1)}{2 + 1}$$

$$\Rightarrow \quad 2c - 3 = \frac{(4 - 6 + 2) - (1 + 3 + 2)}{3}$$

$$\Rightarrow \quad 2c - 3 = -\frac{6}{3}$$

$$\Rightarrow \quad 2c = 1$$

$$\Rightarrow \quad c = \frac{1}{2} \in (-1, 2)$$

Hence, Lagrange's mean value theorem is verified.

*********** END **********