Algebraic Identities Ex 4.5 Q1

Answer :

In the given problem, we have to find Product of equations

(i)Given $(3x+2y+2z)(9x^2+4y^2+4z^2-6xy-4yz-6zx)$

We shall use the identity

$$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$
$$=(3x)^3+(2y)^3+(2z)^3-3(3x)(2y)(2z)$$
$$=(3x)\times(3x)\times(3x)+(2y)\times(2y)\times(2y)+(2z)\times(2z)\times(2z)-3(3x)(2y)(2z)$$
$$=27x^3+8y^3+8z^3-36xyz$$

Hence the product of $(3x+2y+2z)(9x^2+4y^2+4z^2-6xy-4yz-6zx)$ is $\boxed{27x^3+8y^3+8z^3-36xyz}$

(ii) Given $(4x-3y+2z)(16x^2+9y^2+4z^2+12xy+6yz-8zx)$

We shall use the identity

$$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$
$$=(4x)^3+(3y)^3+(2z)^3-3(4x)(3y)(2z)$$
$$=(4x)\times(4x)\times(4x)+(-3y)\times(-3y)\times(-3y)+(2z)\times(2z)\times(2z)-3(4x)(-3y)(2z)$$
$$=64x^3-27y^3+8z^3+72xyz$$

Hence the product of $(4x-3y+2z)(16x^2+9y^2+4z^2+12xy+6yz-8zx)$ is

$\boxed{64x^3-27y^3+8z^3+72xyz}$

(iii) Given $(2a-3b-2c)(4a^2+9b^2+4c^2+6ab-6bc+8ca)$

We shall use the identity

$$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$
$$=(2a)^3+(3b)^3+(2c)^3-3(2a)(3b)(2c)$$
$$=(2a)\times(2a)\times(2a)+(-3b)\times(-3b)\times(-3b)+(-2c)\times(-2c)\times(-2c)-3(2a)(-3b)($$
$$=8a^3-27b^3-8c^3-36abc$$

Hence the product of $(2a-3b-2c)(4a^2+9b^2+4c^2+6ab-6bc+8ca)$ is $\boxed{8a^3-27b^3-8c^3-36abc}$

(iv) Given $(3x-4y+5z)(9x^2+16y^2+25z^2+12xy-15zx+20yz)$

We shall use the identity

$$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$
$$=(3x)^3+(-4y)^3+(5z)^3-3(3x)(4y)(5z)$$
$$=(3x)\times(3x)\times(3x)+(-4y)\times(-4y)\times(-4y)+(5z)\times(5z)\times(5z)-3(3x)(-4y)(5z)$$
$$=27x^3-64y^3+125z^3+180xyz$$

Hence the product of $(3x-4y+5z)(9x^2+16y^2+25z^2+12xy-15zx+20yz)$ is

$\boxed{27x^3-64y^3+125z^3+180xyz}$

Algebraic Identities Ex 4.5 Q2

Answer :

In the given problem, we have to find value of $x^3+y^3+z^3-3xyz$

Given $x+y+z=8, xy+yz+zx=20$

We shall use the identity

$$(x+y+z)^2 = x^2+y^2+z^2+2(xy+yz+za)$$
$$(x+y+z)^2 = x^2+y^2+z^2+2(20)$$
$$64 = x^2+y^2+z^2+40$$
$$64-40 = x^2+y^2+z^2$$
$$24 = x^2+y^2+z^2$$

We know that

$$x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$
$$x^3+y^3+z^3-3xyz = (x+y+z)\left[\left(x^2+y^2+z^2\right)-(xy+yz+zx)\right]$$

Here substituting $x+y+z=8, xy+yz+zx=20, x^2+y^2+z^2=24$ we get

$$x^3+y^3+z^3-3xyz = 8\left[(24-20)\right]$$
$$= 8\times 4$$
$$= 32$$

Hence the value of $x^3+y^3+z^3-3xyz$ is $\boxed{32}$.

Algebraic Identities Ex 4.5 Q3

Answer :

In the given problem, we have to find value of $a^3+b^3+c^3-3abc$

Given $a+b+c=9, ab+bc+ca=26$

We shall use the identity

$$(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$$
$$(a+b+c)^2 = a^2+b^2+c^2+2(26)$$
$$(9)^2 = a^2+b^2+c^2+52$$
$$81-52 = a^2+b^2+c^2$$
$$29 = a^2+b^2+c^2$$

We know that

$$a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$
$$a^3+b^3+c^3-3abc = (a+b+c)\left[\left(a^2+b^2+c^2\right)-(ab+bc+ca)\right]$$

Here substituting $a+b+c=9, ab+bc+ca=26, a^2+b^2+c^2=29$ we get,

$$a^3+b^3+c^3-3abc = 9\left[(29-26)\right]$$
$$= 9\times 3$$
$$= 27$$

Hence the value of $a^3+b^3+c^3-3abc$ is $\boxed{27}$.

*********** END **********