Trigonometric Ratios Ex 5.1 Q16

Answer :

Given: $\tan\theta = \frac{1}{\sqrt{7}}$ ......(1)

To show that $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{3}{4}$

Now, we know that

Since $\tan\theta = \frac{\text{Perpendicular side opposite to}\angle\theta}{\text{Base side adjacent to}\angle\theta}$ ......(2)

Therefore,

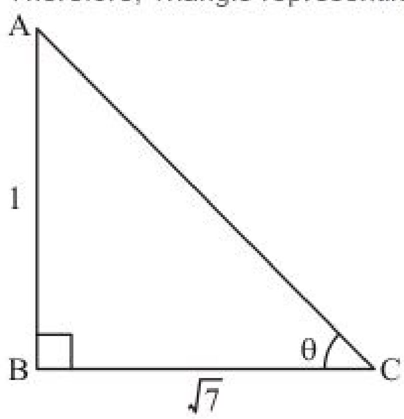
Comparing Equation (1) and (2)

We get,

Perpendicular side opposite to $\angle\theta = 1$

Base side adjacent to $\angle\theta = \sqrt{7}$

Therefore, Triangle representing angle θ is as shown below



Hypotenuse AC is unknown and it can be found by using Pythagoras theorem

Therefore by applying Pythagoras theorem

We get,

$$AC^2 = AB^2 + BC^2$$

Therefore by substituting the values of known sides

We get,

$$AC^2 = (1)^2 + \left(\sqrt{7}\right)^2$$

Therefore,

$$AC^2 = 1 + 7$$
$$AC^2 = 8$$
$$AC = \sqrt{8}$$
$$AC = \sqrt{2 \times 2 \times 2}$$

Therefore,

$$AC = 2\sqrt{2} \; \cdots\cdots \text{(3)}$$

Now, we know that

$$\sin\theta = \frac{\text{Perpendicular side opposite to} \angle\theta}{\text{Hypotenuse}}$$

Now from figure (a)

We get,

$$\sin\theta = \frac{AB}{AC}$$

Hypotenuse AC is unknown and it can be found by using Pythagoras theorem

Therefore by applying Pythagoras theorem

We get,

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Therefore,

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Now, we know that

$$\sin\theta = \frac{\text{Perpendicular side opposite to} \angle\theta}{\text{Hypotenuse}}$$

Now from figure (a)

We get,

$$\sin\theta = \frac{AB}{AC}$$

Now, we know that $\sec\theta = \frac{1}{\cos\theta}$

Therefore, from equation (6)

We get,

$$\sec\theta = \frac{1}{\frac{\sqrt{7}}{2\sqrt{2}}}$$

Therefore,

$$\sec\theta = \frac{2\sqrt{2}}{\sqrt{7}} \quad \text{...... (7)}$$

Now, *L.H.S* of the equation to be proved is as follows

$$L.H.S = \frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$$

Substituting the value of $\operatorname{cosec}\theta$ and $\sec\theta$ from equation (6) and (7)

We get,

$$L.H.S = \frac{\left[\left(2\sqrt{2}\right)^2\right] - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{\left[\left(2\sqrt{2}\right)^2\right] + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}$$

$$L.H.S = \frac{(4\times 2) - \frac{(4\times 2)}{7}}{(4\times 2) + \frac{(4\times 2)}{7}}$$

$$L.H.S = \frac{(8) - \frac{(8)}{7}}{(8) + \frac{(8)}{7}}$$

Now by taking L.C.M. in numerator as well as denominator

We get,

$$L.H.S = \frac{\frac{(7\times 8) - 8}{7}}{\frac{(7\times 8) + 8}{7}}$$

Therefore,

$$L.H.S = \frac{\frac{56-8}{7}}{\frac{56+8}{7}}$$

$$L.H.S = \frac{\frac{48}{7}}{\frac{64}{7}}$$

Therefore,

$$L.H.S = \frac{48}{7} \times \frac{7}{64}$$

$$L.H.S = \frac{48}{64}$$

$$L.H.S = \frac{3}{4}$$

$$L.H.S = \frac{3}{4} = R.H.S$$

Therefore,

$$\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{3}{4}$$

Hence proved that

$$\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{3}{4}$$

*********** END **********