

Exercise 7B

Question 1:

 $m = a\cos\theta + b\sin\theta$ and $n = a\sin\theta - b\cos\theta$

$$\text{LHS} = \text{m}^2 + \text{n}^2 = (a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2$$

$$= (a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta)$$

$$+ (a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta)$$

$$= a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$$

$$= a^2 + b^2 = \text{RHS}$$

:. LHS = RHS

Question 2:

 $x = a \sec \theta + b \tan \theta$, and $y = a \tan \theta + b \sec \theta$

LHS =
$$(x^2 - y^2)$$
 = $(a \sec \theta + b \tan \theta)^2$ - $(a \tan \theta + b \sec \theta)^2$
= $(a^2 \sec^2 \theta + b^2 \tan^2 \theta + 2ab \sec \theta \tan \theta)$
- $(a^2 \tan^2 \theta + b^2 \sec^2 \theta + 2ab \tan \theta \sec \theta)$
= $a^2 (\sec^2 \theta - \tan^2 \theta) - b^2 (\sec^2 \theta - \tan^2 \theta)$
= $a^2 - b^2$ = RHS

:. LHS = RHS

Question 3:

$$\begin{split} &\left(\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta\right) = 1 \quad \text{and} \quad \left(\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta\right) = 1 \\ &\text{Now,} \quad \left(\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta\right) = 1 \\ &\left(\text{Squaring both sides, we get}\right) \\ &\frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta - \frac{2xy}{ab}\sin\theta\cos\theta = 1 - - - - - - - (1) \\ &\left(\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta\right) = 1 \\ &\left(\text{Squaring both sides, we get}\right) \\ &\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + \frac{2xy}{ab}\sin\theta\cos\theta = 1 - - - - - - - (2) \\ &\text{Adding (1) \& (2), we get} \\ &\frac{x^2}{a^2}\left(\sin^2\theta + \cos^2\theta\right) + \frac{y^2}{b^2}\left(\sin^2\theta + \cos^2\theta\right) = 2 \\ &\frac{x^2}{a} + \frac{y^2}{b} = 2(\text{proved}) \end{split}$$

******* END *******