Let $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$.

Then, $kA = \begin{bmatrix} ka_1 & kb_1 & kc_1 \\ ka_2 & kb_2 & kc_2 \\ ka_3 & kb_3 & kc_3 \end{bmatrix}$.

$$\therefore |kA| = \begin{vmatrix} ka_1 & kb_1 & kc_1 \\ ka_2 & kb_2 & kc_2 \\ ka_3 & kb_3 & kc_3 \end{vmatrix}$$

$$= k^3 \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \qquad \text{(Taking out common factors } k \text{ from each row)}$$

$$= k^3 |A|$$

$$\therefore |kA| = k^3 |A|$$

Hence, the correct answer is C.

Question 16:

Which of the following is correct?

A. Determinant is a square matrix.

B. Determinant is a number associated to a matrix.

C. Determinant is a number associated to a square matrix.

D. None of these

Answer

Answer: C

We know that to every square matrix, $A = [aij]$ of order *n*. We can associate a number called the determinant of square matrix *A*, where $aij = (i, j)^{th}$ element of *A*.

Thus, the determinant is a number associated to a square matrix.

Hence, the correct answer is C.

Exercise 4.3

Question 1:

Find area of the triangle with vertices at the point given in each of the following:

(i) (1, 0), (6, 0), (4, 3) (ii) (2, 7), (1, 1), (10, 8)

(iii) (−2, −3), (3, 2), (−1, −8)

Answer

(i) The area of the triangle with vertices (1, 0), (6, 0), (4, 3) is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[1(0-3) - 0(6-4) + 1(18-0)\right]$$

$$= \frac{1}{2}\left[-3+18\right] = \frac{15}{2} \text{ square units}$$

(ii) The area of the triangle with vertices (2, 7), (1, 1), (10, 8) is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[2(1-8) - 7(1-10) + 1(8-10)\right]$$

$$= \frac{1}{2}\left[2(-7) - 7(-9) + 1(-2)\right]$$

$$= \frac{1}{2}\left[-14+63-2\right] = \frac{1}{2}\left[-16+63\right]$$

$$= \frac{47}{2} \text{ square units}$$

(iii) The area of the triangle with vertices (−2, −3), (3, 2), (−1, −8) is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[-2(2+8) + 3(3+1) + 1(-24+2)\right]$$

$$= \frac{1}{2}\left[-2(10) + 3(4) + 1(-22)\right]$$

$$=\frac{1}{2}\left[-2(10)+3(4)+1(-22)\right]$$

$$=\frac{1}{2}\left[-20+12-22\right]$$

$$=-\frac{30}{2}=-15$$

Hence, the area of the triangle is $|-15|=15\text{ square units}$.

Question 2:

Show that points

$A(a,b+c),B(b,c+a),C(c,a+b)$ are collinear

Answer

Area of ΔABC is given by the relation,

$$\Delta=\frac{1}{2}\begin{vmatrix}a & b+c & 1\\ b & c+a & 1\\ c & a+b & 1\end{vmatrix}$$

$$=\frac{1}{2}\begin{vmatrix}a & b+c & 1\\ b-a & a-b & 0\\ c-a & a-c & 0\end{vmatrix}\quad\left(\text{Applying }R_2\to R_2-R_1\text{ and }R_3\to R_3-R_1\right)$$

$$=\frac{1}{2}(a-b)(c-a)\begin{vmatrix}a & b+c & 1\\ -1 & 1 & 0\\ 1 & -1 & 0\end{vmatrix}$$

$$=\frac{1}{2}(a-b)(c-a)\begin{vmatrix}a & b+c & 1\\ -1 & 1 & 0\\ 0 & 0 & 0\end{vmatrix}\quad\left(\text{Applying }R_3\to R_3+R_2\right)$$

$$=0\qquad\left(\text{All elements of }R_3\text{ are }0\right)$$

Thus, the area of the triangle formed by points A, B, and C is zero.
Hence, the points A, B, and C are collinear.

Question 3:

Find values of k if area of triangle is 4 square units and vertices are

(i) (k, 0), (4, 0), (0, 2) (ii) (−2, 0), (0, 4), (0, k)

Answer

We know that the area of a triangle whose vertices are (x_1, y_1), (x_2, y_2), and (x_3, y_3) is the absolute value of the determinant (Δ), where

$$\Delta=\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$$

It is given that the area of triangle is 4 square units.

∴Δ = ± 4.

(i) The area of the triangle with vertices (k, 0), (4, 0), (0, 2) is given by the relation,

$$\Delta=\frac{1}{2}\begin{vmatrix}k & 0 & 1\\ 4 & 0 & 1\\ 0 & 2 & 1\end{vmatrix}$$

$$=\frac{1}{2}\left[k(0-2)-0(4-0)+1(8-0)\right]$$

$$=\frac{1}{2}\left[-2k+8\right]=-k+4$$

∴$-k + 4 = \pm 4$

When $-k + 4 = -4$, $k = 8$.

When $-k + 4 = 4$, $k = 0$.

Hence, k = 0, 8.

(ii) The area of the triangle with vertices (−2, 0), (0, 4), (0, k) is given by the relation,

$$\Delta=\frac{1}{2}\begin{vmatrix}-2 & 0 & 1\\ 0 & 4 & 1\\ 0 & k & 1\end{vmatrix}$$

$$=\frac{1}{2}\left[-2(4-k)\right]$$

$$=k-4$$

∴$k - 4 = \pm 4$

When $k - 4 = -4$, $k = 0$.

When $k - 4 = 4$, $k = 8$.

Hence, k = 0, 8.

Question 4:

(i) Find equation of line joining (1, 2) and (3, 6) using determinants

(ii) Find equation of line joining (3, 1) and (9, 3) using determinants

Answer

(i) Let P (x, y) be any point on the line joining points A (1, 2) and B (3, 6). Then, the points A, B, and P are collinear. Therefore, the area of triangle ABP will be zero.

$$\therefore \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2}\left[1(6-y)-2(3-x)+1(3y-6x)\right]=0$$

$$\Rightarrow 6-y-6+2x+3y-6x=0$$

$$\Rightarrow 2y-4x=0$$

$$\Rightarrow y=2x$$

Hence, the equation of the line joining the given points is $y = 2x$.

(ii) Let P (x, y) be any point on the line joining points A (3, 1) and

B (9, 3). Then, the points A, B, and P are collinear. Therefore, the area of triangle ABP will be zero.

$$\therefore \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2}\left[3(3-y)-1(9-x)+1(9y-3x)\right]=0$$

$$\Rightarrow 9-3y-9+x+9y-3x=0$$

$$\Rightarrow 6y-2x=0$$

$$\Rightarrow x-3y=0$$

Hence, the equation of the line joining the given points is $x - 3y = 0$.

*********** END **********