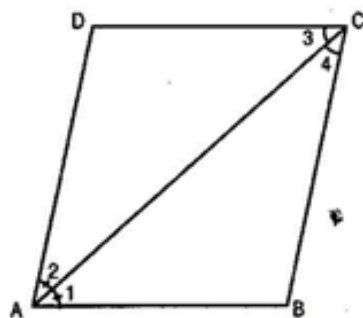




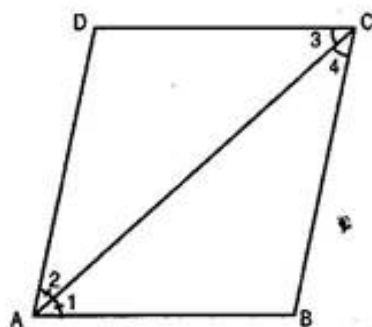
Exercise 8.1

Q6. Diagonal AC of a parallelogram ABCD bisects $\angle A$ (See figure). Show that:



- (i) It bisects $\angle C$ also.
- (ii) ABCD is a rhombus.

Ans. Diagonal AC bisects $\angle A$ of the parallelogram ABCD.



- (i) Since $AB \parallel DC$ and AC intersects them.

$$\therefore \angle 1 = \angle 3 \text{ [Alternate angles](i)}$$

$$\text{Similarly } \angle 2 = \angle 4 \text{(ii)}$$

$$\text{But } \angle 1 = \angle 2 \text{ [Given].....(iii)}$$

$$\therefore \angle 3 = \angle 4 \text{ [Using eq. (i), (ii) and (iii)]}$$

Thus AC bisects $\angle C$.

$$(ii) \angle 2 = \angle 3 = \angle 4 = \angle 1$$

$$\Rightarrow AD = CD \text{ [Sides opposite to equal angles]}$$

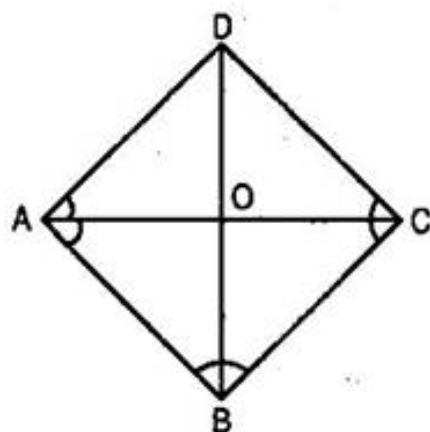
$$\therefore AB = CD = AD = BC$$

Hence ABCD is a rhombus.

Q7. ABCD is a rhombus. Show that the diagonal AC bisects $\angle A$ as well as $\angle C$ and diagonal BD bisects $\angle B$ as well as $\angle D$.

Ans. ABCD is a rhombus. Therefore, $AB = BC = CD = AD$

Let O be the point of bisection of diagonals.



$\therefore OA = OC$ and $OB = OD$

In $\triangle AOB$ and $\triangle AOD$,

$OA = OA$ [Common]

$AB = AD$ [Equal sides of rhombus]

$OB = OD$ (diagonals of rhombus bisect each other]

$\therefore \triangle AOB \cong \triangle AOD$ [By SSS congruency]

$\Rightarrow \angle OAD = \angle OAB$ [By C.P.C.T.]

$\Rightarrow OA$ bisects $\angle A$ (i)

Similarly, $\triangle BOC \cong \triangle DOC$ [By SSS congruency]

$$\Rightarrow \angle OCB = \angle OCD [\text{By C.P.C.T.}]$$

$$\Rightarrow OC \text{ bisects } \angle C \dots\dots\dots(\text{ii})$$

From eq. (i) and (ii), we can say that diagonal AC bisects $\angle A$ and $\angle C$.

Now in $\triangle AOB$ and $\triangle BOC$,

$$OB = OB [\text{Common}]$$

$$AB = BC [\text{Equal sides of rhombus}]$$

$$OA = OC (\text{diagonals of rhombus bisect each other})$$

$$\therefore \triangle AOB \cong \triangle COB [\text{By SSS congruency}]$$

$$\Rightarrow \angle OBA = \angle OBC [\text{By C.P.C.T.}]$$

$$\Rightarrow OB \text{ bisects } \angle B \dots\dots\dots(\text{iii})$$

$$\text{Similarly, } \triangle AOD \cong \triangle COD [\text{By SSS congruency}]$$

$$\Rightarrow \angle ODA = \angle ODC [\text{By C.P.C.T.}]$$

$$\Rightarrow OD \text{ bisects } \angle D \dots\dots\dots(\text{iv})$$

From eq. (iii) and (iv), we can say that diagonal BD bisects $\angle B$ and $\angle D$.

Q8. ABCD is a rectangle in which diagonal AC bisects $\angle A$ as well as $\angle C$. Show that:

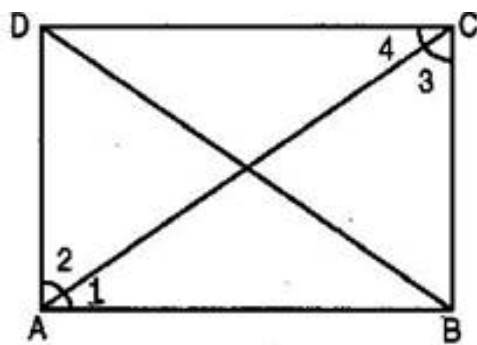
(i) ABCD is a square.

(ii) Diagonal BD bisects both $\angle B$ as well as $\angle D$.

Ans. ABCD is a rectangle. Therefore $AB = DC \dots\dots\dots(\text{i})$

$$\text{And } BC = AD$$

$$\text{Also } \angle A = \angle B = \angle C = \angle D = 90^\circ$$



(i) In $\triangle ABC$ and $\triangle ADC$

$$\angle 1 = \angle 2 \text{ and } \angle 3 = \angle 4$$

[AC bisects $\angle A$ and $\angle C$ (given)]

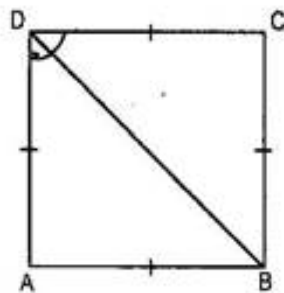
$$AC = AC \text{ [Common]}$$

$\therefore \triangle ABC \cong \triangle ADC$ [By ASA congruency]

$$\Rightarrow AB = AD \text{(ii)}$$

From eq. (i) and (ii), $AB = BC = CD = AD$

Hence ABCD is a square.



(ii) In $\triangle ABC$ and $\triangle ADC$

$$AB = BA \text{ [Since ABCD is a square]}$$

$$AD = DC \text{ [Since ABCD is a square]}$$

$$BD = BD \text{ [Common]}$$

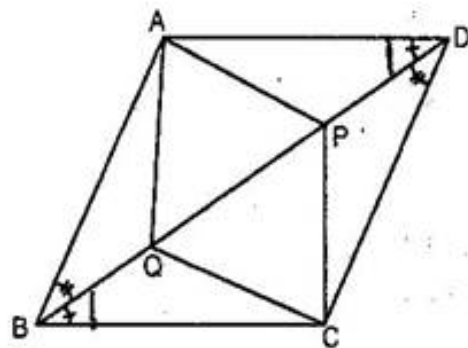
$\therefore \triangle ABD \cong \triangle CBD$ [By SSS congruency]

$$\Rightarrow \angle ABD = \angle CBD \text{ [By C.P.C.T.].....(iii)}$$

$$\text{And } \angle ADB = \angle CDB \text{ [By C.P.C.T.].....(iv)}$$

From eq. (iii) and (iv), it is clear that diagonal BD bisects both $\angle B$ and $\angle D$.

Q9. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that $DP = BQ$ (See figure). Show that:



- (i) $\triangle APD \cong \triangle CQB$
- (ii) $AP = CQ$
- (iii) $\triangle AQB \cong \triangle CPD$
- (iv) $AQ = CP$
- (v) APCQ is a parallelogram.

Ans. (i) In $\triangle APD$ and $\triangle CQB$,
 $DP = BQ$ [Given]

$\angle ADP = \angle QBC$ [Alternate angles ($AD \parallel BC$ and BD is transversal)]

$AD = CB$ [Opposite sides of parallelogram]

$\therefore \triangle APD \cong \triangle CQB$ [By SAS congruency]

(ii) Since $\triangle APD \cong \triangle CQB$

$\Rightarrow AP = CQ$ [By C.P.C.T.]

(iii) In $\triangle AQB$ and $\triangle CPD$,

$$BQ = DP[\text{Given}]$$

$$\angle ABQ = \angle PDC [\text{Alternate angles } (AB \parallel CD \text{ and } BD \text{ is transversal})]$$

$$AB = CD[\text{Opposite sides of parallelogram}]$$

$$\therefore \triangle AQB \cong \triangle CPD [\text{By SAS congruency}]$$

$$\text{(iv) Since } \triangle AQB \cong \triangle CPD$$

$$\Rightarrow AQ = CP[\text{By C.P.C.T.}]$$

$$\text{(v) In quadrilateral APCQ,}$$

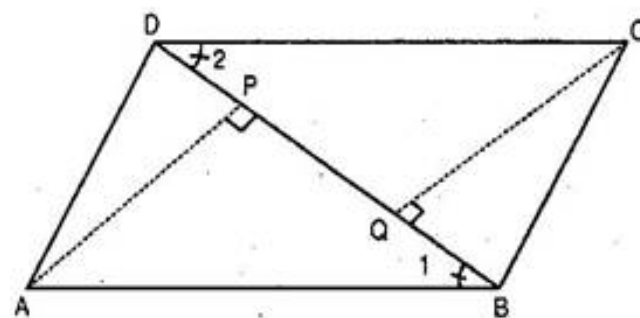
$$AP = CQ[\text{proved in part (i)}]$$

$$AQ = CP[\text{proved in part (iv)}]$$

Since opposite sides of quadrilateral APCQ are equal.

Hence APCQ is a parallelogram.

Q10. ABCD is a parallelogram and AP and CQ are the perpendiculars from vertices A and C on its diagonal BD (See figure). Show that:



$$\text{(i) } \triangle APB \cong \triangle CQD$$

$$\text{(ii) } AP = CQ$$

Ans. Given: ABCD is a parallelogram. $AP \perp BD$ and $CQ \perp BD$

Proof: (i) In $\triangle APB$ and $\triangle CQD$,

$\angle 1 = \angle 2$ [Alternate interior angles]

$AB = CD$ [Opposite sides of a parallelogram are equal]

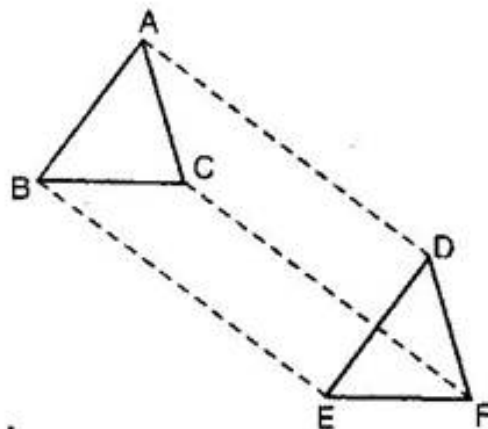
$\angle APB = \angle CQD = 90^\circ$

$\therefore \triangle APB \cong \triangle CQD$ [By ASA Congruency]

(ii) Since $\triangle APB \cong \triangle CQD$

$\therefore AP = CQ$ [By C.P.C.T.]

Q11. An $\triangle ABC$ and $\triangle DEF$, $AB = DE$, $AB \parallel DE$, $BC = EF$ and $BC \parallel EF$. Vertices A, B and C are joined to vertices D, E and F respectively (See figure). Show that:



(i) Quadrilateral ABED is a parallelogram.

(ii) Quadrilateral BEFC is a parallelogram.

(iii) $AD \parallel CF$ and $AD = CF$

(iv) Quadrilateral ACFD is a parallelogram.

(v) $AC = DF$

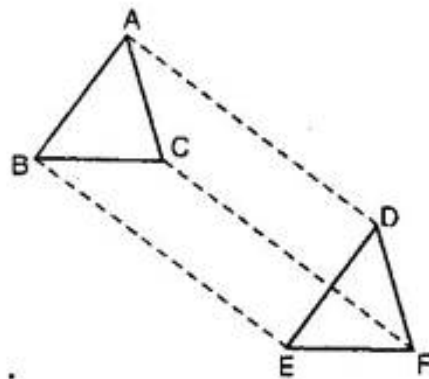
(vi) $\triangle ABC \cong \triangle DEF$

Ans. (i) In $\triangle ABC$ and $\triangle DEF$

$AB = DE$ [Given]

And $AB \parallel DE$ [Given]

$\therefore ABED$ is a parallelogram.



(ii) In $\triangle ABC$ and $\triangle DEF$

$BC = EF$ [Given]

And $BC \parallel EF$ [Given]

$\therefore BEFC$ is a parallelogram.

(iii) As $ABED$ is a parallelogram.

$\therefore AD \parallel BE$ and $AD = BE$ (i)

Also $BEFC$ is a parallelogram.

$\therefore CF \parallel BE$ and $CF = BE$ (ii)

From (i) and (ii), we get

$\therefore AD \parallel CF$ and $AD = CF$

(iv) As $AD \parallel CF$ and $AD = CF$

$\Rightarrow ACFD$ is a parallelogram.

(v) As $ACFD$ is a parallelogram.

$\therefore AC = DF$

(vi) In $\triangle ABC$ and $\triangle DEF$,

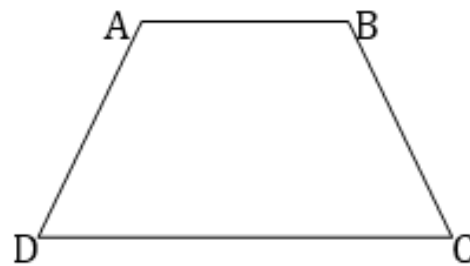
$AB = DE$ [Given]

$BC = EF$ [Given]

$AC = DF$ [Proved]

$\therefore \triangle ABC \cong \triangle DEF$ [By SSS congruency]

Q12. ABCD is a trapezium in which $AB \parallel CD$ and $AD = BC$ (See figure). Show that:



(i) $\angle A = \angle B$

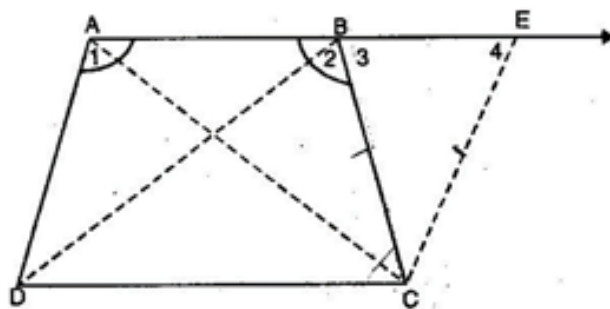
(ii) $\angle C = \angle D$

(iii) $\triangle ABC \cong \triangle BAD$

(iv) Diagonal $AC =$ Diagonal BD

Ans. Given: ABCD is a trapezium.

$AB \parallel CD$ and $AD = BC$



To prove: (i) $\angle A = \angle B$

$$(ii) \angle C = \angle D$$

$$(iii) \triangle ABC \cong \triangle BAD$$

$$(iv) \text{Diag. AC} = \text{Diag. BD}$$

Construction: Draw $CE \parallel AD$ and extend AB to intersect CE at E .

Proof: (i) As $AECD$ is a parallelogram. [By construction]

$$\therefore AD = EC$$

$$\text{But } AD = BC \text{ [Given]}$$

$$\therefore BC = EC$$

$$\Rightarrow \angle 3 = \angle 4 \text{ [Angles opposite to equal sides are equal]}$$

$$\text{Now } \angle 1 + \angle 4 = 180^\circ \text{ [Interior angles]}$$

$$\text{And } \angle 2 + \angle 3 = 180^\circ \text{ [Linear pair]}$$

$$\Rightarrow \angle 1 + \angle 4 = \angle 2 + \angle 3$$

$$\Rightarrow \angle 1 = \angle 2 \text{ [}\because \angle 3 = \angle 4 \text{]}$$

$$\Rightarrow \angle A = \angle B$$

$$(ii) \angle 3 = \angle C \text{ [Alternate interior angles]}$$

$$\text{And } \angle D = \angle 4 \text{ [Opposite angles of a parallelogram]}$$

$$\text{But } \angle 3 = \angle 4 \text{ [}\triangle BCE \text{ is an isosceles triangle]}$$

$$\therefore \angle C = \angle D$$

$$(iii) \text{ In } \triangle ABC \text{ and } \triangle BAD,$$

$$AB = AB \text{ [Common]}$$

$$\angle 1 = \angle 2 \text{ [Proved]}$$

$$AD = BC \text{ [Given]}$$

$$\therefore \triangle ABC \cong \triangle BAD \text{ [By SAS congruency]}$$

$$(iv) \text{ We had observed that,}$$

$$\therefore \triangle ABC \cong \triangle BAD$$

$$\Rightarrow AC = BD \text{ [By C.P.C.T.]}$$

***** END *****