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Differentials Errors and Approximation Ex14.1 Q1

$$\text{Let } x = \frac{\pi}{2},\ x + \Delta x = \frac{22}{14}$$

$$\Delta x = \frac{22}{14} - x$$

$$\Delta x = \left(\frac{22}{14} - \frac{\pi}{2}\right)$$

$$y = \sin x$$

$$\frac{dy}{dx} = \cos x$$

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} = \frac{\cos \pi}{2}$$

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} = 0$$

$$\therefore \quad \Delta y = \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} \times \Delta ax$$

$$= 0 \times \left(\frac{22}{14} - \frac{\pi}{2}\right)$$

$$\Delta y = 0$$

So, there is no change in y.

Differentials Errors and Approximation Ex14.1 Q2

Let $x = 10,\ x + \Delta x = 9.8$

$\Delta x = 9.8 - x$

$= 9.8 - 10$

$\Delta x = -0.2$

$y = \frac{4}{3}\pi x^3$ [volume of sphere]

$\frac{dy}{dx} = 4\pi r^2$

$\left(\frac{dy}{dx}\right)_{x=10} = 4\pi (10)^2$

$\left(\frac{dy}{dx}\right)_{x=10} = 400\pi \text{ cm}^2$

$\Delta y = \left(\frac{dy}{dx}\right)_{x=10} \times \Delta x$

$= 400\pi \times (-0.2)$

$\Delta y = -80\pi \text{ cm}^3$

So, approximate diecocase in volume is $80\pi \text{ cm}^3$.

Differentials Errors and Approximation Ex14.1 Q3

Let $x = 10,\ x + \Delta x = 10 + \frac{k}{100} \times 10$

$x + \Delta x = 10 + 0.k$

$\Rightarrow \quad \Delta x = 10 + 0.k - 10$

$\Delta x = 0.k$

$y = \pi r^2$

$\frac{dy}{dx} = 2\pi r$

$\left(\frac{dy}{dx}\right)_{x=10} = 2\pi (10)$

$\left(\frac{dy}{dx}\right)_{x=10} = 20\pi \text{ cm}$

So,

$\Delta y = \left(\frac{dy}{dx}\right)_{x=10} \times \Delta x$

$= (20\pi) \times (0.k)$

$\Delta y = 2k\pi \text{cm}^2$

Area of the plate increases by $2k\pi \text{ cm}^2$.

Differentials Errors and Approximation Ex14.1 Q4

Let $\text{length}(L) = x$

$$x + \Delta x = x + \frac{x}{100}$$

$$\Delta x = 0.01x$$

Now,

$$y = 6x^2$$

$$\frac{dy}{dx} = 12x \text{ cm}$$

So,

$$\Delta y = \left(\frac{dy}{dx}\right) \times \Delta x$$

$$= (12x)(0.01x)$$

$$\Delta y = 0.12x^2 \text{ cm}^2$$

$$= 6(0.02)x^2$$

$$= 2\% \text{ of } 6x^2$$

Percentage error in area is 2%.

********** END **********