Indefinite Integrals Ex 19.17 Q5

Let $I = \int \frac{1}{\sqrt{(x-\alpha)(\beta - x)}} dx, (\text{as} \;\; \beta > \alpha)$

$$= \int \frac{1}{\sqrt{-x^2 - x(\alpha+\beta) - \alpha\beta}} dx$$

$$= \int \frac{1}{\sqrt{-\left[x^2 - 2x\left(\frac{\alpha+\beta}{2}\right) + \left(\frac{\alpha+\beta}{2}\right)^2 - \left(\frac{\alpha+\beta}{2}\right)^2 + \alpha\beta\right]}} dx$$

$$= \int \frac{1}{\sqrt{-\left[\left(x - \frac{\alpha+\beta}{2}\right)^2 - \left(\frac{\alpha+\beta}{2}\right)^2\right]}} dx$$

$$= \int \frac{1}{\sqrt{\left[\left(\frac{\beta-\alpha}{2}\right)^2 - \left(x - \frac{\alpha+\beta}{2}\right)^2\right]}} dx, \qquad [\because \qquad \beta > \alpha]$$

Let $\left(x - \frac{\alpha+\beta}{2}\right) = t$

$\Rightarrow \qquad dx = dt$

$$I = \int \frac{1}{\sqrt{\left(\frac{\beta-\alpha}{2}\right)^2 - t^2}} dt$$

$$= \sin^{-1}\left(\frac{t}{\frac{\beta-\alpha}{2}}\right) + c \qquad \left[\text{Since} \;\; \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \sin^{-1}\left(\frac{2\left(x - \frac{\alpha+\beta}{2}\right)}{\beta - \alpha}\right) + c$$

$$I = \sin^{-1}\left(\frac{2x - \alpha - \beta}{\beta - \alpha}\right) + c$$

Indefinite Integrals Ex 19.17 Q6

Let $I = \int \frac{1}{\sqrt{7 - 3x - 2x^2}} dx$

$$= \int \frac{1}{\sqrt{-2\left[x^2 + \frac{3}{2}x - \frac{7}{2}\right]}} dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{-\left[x^2 + 2x\left(\frac{3}{4}\right) + \left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2 - \frac{7}{2}\right]}} dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{-\left[\left(x - \frac{3}{4}\right)^2 - \frac{65}{16}\right]}} dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\left(\frac{\sqrt{65}}{4}\right)^2 - \left(x + \frac{3}{4}\right)^2}} dx$$

Let $\left(x + \frac{3}{4}\right) = t$

$\Rightarrow \quad dx = dt$

$$I = \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\left(\frac{\sqrt{65}}{4}\right)^2 - t^2}} dt$$

$$= \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{t}{\frac{\sqrt{41}}{4}}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{4\left(x + \frac{3}{4}\right)}{\sqrt{65}}\right) + c$$

$$I = \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{4x + 3}{\sqrt{65}}\right) + c$$

Indefinite Integrals Ex 19.17 Q7

Let $I = \int \frac{1}{\sqrt{16 - 6x - x^2}} dx$

$$= \int \frac{1}{\sqrt{-\left[x^2 + 6x - 16\right]}} dx$$

$$= \int \frac{1}{\sqrt{-\left[x^2 + 2x(3) + (3)^2 - (3)^2 - 16\right]}} dx$$

$$= \int \frac{1}{\sqrt{-\left[(x + 3)^2 - 25\right]}} dx$$

$$= \int \frac{1}{\sqrt{25 - (x + 3)^2}} dx$$

Let $(x + 3) = t$

$\Rightarrow \quad dx = dt$

$$I = \int \frac{1}{\sqrt{5^2 - t^2}} dt$$

$$= \sin^{-1}\left(\frac{t}{5}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \sin^{-1}\left(\frac{x + 3}{5}\right) + c$$

Indefinite Integrals Ex 19.17 Q8

$7-6x-x^2$ can be written as $7-\left(x^2+6x+9-9\right)$.

Therefore,

$$7-\left(x^2+6x+9-9\right)$$
$$=16-\left(x^2+6x+9\right)$$
$$=16-(x+3)^2$$
$$=(4)^2-(x+3)^2$$

$$\therefore \int \frac{1}{\sqrt{7-6x-x^2}}dx=\int \frac{1}{\sqrt{(4)^2-(x+3)^2}}dx$$

Let $x+3=t$

$\Rightarrow dx=dt$

$$\Rightarrow \int \frac{1}{\sqrt{(4)^2-(x+3)^2}}dx=\int \frac{1}{\sqrt{(4)^2-(t)^2}}dt$$
$$=\sin^{-1}\left(\frac{t}{4}\right)+C$$
$$=\sin^{-1}\left(\frac{x+3}{4}\right)+C$$

Indefinite Integrals Ex 19.17 Q9

We have $\displaystyle\int \frac{dx}{\sqrt{5x^2-2x}}=\int \frac{dx}{\sqrt{5\left(x^2-\frac{2x}{5}\right)}}$

$$=\frac{1}{\sqrt{5}}\int \frac{dx}{\sqrt{\left(x-\frac{1}{5}\right)^2-\left(\frac{1}{5}\right)^2}} \quad \text{(completing the square)}$$

Put $x-\frac{1}{5}=t$. Then $dx=dt$.

Therefore, $\displaystyle\int \frac{dx}{\sqrt{5x^2-2x}}=\frac{1}{\sqrt{5}}\int \frac{dt}{\sqrt{t^2-\left(\frac{1}{5}\right)^2}}$

$$=\frac{1}{\sqrt{5}}\log\left|t+\sqrt{t^2-\left(\frac{1}{5}\right)^2}\right|+C \qquad [\text{by } 7.4\ (4)]$$
$$=\frac{1}{\sqrt{5}}\log\left|x-\frac{1}{5}+\sqrt{x^2-\frac{2x}{5}}\right|+C$$

*********** END **********