Definite Integrals Ex 20.3 Q11

$$\int_0^{\frac{\pi}{2}} \left|\cos 2x\right| dx$$

$$= \int_0^{\frac{\pi}{4}} -\cos 2x \, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} +\cos 2x \, dx$$

$$= \left[\frac{+\sin 2x}{2}\right]_0^{\frac{\pi}{4}} + \left[\frac{-\sin 2x}{2}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left[\sin\frac{\pi}{2} - \sin 0\right] + \frac{1}{2}\left[\sin\pi + \sin\frac{\pi}{2}\right]$$

$$= \frac{1}{2}[1] + \frac{1}{2}[1]$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1$$

$$\therefore \int_0^{\frac{\pi}{2}} \left|\cos 2x\right| dx = 1$$

Definite Integrals Ex 20.3 Q12

$$\int_0^{2\pi} \left|\sin x\right| dx = \int_0^{\pi} \sin x dx + \int_{\pi}^{2\pi} -\sin x dx$$

$$= \left[-\cos x\right]_0^{\pi} + \left[\cos x\right]_{\pi}^{2\pi}$$

$$= [1+1] + [1+1]$$

$$\int_0^{2\pi} \left|\sin x\right| dx = 4$$

Definite Integrals Ex 20.3 Q13

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} |\sin x|\, dx$$

$$= \int_{-\frac{\pi}{4}}^{0} -\sin x\, dx + \int_{0}^{\frac{\pi}{4}} \sin x\, dx$$

$$= [\cos x]_{-\frac{\pi}{4}}^{0} + [-\cos x]_{0}^{\frac{\pi}{4}}$$

$$= \left(1 - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} - 1\right)$$

$$= \left(2 - \sqrt{2}\right)$$

$$\therefore \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} |\sin x|\, dx = 2 - \sqrt{2}$$

Definite Integrals Ex 20.3 Q14

We have,

$$I = \int_{2}^{8} |x - 5|\, dx$$

We have,

$$|x - 5| = \begin{cases} x - 5 & \text{if } x \in (5, 8) \\ -(x - 5) & \text{if } x \in (2, 5) \end{cases}$$

Hence,

$$I = \int_{2}^{5} -(x - 5)\, dx + \int_{5}^{8} (x - 5)\, dx$$

$$= -\left[\frac{x^2}{2} - 5x\right]_2^5 + \left[\frac{x^2}{2} - 5x\right]_5^8$$

$$= -\left[\left(\frac{25}{2} - 25\right) - \left(\frac{4}{2} - 10\right)\right] + \left[\left(\frac{64}{2} - 40\right) - \left(\frac{25}{2} - 25\right)\right]$$

$$= -\left[-\frac{25}{2} + 8\right] + \left[(-8) + \left(\frac{25}{2}\right)\right]$$

$$= \frac{25}{2} - 8 - 8 + \frac{25}{2}$$

$$= 25 - 16 = 9$$

$$\therefore \int_{2}^{8} |x - 5|\, dx = 9$$

*********** END **********