Definite Integrals Ex 20.5 Q9

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 1,\ b = 2$ and $f(x) = x^2$

$$\therefore h = \frac{1}{n} \Rightarrow nh = 1$$

Thus, we have,

$$I = \int_1^2 x^2 dx$$

$$= \lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(1+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[1+(1+h)^2+(1+2h)^2+-----+\left(1+(n-1)h\right)^2\right]$$

$$= \lim_{h\to 0} h\left[1+\left(1+2h+h^2\right)+\left(1+2\times 2h+2\times 2h^2\right)+----\left(1+2\times(n-1)h+(1-n)^2h^2\right)\right]$$

$$= \lim_{h\to 0} h\left[n+2h\{1+2+3---(n-1)\}+h^2\left\{1^2+2^2+3^2+---(n-1)^2\right\}\right]$$

$$\because h = \frac{1}{n} \text{ \& if } h\to 0 \Rightarrow n\to\infty$$

$$= \lim_{n\to\infty} \frac{1}{n}\left[n+\frac{2}{n}\frac{n(n-1)}{2}+\frac{1}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} 1+\frac{n^2}{n^2}\left(1-\frac{1}{n}\right)+\frac{1}{6n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= 1+1+\frac{2}{6} = \frac{7}{3}$$

$$\therefore \int_1^2 x^2 dx = \frac{7}{3}$$

Definite Integrals Ex 20.5 Q10

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 2,\ b = 3$ and $f(x) = 2x^2+1$

$\therefore\ h = \frac{1}{n} \Rightarrow nh = 1$

Thus, we have,

$$I = \int_2^3 (2x^2+1)dx$$

$$= \lim_{h\to 0} h\left[f(2)+f(2+h)+f(2+2h)+----f(2+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[(2\times 2^2+1)\{2(2+h)^2+1\}+\{2(2+2h)^2+1\}+----+\{2(2+(n-1)h)^2+1\}\right]$$

$$= \lim_{h\to 0} h\left[9n+8h(1+2+3---)+2h^2(1^2+2^2+3^2+--)\right]$$

$\because h = \frac{1}{n}$ & if $h\to 0 \Rightarrow n\to\infty$

$$= \lim_{n\to\infty} \frac{1}{n}\left[9n+\frac{8}{n}\frac{n(n-1)}{2}+\frac{2}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} 9+\frac{4}{n^2}n^2\left(1-\frac{1}{n}\right)+\frac{1}{3n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= 9+4+\frac{2}{3} = \frac{41}{3}$$

$$\therefore \int_2^3 (2x^2+1)dx = \frac{41}{3}$$

Definite Integrals Ex 20.5 Q11

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h = \frac{b-a}{n}$$

Here $a = 1,\ b = 2$ and $f(x) = x^2-1$

$\therefore\ h = \frac{1}{n} \Rightarrow nh = 1$

Thus, we have,

$$I = \int_1^2 (x^2-1)dx$$

$$= \lim_{h\to 0} h\left[f(1)+f(1+h)+f(1+2h)+----f(1+(n-1)h)\right]$$

$$= \lim_{h\to 0} h\left[(1^2-1)\{(1+h)^2-1\}+\{(1+2h)^2-1\}+-----+\{(1+(n-1)h)^2-1\}\right]$$

$$= \lim_{h\to 0} h\left[0+2h(1+2+3+---)+h^2(1+2^2+3^2+--)\right]$$

$\because h = \frac{1}{n}$ & if $h\to 0 \Rightarrow n\to\infty$

$$= \lim_{n\to\infty} \frac{1}{n}\left[\frac{2}{n}\frac{n(n-1)}{2}+\frac{1}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$= \lim_{n\to\infty} \frac{1}{n^2}n^2\left(1-\frac{1}{n}\right)+\frac{1}{6n^3}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$= 1+\frac{2}{6} = \frac{4}{3}$$

$$\therefore \int_1^2 (x^2-1)dx = \frac{4}{3}$$

Definite Integrals Ex 20.5 Q12

We have,

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[f(a)+f(a+h)+f(a+2h)+----f(a+(n-1)h)\right]$$

$$\text{where } h=\frac{b-a}{n}$$

Here $a=0,\ b=2$ and $f(x)=x^2+4$

$$\therefore h=\frac{2}{n} \Rightarrow nh=2$$

Thus, we have,

$$I=\int_0^2\left(x^2+4\right)dx$$

$$=\lim_{h\to 0} h\left[f(0)+f(h)+f(2h)+----f(0+(n-1)h)\right]$$

$$=\lim_{h\to 0} h\left[4\left(h^2+4\right)+\left\{(2h)^2+4\right\}+----\left\{(n-1)h^2+4\right\}\right]$$

$$\because h=\frac{2}{n} \ \& \text{ if } h\to 0\Rightarrow n\to\infty$$

$$=\lim_{n\to\infty}\frac{2}{n}\left[4n+\frac{4}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$$

$$=\lim_{n\to\infty} 8+\frac{4}{3n^2}n^3\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)$$

$$=8+\frac{4\times 2}{3}=\frac{32}{3}$$

$$\therefore \int_0^2\left(x^2+4\right)dx=\frac{32}{3}$$

*********** END **********