Arithematic Progressions Ex 19.5 Q3(ii)

(ii) T.P $b+c-a, c+a-b, a+b-c$ are in A.P.

$b+c-a, c+a-b, a+b-c$ are in A.P only if $(c+a-b)-(b+c-a)=(a+b-c)-(c+a-b)$

LHS $\Rightarrow (c+a-b)-(b+c-a)$

$\Rightarrow \quad 2a-2b \qquad$ ---(i)

RHS $\Rightarrow (a+b-c)-(c+a-b)$

$\Rightarrow \quad 2b-2c \qquad$ ---(ii)

Since,

a, b, c are in A.P

$\therefore \quad b-a=c-b$

or $\quad a-b=b-c \qquad$ ---(iii)

From (i), (ii) and (iii)

LHS = RHS

Thus, given numbers

$b+c-a, c+a-b, a+b-c$ are in A.P

Arithematic Progressions Ex 19.5 Q3(iii)

To prove $bc-a^2, ca-b^2, ab-c^2$ are in A.P

$$\left(ca-b^2\right)-\left(bc-a^2\right)=\left(ab-c^2\right)-\left(ca-b^2\right)$$

LHS $= \left(a-b^2-bc+a^2\right)$

$= (a-b)[a+b+c] \qquad$ ---(i)

RHS $= ab-c^2-ca+b^2$

$= (b-c)[a+b+c] \qquad$ ---(ii)

and since a, b, c are in ab

$b-c=a-b$

$\therefore \quad$ LHS = RHS

and

Thus, $bc-a^2, ca-b^2, ab-c^2$ are in A.P

Arithematic Progressions Ex 19.5 Q4

(i) If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$

LHS $= \frac{1}{b} - \frac{1}{a}$

$$= \frac{a-b}{ab} = \frac{c(a-b)}{abc} \qquad \text{---(i)}$$

RHS $= \frac{1}{c} - \frac{1}{b}$

$$= \frac{a(b-c)}{abc} \qquad \text{---(ii)}$$

Since, $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P

$$\frac{b+c}{a} - \frac{c+a}{b} = \frac{c+a}{b} - \frac{a+b}{c}$$

$$\frac{b^2+cb-ac-a^2}{ab} = \frac{c^2+ac-ab-b^2}{bc}$$

$$\Rightarrow \quad \frac{(b-a)(a+b+c)}{ab} = \frac{(c-b)(a+b+c)}{bc}$$

or $$\frac{a(b-c)}{abc} = \frac{c(a-b)}{abc} \qquad \text{---(iii)}$$

From (i), (ii) and (iii)

LHS = RHS

Hence, $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

(ii) If bc, ca, ab are in A.P

Then,

$$ca - bc = ab - ca$$

$$c(a-b) = a(b-c) \qquad \text{---(i)}$$

If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P

$$\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$

$$\Rightarrow \quad c(a-b) = a(b-c) \qquad \text{---(ii)}$$

Thus, the condition necessary to prove bc, ca, ab in A.P is fullfilled.

Thus, bc, ca, ab, are in A.P.

*********** END **********