Co-Ordinate Geometry Ex 14.3 Q42

Answer :

We have a rectangle ABCD formed by joining the points A (−1,−1); B (−1, 4); C (5, 4) and D (5,−1).
The mid-points of the sides AB, BC, CD and DA are P, Q, R, S respectively.
We have to find that whether PQRS is a square, rectangle or rhombus.
In general to find the mid-point $\mathrm{P}(x,y)$ of two points $\mathrm{A}(x_1,y_1)$ and $\mathrm{B}(x_2,y_2)$ we use section formula as,

$$\mathrm{P}(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Therefore mid-point P of side AB can be written as,

$$P(x,y)=\left(\frac{-1-1}{2},\frac{4-1}{2}\right)$$

Now equate the individual terms to get,

$$x=-1$$

$$y=\frac{3}{2}$$

So co-ordinates of P is $\left(-1,\frac{3}{2}\right)$

Similarly mid-point Q of side BC can be written as,

$$Q(x,y)=\left(\frac{5-1}{2},\frac{4+4}{2}\right)$$

Now equate the individual terms to get,

$$x=2$$

$$y=4$$

So co-ordinates of Q is (2, 4)

Similarly mid-point R of side CD can be written as,

$$R(x,y)=\left(\frac{5+5}{2},\frac{4-1}{2}\right)$$

Now equate the individual terms to get,

$$x=5$$

$$y=\frac{3}{2}$$

So co-ordinates of R is $\left(5,\frac{3}{2}\right)$

Similarly mid-point S of side DA can be written as,

$$\mathrm{S}(x,y)=\left(\frac{5-1}{2},\frac{-1-1}{2}\right)$$

Now equate the individual terms to get,

$$x=2$$

$$y=-1$$

So co-ordinates of S is (2,−1)

So we should find the lengths of sides of quadrilateral PQRS.

$$\mathrm{PQ}=\sqrt{(2+1)^2+\left(4-\frac{3}{2}\right)^2}$$

$$=\sqrt{9+\frac{25}{4}}$$

$$= \frac{\sqrt{61}}{2}$$

$$QR = \sqrt{(2-5)^2 + \left(4 - \frac{3}{2}\right)^2}$$

$$= \sqrt{9 + \frac{25}{4}}$$

$$= \frac{\sqrt{61}}{2}$$

$$RS = \sqrt{(5-2)^2 + \left(\frac{3}{2} + 1\right)^2}$$

$$= \sqrt{9 + \frac{25}{4}}$$

$$= \frac{\sqrt{61}}{2}$$

$$SP = \sqrt{(2+1)^2 + \left(-1 - \frac{3}{2}\right)^2}$$

$$= \sqrt{9 + \frac{25}{4}}$$

$$= \frac{\sqrt{61}}{2}$$

All the sides of quadrilateral are equal.

So now we will check the lengths of the diagonals.

$$PR = \sqrt{(5+1)^2 + \left(\frac{3}{2} - \frac{3}{2}\right)^2}$$
$$= 6$$
$$QS = \sqrt{(2-2)^2 + (4+1)^2}$$
$$= 5$$

All the sides are equal but the diagonals are unequal. Hence ABCD is a rhombus.

Co-Ordinate Geometry Ex 14.3 Q43

Answer :

Let A (−3, 2); B (−5,−5); C (2,−3) and D (4, 4) be the vertices of a quadrilateral. We have to prove that the quadrilateral ABCD is a rhombus.

So we should find the lengths of sides of quadrilateral ABCD.

$$AB = \sqrt{(-5+3)^2 + (-5-2)^2}$$
$$= \sqrt{4+49}$$
$$= \sqrt{53}$$
$$BC = \sqrt{(2+5)^2 + (-3+5)^2}$$
$$= \sqrt{4+49}$$
$$= \sqrt{53}$$
$$CD = \sqrt{(4-2)^2 + (4+3)^2}$$
$$= \sqrt{4+49}$$
$$= \sqrt{53}$$
$$AD = \sqrt{(4+3)^2 + (4-2)^2}$$
$$= \sqrt{4+49}$$
$$= \sqrt{53}$$

All the sides of quadrilateral are equal. Hence ABCD is a rhombus.

********** END **********