Co-Ordinate Geometry Ex 14.5 Q1

Answer :

We know area of triangle formed by three points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) is given by

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

(i) The vertices are given as (6, 3), (−3, 5), (4, −2).

$$\Delta = \frac{1}{2}\left|6(5+2) - 3(-2-3) + 4(3-5)\right|$$
$$= \frac{1}{2}\left|6\times 7 - 3\times(-5) + 4\times(-2)\right|$$
$$= \frac{1}{2}\left|42 + 15 - 8\right|$$
$$= \frac{49}{2} \text{ sq units}$$

(ii) The vertices are given as $(at_1^2,\ 2at_1),\ (at_2^2,\ 2at_2),\ (at_3^2,\ 2at_3)$.

$$\Delta = \frac{1}{2}\left|at_1^2(2at_2 - 2at_3) + at_2^2(2at_3 - 2at_1) + at_3^2(2at_1 - 2at_2)\right|$$
$$= \frac{1}{2}\cdot 2a^2\left|(t_1^2t_2 - t_1^2t_3) + (t_2^2t_3 - t_2^2t_1) + (t_3^2t_1 - t_3^2t_2)\right|$$
$$= a^2\left|(t_1^2t_2 - t_2^2t_1) + (t_2^2t_3 - t_1^2t_3) + (t_3^2t_1 - t_3^2t_2)\right|$$
$$= a^2\left|t_1t_2(t_1 - t_2) + t_3(t_2^2 - t_1^2) + t_3^2(t_1 - t_2)\right|$$
$$= a^2\left|(t_1 - t_2)\{t_1t_2 - t_3(t_2 + t_1) + t_3^2\}\right|$$
$$= a^2\left|(t_1 - t_2)\{t_1t_2 - t_3t_2 - t_3t_1 + t_3^2\}\right|$$
$$= a^2\left|(t_1 - t_2)\{t_2(t_1 - t_3) - t_3(-t_3 + t_1)\}\right|$$
$$= a^2\left|(t_1 - t_2)(t_1 - t_3)(t_2 - t_3)\right|$$

or, $\Delta = a^2(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)$ assuming $t_1 > t_2,\ t_2 > t_3,\ t_3 > t_1$

(iii)

The vertices are given as $(a,\ c+a),\ (a,\ c),\ (-a,\ c-a)$.

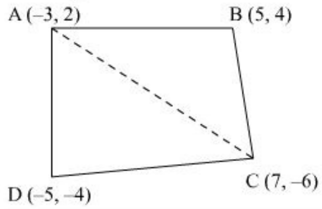
$$\Delta = \frac{1}{2}\left|a(c - c + a) + a(c - a - c - a) - a(c + a - c)\right|$$
$$= \frac{1}{2}\left|a(a) + a(-2a) - a(a)\right|$$
$$= \frac{1}{2}\left|-2a^2\right| = a^2$$

Co-Ordinate Geometry Ex 14.5 Q2

Answer :

(i)

Let the vertices of the quadrilateral be A (−3, 2), B (5, 4), C (7, −6), and D (−5, −4). Join AC to form two triangles ΔABC and ΔACD.



$$\text{Area of a triangle} = \frac{1}{2}\left\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right\}$$

$$\text{Area of } \Delta ABC = \frac{1}{2}\left\{-3(4+6) + 5(-6-2) + 7(2-4)\right\}$$

$$= \frac{1}{2}(-30 - 40 - 14) = -42$$

$\therefore$ Area of $\Delta ABC = 42$ square units

$$\text{Area of } \Delta ACD = \frac{1}{2}\left\{-3(-6+4) + 7(-4-2) - 5(2+6)\right\}$$

$$= \frac{1}{2}\{6 - 42 - 40\} = -38$$

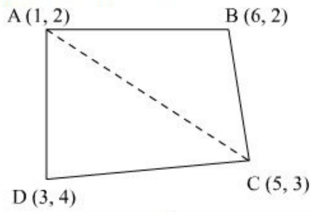
$\therefore$ Area of $\Delta ACD = 38$ square units

Area of $\square ABCD$ = Area of ΔABC + Area of ΔACD

$$= (42 + 38)\text{ square units} = 80 \text{ square units}$$

(ii)

Let the vertices of the quadrilateral be A (1, 2), B (6, 2), C (5, 3), and D (3, 4). Join AC to form two triangles ΔABC and ΔACD.



$$\text{Area of a triangle} = \frac{1}{2}\left\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right\}$$

$$\text{Area of } \Delta ABC = \frac{1}{2}\left\{1(2-3) + 6(3-2) + 5(2-2)\right\}$$

$$= \frac{1}{2}(-1+6) = \frac{5}{2} \text{ square units}$$

$$\text{Area of } \Delta ACD = \frac{1}{2}\left\{1(3-4) + 5(4-2) + 3(2-3)\right\}$$

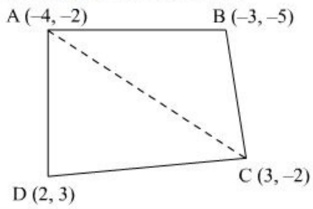
$$= \frac{1}{2}\{-1 + 10 - 3\} = 3 \text{ square units}$$

Area of $\square ABCD$ = Area of ΔABC + Area of ΔACD

$$= \left(\frac{5}{2} + 3\right)\text{square units} = \frac{11}{2} \text{ square units}$$

(iii)

Let the vertices of the quadrilateral be A (−4, −2), B (−3, −5), C (3, −2), and D (2, 3). Join AC to form two triangles ΔABC and ΔACD.



$$\text{Area of a triangle} = \frac{1}{2}\left\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right\}$$

$$\text{Area of } \Delta ABC = \frac{1}{2}\left[(-4)\{(-5)-(-2)\} + (-3)\{(-2)-(-2)\} + 3\{(-2)-(-5)\}\right]$$

$$= \frac{1}{2}(12+0+9) = \frac{21}{2} \text{ square units}$$

$$\text{Area of } \Delta ACD = \frac{1}{2}\left[(-4)\{(-2)-(3)\} + 3\{(3)-(-2)\} + 2\{(-2)-(-2)\}\right]$$

$$= \frac{1}{2}\{20+15+0\} = \frac{35}{2} \text{ square units}$$

$$\text{Area of } \square ABCD = \text{Area of } \Delta ABC + \text{Area of } \Delta ACD$$

$$= \left(\frac{21}{2} + \frac{35}{2}\right) \text{square units} = 28 \text{ square units}$$

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