Higher Order Derivatives Ex 12.1 Q23

$$y = e^{2x}(ax + b)$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = e^{2x}(a) + 2(ax + b)(e^{2x})$$

$$\Rightarrow \quad \frac{dy}{dx} = ae^{2x} + 2y$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 2ae^{2x} + 2\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 2\frac{dy}{dx} + 2ae^{2x} + 4y - 4y = 2\frac{dy}{dx} + 2\frac{dy}{dx} - 4y$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$$

$$\Rightarrow \quad y_2 - 4y_1 + 4y = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q24

$$x = \sin\left(\frac{1}{a}\log y\right)$$

$$\Rightarrow \quad \sin^{-1} x = \frac{1}{a}\log y$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{1}{\sqrt{1-x^2}} = \frac{1}{ay}\frac{dy}{dx}$$

$$\Rightarrow \quad y_1 = \frac{dy}{dx} = \frac{ay}{\sqrt{1-x^2}}$$

differentiating w.r.t.x

$$\Rightarrow \quad y_2 = \frac{d^2y}{dx^2} = a\left[\frac{\sqrt{1-x^2}\,\frac{dy}{dx} + \frac{y \times 2x}{2\sqrt{1-x^2}}}{1-x^2}\right]$$

$$\Rightarrow \quad \left(1-x^2\right)y_2 = a\sqrt{1-x^2}\,\frac{dy}{dx} + \frac{ayx}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad \left(1-x^2\right)y_2 = x\,\frac{dy}{dx} + a\sqrt{1-x^2} \times \frac{ay}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad \left(1-x^2\right)y_2 - xy_1 - a^2y = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q25

$$\log y = \tan^{-1} x$$

differentiating w.r.t. x

$$\Rightarrow \qquad \frac{1}{y}\frac{dy}{dx} = \frac{1}{1+x^2}$$

$$\Rightarrow \qquad \left(1+x^2\right)\frac{dy}{dx} = y$$

differentiating w.r.t.x

$$\Rightarrow \qquad \left(1+x^2\right)\frac{d^2y}{dx^2} + 2x\,\frac{dy}{dx} = \frac{dy}{dx}$$

$$\Rightarrow \qquad \left(1+x^2\right)\frac{d^2y}{dx^2} + \left(2x-1\right)\frac{dy}{dx} = 0$$

Hence proved!

Higher Order Derivatives Ex 12.1 Q26

$$y = \tan^{-1} x$$

differentiating w.r.t. x

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{1+x^2}$$

$$\Rightarrow \quad \left(1+x^2\right)\frac{dy}{dx} = 1$$

differentiating w.r.t.x

$$\Rightarrow \quad \left(1+x^2\right)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0$$

Hence proved!

*********** END **********