Chapter 6 Determinants Ex 6.2 Q3

$$\begin{vmatrix} a & b+c & a^2 \\ b & c+a & b^2 \\ c & a+b & c^2 \end{vmatrix}$$

Apply: $C_2 \rightarrow C_2 + C_1$.

$$= \begin{vmatrix} a & b+c+a & a^2 \\ b & c+a+b & b^2 \\ c & a+b+c & c^2 \end{vmatrix}$$

Take $(a+b+c)$ common from C_2

$$= (b+c+a) \begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix}$$

Apply: $R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 - R_1$

$$= (b+c+a) \begin{vmatrix} a & 1 & a^2 \\ b-a & 0 & b^2-a^2 \\ c-a & 0 & c^2-a^2 \end{vmatrix}$$

$$= (b+c+a)(b-a)(c-a) \begin{vmatrix} a & 1 & a^2 \\ 1 & 0 & b+a \\ 1 & 0 & c+a \end{vmatrix}$$

$$= (b+c+a)(b-a)(c-a)(b-c)$$

Chapter 6 Determinants Ex 6.2 Q4

$$Let\ \Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

$Applying\ R_2 \to R_2 - R_1\ and\ R_3 \to R_3 - R_1 we\ get,$

$$\Delta = \begin{vmatrix} 1 & a & bc \\ 0 & b-a & ca-bc \\ 0 & c-a & ab-ba \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & a & bc \\ 0 & b-a & c(a-b) \\ 0 & c-a & b(a-c) \end{vmatrix}$$

$Taking\ (a-b)\ and\ (a-c)\ common,\ we\ have$

$$\Delta = (a-b)(a-c)\begin{vmatrix} 1 & a & bc \\ 0 & -1 & c \\ 0 & -1 & b \end{vmatrix}$$

$$\Rightarrow \Delta = (a-b)(c-a)(b-c)$$

Chapter 6 Determinants Ex 6.2 Q5

$$Let\ \Delta = \begin{vmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{vmatrix}$$

$Applying\ C_1 \to C_1 + C_2 + C_3\ we\ get,$

$$\Delta = \begin{vmatrix} 3x+\lambda & x & x \\ 3x+\lambda & x+\lambda & x \\ 3x+\lambda & x & x+\lambda \end{vmatrix}$$

$Taking\ (3x+\lambda)\ common,\ we\ have$

$$\Delta = (3x+\lambda)\begin{vmatrix} 1 & x & x \\ 1 & x+\lambda & x \\ 1 & x & x+\lambda \end{vmatrix}$$

$Applying\ R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1,\ we\ get,$

$$\Delta = (3x+\lambda)\begin{vmatrix} 1 & x & x \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix}$$

$$\Rightarrow \Delta = \lambda^2(3x+\lambda)$$

Chapter 6 Determinants Ex 6.2 Q6

$$Let\ \Delta = \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

$Applying\ C_1 \to C_1 + C_2 + C_3\ we\ get,$

$$\Delta = \begin{vmatrix} a+b+c & b & c \\ a+b+c & a & b \\ a+b+c & c & a \end{vmatrix}$$

$Taking\ (a+b+c)\ common,\ we\ have$

$$\Delta = (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & a & b \\ 1 & c & a \end{vmatrix}$$

$Applying\ R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1,\ we\ get,$

$$\Delta = (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & a-b & b-c \\ 0 & c-b & a-c \end{vmatrix}$$

$$\Rightarrow \Delta = (a+b+c)[(a-b)(a-c)-(b-c)(c-b)]$$

$$\Rightarrow \Delta = (a+b+c)\left[a^2 - ac - ab + bc + b^2 + c^2 - 2bc\right]$$

$$\Rightarrow \Delta = (a+b+c)\left[a^2 + b^2 + c^2 - ac - ab - bc\right]$$

Chapter 6 Determinants Ex 6.2 Q7

$$\begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix} = \begin{vmatrix} 2+x & 1 & 1 \\ 2+x & x & 1 \\ 2+x & 1 & x \end{vmatrix} = (2+x)\begin{vmatrix} 1 & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$$

$$= (2+x)\begin{vmatrix} 1 & 1 & 1 \\ 0 & x-1 & 0 \\ 0 & 0 & x-1 \end{vmatrix}$$

$$= (2+x)(x-1)^2$$

*********** END **********