Indefinite Integrals Ex 19.9 Q10

Let $\quad I = \int \frac{1}{\sqrt{1-x^2}\left(\sin^{-1} x\right)^2} dx$ - - - - - (i)

Let $\sin^{-1} x = t$ then,

$d\left(\sin^{-1} x\right) = dt$

$\Rightarrow \quad \frac{1}{\sqrt{1-x^2}} dx = dt$

Putting $\sin^{-1} x = t$ and $\frac{1}{\sqrt{1-x^2}} dx = dt$ in equation (i), we get

$I = \int \frac{dt}{t^2}$

$= \int t^{-2} dt$

$= -1t^{-1} + c$

$= \frac{-1}{t} + c$

$= \frac{-1}{\sin^{-1} x} + c$

$\therefore \quad I = \frac{-1}{\sin^{-1} x} + c$

Indefinite Integrals Ex 19.9 Q11

Let $I = \int \frac{\cot x}{\sqrt{\sin x}} dx$ ----- (i)

Let $\sin x = t$ then,

$d(\sin x) = dt$

$\Rightarrow$ $\cos x \, dx = dt$

Now, $I = \int \frac{\cot x}{\sqrt{\sin x}} dx$

$= \int \frac{\cos x}{\sin x \sqrt{\sin x}} dx$

$= \int \frac{\cos x}{(\sin x)^{\frac{3}{2}}} dx$

$\Rightarrow$ $= \int \frac{\cos x}{(\sin x)^{\frac{3}{2}}} dx$ ------ (ii)

Putting $\sin x = t$ and $\cos x \, dx = dt$ in equation (ii), we get

$I = \int \frac{dt}{t^{\frac{3}{2}}}$

$= \int t^{-\frac{3}{2}} dt$

$= -2t^{-\frac{1}{2}} + c$

$= \frac{-2}{\sqrt{t}} + c$

$= \frac{-2}{\sqrt{\sin x}} + c$

$\therefore$ $I = \frac{-2}{\sqrt{\sin x}} + c$

Indefinite Integrals Ex 19.9 Q12

Let $\quad I = \int \frac{\tan x}{\sqrt{\cos x}}\,dx$

$\therefore \quad I = \int \frac{\sin x}{\cos x\sqrt{\cos x}}\,dx$

$= \int \frac{\sin x}{(\cos x)^{\frac{3}{2}}}\,dx$

$\Rightarrow \quad I = \int \frac{\sin x}{(\cos x)^{\frac{3}{2}}}\,dx \text{------} (i)$

Let $\quad \cos x = t \quad$ then,

$d(\cos x) = dt$

$\Rightarrow \quad -\sin x\,dx = dt$

$\Rightarrow \quad \sin x\,dx = -dt$

Putting $\cos x = t$ and $\sin x\,dx = -dt$ in equation (i), we get

$I = \int \frac{-dt}{t^{\frac{3}{2}}}$

$= -\int t^{-\frac{3}{2}}dt$

$= -\left[-2t^{-\frac{1}{2}}\right] + c$

$= \frac{2}{t^{\frac{1}{2}}} + c$

$= \frac{2}{\sqrt{\cos x}} + c$

$\therefore \quad I = \frac{2}{\sqrt{\cos x}} + c$

Indefinite Integrals Ex 19.9 Q13

Let $\quad I = \int \frac{\cos^3 x}{\sqrt{\sin x}} dx$

$\therefore \quad I = \int \frac{\cos^2 x \cos x}{\sqrt{\sin x}} dx$

$= \int \frac{\left(1 - \sin^2 x\right) \cos x}{\sqrt{\sin x}} dx$

$\therefore \quad I = \int \frac{\left(1 - \sin^2 x\right)}{\sqrt{\sin x}} \cos x \, dx \text{ - - - - - - } (i)$

Let $\quad \sin x = t \quad$ then,

$d\left(\sin x\right) = dt$

$\Rightarrow \quad \cos x \, dx = dt$

Putting $\sin x = t$ and $\cos x \, dx = dt$ in equation (i), we get

$I = \int \frac{1 - t^2}{\sqrt{t}} dt$

$= \int \left(t^{\frac{-1}{2}} - t^2 \times t^{\frac{-1}{2}}\right) dt$

$= \int \left(t^{\frac{-1}{2}} - t^{\frac{3}{2}}\right) dt$

$= 2t^{\frac{1}{2}} - \frac{2}{5} t^{\frac{5}{2}} + c$

$\Rightarrow \quad I = 2\left(\sin x\right)^{\frac{1}{2}} - \frac{2}{5}\left(\sin x\right)^{\frac{5}{2}} + c$

$\therefore \quad I = 2\sqrt{\sin x} - \frac{2}{5}\left(\sin x\right)^{\frac{5}{2}} + c$

Indefinite Integrals Ex 19.9 Q14

Let $\quad I = \int \frac{\sin^3 x}{\sqrt{\cos x}} dx$

$\therefore \quad I = \int \frac{\sin^2 x \sin x}{\sqrt{\cos x}} dx$

$\Rightarrow \quad I = \int \frac{\left(1 - \cos^2 x\right)}{\sqrt{\cos x}} \sin x \, dx \, ------ (i)$

Let $\quad \cos x = t \quad$ then,

$d\left(\cos x\right) = dt$

$\Rightarrow \quad -\sin x \, dx = dt$

$\Rightarrow \quad \sin x \, dx = -dt$

Putting $\cos x = t$ and $\sin x \, dx = -dt$ in equation (i), we get

$$I = \int \frac{\left(1 - t^2\right)}{\sqrt{t}} \times -dt$$

$$= \int \frac{t^2 - 1}{\sqrt{t}} dt$$

$$= \int \left(\frac{t^2}{t^{\frac{1}{2}}} - \frac{1}{t^{\frac{1}{2}}}\right) dx$$

$$= \int \left(t^{2-\frac{1}{2}} - t^{\frac{-1}{2}}\right) dt$$

$$= \int \left(t^{\frac{3}{2}} - t^{-\frac{1}{2}}\right) dt$$

$$= \frac{2}{5} t^{\frac{5}{2}} - 2t^{\frac{1}{2}} + c$$

$\therefore \quad I = \frac{2}{5} \cos^{\frac{5}{2}} x - 2 \cos^{\frac{1}{2}} x + c$

$\therefore \quad I = \frac{2}{5} \cos^{\frac{5}{2}} x - 2\sqrt{\cos x} + c$

*********** END **********