Differentiation Ex 11.8 Q9

Let $\quad u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Put $\quad x = \tan\theta,$

$$u = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$u = \sin^{-1}(\sin 2\theta) \qquad \text{---(i)}$$

Let $\quad v = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$$= \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$v = \cos^{-1}(\cos 2\theta) \qquad \text{---(ii)}$$

Here, $\quad 0 < x < 1$

$\Rightarrow \quad 0 < \tan\theta < 1$

$\Rightarrow \quad 0 < \theta < \frac{\pi}{4}$

So, from equation (i),

$$u = 2\theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$

$$u = 2\tan^{-1}x \qquad \left[\text{Since, } x = \tan\frac{\pi}{2}\right]$$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{2}{1+x^2} \qquad \text{---(iii)}$$

From equation (ii),

$$v = 2\theta \qquad \left[\text{Since, } \cos^{-1}(\cos\theta) = \theta, \text{ if } \theta \in [0, \pi]\right]$$

$$v = 2\tan^{-1}x \qquad [\text{Since, } x = \tan\theta]$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{2}{1+x^2} \qquad \text{---(iv)}$$

Differentiation Ex 11.8 Q10

Let $\quad u = \tan^{-1}\left(\frac{1+ax}{1-ax}\right)$

Put $\quad ax = \tan\theta$

$$u = \tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\frac{\tan\pi}{4}+\tan\theta}{1-\frac{\tan\pi}{4}\tan\theta}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\theta\right)\right)$$

$$= \frac{\pi}{4}+\theta$$

$$u = \frac{\pi}{4}+\tan^{-1}(ax) \qquad \left[\text{Since, } \tan\theta = ax\right]$$

Differentiate it with respect to x using chain rule,

$$\frac{du}{dx} = 0 + \frac{1}{1+(ax)^2}\frac{d}{dx}(ax)$$

$$\frac{du}{dx} = \frac{a}{1+a^2x^2} \qquad \text{---(i)}$$

Now,

Let $\quad v = \sqrt{1+a^2x^2}$

Differentiating it with respect to x using chain rule,

$$\frac{dv}{dx} = \frac{1}{2\sqrt{1+a^2x^2}}\frac{d}{dx}\left(1+a^2x^2\right)$$

$$= \frac{1}{2\sqrt{1+a^2x^2}}\left(2a^2x\right)$$

$$\frac{dv}{dx} = \frac{a^2x}{\sqrt{1+a^2x^2}} \qquad \text{---(ii)}$$

Differentiation Ex 11.8 Q11

Let $u = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Put $x = \sin\theta$,

$$u = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$$
$$= \sin^{-1}(2\sin\theta\cos\theta)$$
$$u = \sin^{-1}(\sin 2\theta) \quad \text{---(i)}$$

Let $v = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

$$= \tan^{-1}\left(\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}\right)$$
$$= \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right)$$
$$v = \tan^{-1}(\tan\theta) \quad \text{---(ii)}$$

Here, $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

$\Rightarrow \quad -\frac{1}{\sqrt{2}} < \sin\theta < \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \left(-\frac{\pi}{4}\right) < \theta < \left(\frac{\pi}{4}\right)$

So, from equation (i),

$$u = 2\theta \qquad \left[\text{Since, } \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$
$$u = 2\sin^{-1}x$$

Differentiating it with respect to x,

$$\frac{du}{dx} = \frac{2}{\sqrt{1-x^2}} \quad \text{---(iii)}$$

From equation (ii),

$$v = \theta \qquad \left[\text{Since, } \tan^{-1}(\tan\theta) = \theta, \text{ if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$$
$$v = \sin^{-1}x \qquad [\text{Since, } x = \sin\theta]$$

Differentiating it with respect to x,

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}} \quad \text{---(iv)}$$

Dividing equation (iii) by (iv)

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{\sqrt{1-x^2}} \times \frac{\sqrt{1-x^2}}{1}$$

$$\frac{du}{dv} = 2$$

*********** END **********