Relations Ex 1.2 Q5

We have, Z be set of integers and

$R = \{(a,b) : a,b \in Z \text{ and } a+b \text{ is even}\}$ be a relation on Z.

We want to prove that R is an equivalence relation on Z.

Now,

Reflexivity: Let $a \in Z$

$$\Rightarrow \quad a+a \text{ is even} \qquad \begin{bmatrix}\text{if a is even} \Rightarrow a+a \text{ is even}\\ \text{if a is odd} \Rightarrow a+a \text{ is even}\end{bmatrix}$$

$$\Rightarrow \quad (a,a) \in R$$

$$\Rightarrow \quad R \text{ is reflexive}$$

Symmetric: Let $a,b \in Z$ and $(a,b) \in R$

$$\Rightarrow \quad a+b \text{ is even}$$
$$\Rightarrow \quad b+a \text{ is even}$$
$$\Rightarrow \quad (b,a) \in R,$$

$$\Rightarrow \quad R \text{ is symmetric}$$

Transitivity: Let $(a,b) \in R$ and $(b,c) \in R$ For some $a,b,c \in Z$

$$\Rightarrow \quad a+b \text{ is even and } b+c \text{ is even}$$
$$\Rightarrow \quad a+c \text{ is even} \qquad \begin{bmatrix}\text{if b is odd, then a and c must be odd} \Rightarrow a+c \text{ is even,}\\ \text{If b is even, then a and c must be even} \Rightarrow a+c \text{ is even}\end{bmatrix}$$
$$\Rightarrow \quad (a,c) \in R$$

$$\Rightarrow \quad R \text{ is transitive}$$

Hence, R is an equivalence relation on Z

Relations Ex 1.2 Q6

Let Z be set of integers

$R = \{(m,n) : m-n$ is divisible by $13\}$ be a relation on Z.

Now,

Reflexivity: Let m $\in Z$

$\Rightarrow \quad m - m = 0$

$\Rightarrow \quad m - m$ is divisible by 13

$\Rightarrow \quad (m,m) \in R,$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let m,n $\in Z$ and $(m,n) \in R$

$\Rightarrow \quad m - n = 13.p$ For some $p \in Z$

$\Rightarrow \quad n - m = 13 \times (-p)$

$\Rightarrow \quad n - m$ is divisible by 13

$\Rightarrow \quad (n - m) \in R,$

so

$\Rightarrow \quad R$ is symmetric

Transitivity: Let $(m,n) \in R$ and $(n,q) \in R$ For some m,n,q $\in$ Z

$\Rightarrow \quad m - n = 13p$ and $n - q = 13s$ For some p,s $\in$ Z

$\Rightarrow \quad m - q = 13(p + s)$

$\Rightarrow \quad m - q$ is divisible by 13

$\Rightarrow \quad (m,q) \in R$

$\Rightarrow \quad R$ is transitive

Hence, R is an equivalence relation on Z

Relations Ex 1.2 Q7

(x, y) R (u, v) $\Leftrightarrow$ xv = yu

TPT Reflexive $\because$ xy = yx

$\therefore$ (x, y) R (x, y)

TPT Symmetric Let (x, y) R (u, v)

TPT (u, v) R (x, y)

Given xv = yu

$\Rightarrow$ yu = xv

$\Rightarrow$ uy = vx

$\therefore$ (u, v) R (x, y)

Transitive Let (x, y) R (u, v) and (u, v) R (p, q) ....(i)

TPT (x, y) R (p, q)

TPT xq = yp

from (1) xv = yu & uq = vp

xvuq = yuvp

xq = yp

$\therefore$ R is transitive

since R is reflexive symmetric & transitive all means it is an equivalence relation.]

Relations Ex 1.2 Q8

We have, $A = \{x \in z: \ 0 \le x \le 12\}$ be a set and

$R = \{(a,b): \ a = b\}$ be a relation on A

Now,

Reflexivity: Let $a \in A$

$\Rightarrow \quad a = a$

$\Rightarrow \quad (a,a) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $a,b \in A$ and $(a,b) \in R$

$\Rightarrow \quad a = b$

$\Rightarrow \quad b = a$

$\Rightarrow \quad (b,a) \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let a, b & $c \in A$

and Let $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow \quad a = b$ and $b = c$

$\Rightarrow \quad a = c$

$\Rightarrow \quad (a,c) \in R$

$\Rightarrow \quad R$ is transitive

Since R is being relfexive,symmetric and transitive, so R is an equivalance relation.

Also, we need to find the set of all elements related to 1.
Since the relation is given by, R={(a,b):a=b}, and 1 is an element of A,
R={(1,1):1=1}
Thus, the set of all elements related to 1 is 1.

*********** END **********