



Chapter 6 Determinants Ex 6.2 Q49

$$\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$, we have:

$$\begin{aligned} \Delta &= \begin{vmatrix} 2(a+b+c) & 2(a+b+c) & 2(a+b+c) \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} \\ &= 2(a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} \end{aligned}$$

Applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$, we have:

$$\Delta = 2(a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ c+a & b-c & b-a \\ a+b & c-a & c-b \end{vmatrix}$$

Expanding along R_1 , we have:

$$\begin{aligned} \Delta &= 2(a+b+c)(1)[(b-c)(c-b) - (b-a)(c-a)] \\ &= 2(a+b+c)[-b^2 - c^2 + 2bc - bc + ba + ac - a^2] \\ &= 2(a+b+c)[ab + bc + ca - a^2 - b^2 - c^2] \end{aligned}$$

It is given that $\Delta = 0$.

$$(a+b+c)[ab + bc + ca - a^2 - b^2 - c^2] = 0$$

$$\Rightarrow \text{Either } a+b+c=0, \text{ or } ab+bc+ca-a^2-b^2-c^2=0.$$

Now,

$$ab+bc+ca-a^2-b^2-c^2=0$$

$$\Rightarrow -2ab-2bc-2ca+2a^2+2b^2+2c^2=0$$

$$\Rightarrow (a-b)^2 + (b-c)^2 + (c-a)^2 = 0$$

$$\Rightarrow (a-b)^2 = (b-c)^2 = (c-a)^2 = 0 \quad \left[(a-b)^2, (b-c)^2, (c-a)^2 \text{ are non-negative} \right]$$

$$\Rightarrow (a-b) = (b-c) = (c-a) = 0$$

$$\Rightarrow a=b=c$$

Hence, if $\Delta = 0$, then either $a+b+c=0$ or $a=b=c$.

Chapter 6 Determinants Ex 6.2 Q50

$$\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} p-a & 0 & c-r \\ 0 & q-b & c-r \\ a & b & r \end{vmatrix} = 0 [R_1 = R_1 - R_3, R_2 = R_2 - R_3]$$

$$\Rightarrow (p-a)[r(q-b) - b(c-r)] + (c-r)[0 - a(q-b)] = 0$$

$$\Rightarrow (p-a)r(q-b) - (p-a)b(c-r) - (c-r)a(q-b) = 0$$

$$\Rightarrow \frac{(p-a)r(q-b)}{(p-a)(q-b)(r-c)} - \frac{(p-a)b(c-r)}{(p-a)(q-b)(r-c)} - \frac{(c-r)a(q-b)}{(p-a)(q-b)(r-c)} = 0$$

$$\Rightarrow \frac{r}{(r-c)} + \frac{b}{(q-b)} + \frac{a}{(p-a)} = 0$$

$$\Rightarrow \frac{r}{(r-c)} + \frac{b-q+q}{(q-b)} + \frac{a+p-p}{(p-a)} = 0$$

$$\Rightarrow \frac{r}{(r-c)} + \frac{q}{(q-b)} + \frac{(b-q)}{(q-b)} + \frac{(a-p)}{(p-a)} + \frac{p}{(p-a)} = 0$$

$$\Rightarrow \frac{r}{(r-c)} + \frac{q}{(q-b)} - 1 - 1 + \frac{p}{(p-a)} = 0$$

$$\Rightarrow \frac{r}{(r-c)} + \frac{q}{(q-b)} + \frac{p}{(p-a)} = 2$$

$$\therefore \frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$$

Chapter 6 Determinants Ex 6.2 Q51

Let us show that $x = 2$ is a root of the given equation:

Putting $x = 2$ in the LHS, we get

$$\begin{vmatrix} 2 & -6 & -1 \\ 2 & -6 & -1 \\ -3 & 4 & 4 \end{vmatrix} = 0$$

$$\therefore R_1 = R_2$$

Hence, $x = 2$ is a root of the given equation.

Now, we see if there are any other roots. For this we need to solve the equation:

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & -6 & -1 \\ x-1 & -3x & x-3 \\ x-1 & 2x & x+2 \end{vmatrix} = 0$$

$$\Rightarrow (x-1) \begin{vmatrix} 1 & -6 & -1 \\ 1 & -3x & x-3 \\ 1 & 2x & x+2 \end{vmatrix} = 0$$

$$\Rightarrow (x-1) \begin{vmatrix} 1 & -6 & -1 \\ 1 & -3x & x-3 \\ 1 & 2x & x+2 \end{vmatrix} = 0$$

$$\Rightarrow (x-1) \begin{vmatrix} 1 & -6 & -1 \\ 0 & -3x+6 & x-3+1 \\ 0 & 2x+6 & x+2+1 \end{vmatrix} = 0$$

$$\Rightarrow (x-1) \begin{vmatrix} 1 & -6 & -1 \\ 0 & -3(x-2) & x-2 \\ 0 & 2(x+3) & x+3 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)(x-2)(x+3) \begin{vmatrix} 1 & -6 & -1 \\ 0 & -3 & 1 \\ 0 & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)(x-2)(x+3) = 0$$

$$\Rightarrow (x-1) = 0 \quad (x-2) = 0 \quad (x+3) = 0$$

$$\Rightarrow x = 1 \quad x = 2 \quad x = -3$$

***** END *****

