Definite Integrals Ex 20.1 Q27

We have,

$$\int x \cos x \, dx = x\int \cos x dx - \int \left(\int \cos x dx\right) \frac{dx}{dx} dx = x \sin x - \int \sin x dx$$

$$\therefore \int_0^{\frac{\pi}{2}} x \cos x \, dx = \left[x \sin x + \cos x\right]_0^{\frac{\pi}{2}} = \left[\frac{\pi}{2} + 0 - 0 - 1\right] = \frac{\pi}{2} - 1$$

$$\therefore \int_0^{\frac{\pi}{2}} x \cos x dx = \frac{\pi}{2} - 1$$

Definite Integrals Ex 20.1 Q28

We have,

$$\int x^2 \cos x \, dx = x^2\int \cos x dx - \int (2x)\left(\int \cos x dx\right) dx = x^2 \sin x - \int \sin x .2x dx$$

$$= x^2 \sin x - 2\left[x\int \sin x - \int\left(\int \sin x dx\right) dx\right]$$

$$= x^2 \sin x - 2\left[-x \cos x + \int \cos x dx\right]$$

$$\therefore \int_0^{\frac{\pi}{2}} x^2 \cos x \, dx = \left[x^2 \sin x + 2x \cos x - 2 \sin x\right]_0^{\frac{\pi}{2}}$$

$$= \left[\frac{\pi^2}{4} + 0 - 2 - 0 - 0 + 0\right]$$

$$= \frac{\pi^2}{4} - 2$$

$$\therefore \int_0^{\frac{\pi}{2}} x^2 \cos x dx = \frac{\pi^2}{4} - 2$$

Definite Integrals Ex 20.1 Q29

We have,

$$\int x^2 \sin x \, dx = x^2\int \sin x dx - \int 2x \left(\int \sin x dx\right) dx = x^2 \cos x + \int 2x \cos x dx$$

$$= x^2 \cos x + 2\left[x\int \cos x dx - \int\left(\int \cos x dx\right) dx\right]$$

$$= -x^2 \cos x + 2\left[x \sin x - \int \sin x dx\right]$$

$$\therefore \int_0^{\frac{\pi}{4}} x^2 \sin x \, dx = \left[-x^2 \cos x + 2x \sin x + 2 \cos x\right]_0^{\frac{\pi}{4}}$$

$$= \frac{-\pi^2}{16} \cdot \frac{1}{\sqrt{2}} + \frac{\pi}{2} \cdot \frac{1}{\sqrt{2}} + 2 . \frac{1}{\sqrt{2}} + 0 - 0 - 2$$

$$= \frac{1}{\sqrt{2}}\left[\frac{-\pi^2}{16} + \frac{\pi}{2} + 2\right] - 2$$

$$= \sqrt{2} + \frac{\pi}{2\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} - 2$$

$$\therefore \int_0^{\frac{\pi}{4}} x^2 \sin x dx = \sqrt{2} + \frac{\pi}{2\sqrt{2}} - \frac{\pi^2}{16\sqrt{2}} - 2$$

Definite Integrals Ex 20.1 Q30

We have,

$$\int x^2 \cos 2x \, dx = x^2 \int \cos 2x dx - \int 2x \left(\int \cos 2x dx\right) dx$$

$$= \frac{x^2 \sin 2x}{2} - \int 2x \times \frac{\sin 2x}{2} dx$$

$$= \frac{x^2 \sin 2x}{2} - \left[x \int \sin 2x dx - \int \left(\int \sin 2x dx\right) dx\right]$$

$$= \frac{x^2 \sin 2x}{2} + \left[\frac{x \cos 2x}{2} - \int \frac{x \cos 2x}{2}\right]$$

$$\therefore \quad \int_0^{\frac{\pi}{2}} x^2 \cos 2x \, dx = \left[\frac{x^2 \sin 2x}{2} + \frac{x \cos 2x}{2} - \frac{\sin 2x}{4}\right]_0^{\frac{\pi}{2}}$$

$$= \left[\frac{\pi^2}{8} \times 0 + \frac{\pi}{4}(-1) - 0 - 0 - 0 + 0\right]$$

$$= \frac{-\pi}{4}$$

$$\therefore \int_0^{\frac{\pi}{2}} x^2 \cos 2x dx = \frac{-\pi}{4}$$

*********** END **********