Rationalisation Ex 3.2 Q5

Answer :

(i) We know that rationalization factor for $3\sqrt{2}+2\sqrt{3}$ and $\sqrt{3}-\sqrt{2}$ are $3\sqrt{2}-2\sqrt{3}$ and $\sqrt{3}+\sqrt{2}$ respectively. We will multiply numerator and denominator of the given expression $\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$ and $\frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}$ by $3\sqrt{2}-2\sqrt{3}$ and $\sqrt{3}+\sqrt{2}$ respectively, to get

$$\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}\times\frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}+\frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}=\frac{\left(3\sqrt{2}\right)^2+\left(2\sqrt{3}\right)^2-2\times3\sqrt{2}\times2\sqrt{3}}{\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2}+\frac{\sqrt{36}+\sqrt{24}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}$$

$$=\frac{18+12-12\sqrt{6}}{18-12}+\frac{6+\sqrt{24}}{3-2}$$

$$=\frac{30-12\sqrt{6}+36+12\sqrt{6}}{6}$$

$$=\frac{66}{6}$$

$$=11$$

Hence the given expression is simplified to $\boxed{11}$.

(ii) We know that rationalization factor for $\sqrt{5}-\sqrt{3}$ and $\sqrt{5}+\sqrt{3}$ are $\sqrt{5}+\sqrt{3}$ and $\sqrt{5}-\sqrt{3}$ respectively. We will multiply numerator and denominator of the given expression $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ by $\sqrt{5}+\sqrt{3}$ and $\sqrt{5}+\sqrt{3}$ respectively, to get

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}+\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}=\frac{\left(\sqrt{5}\right)^2+\left(\sqrt{3}\right)^2+2\times\sqrt{5}\times\sqrt{3}}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}+\frac{\left(\sqrt{5}\right)^2+\left(\sqrt{3}\right)^2-2\times\sqrt{5}\times\sqrt{3}}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$$=\frac{5+3+2\sqrt{15}}{5-3}+\frac{5+3-2\sqrt{15}}{5-3}$$

$$=\frac{5+3+2\sqrt{15}+5+3-2\sqrt{15}}{2}$$

$$=\frac{16}{2}$$

$$=8$$

Hence the given expression is simplified to $\boxed{8}$.

(iii) We know that rationalization factor for $3+\sqrt{5}$ and $3-\sqrt{5}$ are $3-\sqrt{5}$ and $3+\sqrt{5}$ respectively. We will multiply numerator and denominator of the given expression $\frac{7+3\sqrt{5}}{3+\sqrt{5}}$ and $\frac{7-3\sqrt{5}}{3-\sqrt{5}}$ by $3-\sqrt{5}$ and $3+\sqrt{5}$ respectively, to get

$$\frac{7+3\sqrt{5}}{3+\sqrt{5}}\times\frac{3-\sqrt{5}}{3-\sqrt{5}}-\frac{7-3\sqrt{5}}{3-\sqrt{5}}\times\frac{3+\sqrt{5}}{3+\sqrt{5}}=\frac{7\times3-7\times\sqrt{5}+9\times\sqrt{5}-3\times\left(\sqrt{5}\right)^2}{(3)^2-\left(\sqrt{5}\right)^2}-\frac{7\times3+7\times\sqrt{5}-9\times\sqrt{5}-3\times(}{(3)^2-\left(\sqrt{5}\right)^2}$$

$$=\frac{21-7\sqrt{5}+9\sqrt{5}-3\times5}{9-5}-\frac{21+7\sqrt{5}-9\sqrt{5}-3\times5}{9-5}$$

$$=\frac{21+2\sqrt{5}-15}{4}-\frac{21-2\sqrt{5}-15}{4}$$

$$=\frac{6+2\sqrt{5}-6+2\sqrt{5}}{4}$$

$$=\frac{4\sqrt{5}}{4}$$

$$=\sqrt{5}$$

Hence the given expression is simplified to $\boxed{\sqrt{5}}$.

(iv) We know that rationalization factor for $2+\sqrt{3}$, $\sqrt{5}-\sqrt{3}$, and $2-\sqrt{5}$ are $2-\sqrt{3}$, $\sqrt{5}+\sqrt{3}$, and $2+\sqrt{5}$ respectively. We will multiply numerator and denominator of the given expression $\frac{1}{2+\sqrt{3}}, \frac{2}{\sqrt{5}-\sqrt{3}}$ and $\frac{1}{2-\sqrt{5}}$ by $2-\sqrt{3}$, $\sqrt{5}+\sqrt{3}$, and $2+\sqrt{5}$ respectively, to get

$$\begin{aligned}
\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}+\frac{1}{2-\sqrt{5}}\times\frac{2+\sqrt{5}}{2+\sqrt{5}} &= \frac{2-\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2}+\frac{2\sqrt{5}+2\sqrt{3}}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}+\frac{2-\sqrt{5}}{(2)^2-\left(\sqrt{5}\right)^2} \\
&= \frac{2-\sqrt{3}}{1}+\frac{2\sqrt{5}+2\sqrt{3}}{5-3}+\frac{2+\sqrt{5}}{4-5} \\
&= \frac{2-\sqrt{3}}{1}+\frac{2\sqrt{5}+2\sqrt{3}}{2}+\frac{2+\sqrt{5}}{-1} \\
&= 2-\sqrt{3}+\sqrt{5}+\sqrt{3}-\sqrt{5}-2 \\
&= 0
\end{aligned}$$

Hence the given expression is simplified to $\boxed{0}$.

(v) We know that rationalization factor for $\sqrt{5}+\sqrt{3}$, $\sqrt{3}+\sqrt{2}$, and $\sqrt{5}+\sqrt{2}$ are $\sqrt{5}-\sqrt{3}$, $\sqrt{3}-\sqrt{2}$, and $\sqrt{5}-\sqrt{2}$ respectively. We will multiply numerator and denominator of the given expression $\frac{2}{\sqrt{5}+\sqrt{3}}, \frac{1}{\sqrt{3}+\sqrt{2}}$ and $\frac{3}{\sqrt{5}+\sqrt{2}}$ by $2-\sqrt{3}$, $\sqrt{5}+\sqrt{3}$, and $2+\sqrt{5}$ respectively, to get

$$\begin{aligned}
\frac{2}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} &= \frac{2\sqrt{5}-2\sqrt{3}}{5-3}+\frac{\sqrt{3}-\sqrt{2}}{3-2}-\frac{3\sqrt{5}-3\sqrt{2}}{5-2} \\
&= \frac{2\sqrt{5}-2\sqrt{3}}{2}+\frac{\sqrt{3}-\sqrt{2}}{1}-\frac{3\sqrt{5}-3\sqrt{2}}{3} \\
&= \sqrt{5}-\sqrt{3}+\sqrt{3}-\sqrt{2}-\sqrt{5}+\sqrt{2} \\
&= 0
\end{aligned}$$

Hence the given expression is simplified to $\boxed{0}$.

*********** END **********