Indefinite Integrals Ex 19.27 Q5

Let $I = \int e^{2x} \sin x \cos x dx$

$$= \frac{1}{2}\int e^{2x} 2 \sin x \cos x dx$$

$$= \frac{1}{2}\int e^{2x} \sin 2x dx$$

We know that

$$\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2+b^2}\left\{a \sin bx - b \cos bx\right\} + c$$

$$\Rightarrow \quad \int e^{2x} \sin 2x dx = \frac{e^{2x}}{8}\left\{2 \sin 2x - 2 \cos 2x\right\} + c$$

$$\therefore \quad I = \frac{1}{2} \cdot \frac{e^{2x}}{8}\left\{2 \sin 2x - 2 \cos 2x\right\} + c$$

$$\therefore \quad I = \frac{e^{2x}}{8}\left\{\sin 2x - \cos 2x\right\} + c$$

Indefinite Integrals Ex 19.27 Q6

Let $I = \int e^{2x} \sin x \, dx$ $\quad\quad ...(1)$

Integrating by parts, we obtain

$$I = \sin x \int e^{2x} dx - \int \left\{ \left(\frac{d}{dx} \sin x \right) \int e^{2x} dx \right\} dx$$

$$\Rightarrow I = \sin x \cdot \frac{e^{2x}}{2} - \int \cos x \cdot \frac{e^{2x}}{2} dx$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \int e^{2x} \cos x \, dx$$

Again integrating by parts, we obtain

$$I = \frac{e^{2x} \cdot \sin x}{2} - \frac{1}{2} \left[\cos x \int e^{2x} dx - \int \left\{ \left(\frac{d}{dx} \cos x \right) \int e^{2x} dx \right\} dx \right]$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \left[\cos x \cdot \frac{e^{2x}}{2} - \int (-\sin x) \frac{e^{2x}}{2} dx \right]$$

$$\Rightarrow I = \frac{e^{2x} \cdot \sin x}{2} - \frac{1}{2} \left[\frac{e^{2x} \cos x}{2} + \frac{1}{2} \int e^{2x} \sin x dx \right]$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} - \frac{1}{4} I \quad\quad \text{[From (1)]}$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} - \frac{1}{4} I$$

$$\Rightarrow I + \frac{1}{4} I = \frac{e^{2x} \cdot \sin x}{2} - \frac{e^{2x} \cos x}{4}$$

$$\Rightarrow \frac{5}{4} I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4}$$

$$\Rightarrow I = \frac{4}{5} \left[\frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} \right] + C$$

$$\Rightarrow I = \frac{e^{2x}}{5} \left[2 \sin x - \cos x \right] + C$$

Indefinite Integrals Ex 19.27 Q8

Let $I = \int e^x \sin^2 x dx$

$$= \frac{1}{2} \int e^x 2 \sin^2 x dx$$

$$= \frac{1}{2} \int e^x \left(1 - \cos 2x \right) dx$$

$$= \frac{1}{2} \int e^x dx - \frac{1}{2} \int e^x \cos 2x dx$$

$$\because \quad \int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2} \left\{ a \cos bx - b \sin bx \right\} + c$$

$$\therefore \quad I = \frac{1}{2} \left[e^x - \frac{e^x}{5} \left\{ \cos 2x + 2 \sin 2x \right\} \right] + c$$

$$\therefore \quad I = \frac{e^x}{2} - \frac{e^x}{10} \left\{ \cos 2x + 2 \sin 2x \right\} + c$$

Indefinite Integrals Ex 19.27 Q9

Let $I = \int \frac{1}{x^3} \sin(\log x)\, dx$

Let $\log x = t$

$\Rightarrow \quad \frac{1}{x} dx = dt$

$\Rightarrow \quad dx = e^t dt$

$\therefore \quad I = \int e^{-2t} \sin t dt$

We know that,

$$\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2} \{a \sin bx - b \cos bx\} + c$$

$\therefore \quad \int e^{-2t} \sin + dt = \frac{e^{-2t}}{5} \{-2 \sin t - \cos t\} + c$

$\therefore \quad I = \frac{x^{-2}}{5} \{-2 \sin(\log x) - \cos(\log x)\} + c$

Hence,

$$\int \frac{1}{x^3} \sin(\log x)\, dx = \frac{-1}{5x^2} \{2 \sin(\log x) + \cos(\log x)\} + c$$

*********** END **********