Trigonometric Ratios Ex 5.1 Q13

Answer :

Given:

$$\sec\theta = \frac{13}{5}$$

To show that $\dfrac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = 3$

Now, we know that $\cos\theta = \dfrac{1}{\sec\theta}$

Therefore,

$$\cos\theta = \frac{1}{\frac{13}{5}}$$

Therefore,

$$\cos\theta = \frac{5}{13} \quad \ldots\ldots (1)$$

Now, we know that

$$\cos\theta = \frac{\text{Base side adjacent to} \angle\theta}{\text{Hypotenuse}} \quad \ldots\ldots (2)$$

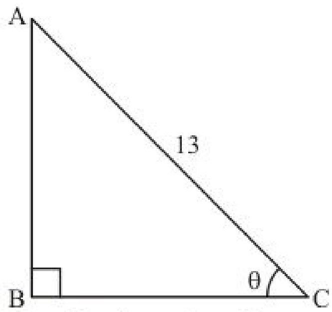
Now, by comparing equation (1) and (2)

We get,

Base side adjacent to $\angle\theta$ = 5

And

Hypotenuse = 13



Therefore from above figure

Base side $BC = 5$

Hypotenuse $AC = 13$

Side *AB* is unknown, It can be determined by using Pythagoras theorem

Therefore by applying Pythagoras theorem

We get,

$$AC^2 = AB^2 + BC^2$$

Therefore by substituting the values of known sides

We get,

$$13^2 = AB^2 + 5^2$$

Therefore,

$$AB^2 = 13^2 - 5^2$$

$$AB^2 = 169 - 25$$

$$AB^2 = 144$$

$$AB = \sqrt{144}$$

Therefore,

$AB = 12$ …… (3)

Now, we know that

$$\sin\theta = \frac{\text{Perpendicular side opposite to } \angle\theta}{\text{Hypotenuse}}$$

Now from figure (a)

We get,

$$\sin\theta = \frac{AB}{AC}$$

Therefore,

$\sin\theta = \dfrac{12}{13}$ …… (4)

Now L.H.S. of the equation to be proved is as follows

$$L.H.S. = \frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$$

Substituting the value of $\cos\theta$ and $\sin\theta$ from equation (1) and (4) respectively

We get,

$$L.H.S. = \frac{2\left(\frac{12}{13}\right) - 3\left(\frac{5}{13}\right)}{4\left(\frac{12}{13}\right) - 9\left(\frac{5}{13}\right)}$$

Therefore,

$$L.H.S. = \frac{2\times12-3\times5}{4\times12-9\times5}$$

$$L.H.S. = \frac{24-15}{48-45}$$

$$L.H.S. = \frac{9}{3}$$

$$L.H.S. = 3$$

Hence proved that,

$$\frac{2\sin\theta-3\cos\theta}{4\sin\theta-9\cos\theta} = 3$$

*********** END **********