Complex Numbers Ex 13.2 Q19

$\mathrm{Re}\,(z^2) = 0, |z|=2$

Let $z=x+iy$

$z^2 = 0$

$\Rightarrow (x+iy)^2 = 0$

$\Rightarrow x^2 + 2ixy - y^2 = 0$

$\Rightarrow x^2 - y^2 = 0 \ldots\ldots (i)$, which is the real part of $(x+iy)^2$.

$|z|=2$

$\Rightarrow \sqrt{x^2 + y^2} = 2$

$\Rightarrow x^2 + y^2 = 4 \ldots\ldots (ii)$

Adding (i) and (ii), we get

$2x^2 = 4$

$\Rightarrow x^2 = 2$

$\Rightarrow x = \pm\sqrt{2}, y = \pm\sqrt{2}$

$$\begin{aligned} x+iy &= \sqrt{2} + i\sqrt{2} \\ &= -\sqrt{2} - i\sqrt{2} \\ &= \sqrt{2} - i\sqrt{2} \\ &= -\sqrt{2} + i\sqrt{2} \end{aligned}$$

Complex Numbers Ex 13.2 Q20

let $z = x + iy$,

$$\frac{z-1}{z+1}$$

$$=\frac{x+iy-1}{x+iy+1}$$

$$=\frac{x-1+iy}{x+1+iy}$$

$$=\frac{(x-1+iy)(x+1-iy)}{(x+1+iy)(x+1-iy)}\text{[Rationalizing the denominator]}$$

$$=\frac{(x-1+iy)(x+1-iy)}{(x+1)^2-(iy)^2}$$

$$=\frac{x^2+x-ixy-x-1+iy+ixy+iy+y^2}{x^2+2x+1+y^2}$$

$$=\frac{x^2-1+2iy+y^2}{x^2+2x+1+y^2}$$

$$=\frac{x^2+y^2-1}{x^2+2x+1+y^2}+i\frac{2y}{x^2+2x+1+y^2}$$

$\because$ It is a purely imaginary no. therefore real part $=0$

$$\frac{x^2+y^2-1}{x^2+2x+1+y^2}=0$$

$$\Rightarrow x^2+y^2-1=0$$

$$\Rightarrow x^2+y^2=1$$

$$\Rightarrow \sqrt{x^2+y^2}=1$$

$$\Rightarrow |z| = 1$$

Complex Numbers Ex 13.2 Q21

Let $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$

$|z_1| = 1 \Rightarrow x_1^2 + y_1^2 = 1$

$$z_2 = \frac{z_1-1}{z_1+1}$$

$$x_2 + iy_2 = \frac{x_1+iy_1-1}{x_1+iy_1+1}$$

$$\Rightarrow x_2 + iy_2 = \frac{x_1-1+iy_1}{x_1+1+iy_1}$$

$$\Rightarrow x_2 + iy_2 = \frac{(x_1-1+iy_1)(x_1+1-iy_1)}{(x_1+1+iy_1)(x_1+1-iy_1)}\text{[Rationalizing the denominator]}$$

$$\Rightarrow x_2 + iy_2 = \frac{(x_1-1)(x_1+1)-iy_1(x_1-1)+iy_1(x_1+1)+y_1^2}{(x_1+1)^2-(iy_1)^2}$$

$$\Rightarrow x_2 + iy_2 = \frac{x_1^2-1+y_1^2-iy_1x_1+iy_1+iy_1x_1+iy_1}{(x_1+1)^2-(iy_1)^2}$$

$$\Rightarrow x_2 + iy_2 = \frac{x_1^2+y_1^2-1+2iy_1}{(x_1+1)^2-(iy_1)^2}$$

$$\Rightarrow x_2 + iy_2 = \frac{1-1+2iy_1}{(x_1+1)^2-(iy_1)^2}\ [\because x_1^2+y_1^2=1]$$

$$\Rightarrow x_2 + iy_2 = \frac{2iy_1}{(x_1+1)^2-(iy_1)^2}\ [\because x_1^2+y_1^2=1]$$

Since there is no real part in the RHS, therefore $x_2 = 0$.

The real part of the $z_2 = 0$.

Complex Numbers Ex 13.2 Q22

Let $z = x + iy$

$|z+1| = z + 2(1+i)$

$\Rightarrow |x+iy+1| = x+iy+2+2i$

$\Rightarrow \sqrt{(x+1)^2 + y^2} = (x+2)+i(y+2)$

Comparing, real and imaginary parts, we get

$x+2 = \sqrt{x^2+2x+1+y^2}$ and $y+2=0$

$y+2=0$

$\Rightarrow y=-2$

& $(x+2)^2 = x^2+2x+1+y^2$

$\Rightarrow x^2+4x+4 = x^2+2x+1+y^2$

$\Rightarrow 2x+3 = y^2$

$\Rightarrow 2x+3 = (-2)^2$

$\Rightarrow 2x+3 = 4$

$\Rightarrow 2x = 1$

$\Rightarrow x = \frac{1}{2}$

$\therefore z = x+iy = \frac{1}{2} - i2$

*********** END **********