Indefinite Integrals Ex 19.13 Q10

Let $I = \int \frac{\sin 2x}{\sqrt{\sin^4 x + 4\sin^2 x - 2}} dx$

Let $\sin^2 x = t$

$\Rightarrow \quad 2\sin x \cos x \, dx = dt$

$\Rightarrow \quad \sin 2x \, dx = dt$

$$I = \int \frac{dt}{\sqrt{t^2 + 4t - 2}}$$

$$= \int \frac{dt}{\sqrt{t^2 + 2t(2) + (2)^2 - (2)^2 - 2}} dx$$

$$= \int \frac{dt}{\sqrt{(t+2)^2 - 6}}$$

Let $t + 2 = u$

$dt = du$

$$= \int \frac{du}{\sqrt{u^2 - \left(\sqrt{6}\right)^2}}$$

$$= \log\left|u + \sqrt{u^2 - 6}\right| + c \qquad \left[\text{Since} \quad \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c\right]$$

$$= \log\left|t + 2 + \sqrt{(t+2)^2 - 6}\right| + c$$

$$I = \log\left|\sin^2 x + 2 + \sqrt{\sin^4 x + 4\sin^2 x - 2}\right| + c$$

Indefinite Integrals Ex 19.13 Q11

$$\int \frac{\sin 2x}{\sqrt{\cos^4 x - \sin^2 x + 2}} dx =$$

$$let\ t = \cos^2 x \rightarrow -dt = 2\cos x \sin x dx$$

$$\int \frac{\sin 2x}{\sqrt{\cos^4 x - \sin^2 x + 2}} dx = \int \frac{-1}{\sqrt{t^2 - (1-t) + 2}} dt$$

$$= \int \frac{-1}{\sqrt{t^2 + t + 1}} dt = \int \frac{-1}{\sqrt{t^2 + t + \frac{1}{4} + \frac{3}{4}}} dt$$

$$= \int \frac{-1}{\sqrt{\left(t + \frac{1}{2}\right)^2 + \frac{3}{4}}} dt = -\log\left|\left(t + \frac{1}{2}\right) + \sqrt{t^2 + t + 1}\right|$$

$$= -\log\left|\left(\cos^2 x + \frac{1}{2}\right) + \sqrt{\cos^4 x + \cos^2 x + 1}\right| + C$$

Indefinite Integrals Ex 19.13 Q12

Let $I = \int \frac{\cos x}{\sqrt{4 - \sin^2 x}} dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x \, dx = dt$

$$= \int \frac{dt}{\sqrt{(2)^2 - t^2}}$$

$$= \sin^{-1}\left(\frac{t}{2}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \sin^{-1}\left(\frac{\sin x}{2}\right) + c$$

Indefinite Integrals Ex 19.13 Q13

Let $I = \int \frac{1}{x^{\frac{2}{3}}\sqrt{x^{\frac{2}{3}} - 4}} dx$

Let $x^{\frac{1}{3}} = t$

$\Rightarrow \quad \frac{1}{3} x^{\frac{1}{3}-1} dx = dt$

$\Rightarrow \quad \frac{1}{3} x^{-\frac{2}{3}} dx = dt$

$\Rightarrow \quad \frac{dx}{x^{\frac{2}{3}}} = 3dt$

$$I = 3\int \frac{dt}{\sqrt{t^2 - (2)^2}}$$

$$= 3\log\left|t + \sqrt{t^2 - 4}\right| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c\right]$$

$$I == 3\log\left|x^{\frac{1}{3}} + \sqrt{x^{\frac{2}{3}} - 4}\right| + c$$

*********** END **********