Differentiation Ex 11.6 Q7

Here,

$$y = e^{x^{e^x}} + x^{e^{e^x}} + e^{x^{x^e}}$$

$$y = u + v + w$$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} \qquad \text{---(i)}$$

Were $u = e^{x^{e^x}}, v = x^{e^{e^x}}, w = e^{x^{x^e}}$

Now, $u = e^{x^{e^x}}$ ---(ii)

Taking log on both the sides,

$$\log x = \log e^{x^{e^x}}$$

$$\log x = x^{e^x} \log e$$

$$\log x = x^{e^x} \qquad \text{---(iii)}$$

$\left\{\begin{matrix}\text{since } \log e - 1, \\ \log a^b = b \log a\end{matrix}\right\}$

Taking log on both the sides,

$$\log\log x = \log x^{e^x}$$

$$\log\log x = e^x \log x$$

Differentiating it with respect to x,

$$\frac{1}{\log x}\frac{d}{dx}(\log x) = e^x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}\left(e^x\right)$$

$$\frac{1}{\log x}\frac{1}{4}\frac{du}{dx} = \frac{e^x}{x} + e^x \log x$$

$$\frac{du}{dx} = 4\log x\left[\frac{e^x}{x} + e^x \log x\right]$$

$$\frac{du}{dx} = e^{x^{e^x}} * x^{e^x}\left[\frac{e^x}{x} + e^x \log x\right] \qquad \text{---(A)}$$

Using equqtion (ii) and (iii)

Now

$$v = x^{e^{e^x}} \qquad \text{---(iv)}$$

Taking log on both the sides,

$$\log v = \log x^{e^{e^x}}$$

$$\log v = e^{e^x} \log x$$

Differentiating it with respect to x,

$$\frac{1}{v}\frac{dv}{dx} = e^{e^x}\frac{d}{dx}(\log x) + \log x \frac{d}{dx}\left(e^{e^x}\right)$$

$$\frac{1}{v}\frac{dv}{dx} = e^{e^x}\left(\frac{1}{x}\right) + \log x e^{e^x}\frac{d}{dx}\left(e^x\right)$$

$$\frac{dv}{dx} = v[e^{e^x}\left(\frac{1}{x}\right) + \log x e^{e^x} e^x]$$

$$\frac{dv}{dx} = x^{e^{e^x}} * e^{e^x}[\frac{1}{x} + e^x \log x] \qquad \text{---(B)}$$

$\{\text{sinx using equation}(4)\}$

Now, $w = e^{x^{x^e}}$ ---(v)

Taking log on both the sides,

$$\log w = \log e^{x^{x^e}}$$

$$\log w = x^{x^e} \log e$$

$$\log w = x^{x^e} \qquad \text{---(vi)}$$

Taking log on both the sides,

$$\log\log w = \log x^{x^e}$$

$$\log\log w = x^e \log x$$

Differentiating it with respect to x,

$$\frac{1}{\log w}\frac{d}{dx}(\log w) = x^{e}\frac{d}{dx}(\log x) + \log x\frac{d}{dx}\left(x^{e}\right)$$
$$\frac{1}{\log w}\left(\frac{1}{w}\right)\frac{dw}{dx} = x^{e}\left(\frac{1}{x}\right) + \log e x^{e-1}$$
$$\frac{dw}{dx} = w\log w[x^{e-1} + e\log x x^{e-1}]$$
$$\frac{dw}{dx} = e^{x^{x^{e}}} x^{x^{e}} x^{e-1}(1 + e\log x) \qquad \text{---(C)} \quad \{\text{Using equation (v), (vi)}\}$$

Using equation (A), (B) and (C) in equation (i),

$$\frac{dy}{dx} = e^{x^{e^{x}}} x^{e^{x}}[\frac{e^{x}}{x} + e^{x}\log x] + x^{e^{e^{x}}} e^{e^{x}}[\frac{1}{x} + e^{x}\log x]$$
$$+e^{x^{x^{e}}} x^{x^{e}} x^{e-1}(1 + e\log x)$$

Differentiation Ex 11.6 Q8

Here,

$$y = (\cos x)^{(\cos x)^{(\cos x)^{\cdots\infty}}}$$
$$y = (\cos x)^{y}$$

Taking log on both the sides,

$$\log y = \log(\cos x)^{y}$$
$$\log y = y\log(\cos x), \{\text{since } \log a^{b} = b\log a\}$$

Differentiating it with respect to x using product rule and chain rule,

$$\frac{1}{y}\frac{dy}{dx} = y\frac{d}{dx}\log(\cos x) + \log\cos x\frac{dy}{dx}$$
$$\frac{1}{y}\frac{dy}{dx} = y\left(\frac{1}{\cos x}\right)\frac{d}{dx}(\cos x) + \log\cos x\frac{dy}{dx}$$
$$\frac{dy}{dx}\left(\frac{1}{y} - \log\cos x\right) = \frac{y}{\cos x}(-\sin x)$$
$$\frac{dy}{dx}\left(\frac{1 - y\log\cos x}{y}\right) = -y\tan x$$
$$\frac{dy}{dx} = -\frac{y^{2}\tan x}{(1 - y\log\cos x)}$$

*********** END **********