Derivatives as a Rate Measurer Ex 13.2 Q28

Let edge of cube be x cm

Here,

$$\frac{dV}{dt} = 9 \text{ cm}^3/\text{sec}$$

To find $\frac{dA}{dt}$ when $x = 10$ cm

We know that

$$V = x^3$$

$$\frac{dV}{dt} = 3x^2\left(\frac{dx}{dt}\right)$$

$$9 = 3(10)^2\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{3}{100} \text{ cm/sec}$$

Now, $A = 6x^2$

$$\frac{dA}{dt} = 12x\frac{dx}{dt}$$

$$= 12(10)\left(\frac{3}{100}\right)$$

$$\frac{dA}{dt} = 3.6 \text{ cm}^2/\text{sec.}$$

Derivatives as a Rate Measurer Ex 13.2 Q29

Given, $\frac{dV}{dt} = 25 \text{ cm}^3/\text{sec}$

To find $\frac{dA}{dt}$ when $r = 5$ cm

We know that,

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{3}\pi\left(3r^2\right)\frac{dr}{dt}$$

$$25 = 4\pi\left(5\right)^2\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{4\pi} \text{ cm/sec}$$

Now, $A = 4\pi r^2$

$$\frac{dA}{dt} = 8\pi r\frac{dr}{dt}$$

$$= 8\pi\left(5\right)\left(\frac{1}{4\pi}\right)$$

$$\frac{dA}{dt} = 10 \text{ cm}^2/\text{sec.}$$

Derivatives as a Rate Measurer Ex 13.2 Q30

Given,

$$\frac{dx}{dt} = -5 \text{ cm/min}$$

$$\frac{dy}{dt} = 4 \text{ cm/min}$$

(i) To find $\frac{dP}{dt}$ when $x = 8$ cm, $y = 6$ cm

$$P = 2\left(x + y\right)$$

$$\frac{dP}{dt} = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$$

$$= 2\left(-5 + 4\right)$$

$$\frac{dP}{dt} = -2 \text{ cm/min}$$

(ii) To find $\frac{dA}{dt}$ when $x = 8$ cm and $y = 6$ cm

$$A = xy$$

$$\frac{dA}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$$

$$= \left(8\right)\left(4\right) + \left(6\right)\left(-5\right)$$

$$= 32 - 30$$

$$\frac{dA}{dt} = 2 \text{ cm}^2/\text{min.}$$

Derivatives as a Rate Measurer Ex 13.2 Q31

Let r be the radius of the given disc and A be its area.

Then, $A = \pi r^2$

$\therefore \quad \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ [by chain rule]

Now, the approximate increase of radius $= dr = \frac{dr}{dt}\Delta t = 0.05 \text{ cm/sec}$

$\therefore$ the approximate rate of increase in area is given by

$$dA = \frac{dA}{dt}(\Delta t) = 2\pi r\left(\frac{dr}{dt}\Delta t\right) = 2\pi(3.2)(0.05) = 0.320\pi \text{ cm}^3/\text{s}$$

*********** END **********