

Exercise 7B

Question 12:

$$\begin{split} (\cos e c \theta - \sin \theta) &= a^3 \text{ and } (\sec \theta - \cos \theta) = b^3 \\ a^3 &= \left(\frac{1}{\sin \theta} - \sin \theta\right) = \left(\frac{1 - \sin^2 \theta}{\sin \theta}\right) = \frac{\cos^2 \theta}{\sin \theta} \\ \Rightarrow a &= \frac{\cos^{\frac{7}{3}} \theta}{\sin^{\frac{7}{3}} \theta} \\ b^3 &= \left(\frac{1}{\cos \theta} - \cos \theta\right) = \frac{\left(1 - \cos^2 \theta\right)}{\cos \theta} = \frac{\sin^2 \theta}{\cos \theta} \\ \Rightarrow b &= \frac{\sin^{\frac{7}{3}} \theta}{\cos^{\frac{7}{3}} \theta} \\ \therefore a^2 b^2 \left(a^2 + b^2\right) &= a^4 b^2 + a^2 b^4 \\ &= a^3 \left(ab^2\right) + \left(a^2 b\right) b^3 \\ &= \frac{\cos^2 \theta}{\sin \theta} \left[\frac{\cos^{\frac{7}{3}} \theta}{\sin^{\frac{7}{3}} \theta} \times \frac{\sin^{\frac{7}{3}} \theta}{\cos^{\frac{7}{3}}}\right] + \left[\frac{\cos^{\frac{4}{3}} \theta}{\sin^{\frac{7}{3}} \theta} \times \frac{\sin^{\frac{7}{3}} \theta}{\cos^{\frac{7}{3}} \theta}\right] \times \frac{\sin^2 \theta}{\cos \theta} \\ &= \frac{\cos^2 \theta}{\sin \theta} \times \sin \theta + \cos \theta \times \frac{\sin^2 \theta}{\cos \theta} = \left(\cos^2 \theta + \sin^2 \theta\right) = 1 \end{split}$$

Question 13:

$$a\cos^{3}\theta + 3a\sin^{2}\theta\cos\theta = m \text{ and } a\sin^{3}\theta + 3a\sin\theta\cos^{2}\theta = n$$

$$LHS = (m+n)^{2/3} + (m-n)^{2/3}$$

$$= \left(a\cos^{3}\theta + 3a\sin^{2}\theta\cos\theta + a\sin^{3}\theta + 3a\sin\theta\cos^{2}\theta\right)^{2/3}$$

$$+ \left(a\cos^{3}\theta + 3a\sin^{2}\theta\cos\theta - a\sin^{3}\theta - 3a\sin\theta\cos^{2}\theta\right)^{2/3}$$

$$\left[\because (a+b)^{3} = a^{3} + b^{3} + 3a^{2}b + 3ab^{2}, \\ (a-b)^{3} = a^{3} - b^{3} - 3a^{2}b + 3ab^{2}\right]$$

$$= \left[\left(a^{\frac{1}{3}}\cos\theta + a^{\frac{1}{3}}\sin\theta\right)^{3}\right]^{\frac{3}{3}} + \left[\left(a^{\frac{1}{3}}\cos\theta - a^{\frac{1}{3}}\sin\theta\right)^{3}\right]^{\frac{2}{3}}$$

$$= \left(a^{\frac{1}{3}}\cos\theta + a^{\frac{1}{3}}\sin\theta\right)^{2} + \left(a^{\frac{1}{3}}\cos\theta - a^{\frac{1}{3}}\sin\theta\right)^{2}$$

$$= a^{2/3}\cos^{2}\theta + a^{2/3}\sin^{2}\theta + 2a^{2/3}\cos\theta\sin\theta + a^{2/3}\cos^{2}\alpha + a^{2/3}\sin^{2}\theta - 2a^{2/3}\sin\theta\cos\theta$$

$$= a^{2/3}\left(\cos^{2}\theta + \sin^{2}\theta\right) + a^{2/3}\left(\cos^{2}\theta + \sin^{2}\theta\right)$$

$$= a^{2/3} + a^{2/3} = 2a^{2/3} = RHS$$

$$\therefore LHS = RHS$$

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