Continuity Ex 9.1 Q5

We have, to check the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{\sin 3(-h)}{-h} = \lim_{h\to 0} -\frac{\sin 3h}{-h} = 3$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0}\frac{\sin 3h}{h} = 3$$

$$f(0) = 1$$

$$\text{LHL} = \text{RHL} \neq f(0)$$

$\Rightarrow$ Function is discontinuous at $x = 0$. It is removable discontinuty.

Continuity Ex 9.1 Q6

We have, to check the continuity of the function at $x = 0$.

$$\text{L.H.L} = \lim_{x\to 0^-} f(x) = \lim_{x\to 0} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} e^{1/-h} = e^{-\infty} = 0$$

$$\text{R.H.L} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} e^{1/h} = e^{\infty} = \infty$$

So, LHL $\neq$ RHL

Hence, the function is discontinuous at $x = 0$. This is discontinuty of I^{st} kind.

Continuity Ex 9.1 Q7

We want, to check the continuity of the given function at $x = 0$.

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0}\frac{1-\cos(-h)}{(-h)^2}$$

$$= \lim_{h\to 0}\frac{1-\cos h}{h^2} \quad [\because \cos(-\theta) = \cos\theta]$$

$$= \lim_{h\to 0}\frac{2\sin^2\frac{h}{2}}{h^2} \quad \left[\because 1-\cos\theta = 2\sin^2\frac{\theta}{2}\right]$$

$$= \lim_{h\to 0} 2.\left(\frac{\sin\frac{h}{2}}{h}\right)^2 = 2\times\frac{1}{4} = \frac{1}{2}$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0}\frac{1-\cos h}{h^2} = \lim_{h\to 0}\frac{2\sin^2\frac{h}{2}}{h^2} = \lim_{h\to 0} 2\left(\frac{\sin^2\frac{h}{2}}{h}\right)^2 = 2\times\frac{1}{4} = \frac{1}{2}$$

$$f(0) = 1$$

$$\text{LHL} = \text{RHL} \neq f(0)$$

Hence, the function is discontinuous at $x = 0$

This is removable discontinuty.

Continuity Ex 9.1 Q8

We want, to check the continuty of the function at $x = 0$.

LHL = $\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{-h - |-h|}{2} = \lim_{h \to 0} \frac{-h - h}{2} = 0$

RHL = $\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{h - (|h|)}{2} = 0$

$f(0) = 2$

Thus, LHL = RHL $\neq f(0)$

Hence, The function is discontinuous at $x = 0$

This is removable discontinuty.

*********** END **********