Definite Integrals Ex 20.2 Q36

We have,

$$\int_0^{\frac{\pi}{2}} \underset{\mathrm{I}}{x^2} \underset{\mathrm{II}}{\sin x}\, dx$$

Using by parts, we get

$$x^2 \int \sin x dx - \int \left(\int \sin x dx \right) \frac{dx^2}{dx} .dx$$

$$= x^2 \cos x + \int \cos x .2x dx$$

Again applying by parts

$$= x^2 \cos x + 2 \left[x \int \cos x dx - \int \left(\int \cos x dx \right) . \frac{dx}{dx} .dx \right]$$

$$= x^2 \cos x + 2 \left[x \sin x - \int \sin x dx \right]$$

$$= \left[x^2 \cos x + 2x \sin x + 2 \cos x \right]_0^{\frac{\pi}{2}}$$

$$= \pi + 0 - 0 - 0 - 2$$

$$= \pi - 2$$

$$\therefore \int_0^{\frac{\pi}{2}} x^2 \sin x dx = \pi - 2$$

Definite Integrals Ex 20.2 Q37

Let $x = \cos 2\theta$
Differentiating w.r.t. x, we get
$dx = -2\sin 2\theta d\theta$

Now, $x = 0 \Rightarrow \theta = \frac{\pi}{4}$
$x = 1 \Rightarrow \theta = 0$

$$\therefore \int_0^1 \sqrt{\frac{1-x}{1+x}}dx = \int_{\frac{\pi}{4}}^{0} \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}\left(-2\sin 2\theta\right)d\theta$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}\left(\ 2\sin 2\theta\right)d\theta \qquad \left[\because \sin 2\theta = 2\sin\theta\cos\theta;\ \text{and}\ \sin^2\theta = \frac{1-\cos 2\theta}{2}\right]$$

$$= 2\int_0^{\frac{\pi}{4}} \frac{\sin\theta}{\cos\theta}.\sin 2\theta d\theta$$

$$= 4\int_0^{\frac{\pi}{4}} \sin^2\theta d\theta$$

$$= 2\int_0^{\frac{\pi}{4}} \left(1-\cos 2\theta\right)d\theta$$

$$= 2\left[\theta - \frac{\sin^2\theta}{2}\right]_0^{\frac{\pi}{4}}$$

$$= 2\left[\frac{\pi}{4} - \frac{1}{2}\right]$$

$$= \frac{\pi}{2} - 1$$

$$\therefore \int_0^1 \sqrt{\frac{1-x}{1+x}}dx = \frac{\pi}{2} - 1$$

Definite Integrals Ex 20.2 Q38

We have,

$$\int_0^1 \frac{1-x^2}{\left(1+x^2\right)^2}dx = \int_0^1 \frac{-x^2\left(1-\frac{1}{x^2}\right)dx}{x^2\left(x+\frac{1}{x}\right)^2} = -\int_0^1 \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)^2}$$

Let $x + \frac{1}{x} = t \Rightarrow 1 - \frac{1}{x^2}dx = dt$

When $x = 0 \Rightarrow t = \infty$
$x = 1 \Rightarrow t = 2$

$$\therefore \int_0^1 \frac{1-x^2}{\left(1+x^2\right)^2}dx = -\int_{\infty}^{2} \frac{dt}{t^2} = \int_2^{\infty} \frac{dt}{t^2} = \left[-\frac{1}{t}\right]_2^{\infty} = \left(\frac{1}{2} - 0\right) = \frac{1}{2}$$

Definite Integrals Ex 20.2 Q39

Put $t = x^5 + 1$, then $dt = 5x^4\ dx$.

Therefore, $\int 5x^4 \sqrt{x^5+1}\ dx = \int \sqrt{t}\ dt = \frac{2}{3}dt = \frac{2}{3}t^{\frac{3}{2}} = \frac{2}{3}\left(x^5+1\right)^{\frac{3}{2}}$

Hence, $\int_{-1}^{1} 5x^4 \sqrt{x^4+1}\ dx = \frac{2}{3}\left[\left(x^5+1\right)^{\frac{3}{2}}\right]_{-1}^{1}$

$$= \frac{2}{3}\left[\left(1^5+1\right)^{\frac{3}{2}} - \left((-1)^5+1\right)^{\frac{3}{2}}\right]$$

$$= \frac{2}{3}\left[2^{\frac{3}{2}} - 0^{\frac{3}{2}}\right] = \frac{2}{3}\left(2\sqrt{2}\right) = \frac{4\sqrt{2}}{3}$$

Alternatively, first we transform the integral and thenevaluate the transformed integral with new limits.
Let $t = x^5 + 1$. Then $dt = 5x^4\ dx$.
Note that, when $x = -1$, $t = 0$ and when $x = 1$, $t = 2$
Thus, as x varies from -1 to 1, t varies from 0 to 2
Therefore $\int_{-1}^{1} 5x^4 \sqrt{x^5+1}\ dx = \int_0^2 \sqrt{t}\ dt$

$$= \frac{2}{3}\left[t^{\frac{3}{2}}\right]_0^2 = \frac{2}{3}\left[2^{\frac{3}{2}} - 0^{\frac{3}{2}}\right] = \frac{2}{3}\left(2\sqrt{2}\right) = \frac{4\sqrt{2}}{3}$$

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