



Indefinite Integrals Ex 19.30 Q47

Consider the integral

$$I = \int \frac{1}{x(x^3 + 8)} dx$$

Rewriting the above integrand, we have,

$$\begin{aligned} I &= \int \frac{x^2}{x^3(x^3 + 8)} dx \\ &= \frac{1}{3} \int \frac{3x^2}{x^3(x^3 + 8)} dx \end{aligned}$$

Now substituting $x^3 = t$, we have,

$$\begin{aligned} 3x^2 dx &= dt \\ \Rightarrow I &= \frac{1}{3} \int \frac{dt}{t(t+8)} \end{aligned}$$

Let us separate the integrand by partial fractions.

Thus,

$$\begin{aligned} \frac{1}{t(t+8)} &= \frac{A}{t} + \frac{B}{t+8} \\ \Rightarrow \frac{1}{t(t+8)} &= \frac{A(t+8) + Bt}{t(t+8)} \end{aligned}$$

$$\Rightarrow 1 = A(t+8) + Bt$$

$$\Rightarrow 1 = At + 8A + Bt$$

Comparing the coefficients, we have,

$$A+B=0 \text{ and } 8A=1$$

$$\Rightarrow A = \frac{1}{8} \text{ and } B = -\frac{1}{8}$$

Therefore,

$$\begin{aligned} I &= \frac{1}{3} \int \frac{dt}{t(t+8)} \\ &= \frac{1}{3} \int \left[\frac{\frac{1}{8}}{t} - \frac{\frac{1}{8}}{t+8} \right] dt \\ &= \frac{1}{3} \times \frac{1}{8} \int \frac{dt}{t} - \frac{1}{3} \times \frac{1}{8} \int \frac{dt}{t+8} \\ &= \frac{1}{24} \log t - \frac{1}{24} \times \log(t+8) + C \\ &= \frac{1}{24} \log x^3 - \frac{1}{24} \times \log(x^3+8) + C \\ &= \frac{3}{24} \log x - \frac{1}{24} \times \log(x^3+8) + C \\ &= \frac{1}{8} \log x - \frac{1}{24} \times \log(x^3+8) + C \end{aligned}$$

Indefinite Integrals Ex 19.30 Q48

$$\text{Let } \frac{3}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{1+x^2}$$

$$\begin{aligned} \Rightarrow 3 &= A(1+x^2) + (Bx+C)(1-x) \\ &= (A-B)x^2 + (B-C)x + (A+C) \end{aligned}$$

Equating similar terms, we get,

$$A-B=0, \quad B-C=0, \quad A+C=3$$

Solving we get,

$$A=C=\frac{3}{2} \quad \text{and} \quad B=\frac{3}{2}$$

Thus,

$$\begin{aligned} I &= \frac{3}{2} \int \frac{dx}{1-x} + \frac{3}{2} \int \frac{xdx}{1+x^2} + \frac{3}{2} \int \frac{dx}{1+x^2} \\ &= -\frac{3}{2} \log|1-x| + \frac{3}{2} \log|1+x^2| + \frac{3}{2} \tan^{-1} x + c \end{aligned}$$

$$I = \frac{3}{4} \left[\log \left| \frac{1+x^2}{(1-x)^2} \right| + 2 \tan^{-1} x + c \right]$$

Indefinite Integrals Ex 19.30 Q49

Let

$$\sin x = t$$

$$\Rightarrow \cos x = dt$$

$$\therefore \int \frac{\cos x}{(1 - \sin x)^3 (2 + \sin x)} = \int \frac{1}{(1 - t)^3 (2 + t)} dt$$

$$\text{Let } f(t) = \frac{1}{(1 - t)^3 (2 + t)}$$

Then, suppose

$$\frac{1}{(1 - t)^3 (2 + t)} = \frac{A}{1 - t} + \frac{B}{(1 - t)^2} + \frac{C}{(1 - t)^3} + \frac{D}{(2 + t)}$$

$$\Rightarrow 1 = A(1 - t)^2(2 + t) + B(1 - t)(2 + t) + C(2 + t) + D(1 - t)^3$$

Put $t = 1$

$$1 = 3C$$

$$\Rightarrow C = \frac{1}{3}$$

Put $t = -2$

$$1 = 27D$$

$$\Rightarrow D = \frac{1}{27}$$

Similarly, we can find that $A = \frac{-1}{27}$ and $B = \frac{+1}{9}$

$$\begin{aligned} \therefore \int \frac{1}{(1 - t)^3 (2 + t)} dt &= \frac{-1}{27} \int \frac{1}{1 - t} dt + \frac{1}{9} \int \frac{dt}{(1 - t)^2} + \frac{1}{3} \int \frac{dt}{(1 - t)^3} + \frac{1}{27} \int \frac{dt}{2 + t} \\ &= \frac{-1}{27} \log|1 - t| + \frac{1}{9(1 - t)} + \frac{1}{6(1 - t)^2} + \frac{1}{27} \log|2 + t| + c \end{aligned}$$

Putting $t = \sin x$, we get

$$\begin{aligned} &\int \frac{\cos x}{(1 - \sin x)^3 (2 + \sin x)} dx \\ &= \frac{-1}{27} \log|1 - \sin x| + \frac{1}{9(1 - \sin x)} + \frac{1}{6(1 - \sin x)^2} + \frac{1}{27} \log|2 + \sin x| + C \end{aligned}$$

Indefinite Integrals Ex 19.30 Q50

Consider the integral

$$I = \int \frac{2x^2 + 1}{x^2(x^2 + 4)} dx$$

Now let us separate the fraction $\frac{2x^2 + 1}{x^2(x^2 + 4)}$ through partial fractions.

Substitute $x^2 = t$, then

$$\frac{2x^2 + 1}{x^2(x^2 + 4)} = \frac{2t + 1}{t(t + 4)}$$

$$\Rightarrow \frac{2t + 1}{t(t + 4)} = \frac{A}{t} + \frac{B}{t + 4}$$

$$\Rightarrow \frac{2t + 1}{t(t + 4)} = \frac{A(t + 4) + Bt}{t(t + 4)}$$

$$\Rightarrow 2t + 1 = A(t + 4) + Bt$$

$$\Rightarrow 2t + 1 = At + 4A + Bt$$

Comparing the coefficients, we have,

$$A + B = 2 \text{ and } 4A = 1$$

$$\Rightarrow A = \frac{1}{4} \text{ and } B = \frac{7}{4}$$

$$\Rightarrow \frac{2x^2 + 1}{x^2(x^2 + 4)} = \frac{1}{4x^2} + \frac{7}{4(x^2 + 4)}$$

Thus, we have,

$$I = \int \frac{2x^2 + 1}{x^2(x^2 + 4)} dx$$

$$= \frac{1}{4} \int \frac{dx}{x^2} + \frac{7}{4} \int \frac{dx}{(x^2 + 4)}$$

$$= -\frac{1}{4x} + \frac{7}{4} \times \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + C$$

$$= -\frac{1}{4x} + \frac{7}{8} \tan^{-1}\left(\frac{x}{2}\right) + C$$

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