



Adjoint and Inverse of Matrix Ex 7.1 Q8(i)

Here, $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

Expanding using 1st row, we get

$$\begin{aligned} |A| &= 1 \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} + 3 \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} \\ &= 1(6 - 1) - 2(4 - 3) + 3(2 - 9) \\ &= 5 - 2 \times 1 + 3 \times (-7) \\ &= 5 - 2 - 21 = -18 \neq 0 \end{aligned}$$

Therefore, A^{-1} exists.

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 5 & C_{21} = -1 & C_{31} = -7 \\ C_{12} = -1 & C_{22} = -7 & C_{32} = 5 \\ C_{13} = -7 & C_{23} = 5 & C_{33} = -1 \end{array}$$

$$\begin{aligned} \therefore \text{adj } A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{bmatrix}^T \\ &= \begin{bmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{bmatrix} \end{aligned}$$

Now,

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\text{Hence, } A^{-1} = \frac{1}{(-18)} \begin{bmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{bmatrix} = \begin{bmatrix} \frac{-5}{18} & \frac{1}{18} & \frac{7}{18} \\ \frac{1}{18} & \frac{7}{18} & \frac{-5}{18} \\ \frac{7}{18} & \frac{-5}{18} & \frac{1}{18} \end{bmatrix}$$

$$\text{Here, } A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$$

Expanding using first column, we get

$$\begin{aligned} |A| &= 1 \begin{vmatrix} -1 & -1 \\ 3 & -1 \end{vmatrix} - 2 \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} + 5 \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} \\ &= 1 \times (1 + 3) - 2(-1 + 2) + 5(3 + 2) \\ &= 4 - 2(1) + 5(5) = 27 \neq 0 \end{aligned}$$

Therefore, A^{-1} exists.

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 4 & C_{21} = +17 & C_{31} = 3 \\ C_{12} = -1 & C_{22} = -11 & C_{32} = +6 \\ C_{13} = 5 & C_{23} = +1 & C_{33} = -3 \end{array}$$

$$\begin{aligned} \therefore \text{adj} A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} 4 & -1 & 5 \\ 17 & -11 & 1 \\ 3 & 6 & -3 \end{bmatrix}^T \\ &= \begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix} \end{aligned}$$

$$\text{Now, } A^{-1} = \frac{1}{|A|} \cdot \text{adj} A$$

$$\therefore A^{-1} = \frac{1}{27} \begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix} = \begin{bmatrix} \frac{4}{27} & \frac{17}{27} & \frac{3}{27} \\ \frac{-1}{27} & \frac{-11}{27} & \frac{6}{27} \\ \frac{5}{27} & \frac{1}{27} & \frac{-3}{27} \end{bmatrix} = \begin{bmatrix} \frac{4}{27} & \frac{17}{27} & \frac{1}{9} \\ \frac{-1}{27} & \frac{-11}{27} & \frac{2}{9} \\ \frac{5}{27} & \frac{1}{27} & \frac{-1}{9} \end{bmatrix}$$

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$$\text{Here, } A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Expanding using first column, we get

$$\begin{aligned} |A| &= 2 \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} + 1 \begin{vmatrix} -1 & -1 \\ 1 & 2 \end{vmatrix} + 1 \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} \\ &= 2(4-1) + 1(-2+1) + 1(1-2) \\ &= 2(3) + 1(-1) + 1(-1) = 6 - 2 = 4 \neq 0 \end{aligned}$$

Therefore, A^{-1} exists

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 3 & C_{21} = +1 & C_{31} = -1 \\ C_{12} = +1 & C_{22} = 3 & C_{32} = +1 \\ C_{13} = -1 & C_{23} = +1 & C_{33} = 3 \end{array}$$

$$\begin{aligned} \therefore \text{adj } A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}^T \\ &= \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix} \end{aligned}$$

$$\text{Now, } A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\therefore A^{-1} = \frac{1}{4} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & \frac{-1}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4} \\ \frac{-1}{4} & \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

$$\text{Here, } A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

Expanding using first column, we get

$$\begin{aligned} |A| &= 2 \begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix} - 0 \begin{vmatrix} 5 & 0 \\ 0 & 3 \end{vmatrix} - 1 \begin{vmatrix} 5 & 1 \\ 0 & 1 \end{vmatrix} \\ &= 2(3 - 0) - 0 - 1(5) \\ &= 2(3) - 1(5) = 1 \neq 0 \end{aligned}$$

Therefore, A^{-1} exists

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 3 & C_{21} = -1 & C_{31} = 1 \\ C_{12} = -15 & C_{22} = 6 & C_{32} = -5 \\ C_{13} = 5 & C_{23} = -2 & C_{33} = 2 \end{array}$$

$$\begin{aligned} \therefore \text{adj } A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} 3 & -15 & 5 \\ -1 & 6 & -2 \\ 1 & -5 & 2 \end{bmatrix}^T \\ &= \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \therefore A^{-1} &= \frac{1}{|A|} \cdot \text{adj } A \\ &= \frac{1}{1} \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \end{aligned}$$

$$\text{Hence, } A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$\text{Here, } A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$$

Expanding using first column, we get

$$\begin{aligned} |A| &= 0 \begin{vmatrix} -3 & 4 \\ -3 & 4 \end{vmatrix} - 1 \begin{vmatrix} 4 & 4 \\ 3 & 4 \end{vmatrix} - 1 \begin{vmatrix} 4 & -3 \\ 3 & -3 \end{vmatrix} \\ &= 0 - 1(6 - 12) - 1(-12 + 9) \\ &= -1(4) - 1(-3) = -1 \neq 0 \end{aligned}$$

Therefore, A^{-1} exists

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 0 & C_{21} = -1 & C_{31} = 1 \\ C_{12} = -4 & C_{22} = 3 & C_{32} = -4 \\ C_{13} = -3 & C_{23} = +3 & C_{33} = -4 \end{array}$$

$$\begin{aligned} \therefore \text{adj } A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} 0 & -4 & -3 \\ -1 & 3 & 3 \\ 1 & -4 & -4 \end{bmatrix}^T \\ &= \begin{bmatrix} 0 & -1 & 1 \\ -4 & 3 & -4 \\ -3 & 3 & -4 \end{bmatrix} \end{aligned}$$

$$\text{Now, } A^{-1} = \frac{1}{|A|} \cdot \text{adj } A$$

$$A^{-1} = \frac{1}{(-1)} \begin{bmatrix} 0 & -1 & 1 \\ -4 & 3 & -4 \\ -3 & 3 & -4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$$

$$\text{Here, } A = \begin{bmatrix} 0 & 0 & -1 \\ 3 & 4 & 5 \\ -2 & -4 & -7 \end{bmatrix}$$

Expanding using first column, we get

$$\begin{aligned} |A| &= 0 \begin{vmatrix} 4 & 5 \\ -4 & -7 \end{vmatrix} - 0 \begin{vmatrix} 3 & 5 \\ -2 & -7 \end{vmatrix} - 1 \begin{vmatrix} 3 & 4 \\ -2 & -4 \end{vmatrix} \\ &= 0 - 0 - 1(-12 + 8) \\ &= -1(-4) = 4 \neq 0 \end{aligned}$$

Therefore, A^{-1} exists

Cofactors of A are:

$$\begin{array}{lll} C_{11} = -8 & C_{21} = +4 & C_{31} = 4 \\ C_{12} = +11 & C_{22} = -2 & C_{32} = -3 \\ C_{13} = -4 & C_{23} = +0 & C_{33} = 0 \end{array}$$

$$\begin{aligned} \therefore \text{adj } A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} -8 & 11 & -4 \\ 4 & -2 & 0 \\ 4 & -3 & 0 \end{bmatrix}^T \\ &= \begin{bmatrix} -8 & 4 & 4 \\ 11 & -2 & -3 \\ -4 & 0 & 0 \end{bmatrix} \end{aligned}$$

$$\therefore A^{-1} = \frac{1}{|A|} \cdot \text{adj } A$$

$$A^{-1} = \frac{1}{4} \begin{bmatrix} -8 & 4 & 4 \\ 11 & -2 & -3 \\ -4 & 0 & 0 \end{bmatrix}$$

$$\text{Hence, } A^{-1} = \frac{1}{4} \begin{bmatrix} -8 & 4 & 4 \\ 11 & -2 & -3 \\ -4 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 1 \\ \frac{11}{4} & \frac{-1}{2} & \frac{-3}{4} \\ -1 & 0 & 0 \end{bmatrix}$$

$$\text{Here, } A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & \sin \alpha & -\cos \alpha \end{bmatrix}$$

Expanding using first column, we get

$$\begin{aligned} |A| &= 1 \begin{vmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & -\cos \alpha \end{vmatrix} - 0 + 0 \\ &= -\cos^2 \alpha + \sin^2 \alpha \\ &= -(\cos^2 \alpha + \sin^2 \alpha) \\ |A| &= -1 \neq 0 \end{aligned}$$

Therefore, A^{-1} exists

Cofactors of A are:

$$\begin{array}{lll} C_{11} = -1 & C_{21} = 0 & C_{31} = 0 \\ C_{12} = 0 & C_{22} = -\cos \alpha & C_{32} = -\sin \alpha \\ C_{13} = 0 & C_{23} = -\sin \alpha & C_{33} = \cos \alpha \end{array}$$

$$\begin{aligned} \therefore \text{adj } A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos \alpha & -\sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix}^T \\ &= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos \alpha & -\sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix} \end{aligned}$$

$$\text{Now, } A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\therefore A^{-1} = \frac{1}{(-1)} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos \alpha & -\sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix}$$

$$\text{Hence, } A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & \sin \alpha & -\cos \alpha \end{bmatrix}$$

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