(a) The given point is (0, 0, 0) and the plane is $3x-4y+12z=3$

$$\therefore d=\left|\frac{3\times0-4\times0+12\times0-3}{\sqrt{(3)^2+(-4)^2+(12)^2}}\right|=\frac{3}{\sqrt{169}}=\frac{3}{13}$$

(b) The given point is (3, − 2, 1) and the plane is $2x-y+2z+3=0$

$$\therefore d=\left|\frac{2\times3-(-2)+2\times1+3}{\sqrt{(2)^2+(-1)^2+(2)^2}}\right|=\left|\frac{13}{3}\right|=\frac{13}{3}$$

(c) The given point is (2, 3, −5) and the plane is $x+2y-2z=9$

$$\therefore d=\left|\frac{2+2\times3-2(-5)-9}{\sqrt{(1)^2+(2)^2+(-2)^2}}\right|=\frac{9}{3}=3$$

(d) The given point is (−6, 0, 0) and the plane is $2x-3y+6z-2=0$

$$d=\left|\frac{2(-6)-3\times0+6\times0-2}{\sqrt{(2)^2+(-3)^2+(6)^2}}\right|=\left|\frac{-14}{\sqrt{49}}\right|=\frac{14}{7}=2$$

Miscellaneous Solutions

Question 1:

Show that the line joining the origin to the point (2, 1, 1) is perpendicular to the line determined by the points (3, 5, −1), (4, 3, −1).

Answer

Let OA be the line joining the origin, O (0, 0, 0), and the point, A (2, 1, 1).

Also, let BC be the line joining the points, B (3, 5, −1) and C (4, 3, −1).

The direction ratios of OA are 2, 1, and 1 and of BC are (4 − 3) = 1, (3 − 5) = −2, and (−1 + 1) = 0

OA is perpendicular to BC, if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\therefore a_1a_2 + b_1b_2 + c_1c_2 = 2 \times 1 + 1\,(-2) + 1 \times 0 = 2 - 2 = 0$

Thus, OA is perpendicular to BC.

Question 2:

If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both of these are $m_1n_2 - m_2n_1,\ n_1l_2 - n_2l_1,\ l_1m_2 - l_2m_1$.

Answer

It is given that l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines. Therefore,

$$l_1l_2+m_1m_2+n_1n_2=0 \qquad ...(1)$$

$$l_1^2+m_1^2+n_1^2=1 \qquad ...(2)$$

$$l_2^2+m_2^2+n_2^2=1 \qquad ...(3)$$

Let l, m, n be the direction cosines of the line which is perpendicular to the line with direction cosines l_1, m_1, n_1 and l_2, m_2, n_2.

$$\therefore ll_1+mm_1+nn_1=0$$
$$ll_2+mm_2+nn_2=0$$
$$\therefore \frac{l}{m_1n_2-m_2n_1}=\frac{m}{n_1l_2-n_2l_1}=\frac{n}{l_1m_2-l_2m_1}$$
$$\Rightarrow \frac{l^2}{(m_1n_2-m_2n_1)^2}=\frac{m^2}{(n_1l_2-n_2l_1)^2}=\frac{n^2}{(l_1m_2-l_2m_1)^2}$$
$$\Rightarrow \frac{l^2}{(m_1n_2-m_2n_1)^2}=\frac{m^2}{(n_1l_2-n_2l_1)^2}=\frac{n^2}{(l_1m_2-l_2m_1)^2}$$

$$= \frac{l \quad + m \quad + n}{\left(m_1n_2 - m_2n_1\right)^2 + \left(n_1l_2 - n_2l_1\right)^2 + \left(l_1m_2 - l_2m_l\right)^2} \qquad \ldots(4)$$

l, *m*, *n* are the direction cosines of the line.

$\therefore l^2 + m^2 + n^2 = 1$ ... (5)

It is known that,

$$\left(l_1^2 + m_1^2 + n_1^2\right)\left(l_2^2 + m_2^2 + n_2^2\right) - \left(l_1l_2 + m_1m_2 + n_1n_2\right)^2$$
$$= \left(m_1n_2 - m_2n_1\right)^2 + \left(n_1l_2 - n_2l_1\right)^2 + \left(l_1m_2 - l_2m_1\right)^2$$

From (1), (2), and (3), we obtain

$$\Rightarrow 1.1 - 0 = \left(m_1n_2 + m_2n_1\right)^2 + \left(n_1l_2 - n_2l_1\right)^2 + \left(l_1m_2 - l_2m_1\right)^2$$

$$\therefore \left(m_1n_2 - m_2n_1\right)^2 + \left(n_1l_2 - n_2l_1\right)^2 + \left(l_1m_2 - l_2m_1\right)^2 = 1 \qquad \ldots(6)$$

Substituting the values from equations (5) and (6) in equation (4), we obtain

$$\frac{l^2}{\left(m_1n_2 - m_2n_1\right)^2} = \frac{m^2}{\left(n_2l_2 - n_2l_1\right)^2} = \frac{n^2}{\left(l_1m_2 - l_2m_1\right)^2} = 1$$
$$\Rightarrow l = m_1n_2 - m_2n_1, m = n_1l_2 - n_2l_1, n = l_1m_2 - l_2m_1$$

Thus, the direction cosines of the required line are $m_1n_2 - m_2n_1,\ n_1l_2 - n_2l_1, \text{and}\ l_1m_2 - l_2m_1.$

Question 3:

Find the angle between the lines whose direction ratios are *a*, *b*, *c* and *b* − *c*, *c* − *a*, *a* − *b*.

Answer

The angle *Q* between the lines with direction cosines, *a*, *b*, *c* and *b* − *c*, *c* − *a*, *a* − *b*, is given by,

$$\cos Q = \left| \frac{a(b-c) + b(c-a) + c(a-b)}{\sqrt{a^2 + b^2 + c^2} + \sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}} \right|$$
$$\Rightarrow \cos Q = 0$$
$$\Rightarrow Q = \cos^{-1} 0$$
$$\Rightarrow Q = 90°$$

Thus, the angle between the lines is 90°.

Question 4:

Find the equation of a line parallel to *x*-axis and passing through the origin.

Answer

The line parallel to *x*-axis and passing through the origin is *x*-axis itself.

Let A be a point on *x*-axis. Therefore, the coordinates of A are given by (*a*, 0, 0), where *a* ∈ R.

Direction ratios of OA are (*a* − 0) = *a*, 0, 0

The equation of OA is given by,

$$\frac{x-0}{a} = \frac{y-0}{0} = \frac{z-0}{0}$$
$$\Rightarrow \frac{x}{1} = \frac{y}{0} = \frac{z}{0} = a$$

Thus, the equation of line parallel to *x*-axis and passing through origin is

$$\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$$

Question 5:

If the coordinates of the points A, B, C, D be (1, 2, 3), (4, 5, 7), (−4, 3, −6) and (2, 9, 2) respectively, then find the angle between the lines AB and CD.

Answer

The coordinates of A, B, C, and D are (1, 2, 3), (4, 5, 7), (−4, 3, −6), and (2, 9, 2) respectively.

The direction ratios of AB are (4 − 1) = 3, (5 − 2) = 3, and (7 − 3) = 4

The direction ratios of CD are (2 −(− 4)) = 6, (9 − 3) = 6, and (2 −(−6)) = 8

It can be seen that, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$

Therefore, AB is parallel to CD.

Thus, the angle between AB and CD is either 0° or 180°.

sQuestion 6:

$$\frac{x-1}{\ } = \frac{y-2}{\ } = \frac{z-3}{\ } \qquad \frac{x-1}{\ } = \frac{y-1}{\ } = \frac{z-6}{\ }$$

If the lines $-3 \quad 2k \quad 2$ and $3k \quad 1 \quad -5$ are perpendicular, find the value of k.

Answer

The direction of ratios of the lines, $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-1}{1}=\frac{z-6}{-5}$, are −3, $2k$, 2 and $3k$, 1, −5 respectively.

It is known that two lines with direction ratios, a_1, b_1, c_1 and a_2, b_2, c_2, are perpendicular, if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$\therefore -3(3k)+2k\times 1+2(-5)=0$$

$$\Rightarrow -9k+2k-10=0$$

$$\Rightarrow 7k=-10$$

$$\Rightarrow k=\frac{-10}{7}$$

Therefore, for $k=-\frac{10}{7}$, the given lines are perpendicular to each other.

Question 7:

Find the vector equation of the plane passing through (1, 2, 3) and perpendicular to the plane $\vec{r}\cdot\left(\hat{i}+2\hat{j}-5\hat{k}\right)+9=0$

Answer

The position vector of the point (1, 2, 3) is $\vec{r}_1=\hat{i}+2\hat{j}+3\hat{k}$

The direction ratios of the normal to the plane, $\vec{r}\cdot\left(\hat{i}+2\hat{j}-5\hat{k}\right)+9=0$, are 1, 2, and −5 and the normal vector is $\vec{N}=\hat{i}+2\hat{j}-5\hat{k}$

The equation of a line passing through a point and perpendicular to the given plane is given by, $\vec{l}=\vec{r}+\lambda\vec{N},\ \lambda\in R$

*********** END **********