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# Minimum degree algorithm

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In [numerical analysis](#) the **minimum degree algorithm** is an [algorithm](#) used to permute the rows and columns of a [symmetric sparse matrix](#) before applying the [Cholesky decomposition](#), to reduce the number of non-zeros in the Cholesky factor. This results in reduced storage requirements and means that the Cholesky factor, or sometimes an incomplete Cholesky factor used as a preconditioner (for example in the preconditioned [conjugate gradient](#) algorithm) can be applied with fewer arithmetic operations.

Minimum degree algorithms are often used in the [finite element method](#) where the reordering of nodes can be carried out depending only on the topology of the mesh, rather than the coefficients in the partial differential equation, resulting in efficiency savings when the same mesh is used for a variety of coefficient values.

Given a linear system

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

where **A** is an  $n \times n$  real symmetric sparse square matrix the Cholesky factor **L** will typically suffer 'fill in', that is have more non-zeros than the upper triangle of **A**. We seek a [permutation matrix](#) **P**, so that the matrix **P<sup>T</sup>AP**, which is also symmetric, has the least possible fill in its Cholesky factor. We solve the reordered system

$$(\mathbf{P}^T \mathbf{A} \mathbf{P}) (\mathbf{P}^T \mathbf{x}) = \mathbf{P}^T \mathbf{b}.$$

The problem of finding the best ordering is an [NP-complete](#) problem and is thus intractable, so heuristic methods are used instead. The minimum degree algorithm is derived from a method first proposed by [Markowitz](#) in 1959 for non-symmetric [linear programming](#) problems, which is loosely described as follows. At each step in [Gaussian elimination](#) row and column permutations are performed so as to minimize the number of off diagonal non-zeros in the pivot row and column. A symmetric version of Markowitz method was described by Tinney and Walker in 1967 and Rose later derived a graph theoretic version of the algorithm where the factorization is only simulated, and this was named the minimum degree algorithm. The graph referred to is the graph with *n* vertices, with vertices *i* and *j* connected by an edge when  $a_{ij} \neq 0$ , and the *degree* is the degree of the vertices. A crucial aspect of such algorithms is a tie breaking strategy when there is a choice of renumbering resulting in the same degree.

A version of the minimum degree algorithm was implemented in the [MATLAB](#) function **symmmd** (where MMD stands for multiple minimum degree), but has now been superseded by a symmetric approximate multiple minimum degree function **symamd**, which is faster. This is confirmed by theoretical analysis, which shows that for graphs on *n* vertices and *m* edges, MMD has a tight [upper bound](#) of  $O(n^2m)$  on its run time, whereas for AMD a tight bound of  $O(nm)$  holds.

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**Numerical linear algebra**

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**Key concepts** [Floating point](#) · [Numerical stability](#)

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