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# Birkhoff interpolation

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- This article **has no lead section**. (*December 2010*)
- This article **may be confusing or unclear to readers**. (*December 2010*)

In **mathematics**, **Birkhoff interpolation** is an extension of [polynomial interpolation](#). It refers to the problem finding a polynomial  $p$  of degree  $d$  such that certain [derivatives](#) have specified values at specified points:

$$p^{(n_i)}(x_i) = y_i \quad \text{for } i = 1, \dots, d,$$

where the data points  $(x_i, y_i)$  and the nonnegative integers  $n_i$  are given. It differs from [Hermite interpolation](#) in that it is possible to specify derivatives of  $p$  at some points without specifying the lower derivatives or the polynomial itself. The name refers to [George David Birkhoff](#), who first studied the problem in [Birkhoff \(1906\)](#).

In contrast to [Lagrange interpolation](#) and [Hermite interpolation](#), a Birkhoff interpolation problem does not always have a unique solution. For instance, there is no quadratic polynomial  $p$  such that  $p(-1) = p(1) = 0$  and  $p'(0) = 1$ . On the other hand, the Birkhoff interpolation problem where the values of  $p'(-1)$ ,  $p(0)$  and  $p'(1)$  are given always has a unique solution ([Passow 1983](#)).

An important problem in the theory of Birkhoff interpolation is to classify those problems that have a unique solution. [Schoenberg \(1966\)](#) formulates the problem as follows. Let  $d$  denote the number of conditions (as above) and let  $k$  be the number of interpolation points. Given a  $d$ -by- $k$  matrix  $E$ , all of whose entries are either 0 or 1, such that exactly  $d$  entries are 1, then the corresponding problem is to determine  $p$  such that

$$p^{(j)}(x_i) = y_{i,j} \quad \text{for all } (i,j) \text{ with } e_{ij} = 1.$$

The matrix  $E$  is called the incidence matrix. For example, the incidence matrices for the interpolation problems mentioned in the previous paragraph are:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

Now the question is: does a Birkhoff interpolation problem with a given incidence matrix have a unique solution for any choice of the interpolation points?

The case with  $k = 2$  interpolation points was tackled by [Pólya \(1931\)](#). Let  $S_m$  denote the sum of the entries in the first  $m$  columns of the incidence matrix:

$$S_m = \sum_{i=1}^k \sum_{j=1}^m e_{ij}.$$

Then the Birkhoff interpolation problem with  $k = 2$  has a unique solution if and only if  $S_m \geq m$  for all  $m$ .

[Schoenberg \(1966\)](#) showed that this is a necessary condition for all values of  $k$ .

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