

Generate integer from 1 to 7 with equal probability

Given a function `foo()` that returns integers from 1 to 5 with equal probability, write a function that returns integers from 1 to 7 with equal probability using `foo()` only. Minimize the number of calls to `foo()` method. Also, use of any other library function is not allowed and no floating point arithmetic allowed.

Solution:

We know `foo()` returns integers from 1 to 5. How we can ensure that integers from 1 to 7 occur with equal probability?

If we somehow generate integers from 1 to a-multiple-of-7 (like 7, 14, 21, ...) with equal probability, we can use modulo division by 7 followed by adding 1 to get the numbers from 1 to 7 with equal probability.

We can generate from 1 to 21 with equal probability using the following expression.

```
5*foo() + foo() - 5
```

Let us see how above expression can be used.

1. For each value of first `foo()`, there can be 5 possible combinations for values of second `foo()`. So, there are total 25 combinations possible.
2. The range of values returned by the above equation is 1 to 25, each integer occurring exactly once.
3. If the value of the equation comes out to be less than 22, return modulo division by 7 followed by adding 1. Else, again call the method recursively. The probability of returning each integer thus becomes $1/7$.

The below program shows that the expression returns each integer from 1 to 25 exactly once.

```
#include <stdio.h>

int main()
{
    int first, second;
```

```
for ( first=1; first<=5; ++first )
    for ( second=1; second<=5; ++second )
        printf ("%d \n", 5*first + second - 5);
return 0;
}
```

Output:

```
1
2
.
.
24
25
```

The below program depicts how we can use foo() to return 1 to 7 with equal probability.

```
#include <stdio.h>

int foo() // given method that returns 1 to 5 with equal
{
    // some code here
}

int my_rand() // returns 1 to 7 with equal probability
{
    int i;
    i = 5*foo() + foo() - 5;
    if (i < 22)
        return i%7 + 1;
    return my_rand();
}

int main()
{
    printf ("%d ", my_rand());
    return 0;
}
```