Multi-Objective Optimization Using Evolutionary Algorithms

First Edition

Kalyanmoy Deb
Professor, Department of Mechanical Engineering
Indian Institute of Technology Kanpur, India

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Preface

Optimization is a procedure of finding and comparing feasible solutions until no better solution can be found. Solutions are termed good or bad in terms of an objective, which is often the cost of fabrication, amount of harmful gases, efficiency of a process, product reliability, or other factors. A significant portion of research and application in the field of optimization considers a single objective, although most real-world problems involve more than one objective. The presence of multiple conflicting objectives (such as simultaneously minimizing the cost of fabrication and maximizing product reliability) is natural in many problems and makes the optimization problem interesting to solve. Since no one solution can be termed as an optimum solution to multiple conflicting objectives, the resulting multi-objective optimization problem resorts to a number of trade-off optimal solutions. Classical optimization methods can at best find one solution in one simulation run, thereby making those methods inconvenient to solve multi-objective optimization problems.

Evolutionary algorithms (EAs), on the other hand, can find multiple optimal solutions in one single simulation run due to their population-approach. Thus, EAs are ideal candidates for solving multi-objective optimization problems. This book provides a comprehensive survey of most multi-objective EA approaches suggested since the evolution of such algorithms. Although a number of approaches were outlined sparingly in the early years of the subject, more pragmatic multi-objective EAs (MOEAs) were first suggested about a decade ago. All such studies exist in terms of research papers in various journals and conference proceedings, which thus force newcomers and practitioners to search different sources in order to obtain an overview of the topic. This fact has been the primary motivation for me to take up this project and to gather together most of the MOEA techniques in one text.

This present book provides an extensive discussion on the principles of multi-objective optimization and on a number of classical approaches. For those readers unfamiliar with multi-objective optimization, Chapters 2 and 3 provide the necessary background. Readers with a classical optimization background can take advantage of Chapter 4 to familiarize themselves with various evolutionary algorithms. Beginning with a detailed description of genetic algorithms, an introduction to three other EAs, namely evolution strategy, evolutionary programming, and genetic programming, is provided. Since the search for multiple solutions is important in multi-objective optimization, a detailed description of EAs, particularly designed to solve multi-modal optimization problems, is also presented. Elite-preservation or emphasizing currently elite solutions is an important operator in an EA. In this book, we classify MOEAs according to whether they preserve elitism or not. Chapter 5 presents a number of non-elitist MOEAs. Each algorithm is described by presenting a step-by-step procedure, showing a hand calculation, discussing advantages and disadvantages of the algorithm, calculating its computational complexity, and finally presenting a computer simulation on a test problem. In order to obtain a comparative evaluation of different algorithms, the same test problem with the same parameter settings is used for most MOEAs presented in the book. Chapter 6 describes a number of elitist MOEAs in an identical manner.

Constraints are inevitable in any real-world optimization problem, including multi-objective optimization problems. Chapter 7 presents a number of techniques specializing in handling constrained optimization problems. Such approaches include simple modifications to the MOEAs discussed in Chapters 5 and 6 to give more specialized new MOEAs.

Whenever new techniques are suggested, there is room for improvement and further research. Chapter 8 discusses a number of salient issues regarding MOEAs. This chapter amply emphasizes the importance of each issue in developing and applying MOEAs in a better manner by presenting the current state-of-the-art research and by proposing further research directions.

Finally, in Chapter 9, the usefulness of MOEAs in real-world applications is demonstrated by presenting a number of applications in engineering design. This chapter also discusses plausible hybrid techniques for combining MOEAs with a local search technique for developing an even better and a pragmatic multi-objective optimization tool.

This book would not have been completed without the dedication of a number of my students, namely Sameer Agrawal, Amrit Pratap, Tushar Goel and Thirunavukkarasu Meyarivan. They have helped me in writing computer codes for investigating the performance of the different algorithms presented in this book and in discussing with me for long hours various issues regarding multi-objective optimization. In this part of the world, where the subject of evolutionary algorithms is still a comparative fad, they were my colleagues and inspirations. I also appreciate the help of Dhiraj Joshi, Ashish Anand, Shamik Chaudhury, Pawan Nain, Akshay Mohan, Saket Awasthi and Pawan Zope. In any case, I must not forget to thank Nidamarthi Srinivas who took up the challenge to code the first viable MOEA based on the non-domination concept. This ground-breaking study on non-dominated sorting GA (NSGA) inspired many MOEA researchers and certainly most of our MOEA research activities at the Kanpur Genetic Algorithms Laboratory (KanGAL), housed at the Indian Institute of Technology Kanpur, India.

The first idea for writing this book originated during my visit to the University of Dortmund during the period 1998–1999 through the Alexander von Humboldt (AvH) Fellowship scheme. The resourceful research environment at the University of Dortmund and the ever-supportive sentiments of AvH organization were helpful in formulating a plan for the contents of this book. Discussions with Eckart Zitzler, Lothar Thiele, Jürgen Branke, Frank Kursawe, Günter Rudolph and Ian Parmee on various issues on multi-objective optimization are acknowledged. Various suggestions given by Marco Laumanns and Eckart Zitzler in improving an earlier draft of this book are highly appreciated. I am privileged to get continuous support and encouragement from two stalwarts in the field of evolutionary computation, namely David E. Goldberg and Hans-Paul Schwefel. The help obtained from Victoria Coverstone-Carroll, Bill Hartmann, Hisao Ishibuchi and Eric Michelssen was also very useful. I also thank David B. Fogel for pointing me towards some of the early multi-objective EA studies.

Besides our own algorithms for multi-objective optimization, this book also presents a number of algorithms suggested by other researchers. Any difference between what is presented here and the original version of these algorithms is purely unintentional. Wherever in doubt, the original source can be referred. However, I would be happy to receive any such comments, which would be helpful to me in preparing the future editions of this book.

The completion of this book came at the expense of my long hours of absence from home. I am indebted to Debjani, Debayan, Dhriti, and Mr and Mrs S. K. Sarkar for their understanding and patience.

Kalyanmoy Deb
Indian Institute Technology Kanpur
deb@iitk.ac.in