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Binary splitting

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In [mathematics](#), **binary splitting** is a technique for speeding up numerical evaluation of many types of [series](#) with rational terms. In particular, it can be used to evaluate [hypergeometric series](#) at rational points. Given a series

$$S(a, b) = \sum_{n=a}^b \frac{p_n}{q_n}$$

where p_n and q_n are integers, the goal of binary splitting is to compute integers $P(a, b)$ and $Q(a, b)$ such that

$$S(a, b) = \frac{P(a, b)}{Q(a, b)}.$$




The splitting consists of setting $m = \lfloor (a + b)/2 \rfloor$ and recursively computing $P(a, b)$ and $Q(a, b)$ from $P(a, m)$, $P(m, b)$, $Q(a, m)$, and $Q(m, b)$. When a and b are sufficiently close, $P(a, b)$ and $Q(a, b)$ can be computed directly from $p_a \dots p_b$ and $q_a \dots q_b$.

Binary splitting requires more memory than direct term-by-term summation, but is asymptotically faster since the sizes of all occurring subproducts are reduced. Additionally, whereas the most naive evaluation scheme for a rational series uses a full-precision division for each term in the series, binary splitting requires only one final division at the target precision; this is not only faster, but conveniently eliminates rounding errors. To take full advantage of the scheme, fast multiplication algorithms such as [Toom–Cook](#) and [Schönhage–Strassen](#) must be used; with ordinary $O(n^2)$ multiplication, binary splitting may render no speedup at all or be slower.

Since all subdivisions of the series can be computed independently of each other, binary splitting lends well to [parallelization](#) and [checkpointing](#).

In a less specific sense, *binary splitting* may also refer to any [divide and conquer algorithm](#) that always divides the problem in two halves.

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Categories: [Computer arithmetic algorithms](#)

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