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Merge sort

Article Talk

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In computer science, merge sort (also commonly spelled **mergesort**) is an $O(n \log n)$ comparison-based sorting algorithm. Most implementations produce a stable sort, which means that the implementation preserves the input order of equal elements in the sorted output. Mergesort is a divide and conquer algorithm that was invented by John von Neumann in 1945.^[1] A detailed description and analysis of bottom-up mergesort appeared in a report by Goldstine and Neumann as early as 1948.[2]

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Merge sort

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An example of merge sort. First divide the list into the smallest unit (1 element), then compare each element with the adjacent list to sort and merge the two adjacent lists. Finally all the elements are sorted and merged.

	_
Class	Sorting algorithm
Data structure	Array
Worst case performand	$O(n \log n)$
Best case performance	$O(n \log n)$ typical,
	O(n) natural variant
Average case performa	$O(n \log n)$

Worst case space O(n) total, O(n) auxiliary complexity

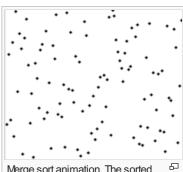
Algorithm [edit]

Conceptually, a merge sort works as follows:

- 1. Divide the unsorted list into *n* sublists, each containing 1 element (a list of 1 element is considered sorted).
- 2. Repeatedly merge sublists to produce new sorted sublists until there is only 1 sublist remaining. This will be the sorted list.

Top-down implementation [edit]

Example C-like code using indices for top down merge sort algorithm that recursively splits the list (called runs in this example) into sublists until sublist size is 1, then merges those sublists to produce a sorted list. The copy back step could be avoided if the recursion alternated between two functions so that the direction of the merge corresponds with the level of recursion



Merge sort animation. The sorted elements are represented by dots.

```
/* array A[] has the items to sort; array B[] is a work array */
TopDownMergeSort(A[], B[], n)
{
    TopDownSplitMerge(A, 0, n, B);
// iBegin is inclusive; iEnd is exclusive (A[iEnd] is not in the set)
TopDownSplitMerge(A[], iBegin, iEnd, B[])
{
    if(iEnd - iBegin < 2)</pre>
                                                  // if run size == 1
        return;
                                                     consider it sorted
```

```
Suomi
ใทย
Türkçe
Українська
Tiếng Wệt
中文
```

```
// recursively split runs into two halves until run size == 1,
    // then merge them and return back up the call chain
    iMiddle = (iEnd + iBegin) / 2;
                                                // iMiddle = mid point
    TopDownSplitMerge(A, iBegin, iMiddle, B); // split / merge left half
    TopDownSplitMerge(A, iMiddle, iEnd, B); // split / merge right half
    TopDownMerge (A, iBegin, iMiddle, iEnd, B); // merge the two half runs
    CopyArray(B, iBegin, iEnd, A);
                                                // copy the merged runs back to A
// left half is A[iBegin :iMiddle-1]
// right half is A[iMiddle:iEnd-1
TopDownMerge(A[], iBegin, iMiddle, iEnd, B[])
    i0 = iBegin, i1 = iMiddle;
    // While there are elements in the left or right runs
    for (j = iBegin; j < iEnd; j++) {</pre>
        // If left run head exists and is <= existing right run head.
        if (i0 < iMiddle && (i1 >= iEnd \mid \mid A[i0] \le A[i1])) {
            B[j] = A[i0];
            i0 = i0 + 1;
        } else {
           B[j] = A[i1];
            i1 = i1 + 1;
        }
   }
}
CopyArray(B[], iBegin, iEnd, A[])
    for(k = iBegin; k < iEnd; k++)</pre>
       A[k] = B[k];
```

Bottom-up implementation [edit]

Example C like code using indices for bottom up merge sort algorithm which treats the list as an array of *n* sublists (called *runs* in this example) of size 1, and iteratively merges sub-lists back and forth between two buffers:

```
void BottomUpMerge(A[], iLeft, iRight, iEnd, B[])
  i0 = iLeft;
  i1 = iRight;
  /* While there are elements in the left or right runs */
  for (j = iLeft; j < iEnd; j++)</pre>
      /* If left run head exists and is <= existing right run head */</pre>
      if (i0 < iRight && (i1 >= iEnd \mid \mid A[i0] \le A[i1]))
        {
          B[j] = A[i0];
          i0 = i0 + 1;
        }
      else
          B[j] = A[i1];
          i1 = i1 + 1;
    }
void CopyArray(B[], A[], n)
    for(i = 0; i < n; i++)</pre>
        A[i] = B[i];
/* array A[] has the items to sort; array B[] is a work array */
```

```
void BottomUpSort(A[], B[], n)
  /* Each 1-element run in A is already "sorted". */
  /* Make successively longer sorted runs of length 2, 4, 8, 16... until whole array
is sorted. */
 for (width = 1; width < n; width = 2 * width)</pre>
     /* Array A is full of runs of length width. */
     for (i = 0; i < n; i = i + 2 * width)
         /* or copy A[i:n-1] to B[] ( if(i+width >= n) ) */
         BottomUpMerge(A, i, min(i+width, n), min(i+2*width, n), B);
     /* Now work array B is full of runs of length 2*width. */
       Copy array B to array A for next iteration. */
     /\ast A more efficient implementation would swap the roles of A and B \ast/
     CopyArray(B, A, n);
     /* Now array A is full of runs of length 2*width. */
}
```

Top-down implementation using lists [edit]

Pseudocode for top down merge sort algorithm which recursively divides the input list into smaller sublists until the sublists are trivially sorted, and then merges the sublists while returning up the call chain.

```
function merge sort(list m)
    // Base case. A list of zero or one elements is sorted, by definition.
   if length(m) \leq 1
        return m
    // Recursive case. First, *divide* the list into equal-sized sublists.
    var list left, right
    var integer middle = length(m) / 2
    for each x in m before middle
         add x to left
    for each x in m after or equal middle
         add x to right
    // Recursively sort both sublists
   left = merge_sort(left)
    right = merge_sort(right)
    // Then merge the now-sorted sublists.
    return merge(left, right)
```

In this example, the merge function merges the left and right sublists.

```
function merge(left, right)
    var list result
    while notempty(left) and notempty(right)
        if first(left) <= first(right)</pre>
            append first(left) to result
            left = rest(left)
        else
            append first (right) to result
            right = rest(right)
    // either left or right may have elements left
    while notempty(left)
        append first(left) to result
        left = rest(left)
    while notempty(right)
        append first (right) to result
        right = rest(right)
    return result
```

Natural merge sort [edit]

A natural merge sort is similar to a bottom up merge sort except that any naturally occurring runs (sorted sequences) in the input are exploited. In the bottom up merge sort, the starting point assumes each run is one item long. In practice, random input data will have many short runs that just happen to be sorted. In the typical case, the natural merge sort may not need as many passes because there are fewer runs to merge. In the best case, the input is already sorted (i.e., is one run), so the natural merge sort need only make one pass through the data. Example:

Tournament replacement selection sorts are used to gather the initial runs for external sorting algorithms.

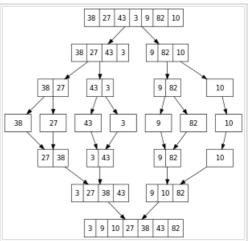
Analysis [edit]

In sorting n objects, merge sort has an average and worst-case performance of $O(n \log n)$. If the running time of merge sort for a list of length n is T(n), then the recurrence T(n) = 2T(n/2) + n follows from the definition of the algorithm (apply the algorithm to two lists of half the size of the original list, and add the n steps taken to merge the resulting two lists). The closed form follows from the master theorem.

In the worst case, the number of comparisons merge sort makes is equal to or slightly smaller than $(n \lceil \lg n \rceil - 2^{\lceil \lg n \rceil} + 1)$, which is between $(n \lg n - n + 1)$ and $(n \lg n + n + 0 (\lg n))$. [3]

For large n and a randomly ordered input list, merge sort's expected (average) number of comparisons approaches $\alpha \cdot n$ fewer than the worst case where

$$\alpha = -1 + \sum_{k=0}^{\infty} \frac{1}{2^k + 1} \approx 0.2645.$$



Arecursive merge sort algorithm used to sort an array of 7 integer values. These are the steps a human would take to emulate merge sort (top-down).

In the *worst* case, merge sort does about 39% fewer comparisons than quicksort does in the *average* case. In terms of moves, merge sort's worst case complexity is $O(n \log n)$ —the same complexity as quicksort's best case, and merge sort's best case takes about half as many iterations as the worst case. [citation needed]

Merge sort is more efficient than quicksort for some types of lists if the data to be sorted can only be efficiently accessed sequentially, and is thus popular in languages such as Lisp, where sequentially accessed data structures are very common. Unlike some (efficient) implementations of quicksort, merge sort is a stable sort.

Merge sort's most common implementation does not sort in place [citation needed]; therefore, the memory size of the input must be allocated for the sorted output to be stored in (see below for versions that need only n/2 extra spaces).

Merge sort also has some demerits. One is its use of 2n locations; the additional n locations are commonly used because merging two sorted sets in place is more complicated and would need more comparisons and move operations. But despite the use of this space the algorithm still does a lot of work: The contents of m are first copied into left and right and later into the list result on each invocation of $merge_sort$ (variable names according to the pseudocode above).

Variants [edit]

Variants of merge sort are primarily concerned with reducing the space complexity and the cost of copying.

A simple alternative for reducing the space overhead to n/2 is to maintain *left* and *right* as a combined structure, copy only the *left* part of m into temporary space, and to direct the *merge* routine to place the merged output into m. With this version it is better to allocate the temporary space outside the *merge* routine, so that only one allocation is needed. The excessive copying mentioned previously is also mitigated, since the last pair of lines

before the return result statement (function merge in the pseudo code above) become superfluous.

One drawback of merge sort, when implemented on arrays, is its O(n) working memory requirement. Several inplace variants have been suggested:

- Katajainen *et al.* present an algorithm that requires a constant amount of working memory: enough storage space to hold one element of the input array, and additional space to hold O(1) pointers into the input array. They achieve an $O(n \log n)$ time bound with small constants, but their algorithm is not stable.^[4]
- Several attempts have been made at producing an *in-place merge* algorithm that can be combined with a standard (top-down or bottom-up) merge sort to produce an in-place merge sort. In this case, the notion of "in-place" can be relaxed to mean "taking logarithmic stack space", because standard merge sort requires that amount of space for its own stack usage. It was shown by Geffert *et al.* that in-place, stable merging is possible in $O(n \log n)$ time using a constant amount of scratch space, but their algorithm is complicated and has high constant factors: merging arrays of length n and m can take 5n + 12n + o(m) moves. Other in-place algorithms include SymMerge, which takes $O((n + m) \log (n + m))$ time in total. Plugging such an algorithm into merge sort increases its complexity to the non-linearithmic, but still quasilinear, $O(n \log^2 n)$.

An alternative to reduce the copying into multiple lists is to associate a new field of information with each key (the elements in m are called keys). This field will be used to link the keys and any associated information together in a sorted list (a key and its related information is called a record). Then the merging of the sorted lists proceeds by changing the link values; no records need to be moved at all. A field which contains only a link will generally be smaller than an entire record so less space will also be used. This is a standard sorting technique, not restricted to merge sort.

A variant named *binary merge sort* uses a *binary insertion sort* to sort groups of 32 elements, followed by a final sort using merge sort. It combines the speed of insertion sort on small data sets with the speed of merge sort on large data sets.^[7]

Use with tape drives [edit]

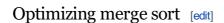
An external merge sort is practical to run using disk or tape drives when the data to be sorted is too large to fit into memory. External sorting explains how merge sort is implemented with disk drives. A typical tape drive sort uses four tape drives. All I/O is sequential (except for rewinds at the end of each pass). A minimal implementation can get by with just 2 record buffers and a few program variables.

Naming the four tape drives as A, B, C, D, with the original data on A, and using only 2 record buffers, the algorithm is similar to Bottom-up implementation, using pairs of tape drives instead of arrays in memory. The basic algorithm can be described as follows:

- 1. Merge pairs of records from A; writing two-record sublists alternately to C and D.
- 2. Merge two-record sublists from C and D into four-record sublists; writing these alternately to A and B.
- 3. Merge four-record sublists from A and B into eight-record sublists; writing these alternately to C and D
- 4. Repeat until you have one list containing all the data, sorted --- in log2(n) passes.

Instead of starting with very short runs, usually a hybrid algorithm is used, where the initial pass will read many records into memory, do an internal sort to create a long run, and then distribute those long runs onto the output set. The step avoids many early passes. For example, an internal sort of 1024 records will save 9 passes. The internal sort is often large because it has such a benefit. In fact, there are techniques that can make the initial runs longer than the available internal memory.^[8]

A more sophisticated merge sort that optimizes tape (and disk) drive usage is the polyphase merge sort.



On modern computers, locality of reference can be of paramount importance in software optimization, because multilevel memory hierarchies are used. Cache-aware versions of the merge sort algorithm, whose operations have been specifically chosen to minimize the movement of pages in and out of a machine's memory cache, have been proposed. For example, the **tiled merge sort** algorithm stops partitioning subarrays when



Merge sort type algorithms allowed large data sets to be sorted on early computers that had small random access memories by modern standards. Records were stored on magnetic tape and processed on banks of magnetic tape drives, such as these IBM729s.

subarrays of size S are reached, where S is the number of data items fitting into a CPU's cache. Each of these subarrays is sorted with an in-place sorting algorithm such as insertion sort, to discourage memory swaps, and normal merge sort is then completed in the standard recursive fashion. This algorithm has demonstrated better performance on machines that benefit from cache optimization. (LaMarca & Ladner 1997)

Kronrod (1969) suggested an alternative version of merge sort that uses constant additional space. This algorithm was later refined. (Katajainen, Pasanen & Teuhola 1996)

Also, many applications of external sorting use a form of merge sorting where the input get split up to a higher number of sublists, ideally to a number for which merging them still makes the currently processed set of pages fit into main memory.

Parallel merge sort [edit]

Merge sort parallelizes well due to use of the divide-and-conquer method. Several parallel variants are discussed in the third edition of Cormen, Leiserson, Rivest, and Stein's *Introduction to Algorithms*. ^[9] The first of these can be very easily expressed in a pseudocode with fork and join keywords:

This algorithm is a trivial modification from the serial version, but its speedup is not impressive: it runs in $\Theta(n)$ time, which is somewhat faster than the serial version's $\Theta(n \log n)$ time, but still has only $\Theta(\log n)$ parallelism. A parallel merge algorithm to not only parallelize the recursive division of the array, but also the merge operation leads to a better parallel sort with parallelism $\Theta(n / \log n)$. Such a sort can perform well in practice when combined with a fast stable sequential sort, such as insertion sort, and a fast sequential merge as a base case for merging small arrays. [10] Merge sort was one of the first sorting algorithms where optimal speed up was achieved, with Richard Cole using a clever subsampling algorithm to ensure O(1) merge. [11] Other sophisticated parallel sorting algorithms can achieve the same or better time bounds with a lower constant. For example, in 1991 David Powers described a parallelized quicksort (and a related radix sort) that can operate in $O(\log n)$ time on a CRCW PRAM with n processors by performing partitioning implicitly. [12] Powers [13] further shows that a pipelined version of Batcher's Bitonic Mergesort at $O(\log^2 n)$ time on a butterfly sorting network is in practice actually faster than his $O(\log n)$ sorts on a PRAM, and he provides detailed discussion of the hidden overheads in comparison, radix and parallel sorting.

Comparison with other sort algorithms [edit]

Although heapsort has the same time bounds as merge sort, it requires only $\Theta(1)$ auxiliary space instead of merge sort's $\Theta(n)$. On typical modern architectures, efficient quicksort implementations generally outperform mergesort for sorting RAM-based arrays. [citation needed] On the other hand, merge sort is a stable sort and is more efficient at handling slow-to-access sequential media. Merge sort is often the best choice for sorting a linked list: in this situation it is relatively easy to implement a merge sort in such a way that it requires only $\Theta(1)$ extra space, and the slow random-access performance of a linked list makes some other algorithms (such as quicksort) perform poorly, and others (such as heapsort) completely impossible.

As of Perl 5.8, merge sort is its default sorting algorithm (it was quicksort in previous versions of Perl). In Java, the Arrays.sort() & methods use merge sort or a tuned quicksort depending on the datatypes and for implementation efficiency switch to insertion sort when fewer than seven array elements are being sorted. [14] Python uses Timsort, another tuned hybrid of merge sort and insertion sort, that has become the standard sort algorithm in Java SE 7, [15] on the Android platform, [16] and in GNU Octave. [17]

Notes [edit]

- 1. ^ Knuth (1998, p. 158)
- 2. ^ Jyrki Katajainen and Jesper Larsson Träff (1997). "A meticulous analysis of mergesort programs".
- 3. ^ The worst case number given here does not agree with that given in Knuth's *Art of Computer Programming, Vol*3. The discrepancy is due to Knuth analyzing a variant implementation of merge sort that is slightly sub-optimal
- 4. ^ Katajainen, Jyrki; Pasanen, Tomi; Teuhola, Jukka (1996). "Practical in-place mergesort". Nordic J. Computing 3

- (1): 27-40. CiteSeerX: 10.1.1.22.8523 ₺.
- 5. A Geffert, Viliam; Katajainen, Jyrki; Pasanen, Tomi (2000). "Asymptotically efficient in-place merging". *Theoretical Computer Science* **237**: 159–181. doi:10.1016/S0304-3975(98)00162-5 &.
- 6. *Kim, Pok-Son; Kutzner, Ame (2004). Stable Minimum Storage Merging by Symmetric Comparisons. European Symp. Algorithms. Lecture Notes in Computer Science 3221. pp. 714–723. doi:10.1007/978-3-540-30140-0_63 &. ISBN 978-3-540-23025-0. CiteSeerX: 10.1.1.102.4612 &.
- 7. ^ "Binary Merge Sort" &.
- 8. ^ Selection sort. Knuth's snowplow. Natural merge.
- 9. ^ Cormen et al. 2009, pp. 797-805
- 10. ^ V. J. Duvanenko, "Parallel Merge Sort", Dr. Dobb's Journal, March 2011 &
- 11. ^ Cole, Richard (August 1988). "Parallel merge sort". SIAM J. Comput. 17 (4): 770-785. doi:10.1137/0217049 &
- 12. ^ Powers, David M. W. Parallelized Quicksort and Radixsort with Optimal Speedup &, Proceedings of International Conference on Parallel Computing Technologies. Novosibirsk. 1991.
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- 14. ^ OpenJDK Subversion ₺
- 16. ^ "Class: java.util.TimSort<T>" ☑. Android JDK Documentation. Retrieved 19 Jan 2015. [dead link]
- 17. A "liboctave/util/oct-sort.cc" & Mercurial repository of Octave source code. Lines 23-25 of the initial comment block. Retrieved 18 Feb 2013. "Code stolen in large part from Python's, listobject.c, which itself had no license header. However, thanks to Tim Peters for the parts of the code I ripped-off."





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External links [edit]

- Animated Sorting Algorithms: Merge Sort ☑ graphical demonstration and discussion of array-based merge sort
- demonstration and discussion of array-based merge sort

 Animated Sorting Algorithms: Merge Sort & Allows stepping through the steps in the algorithm.
- Dictionary of Algorithms and Data Structures: Merge sort ☑
- Mergesort applet ☑ with "level-order" recursive calls to help improve algorithm analysis

v·t·e	Sorting algorithms	[hide]
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Exchange sorts	Bubble sort \cdot Cocktail sort \cdot Odd–even sort \cdot Comb sort \cdot Gnome sort \cdot Quicksort \cdot Stooge sort Bogosort	
Selection sorts	Selection sort · Heapsort · Smoothsort · Cartesian tree sort · Tournament sort · Cycle sort	
Insertion sorts	Insertion sort · Shellsort · Splaysort · Tree sort · Library sort · Patience sorting	
Merge sorts	Merge sort · Cascade merge sort · Oscillating merge sort · Polyphase merge sort · Strand sort	
Distribution sorts	American flag sort · Bead sort · Bucket sort · Burstsort · Counting sort · Pigeonhole sort · Proxm Radix sort · Flashsort	ap sort •
Concurrent sorts	Bitonic sorter · Batcher odd-even mergesort · Pairwise sorting network	
Hybrid sorts	Block sort · Timsort · Introsort · Spreadsort · JSort	
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