

Puzzle 17 | (Ratio of Boys and Girls in a Country where people want only boys)

In a country, all families want a boy. They keep having babies till a boy is born. What is the expected ratio of boys and girls in the country?

Solution:

Assumptions: Probability of having a boy or girl is same. Also, the probability of next kid being a boy doesn't depend on history.

The problem can be solved by counting expected number of girls before a baby boy is born.

Let NG be the expected no. of girls before a boy is born

Let p be the probability that a child is girl and $(1-p)$ be probability that a child is girl.

NG can be written as sum of following infinite series.

$$NG = 0 \cdot p + 1 \cdot p \cdot (1-p) + 2 \cdot p \cdot p \cdot (1-p) + 3 \cdot p \cdot p \cdot p \cdot (1-p) + 4 \cdot p \cdot p \cdot p \cdot p \cdot (1-p) + \dots$$

Putting $p = 1/2$ and $(1-p) = 1/2$ in above formula.

$$NG = 0 \cdot (1/2) + 1 \cdot (1/2)^2 + 2 \cdot (1/2)^3 + 3 \cdot (1/2)^4 + 4 \cdot (1/2)^5 + \dots$$

$$1/2 \cdot NG = 0 \cdot (1/2)^2 + 1 \cdot (1/2)^3 + 2 \cdot (1/2)^4 + 3 \cdot (1/2)^5 + 4 \cdot (1/2)^6 + \dots$$

$$NG - NG/2 = 1 \cdot (1/2)^2 + 1 \cdot (1/2)^3 + 1 \cdot (1/2)^4 + 1 \cdot (1/2)^5 + 1 \cdot (1/2)^6 + \dots$$

Using sum formula of infinite geometrical progression with ratio less than 1

$$NG/2 = (1/4)/(1-1/2) = 1/2$$

$$NG = 1$$

So Expected Number of number of girls = 1

Since the expected number of girls is 1 and there is always a baby boy, the expected ratio of boys and girls is 50:50