Richardson–Lucy deconvolution

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Not to be confused with Modified Richardson iteration.

The **Richardson–Lucy algorithm**, also known as **Lucy–Richardson deconvolution**, is an iterative procedure for recovering a latent image that has been blurred by a known point spread function. It was named after William Richardson and Leon Lucy, who described it independently. [1][2]

Description [edit]

When an image is recorded on a detector such as photographic film or a charge-coupled device, it is generally slightly blurred, with an ideal point source not appearing as a point but being spread out, into what is known as the point spread function. Non-point sources are effectively the sum of many individual point sources, and pixels in an observed image can be represented in terms of the point spread function and the latent image as

$$d_i = \sum_j p_{ij} u_j$$

where p_{ij} is the point spread function (the fraction of light coming from true location j that is observed at position i), u_j is the pixel value at location j in the latent image, and d_i is the observed value at pixel location i. The statistics are performed under the assumption that u_j are Poisson distributed, which is appropriate for photon noise in the data.

The basic idea is to calculate the most likely u_j given the observed d_i and known p_{ij}. This leads to an equation for u_j which can be solved iteratively according to

$$u_j^{(t+1)} = u_j^{(t)} \sum_i \frac{d_i}{c_i} p_{ij}$$

where

$$c_i = \sum_j p_{ij} u_j^{(t)}.$$

It has been shown empirically that if this iteration converges, it converges to the maximum likelihood solution for u_j.[3]

This can also be written more generally (for more dimensions) in terms of convolution,[4]

$$u^{(t+1)} = u^{(t)} \cdot \left(\frac{d}{u^{(t)} \otimes p} \otimes \hat{p} \right)$$

where the division and multiplication are element wise, and $\hat{p}$ is the flipped point spread function, such that

$$\hat{p}_{nm} = p_{(i-n)(j-m)}, 0 \leq n, m \leq i, j$$

In problems where the point spread function p_{ij} is dependent on one or more unknown parameters, the Richardson–Lucy algorithm cannot be used.

References [edit]

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3. ^ Shepp, L. A.; Vardi, Y. (1982), "Maximum Likelihood Reconstruction for Emission Tomography", *IEEE Transactions on Medical Imaging* **1**: 113, doi:10.1109/TMI.1982.4307558
4. ^ Fish D. A.,; Brinicombe A. M., Pike E. R., and Walker J. G. (1995), "Blind deconvolution by means of the Richardson–Lucy algorithm" (PDF), *Journal of the Optical Society of America A* **12** (1): 58–65, Bibcode:1995JOSAA..12...58F, doi:10.1364/JOSAA.12.000058