Shannon–Fano–Elias coding

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In information theory, **Shannon–Fano–Elias coding** is a precursor to arithmetic coding, in which probabilities are used to determine codewords.[1]

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Algorithm description [edit]

Given a discrete random variable *X* of ordered values to be encoded, let $p(x)$ be the probability for any *x* in *X*. Define a function

$$\bar{F}(x) = \sum_{x_i < x} p(x_i) + \frac{1}{2} p(x)$$

Algorithm:

For each *x* in *X*,

Let *Z* be the binary expansion of $\bar{F}(x)$.

Choose the length of the encoding of *x*, $L(x)$, to be the integer $\left\lceil \log_2 \frac{1}{p(x)} \right\rceil + 1$

Choose the encoding of *x*, $code(x)$, be the first $L(x)$ most significant bits after the decimal point of *Z*.

Example [edit]

Let X = {A, B, C, D}, with probabilities p = {1/3, 1/4, 1/6, 1/4}.

For A

$$\bar{F}(A) = \frac{1}{2} p(A) = \frac{1}{2} \cdot \frac{1}{3} = 0.1666...$$

In binary, Z(A) = 0.**001**0101010...

L(A) = $\left\lceil \log_2 \frac{1}{\frac{1}{3}} \right\rceil + 1$ = **3**

code(A) is 001

For B

$$\bar{F}(B) = p(A) + \frac{1}{2} p(B) = \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{4} = 0.4583333...$$

In binary, Z(B) = 0.**011**10101010101...

L(B) = $\left\lceil \log_2 \frac{1}{\frac{1}{4}} \right\rceil + 1$ = **3**

code(B) is 011

For C

$$\bar{F}(C) = p(A) + p(B) + \frac{1}{2} p(C) = \frac{1}{3} + \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{6} = 0.66666...$$

In binary, Z(C) = 0.**1010**10101010...

L(C) = $\left\lceil \log_2 \frac{1}{\frac{1}{6}} \right\rceil + 1$ = **4**

code(C) is 1010

For D

$$\bar{F}(D) = p(A) + p(B) + p(C) + \frac{1}{2}p(D) = \frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{2} \cdot \frac{1}{4} = 0.875$$

In binary, Z(D) = 0.**111**

L(D) = $\left\lceil \log_2 \frac{1}{\frac{1}{4}} \right\rceil + 1$ = **3**

code(D) is 111

Algorithm analysis [edit]

Prefix code [edit]

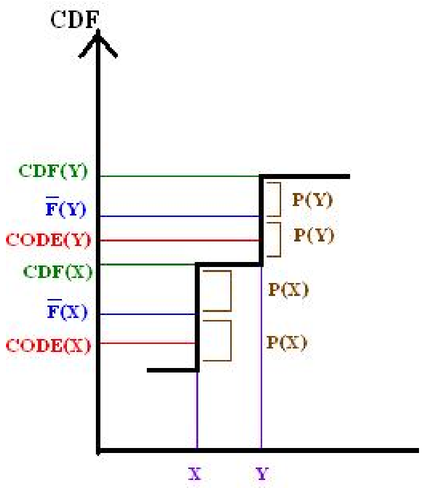
Shannon–Fano–Elias coding produces a binary prefix code, allowing for direct decoding.

Let bcode(x) be the rational number formed by adding a decimal point before a binary code. For example, if code(C)=1010 then bcode(C) = 0.1010. For all x, if no y exists such that

$$bcode(x) \le bcode(y) < bcode(x) + 2^{-L(x)}$$

then all the codes form a prefix code.

By comparing F to the CDF of X, this property may be demonstrated graphically for Shannon–Fano–Elias coding.



By definition of L it follows that

$$2^{-L(x)} \le \frac{1}{2}p(x)$$

And because the bits after L(y) are truncated from F(y) to form code(y), it follows that

$$\bar{F}(y) - bcode(y) \le 2^{-L(y)}$$

thus bcode(y) must be no less than CDF(x).

So the above graph demonstrates that the $bcode(y) - bcode(x) > p(x) \ge 2^{-L(x)}$, therefore the prefix property holds.

Code length [edit]

The average code length is $LC(X) = \sum_{x \epsilon X} p(x)L(x) = \sum_{x \epsilon X} p(x)(\left\lceil \log_2 \frac{1}{p(x)} \right\rceil + 1)$.

Thus for H(X), the Entropy of the random variable X,

$$H(X) + 1 \le LC(X) < H(X) + 2$$

Shannon Fano Elias codes from 1 to 2 extra bits per symbol from X than entropy, so the code is not used in practice.

References [edit]

1. ^ T. M. Cover and Joy A. Thomas (2006). *Elements of information theory* (2nd ed.). John Wiley and Sons. pp. 127–128. ISBN 978-0-471-24195-9.

v · t · e	Data compression methods	[hide]
Lossless	Entropy type	Unary · Arithmetic · Golomb · Huffman (Adaptive · Canonical · Modified) · Range · Shannon · Shannon–Fano · **Shannon–Fano–Elias** · Tunstall · Universal (Exp-Golomb · Fibonacci · Gamma · Levenshtein)
	Dictionary type	Byte pair encoding · DEFLATE · Lempel–Ziv (LZ77 / LZ78 (LZ1 / LZ2) · LZJB · LZMA · LZO · LZRW · LZS · LZSS · LZW · LZWL · LZX · LZ4 · Statistical)
	Other types	BWT · CTW · Delta · DMC · MTF · PAQ · PPM · RLE
Audio	Concepts	Bit rate (average (ABR) · constant (CBR) · variable (VBR)) · Companding · Convolution · Dynamic range · Latency · Nyquist–Shannon theorem · Sampling · Sound quality · Speech coding · Sub-band coding
	Codec parts	A-law · μ-law · ACELP · ADPCM · CELP · DPCM · Fourier transform · LPC (LAR · LSP) · MDCT · Psychoacoustic model · WLPC
Image	Concepts	Chroma subsampling · Coding tree unit · Color space · Compression artifact · Image resolution · Macroblock · Pixel · PSNR · Quantization · Standard test image
	Methods	Chain code · DCT · EZW · Fractal · KLT · LP · RLE · SPIHT · Wavelet
Video	Concepts	Bit rate (average (ABR) · constant (CBR) · variable (VBR)) · Display resolution · Frame · Frame rate · Frame types · Interlace · Video characteristics · Video quality
	Codec parts	Lapped transform · DCT · Deblocking filter · Motion compensation
Theory	Entropy · Kolmogorov complexity · Lossy · Quantization · Rate–distortion · Redundancy · Timeline of information theory	
Compression formats · Compression software (codecs)		

Categories: Lossless compression algorithms