Long division

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This article is about elementary handwritten division. For mathematical definition and properties, see Division (mathematics) and Euclidean division. For software algorithms, see Division algorithm. For other uses, see Long division (disambiguation).

In arithmetic, **long division** is a standard division algorithm suitable for dividing multidigit numbers that is simple enough to perform by hand. It breaks down a division problem into a series of easier steps. As in all division problems, one number, called the dividend, is divided by another, called the divisor, producing a result called the quotient. It enables computations involving arbitrarily large numbers to be performed by following a series of simple steps.[1] The abbreviated form of long division is called short division, which is almost always used instead of long division when the divisor has only one digit. Chunking (also known as the partial quotients method or the hangman method) is a less-efficient form of long division which may be easier to understand.

Contents [hide]

Place in education [edit]

Inexpensive calculators and computers have become the most common way to solve division problems, eliminating a traditional mathematical exercise, and decreasing the educational opportunity to show how to do so by paper and pencil techniques. (Internally, those devices use one of a variety of division algorithms). In the United States, long division has been especially targeted for de-emphasis, or even elimination from the school curriculum, by reform mathematics, though traditionally introduced in the 4th or 5th grades.

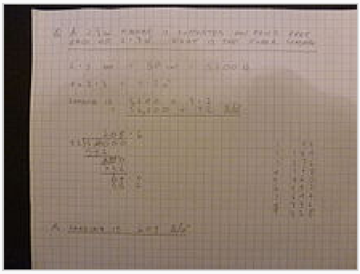
Method [edit]

In English-speaking countries, long division does not use the slash (/) or obelus (÷) signs, instead displaying the dividend, divisor, and (once it is found) quotient in a tableau.

The process is begun by dividing the left-most digit of the dividend by the divisor. The quotient (rounded down to an integer) becomes the first digit of the result, and the remainder is calculated (this step is notated as a subtraction). This remainder carries forward when the process is repeated on the following digit of the dividend (notated as 'bringing down' the next digit to the remainder). When all digits have been processed and no remainder is left, the process is complete.

An example is shown below, representing the division of 500 by 4 (with a result of 125).

```
  125     (Explanations)
4)500
  4        (4 ×  1 = 4)
  10       (5 -  4 = 1)
   8       (4 ×  2 = 8)
   20      (10 -  8 = 2)
   20      (4 ×  5 = 20)
    0      (20 - 20 = 0)
```



An example of long division performed without a calculator.

In the above example, the first step is to find the shortest sequence of digits starting from the left end of the dividend, 500, that the divisor 4 goes into at least once; this shortest sequence in this example is simply the first digit, 5. The largest number that the divisor 4 can be multiplied by without exceeding 5 is 1, so the digit 1 is put above the 5 to start constructing the quotient. Next, the 1 is multiplied by the divisor 4, to obtain the largest whole number (4 in this case) that is a multiple of the divisor 4 without exceeding the 5; this product of 1 times 4 is 4, so 4 is placed underneath the 5. Next the 4 under the 5 is subtracted from the 5 to get the remainder, 1, which is placed under the 4 under the 5. This remainder 1 is necessarily smaller than the divisor 4. Next the first as-yet unused digit in the dividend, in this case the first digit 0 after the 5, is copied directly underneath itself and next to the remainder 1, to form the number 10. At this point the process is repeated enough times to reach a stopping point: The largest number by which the divisor 4 can be multiplied without exceeding 10 is 2, so 2 is written above the 0 that is next to the 5 – that is, directly above the last digit in the 10. Then the latest entry to the quotient, 2, is multiplied by the divisor 4 to get 8, which is the largest multiple of 4 that does not exceed 10; so 8 is written below 10, and the subtraction 10 minus 8 is performed to get the remainder 2, which is placed below the 8. This remainder 2 is necessarily smaller than the divisor 4. The next digit of the dividend (the last 0 in 500) is copied directly below itself and next to the remainder 2, to form 20. Then the largest number by which the divisor 4 can be multiplied without exceeding 20 is ascertained; this number is 5, so 5 is placed above the last dividend digit that was brought down (i.e., above the rightmost 0 in 500). Then this new quotient digit 5 is multiplied by the divisor 4 to get 20, which is written at the bottom below the existing 20. Then 20 is subtracted from 20, yielding 0, which is written below the 20. We know we are done now because two things are true: there are no more digits to bring down from the dividend, and the last subtraction result was 0.

If the last remainder when we ran out of dividend digits had been something other than 0, there would have been two possible courses of action. (1) We could just stop there and say that the dividend divided by the divisor is the quotient written at the top with the remainder written at the bottom; equivalently we could write the answer as the quotient followed by a fraction that is the remainder divided by the divisor. Or, (2) we could extend the dividend by writing it as, say, 500.000... and continue the process (using a decimal point in the quotient directly above the decimal point in the dividend), in order to get a decimal answer, as in the following example.

```
    31.75
4)127.00
  12        (12 ÷ 4 = 3)
   07       (0 remainder, bring down next figure)
    4       (7 ÷ 4 = 1 r 3 )
    3.0      (0 is added in order to make 3 divisible by 4; the 0 is accounted
for by adding a decimal point in the quotient.)
    2.8      (7 × 4 = 28)
      20     (an additional zero is brought down)
      20     (5 × 4 = 20)
       0
```

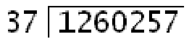
In this example, the decimal part of the result is calculated by continuing the process beyond the units digit, "bringing down" zeros as being the decimal part of the dividend.

This example also illustrates that, at the beginning of the process, a step that produces a zero can be omitted. Since the first digit 1 is less than the divisor 4, the first step is instead performed on the first two digits 12. Similarly, if the divisor were 13, one would perform the first step on 127 rather than 12 or 1.

Basic procedure for long division of *n* ÷ *m* [edit]

1. Find the location of all decimal points in the dividend *n* and divisor *m*.
2. If necessary, simplify the long division problem by moving the decimals of the divisor and dividend by the same number of decimal places, to the right, (or to the left) so that the decimal of the divisor is to the right of the last digit.
3. When doing long division, keep the numbers lined up straight from top to bottom under the tableau.
4. After each step, be sure the remainder for that step is less than the divisor. If it is not, there are three possible problems: the multiplication is wrong, the subtraction is wrong, or a greater quotient is needed.
5. In the end, the remainder, *r*, is added to the growing quotient as a fraction, *r*/*m*.

Example with multi-digit divisor [edit]



A divisor of any number of digits can be used. In this example, 37 is to be divided into 1260257. First the problem is set up as follows:

```
   _______
37)1260257
```

Digits of the number 1260257 are taken until a number greater than or equal to 37 occurs. So 1 and 12 are less than 37, but 126 is greater. Next, the greatest multiple of 37 less than or equal to 126 is computed. So 3 × 37 = 111 < 126, but 4 × 37 > 126. The multiple 111 is written underneath the 126 and the 3 is written on the top where the solution will appear:

```
     3
   _______
37)1260257
   111
```

Note carefully which place-value column these digits are written into. The 3 in the quotient goes in the same column (ten-thousands place) as the 6 in the dividend 1260257, which is the same column as the last digit of 111.

The 111 is then subtracted from the line above, ignoring all digits to the right:

```
     3
   _______
37)1260257
   111
   ---
    15
```

Now the digit from the next smaller place value of the dividend is copied down appended to the result 15:

```
     3
   _______
37)1260257
   111
   ---
    150
```

The process repeats: the greatest multiple of 37 less than or equal to 150 is subtracted. This is 148 = 4 × 37, so a 4 is added to the solution line. Then the result of the subtraction is extended by another digit taken from the dividend:

```
     34
   _______
37)1260257
   111
   ---
```

```
      150
      148
        22
```

The greatest multiple of 37 less than or equal to 22 is 0 × 37 = 0. Subtracting 0 from 22 gives 22, we often don't write the subtraction step. Instead, we simply take another digit from the dividend:

```
     340
37)1260257
   111
    150
    148
      225
```

The process is repeated until 37 divides the last line exactly:

```
     34061
37)1260257
   111
    150
    148
      225
      222
        37
```

Mixed mode long division [edit]

For non-decimal currencies (such as the British £sd system before 1971) and measures (such as avoirdupois) **mixed mode** division must be used. Consider dividing 50 miles 600 yards into 37 pieces:

```
        m -     yd -    ft -    in
        1 -     634      1       9 r. 15"
37)    50 -     600 -    0 -     0
       37     22880     66     348
       13     23480     66     348
   17600      222       37     333
    5280        128     29      15
   22880        111    348      ==
   =====         170   ===
                 148
                  22
                  66
                  ==
```

Each of the four columns is worked in turn. Starting with the miles: 50/37 = 1 remainder 13. No further division is possible, so perform a long multiplication by 1,760 to convert miles to yards, the result is 22,880 yards. Carry this to the top of the yards column and add it to the 600 yards in the dividend giving 23,480. Long division of 23,480 / 37 now proceeds as normal yielding 634 with remainder 22. The remainder is multiplied by 3 to get feet and carried up to the feet column. Long division of the feet gives 1 remainder 29 which is then multiplied by twelve to get 348 inches. Long division continues with the final remainder of 15 inches being shown on the result line.

Non-decimal radix [edit]

The same method and layout is used for binary, octal and hexadecimal. An address range of 0xf412df divided into 0x12 parts is:

```
     0d8f45 r. 5
12 ) f412df
      ea
       a1
       90
       112
       10e
```

```
          4d
          48
           5f
           5a
            5
```

Binary is of course trivial because each digit in the result can only be 1 or 0:

```
         ______1110 r. 11
    1101) 10111001
             1101
             10100
              1101
               1110
               1101
                  11
```

Interpretation of decimal results [edit]

When the quotient is not an integer and the division process is extended beyond the decimal point, one of two things can happen. (1) The process can terminate, which means that a remainder of 0 is reached; or (2) a remainder could be reached that is identical to a previous remainder that occurred after the decimal points were written. In the latter case, continuing the process would be pointless, because from that point onward the same sequence of digits would appear in the quotient over and over. So a bar is drawn over the repeating sequence to indicate that it repeats forever.

Notation in non-English-speaking countries [edit]

China, Japan and India use the same notation as English-speakers. Elsewhere, the same general principles are used, but the figures are often arranged differently.

Latin America [edit]

In Latin America (except Argentina, Mexico, Colombia, Venezuela, Uruguay and Brazil), the calculation is almost exactly the same, but is written down differently as shown below with the same two examples used above. Usually the quotient is written under a bar drawn under the divisor. A long vertical line is sometimes drawn to the right of the calculations.

```
      500 ÷ 4 =  125      (Explanations)
      4                   (4 ×  1 = 4)
      10                  (5 -  4 = 1)
       8                  (4 ×  2 = 8)
       20                 (10 -  8 = 2)
       20                 (4 ×  5 = 20)
        0                 (20 - 20 = 0)
```

and

```
      127 ÷ 4 = 31.75
      124
        30       (a 0 is added in order to make 3 divisible by 4; the 0 is accounted
  for by adding a decimal point in the quotient)
        28       (7 × 4 = 28)
         20      (an additional zero is added)
         20      (5 × 4 = 20)
           0
```

In Mexico, the US notation is used, except that only the result of the subtraction is annotated and the calculation is done mentally, as shown below:

```
      125      (Explanations)
    4)500
      10       (5 -  4 = 1)
       20      (10 -  8 = 2)
```

```
     0      (20 - 20 = 0)
```

In Brazil, Venezuela, Uruguay, Quebec and Colombia, the European notation (see below) is used, except that the quotient is not separated by a vertical line, as shown below:

```
  127|4
 -124 31,75
    30
   -28
     20
    -20
      0
```

Same procedure applies in Mexico, only the result of the subtraction is annotated and the calculation is done mentally.

Europe [edit]

In Spain, Italy, France, Portugal, Lithuania, Romania, Turkey, Greece, Belgium, and Russia, the divisor is to the right of the dividend, and separated by a vertical bar. The division also occurs in the column, but the quotient (result) is written below the divider, and separated by the horizontal line. The same method is used in Iran.

```
  127|4
 -124|31,75
    30
   -28
     20
    -20
      0
```

In France, a long vertical bar separates the dividend and subsequent subtractions from the quotient and divisor, as in the example below of 6359 divided by 17, which is 374 with a remainder of 1.

```
  6359|17
 - 51 |374
   125|
 - 119|
     69|
    -68|
      1|
```

Decimal numbers are not divided directly, the dividend and divisor are multiplied by a power of ten so that the division involves two whole numbers. Therefore, if one were dividing 12,7 by 0,4 (commas being used instead of decimal points), the dividend and divisor would first be changed to 127 and 4, and then the division would proceed as above.

In Germany, the notation of a normal equation is used for dividend, divisor and quotient (cf. first section of Latin American countries above, where it's done virtually the same way):

```
  127 : 4 = 31,75
 -12
   07
   -4
    30
   -28
     20
    -20
      0
```

The same notation is adopted in Denmark, Norway, Macedonia, Poland, Croatia, Slovenia, Hungary, Czech Republic, Slovakia, Vietnam and in Serbia.

In the Netherlands, the following notation is used:

```
    12 / 135 \ 11,25
         12
          15
          12
           30
           24
            60
            60
             0
```

Generalizations [edit]

Rational numbers [edit]

Long division of integers can easily be extended to include non-integer dividends, as long as they are rational. This is because every rational number has a recurring decimal expansion. The procedure can also be extended to include divisors which have a finite or terminating decimal expansion (i.e. decimal fractions). In this case the procedure involves multiplying the divisor and dividend by the appropriate power of ten so that the new divisor is an integer – taking advantage of the fact that $a \div b = (ca) \div (cb)$ – and then proceeding as above.

Polynomials [edit]

A generalised version of this method called polynomial long division is also used for dividing polynomials (sometimes using a shorthand version called synthetic division).

See also [edit]

- Arbitrary-precision arithmetic
- Egyptian multiplication and division
- Elementary arithmetic
- Fourier division
- Polynomial long division
- Shifting nth root algorithm – for finding square root or any nth root of a number
- Short division

References [edit]

1. ^ Weisstein, Eric W., "Long Division", *MathWorld*.

External links [edit]

- Long Division Algorithm
- [1] Long Division and Euclid's Lemma

Categories: Division (mathematics)