Given p and n, find the largest x such that p^x divides n!

Given an integer n and a prime number p, find the largest x such that p^x (p raised to power x) divides n! (factorial)

Examples:

```
Input:  n = 7, p = 3
Output: x = 2
3² divides 7! and 2 is the largest such power of 3.

Input:  n = 10, p = 3
Output: x = 4
3⁴ divides 10! and 4 is the largest such power of 3.
```

We strongly recommend to minimize your browser and try this yourself first.

n! is multiplication of {1, 2, 3, 4, …n}.

How many numbers in {1, 2, 3, 4, ….. n} are divisible by p?
Every p'th number is divisible by p in {1, 2, 3, 4, ….. n}. Therefore in n!, there are $\lfloor n/p \rfloor$ numbers divisible by p. So we know that the value of x (largest power of p that divides n!) is at-least $\lfloor n/p \rfloor$.

Can x be larger than $\lfloor n/p \rfloor$?
Yes, there may be numbers which are divisible by p^2, p^3, …

How many numbers in {1, 2, 3, 4, ….. n} are divisible by p^2, p^3, …?
There are $\lfloor n/(p^2) \rfloor$ numbers divisible by p^2 (Every p^2'th number would be divisible). Similarly, there are $\lfloor n/(p^3) \rfloor$ numbers divisible by p^3 and so on.

What is the largest possible value of x?

So the largest possible power is $\lfloor n/p \rfloor + \lfloor n/(p^2) \rfloor + \lfloor n/(p^3) \rfloor + \ldots\ldots$

Note that we add only $\lfloor n/(p^2) \rfloor$ only once (not twice) as one p is already considered by expression $\lfloor n/p \rfloor$. Similarly, we consider $\lfloor n/(p^3) \rfloor$ (not thrice).

Following is C implementation based on above idea.

```
// C program to find largest x such that p*x divides n!
#include <stdio.h>

// Returns largest power of p that divides n!
int largestPower(int n, int p)
{
    // Initialize result
    int x = 0;

    // Calculate x = n/p + n/(p^2) + n/(p^3) + ....
    while (n)
    {
        n /= p;
        x += n;
    }
    return x;
}

// Driver program
int main()
{
    int n = 10, p = 3;
    printf("The largest power of %d that divides %d! is 
           p, n, largestPower(n, p));
    return 0;
}
```

Output:

```
The largest power of 3 that divides 10! is 4
```

Time complexity of the above solution is $Log_p n$.

Source:

http://e-maxx.ru/algo/factorial_divisors