Given n dice each with m faces, numbered from 1 to m, find the number of ways to get sum X. X is the summation of values on each face when all the dice are thrown.

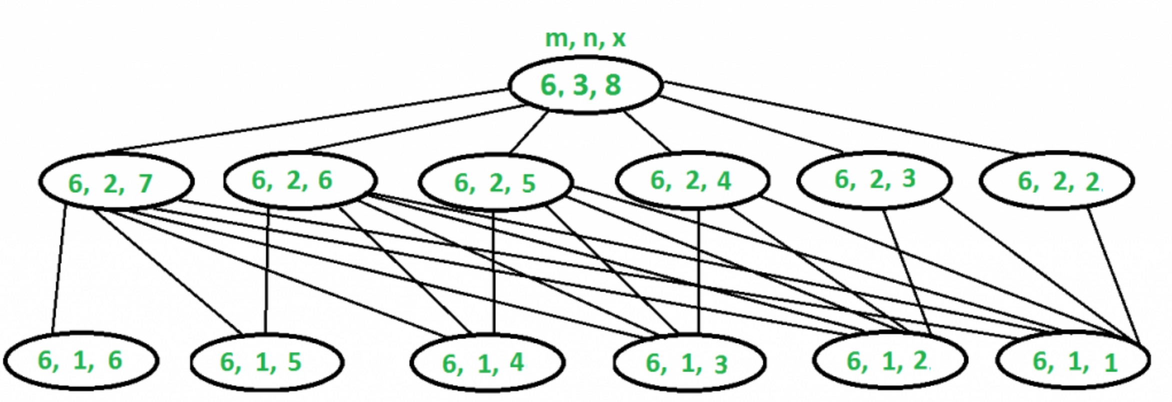
The **Naive approach** is to find all the possible combinations of values from n dice and keep on counting the results that sum to X.

This problem can be efficiently solved using **Dynamic Programming (DP)**.

```
Let the function to find X from n dice is: Sum(m, n, X)
The function can be represented as:
Sum(m, n, X) = Finding Sum (X - 1) from (n - 1) dice plus 1 from nth dice
               + Finding Sum (X - 2) from (n - 1) dice plus 2 from nth dice
               + Finding Sum (X - 3) from (n - 1) dice plus 3 from nth dice
                  ...................................................
                  ...................................................
                  ...................................................
              + Finding Sum (X - m) from (n - 1) dice plus m from nth dice

So we can recursively write Sum(m, n, x) as following
Sum(m, n, X) = Sum(m, n - 1, X - 1) +
               Sum(m, n - 1, X - 2) +
               .................... +
               Sum(m, n - 1, X - m)
```

Why DP approach?
The above problem exhibits overlapping subproblems. See the below diagram. Also, see thisrecursive implementation. Let there be 3 dice, each with 6 faces and we need to find the number of ways to get sum 8:




```
Sum(6, 3, 8) = Sum(6, 2, 7) + Sum(6, 2, 6) + Sum(6, 2, 5) +
               Sum(6, 2, 4) + Sum(6, 2, 3) + Sum(6, 2, 2)

To evaluate Sum(6, 3, 8), we need to evaluate Sum(6, 2, 7) which can
recursively written as following:
Sum(6, 2, 7) = Sum(6, 1, 6) + Sum(6, 1, 5) + Sum(6, 1, 4) +
               Sum(6, 1, 3) + Sum(6, 1, 2) + Sum(6, 1, 1)

We also need to evaluate Sum(6, 2, 6) which can recursively written
as following:
Sum(6, 2, 6) = Sum(6, 1, 5) + Sum(6, 1, 4) + Sum(6, 1, 3) +
               Sum(6, 1, 2) + Sum(6, 1, 1)
..............................................
```

```
.........................................
Sum(6, 2, 2) = Sum(6, 1, 1)
```

Please take a closer look at the above recursion. The sub-problems in RED are solved first time and sub-problems in BLUE are solved again (exhibit overlapping sub-problems). Hence, storing the results of the solved sub-problems saves time.

Following is C++ implementation of Dynamic Programming approach.

```
// C++ program to find number of ways to get sum 'x' with 'n'
// dice where every dice has 'm' faces
#include <iostream>
#include <string.h>
using namespace std;

// The main function that returns number of ways to get sum 'x'
// with 'n' dice and 'm' with m faces.
int findWays(int m, int n, int x)
{
    // Create a table to store results of subproblems.  One extra
    // row and column are used for simpilicity (Number of dice
    // is directly used as row index and sum is directly used
    // as column index).  The entries in 0th row and 0th column
    // are never used.
    int table[n + 1][x + 1];
    memset(table, 0, sizeof(table)); // Initialize all entries as 0

    // Table entries for only one dice
    for (int j = 1; j <= m && j <= x; j++)
        table[1][j] = 1;

    // Fill rest of the entries in table using recursive relation
    // i: number of dice, j: sum
    for (int i = 2; i <= n; i++)
        for (int j = 1; j <= x; j++)
            for (int k = 1; k <= m && k < j; k++)
                table[i][j] += table[i-1][j-k];

    /* Uncomment these lines to see content of table
    for (int i = 0; i <= n; i++)
    {
      for (int j = 0; j <= x; j++)
        cout << table[i][j] << " ";
      cout << endl;
    } */
    return table[n][x];
}

// Driver program to test above functions
int main()
{
    cout << findWays(4, 2, 1) << endl;
    cout << findWays(2, 2, 3) << endl;
    cout << findWays(6, 3, 8) << endl;
    cout << findWays(4, 2, 5) << endl;
    cout << findWays(4, 3, 5) << endl;

    return 0;
}
```

Output:

```
0
2
21
4
```

```
6
```

Time Complexity: O(m * n * x) where m is number of faces, n is number of dice and x is given sum.

We can add following two conditions at the beginning of findWays() to improve performance of program for extreme cases (x is too high or x is too low)

```
// When x is so high that sum can not go beyond x even when we
// get maximum value in every dice throw.
if (m*n <= x)
    return (m*n == x);

// When x is too low
if (n >= x)
    return (n == x);
```

With above conditions added, time complexity becomes O(1) when x >= m*n or when x <= n.

Exercise:
Extend the above algorithm to find the probability to get Sum > X.