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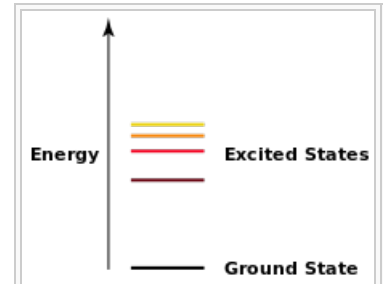
# Ground state

From Wikipedia, the free encyclopedia

The **ground state** of a **quantum mechanical** system is its lowest-energy **state**; the energy of the ground state is known as the **zero-point energy** of the system. An **excited state** is any state with energy greater than the ground state. The ground state of a **quantum field theory** is usually called the **vacuum state** or the **vacuum**.

If more than one ground state exists, they are said to be **degenerate**. Many systems have degenerate ground states. Degeneracy occurs whenever there exists a **unitary operator** which acts non-trivially on a ground state and **commutes** with the **Hamiltonian** of the system.

According to the **third law of thermodynamics**, a system at **absolute zero temperature** exists in its ground state; thus, its **entropy** is determined by the degeneracy of the ground state. Many systems, such as a perfect **crystal lattice**, have a unique ground state and therefore have zero entropy at absolute zero. It is also possible for the highest excited state to have **absolute zero temperature** for systems that exhibit **negative temperature**.



Energy levels for an **electron** in an **atom**: **ground state** and **excited states**. After absorbing **energy**, an electron may **jump** from the ground state to a higher energy excited state.

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## 1D ground state has no nodes

In 1D the ground state of the **Schrödinger equation** has no nodes. This can be proved considering an average energy in the state with a node at  $x = 0$ , i.e.  $\psi(0) = 0$ . Consider the average energy in this state

$$\langle \psi | H | \psi \rangle = \int dx \left( -\frac{\hbar^2}{2m} \psi^* \frac{d^2 \psi}{dx^2} + V(x) |\psi(x)|^2 \right) \text{ where } V(x) \text{ is the potential. Now consider a}$$

small interval around  $x = 0$ , i.e.  $x \in [-\epsilon, \epsilon]$ . Take a new wavefunction  $\psi'(x)$  to be defined as

$\psi'(x) = \psi(x), x < -\epsilon$  and  $\psi'(x) = -\psi(x), x > \epsilon$  and constant for  $x \in [-\epsilon, \epsilon]$ . If epsilon is small enough then this is always possible to do so that  $\psi'(x)$  is continuous. So assuming  $\psi(x) \approx -cx$  around

$x = 0$ , we can write the new function as

$$\psi'(x) = N \begin{cases} |\psi(x)| & |x| > \epsilon \\ c\epsilon & |x| \leq \epsilon \end{cases}$$

where  $N = \frac{1}{\sqrt{1 + |c|^2 \epsilon^3 / 3}}$  is the norm. Note that the kinetic energy density  $|d\psi'/dx|^2 < |d\psi/dx|^2$

everywhere because of the normalization. Now consider the potential energy. For definiteness let us choose  $V(x) \geq 0$ . Then it is clear that outside the interval  $x \in [-\epsilon, \epsilon]$  the potential energy density is smaller for the  $\psi'$  because  $|\psi'| < |\psi|$  there. On the other hand, in the interval  $x \in [-\epsilon, \epsilon]$  we have

$$V_{avg}' = \int_{-\epsilon}^{\epsilon} dx V(x) |\psi'|^2 = \frac{\epsilon^3 |c|^2}{1 + |c|^2 \epsilon^3 / 3} \int_{-\epsilon}^{\epsilon} V(x) \approx \frac{2\epsilon^4 |c|^2}{3} V(0) + \dots$$

which is correct to this order of  $\epsilon$  and  $\dots$  indicate higher order corrections. On the other hand, the potential energy in the  $\psi$  state is

$$V_{avg} = \int_{-\epsilon}^{\epsilon} dx V(x) |\psi|^2 = \int_{-\epsilon}^{\epsilon} dx |c|^2 x^2 V(x) \approx \frac{2\epsilon^4 |c|^2}{3} V(0) + \dots \text{ which is the same as}$$

that of the  $\psi'$  state to the order shown. Therefore, the potential energy unchanged to leading order in  $\epsilon$  by

deforming the state with a node  $\psi$  into a state without a node  $\psi'$ . We can do this by removing all nodes thereby reducing the energy, which implies that the ground state energy must not have a node. This completes the proof.

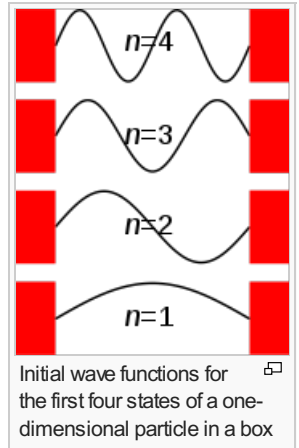
## Examples [\[edit\]](#)

- The **wave function** of the ground state of a **particle in a one-dimensional well** is a half-period **sine wave** which goes to zero at the two edges of the well.

The energy of the particle is given by  $\frac{h^2 n^2}{8mL^2}$ , where *h* is the **Planck**

**constant**, *m* is the mass of the particle, *n* is the energy state (*n* = 1 corresponds to the ground-state energy), and *L* is the width of the well.

- The wave function of the ground state of a hydrogen atom is a spherically-symmetric distribution centred on the **nucleus**, which is largest at the center and reduces **exponentially** at larger distances. The **electron** is most likely to be found at a distance from the nucleus equal to the **Bohr radius**. This function is known as the 1s **atomic orbital**. For hydrogen (H), an electron in the ground state has energy −13.6 eV, relative to the **ionization threshold**. In other words, 13.6 eV is the energy input required for the electron to no longer be **bound** to the atom.
- The exact definition of one **second** of **time** since 1997 has been the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two **hyperfine** levels of the ground state of the **caesium-133** atom at rest at a temperature of 0 K.<sup>[1]</sup>



## Notes [\[edit\]](#)

- ↑ "Unit of time (second)" ↗. *SI Brochure*. **BIPM**. Retrieved 2013-12-22.

## Bibliography [\[edit\]](#)

- Feynman, Richard**; Leighton, Robert; Sands, Matthew (1965). "see section 2-5 for energy levels, 19 for the hydrogen atom". *The Feynman Lectures on Physics* ↗ **3**.

Categories: Quantum mechanics

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