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# Multivariate interpolation

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In [numerical analysis](#), **multivariate interpolation** or **spatial interpolation** is [interpolation](#) on functions of more than one variable.

The function to be interpolated is known at given points  $(x_i, y_i, z_i, \dots)$  and the interpolation problem consist of yielding values at arbitrary points  $(x, y, z, \dots)$ .

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- Irregular grid (scattered data)
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## Regular grid [edit]

For function values known on a [regular grid](#) (having predetermined, not necessarily uniform, spacing), the following methods are available.

### Any dimension [edit]

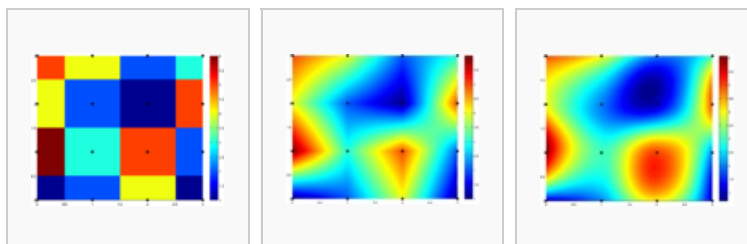
- [Nearest-neighbor interpolation](#)

### 2 dimensions [edit]

- [Barnes interpolation](#)
- [Bilinear interpolation](#)
- [Bicubic interpolation](#)
- [Bézier surface](#)
- [Lanczos resampling](#)
- [Delaunay triangulation](#)
- [Inverse distance weighting](#)
- [Kriging](#)
- [Natural neighbor](#)
- [Spline interpolation](#)

[Bitmap resampling](#) is the application of 2D multivariate interpolation in [image processing](#).

Three of the methods applied on the same dataset, from 16 values located at the black dots. The colours represent the interpolated values.



Nearest neighbor

Bilinear

Bicubic

See also [Padua points](#), for [polynomial interpolation](#) in two variables.

### 3 dimensions [edit]

- [Trilinear interpolation](#)
- [Tricubic interpolation](#)

See also [bitmap resampling](#).

### Tensor product splines for *N* dimensions [\[edit\]](#)

Catmull-Rom splines can be easily generalized to any number of dimensions. The [cubic Hermite spline](#) article will remind you that  $\text{CINT}_x(f_{-1}, f_0, f_1, f_2) = \mathbf{b}(x) \cdot (f_{-1} f_0 f_1 f_2)$  for some 4-vector  $\mathbf{b}(x)$  which is a function of *x* alone, where  $f_j$  is the value at *j* of the function to be interpolated. Rewrite this approximation as

$$\text{CR}(x) = \sum_{i=-1}^2 f_i b_i(x)$$

This formula can be directly generalized to *N* dimensions:<sup>[1]</sup>

$$\text{CR}(x_1, \dots, x_N) = \sum_{i_1, \dots, i_N=-1}^2 f_{i_1 \dots i_N} \prod_{j=1}^N b_{i_j}(x_j)$$

Note that similar generalizations can be made for other types of spline interpolations, including Hermite splines. In regards to efficiency, the general formula can in fact be computed as a composition of successive **CINT**-type operations for any type of tensor product splines, as explained in the [tricubic interpolation](#) article. However, the fact remains that if there are *n* terms in the 1-dimensional **CR**-like summation, then there will be *n*<sup>*N*</sup> terms in the *N*-dimensional summation.

### Irregular grid (scattered data) [\[edit\]](#)


Schemes defined for scattered data on an [irregular grid](#) should all work on a regular grid, typically reducing to another known method.

- [Nearest-neighbor interpolation](#)
- [Triangulated irregular network](#)-based [natural neighbor](#)
- [Triangulated irregular network](#)-based [linear interpolation](#) (a type of [piecewise linear function](#))
- [Inverse distance weighting](#)
- [Kriging](#)
- [Radial basis function](#)
- [Thin plate spline](#)
- [Polyharmonic spline](#) (the thin-plate-spline is a special case of a polyharmonic spline)
- [Least-squares spline](#)

### Notes [\[edit\]](#)

- ↑ [Two hierarchies of spline interpolations. Practical algorithms for multivariate higher order splines](#) [↗](#)

### External links [\[edit\]](#)

- [Example C++ code for several 1D, 2D and 3D spline interpolations \(including Catmull-Rom splines\)](#). [↗](#)
- [Multi-dimensional Hermite Interpolation and Approximation](#) , Prof. Chandrajit Bajaja, [Purdue University](#)

Categories: [Interpolation](#) | [Multivariate interpolation](#)

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