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Cuthill–McKee algorithm

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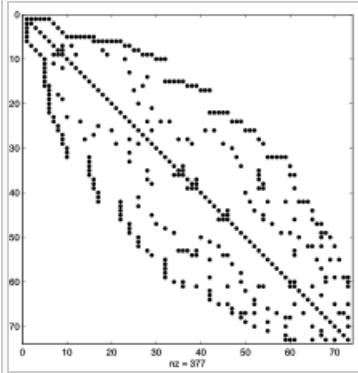
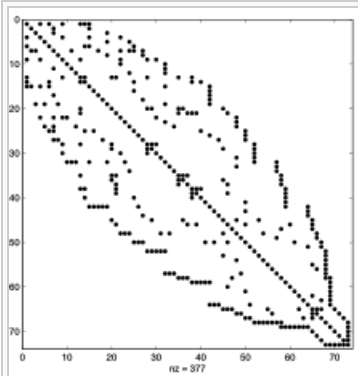
In the mathematical subfield of matrix theory, the **Cuthill–McKee algorithm** (**CM**), named for Elizabeth Cuthill and J. McKee ,[1] is an algorithm to permute a sparse matrix that has a symmetric sparsity pattern into a band matrix form with a small bandwidth. The **reverse Cuthill–McKee algorithm** (**RCM**) due to Alan George is the same algorithm but with the resulting index numbers reversed. In practice this generally results in less fill-in than the CM ordering when Gaussian elimination is applied.[2]



Cuthill-McKee ordering of a matrix



RCM ordering of the same matrix

The Cuthill McKee algorithm is a variant of the standard breadth-first search algorithm used in graph algorithms. It starts with a peripheral node and then generates levels R_i for $i = 1, 2, ..$ until all nodes are exhausted. The set R_{i+1} is created from set R_i by listing all vertices adjacent to all nodes in R_i. These nodes are listed in increasing degree. This last detail is the only difference with the breadth-first search algorithm.

Algorithm [edit]

Given a symmetric $n \times n$ matrix we visualize the matrix as the adjacency matrix of a graph. The Cuthill–McKee algorithm is then a relabeling of the vertices of the graph to reduce the bandwidth of the adjacency matrix.

The algorithm produces an ordered *n*-tuple *R* of vertices which is the new order of the vertices.

First we choose a peripheral vertex (the vertex with the lowest degree) *x* and set *R* := ({*x*}).

Then for $i = 1, 2, \ldots$ we iterate the following steps while |*R*| < *n*

- Construct the adjacency set A_i of R_i (with R_i the *i*-th component of *R*) and exclude the vertices we already have in *R*

$$A_i := \mathrm{Adj}(R_i) \setminus R$$

- Sort A_i with ascending vertex order (vertex degree).
- Append A_i to the Result set *R*.

In other words, number the vertices according to a particular breadth-first traversal where neighboring vertices are visited in order from lowest to highest vertex order.

See also [edit]

- Graph bandwidth
- Sparse matrix

References [edit]

1. ^ E. Cuthill and J. McKee. *Reducing the bandwidth of sparse symmetric matrices* In Proc. 24th Nat. Conf. ACM, pages 157–172, 1969.
2. ^ J. A. George and J. W-H. Liu, Computer Solution of Large Sparse Positive Definite Systems, Prentice-Hall, 1981

- Cuthill–McKee documentation for the Boost C++ Libraries.
- A detailed description of the Cuthill–McKee algorithm.
- symrcm MATLAB's implementation of RCM.