Line search

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In optimization, the **line search** strategy is one of two basic iterative approaches to find a local minimum $\mathbf{x}^*$ of an objective function $f: \mathbb{R}^n \to \mathbb{R}$. The other approach is trust region.

The line search approach first finds a descent direction along which the objective function f will be reduced and then computes a step size that determines how far $\mathbf{x}$ should move along that direction. The descent direction can be computed by various methods, such as gradient descent, Newton's method and Quasi-Newton method. The step size can be determined either exactly or inexactly.

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Example use [edit]

Here is an example gradient method that uses a line search in step 4.

1. Set iteration counter $k=0$, and make an initial guess, $\mathbf{x}_0$ for the minimum
2. Repeat:
3. Compute a descent direction $\mathbf{p}_k$
4. Choose α_k to 'loosely' minimize $h(\alpha) = f(\mathbf{x}_k + \alpha \mathbf{p}_k)$ over $\alpha \in \mathbb{R}_+$
5. Update $\mathbf{x}_{k+1} = \mathbf{x}_k + \alpha_k \mathbf{p}_k$, and $k = k+1$
6. Until $\|\nabla f(\mathbf{x}_k)\|$ < tolerance

At the line search step (4) the algorithm might either *exactly* minimize *h*, by solving $h'(\alpha_k) = 0$, or *loosely*, by asking for a sufficient decrease in *h*. One example of the former is conjugate gradient method. The latter is called inexact line search and may be performed in a number of ways, such as a backtracking line search or using the Wolfe conditions.

Like other optimization methods, line search may be combined with simulated annealing to allow it to jump over some local minima.

Algorithms [edit]

Direct search methods [edit]

In this method, the minimum must first be bracketed, so the algorithm must identify points x_1 and x_2 such that the sought minimum lies between them. The interval is then divided by computing $f(x)$ at two internal points, x_3 and x_4, and rejecting whichever of the two outer points is not adjacent to that of x_3 and x_4 which has the lowest function value. In subsequent steps, only one extra internal point needs to be calculated. Of the various methods of dividing the interval,[1] golden section search is particularly simple and effective, as the interval proportions are preserved regardless of how the search proceeds:

$$\frac{1}{\phi}(x_2 - x_1) = x_4 - x_1 = x_2 - x_3 = \phi(x_2 - x_4) = \phi(x_3 - x_1) = \phi^2(x_4 - x_3)$$ where

$$\phi = \frac{1}{2}(1+\sqrt{5}) \approx 1.618$$

See also [edit]

- Backtracking line search
- Secant method
- Newton–Raphson method

- Pattern search (optimization)
- Nelder–Mead method
- Golden section search

References [edit]

1. ^ Box, M. J.; Davies, D.; Swann, W. H. (1969). *Non-Linear optimisation Techniques*. Oliver & Boyd.

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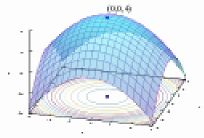

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