

Given a positive integer N, count all possible distinct binary strings of length N such that there are no consecutive 1's.

Examples:

```
Input:  N = 2
Output: 3
// The 3 strings are 00, 01, 10
```

```
Input: N = 3
Output: 5
// The 5 strings are 000, 001, 010, 100, 101
```

This problem can be solved using Dynamic Programming. Let  $a[i]$  be the number of binary strings of length  $i$  which do not contain any two consecutive 1's and which end in 0. Similarly, let  $b[i]$  be the number of such strings which end in 1. We can append either 0 or 1 to a string ending in 0, but we can only append 0 to a string ending in 1. This yields the recurrence relation:

$$a[i] = a[i - 1] + b[i - 1]$$

$$b[i] = a[i - 1]$$

The base cases of above recurrence are  $a[1] = b[1] = 1$ . The total number of strings of length  $i$  is just  $a[i] + b[i]$ .

Following is C++ implementation of above solution. In the following implementation, indexes start from 0. So  $a[i]$  represents the number of binary strings for input length  $i+1$ . Similarly,  $b[i]$  represents binary strings for input length  $i+1$ .

```
// C++ program to count all distinct binary strings
// without two consecutive 1's
#include <iostream>
using namespace std;
```

```
int countStrings(int n)
{
    int a[n], b[n];
    a[0] = b[0] = 1;
    for (int i = 1; i < n; i++)
    {
        a[i] = a[i-1] + b[i-1];
        b[i] = a[i-1];
    }
    return a[n-1] + b[n-1];
}
```

```
// Driver program to test above functions
int main()
{
    cout << countStrings(3) << endl;
    return 0;
}
```

Output:

5

Source:

[courses.csail.mit.edu/6.006/oldquizzes/solutions/q2-f2009-sol.pdf](https://courses.csail.mit.edu/6.006/oldquizzes/solutions/q2-f2009-sol.pdf)