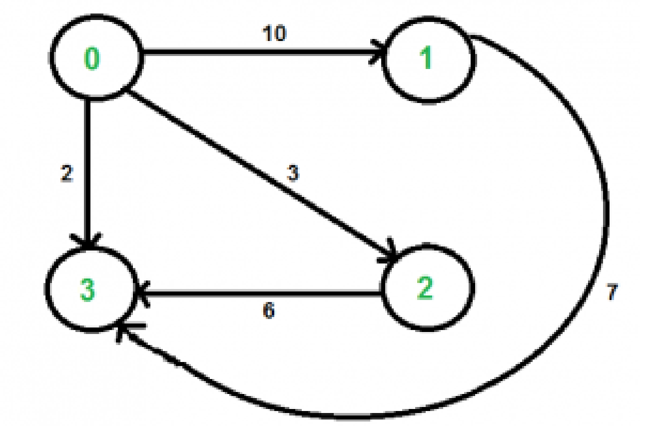
Given a directed and two vertices ‘u’ and ‘v’ in it, find shortest path from ‘u’ to ‘v’ with exactly k edges on the path.

The graph is given as adjacency matrix representation where value of graph[i][j] indicates the weight of an edge from vertex i to vertex j and a value INF(infinite) indicates no edge from i to j.

For example consider the following graph. Let source ‘u’ be vertex 0, destination ‘v’ be 3 and k be 2. There are two walks of length 2, the walks are {0, 2, 3} and {0, 1, 3}. The shortest among the two is {0, 2, 3} and weight of path is 3+6 = 9.



The idea is to browse through all paths of length k from u to v using the approach discussed in the previous post and return weight of the shortest path. A **simple solution** is to start from u, go to all adjacent vertices and recur for adjacent vertices with k as k-1, source as adjacent vertex and destination as v. Following is C++ implementation of this simple solution.

```
// C++ program to find shortest path with exactly k edges
#include <iostream>
#include <climits>
using namespace std;

// Define number of vertices in the graph and inifinite value
#define V 4
#define INF INT_MAX

// A naive recursive function to count walks from u to v with k edges
int shortestPath(int graph[][V], int u, int v, int k)
{
   // Base cases
   if (k == 0 && u == v)             return 0;
   if (k == 1 && graph[u][v] != INF) return graph[u][v];
   if (k <= 0)                       return INF;

   // Initialize result
   int res = INF;

   // Go to all adjacents of u and recur
   for (int i = 0; i < V; i++)
   {
       if (graph[u][i] != INF && u != i && v != i)
       {
           int rec_res = shortestPath(graph, i, v, k-1);
           if (rec_res != INF)
              res = min(res, graph[u][i] + rec_res);
       }
   }
   return res;
}

// driver program to test above function
int main()
{
```

```
    /* Let us create the graph shown in above diagram*/
     int graph[V][V] = { {0, 10, 3, 2},
                        {INF, 0, INF, 7},
                        {INF, INF, 0, 6},
                        {INF, INF, INF, 0}
                      };
    int u = 0, v = 3, k = 2;
    cout << "Weight of the shortest path is " <<
          shortestPath(graph, u, v, k);
    return 0;
}
```

Output:

```
Weight of the shortest path is 9
```

The worst case time complexity of the above function is $O(V^k)$ where V is the number of vertices in the given graph. We can simply analyze the time complexity by drawing recursion tree. The worst occurs for a complete graph. In worst case, every internal node of recursion tree would have exactly V children.

We can optimize the above solution using **Dynamic Programming**. The idea is to build a 3D table where first dimension is source, second dimension is destination, third dimension is number of edges from source to destination, and the value is count of walks. Like other Dynamic Programming problems, we fill the 3D table in bottom up manner.

```
// Dynamic Programming based C++ program to find shortest path with
// exactly k edges
#include <iostream>
#include <climits>
using namespace std;

// Define number of vertices in the graph and inifinite value
#define V 4
#define INF INT_MAX

// A Dynamic programming based function to find the shortest path from
// u to v with exactly k edges.
int shortestPath(int graph[][V], int u, int v, int k)
{
    // Table to be filled up using DP. The value sp[i][j][e] will store
    // weight of the shortest path from i to j with exactly k edges
    int sp[V][V][k+1];

    // Loop for number of edges from 0 to k
    for (int e = 0; e <= k; e++)
    {
        for (int i = 0; i < V; i++)  // for source
        {
            for (int j = 0; j < V; j++) // for destination
            {
                // initialize value
                sp[i][j][e] = INF;

                // from base cases
                if (e == 0 && i == j)
                    sp[i][j][e] = 0;
                if (e == 1 && graph[i][j] != INF)
                    sp[i][j][e] = graph[i][j];

                //go to adjacent only when number of edges is more than 1
                if (e > 1)
                {
                    for (int a = 0; a < V; a++)
                    {
                        // There should be an edge from i to a and a
                        // should not be same as either i or j
                        if (graph[i][a] != INF && i != a &&
```

```
                            j!= a && sp[a][j][e-1] != INF)
                          sp[i][j][e] = min(sp[i][j][e], graph[i][a] +
                                                         sp[a][j][e-1]);
                     }
                 }
            }
         }
    }
    return sp[u][v][k];
}

// driver program to test above function
int main()
{
    /* Let us create the graph shown in above diagram*/
     int graph[V][V] = { {0, 10, 3, 2},
                        {INF, 0, INF, 7},
                        {INF, INF, 0, 6},
                        {INF, INF, INF, 0}
                      };
    int u = 0, v = 3, k = 2;
    cout << shortestPath(graph, u, v, k);
    return 0;
}
```

Output:

```
Weight of the shortest path is 9
```

Time complexity of the above DP based solution is $O(V^3K)$ which is much better than the naive solution.