

## MENU



FIND MEMBERS BY USERNAME OR SKILL

REGISTER

LOG IN

## COMPETE

DESIGN CHALLENGES

DEVELOPMENT CHALLENGES

DATA SCIENCE CHALLENGES

COMPETITIVE PROGRAMMING

## LEARN

GET STARTED

DESIGN

DEVELOPMENT

DATA SCIENCE

COMPETITIVE PROGRAMMING

## COMMUNITY

OVERVIEW

PROGRAMS

FORUMS

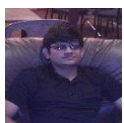
STATISTICS

EVENTS

BLOG

## Forums

Select a Forum ▼

[Search](#) | [Watch Thread](#) | [My Post History](#) | [My Watches](#) | [User Settings](#)View: Flat ([oldest first](#)) | [Threaded](#) | [Tree](#)[Previous Thread](#) | [Next Thread](#)[Forums](#)   [Algorithm Discussions](#)   [Skill-building and Educational Discussion for Algorithm](#)   [Advanced dynamic programming techniques](#)Re: Advanced dynamic programming techniques (response to [post](#) by [anastasov.bg](#)) | [1 edit](#) | Sat, Aug 1, 2009 at 8:44 PM EDT  
[Reply](#)**FameofLight**  
534 posts

I have recently started practicing problem from TOpCoder on DP . I have encountered some amazing problem which teach me something.

Apart from above high level stuff , there are quite easy techniques which used with dp wonders in problem solving .

Like this <http://forums.topcoder.com/?module=Thread&threadID=625789&start=0&mc=11>  
Instead of Going to any bottom up or top down if you go to all possible next state from current state it is more easy to think

Also this one [http://www.topcoder.com/stat?c=problem\\_statement&pm=8538](http://www.topcoder.com/stat?c=problem_statement&pm=8538) which beautifully uses bit masking with dp.

I have also a list of problems from here <http://forums.topcoder.com/?module=Thread&threadID=625789&start=0&mc=11> . I am currently solving problem in green and yellow.

My question you guys have much experience then me , can post some question , analysis or tutorial which will be really helpful to understand other techniques used with dp.

Re: Advanced dynamic programming techniques (response to [post](#) by [Duc](#)) | [Reply](#)

Sat, Jul 11, 2009 at 10:43 PM EDT



[felix\\_halim](#)  
203 posts

The algorithm has been described by [ardiankp](#) above. It's a bit high level but it will make sense after several hours ;). Just to add more details so that it's easier to understand:

Initially, you have a simple DP  $O(N^2 * K)$ :

$s[i] = \text{sum of weights from } w[i] \text{ to } w[n]$

```
for i:=1 to N do
```

```
  dp[1][i] = 0;
```

```
for k:=2 to K do
```

```
  for a:=1 to N do
```

```
    for b:=a+1 to N do
```

```
      dp[k][a] = max(dp[k][a], (b-a) * s[b] + dp[k-1][b]);
```

all = sum of s from  $s[1] \dots s[n]$

minCost = all -  $w[1]$  -  $dp[K][1]$ ;

We want to eliminate the innermost loop "for b:=a+1 to N do".

Each value **b** here is actually a line equation:

$dp[k][a] = (b-a)*s[b] + dp[k-1][b]$ ;

$dp[k][a] = (-s[b]) * a + s[b] * b + dp[k-1][b]$ ;

is a line  $y = m * x + c$

We know all the value for  $m[b] = -s[b]$  and  $c[b] = s[b] * b + dp[k-1][b]$  from the pre-calc and the previous DP iteration. Hence, we have N line equations which are represented by  $m[b]$  and  $c[b]$ .

Support

Since we want the  $y = dp[k][a]$  value to be maximum, we are interested only the line which has the highest y value for all x. The good thing is all lines is already sorted by its slope  $m[b]$  in increasing order. Therefore we can omit the sorting and do the convex-hull-like algorithm in  $O(N)$ .

Having this, we can improve the DP above to  $O(N * K)$ :

$s[i] = \text{sum of weights from } w[i] \text{ to } w[n]$

```
for i:=1 to N do
```

```
  dp[1][i] = 0;
```

```
for k:=2 to K do {
```

```
  select the top-most lines (m,c)
```

```
  from all line equations (1..N)
```

```
  using convex-hull-like algorithm
```

```
  i = 0;
```

```
  for a:=1 to N do {
```

```
    dp[k][a] = max(dp[k][a], m[i] * a + c[i]);
```

```
    advance i as necessary to maintain
```

```
    line i as the top-most line
```

```
}
}
```

```
all = sum of s from s[1] .. s[n]
minCost = all - w[1] - dp[K][1];
```

Hope that helps

Re: Advanced dynamic programming techniques (response to [post](#) by [ardiankp](#)) | [Reply](#)

Fri, Mar 13, 2009 at 10:08 PM EDT

**Duc**  
215 posts

given set of points and gradients, find the minimum y-intercept for each gradient. Ignoring the sorting, this problem can be solved in  $O(n)$ .

Can you describe the algorithm... I dont know how to solve this in  $O(n)$

Re: Advanced dynamic programming techniques (response to [post](#) by [anastasov.bg](#)) | [Reply](#)

Thu, Jan 8, 2009 at 4:01 PM EST



**nima.ahmadi**  
116 posts

If I remember correctly, the task 'Cutting A Greed' from IOI 2006 practice, is a good problem to practice on. The sad thing is that it doesn't come with test cases! Anyway [here](#) you can find the problem statement.

Re: Advanced dynamic programming techniques (response to [post](#) by [ardiankp](#)) | [Reply](#)

Wed, Jan 7, 2009 at 11:31 PM EST

**dragoon**  
130 posts

Hmm... Ok I will code it and let you know my condition. Anyway thanks for the nice problem and nice solution too :)

Re: Advanced dynamic programming techniques (response to [post](#) by [dragoon](#)) | [Reply](#)

Wed, Jan 7, 2009 at 12:13 PM EST



**ardiankp**  
677 posts

it's  $O(N)$  because of the lines are automatically sorted (the gradient is in decreasing order depend on the direction you see)).

Anyway you got the idea :)

**Support**

Re: Advanced dynamic programming techniques (response to [post](#) by [ardiankp](#)) | [Reply](#)

Wed, Jan 7, 2009 at 9:24 AM EST

**dragoon**  
130 posts

Well wont it take  $O(n \lg n)$  in each step? so total run time according to me should be,  $kn \lg n$ ? am i wrong? if so then how to do that in  $O(n)$  instead of  $n \lg n$ ?

Re: Advanced dynamic programming techniques (response to [post](#) by [dragoon](#)) | [Reply](#)

Wed, Jan 7, 2009 at 5:50 AM EST



Let the number of leaves in location  $i$  be  $w_i$ . First, I ignore the first hole since it is always there.

If we do not put any hole, then the total cost =  $w_1 + 2w_2 + 3w_3 + \dots + w_n$ . If we put one hole in position 4, then the total cost will be reduced by  $4 \cdot (w_4 + w_5 + w_6 + \dots + w_n)$ . If we put another

**ardiankp**  
677 posts

hole in position 6, then the total cost will be reduced by  $(6-4) \cdot (w_6 + w_7 + \dots + w_n)$ .

Therefore, let  $s_i$  be the sum of  $w_i$  from  $i$  to  $n$ , if we put holes in position  $a, b, c, d$  then the total reduced cost is

$a \cdot s_a +$   
 $(b-a) \cdot s_b +$   
 $(c-b) \cdot s_c +$   
 $(d-c) \cdot s_d,$   
and our objective is to maximize this sum.

Now, let  $DP[K-1, b]$  be the optimal placing of  $(K-1)$  holes when the first is in  $b$  (e.g.  $DP[3, b] = (c-b) \cdot s_c + (d-c) \cdot s_d$ , for optimal choice of  $c$  and  $d$ ), then  $DP[K, a] =$  the minimum of  $(b-a) \cdot s_b + DP[K-1, b]$ , for all possible  $b$ .

We can see that  $(b-a) \cdot s_b + DP[K-1, b]$  form a line equation with gradient  $-s_b$ , and offset  $b \cdot s_b + DP[K-1, b]$ .

Hence, to calculate  $DP[K, a]$  for all  $a$ , first we built the line equation of all  $b$ , extract the "promising lines" ( $g_i < g_j$  and  $o_i > o_j$ , where  $g$  and  $o$  are gradient and offset respectively). Having all the lines, we can do line-sweep to find the optimal  $b$  for all  $a$  in  $O(n)$ .

Re: Advanced dynamic programming techniques (response to [post](#) by [forest](#)) | [Reply](#) 1 edit | Wed, Jan 7, 2009 at 1:44 AM EST

**dragon**  
130 posts

May be it will sound stupid, but will BS on the max limit for each pile would work?

Edit: Ignore it, ArdianKp's solution seems to be interesting. Can you please enlight us more?

Re: Advanced dynamic programming techniques (response to [post](#) by [forest](#)) | [Reply](#)

Tue, Jan 6, 2009 at 7:57 PM EST



**forest**  
82 posts

up :). Any hints, please, anyone.

## Support

Re: Advanced dynamic programming techniques (response to [post](#) by [gawry](#)) | [Reply](#)

Fri, Apr 4, 2008 at 2:26 AM EDT



**forest**  
82 posts

Another example problem is: <http://vn.spoj.pl/IOITRAIN/problems/NKLEAVES/en>

Really nice problem! Its strict timelimit made me a little nervous but I still like it :]

Could you give a hint how to solve it ?

Re: Advanced dynamic programming techniques (response to [post](#) by [ardiankp](#)) | [Reply](#)

Tue, Mar 25, 2008 at 5:34 PM EDT



**gawry**  
207 posts

Another example problem is: <http://vn.spoj.pl/IOITRAIN/problems/NKLEAVES/en>

Really nice problem! Its strict timelimit made me a little nervous but I still like it :]

Re: Advanced dynamic programming techniques (response to [post](#) by [anastasov.bg](#)) | [Reply](#) Tue, Mar 25, 2008 at 1:11 PM EDT



**ardiankp**  
677 posts

I am no expert in both techniques, but I just try to give few insight.

- 1) There has been a discussion in similar problems applying same technique in <http://forums.topcoder.com/?module=Thread&threadID=579321>. You can check that thread :)
- 2) Try to reduce the problems into: given set of points and gradients, find the minimum y-intercept for each gradient. Ignoring the sorting, this problem can be solved in  $O(n)$ . Another example problem is: <http://vn.spoj.pl/IOITRAIN/problems/NKLEAVES/en/> (thanks to [ktuan](#) who introduce me this problem :)

Advanced dynamic programming techniques | [Reply](#)

Tue, Mar 25, 2008 at 11:10 AM EDT



**anastasov.bg**  
312 posts

I learned dynamic programming mainly from TopCoder (mostly by the tutorial). I find DP to be really helpful in solving a lot of problems and I have encountered some problems, which require some advanced techniques to be solved:

- 1) Reducing the inner cycle (or fully eliminating the inner cycle). As an example of such one is [this](#) [neal\\_wu](#)'s solution.
- 2) Using 'Convex hull' to speed up the transitions. An example of such a problem is 'acquire' from USACO Gold 2008 March.

I don't fully understand the first technique and I don't understand the second at all. I have looked over the solution to 'acquire', but I don't get it.

So here is my request:

Can someone put a link, or give a good description of these techniques (if there are others, mention them) and some problems to practice them on.

Forums   Algorithm Discussions   Skill-building and Educational Discussion for Algorithm   Advanced dynamic programming techniques   [RSS](#)

[Previous Thread](#)   |   [Next Thread](#)

**Support**

**OTHERS**

**SITEMAP**

**ABOUT US**

**CONTACT US**

**HELP CENTER**

**PRIVACY POLICY**

**TERMS**

topcoder is also on

**Support**