Triangulation (geometry)

From Wikipedia, the free encyclopedia

In geometry, a **triangulation** of a planar object is a subdivision into triangles, and by extension the subdivision of higher-dimension geometric objects into simplices. Triangulation of a three-dimensional volume would involve subdividing it into tetrahedra ("pyramids" of various shapes and sizes) packed together.

In most instances, the triangles of a triangulation are required to meet edge-to-edge and vertex-to-vertex.

Different types of triangulation may be defined, depending both on what geometric object is to be subdivided and on how the subdivision is determined.

- A triangulation T of $\mathbb{R}^{n+1}$ is a subdivision of $\mathbb{R}^{n+1}$ into $(n+1)$-dimensional simplices such that any two simplices in T intersect in a common face (a simplex of any lower dimension) or not at all, and any bounded set in $\mathbb{R}^{n+1}$ intersects only finitely many simplices in T. That is, it is a locally finite simplicial complex that covers the entire space.
- A point set triangulation, i.e., a triangulation of a discrete set of points $P \subset \mathbb{R}^{n+1}$, is a subdivision of the convex hull of the points into simplices such that any two simplices intersect in a common face of any dimension or not at all and such that the set of vertices of the simplices coincides with P. Frequently used and studied point set triangulations include the Delaunay triangulation (for points in general position, the set of simplices that are circumscribed by an open ball that contains no input points) and the minimum-weight triangulation (the point set triangulation minimizing the sum of the edge lengths).
- In cartography, a triangulated irregular network is a point set triangulation of a set of two-dimensional points together with elevations for each point. Lifting each point from the plane to its elevated height lifts the triangles of the triangulation into three-dimensional surfaces, which form an approximation of a three-dimensional landform.
- A polygon triangulation is a subdivision of a given polygon into triangles meeting edge-to-edge, again with the property that the set of triangle vertices coincides with the set of vertices of the polygon. Polygon triangulations may be found in linear time and form the basis of several important geometric algorithms, including a simple solution to the art gallery problem. The constrained Delaunay triangulation is an adaptation of the Delaunay triangulation from point sets to polygons or, more generally, to planar straight-line graphs.
- A triangulation of a surface consists of a net of triangles with points on a given surface covering the surface partly or totally.
- In the finite element method, triangulations are often used as the mesh underlying a computation. In this case, the triangles must form a subdivision of the domain to be simulated, but instead of restricting the vertices to input points, it is allowed to add additional Steiner points as vertices. In order to be suitable as finite element meshes, a triangulation must have well-shaped triangles, according to criteria that depend on the details of the finite element simulation; for instance, some methods require that all triangles be right or acute, forming nonobtuse meshes. Many meshing techniques are known, including Delaunay refinement algorithms such as Chew's second algorithm and Ruppert's algorithm.
- In more general topological spaces, triangulations of a space generally refer to simplicial complexes that are homeomorphic to the space.

The concept of a triangulation may also be generalized somewhat to subdivisions into shapes related to triangles. In particular, a pseudotriangulation of a point set is a partition of the convex hull of the points into pseudotriangles, polygons that like triangles have exactly three convex vertices. As in point set triangulations, pseudotriangulations are required to have their vertices at the given input points.

External links [edit]

- Weisstein, Eric W., "Simplicial complex", *MathWorld*.
- Weisstein, Eric W., "Triangulation", *MathWorld*.

Categories: Triangulation (geometry)

Wikipedia The Free Encyclopedia

Main page
Contents
Featured content
Current events
Random article
Donate to Wikipedia
Wikipedia store

Interaction
Help
About Wikipedia
Community portal
Recent changes
Contact page

Tools
What links here
Related changes
Upload file
Special pages
Permanent link
Page information
Wikidata item
Cite this page

Print/export
Create a book
Download as PDF
Printable version

Languages
Беларуская (тарашкевіца)
Français
עברית
Polski
Português
Русский
Українська
Edit links

This page was last modified on 16 April 2015, at 16:00.

Text is available under the Creative Commons Attribution-ShareAlike License; additional terms may apply. By using this site, you agree to the Terms of Use and Privacy Policy. Wikipedia® is a registered trademark of the Wikimedia Foundation, Inc., a non-profit organization.

Privacy policy About Wikipedia Disclaimers Contact Wikipedia Developers Mobile view