Coppersmith–Winograd algorithm

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In linear algebra, the **Coppersmith–Winograd algorithm**, named after Don Coppersmith and Shmuel Winograd, was the asymptotically fastest known algorithm for square matrix multiplication until 2010. It can multiply two $n \times n$ matrices in $O(n^{2.375477})$ time [1] (see Big O notation). This is an improvement over the naïve $O(n^3)$ time algorithm and the $O(n^{2.807355})$ time Strassen algorithm. Algorithms with better asymptotic running time than the Strassen algorithm are rarely used in practice.[why?] It is possible to improve the exponent further; however, the exponent must be at least 2 (because an $n \times n$ matrix has n^2 values, and all of them have to be read at least once to calculate the exact result).

In 2010, Andrew Stothers gave an improvement to the algorithm, $O(n^{2.374})$.[2][3] In 2011, Virginia Williams combined a mathematical short-cut from Stothers' paper with her own insights and automated optimization on computers, improving the bound to $O(n^{2.3728642})$.[4] In 2014, François Le Gall simplified the methods of Williams and obtained an improved bound of $O(n^{2.3728639})$.[5]

The Coppersmith–Winograd algorithm is frequently used as a building block in other algorithms to prove theoretical time bounds. However, unlike the Strassen algorithm, it is not used in practice because it only provides an advantage for matrices so large that they cannot be processed by modern hardware.[6]

Henry Cohn, Robert Kleinberg, Balázs Szegedy and Chris Umans have re-derived the Coppersmith–Winograd algorithm using a group-theoretic construction. They also showed that either of two different conjectures would imply that the optimal exponent of matrix multiplication is 2, as has long been suspected. However, they were not able to formulate a specific solution leading to a better running-time than Coppersmith-Winograd at the time.[7]

References [edit]

1. ^ Coppersmith, Don; Winograd, Shmuel (1990), "Matrix multiplication via arithmetic progressions" (PDF), *Journal of Symbolic Computation* **9** (3): 251, doi:10.1016/S0747-7171(08)80013-2
2. ^ Stothers, Andrew (2010), *On the Complexity of Matrix Multiplication* (PDF).
3. ^ Davie, A.M.; Stothers, A.J. (2013), "Improved bound for complexity of matrix multiplication", *Proceedings of the Royal Society of Edinburgh* **143A**: 351–370, doi:10.1017/S0308210511001648
4. ^ Williams, Virginia (2011), *Breaking the Coppersmith-Winograd barrier* (PDF)
5. ^ Le Gall, François (2014), "Powers of tensors and fast matrix multiplication", *Proceedings of the 39th International Symposium on Symbolic and Algebraic Computation (ISSAC 2014)*, arXiv:1401.7714
6. ^ Robinson, Sara (2005), "Toward an Optimal Algorithm for Matrix Multiplication" (PDF), *SIAM News* **38** (9)
7. ^ Cohn, H.; Kleinberg, R.; Szegedy, B.; Umans, C. (2005). "Group-theoretic Algorithms for Matrix Multiplication". *46th Annual IEEE Symposium on Foundations of Computer Science (FOCS'05)*. p. 379. doi:10.1109/SFCS.2005.39. ISBN 0-7695-2468-0.

Further reading [edit]

- P. Bürgisser, M. Clausen, and M.A. Shokrollahi. *Algebraic complexity theory.* Grundlehren der mathematischen Wissenschaften, No. 315 Springer Verlag 1997.

See also [edit]

- Computational complexity of mathematical operations
- Gauss–Jordan elimination
- Strassen algorithm

v · t · e	Numerical linear algebra [hide]
Key concepts	Floating point · Numerical stability
Problems	Matrix multiplication (algorithms) · Matrix decompositions · Linear equations · Sparse problems
Hardware	CPU cache · TLB · Cache-oblivious algorithm · SIMD · Multiprocessing
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Categories: Numerical linear algebra | Matrix theory | Matrix multiplication algorithms