gvikei's blog

Basic Binary Indexed Tree (English version)

By gvikei, 5 years ago,

Alright, so this is my 2nd post here. The 1st one, as you can see, was written in Vietnamese – my mother tounge, just because I thought that CodeForces Blog could be used for my personal purposes and my entry would not be read by anyone else, except me :D

Due to that, I'm gonna translate that post into English so that anyone can read it and leave feedbacks :) One more thing to say, *I'm not the original author of this article, I just rewrite it to understand BIT better *

PS: As I said, Vietnamese is my mother tounge, and I'm only a high school student, so sorry for my bad English. J

Advantages of BIT:

- Use less memory than RMQ
- Easy to code
- Can be used in many problems about number sequence
- Runtime: O(logN)

Disadvantages : ='= BIT is hard to understand (so that's why I'm writing :D)

Main content:

BIT consists of two operations in an array A[1..N] of numbers

SET (index, value) : To add value to A[index] (or A[index] += value)

GET (index) : To sum up A[1]..A[index] (or Get := A[1] + ... + A[index])

Details:

We know that a natural number can be expressed as a sum of the powers of 2, for example:

Example 1:

22	=	16	+	4	+	2
	=	2^4	+	2^2	+	2 ^1

Applying this idea for BIT, we're going to express the sum of A[1]..A[n] as a sum of sub arrays, each of them has 2^k elements

How to code:

S(i,j) is the sum of A[i]..A[j] (or S[i,j] = A[i] + A[i+1] +…+ A[j]). So with number 22 in Example 1, expressed as a sum of the powers of 2, is gonna be like this:

S(1,22) = S(1,16) + S(17,20) + S(21,22)

To get the positions of sub arrays, use this formula: i - i AND (-i) + 1.

Demo.:

22 - (22 AND (-22)) = 20

20 - (20 AND (-20)) = 16

16 - (16 AND (-16)) = 0

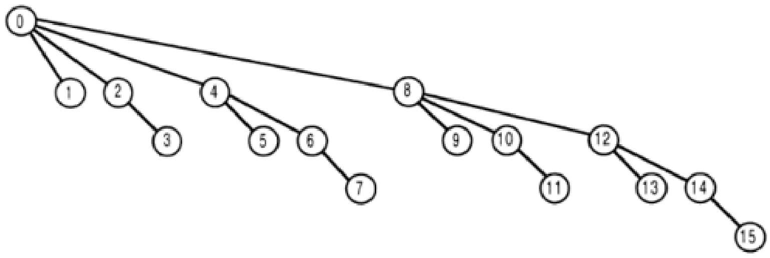
Thus, the structure of BIT T[] will be:

T[i]: Sum of S(i - i AND (-i) + 1 , i)

How does GET work?

Notice: GET (index) sums up A[1] → A[index].

Idea: To get the sum of A[1].. A[index], here we start with A[index] first. Parent of A[index] can be reached by using the following formula: i – i AND (-i)



Pseudo-code:

```
GET (T,i)

s ← 0

while i > 0 do

        s ← s + T[i]

        i ← i – i and (-i)

return s
```

How does SET work?

Notice: SET (index,value) adds value units to A[index]

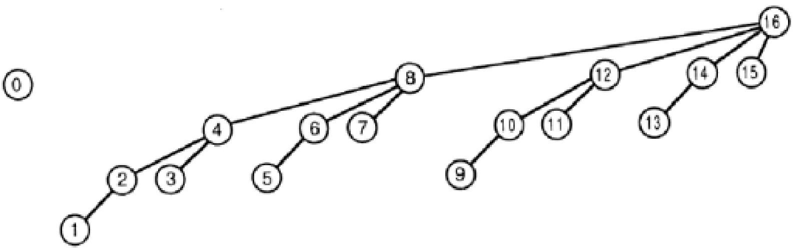
Idea: To increase the value of A[index] with value, is to increase sub arrays which CONTAINS A[index]

Example : We want to add 1 to A[9]. So we also have to add 1 to sub arrays which contains A[9]. Look at the image, here we have A[10], A[12] and A[16] both consist of A[9] J

So how do we find which array contains A[9]?

Use this formula: i + i and (-i)!

With this relationship, the demonstration of BIT will be opposed to the first one in GET procedure:



Pseudo-code

```
SET (T,i,v)

while i ≤ size[T] do

        T[i] ← T[i] + v3        i ← i + i AND (-i)
```

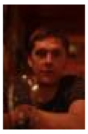
So that's all I know about BIT up to now :(I'ma continue with 2D BIT later. Thank you for reading.

binary indexed tree, sfd, саратов

0 gvikei 5 years ago 25

Comments (25)

Write comment?



iakolzin

5 years ago, # | 0

This data structure is also known as Fenwick tree. Point it out for better understanding what are you talking about.

→ Reply

makagonov

5 years ago, # | +1

"Fenwick tree" is a common name for post-soviet countries mainly. In other countries it is called Binary Indexed Tree.

→ Reply

Xellos

18 months ago, # ^ | 0

Well, a "binary indexed tree" is more general: exactly as the name says, its vertices (e.g. paths to them from the root) are, in some way, represented by the binary representation of their indices.

For example, one version of segment trees is binary indexed as well, when the root has number 1 and vertex i has sons $2i$ and $2i+1$, so the path root — vertex i is given by the binary representation of i, starting from the most significant 1 and up to the least significant bit.

A Fenwick tree just indexes the vertices in a special, compressed way.

BTW, it's called Finnish tree in Slovakia, I think the name stems from some Finnish olympiad that it appeared in once :D

→ Reply

makagonov

5 years ago, # | 0

Could you post some problems (with the possibility to submit them) on this data structure? But not the ones which require to count sum of elements from i to j, some good ones.

→ Reply

juver

5 years ago, # ^ | 0

BIT 3D

→ Reply

Baurzhan

5 years ago, # ^ | 0

problems from acm.timus.ru where i used BIT:

1028,1090,1521,1523(something like 1028)

→ Reply

5 years ago, # ^ | 0

Thanks a lot!