Odds algorithm

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The **odds-algorithm** is a mathematical method for computing optimal strategies for a class of problems that belong to the domain of optimal stopping problems. Their solution follows from the *odds-strategy*, and the importance of the odds-strategy lies in its optimality, as explained below.

The odds-algorithm applies to a class of problems called *last-success*-problems. Formally, the objective in these problems is to maximize the probability of identifying in a sequence of sequentially observed independent events the last event satisfying a specific criterion (a "specific event"). This identification must be done at the time of observation. No revisiting of preceding observations is permitted. Usually, a specific event is defined by the decision maker as an event that is of true interest in the view of "stopping" to take a well-defined action. Such problems are encountered in several situations.

Contents [hide]

Examples [edit]

Two different situations exemplify the interest in maximizing the probability to stop on a last specific event.

1. Suppose a car is advertised for sale to the highest bidder (best "offer"). n potential buyers respond and ask to see the car. Each insists upon an immediate decision from the seller to accept the bid, or not. Define a bid as *interesting*, and coded *1* if it is better than all preceding bids, and coded *0* otherwise. The bids will form a random sequence of 0s and 1s. Only 1s interest the seller, who may fear that each successive 1 might be the last. It follows from the definition that the very last 1 is the highest bid. Maximizing the probability of selling on the last 1 therefore means maximizing the probability of selling *best*.
2. A physician, using a special treatment, may use the code 1 for a successful treatment, 0 otherwise. The physician treats a sequence of n patients the same way, and wants to minimize any suffering, and to treat every responsive patient in the sequence. Stopping on the last 1 in such a random sequence of 0s and 1s would achieve this objective. Since the physician is no prophet, the objective is to maximize the probability of stopping on the last 1.

Definitions [edit]

Consider a sequence of n independent events. Associate with this sequence another sequence $I_1, I_2, \ldots, I_n$ with values 1 or 0. Here $I_k = 1$ stands for the event that the kth observation is interesting (as defined by the decision maker), and $I_k = 0$ for non-interesting. Let $p_k = P(I_k = 1)$ be the probability that the kth event is interesting. Further let $q_k = 1 - p_k$ and $r_k = p_k/q_k$. Note that r_k represents the odds of the kth event turning out to be interesting, explaining the name of the odds-algorithm.

Algorithmic procedure of the odds-algorithm [edit]

The odds-algorithm sums up the odds in reverse order

$$r_n + r_{n-1} + r_{n-2} + \cdots,$$

until this sum reaches or exceeds the value 1 for the first time. If this happens at index *s*, it saves *s* and the corresponding sum

$$R_s = r_n + r_{n-1} + r_{n-2} + \cdots + r_s.$$

If the sum of the odds does not reach 1, it sets *s* = 1. At the same time it computes

$$Q_s = q_n q_{n-1} \cdots q_s.$$

The output is

1. s, the stopping threshold
2. $w = Q_s R_s$, the win probability.

Odds-strategy [edit]

The odds-strategy is the rule to observe the events one after the other and to stop on the first interesting event from index *s* onwards (if any), where *s* is the stopping threshold of output a.

The importance of the odds-strategy, and hence of the odds-algorithm, lies in the following odds-theorem.

Odds-theorem [edit]

The odds-theorem states that

1. The odds-strategy is *optimal*, that is, it maximizes the probability of stopping on the last 1.
2. The win probability of the odds-strategy equals $w = Q_s R_s$
3. If $R_s \geq 1$, the win probability w is always at least $1/e = 0.368\ldots$, and this lower bound is *best possible*.

Features of the odds-algorithm [edit]

The odds-algorithm computes the optimal *strategy* and the optimal *win probability* at the same time. Also, the number of operations of the odds-algorithm is (sub)linear in n. Hence no quicker algorithm can possibly exist for all sequences, so that the odds-algorithm is, at the same time, optimal as an algorithm.

Source [edit]

F. T. Bruss (2000) devised the odds algorithm, and coined its name. It is also known as Bruss-algorithm (strategy). Free implementations can be found on the web.

Applications [edit]

Applications reach from medical questions in clinical trials over sales problems, secretary problems, portfolio selection, (one-way) search strategies, trajectory problems and the parking problem to problems in on-line maintenance and others.

There exists, in the same spirit, an Odds-Theorem for continuous-time arrival processes with independent increments such as the Poisson process (Bruss (2000)). In some cases, the odds are not necessarily known in advance (as in Example 2 above) so that the application of the odds-algorithm is not directly possible. In this case each step can use sequential estimates of the odds. This is meaningful, if the number of unknown parameters is not large compared with the number n of observations. The question of optimality is then more complicated, however, and requires additional studies. Generalizations of the odds-algorithm allow for different rewards for failing to stop and wrong stops as well as replacing independence assumptions by weaker ones (Ferguson (2008)).

See also [edit]

- Secretary problem
- Clinical trial
- House selling problem

References [edit]

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External links [edit]

- Bruss-Algorithmus http://www.p-roesler.de/odds.html

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