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Ant colony optimization algorithms

From Wikipedia, the free encyclopedia
(Redirected from [Ant colony optimization](#))

In [computer science](#) and [operations research](#), the **ant colony optimization algorithm** (**ACO**) is a [probabilistic](#) technique for solving computational problems which can be reduced to finding good paths through [graphs](#).

This algorithm is a member of the **ant colony algorithms** family, in [swarm intelligence](#) methods, and it constitutes some [metaheuristic](#) optimizations. Initially proposed by [Marco Dorigo](#) in 1992 in his PhD thesis,^{[1][2]} the first algorithm was aiming to search for an optimal path in a graph, based on the behavior of [ants](#) seeking a path between their [colony](#) and a source of food. The original idea has since diversified to solve a wider class of numerical problems, and as a result, several problems have emerged, drawing on various aspects of the behavior of ants.



Ant behavior was the inspiration for the metaheuristic optimization technique ⁶³

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Overview ^[edit]

Summary ^[edit]

In the natural world, ants (initially) wander [randomly](#), and upon finding food return to their colony while laying down [pheromone](#) trails. If other ants find such a path, they are likely not to keep travelling at random, but to instead follow the trail, returning and reinforcing it if they eventually find food (see [Ant communication](#)).

Over time, however, the pheromone trail starts to evaporate, thus reducing its attractive strength. The more time it takes for an ant to travel down the path and back again, the more time the pheromones have to evaporate. A short path, by comparison, gets marched over more frequently, and thus the pheromone density becomes higher on shorter paths than longer ones. Pheromone evaporation also has the advantage of avoiding the convergence to a locally optimal solution. If there were no evaporation at all, the paths chosen by the first ants would tend to be excessively attractive to the following ones. In that case, the exploration of the solution space would be constrained..

Thus, when one ant finds a good (i.e., short) path from the colony to a food source, other ants are more likely to follow that path, and **positive feedback** eventually leads to all the ants following a single path. The idea of the ant colony algorithm is to mimic this behavior with "simulated ants" walking around the graph representing the problem to solve.

Common extensions ^[edit]

Here are some of the most popular variations of ACO algorithms.

Elitist ant system ^[edit]

The global best solution deposits pheromone on every iteration along with all the other ants.

Max-min ant system (MMAS) ^[edit]

Added maximum and minimum pheromone amounts

τ

max

,

τ

min

. Only global best or iteration best tour deposited pheromone <MAZ>. All edges are initialized to

τ

max

 and reinitialized to

τ

max

 when nearing stagnation.^[3]

Ant colony system ^[edit]

It has been presented above.^[4]

Rank-based ant system (ASrank) ^[edit]

All solutions are ranked according to their length. The amount of pheromone deposited is then weighted for each solution, such that solutions with shorter paths deposit more pheromone than the solutions with longer paths.

Continuous orthogonal ant colony (COAC) ^[edit]

The pheromone deposit mechanism of COAC is to enable ants to search for solutions collaboratively and effectively. By using an orthogonal design method, ants in the feasible domain can explore their chosen regions rapidly and efficiently, with enhanced global search capability and accuracy.

The orthogonal design method and the adaptive radius adjustment method can also be extended to other optimization algorithms for delivering wider advantages in solving practical problems.^[5]

Recursive ant colony optimization ^[edit]

It is a recursive form of ant system which divides the whole search domain into several sub-domains and solves the objective on these subdomains.^[6] The results from all the subdomains are compared and the best few of them are promoted for the next level. The subdomains corresponding to the selected results are further subdivided and the process is repeated till an output of desired precision is obtained. This method has been tested on ill-posed geophysical inversion problems and works well.^[7]

Convergence ^[edit]

For some versions of the algorithm, it is possible to prove that it is convergent (i.e., it is able to find the global optimum in finite time). The first evidence of a convergence ant colony algorithm was made in 2000, the graph-based ant system algorithm, and then algorithms for ACS and MMAS. Like most [metaheuristics](#), it is very difficult to estimate the theoretical speed of convergence. In 2004, Zlochin and his colleagues^[8] showed that COA-type algorithms could be assimilated methods of [stochastic gradient descent](#), on the [cross-entropy](#) and [estimation of distribution algorithm](#). They proposed these [metaheuristics](#) as a **"research-based model"**. A performance analysis of continuous ant colony algorithm based on its various parameter suggest its sensitivity of convergence on parameter tuning.^[9]

Example pseudo-code and formula [\[edit\]](#)

```
procedure ACO MetaHeuristic
  while (not_termination)
    generateSolutions()
    daemonActions()
    pheromoneUpdate()
  end while
end procedure
```

Edge selection [\[edit\]](#)

An ant is a simple computational agent in the ant colony optimization algorithm. It iteratively constructs a solution for the problem at hand. The intermediate solutions are referred to as solution states. At each iteration of the algorithm, each ant moves from a state x to state y , corresponding to a more complete intermediate solution. Thus, each ant k computes a set $A_k(x)$ of feasible expansions to its current state in each iteration, and moves to one of these in probability. For ant k , the probability p_{xy}^k of moving from state x to state y depends on the combination of two values, viz., the *attractiveness* η_{xy} of the move, as computed by some heuristic indicating the *a priori* desirability of that move and the *trail level* τ_{xy} of the move, indicating how proficient it has been in the past to make that particular move.

The *trail level* represents a posteriori indication of the desirability of that move. Trails are updated usually when all ants have completed their solution, increasing or decreasing the level of trails corresponding to moves that were part of "good" or "bad" solutions, respectively.

In general, the k th ant moves from state x to state y with probability

$$p_{xy}^k = \frac{(\tau_{xy}^\alpha)(\eta_{xy}^\beta)}{\sum_{z \in \text{allowed}_x} (\tau_{xz}^\alpha)(\eta_{xz}^\beta)}$$

where

τ_{xy} is the amount of pheromone deposited for transition from state x to y , $0 \leq \alpha$ is a parameter to control the influence of τ_{xy} , η_{xy} is the desirability of state transition xy (*a priori* knowledge, typically $1/d_{xy}$ where d is the distance) and $\beta \geq 1$ is a parameter to control the influence of η_{xy} . τ_{xz} and η_{xz} represent the attractiveness and trail level for the other possible state transitions.

Pheromone update [\[edit\]](#)

When all the ants have completed a solution, the trails are updated by $\tau_{xy} \leftarrow (1 - \rho)\tau_{xy} + \sum_k \Delta\tau_{xy}^k$

where τ_{xy} is the amount of pheromone deposited for a state transition xy , ρ is the *pheromone evaporation coefficient* and $\Delta\tau_{xy}^k$ is the amount of pheromone deposited by k th ant, typically given for a **TSP** problem (with moves corresponding to arcs of the graph) by

$$\Delta\tau_{xy}^k = \begin{cases} Q/L_k & \text{if ant } k \text{ uses curve } xy \text{ in its tour} \\ 0 & \text{otherwise} \end{cases}$$

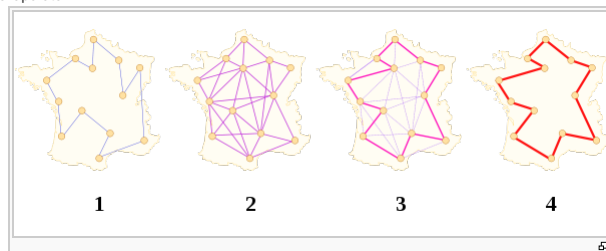
where L_k is the cost of the k th ant's tour (typically length) and Q is a constant.

Applications [\[edit\]](#)

Ant colony optimization algorithms have been applied to many combinatorial optimization problems, ranging from quadratic assignment to **protein** folding or **routing vehicles** and a lot of derived methods have been adapted to dynamic problems in real variables, stochastic problems, multi-targets and parallel implementations. It has also been used to produce near-optimal solutions to the **travelling salesman problem**. They have an advantage over **simulated annealing** and **genetic algorithm** approaches of similar problems when the graph may change dynamically; the ant colony algorithm can be run continuously and adapt to changes in real time. This is of interest in **network routing** and urban transportation systems.

The first ACO algorithm was called the ant system^[10] and it was aimed to solve the travelling salesman problem, in which the goal is to find the shortest round-trip to link a series of cities. The general algorithm is relatively simple and based on a set of ants, each making one of the possible round-trips along the cities. At each stage, the ant chooses to move from one city to another according to some rules:

1. It must visit each city exactly once;
2. A distant city has less chance of being chosen (the visibility);
3. The more intense the pheromone trail laid out on an edge between two cities, the greater the probability that that edge will be chosen;
4. Having completed its journey, the ant deposits more pheromones on all edges it traversed, if the journey is short;
5. After each iteration, trails of pheromones evaporate.



Scheduling problem [\[edit\]](#)

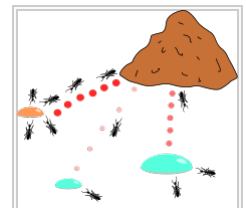
- Job-shop scheduling problem (JSP)^[11]
- Open-shop scheduling problem (OSP)^{[12][13]}
- Permutation flow shop problem (PFSP)^[14]
- Single machine total tardiness problem (SMTTP)^[15]
- Single machine total weighted tardiness problem (SMTWTP)^{[16][17][18]}
- Resource-constrained project scheduling problem (RCPSP)^[19]
- Group-shop scheduling problem (GSP)^[20]
- Single-machine total tardiness problem with sequence dependent setup times (SMTTPDST)^[21]
- Multistage flowshop scheduling problem (MFSP) with sequence dependent setup/changeover times^[22]

Vehicle routing problem [\[edit\]](#)

- Capacitated vehicle routing problem (CVRP)^{[23][24][25]}
- Multi-depot vehicle routing problem (MDVRP)^[26]
- Period vehicle routing problem (PVRP)^[27]
- Split delivery vehicle routing problem (SDVRP)^[28]
- Stochastic vehicle routing problem (SVRP)^[29]
- Vehicle routing problem with pick-up and delivery (VRPPD)^{[30][31]}
- Vehicle routing problem with time windows (VRPTW)^{[32][33][34]}
- Time dependent vehicle routing problem with time windows (TDVRPTW)^[35]
- Vehicle routing problem with time windows and multiple service workers (VRPTWMS)

Assignment problem [\[edit\]](#)

- Quadratic assignment problem (QAP)^[36]
- Generalized assignment problem (GAP)^{[37][38]}
- Frequency assignment problem (FAP)^[39]
- Redundancy allocation problem (RAP)^[40]



Knapsack problem: The ants prefer the smaller drop of honey over the more abundant, but less nutritious, sugar ^[5]

Set problem [\[edit\]](#)

- [Set cover problem \(SCP\)](#)^{[41][42]}
- [Partition problem \(SPP\)](#)^[43]
- [Weight constrained graph tree partition problem \(WCGTPP\)](#)^[44]
- [Arc-weighted l-cardinality tree problem \(AWICTP\)](#)^[45]
- [Multiple knapsack problem \(MKP\)](#)^[46]
- [Maximum independent set problem \(MIS\)](#)^[47]

Device sizing problem in nanoelectronics physical design [\[edit\]](#)

- [Ant colony optimization \(ACO\) based optimization of 45 nm CMOS-based sense amplifier circuit could converge to optimal solutions in very minimal time.](#)^[48]
- [Ant colony optimization \(ACO\) based reversible circuit synthesis could improve efficiency significantly.](#)^[49]

Image processing [\[edit\]](#)

ACO algorithm is used in image processing for image edge detection and edge linking.^{[50][51]}

- **Edge detection:**

The graph here is the 2-D image and the ants traverse from one pixel depositing pheromone.The movement of ants from one pixel to another is directed by the local variation of the image's intensity values. This movement causes the highest density of the pheromone to be deposited at the edges.

The following are the steps involved in edge detection using ACO:^{[52][53]}

Step1: Initialization:

Randomly place K ants on the image $I_{M_1M_2}$ where $K = (M_1 * M_2)^{\frac{1}{2}}$. Pheromone matrix $\tau_{(i,j)}$ are initialized with a random value. The major challenge in the initialization process is determining the heuristic matrix.

There are various methods to determine the heuristic matrix. For the below example the heuristic matrix was calculated based on the local statistics: the local statistics at the pixel position (i,j).

$$\eta_{(i,j)} = \frac{1}{Z} * Vc * I_{(i,j)}$$

Where I is the image of size $M_1 * M_2$

$$Z = \sum_{i=1:M_1} \sum_{j=1:M_2} Vc(I_{i,j}), \text{ which is a normalization factor}$$

$$Vc(I_{i,j}) = f(|I_{(i-2,j-1)} - I_{(i+2,j+1)}| + |I_{(i-2,j+1)} - I_{(i+2,j-1)}| + |I_{(i-1,j-2)} - I_{(i+1,j+2)}| + |I_{(i-1,j+1)} - I_{(i+1,j-1)}| + |I_{(i-1,j)} - I_{(i+1,j)}| + |I_{(i-1,j+1)} - I_{(i-1,j-1)}| + |I_{(i-1,j+2)} - I_{(i-1,j-2)}| + |I_{(i,j-1)} - I_{(i,j+1)}|)$$

$f(\cdot)$ can be calculated using the following functions:

$$f(x) = \lambda x, \quad \text{for } x \geq 0; \quad (1)$$

$$f(x) = \lambda x^2, \quad \text{for } x \geq 0; \quad (2)$$

$$f(x) = \begin{cases} \sin(\frac{\pi x}{2\lambda}), & \text{for } 0 \leq x \leq \lambda; \\ 0, & \text{else} \end{cases} \quad (3)$$

$$f(x) = \begin{cases} \pi x \sin(\frac{\pi x}{2\lambda}), & \text{for } 0 \leq x \leq \lambda; \\ 0, & \text{else} \end{cases} \quad (4)$$

The parameter λ in each of above functions adjusts the functions' respective shapes.

Step 2 Construction process:

The ant's movement is based on [4-connected pixels](#) or [8-connected pixels](#). The probability with which the ant moves is given by the probability equation $P_{x,y}$

Step 3 and Step 5 Update process:

The pheromone matrix is updated twice. in step 3 the trail of the ant (given by $\tau_{(x,y)}$) is updated where as in step 5 the evaporation rate of the trail is updated which is given by the below equation.

$$\tau_{new} \leftarrow (1 - \psi)\tau_{old} + \psi\tau_0, \text{ where } \psi \text{ is the pheromone decay coefficient } 0 < \tau < 1$$

Step 7 Decision Process:

Once the K ants have moved a fixed distance L for N iteration, the decision whether it is an edge or not is based on the threshold T on the pheromone matrix. Threshold for the below example is calculated based on [Otsu's method](#).

Image Edge detected using ACO:

The above images are generated using different functions given by the equation (1) to (4).^[54]



- **Edge linking:**^[55]

ACO has also been proven effective in edge linking algorithms too.

Others [\[edit\]](#)

- [Classification](#)^[66]
- [Connection-oriented network routing](#)^[57]
- [Connectionless network routing](#)^{[58][59]}
- [Data mining](#) ^{[56][60][61][62]}
- [Discounted cash flows in project scheduling](#)^[63]
- [Distributed information retrieval](#)^{[64][65]}
- [Grid workflow scheduling problem](#)^[66]
- [Intelligent testing system](#)^[67]
- [System identification](#)^{[68][69]}
- [Protein folding](#)^{[70][71][72]}
- [Power electronic circuit design](#)^[73]

Definition difficulty [\[edit\]](#)



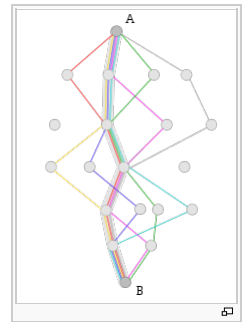
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With an ACO algorithm, the shortest path in a graph, between two points A and B, is built from a combination of several paths. It is not easy to give a precise definition of what algorithm is or is not an ant colony, because the definition may vary according to the authors and uses. Broadly speaking, ant colony algorithms are regarded as [populated metaheuristics](#) with each solution represented by an ant moving in the search space. Ants mark the best solutions and take account of previous markings to optimize their search. They can be seen as [probabilistic multi-agent](#) algorithms using a [probability distribution](#) to make the transition between each [iteration](#). In their versions for combinatorial problems, they use an iterative construction of solutions. According to some authors, the thing which distinguishes ACO algorithms from other relatives (such as algorithms to estimate the distribution or particle swarm optimization) is precisely their constructive aspect. In combinatorial problems, it is possible that the best solution eventually be found, even though no ant would prove effective. Thus, in the example of the Travelling salesman problem, it is not necessary that an ant actually travels the shortest route: the shortest route can be built from the strongest segments of the best solutions. However, this definition can be problematic in the case of problems in real variables, where no structure of 'neighbours' exists. The collective behaviour of [social insects](#) remains a source of inspiration for researchers. The wide variety of algorithms (for optimization or not) seeking self-organization in biological systems has led to the concept of "[swarm intelligence](#)", which is a very general framework in which ant colony algorithms fit.

Stigmergy algorithms [\[edit\]](#)

There is in practice a large number of algorithms claiming to be "ant colonies", without always sharing the general framework of optimization by canonical ant colonies (COA). In practice, the use of

an exchange of information between ants via the environment (a principle called "**stigmergy**") is deemed enough for an algorithm to belong to the class of ant colony algorithms. This principle has led some authors to create the term "value" to organize methods and behavior based on search of food, sorting larvae, division of labour and cooperative transportation.^[74]



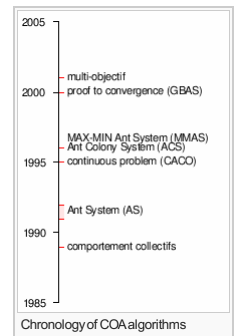
Related methods ^[edit]

- Genetic algorithms** (GA) maintain a pool of solutions rather than just one. The process of finding superior solutions mimics that of evolution, with solutions being combined or mutated to alter the pool of solutions, with solutions of inferior quality being discarded.
- Simulated annealing** (SA) is a related global optimization technique which traverses the search space by generating neighboring solutions of the current solution. A superior neighbor is always accepted. An inferior neighbor is accepted probabilistically based on the difference in quality and a temperature parameter. The temperature parameter is modified as the algorithm progresses to alter the nature of the search.
- Reactive search optimization** focuses on combining machine learning with optimization, by adding an internal feedback loop to self-tune the free parameters of an algorithm to the characteristics of the problem, of the instance, and of the local situation around the current solution.
- Tabu search** (TS) is similar to simulated annealing in that both traverse the solution space by testing mutations of an individual solution. While simulated annealing generates only one mutated solution, tabu search generates many mutated solutions and moves to the solution with the lowest fitness of those generated. To prevent cycling and encourage greater movement through the solution space, a tabu list is maintained of partial or complete solutions. It is forbidden to move to a solution that contains elements of the tabu list, which is updated as the solution traverses the solution space.
- Artificial immune system** (AIS) algorithms are modeled on vertebrate immune systems.
- Particle swarm optimization** (PSO), a **swarm intelligence** method
- Intelligent water drops** (IWD), a swarm-based optimization algorithm based on natural water drops flowing in rivers
- Gravitational search algorithm (GSA), a **swarm intelligence** method
- Ant colony clustering method (ACCM), a method that make use of clustering approach,extending the ACO.
- Stochastic diffusion search** (SDS), an agent-based probabilistic global search and optimization technique best suited to problems where the objective function can be decomposed into multiple independent partial-functions

History ^[edit]

Chronology of ant colony optimization algorithms.

- 1959, **Pierre-Paul Grassé** invented the theory of **stigmergy** to explain the behavior of nest building in **termites**;^[75]
- 1983, Deneubourg and his colleagues studied the **collective behavior** of ants;^[76]
- 1988, and Moysen Manderick have an article on **self-organization** among ants;^[77]
- 1989, the work of Goss, Aron, Deneubourg and Pasteels on the **collective behavior of Argentine ants**, which will give the idea of ant colony optimization algorithms;^[78]
- 1989, implementation of a model of behavior for food by Ebling and his colleagues;^[79]
- 1991, M. Dorigo proposed the **ant system** in his doctoral thesis (which was published in 1992^[2]). A technical report extracted from the thesis and co-authored by V. Maniezzo and A. Colomi ^[80] was published five years later;^[10]
- 1996, publication of the article on ant system;^[10]
- 1996, Hoos and Stützle invent the **max-min ant system**;^[3]
- 1997, Dorigo and Gambardella publish the **ant colony system**;^[4]
- 1997, Schoonderwoerd and his colleagues developed the first application to **telecommunication networks**;^[81]
- 1998, Dorigo launches first conference dedicated to the ACO algorithms;^[82]
- 1998, Stützle proposes initial **parallel implementations**;^[83]
- 1999, Bonabeau, Dorigo and Theraulaz publish a book dealing mainly with artificial ants ^[84]
- 2000, special issue of the Future Generation Computer Systems journal on ant algorithms^[85]
- 2000, first applications to the **scheduling**, scheduling sequence and **the satisfaction of constraints**;
- 2000, Gufjahr provides the first evidence of **convergence** for an algorithm of ant colonies^[86]
- 2001, the first use of COA algorithms by companies (**Eurobios** [↗] and **AntOptima** [↗]);
- 2001, IREDA and his colleagues published the first **multi-objective** algorithm ^[87]
- 2002, first applications in the design of schedule, Bayesian networks;
- 2002, Bianchi and her colleagues suggested the first algorithm for **stochastic** problem;^[88]
- 2004, Zlochin and Dorigo show that some algorithms are equivalent to the **stochastic gradient descent**, the **cross-entropy method** and **algorithms to estimate distribution** ^[8]
- 2005, first applications to **protein folding** problems.
- 2012, Prabhakar and colleagues publish research relating to the operation of individual ants communicating in tandem without pheromones, mirroring the principles of computer network organization. The communication model has been compared to the **Transmission Control Protocol**.^[89]



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