




Interaction

- [Help](#)
- [About Wikipedia](#)
- [Community portal](#)
- [Recent changes](#)
- [Contact page](#)

Print/export


- Create a book
- Download as PDF
- Printable version

Languages
Français
Русский
 Edit links

Article [Talk](#)

Read Edit

More▼

Search 

Insertion [\[edit\]](#)

The basic procedure used to insert each σ_i is called **Schensted insertion** or **row-insertion** (to distinguish it from a variant procedure called column-insertion). Its simplest form is defined in terms of "incomplete standard tableaux": like standard tableaux they have distinct entries, forming increasing rows and columns, but some values (still to be inserted) may be absent as entries. The procedure takes as arguments such a tableau T and a value x not present as entry of T ; it produces as output a new tableau denoted $T \leftarrow x$ and a square s by which its shape has grown. The value x appears in the first row of $T \leftarrow x$, either having been added at the end (if no entries larger than x were present), or otherwise replacing the first entry $y > x$ in the first row of T . In the former case s is the square where x is added, and the insertion is completed; in the latter case the replaced entry y is similarly inserted into the second row of T , and so on, until at some step the first case applies (which certainly happens if an empty row of T is reached).

More formally, the following [pseudocode](#) describes the row-insertion of a new value x into T .^[1]

1. Set $i = 1$ and j to one more than the length of the first row of T .
2. While $j > 1$ and $x < T_{i,j-1}$, decrease j by 1. (Now (i, j) is the first square in row i with either an entry larger than x in T , or no entry at all.)
3. If the square (i, j) is empty in T , terminate after adding x to T in square (i, j) and setting $s = (i, j)$.
4. Swap the values x and $T_{i,j}$. (This inserts the old x into row i , and saves the value it replaces for insertion into the next row.)
5. Increase i by 1 and return to step 2.

The shape of T grows by exactly one square, namely s .

Correctness [\[edit\]](#)

The fact that $T \leftarrow x$ has increasing rows and columns, if the same holds for T , is not obvious from this procedure (entries in the same column are never even compared). It can however be seen as follows. At all times except immediately after step 4, the square (i, j) is either empty in T or holds a value greater than x ; step 5 re-establishes this property because (i, j) now is the square immediately below the one that originally contained x in T . Thus the effect of the replacement in step 4 on the value $T_{i,j}$ is to make it smaller; in particular it cannot become greater than its right or lower neighbours. On the other hand the new value is not less than its left neighbour (if present) either, as is ensured by the comparison that just made step 2 terminate. Finally to see that the new value is larger than its upper neighbour $T_{i-1,j}$ if present, observe that $T_{i-1,j}$ holds after step 5, and that decreasing j in step 2 only decreases the corresponding value $T_{i-1,j}$.

Constructing the tableaux [\[edit\]](#)

The full Schensted algorithm applied to a permutation σ proceeds as follows.

1. Set both P and Q to the empty tableau
2. For i increasing from 1 to n compute $P \leftarrow \sigma_i$ and the square s by the insertion procedure; then replace P by $P \leftarrow \sigma_i$ and add the entry i to the tableau Q in the square s .
3. Terminate, returning the pair (P, Q) .

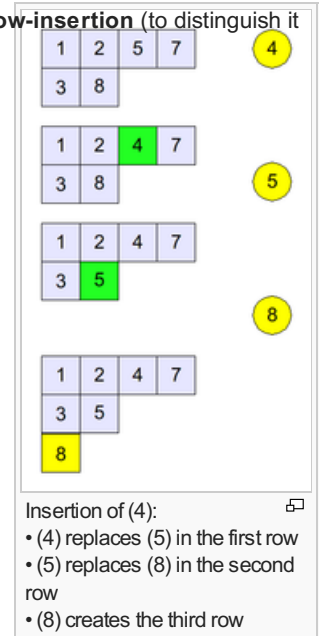
The algorithm produces a pair of standard Young tableaux.

Invertibility of the construction [\[edit\]](#)

It can be seen that given any pair (P, Q) of standard Young tableaux of the same shape, there is an inverse procedure that produces a permutation that will give rise to (P, Q) by the Schensted algorithm. It essentially consists of tracing steps of the algorithm backwards, each time using an entry of Q to find the square where the inverse insertion should start, moving the corresponding entry of P to the preceding row, and continuing upwards through the rows until an entry of the first row is replaced, which is the value inserted at the corresponding step of the construction algorithm. These two inverse algorithms define a bijective correspondence between permutations of n on one side, and pairs of standard Young tableaux of equal shape and containing n squares on the other side.

Properties [\[edit\]](#)

One of the most fundamental properties, but not evident from the algorithmic construction, is symmetry:



- If the Robinson–Schensted correspondence associates tableaux (P, Q) to a permutation σ , then it associates (Q, P) to the inverse permutation σ^{-1} .

This can be proven, for instance, by appealing to [Viennot's geometric construction](#).

Further properties, all assuming that the correspondence associates tableaux (P, Q) to the permutation $\sigma = (\sigma_1, \dots, \sigma_n)$.

- In the pair of tableaux (P', Q') associated to the reversed permutation $(\sigma_n, \dots, \sigma_1)$, the tableau P' is the transpose of the tableau P , and Q' is a tableau determined by Q , independently of P (namely the transpose of the tableau obtained from Q by the [Schützenberger involution](#)).
- The length of the [longest increasing subsequence](#) of $\sigma_1, \dots, \sigma_n$ is equal to the length of the first row of P (and of Q).
- The length of the longest decreasing subsequence of $\sigma_1, \dots, \sigma_n$ is equal to the length of the first column of P (and of Q), as follows from the previous two properties.
- The descent set $\{i : \sigma_i > \sigma_{i+1}\}$ of σ equals the descent set $\{i : i+1 \text{ is in a row strictly below the row of } i\}$ of Q .
- Identify subsequences of π with their sets of indices. It is a theorem of Greene that for any $k \geq 1$, the size of the largest set that can be written as the union of at most k increasing subsequences is $\lambda_1 + \dots + \lambda_k$. In particular, λ_1 equals the largest length of an increasing subsequence of π .
- If σ is an [involution](#), then the number of fixed points of σ equals the number of columns of odd length in λ .

See also [\[edit\]](#)

- [Viennot's geometric construction](#), which provides a diagrammatic interpretation of the correspondence.

Notes [\[edit\]](#)

- ↑ Adapted from [D. E. Knuth](#) (1973), *The Art of Computer Programming* **3**, pp. 50–51

References [\[edit\]](#)

- [Fulton, William](#) (1997), *Young Tableaux*, London Mathematical Society Student Texts **35**, [Cambridge University Press](#), ISBN 978-0-521-56144-0, MR 1464693 [↗](#).
- [Knuth, Donald E.](#) (1970), "Permutations, matrices, and generalized Young tableaux" [↗](#), *Pacific Journal of Mathematics* **34**: 709–727, doi:10.2140/pjm.1970.34.709 [↗](#), MR 0272654 [↗](#)
- [Robinson, G. de B.](#) (1938), "On the Representations of the Symmetric Group", *American Journal of Mathematics* **60** (3): 745–760, doi:10.2307/2371609 [↗](#), JSTOR 2371609 [↗](#), Zbl 0019.25102 [↗](#).
- [Sagan, B. E.](#) (2001), *The Symmetric Group*, Graduate Texts in Mathematics **203**, New York: Springer-Verlag, ISBN 0-387-95067-2.
- [Schensted, C.](#) (1961), "Longest increasing and decreasing subsequences", *Canadian Journal of Mathematics* **13**: 179–191, doi:10.4153/CJM-1961-015-3 [↗](#), MR 0121305 [↗](#).
- [Stanley, Richard P.](#) (1999), *Enumerative Combinatorics, Vol. 2* [↗](#), Cambridge Studies in Advanced Mathematics **62**, [Cambridge University Press](#), ISBN 978-0-521-56069-6, MR 1676282 [↗](#).
- [Zelevinsky, A. V.](#) (1981), "A generalization of the Littlewood–Richardson rule and the Robinson–Schensted–Knuth correspondence", *Journal of Algebra* **69** (1): 82–94, doi:10.1016/0021-8693(81)90128-9 [↗](#), MR 613858 [↗](#).

Further reading [\[edit\]](#)

- [Green, James A.](#) (2007). *Polynomial representations of GL_n* . Lecture Notes in Mathematics **830**. With an appendix on Schensted correspondence and Littelmann paths by K. Erdmann, J. A. Green and M. Schocker (2nd corrected and augmented ed.). Berlin: [Springer-Verlag](#). ISBN 3-540-46944-3. Zbl 1108.20044 [↗](#).

External links [\[edit\]](#)

- [van Leeuwen, M.A.A.](#) (2001), "Robinson–Schensted correspondence" [↗](#), in [Hazewinkel, Michiel](#), *Encyclopedia of Mathematics*, Springer, ISBN 978-1-55608-010-4

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