Puzzle 16 | (100 Doors)

There are 100 doors in a row, all doors are initially closed. A person walks through all doors multiple times and toggle (if open then close, if close then open) them in following way:

In first walk, the person toggles every door

In second walk, the person toggles every second door, i.e., 2^{nd}, 4^{th}, 6^{th}, 8^{th}, …

In third walk, the person toggles every third door, i.e. 3^{rd}, 6^{th}, 9^{th}, …

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In 100^{th} walk, the person toggles 100^{th} door.

Which doors are open in the end?

Solution:

A door is toggled in i^{th} walk if i divides door number. For example the door number 45 is toggled in 1^{st}, 3^{rd}, 5^{th}, 9^{th} and 15^{th} walk.

The door is switched back to initial stage for every pair of divisors. For example 45 is toggled 6 times for 3 pairs (5, 9), (15, 3) and (1, 45).

It looks like all doors would become closes at the end. But there are door numbers which would become open, for example 16, the pair (4, 4) means only one walk. Similarly all other perfect squares like 4, 9, ….

So the answer is 1, 4, 9, 16, 25, 36, 49, 64, 81 and 100.