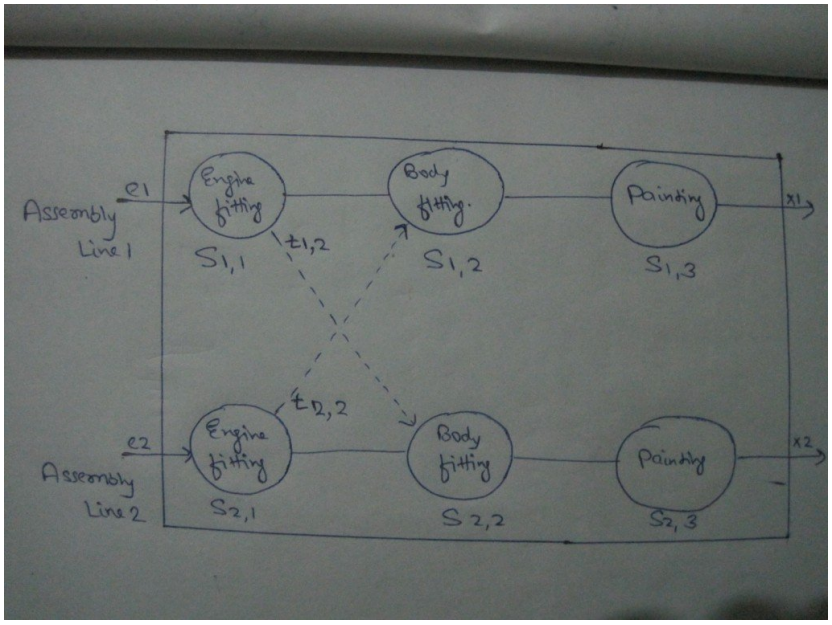
A car factory has two assembly lines, each with n stations. A station is denoted by $S_{i,j}$ where i is either 1 or 2 and indicates the assembly line the station is on, and j indicates the number of the station. The time taken per station is denoted by $a_{i,j}$. Each station is dedicated to some sort of work like engine fitting, body fitting, painting and so on. So, a car chassis must pass through each of the n stations in order before exiting the factory. The parallel stations of the two assembly lines perform the same task. After it passes through station $S_{i,j}$, it will continue to station $S_{i,j+1}$ unless it decides to transfer to the other line. Continuing on the same line incurs no extra cost, but transferring from line i at station j – 1 to station j on the other line takes time $t_{i,j}$. Each assembly line takes an entry time e_i and exit time x_i which may be different for the two lines. Give an algorithm for computing the minimum time it will take to build a car chassis.

The below figure presents the problem in a clear picture:



The following information can be extracted from the problem statement to make it simpler:

- Two assembly lines, 1 and 2, each with stations from 1 to n.
- A car chassis must pass through all stations from 1 to n in order(in any of the two assembly lines). i.e. it cannot jump from station i to station j if they are not at one move distance.
- The car chassis can move one station forward in the same line, or one station diagonally in the other line. It incurs an extra cost ti, j to move to station j from line i. No cost is incurred for movement in same line.
- The time taken in station j on line i is $a_{i, j}$.
- $S_{i, j}$ represents a station j on line i.

Breaking the problem into smaller sub-problems:
We can easily find the ith factorial if (i-1)th factorial is known. Can we apply the similar funda here?
If the minimum time taken by the chassis to leave station $S_{i, j-1}$ is known, the minimum time taken to leave station $S_{i, j}$ can be calculated quickly by combining $a_{i, j}$ and $t_{i, j}$.

T1(j) indicates the minimum time taken by the car chassis to leave station j on assembly line 1.

T2(j) indicates the minimum time taken by the car chassis to leave station j on assembly line 2.

Base cases:
The entry time e_i comes into picture only when the car chassis enters the car factory.

Time taken to leave first station in line 1 is given by:
T1(1) = Entry time in Line 1 + Time spent in station $S_{1,1}$

$T1(1) = e_1 + a_{1,1}$

Similarly, time taken to leave first station in line 2 is given by:

$T2(1) = e_2 + a_{2,1}$

Recursive Relations:

If we look at the problem statement, it quickly boils down to the below observations:

The car chassis at station $S_{1,j}$ can come either from station $S_{1, j-1}$ or station $S_{2, j-1}$.

Case #1: Its previous station is $S_{1, j-1}$

The minimum time to leave station $S_{1,j}$ is given by:

$T1(j)$ = Minimum time taken to leave station $S_{1, j-1}$ + Time spent in station $S_{1, j}$

$T1(j) = T1(j-1) + a_{1, j}$

Case #2: Its previous station is $S_{2, j-1}$

The minimum time to leave station S1, j is given by:

$T1(j)$ = Minimum time taken to leave station $S_{2, j-1}$ + Extra cost incurred to change the assembly line + Time spent in station $S_{1, j}$

$T1(j) = T2(j-1) + t_{2, j} + a_{1, j}$

The minimum time T1(j) is given by the minimum of the two obtained in cases #1 and #2.

$T1(j) = \min((T1(j-1) + a_{1, j}), (T2(j-1) + t_{2, j} + a_{1, j}))$

Similarly the minimum time to reach station S2, j is given by:

$T2(j) = \min((T2(j-1) + a_{2, j}), (T1(j-1) + t_{1, j} + a_{2, j}))$

The total minimum time taken by the car chassis to come out of the factory is given by:

T_{min} = min(Time taken to leave station $S_{i,n}$ + Time taken to exit the car factory)

$T_{min} = \min(T1(n) + x_1, T2(n) + x_2)$

Why dynamic programming?

The above recursion exhibits overlapping sub-problems. There are two ways to reach station $S_{1, j}$:

1. From station $S_{1, j-1}$
2. From station $S_{2, j-1}$

So, to find the minimum time to leave station $S_{1, j}$ the minimum time to leave the previous two stations must be calculated(as explained in above recursion).

Similarly, there are two ways to reach station $S_{2, j}$:

1. From station $S_{2, j-1}$
2. From station $S_{1, j-1}$

Please note that the minimum times to leave stations $S_{1, j-1}$ and $S_{2, j-1}$ have already been calculated.

So, we need two tables to store the partial results calculated for each station in an assembly line. The table will be filled in bottom-up fashion.

Note:

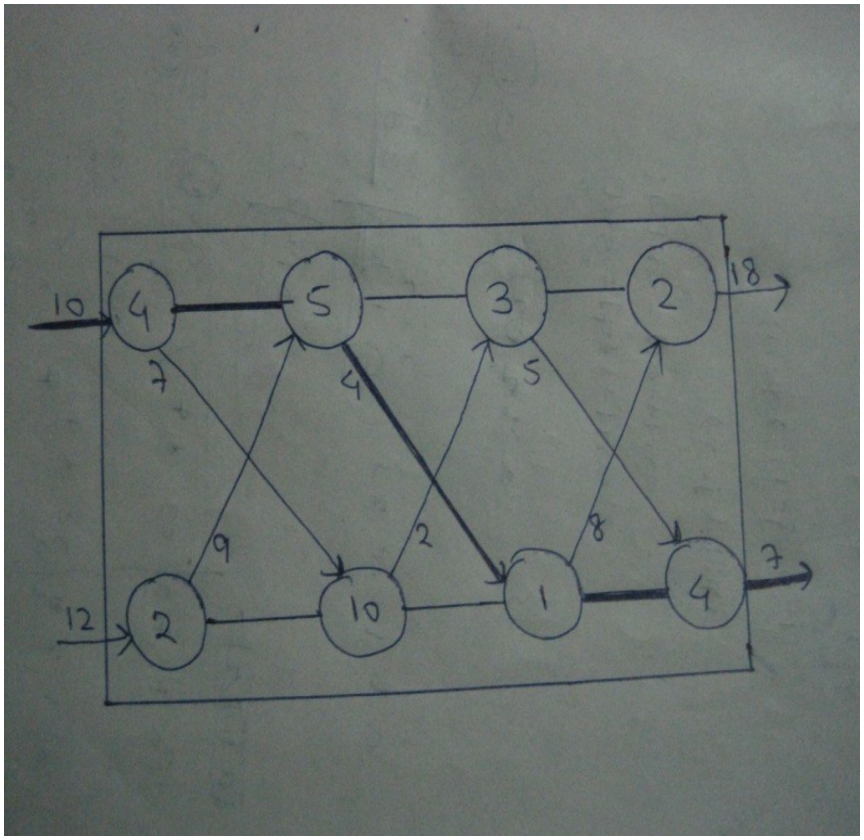
In this post, the word “leave” has been used in place of “reach” to avoid the confusion. Since the car chassis must spend a fixed time in each station, the word leave suits better.

Implementation:

```
// A C program to find minimum possible time by the car chassis to complete
#include <stdio.h>
#define NUM_LINE 2
#define NUM_STATION 4

// Utility function to find minimum of two numbers
int min(int a, int b) { return a < b ? a : b; }

int carAssembly(int a[][NUM_STATION], int t[][NUM_STATION], int *e, int *x)
{
    int T1[NUM_STATION], T2[NUM_STATION], i;

    T1[0] = e[0] + a[0][0]; // time taken to leave first station in line 1
    T2[0] = e[1] + a[1][0]; // time taken to leave first station in line 2

    // Fill tables T1[] and T2[] using the above given recursive relations
    for (i = 1; i < NUM_STATION; ++i)
    {
        T1[i] = min(T1[i-1] + a[0][i], T2[i-1] + t[1][i] + a[0][i]);
        T2[i] = min(T2[i-1] + a[1][i], T1[i-1] + t[0][i] + a[1][i]);
    }

    // Consider exit times and retutn minimum
    return min(T1[NUM_STATION-1] + x[0], T2[NUM_STATION-1] + x[1]);
}

int main()
{
    int a[][NUM_STATION] = {{4, 5, 3, 2},
                {2, 10, 1, 4}};
    int t[][NUM_STATION] = {{0, 7, 4, 5},
                {0, 9, 2, 8}};
    int e[] = {10, 12}, x[] = {18, 7};

    printf("%d", carAssembly(a, t, e, x));

    return 0;
}
```

Output:

```
35
```



The bold line shows the path covered by the car chassis for given input values.

Exercise:

Extend the above algorithm to print the path covered by the car chassis in the factory.

References:

Introduction to Algorithms 3rd Edition by Clifford Stein, Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest