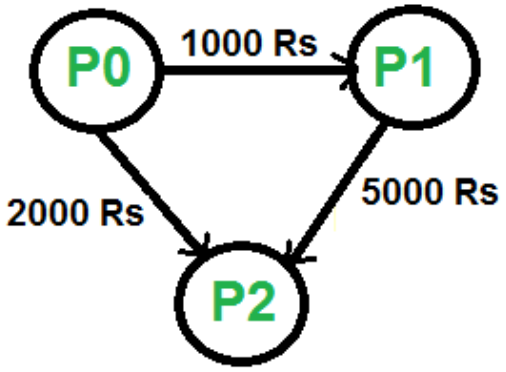
Minimize Cash Flow among a given set of friends who have borrowed money from each other

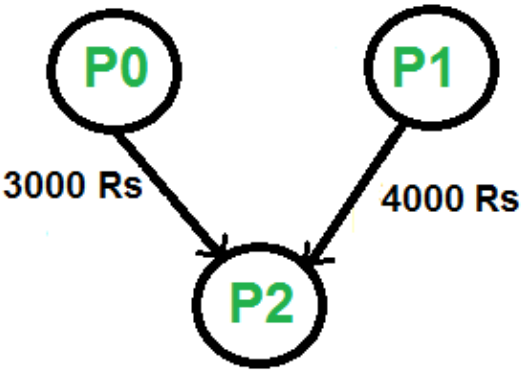
Given a number of friends who have to give or take some amount of money from one another. Design an algorithm by which the total cash flow among all the friends is minimized.

Example:
Following diagram shows input debts to be settled.



P0 has to pay 1000 Rs to P1
P0 also has to pay 2000 Rs to P2
P1 has to pay 5000 Rs to P2.

Above debts can be settled in following optimized way



P1 pays 4000 Rs to P2
P0 pays 3000 Rs to P2

We strongly recommend to minimize your browser and try this yourself first.
The idea is to use Greedy algorithm where at every step, settle all amounts of one person and recur for remaining n-1 persons.
How to pick the first person? To pick the first person, calculate the net amount for every person where net amount is obtained by subtracting all debts (amounts to pay) from all credits (amounts to be paid). Once net amount for every person is evaluated, find two persons with maximum and minimum net amounts. These two persons are the most creditors and debtors. The person with minimum of two is our first person to be settled and removed from list. Let the minimum of two amounts be x. We pay 'x' amount from the maximum debtor to maximum creditor and settle one person. If x is equal to the maximum debit, then maximum debtor is settled, else maximum creditor is settled.

The following is detailed algorithm.

Do following for every person Pi where i is from 0 to n-1.

1) Compute the net amount for every person. The net amount for person 'i' can be computed be subtracting sum of all debts from sum of all credits.

2) Find the two persons that are maximum creditor and maximum debtor. Let the maximum amount to be credited maximum creditor be maxCredit and maximum amount to be debited from maximum debtor be maxDebit. Let the maximum debtor be Pd and maximum creditor be Pc.

3) Find the minimum of maxDebit and maxCredit. Let minimum of two be x. Debit 'x' from Pd and credit this amount to Pc

4) If x is equal to maxCredit, then remove Pc from set of persons and recur for remaining (n-1) persons.

5) If x is equal to maxDebit, then remove Pd from set of persons and recur for remaining (n-1) persons.

Thanks to Balaji S for suggesting this method in a comment here.

The following is C++ implementation of above algorithm.

```
// C++ program to fin maximum cash flow among a set of persons
#include<iostream>
using namespace std;

// Number of persons (or vertices in the graph)
#define N 3

// A utility function that returns index of minimum value in arr[]
int getMin(int arr[])
{
    int minInd = 0;
    for (int i=1; i<N; i++)
        if (arr[i] < arr[minInd])
            minInd = i;
    return minInd;
}

// A utility function that returns index of maximum value in arr[]
int getMax(int arr[])
{
    int maxInd = 0;
    for (int i=1; i<N; i++)
        if (arr[i] > arr[maxInd])
            maxInd = i;
    return maxInd;
}

// A utility function to return minimum of 2 values
int minOf2(int x, int y)
{
    return (x<y)? x: y;
}

// amount[p] indicates the net amount to be credited/debited
// to/from person 'p'
// If amount[p] is positive, then i'th person will amount[i]
// If amount[p] is negative, then i'th person will give  -amount[i]
void minCashFlowRec(int amount[])
{
    // Find the indexes of minimum and maximum values in amount[]
    // amount[mxCredit] indicates the maximum amount to be given
    //                  (or credited) to any person .
    // And amount[mxDebit] indicates the maximum amount to be taken
    //                  (or debited) from any person.
    // So if there is a positive value in amount[], then there must
```

```
    // be a negative value
    int mxCredit = getMax(amount), mxDebit = getMin(amount);

    // If both amounts are 0, then all amounts are settled
    if (amount[mxCredit] == 0 && amount[mxDebit] == 0)
        return;

    // Find the minimum of two amounts
    int min = minOf2(-amount[mxDebit], amount[mxCredit]);
    amount[mxCredit] -= min;
    amount[mxDebit] += min;

    // If minimum is the maximum amount to be
    cout << "Person " << mxDebit << " pays " << min
         << " to " << "Person " << mxCredit << endl;

    // Recur for the amount array.  Note that it is guaranteed that
    // the recursion would terminate as either amount[mxCredit]
    // or  amount[mxDebit] becomes 0
    minCashFlowRec(amount);
}

// Given a set of persons as graph[] where graph[i][j] indicates
// the amount that person i needs to pay person j, this function
// finds and prints the minimum cash flow to settle all debts.
void minCashFlow(int graph[][N])
{
    // Create an array amount[], initialize all value in it as 0.
    int amount[N] = {0};

    // Calculate the net amount to be paid to person 'p', and
    // stores it in amount[p]. The value of amount[p] can be
    // calculated by subtracting debts of 'p' from credits of 'p'
    for (int p=0; p<N; p++)
       for (int i=0; i<N; i++)
          amount[p] += (graph[i][p] -  graph[p][i]);

    minCashFlowRec(amount);
}

// Driver program to test above function
int main()
{
    // graph[i][j] indicates the amount that person i needs to
    // pay person j
    int graph[N][N] = { {0, 1000, 2000},
                        {0, 0, 5000},
                        {0, 0, 0},};

    // Print the solution
    minCashFlow(graph);
    return 0;
}
```

Run on IDE

Output:

```
Person 1 pays 4000 to Person 2
Person 0 pays 3000 to Person 2
```

Algorithmic Paradigm: Greedy

Time Complexity: $O(N^2)$ where N is the number of persons.