Fast multipole method

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The **fast multipole method (FMM)** is a numerical technique that was developed to speed up the calculation of long-ranged forces in the n-body problem. It does this by expanding the system Green's function using a multipole expansion, which allows one to group sources that lie close together and treat them as if they are a single source.[1]

The FMM has also been applied in accelerating the iterative solver in the method of moments (MOM) as applied to computational electromagnetics problems.[2] The FMM was first introduced in this manner by Greengard and Rokhlin[3] and is based on the multipole expansion of the vector Helmholtz equation. By treating the interactions between far-away basis functions using the FMM, the corresponding matrix elements do not need to be explicitly stored, resulting in a significant reduction in required memory. If the FMM is then applied in a hierarchical manner, it can improve the complexity of matrix-vector products in an iterative solver from $\mathcal{O}(N^2)$ to $\mathcal{O}(N)$ in finite arithmetic, i.e., given a tolerance ϵ, the matrix-vector product is guaranteed to be within a tolerance ϵ. The dependence of the complexity on the tolerance ϵ is $\mathcal{O}(\log(1/\epsilon))$, i.e., the complexity of FMM is $\mathcal{O}(\log(1/\epsilon)N)$. This has expanded the area of applicability of the MOM to far greater problems than were previously possible.

The FMM, introduced by Rokhlin and Greengard, has been said to be one of the top ten algorithms of the 20th century.[4] The FMM algorithm reduces the complexity of matrix-vector multiplication involving a certain type of dense matrix which can arise out of many physical systems.

The FMM has also been applied for efficiently treating the Coulomb interaction in Hartree–Fock and density functional theory calculations in quantum chemistry.

See also   [edit]

| | √x | Mathematics portal | | Physics portal | | Astronomy portal |

References   [edit]

1. ^ Rokhlin, Vladimir (1985). "Rapid Solution of Integral Equations of Classic Potential Theory." J. Computational Physics Vol. 60, pp. 187-207.
2. ^ Nader Engheta, William D. Murphy, Vladimir Rokhlin, and Marius Vassiliou (1992), "The Fast Multipole Method for Electromagnetic Scattering Computation," IEEE Transactions on Antennas and Propagation 40, 634-641.
3. ^ http://www-theor.ch.cam.ac.uk/people/ross/thesis/node97.html
4. ^ Barry A Cipra (May 16, 2000). "The Best of the 20th Century: Editors Name Top 10 Algorithms". SIAM News (Society for Industrial and Applied Mathematics) 33 (4): 2. Retrieved December 23, 2010.

External links   [edit]

- Gibson, Walton C. *The Method of Moments in Electromagnetics*. Chapman & Hall/CRC, 2008. ISBN 978-1-4200-6145-1
- FEKO from EM Software & Systems includes the Multilevel FMM as solution option.
- Serenity A high-fidelity Radar Cross Section (RCS) code that uses the Moment Method and the FMM.
- Abstract of Greengard and Rokhlin's original paper
- A short course on fast multipole methods by Rick Beatson and Leslie Greengard.
- JAVA Animation of the Fast Multipole Method Nice animation of the Fast Multipole Method with different adaptations.

Free software   [edit]

- Puma-EM A high performance, parallelized, open source Method of Moments / Multilevel Fast Multipole Method electromagnetics code
- KIFMM3d The Kernel-Independent Fast Multipole 3d Method (kifmm3d) is a new FMM implementation which does not require the explicit multipole expansions of the underlying kernel, and it is based on kernel evaluations.

- FastFieldSolvers ⧉ FastFieldSolvers consists of tools developed at M.I.T. for the solution of Maxwell equations and extraction of circuit parasites (inductance and capacitance).
- ExaFMM ⧉ ExaFMM is a CPU/GPU capable 3D FMM code for Laplace/Helmholtz kernels that focuses on parallel scalability.
- ScalFMM ⧉ ScalFMM is a C++ software library developed at Inria Bordeaux with high emphasis on genericity and parallelization (using OpenMP/MPI).

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