Comp 7405 Assignment 1

Darasirikul Varis
3035754210

1). Non dividend pay Stock = S
Risk free rate = r
time = $t = 0$
Delivery time = T
Specified price = F

With Forward Contract - no arbitage

$$F = S(0)e^{rT}$$

With Time Value of Money rule
Future stock price will be

$$S(T) = S(0)\left(1+\frac{r}{m}\right)^{mt}$$

Since we assume that this stock is risk free and No arbitrage, we will get

$$S(T) = F$$

then $$F = S(0)\left(1+\frac{r}{m}\right)^{mt}$$

since $$\lim_{m\to\infty}\left(1+\frac{r}{m}\right)^{m} = e^{r}$$

we will get $$F = S(0)e^{rT}$$

Comp 7405 Assignment 1

Davasirikul Varis
3035754210

2) time = t
Strike price = K
Maturity = T

2.1) Lower bound on option value

$$C \geq S(0) - Ke^{-rt}$$

Vertical spread is created by
long call at Strike = K_1
Short call at Strike = K_2
where $K_2 \geq K_1$

we will get

$$C_t^T(K_1) \geq S(0) - K_1 e^{-rt} \quad \text{(long call)}$$
$$C_t^T(K_2) \geq S(0) - K_2 e^{-rt} \quad \text{(Short call)}$$

After Compare different we will get

$$C_t^T(K_1) - C_t^T(K_2) \geq K_2 e^{-rt} - K_1 e^{-rt}$$

rearrange : $C_t^T(K_1) - C_t^T(K_2) \geq -e^{-rt}(K_1 - K_2)$ ①

since $K_2 \geq K_1$ we will get

$$C_t^T(K_1) - C_t^T(K_2) \geq 0 \quad ②$$

rearrange since $K_2 \geq K_1$

$$K_1 - K_2 \leq 0 \quad ③$$

② ÷ ③ ; $\dfrac{C_t^T(K_1) - C_t^T(K_2)}{K_1 - K_2} < 0$ ④

Comp 7405 Assignment 1

Darasirikul Varis
3035754210

① ÷ ③ ; $\frac{C_t^T(K_1) - C_t^T(K_2)}{K_1 - K_2} > -e^{-rT}$ ⑤

If we combine ④ and ⑤ then

$$0 > \frac{C_t^T(K_1) - C_t^T(K_2)}{K_1 - K_2} > -e^{-rT}$$

2.2) Assume that initial Value is Negative

$$V_0 < 0$$

Butterfly spread will provide a strictly positive cash flow at

$t = 0$

also $V_T = 0$ since it is provide plus cash flow

$V_T = 0$ when $S_T \in [0, K-\Delta K] \cup [K+\Delta K, \infty)$
$V_T > 0$ when $S_T \in (K-\Delta K, K+\Delta K)$

From the above condition this is a free lunch, so it goes against the no-arbitage condition