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Comp 7405 Assignment 2 (10 marks)

1. (2 marks) Implement the Black-Scholes formulas for C(S; t) and P(S; t), and calculate the values of both call and put options with following parameters:

```python
import numpy as np
from scipy.stats import norm
# 1. (2 marks) Implement the Black-Scholes formulas for C(S; t) and P(S; t),
# and calculate the values of both call and put options with following parameters:
# Data
# S = 50, K = 50, t = 0, T = 0:5, sigma = 20%, and r = 1%.
# S = 50, K = 60, t = 0, T = 0:5, sigma = 20%, and r = 1%.
# S = 50, K = 50, t = 0, T = 1:0, sigma = 20%, and r = 1%.
# S = 50, K = 50, t = 0, T = 0:5, sigma = 30%, and r = 1%.
# S = 50, K = 50, t = 0, T = 0:5, sigma = 20%, and r = 2%.
data = [
    {
        "S": 50,
        "K": 50,
        "t": 0,
        "T": 0.5,
        "sigma": 0.2,
        "r": 0.01
    },
    {
        "S": 50,
        "K": 60,
        "t": 0,
        "T": 0.5,
        "sigma": 0.2,
        "r": 0.01
    },
    {
        "S": 50,
        "K": 50,
        "t": 0,
        "T": 1,
        "sigma": 0.2,
        "r": 0.01
    },
    {
        "S": 50,
        "K": 50,
        "t": 0,
        "T": 0.5,
        "sigma": 0.3,
        "r": 0.01
    },
    {
        "S": 50,
        "K": 50,
        "t": 0,
        "T": 0.5,
        "sigma": 0.2,
        "r": 0.02
    },
]
```

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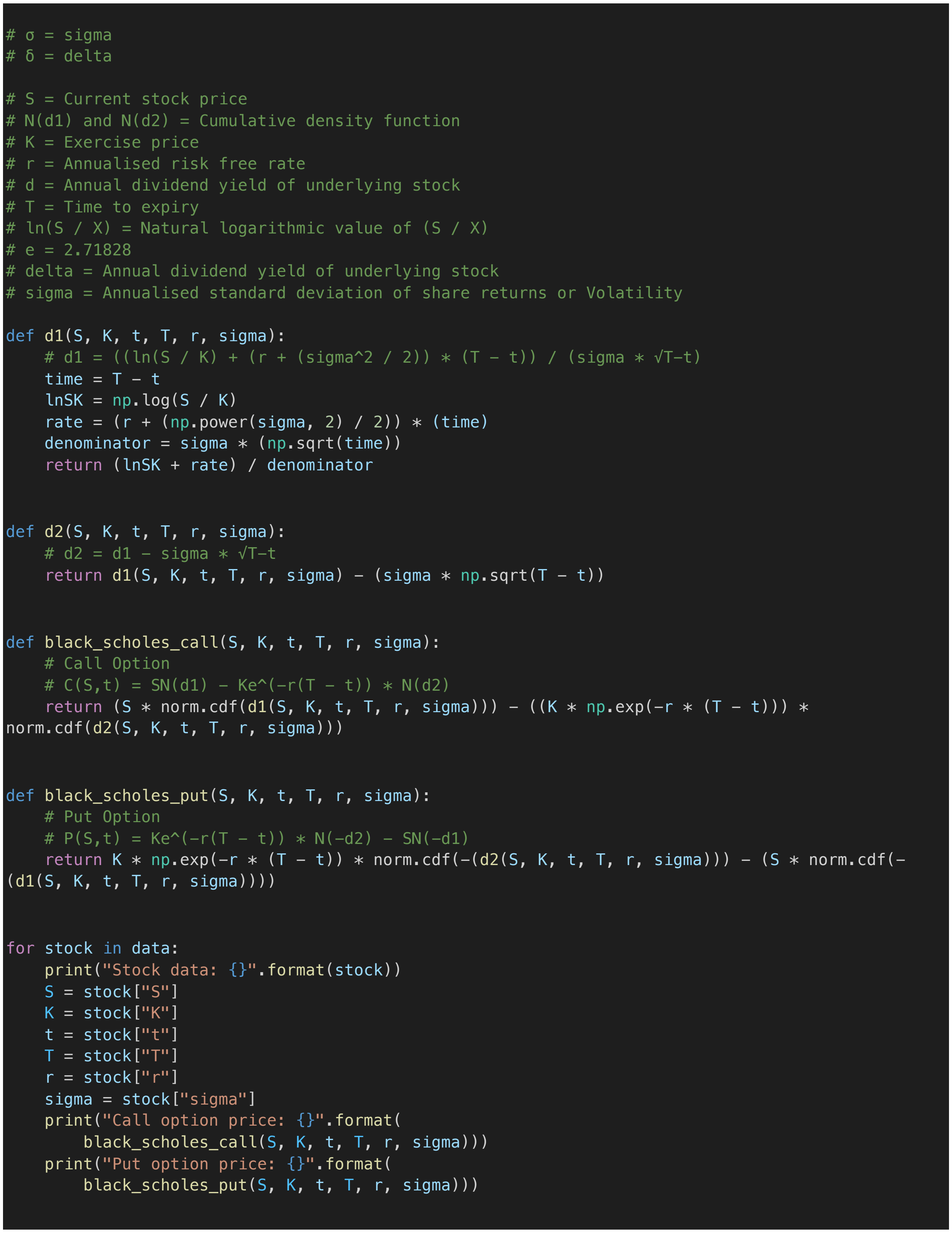

```
# σ = sigma
# δ = delta

# S = Current stock price
# N(d1) and N(d2) = Cumulative density function
# K = Exercise price
# r = Annualised risk free rate
# d = Annual dividend yield of underlying stock
# T = Time to expiry
# ln(S / X) = Natural logarithmic value of (S / X)
# e = 2.71828
# delta = Annual dividend yield of underlying stock
# sigma = Annualised standard deviation of share returns or Volatility

def d1(S, K, t, T, r, sigma):
    # d1 = ((ln(S / K) + (r + (sigma^2 / 2)) ∗ (T − t)) / (sigma ∗ √T−t)
    time = T − t
    lnSK = np.log(S / K)
    rate = (r + (np.power(sigma, 2) / 2)) ∗ (time)
    denominator = sigma ∗ (np.sqrt(time))
    return (lnSK + rate) / denominator


def d2(S, K, t, T, r, sigma):
    # d2 = d1 − sigma ∗ √T−t
    return d1(S, K, t, T, r, sigma) − (sigma ∗ np.sqrt(T − t))


def black_scholes_call(S, K, t, T, r, sigma):
    # Call Option
    # C(S,t) = SN(d1) − Ke^(−r(T − t)) ∗ N(d2)
    return (S ∗ norm.cdf(d1(S, K, t, T, r, sigma))) − ((K ∗ np.exp(−r ∗ (T − t))) ∗
norm.cdf(d2(S, K, t, T, r, sigma)))


def black_scholes_put(S, K, t, T, r, sigma):
    # Put Option
    # P(S,t) = Ke^(−r(T − t)) ∗ N(−d2) − SN(−d1)
    return K ∗ np.exp(−r ∗ (T − t)) ∗ norm.cdf(−(d2(S, K, t, T, r, sigma))) − (S ∗ norm.cdf(−
(d1(S, K, t, T, r, sigma))))


for stock in data:
    print("Stock data: {}".format(stock))
    S = stock["S"]
    K = stock["K"]
    t = stock["t"]
    T = stock["T"]
    r = stock["r"]
    sigma = stock["sigma"]
    print("Call option price: {}".format(
        black_scholes_call(S, K, t, T, r, sigma)))
    print("Put option price: {}".format(
        black_scholes_put(S, K, t, T, r, sigma)))
```

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Result of (1)
Stock data: {'S': 50, 'K': 50, 't': 0, 'T': 0.5, 'sigma': 0.2, 'r': 0.01}
Call option price: 2.9380121169138036
Put option price: 2.6886360765479225

Stock data: {'S': 50, 'K': 60, 't': 0, 'T': 0.5, 'sigma': 0.2, 'r': 0.01}
Call option price: 0.3870694028577839
Put option price: 10.08781815441872

Stock data: {'S': 50, 'K': 50, 't': 0, 'T': 1, 'sigma': 0.2, 'r': 0.01}
Call option price: 4.216659345054804
Put option price: 3.7191510325132064

Stock data: {'S': 50, 'K': 50, 't': 0, 'T': 0.5, 'sigma': 0.3, 'r': 0.01}
Call option price: 4.338822781168002
Put option price: 4.089446740802124

Stock data: {'S': 50, 'K': 50, 't': 0, 'T': 0.5, 'sigma': 0.2, 'r': 0.02}
Call option price: 3.060327056727921
Put option price: 2.5628187441863233

Base on the result, you will see that
Strike price is increased (50 -> 60)
Call option price will be decreased
Put option price will be increased

Maturity is increased (0.5 -> 1)
Call option will be increased
Put option will be increased

Volatility is increased (0.2 -> 0.3)
Call option will be increased
Put option will be increased

Risk free rate is increased (0.01 -> 0.02)
Call option will be increased
Put option will be decreased

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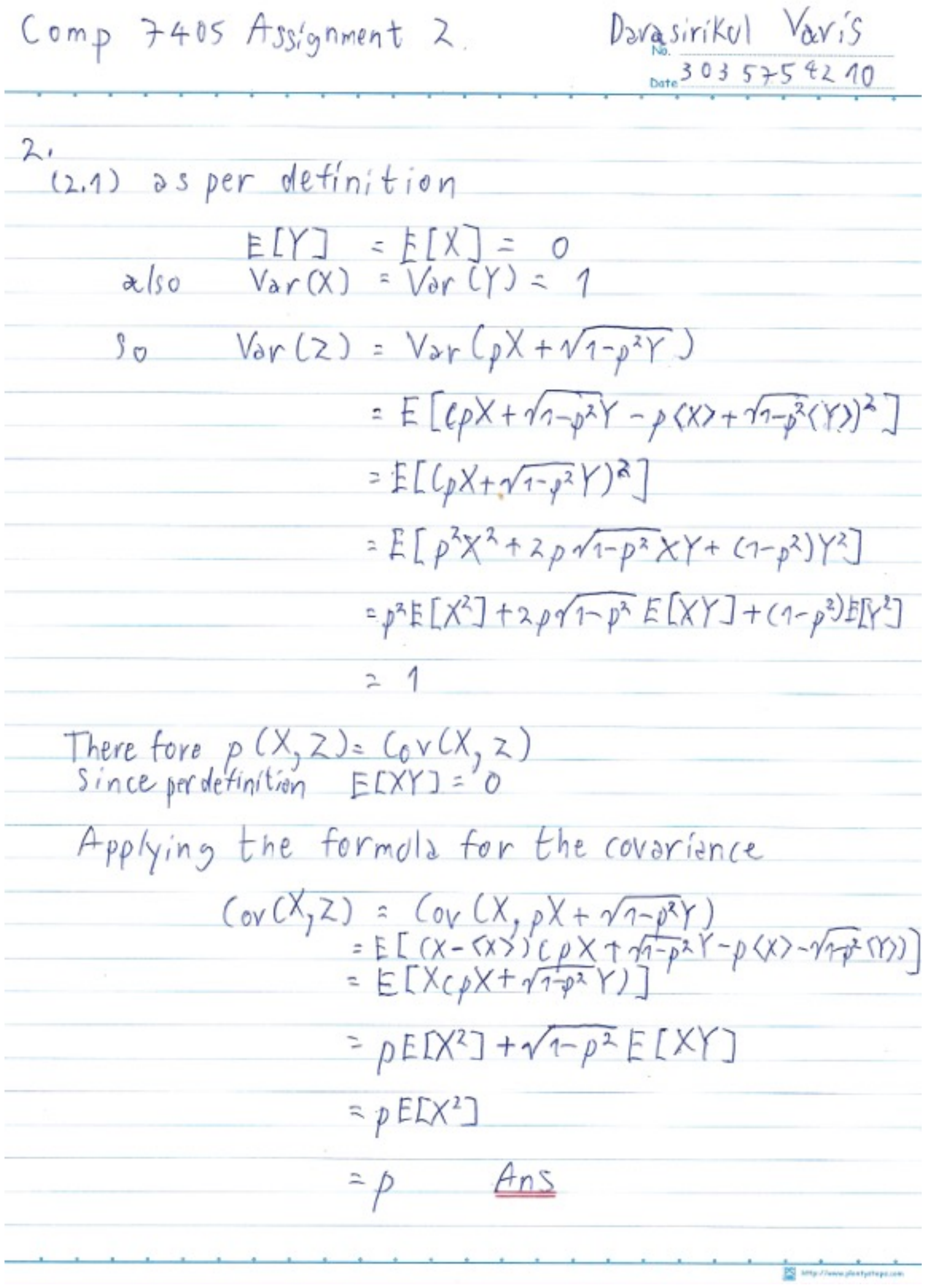
2.
(2.1)



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No. 3035754210
Date

2.
(2.1) as per definition

$$E[Y] = E[X] = 0$$

also $$Var(X) = Var(Y) = 1$$

so $$Var(Z) = Var(\rho X + \sqrt{1-\rho^2}Y)$$

$$= E[(\rho X + \sqrt{1-\rho^2}Y - \rho\langle X\rangle + \sqrt{1-\rho^2}\langle Y\rangle)^2]$$

$$= E[(\rho X + \sqrt{1-\rho^2}Y)^2]$$

$$= E[\rho^2X^2 + 2\rho\sqrt{1-\rho^2}XY + (1-\rho^2)Y^2]$$

$$= \rho^2E[X^2] + 2\rho\sqrt{1-\rho^2}E[XY] + (1-\rho^2)E[Y^2]$$

$$= 1$$

There fore $\rho(X,Z) = Cov(X,Z)$
Since per definition $E[XY] = 0$

Applying the formula for the covariance

$$Cov(X,Z) = Cov(X, \rho X + \sqrt{1-\rho^2}Y)$$

$$= E[(X - \langle X\rangle)(\rho X + \sqrt{1-\rho^2}Y - \rho\langle X\rangle - \sqrt{1-\rho^2}\langle Y\rangle)]$$

$$= E[X(\rho X + \sqrt{1-\rho^2}Y)]$$

$$= \rho E[X^2] + \sqrt{1-\rho^2}E[XY]$$

$$= \rho E[X^2]$$

$$= \rho$$ Ans

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(2.2)

```
import numpy as np

# (2.2) Write a short program to numerically verify ρ(X,Z) = ρ□


def generate_standard_normal_random_variable(size):
    # (a) write a standard normal random variable generator.
    return np.random.standard_normal(size=(size, 2))


def generate_Z(X, Y):
    # Please use 0.5 as the correlation ρ□.
    p = 0.5
    # Z formula Z = ρX + ((√1−ρ^2) ∗ Y)
    return (p ∗ X) + (np.sqrt(1 − np.power(p, 2)) ∗ Y)


def calculate_correlation_coefficient(X_list, Z_list):
    # snr_list = [[X, Y]]
    # Z_list = [Z]
    # Calculate ρ(X,Z)
    # ρ(X,Z) = Cov(X,Z) / √Var(X)Var(Z)
    # X_list = snr_list[:,0]
    return np.cov(X_list, Z_list, bias=True)[0][1] / np.sqrt(np.var(X_list) ∗ np.var(Z_list))


def correlated_normal_random_variables():
    # (b) generate 200 samples of X and Y.
    # standard normal random variable size 200
    # snr = [[X,Y]]
    snr_list = generate_standard_normal_random_variable(200)

    # (c) generate the samples of Z using the formula and the samples of X and Y .
    Z_list = [generate_Z(snr[0], snr[1]) for snr in snr_list]


    # (d) calculate the sample correlation coefficient  ρ(X,Z) based on the samples
    # of X and Z, and compare it with the theoretical value 0.5.
    X_list = snr_list[:, 0]

    print("p(X, Z): {}".format(calculate_correlation_coefficient(X_list, Z_list)))


correlated_normal_random_variables()
```

Result of 2.2:

p(X, Z) : 0.5315036877607728

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3.
(3.1)

```
import numpy as np
from scipy.stats import norm

# σ = sigma
# δ = delta


def calculate_d1_d2(S, K, t, T, r, q, sigma):
    time = T - t
    lnSK = np.log(S / K)
    rate = r - q
    denominator = sigma * (np.sqrt(time))
    plus_sigma = (1 / 2) * sigma * np.sqrt(time)

    # d1 = ((ln(S / K) + (r - q)(T - t)) / (sigma * √T-t)) + ((1/2) * sigma * √T-t)
    d1 = ((lnSK + (rate * time)) / denominator) + plus_sigma

    # d1 = ((ln(S / K) + (r - q)(T - t)) / (sigma * √T-t)) - ((1/2) * sigma * √T-t)
    d2 = ((lnSK + (rate * time)) / denominator) - plus_sigma

    return d1, d2


def black_scholes_call(S, K, t, T, r, q, sigma):
    time = T - t
    d1, d2 = calculate_d1_d2(S, K, t, T, r, q, sigma)
    # Call Option
    # C(S,t) = Se^(-q(T - t))N(d1) - Ke^(-r(T - t)) * N(d2)
    return (S * np.exp(-q * (time)) * norm.cdf(d1)) - ((K * np.exp(-r * (time))) * norm.cdf(d2))


def black_scholes_put(S, K, t, T, r, q, sigma):
    time = T - t
    d1, d2 = calculate_d1_d2(S, K, t, T, r, q, sigma)
    # Call Option
    # P(S,t) = Ke^(-r(T - t)) * N(-d2) - Se^(-q(T - t))N(-d1)
    return ((K * np.exp(-r * (time))) * norm.cdf(-d2)) - (S * np.exp(-q * (time)) * norm.cdf(-
d1))


def black_scholes_vega(S, K, t, T, r, q, sigma):
    d1, d2 = calculate_d1_d2(S, K, t, T, r, q, sigma)
    time = T - t
    # Note that the formulas for ∂C(σ) / ∂σ and ∂P(σ) / ∂σ also need to change to:
    # ∂C(σ) / ∂σ = ∂P(σ) / ∂σ = Se^(-q(T - t)) * √T-t * N'(d1)
    return S * np.exp(-q * (time)) * np.sqrt(time) * norm.pdf(d1)


# (3.1)
# Implement the algorithm presented in Lecture 4 to calculate implied volatilities
# with the extended Black-Scholes formulas (1)-(2).
# ==========================================================================
```

```python
def calculate_implied_volatility(S, K, t, T, r, q, option_type, C_true):
    time = T - t
    # The initial guess σ^ changes to:
    #
    # σ^ = √2|(lnS0 / K + (r - q)(T - t)) / T - t |
    sigma_hat = np.sqrt(2 * np.abs(np.log(S / K) + ((r - q) * (time))))
    tol = 1e-8
    nmax = 100
    sigma_diff = 1
    n = 1
    sigma = sigma_hat
    # C_true = black_scholes_call(S, K, t, T, r, q, sigma_true) if option_type == 'C' else
black_scholes_put(S, K, t, T, r, q, sigma_true)

    while (sigma_diff >= tol and n < nmax):
        C = black_scholes_call(S, K, t, T, r, q, sigma) if option_type == 'C' else
black_scholes_put(
            S, K, t, T, r, q, sigma)
        Cvega = black_scholes_vega(S, K, t, T, r, q, sigma)

        if Cvega == 0:
            return np.nan

        increment = (C - C_true) / Cvega
        sigma = sigma - increment
        n = n+1
        sigmadiff = abs(increment)

    return sigma
```

(3.2)

```python
import csv
import numpy as np
import pandas as pd
from datetime import datetime
import matplotlib.pyplot as plt
from question_3_1 import calculate_implied_volatility

market_data = []
instruments_data = []


def convert_local_time(date_time_string):
    return datetime.strptime(
        date_time_string, '%Y-%b-%d %H:%M:%S.%f').time().strftime("%H:%M:%S")


def create_date(date_time_string):
    return datetime.strptime(
        date_time_string, '%Y-%b-%d %H:%M:%S.%f')


with open('./instruments.csv') as csv_file:
    for row in csv.DictReader(csv_file, skipinitialspace=True):
        d_row = {}
        for key, value in row.items():
            if (key != 'Type' and key != 'OptionType' and key != 'Symbol' and value != ''):
                d_row[key] = float(value)
            else:
                d_row[key] = value

        instruments_data.append(d_row)


with open('./marketdata.csv') as csv_file:
    for row in csv.DictReader(csv_file, skipinitialspace=True):
        local_time = convert_local_time(row['LocalTime'])
        d_row = {}
        for key, value in row.items():
            if (key != 'LocalTime' and key != 'Symbol'):
                d_row[key] = float(value)
            else:
                d_row[key] = value

        market_data.append(d_row)
```

(3.2.1)

```
# calculate the bid/ask implied volatilities of all instruments at
# 09:31:00, 09:32:00, 09:33:00.
# Specifically, you take snapshots of the given market data at
# 09:31:00, 09:32:00, 09:33:00, respectively.
# Then for each snapshot,
# you calculate the bid implied volatility and ask implied volatility of each instrument.
# Put your results in three separate csv files
# using the names "31.csv", "32.csv", and "33.csv"
# (this is to make our tutor's life easier, thank you).
# The csv files should have the following format:
# ----------------------------------------------
# Strike | BidVolP | AskVolP | BidVolC | AskVolC
#   1.9  |  ....   |  ....   |  ....   |  ....

def compute_latest_data(data_list, exit_key, exit_value, spot_key, spot_value):
    result = []
    for index, data in enumerate(data_list):
        exit_time = convert_local_time(data[exit_key])
        result_index = next((index for (index, d) in enumerate(
            result) if d["Symbol"] == data['Symbol']), None)

        if result_index is None:
            result.append(data)
        else:
            result.pop(result_index)
            result.append(data)

        if exit_time == exit_value:
            break

    last_spot_index = 0
    for index, value in enumerate(result):
        if value[spot_key] == spot_value:
            last_spot_index = index

    result = result[:(last_spot_index + 1)]

    return result
```

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```python
def compute_implied_volatility(data, instruments, T, r, q):
    # Sample {'LocalTime': '2016-Feb-16 09:32:00.907981', 'Symbol': 10000566.0, 'Last': 0.0027,
'Bid1': 0.0026, 'BidQty1': 1.0, 'Ask1': 0.0035, 'AskQty1': 6.0}
    implied_volatility = []
    equity_price = data[-1]
    for index, market in enumerate(data):
        instrument = next(
            filter(lambda v: v['Symbol'] == market['Symbol'], instruments), None)

        if instrument['Type'] != 'Option':
            continue

        computed_data = {}
        K = instrument['Strike']

        bid_implied_volatility = calculate_implied_volatility(
            equity_price['Last'], K, 0, T, r, q, instrument['OptionType'], market['Bid1'])
        ask_implied_volatility = calculate_implied_volatility(
            equity_price['Last'], K, 0, T, r, q, instrument['OptionType'], market['Ask1'])

        bid_implied_volatility = 'NaN' if np.isnan(
            bid_implied_volatility) else bid_implied_volatility
        ask_implied_volatility = 'NaN' if np.isnan(
            ask_implied_volatility) else ask_implied_volatility

        if instrument['OptionType'] == 'P':
            # Calculate Implied Volatility
            computed_data = {
                'Strike': K,
                'BidVolP': bid_implied_volatility,
                'AskVolP': ask_implied_volatility,
                'BidVolC': '',
                'AskVolC': '',
                'Symbol': market['Symbol'],
                'LocalTime': market['LocalTime'],
            }
        if instrument['OptionType'] == 'C':
            # Calculate Implied Volatility
            computed_data = {
                'Strike': K,
                'BidVolP': '',
                'AskVolP': '',
                'BidVolC': bid_implied_volatility,
                'AskVolC': ask_implied_volatility,
                'Symbol': market['Symbol'],
                'LocalTime': market['LocalTime'],
            }

        implied_volatility.append(computed_data)

    return implied_volatility
```

```python
def create_result_file(result_file_name, iv_data):
    with open(result_file_name, "w") as f:
        wr = csv.DictWriter(
            f, delimiter=",", fieldnames=list(iv_data[0].keys()))
        wr.writeheader()
        wr.writerows(iv_data)


def calculate_bid_ask_implied_volatilities_all_instruments():
    # Compute 09:31:00 Data
    options_31 = compute_latest_data(
        market_data, 'LocalTime', '09:31:00', 'Symbol', '510050')

    # Compute 09:32:00 Data
    options_32 = compute_latest_data(
        market_data, 'LocalTime', '09:32:00', 'Symbol', '510050')

    # Compute 09:33:00 Data
    options_33 = compute_latest_data(
        market_data, 'LocalTime', '09:33:00', 'Symbol', '510050')
    q = 0.2  # 20%
    r = 0.04  # 4%
    # Time to maturity
    T = (24 - 16) / 365

    iv_31 = compute_implied_volatility(options_31, instruments_data, T, r, q)
    iv_32 = compute_implied_volatility(options_32, instruments_data, T, r, q)
    iv_33 = compute_implied_volatility(options_33, instruments_data, T, r, q)

    # Create implied volatility result
    create_result_file("31.csv", iv_31)
    create_result_file("32.csv", iv_32)
    create_result_file("33.csv", iv_33)

    return iv_31, iv_32, iv_33


iv_31, iv_32, iv_33 = calculate_bid_ask_implied_volatilities_all_instruments()
```

(3.2.2)

```python
# (3.2.2)
# Put the results into three different plots one for each time point.
# For each plot, the x-axis should be the strike levels,
# and the y-axis should be implied volatilities.

def compute_x_y(iv_data):
    x = []
    y = []
    for iv in iv_data:
        x.append(iv['Strike'])

        vola = 0
        counter = 0
        if iv['BidVolP'] != '' and iv['BidVolP'] != 'NaN':
            vola += iv['BidVolP']
            counter += 1
        if iv['AskVolP'] != '' and iv['AskVolP'] != 'NaN':
            vola += iv['AskVolP']
            counter += 1

        if iv['BidVolC'] != '' and iv['BidVolC'] != 'NaN':
            vola += iv['BidVolC']
            counter += 1

        if iv['AskVolC'] != '' and iv['AskVolC'] != 'NaN':
            vola += iv['AskVolC']
            counter += 1

        if (counter != 0):
            vola = vola / counter

        y.append(vola)

    return x, y


def plot_iv_data(iv_31, iv_32, iv_33):
    print(pd.DataFrame(iv_31))
    x_31, y_31 = compute_x_y(iv_31)
    x_32, y_32 = compute_x_y(iv_32)
    x_33, y_33 = compute_x_y(iv_33)

    # For 31.csv
    plt.plot(x_31, y_31)
    # naming the x-axis
    plt.xlabel('Strike Level')
    # naming the y-axis
    plt.ylabel('Implied volatilities.')
    # plot title
    plt.title('31.csv Data')
    plt.show()

    # For 32.csv
    plt.plot(x_32, y_32)
    # naming the x-axis
    plt.xlabel('Strike Level')
    # naming the y-axis
    plt.ylabel('Implied volatilities.')
    # plot title
    plt.title('32.csv Data')
    plt.show()

    # For 33.csv
    plt.plot(x_33, y_33)
    # naming the x-axis
    plt.xlabel('Strike Level')
    # naming the y-axis
    plt.ylabel('Implied volatilities.')
    # plot title
    plt.title('33.csv Data')
    plt.show()

plot_iv_data(iv_31, iv_32, iv_33)
```

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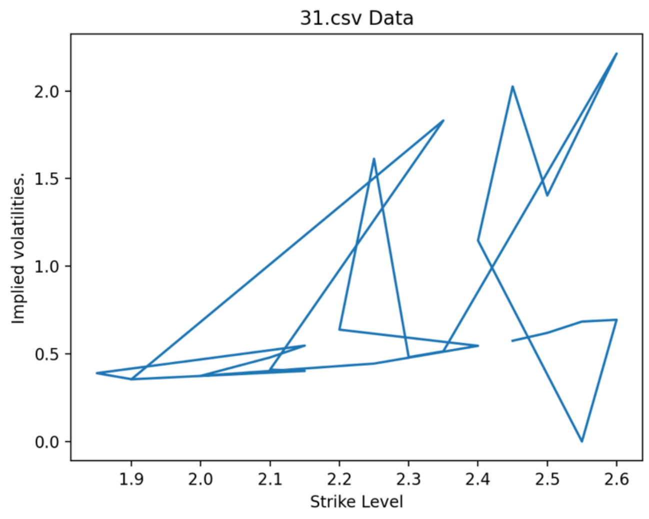

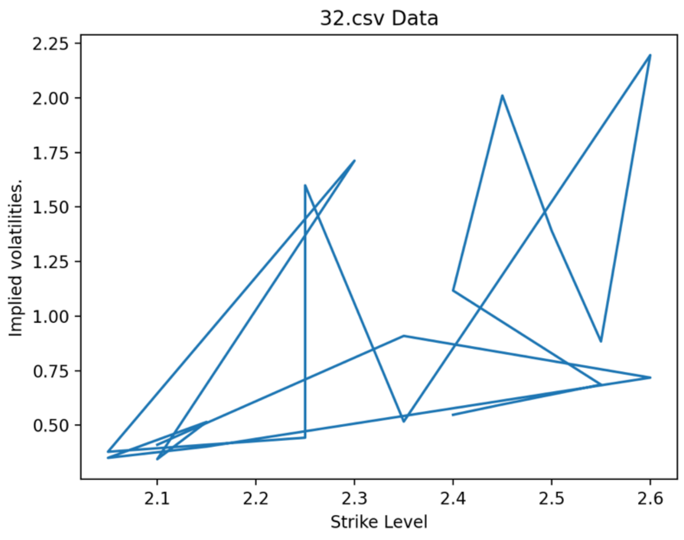

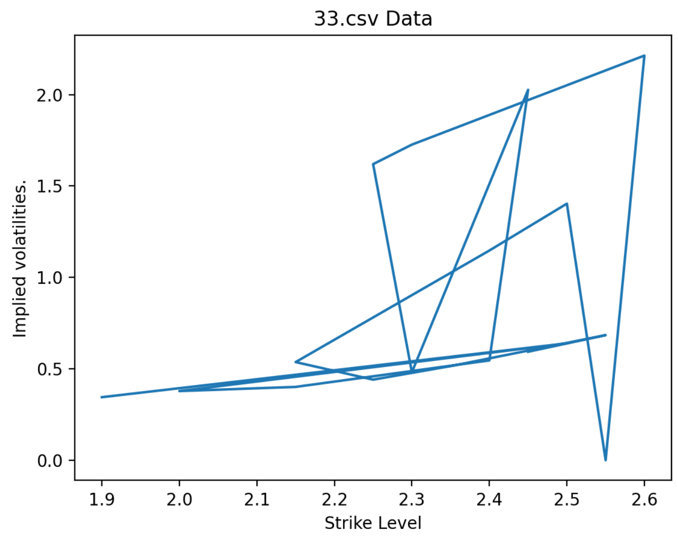

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(3.3)

```python
import csv
import numpy as np
from datetime import datetime
from question_1 import black_scholes_call, black_scholes_put


market_data = []
instruments_data = []

with open('./marketdata.csv') as csv_file:
    for row in csv.DictReader(csv_file, skipinitialspace=True):
        d_row = {}
        for key, value in row.items():
            if (key != 'LocalTime' and key != 'Symbol'):
                d_row[key] = float(value)
            else:
                d_row[key] = value

        market_data.append(d_row)


with open('./instruments.csv') as csv_file:
    for row in csv.DictReader(csv_file, skipinitialspace=True):
        d_row = {}
        for key, value in row.items():
            if (key != 'Type' and key != 'OptionType' and key != 'Symbol' and value != ''):
                d_row[key] = float(value)
            else:
                d_row[key] = value

        instruments_data.append(d_row)

r = 0.04  # 4%
q = 0.2  # 20%

# (3.3)
# The trading unit for buying/selling an option is 10000, and the transaction
# cost is about 3.3 RMBs per unit. Using the non-arbitrage conditions you
# have learned so far, check whether you see any arbitrage opportunities in
# the data. You can consider two cases: one without any transaction cost, and
# the other one with the real transaction cost. You can assume there is no
# transaction cost for A50ETF. Write down your findings and submit them.


def call_put_parity(S, K, t, T, r, q):
    # C(S,t) - P(S,t) = Se^(-q(T - t)) - Ke^(-r(T - t))
    time = T - t
    return (S * np.exp(-q * (time))) - (K * np.exp(-r * (time)))


def write_result_to_text_file(text, file_name):
    # Open a file with access mode 'a'
    with open(file_name, "a") as file_object:
        # Append 'hello' at the end of file
        file_object.write(text)
        file_object.write("\n")
```

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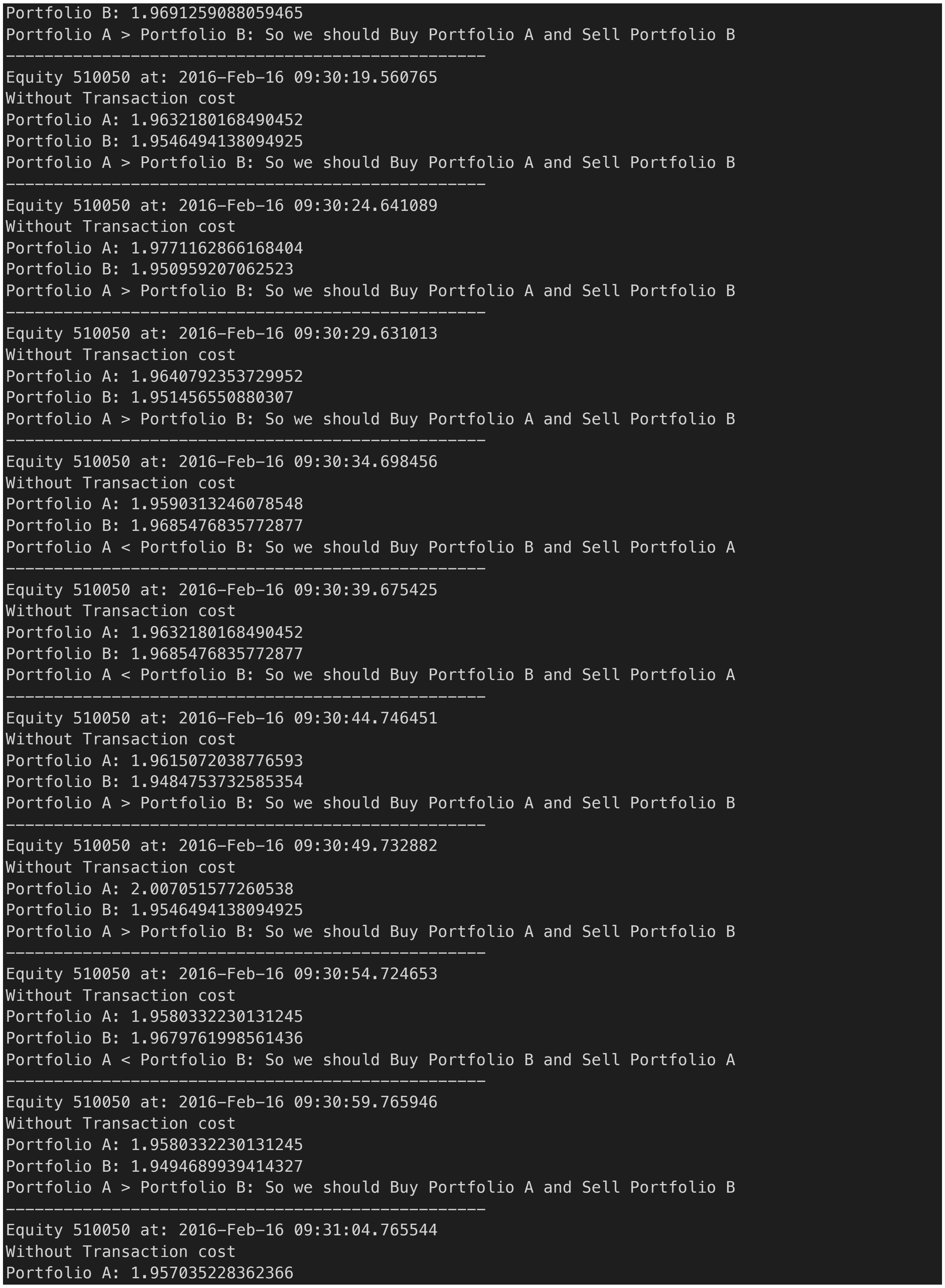
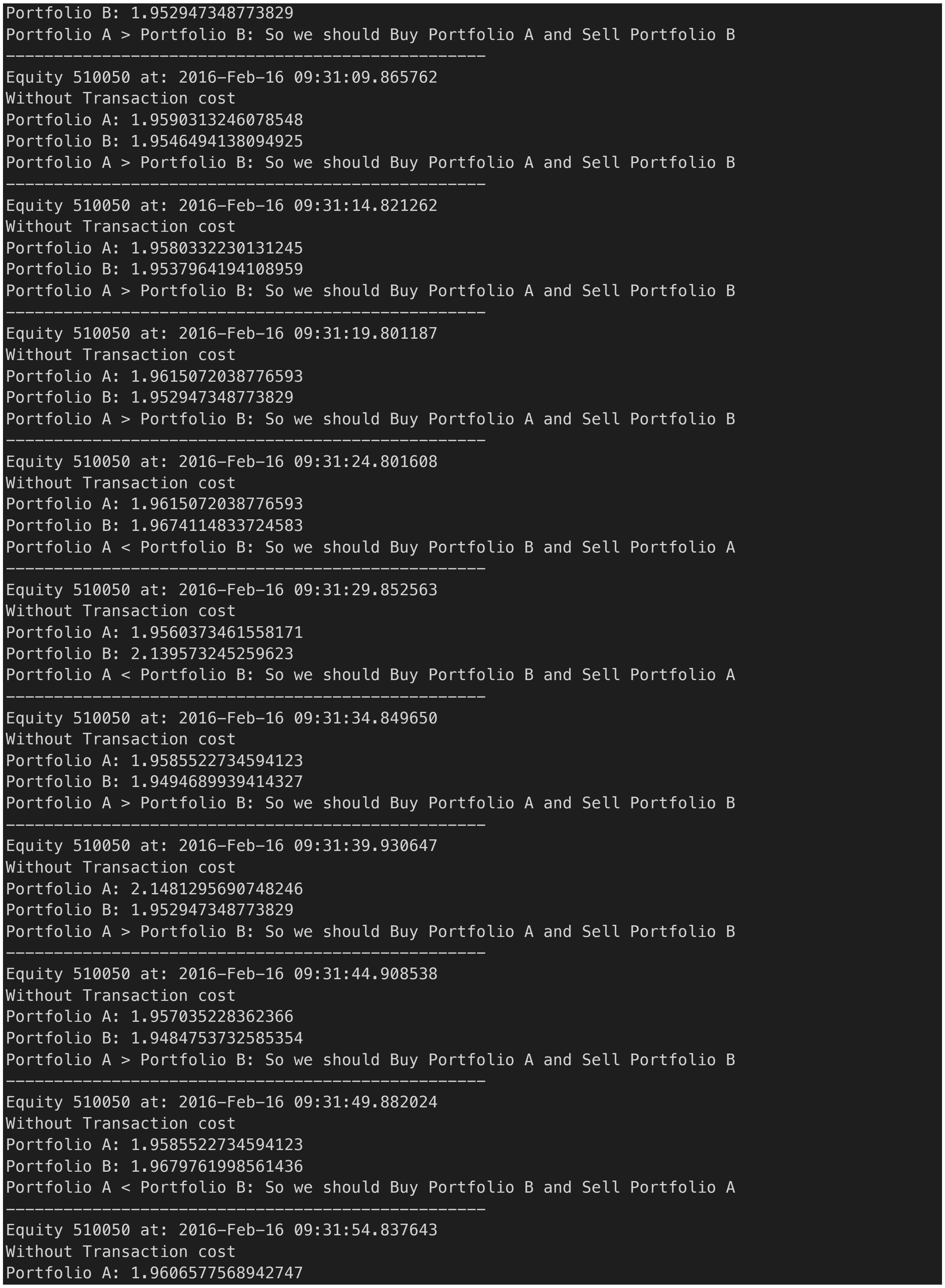
```
# Calculate call-put parity from 31.csv ()


def portfolio_estimation():
    filtered_data = []
    temp_object = {}

    for market in market_data:
        instrument = next(
            filter(lambda v: v['Symbol'] == market['Symbol'], instruments_data), None)

        if instrument['Symbol'] == '510050':
            temp_object['Last'] = market['Last']
            temp_object['Bid1'] = market['Bid1']
            temp_object['BidQty1'] = market['BidQty1']
            temp_object['Ask1'] = market['Ask1']
            temp_object['AskQty1'] = market['AskQty1']
            temp_object['LocalTime'] = market['LocalTime']
            filtered_data.append(temp_object)
            temp_object = {}
            continue

        if instrument['OptionType'] == 'C':
            temp_object['callLast'] = market['Last']
            temp_object['callBid'] = market['Bid1']
            temp_object['callBidQty'] = market['BidQty1']
            temp_object['callAsk'] = market['Ask1']
            temp_object['callAskQty'] = market['AskQty1']
            temp_object['callStrike'] = instrument['Strike']

        if instrument['OptionType'] == 'P':
            temp_object['putLast'] = market['Last']
            temp_object['putBid'] = market['Bid1']
            temp_object['putBidQty'] = market['BidQty1']
            temp_object['putAsk'] = market['Ask1']
            temp_object['putAskQty'] = market['AskQty1']
            temp_object['putStrike'] = instrument['Strike']

    with open("question_3_3_generated_data_table.csv", "w") as f:
        wr = csv.DictWriter(
            f, delimiter=",", fieldnames=list(filtered_data[0].keys()))
        wr.writeheader()
        wr.writerows(filtered_data)

    # Time to maturity
    T = (24 - 16) / 365

    sigma = 0.2

    # Clean up portfolio compare result file
    portfolio_compare_result_file = 'question_3_3_portfolio_compare.txt'
    open(portfolio_compare_result_file, 'w').close()

    for data in filtered_data:
        # Call - Put parity formula
        # C(S,t) - P(S,t) = Se^(-q(T - t)) - Ke^(-r(T - t))
        #
        # Portfolio A = C(S,t) + Ke^(-r(T - t))
        port_a = black_scholes_call(
            data['Last'], data['callStrike'], 0, T, r, sigma) + (data['callStrike'] * np.exp(-r *
T))

        # Portfolio B = P(S,t) + Se^(-q(T - t))
        port_b = black_scholes_put(
            data['Last'], data['putStrike'], 0, T, r, sigma) + (data['Last'] * np.exp(-q * T))

        write_result_to_text_file('Equity 510050 at: {}'.format(
            data['LocalTime']), portfolio_compare_result_file)

        write_result_to_text_file(
            'Without Transaction cost', portfolio_compare_result_file)
        write_result_to_text_file('Portfolio A: {}'.format(
            port_a), portfolio_compare_result_file)
        write_result_to_text_file('Portfolio B: {}'.format(
            port_b), portfolio_compare_result_file)
        if port_a == port_b:
            write_result_to_text_file(
                'Portfolio A == Portfolio B: No Arbitrage oppotunity',
portfolio_compare_result_file)
            write_result_to_text_file(
                '---------------------------------------------------',
portfolio_compare_result_file)
            continue

        if port_a > port_b:
            write_result_to_text_file(
                'Portfolio A > Portfolio B: So we should Buy Portfolio A and Sell Portfolio B',
portfolio_compare_result_file)

        if port_a < port_b:
            write_result_to_text_file(
                'Portfolio A < Portfolio B: So we should Buy Portfolio B and Sell Portfolio A',
portfolio_compare_result_file)

        write_result_to_text_file(
            '---------------------------------------------------', portfolio_compare_result_file)

    return None


portfolio_estimation()
```

Results and analysis

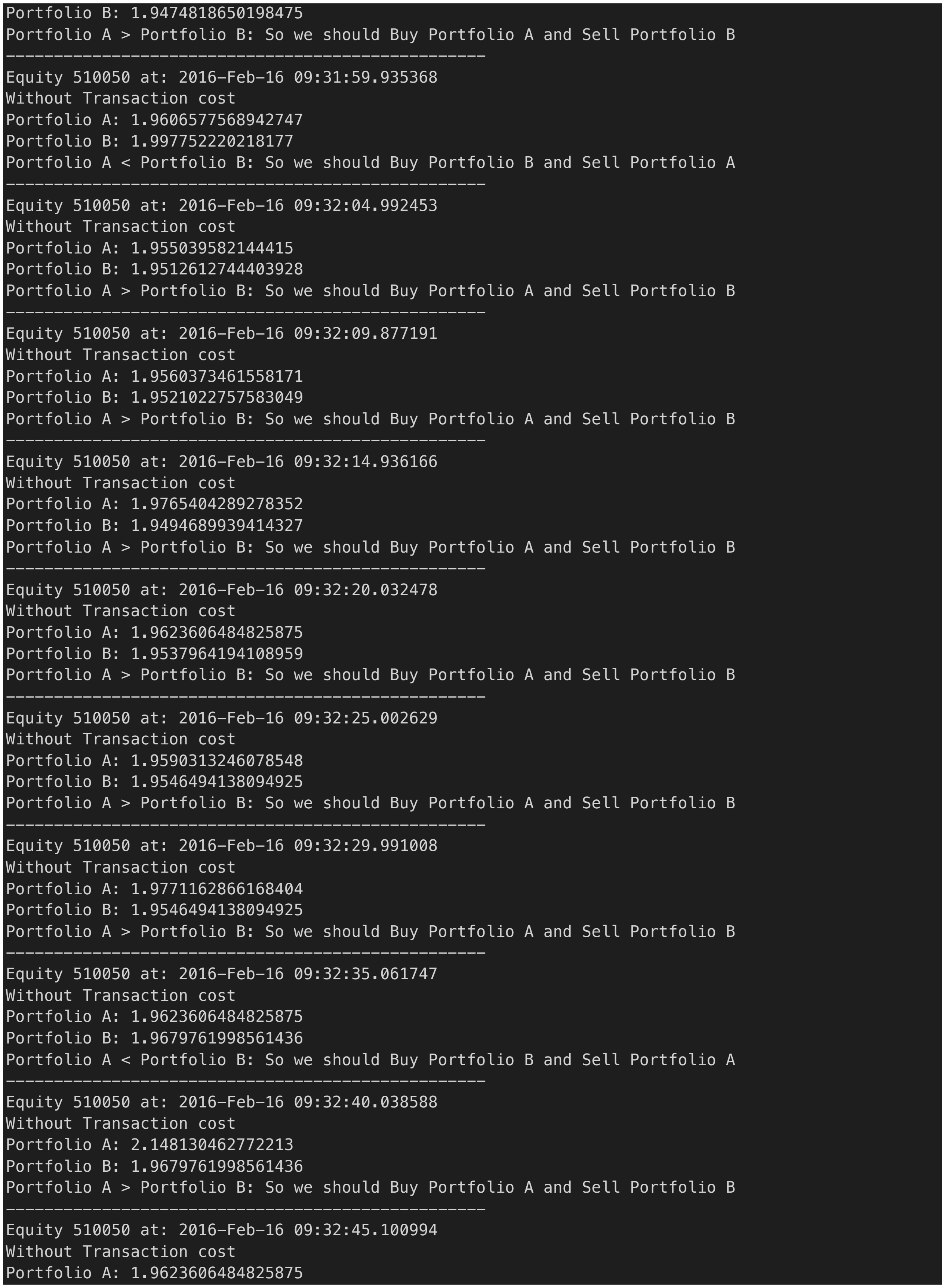
```
Equity 510050 at: 2016-Feb-16 09:30:04.520358
Without Transaction cost
Portfolio A: 2.0983792311040372
Portfolio B: 1.951456550880307
Portfolio A > Portfolio B: So we should Buy Portfolio A and Sell Portfolio B
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:30:09.587622
Without Transaction cost
Portfolio A: 1.9600295278877207
Portfolio B: 1.9691259088059465
Portfolio A < Portfolio B: So we should Buy Portfolio B and Sell Portfolio A
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:30:14.591004
Without Transaction cost
Portfolio A: 2.1481324145450515
```

Darasirikul Varis
Student id: 3035754210


```
Portfolio B: 1.9691259088059465
Portfolio A > Portfolio B: So we should Buy Portfolio A and Sell Portfolio B
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:30:19.560765
Without Transaction cost
Portfolio A: 1.9632180168490452
Portfolio B: 1.9546494138094925
Portfolio A > Portfolio B: So we should Buy Portfolio A and Sell Portfolio B
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:30:24.641089
Without Transaction cost
Portfolio A: 1.9771162866168404
Portfolio B: 1.950959207062523
Portfolio A > Portfolio B: So we should Buy Portfolio A and Sell Portfolio B
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:30:29.631013
Without Transaction cost
Portfolio A: 1.9640792353729952
Portfolio B: 1.951456550880307
Portfolio A > Portfolio B: So we should Buy Portfolio A and Sell Portfolio B
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:30:34.698456
Without Transaction cost
Portfolio A: 1.9590313246078548
Portfolio B: 1.9685476835772877
Portfolio A < Portfolio B: So we should Buy Portfolio B and Sell Portfolio A
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:30:39.675425
Without Transaction cost
Portfolio A: 1.9632180168490452
Portfolio B: 1.9685476835772877
Portfolio A < Portfolio B: So we should Buy Portfolio B and Sell Portfolio A
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:30:44.746451
Without Transaction cost
Portfolio A: 1.9615072038776593
Portfolio B: 1.9484753732585354
Portfolio A > Portfolio B: So we should Buy Portfolio A and Sell Portfolio B
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:30:49.732882
Without Transaction cost
Portfolio A: 2.007051577260538
Portfolio B: 1.9546494138094925
Portfolio A > Portfolio B: So we should Buy Portfolio A and Sell Portfolio B
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:30:54.724653
Without Transaction cost
Portfolio A: 1.9580332230131245
Portfolio B: 1.9679761998561436
Portfolio A < Portfolio B: So we should Buy Portfolio B and Sell Portfolio A
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:30:59.765946
Without Transaction cost
Portfolio A: 1.9580332230131245
Portfolio B: 1.9494689939414327
Portfolio A > Portfolio B: So we should Buy Portfolio A and Sell Portfolio B
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:31:04.765544
Without Transaction cost
Portfolio A: 1.957035228362366
```



```
Portfolio B: 1.952947348773829
Portfolio A > Portfolio B: So we should Buy Portfolio A and Sell Portfolio B
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:31:09.865762
Without Transaction cost
Portfolio A: 1.9590313246078548
Portfolio B: 1.9546494138094925
Portfolio A > Portfolio B: So we should Buy Portfolio A and Sell Portfolio B
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:31:14.821262
Without Transaction cost
Portfolio A: 1.9580332230131245
Portfolio B: 1.9537964194108959
Portfolio A > Portfolio B: So we should Buy Portfolio A and Sell Portfolio B
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:31:19.801187
Without Transaction cost
Portfolio A: 1.9615072038776593
Portfolio B: 1.952947348773829
Portfolio A > Portfolio B: So we should Buy Portfolio A and Sell Portfolio B
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:31:24.801608
Without Transaction cost
Portfolio A: 1.9615072038776593
Portfolio B: 1.9674114833724583
Portfolio A < Portfolio B: So we should Buy Portfolio B and Sell Portfolio A
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:31:29.852563
Without Transaction cost
Portfolio A: 1.9560373461558171
Portfolio B: 2.139573245259623
Portfolio A < Portfolio B: So we should Buy Portfolio B and Sell Portfolio A
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:31:34.849650
Without Transaction cost
Portfolio A: 1.9585522734594123
Portfolio B: 1.9494689939414327
Portfolio A > Portfolio B: So we should Buy Portfolio A and Sell Portfolio B
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:31:39.930647
Without Transaction cost
Portfolio A: 2.1481295690748246
Portfolio B: 1.952947348773829
Portfolio A > Portfolio B: So we should Buy Portfolio A and Sell Portfolio B
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:31:44.908538
Without Transaction cost
Portfolio A: 1.957035228362366
Portfolio B: 1.9484753732585354
Portfolio A > Portfolio B: So we should Buy Portfolio A and Sell Portfolio B
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:31:49.882024
Without Transaction cost
Portfolio A: 1.9585522734594123
Portfolio B: 1.9679761998561436
Portfolio A < Portfolio B: So we should Buy Portfolio B and Sell Portfolio A
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:31:54.837643
Without Transaction cost
Portfolio A: 1.9606577568942747
```



```
Portfolio B: 1.9474818650198475
Portfolio A > Portfolio B: So we should Buy Portfolio A and Sell Portfolio B
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:31:59.935368
Without Transaction cost
Portfolio A: 1.9606577568942747
Portfolio B: 1.997752220218177
Portfolio A < Portfolio B: So we should Buy Portfolio B and Sell Portfolio A
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:32:04.992453
Without Transaction cost
Portfolio A: 1.955039582144415
Portfolio B: 1.9512612744403928
Portfolio A > Portfolio B: So we should Buy Portfolio A and Sell Portfolio B
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:32:09.877191
Without Transaction cost
Portfolio A: 1.9560373461558171
Portfolio B: 1.9521022757583049
Portfolio A > Portfolio B: So we should Buy Portfolio A and Sell Portfolio B
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:32:14.936166
Without Transaction cost
Portfolio A: 1.9765404289278352
Portfolio B: 1.9494689939414327
Portfolio A > Portfolio B: So we should Buy Portfolio A and Sell Portfolio B
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:32:20.032478
Without Transaction cost
Portfolio A: 1.9623606484825875
Portfolio B: 1.9537964194108959
Portfolio A > Portfolio B: So we should Buy Portfolio A and Sell Portfolio B
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:32:25.002629
Without Transaction cost
Portfolio A: 1.9590313246078548
Portfolio B: 1.9546494138094925
Portfolio A > Portfolio B: So we should Buy Portfolio A and Sell Portfolio B
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:32:29.991008
Without Transaction cost
Portfolio A: 1.9771162866168404
Portfolio B: 1.9546494138094925
Portfolio A > Portfolio B: So we should Buy Portfolio A and Sell Portfolio B
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:32:35.061747
Without Transaction cost
Portfolio A: 1.9623606484825875
Portfolio B: 1.9679761998561436
Portfolio A < Portfolio B: So we should Buy Portfolio B and Sell Portfolio A
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:32:40.038588
Without Transaction cost
Portfolio A: 2.148130462772213
Portfolio B: 1.9679761998561436
Portfolio A > Portfolio B: So we should Buy Portfolio A and Sell Portfolio B
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:32:45.100994
Without Transaction cost
Portfolio A: 1.9623606484825875
```

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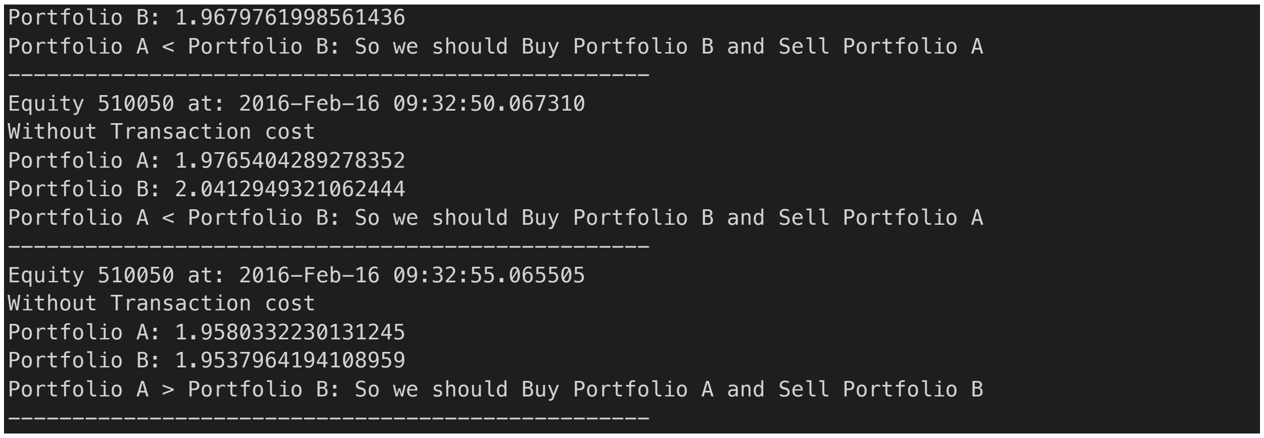
Student id: 3035754210



```
Portfolio B: 1.9679761998561436
Portfolio A < Portfolio B: So we should Buy Portfolio B and Sell Portfolio A
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:32:50.067310
Without Transaction cost
Portfolio A: 1.9765404289278352
Portfolio B: 2.0412949321062444
Portfolio A < Portfolio B: So we should Buy Portfolio B and Sell Portfolio A
--------------------------------------------------
Equity 510050 at: 2016-Feb-16 09:32:55.065505
Without Transaction cost
Portfolio A: 1.9580332230131245
Portfolio B: 1.9537964194108959
Portfolio A > Portfolio B: So we should Buy Portfolio A and Sell Portfolio B
--------------------------------------------------
```

(Result file is at question_3_3_portfolio_compare.txt)