Comp 7405 Assignment 2

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2.

(2.1) as per definition

$$E[Y] = E[X] = 0$$

also $\quad Var(X) = Var(Y) = 1$

So $\quad Var(Z) = Var(pX + \sqrt{1-p^2}Y)$

$$= E[(pX + \sqrt{1-p^2}Y - p\langle X\rangle + \sqrt{1-p^2}\langle Y\rangle)^2]$$

$$= E[(pX + \sqrt{1-p^2}Y)^2]$$

$$= E[p^2X^2 + 2p\sqrt{1-p^2}XY + (1-p^2)Y^2]$$

$$= p^2E[X^2] + 2p\sqrt{1-p^2}E[XY] + (1-p^2)E[Y^2]$$

$$= 1$$

There fore $p(X, Z) = Cov(X, Z)$

Since per definition $\quad E[XY] = 0$

Applying the formula for the covariance

$$Cov(X, Z) = Cov(X, pX + \sqrt{1-p^2}Y)$$

$$= E[(X - \langle X\rangle)(pX + \sqrt{1-p^2}Y - p\langle X\rangle - \sqrt{1-p^2}\langle Y\rangle)]$$

$$= E[X(pX + \sqrt{1-p^2}Y)]$$

$$= pE[X^2] + \sqrt{1-p^2}E[XY]$$

$$= pE[X^2]$$

$$= p$$

Ans