

THE UNIVERSITY OF HONG KONG
M.Sc. DECEMBER 2020 EXAMINATIONS

DEPARTMENT OF INDUSTRIAL AND MANUFACTURING SYSTEMS ENGINEERING
(IELM7018) FINANCIAL ENGINEERING

Important notice:

By submitting this assessment through Moodle, you declare that the submitted assessment represents your own work and you have not committed plagiarism and/or collusion in the submission.

Please write your own solution in English on A4 size papers, scan them to a single PDF file, and submit the PDF onto the HKU Moodle by 5:00 pm on December 16, 2020.

Short Answer Questions (40 marks)

- (a) If the Treasury yield curve is downward sloping, you would expect that the farthest T-bill futures contracts would be at rates _____ than the nearby contract months.

- A. higher
- B. lower
- C. no different

and explain why. (4 marks)

- (b) The purchaser of a put option on a T-bond is hoping that interest rates will _____

- A. rise
- B. fall
- C. stay constant

and explain why. (4 marks)

- (c) Suppose that zero interest rates with continuous compounding are as follows:

Maturity (months)	Rate (% per annum)
3	6.0
6	6.2
9	6.4
12	6.5
15	6.6
18	6.7

Calculate forward interest rates for the second, fourth, and sixth quarters. (4 marks)

- (d) You are considering a project with the following net after-tax cash flows (in millions of dollars):

Years from Now	After-Tax Cash Flow
0	-500
1-10	160

The project's beta is 2. Assuming that $r_f = 9\%$ and $E(r_M) = 15\%$; what is the net present value of the project? (4 marks)

- (e) Following the above question, what is the highest possible beta estimate for the project before its NPV becomes negative? (4 marks)
- (f) Suppose that you enter into a short futures contract to sell July gold for \$20 per ounce. The size of the contract is 4,000 ounces. The initial margin is \$6,000, and the maintenance margin is \$5,000. What change in the futures price will lead to a margin call? (4 marks)

- (g) A 5-year bond with face value \$100 provides a coupon of 8% semiannually and has a cash price of \$105. What is the bond's (continuously compounding) yield? **(4 marks)**
- (h) A deposit account pays 14% per annum with continuous compounding, but interest is actually paid quarterly. How much interest will be paid each quarter on a \$10,000 deposit? **(4 marks)**
- (i) On November 1, 2018 your savings account had \$500,000 in it and for every month thereafter you deposit \$738 into your savings account. What annual rate of interest must be earned for your account to have \$1,500,000 in it on November 1, 2021 (after the last \$738 is deposited into the account)? **(4 marks)**
- (j) Explain why an FRA is equivalent to the exchange of a floating rate of interest for a fixed rate of interest? **(4 marks)**

Long Answer Questions (60 marks)

Question 1 (12 marks)

- (a) You are a distributor of gold and need to make deliveries of 10,000 kilogram (kg) one month from now. You currently have no gold in inventory. The current spot price of wheat is \$8 per kg and the futures price for delivery in one month is \$8.2. You would like to hedge the uncertainty about the spot price one month from now.
- i) If your storage cost is \$0.15 per kg (paid at the end of month), what would you do? (assume the monthly compound interest rate is r) **(3 marks)**
 - ii) Suppose that in the short run, your storage cost increases to \$0.25 per kg. What would you do? (assume the monthly compound interest rate is r) **(3 marks)**
- (b) Spot price for silver is \$154.70 per ton and the 12-month silver futures is traded at \$150.00. The 1-year interest rate is 4% (annually compounded).
- i) What is the net convenience yield (annually compound) on silver for the 12 month period? **(3 marks)**
 - ii) You need 1,000 tons of silver in 12 months. How would you lock into a price today using the futures contracts? (The size for each silver futures contract is 100 tons.) **(3 marks)**

Question 2 (12 marks)

A mutual fund currently has \$50M in Hang Seng index and \$50M in one-year zero-coupon bonds. Assume that the one-year interest rate is 6% (annually compound). Assume that the current quote on the Heng Seng index is 1,125, each futures contract is written on 250 units of the index and the dividend yield on the index is approximately 3% per year, i.e., \$1,000 invested in the index yields \$30 in dividends at the end of the year.

- (a) Suppose you invest $\$1,125 \times 250$ in one-year zero-coupon bonds and at the same time enter into a single futures contract with long position on Heng Seng index with one year to maturity.
- i) Assume that in one year the index finishes at 1,000. What is the total value of your position? How does this compare with buying 250 units of the index and holding them for a year? **(3 marks)**
 - ii) Assume that in one year the index finishes at 1,200. Repeat the analysis. **(3 marks)**
- (b) If this mutual fund decides to switch to a 70/30 stock/bond mix (\$70M in Heng Seng index and \$30M in one year zero-coupon bonds) for a period of one year,
- i) how would you implement this strategy using Heng Seng index futures? How many contracts with one year to maturity would you need? **(3 marks)**

- ii) Assume that the index finishes the year at 1,200, describe the plan's portfolio in one year and one day from now (right after the futures expire). What is the stock/bond mix? **(3 marks)**

Question 3 (12 marks)

You are a bond trader and see on your screen the following information on three bonds with annual coupon payments and par value of \$100:

Bond	Coupon rate (%)	Maturity (year)	Yield(%)
A	0	1	4.00
B	4	2	4.50
C	5	3	5.00

Coupon payments are annual.

- (a) What are the prices of the above bonds? **(3 marks)**
- (b) Construct the current term-structure of spot interest rates. **(3 marks)**
- (c) Explain how you would synthetically replicate a zero-coupon bond with a maturity of 3 years and a par value of \$100. **(3 marks)**
- (d) What should be the price of the bond so that there is no arbitrage? **(3 marks)**

Question 4 (12 marks)

- (a) Alibaba stock price is currently \$100. At the end of 3 months it will be either \$120 or \$83.33. The risk-free interest rate (quarterly compound) is 3% per annum. What is the value of a 3-month European call option with a strike price of \$100? Calculate your answer to this problem using
- i) replication. **(4 marks)**
- ii) the risk-neutral method. **(4 marks)**
- (b) HSBC is currently trading at \$90.29 per share. You believe that HSBC will have an expected return of 7% with volatility of 26.1% per year, while annual interest rates are at 0.95%. What is the price of an European put on HSBC with a strike price of \$90 and maturity of 1 year? (Hint: The price of an European call is given by

$$C(S, K, T) = SN(d_1) - KR^{-T}N(d_2)$$

where $d_1 = \frac{\ln(S/(KR^{-T}))}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T}$, $d_2 = d_1 - \sigma\sqrt{T}$, $R = 1 + 0.95\% = 1.0095$. Use the attached Normal distribution table to find the values of $N(d_1)$ and $N(d_2)$.) **(4 marks)**

Question 5 (12 marks)

- (a) Which of the following portfolios can not be on the Markowitz efficient frontier? Explain briefly.

Portfolio	Expected Return	Standard Deviation
Q	15%	22.5%
R	15.75%	24.75%
S	16%	25%
T	17.25%	27.75%

(4 marks)

- (b) You need to invest \$30M in two assets: a risk-free asset with an expected return of 6% and a risky asset with an expected return of 15% and a standard deviation of 35%. You face a cap of 25% on the portfolio's standard deviation. What is the maximum expected return you can achieve on your portfolio? **(4 marks)** What is the corresponding Sharpe ratio of the portfolio with the maximum expected return? **(4 marks)**

Appendix

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997

- End of Paper -