**Question 1:** Note: In this question, all columns will be written in their transposed form, as rows, to make the typography simpler. Matrix M has three rows and three columns, and the columns form an orthonormal basis. One of the columns is [2/7,3/7,6/7], and another is [6/7, 2/7, -3/7]. Let the third column be [x,y,z]. Since the length of the vector [x,y,z] must be 1, there is a constraint that x2+y2+z2 = 1. However, there are other constraints, and these other constraints can be used to deduce facts about the ratios among x, y, and z. Compute these ratios.

**Answer:** Let C1 be [2/7,3/7,6/7], C2 be [6/7, 2/7, -3/7] and C3 be [x, y, z]

The dot product of any two columns must be zero.

C1.C2 = (2/7 \* 6/7) + (3/7 \* 2/7) + (6/7 \* -3/7) = 0

C2.C3 = (6/7 \* x) + (2/7 \* y) + (-3/7 \* z) = 0 → 6x +2y -3z = 0 – Eq 1

C3.C1 = (x \* 2/7) + (y \* 3/7) + (z \* 6/7) = 0 → 2x + 3y + 6z = 0 – Eq 2 2 \* Eq 1 + Eq 2 → 12x + 4y -6z + 2x + 3y +6z = 0 → 14x + 7y = 0 → y = -2x 3 \* Eq 2 – Eq 1 → 6x + 9y + 18z – 6x – 2y + 3z = 0 → 7y + 21z = 0 → y = -3z x: y: z = -2: 1: -3

**Question 2**: Find the eigenvalues and eigenvectors of the following matrix:



You should assume the first component of an eigenvector is 1. Then, find out One eigenvalue and One eigenvector.

Let the given matrix be A = 2/ 3 3 /10 and the eigen vector be of the form 1 /e

Ax = λx ➔ 2 /3 3 /10 \* 1 /e = λ \* 1 /e → 2 + 3e = λ and 3 + 10e = λe → 3 + 10e = (2 + 3e)e 3e^2 – 8e + 3 = 0 → e = 3, -1/3

The eigen vectors are 𝟏 /𝟑 𝒂𝒏𝒅 𝟏 −𝟏/𝟑

The eigen values are 2 + 3e = λ → λ = 2 + 3\*3 = 11 and λ = 2 + 3\*(-1/3) = 1

**Question 3**: Suppose [1,3,4,5,7] is an eigenvector of some matrix. What is the unit eigenvector in the same direction? Find out the components of the unit eigenvector.

**Answer:** Given the eigen vector of some matrix be M = [1,3,4,5,7]

To get the unit eigen vector of given matrix, we need to divide each component by square root of sum of squares in the same direction.

Sum of squares = 1 ^2 + 3^2 + 4 ^2 + 5^2 +7 ^2 = 100 and its square root is 10 Unit Eigen Vector = [1/10,3/10,4/10,5/10,7/10]

**Question 4**: Suppose we have three points in a two dimensional space: (1,1), (2,2), and (3,4). We want to perform PCA on these points, so we construct a 2-by-2 matrix, call it N, whose eigenvectors are the directions that best represent these three points. Construct the matrix N and identify, its elements.

The given three points in a 2- D space are (1,1), (2,2), and (3,4).

We should construct a matrix whose rows correspond to points and columns correspond to dimensions of the space.

Then the matrix will be M = 1 1 2 2 3 4 M^T M = 1 2 3 1 2 4 \* 1 1 2 2 3 4 = 14 17 17 21

**Question 5**: Consider the diagonal matrix M =



Compute its Moore-Penrose pseudoinverse.

**Answer:** Moore-Penrose pseudoinverse means the matrix having diagonal elements replaced by 1 and divided by corresponding elements of given matrix and the other elements will be zero.

Moore-Penrose pseudoinverse of given matrix is 𝟏 𝟎 𝟎 𝟎 𝟏/𝟐 𝟎 𝟎 𝟎 𝟎

**Question 6**: When we perform a CUR decomposition of a matrix, we select rows and columns by using a particular probability distribution for the rows and another for the columns. Here is a matrix that we wish to decompose:



Calculate the probability distribution for the rows.

**Answer:**

Probability with which we choose now = sum of squares of elements in the rows /sum of squares of elements in the matrix

Sum of squares of elements in the matrix = 12\*13\*25/6 = 3900/6 = 650

P(R1) = 1 ^2 + 2 ^2 + 3 ^2 /650 = 14/650 = 0.02

P(R2) = 4^ 2 + 5^2 + 6^2 /650 = 77/650 = 0.12

P(R3) = 7^2 + 8^2 + 9^2 /650 = 194/650 = 0.298

P(R4) = 10^ 2 + 11^2 + 12^2/ 650 = 365/650 =0.56