Given,

Cost of building power plant in Diablo Canyon = 10M USD

Cost of building power plant in Roy Rogers City = 20M USD

Cost of hiring the geologist = 1M USD

Probability of Earthquake without expert opinion = 0.20

⇒ Probability of no earthquake without expert opinion = 0.80

Geologist will predict an earthquake on 95% of occasions for which an earthquake will occur. AND

She will predict an earthquake will not occur on 90% of occasions for which an earthquake will not occur.

⇒ She will predict an earthquake on 5% of occasions for which an earthquake will not occur AND

She will predict an earthquake will not occur on 10% of occasions for which an earthquake will occur.

Let,

probability of earthquake = P(Earthquake) = 0.2

probability of no earthquake = P(No Earthquake) = 0.8

probability of geologist saying earthquake will occur and earthquake occurs

$$= P(\text{Positive} \mid \text{Earthquake}) = 0.95$$

probability of geologist saying earthquake will occur and no earthquake occurs

$$= P(\text{Negative} \mid \text{Earthquake}) = 0.05$$

probability of geologist saying earthquake will not occur and no earthquake occurs

$$= P(\text{Negative} \mid \text{No Earthquake}) = 0.90$$

probability of geologist saying earthquake will not occur and earthquake occurs

$$= P(\text{Positive} \mid \text{No Earthquake}) = 0.10$$

So,

P(Earthquake)	0.2
P(No Earthquake)	0.8

P(Positive | Earthquake)	0.95
P(Positive | No Earthquake)	0.10
P(Negative | Earthquake)	0.05
P(Negative | No Earthquake)	0.90

	Earthquake	No Earthquake	Total
Positive	$0.95 \times 0.2 = (0.19)$	$0.10 \times 0.8 = (0.08)$	(0.27)
Negative	$0.05 \times 0.2 = (0.01)$	$0.90 \times 0.8 = (0.72)$	(0.73)
Total	(0.20)	(0.80)	1

Now, as per Baye's Theorem,

$$P(\text{Earthquake} \mid \text{Positive}) = \frac{P(\text{Positive} \mid \text{Earthquake}) \times P(\text{Earthquake})}{P(\text{Positive} \mid \text{Earthquake}) \times P(\text{Earthquake}) + P(\text{Positive} \mid \text{NoEarthquake}) \times P(\text{NoEarthquake})}$$

$$= \frac{0.19}{0.27} \simeq 0.703704$$

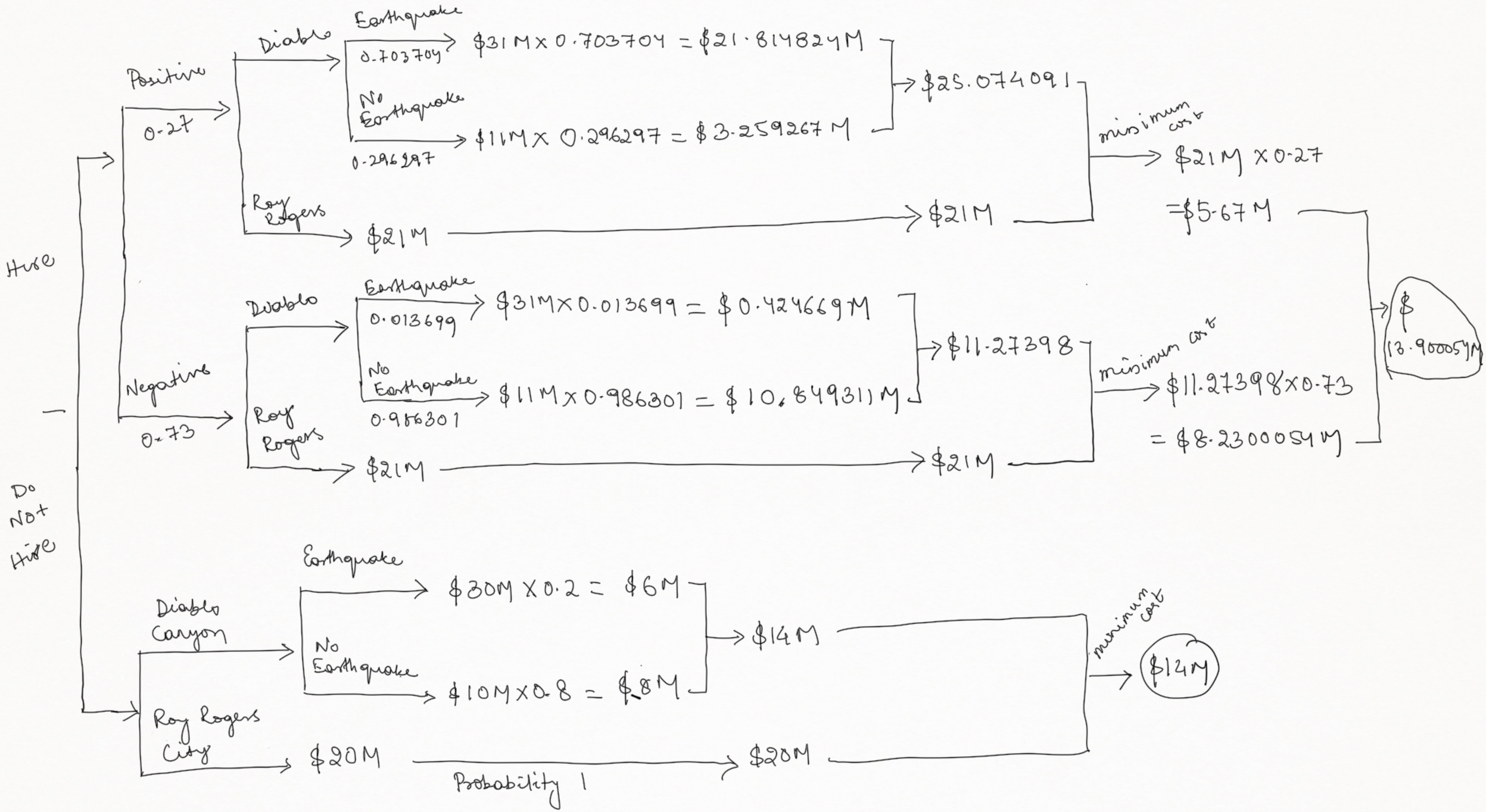
$$P(\text{Earthquake} \mid \text{Negative}) = \frac{P(\text{Negative} \mid \text{Earthquake}) \times P(\text{Earthquake})}{P(\text{Negative} \mid \text{Earthquake}) \times P(\text{Earthquake}) + P(\text{Negative} \mid \text{NoEarthquake}) \times P(\text{NoEarthquake})}$$

$$= \frac{0.01}{0.73} \simeq 0.013699$$

Similarly,

$$P(\text{NoEarthquake} \mid \text{Positive}) = \frac{0.08}{0.27} \simeq 0.296297$$

$$P(\text{NoEarthquake} \mid \text{Negative}) = \frac{0.72}{0.73} \simeq 0.986301$$


Hire
Positive
0.27
Diablo
Earthquake
0.703704
$31M x 0.703704 = $21.814824M
No Earthquake
0.296297
$11M x 0.296297 = $3.259267M
$25.074091
Roy Rogers
$21M
$21M
minimum cost
$21M x 0.27
=$5.67M
Negative
0.73
Diablo
Earthquake
0.013699
$31M x 0.013699 = $0.424669M
No Earthquake
0.986301
$11M x 0.986301 = $10.849311M
$11.27398
Roy Rogers
$21M
$21M
minimum cost
$11.27398 x 0.73
= $8.230054M
$13.900054M
Do Not Hire
Earthquake
$30M x 0.2 = $6M
Diablo Canyon
No Earthquake
$10M x 0.8 = $8M
$14M
Roy Rogers City
$20M
Probability 1
$20M
minimum cost
$14M

From the above decision tree, if the geologist is hired,

expected cost = $13.9M (approx),

and if the geologist is not hired,

expected cost = $14M

So, the geologist should be hired.